Introduction
How to use this Guide

I. Selection of work and pacing
You will need to adjust your approach to using Book 7 of the International Secondary Maths series according to what went before. Clearly, those teachers who have used Book 6 in the previous year will find it easier.

However, in some school systems there are a lot of new admissions to Class VII and you may be faced with students who have had a variety of previous mathematical experiences.

Book 7 attempts to address both of these situations. Even when previous work is assumed, there are notes in the text reminding students about the key facts they need to know, so a newcomer to the topic may quickly connect with it.

This does mean that there is a degree of repetition in the book that may not be necessary for all students. Although revision is good, too much can become boring and lose the interest of more able students. The teacher must use professional judgement about selection of material and pacing. Some assistance is given in this Guide, chapter by chapter, but the best teachers quickly assess the abilities of their students and adapt tasks accordingly.

II. Integrated mathematics
This textbook series deliberately exploits connections between the different branches of mathematics: that is why it is not segregated into arithmetic, algebra, geometry, etc. Some explanation of this is given in this Guide, chapter by chapter, under the sub-heading Links.

III. Lesson planning
The subject teacher is best placed to write a lesson plan, taking into account the students' abilities and mathematical experience. Help is given here in the form of suggestions: these should assist you to write your own plans in the format required by the management of your school. The headings used here are as follows:

Objectives
- General objectives
- Specific objectives (sometimes called learning outcomes)

These are student-focused, i.e. they state what we desire the students to know, to learn, to do, by the end of the lesson.
Method
Also known as Strategy, Procedure, Techniques, or Methodology.
These are teacher focused, i.e. they state what the teacher should do to enable the students to learn.

Resources
Materials needed other than the textbook.
Photocopiable sheets to supplement the text are provided in this Guide, but other items will be needed from time to time.

Assignment
Keeping in mind professional judgement, advice is given about the suitability of certain exercises for homework, classwork, diagnostics, revision, etc.

Vocabulary
Keywords highlighted: ensure students understand their meanings.

IV. Bloom’s Taxonomy
Although the hierarchy of cognitive skills is now fairly well known (Bloom’s Taxonomy), teaching mathematics often requires operating at a number of levels simultaneously. Teachers may find this simplified version of more practical help than the rigorous theoretical framework:

Basic: Remember → Understand → Apply
Higher: Analyse → Create → Evaluate

In the Basics, students learn (memorize) facts until they can Remember (recall), Understand (explain, give examples, answer routine questions), and Apply (use the facts in standard situations).

For Higher skills, students deduce, distinguish, construct, organize, plan, modify, develop, draw relevant diagrams, etc. when faced with a new situation. These higher skills may collectively be called Problem-solving.

The textbook contains material at both Basic and Higher levels. Always ensure that all students can do the basics and push as many as possible towards the problem-solving higher levels of cognitive activity. Advice on this is provided in the Guide under Objectives.

V. The Exercises
Each chapter of the book has exercises coded as follow:

Exercises A, B, C are on the chapter topic by topic.
Exercises M are miscellaneous on the chapter theme.
Exercises X are challenging questions for more able students.

VI. Useful Sheets
Specimen Examination Paper [first half year]
Specimen Examination Paper [whole year]
3:4:5 Triangles
Coordinate Grids
Squared paper (9 mm)
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</table>
Although probability was introduced in Book 6, it is introduced here as a new concept. For this older age-group, more formal definitions are used. At the start of the year it is an appropriate time to emphasize greater precision in the use of mathematical language and logic words.

### LESSON PLANNING

#### Objectives

| General | To understand the meaning of probability and its scale, correctly calculating probabilities of outcomes to simple random events using correct notation |
| Specific | 1. To know and use correctly these words: event, outcome, equally likely, fair, probability |
| | 2. To calculate the probability (using fractions or decimals as appropriate) of the outcomes to simple random events |
| | 3. To use correctly the P( ) notation |
| | 4. To use tables to work out probabilities of outcomes of two independent events |

#### Pacing

2 lessons, 1 homework

#### Links

Fractions, decimals, logic

#### Method

Oral opening “Tell me something impossible”, leading to the scale of probability. Estimation examples, calculation examples using symmetry/fairness (balls in bag, etc.). “Fair” dice. Clarify “event” and “outcome”, with examples.

Introduce P( ) notation. “Mathematicians are lazy.” Students may have trouble with this because P is not a symbol for a number. It is the first time they will have seen a letter used functionally, so give this time to sink in.

P(red) is short for “The probability that the outcome is red”. Saves writing!

For two events (e.g. two dice), “What are the outcomes?” 2, 3, …, 12. But they are not equally likely. The table ensures all outcomes listed are equally likely.

#### Resources

A large dice, to demonstrate (from symmetry) that the six outcomes are equally likely (unless it’s weighted inside!).

Real coloured balls, marbles, etc. in bags to show that to be “fair” the objects have to be the same size (and you mustn’t peep when choosing!).

#### Assignments

EX 1A easy. Homework #6–10

EX 1B classwork (with discussion allowed)
**Vocabulary**
- event, outcome, equally likely, fair
- probability of $= P(\ )$
- and, or, at least, not less than

**ANSWERS**

**Exercises**

**EX 1A**

1. a) $\frac{3}{8}$  
   b) $\frac{1}{8}$  
   c) $\frac{1}{2}$  
   d) $\frac{7}{8}$

2. a) $\frac{1}{6}$  
   b) $\frac{1}{3}$  
   c) $\frac{1}{2}$  
   d) 0  
   e) $\frac{1}{2}$  
   f) 0

3. a) close to 1 (in most places)  
   b) close to 0  
   c) close to 1  
   d) 0  
   e) $\frac{1}{2}$  
   f) $\frac{2}{11}$  
   g) 0 or 1

4. a) $\frac{1}{3}$  
   b) $\frac{2}{3}$  
   c) $\frac{3}{5}$

5. a) $\frac{4}{9}$  
   b) $\frac{7}{9}$  
   c) 1  
   d) 0

6. a) $\frac{1}{8}$  
   b) $\frac{9}{16}$  
   c) $\frac{3}{16}$  
   d) $\frac{1}{8}$

7. a) $\frac{2}{11}$  
   b) $\frac{9}{11}$  
   c) $\frac{4}{11}$  
   d) $\frac{6}{11}$

8. Behram, because win, lose, draw are not equally likely.

9. $\frac{36}{61}$

10. a) $\frac{1}{4}$  
    b) $\frac{1}{4}$  
    c) $\frac{1}{4}$  
    d) $\frac{1}{2}$  
    e) 0

**EX 1B**

1. | 1 2 3 4 5 6 |
<table>
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<tbody>
<tr>
<td>1 0 1 2 3 4</td>
</tr>
<tr>
<td>2 1 0 1 2 3</td>
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<tr>
<td>3 2 1 0 1 2</td>
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<td>4 3 2 1 0 1</td>
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<tr>
<td>5 4 3 2 1 0</td>
</tr>
<tr>
<td>6 5 4 3 2 1</td>
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</tbody>
</table>

   a) $\frac{1}{6}$  
   b) $\frac{1}{4}$  
   c) $\frac{1}{9}$  
   d) $\frac{11}{36}$  
   e) $\frac{4}{9}$
Chapter 1 Chance

2. | 1 | 2 | 3 | 4 | 5 | 6 |
---|---|---|---|---|---|
R | R1 | R2 | R3 | R4 | R5 | R6 |
B | B1 | B2 | B3 | B4 | B5 | B6 |
G | G1 | G2 | G3 | G4 | G5 | G6 |

\[ \frac{1}{18} \quad \frac{1}{6} \quad \frac{2}{3} \quad 0 \quad \frac{2}{9} \]

3. | R | W | B | G |
---|---|---|---|
R | RR | WR | BR | GR |
W | RW | WW | BW | GW |
B | RB | WB | BB | GB |

\[ \frac{1}{4} \quad \frac{1}{2} \quad \frac{1}{12} \quad \frac{1}{6} \quad 0 \quad 1 \]

4. | R | G | B |
---|---|---|
R | RR | GR | BR |
W | RW | GW | BW |

\[ \frac{1}{2} \quad \frac{1}{3} \quad \frac{1}{6} \quad \frac{1}{6} \quad \frac{1}{3} \]

5. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
---|---|---|---|---|---|---|---|---|---|---|
3 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 |
4 | 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 |
5 | 50 | 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 |
6 | 60 | 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 |

\[ \frac{1}{10} \quad \frac{21}{40} \quad \frac{11}{20} \quad \frac{1}{5} \quad \frac{9}{40} \]

6. | 3 | 4 | 5 | 6 |
---|---|---|---|
3 | 6 | 7 | 8 | 9 |
4 | 7 | 8 | 9 | 10 |
5 | 8 | 9 | 10 | 11 |
6 | 9 | 10 | 11 | 12 |

\[ \frac{1}{8} \quad \frac{1}{4} \quad \frac{1}{8} \quad 0 \quad \frac{1}{3} \]

7. | Ph | Ch | Bi |
---|---|---|---|
Ma | PhMa | ChMa | BiMa |
En | PhEn | ChEn | BiEn |
Ac | PhAc | ChAc | BiAc |
Co | PhCo | ChCo | BiCo |

\[ \frac{1}{4} \quad \frac{1}{4} \]

[move to the next page]
8. | Red  | 2 | 3 | 5 |
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</table>

   a) \( \frac{1}{6} \)  
   b) \( \frac{1}{12} \)  
   c) \( \frac{3}{4} \)  
   d) \( \frac{1}{3} \)  
   e) 1

Blue

9. | Red  | 1 | 2 | 3 | 4 | 5 | 6 |
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<td>10</td>
<td>12</td>
<td>14</td>
<td>16</td>
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</tr>
</tbody>
</table>

   a) \( \frac{1}{12} \)  
   b) 0  
   c) \( \frac{1}{36} \)  
   d) \( \frac{1}{2} \)  
   e) \( \frac{7}{36} \)

10. a) \( \frac{1}{36} \)  
    b) \( \frac{1}{3} \)  
    c) \( \frac{1}{9} \)  
    d) \( \frac{5}{12} \)  
    e) \( \frac{5}{12} \)

EX 1X

1. H H H   H T H   T H H   T T H
   H H T   H T T   T H T   T T T

   a) \( \frac{1}{8} \)  
   b) \( \frac{3}{8} \)  
   c) \( \frac{3}{8} \)  
   d) \( \frac{1}{8} \)  
   e) \( \frac{1}{2} \)

2. a) \( \frac{1}{2} \)  
    b) \( \frac{1}{10} \) each  
    c) \( \frac{7}{10} \)  
    d) \( \frac{3}{10} \)  
    e) \( \frac{3}{10} \)

3. a) The total is more than 100%.
    b) green 5%
    c) i. 0.45
        ii. 0.65
Avoid the temptation to rush straight into the formula. The key to understanding for most students is visual. You need visual aids to teach this well.

**LESSON PLANNING**

**Objectives**

<table>
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<th>General</th>
<th>To calculate correctly the areas of parallelograms, triangles, and trapeziums from given data</th>
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<tr>
<td>Specific</td>
<td>1. To understand that the formula for the area of a rectangle can be adapted to give us formulas for parallelogram, triangle, and trapezium</td>
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<td></td>
<td>2. To calculate areas of these shapes; to calculate lengths given areas</td>
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</table>

**Pacing**

At least 4 lessons, 1 homework

**Links**

Symmetry, coordinates

**Method**

Practical demonstration. Prepare visual aids. Make sure they are large enough for all to see. (Just drawing on the board has limited impact because the shapes do not move.) Ideas follow:
a) Parallelogram

Take a large cardboard parallelogram. Cut the right-angled triangle from one end and move it to the other.

*Is the area the same? Yes*

*Is the height the same? Yes*

*Now look at the base.* (Show “before” and “after”)

*Is it the same length? Yes*

So the area of the parallelogram = area of the rectangle = base \( \times \) height

b) Parallelogram

Take a stack of exercise books and pile them up neatly facing the class. What is the
area of the facing rectangle? Point out the base and height. Now shear the stack. 

Is the area the same? Yes

If some bright child points out the edges are not straight now (correctly!), then say, “Imagine the books are really thin.”

c) Triangle
Take two large congruent cardboard triangles. Place them together to show that their areas are the same. Now Blu-tack them to the board (or pin them to a pinboard) to make a parallelogram.

The area of one triangle is half the area of the parallelogram.

d) Triangle
Demonstrate the rotational symmetry of the parallelogram by drawing around the outline of a large cardboard cutout, and then rotating it to fit the outline. Then draw a diagonal on your parallelogram and repeat, showing how the two triangles change places. Hence, area of triangle is half that of the parallelogram.

e) Trapezium
Take two large, congruent, cardboard trapeziums. Fit them together to show that their areas are the same. Then (by rotating one) place them to form a parallelogram (whose area formula is known).

Area parallelogram $= (a + b)h$

Area trapezium $= \frac{1}{2} (a + b)h$

Note: Take only one shape per lesson, and set the exercise in the text immediately.

Resources
Demo material (See Method.)
Coordinate grids (photocopiable sheets available here after the answers)
Calculators for EX 2C, 2M, 2X

Assignments
EX 2A, 2B, 2C in class
EX 2M homework [all quite time-consuming]
Vocabulary
rectangle, parallelogram, triangle, trapezium
perpendicular height, altitude
rotational symmetry, line symmetry
quadrilateral, hexagon, rhombus, isosceles

ANSWERS

Exercises

EX 2A

1. a) 20 cm²  b) 18 cm²  c) 120 cm²  d) 54 cm²
2. a) 24 cm²  b) 55 cm²  c) 80 cm²  d) 36 cm²
3. a) 45 cm²  b) 36 cm²  c) 35 cm²  d) 66 cm²
4. a) 12 cm²  b) 24 cm²  c) 32 cm²  d) 34 cm²
5. a) 6 unit²  b) 20 unit²
6. a) 27 unit²  b) 4 unit²
7. a) \( a = 10 \text{ cm} \)  
b) \( b = 5 \text{ cm} \)  
c) \( c = 3 \text{ cm} \)  
d) \( x = 2 \text{ cm}, y = 12 \text{ cm} \)
8. a) 12 cm  
b) 16 cm  
9. 53 cm\(^2\)  
10. a) \( x = 9 \text{ cm} \)  
b) \( y = 4.8 \text{ cm} \)

**EX 2B**

1. a) 27 cm\(^2\)  
b) 3 cm\(^2\)  
c) 48 cm\(^2\)  
d) 27.5 cm\(^2\)  
2. a) 84 cm\(^2\)  
b) 42 cm\(^2\)  
c) 15 cm\(^2\)  
d) 31.5 cm\(^2\)  
3. a) \( h = 4 \text{ cm} \)  
b) \( b = 16 \text{ cm} \)  
c) \( p = 6 \text{ cm} \)  
d) \( x = 2 \text{ cm} \)  
4. a) \( f = 10 \text{ cm} \)  
b) \( g = 6 \text{ cm} \)  
c) \( h = 30 \text{ cm} \)  
d) \( i = 3 \text{ cm} \)  
5. 

   ![Diagram](attachment:triangle.png) 
   a) 6 unit\(^2\)  
   b) 7.5 unit\(^2\)  

6. 

   ![Diagram](attachment:rectangle.png) 
   a) 4 unit\(^2\)  
   b) 28 unit\(^2\)
7. $39 \text{ cm}^2$

8. a) $l = 12 \text{ cm}$  
    b) $d = 8 \text{ cm}$

9.

10. Area $A = 24 \text{ cm}^2$, Area $B = 50 \text{ cm}^2$, Area $C = 33 \text{ cm}^2$, total area $= 107 \text{ cm}^2$

EX 2C

1. a) $17.5 \text{ cm}^2$  
    b) $12 \text{ cm}^2$  
    c) $84 \text{ cm}^2$  
    d) $30.5 \text{ cm}^2$

2. a) $42 \text{ cm}^2$  
    b) $42.5 \text{ cm}^2$  
    c) $155.25 \text{ cm}^2$  
    d) $96.6 \text{ cm}^2$

3. a) $8 \text{ unit}^2$  
    b) $19.5 \text{ unit}^2$
4. a) 21 unit²
   b) 30 unit²

5. a) Area $A = 132.3 \text{ m}^2$, Area $B = 193.2 \text{ m}^2$, Area $C = 112 \text{ m}^2$
   b) 437.5 m²
   c) Rs 21,875

6. 15.45 m²

7. a) 60 cm²
   b) 156.24 cm²
   c) 52.5 cm²
   d) 104.5 cm²

8. 17.405 m²

9. a)
   b) 50.5 unit²
10. a)  

b)  

c)  

d)  

[(c) and (d) interchangeable]

EX 2M

1. a) $p = 6 \text{ cm}$  
b) $q = 4 \text{ cm}$  
c) $r = 3 \text{ cm}$  
d) $s = 4 \text{ cm}$

2. a) $21.6 \text{ cm}^2$  
b) $81.78 \text{ m}^2$  
c) $216 \text{ cm}^2$  
d) $51.12 \text{ m}^2$

3. a) $69.9 \text{ cm}^2$  
b) $14.025 \text{ m}^2$  
c) $29.9 \text{ cm}^2$  
d) $86.94 \text{ m}^2$

4. 

b) 12 unit$^2$
   28 unit$^2$
   18 unit$^2$
   20 unit$^2$
   7 unit$^2$
Total = 85 unit$^2$

5. 66 cm$^2$

6. 49 cm$^2$

7. a) 16 cm$^2$  
b) 2 cm$^2$  
c) 1.5 cm$^2$  
d) 8 cm$^2$
Chapter 2 Areas

8. 

![Graph with points A,B,C,D,E,F,G]

\[
\begin{align*}
7 \text{ unit}^2 \\
14 \text{ unit}^2 \\
54 \text{ unit}^2 \\
\text{Total} & = 75 \text{ unit}^2
\end{align*}
\]

9. 14.5 cm²

10. 16 cm²

**EX 2X**

1. 38 unit²

2. 35.935 cm²

3. 1584 mm²
Despite the chapter title this is really about algebra. The arrow diagrams are a method of enabling students to understand linear algebraic expressions and their equivalents.

**LESSON PLANNING**

**Objectives**

<table>
<thead>
<tr>
<th>General</th>
<th>To understand simple linear functions</th>
</tr>
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</table>
| Specific| 1. To draw the arrow diagram that represents a function  
2. To calculate input values, output values, or state the rule of a function, given sufficient data  
3. To represent correctly function rules using algebraic expressions  
4. To use arrow diagrams to obtain equivalent algebraic expressions  
5. To evaluate linear algebraic expressions for given values of an unknown  
6. To use the distributive law (instinctively)  
7. To interpret and devise “Think of a number” puzzles, using algebra (Higher skills) |

<table>
<thead>
<tr>
<th>Pacing</th>
<th>3 lessons, 1 homework</th>
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<table>
<thead>
<tr>
<th>Links</th>
<th>BODMAS</th>
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**Method**

Straightforward boardwork: examples and eliciting feedback until students have grasped the technique.

Exercise 3A is relatively easy, even for students who have never seen an arrow diagram before. Define and classify input, output, and rule. The algebraic expression of a rule is a function. This word is not used in the text, but should be taught, e.g.  

\[ n \rightarrow 2n + 3 \]  

represents the function “double and then add 3”.

Use EX 3A.

Evaluation and equivalents can be taken together as in the text. Having established arrow diagrams the next step is to wean them off, but go back whenever there is confusion.

For example, to evaluate \( 2x - 5 \), think of the arrow diagram that would produce it:

\[ x \rightarrow 2 \times 2 \rightarrow 2x \rightarrow -5 \rightarrow 2x - 5 \]

That is, first double, then subtract 5.

Use EX 3B, etc.
The puzzle section is fun and self-explanatory. Students need to talk, and try their puzzles out on each other.

Use EX 3C.

**Assignments**
EX 3C is best done in class. Any part of the rest may be set as homework.

**Vocabulary**
function, input, output, rule
becomes ($\rightarrow$)
reverse the flow, inverse operation

**ANSWERS**

**Exercises**

**EX 3A**

1. a)  
   \[
   \begin{array}{c}
   1 \\
   \rightarrow +10 \\
   \rightarrow 11
   \end{array}
   \]

   b)  
   \[
   \begin{array}{c}
   0.5 \\
   \rightarrow \times2 \\
   \rightarrow 1 \\
   \rightarrow +1 \\
   \rightarrow 2
   \end{array}
   \]

   c)  
   \[
   \begin{array}{c}
   18 \\
   \rightarrow \times2 \\
   \rightarrow 36 \\
   \rightarrow +9 \\
   \rightarrow 4 \\
   \rightarrow +1 \\
   \rightarrow 5
   \end{array}
   \]

   d)  
   \[
   \begin{array}{c}
   2 \\
   \rightarrow \frac{1}{2} \\
   \rightarrow 2 \frac{1}{2} \\
   \rightarrow \times2 \\
   \rightarrow 5
   \end{array}
   \]

2. a)  
   \[
   \begin{array}{c}
   11 \\
   \rightarrow +6 \\
   \rightarrow 17
   \end{array}
   \]

   b)  
   \[
   \begin{array}{c}
   2 \\
   \rightarrow +5 \\
   \rightarrow 7 \\
   \rightarrow \times2 \\
   \rightarrow 14
   \end{array}
   \]

   c)  
   \[
   \begin{array}{c}
   18 \\
   \rightarrow -8 \\
   \rightarrow 10 \\
   \rightarrow +5 \\
   \rightarrow 2 \\
   \rightarrow +1 \\
   \rightarrow 3
   \end{array}
   \]

   d)  
   \[
   \begin{array}{c}
   6 \\
   \rightarrow \times2 \\
   \rightarrow 12 \\
   \rightarrow +4 \\
   \rightarrow 3
   \end{array}
   \]

3. a)  
   \[
   \begin{array}{c}
   6 \\
   \rightarrow \rightarrow \rightarrow 8 \\
   10 \\
   \rightarrow \rightarrow \rightarrow 10 \\
   4 \\
   \rightarrow \rightarrow \rightarrow 7 \\
   40 \\
   \rightarrow \rightarrow \rightarrow 25
   \end{array}
   \]

   b)  
   \[
   \begin{array}{c}
   12 \\
   \rightarrow \rightarrow \rightarrow 2 \\
   11 \\
   \rightarrow \rightarrow \rightarrow 1.5 \\
   9 \\
   \rightarrow \rightarrow \rightarrow 0.5 \\
   6 \\
   \rightarrow \rightarrow \rightarrow -1
   \end{array}
   \]
Chapter 3 Arrow Diagrams

4. a) \[ a \rightarrow \times 2 \rightarrow 2a \rightarrow ^{-1} \rightarrow 2a - 1 \]
   
   b) \[ b \rightarrow +1.5 \rightarrow b + 1.5 \rightarrow \times 2 \rightarrow 2(b + 1.5) \]
   
   c) \[ c \rightarrow +1 \rightarrow c + 1 \rightarrow ^{+2} \rightarrow \frac{c + 1}{2} \]
   
   d) \[ d \rightarrow ^{÷3} \rightarrow \frac{d}{3} \rightarrow ^{-8} \rightarrow \frac{d}{3} - 8 \]

5. a) \[ a \rightarrow \times 3 \rightarrow 3a \rightarrow ^{-5} \rightarrow 3a - 5 \]
   
   b) \[ b \rightarrow \times 0.5 \rightarrow 0.5b \rightarrow ^{+6} \rightarrow 0.5b + 6 \]
   
   c) \[ c \rightarrow ^{-1} \rightarrow c - 1 \rightarrow \times 2 \rightarrow 2(c - 1) \]
   
   d) \[ d \rightarrow ^{+6} \rightarrow d + 6 \rightarrow ^{÷5} \rightarrow \frac{d + 6}{5} \]

6. AQ BS CP DR
7. AQ BR CP DS
8. AR BQ CS DP
9. a) \[ r \rightarrow 3r \rightarrow 3r - 7 \]
   b) \[ s \rightarrow \frac{s}{5} \rightarrow \frac{s}{5} + 1 \]
   c) \[ t \rightarrow t + 2 \rightarrow \frac{t + 2}{3} \]
   d) \[ u \rightarrow u - 4 \rightarrow 2(u - 4) \]

10. a) \[ n \rightarrow 3n \rightarrow 3n - 2 \rightarrow 2(3n - 2) \rightarrow 2(3n - 2) - 3 \]

   b) \[ n \rightarrow 6n \rightarrow 6n - 7 \]

   c) input \quad output

   \begin{align*}
   1 & \rightarrow -1 \\
   2 & \rightarrow 5 \\
   3 & \rightarrow 11
   \end{align*}

   Same for (a) and (b), whatever the input.

**EX 3B**

1. a) 1 \hspace{1cm} b) -4 \hspace{1cm} c) 1 \hspace{1cm} d) 1
2. a) 3 \hspace{1cm} b) 9 \hspace{1cm} c) 18 \hspace{1cm} d) 63
3. a) 2 \hspace{1cm} b) 8 \hspace{1cm} c) 8 \hspace{1cm} d) 2
4. Outputs are equivalent

5. Outputs are equivalent

6. a) $3(a - 1) = 3a - 3$
   b) $4b + 20 = 4(b + 5)$
   c) $5(c - 4) = 5c - 20$
   d) $12d - 18 = 6(2d - 3)$

7. a) $\frac{2a - 12}{2} = a - 6$
   b) $b - 5 = \frac{3b - 15}{3}$
   c) $\frac{16c - 20}{4} = 4c - 5$
   d) $\frac{7d + 7}{7} = d + 1$

8. a) $5x + 5$
   b) $12x - 18$
   c) $7x - 3.5$
   d) $0.5x + 1$
   e) $24x + 6$
   f) $1.5x + 15$
   g) $4 + 2x$
   h) $7 - 3.5x$
   i) $2x + 2y$
   j) $8x + 4y$
   k) $8 - 16x$
   l) $63 + 27x$

9. a) $x + 5$
   b) $x - 8$
   c) $2x - 0.5$
   d) $1.5x + 2$
   e) $2x - 1.5$
   f) $2x + 0.25$
   g) $1 + \frac{x}{2}$
   h) $4 - \frac{x}{7}$
   i) $2x + \frac{y}{2}$
   j) $\frac{x - y}{2 - 4}$
   k) $2 - \frac{x}{4}$
   l) $6 - \frac{x}{3}$

10. AH BI CE DF G (odd)

EX 3C

1. a) $n$
   b) $3n$
   c) $3n + 6$
   d) $\frac{3n + 6}{3} = n + 2$
   e) $n + 2 + 4 = n + 6$
   f) $n + 6 - n = 6$
   g) 6 (Ans)
2. • Think of a number.
   • Double it.
   • Add 40.
   • Divide by 2.
   • Take away the number first thought of.
   • Your answer is 20.

3. • \( n \)
   • \( 5n \)
   • \( 5n + 25 \)
   • \( \frac{5n + 25}{5} = n + 5 \)
   • \( n + 5 - 5 = n \)
   • \( n \) (Ans)

4. • Think of a number.
   • Multiply by 3.
   • Add 24.
   • Divide by 3.
   • Take away the number first thought of.
   • Your answer is 8.

EX 3X

1. a) \( 8 - 8y \) b) \( 2a - a^3 + 2a^3 \)
   c) \( 4x^2 - 8y + 2xy \) d) \( 3c + 2 \)

2. a) \( -1 \) b) \( 5.24 \) c) \( 6 \) d) \( 15 \)

3. Differences are always multiples of 3.
   Difference = \( (5n + 2) - (2n + 5) = 3n - 3 = 3(n - 1) \),
   which is always a multiple of 3.
This is largely revision, but there are more questions here on 2 and 3 d.p. Also, significant figures are introduced for the first time, and can cause difficulty. Give time to this topic.

### LESSON PLANNING

#### Objectives

<table>
<thead>
<tr>
<th>General</th>
<th>To round off numbers up to 3 d.p. or 3 s.f., and use of 1 s.f. estimation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specific</td>
<td>1. To round off correctly to the nearest whole number, 1 d.p., 2 d.p., or 3 d.p.</td>
</tr>
<tr>
<td></td>
<td>2. To be able to identify the significant digits in a number, and to round off correctly up to 3 s.f.</td>
</tr>
<tr>
<td></td>
<td>3. To use 1 s.f. estimations to simplify a calculation and find the approximate answer</td>
</tr>
<tr>
<td></td>
<td>4. To use the ( \approx ) sign correctly when estimating</td>
</tr>
<tr>
<td></td>
<td>5. (Higher skills) To solve word problems involving estimation and/or rounding off</td>
</tr>
</tbody>
</table>

#### Pacing

- 2 lessons
- 1 homework

#### Method

- Oral introduction. Remind students that
  - tenths have 1 d.p.
  - hundredths have 2 d.p.
  - thousandths have 3 d.p.

- Revise rounding off technique, following the textbook.

- Significant figures is not so easy.
  - “What/who is significant to you?”
  - “What does ‘significant’ mean?” Important, valuable.
  - “In this number which digit has the greatest value?”

  e.g. 4215
  
  6.35
  
  12.999
  
  0.024

- Only move on when the most significant digit can be generally identified. Then follow the text for counting the figures and rounding off.

- Use EX 4A.

- We normally round off at the end of a calculation.
We normally estimate before the calculation, as a check on our answer. Give examples, estimate to 1 s.f. each item, then calculate as carefully as possible—no wild approximations allowed. Introduce the \( \approx \) sign. Set EX 4B.

**Assignments**
Can use EX 4B (even numbers) for homework.
EX 4A and EX 4B (odd numbers) in class

**Vocabulary**
significant figures (s.f.), digits
approximately equal (=)

### ANSWERS

**Exercises**

**EX 4A**

[Some of these can be used as an oral exercise.]

1. a) 248 m  b) 126 m  c) 73°  d) 19 kg
   e) 75 s  f) 48 km/h  g) 2476 cm²  h) 1 mm
   i) 251 ml  j) 477 km²  k) 3 m³  l) 3189 g

2. a) 1  b) 3  c) 3  d) 2
   e) 1  f) 6  g) 3  h) 3
   i) 3  j) 1  k) 0  l) 3

3. a) 7.6  b) 6.5  c) 28.6  d) 0.1
   e) 9.8  f) 9.0  g) 7.8  h) 10.0

4. a) 6.43  b) 0.04  c) 2.04  d) 0.01
   e) 3.14  f) 7.04  g) 16.05  h) 201.12

5. a) 9.877  b) 8.765  c) 7.654  d) 6.012
   e) 15.038  f) 12.040  g) 0.030  h) 2.718

6. a) 2  b) 2  c) 3  d) 3
   e) 3  f) 2  g) 3  h) 4
   i) 4  j) 4  k) 4  l) 3

7. a) 3  b) 2  c) 0.1  d) 0.008
   e) 0.0007  f) 0.7  g) 1  h) 0.08

8. a) 3.8  b) 2.1  c) 0.61  d) 0.0059
   e) 0.00018  f) 0.80  g) 1.5  h) 0.072
9. a) 84.1  
   b) 8.42  
   c) 0.841  
   d) 0.488  
   e) 48.5  
   f) 4.86  
   g) 0.402  
   h) 0.512  
10. a) 0.057  
    b) 0.06  
    c) 0.1  
    d) 0.0573  
    e) 0.057  
    f) 0.06

**EX 4B**

[Calculators strictly forbidden]

<table>
<thead>
<tr>
<th></th>
<th>a) 1200</th>
<th>b) 12000</th>
<th>c) 270000</th>
<th>d) 9000000</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.</td>
<td>a) 14000</td>
<td>b) 1000</td>
<td>c) 4</td>
<td>d) 9</td>
</tr>
<tr>
<td></td>
<td>a) 2</td>
<td>b) 3</td>
<td>c) 10</td>
<td>d) 400</td>
</tr>
<tr>
<td></td>
<td>a) 0.5</td>
<td>b) 1.5</td>
<td>c) 0.1</td>
<td>d) 0.5</td>
</tr>
<tr>
<td></td>
<td>a) 4</td>
<td>b) 3</td>
<td>c) 6</td>
<td>d) 20</td>
</tr>
<tr>
<td></td>
<td>a) 141.183</td>
<td>b) 7.995</td>
<td>c) 51.3</td>
<td>d) 1.9375</td>
</tr>
</tbody>
</table>

7. 20  
8. 2400 m²  
9. 1 mm

10. a) 8000  
    b) Below

**EX 4X**

<table>
<thead>
<tr>
<th></th>
<th>a) 0.2; 0.19</th>
<th>b) 12.2; 10</th>
<th>c) 0.25; 0.24</th>
<th>d) 0.14; 0.12</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.</td>
<td>a) 0.9 cm; 0.725 cm</td>
<td>b) 125 cm³; 97.336 cm³</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td>Calculator answers</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2a) 15459  
3a) 2  
    b) 1381  
    c) 6  
    d) 9

3b) 3  
3c) 12  
3d) 401

Estimates can be trusted to give the correct position of the decimal point. Sometimes they are very close to the exact answer, but not always.
Most of this chapter should have been met before. However, many students name angles incorrectly: make sure that they get this straight. Use the mathematical word “vertex” rather than corner. When measuring with a protractor, establish the routine of finding the zero of the scale to be used.

**LESSON PLANNING**

**Objectives**

<table>
<thead>
<tr>
<th>General</th>
<th>To measure and calculate angles, using simple angle facts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specific</td>
<td>1. To name angles correctly</td>
</tr>
<tr>
<td></td>
<td>2. To distinguish between acute, obtuse, right, straight, and reflex angles</td>
</tr>
<tr>
<td></td>
<td>3. To measure angles with a protractor (to an accuracy of ± 1°)</td>
</tr>
<tr>
<td></td>
<td>4. To identify interior and exterior angles of a triangle</td>
</tr>
<tr>
<td></td>
<td>5. To know the facts about adjacent angles on a straight line, angle-sum of a triangle, and vertically opposite angles</td>
</tr>
<tr>
<td></td>
<td>6. To know the 8 main points of the compass, and 3-figure bearings, and use them to solve word problems</td>
</tr>
</tbody>
</table>

| Pacing | 3 lessons, 1 homework |
| Links | Algebra (using small letters for the size of angles) |

**Method**

Child standing with arms outstretched:

The two arms form an angle!

His/her head is the vertex!!

Like people, angles have short names and full names.

Follow the text for naming.

Types of angles should take only a few minutes. The text can be referred to if anyone has forgotten.

Spend time on measuring. Show how to choose the correct protractor scale to read, using either a board protractor or on an OHP. Many students are careless about this.

Use EX 5A.

For the angle facts, if they haven’t seen it before, ripping off three vertices of a triangle and placing them on a line is the classic demo for the angle-sum being 180°.
For vertically opposite, crossed rulers or a pair of scissors work well. Moving them shows how opposite pairs match.

Bearings can be introduced as the modern digital way of showing direction, suitable for computerised controls in ships and aeroplanes. Follow the text. Use EX 5B.

**Resources**

- Protractors, sharp pencils (H preferred to HB)
- Large demo protractor, or OHP
- Cardboard triangle to show angle-sum (See Method.)
- Two large rulers to show vert opp \( \angle \)s (See Method.)
- Compass card (borrowed from Geography department?)

**Assignments**

Possible homeworks: EX 5A #6–10 or EX 5B #6–10

**Vocabulary**

- vertex, arm
- acute, right, obtuse, straight, reflex, revolution
- interior, exterior, adjacent
- angle-sum of a triangle
- vertically opposite
- points of the compass, 3-figure bearing

**ANSWERS**

**Exercises**

**EX 5A**

1. a) \(64^\circ, 32^\circ, 72^\circ, 5^\circ\)  
   b) \(100^\circ, 127^\circ, 172^\circ\)  
   c) \(180^\circ \text{ straight}, 200^\circ \text{ reflex}, 90^\circ \text{ right}, 300^\circ \text{ reflex}\)
2. a) \(90^\circ\)  
   b) \(306^\circ, 205^\circ, 280^\circ\)  
   c) \(73^\circ, 46^\circ, 65^\circ\)  
   d) \(181^\circ, 170^\circ\)
3. a) $\angle ACB$ or $\angle BCA$  b) $\angle FBG$ or $\angle GBF$
   c) $\angle CAD$ or $\angle DAC$  d) $\angle QSR$ or $\angle RSQ$

4. a) 90°  b) 50°  c) 40°  d) 120°

5. a) 30°  b) 70°  c) 60°  d) 90°

6. a) $\angle KHB$ or $\angle BHK$  b) $\angle CGH$ or $\angle CGK$ or $\angle HGC$ or $\angle KGC$
   c) $\angle DCF$ or $\angle GCF$ or $\angle GCE$ or $\angle DCE$ or $\angle FCD$ or $\angle FCG$ or $\angle ECG$ or $\angle ECD$
   d) $\angle CFG$ or $\angle CFH$ or $\angle CFK$ or $\angle GFC$ or $\angle HFC$ or $\angle KFC$

7. a) 54°  b) 128°  c) 18°  d) 144°

8. a) acute; estimate; 24°  b) obtuse; estimate; 101°
   c) acute; estimate; 81°  d) obtuse; estimate; 163°

9. a) 336°  b) 206°  c) 296°  d) 257°

10. a)
EX 5B

1. a) \( \angle CBD = 153^\circ \) b) \( \angle PQT = 120^\circ \)  
   c) \( \angle XYW = 140^\circ \) d) \( \angle LNM = 80^\circ \)  
2. a) \( \angle ACB = 60^\circ \) b) \( \angle GCB = 55^\circ \)  
   c) \( \angle ABD = 35^\circ \) d) \( \angle ABD = 126^\circ \)  
3. \( a = 65, b = 25, c = 25, d = 155, e = 5, f = 175 \)  
4. \( a = 25, b = 85, c = 95, d = 65, e = 65, f = 90 \)  
5. a) 270° b) 135° c) 045° d) 225°  
6. a) S b) S60°E c) NW d) N  
7. a) 080° b) 340°  
8. a) 040° b) 120° c) 80° d) 100°  
9. a) 120° b) 45°  
10. \( a = 45, b = 80, c = 80, d = 100, e = 38, f = 40, g = 102 \)

EX 5X

1. [drawing]  
2. 80° and 80°, or 20° and 140°  
3. [research and poster]
The exercises revise the first five chapters and work covered in previous books.

ANSWERS

Exercises

EX 6A

1. a) 0.229; 0.229 + 0.229 = 0.458
   b) 5.7155; 5.7155 ÷ 1.61 = 3.55
   c) 2.4; 2.4 × 1.061 = 2.5464
   d) 80.24; 80.24 – 5.44 = 74.8

2. a) 0.08 mm² b) 175 mm² c) 8.41 cm² d) 0.85 cm²

3. a) \(\frac{1}{2}\) b) \(\frac{1}{3}\) c) \(\frac{1}{6}\) d) 0
   e) \(\frac{2}{3}\)

4. a) 9 cm² b) 260 cm² c) 35 cm² d) 55 cm²

5. a) \(\overline{14} + 76 = \overline{90}\)
   b) \(\overline{0.7} \times 10 = \overline{7} + 1 = \overline{8}\)
   c) \(\overline{21} \times 2 = \overline{42} + 7 = \overline{6} \div 3 = \overline{3}\)
   d) \(\overline{1} + \frac{1}{4} = \overline{1\frac{1}{4}} \times 4 = \overline{5}\)

6. a) False. In a kite there is one line of symmetry.
   b) True
   c) False. In a parallelogram opposite pairs of angles are equal.
   d) True
7. a) 57  
   b) 186  
   c) 105  
   d) 1688  
8. a) 0.44  
   b) 0.38  
   c) 5.47  
   d) 0.19  
   e) 10.20  
   f) 3.01  
9. a) $\angle DBC$ and $\angle CBD$  
    b) $\angle COD$ and $\angle DOC$  
    c) $\angle BCA$ and $\angle ACB$  
    d) $\angle BDC$ and $\angle CDB$  
10. 
<table>
<thead>
<tr>
<th>Red</th>
<th>White</th>
<th>Blue</th>
<th>Yellow</th>
</tr>
</thead>
<tbody>
<tr>
<td>Head</td>
<td>HR</td>
<td>HW</td>
<td>HB</td>
</tr>
<tr>
<td>Tail</td>
<td>TR</td>
<td>TW</td>
<td>TB</td>
</tr>
</tbody>
</table>
   a) $\frac{1}{8}$  
   b) $\frac{1}{2}$  
   c) $\frac{3}{4}$  
   d) $\frac{1}{4}$  
   e) 0  

EX 6B  
1. a) 1  
   b) 1  
   c) 1  
   d) 4  
2. a) 26 cm$^2$  
   b) 20 cm$^2$  
   c) 9 cm$^2$  
   d) 27 cm$^2$  
3. AR  BS  CQ  DP  
4. a) 13  
   b) 20  
   c) 50  
   d) 10  
5. a) 4  
   b) 3  
   c) 2  
   d) 4  
6. 11, 13, 17, 19, 23, 29  
7. a) $\angle DCA = 143^\circ$  
    b) $\angle ABE = 68^\circ$  
    c) $\angle ABD = 137^\circ$  
    d) $\angle BDC = 82^\circ$  
8. 
<table>
<thead>
<tr>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
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<td>9</td>
<td>10</td>
<td>11</td>
<td>12</td>
<td>13</td>
</tr>
</tbody>
</table>
   a) $\frac{1}{18}$  
   b) $\frac{5}{12}$  
   c) $\frac{1}{2}$  
   d) 0  
   e) $\frac{1}{3}$  
   d) $\frac{1}{3}$  
   9. a) 92 cm$^2$  
   b) 42 cm$^2$  
   c) 45 cm$^2$  
   d) 36 cm$^2$  
10. (–1, 2), (1, 2), (1, 4) and (–1, 4)  

EX 6C  
1. a) 4  
   b) 5  
   c) $\frac{1}{3}$  
   d) 7  
2. 21, 28, 35, 42, 49  
3. a) $\frac{5 \times 4}{9}$  
   b) $\frac{2}{5}$  
   c) $\frac{334}{45}$  
   d) $\frac{1\times 4}{45}$
4.  a) 2.7  b) 1.5  c) 2.2  d) 0.063  
    e) 0.00012  f) 0.80  g) 1.6  h) 0.083
5.  \(a = 60, b = 30, c = 30, d = 150, e = 15, f = 165\)
6.  \[
\begin{align*}
\text{a)} & \quad \text{Area} = 6 \text{ unit}^2 \\
\text{b)} & \quad \text{Area} = 20 \text{ unit}^2
\end{align*}
\]
7.  a) median = 7.5  range = 8  
    b) median = 6  range = 98  
    c) median = 2.6  range = 5.7  
    d) median = 1.0  range = 1.0
8.  a) \(5a - b\)  b) \(5a - 6b\)  c) \(30a\)  d) \(30a\)  
    e) \(5p - 2q\)  f) \(6p^3\)  g) \(p + p^3 - 2q\)  e) \(6p^3 q^3\)
9.  a) 41.4  b) 504.03  c) 14.7  d) 7.29
10.  \(OA\) is 058°; \(OB\) is 160°; \(OC\) is 300°; \(OD\) is 330°

EX 6D
1.  a) \(a = 50, b = 80, c = 80, d = 100, e = 40, f = 8, g = 132\)
2.  a) 0.1 each  b) 0.5  c) 0  d) 0.5
3.  68 cm²
4.  a) \(2 \times 3^2\)  b) \(2^3 \times 3\)  c) \(2 \times 31\)  d) \(2^4 \times 7\)
5.  a) \[\frac{1}{4} = \frac{3}{12} = \frac{9}{36} = \frac{25}{100} = 25\%\]
    b) \[\frac{9}{144} = \frac{3}{48} = \frac{1}{16}\]
    c) \[\frac{7}{50} = \frac{14}{100} = 0.14\]
    d) \[12.5\% = \frac{12.5}{100} = \frac{125}{1000} = \frac{1}{8}\]
6. a) $3x + 3$  
   b) $0.2x - 0.4$  
   c) $1.7x + 17$  
   d) $8x - 4y$  
   e) $63 - 14x$  
   f) $8 + 16y$  
   g) $x + 2$  
   h) $3n - 2$  
   i) $3p + \frac{1}{2}$  
   j) $2x - \frac{y}{2}$  
   k) $8 - \frac{q}{3}$  
   l) $0.9x - 0.7$

7. a) 
   b) 
   c) 
   d) 
   e) 
   f)
8. \[ \text{Area} = 8 + 12 + 28 + 16 + 8 = 72 \text{ unit}^2 \]

9. 

\[
\begin{array}{ccc} 
\frac{1}{12} & \frac{1}{3} & \frac{1}{3} \\
\frac{1}{2} & \frac{1}{4} & 0 \\
\frac{1}{6} & \frac{1}{6} & \frac{5}{12} \\
\end{array}
\]

10. a) 48 b) 105 c) 54 d) 21 e) 8 f) 4

EX 6X

1. 20 cm\(^2\)

2. a) 11 b) 10.25 c) 15.75 d) 15

3. a) The total percentage is only 95%.
   b) Green should be 20%.
   c) i. 0.45  
      ii. 0.55  
      iii. 0.2  
      iv. 0.65
Chapter 7

Percentages

The meat of this chapter is the second part, the first being revision of basics. There is a limit to how much the teacher can explain problems. Endlessly doing examples on the board is not efficient. Calculators are essential in this chapter. Rounding-off instructions must be followed precisely.

**LESSON PLANNING**

<table>
<thead>
<tr>
<th>Objectives</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>General</strong></td>
<td>To solve problems using percentages</td>
</tr>
</tbody>
</table>
| **Specific** | 1. To change fractions and decimals to percentages, and *vice versa*
| | 2. To find the percentage of a quantity
| | 3. To find what percentage one quantity is of another
| | 4. To use the fact that if the percentage of part is known, then the rest must make the total up to 100%
| | 5. To solve problems of proportions by scaling percentages up or down
| | 6. To interpret pie charts by measuring angles and calculating percentages of items represented |

**Pacing**

2 lessons, 1 homework

**Links**

Fractions, decimals, direct proportion

**Method**

Follow the text, very quickly, at the start. A good class may not even need these reminders. Settle class down to work on EX 7A. Allow discussion.

Working in pairs (carefully chosen) frees you to circulate providing hints and advice.

For the problem-solving EX 7B, the few examples in the text should be enough to show that choosing the best strategy is the challenge here. Students learn this by engaging with the material, not by watching you solve them on the board. Give them time, allow limited talking (i.e. about the work, not chit chat!). High flyers who finish quickly can be given EX 7X to tackle.

**Assignments**

Suitable homework EX 7B #9 and 10

**Vocabulary**

scale up and down
proportions
ANSWERS

Exercises

EX 7A

1. a) \( \frac{1}{2} \)  
   b) \( \frac{1}{5} \)  
   c) \( \frac{7}{10} \)  
   d) \( \frac{9}{20} \)

2. a) About 25% of the cake is chocolate.  
   b) 75% of that cold drink is just water.  
   c) I made only 12.5% mistakes.  
   d) Nearly 10% of my time was wasted.

3. a) Rs 18  
   b) Rs 30 000  
   b) Rs 200  
   d) Rs 20

4. | Decimal | Percentages | Fraction |
<table>
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<tbody>
<tr>
<td>0.55</td>
<td>55%</td>
<td>( \frac{55}{100} )</td>
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<tr>
<td>0.08</td>
<td>8%</td>
<td>( \frac{4}{50} )</td>
</tr>
<tr>
<td>0.27</td>
<td>27%</td>
<td>( \frac{27}{100} )</td>
</tr>
<tr>
<td>0.8</td>
<td>80%</td>
<td>( \frac{4}{5} )</td>
</tr>
</tbody>
</table>

5. 3%, 0.2, 21%, \( \frac{24}{100} \), 25%, 0.3, \( \frac{5}{8} \)

6. a) 0.28; 28%  
   b) 0.22; 22%  
   c) 0.6; 60%  
   d) 0.3; 30%

7. a) 0.43; 43%  
   b) 0.44; 44%  
   c) 0.64; 64%  
   d) 0.88; 88%

8. a) 87g  
   b) 232.5 kg  
   c) Rs 600  
   d) 450 ml

9. 0.19, \( \frac{1}{5} \), 21%, 0.22, 23%, \( \frac{2}{7} \), \( \frac{1}{3} \), \( \frac{3}{8} \)

10. sugars 21%; fats 3%; fibre 22%

EX 7B

1. a) Rs 700  
  b) Rs 14 700

2. Rs 3960

3. Rs 4680

4. Rs 92.7 million
5. a) 35.3%  b) 54.6%
6. a) 50%  b) 75%  c) 87.5%  d) 25%
7. of 10 20 50 100 200
   | 8% | 1.6 | 4 | 8 | 16 |
   | 10%| 1   | 2 | 5 | 10 | 20 |
   | 15%| 1.5 | 3 | 7.5 | 15 | 30 |
   | 50%| 5   | 10 | 25 | 50 | 100 |
8. [A 15.33%; B 15.5%] B has higher fat content.
9. milk 43.1%; cream 34.7%; sugar 9.7%; egg 8.3%; vanilla 4.2%
10. 17.7%

**EX 7X**
1. 78.1%
2. a) 37.4  b) 122.4  c) 122.4; same as (b)  d) 670 \times 1.23 = 824.1
3. a) 112 cm²  b) 16.8 \times 9.6 = 161.28 cm²  c) 1.44  d) 44% increase
Chapter 8

Equations

This is a short chapter that can be completed quickly, but it prepares the ground for “unwrapping” the variable in more formal methods of equation solution to be explained later.

LESSON PLANNING

Objectives

General  To solve simple linear equations

Specific  1. To create an arrow diagram representing an equation with known output and unknown input
  2. To write down the equation represented by an arrow diagram
  3. To solve equations by reversing the flow of the arrow diagrams
  4. To solve equations by mentally referring to their arrow diagrams

Pacing  1 lesson, 1 homework

Method  Provided that arrow diagrams have been assimilated from Chapter 3, this is an easy topic to introduce. Solving the equation is just finding the input value of an arrow diagram. Keep explanations minimal. Set the exercise in class. The later questions will require a calculator.

Assignments  Homework EX 7A #6, 8, 10

Vocabulary  equation, solve

ANSWERS

Exercises

EX 8A

1. AR  BP  CS  DQ

2. a) $x \rightarrow -7 \rightarrow x \times 8 \rightarrow 24$
   b) $x \times 8 \rightarrow -7 \rightarrow 24$
   c) $x \times 7 \rightarrow +8 \rightarrow 24$
   d) $x + 8 \rightarrow x \times 7 \rightarrow 24$
3. a) \(2n - 5 = 20\)  
   b) \(3(n + 5) = 21\)  
   c) \(4n + 2 = 22\)  
   d) \(\frac{n + 6}{2} = 4.5\)

4. a) \(4n + 4 = 12\)  
   b) \(2(n - 7) = 10\)  
   c) \(\frac{n + 5}{2} = 7\)  
   d) \(\frac{n}{3} + 21 = 26\)

5. a) \(n \times 4 \rightarrow +4 \rightarrow 12\)  
   \(n = 2\)  
   b) \(n \times -7 \rightarrow \times 2 \rightarrow 10\)  
   \(n = 12\)  
   c) \(n \times +5 \rightarrow +2 \rightarrow 7\)  
   \(n = 9\)  
   d) \(n \div 3 \rightarrow +21 \rightarrow 26\)  
   \(n = 15\)

6. a) \(n \times 6 \rightarrow -5 \rightarrow 13\)  
   \(n = 3\)  
   b) \(a \times +4 \rightarrow \times 6 \rightarrow 18\)  
   \(a = -1\)  
   c) \(p \times 2 \rightarrow -7 \rightarrow 9\)  
   \(p = 8\)  
   d) \(t \times +1 \rightarrow \div 2 \rightarrow 8.5\)  
   \(t = 16\)

7. a) \(x \div 2 \rightarrow +72 \rightarrow 89\)  
   \(x = 34\)  
   b) \(x \div 72 \rightarrow +2 \rightarrow 89\)  
   \(x = 106\)  
   c) \(x \times 2 \rightarrow -72 \rightarrow 89\)  
   \(x = 80.5\)  
   d) \(x \times 72 + 2 \rightarrow 89\)  
   \(x = 1.2\) (2 s.f.)

8. a) \(p = 0.5\)  
   b) \(q = 6\)  
   c) \(r = 12\)  
   d) \(s = 426.6\)

9. a) \(t = 2\)  
   b) \(u = 1.5\)  
   c) \(v = -5.3\)  
   d) \(w = 2.8\)

10. a) \(x = 2.2\)  
    b) \(y = 7\)  
    c) \(u = 2\)  
    d) \(v = 1.8\) (2 s.f.)
EX 8X

1. \( x = 11, y = 7 \)

2. • Think of a number: \( n \)
   • Divide by 2: \( \frac{n}{2} \)
   • Add 16: \( \frac{n}{2} + 16 \)
   • Multiply by 3: \( 3\left(\frac{n}{2} + 16\right) = \frac{3n}{2} + 48 \)
   • Subtract 10: \( \frac{3n}{2} + 38 \)
   • Double it: \( 2\left(\frac{3n}{2} + 38\right) = 3n + 76 \)
   • Subtract 1: \( 3n + 75 \)
   • Divide by 3: \( \frac{3n + 75}{3} = n + 25 \)
   • Take away first number: 25

3. [many possible equations and puzzles]
Chapter 9
Quadrilaterals

The first part ought to be revision. As soon as this is clearly established, move on to the Higher skills problem solving EX 9B. The last few questions will be challenging for most students at this stage.

LESSON PLANNING

Objectives

General To solve problems involving quadrilaterals, making use of knowledge of their properties

Specific 1. To recognize the square, rectangle, parallelogram, trapezium, rhombus, and kite (convex and concave)
2. To know about equal or parallel sides, equal angles, equal diagonals, diagonals intersecting at 90°, diagonals bisecting each other, line symmetry, and rotational symmetry properties of the special quadrilaterals
3. To know that the angle-sum of a quadrilateral is 360°
4. To apply all the above to problem-solving (Higher skills)

Pacing 2 lessons, 1 homework

Links Equations

Method Begin class discussion using the start of the chapter diagrams A to G to facilitate this oral work. Having identified each, move on to properties. Take time. Ask about:

i equal sides
ii parallel sides
iii equal angles
iv right angles
v equal diagonals
vi perpendicular diagonals
vii bisecting diagonals
viii line symmetry
ix rotational symmetry

Explain order, i.e. number of ways to fit its own outline. [This is not given in the text but is expected in the exercise.] Square has order 4; rectangle, parallelogram, and rhombus have only order 2.

Use EX 9A to reinforce the concepts.
For the second part, follow the text examples for some strategies to use in problem-solving. The 360° angles-sum fact must be learnt well. Equal length marks must be explained. The rest can be for reference. Use EX 9B.

<table>
<thead>
<tr>
<th>Resources</th>
<th>Photocopiable sheet in this Guide for making triangles (see EX 9A #9)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assignments</td>
<td>Homework EX 9B #6–9</td>
</tr>
<tr>
<td>Vocabulary</td>
<td>square, rectangle, parallelogram, trapezium, rhombus, kite (convex, concave), diagonal, exterior angle, symmetry</td>
</tr>
</tbody>
</table>

**ANSWERS**

**Exercises**

**EX 9A**

1. a) rectangle; rhombus  
   b) kite  
   c) parallelogram; trapezium

2. a) square  
   b) rectangle; rhombus  
   c) kite; trapezium

3. a) kite  

b) parallelogram

4. square

5. a) Diagonals intersect at right angles.  
   b) Diagonals bisect each other.

6. a) square; rectangle  
   b) square; parallelogram; rectangle; rhombus  
   c) kite

7. a) square; parallelogram; rectangle; rhombus  
   b) square; kite; rhombus

8. a) square; parallelogram; rectangle; rhombus  
   b) trapezium  
   c) kite (and any scalene quadrilateral)
9. a) Rectangle, Parallelogram, Kite

b) Rectangle, Rectangle, Rhombus, Parallelogram
10. a) Right angled Isosceles $\triangle$s

b) Isosceles $\triangle$s

c) Scalene $\triangle$s

d) Scalene $\triangle$s
EX 9B

1. a) $a = 100, b = 71$  
   b) $c = 240$
2. a) $d = 128, c = 128$  
   b) $f = 40, g = 105$
3. a) $h = 68, i = 112, j = 112$  
   b) $k = 95, l = 70, m = 85, n = 150$
4. a) $p = 87$  
   b) $q = 147$
5. a) $r = 60$  
   b) $s = 110, t = 75$
6. a) $x = 20$  
   b) $y = 30$
7. a) $a = 24$  
   b) $b = 30$
8. a) $c = 45$  
   b) $d = 10$
9. a) $e = 60, f = 60$  
   b) $g = 36, h = 144$
10. a) $j = 25$  
    b) $k = 10$

EX 9X

1. a) $x = 105, y = 80$  
   b) $x = 9$  
   c) $x = 30$
2. a) kite  
   b) square  
   c) kite  
   d) parallelogram
3. a) $540^\circ$  
   b) $720^\circ$  
   c) $1440^\circ$  
   d) $180(n - 2)^\circ$
Right-Angled Triangles (3 : 4 : 5)
Chapter 10  Negative Numbers

This topic needs a good introduction so that the rule of signs is not just a magic trick but has a sound basis in common sense. Students usually have far more difficulty with addition and subtraction because the sign of the answer depends on the quantities.
For example, \(-5 + 7 = 2\) but \(-7 + 5 = -2\)
It really has to be counted up or down a number line. The strategy of the chapter is to tackle this more difficult part first.

LESSON PLANNING

Objectives

General  To use negative numbers correctly in all four operations, algebraic expressions and equations

Specific  
1. To add or subtract correctly using positive and/or negative numbers
2. To decide which of two numbers is less or more and to use < and > signs correctly
3. To multiply and divide positive and/or negative numbers correctly, following the rule of signs
4. To evaluate algebraic expressions where the unknown is a negative number
5. To solve equations involving negative numbers, where the unknown may be negative

Pacing  2 lessons, may be 1 homework

Method  
Use a classroom number line display (See Resources.)
Explain that adding implies going up the scale.
Subtraction implies going down.
If the positive scale is “HOT” and the negative side is “COLD” then it provides explanations as follows:
For example,  
++ 5 means “adding something HOT”. Go up.
+- 5 means “adding something COLD”. Go down
−+ 5 means “take away something HOT”. Go down
−− 5 means “take away something COLD”. Go up
Other explanatory models are in the text, leading to the rule for adjacent signs.
Use EX 10A immediately to reinforce. After a few minutes read out the first few answers so students have fast feedback. Then you can circulate and troubleshoot.
For multiplication and division follow the text to establish the rule of signs for multiplication/division.
Use EX 10B without prior explanation of the algebra questions coming up at the end of the exercise.

**Resources**
Large number line display similar in design to the one printed in the text, i.e. vertical, with red (hot) for positive numbers and blue (cold) for the negatives. If space does not permit, then it can be fixed horizontally, but vertical is preferable. Every mathematics classroom should have one.

**Assignments**
EX 10A #7-10 or EX 10B (even numbers) suitable for homework

**Vocabulary**
positive, negative
adjacent
less than, greater than

**ANSWERS**

**Exercises**

**EX 10A**

1. a) 1  
   b) 7  
   c) –2  
   d) 7
2. a) –8  
   b) –4  
   c) –12  
   d) –8
3. a) 4  
   b) –3  
   c) –3  
   d) –13
4. a) –2  
   b) 2  
   c) 1  
   d) 5
5. a) 0  
   b) 0  
   c) 0  
   d) 0
6. a) 2  
   b) 6  
   c) 0  
   d) –1
7. a) –14  
   b) 12  
   c) 21  
   d) –4
8. a) \( a = –2 \)  
   b) \( b = –5 \)  
   c) \( c = 7 \)  
   d) \( d = 5 \)
9. a) \( x = 6 \)  
   b) \( x = –10 \)  
   c) \( x = –7 \)  
   d) \( x = 6 \)
10. a) \( 9 – – 2 > 12 + –2 \)  
    b) \( –6 + –1 < 7 – 2 \)  
    c) \( –8 – 2 < 5 –8 \)  
    d) \( 5 + –8 > –7 – –3 \)

**EX 10B**

1. a) –10  
   b) –10  
   c) 10  
   d) –10
2. a) –63  
   b) 63  
   c) –63  
   d) –63
3. a) 24  
   b) 40  
   c) –60  
   d) –60
4. a) –4  
   b) –2  
   c) 7  
   d) –4
5. a) –15  
   b) –6  
   c) –100  
   d) 64
6. a) –4  b) 6  c) –14  d) 16  
7. a) –26  b) –4  c) –1  d) –5  
8. a) x = –3  b) x = –2  c) x = –5  d) x = –1  
9. a) 10  b) 20  c) 26  d) –8  
10. a) –4  b) –2  c) –6  d) 2

**EX 10X**

1.  
   \[
   \begin{array}{c|cccccccc}
   \times & -4 & -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 \\
   \hline
   4 & -16 & -12 & -8 & -4 & 0 & 4 & 8 & 12 & 16 \\
   3 & -12 & -9 & -6 & -3 & 0 & 3 & 6 & 9 & 12 \\
   2 & -8 & -6 & -4 & -2 & 0 & 2 & 4 & 6 & 8 \\
   1 & -4 & -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 \\
   0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
   -1 & 4 & 3 & 2 & 1 & 0 & -1 & -2 & -3 & -4 \\
   -2 & 8 & 6 & 4 & 2 & 0 & -2 & -4 & -6 & -8 \\
   -3 & 12 & 9 & 6 & 3 & 0 & -3 & -6 & -9 & -12 \\
   -4 & 16 & 12 & 8 & 4 & 0 & -4 & -8 & -12 & -16 \\
   \end{array}
   \]

2. a) –4  
   b) \[\begin{array}{c}
   -4 \quad -2 \quad 2 \quad 4 \quad 8 \\
   \end{array}\]

3. 3 or –7
Chapter 11

Averages

This chapter aims to introduce statistical thinking. Why should one particular average be more suitable than another? Why, indeed, are there three ways to calculate an average? The full answers are not given here, but familiarity with the calculations should give a "feel" for how they work, which will be beneficial in later work.

LESSON PLANNING

Objectives

| General | To find the correct mean, median, or mode from given data, and understand what they represent |
| Specific | 1. To distinguish between the three types of average, and have some understanding of why we need all three |
| | 2. To calculate a mean from given data (ungrouped or grouped); to calculate totals when given the mean, and solve related problems |
| | 3. To calculate the median from given data (ungrouped) and interpret its meaning |
| | 4. To state the mode from grouped data and interpret its meaning |
| | 5. (Higher skills) To answer word problems involving these measures of average |

Pacing | 3 lessons, 1 homework |

Method

Initiate discussion on "representation". How do we choose someone to represent a group? Vote? How do we choose a number to fairly represent a set of numbers? Heading to median, mode, mean. Follow the text.

[These are all averages in mathematics; in common speech the mean is usually called the average.]

Deal with calculating the mean, and how to round it off (next level of accuracy). The arrow diagram clearly shows how to work back from a mean to a total.

Start the students on EX 11A and let them move to EX 11B when ready. This will spread the class out, but not all will be able to tackle the higher level problems towards the end of EX 11B. A lot of practice is needed for this topic. Just give them enough time.

Assignments

If you need a homework assignment as the students systematically work though, you could set them the next 4 questions. Alternatively, avoid #6 and 7 (in both exercises) in class, and set for homework.
Vocabulary average, mean, median, mode
grouped data, frequency

ANSWERS
Exercises

EX 11A

1. a) An average is a number we use when we need one number to represent a set of numbers.
   
b) If we arrange the items of data in order and choose the middle one as the average, it is called the median.
   
c) The average found by choosing the most frequent number is called the mode.
   
d) The most useful average mathematically is the mean because every number in the set of data is used in the calculation.
   
e) The number used to show the spread of the data is called the range. It is not an average.

2. a) 6.4 b) 6.0

3. a) 24.9 grapes b) 22.6 grapes

4. 6.0

5. a)  

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<th>f</th>
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<td></td>
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   b) 13.1 pencils

6. 58.6 kg

7. a)  

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<td>30</td>
<td>60</td>
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</tr>
</tbody>
</table>

b) 30 girls
c) 2.0 games
d) 1.8 games
8. Averages

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f$</th>
<th>$xf$</th>
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<tbody>
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<tr>
<td>80</td>
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<td>90</td>
<td>10</td>
<td>900</td>
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<tr>
<td>100</td>
<td>12</td>
<td>1200</td>
</tr>
<tr>
<td>110</td>
<td>15</td>
<td>1650</td>
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<tr>
<td>120</td>
<td>10</td>
<td>1200</td>
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<tr>
<td>130</td>
<td>6</td>
<td>780</td>
</tr>
<tr>
<td>140</td>
<td>4</td>
<td>560</td>
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</table>

Mean = 108.7 cm

9. Averages

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<td>3</td>
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<td>12</td>
</tr>
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<td>4</td>
<td>6</td>
<td>24</td>
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<td>5</td>
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<td>6</td>
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<td>8</td>
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<td>112</td>
</tr>
<tr>
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<td>5</td>
<td>45</td>
</tr>
<tr>
<td>10</td>
<td>1</td>
<td>10</td>
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</table>

Mean = 6.2 marks

10. Averages

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<tbody>
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<td>175.2</td>
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<tr>
<td>14.7</td>
<td>16</td>
<td>235.2</td>
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<tr>
<td>14.8</td>
<td>17</td>
<td>251.6</td>
</tr>
<tr>
<td>14.9</td>
<td>28</td>
<td>417.2</td>
</tr>
<tr>
<td>15.0</td>
<td>31</td>
<td>465.0</td>
</tr>
<tr>
<td>15.1</td>
<td>10</td>
<td>151.0</td>
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<tr>
<td>15.2</td>
<td>2</td>
<td>30.4</td>
</tr>
<tr>
<td>15.3</td>
<td>1</td>
<td>15.3</td>
</tr>
</tbody>
</table>

a) Mean = 14.87 mm

b) The machine is producing screws below the specified length on average. The range of 0.8 mm is also not acceptable.
EX 11B

1. 6.7

2.  

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f$</th>
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</tr>
</thead>
<tbody>
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<td>9</td>
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<td>1</td>
<td>27</td>
</tr>
<tr>
<td></td>
<td>17</td>
<td>347</td>
</tr>
</tbody>
</table>

   a) 20.4 fish  
   b) 19.8 fish

3.  
   a) B (35 kg)  
   b) C (28 kg)  

   c) Mean of A $= \frac{30}{20} = 1.5$ kg  
   Mean of B $= \frac{35}{20} = 1.8$ kg  

   Mean of C $= \frac{28}{15} = 1.9$ kg  

   Highest Mean. C wins prize fairly.

4. 57.0 kg

5. 13.4 marks

6.  

<table>
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<tr>
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</tr>
</thead>
<tbody>
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<td>60</td>
<td>1</td>
<td>60</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>764</td>
</tr>
</tbody>
</table>

   a) Mean = Rs 50.9  
   Median = Rs 50  
   Mode = Rs 48  

   b) Mode unsuitable because there are only 15 items of data.

   c) Mean would go down to Rs 50.3 and Median to Rs 49.

7. Boys' Mean = 2.52 kg

   Girls' Mean = 2.31 kg

   On average boys are 0.21 kg heavier.
8. 

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f$</th>
<th>$xf$</th>
</tr>
</thead>
<tbody>
<tr>
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<td>1</td>
<td>50</td>
</tr>
<tr>
<td>100</td>
<td>6</td>
<td>600</td>
</tr>
<tr>
<td>150</td>
<td>14</td>
<td>2100</td>
</tr>
<tr>
<td>200</td>
<td>4</td>
<td>800</td>
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<tr>
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<td>2</td>
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<tr>
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<td>300</td>
</tr>
<tr>
<td></td>
<td>28</td>
<td>4350</td>
</tr>
</tbody>
</table>

Mean = Rs 155.4
Median = Rs 150
Mode = Rs 150 

b) the Mean
c) the Median or the Mode
d) Mean goes to Rs 153.8, Median and Mode unchanged.

9. 209 lines (rounded up to the nearest whole line)

10. 5 packets, 6.3 chocolates each (1 d.p.)

**EX 11X**

1. a) 10.5 years b) 11 years c) 10 years

2. mark 52, Mean 62.2; or
   mark 76, Mean 67.0

3. girls C, F, G;
   boys A, B, D, E
Chapter 12

Revision Exercises

These exercises revise the first 11 chapters, with just a few easy questions on topics which should have been covered earlier.

ANSWERS

Exercises

EX 12A

1. a) \(\frac{41}{9}\)  
   b) \(\frac{12}{5}\)  
   c) \(\frac{344}{45}\)  
   d) \(\frac{19}{45}\)

2. 0.6

3. a) 148 g  
   b) 336 kg  
   c) Rs 600  
   d) 480 ml

4. 

5. a) 3  
   b) 1  
   c) 6  
   d) 2

6. a) 51.3 matchsticks  
   b) down

7. • Think of a number: \(n\)  
   • Multiply by 3: \(3n\)  
   • Add 15: \(3n + 15\)  
   • Divide by 3: \(\frac{3n + 15}{3} = n + 5\)  
   • Double it: \(2(n + 5) = 2n + 10\)  
   • Subtract 10: \(2n\) This is double the first number.

Area \(ABCD = 60 \text{ unit}^2\)

Area \(PQRS = 20 \text{ unit}^2\)
8. Rs 16,400
9. a) 15, 60 and 85  
   b) 36, 9, 2, 72, 12
10. a) $\angle ROS, \angle SOR$  
    b) $\angle QPR, \angle RPQ$  
    c) $\angle CBD, \angle DBC$  
    d) $\angle ABD, \angle DBA$

**EX 12B**

1. a) 0.2  
   b) 0.3  
   c) 0.5  
   d) 0  
   e) 0.8
2. a) 46 mm$^2$  
    b) 30.6 cm$^2$  
    c) 3600 mm$^2$  
    d) 90 cm$^2$
3. cricket 55.6%; football 25%; hockey 19.4% (1 d.p.)
   Total is 100% (checks)
4. 88 cm$^2$
5. a) –18  
    b) –2  
    c) 3  
    d) 54
6. a) no symmetry  
    b) line symmetry (reflection in the baseline)
7. a) 0.36; 36%  
    b) 0.26; 26%  
    c) 0.8; 80%  
    d) 0.7; 70%
8. a) $5n - 2 = 13$  
    b) $5(n + 3) = 65$  
    c) $2n + 4 = 204$  
    d) $\frac{n + 2}{6} = 10$
9. a) Rectangle; Rhombus  
    b) Rectangle; Rhombus; Parallelogram
10. 134.2 cm (1 d.p.)

**EX 12C**

1. | Blue | a) $\frac{1}{6}$ | b) $\frac{1}{3}$ |
<table>
<thead>
<tr>
<th></th>
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</tr>
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<tbody>
<tr>
<td>Red</td>
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<td>c) 0</td>
</tr>
<tr>
<td></td>
<td>2 5 7 9</td>
<td></td>
</tr>
<tr>
<td></td>
<td>6 9 11 13</td>
<td></td>
</tr>
</tbody>
</table>
2. a) 51.03 cm$^2$  
    b) 33.82 cm$^2$  
    c) 51.3 cm$^2$  
    d) 2.56 m$^2$
3. a) \[ n \times 5 \rightarrow \text{ } n = 2 \]
b) \[ a + 3 \rightarrow \times 5 \rightarrow a = 4 \]
c) \[ p \times 7 \rightarrow -2 \rightarrow p = -3 \]
d) \[ t \rightarrow -3 \rightarrow \div 2 \rightarrow t = 10 \]

4. a) \[ x = 81, y = 99, z = 99 \]
b) \[ p = 32, q = 116; r = 33, s = 75 \]

5. AR BP CQ DS

6. 7A: Mean 3.1 (1 d.p.)

7. a) \[ a + b \]
b) \[ 5a - b \]
c) \[ 5x - y \]
d) \[ 2x + 4y \]

8. a) \[ 6.7; 7.32 \text{ (3 s.f.)} \]
b) \[ 2; 2.04 \text{ (3 s.f.)} \]
c) \[ 10; 11.4 \text{ (3 s.f.)} \]
d) \[ 100; 105 \text{ (3 s.f.)} \]

9. a) \[ 1 \]
b) \[ 7 \]
c) \[ -5 \]
d) \[ -4 \]

10. a) \[ 000^\circ \]
b) \[ 210^\circ \]
c) \[ 300^\circ \]

EX 12D

1. a) median 45, range 51
   b) median 55, range 27
   c) median 53, range 51

2. a) Area \( \Delta = 8 \text{ unit}^2 \)
   b) Area parallelogram = 12 unit^2

3. a) Kite; Trapezium
   b) Parallelogram; Rectangle; Rhombus
   c) None
   d) Square
4. a) $p + 4 \rightarrow -2 \quad p = -6$
b) $p - 2 \rightarrow -4 \quad p = -2$
c) $p - 4 \rightarrow -5 \quad p = -1$
d) $p + 5 \rightarrow 4 \quad p = -1$

5. a) $6x - 3$ b) $2.5x + 25$ c) $4 - 12x$ d) $x + 3$
e) $3 - x$ f) $1.3x + 2$

6. $\frac{1}{4}, 26\%, \frac{4}{15}, \frac{3}{10}, \frac{151}{500}, 31\%, \frac{5}{16}, 0.32$

7. a) $w = 60, x = 60$ b) $y = 45, z = 45$

8. | TL | 0 | 1 | 2 | 4 |
<table>
<thead>
<tr>
<th></th>
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<tr>
<td>4</td>
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<td>5</td>
<td>6</td>
<td>8</td>
</tr>
</tbody>
</table>

9. a) $-5$ b) $10$ c) $16$ d) $15$

10. 7.4 marks

EX 12X

1. 8 lines of symmetry; rotational symmetry of order 8 about the centre.

2. a) $6a^2b^2c^2d^5$
b) $3abc + a^2b + 2ab^2$
c) $1.7x^2 + 1.7x$
d) $5y^3$

3. In general, Maths marks are higher (median 65 compared with 60). However, the Maths marks have a wider spread (range of 60) compared with English (range of only 32).
As well as providing opportunity for using geometrical instruments, this chapter demands that students read detailed instructions in the questions, and follow them exactly. Not all students find this congenial!

LESSON PLANNING

Objectives

General  To perform ruler and compasses constructions accurately, following instructions

Specific  1. To develop facility in using the equipment, and avoid unsafe behaviour
             2. To be able to construct the bisector of a given angle, and the perpendicular bisector of a given line segment
             3. To be able to construct the perpendicular to a given line at a point on the line, or from a point off the line
             4. To be able to construct diagrams involving the above standard techniques by carefully following written instructions

Pacing  Maybe 4 lessons, 1 homework

Links  Properties of quadrilaterals

Method  Practical sessions, after introduction.

Safety point: Students must use a different pencil for ordinary writing, never the one in the compasses with the point raised, which is very risky. One push from behind and an eye could be lost.

Language point: The piece of equipment is a pair of compasses (or just compasses), not a compass, which is a navigational instrument for finding directions. Compasses are like trousers: they come in pairs and have two legs!

Practical point: A full length pencil in a pair of compasses limits the ability to turn smoothly because the top of the pencil contacts the hand. Advise students to snap the pencil in half and sharpen a point on each. Keep one for the compasses and one spare.

Use board compasses to demonstrate the two bisections. Emphasize the symmetry: that’s how they will remember the correct sequence in each case. Set EX 13A.

Again, using large board compasses, show how the two basic techniques can be developed to draw perpendiculars to a line. Follow the text. Set EX 13B.
Do not waste time showing repeatedly how to do these constructions. Students need to learn it by doing. Expect lots of distress. You will need to circulate and troubleshoot. Also, allow students to help each other. Although there are only two exercises, shorter than usual, they are very time-consuming.

During circulation, you may like to use the terms inscribed circle, circumcircle, incentre, circumcentre, and medians with the more able students when they reach the relevant questions.

<table>
<thead>
<tr>
<th>Resources</th>
<th>Sets of compasses, protractors</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Board compasses</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Assignments</th>
<th>Homework from the exercises not recommended</th>
</tr>
</thead>
</table>

| Vocabulary | (pair of) compasses                          |
|           | angle bisector, perpendicular bisector, line segment, perpendicular to a line (shortest distance) |

**ANSWERS**

**Exercises**

**EX 13A**

1. a) ![Diagram 1a](#)  
   ![Diagram 1b](#)

2. a) ![Diagram 2a](#)  
   ![Diagram 2b](#)
3.

a) \[ \angle 50^\circ \]

b) \[ \angle 70^\circ \]

4. [Inscribed circle]

5. [Circumcircle]

6. The centre of the circle is the midpoint of the largest side. The triangle fits into the semicircle. The largest side is a diameter of the circle.
7. 

[Medians]
The three lines meet at one point.

8. c) Yes
d) Yes, because if it is not symmetrical, they will not meet at a point.

EX 13B

1. 

2. 

45°
45°
3. \( \angle C = 45^\circ \)  
(\( f \)) \( BC = 7.1 \text{ cm} \)

4. \( \angle C = 135^\circ \)  

5. \( \angle C = 135^\circ \)
6. [Diagrams of constructions with lines and angles marked.]

7. [Diagrams of constructions with lines and angles marked.]

8. [Diagrams of constructions with lines and angles marked.]

[It is very difficult to get accurate answers here.]

d) $BC = 7.5\text{ cm}$

e) $\text{perp} = 3.4\text{ cm}$

f) $12.9\text{ cm}^2$ (1 d.p.)

g) $12.8\text{ cm}^2$ (1 d.p.)
EX 13X

1.

2.
3. \((1.7, 0)\)
It is most helpful if a school has a calculator policy and specifies the make and model for students to bring to lessons. Unfortunately few do, so you could well be faced with a class bringing calculators ranging from super computers to antiques, not to mention calculators on smartphones or wrist watches. It is hopeless trying to teach key sequences: students just have to work it out for themselves.

**LESSON PLANNING**

**Objectives**

| General | To use calculator memory, brackets, square, and square-root keys effectively in calculations |
| Specific | 1. To know how to use brackets, memory keys, and fraction display (if available) in calculations |
| | 2. To enter negative numbers correctly in calculations |
| | 3. To use square and square-root keys correctly |
| | 4. To perform complex calculations involving more than one of the above |

| Pacing | 2 lessons, 1 homework |
| Links | Algebraic expressions, BODMAS |

**Method**

The best way to handle diversity of machines is to do group work. Arrange for those with similar models to be in the same group where possible.

Ask everyone to enter $3 + 5 \times 2 =$

Those who get the answer 16 have very old models. Best to lend them a calculator for the lesson and request they acquire something more up to date.

**First group task:** Find out how to enter and recall memory. There are hints in the text.

**Second group task:** Find out if your calculator has fraction mode $\frac{3 + 6}{12 - 1}$.

Can you enter $\frac{3 + 6.9}{12 - 1}$ exactly as it appears on paper?

**Third group task:** Can you calculate the same amount $\frac{3 + 6.9}{12 - 1}$ without using fraction mode, but using brackets instead?

**Fourth group task:** Find out if your calculator has a way of automatically memorising the last answer. Try $\text{ANS}$ when ready, allow the groups to proceed with EX 14A.

**Fifth task:** Find out how to square a number.

Can you square -3 and obtain 9?
Sixth task: Can you find a square root?
Show that the square-root of 36 is 6.
When ready, allow the groups to proceed with EX 14B.
For any individual or group that works quickly, the cross figure puzzle in EX 14X is quite a challenge.

Resources
Have a few spare calculators ready for the students who forget to bring them, or who bring obsolete models. [There are usually a few lying in lost and found.]

Assignments
Not an easy topic for choosing a homework assignment. If necessary, use EX 14A #7, 9.

Vocabulary
Memory, recall, square, square-root, fraction display

ANSWERS

EXercises

EX 14A
1. a) 5 b) 18 c) 11 d) 32
2. a) 21 b) 27 c) 5 d) 2
3. AR BP CS DQ
4. a) 31 b) 2 c) 16 d) 60.8
5. a) 19 b) 0.5 c) 2.15 d) 6
6. a) 0.85 b) 4.8 c) 0.5 d) 18.4
7. a) 18.1 b) 0.908 c) 8.03 d) 12.4
8. a) 50.9 b) 85.2 c) 2.4 d) 1.6
9. a) 1.3 b) 60 c) 880 d) 10
10. a) 577 b) 44 c) 5 d) 5

EX 14B
1. a) 19 b) 49 c) -5 d) 13
2. a) 7 b) 5 c) 48 d) 1
3. a) 36 b) 9 c) 3 d) 9
4. C
5. & 6 [answers given]
6. a) -0.7 b) -18.6 c) 11 d) 14.92
7. a) 19 b) 12.36 c) 22.6 d) 3.8
9. a) 0.5  
   b) 2  
   c) 4  
   d) 0.53  
10. a) –9  
    b) 56.72  
    c) –22.12  
    d) –53.66  

EX 14X

1.  

2. a) 13.4 (3 s.f.)  
    b) 24.9 (3 s.f.)  
    c) 4.49 (3 s.f.)
Some new angle facts are introduced, and old ones revised. This is the point where we begin to insist on reasoning. Students often obtain correct numerical answers for the wrong reason. You do need to focus on the reasons given.

LEsson PLANNING

Objectives

General
To deduce the size of angles by correct reasoning using simple angle facts

Specific
1. To recognize parallel lines and transversals
2. To use alternate and corresponding angles in calculations
3. To state the correct reason for each step in a calculation, using reason codes for corr\(\angle\)s, alt \(\angle\)s, vert opp \(\angle\)s, adj \(\angle\)s on a st line, angle-sum of \(\Delta\), and angle-sum of quad
4. Solving problems (Higher level skills) involving more than one of the above by correct reasoning

Pacing
2 or 3 lessons, 1 homework

Links
Rotational symmetry, translation

Method
- Explain parallel lines and transversal. Follow the text.
  Some other words beginning with trans:
  transport  carrying across
  transmit  sending across
  transatlantic  across the Atlantic

  The transversal cuts across the parallels.
- Demonstrate corresponding angles using cardboard angles on a stick:

  Card on stick

  Draw the parallels on the board and show the equal angles by placing the card angle at the top and sliding it down to the lower corresponding position.
• Demonstrate alternate angles using a piece of thick wire in a Z-shape. Rotate to show the equal angles. Put the wire over parallel lines on the board:

![Z-shape diagram]

Use EX 15A.

• Give reason codes for alt and corr $\angle$s. Ask for recall of other angles facts: give their codes as they are mentioned. They are in the text for reference, but should be memorised, at least approximately. (Be flexible about this.)

Use EX 15B.

<table>
<thead>
<tr>
<th>Resources</th>
<th>Card on stick for demo (See Method.)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Wire in Z-shape for demo (See Method.)</td>
</tr>
</tbody>
</table>

| Assignments        | Homework EX 15A #6-10               |

| Vocabulary         | parallel, transversal               |
|                    | alternate, corresponding, vertically opposite, adjacent angles on a straight line, angle-sum |

### ANSWERS

#### Exercises

**EX 15A**

1. a) $p$ and $q$ are corresponding angles.
   b) $r$ and $q$ are alternate angles.
   c) $t$ and $v$ are alternate angles.
   d) $s$ and $t$ are corresponding angles.

2. a) $u$ and $v$ are alternate angles.
   b) $u$ and $w$ are corresponding angles.
   c) $x$ and $v$ are corresponding angles.
   d) The line $PQ$ is the transversal.

3. a) $x = 35$, corresponding
   b) $x = 25$, alternate
   c) $x = 100$, alternate
   d) $x = 120$, corresponding
4.\[30°\] 
5. \[50°\] 
6. \[40°\] 
7. \[115°\] 
8. \[x = 10, \text{ alternate}\]
9. \[2x = 70, \text{ alternate}\]
10. a) \[a = 115, \text{ alternate}\] b) \[b = 115, \text{ corresponding}\]

EX 15B
1. \[a = 75 (\text{corr } \angle s); b = 60 (\angle \text{s on st line}); c = 45 (\text{angle-sum of } \triangle)\]
2. \[a = 82 (\angle \text{s on st line}); b = 82 (\text{corr } \angle s); c = 72 (\text{alt } \angle s)\]
3. \[a = 115 (\text{alt } \angle s); b = 115 (\text{corr } \angle s); b = 115 (\text{vert opp } \angle s)\]
4.  a)  \( p = 119 \) (adj \( \angle s \) on st line) \[ x = 119 \] (alt \( \angle s \))
   b)  \( q = 61 \) (corr \( \angle s \)) \[ x = 119 \] (adj \( \angle s \) on st line)

5.  \[
\begin{align*}
\text{Diagram of angles.} & \quad a = 68 \text{ (adj } \angle \text{s on st line)} \\
& \quad b = 64 \text{ (angle-sum of } \Delta) \\
& \quad x = 64 \text{ (corr } \angle \text{s)} \\
& \quad \text{[substitute letters for } a \text{ and } b \text{ allowed]}
\end{align*}
\]

6.  \[
\begin{align*}
\text{Diagram of angles.} & \quad a = 58 \text{ (corr } \angle \text{s)} \\
& \quad y = 58 \text{ (vert opp } \angle \text{s)} \\
& \quad b = 58 \text{ (corr } \angle \text{s or alt } \angle \text{s)} \\
& \quad x = 122 \text{ (adj } \angle \text{s on st line)} \\
& \quad \text{[many other methods]}
\end{align*}
\]

7.  \[
\begin{align*}
\text{Diagram of angles.} & \quad a = 25 \text{ (adj } \angle \text{s on st line)} \\
& \quad x = 63 \text{ (angle-sum of } \Delta) \\
& \quad b = 25 \text{ (vert opp } \angle \text{s or adj } \angle \text{s on st line)} \\
& \quad c = 63 \text{ (alt } \angle \text{s)} \\
& \quad y = 92^{\circ} \text{ (angle-sum of } \Delta) \\
& \quad \text{[many other methods]}
\end{align*}
\]

8.  \[
\begin{align*}
\text{Diagram of angles.} & \quad a = 36 \text{ (alt } \angle \text{s)} \\
& \quad b = 26 \text{ (alt } \angle \text{s)} \\
& \quad x = a + b = 62
\end{align*}
\]

9.  \[
\begin{align*}
\text{Diagram of angles.} & \quad y = 55 \text{ (alt } \angle \text{s)} \\
& \quad a = 48 \text{ (adj } \angle \text{s on st line)} \\
& \quad x = 48 \text{ (alt } \angle \text{s)}
\end{align*}
\]
10.  
   a) \( \angle ABD = \angle BDC = 50^\circ \) or \( \angle BAD = \angle ADE = 40^\circ \)
       \[ \therefore AB \text{ is parallel to } EC \text{ (alt } \angle \text{s)} \]
   b) \( \angle EAD = 89^\circ \) (angle-sum of \( \Delta \))
       \( \angle ADB = 90^\circ \) (angle-sum of \( \Delta \) or adj s on st line)
       \[ \angle EAD \neq \angle ADB \]
       \[ \therefore AE \text{ is not parallel to } EC \]

**EX 15X**

1. The interior angles are either 50° or 130°.
2. \( AB \) is not parallel to \( CD \) (although it looks like it is). Using the \( y \)-axis as transversal, measure alternate angles. There is one degree difference, showing that \( AB \) is not quite parallel to \( CD \) and the angles are not quite alternate.
3. \( \angle ADC = 60^\circ \) (alt \( \angle \)s)
   \( \angle ADB = 62^\circ \) (angle-sum of \( \Delta \))
   \( \angle BDE = 58^\circ \) (angle-sum of \( \Delta \))
   \[ \angle ADC + \angle ADB + \angle BDE \]
   \[ = 60^\circ + 62^\circ + 58^\circ \]
   \[ = 180^\circ, \text{ a straight angle } \angle CDE \]
   i.e. \( CDE \) is a straight line.
Chapter 16

Three Dimensions

Many students have difficulty visualising in 3D from a drawing. Models really help. This chapter goes further than Book 6 Chapter 17, making a clearer connection between the formulas for volume of a cuboid and a prism.

LESSON PLANNING

Objectives

General  
To represent 3D objects on paper and calculate volumes and surface areas of cuboids and prisms

Specific  
1. To draw cubes and cuboids on squared paper, using mathematical conventions of representation
2. To answer simple questions about vertices, faces, edges, and right angles in cuboids
3. To calculate volumes of cuboids and prisms; to calculate lengths or areas given volumes
4. To calculate volumes of composite shapes; to answer word problems involving volume
5. To draw nets of 3D shapes and use them in surface area calculations

Pacing  
3 lessons, 1 homework

Links  
Arrow diagrams, algebraic formulas

Method  
Reinforce the text introduction to the chapter. Issue squared paper (provided here) if necessary.
Set EX 16A.
Have models available to demonstrate clearly what a prism is, and that a cuboid is just a special case of a prism.
Show (using text example) how the volume formula for a prism makes sense.
Start on EX 16B straightaway. From #4 (b) onwards when the inverse is used refer students to the arrow diagrams just before the exercise. (Not everybody will need this.)
Introduce nets as an efficient method of calculating surface area.
For EX 16C it is suggested to have paper nets for #1 and #2 as some students will not be able to visualize it from the page alone.
Chapter 16 Three Dimensions

Resources
Ready-made models of cubes, cuboids and prisms, and other non-prism solids.
A variety of nets, especially of cubes. Squared paper (photocopiable sheets provided after the Answers in this chapter.)

Assignments
EX 16A #3–10 is revision: could be used diagnostically, or for homework. EX 16B #2 could be set for homework. Or EX 16C #2.

Vocabulary
prism, cross-section, surface area, volume

ANSWERS

Exercises

EX 16A
1. [suitable drawings of cuboids]
2. [suitable drawings of cuboids]
3. a) 6 b) 12 c) 8
4. All edges of a cube are equal in length.
5. No, because we already have two different measurements and they have to be equal for a cube.
6. a) Yes, 40 cm by 40 cm is possible.
   b) No, it could be 40 cm by 40 cm by 50 cm, for example.
7. 180 cm
8. a) 90° b) acute or obtuse, sometimes 90°
9. a) 16 cm³ b) 9 cm³ c) 45 cm³ d) 24 cm³
10. a) 24 cm³ b) 600 cm³ c) 1798.9 cm³ d) 57100 cm³

EX 16B
1. a) Yes b) No c) Yes d) Yes
2. a) 60 cm³ b) 30 cm³ c) 320 cm³ d) 60 cm³
3. a) 20 cm³; 160 cm³ b) 84 cm³; 1680 cm³ c) 45 cm³; 270 cm³ d) 14 cm³; 84 cm³
4. a) 17.6 cm³ b) 20 cm
5. 4 cm
6. 300 m²
7. [many possible answers] Most sensible is to double the shortest measurement, giving 12 cm by 10 cm by 8 cm.
8. 4 m
9. 7.5 m
10. 1.5 m

EX 16C
1. Q
2. 
   1
   2
   3
   4
3. a) pyramid b) prism (triangular cross-section)
   c) cube d) prism (trapezium cross-section)
4. [many possible nets]
   a) 214 cm$^2$ b) 363 cm$^2$
   c) 470.5 cm$^2$ d) 171.5 cm$^2$
5. 152 cm$^2$
6. a) 
   b) 
   c) 
   d) [or equivalents]
7. a) 48 cm$^2$ b) 2 cm c) 48 cm$^3$
8. 

![Diagram of a 3D shape with dimensions 8 cm x 8 cm x 9 cm.]

9. 

![Diagram of a dice with labels A and B and a red dot.]

10. 

![Diagram of a dice with black dots and labels.]

EX 16X

1. 252 cm²
2. 231 cm³
3. 

![Diagram of a shape with blue and red labels.]

[or equivalents]
Chapter 17

Revision Exercises

ANSWERS

Exercises

EX 17A

1. a) b) c) d)

2. a) $2^2 \times 5$ b) $2 \times 19$ c) $3 \times 17$ d) $2^5 \times 3$

3. a) 9.2 (1 d.p.) b) 9.3 (1 d.p.)

4. a) $28 - x$ b) Rs 130 + $x$ c) $6x$ d) $\frac{x}{5}$

5. a) 144 cm b) 147 cm c) 26 cm d) 16 cm

6. a) $5\frac{3}{4}$ b) $6\frac{1}{2}$ c) 3 d) $8\frac{3}{4}$

7. a) $x = 49$ (corr $\angle$s) b) $x = 62$ (alt $\angle$s) c) $x = 105$ (alt $\angle$s) d) $x = 141$ (corr $\angle$s)
Chapter 17 Revision Exercises

8. a) 3 unit²  b) 37.5 unit²

9. a)  

b)  

10. a) 21 → 6  b) −7 → 3
    −3 → −2  13 → −5
    −6 → −3  1 → −1/5
    3 → 0  −0.5 → 0.4

EX 17B

1. 228 cm

2. a)  
    b)  
    c)  

3. a) 0.1  b) 0.076  c) 0.076  d) 0.0756
4. Maths: median 58, range 68  
   English: median 50, range 35  
   In general, the Maths results are better, and are more spread out than the English results.

5. a) 29  b) 1.7  c) 16  d) 7.5

6. |       | Red |       |   |
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a) \(\frac{1}{12}\)  b) \(\frac{1}{12}\)  c) \(\frac{1}{36}\)  d) \(\frac{1}{6}\)

7. a) \(A \rightarrow H\)  b) \(G \rightarrow B\)  c) \(D \rightarrow I\)
   d) \(C \rightarrow C\)  e) \(BC \rightarrow GC\)  f) \(CI \rightarrow CD\)
   g) \(BD \rightarrow GI\)  h) \(\angle BAD \rightarrow \angle GHI\)

8. a) 24%  b) 8%  c) 35%  d) 18%

9. AQ, BS, CR, DP

10. a) 1, 3, 5, 15  b) 1, 2, 3, 4, 6, 8, 12, 24
    c) 1, 2, 3, 5, 6, 10, 15, 30  d) 1, 2, 4, 8, 16, 32

EX 17C

1. a) \(96 + x + x + 4 + x - 28 = 360\)

   \[72 + 3x = 360\]

   \[x = 96\]

b) 96, 96, 100, 68  c) kite

2. \(a = 69\) (adj \(\angle\)s on st line)
   \(b = 60\) (angle-sum of \(\Delta\))
   \(x = 60\) (corr \(\angle\)s)

3. 31 mm
Chapter 17 Revision Exercises

4.  
   a) \( x \times 4 - 3 \rightarrow -7 \quad x = -1 \)
   b) \( x + 7 \rightarrow 5 \rightarrow -25 \quad x = -12 \)
   c) \( x \times 6 - 1 \rightarrow 2 \rightarrow -12.5 \quad x = -4 \)
   d) \( x + 2 \rightarrow 3 \rightarrow -18 \quad x = -8 \)

5. 
   a) \[
   \begin{align*}
   x &\rightarrow 0 \\
   0 &\rightarrow 0 \\
   \end{align*}
   
   b) \[
   \begin{align*}
   x &\rightarrow 0 \\
   0 &\rightarrow 0 \\
   \end{align*}
   
   c) \[
   \begin{align*}
   x &\rightarrow 0 \\
   0 &\rightarrow 0 \\
   \end{align*}
   
   d) \[
   \begin{align*}
   x &\rightarrow 0 \\
   0 &\rightarrow 0 \\
   \end{align*}
   
6.  
   a) 30  
   b) 2  
   c) 10  
   d) 1000  

7.  
   a) 2.5%  
   b) 3.8%  

8.  
   a) \( 3 - 3x \)  
   b) \( -7x + 28 \)  
   c) \( 5x - 2.5 \)  
   d) \( -0.8 + 0.4x \)  
   e) \( 2 + 2x \)  
   f) \( -4x + 2 \)  
   g) \( -2x - 1 \)  
   h) \( -1 - 3x \)
9. 280 cm²
10. a) 210°  b) 030°  c) 300°  d) 270°

EX 17D

1. 

2.

3. 5, 13, 17, 29
4. \(a = 59\) (adj \(\angle\) on st line); \(b = 78\) (adj \(\angle\) on st line);
\(c = 132\) (angle-sum of quad); \(d = 48\) (adj \(\angle\) on it line)

5. a) 

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<td>6</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>32</strong></td>
<td><strong>51</strong></td>
</tr>
</tbody>
</table>

Mean = \(\frac{51}{32}\) = 1.6 cars (1 d.p.)

b) 1.7 cars (1 d.p.)
6. a) HCF = 10  
    b) LCM = 900

7. a) \( \frac{2}{7} = \frac{4}{14} = \frac{12}{42} \)  
    b) \( \frac{3}{5} = \frac{9}{15} = \frac{18}{30} \)  
    c) \( \frac{5}{8} = \frac{25}{40} = \frac{15}{24} \)  
    d) \( \frac{1}{6} = \frac{2}{12} = \frac{5}{30} \)

8. a) 61 (2 s.f.)  
    b) -6.2 (2 s.f.)  
    c) 1.3 (2 s.f.)  
    d) 7.2 (2 s.f.)

9. a) \( x + 4 \)  
    b) \( 3c + 7 \)  
    c) \( \frac{d}{3} - 6 \)  
    d) \( y^2 + 2 \)

10. a)
    \[
    \begin{array}{ccc}
        4 & \frac{1}{2} & 4\frac{1}{2} \\
        3\frac{1}{2} & 3 & 2\frac{1}{2} \\
        1\frac{1}{2} & 5\frac{1}{2} & 2 \\
    \end{array}
    \]

    b)
    \[
    \begin{array}{ccc}
        1\frac{1}{2} & 1 & 1\frac{1}{2} \\
        1\frac{1}{3} & 1\frac{1}{3} & 1\frac{1}{3} \\
        1\frac{1}{6} & 1\frac{2}{3} & 1\frac{1}{6} \\
    \end{array}
    \]

    c)
    \[
    \begin{array}{ccc}
        \frac{4}{5} & 1\frac{1}{5} & \frac{7}{10} \\
        \frac{4}{5} & \frac{9}{10} & 1 \\
        1\frac{1}{10} & \frac{3}{5} & 1 \\
    \end{array}
    \]

    d)
    \[
    \begin{array}{ccc}
        2 & 0.1 & 1.2 \\
        0.3 & 1.1 & 1.9 \\
        1 & 2.1 & 0.2 \\
    \end{array}
    \]

EX 17X

1. a) [Start with a large circle and draw radii at 40° between.]
    b) [36° between radii]

2. 
\[
\begin{array}{cccccccccccc}
    2 & 3 & 5 & 7 & 11 & 13 & 17 & 19 & 23 & 29 \\
    31 & 37 & 41 & 43 & 47 & 53 & 59 & 61 & 67 & 71 \\
    73 & 79 & 83 & 89 & 97 & 101 & 103 & 107 & 109 & 113 \\
    127 & 131 & 137 & 139 & 149 & 151 & 157 & 163 & 167 & 173 \\
    179 & 181 & 191 & 193 & 197 & 199 \\
\end{array}
\]

3. a) \(-0.3\)  
    b) 5  
    c) 0.5  
    d) 10
Instructions: The time allowed is 1 hour.
You may use a calculator.
You will also need pen, pencil, rubber, ruler, protractor, compasses.
Try to answer all the questions.
Check your work carefully.
The marks for each question are shown in brackets. [Max marks = 100]

1. Write down the answers to the following:
   a) \(-7 - -3\)  
   b) \(0 + -1 + 2\)  
   c) \(2 \times -17\)  
   d) \(-42 \div -7\)  

2. A bag contains 10 cards. On each there is a number:
   17  18  19  20  21  22  23  24  25  26
The cards are shuffled in the bag and one card is taken out at random. Find:
   a) \(P(\text{There is a 7 on the card})\)  
   b) \(P(\text{there is a 2 on the card})\)  
   c) \(P(\text{The number is a multiple of 3})\)  
   d) \(P(\text{The number is prime})\)  

3. Use your protractor to measure the acute angle accurately. State the angle's full name and size in degrees.  

4. Write down equivalent expressions for the following:
   a) \(\frac{6x + 24}{6}\)  
   b) \(7(1 - 5x)\)  
   c) \(3.5(2x + 3)\)  
   d) \(\frac{40 - 16x}{8} + x\)
5. Bisect this angle, using ruler and compass construction. [Leave all construction lines visible.]

6. Calculate the areas of the diagrams shown:
   a) 
   b) 

7. Calculate:
   a) 31.6% of 552 g (to nearest gram)
   b) 47.3 of 890 kg (to 1 d.p.)
   c) 3% of Rs 25 125 (to nearest rupee)
   d) 97.5% of 0.61 l (to nearest ml)

8. Calculate the size of each angle in this kite:

   (not to scale)
9. The number of siblings of each student in Class VII-S was surveyed. This is what was found:

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<tr>
<td>5</td>
<td>1</td>
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</tbody>
</table>

Complete the table and use it to calculate the mean number of siblings. \([\text{4}]\)

10. In the diagram, state the reasons (codes) for the following facts:
   a) \( a = x \)
   b) \( a = b \)
   c) \( x = b \) \([\text{3}]\)

11. Two ordinary fair dice are rolled together and the total score recorded. Make a table showing all possible outcomes. Use your table to find.
   a) \( P(10 \text{ or more}) \)
   b) \( P(\text{even}) \)
   c) \( P(5 \text{ or 6}) \)
   d) \( P(1) \) \([\text{5}]\)

12. Calculate the total area of the side of a building, as shown:

![Diagram](not to scale)

13. Solve these equations:
   a) \( 2p - 5 = 3 \)
   b) \( \frac{q + 1}{2} = 7.5 \)
   c) \( 2(r - 8) = -10 \)
   d) \( 0.3(156s - 48) = 72 \) (to 2 s.f.) \([\text{8}]\)
14. Write down the 3-figure bearings of these directions:
   a) South b) North c) North-West d) East

15. Arrange these in order, smallest first:
   \[
   \frac{2}{9}, \frac{11}{50}, 23\%, \frac{1}{5}, 0.21
   \]

16. In the diagram, calculate the size of \( x \):

17. Find the value of these expressions when \( n = -1.5 \):
   a) \(-6 + 4n\)  
   b) \(-6(n + 0.5)\)
   c) \(\frac{3n + 10}{5}\)  
   d) \(\frac{14 - 2n}{-34}\)

18. The mean height of 8 girls is 141.5 cm. If the tallest girl (149 cm) is absent one day and two new girls (138 cm and 147 cm) join the group, what is the new mean?

19. [Protractor not allowed]
   Draw and label a line \( AC = 10 \) cm.
   Construct a right-angle at \( A \).
   \( AB = 7.5 \) cm. Label \( B \).
   Join \( BC \).
   Construct the perpendicular bisector of \( BC \).
   [Leave all construction lines visible.]

20. Find the value of \( x \) and of \( y \), giving reasons for each stage of your solution.
Answers and Mark Scheme

[Max. Marks = 100]

1. a) –4 b) –3
   c) –34 d) 6 [1 each = 4]

2. a) 0.1 or \( \frac{1}{10} \) b) 0.7 or \( \frac{7}{10} \)
   c) 0.3 or \( \frac{3}{10} \) d) 0.3 or \( \frac{3}{10} \) [1 each = 4]

3. \( \angle AOC = 36^\circ \) or \( \angle COA = 36^\circ \) [Name 1, size 1, = 2, allow ±1°)

4. a) \( x + 4 \) b) \( 7 - 35x \)
   c) \( 7x + 10.5 \) d) \( 5 - x \) [1 each = 4]

5. [1 first arc
   1 next 2 arcs
   1 bisector line
   = 3]

6. a) \( \frac{1}{2} \times 6 \times 8 = 24 \text{ cm}^2 \) b) \( 7 \times 8.5 = 59.5 \text{ cm}^2 \) [2 each = 4, ignore units]

7. a) 174 g b) 421.0 kg
   c) Rs 754 d) 595 ml [1 each = 4]

8. \( 7a + 7a + 4a + 90 = 360 \)
   \[ 18a = 270 \]
   \[ a = 15 \]

   Angles are 60°, 105°, 90°, 105° [1 each = 4, any method]

9.  

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<th>( f )</th>
<th>( xf )</th>
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   Mean = \( \frac{67}{30} \approx 2.2 \) siblings [Table 2
   Mean 2

   Allow 1 for round off error or failing to round to 1 d.p. = 4]

10. a) (vert opp \( \angle s \)) b) (alt \( \angle s \))
    c) (corr \( \angle s \)) [1 each = 3]
11. |   | 1 | 2 | 3 | 4 | 5 | 6 |
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</table>

a) $\frac{1}{6}$  
b) $\frac{1}{2}$  
c) $\frac{1}{4}$  
d) 0  
[Table 1 + 1 each = 5]

12. Area = $\frac{1}{2} (8.2 + 14.6) 21.4 + \frac{1}{2} (14.6 + 8.2) 12.6$

= 243.96 + 143.64

= 387.6 m²  
[3 each area + 1 final answer = 7]

13. a) $p = 4$  
b) $q = 14$  
c) $r = 3$  
d) $s = 1.8$ (2 s.f.)  
[1, 2, 3 = 8, any method]

14. a) 180°  
b) 000°  
c) 315°  
d) 270°  
[1 each = 4]

15. $\frac{1}{5}$ 0.21 $\frac{11}{50}$ $\frac{2}{8}$ 23%

20% 21% 22% 22.2% 23%  
[1 each in correct position = 5]

16. Interior angles are 53°, 116°, 110° and $x$ (adj ∠s on st line)

$x = 81$ (angle sum of quad)  
[Interior ∠s 1 each, ans 2 = 5]

17. a) −12  
b) 6  
c) 1.1  
d) −0.5  
[2 each = 8]

18. Total girls = $141.5 \times 8 = 1132$

$1132 - 149 + 138 + 147 = 1268$ new total

New mean = $\frac{1268}{9} = 140.9$ cm (1 d.p.)

Deduct 1 if final answer not rounded to 1 d.p.
19. [ABC labelled 2
construction of rt $\angle$ 3
construction of perp bis 3  = 8]

20. $\angle ECD = 20^\circ$ (adj $\angle$s on st line)
x = 65 (angle sum of $\triangle$)
$\angle ABC = x^\circ = 65^\circ$ (alt $\angle$s)
y = 95 (alt $\angle$s)

[2] Allow other
[2] valid methods
[2] = 8
Chapter 18  Sequences

This is an interesting chapter with scope for pushing students to higher level reasoning, especially in dealing with design sequences.

LESSON PLANNING

Objectives
General To be familiar with sequences of numbers and designs, and their \( n \)th term formulas

Specific 1. To know the structure of linear sequences and how to find their \( n \)th term formulas
2. To predict terms of sequences (numbers and designs) using \( n \)th term and other information
3. To recognize a sequence of squares (with or without adjustment) and answer related questions
4. To be aware of other possible rules for generating sequences

Pacing 3 or 4 lessons, 1 homework

Links A topic combining algebraic, numerical, and visual approaches. Multiples.

Method Make a game of it: “What am I thinking?”
Have a list of 6 to 8 prepared sequences.
“What is the missing number?” “What is the next number/design?” Include a Fibonacci and a time sequence, e.g. 8 10 12 2 4 ?
From there move on to special cases of “equal steps”.
Follow the text.
Introduce the idea of a formula for the \( n \)th term value.
Make it clear that \( n \) is not the term value. It is for counting the terms, giving the term position, but the \( n \)th term formula gives the value of the term, e.g.
Term values: 10 20 30 40 50 ... 10\( n \)
Term position(\( n \)) 1 2 3 4 5 ... \( n \)
Follow the text for multiples and adjusted multiples.
[No need to call these linear sequences at this stage, although that is what they are.]
Use EX 18A.
The second half is much more difficult to get across. Expect students to struggle.
However, it is sufficient at this stage if they know that sequences without equal steps do exist and have \( n \)th term formulas.
Use EX 18B. There are many hints to lead students towards the correct answers.

Resources
Squared paper (for the design questions)

Assignments
EX 18B #5, 6, 7 make a creative homework challenge (but don’t be surprised if they can not do them!).

Vocabulary
Sequence, term, $n$th term formula, steps, adjust

ANSWERS

Exercises

EX 18A

1. a) 32 35  
   b) 300 350  
   c) –40 –60  
   d) $\frac{2}{11}$ 0

2. a) $20n$  
   b) $7n$  
   c) $−2n$  
   d) 1.5$n$

3. a) $4n + 3$  
   b) $n + 5$  
   c) $5n + 1$  
   d) $6n – 1$

4. a) $2n – 1$  
   b) $2n + 1$  
   c) $\frac{1}{2}n + \frac{1}{2}$  
   d) 0.6$n – 0$

5. a) $9 – 4n$  
   b) $17 – 5n$  
   c) $130 – 30n$  
   d) $2.5 – 0.7n$

6. a) 5 7 9 11  
   b) 13 33 57 77  
   c) 2.0 2.3 2.6 2.9  
   d) 9 2 –5 –2

7. a)  
   b) 4  
   c) $4n + 1$  
   d) 401

Design 4  
Design 5

8. a)  
   b)  
   c)  
   d)  

Wall 4  
Wall 5
Chapter 18 Sequences

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EX 18B

1. a) n rows, n + 2 columns
   b) n rows, n + 2 columns
   c) n(n + 2)
   d) Design 10

2. a) n^2 + 2
   b) 51
   c) 51
   d) Design 9

3. a) (n + 1)^2
   b) (n + 3)^2

4. a) n(n + 1) or n^2 + n
   b) \frac{1}{2}n(n + 1) or \frac{1}{2}(n^2 + n)
   c) 1 3 6 10 15 21 28 36 45 55
   d) 5050

5. a) 18 20 [primes + 1]
   b) 34 38 [primes \times 2]
   c) 16 18 [primes – 1]
   d) 8.5 9.5 [primes ÷ 2]

6. a) 4 29
   b) –1 1
   c) 1.3 10.9
   d) b 2a + 3b

7. a) days in each month of a leap year; 31
   b) intervals of 1 h 45 min (24 h clock); 0545
   c) bearings at 100° intervals
   d) banknotes (rupees) currently available in Pakistan; 5000
8. a) $4n + 6$  
    b) Design 9  
    c) $8n + 28$  
    d) $360 \text{ m}^2; 728 \text{ m}^2$

9. a) double the previous term; 160  
    b) previous term $\times 3; 243$  
    c) previous term $\times 1.5; 121.5$  
    d) The multiplying factor is the same for the step sequence as the original.

10. a) \begin{align*}
    n &= 2 \\
    \text{Shaded} &= n + 1 \\
    \text{Unshaded} &= n + 2 \\
    \text{Total} &= 2n + 3
    \end{align*}

b) \begin{align*}
    n &= 2 \\
    \text{Unshaded} &= 2(n + 1) = 2n + 2 \\
    \text{Shaded} &= 1 \\
    \text{Total} &= 2n + 3
    \end{align*}

c) \begin{align*}
    n &= 2 \\
    \text{Unshaded} &= 2n \\
    \text{Shaded} &= 3 \\
    \text{Total} &= 2n + 3
    \end{align*}

**EX 18X**

1. [Calculator exercise; when $n = 1$ it may be simplified first.]

2. [Steps (odds) indicate a square in the formula. Comparing with sequence for $n^2$ gives the formula.]
   
   $n$th term = $n^2 + 10$

3. $n$th term = $(2n + 1)^2 - (4n + 3) = 4n^2 - 2$
This short chapter revises basic work on ratios. The new feature is expressing ratios in the form $1:n$ or $n:1$. This sets the scene for work on scale factors later, and on gradients of straight line graphs.

### LESSON PLANNING

<table>
<thead>
<tr>
<th>Objectives</th>
<th></th>
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</thead>
<tbody>
<tr>
<td>General</td>
<td>To solve problems involving ratios</td>
</tr>
</tbody>
</table>
| Specific   | 1. To use equivalent ratios in situations of direct proportion  
2. To compare two quantities using ratios  
3. To divide a quantity in given ratios  
4. To express ratios in the form $1:n$ or $n:1$ and relate these to scale factors and percentages of the whole |

| Pacing     | 2 lessons, 1 homework |
| Links      | Fractions, decimals, percentages |
| Method     | Set EX 19A after the briefest of introductions. The textbook examples may be referred to if students have difficulty. The second stage of scaling down to $1:n$ or $n:1$ involves no new theory, but leads naturally into the problems in EX 19B. Keep explanations brief and utilize the time available on the exercise. Allow limited discussion during classwork: “pair and share” technique works well. |

| Resources  | Graph paper (standard 2 mm recommended) for EX 19A #8 and EX 19B #3 and 9. |
| Assignments | EX 19B #8–10 makes a good homework. |
| Vocabulary | ratio, direct proportion  
scale up, scale down, scale factor |

### ANSWERS

#### Exercises

**EX 19A**

1. a) $7:5$  
b) $5:12$
2. a) $1:90$  
b) $90:1$
3. 35 red; 45 white
4. a) 1:14 b) 14:15
5. a) 7.5 l b) 80 s
6. a) \( \frac{2}{5} \) b) \( \frac{3}{5} \) c) 1.5 (or 1\( \frac{1}{2} \))
7. a) 1:25 b) 2:25 c) 3:1 d) 1000:1
8. a) A Rs 1440 B Rs 2880 C Rs 5760 
b) A Rs 2240 B Rs 3360 C Rs 4480 
c) A Rs 1890 B Rs 3780 C Rs 4410 
d) A Rs 3600 B Rs 2160 C Rs 4320 
9. 120 chickens
10. a) 50 km/h b) 24 mph c) 83 km/h d) 60 mph

**EX 19B**
1. 1:25; SF = 2.5
2. 1:0.9
3. 1:12; 93.6 l
4. 1:1.6; 76.8 km
5. 142:1; Rs 13,308
6. 0.87:1; 87%
7. 1:4.3 (1 d.p.); True
8. a) True b) True c) True d) True
9. flour 470; eggs 9; milk 850 ml; water 320 ml; butter 130 g (salt 4 pinches)
10. D A C B

**EX 19X**
1. red 10; green 20; blue 70
2. apples 96; coconuts 48
3. a) 43 pink, 57 blue b) 87 pink, 113 blue
Although plotting points on a coordinate grid has been extensively covered, the idea that two quantities (x and y) can have a relationship represented by a graph is newly explored here. We begin with straight line graphs, although not all relationships are linear, because they are the easiest and common in practice.

**LESSON PLANNING**

**Objectives**

**General** To draw the graphs of simple linear equations and recognize their gradients

**Specific**
1. To relate the size of the steps in a linear sequence to the gradient of its graph
2. To know that a straight line graph through the origin (with gradient \( m \)) represents a sequence of multiples of \( m \)
3. To know that a sequence with \( n \)th term formula = \( mn + c \) can be represented by a graph of the form \( y = mx + c \)
4. To be able to plot linear graphs given their equations
5. To be able to determine from its equation whether a point lies on a line or not
6. To write down the gradient of a line from its equation
7. To determine whether a line slopes up or down from its gradient
8. To recognize lines of zero gradient

**Pacing** 4 lessons, 1 homework

**Links** Sequences, linear algebraic expressions

**Method**

The strategy suggested here is to link linear equations with linear sequences. By plotting term value \( y \) against term position \( x \), the \( n \)th term formula gives us the equation.

For example, \( y = 2x - 1 \) represents a sequence with \( n \)th term = \( 2x - 1 \)

The step (2 in this case) represents the gradient of the line. The bigger the step, the greater the gradient, the steeper the line. Follow the text for examples.

Show that sequences of multiples produce straight lines through the origin, but that we can adjust up or down to obtain parallel lines.

Only use positive gradient examples at this stage.

Use EX 20A, which is easy but important.

Ensure most students complete this before moving on.
Demonstrate that a decreasing sequence leads to a downward sloping graph. Negative step; negative gradient.

Mention (for the future) that the gradient can be zero, when all the terms are the same, giving a horizontal line graph, e.g. \( y = 3 \)

from the sequence \( \frac{3}{0} \frac{3}{0} \ldots \)

Distinguish the steepness of a line from its gradient e.g. a gradient of \(-3\) is less than a gradient of \(-2\), but the line is steeper.

Hammer home the concept of the line equation being true for every point on the line, and false for every point off the line. This features in EX 20B and should be used extensively.

<table>
<thead>
<tr>
<th>Resources</th>
<th>Photocopiable coordinate grids (9 mm squares) or commercial graph paper</th>
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<tbody>
<tr>
<td></td>
<td>Prepared graphs on flip-chart or A3 thin card for demonstration saves time.</td>
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<td>Multimedia projector—some excellent graphing software available</td>
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</table>

| Assignments | Aim to complete both exercises by all students.                      |
|            | Suitable for homework EX 20B #1−5                                    |

| Vocabulary | multiples, step gradient, positive, negative, slope, steepness        |

**ANSWERS**

**Exercises**

**EX 20A**

1. a) 5  
   b) 2  
   c) 2  
   d) 3

2. a) Yes  
   b) Yes  
   c) No  
   d) Yes

3. a) \( y = 2x + 1 \)  
   b) \( y = 5x - 6 \)  
   c) \( y = 6x \)  
   d) \( y = x - 5 \)

4. P S Q R

5. a) 9 18 27 36  
   b) 6 8 10 12  
   c) -16 -15 -14 -13  
   d) 2 3.5 5 6.5

6. a) \( A(1, 3) \)  
   b) \( B(2, 16) \)  
   c) \( C(5, -3) \)  
   d) \( D(-1.5, -2) \)

7. a) \( y = 2x \)  
   b) \( y = 2x + 8 \)  
   c) \( y = 2x - 4 \)  
   d) \( y = 2x - 8 \)
8.  a) 

\[ y = 2x + 1 \]

b) \( y = x + 7 \)

c) \( y = x + 7 \)

d) Not on line

9. 

a) \( A(2, 9) \) \( B(-3, 1.5) \)

d) \( P \) is on \( y = 1.5x \)

Check: \( y = 1.5 \times 2 \)

10. a) \( A(-2, -9), B(2, 3) \)

b) e.g. \((-6, -11)\) and \((5, 11)\)

c) gradient \( a = 3 \); gradient \( b = 2 \); 
(a) is steeper

d) \( (4, 9); \) true for \( a \) \( 9 = 3 \times 4 - 3 \);
true for \( b \) \( 9 = 2 \times 4 + 1 \)
EX 20B

1. a) –2  b) Decreasing  c) –2  d) Down
2. a) No  b) Yes  c) Yes  d) No
3. a) $y = -x + 6$  b) $y = 2x + 7$
   c) $y = -1.5x$  d) $y = -3x + 2$
4. a) –1; Down  b) 2; Up
   c) –1.5; Down  d) –3; Down
5. a) 1.5  b) –5  c) $\frac{2}{5}$  d) –1
6. a) 4; Up  b) –1; Down
   c) –3; Down  d) 2; Up
7. a) $y = 3x$  b) $y = 3x - 12$
   c) $y = -2x + 4$  d) $y = -2x - 10$
8. a) 
   
   b) $\frac{1}{2}$
   c) $y = \frac{1}{2}x + 6$
   d) When $x = 1, y = 6 - \frac{1}{2}$
   
   R not on line
9. \(P(-2, 5), Q(10, -19)\)

\(c)\) When \(x = -14, y = -2x - 14 + 1 = 29. T\) is not on the line.

Graph checks \(T\) just off line.

10. \(a)\) (4, 5) and (-8, 2) \hspace{1cm} \(b)\) 0 gradient

\(c)\) \(T(8, 6)\) \hspace{1cm} \(d)\) \(y = 6\) True for \(T\); when \(x = 8, y = \frac{1}{4} \times 8 + 4 = 6\) (true for \(T\))
EX 20X

1. a) QRPS  
   b) PRQS

2.

3. $y = x^2$; parabola; gradient varies

Different—not a parabola; does not increase so fast; negative $x$ meaningless
Alike—variable gradient that increases with $x$
In the text enlargements are introduced as transformations that change the size of an object without distorting its shape.

**LESSON PLANNING**

**Objectives**

**General**
To understand and use enlargement of geometrical diagrams

**Specific**
1. To know that an enlargement is defined by its centre and scale factor (SF)
2. To know that an enlargement preserves angles, i.e. the shape does not distort
3. To know that all lengths on an enlarged shape are scaled up or down uniformly, according to the scale factor
4. To know that the centre of an enlargement is the point from which it seems to grow
5. To know and use the fact that guidelines connecting corresponding points meet at the centre of enlargement
6. To use notation such as $A \rightarrow A'$ to represent corresponding points
7. To calculate scale factors using the fact that corresponding lengths are in proportion, i.e. $SF = \frac{\text{new length}}{\text{old length}}$
8. To know that the inverse of an enlargement is also an enlargement with $SF$ which is the reciprocal of the original enlargement
9. To know that mathematical enlargements can make diagrams larger or smaller: $SF > 1$ larger, $SF < 1$ smaller
10. To change the scale of a map expressed as a ratio to a scale factor, and vice versa; to calculate corresponding distances using this information

**Pacing**
At least 4 lessons (because a lot of drawing is required), 1 homework

**Links**
Ratio, fractions, arrow diagrams (inverse)
accurate drawing and measurement

**Method**
Revise reflection, rotation, translation orally “What do they have in common?” Size is preserved.
Introduce enlargements where shape is preserved, but size changes. Give examples with easy scale factors (e.g. $SF = 2$)
Define centre of enlargement and show how to use guidelines. Follow the text.
Emphasize that enlargement must not distort the shape so corresponding lengths are scaled the same.
Use EX 21A.
Revise some enlargements. Use arrow diagrams to find the scale factor, e.g.

\[
\begin{array}{c}{\times} \frac{3}{1} \\ \overline{\div} \frac{3}{1}\end{array}
\]

\[SF(\text{reverse}) = \frac{1}{3}\] because
\[
\times \frac{1}{3} = \div 3\]
Explain that in Maths, reducing the size is also an enlargement! In such cases SF < 1
Follow the text for maps.

Use EX 21B

<table>
<thead>
<tr>
<th>Resources</th>
<th>Squared paper (9 mm photocopiable available here)</th>
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<tbody>
<tr>
<td></td>
<td>Coordinate grids (for EX 21A #8 and EX 21B #10)</td>
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</table>

<table>
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<tr>
<th>Assignments</th>
<th>Suitable for homework: EX 21A #6 and 9</th>
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</table>

<table>
<thead>
<tr>
<th>Vocabulary</th>
<th>enlargement, centre, scale factor (SF)</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>guidelines, corresponding points</td>
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<tr>
<td></td>
<td>preserve</td>
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<td></td>
<td>map ratios</td>
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<td>(congruent, similar)</td>
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</table>

**ANSWERS**

**Exercises**

**EX 21A**

1.  a) Yes; SF = 2 b) No c) Yes; SF = 3 d) No

2.  a) 

   ![Diagram](image-url)
3. b) 

a) 13.1 cm (approx)  
   c) 26.2 cm (approx)  

Same SF = 2  

[Note: these answers depend on the size of squares being used.]

4. a) $A'B' = 35$ cm  
   b) $\angle A'B'C' = 10^\circ$  
   c) diagonal $B'G' = 40.5$ cm  
   d) perimeter = 163.0 cm

5. a)
Chapter 21 Enlargements

b) 

c)
Chapter 21 Enlargements

6. a) 

b)
Chapter 110

21. Enlargements

c) [Diagram of an enlarged shape with dimensions 7 cm, 5 cm, 3.5 cm, and 2.5 cm marked]

d) [Diagram of another enlarged shape with dimensions marked]

7. [Diagram of a square with dimensions 7 cm, 3.5 cm, 2.5 cm, and 5 cm marked]
9. a) $SF = 1.8; x = 0.8 \text{ cm}$
   b) $SF = 2.4; y = 4 \text{ cm}$
   c) $SF = 1.5; p = 8 \text{ cm}; q = 13.5 \text{ cm}$
   d) $SF = 1.6; r = 11.2 \text{ cm}; s = 20 \text{ cm}$

10. a) $A' (-4, 6)$
    b) $B' (2, 6)$
    c) $C' (1, 4)$
EX 21B

1. a) $Q$  
   b) $R$  
   c) $S$  
   d) $P$

2. a) 2  
   b) 1.5  
   c) $\frac{2}{3}$  
   d) $\frac{1}{3}$

3. a) 5  
   b) $\frac{5}{2}$  
   c) $\frac{4}{3}$  
   d) $\frac{2}{3}$

4. a) 0.4  
   b) 0.625  
   c) 0.25  
   d) 0.8

5. a) 1:50  
   b) SF = 50  
   c) 110 cm desk; 190 cm board  
   d) 90 cm door; 75 cm desk

6. a) 450 m (approx)  
   b) 350 m (approx)  
   c) 1:5000  
   d) 7.5 cm

7. a) 1:400  
   b) 10 m  
   c) 208 m$^2$  
   d) 30 m by 22 m

8. A grid with labeled points A, B, C, D, and E, with axes labeled x and y.

9. a) 1:20000  
   b) 1:50  
   c) 1:10  
   d) 1:25000

10. a) 26 cm  
    b) 62 m  
    c) 20 cm  
    d) 1:1 500 000
EX 21X

1. Place the four pieces of paper like this:

```
A5
A6
A7
A8
```

The vertices aligned on the diagonal demonstrate that they are enlargements.

The scale factor from each piece to the next larger size is 1.4 (approx). The scale factor to the next smaller size is 0.7 (approx).

2. Only (b) is true. The others are false.

Widescreen TV has ratio of width : height = 16 : 9. However, much material broadcast was designed to fit old screens in the ratio 4 : 3 (which is 16 : 12). The pictures have to be distorted to make them fit the widescreen.

3. The area SF is the square of the enlargement SF.
ANSWERS

Exercises

EX 22A

1. \( D(8, 1), E(6, 1), F(8, 4) \)

2. a) \( \frac{1}{18} \)  
   b) \( \frac{1}{3} \)  
   c) \( \frac{2}{3} \)  
   d) 0  
   e) \( \frac{1}{3} \)  
   f) \( \frac{1}{9} \)

3. \( a = 30, b = 60, c = 60, d = 120, e = 40, f = 160 \)

4. a) 4  
   b) -4  
   c) 2  
   d) 10

5. a) 5.52 l  
   b) 2 h

6. a) 1, 2, 4, 8, 16  
   b) 1, 2, 3, 6, 9, 18  
   c) 1, 2, 4, 5, 8, 10, 20, 40  
   d) 1, 2, 3, 5, 6, 9, 10, 15, 18, 30, 45, 90

7. a) \( x = 4 \) cm  
   b) \( y = 8 \) cm, \( z = 5 \) cm

8. 30 cm

9. 36% 0.39 \( \frac{2}{5} \) 0.41 \( \frac{4}{9} \) \( \frac{7}{15} \)

10. a) \( p = 42 \) (alt \( \angle \)s)  
   b) \( q = 42 \) (corr \( \angle \)s)  
   c) \( q = 42 \) (alt \( \angle \)s)  
   d) \( r = 138 \) (adj \( \angle \)s on st line)

EX 22B

1. a) 30 cm\(^2\)  
   b) 20 cm\(^2\)

2. a) 30 - \( x \)  
   b) 15 + \( x \)  
   c) 30\( x \)  
   d) \( \frac{x}{3} \)

3. a) 336  
   b) 84  
   c) -15  
   d) -112

4. a) 6  
   b) \( \frac{2}{3} \)  
   c) \( \frac{2}{9} \)  
   d) \( \frac{1}{3} \)

5. a) Rs 1.4 million  
   b) 11.7%  
   c) 13.2%

6. 30 000 cm\(^2\) (or 3 m\(^2\))
7. 

8. a) 

b) $n$

c) $2n + 6$

d) 60 green; 132 white

9. a) A: Rs 350 000  
    B: Rs 299 000

b) A: Rs 200 000  
    B: Rs 393 000

c) In general, Company A has higher executive salaries than Company B, and they are not so diverse.

10. He should buy 3 doz; 2.4 bottles per guest.
EX 22C

1. a) \( x = 8 \text{ cm} \)  
   b) \( y = 7 \text{ cm} \)

2. a) \( x \times 5 \rightarrow 5x \rightarrow -9 \rightarrow 31 \)
   b) \( x \times 9 \rightarrow 9x \rightarrow +5 \rightarrow 31 \)
   c) \( x \rightarrow +5 \rightarrow x+5 \rightarrow \times 9 \rightarrow 31 \)
   d) \( x \rightarrow -9 \rightarrow x-9 \rightarrow \times 5 \rightarrow 31 \)

3. a) \( \frac{1}{3} \)  
   b) \( \frac{4}{5} \)  
   c) \( \frac{4}{5} \)  
   d) \( \frac{2}{3} \)
   e) \( \frac{1}{2} \)

4. i) \( a = c \)  
   ii) \( b = d \)  
   iii) \( a + b = 180 \)  
   iv) \( c + d = 180 \)  
   \( a = 50, b = 130, c = 50, d = 130 \)

5. \( 256 \text{ cm}^2 \)

6. a) \( 5a + 25 = 5(a + 5) \)  
   b) \( 4(b - 1) = 4b - 4 \)
   c) \( 3(c - 4) = 3c - 12 \)  
   d) \( 10d - 15 = 5(2d - 3) \)

7. a) \( \frac{1}{2} \)  
   b) \( y = \frac{1}{2}x \)
   c) \( 2 = \frac{1}{2}x + 3 \)  
   d) \( y = \frac{1}{2}x - 4 \)

8. a) rectangle, square, parallelogram, rhombus
   b) rectangle, square, parallelogram, rhombus
   c) square, rhombus, kite  
   d) rectangle, square

9. [Diagram of angles measuring 26°]
EX 22D

1. a) e.g. 30 \times 10 \text{ (many other factors)}
   
   b) 201 = 3 \times 67
c) 261 = 9 \times 29
d) 91 = 7 \times 13

2. \quad n
\quad n + 10
\quad 2(n + 10) = 2n + 20
\quad 2n + 20 - 10 = 2n + 10
\quad \frac{2n + 10}{2} = n + 5
\quad n + 5 - n = 5

The answer is always 5.

3. \quad 1 : 1.1; 46.2 l

4. a) Both have 6 faces; the cubes’ faces are squares but the cuboids’ faces are rectangles.
   
   b) Both have 12 edges; the cubes’ edges are equal in length but the cuboids have 4 lengths, 4 widths, and 4 heights.
   
   c) Both have 8 vertices.
   
   d) cuboid, \( V = lbh \); cube, \( V = l^3 \)

5. a) \( x = 24 \)

6. \quad 25^\circ = \angle BAC = \angle CAB = \angle FAC = \angle CAF \text{ (any two)}
\quad 65^\circ = \angle DBF = \angle FBD
\quad 115^\circ = \angle BDE = \angle EDB = \angle CDE = \angle EDC \text{ (any two)}
7. a) \[ y = x + 1 \]
   
   b) 1
   
   c) \[ y = x + 1 \]
   
   d) No, below
   
   \[ 3.2 \neq 2.3 + 1 \]

8. a) \[ 5a - b \]
   
   b) \[ 7p - 8q \]
   
   c) \[ r + s \]
   
   d) \[ -x + y \]

   e) \[ 3a - 16 \]
   
   f) \[ 3b + a - 7 + c \]
   
   g) \[ 2a^2 + 4a + 6 \]
   
   h) \[ 1 + x^2 + x \]

9. A geometric figure with angles labeled 45°, 60°, 70°, and 90°.

10. a) 
    
    b) green tiles = \( n \); white tiles = \( 2n + 2 \)
    
    c) green 10; white 22
    
    d) 31
EX 22X

1. a) $-12$
   
   b) $-12 + 3 - 9 + 3 - 0.5 - 3.5 + 1 - 2.5 \times 2 = -5$

2. [with symmetrical additions]

3. a) 2, 3, 5, 7, 11, 13, 17, 19
   
   b) Even total: avoid choosing 2.
      
      Odd total: choose 2 and any other prime
   
   c) All other even numbers are multiples of 2 and hence not prime.
   
   d) Consecutive numbers are alternately even and odd.
      
      So 3 consecutive numbers must be
      
      either E O E,
      
      or O E O.
      
      As 2 is the only even prime, and the smallest, neither of these is possible.
This chapter develops previously-introduced representations of data and adds time series and timetables. More rigorous attention is given to labelling, interpreting, and accurate drawing than before.

**LESSON PLANNING**

**Objectives**

| General | To draw and interpret correctly bar charts, line graphs, and pie charts; to interpret time series; to use 24 h transport timetable |
| Specific | 1. To draw and interpret line graphs, and bar charts of grouped and ungrouped data, with either horizontal or vertical bars  
2. To draw accurate pie charts by calculating the correct angles required for each sector; to interpret from given pie charts  
3. To interpret from given time series; to be aware of the risk of interpolation  
4. To answer simple logistical transport questions with reference to 24 h timetables |

**Pacing** | 3 lesson, 1 homework |

**Links** | Fractions of a quantity, ratios, use of calculator (with rounding off), accurate drawing (using ruler, compasses, and protractor), changing 24 h clock to am/pm times |

**Method** | Minimal explanation is required because what is not revision in this chapter is just common sense.  
Follow the text for horizontal bar charts, how to label the bars for grouped and ungrouped data, and how to calculate angles (fractions of 360°) for pie charts.  
Get busy on EX 23A as soon as possible: the learning comes with the practice.  
Use the examples in the text of vertical and horizontal timetables. Find the correct line for a bus and/or train. You might want to revise time period calculations. [See Book 6 Ch 23]  
Maximize time spent on EX 23B.  
Time series is easy. Explain that the lines joining the points show the general trend or direction and only the measured points are accurate. EX 23C should be quick and easy for most students. |

**Assignments** | Suitable homework: EX 23B #3 and 6 |
Vocabulary

grouped and ungrouped data
bar chart, line graph, pie chart, time series

ANSWERS

Exercises

EX 23A

1. a) Science 93% b) Eng Lit and History c) 5% marks d) 26% marks

2. a) red b) \( \frac{23}{164} \) c) 88° red; 50° orange d) 190° blue; 14° green; 9° yellow; 9° others; Total = 360°

3. a) ungrouped b) size 7\( \frac{1}{2} \) c) 99 boys d) size 6; 30 boys

4. a) Teacher

\[ \begin{array}{c}
A \\
B \\
C \\
D \\
\end{array} \]

b) C is 3 × A
c) 165 girls
d) 41.3 votes; B

5. a) 8–10 min

\[ \begin{array}{c}
\text{Customers} \\
0 \\
20 \\
40 \\
60 \\
80 \\
100 \\
120 \\
140 \\
160 \\
\end{array} \]

b) 8–10 min
c) 14 min; exact longest and shortest times not known
d) agree

6. a) Beef most; Vegetable least b) \( \frac{14}{45} \) c) \( \frac{17}{90} \) d) Beef 1071; Chicken 667; Vegetable 405

7. a) 7 b) most April; least November c) December and February d) 40
8. a) 150 days  
   b) 0–10 °C  
   c) 75; we are not given raw data.

     | Temp °C | \( f \) |
     |--------|--------|
     | 0–10   |  5     |
     | 10–20  |  60    |
     | 20–30  | 150    |
     | 30–40  |  140   |
     | 40–50  |   10   |
     | Total  |  365   |

9. a) \( \frac{5}{64} \)  
   b) Angles are:  
      Food 84°,  
      Rent 197°,  
      Transport 45°,  
      Clothes 28°,  
      Other 6°.

10. a) 31; probably a knockout with 32 competing teams

EX 23B

1. a) 1 h 6 min  
   b) 30 min  
   c) 1 h 30 min  
   d) 12 min

2. a) 40 min  
   b) 30 min  
   c) 1 min  
   d) 28 min

3. a) The 0857  
   b) The 1137  
   c) The 0857  
   d) The 0737

4. a) 0909  
   b) 0829  
   c) 0749  
   d) 0909

5. a) 0947  
   b) 1239; 2 h 10 min  
   c) 0700 Balaware to Devpur, 0857 Devpur to Liman  
   d) 0820 Balaware to Exitan, 1209 Exitan to Liman
6. a) 20 min  b) 29 min  c) 13 min  d) 3 min
7. a) The 1702  b) The 1017  c) The 1332  d) The 1227
8. a) 22 min  b) 19 min  c) 3 min  d) 1325
9. a) 1558  b) 1033  c) 1243  d) 1033
10. a) 1154  b) 8 min
     c) 1017 Prince Rd to Mall Rd; 1245 Mall Rd to Hightown
     d) The 1540; 1 h 20 min; 8 min

EX 23C
1. a) 4 days  b) 5 days
    c) 24th–29th  d) 18th; 3.5 h
2. a) 40 °C; Thu 15th am  b) Thu 15th  c) Fri 16th pm to Mon 19th am
    d) 2 °C
3. a) Sun  b) 105
    c) Wed & Thu  d) Sat, Fri, Tue

EX 23X
1. By using the median of each group to represent it we obtain a mean of 27.5 °C (1 d.p.)
2. Liman 1300 1340 1420 1500 1600
   Fourways 1312 1352 1432 – 1612
   Exitan 1330 1410 1450 – 1630
   Devpur 1402 1442 1522 – 1702
   Crendon 1418 1458 1538 1609 1718
   Balaware 1439 1519 1549 1630 1739
3. a) ![Graph](image)
    b) 7450 m (estimated)
Chapter 24

Harder Equations

In this series, algebra is built up in stages. We deal with more difficult linear equations in this chapter. First, we introduce the concept of the equation as a balance. Then we bring in negative terms and negative solutions. We avoid all cook book recipes for solution such as “change sides, change sign” because these do not aid understanding of the method.

LESSON PLANNING

Objectives

General  To solve linear equations

Specific  1. To use the concept of a balance as an aid to solve linear equations
        2. To understand that the solution value makes the equation a true statement, all other values make it false
        3. To check solutions to equations by substituting (separately) into the right and left hand sides

Pacing  3 lessons, 1 homework

Method

• Demonstrate the balance method on the board using only positive numbers and unknowns to start with as examples. When balancing, physically rub out the balancing items, e.g. for \(3x + 7 = 2x + 8\)

Rub out 2\(x\) symbols on each side.
Then, obviously \(x = 1\)
Always do a check, to see if the equation is true with your solution value, e.g. for the above:
left = \(3x + 7 = 3 \times 1 + 7 = 10\)
right = \(2x + 8 = 2 \times 1 + 8 = 10\)
checks OK.
Use EX 24A.

• When there are negative terms we cannot draw pictures, but the balancing technique can be used by adding to both sides to neutralize the negatives. Use the text examples; set EX 24B.
Insist on working down the page with the equals signs vertically aligned. The problems in #8, 9, and 10 should not be avoided.

**Assignments**

Suitable homework: EX 24B #3, 4, 5

**Vocabulary**

balance, solve, true statement

**ANSWERS**

**Exercises**

**EX 24A**

1. a) \(3a + 2 = 2a + 16; a = 14\)  
   b) \(b + 12 = 4b + 3; b = 3\)  
   c) \(2c + 19 = 7c + 14; c = 1\)  
   d) \(2d + 36 = 4d + 18; d = 9\)

2. a) \(a = 1\)  
   b) \(b = 2\)  
   c) \(c = 3\)  
   d) \(d = 5\)

3. a) \(p = 3\)  
   b) \(q = 7\)  
   c) \(r = 1\)  
   d) \(s = 15\)

4. a) \(u = 9\)  
   b) \(v = 3\)  
   c) \(w = 0.5\)  
   d) \(x = 0.3\)

5. a) \(x = \frac{1}{2}\)  
   b) \(x = \frac{2}{3}\)  
   c) \(x = \frac{4}{5}\)  
   d) \(x = \frac{1}{7}\)

6. a) \(y = 0.5\)  
   b) \(y = 0.2\)  
   c) \(y = 0.7\)  
   d) \(y = 0.9\)

7. a) \(m = 5\)  
   b) \(n = 6\)  
   c) \(p = 7\)  
   d) \(q = 9\)

8. a) \(a = 5\)  
   b) \(b = 0.5\)  
   c) \(c = 2\)  
   d) \(d = 8\)

9. a) \(y = 5\)  
   b) \(y = 7\)  
   c) \(y = 4\)  
   d) \(y = 1.5\)

10. a) \(2n + 23\)  
     b) \(5n + 2\)  
     c) \(2n + 23 = 5n + 2; n = 7\)  
     d) Ahmed \(2 \times 7 + 23 = 37\); Bilal \(5 \times 7 + 2 = 37\)

**EX 24B**

1. a) \(3n + 2 = 7n – 18\)  
   b) Take \(2x\) away from each side.

   \[2 = 4n – 18\]  
   Add 7 to each side.

   \[20 = 4n\]  
   Divide by 3

   \[5 = n\]  
   \[n = 5\]

   c) \(6m – 3 = 4m + 7\)  
   d) \(9y – 1 = 6 – y\)

   \[2m – 3 = 7\]  
   \[10y – 1 = 6\]

   \[2m = 10\]  
   \[10y = 7\]

   \[m = 5\]  
   \[y = 0.7\]
2. a) \( p = 3 \)  
   b) \( q = 4 \)  
   c) \( r = 5 \)  
   d) \( s = 12 \)

3. a) \( x = 8 \)  
   b) \( x = 1 \)  
   c) \( y = 3 \)  
   d) \( y = 5 \)

4. a) \( k = \frac{1}{2} \)  
   b) \( l = \frac{1}{3} \)  
   c) \( m = \frac{3}{5} \)  
   d) \( n = \frac{9}{2} \)

5. a) \( a = 0.2 \)  
   b) \( b = 0.6 \)  
   c) \( c = 0.8 \)  
   d) \( d = 1.7 \)

6. a) \( w = 5 \)  
   b) \( x = 25 \)  
   c) \( y = 2.6 \)  
   d) \( z = 8 \)

7. a) \( a = -2 \)  
   b) \( b = -3 \)  
   c) \( c = -7 \)  
   d) \( d = -5 \)

8. a) \( 2n + 3 = 5n - 9; n = 4 \)  
   b) \( 5t - 9 = t + t + 6; t = 5 \); Sonia 11, Taymur 5

9. a) \( m + 1000 = 2(m - 250); m = 1500 \); Rs 1500 each  
   b) \( 2(d + 8) = 100 - d; d = 28 \); February

10. a) \( 4q = 3(q + 3); q = 9 \); Qirat 9, Rida 10, Sarah 12  
    b) \( b + 13 = 2(b + 1); b = 11 \); Bryant 11, Jibran 12

EX 24X

1. 5 : 4

2. Ali 4, Babar 12

3. ii) \( 3(x - 1) + 1 \)  
   iv) \( 3x - 2 \)  
   v) \( 4x - 1 - x + 2 \)
Although the prism has been discussed in Chapter 16, this short chapter attempts to clarify the meaning of cross-section. [It also brings the concept of symmetry into three dimensions in EX 25X #2 and 3, as an optional extra.]

LESSON PLANNING

Objectives

<table>
<thead>
<tr>
<th>General</th>
<th>To visualize and draw cross-sections of solids</th>
</tr>
</thead>
</table>
| Specific | 1. To draw a sequence of cross-sections of a solid which is non-uniform  
2. To identify uniform cross-section and know that this defines a prism  
3. To identify planes of symmetry [optional] |

Pacing | 1 lesson |

Links | Sequences |

Method | This is a very easy chapter which comes as light relief towards the end of the academic year. Just set EX 25A and let them get on with it!  
Allow discussion |

Assignments | Ex 25X, the optional extra exercise, is not so difficult in this chapter. If homework is needed, #1 is suitable. |

Vocabulary | prism, cross-section, sequence  
[plane of symmetry] |

ANSWERS

Exercises

EX 25A

1. D A B C
2. B A D C
3. 

[Diagram of cross-sections]

128
4. a stool (such as ones used in laboratories)
5. None are prisms.
6. box and chocolate
7. C B A F E D
8. 3. a) 5  b) 1  c) 3  d) 2
9. If a prism has \( n \) lines of symmetry in its cross-section, then it has \( n + 1 \) planes of symmetry.
10. EX 25X
Chapter 26 Units

This is an important chapter, but more for general knowledge than for developing mathematical ability.

LESSON PLANNING

Objectives

<table>
<thead>
<tr>
<th>General</th>
<th>To know the connections between units and how to use them correctly</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specific</td>
<td>1. To know the commonly used units of length, area, volume, and mass (“weight”) and to calculate with them correctly</td>
</tr>
<tr>
<td></td>
<td>2. To know the land measures of are and hectare, their symbols, their connection to m², etc.</td>
</tr>
<tr>
<td></td>
<td>3. To know and use the connections between the mass and volume of water</td>
</tr>
<tr>
<td></td>
<td>4. To convert from old units when given approximate SI units</td>
</tr>
<tr>
<td></td>
<td>5. To understand that SI is a coherent system, write the unit symbol prefixes correctly, and grasp the concept of orders of magnitude</td>
</tr>
</tbody>
</table>

Pacing | 4 lessons, 2 homeworks

Links | Estimation, rounding off
| General science topics
| International travel

Method | This topic can be quite tedious. Not all the unit relationships need to be memorised. Fortunately, quite a few can be worked out quickly from those already known, or from a few key visual images.

Follow the text for revision of the simple relationships and definition of are and hectare. Students may be puzzled that areas are usually in square units (m², cm², km² etc) but are and hectare are not.

Explain that they are units of area in themselves, so a power of 2 is not necessary.

Set EX 26A. Allow referring back to the text, and use of calculators.

Start a class discussion to elicit other units they may have met on their travels or otherwise. Make a list of these. Then refer to the text for approximation.

Set EX 26B.

The text lays out the most relevant information about SI but there is no need to go through it in detail. A good class project is to research all the units named after people: Volt, Kelvin, Hertz, etc (They all have capital letters for their units symbols.)
In class, focus on section 6, the correct method of writing units. Exercise 26C reinforces this, and knowledge of the most often used prefixes.

**Assignments**

Research units named after people (see above)
EX 26M #6-10
Both suitable for homework

**ANSWERS**

**Exercises**

**EX 26A**

1. a) 100 cm
   b) 100 cm
   c) 100 cm

   100 rows of 100 cm² = 10 000 cm²

2. a) 11 250 cm²
   b) 1.125 m² checks

3. a) 60 270 cm³
   b) 0.06027 m³ checks

4. a) 5360 kg
   b) 5.36 t checks

5. 906.5 km²

6. a) 144 000 cm³
   b) 134.4 l
   c) 142.8 kg
   d) 140.2 kg

7. a) 12 000 cm², 1.2 m²
   b) 6500 cm³, 0.65 m³
   c) 1800 cm², 0.18 m²
   d) 52 000 cm³, 5.2 m³

8. a) 3.565 l
   b) 7.32 l
   c) 1.2696 l
   d) 3.3 l

9. b) (108 000 cm³)
   a) (40 000 cm³)
   d) (4500 cm³)
   c) (1500 cm³)

10. a) 600 l
    b) 7500 cm² or 0.75 m²
    c) 1.8 m², 13 pieces, 0.6 m² left over
    d) 475 cm³

**EX 26B**

1. a) 256 km
   b) 64 km
   c) 152, 128 km
   d) 120 km, 58 km, 40 km

2. a) 3.15 m²
   b) shorter 19.8 m
   c) 10 cm
   d) 1.05 m
3. a) 50  
   b) 65  
   c) 80  
   d) 95
4. a) 70 kg  
   b) 79 kg  
   c) 83 kg  
   d) 64, 72, 76 kg
5. a) 88 l  
   b) 72 l  
   c) 64 l  
   d) 48.8 l
6. a) 4.5 l  
   b) 22.5 l  
   c) 45 l  
   d) 90 l
7. 0.2 ha (1 d.p.)
8. a) 75 l/s  
   b) 67 l/s
9. a) 20 °C  
   b) 0 °C  
   c) 212 °F  
   d) –40 °F
   [Interesting to note that –40 °C = –40 °F.]
10. a) 20.2 m²  
    b) 4.3 m²  
    c) 10.8 m²  
    d) 3.8 m²

EX 26C

1. c)  b) a)  d)
2. a) 2500 g  
    b) 0.25 g  
    c) 600 g  
    d) 0.075 g
3. a) 0.776 l  
    b) 0.7 l  
    c) 5 l  
    d) 0.045 l
4. c) b)  d) a)
5. a) 185 kPa  
    b) 8.3 kHz  
    c) 70 cm  
    d) 2.3 MN
6. d)  c) b)  a)
7. a) 6 300 000 J  
    b) 250 000 J  
    c) 400 000 J  
    d) 0.885 J
8. a) min, h  
    b) ha  
    c) 1.95 t  
    d) dB
9. a) 37 000 000  
    b) 4 800 000 000  
    c) 90 000 000  
    d) 1 400 000
10. a) ms  
     b) Mt  
     c) Mm  
     d) mm

EX 26M

1. a) 5500 cm²  
    b) 0.55 m²  
    Check
2. 553.4 km²
3. a) 2 tins  
    b) 6 tins  
    c) 1 tin  
    d) No change  
    [Note: Rounding up is necessary here.]
4. a) 3936  
    b) 2397  
    c) 7816  
    d) 128
5. a) 79 kg  
    b) 81 kg  
    c) 87 kg  
    d) 93 kg
   Estimates are too high.
6.  2 ha (nearest whole number)
7.  a) 2.4 m   b) 8.9 m$^3$ (2 s.f.)   c) 13 m$^2$ (2 s.f.)  
    d) 3.2 a (2 s.f.)
8.  a) 0.884 l  b) 0.8 l  c) 2.5 l  d) 0.036 l
9.  c)  d) a)  b)
10. a) 300 000  b) 48 000 000  c) 2 500 000 000  
    d) 20 000 000

**EX 26X**

1.  a) 1 zettametre is enormous = 1 Zm = 10$^{21}$ m.
    1 zeptometre is incredibly small = 1 zm = 10$^{-21}$ m.
    b) 1 Ym = 1 yottametre = 1000 Zm = 10$^{24}$ m
    1 ym = 1 yoctometre = $\frac{1}{1000}$th of 1 zm = 10$^{-24}$ m
    c) Standard form uses indices on a base of 10 to indicate zeros. Scientists use it because it avoids having to use prefixes.

2.  Primidi, Duodi, Tridi, Quartidi, Quintidi, Sextidi, Septidi, Octidi, Nonidi, De'cadi

3.  Centigrade has been in use since 1743. In 1948 it was renamed the degree Celsius, because it was confused with an angle unit of the same name. However, even today, some people persist in calling it Centigrade.

    Fahrenheit was a physicist who designed a temperature scale with 32 °F at the freezing point of water, and 96 °F for human body temperature, so he could easily divide up the temperatures between by repeatedly dividing the scale by 2.

    $[96 - 32 = 64 = 2^9]$

    He found that water boils at around 212 °F on his scale.
Chapter 27

Revision Exercises

ANSWERS

Exercises

EX 27A

1. a) \( \frac{2}{5}, 0.4 \)  
b) \( \frac{1}{5}, 0.2 \)  
c) \( \frac{4}{5}, 0.8 \)  
d) \( \frac{3}{5}, 0.6 \)

2. Multiply by 3; add 12; divide by 3; take away the number first thought of; the answer is 4.

3. a) \( 3n - 2 = 13 \)  
b) \( 2(n + 5) = 22 \)  
c) \( 4n + 2 = 30 \)  
d) \( \frac{n + 15}{5} = 15 \)

4. Incircle of \( \triangle ABC \)

5. 6 cm

6. a) 23, up  
b) -2, down  
c) -5, down  
d) 9, up

7. a) \( a = -2 \)  
b) \( b = -5 \)  
c) \( c = -3 \)  
d) \( d = -9 \)

8. a) \( l = 12 \text{ cm} \)  
b) \( d = 6 \text{ cm} \)

9. a) \( 120^\circ \)  
b) \( 15^\circ \)

10. a) \( 35.98 \) (2 d.p.)  
b) The mean is slightly less than the correct length, and the range is only 0.5 mm, so the machine is acceptably accurate.

EX 27B

1.
2. a) 

b) 

3. a) 28 800 cm³  
   b) 0.0288 m³

4.

```
<table>
<thead>
<tr>
<th>Input</th>
<th>Output (upper branch)</th>
<th>Output (lower branch)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>−2</td>
<td>−2</td>
</tr>
<tr>
<td>−2</td>
<td>−10</td>
<td>−10</td>
</tr>
<tr>
<td>7</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>10</td>
<td>14</td>
<td>14</td>
</tr>
<tr>
<td>0</td>
<td>−6</td>
<td>−6</td>
</tr>
</tbody>
</table>
```

5. a) \( x = 38 \)  
   b) \( y = 120, z = 89 \)
6.

7. a) 3  
   b) \( y = 3x \)  
   c) \( y = 3x - 4 \)  
   d) \( y = 3x + 6 \)

8. a) 36 l  
   b) 40.5 l

9. 1.6 mm

10. b) 18.6 cm²
    d) \( BC = 8.9 \) cm
    Perp = 4.2 cm
    Area = 18.7 cm²

    [It is quite difficult to obtain exactly the same area. A range of ±0.2 cm² is acceptable.]

EX 27C

1. a) 6:5  
   b) 5:11

2. \( C B A D \)

3. a) \( \angle PRQ, \angle QRP \)  
    b) \( \angle FBG, \angle GBF \)  
    c) \( \angle MKN, \angle NKM \)  
    d) \( \angle VXW, \angle WXV \)

4. a) 43  
    b) 2.8  
    c) 35  
    d) 16

5. a) 1:50  
    b) 50  
    c) 1.5 m  
    d) 1 m

6. [Diagram of a geometric figure with labeled points and angles]
7. a) \(-11\)  
b) \(18\)  
c) \(21\)  
d) \(-2\)

8. a) 85 days  
b) 0 to 10 °C  
c) 60 days; data grouped

d) 

<table>
<thead>
<tr>
<th>Temp °C</th>
<th>(f)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–10</td>
<td>10</td>
</tr>
<tr>
<td>10–20</td>
<td>120</td>
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<tr>
<td>20–30</td>
<td>150</td>
</tr>
<tr>
<td>30–40</td>
<td>80</td>
</tr>
<tr>
<td>40–50</td>
<td>5</td>
</tr>
<tr>
<td>Total</td>
<td>365</td>
</tr>
</tbody>
</table>

9. a) 

\[ p \xrightarrow{\times 2} 2p \xrightarrow{-5} 2p - 5 \]

b) 

\[ q \xrightarrow{+5} \frac{q}{5} \xrightarrow{-2} \frac{q}{5} - 2 \]

c) 

\[ r \xrightarrow{+2} r + 2 \xrightarrow{+5} \frac{r + 2}{5} \]

d) 

\[ s \xrightarrow{-2} s - 2 \xrightarrow{\times 5} 5(s - 2) \]

10. One possible net is as follows:

\[ 
\begin{array}{c}
\bullet \\
\bullet \\
\bullet \\
\bullet \\
\bullet \\
\bullet \\
\bullet \\
\end{array}
\]

EX 27D

1. d) c) a) b)

2. a) 14.3 fish per tank (1 d.p.)

b) 13.9 fish per tank (1 d.p.)
3. a) Friday  
   b) 90 
   c) Monday, Tuesday  
   d) Monday, Tuesday, Thursday 
4. a) square  
   b) 4 
5. a) 1017  
   b) 1349 
   c) Chandpur 0810 – Crescendo 0847  
      Crescendo 1007 – Silverbeam 1109 
   d) Chandpur 0810 – Lunabad 0819  
      Lunabad 1039 – Silverbeam 1109 
6. a) 29  
    b) 48  
    c) 85  
    d) –13 
7. a) 49 cm²  
    b) 76.9 cm²  
    c) 68 cm²  
    d) 81 m² 
8. a) True  
    b) True  
    c) False  
    d) False 
9. a) 3n + 3  
    b) 78  
    c) 40  
    d) 6n + 3 
10. a) Yes, alternate ∠s 58° and alternate ∠s 61° 
   b) No, ∠CAD = 59°, ∠ADB = 61°, not alternate ∠s 

EX 27X 
1. a) 9, 8.0 (2 s.f.)  
    b) 10, 9.8 (2 s.f.) 
    c) 1, 1.1 (2 s.f.)  
    d) 1.25, 1.1 (2 s.f.) 
2. a) 80 cm², 96.8 cm²  
    b) 1.21  
    c) area SF = (length and width SF)² 
3. ii) 10  
    iv) 7  
    vi) 2
The intention in this chapter is to explain how multiplication and division work, i.e. what they mean, not just how to do the calculations.

**LESSON PLANNING**

**Objectives**

**General**
To multiply and divide fractions and mixed numbers by whole numbers, fractions and mixed numbers; to change fractions to decimals

**Specific**
1. To multiply a whole number by a fraction and recognize commutativity of multiplication
2. To multiply two or three fractions together; to multiply fractions where one of them is a bracketed sum or difference of fractions or mixed numbers
3. To understand the meaning of dividing by a fraction; to use reciprocals to obtain answers
4. To change fractions to decimals, including recurring decimals, using correct notation
5. To solve word problems based on the above

**Pacing**
4 lessons, 1 or 2 homeworks

**Links**
Priority of operations (BODMAS)
Use of calculator, rounding off

**Method**
The danger here is to rush into mechanical techniques for obtaining answers before understanding of the process is established. Some diagrams are given in the text. Use them, and others, to show what is happening.

The text builds up in stages:

- Whole numbers × fraction
- Fraction × fraction
- Fractions × mixed number
- Triple products, including cases where one factor is a bracket containing a sum or difference
- Division by a fraction, reciprocals
- Changing fractions to decimals, recurring decimals

Deal with each stage at a time, lesson by lesson, and with board examples, before setting the relevant exercise.
Assignments

Best for homework are some of EX 28B, or EX 28M.

Vocabulary

reciprocal

ANSWERS

Exercises

EX 28A

1. a) \(6 \times \frac{1}{2} = 3\)  
b) \(12 \times \frac{2}{3} = 8\)  
c) \(3 \times \frac{5}{6} = 2 \frac{1}{2}\)  
d) \(10 \times \frac{3}{5} = 6\)

2. a) true  
b) true  
c) true  
d) false

3. a) \(12 \times \frac{3}{4} = \frac{3}{4}\) of 12  
b) \(15 \times \frac{2}{5} = \frac{2}{5}\) of 15  
c) \(16 \times \frac{1}{4} = \frac{1}{4}\) of 16  
d) \(25 \times \frac{3}{5} = \frac{3}{5}\) of 25

4. a) 2  
b) 3  
c) 4  
d) 8

5. a) \(3 \frac{1}{2}\)  
b) 15  
c) 8  
d) 75

6. a) \(3 \frac{1}{5}\)  
b) \(8 \frac{1}{2}\)  
c) \(4 \frac{1}{2}\)  
d) \(6 \frac{1}{3}\)

7. a) \(4 \frac{2}{7}\)  
b) \(2 \frac{1}{6}\)  
c) \(6 \frac{2}{5}\)  
d) \(2 \frac{1}{12}\)

8. a) \(1 \frac{1}{4}\) l  
b) 4 m  
c) \(1 \frac{3}{5}\) years  
d) \(27 \frac{1}{2}\) days

9. a) \(12 \frac{3}{4}\) m  
b) \(3 \frac{3}{5}\) t  
c) 25 °C  
d) No, 3 m short

10. a) 20 boxes  
b) \(9 \frac{1}{3}\) l  
c) 1520 m  
d) \(\frac{11}{12}\) 92 [Actually \(91 \frac{2}{3}\), but you can't have part of a pack.]

EX 28B

1. a) \(\frac{6}{25}\)  
b) \(\frac{3}{8}\)  
c) \(\frac{3}{7}\)  
d) \(\frac{1}{6}\)

2. a) \(\frac{1}{6}\)  
b) 2  
c) \(\frac{1}{10}\)  
d) \(\frac{1}{24}\)

3. a) \(\frac{1}{6}\)  
b) \(\frac{4}{75}\)  
c) 9  
d) 5

4. a) \(\frac{3}{20}\)  
b) \(\frac{1}{6}\)  
c) 1  
d) \(2 \frac{1}{2}\)

5. a) \(\frac{2}{5}\)  
b) 3  
c) 2  
d) \(\frac{1}{10}\)
6. a) \( \frac{3}{10} \)  
   b) \( \frac{1}{8} \)  
   c) \( \frac{1}{7} \)  
   d) \( \frac{1}{4} \)

7. a) \( \frac{1}{48} \)  
   b) \( 1\frac{1}{6} \)  
   c) \( \frac{7}{8} \)  
   d) \( \frac{7}{12} \)

8. a) \( 1\frac{2}{3} \)  
   b) 6  
   c) 1  
   d) 210

9. a) 144 l  
   b) 5 l  
   c) 1\( \frac{1}{3} \) m  
   d) Rs 153

10. a) \( \frac{2}{3} \)  
    b) \( \frac{197}{200} \)  
    c) aunt [uncle \( \frac{3}{5} \); aunt \( \frac{4}{5} \)]  
    d) Rs 1800

**EX 28C**

1. a) 14 bottles  
    b) 40 bottles  
    c) 18 bottles  
    d) 32 pieces

2. AS, BR, CP, DQ

3. a) true  
    b) true  
    c) false  
    d) true

4. a) 34  
    b) 45  
    c) 20  
    d) 32

5. a) \( \frac{2}{5} \)  
    b) \( 1\frac{1}{5} \)  
    c) 1  
    d) 21

6. a) 6  
    b) 2\( \frac{1}{8} \)  
    c) 18  
    d) \( 1\frac{1}{2} \)

7. a) \( 1\frac{1}{4} \)  
    b) \( 1\frac{1}{3} \)  
    c) \( 3\frac{1}{2} \)  
    d) \( \frac{19}{90} \)

8. a) \( \frac{3}{8} \)  
    b) \( \frac{32}{49} \)  
    c) \( \frac{1}{27} \)  
    d) \( \frac{1}{21} \)

9. a) \( \frac{1}{6} \)  
    b) 2  
    c) \( \frac{1}{4} \)  
    d) \( 4\frac{1}{2} \)

10. a) 16  
     b) \( 1\frac{7}{8} \)  
     c) \( \frac{3}{4} \)  
     d) \( \frac{1}{2} \)

**EX 28D**

1. a) 0.55  
   b) 0.25  
   c) 0.52  
   d) 0.8

2. a) 0.75  
   b) 0.62  
   c) 0.2  
   d) 0.3

3. a) 0.625  
   b) 0.025  
   c) 0.0375  
   d) 0.04375

4. a) 0.56 (2 s.f.)  
    b) 0.65 (2 s.f.)  
    c) 0.17 (2 s.f.)  
    d) 0.96 (2 s.f.)

5. a) 1.29 (3 s.f.)  
    b) 2.84 (3 s.f.)  
    c) 3.82 (3 s.f.)  
    d) 1.96 (3 s.f.)

6. a) 0.4  
    b) 0.45  
    c) 0.83  
    d) 0.24
7. a) 0.153846  b) 0.384615  
   c) 0.538461  d) 1.076923

8. \[ \frac{1}{7} = 0.142857 \]
   \[ \frac{2}{7} = 0.285714 \]
   \[ \frac{3}{7} = 0.428571 \]
   \[ \frac{4}{7} = 0.571428 \]
   \[ \frac{5}{7} = 0.714285 \]
   \[ \frac{6}{7} = 0.857142 \]

Same set of 6 digits recur; no 3, 6 or 9 digits.

9. \[ \frac{1}{9} = 0.1, \frac{2}{9} = 0.2, \frac{3}{9} = 0.3, \text{etc.} \]

   The recurring digit is the same as the numerator.
   
   \[ 0.9 = 1 \] [Cue—Discussion of infinity]


EX 28M

1. a) 4  b) \(7\frac{1}{3}\)  c) 9  d) \(4\frac{1}{3}\)

2. a) \(4\frac{4}{5}\) m  b) \(5\frac{5}{6}\)  
   c) 2 m  d) 11 boxes needed [10\(\frac{2}{3}\) filled]

3. a) \(\frac{4}{5}\)  b) \(\frac{2}{45}\)  c) 16  d) 15

4. a) \(\frac{1}{5}\)  b) \(\frac{5}{2} \frac{5}{5}\)  c) \(\frac{1}{3}\)  d) \(\frac{1}{25}\)

5. a) \(\frac{1}{9}\)  b) 44  c) \(\frac{1}{13}\)  d) \(49\frac{1}{2}\)

6. a) 46  b) 60  c) \(25\frac{1}{2}\)  d) 4

7. a) \(1\frac{3}{7}\)  b) 2  c) 1  d) \(4\frac{1}{2}\)

8. a) \(1\frac{1}{4}\)  b) \(\frac{1}{6}\)  c) \(1\frac{3}{4}\)  d) \(1\frac{3}{10}\)

9. a) 0.529 (3 d.p.)  b) 0.722 (3 d.p.)  
   c) 0.632 (3 d.p.)  d) 0.524 (3 d.p.)

10. a) 0.63  b) 0.769230  c) 0.46  d) 0.27
EX 28X

1. \[
\begin{align*}
\frac{1}{11} &= 0.09 \\
\frac{2}{11} &= 0.18 \\
\frac{3}{11} &= 0.27 \\
\frac{4}{11} &= 0.36 \\
\frac{5}{11} &= 0.45 \\
\frac{6}{11} &= 0.54 \\
\frac{7}{11} &= 0.63 \\
\frac{8}{11} &= 0.72 \\
\frac{9}{11} &= 0.81 \\
\frac{10}{11} &= 0.90
\end{align*}
\]

The recurring digits are \( 9 \times \) the numerator.

2. Let \( n = 0.54 \)

Then \( 100n = 54.54 \)

\[
99n = 100n - n = 54.54 - 0.54 = 54
\]

\( \therefore n = \frac{54}{99} = \frac{6}{11} \) as required

3. a) \( \frac{4}{33} \)    b) \( \frac{47}{99} \)    c) \( \frac{8}{37} \)    d) \( \frac{47}{333} \)
This topic can generate interesting class discussion.

LESSON PLANNING

Objectives

General  To state correctly whether a mathematical statement is true or false

Specific  1. To understand the need for precision in the use of mathematical language
          2. To know that a true statement is always true; a few true examples do not prove it generally true

Pacing  1 lesson, 1 homework

Links  A variety of topics

Method  Use EX 29A as the basis for class discussion. Related questions can be posed, e.g. for #2, “Would it be true for water?”; for #3, “What about a rhombus?”

Assignments  Set EX 29B for homework.

Vocabulary  true, false

ANSWERS

Exercises

EX 29A

1. True
2. False (It would be true for water.)
3. True
4. False
5. False (1001 = 7 × 143)
6. True
7. False (x = –2 is also a solution.)
8. True
9. False (e.g. if the working week is 6 days)
10. True (Fibonacci sequences can begin with any two numbers.)

**EX 29B**

1. Put “whole” before “number”. (It is false for decimals.)
2. Add “provided the number is not 1”.
3. Put “positive” before “number”. (It is false for negatives.)
4. Put “some” before “rectangles”. (A square is a special kind of rectangle.)
5. Add “or 0”.
6. or insert “not” before “the”.

7. Add “interior” before “angles”.
8. Add “regular” before “octagon”.
9. Replace “parallelogram” with “triangle” or delete \(
\frac{1}{2} \).
10. Replace “negative” with “zero”.

**EX 29X**

1. False when the number is 0 or 1.
2. False for a trapezium that is not isosceles.
3. False. The median is best when the group contains a few scores much higher or lower than the rest.
This chapter develops the straight line equation more formally than before.

**LESSON PLANNING**

**Objectives**

**General** To know and use correctly the straight line equation \( y = mx + c \); gradients and intercepts.

**Specific**

1. To know the definition of gradient as \( \frac{\text{increase in } y}{\text{increase in } x} \)
2. To know the equations of horizontal \((y = c)\) and vertical \((x = k)\) lines.
3. To use the negative reciprocal to find the gradient of a line perpendicular to a given line.
4. To read a \( y \)-intercept from a graph and connect it with the line equation.
5. To use the general line equation, \( y = mx + c \) in problems.
6. To plot straight lines given relevant data.
7. To sketch straight lines given their equations, stating gradient and \( y \)-intercept.

**Pacing** 3 lessons, 2 homeworks

**Links** Rotation, translation

**Method** Remind students about gradients. Hold a ruler to demonstrate negative and positive:
Use the text for the formal definition and examples, including the two special cases.

Now, rotating a line through 90° produces a perpendicular line. Use text to show the relationship between their gradients. Define the term negative reciprocal.

Now, show that \( y = 2x \) (say) is parallel to \( y = 2x + 3 \) (say). The translation is 3 units upwards. Relate this to the equation, leading on to \( y = mx + c \) in general.

Another example is in the text.

Use EX 30A.

Drawing graphs from given data is relatively easy.

Do one example only, showing how to make a table and then plot the points, and set EX 30B straightaway. This exercise will take some time.

**Resources**

Squared paper (photocopiable on page 77 of this Guide)

**Assignments**

Homework suggestions: EX 30A #7, 8, EX 30B #3, 4, 6

**Vocabulary**

perpendicular, gradient, \( y \)-intercept

**ANSWERS**

**Exercises**

**EX 30A**

1. a) 6  b) 2  c) \(-\frac{2}{3}\)  d) \(-\frac{4}{5}\)

2. a) \(\frac{5}{3}\)  b) \(\frac{1}{2}\)  c) \(\frac{1}{4}\)  d) 0

3. a) 1  b) \(-1\)

4. a) 5, 0, \(y = 5x\)  b) \(\frac{1}{3}, 2, y = \frac{1}{3}x + 2\)

   c) \(-\frac{1}{2}, -2, y = -\frac{1}{2}x - 2\)  d) \(-3, 12, y = -3x + 12\)

5. \(\text{gcd}(b) \times \text{gcd}(d) = \frac{1}{3} \times -3 = -1\)

6. a) 0  b) \(-\frac{1}{3}\)  c) \(-\frac{4}{9}\)  d) \(-\frac{6}{5}\)

   e) \(-6\)  f) \(\infty\)

7. a) \(y = x + 2\)  b) \(y = 3x + 3\)

   c) \(y = -x + 2.5\)  d) \(y = \frac{-1}{2}x + 2\)
8. a) $y = x - 2$
   b) $y = 2x$
   c) $y = -1.33x$
   d) $y = 3x$

9. a) $y = 5x$
   b) $y = -6x$
   c) $y = 3x$
   d) $y = 4x$

10. a) $y = 2$
    b) $y = -3$
    c) $x = 5$
    d) $x = -4$
EX 30B

1. \[ y = 2x - 1 \]
   gd:\(2\), \(y\)-intercept = -1

2. \[ y = -\frac{1}{2}x + \frac{1}{2} \]
   gd:\(-\frac{1}{2}\), \(y\)-intercept = \(\frac{1}{2}\)
3. 

<table>
<thead>
<tr>
<th>x</th>
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</thead>
<tbody>
<tr>
<td>-2</td>
<td>-9</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>11</td>
</tr>
</tbody>
</table>

4. a) 

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>-19</td>
</tr>
<tr>
<td>-2</td>
<td>-15</td>
</tr>
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<td>-11</td>
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<td>5</td>
</tr>
<tr>
<td>4</td>
<td>9</td>
</tr>
<tr>
<td>5</td>
<td>13</td>
</tr>
</tbody>
</table>

b) 1 
c) 4 
d) \( gdt = \frac{4}{1} = 4 \)

5. A Yes  B Yes  C Yes  D No
6. a)  

   ![Graph of a straight line]

   b)  

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>R</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

7. a) –1  

   b) 1  

   c) \( y = x + 1 \)  

   d) \( P(-1, 0) \)

8. \( y = \frac{2}{3}x + 1 \)

9. 

<table>
<thead>
<tr>
<th>x</th>
<th>-9</th>
<th>-6</th>
<th>-3</th>
<th>0</th>
<th>3</th>
<th>6</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>-1</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

   b) Yes; 3, 1  

   c) \( \frac{1}{3} \)  

   d) \( \frac{1}{3} \)

10. a) \( y = \frac{1}{2}x - 1, \frac{1}{2}, -1 \)  

    b) \( \frac{2}{3}, 3 \)  

    c) \( y = \frac{-1}{5}x + 3, P(100, -17) \)  

    d) \( \frac{gdt(L_1)}{gdt(L_2)} = \frac{5}{2}, \frac{gdt(L_1)}{gdt(L_2)} = 2 \) \( gdt(L_1) \) larger

EX 30X

1. \( P(1, 4) \)

2. (5, 2)

3. \( y = 2x + 5, y = 5x + 2, x = 1 \)
This chapter introduces the mathematically interesting number we call π. It shows that mathematics is not just for working out useful things but has its own beauty.

**LESSON PLANNING**

**Objectives**

<table>
<thead>
<tr>
<th>General</th>
<th>To solve problems using $C = \pi d$ with various approximations for $\pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specific</td>
<td>1. To know the definitions of radius, diameter, and circumference</td>
</tr>
<tr>
<td></td>
<td>2. To discover the experimental result that $\frac{C}{d} \approx 3$</td>
</tr>
<tr>
<td></td>
<td>3. To know that $\pi$ can only be written as an approximation</td>
</tr>
<tr>
<td></td>
<td>4. To solve problems involving the circumferences of circles, and fractions of circles</td>
</tr>
</tbody>
</table>

**Pacing**

2 lessons, 1 homeworks

**Method**

Begin with a practical session, dividing the class into groups. First, using an object, demonstrate the meanings of radius, diameter, and circumference. The text can be used for reference. Tables should be drawn as shown for EX 31A. At the end, discuss the group results in a plenary session. Expect (with good measurement) that $\frac{C}{d}$ is always just over 3.

Give the actual value of $\pi$ to many d.p. This emphasizes its essential nature. Only then give a sequence of approximations. Get students to find their own calculator approximation for $\pi$. “How accurate is it?”

Do a few easy $C = \pi d$ calculations, taking $\pi$ as 3. Then set EX 31B.

**Resources**

For the practical session a selection of round objects is needed, e.g. round cheese boxes, cylindrical tins, toilet roll tubes, coins, wheels, etc, with strings, tapes, and (long) rulers for measuring.

**Assignments**

Suggested homework: EX 31B #10

**Vocabulary**

circumference, radius, diameter, $\pi$
ANSWERS

Exercises

EX 31A

The $C/d$ column should give answers just over 3. [There will be quite a lot of variation in practice because of inaccurate measuring.]

EX 31B

1. a) 15 cm  
   b) 24 cm  
   c) 6 m  
   d) 1.5 m
2. a) 18.6 cm  
   b) 12.4 m  
   c) 27.9 cm  
   d) 3.1 m
3. a) 6 cm  
   b) 9.3 cm  
   c) 1.3 m  
   d) 1.7 m
4. 141 cm
5. 1.11 m
6. 18.28 m (nearest cm)
7. 8.57 m (nearest cm)
8. 471 mm (nearest mm)
9. 35 cm (nearest cm)
10. a) 3.142  
    b) 44 cm  
    c) 15.4 m  
    d) 2.1 m

EX 31X

1. 1
2. March 14th (3/14 in month/day format)  
   22nd July (22/7 in day/month format)
3. a) 1 d.p.  
    b) 1 d.p.  
    c) 1 d.p.  
    d) 2 d.p.
32

Revision Exercises

1. EX 32A

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<td>2</td>
<td>1</td>
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</tr>
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</table>

2. a) 159 m  
   b) 57 km/h  
   c) 4 m³  
   d) 1 mm

3. 96 cm²

4. a) −5  
   b) −4  
   c) 5  
   d) −2

5. construction

6. 1200 m²

7. a) days in the month, 31  
   b) Time sequence of 45 min intervals, 0215  
   c) Bearings with 50° rotation from previous; 140°  
   d) Starting with triple 1 add previous 3 terms; 31

8. 32 cm

9. a) \( y = 6; \)  \( \text{LHS} = 0.2(6 + 2) + 0.1(6 + 1) = 0.2 \times 8 + 0.1 \times 7 = 2.3 = \text{RHS} \)
   b) \( y = 5; \)  \( \text{LHS} = 0.7(10 + 1) = 7.7 \)  \( \text{RHS} = 5 + 2.7 = 7.7 \)
   c) \( y = 9; \)  \( \text{LHS} = 2(11) + 9 = 31 \)  \( \text{RHS} = 2(16) − 1 = 31 \)
   d) \( y = 2.5; \)  \( \text{LHS} = 5(10 + 1) = 55 \)  \( \text{RHS} = 10(5.5) = 55 \)

10. Mean = 450.0 (1 d.p.); pass (The range is only 0.5 g.)
EX 32B

1. \[ y = -\frac{1}{2}x + 3 \]

2. a) \[ \quad \quad \quad +6 \quad \quad \quad +19 \]
   b) \[ \quad \quad \quad +5 \quad \quad \quad \times2 \quad \quad \quad +16 \]
   c) \[ \quad \quad \quad \times8 \quad \quad \quad +4 \quad \quad \quad +14 \]
   d) \[ -8 \quad \quad \quad +5 \quad \quad \quad +3 \quad \quad \quad +1 \quad \quad \quad 4 \]

3. a) \[ \frac{28}{9} \]
   b) \[ \frac{2}{5} \]
   c) \[ \frac{428}{45} \]
   d) \[ \frac{444}{45} \]

4. 9.3 (1 d.p.)

5. a) 118 (3 s.f.)
   b) 8
   c) 145 (3 s.f.)
   d) 18.9

6. 

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<td>2</td>
<td>4</td>
<td>6</td>
</tr>
</tbody>
</table>

   a) \[ \frac{1}{36} \]
   b) \[ \frac{1}{12} \]
   c) \[ \frac{1}{36} \]
   d) \[ \frac{5}{6} \]

7. a) \[ y = 6x + 1 \]
   b) \[ y = 6x - 3 \]
   c) \[ y = -x + 2 \]
   d) \[ y = -\frac{8}{5}x - 4 \]

8. a) 20 min
   b) 3 min
   c) 1328
   d) 1457

9. b) c) d) a)

10. \[ \frac{1}{15} = 0.06 \]
    \[ \frac{2}{15} = 0.13 \]
    \[ \frac{3}{15} = 0.2 \]
    \[ \frac{4}{15} = 0.26 \]
    \[ \frac{5}{15} = 0.3 \]
    \[ \frac{6}{15} = 0.4 \]
    \[ \frac{7}{15} = 0.46 \]
    \[ \frac{8}{15} = 0.53 \]
    \[ \frac{9}{15} = 0.6 \]

Chapter 32 Revision Exercises
\[
\begin{align*}
\frac{10}{15} &= 0.6 & \frac{11}{15} &= 0.7\bar{3} & \frac{12}{15} &= 0.8 \\
\frac{13}{15} &= 0.8\bar{6} & \frac{14}{15} &= 0.9\bar{3} & \frac{15}{15} &= 1
\end{align*}
\]

**EX 32C**

1. a) 36 cm²  
   b) 99 cm²  
2. a) 28 cm²  
   b) 5 cm²  
3. a) VII-Q, 48 kg  
   b) VII-P, 30 kg  
   c) VII-R has the highest mean of 2.5 kg (1 d.p.).
4. 
   \[
   \begin{align*}
   \text{40°} \\
   \text{40°} \\
   \text{40°} \\
   \text{40°}
   \end{align*}
   \]
5. a) \(n + 3\)  
   b) \(0.2n + 1.4\)  
   c) \(-30n + 110\)  
   d) \(-\frac{1}{2}n + 5\)
6. a) \(4x + 20 = 4(x + 5)\)  
   b) \(7(x - 2) = 7x - 14\)  
   c) \(9(x - 4) = 9x - 36\)  
   d) \(30x - 24 = 6(5x - 4)\)
7. 0.52 ha
8. a) \(\frac{1}{5}\)  
   b) 1  
   c) \(\frac{1}{9}\)  
   d) \(\frac{2}{17}\)
9. 26 cm²
10. 44.7% (1 d.p.)

**EX 32D**

<table>
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<tr>
<th></th>
<th>2</th>
<th>3</th>
<th>-5</th>
</tr>
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<td>4</td>
<td>5</td>
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</tr>
<tr>
<td>4</td>
<td>6</td>
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<td>-1</td>
</tr>
<tr>
<td>6</td>
<td>8</td>
<td>9</td>
<td>1</td>
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a) \(\frac{1}{9}\)  
   b) \(\frac{1}{3}\)  
   c) \(\frac{2}{9}\)  
   d) 0
2. a) (Hint: Tracing paper may be helpful here.)

b) (Hint: Start by drawing radii separated by 120°.)

c) (Hint: Start by drawing 5 radii separated by 72°.)

d) (Hint: Start by drawing 5 radii separated by 72°.)
3. a) 4  
  b) \( \frac{5}{3} \)  
  c) \( \frac{7}{5} \) or 1.4  
  d) \( \frac{4}{5} \) or 0.8  
4. a) 2182 kg  
  b) 2.182 t  
5. a) False  
  b) False  
  c) True  
  d) True  
6. a) 79.92  
  b) 18.318  
  c) 14.5625  
  d) 3.66  
7. a) 18°, 54°, 144°, 144°  
  b) 18°, 18°, 54°  
8. 

\[ a = 22° \text{ (alt \( \angle \)'s) } \]
\[ b = 44° \text{ (alt \( \angle \)'s) } \]
\[ x = a + b = 22 + 44 = 66° \]  
9. 
\[ A(-2, -3) \]
\[ B(2, 7) \]
\[ C \text{ is on } y = 2.5x \]
\[ 3 = 2.5x \]  
10. a) circle  
  b) square  
  c) rectangle  
  d) trapezium
EX 32X

1. Let \( n = 0.589 \)

\[
\begin{align*}
n &= 0.589589589\ldots \\
1000n &= 589.589589589\ldots \\
999n &= 1000n - n \\
&= 589.589 - 0.589 \\
&= 589 \\
\frac{n}{999} &= \frac{589}{999} \\
0.589 &= \frac{589}{999} \text{ as required}
\end{align*}
\]

2. There are various memory tricks to aid memorization. Although \( \pi \) has no repeating patterns there are some repeated digits. If we space out like this it is easier to memorize the first 22 d.p.:

\[3.14 \ 1592 \ 6535 \ 8979 \ 3238 \ 4626\ldots\]

3. \[3^2 \times 7 \times 11 \times 13 \times 17\]
Instructions: Time allowed 1 hour.
You may use a calculator.
You will also need: pen, pencil, rubber, ruler, compasses, graph paper.
Try to answer all the questions.
Check your work carefully.
The marks for each question are shown in brackets. [Total marks = 100]

1. a) In this sequence of designs find a formula for the number of spots in the $n$th design:
   Design 1 Design 2 Design 3
   b) Draw Design 5
c) How many spots would there be in Design 7?
d) Which design has 122 spots? [4]

2. Calculate the total area of this shape, made up of a triangle and parallelogram.
3. This recipe has to be scaled up to enable 14 people to be served. Make a list of the quantity of each ingredient needed. Round off sensibly.

<table>
<thead>
<tr>
<th>Ingredients</th>
<th>Quantity</th>
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<tbody>
<tr>
<td>Flour</td>
<td>165 g</td>
</tr>
<tr>
<td>Salt</td>
<td>pinch</td>
</tr>
<tr>
<td>Eggs</td>
<td>3</td>
</tr>
<tr>
<td>Milk</td>
<td>300 ml</td>
</tr>
<tr>
<td>Water</td>
<td>110 ml</td>
</tr>
<tr>
<td>Butter</td>
<td>60 g</td>
</tr>
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</table>

4. Out of a salary of Rs 52,000 per month, a clerical worker spends 24% on rent, 14% on clothes, 30% on food, 12% on transport, 2% on taxes, 8% on children’s education, and saves the rest. How much is saved each month?

5. Write down the gradient of each line:

(a) 
(b) 
(c) 
(d) 

6. Solve these equations. Check solutions. Show working.

a) \( 6(x - 7) + 7 = 13 \)

b) \( 9(x + 3) = 32 - x \)

c) \( 5(x - 0.1) = 4(x + 0.1) + 0.2 \)

d) \( 0.4(x + 1) = 0.3(x - 2) \)
7. Calculate the size of the three angles marked with letters in this kite

\[ \begin{align*}
\angle 1 &= 2x^\circ \\
\angle 2 &= 6x^\circ \\
\angle 3 &= x^\circ
\end{align*} \]

8. What is the scale factor of:
   a) the enlargement from \( A \) to \( B \)?
   b) the enlargement from \( B \) to \( C \)?
   c) the enlargement from \( C \) to \( B \)?
   d) the enlargement from \( C \) to \( A \)?

9. In a seven-a-side football match the school team players had a mean weight of 62.1 kg. However, the goalie Tariq (51.1 kg) was injured. He was replaced by substitute Yusuf (52.5 kg). What was the new mean weight of the school team?
10. The graph shows the temperature of a patient admitted to hospital, recorded twice daily (7 am and 7 pm):

![Temperature Graph]

a) What was the highest temperature recorded, and on which days?
b) When did the biggest fall in temperature occur?
c) What was the patient’s temperature at midday on Thursday 16th? Can we be sure of that?
d) What was the difference between the patient’s temperature on admission and when discharged from the hospital?


12. Work out the value of \( a \), \( b \) and \( c \). Give a reason in each case.

![Angle Diagram]

13. Calculate:

a) \( 16 \times \frac{5}{8} = \)

b) \( \frac{9}{10} \times \frac{1}{19} \times 20 = \)

c) \( \frac{16}{27} \times (2 \frac{1}{2} - 1 \frac{3}{8}) \times 1 \frac{5}{7} = \)

d) \( 1 \frac{7}{16} \div 2 \frac{3}{8} = \)
14. On graph paper, draw coordinate axes labelled from –8 to 8 in both directions. Draw an accurate graph of the line for which this data is true:

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<th>y</th>
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<tr>
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<tr>
<td>4</td>
<td>–8</td>
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Using information obtained from your graph write down its equation. [6]

15. A bag contains equal-sized buttons of different colours in the following proportions:

<table>
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<th>Red</th>
<th>Green</th>
<th>Brown</th>
<th>Black</th>
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<tbody>
<tr>
<td>5%</td>
<td>25%</td>
<td>30%</td>
<td>45%</td>
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</table>

a) State why there must be an error in this information.

b) The green proportion is incorrect. Correct it.

c) A randomly selected button is taken from the bag. Find:
   i) \( P(\text{red or green}) \)
   ii) \( P(\text{brown or black}) \)
   iii) \( P(\text{not red}) \)
   iv) \( P(\text{pink}) \)
   v) \( P(\text{not yellow}) \)

d) If there are 340 buttons in the bag, how many are black? [6]

16. A small child’s bicycle travels 880 m whilst its wheels turn 1000 times. To the nearest cm, what is the diameter of the bicycle wheels? [6]

17. Copy and complete the net of this cube whose base is labelled X and the face opposite Y is labelled Z.

18. Find the value of the following expressions when \( x = -6 \)

   a) \(-4 + 6x\) 
   b) \(-2(x + 7)\) 
   c) \(\frac{3x - 5}{10}\) 
   d) \(\frac{12x - 2x}{-4}\) 
   e) \(\frac{9(8-x)}{-21}\) 
   f) \(\frac{-3(x - 1)+1}{-11}\) [6]
19. Use your calculator to work these out. Round off in each case as stated:
   a) \( \frac{4.63 \times 7.98}{1.76 + 2.11} \) (to 3 s.f.)
   b) \( (98.4 - 1.36) \times (27.6 - 12.4) \) (to 3 s.f.)
   c) \( \frac{2.6}{1.4} \times \frac{7.5}{1.9} \times \frac{0.5}{1.5} \) (to 1 d.p.)
   d) \( 3.14 \times 6.3^2 \) (to 2 d.p.) [4]

20. a) Add together 7.2 kg, 720 g, 7520 mg and give your answer in grams.
    b) Add together 1 km, 450 cm, 7.45 m, and give your answer in metres.
    c) Add together 2 km², 420 ha, 2000 m², and give your answer in ha.
    d) Find the difference between 1.4 kW and 750 W, and give your answer in watts. [4]
Answers and Mark Scheme

1. a) \( n^2 + 1 \)  
   b) \( \)  
   c) 50  
   d) 11  
   [1 each = 4]

2. parallelogram = \( 16 \times 8 = 128 \)  
   triangle = \( \frac{1}{2} \times 25 \times 10 = 125 \)  
   total = 253 cm\(^2\)  
   [1 each shape = 2, total 2 = 4]

3. flour 385 g [allow 380 or 390]  
   salt large pinch [allow pinch]  
   eggs 7  
   milk 700 ml  
   water 255 ml [allow 260]  
   butter 140 g  
   [4 –1 each error = 4]

4. 10% saved Rs 5200  
   [% 2 + Ans 2 = 4]

5. a) \( \frac{1}{3} \)  
   b) –3  
   c) 0  
   d) 1  
   [1 each = 4]

6. a) \( 6x - 42 + 7 = 13 \)  
   \[ 6x = 48 \]  
   \[ x = 8 \]  
   LHS = 6(1) + 7 = 13 = RHS  
   b) \( 9x + 27 = 32 - x \)  
   \[ 10x = 5 \]  
   \[ x = 0.5 \text{ or } \frac{1}{2} \]  
   LHS = 9(3.5) = 31.5  
   RHS = 32 – 0.5 = 31.5  
   [1 each answer + 1 for working shown (and attempt to check shown) = 8]
   c) \( 5x - 0.5 = 4x + 0.4 + 0.2 \)  
   \[ x = 1.1 \]  
   LHS = 5(1) = 5  
   RHS = 4(1.2) + 0.2 = 4.8 + 0.2 = 5  
   d) \( 0.4x + 0.4 = 0.3x - 0.6 \)  
   \[ 0.1x = -1 \]  
   \[ x = -10 \]  
   LHS = 0.(-9) = -3.6  
   RHS = 0.3(-12) = -3.6
7. \[6x + 6x + 2x + x = 15x = 360\]
   \[x = 24\]
   Angles are 24°, 48°, 144°, 144°
   [Method 2 1 angle each = 6]

8. a) 2 b) \(\frac{3}{2}\) (or 1.5) c) \(\frac{2}{3}\) d) \(\frac{1}{2}\) (or 0.5)
   [1 each = 4]

9. Total weight = 62.1 \times 7 = 434.7 kg
   New total = 434.7 – 51.1 + 52.5 = 436.1
   New mean = 436.1 \div 7 = 62.3 kg
   [Total 2 New total 2 New Mean 2 = 6]

10. a) 40 °C, Thu and Fri b) between Fri evening and Sat morning c) 39.5 °C; no d) 2.5°
    [1 each = 4]

11. [Bisect 180° 2 Bisect 90° 2 Select 135° 2 (marked) = 6]

12. a) 70 (adj \(\angle\)s on st line) b) 64 (angle-sum of \(\Delta\)) c) 64 (corr \(\angle\)s)
    [1 each answer 1 each reason = 6]

13. a) 10 b) \(\frac{12}{15} \times \frac{1}{15} \times 2 \theta = 2\)
   c) \(\frac{3}{2} \times \frac{1}{6} \times \frac{12}{7} = \frac{8}{7} = 1 \frac{1}{7}\)
   d) \(\frac{23}{12} \div \frac{19}{8} = \frac{23}{12} \times \frac{8}{19} = \frac{23}{38}\)
    [1 each = 4]
14. \[ gdt = \frac{-3}{2}, \text{y-intercept} = -2 \]
Equation is \[ y = \frac{-3}{2}x - 2 \]

15. 
   a) The percentages total > 100% 
   b) 20% 
   c) i) 0.25 ii) 0.75 iii) 0.95 iv) 0 v) 1 
   d) 153 

16. 
   circumference = \( \frac{880}{1000} = 0.88 \text{ m} = 88 \text{ cm} \) 
   diameter = \( \frac{88}{\pi} = 28 \text{ cm} \) (nearest cm) 

17. 

18. 
   a) –40 
   b) –2 
   c) –2 
   d) –6 
   e) –6 
   f) –2 

19. 
   a) 9.55 (3 s.f.) 
   b) 1480 (3 s.f.) 
   c) 2.4 (1 d.p.) 
   d) 124.63 (2 d.p.) 

20. 
   a) 7927.52 g 
   b) 1011.95 m 
   c) 620.2 ha 
   d) 650 W 

[6 – 1 each error = 6 
Allow fractions or percentages for (c).] 
[circ 3, dia 3 = 6] 
[X 2 
Z 2 
= 4] 
[1 each = 4] 
[1 each = 4 must be exact]