7th EDITION

NEW SYLLABUS
MATHEMATICS
TEACHER'S RESOURCE BOOK

Consultant • Dr Yeap Ban Har **Authors** • Dr Joseph Yeo • Teh Keng Seng • Loh Cheng Yee • Ivy Chow

OXFORD
UNIVERSITY PRESS

OXFORD
UNIVERSITY PRESS

CONTENTS

OXFORD UNIVERSITY PRESS

OXFORD
UNIVERSITY PRESS

OXFORD
UNIVERSITY PRESS

Syllabus Matching Grid

Cambridge O Level Mathematics (Syllabus D) 4024/4029. Syllabus for examination in 2018, 2019 and 2020.

Theme or Topic	Subject Content	Reference
1. Number	Identify and use: • Natural numbers • Integers (positive, negative and zero) • Prime numbers • Square numbers • Cube numbers • Common factors and common multiples • Rational and irrational numbers (e.g. π, $\sqrt{2}$) • Real numbers	Book 1: Chapter 1 Chapter 2
2. Set language and notation	• Use set language, set notation and Venn diagrams to describe sets and represent relationships between sets • Definition of sets: e.g. $A = \{x : x \text{ is a natural number}\}$, $B = \{(x, y): y = mx + c\}$, $C = \{x : a \leqslant x \leqslant b\}$, $D = \{a, b, c, \ldots\}$	Book 2: Chapter 14 Book 4: Chapter 2
2. Squares, square roots, cubes and cube roots	Calculate • Squares • Square roots • Cubes and cube roots of numbers	Book 1: Chapter 1 Chapter 2
4. Directed numbers	• Use directed numbers in practical situations	Book 1: Chapter 2
5. Vulgar and decimal fractions and percentages	• Use the language and notation of simple vulgar and decimal fractions and percentages in appropriate contexts • Recognise equivalence and convert between these forms	Book 1: Chapter 2
6. Ordering	• Order quantities by magnitude and demonstrate familiarity with the symbols $=, \neq, <, >, \leqslant, \geqslant$.	Book 1: Chapter 2 Chapter 5
7. Standard form	• Use the standard form $A \times 10^n$, where n is a positive or negative integer, and $1 \leqslant A < 10$.	Book 3: Chapter 4
8. The four operations	Use the four operations for calculations with: • Whole numbers • Decimals • Vulgar (and mixed) fractions including correct ordering of operations and use of brackets.	Book 1: Chapter 2
9. Estimation	• Make estimates of numbers, quantities and lengths • Give approximations to specified numbers of significant figures and decimal places • Round off answers to reasonable accuracy in the context of a given problem	Book 1: Chapter 3
10 Limits of accuracy	• Give appropriate upper and lower bounds for data given to a specified accuracy • Obtain appropriate upper and lower bounds to solutions of simple problems given to a specified accuracy	Book 3: Chapter 3

OXFORD UNIVERSITY PRESS

11. Ratio, proportion, rate	• Demonstrate an understanding of ratio and proportion • Increase and decrease a quantity by a given ratio • Use common measures of rate • Solve problems involving average speed	Book 1: Chapter 9 Book 2: Chapter 1
12. Percentages	• Calculate a given percentage of a quantity • Express one quantity as a percentage of another • Calculate percentage increase or decrease • Carry out calculations involving reverse percentages	Book 1: Chapter 8 Book 3: Chapter 5
13. Use of an electronic calculator	• Use an electronic calculator efficiently • Apply appropriate checks of accuracy • Enter a range of measures including 'time' • Interpret the calculator display appropriately	Book 1: Chapter 2 Chapter 4 Book 2: Chapter 11 Book 3: Chapter 10 Book 4: Chapter 4
14. Time	• Calculate times in terms of the 24-hour and 12-hour clock • Read clocks, dials and timetables	Book 1: Chapter 9
15. Money	• Solve problems involving money and convert from one currency to another	Book 3: Chapter 5
16. Personal and small business finance	• Use given data to solve problems on personal and small business finance involving earnings, simple interest and compound interest • Extract data from tables and charts	Book 3: Chapter 5
17. Algebraic representation and formulae	• Use letters to express generalised numbers and express arithmetic processes algebraically • Substitute numbers for words and letters in formulae • Construct and transform formulae and equations	Book 1: Chapter 4 Chapter 5 Book 2: Chapter 2 Book 3: Chapter 1
18. Algebraic manipulation	• Manipulate directed numbers • Use brackets and extract common factors • Expand product of algebraic expressions • Factorise where possible expressions of the form: $ax + bx + kay + kby$ $a^2x^2 - b^2y^2$ $a^2 + 2ab + b^2$ $ax^2 + bx + c$ • Manipulate algebraic fractions • Factorise and simplify rational expressions	Book 1: Chapter 4 Book 2: Chapter 3 Chapter 4 Chapter 6
19. Indices	• Understand and use the rules of indices • Use and interpret positive, negative, fractional and zero indices	Book 3: Chapter 4

OXFORD UNIVERSITY PRESS

20. Solutions of equations and inequalities	• Solve simple linear equations in one unknown • Solve fractional equations with numerical and linear algebraic denominators • Solve simultaneous linear equations in two unknowns • Solve quadratic equations by factorisation, completing the square or by use of the formula • Solve simple linear inequalities	Book 1: Chapter 5 Book 2: Chapter 2 Chapter 5 Book 3: Chapter 1 Chapter 3
21. Graphical representation of inequalities	• Represent linear inequalities graphically	Book 4: Chapter 1
22. Sequences	• Continue a given number sequence • Recognise patterns in sequences and relationships between different sequences • Generalise sequences as simple algebraic statements	Book 1: Chapter 7
23. Variation	• Express direct and inverse variation in algebraic terms and use this form of expression to find unknown quantities	Book 2: Chapter 1
24. Graphs in practical situations	• Interpret and use graphs in practical situations including travel graphs and conversion graphs • Draw graphs from given data • Apply the idea of rate of change to easy kinematics involving distance-time and speed-time graphs, acceleration and deceleration • Calculate distance travelled as area under a linear speed-time graph	Book 1: Chapter 6 Book 2: Chapter 2 Book 3: Chapter 7
25. Graphs in practical situations	• Construct tables of values and draw graphs for functions of the form ax^n where a is a rational constant, $n = -2, -1, 0, 1, 2, 3$, and simple sums of not more than three of these and for functions of the form ka^x where a is a positive integer • Interpret graphs of linear, quadratic, cubic, reciprocal and exponential functions • Solve associated equations approximately by graphical methods • Estimate gradients of curve by drawing tangents	Book 1: Chapter 6 Book 2: Chapter 1 Chapter 2 Chapter 5 Book 3: Chapter 1 Chapter 7
26. Function notation	• Use function notation, e.g. $f(x) = 3x - 5$; $f : x \mapsto 3x - 5$, to describe simple functions • Find inverse functions $f^{-1}(x)$	Book 2: Chapter 7 Book 3: Chapter 2
27. Coordinate geometry	• Demonstrate familiarity with Cartesian coordinates in two dimensions • Find the gradient of a straight line • Calculate the gradient of a straight line from the coordinates of two points on it • Calculate the length and the coordinates of the midpoint of a line segment from the coordinates of its end points • Interpret and obtain the equation of a straight line graph in the form $y = mx + c$ • Determine the equation of a straight line parallel to a given line • Find the gradient of parallel and perpendicular lines	Book 1: Chapter 6 Book 2: Chapter 2 Book 3: Chapter 6

OXFORD UNIVERSITY PRESS

28. Geometrical terms	• Use and interpret the geometrical terms: point; line; plane; parallel; perpendicular; bearing; right angle, acute, obtuse and reflex angles; interior and exterior angles; similarity and congruence • Use and interpret vocabulary of triangles, special quadrilaterals, circles, polygons and simple solid figures • Understand and use the terms: centre, radius, chord, diameter, circumference, tangent, arc, sector and segment	Book 1: Chapter 10 Chapter 11 Book 2: Chapter 8 Book 3: Chapter 9 to Chapter 13
29. Geometrical constructions	• Measure lines and angles • Construct a triangle, given the three sides, using a ruler and a pair of compasses only • Construct other simple geometrical figures from given data, using a ruler and protractor as necessary • Construct angle bisectors and perpendicular bisectors using a pair of compasses as necessary • Read and make scale drawings • Use and interpret nets	Book 1: Chapter 12 Chapter 14 Book 2: Chapter 8 Book 4: Chapter 8
30. Similarity and congruence	• Solve problems and give simple explanations involving similarity and congruence • Calculate lengths of similar figures • Use the relationships between areas of similar triangles, with corresponding results for similar figures, and extension to volumes and surface areas of similar solids	Book 2: Chapter 8 Book 3: Chapter 11 Chapter 12
31. Symmetry	• Recognise rotational and line symmetry (including order of rotational symmetry) in two dimensions • Recognise symmetry properties of the prism (including cylinder) and the pyramid (including cone) • Use the following symmetry properties of circles: (a) equal chords are equidistant from the centre (b) the perpendicular bisector of a chord passes through the centre (c) tangents from an external point are equal in length	Book 2: Chapter 13 Book 3: Chapter 13
32. Angles	• Calculate unknown angles and give simple explanations using the following geometrical properties: (a) angles at a point (b) angles at a point on a straight line and intersecting straight lines (c) angles formed within parallel lines (d) angle properties of triangles and quadrilaterals (e) angle properties of regular and irregular polygons (f) angle in a semi-circle (g) angle between tangent and radius of a circle (h) angle at the centre of a circle is twice the angle at the circumference (i) angles in the same segment are equal (j) angles in opposite segments are supplementary	Book 1: Chapter 10 Chapter 11 Book 3: Chapter 13
33. Loci	• Use the following loci and the method of intersecting loci for sets of points in two dimensions which are: (a) at a given distance from a given point (b) at a given distance from a given straight line (c) equidistant from two given points (d) equidistant from two given intersecting straight line	Book 4: Chapter 8
34. Measures	• Use current units of mass, length, area, volume and capacity in practical situations and express quantities in terms of larger or smaller units	Book 1: Chapter 13 Chapter 14

OXFORD UNIVERSITY PRESS

35. Mensuration	• Solve problems involving: (a) the perimeter and area of a rectangle and triangle (b) the perimeter and area of a parallelogram and a trapezium (c) the circumference and area of a circle (d) arc length and sector area as fractions of the circumference and area of a circle (e) the surface area and volume of a cuboid, cylinder, prism, sphere, pyramid and cone (f) the areas and volumes of compound shapes	Book 1: Chapter 13 Chapter 14 Book 2: Chapter 12 Book 3: Chapter 10
36. Trigonometry	• Interpret and use three-figure bearings • Apply Pythagoras' theorem and the sine, cosine and tangent ratios for acute angles to the calculation of a side or an angle of a right-angled triangles • Solve trigonometrical problems in two dimensions involving angles of elevation and depression • Extend sine and cosine functions to angles between 90° and 180° • Solve problems using the sine and cosine rules for any triangle and the formula area of triangle = $\frac{1}{2}\ ab \sin C$ • Solve simple trigonometrical problems in three dimensions	Book 2: Chapter 10 Chapter 11 Book 3: Chapter 8 Chapter 9
37. Vectors in two dimensions	• Describe a translation by using a vector represented by $\begin{pmatrix} x \\ y \end{pmatrix}$, $\overrightarrow{AB}$ or $\mathbf{a}$ • Add and subtract vectors • Multiple a vector by a scalar • Calculate the magnitude of a vector $\begin{pmatrix} x \\ y \end{pmatrix}$ as $\sqrt{x^2 + y^2}$ • Represent vectors by directed line segments • Use the sum and difference of two vectors to express given vectors in terms of two coplanar vectors • Use position vectors	Book 4: Chapter 7
38. Matrices	• Display information in the form of a matrix of any order • Solve problems involving the calculation of the sum and product (where appropriate) of two matrices, and interpret the results • Calculate the product of a matrix and a scalar quantity • Use the algebra of 2×2 matrices including the zero and identity 2×2 matrices • Calculate the determinant $\lvert\mathbf{A}\rvert$ and inverse $\mathbf{A}^{-1}$ of a non-singular matrix $\mathbf{A}$	Book 4: Chapter 5
39. Transformations	• Use the following transformations of the plane: reflection (M), rotation (R), translation (T), enlargement (E) and their combinations • Identify and give precise descriptions of transformations connecting given figures • Describe transformations using coordinates and matrices	Book 2: Chapter 9 Book 4: Chapter 6
40. Probability	• Calculate the probability of a single event as either a fraction or a decimal • Understand that the probability of an event occurring = 1 – the probability of the event not occurring • Understand relative frequency as an estimate of probability • Calculate the probability of simple combined events using possibility diagrams and tree diagrams where appropriate	Book 2: Chapter 15 Book 4: Chapter 3

OXFORD UNIVERSITY PRESS

41. Categorical, numerical and grouped data	• Collect, classify and tabulate statistical data • Read, interpret and draw simple inferences from tables and statistical diagrams • Calculate the mean, median, mode and range for individual and discrete data and distinguish between the purposes for which they are used • Calculate an estimate of the mean for grouped and continuous data • Identify the modal class from a grouped frequency distribution	Book 1: Chapter 15 Book 2: Chapter 17 Book 4: Chapter 4
42. Statistical diagrams	• Construct and interpret bar charts, pie charts, pictograms, simple frequency distributions, frequency polygons, histograms with equal and unequal intervals and scatter diagrams • Construct and use cumulative frequency diagrams • Estimate and interpret the median, percentiles, quartiles and interquartile range for cumulative frequency diagrams • Calculate with frequency density • Understand what is meant by positive, negative and zero correlation with reference to a scatter diagram • Draw a straight line of best fit by eye	Book 1: Chapter 15 Book 2: Chapter 16 Book 4: Chapter 4

OXFORD UNIVERSITY PRESS

Secondary 4 Mathematics Scheme of Work

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
1	**1** **Linear Inequalities in Two Variables**	**1.1 Linear Inequalities in Two Variables** (pp. 3 – 13)	• Illustrate linear inequalities in two variables • Differentiate linear inequalities in two variables from linear equations in two variables • Graph linear inequalities in two variables	Represent linear inequalities graphically	Investigation – Linear Inequalities in Two Variables (pp. 4 – 6)			Investigation – Linear Inequalities in Two Variables (pp. 4 – 6)
1		**1.2 Application of Systems of Linear Inequalities in Two Variables in Real-World Contexts** (pp. 13 – 16)	• Solve problems involving linear inequalities in two variables • Solve a system of linear inequalities in two variables • Solve problems involving systems of linear inequalities in two variables					
1		**Miscellaneous**					Solutions for Challenge Yourself	

OXFORD UNIVERSITY PRESS

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
2	**2** **Further Sets**	**2.1 Applications of Venn Diagrams in Problem Sums** (pp. 23 – 29)	• Solve problems on classification and cataloguing • Express problems in set notations and draw Venn diagrams to obtain solutions	Use set language, set notation and Venn diagrams to describe sets and represent relationships between sets Definition of sets: e.g. $A = \{x : x$ is a natural number$\}$, $B = \{(x, y)$: $y = mx + c\}$, $C = \{x : a \leqslant x \leqslant b\}$, $D = \{a, b, c, \ldots\}$				
2		**2.2 Formulas in Set Theory** (pp. 29 – 33)			Class Discussion (p. 30) Thinking Time (p. 31)			Thinking Time (p. 31)
2		**Miscellaneous**					Solutions for Challenge Yourself	

OXFORD UNIVERSITY PRESS

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
3	**3** **Probability of Combined Events**	**3.1 Probability of Single Events** (pp. 37 – 40)		Calculate the probability of a single event as either a fraction or a decimal	Thinking Time (p. 38)			Thinking Time (p. 38)
3		**3.2 Simple Combined Events, Possibility Diagrams and Tree Diagrams** (pp. 41 – 50)	• Calculate the probability of simple combined events using possibility diagrams and tree diagrams	Calculate the probability of simple combined events using possibility diagrams and tree diagrams where appropriate				
4		**3.3 Addition Law of Probability and Mutually Exclusive Events** (pp. 51 – 55)	• Use the Addition Law of Probability to solve problems involving mutually exclusive events		Investigation – Mutually Exclusive and Non-Mutually Exclusive Events (p. 51)			Investigation – Mutually Exclusive and Non-Mutually Exclusive Events (p. 51)
4		**3.4 Multiplication Law of Probability and Independent Events** (pp. 55 – 68)	• Use the Multiplication Law of Probability to solve problems involving independent and dependent events		Class Discussion – Choosing a Diagram to Represent the Sample Space (p. 55) Investigation – Dependent Events (p. 60) Performance Task (pp. 62 – 64)			Class Discussion – Choosing a Diagram to Represent the Sample Space (p. 55) Investigation – Dependent Events (p. 60) Performance Task (pp. 62 – 64)
4		**Miscellaneous**					Solutions for Challenge Yourself	

OXFORD UNIVERSITY PRESS

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
5	**4** **Statistical Data Analysis**	**4.1 Cumulative Frequency Table and Curve** (pp. 75 – 85)	• State the features of cumulative frequency curves • Interpret and analyse cumulative frequency curves	Construct and use cumulative frequency diagrams	Class Discussion – Constructing a Table of Cumulative Frequencies (p. 75)			Class Discussion – Constructing a Table of Cumulative Frequencies (p. 75)
5		**4.2 Median, Quartiles, Percentiles, Range and Interquartile Range** (pp. 86 – 99)	• Estimate the median, quartiles and percentiles from cumulative frequency curves • Calculate the quartiles for a set of discrete data	Calculate the mean, median, mode and range for individual and discrete data and distinguish between the purposes for which they are used Estimate and interpret the median, percentiles, quartiles and interquartile range for cumulative frequency diagrams				
5		**4.3 Box-and-Whisker Plots** (pp. 100 – 110)	• Interpret and analyse box-and-whisker plots		Class Discussion – Vertical Box-and Whisker Plots (p. 102)	Internet Resources – Outliers (p. 106)	Internet Resources – Outliers (p. 106)	Class Discussion – Vertical Box-and Whisker Plots (p. 102) Exercise 4C Questions 11 and 12 (pp. 109 – 110)

OXFORD UNIVERSITY PRESS

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
6		**4.4 Standard Deviation** (pp. 111 – 128)	• Calculate the standard deviation • Use the mean and standard deviation to compare two sets of data		Investigation – Are Averages Adequate for Comparing Distributions? (p. 111) Investigation – Obtaining a Formula for a New Measure of Spread (pp. 112 – 114) Thinking Time (p. 121) Class Discussion – Matching Histograms with Data Sets (p. 122) Class Discussion – Can We Always Trust the Statistics We Read? (pp. 123 – 125)			Investigation – Are Averages Adequate for Comparing Distributions? (p. 111) Investigation – Obtaining a Formula for a New Measure of Spread (pp. 112 – 114) Thinking Time (p. 121) Class Discussion – Matching Histograms with Data Sets (p. 122) Class Discussion – Can We Always Trust the Statistics We Read? (pp. 123 – 125)
6		**Miscellaneous**					Solutions for Challenge Yourself	

OXFORD UNIVERSITY PRESS

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
6	**5** **Matrices**	**5.1 Introduction** (pp. 137 – 142)	• Display information in the form of a matrix of any order • Interpret the data in a given matrix	Display information in the form of a matrix of any order	Class Discussion – Defining a Matrix (p. 137) Thinking Time (p. 139) Thinking Time (p. 139)			Class Discussion – Defining a Matrix (p. 137) Thinking Time (p. 139) Thinking Time (p. 139)
6		**5.2 Addition and Subtraction of Matrices** (pp. 143 – 147)	• Add and subtract two matrices of the same order	Solve problems involving the calculation of the sum and product (where appropriate) of two matrices, and interpret the results	Class Discussion – Addition of Matrices (p.143) Class discussion – Subtraction of Matrices (p. 144) Thinking Time (p. 146)			Class Discussion – Addition of Matrices (p.143) Class discussion – Subtraction of Matrices (p. 144) Thinking Time (p. 146)
7		**5.3 Matrix Multiplication** (pp. 148 – 157)	• Multiply a matrix by a scalar • Multiply two matrices	Calculate the product of a matrix and a scalar quantity	Class Discussion – Multiplying a Matrix by a Scalar (p. 148) Class Discussion – Multiplying a Matrix with another Matrix (pp. 151 – 152) Thinking Time (p. 152) Thinking Time (p. 156)			Class Discussion – Multiplying a Matrix by a Scalar (p. 148) Class Discussion – Multiplying a Matrix with another Matrix (pp. 151 – 152) Thinking Time (p. 152) Thinking Time (p. 156)

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
7		**5.4 Determinant of a Matrix** (p. 158)		Use the algebra of 2×2 matrices including the zero and identity 2×2 matrices Calculate the determinant $\|\mathbf{A}\|$ and inverse $\mathbf{A}^{-1}$ of a non-singular matrix $\mathbf{A}$				
7		**5.5 Inverse of a Matrix** (pp. 158 – 164)	• Find the inverse of a 2×2 matrix		Thinking Time (p. 160)			Thinking Time (p. 160)
8		**5.6 Applications of Matrices** (pp. 164 – 176)	• Solve problems involving addition, subtraction and multiplication of matrices		Class Discussion – Interesting Properties of Matrices (pp. 167 – 168) Investigation – Encoding and Decoding Messages (pp. 169 – 171) Journal Writing (pp. 171 – 172)			Class Discussion – Interesting Properties of Matrices (pp. 167 – 168) Investigation – Encoding and Decoding Messages (pp. 169 – 171) Journal Writing (pp. 171 – 172)
8		**Miscellaneous**					Solutions for Challenge Yourself	

OXFORD UNIVERSITY PRESS

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
8	**6** **Further Geometrical Transformations**	**6.1 Enlargement** (pp. 183 – 193)	• Enlarge a figure with a whole negative and diminishing scale factor • Find the centre and scale factor of enlargement given the original figure and its enlarged image		Class Discussion – Enlargement in our surroundings (p. 187)			Class Discussion – Enlargement in our surroundings (p. 187)
9		**6.2 Transformation Matrices for Reflection and Rotation** (pp. 194 – 199)	• Link transformations with Matrices	Describe transformations using coordinates and matrices				
9		**6.3 Transformation Matrix for Enlargement** (pp. 200 – 202)						
10		**6.4 Inverse Transformations and Combined Transformations** (pp. 203 – 212)	• Find the image figure of an object under a combination of transformations	Use the following transformations of the plane: reflection (M), rotation (R), translation (T), enlargement (E) and their combinations Identify and give precise descriptions of transformations connecting given figures	Thinking Time (p. 206)			Thinking Time (p. 206)
10		**Miscellaneous**					Solutions for Challenge Yourself	

OXFORD UNIVERSITY PRESS

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
10	**7** **Vectors**	**7.1 Vectors in Two Dimensions** (pp. 217 – 227)	• Use vector notations • Represent vectors as directed line segments • Represent vectors in column vector form	Describe a translation by using a vector represented by $\begin{pmatrix} x \\ y \end{pmatrix}$, $\overrightarrow{AB}$ or **a** Calculate the magnitude of a vector $\begin{pmatrix} x \\ y \end{pmatrix}$ as $\sqrt{x^2 + y^2}$ Represent vectors by directed line segments	Class Discussion – Scalar and Vector Quantities (p. 217) Thinking Time (p. 217) Class Discussion – Equal Vectors (p. 220) Thinking Time (p. 221)			Class Discussion – Scalar and Vector Quantities (p. 217) Thinking Time (p. 217) Class Discussion – Equal Vectors (p. 220) Thinking Time (p. 221)
11		**7.2 Addition of Vectors** (pp. 228 – 234)	• Add vectors	Add and subtract vectors	Thinking Time (p. 231) Class Discussion – The Zero Vector (p. 233)			Thinking Time (p. 231) Class Discussion – The Zero Vector (p. 233)
11		**7.3 Vector Subtraction** (pp. 235 – 246)	• Subtract vectors		Thinking Time (p. 238)			Thinking Time (p. 238)
11		**7.4 Scalar Multiples of a Vector** (pp. 246 – 250)	• Multiply a vector by a scalar	Multiply a vector by a scalar	Thinking Time (p. 247) Class Discussion – Graphical Representation of Vectors (p. 250)			Thinking Time (p. 247) Class Discussion – Graphical Representation of Vectors (p. 250)

OXFORD UNIVERSITY PRESS

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
12		**7.5 Expression of a Vector in Terms of Two Other Vectors** (pp. 250 – 252)	• Express a vector in terms of two non-zero and non-parallel coplanar vectors	Use the sum and difference of two vectors to express given vectors in terms of two coplanar vectors	Class Discussion – Expressing a Vector in Terms of Two Other Vectors (p. 252)			Class Discussion – Expressing a Vector in Terms of Two Other Vectors (p. 252)
12		**7.6 Position Vectors** (pp. 253 – 256)	• Express a vector in terms of position vectors • Express translation by a vector	Use position vectors				
13		**7.7 Applications of Vectors** (pp. 257 – 267)	• Solve geometric problems involving the use of vectors		Class Discussion – Real-Life Examples of Resultant Vectors (p. 258)			
13		**Miscellaneous**					Solutions for Challenge Yourself	

OXFORD UNIVERSITY PRESS

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
13	**8** **Loci**	**8.1 Introduction to Loci** (p. 275)			Class Discussion – Introduction to Loci (p. 275)			
13		**8.2 Locus Theorems** (pp. 276 – 281)	• Construct simple loci of points in two dimensions		Investigation – Locus Theorem 1 (p. 277) Investigation – Locus Theorem 2 (p. 278) Investigation – Locus Theorem 3 (p. 279) Investigation – Locus Theorem 4 (p. 280)			Investigation – Locus Theorem 1 (p. 277) Investigation – Locus Theorem 2 (p. 278) Investigation – Locus Theorem 3 (p. 279) Investigation – Locus Theorem 4 (p. 280)
14		**8.3 Intersection of Loci** (pp. 282 – 287)	• Solve problems involving intersection of loci	Use the following loci and the method of intersecting loci for sets of points in two dimensions which are: (a) at a given distance from a given point (b) at a given distance from a given straight line (c) equidistant from two given points (d) equidistant from two given intersecting straight line				
14		**8.4 Further Loci** (pp. 288 – 296)						
14		**Miscellaneous**					Solutions for Challenge Yourself	

OXFORD UNIVERSITY PRESS

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
15	**9** **Revision: Numbers and Algebra**	**9.1 Numbers and Percentages** (pp. 303 – 307)						
15		**9.2 Proportion, Ratio, Rate and Speed** (pp. 307 – 310)						
16		**9.3 Algebraic Manipulation and Formulae** (pp. 311 – 318)						
16		**9.4 Equations and Inequalities** (pp. 319 – 325)						
17		**9.5 Functions and Graphs** (pp. 326 – 333)						
17		**9.6 Graphs in Practical Situations** (pp. 334 – 342)						
18		**9.7 Sets** (pp. 342 – 347)						
18		**9.8 Matrices** (pp. 348 – 356)						

OXFORD UNIVERSITY PRESS

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
19	**10** **Revision: Geometry and Measurement**	**10.1 Angles, Triangles and Polygons** (pp. 359 – 364)						
19		**10.2 Congruence and Similarity** (pp. 365 – 369)						
20		**10.3 Pythagoras' Theorem and Trigonometry** (pp. 370 – 377)						
20		**10.4 Mensuration** (pp. 378 – 386)						
21		**10.5 Geometrical Transformation and Symmetry** (pp. 387 – 389)						
21		**10.6 Coordinate Geometry** (pp. 390 – 395)						
22		**10.7 Vectors in Two Dimensions** (pp. 395 – 402)						
22		**10.8 Properties of Circles** (pp. 403 – 408)						
23	**11** **Revision: Probability and Statistics**	**11.1 Probability** (pp. 411 – 418)						
23		**11.2 Statistics** (pp. 419 – 432)						

OXFORD UNIVERSITY PRESS

OXFORD UNIVERSITY PRESS

Chapter 1 Linear Inequalities in Two Variables

TEACHING NOTES

Suggested Approach

Students have learned how to solve linear inequalities in one variable in Book 1. Teachers may wish to recap on the properties of linear inequalities. In this chapter, students will be required to draw many graphs, so teachers should revise the skills on graph drawing, such as appropriate choice of scales and labelling of axes. This chapter exposes students to real-life problems involving systems of linear inequalities in two variables.

Section 1.1: Linear Inequalities in Two Variables

Teachers should go through the Investigation (Linear Inequalities in Two Variables) on page 4 of the textbook with students, and highlight the difference between the graphs of $ax + by = c$, $ax + by < c$ and $ax + by > c$.

Students should be given ample practice on drawing graphs of linear inequalities in two variables and writing linear inequalities in two variables from graphs. Therefore, the solution to a system of linear inequalities in two variables lies in the unshaded region.

Section 1.2: Application of Systems of Linear Inequalities in Two Variables in Real-World Contexts

Teachers should make use of Worked Example 4 (Solving Real-life Problems involving Systems of Linear Inequalities in Two Variables) on page 13 of the textbook to illustrate to students that such problems usually involve the following three parts:

(a) Writing inequalities that satisfy the given conditions.

(b) Drawing the graphs of the inequalities.

(c) Using the unshaded region to find the maximum or minimum value (or values relating to the maximum or minimum value).

Challenge Yourself

Students are likely to be unfamiliar with the investment of funds at their age. Teachers may provide a simple introduction and use the opportunity to show students how investors can make use of linear inequalities in two variables to make decisions on the investment of funds.

OXFORD UNIVERSITY PRESS

Worked Solutions

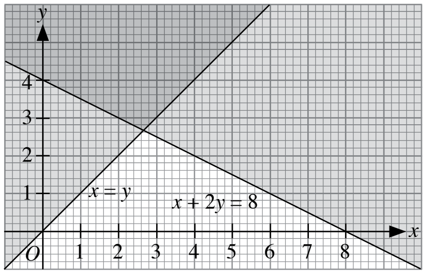
Investigation (Linear Inequalities in Two Variables)

	Case 1		**Case 2**		**Case 3**	
	Coordinates of Point	$x + 2y$	Coordinates of Point	$x + 2y$	Coordinates of Point	$x + 2y$
	$(-2, 3)$	$-2 + 2(3) = 4$	$(-3, 3)$	$-3 + 2(3) = 3$	$(-3, 4)$	$-3 + 2(4) = 5$
	$(0, 2)$	$0 + 2(2) = 4$	$(-2, 1)$	$-2 + 2(1) = 0$	$(-1, 3)$	$-1 + 2(3) = 5$
	$(2, 1)$	$2 + 2(1) = 4$	$(-1, 0)$	$-1 + 2(0) = 1$	$(0, 4)$	$0 + 2(4) = 8$
	$(4, 0)$	$4 + 2(0) = 4$	$(1, -1)$	$1 + 2(-1) = -1$	$(2, 3)$	$2 + 2(3) = 8$
	$(6, -1)$	$6 + 2(-1) = 4$	$(4, -2)$	$4 + 2(-2) = 0$	$(5, 2)$	$5 + 2(2) = 9$
Observation	The values of $x + 2y$ are always equal to 4.		The values of $x + 2y$ are always less than 4.		The values of $x + 2y$ are always greater than 4.	

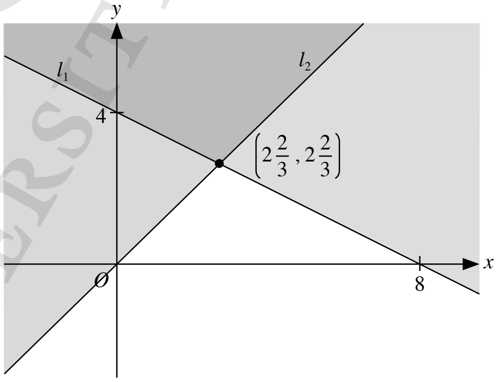
Table 1.1

Note to teachers: The above table only shows five values of x + 2y for Case 2 and Case 3. Teachers may divide the class into some groups and tell each group to find the values of x + 2y for different sets of points above and/or below the line, so that all points in Fig. 1.1(b) and Fig. 1.1(c) are covered.

Practise Now 1



Practise Now 2



Equation of l_1:

$$\frac{y - 2\frac{2}{3}}{x - 2\frac{2}{3}} = \frac{4 - 0}{0 - 8}$$

$$-2y + \frac{16}{3} = x - 2\frac{2}{3}$$

$$2y + x = 8$$

The unshaded region lies below l_1. Hence, $2y + x \leqslant 8$ defines a part of the unshaded region.

Equation of l_2: $y = x$

The unshaded region lies below l_2. Hence, $y < x$ defines a part of the unshaded region.

$\therefore$ The unshaded region is defined by two inequalities:

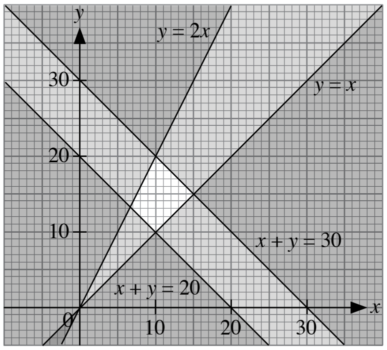
$2y + x \leqslant 8$ and $y < x$.

OXFORD UNIVERSITY PRESS

Practise Now 3

Draw the lines $y = x$, $y = 2x$, $x + y = 20$ and $x + y = 30$.
Shade the regions not required by the inequalities:
$y \geqslant x$, $y \leqslant 2x$, $20 \leqslant x + y \leqslant 30$

(i) Below $y = x$
(ii) Above $y = 2x$
(iii) Below $x + y = 20$
(iv) Above $x + y = 30$

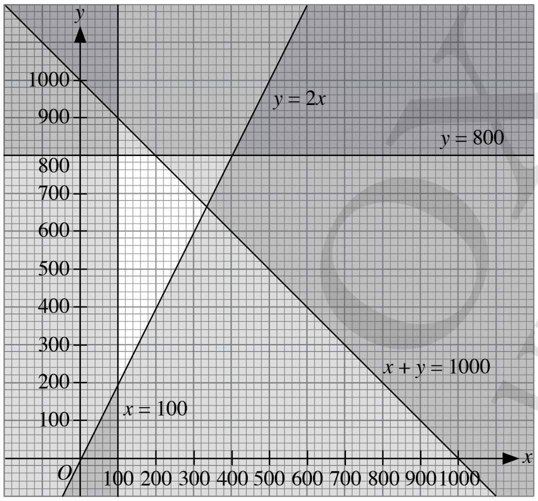


$x + 2y$ must be satisfied by the unshaded region.
If $x = 10$, $y = 20$, we obtain the greatest value of $x + 2y$.
Greatest value of $x + 2y = 10 + 2(20)$
$= 50$
If $x = 10$, $y = 10$, we obtain the least value of $x + 2y$.
Least value of $x + 2y = 10 + 2(10)$
$= 30$

Practise Now 4

(a) $x + y \leqslant 1000$, $y \geqslant 2x$, $x \geqslant 100$, $y \leqslant 800$

(b)

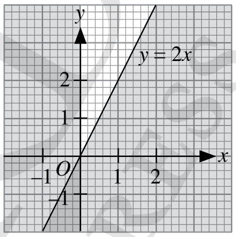


(c) Let the profit be $\$P$.
$P = 6x + 5y$
Substitute the vertices of the unshaded region into $P = 6x + 5y$.
$x = 100$, $y = 800$: $P = 6(100) + 5(800)$
$= 4600$
$x = 200$, $y = 800$: $P = 6(200) + 5(800)$
$= 5200$
$x = 330$, $y = 670$: $P = 6(330) + 5(670)$
$= 5330$
$x = 100$, $y = 200$: $P = 6(100) + 5(200)$
$= 1600$
$\therefore$ The shopkeeper should order 330 cans of *Coola* and 670 cans of *Shiok* to give the maximum profit.

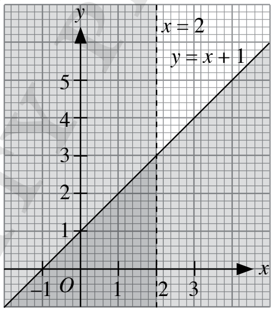
OXFORD UNIVERSITY PRESS

Exercise 1A

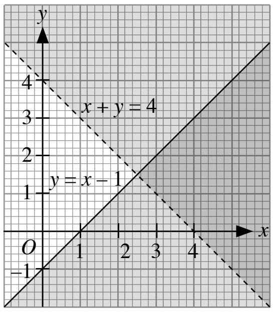
1. **(a)**



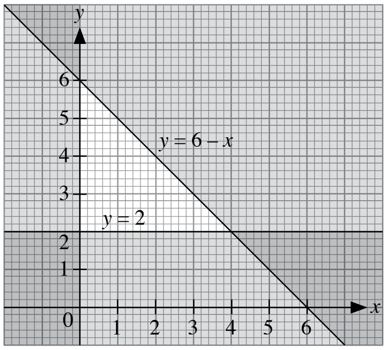
(b)



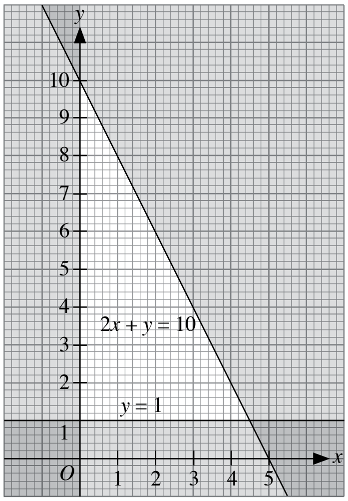
(c)



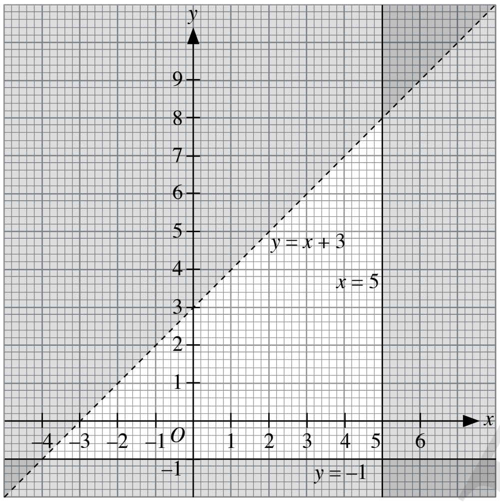
(d)



(e)



(f)



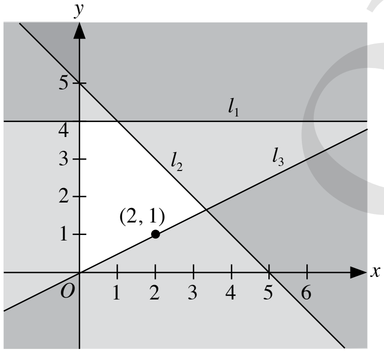
2. **(a)** $x \geqslant 0, y \geqslant 0, y \leqslant 3, x + y \leqslant 4$

(b) $x \leqslant 3, 2y > x, 3x + 2y \geqslant 6, 3y \leqslant x + 9$

(c) $x > 4, 4y \geqslant x, 4x + y \geqslant 4, 2y \leqslant x + 4$

(d) $2y > x, 2x + y \leqslant 4, y \leqslant 2x + 2$

3. **(a)**



Equation of l_1: $y = 4$

Equation of l_2: $y = –x + 5$

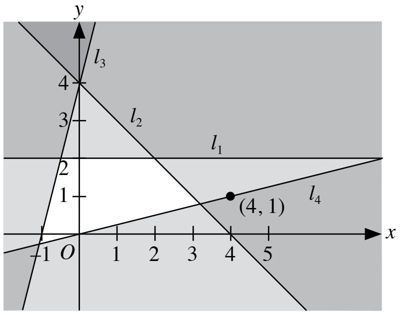
$x + y = 5$

Equation of l_3: $y = \frac{x}{2}$

$2y = x$

$\therefore$ The inequalities are $x \geqslant 0, y \leqslant 4, x + y \leqslant 5$ and $2y \geqslant x$.

(b)



Equation of l_1: $y = 2$

Equation of l_2: $y = –x + 4$

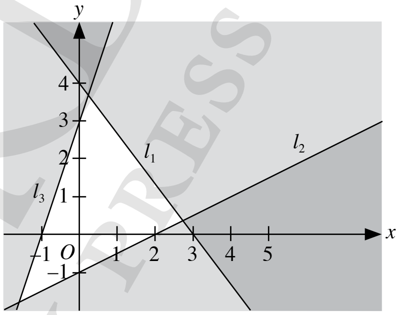
$x + y = 4$

Equation of l_3: $y = 4x + 4$

Equation of l_4: $y = \frac{x}{4}$

$\therefore$ The inequalities are $y = 2, x + y \leqslant 4, y \leqslant 4x + 4$ and $4y \geqslant x$.

(c)



Equation of l_1: $y = -\frac{4}{3}x + 4$

$3y = –4x + 12$

$4x + 3y = 12$

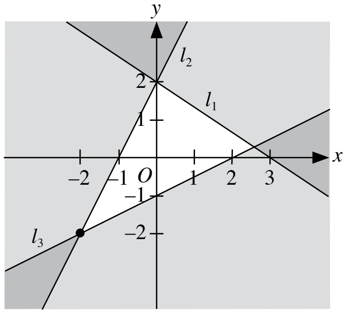
Equation of l_2: $y = \frac{1}{2}x - 1$

$2y = x – 2$

Equation of l_3: $y = 3x + 3$

$\therefore$ The inequalities are $4x + 3y \leqslant 12$, $2y \geqslant x - 2$ and $y \leqslant 3x + 3$.

(d)



Equation of l_1: $y = -\frac{2}{3}x + 2$

$3y = –2x + 6$

$2x + 3y = 6$

Equation of l_2: $y = 2x + 2$

Equation of l_3: $y = \frac{x}{2} - 1$

$2y = x – 2$

$\therefore$ The inequalities are $2x + 3y \leqslant 6, y \leqslant 2x + 2$ and $2y \geqslant x – 2$.

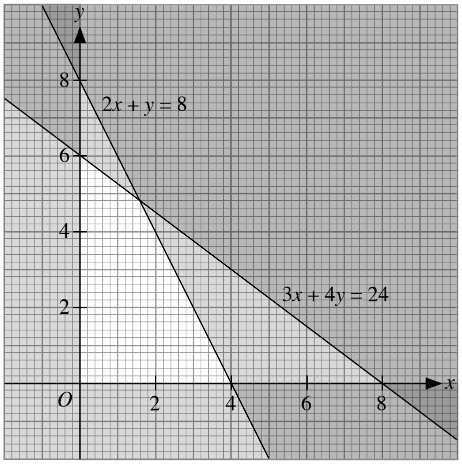
OXFORD UNIVERSITY PRESS

4. Draw the lines $3x + 4y = 24$ and $2x + y = 8$.

Shade the regions not required by the inequalities:

$x \geqslant 0, y \geqslant 0, 3x + 4y \leqslant 24$ and $2x + y \leqslant 8$

- **(i)** Below the x-axis
- **(ii)** Left of the y-axis
- **(iii)** Above $3x + 4y = 24$
- **(iv)** Above $2x + y = 8$



$4x + y$ must be satisfied by the unshaded region.

If $x = 4, y = 0$, we obtain the greatest value of $4x + y$.

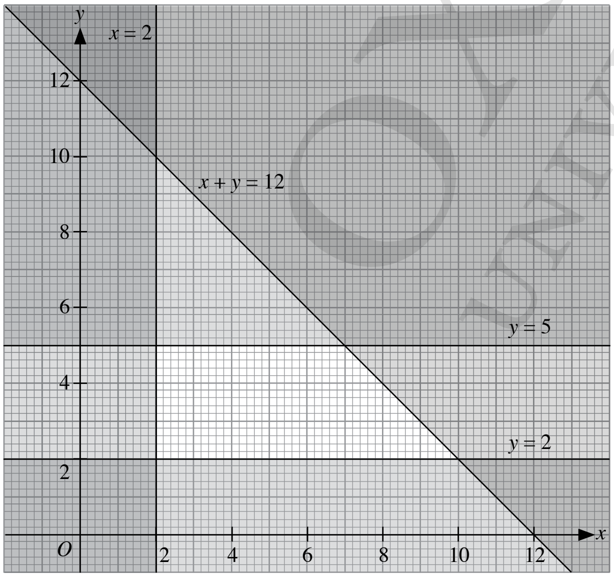
$$\text{Greatest value of } 4x + y = 4(4) + 0 = 16$$

5. Draw the lines $x = 2, y = 2, y = 5$ and $x + y = 12$.

Shade the regions not required by the inequalities:

$x \geqslant 2, y \geqslant 2, y \leqslant 5$ and $x + y \leqslant 12$

- **(i)** Left of $x = 2$
- **(ii)** Below $y = 2$
- **(iii)** Above $y = 5$
- **(iv)** Above $x + y = 12$



Substitute the coordinates of the vertices of the unshaded region into $P = x + 3y$.

$x = 2, y = 2$: $P = 2 + 3(2)$
$= 8$

$x = 2, y = 5$: $P = 2 + 3(5)$
$= 17$

$x = 7, y = 5$: $P = 7 + 3(5)$
$= 22$

$x = 10, y = 2$: $P = 10 + 3(2)$
$= 16$

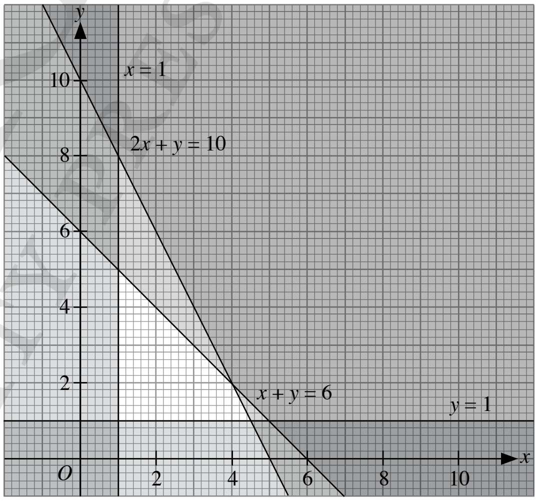
$\therefore$ The greatest value of P is 22 and the least value of P is 8.

6. Draw the lines $x = 1, y = 1, x + y = 6$ and $2x + y = 10$.

Shade the regions not required by the inequalities:

$x \geqslant 1, y \geqslant 1, x + y \leqslant 6$ and $2x + y \leqslant 10$

- **(i)** Left of $x = 1$
- **(ii)** Below $y = 1$
- **(iii)** Above $x + y = 6$
- **(iv)** Above $2x + y = 10$



Substitute the coordinates of the vertices of the unshaded region into $P = 3x + y$.

$x = 1, y = 1$: $P = 3(1) + 1$
$= 4$

$x = 1, y = 5$: $P = 3(1) + 5$
$= 8$

$x = 4, y = 2$: $P = 3(4) + 2$
$= 14$

$x = 4\frac{1}{2}, y = 1$: $P = 3\left(4\frac{1}{2}\right) + 1$
$= 14\frac{1}{2}$

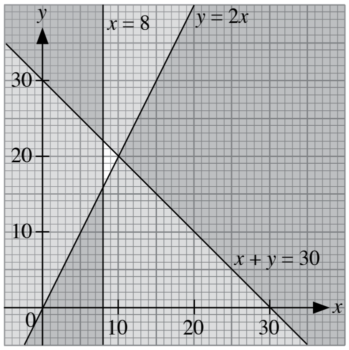
$\therefore$ The greatest value of P is $14\frac{1}{2}$ and the least value of P is 4.

OXFORD UNIVERSITY PRESS

Exercise 1B

1. **(a)** $x + y \leqslant 30, x \geqslant 8, y \leqslant 2x$

(b)



(c) Let the length of ribbon that Kate has be L cm.

$L = 30x + 15y$

Substitute the vertices of the unshaded region into $L = 30x + 15y$.

$x = 8, y = 16$: $L = 03(8) + 15(16)$
$= 480$

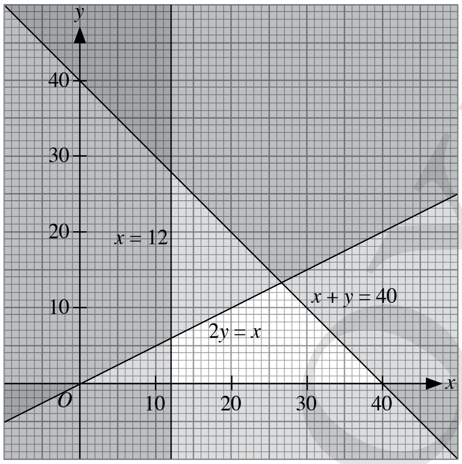
$x = 8, y = 22$: $L = 30(8) + 15(22)$
$= 570$

$x = 10, y = 20$: $L = 30(10) + 15(20)$
$= 600$

$\therefore$ The maximum possible length of ribbon that Kate has is 600 cm.

2. **(a)** $x + y \leqslant 40, x \geqslant 12, x \geqslant 2y$

(b)



(c) Let the weight of dough be W g.

$W = 6x + 8y$

Substitute the vertices of the unshaded region into $W = 6x + 8y$.

$x = 12, y = 0$: $W = 6(12) + 8(0)$
$= 72$

$x = 12, y = 6$: $W = 6(12) + 8(6)$
$= 120$

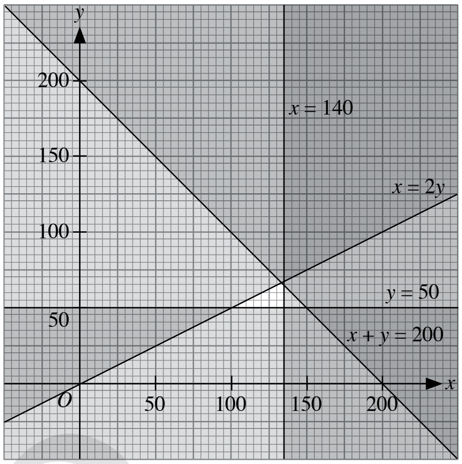
$x = 27, y = 13$: $W = 6(27) + 8(13)$
$= 266$

$x = 40, y = 0$: $W = 6(40) + 8(0)$
$= 240$

$\therefore$ The maximum possible weight of dough that the chef has is 266 g.

3. **(a)** $x + y \leqslant 200, x \geqslant 2y, y \geqslant 50, x \leqslant 140$

(b)



(c) Let the profit be \$$P$.

$P = 10x + 8y$

Substitute the vertices of the unshaded region into $P = 10x + 8y$.

$x = 100, y = 50$: $P = 10(100) + 8(50)$
$= 1400$

$x = 132.5, y = 65$: $P = 10(132.5) + 8(65)$
$= 1845$

$x = 140, y = 60$: $P = 10(140) + 8(60)$
$= 1880$

$x = 140, y = 50$, $P = 10(140) + 8(50)$
$= 1800$

$\therefore$ The supermarket manager should order 140 bottles of *Power Clean* and 60 bottles of *Disappear* to give the maximum profit.

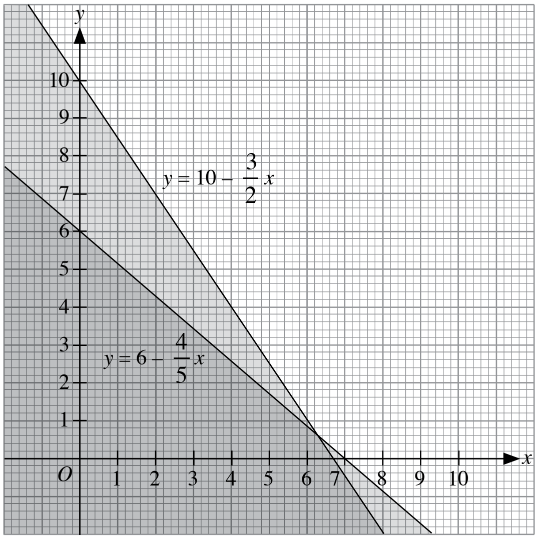
4. **(a)** $40x + 50y \geqslant 300$

$$y \geqslant 6 - \frac{4}{5}x$$

$3000x + 2000y \geqslant 20\ 000$

$$y \geqslant 10 - \frac{3}{2}x$$

(b)



(c) Least number of Gigantic ships and Jumbo ships can be (8, 0), (7, 1), (6, 2), (5, 3) and (4, 4).

$\therefore$ Least number of ships is 8.

OXFORD UNIVERSITY PRESS

5.

	Total weight (kg)	**Weight of *A* (kg)**	**Weight of *B* (kg)**
Fragrant	x	$\frac{4x}{5}$	$\frac{x}{5}$
Instant	y	$\frac{2y}{5}$	$\frac{3y}{5}$

(a) $\frac{4x}{5} + \frac{2y}{5} \leqslant 3200$

$4x + 2y \leqslant 16\,000$

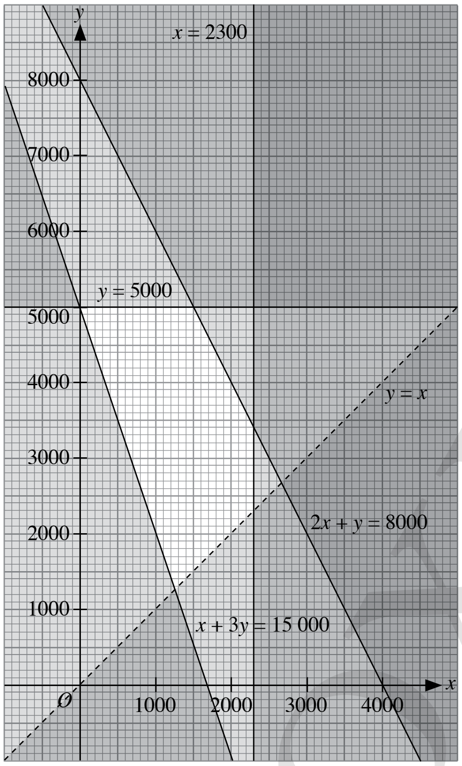
$2x + y \leqslant 8000$

$\frac{x}{5} + \frac{3y}{5} \geqslant 3000$

$x + 3y \geqslant 15\,000$

(b) $y > x, x \leqslant 2300, y \leqslant 5000$

(c)



(d) Using the vertices of the unshaded region,

at (0, 5000), $P = 0 + 5000$

$= 5000$

at (1500, 5000), $P = 1500 + 5000$

$= 6500$

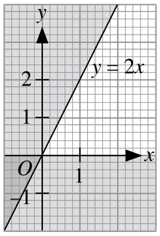
at (1800, 4400), $P = 1800 + 4400$

$= 6200$

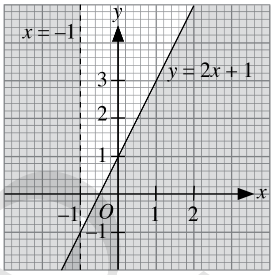
∴ The dealer should produce 1500 kg of *Fragrant* and 5000 kg of *Instant* to maximise the profit.

Review Exercise 1

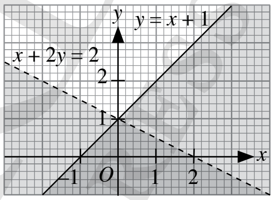
1. (a)



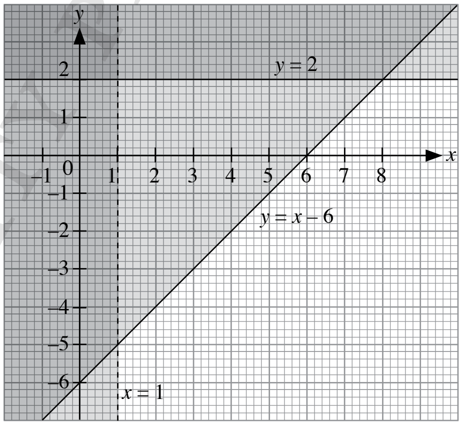
(b)



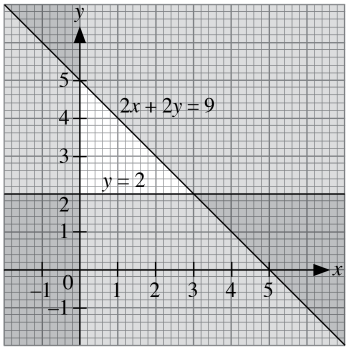
(c)



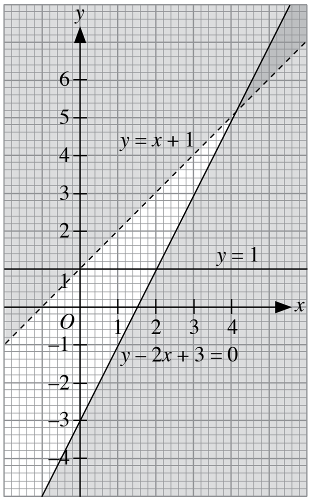
(d)



(e)



(f)



2. **(a)** $y \geqslant 0, x \leqslant 2, y \leqslant 2, 2x + y \geqslant 2$

(b) $x \leqslant 3, 2y \leqslant x + 4, 3x + 2y \geqslant 6, 2y \geqslant x$

(c) $x + 2y \leqslant 10, 2y - 4x + 3 \leqslant 0, 2y - x + 3 \geqslant 0$

(d) Equation of CD: $y = -3x + 3$

$$3x + y = 3$$

Equation of AD: $\dfrac{y-3}{x-0} = \dfrac{7-3}{3-0}$

$$\frac{y-3}{x} = \frac{4}{3}$$

$$3y - 9 = 4x$$

$$3y = 4x + 9$$

Equation of AB: $\dfrac{y-2}{x-6} = \dfrac{7-2}{3-6}$

$$\frac{y-2}{x-6} = -\frac{5}{3}$$

$$3y - 6 = -5x + 30$$

$$5x + 3y = 36$$

Equation of BC: $\dfrac{y-0}{x-1} = \dfrac{2-0}{6-1}$

$$\frac{y}{x-1} = \frac{2}{5}$$

$$5y = 2x - 2$$

$$5y + 2 = 2x$$

$\therefore$ The inequalities are $3x + y \geqslant 3$, $3y \leqslant 4x + 9$, $5x + 3y \leqslant 36$ and $5y + 2 \geqslant 2x$.

(e) Equation of AB: $\dfrac{y-5}{x-1} = \dfrac{5-3}{1-5}$

$$\frac{y-5}{x-1} = -\frac{1}{2}$$

$$2y - 10 = 1 - x$$

$$x + 2y = 11$$

Equation of BC: $\dfrac{y-3}{x-5} = \dfrac{3-(-1)}{5-\left(-\dfrac{1}{2}\right)}$

$$\frac{y-3}{x-5} = \frac{8}{11}$$

$$11y - 33 = 8x - 40$$

$$11y + 7 = 8x$$

Equation of CD: $\dfrac{y-3}{x-(-1)} = \dfrac{3-(-1)}{-1-\left(-\dfrac{1}{2}\right)}$

$$\frac{y-3}{x+1} = -8$$

$$y - 3 = -8x - 8$$

$$8x + y + 5 = 0$$

Equation of AD: $\dfrac{y-5}{x-1} = \dfrac{5-3}{1-(-1)}$

$$y - 5 = x - 1$$

$$y = x + 4$$

$\therefore$ The inequalities are $x + 2y \leqslant 11$, $11y + 7 \geqslant 8x$, $8x + y + 5 \geqslant 0$ and $y \leqslant x + 4$.

(f) Equation of OA: $y = 7x$

Equation of AB: $\dfrac{y-4}{x-4} = \dfrac{7-4}{1-4}$

$$y - 4 = 4 - x$$

$$x + y = 8$$

Equation of BC: $\dfrac{y-2}{x-5} = \dfrac{2-4}{5-4}$

$$\frac{y-2}{x-5} = -2$$

$$y - 2 = 10 - 2x$$

$$2x + y = 12$$

Equation of CD: $\dfrac{y-2}{x-5} = \dfrac{2-(-3)}{5-2}$

$$\frac{y-2}{x-5} = \frac{5}{3}$$

$$3y - 6 = 5x - 25$$

$$3y + 19 = 5x$$

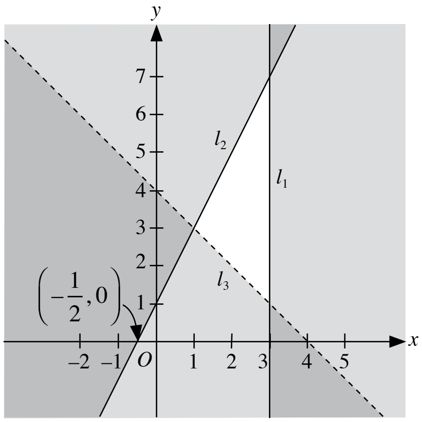
Equation of OD: $y = -\dfrac{3}{2}x$

$$3x + 2y = 0$$

$\therefore$ The inequalities are $y \leqslant 7x$, $x + y \leqslant 8$, $2x + y \leqslant 12$, $3y + 19 \geqslant 5x$ and $3x + 2y \geqslant 0$.

OXFORD UNIVERSITY PRESS

(g)



Equation of l_1: $x = 3$

Equation of l_2: $y = 2x + 1$

Equation of l_3: $y = -x + 4$

$x + y = 4$

$\therefore$ The inequalities are $x \leqslant 3$, $y \leqslant 2x + 1$ and $x + y > 4$.

3. Draw the lines $x = 0$, $y = 0$, $x = 3y$ and $y = 4x - 11$.

Shade the regions not required by the inequalities:

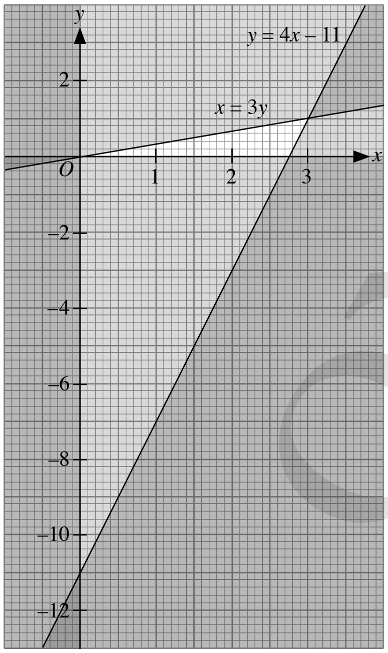
$x \geqslant 0$, $y \geqslant 0$, $x \geqslant 3y$ and $y \geqslant 4x - 11$

(i) Left of y-axis

(ii) Below x-axis

(iii) Above $x = 3y$

(iv) Above $y = 4x - 11$



From the vertices of the unshaded region,

at $(0, 0)$, $6y - x = 0$

at $(3, 1)$, $6y - x = 6 - 3$

$= 3$

at $\left(2\frac{2}{3}, 0\right)$, $6y - x = -2\frac{3}{4}$

$\therefore$ The greatest value of $6y - x$ is 3.

OXFORD UNIVERSITY PRESS

4. Draw the lines $x + 4y = 12$, $2y = 3x + 6$, $y = x - 2$ and $y = 3x - 10$.

Shade the regions not required by the inequalities:

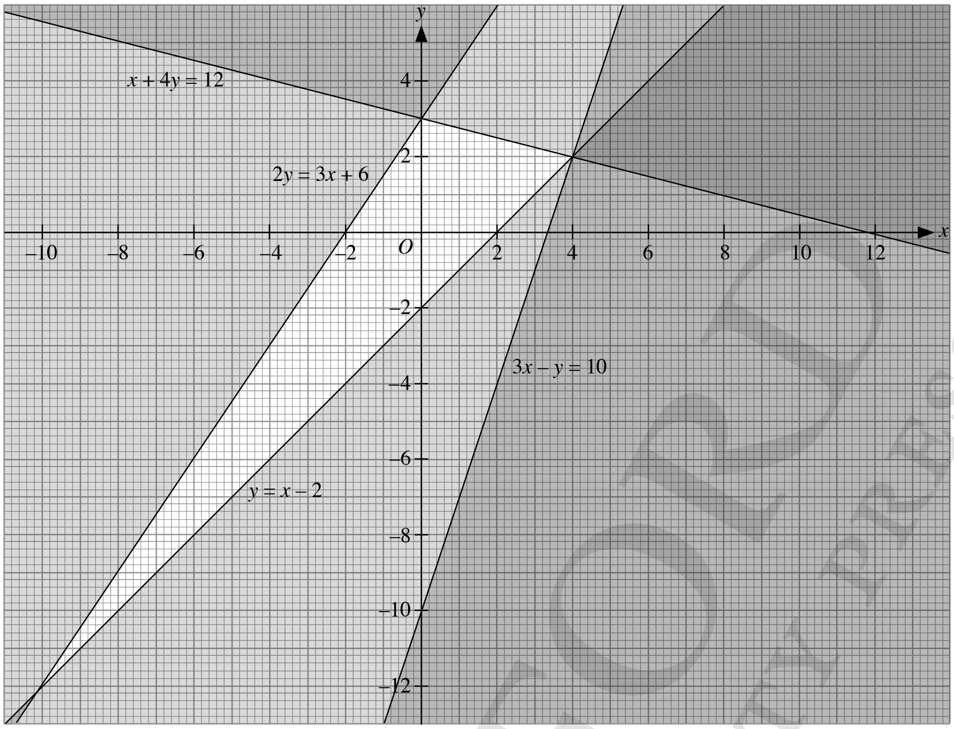
$x + 4y \leqslant 12$, $2y \leqslant 3x + 6$, $y \geqslant x - 2$ and $3x - y \leqslant 10$

(i) Above $x + 4y = 12$

(ii) Above $2y = 3x + 6$

(iii) Below $x - 3$

(iv) Below $y = 3x - 10$



From the vertices of the unshaded region,

at $(-10, -12)$, $x - y = -10 - (-12)$

$= 2$

at $(-2, 0)$, $x - y = -2$

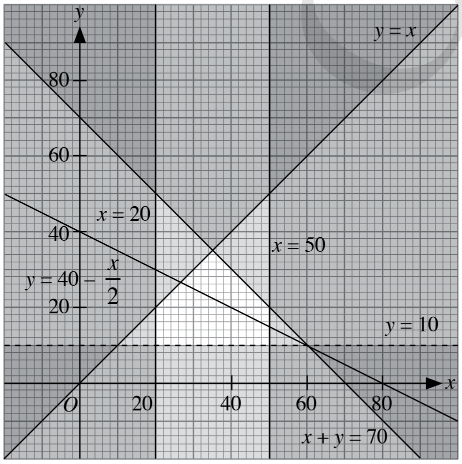
at $(4, 2)$, $x - y = 2 - 2$

$= 2$

$\therefore$ The greatest value of $x - y$ is 2.

5. **(a)** $y > 10$, $20 \leqslant x \leqslant 50$, $x + y < 70$, $x \geqslant y$

(b)



(c) Let the profit $P = 6x + 12y$.

$6x + 12y \geqslant 480$

$x + 2y \geqslant 80$

$y \geqslant 40 - \frac{x}{2}$

Draw the line $y = 40 - \frac{x}{2}$. Minimum number of ducks occurs at $x = 50$.

When $x = 50$, $y = 40 - \frac{x}{2}$

$= 15$

$\therefore$ Minimum number of ducks is 15.

OXFORD UNIVERSITY PRESS

6. (a) $x + y \geqslant 10$,

$240x + 160y \leqslant 2400$,

$3x + 2y \leqslant 30$

$2y \leqslant 30 - 3x$

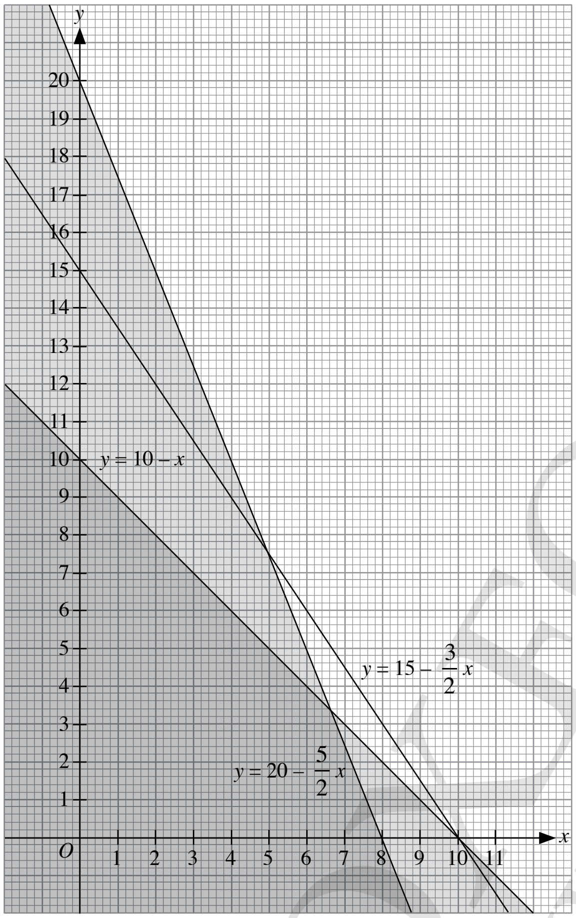
$$y \leqslant 15 - \frac{3}{2}x$$

$200x + 80y \geqslant 1600$

$5x + 2y \geqslant 40$

$$y \geqslant 20 - \frac{5}{2}x$$

(b)



(c) From the unshaded region,

at (6.7, 3.3), $x + y = 6.7 + 3.3$

$= 10$

at (10, 0), $x + y = 10 + 10$

$= 10$

at (5, 7.5), $x + y = 5 + 7.5$

$= 12.5$

$\therefore$ Maximum weight of the mixture of potato chips that contains the desired amount of calories and vitamins is 12.5 kg.

Challenge Yourself

Let \$$x$ be the amount of money (in thousands) invested in the government bond fund

and \$$y$ be the amount of money (in thousands) invested in the bank's fund.

Hence \$$(1000 - x - y)$ is the amount of money (in thousands) invested in the high-risk account.

Let \$$z$ be the total interest (in thousands) returned from all the investments.

Hence $z = 0.05x + 0.07y + 0.1(1000 - x - y)$

$= 100 - 0.05x - 0.03y$

Our goal is to find the value of x and of y that maximises the value of z.

(i) The investment in each fund cannot be less than 0.

$x \geqslant 0, y \geqslant 0, 1000 - x - y \geqslant 0$

$y \leqslant 1000 - x$

(ii) The banker decides not to invest more than \$100 000 in the high-risk account:

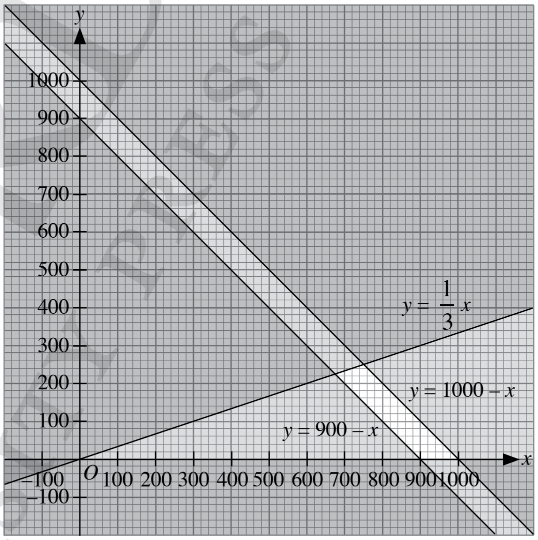
$1000 - x - y \leqslant 100$

$y \geqslant 900 - x$

(iii) He needs to invest at least three times as much in the government bond fund as in the bank's fund:

$x \geqslant 3y$

$$y \leqslant \frac{1}{3}x$$



In order to find the maximum value of z, substitute the coordinates of the vertices of the unshaded region into the equation for z.

At (750, 250), $z = 100 - 0.05(750) - 0.03(250)$

$= 55$

At (675, 225): $z = 100 - 0.05(675) - 0.03(225)$

$= 59.5$

At (1000, 0): $z = 100 - 0.05(1000) - 0.03(0)$

$= 50$

At (900, 0): $z = 100 - 0.05(900) - 0.03(0)$

$= 55$

The maximum return on the investment occurs at the point (675, 225), i.e. when

\$675 000 is invested in the government bond fund,

\$225 000 is invested in the bank's fund, and

\$100 000 is invested in the high-risk account.

OXFORD UNIVERSITY PRESS

Chapter 2 Further Sets

TEACHING NOTES

Suggested Approach

Students have previously learnt how to solve problems involving set notations and Venn diagrams in Book 2. Teachers may wish to begin the lesson by going through the Chapter opener on page 21 of the textbook and getting students to recall the use of set notations and Venn diagrams in order to solve the problem.

Section 2.1: Applications of Venn Diagrams in Problem Sums

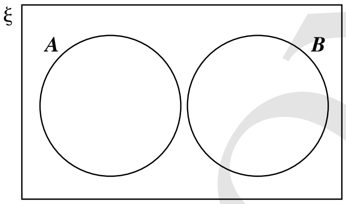
Teachers can introduce a problem that is related to something students are familiar with to pique their interest. For instance, teachers can address the students by asking, "In this class of 35 students, 20 of you like Maths and 18 of you like Science. How many of you like both subjects if all of you like at least one subject?". Following which, teachers can go through Worked Examples 1 and 2 to further reinforce the concept of the use of Venn diagrams to consolidate information provided about sets.

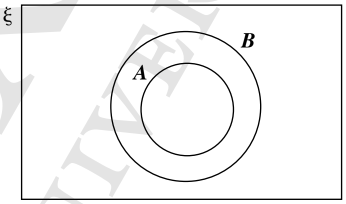
Some students might encounter difficulties with the interpretation of information, as deriving the accurate information from statements involving sets might be confusing.
For example, teachers can ask students to differentiate the following statements:
(a) There are 20 elements in A → (maybe in B as well)
(b) There are 20 elements in A only → (definitely not in B)
Teachers should remind students to read statements carefully in order to distinguish their meanings.

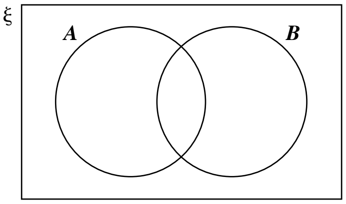
Section 2.2: Formulas in Set Theory

In the introduction of the formula $n(A \cup B) = n(\mathrm{A}) + n(B) - n(A \cap B)$, teachers can illustrate to students how the formula can be easily derived with the use of a Venn diagram, and hence, using Venn diagrams to solve problems of classification is still useful and relevant. Although the formula $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$ is given, teachers can stress that students need not memorise it. However, teachers can choose to get students to verify the formula in order to understand it (see Class Discussion on Page 30).

Teachers can also highlight to students that problems involving the greatest and least possible values of $n(A \cap B)$, $n(A \cup B)$, etc. always occur in the following three cases:







Challenge Yourself

Students should be able to see that this question can be easily solved with the use of a Venn diagram.

OXFORD UNIVERSITY PRESS

WORKED SOLUTIONS

Class Discussion (Page 30)

Teachers should note that you need not devote too much time to this activity, but it would be good to let students work through the verification and share with other students how they did so.

Thinking Time (Page 31)

n$(A \cap B)$ will have the greatest value when $B \subseteq A$.

$\Rightarrow$ n$(A \cap B)$ = n(B) = 10

n$(A \cap B)$ will have the lowest value when $A \cup B = \varnothing$

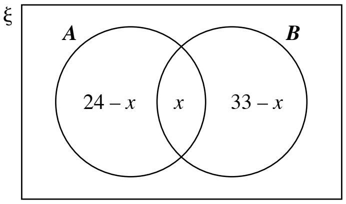
$\Rightarrow$ n$(A \cap B)$ = 0

Practise Now 1

Let A = {respondents who enjoy Italian food} and

B = {respondents who enjoy Chinese food}

There are 24 people in A, of whom $(24 - x)$ are only in A. Similarly, there must be $(33 - x)$ who are only in B.



Since all 50 people enjoy at least one of the two types of cuisine, $A \cup B = \xi$.

$(24 - x) + x + (33 - x) = 50$

$x = 7$

$\therefore$ There are 7 people who enjoy both types of cuisine.

Practise Now 2

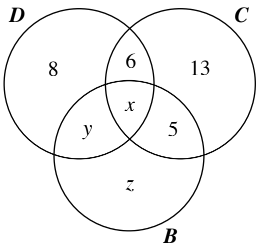
Let D = {members of the dance group},

C = {members of the choir},

B = {members of the band}

$\therefore$ n(D) = 20, n(C) = 26, n(B) = 15

Let x represent the number of students who are in all three groups, y represent the number of students who are in the dance group and band only and z represent the number of students who are in the band only.



(i) n(C) = 26

$x + 6 + 5 + 13 = 26$

$= 2$

$\therefore$ 2 students are involved in all three activities.

(ii) n(D) = 20

$y + 8 + 6 + 2 = 20$

$y = 4$

$\therefore$ 4 students are members of the dance group and band only.

(iii) n(B) = 15

$z + 4 + 2 + 5 = 15$

$z = 4$

$\therefore$ 4 students are members of the band only.

(iv) n$(D \cup B \cup T)$ = 8 + 6 + 2 + 4 + 13 + 5 + 4

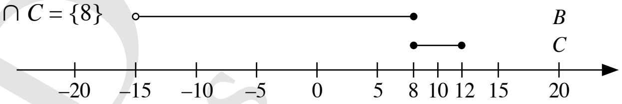
= 42

$\therefore$ There are 42 students altogether.

Practise Now 3

(i) A' = {–20, 20}

(ii) $B \cap C$ = {8}



(iii) $B \cup C = \{x: -15 < x \leqslant 12\}$

(iv) $B \cap A' = \varnothing$

Practise Now 4

A is the set of points on the straight line $y = 5x - 2$ with gradient 5 and y-intercept –2.

B is the set of points on the line $y = ax + b$.

Since n$(A \cap B)$ = 0, the two lines do not intersect.

Therefore, they must be parallel, i.e. $a = 5$.

b can take any value except –2, one possible value being 1.

Practise Now (Page 30)

n$(A \cup B)$ = n(A) + n(B) – n$(A \cap B)$

12 = 5 + 10 – n$(A \cap B)$

n$(A \cap B)$ = 3

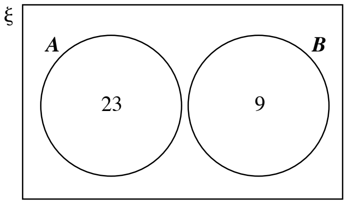
Practise Now 5

(i) n$(A \cup B)$ will have the greatest value when $A \cap B = \varnothing$

n$(A \cup B)$ = n(A) + n(B)

= 23 + 9

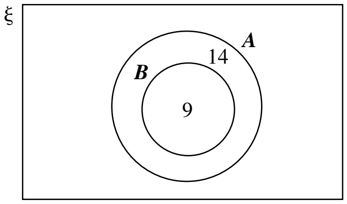
= 32



(ii) n$(A \cup B)$ = will have the least value when $B \subseteq A$.

n$(A \cup B)$ = n(A)

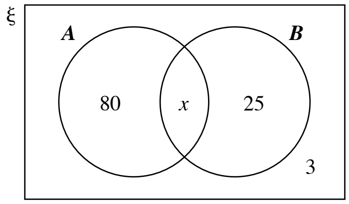
= 23



OXFORD UNIVERSITY PRESS

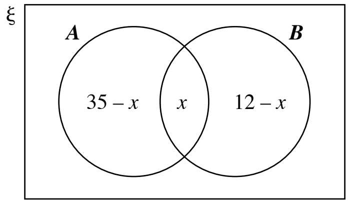
Exercise 2A

1. Let A = {pupils who can play the guitar} and
B = {pupils who can play the piano}
There are $(80 + x)$ people in A, of whom 80 are only in A. Similarly, there must be $(25 + x)$ who are in B.



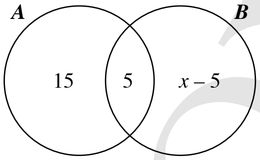
Since 3 pupils can play neither instrument, $A \cup B = 117$.
$80 + x + 25 = 117$
$x = 12$
$\therefore$ There are 12 pupils who can play both instruments.

2. Let A = {leaders who can speak English} and
B = {leaders who can speak Mandarin}
There are 35 people in A, of whom $(35 - x)$ are only in A. Similarly, there must be $(12 - x)$ who are only in B.



Since all 40 people speak at least one of the two languages, $A \cup B = \xi$.
$(35 - x) + x + (12 - x) = 40$
$x = 7$
$\therefore$ There are 7 of them who can speak both languages.

3. Let $n(B) = x$



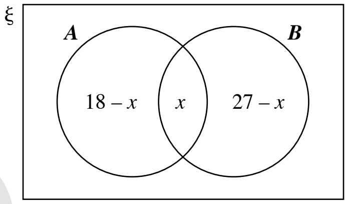
$n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$44 = 20 + x - 5$
$x = 29$
$\therefore n(B) = 29$

4. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$= 10 + 6 - 3$
$= 13$

5. **(i)** $n(A \cap B)$ will have the greatest value when $B \subseteq A$.
$n(A \cap B) = n(B)$
$= 17$
$n(A \cap B)$ will have the least value when the difference between $n(A) + n(B)$ and $n(\xi)$ is the lowest possible
$n(A \cap B) = [n(A) + n(B)] - n(\xi)$
$= (24 + 17) - 40$
$= 1$

(ii) $n(A \cup B)$ will have the greatest value when $n(A \cup B) = n(\xi) = 40$.
$n(A \cup B)$ = will have the least value when $B \subseteq A$.
$n(A \cup B) = n(A)$
$= 24$

6. **(a)** Let A = {families who have a car} and
B = {families who have a motorcycle}
There are 18 families in A, of whom $(18 - x)$ are only in A. Similarly, there must be $(27 - x)$ who are only in B.



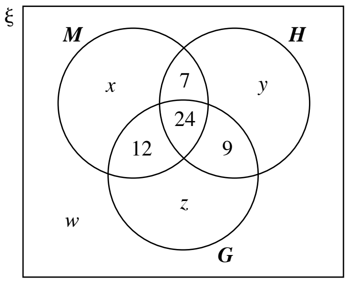
(b) Since all 35 families possess at least one of the vehicles, $A \cup B = \xi$.
(i) $(18 - x) + x + (27 - x) = 35$
$x = 10$
$\therefore$ 10 families have both cars and motorcycles.
(ii) $18 - x = 18 - 10$
$= 8$
$\therefore$ 8 families have cars but not motorcycles.

7. Let A = {pupils with POSBank's savings accounts} and
B = {pupils with commercial banks' savings accounts}
There are 60% of pupils in A, of whom $(60 - x)$% are only in A. Similarly, there must be $(72 - x)$% who are only in B.
Since all pupils have at least one savings account, $A \cup B = \xi$.
$(60 - x) + x + (72 - x) = 100$
$x = 32$
$\therefore$ 32% of the pupils have savings accounts in both POSBank and commercial banks.

8. Let M = {pupils who failed Mathematics},
H = {pupils who failed History} and
G = {pupils who failed Geography}
$\therefore n(M) = 46, n(H) = 52, n(G) = 50$
Number of pupils who failed Mathematics and History only
$= 31 - 24$
$= 7$
Number of pupils who failed History and Geography only
$= 33 - 24$
$= 9$
Number of pupils who failed Mathematics and Geography only
$= 36 - 24$
$= 12$

OXFORD UNIVERSITY PRESS

Let x, y and z represent the number of pupils who failed Mathematics only, History only and Geography only respectively; and w represent the number of pupils who did not fail any of these subjects.



$n(M) = 46$

$x + 12 + 24 + 7 = 46$

$x = 3$

$n(H) = 52$

$y + 7 + 24 + 9 = 52$

$y = 12$

$n(G) = 50$

$z + 12 + 24 + 9 = 50$

$z = 5$

Number of pupils who failed at least one subject

$= n(M \cup H \cup G)$

$= 3 + 7 + 24 + 12 + 12 + 9 + 5$

$= 72$

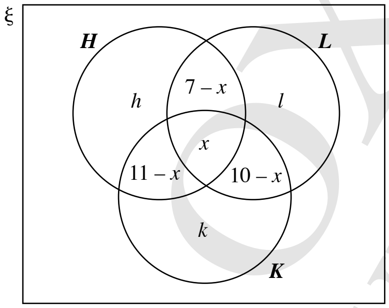
9. **(i)** Let H = {men with heart disease},

L = {men with lung disease} and

K = {men with kidney disease}

$\therefore$ $n(H) = 30$, $n(L) = 30$, $n(K) = 33$

Let x represent the number of men with all three diseases, and h, l and k represent the number of men with heart disease only, lung disease only and kidney disease only, respectively.



$n(H) = 30$

$h + (7 - x) + (11 + x) + x = 30$

$h = 12 + x$

$n(L) = 30$

$l + (7 - x) + x + (10 - x) = 30$

$l = 13 + x$

$n(K) = 33$

$k + (11 - x) + x + (10 - x) = 33$

$k = 12 + x$

Since all of them have defects in at least one of these organs, $H \cup L \cup K = \xi$.

$(12 - x) + (7 - x) + x + (11 - x)$

$+ (10 - x) + (12 + x) + (13 + x) = 68$

$65 - x = 68$

$x = 3$

$\therefore$ 3 of the elderly men suffered from all the three diseases.

(ii) $l = 13 + x$

$= 13 + 3$

$= 16$

$\therefore$ 16 elderly men had only lung disease.

10. Let O = {people who liked orange flavour},

L = {people who liked lemon flavour} and

G = {people who liked grape flavour}

$\therefore$ $n(O) = 85$, $n(L) = 90$, $n(G) = 65$

Number of people who liked orange and lemon only $= 45 - 15$

$= 30$

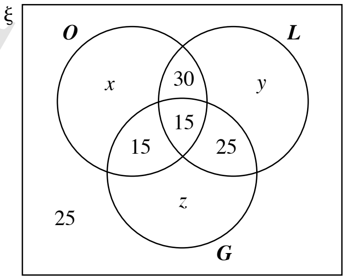
Number of people who liked lemon and grape only $= 40 - 15$

$= 25$

Number of people who liked orange and grape only $= 30 - 15$

$= 15$

Let x, y and z represent the number of people who liked orange only, lemon only and grape only respectively.



(i) $n(O) = 85$

$x + 30 + 15 + 15 = 85$

$x = 25$

$n(L) = 90$

$y + 30 + 15 + 25 = 90$

$y = 20$

$n(G) = 65$

$z + 15 + 15 + 25 = 65$

$z = 10$

Total number of people interviewed

$= 25 + 30 + 15 + 15 + 20 + 25 + 10 + 25$

$= 165$

(ii) Number of people who liked orange alone = 25

(iii) Number of people who liked lemon alone = 20

(iv) Number of people who liked grape alone = 10

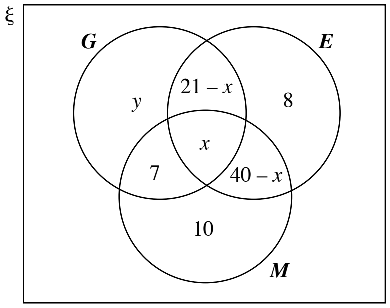
11. **(a)** Let G = {boys who passed Geography}

E = {boys who passed English}

M = {boys who passed Mathematics}

OXFORD UNIVERSITY PRESS

Let x represent the number of boys who passed all three subjects and y represent the number of boys who passed Geography only.



(b) **(i)** $n(E) = 54$

$(21 - x) + (40 - x) + x + 8 = 54$

$x = 15$

Since none of them failed all three subjects,

$G \cup E \cup M = \xi$.

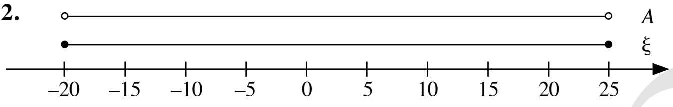
$y + 6 + 15 + 7 + 25 + 8 + 10 = 80$

$y = 9$

$\therefore$ 9 boys passed Geography only.

(ii) 15 boys passed all three subjects.

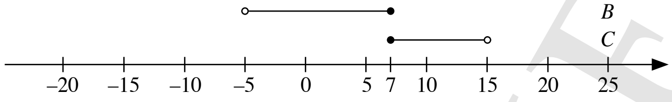
12.



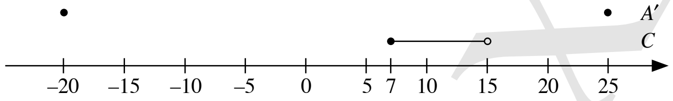
(i) $A' = \{-20, 20\}$

(ii) $B \cap C = \{7\}$

(iii) $B \cup C = \{x: -5 < x < 15\}$



(iv) $C \cap A' = \varnothing$



13. A is the set of points on the straight line $y = \frac{2}{5}x - 7$ with gradient $\frac{5}{2}$ and y-intercept –7.

B is the set of points on the line $y = ax - b$.

Since $n(A \cap B) = 0$, the two lines do not intersect.

Therefore, they must be parallel, i.e. $a = \frac{5}{2}$.

b can take any value except 7, one possible value being 5.

14. **(i)** Since A and B are disjoint sets, $n(A \cap B) = 0$

(ii) $n(A \cup B) = n(A) + n(B)$

$= 84 + 16$

$= 100$

15. **(i)** $n(A \cap B) = n(A)$

$= 42$

(ii) $n(A \cup B) = n(B)$

$= 75$

16. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$r = p + q - n(A \cap B)$

$\therefore n(A \cap B) = p + q - r$

17. **(i)** $n(A) = n(A \cap B') + n(A \cap B)$

$= 28 + 5$

$= 33$

(ii) $n(B) = n(A' \cap B) + n(A \cap B)$

$= 23 + 5$

$= 28$

(iii) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 33 + 28 - 5$

$= 56$

18. Greatest possible value of $n(A \cup B)'$ is obtained when $B \subseteq A$.

$n(A \cup B) = n(A)$

$= 50$

$n(A \cup B)' = n(\xi) - n(A \cup B)$

$= 80 - 50$

$= 30$

19. Least possible value of $n(A \cap B)'$ is obtained when $A \subseteq B$.

$n(A \cap B) = n(A)$

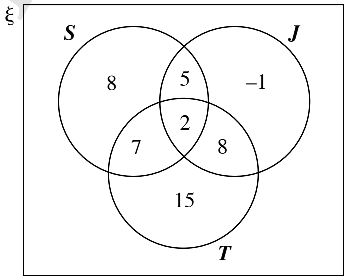
$= 32$

$n(A \cap B)' = n(\xi) - n(A \cap B)$

$= 68 - 32$

$= 36$

20.



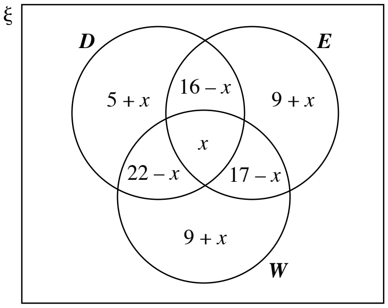
From the Venn diagram, the numbers reported by the prefect will imply that a negative result occurs for the number of pupils taking part in judo only, and this is not possible. Hence, the information was not correct.

21. Let D = {youths who take up dancing},

E = {youths who take up sewing} and

W = {youths who take up swimming}

$\therefore n(D) = 43, n(E) = 42, n(W) = 48$



$n(\xi) = 90$

$(5 + x) + (16 - x) + x + (22 - x) + (9 + x) + (17 - x) + (9 + x) = 90$

$78 + x = 90$

$x = 12$

OXFORD UNIVERSITY PRESS

22. Let M = {students who failed their Maths paper} and
G = {students who failed their Geography paper}

$\therefore$ n(M) = 15, n(G) = 19

(i) Greatest number of students that failed both papers is obtained when $M \subseteq G$.

n($M \cap G$) = n(M)
= 15

(ii) Least number of students that failed both papers is obtained when $M \cap G = \varnothing$

$\Rightarrow$ n($M \cap G$) = 0

(iii) Maximum number of students who failed only one paper is obtained when $M \cap G = \varnothing$

n($M \cup G$) = n(M) + n(G)
= 15 + 19
= 34

Review Exercise 2

1. Let M = {houses with mango trees} and
R = {houses with rambutan trees}

$\therefore$ n(M) = 32, n(R) = 28

n($M \cup R$)′ = 2

n($M \cup R$) = n(ξ) – n($M \cup R$)’
= 44 – 2
= 42

n($M \cup R$) = n(M) + n(R) – n($M \cap R$)
42 = 32 + 28 – n($M \cap R$)

n($M \cap R$) = 18

$\therefore$ 18 houses have both mango and rambutan trees.

2. Let T = {employees who drink tea} and
C = {employees who drink coffee}

n(T) = 28

n($T \cap C'$) = 10

n($T \cap C$) = n(T) – n($T \cap C$’)
= 28 – 10
= 18

Since all employees drink either tea or coffee,

n(ξ) = 43

n($T \cup C$) = n(T) + n(C) – n($T \cap C$)
43 = 28 + n(C) – 18
n(C) = 33

n($C \cap T'$) = 33 – 18
= 15

$\therefore$ 15 employees drink coffee but not tea.

3. Let A = {students who take part in athletics} and
S = {students who take part in swimming}

$\therefore$ n(A) = 170, n(S) = 192

(i) Since no student can take part in both activities, n($A \cap S$) = 0

n($A \cup S$) = n(A) + n(S) – n($A \cap S$)
= 170 + 192 – 0
= 362

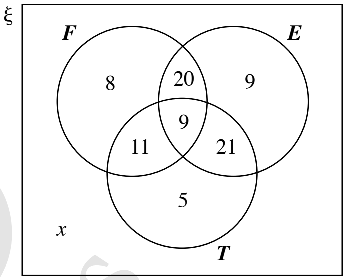
(ii) n($A \cap S$) = 35

n($A \cup$ S) = n(A) + n(S) – n($A \cap S$)
= 170 + 192 – 35
= 327

4. Let F = {viewers who watch Channel 5},
E = {viewers who watch Channel 8} and
T = {viewers who watch Channel 12}

Let n(ξ) = 100.

$\therefore$ n(F) = 48, n(E) = 59, n(T) = 46



(i) From the Venn diagram, number of viewers who watched Channel 5 and 8 but not 12 = 20

$\therefore$ 20% of viewers watched Channel 5 and 8 but not 12.

(ii) From the Venn diagram, number of viewers who watched exactly two channels
= 11 + 20 + 21
= 52

$\therefore$ 52% of viewers watched exactly two channels.

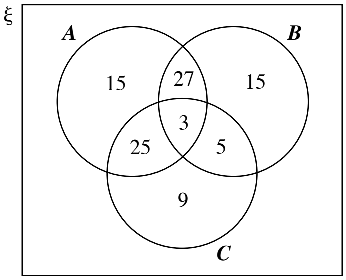
(iii) Let x represent the number of viewers who did not watch any of the channels.

x + 8 + 11 + 20 + 9 + 21 + 9 + 5 = 100
x = 17

$\therefore$ 17% of viewers did not watch any of the channels.

5. Let A = {students who attempted Part A},
B = {students who attempted Part B} and
C = {students who attempted Part C}

$\therefore$ n(A) = 70, n(B) = 50, n(C) = 42



(i) From the Venn diagram, number of students who attempted Part A but not B and C = 15

(ii) From the Venn diagram, number of students who attempted Part B but not A and C = 15

(iii) From the Venn diagram, number of students who attempted at least two parts
= 27 + 3 + 25 + 5
= 60

OXFORD UNIVERSITY PRESS

6. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$45 = 28 + n(B) - 12$

$n(B) = 29$

7. **(i)** Greatest value of $n(A \cap B)$ is obtained when $B \subseteq A$.

$n(A \cap B) = n(B)$

$= 12$

Lowest value of $n(A \cap B) = n(A) + n(B) - n(\xi)$

$= 15 + 12 - 25$

$= 2$

(ii) Greatest value of $n(A \cup B)$ is obtained when $n(A \cap B)$ is the smallest.

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 15 + 12 - 2$

$= 25$

Least value of $n(A \cup B)$ is obtained when $B \subseteq A$.

$n(A \cup B) = n(A)$

$= 15$

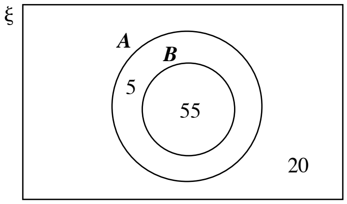
8. Greatest possible value of $n(A \cup B)'$ is obtained when $n(A \cup B)$ is lowest, i.e. when $B \subseteq A$.

$n(A \cup B)' = n(\xi) - n(A)$

$= 20 - 13$

$= 7$

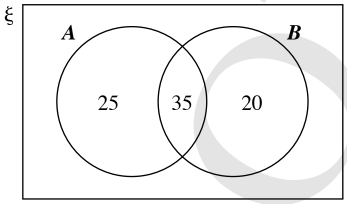
9. Greatest possible value of $n(A \cap B)$ is obtained when $B \subseteq A$.



$n(A \cap B) = n(B)$

$= 55$

Least possible value of $n(A \cap B)$ is obtained when the intersection of A and B is the smallest.



$n(A \cap B) = n(A) + n(B) - n(\xi)$

$= 60 + 55 - 80$

$= 35$

Challenge Yourself

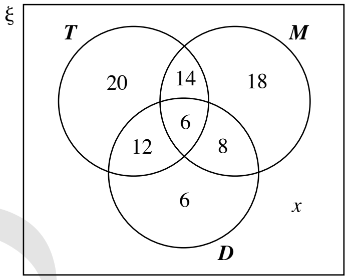
Let T = {members who attended the trading category},

M = {members who attended the manufacturing category} and

D = {members who attended the design category}

$\therefore n(T) = 52, n(M) = 46, n(D) = 32$

Let x represent the number of members who did not attend any of the conventions.



$x + 20 + 12 + 14 + 6 + 8 + 18 + 6 = 150$

$x = 150 - 84$

$= 66$

Percentage of members who did not attend any of the conventions

$= \frac{66}{150} \times 100\%$

$= 44\%$

OXFORD UNIVERSITY PRESS

Chapter 3 Probability of Combined Events

TEACHING NOTES

Suggested Approach

As the pupils are already familiar with the concept of probability that they learnt in Secondary Two, it would be easier to approach this topic. A quick revision is suggested with the summary list on the first page of this chapter.

Following the revision, teachers can get students to think about how games in sports such as tennis, football or hockey are started. Teachers can prompt students to notice that generally, a coin or something else with two sides is used, and a player from each team will choose either face, determining who has the first advantage based on the outcome of the toss. Why is the coin the norm in most cases? Why not use a die or any other objects?

Teachers can then get the whole class to throw a coin 20 times each and record the number of occurrences of heads and tails. Students can then tally the number of heads and tails to draw a distinction between the theoretical and actual probabilities occurring in an event. Teachers can urge students to think about whether the outcome of one toss affects the outcome of the next toss. Teachers may then discuss some cases where probability is useful in making real-life predictions, and demonstrate why learning about the probability of combined events, and not just single events, is important in the real-world context.

Section 3.1: Probability of Single Events
As the pupils are already familiar with set notation, teachers can introduce the concept of sample space and events using set notation.

Section 3.2: Simple Combined Events, Possibility Diagrams and Tree Diagrams
In this section, we introduce the possibility diagram and tree diagram when two events are taking place. Possibility diagrams and tree diagrams are very useful for solving problems involving two events. Teachers can go through the Worked Examples or work out the Practise Now questions on the board for the class and let students read the Worked Examples themselves.

Section 3.3: Addition Law of Probability and Mutually Exclusive Events
Go through the Investigation on Mutually Exclusive and Non-Mutually Exclusive Events on page 51 so that pupils can distinguish the difference between them. The main concept is that Mutually Exclusive Events cannot occur at the same time and P(A or B) or P($A \cup B$) = P(A) + P(B).

Section 3.4: Multiplication Law of Probability and Independent Events
Discuss the concept of independent events and dependent events using simple everyday life examples such as the following:

(i) Throwing a coin followed by tossing a die. Will the first event affect the result of the second event?

(ii) Tossing a white die followed by tossing a red die. Will the first event affect the result of the second event?

(iii) A bag has 5 red marbles and 7 blue marbles. All the marbles are identical except for their colour. A marble is selected, its colour is noted and it is put back into the bag. A second marble is then picked and its colour noted. Will the first event affect the result of the second event?

(iv) A bag has 8 red marbles and 9 blue marbles. All the marbles are identical except for their colour. A marble is selected, its colour is noted and it is put aside. A second marble is then picked and its colour is noted. Will the first event affect the result of the second event?

Teachers can work through the Investigation on Dependent Events on page 60 for a better understanding of dependent events. Teachers can also use some of the questions in Practise Now 9 and 10 to show how problems involving independent and dependent events can solved, and teachers can get students to work with tree diagrams as well as possibility diagrams.

If time permits, ask the pupils to work on the Performance Task on page 62 for enrichment.

OXFORD UNIVERSITY PRESS

Challenge Yourself

For question 2, teachers may advise the students to search on the Internet by keying in ‘Monte Hall problem’ to understand more on this question.

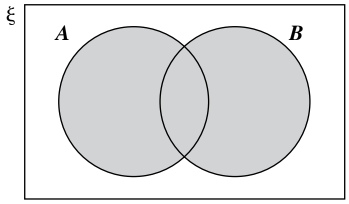
Question 3 involves the concept of the probability of the complement of any event, i.e. $P(E') = 1 - P(E)$, to find out the probability of at least two students having their birthday falling on the same day of the year.

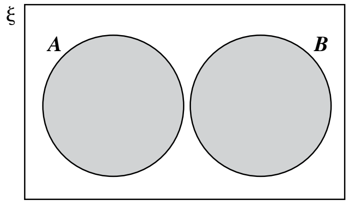
OXFORD UNIVERSITY PRESS

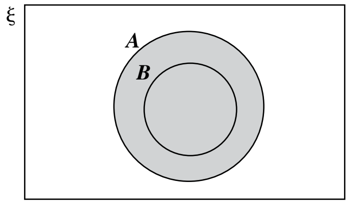
WORKED SOLUTIONS

Thinking Time (Page 38)

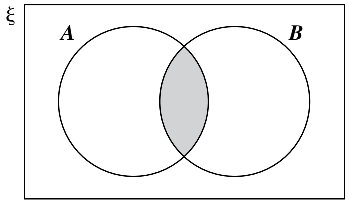
P($A \cup B$) refers to the probability of an event landing in A or in B or in both A and B.

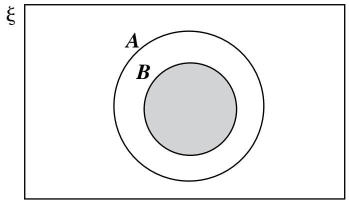






P($A \cap B$) refers to the probability of an event landing in A and B.





Investigation (Mutually Exclusive and Non-Mutually Exclusive Events)

1. The sample space is {1, 2, 3, 4, 5, 6, 7, 8}.

Part 1: Mutually Exclusive Events

2. $A = \{2, 3, 5, 7\}$ and P(A) = $\frac{4}{8} = \frac{1}{2}$

3. $B = \{4, 8\}$ and P(B) = $\frac{2}{8} = \frac{1}{4}$

4. No

5. $A \cup B = \{2, 3, 4, 5, 7, 8\}$ and P($A \cup B$) = $\frac{6}{8} = \frac{3}{4}$

6. Yes, since the two events are mutually exclusive.

Part 2: Non-Mutually Exclusive Events

7. $C = \{1, 3, 5, 7\}$ and P(C) = $\frac{4}{8} = \frac{1}{2}$

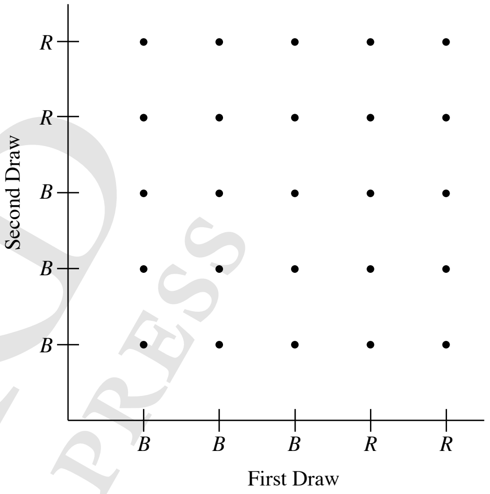
8. Yes

9. $A \cup C = \{1, 2, 3, 5, 7\}$ and P($A \cup C$) = $\frac{5}{8}$

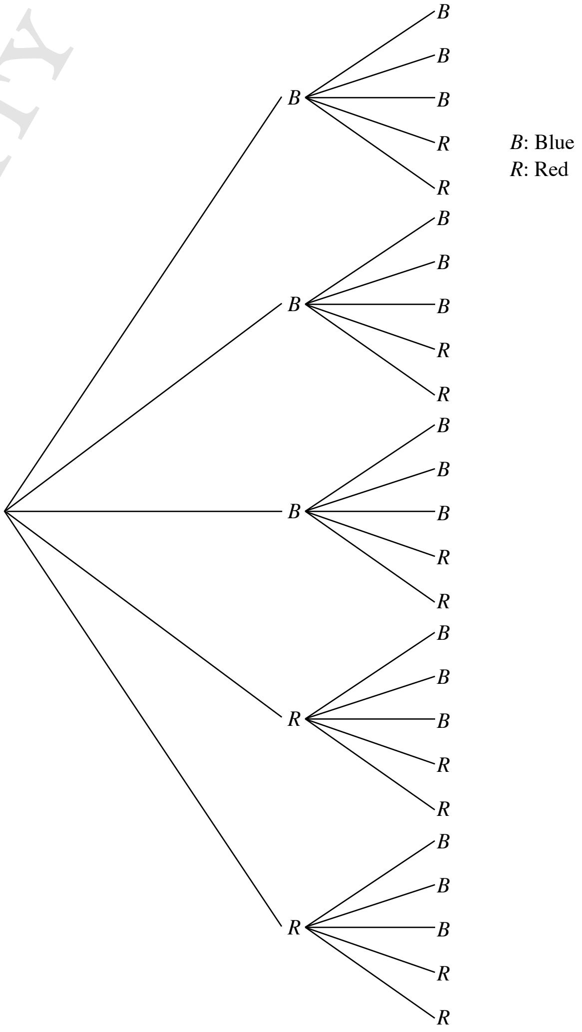
10. No, since the two events are not mutually exclusive.

Class Discussion (Choosing a Diagram to Represent the Sample Space)

1. (a)



(b)

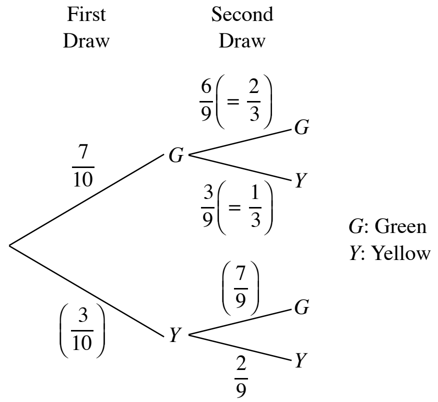


OXFORD UNIVERSITY PRESS

2. It is easier to represent the sample space on the possibility diagram than the tree diagram.

Investigation (Dependent Events)

1.



2. **(i)** Probability = $\frac{1}{3}$

(ii) Probability = $\frac{2}{9}$

3. No

Yes. If the first marble drawn is green, then there will still be 3 yellow marbles in the bag. However, if the first marble drawn is yellow, then there will only be 2 yellow marbles left.

4. P(*GY* or *YY*)

$= \left(\frac{7}{10} \times \frac{3}{9}\right) + \left(\frac{3}{10} \times \frac{2}{9}\right)$

$= \frac{3}{10}$

No

5. **(i)** Event *B* is dependent on event *A* since the probability of *B* happening depends on the outcome of *A*.

(ii) No

Performance Task (Page 62)

(a) Area of the unit circle

$= \pi(1)^2$

= 3.14 square units (to 3 s.f.)

(b) The number of points within the unit circle

(c) =D2/E2*4

(d) The mean area of the unit circle is 3.20 square units (to 3 s.f.).

Yes, it is close enough to the value of π.

Practise Now 1

(a) Let *S* represent the sample space.

S = {22, 23, 25, 32, 33, 35, 52, 53, 55}

(b) **(i)** Let *A* be the event that the two-digit number formed is prime.

A = {23, 53}

$P(A) = \frac{2}{9}$

(ii) Let *B* be the event that the two-digit number contains the digit '2'.

B = {22, 23, 25, 32, 52}

$P(B) = \frac{5}{9}$

(iii) Let *C* be the event that the two-digit number is divisible by 4.

C = {32, 52}

$P(C) = \frac{2}{9}$

(iv) Let *D* be the event that the two-digit number is divisible by 13.

D = {52}

$P(D) = \frac{1}{9}$

(v) Let *E* be the event that the two-digit number is not divisible by 13.

E = {22, 23, 25, 32, 33, 35, 53, 55}

$P(E) = \frac{8}{9}$

Alternatively,

$P(D') = 1 - P(D)$

$= 1 - \frac{1}{9}$

$= \frac{8}{9}$

Practise Now 2

Let *S* represent the sample space.

$S = \{C, L, E_1, V, E_2, R\}$

(i) Let *A* be the event that the letter chosen is an 'E'.

$A = \{E_1, E_2\}$

$P(A) = \frac{2}{6}$

$= \frac{1}{3}$

(ii) Let *B* be the event that the letter chosen is a 'C' or a 'R'.

B = {C, R}

$P(B) = \frac{2}{6}$

$= \frac{1}{3}$

(iii) Let *C* be the event that the letter chosen is a 'K'.

C = { }

$P(C) = \frac{0}{6}$

= 0

(iv) Let *D* be the event that the letter chosen is a consonant.

D = {C, L, V, R}

$P(D) = \frac{4}{6}$

$= \frac{2}{3}$

OXFORD UNIVERSITY PRESS

Practise Now 3

1. **(a)**

6-sided Die

Tetrahedral Die

	1	**2**	**3**	**4**	**5**	**6**
1	1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
2	2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
5	5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
6	6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

(b) **(i)** P(both dice show the same number) $= \frac{4}{24}$

$= \frac{1}{6}$

(ii) P(the number shown on the tetrahedral die is greater than the number shown on the 6-sided die)

$= \frac{10}{24}$

$= \frac{5}{12}$

(iii) P(the numbers shown on both dice are prime numbers)

$= \frac{6}{24}$

$= \frac{1}{4}$

2.

Second Card

First Card

	1	**2**	**3**	**4**	**5**
1	1, 1	1, 2	1, 3	1, 4	1, 5
2	2, 1	2, 2	2, 3	2, 4	2, 5
3	3, 1	3, 2	3, 3	3, 4	3, 5
4	4, 1	4, 2	4, 3	4, 4	4, 5
5	5, 1	5, 2	5, 3	5, 4	5, 5

(i) P(number shown on the second card is greater than the number shown on the first card)

$= \frac{10}{25}$

$= \frac{2}{5}$

(ii) P(sum of the two numbers shown is greater than 7)

$= \frac{6}{25}$

(iii) P(product of the two numbers shown is greater than 10)

$= \frac{8}{25}$

Practise Now 4

1. **(a)**

Tetrahedral Die

6-sided Die

+	**1**	**2**	**5**	**6**
1	2	3	6	7
2	3	4	7	8
3	4	5	8	9
4	5	6	9	10
5	6	7	10	11
6	7	8	11	12

Tetrahedral Die

6-sided Die

×	**1**	**2**	**5**	**6**
1	1	2	5	6
2	2	4	10	12
3	3	6	15	18
4	4	8	20	24
5	5	10	25	30
6	6	12	30	36

(b) **(i)** P(sum of the scores is even)

$= \frac{12}{24}$

$= \frac{1}{2}$

(ii) P(sum of the scores is divisible by 3)

$= \frac{8}{24}$

$= \frac{1}{3}$

(iii) P(sum of the scores is a perfect square)

$= \frac{4}{24}$

$= \frac{1}{6}$

(iv) P(sum of the scores is less than 2)

= 0

(c) **(i)** P(product of the scores is odd)

$= \frac{6}{24}$

$= \frac{1}{4}$

(ii) P(product of the scores is larger than 12)

$= \frac{8}{24}$

$= \frac{1}{3}$

(iii) P(product of the scores is a prime number)

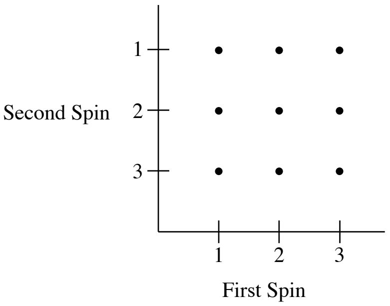
$= \frac{5}{24}$

(iv) P(product of the scores is less than 37)

= 1

OXFORD UNIVERSITY PRESS

2. (a)



(i) P(each score is a '1')

$= \frac{1}{9}$

(ii) P(at least one of the scores is a '3')

$= \frac{5}{9}$

(b) (i)

	1	**2**	**3**
1	1	2	3
2	2	2	3
3	3	3	3

(ii) P(final score is even)

$= \frac{3}{9}$

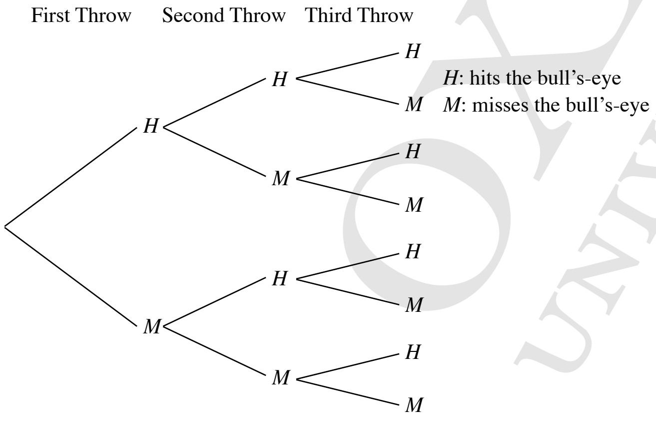
$= \frac{1}{3}$

(iii) P(final score is a prime number)

$= \frac{8}{9}$

Practise Now 5

1.



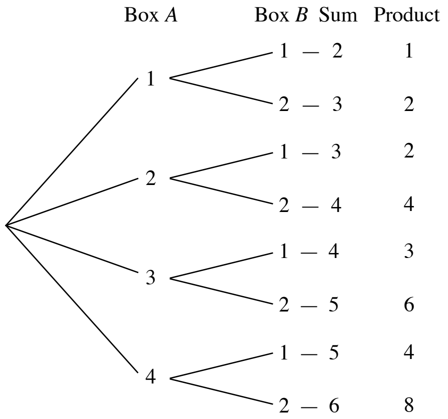
(i) P(misses the bull's-eye once)

$= \frac{3}{8}$

(ii) P(hits the bull's-eye at least once)

$= \frac{7}{8}$

2. (a)



(b) (i) P(at least one '1' is obtained)

$= \frac{5}{8}$

(ii) P(the sum of the two numbers is 3)

$= \frac{2}{8}$

$= \frac{1}{4}$

(iii) P(the product of two numbers is at least 4)

$= \frac{4}{8}$

$= \frac{1}{2}$

(iv) P(the sum is equal to the product)

$= \frac{1}{8}$

Practise Now 6

1. (i) P(picture card or Ace)

= P(picture card) + P(Ace)

$= \frac{12}{52} + \frac{4}{52}$

$= \frac{16}{52}$

$= \frac{4}{13}$

(ii) P(Ace or card bearing a number which is divisible by 3)

= P(Ace) + P(card bearing a number which is divisible by 3)

$= \frac{4}{52} + \frac{12}{52}$

$= \frac{4}{52} + \frac{12}{52}$

$= \frac{16}{52}$

$= \frac{4}{13}$

(iii) P(King or Queen)

= P(King) + P(Queen)

$= \frac{4}{52} + \frac{4}{52}$

OXFORD UNIVERSITY PRESS

$= \frac{8}{52}$

$= \frac{2}{13}$

(iv) P(Jack or Ace)

$= \text{P(Jack)} + \text{P(Ace)}$

$= \frac{4}{52} + \frac{4}{52}$

$= \frac{8}{52}$

$= \frac{2}{13}$

P(neither Jack or Ace)

$= 1 - \frac{2}{13}$

$= \frac{11}{13}$

Practise Now 7

1. (i) P(P or Q wins)

$= \text{P}(P \text{ wins}) + \text{P}(Q \text{ wins})$

$= \frac{1}{5} + \frac{1}{6}$

$= \frac{11}{30}$

(ii) P(Q or R or S wins)

$= \text{P}(Q \text{ wins}) + \text{P}(P \text{ wins}) + \text{P}(S \text{ wins})$

$= \frac{1}{6} + \frac{1}{7} + \frac{1}{8}$

$= \frac{73}{168}$

(iii) P(P or Q or R or S wins)

$= \text{P}(P \text{ wins}) + \text{P}(Q \text{ wins}) + \text{P}(P \text{ wins}) + \text{P}(S \text{ wins})$

$= \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8}$

$= \frac{533}{840}$

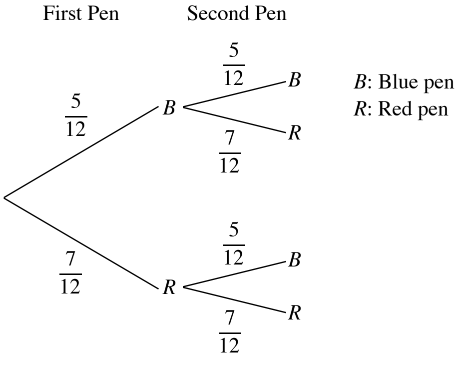
P(none wins)

$= 1 - \frac{533}{840}$

$= \frac{307}{840}$

Practise Now 8

1.



(i) P(first pen selected is R)

$= \frac{7}{12}$

(ii) P(second pen selected is B, given that the first pen selected is B)

$= \frac{5}{12}$

(iii) P(first pen selected is B and the second pen selected is B)

$= \frac{5}{12} \times \frac{5}{12}$

$= \frac{25}{144}$

(iv) P(second pen selected is B)

$= \text{P}(B, B) + \text{P}(R, B)$

$= \frac{5}{12} \times \frac{5}{12} + \frac{7}{12} \times \frac{5}{12}$

$= \frac{5}{12}$

(v) P(no blue pen was selected)

$= \frac{7}{12} \times \frac{7}{12}$

$= \frac{49}{144}$

Practise Now 9

1. In the 'Administrative' Department, there are 6 men and 8 women and in the Technical Department, there are 12 men and 4 women.

(i) P(both the chairman and chairwoman are from the 'Technical' Department)

$= \frac{12}{18} \times \frac{4}{12}$

$= \frac{2}{9}$

(ii) P(the chairman is from the 'Administrative' Department and the chairwoman is from the 'Technical' Department)

$= \frac{6}{18} \times \frac{4}{12}$

$= \frac{1}{9}$

2. (i) P(both laptops break down)

$= 0.1 \times 0.35$

$= 0.035$

(ii) P(Laptop X breaks down but Laptop Y does not)

$= 0.1 \times (1 - 0.35)$

$= 0.065$

(iii) P(exactly one of the laptops breaks down)

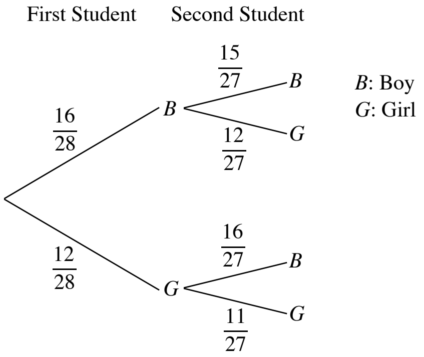
$= [0.1 \times (1 - 0.35)] + [(1 - 0.1) \times 0.35]$

$= 0.38$

OXFORD UNIVERSITY PRESS

Practise Now 10

1.



(i) P(first student is B and the second student is G)

$= P(B, G)$

$= \frac{16}{28} \times \frac{12}{27}$

$= \frac{16}{63}$

(ii) P(one student is B while the other student is G)

$= P(B, G) + P(G, B)$

$= \frac{16}{28} \times \frac{12}{27} + \frac{12}{28} \times \frac{16}{27}$

$= \frac{32}{63}$

(iii) P(at least one of the students is G)

$= P(B, G) + P(G, B) + P(G, G)$

$= \frac{16}{28} \times \frac{12}{27} + \frac{12}{28} \times \frac{16}{27} + \frac{12}{28} \times \frac{11}{27}$

$= \frac{43}{63}$

Alternatively,

P(at least one of the students is G)

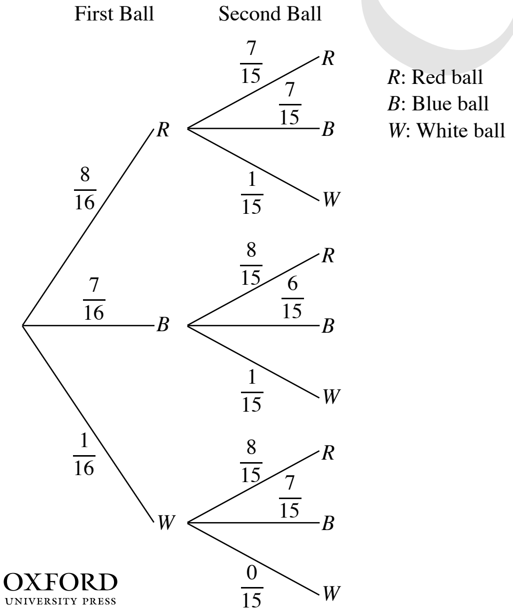
$= 1 - P$(both students are B)

$= 1 - (B, B)$

$= 1 - \frac{16}{28} \times \frac{15}{27}$

$= \frac{43}{63}$

2.



(i) P(first ball is R and the second ball is B)

$= P(R, B)$

$= \frac{8}{16} \times \frac{7}{15}$

$= \frac{7}{30}$

(ii) P(one ball is R while the other ball is B)

$= P(R, B) + P(B, R)$

$= \frac{8}{16} \times \frac{7}{15} + \frac{7}{16} \times \frac{8}{15}$

$= \frac{7}{15}$

(iii) P(two balls are of the same colour)

$= P(R, R) + P(B, B) + P(W, W)$

$= \frac{8}{16} \times \frac{7}{15} + \frac{7}{16} \times \frac{6}{15} + \frac{1}{16} \times \frac{0}{15}$

$= \frac{49}{120}$

Exercise 3A

1. Let S represent the sample space.

$S = \{(H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6), (T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6)\}$

2. Let S represent the sample space.

Let F be the event that the pen drawn is faulty.

Let N be the event that the pen drawn is not faulty.

$S = \{F_1, F_2, F_3, N_1, N_2, N_3, N_4\}$

Probability that the pen drawn is not faulty $= \frac{4}{7}$

After drawing the first pen, $S = \{F_1, F_2, F_3, N_1, N_2, N_3\}$

Probability that the second pen drawn is faulty $= \frac{3}{6}$

$= \frac{1}{2}$

3. Let S represent the sample space.

$S = \{\mathrm{B}, \mathrm{I}_1, \mathrm{I}_2, \mathrm{I}_3, \mathrm{L}, \mathrm{O}, \mathrm{P}, \mathrm{S}_1, \mathrm{S}_2, \mathrm{T}, \mathrm{Y}\}$

(i) P(the letter on the chosen card is a ‘S’)

$= \frac{2}{11}$

(ii) P(the letter on the chosen card is a ‘P’ or an ‘I’)

$= \frac{4}{11}$

(iii) P(the letter on the chosen card is a vowel)

$= \frac{4}{11}$

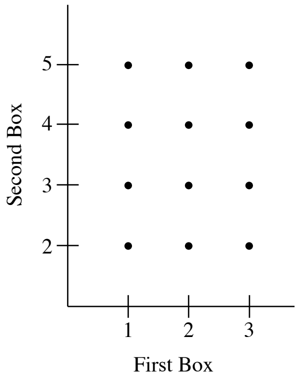
(iv) P(the letter on the chosen card is a consonant)

$= 1 - \frac{4}{11}$

$= \frac{7}{11}$

OXFORD UNIVERSITY PRESS

4. (a)



(b) (i) P(cards bear the same number)

$= \frac{2}{12}$

$= \frac{1}{6}$

(ii) P(numbers on the cards are different)

$= \frac{10}{12}$

$= \frac{5}{6}$

Alternatively,

P(numbers on the cards are different)

= 1 − P(numbers on the cards are the same)

$= 1 - \frac{1}{6}$

$= \frac{5}{6}$

(iii) P(larger of the two numbers on the card is 3)

$= \frac{3}{12}$

$= \frac{1}{4}$

5. (a)

First Number

Second Number

+	**0**	**1**	**2**	**3**	**4**	**5**
0	0	1	2	3	4	5
1	1	2	3	4	5	6
2	2	3	4	5	6	7
3	3	4	5	6	7	8
4	4	5	6	7	8	9
5	5	6	7	8	9	10

(b) 36 possible outcomes

(c) (i) P(sum of the two numbers is 7)

$= \frac{4}{36}$

$= \frac{1}{9}$

(ii) P(sum of the two numbers is a prime number)

$= \frac{17}{36}$

(iii) P(sum of the two numbers is not a prime number)

$= 1 - \frac{17}{36}$

$= \frac{19}{36}$

(iv) P(sum of the two numbers is even)

$= \frac{18}{36}$

$= \frac{1}{2}$

(v) P(sum of the two numbers is not even)

$= 1 - \frac{1}{2}$

$= \frac{1}{2}$

(d) The sum of 7 is more likely to occur.

6. (a)

x, y

+	**4**	**5**	**6**
7	11	12	13
8	12	13	14
9	13	14	15

x, y

×	**4**	**5**	**6**
7	28	35	42
8	32	40	48
9	36	45	54

(b) (i) P(sum $x + y$ is prime)

$= \frac{4}{9}$

(ii) P(sum $x + y$ is greater than 12)

$= \frac{6}{9}$

$= \frac{2}{3}$

(iii) P(sum $x + y$ is at most 14)

$= \frac{8}{9}$

(c) (i) P(product xy is odd)

$= \frac{2}{9}$

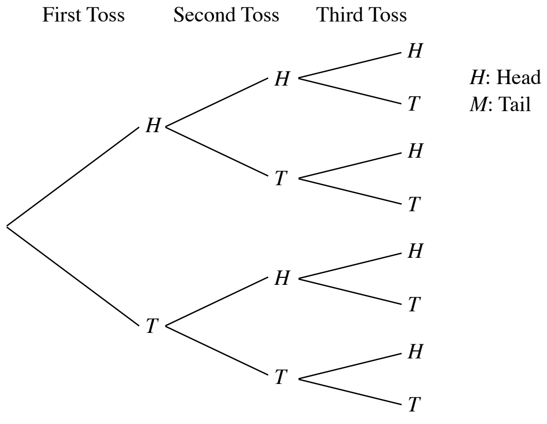
(ii) P(product xy is even)

$= \frac{7}{9}$

(iii) P(product xy is at most 40)

$= \frac{5}{9}$

7.



OXFORD UNIVERSITY PRESS

(i) P(three heads)

= $\frac{1}{8}$

(ii) P(exactly two heads)

= $\frac{3}{8}$

(iii) P(at least two heads)

= $\frac{4}{8}$

= $\frac{1}{2}$

Alternatively,

P(at least two heads)

= P(exactly two heads) + P(three heads)

= $\frac{3}{8} + \frac{1}{8}$

= $\frac{4}{8}$

= $\frac{1}{2}$

8. Let S represent the sample space.

$S = \{RB, BB, WB, RR, BR, WR\}$

(i) P(marbles selected are of the same colour)

= $\frac{2}{6}$

= $\frac{1}{3}$

(ii) P(marbles selected are blue and red)

= $\frac{2}{6}$

= $\frac{1}{3}$

(iii) P(marbles selected are of different colours)

= $1 - \frac{1}{3}$

= $\frac{2}{3}$

9. **(a)** Let S represent the sample space.

$S = \{11, 12, 13, 21, 22, 23, 31, 32, 33\}$

(b) **(i)** P(number formed is divisible by 3)

= $\frac{3}{9}$

= $\frac{1}{3}$

(ii) P(number formed is a perfect square)

= 0

(iii) P(number formed is a prime number)

= $\frac{4}{9}$

(iv) P(number formed is a composite number)

= $\frac{5}{9}$

10. Let S represent the sample space.

$S = \{BBB, BGB, BBG, GBB, BGG, GBG, GGB, GGG\}$

(i) P(three grandsons)

= $\frac{1}{8}$

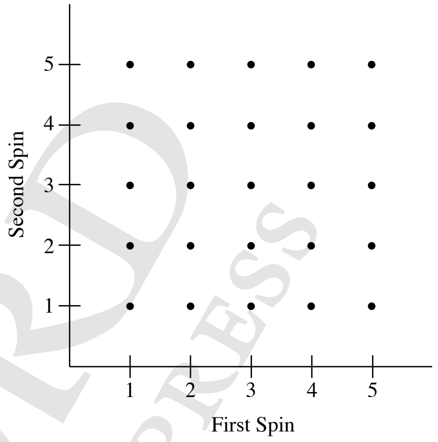
(ii) P(two grandsons and one granddaughter)

= $\frac{3}{8}$

(iii) P(one grandson and two granddaughters)

= $\frac{3}{8}$

11.



(a) **(i)** P(numbers on the spinners whose sum is 6)

= $\frac{5}{25}$

= $\frac{1}{5}$

(ii) P(the same numbers on both spinners)

= $\frac{5}{25}$

= $\frac{1}{5}$

(iii) P(different numbers on both spinners)

= $1 - \frac{1}{5}$

= $\frac{4}{5}$

(iv) P(two different prime numbers)

= $\frac{6}{25}$

(b) P(first number less than second number)

= $\frac{10}{25}$

= $\frac{2}{5}$

12. **(a)**

Die

		1	**2**	**3**	**4**	**5**	**6**
Coin	***H***	1	2	3	4	5	6
	T	2	4	6	8	10	12

OXFORD UNIVERSITY PRESS

(b) **(i)** P(player’s score is odd)

$= \frac{3}{12}$

$= \frac{1}{4}$

(ii) P(player’s score is even)

$= 1 - \frac{1}{4}$

$= \frac{3}{4}$

(iii) P(player’s score is a prime number)

$= \frac{3}{12}$

$= \frac{1}{4}$

(iv) P(player’s score is less than or equal to 8)

$= \frac{10}{12}$

$= \frac{5}{6}$

(v) P(player’s score is a multiple of 3)

$= \frac{4}{12}$

$= \frac{1}{3}$

13. **(a)**

First Die

Second Die

–	**0**	**1**	**2**	**3**	**4**	**5**
0	0	1	2	3	4	5
1	1	0	1	2	3	4
2	2	1	0	1	2	3
3	3	2	1	0	1	2
4	4	3	2	1	0	1
5	5	4	3	2	1	0

(b) **(i)** P(difference of the two numbers is 1)

$= \frac{10}{36}$

$= \frac{5}{18}$

(ii) P(difference of the two numbers is non-zero)

$= \frac{30}{36}$

$= \frac{5}{6}$

(iii) P(difference of the two numbers is odd)

$= \frac{18}{36}$

$= \frac{1}{2}$

(iv) P(difference of the two numbers is a prime number)

$= \frac{16}{36}$

$= \frac{4}{9}$

(v) P(difference of the two numbers is more than 2)

$= \frac{12}{36}$

$= \frac{1}{3}$

14.

Second Ball

7	1, 7	2, 7	4, 7	5, 7	
5	1, 5	2, 5	4, 5		7, 5
4	1, 4	2, 4		5, 4	7, 4
2	1, 2		4, 2	5, 3	7, 2
1		2, 1	4, 1	5, 1	7, 1
	1	**2**	**4**	**5**	**7**

First Ball

(i) P(numbers obtained on both balls are prime)

$= \frac{6}{20}$

$= \frac{3}{10}$

(ii) P(sum of the numbers obtained is odd)

$= \frac{11}{20}$

(iii) P(product of the numbers obtained is greater than 20)

$= \frac{4}{20}$

$= \frac{1}{5}$

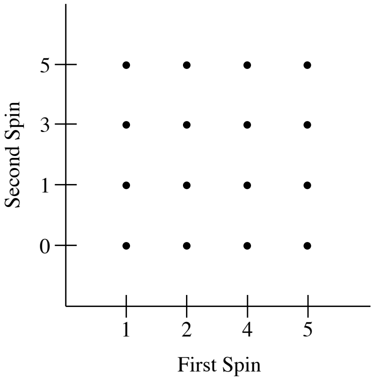
(iv) P(difference in the numbers obtained is less than 7)

$= 1$

(v) P(product of the numbers obtained is divisible by 9)

$= 0$

15.



OXFORD UNIVERSITY PRESS

(i) P(scores on both cards are the same)

$= \frac{2}{16}$

$= \frac{1}{8}$

(ii) P(scores on both cards are prime)

$= \frac{4}{16}$

$= \frac{1}{4}$

(iii) P(sum of the scores is odd)

$= \frac{8}{16}$

$= \frac{1}{2}$

(iv) P(sum of the scores is divisible by 5)

$= \frac{4}{16}$

$= \frac{1}{4}$

(v) P(sum of the scores is 6 or less)

$= \frac{11}{16}$

(vi) P(product of the scores is not 0)

$= \frac{12}{16}$

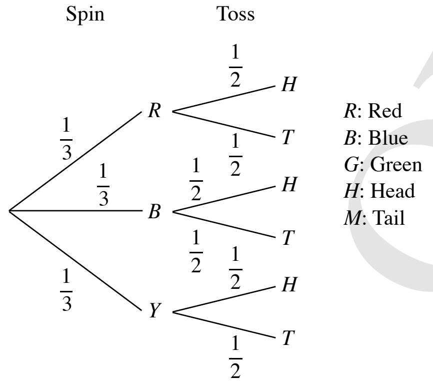
$= \frac{3}{4}$

(vii) P(product of the scores is greater than 11)

$= \frac{4}{16}$

$= \frac{1}{4}$

16.



(i) P(red on the spinner and tail on the coin)

$= \frac{1}{3} \times \frac{1}{2}$

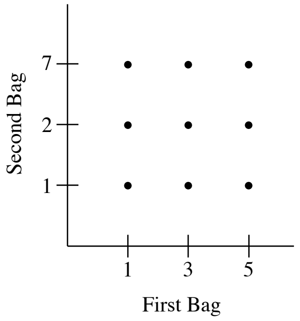
$= \frac{1}{6}$

(ii) P(blue or yellow on the spinner and head on the coin)

$= \left(\frac{1}{3} \times \frac{1}{2}\right) + \left(\frac{1}{3} \times \frac{1}{2}\right)$

$= \frac{1}{3}$

17.



(i) P(two numbers obtained are both odd)

$= \frac{6}{9}$

$= \frac{2}{3}$

(ii) P(two numbers obtained are prime)

$= \frac{4}{9}$

(iii) P(sum of the numbers greater than 4)

$= \frac{6}{9}$

$= \frac{2}{3}$

(iv) P(sum of the numbers is even)

$= \frac{6}{9}$

$= \frac{2}{3}$

(v) P(product is prime)

$= \frac{4}{9}$

(vi) P(product is greater than 20)

$= \frac{2}{9}$

(vii) P(product is divisible by 7)

$= \frac{3}{9}$

$= \frac{1}{3}$

18. (a) (i) P(land with the face printed '4' down)

$= \frac{1}{4}$

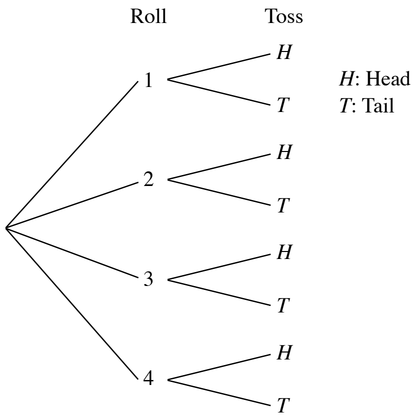
(ii) P(land such that the sum of the three upper faces is odd)

$= \frac{2}{4}$

$= \frac{1}{2}$

OXFORD UNIVERSITY PRESS

(b)



19. P(first component tested is defective) = $\frac{1}{7}$

20.

Kate's Room

Nora's Room	1A	1B	1C	2A	2B	2C
1A		1	2	3	4	5
1B	1		3	4	5	6
1C	2	3		5	6	7
2A	3	4	5		7	8
2B	4	5	6	7		9
2C	5	6	7	8	9	

(a) **(i)** P(stay next to each other)

$= \frac{8}{30}$

$= \frac{4}{15}$

(ii) P(stay on different storeys)

$= \frac{18}{30}$

$= \frac{3}{5}$

(iii) P(do not stay next to each other)

$= 1 - \frac{4}{15}$

$= \frac{11}{15}$

(b) P(Kate stays on the second floor and next to Nora)

$= \frac{4}{15}$

21. There are $4 \times 4 \times 6 = 96$ outcomes.

Let the event of the score on the 6-sided die greater than the sum of the scores of the two tetrahedral dice be A.

20 outcomes for event A:

{611, 612, 613, 614, 621, 622, 623, 631, 632, 641, 511, 512, 513, 521, 522, 531, 411, 412, 421, 311}

$\therefore$ P(A) = $\frac{20}{96}$

$= \frac{5}{24}$

Exercise 3B

1. Let S represent the sample space.

$S = \{11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21\}$

(i) P(number is even)

$= \frac{5}{11}$

(ii) P(number is prime)

$= \frac{4}{11}$

(iii) P(number is either even or prime)

= P(number is even) + P(number is prime)

$= \frac{5}{11} + \frac{4}{11}$

$= \frac{9}{11}$

(iv) P(number is divisible by 3)

$= \frac{4}{11}$

(v) P(number is neither even nor prime)

= 1 – P(number is either even or prime)

$= 1 - \frac{9}{11}$

$= \frac{2}{11}$

2. Let S represent the sample space.

$S = \{R_1, R_2, R_3, R_4, R_5, R_6, R_7, G_1, G_2, G_3, G_4, G_5, B_1, B_2, B_3\}$

(i) P(red marble)

$= \frac{7}{15}$

(ii) P(green marble)

$= \frac{5}{15}$

$= \frac{1}{3}$

(iii) P(either red or green marble)

= P(red marble) + P(green marble)

$= \frac{7}{15} + \frac{5}{15}$

$= \frac{12}{15}$

$= \frac{4}{5}$

(iv) P(neither red nor green marble)

= 1 – P(either red or green marble)

$= 1 - \frac{4}{5}$

$= \frac{1}{5}$

OXFORD UNIVERSITY PRESS

3. Let S represent the sample space.

$S = \{A, C, E_1, E_2, I, L_1, L_2, L_3\ M, S, T, U_1, U_2, U_3, V, X, Y\}$

(i) P(letter ‘U’)

$= \dfrac{3}{17}$

(ii) P(letter ‘E’)

$= \dfrac{2}{17}$

(iii) P(letter ‘U’ or ‘E’)

= P(letter ‘U’) + P(letter ‘E’)

$= \dfrac{3}{17} + \dfrac{2}{17}$

$= \dfrac{5}{17}$

(iv) P(consonant)

$= \dfrac{10}{17}$

(v) P(letter ‘U’ or consonant)

= P(letter ‘U’) + P(consonant)

$= \dfrac{3}{17} + \dfrac{10}{17}$

$= \dfrac{13}{17}$

(vi) P(letter ‘U’ or ‘E’ or ‘L’)

= P(letter ‘U’) + P(letter ‘E’) + P(letter ‘L’)

$= \dfrac{13}{17} + \dfrac{2}{17} + \dfrac{13}{17}$

$= \dfrac{8}{17}$

4. **(i)** P(team wins or loses a particular match)

= P(team wins) + P(team loses)

$= \dfrac{7}{10} + \dfrac{2}{15}$

$= \dfrac{5}{6}$

(ii) P(team neither wins nor loses a particular match)

= P(match ends in a draw)

$= 1 - \dfrac{5}{6}$

$= \dfrac{1}{6}$

5. **(i)** P(King or Jack)

= P(King) + P(Jack)

$= \dfrac{4}{52} + \dfrac{4}{52}$

$= \dfrac{8}{52}$

$= \dfrac{2}{13}$

(ii) P(Queen or card bearing a prime number)

= P(Queen) + P(card bearing a prime number)

$= \dfrac{4}{52} + \dfrac{16}{52}$

$= \dfrac{20}{52}$

$= \dfrac{5}{13}$

(iii) P(card bearing a number that is divisible by 3 or 5)

$= \dfrac{20}{52}$

$= \dfrac{5}{13}$

(iv) P(neither King nor Jack)

= 1 – P(King or Jack)

$= 1 - \dfrac{2}{13}$

$= \dfrac{11}{13}$

6. **(i)** P(4 or 5 strokes)

= P(4 strokes) + P(5 strokes)

$= \dfrac{1}{14} + \dfrac{2}{7}$

$= \dfrac{5}{14}$

(ii) P(4, 5 or 6 strokes)

= P(4 strokes) + P(5 strokes) + P(6 strokes)

$= \dfrac{1}{14} + \dfrac{2}{7} + \dfrac{3}{7}$

$= \dfrac{11}{14}$

(iii) P(more than 6 strokes)

$= 1 - \dfrac{11}{14}$

$= \dfrac{3}{14}$

7. **(i)** P(Alpha or Gamma wins)

= P(Alpha wins) + P(Gamma wins)

$= \dfrac{4}{15} + \dfrac{1}{5}$

$= \dfrac{7}{15}$

(ii) P(Alpha, Beta or Gamma wins)

= P(Alpha wins) + P(Beta wins) + P(Gamma wins)

$= \dfrac{4}{15} + \dfrac{1}{10} + \dfrac{1}{5}$

$= \dfrac{17}{30}$

(iii) P(neither Alpha nor Gamma wins)

= 1 – P(Alpha or Gamma wins)

$= 1 - \dfrac{7}{15}$

$= \dfrac{8}{15}$

OXFORD UNIVERSITY PRESS

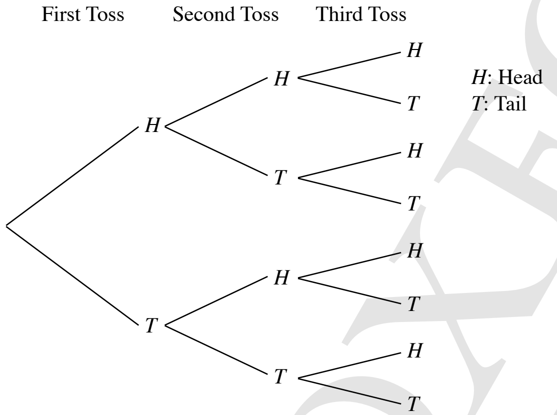
(iv) P(none wins)
$= 1 - \text{P(Alpha, Beta or Gamma wins)}$
$= 1 - \frac{17}{30}$
$= \frac{13}{30}$

8. **(i)** P(one of them wins the award)
= P(Priya wins or Rui Feng wins or Amirah wins)
= P(Priya wins) + P(Rui Feng wins) + P(Amirah wins)
$= \frac{1}{3} + \frac{1}{8} + \frac{1}{20}$
$= \frac{61}{120}$

(ii) P(none of them wins the award)
= 1 − P(one of them wins the award)
$= 1 - \frac{61}{120}$
$= \frac{59}{120}$

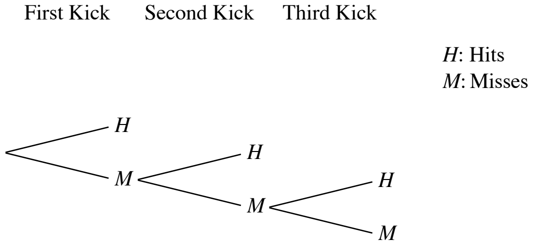
(iii) P(Priya and Rui Feng will not win the award)
= 1 − P(Priya wins) − P(Rui Feng wins)
$= 1 - \frac{1}{3} - \frac{1}{8}$
$= \frac{13}{24}$

9. **(a)**



(b) **(i)** Mutually exclusive
(ii) Not mutually exclusive
(iii) Not mutually exclusive
(iv) Mutually exclusive
(v) Not mutually exclusive
(vi) Not mutually exclusive

10. **(a)**

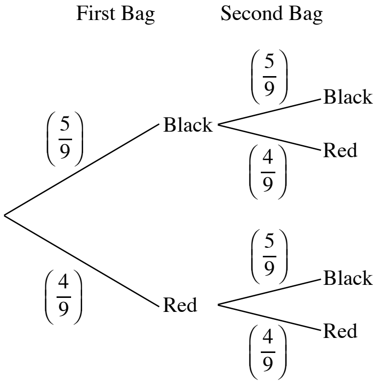


4 outcomes

(b) Not mutually exclusive as Event *A* and *B* can happen at the same time.

Exercise 3C

1. **(a)**



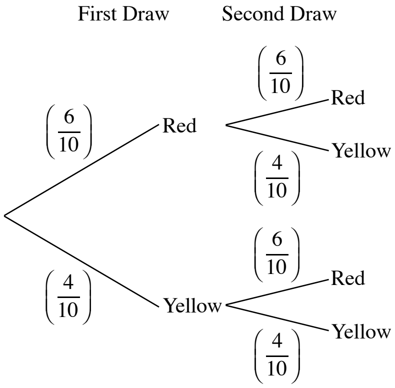
(b) **(i)** P(black marble from the first bag)
$= \frac{5}{9}$

(ii) P(red marble from the second bag, given that a black marble is drawn from the first bag)
$= \frac{4}{9}$

(iii) P(black marble from the first bag, red marble from the second bag)
$= \frac{5}{9} \times \frac{4}{9}$
$= \frac{20}{81}$

(iv) P(red marble from the second bag)
= P(red, red) + P(black, red)
$= \left(\frac{4}{9} \times \frac{4}{9}\right) + \left(\frac{5}{9} \times \frac{4}{9}\right)$
$= \frac{4}{9}$

2. **(a)**



(b) **(i)** P(two red balls)
$= \frac{6}{10} \times \frac{6}{10}$
$= \frac{9}{25}$

(ii) P(one ball of each colour)

= P(red, yellow) + P(yellow, red)

$= \left(\frac{6}{10} \times \frac{4}{10}\right) + \left(\frac{4}{10} \times \frac{6}{10}\right)$

$= \frac{12}{25}$

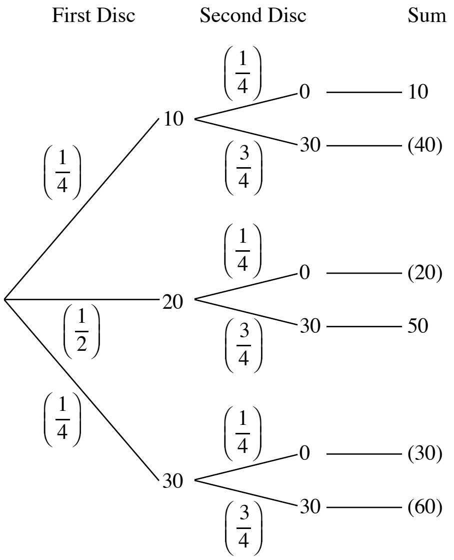
(iii) P(yellow ball on the second draw)

= P(red, yellow) + P(yellow, yellow)

$= \left(\frac{6}{10} \times \frac{4}{10}\right) + \left(\frac{4}{10} \times \frac{4}{10}\right)$

$= \frac{2}{5}$

3. **(a)**



(b) **(i)** P(first number ⩽ second number)

$= \left(\frac{1}{4} \times \frac{3}{4}\right) + \left(\frac{1}{2} \times \frac{3}{4}\right) + \left(\frac{1}{4} \times \frac{3}{4}\right)$

$= \frac{3}{4}$

(ii) P(second number is zero)

$= \left(\frac{1}{4} \times \frac{1}{4}\right) + \left(\frac{1}{2} \times \frac{1}{4}\right) + \left(\frac{1}{4} \times \frac{1}{4}\right)$

$= \frac{1}{4}$

(c) **(i)** P(gets \$2)

$= \left(\frac{1}{4} \times \frac{3}{4}\right) + \left(\frac{1}{2} \times \frac{1}{4}\right) + \left(\frac{1}{4} \times \frac{1}{4}\right)$

$= \frac{3}{8}$

(ii) P(gets \$5)

$= \left(\frac{1}{2} \times \frac{3}{4}\right) + \left(\frac{1}{4} \times \frac{3}{4}\right)$

$= \frac{9}{16}$

(iii) P(gets \$2 or \$5)

= P(gets \$2) + P(gets \$5)

$= \frac{3}{8} + \frac{9}{16}$

$= \frac{15}{16}$

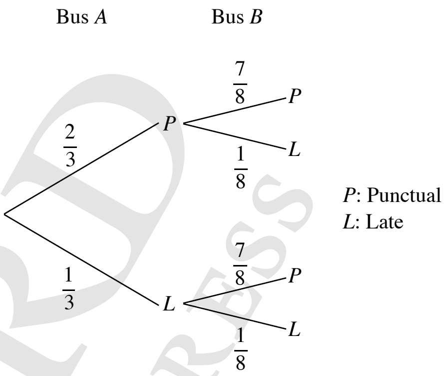
(iv) P(gets nothing)

= 1 − P(gets \$2 or \$5)

$= 1 - \frac{15}{16}$

$= \frac{1}{16}$

4.



(i) P(both buses are punctual)

$= \frac{2}{3} \times \frac{7}{8}$

$= \frac{7}{12}$

(ii) P(Bus *A* is late, Bus *B* is punctual)

$= \frac{1}{3} \times \frac{7}{8}$

$= \frac{7}{24}$

(iii) P(exactly one bus is late)

$= \left(\frac{1}{3} \times \frac{7}{8}\right) + \left(\frac{2}{3} \times \frac{1}{8}\right)$

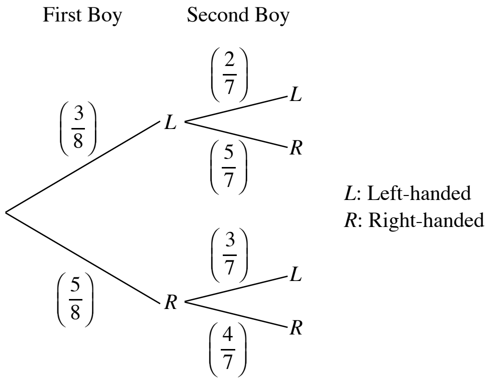
$= \frac{3}{8}$

5. P(boy is left-handed) $= \frac{3}{8}$

(a) P(second boy is left-handed, given that the first boy is left-handed)

$= \frac{2}{7}$

(b)



OXFORD UNIVERSITY PRESS

(c) **(i)** P(first boy is right-handed, second boy is left-handed)

$= \text{P}(R, L)$

$= \frac{5}{8} \times \frac{3}{7}$

$= \frac{15}{56}$

(ii) P(both boys are left-handed)

$= \text{P}(L, L)$

$= \frac{3}{8} \times \frac{2}{7}$

$= \frac{3}{28}$

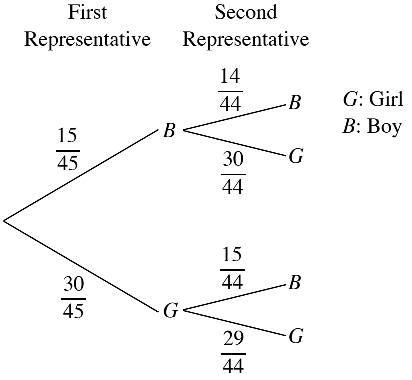
(iii) P(second boy chosen is left-handed)

$= \text{P}(R, L) + \text{P}(L, L)$

$= \frac{15}{56} + \frac{3}{28}$

$= \frac{3}{8}$

6.



(i) P(first representative is a girl)

$= \frac{30}{45}$

$= \frac{2}{3}$

(ii) P(second representative is a girl, given that the first representative is a boy)

$= \frac{30}{44}$

$= \frac{15}{22}$

(iii) P(first representative is a boy and second representative is a girl)

$= \frac{15}{45} \times \frac{30}{44}$

$= \frac{5}{22}$

(iv) P(one boy and one girl)

$= \left(\frac{15}{45} \times \frac{30}{44}\right) + \left(\frac{30}{45} \times \frac{15}{44}\right)$

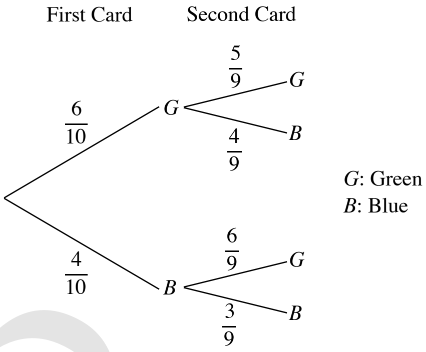
$= \frac{5}{11}$

7. **(a)** P(green card)

$= \frac{6}{10}$

$= \frac{3}{5}$

(b)



(i) P(two green cards)

$= \text{P}(G, G)$

$= \frac{6}{10} \times \frac{5}{9}$

$= \frac{1}{3}$

(ii) P(one card of each colour)

$= \text{P}(G, B) + \text{P}(B, G)$

$= \left(\frac{6}{10} \times \frac{4}{9}\right) + \left(\frac{4}{10} \times \frac{6}{9}\right)$

$= \frac{8}{15}$

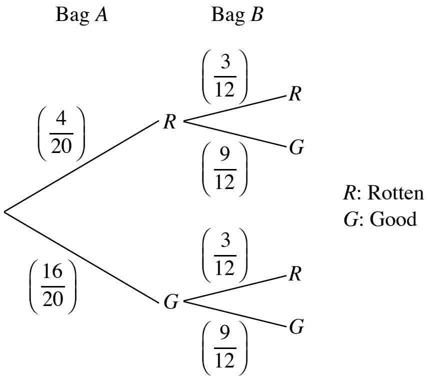
(iii) P(at least one blue card)

$= 1 - \text{P(two green cards)}$

$= 1 - \frac{1}{3}$

$= \frac{2}{3}$

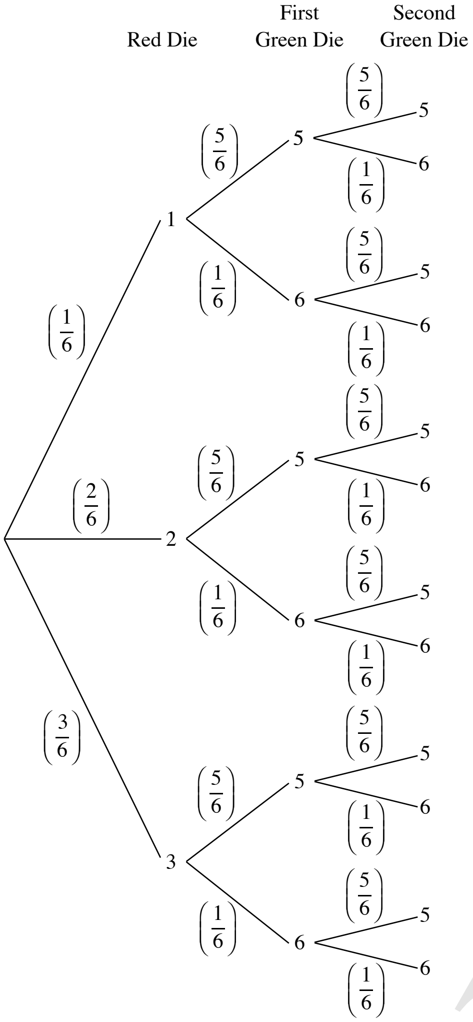
8. **(a)**



(b) Yes. Since the selections of the rotten potatoes from the two bags are independent events, hence the Multiplication Law of Probability applies.

OXFORD UNIVERSITY PRESS

9. (a)



(b) (i) P(2, 5, 6)

$$= \frac{2}{6} \times \frac{5}{6} \times \frac{1}{6}$$

$$= \frac{5}{108}$$

(ii) P(3, 6, 6)

$$= \frac{3}{6} \times \frac{1}{6} \times \frac{1}{6}$$

$$= \frac{1}{72}$$

(iii) P(exactly two sixes)

= P(1, 6, 6) + P(2, 6, 6) + P(3, 6, 6)

$$= 1 \times \frac{1}{6} \times \frac{1}{6}$$

$$= \frac{1}{36}$$

(iv) P(a sum of 12)

= P(1, 5, 6) + P(1, 6, 5) + P(2, 5, 5)

$$= \left(\frac{1}{6} \times \frac{5}{6} \times \frac{1}{6}\right) + \left(\frac{1}{6} \times \frac{1}{6} \times \frac{5}{6}\right) + \left(\frac{2}{6} \times \frac{5}{6} \times \frac{5}{6}\right)$$

$$= \frac{5}{18}$$

(v) P(a sum which is divisible by 3)

= P(a sum of 12 or a sum of 15)

= P(a sum of 12) + P(3, 6, 6)

$$= \frac{5}{18} + \left(\frac{3}{6} \times \frac{1}{6} \times \frac{1}{6}\right)$$

$$= \frac{7}{24}$$

10. (i) P(will not buy a sack of rice in a particular month)

$$= 1 - \frac{4}{9}$$

$$= \frac{5}{9}$$

(ii) P(will not buy a sack of rice in two consecutive months)

$$= \frac{5}{9} \times \frac{5}{9}$$

$$= \frac{25}{81}$$

(iii) P(buys a sack of rice in one of two particular months)

$$= \left(\frac{4}{9} \times \frac{5}{9}\right) + \left(\frac{5}{9} \times \frac{4}{9}\right)$$

$$= \frac{40}{81}$$

11. (i) P(three representatives are females)

$= \text{P}(F, F, F)$

$$= \frac{36}{76} \times \frac{35}{90} \times \frac{52}{90}$$

$$= \frac{91}{855}$$

(ii) P(representative from the front office is male while the other two representatives are females)

$= \text{P}(M, F, F)$

$$= \frac{40}{76} \times \frac{35}{90} \times \frac{52}{90}$$

$$= \frac{182}{1539}$$

(iii) P(exactly one representative is male)

$= \text{P}(M, F, F) + \text{P}(F, M, F) + \text{P}(F, F, M)$

$$= \left(\frac{40}{76} \times \frac{35}{90} \times \frac{52}{90}\right) + \left(\frac{36}{76} \times \frac{55}{90} \times \frac{52}{90}\right) + \left(\frac{36}{76} \times \frac{35}{90} \times \frac{38}{90}\right)$$

$$= \frac{5591}{15\,390}$$

12. (a) (i) P(two black shirts)

$$= \frac{8}{16} \times \frac{7}{15}$$

$$= \frac{7}{30}$$

(ii) P(one shirt is black and one shirt is white)

$$= \left(\frac{8}{16} \times \frac{6}{15}\right) + \left(\frac{6}{16} \times \frac{8}{15}\right)$$

$$= \frac{2}{5}$$

OXFORD UNIVERSITY PRESS

(iii) P(two shirts are of the same colour)

$$= \left(\frac{8}{16} \times \frac{7}{15}\right) + \left(\frac{6}{16} \times \frac{5}{15}\right) + \left(\frac{2}{16} \times \frac{1}{15}\right)$$

$$= \frac{11}{30}$$

(b) P(all three shirts are black)

$$= \frac{8}{16} \times \frac{7}{15} \times \frac{6}{14}$$

$$= \frac{1}{10}$$

(c) No, since selections of the three black shirts are dependent events.

13. (i) P(first card bears the letter 'O')

= P(O)

$$= \frac{3}{10}$$

(ii) P(two cards bear the letter 'P' and 'O' in that order)

= P(P, O)

$$= \frac{2}{10} \times \frac{3}{9}$$

$$= \frac{1}{15}$$

(iii) P(two cards bear the letter 'P' and 'O' in any order)

= P(P, O) + P(O, P)

$$= \left(\frac{2}{10} \times \frac{3}{9}\right) + \left(\frac{3}{10} \times \frac{2}{9}\right)$$

$$= \frac{2}{15}$$

(iv) P(two cards bear the same letter)

= P(R, R) + P(O, O) + P(P, P)

$$= \left(\frac{2}{10} \times \frac{1}{9}\right) + \left(\frac{3}{10} \times \frac{2}{9}\right) + \left(\frac{2}{10} \times \frac{1}{9}\right)$$

$$= \frac{1}{9}$$

14. (a) P(ball numbered '8')

$$= \frac{1}{5}$$

(b) (i) P(number on each ball is even)

= P(2, 8) + P(8, 2)

$$= \left(\frac{1}{5} \times \frac{1}{4}\right) + \left(\frac{1}{5} \times \frac{1}{4}\right)$$

$$= \frac{1}{10}$$

(ii) P(sum of the numbers on the balls is more than 10)

= P(2, 9) + P(9, 2) + P(5, 8) + P(8, 5) + P(5, 9) + P(9, 5)
+ P(8, 9) + P(9, 8)

$$= 8\left(\frac{1}{5} \times \frac{1}{4}\right)$$

$$= \frac{2}{5}$$

(iii) P(number on each ball is not a prime number)

= P(1, 8) + P(8, 1) + P(1, 9) + P(9, 1) + P(8, 9) + P(9, 8)

$$= 6\left(\frac{1}{5} \times \frac{1}{4}\right)$$

$$= \frac{3}{10}$$

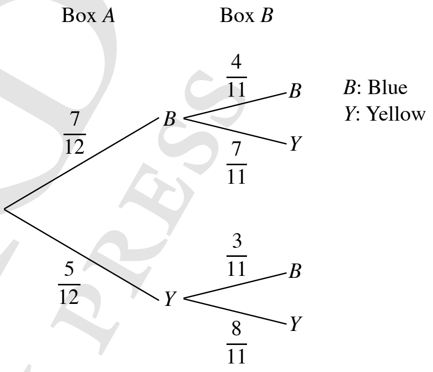
(iv) P(only one ball is odd number)

= P(1, 2) + P(2, 1) + P(1, 8) + P(8, 1) + P(2, 5)
+ P(5, 2) + P(2, 9) + P(9, 2) + P(5, 8) + P(8, 5)
+ P(8, 9) + P(9, 8)

$$= 12\left(\frac{1}{5} \times \frac{1}{4}\right)$$

$$= \frac{3}{5}$$

15. (a)



(b) (i) P(Box A has more yellow balls than blue balls)

= 0

(ii) P(Box A has exactly 7 blue and 5 yellow balls)

= P(B, B) + P(Y, Y)

$$= \left(\frac{7}{12} \times \frac{4}{11}\right) + \left(\frac{5}{12} \times \frac{8}{11}\right)$$

$$= \frac{17}{33}$$

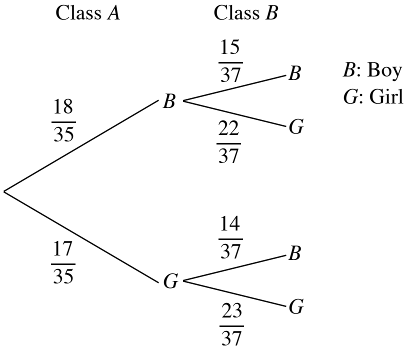
(iii) P(Box A has twice as many blue balls as yellow balls)

= P(Y, B)

$$= \frac{5}{12} \times \frac{3}{11}$$

$$= \frac{5}{44}$$

16.



OXFORD UNIVERSITY PRESS

(i) P(student was initially from Class A)

$= \frac{1}{37}$

(ii) P(student is a boy)

$= P(B, B) + P(G, B)$

$= \left(\frac{18}{35} \times \frac{15}{37}\right) + \left(\frac{17}{35} \times \frac{14}{37}\right)$

$= \frac{508}{1295}$

17. (i) P(volcanic eruptions in one of the three countries)

$= 0.03 \times 0.12 \times 0.3$

$= 0.001\ 08$

(ii) P(no volcanic eruptions)

$= (1 - 0.03) \times (1 - 0.12) \times (1 - 0.3)$

$= 0.598$ (to 3 s.f.)

(iii) P(at least one volcanic eruptions)

= 1 − P(no volcanic eruptions)

$= 1 - 0.598$

$= 0.402$ (to 3 s.f.)

(iv) P(exactly two volcanic eruptions)

$= (0.97 \times 0.12 \times 0.3) + (0.03 \times 0.88 \times 0.3) + (0.03 \times 0.12 \times 0.7)$

$= 0.0454$ (to 3 s.f.)

18. (i) P(a red and two blue balls in that order)

$= \frac{10}{26} \times \frac{8}{24} \times \frac{3}{65}$

$= \frac{3}{65}$

(ii) P(a red, a yellow and a blue ball in that order)

$= \frac{10}{26} \times \frac{7}{25} \times \frac{9}{24}$

$= \frac{21}{520}$

(iii) P(three balls of different colours)

= P(red, yellow, blue) + P(red, blue, yellow)
+ P(yellow, red, blue) + P(yellow, blue, red)
+ P(blue, yellow, red) + P(blue, red, yellow)

$= 6 \times \left(\frac{10}{26} \times \frac{7}{25} \times \frac{9}{24}\right)$

$= \frac{63}{260}$

19. (a) (i) P(game ends on the third roll)

$= \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6}$

$= \frac{25}{216}$

(ii) P(game ends on the fourth roll)

$= \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6}$

$= \frac{125}{1296}$

(iii) P(game ends by the fourth roll)

$= \frac{1}{6} + \left(\frac{5}{6} \times \frac{1}{6}\right) + \left(\frac{5}{6} \times \frac{5}{6} \times \frac{1}{6}\right) + \left(\frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6}\right)$

$= \frac{671}{1296}$

(b) (i) P(game ends on the third roll)

$= \left(\frac{5}{6} \times \frac{1}{6} \times \frac{1}{6}\right) + \left(\frac{1}{6} \times \frac{5}{6} \times \frac{1}{6}\right)$

$= \frac{5}{108}$

(ii) P(game ends on the third roll and the sum of the scores is odd)

$= \left(\frac{1}{2} \times \frac{1}{6} \times \frac{1}{6}\right) + \left(\frac{1}{6} \times \frac{1}{2} \times \frac{1}{6}\right)$

$= \frac{1}{36}$

Review Exercise 3

1. (i) P(number ‘3’ followed by a head)

$= \frac{1}{6} \times \frac{1}{2}$

$= \frac{1}{12}$

(ii) P(even number followed by a tail)

$= \frac{3}{6} \times \frac{1}{2}$

$= \frac{1}{4}$

2. (i) P(same number)

$= 1 \times \frac{1}{6}$

$= \frac{1}{6}$

(ii) P(two even numbers)

$= \frac{3}{6} \times \frac{3}{6}$

$= \frac{1}{4}$

(iii) P(two odd numbers)

$= \frac{3}{6} \times \frac{3}{6}$

$= \frac{1}{4}$

(iv) P(one odd and one even number)

$= \left(\frac{3}{6} \times \frac{3}{6}\right) + \left(\frac{3}{6} \times \frac{3}{6}\right)$

$= \frac{1}{2}$

3. (i) P(number is greater than 28)

$= \frac{22}{50}$

$= \frac{11}{25}$

OXFORD UNIVERSITY PRESS

(ii) P(number includes the digit '3')

$$= \frac{14}{50}$$

$$= \frac{7}{25}$$

(iii) P(number is prime)

$$= \frac{15}{50}$$

$$= \frac{3}{10}$$

(iv) P(number is divisible by 4)

$$= \frac{12}{50}$$

$$= \frac{6}{25}$$

4. **(i)** P(two people born in the same month)

$$= 1 \times \frac{1}{12}$$

$$= \frac{1}{12}$$

(ii) P(three people born in the same month)

$$= 1 \times \frac{1}{12} \times \frac{1}{12}$$

$$= \frac{1}{144}$$

P(three people not born in the same month)

= 1 – P(three people born in the same month)

$$= 1 - \frac{1}{144}$$

$$= \frac{143}{144}$$

(iii) P(four people born in the same month)

$$= 1 \times \frac{1}{12} \times \frac{1}{12} \times \frac{1}{12}$$

$$= \frac{1}{1728}$$

5. **(i)** P(Huixian will catch her bus on a particular day)

$$= 1 - \frac{1}{7}$$

$$= \frac{6}{7}$$

(ii) P(Huixian will miss her bus on two particular consecutive days)

$$= \frac{1}{7} \times \frac{1}{7}$$

$$= \frac{1}{49}$$

(iii) P(Huixian will miss her bus on one of two particular consecutive days)

$$= \left(\frac{1}{7} \times \frac{6}{7}\right) + \left(\frac{6}{7} \times \frac{1}{7}\right)$$

$$= \frac{12}{49}$$

(iv) P(Huixian will catch her bus on three particular consecutive days)

$$= \frac{6}{7} \times \frac{6}{7} \times \frac{6}{7}$$

$$= \frac{216}{343}$$

6. **(i)** P(one of them wins the gold medal)

= P(Rui Feng wins) + P(Michael wins) + P(Khairul wins)

$$= \frac{1}{2} + \frac{1}{6} + \frac{1}{8}$$

$$= \frac{19}{24}$$

(ii) P(none of them wins the gold medal)

$$= 1 - \frac{19}{24}$$

$$= \frac{5}{24}$$

(iii) P(Rui Feng fails to win the gold medal)

$$= 1 - \frac{1}{2}$$

$$= \frac{1}{2}$$

7.

Second Disc

6	16	26	36	46	56	
5	15	25	35	45		65
4	14	24	34		54	64
3	13	23		43	53	63
2	12		32	42	52	62
1		21	31	41	51	61
	1	**2**	**3**	**4**	**5**	**6**

First Disc

(i) P(number formed is divisible by 2)

$$= \frac{15}{30}$$

$$= \frac{1}{2}$$

(ii) P(number formed is divisible by 5)

$$= \frac{5}{30}$$

$$= \frac{1}{6}$$

(iii) P(number formed is a prime number)

$$= \frac{7}{30}$$

(iv) P(number formed is a perfect square)

$$= \frac{4}{30}$$

$$= \frac{2}{15}$$

OXFORD UNIVERSITY PRESS

8. **(i)** P(first two cards are letter 'O')

$$= \left(\frac{1}{6}\times\frac{1}{5}\right)+\left(\frac{1}{5}\times\frac{1}{6}\right)$$

$$= \frac{1}{15}$$

(ii) P(second card is letter 'F')

$$= \frac{5}{6}\times\frac{1}{5}$$

$$= \frac{1}{6}$$

(iii) P(word 'FOLLOW' is obtained, in that order)

$$= \frac{1}{6}\times\frac{2}{5}\times\frac{2}{4}\times\frac{1}{3}\times\frac{1}{2}\times 1$$

$$= \frac{1}{180}$$

9. **(i)** P(Khairul selects a dark chocolate)

$$= \frac{y}{x+y}$$

(ii) P(Khairul selects a white chocolate, Priya selects a dark chocolate)

$$= \frac{x}{x+y}\times\frac{y}{x+y-1}$$

$$= \frac{xy}{(x+y)(x+y-1)}$$

(iii) P(chocolate selected by them are of different types)

$$= \left(\frac{x}{x+y}+\frac{y}{x+y-1}\right)+\left(\frac{y}{x+y}\times\frac{x}{x+y-1}\right)$$

$$= \frac{2xy}{(x+y)(x+y-1)}$$

10. P(traffic jam)

$$= \left(\frac{1}{4}\times\frac{2}{5}\right)+\left(\frac{3}{4}\times\frac{1}{5}\right)$$

$$= \frac{1}{4}$$

11. **(a)** **(i)** P(one girl, one boy)

$$= \left(\frac{10}{30}\times\frac{20}{29}\right)+\left(\frac{20}{30}\times\frac{10}{29}\right)$$

$$= \frac{40}{87}$$

(ii) P(no girls)

$$= \frac{20}{30}\times\frac{19}{29}$$

$$= \frac{38}{87}$$

(b) **(i)** P(both travel to school by bus)

$$= \frac{6}{10}\times\frac{5}{9}$$

$$= \frac{1}{3}$$

(ii) P(both travel to school by different means of transportation)

$$= \left(\frac{6}{10}\times\frac{4}{9}\right)+\left(\frac{4}{10}\times\frac{6}{9}\right)$$

$$= \frac{8}{15}$$

(iii) P(at least one travels to school by bus)

$$= \left(\frac{6}{10}\times\frac{4}{9}\right)+\left(\frac{4}{10}\times\frac{6}{9}\right)+\left(\frac{6}{10}\times\frac{5}{9}\right)$$

$$= \frac{13}{15}$$

12. **(i)** P(next two days also wet)

$$= 0.6\times 0.6$$

$$= 0.36$$

(ii) P(Tuesday is wet, Wednesday is fine)

$$= 0.6\times 0.4$$

$$= 0.24$$

(iii) P(one day fine, one day wet)

$$= (0.6\times 0.4) + (0.4\times 0.2)$$

$$= 0.32$$

(iv) P(two of the three days are wet)

$$= (0.6\times 0.6\times 0.4) + (0.6\times 0.4\times 0.2) + (0.4\times 0.2\times 0.6)$$

$$= 0.24$$

13. **(i)** P(first two sweets are different)

$$= \left(\frac{2}{10}\times\frac{3}{9}\right)+\left(\frac{3}{10}\times\frac{2}{9}\right)+\left(\frac{2}{10}\times\frac{5}{9}\right)+\left(\frac{5}{10}\times\frac{2}{9}\right)+\left(\frac{3}{10}\times\frac{5}{9}\right)+\left(\frac{5}{10}\times\frac{3}{9}\right)$$

$$= \frac{31}{45}$$

(ii) P(three sweets are the same)

$$= \left(\frac{3}{10}\times\frac{2}{9}\times\frac{1}{8}\right)+\left(\frac{5}{10}\times\frac{4}{9}\times\frac{3}{8}\right)$$

$$= \frac{11}{120}$$

(iii) P(first two sweets are the same, third sweet is a toffee)

$$= \left(\frac{2}{10}\times\frac{1}{9}\times\frac{5}{8}\right)+\left(\frac{3}{10}\times\frac{2}{9}\times\frac{5}{8}\right)+\left(\frac{3}{10}\times\frac{4}{9}\times\frac{3}{8}\right)$$

$$= \frac{5}{36}$$

14. **(i)** P(all three airplanes land at Terminal 2)

$$= \frac{1}{4}\times\frac{1}{3}\times\frac{1}{6}$$

$$= \frac{1}{72}$$

(ii) P(exactly two airplanes land at Terminal 1)

$$= \left(\frac{3}{4}\times\frac{2}{3}\times\frac{1}{6}\right)+\left(\frac{3}{4}\times\frac{1}{3}\times\frac{5}{6}\right)+\left(\frac{1}{4}\times\frac{2}{3}\times\frac{5}{6}\right)$$

$$= \frac{31}{72}$$

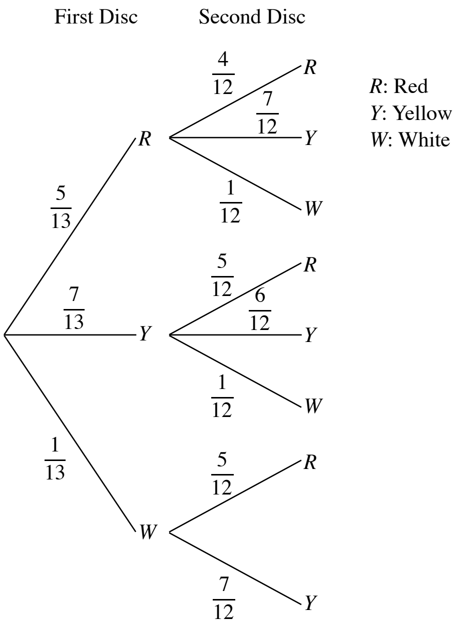
(iii) P(exactly one airplane lands at Terminal 1)

$$= \left(\frac{3}{4}\times\frac{1}{3}\times\frac{1}{6}\right)+\left(\frac{1}{4}\times\frac{1}{3}\times\frac{5}{6}\right)+\left(\frac{1}{4}\times\frac{2}{3}\times\frac{1}{6}\right)$$

$$= \frac{5}{36}$$

OXFORD UNIVERSITY PRESS

15.



(i) P(two red discs)

$= \frac{5}{13} \times \frac{4}{12}$

$= \frac{5}{39}$

(ii) P(a red and a yellow disc in that order)

$= \frac{5}{13} \times \frac{7}{12}$

$= \frac{35}{136}$

(iii) P(two white discs)

$= 0$

(iv) P(two discs of different colours)

= 1 – P(two discs of the same colour)

$= 1 - \left[\left(\frac{5}{13} \times \frac{4}{12}\right) + \left(\frac{7}{13} \times \frac{6}{12}\right)\right]$

$= \frac{47}{78}$

16. (a) (i) P(all three men hit the target)

$= \frac{2}{3} \times \frac{3}{5} \times \frac{4}{7}$

$= \frac{8}{35}$

(ii) P(all three men miss the target)

$= \frac{1}{3} \times \frac{2}{5} \times \frac{3}{7}$

$= \frac{2}{35}$

(iii) P(exactly two of them hit the target)

$= \left(\frac{2}{3} \times \frac{3}{5} \times \frac{3}{7}\right) + \left(\frac{2}{3} \times \frac{2}{5} \times \frac{4}{7}\right) + \left(\frac{1}{3} \times \frac{3}{5} \times \frac{4}{7}\right)$

$= \frac{46}{105}$

(iv) P(at least one of them hits the target)

= 1 – P(none of them hits the target)

$= 1 - \left(\frac{1}{3} \times \frac{2}{5} \times \frac{3}{7}\right)$

$= \frac{33}{35}$

(b) (i) P(game ends after two shots)

$= \frac{1}{3} \times \frac{3}{5}$

$= \frac{1}{5}$

(ii) P(game ends after three shots)

$= \frac{1}{3} \times \frac{2}{5} \times \frac{4}{7}$

$= \frac{8}{105}$

(iii) P(game ends by the third shot)

= P(game ends after the first shot)

\+ P(game ends after the second shot)

\+ P(game ends after the third shot)

$= \frac{2}{3} + \frac{1}{5} + \frac{8}{105}$

$= \frac{33}{35}$

Challenge Yourself

1.

Second Die

6	1, 6	2, 6	3, 6	4, 6	5, 6	6, 6
5	1, 5	2, 5	3, 5	4, 5	5, 5	6, 5
4	1, 4	2, 4	3, 4	4, 4	5, 4	6, 4
3	1, 3	2, 3	3, 3	4, 3	5, 3	6, 3
2	1, 2	2, 2	3, 2	4, 2	5, 2	6, 2
1	1, 1	2, 1	3, 1	4, 1	5, 1	6, 1
	1	**2**	**3**	**4**	**5**	**6**

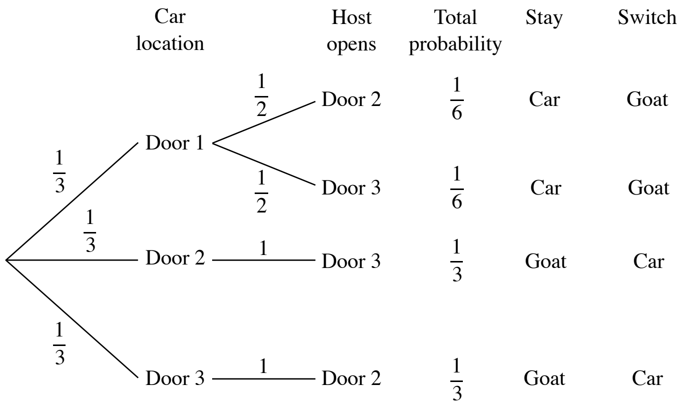
First Die

There are at least 11 outcomes with at least a ‘3’ and the probability that both of the scores are ‘3’s is $\frac{1}{11}$.

2. Contestants who switch doors have a $\frac{2}{3}$ chance of winning the car, while contestants who stick to their choice have only a $\frac{1}{3}$ chance. By definition, the conditional probability of winning by switching, given the contestant initially picks door 1 and the host opens door 3, is the probability for the event “car is behind door 2 and host opens door 3” divided by the probability for “host opens door 3”. These probabilities can be determined by referring to the probability tree as shown. The conditional probability of winning by switching is $\frac{\frac{1}{3}}{\frac{1}{3}+\frac{1}{6}} = \frac{2}{3}$.

OXFORD UNIVERSITY PRESS

The tree diagram showing the probability of every possible outcome if the player initially picks Door 1 is shown below.



3. **(a)** 1

(b) $1 - \frac{364}{365} \times \frac{363}{365} \times \frac{362}{365} \times \ldots \times \frac{326}{365} = 0.8912$

Yes

(c) $1 - \frac{364}{365} \times \frac{363}{365} \times \frac{362}{365} \times \ldots \times \frac{324 - x + 1}{365} > \frac{1}{2}$

$\frac{364}{365} \times \frac{363}{365} \times \frac{362}{365} \times \ldots \times \frac{364 - x + 1}{365} < \frac{1}{2}$

Least number of students is 22.

OXFORD UNIVERSITY PRESS

Chapter 4 Statistical Data Analysis

TEACHING NOTES

Suggested Approach

This chapter serves as an introduction to the important concepts of cumulative frequency curves, box-and-whisker plots and standard deviation.

Before actually plotting cumulative frequency curves, it would be good for the teachers to revise the choice of scales and labelling of scales on both axes. Students are often weak in these areas. Students should be encouraged to draw the curves free hand as well as to use curved rules to assist them.

Teachers should ask students to remember the general shapes of cumulative frequency curves and box-and-whisker plots. This will enable students to identify and rectify errors when they sketch or plot a few points wrongly and the shapes of their curves/ plots look odd.

Section 4.1: Cumulative Frequency Table and Curve

Teachers can point out to students that cumulative frequency curves can be used to interpret the data readily; for example, finding the percentage of students who passed the test when given that the passing mark = x marks or the marks above which $y\%$ of the students scored and so on.

For more advanced students, teachers can choose to discuss with them how a set of marks can be moderated by changing the passing mark such that $p\%$ of students pass, or the range of marks for a certain grade in order for $q\%$ of students to obtain that grade etc.

Section 4.2: Median, Quartiles, Percentiles, Range and Interquartile Range

From the cumulative frequency curve, median, quartiles and percentiles can be found with ease. It should be pointed out to the students that the values obtained from the curves are merely estimates. For ungrouped data consisting of an even number of values, say, 40 values, the median is the mean of the two middle values ($a = 20^{\text{th}}$ value and $b = 21^{\text{st}}$ value), when the values are arranged in order of magnitude, i.e. median = $\frac{a+b}{2}$.

From the cumulative frequency curve representing the data, the reading from the horizontal axis corresponding to the $\left(\frac{40}{2}\right)^{\text{th}} = 20^{\text{th}}$ value will be taken to be the estimate of the median. Thus, in general, from the cumulative frequency diagram representing a set of data of n values, the estimates of lower quartile, median, upper quartile and p^{th} percentile are readings corresponding to $\left(\frac{n}{4}\right)^{\text{th}}$, $\left(\frac{n}{2}\right)^{\text{th}}$, $\left(\frac{3n}{4}\right)^{\text{th}}$ and $\left(\frac{pn}{100}\right)^{\text{th}}$ values respectively.

An **average** is a number that summarises a set of data. In many cases, the average provides sufficient information, but it fails to give an indication of whether the data are clustered together or spread across a wide range of values. Interquartile range is one measure which roughly provides such an indication. There are other such measures which are known as measures of dispersion.

Teachers should also encourage students to search online for more real-life examples related to cumulative frequency curves and box-and-whisker plots.

Section 4.3: Box-and-Whisker Plots

Teachers may group 2 to 3 students in a group to do the Class Discussion on Vertical Box-and-Whisker Plots on page 102 of the textbook.

For better students, teachers may suggest that they check the Internet for more information about box-and-whisker plots and how they can be plotted by IT softwares.

Teachers can further take the chance to discuss about the comparison of cumulative frequency curves and box-and-whisker plots. Readings obtained from the curves are merely estimates whereas readings obtained from the box-and-whisker plots are exact values.

OXFORD UNIVERSITY PRESS

Section 4.4: Standard Deviation

Teachers can use use the Investigation activity on page 111 to introduce the need to have a statistical measure — standard deviation — to describe the distribution of a set of data. Then, guide the students to go through the Investigation activity on page 112 to obtain the formula for a new measure of spread. With this, there should be more opportunities for the students to compare the means and standard deviations of two sets of data by referring to the context of the questions.

Teachers should take time to go through the use of calculator to find the mean and standard deviation for a set of data. Also, teachers should engage the class in the Class Discussion on page 123 to allow the students to discuss about examples of inappropriate representations of data from newspapers and other sources, e.g. whether certain representations are misleading.

Challenge Yourself

For question 1, teachers may guide the students to use the hint provided to choose the set that has the same standard deviation as that of set *A*. By analysing all the differences between the data in each set, students should be able to explain why the sets are chosen.

For question 2, teachers may challenge the students to give an example of a pair of asymmetrical sets with the same mean, standard deviation and data size.

OXFORD UNIVERSITY PRESS

WORKED SOLUTIONS

Class Discussion (Constructing a Table of Cumulative Frequencies)

1.

Number of hours, t	**Cumulative Frequency**
$t \leqslant 2$	3
$t \leqslant 4$	3 + 5 = 8
$t \leqslant 6$	8 + 16 = 24
$t \leqslant 8$	24 + 12 = 36
$t \leqslant 10$	36 + 4 = 40

2. **(i)** Number of students who surf the Internet for 6 hours or less = 24

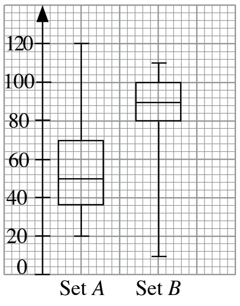
(ii) Number of students who surf the Internet for more than 8 hours = 40 – 36 = 4

(iii) Number of students who surf the Internet for more than 4 hours but not more than 10 hours = 40 – 8 = 32

3. It represents the total number of students. It is the sum of all the frequencies.

Class Discussion (Vertical Box-and-Whisker Plots)

1.

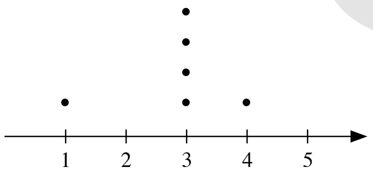


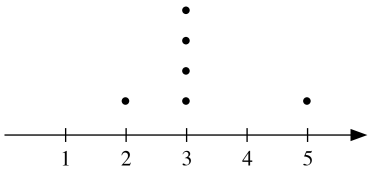
2. The height of the rectangular box represents the interquartile range. Set *A* has a larger interquartile range as compared to Set *B*.

3. Set *A* has a wider spread of data as compared to Set *B*.

Investigation (Are Averages Adequate for Comparing Distributions?)

1.





2. No, as these three averages are not able to describe the distribution of the set of data.

Investigation (Obtaining a Formula for a New Measure of Spread)

Part 1: Mean Temperatures

1. Mean temperature of City *A*

$$= \frac{25 + 24 + 26 + 33 + 31 + 29}{6}$$

= 28 °C

Mean temperature of City *B*

$$= \frac{21 + 15 + 23 + 36 + 41 + 32}{6}$$

= 28 °C

2. Yes

3. The spread of the temperatures of City *A* is less wide as compared to the spread of the temperatures in City *B*.

Part 2: Spread of the Temperatures

4.

x	$x - \bar{x}$
25	25 – 28 = –3
24	24 – 28 = –4
26	26 – 28 = –2
33	33 – 28 = 5
31	31 – 28 = 3
29	29 – 28 = 1
Sum	$\Sigma(x - \bar{x}) = 0$

5. The data of City *B* is more spread out.

6.

x	$x - \bar{x}$
21	21 – 28 = –7
15	15 – 28 = –13
23	23 – 28 = –5
36	36 – 28 = 8
41	41 – 28 = 13
32	32 – 28 = 4
Sum	$\Sigma(x - \bar{x}) = 0$

The value of $\Sigma(x - \bar{x})$ from City *B* is the same as the value of $\Sigma(x - \bar{x})$ from City *A*.

This is not a good measure of spread since it does not show that the spread of the temperatures in City *B* is wider than that in City *A*.

OXFORD UNIVERSITY PRESS

7. For City A,

x	$(x-\overline{x})^2$
25	$(25-28)^2 = 9$
24	$(24-28)^2 = 16$
26	$(26-28)^2 = 4$
33	$(33-28)^2 = 25$
31	$(31-28)^2 = 9$
29	$(29-28)^2 = 1$
Sum	$\Sigma(x-\overline{x})^2 = 64$

For City B,

x	$(x-\overline{x})^2$
21	$(21-28)^2 = 49$
15	$(15-28)^2 = 169$
23	$(23-28)^2 = 25$
36	$(36-28)^2 = 64$
41	$(41-28)^2 = 169$
32	$(32-28)^2 = 16$
Sum	$\Sigma(x-\overline{x})^2 = 492$

The value of $\Sigma(x-\overline{x})^2$ from City B is greater than the value of $\Sigma(x-\overline{x})^2$ from City A.

This is a good measure of spread since this will remove the negative value of the difference between each data and the mean and hence show that the spread of the temperatures in City B is wider than that in City A.

8. $\Sigma(x-\overline{x})^2$ will increase when there are more data values.

No, it does not mean that the spread will increase when there are more data values.

9. For City A,

$$\frac{\Sigma\left(x-\overline{x}\right)^2}{6} = \frac{64}{6} = 10.7 \text{ (to 3 s.f.)}$$

For City B,

$$\frac{\Sigma\left(x-\overline{x}\right)^2}{6} = \frac{492}{6} = 82$$

This will provide a good indication on how the data are spread about the mean since it takes into account the number of data.

10. For City A,

$$\sqrt{\frac{\Sigma\left(x-\overline{x}\right)^2}{n}} = \sqrt{\frac{64}{6}} = 3.26 \text{ (to 3 s.f.)}$$

11. For City B,

$$\sqrt{\frac{\Sigma\left(x-\overline{x}\right)^2}{n}} = \sqrt{\frac{492}{6}} = 9.06 \text{ (to 3 s.f.)}$$

12. The standard deviation for City B is larger. This means that the temperatures of City B are more widely spread than those of City A.

Thinking Time (Page 121)

The measured mean is 1.5 °C higher than the correct mean since the total error of the measurements is divided by the number of measurements.

There will be no change in the standard deviation since the difference between each data and the mean cancels the 1.5 °C error.

Class Discussion (Matching Histograms with Data Sets)

Histogram *A* represents data set V since the distribution is skewed to the right so its median should match the bar on the left.

Histogram *B* represents data set III since the distribution is skewed to the left so its median should match the bar on the right.

Histogram *C* represents data set VI since its mean and median are close to each other so its distribution is more symmetrical.

Histogram *D* represents data set II since its mean and median are the same so its distribution is more symmetrical. But the data is more widely spread so standard deviation is higher.

Histogram *E* represents data set I since the distribution is skewed to the left so its median should match the bar on the right. Also, its mean and median are close to each other.

Histogram *F* represents data set IV since the distribution is skewed to the right so its median should match the bar on the left.

Class Discussion (Can We Always Trust the Statistics We Read?)

Part 1: The Choice of Averages

1. **(i)** Median starting salary of \$2960 means that 50% of the 2014 SNU cohort who graduated have starting salary less than \$2960.

(ii) 90th percentile starting salary of \$5120 means that 90% of the 2014 SNU cohort who graduated have starting salary less than \$5120.

2. Numerical difference is \$620.

There is a wide distribution of the starting salaries among SNU's fresh graduates, and this could be due to the job types.

3. No. The 'average monthly starting salary' used in the news report gives only the mean starting salary of fresh graduates from SNU which does not take into account many aspects such as median and standard deviation.

4. The choice of averages will depend on the message the writer wants to bring across and it could be biased.

Part 2: The Collection of Statistical Data

5. No. The advertisement did not indicate clearly the sample size and the profile of the respondents.

6. To name 3 toothpaste brands is excessive since most people will usually stick to a particular brand. By asking respondents to name 3 brands, the chances of Superclean toothpaste being recommended will be high. Also, as long as Superclean toothpaste appears among the 3, it would be considered as being recommended.

OXFORD UNIVERSITY PRESS

7. The collection of statistics can be designed in many ways and can be skewed to support whichever analysis that one is trying to present.

Part 3: The Display of Statistical Data

8. The bar chart seems to indicate that the investment company has increased the projected growth in dividend payment by one third. However, the scale on the y-axis is not consistent as it is not equally spaced with a jump from 0 to 6.5.
9. **(i)** Company A since its sector has the largest area.
   (ii) The pie chart shows that the area of the sector for Company A is the largest, suggesting that Company A has the largest market share. However, the figure shows that Company B has the largest percentage of the global smartphone market.
10. Companies may display statistical data using diagrams which are visually misleading to their own advantage. Hence it is important to read the values on the diagrams too.

Practise Now (Page 76)

(a)

Length (x mm)	Cumulative Frequency
$x \leqslant 30$	1
$x \leqslant 35$	4
$x \leqslant 40$	10
$x \leqslant 45$	22
$x \leqslant 50$	32
$x \leqslant 55$	38
$x \leqslant 60$	40

(b) **(i)** Number of insects which are 50 mm or less in length = 32

(ii) Number of insects which are more than 45 mm in length = 40 – 22 = 18

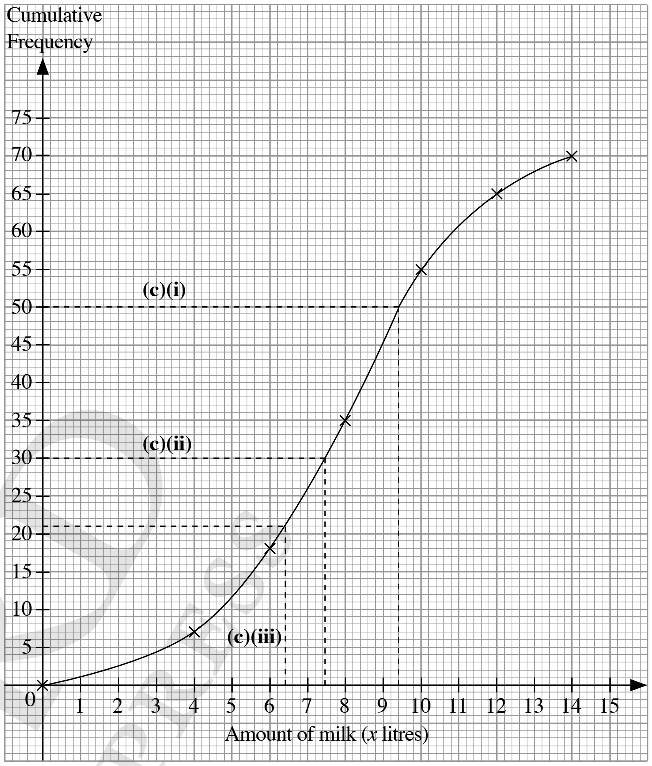
(iii) Number of insects which are more than 35 mm but less than or equal to 50 mm in length = 32 – 4 = 28

Practise Now 1

(a)

Amount of milk (x litres)	Number of cows
$x \leqslant 4$	7
$x \leqslant 6$	18
$x \leqslant 8$	35
$x \leqslant 10$	55
$x \leqslant 12$	65
$x \leqslant 14$	70

(b) **Cumulative Frequency Curve for the amount of milk (in litres) produced**



(c) **(i)** From the curve, the number of cows that produce less than or equal to 9.4 litres of milk is 50.

(ii) From the curve, the number of cows that produce less than or equal to 7.4 litres of milk is 30.

$\therefore$ 70 – 30 = 40 cows produce more than 7.4 litres of milk.

$\therefore$ The fraction of the 70 cows that produce more than 7.4 litres of milk is $\frac{40}{70} = \frac{4}{7}$.

(iii) 70% of the cows means $\frac{70}{100} \times 70 = 49$, i.e. 49 cows produce more than x litres of milk.

$\therefore$ 70 – 49 = 21 cows produce less than or equal to x litres of milk.

From the curve, $x = 6.4$.

Practise Now 2

(i) From the curve, the number of oranges having Vitamin C content less than 32 mg is 180.

(ii) From the curve, the total number of oranges having Vitamin C content of 26 mg or less is 40.

$\therefore$ 200 – 40 = 160 oranges have Vitamin C content of 26 mg or more.

$\therefore$ The fraction of the total number of oranges having Vitamin C content of 26 mg or more is $\frac{160}{200} = \frac{4}{5}$.

(iii) 40% of the oranges means $\frac{40}{100} \times 200 = 80$, i.e. 80 oranges have Vitamin C content of p mg or more.

$\therefore$ 200 – 80 = 120 oranges have Vitamin C content of p mg or less.

From the curve, $p = 29.8$.

OXFORD UNIVERSITY PRESS

Practise Now 3

1. **(i)** Arrange the given data in ascending order.

$$\overbrace{10, 12, \underset{\overset{\uparrow}{Q_1}}{14}, 22, 25}^{\text{lower half}}\underset{\overset{\uparrow}{Q_2}}{,}\overbrace{36, 38, \underset{\overset{\uparrow}{Q_3}}{44}, 45, 59}^{\text{upper half}}$$

For the given data, $n = 10$.

$$\therefore Q_2 = \frac{25 + 36}{2} = 30.5$$

$Q_1 = 14$

$Q_3 = 44$

(ii) Range $= 59 - 10$

$= 49$

(iii) Interquartile range $= Q_3 - Q_1$

$= 44 - 14$

$= 30$

2. **(i)** Arrange the given data in ascending order.

$$\overbrace{9, \underset{\overset{\uparrow}{Q_1}}{15, 16}, 23}^{\text{lower half}}, \underset{\overset{\uparrow}{Q_2}}{26}, \overbrace{32, \underset{\overset{\uparrow}{Q_3}}{35, 47}, 54}^{\text{upper half}}$$

For the given data, $n = 9$.

$\therefore Q_2 = 26$

$$Q_1 = \frac{15 + 16}{2} = 15.5$$

$$Q_3 = \frac{35 + 47}{2} = 41$$

(ii) Range $= 54 - 9$

$= 45$

(iii) Interquartile range $= Q_3 - Q_1$

$= 41 - 15.5$

$= 25.5$

Practise Now 4

1. **(i)** For this set of data, $n = 120$.

$$\therefore \frac{n}{2} = 60, \frac{n}{4} = 30 \text{ and } \frac{3n}{4} = 90$$

From the graph, median = 50,

lower quartile = 17,

upper quartile = 81.

(ii) Interquartile range $= 81 - 17$

$= 64$

(iii) 10% of the total frequency $= \frac{10}{100} \times 120$

$= 12$

From the graph, the 10th percentile = 8.

80% of the total frequency $= \frac{80}{100} \times 120$

$= 96$

From the graph, the 80th percentile = 84.

(iv) 60% of the students $= \frac{60}{100} \times 120 = 72$, i.e. 72 students passed the test.

$\therefore$ 120 – 72 = 48 students failed the test.

From the graph, the passing mark is 34.

Practise Now 5

For this set of data, $n = 300$.

$$\therefore \frac{n}{2} = 150, \frac{n}{4} = 75 \text{ and } \frac{3n}{4} = 225$$

(a) **(i)** From the graph, median mark of School A = 62.

(ii) From the graph, lower quartile = 50,

upper quartile = 68.

$\therefore$ Interquartile range of School $A = 68 - 50$

$= 18$

(b) **(i)** From the graph, median mark of School B = 70.

(ii) From the graph, lower quartile = 55,

upper quartile = 79.

$\therefore$ Interquartile range of School $B = 79 - 55$

$= 24$

(c) School B performed better overall because its median mark is higher as compared to that of School A.

(d) School A's quiz marks were more consistent as its interquartile range is lower as compared to that of School B.

Practise Now 6

Arrange the given data in ascending order.

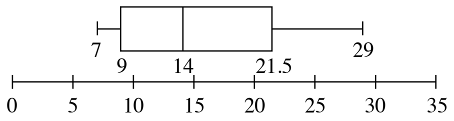
$$7, \underset{\overset{\uparrow}{Q_1}}{9, 9}, 13, \underset{\overset{\uparrow}{Q_2}}{14}, 16, \underset{\overset{\uparrow}{Q_3}}{20, 23}, 29$$

For the given data, $n = 9$.

$\therefore Q_2 = 14$

$$Q_1 = \frac{9 + 9}{2} = 9$$

$$Q_3 = \frac{20 + 23}{2} = 21.5$$



Practise Now 7

(i) From the box-and-whisker plot, the median mark is 28.

(ii) Range = MAX – MIN

$= 70 - 8$

$= 62$

(iii) Interquartile range $= Q_3 - Q_1$

$= 37 - 23$

$= 14$

OXFORD UNIVERSITY PRESS

Practise Now 8

(a) For Brick A,

(i) range = MAX – MIN
= 6300 – 3800
= 2500 psi

(ii) median = 4700 psi

(iii) interquartile range = $Q_3 - Q_1$
= 5700 – 4200
= 1500 psi

(b) For Brick B,

(i) range = MAX – MIN
= 7000 – 2000
= 5000 psi

(ii) median = 5500 psi

(iii) interquartile range = $Q_3 - Q_1$
= 6000 – 3800
= 2200 psi

(c) Brick B because its median compressive strength (psi) is higher as compared to that of Brick A.

Practise Now 9

x	x^2
6	36
9	81
15	225
26	676
10	100
14	196
21	441
3	9
$\Sigma x = 104$	$\Sigma x^2 = 1764$

$$\text{Mean, } \bar{x} = \frac{\Sigma x}{n} = \frac{104}{8} = 13$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2} = \sqrt{\frac{1764}{8} - 13^2} = 7.18 \text{ (to 3 s.f.)}$$

Practise Now (Pg 117)

Standard deviation of their ages = 3.74 years

Practise Now 10

(a)

Marks	Frequency	Mid-value (x)	fx	fx^2
$0 < x \leqslant 4$	3	2	6	12
$4 < x \leqslant 8$	8	6	48	288
$8 < x \leqslant 12$	14	10	140	1400
$12 < x \leqslant 16$	2	14	28	392
$16 < x \leqslant 20$	3	18	54	972
Sum	$\Sigma f = 30$		$\Sigma fx = 276$	$\Sigma fx^2 = 3064$

(i) $$\text{Mean, } \bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{276}{30} = 9.2$$

(ii) $$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \bar{x}^2} = \sqrt{\frac{3064}{30} - 9.2^2} = 4.18 \text{ (to 3 s.f.)}$$

(b) Since the mean mark for Class A is lower than that of Class B, the students of Class A did not perform as well overall in comparison to the students of Class B.
Since the standard deviation of Class A is higher than that of Class B, this indicates that there is a greater spread of marks in Class A, i.e. some students scored very high marks while some scored very low marks.

Practise Now (Page 121)

Standard deviation = 12.1 g

Exercise 4A

1. **(a)**

Marks (m)	Cumulative Frequency
$m \leqslant 10$	3
$m \leqslant 20$	15
$m \leqslant 30$	24
$m \leqslant 40$	35
$m \leqslant 50$	52
$m \leqslant 60$	71
$m \leqslant 70$	91
$m \leqslant 80$	105
$m \leqslant 90$	115
$m \leqslant 100$	120

OXFORD UNIVERSITY PRESS

(b) **(i)** Number of students who scored less than or equal to 30 marks is 24.

(ii) Number of students who scored more than 80 marks is $120 - 105 = 15$.

(iii) Number of students who scored more than 40 marks but not more than 90 marks is $115 - 35 = 80$.

2. **(a)**

Number of Pull-ups (x)	Cumulative Frequency
$x < 6$	69
$x < 8$	132
$x < 10$	160
$x < 12$	184
$x < 16$	203
$x < 20$	217
$x < 25$	230

(b) **(i)** Number of students who qualified for the Gold Award is $230 - 184 = 46$.

(ii) Number of students who qualified for the Silver Award is $184 - 132 = 52$.

(iii) Number of students who qualified for the Bronze Award is $132 - 69 = 63$.

3. **(i)** Number of students whose masses are less than or equal to 65 kg is 26.

(ii) Number of students whose masses are less than or equal to 68.6 kg is 76.

$\therefore$ Number of students whose masses are more than 68.6 kg is $100 - 76 = 24$.

(iii) Number of students whose masses are less than or equal to 64.4 kg is 20.

$\therefore$ $100 - 20 = 80$ students have masses more than 64.4 kg.

$\therefore$ The percentage of the total number of students whose masses are more than 64.4 kg is $\frac{80}{100} \times 100\% = 80\%$.

4. **(i)** Number of loaves of bread having masses less than or equal to 450.4 g is 33.

(ii) Number of loaves of bread which are underweight (i.e. mass is 446.2 g or less) is 3.

Number of loaves of bread which are overweight (i.e. mass more than 453.6 g) is $50 - 47 = 3$.

The number of loaves of bread which are underweight or overweight is $3 + 3 = 6$.

(iii) $\frac{3}{10}$ of the loaves of bread means $\frac{3}{10} \times 50 = 15$, i.e. 15 loaves of bread have masses more than x g.

$\therefore$ $50 - 15 = 35$ loaves of bread have masses less than or equal to x g.

From the curve, $x = 450.6$ g.

5. **(a)** **(i)** For Soil A, the number of earthworms having lengths less than or equal to 46 mm is 130.

For Soil B, the number of earthworms having lengths less than or equal to 46 mm is 90.

(ii) For Soil A, the number of earthworms having lengths more than 76 mm is $500 - 455 = 45$.

$\therefore$ The percentage of earthworms having lengths more than 76 mm is $\frac{45}{500} \times 100\% = 9\%$.

For Soil B, the number of earthworms having lengths more than 76 mm is $500 - 485 = 15$.

$\therefore$ The percentage of earthworms having lengths more than 76 mm is $\frac{15}{500} \times 100\% = 3\%$.

(iii) 18% of the earthworms means $\frac{18}{100} \times 500 = 90$, i.e. 90 earthworms have lengths a mm or less.

$\therefore$ For Soil A, $a = 41$ and for Soil B, $a = 46$.

(b) Soil A produced the longest earthworm among the 1000 earthworms.

(c) Percentage of 'satisfactory' earthworms from Soil A

$= \frac{500 - 320}{500} \times 100\% = 36\%$.

Percentage of 'satisfactory' earthworms from Soil B

$= \frac{500 - 380}{500} \times 100\% = 24\%$

6. **(i)** Number of students who scored less than 45 marks = 57

(ii) Number of students who scored less than 34 marks = 26

$\therefore$ $\frac{26}{120} = \frac{13}{60}$ of the total number of students failed the music examination.

(iii) 27.5% of the number of students means $\frac{27.5}{100} \times 120 = 33$, i.e. 33 students obtained at least a marks in the music examination.

$\therefore$ $120 - 33 = 87$ students obtained less than a marks.

From the curve, $a = 52$.

7. **(i)** Number of bicycles that travelled at a speed less than 18 km/h = 13

(ii) Number of bicycles that travelled at a speed greater than or equal to 29 km/h $= 100 - 80 = 20$

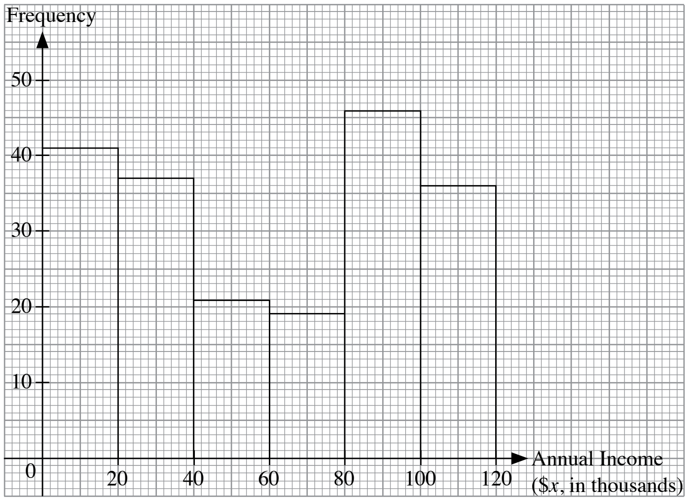
$\therefore$ $\frac{20}{100} = \frac{1}{5}$ of the total number of bicycles travelled at a speed greater than or equal to 29 km/h.

(iii) 40% of the bicycles means $\frac{40}{100} \times 100 = 40$, i.e. 40 bicycles have a speed less than v km/h.

$\therefore$ From the curve, $v = 23$.

OXFORD UNIVERSITY PRESS

8. **(i)**



(ii) The range of annual income the household is most likely to earn is $80 \leqslant x < 100$.

9. **(a)** **(i)** Number of tomatoes with masses more than 56 g
$= 80 - 36 = 44$

$\therefore$ The percentage of grade A tomatoes is $\frac{44}{80} \times 100$
$= 55\%$.

(ii) 15% of the tomatoes means $\frac{15}{100} \times 80 = 12$, i.e. 12 tomatoes are y g or less.

$\therefore$ From the curve, $y = 52$.

(iii) Number of grade B tomatoes which are between grade A and C
$= 80 - 44 - 12$
$= 24$

(b) **(i)**

Masses (x g)	Frequency
$40 < x \leqslant 45$	2
$45 < x \leqslant 50$	6
$50 < x \leqslant 55$	18
$55 < x \leqslant 60$	44
$60 < x \leqslant 65$	10

(ii) Mean mass of a tomato produced at the nursery

$$= \frac{42.5 \times 2 + 47.5 \times 6 + 52.5 \times 18 + 57.5 \times 44 + 62.5 \times 10}{80}$$

$= 55.875$ g

10. **(i)** Examination B is the most challenging as more students scored lower marks as compared to the other two examinations.

(ii) Examination C is the least challenging as fewer students scored lower marks and there is a sharp increase in the number of students in the higher mark range, indicating more students scoring higher marks as compared to the other two examinations.

Exercise 4B

1. **(a)** Arrange the given data in ascending order.

$$\overbrace{2, \underset{\substack{\uparrow\\ Q_1}}{4}, 5}^{\text{lower half}}, \underset{\substack{\uparrow\\ Q_2}}{6}, \overbrace{7, \underset{\substack{\uparrow\\ Q_3}}{8}, 10}^{\text{upper half}}$$

For the given data, $n = 7$.

$\therefore Q_2 = 6$
$Q_1 = 4$
$Q_3 = 8$
Range $= 10 - 2$
$= 8$

Lower quartile, $Q_1 = 4$
Median, $Q_2 = 6$
Upper quartile, $Q_3 = 8$
Interquartile range $= Q_3 - Q_1$
$= 8 - 4$
$= 4$

(b) Arrange the given data in ascending order.

$$\overbrace{51, 54\underset{\substack{\uparrow\\ Q_1}}{,}\ 63, 64}^{\text{lower half}}\underset{\substack{\uparrow\\ Q_2}}{,}\ \overbrace{66, 70\underset{\substack{\uparrow\\ Q_3}}{,}\ 72, 80}^{\text{upper half}}$$

For the given data, $n = 8$.

$\therefore Q_2 = \frac{64 + 66}{2} = 65$

$Q_1 = \frac{54 + 63}{2} = 58.5$

$Q_3 = \frac{70 + 72}{2} = 71$

Range $= 80 - 51$
$= 29$

Lower quartile, $Q_1 = 58.5$
Median, $Q_2 = 65$
Upper quartile, $Q_3 = 71$
Interquartile range $= Q_3 - Q_1$
$= 71 - 58.5$
$= 12.5$

(c) Arrange the given data in ascending order.

$$\overbrace{9, 10\underset{\substack{\uparrow\\ Q_1}}{,}\ 14, 16}^{\text{lower half}}, \underset{\substack{\uparrow\\ Q_2}}{18}, \overbrace{22, 27\underset{\substack{\uparrow\\ Q_3}}{,}\ 32, 40}^{\text{upper half}}$$

For the given data, $n = 9$.

$\therefore Q_2 = 18$

$Q_1 = \frac{10 + 14}{2} = 12$

$Q_3 = \frac{27 + 32}{2} = 29.5$

OXFORD UNIVERSITY PRESS

Range = 40 – 9

= 31

Lower quartile, Q_1 = 12

Median, Q_2 = 18

Upper quartile, Q_3 = 29.5

Interquartile range = $Q_3 – Q_1$

= 29.5 –12

= 17.5

(d) Arrange the given data in ascending order.

$$\overbrace{86, 98, \underset{\overset{\uparrow}{Q_1}}{102}, 138, 164}^{\text{lower half}}\underset{\overset{\uparrow}{Q_2}}{,}\ \overbrace{168, 184, \underset{\overset{\uparrow}{Q_3}}{207}, 244, 250}^{\text{upper half}}$$

For the given data, $n = 10$.

$$\therefore Q_2 = \frac{164 + 168}{2} = 166$$

$Q_1 = 102$

$Q_3 = 207$

Range = 250 – 86

= 164

Lower quartile, Q_1 = 102

Median, Q_2 = 166

Upper quartile, Q_3 = 207

Interquartile range = $Q_3 – Q_1$

= 207 – 102

= 105

(e) Arrange the given data in ascending order.

$$\overbrace{2, 7, 4.9, \underset{\overset{\uparrow}{Q_1}}{,}\ 6.7, 8.5}^{\text{lower half}}, \underset{\overset{\uparrow}{Q_2}}{10.4}, \overbrace{11.8, 13.1\underset{\overset{\uparrow}{Q_3}}{,}\ 15.1, 22.4}^{\text{upper half}}$$

For the given data, $n = 9$.

$\therefore Q_2 = 10.4$

$$Q_1 = \frac{4.9 + 6.7}{2} = 5.8$$

$$Q_3 = \frac{13.1 + 15.1}{2} = 14.1$$

Range = 22.4 – 2.7

= 19.7

Lower quartile, Q_1 = 5.8

Median, Q_2 = 10.4

Upper quartile, Q_3 = 14.1

Interquartile range = $Q_3 – Q_1$

= 14.1 – 5.8

= 8.3

2. Arrange the given data in ascending order.

$$\overbrace{0, 0, \underset{\overset{\uparrow}{Q_1}}{1}, 6, 6}^{\text{lower half}}\underset{\overset{\uparrow}{Q_2}}{,}\ \overbrace{9, 9, \underset{\overset{\uparrow}{Q_3}}{24}, 27, 29}^{\text{upper half}}$$

For the given data, $n = 10$.

$$\therefore Q_2 = \frac{6 + 9}{2} = 7.5$$

$Q_1 = 1$

$Q_3 = 24$

(i) Median, Q_2 = 7.5

Lower quartile, Q_1 = 1

Upper quartile, Q_3 = 24

(ii) Range = 29 – 0

= 29

Interquartile range = $Q_3 – Q_1$

= 24 – 1

= 23

3. **(i)** Median mark = $\frac{42 + 48}{2}$

= 45

(ii) Range = 95 – 9

= 86

(iii) Upper quartile = $\frac{73 + 79}{2}$

= 76

Lower quartile = $\frac{28 + 30}{2}$

= 29

$\therefore$ Interquartile range = 76 – 29

= 47

4. For this set of data, $n = 300$.

$$\therefore \frac{n}{2} = 150,\ \frac{n}{4} = 75 \text{ and } \frac{3n}{4} = 225$$

(a) **(i)** From the graph, median mark = 97,

lower quartile = 88,

upper quartile = 105.

(ii) Interquartile range = 105 – 88

= 17

(b) **(i)** 20% of the total frequency = $\frac{20}{100} \times 300$

= 60

From the graph, the 20th percentile = 85.

(ii) 90% of the total frequency = $\frac{90}{100} \times 300$

= 270

From the graph, the 90th percentile = 110.

5. For this set of data, $n = 56$.

$$\therefore \frac{n}{2} = 28,\ \frac{n}{4} = 14 \text{ and } \frac{3n}{4} = 42$$

(i) From the graph, median height = 50.

(ii) From the graph, upper quartile = 57.

(iii) From the graph, lower quartile = 39.

(iv) Number of plants having heights greater than 57 cm

= 56 – 42

= 14

OXFORD UNIVERSITY PRESS

6. For this set of data, $n = 600$.

$\therefore \frac{n}{2} = 300$, $\frac{n}{4} = 150$ and $\frac{3n}{4} = 450$

(a) (i) From the graph, median length = 35.5.

(ii) From the graph, lower quartile = 32.5,

upper quartile = 38.5.

$\therefore$ Interquartile range = 38.5 – 32.5

= 6

(b) 65% of the leaves means $\frac{65}{100} \times 600 = 390$, i.e. 390 leaves are considered healthy since their lengths are longer than h mm.

$\therefore$ 600 – 390 = 210 leaves have lengths which are shorter than or equal to h mm.

From the curve, $h = 34$.

7. For this set of data, $n = 80$.

$\therefore \frac{n}{2} = 40$, $\frac{n}{4} = 20$ and $\frac{3n}{4} = 60$

(a) (i) From the graph, median mark = 23.5.

(ii) From the graph, upper quartile = 26.5.

(iii) From the graph, lower quartile = 20.

$\therefore$ Interquartile range = 26.5 – 20

= 6.5

(iv) Number of participants who scored more than or equal to 26 marks but less than 30 marks

= (80 – 56) – (80 – 72)

= 16

(b) 37.5% of the students $\frac{37.5}{100} \times 80 = 30$, i.e. 30 students passed the quiz.

$\therefore$ 80 – 30 = 50 students failed the quiz.

From the graph, the passing mark is 25.

8. For this set of data, $n = 800$.

$\therefore \frac{n}{2} = 400$, $\frac{n}{4} = 200$ and $\frac{3n}{4} = 600$

(a) (i) From the graph, median travelling expenses of School A = 42.

(ii) From the graph, median travelling expenses of School B = 58.

(b) (i) From the graph, lower quartile = 30,

upper quartile = 56.

$\therefore$ Interquartile range of the travelling expenses of School A

= 56 – 30

= 26

(ii) From the graph, lower quartile = 48,

upper quartile = 72.

$\therefore$ Interquartile range of the travelling expenses of School B

= 72 – 48

= 24

(c) 80% of the total frequency = $\frac{80}{100} \times 800$

= 640

From the graph, the 80th percentile = 75.

(d) The students from School B spend more on daily travelling expenses because their median travelling expenses are higher as compared to that of the students from School A.

9. (i) For Class A, $n = 40$.

$\therefore \frac{n}{2} = 20$, $\frac{n}{4} = 10$ and $\frac{3n}{4} = 30$

From the graph, lower quartile of Class A = 21,

median score of Class A = 28,

upper quartile of Class A = 34.

(ii) There are 38 students in Class B.

(iii) For Class B, $n = 38$.

$\therefore \frac{n}{2} = 19$, $\frac{n}{4} = 9.5$ and $\frac{3n}{4} = 28.5$

From the graph, lower quartile of Class B = 20,

upper quartile of Class B = 28.5.

$\therefore$ Interquartile range of Class B

= 28.5 – 20

= 8.5

(iv) From the graph, 37 students scored 38 marks or less.

For Class B, number of students who received a gold award

= 38 – 37

= 1

Percentage of students who received a gold award

$= \frac{1}{38} \times 100\%$

= 2.63%

(v) Median score of Class A = 28, median score of Class B = 24

Interquartile range of Class A = 34 – 21 = 13, interquartile range of Class B = 8.5

Class A generally performed better because the median score for Class A is higher than that of Class B. The results of Class A are less consistent since the interquartile range of Class A is higher than that of Class B.

10. For Class A, $n = 500$.

$\therefore \frac{n}{2} = 250$, $\frac{n}{4} = 125$ and $\frac{3n}{4} = 375$

(a) (i) From the graph, median mark of School A = 50.

(ii) 70% of the total frequency = $\frac{70}{100} \times 500$

= 350

From the graph, the 70th percentile = 60.

(iii) From the graph, lower quartile of School A = 33,

upper quartile of School A = 62.

$\therefore$ Interquartile range of School A = 62 – 33

= 29

(iv) Number of cadets who scored less than 43 marks = 195

(v) 60% of the cadets = $\frac{60}{100} \times 500 = 300$, i.e. 300 cadets passed the test.

$\therefore$ 500 – 300 = 200 cadets failed the test.

From the graph, the passing mark is 44.

(b) Percentage of cadets who scored distinctions in School *A*

$= \dfrac{500 - 430}{500} \times 100\%$

$= 14\%$

Percentage of cadets who scored distinctions in School *B*

$= \dfrac{500 - 355}{500} \times 100\%$

$= 29\%$

(c) Median mark of School *A* = 50,

median mark of School *B* = 60.

Agree. The median mark of School *B* is higher as compared to that of School *A*. School *B* has a higher percentage of cadets who scored distinctions as compared to School *A*.

11. (a) Arrange the given data in ascending order.

$$\text{City } X \quad \overbrace{16, 21, \underset{\underset{Q_1}{\uparrow}}{23}, 37, 50}^{\text{lower half}} \underset{\underset{Q_2}{\uparrow}}{,} \overbrace{53, 65, \underset{\underset{Q_3}{\uparrow}}{80}, 81, 100}^{\text{upper half}}$$

For the given data, $n = 10$.

$\therefore Q_2 = \dfrac{50 + 53}{2} = 51.5$

$Q_1 = 23$

$Q_3 = 80$

(i) Range = 100 – 16

= 84

(ii) Median = 51.5

(iii) Interquartile range = 80 – 23

= 57

(b) Arrange the given data in ascending order.

$$\text{City } Y \quad \overbrace{79, 99, \underset{\underset{Q_1}{\uparrow}}{103}, 114, 121}^{\text{lower half}} \underset{\underset{Q_2}{\uparrow}}{,} \overbrace{171, 198, \underset{\underset{Q_3}{\uparrow}}{200}, 235, 308}^{\text{upper half}}$$

For the given data, $n = 10$.

$\therefore Q_2 = \dfrac{121 + 171}{2} = 146$

$Q_1 = 103$

$Q_3 = 200$

(i) Range = 308 – 79

= 229

(ii) Median = 146

(iii) Interquartile range = 200 – 103

= 97

(c) City *Y* has a greater spread as the interquartile range is higher as compared to that of City *X*.

(d) Since the median PSI of City *Y* is higher than that of City *X*, therefore the air quality of City *Y* is worse than the air quality of City *X*.

Since the interquartile range of PSI of City *Y* is higher than that of City *X*, therefore the air quality of City *Y* is less consistent than the air quality of City *X*.

12. For this set of data, $n = 60$.

$\therefore \dfrac{n}{2} = 30,\ \dfrac{n}{4} = 15 \text{ and } \dfrac{3n}{4} = 45$

(a) (i) From the curve, the lower quartile = 10,

the median = 13,

the upper quartile = 15.25.

(ii) Interquartile range = 15.25 – 10

= 5.25

(b) Percentage of clients who waited for not more than 15 minutes at the bank

$= \dfrac{60 - 16}{60} \times 100\%$

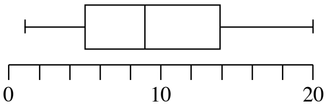
= 73.3% (to 3 s.f.)

(c) The intersection of the two cumulative curves represents the median waiting time as it represents the waiting time for 50% of the clients.

Exercise 4C

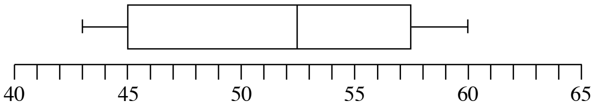
1. (a) Arrange the given data in ascending order.

$$1, \underset{\underset{Q_1}{\uparrow}}{5}, 8, \underset{\underset{Q_2}{\uparrow}}{9}, 11, \underset{\underset{Q_3}{\uparrow}}{14}, 20$$



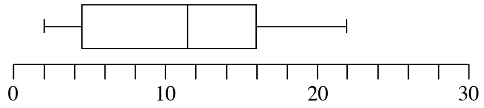
(b) Arrange the given data in ascending order.

$$43, 45 \underset{\underset{Q_1}{\uparrow}}{,} 45, 51 \underset{\underset{Q_2}{\uparrow}}{,} 54, 57 \underset{\underset{Q_3}{\uparrow}}{,} 58, 60$$



(c) Arrange the given data in ascending order.

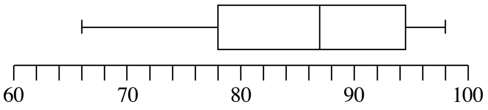
$$2, 3, 3 \underset{\underset{Q_1}{\uparrow}}{,} 6, 8, 11 \underset{\underset{Q_2}{\uparrow}}{,} 12, 15, 15 \underset{\underset{Q_3}{\uparrow}}{,} 17, 21, 22$$



OXFORD UNIVERSITY PRESS

(d) Arrange the given data in ascending order.

$$66, 77, \underset{\underset{Q_1}{\uparrow}}{79}, 82, \underset{\underset{Q_2}{\uparrow}}{87}, 87, \underset{\underset{Q_3}{\uparrow}}{93}, 96, 98$$



2. **(i)** Lower quartile of the temperature = 19°C

Median of the temperature = 21°C

Upper quartile of the temperature = 25°C

(ii) Range = 27 – 16

= 11°C

3. **(i)** Median blood pressure level of the patients = 169 mm of mercury

(ii) Lower quartile = 162 mm of mercury

Upper quartile = 185 mm of mercury

Interquartile range = 185 – 162

= 23 mm

4. **(i)** Lower quartile of the alcohol content of the drivers = 0.05 g/dL

Median of the alcohol content of the drivers = 0.07 g/dL

Upper quartile of the alcohol content of the drivers = 0.086 g/dL

(ii) The highest 25% of the drivers has a larger spread of alcohol content as compared to that of the lowest 25% of the drivers.

5. Arrange the given data in ascending order.

$$168, 180, 185, \underset{\underset{Q_1}{\uparrow}}{\,}\, 192, 192, 195, \underset{\underset{Q_2}{\uparrow}}{\,}\, 195, 196, 198, \underset{\underset{Q_3}{\uparrow}}{\,}\, 200, 205, 213$$

(i) $a = 168$

$$b = Q_1 = \frac{185 + 192}{2} = 188.5$$

$$c = Q_2 = \frac{195 + 195}{2} = 195$$

$$d = Q_3 = \frac{198 + 200}{2} = 199$$

$e = 213$

(ii) $d - b = 199 - 188.5$

$= 10.5$

It represents the interquartile range of the data.

(iii) $e - a = 213 - 168$

$= 45$

It represents the range of the data.

6. **(a)** **(i)** Type *A*

(ii) Type *C*

(b) Type *B*

(c) Type *B*

7. **(a)** For School *A*,

(i) range = 82 – 48

= 34 kg

(ii) median = 63 kg

(iii) interquartile range = $Q_3 - Q_1$

= 67 – 59

= 8 kg

(b) For School *B*,

(i) range = 86 – 45

= 41 kg

(ii) median = 68 kg

(iii) interquartile range = $Q_3 - Q_1$

= 78 – 64

= 14 kg

(c) Agree. School *B* has a higher median mass as compared to that of School *A*.

8. **(a)** For the Geography examination,

(i) range = 88 – 24

= 64

(ii) median = 56

(iii) interquartile range = $Q_3 - Q_1$

= 66 – 40

= 26

(b) For the History examination,

(i) range = 94 – 10

= 84

(ii) median = 42

(iii) interquartile range = $Q_3 - Q_1$

= 66 – 30

= 36

(c) Agree. The lower quartile and the median mark for the Geography examination are higher than those of the History examination. The lower quartile for the Geography examination is slightly lower than the median mark for the History examination.

(d) History examination. Its interquartile range is larger than that of the Geography examination.

9. **(a)** For this set of data, $n = 64$.

$$\therefore\ \frac{n}{2} = 32,\ \frac{n}{4} = 16 \text{ and } \frac{3n}{4} = 48$$

(i) Median = 15 hours

(ii) Lower quartile = 12 hours

Upper quartile = 19 hours

∴ Interquartile range = 19 – 12

= 7 hours

(iii) Number of adults who spent more than 25 hours per week watching television

= 64 – 58

= 6

(b) **(i)** Median = 34 hours

(ii) Lower quartile = 28 hours

Upper quartile = 52 hours

∴ Interquartile range = 52 – 28

= 24 hours

(c) Agree. The median number of hours spent by the teenagers watching television is higher as compared to that of the adults.

OXFORD UNIVERSITY PRESS

(d) The interquartile range of number of hours spent by the teenagers is larger as compared to that of the adults and hence the number of hours spent by the teenagers is more widely spread as compared to the number of hours spent by the adults.

10. (a) For Prestige Country Club,

(i) median age = 40.5 years

(ii) lower quartile = 37.5 years
upper quartile = 42.5 years
∴ interquartile range = 42.5 – 37.5
= 5 years

(b) For Luxury Country Club,

(i) median age = 50 years

(ii) lower quartile = 44 years
upper quartile = 53.5 years
∴ interquartile range = 53.5 – 44
= 9.5 years

(c) The youngest 25% of the members has a larger spread of ages as compared to that of the oldest 25% of the members.

(d) Luxury Country Club as its interquartile range is higher as compared to that of Prestige Country Club.

(e) The median age of Luxury Country Club is higher than the median age of Prestige Country Club so the members from the Luxury Country Club are generally older in age as compared to that of the members from the Prestige Country Club. Also, the interquartile range of the ages of Luxury Country Club members is larger than that of Prestige Country Club members which indicates the wider spread of ages of Luxury Country Club members.

11. (a) For set X,

(i) median = 30

(ii) range = 50 – 10
= 40

(iii) lower quartile = 20
upper quartile = 40
∴ interquartile range = 40 – 20
= 20

For set Y,

(i) median = 21

(ii) range = 37 – 12
= 25

(iii) lower quartile = 17
upper quartile = 24
∴ interquartile range = 24 – 17
= 7

(b) Set X

(c) Set X

(d) Set Y

(e) Cumulative curve B

(f) Cumulative Frequency Curve A matches Histogram P because the steep increase in the gradient of the first half of the curve corresponds to the increase in the frequency for the first half of the bars in the histogram, whereas the gradual increase in the gradient of the second half of the curve corresponds to the decrease in the frequency for the second half of the bars.

Cumulative Frequency Curve B matches Histogram R because it is a straight line with constant gradient that corresponds to the constant frequency of the bars in the histogram.

Cumulative Frequency Curve C matches Histogram Q because the gradient of the curve becomes steeper as the x-axis values increase, which corresponds to the increase in the frequency of the bars in the histogram.

(g) The distribution of the data is symmetrical for Histogram P.
The distribution of the data is skewed to the left for Histogram Q.
The data are distributed equally for Histogram R.

12. Data X matches Histogram B because the lower whisker is much longer than the upper whisker, which matches the relatively many more shorter bars on the left side of the histogram than the right side of the histogram.

Data Y matches Histogram A because the whiskers are of equal length and the median is at the centre of the box, which matches the equal distribution of the bars of the histogram.

Data Z matches Histogram C because the box is short with the median at the centre of the box, indicating that the distribution is concentrated at the centre as shown in Histogram C. The whiskers are of equal length which corresponds to the balanced and symmetrical distribution on the left and right sides of the histogram.

Exercise 4D

1. (a)

x	x^2
3	9
4	16
5	25
7	49
8	64
10	100
13	169
$\Sigma x = 50$	$\Sigma x^2 = 432$

$$\text{Mean, } \bar{x} = \frac{\Sigma x}{n} = \frac{50}{7} = 7.1429 \text{ (to 5 s.f.)}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2} = \sqrt{\frac{432}{7} - 7.1429^2} = 3.27 \text{ (to 3 s.f.)}$$

OXFORD UNIVERSITY PRESS

(b)

x	x^2
28	784
25	625
32	1024
20	400
30	900
19	361
22	484
24	576
27	729
23	529
$\Sigma x = 250$	$\Sigma x^2 = 6412$

$$\text{Mean, } \overline{x} = \frac{\Sigma x}{n} = \frac{250}{10} = 25$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma x^2}{n} - \overline{x}^2} = \sqrt{\frac{6412}{10} - 25^2} = 4.02 \text{ (to 3 s.f.)}$$

(c)

x	x^2
–5	25
–4	16
0	0
1	1
4	16
–2	4
$\Sigma x = -6$	$\Sigma x^2 = 62$

$$\text{Mean, } \overline{x} = \frac{\Sigma x}{n} = \frac{-6}{6} = -1$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma x^2}{n} - \overline{x}^2} = \sqrt{\frac{62}{6} - (-1)^2} = 3.06 \text{ (to 3 s.f.)}$$

2. **(a)** Standard deviation = 11.1 (to 3 s.f.)

(b) Standard deviation = 9.35 (to 3 s.f.)

(c) Standard deviation = 11.9 (to 3 s.f.)

3.

Marks (x)	**Frequency**	fx	fx^2
2	5	10	20
3	7	21	63
4	6	24	96
5	4	20	100
6	9	54	324
7	3	21	147
8	6	48	384
Sum	$\Sigma f = 40$	$\Sigma fx = 198$	$\Sigma fx^2 = 1134$

$$\text{Mean, } \overline{x} = \frac{\Sigma fx}{\Sigma f} = \frac{198}{40} = 4.95$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \overline{x}^2} = \sqrt{\frac{1134}{40} - 4.95^2} = 1.96 \text{ (to 3 s.f.)}$$

4.

Number of goals scored per match (x)	**Frequency**	fx	fx^2
0	10	0	0
1	8	8	8
2	7	14	28
3	6	18	54
4	2	8	32
5	3	15	75
6	1	6	36
Sum	$\Sigma f = 37$	$\Sigma fx = 69$	$\Sigma fx^2 = 233$

$$\text{Mean, } \overline{x} = \frac{\Sigma fx}{\Sigma f} = \frac{69}{37} = 1.86 \text{ (to 3 s.f.)}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \overline{x}^2} = \sqrt{\frac{233}{37} - \left(\frac{69}{37}\right)^2} = 1.68 \text{ (to 3 s.f.)}$$

OXFORD UNIVERSITY PRESS

5.

x	Frequency	Mid-value (x)	fx	fx^2
$0 < x \leqslant 5$	4	2.5	10	25
$5 < x \leqslant 10$	12	7.5	90	675
$10 < x \leqslant 15$	20	12.5	250	3125
$15 < x \leqslant 20$	24	17.5	420	7350
$20 < x \leqslant 25$	16	22.5	360	8100
$25 < x \leqslant 30$	4	27.5	110	3025
Sum	$\Sigma f = 80$		$\Sigma fx = 1240$	$\Sigma fx^2 = 22\,300$

$$\text{Mean, } \bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{1240}{80} = 15.5$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \bar{x}^2} = \sqrt{\frac{22\,300}{80} - 15.5^2} = 6.20 \text{ (to 3 s.f.)}$$

6.

Salary ($)	Frequency	Mid-value (x)	fx	fx^2
$200 < x \leqslant 220$	8	210	1680	352 800
$220 < x \leqslant 240$	23	230	5290	1 216 700
$240 < x \leqslant 260$	16	250	4000	1 000 000
$260 < x \leqslant 280$	3	270	810	218 700
$280 < x \leqslant 300$	10	290	2900	841 000
Sum	$\Sigma f = 60$		$\Sigma fx = 14\,680$	$\Sigma fx^2 = 3\,629\,200$

$$\text{Mean, } \bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{14\,680}{60} = 245 \text{ (to 3 s.f.)}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \bar{x}^2} = \sqrt{\frac{3\,629\,200}{60} - \left(244\frac{2}{3}\right)^2} = 25.0 \text{ (to 3 s.f.)}$$

7. **(a)** Standard deviation = 11.7 (to 3 s.f.)

(b) Standard deviation = 7.23 (to 3 s.f.)

8. **(i)** For Class A,

x	x^2
4	16
6	36
6	36
7	49
8	64
10	100
11	121
12	144
$\Sigma x = 64$	$\Sigma x^2 = 566$

$$\text{Mean, } \bar{x} = \frac{\Sigma x}{n} = \frac{64}{8} = 8$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2} = \sqrt{\frac{566}{8} - 8^2} = 2.60 \text{ (to 3 s.f.)}$$

For Class B,

x	x^2
0	0
1	1
1	1
2	4
3	9
14	196
17	289
25	625
$\Sigma x = 63$	$\Sigma x^2 = 1125$

$$\text{Mean, } \bar{x} = \frac{\Sigma x}{n} = \frac{63}{8} = 7.875$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2} = \sqrt{\frac{1125}{8} - 7.875^2} = 8.87 \text{ (to 3 s.f.)}$$

OXFORD UNIVERSITY PRESS

(ii) The mean scores of the students from Class A and Class B are approximately the same. This means that the students from each class in general did not perform better or worse than the students from the other class. However, the scores of Class B have a higher standard deviation than those of Class A, which indicates that there is a greater spread in the scores of Class B.

9. (i) Mean mark = 10

$$\begin{aligned}\frac{x+5+16+6+10+4}{6} &= 10\\ x+41 &= 60\\ x &= 19\end{aligned}$$

(ii)

x	x^2
4	16
5	25
6	36
10	100
16	256
19	361
$\Sigma x = 60$	$\Sigma x^2 = 794$

$$\begin{aligned}\text{Standard deviation} &= \sqrt{\frac{\Sigma x^2}{n} - \overline{x}^2}\\ &= \sqrt{\frac{794}{6} - 10^2}\\ &= 5.69 \text{ (to 3 s.f.)}\end{aligned}$$

(iii) Priya had the highest score of 19 marks, and performed much better than her friends, given that her score was almost twice that of the mean mark.

10. (i)

x	x^2
23	529
15	225
8	64
13	169
28	784
6	36
15	225
$\Sigma x = 108$	$\Sigma x^2 = 2032$

$$\begin{aligned}\text{Mean, } \overline{x} &= \frac{\Sigma x}{n}\\ &= \frac{108}{7}\\ &= 15.4 \text{ minutes}\end{aligned}$$

$$\begin{aligned}\text{Standard deviation} &= \sqrt{\frac{\Sigma x^2}{n} - \overline{x}^2}\\ &= \sqrt{\frac{2032}{7} - \left(\frac{108}{7}\right)^2}\\ &= 7.23 \text{ minutes (to 3 s.f.)}\end{aligned}$$

(ii)

x	x^2
20	400
12	144
5	25
10	100
25	625
3	9
12	144
$\Sigma x = 87$	$\Sigma x^2 = 1447$

$$\begin{aligned}\text{Mean, } \overline{x} &= \frac{\Sigma x}{n}\\ &= \frac{87}{7}\\ &= 12.4 \text{ minutes (to 3 s.f.)}\end{aligned}$$

$$\begin{aligned}\text{Standard deviation} &= \sqrt{\frac{\Sigma x^2}{n} - \overline{x}^2}\\ &= \sqrt{\frac{1447}{7} - \left(\frac{87}{7}\right)^2}\\ &= 7.23 \text{ minutes (to 3 s.f.)}\end{aligned}$$

(iii) Since the mean time taken for Kate to fall asleep in New Zealand is lower than that in Singapore, therefore the time taken for Kate to fall asleep in New Zealand is shorter than the time taken for her to fall asleep when she is in Singapore.

Both standard deviations are approximately the same which indicates that the spread of the time taken for Kate to fall asleep in New Zealand and Singapore is the same.

11. (i) For Train A,

Time (minutes, x)	Frequency	fx	fx^2
2	3	6	12
3	2	6	18
4	5	20	80
5	12	60	300
6	10	60	360
7	6	42	294
8	1	8	64
9	1	9	81
Sum	$\Sigma f = 40$	$\Sigma fx = 211$	$\Sigma fx^2 = 1209$

$$\begin{aligned}\text{Mean, } \overline{x} &= \frac{\Sigma fx}{\Sigma f}\\ &= \frac{211}{40}\\ &= 5.28 \text{ (to 3 s.f.)}\end{aligned}$$

OXFORD UNIVERSITY PRESS

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \overline{x}^2}$$

$$= \sqrt{\frac{1209}{40} - 5.275^2}$$

$$= 1.55 \text{ (to 3 s.f.)}$$

For Train *B*,

Time (minutes, x)	Frequency	fx	fx^2
2	4	8	16
3	3	9	27
4	9	36	144
5	9	45	225
6	7	42	252
7	5	35	245
8	3	24	192
9	0	0	0
Sum	$\Sigma f = 40$	$\Sigma fx = 199$	$\Sigma fx^2 = 1101$

$$\text{Mean, } \overline{x} = \frac{\Sigma fx}{\Sigma f}$$

$$= \frac{199}{40}$$

$$= 4.98 \text{ (to 3 s.f.)}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \overline{x}^2}$$

$$= \sqrt{\frac{1101}{40} - 4.975^2}$$

$$= 1.67 \text{ (to 3 s.f.)}$$

(ii) Train *A* arrives late more consistently than Train *B* since its standard deviation of the time of arriving after the scheduled time is lower than that of Train *B*.

(iii) Train *B* is more punctual on the whole than Train *A* since its mean time of arriving after the scheduled time is shorter than that of Train *A*.

12. (a)

Time (minutes)	Frequency	Mid-value (x)	fx	fx^2
$20 < t \leqslant 22$	5	21	1680	352 800
$22 < t \leqslant 24$	11	23	5290	1 216 700
$24 < t \leqslant 26$	27	25	4000	1 000 000
$26 < t \leqslant 28$	13	27	810	218 700
$28 < t \leqslant 30$	4	29	2900	841 000
Sum	$\Sigma f = 60$		$\Sigma fx = 1500$	$\Sigma fx^2 = 37\,740$

(i) $$\text{Mean, } \overline{x} = \frac{\Sigma fx}{\Sigma f}$$

$$= \frac{1500}{60}$$

$$= 25$$

(ii) $$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \overline{x}^2}$$

$$= \sqrt{\frac{37\,400}{60} - 25^2}$$

$$= 2$$

(b) The patients in both hospitals have the same waiting time on the whole since their mean waiting time is the same. However, Hillview Hospital has a higher standard deviation, which indicates that there is a greater spread in the waiting time, i.e. some patients have a much longer waiting time than other patients.

13. (a) For City *A*,

Temperature (°C)	Frequency	Mid-value (x)	fx	fx^2
$35 \leqslant x < 40$	1	37.5	37.5	1406.25
$40 \leqslant x < 45$	4	42.5	170	7225
$45 \leqslant x < 50$	12	47.5	570	27 075
$50 \leqslant x < 55$	23	52.5	1207.5	63 393.75
$55 \leqslant x < 60$	7	57.5	402.5	23 143.75
$60 \leqslant x < 65$	3	62.5	187.5	11 718.75
Sum	$\Sigma f = 50$		$\Sigma fx = 2575$	$\Sigma fx^2 = 133\,962.5$

(i) $$\text{Mean, } \overline{x} = \frac{\Sigma fx}{\Sigma f}$$

$$= \frac{2575}{50}$$

$$= 51.5\ °\text{C}$$

(ii) $$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \overline{x}^2}$$

$$= \sqrt{\frac{133\,962.5}{50} - 51.5^2}$$

$$= 5.20\ °\text{C (to 3 s.f.)}$$

OXFORD UNIVERSITY PRESS

For City *B*,

Temperature (°C)	**Frequency**	**Mid-value (x)**	fx	fx^2
$35 \leqslant x < 40$	2	37.5	37.5	1406.25
$40 \leqslant x < 45$	14	42.5	170	7225
$45 \leqslant x < 50$	16	47.5	570	27 075
$50 \leqslant x < 55$	10	52.5	1207.5	63 393.75
$55 \leqslant x < 60$	5	57.5	402.5	23 143.75
$60 \leqslant x < 65$	3	62.5	187.5	11 718.75
Sum	$\Sigma f = 50$		$\Sigma fx = 2430$	$\Sigma fx^2 = 120\,012.5$

(i) Mean, $\bar{x} = \dfrac{\Sigma fx}{\Sigma f}$

$= \dfrac{2430}{50}$

$= 48.6$°C

(ii) Standard deviation $= \sqrt{\dfrac{\Sigma fx^2}{\Sigma f} - \bar{x}^2}$

$= \sqrt{\dfrac{120\,012.5}{50} - 48.6^2}$

$= 6.19$°C (to 3 s.f.)

(b) City *A* is warmer on the whole than City *B* because its mean temperature is higher than that of City *B*.

(c) City *A*'s daily temperature is more consistent as its standard deviation is lower.

14. Mean = 9

$$\frac{10 + 6 + 18 + x + 15 + y}{6} = 9$$

$$10 + 6 + 18 + x + 15 + y = 54$$

$$x + y = 5$$

$$y = 5 - x \quad \text{— (1)}$$

$$10^2 + 6^2 + 18^2 + x^2 + 15^2 + y^2 = 685 + x^2 + y^2$$

Standard deviation = 6

$$\sqrt{\frac{685 + x^2 + y^2}{6} - 9^2} = 6$$

$$\frac{685 + x^2 + y^2}{6} - 9^2 = 36$$

$$\frac{685 + x^2 + y^2}{6} = 117$$

$$685 + x^2 + y^2 = 702$$

$$x^2 + y^2 = 17 \quad \text{— (2)}$$

Substitute (1) into (2):

$$x^2 + (5 - x)^2 = 17$$

$$x^2 + 25 - 10x + x^2 = 17$$

$$2x^2 - 10x + 8 = 0$$

$$2(x^2 - 5x + 4) = 0$$

$$2(x - 1)(x - 4) = 0$$

$$x = 1 \text{ or } x = 4$$

When $x = 1$, $y = 4$.

When $x = 4$, $y = 1$.

$\therefore x = 1, y = 4$ or $x = 4, y = 1$

15. (i) Sets *A* and *C* since the mean of each set is 5.

(ii) Set *C* since the numbers in the set are the closest to each other compared to the numbers in the other two sets.

16. (i) Yes, we can use $\dfrac{\bar{x} + \bar{y}}{2}$ to find the combined mean since the number of students from each school is the same.

$$\frac{\bar{x} + \bar{y}}{2} = \frac{1}{2}\left(\frac{\Sigma fx}{100} + \frac{\Sigma fy}{100}\right)$$

$$= \frac{\Sigma fx + \Sigma fy}{200}$$

$$= \frac{\Sigma(fx + fy)}{200}$$

$$= \overline{(x + y)}$$

which is the combined mean of *x* and *y*.

(ii) No, not possible since the sum of the standard deviations of the masses of both schools $\sqrt{\dfrac{\Sigma fx^2}{100} - \bar{x}^2} + \sqrt{\dfrac{\Sigma fy^2}{100} - \bar{y}^2}$ $\neq \sqrt{\dfrac{\Sigma fx^2 + \Sigma fy^2}{200} - \overline{(x + y)}^2}$ which is the combined standard deviation.

(iii) For Brighthill School and Hogwarts School combined,

Mass (x kg)	**Frequency**	fx	fx^2
40	7	280	11 200
45	26	1170	52 650
50	60	3000	150 000
55	34	1870	102 850
60	25	1500	90 000
65	33	2145	139 425
70	10	700	49 000
75	1	75	5625
80	4	320	25 600
Sum	$\Sigma f = 200$	$\Sigma fx = 11\,060$	$\Sigma fx^2 = 626\,350$

Combined mean mass

$= \dfrac{11\,060}{200}$

$= 55.3$ kg

Combined standard deviation

$= \sqrt{\dfrac{626\,350}{200} - 55.3^2}$

$= 8.58$ kg (to 3 s.f.)

OXFORD UNIVERSITY PRESS

Review Exercise 4

1. **(a)** **(i)** Number of students who take less than 17.5 minutes to travel to school

= 200

(ii) Number of students who take at least 27 minutes to travel to school

= 750 – 600

= 150

$\therefore \frac{150}{750} = \frac{1}{5}$ of the 750 students take at least 27 minutes to travel to school.

(iii) 40% of the 750 students means $\frac{40}{100} \times 750 = 300$, i.e. 300 students take at least x minutes to travel to school.

$\therefore$ 750 – 300 = 450 students take less than x minutes to travel to school.

From the curve, $x = 23$.

(b) 90% of the 750 students means $\frac{90}{100} \times 750 = 675$, i.e. 675 students take less than 31.5 minutes to travel to school.

$\therefore$ 90th percentile = 31.5 minutes

2.

$$5, 9, 9, \underset{\substack{\uparrow\\ Q_1}}{10}, 12, 13, 15, \underset{\substack{\uparrow\\ Q_2}}{\,} 19, 20, 20, \underset{\substack{\uparrow\\ Q_3}}{23}, 30, 84, 120$$

For the given data, $n = 14$.

$\therefore Q_2 = \frac{15 + 19}{2} = 17$

$Q_1 = 10$

$Q_3 = 23$

(i) Median = \$17

(ii) Interquartile range $= Q_3 - Q_1$

$= 23 - 10$

= \$13

(iii) Mean

$$= \frac{5 + 9 + 9 + 10 + 12 + 13 + 15 + 19 + 20 + 20 + 23 + 30 + 84 + 120}{14}$$

= \$27.79 (to the nearest cent)

$$\text{Standard deviation} = \sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2}$$

$$= \sqrt{\frac{5^2 + 9^2 + 9^2 + 10^2 + 12^2 + 13^2 + 15^2 + 19^2 + 20^2 + 20^2 + 23^2 + 30^2 + 84^2 + 120^2}{14} - 27.786^2}$$

= \$31.69 (to the nearest cent)

3.

Number of children (x)	**Frequency**	fx	fx^2
0	5	0	0
1	6	6	6
2	4	8	16
3	3	9	27
4	2	8	32
5	1	5	25
Sum	$\Sigma f = 21$	$\Sigma fx = 36$	$\Sigma fx^2 = 106$

$$\text{Mean, } \bar{x} = \frac{\Sigma fx}{\Sigma f}$$

$$= \frac{36}{21}$$

= 1.71 (to 3 s.f.)

(a) $\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \bar{x}^2}$

$$= \sqrt{\frac{106}{21} - \left(\frac{36}{21}\right)^2}$$

= 1.45 (to 3 s.f.)

(b) For the given data, $n = 21$.

(i) $x_1 = \frac{0 + 1}{2} = 0.5$

$x_2 = 1$

$x_3 = \frac{3 + 3}{2} = 3$

(ii) Interquartile range $= x_3 - x_1$

$= 3 - 0.5$

$= 2.5$

4. **(a)** For this set of data, $n = 124$.

$\therefore \frac{n}{2} = 62, \frac{n}{4} = 31$ and $\frac{3n}{4} = 93$

(i) Median length = 44 mm

(ii) Lower quartile = 37 mm

Upper quartile = 50 mm

(b) Interquartile range = 50 – 37

= 13 mm

(c) **(i)** Number of ears of barley with lengths greater than 55 mm

= 124 – 109

= 15

(ii) Number of ears of barley with lengths not greater than 25 mm or greater than 64 mm

= 4 + 2

= 6

OXFORD UNIVERSITY PRESS

5. **(a)** For Vishal's shots,

x	x^2
47	2209
16	256
32	1024
1	1
19	361
35	1225
$\Sigma x = 150$	$\Sigma x^2 = 5076$

(i) Mean distance, $\bar{x} = \dfrac{\Sigma x}{n}$

$$= \frac{150}{6}$$

$$= 25 \text{ mm}$$

(ii) Standard deviation $= \sqrt{\dfrac{\Sigma x^2}{n} - \bar{x}^2}$

$$= \sqrt{\frac{5076}{6} - 25^2}$$

$$= 14.9 \text{ mm (to 3 s.f.)}$$

For Jun Wei's shots,

x	x^2
20	400
9	81
16	256
43	1849
13	169
4	16
$\Sigma x = 105$	$\Sigma x^2 = 2771$

(i) Mean distance, $\bar{x} = \dfrac{\Sigma x}{n}$

$$= \frac{105}{6}$$

$$= 17.5 \text{ mm}$$

(ii) Standard deviation $= \sqrt{\dfrac{\Sigma x^2}{n} - \bar{x}^2}$

$$= \sqrt{\frac{2771}{6} - 17.5^2}$$

$$= 12.5 \text{ mm (to 3 s.f.)}$$

(b) Vishal's shots are less accurate than Jun Wei's shots since the mean distance of Vishal's shots from the centre of the target is higher. Also, Vishal's shots have a higher standard deviation which indicates the greater spread of the distance of each shot from the centre of the target.

6. For this set of data, $n = 160$.

$\therefore\ \dfrac{n}{2} = 80,\ \dfrac{n}{4} = 40$ and $\dfrac{3n}{4} = 120$

(a) **(i)** From the graph, median mark of School $A = 48$.

(ii) From the graph, lower quartile $= 42$,

upper quartile $= 55$.

$\therefore$ Interquartile range of School $A = 55 - 42$

$= 13$

(b) **(i)** From the graph, median mark of School $B = 57$.

(ii) From the graph, lower quartile $= 39$,

upper quartile $= 72$.

$\therefore$ Interquartile range of School $B = 72 - 39$

$= 33$

(c) Number of students from School B who scored more than 80 marks

$= 160 - 140$

$= 20$

$\therefore\ \dfrac{20}{160} \times 100 = 12.5\%$ of the 160 students from School B scored more than 80 marks.

(d) Students from School B scored better on the whole than students from School A for the same examination as the median mark of School B is higher than that of School A. The interquartile range of School B is higher which indicates a greater spread of the scores of the students from School B.

7.

For Brightworks,

Lifespans (hours)	**Frequency**	**Mid-value (x)**	fx	fx^2
$600 \leqslant t < 700$	2	650	1300	845 000
$700 \leqslant t < 800$	9	750	6750	5 062 500
$800 \leqslant t < 900$	16	850	12 750	10 837 500
$900 \leqslant t < 1000$	21	950	19 950	18 952 500
$1000 \leqslant t < 1100$	29	1050	15750	16 537 500
$1100 \leqslant t < 1200$	18	1150	20 700	23 805 000
$1200 \leqslant t < 1300$	5	1250	6250	7 812 500
Sum	$\Sigma f = 100$		Σfx $= 99\,000$	Σfx^2 $= 100\,010\,000$

(i) Mean, $\bar{x} = \dfrac{\Sigma fx}{\Sigma f}$

$$= \frac{99\,000}{100}$$

$$= 990 \text{ hours}$$

$\therefore\ p = 990$

OXFORD UNIVERSITY PRESS

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \overline{x}^2}$$

$$= \sqrt{\frac{100\,010\,000}{100} - 990^2}$$

$$= 141 \text{ hours (to 3 s.f.)}$$

$\therefore q = 150$

$r = 100 - 8 - 10 - 12 - 16 - 18 - 12$

$= 24$

For Luminia,

Lifespans (hours)	**Frequency**	**Mid-value (x)**	***fx***	**fx^2**
$600 \leqslant t < 700$	8	650	5200	3 380 000
$700 \leqslant t < 800$	10	750	7500	5 625 000
$800 \leqslant t < 900$	12	850	10 200	8 670 000
$900 \leqslant t < 1000$	16	950	15 200	14 400 000
$1000 \leqslant t < 1100$	24	1050	25 200	26 460 000
$1100 \leqslant t < 1200$	18	1150	20 700	23 805 000
$1200 \leqslant t < 1300$	12	1250	15 000	18 750 000
Sum	$\Sigma f = 100$		Σfx $= 99\,000$	Σfx^2 $= 101\,130\,000$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \overline{x}^2}$$

$$= \sqrt{\frac{101\,130\,000}{100} - 989.5^2}$$

$$= 179 \text{ hours (to 3 s.f.)}$$

$\therefore t = 179$

(ii) The light bulbs produced by both companies have the same amount of lifespans in general as the median lifespans are almost the same. However, the light bulbs produced by Lumina have a higher standard deviation which indicates that there is a greater spread of the lifespans of their light bulbs. Hence Lumina is less consistent than Brightworks in producing light bulbs with the same lifespan.

8. (i) For University *A*,

interquartile range $= 3520 - 2760$

$= \$760$

For University *B*,

interquartile range $= 3160 - 2640$

$= \$520$

(ii) Agree. The median starting monthly salary of the fresh graduates from University *A* is higher than that of University *B*.

(iii) University *A* since 25% of its students have a starting monthly salary of more than \$3520 whereas 25% of students from University *B* have a starting monthly salary of more than \$3160.

9. For this set of data, $n = 600$.

$\therefore \frac{n}{2} = 300$, $\frac{n}{4} = 150$ and $\frac{3n}{4} = 450$

(a) (i) From the graph, median mark = 49.

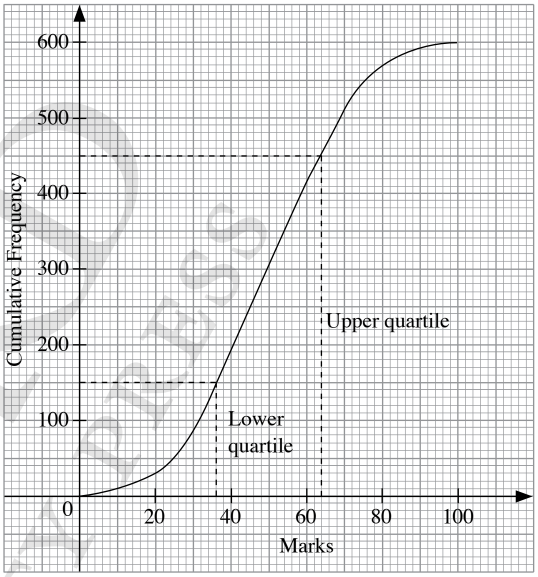
(ii) 60% of the students $= \frac{60}{100} \times 600 = 360$, i.e. 360 students passed the examination.

$\therefore 600 - 360 = 240$ students failed the test.

From the graph, the passing mark is 44.

(b)

Cumulative Frequency Curve
for Marks Scored by 600 Pupils



Interquartile range $= 64 - 36$

$= 28$

(c) Median mark = 68

Interquartile range $= 82 - 58$

$= 24$

The performance of the students in Fermat High School is better than that of Euler High School since the median mark of Fermat High School is higher. Moreover, the performance of the students in Fermat High School is more consistent than that of Euler High School since the standard deviation of Fermat High School is lower.

10. (a)

Distance (cm)	**Frequency**
$180 \leqslant x < 200$	18
$200 \leqslant x < 220$	42
$220 \leqslant x < 240$	15
$240 \leqslant x < 260$	4
$260 \leqslant x < 280$	1

OXFORD UNIVERSITY PRESS

(b)

Distance (cm)	Frequency	Mid-value (x)	fx	fx^2
$180 \leqslant x < 200$	18	190	3420	649 800
$200 \leqslant x < 220$	42	210	8820	1 852 200
$220 \leqslant x < 240$	15	230	3450	793 500
$240 \leqslant x < 260$	4	250	1000	250 000
$260 \leqslant x < 280$	1	270	270	72 900
Sum	$\Sigma f = 80$		Σfx $= 16\,960$	Σfx^2 $= 3\,618\,400$

(i) Mean, $\bar{x} = \dfrac{\Sigma fx}{\Sigma f}$

$$= \frac{16\,960}{80}$$

$$= 212 \text{ cm}$$

(ii) Standard deviation $= \sqrt{\dfrac{\Sigma fx^2}{\Sigma f} - \bar{x}^2}$

$$= \sqrt{\frac{3\,618\,400}{80} - 212^2}$$

$$= 16.9 \text{ cm (to 3 s.f.)}$$

(c) Both cumulative frequency curves will intersect at the point that correspond to the mean standing broad jump distance. The new curve will have a gentler gradient on the whole than the given curve.

11. For this set of data, $n = 200$.

$\therefore\ \dfrac{n}{2} = 100,\ \dfrac{n}{4} = 50$ and $\dfrac{3n}{4} = 150$

(a) **(i)** Median mass = 54.5 kg

(ii) Interquartile range = 59 – 51.5
= 7.5 kg

(iii) Number of Grade 1 eggs
= 200 – 165
= 35

$\therefore\ \dfrac{35}{200} \times 100 = 17.5\%$ of the 200 eggs are Grade 1.

Number of Grade 2 eggs
= 165 – 45
= 120

$\therefore\ \dfrac{120}{200} \times 100 = 60\%$ of the 200 eggs are Grade 2.

Number of Grade 3 eggs
= 45

$\therefore\ \dfrac{45}{200} \times 100 = 22.5\%$ of the 200 eggs are Grade 3.

(b) **(i)** Median mass = 50 kg
Interquartile range = 52 – 45
= 7 kg

(ii) The quality of the eggs from Rainbow Farm is better than that of Skyhi Farm since Rainbow Farm has a higher percentage of Grade 1 eggs. Also, the eggs from Rainbow Farm are heavier, on average, as the median mass of the eggs from Rainbow Farm is higher as compared to the median mass of the eggs from Skyhi Farm.

Challenge Yourself

1. Sets W and X since the spread between the data in each of these sets is the same as Set A.

2. $M = \{-2, -1, 0, 1, 2\}$
$N = \left\{-\sqrt{5}, 0, 0, 0, \sqrt{5}\right\}$
$n = 5$
Mean = 0
Standard deviation = $\sqrt{2}$

OXFORD UNIVERSITY PRESS

Chapter 5 Matrices

TEACHING NOTES

Suggested Approach

Teachers may want to start this chapter by engaging the class to discuss the chapter opener on the use of matrix code to simulate reality and how humans live in this simulated environment known as the Matrix, as seen in the movie. Following that, teachers can describe to students that in Mathematics, a matrix is a rectangular array of numbers which can be used to represent information in the real world.

Section 5.1: Introduction to Matrices

Introduce the idea of matrices by engaging the students to discuss the Class Discussion activity on page 137. This short activity will give them a brief idea on how information can be represented using a matrix. In this section, teachers should spend time reinforcing the meaning of the order of a matrix. Students need to be clear with the order of the matrix before they can proceed to learn the addition, subtraction and multiplication of matrices.

Section 5.2: Addition and Subtraction of Matrices

In this section, both Class Discussion activities on page 143 and 144 enable the students to understand how the addition and subtraction of matrices are applied in our daily life. Teachers should emphasise that for both operations to work, all the matrices involved must be of the same order. To conclude this section, teachers can get students to discuss whether the matrix addition/subtraction is commutative or associative. It would be good if the students can use examples to justify the results.

Section 5.3: Matrix Multiplication

Teachers can go through the Class Discussion activity on page 148 to allow students to have an idea on how multiplication of a matrix by a scalar is applied to our daily life. For this operation, the order of a matrix is not important. For multiplication of a matrix by another matrix, teachers need to inform the students that it is not a direct extension of multiplication unlike matrix addition and subtraction. Using the Class Discussion activity on page 151, teachers can guide the students to understand that for this operation to be possible, the number of columns of the first matrix must be equal to the number of rows of the second matrix and the order of the resultant matrix will be the number of rows of the first matrix by the number of columns of the second matrix. More time should be allocated for the students to master this concept. To conclude this section, teachers should engage the class to discuss the Thinking Time activity on page 156. This activity will help to reinforce the concept of the multiplication of a matrix by another matrix. Students should be encouraged to give examples to justify their result.

Section 5.4: Determinant of a Matrix

In teaching this section, teachers should emphasise to students to always bear in mind the formula of the determinant of a matrix, i.e. ($ad - bc$), and not make careless mistakes such as calculating the determinant using ($ad + bc$) or ($ab - dc$) etc.

OXFORD UNIVERSITY PRESS

Section 5.5: Inverse of a Matrix

To illustrate the idea of the inverse of a matrix, teachers can explain to students the significance of the number "1" as an identity in multiplication. For instance: $5 \times 1 = 5$, $3 \times 1 = 3$, etc. Thus, in the case of a matrix, if $\begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}$, then $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is the identity for a 2×2 matrix. Teachers can go through the Class Discussion on Interesting Properties of Matrices to show that solving simultaneously, the identity matrix $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ can be derived.

Teachers can then show that with regards to $5 \times \frac{1}{5} = 1$, $3 \times \frac{1}{3} = 1$, etc., $\frac{1}{5}$ is the inverse of 5 and $\frac{1}{3}$ is the inverse of 3, and vice versa. Hence, to find the inverse of $\begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}$, students should see that the equation $\begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ can be utilised. Teachers may then introduce the general formula for finding the 2×2 inverse matrix, after students have understood the concept of the inverse matrix.

Section 5.6: Applications of Matrices

Students are generally weak in the explanation on what the elements in a resultant matrix represent after the operations of multiplication. Teachers should give time for the students to discuss on how such questions should be answered in the context of the questions. If time permits, teachers should allocate time for the students to try the Investigation activity on page 169. This activity will allow the students to understand how matrices can be used to encode and decode messages.

Challenge Yourself

For question 1, teachers can guide the students to use the concept of solving algebraic equations to solve for the unknown matrix. However, for question 2, students should be reminded that there is no division of matrices, hence guide the students to use $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ to represent the unknown matrix.

Question 5 is a follow-up from the chapter opener. Students will need to make use all the concepts learnt in this chapter to use the correct matrix operations on **X** so that the different changes can take place.

OXFORD UNIVERSITY PRESS

WORKED SOLUTIONS

Class Discussion (Defining a Matrix)

1. The above matrix has 2 rows and 3 columns.
2. They represent the number of pens of each of the three brands in Shop 2.
3. The elements in the second column represent the number of pens of Brand *B* sold in each of the two shops. Similarly, the elements in the third column represent the number of pens of Brand *C* sold in each of the two shops.
4. $\begin{pmatrix} 16 & 58 \\ 7 & 64 \\ 69 & 76 \end{pmatrix}$

Thinking Time (Page 139)

1. $\begin{pmatrix} 1 & 6 & 7 \\ 2 & 5 & 8 \\ 3 & 4 & 9 \end{pmatrix}$
2. $(0 \quad 0)$ and $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$
3. No, (0) is a zero matrix while 0 is an element.

Thinking Time (Page 139)

1. No, $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ is a 2 × 1 matrix and $\begin{pmatrix} 2 & 0 \\ 3 & 0 \end{pmatrix}$ is a 2 × 2 matrix.
2. No, $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ is a 2 × 2 zero matrix and $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ is a 3 × 3 zero matrix.
3. No, two of the corresponding elements in $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $\begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$ are not equal.

Class Discussion (Addition of Matrices)

1. $$\begin{aligned} \mathbf{M} + \mathbf{N} &= \begin{pmatrix} 16 & 7 & 69 \\ 58 & 64 & 76 \end{pmatrix} + \begin{pmatrix} 70 & 80 & 50 \\ 30 & 20 & 50 \end{pmatrix} \\ &= \begin{pmatrix} 16 + 70 & 7 + 80 & 69 + \underline{50} \\ 58 + \underline{30} & \underline{64 + 20} & \underline{76 + 50} \end{pmatrix} \\ &= \begin{pmatrix} 86 & \underline{87} & \underline{119} \\ \underline{88} & \underline{84} & \underline{126} \end{pmatrix} \end{aligned}$$
2. No, there are some elements in the first matrix that do not have corresponding elements in the second matrix to add.
3. Yes, since all elements in one matrix can be added to their corresponding elements in the other matrix.

Class Discussion (Subtraction of Matrices)

1. $$\begin{aligned} \mathbf{X} - \mathbf{Y} &= \begin{pmatrix} 86 & 87 & 119 \\ 88 & 84 & 126 \end{pmatrix} - \begin{pmatrix} 30 & 24 & 98 \\ 61 & 67 & 117 \end{pmatrix} \\ &= \begin{pmatrix} 86 - 30 & 87 - 24 & 119 - \underline{98} \\ 88 - \underline{61} & \underline{84 - 67} & \underline{126 - 117} \end{pmatrix} \\ &= \begin{pmatrix} 56 & \underline{63} & \underline{21} \\ \underline{27} & \underline{17} & \underline{9} \end{pmatrix} \end{aligned}$$
2. No

Thinking Time (Page 146)

1. Yes
2. Yes
3. No
4. No

Class Discussion (Multiplying a Matrix by a Scalar)

1. $$\begin{aligned} 2\mathbf{P} &= 2\begin{pmatrix} 56 & 63 & 21 \\ 27 & 17 & 9 \end{pmatrix} \\ &= \begin{pmatrix} 2 \times 56 & 2 \times 63 & 2 \times \underline{21} \\ 2 \times \underline{27} & \underline{2 \times 17} & \underline{2 \times 9} \end{pmatrix} \\ &= \begin{pmatrix} 112 & \underline{126} & \underline{42} \\ \underline{54} & \underline{34} & \underline{18} \end{pmatrix} \end{aligned}$$
2. No

Class Discussion (Multiplication of a Matrix with another Matrix)

1. $$\begin{aligned} \begin{pmatrix} 56 & 63 & 21 \\ 27 & 17 & 9 \end{pmatrix}\begin{pmatrix} 1.5 \\ 2 \\ 1.8 \end{pmatrix} &= \begin{pmatrix} (56 \times 1.5) + (63 \times 2) + (21 \times \underline{1.8}) \\ (27 \times \underline{1.5}) + (\underline{17} \times \underline{2}) + (\underline{9} \times \underline{1.8}) \end{pmatrix} \\ &= \begin{pmatrix} \underline{247.8} \\ \underline{90.7} \end{pmatrix} \end{aligned}$$
2. The number of rows of matrix **R** is the same as the number of rows of matrix **P** and the number of columns of matrix **R** is the same as the number of columns of matrix **Q**.
3. The condition is that the number of columns of the first matrix must be equal to the number of rows of the second matrix.

Thinking Time (Page 152)

(a) Yes, since the number of columns of the first matrix is equal to the number of rows of the second matrix, which is 3, and the order is 2 × 2.

(b) Yes, since the number of columns of the first matrix is equal to the number of rows of the second matrix, which is 2, and the order is 3 × 3.

(c) Yes, since the number of columns of the first matrix is equal to the number of rows of the second matrix, which is 2, and the order is 1 × 2 .

OXFORD UNIVERSITY PRESS

(d) No, since the number of columns of the first matrix, which is 2, is not equal to the number of rows of the second matrix, which is 1.

(e) Yes, since the number of columns of the first matrix is equal to the number of rows of the second matrix, which is 1, and the order is 3×3.

(f) Yes, since the number of columns of the first matrix is equal to the number of rows of the second matrix, which is 3, and the order is 1×1.

Thinking Time (Page 156)

No, **AB** is not equal to **BA**.

For example, $\mathbf{A} = (2 \quad 1)$ and $\mathbf{B} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$.

Then matrix **AB** is of order 1×1 while matrix **BA** is of order 2×2.

Thinking Time (Page 160)

1 divided by 0 will give infinity, which is undefined.

Class Discussion (Interesting Properties of Matrices)

1. $$\mathbf{AB} = \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 2 & 0 \end{pmatrix}$$
$$= \begin{pmatrix} \underline{0} & \underline{0} \\ \underline{0} & \underline{0} \end{pmatrix}$$

However, $\mathbf{A} \neq 0$ and $\mathbf{B} \neq 0$.

2. $$\mathbf{PQ} = \begin{pmatrix} 2 & 0 \\ 3 & 0 \end{pmatrix}\begin{pmatrix} 2 & 3 \\ 0 & 0 \end{pmatrix}$$
$$= \begin{pmatrix} \underline{4} & \underline{6} \\ \underline{6} & \underline{9} \end{pmatrix}$$
$$\mathbf{PR} = \begin{pmatrix} 2 & 0 \\ 3 & 0 \end{pmatrix}\begin{pmatrix} 2 & 3 \\ 0 & 3 \end{pmatrix}$$
$$= \begin{pmatrix} \underline{4} & \underline{6} \\ \underline{6} & \underline{9} \end{pmatrix}$$

However, $\mathbf{P} \neq 0$ and $\mathbf{Q} \neq \mathbf{R}$.

3. Pick any 2×2 matrix such as $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and let the identity matrix be $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$. Work out the values of a, b, c and d in the multiplication $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$.

4. Use a general matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ where $ad - bc \neq 0$ and follow the steps outlined on pages 159 to 160. The proof can then be obtained.

Investigation (Encoding and Decoding Messages)

Part A:

1. EDVH PHQW

2. Letter H

Yes, it corresponds to the letter E in the original message.

Part B:

3. Letter A

4. No, letter E and C.

5. $$\begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix}\begin{pmatrix} 18 & 15 \\ 15 & 13 \end{pmatrix} = \begin{pmatrix} 69 & 58 \\ 120 & 101 \end{pmatrix} \rightarrow \begin{pmatrix} 17 & 6 \\ 16 & 23 \end{pmatrix} \rightarrow \begin{pmatrix} \text{Q} & \text{F} \\ \text{P} & \text{W} \end{pmatrix}$$

So, ROOM is encoded as QPFW.

$$\begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix}\begin{pmatrix} 9 \\ 14 \end{pmatrix} = \begin{pmatrix} 41 \\ 73 \end{pmatrix} \rightarrow \begin{pmatrix} 15 \\ 21 \end{pmatrix} \rightarrow \begin{pmatrix} \text{O} \\ \text{U} \end{pmatrix}$$

So, IN is encoded as OU.

$$\begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix}\begin{pmatrix} 2 & 19 & 13 & 14 \\ 1 & 5 & 5 & 20 \end{pmatrix} = \begin{pmatrix} 7 & 62 & 44 & 62 \\ 12 & 105 & 75 & 110 \end{pmatrix}$$
$$\rightarrow \begin{pmatrix} 7 & 10 & 18 & 10 \\ 12 & 1 & 23 & 20 \end{pmatrix} \rightarrow \begin{pmatrix} \text{G} & \text{J} & \text{R} & \text{J} \\ \text{L} & \text{A} & \text{W} & \text{F} \end{pmatrix}$$

So, BASEMENT is encoded as GLJARWJF.

The entire encoded message is JAAY IMQP FWOU GLJA RWJF.

Part C:

6. $$\begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix}\begin{pmatrix} 10 & 7 \\ 13 & 6 \end{pmatrix} = \begin{pmatrix} 7 & 58 \\ -11 & -17 \end{pmatrix} \rightarrow \begin{pmatrix} 7 & 8 \\ 15 & 9 \end{pmatrix} \rightarrow \begin{pmatrix} \text{G} & \text{H} \\ \text{O} & \text{I} \end{pmatrix}$$

So, JMGF is decoded as GOHI.

$$\begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix}\begin{pmatrix} 17 & 15 \\ 4 & 21 \end{pmatrix} = \begin{pmatrix} 30 & 9 \\ -73 & -12 \end{pmatrix} \rightarrow \begin{pmatrix} 4 & 9 \\ 5 & 14 \end{pmatrix} \rightarrow \begin{pmatrix} \text{D} & \text{I} \\ \text{E} & \text{N} \end{pmatrix}$$

So, QDOU is decoded as DEIN.

$$\begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix}\begin{pmatrix} 17 & 6 \\ 16 & 23 \end{pmatrix} = \begin{pmatrix} 18 & -11 \\ -37 & 39 \end{pmatrix} \rightarrow \begin{pmatrix} 18 & 15 \\ 15 & 13 \end{pmatrix} \rightarrow \begin{pmatrix} \text{R} & \text{O} \\ \text{O} & \text{M} \end{pmatrix}$$

So, QPFW is decoded as ROOM.

The above message is GO HIDE IN ROOM.

Journal Writing (Page 171)

1. $$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 5 \\ 4 \end{pmatrix} = \begin{pmatrix} -4 \\ 5 \end{pmatrix}$$

The image of $Q(5, 4)$ is $(-4, 5)$.

2. **(i)** Under 90° clockwise rotation about O, $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ will be transformed to $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$.

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$$

The matrix is $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$.

(ii) Under 180° rotation about O, $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ will be transformed to $\begin{pmatrix} -2 \\ -3 \end{pmatrix}$.

$$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} -2 \\ -3 \end{pmatrix}$$

The matrix is $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$.

OXFORD UNIVERSITY PRESS

(iii) Under reflection in the x-axis, $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ will be transformed to $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$.

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$$

The matrix is $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.

(iv) Under reflection in the y-axis, $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ will be transformed to $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$.

$$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$$

The matrix is $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$.

3. **(i)** $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

(ii) $\begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix}$

(iii) $\begin{pmatrix} 1-2a^2 & -2ab & -2ac \\ -2ab & 1-2b^2 & -2bc \\ -2ac & -2bc & 1-2c^2 \end{pmatrix}$

A transformation matrix defines how to map points from one coordinate space into another coordinate space. By modifying the contents of a transformation matrix, it can perform several standard graphical display operations, including translation, rotation, and scaling.

A matrix that describes a translation operation is $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ t_x & t_y & 1 \end{pmatrix}$.

This operation modifies the x and y coordinates of each point by a specified amount.

A matrix that describes a scaling operation is $\begin{pmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{pmatrix}$. This matrix can stretch or shrink an image by performing a scaling operation. This operation modifies the x and y coordinates by some factor. The magnitude of the x and y factors will result in the new image being either larger or smaller than the original.

A matrix that rotates an image counterclockwise by an angle q is $\begin{pmatrix} \cos q & \sin q & 0 \\ -\sin q & \cos q & 0 \\ 0 & 0 & 1 \end{pmatrix}$. It can rotate an image by a specified angle by performing a rotation operation. You can specify the magnitude and direction of the rotation by specifying factors for both x and y.

Practise Now (Page 138)

1. **(a)** 2 × 4

(b) 2 × 2

(c) 3 × 1

(d) 1 × 4

(e) 1 × 1

(f) 1 × 1

2. **(i)** $\mathbf{M} = \begin{pmatrix} 14 & 3 & 5 & 2 \\ 1 & 8 & 3 & 4 \end{pmatrix}$

(ii) 5 boys like swimming best.

(iii) The sum is 15 and this represents the number of students who like soccer.

(iv) The sum of the elements in the second row of **M** will represent the number of girls in the class and the answer is 16.

Practise Now 1

$$\mathbf{X} = \mathbf{Y}$$

$$\begin{pmatrix} 8a & 5 \\ c & -9 \end{pmatrix} = \begin{pmatrix} 16 & a+b \\ d+3 & 3d \end{pmatrix}$$

Equating the corresponding elements, we have

$8a = 16$ — (1)

$5 = a + b$ — (2)

$c = d + 3$ — (3)

$-9 = 3d$ — (4)

From (1),

$a = \dfrac{16}{8}$

$= 2$

Substitute $a = 2$ into (2),

$5 = 2 + b$

$b = 3$

From (4),

$d = \dfrac{-9}{3}$

$= -3$

Substitute $d = -3$ into (3),

$c = -3 + 3$

$= 0$

$\therefore a = 2, b = 3, c = 0, d = -3$

Practise Now 2

1. **(a)** $\mathbf{P} + \mathbf{Q} = \begin{pmatrix} 5 & 8 \\ 12 & 6 \end{pmatrix} + \begin{pmatrix} 0 & -3 \\ -4 & 7 \end{pmatrix}$

$= \begin{pmatrix} 5 & 5 \\ 8 & 13 \end{pmatrix}$

OXFORD UNIVERSITY PRESS

(b) $$\mathbf{P}-\mathbf{Q}+\mathbf{R} = \left[\begin{pmatrix} 5 & 8 \\ 12 & 6 \end{pmatrix} - \begin{pmatrix} 0 & -3 \\ -4 & 7 \end{pmatrix}\right] + \begin{pmatrix} -7 & 0 \\ 6 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 5 & 11 \\ 16 & -1 \end{pmatrix} + \begin{pmatrix} -7 & 0 \\ 6 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} -2 & 11 \\ 22 & -1 \end{pmatrix}$$

(c) **R** – **S** is not possible because **R** and **S** have different orders.

2. **(i)** $$\mathbf{Q} = \begin{pmatrix} 46 & 40 & 31 \\ 42 & 38 & 35 \end{pmatrix}$$

(ii) $$\mathbf{P}+\mathbf{Q} = \begin{pmatrix} 49 & 28 & 39 \\ 47 & 45 & 21 \end{pmatrix} + \begin{pmatrix} 46 & 40 & 31 \\ 42 & 38 & 35 \end{pmatrix}$$

$$= \begin{pmatrix} 95 & 68 & 70 \\ 89 & 83 & 56 \end{pmatrix}$$

(iii) They represent the total marks scored for the two Mathematics and Science tests by Raj, Ethan and Farhan.

Practise Now 3

1. **(i)** $$3\mathbf{A}+2\mathbf{B} = 3\begin{pmatrix} 2 & -1 \\ -3 & 6 \\ 5 & 8 \end{pmatrix} + 2\begin{pmatrix} 4 & 7 \\ 0 & -9 \\ -10 & 11 \end{pmatrix}$$

$$= \begin{pmatrix} 6 & -3 \\ -9 & 18 \\ 15 & 24 \end{pmatrix} + \begin{pmatrix} 8 & 14 \\ 0 & -18 \\ -20 & 22 \end{pmatrix}$$

$$= \begin{pmatrix} 14 & 11 \\ -9 & 0 \\ -5 & 46 \end{pmatrix}$$

(ii) $$4\mathbf{B}-3\mathbf{A} = 4\begin{pmatrix} 4 & 7 \\ 0 & -9 \\ -10 & 11 \end{pmatrix} - 3\begin{pmatrix} 2 & -1 \\ -3 & 6 \\ 5 & 8 \end{pmatrix}$$

$$= \begin{pmatrix} 16 & 28 \\ 0 & -36 \\ -40 & 44 \end{pmatrix} - \begin{pmatrix} 6 & -3 \\ -9 & 18 \\ 15 & 24 \end{pmatrix}$$

$$= \begin{pmatrix} 10 & 31 \\ 9 & -54 \\ -55 & 20 \end{pmatrix}$$

2. **(a)** $$-2\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 4 \\ 2 \end{pmatrix} = 3\begin{pmatrix} 6 \\ -2 \end{pmatrix}$$

$$\begin{pmatrix} -2x \\ -2y \end{pmatrix} + \begin{pmatrix} 4 \\ 2 \end{pmatrix} = \begin{pmatrix} 18 \\ -6 \end{pmatrix}$$

$$\begin{pmatrix} -2x+4 \\ -2y+2 \end{pmatrix} = \begin{pmatrix} 18 \\ -6 \end{pmatrix}$$

Equating the corresponding elements, we have

$$\begin{aligned} -2x + 4 &= 18 & \text{and} \quad -2y + 2 &= -6 \\ -2x &= 18 - 4 & -2y &= -6 - 2 \\ &= 14 & &= -8 \\ x &= -7 & y &= 4 \end{aligned}$$

(b) $$2\begin{pmatrix} 3 & x \\ 0 & y \end{pmatrix} + \begin{pmatrix} 0 & 4 \\ 6 & -2 \end{pmatrix} = 3\begin{pmatrix} 2 & 5 \\ 2 & 14 \end{pmatrix}$$

$$\begin{pmatrix} 6 & 2x \\ 0 & 2y \end{pmatrix} + \begin{pmatrix} 0 & 4 \\ 6 & -2 \end{pmatrix} = \begin{pmatrix} 6 & 15 \\ 6 & 42 \end{pmatrix}$$

$$\begin{pmatrix} 6 & 2x+4 \\ 6 & 2y-2 \end{pmatrix} = \begin{pmatrix} 6 & 15 \\ 6 & 42 \end{pmatrix}$$

Equating the corresponding elements, we have

$$\begin{aligned} 2x + 4 &= 15 & \text{and} \quad 2y - 2 &= 42 \\ 2x &= 15 - 4 & 2y &= 42 + 2 \\ &= 11 & &= 44 \\ x &= 5.5 & y &= 22 \end{aligned}$$

Practise Now 4

(i) $$5\mathbf{D} = 5\begin{pmatrix} 15 & 25 \\ 21 & 8 \end{pmatrix}$$

$$= \begin{pmatrix} 75 & 125 \\ 105 & 40 \end{pmatrix}$$

(ii) $$\mathbf{E} = \begin{pmatrix} 14 & 10 \\ 18 & 7 \end{pmatrix}$$

(iii) $$5\mathbf{D}+\mathbf{E} = 5\begin{pmatrix} 15 & 25 \\ 21 & 8 \end{pmatrix} + \begin{pmatrix} 14 & 10 \\ 18 & 7 \end{pmatrix}$$

$$= \begin{pmatrix} 75 & 125 \\ 105 & 40 \end{pmatrix} + \begin{pmatrix} 14 & 10 \\ 18 & 7 \end{pmatrix}$$

$$= \begin{pmatrix} 89 & 135 \\ 123 & 47 \end{pmatrix}$$

It represents the total number of adults and children carried by the bus from Monday to Saturday in the mornings and afternoons.

Practise Now 5

(a) $$\begin{pmatrix} 2 & -3 & 5 \\ -7 & 0 & 8 \end{pmatrix}\begin{pmatrix} 4 & -9 \\ -5 & 10 \\ 21 & 6 \end{pmatrix}$$

$$= \begin{pmatrix} 2\times 4+(-3)\times(-5)+5\times 21 & 2\times(-9)+(-3)\times 10+5\times 6 \\ (-7)\times 4+0\times(-5)+8\times 21 & (-7)\times(-9)+0\times 10+8\times 6 \end{pmatrix}$$

$$= \begin{pmatrix} 128 & -18 \\ 140 & 111 \end{pmatrix}$$

(b) $$\begin{pmatrix} 4 & -9 \\ -5 & 10 \\ 21 & 6 \end{pmatrix}\begin{pmatrix} 2 & -3 & 5 \\ -7 & 0 & 8 \end{pmatrix}$$

$$= \begin{pmatrix} 4\times 2+(-9)\times(-7) & 4\times(-3)+(-9)\times 0 & 4\times 5+(-9)\times 8 \\ (-5)\times 2+10\times(-7) & (-5)\times(-3)+10\times 0 & (-5)\times 5+10\times 8 \\ 21\times 2+6\times(-7) & 21\times(-3)+6\times 0 & 21\times 5+6\times 8 \end{pmatrix}$$

$$= \begin{pmatrix} 71 & -12 & -52 \\ -80 & 15 & 55 \\ 0 & -63 & 153 \end{pmatrix}$$

OXFORD UNIVERSITY PRESS

(c) $(2 \quad 3)\begin{pmatrix} 7 & 8 \\ -9 & 4 \end{pmatrix}$

$$\begin{aligned} &= (2 \times 7 + 3 \times (-9) \quad 2 \times 8 + 3 \times 4) \\ &= (-13 \quad 28) \end{aligned}$$

(d) Not possible

(e) $\begin{pmatrix} 7 \\ 8 \\ -5 \end{pmatrix}(-7 \quad 2 \quad 3)$

$$= \begin{pmatrix} 7 \times (-7) & 7 \times 2 & 7 \times 3 \\ 8 \times (-7) & 8 \times 2 & 8 \times 3 \\ -5 \times (-7) & -5 \times 2 & -5 \times 3 \end{pmatrix}$$

$$= \begin{pmatrix} -49 & 14 & 21 \\ -56 & 16 & 24 \\ 35 & -10 & -15 \end{pmatrix}$$

(f) $(-7 \quad 2 \quad 3)\begin{pmatrix} 7 \\ 8 \\ -5 \end{pmatrix}$

$$\begin{aligned} &= (-7 \times 7 + 2 \times 8 + 3 \times (-5)) \\ &= (-48) \end{aligned}$$

Practise Now (Page 158)

$$\mathbf{Q} = \begin{pmatrix} 2 & -1 \\ -2 & 4 \end{pmatrix}$$

$$\begin{aligned} |\mathbf{Q}| &= (2 \times 4) - [-1 \times (-2)] \\ &= 8 - 2 \\ &= 6 \end{aligned}$$

Practise Now 6

$$\mathbf{AB} = \begin{pmatrix} 2 & 1 \\ 13 & 7 \end{pmatrix}\begin{pmatrix} 7 & -1 \\ -13 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \mathbf{I}$$

$$\mathbf{BA} = \begin{pmatrix} 7 & -1 \\ -13 & 2 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 13 & 7 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \mathbf{I}$$

Since $\mathbf{AB} = \mathbf{BA} = \mathbf{I}$, $\mathbf{A}$ is the inverse of $\mathbf{B}$ and $\mathbf{B}$ is the inverse of $\mathbf{A}$ i.e. $\mathbf{A} = \mathbf{B}^{-1}$ and $\mathbf{B} = \mathbf{A}^{-1}$.

Practise Now 7

$$\mathbf{A} = \begin{pmatrix} 6 & 3 \\ 7 & 2 \end{pmatrix}$$

$$\begin{aligned} |\mathbf{A}| &= (6 \times 2) - (3 \times 7) \\ &= -9 \\ &\neq 0 \end{aligned}$$

$\therefore$ $\mathbf{A}^{-1}$ exists.

$$\begin{aligned} \mathbf{A}^{-1} &= \frac{1}{-9}\begin{pmatrix} 2 & -3 \\ -7 & 6 \end{pmatrix} \\ &= \begin{pmatrix} -\frac{2}{9} & -3 \\ \frac{7}{9} & 6 \end{pmatrix} \\ &= \begin{pmatrix} -\frac{2}{9} & \frac{1}{3} \\ \frac{7}{9} & -\frac{2}{3} \end{pmatrix} \end{aligned}$$

$$\mathbf{B} = \begin{pmatrix} 1 & 1 \\ -9 & -3 \end{pmatrix}$$

$$\begin{aligned} |\mathbf{B}| &= [1 \times (-3)] - [1 \times (-9)] \\ &= 6 \\ &\neq 0 \end{aligned}$$

$\therefore$ $\mathbf{B}^{-1}$ exists.

$$\begin{aligned} \mathbf{B}^{-1} &= \frac{1}{6}\begin{pmatrix} -3 & -1 \\ 9 & 1 \end{pmatrix} \\ &= \begin{pmatrix} \frac{-3}{6} & \frac{-1}{6} \\ \frac{9}{6} & \frac{1}{6} \end{pmatrix} \\ &= \begin{pmatrix} -\frac{1}{2} & -\frac{1}{6} \\ \frac{3}{2} & \frac{1}{6} \end{pmatrix} \end{aligned}$$

$$\mathbf{C} = \begin{pmatrix} 2 & 1 \\ 8 & 4 \end{pmatrix}$$

$$\begin{aligned} |\mathbf{C}| &= (2 \times 4) - (1 \times 8) \\ &= 0 \end{aligned}$$

$\therefore$ $\mathbf{C}^{-1}$ does not exist. Hence, $\mathbf{C}$ is a singular matrix.

Practise Now 8

$$\mathbf{A} = \begin{pmatrix} 2 & -2 \\ 7 & -3 \end{pmatrix}$$

$$\begin{aligned} |\mathbf{A}| &= [2 \times (-3)] - [(-2) \times 7] \\ &= 8 \\ &\neq 0 \end{aligned}$$

$\therefore$ $\mathbf{A}^{-1}$ exists.

$$\begin{aligned} \mathbf{A}^{-1} &= \frac{1}{8}\begin{pmatrix} -3 & 2 \\ -7 & 2 \end{pmatrix} \\ &= \begin{pmatrix} \frac{-3}{8} & \frac{2}{8} \\ \frac{-7}{8} & \frac{2}{8} \end{pmatrix} \\ &= \begin{pmatrix} -\frac{3}{8} & \frac{1}{4} \\ -\frac{7}{8} & \frac{1}{4} \end{pmatrix} \end{aligned}$$

OXFORD
UNIVERSITY PRESS

(i)
$$\begin{aligned}
\mathbf{AP} &= \mathbf{B}\\
\mathbf{A}^{-1}\mathbf{AP} &= \mathbf{A}^{-1}\mathbf{B}\\
(\mathbf{A}^{-1}\mathbf{A})\mathbf{P} &= \mathbf{A}^{-1}\mathbf{B}\\
\mathbf{IP} &= \mathbf{A}^{-1}\mathbf{B}\\
\mathbf{P} &= \begin{pmatrix} -\frac{3}{8} & \frac{1}{4} \\ -\frac{7}{8} & \frac{1}{4} \end{pmatrix}\begin{pmatrix} 6 & 5 \\ 3 & 1 \end{pmatrix}\\
&= \begin{pmatrix} -\frac{3}{2} & -\frac{13}{8} \\ -\frac{9}{2} & -\frac{33}{8} \end{pmatrix}
\end{aligned}$$

(ii)
$$\begin{aligned}
\mathbf{QA} &= \mathbf{B}\\
\mathbf{QAA}^{-1} &= \mathbf{BA}^{-1}\\
\mathbf{Q}(\mathbf{AA}^{-1}) &= \mathbf{BA}^{-1}\\
\mathbf{QI} &= \mathbf{BA}^{-1}\\
\mathbf{Q} &= \begin{pmatrix} 6 & 5 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} -\frac{3}{8} & \frac{1}{4} \\ -\frac{7}{8} & \frac{1}{4} \end{pmatrix}\\
&= \begin{pmatrix} -\frac{53}{8} & \frac{11}{4} \\ -2 & 1 \end{pmatrix}
\end{aligned}$$

Practise Now 9

1. **(i)** There are 20 questions in the test.

(ii)
$$\begin{aligned}
\mathbf{XY} &= \begin{pmatrix} 16 & 0 & 4 \\ 12 & 5 & 3 \end{pmatrix}\begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}\\
&= \begin{pmatrix} 16 \times 2 + 0 \times 0 + 4 \times (-1) \\ 12 \times 2 + 5 \times 0 + 3 \times (-1) \end{pmatrix}\\
&= \begin{pmatrix} 28 \\ 21 \end{pmatrix}
\end{aligned}$$

(iii) It represents the total marks awarded to Huixian and Lixin respectively.

2. **(i)**
$$\begin{pmatrix} 70 & 120 & 90 & 80 \\ 120 & 0 & 150 & 140 \\ 0 & 150 & 85 & 60 \\ 200 & 140 & 70 & 0 \\ 80 & 110 & 0 & 95 \end{pmatrix} \text{ and } \begin{pmatrix} 35 \\ 40 \\ 45 \\ 38 \end{pmatrix}$$

$$\begin{aligned}
&\begin{pmatrix} 70 & 120 & 90 & 80 \\ 120 & 0 & 150 & 140 \\ 0 & 150 & 85 & 60 \\ 200 & 140 & 70 & 0 \\ 80 & 110 & 0 & 95 \end{pmatrix}\begin{pmatrix} 35 \\ 40 \\ 45 \\ 38 \end{pmatrix}\\
&= \begin{pmatrix} 70 \times 35 + 120 \times 40 + 90 \times 45 + 80 \times 38 \\ 120 \times 35 + 0 \times 40 + 150 \times 45 + 140 \times 38 \\ 0 \times 35 + 150 \times 40 + 85 \times 45 + 60 \times 38 \\ 200 \times 35 + 140 \times 40 + 70 \times 45 + 0 \times 38 \\ 80 \times 35 + 110 \times 40 + 0 \times 45 + 95 \times 38 \end{pmatrix}\\
&= \begin{pmatrix} 14\,340 \\ 16\,270 \\ 12\,105 \\ 15\,750 \\ 10\,810 \end{pmatrix}
\end{aligned}$$

(ii)
$$\begin{pmatrix} 45 & 42 & 38 & 55 & 52 \end{pmatrix}\begin{pmatrix} 70 & 120 & 90 & 80 \\ 120 & 0 & 150 & 140 \\ 0 & 150 & 85 & 60 \\ 200 & 140 & 70 & 0 \\ 80 & 110 & 0 & 95 \end{pmatrix}$$
$$= \begin{pmatrix} 23\,350 & 24\,520 & 17\,430 & 16\,700 \end{pmatrix}$$

Total number of otahs supplied by the factory to these outlets = 82 000

(iii)
$$\begin{pmatrix} 45 & 42 & 38 & 55 & 52 \end{pmatrix}\begin{pmatrix} 14\,340 \\ 16\,270 \\ 12\,105 \\ 15\,750 \\ 10\,810 \end{pmatrix} = \begin{pmatrix} 3\,217\,000 \end{pmatrix}$$

or

$$\begin{pmatrix} 23\,350 & 24\,520 & 17\,430 & 16\,700 \end{pmatrix}\begin{pmatrix} 35 \\ 40 \\ 45 \\ 38 \end{pmatrix} = \begin{pmatrix} 3\,217\,000 \end{pmatrix}$$

Practise Now 10

(a) The simultaneous equations may be written in matrix form as

$$\begin{pmatrix} 4 & -1 \\ 6 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 18 \end{pmatrix}$$

$$\begin{aligned}
\text{Determinant of } \begin{pmatrix} 4 & -1 \\ 6 & 2 \end{pmatrix} &= (4 \times 2) - [(-1) \times 6]\\
&= 14
\end{aligned}$$

$$\begin{aligned}
\begin{pmatrix} x \\ y \end{pmatrix} &= \begin{pmatrix} 4 & -1 \\ 6 & 2 \end{pmatrix}^{-1}\begin{pmatrix} 5 \\ 18 \end{pmatrix}\\
&= \frac{1}{14}\begin{pmatrix} 2 & 1 \\ -6 & 4 \end{pmatrix}\begin{pmatrix} 5 \\ 18 \end{pmatrix}\\
&= \frac{1}{14}\begin{pmatrix} 28 \\ 42 \end{pmatrix}\\
&= \begin{pmatrix} 2 \\ 3 \end{pmatrix}
\end{aligned}$$

$\therefore x = 2, y = 3$

The graphs of $4x - y = 5$ and $6x + 2y = 18$ intersect at the point $(2, 3)$.

OXFORD UNIVERSITY PRESS

(b) The simultaneous equations may be written in matrix form as

$$\begin{pmatrix} 4 & 1 \\ 12 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 15 \\ 13 \end{pmatrix}$$

Determinant of $\begin{pmatrix} 4 & 1 \\ 12 & 3 \end{pmatrix} = (4 \times 3) - (1 \times 12)$

$= 0$

Hence $\begin{pmatrix} 4 & 1 \\ 12 & 3 \end{pmatrix}$ is a singular matrix and its inverse matrix does not exist. The graphs of $4x + y = 15$ and $12x + 3y = 13$ represent two parallel lines. There is no solution since there is no intersection between the two lines.

(c) The simultaneous equations may be written in matrix form as

$$\begin{pmatrix} 8 & 2 \\ 4 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 14 \\ 7 \end{pmatrix}$$

Determinant of $\begin{pmatrix} 8 & 2 \\ 4 & 1 \end{pmatrix} = (8 \times 1) - (2 \times 4)$

$= 0$

Hence $\begin{pmatrix} 8 & 2 \\ 4 & 1 \end{pmatrix}$ is a singular matrix and its inverse matrix does not exist. The first equation is obtained from the second by multiplying throughout by 2. Thus, the graphs of $8x + 2y = 14$ and $4x + y = 7$ represent the same line, i.e. the two lines coincide. There is an infinite number of solutions. Some solutions include (1, 3) and (2, –1).

Exercise 5A

1. **(a)** 3×2
(b) 1×3
(c) 3×3
(d) 2×1
(e) 1×1
(f) 2×2

2. **(a)** **A** = **B**
(b) **C** ≠ **D** since two of their corresponding elements are not equal.
(c) **P** ≠ **Q** since the matrices are of different order.
(d) **X** ≠ **Y** since the matrices are of different order.

3. **(i)** $\mathbf{F} = \begin{pmatrix} 4 & 0 & 5 & 6 \\ 8 & 7 & 5 & 3 \end{pmatrix}$
(ii) Banana
(iii) The sum of the elements in the first row of **F** is 15 and it represents the total number of boys in the class.
(iv) The sum of the elements in the fourth column of **F** and the answer is 9.

4. **(a)** **P** ≠ **Q** since the two matrices are of different order.
(b) **X** = **Y**

5. **B** = **Q**
C = **O**
D = **I**
E = **H**
F = **P**
G = **L**
J = **N**

6. **(a)** $\begin{pmatrix} 2 & 3 \\ 5 & k \end{pmatrix} = \begin{pmatrix} 2a & b \\ c & 7 \end{pmatrix}$

Equating the corresponding elements, we have
$2 = 2a$
$a = \frac{2}{2}$
$= 1$
$3 = b$
$5 = c$
$k = 7$
$\therefore a = 1, b = 3, c = 5, k = 7$

(b) $\begin{pmatrix} 3 & 5 & b \\ 7 & -3 & c \end{pmatrix} = \begin{pmatrix} a & 5 & 13 \\ d & -a & 6 \end{pmatrix}$

Equating the corresponding elements, we have
$3 = a$
$b = 13$
$7 = d$
$c = 6$
$\therefore a = 3, b = 13, c = 6, d = 7$

(c) $\begin{pmatrix} 2x & 18 \\ 3y & 36 \end{pmatrix} = \begin{pmatrix} 14 & 2k \\ 15 & 6h \end{pmatrix}$

Equating the corresponding elements, we have
$2x = 14$
$x = 7$
$2k = 18$
$k = 9$
$3y = 15$
$y = 5$
$6h = 36$
$h = 6$
$\therefore h = 6, k = 9, x = 7, y = 5$

(d) $(2x - 3 \quad y + 4) = (7 \quad 6)$

Equating the corresponding elements, we have
$2x - 3 = 7$
$2x = 10$
$x = 5$
$y + 4 = 6$
$y = 2$
$\therefore x = 5, y = 2$

(e) $\begin{pmatrix} \frac{1}{2}x & x+4 \\ 5 & 3y \end{pmatrix} = \begin{pmatrix} 3 & h \\ k-9 & 27 \end{pmatrix}$

Equating the corresponding elements, we have

$\frac{1}{2}x = 3$

$x = 6$

$x + 4 = h$

$h = 6 + 4$

$= 10$

$5 = k - 9$

$k = 14$

$3y = 27$

$y = 9$

$\therefore h = 10, k = 14, x = 6, y = 9$

(f) $\begin{pmatrix} 2x-5 & y-4 \\ z+3 & 5k \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

Equating the corresponding elements, we have

$2x - 5 = 0$

$2x = 5$

$x = 2\frac{1}{2}$

$y - 4 = 0$

$y = 4$

$z + 3 = 0$

$z = -3$

$5k = 0$

$k = 0$

$\therefore k = 0, x = 2\frac{1}{2}, y = 4, z = -3$

7. **(i)** $\mathbf{S} = \begin{pmatrix} 0 & 3 & 1 & 7 \\ 3 & 0 & 4 & 2 \\ 1 & 4 & 0 & 5 \\ 7 & 2 & 5 & 0 \end{pmatrix}$

(ii) The total number of goals scored in the match between Team *C* and Team *D* is 5.

(iii) The zeros in **S** represent that there is no match between the same team.

(iv) The sum of the elements in the second row of **S** is 9 and this sum represents the total number of goals scored by Team *B*.

(v) The total number of goals scored in the match between two teams is the same.

Exercise 5B

1. **(a)** $\begin{pmatrix} 3 & 4 \\ 8 & -5 \end{pmatrix} + \begin{pmatrix} 4 & 6 \\ 3 & 0 \end{pmatrix} = \begin{pmatrix} 7 & 10 \\ 11 & -5 \end{pmatrix}$

(b) $\begin{pmatrix} 7 \\ -8 \end{pmatrix} + \begin{pmatrix} 5 \\ -9 \end{pmatrix} = \begin{pmatrix} 12 \\ -17 \end{pmatrix}$

(c) $(2 \quad 8 \quad -3) + (-4 \quad 7 \quad 0) = (-2 \quad 15 \quad -3)$

(d) Not possible because the matrices have different orders.

(e) $\begin{pmatrix} 2 & -3 & 8 \\ 10 & 5 & 4 \end{pmatrix} - \begin{pmatrix} 5 & 6 & 7 \\ -3 & 0 & 12 \end{pmatrix} = \begin{pmatrix} -3 & -9 & 1 \\ 13 & 5 & -8 \end{pmatrix}$

(f) $\begin{pmatrix} 12 \\ -8.3 \\ 4 \end{pmatrix} - \begin{pmatrix} 8 \\ 1.7 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ -10 \\ 4 \end{pmatrix}$

(g) Not possible because the matrices have different orders.

(h) $\begin{pmatrix} 8 & 9 \\ -7 & 6 \end{pmatrix} + \begin{pmatrix} 4 & 0 \\ 8 & 0 \end{pmatrix} = \begin{pmatrix} 12 & 9 \\ 1 & 6 \end{pmatrix}$

2. **(a)** $\begin{pmatrix} 3 \\ 4 \end{pmatrix} + \begin{pmatrix} -1 \\ 5 \end{pmatrix} - \begin{pmatrix} 6 \\ 7 \end{pmatrix} = \begin{pmatrix} -4 \\ 2 \end{pmatrix}$

(b) $\begin{pmatrix} 4 & -1 \\ 3 & 2 \end{pmatrix} + \begin{pmatrix} 3 & 2 \\ -5 & 4 \end{pmatrix} - \begin{pmatrix} -6 & 4 \\ 2 & 1 \end{pmatrix}$

$= \left[\begin{pmatrix} 4 & -1 \\ 3 & 2 \end{pmatrix} + \begin{pmatrix} 3 & 2 \\ -5 & 4 \end{pmatrix}\right] - \begin{pmatrix} -6 & 4 \\ 2 & 1 \end{pmatrix}$

$= \begin{pmatrix} 7 & 1 \\ -2 & 6 \end{pmatrix} - \begin{pmatrix} -6 & 4 \\ 2 & 1 \end{pmatrix}$

$= \begin{pmatrix} 13 & -3 \\ -4 & 5 \end{pmatrix}$

(c) $(1 \quad 3) - (3 \quad 4) + (-2 \quad 6)$

$= [(1 \quad 3) - (3 \quad 4)] + (-2 \quad 6)$

$= (-2 \quad -1) + (-2 \quad 6)$

$= (-4 \quad 5)$

(d) $\begin{pmatrix} 3 & 1 & 5 \\ -7 & 8 & -2 \end{pmatrix} - \begin{pmatrix} 2 & -1 & 0 \\ 5 & -2 & 6 \end{pmatrix} + \begin{pmatrix} 7 & 5 & 8 \\ -2 & 4 & -9 \end{pmatrix}$

$= \left[\begin{pmatrix} 3 & 1 & 5 \\ -7 & 8 & -2 \end{pmatrix} - \begin{pmatrix} 2 & -1 & 0 \\ 5 & -2 & 6 \end{pmatrix}\right] + \begin{pmatrix} 7 & 5 & 8 \\ -2 & 4 & -9 \end{pmatrix}$

$= \begin{pmatrix} 1 & 2 & 5 \\ -12 & 10 & -8 \end{pmatrix} + \begin{pmatrix} 7 & 5 & 8 \\ -2 & 4 & -9 \end{pmatrix}$

$= \begin{pmatrix} 8 & 7 & 13 \\ -14 & 14 & -17 \end{pmatrix}$

(e) Not possible because the matrices have different orders.

(f) $\begin{pmatrix} 4 & -3 \\ 2 & 5 \\ -8 & 9 \end{pmatrix} - \begin{pmatrix} -3 & 2 \\ 7 & -1 \\ 6 & -3 \end{pmatrix} + \begin{pmatrix} 4 & 5 \\ 0 & -6 \\ 2 & 8 \end{pmatrix}$

$= \left[\begin{pmatrix} 4 & -3 \\ 2 & 5 \\ -8 & 9 \end{pmatrix} - \begin{pmatrix} -3 & 2 \\ 7 & -1 \\ 6 & -3 \end{pmatrix}\right] + \begin{pmatrix} 4 & 5 \\ 0 & -6 \\ 2 & 8 \end{pmatrix}$

$= \begin{pmatrix} 7 & -5 \\ -5 & 6 \\ -14 & 12 \end{pmatrix} + \begin{pmatrix} 4 & 5 \\ 0 & -6 \\ 2 & 8 \end{pmatrix}$

$= \begin{pmatrix} 11 & 0 \\ -5 & 0 \\ -12 & 20 \end{pmatrix}$

(g) Not possible because the matrices have different orders.

OXFORD UNIVERSITY PRESS

(h) $(5) - (-6) + (3)$
$= (11) + (3)$
$= (14)$

3. **(i)** $\mathbf{Q} = \begin{pmatrix} 42 & 35 & 38 \\ 33 & 40 & 37 \end{pmatrix}$

(ii) $\mathbf{P} + \mathbf{Q} = \begin{pmatrix} 41 & 38 & 29 \\ 39 & 33 & 36 \end{pmatrix} + \begin{pmatrix} 42 & 35 & 38 \\ 33 & 40 & 37 \end{pmatrix}$

$= \begin{pmatrix} 83 & 73 & 67 \\ 72 & 73 & 73 \end{pmatrix}$

(iii) They represent the total marks scored for the two Mathematics and English tests by Nora, Shirley and Amirah.

4. **(i)** $\mathbf{A} + \mathbf{B}$

$= \begin{pmatrix} 5 & -5 \\ -4 & 9 \end{pmatrix} + \begin{pmatrix} 1 & 3 \\ -2 & 4 \end{pmatrix}$

$= \begin{pmatrix} 6 & -2 \\ -6 & 13 \end{pmatrix}$

(ii) $\mathbf{B} + \mathbf{A}$

$= \begin{pmatrix} 1 & 3 \\ -2 & 4 \end{pmatrix} + \begin{pmatrix} 5 & -5 \\ -4 & 9 \end{pmatrix}$

$= \begin{pmatrix} 6 & -2 \\ -6 & 13 \end{pmatrix}$

(iii) $\mathbf{B} + \mathbf{C}$

$= \begin{pmatrix} 1 & 3 \\ -2 & 4 \end{pmatrix} + \begin{pmatrix} 0 & 2 \\ -1 & 4 \end{pmatrix}$

$= \begin{pmatrix} 1 & 5 \\ -3 & 8 \end{pmatrix}$

(iv) $\mathbf{C} + \mathbf{B}$

$= \begin{pmatrix} 0 & 2 \\ -1 & 4 \end{pmatrix} + \begin{pmatrix} 1 & 3 \\ -2 & 4 \end{pmatrix}$

$= \begin{pmatrix} 1 & 5 \\ -3 & 8 \end{pmatrix}$

(v) $\mathbf{A} + (\mathbf{B} + \mathbf{C})$

$= \begin{pmatrix} 5 & -5 \\ -4 & 9 \end{pmatrix} + \left[\begin{pmatrix} 1 & 3 \\ -2 & 4 \end{pmatrix} + \begin{pmatrix} 0 & 2 \\ -1 & 4 \end{pmatrix}\right]$

$= \begin{pmatrix} 5 & -5 \\ -4 & 9 \end{pmatrix} + \begin{pmatrix} 1 & 5 \\ -3 & 8 \end{pmatrix}$

$= \begin{pmatrix} 6 & 0 \\ -7 & 17 \end{pmatrix}$

(vi) $(\mathbf{A} + \mathbf{B}) + \mathbf{C}$

$= \left[\begin{pmatrix} 5 & -5 \\ -4 & 9 \end{pmatrix} + \begin{pmatrix} 1 & 3 \\ -2 & 4 \end{pmatrix}\right] + \begin{pmatrix} 0 & 2 \\ -1 & 4 \end{pmatrix}$

$= \begin{pmatrix} 6 & -2 \\ -6 & 13 \end{pmatrix} + \begin{pmatrix} 0 & 2 \\ -1 & 4 \end{pmatrix}$

$= \begin{pmatrix} 6 & 0 \\ -7 & 17 \end{pmatrix}$

5. **(i)** $\mathbf{A} - \mathbf{B}$

$= \begin{pmatrix} 3 & 1 \\ 4 & -2 \end{pmatrix} - \begin{pmatrix} 4 & -1 \\ 3 & -4 \end{pmatrix}$

$= \begin{pmatrix} -1 & 2 \\ 1 & 2 \end{pmatrix}$

(ii) $\mathbf{B} - \mathbf{A}$

$= \begin{pmatrix} 4 & -1 \\ 3 & -4 \end{pmatrix} - \begin{pmatrix} 3 & 1 \\ 4 & -2 \end{pmatrix}$

$= \begin{pmatrix} 1 & -2 \\ -1 & -2 \end{pmatrix}$

(iii) $\mathbf{B} - \mathbf{C}$

$= \begin{pmatrix} 4 & -1 \\ 3 & -4 \end{pmatrix} - \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

$= \begin{pmatrix} 4 & -2 \\ 4 & -4 \end{pmatrix}$

(iv) $\mathbf{A} - (\mathbf{B} - \mathbf{C})$

$= \begin{pmatrix} 3 & 1 \\ 4 & -2 \end{pmatrix} - \left[\begin{pmatrix} 4 & -1 \\ 3 & -4 \end{pmatrix} - \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\right]$

$= \begin{pmatrix} 3 & 1 \\ 4 & -2 \end{pmatrix} - \begin{pmatrix} 4 & -2 \\ 4 & -4 \end{pmatrix}$

$= \begin{pmatrix} -1 & 3 \\ 0 & 2 \end{pmatrix}$

(v) $(\mathbf{A} - \mathbf{B}) - \mathbf{C}$

$= \left[\begin{pmatrix} 3 & 1 \\ 4 & -2 \end{pmatrix} - \begin{pmatrix} 4 & -1 \\ 3 & -4 \end{pmatrix}\right] - \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

$= \begin{pmatrix} -1 & 2 \\ 1 & 2 \end{pmatrix} - \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

$= \begin{pmatrix} -1 & 1 \\ 2 & 2 \end{pmatrix}$

6. **(i)** $\mathbf{A} - \mathbf{B}$

$= \begin{pmatrix} 240 & 210 & 195 & 304 & 195 \\ 95 & 120 & 116 & 102 & 100 \\ 100 & 94 & 132 & 132 & 110 \end{pmatrix} - \begin{pmatrix} 24 & 13 & 5 & 11 & 27 \\ 12 & 18 & 9 & 17 & 13 \\ 10 & 14 & 12 & 21 & 8 \end{pmatrix}$

$= \begin{pmatrix} 216 & 197 & 190 & 293 & 168 \\ 83 & 102 & 107 & 85 & 87 \\ 90 & 80 & 120 & 111 & 102 \end{pmatrix}$

(ii) It represents the stocks for Chinese, Malay and Tamil textbooks for Secondary 1, 2, 3, 4 and 5 in the school bookshop sold between 1st December and 1st January.

Exercise 5C

1. **(a)** $2(1 \quad -2 \quad 3) = (2 \quad -4 \quad 6)$

(b) $4\begin{pmatrix} -2 \\ 1 \end{pmatrix} = \begin{pmatrix} -8 \\ 4 \end{pmatrix}$

OXFORD UNIVERSITY PRESS

(c) $\frac{1}{2}\begin{pmatrix}6\\4\\-8\end{pmatrix} = \begin{pmatrix}3\\2\\-4\end{pmatrix}$

(d) $\frac{1}{3}\begin{pmatrix}6 & 15\\21 & -24\end{pmatrix} = \begin{pmatrix}2 & 5\\7 & -8\end{pmatrix}$

(e) $-2\begin{pmatrix}-1 & 0.5 & 3\\-0.8 & 2 & 1.2\end{pmatrix} = \begin{pmatrix}2 & -1 & -6\\1.6 & -4 & -2.4\end{pmatrix}$

(f) $5\begin{pmatrix}1 & 5\\-4 & 3\\-1 & 2\end{pmatrix} = \begin{pmatrix}5 & 25\\-20 & 15\\-5 & 10\end{pmatrix}$

(g) $3\begin{pmatrix}6 & \frac{1}{2} & 1\\0 & 2 & \frac{1}{3}\\5 & -4 & -2\end{pmatrix} = \begin{pmatrix}18 & 1\frac{1}{2} & 3\\0 & 6 & 1\\15 & -12 & -6\end{pmatrix}$

2. **(a)** $2\begin{pmatrix}-1\\5\end{pmatrix} + 3\begin{pmatrix}4\\-3\end{pmatrix} = \begin{pmatrix}-2\\10\end{pmatrix} + \begin{pmatrix}12\\-9\end{pmatrix}$

$= \begin{pmatrix}10\\1\end{pmatrix}$

(b) $2(3\quad 1\quad 5) - 4(-1\quad 3\quad 2) = (6\quad 2\quad 10) - (-4\quad 12\quad 8)$

$= (10\quad -10\quad 2)$

(c) $5\begin{pmatrix}1 & 3\\-4 & 6\end{pmatrix} - 2\begin{pmatrix}-3 & -1\\4 & 2\end{pmatrix} = \begin{pmatrix}5 & 15\\-20 & 30\end{pmatrix} - \begin{pmatrix}-6 & -2\\8 & 4\end{pmatrix}$

$= \begin{pmatrix}11 & 17\\-28 & 26\end{pmatrix}$

(d) $3\begin{pmatrix}0 & 4 & 1\\5 & 0 & -1\end{pmatrix} - 4\begin{pmatrix}-1 & 3 & 0\\-2 & 1 & -1\end{pmatrix}$

$= \begin{pmatrix}0 & 12 & 3\\15 & 0 & -3\end{pmatrix} - \begin{pmatrix}-4 & 12 & 0\\-8 & 4 & -4\end{pmatrix}$

$= \begin{pmatrix}4 & 0 & 3\\23 & -4 & 1\end{pmatrix}$

3. **(i)** $\mathbf{A} + \mathbf{B} = \begin{pmatrix}4 & 4\\2 & 7\end{pmatrix} + \begin{pmatrix}1 & 2\\-1 & 3\end{pmatrix}$

$= \begin{pmatrix}5 & 6\\1 & 10\end{pmatrix}$

(ii) $\mathbf{A} + 2\mathbf{B} = \begin{pmatrix}4 & 4\\2 & 7\end{pmatrix} + 2\begin{pmatrix}1 & 2\\-1 & 3\end{pmatrix}$

$= \begin{pmatrix}4 & 4\\2 & 7\end{pmatrix} + \begin{pmatrix}2 & 4\\-2 & 6\end{pmatrix}$

$= \begin{pmatrix}6 & 8\\0 & 13\end{pmatrix}$

(iii) $\mathbf{A} - \mathbf{B} - \mathbf{C} = \begin{pmatrix}4 & 4\\2 & 7\end{pmatrix} - \begin{pmatrix}1 & 2\\-1 & 3\end{pmatrix} - \begin{pmatrix}1 & 4\\3 & -5\end{pmatrix}$

$= \left[\begin{pmatrix}4 & 4\\2 & 7\end{pmatrix} - \begin{pmatrix}1 & 2\\-1 & 3\end{pmatrix}\right] - \begin{pmatrix}1 & 4\\3 & -5\end{pmatrix}$

$= \begin{pmatrix}3 & 2\\3 & 4\end{pmatrix} - \begin{pmatrix}1 & 4\\3 & -5\end{pmatrix}$

$= \begin{pmatrix}2 & -2\\0 & 9\end{pmatrix}$

(iv) $2\mathbf{A} - 2\mathbf{C} + 3\mathbf{B} = 2\begin{pmatrix}4 & 4\\2 & 7\end{pmatrix} - 2\begin{pmatrix}1 & 4\\3 & -5\end{pmatrix} + 3\begin{pmatrix}1 & 2\\-1 & 3\end{pmatrix}$

$= \left[\begin{pmatrix}8 & 8\\4 & 14\end{pmatrix} - \begin{pmatrix}2 & 8\\6 & -10\end{pmatrix}\right] + \begin{pmatrix}3 & 6\\-3 & 9\end{pmatrix}$

$= \begin{pmatrix}6 & 0\\-2 & 24\end{pmatrix} + \begin{pmatrix}3 & 6\\-3 & 9\end{pmatrix}$

$= \begin{pmatrix}9 & 6\\-5 & 33\end{pmatrix}$

4. **(a)** $a\begin{pmatrix}2\\2\end{pmatrix} + b\begin{pmatrix}2\\-2\end{pmatrix} = \begin{pmatrix}0\\8\end{pmatrix}$

$\begin{pmatrix}2a\\2a\end{pmatrix} + \begin{pmatrix}2b\\-2b\end{pmatrix} = \begin{pmatrix}0\\8\end{pmatrix}$

$\begin{pmatrix}2a + 2b\\2a - 2b\end{pmatrix} = \begin{pmatrix}0\\8\end{pmatrix}$

Equating the corresponding elements, we have

$2a + 2b = 0$

$a = -b$ — (1)

$2a - 2b = 8$

$a - b = 4$ — (2)

Substitute (1) into (2):

$-b - b = 4$

$-2b = 4$

$b = -2$

Substitute $b = -2$ into (1):

$a = -(-2)$

$= 2$

$\therefore a = 2, b = -2$

(b) $3\begin{pmatrix}2x\\y\end{pmatrix} + 3\begin{pmatrix}x\\3y\end{pmatrix} = \begin{pmatrix}18\\36\end{pmatrix}$

$\begin{pmatrix}6x + 3x\\3y + 9y\end{pmatrix} = \begin{pmatrix}18\\36\end{pmatrix}$

$\begin{pmatrix}9x\\12y\end{pmatrix} = \begin{pmatrix}18\\36\end{pmatrix}$

Equating the corresponding elements, we have

$9x = 18$

$x = 2$

$12y = 36$

$y = 3$

$\therefore x = 2, y = 3$

OXFORD UNIVERSITY PRESS

(c) $2\begin{pmatrix}1&2\\3&4\end{pmatrix}-\begin{pmatrix}3&5\\c&6\end{pmatrix}=\begin{pmatrix}a&b\\7&d\end{pmatrix}$

$$\begin{pmatrix}2&4\\6&8\end{pmatrix}-\begin{pmatrix}3&5\\c&6\end{pmatrix}=\begin{pmatrix}a&b\\7&d\end{pmatrix}$$

$$\begin{pmatrix}-1&-1\\6-c&2\end{pmatrix}=\begin{pmatrix}a&b\\7&d\end{pmatrix}$$

Equating the corresponding elements, we have

$a=-1$

$b=-1$

$6-c=7$

$c=-1$

$d=2$

$\therefore a=-1, b=-1, c=-1, d=2$

(d) $2\begin{pmatrix}5&3&2\\1&6&3\end{pmatrix}+\begin{pmatrix}a&b&c\\-2&-4&5\end{pmatrix}=\begin{pmatrix}9&12&6\\d&e&f\end{pmatrix}$

$$\begin{pmatrix}10&6&4\\2&12&6\end{pmatrix}+\begin{pmatrix}a&b&c\\-2&-4&5\end{pmatrix}=\begin{pmatrix}9&12&6\\d&e&f\end{pmatrix}$$

$$\begin{pmatrix}10+a&6+b&4+c\\0&8&11\end{pmatrix}=\begin{pmatrix}9&12&6\\d&e&f\end{pmatrix}$$

Equating the corresponding elements, we have

$10+a=9$

$a=-1$

$6+b=12$

$b=6$

$4+c=6$

$c=2$

$d=0$

$e=8$

$f=11$

$\therefore a=-1, b=6, c=2, d=0, e=8, f=11$

5. **(i)** $12\mathbf{C}=12\begin{pmatrix}680\\720\\635\end{pmatrix}$

$$=\begin{pmatrix}8160\\8640\\7620\end{pmatrix}$$

(ii) $12\mathbf{C}+\mathbf{J}+\mathbf{D}=12\begin{pmatrix}680\\720\\635\end{pmatrix}+\begin{pmatrix}150\\120\\200\end{pmatrix}+\begin{pmatrix}180\\150\\200\end{pmatrix}$

$$=\begin{pmatrix}8160\\8640\\7620\end{pmatrix}+\begin{pmatrix}150\\120\\200\end{pmatrix}+\begin{pmatrix}180\\150\\200\end{pmatrix}$$

$$=\begin{pmatrix}8490\\8910\\8020\end{pmatrix}$$

This matrix represents the annual fees, in dollars, charged by the childcare centres, including the charges for the special programmes in June and December.

(iii) $12\mathbf{C}+\mathbf{J}=12\begin{pmatrix}680\\720\\635\end{pmatrix}+\begin{pmatrix}150\\120\\200\end{pmatrix}$

$$=\begin{pmatrix}8160\\8640\\7620\end{pmatrix}+\begin{pmatrix}150\\120\\200\end{pmatrix}$$

$$=\begin{pmatrix}8310\\8760\\7820\end{pmatrix}$$

Childcare Centre *Z* charges the lowest fees.

6. **(a)** $\begin{pmatrix}4&3\\-1&5\end{pmatrix}\begin{pmatrix}-2\\6\end{pmatrix}=\begin{pmatrix}4\times(-2)+3\times6\\(-1)\times(-2)+5\times6\end{pmatrix}$

$$=\begin{pmatrix}10\\32\end{pmatrix}$$

(b) $\begin{pmatrix}-3&1\\0&8\end{pmatrix}\begin{pmatrix}6&5\\8&7\end{pmatrix}=\begin{pmatrix}(-3)\times6+1\times8&(-3)\times5+1\times7\\0\times6+8\times8&0\times5+8\times7\end{pmatrix}$

$$=\begin{pmatrix}-10&-8\\64&56\end{pmatrix}$$

(c) $\begin{pmatrix}6\\7\end{pmatrix}(-1\quad3)=\begin{pmatrix}6\times(-1)&6\times3\\7\times(-1)&7\times3\end{pmatrix}$

$$=\begin{pmatrix}-6&18\\-7&21\end{pmatrix}$$

(d) $(-1\quad3)\begin{pmatrix}6\\7\end{pmatrix}=((-1)\times6+3\times7)$

$$=(15)$$

(e) Not possible, since the number of columns of the first matrix, which is 1, is not equal to the number of rows of the second matrix, which is 2.

(f) $\begin{pmatrix}-1&2\\8&5\\3&-7\end{pmatrix}\begin{pmatrix}2&3\\-1&4\end{pmatrix}$

$$=\begin{pmatrix}(-1)\times2+2\times(-1)&(-1)\times3+2\times4\\8\times2+5\times(-1)&8\times3+5\times4\\3\times2+(-7)\times(-1)&3\times3+(-7)\times4\end{pmatrix}$$

$$=\begin{pmatrix}-4&5\\11&44\\13&-19\end{pmatrix}$$

(g) $\begin{pmatrix}3&8&0&5\\-1&0&7&6\\4&9&-2&1\end{pmatrix}\begin{pmatrix}2\\-3\\0\\5\end{pmatrix}$

$$=\begin{pmatrix}3\times2+8\times(-3)+0\times0+5\times5\\(-1)\times2+0\times(-3)+7\times0+6\times5\\4\times2+9\times(-3)+(-2)\times0+1\times5\end{pmatrix}$$

$$=\begin{pmatrix}7\\28\\-14\end{pmatrix}$$

(h) $\begin{pmatrix}\frac{1}{2}\end{pmatrix}(1\quad2\quad3\quad4)=\begin{pmatrix}\frac{1}{2}&1&1\frac{1}{2}&2\end{pmatrix}$

OXFORD UNIVERSITY PRESS

7. $$\begin{pmatrix} 1 & 5 \\ 3 & p \end{pmatrix}\begin{pmatrix} q \\ 7 \end{pmatrix} = \begin{pmatrix} 50 \\ 35 \end{pmatrix}$$

$$\begin{pmatrix} 1\times q+5\times 7 \\ 3\times q+p\times 7 \end{pmatrix} = \begin{pmatrix} 50 \\ 35 \end{pmatrix}$$

$$\begin{pmatrix} q+35 \\ 3q+7p \end{pmatrix} = \begin{pmatrix} 50 \\ 35 \end{pmatrix}$$

Equating the corresponding elements, we have

$$\begin{aligned} q+35 &= 50 \\ q &= 50-35 \\ q &= 15 \\ 3q+7p &= 35 \\ 3(15)+7p &= 35 \\ 7p &= 35-45 \\ 7p &= -10 \\ p &= -1\frac{3}{7} \end{aligned}$$

$\therefore p=-1\frac{3}{7}, q=15$

8. (i) $\mathbf{AB} = \begin{pmatrix} 2 & 0 \\ 1 & 5 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 2 & k \end{pmatrix}$

$$= \begin{pmatrix} 2\times 1+0\times 2 & 2\times 0+0\times k \\ 1\times 1+5\times 2 & 1\times 0+5\times k \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 0 \\ 11 & 5k \end{pmatrix}$$

(ii) $\mathbf{BA} = \begin{pmatrix} 1 & 0 \\ 2 & k \end{pmatrix}\begin{pmatrix} 2 & 0 \\ 1 & 5 \end{pmatrix}$

$$= \begin{pmatrix} 1\times 2+0\times 1 & 1\times 0+0\times 5 \\ 2\times 2+k\times 1 & 2\times 0+k\times 5 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 0 \\ 4+k & 5k \end{pmatrix}$$

(iii) $\mathbf{AB} = \mathbf{BA}$

$$\begin{pmatrix} 2 & 0 \\ 11 & 5k \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 4+k & 5k \end{pmatrix}$$

Equating the corresponding elements, we have

$11 = 4+k$

$\therefore k = 7$

9. (i) $\mathbf{AI} = \begin{pmatrix} 8 & -3 \\ 7 & 5 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$$= \begin{pmatrix} 8\times 1+(-3)\times 0 & 8\times 0+(-3)\times 1 \\ 7\times 1+5\times 0 & 7\times 0+5\times 1 \end{pmatrix}$$

$$= \begin{pmatrix} 8 & -3 \\ 7 & 5 \end{pmatrix}$$

(ii) $\mathbf{IA} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 8 & -3 \\ 7 & 5 \end{pmatrix}$

$$= \begin{pmatrix} 1\times 8+0\times 7 & 1\times(-3)+0\times 5 \\ 0\times 8+1\times 7 & 0\times(-3)+1\times 5 \end{pmatrix}$$

$$= \begin{pmatrix} 8 & -3 \\ 7 & 5 \end{pmatrix}$$

Yes, **AI** = **A** = **IA**.

10. 7**A** + 6**B** + 4**C** + 3**D**

11. (i) $\mathbf{A} = \begin{pmatrix} 2 & 0 \\ 1 & 5 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 1 & 0 \\ 2 & 3 \end{pmatrix}$

$\mathbf{AB} = \begin{pmatrix} 2 & 0 \\ 1 & 5 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 2 & 3 \end{pmatrix}$

$$= \begin{pmatrix} 2\times 1+0\times 2 & 2\times 0+0\times 3 \\ 1\times 1+5\times 2 & 1\times 0+5\times 3 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 0 \\ 11 & 15 \end{pmatrix}$$

$\mathbf{BA} = \begin{pmatrix} 1 & 0 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} 2 & 0 \\ 1 & 5 \end{pmatrix}$

$$= \begin{pmatrix} 1\times 2+0\times 1 & 1\times 0+0\times 5 \\ 2\times 2+3\times 1 & 2\times 0+3\times 5 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 0 \\ 7 & 15 \end{pmatrix}$$

AB ≠ BA

(ii) $\mathbf{A} = \begin{pmatrix} 2 & 0 \\ 1 & 5 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 1 & 0 \\ 2 & 7 \end{pmatrix}$

$\mathbf{AB} = \begin{pmatrix} 2 & 0 \\ 1 & 5 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 2 & 7 \end{pmatrix}$

$$= \begin{pmatrix} 2\times 1+0\times 2 & 2\times 0+0\times 7 \\ 1\times 1+5\times 2 & 1\times 0+5\times 7 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 0 \\ 11 & 35 \end{pmatrix}$$

$\mathbf{BA} = \begin{pmatrix} 1 & 0 \\ 2 & 7 \end{pmatrix}\begin{pmatrix} 2 & 0 \\ 1 & 5 \end{pmatrix}$

$$= \begin{pmatrix} 1\times 2+0\times 1 & 1\times 0+0\times 5 \\ 2\times 2+7\times 1 & 2\times 0+7\times 5 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 0 \\ 11 & 35 \end{pmatrix}$$

AB = BA

OXFORD UNIVERSITY PRESS

(iii) No

$$\mathbf{A}^2 = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$= \begin{pmatrix} a\times a + b\times c & a\times b + b\times d \\ c\times a + d\times c & c\times b + d\times d \end{pmatrix}$$

$$= \begin{pmatrix} a^2 + bc & ab + bd \\ ac + cd & bc + d^2 \end{pmatrix}$$

Hence, $\mathbf{A}^2$ is not $\begin{pmatrix} a^2 & b^2 \\ c^2 & d^2 \end{pmatrix}$.

Exercise 5D

1. **(i)** Determinant $= (3\times 6) - (4\times 5)$

$= -2$

$\neq 0$

$\therefore$ The inverse exists.

$$\text{Inverse} = \frac{1}{-2}\begin{pmatrix} 6 & -4 \\ -5 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{6}{-2} & \frac{4}{2} \\ \frac{5}{2} & \frac{3}{-2} \end{pmatrix}$$

$$= \begin{pmatrix} -3 & 2 \\ \frac{5}{2} & -\frac{3}{2} \end{pmatrix}$$

(ii) Determinant $= \left[2\times\left(-\frac{3}{2}\right)\right] - [(-1)\times 3)]$

$= 0$

$\therefore$ The inverse does not exist.

(iii) Determinant $= (3\times 2) - (5\times 1)$

$= 1$

$\neq 0$

$\therefore$ The inverse exists.

$$\text{Inverse} = \begin{pmatrix} 2 & -5 \\ -1 & 3 \end{pmatrix}$$

(iv) Determinant $= (3\times 7) - (10\times 2)$

$= 1$

$\neq 0$

$\therefore$ The inverse exists.

$$\text{Inverse} = \begin{pmatrix} 7 & -10 \\ -2 & 3 \end{pmatrix}$$

(v) Determinant $= [(-8)\times(-2)] - [(-4)\times(-4)]$

$= 0$

$\therefore$ The inverse does not exist.

(vi) Determinant $= (1\times 6) - [3\times(-2)]$

$= 12$

$\neq 0$

$\therefore$ The inverse exists.

$$\text{Inverse} = \frac{1}{12}\begin{pmatrix} 6 & -3 \\ 2 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{6}{12} & \frac{-3}{12} \\ \frac{2}{12} & \frac{1}{12} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{2} & -\frac{1}{4} \\ \frac{1}{6} & \frac{1}{12} \end{pmatrix}$$

(vii) Determinant $= (1\times 1) - (0\times 0)$

$= 1$

$\neq 0$

$\therefore$ The inverse exists.

$$\text{Inverse} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

(viii) Determinant $= [(-1)\times(-3)] - (3\times 1)$

$= 3 - 3$

$= 0$

$\therefore$ The inverse does not exist.

2. $$\mathbf{AB} = \begin{pmatrix} 2 & 3 \\ 5 & 8 \end{pmatrix}\begin{pmatrix} 8 & -3 \\ -5 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\mathbf{BA} = \begin{pmatrix} 8 & -3 \\ -5 & 2 \end{pmatrix}\begin{pmatrix} 2 & 3 \\ 5 & 8 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Since **AB** = **BA** = **I**, **A** is the inverse of **B** and **B** is the inverse of **A** i.e. $\mathbf{A} = \mathbf{B}^{-1}$ and $\mathbf{B} = \mathbf{A}^{-1}$.

3. $$\mathbf{PQ} = \begin{pmatrix} 7 & 9 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 4 & -9 \\ -3 & 7 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\mathbf{QP} = \begin{pmatrix} 4 & -9 \\ -3 & 7 \end{pmatrix}\begin{pmatrix} 7 & 9 \\ 3 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Since **PQ** = **QP** = **I**, **P** is the inverse of **Q** and **Q** is the inverse of **P** i.e. $\mathbf{P} = \mathbf{Q}^{-1}$ and $\mathbf{Q} = \mathbf{P}^{-1}$.

4. **(a)** $$\frac{1}{ps - qr}\begin{pmatrix} s & -q \\ -r & r \end{pmatrix}$$

OXFORD UNIVERSITY PRESS

(b) Determinant $= \left(a \times \frac{1}{a}\right) - (0 \times 0)$

$= 1$

$\neq 0$

$\therefore$ The inverse exists.

Inverse $= \begin{pmatrix} \frac{1}{a} & 0 \\ 0 & a \end{pmatrix}$

(c) Determinant $= [(-a) \times b] - [b \times a]$

$= -2ab$

$\neq 0$

$\therefore$ The inverse exists.

$\therefore$ Inverse $= \frac{1}{-2ab}\begin{pmatrix} b & -b \\ -a & -a \end{pmatrix}$

$= \begin{pmatrix} \frac{b}{-2ab} & \frac{b}{2ab} \\ \frac{a}{2ab} & \frac{a}{2ab} \end{pmatrix}$

$= \begin{pmatrix} -\frac{1}{2a} & \frac{1}{2a} \\ \frac{1}{2b} & \frac{1}{2b} \end{pmatrix}$

(d) Determinant $= (\cos x \times \cos x) - [(-\sin x) \times \sin x]$

$= \cos^2 x + \sin^2 x$

$= 1$

$\neq 0$

$\therefore$ The inverse exists.

Inverse $= \begin{pmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{pmatrix}$

(e) Determinant $= (0 \times 0) - [(-1) \times 1]$

$= 1$

$\neq 0$

$\therefore$ The inverse exists.

Inverse $= \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

(f) Determinant $= (4 \times 1) - (0 \times 0)$

$= 4$

$\neq 0$

$\therefore$ The inverse exists.

Inverse $= \frac{1}{4}\begin{pmatrix} 1 & 0 \\ 0 & 4 \end{pmatrix}$

$= \begin{pmatrix} \frac{1}{4} & 0 \\ 0 & 1 \end{pmatrix}$

(g) Determinant $= \left(0 \times \frac{1}{3}\right) - (0 \times 0)$

$= 0$

$\therefore$ The inverse does not exist.

5. $|\mathbf{A}| = (2 \times 5) - (4 \times 3)$

$= -2$

$\neq 0$

$\therefore$ The inverse exists.

$\mathbf{A}^{-1} = \frac{1}{-2}\begin{pmatrix} 5 & -4 \\ -3 & 2 \end{pmatrix}$

$= \begin{pmatrix} \frac{5}{-2} & \frac{4}{2} \\ \frac{3}{2} & \frac{2}{-2} \end{pmatrix}$

$= \begin{pmatrix} -\frac{5}{2} & 2 \\ \frac{3}{2} & -1 \end{pmatrix}$

$|\mathbf{B}| = (2 \times 6) - (4 \times 3)$

$= 0$

$\therefore \mathbf{B}^{-1}$ does not exist.

6. Determinant $= (2a \times 5) - [(-4) \times (-1)]$

$= 10a - 4$

$10a - 4 = 16$

$10a = 20$

$a = 2$

$\therefore$ Determinant $= 10(2) - 4$

$= 16$

Inverse of matrix $= \frac{1}{16}\begin{pmatrix} 5 & 4 \\ 1 & 2(2) \end{pmatrix}$

$= \frac{1}{16}\begin{pmatrix} 5 & 4 \\ 1 & 4 \end{pmatrix}$

7. Let $\mathbf{C} = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$

$|\mathbf{C}| = (3 \times 4) - (2 \times 1)$

$= 10$

$\mathbf{C}^{-1} = \frac{1}{10}\begin{pmatrix} 4 & -2 \\ -1 & 3 \end{pmatrix}$

(i) $\mathbf{A}\begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 0 & -3 \end{pmatrix}$

$\mathbf{AC} = \begin{pmatrix} 1 & 2 \\ 0 & -3 \end{pmatrix}$

$\mathbf{ACC}^{-1} = \begin{pmatrix} 1 & 2 \\ 0 & -3 \end{pmatrix}\mathbf{C}^{-1}$

$\mathbf{AI} = \begin{pmatrix} 1 & 2 \\ 0 & -3 \end{pmatrix}\mathbf{C}^{-1}$

$\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 0 & -3 \end{pmatrix}\frac{1}{10}\begin{pmatrix} 4 & -2 \\ -1 & 3 \end{pmatrix}$

$= \frac{1}{10}\begin{pmatrix} 2 & 4 \\ 3 & -9 \end{pmatrix}$

OXFORD UNIVERSITY PRESS

(ii) $\begin{pmatrix}3&2\\1&4\end{pmatrix}\mathbf{B} = \begin{pmatrix}3&2\\-4&-6\end{pmatrix}$

$$\begin{aligned}
\mathbf{CB} &= \begin{pmatrix}3&2\\-4&-6\end{pmatrix}\\
\mathbf{C}^{-1}\mathbf{CB} &= \mathbf{C}^{-1}\begin{pmatrix}3&2\\-4&-6\end{pmatrix}\\
\mathbf{IB} &= \mathbf{C}^{-1}\begin{pmatrix}3&2\\-4&-6\end{pmatrix}\\
\mathbf{B} &= \frac{1}{10}\begin{pmatrix}4&-2\\-1&3\end{pmatrix}\begin{pmatrix}3&2\\-4&-6\end{pmatrix}\\
&= \frac{1}{10}\begin{pmatrix}20&20\\-15&-20\end{pmatrix}\\
&= \begin{pmatrix}\frac{20}{10}&\frac{20}{10}\\\frac{-15}{10}&\frac{-20}{10}\end{pmatrix}\\
&= \begin{pmatrix}2&2\\-\frac{3}{2}&-2\end{pmatrix}
\end{aligned}$$

8. $\mathbf{A} = \begin{pmatrix}2&3\\1&2\end{pmatrix}$

$$\begin{aligned}
|\mathbf{A}| &= (2\times2)-(3\times1)\\
&= 1\\
\mathbf{A}^{-1} &= \begin{pmatrix}2&-3\\-1&2\end{pmatrix}\\
\mathbf{B} &= \begin{pmatrix}1&4\\3&2\end{pmatrix}\\
|\mathbf{B}| &= (1\times2)-(4\times3)\\
&= -10\\
\mathbf{B}^{-1} &= \frac{1}{-10}\begin{pmatrix}2&-4\\-3&1\end{pmatrix}\\
&= \frac{1}{-10}\begin{pmatrix}-2&4\\3&-1\end{pmatrix}
\end{aligned}$$

(i)

$$\begin{aligned}
\mathbf{AP} &= \mathbf{B}\\
\mathbf{A}^{-1}\mathbf{AP} &= \mathbf{A}^{-1}\mathbf{B}\\
\mathbf{IP} &= \mathbf{A}^{-1}\mathbf{B}\\
\mathbf{P} &= \begin{pmatrix}2&-3\\-1&2\end{pmatrix}\begin{pmatrix}1&4\\3&2\end{pmatrix}\\
&= \begin{pmatrix}-7&2\\5&0\end{pmatrix}
\end{aligned}$$

(ii)

$$\begin{aligned}
\mathbf{QA} &= \mathbf{B}\\
\mathbf{QAA}^{-1} &= \mathbf{BA}^{-1}\\
\mathbf{QI} &= \mathbf{BA}^{-1}\\
\mathbf{Q} &= \begin{pmatrix}1&4\\3&2\end{pmatrix}\begin{pmatrix}2&-3\\-1&2\end{pmatrix}\\
&= \begin{pmatrix}-2&5\\4&-5\end{pmatrix}
\end{aligned}$$

9. $\mathbf{A} = \begin{pmatrix}3&1\\2&1\end{pmatrix}$

$$\begin{aligned}
|\mathbf{A}| &= (3\times1)-(1\times2)\\
&= 1\\
\mathbf{A}^{-1} &= \begin{pmatrix}1&-1\\-2&3\end{pmatrix}
\end{aligned}$$

(i)

$$\begin{aligned}
\mathbf{AP} &= \mathbf{B}\\
\mathbf{A}^{-1}\mathbf{AP} &= \mathbf{A}^{-1}\mathbf{B}\\
\mathbf{IP} &= \mathbf{A}^{-1}\mathbf{B}\\
\mathbf{P} &= \begin{pmatrix}1&-1\\-2&3\end{pmatrix}\begin{pmatrix}2&5\\-1&-3\end{pmatrix}\\
&= \begin{pmatrix}3&8\\-7&-19\end{pmatrix}
\end{aligned}$$

(ii)

$$\begin{aligned}
\mathbf{QA} &= \mathbf{B}\\
\mathbf{QAA}^{-1} &= \mathbf{BA}^{-1}\\
\mathbf{QI} &= \mathbf{BA}^{-1}\\
\mathbf{Q} &= \begin{pmatrix}2&5\\-1&-3\end{pmatrix}\begin{pmatrix}1&-1\\-2&3\end{pmatrix}\\
&= \begin{pmatrix}-8&13\\5&-8\end{pmatrix}
\end{aligned}$$

10. $\mathbf{A} = \begin{pmatrix}2&1\\5&3\end{pmatrix}$

$$\begin{aligned}
|\mathbf{A}| &= (2\times3)-(1\times5)\\
&= 1\\
\mathbf{A}^{-1} &= \begin{pmatrix}3&-1\\-5&2\end{pmatrix}
\end{aligned}$$

(i)

$$\begin{aligned}
\mathbf{AX} &= \mathbf{B}-\mathbf{A}\\
&= \begin{pmatrix}3&4\\-2&1\end{pmatrix} - \begin{pmatrix}2&1\\5&3\end{pmatrix}\\
&= \begin{pmatrix}1&3\\-7&-2\end{pmatrix}\\
\mathbf{A}^{-1}\mathbf{AX} &= \mathbf{A}^{-1}\begin{pmatrix}1&3\\-7&-2\end{pmatrix}\\
\mathbf{IX} &= \mathbf{A}^{-1}\begin{pmatrix}1&3\\-7&-2\end{pmatrix}\\
\mathbf{X} &= \begin{pmatrix}3&-1\\-5&2\end{pmatrix}\begin{pmatrix}1&3\\-7&-2\end{pmatrix}\\
&= \begin{pmatrix}10&11\\-19&-19\end{pmatrix}
\end{aligned}$$

(ii)

$$\begin{aligned}
\mathbf{YA} &= 3\mathbf{B}+2\mathbf{A}\\
&= 3\begin{pmatrix}3&4\\-2&1\end{pmatrix} + 2\begin{pmatrix}2&1\\5&3\end{pmatrix}\\
&= \begin{pmatrix}9&12\\-6&3\end{pmatrix} + \begin{pmatrix}4&2\\10&6\end{pmatrix}\\
&= \begin{pmatrix}13&14\\4&9\end{pmatrix}
\end{aligned}$$

OXFORD UNIVERSITY PRESS

$$\mathbf{YAA}^{-1} = \begin{pmatrix} 13 & 14 \\ 4 & 9 \end{pmatrix}\mathbf{A}^{-1}$$

$$\mathbf{YI} = \begin{pmatrix} 13 & 14 \\ 4 & 9 \end{pmatrix}\mathbf{A}^{-1}$$

$$\mathbf{Y} = \begin{pmatrix} 13 & 14 \\ 4 & 9 \end{pmatrix}\begin{pmatrix} 3 & -1 \\ -5 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} -31 & 15 \\ -33 & 14 \end{pmatrix}$$

11. **(i)** $\mathbf{A}^2 = \begin{pmatrix} 2 & 3 \\ 0 & 4 \end{pmatrix}\begin{pmatrix} 2 & 3 \\ 0 & 4 \end{pmatrix}$

$$= \begin{pmatrix} 4 & 18 \\ 0 & 16 \end{pmatrix}$$

(ii) $\mathbf{AB} = \begin{pmatrix} 2 & 3 \\ 0 & 4 \end{pmatrix}\begin{pmatrix} \frac{1}{2} & -k \\ 0 & 2a \end{pmatrix}$

$$= \begin{pmatrix} 1 & -2k+6a \\ 0 & 8a \end{pmatrix}$$

$$\begin{pmatrix} 1 & -2k+6a \\ 0 & 8a \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Equating the corresponding elements,

$-2k + 6a = 0$ — (1)

$8a = 1$

$a = \frac{1}{8}$

Subst. $a = \frac{1}{8}$ into (1):

$-2k + 6\left(\frac{1}{8}\right) = 0$

$2k = \frac{6}{8}$

$k = \frac{3}{8}$

(iii) $|\mathbf{A}| = (2 \times 4) - (3 \times 0)$

$= 8$

Since $|\mathbf{A}| = |\mathbf{C}|$,

$(6 \times h) - [2 \times (-3)] = 8$

$6h + 6 = 8$

$6h = 2$

$h = \frac{2}{6}$

$= \frac{1}{3}$

(iv) When $a = 3$, $\mathbf{B} = \begin{pmatrix} \frac{1}{2} & -\frac{3}{8} \\ 0 & 6 \end{pmatrix}$

$|\mathbf{B}| = \left(\frac{1}{2} \times 6\right) - \left[\left(-\frac{3}{8}\right) \times 10\right]$

$= 3$

Since $|\mathbf{B}| = |\mathbf{C}|$,

$(6 \times h) - [2 \times (-3)] = 3$

$6h + 6 = 3$

$6h = -3$

$h = -\frac{1}{2}$

Exercise 5E

1. **(i)** Each team plays 12 matches.

(ii) $\mathbf{PQ} = \begin{pmatrix} 5 & 1 & 6 \\ 8 & 4 & 0 \\ 2 & 3 & 7 \\ 3 & 4 & 5 \end{pmatrix}\begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}$

$$= \begin{pmatrix} 16 \\ 28 \\ 8 \\ 13 \end{pmatrix}$$

(iii) It represents the total points scored by each team.

2. **(a)** The simultaneous equations may be written in matrix form as

$$\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 7 \\ 11 \end{pmatrix}$$

Determinant of $\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} = (1 \times 1) - [(-1) \times 1]$

$= 2$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}^{-1}\begin{pmatrix} 7 \\ 11 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 7 \\ 11 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} 18 \\ 4 \end{pmatrix}$$

$$= \begin{pmatrix} 9 \\ 2 \end{pmatrix}$$

$\therefore x = 9, y = 2$

The graphs of $x - y = 7$ and $x + y = 11$ intersect at the point (9, 2).

OXFORD UNIVERSITY PRESS

(b) The simultaneous equations may be written in matrix form as

$$\begin{pmatrix} 1 & 3 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 6 \\ 4 \end{pmatrix}$$

Determinant of $\begin{pmatrix} 1 & 3 \\ 2 & 1 \end{pmatrix} = (1 \times 1) - (3 \times 2)$

$= -5$

$$\begin{aligned} \begin{pmatrix} x \\ y \end{pmatrix} &= \begin{pmatrix} 1 & 3 \\ 2 & 1 \end{pmatrix}^{-1}\begin{pmatrix} 6 \\ 4 \end{pmatrix} \\ &= \frac{1}{-5}\begin{pmatrix} 1 & -3 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} 6 \\ 4 \end{pmatrix} \\ &= -\frac{1}{5}\begin{pmatrix} -6 \\ -8 \end{pmatrix} \\ &= \begin{pmatrix} 1\frac{1}{5} \\ 1\frac{3}{5} \end{pmatrix} \end{aligned}$$

$\therefore x = 1\frac{1}{5}, y = 1\frac{3}{5}$

The graphs of $x + 3y = 6$ and $2x + y = 4$ intersect at the point $\left(1\frac{1}{5},\ 1\frac{3}{5}\right)$.

(c) The simultaneous equations may be written in matrix form as

$$\begin{pmatrix} 3 & 2 \\ 5 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 7 \\ 3 \end{pmatrix}$$

Determinant of $\begin{pmatrix} 3 & 2 \\ 5 & -1 \end{pmatrix} = [3 \times (-1)] - (2 \times 5)$

$= -13$

$$\begin{aligned} \begin{pmatrix} x \\ y \end{pmatrix} &= \begin{pmatrix} 3 & 2 \\ 5 & -1 \end{pmatrix}\begin{pmatrix} 7 \\ 3 \end{pmatrix} \\ &= \frac{1}{-13}\begin{pmatrix} -1 & -2 \\ -5 & 3 \end{pmatrix}\begin{pmatrix} 7 \\ 3 \end{pmatrix} \\ &= -\frac{1}{13}\begin{pmatrix} -13 \\ -26 \end{pmatrix} \\ &= \begin{pmatrix} 1 \\ 2 \end{pmatrix} \end{aligned}$$

$\therefore x = 1, y = 2$

The graphs of $3x + 2y = 7$ and $5x - y = 3$ intersect at the point $(1, 2)$.

(d) The simultaneous equations may be written in matrix form as

$$\begin{pmatrix} 3 & 6 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 25 \\ 17 \end{pmatrix}$$

Determinant of $\begin{pmatrix} 3 & 6 \\ 3 & 4 \end{pmatrix} = (3 \times 4) - (6 \times 3)$

$= -6$

$$\begin{aligned} \begin{pmatrix} x \\ y \end{pmatrix} &= \begin{pmatrix} 3 & 6 \\ 3 & 4 \end{pmatrix}^{-1}\begin{pmatrix} 25 \\ 17 \end{pmatrix} \\ &= \frac{1}{-6}\begin{pmatrix} 4 & -6 \\ -3 & 3 \end{pmatrix}\begin{pmatrix} 25 \\ 17 \end{pmatrix} \\ &= -\frac{1}{6}\begin{pmatrix} -2 \\ -24 \end{pmatrix} \\ &= \begin{pmatrix} \frac{1}{3} \\ 4 \end{pmatrix} \end{aligned}$$

$\therefore x = \frac{1}{3},\ y = 4$

The graphs of $3x + 6y = 25$ and $3x + 4y = 17$ intersect at the point $\left(\frac{1}{3}, 4\right)$.

(e) The simultaneous equations may be written in matrix form as

$$\begin{pmatrix} 6 & 7 \\ 5 & 6 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$$

Determinant of $\begin{pmatrix} 6 & 7 \\ 5 & 6 \end{pmatrix} = (6 \times 6) - (7 \times 5)$

$= 1$

$$\begin{aligned} \begin{pmatrix} x \\ y \end{pmatrix} &= \begin{pmatrix} 6 & 7 \\ 5 & 6 \end{pmatrix}^{-1}\begin{pmatrix} 4 \\ 3 \end{pmatrix} \\ &= \begin{pmatrix} 6 & -7 \\ -5 & 6 \end{pmatrix}\begin{pmatrix} 4 \\ 3 \end{pmatrix} \\ &= \begin{pmatrix} 3 \\ -2 \end{pmatrix} \end{aligned}$$

$\therefore x = 3,\ y = -2$

The graphs of $6x + 7y = 4$ and $5x + 6y = 3$ intersect at the point $(3, -2)$.

(f) The simultaneous equations may be written in matrix form as

$$\begin{pmatrix} 2 & -5 \\ 3 & -7 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

Determinant of $\begin{pmatrix} 2 & -5 \\ 3 & -7 \end{pmatrix} = [2 \times (-7)] - [(-5) \times 3]$

$= 1$

$$\begin{aligned} \begin{pmatrix} x \\ y \end{pmatrix} &= \begin{pmatrix} 2 & -5 \\ 3 & -7 \end{pmatrix}^{-1}\begin{pmatrix} 1 \\ 2 \end{pmatrix} \\ &= \begin{pmatrix} -7 & 5 \\ -3 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \end{pmatrix} \\ &= \begin{pmatrix} 3 \\ 1 \end{pmatrix} \end{aligned}$$

$\therefore x = 3,\ y = 1$

The graphs of $2x - 5y = 1$ and $3x - 7y = 2$ intersect at the point $(3, 1)$.

OXFORD UNIVERSITY PRESS

(g) simultaneous equations may be written in matrix form as

$$\begin{pmatrix}2&-4\\1&-2\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}8\\6\end{pmatrix}$$

$$\text{Determinant of }\begin{pmatrix}2&-4\\1&-2\end{pmatrix}=[2\times(-2)]-[(-4)\times 1]$$

$$=0$$

Hence $\begin{pmatrix}2&-4\\1&-2\end{pmatrix}$ is a singular matrix and its inverse matrix does not exist. The graphs of $2x-4y=8$ and $x-2y=6$ represent two parallel lines. There is no solution since there is no intersection between the two lines.

(h) The simultaneous equations may be written in matrix form as

$$\begin{pmatrix}4&2\\2&1\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}10\\5\end{pmatrix}$$

$$\text{Determinant of }\begin{pmatrix}4&2\\2&1\end{pmatrix}=(4\times 1)-(2\times 2)$$

$$=0$$

Hence $\begin{pmatrix}4&2\\2&1\end{pmatrix}$ is a singular matrix and its inverse matrix does not exist. The first equation is obtained from the second by multiplying throughout by 2. Thus, the graphs of $4x+2y=10$ and $2x+y=5$ represent the same line, i.e. the two lines coincide.

3. $$\begin{pmatrix}220&430&555&355\\245&485&520&310\\280&430&515&375\end{pmatrix}\begin{pmatrix}130\\115\\90\\75\end{pmatrix}=\begin{pmatrix}154\,625\\157\,675\\160\,325\end{pmatrix}$$

Total amount collected = \$154 625 + \$157 675 + \$160 325
= \$472 625

4. **(i)** $$\begin{pmatrix}85&74&80&60&82\\65&84&70&52&94\\38&42&56&40&56\end{pmatrix}\text{ and }\begin{pmatrix}2.80\\2.40\\2.60\\3.00\\2.50\end{pmatrix}$$

$$\begin{pmatrix}85&74&80&60&82\\65&84&70&52&94\\38&42&56&40&56\end{pmatrix}\begin{pmatrix}2.80\\2.40\\2.60\\3.00\\2.50\end{pmatrix}=\begin{pmatrix}1008.60\\956.60\\612.80\end{pmatrix}$$

(ii) Total takings for the pie company
= \$1008.60 + \$956.60 + \$612.80
= \$2578

5. **(i)** $$\begin{pmatrix}22&32&42&28\\18&26&36&32\\27&24&52&25\end{pmatrix}\text{ and }\begin{pmatrix}0.90\\1.00\\1.10\\1.20\end{pmatrix}$$

$$\begin{pmatrix}22&32&42&28\\18&26&36&32\\27&24&52&25\end{pmatrix}\begin{pmatrix}0.90\\1.00\\1.10\\1.20\end{pmatrix}=\begin{pmatrix}131.60\\120.20\\135.50\end{pmatrix}$$

(ii) $$\begin{pmatrix}26&29&30\end{pmatrix}\begin{pmatrix}131.60\\120.20\\135.50\end{pmatrix}=(10\,972.4)$$

Total amount collected by all three stalls in January
= \$10 972.40

6. **(i)** $$\begin{pmatrix}220&240&180&85\\50&60&210&135\\10&40&200&250\end{pmatrix}\text{ and }\begin{pmatrix}15\\13.5\\12\\10\end{pmatrix}$$

$$\begin{pmatrix}220&240&180&85\\50&60&210&135\\10&40&200&250\end{pmatrix}\begin{pmatrix}15\\13.5\\12\\10\end{pmatrix}=\begin{pmatrix}9550\\5430\\5590\end{pmatrix}$$

(ii) $$\begin{pmatrix}220&240&180&85\\50&60&210&135\\10&40&200&250\end{pmatrix}\begin{pmatrix}1\\1\\1\\1\end{pmatrix}=\begin{pmatrix}725\\455\\500\end{pmatrix}$$

It represents the total number of T-shirts ordered for men, women and children.

(iii) $$\begin{pmatrix}1&1&1\end{pmatrix}\begin{pmatrix}220&240&180&85\\50&60&210&135\\10&40&200&250\end{pmatrix}=\begin{pmatrix}280&340&590&470\end{pmatrix}$$

It represents the total number of T-shirts ordered for each size.

(iv) $$\begin{pmatrix}280&340&590&470\end{pmatrix}\begin{pmatrix}15\\13.5\\12\\10\end{pmatrix}=(20\quad 570)$$

or

$$\begin{pmatrix}1&1&1\end{pmatrix}\begin{pmatrix}9550\\5430\\5590\end{pmatrix}=(20\ 570)$$

Total cost = \$20 570

7. $$\text{Determinant of }\begin{pmatrix}3&4\\2&6\end{pmatrix}=(3\times 6)-(4\times 2)$$

$$=10$$

$$\begin{pmatrix}3&4\\2&6\end{pmatrix}^{-1}=\frac{1}{10}\begin{pmatrix}6&-4\\-2&3\end{pmatrix}$$

The simultaneous equations may be written in matrix form as

$$\begin{pmatrix}3&4\\2&6\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}18\\22\end{pmatrix}$$

$$\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}3&4\\2&6\end{pmatrix}^{-1}\begin{pmatrix}18\\22\end{pmatrix}$$

$$=\frac{1}{10}\begin{pmatrix}6&-4\\-2&3\end{pmatrix}\begin{pmatrix}18\\22\end{pmatrix}$$

$$=\frac{1}{10}\begin{pmatrix}20\\30\end{pmatrix}$$

$$=\begin{pmatrix}2\\3\end{pmatrix}$$

$\therefore x=2,\ y=3$

The graphs of $3x+4y=18$ and $2x+6y=22$ intersect at the point (2, 3).

OXFORD UNIVERSITY PRESS

8. Determinant of $\begin{pmatrix} 5 & 7 \\ 2 & 3 \end{pmatrix} = (5 \times 3) - (7 \times 2)$

$$= 1$$

$$\begin{pmatrix} 5 & 7 \\ 2 & 3 \end{pmatrix}^{-1} = \begin{pmatrix} 3 & -7 \\ -2 & 5 \end{pmatrix}$$

The simultaneous equations may be written in matrix form as

$$\begin{pmatrix} 5 & 7 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 19 \\ 8 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 & 7 \\ 2 & 3 \end{pmatrix}^{-1}\begin{pmatrix} 19 \\ 8 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & -7 \\ -2 & 5 \end{pmatrix}\begin{pmatrix} 19 \\ 8 \end{pmatrix}$$

$$= \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$\therefore x = 1,\ y = 2$

The graphs of $5x + 7y = 19$ and $2x + 3y = 8$ intersect at the point $(1, 2)$.

This method cannot be applied to the lines $3x + 4y = 7$ and $6x + 8y = 9$ as they are parallel and an inverse matrix does not exist.

9. Determinant of $\begin{pmatrix} 7 & -11 \\ 2 & -3 \end{pmatrix} = [7 \times (-3)] - [(-11) \times 2]$

$$= 1$$

$$\begin{pmatrix} 7 & -11 \\ 2 & -3 \end{pmatrix}^{-1} = \begin{pmatrix} -3 & 11 \\ -2 & 7 \end{pmatrix}$$

The simultaneous equations may be written in matrix form as

$$\begin{pmatrix} 7 & -11 \\ 2 & -3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 10 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 7 & -11 \\ 2 & -3 \end{pmatrix}^{-1}\begin{pmatrix} 10 \\ 3 \end{pmatrix}$$

$$= \begin{pmatrix} -3 & 11 \\ -2 & 7 \end{pmatrix}\begin{pmatrix} 10 \\ 3 \end{pmatrix}$$

$$= \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

$\therefore x = 3,\ y = 1$

The graphs of $7x - 11y = 10$ and $2x - 3y = 3$ intersect at the point $(3, 1)$.

This method cannot be applied to the lines $3x + 2y = 7$ and $6x + 4y = 14$ as the lines are identical and an inverse matrix does not exist.

10. The simultaneous equations may be written in matrix form as

$$\begin{pmatrix} 2 & k \\ 4 & -9 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 7 \\ 4 \end{pmatrix}$$

For the above to have no solution, determinant of $\begin{pmatrix} 2 & k \\ 4 & -9 \end{pmatrix} = 0$, i.e.

$$[2 \times (-9)] - (k + 4) = 0$$

$$-18 - 4k = 0$$

$$4k = -18$$

$\therefore k = -4$

11. (i) $\begin{pmatrix} 2 & 6 & 5 & 4 & 5 \\ 3 & 8 & 2 & 3 & 2 \\ 4 & 9 & 3 & 6 & 3 \\ 3 & 5 & 6 & 3 & 4 \end{pmatrix}$ and $\begin{pmatrix} 30 \\ 1.80 \\ 4.80 \\ 3.50 \\ 2.40 \end{pmatrix}$

$$\begin{pmatrix} 2 & 6 & 5 & 4 & 5 \\ 3 & 8 & 2 & 3 & 2 \\ 4 & 9 & 3 & 6 & 3 \\ 3 & 5 & 6 & 3 & 4 \end{pmatrix}\begin{pmatrix} 30 \\ 1.80 \\ 4.80 \\ 3.50 \\ 2.40 \end{pmatrix} = \begin{pmatrix} 120.8 \\ 129.3 \\ 178.8 \\ 147.9 \end{pmatrix}$$

(ii) $(85 \quad 90 \quad 80 \quad 120)$ and $\begin{pmatrix} 120.8 \\ 129.3 \\ 178.8 \\ 147.9 \end{pmatrix}$

$$(85 \quad 90 \quad 80 \quad 120)\begin{pmatrix} 120.8 \\ 129.3 \\ 178.8 \\ 147.9 \end{pmatrix} = (53\,957)$$

(iii) $\begin{pmatrix} 1.3 & 0 & 0 & 0 \\ 0 & 1.25 & 0 & 0 \\ 0 & 0 & 1.20 & 0 \\ 0 & 0 & 0 & 1.15 \end{pmatrix}$ and $\begin{pmatrix} 120.8 \\ 129.3 \\ 178.8 \\ 147.9 \end{pmatrix}$

$$\begin{pmatrix} 1.3 & 0 & 0 & 0 \\ 0 & 1.25 & 0 & 0 \\ 0 & 0 & 1.20 & 0 \\ 0 & 0 & 0 & 1.15 \end{pmatrix}\begin{pmatrix} 120.8 \\ 129.3 \\ 178.8 \\ 147.9 \end{pmatrix} = \begin{pmatrix} 157.04 \\ 161.63 \\ 214.56 \\ 170.09 \end{pmatrix}$$

12. (i) $\begin{pmatrix} 4 & 5 & 6 \\ 3 & 6 & 7 \\ 5 & 8 & 6 \\ 6 & 4 & 5 \end{pmatrix}$ and $\begin{pmatrix} 12 \\ 15 \\ 24 \end{pmatrix}$

$$\begin{pmatrix} 4 & 5 & 6 \\ 3 & 6 & 7 \\ 5 & 8 & 6 \\ 6 & 4 & 5 \end{pmatrix}\begin{pmatrix} 12 \\ 15 \\ 24 \end{pmatrix} = \begin{pmatrix} 267 \\ 294 \\ 324 \\ 252 \end{pmatrix}$$

(ii) $(60 \quad 80 \quad 90 \quad 80)\begin{pmatrix} 267 \\ 294 \\ 324 \\ 252 \end{pmatrix} = (88\,860)$

Total cost of the order is 88 860 cents.

13. (i) $(280 \quad 320 \quad 360)\begin{pmatrix} 1.2 & 0 & 1.4 & 2.6 & 5.2 \\ 0 & 1.6 & 1.6 & 2.8 & 4.7 \\ 1.4 & 1.8 & 0 & 3 & 4.4 \end{pmatrix}$

$= (840 \quad 1160 \quad 904 \quad 2704 \quad 4544)$

(ii) $(840 \quad 1160 \quad 904 \quad 2704 \quad 4544)\begin{pmatrix} 12.50 \\ 5.20 \\ 7.80 \\ 1.40 \\ 1.10 \end{pmatrix} = (32\,367.20)$

Total cost is \$32 367.20.

OXFORD UNIVERSITY PRESS

Review Exercise 5

1. **(a)**
$$\begin{aligned}&\begin{pmatrix}3&2\\4&-1\end{pmatrix}+\begin{pmatrix}5&4\\3&2\end{pmatrix}-\begin{pmatrix}6&3\\1&-2\end{pmatrix}\\
&=\left[\begin{pmatrix}3&2\\4&-1\end{pmatrix}+\begin{pmatrix}5&4\\3&2\end{pmatrix}\right]-\begin{pmatrix}6&3\\1&-2\end{pmatrix}\\
&=\begin{pmatrix}8&6\\7&1\end{pmatrix}-\begin{pmatrix}6&3\\1&-2\end{pmatrix}\\
&=\begin{pmatrix}2&3\\6&3\end{pmatrix}\end{aligned}$$

(b)
$$\begin{aligned}&\begin{pmatrix}2&3&-4\\6&-1&3\end{pmatrix}+\begin{pmatrix}3&1&5\\-3&2&7\end{pmatrix}+\begin{pmatrix}1&0&0\\0&1&1\end{pmatrix}\\
&=\left[\begin{pmatrix}2&3&-4\\6&-1&3\end{pmatrix}+\begin{pmatrix}3&1&5\\-3&2&7\end{pmatrix}\right]+\begin{pmatrix}1&0&0\\0&1&1\end{pmatrix}\\
&=\begin{pmatrix}5&4&1\\3&1&10\end{pmatrix}+\begin{pmatrix}1&0&0\\0&1&1\end{pmatrix}\\
&=\begin{pmatrix}6&4&1\\3&2&11\end{pmatrix}\end{aligned}$$

(c)
$$\begin{aligned}&\begin{pmatrix}2&3\\4&-7\\5&-3\end{pmatrix}-\begin{pmatrix}4&5\\-2&7\\6&-1\end{pmatrix}+\begin{pmatrix}-3&4\\-1&7\\-6&2\end{pmatrix}\\
&=\left[\begin{pmatrix}2&3\\4&-7\\5&-3\end{pmatrix}-\begin{pmatrix}4&5\\-2&7\\6&-1\end{pmatrix}\right]+\begin{pmatrix}-3&4\\-1&7\\-6&2\end{pmatrix}\\
&=\begin{pmatrix}-2&-2\\6&-14\\-1&-2\end{pmatrix}+\begin{pmatrix}-3&4\\-1&7\\-6&2\end{pmatrix}\\
&=\begin{pmatrix}-5&2\\5&-7\\-7&0\end{pmatrix}\end{aligned}$$

(d)
$$\begin{aligned}&\begin{pmatrix}3\\5\end{pmatrix}-\begin{pmatrix}4\\7\end{pmatrix}+\begin{pmatrix}5\\-3\end{pmatrix}\\
&=\left[\begin{pmatrix}3\\5\end{pmatrix}-\begin{pmatrix}4\\7\end{pmatrix}\right]+\begin{pmatrix}5\\-3\end{pmatrix}\\
&=\begin{pmatrix}-1\\-2\end{pmatrix}+\begin{pmatrix}5\\-3\end{pmatrix}\\
&=\begin{pmatrix}4\\-5\end{pmatrix}\end{aligned}$$

(e)
$$\begin{aligned}&(1\quad 3)-(3\quad 2)+(6\quad 5)\\
&=[(1\quad 3)-(3\quad 2)]+(6\quad 5)\\
&=(-2\quad 1)+(6\quad 5)\\
&=(4\quad 6)\end{aligned}$$

(f)
$$\begin{aligned}&(1\quad 0\quad 7)+(3\quad -2\quad 4)-(7\quad 3\quad -5)\\
&=[(1\quad 0\quad 7)+(3\quad -2\quad 4)]-(7\quad 3\quad -5)\\
&=(4\quad -2\quad 11)-(7\quad 3\quad -5)\\
&=(-3\quad -5\quad 16)\end{aligned}$$

2. **(i)**
$$\begin{aligned}2\mathbf{A}+\mathbf{B}&=\mathbf{C}\\
2\begin{pmatrix}2&-1\\1&3\end{pmatrix}+\begin{pmatrix}5&a\\c&4\end{pmatrix}&=\begin{pmatrix}b&6\\4&d\end{pmatrix}\\
\begin{pmatrix}4&-2\\2&6\end{pmatrix}+\begin{pmatrix}5&a\\c&4\end{pmatrix}&=\begin{pmatrix}b&6\\4&d\end{pmatrix}\\
\begin{pmatrix}9&-2+a\\2+c&10\end{pmatrix}&=\begin{pmatrix}b&6\\4&d\end{pmatrix}\end{aligned}$$

Equating the corresponding elements, we have

$$\begin{aligned}-2+a&=6\\
a&=8\\
b&=9\\
2+c&=4\\
c&=2\\
d&=10\end{aligned}$$

$\therefore a=8, b=9, c=2$ and $d=10$

(ii)
$$\begin{aligned}3\mathbf{A}-2\mathbf{B}&=4\mathbf{C}\\
3\begin{pmatrix}2&-1\\1&3\end{pmatrix}-2\begin{pmatrix}5&a\\c&4\end{pmatrix}&=4\begin{pmatrix}b&6\\4&d\end{pmatrix}\\
\begin{pmatrix}6&-3\\3&9\end{pmatrix}-\begin{pmatrix}10&2a\\2c&8\end{pmatrix}&=\begin{pmatrix}4b&24\\16&4d\end{pmatrix}\\
\begin{pmatrix}-4&-3-2a\\3-2c&1\end{pmatrix}&=\begin{pmatrix}4b&24\\16&4d\end{pmatrix}\end{aligned}$$

Equating the corresponding elements, we have

$$\begin{aligned}-3-2a&=24\\
-2a&=27\\
a&=-13\frac{1}{2}\\
4b&=-4\\
b&=-1\\
3-2c&=16\\
-2c&=13\\
c&=-6\frac{1}{2}\\
4d&=1\\
d&=\frac{1}{4}\end{aligned}$$

$\therefore a=-13\frac{1}{2}, b=-1, c=-6\frac{1}{2}$ and $d=\frac{1}{4}$

3. **(a)**
$$\begin{aligned}\begin{pmatrix}1\\3\end{pmatrix}(3\quad 1)&=\begin{pmatrix}1\times 3&1\times 1\\3\times 3&3\times 1\end{pmatrix}\\
&=\begin{pmatrix}3&1\\9&3\end{pmatrix}\end{aligned}$$

(b)
$$\begin{aligned}(2\quad 3)\begin{pmatrix}3\\1\end{pmatrix}&=(2\times 3+3\times 1)\\
&=(9)\end{aligned}$$

(c)
$$\begin{aligned}(1\quad 2\quad 3)\begin{pmatrix}3\\2\\1\end{pmatrix}&=(1\times 3+2\times 2+3\times 1)\\
&=(10)\end{aligned}$$

OXFORD UNIVERSITY PRESS

(d) $$\begin{pmatrix}1\\2\\3\end{pmatrix}(3 \quad 0 \quad 1) = \begin{pmatrix}1\times3 & 1\times0 & 1\times1\\ 2\times3 & 2\times0 & 2\times1\\ 3\times3 & 3\times0 & 3\times1\end{pmatrix}$$
$$= \begin{pmatrix}3 & 0 & 1\\ 6 & 0 & 2\\ 9 & 0 & 3\end{pmatrix}$$

(e) Not possible

(f) $$\begin{pmatrix}-2 & 3\\ -1 & -2\end{pmatrix}\begin{pmatrix}\frac{1}{2}\\ 1\frac{1}{2}\end{pmatrix} = \begin{pmatrix}-2\times\frac{1}{2}+3\times1\frac{1}{2}\\ -1\times\frac{1}{2}+(-2)\times1\frac{1}{2}\end{pmatrix}$$
$$= \begin{pmatrix}3\frac{1}{2}\\ -3\frac{1}{2}\end{pmatrix}$$

(g) $$\begin{pmatrix}0 & 2\\ 3 & 1\\ -1 & 1\end{pmatrix}\begin{pmatrix}2\\1\end{pmatrix} = \begin{pmatrix}0\times2+2\times1\\ 3\times2+1\times1\\ -1\times2+1\times1\end{pmatrix}$$
$$= \begin{pmatrix}2\\7\\-1\end{pmatrix}$$

(h) $$\begin{pmatrix}0 & -2\\ -1 & 1\\ 3 & -1\end{pmatrix}\begin{pmatrix}2 & 1\\ 0 & -4\end{pmatrix}$$
$$= \begin{pmatrix}0\times2+(-2)\times0 & 0\times1+(-2)\times(-4)\\ -1\times2+1\times0 & -1\times1+1\times(-4)\\ 3\times2+(-1)\times0 & 3\times1+(-1)\times(-4)\end{pmatrix}$$
$$= \begin{pmatrix}0 & 8\\ -2 & -5\\ 6 & 7\end{pmatrix}$$

(i) $$\begin{pmatrix}2 & 1 & 3\\ -1 & -1 & 4\end{pmatrix}\begin{pmatrix}1\\2\\-1\end{pmatrix} = \begin{pmatrix}2\times1+1\times2+3\times(-1)\\ -1\times1+(-1)\times2+4\times(-1)\end{pmatrix}$$
$$= \begin{pmatrix}1\\-7\end{pmatrix}$$

(j) $$(1 \quad 3 \quad 2)\begin{pmatrix}3 & -2\\ 1 & 4\\ -1 & 2\end{pmatrix}$$
$$= (1\times3+3\times1+2\times(-1) \quad 1\times(-2)+3\times4+2\times2)$$
$$= (4 \quad 14)$$

4. **(a)** $$\begin{pmatrix}1 & 3 & 2\\ 0 & 1 & -2\end{pmatrix}\begin{pmatrix}1\\a\\2\end{pmatrix} = \begin{pmatrix}5\\b\end{pmatrix}$$
$$\begin{pmatrix}1+3a+4\\ 0+a-4\end{pmatrix} = \begin{pmatrix}5\\b\end{pmatrix}$$
$$\begin{pmatrix}5+3a\\ a-4\end{pmatrix} = \begin{pmatrix}5\\b\end{pmatrix}$$

Equating the corresponding elements, we have

$5 + 3a = 5$

$a = 0$

$a - 4 = b$

$0 - 4 = b$

$b = -4$

$\therefore$ $a = 0$ and $b = -4$

(b) $$\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}2 & 1\\ 3 & 0\end{pmatrix}\begin{pmatrix}2\\5\end{pmatrix} + \begin{pmatrix}3\\2\end{pmatrix}$$
$$= \begin{pmatrix}9\\6\end{pmatrix} + \begin{pmatrix}3\\2\end{pmatrix}$$
$$= \begin{pmatrix}12\\8\end{pmatrix}$$

Equating the corresponding elements, we have

$x = 12$ and $y = 8$

(c) $$\begin{pmatrix}0 & 1\\ -2 & 0\end{pmatrix}\begin{pmatrix}a & -4\\ b & 0\end{pmatrix} = \begin{pmatrix}2 & 3\\ 0 & 1\end{pmatrix} + \begin{pmatrix}0 & -3\\ 6 & 2c\end{pmatrix}$$
$$\begin{pmatrix}b & 0\\ -2a & 8\end{pmatrix} = \begin{pmatrix}2 & 0\\ 6 & 1+2c\end{pmatrix}$$

Equating the corresponding elements, we have

$-2a = 6$

$a = -3$

$b = 2$

$1 + 2c = 8$

$c = 3\frac{1}{2}$

$\therefore$ $a = -3$, $b = 2$ and $c = 3\frac{1}{2}$

5. Determinant of $\begin{pmatrix}4 & 2\\ 7 & -b\end{pmatrix} = 22$

$[4 \times (-b)] - (2 \times 7) = 22$

$-4b - 14 = 22$

$4b = -36$

$b = -9$

$$\begin{pmatrix}4 & 2\\ 7 & -b\end{pmatrix}^{-1} = \frac{1}{22}\begin{pmatrix}9 & -2\\ -7 & 4\end{pmatrix}$$

6. **(a)** Determinant of $\begin{pmatrix}-1 & 0\\ 0 & -1\end{pmatrix} = [(-1)\times(-1)] - 0$

$= 1$

$$\begin{pmatrix}-1 & 0\\ 0 & -1\end{pmatrix}^{-1} = \begin{pmatrix}-1 & 0\\ 0 & -1\end{pmatrix}$$

(b) Determinant of $\begin{pmatrix}-1 & -1\\ 0 & -1\end{pmatrix} = [(-1)\times(-1)] - 0$

$= 1$

$$\begin{pmatrix}-1 & -1\\ 0 & -1\end{pmatrix}^{-1} = \begin{pmatrix}-1 & 0\\ 0 & -1\end{pmatrix}$$

OXFORD UNIVERSITY PRESS

(c) Determinant of $\begin{pmatrix} 2 & 8 \\ 1 & 4 \end{pmatrix} = (2 \times 4) - (8 \times 1)$

$= 0$

$\therefore \begin{pmatrix} 2 & 8 \\ 1 & 4 \end{pmatrix}$ is a singular matrix and has no inverse.

(d) Determinant of $\begin{pmatrix} 3 & 5 \\ 4 & 8 \end{pmatrix} = (3 \times 8) - (5 \times 4)$

$= 4$

$$\begin{pmatrix} 3 & 5 \\ 4 & 8 \end{pmatrix}^{-1} = \frac{1}{8}\begin{pmatrix} 8 & -5 \\ -4 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & -\frac{5}{4} \\ -1 & \frac{3}{4} \end{pmatrix}$$

(e) Determinant of $\begin{pmatrix} \frac{3}{5} & -\frac{4}{5} \\ \frac{4}{5} & \frac{3}{5} \end{pmatrix} = \left(\frac{3}{5} \times \frac{3}{5}\right) - \left[\left(-\frac{4}{5}\right) \times \left(\frac{4}{5}\right)\right]$

$= 1$

$$\begin{pmatrix} \frac{3}{5} & -\frac{4}{5} \\ \frac{4}{5} & \frac{3}{5} \end{pmatrix}^{-1} = \begin{pmatrix} \frac{3}{5} & \frac{4}{5} \\ -\frac{4}{5} & \frac{3}{5} \end{pmatrix}$$

7. **(a)** The simultaneous equations may be written in matrix form as

$$\begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 90 \\ 90 \end{pmatrix}$$

Determinant of $\begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix} = (2 \times 3) - (1 \times 1)$

$= 5$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}^{-1}\begin{pmatrix} 90 \\ 90 \end{pmatrix}$$

$$= \frac{1}{5}\begin{pmatrix} 3 & -1 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 90 \\ 90 \end{pmatrix}$$

$$= \frac{1}{5}\begin{pmatrix} 180 \\ 90 \end{pmatrix}$$

$$= \begin{pmatrix} 36 \\ 18 \end{pmatrix}$$

$\therefore x = 36,\ y = 18$

The graphs of $2x + y = 90$ and $x + 3y = 90$ intersect at the point $(36, 18)$.

(b) The simultaneous equations may be written in matrix form as

$$\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

Determinant of $\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} = (1 \times 1) - [1 \times (-1)]$

$= 2$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}^{-1}\begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} 0 \\ 4 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$\therefore x = 0,\ y = 2$

The graphs of $x + y = 2$ and $y - x = 2$ intersect at the point $(0, 2)$.

(c) The simultaneous equations may be written in matrix form as

$$\begin{pmatrix} 12 & -6 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 24 \\ 4 \end{pmatrix}$$

Determinant of $\begin{pmatrix} 12 & -6 \\ 2 & -1 \end{pmatrix} = [12 \times (-1)] - [(-6) \times 2]$

$= 0$

Hence $\begin{pmatrix} 12 & -6 \\ 2 & -1 \end{pmatrix}$ is a singular matrix and its inverse matrix does not exist. The first equation is obtained from the second by multiplying throughout by 6. Thus, the graphs of $12x - 6y = 24$ and $2x - y = 4$ represent the same line, i.e. the two lines coincide.

(d) The simultaneous equations may be written in matrix form as

$$\begin{pmatrix} 25 & 5 \\ 5 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 9 \\ 2 \end{pmatrix}$$

Determinant of $\begin{pmatrix} 25 & 5 \\ 5 & 1 \end{pmatrix} = (25 \times 1) - (5 \times 5)$

$= 0$

Hence $\begin{pmatrix} 25 & 5 \\ 5 & 1 \end{pmatrix}$ is a singular matrix and its inverse matrix does not exist. The graphs of $25x + 5y = 9$ and $5x + y = 2$ represent two parallel lines. There is no solution since there is no intersection between the two lines.

8. $\mathbf{P} = \begin{pmatrix} 450 & 240 & 120 & 80 & 60 \\ 250 & 140 & 80 & 60 & 20 \\ 280 & 120 & 50 & 30 & 24 \end{pmatrix}$ and $\mathbf{Q} = \begin{pmatrix} 1.00 \\ 1.50 \\ 6.50 \\ 5.50 \\ 4.80 \end{pmatrix}$

$$\mathbf{PQ} = \begin{pmatrix} 450 & 240 & 120 & 80 & 60 \\ 250 & 140 & 80 & 60 & 20 \\ 280 & 120 & 50 & 30 & 24 \end{pmatrix}\begin{pmatrix} 1.00 \\ 1.50 \\ 6.50 \\ 5.50 \\ 4.80 \end{pmatrix}$$

$$= \begin{pmatrix} 2318 \\ 1406 \\ 1065.20 \end{pmatrix}$$

OXFORD UNIVERSITY PRESS

9. **(i)** $\begin{pmatrix} 12 & 8 & 12 & 15 \\ 15 & 0 & 16 & 14 \\ 0 & 20 & 25 & 16 \end{pmatrix}$ and $\begin{pmatrix} 8.40 \\ 7.80 \\ 8.80 \\ 8.20 \end{pmatrix}$

$$\begin{pmatrix} 12 & 8 & 12 & 15 \\ 15 & 0 & 16 & 14 \\ 0 & 20 & 25 & 16 \end{pmatrix}\begin{pmatrix} 8.40 \\ 7.80 \\ 8.80 \\ 8.20 \end{pmatrix} = \begin{pmatrix} 391.80 \\ 381.60 \\ 507.20 \end{pmatrix}$$

(ii) $(22 \quad 18 \quad 25)\begin{pmatrix} 391.80 \\ 381.60 \\ 507.20 \end{pmatrix} = (28\ 168.40)$

Total amount of money the factory collected from the three shops during the period is \$28 168.40.

10. $$\begin{pmatrix} 480 & 460 & 620 & 430 \\ 350 & 450 & 385 & 540 \\ 420 & 520 & 420 & 620 \\ 380 & 452 & 250 & 486 \end{pmatrix}\begin{pmatrix} 10 \\ 20 \\ 50 \\ 100 \end{pmatrix} = \begin{pmatrix} 88\ 000 \\ 85\ 750 \\ 97\ 600 \\ 73\ 940 \end{pmatrix}$$

$$(1 \quad 1 \quad 1 \quad 1)\begin{pmatrix} 88\ 000 \\ 85\ 750 \\ 93\ 820 \\ 73\ 940 \end{pmatrix} = (345\ 290)$$

Total amount collected by the four machines is \$3452.90.

11. **(i)** $\begin{pmatrix} 11 & 2 & 5 \\ 7 & 2 & 11 \\ 4 & 5 & 10 \\ 7 & 4 & 7 \\ 12 & 1 & 9 \\ 9 & 2 & 8 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}$

$$\begin{pmatrix} 11 & 2 & 5 \\ 7 & 2 & 11 \\ 4 & 5 & 10 \\ 7 & 4 & 7 \\ 12 & 1 & 9 \\ 9 & 2 & 8 \end{pmatrix}\begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 35 \\ 23 \\ 17 \\ 25 \\ 37 \\ 29 \end{pmatrix}$$

(ii) $\begin{pmatrix} 18 & 11 & 2 & 5 \\ 20 & 7 & 2 & 11 \\ 19 & 4 & 5 & 10 \\ 18 & 7 & 4 & 7 \\ 22 & 12 & 1 & 9 \\ 19 & 9 & 2 & 8 \end{pmatrix}$ and $\begin{pmatrix} 300 \\ 500 \\ 200 \\ -300 \end{pmatrix}$

$$\begin{pmatrix} 18 & 11 & 2 & 5 \\ 20 & 7 & 2 & 11 \\ 19 & 4 & 5 & 10 \\ 18 & 7 & 4 & 7 \\ 22 & 12 & 1 & 9 \\ 19 & 9 & 2 & 8 \end{pmatrix}\begin{pmatrix} 300 \\ 500 \\ 200 \\ -300 \end{pmatrix} = \begin{pmatrix} 9800 \\ 6600 \\ 5700 \\ 7600 \\ 10\ 100 \\ 8200 \end{pmatrix}$$

Challenge Yourself

1. **(a)** $\mathbf{A} + \begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} -4 \\ 3 \end{pmatrix}$

$$\mathbf{A} = \begin{pmatrix} -4 \\ 3 \end{pmatrix} - \begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

$$\mathbf{A} = \begin{pmatrix} -7 \\ -1 \end{pmatrix}$$

(b) $2\mathbf{A} - \begin{pmatrix} 0 & 6 \\ 9 & -7 \end{pmatrix} = 3\begin{pmatrix} 4 & 8 \\ -3 & 5 \end{pmatrix}$

$$2\mathbf{A} - \begin{pmatrix} 0 & 6 \\ 9 & -7 \end{pmatrix} = \begin{pmatrix} 12 & 24 \\ -9 & 15 \end{pmatrix}$$

$$2\mathbf{A} = \begin{pmatrix} 12 & 24 \\ -9 & 15 \end{pmatrix} + \begin{pmatrix} 0 & 6 \\ 9 & -7 \end{pmatrix}$$

$$2\mathbf{A} = \begin{pmatrix} 12 & 30 \\ 0 & 8 \end{pmatrix}$$

$$\mathbf{A} = \frac{1}{2}\begin{pmatrix} 12 & 30 \\ 0 & 8 \end{pmatrix}$$

$$\mathbf{A} = \begin{pmatrix} 6 & 15 \\ 0 & 4 \end{pmatrix}$$

2. **(i)** $\begin{pmatrix} 5 & 9 \\ 1 & 2 \end{pmatrix}\mathbf{X} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Let $\mathbf{X} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$.

$$\begin{pmatrix} 5 & 9 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 5a + 9c & 5b + 9d \\ a + 2c & b + 2d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Equating the corresponding elements, we have

$5a + 9c = 1$

$5a = 1 - 9c$

$a = \dfrac{1 - 9c}{5}$ — (1)

$a + 2c = 0$

$a = -2c$ — (2)

Substitute (2) into (1):

$\dfrac{1 - 9c}{5} = -2c$

$1 - 9c = -10c$

$c = -1$

Substitute $c = -1$ into (2):

$a = -2(-1)$

$= 2$

$5b + 9d = 0$

$5b = -9d$

$b = \dfrac{-9d}{5}$ — (3)

$b + 2d = 1$

$b = 1 - 2d$ — (4)

OXFORD UNIVERSITY PRESS

Substitute (4) into (3):

$$\frac{-9d}{5} = 1 - 2d$$
$$-9d = 5 - 10d$$
$$d = 5$$

Substitute $d = 5$ into (4):

$$b = 1 - 2(5)$$
$$= -9$$
$$\therefore \mathbf{X} = \begin{pmatrix} 2 & -9 \\ -1 & 5 \end{pmatrix}$$

(ii) $$Y\begin{pmatrix} 5 & 9 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Let $\mathbf{Y} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$.

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 5 & 9 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 5a+b & 9a+2b \\ 5c+d & 9c+2d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Equating the corresponding elements, we have

$$5a + b = 1$$
$$b = 1 - 5a \quad \text{— (1)}$$
$$9a + 2b = 0$$
$$a = \frac{-2b}{9} \quad \text{— (2)}$$

Substitute (2) into (1):

$$b = 1 - 5\left(\frac{-2b}{9}\right)$$
$$9b = 9 + 10b$$
$$b = -9$$

Substitute $b = -9$ into (2):

$$a = \frac{-2(-9)}{9}$$
$$= 2$$
$$5c + d = 0$$
$$d = -5c \quad \text{— (3)}$$
$$9c + 2d = 1$$
$$2d = 1 - 9c \quad \text{— (4)}$$

Substitute (3) into (4):

$$2(-5c) = 1 - 9c$$
$$-10c = 1 - 9c$$
$$c = -1$$

Substitute $c = -1$ into (3):

$$d = -5(-1)$$
$$= 5$$
$$\therefore \mathbf{Y} = \begin{pmatrix} 2 & -9 \\ -1 & 5 \end{pmatrix}$$

(iii) Yes. The multiplication of **X** or **Y** and $\begin{pmatrix} 5 & 9 \\ 1 & 2 \end{pmatrix}$ in either order will give the identity matrix $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. **X** or **Y** is the inverse of $\begin{pmatrix} 5 & 9 \\ 1 & 2 \end{pmatrix}$.

3. **(a)** $\mathbf{X} = \begin{pmatrix} 0 & 0 \\ 2 & 0 \end{pmatrix}$ and $\mathbf{Y} = \begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix}$

(b) Let $\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$.

$$\mathbf{AB} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\mathbf{AC} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

So **AB** = **AC**. But $\mathbf{A} \neq \mathbf{0}$ and $\mathbf{B} \neq \mathbf{C}$.

4. $\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$ is a 2 × 2 idempotent matrix.

$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ is a 3 × 3 idempotent matrix.

5. **(i)** Add $\begin{pmatrix} 5 & -1.2 & 0 \\ 5 & -1.2 & 0 \\ 5 & -1.2 & 0 \\ 5 & -1.2 & 0 \\ 5 & -1.2 & 0 \\ 5 & -1.2 & 0 \\ 5 & -1.2 & 0 \\ 5 & -1.2 & 0 \\ 5 & -1.2 & 0 \\ 5 & -1.2 & 0 \end{pmatrix}$ to **X**.

(ii) Add $0.05\begin{pmatrix} 0 & 0 & 120 \\ 0 & 0 & \cdots \\ 0 & 0 & \cdots \\ 0 & 0 & \cdots \\ 0 & 0 & \cdots \\ 0 & 0 & \cdots \\ 0 & 0 & \cdots \\ 0 & 0 & \cdots \\ 0 & 0 & \cdots \\ 0 & 0 & 100 \end{pmatrix}$ to **X**

or

$$\mathbf{X}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1.05 \end{pmatrix}$$

OXFORD UNIVERSITY PRESS

Chapter 6 Further Geometrical Transformations

TEACHING NOTES

Suggested Approach:

Teachers can get students to recall the transformations they have previously learnt in Book 2: reflection, rotation and translation. Teachers can then explain that students will be building on those concepts to learn non-isometric transformations, and how these can be linked to matrices. As this chapter involves a significant amount of geometrical constructions, teachers should revise the main construction steps with students, using pages 183 to 186 of the textbook.

Section 6.1: Enlargement

Teachers can revise the construction steps needed for the enlargement of a given figure with a positive scale factor. Teachers should then proceed to illustrate the construction steps for negative scale factors, and the construction steps involved in finding the centre of enlargement as well as the scale factor if given the original figure and its image.

Section 6.2: Transformation Matrices for Reflection and Rotation

Teachers can prompt students to observe the pattern of how coordinates of an image can be obtained following simple transformations such as a reflection in the x- or y-axis and the lines $y = x$ or $y = -x$, as well as for rotations of 90°, 180°, 270° and 260° about the origin. Teachers should emphasise that students need not memorise the matrices for these transformations.

Section 6.3: Transformation Matrix for Enlargement

Teachers should encourage students to remember and recognise the enlargement matrix as $\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$, where k is the scale factor and the origin is the centre of enlargement. For more advanced students, teachers may wish to introduce the stretch matrices $\begin{pmatrix} k & 0 \\ 0 & 1 \end{pmatrix}$, $\begin{pmatrix} 1 & 0 \\ 0 & k \end{pmatrix}$ and $\begin{pmatrix} k_1 & 0 \\ 0 & k_2 \end{pmatrix}$ and the shear matrices $\begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix}$ and $\begin{pmatrix} 1 & 0 \\ k & 1 \end{pmatrix}$.

Section 6.4: Inverse Transformations and Combined Transformations

Teachers should highlight to students the importance of identifying which transformation a figure undergoes first, as students tend to make mistakes in this aspect. For example, in the multiplication of matrices to obtain a combined transformation, the product **AB** is a transformation of **B** followed by **A**, and not the other way round. The use of matrices can also be tricky for some students. For example, for the following problem: Find the point (p, q) which is the image of the point $(1, 2)$ transformed by the matrix $\begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}$; errors such as the following are common:

$$\begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}(1 \quad 2) = (p \quad q),\ \begin{pmatrix} 1 \\ 2 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} p \\ q \end{pmatrix},\ (1 \quad 2)\begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix} = (p \quad q),\ \begin{pmatrix} 1 \\ 2 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix} = (p \quad q)$$

Hence, teachers should work on helping students grasp the concept of how to apply matrices correctly to a transformation.

Challenge Yourself

For Questions 2 and 3, students may choose any point, for instance (1, 1), in order to ascertain the image of this particular point under the transformations. From there, the single transformation can be generalised. Teachers should also remind students to think carefully about which transformation should occur first.

OXFORD UNIVERSITY PRESS

WORKED SOLUTIONS

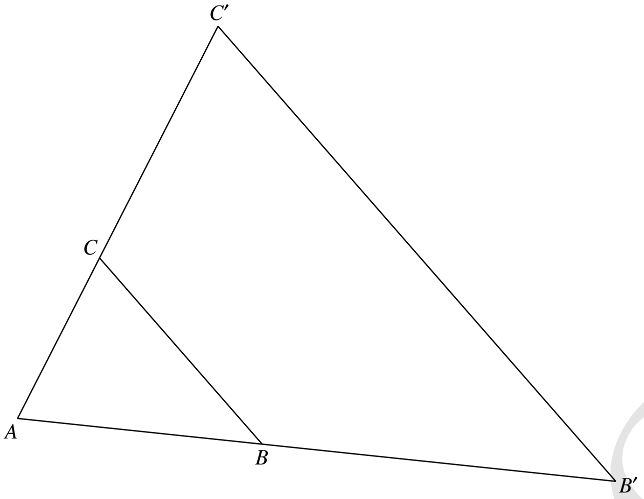
Class Discussion (Enlargement in our surroundings)

Teachers can come up with an example of enlargement in the classroom, for example an A5 notebook compared to an A4 one, before getting students to build on this and discuss with each other.

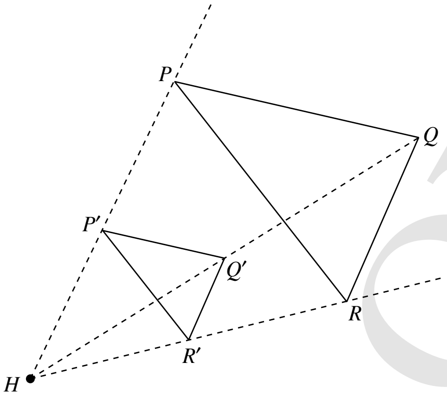
Thinking Time (Page 206)

No, the order of transformation matters.

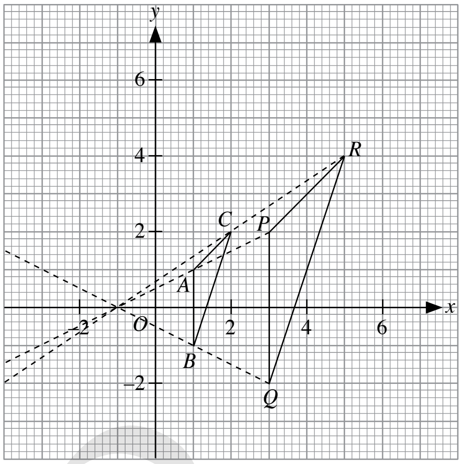
Practise Now 1



Practise Now 2



Practise Now 3

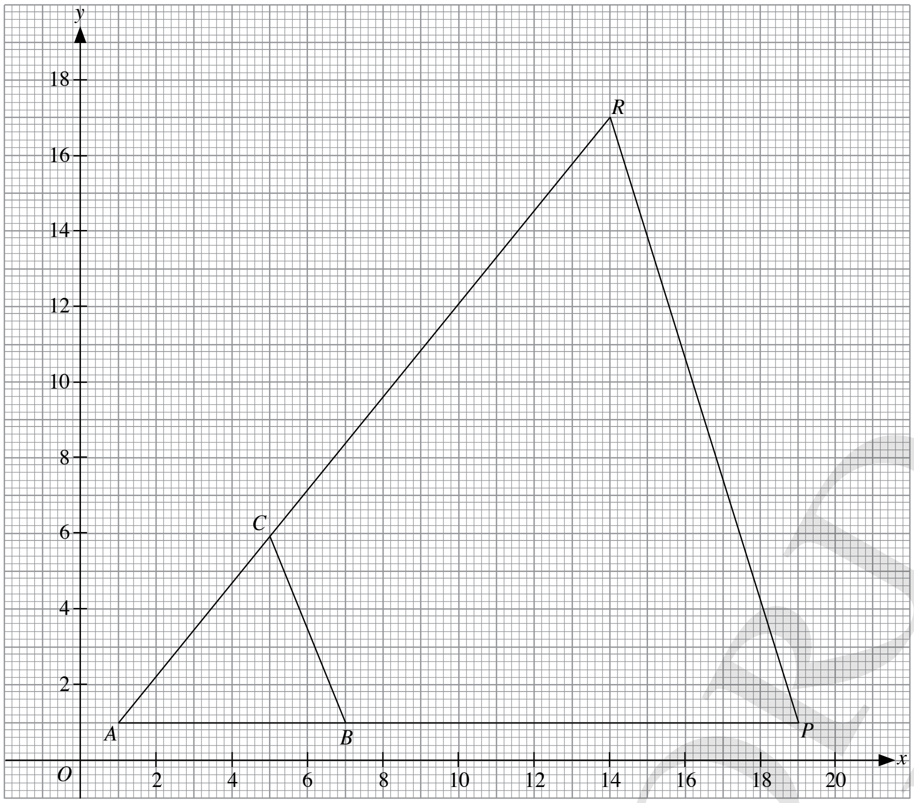


(a) From the graph plotted, coordinates of centre of enlargement are (–1, 0).

(b) Scale factor = $\frac{PQ}{AB} = \frac{4}{2} = 2$

OXFORD UNIVERSITY PRESS

Practise Now 4



(a) From the graph plotted, A is the invariant point and hence, centre of enlargement is (1, 1).

(b) Length of $AB = 6$ units

Height of $\triangle ABC = 5$ units

$$\therefore \text{Area of } \triangle ABC = \frac{1}{2}(6)(5)$$
$$= 15 \text{ units}^2$$
$$\text{Area of } \triangle APR = 3^2 \times 15$$
$$= 135 \text{ units}^2$$

OXFORD UNIVERSITY PRESS

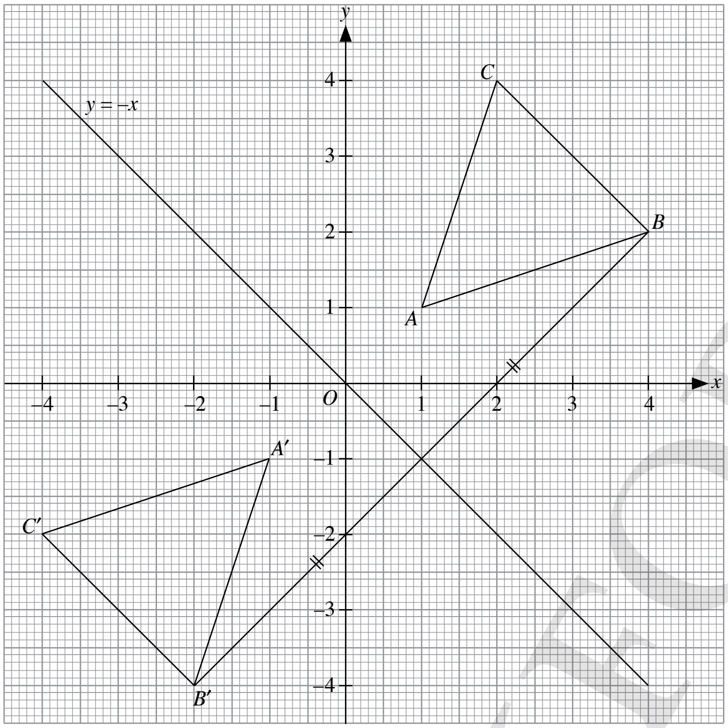
Practise Now 5

Images of the points:

$$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 4 \\ 2 \end{pmatrix} = \begin{pmatrix} -2 \\ -4 \end{pmatrix}$$

$$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 2 \\ 4 \end{pmatrix} = \begin{pmatrix} -4 \\ -2 \end{pmatrix}$$



From the graph, $A'B'C'$ is the reflection of $\triangle ABC$ in the line $y = -x$, i.e. $y + x = 0$. Hence, the matrix $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ represents a reflection in the line $y + x = 0$.

OXFORD UNIVERSITY PRESS

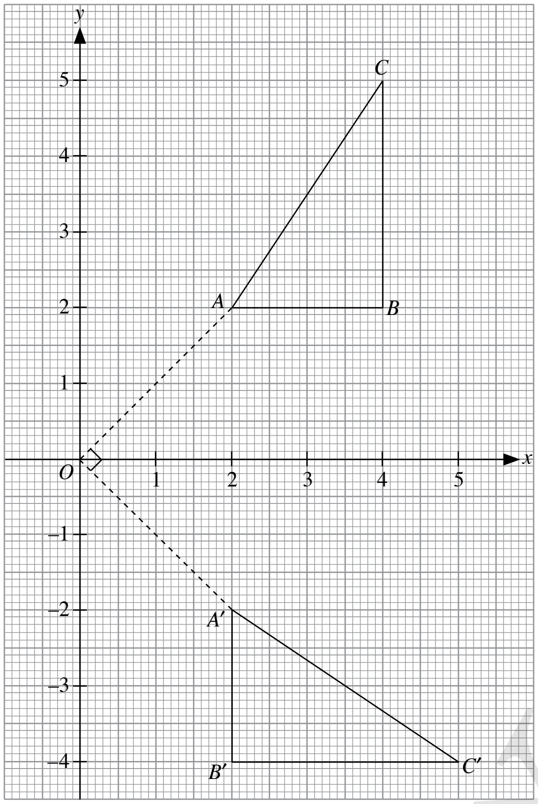
Practise Now 6

Images of the points:

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 2 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 4 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 4 \\ 5 \end{pmatrix} = \begin{pmatrix} 5 \\ -4 \end{pmatrix}$$



Joining OA and OA', it can be seen that A is rotated through 90° clockwise about the origin. Thus, $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ is a matrix representing a 90° clockwise rotation about the origin.

Practise Now 7

Let the transformation matrix be $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$.

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 4 \\ 1 & 3 \end{pmatrix}$$

$$\therefore \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 3 & 4 \\ 1 & 3 \end{pmatrix}$$

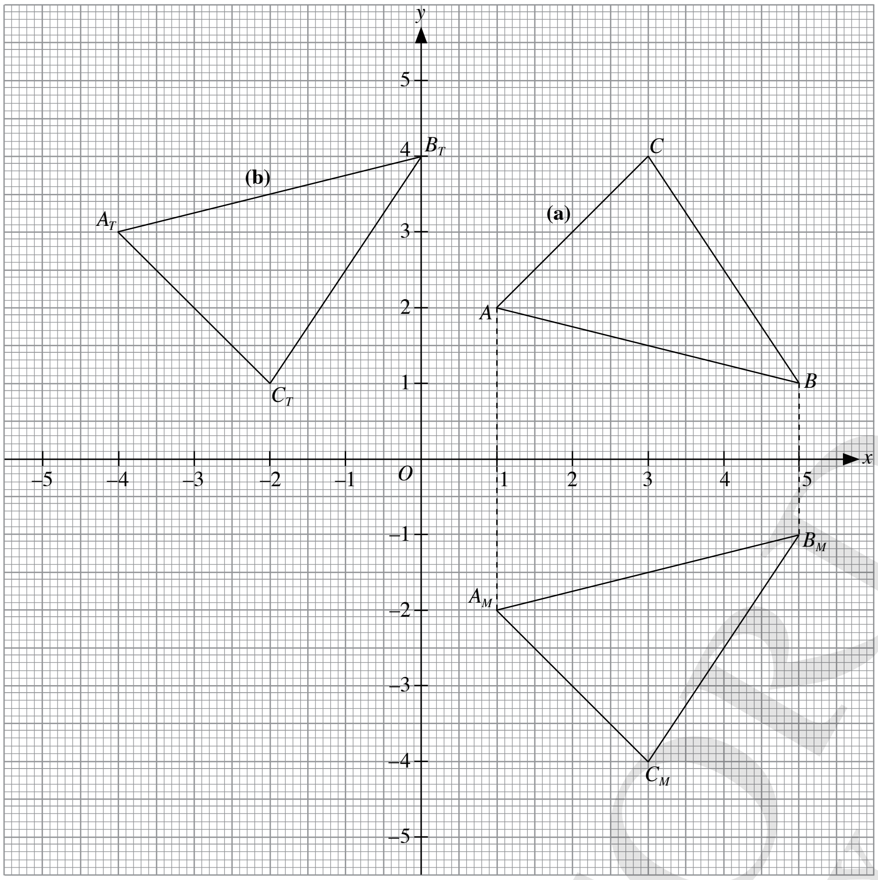
Practise Now 8

$$\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} 1 & 5 & 3 \\ 1 & 4 & 6 \end{pmatrix} = \begin{pmatrix} 3 & 15 & 9 \\ 3 & 12 & 18 \end{pmatrix}$$

The coordinates are $A'(3, 3)$, $B'(15, 12)$ and $C'(9, 18)$.

OXFORD UNIVERSITY PRESS

Practise Now 9



Practise Now 10

(a) Under R, $(a, b) \to (-b, a)$

$$R\begin{pmatrix}2\\3\end{pmatrix} = \begin{pmatrix}-3\\2\end{pmatrix}$$

Under M, $(a, b) \to (a, -b)$

$$M\begin{pmatrix}-3\\2\end{pmatrix} = \begin{pmatrix}-3\\-2\end{pmatrix}$$

$\therefore$ MR(2, 3) would give (–3, –2).

(b) RM$(x, y) = (3, 1)$

Under R, $(a, b) \to (-b, a)$

$$M\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}1\\-3\end{pmatrix}$$

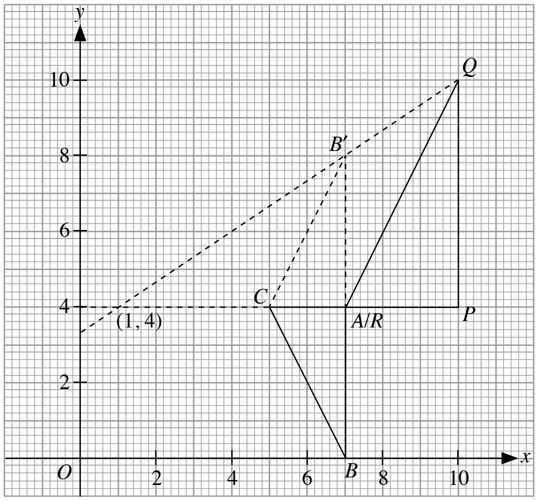
Under M, $(a, b) \to (a, -b)$

$$\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}1\\3\end{pmatrix}$$

$\therefore$ (x, y) is (1, 3).

OXFORD UNIVERSITY PRESS

Practise Now 11

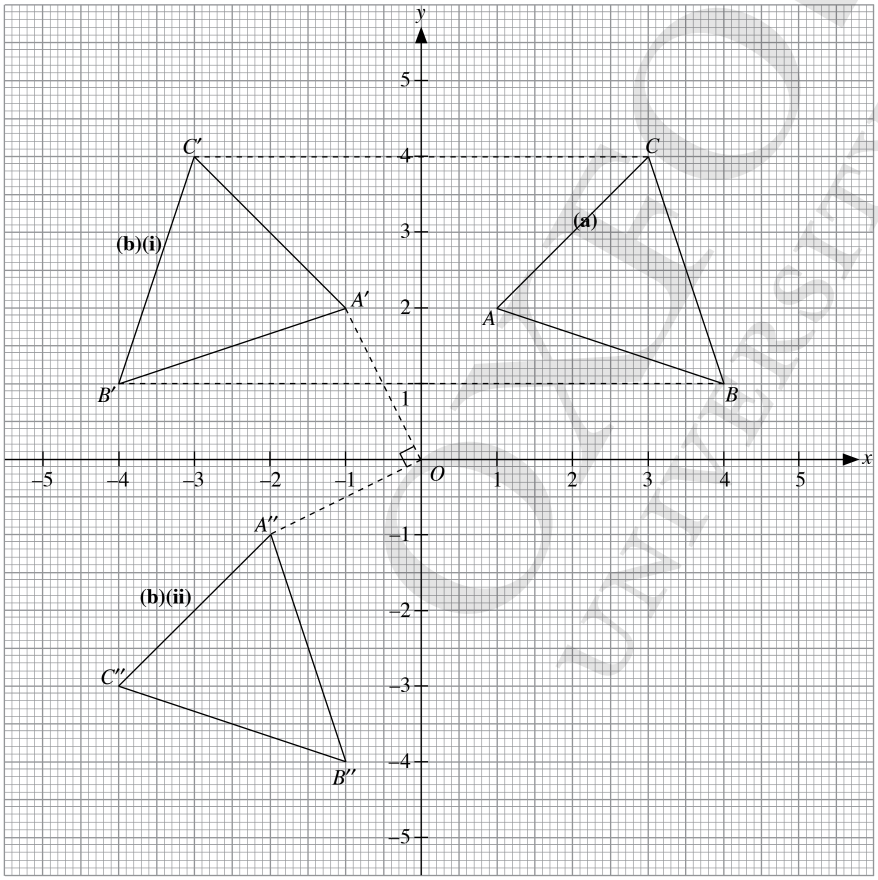


Step 1: $\triangle ABC$ is reflected in the line $y = 4$ to get $\triangle AB'$C

Step 2: $\triangle AB'C$ is enlarged by a scale factor of $\frac{PQ}{AB} = 1\frac{1}{2}$ with enlargement centre (1, 4) to get $\triangle PQR$.

Practise Now 12

(a)(b)

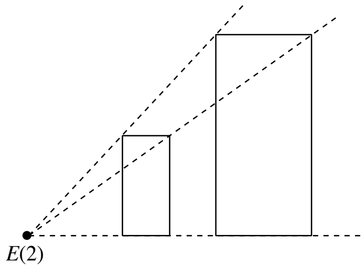


(c) From the graph plotted, a single transformation that will map $\triangle ABC$ onto $\triangle A''B''C''$ is a reflection in the line $y = -x$, i.e. $y + x = 0$.

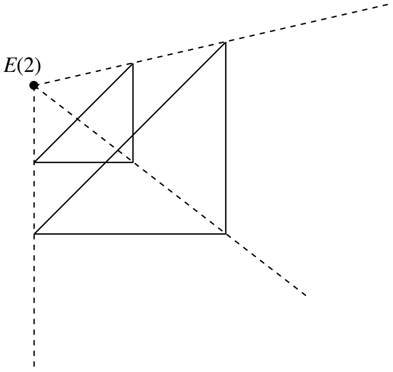
OXFORD
UNIVERSITY PRESS

Exercise 6A

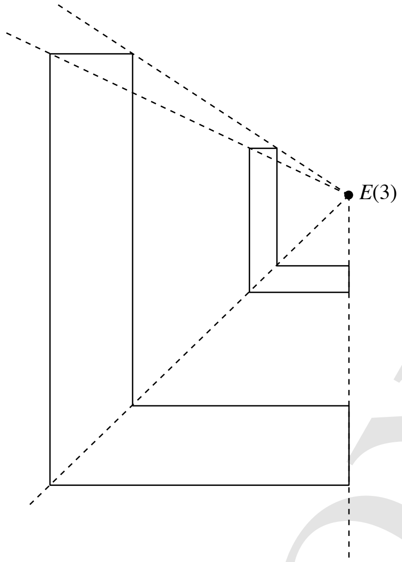
1. **(a)**



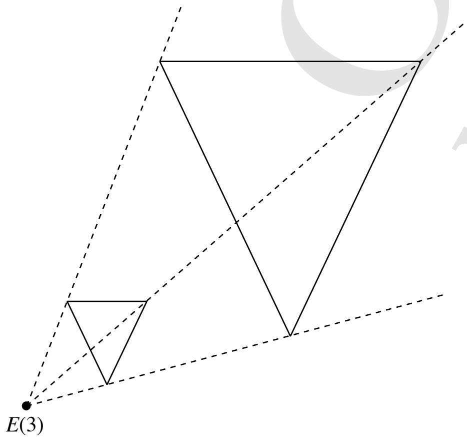
(b)



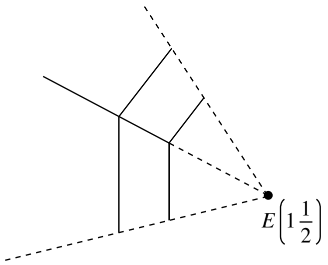
(c)



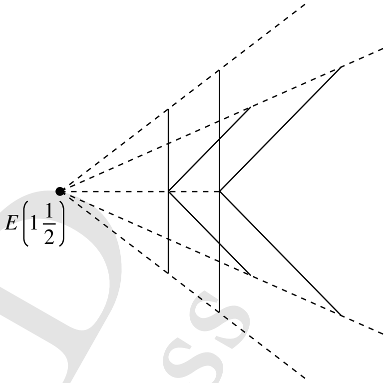
(d)



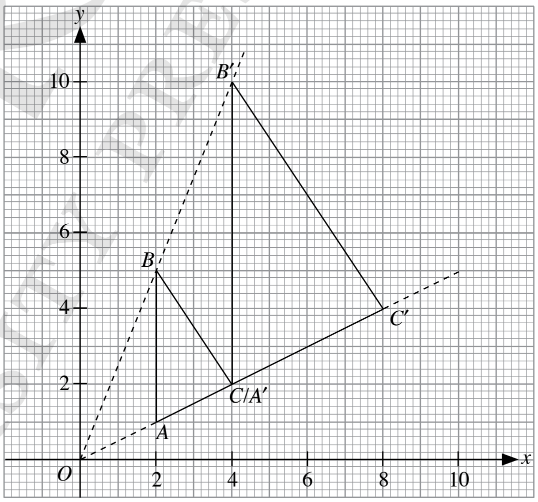
(e)



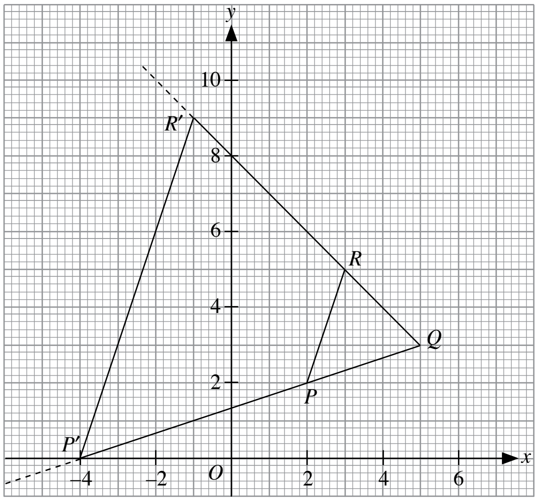
(f)



2.

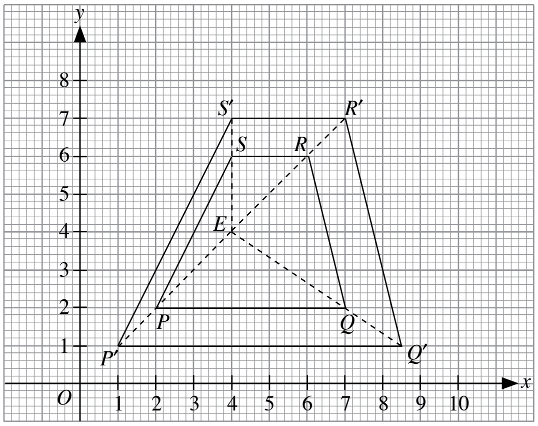


3.

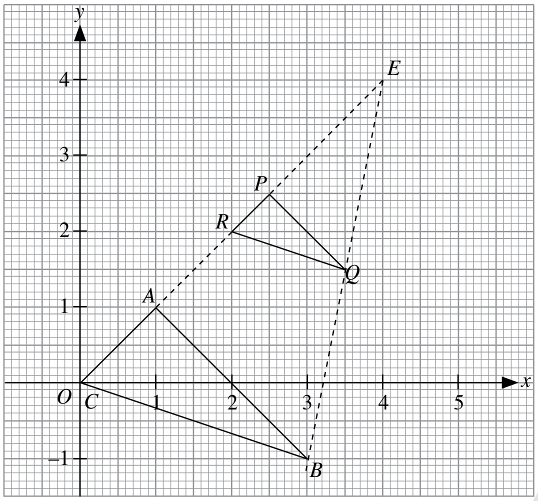


OXFORD UNIVERSITY PRESS

4.

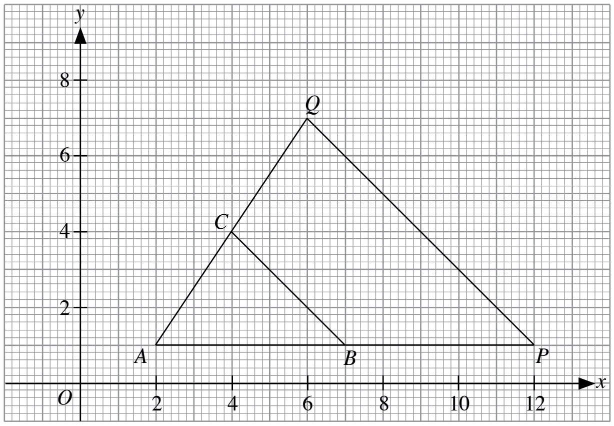


5.



From the graph, the coordinates are $P\left(2\frac{1}{2}, 2\frac{1}{2}\right)$, $Q\left(3\frac{1}{2}, 1\frac{1}{2}\right)$ and $R(2, 2)$.

6.

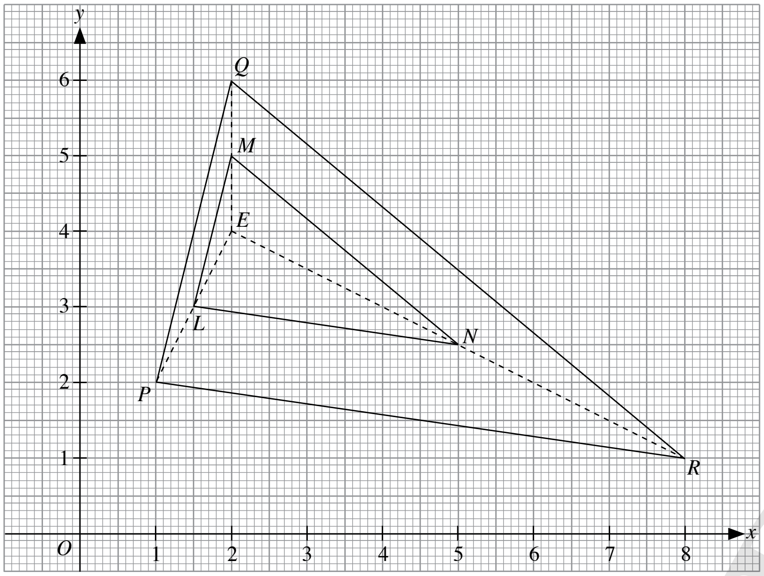


(a) From the graph plotted, A is the invariant point and hence, centre of enlargement is $(2, 1)$.

(b) From the graph, coordinates are $P(12, 1)$ and $Q(6, 7)$.

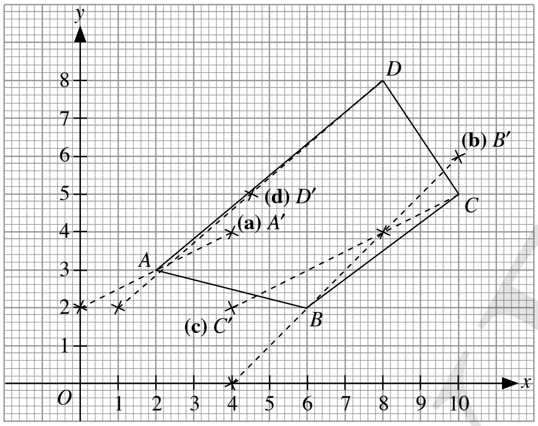
OXFORD UNIVERSITY PRESS

7.



From the graph plotted, the coordinates are $L\left(1\frac{1}{2}, 3\right)$, $M(2, 5)$ and $N\left(5, 2\frac{1}{2}\right)$.

8.



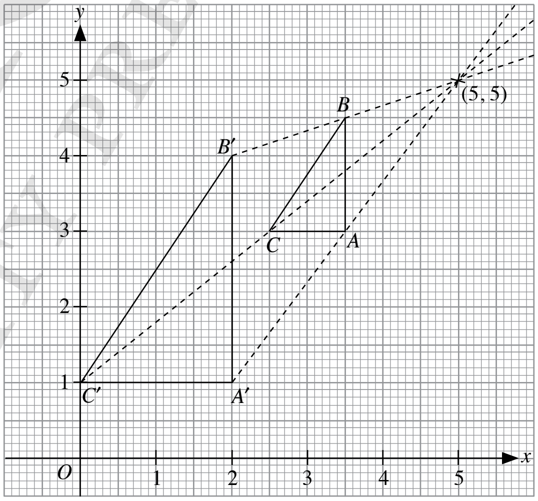
(a) Image of A is $A'(4, 4)$

(b) Image of B is $B'(10, 6)$

(c) Image of C is $C'(4, 2)$

(d) Image of D is $D'\left(4\frac{1}{2}, 5\right)$

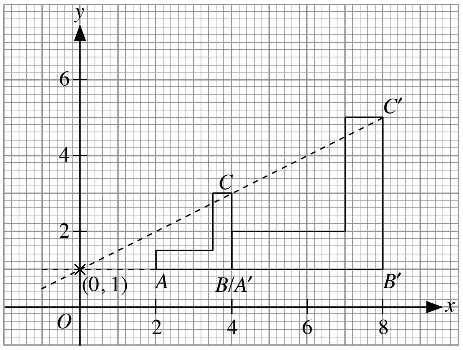
9. (a)



From graph, coordinates of centre of enlargement are (5, 5).

$$\text{Scale factor} = \frac{A'B'}{AB} = \frac{3}{1\frac{1}{2}} = 2$$

(b)

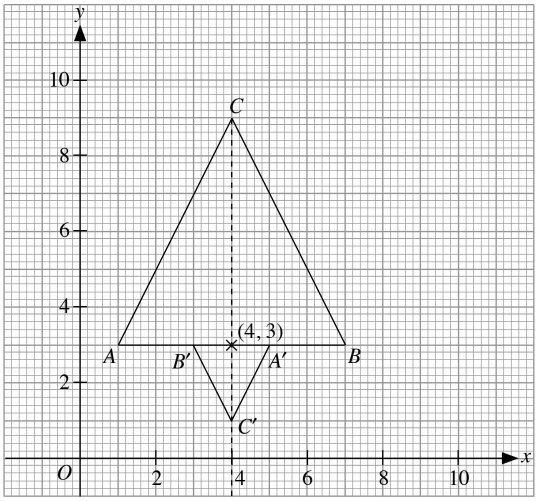


From graph, coordinates of centre of enlargement are (0, 1).

$$\text{Scale factor} = \frac{C'B'}{CB} = \frac{4}{2} = 2$$

OXFORD UNIVERSITY PRESS

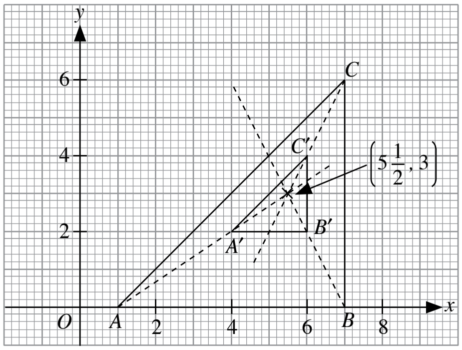
(c)



From graph, coordinates of centre of enlargement are (4, 3).

Scale factor = $-\dfrac{A'B'}{AB} = -\dfrac{2}{6} = -\dfrac{1}{3}$

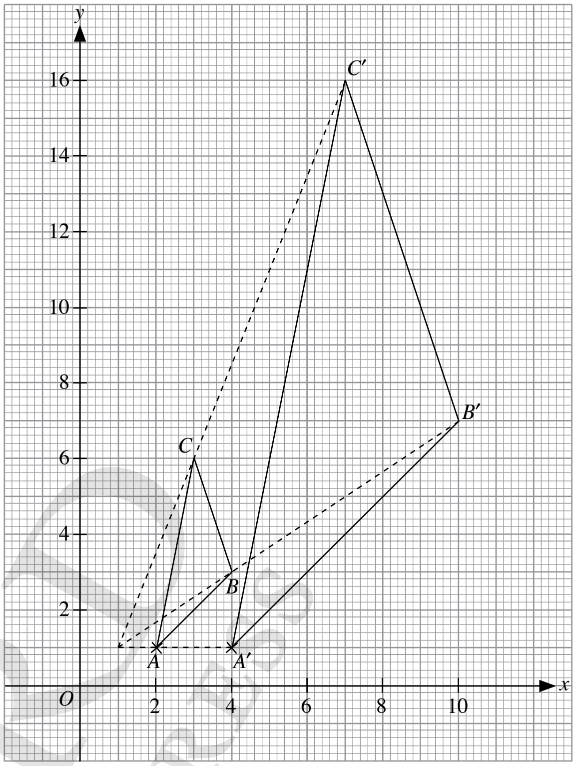
(d)



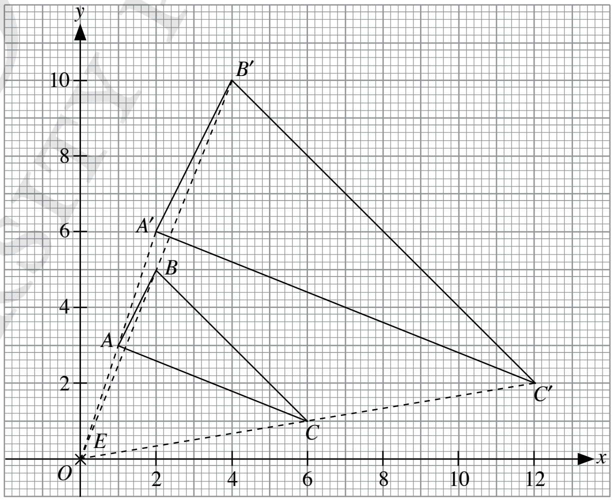
From graph, coordinates of centre of enlargement are $\left(5\dfrac{1}{2}, 3\right)$.

Scale factor = $\dfrac{A'B'}{AB} = \dfrac{2}{6} = \dfrac{1}{3}$

10.

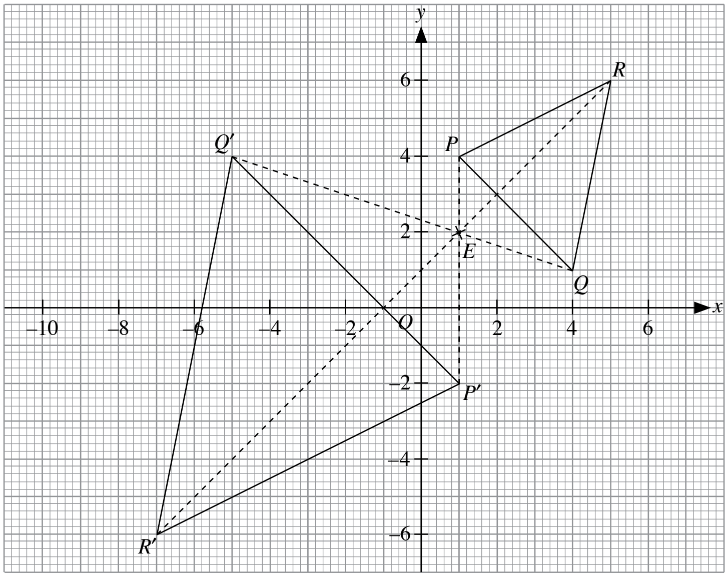


11. (a)

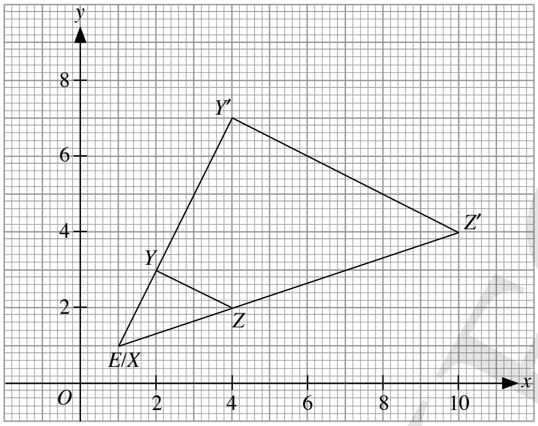


OXFORD UNIVERSITY PRESS

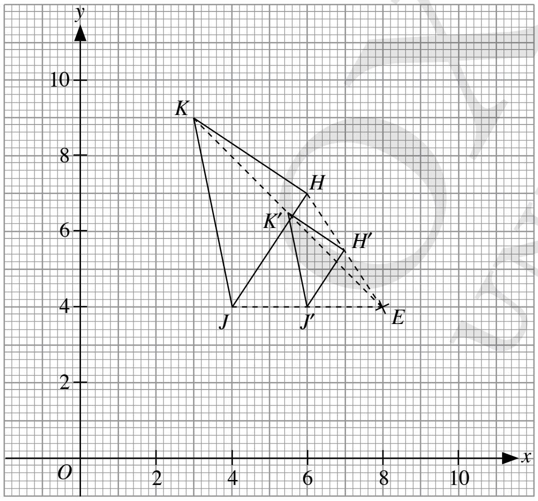
(b)



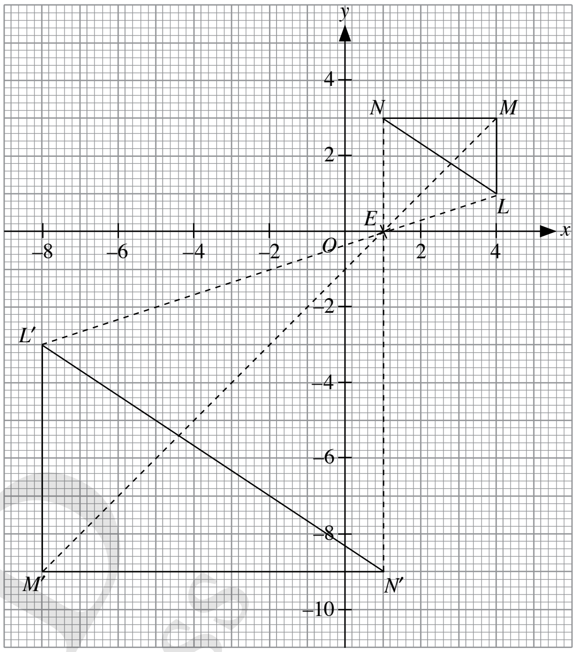
(c)



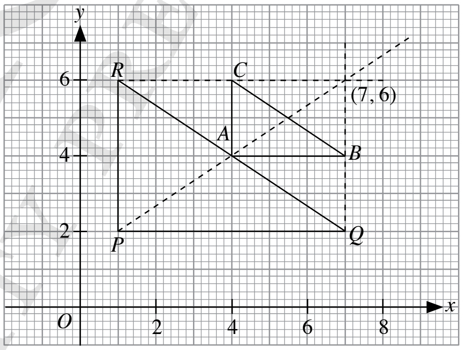
(d)



(e)



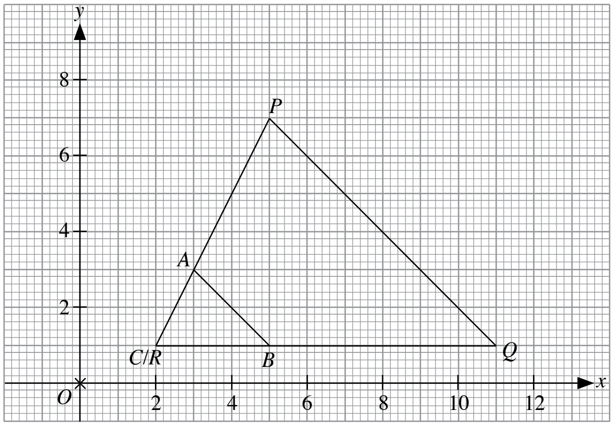
12. (a)



From graph, coordinates of centre of enlargement are (7, 6).

Scale factor = $\dfrac{PQ}{AB} = \dfrac{6}{3} = 2$

(b)

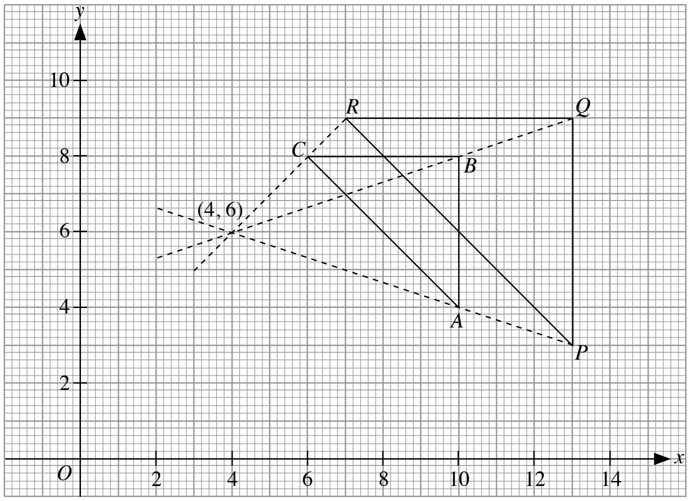


From graph, coordinates of centre of enlargement are (2, 1).

Scale factor = $\dfrac{RQ}{CB} = \dfrac{9}{3} = 3$

OXFORD UNIVERSITY PRESS

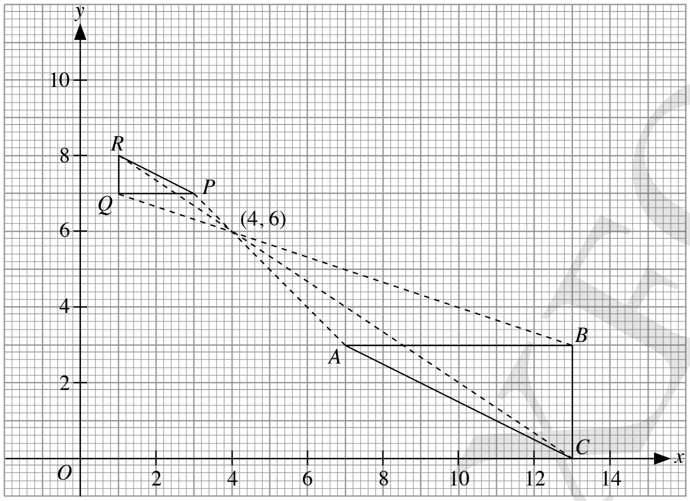
(c)



From graph, coordinates of centre of enlargement are (4, 6).

Scale factor = $\frac{RQ}{CB} = \frac{6}{4} = 1\frac{1}{2}$

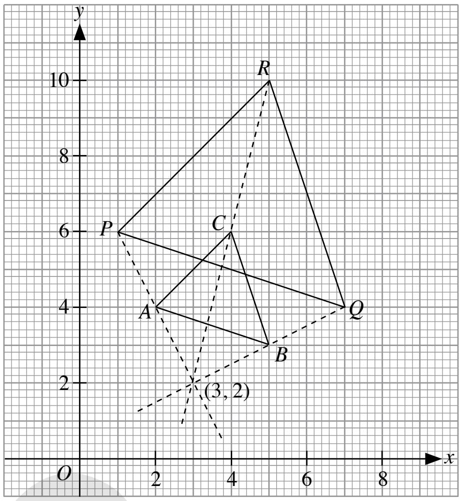
(d)



From graph, coordinates of centre of enlargement are (4, 6).

Scale factor = $-\frac{PQ}{AB} = -\frac{2}{6} = -\frac{1}{3}$

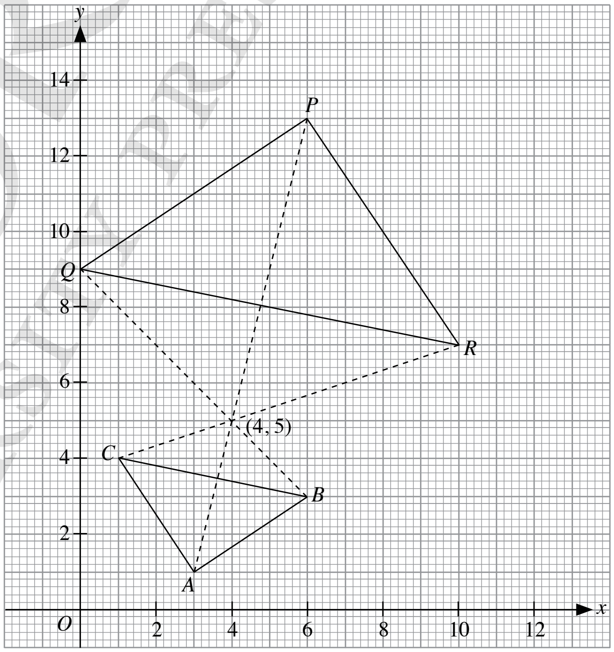
(e)



From graph, coordinates of centre of enlargement are (3, 2).

Scale factor = $\frac{PQ}{AB} = \frac{3}{1\frac{1}{2}} = 2$

(f)



From graph, coordinates of centre of enlargement are (4, 5).

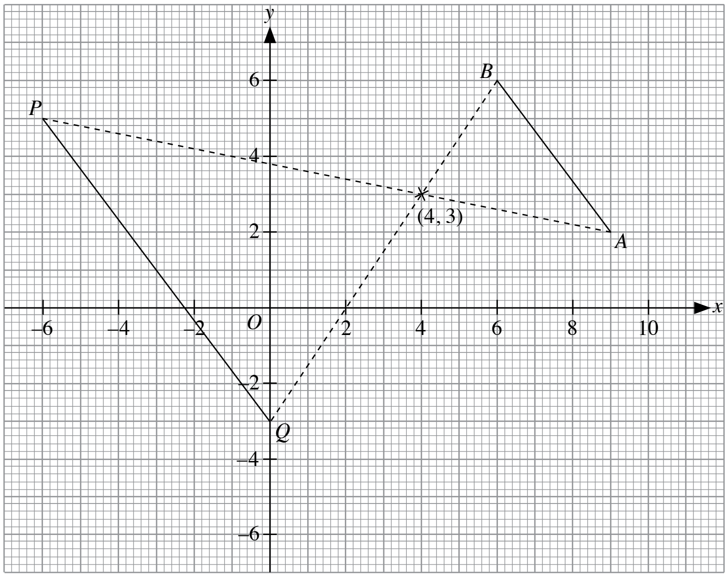
Scale factor = $-\frac{PQ}{AB} = -\frac{3.6}{1.8} = -2$

13. **(a)** The image figure is $\triangle ABC$.

(b) The image figure is rectangle *PBQR*.

OXFORD UNIVERSITY PRESS

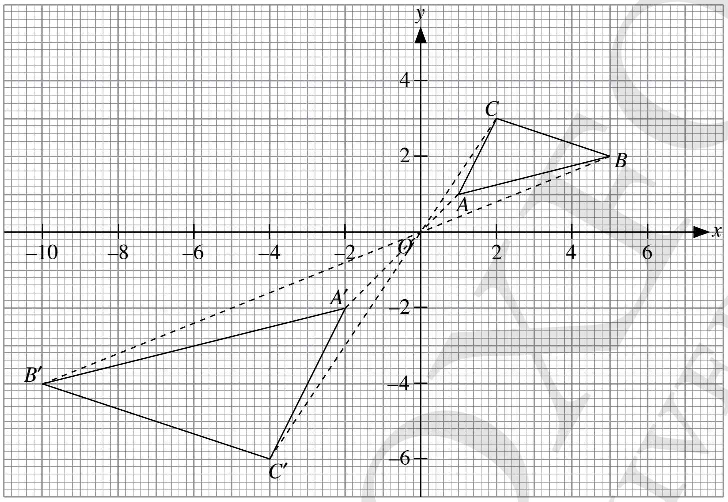
14.



(a) From the graph plotted, the coordinates are $P(-6, 5)$ and $Q(0, -3)$.

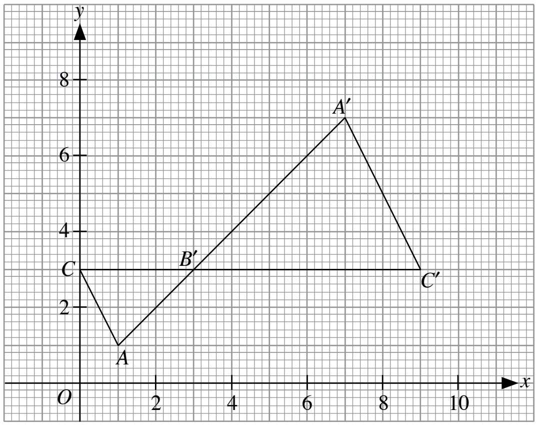
(b) Length of $PQ = 5 \times 2$ units
$= 10$ units

15.



From the graph plotted, the coordinates of $\triangle ABC$ are $A(1, 1)$, $B(5, 2)$ and $C(2, 3)$.

16.



From the graph plotted, the coordinates are $A(1, 1)$ and $C(0, 3)$.

17. Since $PQRS$ is a square, $PQ = RQ = 20$ cm

Scale factor $= \dfrac{PQ}{AB} = \dfrac{20}{10} = 2$

Let the distance of centre of enlargement from point A be x.

Distance of centre of enlargement from point P will be twice that of point A., i.e.

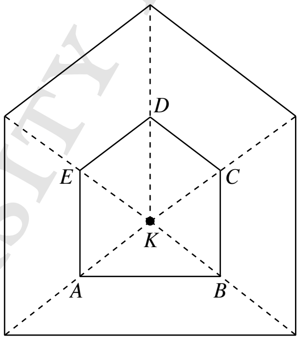
$2x = x + 20$

$x = 20$

$\therefore$ The distance of the centre of enlargement from point A is 20 cm.

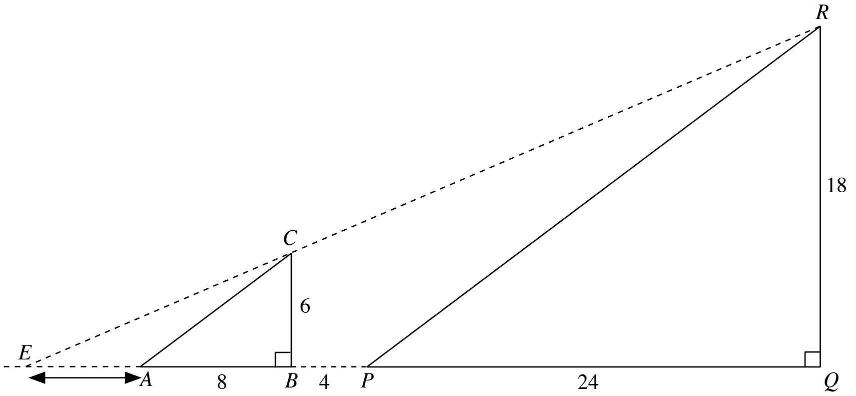
18. Area of image $= k^2 \times$ (area of original figure)

When $k^2 = 4$, $k = 2$



OXFORD UNIVERSITY PRESS

19.

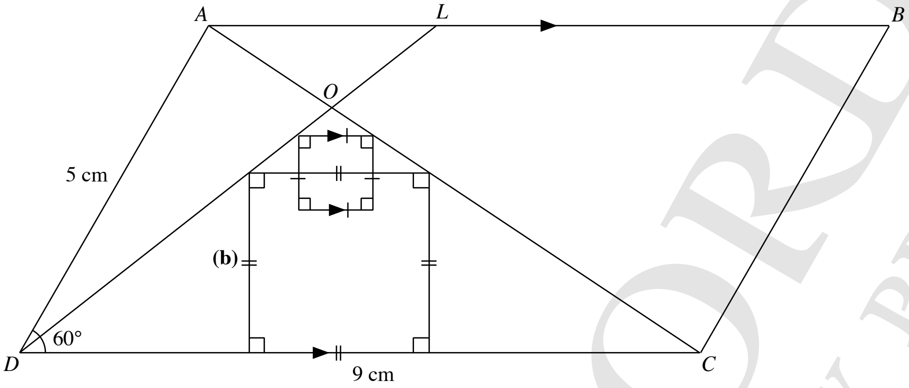


From drawn diagram, $EA = 1.5$ cm

$\therefore$ Length of $EA = 1.5 \times 4$

$= 6$ cm

20.



(a) $\triangle ALO$ is enlarged by a scale factor of –3 at centre O.

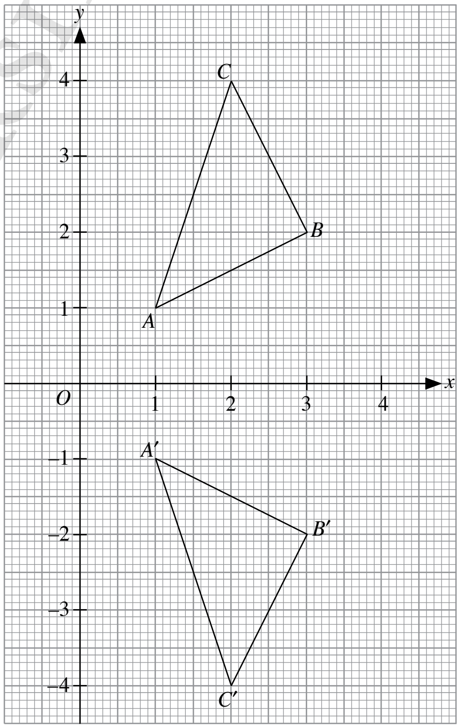
Exercise 6B

1. Images of the points:

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 2 \\ 4 \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$$



From the graph, $\triangle A'B'C'$ is the reflection of $\triangle ABC$ in the x-axis.

Hence, the matrix $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ represents a reflection in the x-axis.

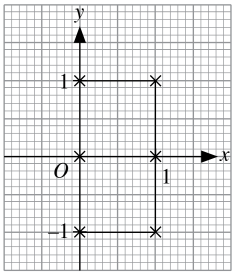
2. Images of the points:

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$



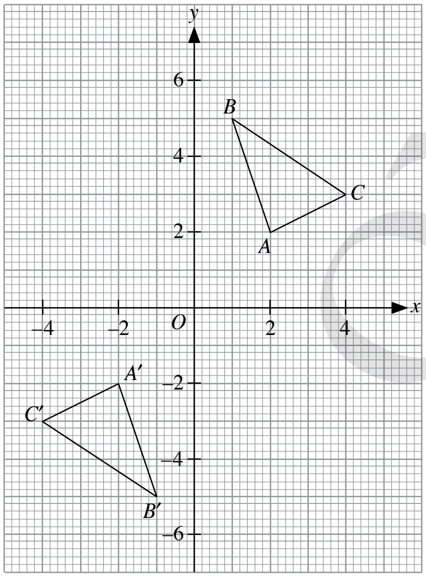
From the graph, the points (0, 0) and (1, 0) are not affected by the transformation. Such a point is called an invariant point.

3. Images of the points:

$$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 2 \\ 2 \end{pmatrix} = \begin{pmatrix} -2 \\ -2 \end{pmatrix}$$

$$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 5 \end{pmatrix} = \begin{pmatrix} -1 \\ -5 \end{pmatrix}$$

$$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 4 \\ 3 \end{pmatrix} = \begin{pmatrix} -4 \\ -3 \end{pmatrix}$$



From the graph, $\triangle ABC$ is rotated 180° about the origin to obtain $A'B'C'$. Hence, the matrix $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ represents a 180° about the origin.

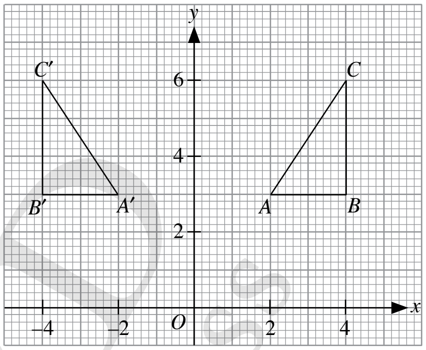
4. Images of the points:

$$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 4 \\ 3 \end{pmatrix} = \begin{pmatrix} -4 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 4 \\ 6 \end{pmatrix} = \begin{pmatrix} -4 \\ 6 \end{pmatrix}$$

$\therefore$ The coordinates are $A'(-2, 3)$, $B'(-4, 3)$ and $C'(-4, 6)$.



From the graph, $\triangle A'B'C'$ is the reflection of ABC in the y-axis. Hence, the matrix $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ represents a reflection in the y-axis, and will map $\triangle A'B'C'$ back onto $\triangle ABC$.

5. Images of the points:

$$\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}\begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 7 \\ 6 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}\begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 12 \\ 11 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}\begin{pmatrix} -2 \\ 5 \end{pmatrix} = \begin{pmatrix} 11 \\ 18 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}\begin{pmatrix} -3 \\ -4 \end{pmatrix} = \begin{pmatrix} -18 \\ -19 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}\begin{pmatrix} -7 \\ -3 \end{pmatrix} = \begin{pmatrix} -23 \\ -19 \end{pmatrix}$$

$\therefore$ The coordinates are $A'(7, 6)$, $B'(12, 11)$, $C'(11, 18)$, $D'(-18, -19)$ and $E'(-23, -19)$.

6. Images of the points:

$$\begin{pmatrix} -2 & 0 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ -3 \end{pmatrix}$$

$$\begin{pmatrix} -2 & 0 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 4 \\ 0 \end{pmatrix} = \begin{pmatrix} -8 \\ 4 \end{pmatrix}$$

$$\begin{pmatrix} -2 & 0 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} -3 \\ 4 \end{pmatrix} = \begin{pmatrix} 6 \\ -7 \end{pmatrix}$$

$$\begin{pmatrix} -2 & 0 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 7 \\ -4 \end{pmatrix} = \begin{pmatrix} -14 \\ 11 \end{pmatrix}$$

$$\begin{pmatrix} -2 & 0 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} -2 \\ -5 \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$$

$\therefore$ The coordinates are $A'(0, -3)$, $B'(-8, 4)$, $C'(6, -7)$, $D'(-14, 11)$ and $E'(4, 3)$.

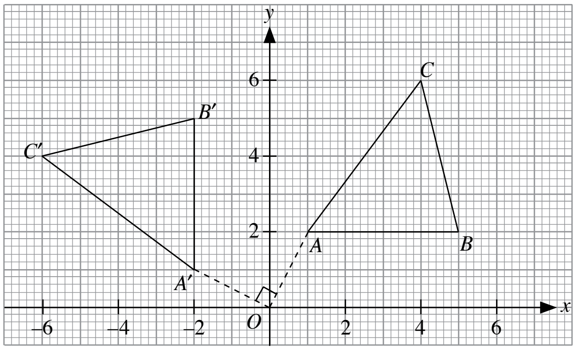
OXFORD UNIVERSITY PRESS

7. Images of the points:

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 5 \\ 2 \end{pmatrix} = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$$

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 4 \\ 6 \end{pmatrix} = \begin{pmatrix} -6 \\ 4 \end{pmatrix}$$



Joining OA and OA', it can be seen that A is rotated through 90° anticlockwise about the origin. ∴ The transformation T is a 90° anticlockwise rotation about the origin.

$$T^2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

T^2 would be a 180° rotation about the origin.

$$\text{Discriminant of } \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = 0 - [(-1) \times 1]$$

$$= 1$$

$$T^{-1} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}^{-1}$$

$$= \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

T^{-1} would be a 90° clockwise rotation about the origin.

8. Let the transformation matrix be $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$.

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 & 3 \\ 2 & -2 \end{pmatrix} = \begin{pmatrix} 4 & 4 \\ 4 & -4 \end{pmatrix}$$

$$\begin{pmatrix} a+2b & 3a-2b \\ c+2d & 3c-2d \end{pmatrix} = \begin{pmatrix} 4 & 4 \\ 4 & -4 \end{pmatrix}$$

Equating the corresponding elements,

$a + 2b = 4$ — (1)

$3a - 2b = 4$ — (2)

$c + 2d = 4$ — (3)

$3c - 2d = -4$ — (4)

Solving (1) and (2) simultaneously, $a = 2, b = 1$.

Solving (3) and (4) simultaneously, $c = 0, d = 2$.

∴ The required matrix is $\begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}$.

9. $$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 7 & -1 \\ -2 & -1 \end{pmatrix}$$

$$\begin{pmatrix} 7 & -1 \\ -2 & -1 \end{pmatrix} = \begin{pmatrix} 7 & -1 \\ -2 & -1 \end{pmatrix}$$

Equating the corresponding elements,

$a + 2b = 7$ — (1)

$-a + b = -1$ — (2)

$c + 2d = -2$ — (3)

$-c + d = -1$ — (4)

Solving (1) and (2) simultaneously, $a = 3, b = 2$.

Solving (3) and (4) simultaneously, $c = 0, d = -1$.

∴ The required matrix is $\begin{pmatrix} 3 & 2 \\ 0 & -1 \end{pmatrix}$.

10. Images of the points:

$$\begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix}\begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 4 \\ -6 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix}\begin{pmatrix} 4 \\ 3 \end{pmatrix} = \begin{pmatrix} 8 \\ -6 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix}\begin{pmatrix} 3 \\ 5 \end{pmatrix} = \begin{pmatrix} 6 \\ -10 \end{pmatrix}$$

∴ The coordinates of the images of the points are (4, –6), (8, –6) and (6, –10).

$$T^2 = \begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix}\begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}$$

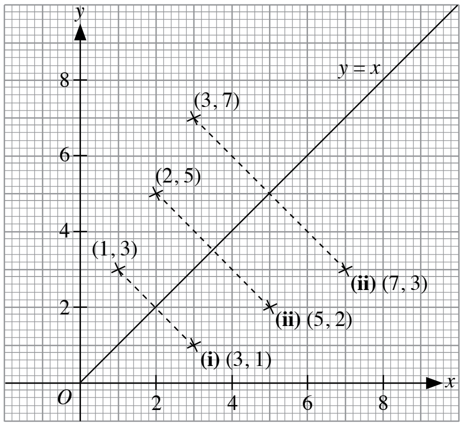
$$\text{Discriminant of } \begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix} = [2 \times (-2)] - 0$$

$$= -4$$

$$T^{-1} = -\frac{1}{4}\begin{pmatrix} -2 & 0 \\ 0 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{pmatrix}$$

11.



(i) From graph, coordinates of the image are (3, 1).

(ii) Under M^{-1}, the point will still be reflected in the line $y = x$. From graph, coordinates of the image are (5, 2).

(iii) Under M^5, 5 successive reflections in the line $y = x$ will give the same image as 1 reflection in the line $y = x$. Coordinates of the image are (7, 3).

(iv) The matrix that represents M^5 and M^{-1} is the same as the matrix that represents M.

Let the transformation matrix representing M be $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$.

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} a + 3b \\ c + 3d \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

Equating the corresponding elements,

$a + 3b = 3$ — (1)

$c + 3d = 1$ — (2)

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 2 \\ 5 \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} 2a + 5b \\ 2c + 5d \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$$

Equating the corresponding elements,

$2a + 5b = 5$ —(3)

$2c + 5d = 2$ — (4)

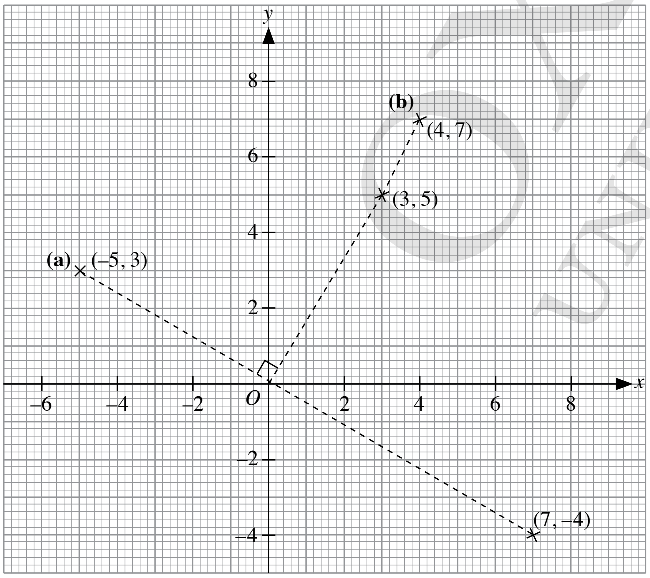
Solving (1) and (3) simultaneously, $a = 0$, $b = 1$.

Solving (2) and (4) simultaneously, $c = 1$, $d = 0$.

$\therefore$ The matrix representing M is $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

Hence, the matrix representing M^5 and M^{-1} is $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

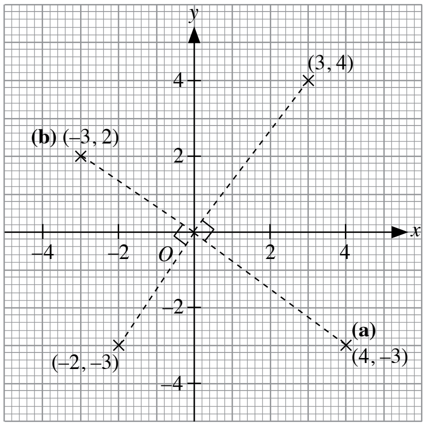
12. (i)



(a) Coordinates of the image are (–5, 3)

(b) Coordinates of the image are (4, 7)

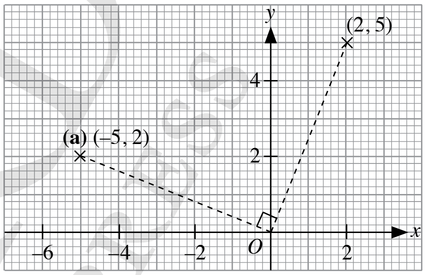
(ii)



(a) Coordinates of the image are (4, –3)

(b) Coordinates of the image are (–3, 2)

(iii)



(a) Coordinates of the image are (–5, 2)

(b) R^8 represents 8 successive anticlockwise rotations of 90° about the origin. Hence, the coordinates of the image of (2, 5) under R^8 is (2, 5).

(iv) (a) Let the transformation matrix representing R be $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$.

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 2 \\ 5 \end{pmatrix} = \begin{pmatrix} -5 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} 2a + 5b \\ 2c + 5d \end{pmatrix} = \begin{pmatrix} -5 \\ 2 \end{pmatrix}$$

Equating the corresponding elements,

$2a + 5b = -5$ — (1)

$2c + 5d = 2$ — (2)

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 2 \\ 5 \end{pmatrix} = \begin{pmatrix} -5 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} 3a + 5b \\ 3c + 5d \end{pmatrix} = \begin{pmatrix} -5 \\ 3 \end{pmatrix}$$

Equating the corresponding elements,

$3a + 5b = -5$ — (3)

$3c + 5d = 3$ — (4)

Solving (1) and (3) simultaneously, $a = 0$, $b = -1$.

Solving (2) and (4) simultaneously, $c = 1$, $d = 0$.

$\therefore$ The matrix representing R is $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$.

OXFORD UNIVERSITY PRESS

(b) R^4 represents 4 successive anticlockwise rotations of 90° about the origin. Hence, the coordinates of the image of the point will be the same as the original point.

Let the transformation matrix representing R^4 be $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$.

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 2 \\ 5 \end{pmatrix} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$$

$$\begin{pmatrix} 2a+5b \\ 2c+5d \end{pmatrix} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$$

Equating the corresponding elements,

$2a + 5b = 2$ — (1)

$2c + 5d = 5$ — (2)

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 3 \\ 5 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$$

$$\begin{pmatrix} 3a+5b \\ 3c+5d \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$$

Equating the corresponding elements,

$3a + 5b = 3$ — (3)

$3c + 5d = 5$ — (4)

Solving (1) and (3) simultaneously, $a = 1, b = 0$.

Solving (2) and (4) simultaneously, $c = 0, d = 1$.

$\therefore$ The matrix representing R is $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

(c) R^9 represents 9 successive anticlockwise rotations of 90° about the origin. The image of the point would be the same as a 90° anticlockwise rotation about the origin. Hence, the transformation matrix will be the same as R i.e. $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$.

Exercise 6C

1. (i) $$\begin{pmatrix} 2\frac{1}{2} & 0 \\ 0 & 2\frac{1}{2} \end{pmatrix}\begin{pmatrix} 1 & 1 & 5 \\ 2 & 4 & 2 \end{pmatrix} = \begin{pmatrix} 2\frac{1}{2} & 2\frac{1}{2} & 12\frac{1}{2} \\ 5 & 10 & 5 \end{pmatrix}$$

The coordinates are $A'\left(2\frac{1}{2}, 5\right)$, $B'\left(2\frac{1}{2}, 10\right)$ and $C'\left(12\frac{1}{2}, 5\right)$.

(ii) Area of $\triangle ABC = \frac{1}{2} \times 4 \times 2$

$= 4$ units^2

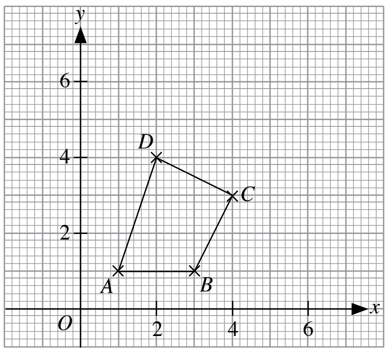
(iii) Scale factor $k = 2\frac{1}{2}$

Area of $\triangle A'B'C' = k^2 \times$ area of $\triangle ABC$

$= \left(2\frac{1}{2}\right)^2 \times 4$

$= 25$ units^2

2.



(i) $$\begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}\begin{pmatrix} 1 & 3 & 4 & 2 \\ 1 & 1 & 3 & 4 \end{pmatrix} = \begin{pmatrix} -2 & -6 & -8 & -4 \\ -2 & -2 & -6 & -8 \end{pmatrix}$$

The coordinates are $A'(-2, -2)$, $B'(-6, -2)$, $C'(-8, -6)$ and $D'(-4, -8)$.

Scale factor $k = -2$

$$\left(\frac{\text{area of image of } ABCD}{\text{area of } ABCD}\right) = k^2 = 4$$

(ii) $$\begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}\begin{pmatrix} 1 & 3 & 4 & 2 \\ 1 & 1 & 3 & 4 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & 1\frac{1}{2} & 2 & 1 \\ \frac{1}{2} & \frac{1}{2} & 1\frac{1}{2} & 2 \end{pmatrix}$$

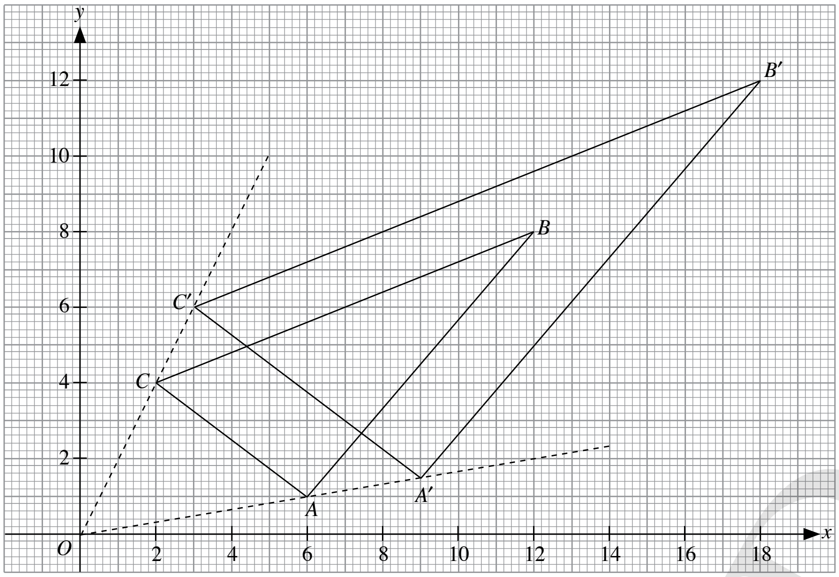
The coordinates are $A'\left(\frac{1}{2}, \frac{1}{2}\right)$, $B'\left(1\frac{1}{2}, \frac{1}{2}\right)$, $C'\left(2, 1\frac{1}{2}\right)$ and $D'(1, 2)$.

Scale factor $k = \frac{1}{2}$

$$\left(\frac{\text{area of image of } ABCD}{\text{area of } ABCD}\right) = k^2 = \frac{1}{4}$$

OXFORD UNIVERSITY PRESS

3.



Let the transformation matrix be $\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$.

$$\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}\begin{pmatrix} 6 \\ 1 \end{pmatrix} = \begin{pmatrix} 9 \\ 1\frac{1}{2} \end{pmatrix}$$

$$\begin{pmatrix} 6k \\ k \end{pmatrix} = \begin{pmatrix} 9 \\ 1\frac{1}{2} \end{pmatrix}$$

Equating the corresponding elements, $k = 1\frac{1}{2}$.

$\therefore$ The required transformation matrix is $\begin{pmatrix} 1\frac{1}{2} & 0 \\ 0 & 1\frac{1}{2} \end{pmatrix}$.

4. Scale factor for transformation of ABC into $\triangle A_1B_1C_1$ is 2.
Scale factor for transformation of $\triangle A_1B_1C_1$ into $\triangle A_2B_2C_2$ is 3.
Scale factor for transformation of $\triangle ABC$ into $\triangle A_2B_2C_2$
$= 2 \times 3$
$= 6$

$\therefore$ The required transformation matrix is $\begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix}$.

5. $$\begin{pmatrix} \frac{1}{3} & 0 \\ 0 & \frac{1}{3} \end{pmatrix}\begin{pmatrix} 3 & 12 & 6 \\ 6 & 6 & 9 \end{pmatrix} = \begin{pmatrix} 1 & 4 & 2 \\ 2 & 2 & 3 \end{pmatrix}$$

$\therefore$ The coordinates of $\triangle A_1B_1C_1$ are $A_1(1, 2)$, $B_1(4, 2)$ and $C_1(2, 3)$.

$$\begin{pmatrix} -6 & 0 \\ 0 & -6 \end{pmatrix}\begin{pmatrix} 1 & 4 & 2 \\ 2 & 2 & 3 \end{pmatrix} = \begin{pmatrix} -6 & -24 & -12 \\ -12 & -12 & -13 \end{pmatrix}$$

$\therefore$ The coordinates of $\triangle A_2B_2C_2$ are $A_2(-6, -12)$, $B_2(-24, -12)$ and $C_2(-12, -18)$.

Scale factor for transformation of $\triangle ABC$ into $\triangle A_2B_2C_2$

$= \frac{1}{3} \times (-6)$

$= -2$

$\therefore$ The required transformation matrix is $\begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$.

Exercise 6D

1. Under R, $(a, b) \rightarrow (-a, b)$

The translation T is represented by $\begin{pmatrix} 2 \\ 4 \end{pmatrix}$.

(a) $$\text{T}\begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} + \begin{pmatrix} 2 \\ 4 \end{pmatrix}$$

$$= \begin{pmatrix} 3 \\ 7 \end{pmatrix}$$

$$\text{R}\begin{pmatrix} 3 \\ 7 \end{pmatrix} = \begin{pmatrix} -3 \\ 7 \end{pmatrix}$$

$\therefore$ The image of the point under the combined transformation represented by RT is $(-3, 7)$.

OXFORD UNIVERSITY PRESS

(b) $R\begin{pmatrix}1\\3\end{pmatrix}=\begin{pmatrix}-1\\3\end{pmatrix}$

$$T\begin{pmatrix}-1\\3\end{pmatrix}=\begin{pmatrix}-1\\3\end{pmatrix}+\begin{pmatrix}2\\4\end{pmatrix}$$

$$=\begin{pmatrix}1\\7\end{pmatrix}$$

$\therefore$ The image of the point under the combined transformation represented by TR is (1, 7).

2. Under a reflection in the x-axis, $(a, b) \rightarrow (a, -b)$
$\Rightarrow$ Coordinates of image of (5, 2) will be (5, –2)
Under a clockwise rotation of 90° about the origin, $(a, b) \rightarrow (b, -a)$
$\Rightarrow$ Coordinates of image of (5, –2) will be (–2, –5).

3. Under a reflection in the y-axis, $(a, b) \rightarrow (-a, b)$
Under a anticlockwise rotation of 90° about the origin, $(a, b) \rightarrow (-b, a)$

(a) First transformation: (2, –3) → (–2, –3)
Second transformation: (–2, –3) → (3, –2)
$\therefore$ The coordinates of the final image are (3, –2).

(b) First transformation: (–4, –1) → (4, –1)
Second transformation: (4, –1) → (1, 4)
$\therefore$ The coordinates of the final image are (1, 4).

4. The translation is represented by $\begin{pmatrix}-4\\3\end{pmatrix}$.

Under a 180° rotation about the origin, $(a, b) \rightarrow (-a, -b)$
Coordinates of image after translation

$$=\begin{pmatrix}7\\-2\end{pmatrix}+\begin{pmatrix}-4\\3\end{pmatrix}$$

$$=\begin{pmatrix}3\\1\end{pmatrix}$$

Coordinates of image after rotation = $\begin{pmatrix}-3\\-1\end{pmatrix}$

$\therefore$ The coordinates of the final image are (–3, –1)

5. E is represented by $\begin{pmatrix}2 & 0\\0 & 2\end{pmatrix}$.

T is represented by $\begin{pmatrix}2\\1\end{pmatrix}$.

(a) $T\begin{pmatrix}2\\1\end{pmatrix}=\begin{pmatrix}2\\1\end{pmatrix}+\begin{pmatrix}2\\1\end{pmatrix}$

$$=\begin{pmatrix}4\\2\end{pmatrix}$$

$$E\begin{pmatrix}4\\2\end{pmatrix}=\begin{pmatrix}2 & 0\\0 & 2\end{pmatrix}\begin{pmatrix}4\\2\end{pmatrix}$$

$$=\begin{pmatrix}8\\4\end{pmatrix}$$

$\therefore$ The coordinates of the image under ET are (8, 4).

(b) $T^2\begin{pmatrix}2\\1\end{pmatrix}=\begin{pmatrix}2\\1\end{pmatrix}+\begin{pmatrix}2\\1\end{pmatrix}+\begin{pmatrix}2\\1\end{pmatrix}$

$$=\begin{pmatrix}6\\3\end{pmatrix}$$

$\therefore$ The coordinates of the image under T^2 are (6, 3).

6. Under M, $(a, b) \rightarrow (-a, b)$

T is represented by $\begin{pmatrix}3\\0\end{pmatrix}$.

(a) $T\begin{pmatrix}2\\3\end{pmatrix}=\begin{pmatrix}2\\3\end{pmatrix}+\begin{pmatrix}2\\3\end{pmatrix}$

$$=\begin{pmatrix}5\\3\end{pmatrix}$$

$$M\begin{pmatrix}5\\3\end{pmatrix}=\begin{pmatrix}-5\\3\end{pmatrix}$$

$\therefore$ The coordinates of MT(2, 3) are (–5, 3).

(b) $TM\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}4\\3\end{pmatrix}$

$$M\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}4\\3\end{pmatrix}-\begin{pmatrix}3\\0\end{pmatrix}$$

$$=\begin{pmatrix}1\\3\end{pmatrix}$$

$$\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}-1\\3\end{pmatrix}$$

$\therefore (x, y) = (-1, 3)$

7. Under a reflection in the line $y = 3$,
(5, 1) → (5, 5)
$\therefore A_1(5, 5)$
Under a reflection in the line $y = 8$,
(5, 5) → (5, 11)
$\therefore A_2(5, 11)$
For (5, 11) to be the image of (5, 1), the point undergoes a reflection in the line $y = 6$.
$\therefore k = 6$

8. Under a reflection in the line $y = 0$, i.e. the x-axis, $(a, b) \rightarrow (a, -b)$
Under a reflection in the line $x = 0$, i.e. the y-axis, $(a, b) \rightarrow (-a, b)$
$\therefore (a, -b) \rightarrow (-a, -b)$
A single transformation to map (a, b) onto $(-a, -b)$ will be a 180° rotation about the origin.
$\therefore P$ will be directly mapped onto P_2 under a half-turn about the origin.

9. (a) $T^2\begin{pmatrix}2\\5\end{pmatrix}=\begin{pmatrix}2\\5\end{pmatrix}+\begin{pmatrix}2\\3\end{pmatrix}+\begin{pmatrix}2\\3\end{pmatrix}$

$$=\begin{pmatrix}6\\11\end{pmatrix}$$

$\therefore$ Coordinates of the image are (6, 11)

OXFORD UNIVERSITY PRESS

(b) Under M, $(a, b) \to (-b, -a)$

Under M^2, $(-b, -a) \to (a, b)$

$\therefore$ Coordinates of the image are (2, 5)

(c) Under M^8, coordinates of the image will be the same as under M^2 and the point itself, i.e. (2, 5).

(d) $$T\begin{pmatrix} 2 \\ 5 \end{pmatrix} = \begin{pmatrix} 2 \\ 5 \end{pmatrix} + \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 4 \\ 8 \end{pmatrix}$$

$$M\begin{pmatrix} 4 \\ 8 \end{pmatrix} = \begin{pmatrix} -8 \\ -4 \end{pmatrix}$$

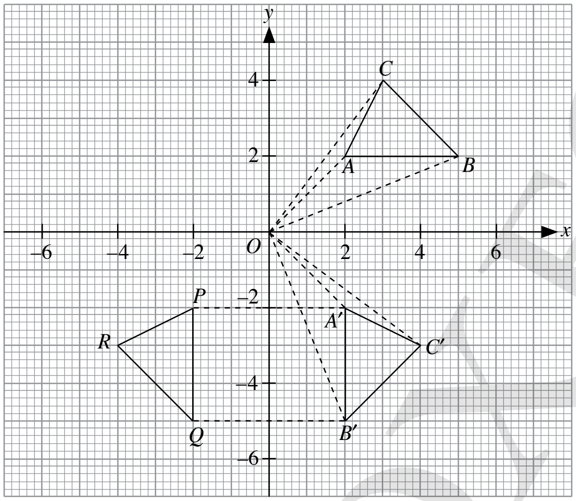
$\therefore$ Coordinates of the image under MT are (–8, –4)

(c) $$M\begin{pmatrix} 2 \\ 5 \end{pmatrix} = \begin{pmatrix} -5 \\ -2 \end{pmatrix}$$

$$T\begin{pmatrix} -5 \\ -2 \end{pmatrix} = \begin{pmatrix} -5 \\ -2 \end{pmatrix} + \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$$

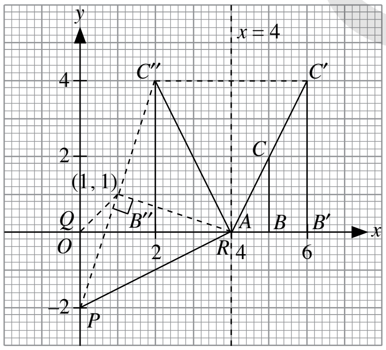
$\therefore$ Coordinates of the image under TM are (–3, 1)

10.

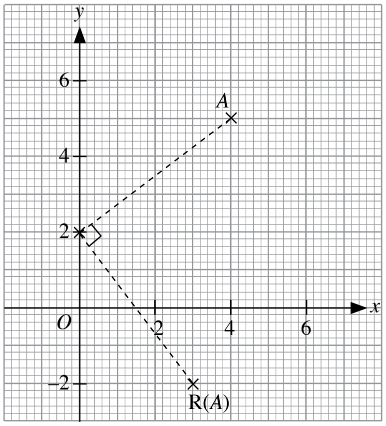


From the graph plotted, $\triangle ABC$ will be mapped onto $\triangle PQR$ under a reflection in the line $y = -x$ i.e. $y + x = 0$.

11.



12.



From graph, coordinates of R(A) are (3, –2).

$$TR(A) = \begin{pmatrix} 3 \\ -2 \end{pmatrix} + \begin{pmatrix} 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 6 \\ -1 \end{pmatrix}$$

$\therefore$ Coordinates of TR(A) are (6, –1)

13. Under R, $(a, b) \to (-b, a)$

T is represented by $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$.

$$T(A) = \begin{pmatrix} 5 \\ 1 \end{pmatrix} + \begin{pmatrix} 3 \\ -2 \end{pmatrix} = \begin{pmatrix} 8 \\ -1 \end{pmatrix}$$

$$RT(A) = \begin{pmatrix} 1 \\ 8 \end{pmatrix}$$

$\therefore B(1, 8)$

$$R(A) = \begin{pmatrix} -1 \\ 5 \end{pmatrix}$$

$$TR(A) = \begin{pmatrix} -1 \\ 5 \end{pmatrix} + \begin{pmatrix} 3 \\ -2 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

$\therefore C(2, 3)$

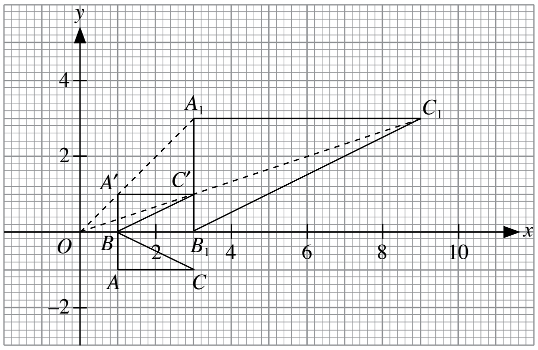
R^2 represents two successive 90° anticlockwise rotations about the origin i.e. 180° rotation about the origin. Under R^2, $(a, b) \to (-a, -b)$

$R^2(A) = (-5, -1)$

$\therefore D(-5, -1)$

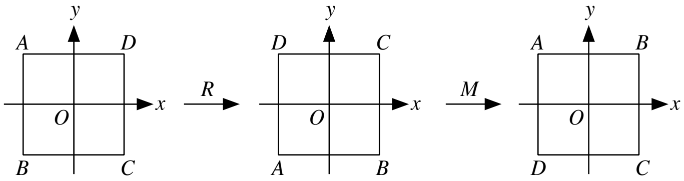
OXFORD UNIVERSITY PRESS

14.



From graph, coordinates of A_1 are (3, 3).

15. **(a)**



(b) A reflection in the line *AOC*

16. Under a reflection in the x-axis, $(a, b) \rightarrow (a, -b)$

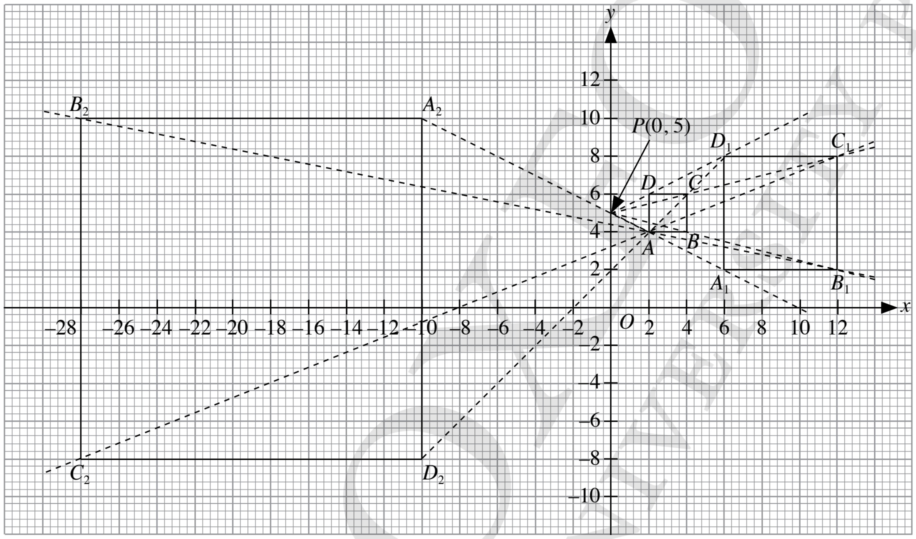
Under a 90° anticlockwise rotation about the origin, $(a, b) \rightarrow (-b, a)$ i.e. $(a, -b) \rightarrow (b, a)$

$\therefore$ Coordinates of the new points are $A'(1, 1)$, $B'(3, 2)$ and $C'(4, -1)$.

Working backwards, $(b, a) \rightarrow (a, b)$

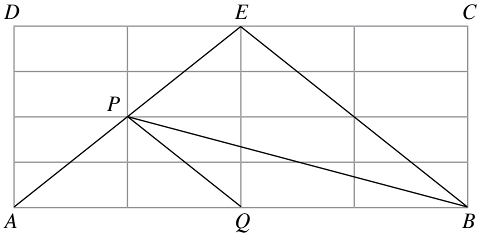
$\therefore$ The coordinates are $P(5, 0)$, $Q(5, 3)$, $R(8, 3)$ and $S(8, 0)$.

17.



From the graph plotted, $C_1(12, 8)$ and $C_2(-28, -8)$.

18.



OXFORD UNIVERSITY PRESS

Review Exercise 6

1. **(a)** Under a reflection in the y-axis, $(a, b) \rightarrow (-a, b)$

$\therefore$ Coordinates of image of P are $(-2, -1)$.

(b) Under a 90° anticlockwise rotation about the origin, $(a, b) \rightarrow (-b, a)$

$\therefore$ Coordinates of image of P are $(1, 2)$.

(c) The translation is represented by $\begin{pmatrix} 1 \\ 5 \end{pmatrix}$.

$$\begin{pmatrix} 2 \\ -1 \end{pmatrix} + \begin{pmatrix} 1 \\ 5 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

$\therefore$ Coordinates of image of P are $(3, 4)$.

2. **(a)** Under a reflection in the x-axis, $(a, b) \rightarrow (a, -b)$

Under a reflection in the y-axis, $(a, b) \rightarrow (-a, b)$ i.e. $(a, -b) \rightarrow (-a, -b)$

$\therefore$ Coordinates of image of K are $(-3, 1)$.

(b) Under a reflection in the y-axis, $(a, b) \rightarrow (-a, b)$

Under a reflection in the x-axis, $(a, b) \rightarrow (a, -b)$ i.e. $(-a, b) \rightarrow (-a, -b)$

$\therefore$ Coordinates of image of K are $(-3, 1)$.

(c) Under a 90° anticlockwise rotation about the origin, $(a, b) \rightarrow (-b, a)$

Under a reflection in the y-axis, $(a, b) \rightarrow (-a, b)$ i.e. $(-b, a) \rightarrow (b, a)$

$\therefore$ Coordinates of image of K are $(-1, 3)$.

(d) Under a 180° rotation about $(2, 0)$, $(3, 1) \rightarrow (1, 1)$

Under a reflection in the line $y = 3$, $(1, 1) \rightarrow (1, 5)$

$\therefore$ Coordinates of image of K are $(1, 5)$.

3. Let the translation vector be $\begin{pmatrix} a \\ b \end{pmatrix}$.

$$\begin{pmatrix} -2 \\ 3 \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$$

$$\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 2 \\ 5 \end{pmatrix} - \begin{pmatrix} -2 \\ 3 \end{pmatrix}$$

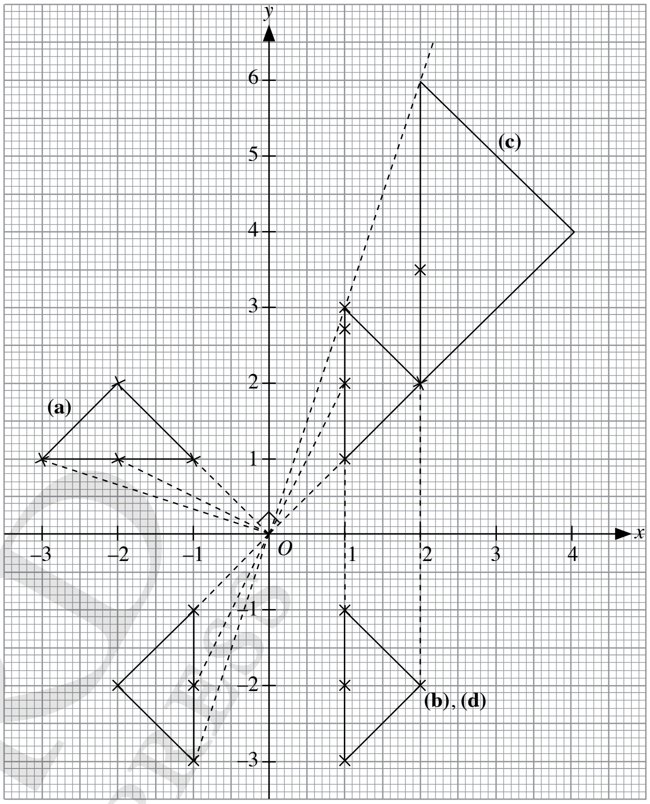
$$= \begin{pmatrix} 4 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ -2 \end{pmatrix} + \begin{pmatrix} 4 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} 9 \\ 0 \end{pmatrix}$$

$\therefore x = 9, y = 0$

4.



5. **(a)** $ABEF$ is enlarged with enlargement centre at A by a scale factor of 2.

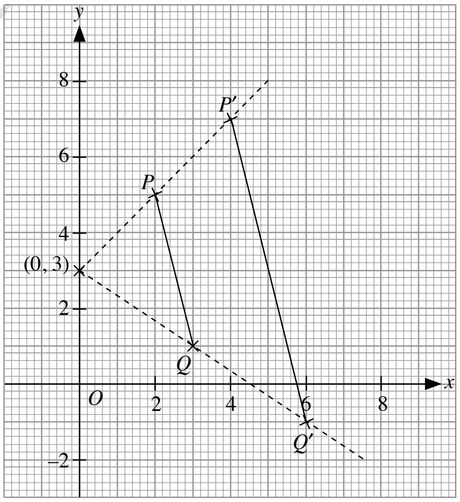
(b) $\triangle ABE$ is translated parallel to AE by length AE.

(c) $\triangle ACI$ is rotated by 180° about point E.

(d) $\triangle ABE$ is first reflected in the line BH, and then rotated by 180° about point E.

(e) $\triangle ACG$ is first reflected in the line FD, and then rotated by 180° about point E.

6.



From the graph plotted, from enlargement centre $(0, 3)$, the image of P at $x = 4$ will give $P'(4, 7)$. Hence, scale factor of enlargement $= 2$.

Coordinates of the image of Q, Q' are $(6, -1)$.

$\therefore p = 7, m = 6$ and $n = -1$

OXFORD UNIVERSITY PRESS

7. H is a reflection in the line $x = 4$. K is an enlargement at centre (4, 2) by scale factor 2.

8. Scale factor, $k = 4$

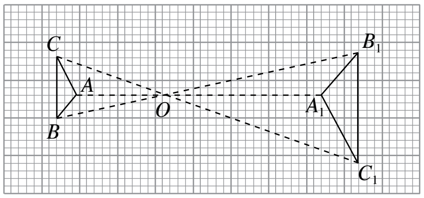
(a) $\frac{B'C'}{BC} = k = 4$

(b) Size of $AB'C'$ = size of ABC

$\therefore \frac{AB'C'}{ABC} = 1$

(c) $\frac{\text{area of } AB'C'}{\text{area of } ABC} = k^2 = 4^2 = 16$

9.



(a) $AB : A_1B_1 = 1 : 2$

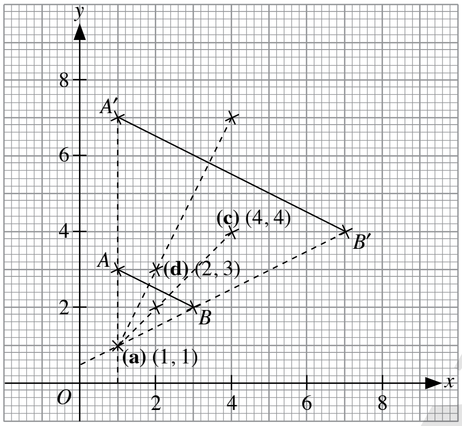
(b) Scale factor of enlargement = –2

Area of $\triangle A_1B_1C_1$: area of $\triangle ABC$

$= (-2)^2 : 1$

$= 4 : 1$

10.



(a) From the graph plotted, coordinates of centre of enlargement are (1, 1).

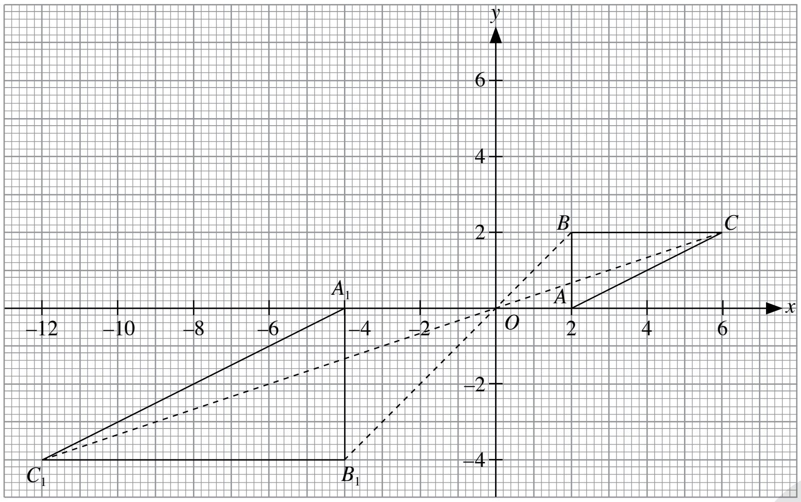
(b) Scale factor = $\frac{A'B'}{AB} = 3$

(c) From graph, coordinates of the image are (4, 4).

(d) From graph, coordinates of the point are (2, 3).

OXFORD UNIVERSITY PRESS

11.

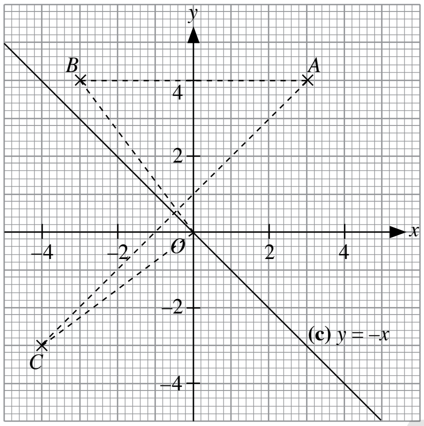


(a) From the graph plotted, the coordinates are $A_1(-4, 0)$, $B_1(-4, -4)$ and $C_1(-12, -4)$.

(b) $\frac{A_1B_1}{AB} = \frac{4}{2} = 2$

(c) $\frac{\text{area of } \triangle ABC}{\text{area of } \triangle A_1B_1C_1} = \frac{1}{2^2} = \frac{1}{4}$

12.



(a) From graph, $B(-3, 4)$. Equation of the invariant line is $x = 0$.

(b) From graph, $C(-4, -3)$.

(c) From graph, C can be obtained from A through a reflection in the line $y = -x$, i.e. $y + x = 0$.

13. (a) Enlargement at centre A with scale factor 4

(b) Translation parallel to AP with length AP

(c) 180° rotation about point P

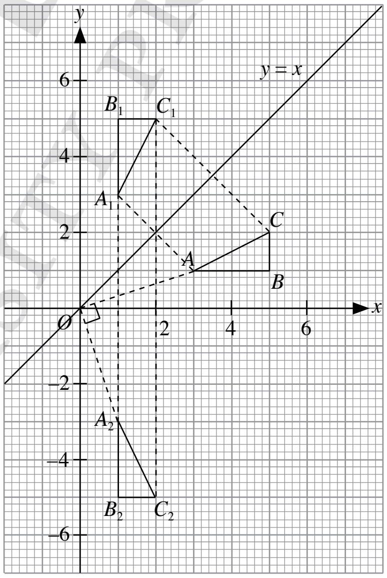
(d) Enlargement at centre A with scale factor 2

(e) Enlargement at centre point X, where $2HX = XK$, with scale factor 3

(f) Enlargement at centre B with scale factor 2

Challenge Yourself

1.



From the graph plotted, $\triangle ABC$ will be mapped onto $\triangle A_2B_2C_2$ under a 90° clockwise rotation about the origin.

2. (a) 180° rotation about the origin

(b) 180° rotation about the origin

3. (a) PQ would be a reflection in the x-axis followed by a 90° clockwise rotation about the origin. Taking the point $(1, 1)$: After Q, the image would be $(1, -1)$; followed by P, the image would be $(-1, -1)$.

$\therefore$ A single transformation equivalent to PQ would be a reflection in the line $y + x = 0$.

(b) QP would be a 90° clockwise rotation about the origin followed by a reflection in the x-axis.

Taking the point $(2, 1)$:

After P, the image would be $(1, -2)$; followed by Q, the image would be $(1, 2)$

$\therefore$ A single transformation equivalent to QP would be a reflection in the line $y = x$.

OXFORD UNIVERSITY PRESS

Chapter 7 Vectors

TEACHING NOTES

Suggested Approach

This topic could be difficult for the average-ability students. It would be easier for the students to understand this topic if vector diagrams on the Cartesian plane are used to explain the different concepts. Teachers should give the students more examples to practise in each section as this will help the students to master the concepts well.

Section 7.1: Vectors in Two Dimensions

Teachers should use the Class Discussion activity on page 217 to allow students to realise the difference between scalar and vector quantities. Opportunities should be given to the students to discuss on other examples of scalars and vectors. The students should be able to explain why their examples are scalars or vectors. With that, teachers should spend some time to go through the different representations of vectors as well as how a vector can be described using a column vector with the aid of a Cartesian plane.

Section 7.2: Addition of Vectors

In this section, introduce the idea of vector addition using the scenario on page 228. With the use of diagrams, students will able to understand both the Triangle Law and Parallelogram Law of Vector Addition better. Students can try to define the zero vector by doing the Class Discussion activity on page 233.

Section 7.3: Vector Subtraction

For this section, introduce the idea of vector subtraction using the addition of negative vectors or Triangle Law of Vector Subtraction with the aid of diagrams. Students should be given more practice to draw the resultant vector from examples of vector subtraction as this will help them to understand the concepts better.

Section 7.4: Scalar Multiples of a Vector

For scalar multiplication, a diagram to illustrate the concept should be used to help the students to develop better understanding. Students should be given time to think about how a positive or negative value of k affects a vector, using Thinking Time on page 247.

Section 7.5: Expression of a Vector in Terms of Two Other Vectors

Recap that the sum or difference of two vectors is also a vector. Hence, any vector can be expressed as the sum or difference of two other vectors. Teachers should encourage the students to use squared papers to find the expression of the required vector as it allows better visualisation of how it can be formed using the sum or difference of the other vectors.

Section 7.6: Position Vectors

Teachers should explain that the position vector of a point must have a fixed starting point and on a Cartesian plane, this would be the origin O. Hence, students should understand that any vector $\overrightarrow{AB}$ on the Cartesian plane can be expressed in terms of the position vector of A and B. They should also know that the vector $\overrightarrow{AB}$ can be regarded as a movement from A to B, which is known as the translation vector.

Section 7.7: Applications of Vectors

In this section, students should be given opportunities to discuss on the real-life examples of resultant vectors. Also, they will solve geometric problems with the use of their knowledge on vectors. Teachers may highlight to students that to find the ratio of the area of two triangles, they must be able to find a relationship between the triangles, for example, similar triangles or triangles with common height. There could also be an intermediate triangle that connects them.

Challenge Yourself

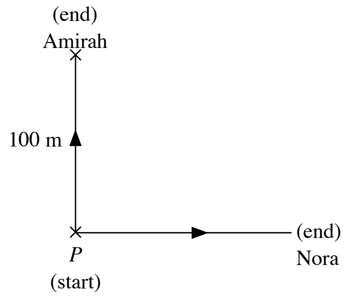
For question 2, advise the students to use cosine rule to find the magnitude of $\overrightarrow{BD}$ in part **(b)**. Highlight to the students that BD is parallel to AE, so $\overrightarrow{BD}$ is $k\overrightarrow{AE}$.

Question 3 is very similar to Practice Now 18, so teachers may use it to guide the students to first prove that $PQRS$ is a parallelogram first and then prove that it is a rectangle by showing that its angles are 90°.

WORKED SOLUTIONS

Class Discussion (Scalar and Vector Quantities)

1.



2. No, since they walked in different directions.

3. Direction

Thinking Time (Page 217)

An example of a scalar is mass (e.g. 100 kg), while an example of a vector is weight (e.g. 100 N).
Other examples of scalars are temperature, volume, energy and time.
Other examples of vectors are acceleration, force and electric field.

Class Discussion (Equal Vectors)

1. Vectors **a** and **b** have the same magnitude and same direction.
Vectors **a** and **c** have the same magnitude but opposite directions.
Vectors **a** and **d** have the same magnitude but different directions.
Vectors **a** and **e** have the same direction but different magnitudes.

2. $\mathbf{a} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$

Both x and y components are the same.

Thinking Time (Page 221)

1. No. The two vectors are in the same direction but the magnitude of $\begin{pmatrix} 4 \\ -6 \end{pmatrix}$ is 2 times the magnitude of $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$.

2. No. The two vectors have the same magnitude but they are in the opposite direction of each other.

Thinking Time (Page 231)

$|\mathbf{a} + \mathbf{b}| = |\mathbf{a}| + |\mathbf{b}|$ if and only if **a** and **b** are parallel vectors in the same direction.
If **a** and **b** are parallel vectors in the same direction, $\mathbf{b} = k\mathbf{a}$ where $k > 0$.

$$\begin{aligned} |\mathbf{a} + \mathbf{b}| &= |\mathbf{a} + k\mathbf{a}| \\ &= |(1 + k)\mathbf{a}| \\ &= (1 + k)|\mathbf{a}| \end{aligned}$$

$$\begin{aligned} |\mathbf{a}| + |\mathbf{b}| &= |\mathbf{a}| + |k\mathbf{a}| \\ &= |\mathbf{a}| + k|\mathbf{a}| \\ &= (1 + k)|\mathbf{a}| \end{aligned}$$

Class Discussion (The Zero Vector)

1. The boat went from Changi Jetty (P) to Pulau Seduku (R) and then went from Pulau Seduku (R) to Changi Jetty (P), so the journey is a zero displacement of the boat from Changi Jetty (P).

2. Since the journey is a zero displacement of the boat from P, $\overrightarrow{PR} + \overrightarrow{RP}$ is a zero vector, i.e. $\overrightarrow{PR} + \overrightarrow{RP} = \mathbf{0}$

Thinking Time (Page 238)

$|\mathbf{a} - \mathbf{b}| = |\mathbf{a}| - |\mathbf{b}|$ if and only if **a** and **b** are parallel vectors in the same direction and $|\mathbf{a}| > |\mathbf{b}|$.
If **a** and **b** are parallel vectors in the same direction and $|\mathbf{a}| > |\mathbf{b}|$, $\mathbf{b} = k\mathbf{a}$ where $0 < k < 1$.

$$\begin{aligned} |\mathbf{a} - \mathbf{b}| &= |\mathbf{a} - k\mathbf{a}| \\ &= |(1 - k)\mathbf{a}| \\ &= (1 - k)|\mathbf{a}| \end{aligned}$$

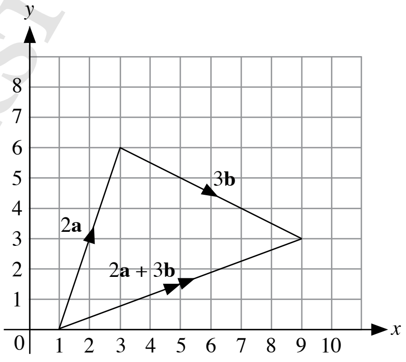
$$\begin{aligned} |\mathbf{a}| - |\mathbf{b}| &= |\mathbf{a}| - |k\mathbf{a}| \\ &= |\mathbf{a}| - k|\mathbf{a}| \\ &= (1 - k)|\mathbf{a}| \end{aligned}$$

Thinking Time (Page 247)

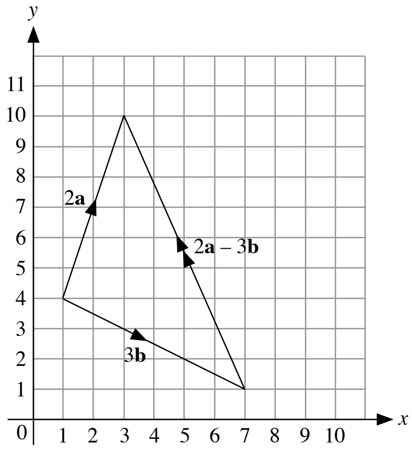
If k is positive, it means that the two vectors are parallel and in the same direction.
If k is negative, it means that the two vectors are parallel and in the opposite directions.

Class Discussion (Graphical Representation of Vectors)

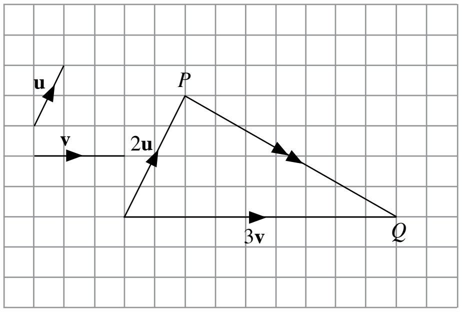
(a) Triangle Law of Vector Addition:



(b) Triangle Law of Vector Subtraction:



Class Discussion (Expressing a Vector in Terms of Two Other Vectors)



$\overrightarrow{PQ} = 3\mathbf{v} - 2\mathbf{u}$

Class Discussion (Real-Life Examples of Resultant Vectors)

An airplane flies 200 km north then 300 km east.

An object is experiencing two perpendicular forces F_1 = 50 N and F_2 = 30 N.

A plane is heading due north with an air speed of 400 km/h when it is blown off course by a wind of 100 km/h from the northeast.

Practise Now (Page 219)

$\overrightarrow{AB} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$

Magnitude of $\overrightarrow{AB} = |\overrightarrow{AB}| = \sqrt{(-3)^2 + 4^2}$ = 5 units

$\mathbf{c} = \begin{pmatrix} -2 \\ -2 \end{pmatrix}$

Magnitude of $\mathbf{c} = |\mathbf{c}| = \sqrt{(-2)^2 + (-2)^2}$ = 2.83 units (to 3 s.f.)

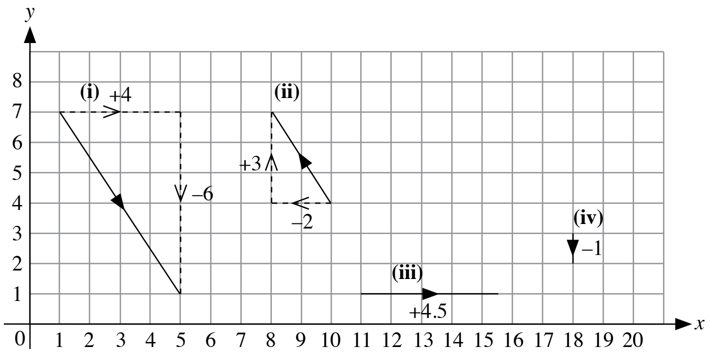
$\overrightarrow{DE} = \begin{pmatrix} 5 \\ 0 \end{pmatrix}$

Magnitude of $\overrightarrow{DE} = |\overrightarrow{DE}| = \sqrt{5^2}$ = 5 units

$\mathbf{f} = \begin{pmatrix} 0 \\ -3 \end{pmatrix}$

Magnitude of $\mathbf{f} = |\mathbf{f}| = \sqrt{(-3)^2}$ = 3 units

Practise Now 1



Practise Now 2

1. **(a)** **(i)** Since $\begin{pmatrix} x+2 \\ 4-y \end{pmatrix} = \begin{pmatrix} 10-x \\ y-5 \end{pmatrix}$, then

$x + 2 = 10 - x$ and $4 - y = y - 5$

$2x = 8$ $\qquad 2y = 9$

$x = 4$ $\qquad y = 4\frac{1}{2}$

$\therefore x = 4$ and $y = 4\frac{1}{2}$

(ii) $\mathbf{a} = \begin{pmatrix} x+2 \\ 4-y \end{pmatrix}$

$= \begin{pmatrix} 4+2 \\ 4-4\frac{1}{2} \end{pmatrix}$

$= \begin{pmatrix} 6 \\ -\frac{1}{2} \end{pmatrix}$

Negative of $\mathbf{a} = -\mathbf{a}$

$= -\begin{pmatrix} 6 \\ -\frac{1}{2} \end{pmatrix}$

$= \begin{pmatrix} -6 \\ \frac{1}{2} \end{pmatrix}$

(iii) $|\mathbf{a}| = \sqrt{6^2 + \left(-\frac{1}{2}\right)^2}$

$= \sqrt{36 + \frac{1}{4}}$

$= \sqrt{\frac{145}{4}}$

$\mathbf{b} = \begin{pmatrix} 10-x \\ y-5 \end{pmatrix}$

$= \begin{pmatrix} 10-4 \\ 4\frac{1}{2}-5 \end{pmatrix}$

$= \begin{pmatrix} 6 \\ -\frac{1}{2} \end{pmatrix}$

OXFORD UNIVERSITY PRESS

$$|\mathbf{b}| = \sqrt{6^2 + \left(-\frac{1}{2}\right)^2}$$

$$= \sqrt{36 + \frac{1}{4}}$$

$$= \sqrt{\frac{145}{4}}$$

$$\therefore\ |\mathbf{a}| = |\mathbf{b}| = \sqrt{\frac{145}{4}}\ \text{(shown)}$$

(b) **(i)**

$$\begin{aligned} |\mathbf{a}| &= |\mathbf{b}| \\ \sqrt{(x+2)^2 + (4-y)^2} &= \sqrt{(10-x)^2 + (y-5)^2} \\ x^2 + 4x + 4 + 16 - 8y + y^2 &= 100 - 20x + x^2 + y^2 - 10y + 25 \\ 4x - 8y + 20 &= -20x - 10y + 125 \\ 2y &= -24x + 105 \\ y &= \frac{-24x + 105}{2} \end{aligned}$$

(ii) **a** may not be equal to **b** because only their magnitudes are equal but they may have different directions.

Practise Now 3

(i) $\overrightarrow{PB} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$

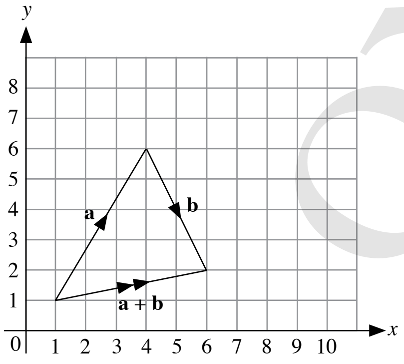
(ii) $\overrightarrow{BQ} = \overrightarrow{AP} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

(iii) $\overrightarrow{PR} = \overrightarrow{BA} = -\overrightarrow{AB} = -\begin{pmatrix} 4 \\ 1 \end{pmatrix} = \begin{pmatrix} -4 \\ -1 \end{pmatrix}$

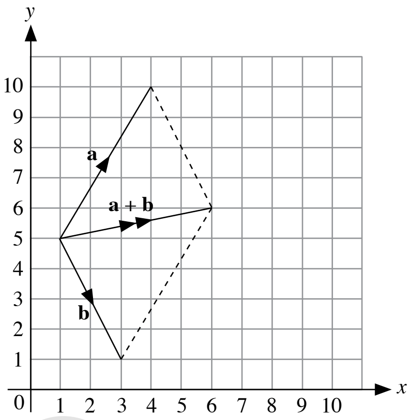
(iv) $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ have the same magnitude but $\overrightarrow{PQ} \neq \overrightarrow{PR}$ because they have opposite directions.

Practise Now 4

1. **(i)** Triangle Law of Vector Addition:



Parallelogram Law of Vector Addition:



(ii) $\mathbf{a} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$ $\quad \mathbf{b} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$ $\quad \mathbf{a} + \mathbf{b} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$

(iii) From **(ii)**,

$$\mathbf{a} + \mathbf{b} = \begin{pmatrix} 3 \\ 5 \end{pmatrix} + \begin{pmatrix} 2 \\ -4 \end{pmatrix}$$

$$= \begin{pmatrix} 5 \\ 1 \end{pmatrix}$$

(iv) $|\mathbf{a}| = \sqrt{3^2 + 5^2} = \sqrt{34}$ units

$|\mathbf{b}| = \sqrt{2^2 + (-4)^2} = \sqrt{20}$ units

$|\mathbf{a} + \mathbf{b}| = \sqrt{5^2 + 1^2} = \sqrt{26}$ units

(v) $|\mathbf{a} + \mathbf{b}| \neq |\mathbf{a}| + |\mathbf{b}|$

From the diagram, the 3 vectors, **a**, **b** and **a** + **b**, form the sides of a triangle. Since the sum of the lengths of any two sides of a triangle is larger than the length of the third side, $|\mathbf{a} + \mathbf{b}| \neq |\mathbf{a}| + |\mathbf{b}|$.

2. **(i)** $\begin{pmatrix} 6 \\ 9 \end{pmatrix} + \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 8 \\ 12 \end{pmatrix}$

(ii) $\begin{pmatrix} 8 \\ -3 \end{pmatrix} + \begin{pmatrix} -10 \\ -5 \end{pmatrix} + \begin{pmatrix} 4 \\ 8 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$

Practise Now 5

(i) $\overrightarrow{PQ} + \overrightarrow{QR} = \overrightarrow{PR}$

(ii) $\overrightarrow{SR} + \overrightarrow{PS} = \overrightarrow{PS} + \overrightarrow{SR} = \overrightarrow{PR}$

(iii) $\overrightarrow{PR} + \overrightarrow{RS} + \overrightarrow{SQ} = \overrightarrow{PQ}$

Practise Now 6

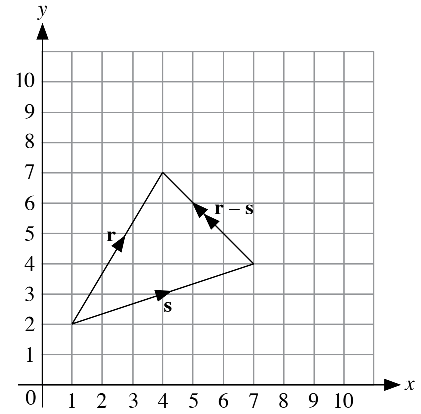
(a) $\begin{pmatrix} 8 \\ -1 \end{pmatrix} + \begin{pmatrix} -8 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

(b) $\begin{pmatrix} -6 \\ -7 \end{pmatrix} + \begin{pmatrix} 6 \\ 7 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

OXFORD UNIVERSITY PRESS

Practise Now 7

(i)



(ii) $\mathbf{r} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$ $\quad \mathbf{s} = \begin{pmatrix} 6 \\ 2 \end{pmatrix}$ $\quad \mathbf{r} - \mathbf{s} = \begin{pmatrix} -3 \\ 3 \end{pmatrix}$

(iii) $|\mathbf{r}| = \sqrt{3^2 + 5^2} = \sqrt{34}$ units

$|\mathbf{s}| = \sqrt{6^2 + 2^2} = \sqrt{40}$ units

$|\mathbf{r} - \mathbf{s}| = \sqrt{(-3)^2 + 3^2} = \sqrt{18}$ units

(iv) $|\mathbf{r} - \mathbf{s}| \neq |\mathbf{r}| - |\mathbf{s}|$

Practise Now 8

(a) $\mathbf{b} - \mathbf{a}$
(b) $\mathbf{a} - \mathbf{b}$
(c) $\mathbf{m} - \mathbf{n}$
(d) $\mathbf{v} + \mathbf{w}$
(e) $-\mathbf{v} - \mathbf{w}$

Practise Now 9

(a) $\overrightarrow{PR} = \overrightarrow{OQ} = \mathbf{q}$

(b) $\overrightarrow{RQ} = \overrightarrow{PO} = -\overrightarrow{OP} = -\mathbf{p}$

(c) $\overrightarrow{OR} = \mathbf{p} + \mathbf{q}$ (Parallelogram Law of Vector Addition)

(d) $\overrightarrow{PQ} = \mathbf{q} - \mathbf{p}$ (Triangle Law of Vector Subtraction)

(e) $\overrightarrow{QP} = \mathbf{p} - \mathbf{q}$ (Triangle Law of Vector Subtraction)

Practise Now 10

(a) $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$

(b) $\overrightarrow{AB} - \overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{CA} = \overrightarrow{CA} + \overrightarrow{AB} = \overrightarrow{CB}$

(c) Not possible to simplify

(d) $\overrightarrow{PQ} - \overrightarrow{PR} = \overrightarrow{PQ} + \overrightarrow{RP} = \overrightarrow{RP} + \overrightarrow{PQ} = \overrightarrow{RQ}$

(e) $\overrightarrow{PQ} - \overrightarrow{RQ} = \overrightarrow{PQ} + \overrightarrow{QR} = \overrightarrow{PR}$

(f) $\overrightarrow{PQ} + \overrightarrow{RP} - \overrightarrow{RS} = \overrightarrow{PQ} + \overrightarrow{RP} + \overrightarrow{SR} = \overrightarrow{SR} + \overrightarrow{RP} + \overrightarrow{PQ} = \overrightarrow{SQ}$

Practise Now 11

(a) (i) $\begin{pmatrix} 1 \\ 3 \end{pmatrix} - \begin{pmatrix} 8 \\ -6 \end{pmatrix} = \begin{pmatrix} -7 \\ 9 \end{pmatrix}$

(ii) $\begin{pmatrix} -2 \\ -3 \end{pmatrix} - \begin{pmatrix} -5 \\ 4 \end{pmatrix} + \begin{pmatrix} -6 \\ 7 \end{pmatrix} = \begin{pmatrix} -3 \\ 0 \end{pmatrix}$

(b) (i) $\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} -5 \\ 7 \end{pmatrix} = \begin{pmatrix} 8 \\ -3 \end{pmatrix}$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 8 \\ -3 \end{pmatrix} - \begin{pmatrix} -5 \\ 7 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 13 \\ -10 \end{pmatrix}$$

$\therefore x = 13, y = -10$

(ii) $\begin{pmatrix} 3 \\ y \end{pmatrix} - \begin{pmatrix} x \\ -9 \end{pmatrix} = \begin{pmatrix} 4 \\ x \end{pmatrix}$

$$\begin{pmatrix} 3 - x \\ y + 9 \end{pmatrix} = \begin{pmatrix} 4 \\ x \end{pmatrix}$$

$$3 - x = 4$$
$$x = -1$$
$$y + 9 = x$$
$$y + 9 = -1$$
$$y = -10$$

$\therefore x = -1, y = -10$

Practise Now 12

1. (a) (i) Since $\begin{pmatrix} 6 \\ -9 \end{pmatrix} = 3\begin{pmatrix} 2 \\ -3 \end{pmatrix}$, then $\begin{pmatrix} 6 \\ -9 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$ are parallel.

(ii) If $\begin{pmatrix} 14 \\ 18 \end{pmatrix} = k\begin{pmatrix} 2 \\ -3 \end{pmatrix}$, then

$14 = 2k$ and $18 = -3k$

$k = 7$ $\quad k = -6$

Since both values of *k* are different, therefore $\begin{pmatrix} 14 \\ 18 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$ are not parallel.

(iii) Since $\begin{pmatrix} -3 \\ 6 \end{pmatrix} = -\frac{3}{4}\begin{pmatrix} 4 \\ -8 \end{pmatrix}$, then $\begin{pmatrix} -3 \\ 6 \end{pmatrix}$ and $\begin{pmatrix} 4 \\ -8 \end{pmatrix}$ are parallel.

(b) A vector in the same direction as $\begin{pmatrix} 4 \\ -3 \end{pmatrix}$ is $2\begin{pmatrix} 4 \\ -3 \end{pmatrix} = \begin{pmatrix} 8 \\ -6 \end{pmatrix}$.

A vector in the opposite direction as $\begin{pmatrix} 4 \\ -3 \end{pmatrix}$ is $-\begin{pmatrix} 4 \\ -3 \end{pmatrix} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$.

2. Since $\begin{pmatrix} 4 \\ -3 \end{pmatrix}$ and $\begin{pmatrix} 12 \\ p \end{pmatrix}$ are parallel vectors, then $\begin{pmatrix} 4 \\ -3 \end{pmatrix} = k\begin{pmatrix} 12 \\ p \end{pmatrix}$.

$4 = 12k$ and $-3 = kp$

$k = \frac{1}{3}$ $\quad p = \frac{-3}{\frac{1}{3}} = -9$

$\therefore p = -9$

OXFORD UNIVERSITY PRESS

Practise Now 13

1. **(i)** $$\mathbf{u} + 3\mathbf{v} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + 3\begin{pmatrix} -2 \\ 1 \end{pmatrix}$$
$$= \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} -6 \\ 3 \end{pmatrix}$$
$$= \begin{pmatrix} -5 \\ 5 \end{pmatrix}$$

(ii) $$3\mathbf{u} - 2\mathbf{v} - \mathbf{w} = 3\begin{pmatrix} 1 \\ 2 \end{pmatrix} - 2\begin{pmatrix} -2 \\ 1 \end{pmatrix} - \begin{pmatrix} 3 \\ -4 \end{pmatrix}$$
$$= \begin{pmatrix} 3 \\ 6 \end{pmatrix} - \begin{pmatrix} -4 \\ 2 \end{pmatrix} - \begin{pmatrix} 3 \\ -4 \end{pmatrix}$$
$$= \begin{pmatrix} 4 \\ 8 \end{pmatrix}$$

2. $$2\mathbf{a} + \mathbf{b} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$$
$$2\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 2 \\ -3 \end{pmatrix} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$$
$$\begin{pmatrix} 2x + 2 \\ 2y - 3 \end{pmatrix} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$$

$2x + 2 = 5$ and $2y - 3 = 3$

$2x = 3$ $\quad$ $2y = 6$

$x = 1\frac{1}{2}$ $\quad$ $y = 3$

$\therefore x = 1\frac{1}{2}$ and $y = 3$

Practise Now 14

(a) The position vector of P is $\overrightarrow{OP} = \begin{pmatrix} 8 \\ -2 \end{pmatrix}$.

The position vector of Q is $\overrightarrow{OQ} = \begin{pmatrix} -1 \\ 7 \end{pmatrix}$.

$\therefore \overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP} = \begin{pmatrix} -1 \\ 7 \end{pmatrix} - \begin{pmatrix} 8 \\ -2 \end{pmatrix} = \begin{pmatrix} -9 \\ 9 \end{pmatrix}$.

(b) $$\overrightarrow{AB} = \begin{pmatrix} -4 \\ 5 \end{pmatrix}$$
$$\overrightarrow{OB} - \overrightarrow{OA} = \begin{pmatrix} -4 \\ 5 \end{pmatrix}$$
$$\overrightarrow{OB} - \begin{pmatrix} 6 \\ -7 \end{pmatrix} = \begin{pmatrix} -4 \\ 5 \end{pmatrix}$$
$$\overrightarrow{OB} = \begin{pmatrix} -4 \\ 5 \end{pmatrix} + \begin{pmatrix} 6 \\ -7 \end{pmatrix}$$
$$\overrightarrow{OB} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$$

$\therefore$ Coordinates of B are $(2, -2)$.

Alternatively,
$$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$$
$$= \begin{pmatrix} 6 \\ -7 \end{pmatrix} + \begin{pmatrix} -4 \\ 5 \end{pmatrix}$$
$$= \begin{pmatrix} 2 \\ -2 \end{pmatrix}$$

$\therefore$ Coordinates of B are $(2, -2)$.

Practise Now 15

Since $ABCD$ is a parallelogram, then
$$\overrightarrow{DC} = \overrightarrow{AB}$$
$$\overrightarrow{OC} - \overrightarrow{OD} = \overrightarrow{OB} - \overrightarrow{OA}$$
$$\overrightarrow{OC} - \begin{pmatrix} 5 \\ -4 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} - \begin{pmatrix} 3 \\ 7 \end{pmatrix}$$
$$\overrightarrow{OC} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} - \begin{pmatrix} 3 \\ 7 \end{pmatrix} + \begin{pmatrix} 5 \\ -4 \end{pmatrix}$$
$$\overrightarrow{OC} = \begin{pmatrix} 1 \\ -9 \end{pmatrix}$$

$\therefore$ Coordinates of C are $(1, -9)$.

Practise Now 16

(a) **(i)** $$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$$
$$= \overrightarrow{AB} + \overrightarrow{AD}$$
$$= 8\mathbf{a} + 4\mathbf{b}$$

(ii) $$\overrightarrow{DF} = \frac{1}{4}\overrightarrow{DC}$$
$$= \frac{1}{4}\overrightarrow{AB}$$
$$= \frac{1}{4}(8\mathbf{a})$$
$$= 2\mathbf{a}$$

(iii) $$DF = \frac{1}{4}DC$$
$$\frac{DF}{DC} = \frac{1}{4}$$
$$\therefore \frac{FC}{DC} = \frac{3}{4}$$

$FC = \frac{3}{4}DC$, i.e.
$$\overrightarrow{FC} = \frac{3}{4}\overrightarrow{DC}$$
$$= \frac{3}{4}\overrightarrow{AB}$$
$$= \frac{3}{4}(8\mathbf{a})$$
$$= 6\mathbf{a}$$

OXFORD UNIVERSITY PRESS

(iv) $\triangle EDF$ and $\triangle BCF$ are similar. (corr. $\angle$s are equal)

$$\therefore \ \frac{EF}{BF} = \frac{DF}{CF} = \frac{1}{3}$$

$$\therefore EF = \frac{1}{3} BF$$

$$\begin{aligned}\overrightarrow{EF} &= \frac{1}{3}\overrightarrow{FB} \\ &= \frac{1}{3}(\overrightarrow{FC} + \overrightarrow{CB}) \\ &= \frac{1}{3}(\overrightarrow{FC} + \overrightarrow{DA}) \\ &= \frac{1}{3}(6\mathbf{a} - 4\mathbf{b}) \\ &= \frac{2}{3}(3\mathbf{a} - 2\mathbf{b})\end{aligned}$$

(b) **(i)** Since $\triangle EDF$ and $\triangle BCF$ are similar, then

$$\begin{aligned}\frac{\text{area of } \triangle EDF}{\text{area of } \triangle BCF} &= \left(\frac{DF}{CF}\right)^2 \\ &= \left(\frac{1}{3}\right)^2 \\ &= \frac{1}{9}\end{aligned}$$

(ii)
$$\begin{aligned}\overrightarrow{DE} &= \overrightarrow{DF} + \overrightarrow{FE} \\ &= \overrightarrow{DF} - \overrightarrow{EF} \\ &= 2\mathbf{a} - \frac{1}{3}(6\mathbf{a} - 4\mathbf{b}) \\ &= 2\mathbf{a} - 2\mathbf{a} + \frac{4}{3}\mathbf{b} \\ &= 1\frac{1}{3}\mathbf{b}\end{aligned}$$

$$\frac{\text{Area of } \triangle EDF}{\text{Area of } \triangle ADF}$$

$= \dfrac{\frac{1}{2} \times DE \times h}{\frac{1}{2} \times AD \times h}$, where h is the common height of $\triangle EDF$ and $\triangle ADF$,

$$\begin{aligned}&= \frac{DE}{AD} \\ &= \frac{1\frac{1}{3}}{4} \\ &= \frac{1}{3}\end{aligned}$$

Practise Now 17

(i) $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP} = 3\mathbf{q} - 9\mathbf{p}$

$\overrightarrow{OR} = \frac{1}{3}\overrightarrow{OP} = \frac{1}{3}(9\mathbf{p}) = 3\mathbf{p}$

$\overrightarrow{OS} = \frac{1}{2}\overrightarrow{OQ} = \frac{1}{2}(3\mathbf{q}) = \frac{3}{2}\mathbf{q}$

$\therefore \ \overrightarrow{RS} = \overrightarrow{OS} - \overrightarrow{PR} = \frac{3}{2}\mathbf{q} - 3\mathbf{p}$

(ii) Since $\overrightarrow{RT} = \frac{1}{4}\overrightarrow{RQ}$, then $\overrightarrow{OT} - \overrightarrow{OR} = \frac{1}{4}(\overrightarrow{OQ} - \overrightarrow{OR})$

$$\begin{aligned}\overrightarrow{OT} - 3\mathbf{p} &= \frac{1}{4}(3\mathbf{q} - 3\mathbf{p}) \\ \overrightarrow{OT} &= \frac{3}{4}\mathbf{q} - \frac{3}{4}\mathbf{p} + 3\mathbf{p} \\ &= \frac{3}{4}\mathbf{q} + \frac{9}{4}\mathbf{p} \\ &= \frac{3}{4}(3\mathbf{p} + \mathbf{q})\end{aligned}$$

$$\begin{aligned}\overrightarrow{TS} &= \overrightarrow{OS} - \overrightarrow{OT} \\ &= \frac{3}{2}\mathbf{q} - \frac{3}{4}(3\mathbf{p} + \mathbf{q}) \\ &= \frac{3}{2}\mathbf{q} - \frac{9}{4}\mathbf{p} - \frac{3}{4}\mathbf{q} \\ &= \frac{3}{4}\mathbf{q} - \frac{9}{4}\mathbf{p} \\ &= \frac{3}{4}(\mathbf{q} - 3\mathbf{p})\end{aligned}$$

Practise Now 18

(i) Let $\overrightarrow{QA} = \mathbf{a}$ and $\overrightarrow{QB} = \mathbf{b}$.

Then $\overrightarrow{AB} = \overrightarrow{QB} - \overrightarrow{QA} = \mathbf{b} - \mathbf{a}$.

Since $\overrightarrow{QP} = 2\overrightarrow{QA} = 2\mathbf{a}$ and $\overrightarrow{QR} = 2\overrightarrow{QB} = 2\mathbf{b}$.

$$\begin{aligned}\text{Then } \overrightarrow{PR} = \overrightarrow{QR} - \overrightarrow{QP} &= 2\mathbf{b} - 2\mathbf{a} \\ &= 2(\mathbf{b} - \mathbf{a}) \\ &= 2\overrightarrow{AB}\end{aligned}$$

Since $\overrightarrow{PR} = 2\overrightarrow{AB}$, then PR is a parallel to AB and $PR = 2AB$.

(ii) Using the same reasoning in **(i)** and $\triangle SPR$, we can show that PR is parallel to DC and $PR = 2DC$.

Since PR is parallel to AB and DC, AB is parallel to DC. Since $PR = 2AB = 2DC$, AB and DC have the same length. Therefore, AB and DC are the opposite sides of the parallelogram $ABCD$.

Exercise 7A

1. **(a)** Magnitude of $\begin{pmatrix} 3 \\ 4 \end{pmatrix} = \sqrt{3^2 + 4^2} = 5$ units

(b) Magnitude of $\begin{pmatrix} -5 \\ 12 \end{pmatrix} = \sqrt{(-5)^2 + 12^2} = 13$ units

(c) Magnitude of $\begin{pmatrix} -7 \\ -2 \end{pmatrix} = \sqrt{(-7)^2 + (-2)^2} = 7.28$ units (to 3 s.f.)

(d) Magnitude of $\begin{pmatrix} 0 \\ -6\frac{1}{2} \end{pmatrix} = \sqrt{0^2 + \left(-6\frac{1}{2}\right)^2} = 6\frac{1}{2}$ units

(e) Magnitude of $\begin{pmatrix} 8 \\ 0 \end{pmatrix} = \sqrt{8^2 + 0^2} = 8$ units

OXFORD UNIVERSITY PRESS

2. **(a)** Negative of $\begin{pmatrix} 12 \\ -7 \end{pmatrix} = -\begin{pmatrix} 12 \\ -7 \end{pmatrix} = \begin{pmatrix} -12 \\ 7 \end{pmatrix}$

(b) Negative of $\begin{pmatrix} -2 \\ 0 \end{pmatrix} = -\begin{pmatrix} -2 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$

(c) Negative of $\begin{pmatrix} 4 \\ 8 \end{pmatrix} = -\begin{pmatrix} 4 \\ 8 \end{pmatrix} = \begin{pmatrix} -4 \\ -8 \end{pmatrix}$

(d) Negative of $\begin{pmatrix} -3 \\ -1.2 \end{pmatrix} = -\begin{pmatrix} -3 \\ -1.2 \end{pmatrix} = \begin{pmatrix} 3 \\ 1.2 \end{pmatrix}$

(e) Negative of $\begin{pmatrix} 0 \\ 3\frac{1}{4} \end{pmatrix} = -\begin{pmatrix} 0 \\ 3\frac{1}{4} \end{pmatrix} = \begin{pmatrix} 0 \\ -3\frac{1}{4} \end{pmatrix}$

3. Since $\begin{pmatrix} a \\ 3 \end{pmatrix} = \begin{pmatrix} -2 \\ a+2b \end{pmatrix}$, then

$a = -2$ — (1)

$3 = a + 2b$ — (2)

Substitute (1) and (2):

$3 = -2 + 2b$

$2b = 5$

$b = 2\frac{1}{2}$

$\therefore a = -2$ and $b = 2\frac{1}{2}$

4. **(a)** $|\overrightarrow{AB}| = \sqrt{7^2 + 0^2}$

$= 7$ units

(b) **(i)** $\overrightarrow{DC} = \overrightarrow{AB} = \begin{pmatrix} 7 \\ 0 \end{pmatrix}$

(ii) $\overrightarrow{DA} = \overrightarrow{CB}$

$= -\overrightarrow{BC}$

$= -\begin{pmatrix} -3 \\ 4 \end{pmatrix}$

$= \begin{pmatrix} 3 \\ -4 \end{pmatrix}$

5. $\overrightarrow{AB} = \begin{pmatrix} -4 \\ 3 \end{pmatrix}$

Magnitude of $\overrightarrow{AB} = |\overrightarrow{AB}| = \sqrt{(-4)^2 + 3^2} = 5$ units

$\overrightarrow{CD} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

Magnitude of $\overrightarrow{CD} = |\overrightarrow{CD}| = \sqrt{1^2 + (-2)^2} = 2.24$ units (to 3 s.f.)

$\mathbf{p} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$

Magnitude of $\mathbf{p} = |\mathbf{p}| = \sqrt{3^2 + 3^2} = 4.24$ units (to 3 s.f.)

$\mathbf{q} = \begin{pmatrix} -2 \\ -1 \end{pmatrix}$

Magnitude of $\mathbf{q} = |\mathbf{q}| = \sqrt{(-2)^2 + (-1)^2} = 2.24$ units (to 3 s.f.)

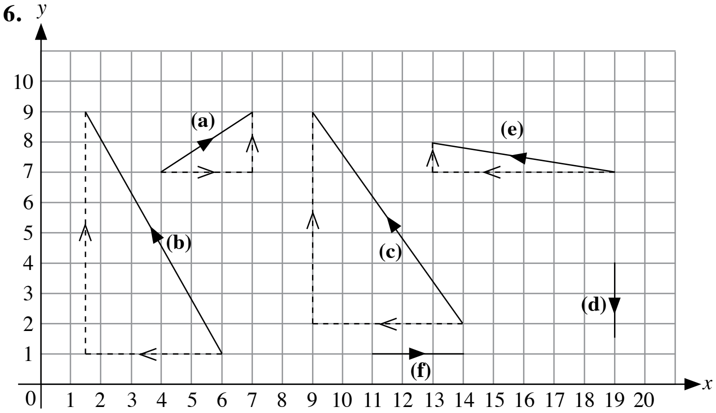
$\overrightarrow{RS} = \begin{pmatrix} -2 \\ 0 \end{pmatrix}$

Magnitude of $\overrightarrow{RS} = |\overrightarrow{RS}| = \sqrt{(-2)^2 + 0^2} = 2$ units

$\overrightarrow{TU} = \begin{pmatrix} 0 \\ 4 \end{pmatrix}$

Magnitude of $\overrightarrow{TU} = |\overrightarrow{TU}| = \sqrt{0^2 + 4^2} = 4$ units

6.



7. **(a)** **(i)** Since $\begin{pmatrix} x-3 \\ 2-y \end{pmatrix} = \begin{pmatrix} 5-x \\ y-9 \end{pmatrix}$, then

$x - 3 = 5 - x$ and $2 - y = y - 9$

$2x = 8$ $\quad 2y = 11$

$x = 4$ $\quad y = 5\frac{1}{2}$

$\therefore x = 4$ and $y = 5\frac{1}{2}$

(ii) $\mathbf{a} = \begin{pmatrix} x-3 \\ 2-y \end{pmatrix}$

$= \begin{pmatrix} 4-3 \\ 2-5\frac{1}{2} \end{pmatrix}$

$= \begin{pmatrix} 1 \\ -3\frac{1}{2} \end{pmatrix}$

Negative of $\mathbf{a} = -\mathbf{a}$

$= -\begin{pmatrix} 1 \\ -3\frac{1}{2} \end{pmatrix}$

$= \begin{pmatrix} -1 \\ 3\frac{1}{2} \end{pmatrix}$

(iii) $|\mathbf{a}| = \sqrt{1^2 + \left(-3\frac{1}{2}\right)^2}$

$= \sqrt{1 + \frac{49}{4}}$

$= \sqrt{\frac{53}{4}}$

$\mathbf{b} = \begin{pmatrix} 5-x \\ y-9 \end{pmatrix}$

$= \begin{pmatrix} 5-4 \\ 5\frac{1}{2}-9 \end{pmatrix}$

$= \begin{pmatrix} 1 \\ -3\frac{1}{2} \end{pmatrix}$

OXFORD UNIVERSITY PRESS

$$|\mathbf{b}| = \sqrt{1^2 + \left(-3\frac{1}{2}\right)^2}$$

$$= \sqrt{1 + \frac{49}{4}}$$

$$= \sqrt{\frac{53}{4}}$$

$$\therefore |\mathbf{a}| = |\mathbf{b}| = \sqrt{\frac{53}{4}} \text{ (shown)}$$

(b) **(i)**

$$\begin{aligned} |\mathbf{a}| &= |\mathbf{b}| \\ \sqrt{(x-3)^2 + (2-y)^2} &= \sqrt{(5-x)^2 + (y-9)^2} \\ x^2 - 6x + 9 + 4 - 4y + y^2 &= 25 - 10x + x^2 + y^2 - 18y + 81 \\ -6x - 4y + 13 &= -10x - 18y + 106 \\ 14y &= -4x + 93 \\ y &= \frac{-4x + 93}{14} \end{aligned}$$

(ii) **a** may not be equal to **b** because only their magnitudes are equal but they may have different directions.

8. **(i)** $|\overrightarrow{AB}| = \sqrt{(-3)^2 + 4^2}$

$= \sqrt{9 + 16}$

$= 5$ units

$|\overrightarrow{CD}| = \sqrt{0^2 + 5^2}$

$= \sqrt{25}$

$= 5$ units

$\therefore |\overrightarrow{AB}| = |\overrightarrow{CD}|$

(ii) $\overrightarrow{AB} \neq \overrightarrow{CD}$ because ony their magnitudes are equal but they have different directions.

9. **(i)** $\overrightarrow{AY} = \begin{pmatrix} 1 \\ -4 \end{pmatrix}$

(ii) $\overrightarrow{YB} = \overrightarrow{XA} = \begin{pmatrix} 2 \\ 6 \end{pmatrix}$

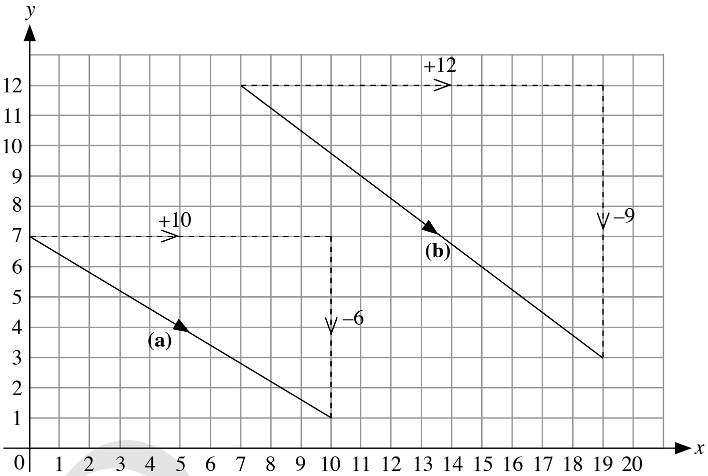
(iii) $\overrightarrow{AC} = \overrightarrow{YX} = -\overrightarrow{XY} = -\begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} -3 \\ -2 \end{pmatrix}$

(iv) $\overrightarrow{AB}$ and $\overrightarrow{AC}$ have the same magnitude but $\overrightarrow{AB} \neq \overrightarrow{AC}$ because they have opposite directions.

10. Since $|\mathbf{a}| = 7$, then

$$\begin{aligned} \sqrt{n^2 + (-3)^2} &= 7 \\ n^2 + (-3)^2 &= 7^2 \\ n^2 + 9 &= 49 \\ n^2 &= 40 \\ n &= \pm\sqrt{40} \end{aligned}$$

11.



12. Since $\mathbf{u} = \mathbf{v}$, then $\begin{pmatrix} 13s \\ 4t \end{pmatrix} = \begin{pmatrix} 6t + 20 \\ 18 - 7s \end{pmatrix}$.

$13s = 6t + 20$ — (1)

$4t = 18 - 7s$ — (2)

From (1): $s = \dfrac{6t + 20}{13}$ — (3)

Substitute (3) into (2):

$$\begin{aligned} 4t &= 18 - 7\left(\frac{6t + 20}{13}\right) \\ 52t &= 234 - 7(6t + 20) \\ 52t &= 234 - 42t - 140 \\ 52t + 42t &= 234 - 140 \\ 94t &= 94 \\ t &= 1 \end{aligned}$$

Substitute $t = 1$ into (3):

$s = \dfrac{6(1) + 20}{13} = 2$

$\therefore s = 2, t = 1$

13. **(a)** **(i)** $\overrightarrow{AB} = \overrightarrow{IJ}$ because $\overrightarrow{AB} = \begin{pmatrix} -2\frac{1}{2} \\ -5 \end{pmatrix}$ and $\overrightarrow{IJ} = \begin{pmatrix} -2\frac{1}{2} \\ -5 \end{pmatrix}$

(ii) $\overrightarrow{DC}$ and $\overrightarrow{HG}$

(b) **(i)** $\overrightarrow{JA}$, $\overrightarrow{GD}$, $\overrightarrow{FE}$

(ii) $\overrightarrow{GF}$, $\overrightarrow{KJ}$, $\overrightarrow{LA}$

(iii) $\overrightarrow{AD}$, $\overrightarrow{JG}$, $\overrightarrow{IH}$

(iv) $\overrightarrow{EG}$

(c) They do not have the same direction.

(d) They have opposite directions.

(e) **(i)** $\overrightarrow{CB}$, $\overrightarrow{DA}$, $\overrightarrow{GJ}$, $\overrightarrow{HI}$ (Any one is correct)

(ii) $\overrightarrow{FE}$, $\overrightarrow{JA}$, $\overrightarrow{GD}$, $\overrightarrow{KL}$ (Any one is correct)

(iii) $\overrightarrow{AL}$, $\overrightarrow{FG}$, $\overrightarrow{JK}$, $\overrightarrow{ED}$ (Any one is correct)

OXFORD UNIVERSITY PRESS

Exercise 7B

1. **(a)** $\begin{pmatrix}2\\4\end{pmatrix}+\begin{pmatrix}7\\5\end{pmatrix}=\begin{pmatrix}9\\9\end{pmatrix}$

(b) $\begin{pmatrix}3\\-5\end{pmatrix}+\begin{pmatrix}-4\\1\end{pmatrix}=\begin{pmatrix}-1\\-4\end{pmatrix}$

(c) $\begin{pmatrix}-9\\-2\end{pmatrix}+\begin{pmatrix}0\\-8\end{pmatrix}+\begin{pmatrix}-3\\7\end{pmatrix}=\begin{pmatrix}-12\\-3\end{pmatrix}$

2. **(a)** $\mathbf{a}+\mathbf{b}=\begin{pmatrix}4\\3\end{pmatrix}+\begin{pmatrix}-7\\2\end{pmatrix}=\begin{pmatrix}-3\\5\end{pmatrix}$

$\mathbf{b}+\mathbf{a}=\begin{pmatrix}-7\\2\end{pmatrix}+\begin{pmatrix}4\\3\end{pmatrix}=\begin{pmatrix}-3\\5\end{pmatrix}$

$\therefore\ \mathbf{a}+\mathbf{b}=\mathbf{b}+\mathbf{a}$

(b)
$$\begin{aligned}(\mathbf{a}+\mathbf{b})+\mathbf{c}&=\left[\begin{pmatrix}4\\3\end{pmatrix}+\begin{pmatrix}-7\\2\end{pmatrix}\right]+\begin{pmatrix}1\\-5\end{pmatrix}\\&=\begin{pmatrix}-3\\5\end{pmatrix}+\begin{pmatrix}1\\-5\end{pmatrix}\\&=\begin{pmatrix}-2\\0\end{pmatrix}\end{aligned}$$

$$\begin{aligned}\mathbf{a}+(\mathbf{b}+\mathbf{c})&=\begin{pmatrix}4\\3\end{pmatrix}+\left[\begin{pmatrix}-7\\2\end{pmatrix}+\begin{pmatrix}1\\-5\end{pmatrix}\right]\\&=\begin{pmatrix}4\\3\end{pmatrix}+\begin{pmatrix}-6\\-3\end{pmatrix}\\&=\begin{pmatrix}-2\\0\end{pmatrix}\end{aligned}$$

$\therefore\ (\mathbf{a}+\mathbf{b})+\mathbf{c}=\mathbf{a}+(\mathbf{b}+\mathbf{c})$

3. **(a)** $\overrightarrow{LM}+\overrightarrow{MN}=\overrightarrow{LN}$

(b) $\overrightarrow{PN}+\overrightarrow{LP}=\overrightarrow{LP}+\overrightarrow{PN}=\overrightarrow{LN}$

(c) $\overrightarrow{LN}+\overrightarrow{NM}+\overrightarrow{MP}=\overrightarrow{LP}$

4. **(a)** $\begin{pmatrix}12\\-6\end{pmatrix}+\begin{pmatrix}-12\\6\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}$

(b) $\begin{pmatrix}5\\7\end{pmatrix}+\begin{pmatrix}-5\\-7\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}$

(c) $\begin{pmatrix}x\\y\end{pmatrix}+\begin{pmatrix}-x\\-y\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}$

5. **(a)** $\begin{pmatrix}9\\1\end{pmatrix}+\begin{pmatrix}-9\\-1\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}$

(b) $\begin{pmatrix}-3\\-7\end{pmatrix}+\begin{pmatrix}3\\7\end{pmatrix}=\mathbf{0}$

(c) $\begin{pmatrix}q\\p\end{pmatrix}+\begin{pmatrix}-q\\-p\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}$

6. **(a)** $\overrightarrow{AB}+\overrightarrow{BA}=\mathbf{0}$

(b) $\overrightarrow{PQ}+\overrightarrow{QR}+\overrightarrow{RP}=\mathbf{0}$

(c) $\overrightarrow{MN}+\overrightarrow{LM}+\overrightarrow{NL}=\overrightarrow{MN}+\overrightarrow{NL}+\overrightarrow{LM}=\mathbf{0}$

7. **(a)** $\mathbf{p}-\mathbf{q}$

(b) $\mathbf{q}-\mathbf{p}$

(c) $\mathbf{b}-\mathbf{a}$

(d) $\mathbf{b}+\mathbf{a}$

(e) $\mathbf{s}-\mathbf{r}$

(f) $\mathbf{r}+\mathbf{s}$

(g) $-\mathbf{m}-\mathbf{n}$

(h) $\mathbf{n}-\mathbf{m}$

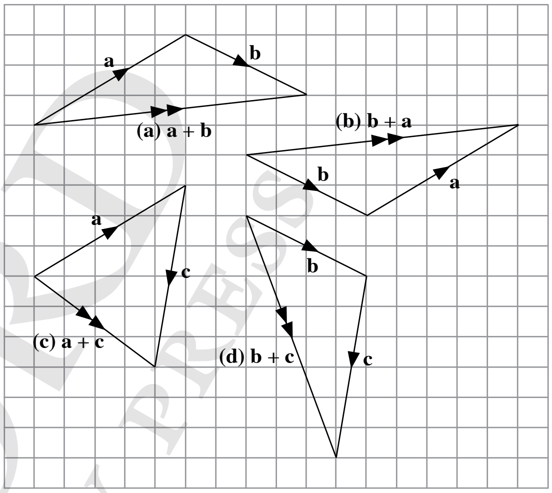
8. **(a)** $\begin{pmatrix}5\\4\end{pmatrix}-\begin{pmatrix}3\\2\end{pmatrix}=\begin{pmatrix}2\\2\end{pmatrix}$

(b) $\begin{pmatrix}-1\\3\end{pmatrix}-\begin{pmatrix}-3\\-4\end{pmatrix}=\begin{pmatrix}2\\7\end{pmatrix}$

(c) $\begin{pmatrix}2\\3\end{pmatrix}+\begin{pmatrix}5\\-2\end{pmatrix}-\begin{pmatrix}7\\-3\end{pmatrix}=\begin{pmatrix}0\\4\end{pmatrix}$

(d) $\begin{pmatrix}4\\7\end{pmatrix}-\begin{pmatrix}-2\\5\end{pmatrix}-\begin{pmatrix}3\\-6\end{pmatrix}=\begin{pmatrix}3\\8\end{pmatrix}$

9.



10. **(a)** $\overrightarrow{PT}+\overrightarrow{TR}=\overrightarrow{PR}$

(b) $\overrightarrow{SQ}+\overrightarrow{QR}=\overrightarrow{SR}$

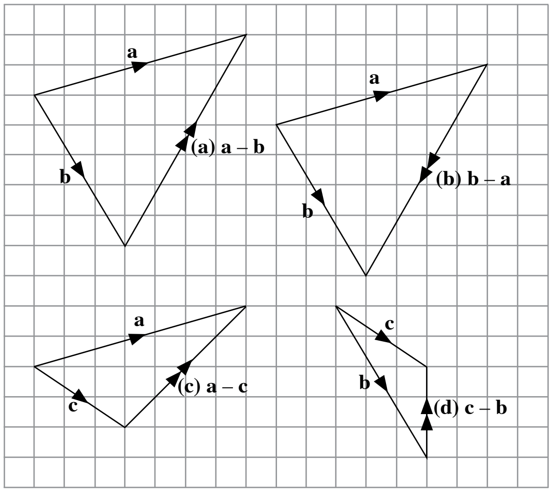
(c) $\overrightarrow{TR}+\overrightarrow{ST}=\overrightarrow{ST}+\overrightarrow{TR}=\overrightarrow{SR}$

(d) $\overrightarrow{SQ}+\overrightarrow{QT}=\overrightarrow{ST}$

(e) $\overrightarrow{SQ}+\overrightarrow{QR}+\overrightarrow{PS}=\overrightarrow{PS}+\overrightarrow{SQ}+\overrightarrow{QR}=\overrightarrow{PR}$

(f) $\overrightarrow{RQ}+\overrightarrow{QT}+\overrightarrow{TP}+\overrightarrow{PS}=\overrightarrow{RS}$

11.



OXFORD UNIVERSITY PRESS

12. **(a)** $\overrightarrow{RT} = \overrightarrow{OS} = \mathbf{s}$

(b) $\overrightarrow{TS} = \overrightarrow{RO} = -\overrightarrow{OR} = -\mathbf{r}$

(c) $\overrightarrow{OT} = \overrightarrow{OR} + \overrightarrow{RT}$

$= \mathbf{r} + \mathbf{s}$

(d) $\overrightarrow{RS} = \overrightarrow{OS} - \overrightarrow{OR}$

$= \mathbf{s} - \mathbf{r}$

(e) $\overrightarrow{SR} = \overrightarrow{OR} - \overrightarrow{OS}$

$= \mathbf{r} - \mathbf{s}$

13. **(a)** $\overrightarrow{RS} + \overrightarrow{ST} = \overrightarrow{RT}$

(b) $\overrightarrow{RS} - \overrightarrow{RT} = \overrightarrow{RS} + \overrightarrow{TR}$

$= \overrightarrow{TR} + \overrightarrow{RS}$

$= \overrightarrow{TS}$

(c) $\overrightarrow{RT} - \overrightarrow{RS} = \overrightarrow{RT} + \overrightarrow{SR}$

$= \overrightarrow{SR} + \overrightarrow{RT}$

$= \overrightarrow{ST}$

(d) Not possible to simplify

(e) $\overrightarrow{RS} - \overrightarrow{TS} = \overrightarrow{RS} + \overrightarrow{ST}$

$= \overrightarrow{RT}$

(f) $\overrightarrow{RS} + \overrightarrow{TR} - \overrightarrow{TU} = \overrightarrow{RS} + \overrightarrow{TR} + \overrightarrow{UT}$

$= \overrightarrow{UT} + \overrightarrow{TR} + \overrightarrow{RS}$

$= \overrightarrow{US}$

14. **(a)** $\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} -3 \\ 2 \end{pmatrix} = \begin{pmatrix} 7 \\ -5 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 7 \\ -5 \end{pmatrix} - \begin{pmatrix} -3 \\ 2 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 10 \\ -7 \end{pmatrix}$

$\therefore x = 10, y = -7$

(b) $\begin{pmatrix} 3 \\ y \end{pmatrix} - \begin{pmatrix} x \\ -8 \end{pmatrix} = \begin{pmatrix} -6 \\ 9 \end{pmatrix}$

$\begin{pmatrix} 3-x \\ y+8 \end{pmatrix} = \begin{pmatrix} -6 \\ 9 \end{pmatrix}$

$3 - x = -6$

$x = 9$

$y + 8 = 9$

$y = 1$

$\therefore x = 9, y = 1$

(c) $\begin{pmatrix} y \\ 3 \end{pmatrix} + \begin{pmatrix} -4 \\ 2x \end{pmatrix} = \begin{pmatrix} 6 \\ x \end{pmatrix}$

$\begin{pmatrix} y-4 \\ 3+2x \end{pmatrix} = \begin{pmatrix} 6 \\ x \end{pmatrix}$

$y - 4 = 6$

$y = 10$

$3 + 2x = x$

$2x - x = -3$

$x = -3$

$\therefore x = -3, y = 10$

(d) $\begin{pmatrix} 2x \\ 5 \end{pmatrix} - \begin{pmatrix} y-3 \\ -10 \end{pmatrix} = \begin{pmatrix} 4 \\ 3y \end{pmatrix}$

$\begin{pmatrix} 2x-y+3 \\ 15 \end{pmatrix} = \begin{pmatrix} 4 \\ 3y \end{pmatrix}$

$2x - y + 3 = 4$

$2x - y = 1$ — (1)

$3y = 15$

$y = 5$ — (2)

Substitute (2) into (1):

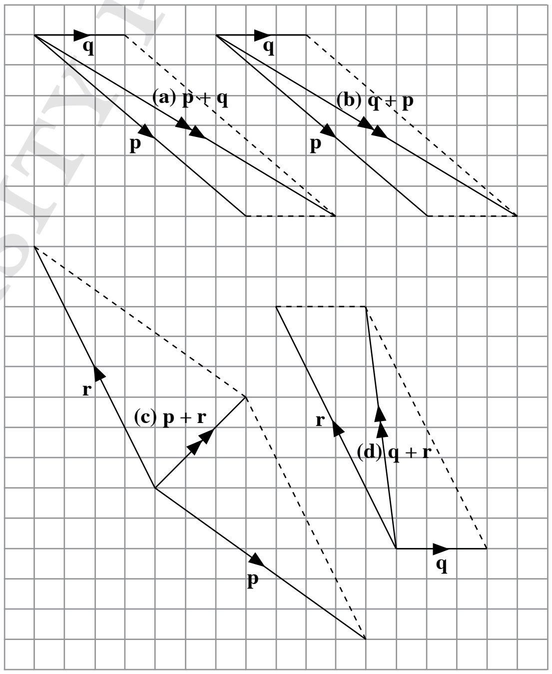
$2x - 5 = 1$

$2x = 6$

$x = 3$

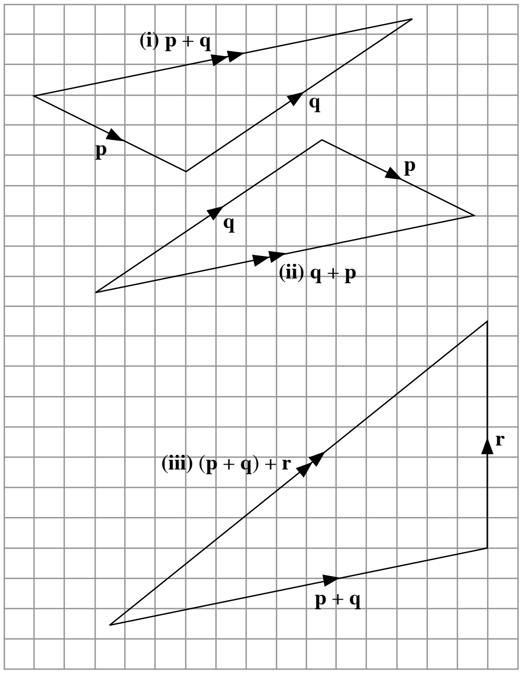
$\therefore x = 3, y = 5$

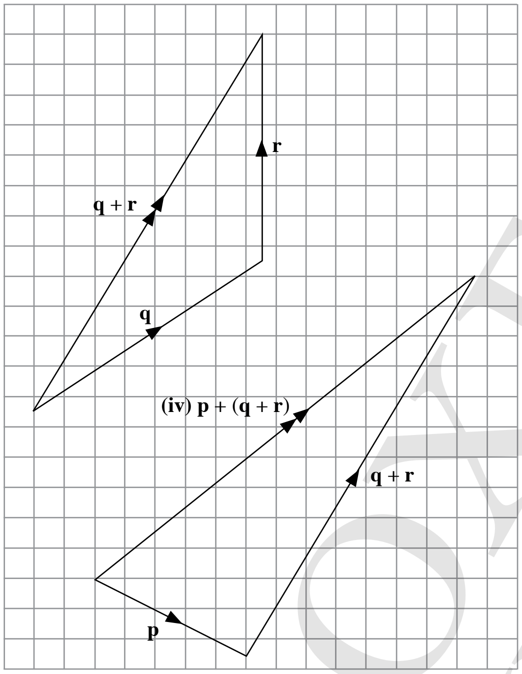
15.



OXFORD
UNIVERSITY PRESS

16. (a)





(b) Yes. Vector addition is commutative.

(c) Yes. Vector addition is associative.

17. (a) (i) $\overrightarrow{PQ} + \overrightarrow{PS} = \overrightarrow{PQ} + \overrightarrow{QR} = \overrightarrow{PR}$

(ii) $\overrightarrow{RO} - \overrightarrow{QO} = \overrightarrow{RO} + \overrightarrow{OQ} = \overrightarrow{RQ}$

(iii) $\overrightarrow{PR} - \overrightarrow{SR} + \overrightarrow{SQ} = \overrightarrow{PR} + \overrightarrow{RS} + \overrightarrow{SQ} = \overrightarrow{PQ}$

(b) (i) $\overrightarrow{SR} = \overrightarrow{PQ} = \mathbf{a}$

(ii) $\overrightarrow{PR} = \overrightarrow{PQ} + \overrightarrow{PS} = \mathbf{a} + \mathbf{b}$

(iii) $\overrightarrow{SQ} = \overrightarrow{SP} + \overrightarrow{PQ} = \overrightarrow{SP} + \overrightarrow{SR} = \mathbf{a} - \mathbf{b}$

18. (a) $\overrightarrow{SK} + \mathbf{u} = \mathbf{0}$

$\therefore \mathbf{u} = \overrightarrow{KS}$

(b) $\overrightarrow{SP} + \overrightarrow{PQ} + \mathbf{u} = \mathbf{0}$

$\therefore \mathbf{u} = \overrightarrow{QS}$

(c) $\overrightarrow{PS} + \overrightarrow{SK} + \overrightarrow{KR} = \mathbf{u}$

$\therefore \mathbf{u} = \overrightarrow{PR}$

(d) $\overrightarrow{PK} + (-\overrightarrow{SK}) = \mathbf{u}$

$\overrightarrow{PK} + \overrightarrow{KS} = \mathbf{u}$

$\therefore \mathbf{u} = \overrightarrow{PS}$

(e) $\overrightarrow{PS} + (-\overrightarrow{RS}) = \mathbf{u}$

$\overrightarrow{PS} + \overrightarrow{SR} = \mathbf{u}$

$\therefore \mathbf{u} = \overrightarrow{PR}$

(f) $\overrightarrow{PQ} + \overrightarrow{QR} + (-\overrightarrow{PR}) = \mathbf{u}$

$\overrightarrow{PQ} + \overrightarrow{QR} + \overrightarrow{RP} = \mathbf{u}$

$\therefore \mathbf{u} = \mathbf{0}$

Exercise 7C

1. (a) Since $\begin{pmatrix} -8 \\ 4 \end{pmatrix} = 4\begin{pmatrix} -2 \\ 1 \end{pmatrix}$, then $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} -8 \\ 4 \end{pmatrix}$ are parallel.

(b) If $\begin{pmatrix} 18 \\ 21 \end{pmatrix} = k\begin{pmatrix} 9 \\ 7 \end{pmatrix}$, then

$18 = 9k$ and $21 = 7k$

$k = 2$ $\quad k = 3$

Since both values of k are different, therefore $\begin{pmatrix} 9 \\ 7 \end{pmatrix}$ and $\begin{pmatrix} 18 \\ 21 \end{pmatrix}$ are not parallel.

(c) Since $\begin{pmatrix} 6 \\ -8 \end{pmatrix} = -2\begin{pmatrix} -3 \\ 4 \end{pmatrix}$, then $\begin{pmatrix} 6 \\ -8 \end{pmatrix}$ and $2\begin{pmatrix} -3 \\ 4 \end{pmatrix}$ are parallel.

2. (a) A vector in the same direction as $\begin{pmatrix} 8 \\ -7 \end{pmatrix}$ is $2\begin{pmatrix} 8 \\ -7 \end{pmatrix} = \begin{pmatrix} 16 \\ -14 \end{pmatrix}$.

A vector in the opposite direction as $\begin{pmatrix} 8 \\ -7 \end{pmatrix}$ is $-\begin{pmatrix} 8 \\ -7 \end{pmatrix} = \begin{pmatrix} -8 \\ 7 \end{pmatrix}$.

(b) A vector in the same direction as $\begin{pmatrix} 3 \\ 9 \end{pmatrix}$ is $2\begin{pmatrix} 3 \\ 9 \end{pmatrix} = \begin{pmatrix} 6 \\ 18 \end{pmatrix}$.

A vector in the opposite direction as $\begin{pmatrix} 3 \\ 9 \end{pmatrix}$ is $-\begin{pmatrix} 3 \\ 9 \end{pmatrix} = \begin{pmatrix} -3 \\ -9 \end{pmatrix}$.

(c) A vector in the same direction as $\begin{pmatrix} -6 \\ -2 \end{pmatrix}$ is $2\begin{pmatrix} -6 \\ -2 \end{pmatrix} = \begin{pmatrix} -12 \\ -4 \end{pmatrix}$.

A vector in the opposite direction as $\begin{pmatrix} -6 \\ -2 \end{pmatrix}$ is $-\begin{pmatrix} -6 \\ -2 \end{pmatrix} = \begin{pmatrix} 6 \\ 2 \end{pmatrix}$.

OXFORD UNIVERSITY PRESS

3. **(a)**
$$\begin{aligned}\mathbf{p} + 2\mathbf{q} &= \begin{pmatrix}5\\2\end{pmatrix} + 2\begin{pmatrix}6\\-3\end{pmatrix}\\ &= \begin{pmatrix}5\\2\end{pmatrix} + \begin{pmatrix}12\\-6\end{pmatrix}\\ &= \begin{pmatrix}17\\-4\end{pmatrix}\end{aligned}$$

(b)
$$\begin{aligned}3\mathbf{p} - \frac{1}{2}\mathbf{q} &= 3\begin{pmatrix}5\\2\end{pmatrix} - \frac{1}{2}\begin{pmatrix}6\\-3\end{pmatrix}\\ &= \begin{pmatrix}15\\6\end{pmatrix} - \begin{pmatrix}3\\-\frac{3}{2}\end{pmatrix}\\ &= \begin{pmatrix}12\\7\frac{1}{2}\end{pmatrix}\end{aligned}$$

(c)
$$\begin{aligned}4\mathbf{p} - 3\mathbf{q} + \mathbf{r} &= 4\begin{pmatrix}5\\2\end{pmatrix} - 3\begin{pmatrix}6\\-3\end{pmatrix} + \begin{pmatrix}-3\\-4\end{pmatrix}\\ &= \begin{pmatrix}20\\8\end{pmatrix} - \begin{pmatrix}18\\-9\end{pmatrix} + \begin{pmatrix}-3\\-4\end{pmatrix}\\ &= \begin{pmatrix}-1\\13\end{pmatrix}\end{aligned}$$

4. **(a)** The position vector of A is $\overrightarrow{OA} = \begin{pmatrix}4\\7\end{pmatrix}$.

(b) The position vector of B is $\overrightarrow{OB} = \begin{pmatrix}-2\\5\end{pmatrix}$.

(c) The position vector of C is $\overrightarrow{OC} = \begin{pmatrix}6\\-1\end{pmatrix}$.

(b) The position vector of D is $\overrightarrow{OD} = \begin{pmatrix}-4\\-9\end{pmatrix}$.

5. The position vector of P is $\overrightarrow{OP} = \begin{pmatrix}3\\-2\end{pmatrix}$.

The position vector of Q is $\overrightarrow{OQ} = \begin{pmatrix}2\\-4\end{pmatrix}$.

The position vector of R is $\overrightarrow{OR} = \begin{pmatrix}2\\3\end{pmatrix}$.

(i) $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP} = \begin{pmatrix}2\\-4\end{pmatrix} - \begin{pmatrix}3\\-2\end{pmatrix} = \begin{pmatrix}-1\\-2\end{pmatrix}$.

(ii) $\overrightarrow{QR} = \overrightarrow{OR} - \overrightarrow{OQ} = \begin{pmatrix}2\\3\end{pmatrix} - \begin{pmatrix}2\\-4\end{pmatrix} = \begin{pmatrix}0\\7\end{pmatrix}$.

(iii) $\overrightarrow{RP} = \overrightarrow{OP} - \overrightarrow{OR} = \begin{pmatrix}3\\-2\end{pmatrix} - \begin{pmatrix}2\\3\end{pmatrix} = \begin{pmatrix}1\\-5\end{pmatrix}$.

(iv) $\overrightarrow{PR} = \overrightarrow{OR} - \overrightarrow{OP} = \begin{pmatrix}2\\3\end{pmatrix} - \begin{pmatrix}3\\-2\end{pmatrix} = \begin{pmatrix}-1\\5\end{pmatrix}$.

6. **(a)** Since $\begin{pmatrix}6\\-3\end{pmatrix} = -\frac{3}{2}\begin{pmatrix}-4\\2\end{pmatrix}$, then $\begin{pmatrix}6\\-3\end{pmatrix}$ and $\begin{pmatrix}-4\\2\end{pmatrix}$ are parallel.

(b) Since $\begin{pmatrix}-5\\15\end{pmatrix} = \frac{5}{3}\begin{pmatrix}-3\\9\end{pmatrix}$, then $\begin{pmatrix}-5\\15\end{pmatrix}$ and $\begin{pmatrix}-3\\9\end{pmatrix}$ are parallel.

(c) If $\begin{pmatrix}7\\-8\end{pmatrix} = k\begin{pmatrix}2\\-3\end{pmatrix}$, then

$7 = 2k$ and $-8 = -3k$

$k = \frac{7}{2}$ $\qquad$ $k = \frac{8}{3}$

Since both values of k are different, therefore $\begin{pmatrix}7\\-8\end{pmatrix}$ and $\begin{pmatrix}2\\-3\end{pmatrix}$ are not parallel.

7. **(a)** Since $\begin{pmatrix}5\\2\end{pmatrix}$ and $\begin{pmatrix}20\\p\end{pmatrix}$ are parallel vectors, then $\begin{pmatrix}5\\2\end{pmatrix} = k\begin{pmatrix}20\\p\end{pmatrix}$.

$5 = 20k$ and $2 = kp$

$k = \frac{1}{4}$ $\qquad$ $p = \dfrac{2}{\frac{1}{4}} = 8$

$\therefore p = 8$

(b) Since $\begin{pmatrix}h\\12\end{pmatrix}$ and $\begin{pmatrix}3\\-9\end{pmatrix}$ are parallel vectors, then

$\begin{pmatrix}h\\12\end{pmatrix} = k\begin{pmatrix}3\\-9\end{pmatrix}$.

$12 = -9k$ and $h = 3k$

$k = -\frac{4}{3}$ $\qquad$ $h = 3\left(-\frac{4}{3}\right) = -4$

$\therefore h = -4$

8. **(a)**
$$\begin{aligned}\mathbf{a} + 2\mathbf{b} &= \begin{pmatrix}8\\9\end{pmatrix}\\ \begin{pmatrix}x\\y\end{pmatrix} + 2\begin{pmatrix}3\\4\end{pmatrix} &= \begin{pmatrix}8\\9\end{pmatrix}\\ \begin{pmatrix}x+6\\y+8\end{pmatrix} &= \begin{pmatrix}8\\9\end{pmatrix}\end{aligned}$$

$x + 6 = 8$ and $y + 8 = 9$

$x = 2$ $\qquad$ $y = 1$

$\therefore x = 2$ and $y = 1$

(b)
$$\begin{aligned}4\mathbf{u} + \mathbf{v} &= 2\begin{pmatrix}\frac{1}{2}\\9\end{pmatrix}\\ 4\begin{pmatrix}2\\y\end{pmatrix} + \begin{pmatrix}x\\2\end{pmatrix} &= \begin{pmatrix}1\\18\end{pmatrix}\\ \begin{pmatrix}8+x\\4y+2\end{pmatrix} &= \begin{pmatrix}1\\18\end{pmatrix}\end{aligned}$$

$8 + x = 1$ and $4y + 2 = 18$

$x = -7$ $\qquad$ $y = 4$

$\therefore x = -7$ and $y = 4$

(c)
$$\begin{aligned}5\mathbf{p} - 2\mathbf{q} &= \begin{pmatrix}3\\23\end{pmatrix}\\ 5\begin{pmatrix}x\\5\end{pmatrix} - 2\begin{pmatrix}6\\y\end{pmatrix} &= \begin{pmatrix}3\\23\end{pmatrix}\\ \begin{pmatrix}5x-12\\25-2y\end{pmatrix} &= \begin{pmatrix}3\\23\end{pmatrix}\end{aligned}$$

$5x - 12 = 3$ and $25 - 2y = 23$

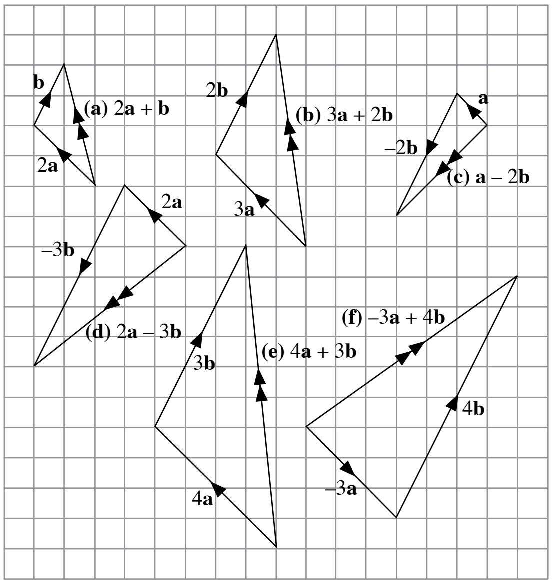
$5x = 15$ $\qquad$ $-2y = -2$

$x = 3$ $\qquad$ $y = 1$

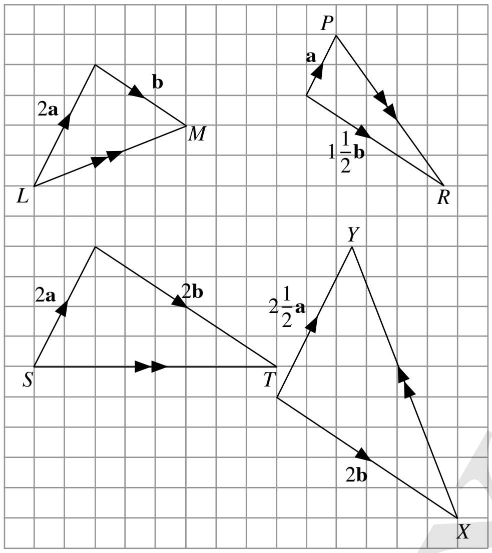
$\therefore x = 3$ and $y = 1$

OXFORD UNIVERSITY PRESS

9.



10.



$\overrightarrow{LM} = 2\mathbf{a} + \mathbf{b}$

$\overrightarrow{PR} = -\mathbf{a} + 1\frac{1}{2}\,\mathbf{b}$

$\overrightarrow{ST} = 2\mathbf{a} + 2\mathbf{b}$

$\overrightarrow{XY} = 2\frac{1}{2}\,\mathbf{a} - 2\mathbf{b}$

11. The position vector of A is $\overrightarrow{OA} = \begin{pmatrix} -3 \\ 8 \end{pmatrix}$.

$$\begin{aligned} \overrightarrow{OB} &= \overrightarrow{OA} + \overrightarrow{AB} \\ &= \begin{pmatrix} -3 \\ 8 \end{pmatrix} + \begin{pmatrix} -2 \\ -4 \end{pmatrix} \\ &= \begin{pmatrix} -5 \\ 4 \end{pmatrix} \end{aligned}$$

$\therefore$ The coordinates of B are (–5, 4).

12. (i) $\overrightarrow{CD} = \frac{2}{3}\begin{pmatrix} 9 \\ -15 \end{pmatrix}$

$$= \begin{pmatrix} 6 \\ -10 \end{pmatrix}$$

(ii) The position vector of A is $\overrightarrow{OA} = \begin{pmatrix} -2 \\ 7 \end{pmatrix}$.

$$\begin{aligned} \overrightarrow{OB} &= \overrightarrow{OA} + \overrightarrow{AB} \\ &= \begin{pmatrix} -2 \\ 7 \end{pmatrix} + \begin{pmatrix} 9 \\ -15 \end{pmatrix} \\ &= \begin{pmatrix} 7 \\ -8 \end{pmatrix} \end{aligned}$$

$\therefore$ The coordinates of B are (7, –8).

(iii) The position vector of D is $\overrightarrow{OD} = \begin{pmatrix} 8 \\ -5 \end{pmatrix}$.

$$\begin{aligned} \overrightarrow{CD} &= \begin{pmatrix} 6 \\ -10 \end{pmatrix} \\ \overrightarrow{OD} - \overrightarrow{OC} &= \begin{pmatrix} 6 \\ -10 \end{pmatrix} \\ \begin{pmatrix} 8 \\ -5 \end{pmatrix} - \overrightarrow{OC} &= \begin{pmatrix} 6 \\ -10 \end{pmatrix} \\ \overrightarrow{OC} &= \begin{pmatrix} 8 \\ -5 \end{pmatrix} - \begin{pmatrix} 6 \\ -10 \end{pmatrix} \\ \overrightarrow{OC} &= \begin{pmatrix} 2 \\ 5 \end{pmatrix} \end{aligned}$$

$\therefore$ The coordinates of C are (2, 5).

13. If $\begin{pmatrix} a \\ b \end{pmatrix} = k\begin{pmatrix} c \\ d \end{pmatrix}$, then

$a = ck$ and $b = dk$

$k = \frac{a}{c}$ $k = \frac{b}{d}$

Since $\begin{pmatrix} a \\ b \end{pmatrix}$ and $\begin{pmatrix} c \\ d \end{pmatrix}$ are parallel, both values of k are the same and hence $\frac{a}{c} = \frac{b}{d}$.

14. Since $\mathbf{u} = k\mathbf{v}$, then $|\mathbf{u}| = k|\mathbf{v}|$.

$$\begin{aligned} \sqrt{(-15)^2 + 8^2} &= k(51) \\ 17 &= k(51) \\ k &= \frac{1}{3} \end{aligned}$$

$$\therefore \mathbf{v} = 3\begin{pmatrix} -15 \\ 8 \end{pmatrix} = \begin{pmatrix} -45 \\ 24 \end{pmatrix}$$

15. (i) $2\overrightarrow{AB} + 5\overrightarrow{CD} = 2\begin{pmatrix} -3 \\ 5 \end{pmatrix} + 5\begin{pmatrix} 1 \\ 4 \end{pmatrix}$

$$\begin{aligned} &= \begin{pmatrix} -6 \\ 10 \end{pmatrix} + \begin{pmatrix} 5 \\ 20 \end{pmatrix} \\ &= \begin{pmatrix} -1 \\ 30 \end{pmatrix} \end{aligned}$$

OXFORD UNIVERSITY PRESS

(ii) Since $\overrightarrow{EF}$ and $\overrightarrow{AB}$ are parallel vectors, then $\begin{pmatrix} k \\ 7.5 \end{pmatrix} = p\begin{pmatrix} -3 \\ 5 \end{pmatrix}$.

$$7.5 = 5p \quad \text{and} \quad k = -3p$$

$$p = \frac{3}{2} \qquad k = -3\left(\frac{3}{2}\right) = -4\frac{1}{2}$$

$$\therefore k = -4\frac{1}{2}$$

(iii) Since $\overrightarrow{CD} = 4\overrightarrow{PQ}$, then $\overrightarrow{PQ}$ and $\overrightarrow{CD}$ are parallel.

16. (i) The position vector of L is $\overrightarrow{OL} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$.

The position vector of M is $\overrightarrow{OM} = \begin{pmatrix} t \\ 6 \end{pmatrix}$.

$$\begin{aligned} \overrightarrow{LM} &= \overrightarrow{OM} - \overrightarrow{OL} \\ &= \begin{pmatrix} t \\ 6 \end{pmatrix} - \begin{pmatrix} -3 \\ 2 \end{pmatrix} \\ &= \begin{pmatrix} t+3 \\ 4 \end{pmatrix} \end{aligned}$$

(ii) If $\overrightarrow{LM}$ and $\mathbf{p} = \begin{pmatrix} 8 \\ 1 \end{pmatrix}$ are parallel vectors, then $\begin{pmatrix} t+3 \\ 4 \end{pmatrix} = k\begin{pmatrix} 8 \\ 1 \end{pmatrix}$.

$$4 = k \quad \text{and} \quad t + 3 = 8k$$

$$k = 4 \qquad t = 8(4) - 3 = 29$$

$$\therefore t = 29$$

(iii) If $|\overrightarrow{LM}| = |\mathbf{p}|$, then

$$\begin{aligned} \sqrt{(t+3)^2 + 4^2} &= \sqrt{8^2 + 1^2} \\ (t+3)^2 + 4^2 &= 8^2 + 1^2 \\ (t+3)^2 &= 64 + 1 - 16 \\ (t+3)^2 &= 49 \\ t + 3 &= \pm 7 \\ t &= \pm 7 - 3 \\ t &= 4 \text{ or } -10 \end{aligned}$$

17. (i) The position vector of P is $\overrightarrow{OP} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$.

$$\begin{aligned} \overrightarrow{OQ} &= \overrightarrow{OP} + \overrightarrow{PQ} \\ &= \begin{pmatrix} 2 \\ -3 \end{pmatrix} + \begin{pmatrix} 8 \\ -2 \end{pmatrix} \\ &= \begin{pmatrix} 10 \\ -5 \end{pmatrix} \end{aligned}$$

$\therefore$ The coordinates of Q are (10, –5).

(ii) Gradient of $PQ = -\frac{2}{8} = -\frac{1}{4}$

(iii) Gradient of $PQ = \frac{y}{x}$

(iv) $\overrightarrow{PQ} = k\begin{pmatrix} x \\ y \end{pmatrix}$, for some real values of k

Exercise 7D

1. Since $ABCD$ is a parallelogram, then

$$\begin{aligned} \overrightarrow{DC} &= \overrightarrow{AB} \\ \overrightarrow{OC} - \overrightarrow{OD} &= \overrightarrow{OB} - \overrightarrow{OA} \\ \overrightarrow{OC} - \begin{pmatrix} 4 \\ 9 \end{pmatrix} &= \begin{pmatrix} 7 \\ 5 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \end{pmatrix} \\ \overrightarrow{OC} &= \begin{pmatrix} 7 \\ 5 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 4 \\ 9 \end{pmatrix} \\ \overrightarrow{OC} &= \begin{pmatrix} 9 \\ 11 \end{pmatrix} \end{aligned}$$

$\therefore$ Coordinates of C are (9, 11).

2. (i)

$$\begin{aligned} \overrightarrow{CM} &= \frac{1}{2}\overrightarrow{DA} \\ &= -\frac{1}{2}\mathbf{q} \end{aligned}$$

(ii)

$$\begin{aligned} \overrightarrow{DB} &= \overrightarrow{DA} + \overrightarrow{AB} \\ &= -\mathbf{q} + \mathbf{p} \end{aligned}$$

(iii)

$$\begin{aligned} \overrightarrow{AM} &= \overrightarrow{AB} + \overrightarrow{BM} \\ &= \overrightarrow{AB} + \frac{1}{2}\overrightarrow{BC} \\ &= \mathbf{p} + \frac{1}{2}\mathbf{q} \end{aligned}$$

(iv)

$$\begin{aligned} \overrightarrow{MD} &= \overrightarrow{MC} + \overrightarrow{CD} \\ &= \frac{1}{2}\mathbf{q} - \mathbf{p} \end{aligned}$$

3. (i)

$$\begin{aligned} \overrightarrow{BC} &= \overrightarrow{BA} + \overrightarrow{AC} \\ &= -\mathbf{u} + \mathbf{v} \\ &= \mathbf{v} - \mathbf{u} \end{aligned}$$

(ii)

$$\begin{aligned} \overrightarrow{AM} &= \frac{1}{2}\overrightarrow{AB} \\ &= \frac{1}{2}\mathbf{u} \end{aligned}$$

(iii)

$$\begin{aligned} \overrightarrow{AN} &= \frac{1}{2}\overrightarrow{AC} \\ &= \frac{1}{2}\mathbf{v} \end{aligned}$$

(iv)

$$\begin{aligned} \overrightarrow{MN} &= \overrightarrow{MA} + \overrightarrow{AN} \\ &= -\frac{1}{2}\mathbf{u} + \frac{1}{2}\mathbf{v} \\ &= \frac{1}{2}(\mathbf{v} - \mathbf{u}) \end{aligned}$$

Alternatively,

$$\begin{aligned} \overrightarrow{MN} &= \frac{1}{2}\overrightarrow{BC} \text{ (similar } \triangle\text{s)} \\ &= \frac{1}{2}(\mathbf{v} - \mathbf{u}) \text{ (from } \textbf{(i)}\text{)} \end{aligned}$$

Since $\overrightarrow{BC} = 2\overrightarrow{MN}$, then BC is parallel to MN and $BC = 2MN$.

OXFORD UNIVERSITY PRESS

4. $$\begin{aligned}\overrightarrow{BM} &= \overrightarrow{BO} + \overrightarrow{OM}\\ &= -\overrightarrow{OB} + \frac{1}{2}\overrightarrow{OA}\\ &= -\mathbf{b} + \frac{1}{2}\mathbf{a}\\ &= \frac{1}{2}\mathbf{a} - \mathbf{b}\end{aligned}$$

5. (i) $$\begin{aligned}\overrightarrow{AB} &= \overrightarrow{AO} + \overrightarrow{OB}\\ &= -\mathbf{a} + \mathbf{b}\\ &= \mathbf{b} - \mathbf{a}\end{aligned}$$

(ii) $$\begin{aligned}\overrightarrow{AC} &= \frac{2}{5}\overrightarrow{AB}\\ &= \frac{2}{5}(\mathbf{b} - \mathbf{a})\end{aligned}$$

(iii) $$\begin{aligned}\overrightarrow{OC} &= \overrightarrow{OA} + \overrightarrow{AC}\\ &= \mathbf{a} + \frac{2}{5}(\mathbf{b} - \mathbf{a})\\ &= \frac{3}{5}\mathbf{a} + \frac{2}{5}\mathbf{b}\\ &= \frac{1}{5}(3\mathbf{a} + 2\mathbf{b})\end{aligned}$$

6. (i) Since *PQRS* is a parallelogram, then

$$\begin{aligned}\overrightarrow{SR} &= \overrightarrow{PQ}\\ \overrightarrow{OR} - \overrightarrow{OS} &= \overrightarrow{OQ} - \overrightarrow{OP}\\ \begin{pmatrix}5\\4\end{pmatrix} - \overrightarrow{OS} &= \begin{pmatrix}4\\2\end{pmatrix} - \begin{pmatrix}1\\0\end{pmatrix}\\ \overrightarrow{OS} &= \begin{pmatrix}5\\4\end{pmatrix} - \begin{pmatrix}4\\2\end{pmatrix} + \begin{pmatrix}1\\0\end{pmatrix}\\ \overrightarrow{OS} &= \begin{pmatrix}2\\2\end{pmatrix}\end{aligned}$$

∴ Coordinates of *S* are (2, 2).

(ii) Since *PRQS* is a parallelogram, then

$$\begin{aligned}\overrightarrow{SQ} &= \overrightarrow{PR}\\ \overrightarrow{OQ} - \overrightarrow{OS} &= \overrightarrow{OR} - \overrightarrow{OP}\\ \begin{pmatrix}4\\2\end{pmatrix} - \overrightarrow{OS} &= \begin{pmatrix}5\\4\end{pmatrix} - \begin{pmatrix}1\\0\end{pmatrix}\\ \overrightarrow{OS} &= \begin{pmatrix}4\\2\end{pmatrix} - \begin{pmatrix}5\\4\end{pmatrix} + \begin{pmatrix}1\\0\end{pmatrix}\\ \overrightarrow{OS} &= \begin{pmatrix}0\\-2\end{pmatrix}\end{aligned}$$

∴ Coordinates of *S* are (0, –2).

7. (i) $$\begin{aligned}\overrightarrow{BC} &= \frac{4}{3}\overrightarrow{BD}\\ &= \frac{4}{3}\mathbf{q}\end{aligned}$$

(ii) $$\begin{aligned}\overrightarrow{AD} &= \overrightarrow{AB} + \overrightarrow{BD}\\ &= -\mathbf{p} + \mathbf{q}\end{aligned}$$

(iii) $$\begin{aligned}\overrightarrow{CA} &= \overrightarrow{CB} + \overrightarrow{BA}\\ &= -\frac{4}{3}\mathbf{q} + \mathbf{p}\\ &= \mathbf{p} - \frac{4}{3}\mathbf{q}\end{aligned}$$

8. (i) $$\begin{aligned}\overrightarrow{MR} &= \overrightarrow{MQ} + \overrightarrow{QR}\\ &= 2\mathbf{b} + \mathbf{a}\\ &= \mathbf{a} + 2\mathbf{b}\end{aligned}$$

(ii) $$\begin{aligned}\overrightarrow{RN} &= \frac{2}{3}\overrightarrow{RS}\\ &= -\frac{2}{3}(4\mathbf{b})\\ &= -\frac{8}{3}\mathbf{b}\end{aligned}$$

(iii) $$\begin{aligned}\overrightarrow{NM} &= \overrightarrow{NR} + \overrightarrow{RM}\\ &= \frac{8}{3}\mathbf{b} - (\mathbf{a} + 2\mathbf{b})\\ &= \frac{2}{3}\mathbf{b} - \mathbf{a}\end{aligned}$$

9. (i) $$\begin{aligned}\overrightarrow{BC} &= \overrightarrow{BA} + \overrightarrow{AC}\\ &= -\mathbf{u} + \mathbf{v}\\ &= \mathbf{v} - \mathbf{u}\end{aligned}$$

(ii) $$\begin{aligned}\overrightarrow{BE} &= \frac{2}{5}\overrightarrow{BC}\\ &= \frac{2}{5}(\mathbf{v} - \mathbf{u})\end{aligned}$$

(iii) $$\begin{aligned}\overrightarrow{AD} &= \overrightarrow{AC} + \overrightarrow{CD}\\ &= \mathbf{v} + \frac{3}{2}\mathbf{u}\end{aligned}$$

(iv) $$\begin{aligned}\overrightarrow{AE} &= \overrightarrow{AB} + \overrightarrow{BE}\\ &= \mathbf{u} + \frac{2}{5}(\mathbf{v} - \mathbf{u})\\ &= \frac{3}{5}\mathbf{u} + \frac{2}{5}\mathbf{v}\\ &= \frac{1}{5}(3\mathbf{u} + 2\mathbf{v})\end{aligned}$$

(v) $$\begin{aligned}\overrightarrow{BD} &= \overrightarrow{BA} + \overrightarrow{AD}\\ &= -\mathbf{u} + \mathbf{v} + \frac{3}{2}\mathbf{u}\\ &= \frac{1}{2}\mathbf{u} + \mathbf{v}\end{aligned}$$

10. (i) $$\begin{aligned}\overrightarrow{PR} &= \overrightarrow{PO} + \overrightarrow{OR}\\ &= -15\mathbf{a} + 15\mathbf{b}\\ &= 15(\mathbf{b} - \mathbf{a})\end{aligned}$$

(ii) $$\begin{aligned}\overrightarrow{PA} &= \frac{1}{4}\overrightarrow{PR}\\ &= \frac{15}{4}(\mathbf{b} - \mathbf{a})\end{aligned}$$

OXFORD UNIVERSITY PRESS

(iii) $$\begin{aligned}\overrightarrow{OA} &= \overrightarrow{OP} + \overrightarrow{PA}\\ &= 15\mathbf{a} + \frac{15}{4}(\mathbf{b} - \mathbf{a})\\ &= \frac{45}{4}\mathbf{a} + \frac{15}{4}\mathbf{b}\\ &= \frac{15}{4}(3\mathbf{a} + \mathbf{b})\end{aligned}$$

(iv) $$\begin{aligned}\overrightarrow{OB} &= \overrightarrow{OP} + \overrightarrow{PB}\\ &= \overrightarrow{OP} + \frac{1}{3}\overrightarrow{PQ}\\ &= 15\mathbf{a} + \frac{1}{3}(15\mathbf{b})\\ &= 15\mathbf{a} + 5\mathbf{b}\\ &= 5(3\mathbf{a} + \mathbf{b})\end{aligned}$$

11. (i) $$\begin{aligned}\overrightarrow{PC} &= \overrightarrow{PO} + \overrightarrow{OC}\\ &= \overrightarrow{PO} + \frac{5}{2}\overrightarrow{OQ}\\ &= -8\mathbf{p} + \frac{5}{2}(8\mathbf{q})\\ &= -8\mathbf{p} + 20\mathbf{q}\\ &= 20\mathbf{q} - 8\mathbf{p}\end{aligned}$$

(ii) $$\begin{aligned}\overrightarrow{PB} &= \frac{1}{4}\overrightarrow{PC}\\ &= \frac{5}{2}(20\mathbf{q} - 8\mathbf{p})\\ &= 5\mathbf{q} - 2\mathbf{p}\end{aligned}$$

(iii) $$\begin{aligned}\overrightarrow{OB} &= \overrightarrow{OP} + \overrightarrow{PB}\\ &= 8\mathbf{p} + 5\mathbf{q} - 2\mathbf{p}\\ &= 6\mathbf{p} + 5\mathbf{q}\end{aligned}$$

(iv) $$\begin{aligned}\overrightarrow{QB} &= \overrightarrow{QO} + \overrightarrow{OB}\\ &= -8\mathbf{q} + 6\mathbf{p} + 5\mathbf{q}\\ &= 6\mathbf{p} - 3\mathbf{q}\end{aligned}$$

12. (a) The position vector of P is $\overrightarrow{OP} = \begin{pmatrix}1\\11\end{pmatrix}$.

The position vector of Q is $\overrightarrow{OQ} = \begin{pmatrix}2\\8\end{pmatrix}$.

The position vector of R is $\overrightarrow{OR} = \begin{pmatrix}-1\\7\end{pmatrix}$.

The position vector of S is $\overrightarrow{OS} = \begin{pmatrix}-2\\8\end{pmatrix}$.

The position vector of T is $\overrightarrow{OT} = \begin{pmatrix}-4\\6\end{pmatrix}$.

(i) $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP} = \begin{pmatrix}2\\8\end{pmatrix} - \begin{pmatrix}1\\11\end{pmatrix} = \begin{pmatrix}1\\-3\end{pmatrix}$

(ii) $\overrightarrow{SR} = \overrightarrow{OR} - \overrightarrow{OS} = \begin{pmatrix}-1\\7\end{pmatrix} - \begin{pmatrix}-2\\8\end{pmatrix} = \begin{pmatrix}1\\-1\end{pmatrix}$

(iii) $\overrightarrow{RQ} = \overrightarrow{OQ} - \overrightarrow{OR} = \begin{pmatrix}2\\8\end{pmatrix} - \begin{pmatrix}-1\\7\end{pmatrix} = \begin{pmatrix}3\\1\end{pmatrix}$

(iv) $\overrightarrow{TQ} = \overrightarrow{OQ} - \overrightarrow{OT} = \begin{pmatrix}2\\8\end{pmatrix} - \begin{pmatrix}-4\\6\end{pmatrix} = \begin{pmatrix}6\\2\end{pmatrix}$

(b) Since $\overrightarrow{TQ} = 2\overrightarrow{RQ}$, $\dfrac{RQ}{TQ} = \dfrac{1}{2}$.

13. (i) $$\begin{aligned}\overrightarrow{BC} &= \overrightarrow{BA} + \overrightarrow{AC}\\ &= \begin{pmatrix}-4\\5\end{pmatrix} + \begin{pmatrix}6\\3\end{pmatrix}\\ &= \begin{pmatrix}2\\8\end{pmatrix}\end{aligned}$$

(ii) $$\begin{aligned}\overrightarrow{AM} &= \overrightarrow{AB} + \overrightarrow{BM}\\ &= \begin{pmatrix}4\\-5\end{pmatrix} + \frac{1}{2}\begin{pmatrix}2\\8\end{pmatrix}\\ &= \begin{pmatrix}5\\-1\end{pmatrix}\end{aligned}$$

(iii) The position vector of A is $\overrightarrow{OA} = \begin{pmatrix}1\\2\end{pmatrix}$.

$$\begin{aligned}\overrightarrow{OD} &= \overrightarrow{OA} + \overrightarrow{AD}\\ &= \overrightarrow{OA} + \overrightarrow{BC}\\ &= \begin{pmatrix}1\\2\end{pmatrix} + \begin{pmatrix}2\\8\end{pmatrix}\\ &= \begin{pmatrix}3\\10\end{pmatrix}\end{aligned}$$

$\therefore$ Coordinates of D are (3, 10).

14. (a) (i) $$\begin{aligned}\overrightarrow{SA} &= \frac{2}{3}\overrightarrow{SR}\\ &= \frac{2}{3}\overrightarrow{PQ}\\ &= \frac{2}{3}\mathbf{b}\end{aligned}$$

(ii) $$\begin{aligned}\overrightarrow{QB} &= \frac{2}{3}\overrightarrow{QR}\\ &= \frac{2}{3}\overrightarrow{PS}\\ &= \frac{2}{3}\mathbf{a}\end{aligned}$$

(iii) $$\begin{aligned}\overrightarrow{PB} &= \overrightarrow{PQ} + \overrightarrow{QB}\\ &= \mathbf{b} + \frac{2}{3}\mathbf{a}\\ &= \frac{2}{3}\mathbf{a} + \mathbf{b}\end{aligned}$$

(iv) $$\begin{aligned}\overrightarrow{QS} &= \overrightarrow{QP} + \overrightarrow{PS}\\ &= -\mathbf{b} + \mathbf{a}\\ &= \mathbf{a} - \mathbf{b}\end{aligned}$$

(v) $$\begin{aligned}\overrightarrow{BA} &= \overrightarrow{BR} + \overrightarrow{RA}\\ &= \frac{1}{3}\mathbf{a} - \frac{1}{3}\mathbf{b}\\ &= \frac{1}{3}(\mathbf{a} - \mathbf{b})\end{aligned}$$

OXFORD UNIVERSITY PRESS

(b) (i) Since $\triangle ABR$ and $\triangle SQR$ are similar (corr. $\angle$s are equal),

$$\overrightarrow{BA} = \frac{1}{3}\overrightarrow{QS}$$

$$\therefore \frac{BA}{QS} = \frac{1}{3}$$

(ii)
$$\frac{\text{Area of } \triangle ABR}{\text{Area of } \triangle SQR} = \left(\frac{BA}{QS}\right)^2$$
$$= \left(\frac{1}{3}\right)^2$$
$$= \frac{1}{9}$$

(iii) Since the area of $PQRS$ is twice the area of $\triangle SQR$,

$$\frac{\text{area of } \triangle ABR}{\text{area of } PQRS} = \frac{1}{18}$$

15. (a) (i)
$$\overrightarrow{RS} = \overrightarrow{RP} + \overrightarrow{PS}$$
$$= -(3\mathbf{a} + 12\mathbf{b}) + 5\mathbf{b}$$
$$= -3\mathbf{a} - 7\mathbf{b}$$

(ii)
$$\overrightarrow{RT} = \frac{2}{3}\overrightarrow{RP}$$
$$= -\frac{2}{3}(3\mathbf{a} + 12\mathbf{b})$$
$$= -2\mathbf{a} - 8\mathbf{b}$$

(iii)
$$\overrightarrow{RQ} = \overrightarrow{RP} + \overrightarrow{PQ}$$
$$= -(3\mathbf{a} + 12\mathbf{b}) + (4\mathbf{a} + \mathbf{b})$$
$$= \mathbf{a} - 11\mathbf{b}$$

(b)
$$\overrightarrow{QT} = \overrightarrow{QR} + \overrightarrow{RT}$$
$$= -(\mathbf{a} - 11\mathbf{b}) + (-2\mathbf{a} - 8\mathbf{b})$$
$$= -3\mathbf{a} + 3\mathbf{b}$$
$$= 3(\mathbf{b} - \mathbf{a}) \text{ (shown)}$$

(c)
$$\overrightarrow{QS} = \overrightarrow{QR} + \overrightarrow{RS}$$
$$= -(\mathbf{a} - 11\mathbf{b}) + (-3\mathbf{a} - 7\mathbf{b})$$
$$= -4\mathbf{a} + 4\mathbf{b}$$
$$= 4(\mathbf{b} - \mathbf{a})$$

(d) (i)
$$\overrightarrow{QT} = \frac{3}{4}\overrightarrow{QS}$$
$$\therefore \frac{QT}{QS} = \frac{3}{4}$$

(ii)
$$\frac{\text{Area of } \triangle PQT}{\text{Area of } \triangle PQS}$$
$$= \frac{\frac{1}{2} \times QT \times h_1}{\frac{1}{2} \times QS \times h_1}$$
, where h_1 is the common height of $\triangle PQT$ and $\triangle PQS$,
$$= \frac{QT}{QS}$$
$$= \frac{3}{4}$$

(iii)
$$\frac{\text{Area of } \triangle PQT}{\text{Area of } \triangle RQT}$$
$$= \frac{\frac{1}{2} \times PT \times h_2}{\frac{1}{2} \times RT \times h_2}$$
, where h_2 is the common height of $\triangle PQT$ and $\triangle RQT$,
$$= \frac{PT}{RT}$$
$$= \frac{1}{2}$$

16. (a) (i)
$$\overrightarrow{OT} = \overrightarrow{OC} + \overrightarrow{CT}$$
$$= \mathbf{q} - 3(\mathbf{p} - \mathbf{q})$$
$$= -3\mathbf{p} + 4\mathbf{q}$$
$$= 4\mathbf{q} - 3\mathbf{p}$$

(ii)
$$\overrightarrow{AT} = \overrightarrow{AO} + \overrightarrow{OT}$$
$$= -\mathbf{p} + 4\mathbf{q} - 3\mathbf{p}$$
$$= -4\mathbf{p} + 4\mathbf{q}$$
$$= 4\mathbf{q} - 4\mathbf{p}$$

(iii)
$$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$$
$$= \mathbf{p} + \mathbf{q}$$

(iv)
$$\overrightarrow{BT} = \overrightarrow{BA} + \overrightarrow{AT}$$
$$= -\mathbf{q} + 4\mathbf{q} - 4\mathbf{p}$$
$$= 3\mathbf{q} - 4\mathbf{p}$$

(v)
$$\overrightarrow{TR} = \frac{3}{4}\overrightarrow{TB}$$
$$= -\frac{3}{4}(3\mathbf{q} - 4\mathbf{p})$$
$$= \frac{3}{4}(4\mathbf{p} - 3\mathbf{q})$$

(b)
$$\overrightarrow{CR} = \overrightarrow{CT} + \overrightarrow{TR}$$
$$= -3(\mathbf{p} - \mathbf{q}) + \frac{3}{4}(4\mathbf{p} - 3\mathbf{q})$$
$$= \frac{3}{4}\mathbf{q}$$

(c) (i)
$$\overrightarrow{CR} = \frac{3}{4}\overrightarrow{OC}$$
$$\therefore \frac{CR}{OC} = \frac{3}{4}$$

(ii) Since $\triangle TCR$ and $\triangle TAB$ are similar (corr. $\angle$s are equal),
$$\frac{\text{area of } \triangle TCR}{\text{area of } \triangle TAB} = \left(\frac{CR}{AB}\right)^2$$
$$= \left(\frac{3}{4}\right)^2$$
$$= \frac{9}{16}$$

17. (a) (i)
$$\overrightarrow{QP} = \overrightarrow{QO} + \overrightarrow{OP}$$
$$= -\mathbf{q} + \mathbf{p}$$
$$= \mathbf{p} - \mathbf{q}$$

(ii)
$$\overrightarrow{QS} = \frac{2}{5}\overrightarrow{QP}$$
$$= \frac{2}{5}(\mathbf{p} - \mathbf{q})$$

OXFORD UNIVERSITY PRESS

(iii) $\overrightarrow{OS} = \overrightarrow{OQ} + \overrightarrow{QS}$

$= \mathbf{q} + \frac{2}{5}(\mathbf{p} - \mathbf{q})$

$= \frac{2}{5}\mathbf{p} + \frac{3}{5}\mathbf{q}$

$= \frac{1}{5}(2\mathbf{p} + 3\mathbf{q})$

(iv) $\overrightarrow{ST} = \overrightarrow{SQ} + \overrightarrow{QT}$

$= -\frac{2}{5}(\mathbf{p} - \mathbf{q}) + \frac{1}{2}\mathbf{q}$

$= -\frac{2}{5}\mathbf{p} + \frac{9}{10}\mathbf{q}$

$= \frac{1}{10}(9\mathbf{q} - 4\mathbf{p})$

(b) **(i)** $\overrightarrow{RS} = \overrightarrow{RP} + \overrightarrow{PS}$

$= \frac{1}{3}\overrightarrow{OP} + \frac{3}{5}\overrightarrow{PQ}$

$= \frac{1}{3}\mathbf{p} + \frac{3}{5}(\mathbf{q} - \mathbf{p})$

$= -\frac{4}{15}\mathbf{p} + \frac{3}{5}\mathbf{q}$

$= \frac{1}{15}(9\mathbf{q} - 4\mathbf{p})$

$= \frac{2}{3} \times \frac{1}{10}(9\mathbf{q} - 4\mathbf{p})$

$= \frac{2}{3}\overrightarrow{ST}$ (shown)

(ii) Since $\overrightarrow{RS} = \frac{2}{3}\overrightarrow{ST}$, therefore R, S and T are collinear and $RS = \frac{2}{3}ST$.

18. **(i)** Let $\overrightarrow{BP} = \mathbf{p}$ and $\overrightarrow{BQ} = \mathbf{q}$.

Then $\overrightarrow{PQ} = \overrightarrow{BQ} - \overrightarrow{BP} = \mathbf{q} - \mathbf{p}$.

Since $\overrightarrow{BA} = 2\overrightarrow{BP} = 2\mathbf{p}$ and $\overrightarrow{BD} = 2\overrightarrow{BQ} = 2\mathbf{q}$,

then $\overrightarrow{AD} = \overrightarrow{BD} - \overrightarrow{BA} = 2\mathbf{q} - 2\mathbf{p}$

$= 2(\mathbf{q} - \mathbf{p})$

$= 2\overrightarrow{PQ}$

Since $\overrightarrow{PQ} = \frac{1}{2}\overrightarrow{AD}$, then PQ is parallel to AD and $PQ = \frac{1}{2}AD$. (shown)

(ii) Applying the same reasoning in **(i)** for $\triangle CSR$ and $\triangle CAD$, we can show that SR is parallel to AD and $SR = \frac{1}{2}AD$.

$\therefore$ PQ is parallel to SR and $PQ = SR$, i.e. $PQRS$ is a parallelogram. (shown)

Review Exercise 7

1. **(a)** Magnitude of $\begin{pmatrix} 5 \\ -12 \end{pmatrix} = \sqrt{5^2 + (-12)^2} = 13$ units

(b) Magnitude of $\begin{pmatrix} -6 \\ 8 \end{pmatrix} = \sqrt{(-6)^2 + 8^2} = 10$ units

(c) Magnitude of $\begin{pmatrix} 5 \\ 2 \end{pmatrix} = \sqrt{5^2 + 2^2} = 5.39$ units (to 3 s.f.)

(d) Magnitude of $\begin{pmatrix} -7 \\ -1 \end{pmatrix} = \sqrt{(-7)^2 + (-1)^2} = 7.07$ units (to 3 s.f.)

(e) Magnitude of $\begin{pmatrix} 0 \\ -3 \end{pmatrix} = \sqrt{0^2 + (-3)^2} = 3$ units

2. $|\overrightarrow{XY}| = 5$

$\sqrt{p^2 + (-2)^2} = 5$

$p^2 + (-2)^2 = 5^2$

$p^2 + 4 = 25$

$p^2 = 21$

$p = \pm\sqrt{21}$

$= \pm 4.58$ (to 3 s.f.)

3. **(i)** $|\overrightarrow{AB}| = \sqrt{(-4)^2 + 2^2}$

$= 4.47$ units (to 3 s.f.)

(ii) $|\overrightarrow{CD}| = 3|\overrightarrow{AB}|$

$\sqrt{p^2 + 12^2} = 3\sqrt{(-4)^2 + 2^2}$

$p^2 + 144 = 9(16 + 4)$

$p^2 = 180 - 144$

$p^2 = 36$

$p = 6$ or $p = -6$ (rejected)

4. Since $\begin{pmatrix} p + q \\ p \end{pmatrix} = \begin{pmatrix} 3 \\ q + 1 \end{pmatrix}$, then

$p + q = 3$ — (1)

$p = q + 1$ — (2)

Substitute (2) into (1):

$q + 1 + q = 3$

$2q = 2$

$q = 1$

Substitute $q = 1$ into (2):

$p = 1 + 1$

$= 2$

$\therefore$ $p = 2$ and $q = 1$

5. **(a)** **(i)** $\overrightarrow{HK}$, $\overrightarrow{GL}$, $\overrightarrow{FE}$, $\overrightarrow{KB}$ (any two)

(ii) $\overrightarrow{CL}$, $\overrightarrow{DE}$

(iii) $\overrightarrow{GH}$, $\overrightarrow{FG}$, $\overrightarrow{KJ}$, $\overrightarrow{LK}$, $\overrightarrow{EL}$ (any two)

(iv) $\overrightarrow{AB}$, $\overrightarrow{CD}$

(v) $\overrightarrow{JH}$, $\overrightarrow{KG}$, $\overrightarrow{CE}$ (any two)

(b) **(i)** $\overrightarrow{LB}$, $\overrightarrow{FL}$ (any one)

(ii) $\overrightarrow{KJ}$, $\overrightarrow{LK}$, $\overrightarrow{EL}$, $\overrightarrow{HI}$, $\overrightarrow{GH}$, $\overrightarrow{FG}$ (any one)

(iii) $\overrightarrow{KB}$, $\overrightarrow{IJ}$, $\overrightarrow{HK}$, $\overrightarrow{GL}$, $\overrightarrow{FE}$ (any one)

OXFORD UNIVERSITY PRESS

(c) **(i)** They do not have the same direction.

(ii) They do not have the same magnitude and direction.

6. **(a)** $\overrightarrow{KN} = \overrightarrow{LM}$

$\overrightarrow{NM} = \underline{\overrightarrow{KL}}$

(b) $\overrightarrow{SR} = \underline{\overrightarrow{UP}}$

$\overrightarrow{RQ} = \underline{\overrightarrow{TU}}$

$\overrightarrow{QP} = \underline{\overrightarrow{ST}}$

(c) $\overrightarrow{AB} = \underline{\overrightarrow{DC}}$

$\overrightarrow{BC} = \underline{\overrightarrow{AD}}$

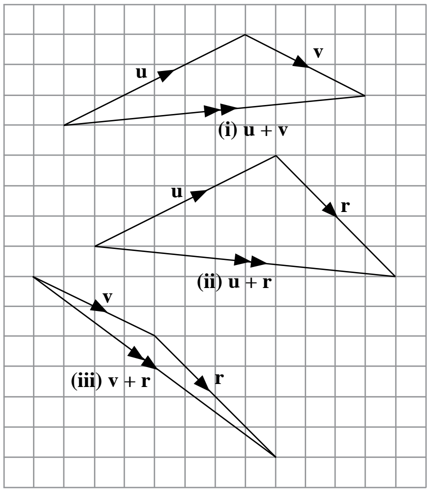
(d) $\overrightarrow{LM} = \underline{\overrightarrow{QP}}$

$\overrightarrow{MN} = \underline{\overrightarrow{RQ}}$

$\overrightarrow{NO} = \underline{\overrightarrow{SR}}$

$\overrightarrow{OP} = \underline{\overrightarrow{LS}}$

7.



8. **(i)** $\overrightarrow{AD} + \overrightarrow{DC} = \overrightarrow{AC}$

(ii) $\overrightarrow{AB} + \overrightarrow{BD} = \overrightarrow{AD}$

(iii) $\overrightarrow{AC} + \overrightarrow{CB} + \overrightarrow{BD} = \overrightarrow{AD}$

(iv) $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \mathbf{0}$

9. **(i)** $\mathbf{x} = \overrightarrow{OP}$

(ii) $\mathbf{x} = \overrightarrow{OR}$

(iii) $\mathbf{x} = \overrightarrow{RS}$

(iv) $\mathbf{x} = \overrightarrow{RP}$

(v) $\mathbf{x} = \overrightarrow{QR}$

(vi) $\mathbf{x} = \overrightarrow{PQ}$

10. **(i)** $\mathbf{a} - \mathbf{b} = \begin{pmatrix} 4 \\ 3 \end{pmatrix} - \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} 2 \\ 2 \end{pmatrix}$

(ii) $\mathbf{b} - \mathbf{c} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} - \begin{pmatrix} -3 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} 5 \\ 0 \end{pmatrix}$

(iii) $\mathbf{a} - (\mathbf{b} + \mathbf{c}) = \begin{pmatrix} 4 \\ 3 \end{pmatrix} - \left[\begin{pmatrix} 2 \\ 1 \end{pmatrix} + \begin{pmatrix} -3 \\ 1 \end{pmatrix}\right]$

$= \begin{pmatrix} 4 \\ 3 \end{pmatrix} - \begin{pmatrix} -1 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} 5 \\ 1 \end{pmatrix}$

(iv) $\mathbf{a} - (\mathbf{b} - \mathbf{c}) = \begin{pmatrix} 4 \\ 3 \end{pmatrix} - \left[\begin{pmatrix} 2 \\ 1 \end{pmatrix} - \begin{pmatrix} -3 \\ 1 \end{pmatrix}\right]$

$= \begin{pmatrix} 4 \\ 3 \end{pmatrix} - \begin{pmatrix} 5 \\ 0 \end{pmatrix}$

$= \begin{pmatrix} -1 \\ 3 \end{pmatrix}$

11. Since $\mathbf{u} = k\mathbf{v}$, then $|\mathbf{u}| = k|\mathbf{v}|$.

$\sqrt{4^2 + (-3)^2} = k(20)$

$5 = k(20)$

$k = \dfrac{1}{4}$

$\therefore\ \mathbf{v} = 4\begin{pmatrix} 4 \\ -3 \end{pmatrix} = \begin{pmatrix} 16 \\ -12 \end{pmatrix}$

12. **(a)** $2\mathbf{p} + 3\mathbf{q} = 2\begin{pmatrix} 5 \\ -12 \end{pmatrix} + 3\begin{pmatrix} 2 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} 10 \\ -24 \end{pmatrix} + \begin{pmatrix} 6 \\ 9 \end{pmatrix}$

$= \begin{pmatrix} 16 \\ -15 \end{pmatrix}$

(b) **(i)** $|\mathbf{p}| = \sqrt{5^2 + (-12)^2}$

$= 13$ units

(ii) $-\mathbf{p} + 2\mathbf{q} = -\begin{pmatrix} 5 \\ -12 \end{pmatrix} + 2\begin{pmatrix} 2 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} -5 \\ 12 \end{pmatrix} + \begin{pmatrix} 4 \\ 6 \end{pmatrix}$

$= \begin{pmatrix} -1 \\ 18 \end{pmatrix}$

$|-\mathbf{p} + 2\mathbf{q}| = \sqrt{(-1)^2 + 18^2}$

$= 18$ units (to the nearest whole number)

OXFORD
UNIVERSITY PRESS

(c) Since $\mathbf{r}$ is parallel to $\mathbf{p}$, then $\mathbf{r} = k\mathbf{p}$.

$$\begin{pmatrix} 20 \\ m \end{pmatrix} = k\begin{pmatrix} 5 \\ -12 \end{pmatrix}$$

$20 = 5k$ and $m = -12k$

$k = 4$ $\quad$ $m = -12(4) = -48$

$\therefore m = -48$

13. (i) The position vector of A is $\overrightarrow{OA} = \begin{pmatrix} -5 \\ 3 \end{pmatrix}$.

The position vector of C is $\overrightarrow{OC} = \begin{pmatrix} 7 \\ 4 \end{pmatrix}$.

$$\begin{aligned} \overrightarrow{AB} &= \begin{pmatrix} 6 \\ 1 \end{pmatrix} \\ \overrightarrow{OB} - \overrightarrow{OA} &= \begin{pmatrix} 6 \\ 1 \end{pmatrix} \\ \overrightarrow{OB} - \begin{pmatrix} -5 \\ 3 \end{pmatrix} &= \begin{pmatrix} 6 \\ 1 \end{pmatrix} \\ \overrightarrow{OB} &= \begin{pmatrix} 6 \\ 1 \end{pmatrix} + \begin{pmatrix} -5 \\ 3 \end{pmatrix} \\ &= \begin{pmatrix} 1 \\ 4 \end{pmatrix} \end{aligned}$$

$\therefore$ Coordinates of B are $(1, 4)$.

$$\begin{aligned} \overrightarrow{AD} &= \begin{pmatrix} 8 \\ -9 \end{pmatrix} \\ \overrightarrow{OD} - \overrightarrow{OA} &= \begin{pmatrix} 8 \\ -9 \end{pmatrix} \\ \overrightarrow{OD} - \begin{pmatrix} -5 \\ 3 \end{pmatrix} &= \begin{pmatrix} 8 \\ -9 \end{pmatrix} \\ \overrightarrow{OD} &= \begin{pmatrix} 8 \\ -9 \end{pmatrix} + \begin{pmatrix} -5 \\ 3 \end{pmatrix} \\ &= \begin{pmatrix} 3 \\ -6 \end{pmatrix} \end{aligned}$$

$\therefore$ Coordinates of D are $(3, -6)$.

(ii)
$$\begin{aligned} \overrightarrow{BC} &= \overrightarrow{OC} - \overrightarrow{OB} \\ &= \begin{pmatrix} 7 \\ 4 \end{pmatrix} - \begin{pmatrix} 1 \\ 4 \end{pmatrix} \\ &= \begin{pmatrix} 6 \\ 0 \end{pmatrix} \end{aligned}$$

$$\begin{aligned} \overrightarrow{CD} &= \overrightarrow{OD} - \overrightarrow{OC} \\ &= \begin{pmatrix} 3 \\ -6 \end{pmatrix} - \begin{pmatrix} 7 \\ 4 \end{pmatrix} \\ &= \begin{pmatrix} -4 \\ -10 \end{pmatrix} \end{aligned}$$

14. (i) The position vector of A is $\overrightarrow{OA} = \mathbf{a} = \begin{pmatrix} 7 \\ 4 \end{pmatrix}$.

The position vector of B is $\overrightarrow{OB} = \mathbf{b}$.

$$\begin{aligned} \overrightarrow{AB} &= \begin{pmatrix} -3 \\ 2 \end{pmatrix} \\ \overrightarrow{OB} - \overrightarrow{OA} &= \begin{pmatrix} -3 \\ 2 \end{pmatrix} \\ \overrightarrow{OB} - \begin{pmatrix} 7 \\ 4 \end{pmatrix} &= \begin{pmatrix} -3 \\ 2 \end{pmatrix} \\ \overrightarrow{OB} &= \begin{pmatrix} -3 \\ 2 \end{pmatrix} + \begin{pmatrix} 7 \\ 4 \end{pmatrix} \\ &= \begin{pmatrix} 4 \\ 6 \end{pmatrix} \end{aligned}$$

$\therefore \mathbf{b} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$

(ii)
$$\begin{aligned} \overrightarrow{OC} &= \overrightarrow{BA} \\ &= -\overrightarrow{AB} \\ &= \begin{pmatrix} 3 \\ -2 \end{pmatrix} \end{aligned}$$

$\therefore$ Coordinates of C are $(3, -2)$.

15. (i)
$$\begin{aligned} \overrightarrow{AC} &= \overrightarrow{AB} + \overrightarrow{BC} \\ &= \mathbf{u} + \frac{3}{2}\mathbf{v} \end{aligned}$$

(ii)
$$\begin{aligned} \overrightarrow{DC} &= \overrightarrow{DA} + \overrightarrow{AC} \\ &= -\mathbf{v} + \mathbf{u} + \frac{3}{2}\mathbf{v} \\ &= \mathbf{u} + \frac{1}{2}\mathbf{v} \end{aligned}$$

(iii)
$$\begin{aligned} \overrightarrow{AQ} &= \overrightarrow{AD} + \overrightarrow{DQ} \\ &= \mathbf{v} + \frac{1}{4}\left(\mathbf{u} + \frac{1}{2}\mathbf{v}\right) \\ &= \frac{1}{4}\mathbf{u} + \frac{9}{8}\mathbf{v} \\ &= \frac{1}{8}(2\mathbf{u} + 9\mathbf{v}) \end{aligned}$$

(iv)
$$\begin{aligned} \overrightarrow{PQ} &= \overrightarrow{PA} + \overrightarrow{AQ} \\ &= -\frac{1}{2}\mathbf{u} + \frac{1}{8}(2\mathbf{u} + 9\mathbf{v}) \\ &= -\frac{1}{4}\mathbf{u} + \frac{9}{8}\mathbf{v} \\ &= \frac{1}{8}(-2\mathbf{u} + 9\mathbf{v}) \\ &= \frac{1}{8}(9\mathbf{v} - 2\mathbf{u}) \end{aligned}$$

OXFORD UNIVERSITY PRESS

16. (a) (i) $\overrightarrow{NM} = \overrightarrow{NP} + \overrightarrow{PM}$

$= -2\mathbf{a} + 2\mathbf{b}$

$= 2\mathbf{b} - 2\mathbf{a}$

(ii) $\overrightarrow{NL} = \frac{1}{2}\overrightarrow{NM}$

$= \frac{1}{2}(2\mathbf{b} - 2\mathbf{a})$

$= \mathbf{b} - \mathbf{a}$

(iii) $\overrightarrow{PK} = \frac{12}{7}\overrightarrow{PL}$

$= \frac{12}{7}(\overrightarrow{PN} + \overrightarrow{NL})$

$= \frac{12}{7}(2\mathbf{a} + \mathbf{b} - \mathbf{a})$

$= \frac{12}{7}(\mathbf{a} + \mathbf{b})$

(iv) $\overrightarrow{PR} = \frac{3}{2}\overrightarrow{PN}$

$= \frac{3}{2}(2\mathbf{a})$

$= 3\mathbf{a}$

(v) $\overrightarrow{PQ} = 2\overrightarrow{PM}$

$= 2(2\mathbf{b})$

$= 4\mathbf{b}$

(b) $\overrightarrow{RQ} = \overrightarrow{RP} + \overrightarrow{PQ}$

$= -3\mathbf{a} + 4\mathbf{b}$

(c) $\overrightarrow{KR} = \overrightarrow{KP} + \overrightarrow{PR}$

$= -\frac{12}{7}(\mathbf{a} + \mathbf{b}) + 3\mathbf{a}$

$= \frac{9}{7}\mathbf{a} - \frac{12}{7}\mathbf{b}$

$= \frac{3}{7}(3\mathbf{a} - 4\mathbf{b})$ (shown)

(d) $\overrightarrow{KR} = \frac{3}{7}\overrightarrow{QR}$

$\therefore \frac{KR}{QR} = \frac{3}{7}$

(e) (i) $\frac{\text{Area of } \triangle PKR}{\text{Area of } \triangle PQR}$

$= \frac{\frac{1}{2} \times KR \times h_1}{\frac{1}{2} \times QR \times h_1}$, where h_1 is the common height of $\triangle PKR$ and $\triangle PQR$,

$= \frac{KR}{QR}$

$= \frac{3}{7}$

(ii) $\frac{\text{Area of } \triangle PKN}{\text{Area of } \triangle PKR}$

$= \frac{\frac{1}{2} \times PN \times h_2}{\frac{1}{2} \times PR \times h_2}$, where h_2 is the common height of $\triangle PKN$ and $\triangle PKR$,

$= \frac{PN}{PR}$

$= \frac{2}{3}$

$\frac{\text{Area of } \triangle PKN}{\text{Area of } \triangle PQR} = \frac{\text{Area of } \triangle PKN}{\text{Area of } \triangle PKR} \times \frac{\text{Area of } \triangle PKR}{\text{Area of } \triangle PQR}$

$= \frac{2}{3} \times \frac{3}{7}$

$= \frac{2}{7}$

17. $\overrightarrow{BP} = 3\overrightarrow{PM}$

$\overrightarrow{OP} - \overrightarrow{OB} = 3(\overrightarrow{OM} - \overrightarrow{OP})$

$\overrightarrow{OP} - \overrightarrow{OB} = 3\overrightarrow{OM} - 3\overrightarrow{OP}$

$\overrightarrow{OP} + 3\overrightarrow{OP} = 3\overrightarrow{OM} + \overrightarrow{OB}$

$4\overrightarrow{OP} = \frac{3}{2}\overrightarrow{OA} + \overrightarrow{OB}$

$4\overrightarrow{OP} = \frac{3}{2}\mathbf{a} + \mathbf{b}$

$\overrightarrow{OP} = \frac{1}{4}\left(\frac{3}{2}\mathbf{a} + \mathbf{b}\right)$

$\overrightarrow{OP} = \frac{1}{8}(3\mathbf{a} + 2\mathbf{b})$

18. (i) $\overrightarrow{OQ} = \overrightarrow{OP} + \overrightarrow{PQ}$

$= 4\mathbf{a} + 4\mathbf{b}$

(ii) $\overrightarrow{OX} = \overrightarrow{OR} + \overrightarrow{RX}$

$= \overrightarrow{PQ} + \frac{1}{2}\overrightarrow{OP}$

$= 4\mathbf{b} + 2\mathbf{a}$

(iii) $\overrightarrow{QS} = \overrightarrow{QR} + \overrightarrow{RS}$

$= \overrightarrow{PO} + 2\overrightarrow{OR}$

$= -4\mathbf{a} + 2(4\mathbf{b})$

$= -4\mathbf{a} + 8\mathbf{b}$

$= 8\mathbf{b} - 4\mathbf{a}$

19. (a) (i) $\overrightarrow{OQ} = \frac{2}{3}\overrightarrow{OB}$

$= \frac{2}{3}\mathbf{b}$

(ii) $\overrightarrow{PQ} = \overrightarrow{PO} + \overrightarrow{OQ}$

$= -2\mathbf{a} + \frac{2}{3}\mathbf{b}$

$= \frac{2}{3}\mathbf{b} - 2\mathbf{a}$

OXFORD UNIVERSITY PRESS

(iii) $\overrightarrow{OM} = \overrightarrow{OA} + \overrightarrow{AM}$

$= \overrightarrow{OA} + \frac{1}{2}\overrightarrow{AB}$

$= \mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a})$

$= \frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$

$= \frac{1}{2}(\mathbf{a} + \mathbf{b})$

(iv) $\overrightarrow{QM} = \overrightarrow{QO} + \overrightarrow{OM}$

$= -\frac{2}{3}\mathbf{b} + \frac{1}{2}(\mathbf{a} + \mathbf{b})$

$= \frac{1}{2}\mathbf{a} - \frac{1}{6}\mathbf{b}$

$= \frac{1}{6}(3\mathbf{a} - \mathbf{b})$

(b) $\overrightarrow{PM} = \overrightarrow{PO} + \overrightarrow{OM}$

$= -2\mathbf{a} + \frac{1}{2}(\mathbf{a} + \mathbf{b})$

$= -\frac{3}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$

$= \frac{1}{2}(\mathbf{b} - 3\mathbf{a})$

$\therefore \frac{PM}{MQ} = \frac{1}{2} \div \frac{1}{6} = 3$

20. (a) (i) $\overrightarrow{QP} = \overrightarrow{QO} + \overrightarrow{OP}$

$= -\mathbf{q} + 2\mathbf{p}$

$= 2\mathbf{p} - \mathbf{q}$

(ii) $\overrightarrow{OR} = \frac{3}{2}\overrightarrow{OP}$

$= \frac{3}{2}(2\mathbf{p})$

$= 3\mathbf{p}$

(iii) $\overrightarrow{SR} = \overrightarrow{SO} + \overrightarrow{OR}$

$= -2\mathbf{q} + 3\mathbf{p}$

$= 3\mathbf{p} - 2\mathbf{q}$

(iv) $\overrightarrow{ST} = \overrightarrow{SQ} + \overrightarrow{QT}$

$= -\mathbf{q} + 3(2\mathbf{p} - \mathbf{q})$

$= 6\mathbf{p} - 4\mathbf{q}$

$= 2(3\mathbf{p} - 2\mathbf{q})$

(b) Since $\overrightarrow{ST} = 2\overrightarrow{SR}$, therefore S, R and T are collinear and $ST = 2SR$.

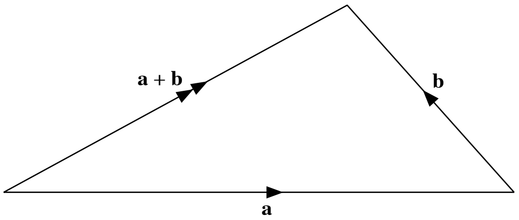
(c) $\frac{\text{Area of } \triangle OPQ}{\text{Area of } \triangle SPT} = \frac{\text{Area of } \triangle OPQ}{\text{Area of } \triangle QPS} \times \frac{\text{Area of } \triangle QPS}{\text{Area of } \triangle SPT}$

$= \frac{1}{1} \times \frac{1}{2}$

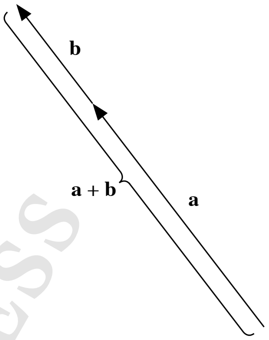
$= \frac{1}{2}$

Challenge Yourself

1. (i)



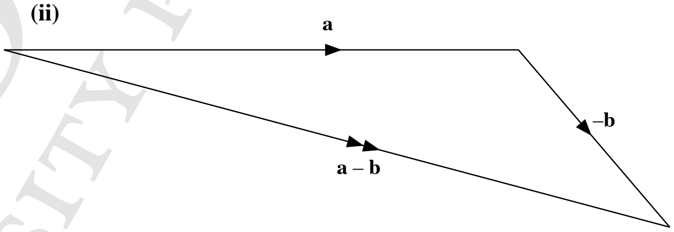
If **a** and **b** are in different directions, $|\mathbf{a} + \mathbf{b}| < |\mathbf{a}| + |\mathbf{b}|$.



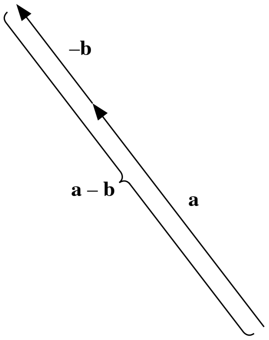
If **a** and **b** are in the same direction, $|\mathbf{a} + \mathbf{b}| = |\mathbf{a}| + |\mathbf{b}|$.

$\therefore |\mathbf{a} + \mathbf{b}| \leqslant |\mathbf{a}| + |\mathbf{b}|$

(ii)



If **a** and **b** are in different directions, $|\mathbf{a} - \mathbf{b}| < |\mathbf{a}| + |\mathbf{b}|$



If **a** and **b** are in opposite directions, $|\mathbf{a} - \mathbf{b}| = |\mathbf{a}| + |\mathbf{b}|$.

$\therefore |\mathbf{a} - \mathbf{b}| \leqslant |\mathbf{a}| + |\mathbf{b}|$

2. (a) (i) $\overrightarrow{EF} = -\mathbf{p}$

(ii) $\overrightarrow{BE} = \overrightarrow{BA} + \overrightarrow{AE}$

$= -\mathbf{p} + \mathbf{q}$

$= \mathbf{q} - \mathbf{p}$

OXFORD UNIVERSITY PRESS

(b) $\angle BOD = 90°$

$OA = OB = OD = OE = 1$ unit

Using Pythagoras' Theorem,

$$|\overrightarrow{BD}| = \sqrt{1^2 + 1^2}$$
$$= \sqrt{2} \text{ units}$$

(c) $\overrightarrow{BD}$ is parallel to $\overrightarrow{AE}$.

The unit vector in the direction of $\overrightarrow{AE}$ is $\dfrac{\overrightarrow{AE}}{|\overrightarrow{AE}|} = \dfrac{\mathbf{q}}{2}$.

$$\overrightarrow{BD} = \frac{\overrightarrow{AE}}{|\overrightarrow{AE}|} \times |\overrightarrow{BD}|$$
$$= \frac{\mathbf{q}}{2} \times \sqrt{2}$$
$$= \frac{\mathbf{q}}{\sqrt{2}}$$

3. Let $\overrightarrow{BP} = \mathbf{p}$ and $\overrightarrow{BQ} = \mathbf{q}$.

Then $\overrightarrow{PQ} = \overrightarrow{PB} + \overrightarrow{BQ} = \mathbf{q} - \mathbf{p}$.

Since $\overrightarrow{BA} = 2\overrightarrow{BP} = 2\mathbf{p}$ and $\overrightarrow{BC} = 2\overrightarrow{BQ} = 2\mathbf{q}$,

then $\overrightarrow{AC} = \overrightarrow{BC} - \overrightarrow{BA}$
$$= 2\mathbf{q} - 2\mathbf{p}$$
$$= 2(\mathbf{q} - \mathbf{p})$$
$$= 2\overrightarrow{PQ}$$

Since $\overrightarrow{AC} = 2\overrightarrow{PQ}$, then PQ is parallel to AC and $AC = 2PQ$.

Using the same reasoning for $\triangle DSR$ and $\triangle DAC$, we can show that SR is parallel to AC and $AC = 2SR$.

$\therefore$ PQ is parallel to SR and $PQ = SR$.

Let $\overrightarrow{CQ} = \mathbf{x}$ and $\overrightarrow{CR} = \mathbf{y}$.

Then $\overrightarrow{QR} = \overrightarrow{QC} + \overrightarrow{CR} = \mathbf{y} - \mathbf{x}$.

Since $\overrightarrow{CB} = 2\overrightarrow{CQ} = 2\mathbf{x}$ and $\overrightarrow{CD} = 2\overrightarrow{CR} = 2\mathbf{y}$,

then $\overrightarrow{BD} = \overrightarrow{CD} - \overrightarrow{CB}$
$$= 2\mathbf{y} - 2\mathbf{x}$$
$$= 2(\mathbf{y} - \mathbf{x})$$
$$= 2\overrightarrow{QR}$$

Since $\overrightarrow{BD} = 2\overrightarrow{QR}$, then QR is parallel to BD and $BD = 2QR$.

Using the same reasoning for $\triangle APS$ and $\triangle ABD$, we can show that PS is parallel to BD and $BD = 2PS$.

$\therefore$ QR is parallel to PS and $QR = PS$.

Since $ABCD$ is a kite, the diagonals intersect at right angles, i.e. AC is perpendicular to BD. Hence PQ and SR are perpendicular to PS and QR.

Since the length of PQ (or SR) is not equal to the length of PS (or QR), therefore $PQRS$ is a rectangle. (shown)

OXFORD UNIVERSITY PRESS

Chapter 8 Loci

TEACHING NOTES

Suggested Approach

Students may not be familiar with the term “loci”, and thus teachers can introduce the concept of how a point can move, and begin the lesson by asking some students to demonstrate walking in a circle around a student in the centre of the circle, or a straight line such that they will always be the same distance from another student etc. Teachers may also wish to briefly revise geometrical constructions as this chapter involves constructing loci and various figures.

Section 8.1: Introduction to Loci

Teachers can make use of the Class Discussion on page 275 to get students to understand the concept behind loci, and how a locus can be obtained through identifying the path of a point under the conditions it must fulfil.

Section 8.2: Locus Theorems

The four Investigations would be useful for students to learn about the Locus Theorems, and thus teachers should spend some time to go through them, to help students better understand how to use these theorems to solve problems.

Section 8.3: Intersection of Loci

The intersection of two loci involves accurate constructions. Hence, teachers should emphasise the importance of using proper techniques in the construction of a circle, perpendicular bisector, angle bisector as well as parallel lines.

Section 8.4: Further Loci

Teachers might need to spend more time on this section as it involves the consolidation and application of past knowledge and careful visualisation. Teachers should also emphasise that dotted lines should be used when the operation sign is “<” or “>”.

Challenge Yourself

Teachers can guide students to identify the shape that the points seem to form. After drawing out the locus, students can check if other points on this locus which were not plotted by them fit the criteria.

OXFORD UNIVERSITY PRESS

WORKED SOLUTIONS

Class Discussion (Introduction to Loci)

Teachers can help to facilitate the discussion among students, and highlight what the term "loci" means.

Investigation (Locus Theorem 1)

Part 1

1. They form a circle.
2. The path is a circle.

Part 2

4. The points form a circle, with O as the centre and a radius of 2 cm.

Investigation (Locus Theorem 2)

Part 1

3. One needs to walk parallel to the wall, 2 m away.

Part 2

5. The points form a straight line, parallel to XY on either side, 3 cm away.

Investigation (Locus Theorem 3)

Part 1

3. One needs to walk in a straight line coinciding with the perpendicular bisector of the two desks.

Part 2

6. The points lie on the perpendicular bisector of XY.

Investigation (Locus Theorem 4)

Part 1

3. One needs to walk in an angle such that the line of path is the angle bisector of the two adjacent walls.

Part 2

6. The points lie on the angle bisector of the angles between the two lines AB and XY.

Practise Now (Page 277)

The locus of point P is a circle with centre Q and a radius of 5 cm.

Practise Now (Page 278)

The loci of point P are two straight lines parallel to and 10 cm away from RS.

Practise Now (Page 279)

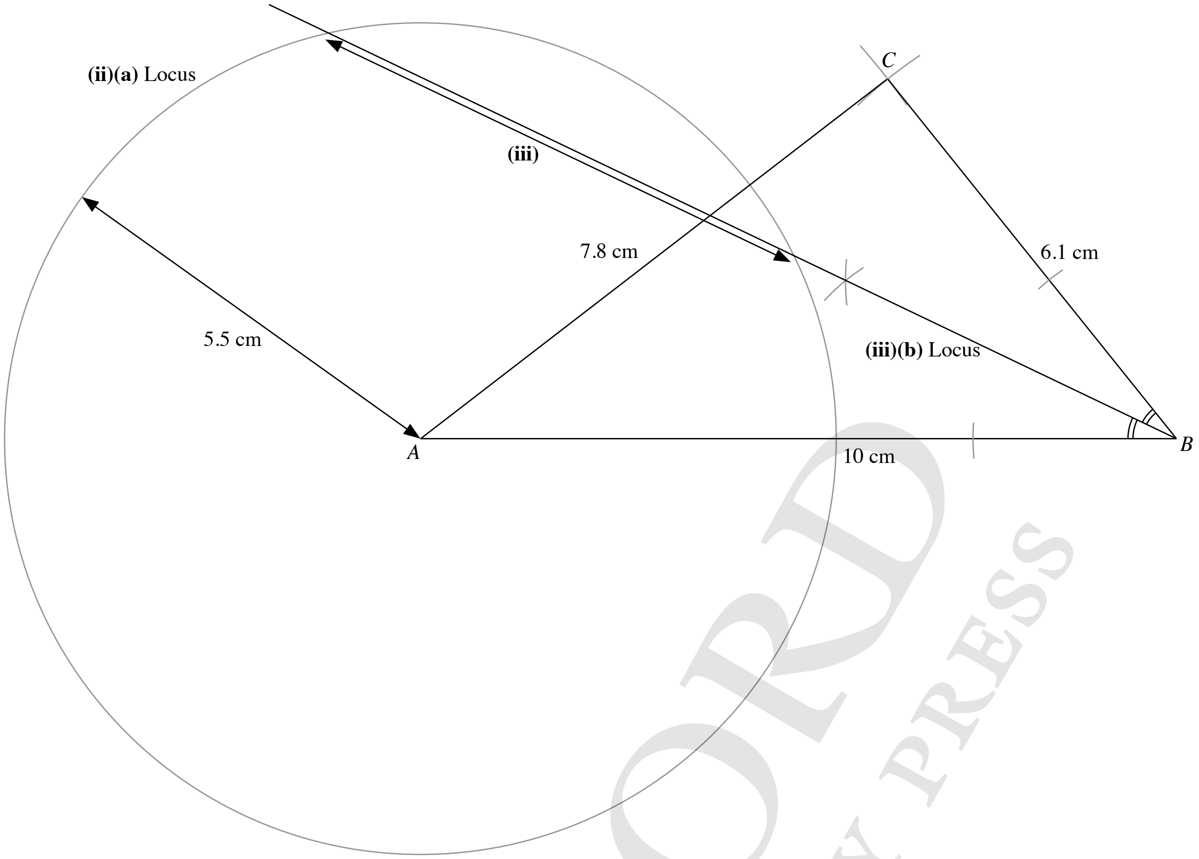
The locus of point C is the perpendicular bisector of the line AB, which cuts AB 6 cm away from each point A and B.

Practise Now (Page 280)

The locus of point C is a pair of straight lines which bisect the angles between the two diagonals of the square.

OXFORD UNIVERSITY PRESS

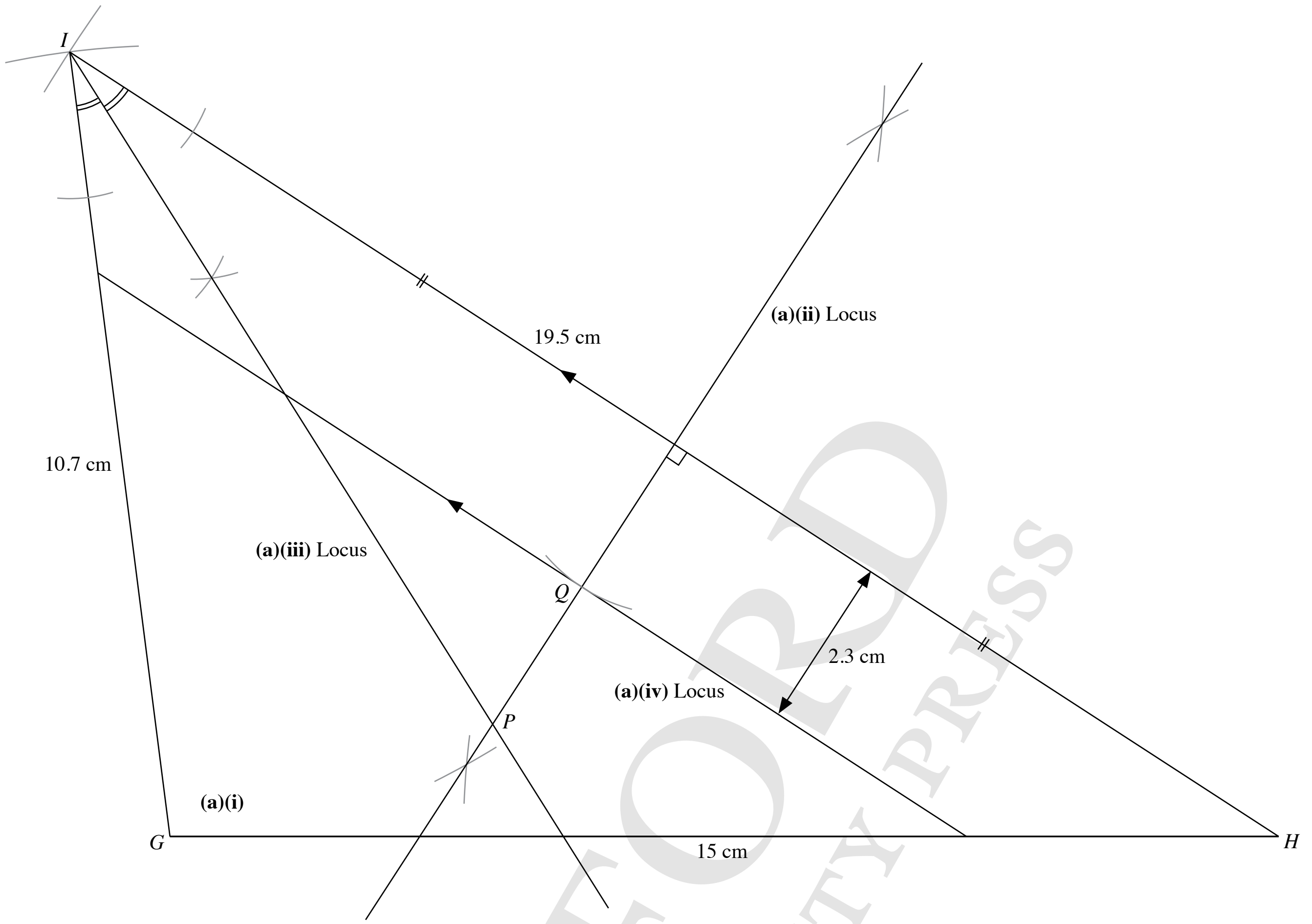
Practise Now 1



(iii) Distance between the 2 points = 6.8 cm

OXFORD UNIVERSITY PRESS

Practise Now 2

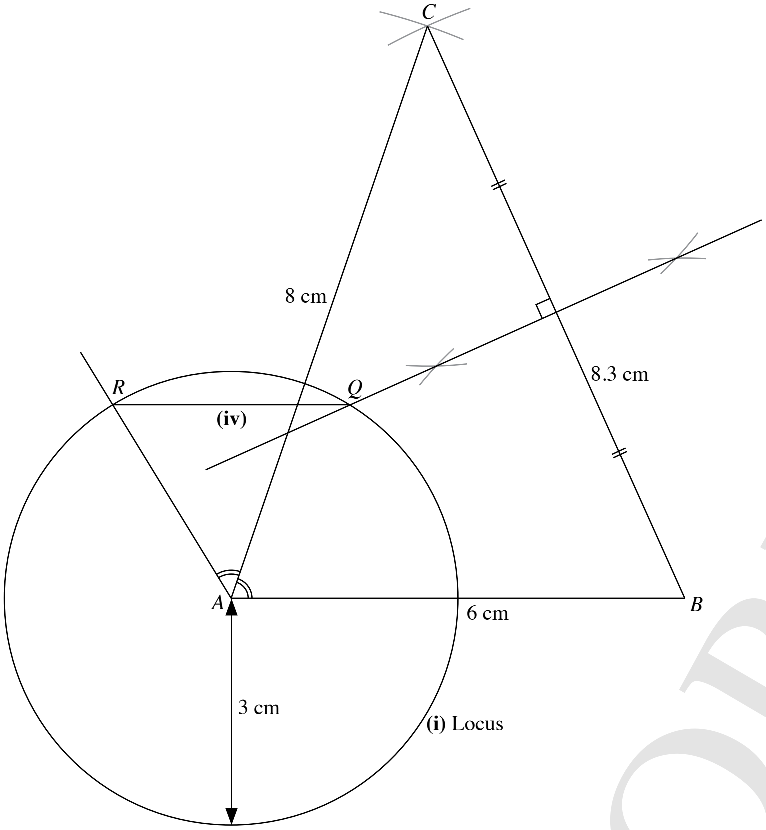


(a) **(i)** $\angle IGH = 97.2°$

(b) **(iii)** From the diagram, $PQ = 2.2$ cm

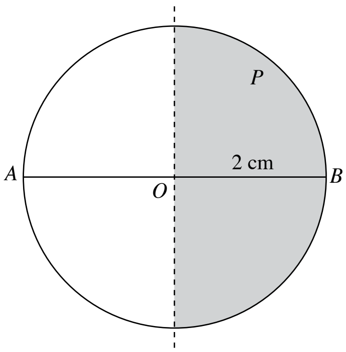
OXFORD UNIVERSITY PRESS

Practise Now 3

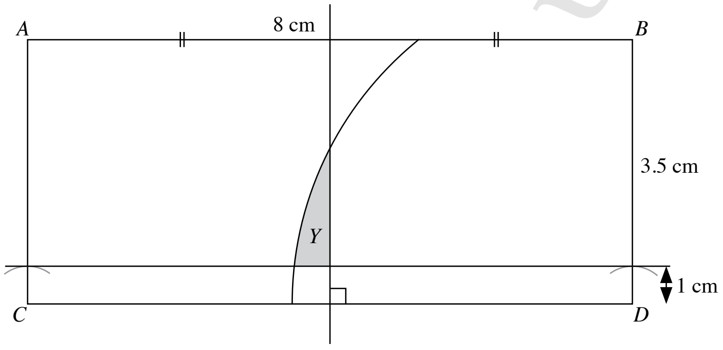


(iv) From the diagram, RQ = 2.8 cm

Practise Now 4

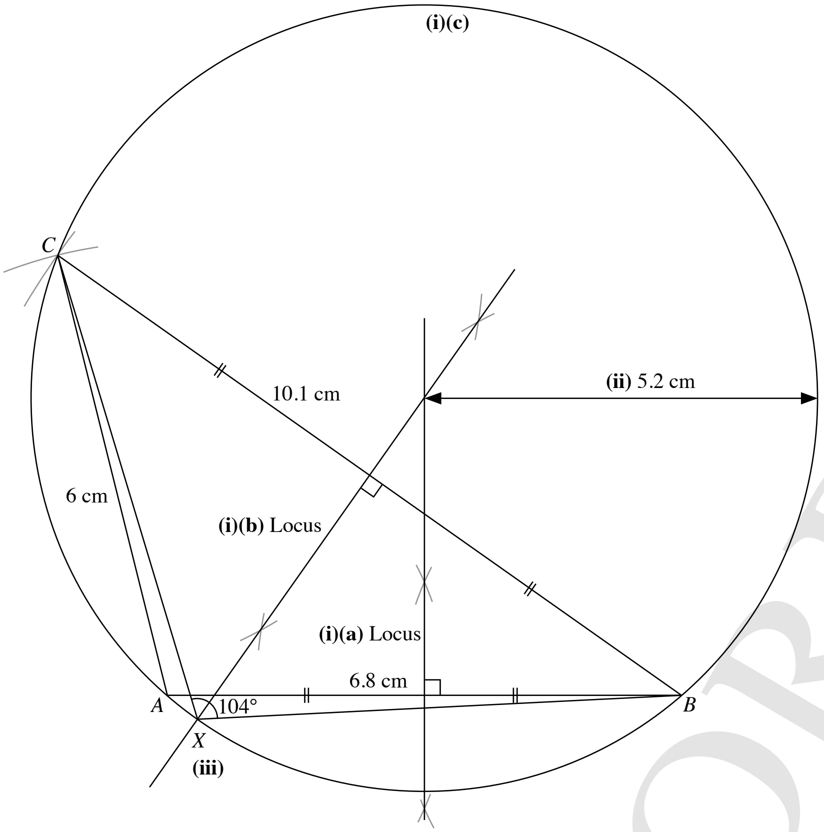


Practise Now 5



OXFORD UNIVERSITY PRESS

Practise Now 6

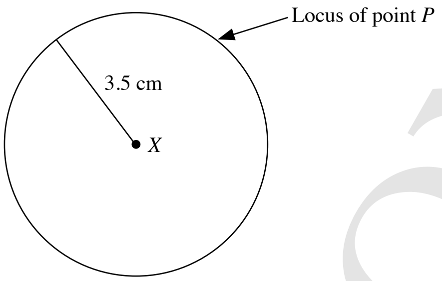


(ii) Radius of circle = 5.2 cm

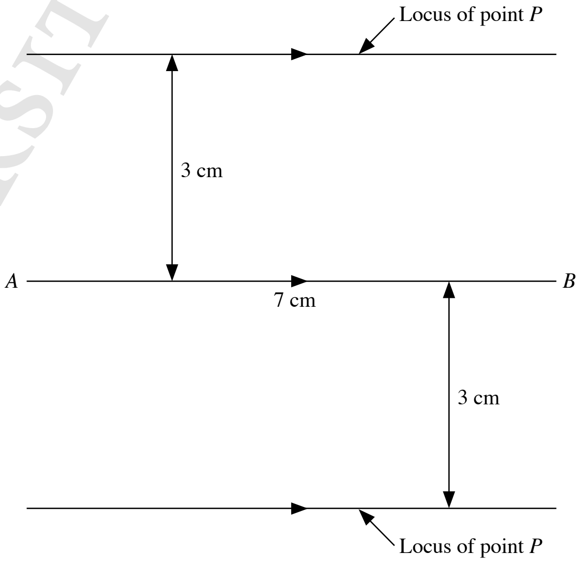
Exercise 8A

1. The locus of point P is a circle with centre O and a radius of 4 cm.

2.



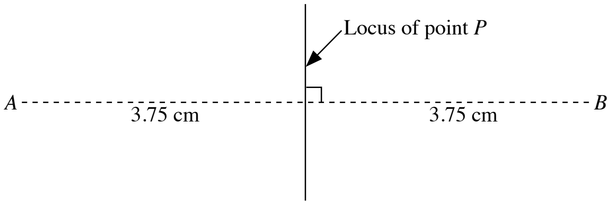
3.



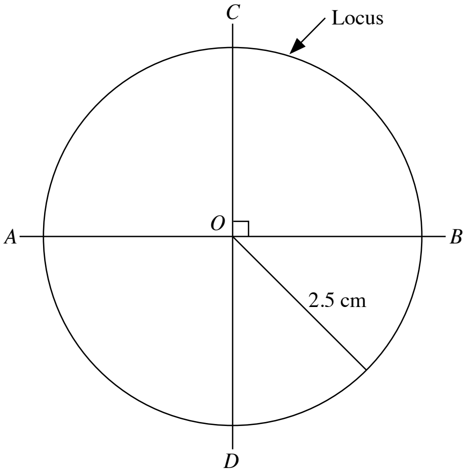
4. The loci of point Q are two straight lines parallel to and 5 cm away from straight line l.

OXFORD UNIVERSITY PRESS

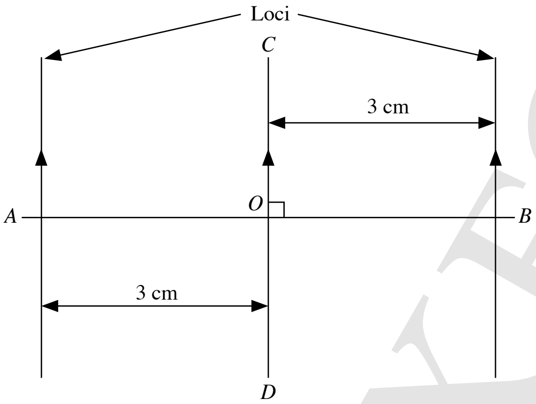
5.



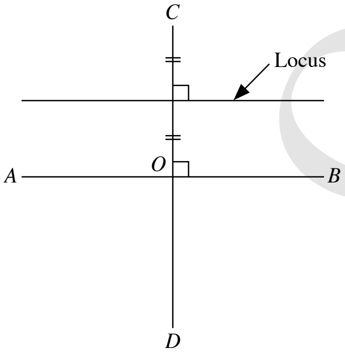
6. **(i)**



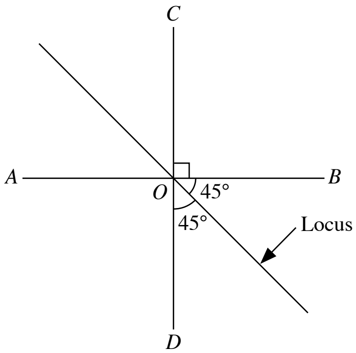
(ii)



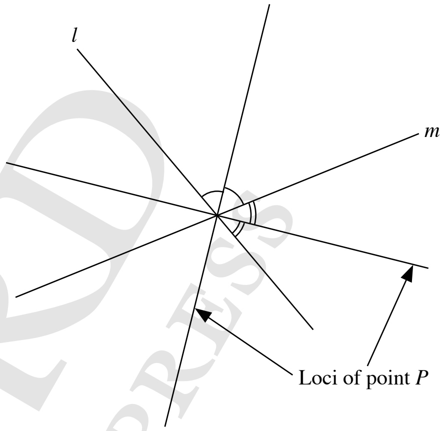
(iii)



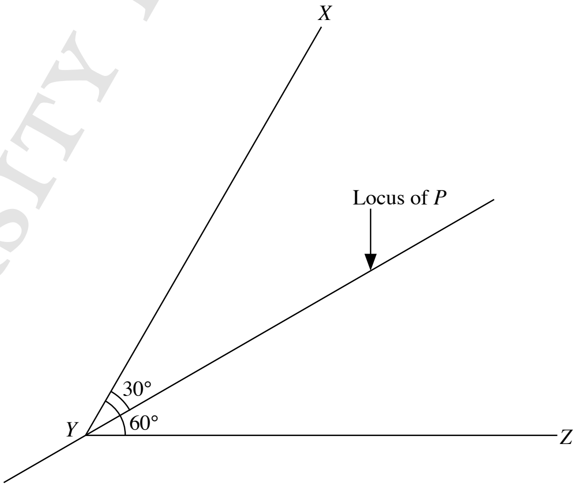
(iv)



7.

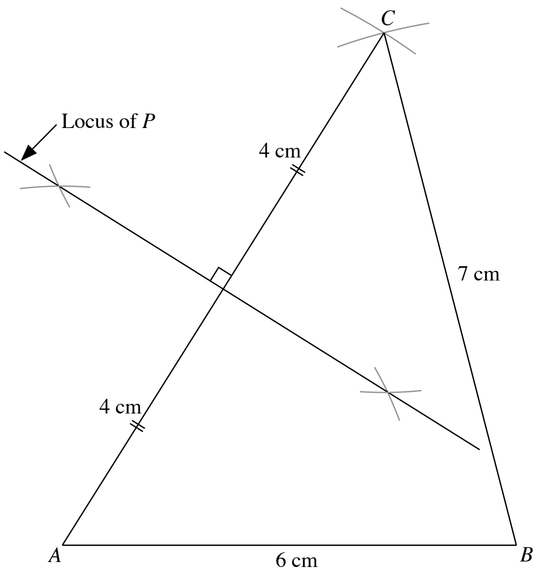


8.

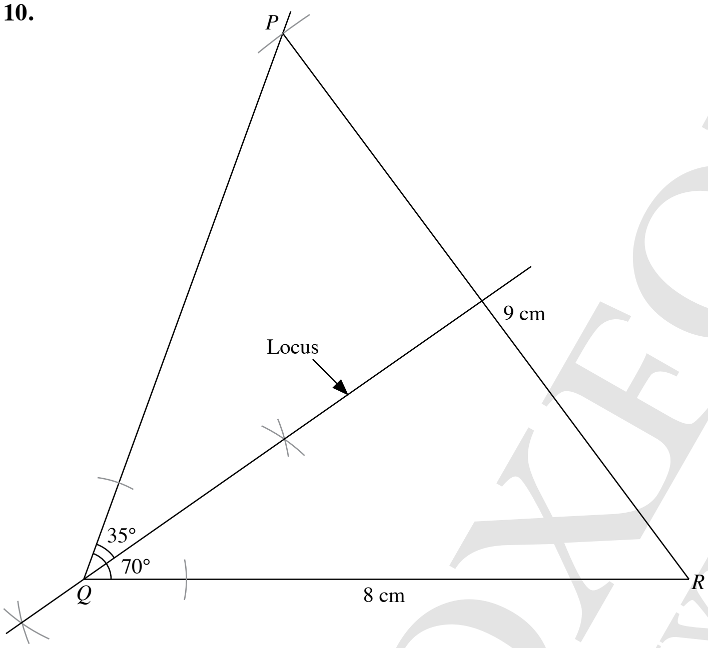


OXFORD UNIVERSITY PRESS

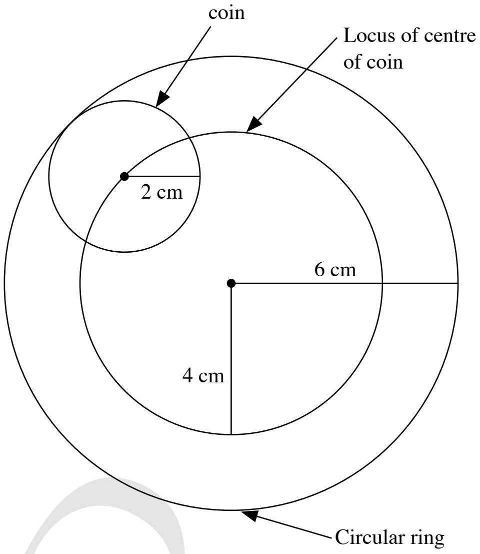
9.



10.



11.

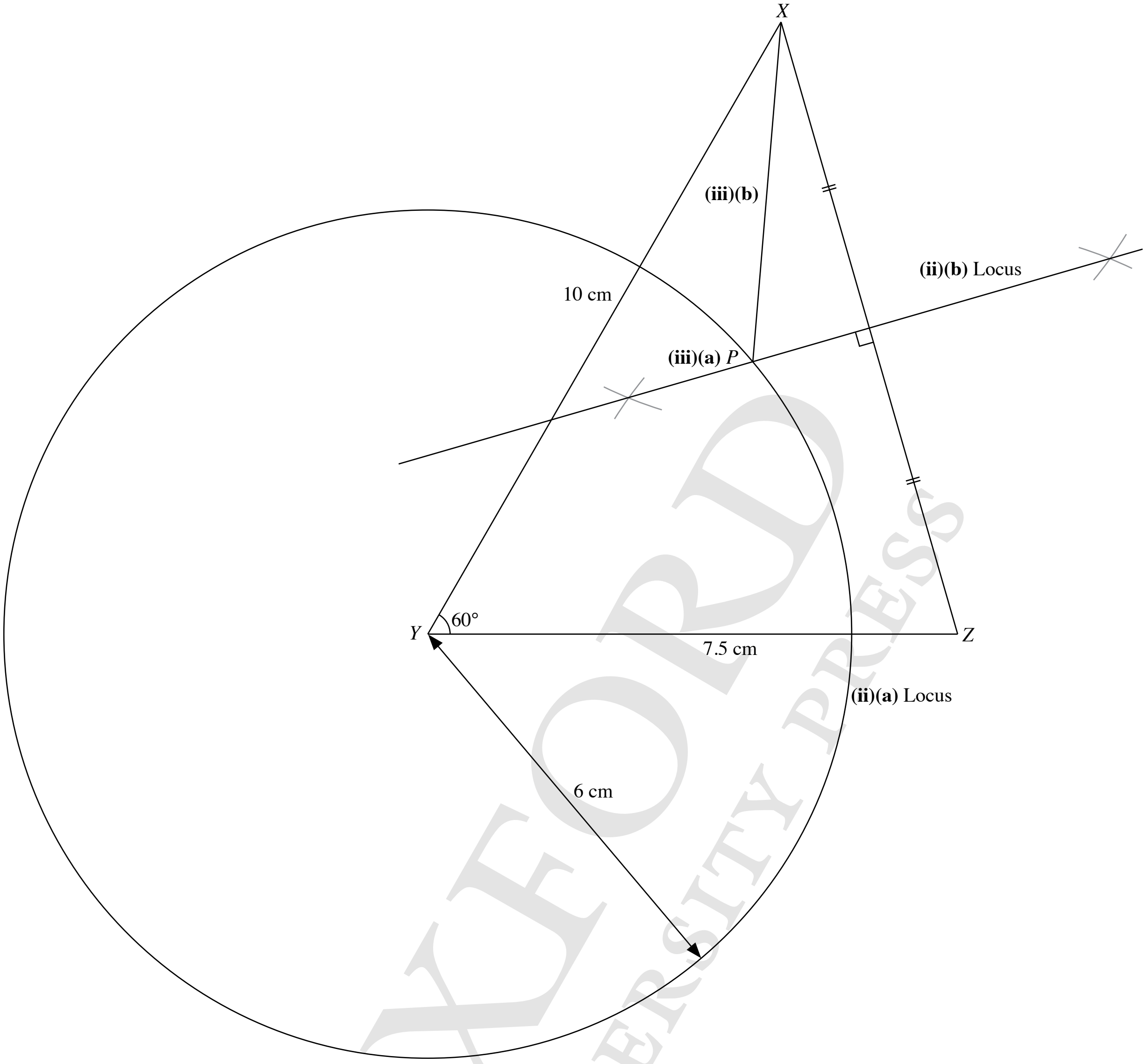


The locus of the centre of the coin is a circle with the same centre as the circular ring, and a radius of 4 cm.

OXFORD UNIVERSITY PRESS

Exercise 8B

1.

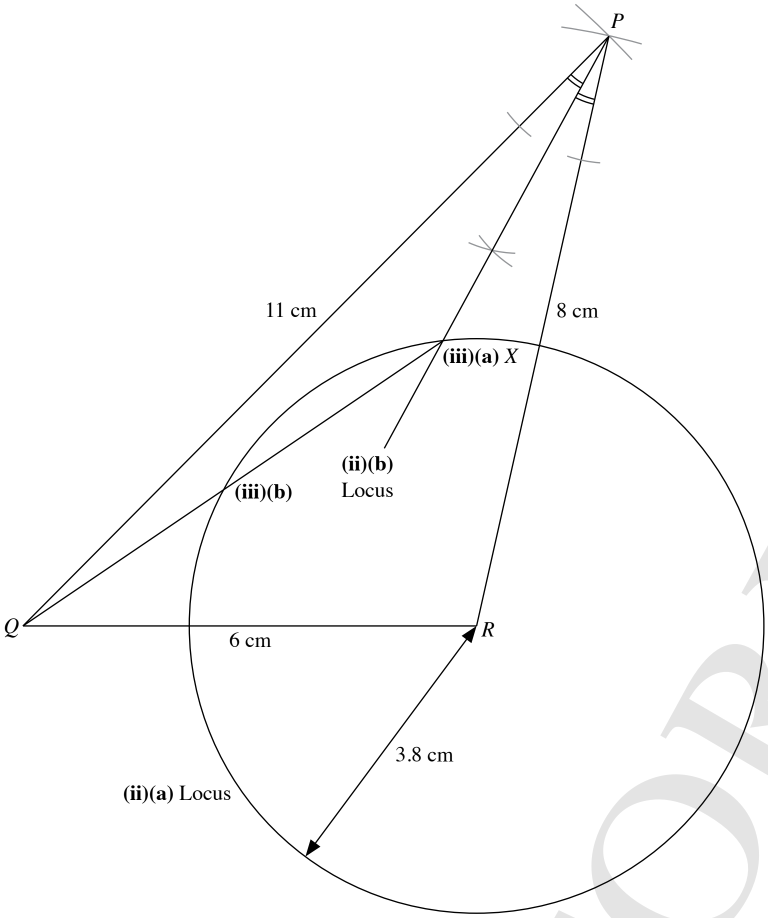


(i) $XZ = 9$ cm

(iii) (b) $PX = 4.8$ cm

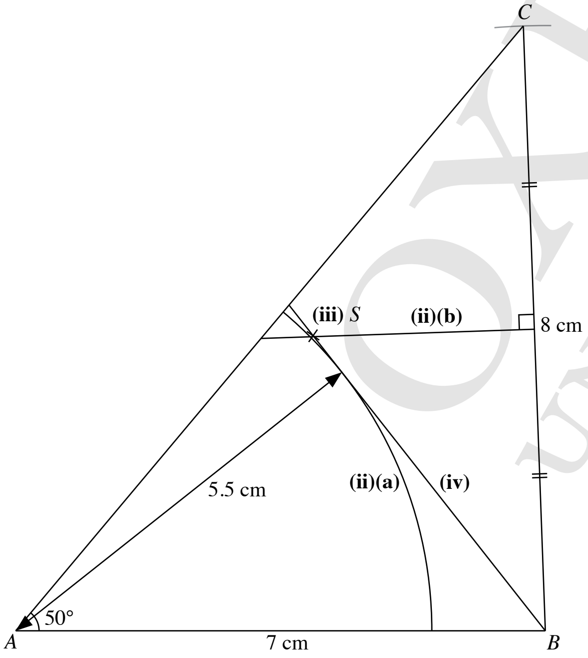
OXFORD UNIVERSITY PRESS

2.



(iii) (b) $XQ = 6.7$ cm

3.



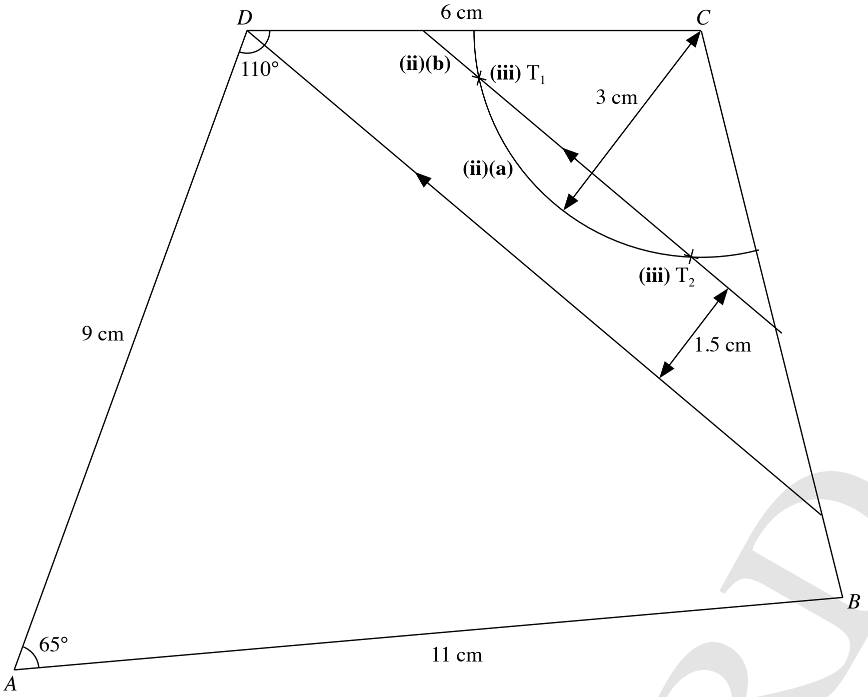
(iv) $BS = 5$ cm

$\therefore$ Distance of seesaw S from $B = 5 \times 10$

$= 50$ m

OXFORD UNIVERSITY PRESS

4.



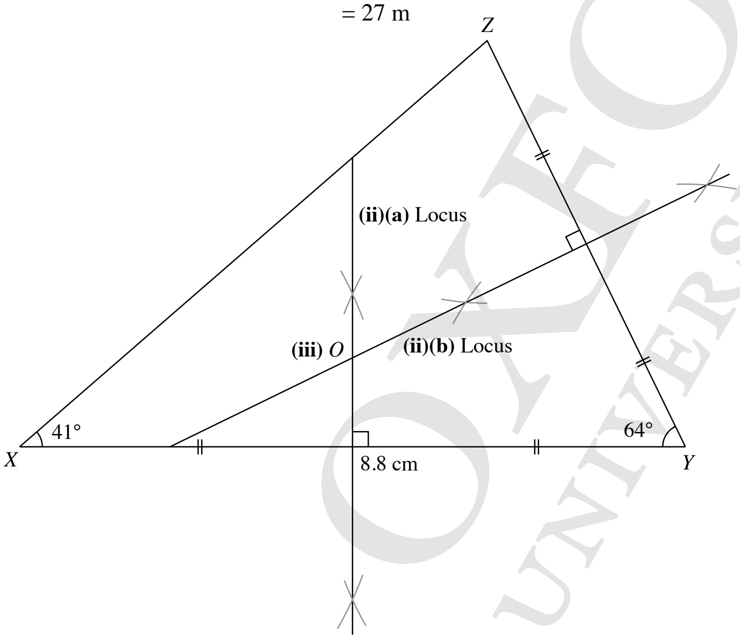
(i) $\angle ABC = 81°$

(iv) From the diagram, T$_1$ and T$_2$ are 2.7 cm apart.

$\therefore$ Distance between T$_1$ and T$_2$ $= 2.7 \times 10$

$= 27$ m

5.

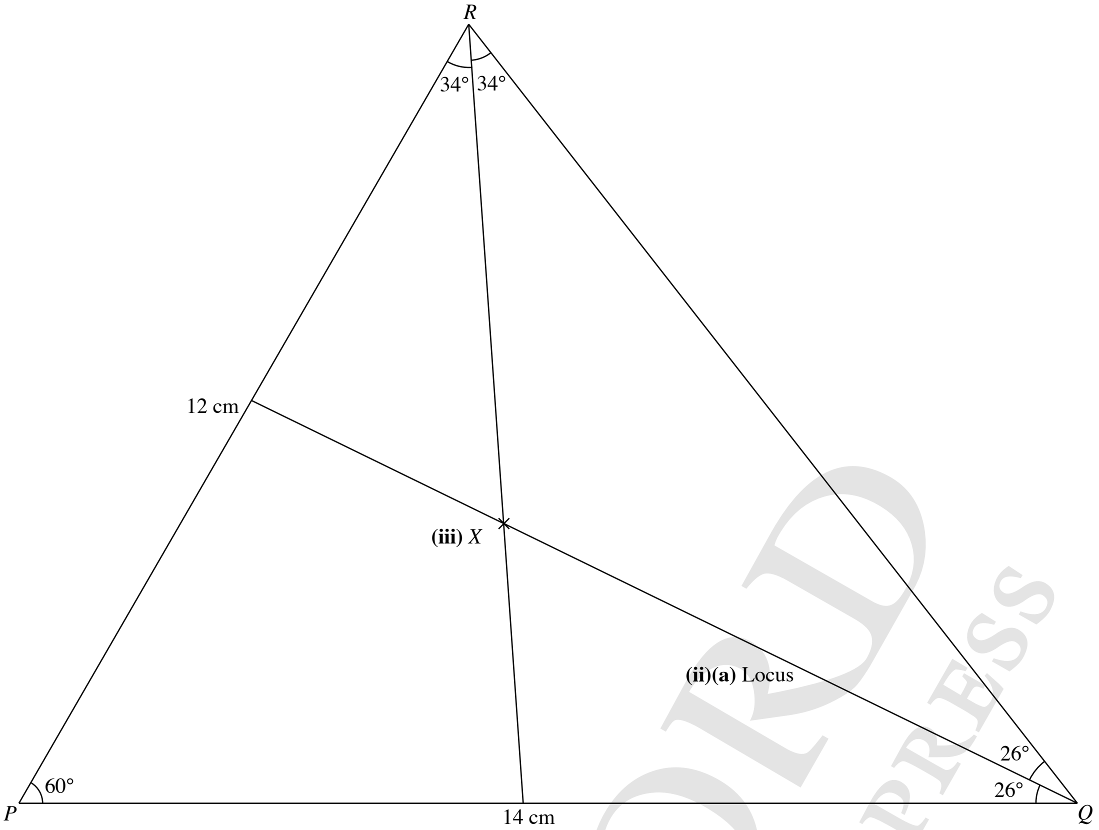


(iv) From the diagram, *O* is 4.5 cm from the corners.

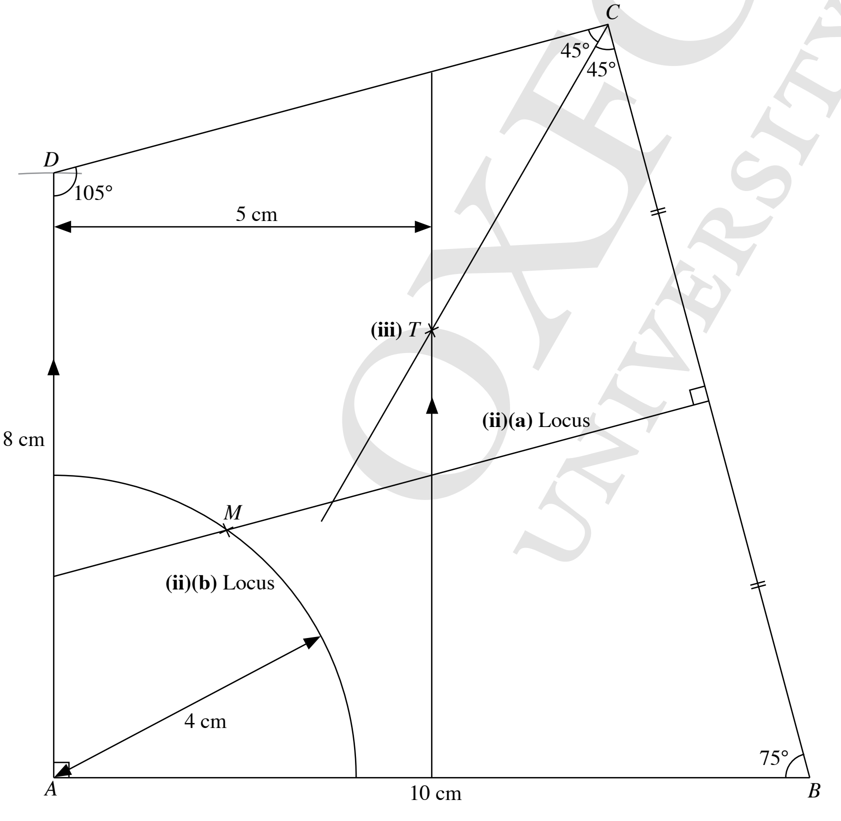
$\therefore$ Distance of hole from *X*, *Y* and *Z* = 4.5 m

OXFORD UNIVERSITY PRESS

6.



7.



(ii) From the diagram, $BM = 8.4$ cm

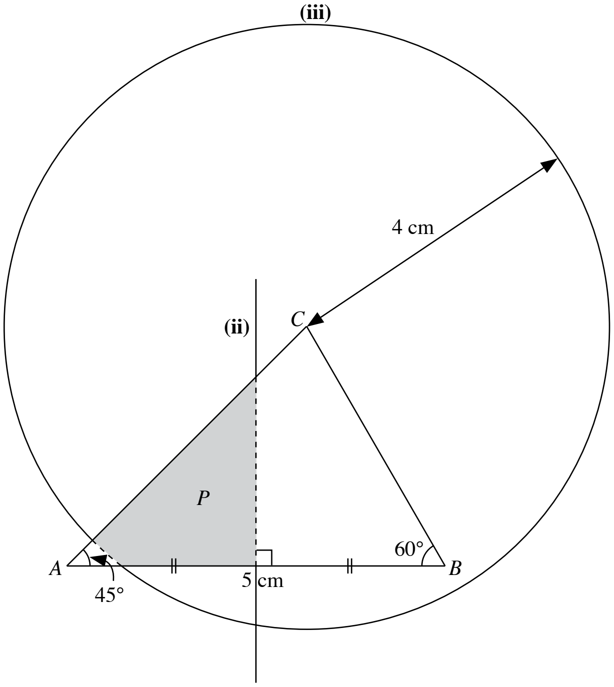
Distance of third mango tree from the other two $= 8.4 \times 10$

$= 84$ m

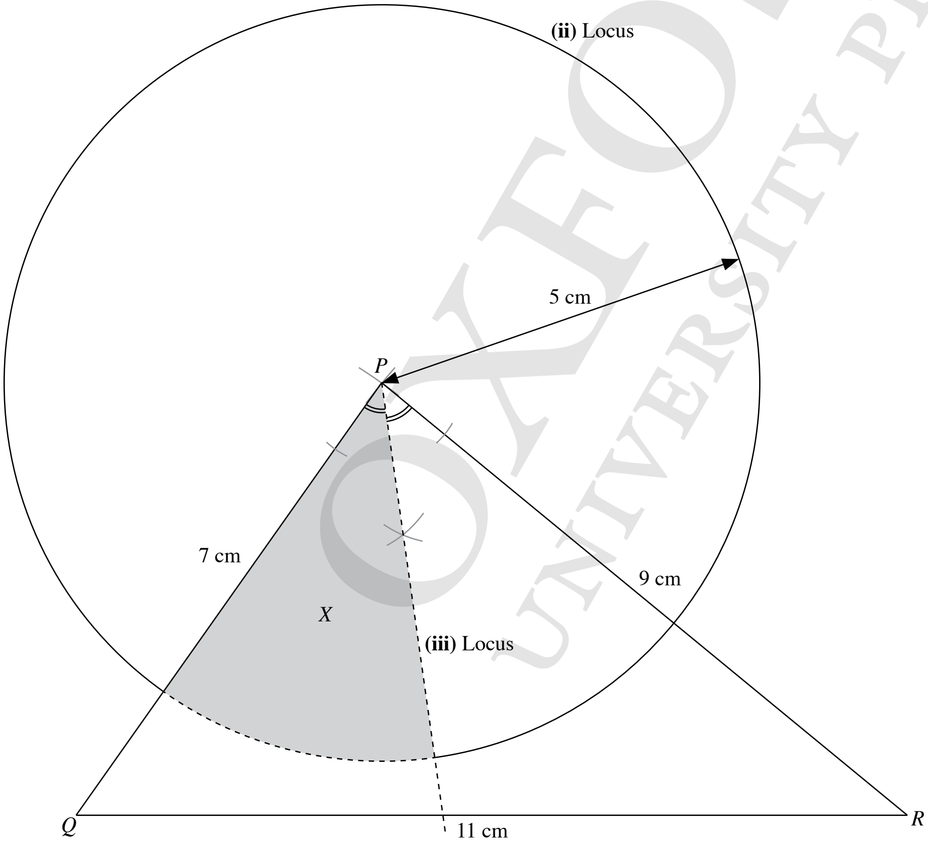
OXFORD UNIVERSITY PRESS

Exercise 8C

1.

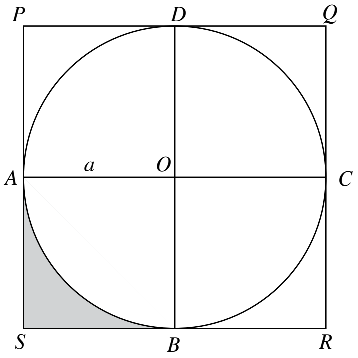


2.

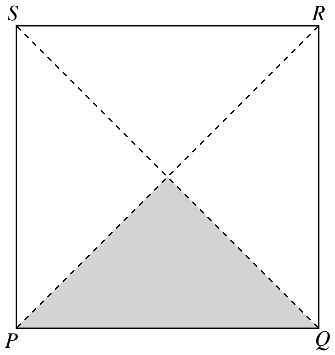


OXFORD UNIVERSITY PRESS

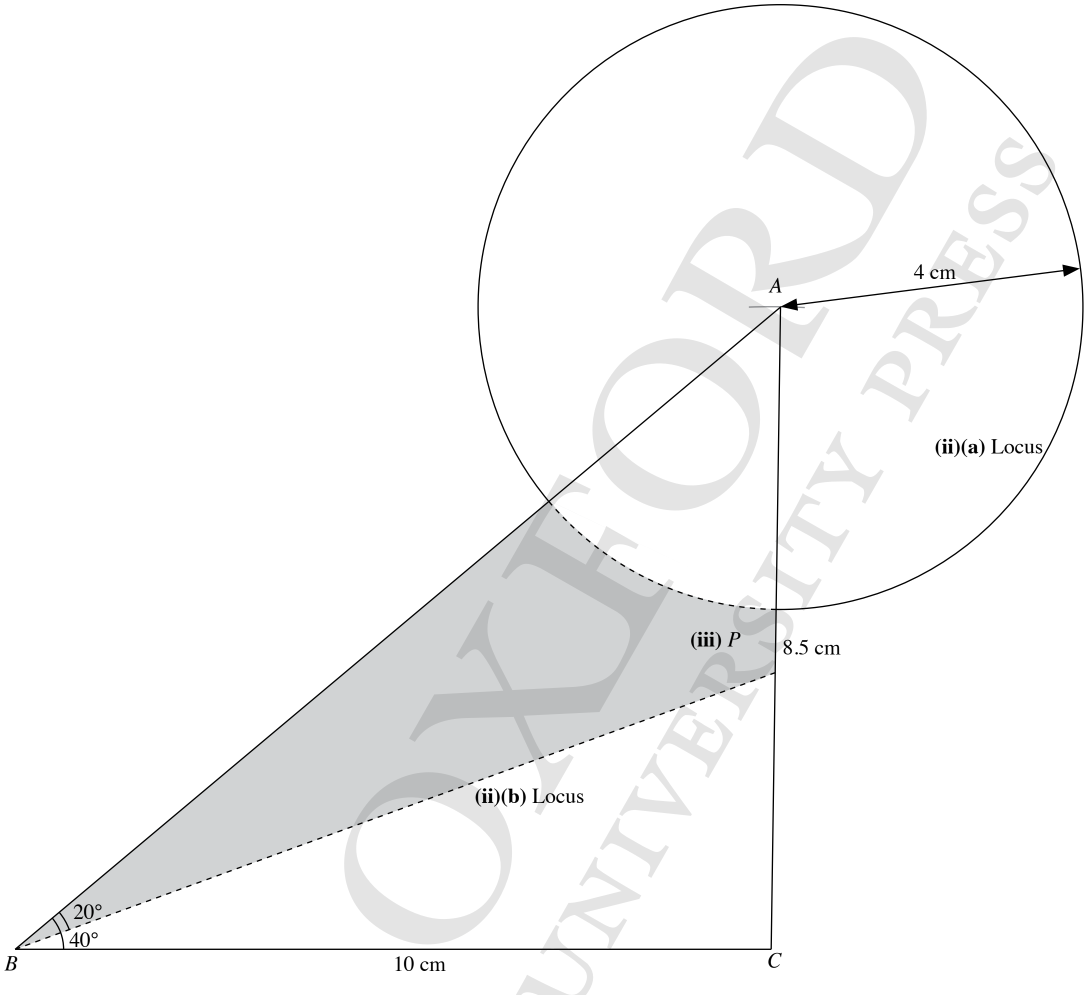
3.



4.



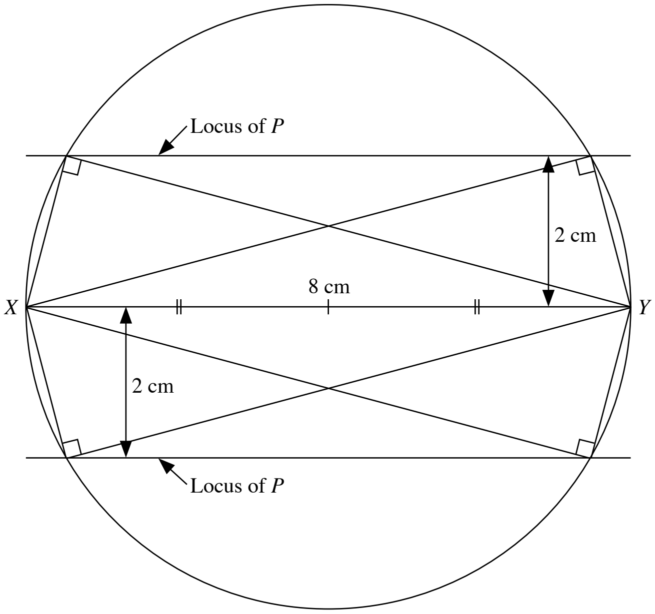
5.



(i) AB = 13.2 cm

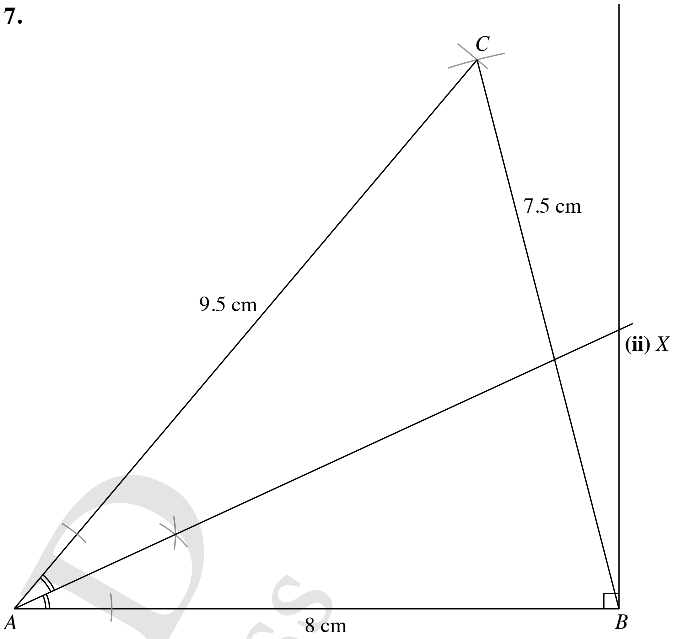
OXFORD UNIVERSITY PRESS

6.



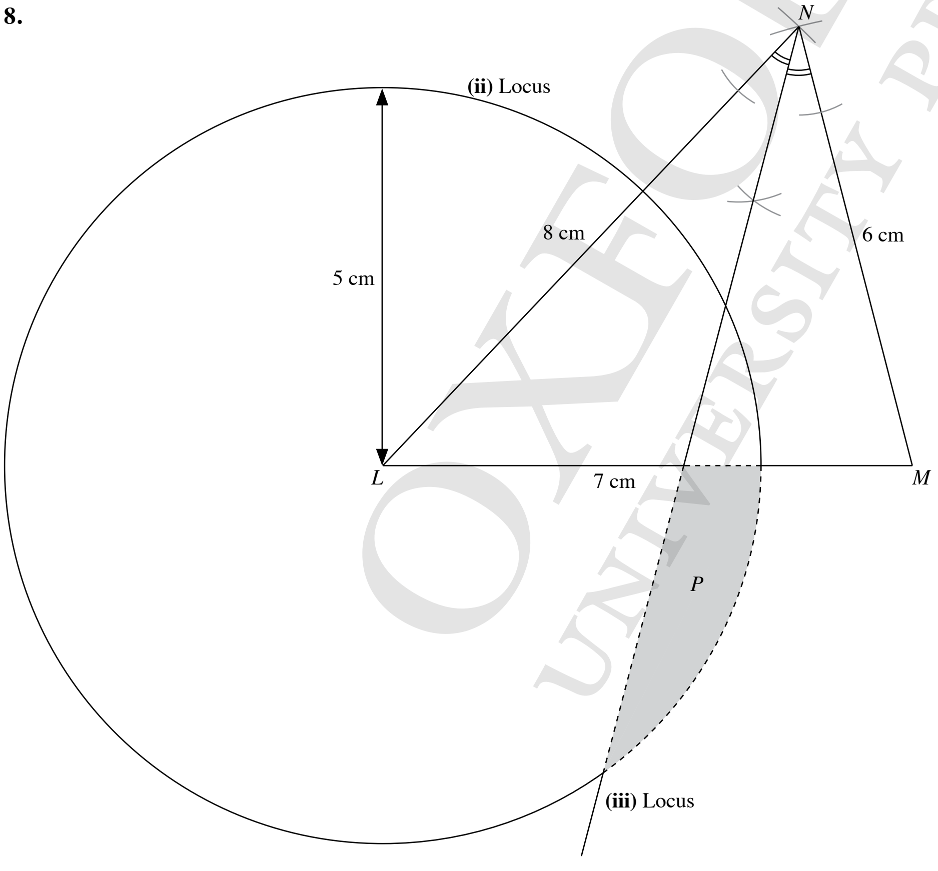
The possible positions of P which makes $\angle XPY$ a right angle are where the circle with diameter XY cuts the loci of P.

7.



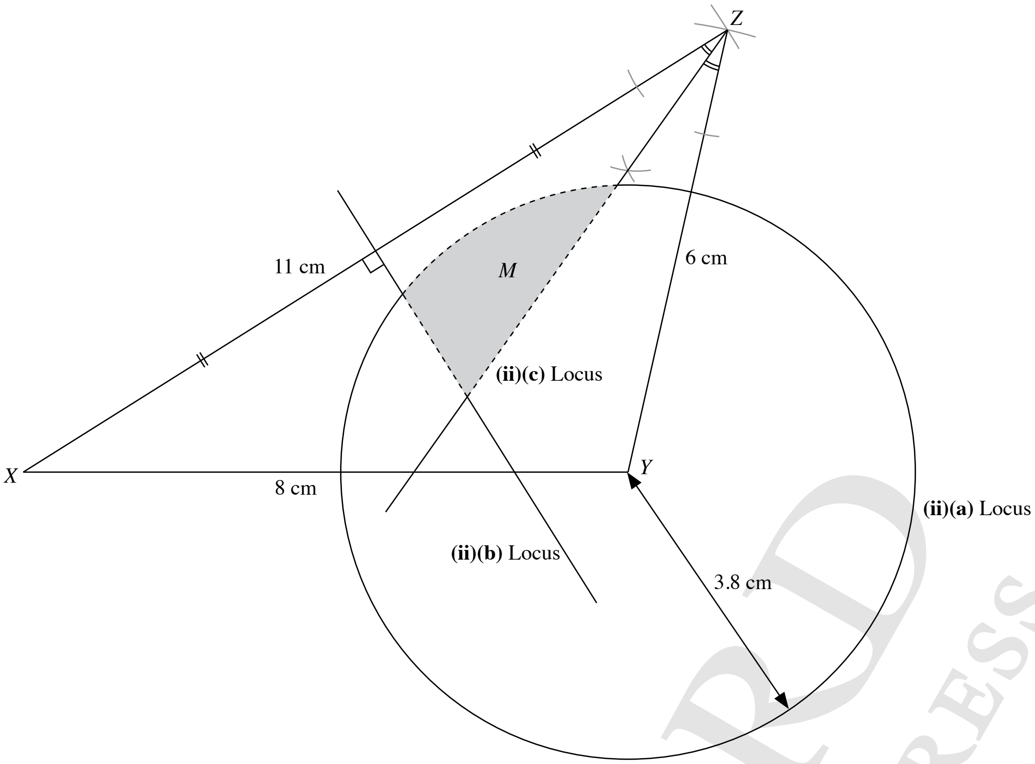
From the diagram, $XB = 3.7$ cm

8.



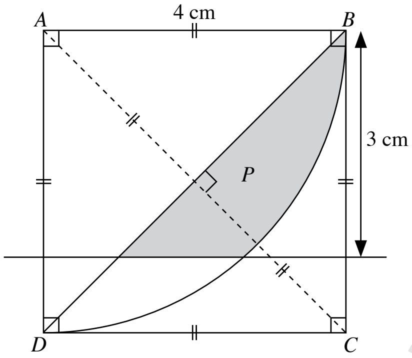
OXFORD UNIVERSITY PRESS

9.



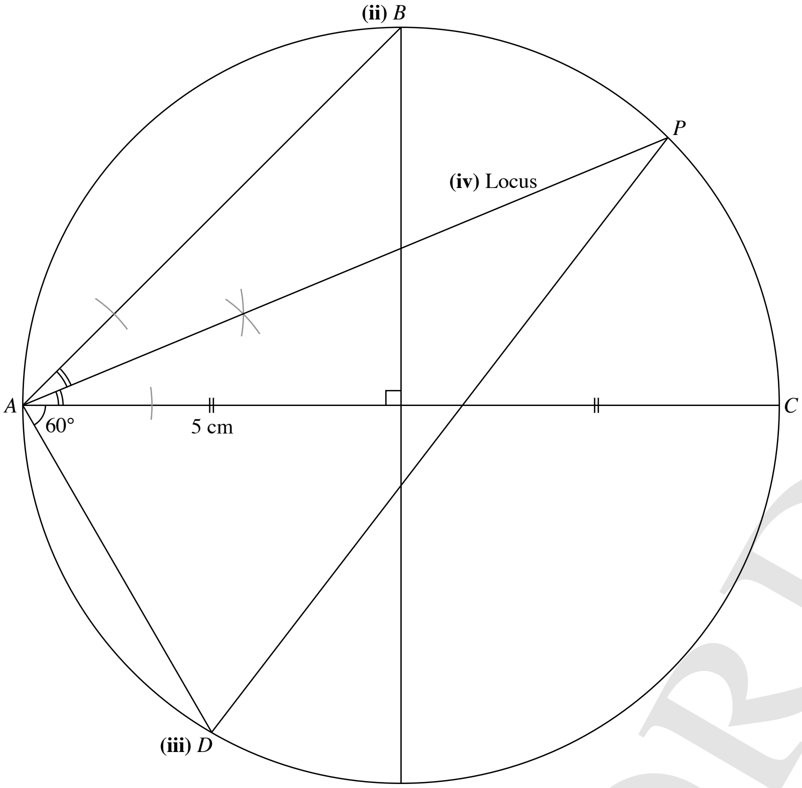
(i) $\angle XYZ = 103°$

10.



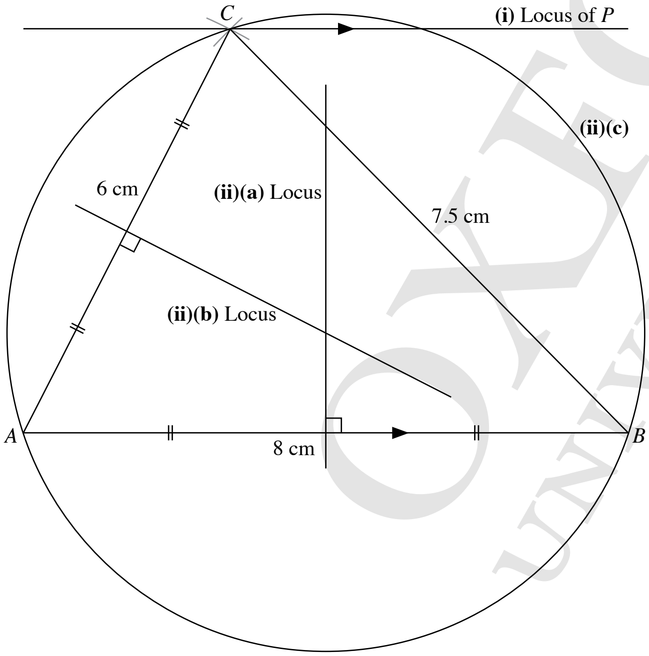
OXFORD UNIVERSITY PRESS

11.



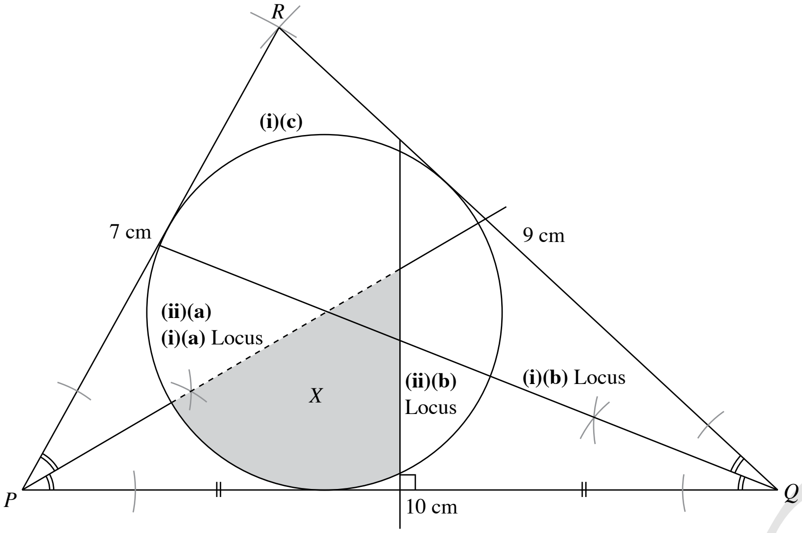
From the diagram, $PD = 9.9$ cm

12.

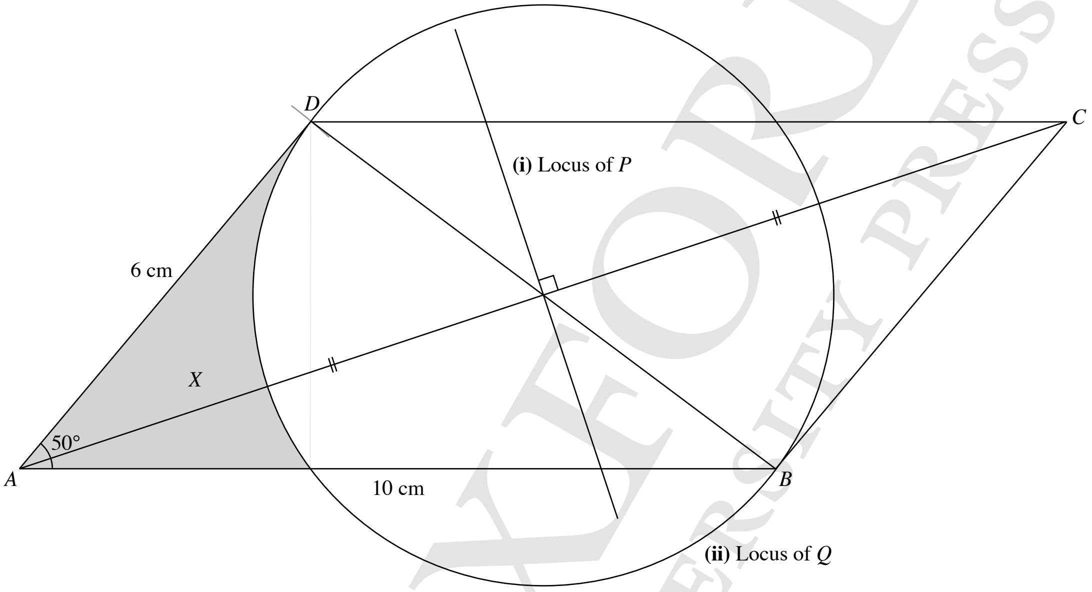


OXFORD UNIVERSITY PRESS

13.



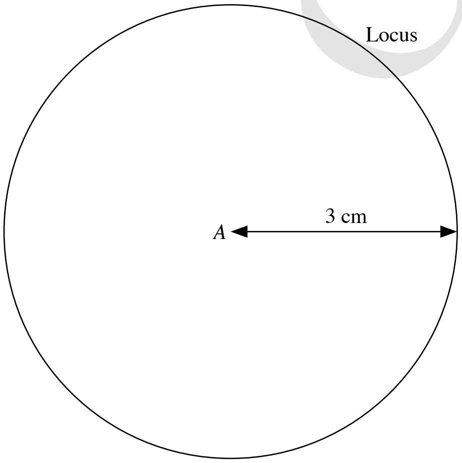
14.



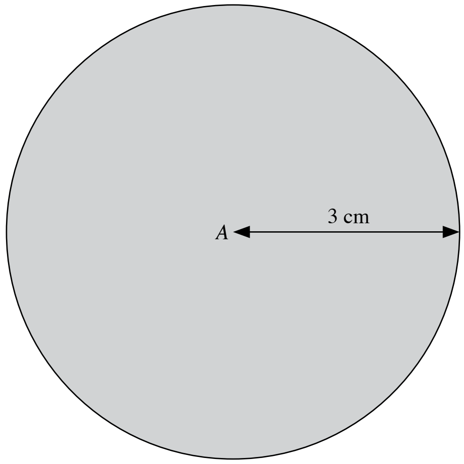
$BD = 7.7$ cm

Review Exercise 8

1. (i)

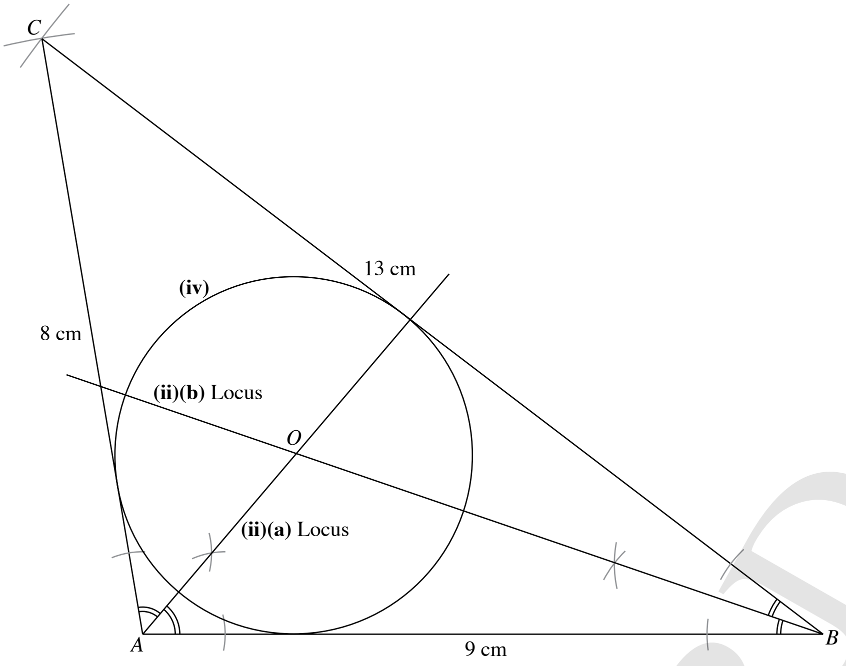


(ii)



OXFORD UNIVERSITY PRESS

2.



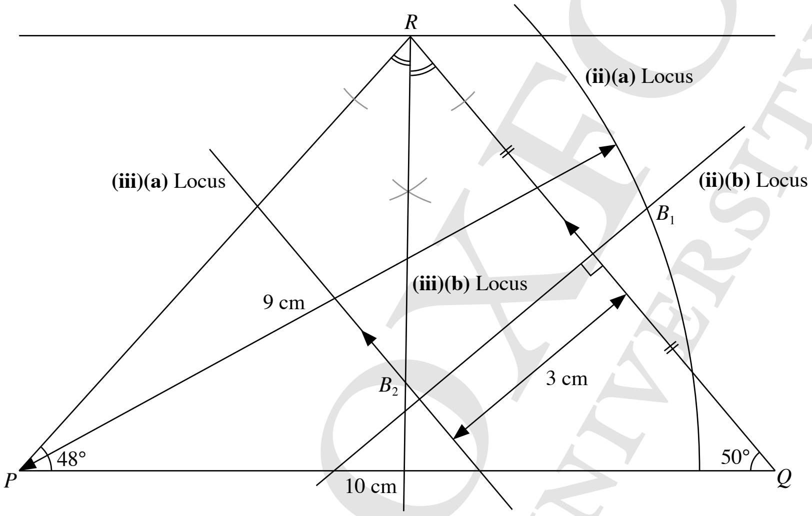
(i) $\angle CAB = 100°$

(v) From the diagram, radius of circle = 2.4 cm

$\therefore$ Radius of circle cut from wooden panel $= \dfrac{2.4}{2}$

$= 1.2$ m

3.



(i) Since the bearing of R from P is 042°,

$\angle QPR = 90° - 42°$

$= 48°$

Since the bearing of Q from R is 140°,

$\angle PQR = 90° - (180° - 140°)$

$= 50°$

From the diagram, $PR = 7.7$ cm

$\therefore$ Width of river $= 5.7 \times \dfrac{10}{2}$

$= 29$ m (to the nearest m)

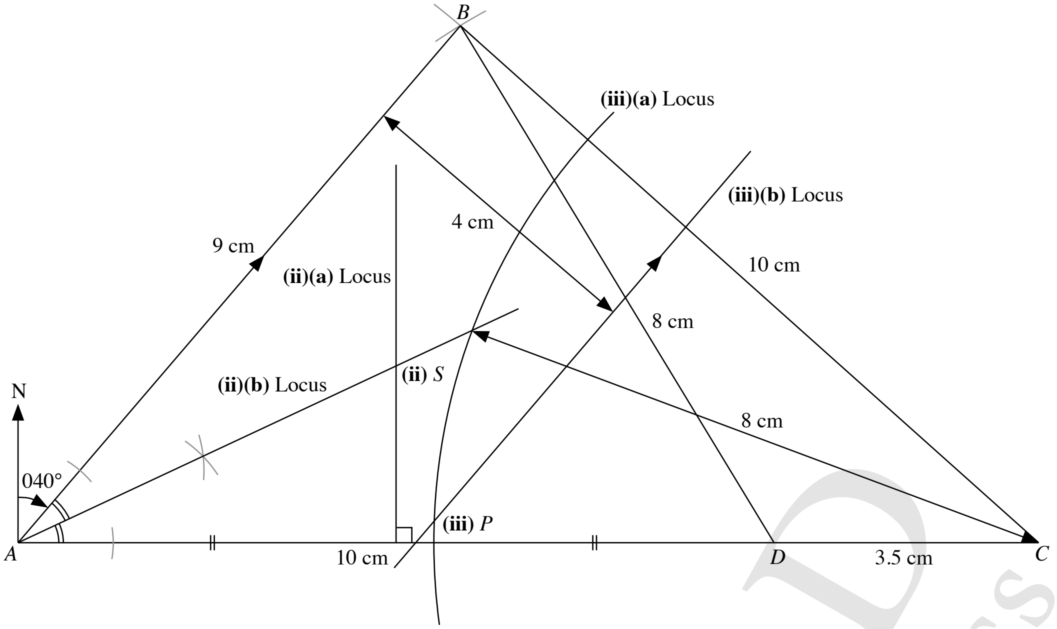
(iv) From the diagram, distance between B_1 and $B_2 = 4$ cm

$\therefore$ Actual distance between the two boats $= 4 \times \dfrac{10}{2}$

$= 20$ m

OXFORD UNIVERSITY PRESS

4.

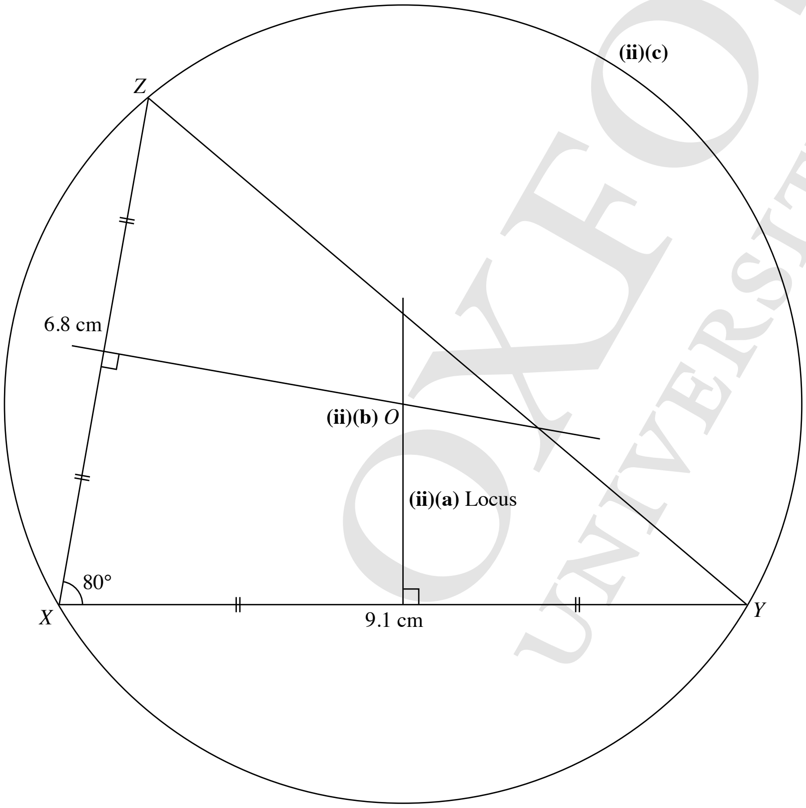


(i) Bearing of B from A = 040°

(iv) From the diagram, AP = 5.5 cm

∴ Actual distance between house A and swimming pool P = 55 m

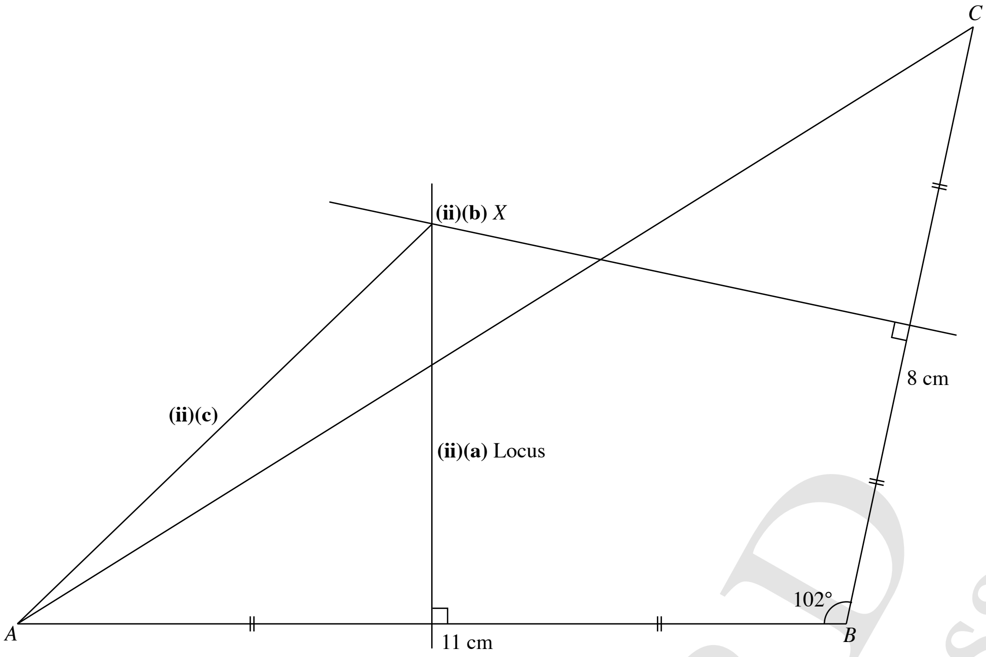
5.



(i) YZ = 10.4 cm

OXFORD UNIVERSITY PRESS

6.

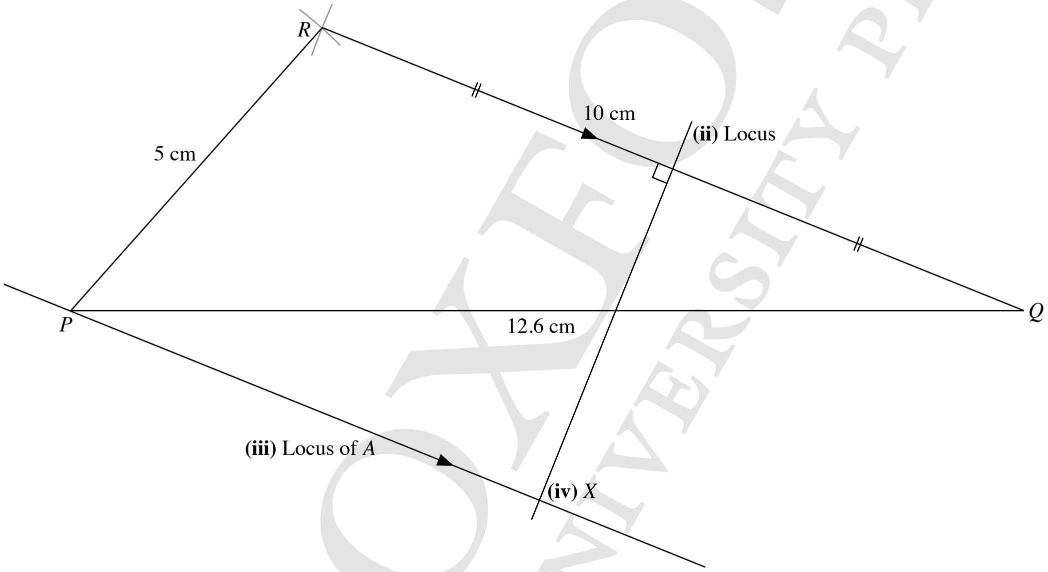


(i) **(a)** $AC = 14.9$ cm

(b) $\angle BAC = 32°$

(ii) **(c)** $AX = 7.6$ cm

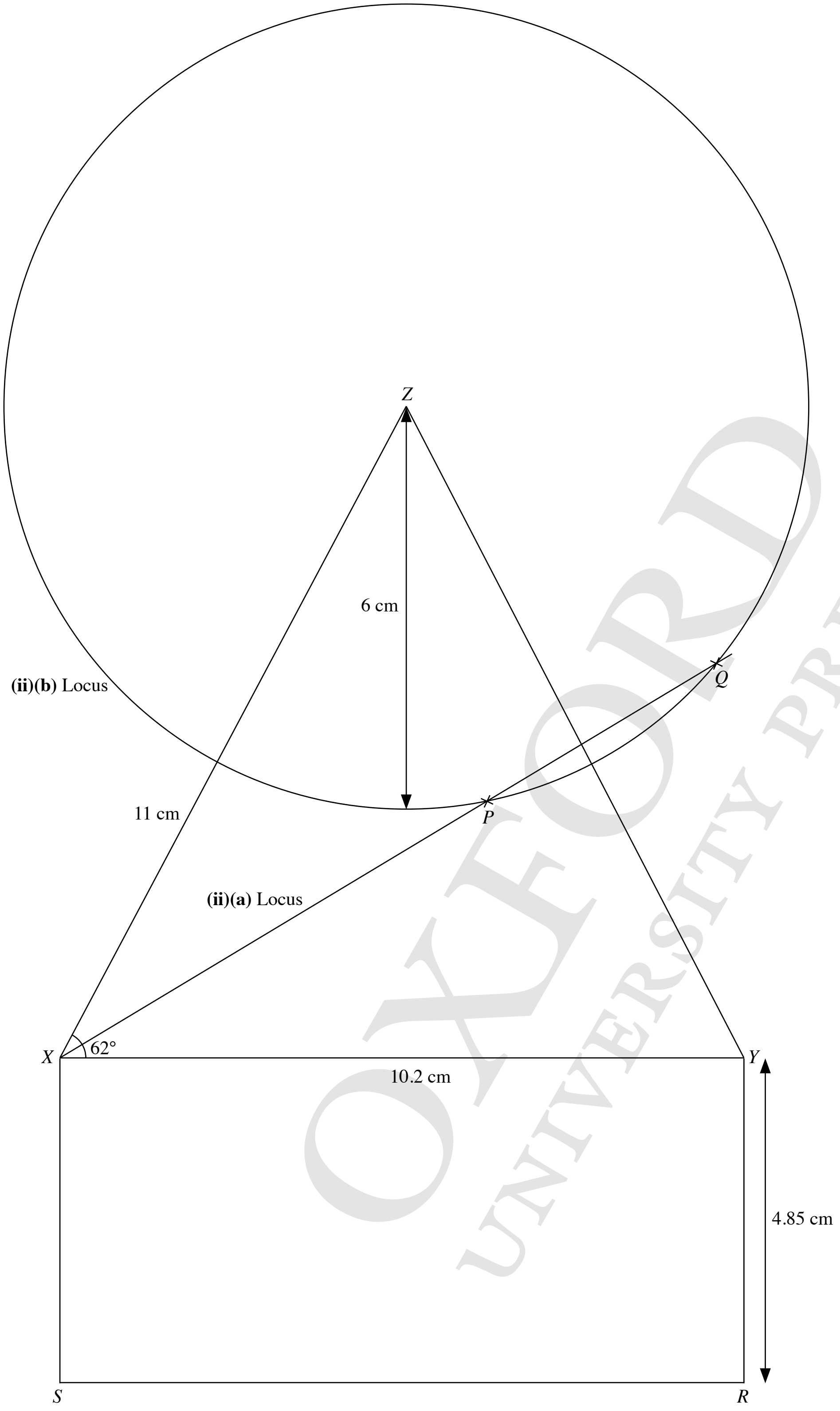
7.



(i) $\angle QRP = 109°$

OXFORD
UNIVERSITY PRESS

8.

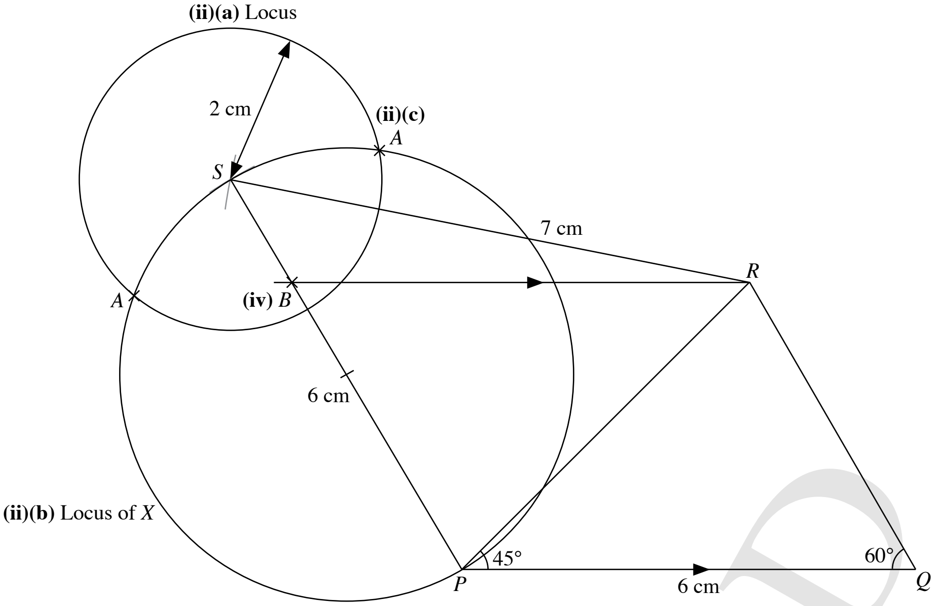


(i) $YZ = 10.9$ cm

(iii) $PQ = 4$ cm

OXFORD
UNIVERSITY PRESS

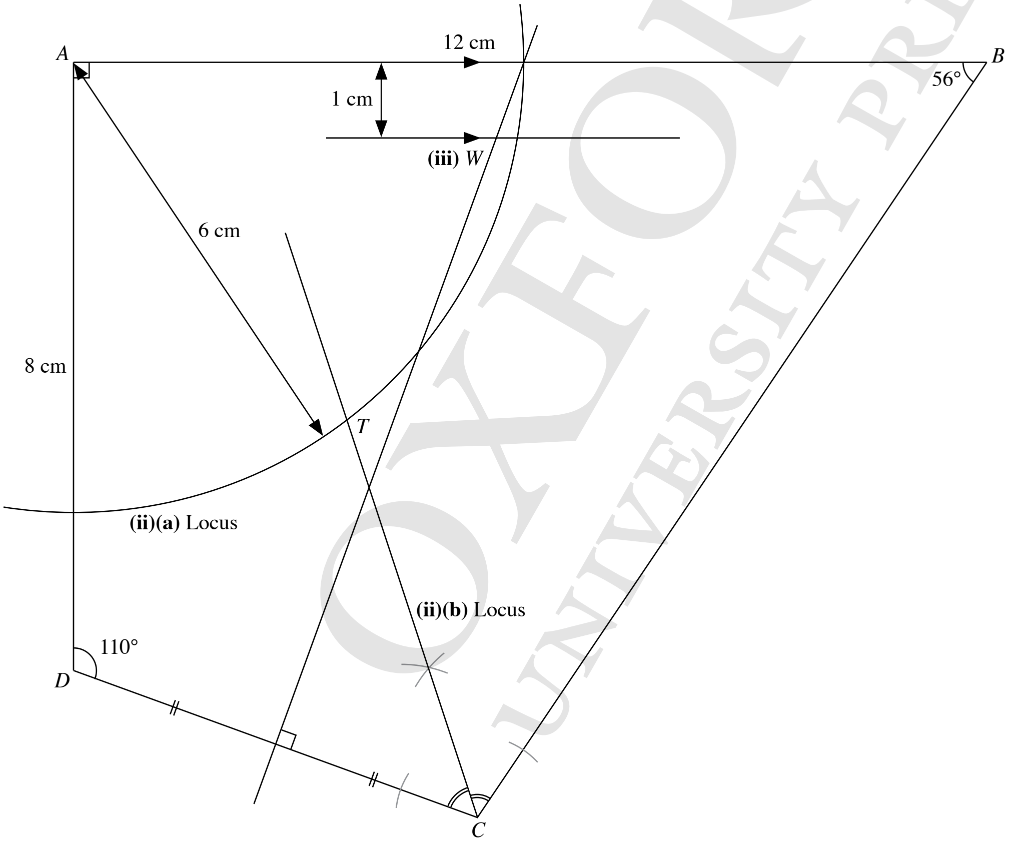
9.



(iii) $PA = 5.7$ cm

(v) $PB = 4.4$ cm

10.



(iv) From the diagram, $BT = 9.5$ cm

$\therefore$ Actual distance of tree from $B = 95$ m

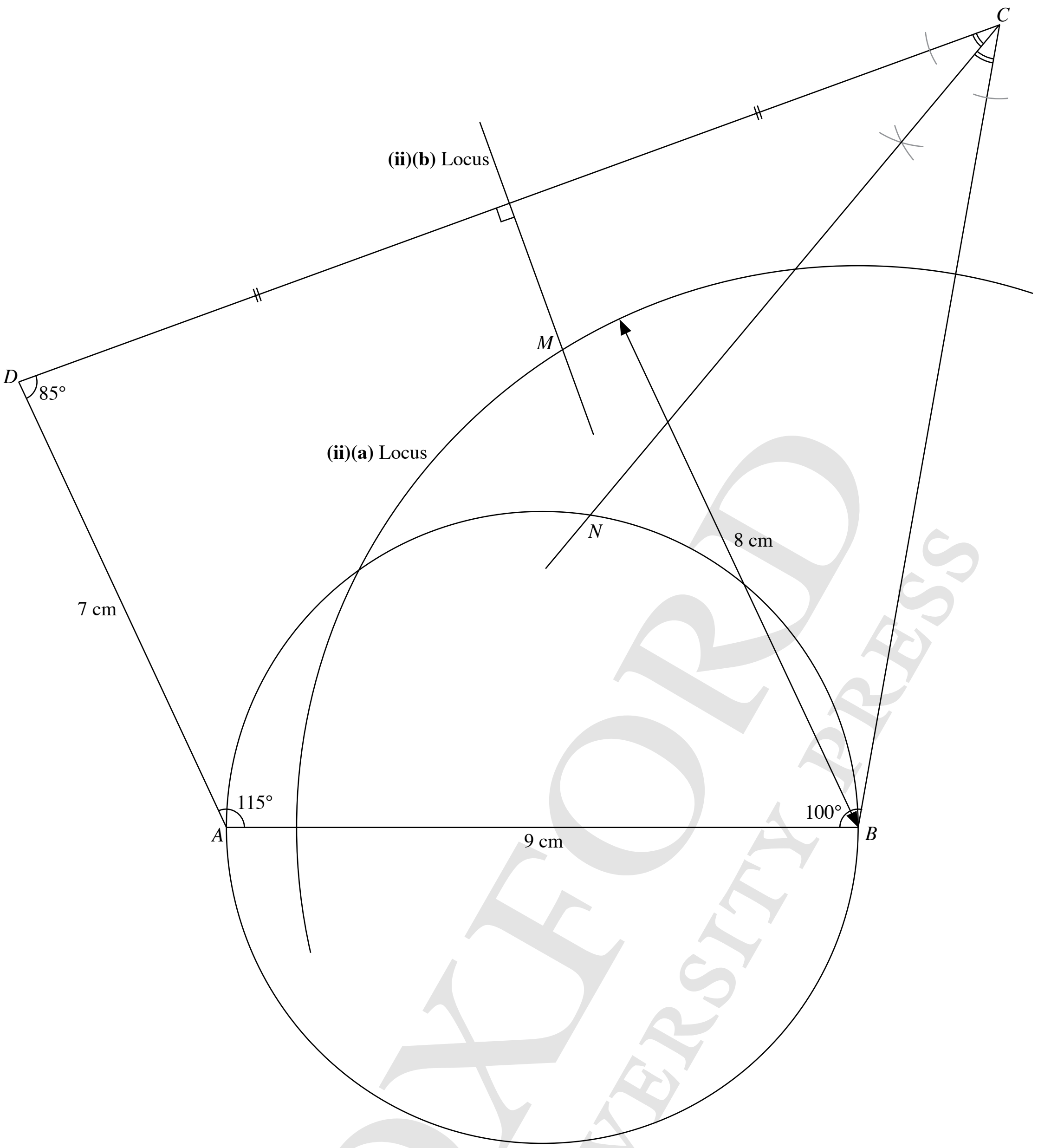
$$\tan 15° = \frac{\text{Height of tree}}{95}$$

Height of tree = 95 tan 15°

= 25 m (to the nearest m)

OXFORD UNIVERSITY PRESS

11.



(iv) From the diagram, $MN = 2.4$ cm

∴ MN on actual playground = 24 m

(v) Scale Actual

1 cm	represents	10 m
1 cm^2	represents	100 m^2
5 cm^2	represents	(5×100)
		$= 500$ m^2

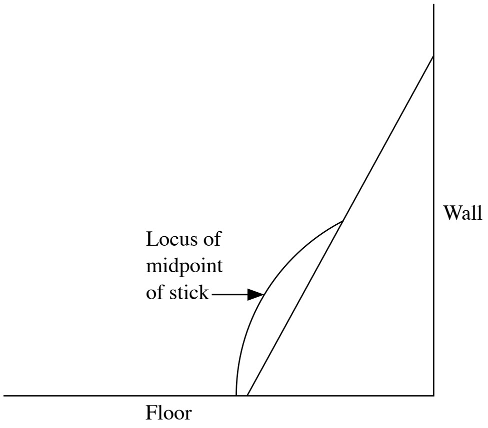
∴ Length of actual square $= \sqrt{500}$ (since length > 0)

$= 22.4$ m (to 3 s.f.)

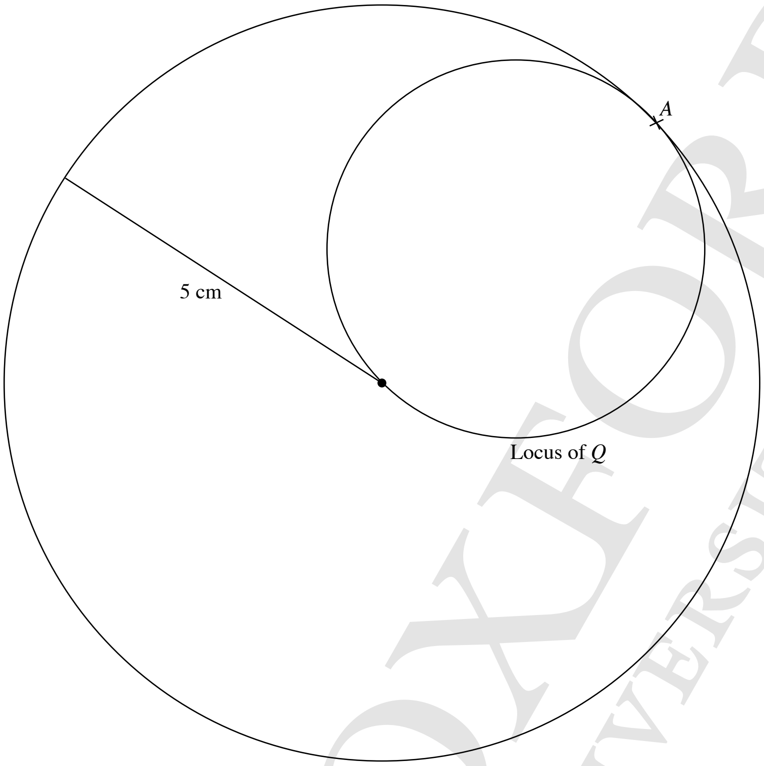
OXFORD UNIVERSITY PRESS

Challenge Yourself

1.

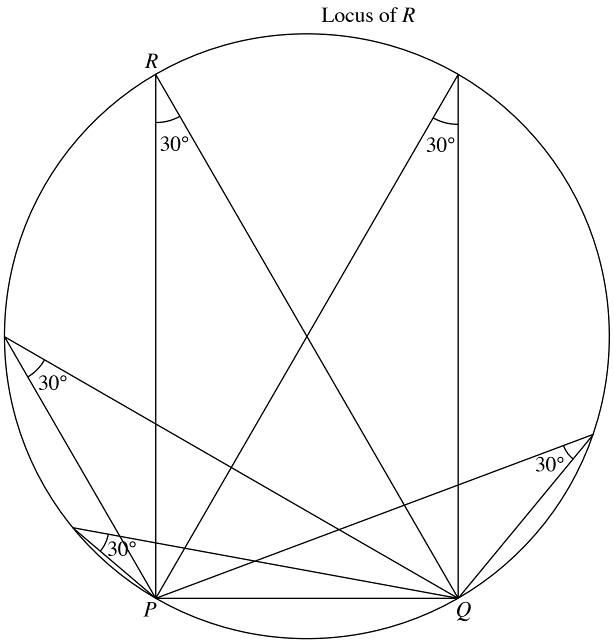


2.



OXFORD UNIVERSITY PRESS

3.



OXFORD UNIVERSITY PRESS

Chapter 9 Revision: Numbers and Algebra

Revision 9A

1. **(i)** 4.274 443 098

(ii) 4.3

2. **(a)** Least possible mass of the piece of metal = 120.5 g

(b) Greatest possible mass of 1 cubic centimetre of the piece of metal $= \dfrac{121.4}{13.5} = 8.99$ (to 3 s.f.)

3. $\tan 79° = \dfrac{AB}{20}$

$AB = 20 \times \tan 79°$

$= 103$ m (to 3 s.f.)

4. **(i)** Difference in the temperatures recorded $= 19.4 - (-89.2)$

$= 108.6$ °C

(ii) Temperature that is mid-way between the two temperatures

$= \dfrac{-89.2 + 19.4}{2}$

$= -34.9$ °C

Alternatively,

$-89.2 + \dfrac{108.6}{2}$

$= -34.9$ °C

5. Fraction of her weekly allowance that is left $= 1 - \dfrac{2}{3} - \left(\dfrac{1}{4} \times \dfrac{1}{3}\right)$

$= \dfrac{1}{4}$

$\dfrac{1}{4}$ of her weekly allowance is $27.

$\therefore$ Her weekly allowance is $27 \times 4 = \$108$

6. **(a)** $792 = 2^3 \times 3^2 \times 11$

(b) $4 = 1 \times 4 = 2 \times 2$

$15 = 1 \times 15 = 3 \times 5$

$4 \times 15 = 60$

60 has 12 factors 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60

$\therefore p = 60$

7. **(a)** $525 = 3 \times 5^2 \times 7$

(b) LCM of 15, x and 35 = 525

Two possible values of x:

$5^2 = 25$, $3 \times 5^2 = 75$

(c) $525k = 3 \times 5^2 \times 7 \times k$

$\therefore k = 3 \times 7 = 21$

8. **(a)** Square root of $a = \sqrt{2^4 \times 3^6}$

$= (2^4 \times 3^6)^{\frac{1}{2}}$

$= 2^2 \times 3^3$

(b) LCM $= 2^4 \times 3^6 \times 5 \times 7$

(c) Greatest number $= 2 \times 3$

9. LCM of 3, 4 and 6 $= 2^2 \times 3$

= 12 hours

$\therefore$ 09 00 + 12 hours = 21 00

10. **(a)** Total cost of the graduation dinner $\approx 400 \times 70$

$= \$28\,000$

(b) Total file size $= \dfrac{680 \times 2.5 \times 10^6}{10^9}$

= 1.7 gigabytes

11. **(i)** $\dfrac{100 - 22.5}{100} \times$ original price = $139.50

$\therefore$ Original price $= \dfrac{139.50 \times 100}{100 - 22.5}$

= $180

(ii) Amount of GST payable $= \dfrac{180}{107} \times 7$

$\approx \$11.78$

12. **(i)** 985 billion $= 985 \times 10^9$

$= 9.85 \times 10^{11}$

(ii) Total trade in 2013 $= \left(985 \times \dfrac{100 - 0.5}{100}\right)$ billion

= $980 billion

13. Radius $= (50 \times 10^{-12})$ m

$= (5 \times 10^{-11})$ m

14. **(i)** $2P - Q = 2(7.8 \times 10^5) - (3.9 \times 10^3)$

$= 1.5561 \times 10^6$

(ii) $0.3(P + 4Q) = 0.3[(7.8 \times 10^5) + 4(3.9 \times 10^3)]$

$= 2.3868 \times 10^5$

(iii) $\dfrac{P}{Q} = \dfrac{7.8 \times 10^5}{3.9 \times 10^3}$

$= 2 \times 10^2$

(iv) $(2PQ)^{-\frac{1}{3}} = [2(7.8 \times 10^5)(3.9 \times 10^3)]^{-\frac{1}{3}}$

$= 5.48 \times 10^{-4}$

15. **(i)** $11^7 \div 11^{-1} = 11^8$

(ii) $\dfrac{1}{121^3} = \dfrac{1}{11^6}$

$= 11^{-6}$

(iii) $\sqrt[5]{11} = 11^{\frac{1}{5}}$

16. **(a)** $36^c \times 2 = 12$

$6^{2c} = 6$

$2c = 1$

$c = \dfrac{1}{2}$

(b) $1 \div 2x^{-5} = 1 \div \dfrac{2}{x^5}$

$= 1 \times \dfrac{x^5}{2}$

$= \dfrac{x^5}{2}$

(c) $8^2 \div 16^{\frac{3}{4}} = \dfrac{1}{2^m}$

$2^6 \div 2^3 = 2^{-m}$

$2^{6-3} = 2^{-m}$

$m = -3$

OXFORD
UNIVERSITY PRESS

(d) $10^n = \dfrac{10^5 \times 10}{(10^2)^3}$

$10^n = \dfrac{10^6}{10^6}$

$10^n = 10^{6-6}$

$n = 0$

17. (i) $4\ 200\ 000\ \text{m}^3 = 4.2 \times 10^6\ \text{m}^3$

$= 4.2 \times 10^6 \times (10^2)^3\ \text{cm}^3$

$= 4.2 \times 10^{12}\ \text{cm}^3$

(ii) Storage capacity of Bedok Reservoir as a percentage of that of MacRitchie Reservoir

$= \dfrac{12.8 \times 10^{12}}{4.2 \times 10^{12}} \times 100\%$

$= 305\%$ (to 3 s.f.)

18. (a) $2.38 \times 10^8 = 238 \times 10^6$

$= 238$ million

$\therefore k = 238$

(b) (i) Number of times $= \dfrac{90.5 \times 10^6}{3.45 \times 10^7}$

$= 2.62$ (to 3 s.f.)

(ii) Average number of mobile phones per person in Indonesia

$= \dfrac{2.37 \times 10^8}{2.38 \times 10^8}$

$= 0.996$ (to 3 s.f.)

19. (a) Let the original rice consumption be C and new rice consumption be C_1.

$C = \dfrac{125}{100} \times C_1$

$C_1 = C \times \dfrac{4}{5}$

$C_1 = 0.8C$

$\therefore$ Percentage of rice consumption to be decreased is 20%.

(b) Let the length and breadth of the original rectangle be l and b respectively.

Area of the new rectangle $= \dfrac{120}{100} l \times \dfrac{80}{100} b$

$= 0.96lb$

Percentage change in its area $= \dfrac{0.96lb - lb}{lb} \times 100\%$

$= -4\%$

20. Commission charged by Chan and Partners

$= \dfrac{5}{100} \times 200\ 000 + \dfrac{1.5}{100} \times (2\ 800\ 000 - 200\ 000)$

$= \$49\ 000$

Commission charged by Lim and Partners

$= \dfrac{3.5}{100} \times 200\ 000 + \dfrac{2.5}{100} \times (2\ 800\ 000 - 200\ 000)$

$= \$72\ 000$

$\therefore$ Mr Koh should engage Chan and Partners to sell his condominium because the commission charged is less.

Revision 9B

1. Number of days the same amount of food will feed all the cows

$= \dfrac{30 \times 20}{40}$

$= 15$

2. 4 workers can manufacture 80 chairs in 12 days.

4 workers can manufacture 80 × 3.75 chairs in 12 × 3.75 days

= 45 days.

$4 \times 6 = 24$ workers can manufacture 300 chairs in $\dfrac{45}{6} = 7.5$ days.

Number of days required = 7.5

3. Let x be the number of postcards that Ethan has.

Then Michael has $4x$ postcards and Nora has $2.5 \times 4x = 10x$ postcards.

$x + 4x + 10x = 180$

$15x = 180$

$x = 12$

Ethan has 12 postcards, Michael has 48 postcards and Nora has 120 postcards.

4. Let $\$x$ be the amount of 1 unit of money.

Then Jun Wei has $\$3x$, Amirah has $\$4x$ and Shirley has $\$5x$.

After Shirley gives Jun Wei \$30, Jun Wei has $\$(3x + 30)$, Amirah has $\$4x$ and Shirley has $\$(5x - 30)$.

$\dfrac{3x + 30}{5x - 30} = \dfrac{9}{11}$

$11(3x + 30) = 9(5x - 30)$

$33x - 45x = -270 - 330$

$12x = 600$

$x = 50$

Jun Wei has $[3(5) + 30] = \$180$ now.

Alternatively,

Let $\$y$ be the sum of money.

Jun Wei has $\$\left(\dfrac{3}{12}x\right)$ originally and $\$\left(\dfrac{9}{30}x\right)$ after Shirley gives him \$30.

$\dfrac{3}{12}x + 30 = \dfrac{9}{30}x$

$x = 600$

Amount of money Jun Wei has now

$= \dfrac{9}{30}(600)$

$= \$180$

5. Interest earned for \$800 $= \dfrac{800 \times 1.2 \times 2}{100} = \$19.20$

Interest earned for \$700 $= \dfrac{700 \times 1.2 \times 1}{100} = \$8.40$

Total amount she has in the bank at the end of 2015

$= \$(800 + 700 + 19.20 + 8.40)$

$= \$1527.60$

6 (i) Amount received = NZ\$(1800 × 0.96)

= NZ\$1728

(ii) Amount received = S\$ $\dfrac{350}{0.98}$

≈ S\$357.14

OXFORD UNIVERSITY PRESS

7. Chargeable income

$$= \$[220\,000 - 3000 - 3(4000) - 2(5000) - 28\,500]$$

$$= \$166\,500$$

$$\text{Total tax payable} = \$13\,950 + \$\left(\frac{17}{100} \times 6500\right)$$

$$= \$15\,055$$

8. $y = kx^n$, where k is a constant

(i) $y = \frac{4}{3}\pi x^3$

$n = 3$

(ii) $y = \frac{d}{x}$

$= dx^{-1}$

$n = -1$

9. $y = \frac{k}{x}$, where k is a constant

When $x = 8$, $y = 2.5$

$2.5 = \frac{k}{8}$

$k = 20$

$y = \frac{20}{x}$

When $x = 10$, $y = \frac{20}{10}$

$= 2$

When $y = 0.8$, $0.8 = \frac{20}{x}$

$x = 25$

x	8	10	25
y	2.5	2	0.8

10. **(a)** **(i)** $V = kr^2$, where k is a constant

When $r = 5$, $V = 240$,

$240 = k(5)^2$

$k = \frac{240}{25}$

$k = 9.6$

$\therefore V = 9.6r^2$

(ii) When $r = 8$, $V = 9.6(8)^2$

$= 614$ cm^3 (to the nearest whole number)

(b) Let the radius of cone P be r_P and the radius of cone Q be r_Q.

$$\frac{\text{Volume of cone } P}{\text{Volume of cone } Q}$$

$$= \frac{9.6r_P^2}{9.6r_Q^2}$$

$$= \frac{9.6(1.25r_Q)^2}{9.6r_Q^2}$$

$$= \frac{25}{16}$$

11. $y = kx^2$, where k is a constant

When $x = 2$, $y = 4k$

When $x = 5$, $y = 25k$

$25k - 4k = 32$

$21k = 32$

$k = \frac{32}{21}$

$\therefore y = \frac{32}{21}x^2$

When $x = 3$, $y = \frac{32}{21}(3^2) = 13\frac{5}{7}$

12. **(a)** $F = \frac{k}{d^2}$, where k is a constant

When $F = 10$, $d = 8$

$10 = \frac{k}{8^2}$

$k = 640$

$F = \frac{640}{d^2}$

(b) When $F = 25$,

$25 = \frac{640}{d^2}$

$d^2 = \frac{640}{25}$

$d = 5.06$ cm (to 3 s.f.)

(c) Let the distance be x when $F = 12$.

$12 = \frac{640}{x^2}$

$x^2 = \frac{640}{12}$

When this distance is doubled, i.e. $d = 2x$, then

$$F = \frac{640}{(2x)^2}$$

$$= \frac{640}{4x^2}$$

$$= \frac{160}{x^2}$$

Replace x^2 by $\frac{640}{12}$.

$$F = \frac{160}{\frac{640}{12}}$$

$$= 160 \times \frac{12}{640}$$

$= 3$ N

13. **(a)** 1: 20 000

20 000 cm = 200 m

= 0.2 km

$n = 0.2$

(b) 1 cm represents 0.2 km.

18 cm represents 0.2 × 18 = 3.6 km.

$\therefore$ Actual distance between the two police stations is 3.6 km.

(c) 1 cm^2 represents 0.2^2 km^2, i.e.

0.04 km^2 is represented by 1 cm^2.

1 km^2 is represented by $\frac{1}{0.04} = 25$ cm^2.

3.2 km^2 is represented by $25 \times 3.2 = 80$ cm^2.

$\therefore$ Area of the nature reserve on the map is 80 cm^2.

OXFORD UNIVERSITY PRESS

14. **(a)** 1 cm to 4 km

6 cm to $4 \times 6 = 24$ km

$\therefore$ Actual distance between the university and polytechnic is 24 km.

(b) 1 cm^2 represents 4^2 km^2.

40 cm^2 represents $16 \times 40 = 640$ km^2.

Hence, on another map,

640 km^2 is represented by 0.4 cm^2, i.e.

0.4 cm^2 represents 640 km^2.

1 cm^2 represents $\dfrac{640}{0.4} = 1600$ km^2.

1 cm represents $\sqrt{1600} = 40$ km = 4 000 000 cm.

$n = 4\,000\,000$

15. **(a)** 1: 20

1 cm represents 20 cm.

24 cm represents $20 \times 24 = 480$ cm.

$\therefore$ Length of the actual lorry is 480 cm.

(b) 1 cm^2 represents 20^2 cm^2

$= 400$ cm^2

$= 0.04$ m^2,

i.e. 0.04 m^2 is represented by 1 cm^2.

1 m^2 is represented by $\dfrac{1}{0.04} = 25$ cm^2.

10 m^2 is represented by $25 \times 10 = 250$ cm^2.

$\therefore$ Area of the load platform of the model lorry is 250 cm^2.

(c) 1 cm^3 represents $20^3 = 8000$ cm^3.

30 cm^3 represents $8000 \times 30 = 240\,000$ cm^3 = 240 litres.

$\therefore$ Volume of the fuel that the actual lorry can hold is 240 litres.

16. **(i)** $700 \text{ km/h} = \dfrac{(700 \times 1000) \text{ m}}{(60 \times 60) \text{ s}}$

$= 194\frac{4}{9}$ m/s

(ii) Flight time $= \dfrac{3240}{700}$

$= 4\frac{22}{35}$ h

= 4 h 38 min (correct to the nearest minute)

17. Distance made by one revolution of the wheel

$= 3.142 \times 58.4$

$= 183.4928$ cm

Distance moved in 1 minute $= \dfrac{25 \times 1000 \times 100}{60}$ cm

Number of revolutions made by the wheel per minute

$= \dfrac{25 \times 1000 \times 100}{60} \div 183.4928$

= 227 (to the nearest whole number)

18. Distance that is still not covered $= 117 - 57$

$= 60$ km

Remaining time $= 6.5 - 2\frac{3}{4}$

$= 3\frac{3}{4}$ h

Average speed required $= \dfrac{60}{3\frac{3}{4}}$

$= 16$ km/h

Speed used for the first part of the journey $= \dfrac{57}{2\frac{3}{4}}$

$= 20\frac{8}{11}$ km/h

Amount of speed to reduce $= 20\frac{8}{11} - 16 = 4\frac{8}{11}$ km/h

19. **(i)** Distance between Burnie and Devonport

$= 2x + 1.5(x - 3)$

$= (3.5x - 4.5)$ km

(ii) Distance between Burnie and Devonport $= 3(x + 1)$ km

(iii) $3.5x - 4.5 = 3(x + 1)$

$3.5x - 3x = 3 + 4.5$

$0.5x = 7.5$

$x = 15$

Distance between Burnie and Devonport $= 3(15 + 1) = 48$ km

(iv) Total distance $= 48 \times 2$

$= 96$ km

Total time $= 3.5 + 3 + 0.5$

$= 7$ h

Average speed of the return journey $= \dfrac{96}{7}$

$= 13\frac{5}{7}$ km/h

20. **(i)** $298\,000 \text{ km/h} = \dfrac{(298\,000 \times 1000) \text{ m}}{(60 \times 60) \text{ s}}$

$= 8.28 \times 10^4$ m/s (to 3 s.f.)

(ii) Difference in the distance transmitted $= 2 \times 372.5$

$= 745$ m

Difference in the times between the two signals

$= \dfrac{745}{8.2778 \times 10^4}$

$= 9.00 \times 10^{-3}$ s (to 3 s.f.)

Revision 9C

1. **(a)** $3(2x - 1) - 4(x - 7) = 6x - 3 - 4x + 28$

$= 2x + 25$

(b) $14 - 3(5 - 4x) + 6x = 14 - 15 + 12x + 6x$

$= 18x - 1$

(c) $7(2y + 3) - 4(3 - y) = 14y + 21 - 12 + 4y$

$= 18y + 9$

(d) $9(5p - 6) + 4(7 - 13p) = 45p - 54 + 28 - 52p$

$= -7p - 26$

(e) $5 - 3(q + r) - 6(3r - 2q) = 5 - 3q - 3r - 18r + 12q$

$= 9q - 21r + 5$

(f) $(a + 2b)^2 - (a - 2b)^2$

$= a^2 + 2(a)(2b) + (2b)^2 - [a^2 - 2(a)(2b) + (-2b)^2]$

$= a^2 + 4ab + 4b^2 - [a^2 - 4ab + 4b^2]$

$= a^2 + 4ab + 4b^2 - a^2 + 4ab - 4b^2$

$= 8ab$

OXFORD UNIVERSITY PRESS

2. **(a)** $5x^2 - 20x^2y = 5x^2(1 - 4y)$

(b) $x^2 - 4xy + 4y^2 = x^2 + 2(x)(-2y) + (-2y)^2$

$= (x - 2y)^2$

(c) $(3x + 4y)^2 - 9z^2 = (3x + 4y)^2 - (3z)^2$

$= (3x + 4y + 3z)(3x + 4y - 3z)$

(d) $6x^2 - 31x + 35 = (3x - 5)(2x - 7)$

(e) $5p^2 + 11p + 2 = (5p + 1)(p + 2)$

3. **(a)** $2[3a - 2(3a - 1) + 4(a + 1)] = 2(3a - 6a + 2 + 4a + 4)$

$= 2(a + 6)$

$= 2a + 12$

(b) $8(x - y) - [x - y - 3(y - z - x)]$

$= 8x - 8y - (x - y - 3y + 3z + 3x)$

$= 8x - 8y - (4x - 4y + 3z)$

$= 8x - 8y - 4x + 4y - 3z$

$= 4x - 4y - 3z$

(c) $2b(c - a) - [3c(a - b) - 3a(b + c)]$

$= 2bc - 2ab - (3ac - 3bc - 3ab - 3ac)$

$= 2bc - 2ab - 3ac + 3bc + 3ab + 3ac$

$= ab + 5bc$

(d) $3(a - c) - \{5(2a - 3b) - [5a - 7(a - b)]\}$

$= 3a - 3c - [10a - 15b - (5a - 7a + 7b)]$

$= 3a - 3c - [10a - 15b - (-2a + 7b)]$

$= 3a - 3c - (10a - 15b + 2a - 7b)$

$= 3a - 3c - (12a - 22b)$

$= 3a - 3c - 12a + 22b$

$= -9a + 22b - 3c$

4. **(a)** $\dfrac{25a^2}{b^2c} \times \dfrac{bc^3}{100a^3} \div \dfrac{15}{c^2} = \dfrac{25a^2}{b^2c} \times \dfrac{bc^3}{100a^3} \times \dfrac{c^2}{15}$

$= \dfrac{25a^2bc^5}{1500a^3b^2c}$

$= \dfrac{c^4}{60ab}$

(b) $\dfrac{8a^5b^2c}{(-2ab)^2} = \dfrac{8a^5b^2c}{4a^2b^2}$

$= 2a^3c$

(c) $\dfrac{2a^2b^3}{3b} \div \dfrac{(2a)^2}{15ab^2} = \dfrac{2a^2b^3}{3b} \times \dfrac{15ab^2}{4a^2}$

$= \dfrac{30a^3b^5}{12a^2b}$

$= \dfrac{5ab^4}{2}$

(d) $\dfrac{a-1}{a-b} \div \dfrac{1-a}{a^2-b^2} = \dfrac{a-1}{a-b} \times \dfrac{a^2-b^2}{1-a}$

$= \dfrac{a-1}{a-b} \times \dfrac{(a+b)(a-b)}{-(a-1)}$

$= -(a + b)$

$= -a - b$

(e) $\dfrac{2x^2+11x+15}{x^2-9} = \dfrac{(2x+5)(x+3)}{(x+3)(x-3)}$

$= \dfrac{2x+5}{x-3}$

5. **(a)** $x^2 + 3y + xy + 3x = x^2 + xy + 3x + 3y$

$= x(x + y) + 3(x + y)$

$= (x + y)(x + 3)$

(b) $ab - bc - ac + c^2 = b(a - c) - c(a - c)$

$= (a - c)(b - c)$

(c) $ax - kx - ah + kh = x(a - k) - h(a - k)$

$= (a - k)(x - h)$

(d) $20ac - 4ad - 15kc + 3kd = 4a(5c - d) - 3k(5c - d)$

$= (5c - d)(4a - 3k)$

(e) $6a^2 + 3ab - 8ka - 4kb = 3a(2a + b) - 4k(2a + b)$

$= (2a + b)(3a - 4k)$

6. **(a)** $ax^2 + bx + c = 0$

$bx = -ax^2 - c$

$b = \dfrac{-ax^2 - c}{x}$

$= -\dfrac{ax^2 + c}{x}$

(b) $\dfrac{1}{a} + \dfrac{b}{2} + \dfrac{3}{c} = k$

$\dfrac{3}{c} = k - \dfrac{1}{a} - \dfrac{b}{2}$

$\dfrac{3}{c} = \dfrac{2ak - 2 - ab}{2a}$

$\dfrac{c}{3} = \dfrac{2a}{2ak - 2 - ab}$

$c = \dfrac{6a}{2ak - 2 - ab}$

(c) $\sqrt{4x^2 - 5k} = 2x + 3$

$4x^2 - 5k = (2x + 3)^2$

$4x^2 - 5k = 4x^2 + 12x + 9$

$-5k = 12x + 9$

$12x = -5k - 9$

$x = \dfrac{-5k - 9}{12}$

$= -\dfrac{5k + 9}{12}$

(d) $v^2 = u^2 + 2as$

$u^2 = v^2 - 2as$

$u = \pm\sqrt{v^2 - 2as}$

(e) $x = \sqrt[3]{\dfrac{a}{b - a}}$

$x^3 = \dfrac{a}{b - a}$

$x^3(b - a) = a$

$bx^3 - ax^3 = a$

$a + ax^3 = bx^3$

$a(1 + x^3) = bx^3$

$a = \dfrac{bx^3}{1 + x^3}$

7. **(a)** $\dfrac{3}{4} + \dfrac{x - 3}{2x} = \dfrac{3x + 2(x - 3)}{4x}$

$= \dfrac{3x + 2x - 6}{4x}$

$= \dfrac{5x - 6}{4x}$

OXFORD UNIVERSITY PRESS

(b) $$\frac{2y+3}{9y^2-1}-\frac{5}{3y-1}=\frac{2y+3}{(3y+1)(3y-1)}-\frac{5}{3y-1}$$
$$=\frac{2y+3-5(3y+1)}{(3y+1)(3y-1)}$$
$$=\frac{2y+3-15y-5}{(3y+1)(3y-1)}$$
$$=\frac{-13y-2}{(3y+1)(3y-1)}$$
$$=-\frac{13y+2}{(3y+1)(3y-1)}$$

(c) $$\frac{3a}{a-3}+\frac{2}{a+4}=\frac{3a(a+4)+2(a-3)}{(a-3)(a+4)}$$
$$=\frac{3a^2+12a+2a-6}{(a-3)(a+4)}$$
$$=\frac{3a^2+14a-6}{(a-3)(a+4)}$$

(d) $$\frac{2}{p^2+4p-5}-\frac{1}{p-1}=\frac{2}{(p+5)(p-1)}-\frac{1}{p-1}$$
$$=\frac{2-(p+5)}{(p+5)(p-1)}$$
$$=\frac{-p-3}{(p+5)(p-1)}$$
$$=-\frac{p+3}{(p+5)(p-1)}$$

(e) $$\frac{5x}{2x-y}+\frac{y}{3x-y}=\frac{5x(3x-y)+y(2x-y)}{(2x-y)(3x-y)}$$
$$=\frac{15x^2-5xy+2xy-y^2}{(2x-y)(3x-y)}$$
$$=\frac{15x^2-3xy-y^2}{(2x-y)(3x-y)}$$

8. **(a)** $$\frac{1}{1-x}+\frac{2}{1+x}+\frac{2x}{x^2-1}=\frac{-1}{x-1}+\frac{2}{x+1}+\frac{2x}{(x+1)(x-1)}$$
$$=\frac{-(1+x)+2(x-1)+2x}{(x+1)(x-1)}$$
$$=\frac{-1-x+2x-2+2x}{(x+1)(x-1)}$$
$$=\frac{3x-3}{(x+1)(x-1)}$$
$$=\frac{3(x-1)}{(x+1)(x-1)}$$
$$=\frac{3}{x+1}$$

(b) $$\frac{3}{x+2}-\frac{x-5}{x^2-4}+\frac{1}{x-2}=\frac{3}{x+2}-\frac{x-5}{(x+2)(x-2)}+\frac{1}{x-2}$$
$$=\frac{3(x-2)-(x-5)+(x+2)}{(x+2)(x-2)}$$
$$=\frac{3x-6-x+5+x+2}{(x+2)(x-2)}$$
$$=\frac{3x+1}{(x+2)(x-2)}$$

(c) $$\frac{1}{2x-3}-\frac{2}{x+2}-\frac{2x-x^2}{2x^2+x-6}$$
$$=\frac{1}{2x-3}-\frac{2}{x+2}-\frac{2x-x^2}{(2x-3)(x+2)}$$
$$=\frac{(x+2)-2(2x-3)-(2x-x^2)}{(2x-3)(x+2)}$$
$$=\frac{x+2-4x+6-2x+x^2}{(2x-3)(x+2)}$$
$$=\frac{x^2-5x+8}{(2x-3)(x+2)}$$

(d) $$\frac{5}{x-2}-\frac{3x+x^2}{x^2-x-2}+\frac{x}{x+1}$$
$$=\frac{5}{x-2}-\frac{3x+x^2}{(x-2)(x+1)}+\frac{x}{x+1}$$
$$=\frac{5(x+1)-(3x+x^2)+x(x-2)}{(x-2)(x+1)}$$
$$=\frac{5x+5-3x-x^2+x^2-2x}{(x-2)(x+1)}$$
$$=\frac{5}{(x-2)(x+1)}$$

9. **(i)** $$\frac{3}{a}=\frac{2}{b}+\frac{1}{c}$$
$$\frac{2}{b}=\frac{3}{a}-\frac{1}{c}$$
$$\frac{2}{b}=\frac{3c-a}{ac}$$
$$\frac{b}{2}=\frac{ac}{3c-a}$$
$$b=\frac{2ac}{3c-a}$$

(ii) When $a=2\frac{1}{5}$ and $c=-3$,

$$b=\frac{2\left(2\frac{1}{5}\right)(-3)}{3(-3)-2\frac{1}{5}}$$
$$=1\frac{5}{28}$$

10. **(a)** $$V=\frac{b^2}{2}(a+3h)$$
$$\frac{2V}{b^2}=a+3h$$
$$3h=\frac{2V}{b^2}-a$$
$$3h=\frac{2V-ab^2}{b^2}$$
$$h=\frac{2V-ab^2}{3b^2}$$

(b) **(i)** When $h = 1.5$ m, $a = 0.2$ and $b = 1.2$,

$$V=\frac{1.2^2}{2}[0.2+3(1.5)]$$
$$=3.384$$

(ii) When $V = 12\text{ m}^3$, $a = 2.5$ and $b = 0.4$,

$$h=\frac{2(12)-(2.5)(0.4)^2}{3(0.4)^2}$$
$$=49.2 \text{ (to 3 s.f.)}$$

OXFORD UNIVERSITY PRESS

(iii) When $V = 15 \text{ m}^3$, $h = 2.4$ m and $a = 0.25$,

$$15 = \frac{b^2}{2}[0.25 + 3(2.4)]$$

$$b = \pm\sqrt{\frac{2(15)}{0.25 + 3(2.4)}}$$

$$= \pm 2.01 \text{ (to 3 s.f.)}$$

11. (i)

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\frac{y^2}{b^2} = \frac{x^2}{a^2} - 1$$

$$\frac{y^2}{b^2} = \frac{x^2 - a^2}{a^2}$$

$$y^2 = \frac{b^2(x^2 - a^2)}{a^2}$$

$$y = \pm\frac{b}{a}\sqrt{x^2 - a^2}$$

(ii) When $x = -5$, $a = 3$ and $b = 2$,

$$y = \pm\frac{2}{3}\sqrt{(-5)^2 - 3^2}$$

$$= \pm 2\frac{2}{3}$$

12. (i) Fraction of the pool $= \dfrac{1}{x}$

(ii) Fraction of the pool $= \dfrac{2}{x + 1.5}$

(iii) Fraction of the pool $= \dfrac{1}{x} + \dfrac{2}{x + 1.5}$

$$= \frac{x + 1.5 + x}{x(x + 1.5)}$$

$$= \frac{2x + 1.5}{x(x + 1.5)}$$

13. (a) $\dfrac{1}{6}, \dfrac{5}{12}, \dfrac{2}{3}, \dfrac{11}{12}, \ldots$

$= \dfrac{2}{12}, \dfrac{5}{12}, \dfrac{8}{12}, \dfrac{11}{12}, \ldots$

$\therefore$ Next two terms are $\dfrac{14}{12} = 1\dfrac{1}{6}$ and $\dfrac{17}{12} = 1\dfrac{5}{12}$.

(b) $0, -1, -8, -27, -64, \ldots$

$= 0, (-1)^3, (-2)^3, (-3)^3, (-4)^3, \ldots$

$\therefore$ n^{th} term is $-(n - 1)^3$.

14. (i) $u_5 = 5^2 - 5 = 20$

$u_6 = 6^2 - 6 = 30$

$\therefore$ Values of the next two terms of the sequence are 20 and 30.

(ii) $u_n = n^2 - n$

(iii)

$$n^2 - n = 110$$

$$n^2 - n - 110 = 0$$

$$(n - 11)(n + 10) = 0$$

$$n = 11 \text{ or } n = -10 \text{ (rejected)}$$

15. (a) Total ages of all boys and girls

$= (x + 1)q + (y + 2)q$

$= q(x + y + 3)$ years

Total ages of the boys $= (x + 1)p$ years

$\therefore$ Average age of the girls $= \dfrac{q(x + y + 3) - p(x + 1)}{y + 2}$ years

(b) Let Ethan's age be a years in 2 years' time. Then Michael's age is $2a$ years in 2 years' time.

In 5 years' time, Ethan's age will be $(a + 3)$ years and Michael's age will be $(2a + 3)$ years.

$$(a + 3) + (2a + 3) = x$$

$$3a + 6 = x$$

$$3a = x - 6$$

$$a = \frac{x - 6}{3}$$

So Michael's present age $= 2a - 2$

$$= 2\left(\frac{x - 6}{3}\right) - 2$$

$$= \frac{2(x - 6) - 6}{3}$$

$$= \frac{2x - 18}{3}$$

16. (i) $S_3 = 9 + 3 + 6 + 9 + 6 + 3 = 36 = 6^2$

$S_4 = 36 + 4 + 8 + 12 + 16 + 12 + 8 + 4 = 100 = 10^2$

$S_5 = 100 + 5 + 19 + 15 + 20 + 25 + 20 + 15 + 10 + 5 = 225 = 15^2$

(ii) $S_n = (1 + 2 + 3 + \ldots + n)^2 = \left[\dfrac{n}{2}(1 + n)\right]^2$

(iii) $S_k = (1 + 2 + 3 + \ldots + k)^2 = \left[\dfrac{k}{2}(1 + k)\right]^2 = 44\ 100$

$$\left[\frac{k}{2}(1 + k)\right]^2 = 44\ 100$$

$$\left[\frac{k}{2}(1 + k)\right]^2 = 210^2$$

$$\frac{k}{2}(1 + k) = 210$$

$$k^2 + k - 420 = 0$$

$$(k - 20)(k + 21) = 0$$

$$k = 20 \text{ or } k = -21 \text{ (rejected)}$$

17. (a) $a = 8, b = 24, c = 24, d = 8$

(b) 8

(c) $3^3 = 27$

(d) **(i)** $12(n - 2)$

(ii) $6(n - 2)^2$

(iii) $(n - 2)^3$

18. (i) $l = 60, m = 25, n = 36$

(ii) $T = S + P - 1$ or $T^2 = 4SP$

(iii) $364 = 169 + P - 1$

$\therefore P = 196$

(iv) 112 is not a perfect square.

(v) 4442 is not a multiple of 4.

19. (a) $p = 4, q = 5$

p and q are the next two terms of the sequence '0, 1, 2, 3, ...'.

(b) $r = 14, s = 20$

(c) $n = v + 3$

(d) **(i)** $2d = nv$

(ii) $2d = n(n - 3)$

$$d = \frac{n(n - 3)}{2}$$

(iii) $n = 30$, $d = \dfrac{30(27)}{2} = 405$

OXFORD UNIVERSITY PRESS

20. (i) $1024 = 2^{10}, 512 = 2^9, 256 = 2^8$

$\therefore p = 2^7 = 128$

(ii) n^{th} term $= 2^{11-n}$

(iii) k^{th} term $= 2^{11-k} = \frac{1}{4}$

$2^{11-k} = 2^{-2}$

$11 - k = -2$

$k = 13$

Revision 9D

1. (i) Largest integer value of $x = 7$

(ii) Smallest integer value of $x = -2$

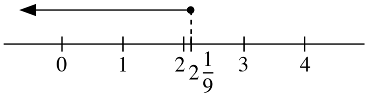
(iii) Largest prime number of $x = 7$

2. (a) $9x - 7 \leqslant 12$

$9x \leqslant 12 + 7$

$9x \leqslant 19$

$x \leqslant 2\frac{1}{9}$

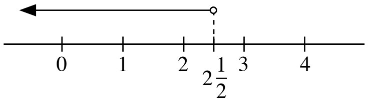


(b) $7 - 2x > 2$

$-2x > 2 - 7$

$-2x > -5$

$x < 2\frac{1}{2}$

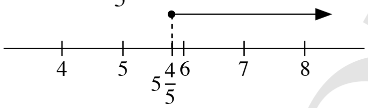


(c) $3 + 5x \geqslant 32$

$5x \geqslant 32 - 3$

$5x \geqslant 29$

$x \geqslant 5\frac{4}{5}$

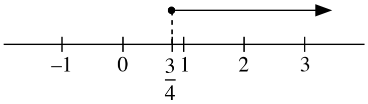


(d) $3x - 4 \geqslant \frac{1}{3}x - 2$

$3x - \frac{1}{3}x \geqslant -2 + 4$

$\frac{8}{3}x \geqslant 2$

$x \geqslant \frac{3}{4}$



(e) $12 < 3x - 1 < 27$

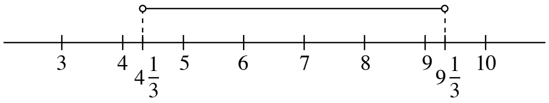
$12 < 3x - 1$ and $3x - 1 < 27$

$12 + 1 < 3x$ and $3x < 27 + 1$

$3x > 13$ and $3x < 28$

$x > 4\frac{1}{3}$ and $x < 9\frac{1}{3}$

$\therefore 4\frac{1}{3} < x < 9\frac{1}{3}$



3. (a) $x + 2y = 8$ — (1)

$3x + 2y = 12$ — (2)

(2) – (1): $3x + 2y - (x + 2y) = 12 - 8$

$3x + 2y - x - 2y = 4$

$2x = 4$

$x = 2$

Substitute $x = 2$ into (1): $2 + 2y = 8$

$2y = 8 - 2$

$2y = 6$

$y = 3$

$\therefore x = 2, y = 3$

(b) $x + y = \frac{5}{6}$ — (1)

$x - y = \frac{1}{6}$ — (2)

(1) + (2): $x + y + x - y = \frac{5}{6} + \frac{1}{6}$

$2x = 1$

$x = \frac{1}{2}$

Substitute $x = \frac{1}{2}$ into (1): $\frac{1}{2} + y = \frac{5}{6}$

$y = \frac{5}{6} - \frac{1}{2}$

$y = \frac{1}{3}$

$\therefore x = \frac{1}{2}, y = \frac{1}{3}$

(c) $3a - 2b = 1$ — (1)

$5a + 3b = -11$ — (2)

(1) × 3: $3(3a - 2b) = 3(1)$

$9a - 6b = 3$ — (3)

(2) × 2: $2(5a + 3b) = 2(-11)$

$10a + 6b = -22$ — (4)

(3) + (4): $9a - 6b + 10a + 6b = 3 + (-22)$

$19a = -19$

$a = -1$

Substitute $a = -1$ into (1): $3(-1) - 2b = 1$

$-3 - 2b = 1$

$-2b = 4$

$b = -2$

$\therefore a = -1, b = -2$

OXFORD UNIVERSITY PRESS

(d) $3p - 4q - 24 = 0$ — (1)

$5p - 6q - 38 = 0$ — (2)

From (1): $3p = 4q + 24$

$$p = \frac{4q + 24}{3} \quad \text{— (3)}$$

Substitute $p = \frac{4q + 24}{3}$ into (2):

$$5\left(\frac{4q + 24}{3}\right) - 6q - 38 = 0$$

$$6\frac{2}{3}q + 40 - 6q - 38 = 0$$

$$\frac{2}{3}q + 2 = 0$$

$$\frac{2}{3}q = -2$$

$$q = -3$$

Substitute $q = -3$ into (1): $3p - 4(-3) - 24 = 0$

$$3p + 12 - 24 = 0$$

$$3p = 12$$

$$p = 4$$

$\therefore p = 4, q = -3$

(e) $\frac{1}{4}x + \frac{3}{5}y = -4$ — (1)

$\frac{1}{5}x + \frac{1}{4}y = -\frac{9}{10}$ — (2)

$(1) \times 4: 4\left(\frac{1}{4}x + \frac{3}{5}y\right) = 4(-4)$

$$x + \frac{12}{5}y = -16 \quad \text{— (3)}$$

$(2) \times 5: 5\left(\frac{1}{5}x + \frac{1}{4}y\right) = 5\left(-\frac{9}{10}\right)$

$$x + \frac{5}{4}y = -\frac{45}{10} \quad \text{— (4)}$$

$(3) - (4): x + \frac{12}{5}y - \left(x + \frac{5}{4}y\right) = -16 - \left(-\frac{45}{10}\right)$

$$x + \frac{12}{5}y - x - \frac{5}{4}y = -16 + \frac{45}{10}$$

$$1\frac{3}{20}y = -11\frac{1}{2}$$

$$y = -10$$

Substitute $y = -10$ into (1): $\frac{1}{4}x + \frac{3}{5}(-10) = -4$

$$\frac{1}{4}x - 6 = -4$$

$$\frac{1}{4}x = 2$$

$$x = 8$$

$\therefore x = 8, y = -10$

4. **(a)**

$$(x - 5)(x + 3) = 7$$

$$(x^2 + 3x - 5x - 15) - 7 = 0$$

$$x^2 - 2x - 22 = 0$$

$a = 1, b = -2, c = -22$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-22)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{92}}{2}$$

$$x = \frac{2 + \sqrt{92}}{2} \quad \text{or} \quad x = \frac{2 - \sqrt{92}}{2}$$

$x = 5.80$ (to 3 s.f.) or $x = -3.80$ (to 3 s.f.)

(b) $(3x - 2)(2x + 7) = 0$

$3x - 2 = 0$ or $2x + 7 = 0$

$$x = \frac{2}{3} \quad \text{or} \quad x = -3\frac{1}{2}$$

(c) $2x^2 + 5x - 12 = 0$

$(2x - 3)(x + 4) = 0$

$2x - 3 = 0$ or $x + 4 = 0$

$$x = 1\frac{1}{2} \quad \text{or} \quad x = -4$$

(d) $4x^2 - 3x = 7$

$4x^2 - 3x - 7 = 0$

$(4x - 7)(x + 1) = 0$

$4x - 7 = 0$ or $x + 1 = 0$

$$x = 1\frac{3}{4} \quad \text{or} \quad x = -1$$

(e) $7x^2 + 15x - 3 = 0$

$a = 7, b = 15, c = -3$

$$x = \frac{-15 \pm \sqrt{15^2 - 4(7)(-3)}}{2(7)}$$

$$x = \frac{-15 \pm \sqrt{309}}{14}$$

$$x = \frac{-15 + \sqrt{309}}{14} \quad \text{or} \quad x = \frac{-15 - \sqrt{309}}{14}$$

$x = 0.184$ (to 3 s.f.) or $x = -2.33$ (to 3 s.f.)

5. **(a)** $3x(x + 4) + 28(x + 2) + 21 = 0$

$$3x^2 + 12x + 28x + 56 + 21 = 0$$

$$3x^2 + 40x + 77 = 0$$

$$(3x + 7)(x + 11) = 0$$

$3x + 7 = 0$ or $x + 11 = 0$

$$x = -2\frac{1}{3} \quad \text{or} \quad x = -11$$

(b)

$$2(4x^2 + 23x) = 105$$

$$8x^2 + 46x = 105$$

$$8x^2 + 46x - 105 = 0$$

$(4x - 7)(2x + 15) = 0$

$4x - 7 = 0$ or $2x + 15 = 0$

$$x = 1\frac{3}{4} \quad \text{or} \quad x = -7\frac{1}{2}$$

OXFORD UNIVERSITY PRESS

(c) $(x-1)^2 - 16 = 0$

$[(x-1)+4][(x-1)-4] = 0$

$(x+3)(x-5) = 0$

$x + 3 = 0$ or $x - 5 = 0$

$x = -3$ or $x = 5$

(d) $\frac{5}{x-1} = 3 + \frac{4}{x}$

$5x = 3x(x-1) + 4(x-1)$

$5x = 3x^2 - 3x + 4x - 4$

$3x^2 - 4x - 4 = 0$

$(3x+2)(x-2) = 0$

$3x + 2 = 0$ or $x - 2 = 0$

$x = -\frac{2}{3}$ or $x = 2$

(e) $\frac{x}{1+x} + \frac{x+1}{1-3x} = \frac{1}{4}$

$\frac{x(1-3x)+(x+1)(1+x)}{(1+x)(1-3x)} = \frac{1}{4}$

$\frac{x-3x^2+x^2+2x+1}{(1+x)(1-3x)} = \frac{1}{4}$

$4(3x - 2x^2 + 1) = (1+x)(1-3x)$

$12x - 8x^2 + 4 = 1 - 3x + x - 3x^2$

$5x^2 - 14x - 3 = 0$

$(5x+1)(x-3) = 0$

$5x + 1 = 0$ or $x - 3 = 0$

$x = -\frac{1}{5}$ or $x = 3$

(f) $\frac{2}{3x} = \frac{x-2}{4(x+3)}$

$8(x+3) = 3x(x-2)$

$8x + 24 = 3x^2 - 6x$

$3x^2 - 14x - 24 = 0$

$(3x+4)(x-6) = 0$

$3x + 4 = 0$ or $x - 6 = 0$

$x = -1\frac{1}{3}$ or $x = 6$

(g) $\frac{5}{x+2} - \frac{5}{x^2-4} = 0$

$\frac{5}{x+2} - \frac{5}{(x+2)(x-2)} = 0$

$\frac{5(x-2)-5}{(x+2)(x-2)} = 0$

$\frac{5x-10-5}{(x+2)(x-2)} = 0$

$5x - 15 = 0$

$5x = 15$

$x = 3$

6. **(i)** When $x = -2$,

$2(-2)^2 + q(-2) - 2 = 0$

$8 - 2q - 2 = 0$

$-2q = -6$

$q = 3$

(ii) $2x^2 + 3x - 2 = 0$

$(2x-1)(x+2) = 0$

$2x - 1 = 0$ or $x + 2 = 0$

$x = \frac{1}{2}$ or $x = -2$

$\therefore$ The other solution is $x = \frac{1}{2}$.

7. **(a)** Smallest possible value of the perimeter of the rectangle

$= 2(8.5 + 5.5)$

$= 28$ cm

(b) Largest possible value of the area of the rectangle

$= 9.5 \times 6.5$

$= 61.75$ cm^2

8. **(a)** $-5 < 12 - 3a < -1$

$-5 < 12 - 3a$ and $12 - 3a < -1$

$3a < 12 + 5$ $\quad$ $-3a < -1 - 12$

$3a < 17$ $\quad$ $-3a < -13$

$a < 5\frac{2}{3}$ $\quad$ $a > 4\frac{1}{3}$

Hence $4\frac{1}{3} < a < 5\frac{2}{3}$.

$\therefore$ Integer value of a is 5.

(b) $b - 5 \leqslant 7$ and $3b - 2 \geqslant 11$

$b \leqslant 7 + 5$ $\quad$ $3b \geqslant 11 + 2$

$b \leqslant 12$ $\quad$ $3b \geqslant 13$

$b \geqslant 4\frac{1}{3}$

Hence $4\frac{1}{3} \leqslant b \leqslant 12$.

$\therefore$ Odd integer values of b are 5, 7, 9, 11.

(c) $\frac{2}{7} < 2x - 1 < \frac{9+2x}{12}$

$\frac{2}{7} < 2x - 1$ and $2x - 1 < \frac{9+2x}{12}$

$2 < 7(2x-1)$ $\quad$ $12(2x-1) < 9 + 2x$

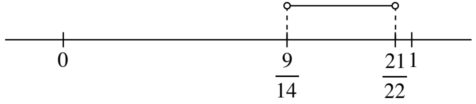
$2 < 14x - 7$ $\quad$ $24x - 12 < 9 + 2x$

$-14x < -7 - 2$ $\quad$ $24x - 2x < 9 + 12$

$-14x < -9$ $\quad$ $22x < 21$

$x > \frac{9}{14}$ $\quad$ $x < \frac{21}{22}$

$\therefore \frac{9}{14} < x < \frac{21}{22}$



OXFORD UNIVERSITY PRESS

9. $-5 \leqslant 4x + 1 \leqslant 2x + 9$

$-5 \leqslant 4x + 1 \quad \text{and} \quad 4x + 1 \leqslant 2x + 9$

$-4x \leqslant 1 + 5 \qquad 4x - 2x \leqslant 9 - 1$

$-4x \leqslant 6 \qquad 2x \leqslant 8$

$x \geqslant -1\frac{1}{2} \qquad x \leqslant 4$

$\therefore -1\frac{1}{2} \leqslant x \leqslant 4$

$-6 \leqslant 2y - 2 \leqslant 8$

$-6 \leqslant 2y - 2 \quad \text{and} \quad 2y - 2 \leqslant 8$

$-2y \leqslant -2 + 6 \qquad 2y \leqslant 8 + 2$

$-2y \leqslant 4 \qquad 2y \leqslant 10$

$y \geqslant -2 \qquad y \leqslant 5$

$\therefore -2 \leqslant y \leqslant 5$

(a) Greatest value of $x - y = 4 - (-2)$

$= 6$

(b) Smallest value of $(x + y)(x - y) = x^2 - y^2$

$= 0^2 - 5^2$

$= -25$

10. **(i)** When $d = 40$,

$40 = 35 + 7t - 2t^2$

$2t^2 - 7t - 35 + 40 = 0$

$(2t - 5)(t - 1) = 0$

$2t - 5 = 0 \quad \text{or} \quad t - 1 = 0$

$t = 2\frac{1}{2} \quad \text{or} \quad t = 1$

$\therefore$ The time(s) when the ball is 40 m above the ground is $2\frac{1}{2}$ s and 1 s.

(ii) When the ball reaches the ground, $d = 0$.

$0 = 35 + 7t - 2t^2$

$2t^2 - 7t - 35 = 0$

$a = 2, b = -7, c = -35$

$$t = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(2)(-35)}}{2(2)}$$

$$t = \frac{7 \pm \sqrt{329}}{4}$$

$$t = \frac{7 + \sqrt{329}}{4} \quad \text{or} \quad t = \frac{7 - \sqrt{329}}{4}$$

$t = 6.28$ (to 3 s.f.) or $t = -2.78$ (rejected)

$\therefore$ The time taken is 6.28 s.

11. **(i)** Greatest possible value of $2x - y = 2(5) - (-1)$

$= 11$

(ii) Least possible value of $2xy = 2(5)(-1)$

$= -10$

(iii) Greatest possible value of $\frac{y}{x} = \frac{7}{1}$

$= 7$

12. Let the number of questions Raj answered correctly be x.

$x(2) + (21 - x)(-1) \leqslant 33$

$2x - 21 + x \leqslant 33$

$3x \leqslant 33 + 21$

$3x \leqslant 54$

$x \leqslant 18$

$\therefore$ Maximum number of questions Raj answered correctly is 18.

13. Let the length of the diagonal be x cm.

Then the length is $(x - 4)$ cm and the breadth is $(x - 6.4)$ cm.

Using Pythagoras' Theorem,

$(x - 4)^2 + (x - 6.4)^2 = x^2$

$x^2 - 8x + 16 + x^2 - 12.8x + 40.96 = x^2$

$x^2 - 20.8x + 56.96 = 0$

$a = 1, b = -20.8, c = 56.96$

$$x = \frac{-(-20.8) \pm \sqrt{(-20.8)^2 - 4(1)(56.96)}}{2(1)}$$

$$x = \frac{20.8 \pm \sqrt{204.8}}{2}$$

$$x = \frac{20.8 + \sqrt{204.8}}{2} \quad \text{or} \quad x = \frac{20.8 - \sqrt{204.8}}{2}$$

$x = 17.6$ (to 3 s.f.) or $x = 3.24$ (rejected since the length and breadth cannot be negative)

$\therefore$ The length of the diagonal is 17.6 cm.

14. **(i)** Since the width is x cm, therefore the length is $(x + 3.4)$ cm.

$x(x + 3.4) = 125$

$x^2 + 3.4x - 125 = 0$

(ii) $a = 1, b = 3.4, c = -125$

$$x = \frac{-3.4 \pm \sqrt{3.4^2 - 4(1)(-125)}}{2(1)}$$

$$x = \frac{-3.4 \pm \sqrt{511.56}}{2}$$

$$x = \frac{-3.4 + \sqrt{511.56}}{2} \quad \text{or} \quad x = \frac{-3.4 - \sqrt{511.56}}{2}$$

$x = 9.61$ (to 2 d.p.) or $x = -13.01$ (rejected)

$\therefore$ The length of the page is $9.61 + 3.4 = 13.01$ cm.

15. Let the distance of Section A of the trail be x km and the distance of Section B of the trail be $(2.4 - x)$ km.

Time for Section A of the trail $= \frac{1000x}{4}$

$= 250x$ s

Time for Section B of the trail $= \frac{1000(2.4 - x)}{5}$

$= \frac{1000(2.4 - x)}{5}$

$= (480 - 200x)$ s

$250x + 480 - 200x = 9(60) + 10$

$50x + 480 = 550$

$50x = 550 - 480$

$50x = 70$

$x = 1.4$

$\therefore$ The distance of Section A of the trail is 1.4 km and the distance of Section B of the trail is 1 km.

OXFORD UNIVERSITY PRESS

16. Let the original number be $10x + y$.

$$\begin{aligned} x + y &= 11 \quad \text{— (1)} \\ 10x + y - (10y + x) &= 27 \\ 10x + y - 10y - x &= 27 \\ 9x - 9y &= 27 \\ x - y &= 3 \quad \text{— (2)} \end{aligned}$$

$$\begin{aligned} \text{(1) + (2): } x + y + x - y &= 11 + 3 \\ 2x &= 14 \\ x &= 7 \end{aligned}$$

$$\begin{aligned} \text{Substitute } x = 7 \text{ into (1): } 7 + y &= 11 \\ y &= 11 - 7 \\ y &= 4 \end{aligned}$$

$x = 7, y = 4$

∴ The original number is 74.

17. (i) $\dfrac{200}{x}$ h

(ii) $\dfrac{200}{x + 5}$ h

(iii)

$$\begin{aligned} \frac{200}{x} - \frac{200}{x + 5} &= 1\frac{1}{4} \\ \frac{200(x + 5) - 200x}{x(x + 5)} &= \frac{5}{4} \\ \frac{200x + 1000 - 200x}{x(x + 5)} &= \frac{5}{4} \\ \frac{1000}{x(x + 5)} &= \frac{5}{4} \\ 4(1000) &= 5x(x + 5) \\ 5x^2 + 25x - 4000 &= 0 \\ x^2 + 5x - 800 &= 0 \text{ (shown)} \end{aligned}$$

(iv) $a = 1, b = 5, c = -800$

$$\begin{aligned} x &= \frac{5 \pm \sqrt{5^2 - 4(1)(-800)}}{2(1)} \\ x &= \frac{-5 \pm \sqrt{3225}}{2} \\ x &= \frac{-5 + \sqrt{3225}}{2} \quad \text{or} \quad x = \frac{-5 - \sqrt{3225}}{2} \end{aligned}$$

$x = 25.9$ (to 3 s.f.) or $x = -30.9$ (rejected)

∴ The speed of the train from Segamat to Singapore is $(25.9 + 5) = 30.9$ km/h.

18. Let the fraction be $\dfrac{x}{x + 2}$.

After the increase, the fraction becomes $\dfrac{x + 3}{x + 2 + 3} = \dfrac{x + 3}{x + 5}$.

$$\begin{aligned} \frac{x + 3}{x + 5} - \frac{x}{x + 2} &= \frac{3}{20} \\ \frac{(x + 3)(x + 2) - x(x + 5)}{(x + 5)(x + 2)} &= \frac{3}{20} \\ \frac{x^2 + 2x + 3x + 6 - x^2 - 5x}{x^2 + 2x + 5x + 10} &= \frac{3}{20} \\ \frac{6}{x^2 + 7x + 10} &= \frac{3}{20} \\ 20(6) &= 3(x^2 + 7x + 10) \\ 120 &= 3x^2 + 21x + 30 \\ 3x^2 + 21x + 30 - 120 &= 0 \\ 3x^2 + 21x - 90 &= 0 \\ x^2 + 7x - 30 &= 0 \\ (x - 3)(x + 10) &= 0 \end{aligned}$$

$x - 3 = 0$ or $x + 10 = 0$

$x = 3$ or $x = -10$ (rejected)

∴ The original fraction is $\dfrac{3}{5}$.

Note to teachers: Since $\dfrac{-10}{-8} = \dfrac{5}{4}$*, the numerator is 1 more than the denominator. The question states that the numerator of the fraction is 2 less than the denominator. Hence x = –10 is rejected.*

19. (a) (i) Cost price of one apple = $\$\dfrac{32}{x}$

(ii) Selling price of one apple $= \dfrac{32}{x} + \dfrac{5}{100}$

$= \$\left(\dfrac{640 + x}{20x}\right)$

(b)

$$\begin{aligned} \left(\frac{640 + x}{20x}\right)(x - 20) &= 35 \\ (640 + x)(x - 20) &= 35(20x) \\ 640x - 12\ 800 + x^2 - 20x &= 700x \\ x^2 + 620x - 700x - 12\ 800 &= 0 \\ x^2 + 80x - 12\ 800 &= 0 \text{ (shown)} \end{aligned}$$

(c) $(x - 160)(x + 80) = 0$

$x - 160 = 0$ or $x + 80 = 0$

$x = 160$ or $x = -80$ (rejected)

The fruitseller bought 160 apples.

20. (a) In 1 minute, Pipe *A* can fill $\dfrac{1}{x}$ of the tank.

In 1 minute, Pipe *B* can fill $\dfrac{1}{x + 40}$ of the tank.

In 1 minute, Pipe *A* and *B* can fill $\dfrac{1}{48}$ of the tank.

$$\begin{aligned} \frac{1}{x} + \frac{1}{x + 40} &= \frac{1}{48} \\ \frac{x + 40 + x}{x(x + 40)} &= \frac{1}{48} \\ \frac{2x + 40}{x^2 + 40x} &= \frac{1}{48} \\ 48(2x + 40) &= x^2 + 40x \\ 96x + 1920 &= x^2 + 40x \\ x^2 - 56x - 1920 &= 0 \text{ (shown)} \end{aligned}$$

(b) (i) $(x - 80)(x + 24) = 0$

$x - 80 = 0$ or $x + 24 = 0$

$x = 80$ or $x = -24$ (rejected)

(ii) –24 cannot be accepted as time cannot be negative.

(c) Time taken by Pipe $B = 80 + 40$

$= 120$ minutes

$= 2$ hours

OXFORD UNIVERSITY PRESS

21. Upper bounds of combined mass
$= 3.75 + 5.75$
$= 9.5$ kg
Lower bounds of combined mass
$= 3.25 + 5.25$
$= 8.5$ kg

22. Upper bound of side of cube = 5.05 cm
Lower bound of side of cube = 4.95 cm
Upper bound of mass of cube = 250.5 g
Lower bound of mass of cube = 249.5 g

Maximum possible density $= \dfrac{250.5}{4.95^3}$
$= 2.07$ g/cm^3 (to 3 s.f.)

Minimum possible density $= \dfrac{249.5}{5.05^3}$
$= 1.94$ g/cm^3 (to 3 s.f.)

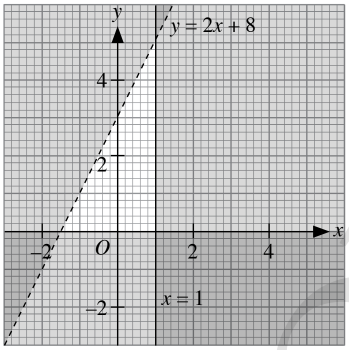
23.

	56	2	4
Upper bound	56.5	2.5	4.5
Lower bound	55.5	1.5	3.5

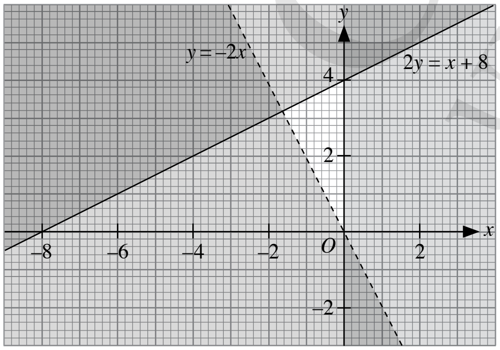
Upper bound of $\dfrac{56 \times 2}{4} = \dfrac{56.5 \times 2.5}{3.5}$
$= 40.4$ (to 3 s.f.)

Lower bound of $\dfrac{56 \times 2}{4} = \dfrac{55.5 \times 1.5}{4.5}$
$= 18.5$

24. (a)



(b)



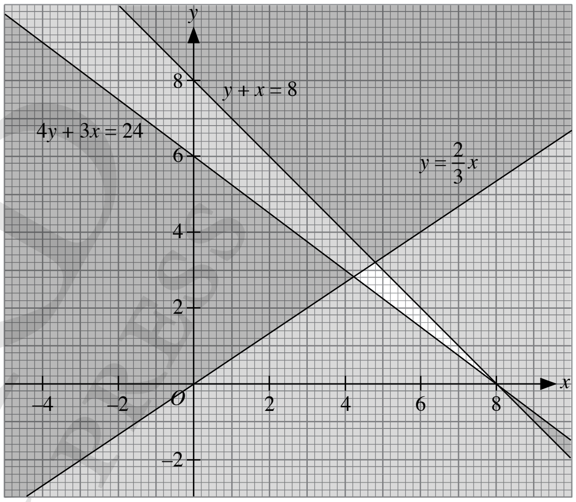
25. (a) Equations of the lines are: $x = 5$, $y = 5$, $y = \dfrac{3}{2}x$ and $y = 5 - x$

$\therefore$ The inequalities which define the unshaded region are $x \leqslant 5$, $y \leqslant 5$, $y \leqslant \dfrac{3}{2}x$ and $y \geqslant 5 - x$.

(b) Equations of the lines are: $y = 4 - \dfrac{1}{4}x$, $2y = 3x - 6$, $y = 2x + 4$ and $y = -x - 2$

$\therefore$ The inequalities which define the unshaded region are $y \leqslant 4 - \dfrac{1}{4}x$, $2y \geqslant 3x - 6$, $y \leqslant 2x + 4$ and $y \geqslant -x - 2$.

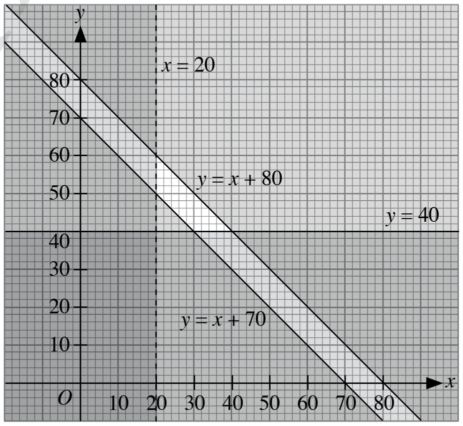
26. (i)



(ii) Greatest value of $(x - y) = 8 - 0$
$= 8$

27. (a) $x > 20$, $y \geqslant 40$, $70 \leqslant x + y \leqslant 80$

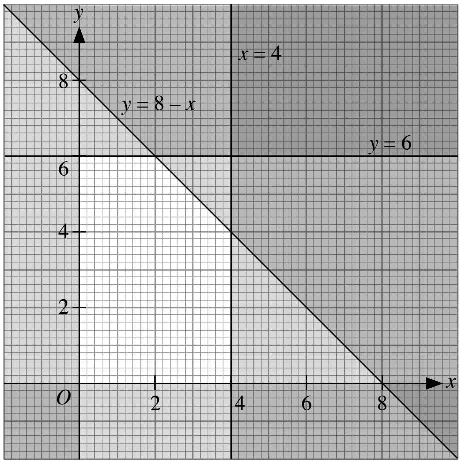
(b)



(c) Total profit = $\$(30x + 20y)$
When $x = 40$ and $y = 40$, total profit
$= 30(40) + 20(40)$
$= \$2000$
When $x = 20$ and $y = 60$, total profit
$= 30(20) + 20(60)$
$= \$1800$
$\therefore$ The bookstore must buy 40 hardback copies and 40 paperback copies to obtain the maximum profit.

OXFORD UNIVERSITY PRESS

28.

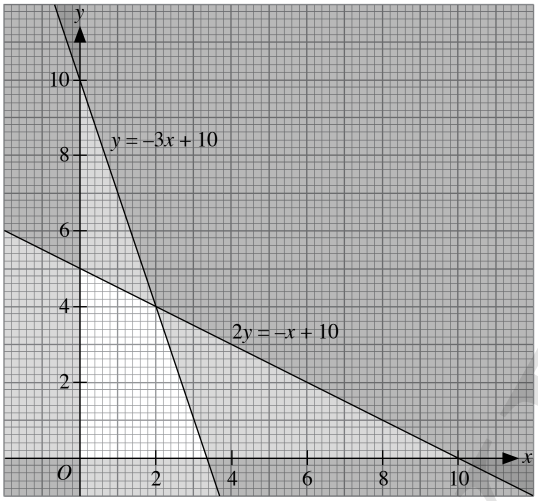


When $x = 2$, $y = 6$: $2(2) + 4(6) = 28$

When $x = 4$, $y = 4$: $2(4) + 4(4) = 24$

$\therefore$ The largest value of $2x + 4y$ is 28.

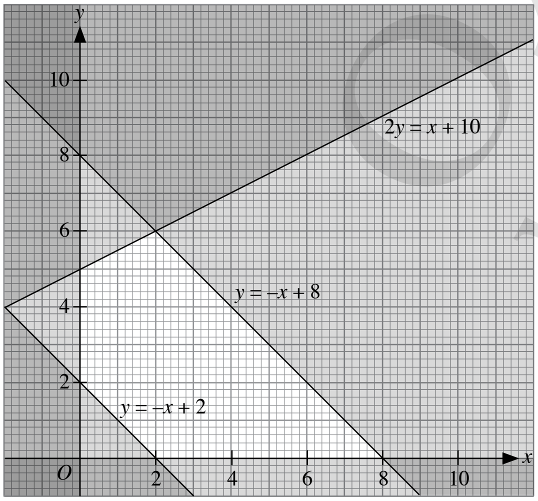
29.



When $x = 2$, $y = 4$: $3(2) + 5(4) = 26$

$\therefore$ The largest value of $3x + 5y$ is 26.

30.



When $x = 8$, $y = 0$: $2(8) – 0 = 16$

When $x = 0$, $y = 5$: $2(0) – 5 = –5$

$\therefore$ The maximum value of $2x – y$ is 16 and the minimum value of $2x – y$ is –5.

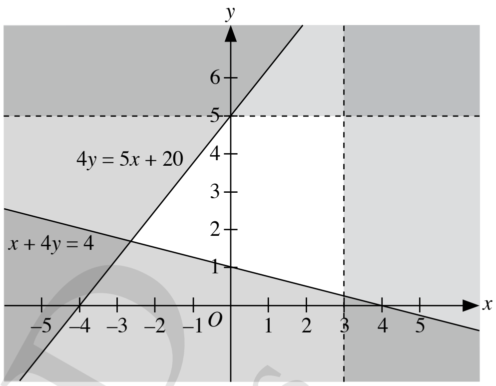
31. Equations of the lines are: $y = 0$, $4y = –3x + 12$ and $y = 3x + 3$

$\therefore$ The inequalities which define the unshaded region R are $y \geqslant 0$, $4y + 3x \leqslant 12$ and $y \leqslant 3x + 3$.

32. **(i)** Equation of the line PQ is $y = -\frac{8}{6}x + 8$ i.e. $3y + 4x = 24$

(ii) $3y + 4x \leqslant 24$

33.



Revision 9E

1. **(a)** It is not a function since the element q in domain A has two images, 17 and 35, in codomain B. Additionally, the element s in domain A does not have an image in codomain B.

(b) It is a function since every element in the domain A has a unique image in the codomain B.

2. **(i)** $f(a) = 7a - \frac{1}{2}$

(ii) $F(a + 1) = 8(a + 1) + 1$

$= 8a + 9$

(iii) $f\left(\frac{a}{7}\right) = 7\left(\frac{a}{7}\right) - \frac{1}{2}$

$= a - \frac{1}{2}$

$F(a – 1) = 8(a – 1) + 1$

$= 8a – 7$

$f\left(\frac{a}{7}\right) + F(a – 1) = a - \frac{1}{2} + (8a – 7)$

$= 9a - 7\frac{1}{2}$

3. $f(x) = 3x^2 - \frac{1}{4}$

$f(–5) = 3(–5)^2 - \frac{1}{4}$

$= 74\frac{3}{4}$

$f\left(\frac{1}{8}\right) = 3\left(\frac{1}{8}\right)^2 - \frac{1}{4}$

$= \frac{3}{64} - \frac{1}{4}$

$= -\frac{13}{64}$

$f(7) = 3(7)^2 - \frac{1}{4}$

$= 146\frac{3}{4}$

OXFORD UNIVERSITY PRESS

4. $f(x) = 2x + 11$

Let $y = 2x + 11$

$f(x) = y$ and $f^{-1}(y) = x$

$2x = y - 11$

$x = \dfrac{y - 11}{2}$

$\therefore f^{-1}(y) = \dfrac{y - 11}{2}$

Hence $f^{-1}(x) = \dfrac{x - 11}{2}$

5. $g(x) = 2x - 5$

For $g(x) = -8$,

$2x - 5 = -8$

$2x = -3$

$x = -\dfrac{3}{2}$

For $g(x) = 0$,

$2x - 5 = 0$

$2x = 5$

$x = \dfrac{5}{2}$

For $g(x) = 3$,

$2x - 5 = 3$

$2x = 8$

$x = 4$

For $g(x) = \dfrac{1}{3}$,

$2x - 5 = \dfrac{1}{3}$

$2x = \dfrac{16}{3}$

$x = \dfrac{8}{3}$

6. $f(x) = ax + b$

$f(4) = 4a + b = 6$ — (1)

$f^{-1}(8) = 5 \Rightarrow f(5) = 5a + b = 8$ — (2)

(2) – (1): $a = 2$

Subst. $a = 2$ into (1):

$b = -2$

$\therefore a = 2, b = -2$

$f(x) = 2x - 2$

Let $y = 2x - 2$

$f(x) = y$ and $f^{-1}(y) = x$

$x = \dfrac{y + 2}{2}$

$\therefore f^{-1}(y) = \dfrac{y + 2}{2}$

Hence $f^{-1}(x) = \dfrac{x + 2}{2}$.

7. $g(x) = \dfrac{5x - 7}{x - 4}$

Let $y = \dfrac{5x - 7}{x - 4}$

$g(x) = y$ and $g^{-1}(y) = x$

$y(x - 4) = 5x - 7$

$xy - 4y = 5x - 7$

$xy - 5x = 4y - 7$

$x(y - 5) = 4y - 7$

$x = \dfrac{4y - 7}{y - 5}$

$\therefore g^{-1}(y) = \dfrac{4y - 7}{y - 5}$

Hence $g^{-1}(x) = \dfrac{4x - 7}{x - 5}$, g^{-1} is not defined when $x = 5$.

8. **(i)** $y = (x + 1)(2 - x)$

At A and C, $y = 0$

$(x + 1)(2 - x) = 0$

$x + 1 = 0$ or $2 - x = 0$

$x = -1$ or $x = 2$

$\therefore$ The coordinates of A are $(-1, 0)$ and the coordinates of C are $(2, 0)$.

At B, $x = 0$

$y = (0 + 1)(2 - 0)$

$= 2$

$\therefore$ The coordinates of B are $(0, 2)$.

(ii) Equation of the line of symmetry is $x = \dfrac{-1 + 2}{2}$

$x = \dfrac{1}{2}$

9. **(i)** At A, $x = 0$

$y = 0^2 + h(0) - 5$

$= -5$

$\therefore$ The coordinates of A are $(0, -5)$.

(ii) At $(1, -2)$,

$-2 = 1^2 + h(1) - 5$

$-2 = 1 + h - 5$

$h = 2$

10. Substitute $(1, 0)$ into $y = x^2 + px + q$.

$0 = 1^2 + p(1) + q$

$p + q = -1$ — (1)

Substitute $(4, 0)$ into $y = x^2 + px + q$.

$0 = 4^2 + p(4) + q$

$4p + q = -16$ — (2)

(2) – (1): $4p + q - (p + q) = -16 - (-1)$

$4p + q - p - q = -16 + 1$

$3p = -15$

$p = -5$

Substitute $p = -5$ into (1): $-5 + q = -1$

$q = -1 + 5$

$q = 4$

$\therefore p = -5, q = 4$

OXFORD UNIVERSITY PRESS

11. **(a)** $y = x^3 + 4$

(b) $y = 4^x$

(c) $y = \dfrac{4}{x}$

12. $n = -2$

13. Substitute $(0, 3)$ into $y = ka^x$.

$3 = ka^0$

$k = 3$

Substitute $(4, 48)$ into $y = 3a^x$.

$48 = 3a^4$

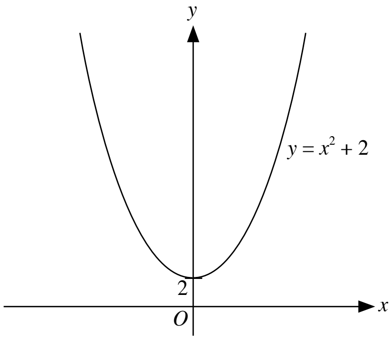
$a^4 = 16$

$a^4 = 2^4$

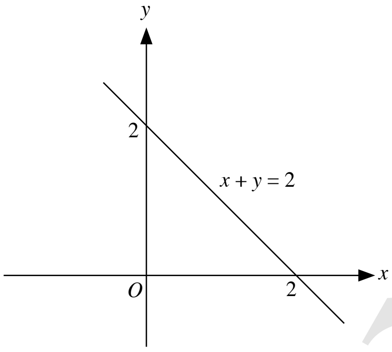
$a = 2$

$\therefore\ a = 2, k = 3$

14. **(a)**



(b)



15. At P, $x = 0$

$y = 9 - 4(0)^2$

$= 9$

$\therefore$ The coordinates of P are $(0, 9)$.

At Q and R, $y = 0$

$0 = 9 - 4x^2$

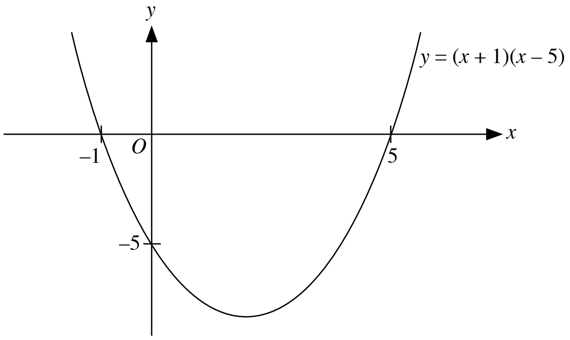
$(3 + 2x)(3 - 2x) = 0$

$3 + 2x = 0$ or $3 - 2x = 0$

$x = -1\frac{1}{2}$ or $x = 1\frac{1}{2}$

$\therefore$ The coordinates of Q are $\left(-1\frac{1}{2}, 0\right)$ and the coordinates of R are $\left(1\frac{1}{2}, 0\right)$.

16. **(a)**



(b) $x = 2$

(c) At $x = 2$, $y = (2 + 1)(2 - 5)$

$= -9$

$\therefore$ The coordinates of the minimum point are $(2, -9)$.

17. **(i)** $y = -(x + 3)^2 + 1$

When $y = 0$,

$(x + 3)^2 = 1$

$x + 3 = \pm 1$

$x + 3 = 1$ or $x + 3 = -1$

$x = -2$ or $x = -4$

$\therefore$ The coordinates of the x-intercepts are $(-2, 0)$ and $(-4, 0)$.

When $x = 0$,

$y = -(0 + 3)^2 + 1$

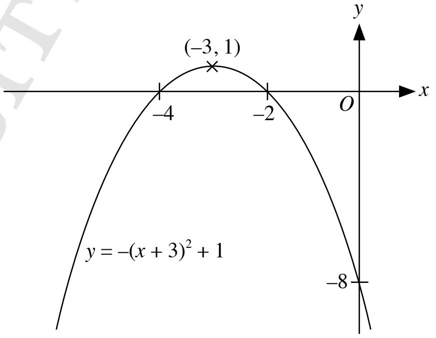
$= -9 + 1$

$= -8$

$\therefore$ The coordinates of the y-intercept are $(0, -8)$.

(ii) Coordinates of the maximum point are $(-3, 1)$.

(iii)



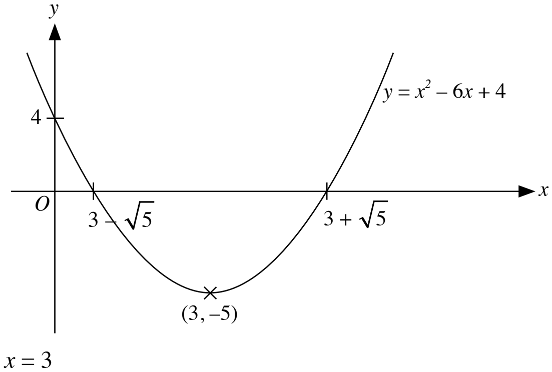
(iv) $x = -3$

18. **(i)** $x^2 - 6x + 4 = x^2 - 6x + (-3)^2 + 4 - (-3)^2$

$= (x - 3)^2 - 5$

(ii) Coordinates of the minimum point are $(3, -5)$.

(iii)



(iv) $x = 3$

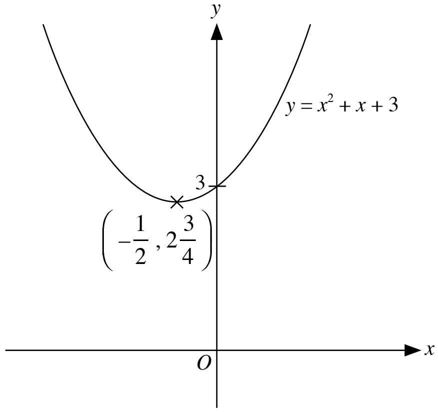
OXFORD UNIVERSITY PRESS

19. (i) $x^2 + x + 3 = x^2 + x + \left(\frac{1}{2}\right)^2 + 3 - \left(\frac{1}{2}\right)^2$

$$= \left(x + \frac{1}{2}\right)^2 + 2\frac{3}{4}$$

(ii) Coordinates of the minimum point are $\left(-\frac{1}{2}, 2\frac{3}{4}\right)$.

(iii)

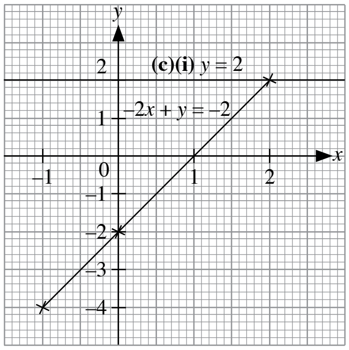


(iv) $x = -\frac{1}{2}$

20. (a)

x	–1	0	2
y	–4	–2	2

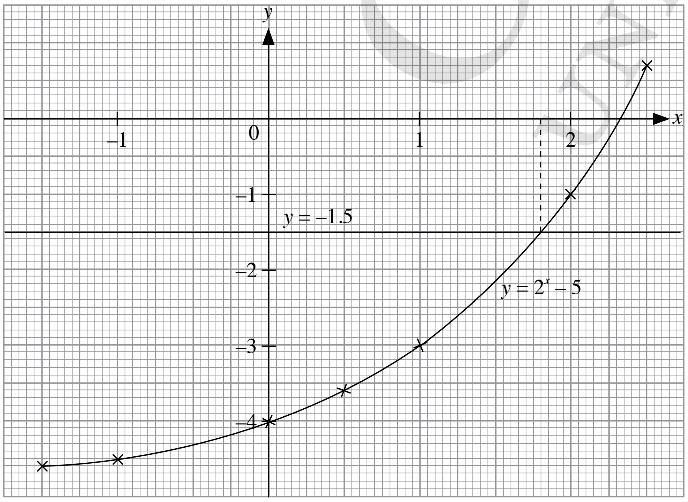
(b)



(c) **(ii)** Area of the trapezium = $\frac{1}{2} \times (2 + 1) \times 2$

$= 3 \text{ units}^2$

21. (a)



(b) $x = 1.8$

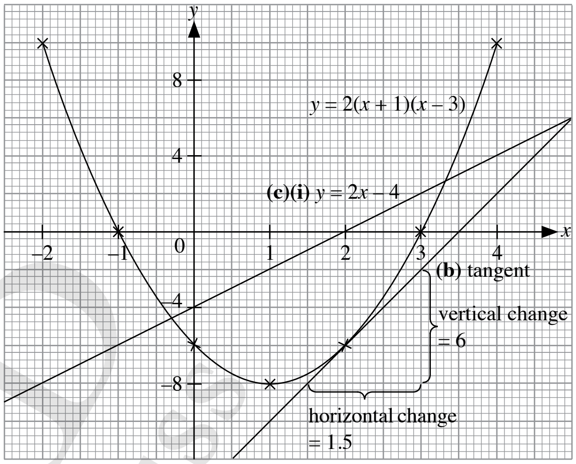
(c) For all real values of x, smallest value of 2^x is 0.

$\therefore$ Smallest value of $2^x - 5$ is –5.

22.

x	–2	–1	0	1	2	3	4
y	10	0	–6	–8	–6	0	10

(a)

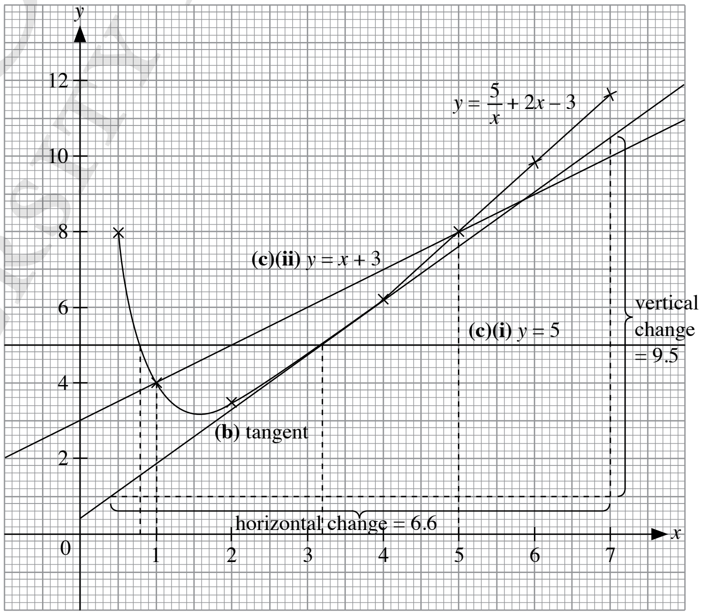


(b) Gradient of tangent = $\frac{6}{1.5} = 4$

(c) **(ii)** $x = –0.3$ or $x = 3.3$

(iii) $-0.3 \leqslant x \leqslant 3.3$

23. (a)



(b) Gradient of tangent = $\frac{9.5}{6.6} = 1.44$ (to 3 s.f.)

(c) **(i)** $\frac{5}{x} + 2x - 8 = 0$

$\frac{5}{x} + 2x - 8 + 5 = 0 + 5$

$\frac{5}{x} + 2x - 3 = 5$

Draw the line $y = 5$.

$x = 0.8$ or $x = 3.2$

OXFORD UNIVERSITY PRESS

(ii) $$\frac{5}{x} + x - 6 = 0$$

$$\frac{5}{x} + x - 6 + x + 3 = 0 + x + 3$$

$$\frac{5}{x} + 2x - 3 = x + 3$$

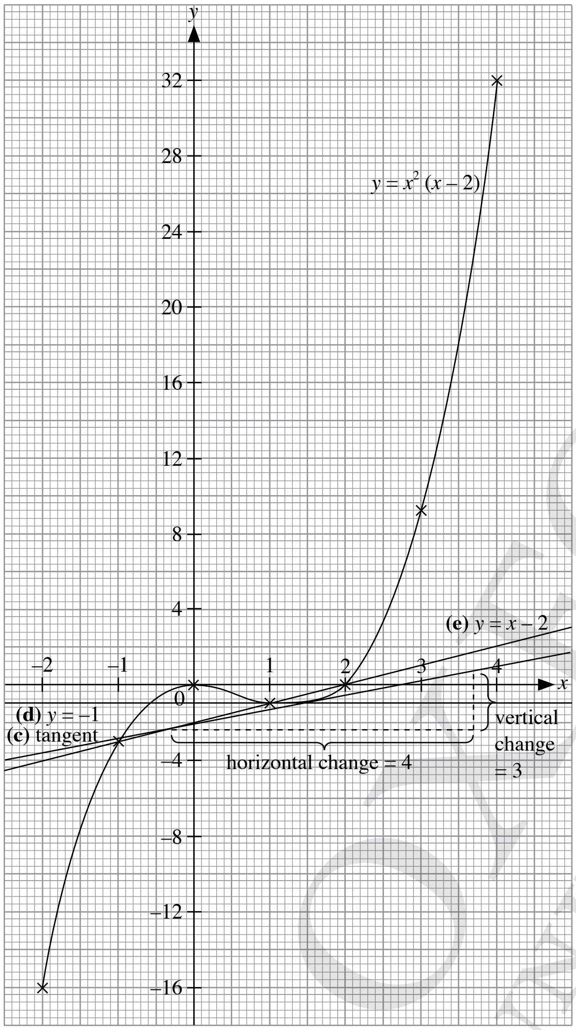
Draw the line $y = x + 3$.

$x = 1$ or $x = 5$

24. (a)

x	–2	–1	0	1	2	3	4
y	–16	–3	0	–1	0	9	32

(b)



(c) Gradient of tangent = $\frac{3}{4} = 0.75$

(d) $x^2(x - 2) = -1$

Draw the line $y = -1$.

$x = -0.6, x = 1$ or $x = 1.6$

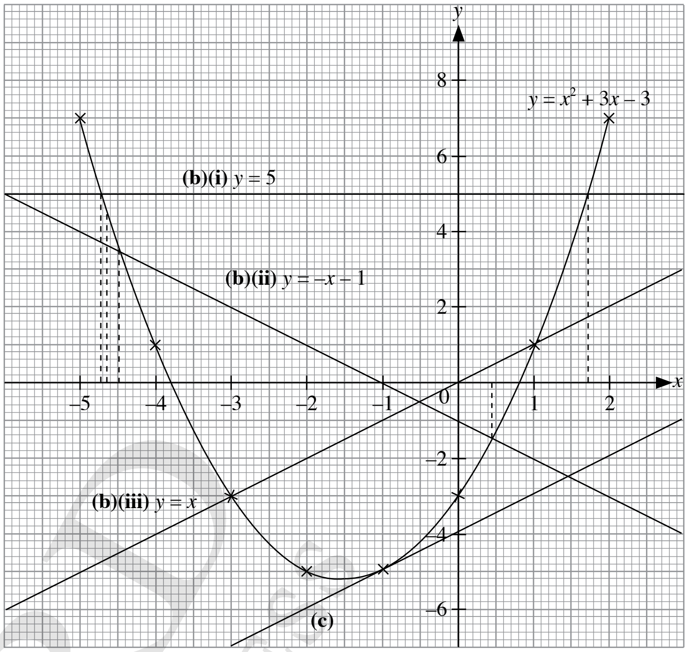
(f) **(i)** $x = -1, x = 1$ and $x = 2$

(ii) $x^2(x - 2) = x - 2$

$x^3 - 2x^2 - x + 2 = 0$

$\therefore A = -2, B = -1, C = 2$

25. (a)



(b) **(i)** $x^2 + 3x = 8$

$x^2 + 3x - 3 = 8 - 3$

Draw the line $y = 5$.

$x = -4.7$ or $x = 1.7$

(ii) $x^2 + 4x = 2$

$x^2 + 4x - x - 3 = 2 - x - 3$

$x^2 + 3x - 3 = -x - 1$

Draw the line $y = -x - 1$.

$x = -4.5$ or $x = 0.45$

(iii) $x^2 + 3x - 3 \leqslant x$

Draw the line $y = x$.

$-3 \leqslant x \leqslant 1$

(c) The coordinates of the point are $(-1, -5)$.

26. (a) Let the height of the box be h m.

Volume of the box = 35 m^3

$x^2h = 35$

$h = \frac{35}{x^2}$

(b) External area, $A = x^2 + (4x)h$

$$= x^2 + (4x)\left(\frac{35}{x^2}\right)$$

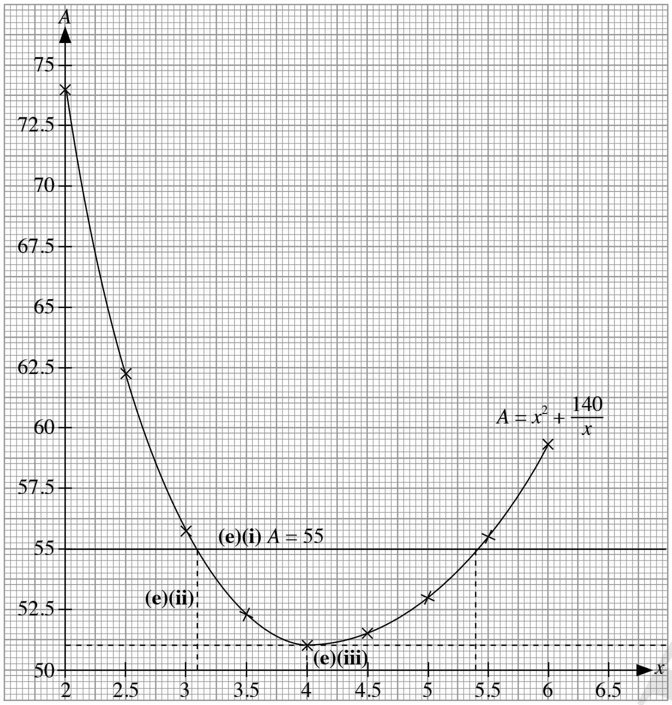
$$= x^2 + \frac{140}{x}$$

(c)

x	2	2.5	3	3.5	4	4.5	5	5.5	6
y	74	62.3	55.7	52.3	51	51.4	53	55.7	59.3

OXFORD UNIVERSITY PRESS

(d)

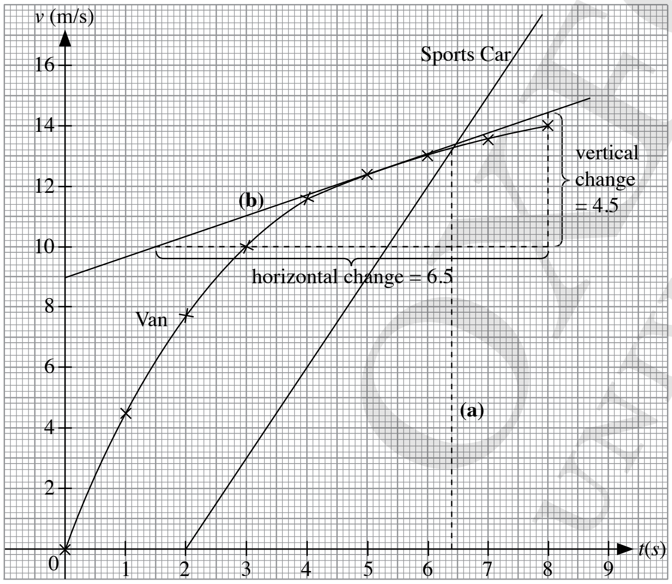


(e) **(i)** $h = 3.1$ m or $h = 5.4$ m

(ii) Minimum value of $A = 51$ m^2

(iii) When $x = 4$, the height is $\frac{35}{4^2} = 2.19$ m (to 3 s.f.)

27.



(a) 6.4 s

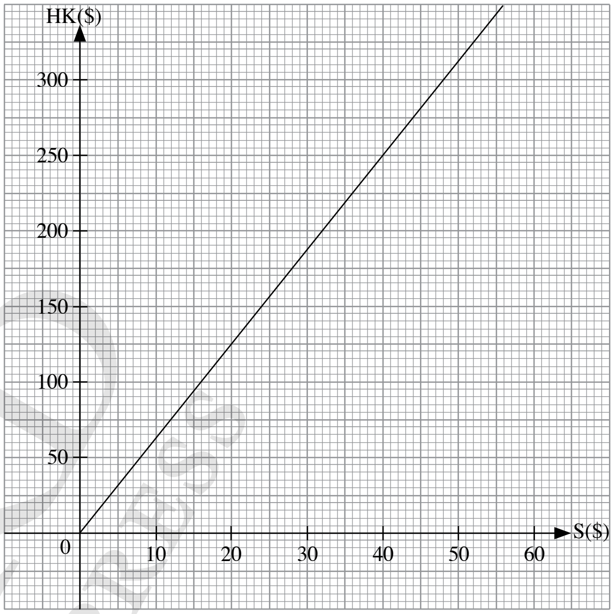
(b) Acceleration = $\frac{4.5}{6.5} = 0.692$ m/s^2 (to 3 s.f.)

Revision 9F

1. **(a)** **(i)** S\$$\left(\frac{55}{100} \times 16\right)$ = S\$8.80

(ii) HK\$$\left(\frac{20}{16} \times 100\right)$ = HK\$125

(b)



$y = \frac{100}{16}x$

$y = \frac{25}{4}x$

$4y = 25x$

2. **(a)** **(i)** When $F = 98$,

$98 = \frac{9}{5}C + 32$

$\frac{9}{5}C = 98 - 32$

$\frac{9}{5}C = 66$

$C = 36\frac{2}{3}$ °C

(ii) When $C = 50$,

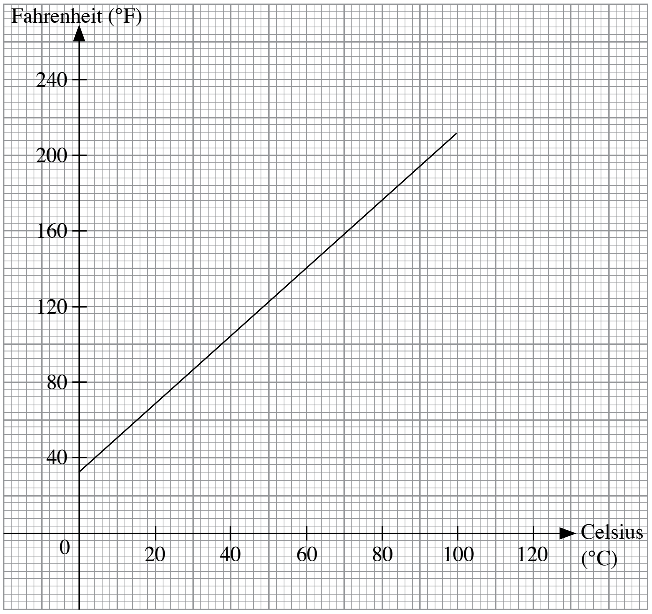
$F = \frac{9}{5}(50) + 32$

$= 90 + 32$

$= 122$ °F

OXFORD UNIVERSITY PRESS

(b)



(c) When $F = 70$,

$$70 = \frac{9}{5}C + 32$$

$$\frac{9}{5}C = 38$$

$$C = 21.1 \text{ (to 3 s.f.)}$$

When $F = 120$,

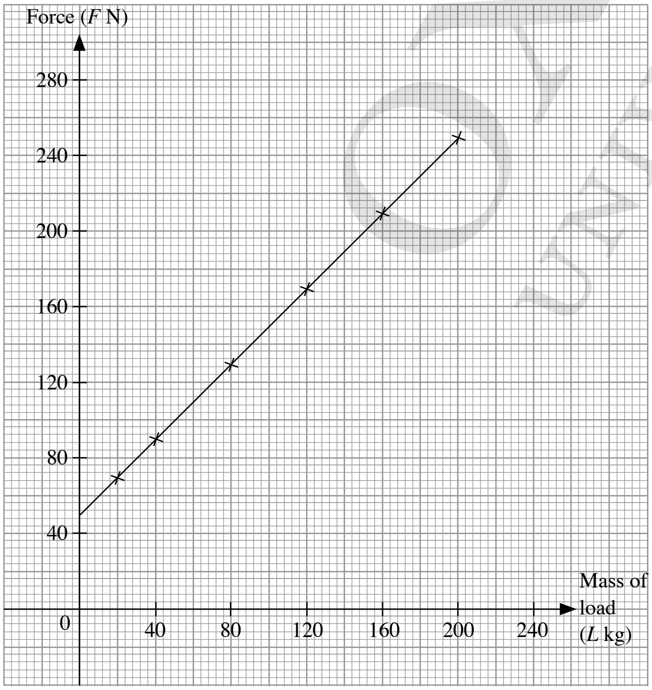
$$120 = \frac{9}{5}C + 32$$

$$\frac{9}{5}C = 88$$

$$C = 48.9 \text{ (to 3 s.f.)}$$

$\therefore$ Increase in $C = 48.9 - 21.1 = 27.8$ °C

3. **(a)**



(b) **(i)** 106 N

(ii) 240 N

(c) Initial force needed = 50 N

4. **(a)** **(i)** \$1.50

(ii) \$4.90

(iii) Company P

(b) Company Q

5. **(i)** Distance = 48 km

(ii) Duration = 2 h

(iii) Distance = 28 km from Q

(iv) Period of time is from 0900 to 1000.

(v) Average Speed

$$= \frac{48}{6}$$

= 8 km/h

6. **(i)** Duration = 20 minutes

(ii) Rate of increase

$$= \frac{188 - 92}{10}$$

= 9.6 mg/d*l* per minute

(iii) Rate of decrease

$$= \frac{188 - 124}{10}$$

= 6.4 mg/d*l* per minute

7. **(i)** Acceleration of the car in the first 20 seconds

$$= \frac{15}{20}$$

$= 0.75 \text{ m/s}^2$

(ii) Distance travelled by the car during the 100 seconds

$$= \frac{1}{2} \times (50 + 100) \times 15$$

= 1125 m

(iii) Distance travelled by the lorry during the 100 seconds

$= 10 \times 100$

= 1000 m

(iv) Let the time when the car overtakes the lorry be t s.

At 20 seconds,

Distance travelled by the car

$$= \frac{1}{2} \times 20 \times 15$$

= 150 m

Distance travelled by the lorry

$= 10 \times 20$

= 200 m

At 70 seconds,

Distance travelled by the car

$$= \frac{1}{2} \times 20 \times 15 + 50 \times 15$$

= 900 m

Distance travelled by the lorry

$= 10 \times 70$

= 700 m

OXFORD UNIVERSITY PRESS

Hence $20 < t < 70$.

$10t = \frac{1}{2} \times 20 \times 15 + (t - 20) \times 15$

$10t = 150 + 15t - 300$

$5t = 150$

$t = 30$ s

8. (i) Duration = 4 minutes

(ii) Rate of increase

$= \frac{12}{15}$

$= 0.8$ °C/minute

(iii) Temperature = 74 °C

(iv) Rate of increase

$= \frac{60}{4}$

$= 15$ °C/minute

9. (i) Total distance travelled by the particle = 1.84 km

$$\frac{1}{2} \times (v + 24) \times 20 + (24 \times 30) + \frac{1}{2} \times 24 \times 40 = 1840$$

$$10v + 240 + 720 + 480 = 1840$$

$$10v = 400$$

$$v = 40$$

(ii) Acceleration during the first 10 seconds

$= -\frac{40 - 24}{20}$

$= -0.8$ m/s^2

(iii) At $t = 10$, speed of the particle

$= 40 - 10(0.8)$

$= 32$ m/s

Distance travelled during the first 10 seconds of its motion

$= \frac{1}{2} \times (40 + 32) \times 10$

$= 360$ m

10. (i) Let the speed be V m/s.

$\frac{V - 6}{3} = \frac{8}{5}$

$5(V - 6) = 8(3)$

$5V - 30 = 24$

$5V = 54$

$V = 10.8$ m/s

(ii) Distance travelled by the particle in the first 12 seconds

$= \frac{1}{2} \times (6 + 14) \times 5 + (14 \times 7)$

$= 148$ m

Average speed during the first 12 seconds of the journey

$= \frac{148}{12}$

$= 12\frac{1}{3}$ m/s

(iii) $\frac{14}{T - 12} = 3\frac{1}{2}$

$\frac{14}{T - 12} = \frac{7}{2}$

$7(T - 12) = 2(14)$

$7T - 84 = 28$

$7T = 112$

$T = 16$

11. (i) Gradient of part **(a)**

$= \frac{10 - 35}{300}$

$= -\frac{1}{12}$

This gradient means that the car used 1 litre of petrol to travel 12 km.

Gradient of part **(b)**

$= \frac{26 - 45}{300}$

$= -\frac{19}{300}$

This gradient means that the car used 19 litres of petrol to travel 300 km.

(ii) The car was moving uphill in part **(a)** while in part **(b)**, the car is moving downhill, thus petrol consumption in part **(a)** is higher.

(iii) Amount of petrol top up = 35 litres

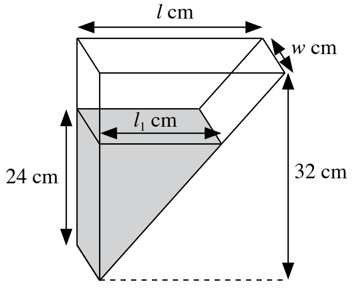
Amount in RMB = 35×6.75

Amount in Singapore dollars

$= \frac{35 \times 6.75}{4.88}$

$\approx$ S\$48.41

12. (i) Let the length of the container be l cm and the width of the container be w cm.



Total volume $= \frac{1}{2} \times l \times 32 \times w$

$= 16lw$ cm^3

When the depth is 24 cm,

$\frac{l_1}{24} = \frac{l}{32}$

$\therefore l_1 = \frac{3}{4}l$

Volume of water $= \frac{1}{2} \times \frac{3}{4}l \times 24 \times w$

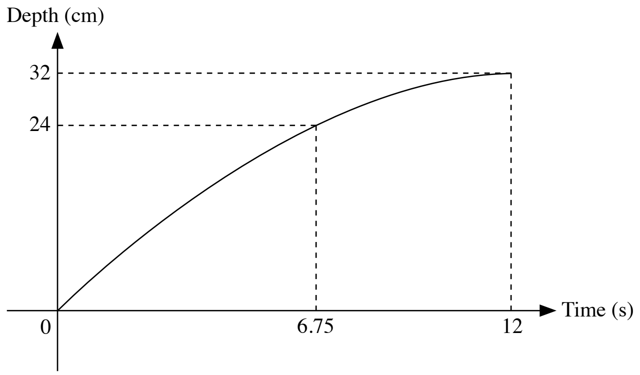
$= 9lw$ cm^3

$16lw$ cm^3 takes 12 seconds.

$\therefore 9lw$ cm^3 takes $\frac{12}{16} \times 9 = 6.75$ seconds.

OXFORD UNIVERSITY PRESS

(ii)



13. (i) Average speed of car $A = \frac{120}{2}$

$= 60$ km/h

Average speed of car $B = \frac{120}{1\frac{5}{6}}$

$= 65\frac{5}{11}$ km/h

(ii) Car B is 40 km from Bangkok Airport and it stopped for 40 minutes.

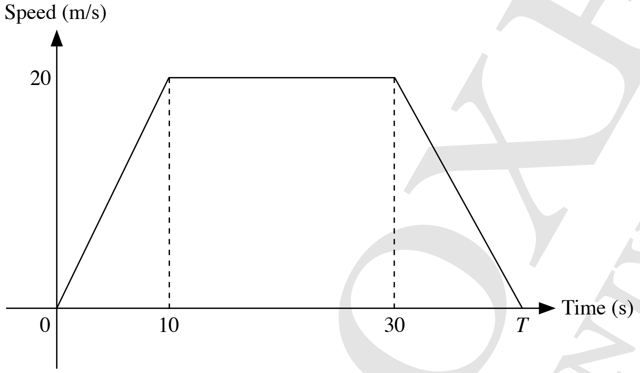
(iii) The two cars are 64 km from Pattaya Town when they met. The time was 1118.

(iv) The cars are 40 km apart.

(v) Speed of car $A = \frac{80}{1}$

$= 80$ km/h

14. (a)



(b) (i) Acceleration for the first 10 seconds $= \frac{20}{10}$

$= 2$ m/s^2

(ii) $\frac{20}{T - 30} = 1.8$

$20 = 1.8(T - 30)$

$20 = 1.8T - 54$

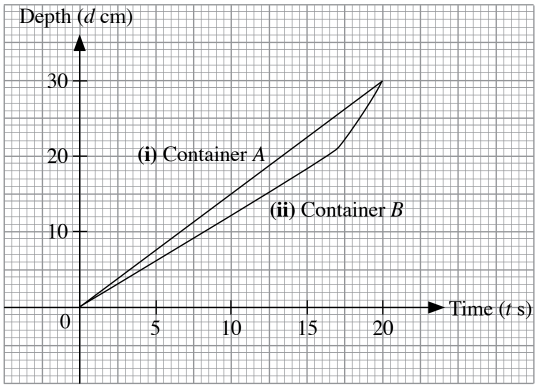
$1.8T = 20 + 54$

$1.8T = 74$

$T = 41.1$ (to 3 s.f.)

$\therefore$ Total time taken is 41.1 s.

15. (a)

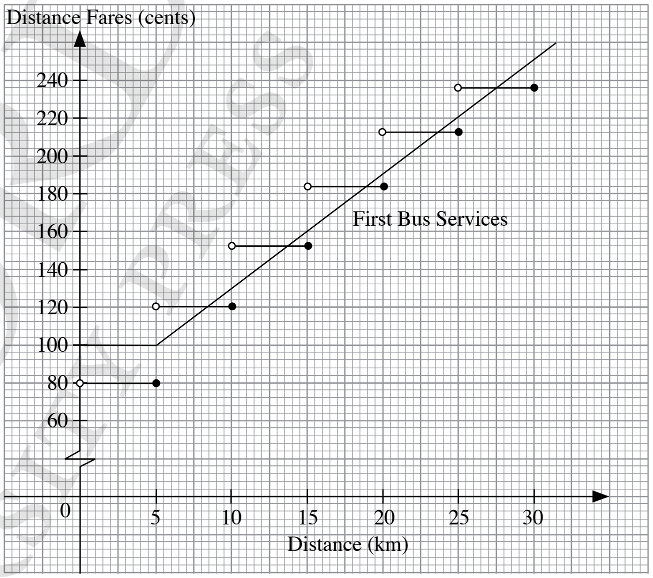


(b) (i) 15 cm

(ii) 12.5 cm

16. (a) 184 cents

(b)



(c) (i) Optimus Bus Services

(ii) Difference in distance fares = 220 – 212

= 8 cents

17. (a) (i) Acceleration $= 8\frac{1}{3}$ m/s^2

(ii) At A, time $= \frac{40}{8\frac{1}{3}} = 4.8$ s

$\therefore$ Coordinates of A are (4.8, 40).

At B,

$40 = -12.5t + 175$

$12.5t = 175 - 40$

$12.5t = 135$

$t = 10.8$

$\therefore$ Coordinates of B are (10.8, 40).

At C,

$0 = -12.5t + 175$

$12.5t = 175$

$t = 14$

$\therefore$ Coordinates of C are (14, 0).

(iii) Deceleration = 12.5 m/s^2

OXFORD UNIVERSITY PRESS

(b) Distance travelled = $\frac{1}{2} \times [(10.8 - 4.8) + 14] \times 40$

$= 400$ m

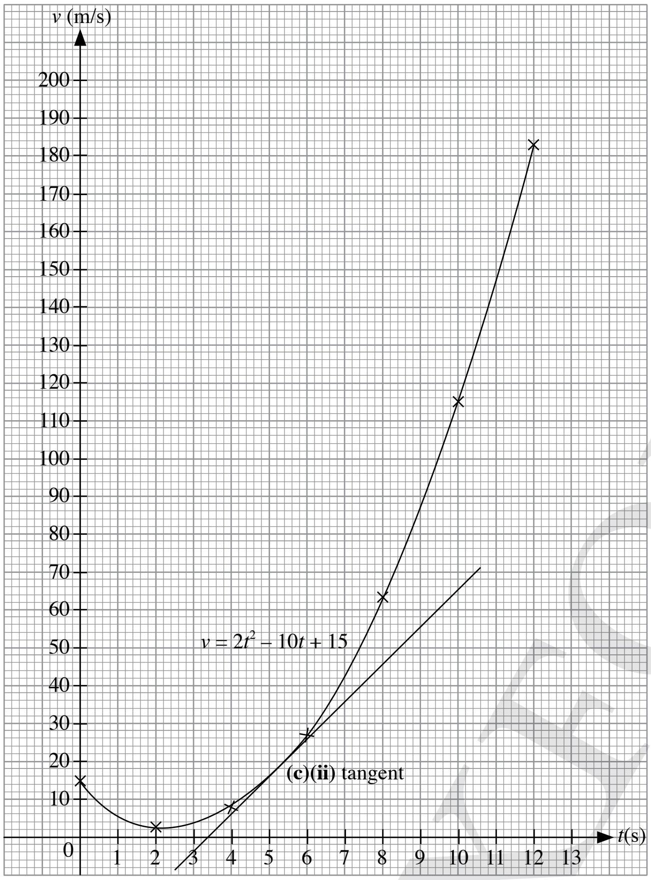
18. (a) $a = 2(4)^2 - 10(4) + 15$

$= 7$

$b = 2(10)^2 - 10(10) + 15$

$= 115$

(b)



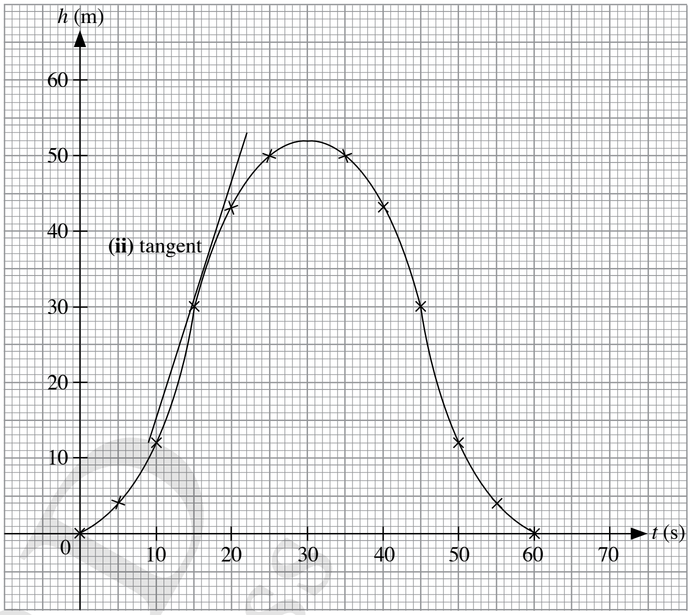
(c) (i) $t = 7.1$ s

(ii) Gradient = $\frac{50}{5}$

$= 10$

This represents the acceleration of the particle at $t = 5$.

19. (i)

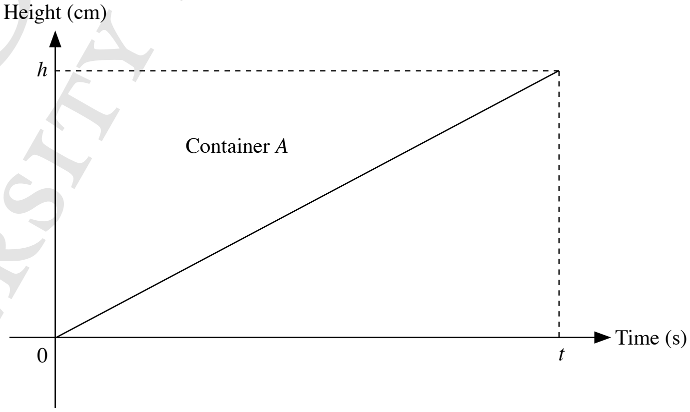


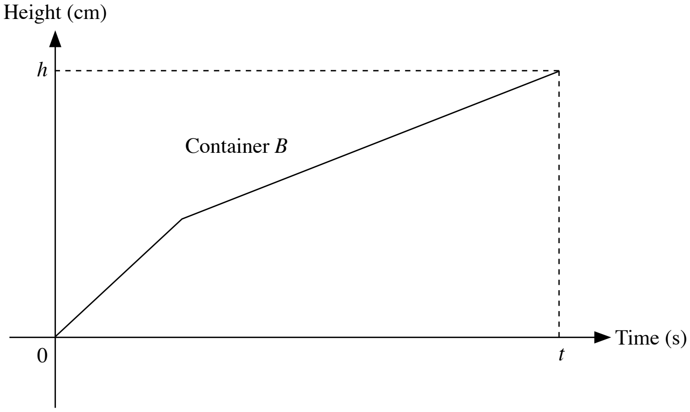
(ii) Gradient = $\frac{31}{10}$

$= 3.1$

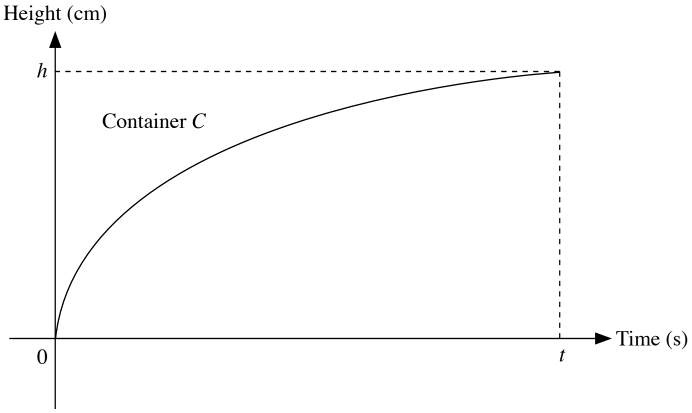
This represents the load is being lifted at a rate of 3.1 m/s at $t = 16$.

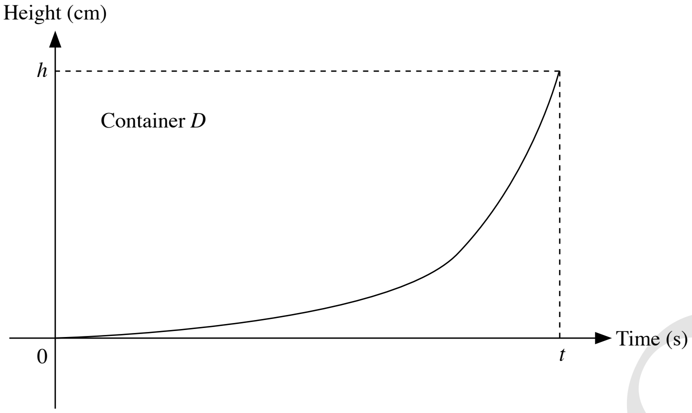
20.





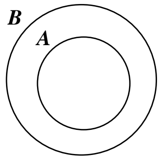
OXFORD UNIVERSITY PRESS



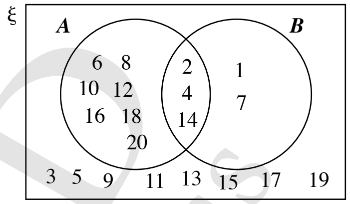


Revision 9G

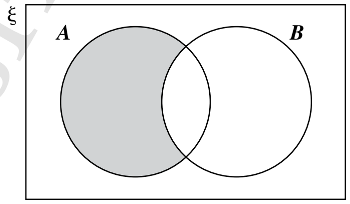
1. $A = \{2, 3, 5, 7\}, B = \{3, 5, 7, 9\}$
   (i) $A \cup B = \{2, 3, 5, 7, 9\}$
   (ii) $A \cap B' = \{2\}$
2. $A = \{\text{a}, \text{e}\}, B = \{\text{a}, \text{c}, \text{f}\}$
   (i) $A \cap B = \{\text{a}\}$
   (ii) $A \cup B = \{\text{a}, \text{c}, \text{e}, \text{f}\}$
   (iii) $A \cap B' = \{\text{e}\}$
3. $A = \{2, 3, 5, 7\}, B = \{2, 4, 6, 8, 10\}, C = \{3, 6, 9\}$
   (i) $A \cap B = \{2\}$
   (ii) $A \cup B' = \{1, 2, 3, 5, 7, 9\}$
   (iii) $A \cap C' = \{2, 5, 7\}$
4. $\xi = \{11, 12, 13, 14, 15, ..., 30, 31, 32, 33\}$
   (i) $A = \{15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32\}$
   (ii) $B = \{11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33\}$
   (iii) $C = \{11, 13, 17, 19, 23, 29, 31\}$
   (iv) $A \cap B = \{15, 17, 19, 21, 23, 25, 27, 29, 31\}$
   (v) $B \cap C = \{11, 13, 17, 19, 23, 29, 31\}$
   (vi) $B \cap C' = \{15, 21, 25, 27, 33\}$
5. **(i)** $A \cap B = A$
   (ii) $A \cup B = B$



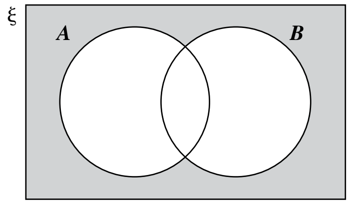
6. $A = \{50, 51, 52, ..., 99, 100\}$
   $B = \{8, 9, 10\}$
   $C = \{64, 81, 100\}$
   (i) $A \cap C = \{64, 81, 100\}$
   (ii) $B = \{8, 9, 10\}$
7. $\xi = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$,
   $A = \{1, 2, 3, 4, 6, 12\}$ and $B = \{1, 3, 5, 7, 9, 11\}$
   (i) $A = \{1, 2, 3, 4, 6, 12\}$
   (ii) $A' \cap B = \{5, 7, 9, 11\}$
   (iii) $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 9, 11, 12\}$
8. $\xi = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$, $A = \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\}$ and $B = \{1, 2, 4, 7, 14\}$
   (a)



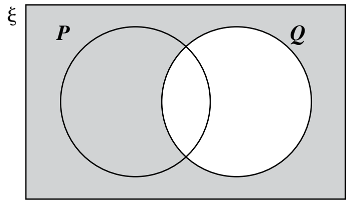
   (b) **(i)** $A \cap B = \{2, 4, 14\}$
   (ii) $A' \cup B = \{1, 2, 3, 4, 5, 7, 9, 11, 13, 14, 15, 17, 19\}$
   (iii) $A' \cap B' = \{3, 5, 9, 11, 13, 15, 17, 19\}$
9. $\xi = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24\}$, $A = \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$, $B = \{3, 6, 9, 12, 15, 18, 21, 24\}$ and $C = \{1, 2, 3, 4, 6, 8, 12, 24\}$
   (i) $C = \{1, 2, 3, 4, 6, 8, 12, 24\}$
   (ii) $A \cap C = \{2, 3\}$
   (iii) $B' \cap C = \{1, 2, 4, 8\}$
   (iv) $A \cap B' = \{2, 5, 7, 11, 13, 17, 19, 23\}$
10. **(i)**



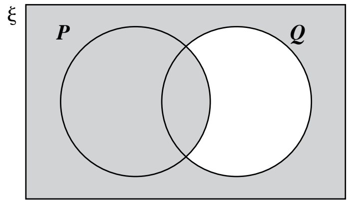
   (ii)



11. **(i)**



   (ii)



OXFORD UNIVERSITY PRESS

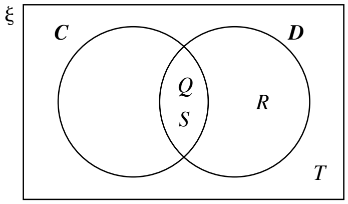
12. **(a)** $A \cap B'$

(b) $A' \cap B'$ or $(A \cup B)'$

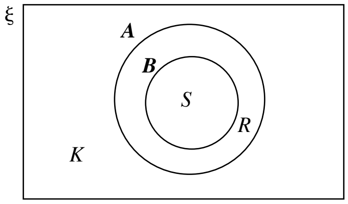
(c) $A \cap B'$

(d) $A' \cup B'$ or $(A \cap B)'$

13.



14. **(i)** and **(iii)**



(ii) $A \subset B$

15. $\xi = \{-10, -9, -8, -7, ..., -1, 0, 1, 2, ..., 12, 13, 14, 15\}$,
$A = \{-10, -9, -8, -7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and
$B = \{-4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$

(i) $(A \cup B)' = \{\ \}$ or $\varnothing$

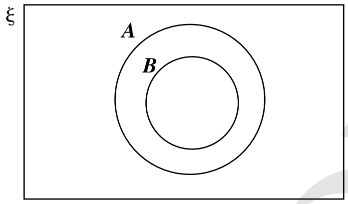
(ii) $A \cap B = \{-4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

(iii) $A \cap B' = \{-10, -9, -8, -7, -6, -5\}$

16. **(a)** **(i)** No, $A \neq B$ as not all rational numbers such as $\frac{3}{4}$, $\frac{5}{6}$, etc are integers.

(ii) No, $A \cup B \neq \xi$ as there are irrational numbers such as $\sqrt{3}$ belonging to ξ but not $A \cup B$.

(b)



(c) **(i)** $\sqrt{2}$, π, etc

(ii) $\frac{1}{3}$, $\frac{3}{4}$, etc

17. **(i)** Greatest possible number of people buying both = 12

(ii) Least possible number of people buying both = 0

(iii) Greatest possible number of people buying only one type = 26

18. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$= 12 + 6 - 4$
$= 14$

19. Number of people who can do both = 33 + 26 – 50
= 9

20. **(i)** Greatest value of $n(A \cap B) = n(B) = 9$
Least value of $n(A \cap B) = 12 + 9 - 20$
$= 1$

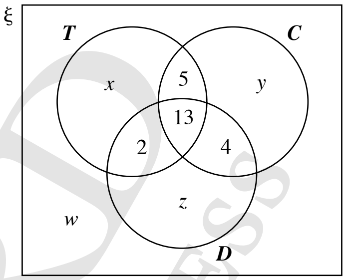
(ii) Greatest value of $n(A \cup B) = n(\xi) = 20$
Least value of $n(A \cup B) = n(A) = 12$

21. **(i)** Since A and B are disjoint sets, $n(A \cap B) = \varnothing$

(ii) $n(A \cup B) = n(A) + n(B)$
$= 11 + 6$
$= 17$

22. Let T = {people who can cook Italian food},
C = {people who can cook Chinese food} and
D = {people who can cook Indian food}
$\therefore$ $n(T) = 30$, $n(C) = 40$, $n(D) = 24$

Let x, y and z represent the number of people who can cook Italian food only, Chinese food only and Indian food only respectively; and w represent the number of people who cannot cook any of these cuisines.



$x + 2 + 5 + 13 = 30$
$x = 10$
$y + 5 + 13 + 4 = 40$
$y = 18$
$z + 2 + 13 + 4 = 24$
$z = 5$
$n(\xi) = 60$

From the Venn diagram, $w = 60 - 10 - 5 - 13 - 2 - 18 - 4 - 5$
$= 3$

$\therefore$ 3 people cannot cook any of the three cuisines.

Revision 9H

1. **(a)** $\begin{pmatrix} -1 & 1 \\ -2 & 5 \end{pmatrix} + \begin{pmatrix} 6 & 0 \\ 1 & 4 \end{pmatrix}$

$= \begin{pmatrix} -1+6 & 1+0 \\ -2+1 & 5+4 \end{pmatrix}$

$= \begin{pmatrix} 5 & 1 \\ -1 & 9 \end{pmatrix}$

(b) Not possible because the third matrix does not have the same order as the first and second matrices.

(c) $\begin{pmatrix} 1 & 2 \end{pmatrix} + \begin{pmatrix} -1 & -3 \end{pmatrix} - \begin{pmatrix} -2 & 5 \end{pmatrix}$

$= \begin{pmatrix} 1+(-1)-(-2) & 2+(-3)-5 \end{pmatrix}$

$= \begin{pmatrix} 2 & -6 \end{pmatrix}$

(d) $(3) + \frac{1}{2}(2) - (0)$

$= (3 + 1 - 0)$

$= (4)$

2. (a) $\begin{pmatrix} 5 \\ -1 \end{pmatrix}\begin{pmatrix} 3 & 4 \end{pmatrix}$

$$= \begin{pmatrix} 5 \times 3 & 5 \times 4 \\ -1 \times 3 & -1 \times 4 \end{pmatrix}$$

$$= \begin{pmatrix} 15 & 20 \\ -3 & -4 \end{pmatrix}$$

(b) $\begin{pmatrix} 3 & 2 \end{pmatrix}\begin{pmatrix} 4 \\ -1 \end{pmatrix}$

$= (3 \times 4 + 2 \times (-1))$

$= (10)$

(c) Not possible since the number of columns in the first matrix is not equal to the number of rows in the second matrix.

(d) $\begin{pmatrix} 2 & -1 & 4 \end{pmatrix}\begin{pmatrix} 0 \\ -3 \\ \frac{1}{2} \end{pmatrix}$

$$= \left(2 \times 0 + (-1) \times (-3) + 4 \times \frac{1}{2} \right)$$

$= (5)$

(e) $\begin{pmatrix} 4 & 1 \\ 0 & 2 \end{pmatrix} + \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$

$$= \begin{pmatrix} 4(1)+1(3) & 4(2)+1(4) \\ 0(1)+2(3) & 0(2)+2(4) \end{pmatrix}$$

$$= \begin{pmatrix} 7 & 12 \\ 6 & 8 \end{pmatrix}$$

3. (a) $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 & -1 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 5 \\ 2 \end{pmatrix}$

$$= \begin{pmatrix} 3(2)+(-1)(3) \\ 0(2)+2(3) \end{pmatrix} + \begin{pmatrix} 5 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} 3 \\ 6 \end{pmatrix} + \begin{pmatrix} 5 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} 8 \\ 8 \end{pmatrix}$$

$\therefore x = 8, y = 8$

(b) $2\begin{pmatrix} x \\ y \end{pmatrix} - 4\begin{pmatrix} 2 \\ 5 \end{pmatrix} = \begin{pmatrix} 4x \\ 3y \end{pmatrix}$

$$\begin{pmatrix} 2x-8 \\ 2y-20 \end{pmatrix} = \begin{pmatrix} 4x \\ 3y \end{pmatrix}$$

$2x - 8 = 4x$ $\qquad$ $2y - 20 = 3y$

$2x - 4x = 8$ $\qquad$ $2y - 3y = 20$

$-2x = 8$ $\qquad$ $-y = 20$

$x = -4$ $\qquad$ $y = -20$

$\therefore x = -4, y = -20$

(c) $x\begin{pmatrix} 2 \\ 4 \end{pmatrix} + y\begin{pmatrix} 5 \\ 7 \end{pmatrix} = \begin{pmatrix} -5 \\ -1 \end{pmatrix}$

$$\begin{pmatrix} 2x+5y \\ 4x+7y \end{pmatrix} = \begin{pmatrix} -5 \\ -1 \end{pmatrix}$$

$2x + 5y = -5$ — (1)

$4x + 7y = -1$ — (2)

$(1) \times 2$: $4x + 10y = -10$ — (3)

$(3) - (2)$: $4x + 10y - 4x - 7y = -10 + 1$

$3y = -9$

$y = -3$

Substitute $y = -3$ into (1):

$2x + 5(-3) = -5$

$2x = -5 + 15$

$2x = 10$

$x = 5$

$\therefore x = 5, y = -3$

4. (a) $\begin{pmatrix} 0 & 2 \\ -1 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

$$\begin{pmatrix} 0 \times x + 2 \times y \\ -1 \times x + 4 \times y \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} 2y \\ -x+4y \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

$2y = 2$

$y = 1$

$-x + 4y = 3$

$-x + 4(1) = 3$

$-x = 3 - 4$

$x = 1$

$\therefore x = 1, y = 1$

(b) $\begin{pmatrix} 2 & 1 \\ 4 & x \end{pmatrix}\begin{pmatrix} y \\ 2 \end{pmatrix} = \begin{pmatrix} 6 \\ 10 \end{pmatrix}$

$$\begin{pmatrix} 2 \times y + 1 \times 2 \\ 4 \times y + x \times 2 \end{pmatrix} = \begin{pmatrix} 6 \\ 10 \end{pmatrix}$$

$$\begin{pmatrix} 2y+2 \\ 4y+2x \end{pmatrix} = \begin{pmatrix} 6 \\ 10 \end{pmatrix}$$

$2y + 2 = 6$

$2y = 4$

$y = 2$

$4y + 2x = 10$

$4(2) + 2x = 10$

$2x = 2$

$x = 1$

$\therefore x = 1, y = 2$

OXFORD UNIVERSITY PRESS

(c) $$\begin{pmatrix} 3 & 1 \\ x & 4 \end{pmatrix}\begin{pmatrix} y \\ 1 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 3\times y+1\times 1 \\ x\times y+4\times 1 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 3y+1 \\ xy+4 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$

$3y + 1 = -2$

$3y = -3$

$y = -1$

$xy + 4 = 1$

$x(-1) = -3$

$x = 3$

$\therefore x = 3, y = -1$

5. $$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \end{pmatrix}\begin{pmatrix} p \\ 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 2q \end{pmatrix}$$

$$\begin{pmatrix} 1\times p+2\times 0+3\times 2 \\ 0\times p+1\times 0+4\times 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 2q \end{pmatrix}$$

$$\begin{pmatrix} p+6 \\ 8 \end{pmatrix} = \begin{pmatrix} 2 \\ 2q \end{pmatrix}$$

$p + 6 = 2$

$p = -4$

$8 = 2q$

$q = 4$

$\therefore p = -4, q = 4$

6. (a) $$\begin{pmatrix} 4 & 2 \\ -1 & p \end{pmatrix}\begin{pmatrix} 2 & 3 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} q & 14 \\ -2 & 7 \end{pmatrix}$$

$$\begin{pmatrix} 4\times 2+2\times 0 & 4\times 3+2\times 1 \\ -1\times 2+p\times 0 & -1\times 3+p\times 1 \end{pmatrix} = \begin{pmatrix} q & 14 \\ -2 & 7 \end{pmatrix}$$

$$\begin{pmatrix} 8 & 14 \\ -2 & -3+p \end{pmatrix} = \begin{pmatrix} q & 14 \\ -2 & 7 \end{pmatrix}$$

$q = 8$

$-3 + p = 7$

$p = 10$

$\therefore p = 10, q = 8$

(b) (i) AB

$$= \begin{pmatrix} 3 & 1 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 2 & 6 \\ 0 & k \end{pmatrix}$$

$$= \begin{pmatrix} 3\times 2+1\times 0 & 3\times 6+1\times k \\ 0\times 2+2\times 0 & 0\times 6+2\times k \end{pmatrix}$$

$$= \begin{pmatrix} 6 & 18+k \\ 0 & 2k \end{pmatrix}$$

BA

$$= \begin{pmatrix} 2 & 6 \\ 0 & k \end{pmatrix}\begin{pmatrix} 3 & 1 \\ 0 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 6 & 14 \\ 0 & 2k \end{pmatrix}$$

(ii) $\mathbf{AB} = \mathbf{BA}$

$$\begin{pmatrix} 6 & 18+k \\ 0 & 2k \end{pmatrix} = \begin{pmatrix} 6 & 14 \\ 0 & 2k \end{pmatrix}$$

$18 + k = 14$

$\therefore k = -4$

7. (i) $2\mathbf{A} + \mathbf{B} = \mathbf{C}$

$$2\begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix} + \begin{pmatrix} 5 & w \\ y & 4 \end{pmatrix} = \begin{pmatrix} x & 6 \\ 4 & z \end{pmatrix}$$

$$\begin{pmatrix} 4 & -2 \\ 2 & 6 \end{pmatrix} + \begin{pmatrix} 5 & w \\ y & 4 \end{pmatrix} = \begin{pmatrix} x & 6 \\ 4 & z \end{pmatrix}$$

$$\begin{pmatrix} 4+5 & -2+w \\ 2+y & 6+4 \end{pmatrix} = \begin{pmatrix} x & 6 \\ 4 & z \end{pmatrix}$$

$$\begin{pmatrix} 9 & -2+w \\ 2+y & 10 \end{pmatrix} = \begin{pmatrix} x & 6 \\ 4 & z \end{pmatrix}$$

$x = 9$

$-2 + w = 6$

$w = 8$

$2 + y = 4$

$y = 2$

$z = 10$

$\therefore w = 8, x = 9, y = 2, z = 10$

OXFORD UNIVERSITY PRESS

(ii) $3\mathbf{A} - 2\mathbf{B} = 4\mathbf{C}$

$$3\begin{pmatrix}2&-1\\1&3\end{pmatrix}-2\begin{pmatrix}5&w\\y&4\end{pmatrix}=4\begin{pmatrix}x&6\\4&z\end{pmatrix}$$

$$\begin{pmatrix}6&-3\\3&9\end{pmatrix}-\begin{pmatrix}10&2w\\2y&8\end{pmatrix}=\begin{pmatrix}4x&24\\16&4z\end{pmatrix}$$

$$\begin{pmatrix}6-10&-3-2w\\3-2y&9-8\end{pmatrix}=\begin{pmatrix}4x&24\\16&4z\end{pmatrix}$$

$$\begin{pmatrix}-4&-3-2w\\3-2y&1\end{pmatrix}=\begin{pmatrix}4x&24\\16&4z\end{pmatrix}$$

$-4 = 4x$

$x = -1$

$-3 - 2w = 24$

$-2w = 27$

$w = -13\frac{1}{2}$

$3 - 2y = 16$

$-2y = 13$

$y = -6\frac{1}{2}$

$1 = 4z$

$z = \frac{1}{4}$

$\therefore\ w = -13\frac{1}{2},\ x = -1,\ y = -6\frac{1}{2},\ z = \frac{1}{4}$

8. (i) $2\mathbf{A} + \mathbf{B} - \mathbf{C}$

$$=2\begin{pmatrix}-2&1\\-1&4\end{pmatrix}+\begin{pmatrix}3&1\\-2&-5\end{pmatrix}-\begin{pmatrix}3&-1\\4&-2\end{pmatrix}$$

$$=\begin{pmatrix}-4&2\\-2&8\end{pmatrix}+\begin{pmatrix}3&1\\-2&-5\end{pmatrix}-\begin{pmatrix}3&-1\\4&-2\end{pmatrix}$$

$$=\begin{pmatrix}-4+3-3&2+1-(-1)\\-2+(-2)-4&8+(-5)-(-2)\end{pmatrix}$$

$$=\begin{pmatrix}-4&4\\-8&5\end{pmatrix}$$

(ii) $\mathbf{B} + \mathbf{AC}$

$$=\begin{pmatrix}3&1\\-2&-5\end{pmatrix}+\begin{pmatrix}-2&1\\-1&4\end{pmatrix}\begin{pmatrix}3&-1\\4&-2\end{pmatrix}$$

$$=\begin{pmatrix}3&1\\-2&-5\end{pmatrix}+\begin{pmatrix}-2\times3+1\times4&-2\times(-1)+1\times(-2)\\-1\times3+4\times4&-1\times(-1)+4\times(-2)\end{pmatrix}$$

$$=\begin{pmatrix}3&1\\-2&-5\end{pmatrix}+\begin{pmatrix}-2&0\\13&-7\end{pmatrix}$$

$$=\begin{pmatrix}1&1\\11&-12\end{pmatrix}$$

(iii) $\mathbf{A} + \mathbf{BC}$

$$=\begin{pmatrix}-2&1\\-1&4\end{pmatrix}+\begin{pmatrix}3&1\\-2&-5\end{pmatrix}\begin{pmatrix}3&-1\\4&-2\end{pmatrix}$$

$$=\begin{pmatrix}-2&1\\-1&4\end{pmatrix}+\begin{pmatrix}3\times3+1\times4&3\times(-1)+1\times(-2)\\-2\times3+(-5)\times4&-2\times(-1)+(-5)\times(-2)\end{pmatrix}$$

$$=\begin{pmatrix}-2&1\\-1&4\end{pmatrix}+\begin{pmatrix}13&-5\\-26&12\end{pmatrix}$$

$$=\begin{pmatrix}11&-4\\-27&16\end{pmatrix}$$

9. $\mathbf{AB} = \mathbf{BA}$

$$\begin{pmatrix}-4&p\\-1&2\end{pmatrix}\begin{pmatrix}q&0\\2&3\end{pmatrix}=\begin{pmatrix}q&0\\2&3\end{pmatrix}\begin{pmatrix}-4&p\\-1&2\end{pmatrix}$$

$$\begin{pmatrix}-4\times q+p\times2&-4\times0+p\times3\\-1\times q+2\times2&-1\times0+2\times3\end{pmatrix}=$$

$$\begin{pmatrix}q\times(-4)+0\times(-1)&q\times p+0\times2\\2\times(-4)+3\times(-1)&2\times p+3\times2\end{pmatrix}$$

$$\begin{pmatrix}-4q+2p&3p\\-q+4&6\end{pmatrix}=\begin{pmatrix}-4q&pq\\-11&2p+6\end{pmatrix}$$

$2p + 6 = 6$

$p = 0$

$-q + 4 = -11$

$q = 15$

$\therefore\ p = 0,\ q = 15$

10. (i) $12\mathbf{A} = 12\begin{pmatrix}120\\95\\102\end{pmatrix}$

$$=\begin{pmatrix}1440\\1140\\1224\end{pmatrix}$$

(ii) $12\mathbf{A} + \mathbf{W} = \begin{pmatrix}1440\\1140\\1224\end{pmatrix}+\begin{pmatrix}60\\75\\62\end{pmatrix}$

$$=\begin{pmatrix}1500\\1215\\1286\end{pmatrix}$$

It represents the fee charged by the three cable television providers for one year including the cost for the World Cup special package.

OXFORD UNIVERSITY PRESS

11. Let $\mathbf{C} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$$\begin{pmatrix} 0.6 & 0.8 \\ 0.8 & -0.6 \end{pmatrix}\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1.2 & 1.6 \\ 1.6 & -1.2 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1.2a + 1.6c & 1.2b + 1.6d \\ 1.6a - 1.2c & 1.6b - 1.2d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Equating the corresponding elements,

$1.2a + 1.6c = 1$ — (1)

$1.2b + 1.6d = 0$ — (2)

$1.6a - 1.2c = 0$ — (3)

$1.6b - 1.2d = 1$ — (4)

(1) × 4: $4.8a + 6.4c = 4$

(3) × 3: $4.8a - 3.6c = 0$

Solving the above two equations simultaneously, $c = 0.4$, $a = 0.3$

(2) × 4: $4.8b + 6.4d = 0$

(4) × 3: $4.8b - 3.6d = 3$

Solving the above two equations simultaneously, $d = -0.3$, $b = 0.4$

$$\therefore \mathbf{C} = \begin{pmatrix} 0.3 & 0.4 \\ 0.4 & -0.3 \end{pmatrix}$$

12. $$\mathbf{AB} = \begin{pmatrix} 1 & 3 \\ -2 & 2 \end{pmatrix}\begin{pmatrix} \frac{1}{4} & -1 \\ 1\frac{1}{4} & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{4} + \frac{15}{4} & -1 + 3 \\ -\frac{2}{4} + \frac{10}{4} & 2 + 2 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & 2 \\ 2 & 4 \end{pmatrix}$$

$$\mathbf{AC} = \begin{pmatrix} 1 & 3 \\ -2 & 2 \end{pmatrix}\begin{pmatrix} 1\frac{3}{4} & -2 \\ \frac{3}{4} & 0 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{7}{4} + \frac{9}{4} & -2 + 0 \\ -\frac{14}{4} + \frac{6}{4} & 4 + 0 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & -2 \\ -2 & 4 \end{pmatrix}$$

$$\mathbf{AB} + \mathbf{AC} = \begin{pmatrix} 4 & 2 \\ 2 & 4 \end{pmatrix} + \begin{pmatrix} 4 & -2 \\ -2 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix}$$

$$= 8\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \mathbf{I} = \mathbf{AA}^{-1}$$

$$\frac{1}{8}(\mathbf{AB} + \mathbf{AC}) = \mathbf{AA}^{-1}$$

$$\frac{1}{8}\mathbf{A}(\mathbf{B} + \mathbf{C}) = \mathbf{AA}^{-1}$$

$$\mathbf{A}^{-1} = \frac{1}{8}(\mathbf{B} + \mathbf{C})$$

$$= \frac{1}{8}\left[\begin{pmatrix} \frac{1}{4} & -1 \\ 1\frac{1}{4} & 1 \end{pmatrix} + \begin{pmatrix} 1\frac{3}{4} & -2 \\ \frac{3}{4} & 0 \end{pmatrix}\right]$$

$$= \frac{1}{8}\begin{pmatrix} 2 & -3 \\ 2 & 1 \end{pmatrix}$$

$$\mathbf{A}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

$$\mathbf{A}^{-1}\mathbf{A}\begin{pmatrix} x \\ y \end{pmatrix} = \mathbf{A}^{-1}\begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{8}\begin{pmatrix} 2 & -3 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

$$= \frac{1}{8}\begin{pmatrix} -8 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$\therefore x = -1, y = 0$

13. $$\mathbf{AB} = \begin{pmatrix} 1 & 2 & 1 \\ 3 & 4 & 2 \end{pmatrix}\begin{pmatrix} 1 & 3 \\ 0 & 2 \\ 2 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 + 0 + 2 & 3 + 4 + 1 \\ 3 + 0 + 4 & 9 + 8 + 2 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & 8 \\ 7 & 19 \end{pmatrix}$$

$$\mathbf{BA} = \begin{pmatrix} 1 & 3 \\ 0 & 2 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 & 1 \\ 3 & 4 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 + 9 & 2 + 12 & 1 + 6 \\ 0 + 6 & 0 + 8 & 0 + 4 \\ 2 + 3 & 4 + 4 & 2 + 2 \end{pmatrix}$$

$$= \begin{pmatrix} 10 & 14 & 7 \\ 6 & 8 & 4 \\ 5 & 8 & 4 \end{pmatrix}$$

OXFORD UNIVERSITY PRESS

14. Determinant of **B** $= (3 \times 2) - (5 \times 1)$

$$= 1$$

$$\mathbf{B}^{-1} = \begin{pmatrix} 2 & -5 \\ -1 & 3 \end{pmatrix}$$

(i)
$$\begin{aligned} \mathbf{PB} &= \mathbf{A} \\ \mathbf{PBB}^{-1} &= \mathbf{AB}^{-1} \\ \mathbf{PI} &= \mathbf{AB}^{-1} \\ \mathbf{P} &= \begin{pmatrix} 7 & 5 \\ 8 & 9 \end{pmatrix}\begin{pmatrix} 2 & -5 \\ -1 & 3 \end{pmatrix} \\ &= \begin{pmatrix} 9 & -20 \\ 7 & -13 \end{pmatrix} \end{aligned}$$

(ii)
$$\begin{aligned} \mathbf{BQ} &= 2\mathbf{A} \\ \mathbf{B}^{-1}\mathbf{BQ} &= \mathbf{B}^{-1}2\mathbf{A} \\ \mathbf{IQ} &= \mathbf{B}^{-1}2\mathbf{A} \\ \mathbf{Q} &= \begin{pmatrix} 2 & -5 \\ -1 & 3 \end{pmatrix}2\begin{pmatrix} 7 & 5 \\ 8 & 9 \end{pmatrix} \\ &= \begin{pmatrix} 2 & -5 \\ -1 & 3 \end{pmatrix}\begin{pmatrix} 14 & 10 \\ 16 & 18 \end{pmatrix} \\ &= \begin{pmatrix} -52 & -70 \\ 34 & 44 \end{pmatrix} \end{aligned}$$

15. (a) The simultaneous equations may be written in matrix form as

$$\begin{pmatrix} 2 & -3 \\ 4 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 8 \\ 4 \end{pmatrix}$$

Determinant of $\begin{pmatrix} 2 & -3 \\ 4 & -1 \end{pmatrix} = [2 \times (-1)] - [(-3) \times 4]$

$$= 10$$

$$\begin{aligned} \begin{pmatrix} x \\ y \end{pmatrix} &= \begin{pmatrix} 2 & -3 \\ 4 & -1 \end{pmatrix}^{-1}\begin{pmatrix} 8 \\ 4 \end{pmatrix} \\ &= \frac{1}{10}\begin{pmatrix} -1 & 3 \\ -4 & 2 \end{pmatrix}\begin{pmatrix} 8 \\ 4 \end{pmatrix} \\ &= \frac{1}{10}\begin{pmatrix} 84 \\ -24 \end{pmatrix} \\ &= \begin{pmatrix} 0.4 \\ -2.4 \end{pmatrix} \end{aligned}$$

$\therefore x = 0.4, y = -2.4$

The graphs of $2x - 3y = 8$ and $4x - y = 4$ intersect at the point $(0.4, -2.4)$.

(b) The simultaneous equations may be written in matrix form as

$$\begin{pmatrix} 5 & -6 \\ 10 & -12 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 12 \\ 24 \end{pmatrix}$$

Determinant of $\begin{pmatrix} 5 & -6 \\ 10 & -12 \end{pmatrix} = [5 \times (-12)] - [(-6) \times 10]$

$$= 0$$

Hence $\begin{pmatrix} 5 & -6 \\ 10 & -12 \end{pmatrix}$ is a singular matrix and its inverse matrix does not exist. The second equation is obtained from the first by multiplying throughout by 2. Thus, the graphs of $5x - 6y = 12$ and $10x - 12y = 24$ represent the same line, i.e. the two lines coincide, and there is an infinite number of solutions.

(c) The simultaneous equations may be written in matrix form as

$$\begin{pmatrix} 6 & -2 \\ 3 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 9 \\ 1 \end{pmatrix}$$

Determinant of $\begin{pmatrix} 6 & -2 \\ 3 & -1 \end{pmatrix} = [6 \times (-1)] - [(-2) \times 3]$

$$= 0$$

Hence $\begin{pmatrix} 6 & -2 \\ 3 & -1 \end{pmatrix}$ is a singular matrix and its inverse matrix does not exist. The graphs of $6x - 2y = 9$ and $3x - y = 1$ represent two parallel lines. There is no solution since there is no intersection between the two lines.

16. (i) $\mathbf{R} = \mathbf{AM}$

$$\begin{aligned} &= \begin{pmatrix} 39 & 5 & 6 \\ 29 & 8 & 13 \end{pmatrix}\begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} \\ &= \begin{pmatrix} 39 \times 2 + 5 \times 0 + 6 \times (-1) \\ 29 \times 2 + 8 \times 0 + 13 \times (-1) \end{pmatrix} \\ &= \begin{pmatrix} 72 \\ 45 \end{pmatrix} \end{aligned}$$

(ii) It represents that Lixin scored 72 marks and Rui Feng scored 45 marks for the driving theory test.

17. (a)
$$\begin{aligned} \begin{pmatrix} 0 & 5 \\ -2 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} &= \begin{pmatrix} -15 \\ 28 \end{pmatrix} \\ \begin{pmatrix} 0 \times x + 5 \times y \\ -2 \times x + 4 \times y \end{pmatrix} &= \begin{pmatrix} -15 \\ 28 \end{pmatrix} \\ \begin{pmatrix} 5y \\ -2x + 4y \end{pmatrix} &= \begin{pmatrix} -15 \\ 28 \end{pmatrix} \end{aligned}$$

$$\begin{aligned} 5y &= -15 \\ y &= -3 \\ -2x + 4y &= 28 \\ -2x + 4(-3) &= 28 \\ -2x &= 40 \\ x &= -20 \end{aligned}$$

$\therefore x = -20, y = -3$

(b) (i)
$$\begin{aligned} \mathbf{AB} &= \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}\begin{pmatrix} 1 \\ 3 \end{pmatrix} \\ &= \begin{pmatrix} 3 \times 1 + 2 \times 3 \\ 1 \times 1 + 4 \times 3 \end{pmatrix} \\ &= \begin{pmatrix} 9 \\ 13 \end{pmatrix} \end{aligned}$$

(ii) Not possible

OXFORD UNIVERSITY PRESS

(iii) $\mathbf{BC} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \begin{pmatrix} 2 & 1 \end{pmatrix}$

$$= \begin{pmatrix} 1\times 2 & 1\times 1 \\ 3\times 2 & 3\times 1 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 1 \\ 6 & 3 \end{pmatrix}$$

(iv) $\mathbf{CB} = \begin{pmatrix} 2 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

$$= (2 \times 1 + 1 \times 3)$$

$$= (5)$$

18. $\mathbf{AB} = \mathbf{A} + \mathbf{B}$

$$\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix} + \begin{pmatrix} a & b \\ 0 & c \end{pmatrix}$$

$$\begin{pmatrix} 2\times a + 0\times 0 & 2\times b + 0\times c \\ 0\times a + 3\times 0 & 0\times b + 3\times c \end{pmatrix} = \begin{pmatrix} 2+a & 0+b \\ 0+0 & 3+c \end{pmatrix}$$

$$\begin{pmatrix} 2a & 2b \\ 0 & 3c \end{pmatrix} = \begin{pmatrix} 2+a & b \\ 0 & 3+c \end{pmatrix}$$

$2a = 2 + a$

$a = 2$

$2b = b$

$b = 0$

$3c = 3 + c$

$c = 1\frac{1}{2}$

$\therefore\ a = 2, b = 0, c = 1\frac{1}{2}$

19. (i) $\mathbf{P} = \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}$

(ii) $\mathbf{RP} = \begin{pmatrix} 12 & 5 & 3 \\ 3 & 8 & 7 \\ 9 & 4 & 4 \end{pmatrix} \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}$

$$= \begin{pmatrix} 12\times 3 + 5\times 1 + 3\times 0 \\ 3\times 3 + 8\times 1 + 7\times 0 \\ 9\times 3 + 4\times 1 + 4\times 0 \end{pmatrix}$$

$$= \begin{pmatrix} 41 \\ 17 \\ 31 \end{pmatrix}$$

(iii) It represents that Wanderers scored 41 points, United scored 17 points and Saints scored 31 points.

20. (i) $\mathbf{K} = 5\mathbf{A}$

$$= 5\begin{pmatrix} 13 & 14 \\ 10 & 12 \end{pmatrix}$$

$$= \begin{pmatrix} 65 & 70 \\ 50 & 60 \end{pmatrix}$$

(ii) $\mathbf{L} = \mathbf{K}\begin{pmatrix} 2.50 \\ 1.80 \end{pmatrix}$

$$= \begin{pmatrix} 65 & 70 \\ 50 & 60 \end{pmatrix} \begin{pmatrix} 2.50 \\ 1.80 \end{pmatrix}$$

$$= \begin{pmatrix} 65\times 2.50 + 70\times 1.80 \\ 50\times 2.50 + 60\times 1.80 \end{pmatrix}$$

$$= \begin{pmatrix} 288.50 \\ 233 \end{pmatrix}$$

(iii) It represents that the total fees paid by the boys for one week is \$288.50 and the total fees paid by the girls for one week is \$233.

21. (i) $\mathbf{PQ} = \begin{pmatrix} 75 & 84 & 135 \\ 88 & 95 & 140 \end{pmatrix} \begin{pmatrix} 0.7 \\ 0.85 \\ 1 \end{pmatrix}$

$$= \begin{pmatrix} 75\times 0.7 + 84\times 0.85 + 135\times 1 \\ 88\times 0.7 + 95\times 0.85 + 140\times 1 \end{pmatrix}$$

$$= \begin{pmatrix} 258.90 \\ 282.35 \end{pmatrix}$$

It represents the cost price for Café *A* and Café *B* respectively on a particular day.

(ii) $\mathbf{R} - \mathbf{Q} = \begin{pmatrix} 3.2 \\ 4 \\ 4.5 \end{pmatrix} - \begin{pmatrix} 0.7 \\ 0.85 \\ 1 \end{pmatrix}$

$$= \begin{pmatrix} 3.2 - 0.7 \\ 4 - 0.85 \\ 4.5 - 1 \end{pmatrix}$$

$$= \begin{pmatrix} 2.5 \\ 3.15 \\ 3.5 \end{pmatrix}$$

It represents the profit on a cup of latte, mocha and iced lemon tea respectively.

OXFORD UNIVERSITY PRESS

(iii) $$\mathbf{P}(\mathbf{R}-\mathbf{Q}) = \begin{pmatrix} 75 & 84 & 135 \\ 88 & 95 & 140 \end{pmatrix}\begin{pmatrix} 2.5 \\ 3.15 \\ 3.5 \end{pmatrix}$$

$$= \begin{pmatrix} 75 \times 2.5 + 84 \times 3.15 + 135 \times 3.5 \\ 88 \times 2.5 + 95 \times 3.15 + 140 \times 3.5 \end{pmatrix}$$

$$= \begin{pmatrix} 924.60 \\ 1009.25 \end{pmatrix}$$

It represents the total profit made by Café *A* and Café *B* respectively on a particular day.

22. (i) $$\mathbf{X} = \mathbf{A}\begin{pmatrix} 10 \\ 20 \\ 50 \end{pmatrix}$$

$$= \begin{pmatrix} 65 & 45 & 46 \\ 60 & 56 & 58 \\ 78 & 56 & 54 \end{pmatrix}\begin{pmatrix} 10 \\ 20 \\ 50 \end{pmatrix}$$

$$= \begin{pmatrix} 65 \times 10 + 45 \times 20 + 46 \times 50 \\ 60 \times 10 + 56 \times 20 + 58 \times 50 \\ 78 \times 10 + 56 \times 20 + 54 \times 50 \end{pmatrix}$$

$$= \begin{pmatrix} 3850 \\ 4620 \\ 4600 \end{pmatrix}$$

It represents the total amount in cents, collected from 10, 20 and 50 cent-coins, on Friday, Saturday and Sunday respectively.

(ii) $$\mathbf{Y} = \mathbf{B}\begin{pmatrix} 1 \\ 2 \\ 5 \\ 10 \end{pmatrix}$$

$$= \begin{pmatrix} 32 & 26 & 18 & 16 \\ 45 & 34 & 20 & 10 \\ 38 & 22 & 25 & 24 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \\ 5 \\ 10 \end{pmatrix}$$

$$= \begin{pmatrix} 32 \times 1 + 26 \times 2 + 18 \times 5 + 16 \times 10 \\ 45 \times 1 + 34 \times 2 + 20 \times 5 + 10 \times 10 \\ 38 \times 1 + 22 \times 2 + 25 \times 5 + 24 \times 10 \end{pmatrix}$$

$$= \begin{pmatrix} 334 \\ 313 \\ 447 \end{pmatrix}$$

It represents the total amount in dollars, collected from \$1, \$2, \$5 and \$10 notes, on Friday, Saturday and Sunday respectively.

(iii) $$\mathbf{S} = \frac{1}{100} \times \begin{pmatrix} 1 & 1 & 1 \end{pmatrix}\mathbf{X}$$

$$= \frac{1}{100} \times \begin{pmatrix} 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} 3850 \\ 4620 \\ 4600 \end{pmatrix}$$

$$= \frac{1}{100} \times (3850 + 4620 + 4600)$$

$$= \frac{1}{100} \times (13\,070)$$

$$= (130.70)$$

It represents the total amount of 10, 20 and 50 cent-coins, in dollars, collected on the three days.

(iv) $$\mathbf{T} = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} 334 \\ 313 \\ 447 \end{pmatrix} + (130.70)$$

$$= (334 + 313 + 447) + (130.70)$$

$$= (1094) + (130.70)$$

$$= (1224.70)$$

It represents the total amount of money, in different denominations, collected on the three days.

23. (i) $$\begin{pmatrix} 50 & 60 & 70 & 40 \\ 30 & 40 & 50 & 30 \\ 40 & 30 & 60 & 50 \end{pmatrix}\begin{pmatrix} 3.20 \\ 3.10 \\ 3.00 \\ 3.30 \end{pmatrix}$$

$$= \begin{pmatrix} 688 \\ 469 \\ 566 \end{pmatrix}$$

(ii) $$\begin{pmatrix} 25 & 12 & 15 \end{pmatrix}\begin{pmatrix} 50 & 60 & 70 & 40 \\ 30 & 40 & 50 & 30 \\ 40 & 30 & 60 & 50 \end{pmatrix}$$

$$= \begin{pmatrix} 2210 & 2430 & 3250 & 2110 \end{pmatrix}$$

(iii) $$\begin{pmatrix} 2210 & 2430 & 3250 & 2110 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

$$= (10\,000)$$

24. (i) $$\begin{pmatrix} 12 & 8 & 11 \\ 18 & 11 & 7 \\ 8 & 9 & 15 \end{pmatrix}\begin{pmatrix} 48 \\ 32 \\ 26 \end{pmatrix}$$

$$= \begin{pmatrix} 1118 \\ 1398 \\ 1062 \end{pmatrix}$$

(ii) $$\begin{pmatrix} 9.80 & 10.40 & 9.90 \end{pmatrix}\begin{pmatrix} 1118 \\ 1398 \\ 1062 \end{pmatrix}$$

$$= (36\,009.40)$$

25. (i) $$\begin{pmatrix} 125 & 320 \\ 200 & 160 \\ 90 & 450 \end{pmatrix}\begin{pmatrix} 0.16 \\ 0.4 \end{pmatrix}$$

$$= \begin{pmatrix} 148 \\ 96 \\ 194.4 \end{pmatrix}$$

(ii) $$\begin{pmatrix} 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} 148 \\ 96 \\ 194.4 \end{pmatrix}$$

$$= (438.40)$$

OXFORD UNIVERSITY PRESS

Chapter 10 Revision: Geometry and Measurement

Revision 10A

1. **(a)** Let n be the number of sides of the polygon.

$$\begin{aligned}\frac{(n-2)\times 180°}{n} &= 168°\\ 180(n-2) &= 168n\\ 180n-360 &= 168n\\ 180n-168n &= 360\\ 12n &= 360\\ n &= 30\end{aligned}$$

(b) Let $x°$ be the size of the exterior angle.

$$\begin{aligned}x°+160°+x° &= 180°\\ 2x &= 20\\ x &= 10\end{aligned}$$

Number of sides of the polygon $= \frac{360°}{10°} = 36$

2. **(a)** Sum of all exterior angles of a hexagon = 360°

Let the exterior angles of the hexagon be $4x°, 5x°, 6x°, 7x°, 7x°$ and $7x°$.

$$\begin{aligned}4x+5x+6x+7x+7x+7x &= 360\\ 36x &= 360\\ x &= 10\end{aligned}$$

Largest interior angle of the hexagon $= 180° - 4\times 10°$
$= 140°$

(b) Sum of all interior angles of a heptagon $= (7-2)\times 180°$
$= 900°$

$$\begin{aligned}126+6x &= 900\\ 6x &= 774\\ x &= 129\end{aligned}$$

3. **(a)** Let the number of sides of the polygon be n.

$$\begin{aligned}(n-2)\times 180° &= 2700°\\ n-2 &= \frac{2700}{180}\\ n-2 &= 15\\ n &= 17\end{aligned}$$

(b)
$$\begin{aligned}36°+55°+65°+(n-3)\times 8.5° &= 360°\\ 156+8.5n-25.5 &= 360\\ 8.5n &= 229.5\\ n &= 27\end{aligned}$$

4. **(i)** $\angle BPQ = \angle ABP$ (alt. $\angle$s, AB // PQ)
$= 72°$

(ii) $\angle PTQ = 180° - 72° - 42°$ ($\angle$ sum of $\triangle PTQ$)
$= 66°$

(iii) $\angle RQP = 180° - 72°$ (int. $\angle$s, BP // RQ)
$= 108°$
$\angle RQT = 108° - 42°$
$= 66°$

5. **(i)** $\angle HKC = 180° - 154°$ (int. $\angle$s, HK // TB)
$= 26°$

(ii) $\angle ACB = 154° - 52°$ (ext. $\angle$ of $\triangle ABC$)
$= 102°$
$\angle QCK = 102°$ (vert. opp. $\angle$s)

(iii) $\angle PQB = 180° - 52°$ (int. $\angle$s, PQ // TB)
$= 128°$

6. $\angle PBC = 180° - 72°$ (int. $\angle$s, BP // CQ)
$= 108°$
$\angle PBT = 180° - 108° - 16°$ (adj. $\angle$s on a str. line)
$= 56°$
$\angle PBH = 108° - 46°$
$= 62°$
$\angle BHK = 180° - (56° + 62°)$ (int. $\angle$s, BT // HK)
$= 62°$

7. **(i)** $\angle ABQ = 60°$ ($\triangle ABQ$ is equilateral)
$\angle ACB = 180° - 82° - 60°$ ($\angle$ sum of $\triangle ABC$)
$= 38°$

(ii) $\angle PQC = 180° - 38° - 101°$ ($\angle$ sum of $\triangle PQC$)
$= 41°$
$\angle AQB = 60°$ ($\triangle ABQ$ is equilateral)
$\angle PQR = 180° - 41° - 60°$ (adj. $\angle$s on a str. line)
$= 79°$

(iii) $\angle ABP = \angle APB = \frac{180° - 82°}{2}$ ($\triangle ABP$ is isosceles)
$= 49°$
$\angle QAB = 60°$ ($\triangle ABQ$ is equilateral)
$\angle ARB = 180° - 60° - 49°$ ($\angle$ sum of $\triangle ARB$)
$= 71°$

8. **(i)** Let $\angle ACB = \angle ABC = x°$.

Hence $\angle ADB = \angle BAD = \frac{1}{2}x°$. (ext. $\angle$ of $\triangle ADB$)

$$\begin{aligned}\therefore\ x° + \frac{1}{2}x° &= 108° \text{ (ext. } \angle \text{ of } \triangle ACD)\\ \frac{3}{2}x &= 108\\ x &= 72\end{aligned}$$

$\therefore\ \angle ACB = 72°$

(ii) $\angle ADB = \frac{72°}{2}$
$= 36°$

9. Sum of all interior angles of a quadrilateral = 360°

Let the interior angles of $ABCD$ be $x°, 2x°, 3x°$ and $4x°$.

$$\begin{aligned}x+2x+3x+4x &= 360\\ 10x &= 360\\ x &= 36\end{aligned}$$

$\therefore$ The interior angles are 36°, 72°, 108° and 144°.

Since $36° + 144° = 180°$ and $72° + 108° = 180°$, there is a pair of parallel lines and hence $ABCD$ is a trapezium.

10. **(i)** $\angle EAB = 180° - 75°$ (int. $\angle$s, AB // ED)
$= 105°$

(ii) Sum of all interior angles of a pentagon $= (5-2)\times 180°$
$= 540°$

$$\begin{aligned}180+155+3x+2x &= 540\\ 5x &= 540-180-155\\ 5x &= 205\\ x &= 41\end{aligned}$$

OXFORD UNIVERSITY PRESS

11. **(i)** $(x + 35) + (y - 22) + (2x - 17) = 180$

$3x + y = 184$ — (1)

$y - 22 = 2x - 17$

$y = 2x + 5$ — (2)

(ii) Substitute (2) into (1): $3x + (2x + 5) = 184$

$5x = 179$

$x = 35.8$

Substitute $x = 35.8$ into (2): $y = 2(35.8) + 5$

$= 76.6$

12. **(i)** $\angle CED = 90°$

$\angle ECD = 180° - 90° - 20°$ ($\angle$ sum of $\triangle ECD$)

$= 70°$

(ii) $\cos 20° = \dfrac{DE}{5}$

$DE = 5 \times \cos 20°$

$= 4.70$ cm (to 3 s.f.)

13. **(i)** $\angle ABC$ is an interior angle of an 18-sided regular polygon.

$\angle ABC = \dfrac{(18 - 2) \times 180°}{18}$

$= 160°$

(ii) $\angle ACB = \dfrac{180° - 160°}{2}$ ($\triangle ABC$ is isosceles)

$= 10°$

$\angle ACD = 160° - 10°$

$= 150°$

14. **(i)** Interior angle of $ABCDEF = \dfrac{(6 - 2) \times 180°}{6}$

$= 120°$

$\angle BAC = \dfrac{180° - 120°}{2}$ ($\triangle ABC$ is isosceles)

$= 30°$

$\angle CAF = 120° - 30°$

$= 90°$

$\angle ATE + \angle TAF = \angle AFE$ (ext. $\angle$ of $\triangle ATF$)

$\angle ATE = 120° - 90°$

$= 30°$

(ii) $\angle BCA = \angle BAC$ ($\triangle ABC$ is isosceles)

$= 30°$

$\angle BCP = 180° - 30°$ (adj. $\angle$s on a str. line)

$= 150°$

15. **(i)** $\angle BAP = \dfrac{(5 - 2) \times 180°}{5}$ (int. $\angle$ of a regular pentagon)

$= 108°$

(ii) $\angle ABC = \dfrac{(6 - 2) \times 180°}{6}$ (int. $\angle$ of a regular hexagon)

$= 120°$

$\angle ABX = \dfrac{120°}{2}$

$= 60°$

(iii) $\angle EXP = \angle ABX$ (corr. $\angle$s, $XP \parallel BA$)

$= 60°$

$\angle EXR = 180° - 60°$ (adj. $\angle$s on a str. line)

$= 120°$

16. **(i)** $\angle BCD = \dfrac{(15 - 2) \times 180°}{15}$ (int. $\angle$ of a regular 15-sided polygon)

$= 156°$

$\angle XCD = 180° - 156°$ (adj. $\angle$s on a str. line)

$= 24°$

$\angle CXD = 180 - 2 \times 24°$ (base $\angle$s of isos. $\triangle$)

$= 132°$

(ii) Since $CB = DE$ and $XC = XD$, $XB = XE$.

Therefore, $\angle XBE = \angle XEB$ and $\triangle XBE$ is isosceles.

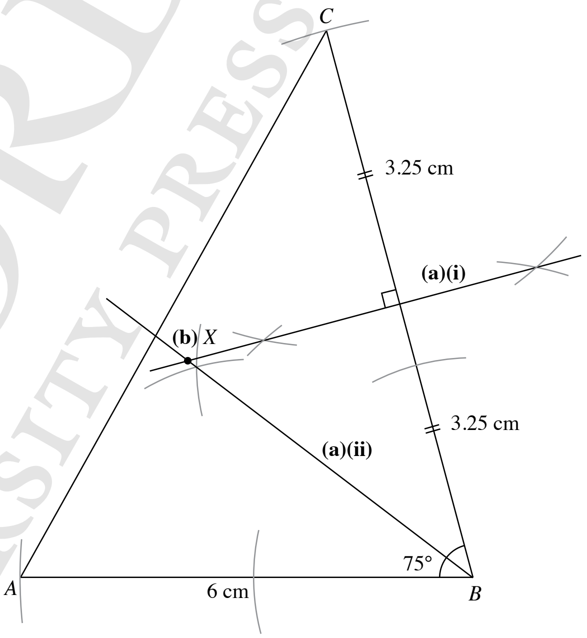
(iii) $\angle CBE = \angle XCD$ (corr. $\angle$s, $CD \parallel BE$)

$= 24°$

$\angle ABE = 156° - 24°$

$= 132°$

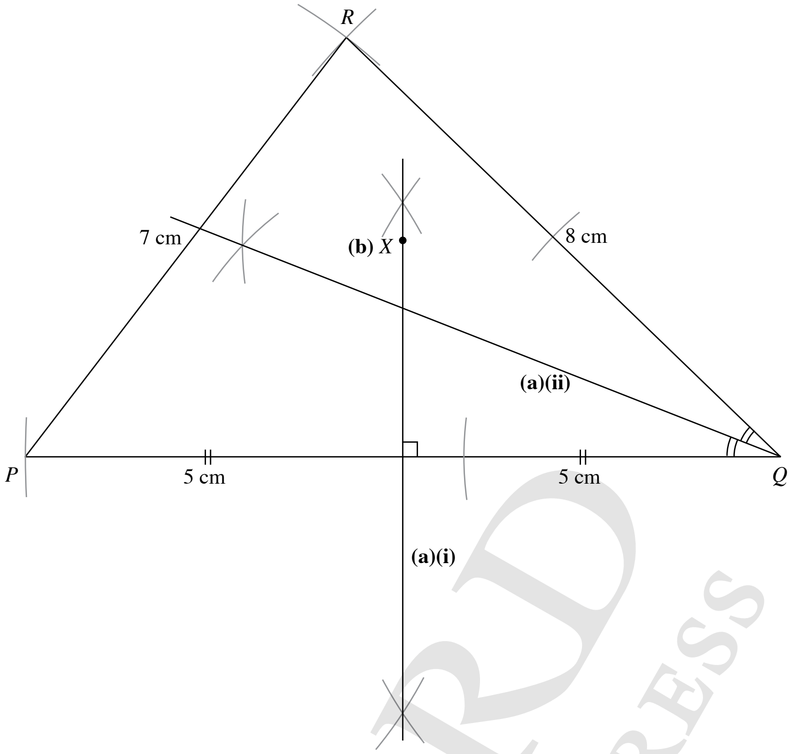
17.



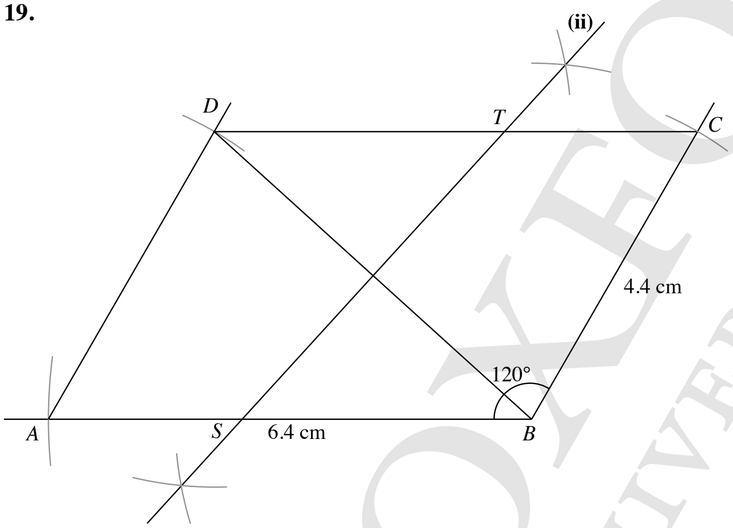
(b) The point X is equidistant from the lines <u>AB</u> and <u>BC</u> and equidistant from the points <u>B</u> and <u>C</u>.

OXFORD UNIVERSITY PRESS

18.



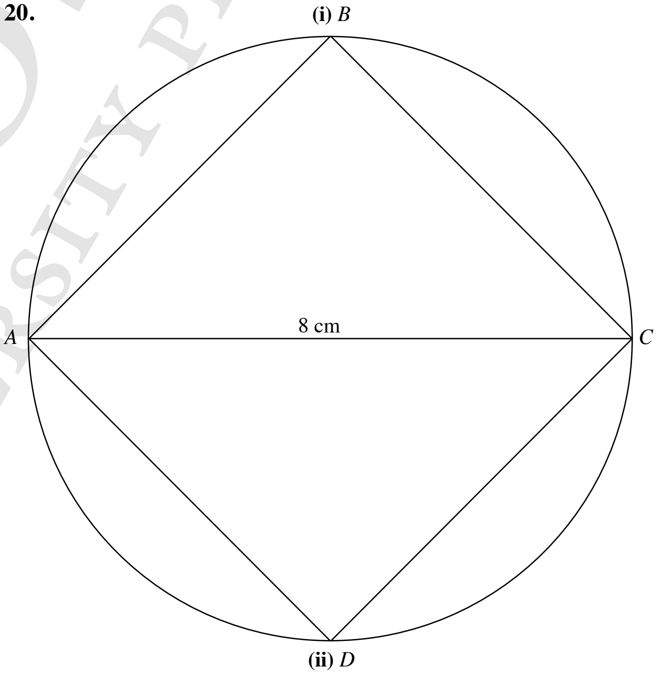
19.



(i) $BD = 5.7$ cm

(ii) $ST = 5.1$ cm

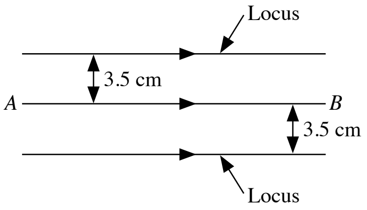
20.



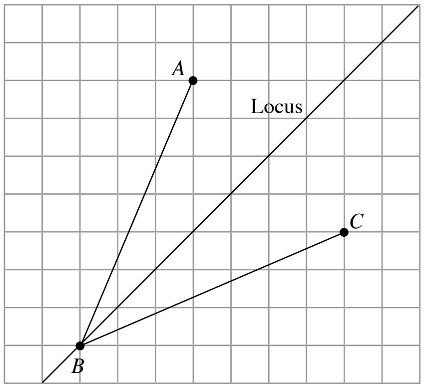
(iii) Square

OXFORD UNIVERSITY PRESS

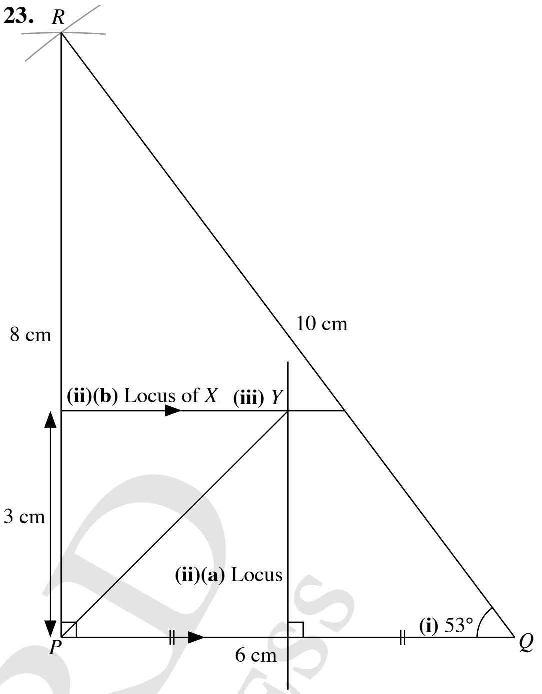
21.



22.



23.



(i) ∠$PQR = 53°$

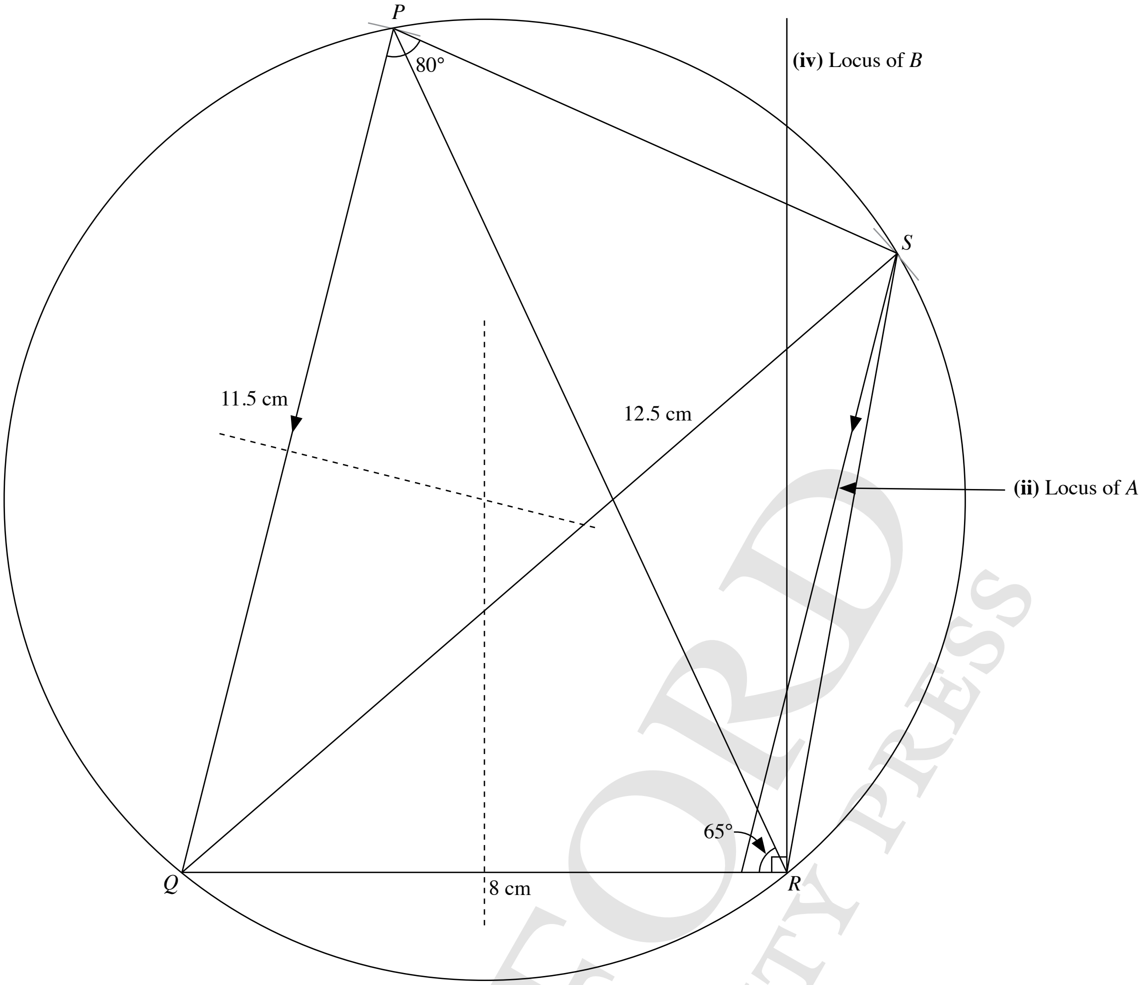
(ii) **(b)** For area of PQX to be 9 cm^2, $\frac{1}{2}(6)(PX) = 9$

$$PX = \frac{2(9)}{6}$$
$$= 3 \text{ cm}$$

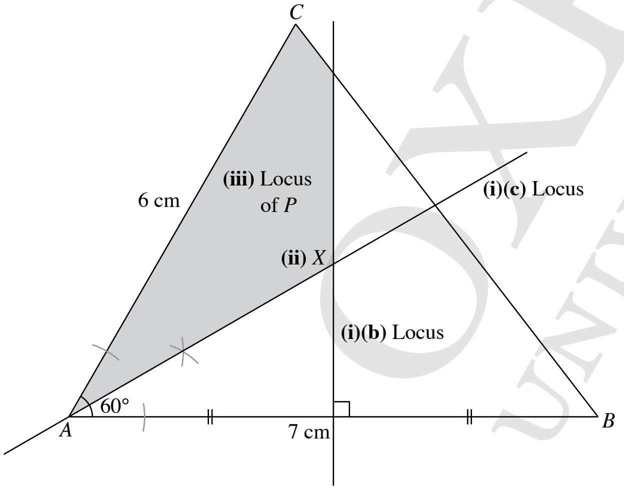
(iii) area of $PQY = 9 \text{ cm}^2$, Y is 3 cm away from the midpoint of PQ.
From diagram, length of $PY = 4.2$ cm

OXFORD UNIVERSITY PRESS

24.

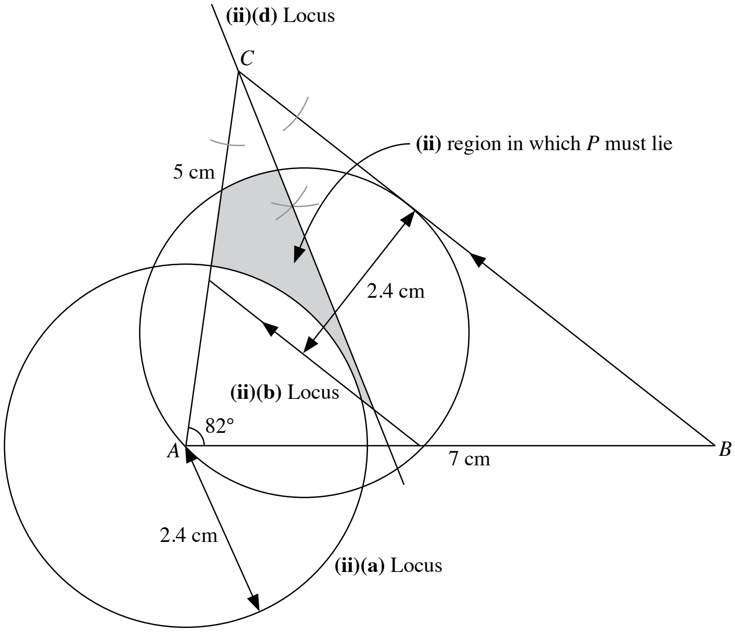


25.



OXFORD UNIVERSITY PRESS

26.



(i) From the diagram, $BC = 8$ cm

Revision 10B

1. Let the capacity of the smaller pond be c litres.

$$\frac{c}{3360} = \left(\frac{1}{2}\right)^3$$

$$\frac{c}{3360} = \frac{1}{8}$$

$$c = \frac{3360}{8}$$

$$= 420$$

$\therefore$ Capacity of the smaller pond is 420 litres.

2. **(i)** Let the height of the large bathtub be h m.

$$\frac{0.75}{h} = \frac{3}{4}$$

$$h = \frac{4 \times 0.75}{3}$$

$$= 1$$

$\therefore$ Height of the large bathtub is 1 m.

(ii) Let the capacity of the small bathtub be c litres.

$$\frac{c}{V} = \left(\frac{3}{4}\right)^3$$

$$\frac{c}{V} = \frac{27}{64}$$

$$c = \frac{27}{64}V$$

$\therefore$ Capacity of the small bathtub is $\frac{27}{64}V$ litres.

3. **(a)** **(i)** Let the smaller radius be r_1 cm and the larger radius be r_2 cm.

$$\left(\frac{r_1}{r_2}\right)^2 = \frac{640}{1210}$$

$$\frac{r_1}{r_2} = \sqrt{\frac{640}{1210}}$$

$$= \frac{8}{11}$$

$\therefore$ The ratio of the smaller radius to the larger radius is 8 : 11.

(ii) Let the smaller volume be V_1 cm^3 and the larger volume be V_2 cm^3.

$$\frac{V_1}{V_2} = \left(\frac{8}{11}\right)^3$$

$$= \frac{512}{1331}$$

$\therefore$ The ratio of the smaller volume to the larger volume is 512 : 1331.

(b) Let the cost of painting the larger sphere be \$$c$.

$$\frac{3.20}{c} = \frac{640}{1210}$$

$$c = \frac{1210 \times 3.20}{640}$$

$$= 6.05$$

$\therefore$ The cost of painting the larger sphere is \$6.05.

4. **(i)** Let the smaller height be h_1 cm and the larger height be h_2 cm.

$$\left(\frac{h_1}{h_2}\right)^3 = \frac{108}{500}$$

$$\frac{h_1}{h_2} = \sqrt[3]{\frac{108}{500}}$$

$$= \frac{3}{5}$$

$\therefore$ The ratio of their heights is 3 : 5.

OXFORD UNIVERSITY PRESS

(ii) Let the smaller total surface area be A_1 cm^2 and the larger total surface area be A_2 cm^2.

$$\begin{aligned}\frac{A_1}{A_2} &= \left(\frac{3}{5}\right)^2\\ &= \frac{9}{25}\end{aligned}$$

∴ The ratio of their total surface areas is 9 : 25.

5. Let the volume of the similar smaller conical flask be V cm^3.

$$\begin{aligned}\frac{V}{845} &= \left(\sqrt{\frac{32}{50}}\right)^3\\ \frac{V}{845} &= \left(\frac{4}{5}\right)^3\\ V &= \frac{64 \times 845}{125}\\ &= 432.64\end{aligned}$$

∴ The volume of the similar smaller conical flask is 432.64 cm^3.

6. (i)
$$\begin{aligned}\frac{V_1}{V_2} &= \left(\frac{300}{20}\right)^3\\ &= \frac{3375}{1}\end{aligned}$$

∴ The ratio of $V_1 : V_2$ is 3375 : 1.

(ii)
$$\begin{aligned}\frac{A_1}{A_2} &= \left(\frac{300}{20}\right)^2\\ &= \frac{225}{1}\end{aligned}$$

∴ The ratio of $A_1 : A_2$ is 225 : 1.

7. (i)
$$\begin{aligned}\text{Actual height} &= \frac{34 \times 600}{100}\\ &= 204 \text{ m}\end{aligned}$$

(ii) Let the cost of painting the actual bridge be \$$c$.

$$\begin{aligned}\frac{4}{c} &= \left(\frac{1}{600}\right)^2\\ c &= 4 \times 600^2\\ &= 1\,440\,000\end{aligned}$$

∴ Cost of painting the actual bridge is \$1 440 000.

(iii) Let the mass of the section in the model be m tonnes.

$$\begin{aligned}\frac{m}{432} &= \left(\frac{1}{600}\right)^3\\ m &= \frac{432}{600^3}\\ &= 0.000\,002\end{aligned}$$

∴ Mass of the section in the model is 0.000 002 tonnes.

8. (i) Let the common height of △ACR and △BCR be h cm.

$$\begin{aligned}\frac{\text{Area of } \triangle ACR}{\text{Area of } \triangle BCR} &= \frac{\frac{1}{2} \times h \times RA}{\frac{1}{2} \times h \times BR}\\ \frac{\text{Area of } \triangle ACR}{98} &= \frac{3}{4}\\ \text{Area of } \triangle ACR &= \frac{3 \times 98}{4}\\ &= 73.5 \text{ cm}^2\end{aligned}$$

(ii) △CPQ and △CBR are similar.

$$\begin{aligned}\frac{\text{Area of } \triangle CPQ}{\text{Area of } \triangle CBR} &= \left(\frac{5}{7}\right)^2\\ \frac{\text{Area of } \triangle CPQ}{98} &= \frac{25}{49}\\ \text{Area of } \triangle CPQ &= \frac{25 \times 98}{49}\\ &= 50 \text{ cm}^2\end{aligned}$$

9. (i) △APQ and △ABC are similar.

$$\begin{aligned}\frac{AP}{AB} &= \frac{AQ}{AC}\\ \frac{2}{8} &= \frac{AQ}{12}\\ AQ &= \frac{2 \times 12}{8}\\ &= 3\end{aligned}$$

∴ $QC = 12 - 3 = 9$ cm

(ii) Let the common height of △APQ and △PQB be h cm.

$$\begin{aligned}\frac{\text{Area of } \triangle APQ}{\text{Area of } \triangle PQB} &= \frac{\frac{1}{2} \times h \times AP}{\frac{1}{2} \times h \times PB}\\ &= \frac{2}{6}\\ &= \frac{1}{3}\end{aligned}$$

(iii) △APQ and △ABC are similar.

$$\begin{aligned}\frac{\text{Area of } \triangle APQ}{\text{Area of } \triangle ABC} &= \left(\frac{2}{8}\right)^2\\ &= \frac{1}{16}\end{aligned}$$

$$\begin{aligned}\therefore \frac{\text{Area of } PBCQ}{\text{Area of } \triangle ABC} &= \frac{16 - 1}{16}\\ &= \frac{15}{16}\end{aligned}$$

10. (i) △ABC and △APQ are similar.

$$\begin{aligned}\frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle APQ} &= \left(\frac{6}{9}\right)^2\\ \frac{20}{\text{Area of } \triangle APQ} &= \frac{4}{9}\\ \text{Area of } \triangle APQ &= \frac{9 \times 20}{4}\\ &= 45 \text{ cm}^2\end{aligned}$$

(ii)
$$\begin{aligned}\text{Area of } BPQC &= 45 - 20\\ &= 25 \text{ cm}^2\end{aligned}$$

11. (i) Let the common height of △APC and △ABP be h cm.

$$\begin{aligned}\frac{\text{Area of } \triangle APC}{\text{Area of } \triangle ABP} &= \frac{\frac{1}{2} \times h \times PC}{\frac{1}{2} \times h \times BP}\\ \frac{36}{\text{Area of } \triangle ABP} &= \frac{3}{2}\\ \text{Area of } \triangle ABP &= \frac{36 \times 2}{3}\\ &= 24 \text{ cm}^2\end{aligned}$$

OXFORD UNIVERSITY PRESS

(ii) Area of $\triangle ABC = 24 + 36$

$$= 60 \text{ cm}^2$$

$$\frac{\text{Area of } \triangle CPQ}{\text{Area of } \triangle CBA} = \left(\frac{3}{2+3}\right)^2$$

$$\frac{\text{Area of } \triangle CPQ}{60} = \left(\frac{3}{5}\right)^2$$

$$\text{Area of } \triangle CPQ = \frac{60 \times 9}{25}$$

$$= 21.6 \text{ cm}^2$$

Alternatively,

Let the common height of $\triangle APQ$ and $\triangle CPQ$ be h_1 cm.

$$\frac{\text{Area of } \triangle APQ}{\text{Area of } \triangle CPQ} = \frac{\frac{1}{2} \times h_1 \times AQ}{\frac{1}{2} \times h_1 \times QC}$$

$$\frac{36 - \text{Area of } \triangle CPQ}{\text{Area of } \triangle CPQ} = \frac{2}{3}$$

$$108 - 3(\text{Area of } \triangle CPQ) = 2(\text{Area of } \triangle CPQ)$$

$$5(\text{Area of } \triangle CPQ) = 108$$

$$\text{Area of } \triangle CPQ = 21.6 \text{ cm}^2$$

12. Let X be the point of intersection of DC and AP.

$\triangle ADX$ and $\triangle PCX$ are similar.

$$\frac{\text{Area of } \triangle ADX}{\text{Area of } \triangle PCX} = \left(\frac{4}{6}\right)^2$$

$$= \frac{4}{9}$$

$\triangle XCP$ and $\triangle ABP$ are similar.

$$\frac{\text{Area of } \triangle XCP}{\text{Area of } \triangle ABP} = \left(\frac{6}{10}\right)^2$$

$$= \frac{9}{25}$$

$$\therefore \frac{\text{Area of } \triangle XCP}{\text{Area of } AXCB} = \frac{9}{16}$$

$$\frac{\text{Area of } ABCD}{\text{Area of } \triangle ABP} = \frac{4+16}{25}$$

$$\frac{40}{\text{Area of } \triangle ABP} = \frac{20}{25}$$

$$\text{Area of } \triangle ABP = \frac{25 \times 40}{20}$$

$$= 50 \text{ cm}^2$$

13. (i) $\triangle APQ$ and $\triangle ACB$ are similar.

$$\frac{AP}{AC} = \frac{PQ}{CB}$$

$$\frac{3}{5} = \frac{PQ}{4}$$

$$PQ = \frac{4 \times 3}{5}$$

$$= 2.4 \text{ cm}$$

(ii) Let the common height of $\triangle ABP$ and $\triangle BPC$ be h cm.

$$\frac{\text{Area of } \triangle ABP}{\text{Area of } \triangle BPC} = \frac{\frac{1}{2} \times h \times AP}{\frac{1}{2} \times h \times PC}$$

$$= \frac{3}{2}$$

The ratio is 3 : 2.

(iii) $\triangle APQ$ and $\triangle ACB$ are similar.

$$\frac{\text{Area of } \triangle APQ}{\text{Area of } \triangle ACB} = \left(\frac{3}{5}\right)^2$$

$$= \frac{9}{25}$$

$$\therefore \frac{\text{Area of } \triangle APQ}{\text{Area of } PQBC} = \frac{9}{16}$$

The ratio is 9 : 16.

14. (i) $\angle APD = \angle BPC$ (vert. opp. $\angle$s)

$AP = BP$ (given)

$DP = CP$ (given)

$\triangle APD \equiv \triangle BPC$ (SAS)

(ii) $\triangle ABD$ and $\triangle BAC$ or $\triangle ADC$ and $\triangle BCD$

(iii) $\triangle ABP$ and $\triangle DCP$

15. (a) $\angle PQX = \angle RYX$ (alt. $\angle$s, PQ // YR)

$\angle PXQ = \angle RXY$ (vert. opp. $\angle$s)

$\therefore$ $\triangle PQX$ and $\triangle RYX$ are similar. (2 pairs of corr. $\angle$s are equal)

(b) (i) $\triangle PQX$ and $\triangle RYX$ are similar.

$$\frac{\text{Area of } \triangle RYX}{\text{Area of } \triangle PQX} = \left(\frac{RY}{PQ}\right)^2$$

$$= \left(\frac{3}{4}\right)^2$$

$$= \frac{9}{16}$$

$\therefore$ The ratio is 9 : 16.

(ii) Let the common height of $\triangle PQX$ and $\triangle QXR$ be h cm.

$$\frac{\text{Area of } \triangle PQX}{\text{Area of } \triangle QXR} = \frac{\frac{1}{2} \times h \times PX}{\frac{1}{2} \times h \times XR}$$

$$= \frac{4}{3}$$

$$\frac{\text{Area of } \triangle QXR}{\text{Area of } PQRS} = \frac{3}{7+7}$$

$$= \frac{3}{14}$$

$\therefore$ The ratio is 3 : 14.

16. (a) (i)

$$\frac{PX}{RX} = \frac{XQ}{XS}$$

$$\frac{6}{10} = \frac{8}{XS}$$

$$XS = \frac{8 \times 10}{6}$$

$$= 13.3 \text{ cm (to 3 s.f.)}$$

(ii)

$$\frac{PX}{RX} = \frac{PQ}{RS}$$

$$\frac{6}{10} = \frac{PQ}{16}$$

$$PQ = \frac{6 \times 16}{10}$$

$$= 9.6 \text{ cm}$$

OXFORD UNIVERSITY PRESS

(b) $\frac{\text{Area of } \triangle PXQ}{\text{Area of } \triangle RXS} = \left(\frac{6}{10}\right)^2$

$\frac{k}{\text{Area of } \triangle RXS} = \frac{9}{25}$

$\text{Area of } \triangle RXS = \frac{25}{9} k \text{ cm}^2$

17. **(i)** $\angle ABC = \angle DBA$ (common $\angle$)

$\angle BAC = \angle BDA$ (given)

$\triangle ABC$ and $\triangle DBA$ are similar. (2 pairs of corr. $\angle$s are equal)

(ii) $\frac{AB}{DB} = \frac{BC}{BA}$

$\frac{6}{BD} = \frac{4}{6}$

$BD = \frac{6 \times 6}{4}$

$= 9$ cm

18. $\triangle ACE$ and $\triangle DCB$ are similar.

$\frac{AE}{DB} = \frac{CE}{CB}$

$\frac{10}{4} = \frac{CE}{2}$

$CE = \frac{10 \times 2}{4}$

$= 5$ cm

19. **(a)** $\angle ABG = \angle CEG$ (alt. $\angle$s, $BA \mathbin{//} CE$)

$\angle AGB = \angle CGE$ (vert. opp. $\angle$s)

$\therefore$ $\triangle ABG$ and $\triangle CEG$ are similar. (2 pairs of corr. $\angle$s are equal)

(b) Triangle *EBC* or Triangle *BFA*

(c) Triangle *ABC* and Triangle *CDA*

$\angle ABC = \angle CDA$ (opp. $\angle$s of parallelogram)

$AB = CD$ (opp. sides of parallelogram *ABCD*)

$DA = BC$ (opp. sides of parallelogram *ABCD*)

$\therefore$ Triangle *ABC* and triangle *CDA* are congruent. (SAS)

(d) **(i)** $\triangle ABG$ and $\triangle CEG$ are similar.

$\therefore \frac{AB}{CE} = \frac{AG}{CG} = \frac{1}{2}$

(ii) Let the common height of $\triangle ABG$ and $\triangle ABC$ be h cm.

$$\frac{\text{Area of } \triangle ABG}{\text{Area of } \triangle ABC} = \frac{\frac{1}{2} \times h \times AG}{\frac{1}{2} \times h \times AC}$$

$$= \frac{1}{3}$$

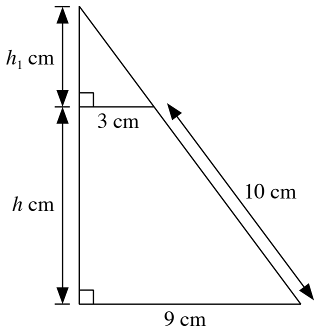
(iii) $\triangle AFG$ and $\triangle CBG$ are similar.

$$\frac{\text{Area of } \triangle AFG}{\text{Area of } \triangle CBG} = \left(\frac{AG}{CG}\right)^2$$

$$= \left(\frac{1}{2}\right)^2$$

$$= \frac{1}{4}$$

20.



(i) Let the height of the solid be h cm.

Using Pythagoras' Theorem, $h = \sqrt{10^2 - 6^2}$

$= 8$

$\therefore$ The height of the solid is 8 cm.

(ii) $\frac{h_1}{h_1 + 8} = \frac{3}{9}$

$9h_1 = 3(h_1 + 8)$

$9h_1 - 3h_1 = 24$

$6h_1 = 24$

$h_1 = 4$ cm

Volume of the frustum $= \frac{1}{3}\pi[9^2(12) - 3^2(4)]$

$= 980 \text{ cm}^3$ (to 3 s.f.)

Revision 10C

1. **(i)** $\sin x = \sin 35°$

$x = 35°$ or $x = 180° - 35°$

$= 145°$

(ii) $\cos x = -\cos 25°$

$x = 180° - 25°$

$= 155°$

(iii) $\sin x = 0.5$

$x = 30°$ or $x = 180° - 30°$

$= 150°$

(iv) $2 \cos x = -1$

$\cos x = -\frac{1}{2}$

$x = 180° - 60°$

$= 120°$

2. **(i)** Using Pythagoras' Theorem,

$5^2 = QS^2 + x^2$

$QS^2 = 25 - x^2$

$QS = \sqrt{25 - x^2}$

(ii) $\sin \angle QRS = \frac{QS}{QR}$

$= \frac{\sqrt{25 - x^2}}{5}$

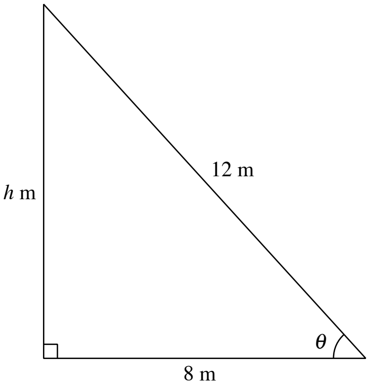
(iii) $\cos \angle PQS = -\cos \angle SQR$

$= -\frac{QS}{QR}$

$= -\frac{\sqrt{25 - x^2}}{5}$

OXFORD UNIVERSITY PRESS

3. **(a)**



(i) Let θ be the angle that the ladder makes with the ground.

$\cos \theta = \dfrac{8}{12}$

$\theta = \cos^{-1}\left(\dfrac{8}{12}\right)$

$= 48.2°$ (to 1 d.p.)

(ii) Let h m be the height of the ladder above the ground.

Using Pythagoras' Theorem,

$12^2 = h^2 + 8^2$

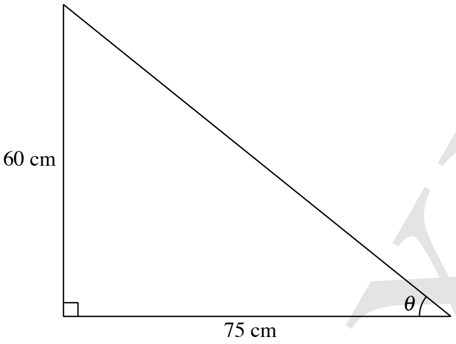
$h^2 = 144 - 64$

$h^2 = 80$

$h = \sqrt{80}$

$= 8.94$ m (to 3 s.f.)

(b)



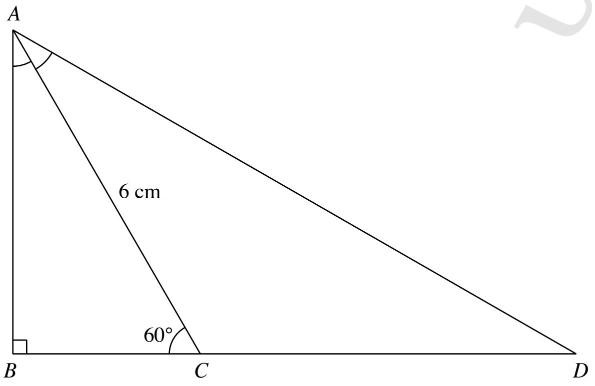
Let θ be the angle of elevation of the tower from the man.

$\tan \theta = \dfrac{60}{75}$

$\theta = \tan^{-1}\left(\dfrac{60}{75}\right)$

$= 38.7°$ (to 1 d.p.)

4.



$\sin 60° = \dfrac{AB}{6}$

$AB = 6 \times \sin 60°$

$= 5.20$ cm (to 3 s.f.)

$\angle BAC = 180° - 90° - 60°$ ($\angle$ sum of $\triangle ABC$)

$= 30°$

Since AC bisects $\angle BAD$, $\therefore$ $\angle BAC = \angle CAD = 30°$.

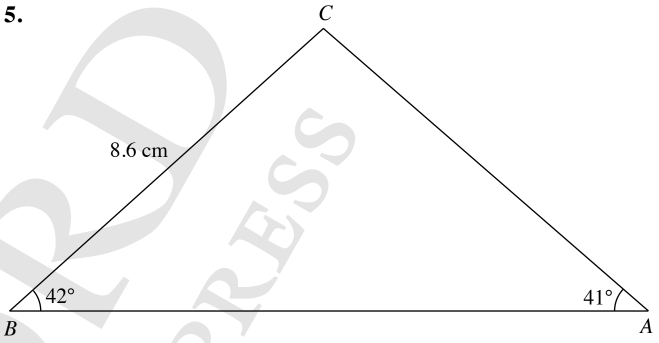
$\cos 60° = \dfrac{AB}{AD}$

$= \dfrac{5.191\,615}{AD}$

$AD = \dfrac{5.191\,615}{\cos 60°}$

$= 10.4$ cm (to 3 s.f.)

5.



(i) $\angle ACB = 180° - 42° - 41°$ ($\angle$ sum of $\triangle ABC$)

$= 97°$

Using sine rule,

$\dfrac{8.6}{\sin 41°} = \dfrac{AB}{\sin 97°}$

$AB = \dfrac{8.6 \times \sin 97°}{\sin 41°}$

$= 13.0$ cm (to 3 s.f.)

(ii) Area of $\triangle ABC = \dfrac{1}{2} \times 8.6 \times 13.011 \times \sin 42°$

$= 37.4$ cm^2 (to 3 s.f.)

6. **(a)** Let C be the point vertically below M.

$\angle MBC = 34.6°$ (alt. $\angle$s, // lines)

$\tan 34.6° = \dfrac{250}{BC}$

$BC = \dfrac{250}{\tan 34.6°}$

$= 362.4$ m (to 4 s.f.)

$\angle MAC = 58.5°$ (alt. $\angle$s, // lines)

$\tan 58.5° = \dfrac{250}{AC}$

$AC = \dfrac{250}{\tan 58.5°}$

$= 153.2$ m (to 4 s.f.)

Distance between the two ships $= 362.4 - 153.2$

$= 209$ m (to 3 s.f.)

OXFORD UNIVERSITY PRESS

(b) Let a m be the length from the top of the window to the foot of the window and b m be the length from the bottom of the window to the foot of the window.

$$\tan 43° = \frac{a}{7.2}$$
$$a = 7.2 \times \tan 43°$$
$$= 6.714 \text{ (to 4 s.f.)}$$
$$\tan 32° = \frac{b}{7.2}$$
$$b = 7.2 \times \tan 32°$$
$$= 4.499 \text{ (to 4 s.f.)}$$

Height of the window = 6.714 – 4.499

= 2.22 m (to 3 s.f.)

7. **(a)** **(i)** $\angle PRQ = 180° - 101° - 49°$ ($\angle$ sum of $\triangle PQR$)

$= 30°$

Bearing of Q from $R = 180° - 30°$

$= 150°$

(ii) Bearing of R from $Q = 360° - 30°$

$= 330°$

(b) Using sine rule,

$$\frac{1.45}{\sin 30°} = \frac{PR}{\sin 101°}$$
$$PR = \frac{1.45 \times \sin 101°}{\sin 30°}$$
$$= 2.85 \text{ km (to 3 s.f.)}$$

8. **(i)** Bearing of C from $A = 42° + 60°$

$= 102°$

(ii) Bearing of C from $B = 180° - (60° - 42°)$

$= 162°$

9. **(a)** **(i)** $\tan \angle ACD = \frac{4.8}{7.6}$

$$\angle ACD = \tan^{-1}\left(\frac{4.8}{7.6}\right)$$
$$= 32.3° \text{ (to 1 d.p.)}$$

(ii) Using Pythagoras' Theorem,

$$AC^2 = 4.8^2 + 7.6^2$$
$$AC = \sqrt{4.8^2 + 7.6^2}$$
$$= 8.99 \text{ cm (to 3 s.f.)}$$

(iii) $\cos 54.6° = \frac{AC}{AB}$

$$AB = \frac{8.988\ 88}{\cos 54.6°}$$
$$= 15.5 \text{ cm (to 3 s.f.)}$$

(b) Area of $\triangle ACE = \frac{1}{2} \times 7 \times 8.988\ 88 \times \sin 54.6°$

$= 25.6 \text{ cm}^2$ (to 3 s.f.)

10. **(i)** $\tan 18° = \frac{TB}{25}$

$$TB = 25 \times \tan 18°$$
$$= 8.12 \text{ m (to 3 s.f.)}$$

(ii) $\angle ACB = 90° - 36°$

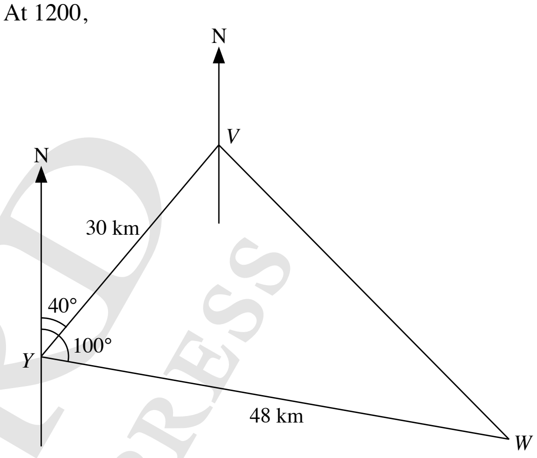
$= 54°$

$$\tan 54° = \frac{AB}{25}$$
$$AB = 25 \times \tan 54°$$
$$= 34.41 \text{ m (to 4 s.f.)}$$

Let θ be the angle of elevation of T from A.

$$\tan \theta = \frac{8.123}{34.41}$$
$$\theta = \tan^{-1}\left(\frac{8.123}{34.41}\right)$$
$$= 13.3° \text{ (to 1 d.p.)}$$

11. At 1200,



(i) Using cosine rule,

$$VW^2 = 30^2 + 48^2 - 2(30)(48) \cos 60°$$
$$VW = 42 \text{ km}$$

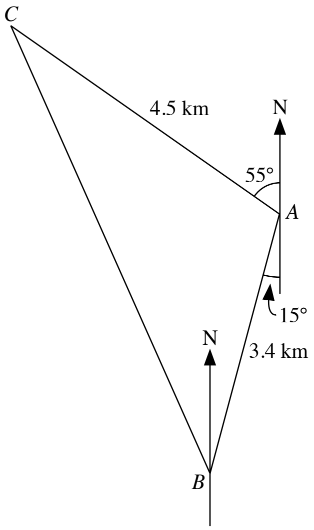
(ii) Using sine rule,

$$\frac{42}{\sin 60°} = \frac{48}{\sin \angle YVW}$$
$$\angle YVW = \sin^{-1}\left(\frac{48 \times \sin 60°}{42}\right)$$
$$= 82° \text{ (to the nearest degree)}$$

(iii) Bearing of ship W from ship $V = 180° - (82° - 40°)$

$= 138°$

12.



(i) Using cosine rule,

$$BC^2 = 4.5^2 + 3.4^2 - 2(4.5)(3.4) \cos 110°$$
$$BC = \sqrt{4.5^2 + 3.4^2 - 2(4.5)(3.4) \cos 110°}$$
$$= 6.50 \text{ km (to 3 s.f.)}$$

OXFORD UNIVERSITY PRESS

(ii) Using sine rule,

$$\frac{6.501}{\sin 110°} = \frac{4.5}{\sin \angle ABC}$$

$$\angle ABC = \sin^{-1}\left(\frac{4.5 \times \sin 110°}{6.501}\right)$$

$$= 40.58° \text{ (to 2 d.p.)}$$

Bearing of C from $B = 360° - (40.58° - 15°)$

$= 334.4°$ (to 1 d.p.)

(iii) Area of $\triangle ABC = \frac{1}{2} \times 4.5 \times 3.4 \times \sin 110°$

$= 7.19 \text{ km}^2$ (to 3 s.f.)

13. (i) $\cos \angle ACD = \frac{24}{43}$

$$\angle ACD = \cos^{-1}\left(\frac{24}{43}\right)$$

$$= 56.1° \text{ (to 1 d.p.)}$$

Bearing of C from $A = 056.1°$ (alt. $\angle$s, // lines)

(ii) $\angle AXB = 180° - 40° - 108°$ ($\angle$ sum of $\triangle ABX$)

$= 32°$

Using sine rule,

$$\frac{25}{\sin 32°} = \frac{AX}{\sin 40°}$$

$$AX = \frac{25 \times \sin 40°}{\sin 32°}$$

$$= 30.32 \text{ m (to 4 s.f.)}$$

$$CX = 43 - 30.32$$

$$= 12.7 \text{ m (to 3 s.f.)}$$

(iii) Using cosine rule,

$BC^2 = 25^2 + 43^2 - 2(25)(43) \cos 108°$

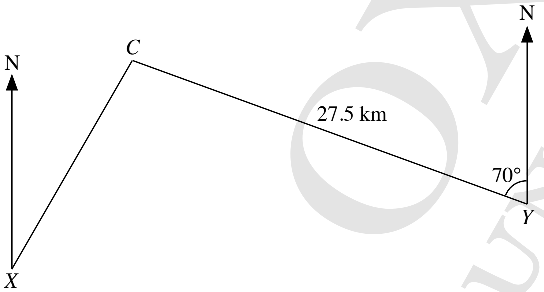
$BC = 56.0$ m (to 3 s.f.)

(iv) Area of the quadrilateral $ABCD$

$$= \left(\frac{1}{2} \times 25 \times 43 \times \sin 108°\right) + \left(\frac{1}{2} \times 43 \times 24 \times \sin 56.1°\right)$$

$= 939 \text{ m}^2$ (to 3 s.f.)

14. (i)



(ii) Actual distance of the cruise ship from Harbour X

$= 3 \times 15$

$= 15$ km

(iii) Bearing of cruise ship from Harbour $X = 030°$

15. (a) Using Pythagoras' Theorem,

$$AC^2 + BC^2 = AB^2$$

$$(4x + 1)^2 + (3x - 2)^2 = 18^2$$

$$16x^2 + 8x + 1 + 9x^2 - 12x + 4 = 324$$

$$16x^2 + 9x^2 + 8x - 12x + 1 + 4 - 324 = 0$$

$$25x^2 - 4x - 319 = 0 \text{ (shown)}$$

(b) $a = 25, b = -4, c = -319$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(25)(-319)}}{2(25)}$$

$$= \frac{4 \pm \sqrt{31\,916}}{50}$$

$$= 3.65 \text{ or } -3.49 \text{ (to 2 d.p.)}$$

(c) (i) Perimeter of $\triangle ABC = 18 + [4(3.653) + 1] + [3(3.653) - 2]$

$= 42.6$ cm (to 3 s.f.)

(ii) Area of $\triangle ABC = \frac{1}{2} \times [4(3.653) + 1] \times [3(3.653) - 2]$

$= 69.9 \text{ cm}^2$ (to 3 s.f.)

16. (i) $\tan \angle ABW = \frac{6}{15}$

$$\angle ABW = \tan^{-1}\left(\frac{6}{15}\right)$$

$$= 21.8° \text{ (to 1 d.p.)}$$

(ii) Using Pythagoras' Theorem,

$BD^2 = 15^2 + 8^2$

$BD = \sqrt{15^2 + 8^2}$

$= 17$ m

$\tan \angle BDX = \frac{6}{17}$

$$\angle BDX = \tan^{-1}\left(\frac{6}{17}\right)$$

$$= 19.4° \text{ (to 1 d.p.)}$$

(iii) Using Pythagoras' Theorem,

$AZ^2 = 6^2 + 8^2$

$AZ = \sqrt{6^2 + 8^2}$

$= 10$ m

Using Pythagoras' Theorem,

$ZC^2 = 6^2 + 15^2$

$ZC = \sqrt{6^2 + 15^2}$

$= 16.16$ m (to 4 s.f.)

Using Pythagoras' Theorem,

$AC^2 = 15^2 + 8^2$

$AC = \sqrt{15^2 + 8^2}$

$= 17$ m

Using cosine rule,

$$AC^2 = AZ^2 + ZC^2 - 2(AZ)(ZC) \cos \angle AZC$$

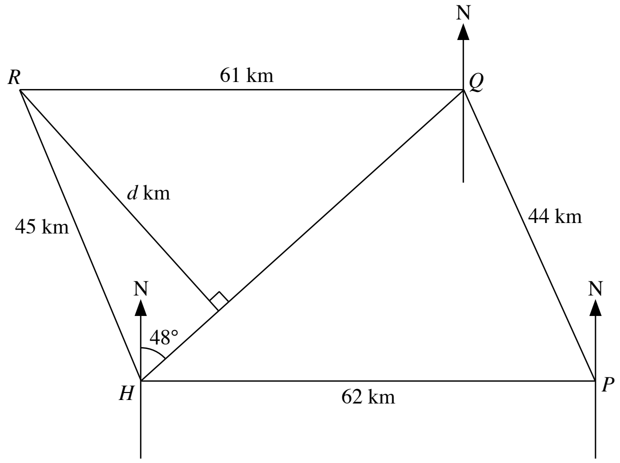
$$\cos \angle AZC = \frac{100 + 261 - 289}{2(10)(16.16)}$$

$$\angle AZC = \cos^{-1}\left[\frac{100 + 261 - 289}{2(10)(16.16)}\right]$$

$$= 77.1° \text{ (to 1 d.p.)}$$

OXFORD UNIVERSITY PRESS

17.



(a) **(i)** Using sine rule,

$$\frac{44}{\sin(90° - 48°)} = \frac{62}{\sin \angle HQP}$$

$$\angle HQP = \sin^{-1}\left(\frac{62 \times \sin 42°}{44}\right)$$

$$= 70.54° \text{ (to 2 d.p.)}$$

Bearing of Q from $P = 360° - (70.54° - 48°)$

$= 337.5°$ (to 1 d.p.)

(ii) $\angle HPQ = 180° - 42° - 70.54°$ ($\angle$ sum of $\triangle HPQ$)

$= 67.46°$

Using sine rule,

$$\frac{44}{\sin 42°} = \frac{HQ}{\sin 67.46°}$$

$$HQ = \frac{44 \times \sin 67.46°}{\sin 42°}$$

$$= 60.7 \text{ km (to 3 s.f.)}$$

(b) Time taken to travel from H to $R = \frac{45}{15}$

$= 3$ h

Time it returns to $H = 1115 + 6$ h $+ 40$ min

$= 1755$

(c) **(i)** Using cosine rule,

$$45^2 = 61^2 + 60.734^2 - 2(61)(60.734) \cos \angle HQR$$

$$\cos \angle HQR = \frac{61^2 + 60.734^2 + 45^2}{2(61)(60.734)}$$

$$\angle HQR = \cos^{-1}\left[\frac{61^2 + 60.734^2 + 45^2}{2(61)(60.734)}\right]$$

$$= 43.4° \text{ (to 1 d.p.)}$$

(ii) Let d km be the shortest distance from R to HQ.

$$\sin 43.39° = \frac{d}{61}$$

$$d = 61 \sin 43.39°$$

$$= 41.9 \text{ (to 3 s.f.)}$$

$\therefore$ The shortest distance from R to HQ is 41.9 km.

(iii) Area of $HPQR$

$$= \left(\frac{1}{2} \times 62 \times 44 \times \sin 67.46°\right) +$$

$$\left(\frac{1}{2} \times 61 \times 60.734 \times \sin 43.39°\right)$$

$= 2530$ km^2 (to 3 s.f.)

18. **(a)** **(i)** $\angle PQR = 180° - 63°$ (adj. $\angle$s on a str. line)

$= 117°$

$\angle PRQ = 180° - 117° - 18°$ ($\angle$ sum of $\triangle PQR$)

$= 45°$

Using sine rule,

$$\frac{250}{\sin 45°} = \frac{QR}{\sin 18°}$$

$$QR = \frac{250 \times \sin 18°}{\sin 45°}$$

$$= 109 \text{ m (to 3 s.f.)}$$

(ii) Bearing of P from $R = 180° + 18°$

$= 198°$

(b) Let θ be the angle of elevation of X from P.

$$\tan \theta = \frac{32}{250}$$

$$\theta = \tan^{-1}\left(\frac{32}{250}\right)$$

$$= 7.3° \text{ (to 1 d.p.)}$$

19. **(a)** **(i)** Let h m be the height of the lighthouse.

$$\tan 40° = \frac{h}{40}$$

$$h = 40 \times \tan 40°$$

$$= 33.6 \text{ (to 3 s.f.)}$$

$\therefore$ The height of the lighthouse is 33.6 m.

(ii) Let θ be the angle of elevation of the top of the lighthouse from C.

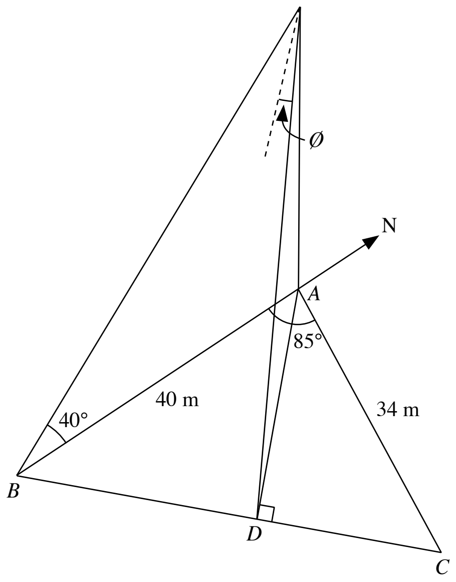
$$\tan \theta = \frac{33.56}{34}$$

$$\theta = \tan^{-1}\left(\frac{33.56}{34}\right)$$

$$= 44.6° \text{ (to 1 d.p.)}$$

OXFORD UNIVERSITY PRESS

(b)



$\angle BAC = 180° - 95°$ (adj. ∠s on a str. line)

$= 85°$

Using cosine rule,

$BC^2 = 40^2 + 34^2 - 2(40)(34)\cos 85°$

$BC = 50.19$ m (to 4 s.f.)

Using sine rule,

$$\frac{\sin \angle ABC}{34} = \frac{\sin 85°}{50.19}$$

$$\sin \angle ABC = \frac{34 \times \sin 85°}{50.19}$$

$$= 0.6748 \text{ (to 4 s.f.)}$$

$$\sin \angle ABC = \frac{AD}{40}$$

$$AD = 40 \times 0.6748$$

$$= 26.992 \text{ m}$$

Let Ø be the angle of depression.

$$\tan Ø = \frac{33.564}{26.992}$$

$$Ø = 51.2° \text{ (to 1 d.p.)}$$

20. **(i)** Using cosine rule,

$$XY^2 + YZ^2 - 2(XY)(YZ)\cos \angle XYZ = XZ^2$$

$$(2x+1)^2 + (2x)^2 - 2(2x+1)(2x)\left(-\frac{1}{21}\right) = (3x)^2$$

$$4x^2 + 4x + 1 + 4x^2 + \frac{8}{21}x^2 + \frac{4}{21}x = 9x^2$$

$$x^2 - 4x - 1 - \frac{8}{21}x^2 - \frac{4}{21}x = 0$$

$$21x^2 - 84x - 21 - 8x^2 - 4x = 0$$

$$13x^2 - 88x - 21 = 0 \text{ (shown)}$$

(ii) $13x^2 - 88x - 21 = 0$

$(13x + 3)(x - 7) = 0$

$x = 7$ or $x = -\frac{3}{13}$ (rejected)

$x = -\frac{3}{13}$ has to be rejected as length cannot be negative.

(iii) $\cos \angle XYZ = -\frac{1}{21}$

$$\sin \angle XYZ = \frac{\sqrt{21^2 - 1^2}}{21} = \frac{\sqrt{440}}{21}$$

$$\text{Area of } \triangle XYZ = \frac{1}{2} \times 15 \times 14 \times \frac{\sqrt{440}}{21}$$

$$= 105 \text{ cm}^2 \text{ (to 3 s.f.)}$$

Revision 10D

1. **(a)** $\angle POQ = \frac{3}{20} \times 360°$

$= 54°$

(b) **(i)** Let the radius of the circle be r cm.

$\pi r^2 = 616$

$$r^2 = \frac{616}{\pi}$$

$$r = \sqrt{\frac{616}{\pi}}$$

$= 14.0$ (to 3 s.f.)

∴ The radius of the circle is 14.0 cm.

(ii) Area of the shaded sector $POQ = \frac{3}{20} \times 616$

$= 92.4 \text{ cm}^2$

2. **(i)** Perimeter of the sector $= \frac{3\pi}{10} \times 14 + 14 + 14$

$= (28 + 4.2\pi)$ cm

(ii) Area of the sector $= \frac{1}{2} \times 14^2 \times \frac{3\pi}{10}$

$= 29.4\pi \text{ cm}^2$

3. **(i)** Area of the circle $= \pi(8^2)$

$= 201 \text{ cm}^2$ (to the nearest cm^2)

(ii) Let the length of the square be x cm.

Using Pythagoras' Theorem,

$8^2 + 8^2 = x^2$

$x^2 = 128$

Area of the square $= x^2 = 128 \text{ cm}^2$

Area of the shaded portion $= \pi(8^2) - 128$

$= 73 \text{ cm}^2$ (to the nearest cm^2)

4. **(i)** Volume of metal used $= \frac{4}{3} \times \pi \times 11^3 - \frac{4}{3} \times \pi \times 9.5^3$

$= 1980 \text{ cm}^3$ (to 3 s.f.)

(ii) Mass of the sphere $= 1983.9 \times 10.7$

$= 21\ 200$ g (to 3 s.f.)

5. **(i)** Volume of the 18 spherical marbles

$= 18 \times \frac{4}{3} \times \pi \times 1^3$

$= 24\pi \text{ cm}^3$

Rise in water level $= 24\pi \div (\pi \times 8^2)$

$= \frac{3}{8}$ cm

(ii) Total surface area of the cylinder that is in contact with water

$$= \left(2 \times \pi \times 8 \times 9\frac{3}{8}\right) + (\pi \times 8^2)$$

$= 214\pi \text{ cm}^2$

OXFORD UNIVERSITY PRESS

6. **(i)** $\left(4 \times \frac{1}{2} \times h \times 4\right) + (4 \times 4) = 176$

$$8h + 16 = 176$$
$$8h = 160$$
$$h = 20$$

$\therefore$ The slant height is 20 cm.

(ii) Let the height of the pyramid be H cm.

Using Pythagoras' Theorem,

$$H^2 + 2^2 = 20^2$$
$$H^2 = 400 - 4$$
$$H^2 = 396$$
$$H = \sqrt{396}$$

Volume of the pyramid $= \frac{1}{3} \times (4 \times 4) \times \sqrt{396}$

$= 106 \text{ cm}^3$ (to 3 s.f.)

7. **(a)** Let the radius of each ball bearing be r cm.

$$300 \times \frac{4}{3} \times \pi \times r^3 = \pi \times 7^2 \times 2.8$$
$$400r^3 = 137.2$$
$$r^3 = \frac{137.2}{400}$$
$$r = \sqrt[3]{\frac{137.2}{400}}$$
$$= 0.7$$

$\therefore$ The radius of each ball bearing is 7 mm.

(b) Volume of the metallic sphere $= \frac{4}{3} \times \pi \times \left(10\frac{1}{2}\right)^3$

$= 1543.5\pi \text{ cm}^3$

Volume of each small cone $= \frac{1}{3} \times \pi \times \left(3\frac{1}{2}\right)^2 \times 3$

$= 12.25\pi \text{ cm}^3$

Number of cones that can be made $= \frac{1543.5\pi}{12.25\pi}$

$= 126$

8. **(a)** **(i)** Length of the arc of sector $OAD = \frac{120°}{360°} \times 2 \times \pi \times 64$

$= 134$ cm (to 3 s.f.)

(ii) Area of sector $OAD = \frac{120°}{360°} \times \pi \times 64^2$

$= 4290 \text{ cm}^2$ (to 3 s.f.)

(iii) Area of segment $ABC = \frac{1}{4} \times \pi \times 64^2 - \frac{1}{2} \times 64^2$

$= 1170 \text{ cm}^2$ (to 3 s.f.)

(b) Volume of concrete needed to build the logo

$= \left(\frac{1}{4} \times \pi \times 64^2 + \frac{120°}{360°} \times \pi \times 64^2\right) \times 5$

$= 37\,500 \text{ cm}^3$

$= 0.0375 \text{ m}^3$ (to 3 s.f.)

(c) Total cost of building this logo $= 0.037\,53 \times 250.50$

$\approx \$9.40$

9. Volume of water in cylindrical jar $A = \pi \times x^2 \times 20$

$= 20x^2\pi \text{ cm}^3$

Let the height of the water in jar B be h cm.

$$\pi \times \left(\frac{5x}{2}\right)^2 \times h = 20x^2\pi$$
$$h = \frac{20x^2\pi}{6.25x^2\pi}$$
$$= 3.2$$

$\therefore$ The height of the water in jar B is 3.2 cm.

10. Total surface area of the figure

$= \left(\frac{3}{4} \times 4 \times \pi \times 7^2\right) + \left(2 \times \frac{1}{2} \times \pi \times 7^2\right)$

$= 616 \text{ cm}^2$ (to 3 s.f.)

11. **(i)** Let the perpendicular distance from O to AB be d cm and the midpoint of AB be M.

$$\sin 1.1 = \frac{AM}{1.8}$$
$$AM = 1.604 \text{ (to 4 s.f.)}$$

$\therefore AB = 2 \times 1.604 = 3.208$ cm

Perimeter of the shaded region

$= (2.2 \times 1.8) + \left(\frac{1}{2} \times \pi \times 3.208\right)$

$= 9.00$ cm (to 3 s.f.)

(ii) Area of the shaded region

$= \left(\frac{1}{2} \times \pi \times 1.604^2\right) - \left(\frac{1}{2} \times 1.8^2 \times 2.2 - \frac{1}{2} \times 1.8^2 \times \sin 2.2\right)$

$= 1.79 \text{ cm}^2$ (to 3 s.f.)

12. Area of the shaded region $= \frac{1}{2} \times 14^2 \times 2.2 - \frac{1}{2} \times 7^2 \times 2.2$

$= 161.7 \text{ cm}^2$

13. Let the radius of the semicircle be r cm.

$$\frac{1}{2}\pi r^2 = 77$$
$$r^2 = \frac{154}{\pi}$$
$$r = \sqrt{\frac{154}{\pi}}$$

Using Pythagoras' Theorem,

$$BD^2 = OB^2 + OD^2$$
$$= r^2 + r^2$$
$$= 2 \times \frac{154}{\pi}$$
$$= \frac{308}{\pi}$$
$$BD = \sqrt{\frac{308}{\pi}} \text{ cm}$$

Area of the shaded region

$= \left(\frac{1}{2} \times \sqrt{\frac{308}{\pi}} \times \sqrt{\frac{308}{\pi}}\right) - \left(\frac{1}{2} \times 77 - \frac{1}{2} \times \sqrt{\frac{154}{\pi}} \times \sqrt{\frac{154}{\pi}}\right)$

$= 35.0 \text{ cm}^2$

OXFORD UNIVERSITY PRESS

14. Area of the cross section which is filled with oil

$= \frac{1}{2} \times r^2(1.8 - \sin 1.8)\text{ cm}^2$

Percentage of the area of the cross section which is filled with oil

$= \dfrac{\frac{1}{2} \times r^2(1.8 - \sin 1.8)}{\pi r^2} \times 100\%$

$= \dfrac{1.8 - \sin 1.8}{2\pi} \times 100\%$

= 13% (to the nearest integer)

15. **(a)** **(i)** Volume of the smallest box required

$= (7 \times 4) \times (7 \times 2) \times (7 \times 10)$

$= 27\,440\text{ cm}^3$

(ii) Volume of the spheres $= \frac{4}{3} \times \pi \times 3.5^3 \times 80$

$= 4573\frac{1}{3}\pi\text{ cm}^3$

Percentage of the total volume of the box filled by the spheres

$= \dfrac{4573\frac{1}{3}\pi}{27\,440} \times 100\%$

= 52.4% (to 3 s.f.)

(b) Total surface area of the 80 spheres $= 80 \times 4 \times \pi \times 3.5^2$

$= 3920\pi\text{ cm}^2$

Volume of paint used for 80 spheres $= 3920\pi \times 0.0002$

$= 0.784\pi\text{ cm}^3$

Number of boxes of spheres that can be painted $= \dfrac{1000}{0.784\pi}$

$= 406$

16. **(i)** Arc length $PQ = 12 \times 1.2$

$= 14.4$ cm

Arc length $RS = 15 \times 1.2$

$= 18$ cm

Perimeter of the shaded region $B = 14.4 + 18 + 3 + 3$

$= 38.4$ cm

(ii) Area of region $A = \frac{1}{2} \times 12^2 \times 1.2$

$= 86.4\text{ cm}^2$

Area of region $B = \frac{1}{2} \times 15^2 \times 1.2 - 86.4$

$= 48.6\text{ cm}^2$

Difference between the areas of the regions A and B

$= 86.4 - 48.6$

$= 37.8\text{ cm}^2$

17. **(i)** Arc $AB = \dfrac{45°}{360°} \times 2 \times \pi \times 7$

= 5.50 cm (to 3 s.f.)

(ii) Area of sector $PQR = \dfrac{45°}{360°} \times \pi \times 7^2$

$= 19.2\text{ cm}^2$ (to 3 s.f.)

(iii) Volume of the prism $= 19.24 \times 8$

$= 154\text{ cm}^3$ (to 3 s.f.)

(iv) Total surface area of the prism

$= (19.24 \times 2) + (8 \times 7 \times 2) + (5.497 \times 8)$

$= 194\text{ cm}^2$ (to 3 s.f.)

18. **(i)** Let the radius of cylinder B and cone A be r cm.

Using similar triangles,

$\frac{r}{8} = \frac{6}{18}$

$r = 2\frac{2}{3}$

Volume of the rocket model

$= \left(\frac{1}{3} \times \pi \times 8^2 \times 18\right) + \left[\pi \times \left(2\frac{2}{3}\right)^2 \times 24\right]$

$= 1740\text{ cm}^3$ (to 3 s.f.)

(ii) Let the slant height be l cm.

Using Pythagoras' Theorem,

$l^2 = 8^2 + 18^2$

$l^2 = 388$

$l = \sqrt{388}$

Total curved surface area

$= (\pi \times 8 \times \sqrt{388}\,) + \left(2 \times \pi \times 2\frac{2}{3} \times 24\right)$

$= 897\text{ cm}^2$ (to 3 s.f.)

19. **(a)** Volume of cone $= \frac{1}{3} \times \pi \times r^2 \times x$

$= \frac{1}{3}\pi r^2 x$

Volume of cylinder $= \pi \times r^2 \times 2x$

$= 2\pi r^2 x$

$\dfrac{\text{Volume of cone}}{\text{Volume of cylinder}} = \dfrac{\frac{1}{3}\pi r^2 x}{2\pi r^2 x}$

$= \frac{1}{6}$

(b) **(i)** $\pi r^2(12) = 485$

$r^2 = \dfrac{485}{12\pi}$

$r = \sqrt{\dfrac{485}{12\pi}}$

= 3.59 cm (to 3 s.f.)

(ii) Let the slant height be l cm.

Using Pythagoras' Theorem,

$l^2 = \dfrac{485}{12\pi} + 36$

$l = 6.990$ (to 4 s.f.)

Curved surface area of the cone $= \pi \times 3.586\,78 \times 6.990$

$= 78.8\text{ cm}^2$ (to 3 s.f.)

(iii) Volume of paint needed

$= [78.7688 + (2 \times \pi \times 3.586\,78 \times 12)$

$+ (2 \times \pi \times 3.586\,78^2)] \times 0.03$

$= 12.9\text{ cm}^3$ (to 3 s.f.)

OXFORD UNIVERSITY PRESS

20. **(i)** Depth of water in the cylindrical tank $= \dfrac{5 \times 8 \times 0.045}{\pi \times 1.2^2}$

$= 0.398$ m

(ii) Decrease in the water level in the cylindrical tank

$$= \frac{5 \times \frac{1}{2} \times \frac{4}{3} \times \pi \times 26^3}{\pi \times 120^2}$$

$= 4.07$ cm (to 3 s.f.)

(iii) Time taken to drain all the remaining water

$$= \frac{(500 \times 800 \times 4.5) - \left(5 \times \frac{1}{2} \times \frac{4}{3} \times \pi \times 26^3\right)}{2500}$$

$= 646$ min (to the nearest minute)

Revision 10E

1. **(i)** Under R, y-coordinate of the point will become negative.

$\therefore$ Image of the point is (2, 5).

(ii) Under T^2, image of point $= 2\begin{pmatrix} 2 \\ -1 \end{pmatrix} + \begin{pmatrix} 2 \\ -5 \end{pmatrix}$

$= \begin{pmatrix} 4 \\ -2 \end{pmatrix} + \begin{pmatrix} 2 \\ -5 \end{pmatrix}$

$= \begin{pmatrix} 6 \\ -7 \end{pmatrix}$

$\therefore$ Image of the point is (6, –7).

(iii) Under R, image of point is (2, 5)

Under TR, image of point $= \begin{pmatrix} 2 \\ -1 \end{pmatrix} + \begin{pmatrix} 2 \\ 5 \end{pmatrix}$

$= \begin{pmatrix} 4 \\ 4 \end{pmatrix}$

$\therefore$ Image of the point is (4, 4).

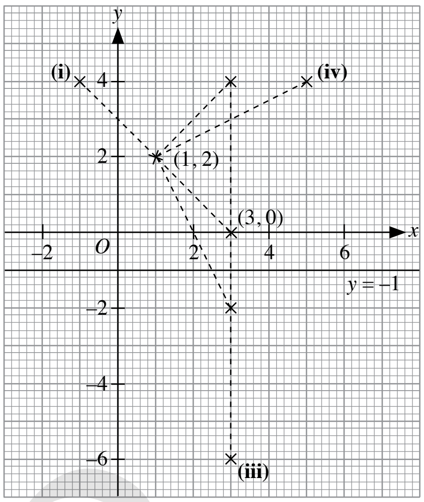
(iv) Under T, image of point $= \begin{pmatrix} 2 \\ -1 \end{pmatrix} + \begin{pmatrix} 2 \\ -5 \end{pmatrix}$

$= \begin{pmatrix} 4 \\ -6 \end{pmatrix}$

Under RT, y-coordinate of the point under T will become negative.

$\therefore$ Image of the point is (4, 6).

2.



(i) R^2 represents a 180° rotation about (1, 2). From the graph, the image of the point is (–1, 4).

(ii) M^2 represents two reflections about the line $y = -1$. The resultant image of the point will be the same point, i.e. (3, 0).

(iii) Under MR, the point will undergo a 90° anticlockwise rotation about (1, 2) followed by a reflection in the line $y = -1$. From the graph, the image of the point is (3, –6).

(iv) Under RM, the point will undergo a reflection in the line $y = -1$ followed by a 90° anticlockwise rotation about (1, 2). From the graph, the image is (5, 4).

3. Let $T = \begin{pmatrix} a \\ b \end{pmatrix}$.

$\begin{pmatrix} 3 \\ 1 \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} 2 \\ -3 \end{pmatrix}$

$\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} - \begin{pmatrix} 2 \\ -3 \end{pmatrix}$

$= \begin{pmatrix} 1 \\ 4 \end{pmatrix}$

$\therefore$ T represents a translation of $\begin{pmatrix} 1 \\ 4 \end{pmatrix}$

(i) Under T, image of point $= \begin{pmatrix} 1 \\ 4 \end{pmatrix} + \begin{pmatrix} 4 \\ -1 \end{pmatrix}$

$= \begin{pmatrix} 5 \\ 3 \end{pmatrix}$

$\therefore$ Image of the point is (5, 3).

(ii) Under M, x-coordinate of image and point (4, –1) will be equidistant from the line $x = 1$.

$\therefore$ Image of the point is (–2, –1).

(iii) Under TM, image of point $= \begin{pmatrix} 1 \\ 4 \end{pmatrix} + \begin{pmatrix} -2 \\ -1 \end{pmatrix}$

$= \begin{pmatrix} -1 \\ 3 \end{pmatrix}$

$\therefore$ Image of the point is (–1, 3).

(iv) Under MT, x-coordinate of image and point (5, 3) will be equidistant from the line $x = 1$.

$\therefore$ Image of the point is (–3, 3).

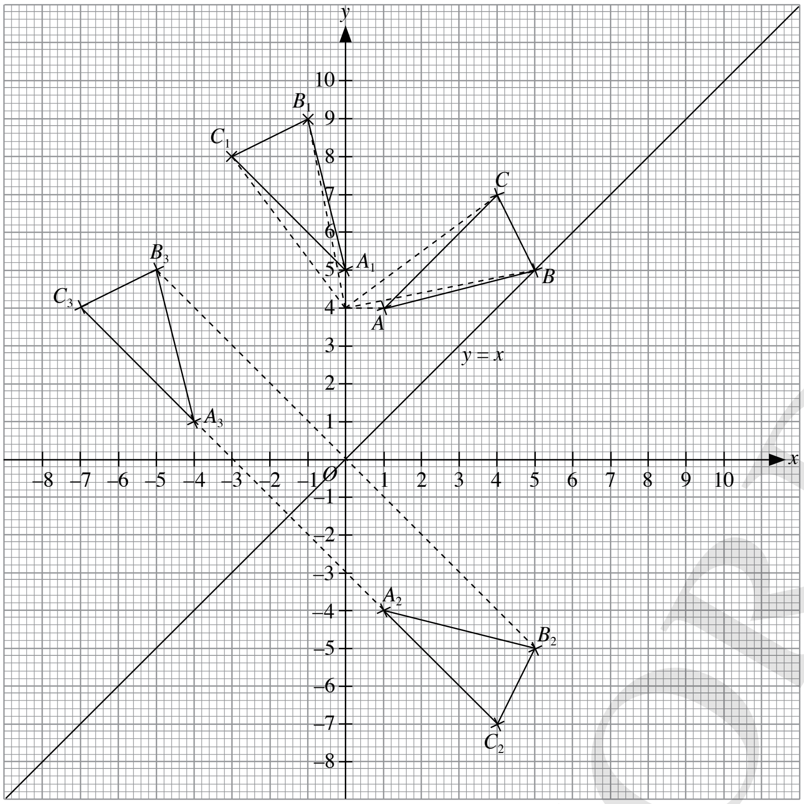
OXFORD UNIVERSITY PRESS

4. **(i)** $\triangle OAB$ undergoes an anticlockwise rotation of 144° about O.

(ii) $\triangle AEO$ undergoes a reflection in the line AO.

(iii) $\triangle AEO$ undergoes a reflection in the line OD.

5.

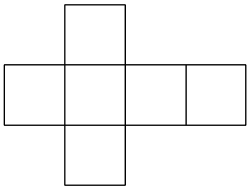


(e) From the graph, $A_1(0, 5)$, $B_1(-1, 9)$ and $C_1(-3, 8)$; $A_3(-4, 1)$, $B_3(-5, 5)$ and $C_3(-7, 4)$.
Comparing the corresponding points, $A_1B_1\,C_1$ can be mapped onto $A_3B_3\,C_3$ by a translation of $\begin{pmatrix} -4 \\ -4 \end{pmatrix}$.

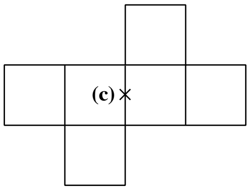
6. **(i)** $\triangle OAB$ undergoes a clockwise rotation of 120° about O.

(ii) $\triangle OBC$ first undergoes a reflection in line AD, followed by a anticlockwise rotation of 120° about O.

7. **(a)**



(b)



8. **(a)** From the graph, roots of the equation are $x = 1$ and $x = 4$.

$$\begin{aligned} y &= (x-1)(x-4) \\ &= x^2 - 4x - x + 4 \\ &= x^2 - 5x + 4 \\ &= -4 + 5x - x^2 \end{aligned}$$

$\therefore\ a = -4$ and $b = 5$

(b) Equation of line of symmetry is $x = 2.5$

9. **(a)** Number of axes of symmetry = 6

(b) **(i)** Sum of interior angles of hexagon $= (6-2) \times 180°$
$= 720°$

$$\angle ABC = \frac{720°}{6} = 120°$$

(ii) Size of each exterior angle of hexagon $= \frac{360°}{6}$
$= 60°$

$$\angle BAC = \angle BCA = \frac{180° - 120°}{2} \text{ (base }\angle\text{s of isos. }\triangle) = 30°$$

$$\angle ACD = 180° - 60° - 30° \text{ (adj. }\angle\text{s on a str. line)} = 90°$$

10. **(a)** Equations of line of symmetry are $y = 0$ and $x = 2.5$

(b) Order of rotational symmetry = 2
Coordinates of centre of rotation = (2.5, 0)

OXFORD UNIVERSITY PRESS

Revision 10F

1. **(a)** Gradient $= \dfrac{10-2}{9-1}$

$= 1$

(b) Since the gradient is 5, therefore $y = 5x + c$.

Given that the line passes through $(2, 3)$,

$3 = 5(2) + c$

$c = -7$

$\therefore$ Equation of the line is $y = 5x - 7$.

2. $\dfrac{-3-k}{4-5} = 2$

$-3 - k = 2(-1)$

$-3 - k = -2$

$k = -1$

Since the gradient is 2, therefore $y = 2x + c$.

Given that the line passes through $(4, -3)$,

$-3 = 2(4) + c$

$c = -11$

$\therefore$ $k = -1$ and equation of the line is $y = 2x - 11$.

3. **(a)** At E, $y = 0$.

$3x + 4(0) = 24$

$3x = 24$

$x = 8$

$\therefore$ Coordinates of E are $(8, 0)$.

At F, $x = 0$.

$3(0) + 4y = 24$

$4y = 24$

$y = 6$

$\therefore$ Coordinates of F are $(0, 6)$.

Length of $EF = \sqrt{(8-0)^2 + (0-6)^2}$

$= 10$ units

(b) $3x + 4y = 24$

$4y = -3x + 24$

$y = -\dfrac{3}{4}x + 6$

So the gradient is $-\dfrac{3}{4}$.

Since the gradient is $-\dfrac{3}{4}$, therefore the equation of the line is $y = -\dfrac{3}{4}x + c$.

Given that the line passes through $(-2, 1)$,

$1 = -\dfrac{3}{4}(-2) + c$

$c = -\dfrac{1}{2}$

$\therefore$ Equation of the line is $y = -\dfrac{3}{4}x - \dfrac{1}{2}$.

4. **(i)** At P, $y = 0$.

$2x + 3(0) = 18$

$2x = 18$

$x = 9$

$\therefore$ Coordinates of P are $(9, 0)$.

At Q, $x = 0$.

$2(0) + 3y = 18$

$3y = 18$

$y = 6$

$\therefore$ Coordinates of Q are $(0, 6)$.

(ii) Length of $PQ = \sqrt{(9-0)^2 + (0-6)^2}$

$= 10.8$ units (to 3 s.f.)

5. At H, $y = 0$.

$\dfrac{x}{4} + \dfrac{0}{6} = 1$

$x = 4$

$\therefore$ Coordinates of H are $(4, 0)$.

At J, $x = 0$.

$\dfrac{0}{4} + \dfrac{y}{6} = 1$

$y = 6$

$\therefore$ Coordinates of J are $(0, 6)$.

Length of $HJ = \sqrt{(4-0)^2 + (0-6)^2}$

$= 7.21$ units (to 3 s.f.)

6. Since the points lie on a straight line,

gradient of AB = gradient of BC

$\dfrac{23-8}{0-(-3)} = \dfrac{k-23}{2-0}$

$5 = \dfrac{k-23}{2}$

$10 = k - 23$

$k = 33$

7. **(a)** Given that the line $3y = k - 2x$ passes through $(-1, -5)$,

$3(-5) = k - 2(-1)$

$k = -17$

(b) $(2k-1)y + (k+1)x = 3$

$y + \dfrac{k+1}{2k-1}x = \dfrac{3}{2k-1}$

$y = -\dfrac{k+1}{2k-1}x + \dfrac{3}{2k-1}$

So the gradient is $-\dfrac{k+1}{2k-1}$.

The gradient of the line $y = 3x - 7$ is 3.

$\therefore -\dfrac{k+1}{2k-1} = 3$

$-(k+1) = 3(2k-1)$

$-k - 1 = 6k - 3$

$-k - 6k = -3 + 1$

$-7k = -2$

$k = \dfrac{2}{7}$

OXFORD UNIVERSITY PRESS

(c) Substitute (–1, 3) and $\left(1, \frac{1}{6}\right)$ into $\frac{x}{a} + \frac{y}{b} = 1$:

$$-\frac{1}{a} + \frac{3}{b} = 1 \quad \text{— (1)}$$

$$\frac{1}{a} + \frac{\frac{1}{6}}{b} = 1$$

$$\frac{1}{a} + \frac{1}{6b} = 1 \quad \text{— (2)}$$

$$(1) + (2): \frac{3}{b} + \frac{1}{6b} = 2$$

$$\frac{18 + 1}{6b} = 2$$

$$12b = 19$$

$$b = 1\frac{7}{12}$$

Substitute $b = 1\frac{7}{12}$ into (1):

$$-\frac{1}{a} + \frac{3}{1\frac{7}{12}} = 1$$

$$-\frac{1}{a} + 1\frac{17}{19} = 1$$

$$-\frac{1}{a} = -\frac{17}{19}$$

$$a = 1\frac{2}{17}$$

$\therefore a = 1\frac{2}{17}$ and $b = 1\frac{2}{17}$

$$\text{Gradient of the line} = \frac{3 - \frac{1}{6}}{-1 - 1}$$

$$= -1\frac{5}{12}$$

8. **(i)** $3x + 4y = 35$

$$4y = -3x + 35$$

$$y = -\frac{3}{4}x + \frac{35}{4}$$

$\therefore$ The gradient of AB is $-\frac{3}{4}$.

(ii) Let the coordinates of K be (d, d).

$3d + 4d = 35$

$7d = 35$

$d = 5$

$\therefore$ Coordinates of K are (5, 5).

9. **(a)** **(i)** $\tan \angle ABO = \frac{4}{3}$

$$\angle ABO = \tan^{-1}\left(\frac{4}{3}\right)$$

$$= 53.1° \text{ (to 1 d.p.)}$$

(ii) The gradient is $-\frac{4}{3}$ and the y-intercept is 4.

$\therefore$ Equation of the line is $y = -\frac{4}{3}x + 4$.

(b) The gradient is $\frac{4}{3}$ and the y-intercept is 4.

$\therefore$ Equation of the line is $y = \frac{4}{3}x + 4$.

10. $5x + 3y = 2$ — (1)

$x - y = 6$ — (2)

From (2): $x = 6 + y$ — (3)

Substitute (3) into (1): $5(6 + y) + 3y = 2$

$$30 + 5y + 3y = 2$$

$$8y = -28$$

$$y = -\frac{7}{2}$$

Substitute $y = -\frac{7}{2}$ into (3): $x = 6 - \frac{7}{2}$

$$= \frac{5}{2}$$

$\therefore$ The point of intersection is $\left(\frac{5}{2}, -\frac{7}{2}\right)$.

Since the gradient is $-1\frac{1}{2}$, therefore the equation of the line is $y = -1\frac{1}{2}x + c$.

Given that the line passes through $\left(\frac{5}{2}, -\frac{7}{2}\right)$,

$$-\frac{7}{2} = -1\frac{1}{2}\left(\frac{5}{2}\right) + c$$

$$c = \frac{1}{4}$$

$\therefore$ Equation of the line is $y = -1\frac{1}{2}x + \frac{1}{4}$.

11. **(a)** Gradient of $AB = \frac{4 - 2}{3 - (-3)}$

$$= \frac{1}{3}$$

Since the gradient is $\frac{1}{3}$, therefore the equation of the line is $y = \frac{1}{3}x + c$.

Given that the line passes through (3, 4),

$$4 = \frac{1}{3}(3) + c$$

$$c = 3$$

$\therefore$ Equation of the line is $y = \frac{1}{3}x + 3$.

(b) **(i)** Base of $\triangle ABC$ = length of BC

$$= 4 - (-2)$$

$$= 6 \text{ units}$$

Height of $\triangle ABC = 3 - (-3)$

$$= 6 \text{ units}$$

Area of $\triangle ABC = \frac{1}{2} \times 6 \times 6$

$$= 18 \text{ units}^2$$

OXFORD UNIVERSITY PRESS

(ii) Gradient of $AC = \dfrac{-2-2}{3-(-3)}$

$$= -\frac{2}{3}$$

Since the gradient of AC is $-\dfrac{2}{3}$, therefore the equation of AC is $y = -\dfrac{2}{3}x + c$.

Given that the line passes through $(3, -2)$,

$$-2 = -\frac{2}{3}(3) + c$$

$$c = 0$$

$\therefore$ Equation of AC is $y = -\dfrac{2}{3}x$.

12. $\sqrt{(4-k-2)^2 + (1-k+2)^2} = \sqrt{13-4k}$

$$(2-k)^2 + (3-k)^2 = 13 - 4k$$

$$4 - 4k + k^2 + 9 - 6k + k^2 = 13 - 4k$$

$$2k^2 - 10k + 13 + 4k - 13 = 0$$

$$2k^2 - 6k = 0$$

$$2k(k-3) = 0$$

$\therefore k = 0$ or $k = 3$

13. (a) (i) Length of $AB = 4 - 2 = 2$ units

Length of $BC = \sqrt{(6-4)^2 + (5+1)^2} = 6.32$ units (to 3 s.f.)

Length of $AC = \sqrt{(6-2)^2 + (5+1)^2} = 7.21$ units (to 3 s.f.)

(ii) Area of $\triangle ABC = \dfrac{1}{2} \times 6 \times 2$

$$= 6 \text{ units}^2$$

(b) Let the height of $\triangle ABD$ be h units.

$$\frac{1}{2} \times h \times 2 = 3$$

$$h = 3$$

$$t = -1 \pm 3$$

$\therefore t = 2$ or $t = -4$

14. (a) Let the coordinates of R be $(0, y)$.

$$PR = QR$$

$$\sqrt{(0+4)^2 + (y-2)^2} = \sqrt{(0-7)^2 + (y-5)^2}$$

$$(0+4)^2 + (y-2)^2 = (0-7)^2 + (y-5)^2$$

$$16 + y^2 - 4y + 4 = 49 + y^2 - 10y + 25$$

$$6y = 54$$

$$y = 9$$

$\therefore$ Coordinates of R are $(0, 9)$.

(b) Let the coordinates of S be $(x, 0)$.

$$PS = QS$$

$$\sqrt{(x+4)^2 + (0-2)^2} = \sqrt{(x-7)^2 + (0-5)^2}$$

$$(x+4)^2 + (0-2)^2 = (x-7)^2 + (0-5)^2$$

$$x^2 + 8x + 16 + 4 = x^2 - 14x + 49 + 25$$

$$22x = 54$$

$$x = \frac{27}{11}$$

$\therefore$ Coordinates of S are $\left(2\dfrac{5}{11}, 0\right)$.

15. $AB^2 = (-2+2)^2 + (1-6)^2 = 25$

$BC^2 = (4+2)^2 + (1-1)^2 = 36$

$AC^2 = (4+2)^2 + (1-6)^2 = 61$

Since $AC^2 = AB^2 + BC^2$, by the converse of Pythagoras' Theorem, $\triangle ABC$ is a right-angled triangle.

Area of $\triangle ABC = \dfrac{1}{2} \times 6 \times 5$

$$= 15 \text{ units}^2$$

Length of $AC = \sqrt{61}$ units

Let the length of the perpendicular from B to AC be h units.

$$\frac{1}{2} \times \sqrt{61} \times h = 15$$

$$h = 3.84 \text{ (to 3 s.f.)}$$

$\therefore$ The length of the perpendicular from B to AC is 3.84 units.

16. (i) Area of $\triangle ABC = \dfrac{1}{2} \times 7 \times 7$

$$= 24.5 \text{ units}^2$$

(ii) Length of $BC = \sqrt{(-1-1)^2 + (9-2)^2}$

$$= \sqrt{4+49}$$

$$= \sqrt{53}$$

$$= 7.28 \text{ units (to 3 s.f.)}$$

(iii) Let the shortest distance from A to BC be d units.

$$\frac{1}{2} \times \sqrt{53} \times d = 24.5$$

$$d = 6.73 \text{ units}$$

17. (a) $AB^2 = (0+1)^2 + (3-0)^2 = 10$

$BC^2 = (1-0)^2 + (1-3)^2 = 5$

$AC^2 = (1+1)^2 + (1-0)^2 = 5$

Since $AB^2 = AC^2 + BC^2$, by the converse of Pythagoras' Theorem, $\triangle ABC$ is a right-angled triangle.

(b) Coordinates of D are $(2, 4)$.

18. (i) Length of $OA = \sqrt{(-4-0)^2 + (3-0)^2}$

$$= \sqrt{16+9}$$

$$= \sqrt{25}$$

$$= 5 \text{ units}$$

$\therefore$ Length of $OB = 3 \times 5 = 15$ units

(ii) Coordinates of B are $(-12, 9)$.

(iii) Area of $ABQP = \dfrac{1}{2} \times (3+9) \times 8$

$$= 48 \text{ units}^2$$

(iv) Length of $AQ = \sqrt{(-4+12)^2 + (3-0)^2}$

$$= \sqrt{64+9}$$

$$= \sqrt{73}$$

$$= 8.54 \text{ units (to 3 s.f.)}$$

OXFORD UNIVERSITY PRESS

19. **(i)** Gradient of $XY = \dfrac{10-2}{3-1}$

$$= 4$$

Since the gradient is 4, therefore the equation of XY is $y = 4x + c$.

Given that the line passes through (1, 2),

$2 = 4(1) + c$

$c = -2$

$\therefore$ Equation of XY is $y = 4x - 2$.

(ii) Coordinates of Z are (5, 2).

(iii) Length of $XY = \sqrt{(3-1)^2 + (10-2)^2}$

$$= \sqrt{4 + 64}$$

$$= \sqrt{68} \text{ units}$$

Let the length of the perpendicular from Z to XY be d units.

$$\frac{1}{2} \times \sqrt{68} \times d = \frac{1}{2} \times 4 \times 8$$

$$d = 3.88 \text{ (to 3 s.f.)}$$

$\therefore$ The length of the perpendicular from Z to XY is 3.88 units.

20. Distance between A and any point (x, y) on the line l

$$= \sqrt{(1-x)^2 + (2-x-5)^2}$$

$$= \sqrt{(1-x)^2 + (-x-3)^2}$$

$$= \sqrt{1 - 2x + x^2 + x^2 + 6x + 9}$$

$$= \sqrt{2x^2 + 4x + 10}$$

$$= \sqrt{2(x^2 + 2x + 5)} \text{ (shown)}$$

21. **(a)** Gradient of $y = 3x - 2 = 3$

$\therefore$ Gradient of line parallel to $y = 3x - 2$ is 3.

(b) $2y = -\dfrac{3}{4}x - 6$

$$y = -\frac{3}{8}x - 3$$

$\therefore$ Gradient of line parallel to $2y = -\dfrac{3}{4}x - 6$ is $-\dfrac{3}{8}$.

22. **(a)** Gradient of $y = 4x + 1 = 4$

Gradient of perpendicular line $= -\dfrac{1}{4}$

(b) $4y = -\dfrac{1}{2}x + 6$

$$y = -\frac{1}{8}x + \frac{6}{4}$$

Gradient of perpendicular line $= -\dfrac{1}{-\frac{1}{8}}$

$$= 8$$

23. $4y = 2x - 16$

$$y = \frac{2}{4}x - \frac{16}{4}$$

$$= \frac{1}{2}x - 4 \quad \text{— (1)}$$

$5y = -10x + 2$

$$y = -2x + \frac{2}{5} \quad \text{— (2)}$$

Gradient of (1) × gradient of (2) $= \dfrac{1}{2}(-2)$

$$= -1$$

$\therefore$ Both lines are perpendicular to each other.

24. $2x + 6y = 1$

$$6y = -2x + 1$$

$$y = -\frac{2}{6}x + \frac{1}{6}$$

$$= -\frac{1}{3}x + \frac{1}{6}$$

Gradient of perpendicular line $= -\dfrac{1}{-\frac{1}{3}}$

$$= 3$$

Given that the line passes through (–3, 4),

$4 = 3(-3) + c$

$c = 13$

$\therefore$ Equation of the line is $y = 3x + 13$

25. **(a)** Gradient of $AB = \dfrac{6-1}{12-2}$

$$= \frac{5}{10}$$

$$= \frac{1}{2}$$

Since $ABC = 90°$, BC is perpendicular to AB.

Gradient of $BC = -\dfrac{1}{\frac{1}{2}}$

$$= -2$$

Given that BC passes through (12, 6),

$6 = -2(12) + c$

$c = 30$

$\therefore$ Equation of BC is $y = -2x + 30$, i.e. $y + 2x = 30$

Since AB is parallel to DC, gradient of $DC = \dfrac{1}{2}$.

Given that DC passes through (6, 9),

$9 = \dfrac{1}{2}(6) + d$

$d = 6$

$\therefore$ Equation of DC is $y = \dfrac{1}{2}x + 6$, i.e. $2y - x = 12$

OXFORD UNIVERSITY PRESS

(b) Point C is the intersection between BC and DC.

$$y + 2x = 30 \quad \text{— (1)}$$

$$y = \frac{1}{2}x + 6 \quad \text{— (2)}$$

Subst. (2) into (1):

$$\frac{1}{2}x + 6 + 2x = 30$$

$$2\frac{1}{2}x = 24$$

$$x = 9\frac{3}{5}$$

Subst. $x = 9\frac{3}{5}$ into (2):

$$y = \frac{1}{2}\left(9\frac{3}{5}\right) + 6$$

$$= 10\frac{4}{5}$$

∴ Coordinates of C are $\left(9\frac{3}{5}, 10\frac{3}{5}\right)$.

Revision 10G

1. $|\mathbf{a}| = |\mathbf{b}|$

$$\sqrt{12^2 + 5^2} = \sqrt{s^2 + 0^2}$$

$\therefore s = 13$

2. **(i)**

$$\overrightarrow{XY} = \overrightarrow{OY} - \overrightarrow{OX}$$

$$= \begin{pmatrix} 2 \\ 4 \end{pmatrix} - \begin{pmatrix} -1 \\ -2 \end{pmatrix}$$

$$= \begin{pmatrix} 3 \\ 6 \end{pmatrix}$$

(ii) $|\overrightarrow{XY}| = \sqrt{3^2 + 6^2}$

$= 6.71$ units (to 3 s.f.)

(iii)

$$2\overrightarrow{XZ} = 5\overrightarrow{XY}$$

$$2(\overrightarrow{OZ} - \overrightarrow{OX}) = 5(\overrightarrow{OY} - \overrightarrow{OX})$$

$$2\overrightarrow{OZ} - 2\begin{pmatrix} -1 \\ -2 \end{pmatrix} = 5\begin{pmatrix} 2 \\ 4 \end{pmatrix} - 5\begin{pmatrix} -1 \\ -2 \end{pmatrix}$$

$$2\overrightarrow{OZ} - \begin{pmatrix} -2 \\ -4 \end{pmatrix} = \begin{pmatrix} 10 \\ 20 \end{pmatrix} - \begin{pmatrix} -5 \\ -10 \end{pmatrix}$$

$$2\overrightarrow{OZ} = \begin{pmatrix} 10 \\ 20 \end{pmatrix} - \begin{pmatrix} -5 \\ -10 \end{pmatrix} + \begin{pmatrix} -2 \\ -4 \end{pmatrix}$$

$$2\overrightarrow{OZ} = \begin{pmatrix} 13 \\ 26 \end{pmatrix}$$

$$\overrightarrow{OZ} = \begin{pmatrix} \frac{13}{2} \\ 13 \end{pmatrix}$$

∴ Coordinates of Z are $\left(6\frac{1}{2}, 13\right)$.

3. **(i)** $|\overrightarrow{AB}| = \sqrt{(-5)^2 + 0^2}$

$= 5$ units

(ii) $|\overrightarrow{AB}| = |\overrightarrow{PQ}|$

$$5 = \sqrt{t^2 + (-3)^2}$$

$$25 = t^2 + 9$$

$$t^2 = 16$$

$$t = \pm 4$$

4. **(i)** Since M is the midpoint of QR, therefore

$$\overrightarrow{QM} = \overrightarrow{MR}$$

$$\overrightarrow{OM} - \overrightarrow{OQ} = \overrightarrow{OR} - \overrightarrow{OM}$$

$$\overrightarrow{OM} + \overrightarrow{OM} = \overrightarrow{OR} + \overrightarrow{OQ}$$

$$2\overrightarrow{OM} = \begin{pmatrix} 2 \\ 6 \end{pmatrix} + \begin{pmatrix} 4 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} 6 \\ 8 \end{pmatrix}$$

$$\overrightarrow{OM} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

(ii)

$$4\overrightarrow{PN} = \overrightarrow{NM}$$

$$4(\overrightarrow{ON} - \overrightarrow{OP}) = \overrightarrow{OM} - \overrightarrow{ON}$$

$$4\overrightarrow{ON} - 4\overrightarrow{OP} = \overrightarrow{OM} - \overrightarrow{ON}$$

$$4\overrightarrow{ON} + \overrightarrow{ON} = \overrightarrow{OM} + 4\overrightarrow{OP}$$

$$5\overrightarrow{ON} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} + 4\begin{pmatrix} -2 \\ -1 \end{pmatrix}$$

$$= \begin{pmatrix} 3 \\ 4 \end{pmatrix} + \begin{pmatrix} -8 \\ -4 \end{pmatrix}$$

$$= \begin{pmatrix} -5 \\ 0 \end{pmatrix}$$

$$\overrightarrow{ON} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

5. **(i)**

$$\overrightarrow{AX} = \frac{1}{3}\overrightarrow{AB}$$

$$= \frac{1}{3}\overrightarrow{OC}$$

$$= \frac{1}{3}(6\mathbf{q})$$

$$= 2\mathbf{q}$$

(ii)

$$\overrightarrow{OX} = \overrightarrow{OA} + \overrightarrow{AX}$$

$$= 6\mathbf{p} + 2\mathbf{q}$$

(iii)

$$\overrightarrow{OY} = \overrightarrow{OC} + \overrightarrow{CY}$$

$$= \overrightarrow{OC} + \frac{1}{2}\overrightarrow{OA}$$

$$= 6\mathbf{q} + \frac{1}{2}(6\mathbf{p})$$

$$= 3\mathbf{p} + 6\mathbf{q}$$

OXFORD UNIVERSITY PRESS

(iv) $\overrightarrow{XY} = \overrightarrow{XO} + \overrightarrow{OY}$

$= -(6\mathbf{p} + 2\mathbf{q}) + 3\mathbf{p} + 6\mathbf{q}$

$= 4\mathbf{q} - 3\mathbf{p}$

(v) $\overrightarrow{AY} = \overrightarrow{AO} + \overrightarrow{OY}$

$= -6\mathbf{p} + 3\mathbf{p} + 6\mathbf{q}$

$= 6\mathbf{q} - 3\mathbf{p}$

6. $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$

$$= \begin{pmatrix} 5 \\ 7 \end{pmatrix} - \begin{pmatrix} 6 \\ -1 \end{pmatrix}$$

$$= \begin{pmatrix} -1 \\ 8 \end{pmatrix}$$

$\overrightarrow{RQ} = \overrightarrow{OQ} - \overrightarrow{OR}$

$$= \begin{pmatrix} 5 \\ 7 \end{pmatrix} - \begin{pmatrix} 2 \\ 6 \end{pmatrix}$$

$$= \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

$\overrightarrow{MN} = \overrightarrow{MP} + \overrightarrow{PN}$

$$= \frac{1}{2}\overrightarrow{OP} + \frac{1}{2}\overrightarrow{PQ}$$

$$= \frac{1}{2}\begin{pmatrix} 6 \\ -1 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} -1 \\ 8 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{5}{2} \\ \frac{7}{2} \end{pmatrix}$$

$\overrightarrow{XY} = \overrightarrow{XR} + \overrightarrow{RY}$

$$= \frac{1}{2}\overrightarrow{QR} + \frac{1}{2}\overrightarrow{RO}$$

$$= -\frac{1}{2}\begin{pmatrix} 3 \\ 1 \end{pmatrix} - \frac{1}{2}\begin{pmatrix} 2 \\ 6 \end{pmatrix}$$

$$= \begin{pmatrix} -\frac{5}{2} \\ -\frac{7}{2} \end{pmatrix}$$

Since $\overrightarrow{XY} = -\overrightarrow{MN}$, therefore $\overrightarrow{XY}$ and $\overrightarrow{MN}$ are parallel vectors in opposite directions and $XY = MN$.

7. $|\overrightarrow{PQ}| = \sqrt{13^2 + 0^2}$

$= 13$ units

$|\overrightarrow{RS}| = \sqrt{(-5)^2 + 12^2}$

$= 13$ units

Hence, $|\overrightarrow{PQ}| = |\overrightarrow{RS}| = 13$ units. (shown)

$\overrightarrow{PQ} \neq \overrightarrow{RS}$ since they do not have the same direction.

8. **(i)** $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$

$$= \begin{pmatrix} 8 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} 8 \\ -2 \end{pmatrix}$$

$|\overrightarrow{PQ}| = \sqrt{8^2 + (-2)^2}$

$= 8.25$ units (to 3 s.f.)

(ii) $\overrightarrow{QR} = \overrightarrow{OR} - \overrightarrow{OQ}$

$$\begin{pmatrix} -2 \\ 4 \end{pmatrix} = \overrightarrow{OR} - \begin{pmatrix} 8 \\ 0 \end{pmatrix}$$

$$\overrightarrow{OR} = \begin{pmatrix} -2 \\ 4 \end{pmatrix} + \begin{pmatrix} 8 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 6 \\ 4 \end{pmatrix}$$

$\therefore$ Coordinates of R are $(6, 4)$.

(iii) $\overrightarrow{PR} = \overrightarrow{OR} - \overrightarrow{OP}$

$$= \begin{pmatrix} 6 \\ 4 \end{pmatrix} - \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} 6 \\ 2 \end{pmatrix}$$

$|\overrightarrow{PR}| = \sqrt{6^2 + 2^2}$

$= 6.32$ units (to 3 s.f.)

9. **(i)** $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$= -3\mathbf{a} + 4\mathbf{b}$

$= 4\mathbf{b} - 3\mathbf{a}$

(ii) $\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC}$

$= -3\mathbf{a} + \mathbf{b}$

$= \mathbf{b} - 3\mathbf{a}$

(iii) $\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AD}$

$$= \overrightarrow{OA} + \frac{2}{3}\overrightarrow{AC}$$

$$= 3\mathbf{a} + \frac{2}{3}(\mathbf{b} - 3\mathbf{a})$$

$$= \mathbf{a} + \frac{2}{3}\mathbf{b}$$

$$= \frac{1}{3}(3\mathbf{a} + 2\mathbf{b})$$

(iv) $\overrightarrow{OE} = \overrightarrow{OA} + \overrightarrow{AE}$

$$= \overrightarrow{OA} + \frac{1}{3}\overrightarrow{AB}$$

$$= 3\mathbf{a} + \frac{1}{3}(4\mathbf{b} - 3\mathbf{a})$$

$$= 2\mathbf{a} + \frac{4}{3}\mathbf{b}$$

$$= \frac{2}{3}(3\mathbf{a} + 2\mathbf{b})$$

OXFORD UNIVERSITY PRESS

10. (a) $|\overrightarrow{AB}| = \sqrt{6^2 + 0^2}$

$= 6$ units

(b) (i)

$$\begin{aligned}\overrightarrow{CB} &= -\overrightarrow{AD}\\ &= -\begin{pmatrix}2\\4\end{pmatrix}\\ &= \begin{pmatrix}-2\\-4\end{pmatrix}\end{aligned}$$

(ii)

$$\begin{aligned}\overrightarrow{EC} &= \frac{2}{3}\overrightarrow{DC}\\ &= \frac{2}{3}\overrightarrow{AB}\\ &= \frac{2}{3}\begin{pmatrix}6\\0\end{pmatrix}\\ &= \begin{pmatrix}4\\0\end{pmatrix}\end{aligned}$$

(iii) $\triangle FDE$ and $\triangle BCE$ are similar triangles (corresponding angles are equal).

So

$$\begin{aligned}\overrightarrow{FE} &= \frac{1}{2}\overrightarrow{EB}\\ &= \frac{1}{2}(\overrightarrow{EC} + \overrightarrow{CB})\\ &= \frac{1}{2}\left[\begin{pmatrix}4\\0\end{pmatrix} + \begin{pmatrix}-2\\-4\end{pmatrix}\right]\\ &= \frac{1}{2}\begin{pmatrix}2\\-4\end{pmatrix}\\ &= \begin{pmatrix}1\\-2\end{pmatrix}\end{aligned}$$

11. (i)

$$\begin{aligned}\overrightarrow{OX} &= \overrightarrow{OP} + \overrightarrow{PX}\\ &= \overrightarrow{OP} + \frac{1}{2}\overrightarrow{PQ}\\ &= \overrightarrow{OP} + \frac{1}{2}(\overrightarrow{OQ} - \overrightarrow{OP})\\ &= \mathbf{p} + \frac{1}{2}(\mathbf{q} - \mathbf{p})\\ &= \mathbf{p} + \frac{1}{2}\mathbf{q} - \frac{1}{2}\mathbf{p}\\ &= \frac{1}{2}\mathbf{p} + \frac{1}{2}\mathbf{q}\\ &= \frac{1}{2}(\mathbf{p} + \mathbf{q})\end{aligned}$$

(ii)

$$\begin{aligned}\overrightarrow{RS} &= 2\overrightarrow{SX}\\ \overrightarrow{OS} - \overrightarrow{OR} &= 2(\overrightarrow{OX} - \overrightarrow{OS})\\ \overrightarrow{OS} - \overrightarrow{OR} &= 2\overrightarrow{OX} - 2\overrightarrow{OS}\\ \overrightarrow{OS} + 2\overrightarrow{OS} &= 2\overrightarrow{OX} + \overrightarrow{OR}\\ 3\overrightarrow{OS} &= 2\left[\frac{1}{2}(\mathbf{p} + \mathbf{q})\right] + \mathbf{r}\\ 3\overrightarrow{OS} &= \mathbf{p} + \mathbf{q} + \mathbf{r}\\ \overrightarrow{OS} &= \frac{1}{3}(\mathbf{p} + \mathbf{q} + \mathbf{r})\end{aligned}$$

12. (a)

$$\begin{aligned}\overrightarrow{PQ} &= \overrightarrow{OQ} - \overrightarrow{OP}\\ &= \begin{pmatrix}3\\3\end{pmatrix} - \begin{pmatrix}0\\2\end{pmatrix}\\ &= \begin{pmatrix}3\\1\end{pmatrix}\end{aligned}$$

(b)

$$\begin{aligned}\overrightarrow{QR} &= 2\overrightarrow{PQ}\\ \overrightarrow{OR} - \overrightarrow{OQ} &= 2\begin{pmatrix}3\\1\end{pmatrix}\\ \overrightarrow{OR} - \begin{pmatrix}3\\3\end{pmatrix} &= \begin{pmatrix}6\\2\end{pmatrix}\\ \overrightarrow{OR} &= \begin{pmatrix}6\\2\end{pmatrix} + \begin{pmatrix}3\\3\end{pmatrix}\\ \overrightarrow{OR} &= \begin{pmatrix}9\\5\end{pmatrix}\end{aligned}$$

$\therefore$ Coordinates of R are $(9, 5)$.

(c) (i)

$$\begin{aligned}|\overrightarrow{PS}| &= \sqrt{6^2 + (-1)^2}\\ &= \sqrt{37}\\ &= 6.08 \text{ units (to 3 s.f.)}\end{aligned}$$

(ii) Gradient of the line $PS = -\frac{1}{6}$

(iii) Equation of the line PS: $y = -\frac{1}{6}x + 2$

(iv) Since TQ is parallel to the y-axis and the coordinates of Q are $(3, 3)$, hence the x-coordinate of T is 3.

Substitute $x = 3$ into the equation of the line PS:

$$\begin{aligned}y &= -\frac{1}{6}(3) + 2\\ &= 1\frac{1}{2}\end{aligned}$$

$\therefore$ Coordinates of T are $\left(3, 1\frac{1}{2}\right)$.

13. (i)

$$\begin{aligned}|\mathbf{b}| &= \sqrt{(-5)^2 + 12^2}\\ &= 13 \text{ units}\end{aligned}$$

(ii)

$$\begin{aligned}3\mathbf{a} + 2\mathbf{b} &= 3\begin{pmatrix}4\\1\end{pmatrix} + 2\begin{pmatrix}-5\\12\end{pmatrix}\\ &= \begin{pmatrix}12\\3\end{pmatrix} + \begin{pmatrix}-10\\24\end{pmatrix}\\ &= \begin{pmatrix}2\\27\end{pmatrix}\end{aligned}$$

(iii)

$$\begin{aligned}2\mathbf{a} - \mathbf{b} &= 2\mathbf{c}\\ 2\begin{pmatrix}4\\1\end{pmatrix} - \begin{pmatrix}-5\\12\end{pmatrix} &= 2\begin{pmatrix}p\\q\end{pmatrix}\\ 2\begin{pmatrix}p\\q\end{pmatrix} &= \begin{pmatrix}8\\2\end{pmatrix} - \begin{pmatrix}-5\\12\end{pmatrix}\\ \begin{pmatrix}p\\q\end{pmatrix} &= \begin{pmatrix}\frac{13}{2}\\-5\end{pmatrix}\end{aligned}$$

$\therefore p = \frac{13}{2}, q = -5$

OXFORD UNIVERSITY PRESS

14. (i) $$\begin{aligned}\overrightarrow{OB} &= 3\overrightarrow{OA}\\ &= 3\begin{pmatrix}-3\\4\end{pmatrix}\\ &= \begin{pmatrix}-9\\12\end{pmatrix}\end{aligned}$$

(ii) $$\begin{aligned}\overrightarrow{AC} &= \overrightarrow{OC} - \overrightarrow{OA}\\ &= \begin{pmatrix}2\\16\end{pmatrix} - \begin{pmatrix}-3\\4\end{pmatrix}\\ &= \begin{pmatrix}5\\12\end{pmatrix}\end{aligned}$$

$$\begin{aligned}|\overrightarrow{AC}| &= \sqrt{5^2 + 12^2}\\ &= 13 \text{ units}\end{aligned}$$

(iii) $$\begin{aligned}\overrightarrow{OD} &= \begin{pmatrix}-3\\4\end{pmatrix} + \begin{pmatrix}5\\-2\end{pmatrix}\\ &= \begin{pmatrix}2\\2\end{pmatrix}\end{aligned}$$

$\therefore$ Coordinates of D are $(2, 2)$.

15. (a) $$\begin{aligned}|\overrightarrow{QR}| &= \sqrt{1^2 + 5^2}\\ &= 5 \text{ units (to the nearest whole number)}\end{aligned}$$

(b) (i) $$\begin{aligned}\overrightarrow{SP} &= \overrightarrow{RQ}\\ &= -\overrightarrow{QR}\\ &= -\begin{pmatrix}1\\5\end{pmatrix}\\ &= \begin{pmatrix}-1\\-5\end{pmatrix}\end{aligned}$$

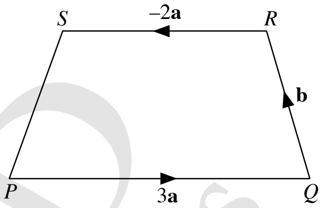
(ii) $$\begin{aligned}\overrightarrow{ST} &= \frac{1}{3}\overrightarrow{SR}\\ &= \frac{1}{3}\overrightarrow{PQ}\\ &= \frac{1}{3}\begin{pmatrix}6\\0\end{pmatrix}\\ &= \begin{pmatrix}2\\0\end{pmatrix}\end{aligned}$$

(iii) $$\begin{aligned}\overrightarrow{RT} &= \frac{2}{3}\overrightarrow{RS}\\ &= -\frac{2}{3}\overrightarrow{PQ}\\ &= -\frac{2}{3}\begin{pmatrix}6\\0\end{pmatrix}\\ &= \begin{pmatrix}-4\\0\end{pmatrix}\end{aligned}$$

(iv) Since $\triangle SPT$ and $\triangle RUT$ are similar (corresponding angles are equal),

$$\begin{aligned}\therefore\ \overrightarrow{UT} &= \frac{2}{1}\overrightarrow{TP}\\ &= 2(\overrightarrow{TS} + \overrightarrow{SP})\\ &= 2\begin{pmatrix}-2\\0\end{pmatrix} + 2\begin{pmatrix}-1\\-5\end{pmatrix}\\ &= \begin{pmatrix}-6\\-10\end{pmatrix}\end{aligned}$$

16. (i) $PQRS$ is a trapezium as $\overrightarrow{PQ} = \frac{3}{2}\overrightarrow{SR}$ so $\overrightarrow{PQ}$ is parallel to $\overrightarrow{SR}$.



(ii) $$\begin{aligned}\overrightarrow{SP} &= \overrightarrow{SR} + \overrightarrow{RQ} + \overrightarrow{QP}\\ &= 2\mathbf{a} - \mathbf{b} - 3\mathbf{a}\\ &= -(\mathbf{a} + \mathbf{b})\end{aligned}$$

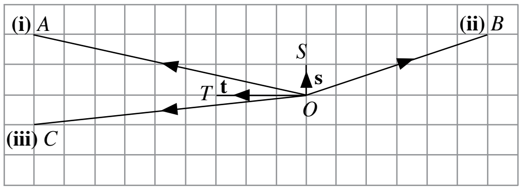
17. (a) (i) $$\begin{aligned}\overrightarrow{OD} &= \frac{1}{2}\overrightarrow{OB}\\ &= \frac{1}{2}(\overrightarrow{OA} + \overrightarrow{AB})\\ &= \frac{1}{2}(5\mathbf{p} - \mathbf{q} + \mathbf{p} + 3\mathbf{q})\\ &= \frac{1}{2}(6\mathbf{p} + 2\mathbf{q})\\ &= 3\mathbf{p} + \mathbf{q}\end{aligned}$$

(ii) $$\begin{aligned}\overrightarrow{AC} &= \overrightarrow{AO} + \overrightarrow{OC}\\ &= -(5\mathbf{p} - \mathbf{q}) + \mathbf{p} + 3\mathbf{q}\\ &= -4\mathbf{p} + 4\mathbf{q}\\ &= 4\mathbf{q} - 4\mathbf{p}\end{aligned}$$

(iii) $$\begin{aligned}\overrightarrow{AM} &= \overrightarrow{AB} + \overrightarrow{BM}\\ &= \mathbf{p} + 3\mathbf{q} + \frac{1}{2}(-5\mathbf{p} + \mathbf{q})\\ &= -1\frac{1}{2}\mathbf{p} + 3\frac{1}{2}\mathbf{q}\\ &= \frac{1}{2}(7\mathbf{q} - 3\mathbf{p})\end{aligned}$$

(b) $$\begin{aligned}\overrightarrow{AE} &= \overrightarrow{AO} + \overrightarrow{OE}\\ &= -(5\mathbf{p} - \mathbf{q}) + 2(\mathbf{p} + 3\mathbf{q})\\ &= 7\mathbf{q} - 3\mathbf{p}\end{aligned}$$

18. (a)



OXFORD UNIVERSITY PRESS

(b) $\overrightarrow{OP} = -5\mathbf{s} - 2\mathbf{t}$

$\overrightarrow{OQ} = -5\mathbf{s} + 2\frac{2}{3}\mathbf{t}$

$\overrightarrow{OR} = 7\mathbf{s} - \mathbf{t}$

$\overrightarrow{OU} = -4\mathbf{s} + \frac{2}{3}\mathbf{t}$

19. (a) (i) $\overrightarrow{QS} = \overrightarrow{QR} + \overrightarrow{RS}$

$= 6\mathbf{b} + 2\mathbf{a}$

$= 2(\mathbf{a} + 3\mathbf{b})$

(ii) $\overrightarrow{QM} = \frac{1}{2}\overrightarrow{QS}$

$= \frac{1}{2}(6\mathbf{b} + 2\mathbf{a})$

$= 3\mathbf{b} + \mathbf{a}$

$= \mathbf{a} + 3\mathbf{b}$

(iii) $\overrightarrow{PN} = \overrightarrow{PQ} + \frac{1}{2}\overrightarrow{QM}$

$= -2\mathbf{a} + \frac{1}{2}(\mathbf{a} + 3\mathbf{b})$

$= -1\frac{1}{2}\mathbf{a} + 1\frac{1}{2}\mathbf{b}$

$= \frac{3}{2}(\mathbf{b} - \mathbf{a})$

(iv) $\overrightarrow{ML} = \overrightarrow{MQ} + \overrightarrow{QL}$

$= -(\mathbf{a} + 3\mathbf{b}) + \frac{2}{3}(6\mathbf{b})$

$= -\mathbf{a} + \mathbf{b}$

$= \mathbf{b} - \mathbf{a}$

(b) Since $\overrightarrow{PN} = \frac{3}{2}\overrightarrow{ML}$, therefore PN is parallel to ML and $PN = \frac{3}{2}ML$.

(c) Trapezium

(d) (i) $\frac{\text{Area of } \triangle PNQ}{\text{Area of } \triangle PSN} = \frac{\frac{1}{2} \times NQ \times h}{\frac{1}{2} \times SN \times h}$, where h is the common height of $\triangle PNQ$ and $\triangle PSN$,

$= \frac{NQ}{SN}$

$= \frac{1}{3}$

(ii) Since $\triangle PSN$ and $\triangle QML$ are similar (corresponding angles are equal),

$\frac{\text{Area of } \triangle PSN}{\text{Area of } \triangle QML} = \left(\frac{PN}{ML}\right)^2$

$= \left(\frac{3}{2}\right)^2$

$= \frac{9}{4}$

20. (a) (i) $\overrightarrow{QR} = \overrightarrow{QP} + \overrightarrow{PR}$

$= -\mathbf{a} + \mathbf{b}$

$= \mathbf{b} - \mathbf{a}$

(ii) $\overrightarrow{QM} = \frac{1}{2}\overrightarrow{QR}$

$= \frac{1}{2}(\mathbf{b} - \mathbf{a})$

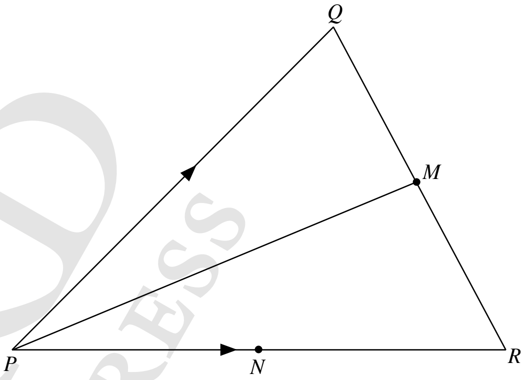
(iii) $\overrightarrow{PM} = \overrightarrow{PQ} + \overrightarrow{QM}$

$= \mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a})$

$= \frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$

$= \frac{1}{2}(\mathbf{a} + \mathbf{b})$

(b)



(c) (i) $\overrightarrow{PS} = 2\overrightarrow{PM}$

$= 2 \times \frac{1}{2}(\mathbf{a} + \mathbf{b})$

$= \mathbf{a} + \mathbf{b}$

(ii) $\overrightarrow{QS} = \overrightarrow{QP} + \overrightarrow{PS}$

$= -\mathbf{a} + \mathbf{a} + \mathbf{b}$

$= \mathbf{b}$

(iii) $\overrightarrow{RS} = \overrightarrow{RP} + \overrightarrow{PS}$

$= -\mathbf{b} + \mathbf{a} + \mathbf{b}$

$= \mathbf{a}$

(d) Parallelogram

Revision 10H

1. Let $OA = OB$ be r cm.

Using Pythagoras' Theorem,

$$OT^2 = OA^2 + AT^2$$
$$(r + 3.2)^2 = r^2 + 5^2$$
$$r^2 + 6.4r + 10.24 = r^2 + 25$$
$$6.4r = 25 - 10.24$$
$$6.4r = 14.76$$
$$r = 2.31 \text{ (to 3 s.f.)}$$

∴ The radius of the circle is 2.31 cm.

2. (i) $\angle OAT = \angle OBT = 90°$ (radius $\perp$ tangent)

Area of the quadrilateral $ATBO = 2 \times \frac{1}{2} \times 5 \times 17$

$= 85 \text{ cm}^2$

OXFORD UNIVERSITY PRESS

(ii) $\tan \angle TOA = \frac{17}{5}$

$\angle TOA = \tan^{-1}\left(\frac{17}{5}\right)$

$= 73.61°$ (to 2 d.p.)

Length of the minor arc $APB = \frac{73.61° \times 2}{360°} \times 2 \times \pi \times 5$

$= 12.8$ cm (to 3 s.f.)

3. Using Pythagoras' Theorem,

$16^2 = 9^2 + d^2$

$d^2 = 16^2 - 9^2$

$= 175$

$d = \sqrt{175}$

$= 13.2$ cm (to 3 s.f.)

The perpendicular distance from the centre of the circle to the chord is 13.2 cm.

4. Using Pythagoras' Theorem,

$r^2 = 7.5^2 + 6^2$

$= 92.25$

$r = \sqrt{92.25}$

$= 9.60$ cm (to 3 s.f.)

The radius of the circle is 9.60 cm.

5. **(i)** $\angle TAB = \angle TBA = 62°$ ($TA = TB$)

$\angle ATB = 180° - 62° - 62°$ ($\angle$ sum of $\triangle ATB$)

$= 56°$

(ii) $\angle BCA = \angle BAT = 62°$ ($\angle$s in alt. segment)

(iii) $\angle CBA = \angle BAT = 62°$ (alt. $\angle$s, $CB \parallel AT$)

$\angle BAC = 180° - 62° - 62°$ ($\angle$ sum of $\triangle ABC$)

$= 56°$

6. **(i)** $\angle OAC = \angle OCA$ (base $\angle$s of isos. $\triangle OAC$,

$= 58°$ $\quad OA = OC$ = radius of circle)

$\angle AOC = 180° - 58° - 58°$ ($\angle$ sum of $\triangle OAC$)

$= 64°$

(ii) $\angle BAC = 90°$ (rt. $\angle$ in semicircle)

$\angle ABC = 180° - 90° - 58°$ ($\angle$ sum of $\triangle ABC$)

$= 32°$

Alternatively,

$\angle AOB = 180° - 64°$ (adj. $\angle$s on a str. line)

$= 116°$

$\angle ABO = \angle BAO = \frac{180° - 116°}{2}$ (base $\angle$s of isos. $\triangle OAB$, $OA = OB$ = radius of circle)

$= 32°$

$\angle ABC = \angle ABO = 32°$

(iii) $\angle OAT = 90°$ (radius $\perp$ tangent)

$\angle CAT = 90° - 58°$

$= 32°$

(iv) $\angle ATO = 180° - 90° - 64°$ ($\angle$ sum of $\triangle OAT$)

$= 26°$

$\angle ATC = \angle ATO = 26°$

Alternatively,

$\angle ATC = 58° - 32°$ (ext. $\angle$ of $\triangle ATC$)

$= 26°$

7. $\angle OAB = \angle OBA$ (base $\angle$s of isos. $\triangle OAB$, $OA = OB$ = radius of circle)

$= \frac{180° - 110°}{2}$

$= 35°$

$\angle BAD = 180° - 84°$ ($\angle$s in opp. segments)

$= 96°$

$\angle OAD = 96° - 35°$

$= 61°$

8. **(i)** $\angle AOC = 2 \times 66°$ ($\angle$ at centre = 2$\angle$ at circumference)

$= 132°$

(ii) $\angle ABC = 180° - 66°$ ($\angle$s in opp. segments)

$= 114°$

$\angle BAC = 180° - 40° - 114°$ ($\angle$ sum of $\triangle ABC$)

$= 26°$

(iii) $\angle ACO = \frac{180° - 132°}{2}$ (base $\angle$s of isos. $\triangle OAC$, $OA = OC$ = radius of circle)

$= 24°$

(iv) $\angle OAT = 90°$ (radius $\perp$ tangent)

$\angle TAB = 90° - 24° - 26°$

$= 40°$

9. **(i)** $\angle ADB = 90°$ (rt. $\angle$ in semicircle)

$\angle ABD = 180° - 90° - 63°$ ($\angle$ sum of $\triangle ABD$)

$= 27°$

(ii) $\angle CDB = \angle ABD = 27°$ (alt. $\angle$s, $DC \parallel AB$)

$\angle DCB = 180° - 63°$ ($\angle$s in opp. segments)

$= 117°$

$\angle CBD = 180° - 27° - 117°$ ($\angle$ sum of $\triangle CBD$)

$= 36°$

(iii) $\angle OBC = 27° + 36°$

$= 63°$

$\angle BOC = 180° - 63° - 63°$ ($\angle$ sum of isos. $\triangle BOC$)

$= 54°$

10. **(i)** $\angle ACB = 38° + 23°$ (ext. $\angle$ of $\triangle ACE$)

$= 61°$

$\angle BDA = \angle ACB$ ($\angle$s in same segment)

$= 61°$

(ii) $\angle AXD = 180° - 61° - 23°$ ($\angle$ sum of $\triangle ADX$)

$= 96°$

$\angle BXC = \angle AXD$ (vert. opp. $\angle$s)

$= 96°$

11. **(i)** $\angle BAC = \frac{98°}{2}$ ($\angle$ at centre = 2$\angle$ at circumference)

$= 49°$

(ii) $\angle BOP = 180° - 98°$ (adj. $\angle$s on a str. line)

$= 82°$

$\angle APC = 22° + 82°$ (ext. $\angle$ of $\triangle BOP$)

$= 104°$

$\angle ACP = 180° - 49° - 104°$ ($\angle$ sum of $\triangle ACP$)

$= 27°$

OXFORD UNIVERSITY PRESS

12. (i) $\angle DAC = \angle DBC$ ($\angle$s in same segment)
$= 68°$
$\angle BAC = 90° - 68°$ (complementary $\angle$s)
$= 22°$
$\angle BDC = \angle BAC$ ($\angle$s in same segment)
$= 22°$
$\angle ADC = 180° - 115°$ (adj. $\angle$s on a str. line)
$= 65°$
$\angle ADB = 65° - 22°$
$= 43°$
$\angle ACB = \angle ADB$ ($\angle$s in same segment)
$= 43°$

Alternatively,
$\angle ADC = 180° - 115°$ (adj. $\angle$s on a str. line)
$= 65°$
$\angle ABC = 180° - 65°$ ($\angle$s in opp. segments)
$= 115°$
$\angle ABD = 115° - 68°$
$= 47°$
$\angle BAD = 180° - 90°$ (adj. $\angle$s on a str. line)
$= 90°$
$\angle ADB = 180° - 90° - 47°$ ($\angle$ sum of $\triangle ADB$)
$= 43°$
$\angle ACB = \angle ADB$ ($\angle$s in same segment)
$= 43°$

(ii) $\angle DAC = \angle CBD$ ($\angle$s in same segment)
$= 68°$
$\angle ACD = 180° - 68° - 65°$ ($\angle$ sum of $\triangle ACD$)
$= 47°$

13. (i) $\angle OAB = \angle OBA = \angle AOB = \dfrac{180°}{3}$ (equilateral $\triangle OAB$)
$= 60°$
$\angle BCD = 180° - 60°$ ($\angle$s in opp. segments)
$= 120°$

Alternatively,
Reflex $\angle BOD = 180° + 60°$
$= 240°$
Reflex $\angle BOD = 2 \times \angle BCD$ ($\angle$ centre $= 2\angle$ at circumference)
$\angle BCD = \dfrac{240°}{2}$
$= 120°$

(ii) $\angle ABC = 60° + 50°$
$= 110°$
$\angle ODC = 180° - 110°$ ($\angle$s in opp. segments)
$= 70°$

(iii) $\angle DOB = 60° + 60°$ (ext. $\angle$ of $\triangle OAB$)
$= 120°$
$\angle OBD = \angle ODB = \dfrac{180° - 120°}{2}$ (base $\angle$s of isos. $\triangle OBD$, $OB = OD =$ radius of circle)
$= 30°$
$\angle CBD = 50° - 30°$
$= 20°$

(iv) $\angle COD = 180° - 70° - 70°$ ($\angle$ sum of isos. $\triangle OCD$)
$= 40°$

14. (i) $\angle TAB = \angle TBA = \dfrac{180° - 48°}{2}$ (base $\angle$s of isos. $\triangle TAB$, $TA = TB$)
$= 66°$
$\angle PBA = 66° - 42°$
$= 24°$

(ii) $\angle AQP = \angle PBA$ ($\angle$s in same segment)
$= 24°$
$\angle AXP = 24° + 44°$ (ext. $\angle$ of $\triangle QAX$)
$= 68°$

15. $\angle APB = 90°$ (rt. $\angle$ in semicircle)
$\angle BPT = 118° - 90°$
$= 28°$
$\angle ABP = 28° + 34°$ (ext. $\angle$ of $\triangle BPT$)
$= 62°$

16. $\angle OAQ = 90°$ (radius $\perp$ tangent)
$\angle OAB = 90° - (3x + 4y)°$ (complementary $\angle$s)
$\angle OAB = \angle OBA$ (base $\angle$s of isos. $\triangle OAB$, $OA = OB$ = radius of circle)
$90° - (3x + 4y)° = 40°$
$(3x + 4y)° = 50°$
$3x + 4y = 50$ — (1)
$\angle AOB = 180° - 40° - 40°$ ($\angle$ sum of isos. $\triangle OAB$)
$= 100°$
$(7x + 6y)° = 100°$
$7x + 6y = 100$ — (2)
$(1) \times 7$: $21x + 28y = 350$ — (3)
$(2) \times 3$: $21x + 18y = 300$ — (4)
$(3) - (4)$: $21x + 28y - 21x - 18y = 350 - 300$
$10y = 50$
$y = 5$
Substitute $y = 5$ into (1):
$3x + 4(5) = 50$
$x = 10$
$\therefore x = 10, y = 5$

17. $\angle BAD = 180° - 132°$ (adj. $\angle$s on a str. line)
$= 48°$
$\angle BCD = 180° - 48°$ ($\angle$s in opp. segments)
$= 132°$
$\angle CBD = \angle CDB = \dfrac{180° - 132°}{2}$ (base $\angle$s of isos. $\triangle BCD$, $CB = CD$)
$= 24°$
$\angle BDA = 90°$ (rt. $\angle$ in semicircle)
$\angle ADC = 90° + 24°$
$= 114°$

18. (i) Reflex $\angle AOC = 132° \times 2$ ($\angle$ at centre $= 2\angle$ at circumference)
$= 264°$
$\angle AOC = 360° - 264°$ ($\angle$s at a point)
$= 96°$

(ii) $\angle APC = 180° - 96°$ ($\angle$s in opp. segments)
$= 84°$

OXFORD UNIVERSITY PRESS

19. **(i)** $\angle QRP = \angle APQ = 37°$ ($\angle$s in alt. segment)

(ii) $\angle QPR = 180° - 62° - 37°$ ($\angle$ sum of $\triangle$)

$= 81°$

(iii) $\angle QPS = 81° + 25°$

$= 106°$

$\angle QRS = 180° - 106°$ ($\angle$s in opp. segments)

$= 74°$

$\angle PRS = 74° - 37°$

$= 37°$

20. $(x + 10y)° + (14y + 2x)° = 180°$ ($\angle$s in opp. segments)

$3x + 24y = 180$

$x + 8y = 60 \quad$ — (1)

$\angle DCB = \angle DAB = 90°$ (rt. $\angle$ in semicircle)

$(x + 14y + 90)° = 180°$ ($\angle$ sum of $\triangle$)

$x + 14y = 90 \quad$ — (2)

(2) – (1): $6y = 30$

$y = 5$

Subst. $y = 5$ into (1):

$x + 8(5) = 60$

$x = 60 - 40$

$= 20$

$\therefore x = 20$ and $y = 5$

(i) $\angle COB = 2x°$ ($\angle$ at centre = $2\angle$ at circumference)

$= 2 \times 20°$

$= 40°$

(ii) $\angle ACD = \angle ABD = 2x°$ ($\angle$s in same segment)

$= 2 \times 20°$

$= 40°$

(iii) $\angle KAB = \angle BDA = 10y°$ ($\angle$s in alt. segment)

$= 10 \times 5°$

$= 50°$

OXFORD UNIVERSITY PRESS

Chapter 11 Revision: Probability and Statistics

Revision 11A

1. **(i)** P(a voter who voted for Candidate A is chosen) = $\dfrac{x}{36 + x}$

(ii)
$$\begin{aligned}\frac{x}{36+x} &= \frac{2}{5}\\ 5x &= 2(36 + x)\\ 5x &= 72 + 2x\\ 3x &= 72\\ x &= 24\end{aligned}$$

2. **(i)** P(blue marble) = $\dfrac{4}{16}$

$= \dfrac{1}{4}$

(ii) Let x be the number of blue marbles to be added.
$$\begin{aligned}\frac{x+4}{16+x} &= \frac{1}{3}\\ 3(x + 4) &= 16 + x\\ 3x + 12 &= 16 + x\\ 2x &= 4\\ x &= 2\end{aligned}$$

(iii) Let y be the number of red or green marbles to be added.
$$\begin{aligned}\frac{4}{16+y} &= \frac{1}{6}\\ 24 &= 16 + y\\ y &= 8\end{aligned}$$

3. **(a)**

First Die

Second Die

×	3	4	5	6	7	8
3	9	12	15	18	21	24
4	12	16	20	24	28	32
5	15	20	25	30	35	40
6	18	24	30	36	42	48
7	21	28	35	42	49	56
8	24	32	40	48	56	64

(b) **(i)** P(product is odd) = $\dfrac{9}{36}$

$= \dfrac{1}{4}$

(ii) P(product is less than or equal to 23) = $\dfrac{12}{36}$

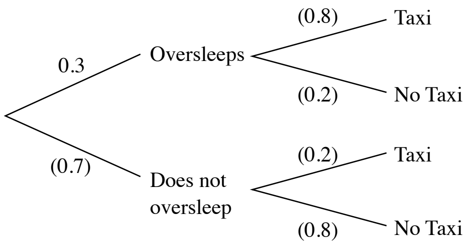
$= \dfrac{1}{3}$

(iii) P(product is prime) = 0

(iv) P(product is divisible by 14) = $\dfrac{6}{36}$

$= \dfrac{1}{6}$

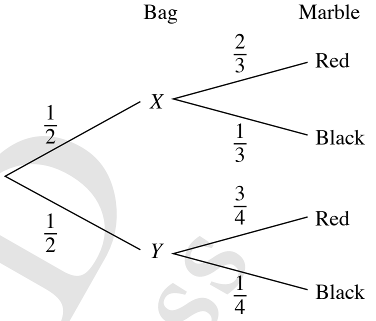
4. **(i)**



(ii) P(Michael takes a taxi to work) = $(0.3 \times 0.8) + (0.7 \times 0.2)$

$= 0.38$

5. **(i)**



(ii) P(a red marble) = $\left(\dfrac{1}{2} \times \dfrac{2}{3}\right) + \left(\dfrac{1}{2} \times \dfrac{3}{4}\right)$

$= \dfrac{17}{24}$

6. **(a)**

Second Pencil Case

First Pencil Case

×	E	P	P	R
E	EE	EP	EP	ER
E	EE	EP	EP	ER
P	PE	PP	PP	PR

(b) **(i)** P(two pens) = $\dfrac{2}{12}$

$= \dfrac{1}{6}$

(ii) P(at least one eraser) = $\dfrac{9}{12}$

$= \dfrac{3}{4}$

7. **(i)** P(both cards contain vowels) = $\dfrac{4}{10} \times \dfrac{3}{9}$

$= \dfrac{2}{15}$

(ii) P(second card contains a vowel)

= P(first card contains a consonant, second card contains a vowel) + P(both cards contain vowels)

$= \left(\dfrac{6}{10} \times \dfrac{4}{9}\right) + \dfrac{2}{15}$

$= \dfrac{2}{5}$

OXFORD UNIVERSITY PRESS

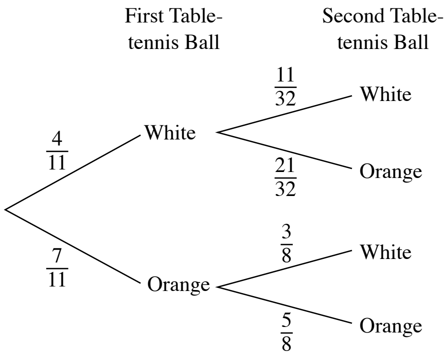
(iii) P(both cards are the same letter)

$= \text{P}(\text{E, E}) + \text{P}(\text{C, C}) + \text{P}(\text{L, L})$

$= \left(\frac{4}{10} \times \frac{3}{9}\right) + \left(\frac{2}{10} \times \frac{1}{9}\right) + \left(\frac{2}{10} \times \frac{1}{9}\right)$

$= \frac{8}{45}$

8.



(i) P(both balls are orange) $= \frac{7}{11} \times \frac{5}{8}$

$= \frac{35}{88}$

(ii) P(both balls are of the same colour) $= \left(\frac{7}{11} \times \frac{5}{8}\right) + \left(\frac{4}{11} \times \frac{11}{32}\right)$

$= \frac{23}{44}$

(iii) P(both balls are of different colours) $= 1 - \frac{23}{44}$

$= \frac{21}{44}$

9. (i) P(a player wins the grand prize on the first spin) $= \frac{30}{360}$

$= \frac{1}{12}$

(ii) P(a player wins the consolation prize on the second spin)

$= \frac{90}{360} \times \frac{60}{360}$

$= \frac{1}{24}$

(iii) P(a player spins the pointer three times and wins nothing)

$= \frac{90}{360} \times \frac{90}{360} \times \frac{180}{360}$

$= \frac{1}{32}$

10. (i) P(ball drawn is white) $= \frac{3}{6}$

$= \frac{1}{2}$

(ii) P(two balls drawn are of different colours)

$= 1 - \left(\frac{3}{6} \times \frac{3}{6}\right) - \left(\frac{2}{6} \times \frac{2}{6}\right) - \left(\frac{1}{6} \times \frac{1}{6}\right)$

$= \frac{11}{18}$

(iii) P(two balls drawn are of the same colour)

$= \left(\frac{3}{6} \times \frac{2}{5}\right) + \left(\frac{2}{6} \times \frac{1}{5}\right)$

$= \frac{4}{15}$

11. (i) P(first two cards will each have the letter ‘L’) $= \frac{2}{4} \times \frac{1}{3}$

$= \frac{1}{6}$

(ii) P(second card will have the letter ‘B’) $= \frac{3}{4} \times \frac{1}{3}$

$= \frac{1}{4}$

(iii) P(cards are drawn in the order ‘BALL’) $= \frac{1}{4} \times \frac{1}{3}$

$= \frac{1}{12}$

12. (i) P(Rui Feng passes the test and Farhan fails the test) $= \frac{2}{3} \times \frac{1}{6}$

$= \frac{1}{9}$

(ii) P(both of them fail the test) $= \frac{1}{3} \times \frac{1}{6}$

$= \frac{1}{18}$

(iii) P(at least one of them passes the test)

$= 1 - \text{P(both of them fail the test)}$

$= 1 - \frac{1}{18}$

$= \frac{17}{18}$

13. (i) P(sum of the three numbers is 3) $= \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6}$

$= \frac{1}{216}$

(ii) P(sum of the three numbers is 4)

$= \text{P}(1, 1, 2) + \text{P}(1, 2, 1) + \text{P}(2, 1, 1)$

$= 3\left(\frac{1}{6} \times \frac{1}{6} \times \frac{1}{6}\right)$

$= \frac{1}{72}$

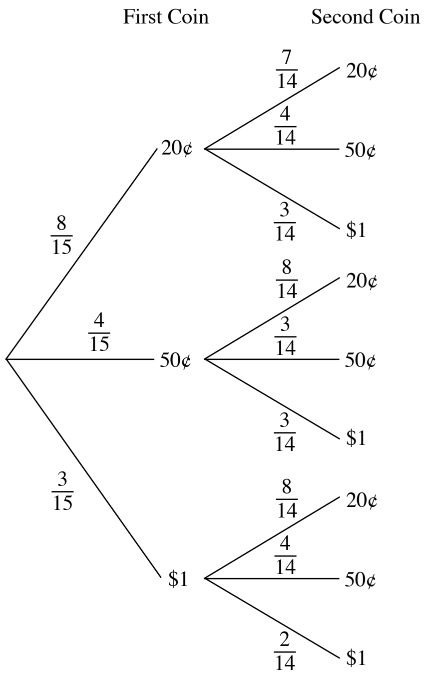
(iii) P(three dice show different numbers)

$= 1 \times \frac{5}{6} \times \frac{4}{6}$

$= \frac{5}{9}$

OXFORD UNIVERSITY PRESS

14. (a)



(b) **(i)** P(first coin is 20¢, second coin is $1) $= \frac{8}{15} \times \frac{3}{14}$

$= \frac{4}{35}$

(ii) P(two coins are in different denominations)

$= 1 - \left[\left(\frac{8}{15}\times\frac{7}{14}\right)+\left(\frac{4}{15}\times\frac{3}{14}\right)+\left(\frac{3}{15}\times\frac{2}{14}\right)\right]$

$= \frac{68}{105}$

(iii) P(second coin is 50¢)

$= \left(\frac{8}{15}\times\frac{4}{14}\right) + \left(\frac{4}{15}\times\frac{3}{14}\right) + \left(\frac{3}{15}\times\frac{4}{14}\right)$

$= \frac{4}{15}$

(iv) P(total value more than $1)

$= \left(\frac{8}{15}\times\frac{3}{14}\right) + \left(\frac{4}{15}\times\frac{3}{14}\right) + \frac{3}{15}$

$= \frac{13}{35}$

15 **(i)** P(boy selected is a Science student, girl selected is an Arts student)

$= \frac{7}{25} \times \frac{14}{23}$

$= \frac{98}{575}$

(ii) P(both are Arts students) $= \frac{18}{25} \times \frac{14}{23}$

$= \frac{252}{575}$

(iii) P(exactly one of them is a Science student)

$= \left(\frac{7}{25}\times\frac{14}{23}\right) + \left(\frac{18}{25}\times\frac{9}{23}\right)$

$= \frac{52}{115}$

16. **(i)** P(none of them wins the competition) $= \frac{1}{3}$

(ii) P(Shirley wins the competition) $= \frac{2}{3} - \frac{1}{3} - \frac{1}{8}$

$= \frac{5}{24}$

(iii) P(Nora or Shirley wins the competition) $= \frac{1}{3} + \frac{5}{24}$

$= \frac{13}{24}$

17. **(i)** $x(1-x) + (1-x)x = \frac{3}{8}$

$2x(1-x) = \frac{3}{8}$

$16x(1-x) = 3$

$16x^2 - 16x + 3 = 0$ (shown)

(ii) $16x^2 - 16x + 3 = 0$

$(4x-1)(4x-3) = 0$

$x = \frac{1}{4}$ or $x = \frac{3}{4}$

$x = \frac{1}{4}$ is rejected since the probability of obtaining a head is higher than the probability of obtaining a tail.

(iii) P(two heads) $= \frac{3}{4} \times \frac{3}{4}$

$= \frac{9}{16}$

18. **(i)** P(both cards bear the letter 'E') $= \frac{3}{10} \times \frac{3}{10}$

$= \frac{9}{100}$

(ii) P(both cards do not bear the letter 'E') $= \frac{7}{10} \times \frac{6}{9}$

$= \frac{7}{15}$

(iii) P(one card bears the letter 'E', the other card does not bear the letter 'E')

$= \left(\frac{3}{10}\times\frac{7}{10}\right) + \left(\frac{7}{10}\times\frac{3}{9}\right)$

$= \frac{133}{300}$

OXFORD UNIVERSITY PRESS

19. **(a)** P(red marble) $= \dfrac{x}{24 + x + y}$

$$\frac{x}{24 + x + y} = \frac{1}{5}$$

$$5x = 24 + x + y$$

$$y = 4x - 24 \quad \text{— (1)}$$

P(blue marble) $= \dfrac{y}{24 + x + y}$

$$\frac{y}{24 + x + y} = \frac{2}{5}$$

$$5y = 2(24 + x + y)$$

$$2x = 3y - 48 \quad \text{— (2)}$$

Substitute (1) into (2):

$$2x = 3(4x - 24) - 48$$

$$2x = 12x - 72 - 48$$

$$10x = 120$$

$$x = 12$$

$$y = 4(12) - 24$$

$$= 24$$

$\therefore x = 12, y = 24$

(b) **(i)** P(two marbles of the same colour)

$$= \left(\frac{24}{60} \times \frac{23}{59}\right) + \left(\frac{24}{60} \times \frac{23}{59}\right) + \left(\frac{12}{60} \times \frac{11}{59}\right)$$

$$= \frac{103}{295}$$

(ii) P(white marble followed by red marble) $= \dfrac{24}{60} \times \dfrac{12}{59}$

$$= \frac{24}{295}$$

20. **(i)** P(Bag *P* contains 7 yellow balls and 7 blue balls)

= P(blue ball selected from Bag *P* and yellow ball selected from Bag *Q*)

$$= \frac{8}{14} \times \frac{5}{15}$$

$$= \frac{4}{21}$$

(ii) P(Bag *Q* contains 6 yellow balls and 8 blue balls)

= P(yellow ball selected from Bag *P* and blue ball selected from Bag *Q*)

$$= \frac{6}{14} \times \frac{9}{15}$$

$$= \frac{9}{35}$$

(iii) P(Bag *P* contains 6 yellow balls and 8 blue balls)

= P(yellow ball selected from Bag *P* and yellow ball selected from Bag *Q*) or

P(blue ball selected from Bag *P* and blue ball selected from Bag *Q*)

$$= \left(\frac{6}{14} \times \frac{6}{15}\right) + \left(\frac{8}{14} \times \frac{10}{15}\right)$$

$$= \frac{58}{105}$$

(iv) P(Bag *Q* contains 5 yellow balls and 10 blue balls) = 0

(v) P(Bag *P* contains 4 yellow balls and 10 blue balls) = 0

(vi) P(Bag *Q* has more blue balls than yellow balls) = 1

21. P(point lies in the blue region) $= \dfrac{\pi(3p)^2 - \pi(2p)^2}{\pi(3p)^2}$

$$= \frac{5\pi p^2}{9\pi p^2}$$

$$= \frac{5}{9}$$

22. $\cos 30° = \dfrac{AB}{35}$

$$AB = 35 \cos 30°$$

P(point lies in the green region)

$$= \frac{\frac{1}{2}\pi(17.5)^2 - \frac{1}{2}(35)(35 \cos 30°) \sin 30°}{\frac{1}{2}\pi(17.5)^2}$$

= 0.449 (to 3 s.f.)

23. **(i)** P(red region) $= \dfrac{\frac{1}{2}[\pi(6^2) - \pi(3^2)]}{\pi(6^2)}$

$$= \frac{3}{8}$$

(ii) P(blue or yellow region) $= \dfrac{\frac{1}{4}[\pi(6^2) - \pi(3^2)] + \pi(3^2)}{\pi(6^2)}$

$$= \frac{7}{16}$$

(iii) P(a region that is not yellow) $= 1 - \dfrac{\frac{1}{4}[\pi(6^2) - \pi(3^2)]}{\pi(6^2)}$

$$= \frac{13}{16}$$

Revision 11B

1. **(a)**

$$7 + 5 + 6 = 2 + a + 8 + 7$$

$$18 = 17 + a$$

$$a = 1$$

$$7 + 5 + 6 + 2 = a + 8 + 7$$

$$20 = 15 + a$$

$$a = 5$$

$\therefore$ The possible values of *a* are 1, 2, 3, 4 and 5.

(b) **(i)**

$$\frac{15 + 42 + 15 + 21 + x + 2x}{6} = 18$$

$$15 + 42 + 15 + 21 + x + 2x = 108$$

$$3x = 15$$

$$x = 5$$

(ii) Standard deviation

$$= \sqrt{\frac{15^2 + 42^2 + 15^2 + 21^2 + 5^2 + 10^2}{6} - 18^2}$$

$$= \sqrt{\frac{2780}{6} - 18^2}$$

= 11.8 years (to 3 s.f.)

OXFORD UNIVERSITY PRESS

2. **(a)** $225° + 2x° + 3x° = 360°$

$$5x = 360 - 225$$
$$x = 27$$

(b) **(i)** Sales value of food $= \frac{225°}{360°} \times 21\,600$

$= \$13\,500$

(ii) Sales value of stationery $= \frac{54°}{360°} \times 21\,600$

$= \$3240$

3. Arrange the data in ascending order:

21, 28, 30, 32, 33, 43, 55

(i) Median number of push-ups = 32

(ii) Range = 55 – 21

= 34

(iii) Interquartile range = 43 – 28

= 15

4. **(i)** There are 57 students in the group.

(ii) Median mass of the school bags = 6.9 kg

(iii) Modal mass of the school bags = 6.3 kg

(iv) Percentage of school bags that are considered overweight

$= \frac{21}{57} \times 100\%$

$= 36.8\%$ (to 3 s.f.)

5. Since the graph does not start at 0 on the x-axis, it has misled the reader that the number of fresh graduates who has found jobs within three months of graduation in 2014 has tripled as compared to that in 2010.

6. **(i)**

Mass (kg)	**Frequency**	**Mid-value (x)**	***fx***
$41 \leqslant x \leqslant 45$	3	43	129
$46 \leqslant x \leqslant 50$	4	48	192
$51 \leqslant x \leqslant 55$	10	53	530
$56 \leqslant x \leqslant 60$	13	58	754
$61 \leqslant x \leqslant 65$	4	63	252
$66 \leqslant x \leqslant 70$	4	68	272
$71 \leqslant x \leqslant 75$	2	73	146
Sum	$\Sigma f = 40$		$\Sigma fx = 2275$

(ii) Mean mass, $\overline{x} = \frac{\Sigma fx}{\Sigma f}$

$= \frac{2275}{40}$

$= 56.875$ kg

7. **(a)** Number of players who cover more than 10.5 km per match

= 560 – 475

= 85

(b) For this set of data, $n = 560$.

$\therefore \frac{n}{2} = 280, \frac{n}{4} = 140$ and $\frac{3n}{4} = 420$

(i) From the graph, median distance covered = 9.5 km.

(ii) From the graph, upper quartile = 10.1 km.

From the graph, lower quartile = 8.9 km.

Interquartile range = 10.1 – 8.9

= 1.2 km

(c) It means that 50% of the players have covered 9.5 km or less.

8. **(a)** **(i)** Mean fare per passenger

$$= \frac{(45 \times 47) + (64 \times 165) + (85 \times 72) + (105 \times 34) + (115 \times 46) + (125 \times 26)}{47 + 165 + 72 + 34 + 46 + 26}$$

= 79 cents

(ii) Modal fare = 64 cents

(b) **(i)**

Number of children	**Number of families**
1	9
2	11
3	10
4	7
5	4
6	2
7	1

(ii) Percentage of families with fewer than three children

$= \frac{20}{44} \times 100\%$

$= 45.5\%$ (to 3 s.f.)

9. **(a)**

Length (mm)	**Frequency**
$30 < x \leqslant 50$	8
$50 < x \leqslant 70$	40
$70 < x \leqslant 90$	34
$90 < x \leqslant 110$	16
$110 < x \leqslant 130$	2

(b)

Length (mm)	**Frequency**	**Mid-value (x)**	***fx***	***fx*²**
$30 < x \leqslant 50$	8	40	320	12 800
$50 < x \leqslant 70$	40	60	2400	144 000
$70 < x \leqslant 90$	34	80	2720	217 600
$90 < x \leqslant 110$	16	100	1600	160 000
$110 < x \leqslant 130$	2	120	240	28 800
Sum	$\Sigma f = 100$		$\Sigma fx = 7280$	$\Sigma fx^2 = 563\,200$

(i) Mean length of a leaf from the plant $= \frac{\Sigma fx}{\Sigma f}$

$= \frac{7280}{100}$

$= 72.8$ mm

OXFORD UNIVERSITY PRESS

(ii) Standard deviation $= \sqrt{\dfrac{\Sigma fx^2}{\Sigma f} - \overline{x}^2}$

$$= \sqrt{\frac{563\,200}{100} - 72.8^2}$$

$= 18.2$ mm (to 3 s.f.)

(c) The cumulative frequency curve will be steeper than the given curve.

10. (i)

PSI in City *A*	Frequency
$50 < x \leqslant 70$	6
$70 < x \leqslant 90$	7
$90 < x \leqslant 110$	7

PSI in City *B*	Frequency
$50 < x \leqslant 70$	14
$70 < x \leqslant 90$	5
$90 < x \leqslant 110$	1

(ii) Mean for the PSI for city *A*

$$= \frac{(60 \times 6) + (80 \times 7) + (100 \times 7)}{20}$$

$= 81$

Standard deviation for the PSI for city *A*

$$= \sqrt{\frac{(6 \times 60^2) + (7 \times 80^2) + (7 \times 100^2)}{20} - 81^2}$$

$$= \sqrt{\frac{136\,400}{20} - 81^2}$$

$= 16.1$ (to 3 s.f.)

Mean for the PSI for city *B*

$$= \frac{(60 \times 14) + (80 \times 5) + (100 \times 1)}{20}$$

$= 67$

Standard deviation for the PSI for city *B*

$$= \sqrt{\frac{(14 \times 60^2) + (5 \times 80^2) + (1 \times 100^2)}{20} - 67^2}$$

$$= \sqrt{\frac{92\,400}{20} - 67^2}$$

$= 11.4$ (to 3 s.f.)

(iii) Since the mean for the PSI for City *A* is higher than that of City *B*, the air quality in City *A* is worse than that of City *B*. Since the standard deviation for the PSI for City *B* is lower than that of City *A*, the PSI readings for City *B* are less widely spread as compared to those of City *A*.

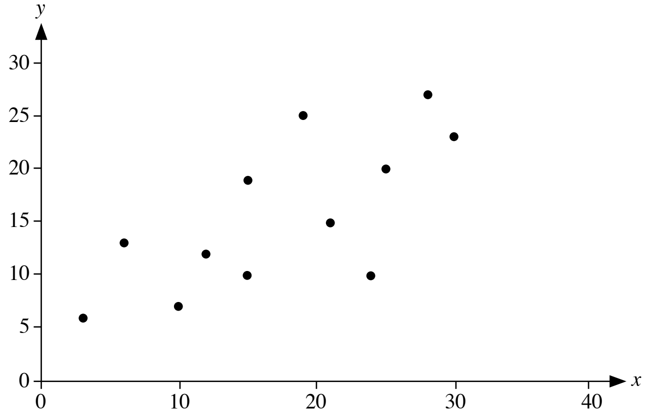
11. (a) Positive, strong correlation

(b) Negative, strong correlation

(c) Positive, weak correlation

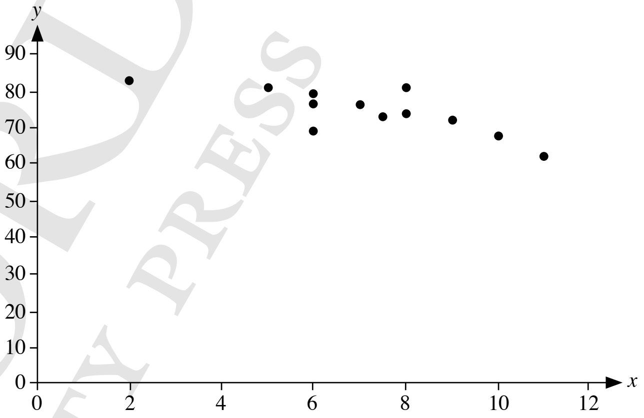
(d) Zero correlation

12. (i)



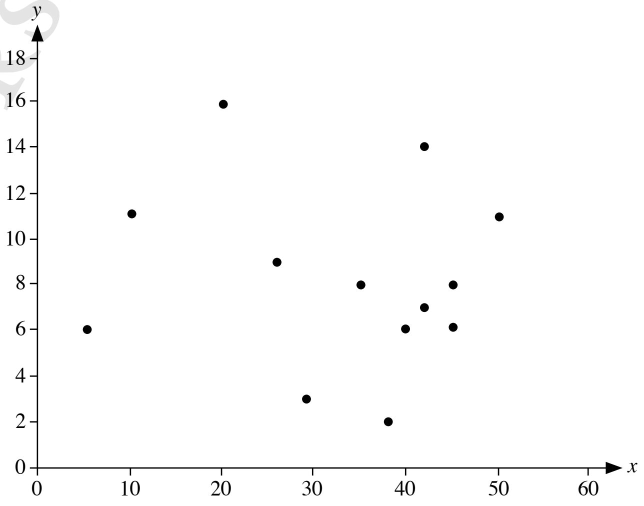
The data have a positive, weak correlation.

(ii)



The data have a negative, strong correlation.

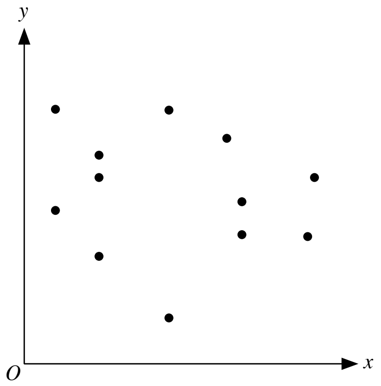
(iii)



The data have zero correlation.

OXFORD UNIVERSITY PRESS

13. **(i)**



(ii) The expected scatter diagram would show zero correlation between the age and height of the University students as by that age, most people would have stopped growing.

14. **(i)**

	Diameter (d cm)	Number of Tree Trunks
Group I	$0 < d < 50$	20
Group II	$50 \leqslant d < 60$	18
Group III	$60 \leqslant d < 70$	25
Group IV	$70 \leqslant d < 100$	37

(ii) Mean of the diameter of a tree trunk

$$= \frac{(25 \times 20) + (55 \times 18) + (65 \times 25) + (85 \times 37)}{100}$$

= 62.6 cm

Standard deviation of the diameter of a tree trunk

$$= \sqrt{\frac{(20 \times 25^2) + (18 \times 55^2) + (25 \times 65^2) + (37 \times 85^2)}{100} - 62.6^2}$$

$$= \sqrt{\frac{439\ 900}{100} - 62.6^2}$$

= 21.9 cm (to 3 s.f.)

(iii) P(diameters of both tree trunks lie in $50 \leqslant d < 60$)

$$= \frac{18}{100} \times \frac{17}{99}$$

$$= \frac{17}{550}$$

15. **(a)** For this set of data, $n = 500$.

$\therefore \ \frac{n}{2} = 250, \ \frac{n}{4} = 125$ and $\frac{3n}{4} = 375$

(i) From the graph, median = 88 g.

(ii) From the graph, upper quartile = 108 g.

From the graph, lower quartile = 60 g.

Interquartile range = 108 – 60

= 48 g

(iii) 80% of the total frequency $= \frac{80}{100} \times 500$

$= 400$

From the graph, the 80th percentile = 114 g

(b) The 80th percentile means that 80% of the starfruits have a mass of 114 g or less.

(c) From the graph, 500 – 150 = 350 starfruits have a mass of 104 g or less.

∴ Value of this mass is 104 g.

(d) **(i)** Median of the delivery to Supermarket Q

= 92 g

The delivery to Supermarket Q has a bigger mass since its median is higher than that of the delivery to Supermarket P.

(ii) Interquartile range of the delivery to Supermarket Q

= 130 – 72

= 58 g

The delivery to Supermarket Q has a bigger spread of mass since its interquartile range is higher than that of the delivery to Supermarket P.

16. **(i)**

Length of Service (x years)	Frequency
$0 < x \leqslant 2$	48
$2 < x \leqslant 4$	54
$4 < x \leqslant 6$	51
$6 < x \leqslant 8$	22
$8 < x \leqslant 10$	9

(ii) Total number of workers in the group = 48 + 54 + 51 + 22 + 9

= 184

(iii) Mean number of years of service per worker

$$= \frac{(1 \times 48) + (3 \times 54) + (5 \times 51) + (7 \times 22) + (9 \times 9)}{184}$$

= 3.80 (to 3 s.f.)

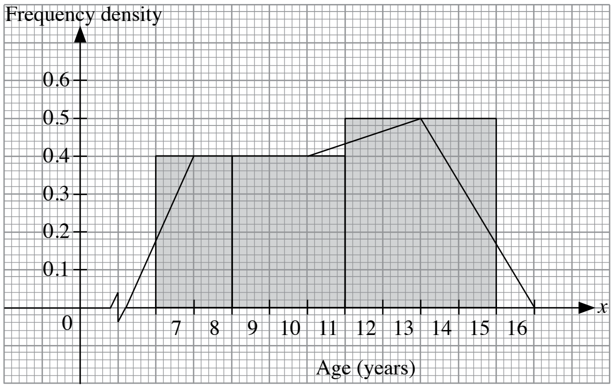
(iv) The actual value of each data is not used to calculate the mean. Only the mid-value for each grouped data is used to find the mean and hence the answer to **(iii)** is an estimate of the mean.

(v) Greatest possible mean number of years of service

$$= \frac{(2 \times 48) + (4 \times 54) + (6 \times 51) + (8 \times 22) + (10 \times 9)}{184}$$

= 4.80 (to 3 s.f.)

17.



OXFORD UNIVERSITY PRESS

18. (i) Mean of the marks

$$= \frac{(10 \times 12) + (30 \times 25) + (50 \times 30) + (70 \times 55) + (90 \times 38)}{160}$$

= 60.25

Standard deviation of the marks

$$= \sqrt{\frac{\begin{array}{l}(12 \times 10^2) + (25 \times 30^2) + (30 \times 50^2)\\ + (55 \times 70^2) + (38 \times 90^2)\end{array}}{160} - 60.25^2}$$

$$= \sqrt{\frac{676\,000}{160} - 60.25^2}$$

= 24.4 (to 3 s.f.)

(ii) The students performed better in the end-of-year Mathematics examination since its mean is higher than that of the mid-year examination.

(iii) The marks in the mid-year Mathematics examination are more consistent since its standard deviation is lower than that of the end-of-year examination.

19. (i) Number of houses in this estate that do not have any children living in them

= 100 – 13 – 37 – 17 – 5 – 7

= 21

(ii) As the number of children living in each of the 7 houses (with more than 4 children) is not known, it is impossible to calculate the mean number of children living in each house in the housing estate.

(iii) Mean number of children in 'crowded houses'

$$= \frac{200 - [(1 \times 13) + (2 \times 37) + (3 \times 17) + (4 \times 5)]}{7}$$

= 6

20. (a) (i) Median = 81

(ii) Range = 140 – 20

= 120

(iii) Interquartile range = 98 – 46

= 52

(b) (i) Median = 75

(ii) Range = 122 – 28

= 94

(iii) Interquartile range = 97 – 56

= 41

(c) Company *A*'s overall sales performance was better since its median number of tablets sold is higher than that of Company *B*.

(d) Company *B*'s sales performance showed more consistency since its interquartile range is lower than that of Company *A*.

21. (a)

Running Club	Lower Quartile	Median	Upper Quartile	Interquartile Range
Swift	500	516	548	48
Ninja	490	510	532	42

(b) (i) Disagree since the median timing of Swift Running Club is higher than that of Ninja Running Club.

(ii) Agree since the interquartile range of Ninja Running Club's timing is lower than that of Swift Running Club.

22. (i) Mean of the lifespans of the batteries

$$= \frac{\begin{array}{c}(5 \times 3) + (7 \times 13) + (9 \times 42) + (11 \times 80)\\ + (13 \times 16) + (15 \times 6)\end{array}}{160}$$

= 10.4 h (to 3 s.f.)

Standard deviation of the lifespans of the batteries

$$= \sqrt{\frac{\begin{array}{l}(3 \times 5^2) + (13 \times 7^2) + (42 \times 9^2)\\ + (80 \times 11^2) + (16 \times 13^2) + (6 \times 15^2)\end{array}}{160} - 10.3875^2}$$

$$= \sqrt{\frac{17\,848}{160} - 10.3875^2}$$

= 1.91 h (to 3 s.f.)

(ii) Since the mean of the lifespans of the batteries manufactured by factory *B* is higher than that of factory *A*, therefore the lifespans of the batteries manufactured by factory *B* are longer than the lifespans of the batteries manufactured by factory *A*.

Since the standard deviation of the lifespans of the batteries manufactured by factory *B* is lower than that of factory *A*, therefore the lifespans of the batteries manufactured by factory *B* are more consistent than the lifespans of the batteries manufactured by factory *A*.

(iii) Battery manufactured by factory *B* since the lifespans of the batteries manufactured by factory *B* are longer and more consistent than the lifespans of the batteries manufactured by factory *A*.

23. (a)

Number of times	Number of employees
1	5
2	3
3	2
4	8
5	6
6	4
7	3
8	2
9	2

OXFORD UNIVERSITY PRESS

(b) There are 35 employees altogether. The employee in the 18th position took medical leave 4 times.

$\therefore$ Median number of times the employees took medical leave in 2014 is 4.

(c) Mean $= \dfrac{\begin{array}{c}(1 \times 5) + (2 \times 3) + (3 \times 2) + (4 \times 8) + (5 \times 6) \\ + (6 \times 4) + (7 \times 3) + (8 \times 2) + (9 \times 2)\end{array}}{35}$

$$= \frac{158}{35}$$

$$= 4.51 \text{ (to 3 s.f.)}$$

Standard deviation

$$= \sqrt{\frac{\begin{array}{l}(5 \times 1^2) + (3 \times 2^2) + (2 \times 3^2) + (8 \times 4^2) \\ + (6 \times 5^2) + (4 \times 6^2) + (3 \times 7^2) \\ + (2 \times 8^2) + (2 \times 9^2)\end{array}}{35} - \left(\frac{158}{35}\right)^2}$$

$$= \sqrt{\frac{894}{35} - \left(\frac{158}{35}\right)^2}$$

$$= 2.27 \text{ (to 3 s.f.)}$$

(d) **(i)** There will be no change in the standard deviation since the difference between each data and the mean cancels the overcount error.

(ii) The erroneous mean is 1 higher than the correct mean since the total overcount (35) is divided by the total number of employees (35).

24. **(i)** Mean speed

$$= \frac{\begin{array}{c}80 + 83 + 70 + 64 + 71 + 75 + 61 + 80 + 79 + 68 \\ + 85 + 73 + 67 + 88 + 72 + 62 + 69 + 74 + 75 + 78\end{array}}{20}$$

$$= \frac{1474}{20}$$

$$= 73.7 \text{ km/h}$$

Standard deviation

$$= \sqrt{\frac{\begin{array}{l}80^2 + 83^2 + 70^2 + 64^2 + 71^2 + 75^2 + 61^2 \\ + 80^2 + 79^2 + 68^2 + 85^2 + 73^2 + 67^2 + 88^2 \\ + 72^2 + 62^2 + 69^2 + 74^2 + 75^2 + 78^2\end{array}}{20} - 73.7^2}$$

$$= \sqrt{\frac{109\ 698}{20} - 73.7^2}$$

$$= 7.29 \text{ km/h (to 3 s.f.)}$$

(ii) Since the mean speed of the group of cars on Sunday is higher than that of the group of cars on Saturday, the group of cars on Sunday was generally travelling at a speed faster than that of the group of cars on Saturday.

Since the standard deviation of the speed of the group of cars on Sunday is lower than that of the group of cars on Saturday, the speeds of the group of cars on Sunday are more consistent than the speeds of the group of cars on Saturday.

(iii) No, since the sample size is only 20.

OXFORD UNIVERSITY PRESS

Problems in Real-World Contexts

1. **(a)** Gradient, $m = \dfrac{76 - 4}{0 - 20}$

$= -3.6$

y-intercept, $c = 76.5$

Equation of the straight line: $T = -3.6t + 76.5$

(b) **(i)** When $T = 56$,

$56 = -3.6t + 76.5$

$3.6t = 20.5$

$t = 5.69$ (3 s.f.)

$\therefore$ The time taken for the temperature of the tea to decrease to 56 °C is 5.69 min.

(ii) When $t = 25$,

$T = -3.6(25) + 76.5$

$= -13.5$ °C

$\therefore$ The temperature of the tea after 25 minutes is −13.5 °C. It is not a good estimate as it is not possible for the tea to cool to a temperature below 0 °C if the tea was cooled under room temperature.

(c) **(i)** When $t = 0$, $T = 78$.

$78 = a(2.72)^{-0.09(0)} + 21$

$78 = a + 21$

$a = 78 - 21$

$= 57$

(ii) When $t = 50$,

$T = 57(2.72)^{-0.09(50)} + 21$

$= 21.6$ °C (to 3 s.f.)

This temperature represents the surrounding temperature.

2. **(a)** Let r m be the radius of the base of the tree trunk.

$2\pi r = 2.5$

$r = \dfrac{2.5}{2\pi}$

$= 0.398$ (to 3 s.f.)

$\therefore$ Radius of the base of the tree trunk is 0.398 m.

(b) Let r_1 m be the radius of the top surface.

$2\pi r_1 = 0.95$

$r_1 = \dfrac{0.95}{2\pi}$

$= 0.151$ (to 3 s.f.)

$\therefore$ Radius of the top surface is 0.151 m.

Let h_1 m be the height of the smaller cone and h m be the height of the larger cone.

Using similar triangles,

$$\frac{h_1}{h} = \frac{r_1}{r}$$

$$\frac{h_1}{1.8 + h_1} = \frac{0.1512}{0.3979}$$

$$\frac{h_1}{1.8 + h_1} = 0.38$$

$$h_1 = 0.38(1.8 + h_1)$$

$$h_1 = 0.684 + 0.38h_1$$

$$h_1 - 0.38h_1 = 0.684$$

$$0.62h_1 = 0.684$$

$$h_1 = 1\frac{16}{155}$$

Volume of the tree trunk

$= \dfrac{1}{3}\pi r^2 h - \dfrac{1}{3}\pi r_1{}^2 h_1 + \pi r^2(2.2)$

$= \dfrac{1}{3}\pi\left(\dfrac{2.5}{2\pi}\right)^2\left(2\dfrac{28}{31}\right) - \dfrac{1}{3}\pi\left(\dfrac{0.95}{2\pi}\right)^2\left(1\dfrac{16}{155}\right)$

$+ \pi\left(\dfrac{2.5}{2\pi}\right)^2 (2.2)$

$= 1.55$ m^3 (to 3 s.f.)

(c) $\dfrac{\text{Mass of tree trunk}}{\text{Volume of tree trunk}} \leqslant 600$

Mass of the tree trunk $\leqslant 1.549 \times 600$

$\therefore$ Mass of the tree trunk $\leqslant$ 929 kg (to 3 s.f.)

Hence, the best possible choice of truck will be the one that has a maximum load capacity of 1 tonne since the mass of the tree trunk is not more than 0.929 tonnes.

3. **(i)** For Singapore,

Net population density $= \dfrac{\text{Total population}}{\text{Total developable land area}}$

$= \dfrac{5\ 300\ 000}{500}$

$= 10\ 600$ people/km^2

For Hong Kong,

Net population density $= \dfrac{\text{Total population}}{\text{Total developable land area}}$

$= \dfrac{7\ 000\ 000}{317}$

$= 22\ 080$ people/km^2

One possible explanation is that the total developable land area, 317 km^2, is 316.5 km^2 rounded off to the nearest whole number. Hence,

Net population density $= \dfrac{\text{Total population}}{\text{Total developable land area}}$

$= \dfrac{7\ 000\ 000}{316.5}$

$= 22\ 116.9\ldots$ people/km^2

The net population density obtained is truncated to become 22 110 people/km^2 as shown in the table.

(ii) For Singapore,

Gross population density $= \dfrac{\text{Total population}}{\text{Total land area}}$

$= \dfrac{5\ 300\ 000}{710}$

$= 7500$ people/km^2 (to 2 s.f.)

OXFORD UNIVERSITY PRESS

For Hong Kong,

$$\text{Gross population density} = \frac{\text{Total population}}{\text{Total land area}} = \frac{7\,000\,000}{1108} = 6300 \text{ people/km}^2 \text{ (to 2 s.f.)}$$

Singapore has a higher gross population density.

(iii) The net population density is a better measure of the population density of a city because gross population density includes undevelopable land area that is not suitable for residential and commercial purposes. Therefore, it would be inaccurate to measure population density by dividing the total population by the total land area.

4. (i) It will be reflected by a horizontal line in the quarter.

(ii) Yes. If we were to compare the gradient of the curve before and after the first quarter of 2013, the tablet sales have been increasing at a much faster rate after the first quarter of 2013. However, there is a sign that the sales are slowing down after the third quarter of 2014, which is still better than the sales before 2013.

(iii) Even when there is a drop in sales in a particular quarter, it may not be noticeable since the sales performance is presented using a cumulative frequency curve.

(iv) A cumulative sales curve is definitely an increasing curve (unless zero sales were made which is unlikely) since cumulative frequency is a 'running total' of the sales. This curve gives the impression that sales have been increasing.

5. (a) (i) \$2 781 606 847 – \$2 185 327 302 = \$596 234 545

(ii) No, since the prices of movie tickets in 1997 and 2009 are different.

(b) (i) Box office earnings of Blue Planet

$= \$2\,781\,606\,847 \times 1.035^4$

$= \$3\,191\,957\,836$

(ii) Box office earnings of The Shipwreck

$= \$2\,185\,372\,302 \times 1.035^{16}$

$= \$3\,789\,405\,063$

(c) \$3 789 405 063 – \$3 191 957 836 = \$597 447 227

The Shipwreck made more money by the amount of \$597 447 227.

This comparison is a fairer one as these earnings take into account the inflation rate of the movie tickets. These earnings are the amounts that these two movies would have raked in if they were released in 2013.

(d) Box office earnings of The President in 2013

$= \$205\,400\,000 \times 1.035^{22}$

6. (a) (i) It represents a direct flight available between the two cities.

(ii) Flight does not exist between the same city.

(b)

$$\mathbf{M}^2 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 0 & 2 & 1 & 0 \\ 2 & 0 & 1 & 2 \end{pmatrix}$$

'1' represents a direct flight between the two cities while '2' represents an indirect flight with stopovers between two cities. '0' represents non-existent flight between the same city.

(c)

$$\mathbf{M}^2 + \mathbf{M} = \begin{pmatrix} 1 & 0 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 0 & 2 & 1 & 0 \\ 2 & 0 & 1 & 2 \end{pmatrix} + \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 \\ 1 & 2 & 1 & 1 \\ 2 & 1 & 2 & 2 \end{pmatrix}$$

It is possible to travel from one city to another city either directly or with stopovers.

(d) $\mathbf{M}^0$ will be the identity matrix, $\mathbf{I}$, since $\mathbf{M}^1\mathbf{M}^{-1} = \mathbf{I}$ so $\mathbf{M}^0 = \mathbf{I}$.

7. (a) Probability of a particular number winning the first prize in the draw

$= \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10}$

$= \frac{1}{10\,000}$

(b) Probability of a particular number winning any prize in a 4-D draw

$= 1 - \left(\frac{9999}{10\,000}\right)^{23}$

$= 0.002\,297$ (to 4 s.f.)

(c) No. The statement should read: "The probability of a particular number winning **a particular** prize in a 4-D draw is one out of 10 000 ..."

(d) Probability of a particular number winning both the first and second prizes in the same 4-D draw

$= \frac{1}{10\,000 \times 10\,000}$

$= \frac{1}{100\,000\,000}$

(e) Probability of any number winning both the first and second prizes in the same 4-D draw

$= \frac{10\,000}{10\,000 \times 10\,000}$

$= \frac{1}{10\,000}$

(f) No. The statement should read: "... the probability of **a particular** number appearing twice in the same draw is one out of 10 000 times 10 000, or one in 100 million."

8. (a) (i) 2012

(ii) The greatest number of cases of dengue fever in the first 10 weeks cannot be observed easily from the above table. Although the data for 2014 contain big numbers such as 436, 479 and 402, the data also contain small numbers like 186, 193 and 209.

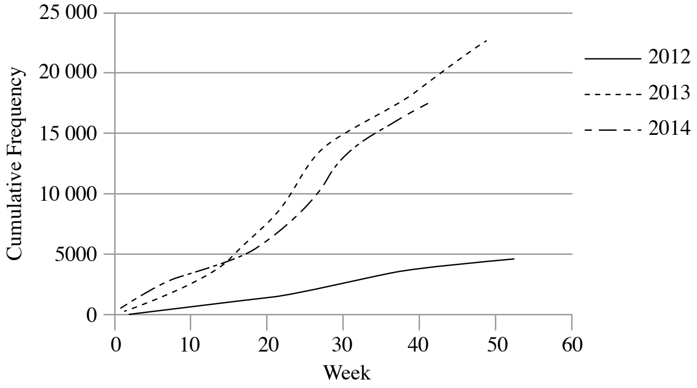
The numbers of cases of dengue fever in the first 10 weeks of 2013 and 2014 have to be added respectively and compared.

OXFORD UNIVERSITY PRESS

(b) The line graphs shown do not allow easy comparison between 2013 and 2014.

An alternative way of presenting the data to allow easier comparison between two or more sets of data is to use cumulative frequency curves as shown below:

Cumulative Frequency Curves for the Number of Cases of Dengue Fever

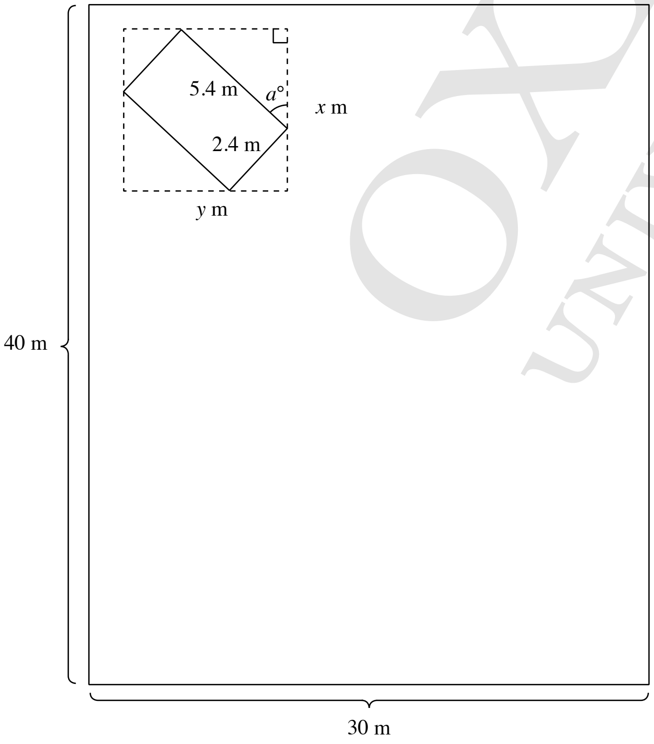


(c) **(i)** 2013

(ii) For the first 15 weeks or so, there are more cases of dengue fever in 2014 than 2013. But from the 15th to the 42nd week, 2014 has fewer cases of dengue fever than 2013.

(iii) It is not possible to predict the number of cases of dengue fever for the remaining 10 weeks of 2014 because it is possible for the remaining 10 weeks of 2014 to continue with the trend of fewer cases of dengue fever than 2013, or on the other hand, it is possible for the remaining 10 weeks of 2014 to have more cases of dengue fever than in 2013, just like the trend for the first 15 weeks or so.

9.



Parking angle, $a°$	**x m**	**Number of car park lots on one side**	**y m**
0°	5.4 cos 0° + 2.4 cos 90° = 5.4 m	$\frac{40}{5.4} = 7$	5.4 cos 90° + 2.4 cos 0° = 2.4 m
30°	5.4 cos 30° + 2.4 cos 60° = 5.88 m	$\frac{40}{5.88} = 6$	5.4 cos 60° + 2.4 cos 30° = 4.78 m
45°	5.4 cos 45° + 2.4 cos 45° = 5.52 m	$\frac{40}{5.52} = 7$	5.4 cos 45° + 2.4 cos 45° = 5.52 m
60°	5.4 cos 60° + 2.4 cos 30° = 4.78 m	$\frac{40}{4.78} = 8$	5.4 cos 30° + 2.4 cos 60° = 5.88 m
90°	5.4 cos 90° + 2.4 cos 0° = 2.4 m	$\frac{40}{2.4} = 16$	5.4 cos 0° + 2.4 cos 90° = 5.4 m

For parking lots on one side, one-way traffic flow:

Minimum width for the parking lots and aisle	**Number of parking lots**
2.4 + 3.6 = 6 m	7 × 5 = 35
4.78 + 3.6 = 8.38 m	6 × 3 = 18
5.52 + 4.2 = 9.72 m	7 × 3 = 21
5.88 + 4.8 = 10.68 m	8 × 2 = 16
5.4 + 6.0 = 11.4 m	16 × 2 = 32

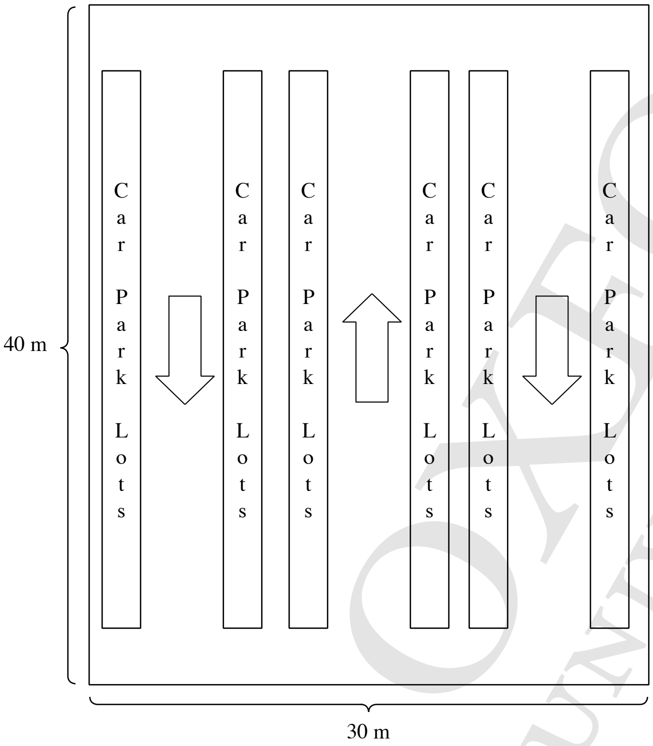
For parking lots on both sides, one-way traffic flow:

Minimum width for the parking lots and aisle	**Number of parking lots**
2.4 + 3.6 + 2.4 = 8.4 m	14 × 3 = 42
4.78 + 4.2 + 4.78 = 13.76 m	12 × 2 = 24
5.52 + 4.8 + 5.52 = 15.84 m	14
5.88 + 4.8 + 5.88 = 16.56 m	16
5.4 + 6 + 5.4 = 16.8 m	32

For parking lots on one or both sides, two-way traffic flow:

Minimum width for the parking lots and aisle	**Number of parking lots**
2.4 + 6 + 2.4 = 10.8 m	14 × 2 = 28
4.78 + 6.3 + 4.78 = 15.86 m	12
5.52 + 6.3 + 5.52 = 17.34 m	14
5.88 + 6.6 + 5.88 = 18.36 m	16
5.4 + 6.6 + 5.4 = 17.4 m	32

The maximum number of parking lots is 42 when parking lots are arranged parallel, on both sides, allowing one-way traffic flow, such that there are 7 cars on each side.



∴ The maximum number of additional car park labels that can be issued is 42.

OXFORD UNIVERSITY PRESS

NOTES

OXFORD UNIVERSITY PRESS