

Teaching Guide

# International Secondary **Maths**

# 8

COLIN **WRIGLEY**



**OXFORD**  
UNIVERSITY PRESS



# Introduction

## How to use this Guide

### **I. Selection of work and pacing**

Book 8 of the *International Secondary Maths Series* is designed for students of Class VIII or equivalent, i.e. they would normally be 12+ years old at the start of the academic year.

Teachers should use their discretion about how much time to spend on each topic, and the extent to which they use the Revision Exercises. There is plenty of material in Book 8: my experience has been that students learn more quickly when actively engaged in answering questions rather than passively watching the teacher solve numerous examples on the board. Advice is given in this Guide, chapter by chapter, but you should adjust according to the feedback you receive from your students.

### **II. Integrated mathematics**

This textbook series exploits links between the different branches of mathematics, treating the subject as a whole rather than separating into arithmetic, algebra, geometry, data handling, etc. There are also links to the sciences and to daily life. Brief advice about any such connections is listed here in this Guide, chapter by chapter.

### **III. Lesson planning**

Schools have different requirements as regards formal lesson planning. Under this heading in the Guide, suggestions have been made to assist teachers to write their own plans in whatever format is required.

#### ***Objectives***

- General objectives
- Specific objectives

These are student-focused, i.e. they identify what it is that we intend the students to gain from the lessons.

#### ***Method***

Also known as Strategy, Procedure, Techniques, or Methodology. These notes are teacher focused, i.e. they describe what the teacher should do to facilitate the students' learning.

#### ***Resources***

Materials needed other than the textbook and basic mathematical equipment. This Guide contains some photocopiable sheets to supplement the textbook, but other items will be needed occasionally.

### **Assignment**

Suggestions for which exercises are more suitable for homework.

### **Vocabulary**

Key words, knowledge of which is vital to understanding the topic.

## **IV. Bloom's Taxonomy**

Book 8 contains material relating to all stages of the cognitive skills hierarchy. In mathematics we often use multiple stages simultaneously. The following may be useful to teachers aiming to develop their students' abilities at the higher levels:

**Basics:** Remember → Understand → Apply

**Higher:** Analyse → Create → Evaluate

In the Basics, students learn facts until they can *Remember* (recall), *Understand* (explain, give examples, answer routine questions), and *Apply* (use the facts in standard situations).

For Higher skills, students deduce, distinguish, construct, organize, modify, develop, draw relevant diagrams, etc. when faced with a new situation. This can be described as *Problem-solving*.

## **V. The Exercises**

Each chapter has exercises coded as follows:

**Exercises A, B, C**, etc. after each section of the chapter

**Exercises M** are miscellaneous questions (if there are multiple sections)

**Exercises X** are challenging questions (usually involving higher levels of cognitive skills) for the more able students only.

**Revision Exercises** appear at regular intervals. The questions are deliberately not graded, i.e. they are a random mix of simple and difficult. Students seem to find this more interesting.

## **VI. Useful Sheets**

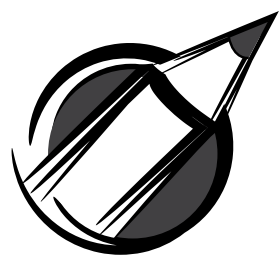
Photocopiable material is available as follows:

squared paper (9 mm)

coordinate grid

large circle

Pythagoras' theorem



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## True or False

At the start of the academic year it is essential to revise some key facts. It is also a good time to stress the importance in mathematics of using precise language.

**LESSON PLANNING****Objectives**

|          |                                                                                                                                                                                                                                                                                                                                                                                               |
|----------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| General  | To read statements carefully and decide whether they are true or false                                                                                                                                                                                                                                                                                                                        |
| Specific | <ol style="list-style-type: none"><li>1. To explain that a statement is true only if it is always true: it is not enough for it to be true for a few examples</li><li>2. To demonstrate an awareness that even one word can change the meaning of a mathematical statement</li><li>3. EX 1X (optional) shows that we need better evidence than a pattern to state something is true</li></ol> |

|        |                      |
|--------|----------------------|
| Pacing | 1 lesson, 1 homework |
|--------|----------------------|

|       |                                |
|-------|--------------------------------|
| Links | Many topics covered in Books 7 |
|-------|--------------------------------|

|        |                                                                                                                                                                                                                                                                                                                                                                                               |
|--------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Method | <p>Divide the students into small groups. Each group answers EX 1A. Discussion allowed. Then check the answers. Choose a student from a group that has the most correct answers to explain each answer to the rest of the class. The group with the highest score out of 10 is the winner.</p> <p>Use EX 1B for homework. Discuss the variety of possible answers during the next lesson.</p> |
|--------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

|             |                                                                                                         |
|-------------|---------------------------------------------------------------------------------------------------------|
| Assignments | Although EX 1X is an optional extra for more advanced students, it can be a useful homework assignment. |
|-------------|---------------------------------------------------------------------------------------------------------|

|            |             |
|------------|-------------|
| Vocabulary | true, false |
|------------|-------------|

**ANSWERS****Exercises****EX 1A**

1. true
2. false
3. false (e.g. when  $SF = \frac{1}{2}$ )

4. true
5. true
6. false
7. true
8. false
9. true
10. false

**EX 1B**

1. "Sides" is ambiguous, use "faces" instead.
2. Substitute square, rhombus, or kite for rectangle.
3. 0.06 (2 d.p. required)
4.  $-3^{\circ}\text{C}$  (must be negative)
5.  $\frac{1}{2}$  base  $\times$  height OR substitute parallelogram for triangle
6.  $180^{\circ}$  (not  $90^{\circ}$ )
7.  $2n + 1$
8. 42
9. 62.2 cm total
10.  $1\,000\,000\text{ cm}^3$  (1  $\text{m}^3 = 100\text{ cm} \times 100\text{ cm} \times 100\text{ cm}$ )

**EX 1X**

1. Numbers of areas are 2, 4, 8, 16, 31. [30 can be obtained by placing the spots symmetrically, but 32 is not possible.]

# Chapter 2 Transformations

Following the theme of Chapter 1 we revisit the transformations already introduced in Book 7, define them more precisely, and start to combine them together.

## LESSON PLANNING

### Objectives

|          |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |
|----------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| General  | To recognize, and represent on diagrams, the transformations of reflection, rotation, translation, and enlargement, singly and in combination, using correct notation                                                                                                                                                                                                                                                                                                                                                                                                                                                              |
| Specific | <ol style="list-style-type: none"><li>1. To define accurately the transformations of reflection, rotation, translation, and enlargement</li><li>2. To use the “vertical dash” notation to represent image points, e.g. <math>A \rightarrow A'</math></li><li>3. To identify what features of each type of transformation remain unchanged</li><li>4. To understand what is meant by the phrase “under the transformation”</li><li>5. To memorise the meanings of R, M, T, E, and I used in transformation algebra</li><li>6. To use and interpret the algebra of transformations in combination, e.g. <math>MT(a)</math></li></ol> |
| Pacing   | 4 lessons, 1 or 2 homeworks                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
| Links    | Coordinates                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |

### Method

According to the students’ previous knowledge, take them through the text in the book, as quickly as seems advisable. Although most of it is revision, pause long enough on the precision now required as follows:

- Reflections are defined by the mirror line.
- Rotations are defined by centre and angle.
- Translations are defined by  $x$  and  $y$  distances moved.
- Enlargements are defined by centre and scale factor.

Ensure familiarity with the “vertical dash” notation for image points  $A \rightarrow A'$  ( $A$  goes to  $A$  dash) etc., and use the word “image”.

Set EX 2A. Encourage reference back to the text when unsure, rather than asking questions out loud—this is a useful skill to develop. Students who are puzzled by reference to an image “under” a transformation may be satisfied to hear that “under” is short for “under the control of”. [This term first appears in EX 2A #4.]

For the transformational algebra students will need time to adjust to the idea that a letter can stand for a shape or a transformation, not just numbers as in conventional algebra. Just reading out its meaning can be helpful.

For example,  $(b) = M(a)$  means “(b) is the reflection of shape (a)”, and  $RM(b)$  means “shape (b) is reflected, and then rotated”.

Work out several examples on the board until the concept is clear. Then ensure they memorize the key symbols M, R, T, E, and I explained in the text. Use EX 2B.

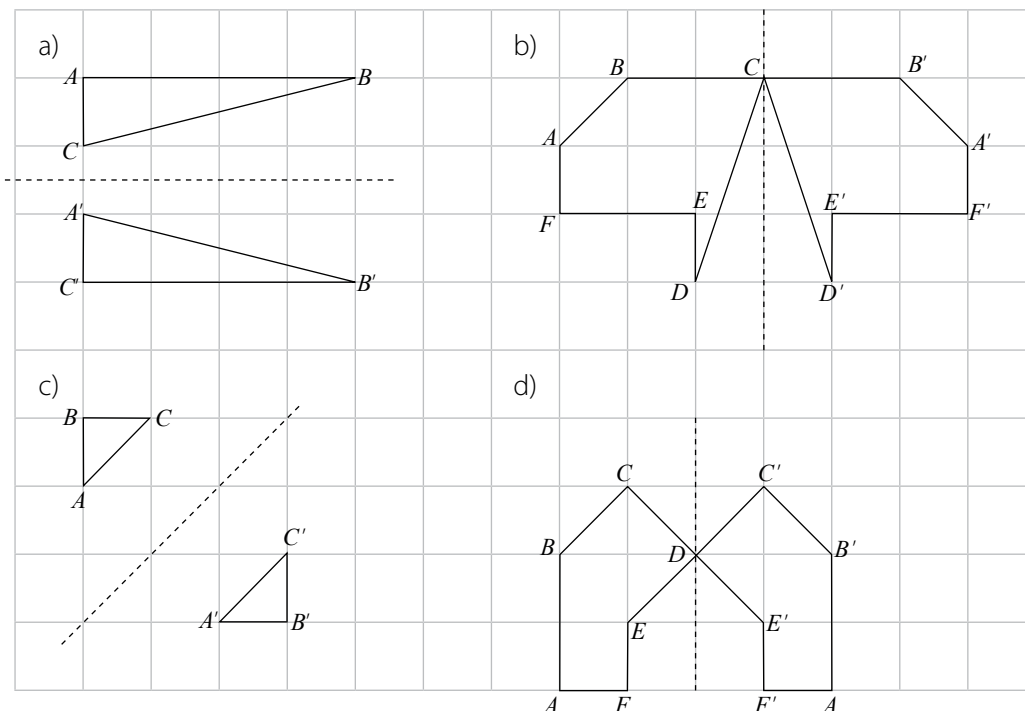
|                    |                                                                                                         |
|--------------------|---------------------------------------------------------------------------------------------------------|
| <b>Resources</b>   | Squared paper and coordinate grids (photocopiable from the Guide—see after the answers to this chapter) |
| <b>Assignments</b> | In each exercise, #10 is a suitable homework question. Easier options are EX 2A #7 and EX 2B #7.        |
| <b>Vocabulary</b>  | transformation, reflection, rotation, translation, enlargement, image, describe fully                   |

## ANSWERS

### Exercises

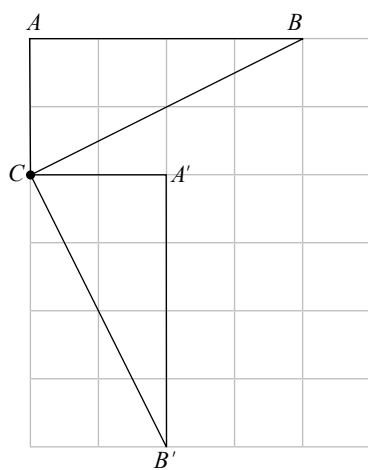
#### EX 2A

- $PQ$
  - $PR$
  - $AB$
  - $TV$
- $A$ ,  $90^\circ$
  - $U$ ,  $-90^\circ$  (or  $270^\circ$ )
  - $B$ ,  $180^\circ$
  - $(-1, 0)$ ,  $180^\circ$
- $A$ , 2
  - $(0, -2)$ , 2
  - $(12, 0)$ , 0.5
  - $(-6, -2)$ , 3
- 

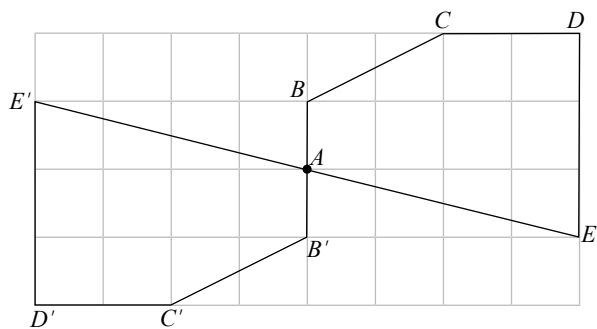


5.

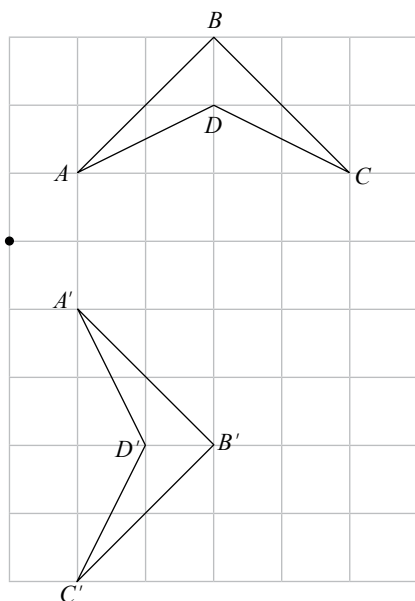
a)



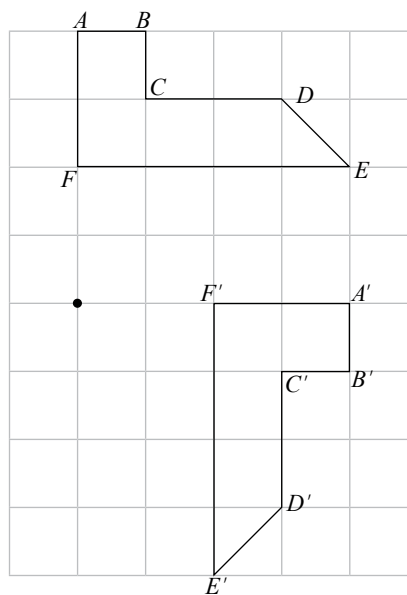
b)



c)

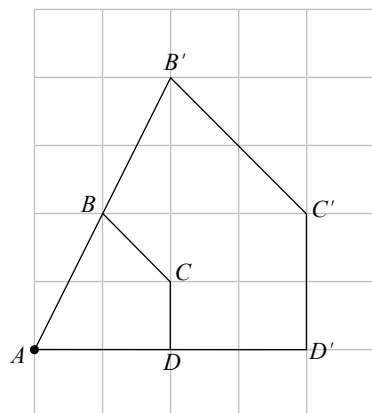


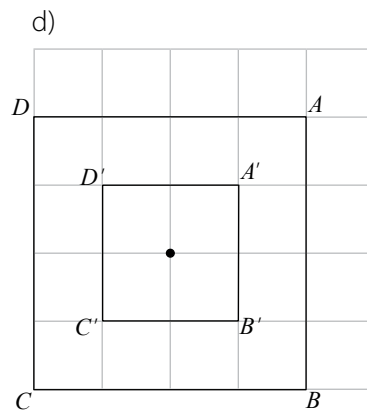
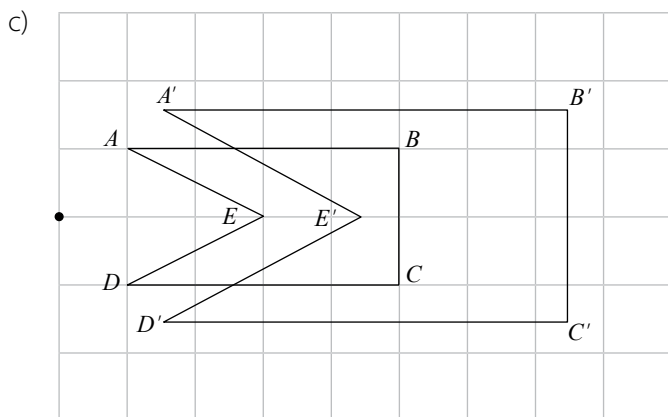
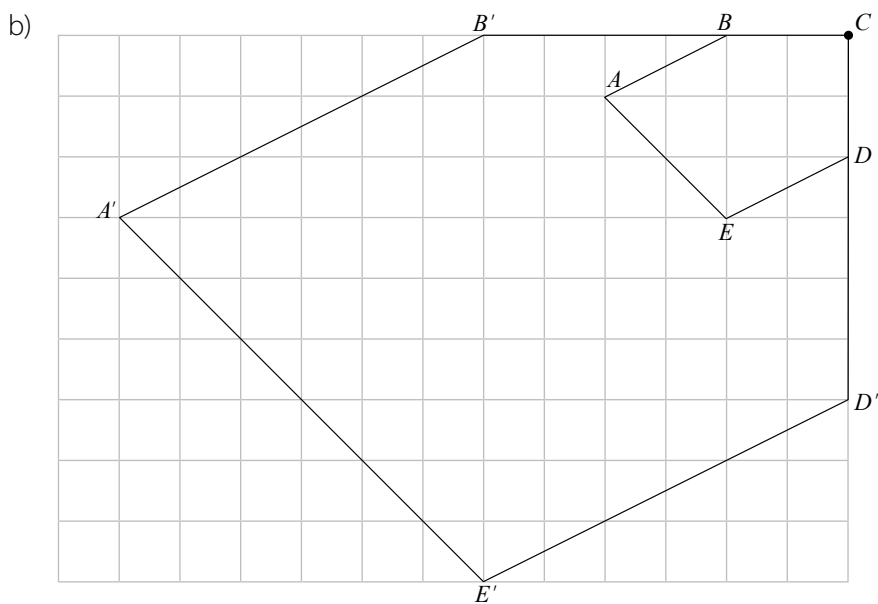
d)



6.

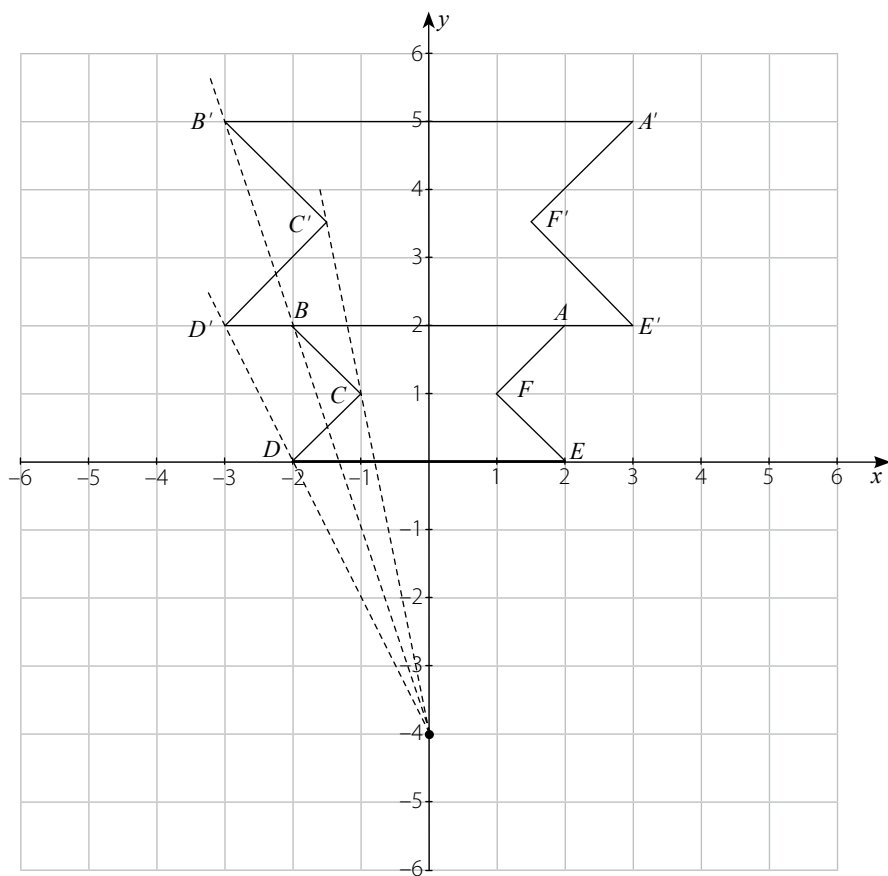
a)



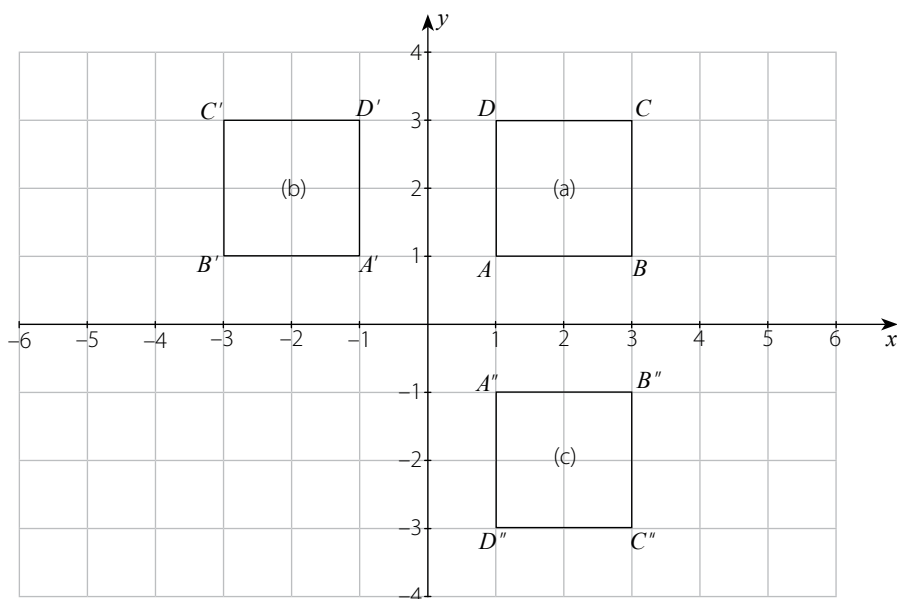


7. a) reflection in the  $y$ -axis  
 b) rotation  $180^\circ$  about  $(-4.5, 0)$   
 c) translation 9 units right  
 d) rotation  $180^\circ$  about  $(0, 0)$
8. a) 2                                      b) 3                                      c)  $\frac{1}{2}$                                       d)  $\frac{2}{3}$
9. a) rotation  $90^\circ$  about  $(6, 2)$   
 b) reflection in the  $y$ -axis  
 c) reflection in the  $y$ -axis  
 d) translation 13 units left

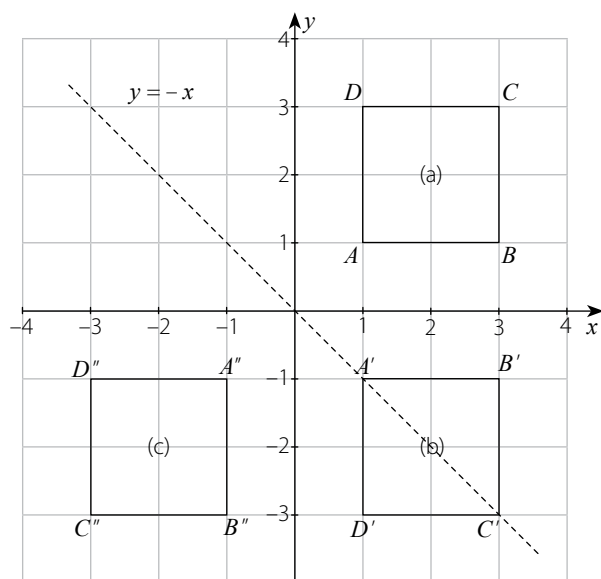
10.

d) centre  $(0, -4)$ **EX 2B**

1.

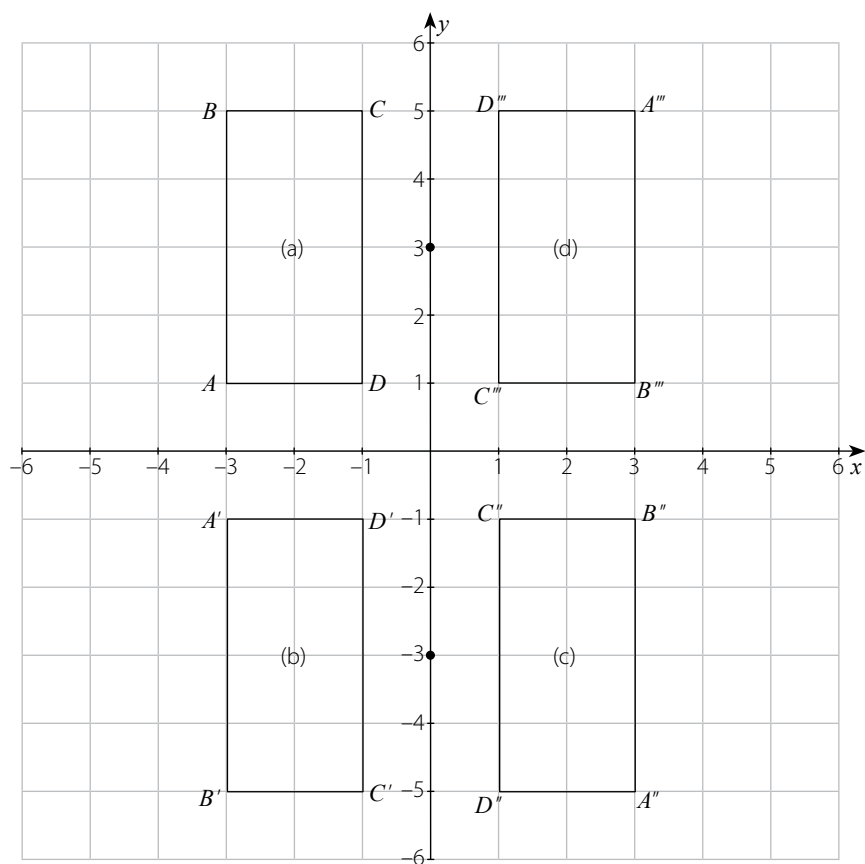
d) reflection in the  $x$ -axis

2.



d) reflection in the line  $y = -x$

3.



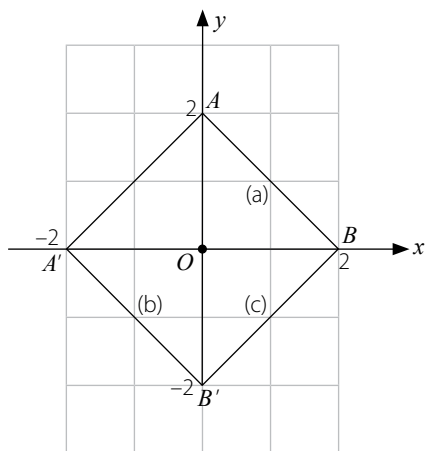
d) rotation  $180^\circ$  about  $(0, 3)$

4.

a) rotation of  $240^\circ$  (or  $-120^\circ$ ) about  $S$

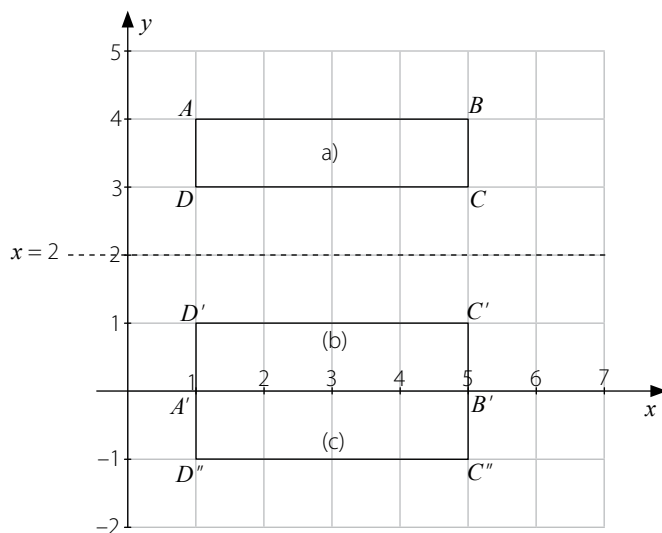
b)  $R^3$  is a rotation of  $360^\circ$  about  $S$  which returns the shape to its original position.

5.

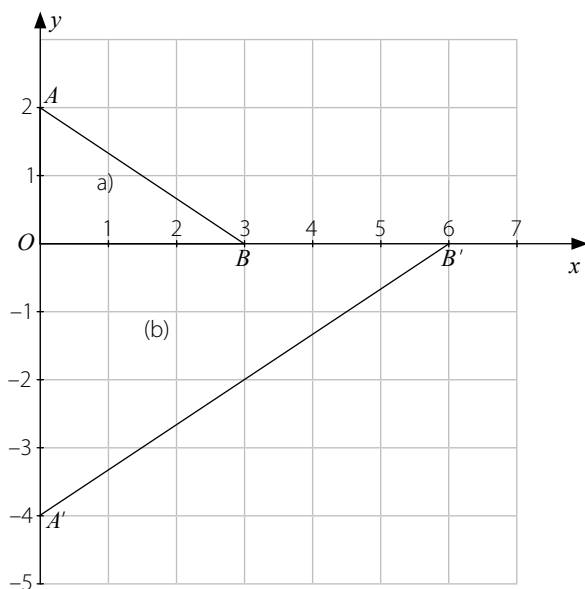
d) reflection in the  $x$ -axis

[Note: this is easier to see by using a cardboard triangle labelled  $AOB$ .]

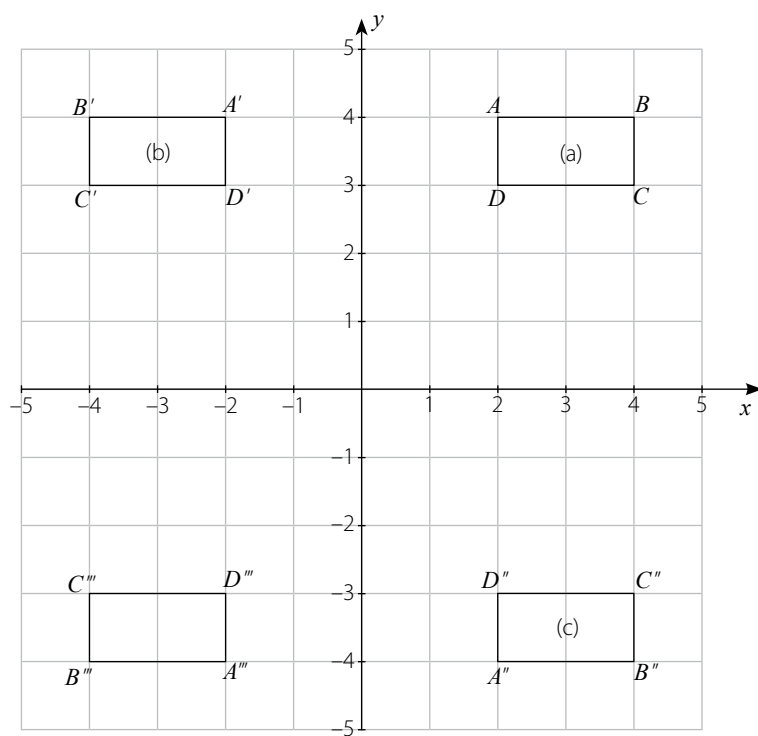
6.

c) reflection in the line  $x = 2$ d)  $M(c) = (b)$ 7.  $45^\circ$  [ $8 \times 45^\circ = 360^\circ$ ]

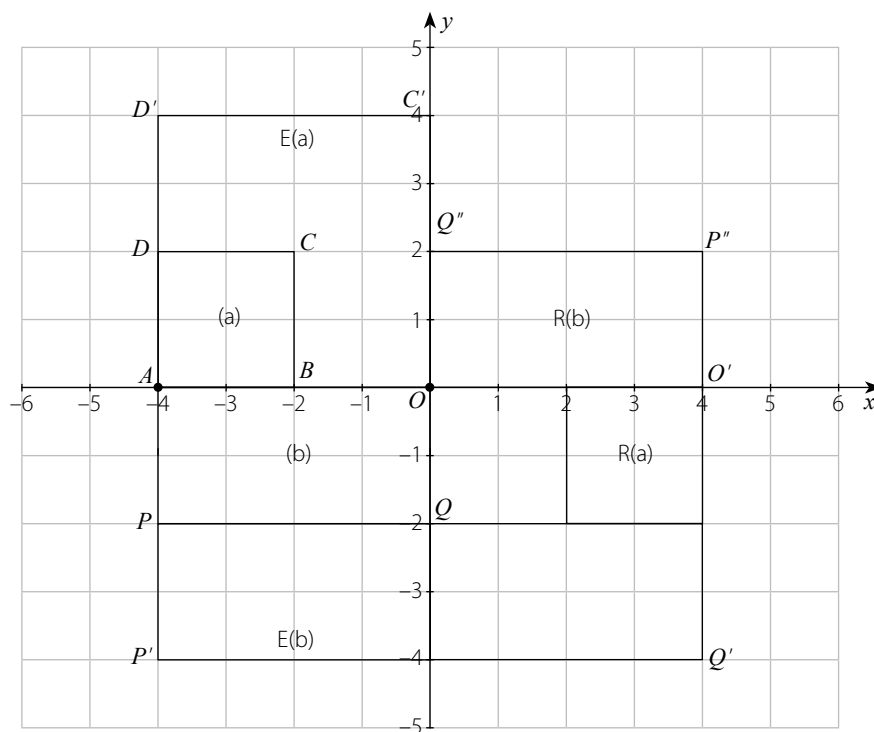
8.

c)  $A'(0, -4)$ d)  $O$

9.

c)  $M_2(b)$  and  $M_1(c)$  are identical.d) Hence  $M_1M_2 = M_2M_1$  is true.

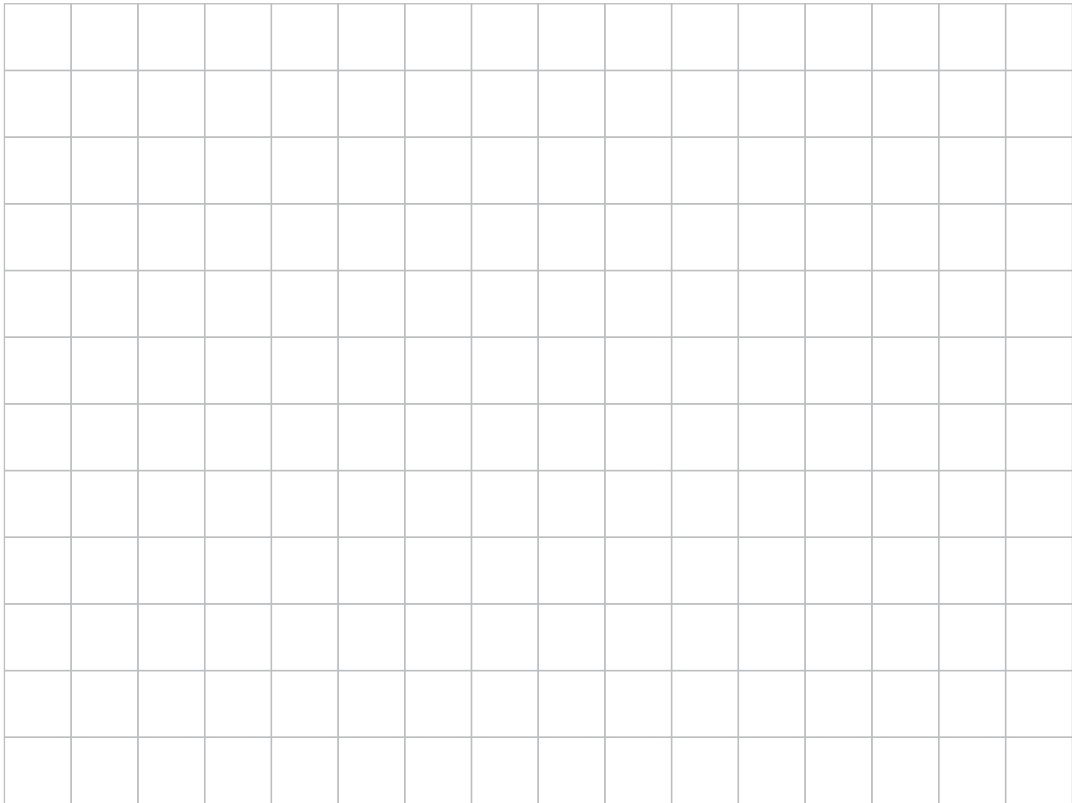
10.

d) False:  $R(a)$  is contained in  $E(b)$  but not  $R(b)$ .

**EX 2X**

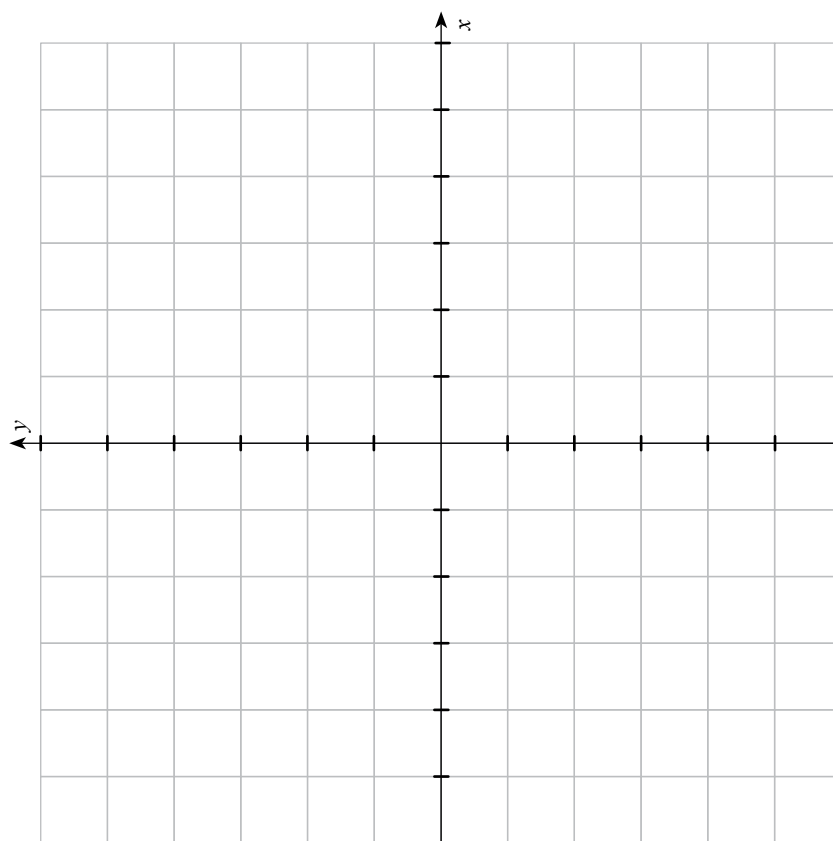
1. Tracing paper recommended  
 $(1, 1), (1, 0), (1.5, -0.5), (2.5, -1.5)$
2. reflection in the  $x$ -axis
3.  $x' = -x, y' = y$

Squared Paper (9 mm)



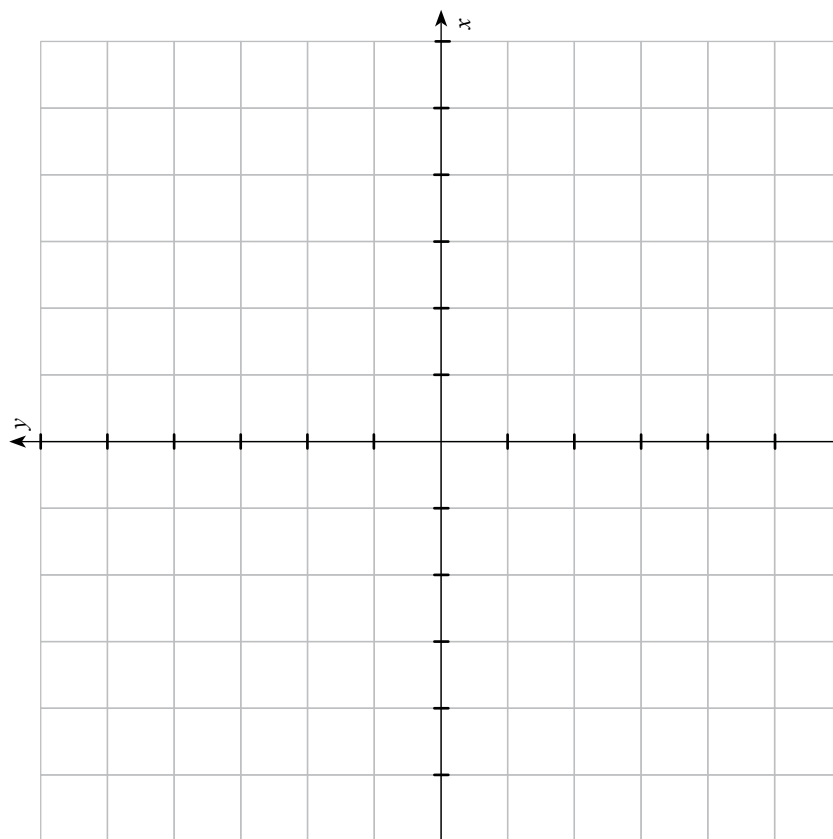
# Coordinate Grid

Question No:



# Coordinate Grid

Question No:



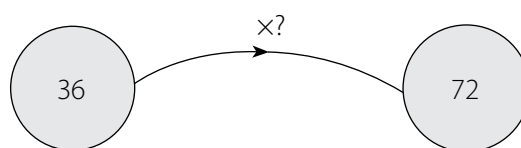
# Chapter 3 Percentage Change

The real beauty of arrow diagrams as an aid to understanding is seen here. This chapter uses scale factors on arrow diagrams to solve some difficult problems.

## LESSON PLANNING

### Objectives

|                 |                                                                                                                                                                                                                                                                                                                                                                                                             |
|-----------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>General</b>  | To solve problems involving changes in percentage, including multiple changes                                                                                                                                                                                                                                                                                                                               |
| <b>Specific</b> | <ol style="list-style-type: none"> <li>1. To state the scale factor equivalent of a percentage increase or decrease</li> <li>2. To use arrow diagrams and scale factors to solve problems, including reversing the flow and multiple changes</li> <li>3. To understand that percentages of different quantities cannot be added or subtracted</li> </ol>                                                    |
| <b>Pacing</b>   | 3 lessons, 1 homework                                                                                                                                                                                                                                                                                                                                                                                       |
| <b>Method</b>   | <ul style="list-style-type: none"> <li>Initially, focus on the practical skill of changing from percentage change to scale factor, with plenty of oral examples. Refer to the text introduction of this chapter. Attempt EX 3A soon after, which reinforces this technique.</li> <li>The next part requires more time. Remind students of how to find a multiplying factor from given data, e.g.</li> </ul> |



Clearly, the multiplying (scale) factor is 2

$$\frac{72}{36} = 2$$

$$\text{In general, SF} = \frac{\text{output}}{\text{input}}$$

Then, follow the text for examples where reverse flows and double diagrams with scale factors are used to find the original amount quickly with successive changes.

Ensure everyone has a calculator. Attempt EX 3B. Allow discussion. Encourage sketches of arrow diagrams. Students who can work without them should be allowed to do so.

|                  |                       |
|------------------|-----------------------|
| <b>Resources</b> | Calculators essential |
|------------------|-----------------------|

**Assignments** Homework: EX 3B #7, 8

**Vocabulary** increase, decrease, scale factor, p. a. (per annum)

## ANSWERS

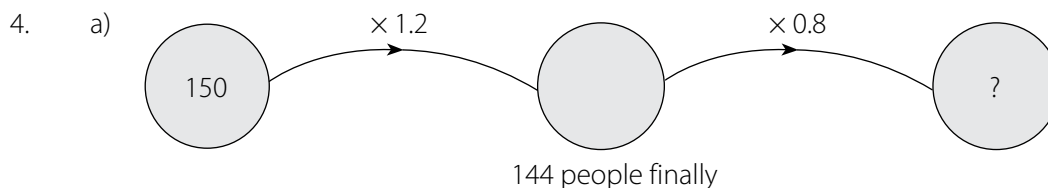
### Exercises

#### EX 3A

- |     |                   |                    |            |           |
|-----|-------------------|--------------------|------------|-----------|
| 1.  | a) 1.24           | b) 1.06            | c) 1.09    | d) 1.1    |
| 2.  | a) 0.92           | b) 0.89            | c) 0.78    | d) 0.6    |
| 3.  | a) 45%            | b) 6%              | c) 83%     | d) 70%    |
| 4.  | a) 9%             | b) 13%             | c) 25%     | d) 30%    |
| 5.  | a) 1.02           | b) 0.98            | c) 1.8     | d) 0.96   |
| 6.  | a) 37% increase   | b) 27% decrease    |            |           |
|     | c) 46% decrease   | d) 54% increase    |            |           |
| 7.  | a) 1.125          | b) 1.035           | c) 1.025   | d) 1.0725 |
| 8.  | a) 0.935          | b) 0.965           | c) 0.895   | d) 0.995  |
| 9.  | a) 14.5% increase | b) 21.5% increase  |            |           |
|     | c) 6.5% decrease  | d) 19.86% decrease |            |           |
| 10. | a) Rs 4624        | b) Rs 561          | c) Rs 6630 | d) Rs 255 |

#### EX 3B

- |    |                                     |                          |              |            |
|----|-------------------------------------|--------------------------|--------------|------------|
| 1. | a) Rs 23 400                        | b) 40.47 km <sup>2</sup> | c) 5.4175 kg | d) 9 h     |
| 2. | a) Rs 560                           | b) 5.65 m                | c) 69.1 ha   | d) Rs 9000 |
| 3. | a) SF = 1.13, 13% increase          |                          |              |            |
|    | b) SF = 1.24, 24% increase          |                          |              |            |
|    | c) SF = 0.9, 10% decrease           |                          |              |            |
|    | d) Rs 3600, SF = 0.75, 25% decrease |                          |              |            |



- b) The 20% increase and decrease are of different quantities.

5.  $n = 800$
6. 26.54% increase
7. 2500 people

8.

| Time        | Price (Rs) | Increase (Rs) | Increase %    |
|-------------|------------|---------------|---------------|
| 2 years ago | 1250       | —             | —             |
| 1 year ago  | 1360       | 110           | 8.8%          |
| Today       | 1475       | 115           | 8.5% (2 s.f.) |

9. a) Rs 64 500                      b) Rs 69 337.50  
     c) Rs 80 128.15 (2 d.p.)      d) Rs 254 871.07 (2 d.p.)
10. Rs 3 million

### EX 3X

1. a) Rs 105 000                      b) Rs 110 250  
     c) Rs 162 889.46 (2 d.p.)      d)  $\text{Rs } 100\,000 \times 1.05^n$
2. a) 3 years                              b) Rs 29 662.50  
     c) Rs 109 662.50                  d) Rs 9 662.50
3. 250

# Chapter 4

## Forming Equations

This chapter brings together a number of skills developed in Book 7, i.e. writing algebraic expressions from given information, constructing a true algebraic statement (equation), and using the solution to answer specific questions.

Students may find some of the questions so easy that they can write the answers immediately. For once, this is to be discouraged. The whole point is to use algebra. By solving the easy questions algebraically a technique is learned that can be used for more difficult situations in future.

### LESSON PLANNING

#### Objectives

|          |                                                                                                                                                                                                                                                                                                                                      |
|----------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| General  | To solve problems using simple linear equations                                                                                                                                                                                                                                                                                      |
| Specific | <ol style="list-style-type: none"><li>1. To form algebraic expressions from given information</li><li>2. To combine algebraic expressions into an equation which is a true statement about the given information</li><li>3. To solve the constructed equation and interpret the solution in terms of the original question</li></ol> |

|        |                       |
|--------|-----------------------|
| Pacing | 2 lessons, 1 homework |
|--------|-----------------------|

|        |                                                                                                                                                                                                                                                                                                                                                                                                                                                       |
|--------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Method | <p>Set EX 4A as a diagnostic exercise. It ought not to cause too much difficulty, but individual students may need help. All questions should be attempted.</p> <p>When ready, allow students to move to EX 4B. At this point allow discussion. The exercises are self-explanatory, guiding students in how to form the equations. Little input is required from the teacher. Fast workers who complete all correctly may be challenged by EX 4X.</p> |
|--------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

|             |                                                        |
|-------------|--------------------------------------------------------|
| Assignments | Suitable homework options: EX 4A #9, 10 or EX 4B #7, 8 |
|-------------|--------------------------------------------------------|

|            |                                       |
|------------|---------------------------------------|
| Vocabulary | algebraic expression, equation, solve |
|------------|---------------------------------------|

## ANSWERS

### Exercises

#### EX 4A

- $2n$
  - $n + 1$
  - $n - 2$
  - $10(n + 1)$
- $4c - 2$
- $\frac{760}{x}$
  - $\frac{700}{x - 1}$
  - $3x - 2$
  - 4, 8
- $6l$  cm
  - $6l = 120, l = 20$
  - $800 \text{ cm}^2$
- $12c + 5$
  - $12c + 5 = 89, c = 7, 7 \text{ cartons used}$
- Rs  $r + 450$
  - Rs  $2r + 450$
  - $2r + 450 = 950, r = 250, \text{ Rida Rs } 250, \text{ Sonia Rs } 700$
- $45n - 5$
  - $45n - 5 = 490, n = 11, 11 \text{ buses}$
- Rs  $s + 1100$
  - Rs  $3s + 2200$
  - $3s + 2200 = 15\,850, s = 4550, \text{ shoes Rs } 4550, \text{ trainers Rs } 5650$
  - $2 \times 5650 + 4550 = 15\,850$ —checks
- $4n$
  - Rs  $100n$
  - Rs  $180n$
  - $180n = 360, n = 2, 2 \times \text{Rs } 100 \text{ notes}, 8 \times \text{Rs } 20 \text{ notes}$
- Rida  $n - 4$  years; Sonia  $2(n - 4)$  years
  - $n - 4 + 2(n - 4) = 36, n = 13, \text{ Zain } 13, \text{ Rida } 9, \text{ Sonia } 18$

#### EX 4B

- $3x + 26$
  - $5x - 2$
  - $3x + 26 = 5x - 2, x = 14, \text{ The girls are both } 14.$
- Isaam  $5n - 1, \text{ Jehanzeb } 3n + 5$
  - $5n - 1 = 3n + 5, n = 3. \text{ They first thought of } 3.$
- Maya  $4n + 1, \text{ Qirat } 7n + 5$
  - $7n + 5 = 2(4n + 1), n = 3, \text{ original number } 3.$
- $x = 2$
  - $81 \text{ cm}^2$
- $x = 3$
  - $11 \text{ cm each}$
- Yes,  $x = 4; 7 \text{ cm}, 5 \text{ cm}, 7 \text{ cm}, 5 \text{ cm}$

7.  $150 = 2x + 10$ ,  $x = 70$ ,  $70 \text{ m}^3/\text{h}$ ,  $80 \text{ m}^3/\text{h}$
8.  $7t + 20 = 685$ ,  $t = 95$ , Mon 1 h 35 min, Tue 1 h 45 min, Wed 3 h 10 min,  
Thu 3 h 20 min, Fri 1 h 35 min
9. a)  $\frac{l}{2} \text{ cm}$                       b)  $4l = 240$ ,  $l = 60$ , area  $2700 \text{ cm}^2$
10. a)  $\frac{l-15}{10} \text{ cm}$                       b)  $\frac{l-15}{10} = 3$ ,  $l = 45$ , ribbon is 45 cm

**EX 4X**

1. 8 or 0
2.  $53 \text{ m}^3/\text{h}$
3.  $1\frac{2}{3}$  times older

# Chapter 5 Direct Proportion

This topic has been dealt with in Book 6 and Book 7 using ratios. Here we take a look at it using scale factors, with obvious mathematical connections to early topics in this book.

## LESSON PLANNING

### Objectives

|             |                                                                                                                                                                                                                                                                                                                    |
|-------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| General     | To solve problems of direct proportion by using scale factors or the unitary method                                                                                                                                                                                                                                |
| Specific    | <ol style="list-style-type: none"> <li>To recognize a situation of direct proportion when two quantities increase at the same rate (scale factor)</li> <li>To solve problems using scale factors</li> <li>To scale down proportional quantities so that one of them is a unit, and hence solve problems</li> </ol> |
| Pacing      | 1 lesson, 1 homework                                                                                                                                                                                                                                                                                               |
| Links       | Enlargements, percentage change                                                                                                                                                                                                                                                                                    |
| Method      | Minimise the introduction. Set EX 5A. Allow any method that gives correct answers.                                                                                                                                                                                                                                 |
| Assignments | Suitable homework: EX 5A #9, 10                                                                                                                                                                                                                                                                                    |
| Vocabulary  | proportion, unit                                                                                                                                                                                                                                                                                                   |

## ANSWERS

### Exercises

#### EX 5A

- \$ 304.72
- 18 kg
  - 200 m<sup>2</sup>
  - 20 m<sup>2</sup>
  - 450 kg
- 2118
  - not quite (7 pages give 2471 words.)
- 20 m
  - 15 m
  - 15 m

d)

|                     |    |    |    |    |
|---------------------|----|----|----|----|
| Building height (m) | 6  | 9  | 12 | 15 |
| Shadow length (m)   | 10 | 15 | 20 | 25 |



# Chapter 6 Ratios, Fractions, Graphs

This chapter makes connections between previous work on ratios, proportion, and fractions with the graph of two quantities in direct proportion.

## LESSON PLANNING

### Objectives

|          |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |
|----------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| General  | To solve direct proportion problems using straight line graphs through the origin, and their equations                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |
| Specific | <ol style="list-style-type: none"> <li>1. To express ratios using whole numbers and fractions in the form <math>1:n</math> (or <math>n:1</math>) and interpret them correctly</li> <li>2. To recognize direct proportion situations and use ratios to complete tables of values</li> <li>3. To recognize that the unit value gives the gradient of a graph representing direct proportion of two quantities</li> <li>4. To use the straight line equation through the origin <math>y = mx</math> to solve direct proportion questions</li> <li>5. To be aware of the risks in interpolation and extrapolation</li> </ol> |
| Pacing   | 3 lessons at least, 1 homework                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |

### Method

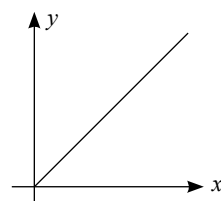
The first part is basically revision. Follow the text introduction as a reminder of different ways of expressing the same ratio.

Set EX 6A almost at once.

The graph section is important. Use the text example to show that quantities in proportion have equivalent ratios in the table of values (both horizontally and vertically) and that the unit value gives the gradient of its graph.

Remind students of  $y = mx + c$ . In all cases of direct proportion  $c = 0$ , so  $y = mx$  and we have a straight line through the origin.

Point out the convention that in tables the upper line is for the horizontal axis, the lower line for the vertical axis.



When the gradient is known, the line equation is known. We can use it to calculate other values. Beware, however:

Interpolation (between given points): If the quantities are whole numbers, decimal answers are not appropriate.

Extrapolation (i.e. outside the range of the given points): The direct proportion relationship may not be valid, although often it is. Caution is required.

Use EX 6B. This will take time as there is a lot of graph plotting. Insist on correctly-labelled scales on the axes.

|                    |                                                                                                      |
|--------------------|------------------------------------------------------------------------------------------------------|
| <b>Resources</b>   | Squared paper (9 mm) is suitable for the graphs, no need for greater accuracy                        |
| <b>Assignments</b> | EX 6A #9 is a good homework question and of general importance. Alternatively, EX 6B #9 is suitable. |
| <b>Vocabulary</b>  | ratio, fraction, gradient, Interpolation, extrapolation, respectively                                |

## ANSWERS

### Exercises

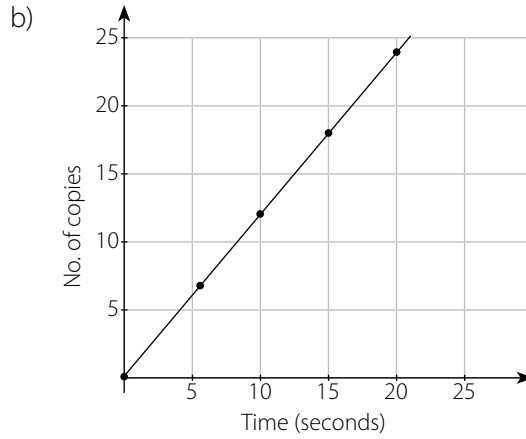
#### EX 6A

- 1:3
  - $\frac{1}{4}$
  - $\frac{1}{3}$
  - $\frac{3}{4}$
- 9:8
  - $1:\frac{8}{9}$
  - $\frac{8}{17}$
  - $\frac{8}{9}$
- $\frac{3}{8}$
  - $\frac{5}{8}$
  - 15 kg
  - 16 kg
- 20 l black; 32 l white
  - $\frac{5}{13}$
  - 1:1.6
  - 1.6
- 1:7
  - 2:1
  - $\frac{1}{10}$
  - 3 games
- 1.25
  - $\frac{5}{9}$
  - 1:1
  - Rs 1310
- 3:7
  - 35 girls
  - $\frac{8}{13}$
  - 1:2
- C
  - $\frac{6}{25}$
  - $\frac{1}{7}$
  - A 39 600
  - B 33 600
  - C 28 800
  - D 12 000
  - E 4800
  - F 1200
- width 20 cm, height 30 cm
  - 1:4
  - 1:9
  - $1:n^2$
- checkout #3
  - $\frac{1}{7}$
  - 5:2:7
  - $\frac{2}{5}$

EX 6B

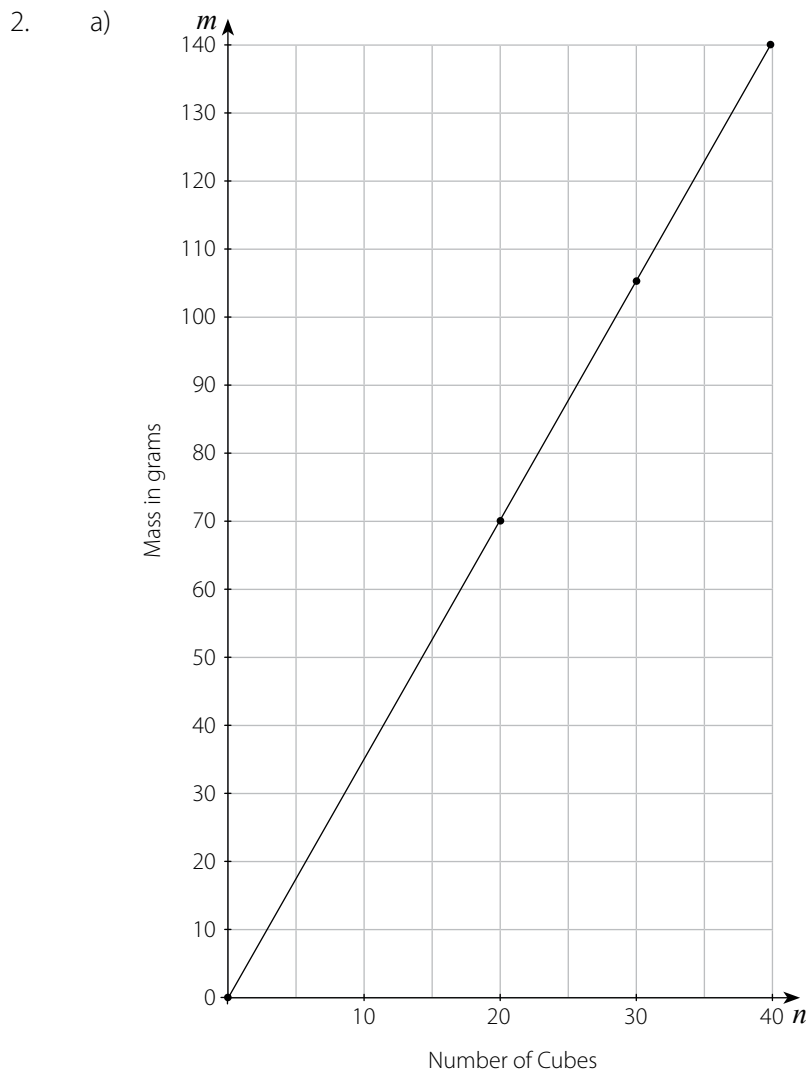
1. a)

|                |   |   |    |    |    |
|----------------|---|---|----|----|----|
| Time (seconds) | 0 | 5 | 10 | 15 | 20 |
| No. of copies  | 0 | 6 | 12 | 18 | 24 |



c) 1.2

d)  $n = 1.2t$



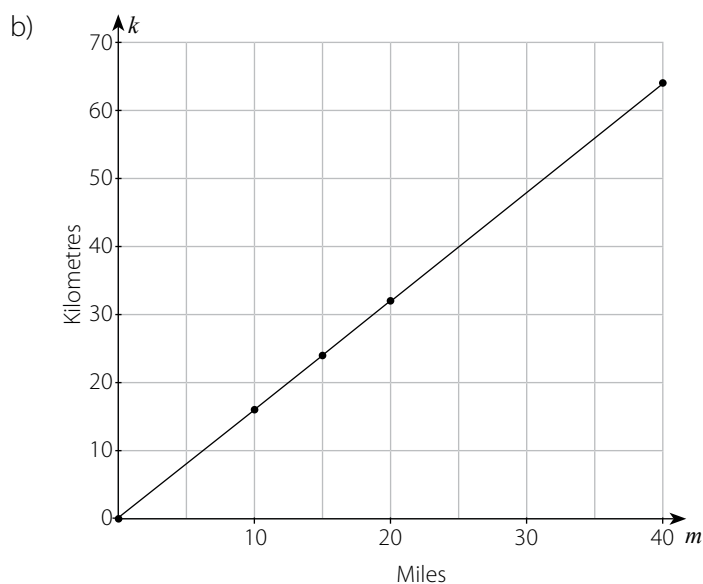
b) Yes, straight line through the origin

c)  $m = 3.5n$

d) 1000

3. a)

|            |   |    |    |    |    |
|------------|---|----|----|----|----|
| miles      | 0 | 10 | 15 | 20 | 40 |
| kilometres | 0 | 16 | 24 | 32 | 64 |

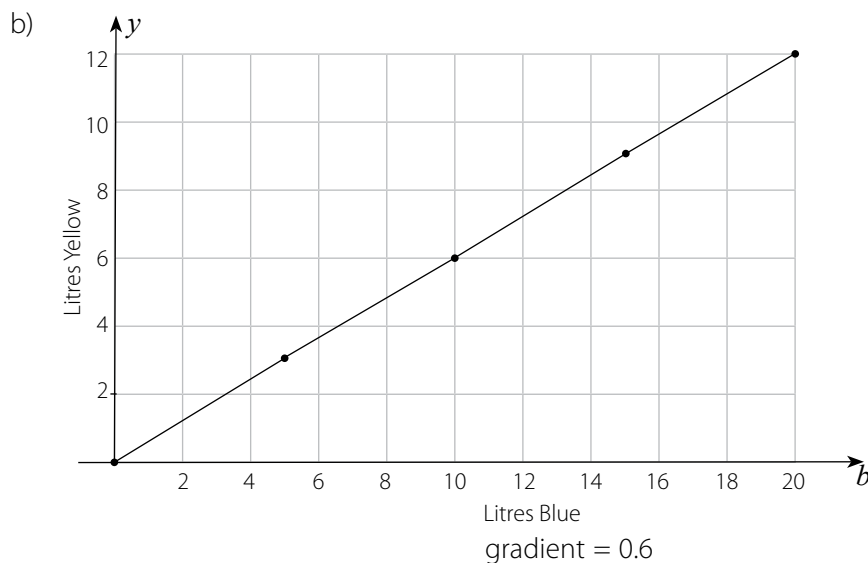


- c) 48 km; 19 miles (approx)  
d)  $k = 1.6m$ ; 115 km

4. a) Rs 115                      b)  $r = 115d$                       c) Rs 1955; \$85  
d) Straight line through the origin, gradient 115.

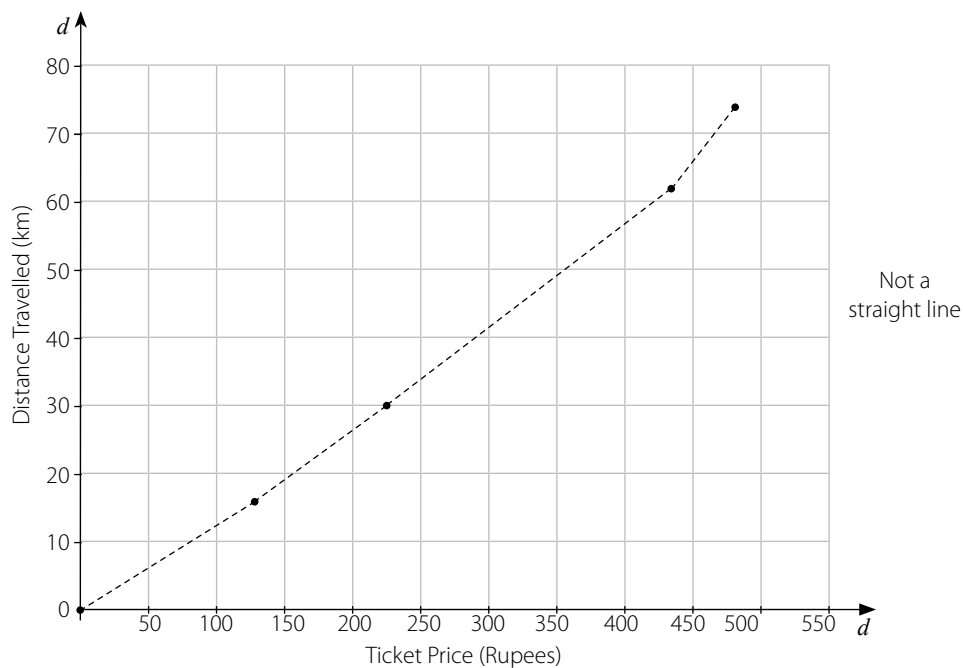
5. a)  $d = \frac{4t}{3}$                       b) 3 m                      c) 2 h 6 min                      d)  $1\frac{1}{3}$  m/h

6. a) 5:3



- c) 4.2 l (approx)                      d)  $y = 0.6b$ ; 25.8 l  
7. a)  $s = 1.5i$                       b) 4.5 tonnes                      c) 1.5                      d) 40%

8. a)



- b) Any pair, e.g.  $16 : 30$  and  $128 : 225$   
 $= 1 : 1.875$   $= 1 : 1.758$  approx

Corresponding ratios not equal

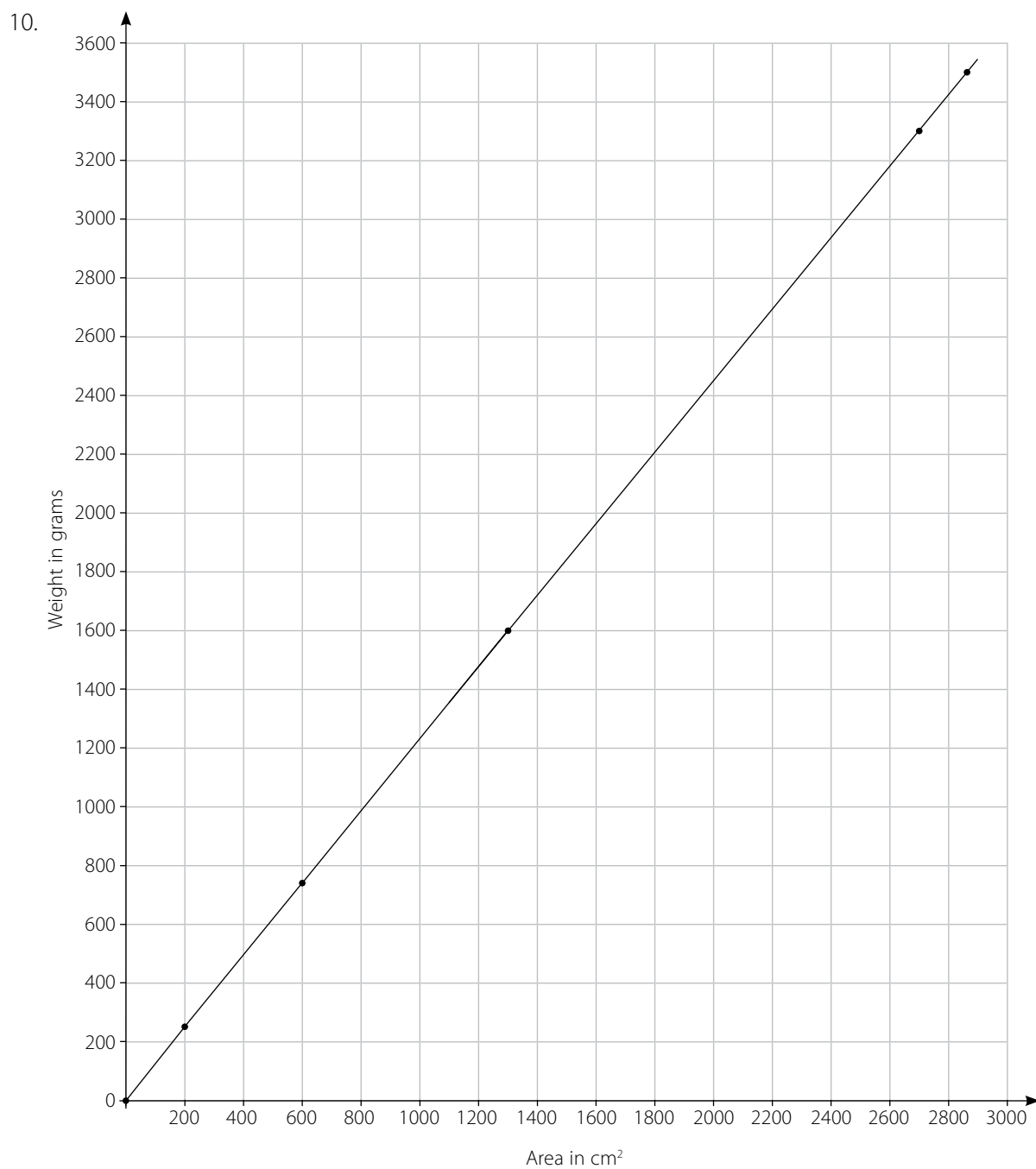
- c) cost of each km to Crendon is Rs 8  
to Devpur is Rs 7.5  
to Exitan is Rs 7  
to Fourways is Rs 6.5

- d) The company gives cheaper rates on the longer journeys.

9.

|       |   |     |      |      |      |    |      |
|-------|---|-----|------|------|------|----|------|
| knots | 0 | 5   | 10   | 15   | 20   | 25 | 30   |
| km/h  | 0 | 9.2 | 18.4 | 27.6 | 36.8 | 46 | 55.2 |

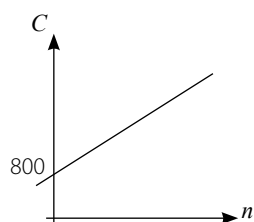
- b)  $y = 1.84x$       c) 31.28 km/h      d) 43 knots (nearest knot)



- a) 1600 g (approx)      b) 3300 g (approx)  
c) 200  $\text{cm}^2$  (approx)      d) 2840  $\text{cm}^2$  (approx)

### EX 6X

1.



Yes; No;  $C = 8.3n + 800$   
Heavy users pay at a reduced rate;  
Rs 800.

2. 3 : 8

(Hint: double first ratio, treble the second.)

3. 1 h 20 min. In practice 6 men could interfere with each other's work and so it could take longer. [This is an example of inverse proportion.]

# Chapter 7 Probability

Although chance has been introduced before in Books 6 and 7, this chapter emphasizes the systematic organization of data in order to calculate probabilities correctly. Simply guessing and hoping that all possible outcomes have been thought of is not satisfactory.

It is assumed that students know the structure of a standard 52-card pack. Not all may know. Others may never have seen a fruit machine, even in films. One should know that those who organize gambling are well aware of the laws of probability and make sure that in the long run they always win and the customer loses. Mathematicians rarely gamble!

## LESSON PLANNING

### Objectives

|             |                                                                                                                                                                                                                                                                                                                                                                    |
|-------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| General     | To calculate and compare probabilities by listing all the outcomes in a systematic manner                                                                                                                                                                                                                                                                          |
| Specific    | <ol style="list-style-type: none"> <li>1. To list the equally likely outcomes of up to three independent events</li> <li>2. To calculate probabilities from listings of equally likely outcomes</li> <li>3. To compare probabilities of two events to decide which is more likely; to put a number of events in order of likelihood</li> </ol>                     |
| Pacing      | 2 lessons, 1 homework                                                                                                                                                                                                                                                                                                                                              |
| Links       | Fractions                                                                                                                                                                                                                                                                                                                                                          |
| Method      | <p>Use text introduction to remind students of basic definitions and facts about chance. Emphasize systematic listing. "Why?" To ensure that no outcome is accidentally omitted. Get straight into EX 7A.</p> <p>To compare probabilities they have to be worked out carefully. Sometimes the two events are almost (but not quite) equally likely. Use Ex 7B.</p> |
| Resources   | This is a theory chapter. No experiments are to be done. However, a large digital counter would be useful to demonstrate systematic sequencing of 3 events.                                                                                                                                                                                                        |
| Assignments | Suitable homework EX 7B #5                                                                                                                                                                                                                                                                                                                                         |
| Vocabulary  | probability, outcome, event, equally likely, systematic, fair, random                                                                                                                                                                                                                                                                                              |

# ANSWERS

## Exercises

### EX 7A

1. a) 

|   |   |   |   |   |   |
|---|---|---|---|---|---|
| H | H | H | T | H | H |
| H | H | T | T | H | T |
| H | T | H | T | T | H |
| H | T | T | T | T | T |
- b) i)  $\frac{1}{8}$       ii)  $\frac{3}{8}$       iii)  $\frac{1}{2}$       iv) 0
2. a) 

|   |   |   |   |   |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|---|---|---|---|---|
| R | R | R | R | Y | R | B | R | R | B | Y | R |
| R | R | B | R | Y | B | B | R | B | B | Y | B |
| R | R | Y | R | Y | Y | B | R | Y | B | Y | Y |
| R | R | G | R | Y | G | B | R | G | B | Y | G |
| R | B | R | R | G | R | B | B | R | B | G | R |
| R | B | B | R | G | B | B | B | B | B | G | B |
| R | B | Y | R | G | Y | B | B | Y | B | G | Y |
| R | B | G | R | G | G | B | B | G | B | G | G |
| Y | R | R | Y | Y | R | G | R | Y | G | Y | R |
| Y | R | B | Y | Y | B | G | R | B | G | Y | B |
| Y | R | Y | Y | Y | Y | G | R | Y | G | Y | Y |
| Y | R | G | Y | Y | G | G | R | G | G | Y | G |
| Y | B | R | Y | G | R | G | B | R | G | G | R |
| Y | B | B | Y | G | B | G | B | B | G | G | B |
| Y | B | Y | Y | G | Y | G | B | Y | G | G | Y |
| Y | B | G | Y | G | G | G | B | G | G | G | G |
- b) i)  $\frac{1}{64}$       ii)  $\frac{37}{64}$       iii)  $\frac{3}{16}$       iv)  $\frac{27}{64}$
3. a) 

|   |   |   |   |   |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|---|---|---|---|---|
| H | H | A | H | T | A | T | H | A | T | T | A |
| H | H | K | H | T | K | T | H | K | T | T | K |
| H | H | Q | H | T | Q | T | H | Q | T | T | Q |
- b) i)  $\frac{1}{4}$       ii)  $\frac{1}{12}$       iii)  $\frac{2}{3}$       iv) 0

4. a)

|    |    |    |    |    |    |    |
|----|----|----|----|----|----|----|
| A1 | B1 | C1 | D1 | E1 | F1 | G1 |
| A2 | B2 | C2 | D2 | E2 | F2 | G2 |
| A3 | B3 | C3 | D3 | E3 | F3 | G3 |
| A4 | B4 | C4 | D4 | E4 | F4 | G4 |
| A5 | B5 | C5 | D5 | E5 | F5 | G5 |
| A6 | B6 | C6 | D6 | E6 | F6 | G6 |

b)

|   | 1  | 2  | 3  | 4  | 5  | 6  |
|---|----|----|----|----|----|----|
| A | A1 | A2 | A3 | A4 | A5 | A6 |
| B | B1 | B2 | B3 | B4 | B5 | B6 |
| C | C1 | C2 | C3 | C4 | C5 | C6 |
| D | D1 | D2 | D3 | D4 | D5 | D6 |
| E | E1 | E2 | E3 | E4 | E5 | E6 |
| F | F1 | F2 | F3 | F4 | F5 | F6 |
| G | G1 | G2 | G3 | G4 | G5 | G6 |

- c) i)  $\frac{1}{42}$   
 ii)  $\frac{1}{6}$   
 iii)  $\frac{2}{3}$   
 iv) 0

5.

|   | Blue Dice |    |    |    |    |    |
|---|-----------|----|----|----|----|----|
|   | 1         | 2  | 3  | 4  | 5  | 6  |
| 1 | 3         | 4  | 5  | 6  | 7  | 8  |
| 2 | 5         | 6  | 7  | 8  | 9  | 10 |
| 3 | 7         | 8  | 9  | 10 | 11 | 12 |
| 4 | 9         | 10 | 11 | 12 | 13 | 14 |
| 5 | 11        | 12 | 13 | 14 | 15 | 16 |
| 6 | 13        | 14 | 15 | 16 | 17 | 18 |

- a)  $\frac{1}{12}$   
 b)  $\frac{1}{36}$   
 c)  $\frac{1}{36}$   
 d) 0

6. a) 24

b)

|   | A  | K  | Q  | J  |
|---|----|----|----|----|
| 1 | 1A | 1K | 1Q | 1J |
| 2 | 2A | 2K | 2Q | 2J |
| 3 | 3A | 3K | 3Q | 3J |
| 4 | 4A | 4K | 4Q | 4J |
| 5 | 5A | 5K | 5Q | 5J |
| 6 | 6A | 6K | 6Q | 6J |

- c)  $\frac{1}{12}$   
 d)  $\frac{1}{8}$

|    |         |         |         |         |
|----|---------|---------|---------|---------|
| 7. | H H H H | H T H H | T H H H | T T H H |
|    | H H H T | H T H T | T H H T | T T H T |
|    | H H T H | H T T H | T H T H | T T T H |
|    | H H T T | H T T T | T H T T | T T T T |

a)  $\frac{3}{8}$

b)  $\frac{5}{16}$

c)  $\frac{1}{16}$

d)  $\frac{5}{16}$

|       |           |            |            |            |
|-------|-----------|------------|------------|------------|
| 8.    | 1 1 1 (3) | 1 2 1 (4)  | 1 3 1 (5)  | 1 4 1 (6)  |
|       | 1 1 2 (4) | 1 2 2 (5)  | 1 3 2 (6)  | 1 4 2 (7)  |
|       | 1 1 3 (5) | 1 2 3 (6)  | 1 3 3 (7)  | 1 4 3 (8)  |
|       | 1 1 4 (6) | 1 2 4 (7)  | 1 3 4 (8)  | 1 4 4 (9)  |
| <hr/> |           |            |            |            |
|       | 2 1 1 (4) | 2 2 1 (5)  | 2 3 1 (6)  | 2 4 1 (7)  |
|       | 2 1 2 (5) | 2 2 2 (6)  | 2 3 2 (7)  | 2 4 2 (8)  |
|       | 2 1 3 (6) | 2 2 3 (7)  | 2 3 3 (8)  | 2 4 3 (9)  |
|       | 2 1 4 (7) | 2 2 4 (8)  | 2 3 4 (9)  | 2 4 4 (10) |
| <hr/> |           |            |            |            |
|       | 3 1 1 (5) | 3 2 1 (6)  | 3 3 1 (7)  | 3 4 1 (8)  |
|       | 3 1 2 (6) | 3 2 2 (7)  | 3 3 2 (8)  | 3 4 2 (9)  |
|       | 3 1 3 (7) | 3 2 3 (8)  | 3 3 3 (9)  | 3 4 3 (10) |
|       | 3 1 4 (8) | 3 2 4 (9)  | 3 3 4 (10) | 3 4 4 (11) |
| <hr/> |           |            |            |            |
|       | 4 1 1 (6) | 4 2 1 (7)  | 4 3 1 (8)  | 4 4 1 (9)  |
|       | 4 1 2 (7) | 4 2 2 (8)  | 4 3 2 (9)  | 4 4 2 (10) |
|       | 4 1 3 (8) | 4 2 3 (9)  | 4 3 3 (10) | 4 4 3 (11) |
|       | 4 1 4 (9) | 4 2 4 (10) | 4 3 4 (11) | 4 4 4 (12) |

a)  $\frac{27}{64}$

b)  $\frac{5}{32}$

c) 0

d)  $\frac{37}{64}$

9.

|          |   | Clubs |    |    |    |     |
|----------|---|-------|----|----|----|-----|
|          |   | A     | K  | Q  | J  | 10  |
| Diamonds | A | AA    | AK | AQ | AJ | A10 |
|          | K | KA    | KK | KQ | KJ | K10 |
|          | Q | QA    | QK | QQ | QJ | Q10 |
|          | J | JA    | JK | JQ | JJ | J10 |

a)  $\frac{1}{20}$

b)  $\frac{1}{5}$

c)  $\frac{3}{5}$

d) 0

10. a) 24

- b) 

|   |   |   |
|---|---|---|
| R | H | A |
| R | H | K |
| R | H | Q |
| R | T | A |
| R | T | K |
| R | T | Q |

|   |   |   |
|---|---|---|
| G | H | A |
| G | H | K |
| G | H | Q |
| G | T | A |
| G | T | K |
| G | T | Q |

|   |   |   |
|---|---|---|
| B | H | A |
| B | H | K |
| B | H | Q |
| B | T | A |
| B | T | K |
| B | T | Q |

|   |   |   |
|---|---|---|
| Y | H | A |
| Y | H | K |
| Y | H | Q |
| Y | T | A |
| Y | T | K |
| Y | T | Q |
- c)  $\frac{1}{24}$                       d)  $\frac{3}{4}$

**EX 7B**

1.  $P(A) = \frac{1}{3}$                        $P(B) = \frac{1}{12}$                       A is more likely.
2.  $P(A) = \frac{1}{4}$                        $P(B) = \frac{1}{5}$                       A is more likely.
3.  $P(A) = \frac{2}{7} \approx 0.29$                        $P(B) = \frac{3}{10} = 0.3$                       B is more likely.
4.  $P(M) = \frac{2}{11}$                        $P(I) = \frac{2}{10}$                       Obtaining I is more likely.
5.  $P(A) = \frac{3}{12} = 0.25$                        $P(B) = \frac{7}{20} = 0.35$   
 $P(C) = \frac{9}{35} = 0.257\dots$                        $P(D) = \frac{8}{25} = 0.32$                       Design B gives the best chance of winning a prize.

6.  $P(A) = \frac{1}{6}$
- | B  | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 | 11 | 12 |
|----|----|----|----|----|----|----|----|----|----|----|----|----|
| 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 | 11 | 12 | 13 |
| 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 | 11 | 12 | 13 | 14 |
| 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 | 11 | 12 | 13 | 14 | 15 |
| 4  | 5  | 6  | 7  | 8  | 9  | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| 5  | 6  | 7  | 8  | 9  | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |
| 6  | 7  | 8  | 9  | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
| 7  | 8  | 9  | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |
| 8  | 9  | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 9  | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 |
| 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 |
| 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 |

$P(B) = \frac{67}{72}$                       B is more likely.

7.  $P(A) = \frac{1}{3}$                        $P(B) = \frac{9}{13}$                       B is more likely.

8. P R Q S

9. a)  $P(\text{point down}) = \frac{168}{200} = 0.84$

b)  $0.16 + 0.84 = 1$ ; Certainty—no other possible outcomes.

c) No

10.  $P(1) = \frac{9}{50} = 0.18$                        $P(2) = \frac{9}{50} = 0.18$

$P(3) = \frac{6}{50} = 0.12$                        $P(4) = \frac{8}{50} = 0.16$

$P(5) = \frac{8}{50} = 0.16$                        $P(6) = \frac{10}{50} = 0.20$

The high probability for 6 and low probability for 3 is some evidence that the dice is weighted. However, it is not strong evidence. More trials would be necessary.

#### EX 7X

1. a)  $\frac{1}{7776}$

b)  $\frac{5}{7776}$

c)  $\frac{15}{7776} = \frac{5}{2592}$

d)  $\frac{6}{7776} = \frac{1}{1296}$

2. a) 9

b) 27

c) 81

d)  $3''$

3. a) 0.04

b) 0.64

c) 0.16

d) 0.16

Total is 1.

# Chapter 8

## Revision Exercises

These exercises contain questions on concepts learnt in Book 7 as well as on the first seven chapters of Book 8.

### ANSWERS

#### Exercises

##### EX 8A

1.  $n$

$5n$

$5n + 20$

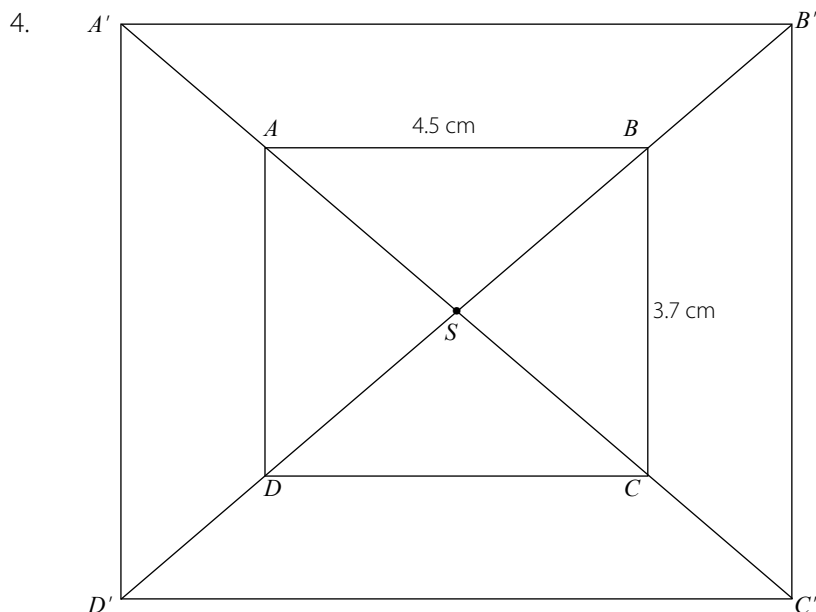
$$\frac{5n + 20}{5} = n + 4$$

$2n + 8$

$2n$

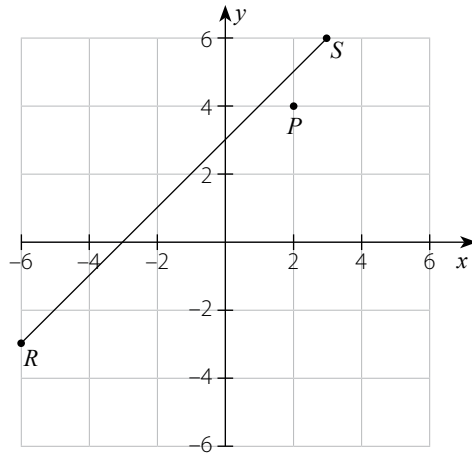
2. a) false                      b) true                      c) true                      d) false

3. a) 45 min                      b) 1 h 8 min                      c) 3 h 41 min                      d) No



$A'B' = 9$  cm,  $B'C' = 7.4$  cm,  
double the original

5. a)



b)  $\text{gdt } RS = 1$

c)  $y = x + 3$

d) No

$4 \neq 2 + 3$

6. a)  $a = 12 \text{ cm}$

b)  $b = 4 \text{ cm}$

c)  $c = 8 \text{ cm}$

d)  $x = 4 \text{ cm}, y = 24 \text{ cm}$

7. a)  $y = x + 4$

b)  $y = 3x + 6$

c)  $y = -x + 3.5$

d)  $y = -\frac{1}{2}x + 4$

8. a) reflection in the line  $y = -0.5$

b) rotation  $180^\circ$  about  $(2.5, -0.5)$

c) rotation  $180^\circ$  about  $(0, -0.5)$

d) translation 5 units left

9. a)  $\frac{11}{48}$

b)  $\frac{1}{4}$

c)  $2\frac{1}{2}$

d) 1

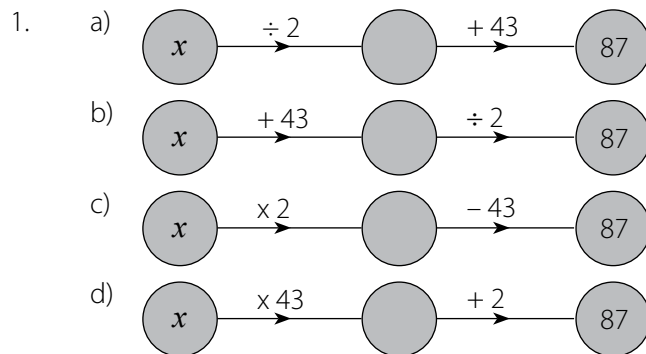
10. a) 3:2

b) 14 girls

c)  $\frac{3}{7}$

d) 1:0.8

**EX 8B**



$x = 88$

$x = 131$

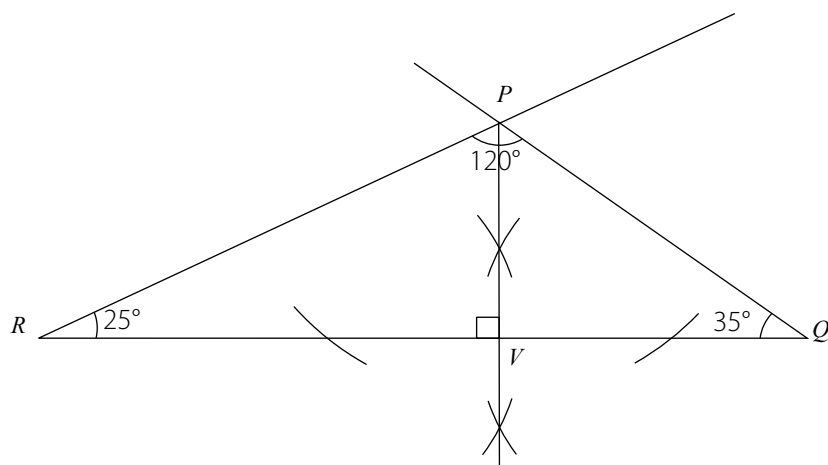
$x = 65$

$x = 1.98 \text{ (3 s.f.)}$

2. a) 3.1%

b) 4.4%

3.



4.  $60^\circ$

5. a)  $\frac{2a-14}{2} = a-7$

b)  $b-6 = \frac{3b-18}{3}$

c)  $\frac{20c-16}{4} = 5c-4$

d)  $\frac{9d+9}{9} = d+1$

6.  $59.4 \text{ cm}^2$

7. a)  $b = 4k$

b)  $7 \text{ kg}$

c)  $4$

d)  $20\%$

8. a)  $x = -7$

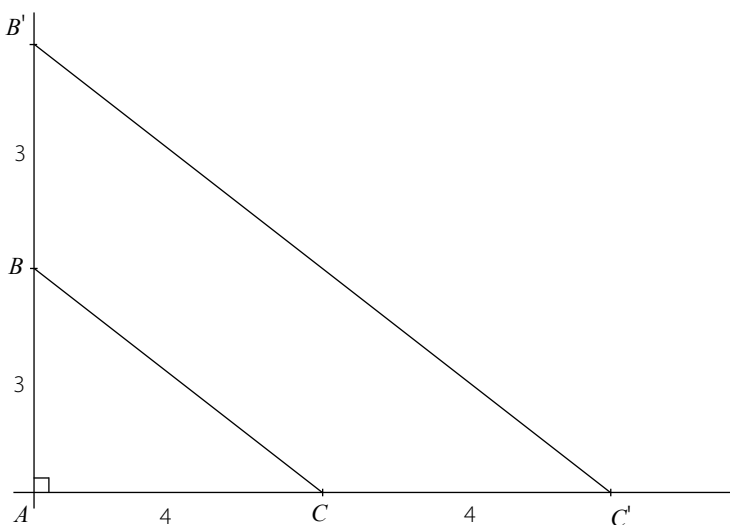
b)  $x = -\frac{1}{3}$

c)  $x = -6.8$

d)  $x = -23$

9. a)  $B'$

b)



c)  $BC = 5 \text{ cm}$      $B'C' = 10 \text{ cm}$     Scale factor 2 applies.

10. a)  $1.045$

b)  $1.0625$

c)  $0.92$

d)  $0.88$

**EX 8C**

- square, rectangle, parallelogram, rhombus
  - square, rhombus, kite
- $015^\circ$
  - $285^\circ$
- $43^\circ$
  - $137^\circ$
  - $14^\circ$
  - $131^\circ$
- |   |   |   |   |   |   |                  |                  |
|---|---|---|---|---|---|------------------|------------------|
| H | H | H | T | H | H |                  |                  |
| H | H | T | T | H | T | a) $\frac{1}{8}$ | b) $\frac{1}{2}$ |
| H | T | H | T | T | H | c) $\frac{7}{8}$ | d) 0             |
| H | T | T | T | T | T |                  |                  |
- 1
  - $y = -x$
  - $y = -x + 2$
  - $y = -x - 3$
- 653 000 (nearest thousand)
- 1.2 (2 s.f.)
  - 11 (2 s.f.)
  - 14 (2 s.f.)
  - 34 (2 s.f.)
- 7:50
  - 3:25
  - 3:1
  - 1:100
- 0.28; 28%
  - 0.38; 38%
  - 0.6; 60%
  - 0.1; 10%
- $P(A) = \frac{1}{2}$ ,  $P(B) = \frac{8}{13}$ , B is more likely.

**EX 8D**

- $35n - 3$
  - $35n - 3 = 207$ ,  $n = 6$ , 6 buses.
- 8
  - 8
  - 11
  - 0
- 0.86; 86%
  - 0.89; 89%
  - 0.91; 91%
  - 0.13; 13%
- $\frac{1}{7} = \frac{2}{14} = \frac{12}{84}$
  - $\frac{4}{5} = \frac{12}{15} = \frac{24}{30}$
  - $\frac{5}{8} = \frac{30}{48} = \frac{60}{96}$
  - $\frac{1}{6} = \frac{5}{30} = \frac{6}{36}$
- $30^\circ\text{C}$
  - 15 Jan
  - 11–14 Jan
  - 20 degrees
- 5
  - most in July; least in September
  - May and October
  - 38 pizzas
- $160 = x + x + 20$ ,  $x = 70$ ,  $70 \text{ m}^3/\text{h}$  and  $90 \text{ m}^3/\text{h}$
- $\frac{2}{11}$
  - $\frac{1}{11}$
  - $\frac{7}{11}$
  - 0

9. a)  $48 \text{ cm}^2$                       b)  $0.2 \text{ cm}$                       c)  $12 \text{ cm}^3$   
10. a)  $3\,600\,000 \text{ J}$                       b)  $350\,000 \text{ J}$                       c)  $300\,000 \text{ J}$                       d)  $0.915 \text{ J}$

**EX 8X**

1. a)  $(1, 2)$                       b)  $(0, 0)$                       c)  $(2, 0)$                       d)  $(3.5, 3.5)$   
2. a)  $90^\circ$                       b)  $15^\circ$                       c)  $112.5^\circ$                       d)  $54^\circ$   
3.  $20\pi \text{ cm}$

# Chapter 9 Brackets

This chapter revisits algebra, taking skills to a further level. Removing brackets preceded by negative multipliers and very simple factorisations are met for the first time.

## LESSON PLANNING

### Objectives

|          |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |
|----------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| General  | To solve equations involving expressions with brackets (positive and negative multipliers); to factorise expressions by extracting a common factor                                                                                                                                                                                                                                                                                                                                           |
| Specific | <ol style="list-style-type: none"> <li>1. To remove the brackets from an expression preceded by a positive or negative multiplier</li> <li>2. To know the meaning of <math>+( \quad )</math> or <math>-( \quad )</math> and how to remove the brackets</li> <li>3. To simplify expressions containing brackets</li> <li>4. To solve equations containing brackets</li> <li>5. To factorise expressions by taking out a common factor</li> <li>6. To use algebra to solve problems</li> </ol> |
| Pacing   | 2 lessons, 1 homework                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
| Links    | Negative numbers and the rule of signs; sequences                                                                                                                                                                                                                                                                                                                                                                                                                                            |

### Method

Board demonstration of examples is required. Use the text and similar examples to revise combining like terms and removing brackets (positive multipliers only). Now extend to negative multipliers. The text uses the “negative reverses all signs” rule. Another approach, preferred by many teachers, is to multiply out each term using the rule of signs:

Method 1:

$$\begin{aligned}
 & -2(x - 5) \\
 & \quad \swarrow \quad \searrow \\
 & = -2x + 10
 \end{aligned}$$

Method 2 (as in the text):

$$\begin{aligned}
 & -2(x - 5) \\
 & \quad \uparrow \quad \uparrow \\
 & \quad +ve \quad -ve
 \end{aligned}$$

(Multiply and reverse)

$$= -2x + 10$$

Care should be taken to explain brackets with a simple + or – in front.

$$+ ( \quad ) \text{ means } + 1 ( \quad )$$

$$- ( \quad ) \text{ means } - 1 ( \quad )$$

Compare this with

$$+ \quad n \quad \text{means} \quad + 1 \quad n$$

$$- \quad n \quad \text{means} \quad - 1 \quad n$$

For example:

$$\begin{aligned} & 5 - (2x + 3) - x \\ &= 5 - 2x - 3 - x \\ &= 2 - 3x \end{aligned}$$

Another example:

$$\begin{aligned} & 5 + (3 - 2x) + x \\ &= 5 + 3 - 2x + x \\ &= 8 - x \end{aligned}$$

Set EX 9A, but when the students reach the equations, warn them that some solutions are fractional or negative. At this stage they should be able to handle that. Higher order skills are needed for the Fibonacci sequence questions #9, 10.

Factorising may be presented as the reverse process to removing brackets. A few examples should suffice—EX 9B is quite easy.

|                    |                                               |
|--------------------|-----------------------------------------------|
| <b>Assignments</b> | EX 9A is suitable for homework.               |
| <b>Vocabulary</b>  | rule of signs, negative multiplier, factorise |

## ANSWERS

### Exercises

#### EX 9A

- |                                |                                   |                                   |                        |
|--------------------------------|-----------------------------------|-----------------------------------|------------------------|
| 1. a) $12x + 13$               | b) $11y - 3$                      | c) $7u + 5$                       | d) $v - 6$             |
| 2. a) $8a + 15$                | b) $b - 8$                        |                                   |                        |
| c) $-2c + 6$                   | d) $-5d + 12$                     |                                   |                        |
| 3. a) $17i - 10$               | b) $-4j + 38$                     | c) $k + 1$                        | d) $2l - 2$            |
| 4. a) $5p + 8q$                | b) $4p - q$                       | c) $22p - 16q$                    | d) $p - 9q$            |
| 5. a) $x = 1$                  | b) $x = 2$                        | c) $x = 1$                        | d) $x = 2$             |
| 6. a) $x = 1.5$                | b) $x = 3$                        | c) $x = 6$                        | d) $x = 3.5$           |
| 7. a) $x = \frac{11}{12}$      | b) $x = 1$                        |                                   |                        |
| c) $x = \frac{7}{4}$ (or 1.75) | d) $x = -\frac{7}{25}$ (or -0.28) |                                   |                        |
| 8. a) $x = 1$                  | b) $x = -\frac{1}{3}$             | c) $x = 8.5$ (or $\frac{17}{2}$ ) | d) $x = \frac{11}{12}$ |

9. a)  $2 + x$ ,  $2 + 2x$ ,  $4 + 3x$ ,  $6 + 5x$   
 b)  $-3 + x$ ,  $-3 + 2x$ ,  $-6 + 3x$ ,  $-9 + 5x$   
 c)  $x + 1$ ,  $x + 2$ ,  $2x + 3$ ,  $3x + 5$   
 d)  $3x$ ,  $5x$ ,  $8x$ ,  $13x$
10. a)  $8 + 3x = 23; x = 5$ ; 4, 5, 9, 13, 22  
 b)  $3 + 5x = 53; x = 10$ ; 1, 10, 11, 21, 32, 53  
 c) Let  $x$  be the first term;  $2x + 21 = 19; x = -1$ ; -1, 7, 6, 13, 19, 22  
 d) Let  $x$  be the second term;  $6 + 3x = 0; x = -2$ ; 3, -2, 1, -1, 0, -1, -1, -2, -3, -5

**EX 9B**

1. a)  $4(x + 2)$  b)  $6(x + 3)$  c) not possible d)  $7(3 - n)$
2. a)  $3(a - 4b)$  b)  $4(c + 3d)$   
 c)  $2(p + 25q)$  d) not possible
3. a)  $2(1 - u)$  b)  $3(1 + v)$  c) not possible d)  $7(1 - 2x)$
4. a)  $5(2x - 1)$  b) not possible c)  $5(3x + 4)$  d)  $5(5 - 6x)$
5. a)  $2(x + y + 1)$  b)  $3(x - y + 2)$   
 c) not possible d)  $5(x - 2y + 3)$
6. a) not possible b)  $2(2n + 3m + 4)$   
 c)  $3(2n + 3m + 1)$  d)  $7(2n - 3m - 1)$
7. a) false b) true c) false d) false
8. a) true b) true c) false d) true
9. a) true b) true c) true d) true
10. a)  $0.2(1 + 6x)$  b)  $0.1(3 + 4x)$   
 c)  $0.4(1 - 4x)$  d)  $0.5(x - 3)$

**EX 9X**

1. a)  $-9x - 46$  b)  $-2x + 4y - 3$   
 c)  $2x - 2$  d)  $3 - x - x^2$
2. a)  $x = 3$  b)  $x = 2$   
 c)  $x = -\frac{3}{4}$  (or  $-0.75$ ) d)  $x = -4$  or  $4$
3. a)  $-3a + 5b$  b) 1, 3, -2, 5, -7, 12  
 c) 1.5, -2, 3.5, -5.5, 9, -14.5

# Chapter 10 Angle Properties of Polygons

This chapter develops the  $180^\circ$  angle-sum of triangles and  $360^\circ$  of quadrilaterals, generalising to any polygon. Properties of the exterior angle are introduced for the first time.

## LESSON PLANNING

### Objectives

|          |                                                                                                                                                                                                                                                                                                                                                                                                                                    |
|----------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| General  | To solve geometric problems involving facts about the angle-sum and/or exterior angles of polygons                                                                                                                                                                                                                                                                                                                                 |
| Specific | <ol style="list-style-type: none"> <li>1. To discover the formula for the angle-sum of a polygon by dividing it into triangles</li> <li>2. To know that the sum of the exterior angles of any polygon is equivalent to one revolution</li> <li>3. To know that the exterior angle of a triangle is equal to the sum of the interior opposite angles</li> <li>4. To solve geometric problems using the above angle facts</li> </ol> |
| Pacing   | 3 lessons, 1 homework                                                                                                                                                                                                                                                                                                                                                                                                              |
| Links    | Simple linear equations                                                                                                                                                                                                                                                                                                                                                                                                            |

### Method

The Book 6 demonstration of the angle-sum of a triangle may be worth repeating. The text provides an easy formal proof. It is good for students to engage with a proof at this stage, even if not all can fully grasp the logic. Talk them through it line by line.

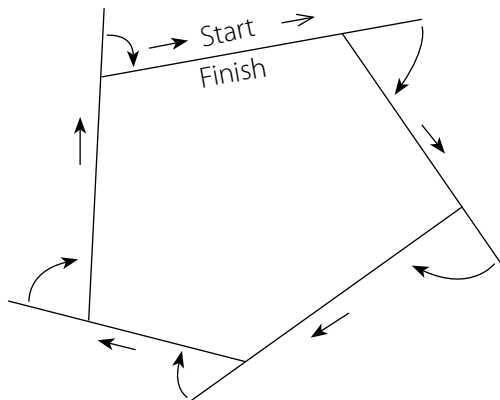
Move straight on to quadrilaterals, which is revision, by cutting into two triangles. Then do a series of polygons with different numbers of sides. Do not give the formula. Emphasize the procedure of making triangles.

Set EX 10A. The formula emerges in #6. It may be used thereafter (without drawing triangles). Encourage the use of reason codes to justify each stage in the calculations, but do not become obsessive about them. Many students at this stage can obtain accurate answers quickly, and pedantic insistence on reasons for each step can hinder their thinking. Also, accept equivalent statements: the codes are just useful abbreviated phrases and there are other ways of expressing the same basic facts.

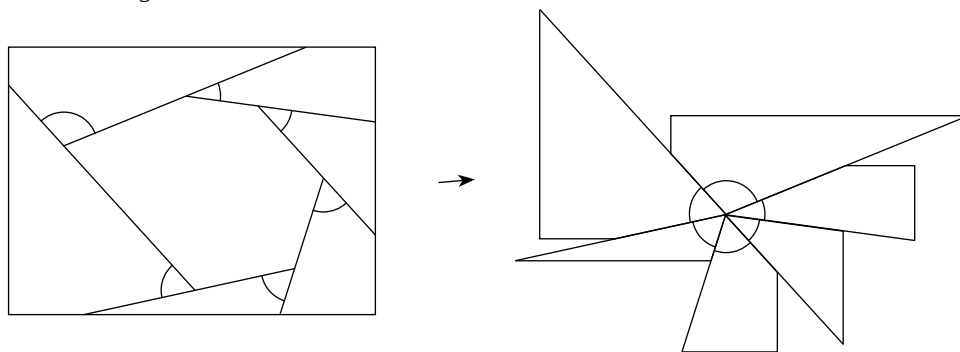
For the exterior angle property there should be a demonstration.

Here are four useful ways of doing this:

- i) Chalk a large polygon on the floor. Ask a student to walk around its perimeter keeping one arm out in front throughout. It will be seen that he returns to the starting point having completed one full turn.
- ii) Chalk a large polygon on the board, complete with exterior angles taken in order. Use a large ruler laid initially along one side and slide it around the perimeter, turning at each vertex. It should be seen that the ruler turns one revolution to return to the start.
- iii) Similar to (ii) but ask the students to draw a large polygon in their books and push a ruler or pencil around perimeter.



- iv) On a sheet of paper or thin card draw a polygon with exterior angles. Mark them and extend to the edge of the paper. Then cut out the angles and fit them together to make  $360^\circ$ .



- For the exterior angle of a triangle follow the text: it is another example of a formal proof that students should be exposed to.  
Use EX 10B.

|                    |                                                  |
|--------------------|--------------------------------------------------|
| <b>Assignments</b> | EX 10A #10 for homework                          |
| <b>Vocabulary</b>  | angle-sum, interior, exterior<br>regular polygon |

# ANSWERS

## Exercises

### EX 10A

1.
  - a)  $a = 80$  (adj  $\angle$ s on a st line),  $b = 28$  (angle-sum of  $\Delta$ )
  - b)  $c = 56$  (adj  $\angle$ s on a st line),  $d = 62$  (angle-sum of isos  $\Delta$ )
  - c)  $a = 97$  (angle-sum of  $\Delta$ ),  $b = 83$  (adj  $\angle$ s on a st line),  $c = 15$  (angle-sum of  $\Delta$ )
  - d)  $d = 70$ , (alt  $\angle$ s),  $x = 50$  (angle-sum of  $\Delta$ )
2.
  - a)  $a = 92$  (angle-sum of quad)
  - b)  $a = 60$  (adj  $\angle$ s on a st line),  $b = 110$  (angle-sum of a quad)
  - c)  $a = 59$  (angle-sum of  $\Delta$ ),  $b = 121$  (adj  $\angle$ s on a st line),  
 $c = 59$  (angle-sum of quad) or (alt  $\angle$ s)
  - d)  $a = 70$  (isos  $\Delta$ ),  $b = 40$  (angle-sum of  $\Delta$ ),  
 $c = 140$  (adj  $\angle$ s on st line),  $d = 50$  (angle-sum of quad)
3.  $x = 10$ ;  $10^\circ$ ,  $100^\circ$ ,  $70^\circ$
4.  $x = 25$ ;  $50^\circ$ ,  $30^\circ$ ,  $140^\circ$ ,  $140^\circ$ ; kite
5.
 

|                |                |                  |                |
|----------------|----------------|------------------|----------------|
| a) $144^\circ$ | b) $150^\circ$ | c) $128.6^\circ$ | d) $171^\circ$ |
|----------------|----------------|------------------|----------------|
6.
 

|                    |                     |  |  |
|--------------------|---------------------|--|--|
| a) $n - 2$         | b) $180(n - 2)$     |  |  |
| c) $25\,560^\circ$ | d) $179\,640^\circ$ |  |  |
7.
 

|                         |                         |
|-------------------------|-------------------------|
| a) $x = 140$ , $y = 40$ | b) $x = 120$ , $y = 60$ |
|-------------------------|-------------------------|
8. a)  $a = 50$ ,  $b = 110$ ,  $c = 70$ ,  $d = 20$ ,  $e = 60$
9.
 

|       |       |
|-------|-------|
| a) 27 | b) 90 |
|-------|-------|
10.
 

|                                                |                    |
|------------------------------------------------|--------------------|
| b) 6                                           | c) $1080^\circ$    |
| d) angles around the inside point, $360^\circ$ | e) $n$ , $180n$    |
| f) $180n - 360$ ;                              | $180(n - 2)$ ; yes |

### EX 10B

1.
 

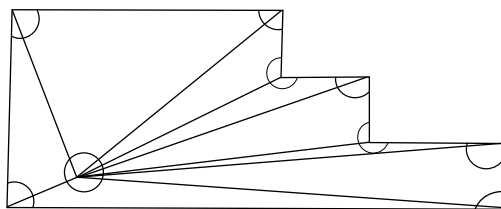
|                                          |                                         |                                           |
|------------------------------------------|-----------------------------------------|-------------------------------------------|
| a) $a = 81$ (ext $\angle$ of $\Delta$ )  | b) $b = 60$ (ext $\angle$ of $\Delta$ ) | c) $c = 20$ (angle-sum of isos $\Delta$ ) |
| d) $d = 130$ (ext $\angle$ of $\Delta$ ) |                                         |                                           |
2.
 

|                                                                                                                       |
|-----------------------------------------------------------------------------------------------------------------------|
| a) $a = 90$ (ext $\angle$ of $\Delta$ ), $b = 90$ (adj $\angle$ s on st line), $c = 45$ (angle-sum of isos $\Delta$ ) |
| b) $x = 30$ (ext $\angle$ of $\Delta$ ), $y = 30$ (vert opp $\angle$ s)                                               |

3. a)  $x = 105$  (angle-sum of quad)  $y = 75$  (adj  $\angle$ s on st line)  
 b)  $x = 80$  (angle-sum of quad)  $y = 100$  (adj  $\angle$ s on st line)
4. a)  $60^\circ$                                       b)  $90^\circ$                                       c)  $30^\circ$                                       d)  $20^\circ$
5. a) ext  $\angle = 36^\circ$ , int  $\angle = 144^\circ$   
 b) angle-sum =  $1440^\circ$ , int  $\angle = 144^\circ$
6. a)  $12^\circ$                                       b)  $168^\circ$                                       c)  $174^\circ$                                       d)  $175^\circ$
7.  $x = 35$ ;  $165^\circ$ ,  $40^\circ$ ,  $110^\circ$ ,  $80^\circ$ ,  $145^\circ$
8. a) 6                                      b) 5                                      c) 15                                      d) 12
9.  $\angle STV = 130^\circ$
10.  $\angle CDV = 130^\circ$

**EX 10X**

1. It is true, e.g. for the given diagram:



[The inside point needs to be chosen so that lines can be joined to all vertices without crossing the polygon perimeter.]

In general,  $n$  triangles for polygon with  $n$  sides,

$$\Rightarrow \text{angle-sum} = 180n - 360 = 180(n - 2)$$

2. If true with  $n$  sides, then

$$\text{angle-sum, } 810 = 180(n - 2)$$

$$810 = 180n - 360$$

$$1170 = 180n$$

$$n = \frac{1170}{180} = 6.5 \text{ (not possible)}$$

Hence, initial statement false.

3.  $179.99964^\circ$

# Chapter 11 Distance, Speed, Time

The three transformations of the distance, speed, time relationship are introduced instinctively rather than by algebraic formula. This topic is easy to grasp, but the calculator entries to deal with time can be frustrating for some. It is a good challenge for efficient use of the calculator.

## LESSON PLANNING

### Objectives

**General** To answer simple questions involving distance, speed (constant or average), and time

**Specific**

1. To know that **speed** =  $\frac{\text{distance}}{\text{time}}$  and to use this to calculate speeds for uniform motion and average speeds for variable motion
2. To know that **distance** = **speed** × **time** and to use this in calculations
3. To know that **time** =  $\frac{\text{distance}}{\text{speed}}$  and to use this in calculations
4. To be aware of the non-decimal nature of units of time and to be able to convert decimals of time to h, min, s, as appropriate

**Pacing** 3 lessons, 1 homework

**Links** Rounding off, efficient use of calculator

### Method

Ensure that everyone has a calculator as they are essential for this topic.

Follow the text to explain why **speed** =  $\frac{\text{distance}}{\text{time}}$ , i.e. use the units.

Distinguish **constant speed** for uniform motion (rare) from **average speed** for variable motion (common). Use simple examples, such as the one in the text, to explain why **distance** = **speed** × **time**.

Set EX 11A which is relatively easy.

The next section involves time units. Before explaining **time** =  $\frac{\text{distance}}{\text{speed}}$ , do some class warm-ups practising time conversions:

1) Change 3.62 h to hours and nearest minute.

2) Change 4.14 h to hours and nearest minute.

3) Change 1 h 17 min to hours.

4) Change 6 h 9 min to hours.

Answers: 1) 3 h 37 min    2) 4 h 8 min    3) 1.28 h    4) 6.15 h

When the students have grasped this, set EX 11B.

Allow discussion with neighbour "pair and share".

Try to seat students with similar calculators next to each other.

|                    |                                               |
|--------------------|-----------------------------------------------|
| <b>Resources</b>   | spare calculators                             |
| <b>Assignments</b> | EX 11A #9, 10 suitable for homework           |
| <b>Vocabulary</b>  | uniform motion, constant speed, average speed |

## ANSWERS

### Exercises

#### EX 11A

- 240 m
  - 300 m
  - 360 m
  - 390 m
- 3 km
  - 3.5 km
  - 5 km
  - 10 km
- 500 km/h
  - 400 km/h
  - 400 km/h
  - 380 km/h
- 6.7 m/s
  - 6.7 m/s
  - 7.1 m/s
  - 6.7 m/s
- 126 m
- 60 km/h
  - 16.7 m/s
- 1.8 km
  - 1.2 km
  - 0.72 km (or 720 m)
  - 0.06 km (or 60 m)
- 1.5 km
  - 3 km
  - 4.5 km
  - 1 km
- Kew Gardens to Richmond, 11.67 m/s (2 d.p.)
  - Turnham Green to Gunnersbury, 32.4 km/h
- 523 km/h
  - 565 km/h
  - 480 km/h
  - 336 km/h

#### EX 11B

- 55 min
  - 1 h 16 min
  - 34 min
  - 2 h 30 min
- 3 min
  - 2 min
  - 4 h 41 min
  - 3 h 8 min
- 60 km/h
  - 84 km/h
  - 90 km/h
  - 77.1 km/h

4.
  - a) 7560 m (or 7.56 km)
  - b) 7200 m (or 7.2 km)
  - c) 12 150 m (or 12.15 km)
  - d) 24 180 m (or 24.18 km)
5.
  - a) 591 km/h
  - b) 589 km/h
  - c) 604 km/h
  - d) 444 km/h
6.
  - a) 200 m
  - b) 100 m
  - c) 400 m
  - d) 400 m hurdles
7. 58 min
8. 100.5 m, 4.8 m/s (2 s.f.)
9.
  - a) Aisha, Bryant
  - b) Chantal, Qais
  - c) Qais, Chantal
  - d) 19.2 km/h
10. ferry 13.4 min (including waiting); road 15.9 min; ferry best

**EX 11X**

1. 40 km/h
2.
  - a) 6.8 m/s
  - b) 6.6 m/s
  - c) 22.8 km/h
3. Draw a distance-time graph (see EX 11B #9). Gradient gives speed.

# Chapter 12 Algebraic Formulas

This chapter builds upon previous work on how algebraic expressions are constructed, defines a formula, and uses the techniques used in solving equations to change the subject of a formula. Calculators are essential for the numerical evaluations.

## LESSON PLANNING

### Objectives

|                    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |
|--------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| General            | To substitute values in a given formula and to change the subject of a formula.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |
| Specific           | <ol style="list-style-type: none"> <li>1. To recognize a formula and the subject of a formula</li> <li>2. To substitute values in a given formula and obtain the value of the subject</li> <li>3. To find the value of a variable on the right-hand side of a formula by substitution of the other values and manipulation</li> <li>4. To change the subject of a given formula by algebraic manipulation; to evaluate the new subject after substituting given values</li> </ol>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |
| Pacing             | 3 lessons, 1 homework                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |
| Links              | Area, volume, laws of physics                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
| <b>Method</b>      | <p>The text explains what a formula is. Take the students through the text. Explain that a formula is a mathematical sentence, i.e. it has a subject and a verb. The verb is "equals"; the subject is the letter on the left-hand side.</p> <p>EX 12A should be relatively easy if previous work on the meaning of algebraic expressions, brackets, use of calculators, etc. has been well learned. It is a good revision of all these topics.</p> <p>For changing the subject, begin by single value calculations of one of the right-hand side letters. Move on to changing the subject as a way of reducing work when many similar calculations are required. Use examples given in the text and other similar examples.</p> <p>Whilst explaining, write the operation being applied to both sides at the side of each line. It is good to get students to do this for the first few questions, but by the end of the exercise it should not be necessary.</p> <p>Use EX 12B.</p> |
| <b>Rescuers</b>    | Calculators (essential)                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |
| <b>Assignments</b> | EX 12B #6, 7 for homework.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |
| <b>Vocabulary</b>  | formula, subject, substitute                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |

**ANSWERS****Exercises****EX 12A**

- |     |                        |                          |                                  |                       |
|-----|------------------------|--------------------------|----------------------------------|-----------------------|
| 1.  | a) $18 \text{ cm}^2$   | b) $6 \text{ cm}^2$      | c) $52.5 \text{ cm}^2$           | d) $18 \text{ cm}^2$  |
| 2.  | a) $480 \text{ cm}^3$  | b) $389.34 \text{ cm}^3$ | c) $0.768 \text{ m}^3$           | d) $588 \text{ cm}^3$ |
| 3.  | a) $42 \text{ cm}^2$   | b) $48 \text{ m}^2$      | c) $40.32 \text{ cm}^2$          | d) $0.54 \text{ m}^2$ |
| 4.  | a) $10 \text{ m}$      | b) $11.25 \text{ m}$     |                                  |                       |
|     | c) $10 \text{ m}$      | d) $0$ (ground level)    |                                  |                       |
| 5.  | a) $22 \text{ min}$    | b) $0.2 \text{ s}$       | c) $8 \text{ h } 28 \text{ min}$ | d) $0.4 \text{ s}$    |
| 6.  | a) $12 \text{ cm}$     | b) $11 \text{ cm}$       | c) $20.4 \text{ cm}$             | d) $36.8 \text{ cm}$  |
| 7.  | a) $24 \text{ m/s}$    | b) $32.5 \text{ m/s}$    | c) $51.4 \text{ m/s}$            | d) $43 \text{ m/s}$   |
| 8.  | a) $2.5$               | b) $3.7$                 | c) $361.88$                      | d) $14.44$            |
| 9.  | a) $115.31$            | b) $0.457$               | c) $6$                           | d) $186$              |
| 10. | a) $415 \text{ cm}^2$  | b) $704 \text{ cm}^2$    |                                  |                       |
|     | c) $1068 \text{ cm}^2$ | d) $1508 \text{ cm}^2$   |                                  |                       |

**EX 12B**

- |    |                             |                       |                     |                    |
|----|-----------------------------|-----------------------|---------------------|--------------------|
| 1. | $2.5$                       |                       |                     |                    |
| 2. | $4$                         |                       |                     |                    |
| 3. | $6$                         |                       |                     |                    |
| 4. | $h = \frac{2A}{b}$          |                       |                     |                    |
|    | a) $10.2 \text{ cm}$        | b) $7.73 \text{ cm}$  |                     |                    |
|    | c) $6.95 \text{ m}$         | d) $66.7 \text{ mm}$  |                     |                    |
| 5. | $a = \frac{2(s - ut)}{t^2}$ |                       |                     |                    |
|    | a) $18$                     | b) $72$               | c) $18.9$           | d) $18.7$          |
| 6. | $t = \frac{50 - h}{4.5}$    |                       |                     |                    |
|    | a) $2 \text{ min}$          | d) $8 \text{ min}$    | c) $11 \text{ min}$ | d) $50 \text{ cm}$ |
| 7. | $x = y(p - z)$              | $y = \frac{x}{p - z}$ |                     |                    |
|    | a) $4.4$                    | b) $6.55$             | c) $28.6$           | d) $2.30$          |
| 8. | a) $-6.17$                  | b) $0.5$              | c) $-2$             | d) $3.8$           |

9. a) 0.567                                      b) 6                                      c) 185                                      d) 0.4  
 10. a) -3.5                                      b) -8                                      c) 5                                      d) 0

### EX 12X

1.  $T = \pi \sqrt{\frac{l}{2.5}}$

|     |      |      |      |      |
|-----|------|------|------|------|
| $l$ | 0.5  | 0.6  | 0.7  | 0.8  |
| $T$ | 1.40 | 1.54 | 1.67 | 1.78 |

(3 s.f.)

2. a)  $a = \frac{v^2 - u^2}{2s}$   
 b)  $s = \frac{v^2 - u^2}{2a}$   
 c)  $u = \pm \sqrt{v^2 + 2as}$   
 d)  $v = \pm \sqrt{u^2 + 2as}$

3. a) 35.8 m                                      b) 41 km/h

[This formula cannot be rearranged; a method of "trial and improvement" has to be used.]

When  $v = 41$ ,  $d = 19.4$ ; when  $v = 42$ ,  $d = 20.18$  (too much)

# Chapter

The students should know the standard locus constructions after completing this chapter. Also, the 3D muscles of the mind are flexed, and it is yet a further opportunity for practising precise use of mathematical language.

## LESSON PLANNING

## Objectives

|          |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |
|----------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| General  | To draw and describe simple loci and answer questions related to loci                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |
| Specific | <ol style="list-style-type: none"> <li>1. To define the term locus</li> <li>2. To illustrate a locus on a coordinate grid by shading unwanted regions</li> <li>3. To construct the locus of a point that is a given distance from a fixed point</li> <li>4. To construct the locus of points that are a given distance from a fixed line or line segment</li> <li>5. To construct the locus of points equidistant from two fixed points</li> <li>6. To construct the locus of points equidistant from two (non-parallel) lines</li> <li>7. To solve problems involving a locus or more than one locus</li> <li>8. To understand that loci may be defined in space (3-D)</li> </ol> |
| Pacing   | 4 lessons, 2 homeworks                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |
| Links    | Constructions, scale drawing, straight line graphs                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |

## Method

The text defines a locus and gives one example each of a locus which is a line, a curve, and a region. Other examples may also be demonstrated on the board.

The next example is the perpendicular bisector, i.e. the locus of points equidistant from two fixed points. Mark two points on the board:

 $\times A$  $\times B$ 

Now elicit class response for equidistant points, chosen at random:

$\times A$

- Yes!
- No!

$\times B$

- Yes!
- No!

- No!

"Where are all the Yes! points?"

This leads into the perpendicular bisector and it is time to remind the students of that construction, using ruler and compasses. This is an example of a locus as a **line**.

Next comes shading when the locus is a **region**. Follow the text example, and the convention of shading out the unwanted parts of the diagram.

Use EX 13A. Drawing diagrams will take some time. Insist on good quality diagrams.

The standard loci are all covered in this exercise, so it is important that all students complete it.

Give notes on each of the basic loci afterwards (See specific objectives.) Do not provide this before they work through the exercise.

The moving point locus example is more difficult, but some physical demonstrations can help. The same holds for the 3-D loci examples. Do not give the answers of the examples. Use EX 13B and allow discussion with the aim of getting students to puzzle them out. Keep equipment handy for them to use if necessary.

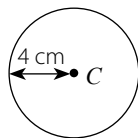
|                   |                                                                                                                                                                                                                                 |
|-------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>Resources</b>  | Ruler, compasses, protractor, sharp pencil<br>Demonstration items: large ball, large cylinder, rope, string or long ruler, sheets of thin card (planes), cones                                                                  |
| <b>Assignment</b> | Suitable for homework Ex 13A #9, 10; EX 13B #10 (coordinate grid required)<br>Advanced students may like to research cycloids, i.e. the locus of a point on a moving wheel, and make a classroom poster and/or a working model. |
| <b>Vocabulary</b> | locus (plural loci), region, shaded, boundary<br>equidistant, line, line segment                                                                                                                                                |

## ANSWERS

### Exercises

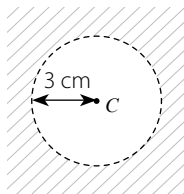
#### EX 13A

1. a)



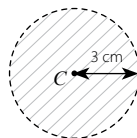
Locus of  $P$  is the circumference only.

b)



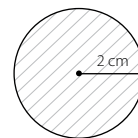
Locus inside circle

c)

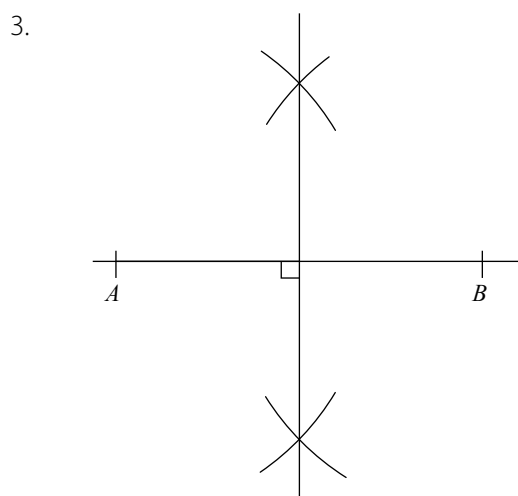
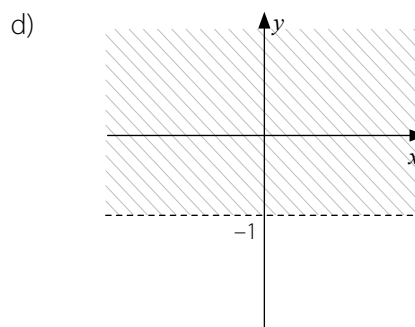
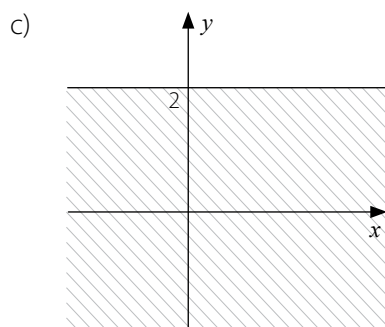
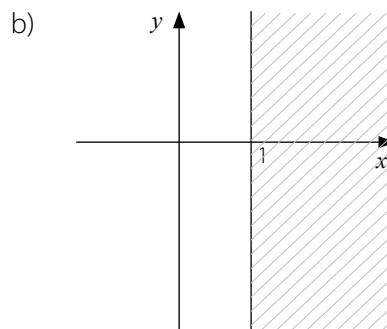
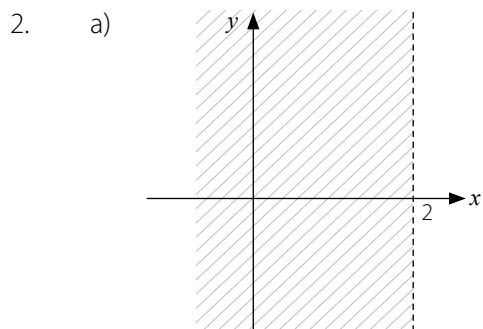


Locus outside

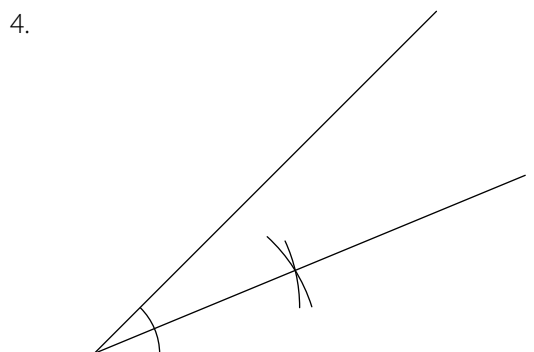
d)



Locus outside or on circumference

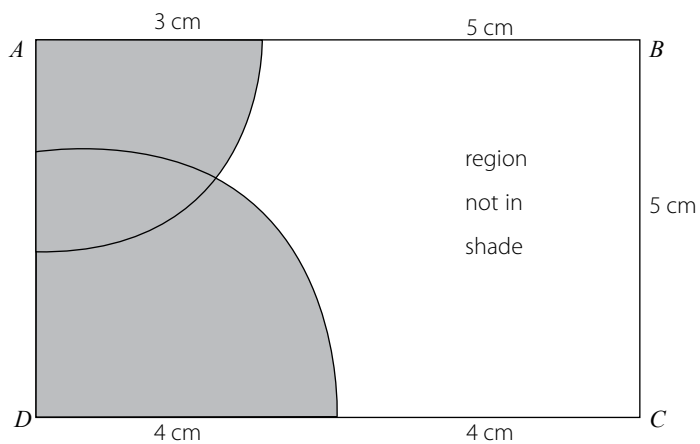


Locus is the perpendicular bisector.

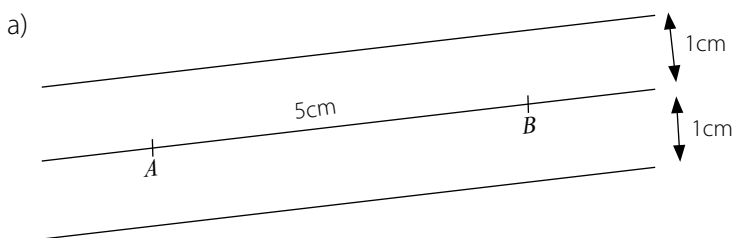


Locus is the  
angle bisector.

5.

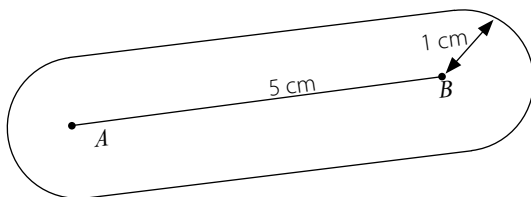


6.

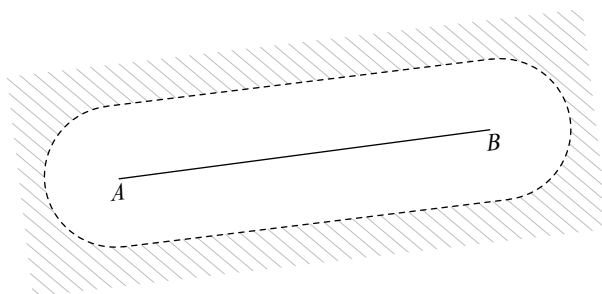


Locus is the two parallel lines.

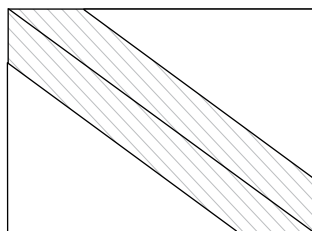
b)



c)



7.



a)



The graph shows a Cartesian coordinate system with x and y axes. A dashed line is drawn through the points (0, 1) and (1, 0). The region below and to the left of this line is shaded with diagonal lines, representing the solution set for the inequality  $x + y < 1$ .

c)



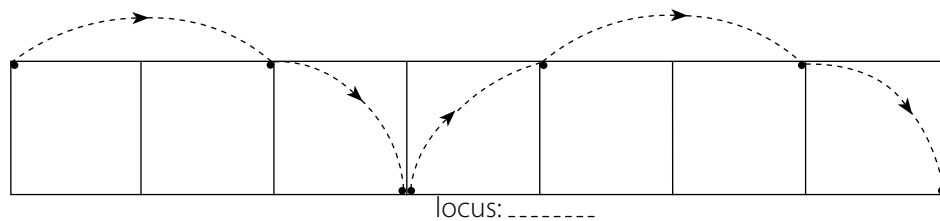
A Cartesian coordinate system with x and y axes. The region to the left of the y-axis (where  $x < 0$ ) is shaded with diagonal lines.

The diagram shows a rectangular field with vertices labeled  $P$  (top-left),  $Q$  (top-right),  $R$  (bottom-right), and  $S$  (bottom-left). The top side  $PQ$  is labeled  $8\text{ m}$ . The right side  $QR$  is labeled  $6\text{ m}$ . A shaded region is shown in the bottom-left corner, bounded by the sides  $PS$  and  $SQ$  and a quarter-circle arc with center  $A$ . The area to the right of the shaded region and below the top boundary is labeled "suitable for planting".

**EX 13B**

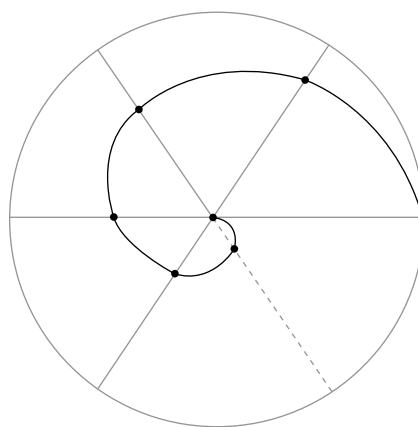
1. a) the surface of an infinite cylinder  
b) a cylinder with hemispheres capping each end
2. a plane perpendicular to AB cutting the line AB 20 cm below A

3. a)



4. a) 6 s

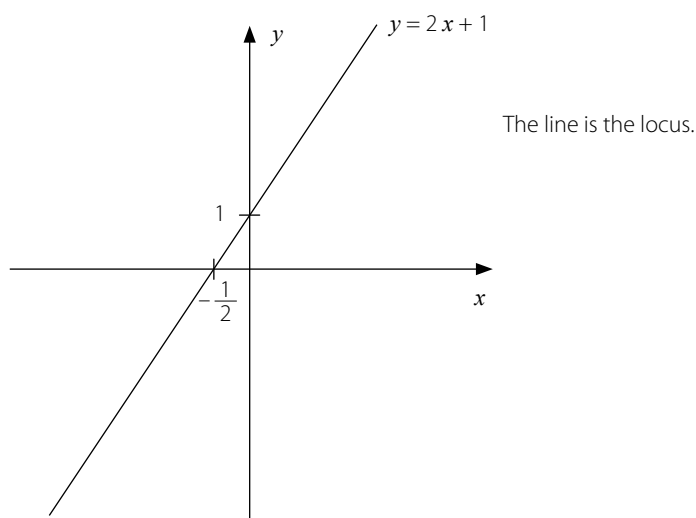
b)



5. practical work

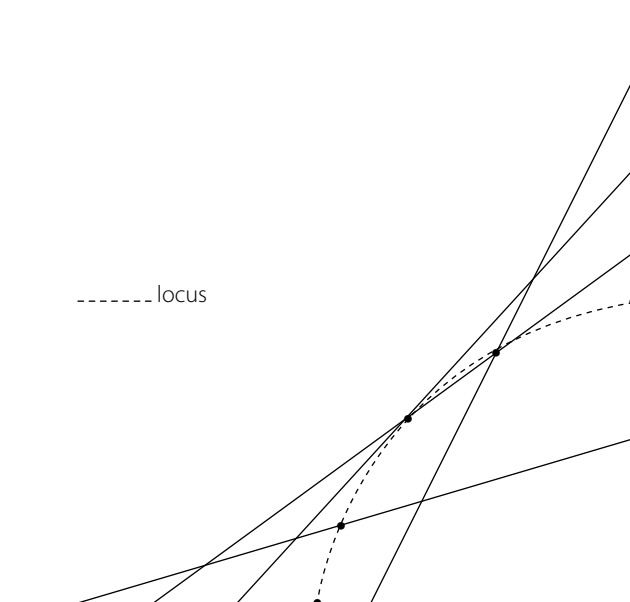
6. two planes parallel to  $P$ , one 8 cm above  $P$ , the other 8 cm below  $P$ 

7.

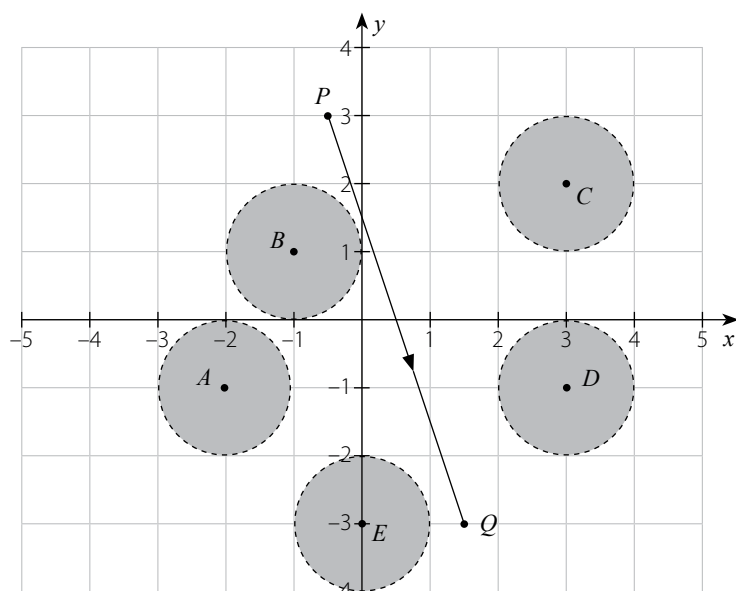


8. a) a horizontal disc, i.e. the interior and circumference of a circle  
b) a cone

9.

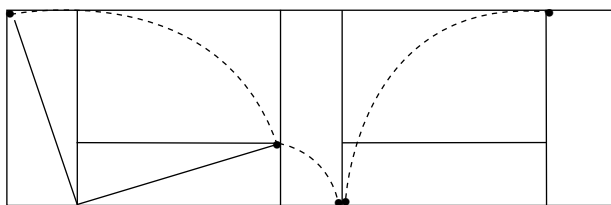


10.

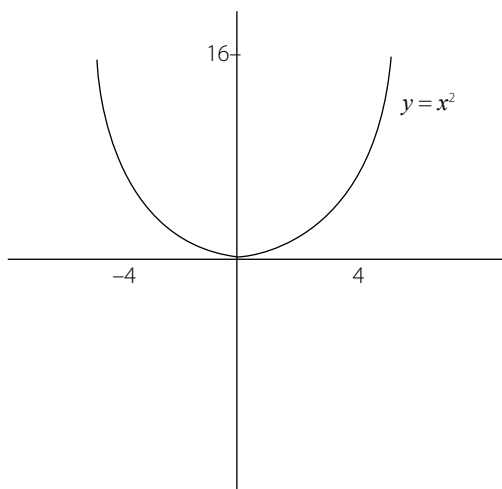


$PQ$  is possible in a straight line.

1.

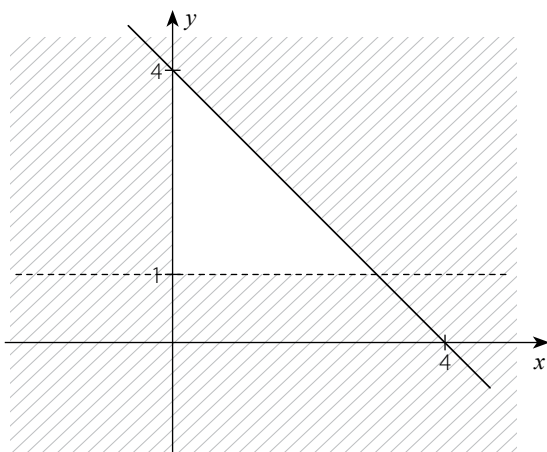


2.



Searchlights rely on the paraboloid shape to produce parallel beams of light from a focal point.

3.



# Chapter 14 Distributions

The word “distribution” appears for the first time. This chapter uses stem-and-leaf diagrams and basic statistics to show that we can describe how a set of data is distributed, i.e. its position in general on a measure or scale, and how spread out the scores are. Quite a few technical points are dealt with regarding the graphs.

## LESSON PLANNING

### Objectives

|         |                                                                                                                                                                                           |
|---------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| General | To represent discrete or continuous data correctly using stem-and-leaf tables, frequency tables, barcharts and frequency polygons; to calculate the statistics of distributions correctly |
|---------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

|          |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
|----------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Specific | <ol style="list-style-type: none"><li>1. To devise stem-and-leaf tables from given data; to read median, mode, and range from the table</li><li>2. To label correctly barchart columns for discrete and continuous data</li><li>3. To draw and label correctly frequency polygons from given data; to interpret such graphs</li><li>4. To estimate the mean from grouped data; to round off means to an accuracy of one additional significant figure more than the raw data</li></ol> |
|----------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

|        |                        |
|--------|------------------------|
| Pacing | 4 lessons, 2 homeworks |
|--------|------------------------|

|        |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |
|--------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Method | <p>There is a lot of detail to learn here. Explain that the most important thing is that this is all about mathematical ways to show how data is distributed.</p> <p>“Are the scores high or low?” Need an average.</p> <p>“Are the scores spread out or close together?” Need the range.</p> <p>These statistics describe a distribution.</p> <p>Stem and leaf technique may be described as a quick method of getting a sense of the distribution.</p> <p>Use the text example:</p> <p>“Which results are higher, Maths or Science?”</p> <p>“Which results are more spread out, Maths or Science?”</p> <p>If the concept is not clear, use the stem-and-leaf table to see what is going on. Follow the text.</p> <p>Calculate median and range from the tables. Easy.</p> <p>Set EX 14A to reinforce the work so far.</p> |
|--------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|



3. a) Mon 1.2833 h (5 s.f.)

Tue 1.2167 h (5 s.f.)

Wed 1.8

Thu 2.05

Fri 1.95

Sat 2.2

b) 1 h 45 min

c) 1 h 53 min

d) 59 min

4. a) 12 y 3 months (nearest completed month)

b) 12 y 3 months

c) 10 months

5. a) 704 kg

b) 64.4 kg (1 d.p.)

c) 58 kg

6. a) 18

b) 167.5 cm

c) 165.6 cm

d) 33 cm

7. a)

| $x$    | $f$ | $xf$ |
|--------|-----|------|
| 0      | 2   | 0    |
| 1      | 1   | 1    |
| 2      | 1   | 2    |
| 3      | 2   | 6    |
| 4      | 4   | 16   |
| 5      | 6   | 30   |
| 6      | 4   | 24   |
| 7      | 2   | 14   |
| 8      | 1   | 8    |
| Totals | 23  | 101  |

$$\text{Mean} = \frac{101}{23} = 4.4 \text{ (1 d.p.)}$$

b) 2

c) 5

d) 6

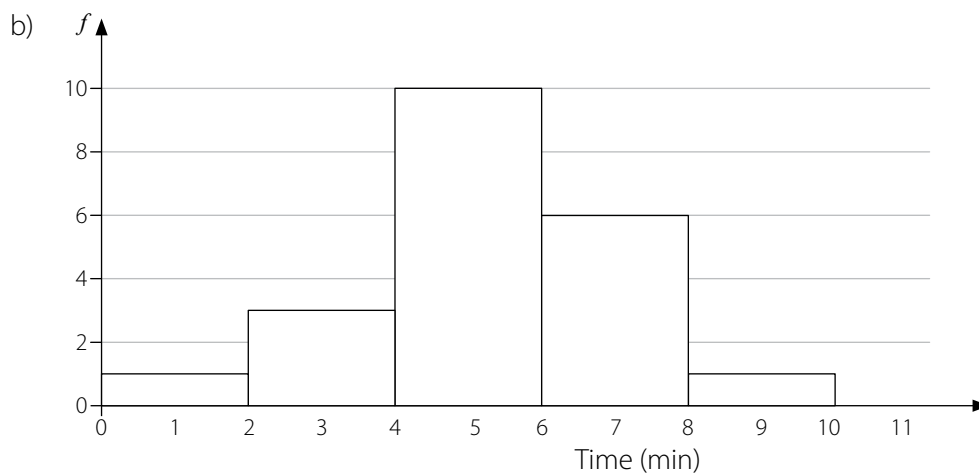
8. a) 16

b)

| Petals $x$ | $f$ | $xf$ |
|------------|-----|------|
| 5          | 4   | 20   |
| 6          | 4   | 24   |
| 7          | 1   | 7    |
| 8          | 3   | 24   |
| 9          | 1   | 9    |
| 10         | 3   | 30   |
|            | 16  | 114  |

$$\text{Mean} = \frac{114}{16} = 7.1 \text{ (1 d.p.)}$$

9. a) 21



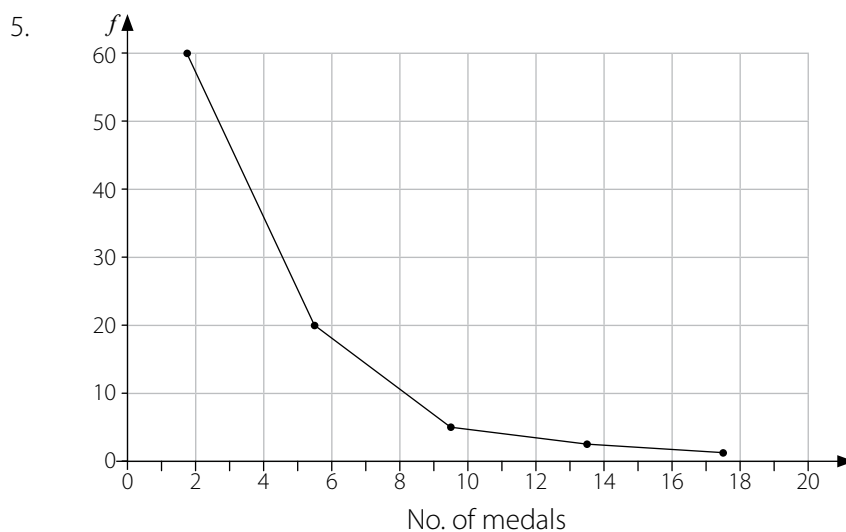
c) 8–10 min (not known precisely)

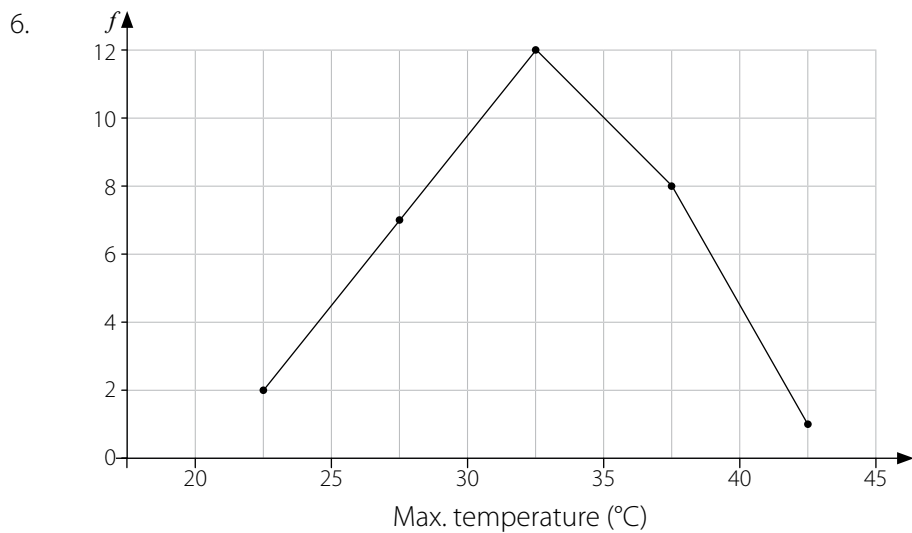
d) Grouped data: exact times not known.

10. a) 35 d) Grouped data: exact numbers of grapes not known  
c) 5–6 grapes d) 9 bunches

#### EX 14B

1. a) discrete b) continuous c) continuous d) discrete  
2. a) 5.5, 8.5, 11.5, 14.5, 17.5, 21.5  
b) 2, 3, 4, 5, 6 c) 5.5, 10.5, 15.5, 20.5, 25.5 d) 0, 3, 6, 9, 12  
3. a) 21 b) No. (It lies in the interval 58–59 min)  
c) 60–61 mm d) 4  
4. a) 30 b) 0–1 pet  
c) 8 or 9 d) 7 students



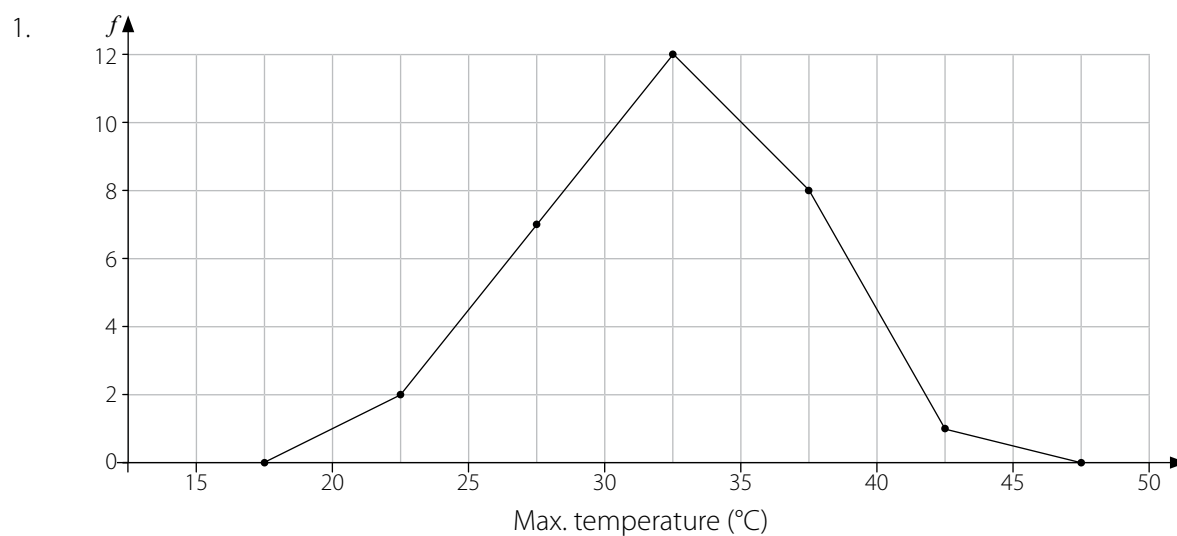


- 7.
- 48 boys, 48 girls
  - 165–170 cm, boys; 170–175 cm, girls
  - girls generally taller
  - | <u>Boys</u> | <u>Girls</u> |
|-------------|--------------|
|-------------|--------------|

| Height (cm) | $f$       |
|-------------|-----------|
| 150–155     | 2         |
| 155–160     | 4         |
| 160–165     | 6         |
| 165–170     | 12        |
| 170–175     | 10        |
| 175–180     | 6         |
| 180–185     | 4         |
| 185–190     | 4         |
|             | <u>48</u> |

| Height (cm) | $f$       |
|-------------|-----------|
| 150–155     | 2         |
| 155–160     | 2         |
| 160–165     | 4         |
| 165–170     | 6         |
| 170–175     | 16        |
| 175–180     | 10        |
| 180–185     | 6         |
| 185–190     | 2         |
|             | <u>48</u> |

- 33.7 °C (1 d.p.)
- 3.6 medals (1 d.p.)
- 170.6 cm, boys; 172.5 cm, girls (1 d.p.)

**EX 14X**

2. a) The data is discrete, but covers a wide range with large intervals. It may be treated as continuous.

Boundaries: 0, 200, 400, 600, 800, 1000

Mid-interval values: 100, 300, 500, 700, 900

- b) The data is continuous but rounded off, so may be treated as discrete.

Boundaries: 20.5, 25.5, 30.5, 35.5, 40.5, 45.5, 50.5

Mid-interval values: 23, 28, 33, 38, 43, 48

3. 6, 10, 14, 18, 22

# Chapter 15 Solving Equations

Students are introduced to a higher level of solving linear equations. Questions here may involve a negative multiplier at the final stage. Also, fractions and decimal multipliers appear in the equations. Finally, for the first time, we have a few equations where more than one solution is possible.

This is a practice chapter. Do not waste too much time demonstrating on the board. The last questions are challenging even for talented students. Don't expect miracles!

## LESSON PLANNING

### Objectives

|          |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |
|----------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| General  | To solve linear equations involving fractions, decimals, and/or negative multiplier; to solve very simple quadratic equations                                                                                                                                                                                                                                                                                                                                                                                           |
| Specific | <ol style="list-style-type: none"> <li>1. To know that dividing by a negative multiplier changes the sign of the solution (rule of signs)</li> <li>2. To simplify linear equations involving decimals or fractions by multiplying each term by a suitable factor</li> <li>3. To solve such linear equations, leaving the solutions as improper fractions if necessary</li> <li>4. To know that quadratic equations (with a squared variable) have two solutions; to solve simple examples of these equations</li> </ol> |
| Pacing   | 4 lessons, 1 homework                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |
| Links    | Rule of signs, brackets, area formula                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |

### Method

Use the examples given in the text to show a quick method of dealing with a negative multiplier in the last line.

Also, show how to clear out fractions.

Emphasize that **each term** has to be multiplied by the **same factor**. Show how it is calculated.

"How would you deal with decimals, e.g.

$$0.1(x + 2) - 0.2(x - 3) = 4x?"$$

Answer: The decimals are tenths, so scale up by 10

$$(x + 2) - 2(x - 3) = 40x \quad \text{etc.}$$

Set the class to work on EX 15A. This exercise is graded. It is suggested that you post the answers to each part so that students receive immediate feedback.

Circulate around the class, troubleshooting and providing hints.

Ex 15B has harder questions, again graded. Before setting this, use the examples given in the text to show that some equations have more than one possible solution. You can introduce the  $\pm$  sign here if you wish.

|                    |                                                                                                    |
|--------------------|----------------------------------------------------------------------------------------------------|
| <b>Assignments</b> | EX 15B #5 is suitable for homework and is a good test of whether the techniques have been grasped. |
| <b>Vocabulary</b>  | quadratic equations                                                                                |

## ANSWERS

### Exercises

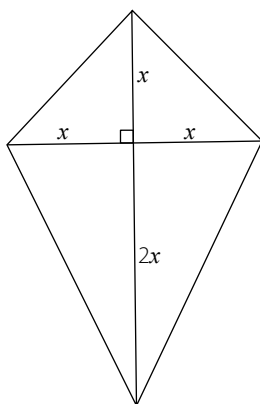
#### EX 15A

- |                           |                        |                       |                        |
|---------------------------|------------------------|-----------------------|------------------------|
| 1. a) $x = -8$            | b) $x = 7$             | c) $x = 25$           | d) $x = -3.2$          |
| 2. a) $x = -1$            | b) $x = 18$            | c) $x = -6$           | d) $x = 1.2$           |
| 3. a) $x = -\frac{1}{2}$  | b) $x = \frac{2}{3}$   | c) $x = -\frac{3}{4}$ | d) $x = -\frac{10}{3}$ |
| 4. a) $x = -\frac{3}{11}$ | b) $x = 11$            | c) $x = -\frac{2}{3}$ | d) $x = \frac{3}{5}$   |
| 5. a) $x = \frac{1}{5}$   | b) $x = 1$             | c) $x = \frac{9}{4}$  | d) $x = -\frac{35}{3}$ |
| 6. a) $x = -3$            | b) $x = 8.75$          |                       |                        |
| c) $x = -0.\dot{3}$       | d) $x = 1.72$ (3 s.f.) |                       |                        |
| 7. a) $x = \frac{1}{7}$   | b) $x = -4$            |                       |                        |
| c) $x = \frac{1}{5}$      | d) $x = -\frac{13}{7}$ |                       |                        |
| 8. a) $x = \frac{14}{3}$  | b) $x = \frac{15}{61}$ | c) $x = 6$            | d) $x = 3$             |
| 9. a) $x = \frac{1}{2}$   | b) $x = \frac{1}{2}$   | c) $x = -3$           | d) $x = \frac{1}{4}$   |
| 10. a) $x = 1$            | b) $x = \frac{13}{8}$  | c) $x = \frac{3}{4}$  | d) $x = \frac{11}{12}$ |

#### EX 15B

- |                             |                         |                        |                        |
|-----------------------------|-------------------------|------------------------|------------------------|
| 1. a) $x = \pm 3$           | b) $x = \pm 4$          | c) $x = \pm 5$         | d) $x = \pm 9$         |
| 2. a) $x = \pm 10$          | b) $x = \pm 7$          | c) $x = \pm 8$         | d) $x = \pm 6$         |
| 3. a) $x = \pm \frac{3}{5}$ | b) $x = \pm 0.1$        | c) $x = \pm 0.3$       | d) $x = \pm 0.2$       |
| 4. a) $x = -2.08$ (3 s.f.)  | b) $x = 0.167$ (3 s.f.) |                        |                        |
| c) $x = -41$                | d) $x = 2.125$          |                        |                        |
| 5. a) $x = -32$             | b) $x = \frac{79}{7}$   | c) $x = \frac{43}{27}$ | d) $x = \frac{82}{93}$ |
| 6. a) 2                     | b) 3                    |                        |                        |

- b)

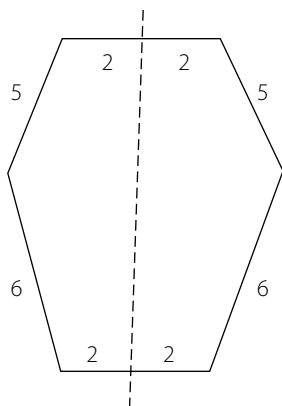


$$300 = 3x^2$$

$$x = 10 \text{ or } x = -10 \text{ (reject)}$$

Diagonals 20 cm, 30 cm

1.    a)  $x = \frac{269}{83}$                       b)  $x = \frac{1101}{64}$   
      c)  $x = \frac{169}{83}$                       d)  $x = \frac{1160}{345} = 3.36$  (3 s.f.)
2.    10
3.     $x = 2.5$ , lengths are 4, 5, 6, 4, 6, 5 cm, taken in order. 1 lin

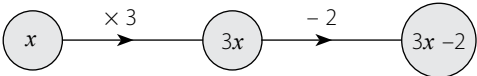


# Chapter 16 Revision Exercises

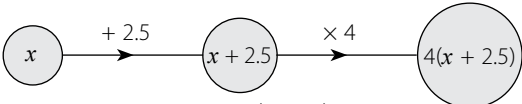
## ANSWERS

### Exercises

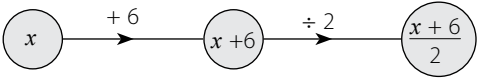
#### EX 16A

1.
  - a)
 

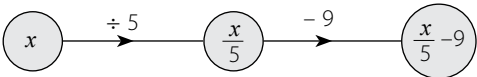
$$x \rightarrow 3x \rightarrow 3x - 2$$

$$x \rightarrow 3x - 2$$
  - b)
 

$$x \rightarrow x + 2.5 \rightarrow 4(x + 2.5)$$

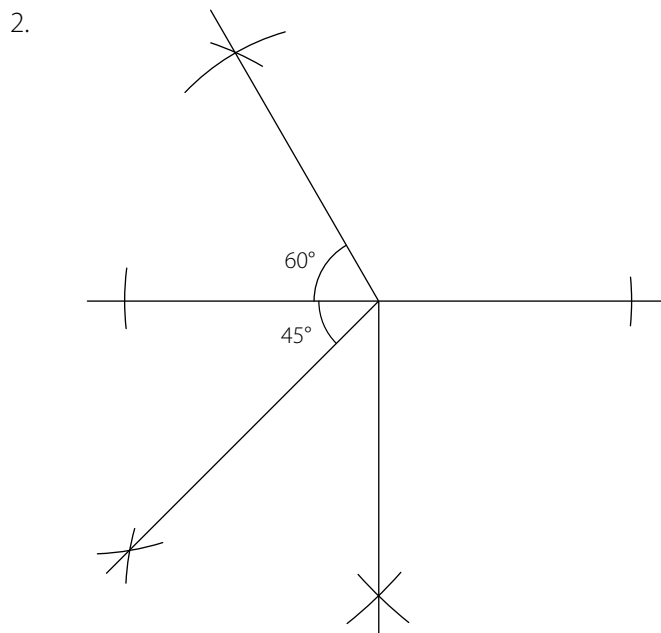
$$x \rightarrow 4(x + 2.5)$$
  - c)
 

$$x \rightarrow x + 6 \rightarrow \frac{x + 6}{2}$$

$$x \rightarrow \frac{x + 6}{2}$$
  - d)
 

$$x \rightarrow \frac{x}{5} \rightarrow \frac{x}{5} - 9$$

$$x \rightarrow \frac{x}{5} - 9$$



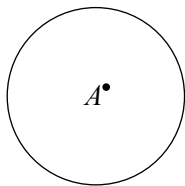


8. a)  $4n + 4$  b)  $n + 7$  c)  $5n + 3$  d)  $6n + 2$   
 9. a) 6.5 m/s b) 6.5 m/s c) 7.3 m/s d) 6.8 m/s  
 10. a) 12 y 2 months b) 12 y 3 months c) 7 months

### EX 16C

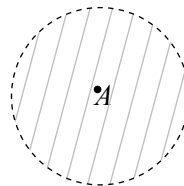
1. a)  $x = -2.5$  b)  $x = -\frac{17}{7} = -2.43$  (3 s.f.)  
 c)  $x = -\frac{13}{9} = -1.44$  (3 s.f.) d)  $x = \frac{14}{9} = 1.56$  (3 s.f.)

2. a)



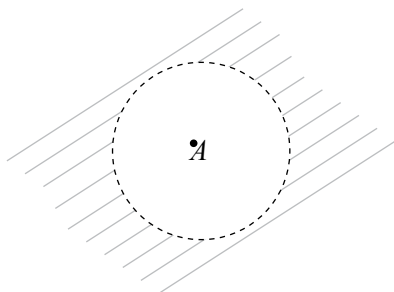
$P$  is on circumference only.

b)



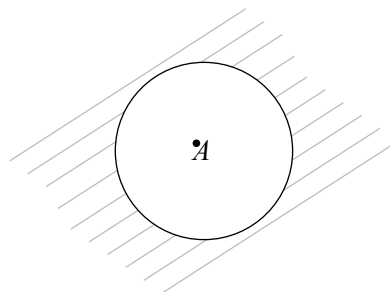
$P$  is outside the circle.

c)



$P$  is inside the circle.

d)



$P$  is inside or on circumference.

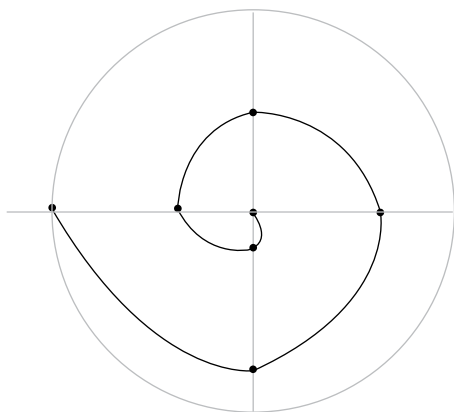
3. a)  $12.8 \text{ cm}^3$  b) 15 cm  
 4. a) 38 b) 1 book  
 c) 8 books d) 8 students  
 5. a)  $5p + 7q$  b)  $p + q$   
 c)  $15p + 4q$  d)  $-p + 15q$   
 6. a)  $a = 110^\circ$  (adj  $\angle$ s on st line)  
 $x = 110^\circ$  (alt  $\angle$ s)  
 b)  $b = 70^\circ$  (alt  $\angle$ s)  
 $x = 110^\circ$  (adj  $\angle$ s on st line)  
 c)  $y = 110^\circ$  (adj  $\angle$ s on st line)  
 $y = a = 110^\circ$  (vert opp  $\angle$ s)  
 $y = x = 110^\circ$  (corr  $\angle$ s)

7. a) 12 m                      b) 15 m                      c) 4 m
8.  $2(l + 3l) = 240$ ;  $l = 30$ ; area  $2700 \text{ cm}^2$
9. a) true                      b) false                      c) true                      d) true
10. a) 1%                      b) 22%                      c) 43%                      d) 20%

**EX 16D**

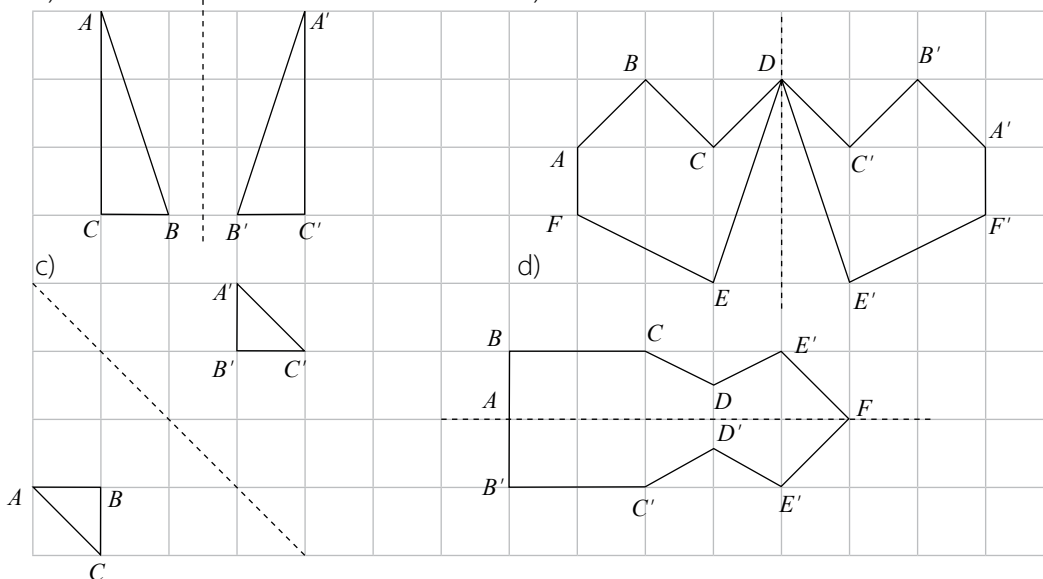
1. a)  $51\frac{3}{7}^\circ$                       b)  $36^\circ$                       c)  $120^\circ$                       d)  $176^\circ$
2. a) 35.4                      b) 2
- c) 6.62 (3 s.f.)                      d) 47.1 (3 s.f.)

3.

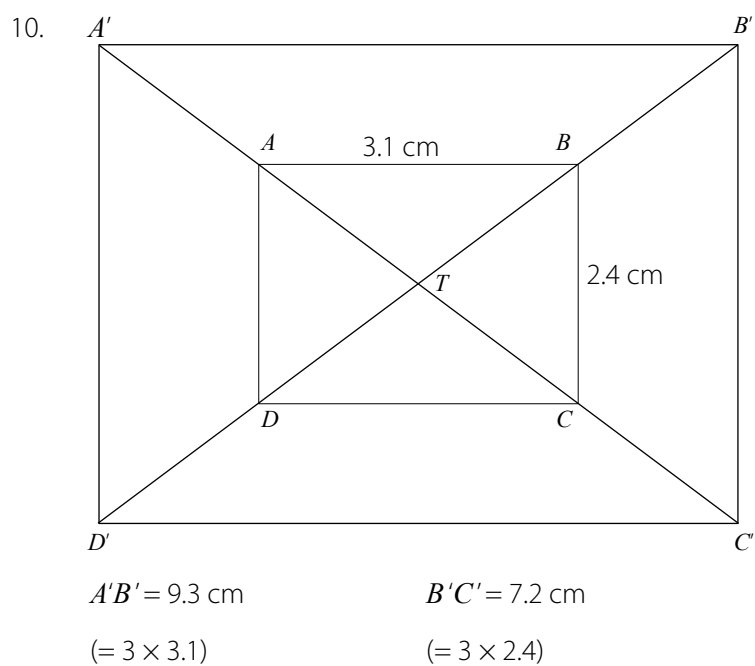


4. a)  $x = -\frac{1}{5}$                       b)  $x = -22$
- c)  $x = \frac{6}{7}$                       d)  $x = -\frac{12}{5}$
5. Rs 88.4 million

6. a)                      b)



7.      a) Q                                  b) S                                  c) P                                  d) R
8.      a)  $\frac{3}{7} = \frac{6}{14} = \frac{12}{28}$                   b)  $\frac{2}{5} = \frac{6}{15} = \frac{24}{60}$
- c)  $\frac{3}{8} = \frac{15}{40} = \frac{21}{56}$                   b)  $\frac{2}{12} = \frac{6}{36} = \frac{5}{30}$
9.      a) 11 880 m or 11.88 km
- b) 3600 m or 3.6 km
- c) 2550 m or 2.55 km
- d) 6.6 km



## EX 16X

1.  $100^\circ, 120^\circ, 80^\circ, 140^\circ, 100^\circ$
2. 8
3.  $r = \sqrt{\frac{A}{\pi}}$  (–ve value rejected); 24.0 cm (3 s.f.)

# Specimen Examination Paper

[for the first half of the year]

Instructions      The time allowed is 1 hour.  
                          You may use a calculator.  
                          You will also need: pen, pencil, rubber, ruler, protractor, compasses.  
                          Try to answer all the questions.

The marks for each question are shown in brackets.

[Full marks = 100]

1. State True or False for the following statements:

a) The mean of 4, 5, and 9 is 6.

b) The interior angles of a kite add up to  $450^\circ$ .

c)  $\frac{5}{6} - \frac{1}{3} = \frac{4}{3}$

d) The area of a trapezium =  $\frac{1}{2}(a + b)h$  where  $a$  and  $b$  are the lengths of the parallel sides and  $h$  is the distance between them.

[4]

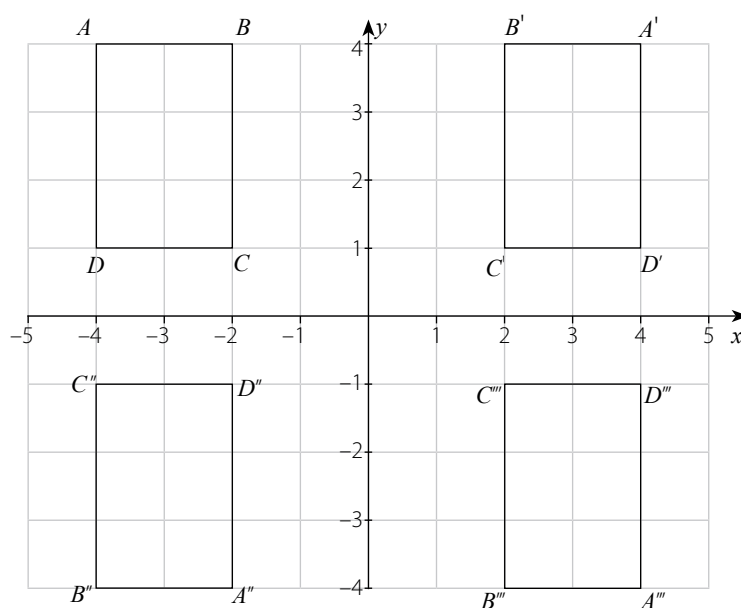
2. In the diagram the rectangle  $ABCD$  is moved to new positions under various transformations. In each case state the type of transformation, specifying it fully:

a)  $ABCD \rightarrow A'B'C'D'$

b)  $ABCD \rightarrow B'A'D'C'$

c)  $ABCD \rightarrow A''B''C''D''$

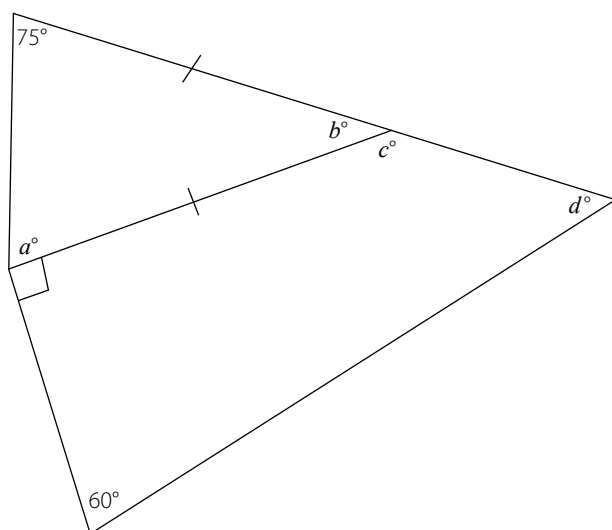
d)  $ABCD \rightarrow A'''B'''C'''D'''$



[4]

3. Write down the correct scale factor to use to make a number  
a) increase by 5%                      b) decrease by 6% [2]
4. In a bowl of fruit there are  $n$  apples. Write down expressions for:  
a) the number of oranges, if there are three times as many oranges as apples  
b) the number of bananas, if there are two more bananas than apples  
c) the number of melons, if there are three fewer melons than apples  
d) the number of grapes, if there are seven times as many grapes than melons [4]
5. In June 2013, Rs 15 000 could be exchanged for \$ 151.60. How many dollars could have been exchanged then for Rs 45 000? [3]
6. A metre stick has 10 cm painted red, 15 cm painted blue, and the rest white.  
a) What is the ratio of red length to blue length? (simplest form)  
b) What fraction of the length of the stick is painted blue?  
c) What is the ratio of blue length to white length (in the form 1: $n$ )?  
d) What fraction of the length painted blue is the length painted red? [4]
7. A set of cards is made with the letters of the word MATHEMATICS.  
Another set is made with letters of the word SCHOOL.  
One card is selected at random from each set.  
Which is more likely:  
Obtaining a vowel from the MATHEMATICS set,  
or, obtaining a vowel from the SCHOOL set?  
Give a reason for your answer. [4]
8. a) Simplify the expression                       $8x + 6 - 5(x + 2)$   
b) Factorise the expression                       $4x - 8y + 12$  [4]

9. Calculate the size of  $a$ ,  $b$ ,  $c$ , and  $d$ , giving reasons:



[8]

10. A train is travelling at an average speed of 52 km/h. Calculate, to the nearest minute, how long it takes to travel

- a) 104 km                      b) 204 km                      c) 304 km

[3]

11.  $R$  is a rotation of  $60^\circ$  about the origin.

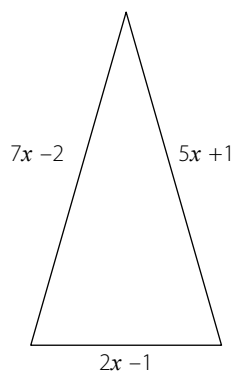
- a) if  $R^n = I$ , state the value of  $n$ .  
b) Describe fully the transformation  $R^2$ .

[5]

12. The population of Megapur increased by 12% in one year. The following year it increased by 14% to 10 214 400 people. What was the population at the start of the two-year period?

[7]

13. Find the value of  $x$  that makes this triangle isosceles, and state the lengths of the sides.



[ not to scale  
lengths in cm ]

[5]

14. A blend of tea is made by mixing low and high quality teas in the ratio 4:1.
- If  $l$  kg of low quality tea is mixed with  $h$  kg of high quality tea, write an equation connecting  $h$  and  $l$ .
  - If we have 120 kg of low quality tea, how much high quality tea should be mixed with it to create the same blend?
  - Without drawing the graph of  $h$  against  $l$ , state its gradient.
  - What percentage of the mixture is high quality tea? [4]
15. The exterior angles of a pentagon are  $2x$ ,  $2x + 10$ ,  $3x$ ,  $5x$ , and  $6x - 10$  degrees. Form an equation to find  $x$  and hence find each interior angle of the pentagon. [5]
16. Given that  $v = u + at$ :
- Make  $u$  the subject of the formula.
  - Make  $a$  the subject of the formula.
  - Find the value of  $t$  when  $v = 15$ ,  $u = 10$ , and  $a = 2.5$ . [4]
17. Draw sketches to illustrate the locus of  $P$  if
- $A$  is a fixed point and  $AP \leq 2$  cm.
  - $AB$  is a fixed line 4 cm long, and  $P$  is 2 cm from  $AB$ .
  - $A$  and  $B$  are fixed points 4 cm apart, and  $P$  is at an equal distance from  $A$  and  $B$ .
  - $A$  is a fixed point 4 cm and  $AP < 2$  cm [8]
18. A survey was done to find the number of applications people kept on their smart phones. The resulting data was grouped as follows:

| No. of apps | Frequency |
|-------------|-----------|
| 6–10        | 1         |
| 11–15       | 3         |
| 16–20       | 10        |
| 21–25       | 36        |
| 26–30       | 46        |
| 31–35       | 2         |

- How many people were questioned in this survey?
- Estimate the mean number of applications people kept on their phones. Why is it an estimate and not exact? [6]

19. Solve these equations:

a)  $-7x = 21$

b)  $3 - (x + 1) = 7$

c)  $\frac{1}{2}(x + 3) = 5 - (x - 1)$

d)  $2.5(x - 3) = 5(1 - 2x)$

[8]

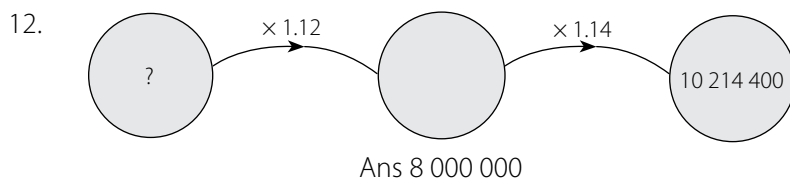
20. b) The area of a triangle is  $200 \text{ cm}^2$ . Its base is exactly four times its perpendicular height. Form an equation and solve it to find the base and height of the triangle.

[8]



11. a)  $n = 6$   
 b) rotation of  $120^\circ$  about the origin

$$\begin{bmatrix} \text{(a) } 3 \\ \text{(b) } 2 \end{bmatrix} = 5$$



$$\begin{bmatrix} \text{scale factors 2 each} \\ \text{dividing 2} \\ \text{answer 1} \end{bmatrix} = 7$$

13.  $7x - 2 = 5x + 1$   
 $x = 1.5$   
 Lengths: 8.5, 8.5, 2 cm

$$\begin{bmatrix} \text{equation 2} \\ \text{solution 1} \\ \text{lengths 2} \end{bmatrix} = 5$$

14. a)  $h = \frac{1}{4}l$  or  $l = 4h$   
 b) 30 kg  
 c)  $\frac{1}{4}$   
 d) 20%

$$[1 \text{ each} = 4]$$

15.  $2x + 2x + 10 + 3x + 5x + 6x - 10 = 360$   
 $x = 20$

Exterior  $\angle$ s are  $40^\circ, 50^\circ, 60^\circ, 100^\circ, 110^\circ$ .

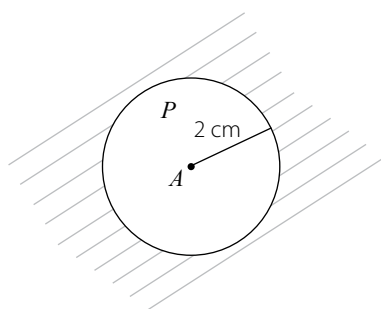
Interior  $\angle$ s are  $140^\circ, 130^\circ, 120^\circ, 80^\circ, 70^\circ$ .

$$\begin{bmatrix} \text{equation 1} \\ \text{solution 1} \\ \text{ext } \angle\text{s } 1 \\ \text{int } \angle\text{s } 2 \end{bmatrix} = 5$$

16. a)  $u = v - at$   
 b)  $a = \frac{v-u}{t}$   
 c)  $t = 2$

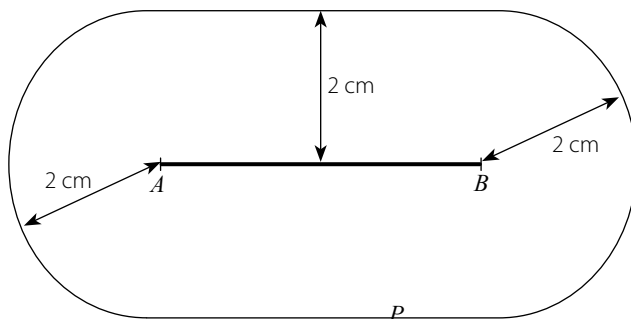
$$\begin{bmatrix} \text{(a) } 1 \\ \text{(b) } 1 \\ \text{(c) } 2 \end{bmatrix} = 4$$

17. a)



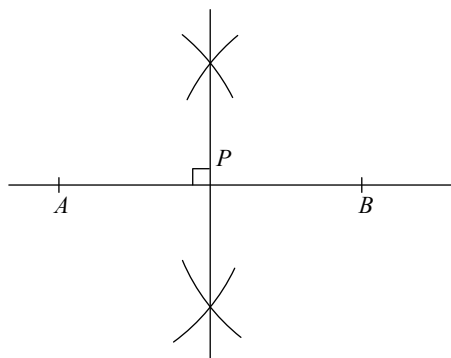
$$\begin{bmatrix} \text{circle 1} \\ \text{shaded outside 1} \end{bmatrix} = 2$$

- b)



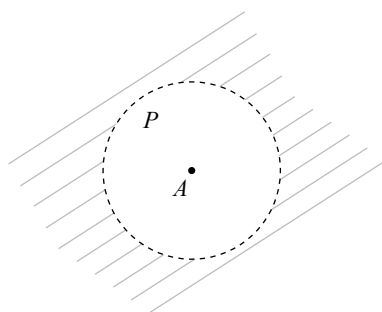
$$\begin{bmatrix} 1 \text{ general shape} \\ 1 \text{ labelling} \end{bmatrix} = 2$$

c)



|                                   |   |     |
|-----------------------------------|---|-----|
| perp bisector                     | 1 | = 2 |
| (Construction need not be shown.) |   |     |
| showing rt $\angle$               | 1 |     |

d)



|                |   |     |
|----------------|---|-----|
| broken circle  | 1 | = 2 |
| shaded outside | 1 |     |

18. a) 98

$$b) 8 \times 1 + 13 \times 3 + 18 \times 10 + 23 \times 36 + 28 \times 46 + 33 \times 2 = 2409$$

$$\text{Mean} = \frac{2409}{98} = 24.6 \text{ (1 d.p.)}$$

Estimated, because grouping does not permit use of original data.

|    |                     |     |   |
|----|---------------------|-----|---|
| a) | 2                   | = 6 |   |
| b) | mid – intervals     |     | 1 |
|    | total               |     | 1 |
|    | mean                |     | 1 |
|    | reason for estimate | 1   |   |

19. a)  $x = -3$ 

$$b) 3 - x - 1 = 7$$

$$x = -5$$

$$c) x + 3 = 10 - 2(x - 1) \text{ or } \frac{1}{2}x + \frac{3}{2} = 5 - x + 1$$

$$x = 3$$

$$d) x - 3 = 2(1 - 2x) \text{ or } 25(x - 3) = 50(1 - 2x) \text{ or } 5(x - 3) = 10(1 - 2x)$$

$$x = 1$$

|                    |   |     |
|--------------------|---|-----|
| (a)                | 2 | = 8 |
| (b)(c)(d)          |   |     |
| Correct first line | 1 |     |
| Solution           | 1 |     |

20.

$$A = \frac{1}{2}bh$$

$$200 = \frac{1}{2}(4h)h$$

$$h = 10$$

$$b = 40$$

|                                    |   |     |
|------------------------------------|---|-----|
| equation                           | 4 | = 8 |
| solution                           | 4 |     |
| (Allow alternative valid methods.) |   |     |

# Chapter 17 Vectors

This short chapter is an early introduction to the concept of a vector, to be dealt with much more rigorously and formally later. It shows that there is more than one kind of algebra (letters for arrows, not just for numbers) and gives a preview of the 2-dimensional nature of vectors. Students of this age are more receptive to unusual ideas, so introducing the concept here pays dividends later.

Note that the law of vector addition is not explicit, although EX 17A may lead to speculation about components.

## LESSON PLANNING

### Objectives

|           |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |
|-----------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| General   | To write correct notation for vectors, understand that they can represent translations, and use components on diagrams and in calculations                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |
| Specific  | <ol style="list-style-type: none"> <li>1. To know that vectors have magnitude and direction, and can represent quantities with magnitude and direction</li> <li>2. To use underlining to indicate vectors when writing; to recognize that bold typeface is used to indicate vectors in printed text</li> <li>3. To draw vectors on squared paper given their components; to state their components given their diagrams</li> <li>4. To state the components of vectors representing translations between points on a coordinate grid</li> <li>5. To solve problems involving vectors in combination</li> </ol> |
| Pacing    | 2 lessons, 1 homework                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |
| Links     | Coordinates, translations                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |
| Method    | <p>"We are going to do arrow mathematics!" Letters represent arrows (not numbers). We underline them (so no confusion). Show how textbooks print them.</p> <p>Long arrows represent large quantities, e.g. force. Direction is also shown. Follow the text definition and notation in the introduction to the chapter.</p> <p><b>Components:</b> Use the text. Go through every single example. Emphasize <math>x</math> and <math>y</math>, positive and negative directions, same as for coordinates.</p> <p>Use EX 17A.</p>                                                                                 |
| Resources | Squared paper, coordinate grids                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |

**Assignments** EX 17A #5. The rest of the exercise can be carried over to the next lessons and completed in class.

**Vocabulary** translation, vector, magnitude, direction, component

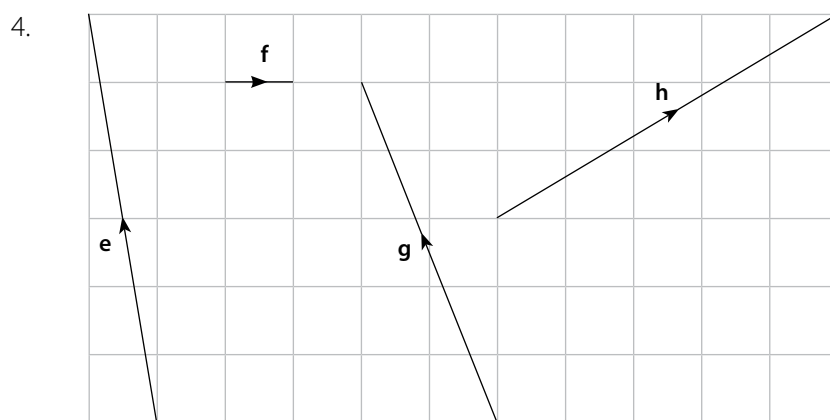
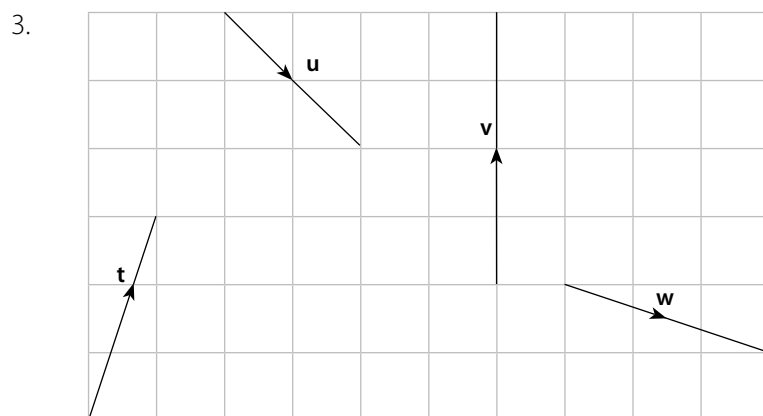
## ANSWERS

### Exercises

#### EX 17A

1.  $\mathbf{a} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$   $\mathbf{b} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$   $\mathbf{c} = \begin{pmatrix} 0 \\ -3 \end{pmatrix}$   $\mathbf{d} = \begin{pmatrix} 3 \\ -6 \end{pmatrix}$

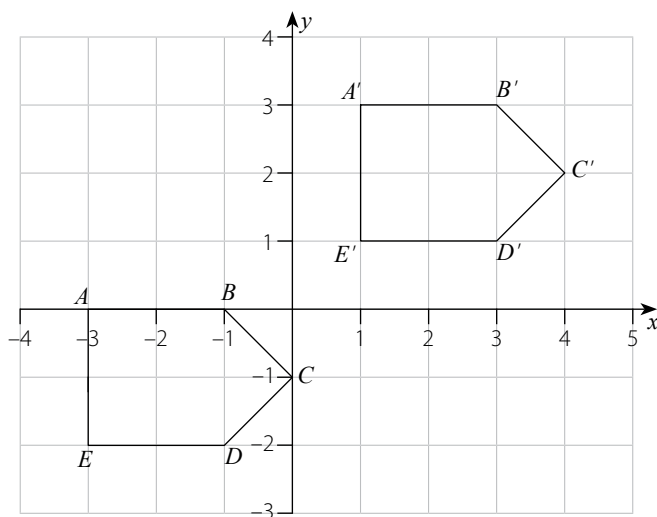
2.  $\mathbf{a} = \begin{pmatrix} 8 \\ 1 \end{pmatrix}$   $\mathbf{b} = \begin{pmatrix} -8 \\ 1 \end{pmatrix}$   $\mathbf{c} = \begin{pmatrix} 6 \\ 2 \end{pmatrix}$   $\mathbf{d} = \begin{pmatrix} -7 \\ 0 \end{pmatrix}$



5.

a)

b)



c)  $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$

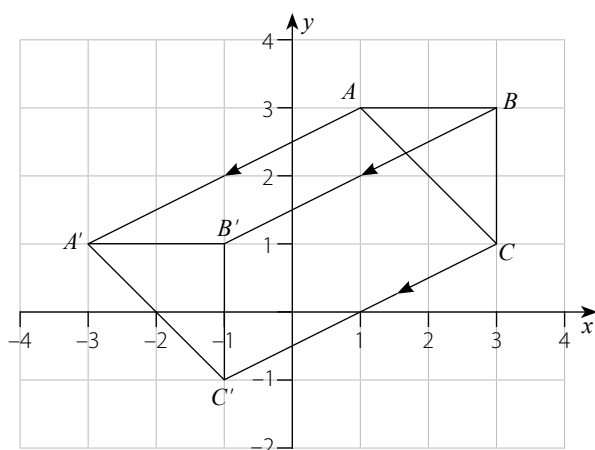
d) They all have the same components.

6.

a)

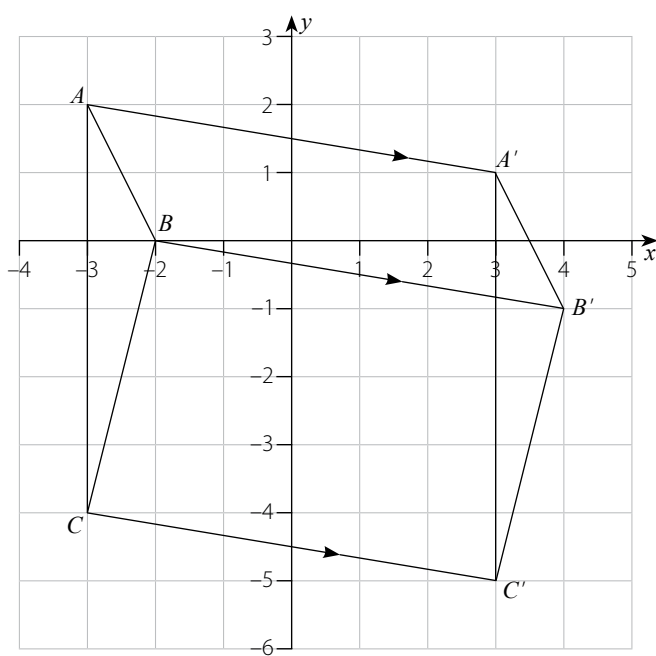
b)

c)



d)  $D'$  is  $(3 - 4, 5 - 2)$   
 $= (-1, 3)$

7.



a)  $A'(3, 1)$

b)  $B'(4, -1)$

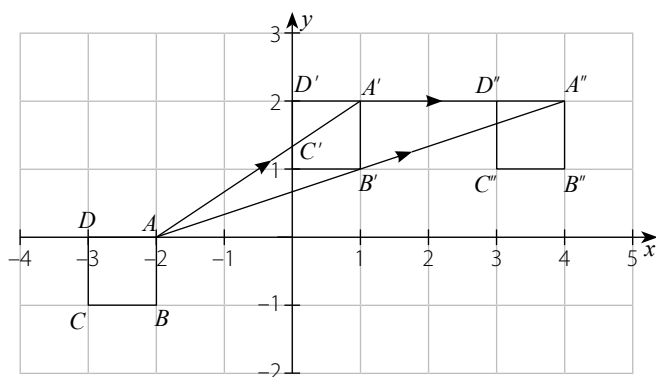
c)  $C'(3, -5)$

8.

a)

b)

c)



d)  $A \rightarrow A'$  is  $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$

$$\begin{pmatrix} 3 \\ 2 \end{pmatrix} + \begin{pmatrix} 3 \\ 0 \end{pmatrix} = \begin{pmatrix} 6 \\ 2 \end{pmatrix}$$

i.e.  $A \rightarrow A'$  followed by  $A' \rightarrow A''$   
is equivalent to  $A \rightarrow A''$

9.

a)

$$\mathbf{u} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$$

$$\mathbf{v} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$$

b)  $\mathbf{w} = \begin{pmatrix} 7 \\ 1 \end{pmatrix}$

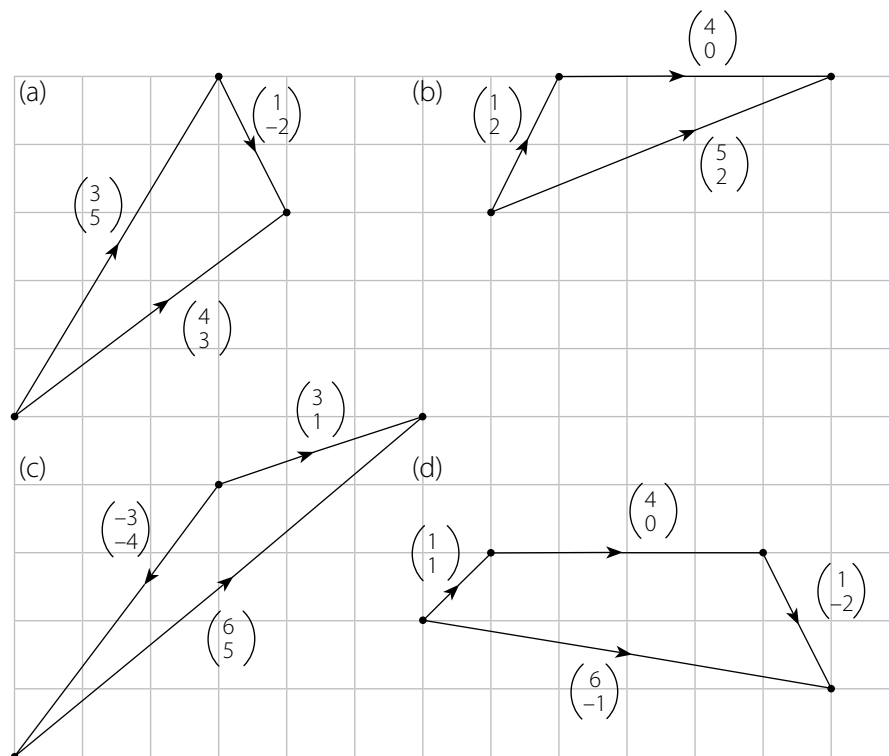
c)

$\mathbf{u} + \mathbf{v} = \mathbf{w}$  is true for the components, i.e.

$$4 + 3 = 7 \text{ and } 3 - 2 = 1$$

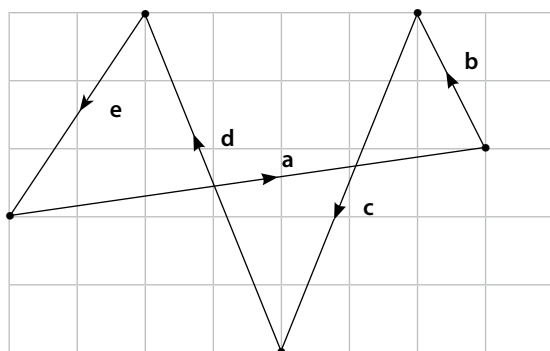
$\mathbf{u}$  followed by  $\mathbf{v}$  is equivalent to  $\mathbf{w}$  directly.

10.



## EX 17X

1.



$$7 - 1 - 2 - 2 - 2 = 0 \text{ and } 1 + 2 - 5 + 5 - 3 = 0$$

Sequence of vectors finishes at the start, i.e. no equivalent translation, or  $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ .

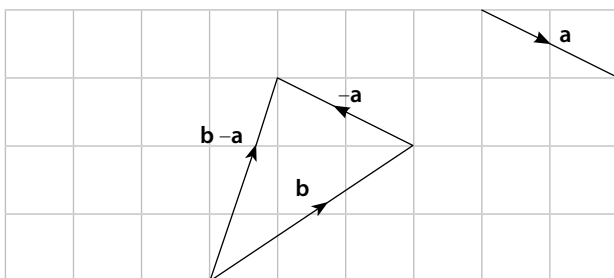
2.

$-\mathbf{a}$  is vector  $\mathbf{a}$  reversed in direction.



$$\mathbf{a} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}, \quad -\mathbf{a} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$

$$\mathbf{b} - \mathbf{a} = \mathbf{b} + (-\mathbf{a})$$



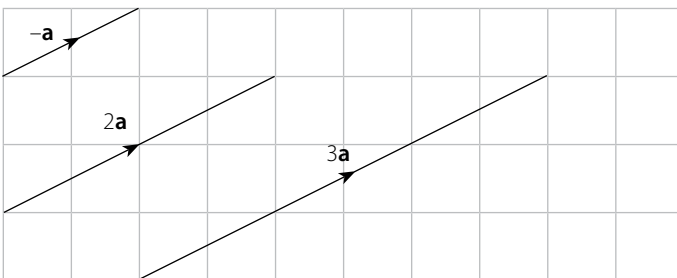
3.

$$2\mathbf{a} = \mathbf{a} + \mathbf{a}$$

The components will double:  $2\mathbf{a} = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$

$$3\mathbf{a} = \mathbf{a} + \mathbf{a} + \mathbf{a}$$

The components will triple:  $3\mathbf{a} = \begin{pmatrix} 6 \\ -3 \end{pmatrix}$



The multiplier preserves the direction but scales up the length.

# Chapter 18 Area of a Circle

The formula for the area of a circle should be explained with justification and not just be stated.

## LESSON PLANNING

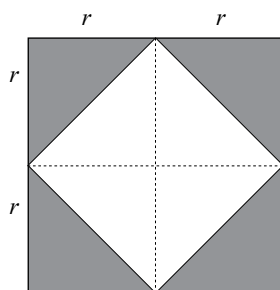
### Objectives

|          |                                                                                                                                                                                                                                                                                                                                                                                                |
|----------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| General  | To solve problems involving the area of a circle and the volume and surface area of a cylinder                                                                                                                                                                                                                                                                                                 |
| Specific | <ol style="list-style-type: none"> <li>1. To understand that <math>\pi r^2</math> is a reasonable formula for calculating the area of a circle</li> <li>2. To use the formula to solve problems involving circles</li> <li>3. To recognize that a cylinder is a prism with circular cross-section</li> <li>4. To solve problems involving the volume and surface area of a cylinder</li> </ol> |
| Pacing   | 3 lessons, 1 homework                                                                                                                                                                                                                                                                                                                                                                          |
| Links    | Manipulation of formulas                                                                                                                                                                                                                                                                                                                                                                       |

### Method

Follow the text introduction to the chapter to show that the area of a circle is between  $2r^2$  and  $4r^2$ . [Revise use of double < to mean “between”]

As an alternative to the derivation of  $2r^2$  in the text, it can be shown that the inner square has half the area of the large square, as can be observed by dividing it into quarters:



Now give the actual formula  $A = \pi r^2$ .

To show that  $\pi$  is the correct value between 2 and 4, do practical work. Divide the students into groups. Give one cardboard circle to each group. Ask the students to cut and re-arrange pieces to form something close to a rectangle. They should then cut each piece in half and try again.

Elicit the length and width (approximately) of the rectangle:

$\frac{1}{2}$  of  $2\pi r$  and  $r$

$$\text{Area} = \pi r \times r = \pi r^2$$

The more cutting we do, the closer this gets to a true rectangle. This is why the formula is correct.

Now use  $A = \pi r^2$  in EX 18A.

The cylinder is to be explained as a prism with circular cross-section. This gives its volume formula. The surface area formulas for open and closed cylinders are not to be memorized, but the method of finding them should be learnt.

Problems with circles and cylinders are in EX 18B.

|                    |                                                                           |
|--------------------|---------------------------------------------------------------------------|
| <b>Resources</b>   | Circles for cutting up (photocopiable in this Guide after the answers)    |
| <b>Assignments</b> | Suitable for homework EX 18B #3, 6                                        |
| <b>Vocabulary</b>  | circle, diameter, radius, circumference<br>cylinder, cross-section, prism |

## ANSWERS

### Exercises

#### EX 18A

- |     |                                 |                                |                         |                        |
|-----|---------------------------------|--------------------------------|-------------------------|------------------------|
| 1.  | a) $78.5 \text{ cm}^2$          | b) $113 \text{ cm}^2$          | c) $3.14 \text{ m}^2$   | d) $9.08 \text{ m}^2$  |
| 2.  | a) $201 \text{ cm}^2$           | b) $227 \text{ mm}^2$          | c) $254 \text{ m}^2$    | d) $69.4 \text{ cm}^2$ |
| 3.  | a) $109 \text{ cm}^2$           | b) $191 \text{ cm}^2$          | c) $3.90 \text{ m}^2$   | d) $5.89 \text{ m}^2$  |
| 4.  | a) $3 \text{ m}^2$              | b) $12 \text{ m}^2$            | c) $27 \text{ m}^2$     | d) $300 \text{ m}^2$   |
| 5.  | a) $1390 \text{ cm}^2$ (3 s.f.) | b) $3850 \text{ cm}^2$         |                         |                        |
|     | c) $154 \text{ cm}^2$           | d) $9.63 \text{ m}^2$ (3 s.f.) |                         |                        |
| 6.  | a) $12 \text{ cm}^2$ (2 s.f.)   | b) $110 \text{ cm}^2$ (2 s.f.) |                         |                        |
|     | c) $78 \text{ m}^2$ (2 s.f.)    | d) $310 \text{ m}^2$ (2 s.f.)  |                         |                        |
| 7.  | a) $1 \text{ cm}$               | b) $2 \text{ cm}$              | c) $10 \text{ m}$       | d) $30 \text{ cm}$     |
| 8.  | a) $11.3 \text{ cm}$            | b) $12.6 \text{ cm}$           | c) $13.8 \text{ cm}$    | d) $17.8 \text{ m}$    |
| 9.  | a) $29.0 \text{ cm}^2$          | b) $29.9 \text{ cm}^2$         | c) $28.5 \text{ cm}^2$  | d) $0.877 \text{ m}^2$ |
| 10. | a) $1.33 \text{ cm}^2$          | b) $1.54 \text{ cm}^2$         | c) $0.636 \text{ cm}^2$ | d) $31.8 \text{ cm}^2$ |

#### EX 18B

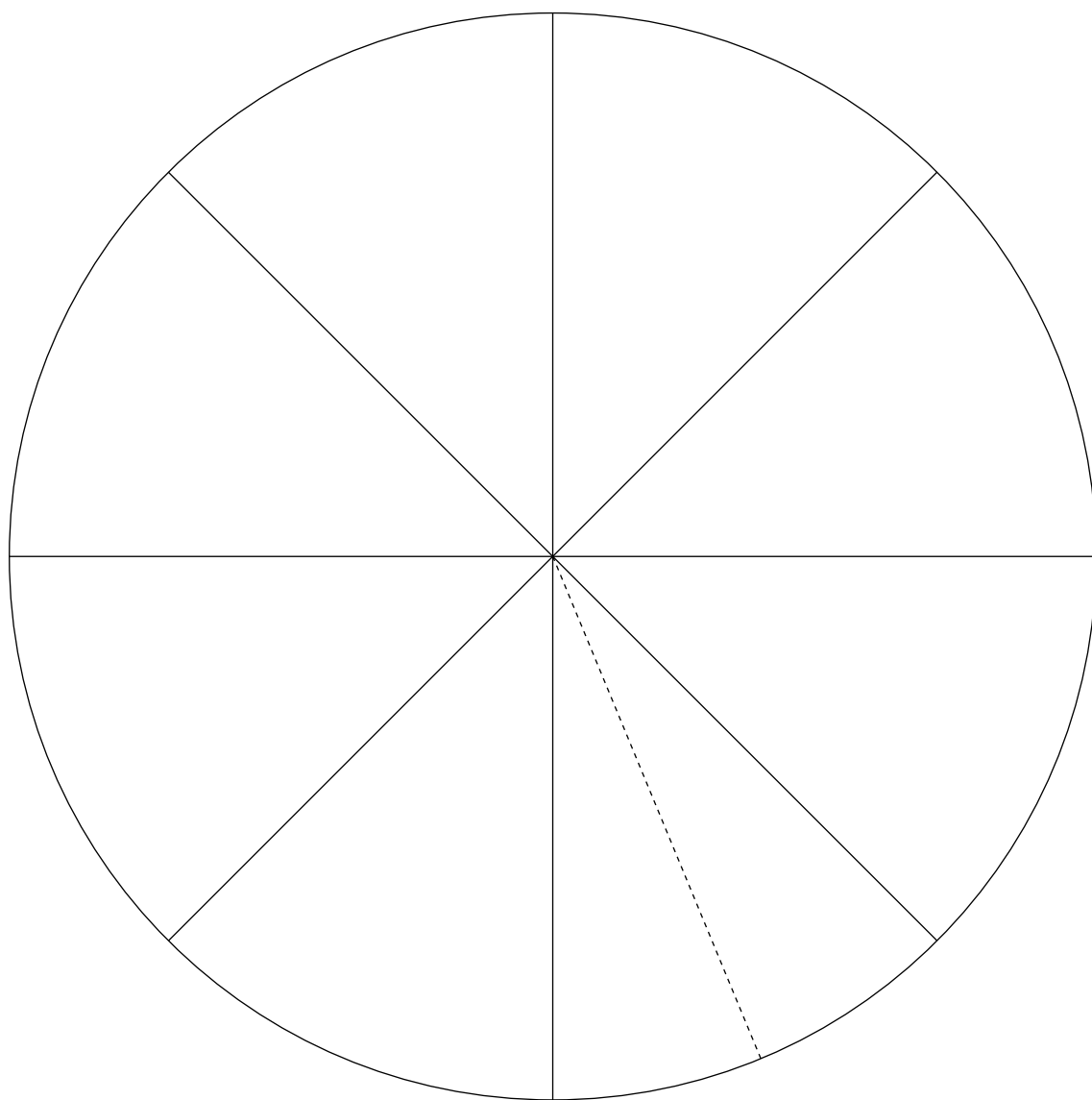
- $5.37 \text{ cm}^2$  (3 s.f.)
- $88.0 \text{ m}^2$  (3 s.f.)
- $1.64 \text{ m}^2$  (3 s.f.)
- $28.6 \text{ m}^2$  (3 s.f.)
- |                                |                                |
|--------------------------------|--------------------------------|
| a) $363 \text{ cm}^3$ (3 s.f.) | b) $286 \text{ cm}^2$ (3 s.f.) |
|--------------------------------|--------------------------------|

6. a)  $486 \text{ cm}^3$  (3 s.f.)                      b)  $303 \text{ cm}^2$  (3 s.f.)
7.  $1490 \text{ cm}^2$  (3 s.f.)
8.  $98 \text{ cm}$
9. a)  $52.8 \text{ cm}^3$  (3 s.f.)                      b) Rs 12 (nearest rupee)
10.  $120.8 \text{ cm}^2$  (1 d.p.)

### EX 18X

1.  $153 \text{ cm}^2$  (3 s.f.)
2.  $C^2 = (2\pi r)^2 = 4\pi^2 r^2$   
and  $4\pi A = 4\pi(\pi r^2) = 4\pi^2 r^2$   
 $C^2 = 4\pi A$
3.  $2 \text{ cm}$

## Large Circle



# Chapter 19 Tree Diagrams

A tree diagram is another way of systematically listing outcomes when there are two or more events, or repeated events, or the outcomes are not equally likely.

The events should be independent but at this stage it is not necessary to delve into the precise nature of statistical independence: it is enough to know that each event does not influence the outcomes of the others.

## LESSON PLANNING

### Objectives

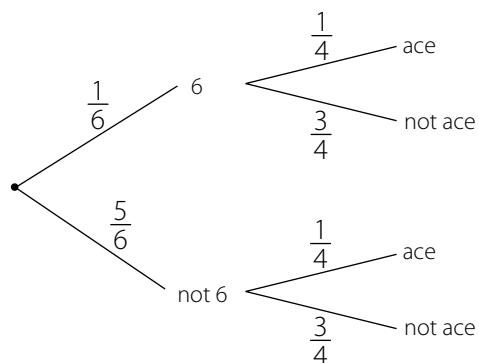
|                    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |
|--------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| General            | To use tree diagrams to solve probability problems                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |
| Specific           | <ol style="list-style-type: none"> <li>1. To draw a tree diagram, labelling the outcomes and assigning probabilities correctly</li> <li>2. To combine probabilities by multiplying the probabilities on branches, and adding probabilities if more than one sequence satisfies the intended outcome</li> </ol>                                                                                                                                                                                                                                                                        |
| Pacing             | 1 lesson, 1 homework                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |
| Links              | Fractions                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |
| <b>Method</b>      | <p>Explain that the trees have all fallen over! We work horizontally.</p> <p>Show how we make branches for each outcome and assign probabilities which must total 1.</p> <p>Then attach the second event's outcomes and assign those probabilities, etc., until all possibilities have been recorded.</p> <p>Use the text examples to show how to multiply across the branches, and add downwards.</p> <p>The exercise is straightforward.</p> <p>Set Ex 19A. Explain that if fractions are given, the answers should be in fractions; if decimals are given, answer in decimals.</p> |
| <b>Assignments</b> | Ex 19A #7 should be assigned for homework, and #8 for a more challenging option.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |
| <b>Vocabulary</b>  | independent, outcome, event, fair                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |

## ANSWERS

## Exercises

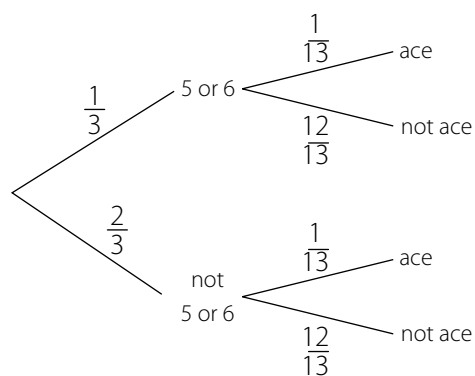
## EX 19A

1. a)



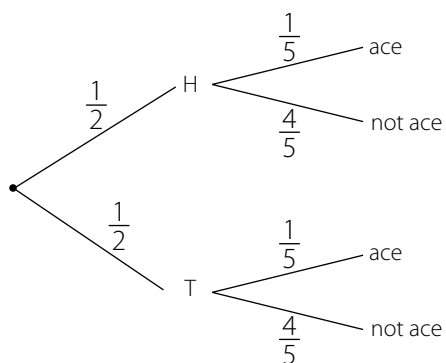
- b) (i)  $\frac{1}{8}$   
 (ii)  $\frac{5}{8}$   
 (iii)  $\frac{3}{8}$

2. a)



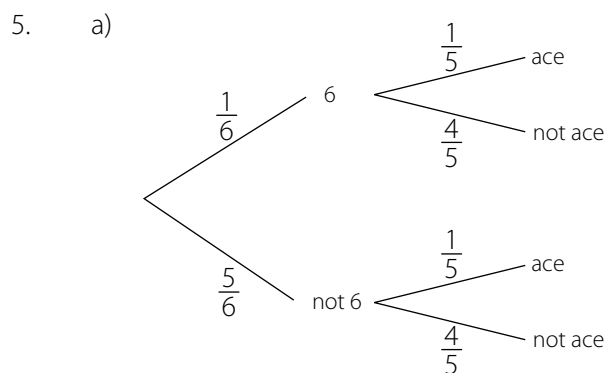
- b) (i)  $\frac{1}{39}$   
 (ii)  $\frac{14}{39}$

3.

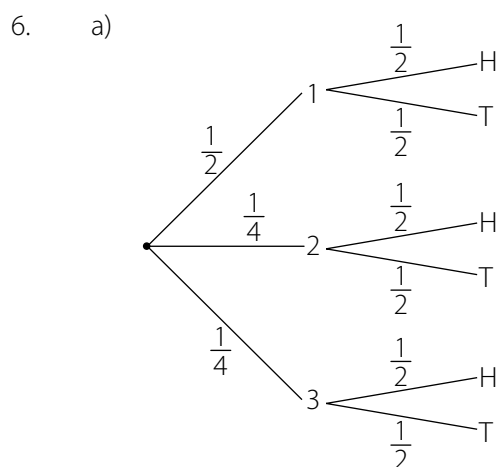


- a)  $\frac{1}{10}$   
 b)  $\frac{3}{5}$   
 c)  $\frac{2}{5}$

4. a)  $P(SSS) = \frac{1}{162}$   
 $P(SSF) = \frac{8}{162} = \frac{4}{81}$   
 $P(SFS) = \frac{5}{162}$   
 $P(SFF) = \frac{40}{162} = \frac{20}{81}$   
 $P(FSS) = \frac{2}{162} = \frac{1}{81}$   
 $P(FSF) = \frac{16}{162} = \frac{8}{81}$   
 $P(FFS) = \frac{10}{162} = \frac{5}{81}$   
 $P(FFF) = \frac{80}{162} = \frac{40}{81}$
- b) Total =  $\frac{162}{162} = 1$

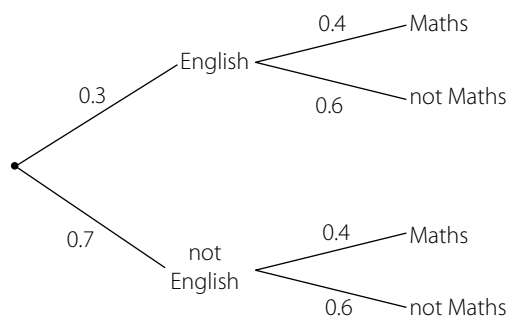


b)  $\frac{2}{3}$



- b) (i)  $\frac{1}{4}$   
(ii)  $\frac{1}{8}$

7. a)



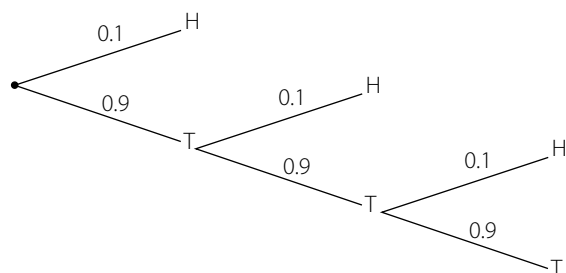
b) (i) 0.12

(ii) 0.46

(iii) 0.42

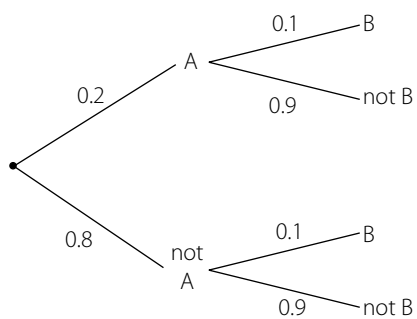
c) Checks

8. a)

b) (i)  $P(\text{win}) = 0.1 + 0.9 \times 0.1 + 0.9 \times 0.9 \times 0.1 = 0.271$ (ii)  $P(\text{lose}) = 0.9^3 = 0.729$ 

(iii) checks

9.



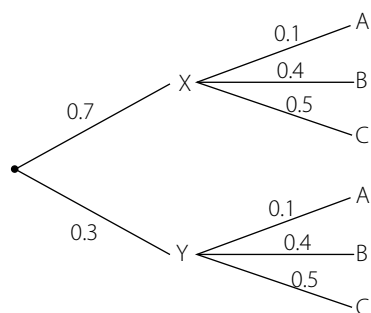
a) 0.18

b) 0.08

c) 0.02

d) 0.72

10. a)



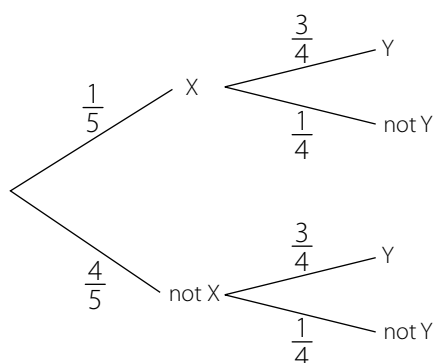
b) (i) 0.35

(ii) 0.12

(iii) 0.15

# EX 19X

1.



$$P(X \text{ and } Y) = \frac{3}{20}$$

$$\begin{aligned} 2. \quad P(\text{lose}) &= \left(\frac{5}{6}\right)^{10} = 0.1615\dots \\ P(\text{win}) &= 1 - 0.1615 = 0.8385\dots \\ &= 0.84 \text{ (2 d.p.)} \end{aligned}$$

$$\begin{aligned} 3. \quad P(\text{all wrong}) &= \left(\frac{99}{100}\right)^{30} = 0.7397 \\ P(\text{someone guessed it}) &= 1 - 0.7397 = 0.26 \text{ (2 s.f.)} \\ \text{result not valid} \end{aligned}$$

We have assumed that each number is equally likely to be chosen, but people choose numbers for all sorts of non-random reasons.

# Chapter 20 Simultaneous Equations: Graphical Method

In this chapter, after establishing a technique for drawing a line that is quicker than making a table of values, we consider the possibilities when there are two lines. If they cross, we use graphs to solve them simultaneously.

## LESSON PLANNING

### Objectives

|          |                                                                                                                                                                                                                                                                                                                                                                                                                                                     |
|----------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| General  | To use alternative forms of the straight line equation as aids to sketching; to solve simultaneous equations by drawing accurate graphs                                                                                                                                                                                                                                                                                                             |
| Specific | <ol style="list-style-type: none"> <li>1. To find the <math>x</math> and <math>y</math>-intercepts of a line from the intercept form of its equation; to sketch the graph using intercepts</li> <li>2. To determine whether two lines are overlapping (i.e. the same line), parallel or intersecting</li> <li>3. To find the simultaneous solution of two equations in the case where their graphs intersect, by drawing accurate graphs</li> </ol> |
| Pacing   | 3 lessons, 1 homework                                                                                                                                                                                                                                                                                                                                                                                                                               |

### Method

Remind students of  $y = mx + c$

Other forms of the straight line exist, e.g.  $ax + by = c$  intercept form.

In this form we can immediately state both intercepts and hence draw a sketch of its graph.

Use text examples and other similar examples to demonstrate the “cover up” method for finding intercepts.

Practise the technique by working through EX 20A.

“Consider the possibilities of two lines”:

- If parallel, their gradients will be the same.
- If equivalent, this may be shown by algebraic manipulation.
- If intersecting, graphs may be drawn to find the point of intersection. This is the point where both equations are true simultaneously.

Use EX 20B.

|                  |                             |
|------------------|-----------------------------|
| <b>Resources</b> | Accurate graph paper (2 mm) |
|------------------|-----------------------------|

|                    |                           |
|--------------------|---------------------------|
| <b>Assignments</b> | EX 20B #4, 8 for homework |
|--------------------|---------------------------|

|                   |                                     |
|-------------------|-------------------------------------|
| <b>Vocabulary</b> | gradient form, intercept form       |
|                   | parallel, overlapping, intersecting |
|                   | simultaneous equations              |

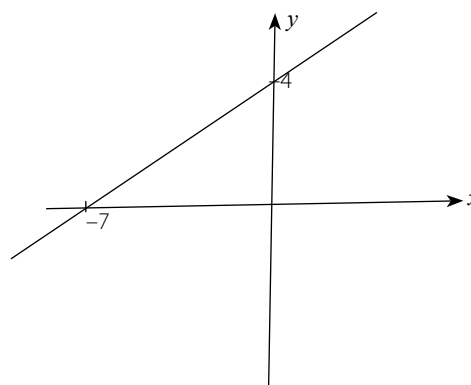
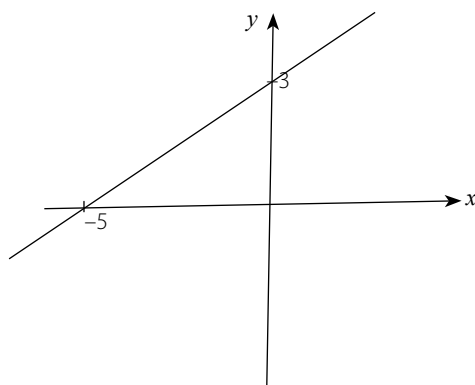
## ANSWERS

### Exercises

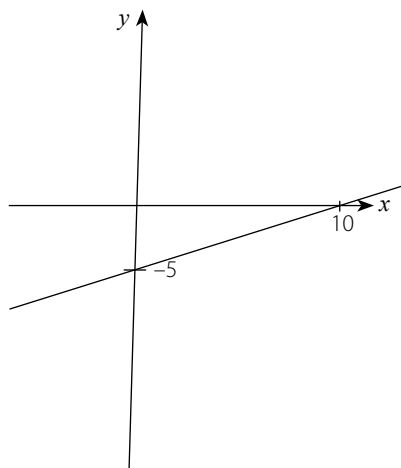
#### EX 20A

1. a)  $3x - 5y = -15$       b)  $-4x + 7y = 28$   
     or  $4x - 7y = -28$   
     c)  $-x + 2y = -10$       d)  $3x - y = 6$   
     or  $x - 2y = 10$

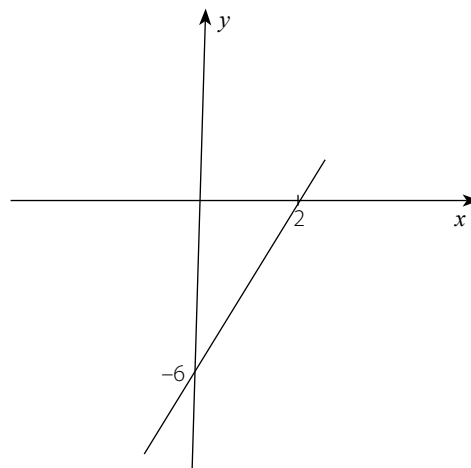
2. a)  $x = -5, y = 3$       b)  $x = -7, y = 4$



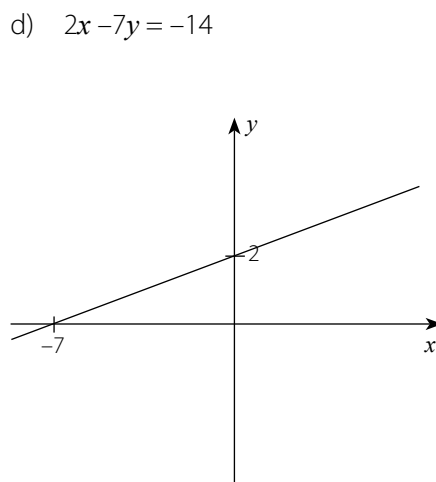
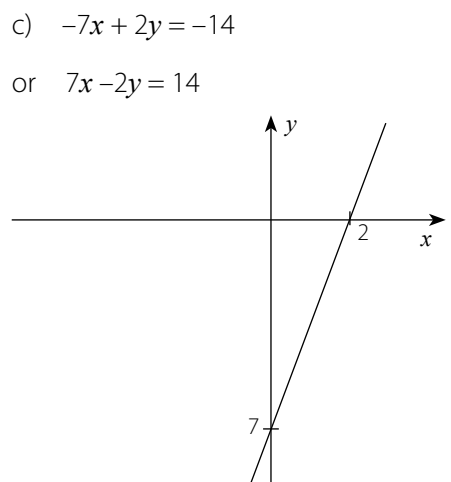
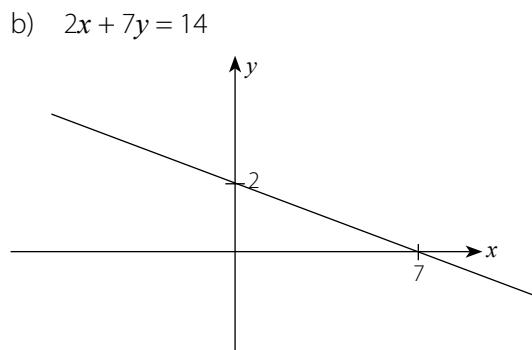
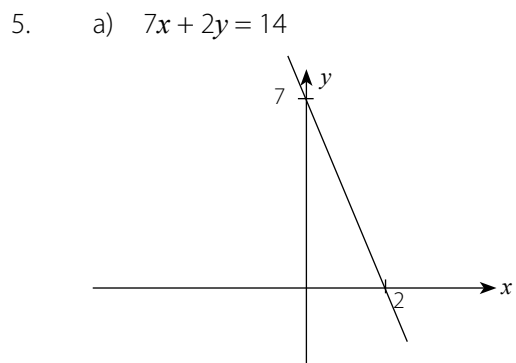
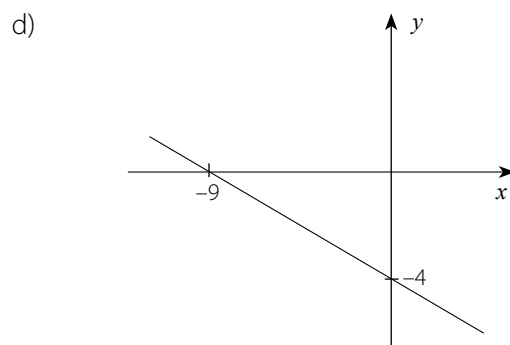
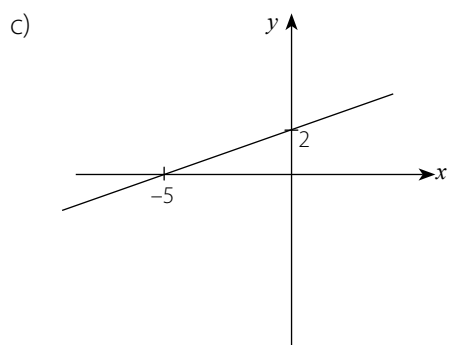
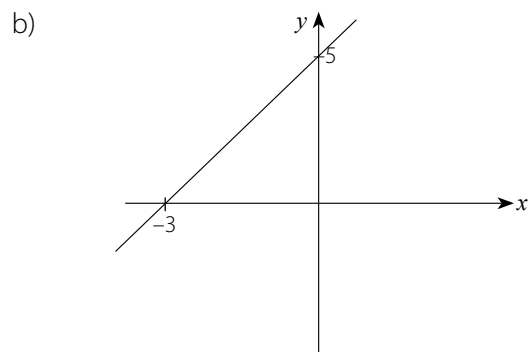
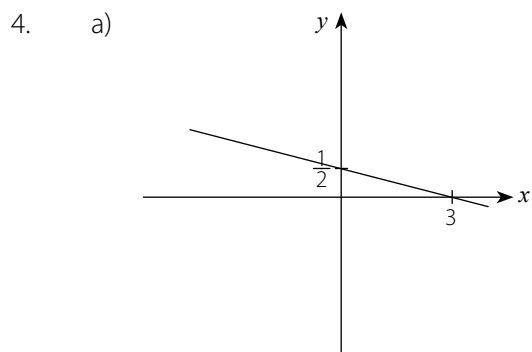
- c)  $x = 10, y = -5$



- d)  $x = 2, y = -6$

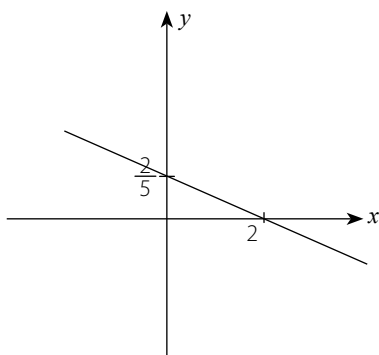


3. a) Q      b) S      c) R      d) P

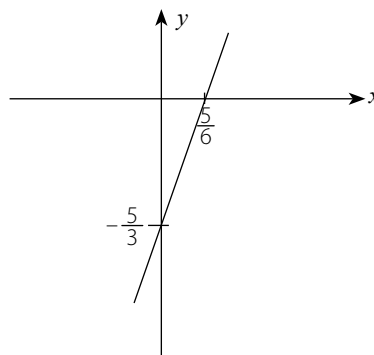


6. a)  $y = -\frac{1}{5}x + \frac{2}{5}$  gradient =  $-\frac{1}{5}$   
 b)  $y = 2x - \frac{5}{3}$  gradient = 2  
 c)  $y = 2x + 12$  gradient = 2  
 d)  $y = -\frac{3}{4}x - 2$  gradient =  $-\frac{3}{4}$

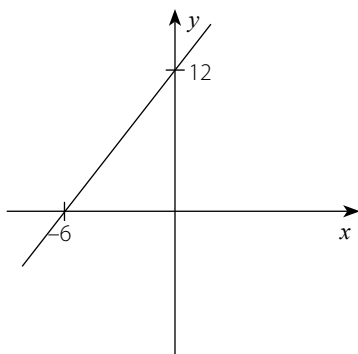
7. a)  $x = 2, y = \frac{2}{5}$



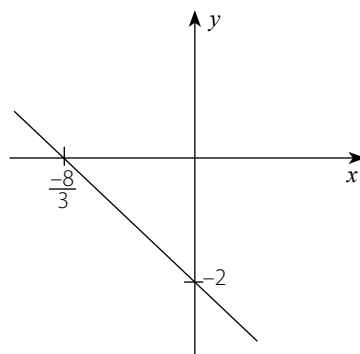
b)  $x = \frac{5}{6}, y = -\frac{5}{3}$



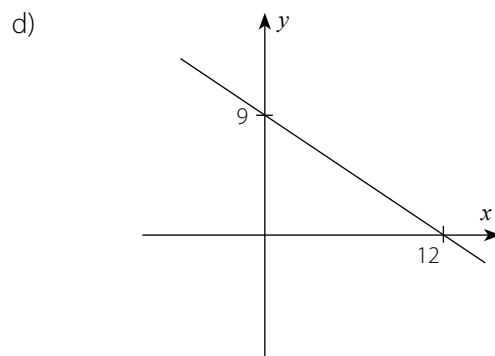
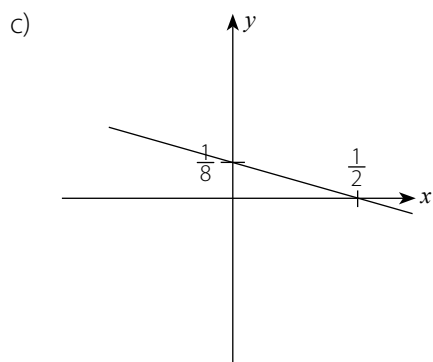
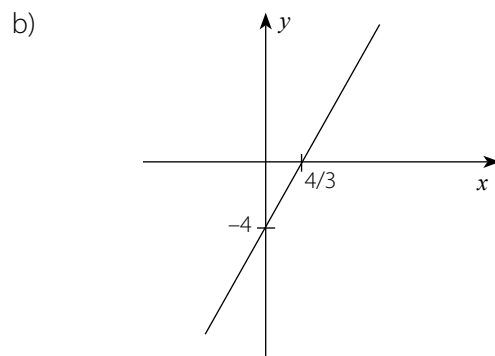
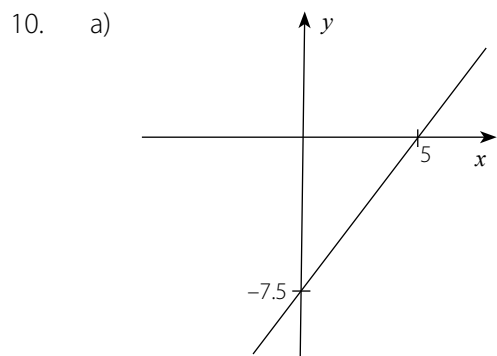
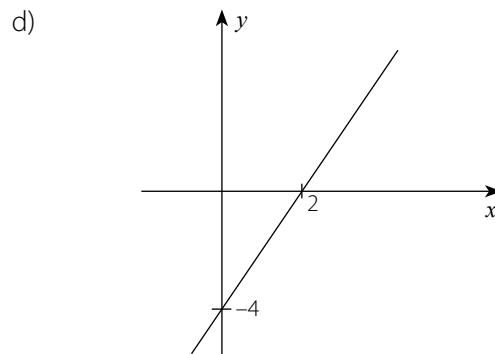
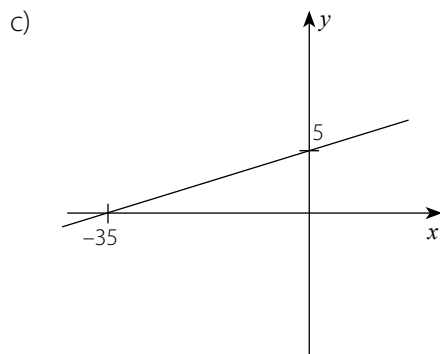
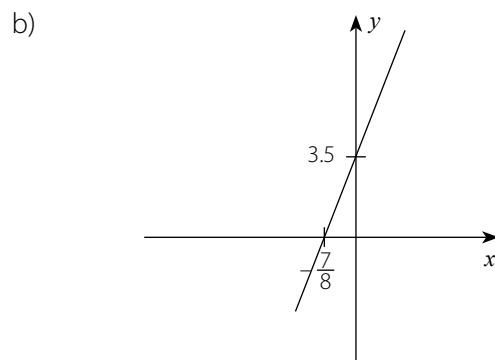
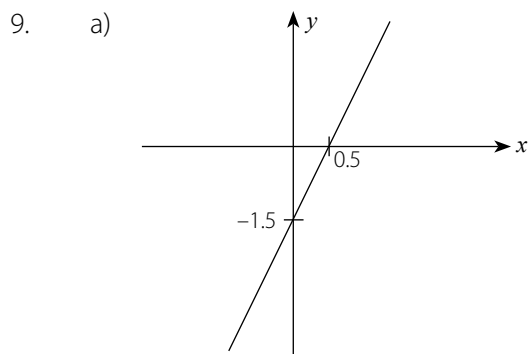
c)  $x = -6, y = 12$



d)  $x = -\frac{8}{3}, y = -2$



8. a) Intercept form:  $9x - 12y = 2$   
 b) Intercept form:  $-2x + 3y = 5$   
 Gradient form:  $y = \frac{2}{3}x + \frac{5}{3}$  (both easy)  
 c) Gradient form:  $y = -\frac{5}{2}x + \frac{1}{2}$   
 Intercept form:  $5x + 2y = 1$  (both easy)  
 d) Intercept form:  $2x - 5y = 20$   
 Gradient form:  $y = \frac{2}{5}x + 4$  (both easy)



## EX 20B

1. a)  $y = 2x - 5$

$gdt = 2$

$3y = 6x + 21$

$y = 2x + 7$

$gdt = 2, \therefore \text{parallel}$

b)  $-x + y = 2$

$y = x + 2$

$gdt = 1$

$1 + y = x$

$y = x - 1$

$gdt = 1, \therefore \text{parallel}$

c)  $x - 2y = 3$

$-2y = -x + 3$

$y = \frac{1}{2}x - \frac{3}{2}$

$gdt = \frac{1}{2}$

$2(y - 1) = x$

$2y - 2 = x$

$2y = x + 2$

$y = \frac{1}{2}x + 1$

$gdt = \frac{1}{2}, \therefore \text{parallel}$

d)  $5x + 5 = y$

$y = 5x + 5$

$gdt = 5$

$-5x + y = 1$

$y = 5x + 1$

$gdt = 5, \therefore \text{parallel}$

2. a)  $y = x + 3$  and

$9 - 3y + 3x = 0$

$-3y = -3x - 9$

$y = x + 3$

$\therefore \text{same line}$

b)  $x = 2(y + 3)$  and

$2y = x - 6$

$\frac{x}{2} = y + 3$

$y = \frac{x}{2} - 3$

$y = \frac{x}{2} - 3,$

$\therefore \text{same line}$

OR  $x = 2y + 6$

$x - 6 = 2y$

$2y = x - 6,$

$\therefore \text{same line}$

c)  $x = 1 - y$  and

$$x + y = 1$$

$$y = 1 - x$$

$$y = 1 - x$$

$\therefore$  same line

d)  $y = \frac{1}{3}x + 2$  and

$$3y = x + 6$$

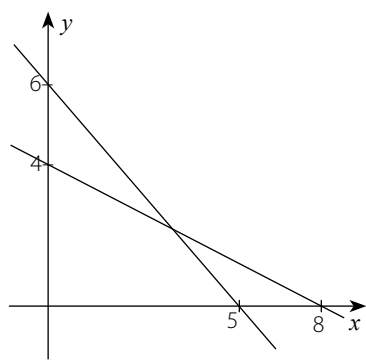
$$-x + 3y = 6$$

$$3y = x + 6$$

$\therefore$  same line

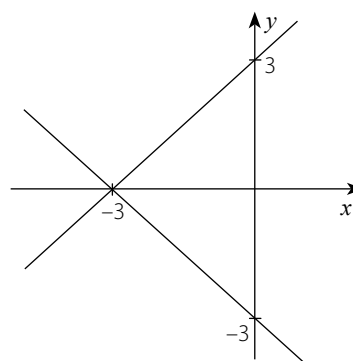
3.

a)



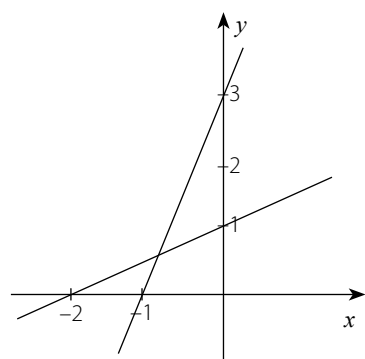
drawn accurately (2.9, 2.6)

b)



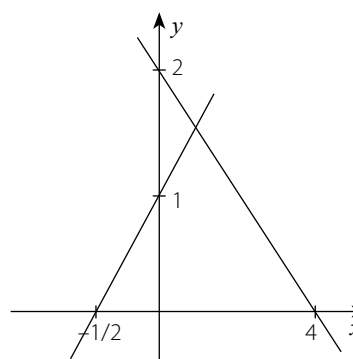
(-3, 0)

c)



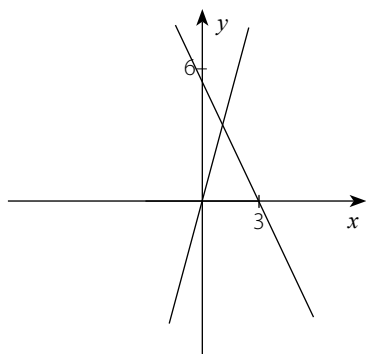
(-0.8, 0.6)

d)



(0.4, 1.8)

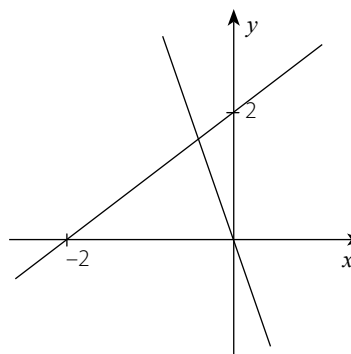
4. a)



drawn accurately

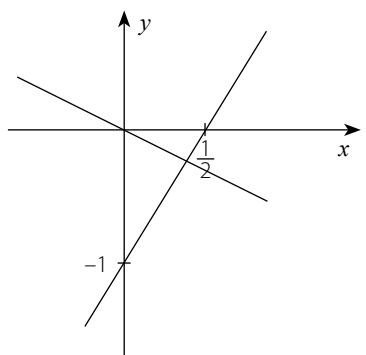
(1.2, 3.6)

b)



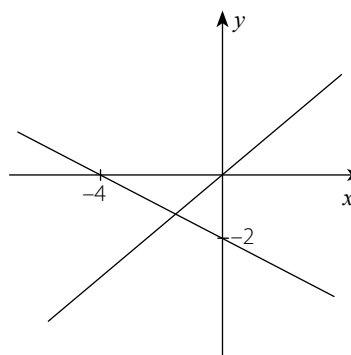
(-0.7, 1.3)

c)



(0.4, -0.2)

d)



(-1.3, -1.3)

5.

a)

$$y = \frac{1}{3}x + 1$$

$$\text{and } y = \frac{1}{3}x + 1$$

same line

infinite solutions

b)

$$y = \frac{1}{4}x + 5$$

$$\text{and } y = -4x + 8$$

not parallel or same

one solution

c)

$$y = 1.5x - 7$$

$$\text{and } y = 1.5x - 7$$

same line

infinite solutions

d)

$$y = \frac{1}{3}x - 2$$

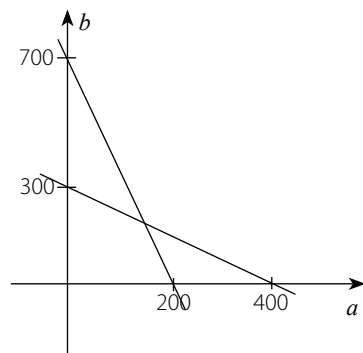
$$\text{and } y = -\frac{1}{3}x - 2$$

not parallel or same

one solution

6. a)  $3a + 4b = 1200$

$7a + 2b = 1400$



drawn accurately

(145, 191)

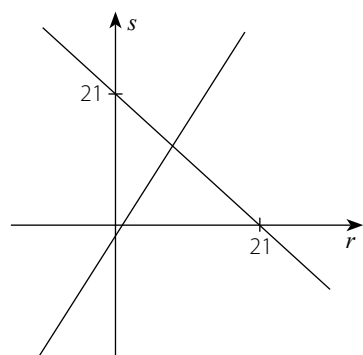
Each apple weighs 145 g.

Each banana weighs 191 g.

[assuming apples have equal weight; also bananas]

7. a)  $r = 2s$

b)  $r + s = 21$



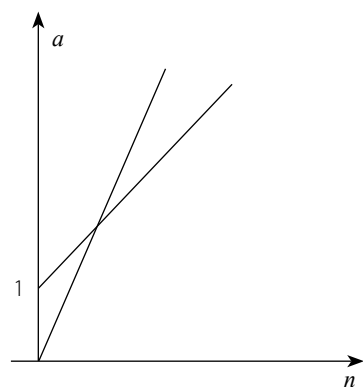
drawn accurately

(14, 7)

Rita is 14.

Salman is 7.

8. a)  $a = 3n, a - n = 1$



drawn accurately

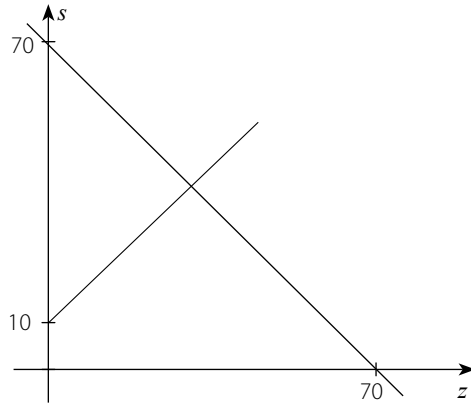
(0.5, 1.5)

Nimra eats  $\frac{1}{2}$  chapati.

Aisha eats  $1\frac{1}{2}$  chapatis.

9.  $s = z + 10$

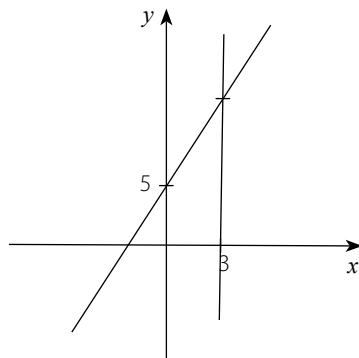
$s + z = 70$



drawn accurately  
(30, 40)

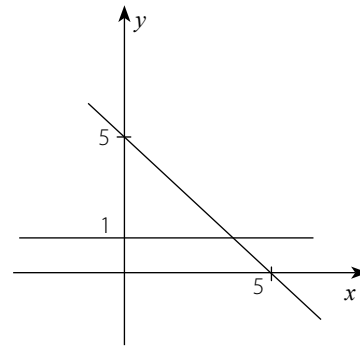
Zainab has Rs 30  
Sonia has Rs 40

10. a)



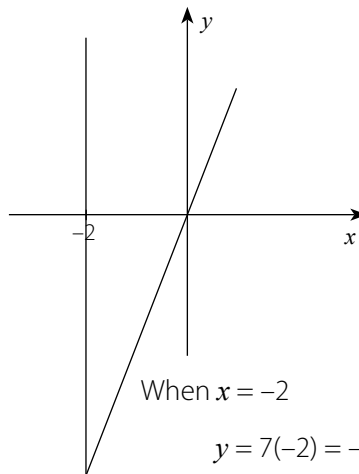
When  $x = 3$   
 $y = 2(3) + 5$   
 $= 11$   
 $(3, 11)$

b)



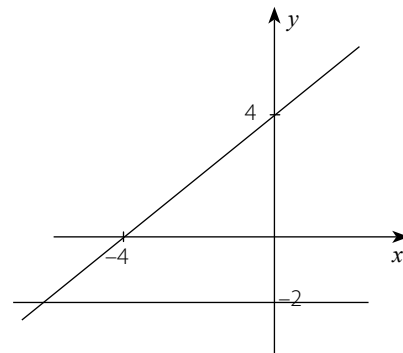
When  $y = 1$   
 $x + 1 = 5$   
 $x = 4$   
 $(4, 1)$

c)



When  $x = -2$   
 $y = 7(-2) = -14$   
 $(-2, -14)$

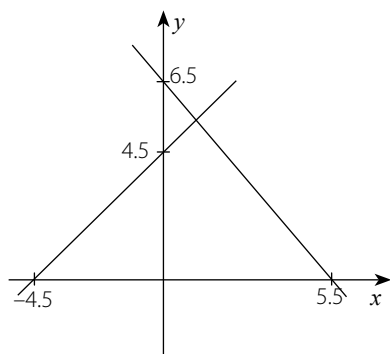
d)



When  $y = -2$   
 $x + 4 = -2$   
 $x = -6$   
 $(-6, -2)$

## EX 20X

1.

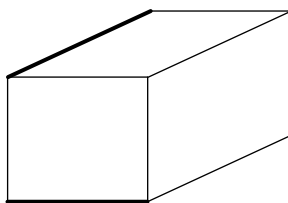


drawn accurately

(0.9, 5.4)

2. parallel, overlapping, skew (i.e. neither parallel nor intersecting) [Hint: Look at the lines formed between the ceiling, floor, walls of a room.]

skew lines



3.  $b + 3a = 44$

$b - a = 2$  or  $a - b = 2$  (We don't know who is older.)

(10.5, 12.5)

(11.5, 9.5)

Either Ali is  $10\frac{1}{2}$ , Bashir is  $12\frac{1}{2}$ ,

or Ali is  $11\frac{1}{2}$ , Bashir is  $9\frac{1}{2}$ .

# Chapter 21 Simultaneous Equations: Algebraic Methods

This chapter is pure algebra. The two algebraic methods of solving simultaneous equations are explained. There is one exercise on each method and a miscellaneous exercise where students have to choose the most appropriate method to apply.

## LESSON PLANNING

### Objectives

|          |                                                                                                                                                                                                                                                                                                                                                                                               |
|----------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| General  | To solve simultaneous equations by substitution and elimination methods                                                                                                                                                                                                                                                                                                                       |
| Specific | <ol style="list-style-type: none"> <li>To solve simultaneous equations by substitution method, with labelling to communicate the process, and final check</li> <li>To solve simultaneous equations by elimination method, with labelling to communicate the process, and final check</li> <li>To identify the most suitable method of solution when solving simultaneous equations</li> </ol> |
| Pacing   | 4 lessons, 2 homeworks                                                                                                                                                                                                                                                                                                                                                                        |

### Method

Start with the substitution method, but first elicit from students the drawbacks of the graphical method used in the last chapter, i.e. slow method, accuracy depending on the graph paper.

Substitution works well for equations in gradient form, or at least one of them.

At the intersecting point the  $x$  and  $y$  values are the same in both equations simultaneously. That is why we may substitute. "This  $x$  is the same as that  $x$ ", etc. Follow the text examples.

Insist on labelling the equations, and stating each step at the side.

Explicitly teach that the equations are used alternately:

Either                      Sub ① in ② (leads to one value)

                                    Sub in ① (obtain other value)

                                    Check in ②

Or                              Sub ② in ①

                                    Sub in ②

                                    Check in ①

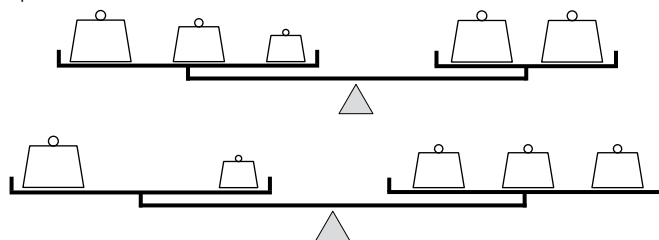
Train students to look out for a simple  $x$  or  $y$  in one equation and make it the subject for substitution in the other equations. If not available, a simple multiple will do, e.g.

$$2x = 1 - 4y$$

$$x = \frac{1}{2} - 2y \text{ may be substituted.}$$

Set EX 21A.

For elimination, the concept of an equation being a balance comes to the fore. Multiplying both sides by a scale factor preserves the equality. Adding two equations together may seem strange, but we have two sets of balancing quantities:



Placing all the items on the left on one pan and all the items on the right on the other pan will preserve the balance. Similarly, subtracting the same quantity from each side preserves the equality.

Recipes for solution involving rules for signs should be avoided as they do not enhance understanding of the concept.

The elimination method would be better named the elimination and substitution method because the second value is obtained by substitution.

Follow the text examples. Note the labelling, and the alternate use of equations again:

First, use elimination.

Then Either Sub in ①, check in ②

Or Sub in ② check in ①

Set EX 21B.

When choice of method is allowed, substitution is preferred if one equation is simple. If not, then use elimination. Throughout this chapter students should not become so attentive to the solving process that they lose sight of the goal. Each equation is true for all the points on its line. For intersecting lines there is only one point that is simultaneously true for both equations.

Set EX 21M.

|                    |                                                          |
|--------------------|----------------------------------------------------------|
| <b>Assignments</b> | Suitable for homework: any selection from the exercises. |
| <b>Vocabulary</b>  | substitution, elimination, simultaneously true           |

## ANSWERS

### Exercises

#### EX 21A

1.  $x = 1, y = -1$
2.  $x = 1, y = -1$
3.  $x = -4, y = 2$
4.  $x = 11, y = 2$
5.  $x = 2, y = 1$
6.  $x = 2.5, y = 1.5$
7.  $x = -3, y = 7$
8.  $x = 3.5, y = 11$
9.  $x = 0.25, y = 0.6$
10.  $x = 0.0625$  or  $\frac{1}{16}, y = 0.625$  or  $\frac{5}{8}$

#### EX 21B

1.  $x = 2, y = 4$
2.  $x = -1, y = 6$
3.  $x = 3, y = -3$
4.  $x = -4, y = 1$
5.  $x = 5, y = -3$
6.  $x = 4, y = 4$
7.  $x = 0, y = 1$
8.  $x = \frac{1}{2}, y = 1$
9.  $x = -\frac{1}{3}, y = \frac{2}{3}$
10.  $x = 1.8, y = 2.8$

#### EX 21M

1.  $x = 12, y = 23$
2.  $x = 1, y = 9$
3.  $x = -2, y = 6$
4.  $x = 4, y = 0$

5.  $x = 0, y = -2$

6.  $x = \frac{1}{2}, y = \frac{1}{2}$

7.  $x = \frac{3}{2}, y = \frac{1}{2}$

8.  $x = 1.6, y = -0.6$

9.  $x = 0.875, y = 0.775$

10.  $x = 1.03, y = 0.5$

**EX 21X**

1. right-angled triangle [graphical method]

2.  $x = 1.6, y = 2.6$  [Scale first equation by  $xy$ .]

3.  $x = 0, y = 0$  and  $x = 1, y = 1$  [graphical method]

# Chapter 22 Pythagoras' Theorem

This is a short chapter but an important one. It is good for students to know that a lot of mathematics is very ancient. It also shows that algebra is not disconnected from geometry.

## LESSON PLANNING

### Objectives

|                    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |
|--------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| General            | To have an intuitive understanding of the truth of Pythagoras' theorem; to use the theorem in calculations                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
| Specific           | <ol style="list-style-type: none"> <li>1. To observe and understand that there are proofs of Pythagoras' theorem</li> <li>2. To memorize and use the three simplest Pythagorean triples, and multiples of them, to solve problems</li> <li>3. To use Pythagoras' theorem to solve problems involving right-angled triangles (to specified accuracy)</li> </ol>                                                                                                                                                                                                                                                                    |
| Pacing             | 2 lessons, 1 homework                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |
| Links              | Scale factors, enlargements, using square roots on a calculator                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |
| <b>Method</b>      | <p>For once, state the formula first, then attempt to justify it. Talk the students through the proof in the text. (They will not all follow it completely, but they should see it done.)</p> <p>Another demo is the classical section with moveable pieces. An enlarged version of this can be used as a class display, or copies could be provided for all the students as a practical activity.</p> <p>The easy whole number solutions to Pythagoras' theorem (Pythagorean triples) must be memorized.</p> <p>Encourage efficient use of the calculator in entering items directly, e.g. <math>\sqrt{3.2^2 + 1.6^2}</math></p> |
| <b>Resources</b>   | Classic section (photocopiable, after the answers in this chapter of the Guide)                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |
| <b>Assignments</b> | Either the practical work suggested above, or EX 22A #6, 8, 10                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |
| <b>Vocabulary</b>  | Pythagoras, theorem, hypotenuse, triple                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |

# ANSWERS

## Exercises

### EX 22A

- |     |                                  |                     |            |            |
|-----|----------------------------------|---------------------|------------|------------|
| 1.  | a) 12.2 cm                       | b) 15.3 cm          | c) 12.2 cm | d) 26.0 cm |
| 2.  | a) 17.2 cm                       | b) 21.3 cm          | c) 5.17 cm | d) 1.73 cm |
| 3.  | a) 5 cm                          | b) 24 cm            | c) 5 cm    | d) 6 cm    |
| 4.  | a) 7 cm                          | b) 12 cm            | c) 10 cm   | d) 14 cm   |
| 5.  | 20.25 m                          |                     |            |            |
| 6.  | a) 12 cm <sup>2</sup>            | b) 16.92 m          |            |            |
| 7.  | a) 96.5 cm <sup>2</sup> (3 s.f.) | b) 12.6 cm (3 s.f.) |            |            |
| 8.  | a) 2.2                           | b) 3.2              | c) 2.8     | d) 7.6     |
| 9.  | a) 9                             | b) 9.8 (2 s.f.)     |            |            |
|     | c) 10                            | d) 8.5 (2 s.f.)     |            |            |
| 10. | a) 52.1 cm (3 s.f.)              | b) 54 cm            |            |            |

### EX 22X

- |    |                  |                                              |                |               |
|----|------------------|----------------------------------------------|----------------|---------------|
| 1. | 49.5 cm (3 s.f.) | [Hint: Draw the net of the cube.]            |                |               |
| 2. | 12.7 cm (3 s.f.) | [Hint: Find the diagonal of the base first.] |                |               |
| 3. | a) $\sqrt{13}$   | b) $\sqrt{26}$                               | c) $\sqrt{21}$ | d) $\sqrt{8}$ |

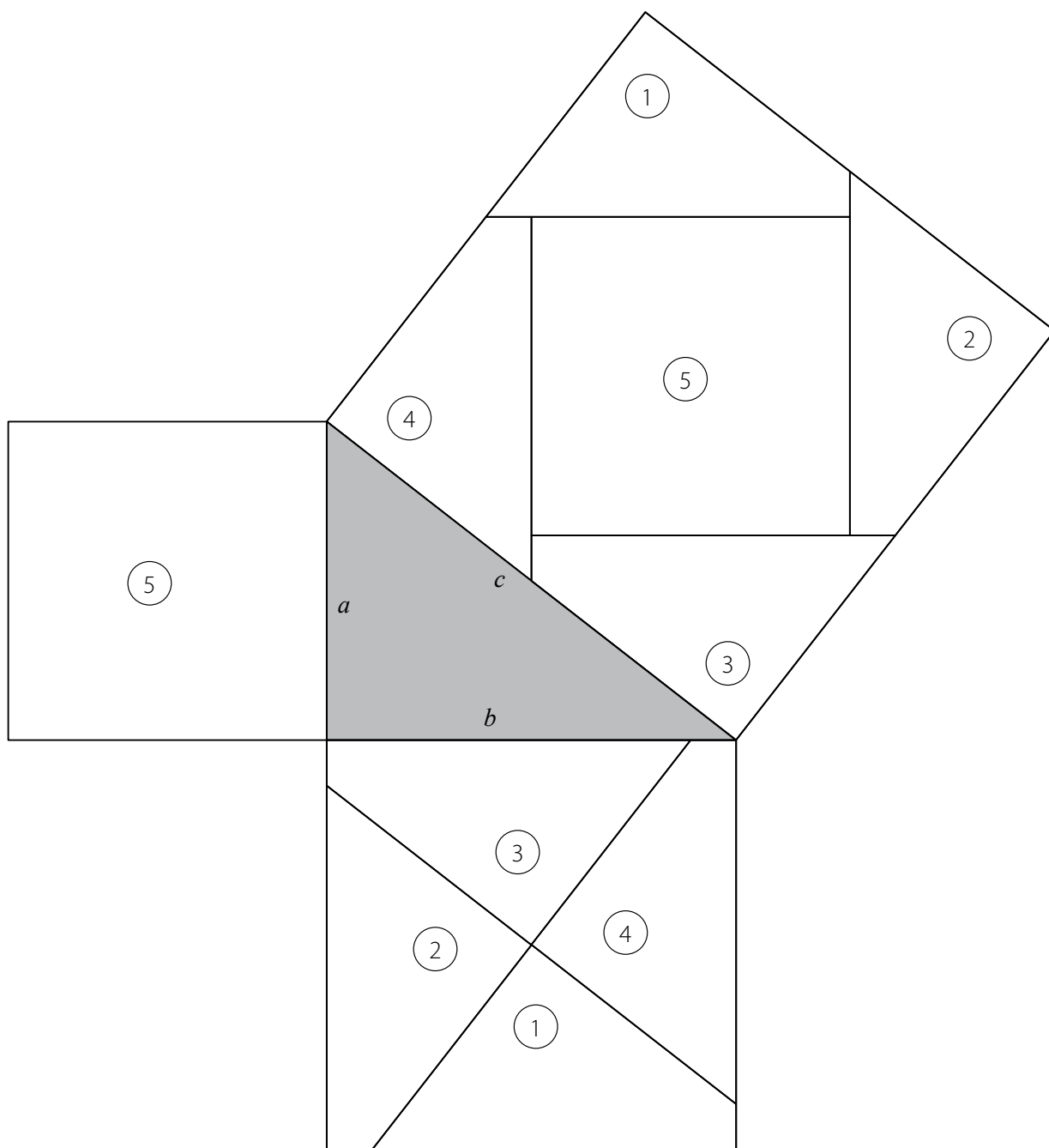
## Pythagoras' Theorem

$$\text{Area } \textcircled{1} + \textcircled{2} + \textcircled{3} + \textcircled{4} = b^2$$

$$\text{Area } \textcircled{5} = a^2$$

$$\text{Area } \textcircled{1} + \textcircled{2} + \textcircled{3} + \textcircled{4} + \textcircled{5} = c^2$$

$$\therefore a^2 + b^2 = c^2$$



# Chapter 23 Similar Figures

Similar figures are used as a way of consolidating work on enlargements, scale-factors, ratios, and their interconnectedness. The emphasis here is on calculations, not on rigorous definitions of similarity.

## LESSON PLANNING

### Objectives

|          |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |
|----------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| General  | To recognize similar figures and use ratios and scale factors to calculate lengths                                                                                                                                                                                                                                                                                                                                                                                                                    |
| Specific | <ol style="list-style-type: none"> <li>1. To find pairs of corresponding sides in similar figures</li> <li>2. To find the scale factor (SF) of enlargement of similar figures by using ratios of corresponding sides</li> <li>3. To calculate lengths using the scale factor of enlargement</li> <li>4. To solve area problems of similar figures by using the square of the scale factor</li> <li>5. To solve volume problems of similar 3D figures by using the cube of the scale factor</li> </ol> |
| Pacing   | 3 lessons, 1 homework                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |
| Links    | Drawing to scale, inverse operations, units                                                                                                                                                                                                                                                                                                                                                                                                                                                           |

### Method

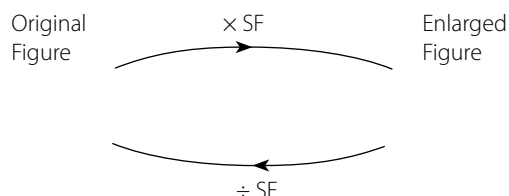
Remind students of the features of enlargements.

- All corresponding lengths are increased by the same scale factor.
- All corresponding angles are equal.

Explain “corresponding” in this context.

Then follow the text for obtaining a SF from a ratio of corresponding sides.

The text example uses SF as a multiplying factor, but to obtain a length on the original diagram the inverse would have to be used, i.e.

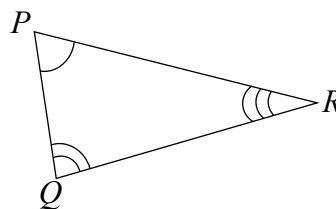
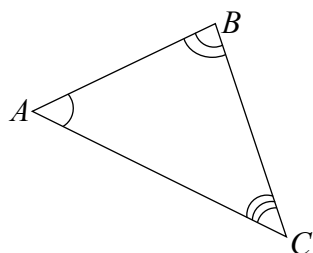


Explain the use of “figure” in this context.

Set EX 23A.

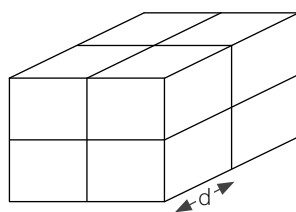
A useful practical hint to identify corresponding pairs, especially in triangles that have been reflected and/or rotated, is to mark corresponding angles, e.g. if  $\triangle ABC$  is similar to  $\triangle PQR$ ,

then  $\angle A = \angle P$ ,  $\angle B = \angle Q$ , and  $\angle C = \angle R$   
                     (single arc)                      (double arc)                      (triple arc)



It is easier to “navigate”. Then corresponding sides are  $AB$  and  $PQ$  (opposite triple arcs) etc.

The square and cube of the length SF for similar areas and volumes can be demonstrated simply with squares (as in the text) and cubes (with models).



small cube : large cube

$d : 2d$  (length ratio)

$1 : 2$

But volume ratio is 1:8

EX 23B explores this theme. After #6, which has interesting links with biology, the questions become difficult. You might wish to stop there.

|                    |                                                             |
|--------------------|-------------------------------------------------------------|
| <b>Resources</b>   | 8 small cubes to demonstrate (see above)                    |
| <b>Assignments</b> | EX 23A #8, 9 for homework                                   |
| <b>Vocabulary</b>  | corresponding, figures (used rather differently from usual) |

## ANSWERS

### Exercises

#### EX 23A

- $x = 12$  cm,  $y = 5.8$  cm
- $x = 8.33$  cm (3 s.f.),  $y = 3.67$  cm (3 s.f.)
- $x = 20$  cm,  $y = 3.6$  cm
- $a = 16.8$  cm,  $b = 20.5$  cm,  $c = 19.2$  cm

5.  $x = 25$  cm,  $y = 7$  cm
6.  $\angle PQR = \angle PQ'R'$  and  $\angle PRQ = \angle PR'Q'$  (or equivalent)  
 $QR$  is parallel to  $Q'R'$  (corr  $\angle$ s)
7.  $a = 7$  cm,  $b = c = 9.33$  cm (3 s.f.)
8. a) 2.5  
 b)  $x = 2.4$  cm,  $y = 12.5$  cm,  $z = 9$  cm  
 c)  $\angle AD'C'$   
 d)  $\angle BCD$
9. a)  $A = \angle R$ ,  $\angle B = \angle Q$ ,  $\angle C = \angle P$   
 b)  $BC$  and  $PQ$ , SF = 1.25  
 c)  $x = 6.25$  cm,  $y = 6$  cm
10. a)  $\angle A = \angle D$  (alt  $\angle$ s),  $\angle B = \angle C$  (alt  $\angle$ s),  $\angle COD = \angle AOB$  (vert opp  $\angle$ s)  
 b)  $OA$  and  $OD$ , SF = 1.5  
 c)  $x = 6$  cm,  $y = 5$  cm  
 d) right-angled (3, 4, 5 Pythag triple)

**EX 23B**

1. a)  $x = 15$                                       b) 2:3  
     c) 4:9                                              d)  $200 \text{ cm}^2, 450 \text{ cm}^2$ ;  $200:450 = 4:9$       checks
2. a) 2                                                  b) 6 cm                                      c)  $12 \text{ cm}^2, 48 \text{ cm}^2$                       d)  $4, 4 = 2^2$
3. a) 1.2                                              b) 1.44                                      c)  $706.8 \text{ cm}^2, 1017.9 \text{ cm}^2$  (1 d.p.)  
     d) 1.44 (3 s.f.), same as (b)
4. a) true                                              b) true                                      c) false                                      d) false
5. a) 37 cm                                              b) 36 cm  
     c)  $1.38 \text{ m}^2$  (3 s.f.)                              d) 752 g (3 s.f.)
6. a) 20 cm                                              b) 100                                      c) 1000  
     b) 1000 times mass, only 100 times oxygen, not viable
7. a) 35 m by 30 m                                      b)  $210 \text{ cm}^2$                                       c)  $186 \text{ m}^2$   
     d)  $65 \text{ cm}^3$  (or 0.065 l)

8. a)  $384 \text{ cm}^2, 512 \text{ cm}^3$   
 b)  $486 \text{ cm}^2, 729 \text{ cm}^3$   
 c) 1.125, 1.266 (4 s.f.), 1.424 (4 s.f.)  
 d)  $1.125^2 = 1.266, 1.125^3 = 1.424$  Checks
9. a) 12.6 cm                      b) 12 cm  
 c)  $260 \text{ cm}^2$  (3 s.f.)              d)  $1320 \text{ cm}^2$  (3 s.f.)
10. a) 8:27                              b) 2:3, 4:9  
 c) 1.5, 2.25                        d)  $216 \text{ cm}^3$

**EX 23X**

1.  $x = 6 \text{ cm}$
2.  $x = \sqrt{6} = 2.45$  (3 s.f.)
3. length ratio = 3:7  
 width ratio 2:3  
 not equal,  $\therefore$  not similar

# Chapter 24 Revision Exercises

## ANSWERS

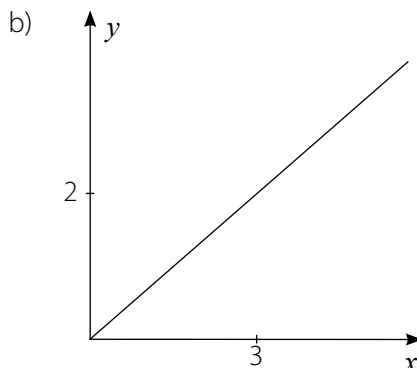
### Exercises

#### EX 24A

1. a)  $3.1 \text{ cm}^2$                       b)  $49.6 \text{ cm}^2$                       c)  $310 \text{ m}^2$                       d)  $1240 \text{ m}^2$
2. a)  $x = \frac{1}{3}$                               b)  $x = 1.5$
3. a) 2.75                              b)  $\frac{11}{15}$                               c) 19:11                              d) Rs 1600
4. a) 73% increase                      b) 34% decrease  
c) 19% decrease                      d) 77% increase
5. a)  $18^\circ$                               b)  $5^\circ$                               c)  $165^\circ$                               d)  $170^\circ$
6. a)  $x = -2.6$                               b)  $x = \frac{1}{9} = 0.\dot{1}$   
c)  $x = 7$                               d)  $x = -\frac{8}{3} = -2.\dot{6}$
7. a) 18.8 cm                              b) 17.2 cm
8. a) true                              b) false                              c) false                              d) true
9. a)  $2a + 3b = 740$ ;  $a + 2b = 440$                               b)  $a = 160$ ,  $b = 140$ , Rs 1040
10.  $0.25 \text{ cm}^2$

#### EX 24B

1. 26.56%
2. a) 3:2  
c)  $\text{gdt} = \frac{2}{3}$   
 $y = \frac{2}{3}x$   
d) 24 l



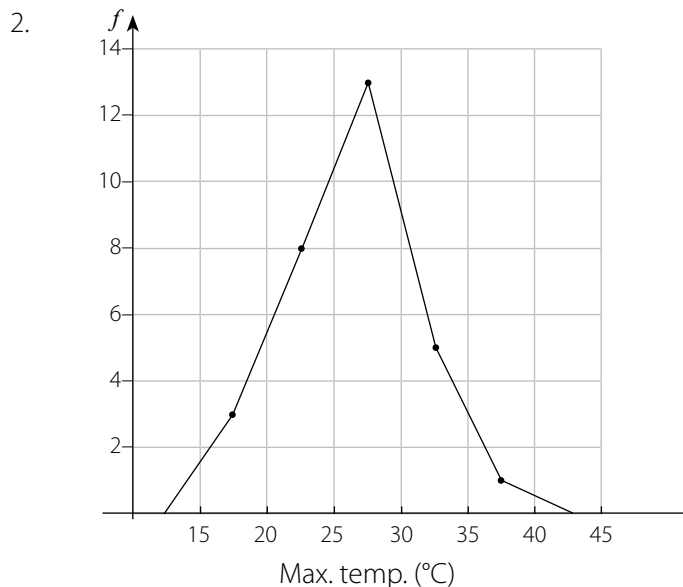
3. a)  $x = -4, y = -3$       b)  $x = 1, y = -1$
4. a)  $w = \frac{p}{2} - l$  OR  $w = \frac{p - 2l}{2}$   
 b) 2 cm      c) 4.6 cm      d)  $w = -1 < 0$ , impossible.
5. 50 km/h      b) 13.9 m/s (3 s.f.)
6. a) False; corresponding angles are equal.      b) true      c) true  
 d) False; the mean is 8.0.
7. a)  $3(2x + 4y + z)$       b)  $7(1 - 2y + 3z)$   
 c) not possible      d)  $5(5 - 6x^2)$
8. two parallel vertical planes on either side of  $P$  and 20 cm away from it
9. a)  $r = s + 2000$ ,  $r + s = 98000$   
 b)  $r = 48000$ ,  $s = 50000$ , Rashid spent Rs 48000, Sonia spent Rs 50000.
10. a)  $x = 3, y = -1$       b)  $x = 0.1, y = -0.2$

**EX 24C**

1. a)

|   | A  | K  | Q  | J  | T  |
|---|----|----|----|----|----|
| 1 | 1A | 1K | 1Q | 1J | 1T |
| 2 | 2A | 2K | 2Q | 2J | 2T |
| 3 | 3A | 3K | 3Q | 3J | 3T |
| 4 | 4A | 4K | 4Q | 4J | 4T |
| 5 | 5A | 5K | 5Q | 5J | 5T |
| 6 | 6A | 6K | 6Q | 6J | 6T |

- b)  $\frac{2}{15}$   
 c)  $\frac{1}{10}$   
 d)  $\frac{1}{6}$

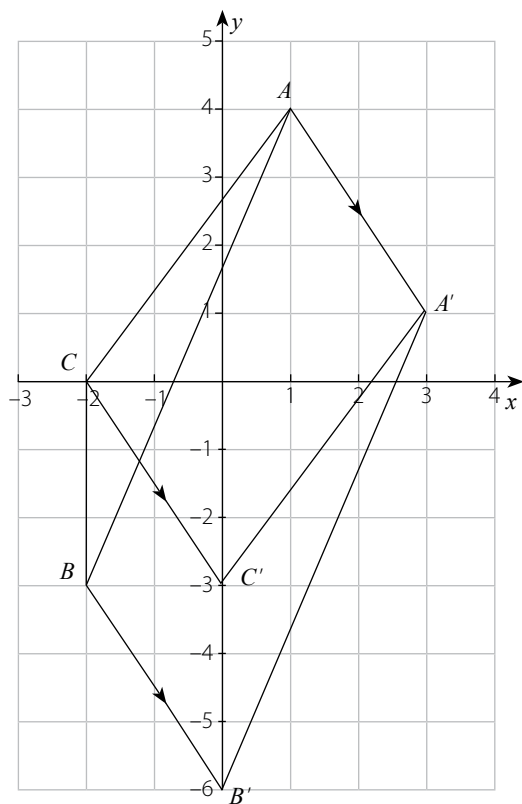


3. a)  $180^\circ$                       b)  $360^\circ$   
      c)  $360^\circ$                       d) regular hexagon
4. a)  $t = 10 - \frac{h}{5}$                       OR     $t = \frac{50 - h}{5}$   
      b) 6 min                      c) 50 cm                      d) 10 min
5.  $236 \text{ cm}^3$  (3 s.f.)                      b) Rs 4.56 (nearest paisa)

6. a)

b)

c)

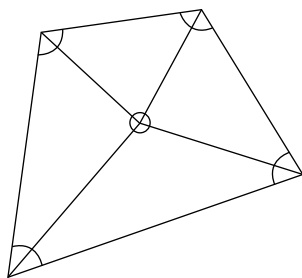
d)  $D'(5, 3)$ 

7. a)  $s + 52$                       b)  $s + 52 + s = 544$ ,  $s = 246$ ,  
      Riaz spent Rs 298, Sohrab spent Rs 246.

8. 6

9. a)  $n$ 

b)



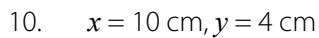
Sum of interior angles  
 = "4 triangles – circle"  
 =  $4 \times 180 - 360$



- 7.



- 9.



## 3. 11

# Chapter 25 Graphs of Functions

Three new functions are introduced: the quadratic, cubic, and reciprocal functions, and their graphs.

## LESSON PLANNING

### Objectives

|          |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |
|----------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| General  | To draw graphs of quadratic, cubic, and reciprocal functions; to recognize distinguishing features of their graphs                                                                                                                                                                                                                                                                                                                                                                  |
| Specific | <ol style="list-style-type: none"> <li>1. To recognize from their equations quadratic, cubic, and reciprocal functions</li> <li>2. To identify the general shapes of quadratic, cubic, and reciprocal graphs</li> <li>3. To draw quadratic, cubic, and reciprocal graphs by plotting points calculated in a table of values</li> <li>4. To read from the graphs key features such as lowest point, equation of the line of symmetry, centre of rotational symmetry, etc.</li> </ol> |
| Pacing   | 2 lessons, 1 homework                                                                                                                                                                                                                                                                                                                                                                                                                                                               |

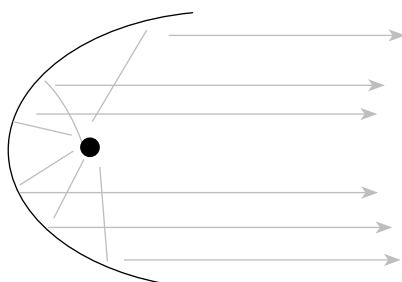
### Method

Quickly remind students of

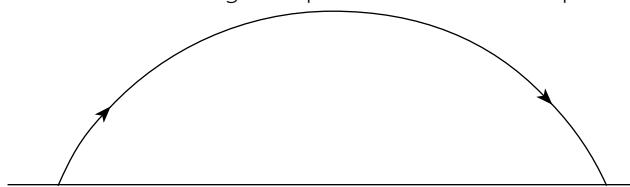
- **straight line** equations: no term in  $xy$ ; no higher powers
- bring in an  $x^2$  term, and the graph curves

Use text examples. Go through all three, showing how the table of values is created. Point out the turning point (highest or lowest) and the line of symmetry, and explain that this curve is called a **parabola**.

Interesting fact: parabolas create parallel beams of light.



Another interesting fact: parabolas are the shape of missile paths.



It's an important shape.

**Cubic graphs** have an  $x^3$  term.

Follow the text for general shape possibilities.

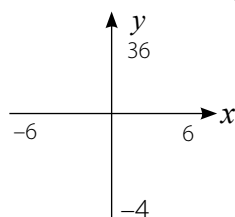
**Reciprocal graphs** have no higher powers but they do have an  $xy$  term (when written without any fractions).

Cubics and reciprocals may be plotted similarly from tables of values.

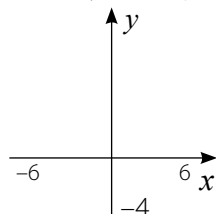
Attempt EX 25A. This will take some time.

Practical hints to give out:

- In constructing tables of values you make fewer mistakes working across the lines.
- Bracket the rows needed to make the  $y$  value.
- Before drawing axes for your graph, do a quick sketch of the range in each direction. For example,



In this case you would place the  $y$ -axis in the middle, but the  $x$ -axis below the middle of your page as shown below:



- When choosing scales, choose steps of 1, 2, 5, 10, 20, 50, 100, etc. to fit the data. Avoid multiples such as 3, 15, 25, etc., as it is hard to estimate between the fixed points.
- It is the most powerful term that determines the shape of the graph.

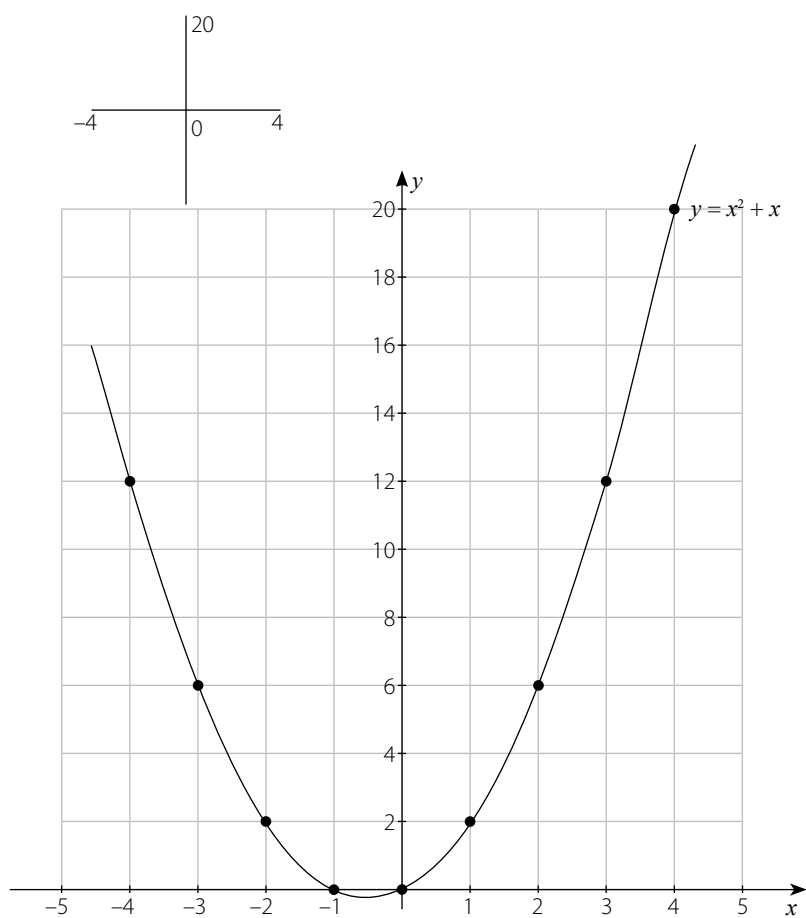
|                    |                                                                                                                                        |
|--------------------|----------------------------------------------------------------------------------------------------------------------------------------|
| <b>Resources</b>   | Squared paper (9 mm) is accurate enough for this chapter. There is no need for commercial graph paper. Have plenty available in class. |
| <b>Assignments</b> | EX 25A #7, 8 and/or 9 suitable for homework.                                                                                           |
| <b>Vocabulary</b>  | quadratic, cubic, reciprocal<br>parabola, line of symmetry                                                                             |

# ANSWERS

## Exercises

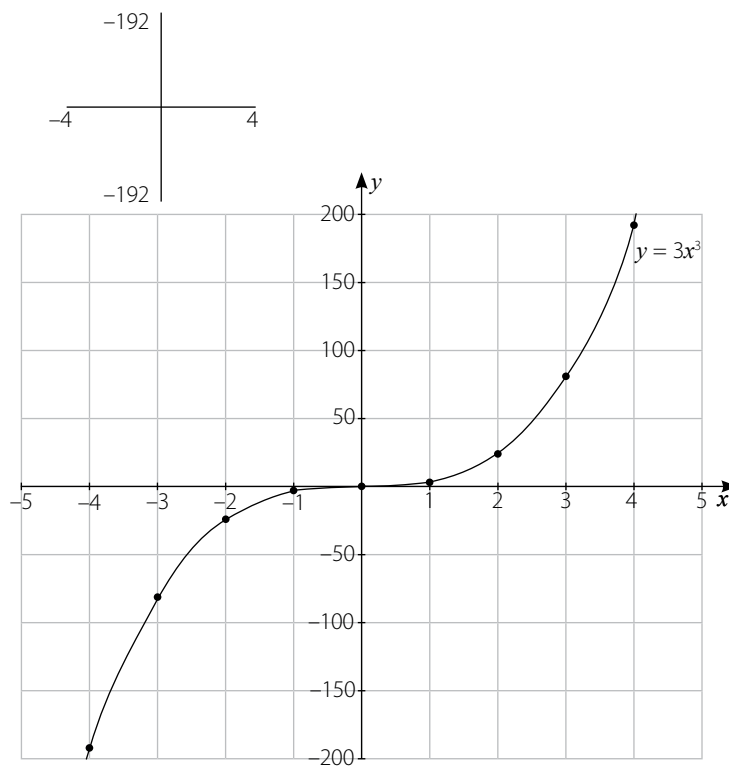
### EX 25A

- |    |    |   |    |   |    |   |    |   |
|----|----|---|----|---|----|---|----|---|
| 1. | a) | R | b) | P | c) | S | d) | Q |
| 2. | a) | P | b) | R | c) | Q | d) | S |
| 3. | a) |   |    |   |    |   |    |   |



b)  $x = -0.5$

4. a)



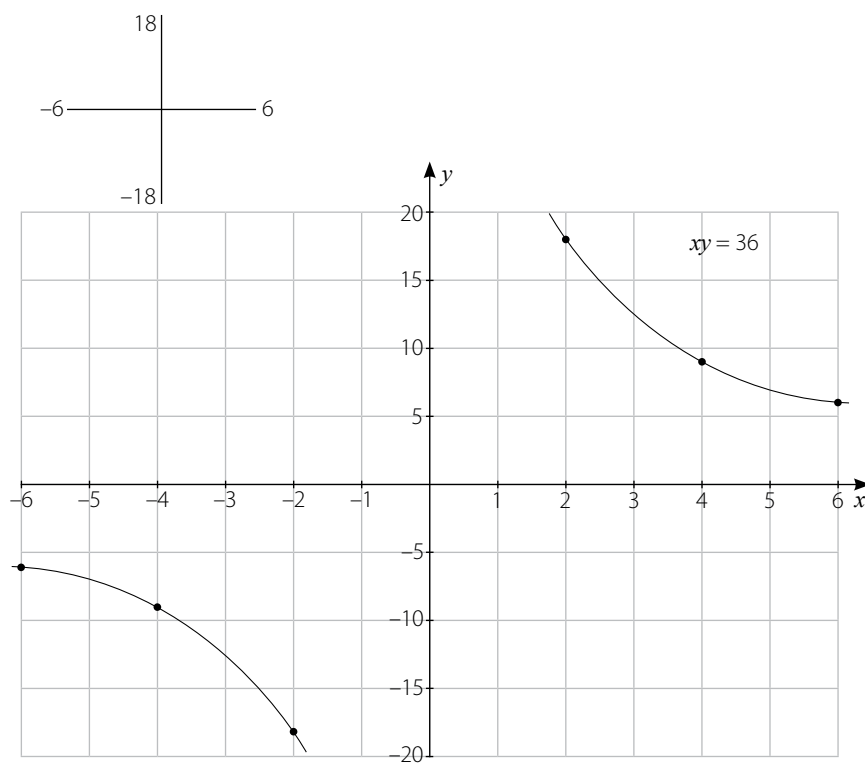
b) (0,0)

5. a)  $y = \frac{36}{x}$

b)

|     |    |    |     |   |    |   |   |
|-----|----|----|-----|---|----|---|---|
| $x$ | -6 | -4 | -2  | 0 | 2  | 4 | 6 |
| $y$ | -6 | -9 | -18 |   | 18 | 9 | 6 |

c)

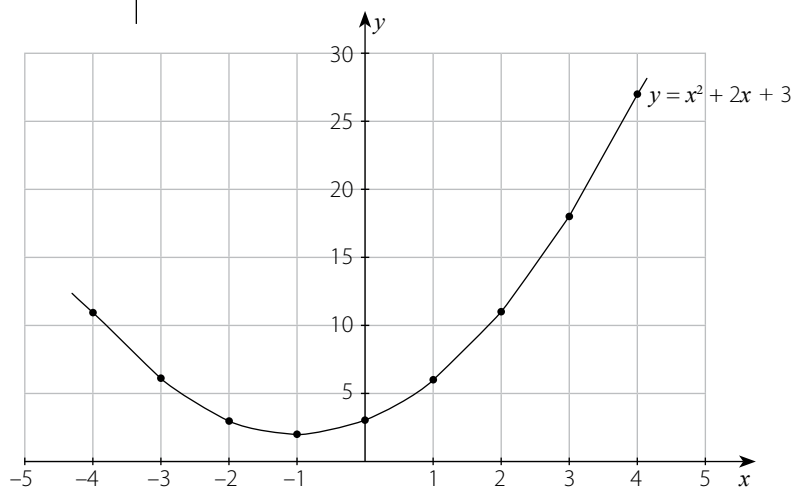
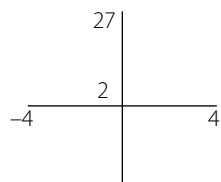


d) (0,0)

6. a)

|       |    |    |    |    |   |   |    |    |    |
|-------|----|----|----|----|---|---|----|----|----|
| $x$   | -4 | -3 | -2 | -1 | 0 | 1 | 2  | 3  | 4  |
| $x^2$ | 16 | 9  | 4  | 1  | 0 | 1 | 4  | 9  | 16 |
| $2x$  | -8 | -6 | -4 | -2 | 0 | 2 | 4  | 6  | 8  |
| 3     | 3  | 3  | 3  | 3  | 3 | 3 | 3  | 3  | 3  |
| $y$   | 11 | 6  | 3  | 2  | 3 | 6 | 11 | 18 | 27 |

b)



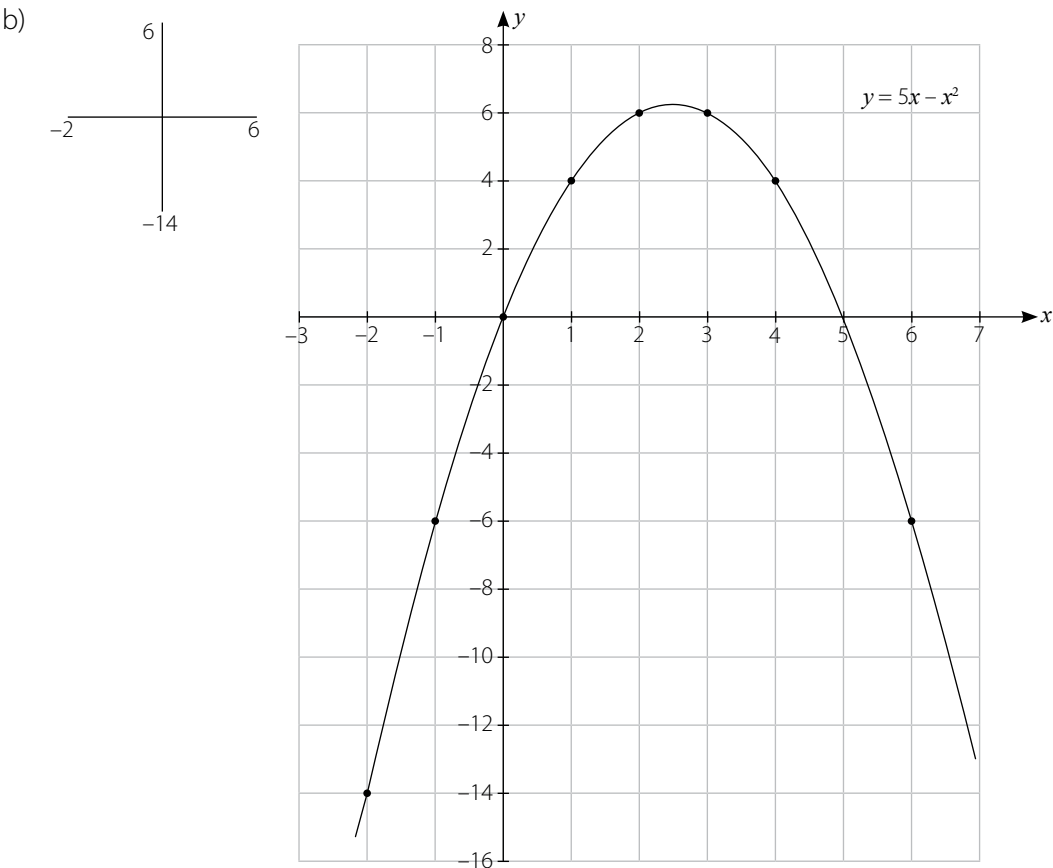
d) (0,0)

c) (-1, 2)

d)  $x = -1$ 

7. a)

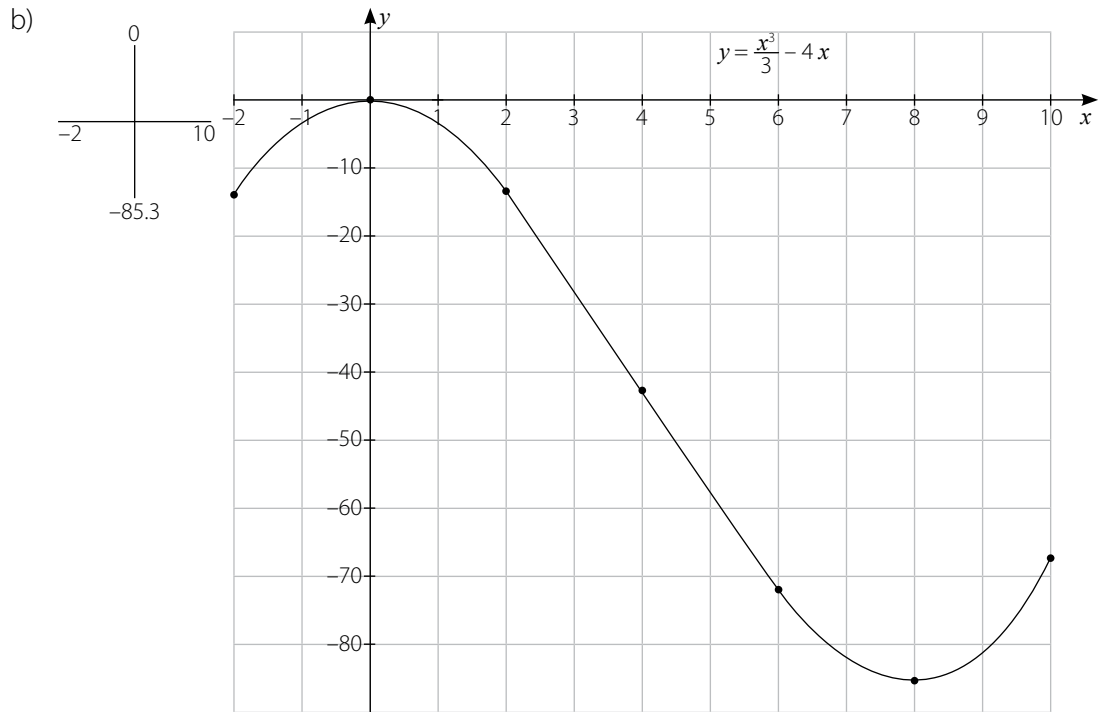
|        |     |    |   |    |    |    |     |     |     |
|--------|-----|----|---|----|----|----|-----|-----|-----|
| $x$    | -2  | -1 | 0 | 1  | 2  | 3  | 4   | 5   | 6   |
| $x^2$  | 4   | 1  | 0 | 1  | 4  | 9  | 16  | 25  | 36  |
| $5x$   | -10 | -5 | 0 | 5  | 10 | 15 | 20  | 25  | 30  |
| $-x^2$ | -4  | -1 | 0 | -1 | -4 | -9 | -16 | -25 | -36 |
| $y$    | -14 | -6 | 0 | 4  | 6  | 6  | 4   | 0   | -6  |



c)  $x = 2.5$                       b)  $(2.5, 6.25)$  from the equation

8. a)

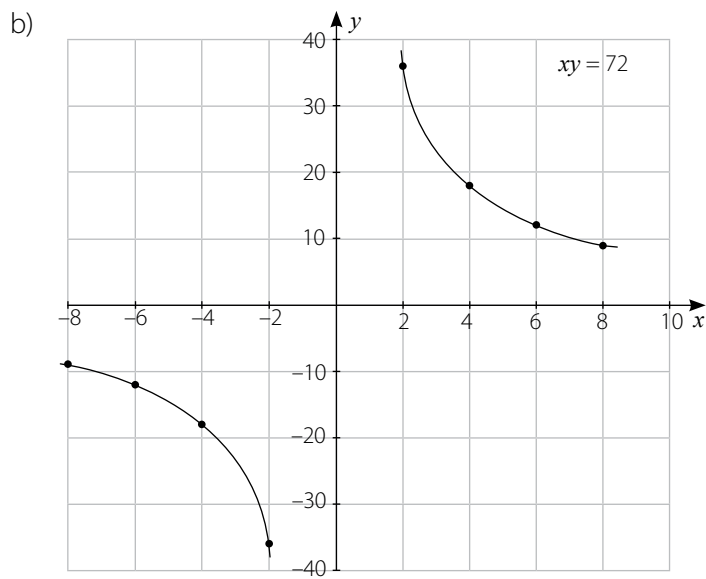
|                 |       |   |       |       |      |       |       |
|-----------------|-------|---|-------|-------|------|-------|-------|
| $x$             | -2    | 0 | 2     | 4     | 6    | 8     | 10    |
| $x^2$           | 4     | 0 | 4     | 16    | 36   | 64    | 100   |
| $x^3$           | -8    | 0 | 8     | 64    | 216  | 512   | 1000  |
| $\frac{x^3}{3}$ | -2.7  | 0 | 2.7   | 21.3  | 72   | 170.7 | 333.3 |
| $-4x^2$         | -16   | 0 | -16   | -64   | -144 | -256  | -400  |
| $y$             | -18.7 | 0 | -13.3 | -42.7 | -72  | -85.3 | -66.7 |



c) (0,0) and (8,-85.3)      b) No

9.

|     |    |     |     |     |   |    |    |    |   |
|-----|----|-----|-----|-----|---|----|----|----|---|
| $x$ | -8 | -6  | -4  | -2  | 0 | 2  | 4  | 6  | 8 |
| $y$ | -9 | -12 | -18 | -36 |   | 36 | 18 | 12 | 9 |

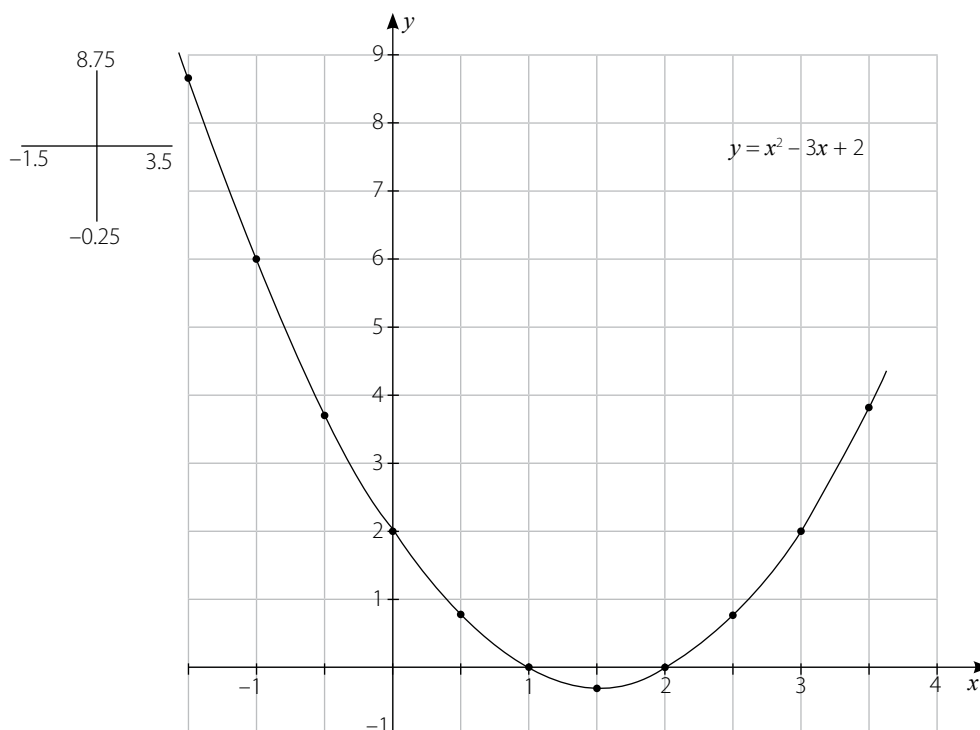


b) rotational symmetry centre (0, 0), order 2

10. a)

|       |      |    |      |   |      |    |       |    |      |    |       |
|-------|------|----|------|---|------|----|-------|----|------|----|-------|
| $x$   | -1.5 | -1 | -0.5 | 0 | 0.5  | 1  | 1.5   | 2  | 2.5  | 3  | 3.5   |
| $x^2$ | 2.25 | 1  | 0.25 | 0 | 0.25 | 1  | 2.25  | 4  | 6.25 | 9  | 12.25 |
| $-3x$ | 4.5  | 3  | 1.5  | 0 | -1.5 | -3 | -4.5  | -6 | -7.5 | -9 | -10.5 |
| 2     | 2    | 2  | 2    | 2 | 2    | 2  | 2     | 2  | 2    | 2  | 2     |
| $y$   | 8.75 | 6  | 3.75 | 2 | 0.75 | 0  | -0.25 | 0  | 0.75 | 2  | 3.75  |

b)

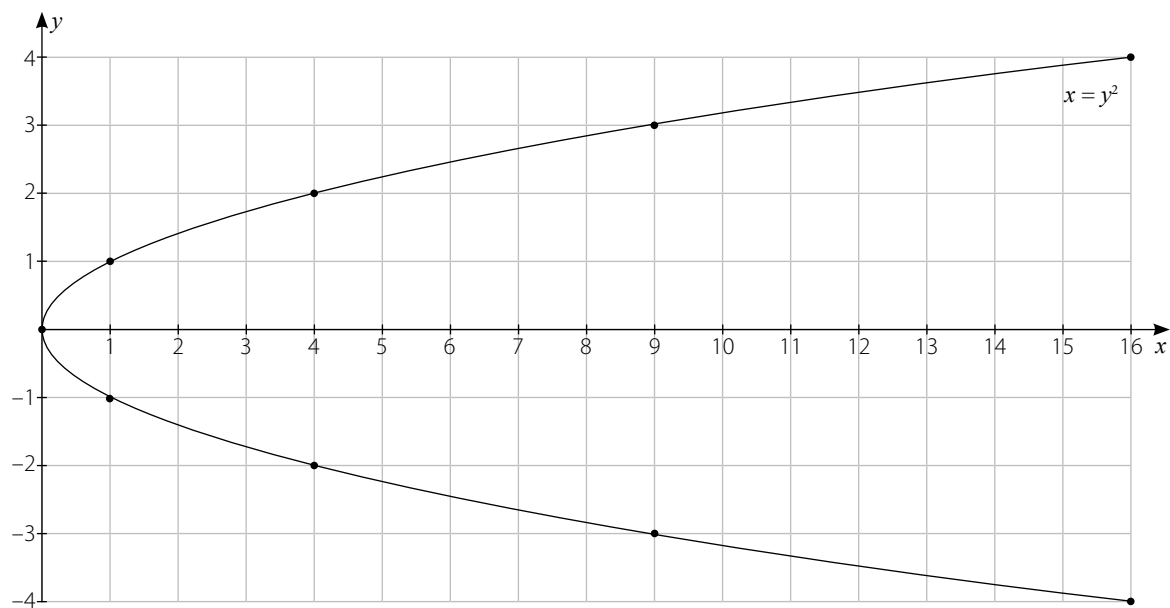
c) When  $y = 2$ ,  $x = 0$  or  $3$ d) line of symmetry at  $x = 1.5$ **EX 25X**

1. **Quadratus** is Latin for square which clearly has 4 sides. A square of side  $x$  has area  $x^2$ . (We say "squared" for a power of 2 for this reason.) Quadratic equations have a term in  $x^2$  as their highest power.

2. Graphically,  $y = \frac{x^3}{4} + \frac{x^2}{2}$  and  $y = x + 2$  intersect at  $(-2, 0)$  and  $(2, 4)$ .

3.

|       |    |    |    |    |   |   |   |   |    |
|-------|----|----|----|----|---|---|---|---|----|
| $x$   | 16 | 9  | 4  | 1  | 0 | 1 | 4 | 9 | 16 |
| $y$   | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4  |
| $y^2$ | 16 | 9  | 4  | 1  | 0 | 1 | 4 | 9 | 16 |



Parabola on its side.

[By reversing  $x$  and  $y$ , the equation becomes  $y = x^2$ , reflecting it in the line  $y = x$ .]

# Chapter 26 Intervals and Inequalities

This is an easy chapter, with yet another type of graph, and it revises the meaning of locus.

## LESSON PLANNING

### Objectives

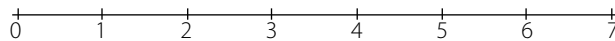
|          |                                                                                                                                                                                                                                                                                                                                                                                                            |
|----------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| General  | To graph intervals and inequalities and the locus of points with inequality conditions                                                                                                                                                                                                                                                                                                                     |
| Specific | <ol style="list-style-type: none"> <li>1. To illustrate an inequality on a number line using standard conventions</li> <li>2. To combine two inequalities into an interval (or split interval) and illustrate it on a number line</li> <li>3. To read set notation used to define the locus of points with inequality conditions; to draw the relevant graph using standard shading conventions</li> </ol> |
| Pacing   | 2 lessons, 1 homework                                                                                                                                                                                                                                                                                                                                                                                      |
| Links    | Straight line equations, especially horizontal and vertical                                                                                                                                                                                                                                                                                                                                                |

### Method

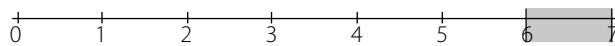
This is straightforward to teach: students just need to know the standard conventions. Follow the text examples. If there is difficulty combining inequalities, try the “cover up” technique. For instance, taking the textbook example

$$x < 6 \text{ and } x > 1$$

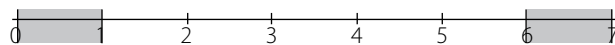
- (A) Draw a line including 1 and 6



- (B) Take one inequality e.g.  $x < 6$ . Cover up unwanted values.



- (c) Now consider the other inequality  $x > 1$  and again cover up unwanted values.



- (D) Finally, consider the boundary values 1 and 6. Is 1 included? No. Is 6 included? No.

So open spots are required, and the solution is:



Use EX 26A.

The locus questions are relatively easy once students can read the notation.

Explain:

- A locus is a set of points.
- The curly brackets mean “the set of”.
- $(x, y)$  means points.
- the colon : is read “such that”.
- After the colon come the conditions restricting the point. Read any commas as “and”. Put some examples on the board and ask students to read them out, e.g.

$$A = \{(x, y) : y > 3x, x \geq 0\}$$

means “A is the set of points such that  $y$  is greater than  $3x$  and  $x$  is greater than or equal to zero.”

When the students have grasped it, set EX 26B.

[Note that at this stage we assume all variables are continuous and real. The restriction to integers is only made in EX 26X, where it is explained.]

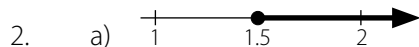
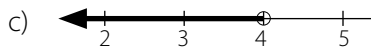
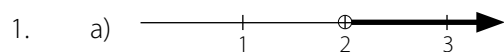
Follow the text for the conventions of shading when drawing the graph, including the “shaded” boundary line when it is not included in the acceptable region.

|                    |                                                                |
|--------------------|----------------------------------------------------------------|
| <b>Assignments</b> | EX 26B #8, 9, 10 for homework                                  |
| <b>Vocabulary</b>  | inequality, interval, acceptable, line graph, locus, condition |

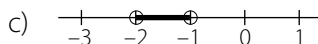
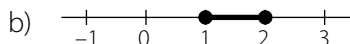
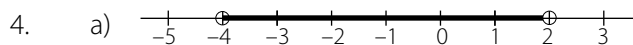
## ANSWERS

### Exercises

#### EX 26A

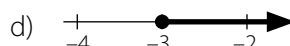
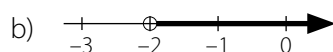


3. a)  $x \geq -2$                       b)  $x < 0$

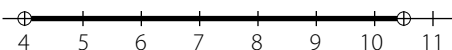
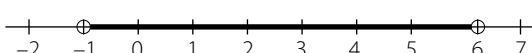
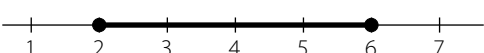
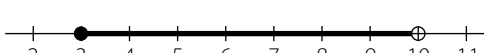
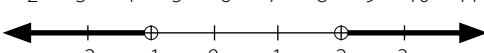
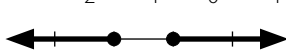
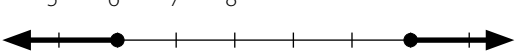
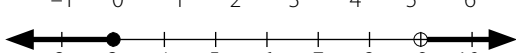
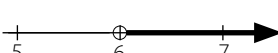
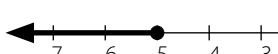
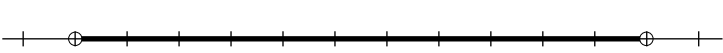
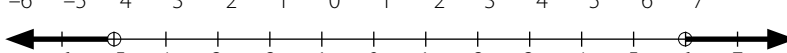


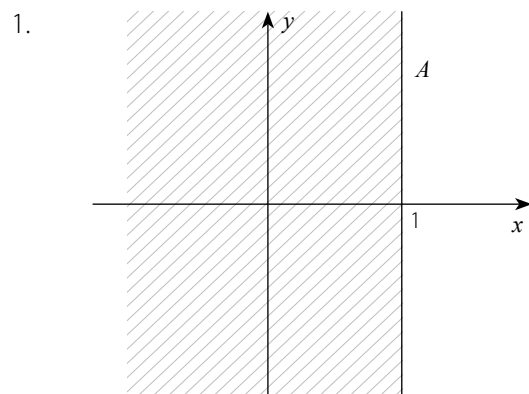
5. a)  $4 < x < 10.5$                       b)  $-1 < x < 6$

c)  $2 \leq x \leq 6$                       d)  $3 \leq x < 10$

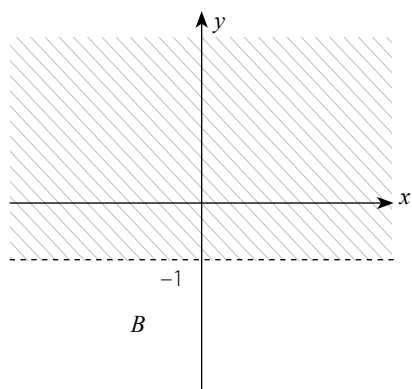


c)  $x \leq 1$                       d)  $x > 2$

6. a)  b)   
 c)  d) 
7. a)  b)   
 c)  d) 
8. a)  $-2 < x < 4$  b)  $-3 \leq x \leq 0$   
 c)  $x < -4$  or  $x > 1$  d)  $x < 9$  or  $x \geq 12$
9. a)  b)   
 c)  d) 
10. a)  $x \geq 8.5$  b)  $-4 < x \leq -1$   
 c)  $x \leq 100$  d)  $x \leq -2$  or  $x \geq 2$

**EX 26B**

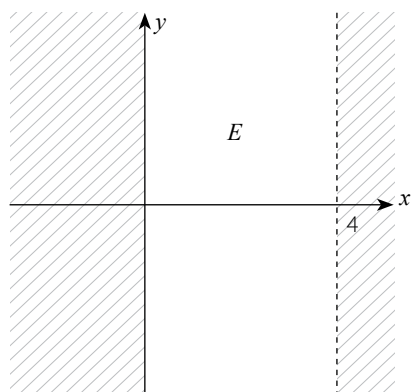
2.



3.  $C = \{(x, y): x < 5\}$

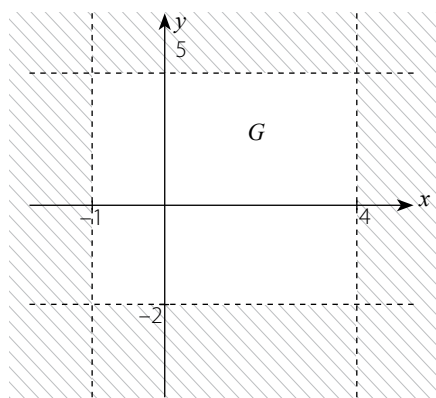
4.  $D = \{(x, y): x \geq -2, y \leq 2\}$

5.

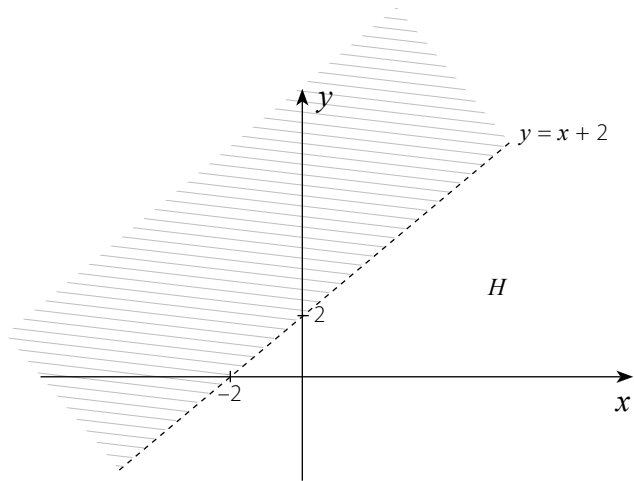


6.  $F = \{(x, y): -3 < x < 3, y \geq -4\}$

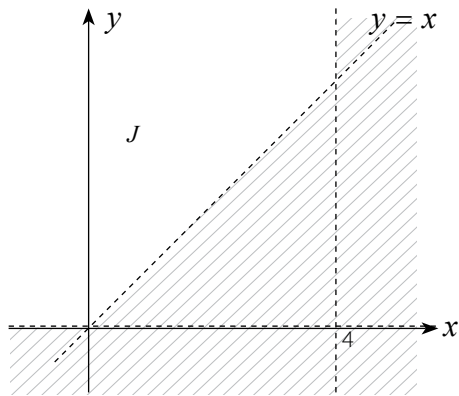
7.



8.



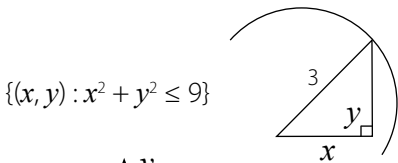
9.



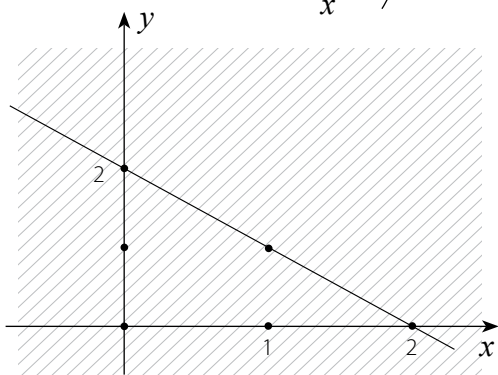
10.  $K = \{(x, y) : x \geq 0, y \geq 0, x + y \leq 4\}$

**EX 26X**

1.

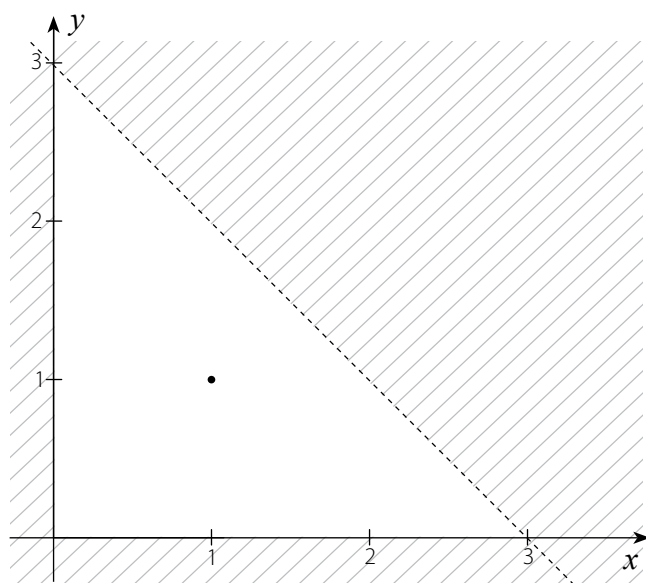


2.



The locus is the set of 6 points marked with a spot.

3.



$$P = \{(1, 1)\}$$

# Chapter 27 Algebraic Products

This is a skill-based chapter with almost no higher level thinking required. Basic algebra is so important later that it is essential to master its elementary techniques before moving on.

## LESSON PLANNING

### Objectives

|             |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |
|-------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| General     | To multiply algebraic expressions correctly, including the cases where factors are expressions in brackets                                                                                                                                                                                                                                                                                                                                                                                                                                 |
| Specific    | <ol style="list-style-type: none"> <li>To multiply two monomials together</li> <li>To multiply a monomial by a binomial expression in a bracket</li> <li>To multiply two binomial expressions together</li> <li>To simplify expressions by collecting like terms after obtaining all the products</li> </ol>                                                                                                                                                                                                                               |
| Pacing      | 3 lessons, 1 homework                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |
| Method      | <p>The first part is mostly revision, but it would be advisable to go through the chapter introduction in the text, very briefly. Set EX 27A #1–4.</p> <p>After a short while, read out the answers. Proceed according to feedback. If the concept is clear and understood, then allow the students to complete the exercise.</p> <p>The product of two brackets is explained in the text using the well-known FOIL technique.</p> <p>Use the examples in the text, setting out in four lines for clarity, as shown.</p> <p>Set EX 27B</p> |
| Assignments | EX 27A #8, 9, 10 for homework                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |
| Vocabulary  | product                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |

## ANSWERS

### Exercises

#### EX 27A

- |                 |           |             |            |
|-----------------|-----------|-------------|------------|
| 1. a) $xy$      | b) $-2p$  | c) $-10q$   | d) $2xy$   |
| 2. a) $-12x^2y$ | b) $8p^3$ | c) $10xy$   | d) $3x^3y$ |
| 3. a) $-21x$    | b) $-21x$ | c) $-21x^2$ | d) $-7x^3$ |

4. a)  $2x - 2y$  b)  $2x - 2x^2$   
 c)  $-6x - 3x^2$  d)  $-3x + x^2$
5. a)  $-4a + b$  b)  $-16a + 3 - 2a^2$   
 c)  $ab - 4b^2 + 18a$  d)  $ab + 5a^2b + 12a$
6. a)  $3x^3 + 6x^2y$  b)  $-5x^2y - 10xy^2$   
 c)  $-6x^2y^2 - 12x^3y$  d)  $7xy^3 - 7y^3$
7. a)  $-x^2y - 4xy^2 + 5x^3y^2$  b)  $8xy + 5y^2 - 6x^2$   
 c)  $25x^2y + 6xy^2 - 21x^3y$  d)  $-14x^2 - 3xy - 4y^2$
8. a)  $3x(5y - 2x) = 15xy - 6x^2$  b)  $-4xy(x + 2y) = -4x^2y - 8xy^2$   
 c)  $-7pq \times 3p^2q = -21p^3q^2$  d)  $-2pq^2 \times -7p = 14p^2q^2$
9. a)  $0.3x(1 + x) = 0.3x + 0.3x^2$  b)  $-2x(x - y) = -2x^2 + 2xy$   
 c)  $0.5pqr \times 0.4qr = 0.2pq^2r^2$  d)  $\frac{1}{4}pq \times -\frac{1}{2}p^2q = \frac{1}{8}p^3q^2$
10. a)  $2x^2 + 2y^2 - x + y$  b)  $p^2 + 2p + q^2 + q + pq$   
 c)  $0$  d)  $-2n^3 - n^4 + n$

**EX 27B**

1.  $(x + 2)(x + 3)$   
 $=$   $x^2$  F  
 $+ 3x$  O  
 $+ 2x$  I  
 $+ 6$  L  
 $=$   $x^2 + 5x + 6$
2. a)  $x^2 + 7x + 10$  b)  $x^2 + 4x + 3$   
 c)  $x^2 - 4x - 21$  d)  $x^2 + 2x - 8$
3. a)  $x^2 - x - 2$  b)  $x^2 + 3x - 18$   
 c)  $2x^2 + 7x + 3$  d)  $2x^2 + 3x + 1$
4. a)  $2x^2 + 5x + 3$  b)  $4x^2 + 12x + 5$   
 c)  $3x^2 - 5x + 2$  d)  $2x^2 + 7x - 4$
5. a)  $a^2 + 3ab + 2b^2$  b)  $a^2 - 4ab + 3b^2$   
 c)  $6a^2 + ab - b^2$  b)  $3a^2 + 5ab - 2b^2$

6. a)  $1 + 2x + x^2$  b)  $1 - 4x + 4x^2$   
c)  $a^2 + 2ab + b^2$  d)  $a^2 - 2ab + b^2$
7. a)  $2x + 2x^2$  b)  $x + 1$   
c)  $x - 1$  d)  $-2x^2$
8. a)  $2x^2 + 4x + 6$  b)  $x^2 + 3x + 9$   
c)  $-2x^2 - 7x - 12$  d)  $-8x^2 - 4x - 3$
9. a)  $2x^2 + 17x + 23$  b)  $2x^2 - 11x - 42$   
c)  $-3x - 33$  d)  $25x + 26$
10. a)  $a^2b - ab^2 + 2a^2 + 3ab + b^2$   
b)  $ab^3 - a^2b^2 - 3a^2 + 4ab - b^2$   
c)  $p^2q + 2p^2 + 4pq + q^2$   
d)  $-p^2 - 2p + q$

**EX 27X**

1.  $x^2 - 10x - 94$
2.  $4ab$
3.  $x^3 - 5x^2 - x + 5$

# Chapter 28 Sequences

Quite different from the previous chapter, this topic requires creative thinking. Treat it as an enjoyable interlude that uses the techniques of much previous work.

## LESSON PLANNING

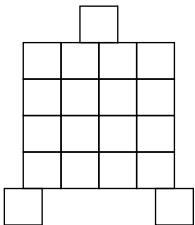
### Objectives

|             |                                                                                                                                                                                                                                                                                                                                                                                      |
|-------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| General     | To investigate sequences of numbers and designs, especially linear, quadratic, and Fibonacci sequences                                                                                                                                                                                                                                                                               |
| Specific    | <ol style="list-style-type: none"> <li>1. To use the method of differences to find <math>n</math>th term formulas for linear and quadratic sequences</li> <li>2. To answer questions requiring use of the <math>n</math>th term formula</li> <li>3. To draw designs to continue a given sequence</li> <li>4. To solve problems relating to Fibonacci and other sequences</li> </ol>  |
| Pacing      | 1 lesson, 1 homework                                                                                                                                                                                                                                                                                                                                                                 |
| Links       | <ul style="list-style-type: none"> <li>• Finding a formula to fit given data</li> <li>• Substitution in a formula</li> <li>• Problem solving</li> <li>• Removing brackets in algebraic expressions</li> <li>• Linear and quadratic functions</li> <li>• Other functions</li> <li>• Obtaining a formula from a design</li> <li>• Stating conclusions derived from evidence</li> </ul> |
| Method      | <p>If necessary, revise the method of finding the <math>n</math>th term of a linear sequence. This is laid out in the text. The quadratic ones are difficult and not many students will grasp it at this stage. If they do, then regard it as a positive indicator of high ability!</p> <p>Use EX 28A.</p> <p>It is not necessary for everyone to complete all the questions.</p>    |
| Assignments | EX 28A #9 suitable for homework                                                                                                                                                                                                                                                                                                                                                      |
| Vocabulary  | <p>linear, quadratic, Fibonacci</p> <p>differences, sequence, <math>n</math>th term formula</p>                                                                                                                                                                                                                                                                                      |

# ANSWERS

## Exercises

### EX 28A

1. a) linear (equal steps)      b)  $2n + 7$       c) 187      d) 100th
2. a) 4      b)  $4n + 3$       c) 203      d) 7
3. a) quadratic (2 rows required)      b)  $(n + 1)^2 + 1$   
     c) 65, 82      d) 442
4. a)  $(n + 4)^2 - 2$       b) 98, 119      c)  $n^2 + 8n + 14$       d) Yes
5. a)  $1.6 - 0.2n$       b)  $-0.6$       c) 8th      d) 0
6. a)       b)  $n^2 + 3$       c) 228      d) 17

7. a) 

|   |   |    |    |     |     |
|---|---|----|----|-----|-----|
| 3 | 9 | 27 | 81 | 243 | 729 |
|   | 6 | 18 | 54 | 162 | 486 |
|   |   | 12 | 36 | 108 | 324 |

It is neither linear nor quadratic.

- b) Each term is 3 times the one before it.
- c)  $3^n$
- d) 14    348    907
8. a) 

|   |    |    |     |     |      |      |
|---|----|----|-----|-----|------|------|
| 3 | 18 | 83 | 258 | 627 | 1298 | 2403 |
|   | 15 | 65 | 175 | 369 | 671  | 1105 |
|   |    | 50 | 110 | 194 | 302  | 434  |
|   |    |    | 60  | 84  | 108  | 132  |
|   |    |    |     | 24  | 24   | 24   |
- b) Formula contains  $n^4$       Adjust +2  
 $n$ th term =  $n^4 + 2$       checks
- c) 4098      d) 810 002

9. a) 47, 76, 123
- b)
- |   |    |   |    |    |    |    |    |    |     |
|---|----|---|----|----|----|----|----|----|-----|
| 1 | 3  | 4 | 7  | 11 | 18 | 29 | 47 | 76 | 123 |
| 2 | 1  | 3 | 4  | 7  | 11 | 18 | 29 | 47 |     |
|   | -1 | 2 | 1  | 3  | 4  | 7  | 11 | 18 |     |
|   |    | 3 | -1 | 2  | 1  | 3  | 4  | 7  |     |
|   |    |   | -4 | 3  | -1 | 2  | 1  | 3  |     |
- c) Each row is also a Fibonacci sequence.
- d)
- |   |   |   |    |    |    |    |
|---|---|---|----|----|----|----|
| 2 | 4 | 6 | 10 | 16 | 26 | 42 |
| 2 | 2 | 4 | 6  | 10 | 16 |    |
|   | 0 | 2 | 2  | 4  | 6  |    |
|   |   | 2 | 0  | 2  | 2  |    |
|   |   |   | -2 | 2  | 0  |    |

True; always true for Fibonacci sequences (although this is not a proof).

10. a)  $n(n+4)$
- b) 5 12 21 32 45 ...
- c)  $(n+2)^2 - 4$
- d)  $(n+2)^2 - 4 = (n+2)(n+2) - 4 =$
- $$= n^2 + 4n + 4 - 4$$
- $$= n^2 + 4n$$
- $$= n(n+4). \text{ Same as (a)}$$

### EX 28X

1. 93rd
2. 21st [Hint: Subtract 11 from each term.]
3. 4 rows:
- |     |     |     |      |      |      |      |      |
|-----|-----|-----|------|------|------|------|------|
| 74  | 249 | 618 | 1289 | 2394 | 4089 | 6554 | 9993 |
| 175 | 369 | 671 | 1105 | 1695 | 2465 | 3439 |      |
| 194 | 302 | 434 | 590  | 770  | 974  |      |      |
| 108 | 132 | 156 | 180  | 204  |      |      |      |
|     | 24  | 24  | 24   | 24   |      |      |      |

# Chapter 29 Scatter Diagrams

The aim of the chapter is to introduce students to correlation. The concept of correlation is the focus, rather than formulas for its precise evaluation.

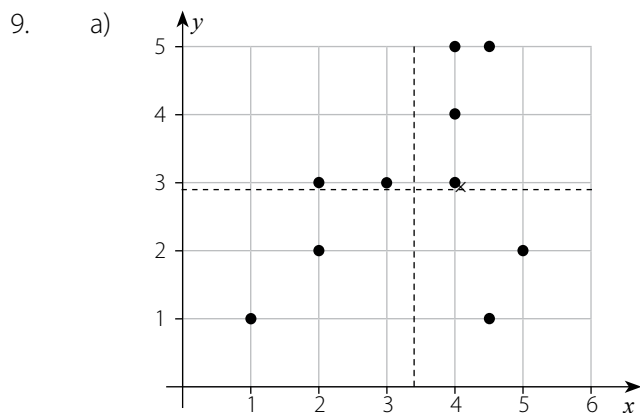
## LESSON PLANNING

### Objectives

|          |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
|----------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| General  | To explore the concept of correlation using scatter diagrams and quartering                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |
| Specific | <ol style="list-style-type: none"><li>1. To demonstrate intuitive understanding of correlation; that it may be strong or weak; and that it lies on a scale from <math>-1</math> to <math>1</math></li><li>2. To draw scatter diagrams from given data, including the mean point</li><li>3. To use quartering through the mean point to determine a description of the correlation</li><li>4. To use a line of best fit through the mean point to determine a description of the correlation</li><li>5. To estimate the value of one quantity if the other is known, by using a line of best fit</li></ol>                                                                                                                                                                                                                                                                                                                                                                                                                                                              |
| Pacing   | 2 lessons, 1 homework                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |
| Links    | Coordinates                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |
| Method   | <p>This is a new topic so begin slowly and carefully. Ask questions such as the ones in the introduction to the chapter in the text. Develop the idea that sometimes two given quantities go together really well; sometimes they go in opposite directions; sometimes the relationship is quite random.</p> <p>Then explain the term “correlation” and the meaning of <math>-1</math>, <math>0</math>, and <math>+1</math> values. Give examples.</p> <p>Scatter diagrams (sometimes abbreviated to “scattergrams”) can be described as a way to record 2-dimensional data about individuals. Use the text example, ensuring that the meaning of each spot is understood.</p> <p>Explain the technique of quartering through the mean point and making the summary table, as a way of observing how the quantities relate to each other. At this point, pause, do EX 29A #1–3 orally, or ask students to write answers and then review.</p> <p>Give out squared paper. Explain how to find the line of best fit.</p> <p>Set the rest of the exercise EX 29A #4–9.</p> |



5. a) 68 kg                      b) 157 cm                      c) strong positive  
d) Quite reliable, because of the strength of the correlation (as most points lie close to the line).
6. a) 4                              b) 5                              c) weak positive  
d) Unreliable, because of the weakness of the correlation (as many points lie far away from the line.)
7. a) weak negative              b) weak positive  
c) zero                              d) strong positive
8. a) weak positive              b) weak negative  
c) strong negative              d) zero

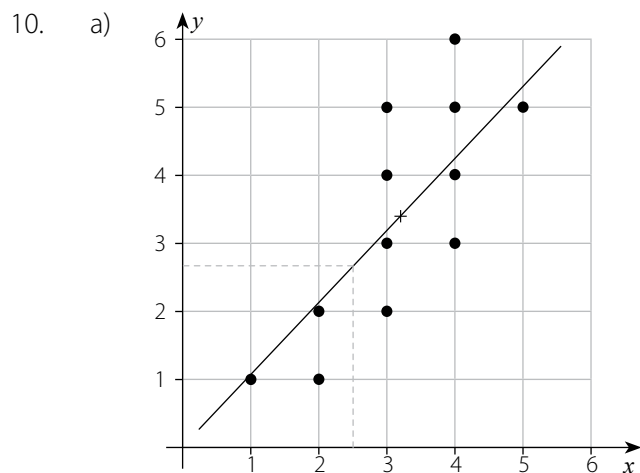


b)  $x = 3.4, y = 2.9$

c)

|   |   |                     |
|---|---|---------------------|
| 2 | 4 | very weak positive, |
| 2 | 2 | close to zero       |

- d) very difficult, owing to unclear correlation  
more data required



b)  $x = 3.2, y = 3.4$

c)  $y = 2.6$  (approx)

- c) strong positive

**EX 29X**

1. The method used in spreadsheets, etc. is to measure how far above the line each point is (negative below), to square these amounts (so the result is positive), and to find their total. The best-fitting line gives the minimum possible total of the squares.
2. If  $x$  and  $y$  go together, i.e. have a strong positive correlation, then there are a number of possibilities:
  - $x$  causes  $y$ .
  - $y$  causes  $x$ .
  - Something else causes  $x$  and  $y$  to increase together.
  - There may be other factors involved.
  - $x$  and  $y$  may be measuring the same thing (but the researcher has not realised it).
3. Correlation is close to 1 (or  $-1$ ) if the points are close to the line of best fit. The gradient of the line is a measure of its slope.

# Chapter 30 Rational Numbers

This is an early introduction to the notion that numbers fall into various categories, and that each extension of the number system enables more types of calculation to take place. The emphasis is upon structure rather than calculation.

## LESSON PLANNING

### Objectives

|          |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |
|----------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| General  | To gain insight into the historical development of the number system and recognize the major types of numbers                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |
| Specific | <ol style="list-style-type: none"><li>1. To understand and define the terms natural numbers, integers, and rational numbers and the relationship between them</li><li>2. To use correctly the standard symbols <math>\mathbb{N}</math>, <math>\mathbb{Z}</math>, and <math>\mathbb{Q}</math> for the sets of naturals, integers, and rationals respectively</li><li>3. To demonstrate understanding that certain operations are not possible in <math>\mathbb{N}</math>, <math>\mathbb{Z}</math>, and <math>\mathbb{Q}</math></li><li>4. To give some examples of irrational numbers</li><li>5. To identify the subset symbol <math>\subset</math> and use it correctly</li></ol> |
| Pacing   | 1 lesson, 1 homework                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |
| Links    | Rule of signs                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |

### Method

Adopt a historical/personal approach. Students may be prompted to recall experiences such as

- early counting experiences, e.g. counting fingers, counting the steps while climbing stairs, etc. [natural numbers]
- taking away everything, leaving nothing, e.g. 8 apples in a fruit bowl, 8 removed, bowl empty. How many left over? [need for zero]
- temperature, above and below freezing [need for negatives]
- measuring [need for numbers between whole numbers, i.e. fractions and decimals]

Explain that each extension to the set of numbers includes all the previous ones, e.g.

3 is a natural.

It is also an integer.

It is also a rational  $3 = \frac{3}{1}$ .

$$\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q}$$

See the Venn diagram in the text.

Set EX 30A

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**Assignments**    EX 30A #9, 10 for homework.

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**Vocabulary**    natural numbers, integers, rational numbers, irrational numbers, counting, measuring, sets, subsets, reciprocal

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## ANSWERS

### Exercises

#### EX 30A

1.    a) naturals                      b) rationals                      c) integers                      d) rationals
2.    a) infinity                      b) infinity                      c) infinity                      d) infinity  
[i.e. no limit]
3.    a) 1                                  b) 7  
         c) -8                              d) impossible
4.    a) 42                                  b) 10  
         c) 416                              d) impossible
5.    a)  $\frac{3}{4}$                                   b)  $\frac{1}{4}$                                   c)  $\frac{1}{8}$                                   d)  $-\frac{1}{4}$
6.    a)  $-\frac{5}{2}$                                   b)  $\frac{19}{5}$                                   c)  $\frac{14001}{10}$                                   d)  $-\frac{6}{5}$
7.    a)  $\frac{5}{7}$                                   b)  $\frac{2}{7}$                                   c) The answer is always 1  
         d) To divide by a rational number, multiply by its reciprocal.
8.    a) false                                  b) true                                  c) false                                  d) true
9.    a) true                                  b) False, ( $\pi$  is irrational.)  
         c) true                                  d) true
10.    A    \_\_\_\_\_     $\mathbb{Q}$   
         B    \_\_\_\_\_     $\mathbb{N}$   
         C    \_\_\_\_\_    none of these  
         D    \_\_\_\_\_     $\mathbb{Z}$

**EX 30X**

1. Division by zero is meaningless. It has no reciprocal.

The set of rational numbers is obtained by dividing pairs of integers, excluding cases where the denominator is zero.

2. a)  $n = 0.\dot{1}$                       b)  $n = 0.\dot{1}\dot{2}$   
 $10n = 1.\dot{1}$                                $100n = 12.\dot{1}\dot{2}$   
 Subtract:  $9n = 1$                        $99n = 12$   
 $n = \frac{1}{9}$                                        $n = \frac{12}{99} = \frac{4}{33}$   
 rational                                      rational
- c)  $n = 0.\dot{1}\dot{2}\dot{3}$                               d)  $n = 0.1\dot{6}$   
 $1000n = 123.\dot{1}\dot{2}\dot{3}$                                $10n = 1.6\dot{6}$   
 $999n = 123$                                $9n = 1.5$   
 $n = \frac{123}{999} = \frac{41}{333}$                                $18n = 3$   
 rational                                       $n = \frac{1}{6}$       rational

3. No. All real numbers can be represented on a number line (denoted by  $\mathbb{R}$ ). These are the rationals and the irrationals and others such as transcendental numbers ( $\pi$  is one of these). Also, there are 2-dimensional numbers (called complex numbers) that can be represented on a plane coordinate grid.

# Chapter 31 Factorising

This chapter develops facility in algebraic manipulation, with a view to setting the scene for factorisation of trinomials which is dealt with in detail in Book 9.

## LESSON PLANNING

### Objectives

|             |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |
|-------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| General     | To factorise simple algebraic expressions where possible                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
| Specific    | <ol style="list-style-type: none"> <li>1. To factorise binomials by extraction of the common factor</li> <li>2. To factorise 4-term expressions by extraction of a common factor, or by grouping the terms in pairs</li> <li>3. To factorise simple trinomials by splitting one term into two and then treating it as a 4-term expression</li> <li>4. To recognize that not all expressions can be factorised</li> </ol>                                                                                                                                        |
| Pacing      | 1 lesson, 1 homework                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |
| Method      | <p>First, explain that factorising means writing the expression as a product of factors. We factorise because there are equations that can only be solved by factorisation, to be dealt with later once the factorisation skills are well developed. Use the text examples and go through all of them with the class so that the concept is clearly understood.</p> <p>Set the exercise EX 31A #1–5.</p> <p>Before moving on to the second half do some more 4-term examples. For #6–9 encourage students to use the FOIL technique to check their answers.</p> |
| Assignments | EX 31A #10 for homework.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
| Vocabulary  | factorise, product                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |

## ANSWERS

### Exercises

#### EX 31A

1.    a)  $2(x - 3y)$                       b)  $-3q(p + 2)$   
       c)  $7a(a - 2b)$                       d) not possible

2. a)  $-13x(y - 2)$  or  $13x(-y + 2)$   
 b)  $17q(2pq + 1)$   
 c)  $-5a(a - 2)$  or  $5a(-a + 2)$   
 d) not possible
3. a)  $x(x - 7)$                                       b)  $-x^2(x - 2)$  or  $x^2(-x + 2)$   
 c)  $2x(x^3 + 3)$                                       d) not possible
4. a)  $x(1 - 3x)$                                       b)  $x^2(1 - 2x)$   
 c)  $-3x(x^2 + 2)$                                       d)  $-8x(1 - 2x^2)$
5. a)  $a(1 + b + 3a)$                                       b)  $2b(1 - 2b + 3b^2)$   
 c)  $9c(1 - 2a + 3b)$                                       d)  $d(1 + d - d^2)$
6. a)  $(x + 2)(x + y)$                                       b)  $(x + 2)(x^2 + 1)$   
 c)  $(2 - y)(x - y)$                                       d)  $(2x + 1)(2y - 1)$
7. a)  $(2a + 1)(a - b)$                                       b)  $(1 - 3b)(a + b)$   
 c)  $(a - b)(a^2 + b)$                                       d)  $(4b - 3)(b - a)$
8. a)  $(1 + x)(3 + y)$                                       b)  $(uv + 1)(uv^2 - 1)$   
 c)  $(c + 3)(a + b)$                                       d)  $(r - 5)(p - 2q)$
9. a)  $(x - 1)(x + 2)$                                       b)  $(x + 5)(x - 3)$   
 c)  $(x + 4)(x + 2)$                                       d)  $(x - 1)(x - 2)$
10. a)  $(2a - b)(a - b)$                                       b)  $(a - 3b)(a + b)$   
 c)  $(a + b)(a + 2b)$                                       d)  $(a + 2b)(a + 3b)$

**EX 31X**

1. a)  $\left(\frac{x}{2} + 1\right) \left(\frac{x}{3} + 1\right)$                                       b)  $(2x + 3)(3x + 4)$   
 c)  $(0.2x + 1)(0.4x - 1)$                                       d)  $(x + 15)(2x - 1)$
2. a)  $(a + b)^2$                                       b)  $(a - b)^2$   
 c)  $(a + b)(a - b)$                                       d) not possible without another type of number
3.  $(a + 2b + 3c)(a + b + c)$

[Group together terms with 1, 2, and 3 as coefficients.]

# Chapter 32 Revision Exercises

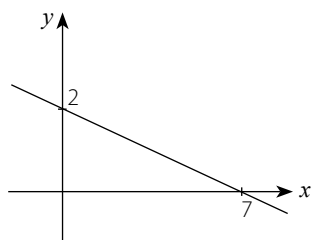
## ANSWERS

### Exercises

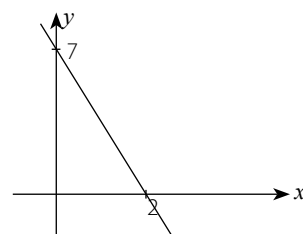
#### EX 32A

1. 58 s
2. a) 1:2                                      b) 1:3                                      c)  $\frac{1}{6}$                                       d) 5
3. a)  $2 - 0.3n$                                       b)  $-1$                                       c) 100th                                      d)  $-11$
4. a) 434 kg                                      b) 62.3 kg (1 d.p.)                                      c) 59 kg
5.  $x = 1, y = 1$
6. a)  $\left. \begin{array}{l} y = \frac{1}{3}x + 3 \\ y = 6 \end{array} \right\}$  1 solution
- b)  $\left. \begin{array}{l} y = 2x + 3 \\ y = 2x + 3 \end{array} \right\}$  infinite solutions  
(same line)
- c)  $\left. \begin{array}{l} y = \frac{1}{4}x + 2 \\ y = -\frac{1}{4}x + 2 \end{array} \right\}$  1 solution
- d)  $\left. \begin{array}{l} y = 2x - 3 \\ y = 2x - 4 \end{array} \right\}$  no solutions  
(parallel lines)
7.  $x = 2, y = -1$
8. a)  $2(a + c + 1)$                                       b)  $3(p - 2q + 3)$                                       c) not possible  
d)  $-5(2x - 4y + 3)$  or  $5(-2x + 4y - 3)$

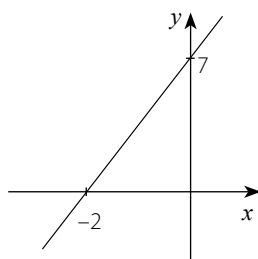
9. a)  $2x + 7y = 14$



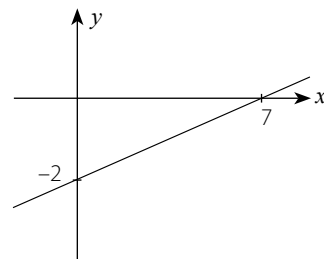
b)  $7x + 2y = 14$



c)  $7x - 2y = -14$



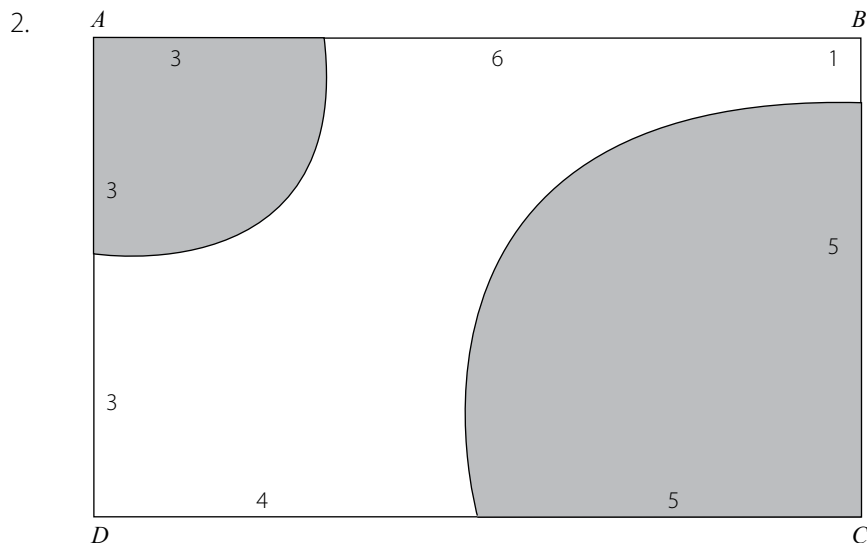
d)  $2x - 7y = 14$



10. 50

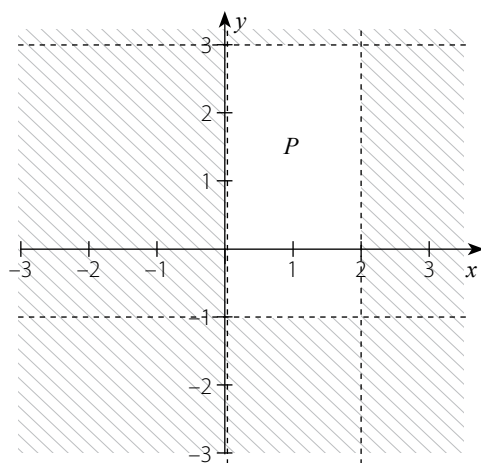
**EX 32B**

1. a)  $d = 0.2t$  or  $d = \frac{1}{5}t$       b) 1.05 m  
 c) 9 min      d) 0.2 m/min

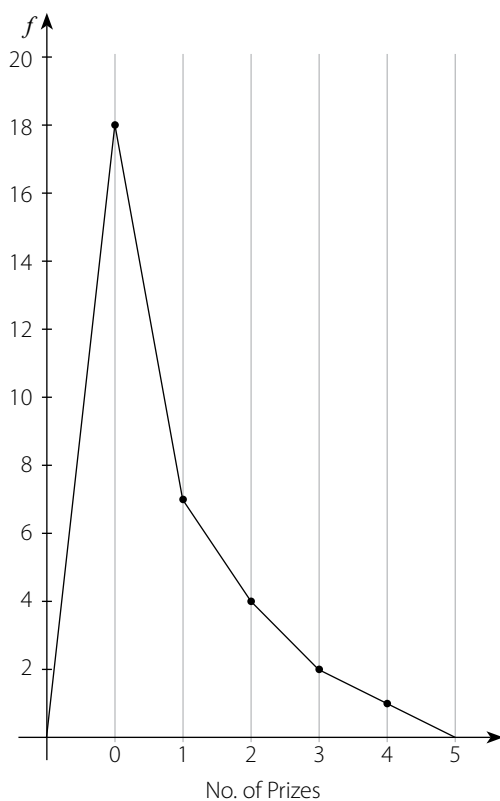


3. a) 556.8 km/h      b) 394.4 km/h  
 c) 336 km/h      d) 390.3 km/h (1 d.p.)
4. a)  $x = -20$       b)  $x = \frac{19}{11}$

5.



6.



7.

a) 13.5 s

b) 30

c) quite strong negative

d) Reasonably reliable; most points are quite close to the line of best fit.

8.

a)  $49.6 \text{ cm}^2$ b)  $28.26 \text{ cm}^2$ c)  $154 \text{ cm}^2$ d)  $1875 \text{ m}^2$ 

9.

a)  $1440^\circ$ b)  $1080^\circ$ c)  $8640^\circ$ d)  $180(n-2)$

10.

|     |      |    |    |    |    |    |
|-----|------|----|----|----|----|----|
|     | Blue |    |    |    |    |    |
|     | 1    | 2  | 3  | 4  | 5  | 6  |
| Red | 1    | 1  | 0  | -1 | -2 | -4 |
|     | 2    | 3  | 2  | 1  | 0  | -2 |
|     | 3    | 5  | 4  | 3  | 2  | 1  |
|     | 4    | 7  | 6  | 5  | 4  | 3  |
|     | 5    | 9  | 8  | 7  | 6  | 5  |
|     | 6    | 11 | 10 | 9  | 8  | 7  |

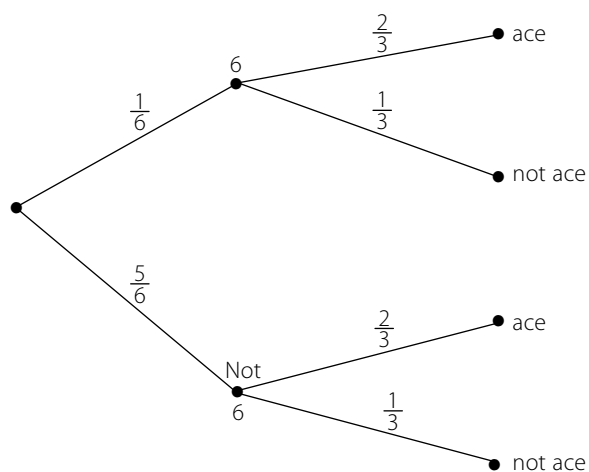
- a)  $\frac{1}{12}$   
 b)  $\frac{1}{6}$   
 c)  $\frac{1}{4}$   
 d) 0

**EX 32C**

1. a) 4 cm                      b) 4.5 m                      c) 30 m<sup>2</sup>                      d) 3 kg  
 2. a)  $-4x - 5y$                       b)  $-19x - 4 - 5x^2$   
     c)  $7x^2 + 3xy + 12x$                       d)  $xy + 8x^2y + 27x$

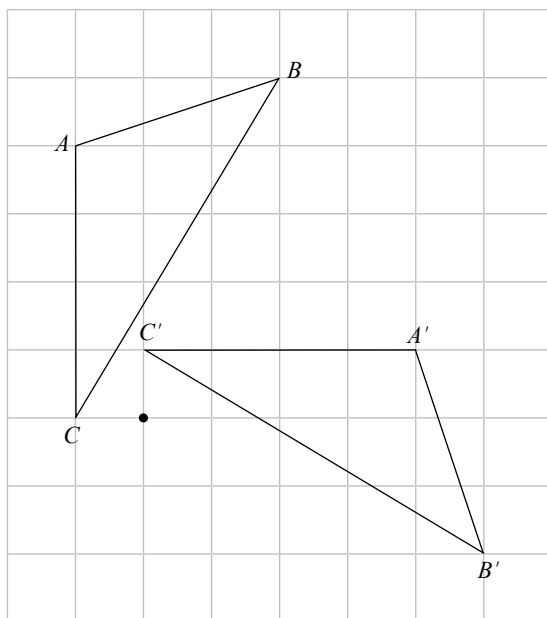
3. 66 m

4. a)

b)  $\frac{1}{9}$ 

5. a) 55 min                      b) 0.18 s                      c) 1 min 18 s                      d) 0.05 s  
 6. a)  $\frac{43}{30}$  or  $1\frac{13}{30}$                       b)  $-\frac{7}{30}$                       c)  $\frac{1}{2}$                       d)  $\frac{18}{25}$

7.



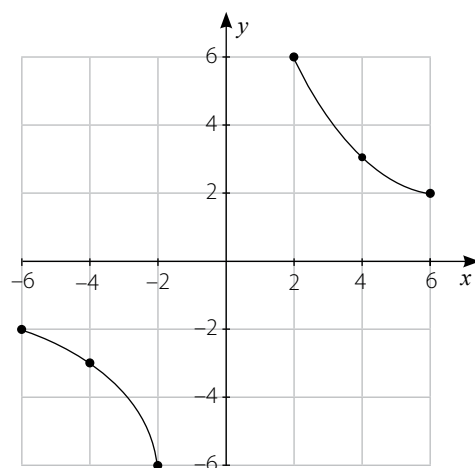
8.

a)  $y = \frac{12}{x}$

b)

|     |    |    |    |   |   |   |   |
|-----|----|----|----|---|---|---|---|
| $x$ | -6 | -4 | -2 | 0 | 2 | 4 | 6 |
| $y$ | -2 | -3 | -6 |   | 6 | 3 | 2 |

c)



d) (0,0)

9.

a)  $2p^2 + 3pq + q^2$

b)  $3p^2 - 4pq + q^2$

c)  $6p^2 - pq - q^2$

d)  $4p^2 + 7pq - 2q^2$

10.  $p(\text{win at A}) = \frac{9}{42} = 0.214$

$p(\text{win at B}) = \frac{8}{36} = 0.222$

$p(\text{win at C}) = \frac{8}{42} = 0.190$

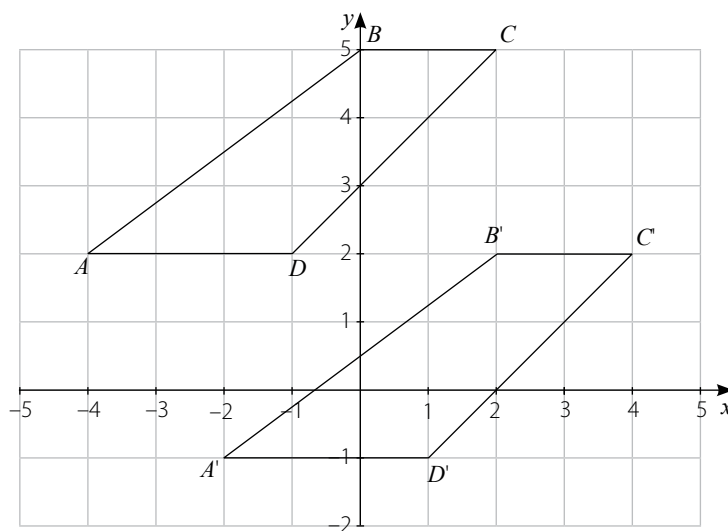
$p(\text{win at D}) = \frac{10}{49} = 0.204 \quad (\text{to 3 s.f.})$

best chance at table B

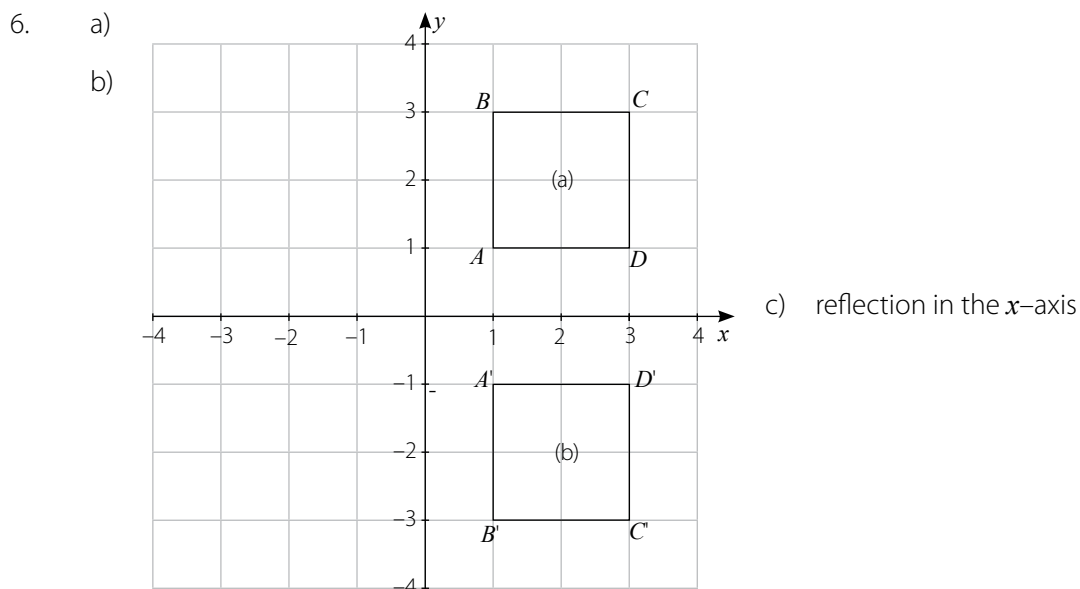
## EX 32D

1. a)  $\text{ext } \angle = 45^\circ, \text{int } \angle = 135^\circ$   
 b)  $\text{angle-sum} = 180 \times 6 = 1080, \text{int } \angle = 135^\circ$
2. a)  $3 < x < 9.5$                       b)  $-2 < x < 5$   
 c)  $1 \leq x \leq 5$                       d)  $2.5 \leq x < 9$
3. a)  $A'(-2, -1), B'(2, 2), C'(4, 2), D'(1, -1)$

b)



4. a)  $p(1 - q + pq)$                       b)  $3p(1 + 2p - 3p^2)$   
 c)  $5p(1 - 2q + 3r)$                       d)  $p(1 - p + p^2)$
5. a)  $x = -\frac{1}{9}$                       b)  $x = -3$   
 c)  $x = \frac{4}{3}$                       d)  $x = \frac{58}{9}$  or  $6\frac{4}{9}$



7.  $x = 12 \text{ cm}, y = 6 \text{ cm}$

8.  $s = \frac{v^2 - u^2}{2a}$

a) 7.5

b) 12

c) 7.2

d) 0

9. a)  $x = \frac{8}{7}$

b)  $x = \frac{3}{2}$

c)  $x = \frac{12}{11}$

d)  $x = \frac{7}{22}$

10. a)  $143 \text{ cm}^3$  (3 s.f.)

b)  $151 \text{ cm}^3$  (3 s.f.)

### EX 32X

1. Each number is the sum of the two immediately above it.

$$a = 3, \quad b = 4, \quad c = 10, \quad d = 15$$

$$1 \quad 7 \quad 21 \quad 35 \quad 35 \quad 21 \quad 7 \quad 1$$

$$1 \quad 8 \quad 28 \quad 56 \quad 70 \quad 56 \quad 28 \quad 8 \quad 1$$

Blaise Pascal was a French philosopher and mathematician of the 17th century. His most important contribution to mathematics was his work on probability theory.

2. a)  $x^3 - 5x - 2$

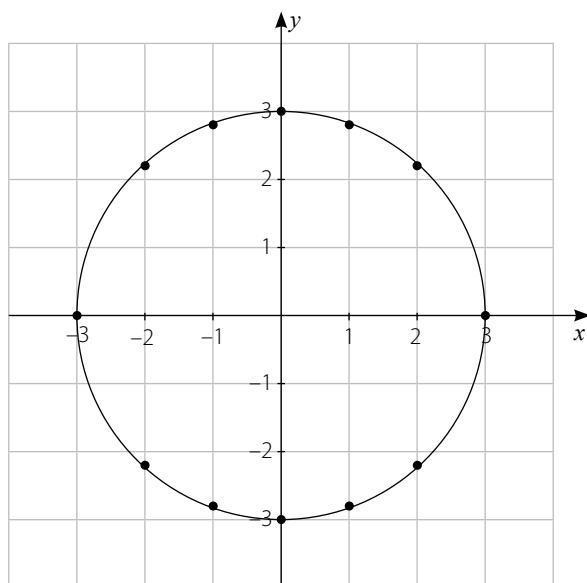
b)  $2x^3 + 3x^2 + 3x + 1$

c)  $x^3 - 1$

d)  $x^3 + 1$

3.  $y = \pm \sqrt{9 - x^2}$

|     |    |           |           |         |           |           |   |
|-----|----|-----------|-----------|---------|-----------|-----------|---|
| $x$ | -3 | -2        | -1        | 0       | 1         | 2         | 3 |
| $y$ | 0  | $\pm 2.2$ | $\pm 2.8$ | $\pm 3$ | $\pm 2.8$ | $\pm 2.2$ | 0 |



circle with centre at the origin, and radius 3

# Specimen Examination Paper

[for the whole year's work]

## Instructions

The time allowed is  $1\frac{1}{2}$  hours.

You may use a calculator.

You will also need: pen, pencil, rubber, ruler, compasses.

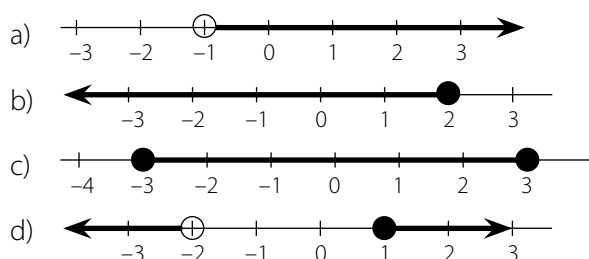
Try to answer all the questions.

Check your work carefully.

The marks for each question are shown in brackets.

[Full marks = 100]

1. Write down the inequalities represented by each of these line graphs:



[4]

2. Factorise the following expressions:

a)  $x^2 - 5x$

b)  $42p^2q + 7pq$

c)  $ab + 2a + 2b^2 + 4b$

[4]

3. a) How long does it take for a train travelling at 45 km/h to travel 52 km?  
(Answer to the nearest minute.)

- b) How far can a bus travel in 2 h 25 min at an average speed of 30 km/h?

[4]

4. The stem-and-leaf diagram below shows the test results of some students:

| Test | Results         |
|------|-----------------|
| 3    | 2               |
| 4    | 4 7 7           |
| 5    | 1 4 6 6 8       |
| 6    | 1 1 1 2 4 6 8 9 |
| 7    | 3 5 5 7         |
| 8    | 2 4             |
| 9    | 3               |

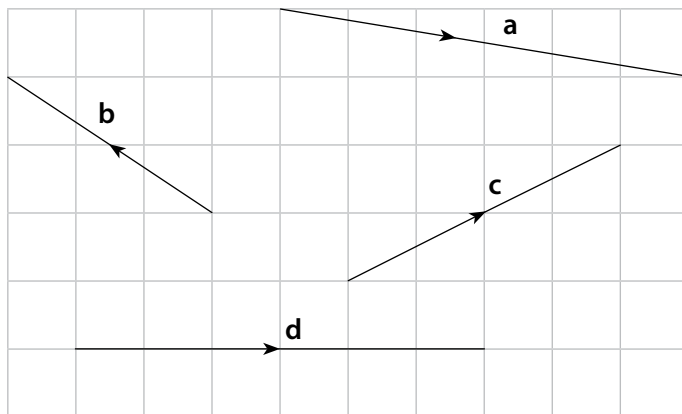
a) How many students' results are recorded here?

b) State the median test result.

c) What is the range of these test results?

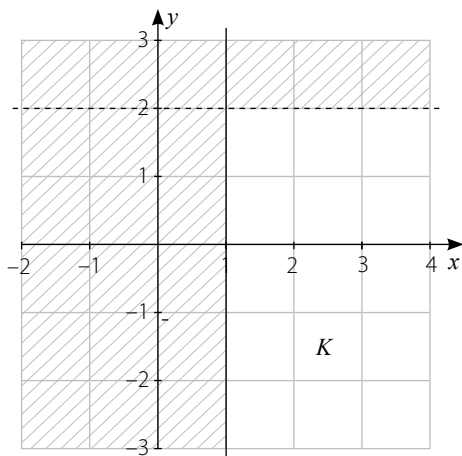
[4]

5. State the components of each vector shown in the diagram.



[4]

6. a) Using set notation state the locus of the points in the region marked  $K$ .



b) Describe the locus of points 50 cm from a fixed point  $A$  in 3-dimensional space.

[4]

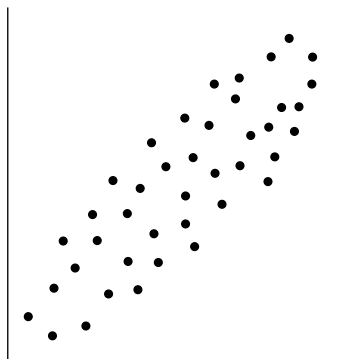
7. a) A regular polygon has an interior angle-sum of  $1980^\circ$ . How many sides does it have?

b) Another regular polygon has interior angles of  $156^\circ$ . How many sides does it have?

[4]

8. Describe the correlations in each of these scatter diagrams:

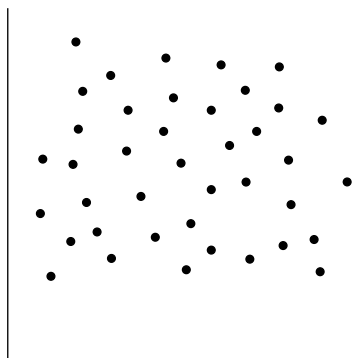
a)



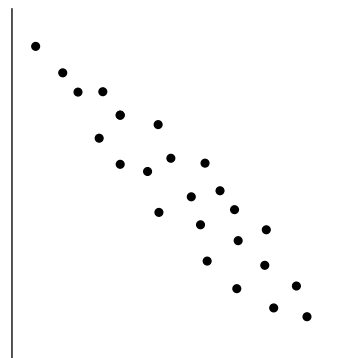
b)



c)



d)



[4]

9. Simplify these products:

a)  $-5x \times x^2$

b)  $2x(y-1)$

c)  $(x+3)(x-1)$

[4]

10. State whether true or false:

a)  $\pi$  is a rational number.

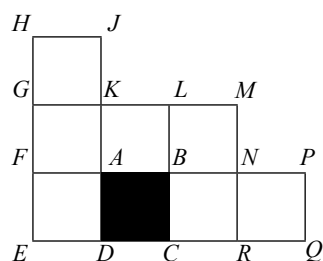
b)  $-3$  is an integer.

c) The reciprocal of  $9.5$  is  $\frac{2}{19}$ .

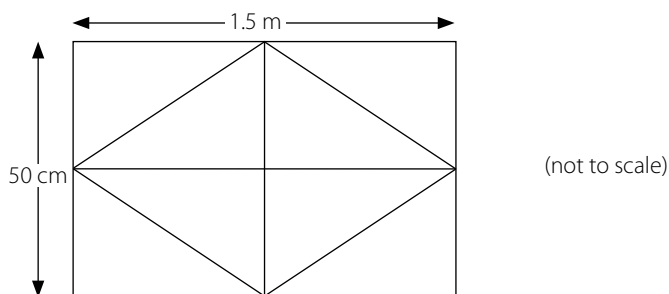
d)  $0.\dot{2}$  is a rational number.

[4]

11. Complete the statements below about transformations of  $ABCD$ .



- Under rotation of  $90^\circ$  about  $F$ ,  $ABCD \rightarrow ?$
  - Under reflection in the line  $FP$ ,  $ABCD \rightarrow ?$
  - Under translation  $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$ ,  $ABCD \rightarrow ?$
  - Under enlargement, centre  $D$ , scale factor 2,  $ABCD \rightarrow ?$  [4]
12. Sketch the lines whose equations are given, clearly marking the points of intersection with each axis on your diagrams.
- $2x + 3y = 6$
  - $3x - 5y = 15$
  - $y = 2x - 4$  [6]
13. The diagram shows the design for a metal gate. Calculate to the nearest centimeter the length of steel required to construct it.



14. Solve these pairs of simultaneous equations:

a)  $y = x + 5$

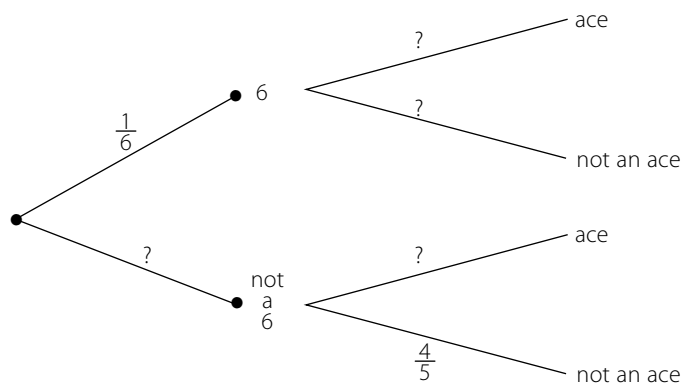
$3x = 2y - 8$

b)  $2x + 5y = 5$

$3x - 4y = 19$  [6]

15. A fair dice is rolled, and one card is selected at random from a set of five cards: ace, king, queen, jack, ten.

a) Copy and complete the tree diagram, showing the probabilities:



b) State  $P(6, \text{ but not an ace})$  and  $P(\text{ace, but not a } 6)$ .

[6]

16. Given the formula  $r = \frac{2a+b}{5}$

a) Rearrange the formula to make  $b$  the subject.

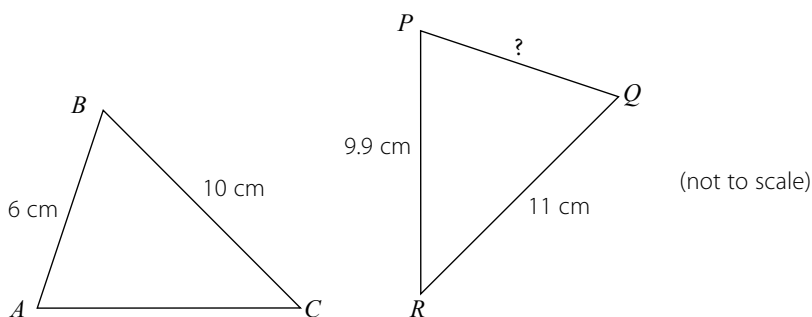
b) Find the value of  $b$  when  $r = 6, a = 1$ .

c) Find the value of  $b$  when  $r = 4.1, a = 1.55$ .

[6]

17. The two triangles are similar  $\angle A = \angle P, \angle B = \angle Q, \angle C = \angle R$ .

Find  $PQ$  and  $AC$ .



[6]

18. a) Copy and complete the table of values for the curve whose equation is  $y = 4x - x^2$ :

|        |    |   |   |   |    |   |     |
|--------|----|---|---|---|----|---|-----|
| $x$    | -1 | 0 | 1 | 2 | 3  | 4 | 5   |
| $x^2$  |    |   |   | 4 |    |   |     |
| $4x$   |    | 0 |   |   | 12 |   |     |
| $-x^2$ |    |   |   |   |    |   | -25 |
| $y$    |    |   |   |   |    |   |     |

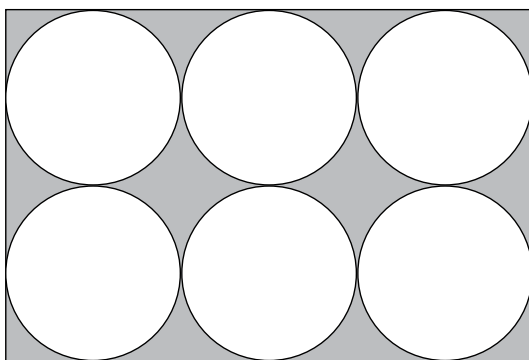
- b) Sketch the curve.  
c) State the name given to this kind of curve. [6]

19. Consider the sequence

6    9    14    21    30    41...

- a) Use the method of differences to find out what type of sequence it is.  
b) Find the formula for the  $n$ th term.  
c) What position in the sequence is the term 105? [6]

20. Six tennis balls are placed in a rectangular box. The plan view is shown in the diagram.



- a) If the radius of each ball is  $r$  cm, find a formula for the total area of the six circles in the diagram.  
b) Find a formula for the total area of the rectangle in terms of  $r$ .  
c) Find a formula for the shaded area.  
d) Factorise your shaded area formula. [8]

## Answers and Mark Scheme

1.
  - a)  $x > -1$
  - b)  $x \leq 2$
  - c)  $-3 \leq x \leq 2$
  - d)  $x < -2$  or  $x > 1$

[1 each = 4]
2.
  - a)  $x(x-5)$
  - b)  $7pq(6p+1)$
  - c)  $(b+2)(a+2b)$
$$\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} = 4$$
3.
  - a) 1 h 9 min
  - b) 87.5 km

[2 each = 4]
4.
  - a) 24
  - b) 61.5
  - c) 61
$$\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} = 4$$
5.
 
$$a = \begin{pmatrix} 6 \\ -1 \end{pmatrix} \quad b = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$$

$$c = \begin{pmatrix} 4 \\ 2 \end{pmatrix} \quad d = \begin{pmatrix} 6 \\ 0 \end{pmatrix}$$

[1 each = 4]
6.
  - a)  $K = \{(x, y): x \geq 1, y < 2\}$
  - b) the surface of a sphere, centre  $A$ , radius 50 cm

[2 each = 4]
7.
  - a) 13
  - b) 15

[2 each = 4]
8.
  - a) strong positive
  - b) weak negative
  - c) zero
  - d) strong negative

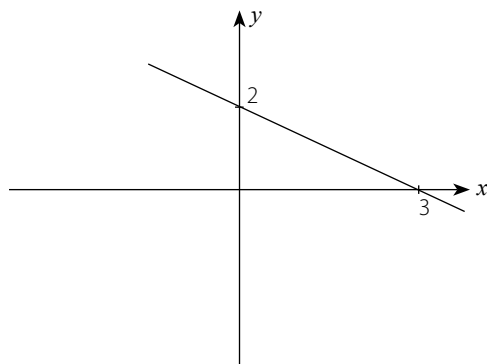
[1 each = 4]
9.
  - a)  $-5x^3$
  - b)  $2xy - 2x$
  - c)  $x^2 + 2x - 3$
$$\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} = 4$$
10.
  - a) false
  - b) true
  - c) true
  - d) true

[1 each = 4]

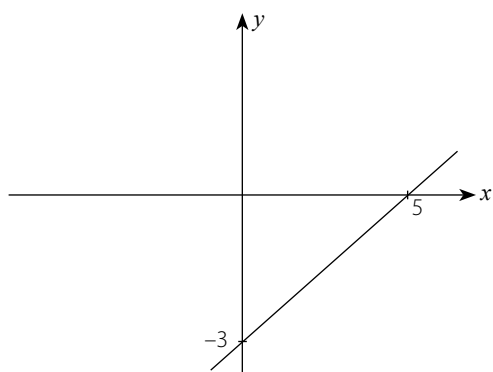
11. a) *GHJK*                      b) *ABLK*  
       c) *HJKG*                      d) *KMRD*

[1 each = 4]

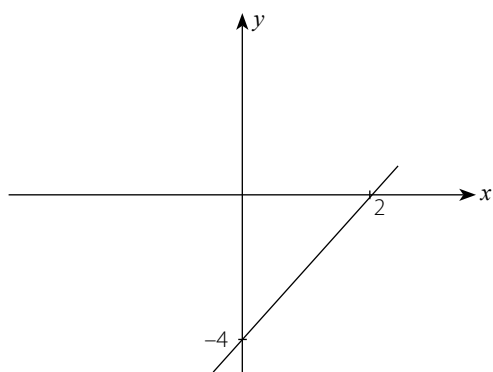
12. a)



- b)



- c)



[1 each intercept = 6]

13. Diagonal =  $\sqrt{0.25^2 + 0.75^2} = 0.7906 \text{ m}$   
 Total length =  $4 \times 0.7906 + 3 \times 1.5 + 3 \times 0.5$   
                   = 9.1623  
                   = 9.16 m           (nearest cm)

$$\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} = 6$$

14. a)  $3x = 2(x + 5) - 8$   
        $x = 2$   
        $y = 7$

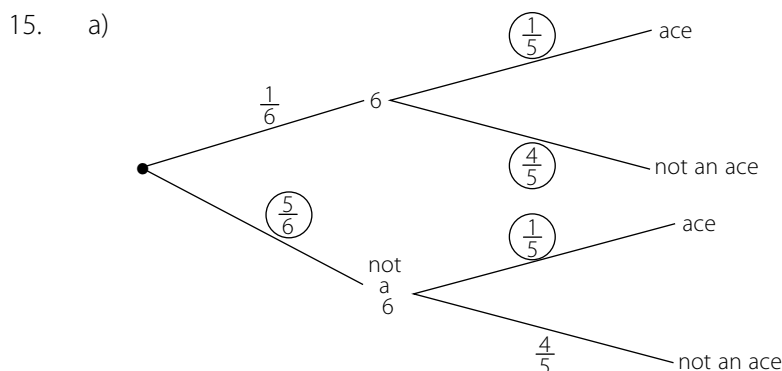
$$\begin{bmatrix} \text{sub 1} \\ \text{ans 1} \\ \text{ans 1} \end{bmatrix} = 3$$

b) ①  $x \times 3$ :  $6x + 15y = 15$   
 ②  $x \times 2$ :  $6x - 8y = 38$   
 Subtract:  $23y = -23$   
 $y = -1$   
 $x = 5$

$$\left[ \begin{array}{l} \text{Method 1} \\ \text{Ans 1} \\ \text{Ans 1} \end{array} \right] = 3$$

OR ①  $x \times 4$ :  $8x + 20y = 20$   
 ②  $x \times 5$ :  $15x - 20y = 95$   
 Add:  $23x = 115$   
 $x = 5$   
 $y = -1$

$$\left[ \begin{array}{l} \text{Method 1} \\ \text{Ans 1} \\ \text{Ans 1} \end{array} \right] = 3$$



$$\left[ \begin{array}{l} 1 \text{ each prob} = 4 \\ 1 \text{ each ans} = 2 \\ \hline = 6 \end{array} \right]$$

b)  $P(6, \text{ but not ace}) = \frac{2}{15}$ ,  $P(\text{ace, but not } 6) = \frac{1}{6}$

16. a)  $b = 5r - 2a$

b) 28

c) 17.4

[2 each = 6]

17.  $SF = \frac{11}{10} = 1.1$

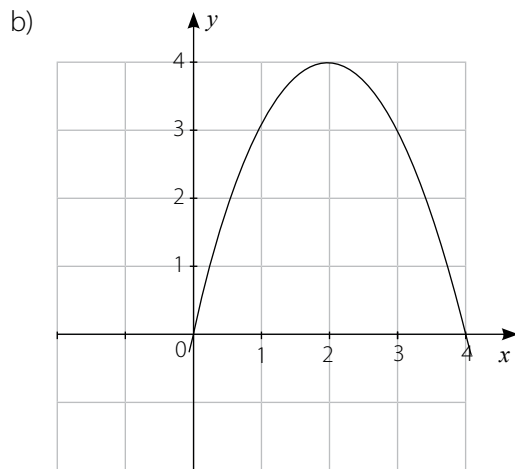
$PQ = 6 \times 1.1 = 6.6 \text{ cm}$

$AC = \frac{9.9}{1.1} = 9 \text{ cm}$

$$\left[ \begin{array}{l} \text{SF } 2 \\ 2 \\ 2 = 6 \end{array} \right]$$

18. a)

|                                          |    |   |    |    |    |     |     |
|------------------------------------------|----|---|----|----|----|-----|-----|
| $x$                                      | -1 | 0 | 1  | 2  | 3  | 4   | 5   |
| $x^2$                                    | 1  | 0 | 1  | 4  | 9  | 16  | 25  |
| $y \begin{cases} 4x \\ -x^2 \end{cases}$ | -4 | 0 | 4  | 8  | 12 | 16  | 20  |
|                                          | -1 | 0 | -1 | -4 | -9 | -16 | -25 |
| $y$                                      | -5 | 0 | 3  | 4  | 3  | 0   | -5  |



c) parabola

$$\left[ \begin{array}{l} \text{a) deduct 1 each error: 3} \\ \text{b) 1 shape, 1 intercepts: 2} \\ \text{c) 1} \end{array} \right] = 6$$

19. a) 6      9      14      21      30      21  
          3      5      7      9      11  
          2      2      2      2  
          quadratic type

b)  $n$ th term =  $n^2 + 5$

c) 10th

$$\left[ \begin{array}{l} 2 \text{ rows 1} \\ \text{type 1} \\ 2 \\ 2 \end{array} \right] = 6$$

20. a)  $6\pi r^2$   
      b)  $24r^2$   
      c)  $24r^2 - 6\pi r^2$   
      d)  $6r^2(4 - \pi)$

[2 each = 8]