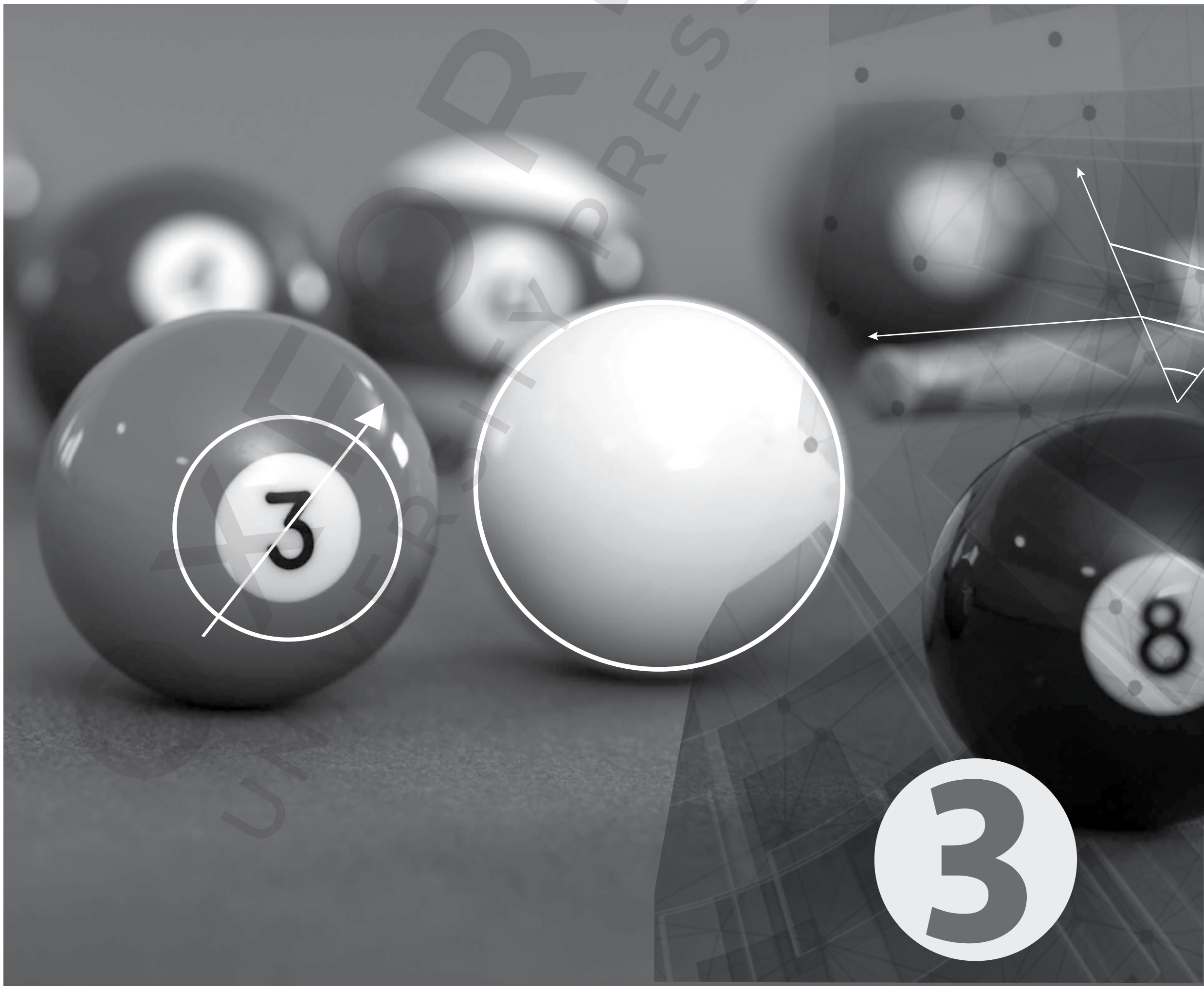
UPDATED
7th EDITION

NEW SYLLABUS
MATHEMATICS
TEACHER'S RESOURCE BOOK

A Comprehensive Mathematics Programme for Grade 8



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Syllabus Matching Grid
National Curriculum of Pakistan 2022 with New Syllabus Mathematics 3 (Updated 7th Edition)

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Scheme Of Work – New Syllabus Mathematics Book 3 Updated 7th Edition

Week (5 classes x 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
1	**1 Number Operations and Direct and Inverse Proportions**	**1.1 Real Numbers (pp. 3 – 6)**	Recognise real numbers Represent real numbers on number line. Differentiate between rational and irrational numbers.	Identify and use rational and irrational numbers (e.g. π, $\sqrt{2}$)	Investigation – Some Interesting Facts about the Irrational Number π (p. 3)	Investigation – Some Interesting Facts about the Irrational Number π (p. 3)		
1		**1.2 Square Roots and Cube Roots (pp. 6 – 10)**	- Find square roots and cube roots using prime factorisation, Mental estimation and calculators	Identify and use square numbers and cube numbers Calculate squares, square roots, cubes and cube roots of numbers				
2		**1.3 Approximation (pp. 11-12)**		Make estimates of numbers, quantities and lengths	Class Discussion – Actual and Approximated Values (p. 59)			Class Discussion – Actual and Approximated Values (p. 11) Ex 1A Q18-20 (p. 19)
2		**1.4 Significant Figures (pp. 12 - 16)**	- Round off numbers to a required number of decimal places and significant figures	Give approximations to specified numbers of significant figures and decimal places	Investigation – Rounding in Real Life (p. 67) Journal Writing (p. 67)			Practise Now 7 Q2 (p. 14) Practise Now 10 Q2 (p. 16) Investigation – Rounding in Real Life (p. 67) Journal Writing (p. 67)

Week (5 classes x 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
3		**1.5 Rounding and Truncation Errors (pp. 16 – 18)**	• Explain the problem of rounding and truncation errors		Investigation – The Missing 0.1% Votes (p. 16)			Investigation – The Missing 0.1% Votes (p. 68) Thinking Time (p. 70) Ex 3B Q 3, 8 – 9, 10(iii) (pp. 70 – 71)
					Thinking Time (p. 17) Investigation – Rounding and Truncation Errors in Calculators (p. 18)			
3		**1.6 Direct Proportion (pp. 20 - 27)**	• Explain the concept of direct proportion • Solve problems involving direct proportion	Demonstrate an understanding of ratio and proportion	Investigation – Direct Proportion (p. 20)			Investigation – Direct Proportion (p. 20)
4			• Explain the concept of direct proportion using tables, equations and graphs • Solve problems involving direct proportion	Express direct variation in algebraic terms and use this form of expression to find unknown quantities Construct tables of values and draw graphs for functions of the form ax^n where a is a rational constant, n = –2, –1, 0, 1, 2, 3, and simple sums of not more than three of these and for functions of the form ka^x where a is a positive integer	Thinking Time (p. 21) Investigation – Graphical Representation of Direct Proportion (pp. 21-22) Thinking Time (p. 22)			Thinking Time (p. 21) Thinking Time (p. 22) Ex 1B Q 11(iv) (p. 27)

Week (5 classes x 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
4		**1.7 Inverse Proportion (pp. 27 – 33)**	• Explain the concept of inverse proportion • Solve problems involving inverse proportion	Demonstrate an understanding of ratio and proportion	Investigation – Inverse Proportion (p. 27)			Investigation – Inverse Proportion (p. 27)
			• Explain the concept of inverse proportion using tables, equations and graphs • Solve problems involving inverse proportion	Express inverse variation in algebraic terms and use this form of expression to find unknown quantities Construct tables of values and draw graphs for functions of the form ax^n where a is a rational constant, n = –2, –1, 0, 1, 2, 3, and simple sums of not more than three of these and for functions of the form ka^x where a is a positive integer	Thinking Time (p. 28) Investigation – Graphical Representation of Inverse Proportion (pp. 28 – 30) Thinking Time (p. 31)			Thinking Time (p. 28) Thinking Time (p. 31) Just For Fun (p. 31)
4		**Miscellaneous**					Solutions for Challenge Yourself	
5	**2 Financial Transaction**	**2.1 Financial Transaction (p. 39)**	• Solve problems involving profit, loss, and discount	Express one quantity as a percentage of another Calculate percentage increase or decrease Use given data to solve problems on personal and small business finance, involving earnings				

Week (5 classes x 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
5		**2.2 Markup (pp. 40 - 41)**	• Solve problems involving Markup/Profit	Carry out calculations involving Markup, rate, Principal amount				
5		**2.3 Simple Interest and Compound Interest (pp. 42 – 47)**	• Solve problems involving simple interest, compound interest and hire purchase • Explain what percentage point is	Use given data to solve problems on personal and small business finance, involving simple interest and compound interest	Class Discussion – Body Mass Index (p. 42) Performance Task (p. 44) Investigation – Simple Interest and Compound Interest (p. 44 - 45)			Performance Task (p. 44) Investigation – Simple Interest and Compound Interest (p. 44 - 45)
6		**2.4 Hire Purchase (pp. 49 - 51)**	Explain Hire Purchase	Solve problems involving Hire purchase.				
6		**2.5 Money Exchange (pp. 52 - 53)**	• Convert one currency to another	Solve problems involving money and convert from one currency to another				
7		**2.6 Insurance, Partnership, and Inheritance (pp. 54 -58)**	Explain insurance, partnership, and inheritance, and solve real world word problems	Solve problems involving insurance, partnership, and inheritance.				
7		**Miscellaneous**					Solution for Challenge Yourself	
8	**3 Further Expansion and Factorisation of Algebraic Expressions**	**3.1 General Term of a Number Sequence (pp. 63 - 66)**	Determine the next few terms and find a formula for the general term of a number sequence	Recognise patterns in sequences and relationships between different sequences	Class Discussion – Generalising Simple Sequences (p. 64)			

Week (5 classes x 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
8		**3.2 Number Patterns (pp. 67 - 68)**	Solve problems involving number sequences and number patterns	Generalise sequences as simple algebraic statements	Worked Example 2 (p. 67)			
8		**3.3 Number Patterns in Real-World Contexts (pp. 69 - 76)**			Investigation (p. 69)			
9		**3.4 Operations on Polynomials (pp. 77 - 78)**	Apply four operations on Polynomials	Solve problems involving four operations with polynomials				
9		**3.5 Expansion and Factorisation of Algebraic Expressions (pp. 79 -85)**	• Expand and simplify algebraic expressions • Use a multiplication frame to factorise algebraic expressions	Expand product of algebraic expressions	Thinking Time (p. 82)			
10		**3.6 Expansion Using Special Algebraic Identities (pp. 86 - 89)**	• Recognise and apply the three special algebraic identities to expand algebraic expressions		Class Discussion – Special Algebraic Identities (p. 86)			
11		**3.7 Cubes of the Sums and Difference of Two Terms Expansion (pp. 90 - 96)**	Recognise and apply the expansions of cubes of the sums, and difference of two terms	Manipulate the algebraic expressions				

Week (5 classes x 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
11		**3.8 Factorisation Using Special Algebraic Identities (pp. 97 - 99)**	• Recognise and apply the three special algebraic identities to factorise algebraic expressions	Factorise where possible expressions of the form: $a^2 + 2ab + b^2$ $ax^2 + bx + c$ $ax + bx + kay + kby$ $a^2x^2 – b^2y^2$				
12		**3.9 Factorisation by Grouping (pp. 100 - 103)**	• Factorise algebraic expressions by grouping		Thinking Time (p. 102) Class Discussion – Equivalent Expressions (p. 103)			Class Discussion – Equivalent Expressions (p. 103)
12		**Miscellaneous**					Solutions for Challenge Yourself	
12	**4 Graphs of Linear Equations and Simultaneous Linear Equations**	**4.1 Gradient of a Straight Line (pp. 107-115)**	• Find the gradient of a straight line • State the y-intercept of a straight line	Find the gradient of a straight line Calculate the gradient of a straight line from the coordinates of two points on it	Investigation – Equation of a Straight Line (p. 107) Class Discussion – Gradients of Straight Lines (p. 112) Class Discussion – Gradients in the Real World (p. 112)	Investigation – Equation of a Straight Line (p. 107)		

Week (5 classes x 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
13		**4.2 Further Applications of Linear Graphs in Real-World Contexts (pp. 116 - 119)**		Apply the idea of rate of change to easy kinematics involving distance-time and speed-time graphs	Worked Example 2 (p. 116)			
13		**4.3 Graphs of Linear Equations in the form ax + by= k** **(pp. 120 - 123)**	• Draw graphs of linear equations in the form $ax + by = k$	Draw graphs from given data	Investigation – Graphs of $ax + by = k$ (p. 120)	Investigation – Graphs of $ax + by = k$ (p. 120)		Investigation – Graphs of $ax + by = k$ (p. 120)
14		**4.4 Solving Simultaneous Linear Equations Using Graphical Method (pp. 124 - 128)**	• Solve simultaneous linear equations in two variables using the graphical method	Solve simultaneous linear equations in two unknowns Solve associated equations approximately by graphical methods	Investigation – Solving Simultaneous Linear Equations Graphically (p.124) Class Discussion – Choices of Appropriate Scales for Graphs and Accuracy of Graphs (p. 125) Class Discussion – Coincident Lines and Parallel Lines (p. 127) Thinking Time (p. 127)	Investigation – Solving Simultaneous Linear Equations Graphically (p. 124) Class Discussion – Coincident Lines and Parallel Lines (p. 127)		Investigation – Solving Simultaneous Linear Equations Graphically (p. 124) Class Discussion – Choices of Appropriate Scales for Graphs and Accuracy of Graphs (p. 125) Class Discussion – Coincident Lines and Parallel Lines (p. 127) Thinking Time (p. 127)

Week (5 classes x 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
14		**4.5 Solving Simultaneous Linear Equations Using Algebraic Methods (pp. 129-138)**	• Solve simultaneous linear equations in two variables using the elimination method • Solve simultaneous linear equations in two variables using the substitution method	Construct and transform formulae and equations Solve simultaneous linear equations in two unknowns	Thinking Time (p. 129) Thinking Time (p. 132) Thinking Time (p. 134) Thinking Time (p.135)			Thinking Time (p. 134) Thinking Time (p. 135)
15		**4.6 Applications of Simultaneous Equations in Real-World Contexts (pp. 139 – 144)**	• Formulate a pair of linear equations in two variables to solve mathematical and real-life problems		Thinking Time (p. 144)	Just For Fun (p. 143)		Just For Fun (p. 143)
15		**Miscellaneous**					Solutions for Challenge Yourself	
16	**5 Indices and Standard Form**	**5.1 Indices (pp. 152 - 153)**	• State and apply the 5 laws of indices	Understand and use the rules of indices Use and interpret positive, negative, fractional and zero indices	Investigation – Indices (p. 152) Class Discussion – Comparing Numbers written in Index Form (p. 153)			Class Discussion – Comparing Numbers written in Index Form (p. 153)

Week (5 classes x 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
16		**5.2 Laws of Indices (pp. 153 - 161)**	• State and apply the 5 laws of indices		Investigation – Law 1 of Indices (pp. 153 - 154) Investigation – Law 2 of Indices (p. 155) Investigation – Law 3 of Indices (p. 156) Investigation – Law 4 of Indices (p. 157) Class Discussion – Simplification using the Law of Indices (p. 158) Investigation – Law 5 of Indices (p. 159) Journal Writing (p. 160) Class Discussion – Is $(a + b)^n = a^n + b^n$? Is $(a - b)^n$ (p. 161)			Class Discussion – Simplification using the Law of Indices (p. 158) Journal Writing (p. 160) Class Discussion – Is $(a + b)^n = a^n + b^n$? Is $(a - b)^n$
16		**5.3 Zero and Negative Indices (pp. 162 - 167)**	• State and use the definitions of zero, negative and rational indices		Investigation – Zero Index (pp. 162 - 163) Thinking Time (p. 164) Investigation – Negative Indices (p. 164) Thinking Time (p. 166)			Investigation – Zero Index (pp. 162 - 163) Thinking Time (p. 164) Investigation – Negative Indices (p. 164) Thinking Time (p. 166)

Week (5 classes x 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
17		**5.4 Rational Indices (pp. 167 - 174)**	• State and use the definitions of zero, negative and rational indices		Class Discussion – Rational Indices (p. 169) Thinking Time (p. 169) Investigation – Rational Indices (p. 170) Thinking Time (p. 171)			Class Discussion – Rational Indices (p. 169) Thinking Time (p. 169) Thinking Time (p. 171)
17		**5.5 Standard Form (pp. 175 - 182)**	• Use the standard form to represent very large or very small numbers	Use the standard form $A \times 10^n$, where n is a positive or negative integer, and $1 \leq A < 10$.	Class Discussion – Standard Form (p. 175) Performance Task (p. 179) Thinking Time (p. 181)	Information (p. 177) Internet Resources (p. 177) Performance Task (p. 179)		Class Discussion – Standard Form (pp. 175) Performance Task (p. 179)
17		**Miscellaneous**					Solutions for Challenge Yourself	
18	**6 Linear Inequalities in One Variable**	**6.1 Simple Inequalities (pp. 187 - 190)**	• Solve simple linear inequalities	Solve simple linear inequalities	Journal Writing (p. 190) Investigation – Properties of Inequalities (p. 187)			Investigation – Properties of Inequalities (p. 187)

Week (5 classes x 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
18		**6.2 Inequalities (pp. 191 - 196)**	• Solve linear inequalities in one variable and represent the solution on a number line	Solve simple linear inequalities	Investigation Properties of Inequalities (p. 191) Investigation – Inequalities (p. 192) Journal Writing (p. 193 Thinking Time (p. 194)			Journal Writing (p. 193) Thinking Time (p. 194)
19		**6.3 Problem Solving involving Inequalities (pp. 197 - 198)**	• Apply linear inequalities to solve word problems					
19		**Miscellaneous**					Solutions for Challenge Yourself	
19	**7 Pythagoras' Theorem**	**7.1 Pythagoras' Theorem (pp. 202 - 211)**	• Solve problems using Pythagoras' Theorem	Apply Pythagoras' theorem to the calculation of a side or an angle of a right-angled triangle	Investigation – Pythagoras' Theorem – The Secret of the Rope-Stretchers (pp. 203 - 204)	Investigation – Pythagoras' Theorem – The Secret of the Rope-Stretchers (pp. 203 - 204)		Investigation – Pythagoras' Theorem – The Secret of the Rope-Stretchers (pp. 203 - 204)
20					Performance Task (p. 205)	Performance Task (p. 205) Internet Resources (p. 205)		Performance Task (p. 205)
21		**7.2 Applications of Pythagoras' Theorem in Real-World Contexts (pp. 212 - 218)**	• Solve problems using Pythagoras' Theorem					

Week (5 classes x 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
21		**7.3 Converse of Pythagoras' Theorem (pp. 219 -220)**	• Determine whether a triangle is a right- angled triangle given the lengths of three sides					
21		**Miscellaneous**					Solutions for Challenge Yourself	
22	**8 Arc Length, Area of Sector**	**8.1 Length of Arc (pp. 225 - 235)**	• Find the arc length of a circle by expressing the arc length as a fraction of the circumference of the circle	Understand and use the terms: centre, radius, diameter, circumference, arc, sector Solve problems involving arc length as a fraction of the circumference of a circle	Investigation – Arc Length (pp. 226 - 227)	Investigation – Arc Length (pp. 226 - 227)		Investigation – Arc Length (pp. 226 - 227)
22		**8.2 Area of Sector (pp. 236 - 242)**	• Find the area of the sector of a circle by expressing the area of a sector as a fraction of the area of the circle • Find the area of a segment of a circle	Understand and use the terms: centre, radius, diameter, circumference, arc, sector Solve problems involving sector area as a fraction of the area of a circle	Investigation – Area of Sector (pp. 236 - 237)	Investigation – Area of Sector (pp. 236 - 237)		Investigation – Area of Sector (pp. 236 - 237)
22		**Miscellaneous**					Solutions for Challenge Yourself	

Week (5 classes x 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
23	**9 Volume and Surface Area of Pyramids, Cones and Spheres**	**9.1 Volume and Surface Area of Pyramids (pp. 246 - 257)**	• Identify and sketch pyramids • Draw and use nets of pyramids to visualise their surface area • Use formulae to calculate the volume and the surface area of pyramids	Solve problems involving the surface area and volume of a pyramid	Class Discussion – What are Pyramids? (p. 246) Thinking Time (p. 248) Journal Writing (p. 248) Investigation – Volume of Pyramids (pp. 249 - 250)	Internet Resources (p. 250)		Class Discussion – What are Pyramids? (p. 246) Thinking Time (p. 248) Journal Writing (p. 248)
23		**9.2 Volume and Surface Area of Cones (pp. 257 - 265)**	• Identify and sketch cones • Draw and use nets of cones to visualise their surface area • Use formulae to calculate the volume and the surface area of cones	Solve problems involving the surface area and volume of a cone	Class Discussion – What are Cones? (p. 257) Journal Writing (p. 258) Investigation – Comparison between a Cone and a Pyramid (p. 259) Thinking Time (p. 260) Investigation – Curved Surface Area of Cones (pp. 262 - 263) Thinking Time (p. 263)			Class Discussion – What are Cones? (p. 257) Journal Writing (p. 258) Investigation – Comparison between a Cone and a Pyramid (p. 259) Thinking Time (p. 260) Investigation – Curved Surface Area of Cones (pp. 262 - 263)

Week (5 classes x 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
24		**9.3 Volume and Surface Area of Sphers (pp. 267 - 272)**	• Identify and sketch spheres • Use formulae to calculate the volume and the surface area of spheres	Solve problems involving the surface area and volume of a sphere	Thinking Time (p. 267) Class Discussion – Is the King's Crown Made of Pure Gold? (pp. 267 - 268) Investigation – Volume of Spheres (pp. 268 - 269) Investigation – Surface Area of Spheres (p. 270) Thinking Time (p. 271)			Thinking Time (p. 267) Class Discussion – Is the King's Crown Made of Pure Gold? (pp. 267 - 268) Investigation – Surface Area of Spheres (p. 270)
24		**9.4 Volume and Surfrace Are of Composite Solids (pp. 273 - 277)**	• Solve problems involving the volume and the surface area of composite solids made up of pyramids, cones, spheres, prisms and cylinders	Solve problems involving the surface area and volume of compound shapes				
24		**Miscellaneous**					Solutions for Challenge Yourself	

Week (5 classes x 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
25	**10 Congruence and Similarity Tests**	**10.1 Congruence Tests (pp. 283 - 298)**	- Apply the four congruence tests to determine whether two or more triangles are congruent	Solve problems and give simple explanations involving similarity and congruence	Investigation – SSS Congruence Test (p. 284) Investigation – SAS Congruence Test (pp. 286 – 287) Investigation – AAS Congruence Test (pp. 290 - 291) Investigation – RHS Congruence Test (p. 292) Class Discussion – Consolidation for Congruence Tests (p. 294)			Investigation – SSS Congruence Test (p. 284) Investigation – SAS Congruence Test (pp. 286 - 287) Investigation – AAS Congruence Test (pp. 290 - 291) Investigation – RHS Congruence Test (p. 292) Class Discussion – Consolidation for Congruence Tests (p. 294)

Week (5 classes x 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
25		**10.2 Similarity Tests (pp. 299 - 313)**	• Apply the three similarity tests to determine whether two or more triangles are similar		Investigation – AA Similarity Test (p. 300) Thinking Time (p. 300) Investigation – SSS Similarity Test (p. 303) Thinking Time (p. 304) Investigation – SAS Similarity Test (pp. 305 – 306) Thinking Time (p. 306)			Investigation – AA Similarity Test (p. 300) Thinking Time (p. 300) Investigation – SSS Similarity Test (p. 303) Thinking Time (p. 304) Investigation – SAS Similarity Test (pp. 305 – 306) Thinking Time (p. 306)
26		**10.3 Applications of Congruent and Similar Triangles (pp. 314 – 318)**	• Solve problems involving congruent and/or similar triangles					
26		**Miscellaneous**					Solutions for Challenge Yourself	
26	**11 Geometrical Constructions**	**11.1 Construction of Triangles (pp. 326 -330)**	• Construct triangles and solve related problems	Construct a triangle, given the three sides, using a ruler and a pair of compasses only				Just for Fun (p. 328) Ex 11A Q 13 – 14 (p. 306)

Week (5 classes x 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
27		**11.2 Construction of Quadrilaterals (pp. 331 - 345)**	• Construct quadrilaterals and solve related problems	Construct other simple geometrical figures from given data, using a ruler and protractor as necessary		Internet Resources (p. 333)		Worked Example 5 (p. 331) Practise Now 5 Q 2 (p. 333) Ex 11B Q 1 – 3, 5, 8 – 9, 19(ii), 25 (pp. 344 - 345)
27		**Miscellaneous**					Solutions for Challenge Yourself	
28	**12 Further Geometrical Transformations**	**12.1 Rotation (pp. 349 - 352)**	• Rotate an object and find the centre of rotation by construction		Thinking Time (p. 350)			Thinking Time (p. 350)
		12.2 Enlargement (pp. 353 - 362)	• Find the centre and scale factor of enlargement given the original figure and its enlarged image		Class Discussion – Enlargement in our surroundings (p. 357)	Internet Resource (Pp 357)		Class Discussion – Enlargement in our surroundings (p. 357)
28		**Miscellaneous**					Solutions for Challenge Yourself	

Week (5 classes x 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
29	**13** **Statistics**	**13.1 Frequency Polygons (pp. 366 - 374)**	• Construct and interpret data from histograms • Evaluate the purposes and appropriateness of the use of different statis-tical diagrams • Explain why some statistical diagrams can lead to a mis-interpretation of data	Construct and interpret histograms with equal and unequal intervals Construct and interpret frequency polygons	Main Text (p. 369)			
29		**13.2 Standard Deviation and Variance (pp. 375 - 388)**	• Calculate the standard deviation • Use the mean and standard deviation to compare two sets of data		Investigation – Are Averages Adequate for Comparing Distributions? (p. 375) Investigation – Obtaining a Formula for a New Measure of Spread (pp. 376 - 377) Thinking Time (p. 384) Class Discussion – Matching Histograms with Data Sets (p. 385)			Investigation – Are Averages Adequate for Comparing Distributions? (p. 375) Investigation – Obtaining a Formula for a New Measure of Spread (pp. 376 - 377) Thinking Time (p. 384) Class Discussion – Matching Histograms with Data Sets (p. 385)
29		**Miscellaneous**					Solutions for Challenge Yourself	

Week (5 classes x 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
30	**14 Probability of Combined Events**	**14.1 Probability of Single Events (pp. 392 - 395)**		Calculate the probability of a single event as either a fraction or a decimal	Thinking Time (p. 393)			Thinking Time (p. 393)
30		**14.2 Simple Combined Events, Possibility Diagrams and Tree Diagrams (pp. 396 - 405)**	• Calculate the probability of simple combined events using possibility diagrams and tree diagrams	Calculate the probability of simple combined events using possibility diagrams and tree diagrams where appropriate				
31		**14.3 Addition Law of Probability and Mutually Exclusive Events (pp. 406 - 410)**	• Use the Addition Law of Probability to solve problems involving mutually exclusive events		Investigation – Mutually Exclusive and Non-Mutually Exclusive Events (p. 406)			Investigation – Mutually Exclusive and Non-Mutually Exclusive Events (p. 406)
31		**14.4 Multiplication Law of Probability and Independent Events (pp. 410 - 423)**	• Use the Multiplication Law of Probability to solve problems involving independent and dependent events		Class Discussion – Choosing a Diagram to Represent the Sample Space (p. 410) Investigation – Dependent Events (p. 415) Performance Task (pp. 417 - 419)			Class Discussion – Choosing a Diagram to Represent the Sample Space (p. 410) Investigation – Dependent Events (p. 415) Performance Task (pp. 417 - 419)

Week (5 classes x 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
31		**Miscellaneous**					Solutions for Challenge Yourself	
31	**15 Sets**	**15.1 Sets (pp. 428 - 431)**	Describe a set in words, list all the elements in a set, and describe the elements in a set • Find the Power set of a set	Use set language, set notation and Venn diagrams to describe sets and represent relationships between sets Definition of sets: e.g. $A = \{x : x \text{ is a natural number}\}$, $B = \{(x, y): y = mx + c\}$, $C = \{x : a \leq x \leq b\}$, $D = \{a, b, c, \ldots\}$				
32		**15.2 Commutative, Associative, and Distributive Laws (pp. 431 - 434)**	• Describe Operations on sets and verify Commutative, Associative and Distributive laws					
32		**15.3 De Motrgan's Laws (pp. 434 - 436)**	• Verify De Morgan's Laws and represent through Venn Diagram		Text on p. 435 Ex 15A Q10 (p. 437)			Text on p. 435 Ex 15A Q10 (p. 437)
32		**Miscellaneous**					Solutions for Challenge Yourself	

Chapter 1 Number Operations and Direct and Inverse Proportions

TEACHING NOTES

Suggested Approach

Teachers can give real-life example when real numbers and approximation is used before getting them to discuss occasions when real numbers and approximation is used in their daily life. Students will also be learning the five rules to identify significant numbers. Furthermore, students will also learn to represent direct and indirect proportions as graphs..

Section 1.1: Real Numbers

Traditionally, real numbers are classified as either rational or irrational numbers. Another way to classify real numbers is according to whether their decimal forms are terminating, recurring, or non-recurring (see page 3 of the textbook). If teachers show students the first million digits of π (see page 3 of the textbook), many students may be surprised that π has so many digits! This suggests that students do not know that π has an infinite number of decimal places. Teachers may wish to celebrate Pi Day with students on March 14 by talking about π or singing the Pi song.

Section 1.2: Square Roots and Cube Roots

Students will recall the previous concept of square and cute numbers and their square and cute roots. Teachers will then use the same approach for square and cube root of real numbers as previously done.

Section 1.3: Approximation

To make learning of mathematics relevant, students should know some reasons why they need to use approximations in their daily lives (see Class Discussion: Actual and Approximated Values).

Teachers should do a recap with students on what they have learnt in previous grades, i.e. how to round off numbers to the nearest tenth, whole number and 10 etc.

Section 1.4: Significant Figures

Through the example on measuring cylinders on page 12 of the textbook, students will learn that a number is more accurate when it is given to a greater number of significant figures.

After learning how to round off numbers to a specified number of significant figures, teachers can arouse students' interest in this topic by bringing in real-life situations where they cannot just round off a number using the rules they have learnt.

Section 1.5: Rounding and Truncation Errors

The investigation on page 16 of the textbook highlights the importance of giving intermediate values correct to four significant figures if we want the final answer to be accurate to three significant figures. Otherwise, a rounding error may occur.

Students should also learn that there is a difference between 'approximately 2.5 million' and 'equal to 2.5 million (to 2 s.f.)' (see the thinking time on page 17 of the textbook).

Teachers should tell students the difference between rounding off a number to, say, 3 significant figures and truncating the same number to 3 significant figures.

Section 1.6: Direct Proportion

When introducing direct proportion, rates need not be stated explicitly. Rates can be used implicitly (see Investigation: Direct Proportion). By showing how one quantity increases proportionally with the other quantity, the concept should be easily relatable. More examples of direct proportion should be discussed and explored to test and enhance thinking and analysis skills.

Teachers should easily state the direct proportion formula between two quantities and the constant k. It is important to highlight the condition $k \neq 0$ as the relation would not hold if $k = 0$ (see Thinking Time on page 21). Through studying how direct proportion means graphically (see Investigation: Graphical Representation of Direct Proportion), students will gain an understanding on how direct proportion and linear functions are related, particularly the positive gradient of the straight line and the graph passing through the origin. The graphical representation will act as a test to determine if two variables are directly proportional.

Section 1.7: Inverse Proportion

The other form of proportion, inverse proportion, can be explored and studied by students (see Investigation: Inverse Proportion). When one variable increases, the other variable decreases proportionally. It is the main difference between direct and inverse proportion and must be emphasised clearly.

Students should be tasked with giving real-life examples of inverse proportion and explaining how they are inversely proportional (see Class Discussion: Real-Life Examples of Quantities in Inverse Proportion).

Teachers should present another difference between both kinds of proportions by reminding students that $\frac{y}{x}$ is a constant in direct proportion while xy is a constant in inverse proportion (see page 20 of the textbook).

Similar to direct proportion teachers can write the inverse proportion formula between two quantities and the constant k. It is important to highlight the condition $k \neq 0$ as the relation would not hold if $k = 0$ (see Thinking Time on page 28).

Although plotting y against x gives a hyperbola, and does not provide any useful information, teachers can show by plotting y against $\frac{1}{x}$ and showing direct proportionality between the two variables (see Investigation: Graphical Representation of Inverse Proportion).

WORKED SOLUTIONS

Investigation (Some Interesting Facts about the Irrational Number π)

1. The 1 000 000th digit of π is 1.
2. The 5 000 000 000 000th digit of π is 2.
3. Lu Chao, a graduate student from China, took 24 hours and 4 minutes to recite π to 67 890 decimal places in 2005.

Class Discussion (Actual and Approximated Values)

1. The actual values indicated in the article include '7 267 582 passengers', and 'one terminal' while approximated values include 'over 25 airlines' and '12 million passengers'. Actual values are exact numbers while approximated values are values which are usually rounded off.
2. **(a)** It is not necessary to specify the actual number of airlines, as an approximation is sufficient to show that Jinnah International Airport carters many airlines.
   (b) A headline serves as a brief summary of the article to draw readers' attentions, thus it is more appropriate to use an approximated value instead of the actual value.

Investigation (The Missing 0.1% Votes)

1. The percentage of votes for each candidate given is correct to 3 significant figures. Due to rounding errors in the intermediate steps, there is a follow-through error, resulting in the missing 0.1% of the votes. If the final answer is correct to 2 significant figures, we will obtain 100%. Hence, the final answer can only be accurate to 2 significant figures.
2. Percentage of votes for Bilal $= \frac{188}{301} \times 100\%$
   $= 62.5\%$ (to 3 s.f.)

   Percentage of votes for Rizwan $= \frac{52}{301} \times 100\%$
   $= 17.3\%$ (to 3 s.f.)

   Percentage of votes for Anosha $= \frac{61}{301} \times 100\%$
   $= 20.3\%$ (to 3 s.f.)

   Total percentage of votes $= 62.5\% + 17.3\% + 20.3\%$
   $= 100.1\%$

   The percentage of votes for each candidate given is correct to 3 significant figures. Due to rounding errors in the intermediate steps, which results in a follow through error, the total percentage of votes is 100.1%. If the final answer is correct to 2 significant figures, we will obtain 100%. Hence, the final answer can only be accurate to 2 significant figures.

Thinking Time (Page 17)

1. **(i)** When the population of City *A* is approximately 2.5 million, it is possible for the exact population size to be 2.47 million.
   (ii) When the population of City *A* is approximately 2.5 million, it is possible for the exact population size to be 2.6 million.
2. **(i)** When the population of City *B* is equal to 2.5 million (to 2 s.f.), it is possible for the exact population size to be 2.47 million as it is equal to 2.5 million when rounded off to 2 significant figures.
   (ii) When the population of City *B* is equal to 2.5 million (to 2 s.f.), it is not possible for the exact population size to be 2.6 million as it is still equal to 2.6 million when rounded off to 2 significant figures.

Note: There is a difference between 'approximately 2.5 million' and 'equal to 2.5 million (to 2 s.f.)'.

Investigation (Direct Proportion)

1. The fine will increase if the number of days a book is overdue increases.
2. $\frac{\text{Fine when a book is overdue for 6 days}}{\text{Fine when a book is overdue for 3 days}} = \frac{90}{45}$
   $= 2$

   The fine will be doubled if the number of days a book is overdue is doubled.
3. $\frac{\text{Fine when a book is overdue for 6 days}}{\text{Fine when a book is overdue for 2 days}} = \frac{90}{30}$
   $= 3$

   The fine will be tripled if the number of days a book is overdue is tripled.
4. $\frac{\text{Fine when a book is overdue for 5 days}}{\text{Fine when a book is overdue for 10 days}} = \frac{75}{150}$
   $= \frac{1}{2}$

   The fine will be halved if the number of days a book is overdue is halved.
5. $\frac{\text{Fine when a book is overdue for 3 days}}{\text{Fine when a book is overdue for 9 days}} = \frac{45}{135}$
   $= \frac{1}{3}$

   The fine will be reduced to $\frac{1}{3}$ of the original number if the number of days a book is overdue is reduced to $\frac{1}{3}$ of the original number.

Thinking Time (Page 21)

If we substitute $k = 0$ into $y = kx$, then $y = 0$.
This implies that for all values of x, $y = 0$.
y cannot be directly proportional to x in this case.

Investigation (Graphical Representation of Direct Proportion)

$y = 15x$ in this context means that for any additional number of a day a book is overdue, the fine will increase by PKR 15.

1.

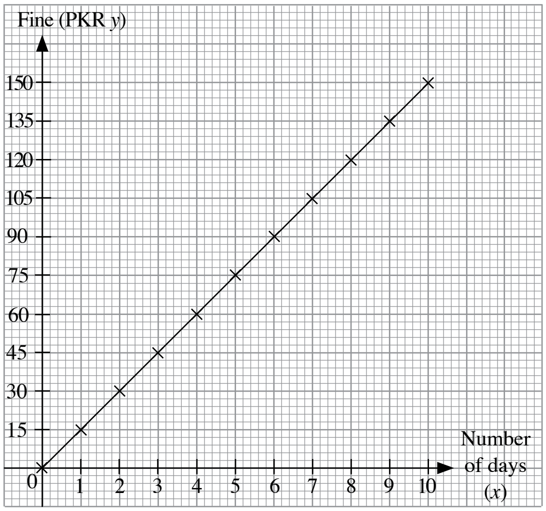


Fig. 1.1

2. The graph is a straight line.
3. The graph passes through the origin.

Thinking Time (Page 22)

1. Since y is directly proportional to x, $y = kx$

$$x = \frac{1}{k}\ y$$

Since $k \neq 0$, then we can rename $\frac{1}{k} = k_1$ where k_1 is another constant.

Hence, $x = k_1y$, where $k_1 \neq 0$ and x is directly proportional to y.

2. $x = k_1y$ is the equation of a straight line. When $y = 0$, $x = 0$.
We will get a straight line of x against y that passes through the origin.
3. If the graph of y does not pass through the origin, then $y = kx + c$, when $c \neq 0$. Since x and y are not related in the form $y = kx$, y is not directly proportional to x.
4. As x increases, y also increases. This does not necessarily conclude that y is directly proportional to x. It is important that when x increases, y increases **proportionally**. Also, when $x = 0$, $y = 0$.
$y = kx + c$ is an example of how x increases and y increases, but y is not directly proportional to x.

Investigation (Inverse Proportion)

1. The time taken decreases when the speed of the car increases.
2. $$\frac{\text{Time taken when speed of the car is 40 km/h}}{\text{Time taken when speed of the car is 20 km/h}} = \frac{3}{6}$$
$$= \frac{1}{2}$$
The time taken will be halved when the speed of the car is doubled.
3. $$\frac{\text{Time taken when speed of the car is 60 km/h}}{\text{Time taken when speed of the car is 20 km/h}} = \frac{2}{6}$$
$$= \frac{1}{3}$$
The time taken will be reduced to $\frac{1}{3}$ of the original number when the speed of the car is tripled.
4. $$\frac{\text{Time taken when speed of the car is 30 km/h}}{\text{Time taken when speed of the car is 60 km/h}} = \frac{4}{2}$$
$$= 2$$
The time taken will be doubled when the speed of the car is halved.
5. $$\frac{\text{Time taken when speed of the car is 40 km/h}}{\text{Time taken when speed of the car is 120 km/h}} = \frac{3}{1}$$
$$= 3$$
The time taken will be tripled when the speed of the car is reduced to $\frac{1}{3}$ of its original speed.

Thinking Time (Page 28)

If we substitute $k = 0$ into $y = \frac{k}{x}$, then $y = 0$.

This implies that for all values of x, $y = 0$

y cannot be inversely proportional to x in this case.

Investigation (Graphical Representation of Inverse Proportion)

1. We would obtain a graph of a hyperbola.
2.

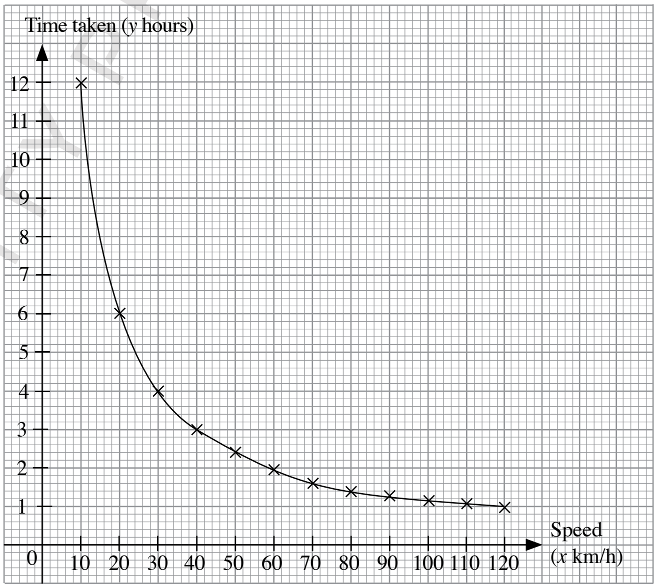


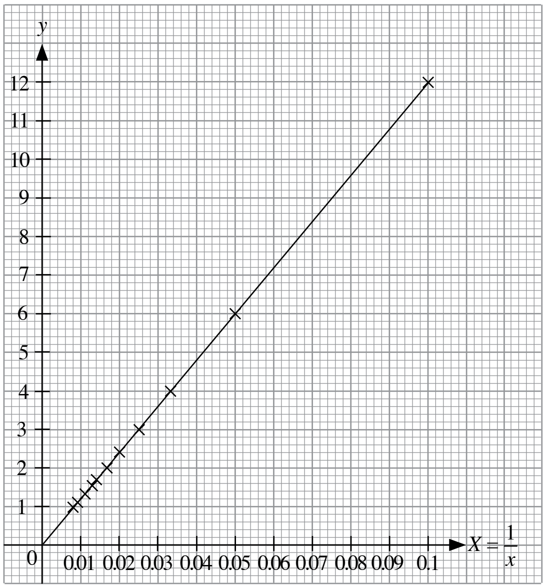
Fig. 1.4

3. When $x = 20$, $y = 6$.
When $x = 40$, $y = 3$.
Change in value of $y = \frac{3}{6}$
$= \frac{1}{2}$
The value of y will be halved when the value of x is doubled.

Speed (x km/h)	10	20	30	40	50	60
$X = \frac{1}{x}$	0.1	0.05	0.033	0.025	0.02	0.017
Time taken (y hours)	12	6	4	3	2.4	2

Speed (x km/h)	70	80	90	100	110	120
$\mathbf{X = \frac{1}{x}}$	0.014	0.013	0.011	0.01	0.009	0.008
Time taken (y hours)	1.7	1.5	1.3	1.2	1.1	1

4.



5. The graph is a straight line that passes through the origin.

$\frac{y}{X}$ is a constant.

6. y is directly proportional to X.

7. $y = k$X, where k is a constant. $\frac{y}{X}$ is a constant and y is directly proportional to X.

Thinking Time (Page 26)

Since y is inversely proportional to x, $y = \frac{k}{x}$

$$x = \frac{k}{y}$$

Hence, $x = \frac{k}{y}$, where $k \neq 0$ and x is inversely proportional to y.

Practise Now 1

	326
3	$\overline{10}\ \overline{62}\ \overline{76}$
+ 3	-9
62	162
+2	-124
646	3876
	- 3876
	0

$\sqrt{106276} = 326$

Practise Now 2

(a)

	4.7
4	$\overline{22}.\overline{09}$
+4	16
87	609
	609
	0

$\sqrt{22.09} = 4.7$

(b)

	9.25
9	$\overline{85}.\overline{56}\ \overline{25}$
+9	-81
182	456
+ 2	- 364
1845	9225
	- 9225
	0

$\sqrt{85.5625} = 9.25$

Practise Now 3

(a) $\sqrt{\frac{16}{121}} = \frac{\sqrt{16}}{\sqrt{121}}$

$$= \frac{4}{11}$$

(b) $\frac{350464}{54756} = \frac{\sqrt{350464}}{\sqrt{54756}}$

$$= \frac{592}{234}$$

$$= \frac{296}{117} = 2\frac{62}{117}$$

Practise Now 4

(a) $29^3 = 29 \times 29 \times 29$

$$= 24389$$

(b) **(i)** $\sqrt[3]{2744}$

$$= \sqrt[3]{2 \times 2 \times 2 \times 7 \times 7 \times 7}$$

$$= 2 \times 7$$

$$= 14$$

2	45564
2	1372
2	686
7	343
7	49
7	7
	1

(ii) $\sqrt[3]{74088}$

$= \sqrt[3]{2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 7 \times 7 \times 7}$

$= 2 \times 3 \times 7$

$= 42$

2	74088
2	37044
2	18522
3	9761
3	3087
3	1029
7	343
7	49
7	7
	1

Practise Now 5

(a) The number 192 has 3 significant figures.
(b) The number 83.76 has 4 significant figures.
(c) The number 3 has 1 significant figure.
(d) The number 4.5 has 2 significant figures.

Practise Now 6

(a) The number 506 has 3 significant figures.
(b) The number 1.099 has 4 significant figures.
(c) The number 3.0021 has 5 significant figures.
(d) The number 70.8001 has 6 significant figures.

Practise Now 7

1. **(a)** The number 0.10 has 2 significant figures.
  (b) The number 0.500 has 3 significant figures.
  (c) The number 41.0320 has 6 significant figures.
  (d) The number 6.090 has 4 significant figures.

2. 4.10 cm is more accurate because 4.10 cm is measured to 3 significant figures, while 4.1 cm is measured to 2 significant figures.

Practise Now 8

(a) The number 0.021 has 2 significant figures.
(b) The number 0.603 has 3 significant figures.
(c) The number 0.001 73 has 3 significant figures.
(d) The number 0.1090 has 4 significant figures.

Practise Now 9

(a) 3800 m, which is corrected to the nearest 10 m, has 3 significant figures.
(b) 25 000 km, which is corrected to the nearest km, has 5 significant figures.
(c) 100 000 g, which is corrected to the nearest 10 000 g, has 2 significant figures.

Practise Now 10

1. **(a)** 3748 = 3750 (to 3 s.f.)
  (b) 0.004 709 89 = 0.004 710 (to 4 s.f.)
  (c) 4971 = 5000 (to 2 s.f.)
  (d) 0.099 99 = 0.10 (to 2 s.f.)
  0.099 99 = 0.100 (to 3 s.f.)

2. Since 67 0X1 (to 3 s.f.), then the possible values of X are 5, 6, 7, 8 or 9.
If 67 0X1 is a perfect square, then by trial and error, $X = 8$.

Practise Now 11

(i) Length of square $= \sqrt{105}$
$= 10.2$ m (to 3 s.f.)

(ii) Perimeter of square $= 10.25 \times 4$
$= 41.0$ m (to 3 s.f.)

Practise Now 12

1. **(i)** Since y is directly proportional to x,
then $y = kx$, where k is a constant.
When $x = 2$, $y = 10$,
$10 = k \times 2$
$\therefore k = 5$
$\therefore y = 5x$

(ii) When $x = 10$,
$y = 5 \times 10$
$= 50$
Alternatively,
when $x = 10$, (x increased by 5 times)
$y = 5 \times 10$ (y increased by 5 times)
$= 50$

We can also use $\frac{y_2}{y_1} = \frac{x_2}{x_1}$,

i.e. $\frac{y}{10} = \frac{10}{2}$
$y = 5 \times 10$
$= 50$

(iii) When $y = 60$,

$$60 = 5x$$

$$\therefore x = \frac{60}{5}$$

$$= 12$$

2. Since y is directly proportional to x,

$$\frac{y_2}{y_1} = \frac{x_2}{x_1}$$

$$\frac{y}{5} = \frac{7}{2}$$

$$y = \frac{7}{2} \times 5$$

$$= 17.5$$

3.

x	4	5	7	8	9.5
y	24	30	42	48	57

Since y is directly proportional to x,

then $y = kx$, where k is a constant.

When $x = 5$, $y = 30$,

$$30 = k \times 5$$

$$\therefore k = 6$$

$$\therefore y = 6x$$

When $y = 48$,

$$48 = 6 \times x$$

$$x = \frac{48}{6}$$

$$= 8$$

When $y = 57$,

$$57 = 6 \times x$$

$$x = \frac{57}{6}$$

$$= 9.5$$

When $x = 4$,

$$y = 6 \times 4$$

$$= 24$$

When $x = 7$,

$$y = 6 \times 7$$

$$= 42$$

Practise Now 13

(i) Since C is directly proportional to d,

then $C = kd$, where k is a constant.

When $d = 60$, $C = 1000$,

$$1000 = k \times 60$$

$$\therefore k = \frac{50}{3}$$

$$\therefore C = \frac{50}{3}d$$

(ii) When $d = 45$,

$$C = \frac{50}{3} \times 45$$

$$= 75$$

$\therefore$ The cost of transporting goods is PKR 750.

(iii) When $C = 1200$,

$$1200 = \frac{50}{3} \times d$$

$$d = 120 \times \frac{3}{50}$$

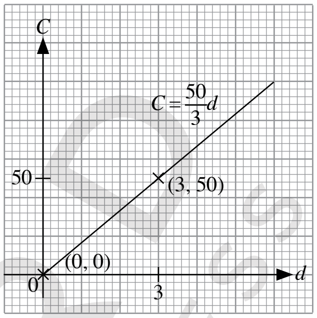
$$= 72$$

$\therefore$ The distance covered is 72 km.

(iv) $C = \frac{50}{3}d$

When $d = 0$, $C = 0$.

When $d = 3$, $C = 50$.



Practise Now 14

(i) Total monthly cost of running the kindergarten

= PKR 5000 + 200 × PKR 41

= PKR 13 200

(ii) Variable amount = PKR 20 580 – PKR 5000

= PKR 15 580

Number of children enrolled $= \frac{15\,580}{41}$

$= 380$

(iii) Variable amount = n × PKR 41

= PKR 41n

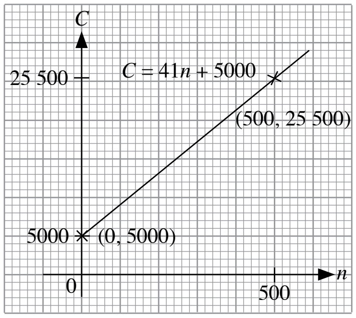
Total monthly cost = variable amount + fixed amount

$\therefore C = 41n + 5000$

(iv) $C = 41n + 5000$

When $n = 0$, $C = 5000$.

When $n = 500$, $C = 25\,500$.



C is *not* directly proportional to n because the line does not pass through the origin.

Practise Now 15

1. **(i)** When $x = 8$, (x increased by 4 times)

$y = \frac{5}{4}$ (y decreased by 4 times)

$= 1.25$

Alternatively,

$x_2y_2 = x_1y_1$

$8 \times y = 2 \times 5$

$y = \frac{10}{8}$

$= 1.25$

(ii) Since y is inversely proportional to x,

then $y = \frac{k}{x}$, where k is a constant.

When $x = 2$, $y = 5$,

$5 = \frac{k}{2}$

$\therefore k = 10$

$\therefore y = \frac{10}{x}$

(iii) When $y = 10$,

$10 = \frac{10}{x}$

$\therefore x = \frac{10}{10}$

$= 1$

2. Since y is inversely proportional to x,

$x_2y_2 = x_1y_1$

$3 \times y = 2 \times 9$

$y = \frac{18}{3}$

$= 6$

3.

x	0.5	1	2	3	5
y	8	4	2	$1\frac{1}{3}$	0.8

Since y is inversely proportional to x,

then $y = \frac{k}{x}$, where k is a constant.

When $x = 2$, $y = 2$,

$2 = \frac{k}{2}$

$\therefore k = 4$

$\therefore y = \frac{4}{x}$

When $y = 4$,

$4 = \frac{4}{x}$

$x = \frac{4}{4}$

$= 1$

When $y = 0.8$,

$0.8 = \frac{4}{x}$

$x = \frac{4}{0.8}$

$= 5$

When $x = 0.5$,

$y = \frac{4}{0.5}$

$= 8$

When $x = 3$,

$y = \frac{4}{3}$

$= 1\frac{1}{3}$

Practise Now 16

(i) Since I is inversely proportional to R,

then $I = \frac{k}{R}$, where k is a constant.

When $R = 0.5$, $I = 12$,

$12 = \frac{k}{0.5}$

$\therefore k = 6$

$\therefore I = \frac{6}{R}$

When $R = 3$,

$I = \frac{6}{3}$

$= 2$

$\therefore$ The current flowing through the wire is 2 A.

(ii) When $I = 3$,

$3 = \frac{6}{R}$

$R = \frac{6}{3}$

$= 2$

$\therefore$ The resistance of the wire is 2 Ω.

Practise Now 17

Men	Days	Wall
↓ 50	↑ 50	↑ 300
25	x	60

$25 \times x \times 300 = 50\ 20\ 60$

$$x = \frac{50 \times 20 \times 60}{25 \times 300}$$

$x = 8$

8 days will be required.

Exercise 1A

1. **(i)**

	451
4	$\overline{20}\ \overline{34}\ \overline{01}$
+4	-16
85	434
+ 5	- 425
1845	901
	-901
	0

$\sqrt{203401} = 451$

(ii)

	325
3	$\overline{10}\ \overline{56}\ \overline{25}$
+ 3	-9
6 2	156
+ 2	- 124
645	3225
	-3225
	0

$\sqrt{105625} = 325$

(iii)

	427
4	$\overline{18}\ \overline{23}\ \overline{29}$
+ 4	-16
8 2	223
+ 2	- 164
645	5929
	-5929
	0

$\sqrt{182329} = 427$

2. **(a)** **(i)** 283 $= 28 \times 28 \times 28$
$= 21952$

(ii) 333 $= 33 \times 33 \times 33$
$= 35937$

(iii) 453 $= 56 \times 56 \times 56$
$= 91125$

(b) **(i)**

23	12167
23	529
23	23
	1

$\sqrt[3]{12167} = \sqrt[3]{23 \times 23 \times 23}$
$= 23$

(ii)

5	42875
5	8575
5	1715
7	343
7	49
7	7
	1

$\sqrt[3]{42875} = \sqrt[3]{5 \times 5 \times 5 \times 7 \times 7 \times 7}$
$= 2 \times 7 = 35$

(iii)

2	74088
2	37044
2	18522
3	9261
3	3087
3	1029
7	343
7	49
7	7
	1

$\sqrt[3]{74088} = \sqrt[3]{2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 7 \times 7 \times 7}$
$= 2 \times 3 \times 7$
$= 42$

3. **(a)** The number 39 018 has 5 significant figures.
(b) The number 0.028 030 has 5 significant figures.
(c) 2900, which is corrected to the nearest 10, has 3 significant figures.

4. **(a)** 728 = 730 (to 2 s.f.)
(b) 503.88 = 503.9 (to 4 s.f.)
(c) 0.003 018 5 = 0.003 019 (to 4 s.f.)

(d) 6396 = 6400 (to 2 s.f.)

6396 = 6400 (to 3 s.f.)

(e) 9.9999 = 10.0 (to 3 s.f.)

(f) 8.076 = 8.08 (to 3 s.f.)

5. Possible values of $x = 4, 5$ or 6

6. **(a)** $\frac{1}{99}$ = 0.010 10 (to 4 s.f.)

(b) 871 × 234 = 203 814

= 200 000 (to 2 s.f.)

(c) $\frac{21^2}{0.219}$ = 2013.698 63

= 2013.7 (to 5 s.f.)

(d) $\frac{3.91^3 - 2.1}{6.41}$ = 9.0 (to 2 s.f.)

7. Greatest number of sweets that can be bought

$= \frac{\text{PKR 2}}{\text{PKR 0.30}}$

= 6 (to the nearest whole number)

8. **(i)** Length of square = $\sqrt{264}$

= 16.2 cm (to 3 s.f.)

(ii) Perimeter of square = 16.25 × 4

= 65.0 cm (to 3 s.f.)

9. **(i)** Radius of circle = $\frac{136}{2\pi}$

= 21.6 m (to 3 s.f.)

(ii) Area of circle = $\pi(21.65)^2$

= 1470 m^2 (to 3 s.f.)

10. Area of square garden = 331.24m^2

Length of it, one side = $\sqrt{33.124m}$

	18.2
1	$\overline{03}\ \overline{31}.\overline{24}$
+ 1	.01
2 8	231
+ 8	- 224
362	724
	-724
	0

Length of its one side = 18.2m

11. Area of squared wall = $\frac{289}{64}$

Height of wall = $\sqrt{\frac{289}{64}}$ m

$= \frac{\sqrt{289}}{\sqrt{64}}$ m

$= \frac{17}{8}$m

$= 1\frac{1}{8}$m

Volume of a cubical tank = 1331m^3

Height of the tank = $\sqrt[3]{1231}$ m

11	1331
11	121
11	11
	1

height of the tank = 11m

13. Area of square blanket = 105625 cm^2

Length of each side = $\sqrt{105625}$ cm

= 325 cm

14. Area of square wall = 24.9001 m^2.

$= \sqrt{24.9001}$

$= \frac{\sqrt{24.9001}}{\sqrt{1000}}$

	499
4	$\overline{24}\ \overline{90}\ \overline{01}$
+ 4	-16
8 9	890
+ 9	- 810
989	8901
	-8901
	0

$= \frac{499}{100}$

= 4.99

Length of wall is 4.99 m

15. Volume of a cube = 4913 cm^3

Length of each side = $\sqrt[3]{4913}$ cm

$= \sqrt[3]{17 \times 17 \times 17}$ cm

= 17 cm

16. Let's try to find out the cube root of 4608.

2	4608
2	2304
2	1152
2	576
2	288
8	144
2	72
2	36
2	18
3	9
3	3
	1

$$\sqrt[3]{4608} = \sqrt[3]{\overline{2 \times 2 \times 2} \times \overline{2 \times 2 \times 2} \times \overline{2 \times 2 \times 2} \times 3 \times 3}$$

$$= 2 \times 2 \times 2 \times \sqrt[3]{3 \times 3}$$

$$= 8\sqrt[3]{3 \times 3}$$

$$= 8\sqrt[3]{9}$$

To make this product a perfect cube, 3 is required to be multiplied with 4608.

17. Length of a side of a cube = 12m

Volume of the cube = l^3

$= 12^3$ m

$= 1728$ m^3

18. Since 21 X09 = 22 000 (to 2 s.f.), then the possible values of X are 5, 6, 7, 8 or 9.

If 21 X09 is a perfect square, then by trial and error, $X = 6$.

19. Largest possible number of people at the concert = 21 249

Smallest possible number of people at the concert = 21 150

20. **(i)** 987 654 321 + 0.000 007 – 987 654 321 = 0.000 007

(ii) 987 654 321 + 0.000 007 – 987 654 321 = 0

(iii) No, the answers for **(i)** and **(ii)** are different. This is because the calculator truncates the value of 987 654 321 + 0.000 007 to give 987 654 321. Hence, the answer for **(ii)** is 0.

Exercise 1B

1. **(i)** Since y is directly proportional to x, then $y = kx$, where k is a constant.

When $x = 4.5$, $y = 3$,

$3 = k \times 4.5$

$\therefore k = \frac{2}{3}$

$\therefore y = \frac{2}{3}x$

(ii) When $y = 6$,

$6 = \frac{2}{3}x$

$x = 6 \times \frac{3}{2}$

$= 9$

(iii) When $x = 12$,

$y = \frac{2}{3} \times 12$

$= 8$

2. **(i)** Since Q is directly proportional to P, then $Q = kP$, where k is a constant.

When $P = 4$, $Q = 28$,

$28 = k \times 4$

$\therefore k = 7$

$\therefore Q = 7P$

(ii) When $P = 5$,

$Q = 7 \times 5$

$= 35$

(iii) When $Q = 42$,

$42 = 7 \times P$

$P = 6$

3. Since z is directly proportional to x,

$\frac{x_2}{z_2} = \frac{x_1}{z_1}$

$\frac{x}{18} = \frac{3}{12}$

$x = \frac{3}{12} \times 18$

$= 4.5$

4. Since B is directly proportional to A,

$\frac{B_2}{A_2} = \frac{B_1}{A_1}$

$\frac{B}{24} = \frac{3}{18}$

$B = \frac{3}{18} \times 24$

$= 4$

5. **(a)**

x	4	20	24	36	44
y	1	5	6	9	11

Since y is directly proportional to x, then $y = kx$, where k is a constant.

When $x = 24$, $y = 6$,

$6 = k \times 24$

$\therefore k = \frac{1}{4}$

$\therefore y = \frac{1}{4}x$

When $y = 9$,

$9 = \frac{1}{4} \times x$

$x = 9 \times 4$

$= 36$

When $y = 11$,

$11 = \frac{1}{4} \times x$

$x = 11 \times 4$

$= 44$

When $x = 4$,

$y = \frac{1}{4} \times 4$

$= 1$

When $x = 20$,

$y = \frac{1}{4} \times 20$

$= 5$

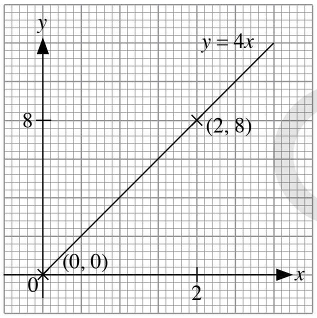
(b)

x	2	3	5.5	8	9.5
y	2.4	3.6	6.6	9.6	11.4

Since y is directly proportional to x,
then $y = kx$, where k is a constant.
When $x = 3$, $y = 3.6$,
$3.6 = k \times 3$
$\therefore k = 1.2$
$\therefore y = 1.2x$
When $y = 9.6$,
$9.6 = 1.2 \times x$
$x = \frac{9.6}{1.2}$
$= 8$
When $y = 11.4$,
$11.4 = 1.2 \times x$
$x = \frac{11.4}{1.2}$
$= 9.5$
When $x = 2$,
$y = 1.2 \times 2$
$= 2.4$
When $x = 5.5$,
$y = 1.2 \times 5.5$
$= 6.6$

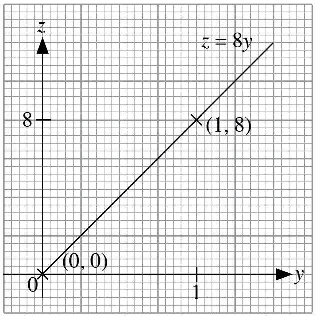
6. **(i)** Since y is directly proportional to x,
then $y = kx$, where k is a constant.
When $x = 5$, $y = 20$,
$20 = k \times 5$
$\therefore k = 4$
$\therefore y = 4x$

(ii) $y = 4x$
When $x = 0$, $y = 0$.
When $x = 2$, $y = 8$.



7. **(i)** Since z is directly proportional to y,
then $z = ky$, where k is a constant.
When $y = 6$, $z = 48$,
$48 = k \times 6$
$\therefore k = 8$
$\therefore z = 8y$

(ii) $z = 8y$
When $y = 0$, $y = 0$.
When $x = 1$, $y = 8$.

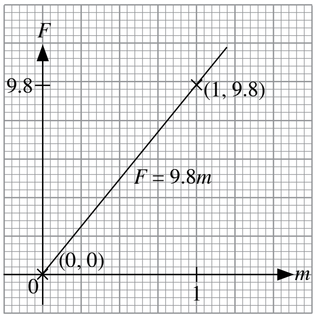


8. **(i)** Since F is directly proportional to m,
then $F = km$, where k is a constant.
When $m = 5$, $F = 49$,
$49 = k \times 5$
$\therefore k = 9.8$
$\therefore F = 9.8m$

(ii) When $m = 14$,
$F = 9.8 \times 14$
$= 137.2$

(iii) When $F = 215.6$,
$215.6 = 9.8 \times m$
$m = \frac{215.6}{9.8}$
$= 22$

(iv) $F = 9.8m$
When $m = 0$, $F = 0$.
When $m = 1$, $F = 9.8$.



9. **(i)** Since P is directly proportional to T,
then $P = kT$, where k is a constant.
When $T = 10$, $y = 25$,
$25 = k \times 10$
$\therefore k = 2.5$
$\therefore P = 2.5T$

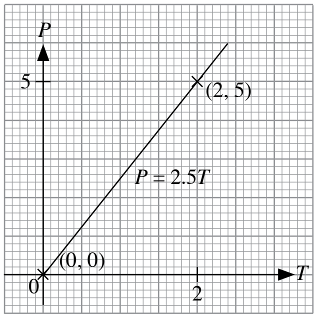
(ii) When $T = 24$,
$P = 2.5 \times 24$
$= 60$

(iii) When $P = 12$,
$12 = 2.5 \times T$
$T = \frac{12}{2.5}$
$= 4.8$

(iv) $P = 2.5T$

When $T = 0$, $P = 0$.

When $T = 2$, $P = 5$.



10. (i) Since V is directly proportional to R,

then $V = kR$, where k is a constant.

When $R = 6$, $V = 9$,

$$9 = k \times 6$$

$$\therefore k = 1.5$$

$$\therefore V = 1.5R$$

(ii) When $R = 15$,

$$V = 1.5 \times 15$$
$$= 22.5$$

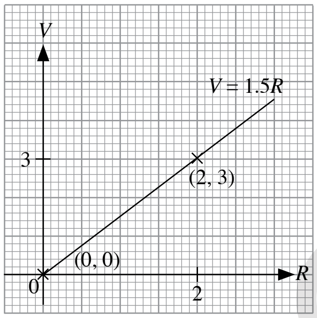
(iii) When $V = 15$,

$$15 = 1.5 \times R$$
$$R = \frac{15}{1.5}$$
$$= 10$$

(iv) $V = 1.5R$

When $R = 0$, $V = 0$.

When $R = 2$, $V = 3$.



11. (i) Total income for that month

= PKR 6000 + PKR 80 × 95

= PKR 13 600

(ii) Variable amount = PKR 13 600 – PKR 6000

= PKR 7600

Number of tyres he sells in that month $= \frac{7600}{80}$

$= 95$

(iii) Variable amount $= n \times$ PKR 80

= PKR $80n$

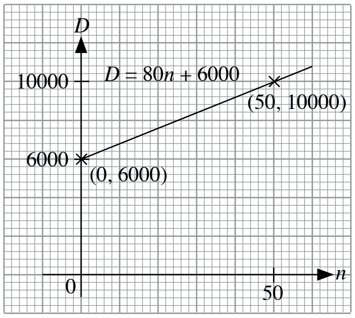
Total income = variable amount + fixed amount

$$\therefore D = 80n + 6000$$

(iv) $D = 80n + 6000$

When $n = 0$, $D = 6000$.

When $n = 50$, $D = 10000$.



D is *not* directly proportional to n because the line does not pass through the origin.

12. Let the mass of ice produced be m tonnes,

the number of hours of production be T hours.

Since m is directly proportional to T,

then $m = kT$, where k is a constant.

When $T = \frac{30}{60} - \frac{10}{60} = \frac{1}{3}$, $m = 20$,

$$20 = k \times \frac{1}{3}$$

$$\therefore k = 60$$

$$\therefore m = 60T$$

When $T = 1.75 - \frac{10}{60}$,

$$m = 60\ \ 1.75 - \frac{1}{6}$$
$$= 95$$

$\therefore$ The mass of ice manufactured is 95 tonnes.

Exercise 1C

1. (i) Since x is inversely proportional to y,

$$y_2x_2 = y_1x_1$$
$$25 \times x = 5 \times 40$$
$$x = \frac{5 \times 40}{25}$$
$$= 8$$

(ii) Since x is inversely proportional to y,

then $x = \frac{k}{y}$, where k is a constant.

When $y = 5$, $x = 40$,

$$40 = \frac{k}{5}$$

$$\therefore k = 200$$

$$\therefore x = \frac{200}{y}$$

(iii) When $x = 400$,

$$400 = \frac{200}{y}$$
$$y = \frac{200}{400}$$
$$= 0.5$$

2. **(i)** Since Q is inversely proportional to P,

then $Q = \frac{k}{P}$, where k is a constant.

When $P = 2$, $Q = 0.25$,

$$0.25 = \frac{k}{2}$$

$$\therefore k = 0.5$$

$$\therefore Q = \frac{0.5}{P}$$

$$= \frac{1}{2P}$$

(ii) When $P = 5$,

$$Q = \frac{1}{2(5)}$$

$$= 0.1$$

(iii) When $Q = 0.2$,

$$0.2 = \frac{1}{2P}$$

$$2P = \frac{1}{0.2}$$

$$= 5$$

$$P = 2.5$$

3.

Men	**Hours**	**Days**
↓ 100	↑ 8	↑ 35
x	10	25

$$x \times 10 \times 25 = 100 \times 8 \times 35$$

$$x = \frac{100 \times 8 \times 35}{10 \times 25}$$

$$x = \frac{28000}{250}$$

$$= 112$$

112 men are required to complete the job.

4. Since z is inversely proportional to x,

$$x_2 z_2 = x_1 z_1$$

$$x \times 70 = 7 \times 5$$

$$x = \frac{7 \times 5}{70}$$

$$= 0.5$$

5. Since B is inversely proportional to A,

$$A_2 B_2 = A_1 B_1$$

$$1.4 \times B = 2 \times 3.5$$

$$B = \frac{2 \times 3.5}{1.4}$$

$$= 5$$

6. **(a)**

x	0.5	2	2.5	3	8
y	24	6	4.8	4	1.5

Since y is inversely proportional to x,

then $y = \frac{k}{x}$, where k is a constant.

When $x = 3$, $y = 4$,

$$4 = \frac{k}{3}$$

$$\therefore k = 12$$

$$\therefore y = \frac{12}{x}$$

When $y = 24$,

$$24 = \frac{12}{x}$$

$$x = \frac{12}{24}$$

$$= 0.5$$

When $y = 1.5$,

$$1.5 = \frac{12}{x}$$

$$x = \frac{12}{1.5}$$

$$= 8$$

When $x = 2$,

$$y = \frac{12}{2}$$

$$= 6$$

When $x = 2.5$,

$$y = \frac{12}{2.5}$$

$$= 4.8$$

(b)

x	3	4	4.5	14.4	25
y	12	9	8	2.5	1.44

Since y is inversely proportional to x,

then $y = \frac{k}{x}$, where k is a constant.

When $x = 4$, $y = 9$,

$$9 = \frac{k}{4}$$

$$\therefore k = 36$$

$$\therefore y = \frac{36}{x}$$

When $y = 8$,

$$8 = \frac{36}{x}$$

$$x = \frac{36}{8}$$

$$= 4.5$$

When $y = 2.5$,

$2.5 = \frac{36}{x}$

$x = \frac{36}{2.5}$

$= 14.4$

When $x = 3$,

$y = \frac{36}{3}$

$= 12$

When $x = 25$,

$y = \frac{36}{25}$

$= 1.44$

7. **(i)** Since f is inversely proportional to λ,

then $f = \frac{k}{\lambda}$, where k is a constant.

When $\lambda = 3000$, $f = 100$,

$100 = \frac{k}{3000}$

$\therefore k = 300\ 000$

$\therefore f = \frac{300\ 000}{\lambda}$

When $\lambda = 500$,

$f = \frac{300\ 000}{500}$

$= 600$

$\therefore$ The frequency of the radio wave is 600 kHz.

(ii) When $f = 800$,

$800 = \frac{300\ 000}{\lambda}$

$\lambda = \frac{300\ 000}{800}$

$= 375$

$\therefore$ The wavelength of the radio wave is 375 m.

8. **(i)** Since t is inversely proportional to N,

then $t = \frac{k}{N}$, where k is a constant.

When $N = 3$, $t = 8$,

$8 = \frac{k}{3}$

$\therefore k = 24$

$\therefore t = \frac{24}{N}$

(ii) When $N = 6$,

$t = \frac{24}{6}$

$= 4$

$\therefore$ The number of hours needed by 6 men is 4 hours.

(iii) When $t = \frac{3}{4}$,

$\frac{3}{4} = \frac{24}{N}$

$N = 24 \times \frac{4}{3}$

$= 32$

$\therefore$ 32 men need to be employed.

Review Exercise 1

1. **(i)** Since y is directly proportional to x,

then $y = kx$, where k is a constant.

When $x = 2$, $y = 6$,

$6 = k \times 2$

$\therefore k = 3$

$\therefore y = 3x$

(ii) When $x = 11$,

$y = 3 \times 11$

$= 33$

(iii) When $y = 12$,

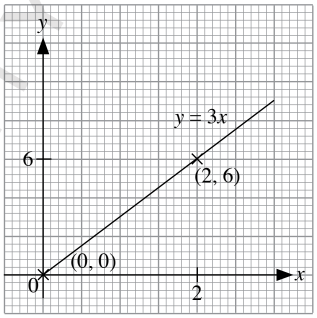
$12 = 3 \times x$

$x = 4$

(iv) $y = 3x$

When $x = 0$, $y = 0$.

When $x = 2$, $y = 6$.



2. **(i)** Since A is directly proportional to B,

then $A = kB$, where k is a constant.

When $B = \frac{5}{6}$, $A = 1\frac{2}{3}$,

$1\frac{2}{3} = k \times \frac{5}{6}$

$\therefore k = 2$

$\therefore A = 2B$

When $B = \frac{1}{3}$,

$A = 2 \times \frac{1}{3}$

$= \frac{2}{3}$

(ii) When $A = 7\frac{1}{2}$,

$7\frac{1}{2} = 2 \times B$

$B = 3\frac{3}{4}$

3. **(i)** Since y is inversely proportional to x,

then $y = \frac{k}{x}$, where k is a constant.

When $x = 3$, $y = 4$,

$4 = \frac{k}{3}$

$\therefore k = 12$

$\therefore y = \frac{12}{x}$

(ii) When $x = 6$,

$y = \frac{12}{6}$

$= 2$

(iii) When $y = 24$,

$24 = \frac{12}{x}$

$x = \frac{12}{24}$

$= 0.5$

4. **(i)** Since z is inversely proportional to $w + 3$,

then $z = \frac{k}{w + 3}$, where k is a constant.

When $w = 3$, $z = 4$,

$4 = \frac{k}{3 + 3}$

$\therefore k = 24$

$\therefore z = \frac{24}{w + 3}$

When $w = 9$,

$z = \frac{24}{9 + 3}$

$= 2$

(ii) When $z = 2.4$,

$2.4 = \frac{24}{w + 3}$

$w + 3 = \frac{24}{2.4}$

$= 10$

$w = 7$

5. **(i)** Total monthly charges

= PKR 981 + PKR 0.86 × 300

= PKR 1239

(ii) Variable amount = PKR 2056 – PKR 981

= PKR 1075

Duration of usage $= \frac{1075}{0.86}$

= 1250 minutes

(iii) Variable amount = n × PKR 0.86

= PKR 0.86n

Total income = variable amount + fixed amount

$\therefore C = 0.86n + 981$

$C - 981 = 0.86n$

Since $\frac{C - 981}{n} = 0.86$ is a constant, then $C - 981$ is directly proportional to n.

6. **(i)** Since G is directly proportional to h,

then $G = kh$, where k is a constant.

When $h = 40$, $G = 2200$,

$2200 = k \times 40$

$\therefore k = 55$

$\therefore G = 55h$

(ii) When $h = 22$,

$G = 55 \times 22$

$= 1210$

$\therefore$ The gravitational potential energy of the objects is 1210 J.

(iii) When $G = 3025$,

$3025 = 55 \times h$

$h = \frac{3025}{55}$

$= 55$

$\therefore$ The height of the object above the surface of the Earth is 55 m.

7. **(i)** Since P is inversely proportional to V,

then $P = \frac{k}{V}$, where k is a constant.

When $V = 4000$, $P = 250$,

$250 = \frac{k}{4000}$

$\therefore k = 1\,000\,000$

$\therefore P = \frac{1\,000\,000}{V}$

When $V = 5000$,

$P = \frac{1\,000\,000}{5000}$

$= 200$

$\therefore$ The pressure of the gas is 200 Pa.

(ii) When $P = 125$,

$125 = \frac{1\,000\,000}{V}$

$V = \frac{1\,000\,000}{125}$

$= 8000$

$\therefore$ The volume of the gas is 8000 dm^3.

Challenge Yourself

1. **(a)** Since A is directly proportional to C,

then $A = k_1C$, where k_1 is a constant.

Since B is directly proportional to C,

then $A = k_2C$, where k_2 is a constant.

$$A + B = k_1C + k_2C$$
$$= (k_1 + k_2)C$$

Since $\frac{A+B}{C} = k_1 + k_2$ is a constant, then $A + B$ is directly proportional to C.

(b) From **(a)**,

$$A - B = k_1C - k_2C$$
$$= (k_1 - k_2)C$$

Since $\frac{A+B}{C} = k_1 - k_2$ is a constant, then $A - B$ is directly proportional to C.

(c) $AB = (k_1C)(k_2C)$

$$= k_1k_2C^2$$
$$\sqrt{AB} = \sqrt{k_1k_2C^2}$$
$$= \sqrt{k_1k_2}\,C$$

Since $\frac{\sqrt{AB}}{C} = \sqrt{k_1k_2}$ is a constant, then $\sqrt{AB}$ is directly proportional to C.

2. **(i)** Since T is directly proportional to B and inversely proportional to P, then

$T = \frac{kB}{P}$, where k is a constant.

When $B = 3$, $P = 18$, $T = 20$,

$$20 = \frac{k \times 3}{18}$$
$$= \frac{k}{6}$$
$$\therefore k = 120$$
$$\therefore T = \frac{120B}{P}$$

(ii) When $B = 4$, $P = 16$,

$$T = \frac{120 \times 4}{16}$$
$$= 30$$

$\therefore$ The number of days needed is 30.

(iii) When $B = 10$, $T = 24$,

$$24 = \frac{120 \times 10}{P}$$
$$P = \frac{120 \times 10}{24}$$
$$= 50$$

$\therefore$ 50 painters need to be employed.

Chapter 2 Financial Transactions

TEACHING NOTES

Suggested Approach

Teachers can get students to discuss examples of percentages, which are used in everyday life. Although the concepts covered in this chapter are applicable to the real world, students might not have encountered the need to be familiar with them and hence might not identify with the situations easily. Teachers should prepare more relatable material, such as advertisements on discounted products, to allow students to appreciate the application of mathematics in practical situations.

Section 2.1: Financial Transactions

The definitions of profit and loss should be made clear to students, whereby:

$$\text{Profit} = \text{Selling price} - \text{Cost price}$$

$$\text{Loss} = \text{Cost price} - \text{Selling price}.$$

Teachers should also emphasise the difference between the expression of profit and loss as a percentage of the cost price and the calculation of percentage gain or loss in terms of the selling price, that may occur in some business transactions.

$$\frac{\text{Profit}}{\text{Cost price}} \times 100\% \qquad \frac{\text{Loss}}{\text{Cost price}} \times 100\%$$

$$\text{Percentage gain} = \frac{\text{Profit}}{\text{Selling price}} \times 100\% \qquad \text{Percentage loss} = \frac{\text{Loss}}{\text{Selling price}} \times 100\%$$

Thus, teachers should remind students to read the questions carefully in order to ascertain the correct percentage to report.

These real-world concepts would be useful for students when they start to work and plan their finances. However, teachers should note that students may not encounter terms such as discount, and thus should explain the term and clearly before going through the topic.

Section 2.3: Simple Interest and Compound Interest

Teachers may apply the prevailing interest rate in the region to an investment example using both simple interest and compound interest, in order to illustrate the effect of the significant difference in the final amount. Teachers should highlight to students that the computation of interest would be different depending on whether a simple or compounded interest is charged, and hence students need to be careful when faced with such questions.

Section 2.4: Hire Purchase

To assert the real-world context of this section, teachers may show students some advertisements on posters or other promotional material that feature the availability of a hire purchase alternative. Teachers can also suggest to students to think about whether Shop *A* having a cheaper interest rate for the hire purchase of the exact same item as compared to Shop *B* implies that a buyer should get the item from Shop *A*. There may be other hidden terms and conditions that make Shop *B*'s item more attractive, such as a longer period of warranty for instance.

Section 2.5: Money Exchange

Teachers may wish to conduct a class exercise by asking students to find out the current exchange rates of the local currency against prominent currencies such as the US Dollar, Euro, Sterling Pound, Japanese Yen, etc. compared to five to ten years ago. Based on the the trend, students can try to predict which currency would be a good investment to make. Teachers can highlight to students that when exchanging money, the money changer would offer both a buying and selling rate, and ensure that students are clear about the difference. Teachers can then explain why the exchange of one currency to another, and back to the previous currency, will usually result in a loss.

WORKED SOLUTIONS

Class Discussion (Body Mass index)

1. –
2. Medical practitioners make use of the BMI to determine which risk category you belong to as shown in the Table 2.1 of the textbook. With this information, they will outline patients' health risks with increasing obesity and provide the necessary advices such as to start to eat more healthily and increase the activity level to lose weight.
3. Other real-life applications of rates include rate of flow of tap water, mobile phone charges and housing loan rate.

Performance Task (Page 44)

Teachers may wish to ask the students to search on the internet to find out the different interest rates as well as charges offered by the different credit card companies such as HBL, UBL, Meezan Bank, Standard Chartered and etc. Students will then present the findings to the class.

Investigation (Simple Interest and Compound Interest)

1. Interest $= \frac{PRT}{100}$

$= \frac{1000 \times 2 \times 3}{100}$

= PKR 60

Total amount after 3 years = PKR 1000 + PKR 60

= PKR 1060

2. **1^{st} year:** Principal P_1 = PKR 1000

Interest I_1 = PKR 1000 × 2%

$= \underline{\text{PKR 20}}$

Total amount at the end of the 1^{st} year,

$A_1 = P_1 + I_1$

$= \text{PKR 1000} + \underline{\text{PKR 20}}$

= PKR 1020

2^{nd} year: Principal $P_2 = A_1$ = PKR 1020

Interest $I_2 = \underline{\text{PKR 1020}} \times 2\%$

$= \underline{\text{PKR 20.40}}$

Total amount at the end of the 2^{nd} year,

$A_2 = P_2 + I_2$

$= \text{PKR 1020} + \underline{\text{PKR 20.40}}$

$= \underline{\text{PKR 1040.40}}$

3^{rd} year: Principal $P_3 = A_2 = \underline{\text{PKR 1040.40}}$

Interest $I_3 = \underline{\text{PKR 1040.40}} \times 2\%$

$= \underline{\text{PKR 20.808}}$

Total amount at the end of the 3^{rd} year,

$A_3 = P_3 + I_3$

$= \text{PKR 1040.40} + \underline{\text{PKR 20.808}}$

$= \underline{\text{PKR 1061.21}}$ (to the nearest paisa)

3. Interest offered by Bank B = PKR 1061.21 – PKR 1000

= PKR 61.21

Difference in amount of interest offered by Bank A and Bank B

= PKR 61.21 – PKR 60

= PKR 1.21

∴ Bank B offers a higher interest of PKR 1.21.

Practise Now 1

1. **(a)** Required percentage $= \frac{\text{PKR 2400} - \text{PKR 1800}}{\text{PKR 1800}} \times 100\%$

$= \frac{\text{PKR 600}}{\text{PKR 1800}} \times 100\%$

$= 33\frac{1}{3}\ \%$

(b) Required percentage $= \frac{\text{PKR 6000} - \text{PKR 5000}}{\text{PKR 5000}} \times 100\%$

$= \frac{\text{PKR 1000}}{\text{PKR 5000}} \times 100\%$

= 20%

2. **(a)** Selling price of chain $= \frac{127}{100} \times \text{PKR 500}$

= PKR 635

(b) Selling price of car $= \frac{94}{100} \times \text{PKR 78 400}$

= PKR 73 696

3. Percentage discount $= \frac{\text{PKR 100} - \text{PKR 88}}{\text{PKR 100}} \times 100\%$

$= \frac{\text{PKR 12}}{\text{PKR 100}} \times 100\%$

= 12%

4. Sale price of toy car $= \frac{94}{100} \times \text{PKR 600}$

= PKR 564

Practise Now 2

1. P = PKR 180

R = 6%

T = 1 year 8 month $= \frac{20}{12}$ yrs

Profit $= \frac{PRT}{100}$

$= \frac{180 \times 6 \times 20}{100 \times 12}$

= PKR 18

Practice Now 3

(a) Amount of interest the man has to pay at the end of 1 year

$= \text{PKR 150 000} \times \frac{5.5}{100}$

= PKR 8250

Amount of interest the man has to pay at the end of 3 years

= PKR 8250 × 3

= PKR 24 750

Total amount he owes the bank

= PKR 150 000 + PKR 24 750

= PKR 174 750

(b) Total amount of interest Shirley earns
= PKR 67 200 – PKR 60 000
= PKR 7200
Amount of interest Shirley earns per last year

$= \text{PKR } 60\,000 \times \frac{3}{100}$

= PKR 1800
Time taken for her investment to grow to PKR 67 200

$= \frac{\text{PKR } 7200}{\text{PKR } 1800}$

= 4 years

Practise Now 4

1. P = PKR 30 000, $R = 5$, $n = 4$
At the end of 4 years, total amount accumulated is

$$A = P\left(1+\frac{R}{100}\right)^n$$

$$= 30\,000\left(1+\frac{5}{100}\right)^4$$

= PKR 36465.19 (to the nearest paisa)

Total interest, $I = A - P$
= PKR 36 465.19 – PKR 3000
= PKR 6465.19

2. **(a)** P = PKR 15 000, $R = 2$, $n = 2$
At the end of 2 years, total amount accumulated is

$$A = P\left(1+\frac{R}{100}\right)^n$$

$$= 15\,000\left(1+\frac{2}{100}\right)^2$$

= PKR 15606 (to the nearest rupee)

Total interest, $I = A - P$
= PKR 15 606 – PKR 15 000
= PKR 606

(b) Since interest is calculated monthly,

P = PKR 15 000, $R = \frac{2}{12} = \frac{1}{6}$, $n = 2 \times 12 = 24$

At the end of 2 years, total amount accumulated is

$$A = P\left(1+\frac{R}{100}\right)^n$$

$$= 15\,000\left(1+\frac{\left(\frac{1}{6}\right)}{100}\right)^{24}$$

= PKR 15 611.6 (to the nearest paisa)

Total interest, $I = A - P$
= PKR 15611.6 – PKR 15 000
= PKR 611.6

3. Since interest is calculated yearly,
P = PKR 4000, $A = 4243.60$, $n = 2$
At the end of 2 years, total amount accumulated is

$$A = P\left(1+\frac{R}{100}\right)^n$$

$$\text{PKR } 4243.60 = \text{PKR } 4000\left(1+\frac{R}{100}\right)^2$$

$$\frac{4243.60}{4000} = \left(1+\frac{R}{100}\right)^2$$

$$1+\frac{R}{100} = \sqrt{1.0609}$$

$$\frac{R}{100} = \sqrt{1.0609} - 1$$

$$R = 100(\sqrt{1.0609} - 1)$$

$$= 3$$

$\therefore$ The interest rate is 3%.

Practise Now 5

(i) Downpayment $= \frac{20}{100} \times \text{PKR } 90\,000$
= PKR 18 000
Remaining amount = PKR 90 000 – PKR 18 000
= PKR 72 000
Amount of interest Amirah owes at the end of 1 year

$= \text{PKR } 72\,000 \times \frac{10}{100}$

= PKR 7200
Amount of interest Amirah owes at the end of 4 years
= PKR 7200 × 4
= PKR 28 800
Total amount to be paid in monthly instalments
= PKR 72 000 + PKR 28 800
= PKR 100 800

Monthly instalment $= \frac{\text{PKR } 100\,800}{48}$ (4 years = 48 months)
= PKR 2100

(ii) Total amount Amirah pays for the air conditioner
= PKR 100 800 + PKR 18 000
= PKR 118 800

(iii) She has to pay PKR (118 800 – 90 000) = PKR 28 800 more for buying the air conditioner on hire purchase.

Practise Now 6

AED 1 = PKR 72.50

1. **(a)** AED 4200 = PKR 72.50 × 4200
= PKR 304 500

(b) PKR 72.50 = AED 1

PKR 1= AED $\frac{1}{72.50}$

PKR 9500= AED $\frac{1}{72.50} \times 9500$

= AED 131.03 $\simeq$ AED 131

Exercise 2A

1. Profit = selling price - cost price

= PKR 3500 - PKR 300

= PKR 500

$$\text{Profit \%} = \frac{\text{Profit}}{\text{Lost price}} \times 100\%$$

$$= \frac{\text{PKR } 500}{\text{PKR } 3500} \times 100\%$$

= 14.28%

2. Loss = Cost price - selling price

= PKR 750000 - PKR 200000

= PKR 55000

$$\text{Loss \%} = \frac{\text{Loss}}{\text{Lost price}} \times 100\%$$

$$= \frac{\text{PKR } 550000}{\text{PKR } 750000} \times 100\%$$

= 73.33%

3.

	Principal	**Interest rate**	**Time**	**Simple Interest**	**Amount**
(a)	PKR 12 000	8%	7 years	PKR 6720	PKR 18 720
(b)	PKR 500	11%	4 years	PKR 220	PKR 720
(c)	PKR 300	9%	4 years	PKR 108	PKR 408
(d)	PKR 3000	4%	10 years	PKR 1200	PKR 4200
(e)	PKR 3600	5%	2 years	PKR 360	PKR 3960
(f)	PKR 1800	7%	18 months	PKR 189	PKR 1989
(g)	PKR 4500	6%	2 years	PKR 540	PKR 5040
(h)	PKR 1200	5%	18 months	PKR 90	PKR 1290

4. Amount of interest paid = PKR 550

Let the sum of money borrowed be PKR x.

$$\text{PKR } 550 = 12\% \times \frac{5}{12} \times \text{PKR } x$$

$$x = 550 \div \frac{12}{100} \div \frac{5}{12}$$

= 11 000

∴ The sum of money was PKR 11 000.

5. Total interest $= \frac{2.25}{100} \times 25 \times \text{PKR } 6400$

= PKR 3600

6. Annual interest on PKR 800 investment $= \frac{6}{100} \times \text{PKR } 800$

= PKR 48

Annual interest on PKR 1200 investment $= \frac{7}{100} \times \text{PKR } 1200$

= PKR 84

Total annual interest = PKR 48 + PKR 84

= PKR 132

7. Amount of interest earned per year $= \text{PKR } 1250 \times \frac{6}{100}$

= PKR 75

Time taken for interest to grow to PKR 750 $= \frac{\text{PKR } 750}{\text{PKR } 75}$

= 10 years

8. Interest rate $= \frac{\text{PKR } 119}{\text{PKR } 4800} \times 100\% \div \frac{7}{12}$

$= 4\frac{1}{4}\ \%$

9. Amount of interest Rizwan has to pay at the end of 1 year

$= \text{PKR } 480\ 000 \times \frac{6}{100}$

= PKR 28 800

Amount of interest Rizwan has to pay at the end of 2 years

= PKR 28 800 × 2

= PKR 57 600

Total amount of money he has to pay at the end of 2 years

= PKR 480 000 + PKR 57 600

= PKR 537 600

10. Total amount of interest the man earns

= PKR 189 000 – PKR 168 000

= PKR 21 000

Amount of interest the man earns per last year

$= \text{PKR } 168\ 000 \times \frac{5}{100}$

= PKR 8400

Time taken for his investment to grow to PKR 168 000

$= \frac{\text{PKR } 21\ 000}{\text{PKR } 8400}$

$= 2\frac{1}{2}$ years

11. P = PKR 500 000, $R = 8$, $n = 3$

At the end of 3 years, total amount accumulated is

$$A = P\left(1 + \frac{R}{100}\right)^n$$

$$= 500\ 000\left(1 + \frac{8}{100}\right)^3$$

= PKR 629 856

Total interest, $I = A - P$

= PKR 629 856 – PKR 500 000

= PKR 129 856

12. (a) P = PKR 450, $R = 10$, $n = 2$

At the end of 2 years, total amount accumulated is

$$A = P\left(1 + \frac{R}{100}\right)^n$$

$$= 450\left(1 + \frac{10}{100}\right)^2$$

= PKR 544.50

Total interest, $I = A - P$

= PKR 544.50 – PKR 450

= PKR 94.50

(b) P = PKR 700, $R = 11$, $n = 3$

At the end of 3 years, total amount accumulated is

$$A = P\left(1 + \frac{R}{100}\right)^n$$

$$= 700\left(1 + \frac{11}{100}\right)^3$$

= PKR 957.34 (to the nearest paisa)

Total interest, $I = A - P$

$= \text{PKR } 957.34 - \text{PKR } 700$

$= \text{PKR } 257.34$

(c) $P = \text{PKR } 5000,\ R = 11\frac{3}{4},\ n = 2$

At the end of 2 years, total amount accumulated is

$$A = P\left(1 + \frac{R}{100}\right)^n$$

$$= 5000\left(1 + \frac{\left(11\frac{3}{4}\right)}{100}\right)^2$$

$= \text{PKR } 6244.03$ (to the nearest paisa)

Total interest, $I = A - P$

$= \text{PKR } 6244.03 - \text{PKR } 5000$

$= \text{PKR } 1244.03$

(d) $P = \text{PKR } 1200,\ R = 4,\ n = 3$

At the end of 3 years, total amount accumulated is

$$A = P\left(1 + \frac{R}{100}\right)^n$$

$$= 1200\left(1 + \frac{4}{100}\right)^3$$

$= \text{PKR } 1349.84$ (to the nearest paisa)

Total interest, $I = A - P$

$= \text{PKR } 1349.84 - \text{PKR } 1200$

$= \text{PKR } 149.84$

(e) $P = \text{PKR } 10\,000,\ R = 7\frac{1}{2},\ n = 2$

At the end of 2 years, total amount accumulated is

$$A = P\left(1 + \frac{R}{100}\right)^n$$

$$= 10\,000\left(1 + \frac{\left(7\frac{1}{2}\right)}{100}\right)^2$$

$= \text{PKR } 11\,556.25$

Total interest, $I = A - P$

$= \text{PKR } 11\,556.25 - \text{PKR } 10\,000$

$= \text{PKR } 1556.25$

13. $P = \text{PKR } 5000,\ R = 5\frac{1}{4},\ n = 3$

At the end of 3 years, total amount accumulated is

$$A = P\left(1 + \frac{R}{100}\right)^n$$

$$= 5000\left(1 + \frac{\left(5\frac{1}{4}\right)}{100}\right)^3$$

$= \text{PKR } 5829.57$ (to the nearest paisa)

14. Amount of interest from end March to June

$$= \frac{3}{100} \times \frac{3}{12} \times \text{PKR } 6000$$

$= \text{PKR } 45$

Amount of interest from end June to December

$$= \frac{3}{100} \times \frac{6}{12} \times \text{PKR } (6000 + 4000)$$

$= \text{PKR } 150$

Total amount in the bank at the end of the year

$= \text{PKR } 6000 + \text{PKR } 45 + \text{PKR } 4000 + \text{PKR } 150$

$= \text{PKR } 10195$

15. Initially, at 3.5% interest rate, interest received

$$= \text{PKR } 6400 \times \frac{3.5}{100} \times \frac{1}{2}$$

$= \text{PKR } 112$

At new 4% interest rate, interest received

$$= \text{PKR } 6400 \times \frac{4}{100} \times \frac{1}{2}$$

$= \text{PKR } 128$

Difference in amount of interest $= \text{PKR } 128 - \text{PKR } 112$

$= \text{PKR } 16$

16. Interest received during first 2 years

$$= \text{PKR } 400\,000 \times \frac{7\frac{1}{4}}{100} \times 2$$

$= \text{PKR } 58\,000$

Interest received during next 5 years

$$= \text{PKR } 400\,000 \times \frac{7.6}{100} \times 5$$

$= \text{PKR } 152\,000$

Total amount at the end of 7 years

$= \text{PKR } 400\,000 + \text{PKR } 58\,000 + \text{PKR } 152\,000$

$= \text{PKR } 610\,000$

17. Let the sum of money deposited by Daniyal be PKR x.

$$\frac{3\frac{3}{4}}{100}x - \frac{3\frac{1}{2}}{100}x = 50$$

$$\frac{\frac{1}{4}}{100}x = 50$$

$$x = 20\,000$$

$\therefore$ The sum of money Daniyal deposits is PKR 20 000.

18. Interest received at 2.75% interest rate

$$= \frac{2.75}{100} \times \text{PKR } 20\,000$$

$= \text{PKR } 550$

Interest received at x% interest rate $= \text{PKR } 550 - \text{PKR } 50$

$= \text{PKR } 500$

New simple interest, $x\% = \frac{\$500}{\$20\,000} \times 100\%$

$= 2.5\%$

$\therefore x = 2.5$

19. **(a)** Since interest is calculated monthly,

P = PKR 150 000, $R = \dfrac{5.68}{12}$, $n = 6 \times 12 = 72$

At the end of 6 years, total amount accumulated is

$$A = P\left(1+\frac{R}{100}\right)^n$$

$$= 150\ 000\left(1+\frac{\left(\frac{5.68}{12}\right)}{100}\right)^{72}$$

= PKR 21 0741.3 (to the nearest paisa)

(b) Since interest is calculated half-yearly,

P = PKR 15 000, $R = \dfrac{5.68}{2} = 2.84$, $n = 6 \times 2 = 12$

At the end of 6 years, total amount accumulated is

$$A = P\left(1+\frac{R}{100}\right)^n$$

$$= 150\ 000\left(1+\frac{2.84}{100}\right)^{12}$$

= PKR 20 991.14 (to the nearest paisa)

20. Since interest is calculated yearly,

P = PKR 5000, A = PKR 5800, $n = 5$

At the end of 5 years, total amount accumulated is

$$A = P\left(1+\frac{R}{100}\right)^n$$

$$\text{PKR } 5800 = \text{PKR } 5000\left(1+\frac{R}{100}\right)^5$$

$$\frac{5800}{5000} = \left(1+\frac{R}{100}\right)^5$$

$$1 + \frac{R}{100} = \sqrt[5]{1.16}$$

$$\frac{R}{100} = \sqrt[5]{1.16} - 1$$

$$R = 100(\sqrt[5]{1.16} - 1)$$

$= 3.01$ (to 3 s.f.)

$\therefore$ The interest rate is 3.01%.

21. Since interest is calculated quarterly,

P = PKR 96.60, $R = \dfrac{4.2}{4} = 1.05$, $n = 1 \times 4 = 4$

At the end of the first years, total amount accumulated is

$$A = P\left(1+\frac{R}{100}\right)^n$$

$$P + \text{PKR } 96.60 = P\left(1+\frac{1.05}{100}\right)^4$$

$$P + \text{PKR } 96.60 = 1.0105^4P$$

$$(1.0105^4 - 1)P = \text{PKR } 96.60$$

$$P = \text{PKR } \frac{96.60}{1.0105^4 - 1}$$

= PKR 2264.09 (to the nearest paisa)

22. Since interest is calculated monthly,

P = PKR 800, $R = \dfrac{12\frac{1}{2}}{12} = \dfrac{25}{24}$, $n = 12$

At the end of 1 year, total amount accumulated is

$$A = P\left(1+\frac{R}{100}\right)^n$$

$$= 800\left(1+\frac{\left(\frac{25}{24}\right)}{100}\right)^2$$

= PKR 905.93 (to the nearest paisa)

Total interest, $I = A - P$

= PKR 905.93 – PKR 800

= PKR 105.93

23. Since interest is calculated daily,

P = PKR 90 000, $R = \dfrac{2}{365}$, $n = 3$

At the end of 3 days, total amount accumulated is

$$A = P\left(1+\frac{R}{100}\right)^n$$

$$= 90\ 000\left(1+\frac{\left(\frac{2}{365}\right)}{100}\right)^3$$

= PKR 90 014.8 (to the nearest paisa)

24. **(i)** Kiran should invest in Company B since the interest earned is higher.

(ii) For Company A,

$$I = \frac{PRT}{100}$$

$$= \frac{8000 \times 4.9 \times 4}{100}$$

= PKR 1568

For Company B,

Since interest is calculated half-yearly,

P = PKR 8000, $R = \dfrac{4.8}{2} = 2.4$, $n = 4 \times 2 = 8$

At the end of 6 years, total amount accumulated is

$$A = P\left(1+\frac{R}{100}\right)^n$$

$$= 8000\left(1+\frac{2.4}{100}\right)^8$$

= PKR 9671.41 (to the nearest cent)

Total interest, $I = A - P$

= PKR 9671.41 – PKR 8000

= PKR 1671.41

Difference in interest earned = PKR 1671.41 – PKR 1568

= PKR 103.41

Exercise 2B

1. **(a)** **(i)** Amount paid by hire purchase

= PKR (400 × 10) + PKR 500

= PKR 4500

Additional amount paid = PKR 4500 – PKR 3600

= PKR 900

(ii) Percentage of cash price = $\frac{\text{PKR } 900}{\text{PKR } 3600} \times 100\%$

= 25%

(b) **(i)** Amount paid by hire purchase

= PKR (7500 × 12) + PKR 15 000

= PKR 105 000

Additional amount paid = PKR 105 000 – PKR 90 000

= PKR 15 000

(ii) Percentage of cash price = $\frac{\text{PKR } 15\,000}{\text{PKR } 90\,000} \times 100\%$

= $16\frac{2}{3}$ %

(c) **(i)** Amount paid by hire purchase

= PKR (500 × 36) + PKR 10 000

= PKR 28 000

Additional amount paid = PKR 28 000 – PKR 25 000

= PKR 3000

Percentage of cash price = $\frac{\$3000}{\$25\,000} \times 100\%$

= 12%

2. **(a)** **(i)** Amount paid by hire purchase

= PKR (900 × 24) + PKR $\left(\frac{10}{100} \times 20\,000\right)$

= PKR 21 600 + PKR 2000

= PKR 23 600

(ii) Additional amount = PKR 23 600 – PKR 20000

= PKR 3600

Percentage saved by paying cash

= $\frac{3600}{20\,000} \times 100\%$

= 18%

(b) **(i)** Amount paid by hire purchase

= PKR (1800 × 20) + PKR $\left(\frac{15}{100} \times 35\,000\right)$

= PKR 36 000 + PKR 5250

= PKR 41250

(ii) Additional amount = PKR 41250 – PKR 35 000

= PKR 6250

Percentage saved by paying cash

= $\frac{6250}{35\,000} \times 100\%$

= 17.9% (to 3 s.f.)

(c) **(i)** Amount paid by hire purchase

= PKR (5200 × 30) + PKR $\left(\frac{25}{100} \times 160\,000\right)$

= PKR 156 000 + PKR 40 000

= PKR 196 000

(ii) Additional amount = PKR 196 000 – PKR 160 000

= PKR 36 000

Percentage saved by paying cash

= $\frac{36\,000}{160\,000} \times 100\%$

= 22.5%

3. **(a)** **(i)** Downpayment = PKR 1000

Remaining amount = PKR 8000 – PKR 1000

= PKR 7000

Amount of interest owed at the end of 1 year

= PKR 7000 × $\frac{8}{100}$

= PKR 560

Total amount to be paid in monthly instalments

= PKR 7000 + PKR 560

= PKR 7560

Monthly instalment = $\frac{\text{PKR } 7560}{12}$

= PKR 630

(ii) Difference as percentage of cash price

= $\frac{560}{8000} \times 100\%$

= 7%

(b) **(i)** Downpayment = PKR 32 000

Remaining amount = PKR 80 000 – PKR 32 000

= PKR 48 000

Amount of interest owed at the end of 1 year

= PKR 48 000 × $\frac{10}{100}$

= PKR 4800

Amount of interest owed at the end of 2 years

= PKR 4800 × $2\frac{1}{2}$

= PKR 12 000

Total amount to be paid in monthly instalments

= PKR 48 000 + PKR 12 000

= PKR 60 000

Monthly instalment = PKR $\frac{60\,000}{24}$

= PKR 2500

(ii) Difference as percentage of cash price

= $\frac{12\,000}{80\,000} \times 100\%$

= 15%

(c) **(i)** Downpayment = PKR 2000

Remaining amount = PKR 12 000 – PKR 2000

= PKR 10 000

Amount of interest owed at the end of 1 year

$= \text{PKR } 10\,000 \times \frac{15}{100}$

= PKR 1500

Amount of interest owed at the end of 1 years

= PKR 1500 × 1

= PKR 1500

Total amount to be paid in monthly instalments

= PKR 10 000 + PKR 2000

= PKR 12 000

Monthly instalment $= \text{PKR } \frac{\text{PKR } 12\,000}{12}$

= PKR 1000

(ii) Difference as percentage of cash price

$= \frac{2000}{12\,000} \times 100\%$

$= 16\frac{2}{3}\,\%$

4. **(a)** Percentage discount $= \frac{\text{PKR } (21\,980 - 17\,980)}{\text{PKR } 21\,980} \times 100\%$

= 18.2% (to 1 d.p.)

(b) Hire purchase price = PKR 550 × 38

= PKR 20 900

Difference = PKR 21 980 – PKR 20 900

= PKR 1080

(c) Total amount of interest = PKR 20 900 – PKR 17 980

= PKR 2920

Amount of interest at the end of 1 year

$= \frac{\text{PKR } 2920}{38} \times 12$

= PKR 922.10 (to the nearest paisa)

Let the rate of simple interest be x%.

$\text{PKR } 922.1 = \text{PKR } 17\,980 \times \frac{x}{100}$

$x = \frac{922.1(100)}{17980}$

= 5.13 (to 3 s.f.)

∴ The rate of simple interest charged for hire purchase is 5.13%.

5. **(i)** Downpayment

$= \frac{15}{100} \times \text{PKR } 32\,000$

= PKR 4800

Remaining amount

= PKR 32 000 – PKR 4800

= PKR 27 200

Amount of interest the man owes at the end of 1 year

$= \text{PKR } 27\,200 \times \frac{9.5}{100}$

= PKR 2584.0

Amount of interest the man owes at the end of 2 years

= PKR 2584 × 2

= PKR 5168

Total amount to be paid in monthly instalments

= PKR 27 200 + PKR 5168

= PKR 32 368

Monthly instalment

$= \frac{\text{PKR } 32368}{24}$ (2 years = 24 months)

= PKR 1348.7 (to the nearest paisa)

(ii) Total amount the man pays for the computer system

= PKR 32 368 + PKR 4800

= PKR 37 168

(iii) He has to pay PKR (37 168 – 3200) = PKR 5168 more for buying the computer system on hire purchase.

6. Downpayment $= \frac{25}{100} \times \text{PKR } x$

= PKR $0.25x$

Remaining amount = PKR x – PKR $0.25x$

= PKR $0.75x$

Amount of interest the man owes at the end of 30 months

$= \text{PKR } 0.75x \times \frac{12}{100} \times \frac{30}{12}$

= PKR $0.975x$

Total amount to be paid in monthly instalments

= PKR 520 × 30

= PKR 15 600

Hence $0.975x = 15\,600$

$x = 16\,000$

Exercise 2C

1. \$1 = PKR 263

\$59 = PKR 263 × 59

= PKR 15517

2. Half of the amount PKR 200000

$= \frac{\text{PKR } 200000}{2} \times 100\%$

= PKR 100000

= PKR 263 = US\$1

$= \text{PKR } 1 = \text{US\$ } \frac{1}{263} \times 100000$

= US\$ 380.23

Next day

PKR 262 = US\$ 1

$\text{PKR } 1 = \text{US\$ } \frac{1}{263}$

$= \text{PKR } 100000 = \text{US\$ } \frac{1}{263} \times 100000$

US \$ 3810.68

Total amount in dollers = US\$ (380.23+381.68)

= US\$ 761.91

3. **(a)** PKR 69.80 = 1 SAR

PKR 1 = $\frac{1}{69.80}$ SAR

PKR 250 000 = $\frac{1}{69.80} \times 250\ 000$ SAR

= 3581.66 SAR

(b) 1 SAR = PKR 69.80

3750 SAR = PKR 69.80 × 3750

= PKR 261 750

4. **(a)** £ 1 = PKR 319.50

£ 320 = PKR 319.50 × 320

= PKR 102 240

(b) PKR 319.50 = £ 1

PKR 1 = £ $\frac{1}{319.50}$

PKR 965 600 = £ $\frac{1}{319.50} \times 965\ 000$

= £ 3022.22

5. PKR 194 = S$ 1

PKR 1 = S$ $\frac{1}{194}$

PKR 520000 = S$ $\frac{1}{263} \times 520000$

= S$ 2680.

The couple spend S$ 2350

The remaining Amount = S$ (2680.41 - 2350)

S$1 = PKR 194

S$0.41 = PKR 194 × 0.41

= PKR 79.54

5. PKR 194 = S$ 1

PKR 1 = S$ $\frac{1}{194}$

PKR 520000 = S$ $\frac{1}{263} \times 520000$

= S$ 2680.

The couple spend S$ 2350

The remaining Amount = S$ (2680.41 - 2350)

S$1 = PKR 194

S$0.41 = PKR 194 × 0.41

= PKR 79.54

Exercise 2D

1. Rate of the premium = $\frac{\text{PKR } 13500}{\text{PKR } 450000} \times 100\%$

= 3%

2. Amount of annual premium = 2.8% of PKR 62000

= $\frac{2.8}{100} \times$ PKR 620000

= PKR 17360

3. Insurance premium paid in

1st year:

4% of PKR 1000000

= 5 $\frac{5}{100} \times 1000000$

= PKR 40000

In 2nd year

= 4% of PKR (1000000 - 5% of 1000000)

= 4% of PKR (1000000 - 50000)

= 4% of PKR 950000

= $\frac{4}{100} \times 950000$

= PKR 38000

In 3rd year

= 4% of PKR (950000 - 5% of 950000)

= 4% of PKR (950000 - 47500)

= $\frac{4}{100} \times 902500$

= PKR 36100

4. Annual premium = 6.5% of PKR 750000

= $\frac{6.5}{100} \times 750000$

= PKR 48750

5. Total amount = PKR 178500

Widow's share = $\frac{1}{8}$ of PKR 1785000

= PKR 223125

Amount left = PKR 1785000 – 223125

2 : 1 × 2

2 : 2

1 : 1

Son's share = PKR $\frac{1561875}{2}$

= PKR 780937.5

Share of each daughter = PKR $\frac{780937.5}{2}$

= PKR 390468.75

7. Total profit = 2 × PKR 547350

= PKR 1094700

8. Amount of premium = 3% of insurance amount

PKR 26400 = $\frac{3}{100}$ × insurance amount

= PKR 88000

9. Each son gets = PKR 782500

Mr Zubair's Total savings = 3 × PKR 782500

= PKR 2347500

10. The amount is to be distributed in the ratio

4 : 3 : 3

A will receive amount as follows

$\frac{3}{4+3+3} \times$ PKR 12850000

= PKR 5140000

B will receive

$\frac{3}{10} \times$ PKR 12850000

= PKR 3855000

C Will receive PKR 3855000.

11. Brother 1 received

$\frac{5}{8} \times$ PKR 413700

= PKR 258562.5

Brother 2 received

PKR 413700 - PKR 258562.5

= PKR 155137.5

12. $A : B : C = 29680 : 37100$

PKR (29680 + 44520 + 37100) = PKR 111300

A invested $\frac{29680}{111300} \times 750000$ = PKR 202500

B invested $\frac{44520}{111300} \times 750000$ = PKR 300000

C invested $\frac{37100}{111300} \times 750000$ = PKR 250000

Review Exercise 2

1. Amount of interest earned for PKR 6000 at the end of 2011

$= \frac{3}{100} \times$ PKR 6000

= PKR 180

Amount of interest earned for PKR 6400 at the end of 2012

$= \frac{3}{100} \times$ PKR 6400

= PKR 192

Total amount Khairul has in the bank at the end of 2013

= PKR 6400 + PKR 180 + PKR 192 + PKR 192

= PKR 6964

2. **(a)** Since interest is calculated monthly,

P = PKR 1 500 000, $R = \frac{4.12}{12}$, $n = 3 \times 12 = 36$

At the end of 3 years, total amount accumulated is

$$A = P\left(1+\frac{R}{100}\right)^n$$

$$= 1\ 500\ 000\left(1+\frac{\left(\frac{4.12}{2}\right)}{100}\right)^{36}$$

= PKR 3 125 298.93 (to the nearest paisa)

(b) Since interest is calculated half-yearly,

P = PKR 1 500 000, $R = \frac{4.12}{2} = 2.06$, $n = 3 \times 2 = 6$

At the end of 3 years, total amount accumulated is

$$A = P\left(1+\frac{R}{100}\right)^n$$

$$= 1\ 500\ 000\left(1+\frac{2.06}{100}\right)^6$$

= PKR 1 695 214.44 (to the nearest paisa)

3. Since interest is calculated half-yearly,

l = PKR 58 000, $R = \frac{4}{2} = 2$, $n = 3 \times 2 = 6$

At the end of the first year, total amount accumulated is

$$A = P\left(1+\frac{R}{100}\right)^n$$

P + PKR 58 000 $= P\left(1+\frac{2}{100}\right)^6$

P + PKR 58 000 = 1.02^6P

$(1.02^6 - 1)P$ = PKR 58 000

P = PKR $\frac{58\ 000}{1.02^6 - 1}$

= PKR 459 724 (to the nearest rupee)

4. **(i)** Downpayment

$= \frac{15}{100} \times$ PKR 4500

= PKR 675

Remaining amount

= PKR 4500 – PKR 675

= PKR 3825

Amount of interest owed at the end of 1 year

= PKR 3825 $\times \frac{12}{100}$

= PKR 459

Amount of interest owed at the end of 18 months

= PKR 459 $\times \frac{18}{12}$

= PKR 688.5

Total amount to be paid in monthly instalments

= PKR 3825 + PKR 688.5

= PKR 4513.5

Monthly instalment

$= \frac{\text{PKR } 4513.5}{18}$

= PKR 250.75

(ii) Total amount the man pays for the printer

= PKR 4500 + PKR 688.5

= PKR 5188.5

5. Premium amount = 2% of PKR 12000000

$= \frac{2}{100} \times$ PKR 12000000

= PKR 240000

6. Annual premium $= \frac{\text{rate of premium}}{100} \times$ life insurance

PKR 17600 $= \frac{\text{rate\%}}{100} \times$ PKR 550000

Rate $= \frac{17600 \times 100}{550000}$

Rate = 3.2%

7. Rate $= \frac{29700 \times 100}{900000}$ %

Rate = 3.3%

8. Rate = $\frac{24.750}{450000} \times 100\%$

Rate = 5.5%

9. A man left PKR 250000

His wife's share = $\frac{1}{8} \times$ PKR 250000

= PKR 31250

His son's and = 2 daughter's share

2 : 1 × 2

2 : 2

1 : 1

His son's share = PKR $\frac{250000 - 31250}{2}$

= PKR $\frac{218750}{2}$ = PKR 109375

Each daughter's share = PKR $\frac{109375}{2}$

= PKR 54687.5

10. 2 sons : 1 daughter

2 × 2 : 1

4 : 1

2 son's share = $\frac{4}{5} \times$ PKR 30000

= PKR 24000

Each son will get $\frac{\text{PKR } 24000}{2}$ = PKR 12000

Daughter's share = $\frac{1}{5} \times$ PKR 30000

= PKR 6000

11. Aslam receives $\frac{5}{12} \times$ PKR 250 000

= PKR 104 166.66

= Pervaiz receive $\frac{4}{12} \times$ PKR 50 000

= PKR 83 333.33

= Sana receive PKR 250000 – PKR 104166.66 – PKR 83333.33

= PKR 62 500

12. Since 5 partners invested equal amount of money hence, each will get equal amount from the annual profit.

Each will get $\frac{\text{PKR}250000}{5}$ = PKR 50000

13. Sibling 1 will receive $\frac{3}{3+4+5} \times$ PKR 144000

= $\frac{3}{12} \times$ PKR 144000

= PKR 48000

Sibling 3 will get $\frac{5}{12} \times$ PKR 144000

= PKR 60000

Challenge Yourself

(i) GST paid by Faiz = $\frac{7}{100} \times$ PKR 50 000

= PKR 8500

(ii) GST paid by Jamil = $\frac{7}{107} \times$ PKR 50 000

= PKR 7264.95

(iii) GST on PKR 50 000 is PKR 8500 which is the same answer as in **(i)**. The shopkeeper is not complaining about it because he rather pays a GST of PKR 8500 than a GST of PKR 7264.95 to the government.

(iv) The amount paid by each customer at Shops *B* and *C* is PKR 8500. As far as the government is concerned, this amount must be inclusive of GST. Another way of looking at this is to ask how the government can keep track of the shops which absorb GST and charge them a different GST amount. All the shops will tell the government that the final transacted amount is inclusive of GST because they can pay a *lower* amount for GST, so this agrees with why the final transacted amount is inclusive of GST regardless of whether the shops charge or absorb GST.

(v) Yes, it makes a difference. The difference is the *original* selling price of the TV *before* the government announces that they will charge GST. Shop *C* has been selling the TV for PKR 50 000 and decides to absorb GST after the announcement, so it still sells the TV for PKR 50 000 (inclusive of GST). If Shop *C* decides not to absorb GST, they will sell the TV for PKR 50 000 (before GST) or PKR 58 500(inclusive of GST), just like what Shop *A* does. Since Shop *B* has been selling the TV for *about* PKR 42 735.29 and decides to charge GST after the announcement, it sells the TV for PKR 50 000 (inclusive of GST) *now*.

Chapter 3 Further Expansion and Factorisation of Algebraic Expressions

TEACHING NOTES

Suggested Approach

Students have done word problems involving number sequences and patterns in previous classes. These word problems required the students to recognise simple patterns from various number sequences and determine either the next few terms or a specific term.

The general form of a quadratic expression in one variable is $ax^2 + bx + c$, where x is the variable and a, b and c are given numbers. In the expression, c is known as the constant term as it does not involve the variable x. When we expand the product of two linear expressions in x, we obtain a quadratic expression in x.

Factorisation is the reverse of expansion. When we expand the product of two linear expressions, we obtain a quadratic expression. By reversing the process, we factorise the quadratic expression into a product of two linear factors.

Teachers can use the Concrete-Pictorial-Approach using the algebra discs to illustrate the process of expansion and factorisation of quadratic expressions. However, the emphasis should be for the students to use a Multiplication Frame when factorising any quadratic expressions.

Section 3.1: General Term of a Number Sequence

Teachers can explain students how to observe a number sequence and look for a pattern so that they can use algebra and find a formula for the general term, $T_n = n^{\text{th}}$ term.

Teachers can get students to work in pairs to find a formula for the general term and hence find a specific term for different number sequences (see Class Discussion: Generalising Simple Sequences). After the students have learnt how to generalise simple sequences, they should know that the aim is not to simply solve the problem but to represent it so that it becomes a general expression which can be used to find specific terms.

Section 3.2: Number Patterns

Teachers can get students to work in pairs to find a formula for the general term and hence find a specific term for different number patterns. They need only to find the formula for the general term and they are able to find n by substituting the value into the formula. They should also learn that with the formula, they can find T_n easily for any n.

Section 3.3: Number Patterns in Real-World Contexts

Teachers may get students to discover number patterns in real-world contexts (e.g. shells, pine cones, rocks, wallpaper, floor tiles) and ask them to represent that number pattern into a general expression.

Through Worked Example 3, students will learn that in the real world, which in this case in Chemistry, the general term of a number sequence is important and advantageous in finding specific terms. In this worked example, finding the general term of the number of hydrogen atoms allowed one to find the member number, number of carbon atom(s) and number of hydrogen atoms easily without going through tedious workings, especially if the value of the specific term is large. For other figures, students should consider drawing the next figure in the sequence so as to identify the pattern.

Section 3.5: Expansion and Factorisation of Algebraic Expressions

In this section, students should have ample practice to expand and factorise slightly more difficult and complicated algebraic expressions. The focus for expansion of algebraic expressions should be on applying the Distributive Law while for factorisation of algebraic expressions, students should be using the Multiplication Frame.

Section 3.6: Expansion Using Special Algebraic Identities

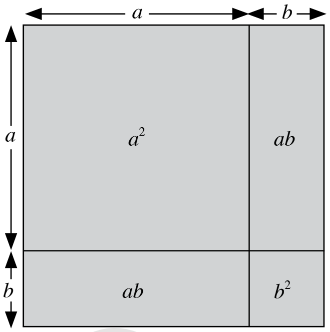
The area of squares and rectangles can be used to show the expansion of the three special algebraic identities. Teachers can also guide students to complete the Class Discussion on page 86 (see Class Discussion: Special Algebraic Identities).

From the Class Discussion activity, students should conclude that these algebraic identities known as **perfect squares**, $(a + b)^2$ and $(a - b)^2$ and the **difference of two squares** $(a + b)(a - b)$, are useful for expanding algebraic expressions.

Special Algebraic Identity 1

Expand $(a + b)^2$.

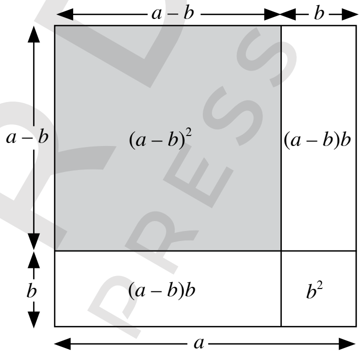
Area of square $= (a + b)^2$
$= a^2 + ab + ab + b^2$
$= a^2 + 2ab + b^2$



Special Algebraic Identity 2

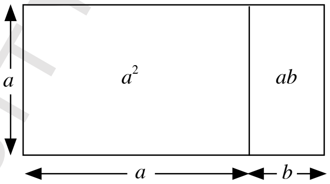
Expand $(a - b)^2$.

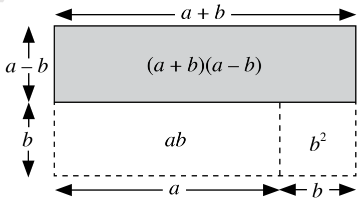
Area of small square
$= (a - b)^2$
$= a^2 - b^2 - 2(a - b)b$
$= a^2 - b^2 - 2ab + 2b^2$
$= a^2 - 2ab + b^2$



Special Algebraic Identity 3

Area of rectangle
$= (a + b)(a - b)$
$= (a^2 - ab) + (ab - b^2)$
$= a^2 - ab + ab - b^2$
$= a^2 - b^2$





As an additional activity, teachers may want to ask students the following:

Is $(a + b)^2 = a^2 + b^2$ and $(a - b)^2 = a^2 - b^2$? Explain your answer.

Below are some common misconceptions regarding expansion that teachers may want to remind students of.

- $(x + 2)^2 = x^2 + 4$ instead of $(x + 2)^2 = x^2 + 4x + 4$
- $(2x - 1)^2 = 4x^2 - 1$ instead of $(2x - 1)^2 = 4x^2 - 4x + 1$

Section 3.7 Factorisation Using Special Algebraic Identities

Since factorisation is the reverse of expansion, when we factorise the quadratic expression using the special algebraic identities, we have

- $a^2 + 2ab + b^2 = (a + b)^2$
- $a^2 - 2ab + b^2 = (a - b)^2$
- $a^2 - b^2 = (a + b)(a - b)$

Teachers should provide ample practice for students to check if the given quadratic expression can be factorised using the special algebraic identities. Get students to learn to identify the '*a*' and '*b*' terms in any given expression.

Section 3.8: Factorisation by Grouping

Students have learnt how to factorise algebraic expressions of the form $ax + ay$ by identifying the common factors (either common numbers or common variables of the terms).

To factorise algebraic expressions of the form $ax + bx + kay + kby$, it may be necessary to regroup the terms of the algebraic expression before being able to identify the common factors. The idea is to identify the common factor(s) in the first two terms and another common factor(s).

For example, to factorise by grouping, we have

$ax + bx + kay + kby$
$= x(a + b) + ky(a + b)$
$= (a + b)(x + ky)$

WORKED SOLUTIONS

Class Discussion (Generalising Simple Sequences)

(a) Hence, $T_n = 3n$.

100th term, $T_{100} = 3 \times 100$

$= 300$

(b) Hence, $T_n = n^2$.

100th term, $T_{100} = 100^2$

$= 10\ 000$

(c) Hence, $T_n = n^3$.

100th term, $T_{100} = 100^3$

$= 1\ 000\ 000$

Investigation (Page 69)

1. Michaelmas Daisy has 55 petals.
2. 4, 6; 7, 10

Thinking Time (Page 82)

$$\begin{aligned}(a + b)(c + d + e) &= a(c + d + e) + b(c + d + e)\\ &= ac + ad + ae + bc + bd + be\end{aligned}$$

Class Discussion (Special Algebraic Identities)

1.
$$\begin{aligned}(a + b)^2 &= (a + b)(a + b)\\ &= a(a + b) + b(a + b)\\ &= a^2 + ab + ab + b^2\\ &= a^2 + 2ab + b^2\end{aligned}$$

2.
$$\begin{aligned}(a - b)^2 &= (a - b)(a - b)\\ &= a(a - b) - b(a - b)\\ &= a^2 - ab - ab + b^2\\ &= a^2 - 2ab + b^2\end{aligned}$$

3.
$$\begin{aligned}(a + b)(a - b) &= a(a - b) + b(a - b)\\ &= a^2 - ab + ab - b^2\\ &= a^2 - b^2\end{aligned}$$

Thinking Time (Page 102)

$$\begin{aligned}5x^2 - 12x - 9 &= 5x^2 - 15x + 3x - 9\\ &= 5x(x - 3) + 3(x - 3)\\ &= (5x + 3)(x - 3)\end{aligned}$$

Class Discussion (Equivalent Expressions)

$A = (x - y)^2 = (x - y)(x - y) = I$

$A = (x - y)^2 = x^2 - 2xy + y^2 = M$

$B = (x + y)(x + y) = (x + y)^2 = G$

$B = (x + y)(x + y) = x^2 + 2xy + y^2 = O$

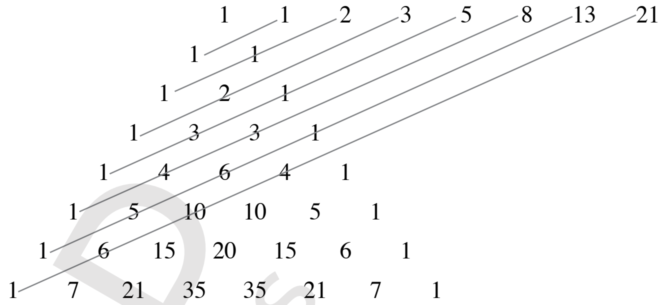
$D = (2w - x)(z - 3y) = 2wz - 6wy + 3xy - xz = F$

$E = -5x^2 + 28x - 24 = 2x - (x - 4)(5x - 6) = L$

$J = x^2 - y^2 = (x + y)(x - y) = K$

Journal Writing (Page 169)

Pascal's Triangle was developed by the French Mathematician Blaise Pascal. It is formed by starting with the number 1. Each number in the subsequent rows is obtained by finding the sum of the number which is diagonally above it to the left and that which is diagonally above it to the right. 0 is used as a substitute in the absence of a number in either of the two positions.



The Fibonacci sequence is a set of numbers that begins with 1 and 1, and each subsequent term is the sum of the previous two terms, i.e. 1, 1, 2, 3, 5, 8, 13, 21, … The sums of the numbers on the diagonals of Pascal's Triangle form the Fibonacci sequence, as illustrated.

Teachers may wish to get students to describe the symmetry in Pascal's Triangle and to identify other patterns that can be observed from the triangle.

Practise Now 1

1. (a) Since the common difference is 4, $T_n = 4n + ?$.

The term before T_1 is $c = T_0$

$= 5 - 4$

$= 1.$

$\therefore$ General term of sequence, $T_n = 4n + 1$

(b) Since the common difference is 5, $T_n = 5n + ?$.

The term before T_1 is $c = T_0$

$= 7 - 5$

$= 2.$

$\therefore$ General term of sequence, $T_n = 5n + 2$

(c) Since the common difference is 6, $T_n = 6n + ?$.

The term before T_1 is $c = T_0$

$= 2 - 6$

$= -4.$

$\therefore$ General term of sequence, $T_n = 6n - 4$

(d) Since the common difference is 3, $T_n = 3n + ?$.

The term before T_1 is $c = T_0$

$= 1 - 3$

$= -2.$

$\therefore$ General term of sequence, $T_n = 3n - 2$

2. (i) 23, 27

(ii) Since the common difference is 4, $T_n = 4n + ?$.

The term before T_1 is $c = T_0$

$= 3 - 4$

$= -1.$

$\therefore$ General term of sequence, $T_n = 4n - 1$

(iii) $T_{50} = 4(50) - 1$
$= 200 - 1$
$= 199$

Practise Now 2

1. **(i)**

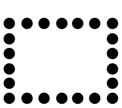


Figure 5

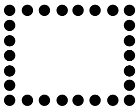


Figure 6

(ii)

Figure Number	Number of Dots
1	$2 + 1 \times 4 = 6$
2	$2 + 2 \times 4 = 10$
3	$2 + 3 \times 4 = 14$
4	$2 + 4 \times 4 = 18$
5	$2 + 5 \times 4 = 22$
6	$2 + 6 \times 4 = 26$
⋮	⋮
n	$2 + n \times 4 = 4n + 2$

(iii) When $n = 2013$,
$4n + 2 = 4(2013) + 2$
$= 8054$
Number of dots in 2013th figure = 8054

2. **(i)** 8th line: $72 = 8 \times 9$
(ii) Since $110 = 10 \times 11 = 10(10 + 1)$,
$k = 10$.

Practise Now 3

(i)

Member Number	Number of carbon atoms	Number of hydrogen atoms
1	2	4
2	3	6
3	4	8
4	5	10
5	6	12
6	7	14
⋮	⋮	⋮
n	$n + 1$	$2n + 2$

(ii) Let $h + 1 = 55$.
$h = 55 - 1$
$= 54$
When $n = h = 54$,
$2n + 2 = 2(54) + 2$
$= 110$
Number of hydrogen atoms the member has = 110

(iii) Let $2k + 2 = 120$.
$2k = 120 - 2$
$= 118$
$k = 59$
When $n = k = 59$,
$n + 1 = 59 + 1$
$= 60$
Number of carbon atoms the member has = 60

Practise Now 4

1. **(a)**

$$\begin{array}{r}
3x^2 - x - 7 \\
5x\overline{)15x^3 - 5x^2 - 35x} \\
15x^3 \phantom{{}- 5x^2 - 35x} \\
\hline
-5x^2 \phantom{{}- 35x} \\
-5x^2 \phantom{{}- 35x} \\
+ \\
\hline
-35x \\
-35x \\
+
\end{array}$$

∴ The quotient is $3x^2 - x - 7$ and remainder is o.

(b)

$$\begin{array}{r}
2x^2 + x - 1 \\
3x+2\overline{)6x^3 + 7x^2 - 2} \\
6x^3 + 4x^2 \phantom{{}- 2} \\
- \quad - \phantom{{}- 2} \\
\hline
3x^2 - x \\
6x^2 + 2x \\
- \quad - \\
\hline
-3x - 2 \\
-3x - 2 \\
+ \quad +
\end{array}$$

∴ The quotient is $2x^2 + x - 1$ and remainder is o.

Practise Now 5

1. **(a)** $5x \times 6y = (5 \times x) \times (6 \times y)$
$= (5 \times 6) \times x \times y$
$= 30xy$

(b) $(-8x) \times 2y = (-8 \times x) \times (2 \times y)$
$= (-8 \times 2) \times x \times y$
$= -16xy$

(c) $x^2yz \times y^2z = (x \times x \times y \times z) \times (y \times y \times z)$
$= (x \times x) \times (y \times y \times y) \times (z \times z)$
$= x^2y^3z^2$

(d) $(-xy) \times (-11x^3y^2)$
$= (-1 \times x \times y) \times (-11 \times x \times x \times x \times y \times y)$
$= [-1 \times (-11)] \times (x \times x \times x \times x) \times (y \times y \times y)$
$= 11x^4y^3$

2. $$\frac{1}{2}a \times \left(-\frac{8}{3}b\right) = \left(\frac{1}{2} \times a\right) \times \left(-\frac{8}{3} \times b\right)$$
$$= \left[\frac{1}{2} \times \left(-\frac{8}{3}\right)\right] \times a \times b$$
$$= -\frac{4}{3}ab$$

Practise Now 6

(a) $-y(5 - 2x) = -5y + 2xy$

(b) $2x(7x + 3y) = 14x^2 + 6xy$

Practise Now 7

(a) $4x(3y - 5z) - 5x(2y - 3z) = 12xy - 20xz - 10xy + 15xz$
$= 12xy - 10xy - 20xz + 15xz$
$= 2xy - 5xz$

(b) $x(2x - y) + 3x(y - 3x) = 2x^2 - xy + 3xy - 9x^2$
$= 2x^2 - 9x^2 - xy + 3xy$
$= -7x^2 + 2xy$

Practise Now 8

(a) $(x + 9y)(2x - y) = x(2x - y) + 9y(2x - y)$
$= 2x^2 - xy + 18xy -9y^2$
$= 2x^2 + 17xy - 9y^2$

(b) $(x^2 - 3)(6x + 7) = x^2(6x + 7) - 3(6x + 7)$
$= 6x^3 + 7x^2 - 18x - 21$

Practise Now 9

$2x(3x - 4y) - (x - y)(x + 3y)$
$= 6x^2 - 8xy - [x(x + 3y) - y(x + 3y)]$
$= 6x^2 - 8xy - (x^2 + 3xy - xy - 3y^2)$
$= 6x^2 - 8xy - (x^2 + 2xy - 3y^2)$
$= 6x^2 - 8xy - x^2 - 2xy + 3y^2$
$= 6x^2 - x^2 - 8xy - 2xy + 3y^2$
$= 5x^2 - 10xy + 3y^2$

Practise Now 10

(a) $(x - 5y)(x + 4y - 1)$
$= x(x + 4y - 1) - 5y(x + 4y - 1)$
$= x^2 + 4xy - x - 5xy - 20y^2 + 5y$
$= x^2 + 4xy - 5xy - x - 20y^2 + 5y$
$= x^2 - xy - x - 20y^2 + 5y$

(b) $(x + 3)(x^2 - 7x - 2)$
$= x(x^2 - 7x -2) + 3(x^2- 7x - 2)$
$= x^3 - 7x^2 - 2x + 3x^2 - 21x - 6$
$= x^3 - 7x^2+ 3x^2- 2x - 21x - 6$
$= x^3 - 4x^2 - 23x - 6$

Practise Now 11

1. **(a)** $x^2 = x \times x$
$-15y^2 = y \times (-15y)$ or $(-y) \times 15y$
$= 3y \times (-5y)$ or $(-3y) \times 5y$

$\times$	x	$-5y$
x	x^2	$-5xy$
$3y$	$3xy$	$-15y^2$

$3xy + (-5xy) = -2xy$
$\therefore\ x^2 - 2xy -15y^2 = (x + 3y)(x - 5y)$

(b) $6x^2 = 6x \times x$ or $3x \times 2x$
$5y^2 = y \times 5y$ or $(-y) \times (-5y)$

$\times$	x	y
$6x$	$6x^2$	$6xy$
$5y$	$5xy$	$5y^2$

$5xy + 6xy = 11xy$
$\therefore\ 6x^2 + 11xy + 5y^2 = (6x + 5y)(x + y)$

2. $3x^2y^2 = 3xy \times xy$
$16 = 1 \times 16$ or $(-1) \times (-16)$
$= 2 \times 8$ or $(-2) \times (-8)$
$= 4 \times 4$ or $(-4) \times (-4)$

$\times$	xy	-2
$3xy$	$3x^2y^2$	$-6xy$
-8	$-8xy$	16

$(-8xy) + (-6xy) = -14xy$
$\therefore\ 3x^2y^2 - 14xy + 16 = (3xy - 8)(xy - 2)$

Practise Now 12

1. **(a)** $(x + 2)^2 = x^2 + 2(x)(2) + 2^2$
$= x^2 + 4x + 4$

(b) $(5x + 3)^2 = (5x)^2 + 2(5x)(3) + 3^2$
$= 25x^2 + 30x + 9$

2. $\left(\frac{1}{2}x + 3\right)^2 = \left(\frac{1}{2}x\right)^2 + 2\left(\frac{1}{2}x\right)(3) + 3^2$
$= \frac{1}{4}x^2 + 3x + 9$

Practise Now 13

1. **(a)** $(1 - 3x)^2 = 1^2 - 2(1)(3x) + (3x)^2$
$= 1 - 6x + 9x^2$

(b) $(2x - 3y)^2 = (2x)^2 - 2(2x)(3y) + (3y)^2$
$= 4x^2 - 12xy + 9y^2$

2. $\left(x - \frac{1}{3}y\right)^2 = x^2 - 2(x)\left(\frac{1}{3}y\right) + \left(\frac{1}{3}y\right)^2$
$= x^2 - \frac{2}{3}xy + \frac{1}{9}y^2$

Practise Now 14

1. **(a)** $(5x + 8)(5x - 8) = (5x)^2 - 8^2$
$= 25x^2 - 64$

(b) $(-2x + 7y)(-2x - 7y) = (-2x)^2 - (7y)^2$
$= 4x^2 - 49y^2$

2. $\left(\frac{x}{4} + y\right)\left(\frac{x}{4} - y\right) = \left(\frac{x}{4}\right)^2 - y^2$
$= \frac{1}{16}x^2 - y$

Practise Now 15

1. **(a)** $1001^2 = (1000 + 1)^2$
$= 1000^2 + 2(1000)(1) + 1^2$
$= 1\,000\,000 + 2000 + 1$
$= 1\,002\,001$

(b) $797^2 = (800 – 3)^2$
$= 800^2 – 2(800)(3) + 3^2$
$= 640\,000 – 4800 + 9$
$= 635\,209$

(c) $305 \times 295 = (305 + 5)(300 – 5)$
$= 300^2 – 5^2$
$= 90\,000 – 25$
$= 89\,975$

Practise Now 16

$(x – y)^2 = 441$
$x^2 – 2xy + y^2 = 441$
Since $xy = 46$,
$\therefore\ x^2 – 2(46) + y^2 = 441$
$x^2 – 92 + y^2 = 441$
$\therefore\ x^2 + y^2 = 533$

Practise Now 17

(a) $(x + 3)$
$= (x)^3 + 3 \times (x)^2 \times 3 + 3 \times (x) \times (3)^2 + (3)^3$
$= x^3 + 3 \times x^2 \times 3 + 3 \times x \times 9 + 27$
$= x^3 + 9x^2 + 27x + 27$

(b) $(x^2 + 2y)^3$
$= (x^2)^3 + 3 \times (x^2)^2 \times 2y + 3 \times x^2 \times (2y)^2 + (2y)^3$
$= x^6 + 3 \times x^4 \times 2y + 3 \times x^2 \times 4y^2 + 8y^3$
$= x^6 + 6x^2y + 12x^2y^2 + 8y^3$

Practise Now 18

$(105)^3$
$(105)^3 = (100 + 5)^3$
$= (100)^3 + 3 \times (100)^2 \times 5 + 3 \times 100 \times (5)^2 + (5)^3$
$= 1000000 + 3 \times 10000 \times 5 + 7500 + 125$
$= 1000000 + 150000 + 7500 + 125$
$= 1157625$

Practise Now 19

$= 8x^3 + 36x^2y + 54xy^2 + 27y^3$
$= (2x)^3 + 3 \times (2x)^2 \times 3y + 3 \times 2x \times (3y)^2 + (3y)^3$
$= (2x + 3y)^3$
$= (2 \times (–1) + 3 \times 2)^3$ [when $x = –1$ and $y = 2$]
$= (–2 + 6)^3$
$= 4^3$
$= 64$

Practise Now 20

$= 1.34 \times .34 + 6 \times 1.34 \times 0.66 + 0.66 \times 0.66 \times 00.66$
$= (1.34)^3 + 3 \times 2 \times 1.34 \times 0.66 + (0.66)^3$
$= (1.34)^3 + 3 \times (1.34 + 0.66) + (0.66)^3$
$= (1.34 + 0.66)^3$
$= 2^3$
$= 8$

Practise Now 21

$(3p – q)^3 + (3p + q)^3 + 18p\,(ap^2 – q^2)$
Let $3p – q = a$ and $3p + q = b$
$a + b = 3p – q + 3p + q = 6p$
and $ab = (3p – q)\,(3p + q) = 9p^2 – q^2$
$= (3p – q)^3 + (3p + q)^3 + 3 \times (9p^2 – q^2) \times 6p$ as:
RHS of the equation $= (3p – q)^3 + (3p + q)^3 + 3\,(ap^2 – q^2) \times 6p$
LHS of the equation $= (a + b)^3$
$= (6p)^3$
$= 216p^3$

Practise Now 22

$2a = 1 – b$
Cubing both sides , $(2a)^3 = (1 – b)^3$
$8a^3 = (1)^3 + (1–b)^3 + 3(1)\,(–b)\,(1+(–b))$
$= (1)^3 + (1–b)^3 + 3(1)\,(–b)\,(1–b)$
$= 1 – b^3 – 3b\,(2a)$ [given that $2a = 1–b$]
$= 1 – b^3 –6ab$
$8a^3 + b^3 + 6ab = 1$ proved

Practise Now 23

$g + 2h = 2gh + 1$
Cubing both sides , $(g + 2h)^3 = (2gh + 1)^3$
$(g)^3 + 8h^3 + 6gh\,(g + 2h) = (2gh)^3 + (1)^3\,3 \times 2gh \times 1(2gh + 1)$
$g^3 + 8h^3 + 6gh\,(g + 2h) = 8g^3h^3 + 1 + 6gh(g + 2h)$
[given that $g + 2h = 2g\,gh + 1$]
$g^3 + 8h^3 = 8g^3h^3$+1 Proved

Practise Now 24

$x^6 – y^3 – 3x^2yz$
$= (x^2)^3 – (y)^3 – 3x^2y(x^2 – y)$
$= (x^2 – y)^3$
$= z^3$ [given that $\times 2 –y = z$]

Practise Now 25

$\frac{1}{k^2} – k^2 = –2$
Cubing both sides ,$(\frac{1}{K^2} –k^2)^3 = (–2)^3$
$(\frac{1}{k^2})^3 – (k^2)^3 – 3 \times (\frac{1}{k^2}) \times (k^2)\,(\frac{1}{k^2} – k^2) = – 8$
$\frac{1}{k^6} – k^6 – 3\,(\frac{1}{k^2} – k^2) = – 8$
$\frac{1}{k^6} – k^6 – 3\,(–2) = – 8$ [given that $\frac{1}{k^2} – k^2 = – 2$
$\frac{1}{k^6} – k^6 – 6 = – 8$
$\frac{1}{k^6} – k^6 = – 8 – 6 = – 14$

Practise Now 26

1. **(a)** $x^2 + 12x + 36 = x^2 + 2(x)(6) + 6^2$

$= (x + 6)^2$

(b) $4x^2 + 20x + 25 = (2x)^2 + 2(2x)(5) + 5^2$

$= (2x + 5)^2$

2. $4x^2 + 2x + \frac{1}{4} = (2x)^2 + 2(2x)\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2$

$= \left(2x + \frac{1}{2}\right)^2$

Practise Now 27

1. **(a)** $4 - 36x + 81x^2 = 2^2 - 2(2)(9x) + (9x)^2$

$= (2 - 9x)^2$

(b) $25x^2 - 10xy + y^2 = (5x^2) - 2(5x)(1) + y^2$

$= (5x - y)^2$

2. $36x^2 - 4xy + \frac{1}{9}y^2 = (6x)^2 - 2(6x)\left(\frac{1}{3}y\right) + \left(\frac{1}{3}y\right)^2$

$= \left(6x - \frac{1}{3}y\right)^2$

Practise Now 28

1. **(a)** $36x^2 - 121y^2 = (6x)^2 - (11y)^2$

$= (6x + 11y)(6x - 11y)$

(b) $-4x^2 + 81 = 81 - 4x^2$

$= 9^2 - (2x)^2$

$= (9 + 2x)(9 - 2x)$

2. $4x^2 - \frac{9}{25}y^2 = (2x)^2 - \left(\frac{3}{5}y\right)^2$

$= \left(2x + \frac{3}{5}y\right)\left(2x - \frac{3}{5}y\right)$

3. $4(x + 1)^2 - 49 = [2(x + 1)]^2 - 7^2$

$= [2(x + 1) + 7][2(x + 1) - 7]$

$= (2x + 9)(2x - 5)$

Practise Now 29

$256^2 - 156^2 = (256 + 156)(256 - 156)$

$= 412 \times 100$

$= 41\ 200$

Practise Now 30

(a) $8x^2y + 4x = 4x(2xy + 1)$

(b) $\pi r^2 + \pi rl = \pi r(r + l)$

(c) $-a^3by + a^2y = a^2y(-ab + 1)$

(d) $3c^2d + 6c^2d^2 + 3c^3 = 3c^2(d + 2d^2 + c)$

Practise Now 31

(a) $2(x + 1) + a(1 + x) = (x + 1)(2 + a)$

(b) $9(x + 2) - b(x + 2) = (x + 2)(9 - b)$

(c) $3c(2x - 3) - 6d(2x - 3) = 3[c(2x - 3) - 2d(2x - 3)]$

$= 3(2x - 3)(c - 2d)$

(d) $7h(4 - x) - (x - 4) = 7h(4 - x) + (4 - x)$

$= (4 - x)(7h + 1)$

Practise Now 32

(a) $xy + 4x + 3y + 12 = x(y + 4) + 3(y + 4)$

$= (y + 4)(x + 3)$

(b) $3by + 4ax + 12ay + bx = 4ax + 12ay + bx + 3by$

$= 4a(x + 3y) + b(x + 3y)$

$= (x + 3y)(4a + b)$

(c) $x^3 - x^2 - 1 + x = x^3 - x^2 + x - 1$

$= x^2(x - 1) + (x - 1)$

$= (x - 1)(x^2 + 1)$

(d) $6xy - 4x - 2z + 3yz = 6xy - 4x + 3yz - 2z$

$= 2x(3y - 2) + z(3y - 2)$

$= (3y - 2)(2x + z)$

Exercise 3A

1. **(a)** Since the common difference is 6, $T_n = 6n + ?$.

The term before T_1 is $c = T_0$

$= 7 - 6$

$= 1.$

$\therefore$ General term of sequence, $T_n = 6n + 1$

(b) Since the common difference is 3, $T_n = 3n + ?$.

The term before T_1 is $c = T_0$

$= -4 - 3$

$= -7.$

$\therefore$ General term of sequence, $T_n = 3n - 7$

(c) Since the common difference is 7, $T_n = 7n + ?$.

The term before T_1 is $c = T_0$

$= 60 - 7$

$= 53.$

$\therefore$ General term of sequence, $T_n = 7n + 53$

(d) Since the common difference is –3, $T_n = -3n + ?$.

The term before T_1 is $c = T_0$

$= 14 + 3$

$= 17.$

$\therefore$ General term of sequence, $T_n = -3n + 17$

2. **(i)** 18, 21

(ii) Since the common difference is 3, $T_n = 3n + ?$.

The term before T_1 is $c = T_0$

$= 3 - 3$

$= 0.$

$\therefore$ General term of sequence, $T_n = 3n$

(iii) $T_{105} = 3(105)$

$= 315$

3. **(i)** 30, 34

(ii) Since the common difference is 4, $T_n = 4n + ?$.

The term before T_1 is $c = T_0$

$= 10 - 4$

$= 6.$

$\therefore$ General term of sequence, $T_n = 4n + 6$

(iii) $T_{200} = 4(200) + 6$
$= 800 + 6$
$= 806$

4. **(i)**

Number of points	1	2	3	4	5	6
Number of segments	1 + 1 = 2	2 + 1 = 3	3 + 1 = 4	4 + 1 = 5	5 + 1 = 6	6 + 1 = 7

(ii) Let the number of points be n.
Number of segments = $n + 1$.
When $n = 49$, number of segments $= 49 + 1$
$= 50$

(iii) $101 = n + 1$
$\therefore n = 101 - 1$
$= 100$

5. **(i)**

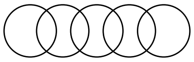


Figure 5

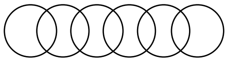


Figure 6

(ii)

Figure Number	Number of Intersection(s) between the Circles
1	0
2	1
3	2
4	3
5	4
6	5
⋮	⋮
n	$n - 1$

(iii) Let $n - 1 = 28$.
$n = 28 + 1$
$= 29$

6. **(a)** When $n = 1$,
$2n^2 + 1 = 2(1)^2 + 1$
$= 2 + 1$
$= 3$
When $n = 2$,
$2n^2 + 1 = 2(2)^2 + 1$
$= 8 + 1$
$= 9$
When $n = 3$,
$2n^2 + 1 = 2(3)^2 + 1$
$= 18 + 1$
$= 19$
When $n = 4$,
$2n^2 + 1 = 2(4)^2 + 1$
$= 32 + 1$
$= 33$
The first four terms of the sequence are 3, 9, 19 and 33.

(b) **(i)** General term of sequence, $T_n = 2n^2 + 1 - 2$
$= 2n^2 - 1$

(ii) $T_{388} = 2(388)^2 - 1$
$= 301\ 088 - 1$
$= 301\ 087$

7. **(i)**

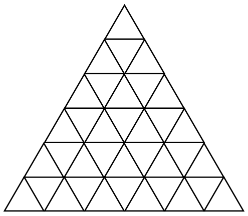


Figure 5

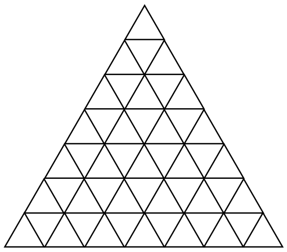


Figure 6

(ii)

Figure Number	Number of Small Triangles
1	4
2	9
3	16
4	25
5	36
6	49
⋮	⋮
n	$(n + 1)^2$

(iii) When $n = 20$,
$(n + 1)^2 = (20 + 1)^2$
$= 21^2$
$= 441$
Number of triangles in 20th figure = 441

(iv) Let $(n + 1)^2 = 121$.
$n + 1 = 11$ or $n + 1 = -11$
$n = 11 - 1$ or $n = -11 - 1$
$= 10$ or $= -12$ (N.A. since $n > 0$)

8. **(i)** 6th line: $54 = 6 \times 9$
(ii) Since $208 = 13 \times 16 = 13(13 + 3)$,
$k = 13$.

9. **(i)** 5th line: $1 + 3 + 5 + 7 + 9 + 11 = 36 = 6^2 = (5 + 1)^2$
(ii) $c = \sqrt{169}$
$= 13$
$d + 1 = 13$
$d = 13 - 1$
$= 12$
$a = 13 + 12$
$= 25$

10. (a) (i)

Number of people	4	6	8	10	12	14
Number of tables	$\frac{4-2}{2} = 1$	$\frac{6-2}{2} = 2$	$\frac{8-2}{2} = 3$	$\frac{10-2}{2} = 4$	$\frac{12-2}{2} = 5$	$\frac{14-2}{2} = 6$

(ii)

Number of tables	1	2	3	4	5	6
Number of people	$2(1) + 2 = 4$	$2(2) + 2 = 6$	$2(3) + 2 = 8$	$2(4) + 2 = 10$	$2(5) + 2 = 12$	$2(6) + 2 = 14$

(b) (i) From (a)(i): When $n = 20$,

$$\frac{n-2}{2} = \frac{20-2}{2}$$
$$= 9$$

∴ 9 tables will be needed to seat 20 people.

(ii) When $n = 30$,

$$\frac{n-2}{2} = \frac{30-2}{2}$$
$$= 14$$

∴ 14 tables will be needed to seat 30 people.

(c) (i) From (a)(ii): When $n = 22$,

$2(22) + 2 = 46$

∴ 46 people can be seated at 22 tables.

(ii) When n = 36,

$2(36) + 2 = 74$

∴ 74 people can be seated at 36 tables.

11. (i)

Number of points on the line segments *AB* (including the points *A* and *B*)	2	3	4	5	6	7
Number of possible line segments	$\frac{2 \times (2-1)}{2}$ $= 1$	$\frac{3 \times (3-1)}{2}$ $= 3$	$\frac{4 \times (4-1)}{2}$ $= 6$	$\frac{5 \times (5-1)}{2}$ $= 10$	$\frac{6 \times (6-1)}{2}$ $= 15$	$\frac{7 \times (7-1)}{2}$ $= 21$

(ii) Number of points including $AB = 18 + 2$
$= 20$

Number of possible line segments $= \frac{20 \times (20-1)}{2}$
$= 190$

12. (i) 1 5 10 10 5 1

(ii)

Row	Sum
1	$1 = 1 = 2^0$
2	$1 + 1 = 2 = 2^1$
3	$1 + 2 + 1 = 4 = 2^2$
4	$1 + 3 + 3 + 1 = 8 = 2^3$
5	$1 + 4 + 6 + 4 + 1 = 16 = 2^4$
6	$1 + 5 + 10 + 10 + 5 + 1 = 32 = 2^5$
$\vdots$	$\vdots$
n	$1 + (n - 1) + \cdots + (n - 1) + 1 = 2^{n-1}$

13. (a)

Figure	1	2	3	4	5	6
Number of black squares (*b*)	1	2	3	4	5	6
Number of white squares (*w*)	$1 \times 2 + 1 = 3$	$2 \times 2 + 1 = 5$	$3 \times 2 + 1 = 7$	$4 \times 2 + 1 = 9$	$5 \times 2 + 1 = 11$	$6 \times 2 + 1 = 13$
Area of whole figure (*b* + *w*)	4	7	10	13	16	19
Perimeter of whole figure (cm)	$2(1 + 4) = 10$	$2(2 + 4) = 12$	$2(3 + 4) = 14$	$2(4 + 4) = 16$	$2(5 + 4) = 18$	$2(6 + 4) = 20$

(b) (i) Number of white squares in Figure 9 $= 9 \times 2 + 1$

$= 19$

(ii) Perimeter of Figure 9 $= 2(9 + 4)$

$= 26$ cm

(iii) Number of white squares in Figure $n = n\,(2 + 1)$

$= 2n + 1$

(iv) Perimeter of Figure $n = 2(n + 4)$

$= (2n + 8)$ cm

14. (i) 8th line: $\dfrac{2}{8 \times 9 \times 10} = \dfrac{1}{8} - \dfrac{2}{9} + \dfrac{1}{10}$

(ii) Based on the pattern, n^{th} line:

$$\frac{2}{n(n + 1)(n + 2)} = \frac{1}{n} - \frac{2}{n + 1} + \frac{1}{n + 2}$$

$$\therefore\ \frac{1}{10} - \frac{2}{11} + \frac{1}{12} = \frac{2}{10 \times 11 \times 12}$$

$$= \frac{2}{1320}$$

$$= \frac{1}{660}$$

(iii)

$$\frac{2}{7980} = \frac{1}{p} - \frac{2}{p + 1} + \frac{1}{p + 2}$$

$$\frac{1}{p} - \frac{2}{p + 1} + \frac{1}{p + 2} = \frac{2}{p(p + 1)(p + 2)}$$

$$\therefore p(p + 1)(p + 2) = 7980$$

$$(p^2 + p)(p + 2) = 7980$$

$$p^3 + 2p^2 + p^2 + 2p - 7980 = 0$$

$$p^3 + 3p^2 + 2p - 7980 = 0$$

$\therefore p = 19$ or

$p = -11 + 17.292$ (5 s.f.) (reject, p is a whole number) or

$p = -11 - 17.292$ (5 s.f.) (reject, $p > 1$)

15. (a) (i) 11, 13

(ii) 24, 28

(iii) 84, 112

(iv) 85, 113

(b) 6th line: $13^2 + 84^2 = 85^2$

7th line: $15^2 + 112^2 = 113^2$

16. (i)

Member Number	Number of carbon atoms	Number of hydrogen atoms
1	3	4
2	4	6
3	5	8
4	6	10
5	7	12
6	8	14
⋮	⋮	⋮
n	$n + 2$	$2n + 2$

(ii) Let $h + 2 = 25$.

$h = 25 - 2$

$= 23$

When $n = h = 23$,

$2n + 2 = 2(23) + 2$

$= 48$

Number of hydrogen atoms the member has = 48

(iii) Let $2k + 2 = 64$.

$2k = 64 - 2$

$= 62$

$k = 31$

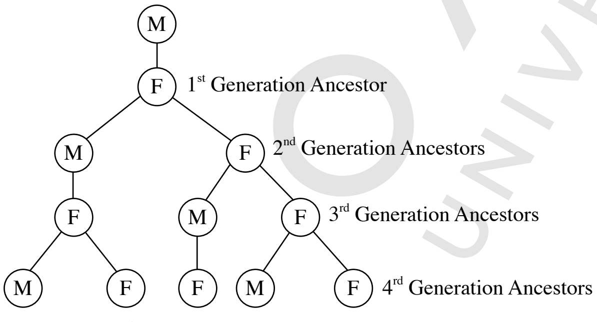
When $n = k = 31$,

$n + 2 = 31 + 2$

$= 33$

Number of carbon atoms the member has = 33

17. (i)

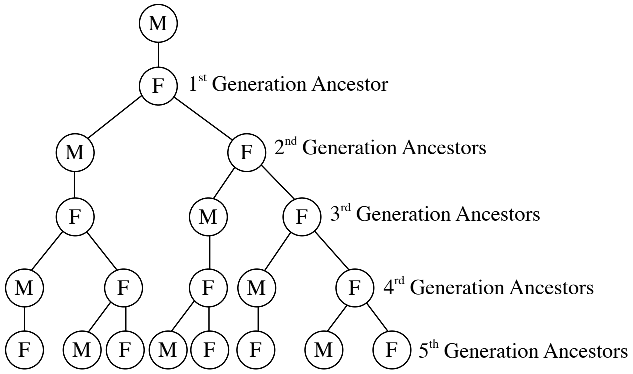


Number of 4th generation ancestors a male bee has = 5

(ii) The number of nth generation ancestors forms a sequence: 1, 2, 3, 5, ... The first two numbers of the sequence are 1 and 2, and each subsequent term is the sum of the previous two terms.

(iii) Number of 5th generation ancestors a male bee has = 3 + 5

= 8



(iv) The sequence for the number of nth generation ancestors is 1, 2, 3, 5, 8, 13, 21, 34, 55, ...

Number of 10th generation ancestors a male bee has = 34 + 55

= 89

Exercise 3B

1. (a)

$$\begin{array}{r} a+b \\ a+b\,\big)\overline{} \\ \\ \hline ab+b^2 \\ ab+b^2 \\ - \quad - \\ \hline 0 \end{array}$$

The quotient is $a + b$ and remainder is o.

(b)

$$\begin{array}{r} a^2+b^2-ab \\ a+b\,\big)\overline{a^3+b^3} \\ a^3+a^2b \\ - \quad - \\ \hline b^3-a^2b \\ b^3+ab^2 \\ - \quad - \\ \hline a^2b-ab^2 \\ a^2b+ab^2 \\ + \quad + \\ \hline 0 \end{array}$$

The quotient is $a^2 + b^2 - ab$ and remainder is 0.

2. (a) $6x \times (-2y) = (6 \times x) \times (-2 \times y)$

$= [6 \times (-2)] \times x \times y$

$= -12xy$

(b) $14x \times \frac{1}{2}y = (14 \times x) \times \left(\frac{1}{2} \times y\right)$

$= \left(14 \times \frac{1}{2}\right) \times x \times y$

$= 7xy$

3. **(a)** $8x(y - 1) = 8xy - 8x$

(b) $-9x(3y - 2z) = -27xy + 18xz$

(c) $3x(2x + 7y) = 6x^2 + 21xy$

(d) $3y(x - 11y) = 3xy - 33y^2$

(e) $-3a(2a + 3b) = -6a^2 - 9ab$

(f) $-4c(2c - 5d) = -8c^2 + 20cd$

(g) $-6h(7k - 3h) = -42hk + 18h^2$

(h) $-8m(-12m - 7n) = 96m^2 + 56mn$

(i) $2p(3p + q + 7r) = 6p^2 + 2pq + 14pr$

(j) $-7s(s - 4t - 3u) = -7s^2 + 28st + 21su$

4. **(a)** $7a(3b - 4c) + 4a(3c - 2b) = 21ab - 28ac + 12ac - 8ab$
$= 21ab - 8ab - 28ac + 12ac$
$= 13ab - 16ac$

(b) $4d(d - 5f) + 2f(3d + 7f) = 4d^2 - 20df + 6df + 14f^2$
$= 4d^2 - 14df + 14f^2$

5. **(a)** $(x + y)(x + 6y) = x(x + 6y) + y(x + 6y)$
$= x^2 + 6xy + xy + 6y^2$
$= x^2 + 7xy + 6y^2$

(b) $(x^2 + 2)(x + 5) = x^2(x + 5) + 2(x + 5)$
$= x^3 + 5x^2 + 2x + 10$

6.

$$\begin{array}{rl} & 5b^2 - 3b + 2 \\ 4b\,\big) & \overline{20b^3 - 12b^2 + 8b} \\ & 20b^3 \\ & - \\ \hline & \quad -12b^2 \\ & \quad -12b^2 \\ & \quad + \\ \hline & \qquad\quad 8b \\ & \qquad\quad 8b \\ & \qquad\quad - \\ \hline & \qquad\quad 0 \end{array}$$

The quotient is $5b^2 - 3b + 2$ and remainder is 0.

7. **(a)** $\left(-\frac{3}{7}x\right) \times \frac{14}{9}y = \left(-\frac{3}{7} \times \frac{14}{9}\right) \times x \times y$
$= -\frac{2}{3}xy$

(b) $9x^3y \times 3x^2y^2 = (9 \times x \times x \times x \times y) \times (3 \times x \times x \times y \times y)$
$= (9 \times 3) \times (x \times x \times x \times x \times x) \times (y \times y \times y)$
$= 27x^5y^3$

(c) $2x^3y \times (-13y^2) = (2 \times x \times x \times x \times y) \times (-13 \times y \times y)$
$= [2 \times (-13)] \times (x \times x \times x) \times (y \times y \times y)$
$= -26x^3y^3$

(d) $(-4xyz) \times (-2x^2y^3z^4)$
$= (-4 \times x \times y \times z) \times (-2 \times x \times x \times y \times y \times y \times z \times z \times z \times z)$
$= [(-4) \times (-2)] \times (x \times x \times x) \times (y \times y \times y \times y) \times (z \times z \times z \times z \times z)$
$= 8x^3y^4z^5$

8. **(a)** $-3xy(x - 2y) = -3x^2y + 6xy^2$

(b) $9x(-3x^2y - 7xz) = -27x^3y - 63x^2z$

(c) $-13x^2y(3x - y) = -39x^3y + 13x^2y^2$

(d) $-5x(-6x - 4x^3y - 3y) = 30x^2 + 20x^4y + 15xy$

9. **(a)** $a(5b + c) - 2a(3c - b) = 5ab + ac - 6ac + 2ab$
$= 5ab + 2ab + ac - 6ac$
$= 7ab - 5ac$

(b) $-2d(4f - 5h) - f(3d + 7h)$
$= -8df + 10dh - 3df - 7fh$
$= -8df - 3df + 10dh - 7fh$
$= -11df + 10dh - 7fh$

(c) $4k(3k + m) - 3k(2k - 5m)$
$= 12k^2 + 4km - 6k^2 + 15km$
$= 12k^2 - 6k^2 + 4km + 15km$
$= 6k^2 + 19km$

(d) $2n(p - 2n) - 4n(n - 2p)$
$= 2np - 4n^2 - 4n^2 + 8np$
$= -4n^2 - 4n^2 + 2np + 8np$
$= -8n^2 + 10np$

10. **(a)** $(a + 3b)(a - b) = a(a - b) + 3b(a - b)$
$= a^2 - ab + 3ab - 3b^2$
$= a^2 + 2ab - 3b^2$

(b) $(3c + 7d)(c - 2d) = 3c(c - 2d) + 7d(c - 2d)$
$= 3c^2 - 6cd + 7cd - 14d^2$
$= 3c^2 + cd - 14d^2$

(c) $(3k - 5h)(-h - 7k) = 3k(-h - 7k) - 5h(-h - 7k)$
$= -3hk - 21k^2 + 5h^2 + 35hk$
$= - 21k^2 - 3hk + 35hk + 5h^2$
$= -21k^2 + 32hk + 5h^2$

(d) $(7m^2 + 2)(m - 4) = 7m^2(m - 4) + 2(m - 4)$
$= 7m^3 - 28m^2 + 2m - 8$

11. **(a)** $5x(x - 6y) + (x + 3y)(3x - 4y)$
$= 5x^2 - 30xy + x(3x - 4y) + 3y(3x - 4y)$
$= 5x^2 - 30xy + 3x^2 - 4xy + 9xy - 12y^2$
$= 5x^2 + 3x^2 - 30xy - 4xy + 9xy - 12y^2$
$= 8x^2 - 25xy - 12y^2$

(b) $(7x - 3y)(x - 4y) + (5x - 9y)(y - 2x)$
$= 7x(x - 4y) - 3y(x - 4y) + 5x(y - 2x) - 9y(y - 2x)$
$= 7x^2 - 28xy - 3xy + 12y^2 + 5xy - 10x^2 - 9y^2 + 18xy$
$= 7x^2 - 10x^2 - 28xy - 3xy + 5xy + 18xy + 12y^2 - 9y^2$
$= -3x^2 - 8xy + 3y^2$

12. **(a)** $(x + 9y)(x + 3y + 1)$
$= x(x + 3y + 1) + 9y(x + 3y + 1)$
$= x^2 + 3xy + x + 9xy + 27y^2 + 9y$
$= x^2 + x + 3xy + 9xy + 9y + 27y^2$
$= x^2 + x + 12xy + 9y + 27y^2$

(b) $(x + 2)(x^2 + x + 1)$
$= x(x^2 + x + 1) + 2(x^2 + x + 1)$
$= x^3 + x^2 + x + 2x^2 + 2x + 2$
$= x^3 + x^2 + 2x^2 + x + 2x + 2$
$= x^3 + 3x^2 + 3x + 2$

13. **(a)** $a^2 = a \times a$
$-4b^2 = b \times (-4b)$ or $(-b) \times 4b$
$= 2b \times (-2b)$ or $(-2b) \times 2b$

×	a	$4b$
a	a^2	$4ab$
$-b$	$-ab$	$-4b^2$

$(-ab) + 4ab = 3ab$
$\therefore\ a^2 + 3ab - 4b^2 = (a - b)(a + 4b)$

(b) $c^2 = c \times c$

$-21d^2 = d \times (-21b)$ or $(-d) \times 21d$

$= 3d \times (-7d)$ or $(-3d) \times 7d$

×	c	$-7d$
c	c^2	$-7cd$
$3d$	$3cd$	$-21d^2$

$3cd + (-7cd) = -4cd$

$\therefore\ c^2 - 4cd - 21d^2 = (c + 3d)(c - 7d)$

(c) $2h^2 = 2h \times h$

$-15k^2 = k \times (-15k)$ or $(-k) \times 15k$

$= 3k \times (-5k)$ or $(-3k) \times 5k$

×	h	$5k$
$2h$	$2h^2$	$10hk$
$-3k$	$-3hk$	$-15k^2$

$(-3hk) + 10hk = 7hk$

$\therefore\ 2h^2 + 7hk - 15k^2 = (2h - 3k)(h + 5k)$

(d) $3m^2 = 3m \times m$

$-12n^2 = n \times (-12n)$ or $(-n) \times 12n$

$= 2n \times (-6n)$ or $(-2n) \times 6n$

$= 3n \times (-4n)$ or $(-3n) \times 4n$

×	m	$-6n$
$3m$	$3m^2$	$-18mn$
$2n$	$2mn$	$-12n^2$

$2mn + (-18mn) = -16mn$

$\therefore\ 3m^2 - 16mn - 12n^2 = (3m + 2n)(m - 6n)$

(e) $3p^2 + 15pq + 18q^2 = 3(p^2 + 5pq + 6q^2)$

$p^2 = p \times p$

$6q^2 = q \times 6q$ or $(-q) \times (-6q)$

$= 2q \times 3q$ or $(-2q) \times (-3q)$

×	p	$3q$
p	p^2	$3pq$
$2q$	$2pq$	$6p^2$

$2pq + 3pq = 5pq$

$\therefore\ 3p^2 + 15pq + 18q^2 = 3(p + 2q)(p + 3q)$

(f) $2r^2t - 9rst + 10s^2t = t(2r^2 - 9rs + 10s^2)$

$2r^2 = 2r \times r$

$10s^2 = s \times 10s$ or $(-s) \times (-10s)$

$= 2s \times 5s$ or $(-2s) \times (-5s)$

×	r	$-2s$
$2r$	$2r^2$	$-4rs$
$-5s$	$-5rs$	$10s^2$

$(-5rs) + (-4rs) = -9rs$

$\therefore\ 2r^2t - 9rst + 10s^2t = t(2r - 5s)(r - 2s)$

14. $$\left(\frac{1}{4}x^2y\right) \times \left(\frac{16}{5}yz^3\right) = \left(\frac{1}{4} \times x \times x \times y\right) \times \left(\frac{16}{5} \times y \times z \times z \times z\right)$$

$$= \left(\frac{1}{4} \times \frac{16}{5}\right) \times (x \times x) \times (y \times y) \times (z \times z \times z)$$

$$= \frac{4}{5}x^2y^2z^3$$

15. (a) $(8x - y)(x + 3y) - (4x + y)(9y - 2x)$

$= 8x(x + 3y) - y(x + 3y) - [4x(9y - 2x) + y(9y - 2x)]$

$= 8x^2 + 24xy - xy - 3y^2 - (36xy - 8x^2 + 9y^2 - 2xy)$

$= 8x^2 + 24xy - xy - 3y^2 - 36xy + 8x^2 - 9y^2 + 2xy$

$= 8x^2 + 8x^2 + 24xy - xy - 36xy + 2xy - 3y^2 - 9y^2$

$= 16x^2 - 11xy - 12y^2$

(b) $(10x + y)(3x + 2y) - (5x - 4y)(-x - 6y)$

$= 10x(3x + 2y) + y(3x + 2y) - [5x(-x - 6y) - 4y(-x - 6y)]$

$= 30x^2 + 20xy + 3xy + 2y^2 - (-5x^2 - 30xy + 4xy + 24y^2)$

$= 30x^2 + 20xy + 3xy + 2y^2 + 5x^2 + 30xy - 4xy - 24y^2$

$= 30x^2 + 5x^2 + 20xy + 3xy + 30xy - 4xy + 2y^2 - 24y^2$

$= 35x^2 + 49xy - 22y^2$

16. (a) $(2x - 3y)(x + 5y - 2)$

$= 2x(x + 5y - 2) - 3y(x + 5y - 2)$

$= 2x^2 + 10xy - 4x - 3xy - 15y^2 + 6y$

$= 2x^2 - 4x + 10xy - 3xy + 6y - 15y^2$

$= 2x^2 - 4x + 7xy + 6y - 15y^2$

(b) $(x + 4)(x^2 - 5x + 7)$

$= x(x^2 - 5x + 7) + 4(x^2 - 5x + 7)$

$= x^3 - 5x^2 + 7x + 4x^2 - 20x + 28$

$= x^3 - 5x^2 + 4x^2 + 7x - 20x + 28$

$= x^3 - x^2 - 13x + 28$

(c) $(x - 1)(x^2 + 2x - 1)$

$= x(x^2 + 2x - 1) - (x^2 + 2x - 1)$

$= x^3 + 2x^2 - x - x^2 - 2x + 1$

$= x^3 + 2x^2 - x^2 - x - 2x + 1$

$= x^3 + x^2 - 3x + 1$

(d) $(3x^2 - 3x + 4)(3 - x)$

$= 3x^2(3 - x) - 3x(3 - x) + 4(3 - x)$

$= 9x^2 - 3x^3 - 9x + 3x^2 + 12 - 4x$

$= -3x^3 + 9x^2 + 3x^2 - 9x - 4x + 12$

$= -3x^3 + 12x^2 - 13x + 12$

17. (a) $x^2y^2 = xy \times xy$

$-15 = 1 \times (-15)$ or $(-1) \times 15$

$= 3 \times (-5)$ or $(-3) \times 5$

×	xy	5
xy	x^2y^2	$5xy$
-3	$-3xy$	-15

$(-3xy) + 5xy = 2xy$

$\therefore\ x^2y^2 + 2xy - 15 = (xy - 3)(xy + 5)$

(b) $12x^2y^2 = 12xy \times xy$ or $6xy \times 2xy$ or $4xy \times 3xy$

$-40 = 1 \times (-40)$ or $(-1) \times 40$

$= 2 \times (-20)$ or $(-2) \times 20$

$= 4 \times (-10)$ or $(-4) \times 10$

$= 5 \times (-8)$ or $(-5) \times 8$

$\times$	$3xy$	-8
$4xy$	$12x^2y^2$	$-32xy$
5	$15xy$	-40

$15xy + (-32xy) = -17xy$

$\therefore\ 12x^2y^2 - 17xy - 40 = (4xy + 5)(3xy - 8)$

(c) $4x^2y^2z - 22xyz + 24z = 2z(2x^2y^2 - 11xy + 12)$

$2x^2y^2 = 2xy \times xy$

$12 = 1 \times 12$ or $(-1) \times (-12)$

$\quad = 2 \times 6$ or $(-2) \times (-6)$

$\quad = 3 \times 4$ or $(-3) \times (-4)$

$\times$	xy	-4
$2xy$	$2x^2y^2$	$-8xy$
-3	$-3xy$	12

$(-3xy) + (-8xy) = -11xy$

$\therefore\ 4x^2y^2z - 22xyz + 24z = 2z(2xy - 3)(xy - 4)$

(d) $2x^2 + \frac{5}{3}xy - 2y^2 = \frac{1}{3}(6x^2 + 5xy + 6y^2)$

$6x^2 = 6x \times x$ or $3x \times 2x$

$-6y^2 = y \times (-6y)$ or $(-y) \times 6y$

$\quad = 2y \times (-3y)$ or $(-2y) \times 3y$

$\times$	$2x$	$3y$
$3x$	$6x^2$	$9xy$
$-2y$	$-4xy$	12

$(-4xy) + 9xy = 5xy$

$\therefore\ 2x^2 + \frac{5}{3}xy - 2y^2 = \frac{1}{3}(3x - 2y)(2x + 3y)$

Exercise 3C

1. **(a)** $(a + 4)^2 = a^2 + 2(a)(4) + 4^2$

$\quad = a^2 + 8a + 16$

(b) $(3b + 2)^2 = (3b)^2 + 2(3b)(2) + 2^2$

$\quad = 9b^2 + 12b + 4$

(c) $(c + 4d)^2 = c^2 + 2(c)(4d) + (4d)^2$

$\quad = c^2 + 8cd + 16d^2$

(d) $(9h + 2k)^2 = (9h)^2 + 2(9h)(2k) + (2k)^2$

$\quad = 81h^2 + 36hk + 4k^2$

2. **(a)** $(m - 9)^2 = m^2 - 2(m)(9) + 9^2$

$\quad = m^2 - 18m + 81$

(b) $(5n - 4)^2 = (5n)^2 - 2(5n)(4) + 4^2$

$\quad = 25n^2 - 40n + 16$

(c) $(9 - 5p)^2 = 9^2 - 2(9)(5p) + (5p)^2$

$\quad = 81 - 90p + 25p^2$

(d) $(3q - 8r)^2 = (3q)^2 - 2(3q)(8r) + (8r)^2$

$\quad = 9q^2 - 48qr + 64r^2$

3. **(a)** $(s - 5)(s + 5) = s^2 - 5^2$

$\quad = s^2 - 25$

(b) $(2t + 11)(2t - 11) = (2t)^2 - 11^2$

$\quad = 4t^2 - 121$

(c) $(7 + 2u)(7 - 2u) = 7^2 - (2u)^2$

$\quad = 49 - 4u^2$

(d) $(w - 10x)(w + 10x) = w^2 - (10x)^2$

$\quad = w^2 - 100x^2$

4. **(a)** $1203^2 = (1200 + 3)^2$

$\quad = 1200^2 + 2(1200)(3) + 3^2$

$\quad = 1\,440\,000 + 7200 + 9$

$\quad = 1\,447\,209$

(b) $892^2 = (900 - 8)^2$

$\quad = 900^2 - 2(900)(8) + 8^2$

$\quad = 810\,000 - 14\,400 + 64$

$\quad = 795\,664$

(c) $1998 \times 2002 = (2000 - 2)(2000 + 2)$

$\quad = 2000^2 - 2^2$

$\quad = 4\,000\,000 - 4$

$\quad = 3\,999\,996$

5. $(x - y)^2 = x^2 - 2xy + y^2$

$\quad = x^2 + y^2 - 2xy$

Since $x^2 + y^2 = 80$ and $xy = 12$,

$\therefore\ (x - y)^2 = 80 - 2(12)$

$\quad = 56$

6. $x^2 - y^2 = (x + y)(x - y)$

Since $x + y = 10$ and $x - y = -4$,

$\therefore\ x^2 - y^2 = 10 \times (-4)$

$\quad = -40$

7. **(a)** $\left(\frac{1}{5}a + 3b\right)^2 = \left(\frac{1}{5}a\right)^2 + 2\left(\frac{1}{5}a\right)(3b) + (3b)^2$

$\quad = \frac{1}{25}a^2 + \frac{6}{5}ab + 9b^2$

(b) $\left(\frac{1}{2}c + \frac{2}{3}d\right)^2 = \left(\frac{1}{2}c\right)^2 + 2\left(\frac{1}{2}c\right)\left(\frac{2}{3}d\right) + \left(\frac{2}{3}d\right)^2$

$\quad = \frac{1}{4}c^2 + \frac{2}{3}cd + \frac{4}{9}d^2$

8. **(a)** $\left(\frac{3}{2}h - 5k\right)^2 = \left(\frac{3}{2}h\right)^2 - 2\left(\frac{3}{2}h\right)(5k) + (5k)^2$

$\quad = \frac{9}{4}h^2 - 15hk + 25k^2$

(b) $\left(-\frac{6}{5}m - 3n\right)^2 = \left(-\frac{6}{5}m\right)^2 - 2\left(-\frac{6}{5}m\right)(3n) + (3n)^2$

$\quad = \frac{36}{25}m^2 + \frac{36}{5}mn + 9n^2$

9. **(a)** $(6p + 5)(5 - 6p) = (5 + 6p)(5 - 6p)$

$\quad = 5^2 - (6p)^2$

$\quad = 25 - 36p^2$

$\quad = -36p^2 + 25$

(b) $\left(9r - \frac{4}{5}q\right)\left(9r + \frac{4}{5}q\right) = (9r)^2 - \left(\frac{4}{5}q\right)^2$

$\quad = 81r^2 - \frac{16}{25}q^2$

(c) $\left(\frac{s}{2} + \frac{t}{3}\right)\left(\frac{t}{3} - \frac{s}{2}\right) = \left(\frac{t}{3} + \frac{s}{2}\right)\left(\frac{t}{3} - \frac{s}{2}\right)$

$\quad = \left(\frac{t}{3}\right)^2 - \left(\frac{s}{2}\right)^2$

$\quad = \frac{t^2}{9} - \frac{s^2}{4}$

$\quad = -\frac{s^2}{4} + \frac{t^2}{9}$

(d) $(u + 2)(u - 2)(u^2 + 4) = (u^2 - 2^2)(u^2 + 4)$
$= (u^2 - 4)(u^2 + 4)$
$= (u^2)^2 - 4^2$
$= u^4 - 16$

10. **(a)** $4(x + 3)^2 - 3(x + 4)(x - 4)$
$= 4[x^2 + 2(x)(3) + 3^2] - 3(x^2 - 4^2)$
$= 4(x^2 + 6x + 9) - 3(x^2 - 16)$
$= 4x^2 + 24x + 36 - 3x^2 + 48$
$= 4x^2 - 3x^2 + 24x + 36 + 48$
$= x^2 + 24x + 84$

(b) $(5x - 7y)(5x + 7y) - 2(x - 2y)^2$
$= (5x)^2 - (7y)^2 - 2[x^2 - 2(x)(2y) + (2y)^2]$
$= 25x^2 - 49y^2 - 2(x^2 - 4xy + 4y^2)$
$= 25x^2 - 49y^2 - 2x^2 + 8xy - 8y^2$
$= 25x^2 - 2x^2 + 8xy - 49y^2 - 8y^2$
$= 23x^2 + 8xy - 57y^2$

11. $\left(\frac{1}{2}x + \frac{1}{2}y\right)^2 = \left(\frac{1}{2}x\right)^2 + 2\left(\frac{1}{2}x\right)\left(\frac{1}{2}y\right) + \left(\frac{1}{2}y\right)^2$
$= \frac{1}{4}x^2 + \frac{1}{2}xy + \frac{1}{4}y^2$
$= \frac{1}{4}x^2 + \frac{1}{4}y^2 + \frac{1}{2}xy$
$= \frac{1}{4}(x^2 + y^2) + \frac{1}{2}xy$

Since $x^2 + y^2 = 14$ and $xy = 5$,
$\therefore \left(\frac{1}{2}x + \frac{1}{2}y\right)^2 = \frac{1}{4}(14) + \frac{1}{2}(5)$
$= 6$

12. $2x^2 - 2y^2 = 125$
$2(x^2 - y^2) = 125$
$2(x + y)(x - y) = 125$

Since $x - y = 2.5$,
$\therefore 2(x + y)(2.5) = 125$
$5(x + y) = 125$
$x + y = 25$

13. $\left(\frac{1}{16}x^2 + \frac{1}{25}y^2\right)\left(\frac{1}{4}x + \frac{1}{5}y\right)\left(\frac{1}{4}x - \frac{1}{5}y\right)$
$= \left(\frac{1}{16}x^2 + \frac{1}{25}y^2\right)\left[\left(\frac{1}{4}x\right)^2 - \left(\frac{1}{5}y\right)^2\right]$
$= \left(\frac{1}{16}x^2 + \frac{1}{25}y^2\right)\left(\frac{1}{16}x^2 - \frac{1}{25}y^2\right)$
$= \left(\frac{1}{16}x^2\right)^2 - \left(\frac{1}{25}y^2\right)^2$
$= \frac{1}{256}x^4 - \frac{1}{625}y^4$

14. **(i)** $(p - 2q)^2 - p(p - 4q)$
$= p^2 - 2(p)(2q) + (2q)^2 - p^2 + 4pq$
$= p^2 - 4pq + 4q^2 - p^2 + 4pq$
$= p^2 - p^2 - 4pq + 4pq + 4q^2$
$= 4q^2$

(ii) Let $p = 5330$ and $q = 10$,
$5310^2 - 5330 \times 5290$
$= [5330 - 2(10)]^2 - 5330[5330 - 4(10)]$
$= 4(10)^2$ (From **(i)**)
$= 400$

15. **(i)** $n^2 - (n - a)(n + a) = n^2 - (n^2 - a^2)$
$= n^2 - n^2 + a^2$
$= a^2$

(ii) Let $n = 16\,947$ and $a = 3$,
$16\,947^2 - 16\,944 \times 16\,950$
$= 16\,947^2 - (16\,947 - 3)(16\,947 + 3)$
$= 3^2$ (From **(i)**)
$= 9$

Exercise 3D

1. **(i)** $a + 4b$
$(a + 4b)^3 = (a)^3 + 3(a)^2 (4b) + 3(a) (4b)^2 + (4b)^3$
$= a^3 + 12\, a^2b + 3(a) (16\, b^2) + 64\, b^3$
$= a^3 + 12\, a^2b + 48\, ab^2 + 64\, b^3$

(ii) $ax + by$
$(ax + by)^3 = (ax)^3 + 3(ax)^2 (by) + 3(ax) (by)^2 + (by)^3$
$= a^3x^3 + 3a^2x^2\, by + 3(ax) (by)^2 + (by)^3$

(iii) $a^2 + b^2$
$(a^2 + b^2)^3 = (a^2)^3 + 3(a^2)^2 (b^2) + 3(a^2) (b^2)^2 + (b^2)^3$
$= a^6 + 3a^4b^2 + 3a^2b^4 + b^6$

(iv) 42
$(42)^3 = (40 + 2)^3$
$= (40)^3 + 3\, (40)^2\, (2) + 3(40)\, (2)^2 + (2)^3$
$= 64000 + 3\times1600 \times2 + 3\times40\times4 + 8$
$= 64000 + 9600 + 482 + 8$
$= 74088$

2. **(i)** $ax - by$
$(ax - by)^3 = (ax)^3 + 3(ax)^2 (-by) + 3(ax) (-by)^2 + (-by)^3$
$= ax^3 + 3\, a^2bx^2y + 3\, ab^2\, xy^2 + b^3\, y^3$

(ii) $a^2 - b^2$
$(a^2 - b^2)^3 = (a^2)^3 + 3(a^2)^2 (-b^2) + 3(a^2) (-b^2)^2 + (-b^2)^3$
$= a^6 + 3a^4\, b^2 + 3\, a^2\, b^4 + b^6$

(iii) $2a - 3bc^2$
$(2a - 3b^c)^3 = (2a)^3 + 3(2a)^2 (-3bc) + 3(2a) (-3bc)^2 + (-3bc)^3$
$= 8a^3 - 36a^2bc + 54\, a\, b^2\, c^2 - 27\, b^3\, c^3$

(iv) $(1 + a - 2b)^3$
let 1 + a = x
$(x - 2b)^3 = (x)^3 + 3(x)^2 (-2b) + 3(x) (-2b)^2 + (-2b)^3$
$= x^3 - 6x^2\, b + 12\, x\, b^2 - 8b^3$ ________ (1)
Substitute $x = 1 + a$ in (1)
$(1 + a - 2b) = (1 + a)^3 - 6(1 + a)^2\, b + 12\, (1 + a)b^2 - 8b^3$
$= 1 + 3a + 3a^2 + a^3 - 6\, (1 + 2a + a^2)\, b + 12b^2 + 12ab^2 - 8b^3$
$= 1 + 3a + 3a^2 + a^3 - 6b - 12ab - 6a^2b + 12b^2 + 12ab^2 - 8b^3$
$= 1 + a^3 + 8b^3 + 3a^2 + 12b^2 - 12b^2 - 6a^2b + 12ab^2 + 3a - 6b$
$-12ab$

(v) $(399)^3$

$(400 - 1)^3 = (400)^3 + 3(400)^2 (-1) + 3(400) (-1)^2 + (1)^3$

$= 64000000 = 480000 + 1200 -1$

$= 63521199$

(v) $(999)^3$

$(1000 - 1)^3 = (1000)^3 + 3(1000)^2 (-1) + 3(1000) (-1)^2 + (1)^3$

$= 1000000000 = 3000000 + 3000 -1$

$= 997002999$

3. $a^3 + 9a^2 + 27a +30$

$= (3)^3 + 9 (3)^2 + 27 (3) + 30$ (substitute $a = 3$)

$= 27 + 9×9 + 27×3 + 30$

$= 27 + 81 +81 + 30$

$= 219$

4. $(31)^3 + 3×(31)^2 × 19 + 3×3 1 × (19)^2 + (19)^3$ ________ (1)

Apply cubes of the sum of two terms, i.e

$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

In the given question $a = 31$ and $l = 19$

$\therefore$ (1) becomes $(31 + 19)^3$

$= (50)^3$

$= 125000$

5. $(x + y)^3 + (x - y)^3 + 6x (x^2 - y^2)$

$x^3 + \cancel{3}x^2y + \cancel{3}xy^2 + y^3 + x^3 - \cancel{3}^2xy + \cancel{3}xy^2 - y^3 + 6x^{3\,-}6xy^2$

$= 8x^3 + \cancel{6}xy^2 - \cancel{6}xy^2$

$= 8x^3$

6. **(i)** $13 × 13 × 13 + 3 × 13 × 13 × 7 + 3 × 13 × 7 × 7 + 7 × 7 × 7$

$= (13)^3 + 3(13)^2 × 7+ 3(13) (7)^2 + (7)^3$

$= (13 + 7)^3$ (apply the formula of cube of sum of two terms ;

$= (20)^3(a + b)^3 = a^3 + 3a^2\mathrm{b} + 3ab^2 + b^3$

$= 8000$

(ii) $0.6 × 0.6 × 0.6 + 2.4 × 2.4 × 2.4 + 3 × 0.6 × 0.6 × 2.4 + 3 × 0.6 × 2.4 × 2.4$

Apply the formula $(a + b)^3 = a^3 + b^3 + 3a^2b +$ we get

$= (0.6)^3 + (2.4)^3 × 3 × (0.6)^2 (2.4) + 3(0.6) (2.4)^2$

$= 0.216 + 13.824 + 3 × 0.36 × 2.4 + 3 × 0.6 × 5.76$

$= 0.216 + 13.824 + 2.592 + 10.368$

$= 27.$

(iii) $51 × 51 × 51 - 3 × 51 × 51 × 46 + 3 × 51 × 46 × 46 - 46 × 46 × 46$

$= (51)^3 - 3(51)^2 (46)+ 3(51) (46)^2 - (46)^3$

$= (51 - 46)^3$ (Apply the formula $a^3 - 3a^2b + 3ab^2 - b^3 = (a-b)^3$

$= (5)^3$

$= 125$

(iv) $31.6 × 31.6 × 31.6 - 3 × 31.6 × 31.6 × 31.6 × 28.6 + 3 × 31.6 × 28.6 × 28.6 × 28.6 - 28.6 × 28.6 × 28.6$

$= (31.6)^3 - 3 (31.6)^2 (28.6) + 3 (31.6) (28.6)^2 - (28.6)^3$

$= (31.6 - 28.6)^3$ (Apply the formula $a^3 - 3a^2b + 3ab^2 - b^3$)

$= (3)^3$

$= 27$

7. $p + 3q - 2 = 0$

$p + 3q = 2$

Take the cube of both sides of equation

$(p + 3q)^3 = (2)^3$

$p^3 + 27 q^3 + 9p^2q + 27pq^2 = 8$

$p^3 + 27 q^3 + 9pq (p + 3q) = 8$

$p^3 + 27 q^3 + 9pq × 2 = 8$ (given : $p + 3q = 2$)

$p^3 + 27 q^3 + 18pq = 8$ (proved)

8. $p = 2q + 4$

$(p - 2q) = 4$

taking the cube of both the sides

$(p - 2q)^3 = (4)^3$

$p^3 - 8q^3 - 6p^2q + 12pq^2 = 64$

$p^3 - 8q^3 - 6pq (p - 2q) = 64$ (given : $p - 2q = 4$)

$p^3 - 8q^3 - 6pq × 4 = 64$

$p^3 - 8q^3 - 24pq = 64$ (proved)

9. $x + 3y = 3xy + 1$

Cubing bith the sides,

$x^3 + 27y^3 + 3×x×3y (x + 3y)$

$x^3 + 27y^3 + 9xy (x + 3y) = x^3 + 27y^3 + 9xy (x + 3y)$

$x^3 + 27y^3 \cancel{+ 9xy (x + 3y)} = 27 x^3 y^3 + 1 \cancel{+ 9xy (x + 3y)}$

$x^3 + 27y^3 = 27 x^3 y^3 + 1$ (proved)

10. $p^3 + q^3 = \mathrm{r}^3$

Taking cube of both the sides

$(p^3 + q^3)3 = (r^3)^3$

$p^9 + 3p^6q^3 + 3p^3q^6 + q^9 = r^9$

$p^9 + q^9 + 3p^3q^3 (p^3 + q^3) = r^9$

$p^9 + q^9 + 3p^3q^3 r^3 = r^9$

11. $x + y = 8$

Taking cube of the equation

$(x + y)^3 = (8)^3$

$x^3 + y^3 + 3x^2y + 3xy^2 = 512$

$x^3 + y^3 + 3xy (x+y) = 512$

$x^3 + y^3 + 3xy × 8 = 512$

$x^3 + y^3 + 24xy = 512$

12. $m + n + 3 = 0$

$m+n = -3$

Taking the cube

$(m+n)^3 = (-3)^3$

$m^3 + n^3 + 3m^2n + 3mn^2 = -27$

$m^3 + n^3 + 3mn (m+n) = -27$

$m^3 + n^3 + 3mn (-3) = -27$

$m^3 + n^3 - 9mn = - 27$

13. $\frac{a^2-1}{a} = 1$

Taking cube of both the sides,

$\left(\frac{a^2-1}{a}\right)^3 = (1)^3$

$\frac{(a^2-1)^3}{a^3} = 1$

$\frac{a^6 - 3a + 3a^2 - 1}{a^3} = 1$

$\frac{a^6 - 3a^3(a^2-1) - 1}{a^3} = 1$

$\frac{a^6-1}{a^3} - \frac{3\cancel{a}^2(a^2)}{\cancel{a}^3} = 1$

$\frac{a^6-1}{a^3} - \frac{3(a^2-1)}{a} = 1$

$\frac{a^6-1}{a^3} = 1 + 3\,\frac{(a^2-1)}{a}$

$\frac{a^6-1}{a^3} = 1 + 3$

$\frac{a^6-1}{a^3} = 4\,\frac{(a^2-1)}{a} = 1$

14. $x - y = 4$ and $xy = 21$

$(x-y)^3 = x^3 - y^3 - 3x^2y + 3xy^2$

$= x^3 - y^3 - 3xy + (x-y)$

$= x^3 - y^3 = (x-y)^3 + 3xy + (x-y)$

$= (4)^3 + 3 \times 21 \times 4$ (substituting $x-y = 4$ and $xy = 21$)

$= 64 + 252$

$= 316$

15. $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

$= 2\,(4 + 1)$

$= 2 \times 5$

$= 10$

Exercise 3E

1. **(a)** $a^2 + 14a + 49 = a^2 + 2(a)(7) + 7^2$

$= (a+7)^2$

(b) $4b^2 + 4b + 1 = (2b)^2 + 2(2b)(1) + 1^2$

$= (2b+1)^2$

(c) $c^2 + 2cd + d^2 = (c+d)^2$

(d) $4h^2 + 20hk + 25k^2 = (2h)^2 + 2(2h)(5k) + (5k)^2$

$= (2h+5k)^2$

2. **(a)** $m^2 - 10m + 25 = m^2 - 2(m)(5) + 5^2$

$= (m-5)^2$

(b) $169n^2 - 52n + 4 = (13n)^2 - 2(13n)(2) + 2^2$

$= (13n-2)^2$

(c) $81 - 180p + 100p^2 = 9^2 - 2(9)(10p) + (10p)^2$

$= (9-10p)^2$

(d) $49q^2 - 42qr + 9r^2 = (7q)^2 - 2(7q)(3r) + (3r)^2$

$= (7q-3r)^2$

3. **(a)** $s^2 - 144 = s^2 - 12^2$

$= (s+12)(s-12)$

(b) $36t^2 - 25 = (6t)^2 - 5^2$

$= (6t+5)(6t-5)$

(c) $225 - 49u^2 = 15^2 - (7u)^2$

$= (15+7u)(15-7u)$

(d) $49w^2 - 81x^2 = (7w)^2 - (9x)^2$

$= (7w+9x)(7w-9x)$

4. **(a)** $59^2 - 41^2 = (59+41)(59-41)$

$= 100 \times 18$

$= 1800$

(b) $7.7^2 - 2.3^2 = (7.7+2.3)(7.7-2.3)$

$= 10 \times 5.4$

$= 54$

5. **(a)** $3a^2 + 12a + 12 = 3(a^2 + 4a + 4)$

$= 3[a^2 + 2(a)(2) + 2^2]$

$= 3(a+2)^2$

(b) $25b^2 + 5bc + \frac{1}{4}c^2 = (5b)^2 + 2(5b)\left(\frac{1}{2}c\right) + \left(\frac{1}{2}c\right)^2$

$= \left(5b + \frac{1}{2}c\right)^2$

(c) $\frac{16}{49}d^2 + \frac{8}{35}df + \frac{1}{25}f^2 = \left(\frac{4}{7}d\right)^2 + 2\left(\frac{4}{7}d\right)\left(\frac{1}{5}f\right) + \left(\frac{1}{5}f\right)^2$

$= \left(\frac{4}{7}d + \frac{1}{5}f\right)^2$

(d) $h^4 + 2h^2k + k^2 = (h^2)^2 + 2(h^2)(k) + k^2$

$= (h^2+k)^2$

6. **(a)** $36m^2 - 48mn + 16n^2 = 4(9m^2 - 12mn + 4n^2)$

$= 4[(3m)^2 - 2(3m)(2n) + (2n)^2]$

$= 4(3m-2n)^2$

(b) $\frac{1}{3}p^2 - \frac{2}{3}pq + \frac{1}{3}q^2 = \frac{1}{3}(p^2 - 2pq + q^2)$

$= \frac{1}{3}[p^2 - 2(p)(q) + q^2]$

$= \frac{1}{3}(p-q)^2$

(c) $16r^2 - rs + \frac{1}{64}s^2 = (4r)^2 - 2(4r)\left(\frac{1}{8}s\right) + \left(\frac{1}{8}s\right)^2$

$= \left(4r - \frac{1}{8}s\right)^2$

(d) $25 - 10tu + t^2u^2 = 5^2 - 2(5)(tu) + (tu)^2$

$= (5-tu)^2$

7. **(a)** $32a^2 - 98b^2 = 2(16a^2 - 49b^2)$

$= 2[(4a)^2 - (7b)^2]$

$= 2(4a+7b)(4a-7b)$

(b) $c^2 - \frac{1}{4}d^2 = c^2 - \left(\frac{1}{2}d\right)^2$

$= \left(c + \frac{1}{2}d\right)\left(c - \frac{1}{2}d\right)$

(c) $\frac{9h^2}{100} - 16k^2 = \left(\frac{3h}{10}\right)^2 - (4k)^2$

$= \left(\frac{3h}{100} + 4k\right)\left(\frac{3h}{100} - 4k\right)$

(d) $m^2 - 64n^4 = m^2 - (8n)^2$
$= (m + 8n)(m - 8n)$

8. **(a)** $(a + 3)^2 - 9 = (a + 3)^2 - 3^2$
$= [(a + 3) + 3][(a + 3) - 3]$
$= a(a + 6)$

(b) $16 - 25(b + 3)^2 = -\{[5(b + 3)]^2 - 4^2\}$
$= -[5(b + 3) + 4][5(b + 3) - 4]$
$= -(5b + 19)(5b + 11)$

(c) $c^2 - (d + 2)^2 = [c + (d + 2)][c - (d + 2)]$
$= (c + d + 2)(c - d - 2)$

(d) $(2h - 1)^2 - 4k^2 = (2h - 1)^2 - (2k)^2$
$= (2h - 1 + 2k)(2h - 1 - 2k)$

(e) $25m^2 - (n - 1)^2 = (5m)^2 - (n - 1)^2$
$= [5m + (n - 1)][5m - (n - 1)]$
$= (5m + n - 1)(5m - n + 1)$

(f) $(p + 1)^2 - (p - 1)^2 = [(p + 1) + (p - 1)][(p + 1) - (p - 1)]$
$= 2p(2)$
$= 4p$

9. **(i)** Let the length of the cube be l cm.
$l^2 = x^2 + 4x + 4$
$= x^2 + 2(x)(2) + 2^2$
$= (x + 2)^2$
$l = \sqrt{(x + 2)^2} \quad (l > 0)$
$= x + 2$
$\therefore$ The length of the cube is $(x + 2)$ cm.

(ii) Volume of the cube
$= l^3$
$= l(l^2)$
$= (x + 2)(x^2 + 4x + 4)$
$= x(x^2 + 4x + 4) + 2(x^2 + 4x + 4)$
$= x^3 + 4x^2 + 4x + 2x^2 + 8x + 8$
$= x^3 + 4x^2 + 2x^2 + 4x + 8x + 8$
$= (x^3 + 6x^2 + 12x + 8)\text{ cm}^3$
$\therefore$ The volume of the cube is $(x^3 + 6x^2 + 12x + 8)\text{ cm}^3$.

10. **(a)** $4(x - 1)^2 - 81(x + 1)^2$
$= [2(x - 1)]^2 - [9(x + 1)]^2$
$= [2(x - 1) + 9(x + 1)][2(x - 1) - 9(x + 1)]$
$= (2x - 2 + 9x + 9)(2x - 2 - 9x - 9)$
$= (11x + 7)(-7x - 11)$
$= -(11x + 7)(7x + 11)$

(b) $16x^2 + 8x + 1 - 9y^2$
$= [(4x)^2 + 2(4x)(1) + 1^2] - (3y)^2$
$= (4x + 1)^2 - (3y)^2$
$= (4x + 1 + 3y)(4x + 1 - 3y)$

(c) $4x^2 - y^2 + 4y - 4$
$= 4x^2 - (y^2 - 4y + 4)$
$= (2x)^2 - [y^2 - 2(y)(2) + 2^2]$
$= (2x)^2 - (y - 2)^2$
$= [2x + (y - 2)][2x - (y - 2)]$
$= 2(x + y - 2)(2x - y + 2)$

(d) $13x^2 + 26xy + 13y^2 - 13$
$= 13(x^2 + 2xy + y^2 - 1)$
$= 13\{[x^2 + 2(x)(y) + y^2] - 1^2\}$
$= 13[(x + y)^2 - 1^2]$
$= 13(x + y + 1)(x + y - 1)$

Exercise 3F

1. **(a)** $45x^2 - 81xy = 9x(5x - 9y)$
(b) $39xy - 15x^2z = 3x(13y - 5xz)$
(c) $xy^2z^2 - x^2y^3 = xy^2(z^2 - xy)$
(d) $-15\pi x^3y - 10\pi x^3 = -5\pi x^3(3y + 2)$

2. **(a)** $6a(x - 2y) + 5(x - 2y) = (x - 2y)(6a + 5)$
(b) $2b(x + 3y) - c(3y + x) = 2b(x + 3y) - c(x + 3y)$
$= (x + 3y)(2b - c)$
(c) $3d(5x - y) - 4f(5x - y) = (5x - y)(3d - 4f)$
(d) $5h(x + 3y) + 10k(x + 3y) = 5[h(x + 3y) + 2k(x + 3y)]$
$= 5(x + 3y)(h + 2k)$

3. **(a)** $ax - 5a + 4x - 20 = a(x - 5) + 4(x - 5)$
$= (x - 5)(a + 4)$
(b) $ax + bx + ay + by = x(a + b) + y(a + b)$
$= (a + b)(x + y)$
(c) $x + xy + 2y + 2y^2 = x(1 + y) + 2y(1 + y)$
$= (1 + y)(x + 2y)$
(d) $x^2 - 3x + 2xy - 6y = x(x - 3) + 2y(x - 3)$
$= (x - 3)(x + 2y)$

4. **(a)** $(x + y)(a + b) - (y + z)(a + b)$
$= (a + b)[(x + y) - (y + z)]$
$= (a + b)(x + y - y - z)$
$= (a + b)(x - z)$

(b) $(c + 2d)^2 - (c + 2d)(3c - 7d)$
$= (c + 2d)[(c + 2d) - (3c - 7d)]$
$= (c + 2d)(c + 2d - 3c + 7d)$
$= (c + 2d)(-2c + 9d)$

(c) $x(2h - k) + 3y(k - 2h)$
$= x(2h - k) - 3y(2h - k)$
$= (2h - k)(x - 3y)$

(d) $6x(4m - n) - 2y(n - 4m)$
$= 2[3x(4m - n) - y(n - 4m)]$
$= 2[3x(4m - n) + y(4m - n)]$
$= 2(4m - n)(3x + y)$

5. **(a)** $3ax + 28by + 4ay + 21bx$
$= 3ax + 4ay + 21bx + 28by$
$= a(3x + 4y) + 7b(3x + 4y)$
$= (3x + 4y)(a + 7b)$

(b) $12cy + 20c - 15 - 9y$
$= 4c(3y + 5) - 3(5 + 3y)$
$= 4c(3y + 5) - 3(3y + 5)$
$= (3y + 5)(4c - 3)$

(c) $dy + fy - fz - dz = y(d + f) - z(f + d)$
$= y(d + f) - z(d + f)$
$= (d + f)(y - z)$

(d) $3x^2 + 6xy - 4xz - 8yz$

$= 3x(x + 2y) - 4z(x + 2y)$

$= (x + 2y)(3x - 4z)$

(e) $2xy - 8x + 12 - 3y = 2x(y - 4) + 3(4 - y)$

$= 2x(y - 4) - 3(y - 4)$

$= (y - 4)(2x - 3)$

(f) $5xy - 25x^2 + 50x - 10y$

$= 5(xy - 5x^2 + 10x - 2y)$

$= 5[x(y - 5x) + 2(5x - y)]$

$= 5[x(y - 5x) - 2(y - 5x)]$

$= 5(y - 5x)(x - 2)$

(g) $x^2y^2 - 5x^2y - 5xy^2 + xy^3$

$= xy(xy - 5x - 5y + y^2)$

$= xy[x(y - 5) + y(-5 + y)]$

$= xy[x(y - 5) + y(y - 5)]$

$= xy(y - 5)(x + y)$

(h) $kx + hy - hx - ky = kx - hx - ky + hy$

$= x(k - h) + y(-k + h)$

$= x(k - h) - y(k - h)$

$= (k - h)(x - y)$

6. **(a)** $144p(y - 5x^2) - 12q(10x^2 - 2y)$

$= 144p(y - 5x^2) + 24q(y - 5x^2)$

$= 24[6p(y - 5x^2) + q(y - 5x^2)]$

$= 24(y - 5x^2)(6p + q)$

(b) $2(5x + 10y)(2y - x)^2 - 4(6y + 3x)(x - 2y)$

$= 10(x + 2y)(2y - x^2) - 12(2y + x)(x - 2y)$

$= 10(x + 2y)(x - 2y)^2 - 12(x + 2y)(x - 2y)$

$= 2(x + 2y)(x - 2y)[5(x - 2y) - 6]$

$= 2(x + 2y)(x - 2y)(5x - 10y - 6)$

7. **(i)** $\frac{1}{3}p^2q + \frac{4}{3}p^2r = \frac{1}{3}p^2(q + 4r)$

(ii) When $p = 1.2$, $q = 36$ and $r = 16$,

$\frac{1}{3} \times 1.2^2 \times 36 + \frac{4}{3} \times 1.2^2 \times 16$

$= \frac{1}{3}(1.2)^2\,[36 + 4(16)]$

$= \frac{1}{3}(1.44)(36 + 64)$

$= \frac{1}{3}(1.44)(100)$

$= \frac{1}{3} \times 144$

$= 48$

8. **(i)** $x^3 + 3x - x^2 - 3$

$= x(x^2 + 3) - (x^2 + 3)$

$= (x^2 + 3)(x - 1)$

(ii) $(x^2 - 3)^3 - (2 - x^2)^2 + 3(x^2 - 3)$

$= (x^2 - 3)^3 + 3(x^2 - 3) - (x^2 - 2)^2$

$= (x^2 - 3)^3 + 3(x^2 - 3) - [(x^2 - 3) + 1]^2$

$= (x^2 - 3)^3 + 3(x^2 - 3) - [(x^2 - 3)^2 + 2(x^2 - 3) + 1]$

$= (x^2 - 3)^3 + 3(x^2 - 3) - (x^2 - 3)^2 - 2(x^2 - 3) - 1$

$= (x^2 - 3)^3 + (x^2 - 3) - (x^2 - 3)^2 - 1$

$= (x^2 - 3)^3 + 3(x^2 - 3) - (x^2 - 3)^2 - 3 - 2(x^2 - 3) + 2$

$= [(x^2 - 3)^3 + 3(x^2 - 3) - (x^2 - 3)^2 - 3] - 2[(x^2 - 3) - 1]$

$= [(x^2 - 3)^2 + 3](x^2 - 3 - 1) - 2(x^2 - 4)$ (From **(i)**)

$= [(x^2)^2 - 2(x^2)(3) + 3^2 + 3](x^2 - 4) - 2(x^2 - 4)$

$= (x^4 - 6x^2 + 12)(x^2 - 4) - 2(x^2 - 4)$

$= (x^4 - 6x^2 + 12 - 2)(x^2 - 4)$

$= (x^4 - 6x^2 + 10)(x^2 - 4)$

Review Exercise 3

1. **(i)** $(3x + 2y)^3$

$= (3x)^3 + 3(3x)^2(2y) + 3(3x)(2y)^2 + (2y)^3$

$= 9x^3 + 54x^2y + 36xy^2 + 8y^3$

(ii) $(a + bc)^3$

$= (a^2)^3 + 3(a^2)^2(bc) + 3(a^2)(bc)^2 + (bc)^3$

$= a^6 + 3a^4bc + 3a^2b^2c^2 + b^3c^3$

(iii) $(-7a + 2b^2)^3$

$= (2b^2 - 7a)^3$

$= (2b^2)^3 - 3(2b^2)^2(7a) + 3(7a)^2(2b^2) - (7a)^3$

$= 8b^6 - 84ab^4 + 294a^2b^2 - 343a^3$

(iv) $(x^2 - y - z)^3$

$= [x^2 - (y + z)]^3$

$= (x^2)^3 - 3(x^2)^2(y + z) + 3x^2(y+z)^2 - (y + z)^3$

$= x^6 - 3x^4(y + z) + 3x^2(y^2 + 2yz + z^2) - y^3 - 3y^2z - 3yz^2 - z^3$

$= x^6 - 3x^4y - 3x^4z + 3x^2y^2 + 6x^2yz + 3x^2z^2 - y^3 - 3y^2z - 3yz^2 - z^3$

(v) $(a - 2b - 3c)^3$

$= [\,a - (2b - 3c)]^3$

$= a^3 - 3a^2(2b + 3c) + 3a(2b + 3c)^2 - (2b + 3c)^3$

$= a^3 - 3a^2(2b + 3c) + 2a(4b^2 + 12bc + 9c^2) - 8b^3 - 36b^2c - 54bc^2$

$= a^3 - 6a^2b - 9a^2c + 8ab^2 + 24abc + 18ac^2 - 8b^3 - 36b^2c - 54bc^2 - 9c^3$

(vi) $(\mathrm{p}^2 - \mathrm{q}^2 - \mathrm{r}^2)^3$

$= [\mathrm{p}^2 - (\mathrm{q}^2 - \mathrm{r}^2)]^3$

$= (\mathrm{p}^2)^3 - 3(\mathrm{p}^2)^2(\mathrm{q}^2 + \mathrm{r}^2) + 3\mathrm{p}^2(\mathrm{q}^2 + \mathrm{r}^2)^2 - (\mathrm{q}^2 + \mathrm{r}^2)^3$

$= \mathrm{p}^6 - 3\mathrm{p}^4(\mathrm{q}^2 + \mathrm{r}^2) + 3\mathrm{p}^2(\mathrm{q}^4 + 2\mathrm{q}^2\mathrm{r}^2 + \mathrm{r}^4) - (\mathrm{q}^2)^3 - 3(\mathrm{q}^2)^2(\mathrm{r}^2) - 3\mathrm{q}^2(\mathrm{r}^2)^2 - (\mathrm{r}^2)^3 3$

b. $(5.83)^3 - 3(5.83)^2(3.83) + 3(5.83)(3.83)^2 - (3.83)^3$

Using cube of the difference of two terms, we get

$= (5.83 - 3.83)^3$

$= (2.00)^3$

$= 8.00$

3. **(a)** $-2a(a - 5b + 7) = -2a^2 + 10ab - 14a$

(b) $(2c + 3d)(3c + 4d) = 2c(3c + 4d) + 3d(3c + 4d)$
$= 6c^2 + 8cd + 9cd + 12d^2$
$= 6c^2 + 17cd + 12d^2$

(c) $(k + 3h)(5h - 4k) = k(5h - 4k) + 3h(5h - 4k)$
$= 5hk - 4k^2 + 15h^2 - 12hk$
$= -4k^2 + 5hk - 12hk + 15h^2$
$= -4k^2 - 7hk + 15h^2$

(d) $(2m + 1)(m^2 + 3m - 1) = 2m(m^2 + 3m - 1) + (m^2 + 3m - 1)$
$= 2m^3 + 6m^2 - 2m + m^2 + 3m - 1$
$= 2m^3 + 6m^2 + m^2 - 2m + 3m - 1$
$= 2m^3 + 7m^2 + m - 1$

4. **(a)** $2p(3p - 5q) - q(2q - 3p) = 6p^2 - 10pq - 2q^2 + 3pq$
$= 6p^2 - 10pq + 3pq - 2q^2$
$= 6p^2 - 7pq - 2q^2$

(b) $-4s(3s + 4r) - 2r(2r - 5s) = -12s^2 - 16sr - 4r^2 + 10sr$
$= -12s^2 - 16sr + 10sr - 4r^2$
$= -12s^2 - 6sr - 4r^2$

(c) $(8t - u)(t + 9u) - t(2u - 7t) = 8t(t + 9u) - u(t + 9u) - t(2u - 7t)$
$= 8t^2 + 72tu - tu - 9u^2 - 2tu + 7t^2$
$= 8t^2 + 7t^2 + 72tu - tu - 2tu - 9u^2$
$= 15t^2 + 69tu - 9u^2$

(d) $(2w + 3x)(w - 5x) - (3w + 7x)(w - 7x)$
$= 2w(w - 5x) + 3x(w - 5x) - [3w(w - 7x) + 7x(w - 7x)]$
$= 2w^2 - 10wx + 3wx - 15x^2 - (3w^2 - 21wx + 7wx - 49x^2)$
$= 2w^2 - 10wx + 3wx - 15x^2 - 3w^2 + 21wx - 7wx + 49x^2$
$= 2w^2 - 3w^2 - 10wx + 3wx + 21wx - 7wx - 15x^2 + 49x^2$
$= -w^2 + 7wx - 34x^2$

5. **(a)** $x^2 = x \times x$
$-63y^2 = y \times (-63y)$ or $(-y) \times 63y$
$= 3y \times (-21y)$ or $(-3y) \times 21y$
$= 7y \times (-9y)$ or $(-7y) \times 9y$

$\times$	x	$9y$
x	x^2	$9xy$
$-7y$	$-7xy$	$-63y^2$

$(-7xy) + 9xy = 2xy$
$\therefore\ x^2 + 2xy - 63y^2 = (x - 7y)(x + 9y)$

(b) $2x^2 = 2x \times x$
$3y^2 = y \times 3y$ or $(-y) \times (-3y)$

$\times$	x	y
$2x$	$2x^2$	$2xy$
$3y$	$3xy$	$3y^2$

$3xy + 2xy = 5xy$
$\therefore\ 2x^2 + 5xy + 3y^2 = (2x + 3y)(x + y)$

(c) $6x^2y^2 = 6xy \times xy$ or $3xy \times 2xy$
$-4 = 1 \times (-4)$ or $(-1) \times 4$
$= 2 \times (-2)$ or $(-2) \times 2$

$\times$	$2xy$	1
$3xy$	$6x^2y^2$	$3xy$
-4	$-8xy$	-4

$(-8xy) + 3xy = -5xy$
$\therefore\ 6x^2y^2 - 5xy - 4 = (3xy - 4)(2xy + 1)$

(d) $3z - 8xyz + 4x^2y^2z = z(3 - 8xy + 4x^2y^2)$
$3 = 3 \times 1$
$4x^2y^2 = xy \times 4xy$ or $(-xy) \times (4xy)$
$= 2xy \times 2xy$ or $(-2xy) \times (-2xy)$

$\times$	1	$-2xy$
3	3	$-6xy$
$-2xy$	$-2xy$	$4x^2y^2$

$(-2xy) + (-6xy) = -8xy$
$\therefore\ 3z - 8xyz + 4x^2y^2z = z(3 - 2xy)(1 - 2xy)$

6. **(a)** $(-x + 5y)^2 = (-x)^2 + 2(-x)(5y) + (5y)^2$
$= x^2 - 10xy + 25y^2$

(b) $(x^2 + y)(x^2 - y) = (x^2)^2 - y^2$
$= x^4 - y^2$

(c) $\left(3x + \frac{4}{5}y\right)^2 = (3x)^2 + 2(3x)\left(\frac{4}{5}y\right) + \left(\frac{4}{5}y\right)^2$

$= 9x^2 + \frac{24}{5}xy + \frac{16}{25}y^2$

(d) $\left(-\frac{1}{4}x - \frac{1}{6}y\right)^2 = \left(-\frac{1}{4}x\right)^2 + 2\left(-\frac{1}{4}x\right)\left(-\frac{1}{6}y\right) + \left(-\frac{1}{6}y\right)^2$

$= \frac{1}{16}x^2 + \frac{1}{12}xy + \frac{1}{36}y^2$

(e) $\left(5x - \frac{7}{4}y\right)\left(5x + \frac{7}{4}y\right) = (5x)^2 - \left(\frac{7}{4}y\right)^2$

$= 25x^2 - \frac{49}{16}y^2$

(f) $\left(\frac{3}{4}xy + \frac{1}{3}z\right)\left(\frac{3}{4}xy - \frac{1}{3}z\right) = \left(\frac{3}{4}xy\right)^2 - \left(\frac{1}{3}z\right)^2$

$= \frac{9}{16}x^2y^2 - \frac{1}{9}z^2$

7. **(a)** $1 - 121x^2 = 1^2 - (11x)^2$
$= (1 + 11x)(1 - 11x)$

(b) $x^2 + 6xy + 9y^2 = x^2 + 2(x)(3y) + (3y)^2$
$= (x + 3y)^2$

(c) $25x^2 - 100xy + 100y^2 = 25(x^2 - 4xy + 4y^2)$
$= 25[x^2 - 2(x)(2y) + (2y)^2]$
$= 25(x - 2y)^2$

(d) $36y^2 - 49(x + 1)^2 = (6y)^2 - [7(x + 1)]^2$
$= [6y + 7(x + 1)][6y - 7(x + 1)]$
$= (6y + 7x + 7)(6y - 7x - 7)$

8. **(a)** $-14xy - 21y^2 = -7y(2x + 3y)$

(b) $9xy^2 - 36x^2y = 9xy(y - 4x)$

(c) $(2x - 3y)(a + b) + (x - y)(b + a)$

$= (2x - 3y)(a + b) + (x - y)(a + b)$

$= (a + b)(2x - 3y + x - y)$

$= (a + b)(3x - 4y)$

(d) $5(x - 2y) - (x - 2y)^2 = (x - 2y)[5 - (x - 2y)]$

$= (x - 2y)(5 - x + 2y)$

(e) $x^2 + 3xy + 2x + 6y = x(x + 3y) + 2(x + 3y)$

$= (x + 3y)(x + 2)$

(f) $3x^3 - 2x^2 + 3x - 2 = x^2(3x - 2) + (3x - 2)$

$= (3x - 2)(x^2 + 1)$

(g) $4cx - 6cy - 8dx + 12dy = 2(2cx - 3cy - 4dx + 6dy)$

$= 2[c(2x - 3y) - 2d(2x - 3y)]$

$= 2(2x - 3y)(c - 2d)$

(h) $5xy - 10x - 12y + 6y^2 = 5x(y - 2) + 6y(-2 + y)$

$= 5x(y - 2) + 6y(y - 2)$

$= (y - 2)(5x + 6y)$

9. $x^3 + x^2 - 4x - 4 = x^2(x + 1) - 4(x + 1)$

$= (x + 1)(x^2 - 4)$

$= (x + 1)(x^2 - 2^2)$

$= (x + 1)(x + 2)(x - 2)$

10. **(a)** $899^2 = (900 - 1)^2$

$= 900^2 - 2(900) + 1^2$

$= 810\ 000 - 1800 + 1$

$= 808\ 201$

(b) $659^2 - 341^2 = (659 + 341)(659 - 341)$

$= 1000 \times 318$

$= 318\ 000$

11. $2(x - y)^2 = 116$

$(x - y)^2 = 58$

$x^2 - 2xy + y^2 = 58$

Since $xy = 24$,

$\therefore\ x^2 - 2(24) + y^2 = 58$

$x^2 - 48 + y^2 = 58$

$x^2 + y^2 = 106$

12. **(i)** $(f + 3)^2 = f^2 + 2(f)(3) + 3^2$

$= f^2 + 6f + 9$

(ii) From **(i)**,

$[(2h + k) + 3]^2 = (2h + k)^2 + 6(2h + k) + 9$

$= (2h)^2 + 2(2h)(k) + k^2 + 12h + 6k + 9$

$= 4h^2 + 4hk + k^2 + 12h + 6k + 9$

Challenge Yourself

1. $(a + b)^2 = a^2 + b^2$

$a^2 + 2ab + b^2 = a^2 + b^2$

$2ab = 0$

$ab = 0$

$\therefore\ \sqrt{ab} = 0$

2. Let $a = h^2 + k^2$ and $b = m^2 + n^2$.

$h^2 + k^2 - m^2 - n^2 = 15$

$h^2 + k^2 - (m^2 + n^2) = 15$

$a - b = 15$

$(h^2 + k^2)^2 + (m^2 + n^2)^2 = 240.5$

$a^2 + b^2 = 240.5$

$(a - b)^2 = a^2 - 2ab + b^2$

$= a^2 + b^2 - 2ab$

$15^2 = 240.5 - 2ab$

$2ab = 240.5 - 225$

$= 15.5$

$(a + b)^2 = a^2 + 2ab + b^2$

$= a^2 + b^2 + 2ab$

$= 240.5 + 15.5$

$= 256$

$\therefore\ h^2 + k^2 + m^2 + n^2 = a + b$

$= \sqrt{256} \quad (h^2 + k^2 + m^2 + n^2 > 0)$

$= 16$

Chapter 4 Graphs of Linear Equations and Simultaneous Linear Equations

TEACHING NOTES

Suggested Approach

In this chapter, students will learn linear equations in the form $ax + by = k$.

They have learnt how to solve simple linear equations. Here, they will be learning how to solve simultaneous linear equations, where a pair of values of x and of y satisfies two linear equations simultaneously, or at the same time. Students are expected to know how to solve them graphically and algebraically and apply this to real-life scenarios by the end of the chapter.

Teachers can build up on past knowledge learnt by students when covering this chapter.

Section 4.1: Gradient of a Straight Line

Teachers should teach students how to take two points on the line and use it to calculate the vertical change (rise) and horizontal change (run), and then the gradient of the straight line.

To make learning more interactive, students can explore how the graph of a straight line in the form $y = mx + c$ changes when either m or c varies (see Investigation: Equation of a Straight Line). Through this investigation, students should be able to observe what happens to the line when m varies. Students should also learn how to differentiate between lines with a positive value of m, a negative value of m and when the value of m is 0.

Section 4.2: Further Applications of Linear Graphs in Real-World Contexts

Teachers can give examples of linear graphs used in many daily situations and explain what each of the graphs is used for. Through Worked Example 2, students will learn how the concepts of gradient and y-intercept can be applied and about their significance in real-world contexts and hence solve similar problems.

Section 4.3: Graphs of Linear Equations in the form $ax + by = k$

Before students start plotting the functions, they should revise the choice of scales and labelling of scales on both axes. Students are often weak in some of these areas. Many errors in students' work arise from their choice of scales. Teachers should spend some time to ensure students learn how to choose an appropriate scale. At this stage however, the choice of scales are specified in most questions.

Section 4.4: Solving Simultaneous Linear Equations Using Graphical Method

It is important teachers state the concept clearly that the point(s) of intersection of two graphs given the solution of a pair of simultaneous equations and this can be illustrated by solving a pair of linear simultaneous equations and then plotting the graphs of these two linear equations to verify the results (see Investigation: Solving Simultaneous Linear Equations Graphically)

Teachers should show clearly that a pair of simultaneous linear equations may have an infinite number of solutions or no solution (see Class Discussion: Coincident Lines and Parallel Lines, and Thinking Time on page 127).

Section 4.5: Solving Simultaneous Linear Equations Using Algebraic Methods

The ability to solve equations is crucial to the study of mathematics. The concept of solving simultaneous linear equations by adding or subtracting both sides of equations can be illustrated using physical examples. An example is drawing a balance and adding or removing coins from both sides of the balance.

Some students make common errors when they are careless in the multiplication or division of both sides of an equation and they may forget that all terms must be multiplied or divided by the same number throughout. The following are some examples.

- $x + 3y = 5$ is taken to imply $2x + 6y = 5$
- $5x + 15y = 14$ is taken to imply $x + 3y = 14$, and then $x = 14 - 3y$

Section 4.6: Applications of Simultaneous Equations in Real-World Contexts

Weaker students may have problems translating words into simultaneous linear equations. Teachers may wish to show more examples and allow more practice for students. Teachers may also want to group students of varying ability together, so that the better students can help the weaker students.

Challenge Yourself

Question 1 can be solved if the Thinking Time activity on page 127 has been discussed. The simultaneous equations in Question 2 can be converted to a familiar form by substituting $\frac{1}{x}$ with a and $\frac{1}{y}$ with b.

Teachers can slowly guide the students for Question 3 if they need help in forming the simultaneous equations.

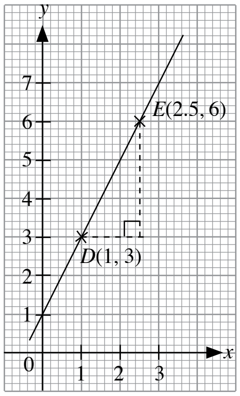
For Questions 4 and 5, teachers can advise students to eliminate one unknown variable and then applying the guess and check method.

WORKED SOLUTIONS

Investigation (Equation of a Straight Line)

1. As the value of c changes, the y-coordinate of the point of intersection of the line with the y-axis changes. The coordinates of the point where the line cuts the y-axis are $(0, c)$.
2. As the value of m increases from 0 to 5, the steepness of the line increases.
3. As the value of m decreases from 0 to –5, the steepness of the line increases.
4. A line with a positive value for m slopes upwards from the left to the right while a line with a negative value for m slopes downwards from the left to the right.

Class Discussion (Gradients of Straight Lines)



(i) Gradient of $DE = \dfrac{3}{1.5}$

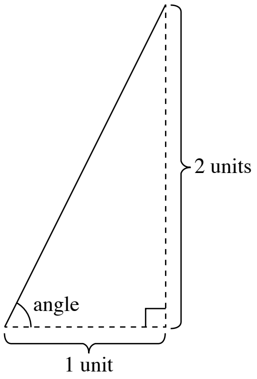
$= 2$

(ii) Yes, gradient of DE = gradient of AB.

(iii) Hence, we can choose *any* two points on a line to find its gradient because the gradient of a straight line is constant.

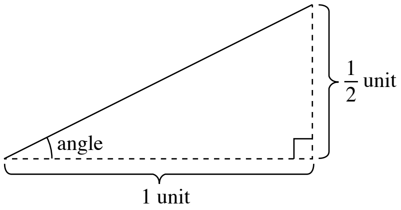
Class Discussion (Gradients in the Real World)

1. Angle of inclination = 45°
2.



Angle of inclination = 63°

3.



Angle of inclination = 27°

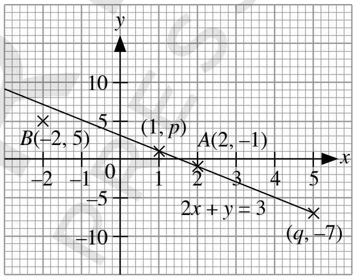
4. A road with a gradient of 1 is generally considered to be steep.

Teachers may wish to get students to name some roads in Pakistan which they think may have an approximate gradient of 1 and to ask students how they can determine the gradients of the roads they have named.

5. A road with a gradient of $\frac{1}{2}$ is generally considered to be steep.

Investigation (Graphs of *ax* + *by* = *k*)

1. **(i)**



(ii) The point $A(2, -1)$ lies on the graph. The point $B(-2, 5)$ does not lie on the graph.

When $x = 2$, $2(2) + y = 3$

$4 + y = 3$

$y = -1$

When $x = -2$, $2(-2) + y = 3$

$-4 + y = 3$

$y = 7 \neq 5$

$A(2, -1)$ satisfies the equation $2x + y = 3$. $B(-2, 5)$ does not satisfy the equation $2x + y = 3$.

(iii) When $x = 1$, $y = p = 1$.

(iv) When $y = -7$, $x = q = 5$.

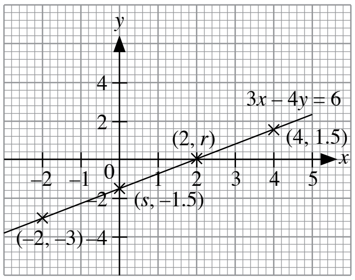
(v) The graph of $y = -2x + 3$ coincides with the graph of $2x + y = 3$.

$2x + y = 3$

$2x - 2x + y = -2x + 3$ (Subtract $2x$ from both sides)

$y = -2x + 3$

2. **(i)**



(ii) When $x = 2$, $y = r = 0$

(iii) When $y = -1.5$, $x = s = 0$

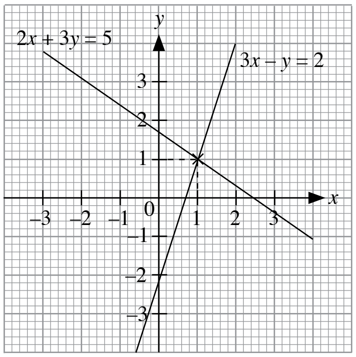
(iv) The coordinates of two other points are $(-2, -3)$ and $(4, 1.5)$.
Other points can be used, as long as they lie on the line.

(v) The graph of $y = \frac{3}{4}x - \frac{3}{2}$ coincides with the graph of $3x - 4y = 6$.

$$3x - 4y = 6$$
$$3x - 3x - 4y = -3x + 6 \qquad \text{(Subtract } 3x \text{ from both sides)}$$
$$-4y = -3x + 6$$
$$\frac{-4y}{-4} = \frac{-3x + 6}{-4} \qquad \text{(Divide both sides by } -4\text{)}$$
$$y = \frac{3}{4}x - \frac{3}{2}$$

Investigation (Solving Simultaneous Linear Equation Graphically)

1. **(i)**



(ii) The coordinates of the point of intersection of the two graphs are (1, 1).

(iii) For $2x + 3y = 5$

When $x = -2$, $2(-2) + 3y = 5$
$y = 3$

When $x = 0$, $2(0) + 3y = 5$
$y = 1\frac{2}{3} \neq -2$

When $x = 1$, $2(1) + 3y = 5$
$y = 1$

When $x = 2$, $2(2) + 3y = 5$
$y = \frac{1}{3} \neq 4$

When $x = 4$, $2(4) + 3y = 5$
$y = -1$

For $3x - y = 2$

When $x = -2$, $3(-2) - y = 2$
$y = -8 \neq 3$

When $x = 0$, $3(0) - y = 2$
$y = -2$

When $x = 1$, $3(1) - y = 2$
$y = 1$

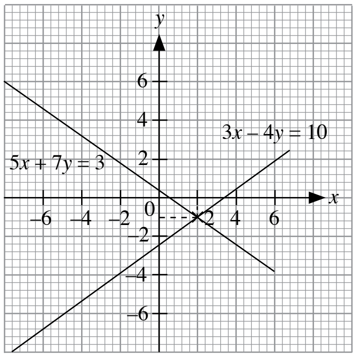
When $x = 2$, $3(2) - y = 2$
$y = 4$

When $x = 4$, $3(4) - y = 2$
$y = 10$

The pair of values satisfying both equations is $x = 1$, $y = 1$.

The pair of values is the same as the point of intersection of the two graphs.

2. **(i)**



(ii) The coordinates of the point of intersection of the two graphs are (2, –1)

(iii) The pair of values of x and y that satisfies both equations are $x = 2$ and $y = -1$.

The coordinates of the point of intersection of the two graphs is the pair of values of x and y that satisfies both the equations.

A coordinates that lies on one line will satisfy the equation of that line. The same applies to the second line. Hence, the coordinates of the point of intersection is the same as the point that lies on both lines and that satisfy both equations.

Class Discussion (Choice of Appropriate Scales for Graphs and Accuracy of Graphs)

1. The graphs should look different to students who have used different scales in both axes.
Teachers should remind students to make a table of values, with at least 3 points, so as to construct the graph of a linear equation. Though two points are sufficient to draw a straight line, the third point will act as a check for the accuracy of the straight line.
It is likely that most students will use 1 cm to 1 unit for both scales. For the better students, prompt them to experiment with other scales, such as 2 cm to 1 unit, 4 cm to 1 unit or 5 cm to 1 unit.

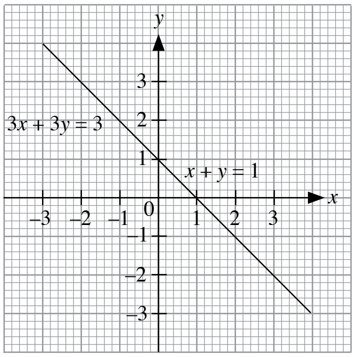
2. **(i)** $y = 2.9$
(ii) $x = -0.6$
If students use 1 cm to 1 unit for both scales, they would discover that the point in ***(i)*** *lies between squares on the graph paper.*

3. By substituting the given value into the linear equation, one can check for the accuracy of the answers.

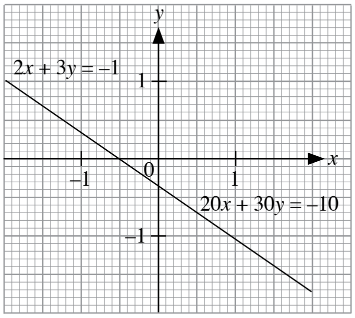
4. Use a larger scale (from 1 cm to 1 unit to 2 cm to 1 unit) and redraw the graph.

Class Discussion (Coincident Lines and Parallel Lines)

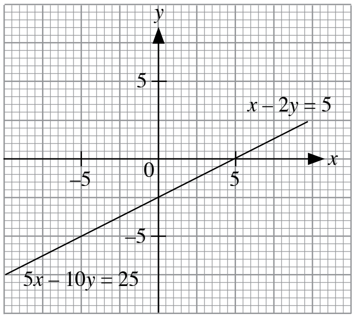
1. **(a)** **(i)**



(ii)



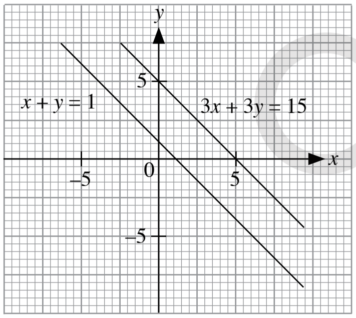
(iii)



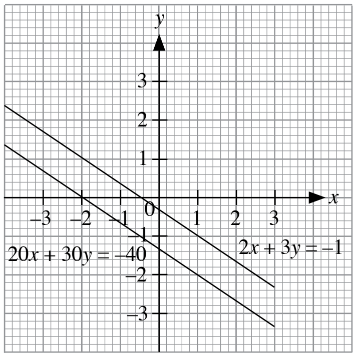
(b) The graphs of each pair of simultaneous equations are a pair of lines that coincide.

(c) Yes, each pair of simultaneous equations has solutions.
The solutions are all the points that lie on the line.

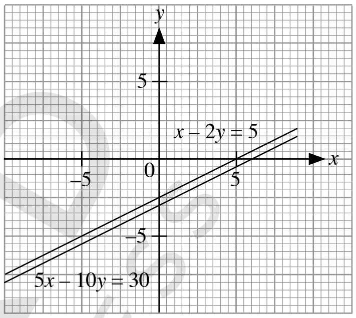
2. **(a)** **(i)**



(ii)



(iii)



(b) The graphs of each pair of simultaneous equations are a pair of parallel lines.

(c) No, each pair of simultaneous equations does not have any solution since they do not intersect and have any point of intersection.

Thinking Time (Page 127)

(a) A pair of simultaneous equations where one equation can be obtained from the other equation through multiplication or division, that is, both equations are equivalent, has infinitely many solutions.

(b) A pair of simultaneous equations where one equation can be contradicted by the other equation has no solution.

Besides the equations in the Class Discussion on the same page, teachers may wish to ask students to come up with their own pairs of simultaneous equations with infinitely many solutions or no solutions.

Thinking Time (Page 129)

The solutions to a linear equation in two variables are the set of x values and y values that satisfy the linear equation. There are infinitely many solutions for all real values of x and y.
For example, the solutions to the equation $2x + y = 13$ is the set $\{(x, y): 2x + y = 13\}$. Some solutions in the set are (1, 11), (2, 9), (3, 7) etc.

Thinking Time (Page 132)

$13x - 6y = 20$ — (1)

$7x + 4y = 18$ — (2)

$7 \times (1)$: $91x - 42y = 140$ — (3)

$13 \times (2)$: $91x + 52y = 234$ — (4)

(3) – (4):

$$(91x - 42y) - (91x + 52y) = 140 - 234$$
$$-94y = -94$$
$$y = 1$$

Substitute $y = 1$ into (1): $13x - 61(1) = 20$

$$13x = 26$$
$$x = 2$$

$\therefore$ The solution is $x = 2$ and $y = 1$.

No. it is not easier to eliminate x first as the LCM of 13 and 7 is larger than 12.

Thinking Time (Page 134)

$7x - 2y = 21$ — (1)

$4x + y = 57$ — (2)

From (2), $x = \frac{57 - y}{4}$ — (3)

Substitute (3) into (1):

$$\left(\frac{57 - y}{4}\right) - 2y = 21$$
$$7(57 - y) - 8y = 84$$
$$399 - 7y - 8y = 84$$
$$15y = 315$$
$$y = 21$$

Substitute $y = 21$ into (3): $x = \frac{57 - 21}{4}$

$$= 9$$

$\therefore$ The solution is $x = 9$ and $y = 21$.

If x is made the subject of equation (1) or (2), we will get the same solution. Making y as the subject of equation is easier since algebraic fractions will not be introduced then.

Thinking Time (Page 135)

$2x + y = 6$ — (1)

$x = 1 - \frac{1}{2}y$ — (2)

$2 \times (2)$: $2x = 2 - y$

$2x + y = 2$ — (3)

Comparing (1) and (3), we notice that the gradients of the 2 equations are the same but with different constants; i.e. they are parallel lines with no solution.

Thinking Time (Page 140)

Let the smaller number be x. Then the greater number is $67 - x$.

$$\therefore (67 - x) - x = 3$$
$$67 - 2x = 3$$
$$2x = 64$$
$$\therefore x = 32$$

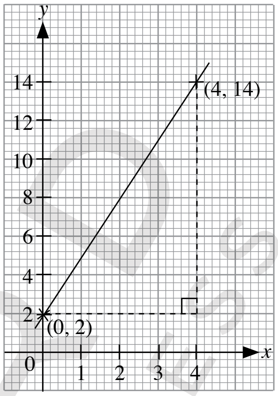
Greater number $= 67 - 32$

$= 35$

The two numbers are 32 and 35.

Practice Now 1

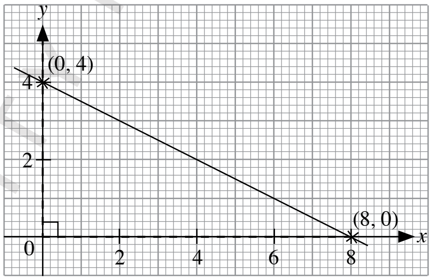
(a)



Gradient $= \frac{12}{4}$

$= 3$

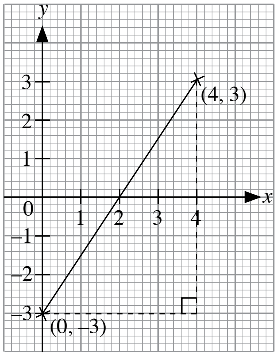
(b)



Gradient $= -\frac{4}{8}$

$= -\frac{1}{2}$

(c)

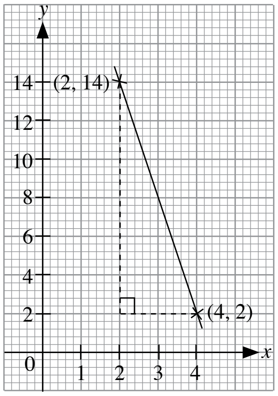


Gradient $= \frac{6}{4}$

$= \frac{3}{2}$

$= 1\frac{1}{2}$

(d)



$$\text{Gradient} = -\frac{12}{2}$$
$$= -6$$

Practise Now 2

(a) Time taken for the technician to repair each computer = 20 minutes

(b) Distance between the technician's workshop and his first customer = 9 km

(c) **(i)** Gradient of $OA = \frac{9}{10}$

The average speed of the technician was $\frac{9}{10}$ km/min.

(ii) Gradient of $AB = 0$

The average speed of the technician was 0 km/min.

(iii) Gradient of $BC = -\frac{4}{5}$

The average speed of the technician was $\frac{4}{5}$ km/min.

(iv) Gradient of $CD = 0$

The average speed of the technician was 0 km/min.

(v) Gradient of $DE = -\frac{5}{7}$

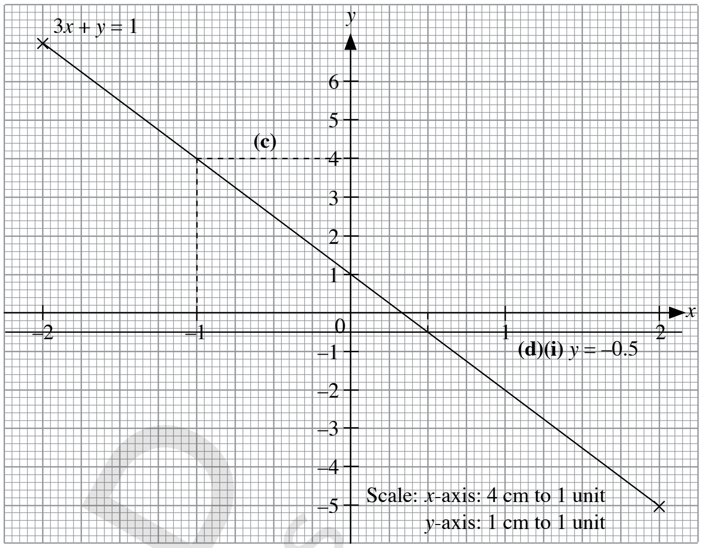
The average speed of the technician was $\frac{5}{7}$ km/min.

Practise Now 3

(a) When $x = -2$, $y = p$,

$$3(-2) + p = 1$$
$$-6 + p = 1$$
$$\therefore p = 7$$

(b)



(c) From the graph in **(b)**,

When $x = -1$,

$q = y = 4$

(d) **(ii)** x-coordinate = 0.5

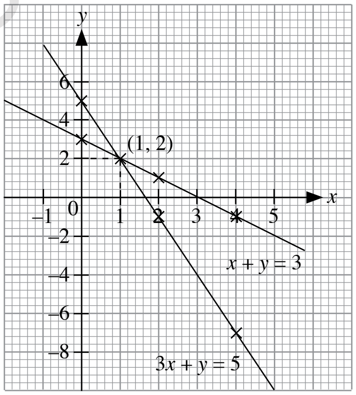
Practise Now 4

1. $x + y = 3$

x	0	2	4
y	3	1	–1

$3x + y = 5$

x	0	2	4
y	5	–1	–7



Scale: x-axis: 1 cm to 1 unit
y-axis: 1 cm to 2 units

The graphs intersect at the point (1, 2).

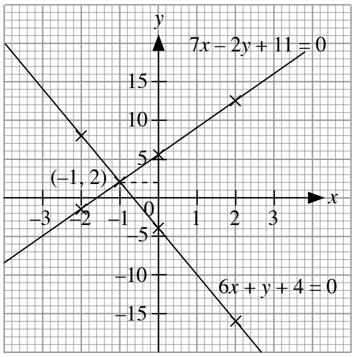
$\therefore$ The solution is $x = 1$ and $y = 2$.

2. $7x - 2y + 11 = 0$

x	–2	0	2
y	–1.5	5.5	12.5

$6x + y + 4 = 0$

x	–2	0	2
y	8	–4	–16



Scale: x-axis: 1 cm to 1 unit
y-axis: 1 cm to 5 units

The graphs intersect at the point (–1, 2).
$\therefore$ The solution is $x = -1$ and $y = 2$.

Practise Now 5

1. **(a)**
$$\begin{aligned} x - y &= 3 \quad \text{— (1)}\\ 4x + y &= 17 \quad \text{— (2)} \end{aligned}$$
(2) + (1):
$$\begin{aligned} (4x + y) + (x - y) &= 17 + 3\\ 4x + y + x - y &= 20\\ 5x &= 20\\ x &= 4 \end{aligned}$$
Substitute $x = 4$ into (2):
$$\begin{aligned} 4(4) + y &= 17\\ 16 + y &= 17\\ y &= 1 \end{aligned}$$
$\therefore$ The solution is $x = 4$ and $y = 1$.

(b)
$$\begin{aligned} 7x + 2y &= 19 \quad \text{— (1)}\\ 7x + 8y &= 13 \quad \text{— (2)} \end{aligned}$$
(2) – (1):
$$\begin{aligned} (7x + 8y) - (7x + 2y) &= 13 - 19\\ 7x + 8y - 7x - 2y &= -6\\ 6y &= -6\\ y &= -1 \end{aligned}$$
Substitute $y = -1$ into (1):
$$\begin{aligned} 7x + 2(-1) &= 19\\ 7x - 2 &= 19\\ 7x &= 21\\ x &= 3 \end{aligned}$$
$\therefore$ The solution is $x = 3$ and $y = -1$.

(c)
$$\begin{aligned} 13x + 9y &= 4 \quad \text{— (1)}\\ 17x - 9y &= 26 \quad \text{— (2)} \end{aligned}$$
(1) + (2):
$$\begin{aligned} (13x + 9y) + (17x - 9y) &= 4 + 26\\ 13x + 9y + 17x - 9y &= 30\\ 30x &= 30\\ x &= 1 \end{aligned}$$
Substitute $x = 1$ into (1):
$$\begin{aligned} 13(1) + 9y &= 4\\ 13 + 9y &= 4\\ 9y &= -9\\ y &= -1 \end{aligned}$$
$\therefore$ The solution is $x = 1$ and $y = -1$.

(d)
$$\begin{aligned} 4x - 5y &= 17 \quad \text{— (1)}\\ x - 5y &= 8 \quad \text{— (2)} \end{aligned}$$
(1) – (2):
$$\begin{aligned} (4x - 5y) - (x - 5y) &= 17 - 8\\ 4x - 5y - x + 5y &= 9\\ 3x &= 9\\ x &= 3 \end{aligned}$$
Substitute $x = 3$ into (2):
$$\begin{aligned} 3 - 5y &= 8\\ -5y &= 5\\ y &= -1 \end{aligned}$$
$\therefore$ The solution is $x = 3$ and $y = -1$.

2.
$$\begin{aligned} 3x - y + 14 &= 0 \quad \text{— (1)}\\ 2x + y + 1 &= 0 \quad \text{— (2)} \end{aligned}$$
(1) + (2):
$$\begin{aligned} (3x - y + 14) + (2x + y + 1) &= 0 + 0\\ 3x - y + 14 + 2x + y + 1 &= 0\\ 5x + 15 &= 0\\ 5x &= -15\\ x &= -3 \end{aligned}$$
Substitute $x = -3$ into (2):
$$\begin{aligned} 2(-3) + y + 1 &= 0\\ y - 5 &= 0\\ y &= 5 \end{aligned}$$
$\therefore$ The solution is $x = -3$ and $y = 5$.

Practise Now 6

(a)
$$\begin{aligned} 2x + 3y &= 18 \quad \text{— (1)}\\ 3x - y &= 5 \quad \text{— (2)} \end{aligned}$$
$3 \times (2)$: $9x - 3y = 15$ — (3)
(1) + (3):
$$\begin{aligned} (2x + 3y) + (9x - 3y) &= 18 + 15\\ 2x + 3y + 9x - 3y &= 33\\ 11x &= 33\\ x &= 3 \end{aligned}$$
Substitute $x = 3$ into (2):
$$\begin{aligned} 3(3) - y &= 5\\ 9 - y &= 5\\ y &= 4 \end{aligned}$$
$\therefore$ The solution is $x = 3$ and $y = 4$.

(b) $4x + y = 11$ — (1)

$3x + 2y = 7$ — (2)

$2 \times (1)$: $8x + 2y = 22$ — (3)

$(3) - (2)$:

$(8x + 2y) - (3x + 2y) = 22 - 7$

$8x + 2y - 3x - 2y = 15$

$5x = 15$

$x = 3$

Substitute $x = 3$ into (1):

$4(3) + y = 11$

$12 + y = 11$

$y = -1$

$\therefore$ The solution is $x = 3$ and $y = -1$.

Practise Now 7

(a) $9x + 2y = 5$ — (1)

$7x - 3y = 13$ — (2)

$3 \times (1)$: $27x + 6y = 15$ — (3)

$2 \times (2)$: $14x - 6y = 26$ — (4)

$(3) + (4)$:

$(27x + 6y) + (14x - 6y) = 15 + 26$

$27x + 6y + 14x - 6y = 41$

$41x = 41$

$x = 1$

Substitute $x = 1$ into (1):

$9(1) + 2y = 5$

$9 + 2y = 5$

$2y = -4$

$y = -2$

$\therefore$ The solution is $x = 1$ and $y = -2$.

(b) $5x - 4y = 17$ — (1)

$2x - 3y = 11$ — (2)

$2 \times (1)$: $10x - 8y = 34$ — (3)

$5 \times (2)$: $10x - 15y = 55$ — (4)

$(3) - (4)$:

$(10x - 8y) - (10x - 15y) = 34 - 55$

$10x - 8y - 10x + 15y = -21$

$7y = -21$

$y = -3$

Substitute $y = -3$ into (2):

$2x - 3(-3) = 11$

$2x + 9 = 11$

$2x = 2$

$x = 1$

$\therefore$ The solution is $x = 1$ and $y = -3$.

Practise Now 8

Method 1:

$\frac{x}{2} - \frac{y}{3} = 4$ — (1)

$\frac{2}{5}x - \frac{y}{6} = 3\frac{1}{2}$ — (2)

$\frac{1}{2} \times (1)$: $\frac{x}{4} - \frac{y}{6} = 2$ — (3)

$(2) - (3)$:

$\left(\frac{2}{5}x - \frac{y}{6}\right) - \left(\frac{x}{4} - \frac{y}{6}\right) = 3\frac{1}{2} - 2$

$\frac{2}{5}x - \frac{y}{6} - \frac{x}{4} + \frac{y}{6} = 1\frac{1}{2}$

$\frac{3}{20}x = 1\frac{1}{2}$

$x = 10$

Substitute $x = 10$ into (1):

$\frac{10}{2} - \frac{y}{3} = 4$

$5 - \frac{y}{3} = 4$

$\frac{y}{3} = 1$

$y = 3$

$\therefore$ The solution is $x = 10$ and $y = 3$.

Method 2:

$\frac{x}{2} - \frac{y}{3} = 4$ — (1)

$\frac{2}{5}x - \frac{y}{6} = 3\frac{1}{2}$ — (2)

$30 \times (1)$: $15x - 10y = 120$ — (3)

$60 \times (2)$: $24x - 10y = 210$ — (4)

$(4) - (3)$:

$(24x - 10y) - (15x - 10y) = 210 - 120$

$24x - 10y - 15x + 10y = 90$

$9x = 90$

$x = 10$

Substitute $x = 10$ into (3):

$15(10) - 10y = 120$

$150 - 10y = 120$

$-10y = -30$

$y = 3$

$\therefore$ The solution is $x = 10$ and $y = 3$.

Practise Now 9

$3y - x = 7$ — (1)

$2x + 3y = 4$ — (2)

From (1), $x = 3y - 7$ — (3)

Substitute (3) into (2):

$2(3y - 7) + 3y = 4$

$6y - 14 + 3y = 4$

$9y = 18$

$y = 2$

Substitute $y = 2$ into (3):

$x = 3(2) - 7$

$= -1$

$\therefore$ The solution is $x = -1$ and $y = 2$.

Practise Now 10

$$\begin{aligned} 3x - 2y &= 8 \quad \text{— (1)} \\ 4x + 3y &= 5 \quad \text{— (2)} \end{aligned}$$

From (1), $3x = 2y + 8$

$$x = \frac{2y + 8}{3} \quad \text{— (3)}$$

Substitute (3) into (2):

$$\begin{aligned} 4\left(\frac{2y + 8}{3}\right) + 3y &= 5 \\ 4(2y + 8) + 9y &= 15 \\ 8y + 32 + 9y &= 15 \\ 17y + 32 &= 15 \\ 17y &= -17 \\ y &= -1 \end{aligned}$$

Substitute $y = -1$ into (3):

$$\begin{aligned} x &= \frac{2(-1) + 8}{3} \\ &= 2 \end{aligned}$$

$\therefore$ The solution is $x = 2$ and $y = -1$.

Practise Now 11

(a) $\dfrac{x - 1}{y - 3} = \dfrac{2}{3}$ — (1)

$\dfrac{x - 2}{y - 1} = \dfrac{1}{2}$ — (2)

From (1),

$$\begin{aligned} 3(x - 1) &= 2(y - 3) \\ 3x - 3 &= 2y - 6 \\ 3x - 2y &= -3 \quad \text{— (3)} \end{aligned}$$

From (2),

$$\begin{aligned} 2(x - 2) &= y - 1 \\ 2x - 4 &= y - 1 \\ y &= 2x - 3 \quad \text{— (4)} \end{aligned}$$

Substitute (4) into (3):

$$\begin{aligned} 3x - 2(2x - 3) &= -3 \\ 3x - 4x + 6 &= -3 \\ -x + 6 &= -3 \\ x &= 9 \end{aligned}$$

Substitute $x = 9$ into (4):

$$\begin{aligned} y &= 2(9) - 3 \\ &= 15 \end{aligned}$$

$\therefore$ The solution is $x = 9$ and $y = 15$.

(b) $3x + 2y = 3$ — (1)

$\dfrac{1}{x + y} = \dfrac{3}{x + 2y}$ — (2)

From (2),

$$\begin{aligned} x + 2y &= 3(x + y) \\ &= 3x + 3y \\ y &= -2x \quad \text{— (3)} \end{aligned}$$

Substitute (3) into (1):

$$\begin{aligned} 3x + 2(-2x) &= 3 \\ 3x - 4x &= 3 \\ x &= -3 \end{aligned}$$

Substitute $x = -3$ into (3):

$$\begin{aligned} y &= -2(-3) \\ &= 6 \end{aligned}$$

$\therefore$ The solution is $x = -3$ and $y = 6$.

Practise Now 12

1. Let the smaller number be x and the greater number be y.

$$\begin{aligned} x + y &= 36 \quad \text{— (1)} \\ y - x &= 9 \quad \text{— (2)} \end{aligned}$$

(1) + (2):

$$\begin{aligned} 2y &= 45 \\ y &= 22.5 \end{aligned}$$

Substitute $y = 22.5$ into (1):

$$\begin{aligned} x + 22.5 &= 36 \\ x &= 13.5 \end{aligned}$$

$\therefore$ The two numbers are 13.5 and 22.5.

2. Let the smaller angle be x and the greater angle be y.

$$\begin{aligned} \frac{1}{3}(x + y) &= 60° \quad \text{— (1)} \\ \frac{1}{4}(y - x) &= 28° \quad \text{— (2)} \end{aligned}$$

$3 \times (1)$: $x + y = 180°$ — (3)

$4 \times (2)$: $y - x = 112°$ — (4)

(3) + (4):

$$\begin{aligned} 2y &= 292° \\ y &= 146° \end{aligned}$$

Substitute $y = 146°$ into (3):

$$\begin{aligned} x + 146° &= 180° \\ x &= 34° \end{aligned}$$

$\therefore$ The two angles are 34° and 146°.

3.

$$\begin{aligned} x + y + 2 &= 2x + 1 \quad \text{— (1)} \\ 2y &= x + 2 \quad \text{— (2)} \end{aligned}$$

From (1),

$y = x - 1$ — (3)

Substitute (3) into (2):

$$\begin{aligned} 2(x - 1) &= x + 2 \\ 2x - 2 &= x + 2 \\ x &= 4 \end{aligned}$$

Substitute $x = 4$ into (3):

$$\begin{aligned} y &= 4 - 1 \\ &= 3 \end{aligned}$$

$$\begin{aligned} \text{Length of rectangle} &= 2(4) + 1 \\ &= 9 \text{ cm} \\ \text{Breadth of rectangle} &= 2(3) \\ &= 6 \text{ cm} \\ \text{Perimeter of rectangle} &= 2(9 + 6) \\ &= 30 \text{ cm} \end{aligned}$$

$\therefore$ The perimeter of the rectangle is 30 cm.

Practise Now 13

Let the numerator of the fraction be x and its denominator be y,

i.e. let the fraction be $\frac{x}{y}$.

$\frac{x+1}{y+1} = \frac{4}{5}$ — (1)

$\frac{x-5}{y-5} = \frac{1}{2}$ — (2)

From (1),

$5(x + 1) = 4(y + 1)$

$5x + 5 = 4y + 4$

$5x - 4y = -1$ — (3)

From (2),

$2(x - 5) = y - 5$

$2x - 10 = y - 5$

$y = 2x - 5$ — (4)

Substitute (4) into (3):

$5x - 4(2x - 5) = -1$

$5x - 8x + 20 = 1$

$-3x = -21$

$x = 7$

Substitute $x = 7$ into (4):

$y = 2(7) - 5$

$= 9$

$\therefore$ The fraction is $\frac{7}{9}$.

Practise Now 14

1. Let the present age of Kiran be x years and that of Kiran's father be y years.

Then in 5 years' time, Kiran's father will be $(y + 5)$ years old and Kiran will be $(x + 5)$ years old.

4 years ago, Kiran's father was $(y - 4)$ years old and and Kiran was $(x - 4)$ years old.

$y + 5 = 3(x + 5)$ — (1)

$y - 4 = 6(x - 4)$ — (2)

From (1),

$y + 5 = 3x + 15$

$y = 3x + 10$ — (3)

Substitute (3) into (2):

$3x + 10 - 4 = 6(x - 4)$

$= 6x - 24$

$3x = 30$

$x = 10$

Substitute $x = 10$ into (3):

$y = 3(10) + 10$

$= 40$

$\therefore$ Kiran's present age is 10 years and Kiran's father's present age is 40 years.

2. Let the amount an adult has to pay be PKR x and the amount a child has to pay be PKR y.

$11x + 5y = 2800$ — (1)

$14x + 9y = 3880$ — (2)

$9 \times (1)$: $99x + 45y = 25200$ — (3)

$5 \times (2)$: $70x + 45y = 19400$ — (4)

(3) – (4):

$(99x + 45y) - (70x + 45y) = 25200 - 19400$

$29x = 5800$

$x = 200$

Substitute $x = 20$ into (1):

$11(200) + 5y = 2800$

$2200 + 5y = 2800$

$5y = 600$

$y = 120$

Total amount a family of 2 adults and 3 children have to pay

= PKR $(2x + 3y)$

= PKR $[2(200) + 3(120)]$

= PKR 760

$\therefore$ The family has to pay PKR 760.

Practise Now 15

Let the tens digit of the original numer be x and its ones digit be y. Then the original number is $10x + y$, the number obtained when the digits of the original number are reversed is $10y + x$.

$x + y = 11$ — (1)

$10x + y - (10y + x) = 9$ — (2)

From (2),

$10x + y - 10y - x = 9$

$9x - 9y = 9$

$x - y = 1$ — (3)

(1) + (3):

$2x = 12$

$x = 6$

Substitute $x = 6$ into (1):

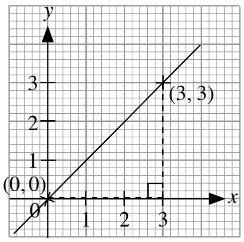
$6 + y = 11$

$y = 5$

$\therefore$ The original number is 65.

Exercise 4A

1. **(a)**

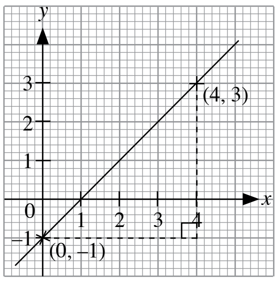


$m = \frac{3}{3}$

$= 1$

$c = 0$

(b)

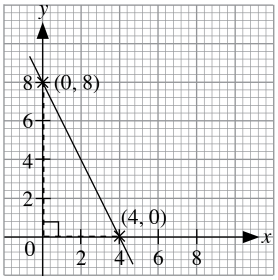


$m = \frac{4}{4}$

$= 1$

$c = -1$

(c)

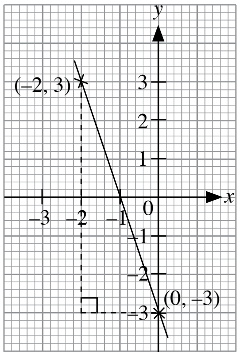


$m = -\frac{8}{4}$

$= -2$

$c = 8$

(d)

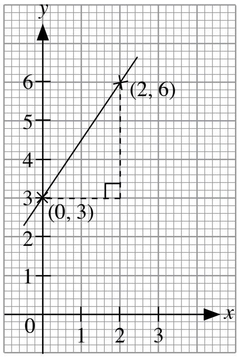


$m = -\frac{6}{2}$

$= -3$

$c = -3$

(e)

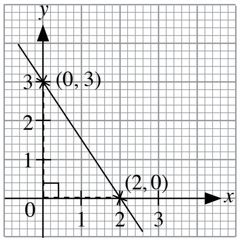


$m = \frac{3}{2}$

$= 1\frac{1}{2}$

$c = 3$

(f)

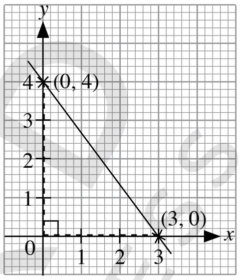


$m = -\frac{3}{2}$

$= -1\frac{1}{2}$

$c = 3$

(g)

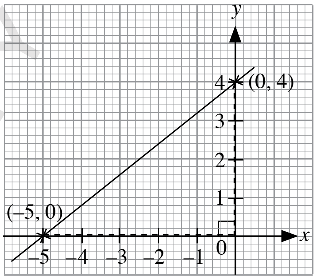


$m = -\frac{4}{3}$

$= -1\frac{1}{3}$

$c = 4$

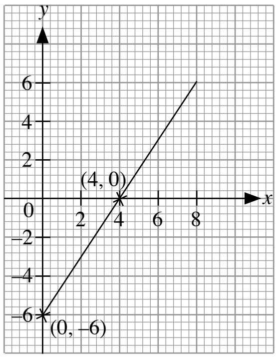
(h)



$m = \frac{4}{5}$

$c = 4$

(i)

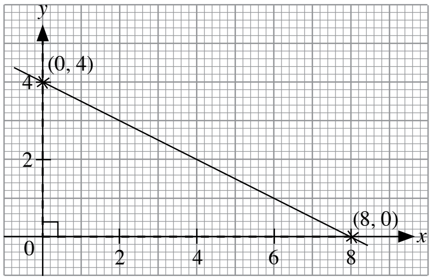


$m = \frac{6}{4}$

$= 1\frac{1}{2}$

$c = -6$

(j)



$m = -\frac{4}{8}$

$= -\frac{1}{2}$

$c = 4$

2. Gradient of Line 1 = 0

Gradient of Line 2 = gradient of Line 5

$= -3$

The slope of Line 3 is undefined.

Gradient of Line 4 = gradient of Line 6

$= \frac{1}{2}$

Exercise 4B

1. **(a)** Hussain left home at 1000 hours.

(b) Distance Hussain travelled before he reached the cafeteria = 50 km

(c) **(i)** Gradient of *OA* = $\frac{50}{1}$

$= 50$

Hussain's average speed was 50 km/h.

(ii) Gradient of *AB* = 0

Hussain's average speed was 0 km/h.

(iii) Gradient of *BC* = $\frac{30}{\frac{1}{2}}$

$= 60$

Hussain's average speed was 60 km/h.

2. **(a)** Distance between Ahsan's home and the post office = 40 km

(b) Total time Ahsan stayed at the post office and at the hawker centre

$= 1 + \frac{1}{2}$

$= 1\frac{1}{2}$ hours

(c) **(i)** Gradient of *OA* = $\frac{40}{2}$

$= 20$

Ahsan's average speed was 20 km/h.

(ii) Gradient of *BC* = $-\frac{20}{1\frac{1}{2}}$

$= -13\frac{1}{3}$

Ahsan's average speed was $13\frac{1}{3}$ km/h.

(iii) Gradient of *DE* = $-\frac{20}{1}$

$= -20$

Ahsan's average speed was 20 km/h.

Exercise 4C

1. (a) $2x + y = 50$

$2x + 10 = 50$ (where $y = 10$; given)

$2x = 50 - 10$

$2x = 40$

$x = \frac{\cancel{40}^{20}}{\cancel{2}}$

$x = 20$

(b) $3y - x = 30$

$3 \times 20 - x = 30$ (given $y = 20$)

$60 - x = 30$

$-x = 30 - 60$

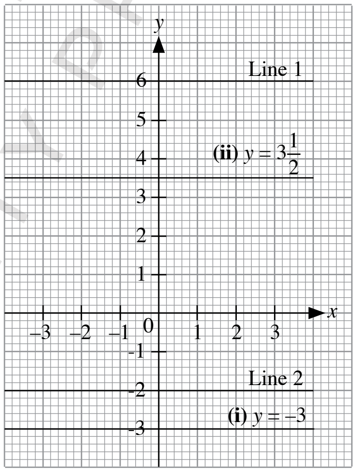
$-x = -60$

$x = 20$

2. **(a)** Line 1: $y = 6$

Line 2: $y = -2$

(b)

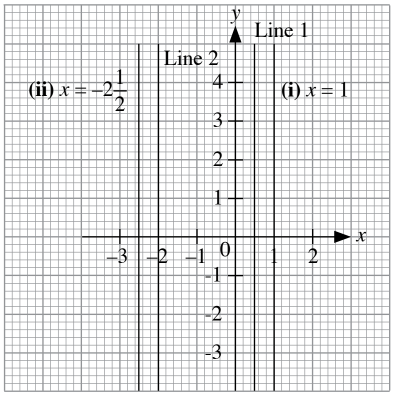


The lines are horizontal. The *y*-coordinates of all the points on the lines are a constant.

3. **(a)** Line 1: $x = 0.5$

Line 2: $x = -2$

(b)



The lines are vertical. The x-coordinates of all the points on the lines are a constant.

4. **(a)** When $x = -5$, $y = p$,

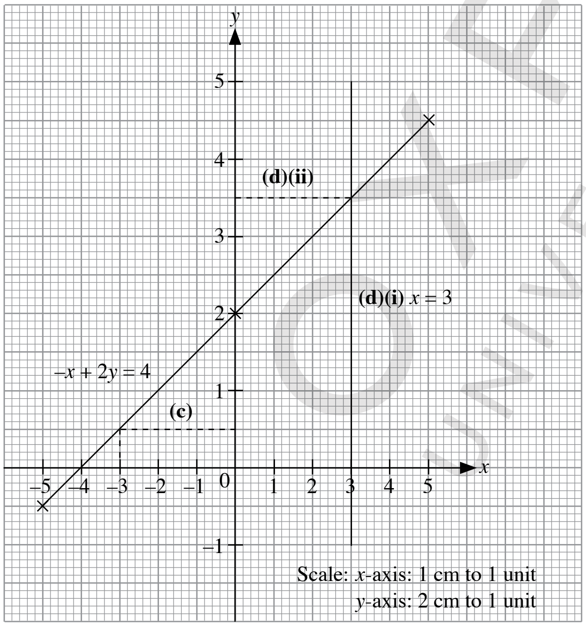
$$-(-5) + 2p = 4$$
$$5 + 2p = 4$$
$$2p = -1$$
$$p = -\frac{1}{2}$$

When $x = 5$, $y = q$,

$$-5 + 2q = 4$$
$$2q = 9$$
$$q = 4\frac{1}{2}$$

$\therefore p = -\frac{1}{2}, q = 4\frac{1}{2}$

(b)



(c) From the graph in **(b)**,

When $y = 0.5$,

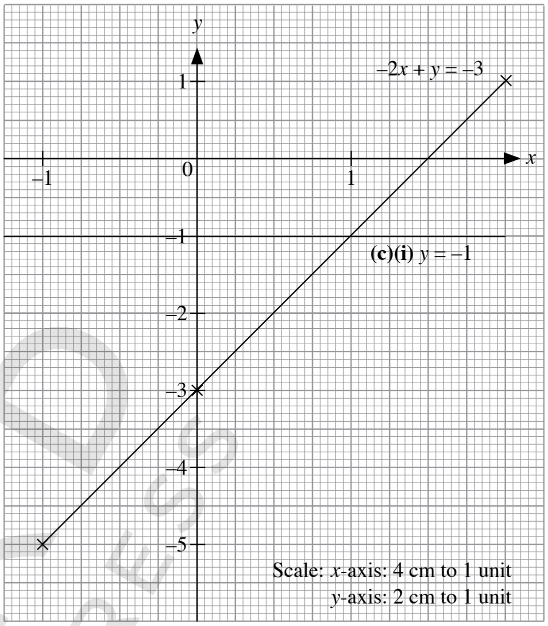
$r = x = -3$

(d) **(ii)** y-coordinate $= 3\frac{1}{2}$

5. **(a)** $-2x + y = -3$

x	–1	0	2
y	–5	–3	1

(b)



(c) **(ii)** Area of trapezium

$$= \frac{1}{2} \times \left(1\frac{1}{2} + 1\right) \times 1$$
$$= 1\frac{1}{4} \text{ units}^2$$

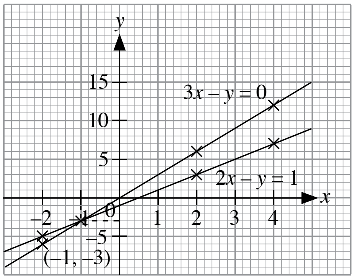
Exercise 4D

1. **(a)** $3x - y = 0$

x	–2	2	4
y	–6	6	12

$2x - y = 1$

x	–2	2	4
y	–5	3	7



Scale: x-axis: 1 cm to 1 unit

y-axis: 1 cm to 5 units

The graphs intersect at the point (–1, –3).

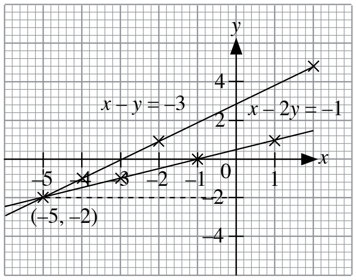
$\therefore$ The solution is $x = -1$ and $y = -3$.

(b) $x - y = -3$

x	–4	–2	0
y	–1	1	3

$x - 2y = -1$

x	–3	–1	1
y	–1	0	1



Scale: x-axis: 1 cm to 1 unit
y-axis: 1 cm to 2 units

The graphs intersect at the point (–5, –2).

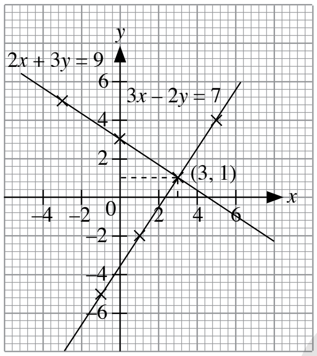
$\therefore$ The solution is $x = -5$ and $y = -2$.

(c) $3x - 2y = 7$

x	–1	1	5
y	–5	–2	4

$2x + 3y = 9$

x	–3	0	3
y	5	3	1



Scale: x-axis: 1 cm to 2 units
y-axis: 1 cm to 2 units

The graphs intersect at the point (3, 1).

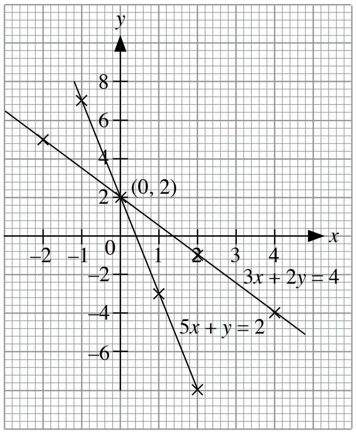
$\therefore$ The solution is $x = 3$ and $y = 1$.

(d) $3x + 2y = 4$

x	–2	2	4
y	5	–1	–4

$5x + y = 2$

x	–1	1	2
y	7	–3	–8



Scale: x-axis: 1 cm to 1 unit
y-axis: 1 cm to 2 units

The graphs intersect at the point (0, 2).

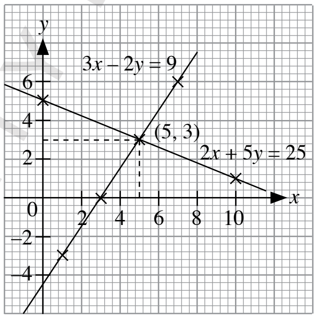
$\therefore$ The solution is $x = 0$ and $y = 2$.

(e) $2x + 5y = 25$

x	0	5	10
y	5	3	1

$3x - 2y = 9$

x	1	3	7
y	–3	0	6



Scale: x-axis: 1 cm to 2 units
y-axis: 1 cm to 5 units

The graphs intersect at the point (5, 3).

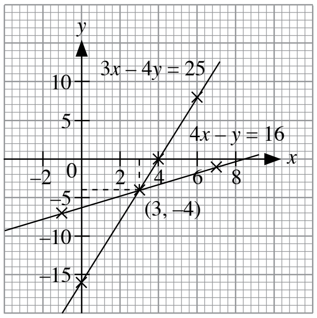
$\therefore$ The solution is $x = 5$ and $y = 3$.

(f) $3x - 4y = 25$

x	–1	3	7
y	–7	–4	–1

$4x - y = 16$

x	0	4	6
y	–16	0	8



Scale: x-axis: 1 cm to 2 units
y-axis: 1 cm to 5 units

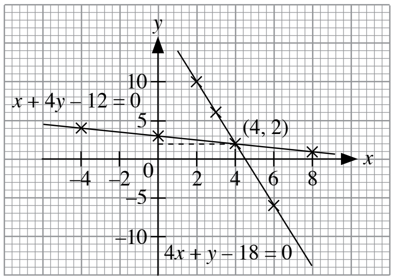
The graphs intersect at the point (3, –4).
∴ The solution is $x = 3$ and $y = -4$.

2. **(a)** $x + 4y - 12 = 0$

x	–4	0	8
y	4	3	1

$4x + y - 18 = 0$

x	2	3	6
y	10	6	–6



Scale: x-axis: 1 cm to 2 units
y-axis: 1 cm to 5 units

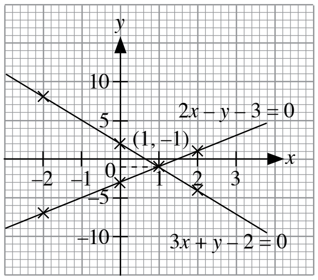
The graphs intersect at the point (4, 2).
∴ The solution is $x = 4$ and $y = 2$.

(b) $3x + y - 2 = 0$

x	–2	0	2
y	8	2	–4

$2x - y - 3 = 0$

x	–2	0	2
y	–7	–3	1



Scale: x-axis: 1 cm to 1 unit
y-axis: 1 cm to 5 units

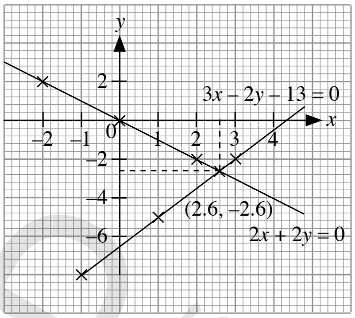
The graphs intersect at the point (1, –1).
∴ The solution is $x = 1$ and $y = -1$.

(c) $3x - 2y - 13 = 0$

x	–1	1	3
y	–8	–5	–2

$2x + 2y = 0$

x	–2	0	2
y	2	0	–2



Scale: x-axis: 1 cm to 1 unit
y-axis: 1 cm to 2 units

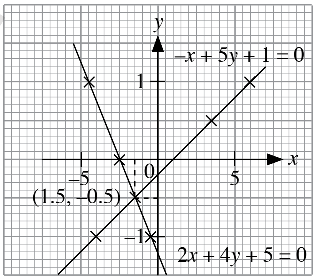
The graphs intersect at the point (2.6, –2.6).
∴ The solution is $x = 2.6$ and $y = -2.6$.

(d) $2x + 4y + 5 = 0$

x	–4.5	–2.5	–0.5
y	1	0	–1

$-x + 5y + 1 = 0$

x	–4	3.5	6
y	–1	0.5	1



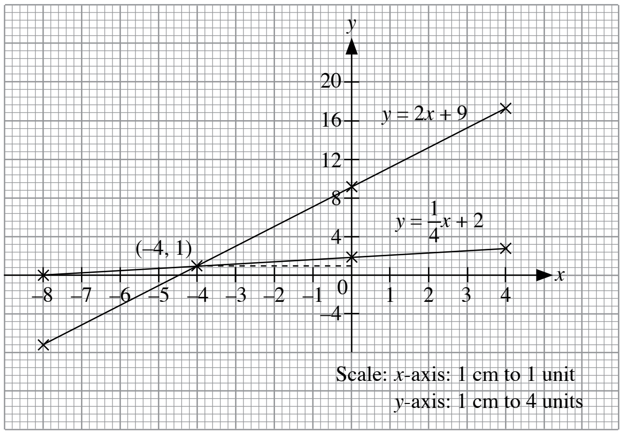
Scale: x-axis: 2 cm to 5 units
y-axis: 2 cm to 1 unit

The graphs intersect at the point (1.5, –0.5).
∴ The solution is $x = 1.5$ and $y = -0.5$.

3. **(a)** **(i)** $y = 2x + 9$

x	–8	0	4
y	–7	9	17

(ii)



(b) **(i)** $y = \frac{1}{4}x + 2$

x	-8	0	4
y	0	2	3

(c) $2x - y = -9$ — (1)

$x - 4y = -8$ — (2)

From (1), $y = 2x + 9$

From (2), $4y = x + 8$

$$y = \frac{1}{4}x + 2$$

From **(a)(ii)**, the graphs intersect at the point $(-4, 1)$.

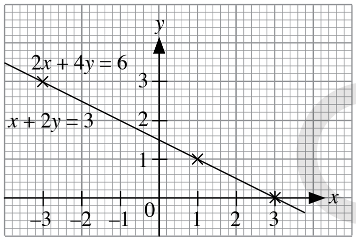
$\therefore$ The solution is $x = -4$ and $y = 1$.

4. **(a)** $x + 2y = 3$

x	-3	1	3
y	3	1	0

$2x + 4y = 6$

x	-3	1	3
y	3	1	0



Scale: x-axis: 1 cm to 1 unit
y-axis: 1 cm to 1 unit

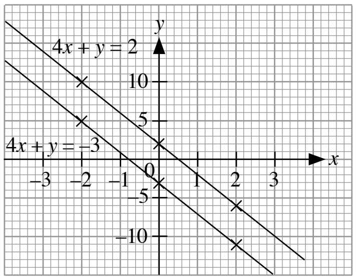
The graphs of each pair of simultaneous equations are identical. The simultaneous equations have an infinite number of solutions.

(b) $4x + y = 2$

x	-2	0	2
y	10	2	-6

$4x + y = -3$

x	-2	0	2
y	5	-3	-11



Scale: x-axis: 1 cm to 1 unit
y-axis: 1 cm to 5 units

The graphs of each pair of simultaneous equations are parallel and have no intersection point.

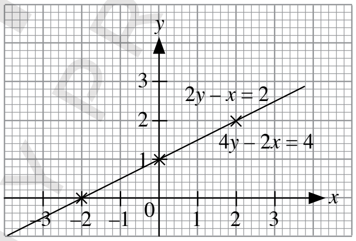
The simultaneous equations have no solution.

(c) $2y - x = 2$

x	-2	0	2
y	0	1	2

$4y - 2x = 4$

x	-2	0	2
y	0	1	2



Scale: x-axis: 1 cm to 1 unit
y-axis: 1 cm to 1 unit

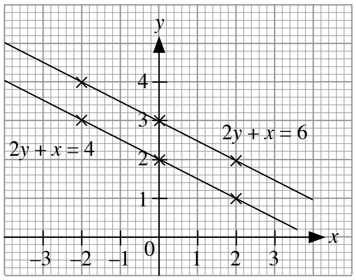
The graphs of each pair of simultaneous equations are identical. The simultaneous equations have an infinite number of solutions.

(d) $2y + x = 4$

x	-2	0	2
y	3	2	1

$2y + x = 6$

x	-2	0	2
y	4	3	2



Scale: x-axis: 1 cm to 1 unit
y-axis: 1 cm to 1 unit

The graphs of each pair of simultaneous equations are parallel and have no intersection point.

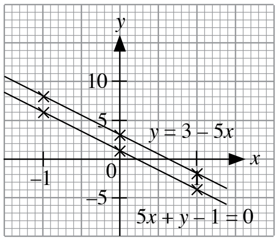
The simultaneous equations have no solution.

5. **(a)** $y = 3 - 5x$

x	–1	0	1
y	8	3	–2

$5x + y - 1 = 0$

x	–1	0	1
y	6	1	–4



Scale: x-axis: 2 cm to 1 unit

y-axis: 1 cm to 5 units

The graphs of each pair of simultaneous equations are parallel and have no intersection point.

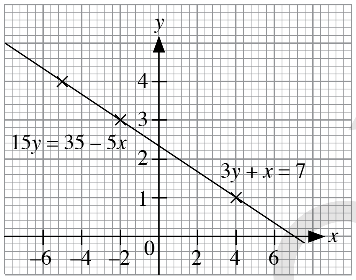
The simultaneous equations have no solutions.

(b) $3y + x = 7$

x	–5	–2	4
y	4	3	1

$15y = 35 - 5x$

x	–5	–2	4
y	4	3	1



Scale: x-axis: 1 cm to 2 units

y-axis: 1 cm to 1 unit

The graphs of each pair of simultaneous equations are identical.

The simultaneous equations have an infinite number of solutions.

Exercise 4E

1. **(a)** $x + y = 16$ — (1)

$x - y = 0$ — (2)

(1) + (2):

$(x + y) + (x - y) = 16 + 0$

$x + y + x - y = 16$

$2x = 16$

$x = 8$

Substitute $x = 8$ into (1):

$8 + y = 16$

$y = 8$

$\therefore$ The solution is $x = 8$ and $y = 8$.

(b) $x - y = 5$ — (1)

$x + y = 19$ — (2)

(2) + (1):

$(x + y) + (x - y) = 19 + 5$

$x + y + x - y = 24$

$2x = 24$

$x = 12$

Substitute $x = 12$ into (2):

$12 + y = 19$

$y = 7$

$\therefore$ The solution is $x = 12$ and $y = 7$.

(c) $11x + 4y = 12$ — (1)

$9x - 4y = 8$ — (2)

(1) + (2):

$(11x + 4y) + (9x - 4y) = 12 + 8$

$11x + 4y + 9x - 4y = 20$

$20x = 20$

$x = 1$

Substitute $x = 1$ into (1):

$11(1) + 4y = 12$

$11 + 4y = 12$

$4y = 1$

$y = \frac{1}{4}$

$\therefore$ The solution is $x = 1$ and $y = \frac{1}{4}$.

(d) $4y + x = 11$ — (1)

$3y - x = 3$ — (2)

(1) + (2):

$(4y + x) + (3y - x) = 11 + 3$

$4y + x + 3y - x = 14$

$7y = 14$

$y = 2$

Substitute $y = 2$ into (1):

$4(2) + x = 11$

$8 + x = 11$

$x = 3$

$\therefore$ The solution is $x = 3$ and $y = 2$.

(e) $3x + y = 5$ — (1)

$x + y = 3$ — (2)

(1) – (2):

$(3x + y) - (x + y) = 5 - 3$

$3x + y - x - y = 2$

$2x = 2$

$x = 1$

Substitute $x = 1$ into (2):

$1 + y = 3$

$y = 2$

$\therefore$ The solution is $x = 1$ and $y = 2$.

(f) $2x + 3y = 5 \quad — (1)$

$2x + 7y = 9 \quad — (2)$

$(2) – (1)$:

$(2x + 7y) – (2x + 3y) = 9 – 5$

$2x + 7y – 2x – 3y = 4$

$4y = 4$

$y = 1$

Substitute $y = 1$ into (1):

$2x + 3(1) = 5$

$2x + 3 = 5$

$2x = 2$

$x = 1$

$\therefore$ The solution is $x = 1$ and $y = 1$.

(g) $7x – 3y = 15 \quad — (1)$

$11x – 3y = 21 \quad — (2)$

$(2) – (1)$:

$(11x – 3y) – (7x – 3y) = 21 – 15$

$11x – 3y – 7x + 3y = 6$

$4x = 6$

$x = 1\frac{1}{2}$

Substitute $x = 1\frac{1}{2}$ into (1):

$7\left(1\frac{1}{2}\right) – 3y = 15$

$10\frac{1}{2} – 3y = 15$

$3y = –4\frac{1}{2}$

$y = –1\frac{1}{2}$

$\therefore$ The solution is $x = 1\frac{1}{2}$ and $y = –1\frac{1}{2}$.

(h) $3y – 2x = 9 \quad — (1)$

$2y – 2x = 7 \quad — (2)$

$(1) – (2)$:

$(3y – 2x) – (2y – 2x) = 9 – 7$

$3y – 2x – 2y + 2x = 2$

$y = 2$

Substitute $y = 2$ into (1):

$3(2) – 2x = 9$

$6 – 2x = 9$

$2x = –3$

$x = –1\frac{1}{2}$

$\therefore$ The solution is $x = –1\frac{1}{2}$ and $y = 2$.

(i) $3a – 2b = 5 \quad — (1)$

$2b – 5a = 9 \quad — (2)$

$(1) + (2)$:

$(3a – 2b) + (2b – 5a) = 5 + 9$

$3a – 2b + 2b – 5a = 14$

$–2a = 14$

$a = –7$

Substitute $a = –7$ into (2):

$2b – 5(–7) = 9$

$2b + 35 = 9$

$2b = –26$

$b = –13$

$\therefore$ The solution is $a = –7$ and $b = –13$.

(j) $5c – 2d = 9 \quad — (1)$

$3c + 2d = 7 \quad — (2)$

$(1) + (2)$:

$(5c – 2d) + (3c + 2d) = 9 + 7$

$5c – 2d + 3c + 2d = 16$

$8c = 16$

$c = 2$

Substitute $c = 2$ into (2):

$3(2) + 2d = 7$

$6 + 2d = 7$

$2d = 1$

$d = \frac{1}{2}$

$\therefore$ The solution is $c = 2$ and $d = \frac{1}{2}$.

(k) $3f + 4h = 1 \quad — (1)$

$5f – 4h = 7 \quad — (2)$

$(1) + (2)$:

$(3f + 4h) + (5f – 4h) = 1 + 7$

$3f + 4h + 5f – 4h = 8$

$8f = 8$

$f = 1$

Substitute $f = 1$ into (1):

$3(1) + 4h = 1$

$3 + 4h = 1$

$4h = –2$

$h = –\frac{1}{2}$

$\therefore$ The solution is $f = 1$ and $h = –\frac{1}{2}$.

(l) $6j – k = 23 \quad — (1)$

$3k + 6j = 11 \quad — (2)$

$(2) – (1)$:

$(3k + 6j) – (6j – k) = 11 – 23$

$3k + 6j – 6j + k = –12$

$4k = –12$

$k = –3$

Substitute $k = –3$ into (2):

$3(–3) + 6j = 11$

$–9 + 6j = 11$

$6j = 20$

$j = 3\frac{1}{3}$

$\therefore$ The solution is $j = 3\frac{1}{3}$ and $k = –3$.

2. **(a)** $7x - 2y = 17$ — (1)
$3x + 4y = 17$ — (2)
$2 \times (1): 14x - 4y = 34$ — (3)
$(3) + (2):$
$(14x - 4y) + (3x + 4y) = 34 + 17$
$14x - 4y + 3x + 4y = 51$
$17x = 51$
$x = 3$
Substitute $x = 3$ into (2):
$3(3) + 4y = 17$
$9 + 4y = 17$
$4y = 8$
$y = 2$
$\therefore$ The solution is $x = 3$ and $y = 2$.

(b) $16x + 5y = 39$ — (1)
$4x - 3y = 31$ — (2)
$4 \times (2): 16x - 12y = 124$ — (3)
$(1) - (3):$
$(16x + 5y) - (16x - 12y) = 39 - 124$
$16x + 5y - 16x + 12y = -85$
$17y = -85$
$y = -5$
Substitute $y = -5$ into (2):
$4x - 3(-5) = 31$
$4x + 15 = 31$
$4x = 16$
$x = 4$
$\therefore$ The solution is $x = 4$ and $y = -5$.

(c) $x + 2y = 3$ — (1)
$3x + 5y = 7$ — (2)
$3 \times (1): 3x + 6y = 9$ — (3)
$(3) - (2):$
$(3x + 6y) - (3x + 5y) = 9 - 7$
$3x + 6y - 3x - 5y = 2$
$y = 2$
Substitute $y = 2$ into (1):
$x + 2(2) = 3$
$x + 4 = 3$
$x = -1$
$\therefore$ The solution is $x = -1$ and $y = 2$.

(d) $3x + y = -5$ — (1)
$7x + 3y = 1$ — (2)
$3 \times (1): 9x + 3y = -15$ — (3)
$(3) - (2):$
$(9x + 3y) - (7x + 3y) = -15 - 1$
$9x + 3y - 7x - 3y = -16$
$2x = -16$
$x = -8$
Substitute $x = -8$ into (1):
$3(-8) + y = -5$
$-24 + y = -5$
$y = 19$
$\therefore$ The solution is $x = -8$ and $y = 19$.

(e) $7x - 3y = 13$ — (1)
$2x - y = 3$ — (2)
$3 \times (2): 6x - 3y = 9$ — (3)
$(1) - (3):$
$(7x - 3y) - (6x - 3y) = 13 - 9$
$7x - 3y - 6x + 3y = 4$
$x = 4$
Substitute $x = 4$ into (2):
$2(4) - y = 3$
$8 - y = 3$
$y = 5$
$\therefore$ The solution is $x = 4$ and $y = 5$.

(f) $9x - 5y = 2$ — (1)
$3x - 4y = 10$ — (2)
$3 \times (2): 9x - 12y = 30$ — (3)
$(1) - (3):$
$(9x - 5y) - (9x - 12y) = 2 - 30$
$9x - 5y - 9x + 12y = -28$
$7y = -28$
$y = -4$
Substitute $y = -4$ into (2):
$3x - 4(-4) = 10$
$3x + 16 = 10$
$3x = -6$
$x = -2$
$\therefore$ The solution is $x = -2$ and $y = -4$.

3. **(a)** $7x - 3y = 18$ — (1)
$6x + 7y = 25$ — (2)
$7 \times (1): 49x - 21y = 126$ — (3)
$3 \times (2): 18x + 21y = 75$ — (4)
$(3) + (4):$
$(49x - 21y) + (18x + 21y) = 126 + 75$
$49x - 21y + 18x + 21y = 201$
$67x = 201$
$x = 3$
Substitute $x = 3$ into (2):
$6(3) + 7y = 25$
$18 + 7y = 25$
$7y = 7$
$y = 1$
$\therefore$ The solution is $x = 3$ and $y = 1$.

(b) $4x + 3y = -5$ — (1)
$3x - 2y = 43$ — (2)
$2 \times (1): 8x + 6y = -10$ — (3)
$3 \times (2): 9x - 6y = 129$ — (4)
$(3) + (4):$
$(8x + 6y) + (9x - 6y) = -10 + 129$
$8x + 6y + 9x - 6y = 119$
$17x = 119$
$x = 7$

Substitute $x = 7$ into (1):
$4(7) + 3y = -5$
$28 + 3y = -5$
$3y = -38$
$y = -11$
$\therefore$ The solution is $x = 7$ and $y = -11$.

(c) $2x + 3y = 8$ — (1)
$5x + 2y = 9$ — (2)
$2 \times (1)$: $4x + 6y = 16$ — (3)
$3 \times (2)$: $15x + 6y = 27$ — (4)
(4) – (3):
$(15x + 6y) - (4x + 6y) = 27 - 16$
$15x + 6y - 4x - 6y = 11$
$11x = 11$
$x = 1$
Substitute $x = 1$ into (2):
$5(1) + 2y = 9$
$5 + 2y = 9$
$2y = 4$
$y = 2$
$\therefore$ The solution is $x = 1$ and $y = 2$.

(d) $5x + 4y = 11$ — (1)
$3x + 5y = 4$ — (2)
$3 \times (1)$: $15x + 12y = 33$ — (3)
$5 \times (2)$: $15x + 25y = 20$ — (4)
(4) – (3):
$(15x + 25y) - (15x + 12y) = 20 - 33$
$15x + 25y - 15x - 12y = -13$
$13y = -13$
$y = -1$
Substitute $y = -1$ into (1):
$5x + 4(-1) = 11$
$5x - 4 = 11$
$5x = 15$
$x = 3$
$\therefore$ The solution is $x = 3$ and $y = -1$.

(e) $4x - 3y = -1$ — (1)
$5x - 2y = 4$ — (2)
$2 \times (1)$: $8x - 6y = -2$ — (3)
$3 \times (2)$: $15x - 6y = 12$ — (4)
(4) – (3):
$(15x - 6y) - (8x - 6y) = 12 - (-2)$
$15x - 6y - 8x + 6x = 14$
$7x = 14$
$x = 2$
Substitute $x = 2$ into (2):
$5(2) - 2y = 4$
$10 - 2y = 4$
$2y = 6$
$y = 3$
$\therefore$ The solution is $x = 2$ and $y = 3$.

(f) $5x - 4y = 23$ — (1)
$2x - 7y = 11$ — (2)
$2 \times (1)$: $10x - 8y = 46$ — (3)
$5 \times (2)$: $10x - 35y = 55$ — (4)
(3) – (4):
$(10x - 8y) - (10x - 35y) = 46 - 55$
$10x - 8y - 10x + 35y = -9$
$27y = -9$
$y = -\frac{1}{3}$
Substitute $y = -\frac{1}{3}$ into (1):
$5x - 4\left(-\frac{1}{3}\right) = 23$
$5x + \frac{4}{3} = 23$
$5x = 21\frac{2}{3}$
$x = 4\frac{1}{3}$
$\therefore$ The solution is $x = 4\frac{1}{3}$ and $y = -\frac{1}{3}$.

4. **(a)** $x + y = 7$ — (1)
$x - y = 5$ — (2)
From (1), $y = 7 - x$ — (3)
Substitute (3) into (2):
$x - (7 - x) = 5$
$x - 7 + x = 5$
$2x = 12$
$x = 6$
Substitute $x = 6$ into (3):
$y = 7 - 6$
$= 1$
$\therefore$ The solution is $x = 6$ and $y = 1$.

(b) $3x - y = 0$ — (1)
$2x + y = 5$ — (2)
From (2), $y = 5 - 2x$ — (3)
Substitute (3) into (1):
$3x - (5 - 2x) = 0$
$3x - 5 + 2x = 0$
$5x = 5$
$x = 1$
Substitute $x = 1$ into (3):
$y = 5 - 2(1)$
$= 3$
$\therefore$ The solution is $x = 1$ and $y = 3$.

(c) $2x - 7y = 5$ — (1)
$3x + y = -4$ — (2)
From (2), $y = -4 - 3x$ — (3)
Substitute (3) into (1):
$2x - 7(-4 - 3x) = 5$
$2x + 28 + 21x = 5$
$23x = -23$
$x = -1$

Substitute $x = -1$ into (3):

$y = -4 - 3(-1)$

$= -1$

$\therefore$ The solution is $x = -1$ and $y = -1$.

(d) $5x - y = 5$ — (1)

$3x + 2y = 29$ — (2)

From (1), $y = 5x - 5$ — (3)

Substitute (3) into (2):

$3x + 2(5x - 5) = 29$

$3x + 10x - 10 = 29$

$13x = 39$

$x = 3$

Substitute $x = 3$ into (3):

$y = 5(3) - 5$

$= 10$

$\therefore$ The solution is $x = 3$ and $y = 10$.

(e) $5x + 3y = 11$ — (1)

$4x - y = 2$ — (2)

From (2), $y = 4x - 2$ — (3)

Substitute (3) into (1):

$5x + 3(4x - 2) = 11$

$5x + 12x - 6 = 11$

$17x = 17$

$x = 1$

Substitute $x = 1$ into (3):

$y = 4(1) - 2$

$= 2$

$\therefore$ The solution is $x = 1$ and $y = 2$.

(f) $3x + 5y = 10$ — (1)

$x - 2y = 7$ — (2)

From (2), $x = 2y + 7$ — (3)

Substitute (3) into (2):

$3(2y + 7) + 5y = 10$

$6y + 21 + 5y = 10$

$11y = -11$

$y = -1$

Substitute $y = -1$ into (3):

$x = 2(-1) + 7$

$= 5$

$\therefore$ The solution is $x = 5$ and $y = -1$.

(g) $x + y = 9$ — (1)

$5x - 2y = 4$ — (2)

From (1), $y = 9 - x$ — (3)

Substitute (3) into (2):

$5x - 2(9 - x) = 4$

$5x - 18 + 2x = 4$

$7x = 22$

$x = 3\frac{1}{7}$

Substitute $x = 3\frac{1}{7}$ into (3):

$y = 9 - 3\frac{1}{7}$

$= 5\frac{6}{7}$

$\therefore$ The solution is $x = 3\frac{1}{7}$ and $y = 5\frac{6}{7}$.

(h) $5x + 2y = 3$ — (1)

$x - 4y = -6$ — (2)

From (2), $x = 4y - 6$ — (3)

Substitute (3) into (1):

$5(4y - 6) + 2y = 3$

$20y - 30 + 2y = 3$

$22y = 33$

$y = 1\frac{1}{2}$

Substitute $y = 1\frac{1}{2}$ into (3):

$x = 4\left(1\frac{1}{2}\right) - 6$

$= 0$

$\therefore$ The solution is $x = 0$ and $y = 1\frac{1}{2}$.

5. **(a)** $x + y = 0.5$ — (1)

$x - y = 1$ — (2)

(1) + (2):

$(x + y) + (x - y) = 0.5 + 1$

$x + y + x - y = 1.5$

$2x = 1.5$

$x = 0.75$

Substitute $x = 0.75$ into (1):

$0.75 + y = 0.5$

$y = -0.25$

$\therefore$ The solution is $x = 0.75$ and $y = -0.25$.

(b) $2x + 0.4y = 8$ — (1)

$5x - 1.2y = 9$ — (2)

$3 \times$ (1): $6x + 1.2y = 24$ — (3)

(3) + (2):

$(6x + 1.2y) + (5x - 1.2y) = 24 + 9$

$6x + 1.2y + 5x - 1.2y = 33$

$11x = 33$

$x = 3$

Substitute $x = 3$ into (1):

$2(3) + 0.4y = 8$

$6 + 0.4y = 8$

$0.4y = 2$

$y = 5$

$\therefore$ The solution is $x = 3$ and $y = 5$.

(c) $10x - 3y = 24.5$ — (1)

$3x - 5y = 13.5$ — (2)

$5 \times$ (1): $50x - 15y = 122.5$ — (3)

$3 \times$ (2): $9x - 15y = 40.5$ — (4)

(3) – (4):

$(50x - 15y) - (9x - 15y) = 122.5 - 40.5$

$50x - 15y - 9x + 15y = 82$

$41x = 82$

$x = 2$

Substitute $x = 2$ into (1):

$10(2) - 3y = 24.5$
$20 - 3y = 24.5$
$3y = -4.5$
$y = -1.5$

$\therefore$ The solution is $x = 2$ and $y = -1.5$.

(d) $6x + 5y = 10.5$ — (1)
$5x - 3y = -2$ — (2)
$3 \times (1)$: $18x + 15y = 31.5$ — (3)
$5 \times (2)$: $25x - 15y = -10$ — (4)
(4) + (3):
$(25x - 15y) + (18x + 15y) = -10 + 31.5$
$25x - 15y + 18x + 15y = 21.5$
$43x = 21.5$
$x = 0.5$

Substitute $x = 0.5$ into (1):

$6(0.5) + 5y = 10.5$
$3 + 5y = 10.5$
$5y = 7.5$
$y = 1.5$

$\therefore$ The solution is $x = 0.5$ and $y = 1.5$.

6. (a) $4x - y - 7 = 0$ — (1)
$4x + 3y - 11 = 0$ — (2)
(2) – (1):
$(4x + 3y - 11) - (4x - y - 7) = 0 - 0$
$4x + 3y - 11 - 4x + y + 7 = 0$
$4y = 4$
$y = 1$

Substitute $y = 1$ into (1):

$4x - 1 - 7 = 0$
$4x = 8$
$x = 2$

$\therefore$ The solution is $x = 2$ and $y = 1$.

(b) $7x + 2y - 33 = 0$ — (1)
$3y - 7x - 17 = 0$ — (2)
(1) + (2):
$(7x + 2y - 33) + (3y - 7x - 17) = 0 + 0$
$7x + 2y - 33 + 3y - 7x - 17 = 0$
$5y = 50$
$y = 10$

Substitute $y = 10$ into (1):

$7x + 2(10) - 33 = 0$
$7x + 20 - 33 = 0$
$7x = 13$
$x = 1\frac{6}{7}$

$\therefore$ The solution is $x = 1\frac{6}{7}$ and $y = 10$.

(c) $5x - 3y - 2 = 0$ — (1)
$x + 5y - 6 = 0$ — (2)
$5 \times (2)$: $5x + 25y - 30 = 0$ — (3)
(3) – (1):
$(5x + 25y - 30) - (5x - 3y - 2) = 0 - 0$
$5x + 25y - 30 - 5x + 3y + 2 = 0$
$28y = 28$
$y = 1$

Substitute $y = 1$ into (2):

$x + 5(1) - 6 = 0$
$x + 5 - 6 = 0$
$x = 1$

$\therefore$ The solution is $x = 1$ and $y = 1$.

(d) $5x - 3y - 13 = 0$ — (1)
$7x - 6y - 20 = 0$ — (2)
$2 \times (1)$: $10x - 6y - 26 = 0$ — (3)
(3) – (2):
$(10x - 6y - 26) - (7x - 6y - 20) = 0 - 0$
$10x - 6y - 26 - 7x + 6y + 20 = 0$
$3x = 6$
$x = 2$

Substitute $x = 2$ into (1):

$5(2) - 3y - 13 = 0$
$10 - 3y - 13 = 0$
$3y = 3$
$y = 1$

$\therefore$ The solution is $x = 2$ and $y = 1$.

(e) $7x + 3y - 8 = 0$ — (1)
$3x - 4y - 14 = 0$ — (2)
$4 \times (1)$: $28x + 12y - 32 = 0$ — (3)
$3 \times (2)$: $9x - 12y - 42 = 0$ — (4)
(3) + (4):
$(28x + 12y - 32) + (9x - 12y - 42) = 0 + 0$
$28x + 12y - 32 + 9x - 12y - 42 = 0$
$37x = 74$
$x = 2$

Substitute $x = 2$ into (1):

$7(2) + 3y - 8 = 0$
$14 + 3y - 8 = 0$
$3y = -6$
$y = -2$

$\therefore$ The solution is $x = 2$ and $y = -2$.

(f) $3x + 5y + 8 = 0$ — (1)
$4x + 13y - 2 = 0$ — (2)
$4 \times (1)$: $12x + 20y + 32 = 0$ — (3)
$3 \times (2)$: $12x + 39y - 6 = 0$ — (4)
(3) – (4):
$(12x + 20y + 32) - (12x + 39y - 6) = 0 - 0$
$12x + 20y + 32 - 12x - 39y + 6 = 0$
$19y = 38$
$y = 2$

Substitute $y = 2$ into (1):

$3x + 5(2) + 8 = 0$

$3x + 10 + 8 = 0$

$3x = -18$

$x = -6$

$\therefore$ The solution is $x = -6$ and $y = 2$.

7. **(a)** $\dfrac{x+1}{y+2} = \dfrac{3}{4}$ — (1)

$\dfrac{x-2}{y-1} = \dfrac{3}{5}$ — (2)

From (1),

$4(x + 1) = 3(y + 2)$

$4x + 4 = 3y + 6$

$4x - 3y = 2$ — (3)

From (2),

$5(x - 2) = 3(y - 1)$

$5x - 10 = 3y - 3$

$5x - 3y = 7$ — (4)

(4) – (3):

$(5x - 3y) - (4x - 3y) = 7 - 2$

$5x - 3y - 4x + 3y = 5$

$x = 5$

Substitute $x = 5$ into (3):

$4(5) - 3y = 2$

$20 - 3y = 2$

$3y = 18$

$y = 6$

$\therefore$ The solution is $x = 5$ and $y = 6$.

(b) $\dfrac{x}{3} - \dfrac{y}{2} = \dfrac{5}{6}$ — (1)

$3x - \dfrac{2}{5}y = 3\dfrac{2}{5}$ — (2)

$9 \times$ (1): $3x - \dfrac{9y}{2} = 7\dfrac{1}{2}$ — (3)

(2) – (3):

$\left(3x - \dfrac{2}{5}y\right) - \left(3x - \dfrac{9y}{2}\right) = 3\dfrac{2}{5} - 7\dfrac{1}{2}$

$3x - \dfrac{2}{5}y - 3x + \dfrac{9y}{2} = -4\dfrac{1}{10}$

$4\dfrac{1}{10}y = -4\dfrac{1}{10}$

$y = -1$

Substitute $y = -1$ into (2):

$3x - \dfrac{2}{5}(-1) = 3\dfrac{2}{5}$

$3x + \dfrac{2}{5} = 3\dfrac{2}{5}$

$3x = 3$

$x = 1$

$\therefore$ The solution is $x = 1$ and $y = -1$.

(c) $\dfrac{x}{4} - \dfrac{3}{8}y = 3$ — (1)

$\dfrac{5}{3}x - \dfrac{y}{2} = 12$ — (2)

$8 \times$ (1): $2x - 3y = 24$ — (3)

$6 \times$ (2): $10x - 3y = 72$ — (4)

(4) – (3):

$(10x - 3y) - (2x - 3y) = 72 - 24$

$10x - 3y - 2x + 3y = 48$

$8x = 48$

$x = 6$

Substitute $x = 6$ into (3):

$2(6) - 3y = 24$

$12 - 3y = 24$

$3y = -12$

$y = -4$

$\therefore$ The solution is $x = 6$ and $y = -4$.

(d) $\dfrac{x-3}{5} = \dfrac{y-7}{2}$ — (1)

$11x = 13y$ — (2)

$26 \times$ (1): $\dfrac{26}{5}(x - 3) = 13(y - 7)$

$\dfrac{26}{5}x - \dfrac{78}{5} = 13y - 91$ — (3)

(2) – (3):

$11x - \left(\dfrac{26}{5}x - \dfrac{78}{5}\right) = 13y - (13y - 91)$

$11x - \dfrac{26}{5}x + \dfrac{78}{5} = 13y - 13y + 91$

$5\dfrac{4}{5}x = 75\dfrac{2}{5}$

$x = 13$

Substitute $x = 13$ into (2):

$11(13) = 13y$

$y = 11$

$\therefore$ The solution is $x = 13$ and $y = 11$.

8. **(a)** $2x + 5y = 12$ — (1)

$4x + 3y = -4$ — (2)

From (1), $2x = 12 - 5y$

$x = \dfrac{12 - 5y}{2}$ — (3)

Substitute (3) into (2):

$4\left(\dfrac{12 - 5y}{2}\right) + 3y = -4$

$24 - 10y + 3y = -4$

$7y = 28$

$y = 4$

Substitute $y = 4$ into (3):

$x = \dfrac{12 - 5(4)}{2}$

$= -4$

$\therefore$ The solution is $x = -4$ and $y = 4$.

(b) $4x - 3y = 25$ — (1)

$6x + 5y = 9$ — (2)

From (1), $4x = 3y + 25$

$$x = \frac{3y + 25}{4} \quad \text{— (3)}$$

Substitute (3) into (2):

$$6\left(\frac{3y + 25}{4}\right) + 5y = 9$$

$$\frac{9y}{2} + \frac{75}{2} + 5y = 9$$

$$9\frac{1}{2}y = -28\frac{1}{2}$$

$$y = -3$$

Substitute $y = -3$ into (3):

$$x = \frac{3(-3) + 25}{4}$$

$$= 4$$

∴ The solution is $x = 4$ and $y = -3$.

(c) $3x + 7y = 2$ — (1)

$6x - 5y = 4$ — (2)

From (1), $3x = 2 - 7y$

$$x = \frac{2 - 7y}{3} \quad \text{— (3)}$$

Substitute (3) into (2):

$$6\left(\frac{2 - 7y}{3}\right) - 5y = 4$$

$$4 - 14y - 5y = 4$$

$$19y = 0$$

$$y = 0$$

Substitute $y = 0$ into (3):

$$x = \frac{2 - 7(0)}{3}$$

$$= \frac{2}{3}$$

∴ The solution is $x = \frac{2}{3}$ and $y = 0$.

(d) $9x + 2y = 5$ — (1)

$7x - 3y = 13$ — (2)

From (1), $9x = 5 - 2y$

$$x = \frac{5 - 2y}{9} \quad \text{— (3)}$$

Substitute (3) into (2):

$$7\left(\frac{5 - 2y}{9}\right) - 3y = 13$$

$$\frac{35}{9} - \frac{14}{9}y - 3y = 13$$

$$4\frac{5}{9}y = -9\frac{1}{9}$$

$$y = -2$$

Substitute $y = -2$ into (3):

$$x = \frac{5 - 2(-2)}{9}$$

$$= 1$$

∴ The solution is $x = 1$ and $y = -2$.

(e) $2y - 5x = 25$ — (1)

$4x + 3y = 3$ — (2)

From (1), $2y = 5x + 25$

$$y = \frac{5x + 25}{2} \quad \text{— (3)}$$

Substitute (3) into (2):

$$4x + 3\left(\frac{5x + 25}{2}\right) = 3$$

$$4x + \frac{15}{2}x + \frac{75}{2} = 3$$

$$11\frac{1}{2}x = -34\frac{1}{2}$$

$$x = -3$$

Substitute $x = -3$ into (3):

$$y = \frac{5(-3) + 25}{2}$$

$$= 5$$

∴ The solution is $x = -3$ and $y = 5$.

(f) $3x - 5y = 7$ — (1)

$4x - 3y = 3$ — (2)

From (1), $3x = 5y + 7$

$$x = \frac{5y + 7}{3} \quad \text{— (3)}$$

Substitute (3) into (2):

$$4\left(\frac{5y + 7}{3}\right) - 3y = 3$$

$$\frac{20}{3}y + \frac{28}{3} - 3y = 3$$

$$3\frac{2}{3}y = -6\frac{1}{3}$$

$$y = -1\frac{8}{11}$$

Substitute $y = -1\frac{8}{11}$ into (3):

$$x = \frac{5\left(-1\frac{8}{11}\right) + 7}{3}$$

$$= -\frac{6}{11}$$

∴ The solution is $x = -\frac{6}{11}$ and $= -1\frac{8}{11}$.

9. (a) $\frac{x}{5} + y + 2 = 0$ — (1)

$\frac{x}{3} - y - 10 = 0$ — (2)

From (1), $y = -\frac{x}{5} - 2$ — (3)

Substitute (3) into (2):

$$\frac{x}{3} - \left(-\frac{x}{5} - 2\right) - 10 = 0$$

$$\frac{x}{3} + \frac{x}{5} + 2 - 10 = 0$$

$$\frac{8}{15}x = 8$$

$$x = 15$$

Substitute $x = 15$ into (3):

$y = -\frac{15}{5} - 2$

$= -5$

∴ The solution is $x = 15$ and $y = -5$.

(b) $\frac{x + y}{3} = 3$ — (1)

$\frac{3x + y}{5} = 1$ — (2)

From (1), $x + y = 9$

$x = 9 - y$ — (3)

Substitute (3) into (2):

$\frac{3(9 - y) + y}{5} = 1$

$27 - 3y + y = 5$

$2y = 22$

$y = 11$

Substitute $y = 11$ into (3):

$x = 9 - 11$

$= -2$

∴ The solution is $x = -2$ and $y = 11$.

(c) $3x - y = 23$ — (1)

$\frac{x}{3} + \frac{y}{4} = 4$ — (2)

From (1), $y = 3x - 23$ — (3)

Substitute (3) into (2):

$\frac{x}{3} + \frac{3x - 23}{4} = 4$

$4x + 9x - 69 = 48$

$13x = 117$

$x = 9$

Substitute $x = 9$ into (3):

$y = 3(9) - 23$

$= 4$

∴ The solution is $x = 9$ and $y = 4$.

(d) $\frac{x}{3} + \frac{y}{2} = 4$ — (1)

$\frac{2}{3}x - \frac{y}{6} = 1$ — (2)

From (1), $2x + 3y = 24$

$2x = 24 - 3y$

$x = \frac{24 - 3y}{2}$ — (3)

Substitute (3) into (1):

$\frac{2}{3}\left(\frac{24 - 3y}{2}\right) - \frac{y}{6} = 1$

$48 - 6y - y = 6$

$7y = 42$

$y = 6$

Substitute $y = 6$ into (3):

$x = \frac{24 - 3(6)}{2}$

$= 3$

∴ The solution is $x = 3$ and $y = 6$.

10. **(a)** $\frac{2}{x + y} = \frac{1}{2x + y}$ — (1)

$3x + 4y = 9$ — (2)

From (1),

$2(2x + y) = x + y$

$4x + 2y = x + y$

$y = -3x$ — (3)

Substitute (3) into (2):

$3x + 4(-3x) = 9$

$3x - 12x = 9$

$-9x = 9$

$x = -1$

Substitute $x = -1$ into (3):

$y = -3(-1)$

$= 3$

∴ The solution is $x = -1$ and $y = 3$.

(b) $\frac{1}{5}(x - 2) = \frac{1}{4}(1 - y)$ — (1)

$\frac{1}{7}\left(x + 2\frac{2}{3}\right) = \frac{1}{3}(3 - y)$ — (2)

$20 \times (1)$:

$4(x - 2) = 5(1 - y)$

$4x - 8 = 5 - 5y$

$4x + 5y = 13$ — (3)

$21 \times (2)$:

$3\left(x + 2\frac{2}{3}\right) = 7(3 - y)$

$3x + 8 = 21 - 7y$

$3x = 13 - 7y$

$x = \frac{13 - 7y}{3}$ — (4)

Substitute (4) into (3):

$4\left(\frac{13 - 7y}{3}\right) + 5y = 13$

$\frac{52}{3} - \frac{28}{3}y + 5y = 13$

$4\frac{1}{3}y = 4\frac{1}{3}$

$y = 1$

Substitute $y = 1$ into (4):

$x = \frac{13 - 7(1)}{3}$

$= 2$

∴ The solution is $x = 2$ and $y = 1$.

(c) $\frac{5x + y}{9} = 2 - \frac{x + y}{5}$ — (1)

$\frac{7x - 3}{2} = 1 + \frac{y - x}{3}$ — (2)

$45 \times (1)$:

$5(5x + y) = 90 - 9(x + y)$

$25x + 5y = 90 - 9x - 9y$

$34x + 14y = 90$

$17x + 7y = 45$ — (3)

$6 \times (2)$:

$3(7x - 3) = 6 + 2(y - x)$

$21x - 9 = 6 + 2y - 2x$

$2y = 23x - 15$

$y = \frac{23x - 15}{2}$ — (4)

Substitute (4) into (3):

$17x + 7\left(\frac{23x - 15}{2}\right) = 45$

$17x + \frac{161}{2}x - \frac{105}{2} = 45$

$97\frac{1}{2}x = 97\frac{1}{2}$

$x = 1$

Substitute $x = 1$ into (4):

$y = \frac{23(1) - 15}{2}$

$= 4$

∴ The solution is $x = 1$ and $y = 4$.

(d) $\frac{x + y}{3} = \frac{x - y}{5}$ — (1)

$\frac{x - y}{5} = 2x - 3y + 5$ — (2)

From (1), $5(x + y) = 3(x - y)$

$5x + 5y = 3x - 3y$

$2x = -8y$

$x = -4y$ — (3)

Substitute (3) into (2):

$\frac{-4y - y}{5} = 2(-4y) - 3y + 5$

$-y = -8y - 3y + 5$

$10y = 5$

$y = \frac{1}{2}$

Substitute $y = \frac{1}{2}$ into (3):

$x = -4\left(\frac{1}{2}\right)$

$= -2$

∴ The solution is $x = -2$ and $y = \frac{1}{2}$.

11. When $x = 3$, $y = -1$,

$3p(3) + q(-1) = 11$

$9p - q = 11$ — (1)

$-q(3) + 5(-1) = p$

$p = -3q - 5$ — (2)

Substitute (2) into (1):

$9(-3q - 5) - q = 11$

$-27q - 45 - q = 11$

$28q = -56$

$q = -2$

Substitute $q = -2$ into (2):

$p = -3(-2) - 5$

$= 1$

∴ The values of p and of q are 1 and –2 respectively.

12. When $x = -11$, $y = 5$,

$p(-11) + 5(5) = q$

$-11p + 25 = q$ — (1)

$q(-11) + 7(5) = p$

$-11q + 35 = p$ — (2)

Substitute (2) into (1):

$-11(-11q + 35) + 25 = q$

$121q - 385 + 25 = q$

$120q = 360$

$q = 3$

Substitute $q = 3$ into (2):

$-11(3) + 35 = p$

$p = 2$

∴ The values of p and of q are 2 and 3 respectively.

13. $8s - 3h = -9$ — (1)

$-29s + 10h = 16$ — (2)

$10 \times (1)$: $80s - 30h = -90$ — (3)

$3 \times (2)$: $-87s + 30h = 48$ — (4)

(3) + (4):

$(80s - 30h) + (-87s + 30h) = -90 + 48$

$7s = 42$

$s = 6$

Substitute $s = 6$ into (1):

$8(6) - 3h = -9$

$48 - 3h = -9$

$3h = 57$

$h = 19$

∴ The height above the ground is 19 m and the time when the cat meets the mouse is 5 s.

Exercise 4F

1. Let the smaller number be x and the greater number be y.

$x + y = 138$ — (1)

$y - x = 88$ — (2)

(1) + (2):

$(x + y) + (y - x) = 138 + 88$

$x + y + y - x = 226$

$2y = 226$

$y = 113$

Substitute $y = 113$ into (1):

$x + 113 = 138$

$x = 25$

∴ The two numbers are 25 and 113.

2. Let the smaller number be x and the greater number be y.

$y - x = 10$ — (1)

$x + y = 4x$ — (2)

From (2), $y = 3x$ — (3)

Substitute (3) into (1):

$3x - x = 10$

$2x = 10$

$x = 5$

Substitute $x = 5$ into (3):

$$\begin{aligned} y &= 3(5) \\ &= 15 \end{aligned}$$

$\therefore$ The two numbers are 5 and 15.

3. Let the cost of a pack of chips be PKR x and the cost of a candy be PKR y.

$$\begin{aligned} x + y &= 42 \quad \text{— (1)} \\ 7x + 4y &= 213 \quad \text{— (2)} \end{aligned}$$

From (1), $y = 42 - x$ — (3)

Substitute (3) into (2):

$$\begin{aligned} 7x + 4(42 - x) &= 213 \\ 7x + 168 - 4x &= 213 \\ 3x &= 45 \\ x &= 15 \end{aligned}$$

Substitute $x = 15$ into (3):

$$\begin{aligned} y &= 42 - 15 \\ &= 27 \end{aligned}$$

$\therefore$ The cost of a pack of chips is PKR 15 and the cost of a candy is PKR 27.

4. Let the cost of 1 kg of potatoes be PKR x and the cost of 1 kg of carrots be PKR y.

$$\begin{aligned} 8x + 5y &= 280 \quad \text{— (1)} \\ 2x + 3y &= 112 \quad \text{— (2)} \end{aligned}$$

$4 \times (2)$: $8x + 12y = 448$ — (3)

(3) – (1):

$$\begin{aligned} (8x + 12y) - (8x + 5y) &= 448 - 280 \\ 8x + 12y - 8x - 5y &= 168 \\ 7y &= 168 \\ y &= 24 \end{aligned}$$

Substitute $y = 24$ into (2):

$$\begin{aligned} 2x + 3(24) &= 112 \\ 2x + 72 &= 112 \\ 2x &= 40 \\ x &= 20 \end{aligned}$$

$\therefore$ 1 kg of potatoes cost PKR 20 and 1 kg of carrots cost PKR 24.

5. Let the first number be x and the second number be y.

$$\begin{aligned} x + 7 &= 2y \quad \text{— (1)} \\ y + 20 &= 4x \quad \text{— (2)} \end{aligned}$$

From (1), $x = 2y - 7$ — (3)

Substitute (3) into (2):

$$\begin{aligned} y + 20 &= 4(2y - 7) \\ &= 8y - 28 \\ 7y &= 48 \\ y &= 6\frac{6}{7} \end{aligned}$$

Substitute $y = 6\frac{6}{7}$ into (3):

$$\begin{aligned} x &= 2\left(6\frac{6}{7}\right) - 7 \\ &= 6\frac{5}{7} \end{aligned}$$

$\therefore$ The two numbers are $6\frac{5}{7}$ and $6\frac{6}{7}$.

6. Let the smaller number be x and the greater number be y.

$$\begin{aligned} x + y &= 48 \quad \text{— (1)} \\ x &= \frac{1}{5}y \quad \text{— (2)} \end{aligned}$$

Substitute (2) into (1):

$$\begin{aligned} \frac{1}{5}y + y &= 48 \\ \frac{6}{5}y &= 48 \\ y &= 40 \end{aligned}$$

Substitute $y = 40$ into (2):

$$\begin{aligned} x &= \frac{1}{5}(40) \\ &= 8 \end{aligned}$$

$\therefore$ The two numbers are 8 and 40.

7. Let the smaller angle be x and the greater angle be y.

$$\begin{aligned} \frac{1}{5}(x + y) &= 24^\circ \quad \text{— (1)} \\ \frac{1}{2}(y - x) &= 14^\circ \quad \text{— (2)} \end{aligned}$$

$5 \times (1)$: $x + y = 120^\circ$ — (3)

$2 \times (2)$: $y - x = 28^\circ$ — (4)

(3) + (4):

$$\begin{aligned} (x + y) + (y - x) &= 120^\circ + 28^\circ \\ x + y + y - x &= 148^\circ \\ 2y &= 148^\circ \\ y &= 74^\circ \end{aligned}$$

Substitute $y = 74^\circ$ into (3):

$$\begin{aligned} x + 74^\circ &= 120^\circ \\ x &= 46^\circ \end{aligned}$$

$\therefore$ The two angles are 46° and 74°.

8. The sides of an equilateral triangle are equal.

$$\begin{aligned} x + y - 9 &= y + 5 \quad \text{— (1)} \\ y + 5 &= 2x - 7 \quad \text{— (2)} \end{aligned}$$

From (1), $x = 14$

$$\begin{aligned} \text{Length of each side} &= 2(14) - 7 \\ &= 21 \text{ cm} \end{aligned}$$

$\therefore$ The length of each side of the triangle is 21 cm.

9.

$$\begin{aligned} 3x - y &= 2x + y \quad \text{— (1)} \\ 3x - y + 2x + y + 2(2x - 3) &= 120 \quad \text{— (2)} \end{aligned}$$

From (2),

$$\begin{aligned} 3x - y + 2x + y + 4x - 6 &= 120 \\ 9x &= 126 \\ x &= 14 \end{aligned}$$

Substitute $x = 14$ into (1):

$$\begin{aligned} 3(14) - y &= 2(14) + y \\ 42 - y &= 28 + y \\ 2y &= 14 \\ y &= 7 \end{aligned}$$

$$\begin{aligned} \text{Area of rectangle} &= [3(14) - 7] \times [2(14) - 3] \\ &= 35 \times 25 \\ &= 875 \text{ cm}^2 \end{aligned}$$

$\therefore$ The area of the rectangle is 875 cm^2.

10. The sides of a rhombus are equal.

$2x + y + 1 = \dfrac{3x - y - 2}{2}$ — (1)

$2x + y + 1 = x - y$ — (2)

From (2), $x = -2y - 1$ — (3)

Substitute (3) into (1):

$$2(-2y - 1) + y + 1 = \frac{3(-2y - 1) - y - 2}{2}$$
$$-4y - 2 + y + 1 = \frac{-7y - 5}{2}$$
$$-6y - 2 = -7y - 5$$
$$y = -3$$

Substitute $y = -3$ into (3):

$x = -2(-3) - 1$
$= 5$

Perimeter of the figure $= 4[5 - (-3)]$
$= 32$ cm

∴ The perimeter of the figure is 32 cm.

11. Let the numerator of the fraction be x and its denominator be y, i.e. let the fraction be $\dfrac{x}{y}$.

$\dfrac{x-1}{y-1} = \dfrac{1}{2}$ — (1)

$\dfrac{x+1}{y+1} = \dfrac{2}{3}$ — (2)

From (1),

$2(x - 1) = y - 1$
$2x - 2 = y - 1$
$y = 2x - 1$ — (3)

Substitute (3) into (2):

$$\frac{x+1}{2x-1+1} = \frac{2}{3}$$
$$3(x + 1) = 4x$$
$$3x + 3 = 4x$$
$$x = 3$$

Substitute $x = 3$ into (3):

$y = 2(3) - 1$
$= 5$

∴ The fraction is $\dfrac{3}{5}$.

12. Let the age of Rani in 2013 be x years old and the age of Jia in 2013 be y years old.

$x + y = 11$ — (1)

$x + 9 = 3y$ — (2)

(1) – (2):

$$(x + y) - (x + 9) = 11 - 3y$$
$$x + y - x - 9 = 11 - 3y$$
$$y - 9 = 11 - 3y$$
$$4y = 20$$
$$y = 5$$

Substitute $y = 5$ into (1):

$x + 5 = 11$
$x = 6$

In 2014,

Age of Rani $= 6 + 1$
$= 7$

Age of Jia $= 5 + 1$
$= 6$

∴ In 2014, the ages of Rani and Jia are 7 years and 6 years respectively.

13. Let the amount an adult has to pay be PKR x and the amount a senior citizen has to pay be PKR y.

$6x + 4y = 22800$ — (1)

$13x + 7y = 45900$ — (2)

From (1), $3x + 2y = 11400$ — (3)

$2 \times$ (2): $26x + 14y = 91800$ — (4)

$7 \times$ (3): $21x + 14y = 79800$ — (5)

(4) – (5):

$$(26x + 14y) - (21x + 14y) = 91800 - 79800$$
$$26x + 14y - 21x - 14y = 12000$$
$$5x = 12000$$
$$x = 2400$$

Substitute $x = 24$ into (3):

$$3(2400) + 2y = 114$$
$$7200 + 2y = 11400$$
$$2y = 4200$$
$$y = 2100$$

Total amount 2 adults and a senior citizen have to pay

= 2(PKR 2400) + PKR 2100

= PKR 6900

∴ The total amount is PKR 6900.

14. Let the number of gift A to buy be x and the number of gift B to buy be y.

$1000x + 800y = 23000$ — (1)

$x + y = 2 + 2 + 13 + 10$
$= 27$ — (2)

From (2), $y = 27 - x$ — (3)

Substitute (3) into (1):

$$1000x + 800(27 - x) = 23000$$
$$1000x + 21600 - 800x = 23000$$
$$200x = 1400$$
$$x = 7$$

Substitute $x = 7$ into (3):

$y = 27 - 7$
$= 20$

∴ Faiz should buy 7 gift A and 20 gift B.

15. Let the number of chickens be x and the number of goats be y.

$x + y = 50$ — (1)

$2x + 4y = 140$ — (2)

From (1), $y = 50 - x$ — (3)

Substitute (3) into (2):

$$2x + 4(50 - x) = 140$$
$$2x + 200 - 4x = 140$$
$$2x = 60$$
$$x = 30$$

Substitute $x = 30$ into (3):

$y = 50 - 30$

$= 20$

Number of more chickens than goats $= 30 - 20$

$= 10$

$\therefore$ There are 10 more chickens than goats.

16. Let the amount Ahsan has be PKR x and the amount Maaz has be PKR y.

$x + y = 80 \quad — (1)$

$\frac{1}{4}x = \frac{1}{6}y \quad — (2)$

From (1), $y = 80 - x \quad — (3)$

Substitute (3) into (2):

$\frac{1}{4}x = \frac{1}{6}(80 - x)$

$3x = 160 - 2x$

$5x = 160$

$x = 32$

Substitute $x = 32$ into (1):

$32 + y = 80$

$y = 48$

$\therefore$ Ahsan received PKR 32 and Maaz received PKR 48.

17. Let the amount deposited in Bank A be PKR x and the amount deposited in Bank B be PKR y.

$x + y = 25\,000 \quad — (1)$

$\frac{0.6}{100}x = \frac{0.65}{100}y \quad — (2)$

From (2), $y = \frac{12}{13}x \quad — (3)$

Substitute (3) into (1):

$x + \frac{12}{13}x = 25\,000$

$\frac{25}{13}x = 25\,000$

$x = 13\,000$

Substitute $x = 13\,000$ into (3):

$y = \frac{12}{13}(13\,000)$

$= 12\,000$

$\therefore$ Rizwan deposited PKR 13 000 in Bank A and PKR 12 000 in Bank B.

18. Let the smaller number be x and the greater number be y.

$\frac{y - 2}{x} = 2 \quad — (1)$

$\frac{5x - 2}{y} = 2 \quad — (2)$

From (1), $y - 2 = 2x$

$x = \frac{y - 2}{x} \quad — (3)$

Substitute (3) into (2):

$\frac{5\left(\frac{y-2}{2}\right) - 2}{y} = 2$

$5\left(\frac{y-2}{2}\right) - 2 = 2y$

$\frac{5}{2}y - 5 - 2 = 2y$

$\frac{1}{2}y = 7$

$y = 14$

Substitute $y = 14$ into (3):

$x = \frac{14 - 2}{2}$

$= 6$

$\therefore$ The two numbers are 6 and 14.

19. Let the tens digit of the original number be x and its ones digit be y. Then the original number is $10x + y$, the number obtained when the digits of the original number are reversed is $10y + x$.

$x + y = \frac{1}{8}(10x + y) \quad — (1)$

$(10x + y) - (10y + x) = 45 \quad — (2)$

From (1),

$8(x + y) = 10x + y$

$8x + 8y = 10x + y$

$2x = 7y$

$x = \frac{7}{2}y \quad — (3)$

From (2),

$10x + y - 10y - x = 45$

$9x - 9y = 45$

$x - y = 5 \quad — (4)$

Substitute (3) into (4):

$\frac{7}{2}y - y = 5$

$\frac{5}{2}y = 5$

$y = 2$

Substitute $y = 2$ into (3):

$x = \frac{7}{2}(2)$

$= 7$

$\therefore$ The original number is 72.

20. Let PKR x be the price of one pear and PKR y be the price of one mango.

$8x + 5y = 1000 + 110$

$8x + 5y = 1110$ _______ (1)

$5x + 4y = 1000 - 175$

$5x + 4y = 825$ ________ (2)

From (1) $8x = 1110 - 5y$

$x = \frac{1110 - 5y}{8}$

Substitute the value of x be (2)

$5\left(\frac{1110 - 5y}{8}\right) + 4y = 825$

$\frac{5550 - 25y}{8} + 4y = 825$

$\frac{5550 - 25y + 32y}{8} = 825$

$5550 + 7y = 825 \times 8$

$5550 + 7y = 6600$

$7y = 6600 - 5550$

$7y = 1050$

$y = 1050$

Substitute $y = 150$ in (1)

$8x + 5 \times 150 = 1110$

$8x = 1110 - 750$

$8x = 360$

$x = 45$

$\therefore$ Cost of one pear = PKR 45

Cost of one mango = PKR 150

21. **(i)** Let the number of shares of Company A Anoshia's mother has be x and the share price of Company B on Day 7 be PKR y.

$$4.6x - 2000y = 7400 \quad \text{— (1)}$$

$$4.8x - 5000(y - 0.5) = -5800 \quad \text{— (2)}$$

From (1),

$$2000y = 4.6x - 7400$$

$$y = \frac{4.6x - 7400}{2000} \quad \text{— (3)}$$

Substitute (3) into (2):

$$4.8x - 5000\left(\frac{4.6x - 7400}{2000} - 0.5\right) = -5800$$

$$4.8x - 11.5x + 18\,500 + 2500 = -5800$$

$$6.7x = 26\,800$$

$$x = 4000$$

$\therefore$ Anoshia's mother has 4000 shares of Company A.

(ii) From **(i)**,

substitute $x = 4000$ into (3):

$$y = \frac{4.6(4000) - 7400}{2000}$$

$$= 5.5$$

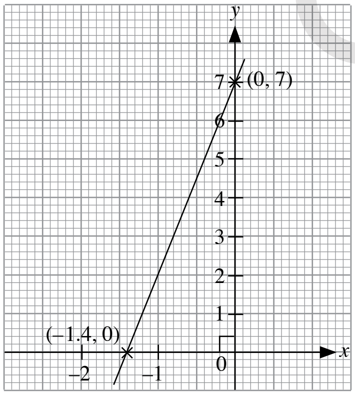
Share price of Company B on Day 12 $= 5.5 - 0.5$

$= 5$

$\therefore$ The share price of Company B on Day 12 is PKR 5.

Review Exercise 4

1. **(a)**

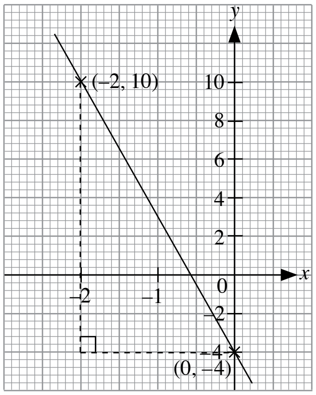


$$m = \frac{7}{1.4}$$

$$= 5$$

$c = 7$

(b)

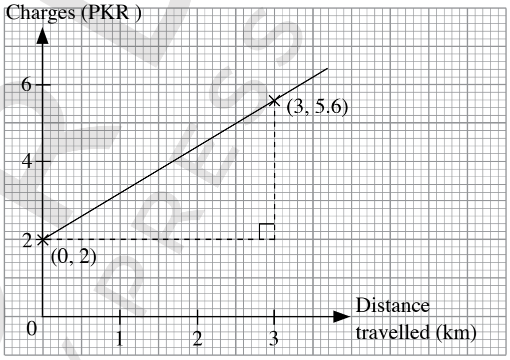


$$m = -\frac{14}{2}$$

$$= -7$$

$c = -4$

2.



$m = y$-intercept

$= 2$

n = gradient of line

$$= \frac{3.6}{3}$$

$= 1.2$

3. **(i)** Price Company A charges for 20 minutes of talk time = PKR 0.80

(ii) Price Company B charges for 50 minutes of talk time = PKR 3.80

(iii) For less than 30 minutes of talk time, Company B charges a lower price than Company A, thus Company B would be able to offer Jamil a better price.

(iv) m_A = gradient of A

$$= \frac{4}{50}$$

$$= \frac{2}{25}$$

m_B = gradient of B

$$= \frac{5}{40}$$

$$= \frac{1}{8}$$

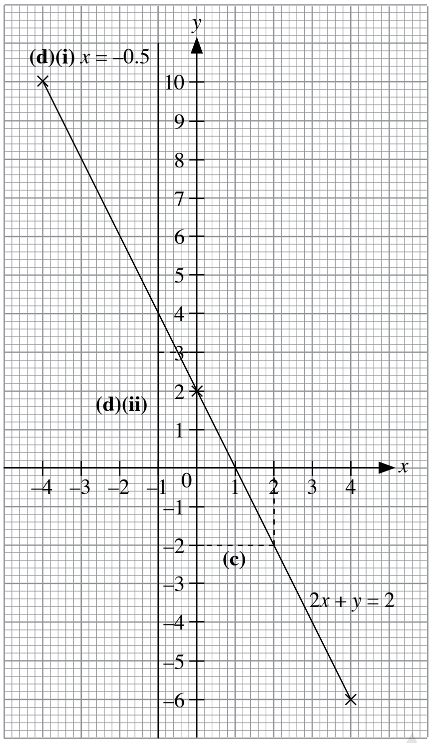
Since $m_B > m_A$, Company B has a greater rate of increase in charges.

(v) At PKR 4 per month,
duration of talk time offered by Company $A = 60$ minutes and
duration of talk time offered by Comapny $B = 52$ minutes.
Since Company A offers more talk time for PKR 4 per month,
Maaz should choose Company A.

4. **(a)** $2x + y = 2$

x	–4	0	4
y	10	2	–6

(b)



(c) From the graph in **(b)**,
When $y = –2$,
$p = x = 2$

(d) **(ii)** The coordinates of the point of intersection are (–0.5, 3).

5. **(a)** **(i)** When $x = –5$, $y = p$,

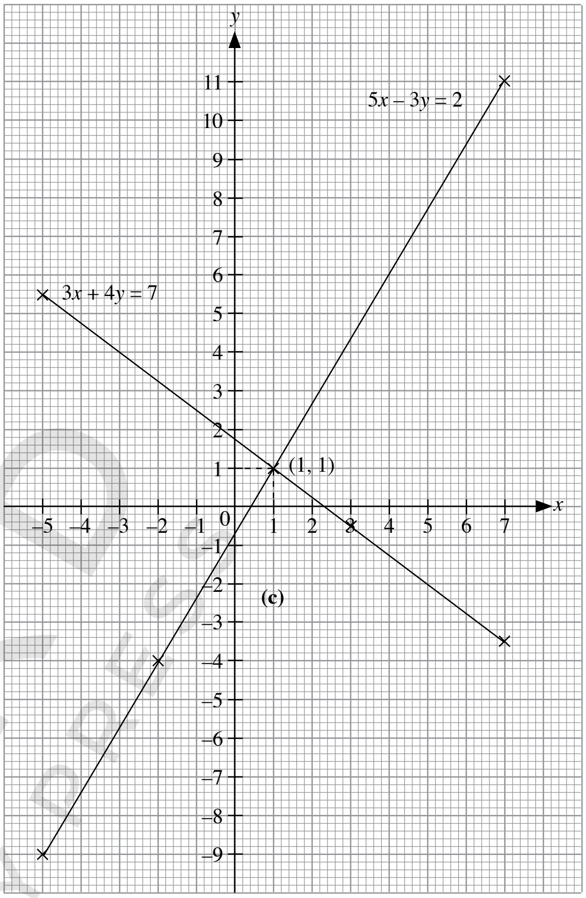
$5(–5) – 3p = 2$
$–25 – 3p = 2$
$3p = –27$
$p = –9$

When $x = 7$, $y = q$,

$5(7) – 3q = 2$
$35 – 3q = 2$
$3q = 33$
$q = 11$

$\therefore p = –9, q = 11$

(ii)



(b) **(i)** $3x + 4y = 7$

x	–5	3	7
y	5.5	–0.5	–3.5

(c) The graphs intersect at the point (1, 1).
$\therefore$ The solution is $x = 1$ and $y = 1$.

6. **(a)** $7x + 2y = 10$ — (1)
$5x + 2y = 6$ — (2)
(1) – (2):

$(7x + 2y) – (5x + 2y) = 10 – 6$
$7x + 2y – 5x – 2y = 4$
$2x = 4$
$x = 2$

Substitute $x = 2$ into (2):

$5(2) + 2y = 6$
$10 + 2y = 6$
$2y = –4$
$y = –2$

$\therefore$ The solution is $x = 2$ and $y = –2$.

(b) $9x + 4y = 28$ — (1)
$4y – 11x = –12$ — (2)
(1) – (2):

$(9x + 4y) – (4y – 11x) = 28 – (–12)$
$9x + 4y – 4y + 11x = 28 + 12$
$20x = 40$
$x = 2$

Substitute $x = 2$ into (1):

$9(2) + 4y = 28$

$18 + 4y = 28$

$4y = 10$

$y = 2\frac{1}{2}$

∴ The solution is $x = 2$ and $y = 2\frac{1}{2}$.

(c) $2x - 5y = 22$ — (1)

$2x - 3y = 14$ — (2)

(1) – (2):

$(2x - 5y) - (2x - 3y) = 22 - 14$

$2x - 5y - 2x + 3y = 8$

$2y = -8$

$y = -4$

Substitute $y = -4$ into (2):

$2x - 3(-4) = 14$

$2x + 12 = 14$

$2x = 2$

$x = 1$

∴ The solution is $x = 1$ and $y = -4$.

(d) $6x - y = 16$ — (1)

$3x + 2y = -12$ — (2)

From (1), $y = 6x - 16$ — (3)

Substitute (3) into (2):

$3x + 2(6x - 16) = -12$

$3x + 12x - 32 = -12$

$15x = 20$

$x = 1\frac{1}{3}$

Substitute $x = 1\frac{1}{3}$ into (3):

$y = 6\left(1\frac{1}{3}\right) - 16$

$= -8$

∴ The solution is $x = 1\frac{1}{3}$ and $y = -8$.

(e) $4x + 3y = 0$ — (1)

$5y + 53 = 11x$ — (2)

From (1), $x = -\frac{3}{4}y$ — (3)

Substitute $x = -\frac{3}{4}y$ into (2):

$5y + 53 = 11\left(-\frac{3}{4}y\right)$

$= -\frac{33}{4}y$

$13\frac{1}{4}y = -53$

$y = -4$

Substitute $y = -4$ into (3):

$x = -\frac{3}{4}(-4)$

$= 3$

∴ The solution is $x = 3$ and $y = -4$.

(f) $5x - 4y = 4$ — (1)

$2x - y = 2.5$ — (2)

From (2), $y = 2x - 2.5$ — (3)

Substitute (3) into (1):

$5x - 4(2x - 2.5) = 4$

$5x - 8x + 10 = 4$

$3x = 6$

$x = 2$

Substitute $x = 2$ into (3):

$y = 2(2) - 2.5$

$= 1.5$

∴ The solution is $x = 2$ and $y = 1.5$.

7. Let the first number be x and the second number be y.

$x + 11 = 2y$ — (1)

$y + 20 = 2x$ — (2)

From (1), $x = 2y - 11$ — (3)

Substitute (3) into (2):

$y + 20 = 2(2y - 11)$

$= 4y - 22$

$3y = 42$

$y = 14$

Substitute $y = 14$ into (3):

$x = 2(14) - 11$

$= 17$

∴ The two numbers are 17 and 14.

8. The parallel sides of a parallelogram are equal.

$x + y + 1 = 3x - 4$ — (1)

$2y - x = x + 2$ — (2)

From (1), $y = 2x - 5$ — (3)

Substitute (3) into (2):

$2(2x - 5) - x = x + 2$

$4x - 10 - x = x + 2$

$2x = 12$

$x = 6$

Substitute $x = 6$ into (3):

$y = 2(6) - 5$

$= 7$

Perimeter of parallelogram $= 2\{[2(7) - 6] + (6 + 7 + 1)\}$

$= 44$ cm

∴ The perimeter of the parallelogram is 44 cm.

9. Let the numerator of the fraction be x and its denominator be y, i.e. let the fraction be $\frac{x}{y}$.

$\frac{x - 1}{y + 2} = \frac{1}{2}$ — (1)

$\frac{x + 3}{y - 2} = 1\frac{1}{4}$ — (2)

From (1),

$2(x - 1) = y + 2$

$2x - 2 = y + 2$

$y = 2x - 4$ — (3)

From (2),

$4(x + 3) = 5(y - 2)$

$4x + 12 = 5y - 10$

$4x - 5y = -22$ — (4)

Substitute (3) into (4):

$4x - 5(2x - 4) = -22$

$4x - 10x + 20 = -22$

$6x = 42$

$x = 7$

Substitute $x = 7$ into (3):

$y = 2(7) - 4$

$= 10$

∴ The fraction is $\frac{7}{10}$.

10. Let the tens digit of the number be x and its ones digit be y.

$x + y = 12$ — (1)

$y = 2x$ — (2)

Substitute (2) into (1):

$x + 2x = 12$

$3x = 12$

$x = 4$

Substitute $x = 4$ into (2):

$y = 2(4)$

$= 8$

∴ The number is 48.

11. (i) Let Hussain's present age be x years old and Hussain's monther's present age be y years old.

$y + 4 = 3(x + 4)$ — (1)

$y - 6 = 7(x - 6)$ — (2)

From (1), $y + 4 = 3x + 12$

$y = 3x + 8$ — (3)

Substitute (3) into (2):

$3x + 8 - 6 = 7(x - 6)$

$3x + 2 = 7x - 42$

$4x = 44$

$x = 11$

∴ Hussain's present age is 11 years.

(ii) From **(i)**,

Substitute $x = 11$ into (3):

$y = 3(11) + 8$

$= 41$

Age of Hussain's mother when he was born $= 41 - 11$

$= 30$

∴ The age of Hussain's mother was 30 years.

12. Let the amount Sarah has be PKR x and the amount Seema has be PKR y.

$2(x - 3) = y + 300$ — (1)

$x + 5 = 2(y - 500)$ — (2)

From (1), $2x - 6 = y + 300$

$y = 2x - 900$ — (3)

Substitute (3) into (2):

$x + 5 = 2(2x - 900 - 500)$

$= 4x - 2800$

$3x = 2805$

$x = 935$

Substitute $x = 935$ into (3):

$y = 2(935) - 9$

$= 1861$

∴ Sarah has PKR 1861 and Seema has PKR 935.

13. Let the number of smartphones be x and the number of tablet computers be y.

$x + y = 36$ — (1)

$895x + 618y = 28\,065$ — (2)

From (1), $y = 36 - x$ — (3)

Substitute (3) into (2):

$895x + 618(36 - x) = 28\,065$

$895x + 22\,248 - 618x = 28\,065$

$277x = 5817$

$x = 21$

Substitute $x = 21$ into (3):

$y = 36 - 21$

$= 15$

∴ The vendor buys 21 smartphones and 15 tablet computers.

14. Let the cost of 1 cup of ice-cream milk tea be PKR x and the cost of 1 cup of citron tea be PKR y.

$5x + 4y = 2680$ — (1)

$7x + 6y = 3860$ — (2)

$3 \times (1)$: $15x + 12y = 8040$ — (3)

$2 \times (2)$: $14x + 12y = 7720$ — (4)

(3) – (4):

$(15x + 12y) - (14x + 12y) = 8040 - 7720$

$15x + 12y - 14x - 12y = 320$

$x = 320$

Substitute $x = 320$ into (1):

$5(320) + 4y = 2680$

$1600 + 4y = 2680$

$6y = 1080$

$y = 270$

Difference in cost = PKR 320 – PKR 180

= PKR 140

∴ The difference in cost is PKR 140.

15. Let x kg be the mass of first type of coffee and y kg be the mass of second type of coffee used for mixture .

$x + y = 20$ kg ______ (1)

The cost of x kg of coffee = PKR 250 x

The cost of y kg of coffee = PKR 350 y

The Cost of 20 kg of coffee = 20 × 280

= PKR 5600

$250\,x + 350y = 5600$ _________ (2)

Now we have ,

$x + y = 20$ ____ (1)

$250x + 350y = 5600$ ______ (2)

Multiply (1) by 350

$350x + 350y = 7000$ ____ (3)

$250x + 350y = 5600$ ____ (4)

Subtract (4) from (3) :

$100\,x = 1400$

$x = 14$ kg

Substitute $x = 14$ in (1) :

$14 + y = 20$

$y = 6$ kg

Mishal used 14kg and 6 kg of each type of coffee respectively.

16. $120x + (175 - 120)y = 2680$ — (1)

$120x + (210 - 120)y = 3240$ — (2)

From (1), $120x + 55y = 2680$ — (3)

From (2), $120x + 90y = 3240$ — (4)

(4) – (3):

$(120x + 90y) - (120x + 55y) = 3240 - 2680$

$120x + 90y - 120x - 55y = 560$

$35y = 560$

$y = 16$

Substitute $y = 16$ into (3):

$120x + 55(16) = 2680$

$120x + 880 = 2680$

$120x = 1800$

$x = 15$

Amount to pay for 140 minutes of talk time

$= 120 \times 15 + (140 - 120) \times 16$

= 2120 paisa

= PKR 21.20

∴ The amount Ahsan has to pay is PKR 21.20.

17. Let the number of students in class 2A be x.

and the number of students in class 2B be y.

$72x + 75y = 75(73.48)$

$= 5511$ — (1)

$x + y = 75$ — (2)

From (2), $y = 75 - x$ — (3)

Substitute (3) into (1):

$72x + 75(75 - x) = 5511$

$72x + 5625 - 75x = 5511$

$3x = 114$

$x = 38$

Substitute $x = 38$ into (3):

$y = 75 - 38$

$= 37$

∴ Class 2A has 38 students and class 2B has 37 students.

Challenge Yourself

1. **(i)** $px - y = 6$ — (1)

$8x - 2y = q$ — (2)

From (1), $2px - 2y = 12$

For the simultaneous equations to have an infinite number of solutions, the two equations should be identical.

$2p = 8$

$p = 4$

$q = 12$

(ii) For the simultaneous equations to have no solution, the two equations should have no point of intersection.

$\therefore p = 4, q \neq 12$

(iii) For the simultaneous equations to have a unique solution, the two equations should have one and only one point of intersection.

$\therefore p \neq 4$ and q is any real number.

2. $\dfrac{4}{x} + \dfrac{15}{y} = 15$ — (1)

$\dfrac{7}{5x} - \dfrac{6}{y} = 3$ — (2)

From (1), $4y + 15x = 15xy$ — (3)

From (2), $7y - 30x = 15xy$ — (4)

(3) = (4):

$4y + 15x = 7y - 30x$

$3y = 45x$

$y = 15x$ — (5)

Substitute (5) into (1):

$$\frac{4}{x} + \frac{15}{15x} = 15$$

$$\frac{4}{x} + \frac{1}{x} = 15$$

$$\frac{5}{x} = 15$$

$$x = \frac{1}{3}$$

Substitute $x = \dfrac{1}{3}$ into (5):

$$y = 15\left(\frac{1}{3}\right)$$

$= 5$

∴ The solution is $x = \dfrac{1}{3}$ and $y = 5$.

3. Let the first number be x and the second number of y.

$11x^2 + 13y^3 = 395$ — (1)

$26y^3 - 218 = 121x^2$ — (2)

$2 \times (1)$: $22x^2 + 26y^3 = 790$ — (3)

(3) – (2):

$(22x^2 + 26y^3) - (26y^2 - 218) = 790 - 121x^2$

$22x^2 + 26y^3 - 26y^3 + 218 = 790 - 121x^2$

$143x^2 = 572$

$x^2 = 4$

$x = 2\ (x > 0)$

Substitute $x = 2$ into (1):

$11(2)^2 + 13y^3 = 395$

$44 + 13y^3 = 395$

$13y^3 = 351$

$y^3 = 27$

$y = 3$

$\therefore$ The solution is $x = 2$ and $y = 3$.

4. **(i)** Let the number of spiders be x, the number of dragonflies be y and the number of houseflies be z.

$x + y + z = 20$ — (1)

$8x + 6y + 6z = 136$ — (2)

$2y + z = 19$ — (3)

From (3), $z = 19 - 2y$ — (4)

Substitute (4) into (1):

$x + y + 19 - 2y = 20$

$y = x - 1$ — (5)

Substitute (4) into (2):

$8x + 6y + 6(19 - 2y) = 136$

$8x + 6y + 114 - 12y = 136$

$8x - 6y = 22$

$4x - 3y = 11$ — (6)

Substitute (5) into (6):

$4x - 3(x - 1) = 11$

$4x - 3x + 3 = 11$

$x = 8$

$\therefore$ The number of spiders is 8.

(ii) From **(i)**,

Substitute $x = 8$ into (5):

$y = 8 - 1$

$= 7$

$\therefore$ The number of dragonflies is 7.

(iii) From **(i)** and **(ii)**,

Substitute $y = 7$ into (4):

$z = 19 - 2(7)$

$= 5$

$\therefore$ The number of houseflies is 5.

5. Let r be rooster , h be hen , and C be the chicks.

$r + h + c = 100$ ________ (1)

$500r + 300h + \frac{100}{3}c =$ ______ (2)

$1500r + 900h + 100c = 30000$ ______ (3)

$15r + 9h + c = 300$ ________ (4)

Subtract (1) from (4)

$14h + 8h = 200$

$7hr + 4h = 100$

$h = \frac{100-72}{4}$

$h = \frac{45 - 7}{4}$

Since h is a positive integer , r must be the multiple of 4.

$25 - \frac{7r}{4} > 0$

$\frac{7r}{4} < 25$

$7r < 100$

$r < 14\frac{2}{7}$

Possible value of r are 4, 8, and 12

r	h	c	$r + h + c$
4	18	78	100
8	11	81	100
12	4	84	100

Chapter 5 Indices and Standard Form

TEACHING NOTES

Suggested Approach

In Book 1, the students have been introduced to writing numbers in index notation. In this chapter, they will learn the laws of indices, zero and negative indices and rational indices.

Teacher should consider using the Investigation activities provided in the textbook to allow students to explore the laws of indices for numbers before moving on to variables. It is not advisable to state all the laws of indices to the students when teaching this chapter. After the students are familiar with laws of indices introduced in Section 5.2, where all the indices are positive integers, teachers can extend it to Section 5.3: Zero and Negative Indices and Section 5.4: Rational Indices.

Teacher should also conduct more discussions on how compound interest and standard form are used in real life.

Section 5.1: Indices

This section gives students a better understanding on the meanings of the base and the index represented in an index notation. Teachers may start on this chapter by giving scenarios where indices are involved and ask the students to represent their answers in index notation, like what they have learnt in Book 1. Teachers should guide students along as they learn how to describe and compare numbers written in index form (see Class Discussion: Comparing Numbers written in Index Form).

Section 5.2: Laws of Indices

Teachers should provide simple numerical examples to illustrate each law of indices. Ample examples should be given to the students to master each law first before moving on to the next law (see Investigation: Law 1 of Indices, Investigation: Law 2 of Indices, Investigation: Law 3 of Indices, Investigation: Law 4 of Indices and Investigation: Law 5 of Indices).

Teachers should clarify any common misconceptions students may have or difficulties they may encounter when working on questions involving the use of a few laws of indices (see Journal writing on page 160 of the textbook).

Section 5.3: Zero and Negative Indices

Teachers may ask the students to explore the meaning of zero and negative indices through activities instead of only asking them to state the definition of such indices (see Investigation: Zero Index and see Investigation: Negative Indices).

It is important to emphasise to the students the meaning of ‘evaluate’ and ‘leaving your answer in positive index form. Teachers should also highlight the importance of recognising where the brackets are placed in a question (see Thinking Time on page 164 of the textbook).

Section 5.4: Rational Indices

In Book 1, students have learnt about the square root and cube root of a number. Teacher may wish to extend on this by introducing the meaning of positive n^{th} root and radical expression.

Teachers should highlight to students to consider the need for the base to be positive in rational indices (see Thinking Time on page 169 of the textbook).

Section 5.5: Standard Form

Teacher may begin this section by getting students to explore how standard forms are being expressed by giving them some examples of very large and small numbers for them to express these numbers in standard form (see Class Discussion: Standard Form). Teachers should highlight the difference between numbers expressed in standard form and numbers not expressed in standard form so that students can better identify which expressions are in standard form.

For the introduction of common prefixes used in our daily lives, teachers may use the range of prefixes used in our daily lives (see page 177 of the textbook) to get the students to give more examples of prefixes that they encounter in their daily lives and to practise reading prefixes.

Some students may find it difficult to manipulate numbers in standard form using a calculator. Teachers should give them time and guide them through some examples on using the calculator to evaluate numbers represented in standard form.

WORKED SOLUTIONS

Investigation (Indices)

Amount of allowance on the 31st day of the month $= 2^{31}$
$= \text{PKR } 2\ 147\ 483\ 548$

Class Discussion (Comparing Numbers written in Index Form)

1. 2^{10} means 2 multiplied by itself 10 times, while 10^2 means 10 multiplied by itself.
2. $2^{10} = (2 \times 2 \times 2 \times 2 \times 2) \times (2 \times 2 \times 2 \times 2 \times 2)$
   $= 32^2 > 10^2$
3. $3^7 = 3(3)^6 = 3(9)^3 > 7^3$
4.

Value of a^b	Value of b^a
$2^3 = 8$	$3^2 = 9$
$2^4 = 16$	$4^2 = 16$
$3^4 = 81$	$4^3 = 64$
$3^5 = 243$	$5^3 = 125$
$4^5 = 1024$	$5^4 = 625$
$4^6 = 4096$	$6^4 = 1296$

(i) If a and b are positive integers such that $b > a$, $a^b = b^a$ when $a = 2$ and $b = 4$.
It is not easy to prove that this is the only solution; students are only expected to use guess and check to find a solution.

(ii) If a and b are positive integers such that $b > a$, $a^b < b^a$ when $a = 1$, i.e. $1^2 < 2^1$, $1^3 < 3^1$, $1^4 < 4^1$, etc and when $a = 2$ and $b = 3$, $a^b < b^a$, i.e. $2^3 < 3^2$.

5. In general, if a and b are positive integers such that $b > a$, then $a^b > b^a$, with some exceptions when $a = 2$ and $b \leqslant 4$ and when $a = 1$.

Investigation (Law 1 of Indices)

1. $7^2 \times 7^3 = (7 \times 7) \times (7 \times 7 \times 7)$
   $= 7 \times 7 \times \ldots \times 7$
   $= 7^5$
   $= 7^{2+3}$
2. $6^4 \times 6^5 = (6 \times 6 \times 6 \times 6) \times (6 \times 6 \times 6 \times 6 \times 6)$
   $= 6 \times 6 \times \ldots \times 6$
   $= 6^9$
   $= 6^{4+5}$
3. $a^3 \times a^4 = (a \times a \times a) \times (a \times a \times a \times a)$
   $= a \times a \times \ldots \times a$
   $= a^7$
   $= a^{3+4}$
4. $a^m \times a^n = \underbrace{(a \times a \times \ldots \times \ldots \times a)}_{m\text{ times}} \times \underbrace{(a \times a \times \ldots \times a)}_{n\text{ times}}$
   $= \underbrace{a \times a \times \ldots \times a}_{m+n\text{ times}}$
   $= a^{m+n}$

Investigation (Law 2 of Indices)

1. $3^5 \div 3^2 = \dfrac{3 \times 3 \times 3 \times 3 \times 3}{3 \times 3}$
   $= 3^3$
   $= 3^{5-2}$
2. $\dfrac{10^6}{10^4} = \dfrac{10 \times 10 \times 10 \times 10 \times 10 \times 10}{10 \times 10 \times 10 \times 10}$
   $= 10^2$
   $= 10^{6-4}$
3. $a^7 \div a^3 = \dfrac{a \times a \times a \times a \times a \times a \times a}{a \times a \times a}$
   $= a^4$
   $= a^{7-3}$
4. $a^m \div a^n = \dfrac{\overbrace{a \times a \times \ldots \times \ldots \times a}^{m\text{ times}}}{\underbrace{a \times a \times \ldots \times a}_{n\text{ times}}}$
   $= \underbrace{a \times a \times \ldots \times a}_{m-n\text{ times}}$
   $= a^{m-n}$

Investigation (Law 3 of Indices)

1. $(2^5)^2 = 2^5 \times 2^5$
   $= 2^{5+5}$ (using Law 1 of indices)
   $= 2^{5 \times 2}$
2. $(10^4)^3 = 10^4 \times 10^4 \times 10^4$
   $= 10^{4+4+4}$ (using Law 1 of indices)
   $= 10^{4 \times 3}$
3. $(a^m)^n = \underbrace{(a^m \times a^m \times \ldots \times a^m)}_{n\text{ times}}$
   $= a^{\overbrace{m+m+\ldots+m}^{n\text{ times}}}$
   $= a^{m \times n}$

Investigation (Law 4 of Indices)

1. $2^3 \times 7^3 = (2 \times 2 \times 2) \times (7 \times 7 \times 7)$
   $= (2 \times 7) \times (2 \times 7) \times (2 \times 7)$
   $= (2 \times 7)^3$
2. $(-3)^2 \times (-4)^2 = (-3) \times (-3) \times (-4) \times (-4)$
   $= [(-3) \times (-4)] \times [(-3) \times (-4)]$
   $= [(-3) \times (-4)]^2$
3. $a^n \times b^n = \underbrace{(a \times a \times \ldots \times a)}_{n\text{ times}} \times \underbrace{(b \times b \times \ldots \times b)}_{n\text{ times}}$
   $= \underbrace{(a \times b) \times (a \times b) \times \ldots \times (a \times b)}_{n\text{ times}}$
   $= (a \times b)^n$

Class Discussion (Simplification using the Laws of Indices)

$(xy^2)^4 \times (3x^2\,y)^4 = (x^4y^{2\times4}) \times (3^4x^{2\times4}y^4)$ (Law 4 and Law 3)
$= (x^4y^8) \times (81x^8y^4)$
$= 81x^{4+8}y^{8+4}$ (Law 1)
$= 81x^{12}y^{12}$

$$
\begin{aligned}
(xy^2)^4 \times (3x^2y)^4 &= [(xy^2) \times (3x^2y)]^4 \quad \text{(Law 4)}\\
&= (3x^{1+2}y^{2+1})^4 \quad \text{(Law 1)}\\
&= (3x^3y^3)^4\\
&= 3^4x^{3\times4}y^{3\times4} \quad \text{(Law 3)}\\
&= 81x^{12}y^{12}
\end{aligned}
$$

Investigation (Law 5 of Indices)

1.
$$
\begin{aligned}
8^3 \div 5^3 &= \frac{8^3}{5^3}\\
&= \frac{8\times8\times8}{5\times5\times5}\\
&= \frac{8}{5}\times\frac{8}{5}\times\frac{8}{5}\\
&= \left(\frac{8}{5}\right)^3
\end{aligned}
$$

2.
$$
\begin{aligned}
(-12)^4 \div (-7)^4 &= \frac{(-12)^4}{(-7)^4}\\
&= \frac{(-12)\times(-12)\times(-12)\times(-12)}{(-7)\times(-7)\times(-7)\times(-7)}\\
&= \frac{(-12)}{(-7)}\times\frac{(-12)}{(-7)}\times\frac{(-12)}{(-7)}\times\frac{(-12)}{(-7)}\\
&= \left[\frac{(-12)}{(-7)}\right]^4
\end{aligned}
$$

3.
$$
\begin{aligned}
a^n \div b^n &= \frac{\overbrace{a\times a\times\ldots\times a}^{n\text{ times}}}{\underbrace{b\times b\times\ldots\times b}_{n\text{ times}}}\\
&= \underbrace{\frac{a}{b}\times\frac{a}{b}\times\ldots\times\frac{a}{b}}_{n\text{ times}}\\
&= \left(\frac{a}{b}\right)^n
\end{aligned}
$$

Journal Writing (Page 160)

1. **(i)** To simplify $\left(\dfrac{2x^2}{y}\right)^3$, Law 3 of indices must be applied to the entire expression.

Nora applied Law 3 of Indices to the algebraic terms x and y but she did not apply the same law to the number.

Farhan applied Law 3 of indices to the number correctly but he applied Law 1 of Indices to the algebraic terms x and y which is the wrong law.

(ii) $\left(\dfrac{2x^2}{y}\right)^3 = \dfrac{2^3\times x^{2\times3}}{y^3} = \dfrac{8x^6}{y^3}$

Class Discussion (Is $(a + b)^n = a^n + b^n$? Is $(a - b)^n = a^n - b^n$?)

$(a + b)^n \neq a^n + b^n$

Example: $a = 3$, $b = 2$, $n = 4$

$(3 + 2)^4 = 5^4 = 625$

$3^4 + 2^4 = 81 + 16 = 97$

$(a - b)^n \neq a^n - b^n$

Example: $a = 3$, $b = 2$, $n = 4$

$(3 - 2)^4 = 1^4 = 1$

$3^4 - 2^4 = 81 - 16 = 65$

Investigation (Zero Index)

1.

Index Form	Value
3^4	81
3^3	27
3^2	9
3^1	3
3^0	1

Table 5.1

2. 81 (i.e. 3^4) must be divided by 3 to get 27 (i.e. 3^3).

3. 27 (i.e. 3^3) must be divided by 3 to get the value of 3^2.

4. 3^2 must be divided by 3 to get the value of 3^1.

5. **(a)** 3^1 must be divided by 3 to get the value of 3^0.

6.

Index Form	Value
$(-2)^4$	16
$(-2)^3$	-8
$(-2)^2$	4
$(-2)^1$	-2
$(-2)^0$	1

Table 5.2

7. No. Any number which is divided by zero is undefined.

Thinking Time (Page 164)

LHS $= -5^0 = -(5^0) = -1$

RHS $= (-5)^0 = 1$

$\therefore -5^0 \neq (-5)^0$

Investigation (Negative Indices)

1.

Index Form	Value
3^2	9
3^1	3
3^0	1
3^{-1}	$\frac{1}{3}$
3^{-2}	$\frac{1}{9}$

Table 5.3

2.

Index Form	Value
$(-2)^2$	4
$(-2)^1$	-2
$(-2)^0$	1
$(-2)^{-1}$	$-\frac{1}{2}$
$(-2)^{-2}$	$\frac{1}{4}$

Table 5.4

3. Undefined. Any number which is divided by zero is undefined.

Thinking Time (Page 166)

1. If a and b are real numbers, and m and n are *integers*, then

Law 1 of Indices: $a^m \times a^n = \underline{a^{m+n}}$ if $a \neq 0$

Law 2 of Indices: $a^m \div a^n = \underline{a^{m-n}}$ if $a \neq 0$

Law 3 of Indices: $(a^m)^n = \underline{a^{mn}}$ if $\underline{a \neq 0}$

Law 4 of Indices: $a^n \times b^n = \underline{(a \times b)^n}$ if $a, b \neq 0$

Law 5 of Indices: $a^n \div b^n = \left(\frac{a}{b}\right)^n$ if $\underline{b \neq 0}$

2. **(i)** In Law 1, it is necessary for $a \neq 0$ because if $0^{-2} \times 0^{-1}$, it is undefined.

(ii) In Law 4, it is necessary for $a, b \neq 0$ because if $0^{-2} \times 0^{-2}$, it is undefined.

3. **(i)** If $m = n$ in Law 2, then LHS $= a^n \div a^n = 1$ and RHS $= a^{n-n} = a^0 = 1$.

So a^0 is a special case of Law 2.

(ii) If $m = 0$, then RHS $= a^{0-n} = \frac{1}{a^n}$ and

LHS $= a^0 \div a^n = 1 \div a^n = \frac{1}{a^n}$

So $a^{-n} = \frac{1}{a^n}$ is a special case of Law 2.

Class Discussion (Rational Indices)

Let $p = 5^{\frac{1}{3}}$. Then $p^3 = (5^{\frac{1}{3}})^3$

$$= 5^{\frac{1}{3}\times 3} \quad \text{(Using Law 3 of Indices)}$$
$$= 5^1$$
$$= 5$$
$$\therefore p = \sqrt[3]{5}$$

In this case, there is only one possible value of p.

Hence, $5^{\frac{1}{3}} = \sqrt[3]{5}$.

Thinking Time (Page 169)

1. If $a < 0$, then $a^{\frac{1}{n}} = \sqrt[n]{a}$ is undefined.

2. If $a = 0$, then $0^{\frac{1}{n}} = \sqrt[n]{0}$ is still undefined when n is a positive integer.

Investigation (Rational Indices)

(a) $5^{\frac{2}{3}} = 5^{2\times\frac{1}{3}}$

$= (5^2)^{\frac{1}{3}}$

$= \sqrt[3]{5^2}$

(b) $5^{\frac{2}{3}} = 5^{\frac{1}{3}\times 2}$

$= (5^{\frac{1}{3}})^2$

$= (\sqrt[3]{5})^2$

Thinking Time (Page 171)

1. If a and b are real numbers, and m and n are *rational numbers*, then

Law 1 of Indices: $a^m \times a^n = \underline{a^{m+n}}$ if $a > 0$

Law 2 of Indices: $a^m \div a^n = \underline{a^{m-n}}$ if $a > 0$

Law 3 of Indices: $(a^m)^n = \underline{a^{mn}}$ if $\underline{a > 0}$

Law 4 of Indices: $a^n \times b^n = \underline{(a \times b)^n}$ if $a, b > 0$

Law 5 of Indices: $a^n \div b^n = \underline{\left(\frac{a}{b}\right)^n}$ if $\underline{b > 0}$

2. **(i)** In Law 1, it is necessary for $a > 0$ otherwise a^m or a^n is not defined.

(ii) In Law 4, it is necessary for $a, b > 0$ otherwise a^n or b^n is not defined.

3. $\sqrt{(-1)\times(-1)}$ is undefined. In Law 4, both $a, b > 0$ but in this case, $a = b = -1 < 0$.

Class Discussion (Standard Form)

1. The powers of 10 are all positive integers.
2. The powers of 10 are all negative integers.
3. **(v)** 2.9×10^4
   (vi) 3×10^8 m/s
   (vii) 3.8×10^{-5} cm
   (viii) 2.99×10^{-23} g

Performance Task (Page 179)

1 GB = 1 073 741 824 bytes

All computer data is stored in a binary format as either a one or a zero. Hence each level is an increment of 2 to the 10th power or 1024. As such, 1 GB = 2^{30}.

2^7 = 128 MB

2^8 = 256 MB

2^9 = 512 MB

Thinking Time (Page 181)

1. 57 910 000 km = 5.791×10^7 km

5 945 900 000 km = 5.9459×10^9 km

2. 3 683 000 m/h = $\frac{3683 \text{ km}}{1 \text{ h}}$ = 3683 km/h = 3.683×10^3 km/h

3. $0.000\,000\,0004 \text{ m} = 4 \times 10^{-10} \text{ m}$

$500\,000\,000\,000\,000\,000\,000\,000\,000 = 5 \times 10^{26}$

$$\begin{aligned}\text{Total volume of air molecules} &= 5 \times 10^{26} \times \pi \times \left(\frac{4 \times 10^{-10}}{2}\right)^2 \\ &= 6.28 \times 10^7 \text{ m}^3 \text{ (to 3 s.f.)}\end{aligned}$$

4. $100 \text{ trillion} = 100 \times 10^{12} = 1 \times 10^{14}$

$1 \times 10^6 = 1 \text{ million} \Rightarrow 2 \times 10^9 = 2 \times 10^6 \times 10^3 = 2000 \text{ million}$

$42\,000\,000 = 4.2 \times 10^7$

Practise Now 1

(a) $$\begin{aligned}4^7 \times 4^5 &= 4^{7+5} \\ &= 4^{12}\end{aligned}$$

(b) $$\begin{aligned}(-3)^6 \times (-3) &= (-3)^{6+1} \\ &= (-3)^7\end{aligned}$$

(c) $$\begin{aligned}a^{12} \times a^8 &= a^{12+8} \\ &= a^{20}\end{aligned}$$

(d) $$\begin{aligned}2xy^4 \times 3x^5y^3 &= 6x^{1+5}y^{4+3} \\ &= 6x^6y^7\end{aligned}$$

Practise Now 2

(a) $$\begin{aligned}9^7 \div 9^3 &= 9^{7-3} \\ &= 9^4\end{aligned}$$

(b) $$\begin{aligned}(-4)^8 \div (-4) &= (-4)^{8-1} \\ &= (-4)^7\end{aligned}$$

(c) $$\begin{aligned}a^{10} \div a^6 &= a^{10-6} \\ &= a^4\end{aligned}$$

(d) $$\begin{aligned}27x^9y^4 \div 9x^6y^3 &= \frac{27x^9y^4}{9x^6y^3} \\ &= 3x^{9-6}y^{4-3} \\ &= 3x^3y\end{aligned}$$

Practise Now 3

1. (a) $$\begin{aligned}(6^3)^4 &= 6^{3\times4} \\ &= 6^{12}\end{aligned}$$

(b) $$\begin{aligned}(k^5)^9 &= k^{5\times9} \\ &= k^{45}\end{aligned}$$

(c) $$\begin{aligned}(3^q)^6 \times (3^4)^q &= 3^{6q} \times 3^{4q} \\ &= 3^{6q+4q} \\ &= 3^{10q}\end{aligned}$$

2. $$\begin{aligned}x^8 \times (x^3)^n \div (x^n)^2 &= x^{10} \\ x^8 \times x^{3n} \div x^{2n} &= x^{10} \\ x^{8+3n-2n} &= x^{10} \\ 8 + n &= 10 \\ n &= 2\end{aligned}$$

Practise Now 4

(a) $3^7 \times 8^7 = 24^7$

(b) $$\begin{aligned}(5b^4)^3 &= 5^3 \times b^{4\times3} \\ &= 125 \times b^{12} \\ &= 125b^{12}\end{aligned}$$

(c) $$\begin{aligned}(-2c^2d^5)^5 &= (-2)^5 \times c^{2\times5} \times d^{5\times5} \\ &= -32 \times c^{10} \times d^{25} \\ &= -32c^{10}d^{25}\end{aligned}$$

(d) $$\begin{aligned}(m^2n)^4 \times (m^4n^3)^5 &= (m^{2\times4}n^4) \times (m^{4\times5}n^{3\times5}) \\ &= (m^8n^4) \times (m^{20}n^{15}) \\ &= m^{8+20}n^{4+15} \\ &= m^{28}n^{19}\end{aligned}$$

(e) $$\begin{aligned}(-p^7q^5)^2 \div (3p^3q^2)^3 &= \frac{(-p^7q^5)^2}{(3p^3q^2)^3} \\ &= \frac{(-1)^2 \times p^{7\times2} \times q^{5\times2}}{3^3 \times p^{3\times3} \times q^{2\times3}} \\ &= \frac{p^{14}q^{10}}{27p^9q^6} \\ &= \frac{p^{14-9}q^{10-6}}{27} \\ &= \frac{p^5q^4}{27}\end{aligned}$$

Practise Now 5

(a) $$\begin{aligned}21^3 \div 7^3 &= \frac{21^3}{7^3} \\ &= \left(\frac{21}{7}\right)^3 \\ &= 3^3\end{aligned}$$

(b) $$\begin{aligned}(26^5)^3 \div 13^{15} &= \frac{26^{15}}{13^{15}} \\ &= \left(\frac{26}{13}\right)^{15} \\ &= 2^{15}\end{aligned}$$

(c) $$\begin{aligned}\left(\frac{p^2}{q}\right)^3 \div \frac{q^7}{p^5} &= \frac{p^6}{q^3} \div \frac{q^7}{p^5} \\ &= \frac{p^6}{q^3} \times \frac{q^7}{p^5} \\ &= \frac{p^{6+5}}{q^{3+7}} \\ &= \frac{p^{11}}{q^{10}}\end{aligned}$$

(d) $$\begin{aligned}\left(\frac{3x^2}{x^3}\right)^3 \div \frac{27x^7}{x^{21}} &= \frac{27x^6}{x^9} \div \frac{27x^7}{x^{21}} \\ &= \frac{27x^6}{x^9} \times \frac{x^{21}}{27x^7} \\ &= \frac{27x^{6+21}}{27x^{9+7}} \\ &= \frac{x^{27}}{x^{16}} \\ &= x^{27-16} \\ &= x^{11}\end{aligned}$$

Practise Now 6

1. **(a)** $2015^0 = 1$

(b) $(-7)^0 = 1$

(c) $$\begin{aligned} 3y^0 &= 3(1) \\ &= 3 \end{aligned}$$

(d) $$\begin{aligned} (3y)^0 &= 3^0 \times y^0 \\ &= 1 \times 1 \\ &= 1 \end{aligned}$$

2. **(a)** $$\begin{aligned} 3^0 \times 3^3 \div 3^2 &= 3^{0+3-2} \\ &= 3^1 \\ &= 3 \end{aligned}$$

(b) $$\begin{aligned} 3^0 + 3^2 &= 1 + 9 \\ &= 10 \end{aligned}$$

Practise Now 7

(a) $$\begin{aligned} 6^{-2} &= \frac{1}{6^2} \\ &= \frac{1}{36} \end{aligned}$$

(b) $$\begin{aligned} (-8)^{-1} &= \frac{1}{(-8)^1} \\ &= -\frac{1}{8} \end{aligned}$$

(c) $$\begin{aligned} \left(\frac{4}{5}\right)^{-3} &= \frac{1}{\left(\frac{4}{5}\right)^3} \\ &= \frac{1}{\left(\frac{64}{125}\right)} \\ &= 1 \div \frac{64}{125} \\ &= 1 \times \frac{125}{64} \\ &= 1\frac{61}{64} \end{aligned}$$

(d) $$\begin{aligned} \left(\frac{1}{9}\right)^{-1} &= \frac{1}{\left(\frac{1}{9}\right)^{-1}} \\ &= 1 \div \frac{1}{9} \\ &= 1 \times 9 \\ &= 9 \end{aligned}$$

Practise Now 8

(a) $$\begin{aligned} a^{-1} \times a^3 \div a^{-2} &= a^{-1+3-(-2)} \\ &= a^4 \end{aligned}$$

(b) $$\begin{aligned} (b^{-5}c^2)^{-3} &= b^{-5\times -3}c^{2\times -3} \\ &= b^{15}c^{-6} \\ &= \frac{b^{15}}{c^6} \end{aligned}$$

(c) $$\begin{aligned} \frac{16d^{-2}e}{(2d^{-1}e)^3} &= \frac{16d^{-2}e}{2^3 \times d^{-1\times 3} \times e^3} \\ &= \frac{16d^{-2}e}{8 \times d^{-3} \times e^3} \\ &= \frac{2d^{-2+3}}{e^{3-1}} \\ &= \frac{2d}{e^2} \end{aligned}$$

(d) $$\begin{aligned} 5f^0 \div 3(f^{-2})^2 &= \frac{5f^0}{3(f^{-2})^2} \\ &= \frac{5}{3f^{-4}} \\ &= \frac{5f^4}{3} \end{aligned}$$

(e) $$\begin{aligned} 18g^{-6} \div 3(g^{-2})^2 &= \frac{18g^{-6}}{3(g^{-2})^2} \\ &= \frac{18g^{-6}}{3g^{-4}} \\ &= \frac{6}{g^{-4+6}} \\ &= \frac{6}{g^2} \end{aligned}$$

(f) $$\begin{aligned} 6h^2 \div 2h^{-2} - h \times h^3 - \frac{4}{h^{-4}} &= \frac{6h^2}{2h^{-2}} - h \times h^3 - \frac{4}{h^{-4}} \\ &= 3h^{2+2} - h^{1+3} - 4h^4 \\ &= 3h^4 - h^4 - 4h^4 \\ &= -2h^4 \end{aligned}$$

Practise Now 9

(a) By prime factorisation, $256 = 4 \times 4 \times 4 \times 4 = 4^4$.

$$\begin{aligned} \therefore \sqrt[4]{256} &= \sqrt[4]{4 \times 4 \times 4 \times 4} \\ &= 4 \end{aligned}$$

(b) By prime factorisation, $1024 = 4 \times 4 \times 4 \times 4 \times 4 = 4^5$.

$$\begin{aligned} \therefore \sqrt[5]{1024} &= \sqrt[5]{4 \times 4 \times 4 \times 4 \times 4} \\ &= 4 \end{aligned}$$

(c) By prime factorisation, $8 = 2 \times 2 \times 2 = 2^3$ and $27 = 3 \times 3 \times 3 = 3^3$.

$$\begin{aligned} \therefore \sqrt[3]{\frac{8}{27}} &= \sqrt[3]{\frac{2 \times 2 \times 2}{3 \times 3 \times 3}} \\ &= \frac{2}{3} \end{aligned}$$

Practise Now 10

(a) $$\begin{aligned} 36^{\frac{1}{2}} &= \sqrt{36} \\ &= 6 \end{aligned}$$

(b) $$\begin{aligned} 8^{-\frac{1}{3}} &= \left(\frac{1}{8}\right)^{\frac{1}{3}} \\ &= \sqrt[3]{\frac{1}{8}} \\ &= \frac{1}{2} \end{aligned}$$

(c) $$\begin{aligned} (-125)^{\frac{1}{3}} &= \left(\frac{1}{-125}\right)^{\frac{1}{3}} \\ &= \frac{1}{\sqrt[3]{-125}} \\ &= -\frac{1}{5} \end{aligned}$$

Practise Now 11

1. **(a)** $$\begin{aligned} 64^{\frac{2}{3}} &= (\sqrt[3]{64}\,)^2 \\ &= 4^2 \\ &= 16 \end{aligned}$$

(b) $$\begin{aligned} 32^{-\frac{3}{5}} &= \left(\frac{1}{32}\right)^{\frac{3}{5}} \\ &= \left(\sqrt[5]{\frac{1}{32}}\right)^3 \\ &= \left(\frac{1}{2}\right)^3 \\ &= \frac{1}{8} \end{aligned}$$

(c) $$\begin{aligned} 100^{1.5} &= 100^{\frac{3}{2}} \\ &= (\sqrt{100}\,)^3 \\ &= (10)^3 \\ &= 1000 \end{aligned}$$

2. **(a)** $$\begin{aligned} \sqrt[3]{a^n} &= (a^n)^{\frac{1}{3}} \\ &= a^{\frac{n}{3}} \end{aligned}$$

(b) $$\begin{aligned} \frac{1}{\sqrt[5]{x^2}} &= \frac{1}{(x^2)^{\frac{1}{5}}} \\ &= \frac{1}{x^{\frac{2}{5}}} \\ &= x^{-\frac{2}{5}} \end{aligned}$$

Practise Now 12

(a) $$\begin{aligned} (m^2)^{\frac{5}{6}} \times m^{\frac{1}{3}} &= m^{2\times\frac{5}{6}} \times m^{\frac{1}{3}} \\ &= m^{\frac{5}{3}} \times m^{\frac{1}{3}} \\ &= m^{\frac{5}{3}+\frac{1}{3}} \\ &= m^2 \end{aligned}$$

(b) $$\begin{aligned} \sqrt[5]{m} \div \sqrt[3]{m^2} &= \frac{\sqrt[5]{m}}{\sqrt[3]{m^2}} \\ &= \frac{m^{\frac{1}{5}}}{(m^2)^{\frac{1}{3}}} \\ &= \frac{m^{\frac{1}{5}}}{m^{\frac{2}{3}}} \\ &= m^{\frac{1}{5}-\frac{2}{3}} \\ &= m^{-\frac{7}{15}} \\ &= \frac{1}{m^{\frac{7}{15}}} \end{aligned}$$

(c) $$\begin{aligned} (m^{-3}n^5)^{-\frac{1}{3}} &= m^{-3\times-\frac{1}{3}}n^{5\times-\frac{1}{3}} \\ &= mn^{-\frac{5}{3}} \\ &= \frac{m}{n^{\frac{5}{3}}} \end{aligned}$$

(d) $$\begin{aligned} \frac{m^{-\frac{1}{3}}n^{-\frac{1}{4}}}{(m^2n^{-\frac{1}{3}})^{-2}} &= \frac{m^{-\frac{1}{3}}n^{-\frac{1}{4}}}{m^{2\times-2}n^{-\frac{1}{3}\times-2}} \\ &= \frac{m^{-\frac{1}{3}}n^{-\frac{1}{4}}}{m^{-4}n^{\frac{2}{3}}} \\ &= m^{-\frac{1}{3}-(-4)}n^{-\frac{1}{4}-\frac{2}{3}} \\ &= m^{\frac{11}{3}}n^{-\frac{11}{12}} \\ &= \frac{m^{\frac{11}{3}}}{n^{\frac{11}{12}}} \end{aligned}$$

(e) $$\begin{aligned} (25m^2n^{-4})^{\frac{1}{2}}(m^3n^{-\frac{2}{5}})^2 &= (25^{\frac{1}{2}} \times m^{2\times\frac{1}{2}} \times n^{-4\times\frac{1}{2}})(m^{3\times 2} \times n^{-\frac{2}{5}\times 2}) \\ &= (5mn^{-2})(m^6n^{-\frac{4}{5}}) \\ &= 5m^{1+6}n^{-2-\frac{4}{5}} \\ &= 5m^7n^{-\frac{14}{5}} \\ &= \frac{5m^7}{n^{\frac{14}{5}}} \end{aligned}$$

(f) $$\begin{aligned} (m^2n^{-\frac{1}{7}}) \times \sqrt[5]{(m^5n^{-5})} &= (m^2n^{-\frac{1}{7}}) \times (m^5n^{-5})^{\frac{1}{5}} \\ &= (m^2n^{-\frac{1}{7}}) \times (m^{5\times\frac{1}{5}}n^{-5\times\frac{1}{5}}) \\ &= (m^2n^{-\frac{1}{7}}) \times (mn^{-1}) \\ &= m^{2+1}n^{-\frac{1}{7}-1} \\ &= m^3n^{-\frac{8}{7}} \\ &= \frac{m^3}{n^{\frac{8}{7}}} \end{aligned}$$

Practise Now 13

(a) $5^x = 125$

$5^x = 5^3$

$x = 3$

(b) $7^y = \frac{1}{49}$

$7^y = \frac{1}{7^2}$

$7^y = 7^{-2}$

$y = -2$

(c) $8^z = 16$

$(2^3)^z = 2^4$

$2^{3z} = 2^4$

$3z = 4$

$z = 1\frac{1}{3}$

Practise Now 14

1. **(a)** $5\,300\,000 = 5.3 \times 10^6$
   (b) $600\,000\,000 = 6 \times 10^8$
   (c) $0.000\,048 = 4.8 \times 10^{-5}$
   (d) $0.000\,000\,000\,21 = 2.1 \times 10^{-10}$
2. **(a)** $1.325 \times 10^6 = 1\,325\,000$
   (b) $4.4 \times 10^{-3} = 0.0044$

Practise Now 15

(a) 1 micrometre $= 10^{-6}$ metres

25.4 micrometres $= 25.4 \times 10^{-6}$ metres

$= 2.54 \times 10^{-5}$ metres

(b) 10 mm = 1 cm

2340 mm $= \frac{1}{10} \times 2340$ cm

$= 234$ cm

$= 2.34 \times 10^2$ cm

(c) 1 terabyte $= 10^{12}$ bytes

4.0 terabytes $= 4.0 \times 10^{12}$ bytes

Practise Now 16

(a) $(1.14 \times 10^5) \times (4.56 \times 10^4)$

$= \boxed{(}\ \boxed{1}\ \boxed{.}\ \boxed{1}\ \boxed{4}\ \boxed{\times 10^x}\ \boxed{5}\ \boxed{)}$

$\boxed{\times}\ \boxed{(}\ \boxed{4}\ \boxed{.}\ \boxed{5}\ \boxed{6}\ \boxed{\times 10^x}\ \boxed{4}\ \boxed{)}\ \boxed{=}$

$= 5.20 \times 10^9$ (to 3 s.f.)

(b) $(4.2 \times 10^{-4}) \times (2.6 \times 10^2)$

$= \boxed{(}\ \boxed{4}\ \boxed{.}\ \boxed{2}\ \boxed{\times 10^x}\ \boxed{-}\ \boxed{4}\ \boxed{)}$

$\boxed{\times}\ \boxed{(}\ \boxed{2}\ \boxed{.}\ \boxed{6}\ \boxed{\times 10^x}\ \boxed{2}\ \boxed{)}\ \boxed{=}$

$= 0.1092$

$= 1.09 \times 10^{-1}$ (to 3 s.f.)

(c) $(2.4 \times 10^8) \div (6 \times 10^4)$

$= \boxed{(}\ \boxed{2}\ \boxed{.}\ \boxed{4}\ \boxed{\times 10^x}\ \boxed{8}\ \boxed{)}\ \boxed{\div}$

$\boxed{(}\ \boxed{6}\ \boxed{\times 10^x}\ \boxed{4}\ \boxed{)}\ \boxed{=}$

$= 4000$

$= 4 \times 10^3$

(d) $\frac{3.5 \times 10^{-5}}{1.4 \times 10^8}$

$= \boxed{(}\ \boxed{3}\ \boxed{.}\ \boxed{5}\ \boxed{\times 10^x}\ \boxed{-}\ \boxed{5}\ \boxed{)}$

$\boxed{\div}\ \boxed{(}\ \boxed{1}\ \boxed{.}\ \boxed{4}\ \boxed{\times 10^x}\ \boxed{8}\ \boxed{)}\ \boxed{=}$

$= 2.5 \times 10^{-13}$

(e) $1.14 \times 10^5 + 4.56 \times 10^4$

$= \boxed{1}\ \boxed{.}\ \boxed{1}\ \boxed{4}\ \boxed{\times 10^x}\ \boxed{5}\ \boxed{\div}$

$\boxed{4}\ \boxed{.}\ \boxed{5}\ \boxed{6}\ \boxed{\times 10^x}\ \boxed{4}\ \boxed{=}$

$= 159\,600$

$= 1.60 \times 10^5$ (to 3 s.f.)

(f) $4 \times 10^4 - 2.6 \times 10^6$

$= \boxed{4}\ \boxed{\times 10^x}\ \boxed{4}\ \boxed{-}\ \boxed{2}\ \boxed{.}\ \boxed{6}\ \boxed{\times 10^x}\ \boxed{6}\ \boxed{=}$

$= -2\,560\,000$

$= -2.56 \times 10^6$

(g) $\frac{2.37 \times 10^{-3} + 3.25 \times 10^{-4}}{4.1 \times 10^5}$

$= \boxed{(}\ \boxed{2}\ \boxed{.}\ \boxed{3}\ \boxed{7}\ \boxed{\times 10^x}\ \boxed{-}\ \boxed{3}\ \boxed{+}\ \boxed{3}\ \boxed{.}\ \boxed{2}\ \boxed{5}$

$\boxed{\times 10^x}\ \boxed{-}\ \boxed{4}\ \boxed{)}\ \boxed{\div}\ \boxed{4}\ \boxed{.}\ \boxed{1}\ \boxed{\times 10^x}\ \boxed{5}\ \boxed{=}$

$= 6.57 \times 10^{-9}$ (to 3 s.f.)

(h) $\frac{6.3 \times 10^6}{1.5 \times 10^2 - 3 \times 10^{-1}}$

$= \boxed{6}\ \boxed{.}\ \boxed{3}\ \boxed{\times 10^x}\ \boxed{6}\ \boxed{\div}\ \boxed{(}\ \boxed{1}\ \boxed{.}\ \boxed{5}$

$\boxed{\times 10^x}\ \boxed{2}\ \boxed{-}\ \boxed{3}\ \boxed{\times 10^x}\ \boxed{-}\ \boxed{1}\ \boxed{)}\ \boxed{=}$

$= 4.21 \times 10^4$ (to 3 s.f.)

Practise Now 17

1 MB $= 10^6$ bytes

512 MB $= 512 \times 10^6$ bytes

$= 5.12 \times 10^8$ bytes

No. of photographs that can be stored $= \frac{5.12 \times 10^8}{640 \times 10^3}$

$= 800$

Exercise 5A

1. **(a)** $2^3 \times 2^7 = 2^{3+7}$

   $= 2^{10}$

   (b) $(-4)^6 \times (-4)^5 = (-4)^{6+5}$

   $= (-4)^{11}$

   (c) $x^8 \times x^3 = x^{8+3}$

   $= x^{11}$

   (d) $(3y^2) \times (8y^7) = 24y^{2+7}$

   $= 24y^9$

2. **(a)** $$\begin{aligned} 5^8 \div 5^5 &= 5^{8-5} \\ &= 5^3 \end{aligned}$$

(b) $$\begin{aligned} (-7)^{11} \div (-7)^4 &= (-7)^{11-4} \\ &= (-7)^7 \end{aligned}$$

(c) $$\begin{aligned} 6x^7 \div x^3 &= 6x^{7-3} \\ &= 6x^4 \end{aligned}$$

(d) $$\begin{aligned} (-15y^9) \div 5y^4 &= \frac{-15y^9}{5y^4} \\ &= -3y^{9-4} \\ &= -3y^5 \end{aligned}$$

3. **(a)** $$\begin{aligned} (9^2)^4 &= 9^{2\times 4} \\ &= 9^8 \end{aligned}$$

(b) $$\begin{aligned} (h^2)^5 &= h^{2\times 5} \\ &= h^{10} \end{aligned}$$

(c) $$\begin{aligned} 3^{14} \times (5^2)^7 &= 3^{14} \times 5^{2\times 7} \\ &= 3^{14} \times 5^{14} \\ &= (3 \times 5)^{14} \\ &= 15^{14} \end{aligned}$$

(d) $$\begin{aligned} 2^3 \times 9^3 &= (2 \times 9)^3 \\ &= 18^3 \end{aligned}$$

(e) $$\begin{aligned} (2k^6)^3 &= 2^3 \times k^{6\times 3} \\ &= 8 \times k^{18} \\ &= 8k^{18} \end{aligned}$$

(f) $$\begin{aligned} (-3x^6y^2)^4 &= (-3)^4 \times x^{6\times 4} \times y^{2\times 4} \\ &= 81 \times x^{24} \times y^8 \\ &= 81x^{24}y^8 \end{aligned}$$

4. **(a)** $$\begin{aligned} 14^{13} \div 7^{13} &= \frac{14^{13}}{7^{13}} \\ &= 2^{13} \end{aligned}$$

(b) $$\begin{aligned} (9^5)^4 \div 3^{20} &= \frac{9^{5\times 4}}{3^{20}} \\ &= \frac{(3^2)^{20}}{3^{20}} \\ &= \frac{3^{2\times 20}}{3^{20}} \\ &= \frac{3^{40}}{3^{20}} \\ &= 3^{40-20} \\ &= 3^{20} \end{aligned}$$

(c) $$\begin{aligned} \left(\frac{m}{2}\right)^5 &= \frac{m^5}{2^5} \\ &= \frac{m^5}{32} \end{aligned}$$

(d) $$\begin{aligned} \left(\frac{3}{n^2}\right)^3 &= \frac{3^3}{n^{2\times 3}} \\ &= \frac{27}{n^6} \end{aligned}$$

(e) $$\begin{aligned} \left(\frac{p^4}{q}\right)^6 &= \frac{p^{4\times 6}}{q^6} \\ &= \frac{p^{24}}{q^6} \end{aligned}$$

(f) $$\begin{aligned} \left(-\frac{x}{y^2}\right)^4 &= \left(-1 \times \frac{x}{y^2}\right)^4 \\ &= (-1)^4 \times \frac{x^4}{y^{2\times 4}} \\ &= 1 \times \frac{x^4}{y^8} \\ &= \frac{x^4}{y^8} \end{aligned}$$

5. **(a)** $$\begin{aligned} h^2k \times h^{11}k^9 &= h^{2+11} \times k^{1+9} \\ &= h^{13}k^{10} \end{aligned}$$

(b) $$\begin{aligned} (-m^7n^3) \times 4m^{11}n^9 &= -1 \times 4 \times m^{7+11} \times n^{3+9} \\ &= -4 \times m^{18} \times n^{12} \\ &= -4m^{18}n^{12} \end{aligned}$$

(c) $$\begin{aligned} 11p^6q^7 \times 2p^3q^{10} &= 11 \times p^{6+3} \times 2 \times q^{7+10} \\ &= 22 \times p^9 \times q^{17} \\ &= 22p^9q^{17} \end{aligned}$$

(d) $$\begin{aligned} h^9k^6 \div h^5k^4 &= \frac{h^9k^6}{h^5k^4} \\ &= h^{9-5}k^{6-4} \\ &= h^4k^2 \end{aligned}$$

(e) $$\begin{aligned} 15m^8n^7 \div 3m^2n &= \frac{15m^8n^7}{3m^2n} \\ &= 5m^{8-2}n^{7-1} \\ &= 5m^6n^6 \end{aligned}$$

(f) $$\begin{aligned} (-10x^5y^6) \div (-2xy^3) &= \frac{-10x^5y^6}{-2xy^3} \\ &= 5x^{5-1}y^{6-3} \\ &= 5x^4y^3 \end{aligned}$$

6. **(a)** $$\begin{aligned} (a^2)^3 \times a^5 &= a^{2\times 3} \times a^5 \\ &= a^6 \times a^5 \\ &= a^{6+5} \\ &= a^{11} \end{aligned}$$

(b) $$\begin{aligned} (b^3)^7 \times (b^4)^5 &= b^{3\times 7} \times b^{4\times 5} \\ &= b^{21} \times b^{20} \\ &= b^{21+20} \\ &= b^{41} \end{aligned}$$

(c) $$\begin{aligned} (c^6)^5 \div (-c^2) &= c^{6\times 5} \div (-c^2) \\ &= \frac{c^{30}}{-c^2} \\ &= \frac{c^{30-2}}{-1} \\ &= -c^{28} \end{aligned}$$

(d) $$\begin{aligned} (-3d^3)^2 \div (2d)^3 &= \frac{(-3d^3)^2}{(2d)^3} \\ &= \frac{(-3)^2 \times d^{3\times 2}}{2^3 \times d^3} \\ &= \frac{9 \times d^6}{8 \times d^3} \\ &= \frac{9}{8} \times d^{6-3} \\ &= \frac{9d^3}{8} \end{aligned}$$

(e)
$$\begin{aligned}(e^3)^5 \div (-e^2)^4 &= \frac{(e^3)^5}{(-e^2)^4}\\ &= \frac{e^{3\times 5}}{(-1)^4 \times e^{2\times 4}}\\ &= \frac{e^{15}}{1\times e^8}\\ &= e^{15-8}\\ &= e^7\end{aligned}$$

(f)
$$\begin{aligned}(4f^6)^3 \div (-2f^3)^3 &= \frac{(4f^6)^3}{(-2f^3)^3}\\ &= \frac{4^3\times f^{6\times 3}}{(-2)^3\times f^{3\times 3}}\\ &= \frac{64\times f^{18}}{-8\times f^9}\\ &= -8\times f^{18-9}\\ &= -8f^9\end{aligned}$$

7. **(a)**
$$\begin{aligned}(ab^2)^3\times(2a^2b)^3 &= a^3\times b^{2\times 3}\times 2^3\times a^{2\times 3}\times b^3\\ &= a^3\times b^6\times 8\times a^6\times b^3\\ &= 8\times a^{3+6}\times b^{6+3}\\ &= 8\times a^9\times b^9\\ &= 8a^9b^9\end{aligned}$$

(b)
$$\begin{aligned}c^2d^2\times(-5c^3d^3)^2 &= c^2d^2\times(-5)^2\times c^{3\times 2}\times d^{3\times 2}\\ &= c^2\times d^2\times 25\times c^6\times d^6\\ &= 25\times c^{2+6}\times d^{2+6}\\ &= 25\times c^8\times d^8\\ &= 25c^8d^8\end{aligned}$$

(c)
$$\begin{aligned}(8e^5f^3)^2\div(\mathrm{e}^3f)^3 &= \frac{(8e^5f^3)^2}{(e^3f)^3}\\ &= \frac{8^2\times e^{5\times 2}\times f^{3\times 2}}{e^{3\times 3}\times f^3}\\ &= \frac{64\times e^{10}\times f^6}{e^9\times f^3}\\ &= 64\times e^{10-9}\times f^{6-3}\\ &= 64\times e\times f^3\\ &= 64ef^3\end{aligned}$$

(d)
$$\begin{aligned}16g^8h^7\div(-2g^3h^2)^3 &= \frac{16g^8h^7}{(-2g^3h^2)^3}\\ &= \frac{16g^8h^7}{(-2)^3\times g^{3\times 3}\times h^{2\times 3}}\\ &= \frac{16g^8h^7}{-8\times g^9\times h^6}\\ &= -2g^{8-9}h^{7-6}\\ &= -2g^{-1}h\\ &= -\frac{2h}{g}\end{aligned}$$

8. **(a)**
$$\begin{aligned}\frac{2a^2}{b}\times\left(\frac{a}{b^2}\right)^2 &= \frac{2a^2}{b}\times\frac{a^2}{b^{2\times 2}}\\ &= \frac{2a^2}{b}\times\frac{a^2}{b^4}\\ &= \frac{2a^{2+2}}{b^{1+4}}\\ &= \frac{2a^4}{b^5}\end{aligned}$$

(b)
$$\begin{aligned}\left(\frac{c}{d^2}\right)^3\times\left(\frac{c^3}{2d}\right)^2 &= \frac{c^3}{d^{2\times 3}}\times\frac{c^{3\times 2}}{2^2\times d^2}\\ &= \frac{c^3}{d^6}\times\frac{c^6}{4\times d^2}\\ &= \frac{c^{3+6}}{4\times d^{6+2}}\\ &= \frac{c^9}{4d^8}\end{aligned}$$

(c)
$$\begin{aligned}\left(\frac{3e^3}{f^2}\right)^4\div\frac{27e^9}{f^{11}} &= \frac{3^4\times e^{3\times 4}}{f^{2\times 4}}\div\frac{27e^9}{f^{11}}\\ &= \frac{81\times e^{12}}{f^8}\div\frac{27e^9}{f^{11}}\\ &= \frac{81\times e^{12}}{f^8}\times\frac{f^{11}}{27\times e^9}\\ &= \frac{81}{27}\,e^{12-9}f^{11-8}\\ &= 3e^3f^3\end{aligned}$$

(d)
$$\begin{aligned}\left(\frac{g^2}{h^3}\right)^6\div\left(\frac{-3g^5}{2h^2}\right)^3 &= \frac{g^{2\times 6}}{h^{3\times 6}}\div\frac{(-3)^3\times g^{5\times 3}}{2^3\times h^{2\times 3}}\\ &= \frac{g^{12}}{h^{18}}\div\frac{-27\times g^{15}}{8\times h^6}\\ &= \frac{g^{12}}{h^{18}}\times\frac{8\times h^6}{-27\times g^{15}}\\ &= -\frac{8}{27}\,g^{12-15}h^{6-18}\\ &= -\frac{8}{27}\,g^{-3}h^{-12}\\ &= -\frac{8}{27g^3h^{12}}\end{aligned}$$

9. **(a)**
$$\begin{aligned}\frac{(2x^2y)^3}{(10xy^3)^2}\times\frac{(5xy^4)^3}{4xy} &= \frac{2^3\times x^{2\times 3}\times y^3}{10^2\times x^2\times y^{3\times 2}}\times\frac{5^3\times x^3\times y^{4\times 3}}{4xy}\\ &= \frac{8\times x^6\times y^3}{100\times x^2\times y^6}\times\frac{125\times x^3\times y^{12}}{4xy}\\ &= \frac{1000\times x^{6+3}\times y^{3+12}}{400\times x^{2+1}\times y^{6+1}}\\ &= \frac{5\times x^9\times y^{15}}{2\times x^3\times y^7}\\ &= \frac{5}{2}\times x^{9-3}\times y^{15-7}\\ &= \frac{5x^6y^8}{2}\end{aligned}$$

(b) $$\begin{aligned}\frac{8x^8y^4}{(2xy^2)^2} \times \frac{(4x^2y^2)^2}{(3xy)^2} &= \frac{8x^8y^4}{2^2 \times x^2 \times y^{2\times2}} \times \frac{4^2 \times x^{2\times2} \times y^{2\times2}}{3^2 \times x^2 \times y^2} \\ &= \frac{8x^8y^4}{4 \times x^2 \times y^4} \times \frac{16 \times x^4 \times y^4}{9 \times x^2 \times y^2} \\ &= \frac{128 \times x^{8+4} \times y^{4+4}}{36 \times x^{2+2} \times y^{4+2}} \\ &= \frac{32 \times x^{12} \times y^8}{9 \times x^4 \times y^6} \\ &= \frac{32}{9} \times x^{12-4} \times y^{8-6} \\ &= \frac{32x^8y^2}{9}\end{aligned}$$

(c) $$\begin{aligned}\frac{(2xy^2)^5}{(4x^2y)^2(xy^3)} &= \frac{2^5 \times x^5 \times y^{2\times5}}{4^2 \times x^{2\times2} \times y^2 \times xy^3} \\ &= \frac{32 \times x^5 \times y^{10}}{16 \times x^4 \times y^2 \times xy^3} \\ &= \frac{32 \times x^5 \times y^{10}}{16 \times x^{4+1} \times y^{2+3}} \\ &= \frac{32 \times x^5 \times y^{10}}{16 \times x^5 \times y^5} \\ &= 2 \times x^{5-5} \times y^{10-5} \\ &= 2 \times x^0 \times y^5 \\ &= 2y^5\end{aligned}$$

(d) $$\begin{aligned}\frac{4x^2y^4 \times 8x^4y^2}{(4x^2y^2)^2} &= \frac{32 \times x^{2+4} \times y^{4+2}}{4^2 \times x^{2\times2} \times y^{2\times2}} \\ &= \frac{32 \times x^6 \times y^6}{16 \times x^4 \times y^4} \\ &= 2 \times x^{6-4} \times y^{6-4} \\ &= 2 \times x^2 \times y^2 \\ &= 2x^2y^2\end{aligned}$$

10. $$\begin{aligned}\frac{(2p^3q^4)^4}{(-3q^5)^2} \div \frac{(4p^2q)^2}{9} &= \frac{p^{a+b}}{q^{a-b}} \\ \frac{2^4 \times p^{3\times4} \times q^{4\times4}}{(-3)^2 \times q^{5\times2}} \div \frac{4^2 \times p^{2\times2} \times q^2}{9} &= \frac{p^{a+b}}{q^{a-b}} \\ \frac{16 \times p^{12} \times q^{16}}{9 \times q^{10}} \div \frac{16 \times p^4 \times q^2}{9} &= \frac{p^{a+b}}{q^{a-b}} \\ \frac{16 \times p^{12} \times q^{16}}{9 \times q^{10}} \times \frac{9}{16 \times p^4 \times q^2} &= \frac{p^{a+b}}{q^{a-b}} \\ \frac{p^{12} \times q^{16}}{q^{10}} \times \frac{1}{p^4 \times q^2} &= \frac{p^{a+b}}{q^{a-b}} \\ \frac{p^{12} \times q^{16}}{p^4 \times q^{10+2}} &= \frac{p^{a+b}}{q^{a-b}} \\ \frac{p^{12-4}}{q^{12-16}} &= \frac{p^{a+b}}{q^{a-b}} \\ \frac{p^8}{q^{-4}} &= \frac{p^{a+b}}{q^{a-b}}\end{aligned}$$

$$\begin{aligned}a + b &= 8 \\ a &= 8 - b \quad \text{— (1)} \\ a - b &= -4 \quad \text{— (2)}\end{aligned}$$

Substitute (1) into (2):

$$\begin{aligned}(8 - b) - b &= -4 \\ 8 - 2b &= -4 \\ 2b &= 12 \\ b &= 6 \\ a &= 8 - 6 \\ &= 2\end{aligned}$$

$\therefore a = 2, b = 6$

Exercise 5B

1. **(a)** $17^0 = 1$

(b) $\left(-\frac{2}{7}\right)^0 = 1$

(c) $$\begin{aligned}4a^0 &= 4(1) \\ &= 4\end{aligned}$$

(d) $$\begin{aligned}-8b^0 &= -8(1) \\ &= -8\end{aligned}$$

(e) $(72cd^2)^0 = 1$

(f) $$\begin{aligned}7(e^8)^0 &= 7(1) \\ &= 7\end{aligned}$$

2. **(a)** $$\begin{aligned}2^0 \times 2^4 &= 2^{0+4} \\ &= 2^4 \\ &= 16\end{aligned}$$

(b) $$\begin{aligned}7^2 \times 7^0 \div 7 &= \frac{7^2 \times 7^0}{7} \\ &= 7^{2+0-1} \\ &= 7\end{aligned}$$

(c) $$\begin{aligned}8^0 - 8^2 &= 1 - 64 \\ &= -63\end{aligned}$$

(d) $$\begin{aligned}6^3 + 6^0 - 6 &= 216 + 1 - 6 \\ &= 211\end{aligned}$$

3. **(a)** $$\begin{aligned}7^{-3} &= \frac{1}{7^3} \\ &= \frac{1}{343}\end{aligned}$$

(b) $$\begin{aligned}(-5)^{-1} &= \frac{1}{(-5)^1} \\ &= -\frac{1}{5}\end{aligned}$$

(c) $$\begin{aligned}\left(\frac{3}{4}\right)^{-2} &= \frac{1}{\left(\frac{3}{4}\right)^2} \\ &= \frac{1}{\left(\frac{9}{16}\right)} \\ &= 1 \div \frac{9}{16} \\ &= 1 \times \frac{16}{9} \\ &= 1\frac{7}{9}\end{aligned}$$

(d) $$\begin{aligned}\left(\frac{5}{3}\right)^{-1} &= \frac{1}{\left(\frac{5}{3}\right)}\\ &= 1 \div \frac{5}{3}\\ &= 1 \times \frac{3}{5}\\ &= \frac{3}{5}\end{aligned}$$

4. **(a)** $$\begin{aligned}(7^2)^{-2} \div 7^{-4} &= \frac{(7^2)^{-2}}{7^{-4}}\\ &= \frac{7^{-4}}{7^{-4}}\\ &= 1\end{aligned}$$

(b) $$\begin{aligned}5^0 - 5^{-2} &= 5^0 - \frac{1}{5^2}\\ &= 1 - \frac{1}{25}\\ &= \frac{24}{25}\end{aligned}$$

(c) $$\begin{aligned}(2^{15})^0 + \left(\frac{3}{5}\right)^{-1} &= 1 + \frac{5}{3}\\ &= 2\frac{2}{3}\end{aligned}$$

(d) $$\begin{aligned}\left(\frac{3}{4}\right)^{-2} \times 3^2 \times 2015^0 &= \left(\frac{4}{3}\right)^2 \times 9 \times 1\\ &= \frac{16}{9} \times 9\\ &= 16\end{aligned}$$

5. **(a)** By prime factorisation, $196 = 2 \times 2 \times 7 \times 7 = 2^2 \times 7^2$.

$$\begin{aligned}\therefore \sqrt{196} &= \sqrt{2 \times 2 \times 7 \times 7}\\ &= 14\end{aligned}$$

(b) By prime factorisation, $125 = 5 \times 5 \times 5 = 5^3$.

$$\begin{aligned}\therefore \sqrt[3]{125} &= \sqrt[3]{5 \times 5 \times 5}\\ &= 5\end{aligned}$$

(c) By prime factorisation, $32 = 2 \times 2 \times 2 \times 2 \times 2 = 2^5$.

$$\begin{aligned}\therefore \sqrt[5]{\frac{1}{32}} &= \sqrt[5]{\frac{1}{2 \times 2 \times 2 \times 2 \times 2}}\\ &= \frac{1}{2}\end{aligned}$$

(d) By prime factorisation, $16 = 2 \times 2 \times 2 \times 2 = 2^4$ and $81 = 3 \times 3 \times 3 \times 3 = 3^4$.

$$\begin{aligned}\therefore \sqrt[4]{\frac{16}{81}} &= \sqrt[4]{\frac{2 \times 2 \times 2 \times 2}{3 \times 3 \times 3 \times 3}}\\ &= \frac{2}{3}\end{aligned}$$

6. **(a)** $$\begin{aligned}81^{\frac{1}{2}} &= \sqrt{81}\\ &= 9\end{aligned}$$

(b) $$\begin{aligned}(-27)^{\frac{1}{3}} &= \sqrt[3]{-27}\\ &= -3\end{aligned}$$

(c) $$\begin{aligned}(16)^{-\frac{1}{4}} &= \left(\frac{1}{16}\right)^{\frac{1}{4}}\\ &= \frac{1}{\sqrt[4]{16}}\\ &= \frac{1}{2}\end{aligned}$$

(d) $$\begin{aligned}4^{1.5} &= 4^{\frac{3}{2}}\\ &= (\sqrt{4}\,)^3\\ &= 2^3\\ &= 8\end{aligned}$$

(e) $$\begin{aligned}8^{-\frac{5}{3}} &= \left(\frac{1}{8}\right)^{\frac{5}{3}}\\ &= \frac{1}{(\sqrt[3]{8}\,)^5}\\ &= \frac{1}{2^5}\\ &= \frac{1}{32}\end{aligned}$$

(f) $$\begin{aligned}(-1000)^{\frac{2}{3}} &= (\sqrt[3]{-1000}\,)^2\\ &= (-10)^2\\ &= 100\end{aligned}$$

7. **(a)** $$\sqrt[4]{a} = a^{\frac{1}{4}}$$

(b) $$\begin{aligned}\sqrt[3]{b^2} &= (b^2)^{\frac{1}{3}}\\ &= b^{2 \times \frac{1}{3}}\\ &= b^{\frac{2}{3}}\end{aligned}$$

(c) $$\begin{aligned}(\sqrt[5]{c}\,)^4 &= (c^{\frac{1}{5}})^4\\ &= c^{\frac{1}{5} \times 4}\\ &= c^{\frac{4}{5}}\end{aligned}$$

(d) $$\begin{aligned}\frac{1}{\sqrt[6]{d}} &= \frac{1}{d^{\frac{1}{6}}}\\ &= d^{-\frac{1}{6}}\end{aligned}$$

(e) $$\begin{aligned}\frac{1}{\sqrt[8]{e^4}} &= \frac{1}{e^{\frac{4}{8}}}\\ &= \frac{1}{e^{\frac{1}{2}}}\\ &= e^{-\frac{1}{2}}\end{aligned}$$

(f) $$\begin{aligned}\frac{1}{(\sqrt[3]{f}\,)^5} &= \frac{1}{f^{\frac{5}{3}}}\\ &= f^{-\frac{5}{3}}\end{aligned}$$

8. **(a)** $$\begin{aligned}11^a &= 1331\\ 11^a &= 11^3\\ a &= 3\end{aligned}$$

(b) $$\begin{aligned}2^b &= \frac{1}{128}\\ 2^b &= \frac{1}{2^7}\\ 2^b &= 2^{-7}\\ b &= -7\end{aligned}$$

(c)
$$\begin{aligned} 9^c &= 3^5 \\ 3^{2c} &= 3^5 \\ 2c &= 5 \\ c &= 2\frac{1}{2} \end{aligned}$$

(d)
$$\begin{aligned} 10^d &= 0.01 \\ 10^d &= \frac{1}{100} \\ 10^d &= 10^{-2} \\ d &= -2 \end{aligned}$$

9. **(a)**
$$\begin{aligned} 5a^4 \times 3a^2 \div a^{-3} &= \frac{5a^4 \times 3a^2}{a^{-3}} \\ &= \frac{5 \times 3 \times a^{4+2}}{a^{-3}} \\ &= \frac{15 \times a^6}{a^{-3}} \\ &= 15 \times a^6 \times a^3 \\ &= 15 \times a^{6+3} \\ &= 15a^9 \end{aligned}$$

(b)
$$\begin{aligned} -24b^{-6} \div (3b^{-3})^2 &= \frac{-24b^{-6}}{(3b^{-3})^2} \\ &= \frac{-24b^{-6}}{3^2 \times b^{-3\times 2}} \\ &= \frac{-24b^{-6}}{9 \times b^{-6}} \\ &= -2\frac{2}{3} \end{aligned}$$

(c)
$$\begin{aligned} (3c)^0 \div (c^{-3}d^5)^{-2} &= \frac{(3c)^0}{(c^{-3}d^5)^{-2}} \\ &= \frac{1}{c^{-3\times -2}d^{5\times -2}} \\ &= \frac{1}{c^6 d^{-10}} \\ &= \frac{d^{10}}{c^6} \end{aligned}$$

(d)
$$\begin{aligned} \frac{(4e^{-6}f^3)^2}{8e^{12}f^6} &= \frac{4^2 \times e^{-6\times 2} \times f^{3\times 2}}{8e^{12}f^6} \\ &= \frac{16 \times e^{-12} \times f^6}{8e^{12}f^6} \\ &= \frac{2 \times f^{6-6}}{e^{12+12}} \\ &= \frac{2 \times f^0}{e^{24}} \\ &= \frac{2}{e^{24}} \end{aligned}$$

(e)
$$\begin{aligned} &(3g^{-3}h^{-1})^2 \times (-4g^3h^{-2})^2 \\ &= (3^2 \times g^{-3\times 2} \times h^{-1\times 2}) \times ((-4)^2 \times g^{3\times 2} \times h^{-2\times 2}) \\ &= (9 \times g^{-6} \times h^{-2}) \times (16 \times g^6 \times h^{-4}) \\ &= 9 \times 16 \times g^{-6+6} \times h^{-2-4} \\ &= 144 \times g^0 \times h^{-6} \\ &= \frac{144}{h^6} \end{aligned}$$

(f)
$$\begin{aligned} (j^2k^{-1})^{-3} \times \left(\frac{j^2}{k^3}\right)^{-3} &= j^{2\times -3} \times k^{-1\times -3} \times \frac{j^{2\times -3}}{k^{3\times -3}} \\ &= j^{-6} \times k^3 \times \frac{j^{-6}}{k^{-9}} \\ &= j^{-6-6} \times k^{3+9} \\ &= j^{-12} \times k^{12} \\ &= \frac{k^{12}}{j^{12}} \end{aligned}$$

(g)
$$\begin{aligned} \frac{(m^5n^3) \times (m^2)^{-2}}{(m^{-1}n)^2} &= \frac{m^5n^3 \times m^{2\times -2}}{m^{-1\times 2} \times n^2} \\ &= \frac{m^5n^3 \times m^{-4}}{m^{-2} \times n^2} \\ &= \frac{m^{5-4} \times n^3}{m^{-2} \times n^2} \\ &= \frac{m \times n^3}{m^{-2} \times n^2} \\ &= m^{1+2} \times n^{3-2} \\ &= m^3n \end{aligned}$$

(h)
$$\begin{aligned} (5p)^3 - 10p \times 7p^2 + \frac{6}{p^{-3}} &= 125p^3 - 10p \times 7p^2 + \frac{6}{p^{-3}} \\ &= 125p^3 - 70p^3 + 6p^3 \\ &= 61p^3 \end{aligned}$$

10. **(a)**
$$\begin{aligned} \sqrt{a} \times \sqrt[3]{a} &= a^{\frac{1}{2}} \times a^{\frac{1}{3}} \\ &= a^{\frac{1}{2}+\frac{1}{3}} \\ &= a^{\frac{3}{6}+\frac{2}{6}} \\ &= a^{\frac{5}{6}} \end{aligned}$$

(b)
$$\begin{aligned} \sqrt[3]{b^2} \div \sqrt[6]{b} &= (b^2)^{\frac{1}{3}} \div b^{\frac{1}{6}} \\ &= \frac{b^{2\times\frac{1}{3}}}{b^{\frac{1}{6}}} \\ &= \frac{b^{\frac{2}{3}}}{b^{\frac{1}{6}}} \\ &= b^{\frac{2}{3}-\frac{1}{6}} \\ &= b^{\frac{4}{6}-\frac{1}{6}} \\ &= b^{\frac{3}{6}} \\ &= b^{\frac{1}{2}} \end{aligned}$$

(c)
$$\begin{aligned} c^{\frac{4}{5}} \times c^{\frac{1}{2}} \div c^{-\frac{2}{5}} &= \frac{c^{\frac{4}{5}+\frac{1}{2}}}{c^{-\frac{2}{5}}} \\ &= c^{\frac{4}{5}+\frac{1}{2}+\frac{2}{5}} \\ &= c^{\frac{8}{10}+\frac{5}{10}+\frac{4}{10}} \\ &= c^{\frac{17}{10}} \end{aligned}$$

(d) $d^{\frac{1}{10}} \div d^{-\frac{1}{5}} \times d^{-\frac{3}{2}} = \dfrac{d^{\frac{1}{10}} \times d^{-\frac{3}{2}}}{d^{-\frac{1}{5}}}$

$$= d^{\frac{1}{10}-\frac{3}{2}-\left(-\frac{1}{5}\right)}$$
$$= d^{\frac{1}{10}-\frac{15}{10}+\frac{2}{10}}$$
$$= d^{-\frac{6}{5}}$$
$$= \frac{1}{d^{\frac{6}{5}}}$$

(e) $(e^{-3}f^{4})^{-\frac{1}{2}} = \dfrac{1}{(e^{-3}f^{4})^{\frac{1}{2}}}$

$$= \frac{1}{e^{-3\times\frac{1}{2}} \times f^{4\times\frac{1}{2}}}$$
$$= \frac{1}{e^{-\frac{3}{2}} \times f^{2}}$$
$$= \frac{e^{\frac{3}{2}}}{f^{2}}$$

(f) $(g^{\frac{2}{3}}h^{-\frac{4}{5}})^{\frac{3}{2}} = g^{\frac{2}{3}\times\frac{3}{2}}h^{-\frac{4}{5}\times\frac{3}{2}}$

$$= gh^{-\frac{6}{5}}$$
$$= \frac{g}{h^{\frac{6}{5}}}$$

11. (a) $(a^{-2}b^{3})^{\frac{1}{3}} \times (a^{4}b^{-5})^{\frac{1}{2}} = (a^{-2\times\frac{1}{3}} \times b^{3\times\frac{1}{3}}) \times (a^{4\times\frac{1}{2}} \times b^{-5\times\frac{1}{2}})$

$$= (a^{-\frac{2}{3}} \times b) \times (a^{2} \times b^{-\frac{5}{2}})$$
$$= a^{-\frac{2}{3}+2} \times b^{1-\frac{5}{2}}$$
$$= a^{\frac{4}{3}} \times b^{-\frac{3}{2}}$$
$$= \frac{a^{\frac{4}{3}}}{b^{\frac{3}{2}}}$$

(b) $(c^{-3}d^{\frac{3}{5}})^{-2} \times (c^{-3}d^{\frac{3}{5}})^{-2} = (c^{-3\times-2} \times d^{\frac{3}{5}\times-2}) \times (c^{\frac{4}{5}\times5}d^{-\frac{2}{5}\times5})$

$$= (c^{6} \times d^{-\frac{6}{5}}) \times (c^{4} \times d^{-2})$$
$$= c^{6+4} \times d^{-\frac{6}{5}-2}$$
$$= c^{10} \times d^{-\frac{16}{5}}$$
$$= \frac{c^{10}}{d^{\frac{16}{5}}}$$

(c) $\dfrac{e^{-\frac{1}{3}}f^{-\frac{1}{4}}}{(e^{2}f^{-\frac{1}{3}})^{-2}} = \dfrac{e^{-\frac{1}{3}}f^{-\frac{1}{4}}}{e^{2\times-2} \times f^{-\frac{1}{3}\times-2}}$

$$= \frac{e^{-\frac{1}{3}}f^{-\frac{1}{4}}}{e^{-4} \times f^{\frac{2}{3}}}$$
$$= e^{-\frac{1}{3}+4}f^{-\frac{1}{4}-\frac{2}{3}}$$
$$= e^{\frac{11}{3}}f^{-\frac{11}{12}}$$
$$= \frac{e^{\frac{11}{3}}}{f^{\frac{11}{12}}}$$

(d) $\left(\dfrac{g^{-2}h^{2}}{25}\right)^{-\frac{1}{2}} = \left(\dfrac{25}{g^{-2}h^{2}}\right)^{\frac{1}{2}}$

$$= \frac{25^{\frac{1}{2}}}{g^{-2\times\frac{1}{2}} \times h^{2\times\frac{1}{2}}}$$
$$= \frac{5}{g^{-1} \times h}$$
$$= \frac{5g}{h}$$

(e) $(4j^{4}k)^{\frac{1}{2}} \div 2h^{3}k^{-\frac{1}{2}} = \dfrac{(4j^{4}k)^{\frac{1}{2}}}{2h^{3}k^{-\frac{1}{2}}}$

$$= \frac{4^{\frac{1}{2}} \times j^{4\times\frac{1}{2}} \times k^{\frac{1}{2}}}{2h^{3}k^{-\frac{1}{2}}}$$
$$= \frac{2 \times j^{2} \times k^{\frac{1}{2}}}{2h^{3}k^{-\frac{1}{2}}}$$
$$= \frac{j^{2} \times k^{\frac{1}{2}+\frac{1}{2}}}{h^{3}}$$
$$= \frac{j^{2}k}{h^{3}}$$

(f) $(m^{3}n^{-\frac{1}{4}})^{4} \div \sqrt[5]{32m^{4}n^{-8}} = \dfrac{(m^{3}n^{-\frac{1}{4}})^{4}}{\sqrt[5]{32m^{4}n^{-8}}}$

$$= \frac{m^{3\times4} \times n^{-\frac{1}{4}\times4}}{(32m^{4}n^{-8})^{\frac{1}{5}}}$$
$$= \frac{m^{12} \times n^{-1}}{32^{\frac{1}{5}} \times m^{4\times\frac{1}{5}} \times n^{-8\times\frac{1}{5}}}$$
$$= \frac{m^{12} \times n^{-1}}{2 \times m^{\frac{4}{5}} \times n^{-\frac{8}{5}}}$$
$$= \frac{m^{12-\frac{4}{5}} \times n^{-1+\frac{8}{5}}}{2}$$
$$= \frac{m^{\frac{56}{5}} \times n^{\frac{3}{5}}}{2}$$
$$= \frac{m^{\frac{56}{5}}n^{\frac{3}{5}}}{2}$$

12. (a) $\left(\dfrac{x^{-4}y^{7}z^{-6}}{x^{3}y^{-1}z^{3}}\right)^{3} \times \left(\dfrac{x^{5}y^{2}z^{-6}}{x^{-3}y^{-5}z^{4}}\right)^{-4}$

$$= \frac{x^{-4\times3} \times y^{7\times3} \times z^{-6\times3}}{x^{3\times3} \times y^{-1\times3} \times z^{3\times3}} \times \frac{x^{5\times-4} \times y^{2\times-4} \times z^{-6\times-4}}{x^{-3\times-4} \times y^{-5\times-4} \times z^{4\times-4}}$$
$$= \frac{x^{-12} \times y^{21} \times z^{-18}}{x^{9} \times y^{-3} \times z^{9}} \times \frac{x^{-20} \times y^{-8} \times z^{24}}{x^{12} \times y^{20} \times z^{-16}}$$
$$= \frac{x^{-12-20} \times y^{21-8} \times z^{-18+24}}{x^{9+12} \times y^{-3+20} \times z^{9-16}}$$
$$= \frac{x^{-32} \times y^{13} \times z^{6}}{x^{21} \times y^{17} \times z^{-7}}$$
$$= x^{-32-21} \times y^{13-17} \times z^{6+7}$$
$$= x^{-53} \times y^{-4} \times z^{13}$$
$$= \frac{z^{13}}{x^{53}y^{4}}$$

(b) $\left(\dfrac{x^3y^{-4}z^7}{x^{-5}y^2}\right)^3 \div \left(\dfrac{x^{-4}yz^{-5}}{x^7y^{-3}}\right)^{-2}$

$$= \frac{x^{3\times3}\times y^{-4\times3}\times z^{7\times3}}{x^{-5\times3}\times y^{2\times3}} \div \frac{x^{-4\times-2}\times y^{-2}\times z^{-5\times-2}}{x^{7\times-2}\times y^{-3\times-2}}$$

$$= \frac{x^9\times y^{-12}\times z^{21}}{x^{-15}\times y^6} \div \frac{x^8\times y^{-2}\times z^{10}}{x^{-14}\times y^6}$$

$$= \frac{x^9\times y^{-12}\times z^{21}}{x^{-15}\times y^6} \times \frac{x^{-14}\times y^6}{x^8\times y^{-2}\times z^{10}}$$

$$= \frac{x^{9-14}\times y^{-12+6}\times z^{21}}{x^{-15+8}\times y^{6-2}\times z^{10}}$$

$$= \frac{x^{-5}\times y^{-6}\times z^{21}}{x^{-7}\times y^4\times z^{10}}$$

$$= x^{-5+7}\times y^{-6-4}\times z^{21-10}$$

$$= x^2\times y^{-10}\times z^{11}$$

$$= \frac{x^2z^{11}}{y^{10}}$$

(c) $\dfrac{ab^n}{bc}\times\dfrac{c^nd}{cd}\div\dfrac{b^{n+2}}{c^{n+3}} = \dfrac{ab^n}{bc}\times\dfrac{c^nd}{cd}\times\dfrac{c^{n+3}}{b^{n+2}}$

$$= \frac{a\times b^n\times c^{n+n+3}\times d}{b^{1+n+2}\times c^2\times d}$$

$$= \frac{a\times b^n\times c^{2n+3}\times d}{b^{n+3}\times c^2\times d}$$

$$= a\times b^{n-(n+3)}\times c^{2n+3-2}$$

$$= a\times b^{-3}\times c^{2n+1}$$

$$= \frac{ac^{2n+1}}{b^3}$$

(d) $\dfrac{(a+b)^n}{bc^2}\div\dfrac{(a+b)^{n+3}}{abc} = \dfrac{(a+b)^n}{bc^2}\times\dfrac{abc}{(a+b)^{n+3}}$

$$= \frac{(a+b)^n\times abc}{(a+b)^{n+3}\times bc^2}$$

$$= \frac{(a+b)^n\times a}{(a+b)^n\times(a+b)^3\times c}$$

$$= \frac{a}{c(a+b)^3}$$

Exercise 5C

1. **(a)** $85\,300 = 8.53 \times 10^4$

(b) $52\,700\,000 = 5.27 \times 10^7$

(c) $0.000\,23 = 2.3 \times 10^{-4}$

(d) $0.000\,000\,094 = 9.4 \times 10^{-8}$

2. **(a)** $9.6 \times 10^3 = 9600$

(b) $4 \times 10^5 = 400\,000$

(c) $2.8 \times 10^{-4} = 0.000\,28$

(d) $1 \times 10^{-6} = 0.000\,001$

3. **(i)** 300 000 000 Hz $= 3 \times 10^8$ Hz

$= 3 \times 10^2 \times 10^6$ Hz

$= 3 \times 10^2$ MHz

(ii) 300 GHz $= 300 \times 10^9$ Hz

$= 3 \times 10^{11}$ Hz

$= 3 \times 10^5 \times 10^6$ Hz

$= 3 \times 10^5$ MHz

4. **(i)** a pm $= 70 \times 10^{-12}$ m

$= 7 \times 10^{-11}$ m

(ii) b nm $= 0.074 \times 10^{-9}$ m

$= 7.4 \times 10^{-1}$ m

(iii) $a : b = 7.0 \times 10^{-11} : 7.4 \times 10^{-11}$

$= 35 : 37$

5. c Mm $= 1500 \times 10^6$ m

$= 1.5 \times 10^9$ m

d Tm $= 5.91 \times 10^{12}$ m

$$\frac{d}{c}\times 100\% = \frac{5.91\times10^{12}}{1.5\times10^9}\times 100\%$$

$= 394\,000\%$

$= 3.94 \times 10^5\ \%$

6. **(a)** $(2.\ 34 \times 10^5) \times (7.12 \times 10^{-4})$

$= \boxed{(}\ \boxed{2}\ \boxed{.}\ \boxed{3}\ \boxed{4}\ \boxed{\times 10^x}\ \boxed{5}\ \boxed{)}$

$\boxed{\times}\ \boxed{(}\ \boxed{7}\ \boxed{.}\ \boxed{1}\ \boxed{2}\ \boxed{\times 10^x}\ \boxed{-}\ \boxed{4}\ \boxed{)}\ \boxed{=}$

$= 166.608$

$= 1.67 \times 10^2$ (to 3 s.f.)

(b) $(5.1 \times 10^{-7}) \times (2.76 \times 10^{-3})$

$= \boxed{(}\ \boxed{5}\ \boxed{.}\ \boxed{1}\ \boxed{\times 10^x}\ \boxed{-}\ \boxed{7}\ \boxed{)}$

$\boxed{\times}\ \boxed{(}\ \boxed{2}\ \boxed{.}\ \boxed{7}\ \boxed{6}\ \boxed{\times 10^x}\ \boxed{-}\ \boxed{3}\ \boxed{)}\ \boxed{=}$

$= 1.4076 \times 10^{-9}$

$= 1.41 \times 10^{-9}$ (to 3 s.f.)

(c) $(13.4 \times 10^4) \div (4 \times 10^5)$

$= \boxed{(}\ \boxed{1}\ \boxed{3}\ \boxed{.}\ \boxed{4}\ \boxed{\times 10^x}\ \boxed{4}\ \boxed{)}$

$\boxed{\div}\ \boxed{(}\ \boxed{4}\ \boxed{\times 10^x}\ \boxed{5}\ \boxed{)}\ \boxed{=}$

$= 0.335$

$= 3.35 \times 10^{-1}$

(d) $\dfrac{3\times10^{-4}}{9\times10^{-8}}$

$= \boxed{(}\ \boxed{3}\ \boxed{\times 10^x}\ \boxed{-}\ \boxed{4}\ \boxed{)}\ \boxed{\div}\ \boxed{(}\ \boxed{9}\ \boxed{\times 10^x}$

$\boxed{-}\ \boxed{8}\ \boxed{)}\ \boxed{=}$

$= 3330$ (to 3 s.f.)

$= 3.33 \times 10^3$

(e) $2.54 \times 10^3 + 3.11 \times 10^4$

$= \boxed{2}\ \boxed{.}\ \boxed{5}\ \boxed{4}\ \boxed{\times 10^x}\ \boxed{3}$

$\boxed{+}\ \boxed{3}\ \boxed{.}\ \boxed{1}\ \boxed{1}\ \boxed{\times 10^x}\ \boxed{4}\ \boxed{=}$

$= 33\,640$

$= 3.36 \times 10^4$ (to 3 s.f.)

(f) $6 \times 10^5 - 3.1 \times 10^7$

$= \boxed{6}\ \boxed{\times 10^x}\ \boxed{5}\ \boxed{-}\ \boxed{3}\ \boxed{.}\ \boxed{1}\ \boxed{\times 10^x}\ \boxed{7}\ \boxed{=}$

$= -30\,400\,000$

$= -3.04 \times 10^7$

(g) $\dfrac{4.37\times10^{-4}+2.16\times10^{-5}}{3\times10^{-3}}$

$=\boxed{(}\,\boxed{4}\,\boxed{.}\,\boxed{3}\,\boxed{7}\,\boxed{\times 10^x}\,\boxed{-}\,\boxed{4}$

$\boxed{+}\,\boxed{2}\,\boxed{.}\,\boxed{1}\,\boxed{6}\,\boxed{\times 10^x}\,\boxed{-}\,\boxed{5}\,\boxed{)}$

$\boxed{\div}\,\boxed{3}\,\boxed{\times 10^x}\,\boxed{-}\,\boxed{3}\,\boxed{=}$

$= 0.153$ (to 3 s.f.)

$= 1.53 \times 10^{-1}$

(h) $\dfrac{2.4\times10^{-10}}{7.2\times10^{-6}-3.5\times10^{-8}}$

$=\boxed{2}\,\boxed{.}\,\boxed{4}\,\boxed{\times 10^x}\,\boxed{-}\,\boxed{1}\,\boxed{0}\,\boxed{\div}\,\boxed{(}\,\boxed{7}\,\boxed{.}\,\boxed{2}$

$\boxed{\times 10^x}\,\boxed{-}\,\boxed{6}\,\boxed{-}\,\boxed{3}\,\boxed{.}\,\boxed{5}\,\boxed{\times 10^x}\,\boxed{-}\,\boxed{8}\,\boxed{)}\,\boxed{=}$

$= 3.35 \times 10^{-5}$ (to 3 s.f.)

7. **(a)** $(1.35 \times 10^{-4})^3$

$=\boxed{(}\,\boxed{1}\,\boxed{.}\,\boxed{3}\,\boxed{5}\,\boxed{\times 10^x}\,\boxed{-}\,\boxed{4}\,\boxed{)}\,\boxed{x^y}\,\boxed{3}\,\boxed{=}$

$= 2.46 \times 10^{-12}$ (to 3 s.f.)

(b) $6(3.4 \times 10^3)^2$

$=\boxed{6}\,\boxed{\times}\,\boxed{(}\,\boxed{3}\,\boxed{.}\,\boxed{4}\,\boxed{\times 10^x}\,\boxed{3}\,\boxed{)}\,\boxed{x^y}\,\boxed{2}\,\boxed{=}$

$= 69\,360\,000$

$= 6.94 \times 10^7$ (to 3 s.f.)

(c) $\sqrt{1.21\times10^8}$

$=\boxed{\sqrt{\ }}\,\boxed{(}\,\boxed{1}\,\boxed{.}\,\boxed{2}\,\boxed{1}\,\boxed{\times 10^x}\,\boxed{8}\,\boxed{)}\,\boxed{=}$

$= 1.1 \times 10^4$

(d) $\sqrt[3]{9.261\times10^6}$

$=\boxed{\text{shift}}\,\boxed{\sqrt[3]{\ }}\,\boxed{(}\,\boxed{9}\,\boxed{.}\,\boxed{2}\,\boxed{6}\,\boxed{1}\,\boxed{\times 10^x}\,\boxed{6}\,\boxed{)}\,\boxed{=}$

$= 210$

$= 2.1 \times 10^2$

(e) $\dfrac{2.3\times10^{-2}\times4.7\times10^3}{2\times10^3}$

$=\boxed{(}\,\boxed{2}\,\boxed{.}\,\boxed{3}\,\boxed{\times 10^x}\,\boxed{-}\,\boxed{2}\,\boxed{\times}\,\boxed{4}\,\boxed{.}\,\boxed{7}$

$\boxed{\times 10^x}\,\boxed{3}\,\boxed{)}\,\boxed{\div}\,\boxed{2}\,\boxed{\times 10^x}\,\boxed{3}\,\boxed{=}$

$= 0.05405$

$= 5.41 \times 10^{-2}$ (to 3 s.f.)

(f) $\dfrac{8\times10^2+2.5\times10^3}{2\times10^{-2}-3.4\times10^{-3}}$

$=\boxed{(}\,\boxed{8}\,\boxed{\times 10^x}\,\boxed{2}\,\boxed{+}\,\boxed{2}\,\boxed{.}\,\boxed{5}\,\boxed{\times 10^x}\,\boxed{3}\,\boxed{)}$

$\boxed{\div}\,\boxed{(}\,\boxed{2}\,\boxed{\times 10^x}\,\boxed{-}\,\boxed{2}\,\boxed{-}\,\boxed{3}\,\boxed{.}\,\boxed{4}\,\boxed{\times 10^x}$

$\boxed{-}\,\boxed{3}\,\boxed{)}\,\boxed{=}$

$= 199\,000$ (to 3 s.f.)

$= 1.99 \times 10^5$

8. **(a)** $2P \times 4Q = 2(7.5 \times 10^3) \times 4(5.25 \times 10^4)$

$= 3.15 \times 10^9$

(b) $Q - P = (5.25 \times 10^4) - (7.5 \times 10^3)$

$= 4.5 \times 10^4$

9. $x + 8y = (2 \times 10^{-3}) + 8(7 \times 10^{-4})$

$= 7.6 \times 10^{-3}$

10. **(a)** $MN = (3.2 \times 10^6) \times (5.0 \times 10^7)$

$= 1.6 \times 10^{14}$

(b) $\dfrac{M}{N} = \dfrac{3.2\times10^6}{5.0\times10^7}$

$= 6.4 \times 10^{-2}$

11. $R = \dfrac{M}{EI}$

$= \dfrac{6\times10^4}{(4.5\times10^8)\times(4\times10^2)}$

$= 3.33 \times 10^{-7}$ (to 3 s.f.)

12. **(i)** 300 000 000 m/s $= 3 \times 10^8$ m/s

(ii) 778.5 million km $= 778.5 \times 10^6$ km

$= 778.5 \times 10^6 \times 10^3$ m

$= 7.785 \times 10^{11}$ m

Time taken $= \dfrac{\text{Distance}}{\text{Speed}}$

$= \dfrac{7.785\times10^{11}}{3\times10^8}$

= 2595 seconds

= 43 minutes 15 seconds

13. **(i)** Distance travelled by rockets in 4 days $= 4.8 \times 10^5$ km

Distance travelled by rockets in 12 days $= \dfrac{4.8\times10^5}{4} \times 12$

$= 1.44 \times 10^6$ km

(ii) Speed $= \dfrac{\text{Distance}}{\text{Time}}$

$= \dfrac{4.8\times10^5}{4}$

$= 1.2 \times 10^5$ km/day

Time taken $= \dfrac{4.8\times10^7}{1.2\times10^5}$

= 400 days

14. **(i)** Increase in population $= 5.45 \times 10^8 - 4.20 \times 10^8$

$= 1.25 \times 10^8$

(ii) $\dfrac{1.17\times10^9}{5.45\times10^8} = 2.15$ (to 3 s.f.)

(iii) $\dfrac{1.23\times10^9}{7.28\times10^8} = 1.69$ (to 3 s.f.)

Review Exercise 5

1. **(a)**
$$\begin{aligned}(a^3b) \times (a^4b^3) &= a^{3+4}b^{1+3}\\ &= a^7b^4\end{aligned}$$

(b)
$$\begin{aligned}(6a^5b^4) \div (2a^3b^2) &= \frac{6a^5b^4}{2a^3b^2}\\ &= 3a^{5-3}b^{4-2}\\ &= 3a^2b^2\end{aligned}$$

(c)
$$\begin{aligned}(-3a^3b^5)^3 &= (-3)^3 \times a^{3\times3} \times b^{5\times3}\\ &= -27a^9b^{15}\end{aligned}$$

(d)
$$\begin{aligned}\left(\frac{2a^2b}{b^3}\right)^3 \div \left(\frac{16a^5}{ab^7}\right) &= \left(\frac{8a^{2\times3}b^3}{b^{3\times3}}\right) \div \left(\frac{16a^5}{ab^7}\right)\\ &= \left(\frac{8a^6b^3}{b^9}\right) \times \left(\frac{ab^7}{16a^5}\right)\\ &= \left(\frac{8a^{6+1}b^{3+7}}{16a^5b^9}\right)\\ &= \left(\frac{8a^7b^{10}}{16a^5b^9}\right)\\ &= \left(\frac{a^{7-5}b^{10-9}}{2}\right)\\ &= \frac{a^2b}{2}\end{aligned}$$

2. **(a)**
$$\begin{aligned}5^{24} \div 5^8 &= 5^{24-8}\\ &= 5^{16}\end{aligned}$$

(b) $\frac{1}{125} = \frac{1}{5^3} = 5^{-3}$

(c) $\sqrt[5]{5} = 5^{\frac{1}{5}}$

3. **(a)**
$$\begin{aligned}5^2 \div 5^{-1} \times 5^0 &= 5^{2-(-1)+0}\\ &= 5^3\\ &= 125\end{aligned}$$

(b)
$$\begin{aligned}2^{-2} - 3^{-2} &= \frac{1}{2^2} - \frac{1}{3^2}\\ &= \frac{1}{4} - \frac{1}{9}\\ &= \frac{9}{36} - \frac{4}{36}\\ &= \frac{5}{36}\end{aligned}$$

(c)
$$\begin{aligned}3^{-2} + \left(\frac{1}{3}\right)^{-1} - (-3)^0 &= \frac{1}{3^2} + 3 - 1\\ &= \frac{1}{9} + 3 - 1\\ &= 2\frac{1}{9}\end{aligned}$$

(d)
$$\begin{aligned}\left(\frac{2}{5}\right)^3 \div \left(\frac{9}{2}\right)^{-2} &= \left(\frac{2^3}{5^3}\right) \div \left(\frac{2}{9}\right)^2\\ &= \left(\frac{8}{125}\right) \div \left(\frac{2^2}{9^2}\right)\\ &= \left(\frac{8}{125}\right) \div \left(\frac{4}{81}\right)\\ &= \frac{8}{125} \times \frac{81}{4}\\ &= 1\frac{37}{125}\end{aligned}$$

4. **(a)** By prime factorisation, $81 = 3 \times 3 \times 3 \times 3 = 3^4$.
$$\begin{aligned}\sqrt[4]{81} &= \sqrt[4]{3\times3\times3\times3}\\ &= 3\end{aligned}$$

(b) By prime factorisation, $27 = 3 \times 3 \times 3 = 3^3$ and $125 = 5 \times 5 \times 5 = 5^3$.
$$\begin{aligned}\sqrt[3]{\frac{27}{125}} &= \sqrt[3]{\frac{3\times3\times3}{5\times5\times5}}\\ &= \frac{3}{5}\end{aligned}$$

(c)
$$\begin{aligned}16^{1.5} &= 16^{\frac{3}{2}}\\ &= (\sqrt{16}\,)^3\\ &= 4^3\\ &= 64\end{aligned}$$

(d)
$$\begin{aligned}1024^{-\frac{3}{5}} &= \left(\frac{1}{1024}\right)^{\frac{3}{5}}\\ &= \left(\frac{1}{\sqrt[5]{1024}}\right)^3\\ &= \left(\frac{1}{4}\right)^3\\ &= \frac{1}{64}\end{aligned}$$

5. **(a)** $\left(\frac{3}{x}\right)^{-4} = \left(\frac{x}{3}\right)^4 = \frac{x^4}{81}$

(b) $3 \div x^{-3} = 3 \div \frac{1}{x^3} = 3 \times x^3 = 3x^3$

6. **(a)**
$$\begin{aligned}(x^3y^{-2}) \times (x^{-3}y^5) &= x^{3-3} \times y^{-2+5}\\ &= x^0 \times y^3\\ &= y^3\end{aligned}$$

(b)
$$\begin{aligned}(5x^2y^3)^0 \div (-2x^{-3}y^5)^{-2} &= \frac{(5x^2y^3)^0}{(-2x^{-3}y^5)^{-2}}\\ &= \frac{1}{(-2x^{-3}y^5)^{-2}}\\ &= (-2x^{-3}y^5)^2\\ &= (-2)^2 \times x^{-3\times2} \times y^{5\times2}\\ &= 4x^{-6}y^{10}\\ &= \frac{4y^{10}}{x^6}\end{aligned}$$

(c)
$$\begin{aligned}\left(\frac{x^2}{y^{-3}}\right)^4 \div \left(\frac{x^5}{y^7}\right)^3 &= \frac{x^8}{y^{-12}} \div \frac{x^{15}}{y^{21}}\\ &= \frac{x^8}{y^{-12}} \times \frac{y^{21}}{x^{15}}\\ &= x^{8-15}y^{21-(-12)}\\ &= x^{-7}y^{33}\\ &= \frac{y^{33}}{x^7}\end{aligned}$$

(d)
$$\begin{aligned}\frac{(3x^{-2}y^5)^2 \times (-2x^3y^{-2})^2}{9x^4y^6} &= \frac{9x^{-4}y^{10} \times 4x^6y^{-4}}{9x^4y^6}\\ &= \frac{36x^{-4+6}y^{10-4}}{9x^4y^6}\\ &= \frac{36x^2y^6}{9x^4y^6}\\ &= \frac{4}{x^2}\end{aligned}$$

7. (a)
$$\begin{aligned}\sqrt[5]{p^3} \times \sqrt[3]{8p} &= p^{\frac{3}{5}} \times (8p)^{\frac{1}{3}}\\ &= p^{\frac{3}{5}} \times 2p^{\frac{1}{3}}\\ &= 2p^{\frac{3}{5}+\frac{1}{3}}\\ &= 2p^{\frac{14}{15}}\end{aligned}$$

(b)
$$\begin{aligned}\left(p^{-3}q^{\frac{3}{5}}\right)^{-\frac{2}{3}} \times \left(p^{\frac{4}{5}}q^{-\frac{2}{5}}\right)^3 &= \left(p^{-3\times-\frac{2}{3}}q^{\frac{3}{5}\times-\frac{2}{3}}\right) \times \left(p^{\frac{4}{5}\times3}q^{-\frac{2}{3}\times3}\right)\\ &= \left(p^2q^{-\frac{2}{5}}\right) \times \left(p^{\frac{12}{5}}q^{-2}\right)\\ &= p^{2+\frac{12}{5}}q^{-\frac{2}{5}-2}\\ &= p^{\frac{22}{5}}q^{-\frac{12}{5}}\\ &= \frac{p^{\frac{22}{5}}}{q^{\frac{12}{5}}}\end{aligned}$$

(c)
$$\begin{aligned}\frac{p^{\frac{2}{3}}q^{-\frac{2}{5}}}{\left(p^2q^{-\frac{1}{5}}\right)^{-3}} &= \frac{p^{\frac{2}{3}}q^{-\frac{2}{5}}}{p^{-6}q^{\frac{3}{5}}}\\ &= \frac{p^{\frac{2}{3}+6}}{q^{\frac{3}{5}+\frac{2}{5}}}\\ &= \frac{p^{\frac{20}{3}}}{q}\end{aligned}$$

(d)
$$\begin{aligned}\left(p^{-\frac{1}{3}}q^2\right)^5 \times \sqrt[3]{27(p^{-3}q^2)} &= p^{-\frac{5}{3}}q^{10} \times (27(p^{-3}q^2))^{\frac{1}{3}}\\ &= p^{-\frac{5}{3}}q^{10} \times 3\left(p^{-3\times\frac{1}{3}}q^{2\times\frac{1}{3}}\right)\\ &= p^{-\frac{5}{3}}q^{10} \times 3p^{-1}q^{\frac{2}{3}}\\ &= 3p^{-\frac{5}{3}-1}q^{10+\frac{2}{3}}\\ &= 3p^{-\frac{8}{3}}q^{\frac{32}{3}}\\ &= \frac{3q^{\frac{32}{3}}}{p^{\frac{8}{3}}}\end{aligned}$$

8. (a)
$$\begin{aligned}4^{-6} \times 4^x &= 1\\ 4^{-6+x} &= 4^0\\ -6+x &= 0\\ x &= 6\end{aligned}$$

(b)
$$\begin{aligned}x^{-3} &= 7\\ \frac{1}{x^3} &= 7\\ 7x^3 &= 1\\ x^3 &= \frac{1}{7}\end{aligned}$$

(c)
$$\begin{aligned}5^{12} \times 5^{-2} \div 5^x &= 25\\ 5^{12-2-x} &= 5^2\\ 5^{10-x} &= 5^2\\ 10-x &= 2\\ x &= 8\end{aligned}$$

9. (a)
$$\begin{aligned}16^a &= 8\\ (2^4)^a &= 2^3\\ 2^{4a} &= 2^3\\ 4a &= 3\\ a &= \frac{3}{4}\end{aligned}$$

(b)
$$\begin{aligned}2015^b &= 2015^0\\ b &= 0\end{aligned}$$

(c)
$$\begin{aligned}\frac{10^c}{10} &= 0.01\\ 10^{c-1} &= \frac{1}{100}\\ 10^{c-1} &= \frac{1}{10^2}\\ 10^{c-1} &= 10^{-2}\\ c-1 &= -2\\ c &= -1\end{aligned}$$

(d)
$$\begin{aligned}\frac{2^{d-6}}{2} &= 2^9\\ 2^{d-6-1} &= 2^9\\ 2^{d-7} &= 2^9\\ d-7 &= 9\\ d &= 16\end{aligned}$$

10. (a) $(6.4 \times 10^6) \times (5.1 \times 10^{-3})$

$= \boxed{(}\ \boxed{6}\ \boxed{.}\ \boxed{4}\ \boxed{\times 10^x}\ \boxed{6}\ \boxed{)}\ \boxed{\times}\ \boxed{(}\ \boxed{5}\ \boxed{.}\ \boxed{1}$

$\boxed{\times 10^x}\ \boxed{-}\ \boxed{3}\ \boxed{)}\ \boxed{=}$

$= 32\ 640$

$= 3.26 \times 10^4$ (to 3 s.f.)

(b) $(2.17 \times 10^{-5}) \div (7 \times 10^4)$

$= \boxed{(}\ \boxed{2}\ \boxed{.}\ \boxed{1}\ \boxed{7}\ \boxed{\times 10^x}\ \boxed{-}\ \boxed{5}\ \boxed{)}$

$\boxed{\div}\ \boxed{(}\ \boxed{7}\ \boxed{\times 10^x}\ \boxed{4}\ \boxed{)}\ \boxed{=}$

$= 3.1 \times 10^{-10}$

(c) $(3.17 \times 10^4) + (2.26 \times 10^5)$

$= \boxed{(}\,\boxed{3}\,\boxed{.}\,\boxed{1}\,\boxed{7}\,\boxed{\times 10^x}\,\boxed{4}\,\boxed{)}$

$\boxed{+}\,\boxed{(}\,\boxed{2}\,\boxed{.}\,\boxed{2}\,\boxed{6}\,\boxed{\times 10^x}\,\boxed{5}\,\boxed{)}\,\boxed{=}$

$= 257\ 700$

$= 2.58 \times 10^5$ (to 3 s.f.)

(d) $(4.15 \times 10^{-3}) - (5.12 \times 10^{-4})$

$= \boxed{(}\,\boxed{4}\,\boxed{.}\,\boxed{1}\,\boxed{5}\,\boxed{\times 10^x}\,\boxed{-}\,\boxed{3}\,\boxed{)}$

$\boxed{-}\,\boxed{(}\,\boxed{5}\,\boxed{.}\,\boxed{1}\,\boxed{2}\,\boxed{\times 10^x}\,\boxed{-}\,\boxed{4}\,\boxed{)}\,\boxed{=}$

$= 3.64 \times 10^{-3}$ (to 3 s.f.)

(e) $\dfrac{5.1 \times 10^{-6} - 2.34 \times 10^5}{4.87 \times 10^{-3} + 9 \times 10^{-2}}$

$= \boxed{(}\,\boxed{5}\,\boxed{.}\,\boxed{1}\,\boxed{\times 10^x}\,\boxed{-}\,\boxed{6}\,\boxed{-}\,\boxed{2}\,\boxed{.}\,\boxed{3}\,\boxed{4}$

$\boxed{\times 10^x}\,\boxed{5}\,\boxed{)}\,\boxed{\div}\,\boxed{(}\,\boxed{4}\,\boxed{.}\,\boxed{8}\,\boxed{7}\,\boxed{\times 10^x}$

$\boxed{-}\,\boxed{3}\,\boxed{+}\,\boxed{9}\,\boxed{\times 10^x}\,\boxed{-}\,\boxed{2}\,\boxed{)}\,\boxed{=}$

$= -2.47 \times 10^6$ (to 3 s.f.)

(f) $\dfrac{8.43 \times 10^7 + 6.8 \times 10^8}{(1.01 \times 10^4)^3}$

$= \boxed{(}\,\boxed{8}\,\boxed{.}\,\boxed{4}\,\boxed{3}\,\boxed{\times 10^x}\,\boxed{7}\,\boxed{+}\,\boxed{6}\,\boxed{.}\,\boxed{8}$

$\boxed{\times 10^x}\,\boxed{8}\,\boxed{)}\,\boxed{\div}\,\boxed{(}\,\boxed{1}\,\boxed{.}\,\boxed{0}\,\boxed{1}\,\boxed{\times 10^x}\,\boxed{4}\,\boxed{)}$

$\boxed{\times x^3}\,\boxed{=}$

$= 7.42 \times 10^{-4}$ (to 3 s.f.)

11. (a) $a - b = 110\ 000\ 000 - 12\ 100\ 000$

$= 1.1 \times 10^8 - 1.21 \times 10^7$

$= 9.79 \times 10^7$

(b) $\sqrt[3]{ab} = \sqrt[3]{(1.1 \times 10^8) \times (1.21 \times 10^7)}$

$= \sqrt[3]{1.331 \times 10^{15}}$

$= 1.1 \times 10^5$

(c) $6c^2 = 6(0.000\ 007)^2$

$= 6(7 \times 10^{-6})^2$

$= 2.94 \times 10^{-10}$

(d) $\dfrac{ac}{b} = \dfrac{(1.1 \times 10^8) \times (7 \times 10^{-6})}{1.21 \times 10^7}$

$= 6.36 \times 10^{-5}$ (to 3 s.f.)

12. (i) $1 \text{ nm} = 10^{-9} \text{ m}$

$7 \text{ nm} = 7 \times 10^{-9} \text{ m}$

Circumference $= \pi d$

$= 3.142(7 \times 10^{-9})$

$= 2.20 \times 10^{-8}$ m (to 3 s.f.)

(ii) Area $= \pi r^2$

$= 3.142\left(\dfrac{7 \times 10^{-9}}{2}\right)^2$

$= 3.85 \times 10^{-17} \text{ m}^2$ (to 3 s.f.)

13. (i) 149 597 870 700 nm $= 1.496 \times 10^{11}$ m (to 4 s.f.)

(ii) Time $= \dfrac{\text{Distance}}{\text{Speed}}$

$= \dfrac{1.496 \times 10^{11}}{3 \times 10^8 \text{ m/s}}$

$= 499$ s (to 3 s.f.)

14. (i) $1 \text{ Mm} = 10^6 \text{ m}$

$240 \text{ Mm} = 240 \times 10^6 \text{ m}$

$= 2.4 \times 10^8 \text{ m}$

(ii) Speed of rocket $= \dfrac{\text{Distance}}{\text{Time}}$

$= \dfrac{1 \text{ m}}{8000 \text{ ns}}$

$= \dfrac{1 \text{ m}}{8000 \times 10^{-9} \text{ s}}$

$= 1.25 \times 10^5 \text{ m/s}$

Time taken $= \dfrac{\text{Distance}}{\text{Speed}}$

$= \dfrac{2.4 \times 10^8 \text{ m}}{1.25 \times 10^5 \text{ m/s}}$

$= 1920$ s

15. (i) Mass of water molecule $= 2(1.66 \times 10^{-24}) + (2.66 \times 10^{-23})$

$= 2.99 \times 10^{-23}$ g (to 3 s.f.)

(ii) Approx. no. of water molecules $= \dfrac{280}{2.99 \times 10^{-23}}$

$= 9.36 \times 10^{24}$ g (to 3 s.f.)

Challenge Yourself

1. $2^{3^4} = (2^{(3^4)} = 2^{81}$

$2^{4^3} = 2^{(4^3)} = 2^{64}$

So, $2^{34} < 2^{43} < 2^{4^3} = 2^{64} < 2^{3^4} = 2^{81}$.

$3^{2^4} = 3^{(2^4)} = 3^{16}$

$3^{4^2} = 3^{(4^2)} = 3^{16}$

So, $3^{16} < 3^{24} < 3^{42}$.

$4^{3^2} = 4^{(3^2)} = 4^9$

$4^{2^3} = 4^{(2^3)} = 4^8$

So, $4^8 < 4^9 < 4^{23} < 4^{30}$.

$3^{16} < 4^{30} = 2^{60} < 2^{3^4} = 2^{81}$

Hence 2^{3^4} is the largest.

2. $3^1, 3^2, 3^3, 3^4, 3^5, 3^6, 3^7, \ldots$

3, 9, 27, 81, 243, 729, 2187, …

We can observe that the last digit of 3^n are in the sequence:

3, 9, 7, 1, 3, 9, 7, 1, …

$\therefore$ Since $2015 \div 4$ gives a remainder of 3, hence the largest digit of 3^{2015} is 7.

3. Let $x = \sqrt{2 + \sqrt{2 + \sqrt{2 + \ldots}}}$.

$$\text{Then } x^2 = \left(\sqrt{2 + \sqrt{2 + \sqrt{2 + \ldots}}}\right)^2$$

$$x^2 = 2 + \sqrt{2 + \sqrt{2 + \ldots}}$$

$$x^2 = 2 + x$$

$$x^2 - x - 2 = 0$$

$$(x - 2)(x + 1) = 0$$

$$x = 2 \text{ or } x = -1$$

Since $\sqrt{2 + \sqrt{2 + \sqrt{2 + \ldots}}} > 0$, hence $x = 2$.

Chapter 6 Linear Inequalities in One Variable

TEACHING NOTES

Suggested Approach

Teachers can begin this chapter by linking to students' prior knowledge of solving simple linear equations in one variable. By replacing the equality sign with inequality signs, teachers can emphasise that the variable can take more than one value.

Students will learn simple inequalities in the form $ax > b$, $ax \geqslant b$, $ax < b$ and $ax \leqslant b$, where a and b are integers, teachers can remind students that "solving an inequality" involves finding all the solutions that satisfy the inequality, which is akin to solving a simple linear equation. To help students better understand linear inequalities and see how it can be applied in our daily lives, teachers may get students to give some real-life examples where inequalities are involved.

Section 6.1: Simple Inequalities

In the investigation on page 187 of the textbook, students are required to work with numerical examples before generalising the conclusions for some properties of inequalities. In this section. Students only need to know how to solve linear inequalities of the form $ax \leqslant b$, $ax \geqslant b$, $ax < b$ and $ax > b$, where a and b are integers and $a > 0$. Teachers should get students to formulate inequalities based on real-world contexts (see the Journal Writing on page 190 of the textbook).

Section 6.2: Inequalities

Teachers should recap with students how to solve simple linear inequalities and to represent the solution on a number line. The use of number lines will help students to visualise and understand the meanings of <, >, ⩽ and ⩾ (see Investigation: Inequalities). Teachers should guide students when solving linear inequalities that involve reversing the inequality signs when multiplying or dividing the inequalities by a negative number as this may be confusing to them. Teachers can use actual numbers to explain how the signs will change when multiplying and dividing by a negative number. Teachers can get students to explore the relationship between the solution of an inequality and that of the corresponding linear equation (see Thinking Time on page 194 of the textbook).

Section 6.3: Problem Solving involving Inequalities

In problem solving involving inequalities, students must work on their mathematical process of interpretation and thinking skills. Teachers can guide students to understand terms such as 'at most', 'at least', 'not more than' and 'not lesser than' and how to form an inequality and solve it to find the answer to the problem.

Challenge Yourself

Since $z = \frac{x}{y}$, teachers can ask students to find the greatest and least possible values of $\frac{x}{y}$ to find the limits in which z must lie.

Teachers can get the students to first factorise the denominator, $x^2 - 14x + 49$ and observe that value of the denominator is more than zero.

WORKED SOLUTIONS

Investigation (Properties of Inequalities)

1.

Cases	Working	Inequality	Is the inequality sign reversed?	Conclusion
Multiplication by a *positive* number on both sides of the inequality $10 > 6$	LHS $= 10 \times 5$ $= 50$ RHS $= 6 \times 5$ $= 30$	$50 > 30$	No	If $x > y$ and $c > 0$, then $cx > cy$.
Division by a *positive* number on both sides of the inequality $10 > 6$	LHS $= 10 \div 5$ $= 2$ RHS $= 6 \div 5$ $= 1.2$	$2 > 1.2$	No	If $x > y$ and $c > 0$, then $\frac{x}{c} > \frac{y}{c}$.

Table 6.2

2. Yes, the conclusions drawn from Table 5.3 apply to $10 \geqslant 6$.
The following conclusions hold for $x \geqslant y$:

- If $x \geqslant y$ and $c > 0$, then $cx \geqslant cy$ and $\frac{x}{c} \geqslant \frac{y}{c}$.

The following conclusions hold for $x < y$:

- If $x < y$ and $c > 0$, then $cx < cy$ and $\frac{x}{c} < \frac{y}{c}$.

The following conclusions hold for $x \leqslant y$:

- If $x \leqslant y$ and $c > 0$, then $cx \leqslant cy$ and $\frac{x}{c} \leqslant \frac{y}{c}$.

Journal Writing (Page 190)

- A bowl of rice contains 5 g of protein. A teenager needs a minimum of 49 g of protein each day. It is given that he only eats rice on a particular day. The inequality which we need to set up to find the least number of bowls of rice he needs to eat in order to meet his minimum protein requirement that day is:

  $$5x \geqslant 49,$$

  where x represents the number of bowls of rice he needs to eat that day.
- The flag-down fare of a courier bike is PKR 5. The bike charges PKR 0.30 for each 385 m it travels. A person has not more than PKR 50 to spend on his bike ride. The inequality which we need to set up to find the maximum distance that he can travel on the bike is:

  $$30x \leqslant 4500,$$

  where x is the number of blocks of 385 m.

Teachers may wish to note that the list is not exhaustive.

Investigation (Properties of Inequalities)

1.

Multiplication by a *negative* number on both sides of the inequality $10 > 6$	LHS $= 10 \times (-5)$ $= -50$ RHS $= 6 \times (-5)$ $= -30$	$-50 < -30$	Yes	If $x > y$ and $d < 0$, then $dx < dy$.
Division by a *negative* number on both sides of the inequality $10 > 6$	LHS $= 10 \div (-5)$ $= -2$ RHS $= 6 \div (-5)$ $= -1.2$	$-2 < -1.2$	Yes	If $x > y$ and $d < 0$, then $\frac{x}{d} < \frac{y}{d}$.

Table 6.3

2. Yes, the conclusions drawn from Table 3.1 apply to $10 \geqslant 6$.
The following conclusions hold for $x \geqslant y$:

- If $x \geqslant y$ and $d < 0$, then $dx \leqslant dy$ and $\frac{x}{d} \leqslant \frac{y}{d}$.

The following conclusions hold for $x < y$:

- If $x < y$ and $d < 0$, then $dx > dy$ and $\frac{x}{d} > \frac{y}{d}$.

The following conclusions hold for $x \leqslant y$:

- If $x \leqslant y$ and $d < 0$, then $dx \geqslant dy$ and $\frac{x}{d} \geqslant \frac{y}{d}$.

Investigation (Inequalities)

1. **(a)** **(ii)** $6 + 2 = 8 < 12 + 2 = 14$
(iii) $6 - 4 = 2 < 12 - 4 = 8$
(b) If $6 < 12$ and a is a real number, then $6 + a < 12 + a$ and $6 - a < 12 - a$.
(c) If $12 > 6$ and a is a real number, then $12 + a > 6 + a$ and $12 - a > 6 - a$.

2. **(a)** **(i)** $-6 < 12$
(ii) $-6 + 2 = -4 < 12 + 2 = 14$
(iii) $-6 - 4 = -10 < 12 - 4 = 8$
(b) If $-6 < 12$ and a is a real number, then $-6 + a < 12 + a$ and $-6 - a < 12 - a$.
(c) If $12 > -6$ and a is a real number, then $12 + a > -6 + a$ and $12 - a > -6 - a$.

3. **(a)** **(i)** $6 > -12$
(ii) $6 + 2 = 8 > -12 + 2 = -10$
(iii) $6 - 4 = 2 > -12 - 4 = -16$
(b) The addition or subtraction of a positive number does not change the inequality sign.

4. Yes, the conclusion applies.

Journal Writing (Page 193)

Other real life applications of inequalities:
BMI, grades, credit limits, text messaging, travel times, weight limits, financial planning, wages and taxes, temperature limits, height limits for vehicles, etc.
Teachers should note that the list is not exhaustive.

Thinking Time (Page 194)

1. $ax + b = c$, where a, b and c are constants and $a > 0$

Step 1: Arrange the terms such that the constants are all on one side of the equation, i.e. $ax = c - b$.

Step 2: Divide both sides by a to solve for x. The equality sign remains.

The steps will not change if $a < 0$.

2. $ax + b > c$, where a, b and c are constants and $a > 0$

Step 1: Arrange the terms such that the constants are all on one side of the inequality, i.e. $ax > c - b$.

Step 2: Divide both sides by a to solve for x. The inequality sign ($>$) remains.

The steps will change if $a < 0$ such that the inequality sign will change to $<$.

3. $ax + b \geqslant c$, where a, b and c are constants and $a > 0$

Step 1: Arrange the terms such that the constants are all on one side of the inequality, i.e. $ax \geqslant c - b$.

Step 2: Divide both sides by a to solve for x. The inequality sign ($\geqslant$) remains.

The step will change if $a < 0$ such that the inequality sign will change to $\leqslant$.

4. The solutions of $ax + b > c$ and $ax + b < c$ do not include the solution of its corresponding linear equation $ax + b = c$.

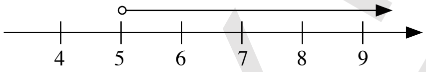
The solutions of $ax + b \geqslant c$ and $ax + b \leqslant c$ include the solution of its corresponding linear equation $ax + b = c$.

Practise Now 1

1. **(a)** $15x > 75$

$$x > \frac{75}{15}$$

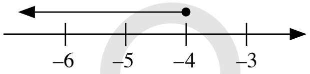
$$\therefore x > 5$$



(b) $4x \leqslant -16$

$$x \leqslant \frac{-16}{4}$$

$$x \leqslant -4$$



2. $6x > 7$

$$x > \frac{7}{6}$$

$$x > 1\frac{1}{6}$$

$\therefore$ The smallest integer value of x is 2.

Practise Now 2

Let the number of buses that are needed to ferry 520 people be x.

Then $45x \geqslant 520$

$$x \geqslant \frac{520}{45}$$

$$x \geqslant 11\frac{5}{9}$$

$\therefore$ The minimum number of buses that are needed to ferry 520 students is 12.

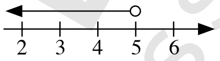
Practise Now 3

1. **(a)** $-6x > -30$

$$6x < 30$$

$$x < \frac{30}{6}$$

$$\therefore x < 5$$

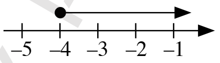


(b) $-8x \leqslant 32$

$$8x \geqslant -32$$

$$x \geqslant -\frac{32}{8}$$

$$\therefore x \geqslant -4$$



2. $-3x < -13$

$$3x > 13$$

$$x > \frac{13}{3}$$

$$x > 4\frac{1}{3}$$

$\therefore$ The smallest integer value of x is 5.

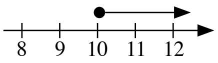
Practise Now 4

(a)

$$x - 3 \geqslant 7$$

$$x - 3 + 3 \geqslant 7 + 3$$

$$x \geqslant 10$$



(b)

$$-2y + 4 > 3$$

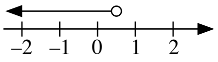
$$-2y + 4 - 4 > 3 - 4$$

$$-2y > -1$$

$$2y < 1$$

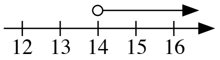
$$\frac{2y}{2} < \frac{1}{2}$$

$$y < \frac{1}{2}$$



Practise Now 5

$$\begin{aligned} 5 - x &< -9 \\ 5 - 5 - x &< -9 - 5 \\ -x &< -14 \\ x &> 14 \end{aligned}$$



(i) Smallest prime value of x is 17

(ii) Smallest perfect cube value of x is $27 = 3^3$

Practise Now 6

1. (a)

$$\begin{aligned} 15x + 1 &< 5(3 + x) \\ 15x + 1 &< 15 + 5x \\ 15x + 1 - 5x &< 15 + 5x - 5x \\ 10x + 1 &< 15 \\ 10x + 1 - 1 &< 15 - 1 \\ 10x &< 14 \\ x &< \frac{14}{10} \\ x &< 1\frac{2}{5} \end{aligned}$$

(b)

$$\begin{aligned} \frac{16y}{3} &\geqslant \frac{y+1}{2} \\ 3 \times 2 \times \frac{16y}{3} &\geqslant 3 \times 2 \times \frac{y+1}{2} \\ 32y &\geqslant 3(y + 1) \\ 32y &\geqslant 3y + 3 \\ 32y - 3y &\geqslant 3y + 3 - 3y \\ 29y &\geqslant 3 \\ y &\geqslant \frac{3}{29} \end{aligned}$$

(c)

$$\begin{aligned} \frac{1}{2}(z - 4) &\leqslant \frac{1}{3}(z + 1) + 2 \\ 2 \times 3 \times \frac{1}{2}(z - 4) &\leqslant 2 \times 3 \times \left[\frac{1}{3}(z+1) + 2\right] \\ 3(z - 4) &\leqslant 2(z + 1) + 6(2) \\ 3z - 12 &\leqslant 2z + 2 + 12 \\ 3z - 12 &\leqslant 2z + 14 \\ 3z - 12 - 2z &\leqslant 2z + 14 - 2z \\ z - 12 &\leqslant 14 \\ z - 12 + 12 &\leqslant 14 + 12 \\ z &\leqslant 26 \end{aligned}$$

2.

$$\begin{aligned} \frac{3}{4}(p - 2) + \frac{1}{2} &> \frac{1}{2}(p - 1) \\ 4 \times \left[\frac{3}{4}(p-2) + \frac{1}{2}\right] &> 4 \times \frac{1}{2}(p - 1) \\ 3(p - 2) + 2 &> 2(p - 1) \\ 3p - 6 + 2 &> 2p - 2 \\ 3p - 4 &> 2p - 2 \\ 3p - 4 - 2p &> 2p - 2 - 2p \\ p - 4 &> -2 \\ p - 4 + 4 &> -2 + 4 \\ p &> 2 \end{aligned}$$

Smallest perfect square value of p is $4 = 2^2$

Practise Now 7

Let x be the marks scored by Seema in her first quiz.

$$\begin{aligned} \frac{x + 76 + 89}{3} &\geqslant 75 \\ 3 \times \frac{x + 76 + 89}{3} &\geqslant 3 \times 75 \\ x + 76 + 89 &\geqslant 225 \\ x + 165 &\geqslant 225 \\ x + 165 - 165 &\geqslant 225 - 165 \\ x &\geqslant 60 \end{aligned}$$

∴ Seema must have scored at least 60 marks for her first quiz.

Practise Now 8

Let x and y be the number of PKR 10 notes and PKR 5 notes respectively.

$x + y = 12$ — (1)

$x \times 10 + y \times 5 < 95$

i.e. $10x + 5y < 95$ — (2)

From (1),

$y = 12 - x$ — (3)

Substitute (3) into (2):

$$\begin{aligned} 10x + 5(12 - x) &< 95 \\ 10x + 60 - 5x &< 95 \\ 5x + 60 &< 95 \\ 5x + 60 - 60 &< 95 - 60 \\ 5x &< 35 \\ x &< \frac{35}{5} \\ x &< 7 \end{aligned}$$

∴ The maximum number of PKR 10 notes that Mishal has is 6.

Exercise 6A

1. (a) If $x > y$, then $5x > 5y$.

(b) If $x < y$, then $\frac{x}{20} < \frac{y}{20}$.

(c) If $x \geqslant y$, then $3x \geqslant 3y$.

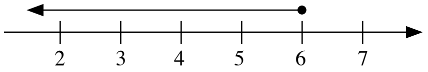
(d) If $x \leqslant y$, then $\frac{x}{10} \leqslant \frac{y}{10}$.

(e) If $15 > 5$ and $5 > x$, then $15 > x$.

(f) If $x < 50$ and $50 < y$, then $x < y$.

2. (a) $3x \leqslant 18$

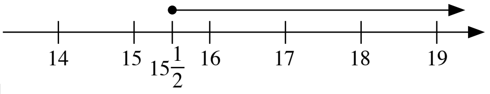
$x \leqslant \frac{18}{3}$



$\therefore x \leqslant 6$

(b) $4x \geqslant 62$

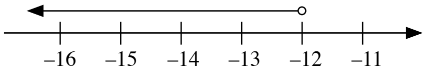
$x \geqslant \frac{62}{4}$



$\therefore x \geqslant 15\frac{1}{2}$

(c) $3y < -36$

$\therefore y < \frac{-36}{3}$

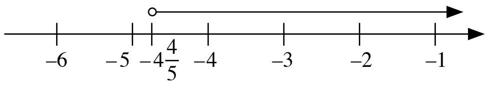


$y < -12$

(d) $5y > -24$

$y > \frac{-24}{5}$

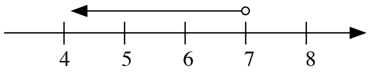
$\therefore y > -4\frac{4}{5}$



(e) $4x < 28$

$x < \frac{28}{4}$

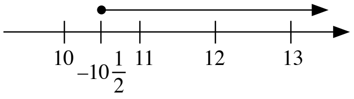
$\therefore x < 7$



(f) $12x \geqslant 126$

$x \geqslant \frac{126}{12}$

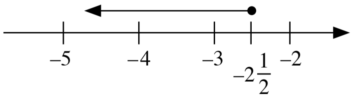
$\therefore x \geqslant 10\frac{1}{2}$



(g) $2y \leqslant -5$

$y \leqslant -\frac{5}{2}$

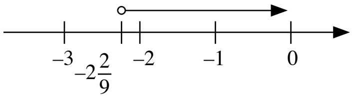
$y \leqslant -2\frac{1}{2}$



(h) $9y > -20$

$y > -\frac{20}{9}$

$\therefore y > -2\frac{2}{9}$



3. Let the number of vans that are needed to ferry 80 people be x.

Then $12x \geqslant 80$

$x \geqslant \frac{80}{12}$

$x \geqslant 6\frac{2}{3}$

$\therefore$ The minimum number of vans that are needed to ferry 80 people is 7.

4. $8 \leqslant 7y$

$7y \geqslant 8$

$y \geqslant \frac{8}{7}$

$y \geqslant 1\frac{1}{7}$

$\therefore$ The smallest rational value of y is $1\frac{1}{7}$.

5. $20x > 33$

$x > \frac{33}{20}$

$x > 1\frac{13}{20}$

$\therefore$ The smallest value of x if x is a prime number is 2.

6. $3x < -105$

$x < \frac{-105}{3}$

$x < -35$

$\therefore$ The greatest odd integer value of x is –37.

7. $5y < 20$ and $2y \geqslant -6$

$y < \frac{20}{5}$ $\quad$ $y \geqslant -\frac{6}{2}$

$y < 4$ $\quad$ $y \geqslant -3$

$\therefore$ The possible values are –3, –2, –1, 0, 1, 2 and 3.

Exercise 6B

1. **(a)** If $x > y$, then $-6x < -6y$

(b) If $x < y$, then $\frac{x}{-30} > \frac{y}{-30}$

(c) If $x \geqslant y$, then $-4x \leqslant -4y$

(d) If $x \leqslant y$, then $\frac{x}{-10} \geqslant \frac{y}{-10}$

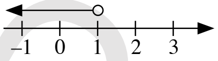
(e) $5 + h < 7 + h$

(f) $5 - k < 7 - k$

2. **(a)** $a + 2 < 3$

$a + 2 - 2 < 3 - 2$

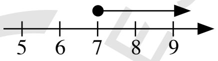
$a < 1$



(b) $b - 3 \geqslant 4$

$b - 3 + 3 \geqslant 4 + 3$

$b \geqslant 7$

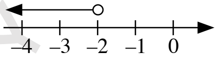


(c) $-c + 3 > 5$

$-c + 3 - 3 > 5 - 3$

$-c > 2$

$c < -2$

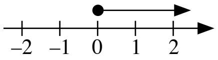


(d) $4 - d \leqslant 4$

$4 - d - 4 \leqslant 4 - 4$

$-d \leqslant 0$

$d \geqslant 0$



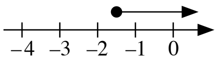
(e) $-2e - 1 \leqslant 2$

$-2e - 1 + 1 \leqslant 2 + 1$

$-2e \leqslant 3$

$e \geqslant \frac{3}{-2}$

$e \geqslant -1\frac{1}{2}$

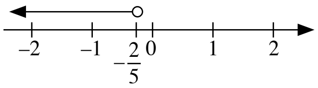


(f) $2 + 5f < 0$

$2 + 5f - 2 < 0 - 2$

$5f < -2$

$f < -\frac{2}{5}$



(g) $g - 7 \geqslant 1 - g$

$g - 7 + g \geqslant 1 - g + g$

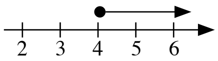
$2g - 7 \geqslant 1$

$2g - 7 + 7 \geqslant 1 + 7$

$2g \geqslant 8$

$g \geqslant \frac{8}{2}$

$g \geqslant 4$

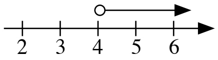


(h) $5h > 4(h + 1)$

$5h > 4h + 4$

$5h - 4h > 4h + 4 - 4h$

$h > 4$



(i) $8j + 3 < 2(7 - j)$

$8j + 3 < 14 - 2j$

$8j + 3 + 2j < 14 - 2j + 2j$

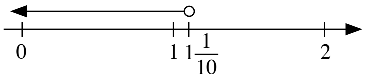
$10j + 3 < 14$

$10j + 3 - 3 < 14 - 3$

$10j < 11$

$j < \frac{11}{10}$

$j < 1\frac{1}{10}$



(j) $4k + 5 \geqslant 2(-2k)$

$4k + 5 \geqslant -4k$

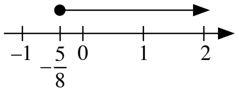
$4k + 5 + 4k \geqslant -4k + 4k$

$8k + 5 \geqslant 0$

$8k + 5 - 5 \geqslant 0 - 5$

$8k \geqslant -5$

$k \geqslant -\frac{5}{8}$



(k) $2(m - 5) \leqslant 2 - m$

$2m - 10 \leqslant 2 - m$

$2m - 10 + m \leqslant 2 - m + m$

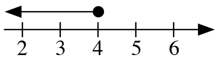
$3m - 10 \leqslant 2$

$3m - 10 + 10 \leqslant 2 + 10$

$3m \leqslant 12$

$m \leqslant \frac{12}{3}$

$m \leqslant 4$



(l) $3(1 - 4n) > 8 - 7n$

$3 - 12n > 8 - 7n$

$3 - 12n + 7n > 8 - 7n + 7n$

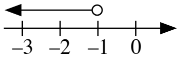
$3 - 5n > 8$

$3 - 5n - 3 > 8 - 3$

$-5n > 5$

$n < \frac{5}{-5}$

$n < -1$



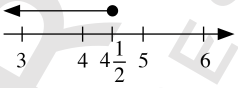
3. $7 + 2x \leqslant 16$

$7 + 2x - 7 \leqslant 16 - 7$

$2x \leqslant 9$

$x \leqslant \frac{9}{2}$

$x \leqslant 4\frac{1}{2}$



(i) Largest integer value of x is 4

(ii) Largest perfect square value of x is $4 = 2^2$

4. $3 - 4x > 3x - 18$

$3 - 4x - 3 > 3x - 18 - 3$

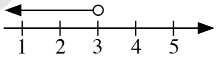
$-4x > 3x - 21$

$-4x - 3x > 3x - 21 - 3x$

$-7x > -21$

$x < \frac{-21}{-7}$

$x < 3$



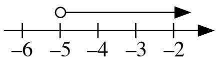
(i) Prime value of x is 2

(ii) Yes, $x = 0$ is less than 3.

5. **(a)** $-5x < 25$

$x > \frac{25}{-5}$

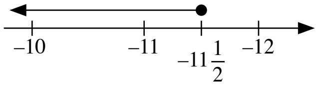
$x > -5$



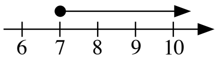
(b) $-12x \geqslant 138$

$x \leqslant \frac{138}{-12}$

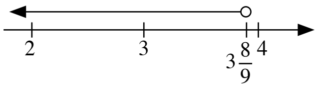
$x \leqslant -11\frac{1}{2}$



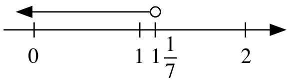
(c) $$\begin{aligned} -y &\leqslant -7 \\ y &\geqslant 7 \end{aligned}$$



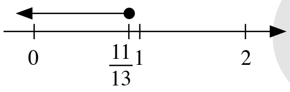
(d) $$\begin{aligned} -9y &> -35 \\ 9y &< 35 \\ y &< \frac{35}{9} \\ y &< 3\frac{8}{9} \end{aligned}$$



(e) $$\begin{aligned} 4(p+1) &< -3(p-4) \\ 4p+4 &< -3p+12 \\ 4p+4+3p &< -3p+12+3p \\ 7p+4 &< 12 \\ 7p+4-4 &< 12-4 \\ 7p &< 8 \\ p &< \frac{8}{7} \\ p &< 1\frac{1}{7} \end{aligned}$$



(f) $$\begin{aligned} 6-(1-2q) &\geqslant 3(5q-2) \\ 6-1+2q &\geqslant 15q-6 \\ 5+2q &\geqslant 15q-6 \\ 5+2q-15q &\geqslant 15q-6-15q \\ 5-13q &\geqslant -6 \\ 5-13q-5 &\geqslant -6-5 \\ -13q &\geqslant -11 \\ q &\leqslant \frac{-11}{-13} \\ q &\leqslant \frac{11}{13} \end{aligned}$$



6. **(a)** $$\begin{aligned} \frac{4a}{3} &\geqslant 2 \\ 3 \times \frac{4a}{3} &\geqslant 3 \times 2 \\ 4a &\geqslant 6 \\ a &\geqslant \frac{6}{4} \\ a &\geqslant 1\frac{1}{2} \end{aligned}$$

(b) $$\begin{aligned} \frac{2b-1}{3} &> \frac{3b}{5} \\ 3 \times 5 \times \frac{2b-1}{3} &> 3 \times 5 \times \frac{3b}{5} \\ 5(2b-1) &> 3(3b) \\ 10b-5 &> 9b \\ 10b-5-9b &> 9b-9b \\ b-5 &> 0 \\ b-5+5 &> 0+5 \\ b &> 5 \end{aligned}$$

(c) $$\begin{aligned} \frac{c+4}{4} &> \frac{c+1}{3} \\ 4 \times 3 \times \frac{c+4}{4} &> 4 \times 3 \times \frac{c+1}{3} \\ 3(c+4) &> 4(c+1) \\ 3c+12 &> 4c+4 \\ 3c+12-4c &> 4c+4-4c \\ -c+12 &> 4 \\ -c+12-12 &> 4-12 \\ -c &> -8 \\ c &< 8 \end{aligned}$$

(d) $$\begin{aligned} \frac{2-d}{2} &< \frac{3-d}{4} + \frac{1}{2} \\ 4 \times \frac{2-d}{2} &< 4 \times \left(\frac{3-d}{4} + \frac{1}{2}\right) \\ 2(2-d) &< (3-d)+2 \\ 4-2d &< 5-d \\ 4-2d+d &< 5-d+d \\ 4-d &< 5 \\ 4-d-4 &< 5-4 \\ -d &< 1 \\ d &> -1 \end{aligned}$$

(e) $$\begin{aligned} \frac{1}{4}(e-2) + \frac{2}{3} &< \frac{1}{6}(e-4) \\ 12 \times \left[\frac{1}{4}(e-2) + \frac{2}{3}\right] &< 12 \times \frac{1}{6}(e-4) \\ 3(e-2)+8 &< 2(e-4) \\ 3e-6+8 &< 2e-8 \\ 3e+2 &< 2e-8 \\ 3e+2-2e &< 2e-8-2e \\ e+2 &< -8 \\ e+2-2 &< -8-2 \\ e &< -10 \end{aligned}$$

(f)

$$\begin{aligned}
\frac{f+1}{2} + \frac{3f+1}{4} &\leqslant \frac{3f-1}{4} + 2\\
4 \times \left(\frac{f+1}{2}+\frac{3f+1}{4}\right) &\leqslant 4 \times \left(\frac{3f-1}{4}+2\right)\\
2(f+1)+(3f+1) &\leqslant (3f-1)+8\\
2f+2+3f+1 &\leqslant 3f-1+8\\
5f+3 &\leqslant 3f+7\\
5f+3-3f &\leqslant 3f+7-3f\\
2f+3-3 &\leqslant 7-3\\
2f &\leqslant 4\\
f &\leqslant \frac{4}{2}\\
f &\leqslant 2
\end{aligned}$$

(g)

$$\begin{aligned}
\frac{1}{5}(3g+4) - \frac{1}{3}(g+1) &\geqslant 1 - \frac{1}{3}(g+5)\\
5 \times 3 \times \left[\frac{1}{5}(3g+4) - \frac{1}{3}(g+1)\right] &\geqslant 5 \times 3 \times \left[1 - \frac{1}{3}(g+5)\right]\\
3(3g+4) - 5(g+1) &\geqslant 15 - 5(g+5)\\
9g+12-5g-5 &\geqslant 15-5g-25\\
4g+7 &\geqslant -10-5g\\
4g+7+5g &\geqslant -10-5g+5g\\
9g+7 &\geqslant -10\\
9g+7-7 &\geqslant -10-7\\
9g &\geqslant -17\\
g &\geqslant \frac{-17}{9}\\
g &\geqslant -1\frac{8}{9}
\end{aligned}$$

(h)

$$\begin{aligned}
4\left(\frac{h}{3}+\frac{3}{4}\right) &< 3\left(\frac{h}{2}-5\right)\\
\frac{4}{3}h + \frac{12}{4} &< \frac{3}{2}h - 15\\
\frac{4}{3}h + 3 - \frac{3}{2}h &< \frac{3}{2}h - 15 - \frac{3}{2}h\\
-\frac{1}{6}h + 3 &< -15\\
-\frac{1}{6}h + 3 - 3 &< -15 - 3\\
-\frac{1}{6}h &< -18\\
6 \times -\frac{1}{6}h &< 6 \times -18\\
-h &< -108\\
h &> 108
\end{aligned}$$

7.

$$\begin{aligned}
\frac{1}{6}(2-p) - 3 &\geqslant \frac{p}{10}\\
6 \times 10 \times \left[\frac{1}{6}(2-p) - 3\right] &\geqslant 6 \times 10 \times \frac{p}{10}\\
10(2-p) - 180 &\geqslant 6p\\
20 - 10p - 180 &\geqslant 6p\\
-10p - 160 &\geqslant 6p\\
-10p - 160 - 6p &\geqslant 6p - 6p\\
-16p - 160 &\geqslant 0\\
-16p - 160 + 160 &\geqslant 0 + 160\\
-16p &\geqslant 160\\
p &\leqslant \frac{160}{-16}\\
p &\leqslant -10
\end{aligned}$$

$\therefore$ The largest possible value of p is -10.

8. **(i)**

$$\begin{aligned}
\frac{1}{3}(2x-7) &\leqslant \frac{3x+2}{2}\\
3 \times 2 \times \frac{1}{3}(2x-7) &\leqslant 3 \times 2 \times \frac{3x+2}{2}\\
2(2x-7) &\leqslant 3(3x+2)\\
4x - 14 &\leqslant 9x + 6\\
4x - 14 - 9x &\leqslant 9x + 6 - 9x\\
-5x - 14 &\leqslant 6\\
-5x - 14 + 14 &\leqslant 6 + 14\\
-5x &\leqslant 20\\
x &\geqslant \frac{20}{-5}\\
x &\geqslant -4
\end{aligned}$$

(ii) Smallest value of x^2 is $(0)^2 = 0$

Review Exercise 6

1. **(a)** $18x < -25$

$$x < \frac{-25}{18}$$

$$\therefore x < -1\frac{7}{18}$$

(b) $10y \geqslant -24$

$$y \geqslant -\frac{24}{10}$$

$$\therefore y \geqslant -2\frac{2}{5}$$

2. $4x \geqslant 11$

$$x \geqslant \frac{11}{4}$$

$$x \geqslant 2\frac{3}{4}$$

$\therefore$ The smallest integer value of x is 3.

3. $3y < -24$

$$y < \frac{-24}{3}$$

$$y < -8$$

$\therefore$ The greatest integer value of y is -9.

4. $5x < 125$

$x < \dfrac{125}{5}$

$x < 25$

$\therefore$ The greatest value of x if x is divisible by 12 is 24.

5. $5y \geqslant 84$

$y \geqslant \dfrac{84}{5}$

$y \geqslant 16\dfrac{4}{5}$

$\therefore$ The smallest value of y if y is a prime number is 17.

6. Let the number of watches that can be bought with PKR 35 000 be x.

Then $1900x \leqslant 35\,000$

$x \leqslant \dfrac{35\,000}{1900}$

$x \leqslant 18\dfrac{8}{19}$

$\therefore$ The maximum number of watches that can be bought with PKR 35 000 is 18.

7. Let the number of minutes Jamil can buy be x.

Then $12.50x \leqslant 250$

$x \leqslant \dfrac{250}{12.5}$

$x \leqslant 20$

$\therefore$ The maximum number of minutes Jamil can buy with PKR 250 is 20.

8. Let the first integer be x.

Then the second integer will be $(x + 1)$.

$x + (x + 1) < 42$

$2x < 41$

$x < \dfrac{41}{2}$

$x < 20.5$

$\therefore$ The largest possible integer x can be is 20.

$20 + 1 = 21$

$21^2 = 441$

$\therefore$ The square of largest possible integer is 441.

9. Let Nadia's age be x years.

Then Kiran's age is $(x - 4)$ years.

$x + (x - 4) \leqslant 45$

$2x \leqslant 45 + 4$

$x \leqslant \dfrac{49}{2}$

$x \leqslant 24.5$

$\therefore$ Maximum possible age of Nadia is 24 years.

$24 - 4 = 20$

$\therefore$ The maximum possible age of Kiran is 20 years.

10. Let the number of ships needed to carry 400 passengers be .

$60x \geqslant 400$

$x \geqslant \dfrac{400}{60}$

$x \geqslant 6\dfrac{2}{3}$

$\therefore$ The minimum number of ships needed to carry 400 passengers is 7.

11. Let the number of pencils that can be bought with PKR 27 be x.

$2.50x \leqslant 27$

$x \leqslant \dfrac{27}{2.5}$

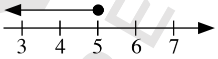
$x \leqslant 10\dfrac{4}{5}$

$\therefore$ The maximum number of pencils that can be bought with PKR 27 is 10.

12. (a) $a - 2 \leqslant 3$

$a - 2 + 2 \leqslant 3 + 2$

$a \leqslant 5$



(b) $2b + 1 < 5 - 4b$

$2b + 1 + 4b < 5 - 4b + 4b$

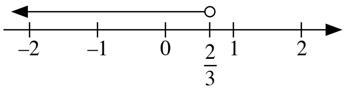
$6b + 1 < 5$

$6b + 1 - 1 < 5 - 1$

$6b < 4$

$b < \dfrac{4}{6}$

$b < \dfrac{2}{3}$



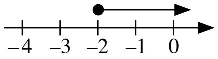
(c) $c \geqslant \dfrac{1}{2}c - 1$

$c - \dfrac{1}{2}c \geqslant \dfrac{1}{2}c - 1 - \dfrac{1}{2}c$

$\dfrac{1}{2}c \geqslant -1$

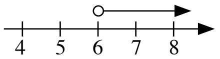
$2 \times \dfrac{1}{2}c \geqslant 2 \times -1$

$c \geqslant -2$



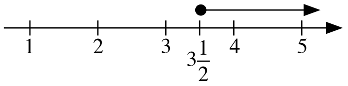
(d)

$$\begin{aligned}\frac{1}{2}d &> 1+\frac{1}{3}d\\ \frac{1}{2}d-\frac{1}{3}d &> 1+\frac{1}{3}d-\frac{1}{3}d\\ \frac{1}{6}d &> 1\\ 6\times\frac{1}{6}d &> 6\times 1\\ d &> 6\end{aligned}$$



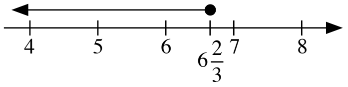
(e)

$$\begin{aligned}2(e-3) &\geqslant 1\\ 2e-6 &\geqslant 1\\ 2e-6+6 &\geqslant 1+6\\ 2e &\geqslant 7\\ e &\geqslant \frac{7}{2}\\ e &\geqslant 3\frac{1}{2}\end{aligned}$$



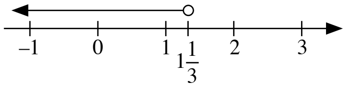
(f)

$$\begin{aligned}5(f-4) &\leqslant 2f\\ 5f-20 &\leqslant 2f\\ 5f-20-2f &\leqslant 2f-2f\\ 3f-20 &\leqslant 0\\ 3f-20+20 &\leqslant 0+20\\ 3f &\leqslant 20\\ f &\leqslant \frac{20}{3}\\ f &\leqslant 6\frac{2}{3}\end{aligned}$$



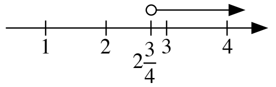
(g)

$$\begin{aligned}-3-g &> 2g-7\\ -3-g-2g &> 2g-7-2g\\ -3-3g &> -7\\ -3-3g+3 &> -7+3\\ -3g &> -4\\ g &< \frac{-4}{-3}\\ g &< 1\frac{1}{3}\end{aligned}$$



(h)

$$\begin{aligned}18-3h &< 5h-4\\ 18-3h-5h &< 5h-4-5h\\ 18-8h &< -4\\ 18-8h-18 &< -4-18\\ -8h &< -22\\ h &> \frac{-22}{-8}\\ h &> 2\frac{3}{4}\end{aligned}$$



13. (a)

$$\begin{aligned}3+\frac{a}{4} &> 5+\frac{a}{3}\\ 4\times 3\times\left(3+\frac{a}{4}\right) &> 4\times 3\times\left(5+\frac{a}{3}\right)\\ 36+3a &> 60+4a\\ 36+3a-4a &> 60+4a-4a\\ 36-a &> 60\\ 36-a-36 &> 60-36\\ -a &> 24\\ a &< -24\end{aligned}$$

(b)

$$\begin{aligned}\frac{4b}{9}-5 &< 3-\frac{2b}{3}\\ 9\times\left(\frac{4b}{9}-5\right) &< 9\times\left(3-\frac{2b}{3}\right)\\ 4b-45 &< 27-6b\\ 4b-45+6b &< 27-6b+6b\\ 10b-45 &< 27\\ 10b-45+45 &< 27+45\\ 10b &< 72\\ b &< 7\frac{1}{5}\end{aligned}$$

(c)

$$\begin{aligned}\frac{4c}{9}-\frac{3}{4} &\geqslant c-\frac{1}{2}\\ 9\times 4\times\left(\frac{4c}{9}-\frac{3}{4}\right) &\geqslant 9\times 4\times\left(c-\frac{1}{2}\right)\\ 16c-27 &\geqslant 36c-18\\ 16c-27-36c &\geqslant 36c-18-36c\\ -20c-27 &\geqslant -18\\ -20c-27+27 &\geqslant -18+27\\ -20c &\geqslant 9\\ c &\leqslant -\frac{9}{20}\end{aligned}$$

(d)

$$\begin{aligned}
\frac{d-2}{3} &< \frac{2d+3}{5} + \frac{5}{8}\\
\frac{d-2}{3} - \frac{2d+3}{5} &< \frac{5}{8}\\
3 \times 5 \times \left(\frac{d-2}{3} - \frac{2d+3}{5}\right) &< 3 \times 5 \times \frac{5}{8}\\
5(d-2) - 3(2d+3) &< \frac{75}{8}\\
5d - 10 - 6d - 9 &< \frac{75}{8}\\
-d - 19 &< \frac{75}{8}\\
-d - 19 + 19 &< \frac{75}{8} + 19\\
-d &< 28\frac{3}{8}\\
d &> -28\frac{3}{8}
\end{aligned}$$

(e)

$$\begin{aligned}
\frac{1}{3}(e+2) &\geqslant \frac{2}{3} + \frac{1}{4}(e-1)\\
\frac{1}{3}(e+2) - \frac{1}{4}(e-1) &\geqslant \frac{2}{3}\\
3 \times 4 \times \left[\frac{1}{3}(e+2) - \frac{1}{4}(e-1)\right] &\geqslant 3 \times 4 \times \frac{2}{3}\\
4(e+2) - 3(e-1) &\geqslant 8\\
4e + 8 - 3e + 3 &\geqslant 8\\
e + 11 &\geqslant 8\\
e + 11 - 11 &\geqslant 8 - 11\\
e &\geqslant -3
\end{aligned}$$

(f)

$$\begin{aligned}
5 - \frac{2f-5}{6} &\leqslant \frac{f+3}{2} + \frac{2(f+1)}{3}\\
6 \times \left(5 - \frac{2f-5}{6}\right) &\leqslant 6 \times \left[\frac{f+3}{2} + \frac{2(f+1)}{3}\right]\\
30 - (2f-5) &\leqslant 3(f+3) + 4(f+1)\\
30 - 2f + 5 &\leqslant 3f + 9 + 4f + 4\\
35 - 2f &\leqslant 7f + 13\\
35 - 2f - 7f &\leqslant 7f + 13 - 7f\\
35 - 9f &\leqslant 13\\
35 - 9f - 35 &\leqslant 13 - 35\\
-9f &\leqslant -22\\
f &\geqslant \frac{-22}{-9}\\
f &\geqslant 2\frac{4}{9}
\end{aligned}$$

Challenge Yourself

1.

$$\begin{aligned}
\sqrt{x} + 2 &= 0\\
\sqrt{x} &= -2
\end{aligned}$$

There is no solution since $\sqrt{x}$ cannot be a negative number.

2. Since $(x+2)^2$ and $(y-3)^2$ cannot be negative.

$$\begin{aligned}
(x+2)^2 &= 0 &&\text{and} & (y-3)^2 &= 0\\
x+2 &= 0 &&\text{and} & y-3 &= 0\\
x &= -2 &&\text{and} & y &= 3
\end{aligned}$$

$$\begin{aligned}
\therefore x + y &= -2 + 3\\
&= 1
\end{aligned}$$

3.

$A + B = 8$ — (1)

$B + C = 11$ — (2)

$B + D = 13$ — (3)

$C + D = 14$ — (4)

$(2) - (3)$: $B + C - B - D = 11 - 13$

$C - D = -2$ — (5)

$(4) + (5)$: $C + D + C - D = 14 + (-2)$

$$\begin{aligned}
2C &= 12\\
\therefore C &= \frac{12}{2}\\
&= 6
\end{aligned}$$

Substitute $C = 6$ into (4): $6 + D = 14$

$$\begin{aligned}
\therefore D &= 14 - 6\\
&= 8
\end{aligned}$$

Substitute $C = 6$ into (2): $B + 6 = 11$

$$\begin{aligned}
\therefore B &= 11 - 6\\
&= 5
\end{aligned}$$

Substitute $B = 5$ into (1): $A + 5 = 8$

$$\begin{aligned}
\therefore A &= 8 - 5\\
&= 3
\end{aligned}$$

4.

$A \times B = 8$ — (1)

$B \times C = 28$ — (2)

$C \times D = 63$ — (3)

$B \times D = 36$ — (4)

$(2) \div (3)$: $\dfrac{B \times C}{C \times D} = \dfrac{28}{63}$

Since C cannot be equal to 0, then $\dfrac{B}{D} = \dfrac{4}{9}$, — (5)

$(4) \times (5)$: $B \times D \times \dfrac{B}{D} = 36 \times \dfrac{4}{9}$

Since D cannot be equal to 0, then $B^2 = 16$.

$$\begin{aligned}
\therefore B &= \pm\sqrt{16}\\
&= 4 \text{ or } -4 \text{ (N.A. since } B > 0)
\end{aligned}$$

Substitute $B = 4$ into (1): $A \times 4 = 8$

$$\begin{aligned}
\therefore A &= \frac{8}{4}\\
&= 2
\end{aligned}$$

Substitute $B = 4$ into (2): $4 \times C = 28$

$$\begin{aligned}
\therefore C &= \frac{28}{4}\\
&= 7
\end{aligned}$$

Substitute $B = 4$ into (4): $4 \times D = 36$

$$\begin{aligned}
\therefore D &= \frac{36}{4}\\
&= 9
\end{aligned}$$

Chapter 7 Pythagoras’ Theorem

TEACHING NOTES

Suggested Approach

There are many ways of proving the Pythagoras’ Theorem. An unofficial tally shows more than 300 ways of doing this. Teachers may use this opportunity to ask students to do a project of finding the best or the easiest method of doing this and get the students to present them to their class (see Performance Task on page 205).

Students should be able to easily recall the previous lesson on similar triangles and apply their understanding in this chapter.

Section 7.1: Pythagoras’ Theorem

Students are expected to know that the longest side of a right-angled triangle is known as the hypotenuse. The condition that the triangle must be a right-angled triangle has to be highlighted.

Teachers may wish to prove the Pythagoras’ Theorem by showing the activity on the pages 203 and 204 (see Investigation: Pythagoras’ Theorem – The Secret of the Rope-Stretchers). Again, it is important to state the theorem applies only to right-angled triangles. The theorem does not hold for other types of triangles.

Section 7.2: Applications of Pythagoras’ Theorem in Real-World Contexts

There are many real-life applications of Pythagoras’ Theorem which the teachers can show to students. The worked examples and exercises should be more than enough for students to appreciate how the theorem is frequently present in real-life. Teachers should always remind students to check before applying the theorem, that the triangle is a right-angled triangle and that the longest side refers to the hypotenuse.

Section 7.3: Converse of Pythagoras’ Theorem

Worked Example 8 provides an example of the converse of Pythagoras’ Theorem. Some students should find the converse of the theorem easily manageable while teachers should take note of students who may have difficulty in following. Students should be guided of the importance of giving reasons to justify their answers.

Challenge Yourself

Question 2 requires the arrangement of 3 right-angled triangles such that their hypotenuses form another triangle. Students should be able to do the rest of the questions if they have understood Pythagoras’ Theorem.

WORKED SOLUTIONS

Investigation (Pythagoras' Theorem – The Secret of the Rope-Stretchers)

Part l:

In all 3 triangles, AB is the hypotenuse.

1, 2, 3, 4.

	BC	AC	AB	BC^2	AC^2	AB^2	$BC^2 + AC^2$
(a)	3 cm	4 cm	5 cm	9 cm^2	16 cm^2	25 cm^2	25 cm^2
(b)	6 cm	8 cm	10 cm	36 cm^2	64 cm^2	100 cm^2	100 cm^2
(c)	5 cm	12 cm	13 cm	25 cm^2	144 cm^2	169 cm^2	169 cm^2

Table 7.1

The value of AB^2 in table 8.1 is the same as the value of $BC^2 + AC^2$.

Part II:

5. In $\triangle ABC$, AB is the hypotenuse.

6. *Any 6 sets of values of BC, AC and AB can be used. Teachers may wish to have students attempt to get integer values for all 3 sides of the triangle.*

7. The value of AB^2 in table 7.2 is the same as the value of $BC^2 + AC^2$.

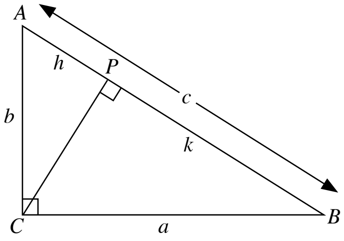
Performance Task (Page 205)

Even though Pythagoras' Theorem was long known years before Pythagoras' time, the theorem was credited to him as he was widely believed to be the first to provide a proof of it, which is shown in Fig. 8.5.

The Babylonians knew about the theorem by the Pythagorean triplets stated found in their remaining text that survived till this day. The Indians were able to list down the Pythagorean triplets, along with a geometrical proof of the Pythagoras' Theorem for a regular right-angled triangle.

The Chinese stated the theorem as the 'Gougu theorem' listed in the Chinese text '*Zhou Bi Suan Jing'* published around the first century B.C. It was also known alternatively as '*Shang Gao Theorem'*, after the Duke of Zhou's astronomer and mathematician, and where the reasoning of Pythagoras' Theorem in '*Zhou Bi Suan Jing'* came from. Some proofs of Pythagoras' Theorem are as follows.

Proof 1: (Using Similar Triangles)



$\angle ACB = \angle BPC = \angle APC = 90°$

Since $\triangle ACB$ is similar to $\triangle APC$,

$$\frac{AB}{AC} = \frac{AC}{AP}$$

i.e. $\frac{c}{b} = \frac{b}{h}$

$b^2 = ch$ — (1)

Since $\triangle ACB$ is similar to $\triangle CPB$,

$$\frac{AB}{CB} = \frac{CB}{PB}$$

i.e. $\frac{c}{a} = \frac{a}{k}$

$a^2 = ck$ — (2)

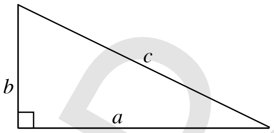
$(1) + (2)$: $b^2 + a^2 = ch + ck$

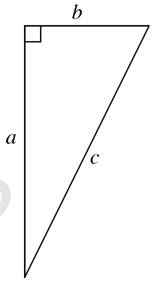
$= c(h + k)$

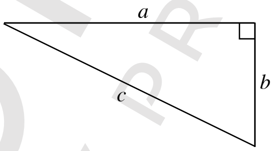
$= c^2$

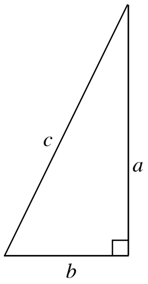
$\therefore a^2 + b^2 = c^2$

Proof 2: (Using four right-angled triangles)

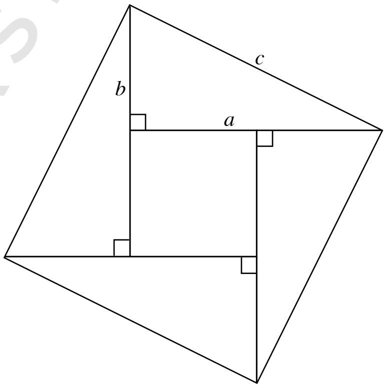








We can arrange the four triangles to form the following diagram.



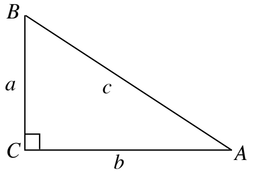
The diagram is a large square of length c units, with a smaller square of length $(a – b)$ units.

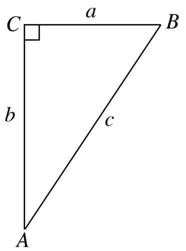
$\therefore$ Area of large square $= 4 \times$ area of a triangle + area of small square

$$c^2 = 4 \times \left(\frac{1}{2} \times a \times b\right) + (a – b)^2$$
$$= 2ab + a^2 – 2ab + b^2$$
$$= a^2 + b^2$$

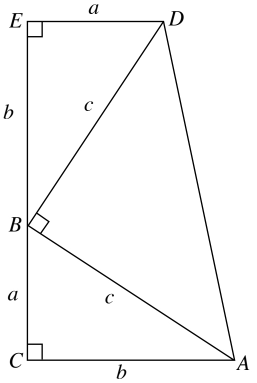
$\therefore a^2 + b^2 = c^2$

Proof 3: (Using a trapezium)





By rotating $\triangle ABC$ 90° clockwise, and placing the second triangle on top of the first one, we can get the following trapezium.



First, we show that $\angle ABD = 90°$.

$\angle ABC + \angle BAC = 180° - 90° = 90°$ (sum of $\angle$ in $\triangle ABC$)

$\angle BDE + \angle DBE = 180° - 90° = 90°$ (sum of $\angle$ in $\triangle BDE$)

Since $\angle BAC = \angle DBE$,

$\angle ABC + \angle DBE = 90°$

$\therefore \angle ABD = 180° - 90°$ (adj. $\angle$s on a str. line)

$= 90°$

Area of trapezium = 2 × Area of $\triangle ABC$ + Area of $\triangle ABD$

$$\begin{aligned}\frac{1}{2} \times (a+b) \times (a+b) &= 2 \times \left(\frac{1}{2} \times a \times b\right) + \left(\frac{1}{2} \times c \times c\right)\\ \frac{1}{2}(a+b)^2 &= ab + \frac{1}{2}c^2\\ (a+b)^2 &= 2ab + c^2\\ c^2 &= (a+b)^2 - 2ab\\ &= a^2 + 2ab + b^2 - 2ab\\ &= a^2 + b^2\end{aligned}$$

$\therefore a^2 + b^2 = c^2$

Practise Now 1

(a) AB is the hypotenuse.

(b) DE is the hypotenuse.

(c) PQ is the hypotenuse.

Practise Now 2

1. In $\triangle ABC$, $\angle C = 90°$.

Using Pythagoras' Theorem,

$$\begin{aligned}AB^2 &= BC^2 + AC^2\\ &= 8^2 + 6^2\\ &= 64 + 36\\ &= 100\end{aligned}$$

$\therefore AB = \sqrt{100}$ (since $AB > 0$)

$= 10$ cm

2. In $\triangle ABC$, $\angle C = 90°$.

Using Pythagoras' Theorem,

$$\begin{aligned}AB^2 &= BC^2 + AC^2\\ &= 7^2 + 24^2\\ &= 49 + 576\\ &= 625\end{aligned}$$

$\therefore AB = \sqrt{625}$ (since $AB > 0$)

$= 25$ cm

Practise Now 3

1. In $\triangle PQR$, $\angle R = 90°$.

Using Pythagoras' Theorem,

$$\begin{aligned}PQ^2 &= QR^2 + PR^2\\ 15^2 &= 12^2 + PR^2\\ PR^2 &= 15^2 - 12^2\\ &= 225 - 144\\ &= 81\end{aligned}$$

$\therefore PR = \sqrt{81}$ (since $PR > 0$)

$= 9$ m

2. In $\triangle PQR$, $\angle R = 90°$.

Using Pythagoras' Theorem,

$$\begin{aligned}PQ^2 &= QR^2 + PR^2\\ 35^2 &= QR^2 + 28^2\\ QR^2 &= 35^2 - 28^2\\ &= 1225 - 784\\ &= 441\end{aligned}$$

$\therefore QR = \sqrt{441}$ (since $QR > 0$)

$= 21$ cm

Practise Now 4

1. **(i)** In $\triangle ABQ$, $\angle B = 90°$.

Using Pythagoras' Theorem,

$$\begin{aligned}AQ^2 &= BQ^2 + AB^2\\ 5^2 &= BQ^2 + 3^2\\ BQ^2 &= 5^2 - 3^2\\ &= 25 - 9\\ &= 16\end{aligned}$$

$\therefore BQ = \sqrt{16}$ (since $PR > 0$)

$= 4$ cm

(ii) In $\triangle ABC$, $\angle B = 90°$.

Using Pythagoras' Theorem,

$$\begin{aligned} AC^2 &= CB^2 + AB^2 \\ &= (4 + 4)^2 + 3^2 \\ &= 8^2 + 3^2 \\ &= 64 + 9 \\ &= 73 \end{aligned}$$

$$\therefore AC = \sqrt{73} \text{ (since } AC > 0)$$
$$= 8.54 \text{ cm (to 3 s.f.)}$$

2. **(i)** In $\triangle GHI$, $\angle I = 90°$.

Using Pythagoras' Theorem,

$$\begin{aligned} GH^2 &= HI^2 + GI^2 \\ 61^2 &= HI^2 + 11^2 \\ HI^2 &= 61^2 - 11^2 \\ &= 3721 - 121 \\ &= 3600 \end{aligned}$$

$$\therefore HI = \sqrt{3600} \text{ (since } HI > 0)$$
$$= 60 \text{ cm}$$

(ii) In $\triangle GRI$, $\angle I = 90°$.

Using Pythagoras' Theorem,

$$\begin{aligned} GR^2 &= RI^2 + GI^2 \\ &= (60 - 21)^2 + 11^2 \\ &= 39^2 + 11^2 \\ &= 1521 + 121 \\ &= 1642 \end{aligned}$$

$$\therefore GR = \sqrt{1642} \text{ (since } GR > 0)$$
$$= 40.5 \text{ cm (to 3 s.f.)}$$

3. **(i)** In $\triangle HKR$, $\angle R = 90°$.

Using Pythagoras' Theorem,

$$\begin{aligned} HK^2 &= KR^2 + HR^2 \\ 19^2 &= 13^2 + HR^2 \\ HR^2 &= 19^2 - 13^2 \\ &= 361 - 169 \\ &= 192 \end{aligned}$$

$$\therefore HR = \sqrt{192} \text{ (since } HR > 0)$$
$$= 13.9 \text{ cm}$$

(ii) In $\triangle PQR$, $\angle R = 90°$.

Using Pythagoras' Theorem,

$$\begin{aligned} PQ^2 &= QR^2 + PR^2 \\ 33^2 &= (QK + 13)^2 + (6 + 13.86)^2 \\ &= (QK + 13)^2 + 19.86^2 \\ (QK + 13)^2 &= 33^2 - 19.86^2 \\ &= 694.58 \end{aligned}$$

$$\therefore QK + 13 = \pm\sqrt{694.58}$$

$$QK = -13 \pm \sqrt{694.58}$$

$QK = 13.4$ cm (to 3 s.f.) or $QK = -39.4$ cm (to 3 s.f.) (rejected, since $QK > 0$)

Practise Now 5

1. Let the length of the cable be x m.

Using Pythagoras' Theorem,

$$\begin{aligned} x^2 &= 24^2 + 14^2 \\ &= 576 + 196 \\ &= 772 \end{aligned}$$

$$\therefore x = \sqrt{772} \text{ (since } x > 0)$$
$$= 27.8 \text{ (to 3 s.f.)}$$

The cable is 27.8 m.

2. Let the vertical height the ladder reached be x m.

Using Pythagoras' Theorem,

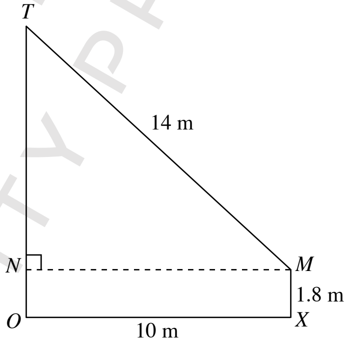
$$\begin{aligned} 2.5^2 &= x^2 + 1.5^2 \\ x^2 &= 2.5^2 - 1.5^2 \\ &= 6.25 - 2.25 \\ &= 4 \end{aligned}$$

$$\therefore x = \sqrt{4} \text{ (since } x > 0)$$
$$= 2$$

The ladder reaches 2 m up the wall.

Practise Now 6

1.



Let the height of the tree be OT.

In $\triangle TNM$, $\angle N = 90°$

Using Pythagoras' Theorem,

$$\begin{aligned} TM^2 &= MN^2 + TN^2 \\ 14^2 &= 10^2 + TN^2 \\ TN^2 &= 14^2 - 10^2 \\ &= 196 - 100 \\ &= 96 \end{aligned}$$

$$\therefore TN = \sqrt{96} \text{ (since } TN > 0)$$
$$= 9.798 \text{ m (to 4 s.f.)}$$

$$\therefore OT = 9.798 + 1.8$$
$$= 11.6 \text{ m (to 3 s.f.)}$$

The height of the tree is 11.6 m.

Practise Now 7

1. In $\triangle ABD$, $\angle A = 90°$.

Using Pythagoras' Theorem,

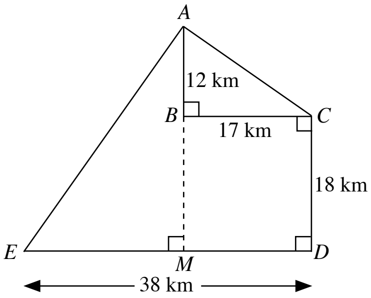
$$\begin{aligned} BD^2 &= DA^2 + BA^2 \\ (2x + 18)^2 &= x^2 + (2x + 12)^2 \\ 4x^2 + 72x + 324 &= x^2 + 4x^2 + 48x + 144 \\ x^2 - 24x - 180 &= 0 \\ (x - 30)(x + 6) &= 0 \\ x = 30 \quad &\text{or} \quad x = -6 \end{aligned}$$

$\therefore x = 30$ (since $x > 0$)

Practise Now 8

(i) $AB = 10 \times 1.2 = 12$ km

$BC = 10 \times 1.7 = 17$ km



In $\triangle ABC$, $\angle B = 90°$.

Using Pythagoras' Theorem,

$$\begin{aligned} AC^2 &= CB^2 + AB^2 \\ &= 17^2 + 12^2 \\ &= 289 + 144 \\ &= 433 \end{aligned}$$

$$\begin{aligned} \therefore AC &= \sqrt{433} \text{ (since } AC > 0) \\ &= 20.8 \text{ km (to 3 s.f.)} \end{aligned}$$

The shortest distance between Port A and Jetty C is 20.8 km.

(ii) Draw a perpendicular line from B to DE cutting DE at M.

In $\triangle AEM$, $\angle M = 90°$.

$$\begin{aligned} AM &= 12 + 18 \\ &= 30 \text{ km} \end{aligned}$$

$$\begin{aligned} EM &= 38 - 17 \\ &= 21 \text{ km} \end{aligned}$$

Using Pythagoras' Theorem,

$$\begin{aligned} AE^2 &= EM^2 + AM^2 \\ &= 21^2 + 30^2 \\ &= 441 + 900 \\ &= 1341 \end{aligned}$$

$$\begin{aligned} \therefore AE &= \sqrt{1341} \text{ (since } AE > 0) \\ &= 36.6 \text{ km (to 3 s.f.)} \end{aligned}$$

The shortest distance between Port A and Island E is 36.6 km.

Practise Now 9

1. (a) AB is the longest side of $\triangle ABC$.

$$\begin{aligned} AB^2 &= 12^2 \\ &= 144 \end{aligned}$$

$$\begin{aligned} BC^2 + AC^2 &= 10^2 + 8^2 \\ &= 100 + 64 \\ &= 164 \end{aligned}$$

Since $AB^2 \neq BC^2 + AC^2$, $\triangle ABC$ is not a right-angled triangle.

(b) PQ is the longest side of $\triangle PQR$.

$$\begin{aligned} PQ^2 &= 34^2 \\ &= 1156 \end{aligned}$$

$$\begin{aligned} QR^2 + PR^2 &= 16^2 + 30^2 \\ &= 256 + 900 \\ &= 1156 \end{aligned}$$

Since $PQ^2 = QR^2 + PR^2$, $\triangle PQR$ is a right-angled triangle where $\angle R = 90°$.

2. (i) XZ is the longest side in $\triangle XYZ$.

$$\begin{aligned} XZ^2 &= 51^2 \\ &= 2601 \end{aligned}$$

$$\begin{aligned} XY^2 + YZ^2 &= 45^2 + 24^2 \\ &= 2025 + 576 \\ &= 2601 \end{aligned}$$

Since $XZ^2 = XY^2 + YZ^2$, $\triangle XYZ$ is a right-angled triangle where $\angle XYZ = 90°$.

(ii) In XYT, $\angle Y = 90°$

Using Pythagoras' Theorem,

$$\begin{aligned} TX^2 &= XY^2 + TY^2 \\ &= 45^2 + (24 - 14)^2 \\ &= 45^2 + 10^2 \\ &= 2025 + 100 \\ &= 2125 \end{aligned}$$

$$\begin{aligned} \therefore TX &= \sqrt{2125} \text{ (since } TX > 0) \\ &= 46.1 \text{ m (to 3 s.f.)} \end{aligned}$$

The distance of the tree from X is 46.1 m.

Exercise 7A

1. (a) Using Pythagoras' Theorem,

$$\begin{aligned} a^2 &= 20^2 + 21^2 \\ &= 400 + 441 \\ &= 841 \end{aligned}$$

$$\begin{aligned} \therefore a &= \sqrt{841} \text{ (since } a > 0) \\ &= 29 \end{aligned}$$

(b) Using Pythagoras' Theorem,

$$\begin{aligned} b^2 &= 12^2 + 35^2 \\ &= 144 + 1225 \\ &= 1369 \end{aligned}$$

$$\begin{aligned} \therefore b &= \sqrt{1369} \text{ (since } b > 0) \\ &= 37 \end{aligned}$$

(c) Using Pythagoras' Theorem,

$c^2 = 10^2 + 12^2$
$= 100 + 144$
$= 244$

$\therefore c = \sqrt{244}$ (since $c > 0$)
$= 15.6$ (to 3 s.f.)

(d) Using Pythagoras' Theorem,

$d^2 = 23^2 + 29^2$
$= 529 + 841$
$= 1370$

$\therefore d = \sqrt{1370}$ (since $d > 0$)
$= 37.0$ (to 3 s.f.)

2. **(a)** Using Pythagoras' Theorem,

$39^2 = a^2 + 15^2$
$a^2 = 39^2 - 15^2$
$= 1521 - 225$
$= 1296$

$\therefore a = \sqrt{1296}$ (since $a > 0$)
$= 36$

(b) Using Pythagoras' Theorem,

$19^2 = b^2 + 14^2$
$b^2 = 19^2 - 14^2$
$= 361 - 196$
$= 165$

$\therefore b = \sqrt{165}$ (since $b > 0$)
$= 12.8$ (to 3 s.f.)

(c) Using Pythagoras' Theorem,

$9.8^2 = c^2 + 6.5^2$
$c^2 = 9.8^2 - 6.5^2$
$= 96.04 - 42.25$
$= 53.79$

$\therefore c = \sqrt{53.79}$ (since $c > 0$)
$= 7.33$ (to 3 s.f.)

(d) Using Pythagoras' Theorem,

$24.7^2 = d^2 + 14.5^2$
$d^2 = 24.7^2 - 14.5^2$
$= 610.09 - 210.25$
$= 399.84$

$\therefore d = \sqrt{399.84}$ (since $d > 0$)
$= 20.0$ (to 3 s.f.)

3. In $\triangle ABC$, $\angle B = 90°$.

Using Pythagoras' Theorem,

$AC^2 = AB^2 + BC^2$
$= 8^2 + 15^2$
$= 64 + 225$
$= 289$

$\therefore AC = \sqrt{289}$ (since $AC > 0$)
$= 17$ cm

4. In $\triangle DEF$, $\angle E = 90°$.

Using Pythagoras' Theorem,

$DF^2 = EF^2 + DE^2$
$= 5.5^2 + 6.7^2$
$= 30.25 + 44.89$
$= 75.14$

$\therefore DF = \sqrt{75.14}$ (since $DF > 0$)
$= 8.67$ m (to 3 s.f.)

5. In $\triangle GHI$, $\angle H = 90°$.

Using Pythagoras' Theorem,

$GI^2 = HI^2 + GH^2$
$65^2 = HI^2 + 33^2$
$HI^2 = 65^2 - 33^2$
$= 4225 - 1089$
$= 3136$

$\therefore HI = \sqrt{3136}$ (since $HI > 0$)
$= 56$ cm

6. In $\triangle MNO$, $\angle N = 90°$.

Using Pythagoras' Theorem,

$MO^2 = MN^2 + NO^2$
$14.2^2 = MN^2 + 11^2$
$MN^2 = 14.2^2 - 11^2$
$= 201.64 - 121$
$= 80.64$

$\therefore MN = \sqrt{80.64}$ (since $MN > 0$)
$= 8.98$ cm (to 3 s.f.)

7. **(i)** In $\triangle PQS$, $\angle Q = 90°$.

Using Pythagoras' Theorem,

$PS^2 = PQ^2 + QS^2$
$53^2 = 45^2 + QS^2$
$QS^2 = 53^2 - 45^2$
$= 2809 - 2025$
$= 784$

$\therefore QS = \sqrt{784}$ (since $QS > 0$)
$= 28$ cm

(ii) In $\triangle QRS$, $\angle S = 90°$.

Using Pythagoras' Theorem,

$QR^2 = QS^2 + SR^2$
$30^2 = 28^2 + SR^2$
$SR^2 = 30^2 - 28^2$
$= 900 - 784$
$= 116$

$\therefore SR = \sqrt{116}$ (since $QS > 0$)
$= 10.8$ cm (to 3 s.f.)

8. H is the midpoint of UV.

$\therefore HV = \dfrac{15.4}{2} = 7.7$ m

$TV = 9.6$ m (isos. $\triangle TUV$)

In $\triangle THV$, $\angle H = 90°$.

Using Pythagoras' Theorem,

$TV^2 = TH^2 + HV^2$

$9.6^2 = TH^2 + 7.7^2$

$TH^2 = 9.6^2 - 7.7^2$

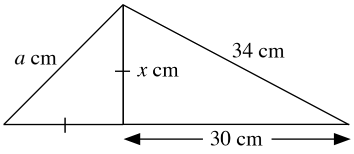
$= 92.16 - 59.29$

$= 32.87$

$\therefore TH = \sqrt{32.87}$ (since $TH > 0$)

$= 5.73$ m (to 3 s.f.)

9. **(a)**



Let the unknown side be x cm.

Using Pythagoras' Theorem on the right-angled triangle on the right,

$34^2 = x^2 + 30^2$

$x^2 = 34^2 - 30^2$

$= 1156 - 900$

$= 256$

Using Pythagoras' Theorem on the right-angled triangle on the left,

$a^2 = x^2 + x^2$

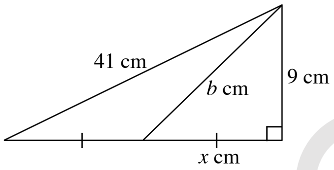
$= 256 + 256$

$= 512$

$\therefore a = \sqrt{512}$ (since $a > 0$)

$= 22.6$ (to 3 s.f.)

(b)



Let the unknown side be x cm.

Using Pythagoras' Theorem on the larger right-angled triangle,

$41^2 = (x + x)^2 + 9^2$

$(2x)^2 = 41^2 - 9^2$

$4x^2 = 1681 - 81$

$4x^2 = 1600$

$x^2 = 400$

Using Pythagoras' Theorem on the smaller right-angled triangle,

$b^2 = x^2 + 9^2$

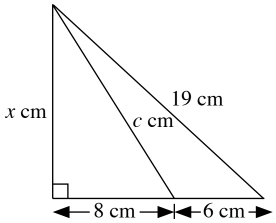
$= 400 + 81$

$= 481$

$\therefore b = \sqrt{481}$ (since $b > 0$)

$= 21.9$ (to 3 s.f.)

(c)



Let the unknown side be x cm.

Using Pythagoras' Theorem on the larger right-angled triangle,

$19^2 = x^2 + (8 + 6)^2$

$19^2 = x + 14^2$

$x^2 = 19^2 - 14^2$

$= 361 - 196$

$= 165$

Using Pythagoras' Theorem on the smaller right-angled triangle,

$c^2 = x^2 + 8^2$

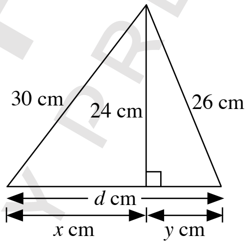
$= 165 + 64$

$= 229$

$\therefore c = \sqrt{229}$ (since $c > 0$)

$= 15.1$

(d)



Let the two unknown sides be x cm and y cm.

Using Pythagoras' Theorem on the right-angled triangle on the left,

$30^2 = x^2 + 24^2$

$x^2 = 30^2 - 24^2$

$= 900 - 576$

$= 324$

Using Pythagoras' Theorem on the right-angled triangle to the right,

$26^2 = y^2 + 24^2$

$y^2 = 26^2 - 24^2$

$= 676 - 576$

$= 100$

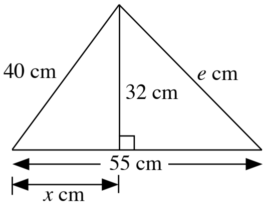
$\therefore d = x + y$

$= \sqrt{324} + \sqrt{100}$ (since $x, y > 0$)

$= 18 + 10$

$= 28$

(e)



Let the unknown side be x cm.

Using Pythagoras' Theorem on the right-angled triangle on the left,

$40^2 = x^2 + 32^2$

$x^2 = 40^2 - 32^2$

$= 1600 - 1024$

$= 576$

$\therefore x = \sqrt{576}$ (since $x > 0$)

$= 24$

Using Pythagoras' Theorem on the right-angled triangle on the right,

$e^2 = (55 - x)^2 + 32^2$

$= (55 - 24)^2 + 32^2$

$= 31^2 + 32^2$

$= 961 + 1024$

$= 1985$

$\therefore e = \sqrt{1985}$ (since $e > 0$)

$= 44.6$ (to 3 s.f.)

10. (a) Using Pythagoras' Theorem on the right-angled triangle with one side 27 cm,

$(2a + a)^2 = 36^2 + 27^2$

$(3a)^2 = 1296 + 729$

$9a^2 = 2025$

$a^2 = 225$

$\therefore a = \sqrt{225}$ (since $a > 0$)

$= 15$

Using Pythagoras' Theorem on the right-angled triangle with one side a cm,

$b^2 = a^2 + 60^2$

$= 225 + 3600$

$= 3825$

$\therefore b = \sqrt{3825}$ (since $b > 0$)

$= 61.8$

(b) Using Pythagoras' Theorem on the larger right-angled triangle,

$39^2 = (3c + 4c)^2 + 25^2$

$(7c)^2 = 39^2 - 25^2$

$49c^2 = 1521 - 625$

$49c^2 = 896$

$c^2 = \frac{128}{7}$

$\therefore c = \sqrt{\frac{128}{7}}$ (since $c > 0$)

$= 4.28$ (to 3 s.f.)

Using Pythagoras' Theorem on the smaller right-angled triangle,

$d^2 = (4c)^2 + 25^2$

$= 16c^2 + 625$

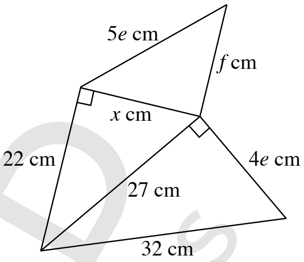
$= 16\left(\frac{128}{7}\right) + 625$

$= 917\frac{4}{7}$

$\therefore d = \sqrt{917\frac{4}{7}}$ (since $d > 0$)

$= 30.3$

(c)



Using Pythagoras' Theorem on the right-angled triangle with side 32 cm,

$32^2 = 27^2 + (4e)^2$

$16e^2 = 32^2 - 27^2$

$= 1024 - 729$

$= 295$

$e^2 = \frac{295}{16}$

$\therefore e = \sqrt{\frac{295}{16}}$ (since $e > 0$)

$= 4.29$ (to 3 s.f.)

Let the unknown side be x cm.

Using Pythagoras' Theorem on the right-angled triangle with side 22 cm,

$27^2 = x^2 + 22^2$

$x^2 = 27^2 - 22^2$

$= 729 - 484$

$= 245$

Using the Pythagoras' Theorem on the right-angled triangle with side $5e$ cm,

$(5e)^2 = f^2 + x^2$

$25e^2 = f^2 + 245$

$25\left(\frac{295}{16}\right) = f^2 + 245$

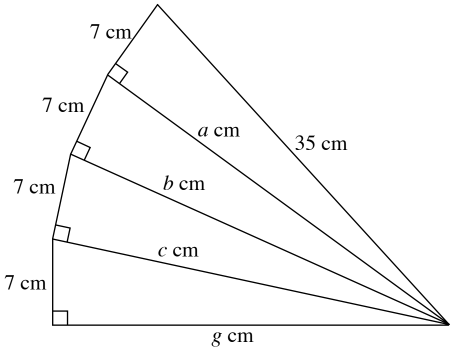
$f^2 = 25 \quad \frac{295}{16} \quad - 245$

$= 215\frac{15}{16}$

$\therefore f = \sqrt{215\frac{15}{16}}$ (since $f > 0$)

$= 14.7$ (to 3 s.f.)

(d)



Let the unknown sides be a cm, b cm and c cm.

Using Pythagoras' Theorem,

$$\begin{aligned}35^2 &= a^2 + 7^2\\ a^2 &= 35^2 - 7^2\\ b^2 + 7^2 &= 35 - 7^2\\ b^2 &= 35^2 - 7^2 - 7^2\\ c^2 + 7^2 &= 35^2 - 7^2 - 7^2\\ c^2 &= 35^2 - 7^2 - 7^2 - 7^2\\ g^2 + 7^2 &= 35^2 - 7^2 - 7^2 - 7^2\\ g^2 &= 35^2 - 7^2 - 7^2 - 7^2 - 7^2\\ &= 1225 - 49 - 49 - 49 - 49\\ &= 1029\end{aligned}$$

$\therefore g = \sqrt{1029}$ (since $g > 0$)

$= 32.1$ (to 3 s.f.)

11. (i) In $\triangle WXY$, $\angle Y = 90°$.

Using Pythagoras' Theorem,

$$\begin{aligned}WX^2 &= XY^2 + WY^2\\ (18 + 14)^2 &= XY^2 + 24^2\\ XY^2 &= 32^2 - 24^2\\ &= 1024 - 576\\ &= 448\end{aligned}$$

$XY = \sqrt{448}$ (since $XY > 0$)

$= 21.17$ m (to 4 s.f.)

$$\begin{aligned}\therefore YQ &= XY - QX\\ &= 21.17 - 9.8\\ &= 11.4 \text{ m (to 3 s.f.)}\end{aligned}$$

(ii) In $\triangle XPY$, $\angle P = 180° - 90°$ (adj. $\angle$s on a str. line)

$= 90°$

Using Pythagoras' Theorem,

$$\begin{aligned}XY^2 &= YP^2 + XP^2\\ 448 &= YP^2 + 14^2\\ YP^2 &= 448 - 14^2\\ &= 448 - 196\\ &= 252\end{aligned}$$

$YP = \sqrt{252}$ (since $YP > 0$)

$= 15.87$ m (to 4 s.f.)

$$\begin{aligned}\therefore \text{Area of } \triangle XPY &= \frac{1}{2} \times 14 \times 15.87\\ &= 111 \text{ m}^2 \text{ (to 3 s.f.)}\end{aligned}$$

12. In $\triangle HBK$, $\angle B = 90°$.

Using Pythagoras' Theorem,

$$\begin{aligned}HK^2 &= BK^2 + BH^2\\ 22^2 &= BK^2 + 15^2\\ BK^2 &= 22^2 - 15^2\\ &= 484 - 225\\ &= 259\end{aligned}$$

$\therefore BK = \sqrt{259}$ (since $BK > 0$)

$= 16.09$ cm (to 4 s.f.)

In $\triangle ABC$, $\angle B = 90°$.

Using Pythagoras' Theorem,

$$\begin{aligned}AC^2 &= AB^2 + BC^2\\ 43^2 &= (AH + 15)^2 + (16.09 + 19)^2\\ (AH + 15)^2 &= 43^2 - 35.09^2\\ AH + 15 &= \sqrt{43^2 - 35.09^2}\\ \therefore AH &= -15 + \sqrt{43^2 - 35.09^2}\\ &= 9.85 \text{ cm (to 3 s.f.)}\end{aligned}$$

or

$$\begin{aligned}AH + 15 &= -\sqrt{43^2 - 35.09^2}\\ AH &= -15 - \sqrt{43^2 - 35.09^2}\\ &= -39.9 \text{ cm (to 3 s.f.)}\end{aligned}$$

(rejected, since $AH > 0$)

13. In $\triangle EPF$, $\angle P = 90°$.

Using Pythagoras' Theorem,

$$\begin{aligned}EF^2 &= PF^2 + PE^2\\ 23^2 &= 13^2 + PE^2\\ PE^2 &= 23^2 - 13^2\\ &= 529 - 169\\ &= 360\end{aligned}$$

$PE = \sqrt{360}$ (since $PE > 0$)

$= 18.97$ m (to 4 s.f.)

In $\triangle DPE$, $\angle DPE = 180° - 90°$ (adj. $\angle$s on a str. line)

$= 90°$

Using Pythagoras' Theorem,

$$\begin{aligned}DE^2 &= PD^2 + PE^2\\ 31^2 &= PD^2 + 360\\ PD &= 31^2 - 360\\ &= 961 - 360\\ &= 601\end{aligned}$$

$PD = \sqrt{601}$ (since $PD > 0$)

$= 24.52$ m (to 4 s.f.)

In $\triangle DGF$, $\angle G = 90°$

Using Pythagoras' Theorem,

$$\begin{aligned}DF^2 &= FG^2 + DG^2\\ (24.52 + 13)^2 &= FG^2 + 32^2\\ FG^2 &= (24.52 + 13)^2 - 32^2\\ FG &= \sqrt{(24.52 + 13)^2 - 32^2} \text{ (since } FG > 0)\\ &= 19.59 \text{ m (to 4 s.f.)}\end{aligned}$$

$\therefore$ Area of the figure

$= \text{Area of } \triangle EPF + \text{Area of } \triangle DPE + \text{Area of } \triangle DGF$

$= \frac{1}{2} \times 13 \times 18.97 \times \frac{1}{2} \times 24.52 \times 18.97 + \frac{1}{2} \times 32 \times 19.59$

$= 669 \text{ m}^2$ (to 3 s.f.)

Exercise 7B

1. Let the length of each cable be x m.

Using Pythagoras' Theorem,

$x^2 = 47^2 + 18^2$

$= 2209 + 324$

$= 2533$

$\therefore x = \sqrt{2533}$ (since $x > 0$)

$= 50.3$ (to 3 s.f.)

The length of each cable is 50.3 m,

2. Let the length of the barricade be x m.

Using Pythagoras' Theorem,

$x^2 = 50^2 + 50^2$

$= 2500 + 2500$

$= 5000$

$\therefore x = \sqrt{5000}$ (since $x > 0$)

$= 70.7$ (to 3 s.f.)

The length of the barricade is 70.7 m.

3. Let the distance Ahsan has to swim be x m.

Using Pythagoras' Theorem,

$x^2 = 50^2 + 30^2$

$= 2500 + 900$

$= 3400$

$\therefore x = \sqrt{3400}$ (since $x > 0$)

$= 58.3$ (to 3 s.f.)

The distance Ahsan has to swim is 58.3 m.

4. Let the vertical height the ladder reached be x m.

Using Pythagoras' Theorem,

$5^2 = x^2 + 1.8^2$

$x^2 = 5^2 - 1.8^2$

$= 25 - 3.24$

$= 21.76$

$\therefore x = \sqrt{21.76}$ (since $x > 0$)

$= 4.66$ (to 3 s.f.)

The ladder reaches 4.66 m up the wall.

5. Let the width of the screen be x inches.

Using Pythagoras' Theorem,

$30^2 = x^2 + 18^2$

$x^2 = 30^2 - 18^2$

$= 900 - 324$

$= 576$

$\therefore x = \sqrt{576}$ (since $x > 0$)

$= 24$

The width of the screen is 24 inches.

6. Let the length of the cable be x m.

Using Pythagoras' Theorem,

$x^2 = 16^2 + (37 - 30^2)$

$= 16^2 + 7^2$

$= 256 + 49$

$= 305$

$\therefore x = \sqrt{305}$ (since $x > 0$)

$= 17.5$ (to 3 s.f.)

The length of the cable is 17.5 m.

7. In $\triangle AED$, $\angle E = 90°$.

Using Pythagoras' Theorem,

$AD^2 = DE^2 + AE^2$

$= 8^2 + 8^2$

$= 64 + 64$

$= 128$

$AD = \sqrt{128}$ (since $AD > 0$)

$= 11.31$ (to 4 s.f.)

In $\triangle BCD$, $\angle C = 90°$.

Using Pythagoras' Theorem,

$DB^2 = BC^2 + DC^2$

$= 14^2 + 14^2$

$= 196 + 196$

$= 392$

$DB = \sqrt{392}$ (since $DB > 0$)

$= 19.80$ (to 4 s.f.)

$\therefore$ Total length $= 11.31 + 19.80$

$= 31.1$ cm (to 3 s.f.)

The total length is 31.1 cm.

8. The diagonals of a rhombus bisect each other and are at right angles to each other.

Let the length of each side of the coaster be x cm.

Using Pythagoras' Theorem,

$x^2 = \left(\frac{10}{2}\right)^2 + \left(\frac{24}{2}\right)^2$

$= 5^2 + 12^2$

$= 25 + 144$

$= 169$

$\therefore x = \sqrt{169}$ (since $x > 0$)

$= 13$

The length of each side of the coaster is 13 cm.

9. **(i)** In $\triangle BKQ$, $\angle B = 90°$.

Using Pythagoras' Theorem,

$KQ^2 = BQ^2 + BK^2$

$21^2 = BQ^2 + 17.2^2$

$BQ^2 = 21^2 - 17.2^2$

$= 441 - 295.84$

$= 145.16$

$\therefore BQ = \sqrt{145.16}$ (since $BQ > 0$)

$= 12.0$ m (to 3 s.f.)

The height above the ground at which the spotlight Q is mounted, BQ, is 12.0 m.

(ii) In $\triangle BHP$, $\angle B = 90°$.

Using Pythagoras' Theorem,

$$\begin{aligned}
HP^2 &= BP^2 + BH^2\\
39^2 &= (12.05 + 12.7)^2 + BH^2\\
BH^2 &= 39^2 - 24.75^2\\
BH &= \sqrt{39^2 - 24.75^2} \quad (\text{since } BH > 0)\\
&= 30.14 \text{ m (to 4 s.f.)}
\end{aligned}$$

$$\begin{aligned}
\therefore HK &= BH - BK\\
&= 30.14 - 17.2\\
&= 12.9 \text{ m (to 3 s.f.)}
\end{aligned}$$

The distance between the projections of the light beams, HK, is 12.9 m.

10. **(i)** In $\triangle PQR$, $\angle Q = 90°$.

Using Pythagoras' Theorem,

$$\begin{aligned}
PR^2 &= RQ^2 + PQ^2\\
&= 1.1^2 + 4.2^2\\
&= 1.21 + 17.64\\
&= 18.85
\end{aligned}$$

$$\begin{aligned}
\therefore PR &= \sqrt{18.85} \quad (\text{since } PR > 0)\\
&= 4.34 \text{ m (to 3 s.f.)}
\end{aligned}$$

The length of the pole is 4.34 m.

(ii) In $\triangle XQY$, $\angle Q = 90°$.

Using Pythagoras' Theorem,

$$\begin{aligned}
XY^2 &= QY^2 + QX^2\\
18.85 &= (YR + 1.1)^2 + (4.2 - 0.9)^2\\
(YR + 1.1)^2 &= 18.85 - 3.3^2\\
YR + 1.1 &= \pm\sqrt{18.85 - 3.3^2}\\
\therefore YR &= -1.1 + \sqrt{18.85 - 3.3^2}\\
YR &= 1.72 \text{ m (to 3 s.f.)}
\end{aligned}$$

or

$$\begin{aligned}
YR &= -1.1 - \sqrt{18.85 - 3.3^2}\\
YR &= -3.92 \text{ m (to 3 s.f.)}
\end{aligned}$$

(rejected, since $YR > 0$)

The distance, YR, is 1.72 m.

11. In $\triangle FGH$, $\angle G = 90°$.

Using Pythagoras' Theorem,

$$\begin{aligned}
FH^2 &= GH^2 + GF^2\\
(4x + 1)^2 &= (x + 1)^2 + (3x + 6)^2\\
16x^2 + 8x + 1 &= x^2 + 2x + 1 + 9x^2 + 36x + 36\\
6x^2 - 30x - 36 &= 0\\
x^2 - 5x - 6 &= 0\\
(x - 6)(x + 1) &= 0
\end{aligned}$$

$x = 6$ or $x = -1$

When $x = 6$,	When $x = -1$,
$FG = 3(6) + 6$	$FG = 3(-1) + 6$
$= 24$ m	$= 3$ m
$GH = 6 + 1$	$GH = -1 + 1$
$= 7$ m	$= 0$ m
	$x = -1$ is rejected since $GH > 0$.

$$\begin{aligned}
\therefore \text{Area of campsite} &= 24 \times 7\\
&= 168 \text{ m}^2
\end{aligned}$$

The area of the campsite is 168 m^2.

12. The side $(x + 2)$ cm is the longest side.

Using Pythagoras' Theorem,

$$\begin{aligned}
(x + 2)^2 &= x^2 + (x + 1)^2\\
x^2 + 4x + 4 &= x^2 + x^2 + 2x + 1\\
x^2 - 2x - 3 &= 0\\
(x - 3)(x + 1) &= 0
\end{aligned}$$

$\therefore x = 3$ or $x = -1$ (rejected, since $x > 0$)

The value of x is 3.

13. **(i)** $HL = 9 - 2 = 7$ cm

$OL = 6$ cm

In $\triangle HLO$, $\angle L = 90°$.

Using Pythagoras' Theorem,

$$\begin{aligned}
OH^2 &= HL^2 + OL^2\\
&= 7^2 + 6^2\\
&= 49 + 36\\
&= 85
\end{aligned}$$

$$\begin{aligned}
\therefore OH &= \sqrt{85} \quad (\text{since } OH > 0)\\
&= 9.22 \text{ cm (to 3 s.f.)}
\end{aligned}$$

The length of the zip is 9.22 cm.

(ii) In $\triangle HMN$, $\angle M = 90°$.

Using Pythagoras' Theorem,

$$\begin{aligned}
HN^2 &= NM^2 + HM^2\\
&= 6^2 + 2^2\\
&= 36 + 4\\
&= 40
\end{aligned}$$

Let the length of NK be x cm,
the length of OK be y cm.

In $\triangle HKN$, $\angle K = 90°$.

Using Pythagoras' Theorem,

$$\begin{aligned}
HN^2 &= NK^2 + HK^2\\
x^2 + HK^2 &= 40\\
HK^2 &= 40 - x^2\\
(\sqrt{85} - OK)^2 &= 40 - x^2\\
85 - 2\sqrt{85}\,y + y^2 &= 40 - x^2\\
y^2 &= 2\sqrt{85}\,y - 45 - x^2
\end{aligned}$$

In $\triangle OKN$, $\angle K = 180° - 90°$ (adj. $\angle$s on a str. line)
$= 90°$

Using Pythagoras' Theorem,

$$\begin{aligned}
ON^2 &= NK^2 + OK^2\\
9^2 &= x^2 + y^2\\
81 &= x^2 + 2\sqrt{85}\,y - 45 - x^2\\
2\sqrt{85}\,y &= 126\\
y &= \frac{63}{\sqrt{85}}\\
\therefore y^2 &= 2\sqrt{85}\,y - 45 - x^2
\end{aligned}$$

$$\frac{63}{\sqrt{85}}^{2} = 2\sqrt{85}\ \frac{63}{\sqrt{85}}\ - 45 - x^2$$

$$x^2 = 126 - 45 - \frac{3969}{85}$$

$$= 34\frac{26}{35}$$

$$\therefore x = \sqrt{34\frac{26}{85}} \text{ (since } x > 0)$$

$$= 5.86 \text{ (to 3 s.f.)}$$

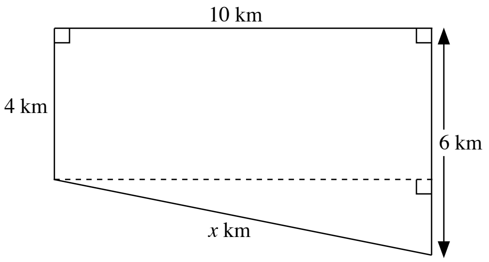
The length of the second zip is 5.86 cm.

14. Distance travelled due North $= 40 \times \frac{6}{60}$

$= 4$ km

Distance travelled due South $= 30 \times \frac{12}{60}$

$= 6$ km



Let the shortest distance be x km.

Using Pythagoras' Theorem,

$$x^2 = 10^2 + (6 - 4)^2$$
$$= 100 + 4$$
$$= 104$$
$$\therefore x = \sqrt{104} \text{ (since } x > 0)$$
$$= 10.2 \text{ (to 3 s.f.)}$$

The shortest distance between the courier and his starting point is 10.2 km.

15. (a) (i) Length of each side of square tabletop

$$= \frac{132}{4}$$
$$= 33 \text{ cm}$$

(ii) Let the radius of the round tabletop be r cm.

$$2\pi r = 132$$
$$2 \times \frac{22}{7} \times r = 132$$
$$\therefore r = 21$$

The radius is 21 cm.

(b) Area of square tabletop $= 33^2$

$= 1089 \text{ cm}^2$

Area of round tabletop $= \pi r^2$

$= \frac{22}{7} \times 21^2$

$= 1386 \text{ cm}^2$

(c) (i) Length of each side of table

$$= \frac{132}{3}$$
$$= 44 \text{ cm}$$

(ii) The height of the equilateral triangle bisects the side opposite it.

Let the height of the equilateral triangle be h cm.

Using Pythagoras' Theorem,

$$44^2 = h^2 + \ \frac{44}{2}^{2}$$
$$h^2 = 44^2 - 22^2$$
$$= 1936 - 484$$
$$= 1452$$
$$h = \sqrt{1452} \text{ (since } h > 0)$$
$$= 38.11 \text{ (to 4 s.f.)}$$

Area of tabletop $= \frac{1}{2} \times 44 \times 38.11$

$= 838 \text{ cm}^2$ (to 3 s.f.)

(d) The tabletop in the shape of a circle should be chosen since it has the greatest area.

Exercise 7C

1. (a) AC is the longest side of $\triangle ABC$.

$$AC^2 = 65^2$$
$$= 4225$$
$$AB^2 + BC^2 = 16^2 + 63^2$$
$$= 256 + 3969$$
$$= 4225$$

Since $AC^2 = AB^2 + BC^2$, $\triangle ABC$ is a right-angled triangle where $\angle B = 90°$.

(b) EF is the longest side of $\triangle DEF$.

$$EF^2 = 27^2$$
$$= 729$$
$$DF^2 + DE^2 = 21^2 + 24^2$$
$$= 441 + 576$$
$$= 1017$$

Since $EF^2 \neq DF^2 + DE^2$, $\triangle DEF$ is not a right-angled triangle.

(c) GH is the longest side in $\triangle GHI$.

$$GH^2 = 7.5^2$$
$$= 56.25$$
$$HI^2 + GI^2 = 7.1^2 + 2.4^2$$
$$= 50.41 + 5.76$$
$$= 56.17$$

Since $GH^2 \neq HI^2 + GI^2$, $\triangle GHI$ is not a right-angled triangle.

(d) MN is the longest side in $\triangle MNO$.

$$MN^2 = \frac{5}{13}^2$$
$$= \frac{25}{169}$$
$$NO^2 + MO^2 = \frac{3}{13}^2 + \frac{4}{13}^2$$
$$= \frac{9}{169} + \frac{16}{169}$$
$$= \frac{25}{169}$$

Since $MN^2 = NO^2 + MO^2$, $\triangle MNO$ is a right-angled triangle where $\angle O = 90°$.

2. PR is the longest side is $\triangle PQR$.

$$PR^2 = 30^2$$
$$= 900$$
$$PQ^2 + QR^2 = 19^2 + 24^2$$
$$= 361 + 576$$
$$= 937$$

Since $PR^2 \neq PQ^2 + QR^2$, $\triangle PQR$ is not a right-angled triangle.

3. $ST = \frac{7}{12}$ cm

$TU = \frac{5}{6}$ cm $= \frac{10}{12}$ cm

$SU = \frac{1}{3}$ cm $= \frac{4}{12}$ cm

TU is the longest side in $\triangle STU$.

$$TU^2 = \frac{10}{12}^2$$
$$= \frac{100}{144}$$
$$SU^2 + ST^2 = \frac{4}{12}^2 + \frac{7}{12}^2$$
$$= \frac{16}{144} + \frac{49}{144}$$
$$= \frac{65}{144}$$

Since $TU^2 \neq SU^2 + ST^2$, $\triangle STU$ is not a right-angled triangle.

4. In $\triangle PQS$, $\angle P = 90°$.

Using Pythagoras' Theorem,

$$SQ^2 = PQ^2 + PS^2$$
$$= 40^2 + 30^2$$
$$= 1600 + 900$$
$$= 2500$$
$$SQ = \sqrt{2500} \text{ (since } SQ > 0)$$
$$= 50 \text{ m}$$
$$\frac{SX}{SQ} = \frac{16}{16+9}$$
$$SX = \frac{16}{25} \times 50$$
$$= 32 \text{ m}$$
$$QX = 50 - 32$$
$$= 18 \text{ m}$$

To show Jamil stops at X is to show RX is perpendicular to QS.

We need to show $\triangle SXR$ and $\triangle QXR$ are right-angled triangles.

RS is the longest side in $\triangle SXR$.

$$RS^2 = 40^2$$
$$= 1600$$
$$SX^2 + RX^2 = 32^2 + 24^2$$
$$= 1024 + 576$$
$$= 1600$$

Since $RS^2 = SX^2 + RX^2$, $\triangle SXR$ is a right-angled triangle where $\angle X = 90°$.

QR is the longest side in $\triangle QXR$.

$$QR^2 = 30^2$$
$$= 900$$
$$RX^2 + QX^2 = 24^2 + 18^2$$
$$= 576 + 324$$
$$= 900$$

Since $QR^2 = RX^2 + QX^2$, $\triangle QXR$ is a right-angled triangle where $\angle X = 90°$.

$\therefore$ Jamil stops at X.

5. Since m and n are positive integers,

$$m^2 + n^2 > m^2 - n^2$$

Also,

$$(m-n)^2 > 0$$
$$m^2 - 2mn + n^2 > 0$$
$$m^2 + n^2 > 2mn$$

c is the longest side in the triangle.

$$c^2 = (m^2 + n^2)$$
$$= m^4 + 2m^2n^2 + n^4$$
$$a^2 + b^2 = (m^2 - n^2)^2 + (2mn)^2$$
$$= m^4 - 2m^2n^2 + n^4 + 4m^2n^2$$
$$= m^4 + 2m^2n^2 + n^4$$

Since $c^2 = a^2 + b^2$, then the triangle is a right-angled triangle.

Review Exercise 7

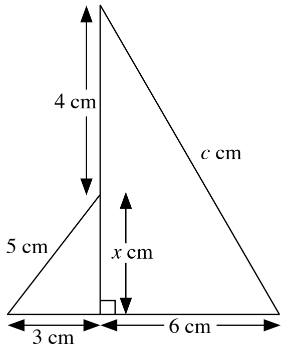
1. **(a)** Using Pythagoras' Theorem,

$$a^2 = 6.3^2 + 9.6^2$$
$$= 39.69 + 92.16$$
$$= 131.85$$
$$\therefore a = \sqrt{131.85} \text{ (since } a > 0)$$
$$= 11.5 \text{ (to 3 s.f.)}$$

(b) Using Pythagoras' Theorem,

$$13.5^2 = b^2 + 8.7^2$$
$$b^2 = 13.5^2 - 8.7^2$$
$$= 182.25 - 75.69$$
$$= 106.56$$
$$\therefore b = \sqrt{106.56} \text{ (since } b > 0)$$
$$= 10.3 \text{ (to 3 s.f.)}$$

(c)



Let the unknown side be x cm.

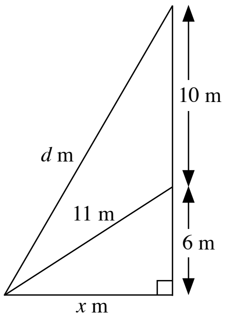
Using Pythagoras' Theorem on the smaller right-angled triangle,

$$5^2 = x^2 + 3^2$$
$$x^2 = 5^2 - 3^2$$
$$= 25 - 9$$
$$= 16$$
$$x = \sqrt{16} \text{ (since } x > 0)$$
$$= 4$$

Using Pythagoras' Theorem on the larger right-angled triangle,

$$c^2 = 6^2 + (x + 4)^2$$
$$= 6^2 + 8^2$$
$$= 36 + 64$$
$$= 100$$
$$\therefore c = \sqrt{100} \text{ (since } c > 0)$$
$$= 10$$

(d)



Let the unknown side be x m.

Using Pythagoras' Theorem on the smaller right-angled triangle,

$$11^2 = x^2 + 6^2$$
$$x = 11^2 - 6^2$$
$$= 121 - 36$$
$$= 85$$

Using Pythagoras' Theorem on the larger right-angled triangle,

$$d^2 = x^2 + (10 + 6)^2$$
$$= 85 + 16^2$$
$$= 85 + 256$$
$$= 341$$
$$\therefore d = \sqrt{341} \text{ (since } c > 0)$$
$$= 18.5 \text{ (to 3 s.f.)}$$

2. **(i)** Let the side of the square be x cm.

Using Pythagoras' Theorem,

$$42.5^2 = x^2 + x^2$$
$$2x^2 = 1806.25$$
$$x = 903.125$$
$$x = \sqrt{903.125} \text{ (since } x > 0)$$
$$= 30.05 \text{ (to 4 s.f.)}$$

$\therefore$ Perimeter of the square $= 4 \times 30.05$

$= 120$ cm (to 3 s.f.)

(ii) Area of the square $= 30.05^2$

$= 903 \text{ cm}^2$ (to 3 s.f.)

3. Let the height of the briefcase be x cm.

Using Pythagoras' Theorem,

$$37^2 = x^2 + 30^2$$
$$x^2 = 37^2 - 30^2$$
$$= 1369 - 900$$
$$= 469$$
$$\therefore x = \sqrt{469} \text{ (since } x > 0)$$
$$= 21.7 \text{ (to 3 s.f.)}$$

The height of the briefcase is 21.7 cm.

4. Let the perpendicular distance from F to GH be x cm.

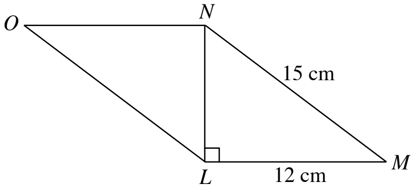
The perpendicular distance from F to GH bisects GH.

Using Pythagoras' Theorem,

$$2^2 = x^2 + 1^2$$
$$x^2 = 2^2 - 1^2$$
$$= 4 - 1$$
$$= 3$$
$$\therefore x = \sqrt{3} \text{ (since } x > 0)$$
$$= 1.73$$

The perpendicular distance from F to GH is 1.73 cm.

5.



Let the length of LN be x cm.

In $\triangle LMN$, $\angle L = 90°$.

Using Pythagoras' Theorem,

$$MN^2 = LN^2 + LM^2$$
$$15^2 = LN^2 + 12^2$$
$$LN^2 = 15^2 - 12^2$$
$$= 225 - 144$$
$$= 81$$
$$LN = \sqrt{81} \text{ (since } LN > 0)$$
$$= 9$$

$\therefore$ Area of stained glass $= 12 \times 9$

$= 108 \text{ cm}^2$

6. **(i)** Let the length of the other diagonal be x cm.

The diagonals of a rhombus bisect and are at right angles to each other.

Using Pythagoras' Theorem,

$$\begin{aligned} 52^2 &= \left(\frac{x}{2}\right)^2 + \left(\frac{48}{2}\right)^2 \\ \frac{x^2}{4} &= 52^2 - 24^2 \\ &= 2704 - 576 \\ &= 2128 \\ x^2 &= 8512 \\ \therefore x &= \sqrt{8512} \text{ (since } x > 0) \\ &= 92.26 \\ &= 92.3 \text{ (to 3 s.f.)} \end{aligned}$$

The length of the other diagonal is 92.3 cm.

(ii) $$\begin{aligned} \text{Area of the floor tile } &= 4 \times \left(\frac{1}{2} \times \frac{92.26}{2} \times \frac{48}{2}\right) \\ &= \frac{1}{2} \times 92.26 \times 48 \\ &= 2210 \text{ cm}^2 \text{ (to 3 s.f.)} \end{aligned}$$

The area of the floor tile is 2210 cm^2.

7. **(i)** In $\triangle ABD$, $\angle A = 90°$.

Using Pythagoras' Theorem,

$$\begin{aligned} BD^2 &= AD^2 + AB^2 \\ &= 48^2 + 36^2 \\ &= 2304 + 1296 \\ &= 3600 \\ \therefore BD &= \sqrt{3600} \text{ (since } BD > 0) \\ &= 60 \text{ cm} \end{aligned}$$

(ii) BC is the longest side in $\triangle BCD$.

$$\begin{aligned} BC^2 &= 87^2 \\ &= 7569 \\ BD^2 + CD^2 &= 60^2 + 63^2 \\ &= 3600 + 3969 \\ &= 7569 \end{aligned}$$

Since $BC^2 = BD^2 + CD^2$, $\triangle BCD$ is a right-angled triangle where $\angle D = 90°$.

8. **(i)** $$\begin{aligned} AP &= 28 - 6 \\ &= 22 \text{ m} \\ CR &= 15 - 6 \\ &= 9 \text{ m} \end{aligned}$$

Area of shaded region $DPQR$

= Area of $ABCD$ – area of $\triangle ADP$ – area of $\triangle CDR$ – area of $PBRQ$

$$\begin{aligned} &= (28 \times 15) - \left(\frac{1}{2} \times 22 \times 15\right) - \left(\frac{1}{2} \times 28 \times 9\right) - 6^2 \\ &= 420 - 165 - 126 - 36 \\ &= 93 \text{ m}^2 \end{aligned}$$

(ii) In $\triangle ADP$, $\angle A = 90°$.

Using Pythagoras' Theorem,

$$\begin{aligned} DP^2 &= AP^2 + AD^2 \\ &= 22^2 + 15^2 \\ &= 484 + 225 \\ &= 709 \text{ m} \\ DP &= \sqrt{709} \text{ (since } DP > 0) \\ &= 26.6 \text{ m (to 3 s.f.)} \end{aligned}$$

(iii) Let the length of AX be x m.

$$\begin{aligned} \frac{1}{2} \times 22 \times 15 &= \frac{1}{2} \times \sqrt{709} \times x \\ x &= \frac{22 \times 15}{\sqrt{709}} \\ &= 12.4 \text{ (to 3 s.f.)} \end{aligned}$$

The length of AX is 12.4 m.

9. **(i)** In $\triangle FTK$, $\angle T = 90°$.

Using Pythagoras' Theorem,

$$\begin{aligned} FK^2 &= KT^2 + FT^2 \\ 18^2 &= 12.5^2 + FT^2 \\ FT^2 &= 18^2 - 12.5^2 \\ &= 324 - 156.25 \\ &= 167.75 \\ \therefore FT &= \sqrt{167.75} \text{ (since } FT > 0) \\ &= 13.0 \text{ m (to 3 s.f.)} \end{aligned}$$

The height of the pole is 13.0 m.

(ii) $$\begin{aligned} \frac{HT}{KT} &= \frac{2}{3+2} \\ HT &= 12.5 \times \frac{2}{5} \\ &= 5 \text{ m} \end{aligned}$$

In $\triangle FTH$, $\angle T = 90°$.

Using Pythagoras' Theorem,

$$\begin{aligned} FH^2 &= HT^2 + FT^2 \\ &= 5^2 + 167.75 \\ &= 25 + 167.75 \\ &= 192.75 \\ \therefore FH &= \sqrt{192.75} \text{ (since } FH > 0) \\ &= 13.9 \text{ m (to 3 s.f.)} \end{aligned}$$

The distance FH is 13.9 m.

10. Let the length of the diagonal be x m.

Using Pythagoras' Theorem,

$$\begin{aligned} x^2 &= 80^2 + 60^2 \\ &= 6400 + 3600 \\ &= 10\,000 \\ x &= \sqrt{10\,000} \text{ (since } x > 0) \\ &= 100 \end{aligned}$$

$$\begin{aligned} \therefore \text{Time taken to complete run } &= \frac{100}{7.5} \\ &= 13\frac{1}{3} \text{ s (to 3 s.f.)} \end{aligned}$$

Farhan takes $13\frac{1}{3}$ s to complete his run.

Challenge Yourself

1. **(a)** $6^2 + 8^2 = 36 + 64$

$= 100$

$= 10^2$

6, 8 and 10 form a Pythagorean Triple.

(b) **(i)** $c^2 = 12^2 + 16^2$

$= 144 + 256$

$= 400$

$\therefore c = \sqrt{400}$ (since $c > 0$)

$= 20$

The Pythagorean Triple is 12, 16 and 20.

(ii) $7^2 + 24^2 = 49 + 576 = 625 = 25^2$

A Pythagorean Triple is 7, 24 and 25.

Alternatively,

$3^2 + 4^2 = 9 + 16 = 25 = 5^2$

Multiply throughout by 25,

$(3 \times 5)^2 + (4 \times 5)^2 = (5 \times 5)^2$

$15^2 + 20^2 = 25^2$

A Pythagorean Triple is 15, 20 and 25.

(c) **(i)** $(3n)^2 + (4n)^2 = 9n^2 + 16n^2$

$= 25n^2$

(ii) $25n^2 = (5n)^2$

Let $n = 7$.

$(3 \times 7)^2 + (4 \times 7)^2 = (5 \times 7)^2$

$21^2 \times 28^2 = 35^2$

The Pythagorean Triple is 21, 28 and 35.

(d) **(i)** When $n = 24$,

$1 + 2n = 1 + 2(24)$

$= 49$

$= 7^2$

$n + 1 = 24 + 1$

$= 25$

The Pythagorean Triple is 7, 24 and 25.

(ii) $1 + 2n = 42$

$2n = 41$

$n = 20\frac{1}{2}$

n is not an integer, so a Pythagorean Triple cannot be obtained.

(iii) When $k = 9$,

$1 + 2n = 9^2$

$2n = 81$

$n = 40$

$n + 1 = 40 + 1$

$= 41$

The Pythagorean Triple is 9, 40 and 41.

2. $\triangle ABC$ is such that $BC^2 = 370$, $AC^2 = 74$ and $AB^2 = 116$.

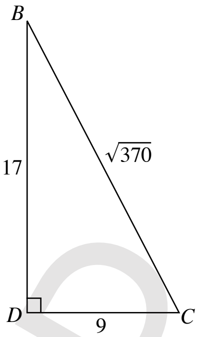
The hint is $370^2 = 9^2 + 17^2$, $74 = 5^2 + 7^2$, $116 = 4^2 + 10^2$.

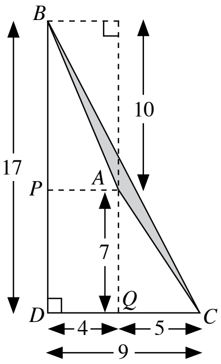
The key is to observe that $17 = 7 + 10$,

$9 = 5 + 4$

So starting with $BC^2 = 9^2 + 17^2$, we have the diagram below.

Then we try to construct the point A as follows.





$\therefore$ Area of $\triangle ABC$

$=$ area of $\triangle BCD$ – area of $\triangle ABP$ – area of $\triangle ACQ$ – area of rectangle $APDQ$

$= \frac{1}{2} \times 9 \times 17 - \frac{1}{2} \times 4 \times 10 - \frac{1}{2} \times 5 \times 7 - 7 \times 4$

$= 11 \text{ units}^2$

3. Let the diameter of A_1, A_2 and A_3 be d_1, d_2 and d_3.

Using Pythagoras' Theorem,

$\therefore d_1^2 = d_2^2 + d_3^2$

$A_1 = \frac{1}{2} \times \pi \times \left(\frac{d_1}{2}\right)^2$

$= \frac{\pi}{8}\ (d_1^2)$

$A_2 + A_3 = \frac{1}{2} \times \pi \times \left(\frac{d_2}{2}\right)^2 + \frac{1}{2} \times \pi \times \left(\frac{d_3}{2}\right)^2$

$= \frac{\pi}{8}\ (d_2^2 + d_3^2)$

$= \frac{\pi}{8}\ (d_1^2)$

Since $A_1 = A_2 + A_3$, the relatioship still holds true.

4. **(i)** Let the length of each side of the equilateral triangle be x cm,
the height of the equilateral triangle be h cm.

Area of equilateral triangle
= Area of square
$= 3^2$
$= 9 \text{ cm}^2$

The height of an equilateral triangle bisects the side.

Using Pythagoras' Theorem,

$$x^2 = h^2 + \left(\frac{x}{2}\right)^2$$

$$h^2 = x^2 - \left(\frac{x}{2}\right)^2$$

$$= x - \frac{x^2}{4}$$

$$= \frac{3}{4}x^2$$

$$\therefore h = \sqrt{\frac{3}{4}x^2} \quad \text{(since } h > 0\text{)}$$

$$= \frac{\sqrt{3}}{2}x$$

$$\therefore \frac{1}{2} \times x \times \frac{\sqrt{3}}{2}x = 9$$

$$\frac{\sqrt{3}}{4}x^2 = 9$$

$$x^2 = \frac{36}{\sqrt{3}}$$

$$\therefore x = \sqrt{\frac{36}{\sqrt{3}}} \quad \text{(since } x > 0\text{)}$$

$$= 4.56 \text{ (to 3 s.f.)}$$

The length of each side of the equilateral triangle is 4.56 cm.

(ii) No. From above, $h = \frac{\sqrt{3}}{2}x$. If x is an integer, h is never an integer and therefore the area of the triangle will not be an integer. Thus, the side of the square is never an integer.

This applies for the converse.

Chapter 8 Arc Length and Sector Area

TEACHING NOTES

Suggested Approach

In this chapter, students will be introduced to circles and how to calculate the arc length and area of the sector of a circle. Teachers may begin the chapter by asking students to identify the different parts of a circle using real-life examples of arcs, sectors and segments of a circle.

Students are expected to know how to apply the formulas of Pythagoras' Theorem when solving problems involving the arc length and sector area.

Section 8.1: Length of Arc

Teachers may begin the chapter by showing students a circle with centre *O* and highlight to students the minor arc, major arc, minor sector, major sector, minor segment and major segment of a circle. Once students are familiar with these terms and are able to identify the parts of a circle, teachers can proceed to guide students on how to derive the formula for the length of an arc of a circle (see Investigation: Arc Length).

Section 8.2: Area of Sector

Teachers can ask students to discover the formula for the area of sector on their own (see Investigation: Area of Sector). Teachers should take note that some students may need some guidance when finding the area of a shaded region involving sectors of circles. Teachers can suggest to students that when tackling such questions, they may need to draw additional lines in the figures given to help them better visualise and work out the solutions.

Challenge Yourself

For Question 1, students need to make an observation from the perimeters of each figure.

WORKED SOLUTIONS

Investigation (Arc Length)

3. The third last column and the last column are equivalent.

4. The third last column and the last column are equivalent.

5. Arc length = $\frac{x°}{360°}$ × circumference, where $x°$ is the angle subtended by the arc at the centre of the circle of radius r.

Investigation (Area of Sector)

3. The third last column and the last column are equivalent.

4. The third last column and the last column are equivalent.

5. Area of a sector of a circle = $\frac{x°}{360°}$ × area of the circle, where $x°$ is the angle subtended by the arc at the centre of the circle of radius r.

Practise Now 1

$$\begin{aligned}\text{External radius of ring, } R &= \frac{40}{2}\\ &= 20 \text{ mm}\end{aligned}$$

$$\begin{aligned}\text{Internal radius of ring, } r &= \frac{33}{2}\\ &= 16.5 \text{ mm}\end{aligned}$$

$$\begin{aligned}\text{Area of ring} &= \pi R^2 - \pi r^2\\ &= \pi(20)^2 - \pi(16.5)^2\\ &= \pi(20^2 - 16.5^2)\\ &= 401 \text{ mm}^2 \text{ (to 3 s.f.)}\end{aligned}$$

Practise Now 2

1. **(i)** $$\begin{aligned}\text{Length of major arc } AYB &= \frac{228°}{360°} \times 2\pi \times 25\\ &= 99.5 \text{ cm (to 3 s.f.)}\end{aligned}$$

(ii) $$\begin{aligned}\text{Perimeter of minor sector} &= \text{length of arc } AXB + OA + OB\\ &= \frac{360° - 228°}{360°} \times 2\pi \times 25 + 25 + 25\\ &= 108 \text{ cm (to 3 s.f.)}\end{aligned}$$

2. $$\begin{aligned}\text{Perimeter of shaded region} &= \text{length of major arc } AOB + OA + OB\\ &= \frac{360° - 150°}{360°} \times 2\pi \times 9 + 9 + 9\\ &= \left(\frac{21}{2}\pi + 18\right) \text{ cm}\end{aligned}$$

3. Length of major arc PXQ = 36 cm

$$\begin{aligned}\frac{360° - 50°}{360°} \times 2\pi \times r &= 36\\ 5.411r &= 36\\ r &= 6.65\end{aligned}$$

Practise Now 3

1. $$\begin{aligned}\angle OQR &= 180° - 90° - 36.9° \ (\angle \text{ sum of a } \triangle)\\ &= 53.1°\end{aligned}$$

$$\begin{aligned}\tan 36.9° &= \frac{RQ}{8}\\ RQ &= 8 \tan 36.9°\\ &= 6.007 \text{ m (to 4 s.f.)}\end{aligned}$$

$$\begin{aligned}\text{Length of arc } RP &= \frac{36.9°}{360°} \times 2\pi \times 8\\ &= 1.64\pi \text{ m}\end{aligned}$$

By Pythagoras' Theorem,

$$\begin{aligned}OQ^2 &= OR^2 + RQ^2\\ &= 8^2 + 6.007^2\\ &= 100.1 \text{ (to 4 s.f.)}\\ OQ &= \sqrt{100.1}\\ &= 10.00 \text{ m (to 4 s.f.)}\\ PQ &= OQ - OP\\ &= 10.00 - 8\\ &= 2.00 \text{ m}\end{aligned}$$

$$\begin{aligned}\therefore \text{ Perimeter of shaded region } PQR &= RQ + PQ + \text{length of arc } RP\\ &= 6.007 + 2.00 + 1.64\pi\\ &= 13.2 \text{ m}\end{aligned}$$

2. $$\begin{aligned}\text{Perimeter of sector} &= \frac{80°}{360°} \times 2\pi \times 10 + 10 + 10\\ &= 34.0 \text{ cm (to 3 s.f.)}\end{aligned}$$

Practise Now 4

$$\begin{aligned}\angle ROQ &= 180° - 90° - 36° \ (\angle \text{ sum of a } \triangle)\\ &= 54°\end{aligned}$$

$$\angle POQ = 2\ \angle ROQ = 2(54°) = 108°$$

$$\begin{aligned}\text{Length of arc } PAQ &= \frac{108°}{360°} \times 2\pi \times 35\\ &= 65.97 \text{ cm (to 4 s.f.)}\end{aligned}$$

$$\begin{aligned}\text{Length of arc } PBQ &= \frac{1}{2} \times \pi \times 56.63\\ &= 88.95 \text{ cm (to 4 s.f.)}\end{aligned}$$

$$\begin{aligned}\therefore \text{ Perimeter of shaded region} &= 65.97 + 88.95\\ &= 15.5 \text{ cm (to 3 s.f.)}\end{aligned}$$

Practise Now 5

(i) Since the length of the minor arc AQB is 33 cm,

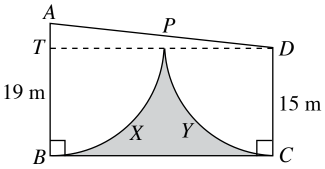
$$\begin{aligned}\frac{\angle AOB}{360°} \times 2\pi \times 15 &= 33\\ \frac{\angle AOB}{360°} &= \frac{33}{30\pi}\\ \angle AOB &= \frac{33}{30\pi} \times 360° = 126.1° \text{ (to 1 d.p.)}\end{aligned}$$

(ii) $$\begin{aligned}\text{Reflex } \angle AOB &= 360° - 126.05° \ (\angle\text{s at a point})\\ &= 233.95°\end{aligned}$$

$$\begin{aligned}\text{Area of major sector } OAPB &= \frac{233.95°}{360°} \times \pi \times 15^2\\ &= 459 \text{ cm}^2 \text{ (to 3 s.f.)}\end{aligned}$$

Practise Now 6

(i) Draw a line DT such that T lies on AB and DT is perpendicular to AB.



$AT = AB - TB$

$= 19 - 15$

$= 4$ m

$AD = AP + PD$

$= 19 + 15$

$= 34$ m

(ii) Using Pythagoras' Theorem,

$DT^2 = 34^2 - 4^2$

$DT = \sqrt{34^2 - 4^2}$

$= \sqrt{1140}$

$= 33.76$ m (to 4 s.f.)

$\angle DAT = 180° - 90° - 6.76°$ ($\angle$ sum of a $\triangle$)

$= 83.24°$

Area of shaded region

= Area of trapezium $ABCD$ – area of sector APB – area of sector DPC

$= \frac{1}{2}(15 + 19)(33.76) - \frac{83.24°}{360°} \times \pi \times 19^2 - \frac{96.76°}{360°} \times \pi \times 15^2$

$= 122$ m^2 (to 3 s.f.)

Exercise 8A

1. **(a)** Area of circle, $\pi r^2 = 616$ mm^2

$r^2 = \frac{616}{\pi}$

$r = \sqrt{\frac{616}{\pi}}$ mm

$\therefore$ Diameter of circle $= 2\sqrt{\frac{616}{\pi}}$

$= 28.0$ mm (to 3 s.f.)

(b) Area of circle, $\pi r^2 = 779\frac{5}{8}$ m^2

$r^2 = \frac{779\frac{5}{8}}{\pi}$

$r = \sqrt{\frac{779\frac{5}{8}}{\pi}}$ m

$\therefore$ Diameter of circle $= 2\sqrt{\frac{779\frac{5}{8}}{\pi}}$

$= 31.5$ m (to 3 s.f.)

(c) Area of circle, $\pi r^2 = 3850$ cm^2

$r^2 = \frac{3850}{\pi}$

$r = \sqrt{\frac{3850}{\pi}}$ cm

$\therefore$ Diameter of circle $= 2\sqrt{\frac{3850}{\pi}}$

$= 70.0$ cm (to 3 s.f.)

2. **(a)** External radius of ring, $R = \frac{15}{2}$

$= 7.5$ cm

Internal radius of ring, $r = \frac{13}{2}$

$= 6.5$ cm

Area of ring $= \pi R^2 - \pi r^2$

$= \pi(7.5)^2 - \pi(6.5)^2$

$= \pi(7.5^2 - 6.5^2)$

$= 44.0$ cm^2 (to 3 s.f.)

(b) External radius of ring, $R = \frac{1.2}{2}$

$= 0.6$ m

Internal radius of ring, $r = \frac{0.9}{2}$

$= 0.45$ m

Area of ring $= \pi R^2 - \pi r^2$

$= \pi(0.6)^2 - \pi(0.45)^2$

$= \pi(0.6^2 - 0.45^2)$

$= 0.495$ m^2 (to 3 s.f.)

3. **(a)** **(i)** Area of big semicircle $= \frac{1}{2} \times \pi \times 14^2$

$= 98\pi$ cm^2

Area of two small semicircles

$= 2 \times \frac{1}{2} \times \pi \times \left(\frac{14}{2}\right)^2$

$= 49\pi$ cm^2

Area of shaded region $= 98\pi - 49\pi$

$= 49\pi$

$= 154$ cm^2 (to 3 s.f.)

(ii) Length of arc of big semicircle

$= \frac{1}{2} \times \pi \times 28$

$= 14\pi$ cm

Length of arc of small semicircles

$= 2 \times \frac{1}{2} \times \pi \times 14$

$= 14\pi$ cm

Perimeter of shaded region

$= 14\pi + 14\pi$

$= 28\pi$

$= 88.0$ cm (to 3 s.f.)

(b) **(i)** Area of square $= 14^2$

$= 196 \text{ cm}^2$

Area of 4 quadrants $= 4 \times \frac{1}{4} \times \pi \times \left(\frac{14}{2}\right)^2$

$= 49\pi \text{ cm}^2$

Area of shaded region $= 196 - 49\pi$

$= 42.1 \text{ cm}^2$

(ii) Perimeter of shaded region

$= 4 \times$ arc length of quadrant

$= 4 \times \frac{1}{4} \times \pi \times 14$

$= 44.0$ cm (to 3 s.f.)

(c) **(i)** Area of shaded region $= \frac{1}{2} \times$ area of circle

$= \frac{1}{2} \times \pi \times 28^2$

$= 1230 \text{ cm}^2$ (to 3 s.f.)

(ii) Arc length of big semicircle

$= \frac{1}{2} \times \pi \times (28 \times 2)$

$= 28\pi$ cm

Arc length of small semicircle

$= \frac{1}{2} \times \pi \times 28$

$= 14\pi$ cm

Perimeter of shaded region

$= 28\pi + 14\pi + 14\pi$

$= 176$ cm (to 3 s.f.)

(d) **(i)** Area of square $= 10^2$

$= 100 \text{ cm}^2$

Area of shaded region

= Area of square – area of 4 quadrants

$= 100 - 4 \times \frac{1}{4} \times \pi \times 3.5^2$

$= 100 - 12.25\pi$

$= 61.5 \text{ cm}^2$ (to 3 s.f.)

(ii) Perimeter of shaded region

$= 4 \times \frac{1}{4} \times \pi \times (3.5 \times 2) + 4 \times (10 - 3.5 \times 2)$

$= 7\pi + 12$

$= 34.0$ cm (to 3 s.f.)

4. **(a)** Area of shaded region

= Area of square – area of 4 circles

$= 56^2 - 4 \times \pi \times \left(\frac{56}{4}\right)^2$

$= 3136 - 784\pi$

$= 673 \text{ cm}^2$ (to 3 s.f.)

(b) Area of unshaded region

$= 4 \times$ area of semicircles

$= 4 \times \frac{1}{2} \times \pi \times \left(\frac{3.5}{2}\right)^2$

$= 6.125\pi \text{ cm}^2$

Area of shaded region

= Area of circle – area of unshaded region

$= \pi \times 3.5^2 - 6.125\pi$

$= 19.2 \text{ cm}^2$ (to 3 s.f.)

(c) Area of shaded region

= Area of big circle – area of small circle

$= \pi \times 14^2 - \pi \times \left(\frac{14}{2}\right)^2$

$= \pi(14^2 - 7^2)$

$= 462 \text{ cm}^2$ (to 3 s.f.)

(d) Area of middle shaded region

$= 48 \times 14 - 2 \times \frac{1}{2} \times \pi \times \left(\frac{14}{2}\right)^2$

$= (672 - 49\pi) \text{ cm}^2$

Area of shaded region

$= 2 \times \frac{1}{2} \times \pi \times \left(\frac{48}{2}\right)^2 + (672 - 49\pi)$

$= 576\pi + (672 - 49\pi)$

$= 2330 \text{ cm}^2$ (to 3 s.f.)

5. **(a)** Diameter of outer circular edge of grass

$= 12 \times 2 + 2 + 2$

$= 28$ m

Circumference of outer circular edge of grass

$= \pi \times 28$

$= 28\pi$ m

(b) Area of land in between $= \pi\left(\frac{28}{2}\right)^2 - \pi(12)^2$

$= \pi(14^2 - 12^2)$

$= 52\pi \text{ m}^2$

6. **(a)** Area of shaded region

= Area of rectangle + area of semicircle

$= 5 \times 4 + \frac{1}{2} \times \pi \times \left(\frac{4}{2}\right)^2$

$= 20 + 2\pi$

$= 26 \text{ m}^2$ (to 2 s.f.)

(b) Total length painted in black

= Circumference of circle + 5 + 4 + 5

$= \pi \times 4 + 14$

$= 27$ m (to 2 s.f.)

7. **(a)** Length of arc $AXB = \frac{82°}{360°} \times 2\pi \times 8$

$= 11.4$ cm (to 3 s.f.)

(b) Length of arc $AXB = \frac{134°}{360°} \times 2\pi \times 14$

$= 32.7$ cm (to 3 s.f.)

(c) Length of arc $AXB = \frac{214°}{360°} \times 2\pi \times 17$

$= 63.5$ cm (to 3 s.f.)

(d) Reflex $\angle AOB = 360° - 46° = 314°$

Length of arc $AXB = \frac{314°}{360°} \times 2\pi \times 9.8$

$= 53.7$ cm (to 3 s.f.)

8. **(a)** **(i)** Length of minor arc $AXB = \frac{76°}{360°} \times 2\pi \times 9$

$= 11.9$ cm (to 3 s.f.)

(ii) Reflex $\angle AOB = 360° - 76° = 284°$

Perimeter of major sector $OAYB$

= length of major arc $AYB + OA + OB$

$= \frac{284°}{360°} \times 2\pi \times 9 + 9 + 9$

$= 62.6$ cm (to 3 s.f.)

(b) **(i)** Length of minor arc $AXB = \frac{112°}{360°} \times 2\pi \times 16$

$= 31.3$ cm (to 3 s.f.)

(ii) Reflex $\angle AOB = 360° - 112° = 248°$

Perimeter of major sector $OAYB$

= length of major arc $AYB + OA + OB$

$= \frac{248°}{360°} \times 2\pi \times 16 + 16 + 16$

$= 101$ cm (to 3 s.f.)

(c) **(i)** $\angle AOB = 360° - 215° = 145°$

Length of minor arc $AXB = \frac{145°}{360°} \times 2\pi \times 17.6$

$= 44.5$ cm (to 3 s.f.)

(ii) Perimeter of major sector $OAYB$

= length of major arc $AYB + OA + OB$

$= \frac{215°}{360°} \times 2\pi \times 17.6 + 17.6 + 17.6$

$= 101$ cm (to 3 s.f.)

9. **(a)** Since the length of minor arc is 26.53 cm,

$\frac{95°}{360°} \times 2\pi \times r = 26.53$

$1.658r = 26.53$

$r = 16.0$ cm (to 3 s.f.)

(b) Since the length of major arc is 104.6 cm,

$\frac{214°}{360°} \times 2\pi \times r = 104.6$

$3.735r = 104.6$

$r = 28.0$ cm (to 3 s.f.)

10. **(a)** Since the length of arc is 12 m,

$\frac{\theta}{360°} \times 2\pi \times 14 = 12$

$0.2443\theta = 12$

$\theta = 49°$ (to the nearest degree)

(b) Since the length of arc is 19.5 m,

$\frac{\theta}{360°} \times 2\pi \times 14 = 19.5$

$0.2443\theta = 19.5$

$\theta = 80°$ (to the nearest degree)

(c) Since the length of arc is 64.2 m,

$\frac{\theta}{360°} \times 2\pi \times 14 = 64.2$

$0.2443\theta = 64.2$

$\theta = 263°$ (to the nearest degree)

(d) Since the length of arc is 84.6,

$\frac{\theta}{360°} \times 2\pi \times 14 = 84.6$

$0.2443\theta = 84.6$

$\theta = 346°$ (to the nearest degree)

11. Distance travelled by the tip of the hour hand $= \frac{45°}{360°} \times 2\pi \times 1.5$

$= 1.18$ m (to 3 s.f.)

12. Since the length of wire is 32 cm,

$\frac{\theta}{360°} \times 2\pi \times 6 + 6 + 6 = 32$

$0.1047\theta + 12 = 32$

$0.1047\theta = 20$

$\theta = 191.0°$ (to 1 d.p.)

13. **(a)** Since the perimeter of minor sector is 77.91 cm,

$\frac{148°}{360°} \times 2\pi \times r + r + r = 77.91$

$2.583r + 2r = 77.91$

$4.583r = 77.91$

$r = 17.0$ cm (to 3 s.f.)

(b) Reflex $\angle AOB = 360° - 44° = 316°$

Since the perimeter of major sector is 278.1 cm,

$\frac{316°}{360°} \times 2\pi \times r + r + r = 278.1$

$5.515r + 2r = 278.1$

$7.515r = 278.1$

$r = 37.0$ cm (to 3 s.f.)

14. Perimeter of arc $AOB = \frac{60°}{360°} \times 2\pi \times 8$

$= \frac{8\pi}{3}$ cm

Perimeter of arc $POQ = \frac{60°}{360°} \times 2\pi \times 17$

$= \frac{17\pi}{3}$ cm

$AP = BQ = 17 - 8 = 9$ cm

Perimeter of shaded region = arc AB + arc $PQ + AP + BQ$

$= \frac{8\pi}{3} + \frac{17\pi}{3} + 9 + 9$

$= \left(18 + \frac{25\pi}{3}\right)$ cm

15. **(i)** Length of minor arc $AOB = \left(\dfrac{\angle AOB}{360°} \times 2\pi r\right)$ cm

Circumference of circle = $(2\pi r)$ cm

Since the length of the minor arc is $\dfrac{7}{24}$ of the circumference of the circle,

$$\frac{\angle AOB}{360°} \times 2\pi r = \frac{7}{24} \times 2\pi r$$
$$\frac{\angle AOB}{360°} = \frac{7}{24}$$
$$\angle AOB = \frac{7}{24} \times 360°$$
$$= 105°$$

(ii) Radius of circle = $\dfrac{14}{2}$ = 7 cm

Length of minor arc = $\dfrac{105°}{360°} \times 2\pi \times 7$

= 12.8 cm (to 3 s.f.)

16. Length of minor arc $OAB = \dfrac{61.82°}{360°} \times 2\pi \times 7.5$

= 8.092 cm (to 4 s.f.)

By Pythagoras' Theorem,

$OP^2 = 7.5^2 + 14^2$

= 252.25

$OP = \sqrt{252.25}$

= 15.88 cm (to 4 s.f.)

$BP = OP - OB = 15.88 - 7.5 = 8.38$ cm

Perimeter of shaded region = arc $OAB + BP + AP$

= 8.092 + 8.38 + 14

= 30.5 cm (to 3 s.f.)

17. Length of minor arc $OPQ = \dfrac{138°}{360°} \times 2\pi \times 26$

= 62.62 cm (to 4 s.f.)

Perimeter of shaded region = arc $OPQ + RP + QR$

= 62.62 + 67.73 + 67.73

= 198 cm (to 3 s.f.)

18. Length of arc = $\dfrac{115.59°}{360°} \times 2\pi \times 13$

= 26.23 cm (to 4 s.f.)

Perimeter of shaded region = 26.23 + 22

= 48.2 cm (to 3 s.f.)

19. **(i)** AP = 16 cm

$OP = OP$ = 9 cm

Using cosine rule,

(ii) Length of arc $ABD = \dfrac{54.54° \times 2}{360°} \times 2\pi \times 9$

= 17.13 cm (to 4 s.f.)

Legnth of arc $ACD = \dfrac{27.27° \times 2}{360°} \times 2\pi \times 16$

= 15.23 cm (to 4 s.f.)

Perimeter of shaded region = 17.13 + 15.23

= 32.4 cm (to 3 s.f.)

20. Let the radius of the circle be r cm.

$\angle OBA = \angle OAB = 30°$

$\angle AOB = 180° - 30° - 30°$ (∠ sum of a △)

= 120°

Length of arc = $\dfrac{360° - 120°}{360°} \times 2\pi \times 7.5$

= 31.42 cm (to 4 s.f.)

Perimeter of shaded region = $\dfrac{15}{2}\sqrt{3} + 31.42$

= 44.4 cm (to 3 s.f.)

21. $\angle POR = 36° + 90°$ (ext. ∠ of △)

= 126°

Length of arc $PR = \dfrac{126°}{360°} \times 2\pi \times 8.229$

= 18.10 cm (to 4 s.f.)

$OR = OP$ = 8.229 cm

TR = 14 + 8.229 = 22.229 cm

Length of arc $QR = \dfrac{36°}{360°} \times 2\pi \times 22.229$

= 13.97 cm (to 4 s.f.)

PQ = 22.229 – 11.33

= 10.899 cm

Perimeter of shaded region = arc PR + arc QR + PQ

= 18.10 + 13.13.97 + 10.899

= 43.0 cm (to 3 s.f.)

Exercise 8B

1. **(a)** Arc length = $\dfrac{72°}{360°} \times 2\pi \times 7$

= 8.80 cm (to 3 s.f.)

Area = $\dfrac{72°}{360°} \times \pi \times 7^2$

= 30.8 cm^2 (to 3 s.f.)

Perimeter = 8.796 + 7 + 7

= 22.8 cm (to 3 s.f.)

(b) Perimeter = 136

$s + 35 + 35 = 136$

$s + 70 = 136$

Arc length, s = 66 mm

$\dfrac{\theta}{360°} \times 2\pi \times 35$ = 66 mm

$\dfrac{7\pi\theta}{36} = 66$

Angle at centre, $\theta = \dfrac{66 \times 36}{7\pi}$

= 108.0° (to 1 d.p.)

Area = $\dfrac{108.04°}{360°} \times \pi \times 35^2$

= 1150 mm^2 (to 3 s.f.)

(c) Area = 1848 mm^2

$\frac{270°}{360°} \times \pi \times r^2 = 1848$

$\frac{3}{4}\pi \times r^2 = 1848$

$r^2 = \frac{2464}{\pi}$

Radius, $r = 28.0$ mm (to 3 s.f.)

Arc length $= \frac{270°}{360°} \times 2\pi \times 28.00$

$= 132$ mm (to 3 s.f.)

Peterimeter $= 131.9 + 28.00 + 28.00$

$= 188$ mm (to 3 s.f.)

(d) Arc length = 220 cm

$\frac{150°}{360°} \times 2\pi \times r = 220$

$\frac{5}{6}\pi r = 220$

Radius, $r = 84.0$ cm (to 3 s.f.)

Area $= \frac{150°}{360°} \times \pi \times 84.03^2$

$= 9240$ cm^2 (to 3 s.f.)

Perimeter $= 220 + 84.03 + 84.03$

$= 388$ cm (to 3 s.f.)

(e) Arc length = 55 m

$\frac{\theta}{360°} \times 2\pi \times 14 = 55$

$\frac{7}{90}\pi\theta = 55$

Angle at centre, $\theta = \frac{55 \times 90}{7\pi}$

$= 225.1°$ (to 1 d.p.)

Area $= \frac{225.09°}{360°} \times \pi \times 14^2$

$= 385$ m^2 (to 3 s.f.)

Perimeter $= 55 + 14 + 14$

$= 83$ m

(f) Area = 154 cm^2

$\frac{75°}{360°} \times \pi \times r^2 = 154$

$\frac{5}{24}\pi \times r^2 = 154$

$r^2 = \frac{739.2}{\pi}$

$r = 15.3$ cm (to 3 s.f.)

Arc length $= \frac{75°}{360°} \times 2\pi \times 15.34$

$= 20.1$ cm (to 3 s.f.)

Perimeter $= 20.08 + 15.34 + 15.34$

$= 50.8$ cm (to 3 s.f.)

2. **(a)** **(i)** Perimeter $= \frac{30°}{360°} \times 2\pi \times 7 + 7 + 7$

$= 17.7$ cm (to 3 s.f.)

(ii) Area $= \frac{30°}{360°} \times \pi \times 7^2$

$= 12.8$ cm^2 (to 3 s.f.)

(b) **(i)** Perimeter $= \frac{360° - 340°}{360°} \times 2\pi \times 3.5 + 3.5 + 3.5$

$= 8.22$ cm (to 3 s.f.)

(ii) Area $= \frac{20°}{360°} \times \pi \times 3.5^2$

$= 2.14$ cm^2 (to 3 s.f.)

(c) **(i)** Perimeter $= \frac{140°}{360°} \times 2\pi \times 6 + 6 + 6$

$= 26.7$ cm (to 3 s.f.)

(ii) Area $= \frac{140°}{360°} \times \pi \times 6^2$

$= 44.0$ cm^2 (to 3 s.f.)

3. Circumference of circle = 88 cm

$2\pi \times r = 88$

$r = 14.01$ cm (to 4 s.f.)

(a) Length of arc $ACB = \frac{60°}{360°} \times 2\pi \times 14.01$

$= 14.7$ cm (to 3 s.f.)

Area of sector $OACB = \frac{60°}{360°} \times \pi \times 14.01^2$

$= 103$ cm^2 (to 3 s.f.)

(b) Length of arc $ACB = \frac{99°}{360°} \times 2\pi \times 14.01$

$= 24.2$ cm (to 3 s.f.)

Area of sector $OACB = \frac{99°}{360°} \times \pi \times 14.01^2$

$= 169$ cm^2 (to 3 s.f.)

(c) Length of arc $ACB = \frac{126°}{360°} \times 2\pi \times 14.01$

$= 30.8$ cm (to 3 s.f.)

Area of sector $OACB = \frac{126°}{360°} \times \pi \times 14.01^2$

$= 216$ cm^2 (to 3 s.f.)

(d) Length of arc $ACB = \frac{216°}{360°} \times 2\pi \times 14.01$

$= 52.8$ cm (to 3 s.f.)

Area of sector $OACB = \frac{216°}{360°} \times \pi \times 14.01^2$

$= 370$ cm^2 (to 3 s.f.)

4. Area of circle = 3850 cm^2

$$\pi \times r^2 = 3850$$

$$r^2 = \frac{3850}{\pi}$$

$$r = 35.00 \text{ cm (to 4 s.f.)}$$

(a) Area of sector $OPSQ = \frac{36°}{360°} \times \pi \times 35.00^2$

$= 385 \text{ cm}^2$ (to 3 s.f.)

Length of arc $PSQ = \frac{36°}{360°} \times 2\pi \times 35.00$

$= 22.0$ cm (to 3 s.f.)

(b) Area of sector $OPSQ = \frac{84°}{360°} \times \pi \times 35.00^2$

$= 898 \text{ cm}^2$ (to 3 s.f.)

Length of arc $PSQ = \frac{84°}{360°} \times 2\pi \times 35.00$

$= 51.3$ cm (to 3 s.f.)

(c) Area of sector $OPSQ = \frac{108°}{360°} \times \pi \times 35.00^2$

$= 1150 \text{ cm}^2$ (to 3 s.f.)

Length arc $PSQ = \frac{108°}{360°} \times 2\pi \times 35.00$

$= 66.0$ cm (to 3 s.f.)

(d) Area of sector $OPSQ = \frac{198°}{360°} \times \pi \times 35.00^2$

$= 2120 \text{ cm}^2$ (to 3 s.f.)

Legnth of arc $= \frac{198°}{360°} \times 2\pi \times 35.00$

$= 121$ cm (to 3 s.f.)

5. **(a)** Area of minor sector = 114 cm^2

$$\frac{150°}{360°} \times \pi \times r^2 = 114$$

$$\frac{5}{12} \pi \times r^2 = 114$$

$$r^2 = \frac{273.6}{\pi}$$

$$r = 9.33 \text{ cm (to 3 s.f.)}$$

(b) Area of major sector = 369 cm^2

$$\frac{360° - 66°}{360°} \times \pi\, r^2 = 369$$

$$\frac{49}{60} \pi \times r^2 = 369$$

$$r^2 = \frac{22\,140}{\pi}$$

$$r = 12.0 \text{ cm (to 3 s.f.)}$$

6. Radius of circle $= \frac{18}{2} = 9$ cm

(a) Area of sector = 42.6 cm^2

$$\frac{\theta}{360°} \times \pi \times 9^2 = 42.6$$

$$\frac{9}{40} \pi\theta = 42.6$$

$$\theta = 60.3° \text{ (to 1 d.p.)}$$

(b) Area of sector = 117.2 cm^2

$$\frac{\theta}{360°} \times \pi \times 9^2 = 117.2$$

$$\frac{9}{40} \pi\theta = 117.2$$

$$\theta = 165.8° \text{ (to 1 d.p.)}$$

(c) Area of sector = 214.5 cm^2

$$\frac{\theta}{360°} \times \pi \times 9^2 = 214.5$$

$$\frac{9}{40} \pi\theta = 214.5$$

$$\theta = 303.5° \text{ (to 1 d.p.)}$$

(d) Area of sector = 18.9 cm^2

$$\frac{\theta}{360°} \times \pi \times 9^2 = 18.9$$

$$\frac{9}{40} \pi\theta = 18.9$$

$$\theta = 26.7° \text{ (to 1 d.p.)}$$

7. **(i)** Length of arc $AB = \frac{45°}{360°} \times 2\pi \times 10$

$= 7.854$ cm (to 4 s.f.)

Length of arc $CD = \frac{45°}{360°} \times 2\pi \times 20$

$= 15.71$ cm (to 4 s.f.)

$AD = BC = 20 - 10 = 10$ cm

Perimeter of shaded region $= 7.854 + 15.71 + 10 + 10$

$= 43.6$ cm (to 3 s.f.)

Area of sector $OAB = \frac{45°}{360°} \times \pi \times 10^2$

$= 39.27 \text{ cm}^2$ (to 4 s.f.)

Area of sector $ODC = \frac{45°}{360°} \times \pi \times 20^2$

$= 157.1 \text{ cm}^2$ (to 4 s.f.)

Area of shaded region $= 157.1 - 39.27$

$= 118 \text{ cm}^2$ (to 3 s.f.)

(ii) Length of arc $AB = \frac{120°}{360°} \times 2\pi \times 5$

$= 10.47$ cm (to 4 s.f.)

Length of arc $CD = \frac{120°}{360°} \times 2\pi \times 8$

$= 16.76$ cm (to 4 s.f.)

$AD = BC = 8 - 5 = 3$ cm

Perimeter of shaded region $= 10.47 + 16.76 + 3 + 3$

$= 33.2$ cm (to 3 s.f.)

Area of sector $OAB = \frac{120°}{360°} \times \pi \times 5^2$

$= 26.18 \text{ cm}^2$ (to 4 s.f.)

$$\text{Area of sector } ODC = \frac{120°}{360°} \times \pi \times 8^2$$
$$= 67.02 \text{ cm}^2 \text{ (to 4 s.f.)}$$
$$\text{Area of shaded region} = 67.02 - 26.18$$
$$= 40.8 \text{ cm}^2 \text{ (to 3 s.f.)}$$

(iii) $$\text{Length of arc } AB = \frac{160°}{360°} \times 2\pi \times 35$$
$$= 97.74 \text{ cm (to 4 s.f.)}$$
$$\text{Length of arc } CD = \frac{160°}{360°} \times 2\pi \times 49$$
$$= 136.8 \text{ cm (to 4 s.f.)}$$
$$AD = BC = 49 - 35 = 14 \text{ cm}$$
$$\text{Perimeter of shaded region} = 97.74 + 136.8 + 14 + 14$$
$$= 263 \text{ cm (to 3 s.f.)}$$
$$\text{Area of sector } OAB = \frac{160°}{360°} \times \pi \times 35^2$$
$$= 1710 \text{ cm}^2 \text{ (to 4 s.f.)}$$
$$\text{Area of sector } ODC = \frac{160°}{360°} \times \pi \times 49^2$$
$$= 3352 \text{ cm}^2 \text{ (to 4 s.f.)}$$
$$\text{Area of shaded region} = 3352 - 1710$$
$$= 1640 \text{ cm}^2 \text{ (to 3 s.f.)}$$

8. **(i)** Since the shaded area POQ is $\frac{5}{18}$ of the area of the whole circle,

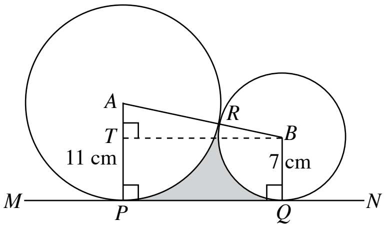
$$\frac{\angle POQ}{360°} \times \pi r^2 = \frac{5}{18} \times \pi r^2$$
$$\frac{\angle POQ}{360°} = \frac{5}{18}$$
$$\angle POQ = \frac{5}{18} \times 360°$$
$$= 100°$$

(ii) $$\text{Area of shaded sector} = 385 \text{ cm}^2$$
$$\frac{100°}{360°} \times \pi \times r^2 = 385$$
$$\frac{5}{18}\pi r^2 = 385$$
$$r^2 = \frac{1386}{\pi}$$
$$r = 21.00 \text{ cm (to 4 s.f.)}$$
$$\text{Diameter of circle} = 21.00 \times 2$$
$$= 42.0 \text{ cm (to 3 s.f.)}$$

9. Perimeter = 38 cm

$$\text{Arc length} + 12 + 12 = 38$$
$$\text{Arc length} = 38 - 12 - 12$$
$$= 14 \text{ cm}$$
$$\frac{\theta}{360°} \times 2\pi \times 12 = 14$$
$$\frac{\pi\theta}{15} = 14$$
$$\theta = 66.85° \text{ (to 2 d.p.)}$$
$$\text{Area of paper used} = \frac{66.85°}{360°} \times \pi \times 12^2$$
$$= 84.0 \text{ cm}^2 \text{ (to 3 s.f.)}$$

10. **(i)** Draw a line BT such that T lies on AP and BT is perpendicular to AP.



$$AT = AP - TP$$
$$= 11 - 7$$
$$= 4 \text{ cm}$$
$$AB = AR + RB$$
$$= 11 + 7$$
$$= 18 \text{ cm}$$

Using Pythagoras' Theorem,

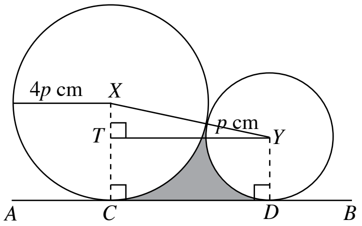
$$BT^2 = 18^2 - 4^2$$
$$BT = \sqrt{18^2 - 4^2}$$
$$= \sqrt{308}$$
$$= 17.55 \text{ cm (to 4 s.f.)}$$
$$\angle ABT = 180° - 90° - 77.16° \text{ } (\angle \text{ sum of a } \triangle)$$
$$= 12.84°$$
$$\angle ABQ = 90° + 12.84° = 102.84°$$

Area of shaded region

= Area of trapezium $ABPQ$ – area of sector APR – area of sector RBQ

$$= \frac{1}{2}(11 + 7)(17.55) - \frac{77.16°}{360°} \times \pi \times 11^2 - \frac{102.84°}{360°} \times \pi \times 7^2$$
$$= 32.5 \text{ cm}^2 \text{ (to 3 s.f.)}$$

11. Draw a line YT such that T lies on XC and YT is perpendicular to XC.



$$XT = XC - TC$$
$$= 4p - p$$
$$= 3p \text{ cm}$$
$$XY = 4p + p$$
$$= 5p \text{ cm}$$

Using Pythagoras' Theorem,

$$YT^2 = XY^2 - XT^2$$
$$= (5p)^2 - (3p)^2$$
$$= 25p^2 - 9p^2$$
$$= 16p^2$$
$$YT = \sqrt{16p^2} = 4p \text{ cm}$$
$$\angle XYT = 180° - 90° - 53.13° \text{ } (\angle \text{ sum of a } \triangle)$$
$$= 36.87°$$

$\angle XYD = 90° + 36.87° = 126.87°$

Area of enclosed region

$= \frac{1}{2}(4p + p)(4p) - \frac{53.13°}{360°} \times \pi \times (4p)^2 - \frac{126.87°}{360°} \times \pi \times p^2$

$= 10p^2 - 7.418p^2 - 1.107p^2$

$= 1.47p^2$ cm^2 (to 3 s.f.)

12. (i) Since $OQ = OA = 16$ cm,

$\angle OQA = \angle OAQ = 66°$

$\angle BOQ = 2 \times 66°$ (ext. $\angle$ of $\triangle$)

$= 132°$

(ii) Length of arc $QB = \frac{132°}{360°} \times 2\pi \times 16$

$= 36.86$ cm (to 4 s.f.)

Length of arc $PYB = \frac{66°}{360°} \times 2\pi \times 32$

$= 36.86$ cm (to 4 s.f.)

$PQ = 32 - 13.02 = 18.98$ cm

Perimeter of shaded region $= 36.86 + 36.86 + 18.98$

$= 92.7$ cm (to 3 s.f.)

(iii) Area of sector $BOQ = \frac{132°}{360°} \times \pi \times 16^2$

$= 294.9$ cm^2 (to 4 s.f.)

Area of sector $APYB = \frac{66°}{360°} \times \pi \times 32^2$

$= 589.8$ cm^2 (to 4 s.f.)

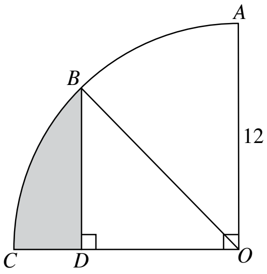
Area of $\triangle AOQ = \frac{1}{2} \times 16 \times 13.02 \times \sin 66°$

$= 95.15$ cm^2 (to 4 s.f.)

Area of shaded region $= 589.8 - 294.9 - 95.15$

$= 200$ cm^2 (to 3 s.f.)

13. (i)



Since B is the midpoint of arc AC,

$\angle BOC = 45°$

$OB = OA = 12$ cm

$\sin 45° = \frac{BD}{12}$

$BD = 12 \sin 45°$

$= 8.49$ cm (to 3 s.f.)

(ii) $\cos 45° = \frac{OD}{12}$

$OD = 12 \cos 45°$

$= 8.485$ cm (to 4 s.f.)

$CD = 12 - 8.485 = 3.515$ cm

Length of arc $CB = \frac{45°}{360°} \times 2\pi \times 12$

$= 9.425$ cm (to 4 s.f.)

Perimeter of shaded region $= 8.485 + 3.515 + 9.425$

$= 21.4$ cm (to 3 s.f.)

(iii) Area of sector $OBC = \frac{45°}{360°} \times \pi \times 12^2$

$= 56.55$ cm^2 (to 4 s.f.)

Area of $\triangle BDO = \frac{1}{2} \times 8.485 \times 8.485$

$= 36.00$ cm^2 (to 4 s.f.)

Area of shaded region $= 56.55 - 36.00$

$= 20.5$ cm^2 (to 3 s.f.)

Review Exercise 8

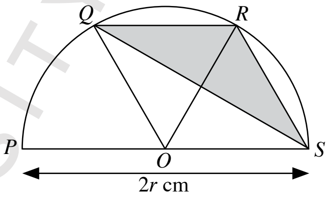
1. (i) Length of arc $BPA = \frac{360° - 120°}{360°} \times 2\pi \times 12$

$= 5.03$ cm (to 3 s.f.)

(ii) Area of sector $OBPA = \frac{240°}{360°} \times \pi \times 12^2$

$= 302$ cm^2 (to 3 s.f.)

2.



Since lengths of arcs $PQ = QR = RS$,

$\angle POQ = \angle QOR = \angle ROS = \frac{\pi}{3}$ rad

Using cosine rule,

$QR^2 = r^2 + r^2 - 2 \times r \times r \times \cos \frac{\pi}{3}$

$= 2r^2 - r^2$

$= r^2$

$QR = r$ cm

$RS^2 = r^2 + r^2 - 2 \times r \times r \times \cos \frac{\pi}{3}$

$= 2r^2 - r^2$

$= r^2$

$RS = r$ cm

$\angle QRS = \angle QRO + \angle ORS$

$= \frac{\pi}{3} + \frac{\pi}{3}$

$= \frac{2\pi}{3}$ rad

Area of shaded region $= \frac{1}{2} \times r \times r \times \sin \frac{2\pi}{3}$

$= 0.433r^2$ cm^2 (to 3 s.f.)

Challenge Yourself

1. **(a)** Perimeter = arc $AB + AB$

$$= \pi \times \frac{14}{2} + 14$$

$$= (7\pi + 14) \text{ cm}$$

(b) Perimeter $= \left(\pi \times \frac{14}{2}\right) + \left(\pi \times \frac{14}{2}\right) + 14$

$$= (7\pi + 14) \text{ cm}$$

(c) Perimeter $= \left(\pi \times \frac{14}{6}\right) + \left(\pi \times \frac{14}{6}\right) + \left(\pi \times \frac{14}{6}\right) + 14$

$$= (7\pi + 14) \text{ cm}$$

(d) Perimeter $= \left(\pi \times \frac{14}{8}\right) + \left(\pi \times \frac{14}{8}\right) + \left(\pi \times \frac{14}{8}\right) + \left(\pi \times \frac{14}{8}\right) + 14$

$$= (7\pi + 14) \text{ cm}$$

All the perimeters are equal.

In general, it does not matter how many identical semicircles are on the line AB, all the perimeters are equal.

2. **(i)** By Pythagoras' Theorem,

$$\begin{aligned} AP^2 &= PB^2 + AB^2 \\ (12 + r)^2 &= (12 - r)^2 + 12^2 \\ 144 + 24r + r^2 &= r^2 - 24r + 144 + 144 \\ 144 + 24r + r^2 &= r^2 - 24r + 288 \\ 24r &= -24r + 144 \\ 48r &= 144 \\ r &= 3 \end{aligned}$$

(ii) $AP = 12 + 3 = 15$ cm

$PB = 12 - 3 = 9$ cm

Using cosine rule,

$$\begin{aligned} \cos \angle PAB &= \frac{15^2 + 12^2 - 9^2}{2 \times 15 \times 12} \\ &= 0.8 \\ \angle PAB &= \cos^{-1} 0.8 \\ &= 0.644 \text{ rad (to 3 s.f.)} \end{aligned}$$

$\therefore \angle PAC = 0.644$ rad

(iii) $\angle PCA = \angle PAC = 0.6435$ rad

$\angle APC = \pi - 0.6435 - 0.6435$ ($\angle$ sum of a $\triangle$)

$= 1.855$ rad

Area of minor sector $RPS = \frac{1}{2} \times 3^2 \times 1.855$

$= 8.346 \text{ cm}^2$ (to 4 s.f.)

Area of minor sector $ARB = \frac{1}{2} \times 12^2 \times 0.6435$

$= 46.33 \text{ cm}^2$ (to 4 s.f.)

Area of minor sector BCS = Area of minor sector ARB

$= 46.33 \text{ cm}^2$

Area of $\triangle APC = \frac{1}{2} \times 15 \times 15 \times \sin 1.855$

$= 108.0 \text{ cm}^2$ (to 4 s.f.)

Area of shaded region $= 108.0 - 46.33 - 46.33 - 8.346$

$= 6.99 \text{ cm}^2$ (to 3 s.f.)

Chapter 9 Volume and Surface Area of Pyramids, Cones and Spheres

TEACHING NOTES

Suggested Approach

In the previous grades, students have learnt to find the volume and surface area of cubes, cuboids, prisms and cylinders. Here, they will learn to determine the volume and surface area of other regular figures, the pyramid, cone and sphere. By the end of this chapter, students are to be familiar with the various formulas in calculating the volume and surface area, as well as the various real-life examples of such figures. When the value of π is not stated, students are to use the value in the calculator. In some problems, students are expected to recall and apply Pythagoras' Theorem.

Section 9.1: Volume and Surface Area of Pyramids

As an introduction, teachers can show students some real-life examples of pyramids and question students on the properties of pyramids (see Class Discussion: What are Pyramids?)

Teachers should go through the part of a pyramid. Following that, students should observe and recognise the various types of pyramids. Teachers should indicate that the pyramids studied in this chapter are right pyramids, where the apex is vertically above the centre of the base and the base is a regular polygon.

The activity in determining the volume of a pyramid is to enable students to appreciate the relation between the volume of a pyramid and its corresponding prism (see Investigation: Volume of Pyramids).

Section 9.2: Volume and Surface Area of Cones

Similar to pyramids, teachers can start off with an activity to introduce cones (see Class Discussion: What are Cones?).

To improve and enhance understanding, students should learn and explain the features of a cone and state the differences between a cone, a cylinder and a pyramid (see Journal Writing on page 258, and Investigation: Comparison between a Cone and a Pyramid).

Proceeding on, students should realise that the volume and total surface area of a cone is analogous to the volume and total surface area of a pyramid. The curved surface area of a cone is one unique calculation that has to be noted.

Section 9.3: Volume and Surface Area of Spheres

Besides the volume and surface area of a sphere, students have to be aware of the volume and total surface area of a hemisphere, or half a sphere as well. Teachers should demonstrate how the volume and surface area of a sphere can be obtained (see Investigation: Volume of Spheres and Investigation: Surface Area of Spheres), and show the simple steps in deriving the volume and total surface area of a hemisphere (see Thinking Time on page 271). This will minimise the formulas students need to recall.

Section 9.4 Volume and Surface Area of Composite Solids

In this section, students are required to make calculations involving the various composite solids made up of regular figures. Besides the ones covered in this chapter, regular figures from previous grades, such as cubes, cuboids, prisms and cylinders may be included. Weaker students may need a revision of their formulas for volume and total surface area.

In calculating the total surface area, students must be careful not to include any sides that are overlapping. It is good practice to state and calculate the volume and total surface area part by part.

Challenge Yourself

A regular tetrahedron is a solid made up of four equilateral triangular faces. Therefore, it is a pyramid regardless of which side it lies on. This information is required for Question 1.

For Question 2, to derive and prove the statement, students should observe that the length from the centre of the top of the hemisphere to the side of the depth of the water in the sphere is also its radius. After applying Pythagoras' Theorem, the statement should follow after a little logical reasoning.

WORKED SOLUTIONS

Class Discussion (What are Pyramids?)

1. The pyramids are made up of one base and four triangular faces joined to the sides of the base. The four triangles are joined by a single point at the other end.
2. The slanted faces of the pyramids are congruent, isosceles triangles.
3. The bases of these pyramids are squares.
4. The vertex of a pyramid is the point where the vertices of the triangle are joined to the vertices of the base. The apex of a pyramid is the point vertically above the base, where the triangles are joined to each other.
5. The cross sections of a pyramid are squares and are not uniform throughout the pyramid.
6. The food pyramids, human pyramid and rice dumplings are pyramids and they have the same features as the pyramids in Fig. 10.1.
7. Three more real-life examples of pyramids are the roof of a house, tents, packets of milk etc.

Thinking Time (Page 248)

The slant edge is the hypotenuse of a right-angled triangle, together with the height of the pyramid and half of the diagonal of the base.

The slant height is the hypotenuse of another right-angled triangle, together with the height of the pyramid and half the side of the base.

The slant faces of regular pyramids are congruent, isosceles triangles.

Journal Writing (Page 248)

Prisms have two polygonal bases that are congruent and parallel to each other while pyramids have only one polygonal base with an apex vertically above it.

The sides of a prism are made up of rectangles while the sides of a pyramid are triangles that are joined at the apex.

The cross-section of a prism is uniform while the cross-section of a pyramid is non-uniform.

Investigation (Volume of Pyramids)

It will take 3 times to fill the prism completely.
Volume of pyramid = $\underline{3}$ × volume of corresponding prism

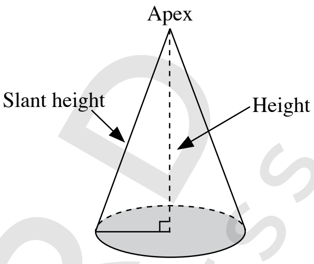
Class Discussion (What are Cones?)

1. The cones have a circular base with a curved surface and an apex opposite the base.
2. The base of a cone and a cylinder is a circle. The sides of a cone and a cylinder are curved surfaces.

   A cone has one circular base while a cylinder has two circular bases. A cone has an apex opposite its base while a cylinder does not have an apex. The cross-section of a cone is non-uniform while the cross-section of a cylinder is uniform.
3. Both the cone and pyramid have one base only. Both the cone and pyramid have an apex. The cross-section of both the cone and pyramid are non-uniform.

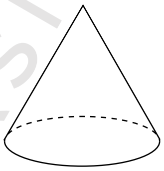
   The base of a cone is a circle while the base of a pyramid is a polygon. The side of a cone is a curved surface while the sides of a pyramid are made up of triangles. The cross-section of a cone is a circle while the cross-section of a pyramid is a polygon.
4. Three more real-life examples of cones are traffic cones, tents and mountains etc.

Journal Writing (Page 258)

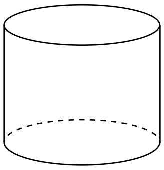


A cone is a solid in which the base is bounded by a simple closed curve and the curved surface tapers into a point called the apex, which is opposite the base. If the apex is vertically above the centre of the circular base, we call the cone a right circular cone.

The perpendicular height (or height) of a cone is the perpendicular distance from the apex to the base of the cone. The slant height of a right circular cone is the distance from the apex to the circumference of the base.

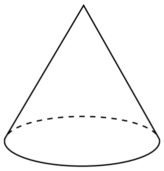


Cone

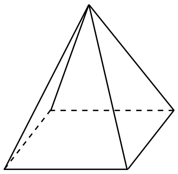


Cylinder

A cone has one circular base while a cylinder has two circular bases.
A cone has an apex opposite its base while a cylinder does not have an apex. The cross-section of a cone is non-uniform while the cross-section of a cylinder is uniform.



Cone



Pyramid

The base of a cone is a circle while the base of a pyramid is a polygon. The side of a cone is a curved surface while the sides of a pyramid are made up of triangles. The cross-section of a cone is a circle while the cross-section of a pyramid is a polygon.

Investigation (Comparison between a Cone and a Pyramid)

1. The polygon will become a circle.
2. The pyramid will become a cone.

Thinking Time (Page 260)

Volume of a cone = $\frac{1}{3}\pi r^2\text{h}$

Volume of a cylinder = $\pi r^2 h$

Since the cone and cylinder have the same base and same height,

$\therefore$ Volume of cone = $\frac{1}{3}$ × volume of a cylinder

Investigation (Curved Surface Area of Cones)

If the number of sectors is increased indefinitely, then the shape in Fig. 10.15(b) will become a $\underline{\text{rectangle}}$ *PQRS*.

Since $PQ + RS$ = circumference of the base circle in Fig. 10.15(a), then the length of the rectangle is $PQ = \underline{\pi r}$.

Since *PS* = slant height of the cone in Fig. 10.15(a), then the breadth of the rectangle is $PS = \underline{l}$.

$$\begin{aligned}\therefore \text{Curved surface area of cone} &= \text{area of rectangle}\\ &= \underline{PQ} \times \underline{PS}\\ &= \underline{\pi rl}\end{aligned}$$

Thinking Time (Page 263)

Total surface of a solid cone

= Curved surface area of cone + base area of cone

= $\pi rl + \pi r^2$

Thinking Time (Page 267)

A hemisphere is half a sphere.

Some real-life examples of hemispheres are bowls, stadium domes, the base of a tilting doll etc.

Class Discussion (Is the King's Crown Made of Pure Gold?)

$$\begin{aligned}\text{Density of the crown} &= \frac{11.6\text{ kg}}{714\text{ cm}^3}\\ &= \frac{(11.6 \times 1000)\text{ g}}{714\text{ cm}^3}\\ &= \frac{11\,600\text{ g}}{714\text{ cm}^3}\\ &= 16.2\text{ g/cm}^3 \text{ (to 3 s.f.)}\end{aligned}$$

Since 16.2 g/cm^3 ≠ 19.3 g/cm^3, the crown was not made of pure gold.

Investigation (Volume of Spheres)

$$\begin{aligned}\text{Volume of cylinder} &= \pi r^2 h\\ &= \pi \times r^2 \times 2r\\ &= \underline{2\pi r^3}\end{aligned}$$

$$\begin{aligned}\text{Volume of sphere} &= \frac{2}{3} \times \text{volume of cylinder}\\ &= \frac{2}{3} \times \underline{2\pi r^3}\\ &= \underline{\frac{4}{3}\pi r^3}\end{aligned}$$

Investigation (Surface Area of Spheres)

Part I:

$$\begin{aligned}\text{Length of second piece of twine} &= 2\pi rh\\ &= 2\pi \times r \times r\\ &= \underline{2\pi r^2}\end{aligned}$$

$$\begin{aligned}\text{Curved surface area of sphere} &= 2 \times \text{length of first piece of twine}\\ &= 2 \times \text{length of second piece of twine}\\ &= 2 \times \underline{2\pi r^2}\\ &= \underline{4\pi r^2}\end{aligned}$$

Part II:

4. 4 circles are covered completely with the orange skin.
5. Surface area of the orange = $4\pi r^2$

Thinking Time (Page 271)

Total surface of a solid hemisphere

= Curved surface area of hemisphere + Base area of hemisphere

= $\frac{1}{2} \times 4\pi r^2 + \pi r^2$

= $2\pi r^2 + \pi r^2$

= $3\pi r^2$

Practise Now 1

1. Volume of triangular pyramid

$$\begin{aligned}&= \frac{1}{3} \times \text{base area} \times \text{height}\\ &= \frac{1}{3} \times 36 \times 7\\ &= 84\text{ cm}^3\end{aligned}$$

2. Volume of the pyramid

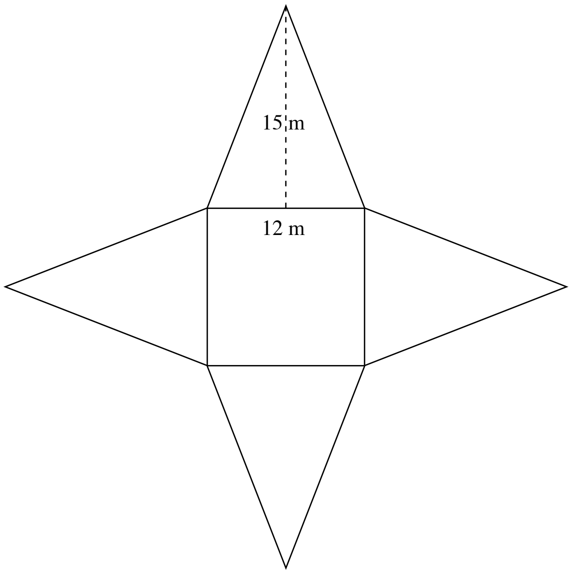
$$\begin{aligned}&= \frac{1}{3} \times \text{base area} \times \text{height}\\ &= \frac{1}{3} \times 229 \times 229 \times 146\\ &= 2\,550\,000\text{ m}^3 \text{ (to 3 s.f.)}\end{aligned}$$

Practise Now 2

$$\begin{aligned}\text{Volume of pyramid} &= \frac{1}{3} \times \text{base area} \times \text{height}\\ 75 &= \frac{1}{3} \times (5 \times 5) \times \text{height}\\ 75 &= \frac{25}{3} \times \text{height}\end{aligned}$$

$\therefore$ Height = 9 m

Practise Now 3



Area of each triangular face $= \frac{1}{2} \times 12 \times 15$

$= 90\text{ m}^2$

Area of square base $= 12 \times 12$

$= 144\text{ m}^2$

$\therefore$ Total surface area of pyramid

$= 4 \times$ area of each triangular face + area of square base

$= 4 \times 90 + 144$

$= 504\text{ m}^2$

Practise Now 4

(i) Total surface area of pyramid

= 4 × area of each triangular face + area of square base

Area of each triangular face

$= \frac{\text{Total surface area of pyramid} - \text{area of square base}}{4}$

$= \frac{161 - (7 \times 7)}{4}$

$= \frac{112}{4}$

$= 28\text{ cm}$

Area of $\triangle VQR = \frac{1}{2} \times 7 \times VB = 28$

$\frac{7}{2} \times VB = 28$

$VB = 8\text{ cm}$

(ii) Let the point where the vertical from V meets the square base be P.

$PB = \frac{1}{2} \times PQ$

$= \frac{1}{2} \times 7$

$= 3.5\text{ cm}$

In $\triangle VPB$, $\angle P = 90°$.

Using Pythagoras' Theorem,

$VB^2 = VP^2 + PB^2$

$8^2 = VP^2 + 3.5^2$

$VP^2 = 8^2 - 3.5^2$

$= 64 - 12.25$

$= 51.75$

$\therefore VP = \sqrt{51.75}$ (since $VP > 0$)

$\therefore$ Volume of pyramid $= \frac{1}{3} \times$ base area $\times$ height

$= \frac{1}{3} \times 7 \times 7 \times \sqrt{51.75}$

$= 117\text{ cm}^3$ (to 3 s.f.)

Practise Now 5

1. Volume of cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times 8^2 \times 17$

$= 362\frac{2}{3}\pi$

$= 1140\text{ cm}^3$ (to 3 s.f.)

2. Volume of cone $= \frac{1}{3}\pi r^2 h$

$84\pi = \frac{1}{3} \times \pi \times 6^2 \times h$

$84\pi = 12\pi h$

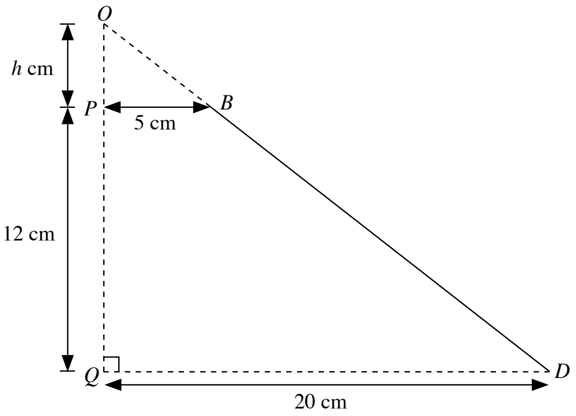
$\therefore h = 7\text{ m}$

The height of the cone is 7 m.

Practise Now 6

Let the height of the smaller cone be h cm.

Then the height of the bigger cone is $(h + 12)$ cm.



Since $\triangle OPB$ is similar to $\triangle OQD$,

$$\frac{OP}{OQ} = \frac{PB}{QD}$$

$$\frac{h}{h+12} = \frac{5}{20}$$

$$\frac{h}{h+12} = \frac{1}{4}$$

$$4h = h + 12$$

$$3h = 12$$

$$h = 4$$

$$\therefore \text{ Height of bigger cone} = 12 + 4$$
$$= 16 \text{ cm}$$

$$\therefore \text{ Volume of frustum} = \text{volume of bigger cone} - \text{volume of smaller cone}$$

$$= \frac{1}{3}\pi R^2 H - \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi(R^2 H - r^2 h)$$

$$= \frac{1}{3}\pi(20^2 \times 16 - 5^2 \times 4)$$

$$= \frac{1}{3}\pi(6300)$$

$$= 2100\pi$$

$$= 6600 \text{ cm}^3 \text{ (to 3 s.f.)}$$

Practise Now 7

1. $\text{Total surface area of cone} = \pi rl + \pi r^2$

$$= \pi \times 9 \times 5 + \pi + 9^2$$
$$= 45\pi + 81\pi$$
$$= 126\pi$$
$$= 396 \text{ cm}^2 \text{ (to 3 s.f.)}$$

2. $\text{Total surface area of cone} = \pi rl + \pi r^2$

$$350 = \pi \times 8 \times l + \pi \times 8^2$$
$$= 8\pi l + 64\pi$$
$$8\pi l = 350 - 64\pi$$
$$\therefore l = \frac{350 - 64\pi}{8\pi}$$
$$= \frac{350 - 64 \times 3.142}{8 \times 3.142}$$
$$= 5.92 \text{ m (to 3 s.f.)}$$

Practise Now 8

1. Let the slant height of the cone be l m.

Using Pythagoras' Theorem, $l = \sqrt{8^2 + 15^2}$

$$= 17$$

$$\therefore \text{ Curved surface area of cone} = \pi rl$$
$$= \pi \times 8 \times 17$$
$$= 136\pi$$
$$= 427 \text{ m}^2 \text{ (to 3 s.f.)}$$

2. Let the height of the cone be h cm.

Using Pythagoras' Theorem, $h = \sqrt{12^2 + 7^2}$

$$= 9.747 \text{ (to 4 s.f.)}$$

$$\therefore \text{ Volume of the cone} = \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3} \times \pi \times 7^2 \times 9.747$$
$$= 500 \text{ cm}^3 \text{ (to 3 s.f.)}$$

Practise Now 9

1. $\text{Radius of ball bearing} = 0.4 \div 2$

$$= 0.2 \text{ cm}$$

$$\text{Volume of ball bearing} = \frac{4}{3}\pi r^3$$
$$= \frac{4}{3} \times \pi \times 0.2^3$$
$$= \frac{4\pi}{375} \text{ cm}^3$$

Mass of 5000 ball bearings

= volume of 5000 ball bearings × density

$$= 5000 \times \frac{4\pi}{375} \times 11.3$$

$$= 1890 \text{ g (to 3 s.f.)}$$

2. $\text{Volume of basketball} = 5600$

$$\frac{4}{3}\pi r^3 = 5600$$
$$r^3 = \frac{4200}{\pi}$$
$$\therefore r = \sqrt[3]{\frac{4200}{\pi}}$$
$$= 11.0 \text{ cm (to 3 s.f.)}$$

The radius of the basketball is 11.0 cm.

Practise Now 10

$$\text{Radius of sphere} = 25 \div 2$$
$$= 12.5 \text{ cm}$$

$$\text{Surface area of sphere} = 4\pi r^2$$
$$= 4 \times \pi \times 12.5^2$$
$$= 6.25\pi$$
$$= 1960 \text{ cm}^2 \text{ (to 3 s.f.)}$$

Practise Now 11

$$\text{Curved surface area of hemisphere} = 200 \text{ cm}^2$$

$$\frac{1}{2} \times 4\pi r^2 = 200$$
$$2\pi r^2 = 200$$
$$r^2 = \frac{100}{\pi}$$
$$\therefore r = \sqrt{\frac{100}{\pi}} \quad (\text{since } r > 0)$$
$$= 5.64 \text{ cm (to 3 s.f.)}$$

Practise Now 12

Height of cone $= \frac{3}{4} \times$ height of cylinder

$= \frac{3}{4} \times 3r$

$= \frac{9}{4}r$

Volume of cone $= \frac{1}{3}\pi r^2 \ \ \frac{9}{4}r$

$= \frac{3}{4}\pi r^3$

Since volume of cone = 10 l = 10 000 cm^3,

then $\frac{3}{4}\pi r^3 = 10\ 000$

$r^3 = \frac{10\ 000 \times 4}{3\pi}$

$= \frac{40\ 000}{3\pi}$

Volume of cylinder $= \pi r^2(3r)$

$= 3\pi r^3$

$= 3\pi \times \frac{40\ 000}{3\pi}$

$= 40\ 000$ cm^3

$= 40$ l

$\therefore$ Amount of water needed to fill container completely = 40 + 10

= 50 l

Practise Now 13

(a) **(i)** Radius of hemisphere $= 30 \div 2$

$= 15$ cm

Height of cone $= 50 - 15$

$= 35$ cm

Volume of solid = volume of cone + volume of hemisphere

$= \frac{1}{3} \times \pi \times 15^2 \times 35 + \frac{1}{2} \times \frac{4}{3} \times \pi \times 15^3$

$= 2625\pi + 2250\pi$

$= 4875\pi$

$= 15\ 300$ cm^3

(ii) Using Pythagoras' Theorem,

Slant height of cone $= \sqrt{15^2 + 35^2}$

$= 38.08$ cm (to 4 s.f.)

Total surface area of solid

= curved surface area of cone

+ curved surface area of hemisphere

$= \pi \times 15 \times 38.08 + 2 \times \pi \times 15^2$

$= 571.2\pi + 450\pi$

$= 1021.2\pi$

$= 3210$ cm^2 (to 3 s.f.)

(b) **(i)** Volume of cylinder $= 4875\pi$

$\pi(12.5^2)h = 4875\pi$

$\therefore h = \frac{4875\pi}{156.25\pi}$

$= 31.2$ cm

The height of the cylinder is 31.2 cm.

(ii) Surface area of the cylinder

$= 2\pi r^2 + 2\pi rh$

$= 2 \times \pi \times 12.5^2 + 2 \times \pi \times 12.5 \times 31.2$

$= 312.5\pi + 780\pi$

$= 1092.5\pi$ cm^2

Exercise 9A

1. Volume of triangular pyramid $= \frac{1}{3} \times$ base area $\times$ height

$= \frac{1}{3} \times 15 \times 4$

$= 20$ cm^3

2. Volume of pyramid $= \frac{1}{3} \times$ base area $\times$ height

$= \frac{1}{3} \times 23 \times 6$

$= 46$ cm^3

3. Base area of pyramid $= \frac{1}{2} \times 7 \times 4$

$= 14$ cm^2

Volume of pyramid $= \frac{1}{3} \times$ base area $\times$ height

$= \frac{1}{3} \times 14 \times 5$

$= 23\frac{1}{3}$ m^3

4. Base area of pyramid $= 10 \times 6$

$= 60$ cm^2

Volume of pyramid $= \frac{1}{3} \times$ base area $\times$ height

$100 = \frac{1}{3} \times 60 \times$ height

$100 = 20 \times$ height

$\therefore$ Height = 5 cm

5. Base area of pyramid $= \frac{1}{2} \times 5 \times 8$

$= 20$ cm^2

Volume of pyramid $= \frac{1}{3} \times$ base area $\times$ height

$50 = \frac{1}{3} \times 20 \times$ height

$50 = \frac{20}{3} \times$ height

$\therefore$ Height = 7.5 cm

6. Volume of pyramid $= \frac{1}{3} \times$ base area $\times$ height

$100 = \frac{1}{3} \times$ base area $\times 12$

$100 = 4 \times$ base area

$\therefore$ Base area = 25 m^2

Let the length of the square base be x.

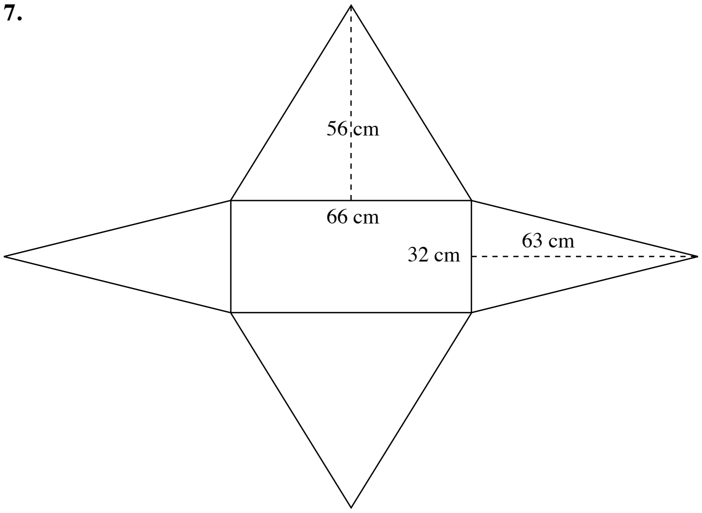
$x^2 = 25$

$\therefore x = \sqrt{25}$ (since $x > 0$)

$= 5$ m

The length of its square base is 5 m.

7.



Area of triangular face $OWX = \frac{1}{2} \times 66 \times 56$

$= 1848 \text{ cm}^2$

Area of triangular face $OXY = \frac{1}{2} \times 32 \times 63$

$= 1008 \text{ cm}^2$

Area of rectangular base $= 66 \times 32$

$= 2112 \text{ cm}^2$

$\therefore$ Total surface area of pyramid

$= 2 \times$ area of OWX $+ 2 \times$ area of OXY + area of square base

$= 2 \times 1848 + 2 \times 1008 + 2112$

$= 7824 \text{ cm}^2$

8. Volume of paper weight $= \frac{1}{3} \times$ base area $\times$ height

$= \frac{1}{3} \times 6 \times 6 \times 7$

$= 84 \text{ cm}^3$

Mass of 4 paper weights = volume of 4 paper weights × density

$= 4 \times 84 \times 3.1$

$= 1041.6$ g

9. Volume of pyramid = mass of pyramid ÷ density

$= 500 \div 6$

$= 83\frac{1}{3} \text{ cm}^3$

Volume of pyramid $= \frac{1}{3} \times$ base area $\times$ height

$83\frac{1}{3} = \frac{1}{3} \times 30 \times \text{height}$

$83\frac{1}{3} = 10 \times \text{height}$

$\therefore$ Height $= 8\frac{1}{3}$ cm

10. **(i)** Let the point where the vertical from V meets the rectangular base be P.

$PQ = 15 \div 2$

$= 7.5$ cm

Using Pythagoras's Theorem, $PV = \sqrt{16^2 - 7.5^2}$

$= 14.13$ cm (to 4 s.f.)

$= 14.1$ cm (to 3 s.f.)

The height of the pyramids is 10.1 cm.

(ii) Volume of pyramid $= \frac{1}{3} \times$ base area $\times$ height

$= \frac{1}{3} \times 15 \times 9 \times 14.13$

$= 636 \text{ cm}^3$ (to 3 s.f.)

11. **(i)** Volume of pyramid $= \frac{1}{3} \times$ base area $\times$ height

$180 = \frac{1}{3} \times 10 \times 8 \times \text{height}$

$180 = \frac{80}{3} \times \text{height}$

$\therefore$ Height $= 6.75$ cm

(ii) Let the slant height from V to PQ be l_1 cm,

the slant height from V to QR be l_2 cm.

Using Pythagoras' Theorem,

$l_1 = \sqrt{6.75^2 + 4^2}$

$= 7.846$ (to 4 s.f.)

$l_2 = \sqrt{6.75^2 + 5^2}$

$= 8.400$ (to 4 s.f.)

$\therefore$ Total surface area of pyramid

= Area of all triangular faces + area of square base

$= 2 \times \quad \frac{1}{2} \times 10 \times 7.846 + \frac{1}{2} \times 8 \times 8.400 \quad + 10 \times 8$

$= 2(39.23 + 33.6) + 80$

$= 145.66 + 80$

$= 226 \text{ cm}^2$ (to 3 s.f.)

12. **(i)** Volume of pyramid $= \frac{1}{3} \times$ base area $\times$ height

$700 = \frac{1}{3} \times 16 \times 14 \times \text{height}$

$700 = \frac{224}{3} \times \text{height}$

$\therefore$ Height $= 9.375$ cm

(ii) Let the slant height from the top of the pyramid to the side with 16 m be l_1 cm,

the slant height from the top of the pyramid to the side with 14 m be l_2 cm.

Using Pythagoras' Theorem,

$l_1 = \sqrt{9.375^2 + 7^2}$

$= 11.70$ (to 4 s.f.)

$l_2 = \sqrt{9.375^2 + 8^2}$

$= 12.32$ (to 4 s.f.)

$\therefore$ Total surface area of pyramid

= Area of all triangular faces + area of square base

$= 2 \times \left(\frac{1}{2} \times 16 \times 11.70 \times \frac{1}{2} \times 14 \times 12.32\right) + 16 \times 14$

$= 2(93.6 + 86.24) + 224$

$= 359.68 + 224$

$= 584 \text{ cm}^2$ (to 3 s.f.)

13. Volume of pyramid $= \frac{1}{3} \times$ base area $\times$ height

$= \frac{1}{3} \times 15 \times 10 \times 20$

$= 1000 \text{ cm}^3$

$$\begin{aligned}\text{Volume of cubical tank } &= l^3\\ &= 30^3\\ &= 27\,000 \text{ cm}^3\end{aligned}$$

Volume of water left in tank after pyramid is removed

$= 27\,000 - 1000$

$= 26\,000 \text{ cm}^3$

Let the depth of the remaining water in the tank be d cm.

$$\begin{aligned}30 \times 30 \times d &= 26\,000\\ 900\,d &= 26\,00\\ \therefore\ d &= 28\frac{8}{9}\end{aligned}$$

The depth of the remaining water is $28\frac{8}{9}$ cm.

14. Let WX be a, XY be b and the height of the pyramid be h.

i.e. $a > b$

Using Pythagoras' Theorem,

$$\begin{aligned}VA &= \sqrt{h^2 + \left(\frac{b}{2}\right)^2}\\ &= \sqrt{h^2 + \frac{b^2}{4}}\\ VB &= \sqrt{h^2 + \left(\frac{a}{2}\right)^2}\\ &= \sqrt{h^2 + \frac{a^2}{4}}\end{aligned}$$

Since $a > b$, $VB > VA$,

$\therefore$ the slant height VA is shorter than VB.

15. **(i)** Let the slant height be l cm.

$$\begin{aligned}\text{Using Pythagoras' Theorem, } l &= \sqrt{8^2 - \left(\frac{8}{2}\right)^2}\\ &= \sqrt{64 - 16}\\ &= \sqrt{48}\\ &= 6.928 \text{ cm (to 4 s.f.)}\\ &= 6.93 \text{ cm (to 3 s.f.)}\end{aligned}$$

(ii) $$\begin{aligned}\text{Base area of tetrahedron } &= \frac{1}{2} \times 8 \times 6928\\ &= 27.712 \text{ cm}^2\end{aligned}$$

Let the height of the tetrahedron be h cm.

$$\begin{aligned}\text{Using Pythagoras's Theorem, } h &= \sqrt{8^2 - \left(\frac{2}{3} \times 6.928\right)^2}\\ &= 6.532 \text{ cm (to 4 s.f.)}\end{aligned}$$

$$\begin{aligned}\therefore\ \text{Volume of tetrahedron } &= \frac{1}{3} \times \text{base area} \times \text{height}\\ &= \frac{1}{3} \times 27.712 \times 6.532\\ &= 60.3 \text{ cm}^3\end{aligned}$$

Exercise 9B

1. **(a)** $$\begin{aligned}\text{Volume of cone } &= \frac{1}{3}\pi r^2 h\\ &= \frac{1}{3} \times \pi \times 6^2 \times 14\\ &= 168\pi\\ &= 528 \text{ cm}^3 \text{ (to 3 s.f.)}\end{aligned}$$

(b) $$\begin{aligned}\text{Volume of cone } &= \frac{1}{3} \times \text{base area} \times \text{height}\\ &= \frac{1}{3} \times 154 \times 5\\ &= 256\frac{2}{3} \text{ cm}^3\end{aligned}$$

(c) $$\begin{aligned}\text{Volume of cone } &= \frac{1}{3}\pi r^2 h\\ &= \frac{1}{3} \times \pi \times \left(\frac{7}{2}\right)^2 \times 14\\ &= 57\frac{1}{6}\pi\\ &= 180 \text{ cm}^3 \text{ (to 3 s.f.)}\end{aligned}$$

(d) $$\begin{aligned}\text{Circumference} &= 132\\ 2\pi r &= 132\\ \therefore\ r &= \frac{132}{2\pi}\\ &= \frac{66}{\pi}\end{aligned}$$

$$\begin{aligned}\text{Volume of cone } &= \frac{1}{3}\pi r^2 h\\ &= \frac{1}{3} \times \pi \times \frac{66}{\pi}^{\quad 2} \quad \times 28\\ &= \frac{40\,656}{\pi}\\ &= 12\,900 \text{ mm}^3\end{aligned}$$

2. $$\begin{aligned}\text{Volume of cone } &= \frac{1}{3}\pi r^2 h\\ 320\pi &= \frac{1}{3} \times \pi \times 8^2 \times h\\ 320\pi &= 21\frac{1}{3}\pi h\\ h &= 15 \text{ cm}\end{aligned}$$

3. $$\begin{aligned}\text{Volume of cone } &= \frac{1}{3} \times \text{base area} \times \text{height}\\ 160 &= \frac{1}{3} \times 20 \times \text{height}\\ 160 &= \frac{20}{3} \times \text{height}\\ \therefore\ \text{Height} &= 24 \text{ m}\end{aligned}$$

4. $$\begin{aligned}\text{Volume of cone } &= \frac{1}{3}\pi r^2 h\\ 132 &= \frac{1}{3} \times \frac{22}{7} \times r^2 \times 14\\ 132 &= \frac{44}{3} \times r^2\\ r^2 &= 9\\ \therefore\ r &= \sqrt{9} \text{ (since } r > 0)\\ &= 3 \text{ cm}\end{aligned}$$

5. **(a)** $$\begin{aligned}\text{Total surface area of cone } &= \pi r l + \pi r^2\\ &= \pi \times 4 \times 7 + \pi \times 4^2\\ &= 28\pi + 16\pi\\ &= 44\pi\\ &= 138 \text{ cm}^2 \text{ (to 3 s.f.)}\end{aligned}$$

(b) Radius of cone $= 28 \div 2$

$= 14$ mm

Total surface area of cone $= \pi rl + \pi r^2$

$= \pi \times 14 \times 30 \times \pi \times 14^2$

$= 420\pi + 196\pi$

$= 616\pi$

$= 1940$ mm^2 (to 3 s.f.)

(c) Circumference $= 132$

$2\pi r = 132$

$\therefore r = \frac{132}{2\pi}$

$= \frac{66}{\pi}$

Total surface area of cone $= \pi rl + \pi r^2$

$= \pi \times \frac{66}{\pi} \times 25 + \pi \times \frac{66}{\pi}^{2}$

$= 1650 + \frac{4356}{\pi}$

$= 3040$ cm^2 (to 3 s.f.)

6. Curved surface area of cone $= 84\pi$ mm^2

$\pi(6)l = 84\pi$

$6\pi l = 84\pi$

$\therefore l = \frac{84\pi}{6\pi}$

$= 14$ mm

The slant height of the cone is 14 mm.

7. Total surface area of cone $= 1000$ cm^2

$\pi(15)l + \pi(15^2) = 1000$

$15\pi l + 225\pi = 1000$

$15\pi l = 1000 - 225\pi$

$\therefore l = \frac{1000 - 225\pi}{15\pi}$

$= \frac{1000 - 225 \times 3.142}{15 \times 3.142}$

$= 6.22$ cm (to 3 s.f.)

The slant height of the cone is 6.22 cm.

8. Curved surface area of cone $= 251$ m^2

$\pi r(5) = 251$

$5\pi r = 251$

$\therefore r = \frac{251}{5\pi}$

$= 16.0$ m (to 3 s.f.)

9. Radius of conical funnel $= 23.2 \div 2$

$= 11.6$ cm

Volume of conical funnel $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times 11.6^2 \times 42$

$= 1883.84\pi$ cm^3

Radius of cylindrical $= 16.2 \div 2$

$= 8.1$ cm

Volume of cylindrical tin $= 1883.84\pi$ cm^3

$\pi(8.1^2)h = 1883.84\pi$

$65.61\pi h = 1883.84\pi$

$\therefore h = \frac{1883.84\pi}{65.61\pi}$

$= 28.7$ cm (to 3 s.f.)

10. Volume of conical block of silver $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times 12^2 \times 16$

$= 768\pi$ cm^3

Radius of a coin $= 1\frac{1}{2} \div 2$

$= \frac{3}{4}$ cm

Volume of a coin $= \pi r^2 h$

$= \pi \times \left(\frac{3}{4}\right)^2 \times \frac{1}{6}$

$= \frac{3}{32}\pi$ cm^3

$\therefore$ Number of coins that can be made

$= \frac{\text{Volume of conical block of silver}}{\text{Volume of a coin}}$

$= \frac{768\pi}{\frac{3}{32}\pi}$

$= 8192$

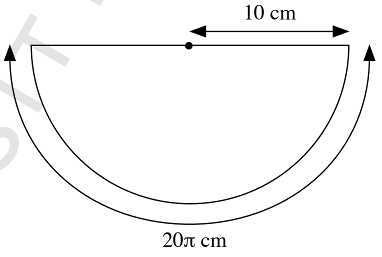
11. Circumference of base of cone $= 2\pi r$

$= 2 \times \pi \times 10$

$= 20\pi$ cm

Observe that $\frac{\text{slant height}}{\text{circumference of cone}} = \frac{20}{20\pi} = \frac{1}{\pi} = \frac{1}{2} \times \frac{1}{2\pi}$.

$\therefore$ The net of the cone is a semicircle, where the slant height is the radius of the semicircle.



12. (i) Let the diameter of the semircircle be d_s cm,
the diameter of the base of the cone be d_c cm.

Circumference of semicircle $= \frac{1}{2}\pi d_s$

$= \frac{1}{2} \times \pi \times 10$

$= 5\pi$ cm

Circumference of base of cone $= 5\pi$ cm

$\pi d_c = 5\pi$

$\therefore d_c = 5$

The diameter of the base of the cone is 5 cm.

(ii) Radius of the cone $= 5 \div 2$

$= 2.5$ cm

Slant height of the cone $= 10 \div 2$

$= 5$ cm

Curved surface area of the cone $= \pi rl$

$= \pi \times 2.5 \times 5$

$= 12.5\pi$

$= 39.3$ cm^2 (to 3 s.f.)

13. Using Pythagoras' Theorem,

$l = \sqrt{5^2 + 12^2}$

$= 13$ cm

Curved surface area of the cone $= \pi rl$

$= \pi \times 5 \times 13$

$= 65\pi$

$= 204$ cm^2 (to 3 s.f.)

14. Using Pythagoras' Theorem,

$h = \sqrt{20^2 - 8^2}$

$= 18.33$ cm (to 4 s.f.)

Volume of cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times 8^2 \times 18.33$

$= 391.04\pi$

$= 1230$ cm^3 (to 3 s.f.)

15. (i) Using Pythagoras' Theorem,

$r^2 = 21^2 - 17^2$

$= 152$

Volume of cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times 152 \times 17$

$= 861\frac{1}{3}\pi$

$= 2710$ mm^3 (to 3 s.f.)

(ii) Total surface area of cone $= \pi rl + \pi r^2$

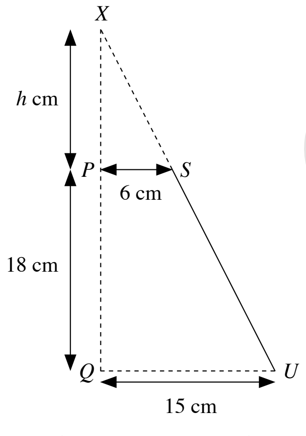
$= \pi \times \sqrt{152} \times 21 + \pi \times 152$

$= 21\sqrt{152}\,\pi + 152\pi$

$= 1290$ mm^2 (to 3 s.f.)

16. Let the height of the smaller cone be h cm.

Then the height of the bigger cone is $(h + 18)$ cm.



Since $\triangle XPS$ is similar to $\triangle XQU$,

$\frac{XP}{XQ} = \frac{PS}{QU}$

$\frac{h}{h + 18} = \frac{6}{15}$

$15h = 6h + 108$

$9h = 108$

$h = 12$

$\therefore$ Height of bigger cone $= 18 + 12$

$= 30$ cm

$\therefore$ Volume of frustum

$=$ volume of bigger cone $-$ volume of smaller cone

$= \frac{1}{3}\pi R^2 H - \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi(R^2 H - r^2 h)$

$= \frac{1}{3}\pi(15^2 \times 30 - 6^2 \times 12)$

$= \frac{1}{3}\pi(6318)$

$= 2016\pi$

$= 6620$ cm^3 (to 3 s.f.)

17. (i) Radius of cone $= 14 \div 2$

$= 7$ cm

Total surface area of solid $= 2\pi rl$

$= 2 \times \pi \times 7 \times 15$

$= 210\pi$

$= 660$ cm^2 (to 3 s.f.)

(ii) Using Pythagoras' Theorem,

$h = \sqrt{15^2 - 7^2}$

$= 13.27$ cm (to 4 s.f.)

Volume of solid $= 2 \times \frac{1}{3}\pi r^2 h$

$= \frac{2}{3} \times \pi \times 7^2 \times 13.27$

$= 1360$ cm^3 (to 3 s.f.)

18. Total surface area of cone $= 1240$ m^2

$\pi(13.5)l + \pi(13.5)^2 = 1240$

$13.5\pi l + 182.25\pi = 1240$

$13.5\pi l = 1240 - 182.25\pi$

$\therefore l = \frac{1240 - 182.25\pi}{13.5\pi}$

$= 15.74$ m (to 4 s.f.)

Using Pythagoras' Theorem,

$h = \sqrt{15.74^2 - 13.5^2}$

$= 8.093$ m (to 4 s.f.)

Volume of cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times 13.5^2 \times 8.093$

$= 1540$ m^3 (to 3 s.f.)

Exercise 9C

1. (a) Volume of sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \pi \times 8^3$

$= 682\frac{2}{3}\pi$

$= 2140$ cm^3 (to 3 s.f.)

(b) Volume of sphere $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \pi \times 14^3$$

$$= 3658\frac{2}{3}\pi$$

$$= 11\ 500 \text{ mm}^3 \text{ (to 3 s.f.)}$$

(c) Volume of sphere $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \pi \times 4^3$$

$$= 85\frac{1}{3}\pi$$

$$= 268 \text{ m}^3 \text{ (to 3 s.f.)}$$

2. (a) Volume of sphere $= 1416 \text{ cm}^3$

$$\frac{4}{3}\pi r^3 = 1416$$

$$r^3 = \frac{1062}{\pi}$$

$$\therefore r = \sqrt[3]{\frac{1062}{\pi}}$$

$$= 6.97 \text{ cm (to 3 s.f.)}$$

(b) Volume of sphere $= 12\ 345 \text{ mm}^3$

$$\frac{4}{3}\pi r^3 = 12\ 345$$

$$r^3 = \frac{37\ 035}{4\pi}$$

$$\therefore r = \sqrt[3]{\frac{37\ 035}{4\pi}}$$

$$= 14.3 \text{ mm (to 3 s.f.)}$$

(c) Volume of sphere $= 780 \text{ m}^3$

$$\frac{4}{3}\pi r^3 = 780$$

$$r^3 = \frac{585}{\pi}$$

$$\therefore r = \sqrt[3]{\frac{585}{\pi}}$$

$$= 5.71 \text{ m (to 3 s.f.)}$$

(d) Volume of sphere $= 972\pi \text{ cm}^3$

$$\frac{4}{3}\pi r^3 = 972\pi$$

$$r^3 = 729$$

$$\therefore r = \sqrt[3]{729}$$

$$= 9 \text{ cm}$$

(e) Volume of sphere $= 498\pi \text{ mm}^3$

$$\frac{4}{3}\pi r^3 = 498\pi$$

$$r^3 = 373\frac{1}{2}$$

$$\therefore r = \sqrt[3]{373\frac{1}{2}}$$

$$= 7.20 \text{ mm (to 3 s.f.)}$$

(f) Volume of sphere $= 15\frac{3}{16}\pi \text{ m}^3$

$$\frac{4}{3}\pi r^3 = 15\frac{3}{16}\pi$$

$$r^3 = \frac{729}{64}$$

$$\therefore r = \sqrt[3]{\frac{729}{64}}$$

$$= 2.25 \text{ m (to 3 s.f.)}$$

3. (a) Surface area of sphere $= 4\pi r^2$

$$= 4 \times \pi \times 12^2$$

$$= 576\pi$$

$$= 1810 \text{ cm}^2 \text{ (to 3 s.f.)}$$

(b) Surface area of sphere $= 4\pi r^2$

$$= 4 \times \pi \times 9^2$$

$$= 324\pi$$

$$= 1020 \text{ mm}^2 \text{ (to 3 s.f.)}$$

(c) Surface area of sphere $= 4\pi r^2$

$$= 4 \times \pi \times 3^2$$

$$= 36\pi$$

$$= 113 \text{ m}^2 \text{ (to 3 s.f.)}$$

4. Total surface area of hemisphere $= \pi r^2 + \frac{1}{2} \times 4\pi r^2$

$$= 3\pi r^2$$

$$= 3 \times \pi \times 7^2$$

$$= 147\pi$$

$$= 147 \times 3.142$$

$$= 462 \text{ cm}^2 \text{ (to 3 s.f.)}$$

5. (a) Surface area of sphere $= 210 \text{ cm}^2$

$$4\pi r^2 = 210$$

$$r^2 = \frac{210}{4\pi}$$

$$\therefore r = \sqrt{\frac{210}{4\pi}} \quad \text{(since } r > 0\text{)}$$

$$= 4.09 \text{ cm (to 3 s.f.)}$$

(b) Surface area of sphere $= 7230 \text{ mm}^2$

$$4\pi r^2 = 7230$$

$$r^2 = \frac{7230}{4\pi}$$

$$\therefore r = \sqrt{\frac{7230}{4\pi}} \quad \text{(since } r > 0\text{)}$$

$$= 24.0 \text{ mm (to 3 s.f.)}$$

(c) Surface area of sphere $= 3163 \text{ m}^2$

$$4\pi r^2 = 3163$$

$$r^2 = \frac{3163}{4\pi}$$

$$\therefore r = \sqrt{\frac{3163}{4\pi}} \quad \text{(since } r > 0\text{)}$$

$$= 15.9 \text{ m (to 3 s.f.)}$$

(d) Surface area of sphere $= 64\pi \text{ cm}^2$

$$4\pi r^2 = 64\pi$$

$$r^2 = 16$$

$$\therefore r = \sqrt{16} \quad \text{(since } r > 0\text{)}$$

$$= 4 \text{ cm}$$

(e) Surface area of sphere $= 911\pi \text{ mm}^2$

$$\begin{aligned} 4\pi r^2 &= 911\pi \\ r^2 &= \frac{911}{4} \\ \therefore r &= \sqrt{\frac{911}{4}} \quad (\text{since } r > 0) \\ &= 15.1 \text{ mm (to 3 s.f.)} \end{aligned}$$

(f) Surface area of sphere $= 49\pi \text{ m}^2$

$$\begin{aligned} 4\pi r^2 &= 49\pi \\ r^2 &= \frac{49}{4} \\ \therefore r &= \sqrt{\frac{49}{4}} \quad (\text{since } r > 0) \\ &= 3.5 \text{ m} \end{aligned}$$

6. Curved surface area of hemisphere $= 364.5\pi \text{ cm}^2$

$$\begin{aligned} \frac{1}{2} \times 4\pi r^2 &= 364.5\pi \\ 2\pi r^2 &= 364.5\pi \\ r^2 &= \frac{364.5}{2} \\ \therefore r &= \sqrt{\frac{364.5}{2}} \quad (\text{since } r > 0) \\ &= 13.5 \text{ cm} \end{aligned}$$

7. Radius of a ball bearing $= 0.7 \div 2$

$= 0.35$ cm

$$\begin{aligned} \text{Volume of a ball bearing} &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3} \times \pi \times 0.35^3 \\ &= 0.1796 \text{ cm}^3 \text{ (to 4 s.f.)} \end{aligned}$$

Mass of a ball bearing $= 0.1796 \times 7.85$

$= 1.40986$ g

$$\begin{aligned} \text{Number of ball bearings} &= \frac{1000}{1.40986} \\ &= 709 \text{ (to the nearest whole number)} \end{aligned}$$

8. Volume of hollow aluminium sphere

$$\begin{aligned} &= \frac{4}{3}\pi R^3 - \frac{4}{3}\pi r^3 \\ &= \frac{4}{3} \times \pi \times 30^3 - \frac{4}{3} \times \pi \times 20^3 \\ &= 36\,000\pi - 10\,666\frac{2}{3}\pi \\ &= 25\,333\frac{1}{3}\pi \text{ cm}^3 \end{aligned}$$

$$\begin{aligned} \text{Mass of hollow aluminium sphere} &= 25\,333\frac{1}{3}\pi \times 2.7 \\ &= 215\,000 \text{ g (to 3 s.f.)} \\ &= 215 \text{ kg} \end{aligned}$$

9. Radius of hemisphere $= 2 \div 2$

$= 1$ cm

$$\begin{aligned} \text{Volume of a hemisphere} &= \frac{1}{2} \times \frac{4}{3}\pi r^3 \\ &= \frac{2}{3}\pi r^3 \\ &= \frac{2}{3} \times \pi \times 1^3 \\ &= \frac{2}{3}\pi \text{ cm}^3 \end{aligned}$$

$$\text{Volume of sphere} = \frac{2}{3}\pi \times 54$$

$$\begin{aligned} \frac{4}{3}\pi R^3 &= 36\pi \\ R^3 &= 27 \\ \therefore R &= \sqrt[3]{27} \\ &= 3 \text{ cm} \end{aligned}$$

10. Radius of sphere $= 26.4 \div 2$

$= 13.2$ cm

$$\begin{aligned} \text{Volume of acid in the sphere} &= \frac{1}{2} \times \frac{4}{3}\pi r^3 \\ &= \frac{2}{3}\pi r^3 \\ &= \frac{2}{3} \times \pi \times 13.2^3 \\ &= 1533.312\pi \text{ cm}^3 \end{aligned}$$

Radius of beaker $= 16 \div 2$

$= 8$ cm

$$\begin{aligned} \text{Volume of acid in the beaker} &= 1533.312\pi \\ \pi R^2 d &= 1533.312\pi \\ \pi \times 8^2 \times d &= 1533.312\pi \\ 64\pi d &= 1533.312\pi \\ \therefore d &= \frac{1533.312}{64} \\ &= 24.0 \text{ cm (to 3 s.f.)} \end{aligned}$$

The depth of the acid in the beaker is 24.0 cm.

11. Radius of cylindrical tin $= 18 \div 2$

$= 9$ cm

$$\begin{aligned} \text{Volume of water in the cylindrical tin} &= \pi r^2 h \\ &= \pi \times 9^2 \times 13.2 \\ &= 1069.2\pi \text{ cm}^3 \end{aligned}$$

Radius of spherical ball bearing $= 9.3 \div 2$

$= 4.65$ cm

$$\begin{aligned} \text{Volume of spherical ball bearing} &= \frac{4}{3}\pi R^3 \\ &= \frac{4}{3} \times \pi \times 4.65^3 \\ &= 134.0595\pi \text{ cm}^3 \end{aligned}$$

Volume of water and spherical ball bearing

$= 1069.2\pi + 134.0595\,\pi$

Volume in the cylindrical tin = 1203.2595π

$$\begin{aligned} \pi \times 9^2 \times H &= 1203.2595\pi \\ 81\pi H &= 1203.2595\pi \\ \therefore H &= \frac{1203.2595}{81} \\ &= 14.86 \text{ cm (to 2 d.p.)} \end{aligned}$$

The new height of water in the tin is 14.86 cm.

12. Volume of sphere = 850 m^3

$$\begin{aligned} \frac{4}{3}\pi r^3 &= 850 \\ r^3 &= \frac{1275}{2\pi} \\ \therefore r &= \sqrt[3]{\frac{1275}{2\pi}} \\ &= 5.876 \text{ m (to 4 s.f.)} \end{aligned}$$

Surface area of sphere $= 4\pi r^2$

$= 4 \times \pi \times 5.876^3$

$= 434 \text{ m}^2$ (to 3 s.f.)

13. Surface area of basketball = 1810 cm^2

$$\begin{aligned} 4\pi r^2 &= 1810 \\ r^2 &= \frac{1810}{4\pi} \\ \therefore r &= \sqrt{\frac{1810}{4\pi}} \text{ (since } r > 0) \\ &= 12.00 \text{ cm (to 4 s.f.)} \end{aligned}$$

Volume of basketball $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \pi \times 12.00^3$

$= 2304\pi$

$= 7240 \text{ cm}^3$ (to 3 s.f.)

14. Curved surface area of hemisphere $= \frac{1}{2} \times 4\pi r^2$

$= 2\pi r^2$ units^2

Flat surface area of hemisphere $= \pi r^2$ units^2

Ratio of red paint to yellow paint $= 2\pi r^2 : \pi r^2$

$= 2 : 1$

15. **(i)** Radius of sphere = radius of cylindrical can

= 3.4 cm

Diameter of sphere = 3.4 × 2

= 6.8 cm

Depth of water in the can when the sphere was placed inside

= 6.8 cm

Surface area of can in contact with water

$= \pi r^2 + 2\pi rh$

$= \pi \times 3.4^2 + 2 \times \pi \times 3.4 \times 6.8$

$= 11.56\pi + 46.24\pi$

$= 57.8\pi$

$= 182 \text{ cm}^2$ (to 3 s.f.)

(ii) Volume of water in the can when the sphere was placed inside

$= \pi r^2 h - \frac{4}{3}\pi r^3$

$= \pi \times 3.4^2 \times 6.8 - \frac{4}{3} \times \pi \times 3.4^3$

$= 3.4^2\pi\left(6.8 - 4\frac{8}{15}\right)$

$= \left(3.4^2 \times 2\frac{4}{15} \times \pi\right) \text{cm}^3$

Let the depth of water in the can before the sphere was placed inside be d cm.

Volume of water $= \left(3.4^2 \times 2\frac{4}{15} \times \pi\right) \text{ cm}^3$

$$\begin{aligned} \pi r^2 d &= 3.4^2 \times 2\frac{4}{15} \times \pi \\ \pi(3.4^2)d &= 3.4^2 \times 2\frac{4}{15} \times \pi \\ \therefore d &= \frac{3.4^2 \times 2\frac{4}{15} \times \pi}{3.4^2\pi} \\ &= 2\frac{4}{15} \end{aligned}$$

The depth of water in the can was $2\frac{4}{15}$ cm.

Exercise 9D

1. Radius of cylinder = 12 ÷ 2

= 6 m

Total surface area of rocket

= Flat surface of cylinder + curved surface area of cylinder + curved surface area of cone

$= \pi \times 6^2 + 2 \times \pi \times 6 \times 42 + \pi \times 6 \times 15$

$= 36\pi + 504\pi + 90\pi$

$= 630\pi$

$= 1980 \text{ m}^2$ (to 3 s.f.)

2. Volume of remaining solid

= Volume of cylinder – volume of cone

$= \pi \times 6^2 \times 15 - \frac{1}{3} \times \pi \times 3^2 \times 15$

$= 540\pi - 45\pi$

$= 495\pi$

$= 1560 \text{ cm}^3$ (to 3 s.f.)

3. **(i)** Volume of the solid

= Volume of hemisphere + volume of cylinder

$= \frac{1}{2} \times \frac{4}{3} \times \pi \times 7^3 + \pi \times 7^2 \times 10$

$= 228\frac{2}{3}\pi + 490\pi$

$= 718\frac{2}{3}\pi$

$= 2260 \text{ cm}^3$ (to 3 s.f.)

(ii) Total surface area of the solid

= Flat surface of cylinder + curved surface area of cylinder + curved surface area of hemisphere

$= \pi \times 7^2 + 2 \times \pi \times 7 \times 10 + \frac{1}{2} \times 4 \times \pi \times 7^2$

$= 49\pi + 140\pi + 98\pi$

$= 287\pi$

$= 902$ cm^2 (to 3 s.f.)

4. (i) Volume of the solid

= Volume of hemisphere + volume of cone

$= \frac{1}{2} \times \frac{4}{3} \times \pi \times 21^3 + \frac{1}{3} \times \pi \times 21^2 \times 28$

$= 6174\pi + 4116\pi$

$= 10\,290\pi$

$= 32\,300$ cm^3 (to 3 s.f.)

(ii) Total surface area of the solid

= Curved surface area of hemisphere + curved surface area of cone

$= \frac{1}{2} \times 4 \times \pi \times 21^2 + \pi \times 21 \times 35$

$= 882\pi + 73.5\pi$

$= 1617\pi$

$= 5080$ cm^3 (to 3 s.f.)

5. $7\ l = 7000$ cm^3

Height of the cone $= \frac{3}{5} \times 4r$

$= \frac{12r}{5}$ cm

Volume of cone $= 7000$ cm^3

$\frac{1}{3} \times \pi \times r^2 \times \frac{12r}{5} = 7000$

$\frac{4}{5}\pi r^3 = 7000$

$r^3 = \frac{8750}{\pi}$

Volume of cylindrical container $= \pi r^2 h$

$= \pi \times r^2 \times 4r$

$= 4\pi r^3$

$= 4\pi \frac{8750}{\pi}$

$= 35\,000$ cm^3

Amount of water needed $= 7000 + 35\,000$

$= 42\,000$ cm^3

$= 42\ l$

6. (i) Radius of cylinder $= 8 \div 2$

$= 4$ m

Total surface area of solid cylinder with conical ends

= 2 × curved surface area of cone + curved surface area of cylinder

$= 2 \times \pi \times 4 \times 6 + 2 \times \pi \times 4 \times 8$

$= 48\pi + 64\pi$

$= 112\pi$

$= 352$ m^2 (to 3 s.f.)

(ii) Using Pythagoras' Theorem,

$h = \sqrt{6^2 - 4^2}$

$= 4.472$ m (to 4 s.f.)

Volume of the solid cylinder with conical ends

= 2 × volume of cone + volume of cylinder

$= 2 \times \frac{1}{3} \times \pi \times 4^2 \times 4.472 + \pi \times 4^2 \times 8$

$= 552$ m^3 (to 3 s.f.)

7. Radius of the cylinder $= 4.7 \div 2$

$= 2.35$ m

Height of cylinder $= 16.5 - 2.35$

$= 14.15$ m

Capacity of tank

= volume of hemisphere + volume of cylinder

$= \frac{1}{2} \times \frac{4}{3} \times \pi \times 2.35^3 + \pi \times 2.35^2 \times 14.15$

$= 273$ m^3 (to 3 s.f.)

8. Volume of cone = volume of ball

$\frac{1}{3} \times \pi \times 4^2 \times h = \frac{4}{3} \times \pi \times 3^2$

$\frac{16}{3}\pi h = 36\pi$

$\therefore h = 36 \times \frac{3}{16}$

$= 6.75$ cm

9. (i) Volume of cone $= 1\frac{1}{5}$ × volume of hemisphere

$\frac{1}{3} \times \pi \times 35^2 \times h = 1\frac{1}{5} \times \frac{1}{2} \times \frac{4}{3} \times \pi \times 35^3$

$408\frac{1}{3}\pi h = 34\,300\pi$

$\therefore h = 34\,300 \div 408\frac{1}{3}$

$= 84$ cm

The height of the cone is 84 cm.

(ii) Using Pythagoras' Theorem,

$l = \sqrt{84^2 + 35^2}$

$= 91$ cm

Total surface area of the solid

= Curved surface area of cone + curved surface area of hemisphere

$= \pi \times 35 \times 91 + \frac{1}{2} \times 4 \times \pi \times 35^2$

$= 3185\pi + 2450\pi$

$= 5635\pi$ cm^2

10. (i) Volume of the solid = volume of pyramid + volume of cuboid

$= \frac{1}{3} \times 30 \times 30 \times 28 + 30 \times 30 \times 40$

$= 8400 + 36\,000$

$= 44\,400$ cm^3

(ii) Using Pythagoras' Theorem,

slant height of pyramid, $l = \sqrt{28^2 + 15^2}$

$= 31.76$ cm (to 4 s.f.)

Total surface area of the solid

= Total surface area of visible sides of cuboid + total surface area of all triangular faces of pyramid

$= (30 \times 30 + 4 \times 30 \times 40) + \left(4 \times \frac{1}{2} \times 30 \times 31.76\right)$

$= 5700 + 1905.6$

$= 7610$ cm^2 (to 3 s.f.)

Review Exercise 9

1. (a) (i) Volume of the solid

= Volume of pyramid + volume of cuboid

$= \frac{1}{3} \times 20 \times 20 \times 24 + 20 \times 20 \times 50$

$= 3200 + 20\ 000$

$= 25\ 200$ cm^3

(ii) Total surface area of the solid

= Total surface area of visible sides of cuboid + total surface area of all triangular faces of pyramid

$= (20 \times 20 + 4 \times 20 \times 50) + \left(4 \times \frac{1}{2} \times 20 \times 26\right)$

$= 4400 + 1040$

$= 5440$ cm^2

(b) (i) Volume of the solid

= 2 × volume of cone + volume of cylinder

$= 2 \times \frac{1}{3} \times \pi \times 0.5^2 \times 1.2 + \pi \times 0.5^2 \times 2.5$

$= \frac{1}{5}\pi + \frac{5}{8}\pi$

$= \frac{33}{40}\pi$

$= 2.59$ m^3 (to 3 s.f.)

(ii) Using Pythagoras' Theorem,

slant height of cone, $l = \sqrt{1.2^2 + 0.5^2}$

$= 1.3$ m

Total surface area of the solid

= 2 × curved surface area of cone + curved surface area of cylinder

$= 2 \times \pi \times 0.5 \times 1.3 + 2 \times \pi \times 0.5 \times 2.5$

$= 1.3\pi + 2.5\pi$

$= 3.8\pi$

$= 11.9$ m^2 (to 3 s.f.)

(c) (i) Volume of the hemisphere

$= \frac{1}{2} \times \frac{4}{3} \times \pi \times 40^3$

$= 42\ 666\frac{2}{3}\pi$

$= 134\ 000$ cm^3 (to 3 s.f.)

(ii) Total surface area of the hemisphere

$= \frac{1}{2} \times 4 \times \pi \times 40^2 + \pi \times 40^2$

$= 3200\pi + 1600\pi$

$= 4800\pi$

$= 15\ 100$ cm^2 (to 3 s.f.)

(d) (i) Volume of the solid

= 2 × volume of hemisphere + volume of cylinder

$= 2 \times \frac{1}{2} \times \frac{4}{3} \times \pi \times 3.5^3 + \pi \times 3.5^2 \times 4$

$= 334$ m^3 (to 3 s.f.)

(ii) Total surface area of the solid

= 2 × curved surface area of hemisphere + curved surface area of cylinder

$= 2 \times \frac{1}{2} \times 4 \times \pi \times 3.5^2 + 2 \times \pi \times 3.5 \times 4$

$= 49\pi + 28\pi$

$= 77\pi$

$= 242$ m^2 (to 3 s.f.)

2. (i) Volume of the structure

= volume of the cone + volume of hemisphere

$= \frac{1}{3} \times \pi \times 5^2 \times 20 + \frac{1}{2} \times \frac{4}{3} \times \pi \times 4^3$

$= 166\frac{2}{3}\pi + 42\frac{2}{3}\pi$

$= 209\frac{1}{3}\pi$

$= 658$ cm^3 (to 3 s.f.)

(ii) Using Pythagoras' Theorem,

slant height, $l = \sqrt{20^2 + 5^2}$

$= 20.62$ cm

Total surface area of the structure

= Total surface area of cone + total surface area of hemisphere

$= (\pi \times 5^2 + \pi \times 5 \times 20.62) + \quad \pi \times 4^2 + \frac{1}{2} \times 4 \times \pi \times 4^2$

$= 128.1\pi + 48\pi$

$= 176.1\pi$

$= 553$ cm^2 (to 3 s.f.)

3. Volume of the rocket

= volume of cone + volume of cylinder

$= \frac{1}{3} \times \pi \times 18^2 \times 49 + \pi \times 18^2 \times 192$

$= 5292\pi + 62\ 208\pi$

$= 67\ 500\pi$ cm^3

$= 0.0675\pi$ m^3

Density of metal $= \frac{2145}{0.0675\pi}$

$= 10\ 115$ kg/m^3 (to the nearest whole number)

4. Surface area of first sphere $= 144\pi$ cm^2

$4\pi r_1^2 = 144\pi$

$r_1^2 = 36$

$\therefore r_1 = \sqrt{36}$ (since $r_1 > 0$)

$= 6$ cm

Volume of first sphere $= \frac{4}{3}\pi r_1^3$

$= \frac{4}{3} \times \pi \times 6^3$

$= 288\pi$ cm^3

Surface area of second sphere = 256π cm^2

$$4\pi r_2^2 = 256\pi$$
$$r_2^2 = 64$$
$$\therefore r_2 = \sqrt{64} \text{ (since } r_2 > 0)$$
$$= 8 \text{ cm}$$

Volume of second sphere $= \frac{4}{3}\pi r_2^3$

$$= \frac{4}{3} \times \pi \times 8^3$$
$$= 682\frac{2}{3}\pi \text{ cm}^3$$

Volume of larger sphere $= 288\pi + 682\frac{2}{3}\pi$

$$\frac{4}{3}\pi R^3 = 970\frac{2}{3}\pi \text{ cm}^3$$
$$R^3 = 728$$
$$\therefore R = \sqrt[3]{728}$$
$$= 8.996 \text{ cm (to 4 s.f.)}$$

Surface area of larger sphere $= 4\pi R^2$

$$= 4 \times \pi \times 8.996^2$$
$$= 1020 \text{ cm}^2 \text{ (to 3 s.f.)}$$

5. **(i)** External radius = $12 \div 2$

$= 6$ cm

Internal diameter = $12 - 2 - 2$

$= 8$ cm

Internal radius = $8 \div 2$

$= 4$ cm

Volume of hollow sphere $= \frac{4}{3}\pi R^3 - \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \pi \times 6^3 - \frac{4}{3} \times \pi \times 4^3$$
$$= 288\pi - 85\frac{1}{3}\pi$$
$$= 202\frac{2}{3}\pi \text{ cm}^3$$

Mass of hollow sphere $= 202\frac{2}{3}\pi \times 5.4$

$$= 3438.159 \text{ g}$$
$$= 3.44 \text{ kg (to 3 s.f.)}$$

(ii) Volume of solid sphere $= 202\frac{2}{3}\pi$ cm^3

$$\frac{4}{3} \times \pi \times r_s^3 = 202\frac{2}{3}\pi$$
$$r_s^3 = 152$$
$$\therefore r_s = \sqrt[3]{152}$$
$$= 5.34 \text{ cm (to 3 s.f.)}$$

6. **(i)** Number of drops = $5000 \div 12.5$

$= 400$

(ii) Volume of one drop of oil = 12.5 mm^3

$$\frac{4}{3} \times \pi \times r^3 = 12.5$$
$$r^3 = \frac{75}{8\pi}$$
$$\therefore r = \sqrt[3]{\frac{75}{8\pi}}$$
$$= 1.44 \text{ mm (to 3 s.f.)}$$

7. Let the radius of the cylinder and sphere be r units.

Surface area of the sphere = $4\pi r^2$ units^2

Curved surface area of cylinder = $2\pi rh$

$$= 2 \times \pi \times r \times 2r$$
$$= 4\pi r^2 \text{ units}^2$$

$\therefore$ Surface area of the sphere

= curved surface area of cylinder (shown)

8. Radius of hemispherical roof = $10 \div 2$

$= 5$ m

Curved surface area of hemispherical roof $= \frac{1}{2} \times 4\pi r^2$

$$= 2 \times \pi \times 5^2$$
$$= 50\pi \text{ m}^2$$

Cost of painting = $50\pi \times$ PKR 1.50

= PKR 235.62 (to the nearest paisa)

9. **(i)** External radius = $50.8 \div 2$

$= 25.4$ cm

Internal diameter = $50.8 - 2.54 - 2.54$

$= 45.72$ cm

Internal radius = $45.72 \div 2$

$= 22.86$ cm

Volume of metal hemispherical bowl

$$= \frac{1}{2} \times \frac{4}{3}\pi R^3 - \frac{1}{2} \times \frac{4}{3}\pi r^3$$
$$= \frac{1}{2} \times \frac{4}{3} \times \pi \times 25.4^3 - \frac{1}{2} \times \frac{4}{3} \times \pi \times 22.86^3$$
$$= 9300 \text{ cm}^3 \text{ (to 4 s.f.)}$$
$$= 0.009\,300 \text{ m}^3$$

Density of metal $= \frac{97.9}{0.009\,300}$

$$= 10\,500 \text{ kg/m}^3 \text{ (to 3 s.f.)}$$

(ii) Volume of liquid in the bowl

$$= \frac{1}{2} \times \frac{4}{3}\pi r^3$$
$$= \frac{1}{2} \times \frac{4}{3} \times \pi \times 22.86^3$$
$$= 25\,020 \text{ cm}^3 \text{ (to 4 s.f.)}$$
$$= 0.02502 \text{ m}^3$$

Mass of the liquid = 31.75×0.02502

$= 0.794$ kg (to 3 s.f.)

$= 794$ g

10. **(i)** Radius of capsule $A = 0.6 \div 2$

$$= 0.3 \text{ cm}$$

Surface area of capsule A

= 2 × curved surface area of hemisphere + curved surface area of cylinder

$$= 2 \times \frac{1}{2} \times 4 \times \pi \times 0.3^2 + 2 \times \pi \times 0.3 \times 2.4$$

$$= 0.36\pi + 1.44\pi$$

$$= 1.8\pi \text{ cm}^2$$

Surface area of capsule $B = 1.8\pi \text{ cm}^2$

$$2 \times \pi \times 0.6^2 + 2 \times \pi \times 0.6 \times h = 1.8\pi$$

$$0.72\pi + 1.2\pi h = 1.8\pi$$

$$1.2\pi h = 1.08\pi$$

$$\therefore h = \frac{1.08}{1.2}$$

$$= 0.9 \text{ cm}$$

(ii) Volume of capsule A

= 2 × volume of hemisphere + volume of cylinder

$$= 2 \times \frac{1}{2} \times \frac{4}{3} \times \pi \times 0.3^3 + \pi \times 0.3^2 \times 2.4$$

$$= 0.036\pi + 0.216\pi$$

$$= 0.252\pi$$

$$= 0.792 \text{ cm}^3 \text{ (to 3 s.f.)}$$

Volume of capsule B

$$= \pi \times 0.6^2 \times 0.9$$

$$= 0.324\pi$$

$$= 1.02 \text{ cm}^3 \text{ (to 3 s.f.)}$$

11. Radius of pillar $= 40 \div 2$

$$= 20 \text{ cm}$$

Since the pillar has the same mass as a solid stone sphere of the same material,

∴ the pillar has the same volume as the solid stone sphere.

Volume of solid stone sphere $= \frac{4}{3}\pi R^3$

$$= \frac{4}{3} \times \pi \times 40^3$$

$$= \frac{256\,000}{3}\pi \text{ cm}^3$$

Volume of pillar $= \frac{256\,000}{3}\pi \text{ cm}^3$

$$\pi \times 20^2 \times h + \frac{1}{2} \times \frac{4}{3} \times \pi \times 20^3 = \frac{256\,000}{3}\pi$$

$$400\pi h + \frac{16\,000}{3}\pi = \frac{256\,000}{3}\pi$$

$$400\pi h = 80\,000\pi$$

$$\therefore h = 200 \text{ cm}$$

12. Radius of cylinder and cone $= 2r \div 2$

$$= r \text{ units}$$

Radius of sphere $= 2r \div 2$

$$= r \text{ units}$$

Volume of cylinder $= \pi \times r^2 \times 2r$

$$= 2\pi r^3 \text{ units}^3$$

Volume of cone $= \frac{1}{3} \times \pi \times r^2 \times 2r$

$$= \frac{2}{3}\pi r^3 \text{ units}^3$$

Volume of sphere $= \frac{4}{3} \times \pi \times r^3$

$$= \frac{4}{3}\pi r^3 \text{ units}^3$$

Ratio of volume of cylinder to volume of cone to volume of sphere

$$= 2\pi r^3 : \frac{2}{3}\pi r^3 : \frac{4}{3}\pi r^3$$

$$= 2 : \frac{2}{3} : \frac{4}{3}$$

$$= 6 : 2 : 4$$

$$= 3 : 1 : 2$$

13. Radius of hemisphere $= 2 \div 2$

$$= 1 \text{ cm}$$

Radius of cone $= 6 \div 2$

$$= 3 \text{ cm}$$

Volume of hemisphere $= \frac{1}{2} \times \frac{4}{3}\pi r^3$

$$= \frac{1}{2} \times \frac{4}{3} \times \pi \times 1^3$$

$$= \frac{2}{3}\pi \text{ cm}^3$$

Volume of cone $= 40 \times \frac{2}{3}\pi$

$$\frac{1}{3} \times \pi \times 3^2 \times h = \frac{80}{3}\pi$$

$$3\pi h = \frac{80}{3}\pi$$

$$\therefore h = \frac{80}{3} \times \frac{1}{3}$$

$$= 8\frac{8}{9} \text{ cm}$$

The height of the chocolate cone is $8\frac{8}{9}$ cm.

14. Radius of cone $= 4.2 \div 2$

$$= 2.1 \text{ cm}$$

Volume of hemisphere $= \frac{1}{2} \times \frac{4}{3}\pi r^3$

$$= \frac{1}{2} \times \frac{4}{3} \times \pi \times 2.1^3$$

$$= 6.174\pi \text{ cm}^3$$

Volume of cone $= 56 - 6.174\pi$

$$\frac{1}{3} \times \pi \times 2.1^2 \times h = 56 - 6.174\pi$$

$$1.47\pi h = 56 - 6.174\pi$$

$$\therefore h = \frac{56 - 6.174\pi}{1.47\pi}$$

$$= 7.93 \text{ cm (to 3 s.f.)}$$

Challenge Yourself

1. Let the side of a face of a tetrahedron which is an equilateral triangle, be x cm,
the slant height of a face of a tetrahedron be l cm, and
the height of the tetrahedron be H cm.
Using Pythagoras' Theorem,

$$l = \sqrt{x^2 - \frac{x}{2}^{\,2}} \quad \text{(since } l > 0\text{)}$$

$$= \frac{\sqrt{3}}{2}x \text{ cm}$$

The centre of a side of a tetrahedron is $\frac{2}{3}$ of its slant height.

Using Pythagoras' Theorem,

$$H = \sqrt{x^2 - \frac{2}{3} \times \frac{\sqrt{3}}{2}x^{\,2}}$$

$$= \sqrt{x^2 - \frac{1}{3}x^2}$$

$$= \sqrt{\frac{2}{3}}\,x \text{ cm}$$

Base area of tetrahedron $= \frac{1}{2} \times x \times \frac{\sqrt{3}}{2}x$

$$= \frac{\sqrt{3}}{4}x^2$$

Volume of tetrahedron = 500 cm^3

$$\frac{1}{3} \times \frac{\sqrt{3}}{4}x^2 \times \sqrt{\frac{2}{3}}\,x = 500$$

$$x^3 = 4243 \text{ (to 4 s.f.)}$$

$$\therefore x = \sqrt[3]{4243}$$

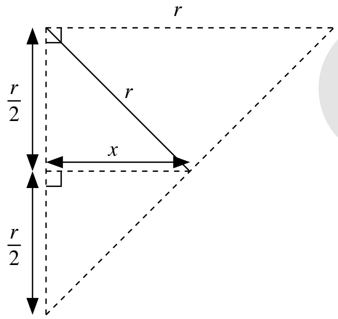
$$= 16.19 \text{ (to 4 s.f.)}$$

Total surface area of tetrahedron $= 4 \times$ base area

$$= 4 \times \frac{\sqrt{3}}{4} \times 16.19^2$$

$$= 454 \text{ cm}^2 \text{ (to 3 s.f.)}$$

2.



Using Pythagoras' Theorem,

$$x = \sqrt{r^2 - \frac{r}{2}^{\,2}}$$

$$= \frac{\sqrt{3}}{2}r$$

Since $x = \frac{\sqrt{3}}{2}r > \frac{r}{2}$, the water in the hemisphere is not a hemisphere on its own.

But the volume of water is **more** than the volume of a hemisphere with $\frac{r}{2}$ as its radius.

Volume of water > volume of hemisphere with radius $\frac{r}{2}$

$$= \frac{1}{2} \times \frac{4}{3} \times \pi \times \left(\frac{r}{2}\right)^3$$

$$= \frac{1}{2} \times \frac{4}{3} \times \pi \times \frac{r^3}{8}$$

$$= \frac{1}{8} \times \frac{1}{2} \times \frac{4}{3} \times \pi \times r^3$$

$$= \frac{1}{8} \times \text{volume of the bowl}$$

$\therefore$ The volume of water is **more** than $\frac{1}{8}$ of the volume of the bowl. (shown)

Chapter 10 Congruence and Similarity Tests

TEACHING NOTES

Suggested Approach

Teachers may want to introduce this topic by asking students to recall what they have learnt in Book 2 on congruent and similar triangles. Then, teachers may get students to investigate whether all the conditions are necessary to prove whether two triangles are congruent or similar.

Section 10.1: Congruence Tests

Teachers may wish to recap with the students, that for congruent triangles, all the corresponding lengths and angles are equal.

For each of the 4 congruent tests that are covered in this section, teachers should ask the students to construct a triangle in as many ways as possible and see what conclusion they can make by comparing these triangles (see Investigation: SSS Congruence Test, Investigation: SAS Congruence Test, Investigation: AAS Congruence Test and RHS Congruence Test).

Teachers should teach students how to match the vertices of two triangles correctly, even if the two triangles are not congruent. Once students have learnt all the 4 congruent tests, they can learn to identify pairs of congruent triangles and prove the congruency (see Class Discussion: Consolidation for Congruence Tests).

Teachers should highlight to students that the 4 congruent tests covered in this chapter are not the only congruence tests.

Section 10.2: Similarity Tests

Teachers may wish to recap with the students, that for similar triangles, all the corresponding lengths and angles are proportional and equal, respectively.

For each of the 3 similarity tests that are covered in this section, teachers should ask the students to construct a triangle in as many ways as possible and see what conclusion they can make by comparing these triangles (see Investigation: AA Similarity Test, Investigation: SSS Similarity Test and Investigation: SAS Similarity Test).

Once students have learnt all the 4 congruent tests and 3 similarity tests, teachers may ask students to compare the congruent tests with the similarity tests (see Thinking Time on page 306). Teachers should highlight to students that the 3 similarity tests covered in this chapter are not the only similarity tests.

Section 10.3: Applications of Congruent and Similar Triangles

Now that students have learnt the congruent and similarity tests, they can apply the concepts to solve problems in mathematics and in real life. For Worked Example 10, teachers should recap with students the properties of a perpendicular bisector and an angle bisector before going through the problem.

Challenge Yourself

For Question 1, let the height of $\triangle PST$ from P to ST be h' units and use similar triangles to solve the problem. Students need to manipulate algebra properly, otherwise they may end up with a long and tedious working. For Question 2, students should identify a pair of similar triangles and let QU be x cm and VS be y cm. Then they can formulate a pair of simultaneous equations involving x and y and solve for x and y to find the length of QU. For Question 3, students should identify two pairs of similar triangles first.

WORKED SOLUTIONS

Investigation (SSS Congruence Test)

5. From this investigation, we can conclude that if the 3 sides of a triangle are equal to the 3 corresponding sides of another triangle, then the two triangles are congruent.

Investigation (SAS Congruence Test)

5. From part 1 of this investigation, if two sides and the included angle of a triangle are given, then only a unique triangle can be constructed.
9. From part 2 of this investigation, if two sides and an angle which is not the included angle of a triangle are given, then there is more than one way to construct the triangle.

Investigation (AAS Congruence Test)

3. From part 1 of this investigation, if two angles and the side of the triangle between the two angles are given, then only a unique triangle can be constructed.
6. From part 2 of this investigation, if two angles and the side of the triangle that is not between the two angles are given, then only a unique triangle can be constructed.
7. It does not matter. Given the values of two angles, we can find the value of the third angle in the triangle.

Investigation (RHS Congruence Test)

3. From the investigation, if the hypotenuse and one side of a right-angled triangle are given, then only a unique triangle can be constructed.

Class Discussion (Consolidation for Congruence Tests)

(a) $A \leftrightarrow A$
$B \leftrightarrow D$
$C \leftrightarrow C$
$AB = AD$ (given)
$BC = DC$ (given)
$AC = AC$ (common side)
$\therefore \triangle ABC \equiv ADC$ (SSS)

(b) $D \leftrightarrow D$
$E \leftrightarrow G$
$F \leftrightarrow F$
$D\hat{E}F = D\hat{G}F$ (given)
$D\hat{F}E = D\hat{F}G = 90°$
$DF = DF$ (common side)
$\therefore \triangle DEF \equiv \triangle DGF$ (AAS)

(c) $P \leftrightarrow P$
$Q \leftrightarrow S$
$R \leftrightarrow R$
$QR = SR$ (given)
$P\hat{R}Q = P\hat{R}S = 90°$
$PR = PR$ (common side)
$\therefore \triangle PQR \equiv \triangle PSR$ (RHS)

(d) $W \leftrightarrow W$
$X \leftrightarrow Z$
$Y \leftrightarrow Y$
$WX = WZ$ (given)
$W\hat{Y}X = W\hat{Y}Z = 90°$
$WY = WY$ (common side)
$\therefore \triangle WXY \equiv WZY$ (SAS)

(e) $A \leftrightarrow C$
$B \leftrightarrow D$
$C \leftrightarrow A$
$AB = CD$ (given)
$B\hat{A}C = D\hat{C}A$ (corr. ∠s, $AB \parallel DC$)
$AC = CA$ (common side)
$\therefore \triangle ABC \equiv \triangle CDA$ (SAS)

(f) $E \leftrightarrow G$
$F \leftrightarrow H$
$G \leftrightarrow E$
$EF = GH$
$EF = HE$
$EG = GE$ (common side)
$\therefore \triangle EFG \equiv \triangle GHE$ (SSS)

(g) $I \leftrightarrow K$
$J \leftrightarrow L$
$K \leftrightarrow I$
$IJ = KL$ (given)
$JK = LI$ (given)
$IK = KI$ (common side)
$\therefore \triangle IJK \equiv \triangle KLI$ (SSS)

(h) $M \leftrightarrow O$
$N \leftrightarrow P$
$O \leftrightarrow M$
$ON = MP$ (given)
$M\hat{O}N = \mathrm{O}\hat{M}P$ (corr. ∠s, $ON \parallel PM$)
$OM = MO$ (common side)
$\therefore \triangle MNO \equiv \triangle OPM$ (SAS)

Investigation (AA Similarity Test)

2. $\angle ACB = 180° - 50° - 30° = 100°$
   $\angle XZY = 180° - 50° - 30° = 100°$
   Yes, $\angle ACB = \angle XZY$
3. Yes, $\dfrac{AB}{XY} = \dfrac{BC}{YZ} = \dfrac{AC}{XZ}$
4. Yes, the two triangles are similar.
5. Yes, these given conditions are enough to prove that the two triangles are similar.

Thinking Time (Page 300)

1. If two angles of a triangle are given, the third unknown angle is a unique angle of the triangle. As such, the AAA Similarity Test is not necessary.
2. Yes, two congruent triangles satisfy the AA Similarity Test. Congruence is a special case of similarity.

Investigation (SSS Similarity Test)

3. $\frac{DE}{PQ} = \frac{2}{4} = \frac{1}{2}$

$\frac{EF}{QR} = \frac{3}{6} = \frac{1}{2}$

$\frac{DF}{PR} = \frac{4}{8} = \frac{1}{2}$

Yes, $\frac{DE}{PQ} = \frac{EF}{QR} = \frac{DF}{PR}$

4. $\angle EDF = \angle QPR$

$\angle DEF = \angle PQR$

$\angle DFE = \angle PRQ$

5. Yes, the two triangles are similar.

6. Yes, these given conditions are enough to prove that the two triangles are similar.

Thinking Time (Page 304)

For both the SSS Congruence Test and the SSS Similarity test, the 3 ratios of the corresponding sides of two triangles must be equal. However, for the SSS Congruence Test, the ratio of the corresponding sides of the two triangles must be equal to 1.

Investigation (SAS Similarity Test)

3. $\frac{PQ}{AB} = \frac{4.5}{3} = 1.5$

$\frac{QR}{BC} = \frac{7.5}{5} = 1.5$

Yes, $\frac{PQ}{AB} = \frac{QR}{BC}$

4. Yes, $\frac{PR}{AC} = \frac{PQ}{AB} = \frac{QR}{BC}$

5. $\angle BAC = \angle QPR$ and $\angle ACB = \angle PRQ$

6. Yes, the two triangles are similar.

7. Yes, these given conditions are enough to prove that the two triangles are similar.

Thinking Time (Page 306)

1. For both the SAS Congruence Test and the SAS Similarity test, the 2 ratios of the corresponding sides of two triangles must be equal and the pair of included angles must also be equal. However, for the SAS Congruence Test, the ratio of the corresponding sides of the two triangles must be equal to 1.

2. Since the given conditions for the AA Similarity Test is enough, there is no need for AAS Similarity Test.

3. Yes. For RHS Similarity Test, if the ratio of the hypotenuse and one side of a right-angled triangle is equal to the ratio of the hypotenuse and one side of another right-angled triangle, then the two triangles are similar. However, this test is not included in the syllabus.

Practise Now 1

1. $A \leftrightarrow \underline{E}$

$B \leftrightarrow \underline{F}$

$C \leftrightarrow \underline{D}$

$AB = \underline{EF} = 5$ m

$BC = \underline{FD} = \underline{11}$ m

$AC = \underline{ED}$ (given)

$\therefore \triangle ABC \equiv \triangle \underline{ABC}$ (\underline{SSS})

2. $W \leftrightarrow W$

$X \leftrightarrow Z$

$Y \leftrightarrow Y$

$WX = WZ$ (given)

$XY = ZY$ (given)

$WY = WY$

$\therefore \triangle WXY \equiv \triangle WZY$ (SSS)

Worked Example 2

$P \leftrightarrow \underline{G}$

$Q \leftrightarrow \underline{H}$

$R \leftrightarrow \underline{F}$

$PQ = \underline{GH} = 9$ mm

$Q\hat{P}R = \underline{H\hat{G}F} = \underline{40°}$

$PR = \underline{GF} = \underline{12}$ mm

$\therefore \triangle PQR \equiv \triangle \underline{GHF}$ (SAS)

Practise Now 2

1. $P \leftrightarrow S$

$Q \leftrightarrow P$

$R \leftrightarrow T$

$PQ = SP$ (given)

$P\hat{Q}R = S\hat{P}T$ (given)

$QR = PT$ (given)

$\therefore \triangle PQR \equiv \triangle SPT$ (SAS)

2. $A \leftrightarrow X$

$B \leftrightarrow Y$

$C \leftrightarrow Z$

$A\hat{B}C = X\hat{Y}Z = 36°$

$BC = YZ = 12$ cm

However, AB is not equal to XY.

$\therefore \triangle ABC$ is not congruent to $\triangle XYZ$.

Practise Now 3

1. **(i)** $A \leftrightarrow C$

$O \leftrightarrow O$

$B \leftrightarrow D$

$AB = CD$ (given)

$O\hat{A}B = O\hat{D}D = 25°$ (alt. $\angle$s)

$OA = OC$

$\therefore \triangle AOB \equiv \triangle COD$ (SAS)

(ii) Since $\triangle AOB \equiv \triangle COD$, then all corresponding angles are equal.

$B\hat{D}C = A\hat{B}O = 25°$

2. **(i)** $P \leftrightarrow R$
$Q \leftrightarrow S$
$S \leftrightarrow Q$
$PQ = RS$ (given)
$P\hat{Q}S = R\hat{S}Q$ (alt. $\angle$s)
$SQ = QS$ (common side)
$\therefore \triangle PQS \equiv \triangle RSQ$ (SAS)

(ii) Since $\triangle PQS \equiv \triangle RSQ$, then all the corresponding sides and angles are equal.
$QR = PS = 7$ cm
$Q\hat{P}S = S\hat{R}Q = 140°$

Worked Example 4

In $\triangle DEF$, $E\hat{F}D = 180° - 80° - 30°$ ($\angle$ sum of a $\triangle$)
$= 70°$
$A \leftrightarrow \underline{F}$
$B \leftrightarrow \underline{E}$
$C \leftrightarrow \underline{D}$
$A\hat{B}C = \underline{F\hat{E}D} = 80°$
$B\hat{A}C = \underline{E\hat{F}D} = \underline{70°}$
$BC = \underline{ED} = \underline{10}$ mm
$\therefore \triangle ABC \equiv \triangle \underline{FED}$ (AAS)

Practise Now 4

(a) $V \leftrightarrow Z$
$W \leftrightarrow Y$
$X \leftrightarrow X$
$V\hat{W}X = Z\hat{Y}X$ (alt. $\angle$s)
$W\hat{X}V = Y\hat{X}Z$ (vert. opp. $\angle$s)
$WX = YX$ (given)
$\therefore \triangle VWX \equiv \triangle ZYX$ (AAS)

(b) $A \leftrightarrow D$
$B \leftrightarrow C$
$C \leftrightarrow B$
$B\hat{A}C = C\hat{D}B = 35°$
$A\hat{C}B = D\hat{B}C$ (given)
$BC = CB$ (same side)
$\therefore \triangle ABC \equiv \triangle DCB$ (AAS)

Worked Example 5

By Pythagoras' Theorem,

$$ST = \sqrt{TU^2 + SU^2}$$
$$= \sqrt{3^2 + 4^2}$$
$$= \sqrt{25}$$
$$= 5 \text{ cm}$$

$P \leftrightarrow \underline{S}$
$Q \leftrightarrow \underline{U}$
$R \leftrightarrow \underline{T}$
$P\hat{Q}R = \underline{S\hat{U}T} = \underline{90°}$
$PR = \underline{ST} = \underline{5}$ cm
$QR = \underline{TU} = \underline{3}$ cm
$\therefore \triangle PQR \equiv \triangle \underline{SUT}$ (RHS)

Practise Now 5

(a) $A \leftrightarrow E$
$B \leftrightarrow D$
$C \leftrightarrow C$
$A\hat{C}B = E\hat{C}D = 90°$
$AB = ED$ (given)
$BC = DC$ (given)
$\therefore \triangle ABC \equiv \triangle EDC$ (RHS)

(b) $X \leftrightarrow Z$
$W \leftrightarrow Y$
$Z \leftrightarrow X$
$X\hat{W}Z = Z\hat{Y}X = 90°$
$WZ = YX$ (given)
$XZ = ZX$ (same side)
$\therefore \triangle XWZ \equiv \triangle ZYX$ (RHS)

Worked Example 6

$A\hat{C}B = A\hat{B}C$ (base $\angle$s of isos. $\triangle$)
$= 70°$

$$Y\hat{X}Z = Y\hat{Z}X = \frac{180° - 40°}{2} \text{ (base } \angle\text{s of isos. } \triangle)$$
$$= 70°$$

$A \leftrightarrow Y$
$B \leftrightarrow \underline{X}$
$C \leftrightarrow \underline{Z}$
$A\hat{B}C = \underline{Y\hat{X}Z} = 70°$
$A\hat{C}B = \underline{Y\hat{Z}X} = \underline{70°}$
$\therefore \triangle ABC$ is similar to $\triangle \underline{YXZ}$ (2 pairs of corr. $\angle$s equal).

Practise Now 6

1. **(a)** $A\hat{B}C = 180° - 60° - 45°$ ($\angle$ sum of a $\triangle$)
$= 75°$
$Y\hat{X}Z = 180° - 60° - 75°$ ($\angle$ sum of a $\triangle$)
$= 45°$
$A \leftrightarrow Y$
$B \leftrightarrow Z$
$C \leftrightarrow X$
$B\hat{A}C = Z\hat{Y}X = 60°$
$A\hat{C}B = Y\hat{X}Z = 45°$
$\therefore \triangle ABC$ is similar to $\triangle YZX$ (2 pairs of corr. $\angle$s equal).

(b) $D\hat{E}F = 180° - 90° - 30°$ (∠ sum of △)

$= 60°$

$Q\hat{P}R = 180° - 90° - 50°$ (∠ sum of a △)

$= 40°$

$D \leftrightarrow P$

$E \leftrightarrow Q$

$F \leftrightarrow R$

Since there are no corresponding pairs of angles that are equal, the two triangles are not similar.

(c) $R \leftrightarrow V$

$S \leftrightarrow U$

$T \leftrightarrow T$

$S\hat{T}R = U\hat{T}V$ (vert. opp. ∠s)

$R\hat{S}T = V\hat{U}T$ (alt. ∠s)

∴ △*RST* is similar to △*VUT* (2 pairs of corr. ∠s equal).

(d) $K \leftrightarrow K$

$L \leftrightarrow N$

$M \leftrightarrow P$

$L\hat{K}M = N\hat{K}P$ (common angle)

$K\hat{L}M = K\hat{N}P$ (corr. ∠s, *LM* // *NP*)

∴ △*KLM* is similar to △*KNP* (2 pairs of corr. ∠s equal).

2. **(i)** $A \leftrightarrow A$

$B \leftrightarrow D$

$C \leftrightarrow E$

$B\hat{A}C = D\hat{A}E$ (common angle)

$A\hat{B}C = A\hat{D}E$ (corr. ∠s, *BC* // *DE*)

∴ △*ABC* is similar to △*ADE* (2 pairs of corr. ∠s equal).

(ii) Since △*ABC* is similar to △*ADE*,

$$\frac{AB}{AD} = \frac{BC}{DE} = \frac{AC}{AE}$$

$$\frac{7}{DE} = \frac{8}{12}$$

$8DE = 84$

$DE = 10.5$ cm

$$\frac{6}{AD} = \frac{8}{12}$$

$8AD = 72$

$AD = 9$ cm

$BD = 9 - 6 = 3$ cm

(iii) $$\frac{AB}{AD} = \frac{6}{3} = 2$$

$$\frac{AC}{CE} = \frac{8}{12-8} = \frac{8}{4} = 2$$

$$\therefore \frac{AB}{BD} = \frac{AC}{CE}$$

Worked Example 7

$A \leftrightarrow T$

$B \leftrightarrow \underline{U}$

$C \leftrightarrow \underline{S}$

$$\frac{AB}{TU} = \frac{7.5}{3} = 2.5$$

$$\frac{AC}{TS} = \frac{10}{\underline{4}} = \underline{2.5}$$

$$\frac{BC}{US} = \frac{15}{6} = \underline{2.5}$$

∴ △*ABC* is similar to △$\underline{TUS}$ (3 ratios of corr. ∠s equal).

Practise Now 7

(a) $A \leftrightarrow Z$

$B \leftrightarrow Y$

$C \leftrightarrow X$

$$\frac{AB}{ZY} = \frac{5}{7.5} = \frac{2}{3}$$

$$\frac{AC}{ZX} = \frac{6}{9} = \frac{2}{3}$$

$$\frac{BC}{YX} = \frac{8}{12} = \frac{2}{3}$$

∴ △*ABC* is similar to △*ZYX* (3 ratios of corr. ∠s equal).

(b) $P \leftrightarrow U$

$Q \leftrightarrow S$

$R \leftrightarrow T$

$$\frac{PQ}{US} = \frac{5}{1.5} = 3\frac{1}{3}$$

$$\frac{PR}{UT} = \frac{3}{1} = 3$$

$$\frac{QR}{ST} = \frac{6}{2} = 3$$

∴ Since the 3 ratios of corresponding angles are not equal, the two triangles are not similar.

Worked Example 8

$D \leftrightarrow \underline{D}$

$E \leftrightarrow \underline{G}$

$F \leftrightarrow \underline{H}$

$E\hat{D}F = \underline{G\hat{D}H}$ (common angle)

$$\frac{DE}{DG} = \frac{1}{2}$$

$$\frac{DF}{DH} = \frac{1}{\underline{2}}$$

$$\therefore \frac{DE}{DG} = \frac{DF}{DH}$$

∴ △*DEF* is similar to △$\underline{DGH}$ (2 ratios of corr. sides and included ∠ equal).

Practise Now 8

(a) $J \leftrightarrow N$

$K \leftrightarrow M$

$L \leftrightarrow L$

$J\hat{L}K = N\hat{L}M$ (vert. opp ∠s)

$$\frac{JL}{NL} = \frac{3}{5.4} = \frac{5}{9}$$

$$\frac{KL}{ML} = \frac{4}{7.2} = \frac{5}{9}$$

$$\therefore \frac{JL}{NL} = \frac{KL}{ML}$$

∴ $\triangle JKL$ is similar to $\triangle NML$ (2 ratios of corr. sides and included ∠ equal).

(b) $A \leftrightarrow Z$

$B \leftrightarrow Y$

$C \leftrightarrow X$

$A\hat{C}B = Z\hat{X}Y = 75°$

$$\frac{AC}{ZX} = \frac{15}{10} = 1.5$$

$$\frac{BC}{YX} = \frac{10}{5} = 2$$

∴ Since the 2 ratios of corresponding sides are not equal, the two triangles are not similar.

Practise Now 9

1. (i) $A \leftrightarrow E$

$B \leftrightarrow C$

$C \leftrightarrow G$

$AC = EG$ (given)

$A\hat{C}B = E\hat{G}C$ (corr. ∠s, BC // EG)

$BC = CG$ (given)

∴ $\triangle ABC \equiv \triangle ECG$ (SAS)

(ii) $A \leftrightarrow F$

$C \leftrightarrow F$

$D \leftrightarrow D$

$A\hat{D}C = E\hat{D}F$ (vert. opp. ∠s)

Since $\triangle ABC$ and $\triangle ECG$ are congruent,

then $B\hat{A}C = C\hat{E}G$,

i.e. $D\hat{A}C = D\hat{E}F$.

∴ $\triangle ACD$ is similar to $\triangle EFD$ (2 pairs of corr. ∠s equal).

(iii) Since $AC = EF = 15$ cm, then $EF = 15 - 9 = 6$ cm

Since $\triangle ACD$ and $\triangle EFD$ are similar, then

$$\frac{DF}{DC} = \frac{EF}{AC}$$

$$\text{i.e. } \frac{DF}{5} = \frac{6}{15}$$

$$\therefore DF = \frac{6}{15} \times 5$$

$$= 2 \text{ cm}$$

2. $A \leftrightarrow X$

$X \leftrightarrow B$

$P \leftrightarrow Q$

$A\hat{X}P = X\hat{B}Q$ (corr. ∠s, XP // BC)

$A\hat{P}X = X\hat{Q}B$ (corr. ∠s, XQ // AC)

∴ $\triangle AXP$ is similar to $\triangle XBQ$ (2 pairs of corr. ∠s equal).

Since $\triangle AXP$ and $\triangle XBQ$ are similar, then

$$\frac{BQ}{XP} = \frac{XB}{AX}$$

$$\text{i.e. } \frac{BQ}{15} = \frac{4}{3}$$

$$\therefore BQ = \frac{4}{3} \times 15$$

$$= 20 \text{ cm}$$

$A \leftrightarrow X$

$B \leftrightarrow B$

$C \leftrightarrow Q$

$A\hat{B}C = X\hat{B}Q$ (common angle)

$A\hat{C}B = X\hat{Q}B$ (corr. ∠s, XP // BC)

∴ $\triangle ABC$ is similar to $\triangle XBQ$ (2 pairs of corr. ∠s equal)

Since $\triangle ABC$ and $\triangle XBQ$ are similar, then

$$\frac{AC}{XQ} = \frac{AB}{XB}$$

$$\text{i.e. } \frac{AC}{16} = \frac{7}{4}$$

$$\therefore AC = \frac{7}{4} \times 16$$

$$= 28 \text{ cm}$$

Practise Now 10

$P \leftrightarrow P$

$A \leftrightarrow B$

$Q \leftrightarrow Q$

$AP = BP$

$AQ = BQ$

$PQ = PQ$ (common side)

∴ $\triangle PAQ$ and $\triangle PBQ$ are congruent (SSS Congruence Test).

∴ $A\hat{P}Q = B\hat{P}Q$

i.e. $A\hat{P}R = B\hat{P}R$

Since $AP = BP$ and PR is a common side, $\triangle PAR$ and $\triangle PBR$ are congruent (SAS Congruence Test).

∴ $AR = RB$

$$A\hat{R}P = B\hat{R}P$$

$$= \frac{180°}{2} \text{ (adj. ∠s on a str. line)}$$

$$= 90°$$

∴ PQ is the perpendicular bisector of AB.

Practise Now 11

1. $A \leftrightarrow A$

$B \leftrightarrow D$

$C \leftrightarrow E$

$$\frac{BC}{DE} = \frac{AC}{AE}$$

i.e. $\frac{BC}{11.2} = \frac{10}{2}$

$\therefore BC = \frac{10}{2} \times 11.2$

$= 56$ m

2. $C \leftrightarrow A$

$D \leftrightarrow B$

$E \leftrightarrow E$

$$\frac{AB}{CD} = \frac{BE}{DE}$$

i.e. $\frac{AB}{1.4} = \frac{18}{2.1}$

$\therefore AB = \frac{18}{2.1} \times 1.4$

$= 12$ m

Exercise 10A

1. (a) Comparing triangle **(ii)** and triangle **(vii)**,

The 3 sides of triangle **(ii)** are equal to the 3 corresponding sides of triangle **(vii)**.

$\therefore$ The two triangles are congruent (SSS).

(b) Comparing triangle **(iii)** and triangle **(v)**,

The 2 sides and the included angle of triangle **(iii)** are equal to the 2 corresponding sides and the corresponding included angle of triangle **(v)**.

$\therefore$ The two triangles are congruent (SAS).

(c) Comparing triangle **(i)** and triangle **(ix)**,

The 2 angles and 1 side of triangle **(i)** are equal to the 2 corresponding angles and the corresponding side of triangle **(ix)**.

$\therefore$ The two triangles are congruent (AAS).

(d) Comparing triangle **(vi)** and triangle **(viii)**,

The hypotenuse and 1 side of triangle **(vi)** are equal to the hypotenuse and 1 side of triangle **(viii)**.

$\therefore$ The two triangles are congruent (RHS).

2. (a) $A \leftrightarrow \underline{P}$

$B \leftrightarrow \underline{Q}$

$B \leftrightarrow \underline{R}$

$AB = \underline{PQ}$ (given)

$BC = \underline{QR} = 8$ cm

$AC = \underline{PR} = 6$ cm

$\therefore \triangle ABC \equiv \triangle \underline{PQR}$ ($\underline{\text{SSS}}$)

(b) $D \leftrightarrow \underline{Z}$

$E \leftrightarrow \underline{Y}$

$F \leftrightarrow \underline{X}$

$DE = \underline{ZY} = 3$ m

$D\hat{E}F = \underline{Z\hat{Y}X} = \underline{70°}$

$EF = \underline{YX} = \underline{5}$ m

$\therefore \triangle DEF \equiv \triangle \underline{ZYX}$ ($\underline{\text{SAS}}$)

(c) $L \leftrightarrow \underline{W}$

$M \leftrightarrow \underline{V}$

$N \leftrightarrow \underline{U}$

$L\hat{M}N = \underline{W\hat{V}U} = \underline{30°}$

$L\hat{N}M = \underline{W\hat{U}V} = \underline{70°}$

$MN = \underline{VU} = \underline{7}$ cm

$\therefore \triangle \underline{NML} \equiv \triangle UVW$ ($\underline{\text{AAS}}$)

(d) $G \leftrightarrow \underline{U}$

$H \leftrightarrow \underline{T}$

$I \leftrightarrow \underline{S}$

$G\hat{H}I = \underline{U\hat{T}S} = \underline{90°}$

$GI = \underline{US} = 13$ mm

$HI = \underline{TS} = \underline{5}$ mm

$\therefore \triangle \underline{IHG} \equiv \triangle STU$ ($\underline{\text{RHS}}$)

3. (a) $A \leftrightarrow E$

$B \leftrightarrow D$

$C \leftrightarrow F$

The 3 sides of $\triangle ABC$ are not equal to the 3 corresponding sides of $\triangle EDF$.

$\therefore \triangle ABC$ is not congruent to $\triangle EDF$.

(b) $X \leftrightarrow Q$

$Y \leftrightarrow R$

$Z \leftrightarrow P$

$X\hat{Z}P = Q\hat{P}R = 40°$

$YZ = RP = 6$ mm

XZ is not equal to QP.

$\therefore \triangle XYZ$ is not congruent to $\triangle QRP$.

(c) $G \leftrightarrow U$

$H \leftrightarrow T$

$I \leftrightarrow S$

$H\hat{G}I = 180° - 75° - 40°$ ($\angle$ sum of a $\triangle$)

$= 65°$

$T\hat{S}U = 180° - 55° - 40°$ ($\angle$ sum of a $\triangle$)

$= 85°$

$HI = ST = 5$ cm

$G\hat{H}I = U\hat{T}S = 40°$

$G\hat{I}H$ is not equal to $U\hat{S}T$ and $H\hat{G}I$ is not equal to $T\hat{U}S$.

$\therefore \triangle GHI$ is not congruent to $\triangle UTS$.

(d) $M \leftrightarrow P$
$N \leftrightarrow Q$
$O \leftrightarrow R$
By Pythagoras' Theorem,
$PQ = \sqrt{12^2 - 5^2}$
$= 10.91$ cm (to 4 s.f.)
$M\hat{N}O = P\hat{Q}R = 90°$
$NO = QR = 5$ cm
MN is not equal to PQ and OM is not equal to RP.
$\therefore \triangle MNO$ is not congruent to $\triangle PQR$.

4. **(a)** $A \leftrightarrow C$
$B \leftrightarrow B$
$D \leftrightarrow D$
$AB = CB$ (given)
$AD = CD$ (given)
$BD = BD$ (common side)
$\therefore \triangle ABD \equiv \triangle CBD$ (SSS)

(b) $A \leftrightarrow C$
$B \leftrightarrow D$
$D \leftrightarrow B$
$AB = CD$ (given)
$AD = CB$ (given)
$BD = DB$ (common side)
$\therefore \triangle ABD \equiv \triangle CDB$ (SSS)

(c) $A \leftrightarrow E$
$B \leftrightarrow D$
$C \leftrightarrow C$
$AC = EC$ (given)
$CB = CD$ (given)
$A\hat{C}B = E\hat{C}D$ (vert. opp. $\angle$s)
$\therefore \triangle ABC \equiv \triangle EDC$ (SAS)

(d) $A \leftrightarrow C$
$B \leftrightarrow D$
$C \leftrightarrow A$
$BC = DA$ (given)
$AC = CA$ (common side)
$B\hat{C}A = D\hat{C}A$ (alt. $\angle$s)
$\therefore \triangle ABC \equiv \triangle CDA$ (SAS)

(e) $A \leftrightarrow C$
$D \leftrightarrow D$
$E \leftrightarrow B$
$AE = CB$ (given)
$A\hat{E}D = C\hat{B}D$ (given)
$E\hat{A}D = B\hat{C}D$ (given)
$\therefore \triangle ADE \equiv \triangle CDB$ (AAS)

(f) $B \leftrightarrow E$
$C \leftrightarrow F$
$D \leftrightarrow D$
$BC = EF$ (given)
$C\hat{B}D = F\hat{E}D$ (given)
$B\hat{D}C = E\hat{D}F$ (vert. opp. $\angle$s)
$\therefore \triangle BCD \equiv \triangle EFD$ (AAS)

(g) $A \leftrightarrow C$
$B \leftrightarrow B$
$D \leftrightarrow D$
$AD = CD$ (given)
$AB = CB$ (given)
$BD = BD$ (common side)
$\therefore \triangle ABD \equiv \triangle CBD$ (SSS)

(h) $A \leftrightarrow C$
$B \leftrightarrow D$
$C \leftrightarrow A$
$BC = DA$ (given)
$AC = CA$ (common side)
$A\hat{B}C = C\hat{D}A = 90°$
$\therefore \triangle ABC \equiv \triangle CDA$ (RHS)

5. **(i)** $R \leftrightarrow V$
$S \leftrightarrow U$
$T \leftrightarrow T$
$RT = VT$ (given)
$ST = UT$ (given)
$R\hat{T}S = V\hat{T}U$ (vert. opp. $\angle$)
$\therefore \triangle RST \equiv \triangle VUT$ (SAS)

(ii) Since $\triangle RST \equiv \triangle VUT$,
$UV = SR = 4$ cm

(iii) Since $\triangle RST \equiv \triangle VUT$,
$U\hat{V}T = S\hat{R}T = 80°$

(iv) RS is parallel to UV.

6. **(i)** $J \leftrightarrow G$
$I \leftrightarrow H$
$H \leftrightarrow I$
$JI = GH$ (given)
$JH = GI$ (given)
$IH = HI$ (common side)
$\therefore \triangle JIH \equiv \triangle GHI$ (SSS)

(ii) Since $\triangle JIH \equiv \triangle GHI$,
$I\hat{G}H = H\hat{J}I = 60°$
$G\hat{H}I = 180° - 60° - 40°$ ($\angle$ sum of a $\triangle$)
$= 80°$

7. **(a)** $A \leftrightarrow C$
$B \leftrightarrow D$
$C \leftrightarrow A$
$BC = DA$ (given)
$AC = CA$ (common side)
$B\hat{C}A = D\hat{A}C$ (alt. $\angle$s)
$\therefore \triangle ABC \equiv \triangle CDA$ (SAS)

(b) $E \leftrightarrow G$
$F \leftrightarrow H$
$G \leftrightarrow E$
$GF = EH$ (given)
$EG = GE$ (common side)
$E\hat{F}G = G\hat{H}E = 90°$
$\therefore \triangle EFG \equiv \triangle GHE$ (RHS)

(c) $I \leftrightarrow K$
$J \leftrightarrow L$
$K \leftrightarrow I$
$IJ = KL$ (given)
$JK = LI$ (given)
$IK = KI$ (common side)
$\therefore \triangle IJK \equiv \triangle KLE$ (SSS)

(d) $M \leftrightarrow O$
$N \leftrightarrow P$
$O \leftrightarrow M$
$MO = OM$ (common side)
$M\hat{N}O = O\hat{P}M = 90°$
$M\hat{O}N = O\hat{M}P$ (alt. $\angle$s)
$\therefore \triangle MNO \equiv \triangle OPM$ (AAS)

(e) $Q \leftrightarrow S$
$R \leftrightarrow T$
$S \leftrightarrow Q$
$QS = SQ$ (common side)
$R\hat{Q}S = T\hat{S}Q$ (alt. $\angle$s)
$Q\hat{S}R = S\hat{Q}T$ (alt. $\angle$s)
$\therefore \triangle QRS \equiv \triangle STQ$ (AAS)

(f) $U \leftrightarrow Q$
$V \leftrightarrow X$
$W \leftrightarrow U$
$VW = XU$ (given)
$UV = QX$ (given)
$U\hat{V}W = Q\hat{X}U = 90°$
$\therefore \triangle UVW \equiv \triangle QXU$ (RHS)

8. $A \leftrightarrow C$
$B \leftrightarrow D$
$C \leftrightarrow A$
$AB = CD$
$BC = DA$
$AC = CA$ (common side)
$\therefore \triangle ABC \equiv \triangle CDA$ (SSS)
$B\hat{A}C = D\hat{C}A$ (alt. $\angle$s)
$A\hat{C}B = C\hat{A}D$ (alt. $\angle$s)
$AC = CA$ (common side)
$\therefore \triangle ABC \equiv \triangle CDA$ (AAS)
$BC = DA$
$AC = CA$ (common side)
$B\hat{A}C = D\hat{C}A$ (alt. $\angle$s)
$\therefore \triangle ABC \equiv \triangle CDA$ (SAS)

Exercise 10B

1. **(a)** Comparing triangle **(i)** and triangle **(iii)**,

The 2 angles of triangle **(i)** are equal to the 2 corresponding angles of triangle **(iii)**.

∴ The two triangles are similar (2 pairs of corr. ∠s equal).

Comparing triangle **(v)** and triangle **(vii)**,

The 2 angles of triangle **(v)** are equal to the 2 corresponding angles of triangle **(vii)**.

∴ The two triangles are similar (2 pairs of corr. ∠s equal).

(b) Comparing triangle **(ii)** and triangle **(vi)**,

$\frac{12}{6} = 2$

$\frac{8}{4} = 2$

$\frac{7.8}{3.9} = 2$

The 3 ratios of the corresponding sides of triangle **(ii)** and triangle **(vi)** are equal.

∴ The two triangles are similar (3 ratios of corr. sides equal).

(c) Comparing triangle **(iv)** and triangle **(viii)**,

$\frac{24}{6} = 4$

$\frac{18}{4.5} = 4$

The ratios of the corresponding sides of triangle **(iv)** and triangle **(viii)** are equal and the pair of included angles are also equal.

∴ The two triangles are similar (2 ratios of corr. sides and included ∠ equal).

2. **(a)** $S\hat{T}U = 180° - 70° - 50°$ ($\angle$sum of a $\triangle$)
$= \underline{60°}$
$A \leftrightarrow \underline{S}$
$B \leftrightarrow \underline{T}$
$C \leftrightarrow \underline{U}$
$B\hat{A}C = \underline{T\hat{S}U} = 70°$
$A\hat{B}C = \underline{S\hat{T}U} = \underline{60°}$
$\therefore \triangle ABC$ is similar to $\triangle \underline{STU}$ ($\underline{2}$ pairs of corr. $\underline{\angle}$s equal)

(b) $X \leftrightarrow \underline{N}$
$Y \leftrightarrow \underline{M}$
$Z \leftrightarrow \underline{L}$
$\frac{XY}{NM} = \frac{24}{\underline{8}} = \underline{3}$
$\frac{XZ}{NL} = \frac{21}{\underline{7}} = \underline{3}$
$\frac{YZ}{ML} = \frac{15}{\underline{5}} = \underline{3}$
$\therefore \triangle XYZ$ is similar to $\triangle \underline{NML}$ ($\underline{3}$ ratios of corr. $\underline{\text{sides}}$ equal)

(c) $D \leftrightarrow \underline{G}$
$E \leftrightarrow \underline{I}$
$F \leftrightarrow \underline{H}$
$D\hat{E}F = \underline{G\hat{I}H} = \underline{90°}$
$\frac{DE}{GI} = \frac{6}{\underline{9}} = \frac{2}{\underline{3}}$
$\frac{EF}{IH} = \frac{4}{\underline{6}} = \frac{2}{\underline{3}}$
$\frac{DE}{GI} = \frac{EF}{\underline{IH}}$
$\therefore \triangle DEF$ is similar to $\triangle \underline{GIH}$ ($\underline{2}$ ratios of corr. $\underline{\text{sides}}$ and $\underline{\text{included}}$ $\angle$ equal).

3. **(a)** In the smaller triangle,

$180^\circ - 60^\circ - 60^\circ = 60^\circ$

In the larger triangle,

$\dfrac{180^\circ - 50^\circ}{2} = 65^\circ$

Two angles of the smaller triangle are not equal to the two corresponding angles of the larger triangle.

$\therefore$ The two triangles are not similar.

(b) $\dfrac{22.5}{45} = 0.5$

$\dfrac{15}{30} = 0.5$

$\dfrac{8}{15} = 0.5333$ (to 4 s.f.)

The 3 ratios of corresponding sides of both triangles are not equal.

$\therefore$ The two triangles are not similar.

(c) Included angle = 110°

$\dfrac{10}{25} = 0.4$

$\dfrac{7}{15} = 0.4667$ (to 4 s.f.)

The ratios of the corresponding sides of both triangles are not equal.

$\therefore$ The two triangles are not similar.

4. **(a)** $A \Leftrightarrow E$

$B \Leftrightarrow D$

$C \Leftrightarrow C$

$A\hat{B}C = E\hat{D}C$ (alt. $\angle$s)

$A\hat{C}B = E\hat{C}D$ (vert. opp. $\angle$s)

$\therefore \triangle ABC$ is similar to $\triangle EDC$ (2 pairs of corr. $\angle$s equal).

(b) $I \Leftrightarrow I$

$J \Leftrightarrow F$

$H \Leftrightarrow G$

$I\hat{J}H = I\hat{F}G$ (corr. $\angle$s, $JH \,//\, FG$)

$J\hat{I}H = F\hat{I}G$ (common angle)

$\therefore \triangle IJH$ is similar to $\triangle IFG$ (2 pairs of corr. $\angle$s equal).

(c) $P \Leftrightarrow T$

$Q \Leftrightarrow S$

$R \Leftrightarrow R$

$P\hat{R}Q = T\hat{R}S$ (vert. opp. $\angle$s)

$\dfrac{PR}{TR} = \dfrac{3}{6} = 0.5$

$\dfrac{QR}{SR} = \dfrac{4}{8} = 0.5$

$\therefore \triangle PQR$ is similar to $\triangle TSR$ (2 ratios of corr. sides and included $\angle$ equal).

(d) $U \Leftrightarrow U$

$V \Leftrightarrow X$

$W \Leftrightarrow Y$

$V\hat{U}W = X\hat{U}Y$ (common angle)

$\dfrac{UV}{UX} = \dfrac{8}{8+4} = \dfrac{8}{12} = \dfrac{2}{3}$

$\dfrac{UW}{UY} = \dfrac{10}{10+5} = \dfrac{10}{15} = \dfrac{2}{3}$

$\therefore \triangle UVW$ is similar to $\triangle UXY$ (2 ratios of corr. sides and included $\angle$ equal).

5. **(a)** $A \Leftrightarrow A$

$B \Leftrightarrow D$

$C \Leftrightarrow E$

$A\hat{C}B = A\hat{E}D$ (corr. $\angle$s, $BC \,//\, DE$)

$B\hat{A}C = D\hat{A}C$ (common angle)

$\therefore \triangle ABC$ is similar to $\triangle ADE$ (2 pairs of corr. $\angle$s equal).

Since $\triangle ABC$ is similar to $\triangle ADE$, then

$$\begin{aligned} \frac{BC}{DE} &= \frac{AC}{AE} \\ \text{i.e. } \frac{x}{12} &= \frac{12}{12+4} \\ \therefore x &= \frac{12}{16} \times 12 \\ &= 9 \end{aligned}$$

$$\begin{aligned} \frac{AB}{AD} &= \frac{AC}{AE} \\ \text{i.e. } \frac{y}{y+6} &= \frac{12}{16} \\ \therefore \frac{y}{y+6} &= \frac{3}{4} \\ 4y &= 3(y+6) \\ 4y &= 3y + 8 \\ y &= 18 \end{aligned}$$

(b) $A \Leftrightarrow E$

$B \Leftrightarrow D$

$C \Leftrightarrow C$

$B\hat{A}C = D\hat{E}C$ (corr. $\angle$s, $AB \,//\, DE$)

$A\hat{C}B = E\hat{C}D$ (common angle)

$\therefore \triangle ABC$ is similar to $\triangle EDC$ (2 pairs of corr. $\angle$s equal).

Since $\triangle ABC$ is similar to $\triangle EDC$, then

$$\begin{aligned} \frac{AC}{EC} &= \frac{BC}{DC} \\ \text{i.e. } \frac{x}{4} &= \frac{6}{5} \\ \therefore x &= \frac{6}{5} \times 4 \\ &= 4.8 \end{aligned}$$

$$\begin{aligned} \frac{DE}{BA} &= \frac{DC}{BC} \\ \text{i.e. } \frac{y}{9} &= \frac{5}{6} \\ \therefore y &= \frac{5}{6} \times 9 \\ &= 7.5 \end{aligned}$$

(c) $A \leftrightarrow A$

$B \leftrightarrow E$

$C \leftrightarrow D$

$A\hat{B}C = A\hat{E}D$ (given)

$B\hat{A}C = E\hat{A}D$ (common angle)

$\therefore \triangle ABC$ is similar to $\triangle AED$ (2 pairs of corr. ∠s equal).

Since $\triangle ABC$ is similar to $\triangle AED$, then

$$\frac{DE}{CB} = \frac{AE}{AB}$$

$$\text{i.e. } \frac{x}{4} = \frac{6+3}{3}$$

$$\therefore x = \frac{9}{3} \times 4$$

$$= 12$$

$$\frac{AB}{AE} = \frac{AC}{AD}$$

$$\text{i.e. } \frac{3}{9} = \frac{6}{3+y}$$

$$\therefore 3(3 + y) = 54$$

$$3 + y = 18$$

$$y = 15$$

(d) $A \leftrightarrow A$

$B \leftrightarrow E$

$C \leftrightarrow F$

$A\hat{B}C = A\hat{E}F$ (corr. ∠s, BC // EF)

$B\hat{A}C = E\hat{A}F$ (common angle)

$\therefore \triangle ABC$ is similar to $\triangle AEF$ (2 pairs of corr. ∠s equal).

Since $\triangle ABC$ is similar to $\triangle AEF$, then

$$\frac{AC}{AF} = \frac{AB}{AE}$$

$$\text{i.e. } \frac{8}{8+x} = \frac{10}{10+4}$$

$$\therefore \frac{8}{8+x} = \frac{10}{14}$$

$$\frac{8}{8+x} = \frac{5}{7}$$

$$56 = 40 + 5x$$

$$5x = 16$$

$$x = 3.2$$

$A \leftrightarrow A$

$C \leftrightarrow F$

$D \leftrightarrow G$

$A\hat{C}D = A\hat{F}G$ (corr. ∠s, CD // FG)

$C\hat{A}D = F\hat{A}G$ (common angle)

$\therefore \triangle ACD$ is similar to $\triangle AFG$ (2 pairs of corr. ∠s equal).

Since $\triangle ACD$ is similar to $\triangle AFG$, then

$$\frac{AD}{AG} = \frac{AC}{AF}$$

$$\text{i.e. } \frac{y}{y+3} = \frac{8}{8+3.2}$$

$$\therefore \frac{y}{y+3} = \frac{8}{11.2}$$

$$11.2y = (8y + 3)$$

$$11.2y = 8y + 24$$

$$3.2y = 24$$

$$y = 7.5$$

6. **(i)** $A \leftrightarrow D$

$B \leftrightarrow C$

$D \leftrightarrow B$

$$\frac{AB}{DC} = \frac{5}{15} = \frac{1}{3}$$

$$\frac{BD}{CB} = \frac{6}{18} = \frac{1}{3}$$

$$\frac{AD}{DB} = \frac{2}{6} = \frac{1}{3}$$

$\therefore \triangle ABD$ is similar to $\triangle DCB$ (3 ratios of corr. sides equal).

(ii) Since $\triangle ACD$ is similar to $\triangle AFG$, then

$D\hat{A}B = C\hat{D}B = 110.5°$

7. $\frac{3}{5}XY = 18$ cm

$XY = 30$ cm

$UY = \frac{2}{5} \times 30 = 12$ cm

$\triangle YUV$ is similar to $\triangle YXZ$ (2 pairs of corr. ∠s equal).

Since $\triangle YUV$ is similar to $\triangle YXZ$, then

$$\frac{XZ}{UV} = \frac{XY}{UY}$$

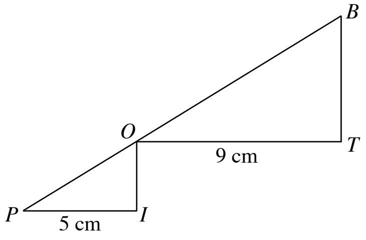
$$\text{i.e. } \frac{18 + WZ}{18} = \frac{30}{12}$$

$$\therefore \frac{18 + WZ}{18} = \frac{5}{2}$$

$$18 + WZ = 45$$

$$WZ = 27 \text{ cm}$$

8.



$\triangle POI$ is similar to $\triangle OBT$.

$OI = 9 – 5 = 4$ cm

$$\frac{BT}{OI} = \frac{OT}{PI}$$

$$\text{i.e. } \frac{BT}{4} = \frac{9}{5}$$

$$\therefore BT = \frac{9}{5} \times 4$$

$$= 7.2 \text{ cm}$$

Length side of square $BLUE$ = 9 + 7.2

= 16.2 cm

9. **(i)** $B \leftrightarrow D$

$A \leftrightarrow B$

$C \leftrightarrow C$

$A\hat{B}C = B\hat{D}C = 90°$

$B\hat{C} = D\hat{B}C$ (common angle)

$\therefore$ $\triangle BAC$ is similar to $\triangle DBC$ (2 pairs of corr. $\angle$s equal).

$B \leftrightarrow D$

$A \leftrightarrow A$

$C \leftrightarrow B$

$A\hat{B}C = A\hat{D}B = 90°$

$B\hat{A}C = D\hat{A}B$ (common angle)

$\therefore$ $\triangle BAC$ is similar to $\triangle DAB$ (2 pairs of corr. $\angle$s equal).

Hence $\triangle BAC$, $\triangle DBC$ and $\triangle DAB$ are similar.

(ii) By Pythagoras' Theorem,

$$AB = \sqrt{3^2 + 4^2}$$
$$= \sqrt{25}$$
$$= 5 \text{ m}$$

Since $\triangle BAC$, $\triangle DBC$ and $\triangle DAB$ are similar, then

$$\frac{BC}{DB} = \frac{AB}{AD}$$

i.e. $$\frac{BC}{4} = \frac{5}{3}$$

$$\therefore BC = \frac{5}{3} \times 4$$
$$= 6\frac{2}{3} \text{ m}$$

By Pythagoras' Theorem,

$$CD = \sqrt{\left(6\frac{2}{3}\right)^2 - 4^2}$$
$$= \sqrt{\frac{256}{9}}$$
$$= 5\frac{1}{3} \text{ m}$$

10. **(i)** $\triangle QTU$ is similar to $\triangle QRS$ (2 pairs of corr. $\angle$s equal).

$$\frac{QT}{QR} = \frac{QU}{QS}$$

i.e. $$\frac{12 + RT}{12} = \frac{8 + 4}{8}$$

$$\therefore \frac{12 + RT}{12} = \frac{3}{2}$$
$$2(12 + RT) = 36$$
$$24 + 2RT = 36$$
$$2RT = 12$$
$$RT = 6 \text{ cm}$$

$\triangle QPU$ is similar to $\triangle SRU$ (2 pairs of corr. $\angle$s equal).

$$\frac{PU}{RU} = \frac{QU}{SU}$$

i.e. $$\frac{PR + 6}{6} = \frac{8 + 4}{4}$$

$$\therefore \frac{PR + 6}{6} = 3$$
$$PR + 6 = 18$$
$$PR = 12 \text{ cm}$$

(ii) $\triangle QPR$ is similar to $\triangle TUR$ (2 pairs of corr. $\angle$s equal).

$$\frac{PQ}{TU} = \frac{QR}{RT}$$

$$\therefore \frac{PQ}{TU} = \frac{12}{6} = 2$$

$\therefore$ The ratio $PQ : TU$ is 2 : 1.

11. **(i)** Since A coincides with T when the triangle is folded, ANT is a straight line.

Hence MN is perpendicular to AT.

(ii) $A \leftrightarrow M$

$R \leftrightarrow N$

$T \leftrightarrow T$

$A\hat{R}T = M\hat{N}T = 90°$

$A\hat{T}R = M\hat{T}N$ (common angle)

$\therefore$ $\triangle ART$ is similar to $\triangle MNT$ (2 pairs of corr. $\angle$s equal).

(iii) By Pythagoras' Theorem,

$$AT = \sqrt{6^2 + 8^2}$$
$$= \sqrt{100}$$
$$= 10 \text{ cm}$$

$$NT = \frac{1}{2} \times 10 = 5 \text{ cm}$$

Since $\triangle ART$ is similar to $\triangle MNT$, then

$$\frac{MN}{AR} = \frac{NT}{RT}$$

i.e. $$\frac{MN}{6} = \frac{5}{8}$$

$$MN = \frac{5}{8} \times 6$$
$$= 3.75 \text{ cm}$$

12. $A \leftrightarrow J$

$J \leftrightarrow A$

$D \leftrightarrow E$

Since $AO = JO$ and $EO = DO$,

$JD = AE$

Since $AO = JO$, $EO = DO$ and $J\hat{O}E = A\hat{O}E$ (vert opp. $\angle$s),

$AD = JE$

$AJ = JA$ (same side)

$\therefore$ $\triangle AJD \equiv \triangle JAE$ (SSS)

Exercise 10C

1. $A \leftrightarrow A'$

$B \leftrightarrow B'$

$C \leftrightarrow C$

$AC = A'C$ (given)

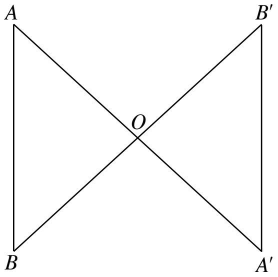
$BC = B'C$ (given)

$A\hat{C}B = A'\hat{C}B'$ (vert. opp. $\angle$s)

$\therefore$ $\triangle ABC \equiv \triangle A'B'C$ (SAS)

Hence, $AB = B'A'$.

2.



$A \leftrightarrow A'$
$O \leftrightarrow O$
$B \leftrightarrow B'$
$A\hat{O}B = A'\hat{O}B'$ (vert. opp. ∠s)
Since AA' and BB' are hinged halfway at O,
$OA = OA'$ and $OB = OB'$
$\therefore \triangle AOB \equiv \triangle A'OB'$ (SAS)
Hence, $AB = A'B'$.

3. $\triangle SOR$ is similar to $\triangle POQ$.

$$\frac{SR}{PQ} = \frac{OR}{OQ}$$

i.e. $\frac{SR}{4} = \frac{30}{15}$

$$\therefore SR = \frac{30}{15} \times 4 = 8 \text{ m}$$

Hence the height of the tree is 8 m.

4. $P \leftrightarrow Q$
$O \leftrightarrow O$
$C \leftrightarrow C$
$PO = QO$ (given)
$OC = OC$ (common side)
$O\hat{C}P = O\hat{C}Q = 90°$
$\therefore \triangle POC$ and $\triangle QOC$ are congruent (RHS Congruence Test).
Hence OC is the angle bisector of $A\hat{O}B$.

5. $A \leftrightarrow A$
$B \leftrightarrow B'$
$C \leftrightarrow C$
$AC = AC$ (common side)
$B\hat{C}A = B'\hat{C}A$ (same line of vision)
$BC = B'C$
$\therefore \triangle AOB \equiv \triangle A'OB'$ (SAS)
Hence $AB = AB'$.

6. Given that the image of the candle is 3 times the length of the candle,

$$\frac{AC}{DE} = \frac{1}{3}$$

$\triangle ABC$ is similar to $\triangle DBE$.

$$\frac{BD}{BA} = \frac{DE}{AC}$$

i.e. $\frac{x}{15} = \frac{3}{1}$

$\therefore x = 45$

7. $P \leftrightarrow Q$
$O \leftrightarrow O$
$M \leftrightarrow M$
$OP = OQ$ (given)
$OM = OM$ (common side)
$P\hat{M}Q = 90°$
$PM = QM$
$\therefore \triangle POM$ and $\triangle QOM$ are congruent (SSS Congruence Test).
Hence OM is the angle bisector of $A\hat{O}B$.

Review Exercise 10

1. **(a)** $A \leftrightarrow R$
$B \leftrightarrow P$
$C \leftrightarrow Q$
$AB = RP = 5$ m
$AC = RQ = 7$ m
$BC = PQ = 6$ m
$\therefore \triangle ABC \equiv \triangle RPQ$ (SSS)

(b) $A \leftrightarrow P$
$B \leftrightarrow Q$
$C \leftrightarrow R$
$AB = PQ = 12$ cm
$AC = PR = 8.9$ cm
$B\hat{A}C = Q\hat{P}R = 80°$
$\therefore \triangle ABC \equiv \triangle PQR$ (SAS)

(c) $A \leftrightarrow P$
$B \leftrightarrow Q$
$C \leftrightarrow R$
$BC = QR = 9$ cm
$A\hat{B}C = P\hat{Q}R = 30°$
AB is not equal to PQ.
$\therefore \triangle ABC$ and $\triangle PQR$ are not congruent.

(d) $P\hat{Q}R = 180° - 75° - 45°$ (∠ sum of a △)
$= 60°$
$A \leftrightarrow P$
$B \leftrightarrow Q$
$C \leftrightarrow R$
$AB = PQ = 65$ mm $= 6.5$ cm
$BC = QR = 89$ mm $= 8.9$ cm
$A\hat{B}C = P\hat{Q}R = 60°$
$\therefore \triangle ABC \equiv \triangle PQR$ (SAS)

2. $J\hat{L}K = 180° - 55° - 50°$ (∠ sum of a △)
$= 75°$
$D \leftrightarrow J$
$E \leftrightarrow L$
$F \leftrightarrow K$
$D\hat{E}F = J\hat{L}K = 75°$
$D\hat{F}E = J\hat{K}L = 50°$
$EF = LK = 3$ cm
$\therefore \triangle DEF \equiv \triangle JLK$ (AAS)

3. **(a)** $A \leftrightarrow D$
$B \leftrightarrow E$
$C \leftrightarrow C$
$AB = DE$ (given)
$A\hat{B}C = D\hat{E}C$ (given)
$B\hat{A}C = E\hat{D}C$ (given)
$\therefore \triangle ABC \equiv \triangle DEC$ (AAS)
$A\hat{C}B = D\hat{C}E$
$BC = EC$
$AC = DC$

(b) $F \leftrightarrow F$
$G \leftrightarrow I$
$H \leftrightarrow J$
$GH = IJ$ (given)
$G\hat{H}F = I\hat{J}F$ (given)
$G\hat{F}H = I\hat{F}J$ (vert. opp. ∠s)
$\therefore \triangle FGH \equiv \triangle FIJ$ (SAS)
$F\hat{G}H = F\hat{I}J$
$FG = FI$
$FH = FJ$

(c) $K \leftrightarrow M$
$L \leftrightarrow N$
$N \leftrightarrow L$
$KN = ML$ (given)
$LN = LN$ (same side)
$L\hat{K}N = N\hat{M}L = 90^\circ$
$\therefore \triangle KLN \equiv \triangle MNL$ (RHS)
$K\hat{L}N = M\hat{N}L$
$K\hat{N}L = M\hat{L}M$
$KL = MN$

(d) $S \leftrightarrow R$
$Q \leftrightarrow P$
$P \leftrightarrow Q$
$QP = PR$ (same side)
$Q\hat{S}P = P\hat{R}Q$ (given)
$S\hat{P}Q = R\hat{Q}P$ (given)
$\therefore \triangle SQP \equiv \triangle RPQ$ (AAS)
$S\hat{Q}P = R\hat{P}Q$
$SQ = RP$
$SP = RQ$

(e) $E \leftrightarrow E$
$B \leftrightarrow C$
$F \leftrightarrow D$
$BE = CE$ (given)
$B\hat{E}F = C\hat{E}D$ (vert. opp. ∠s)
$B\hat{F}E = C\hat{D}E$ (given)
$\therefore \triangle EBF \equiv \triangle ECD$ (AAS)
$E\hat{B}F = E\hat{C}D$
$BF = CD$
$EF = ED$

(f) $F \leftrightarrow F$
$H \leftrightarrow I$
$G \leftrightarrow J$
$GF = JF$ (given)
$G\hat{F}H = J\hat{F}I$ (vert. opp. ∠s)
$H\hat{G}F = J\hat{J}F$ (corr. ∠s, GH // IJI)
$\therefore \triangle FHG \equiv \triangle FIJ$ (AAS)
$\therefore \triangle FH \equiv \triangle FIJ$ (AAS)
$F\hat{H}G = F\hat{I}J$
$FH = FI$
$GH = JI$

4. **(a)** $A\hat{B}C = 180^\circ - 75^\circ - 40^\circ$ (∠ sum of a △)
$= 65^\circ$
$Q\hat{P}R = 180^\circ - 65^\circ - 40^\circ$ (∠ sum of a △)
$= 75^\circ$
$A \leftrightarrow P$
$B \leftrightarrow Q$
$C \leftrightarrow R$
$A\hat{B}C = P\hat{Q}R = 65^\circ$
$B\hat{A}C = Q\hat{P}R = 75^\circ$
$B\hat{C}A = Q\hat{R}P = 40^\circ$
$\therefore \triangle ABC$ is similar to $\triangle PQR$ (3 pairs of corr. ∠s equal).

(b) $A\hat{B}C = 180^\circ - 120^\circ - 25^\circ$ (∠ sum of a △)
$= 35^\circ$
$Q\hat{R}P = 180^\circ - 120^\circ - 45^\circ$ (∠ sum of a △)
$= 15^\circ$
$A \leftrightarrow Q$
$B \leftrightarrow P$
$C \leftrightarrow R$
$A\hat{B}C = Q\hat{P}R = 120^\circ$
$B\hat{A}C$ is not equal to $P\hat{Q}R$ and $B\hat{C}A$ is not equal to $P\hat{R}Q$.
$\therefore \triangle ABC$ is not similar to $\triangle QPR$.

(c) $A \leftrightarrow P$
$B \leftrightarrow Q$
$C \leftrightarrow R$
$\frac{AB}{PQ} = \frac{2}{4} = \frac{1}{2}$
$\frac{BC}{QR} = \frac{6}{12} = \frac{1}{2}$
$\frac{AC}{PR} = \frac{5}{10} = \frac{1}{2}$
$\therefore \triangle ABC$ is similar to $\triangle PQR$ (3 ratios of corr. sides equal).

(d) $A \leftrightarrow P$
$B \leftrightarrow Q$
$C \leftrightarrow R$
$\frac{AB}{PQ} = \frac{1}{2}$
$\frac{BC}{QR} = \frac{4}{8} = \frac{1}{2}$
$\frac{AC}{PR} = \frac{3.5}{7.5} = \frac{7}{15}$
$\therefore$ Since the 3 ratios of corresponding sides are not equal, $\triangle ABC$ is not similar to $\triangle PQR$.

(e) $A \leftrightarrow P$
$B \leftrightarrow R$
$C \leftrightarrow Q$
$B\hat{A}C = Q\hat{P}R = 70°$
$\frac{AB}{PR} = \frac{6}{2} = 3$
$\frac{AC}{PQ} = \frac{5}{3} = \frac{5}{3}$
$\therefore$ Since the 2 ratios of corresponding sides are not equal, $\triangle ABC$ is not similar to $\triangle PRQ$.

(f) $A \leftrightarrow P$
$B \leftrightarrow Q$
$C \leftrightarrow R$
$A\hat{B}C = P\hat{Q}R = 90°$
$\frac{AC}{PR} = \frac{9}{4.5} = 2$
$\frac{BC}{QR} = \frac{7}{3.5} = 2$
$\therefore \triangle ABC$ is similar to $\triangle PQR$ (2 ratios of corr. sides and included $\angle$ equal).

5. **(i)** $O \leftrightarrow O$
$A \leftrightarrow B$
$D \leftrightarrow C$
$AO = BO$ (given)
$DO = CO$ (given)
$A\hat{O}D = B\hat{O}C$ (vert. opp. $\angle$s)
$\therefore \triangle OAD \equiv \triangle OBC$ (SAS)

(ii) Since $\triangle OAD \equiv \triangle OBC$,
then $O\hat{A}D = O\hat{B}C$ and $O\hat{D}A = O\hat{C}A$.

6. **(i)** $P \leftrightarrow S$
$Q \leftrightarrow R$
$R \leftrightarrow Q$
$PQ = SR$ (given)
$QR = RQ$ (common side)
$P\hat{Q}R = S\hat{R}Q$ (corr. $\angle$s, $PQ \mathbin{/\!/} RS$)
$\therefore \triangle PQR \equiv \triangle SRQ$ (SAS)

(ii) Since $\triangle PQR \equiv \triangle SRQ$,
then $QS = PR = 5$ cm
$Q\hat{P}R = Q\hat{S}R = 50°$

7. $P \leftrightarrow Q$
$O \leftrightarrow O$
$C \leftrightarrow C$
$OP = OQ$ (given)
$O\hat{P}C = O\hat{Q}C = 90°$
$OC = OC$ (common side)
$\therefore \triangle POC \equiv \triangle QOC$ (RHS)
Since $\triangle POC \equiv \triangle QOC$,
$P\hat{O}C = \hat{O}C$
$\therefore$ Hence OC is the angle bisector of $A\hat{O}B$.

8. **(a)** $\triangle ABC$ is similar to $\triangle ADE$ (2 pairs of corr. $\angle$s equal).
$\frac{AE}{AC} = \frac{AD}{AB}$
i.e. $\frac{7.4 + a}{7.4} = \frac{5 + 4}{5}$
$\therefore \frac{7.4 + a}{7.4} = \frac{9}{5}$
$5(7.4 + a) = 66.6$
$37 + 5a = 66.6$
$5a = 29.6$
$a = 5.92$

(b) $\triangle ABC$ is similar to $\triangle EDC$ (2 pairs of corr. $\angle$s equal).
$\frac{EC}{AC} = \frac{CD}{CB}$
i.e. $\frac{b}{10} = \frac{11}{7}$
$\therefore 7b = 110$
$b = 15\frac{5}{7}$
$\frac{ED}{AB} = \frac{CD}{CB}$
i.e. $\frac{c}{8} = \frac{11}{7}$
$\therefore 7c = 88$
$c = 12\frac{4}{7}$

(c) $\triangle PXQ$ is similar to $\triangle PAR$ (2 pairs of corr. $\angle$s equal).
$\frac{PR}{PQ} = \frac{PA}{PX}$
i.e. $\frac{6 + d}{6} = \frac{9 + 4}{9}$
$\therefore 54 + 9d = 78$
$9d = 24$
$d = 2\frac{2}{3}$
$\triangle PQY$ is similar to $\triangle PRB$ (2 pairs of corr. $\angle$s equal).
$\frac{PY}{PB} = \frac{PQ}{PR}$
i.e. $\frac{e}{e + 3} = \frac{6}{6 + 2\frac{2}{3}}$
$\therefore 8\frac{2}{3}e = 6e + 18$
$2\frac{2}{3}e = 18$
$e = 6\frac{3}{4}$

9. **(i)** $P \leftrightarrow R$
$Q \leftrightarrow Q$
$S \leftrightarrow P$
$P\hat{Q}S = R\hat{Q}P$ (common angle)
$Q\hat{S}P = Q\hat{P}R = 90°$
$\therefore \triangle PQS$ is similar to $\triangle RQP$ (2 pairs of corr. $\angle$s equal).

(ii) Since $\triangle PQS$ and $\triangle RQP$ are similar, then

$$\frac{QS}{QP} = \frac{QP}{QR}$$

i.e. $\frac{QS}{8} = \frac{8}{10}$

$10QS = 64$

$\therefore QS = 6.4$ cm

10. (a) (i) $B \leftrightarrow A$

$C \leftrightarrow C$

$D \leftrightarrow E$

$C\hat{B}D = C\hat{A}E$ (corr. $\angle$s, BD // AE)

$B\hat{C}D = A\hat{C}E$ (common angle)

$\therefore \triangle BCD$ is similar to $\triangle ACE$ (2 pairs of corr. $\angle$s equal).

$B \leftrightarrow G$

$C \leftrightarrow F$

$D \leftrightarrow E$

Since $C\hat{B}D = C\hat{A}E$, $C\hat{B}D = F\hat{G}E$ (corr. $\angle$s, AC // FG)

$B\hat{C}D = G\hat{F}E$ (corr. $\angle$s, BC // FC)

$\therefore \triangle BCD$ is similar to $\triangle GFE$ (2 pairs of corr. $\angle$s equal).

(ii) Since $\triangle BCD$ and $\triangle ACE$ are similar, then

$$\frac{BD}{AE} = \frac{BC}{AC}$$

i.e. $\frac{BD}{16} = \frac{6}{10 + 6}$

$\therefore 16BD = 96$

$BD = 6$ cm

(b) $BH = 18 + 6 = 24$ cm

$AG = BH = 24$ cm

$\therefore EG = 24 - 16 = 8$ cm

Since $\triangle BCD$ and $\triangle GFE$ are similar, then

$$\frac{FG}{CB} = \frac{EG}{DB}$$

i.e. $\frac{FG}{6} = \frac{8}{6}$

$\therefore FG = 8$ cm

$\therefore FH = 8 + 10 = 18$ cm

(c) $A \leftrightarrow H$

$C \leftrightarrow F$

$E \leftrightarrow D$

$A\hat{C}D = H\hat{F}D$ (corr. $\angle$s, AC // FH)

$A\hat{E}C = H\hat{D}F$ (corr. $\angle$s, DH // AE)

$\therefore \triangle ACE$ is similar to $\triangle HFD$ (2 pairs of corr. $\angle$s equal).

11. (a) (i) $P \leftrightarrow R$

$L \leftrightarrow L$

$Q \leftrightarrow N$

$Q\hat{P}L = N\hat{R}L$ (corr. $\angle$s, PQ // RN)

$P\hat{L}Q = R\hat{L}N$ (vert. opp. $\angle$s)

$\therefore \triangle PLQ$ is similar to $\triangle RLN$ (2 pairs of corr. $\angle$s equal).

(ii) Since $\triangle PLQ$ and $\triangle RLN$ are similar, then

$$\frac{LR}{LP} = \frac{LN}{LQ}$$

i.e. $\frac{LR}{4} = \frac{12 + 4}{8}$

$8LR = 64$

$\therefore LR = 8$ cm

(b) (i) $N \leftrightarrow N$

$Q \leftrightarrow M$

$R \leftrightarrow S$

$N\hat{R}Q = N\hat{S}M$ (corr.$\angle$s, SP // QR)

$Q\hat{N}R = M\hat{N}S$ (common angle)

$\therefore \triangle NQR$ is similar to $\triangle NMS$ (2 pairs of corr. $\angle$s equal).

(ii) Since $\triangle NQR$ and $\triangle NMS$ are similar, then

$$\frac{MS}{QR} = \frac{NM}{NQ}$$

i.e. $\frac{MS}{18} = \frac{12}{24}$

$\frac{MS}{18} = \frac{1}{2}$

$\therefore MS = 9$ cm

(c) $\triangle PLM$ is similar to $\triangle RLQ$ (2 pairs of corr. $\angle$s equal).

$\triangle PQM$ is similar to $\triangle SNM$ (2 pairs of corr. $\angle$s equal).

$\triangle PQM$ is similar to $\triangle RNQ$ (2 pairs of corr. $\angle$s equal).

12. (i) $S \leftrightarrow S$

$T \leftrightarrow T$

$R \leftrightarrow P$

$SR = SP = 9$ cm

$ST = ST$ (given)

$S\hat{T}R = S\hat{T}P = 90°$

$\therefore \triangle STR \equiv \triangle STP$ (RHS)

(ii) $R \leftrightarrow R$

$T \leftrightarrow P$

$U \leftrightarrow Q$

$R\hat{T}U = R\hat{P}Q$ (corr. $\angle$s, TU // PQ)

$T\hat{R}U = P\hat{R}Q$ (common angle)

$\therefore \triangle RTU$ is similar to $\triangle RPQ$ (2 pairs of corr. $\angle$s equal).

Since $\triangle STR \equiv \triangle STP$, $TP = RT$.

Since $\triangle RTU$ and $\triangle RPQ$ are similar, then

$$\frac{RQ}{RU} = \frac{RP}{RT}$$

i.e. $\frac{7 + UQ}{7} = \frac{2}{1}$

$7 + UQ = 14$

$\therefore UQ = 7$ cm

$$\frac{PQ}{TU} = \frac{RP}{RT}$$

i.e. $\frac{PQ}{5} = \frac{2}{1}$

$\therefore PQ = 10$ cm

13. **(i)** $C \leftrightarrow P$

$A \leftrightarrow A$

$N \leftrightarrow N$

$AN = AN$ (common side)

$C\hat{A}N = P\hat{A}N$ (given)

Since CP is a straight line,

$A\hat{N}C = A\hat{N}P = 90°$

$\therefore \triangle CAN \equiv \triangle PAN$ (AAS)

(ii) $C \leftrightarrow C$

$M \leftrightarrow T$

$N \leftrightarrow P$

$M\hat{C}N = T\hat{C}P$ (common angle)

Since $\triangle CAN \equiv \triangle PAN$,

$CN = PN$

$$\frac{CN}{CP} = \frac{1}{2}$$

Since M is the midpoint of CT,

$$\frac{CM}{CT} = \frac{1}{2}$$

$\therefore \triangle CMN$ is similar to $\triangle CTP$ (2 ratios of corr. sides and included $\angle$ equal).

Since $\triangle CMN$ is similar to $\triangle CTP$, $C\hat{N}M = C\hat{P}T$ and MN is parallel to TA.

Hence $MTAN$ is a trapezium.

Challenge Yourself

1. *The following solution gives the shortest working. If the students do not manipulate algebra properly, this can lead to a long and tedious working.*

Let the height of $\triangle PST$ from P to ST be h'.

Using similar triangles,

$$\frac{h'}{h + h'} = \frac{a}{b}$$

i.e. $bh' = a(h + h')$

$bh' - ah' = ah$ — (1)

Area of trapezium $QRST$

= area of $\triangle PST$ – area of $\triangle PQR$

$= \frac{1}{2}b(h + h') - \frac{1}{2}ah'$

$= \frac{1}{2}(bh' - ah' + bh)$

$= \frac{1}{2}(ah + bh)$ (Substitute (1) into the equation)

$= \frac{1}{2}(a + b)h$

2. Given $PQ = QR = RS$, $\triangle PQU$, $\triangle VRU$ and $\triangle VST$ are similar (AA Similarity Test).

Let $QU = x$ cm and $VS = y$ cm.

Then $RU = (5 - x)$ cm and $VR = (5 - y)$ cm.

$$\frac{PQ}{QU} = \frac{VS}{ST}$$

$$\frac{5}{x} = \frac{y}{1}$$

$xy = 5$ — (1)

$$\frac{PQ}{QU} = \frac{VR}{RU}$$

$$\frac{5}{x} = \frac{5 - y}{5 - x}$$

$x(5 - y) = 5(5 - x)$

$5x - xy = 25 - 5x$

$10x - xy = 25$ — (2)

Substitute (1) into (2),

$10x - 5 = 25$

$10x = 30$

$x = 0$

$\therefore QU = 3$ cm

3. It is given that $AB = AC$, $CB = CE$ and $BD = BE$.

Since $CB = CE$ (given), then $DE = 9 - 5 = 4$ cm.

$\triangle CBE$ and $\triangle BDE$ are similar (AA Similarity Test since both triangles are isosceles and $\angle BEC = \angle DEB$).

$$\frac{BE}{BC} = \frac{DE}{DB}$$

$\frac{BE}{9} = \frac{4}{BE}$ since $BD = BE$ (given)

$BE^2 = 36$

$BE = 6$ cm (since $BE > 0$)

$\triangle ABC$ is also similar to $\triangle BDE$ (AA Similarity Test since both triangles are isosceles and $\angle ABC = \angle CBE$).

$$\frac{AC}{BC} = \frac{CE}{BE}$$

$$\frac{AC}{9} = \frac{9}{6}$$

$\therefore AC = \frac{3}{2} \times 9$

$= 13.5$ cm

Chapter 11 Geometrical Constructions

TEACHING NOTES

Suggested Approach

Students have learnt how to draw triangles and quadrilaterals using rulers, protractors and set squares in primary school. Teachers need to reintroduce these construction tools and demonstrate the use of these if students are still unfamiliar with them. When students are comfortable with the use of these construction tools and the compasses, teachers can proceed to the sections on construction of triangles and quadrilaterals.

Section 11.1: Construction of Triangles

Students should be able to construct the following types of triangles at the end of this section:

- Given 2 sides and an included angle
- Given 3 sides
- Given 1 side and 2 angles

As a rule of thumb, students should draw the longest line as a horizontal line. Teachers are to remind their students to mark all angles, vertices, lengths and other markings (same angles, same sides, right angles etc.) clearly. Students should not erase any arcs that they draw in the midst of construction and check their figure at the end.

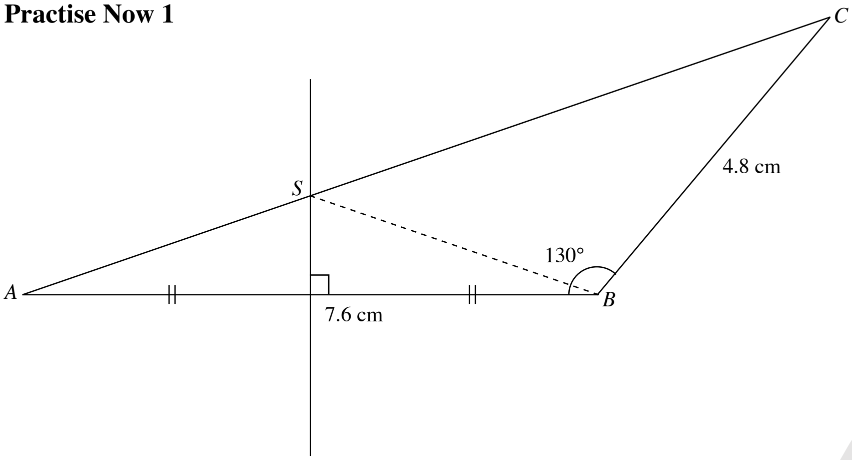
Section 11.2: Construction of Quadrilaterals

Students should be able to construct parallelograms, rhombuses, trapeziums and other quadrilaterals at the end of this section.

As a rule of thumb, students should draw the longest line as a horizontal line. Teachers are to remind their students to mark all angles, vertices, lengths and other markings (same angles, same sides, right angles etc.) clearly. Students should not erase any arcs they draw in the midst of construction and check their figure at the end.

WORKED SOLUTIONS

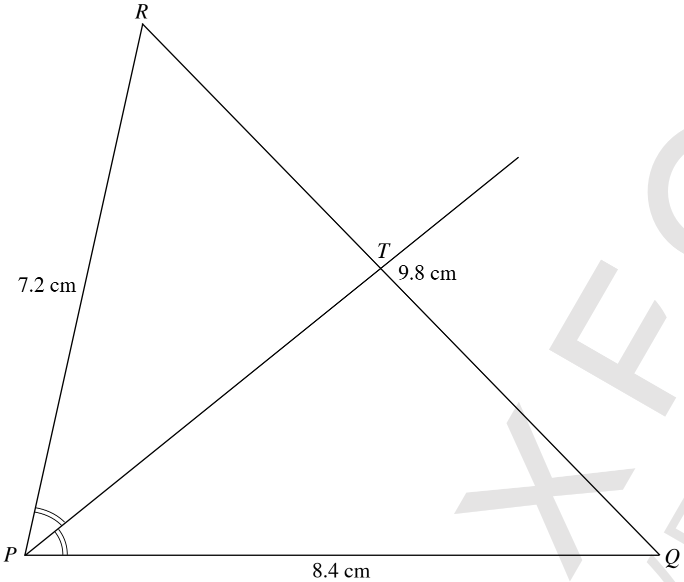
Practise Now 1



(i) Length of $AC = 11.3$ cm

(ii) Length of $BS = 4.0$ cm

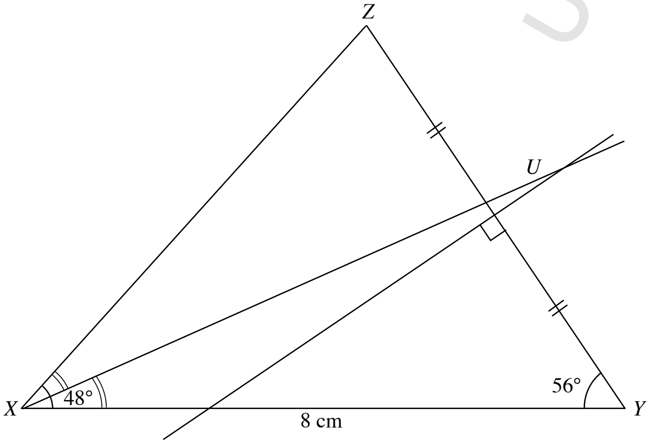
Practise Now 2



(i) Required angle, $Q\hat{P}R = 77°$

(ii) Length of $QT = 5.3$ cm

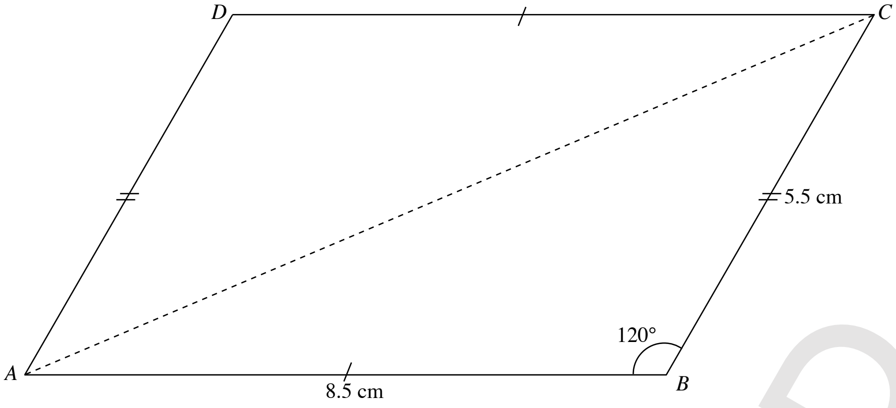
Practise Now 3



(iii) The point *U* is equidistant from the points *Y* and *Z*, and equidistant from the lines *XY* and *XZ*.

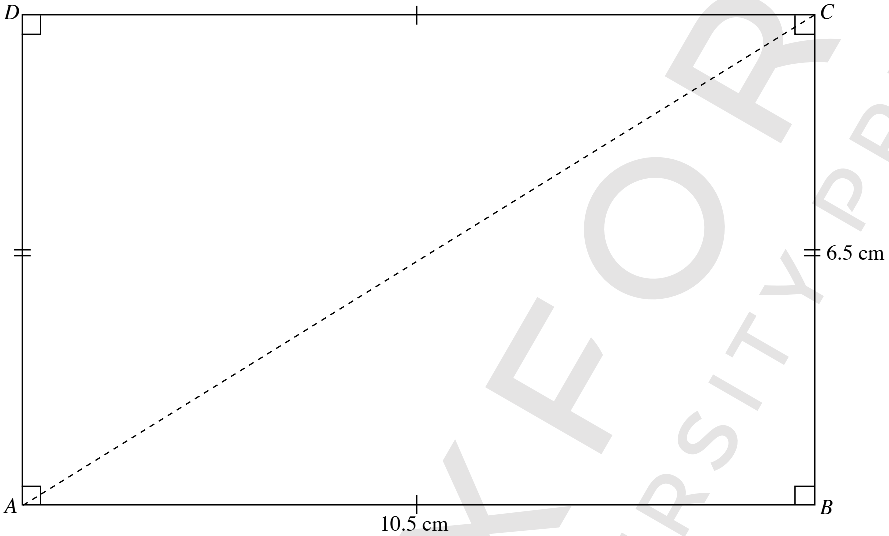
Practise Now 4

1.



Length of *AC* = 12.2 cm

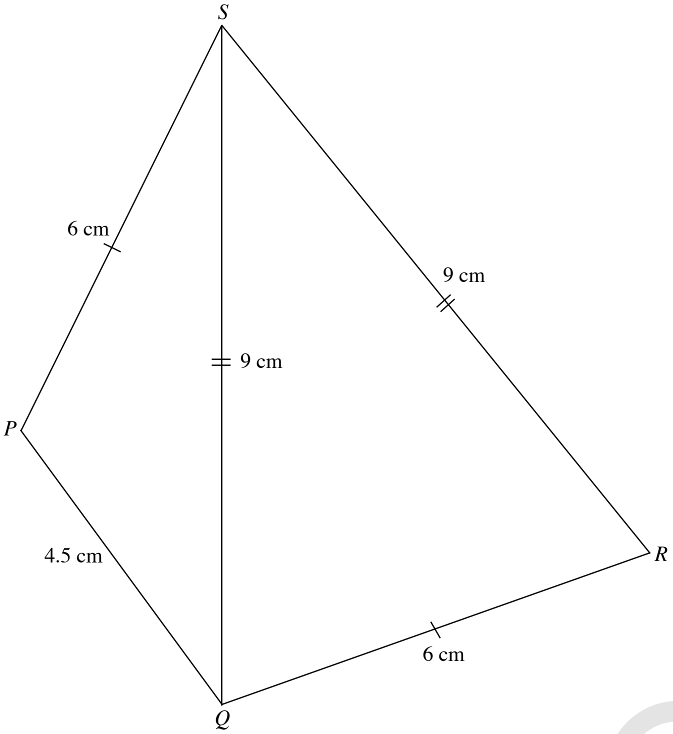
2.



Length of *AC* = 12.3 cm

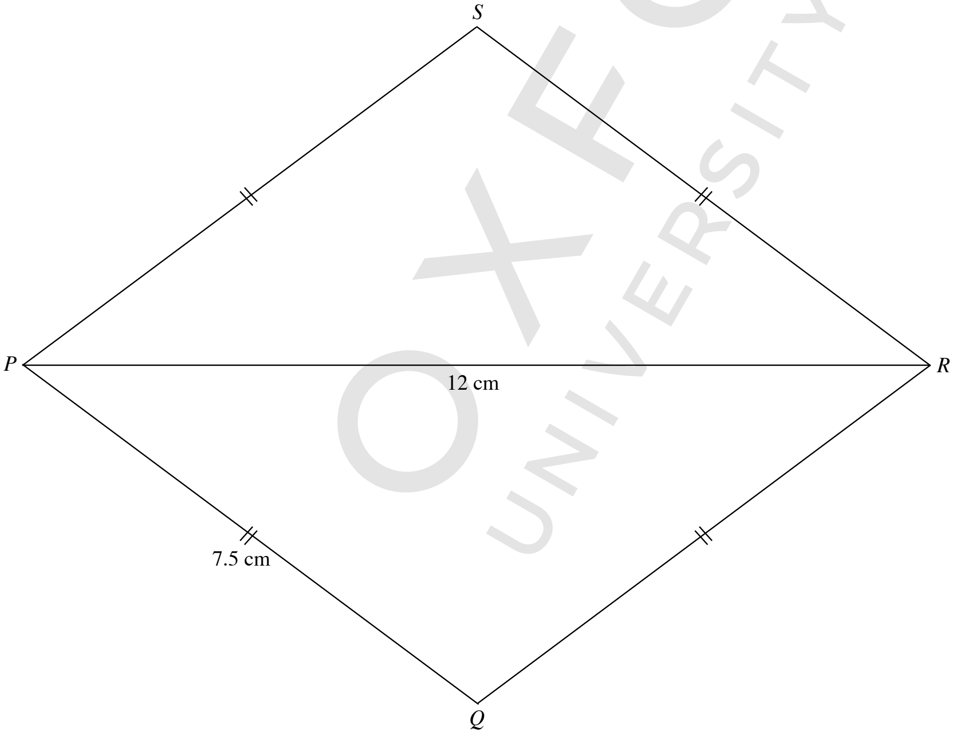
Practise Now 5

1.



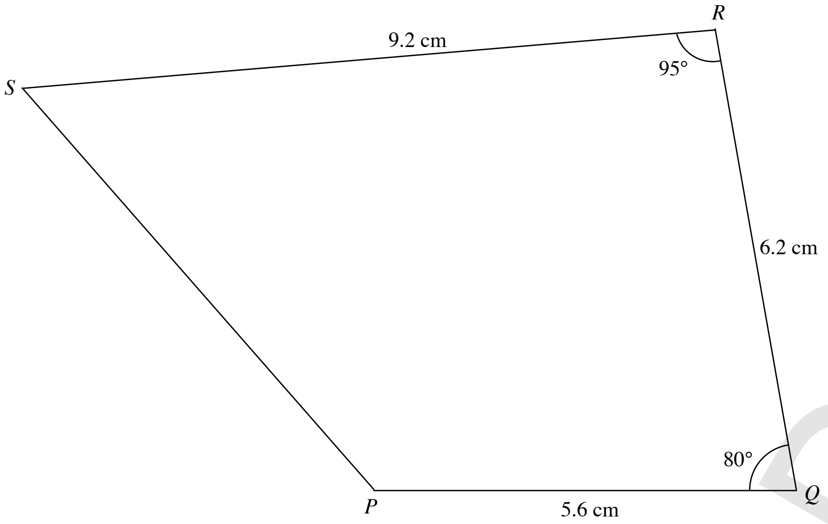
$Q\hat{R}S = 71°$

2.



$Q\hat{R}S = 74°$

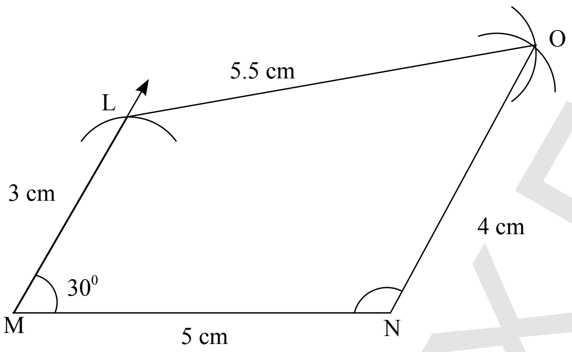
Practise Now 6



(i) Length of *PS* = 7.0 cm

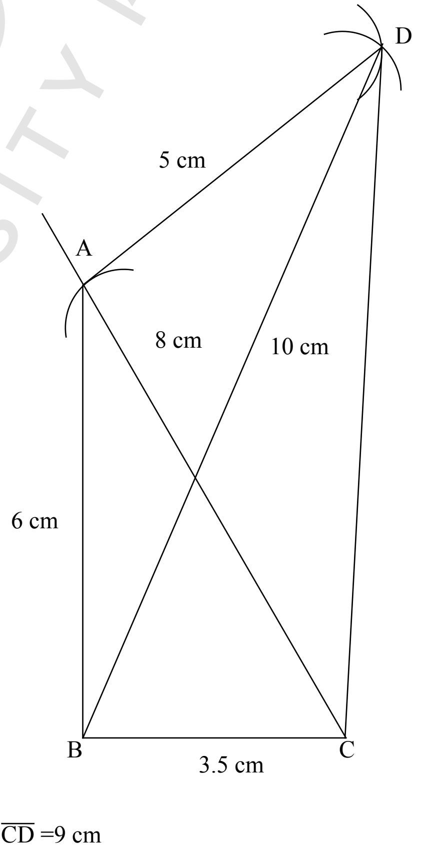
(ii) $P\hat{S}R = 54°$

Practise Now 7



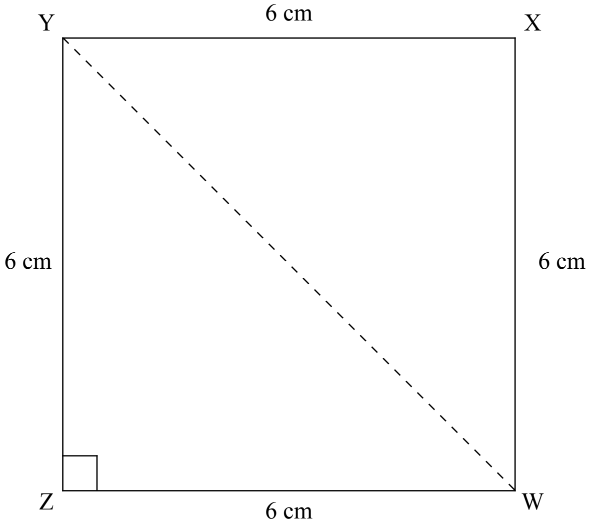
$m\text{M}\hat{\text{N}}\text{O}$ = 118º

Practise Now 8



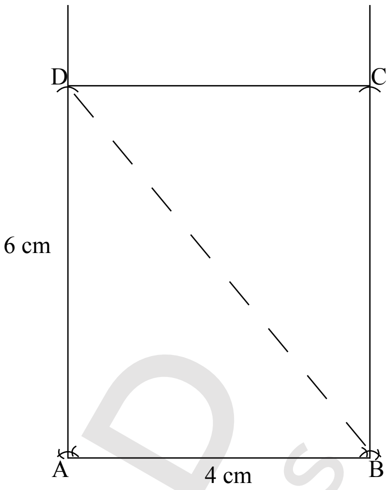
$m\overline{\text{CD}}$ =9 cm

Practise Now 9



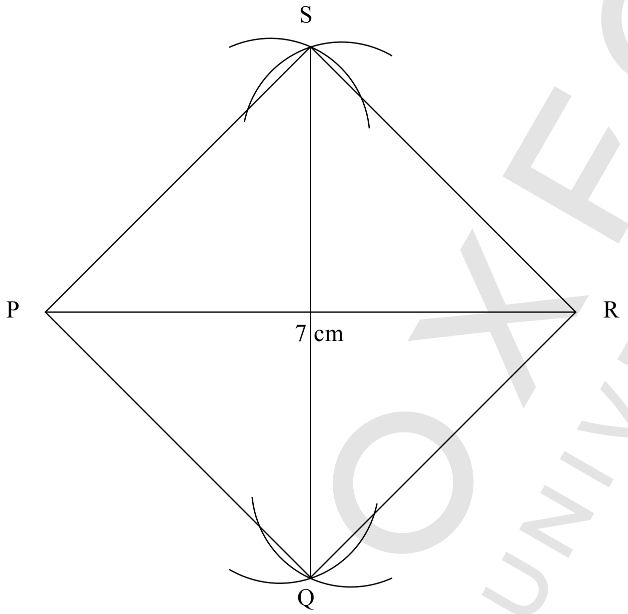
$m\overline{WY}$ = 8.4 cm

Practise Now 11



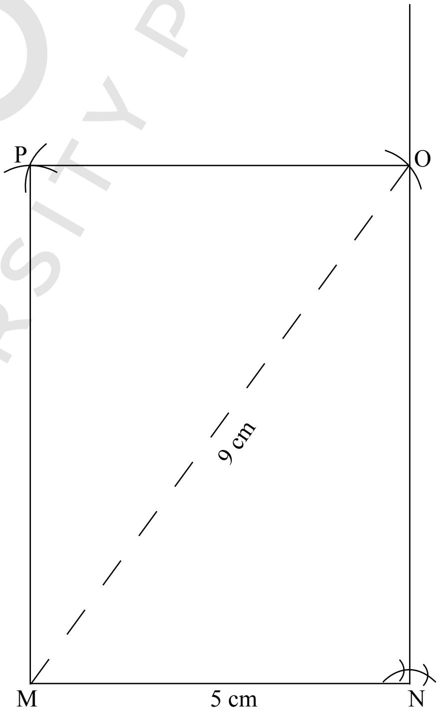
m $\overline{BD}$ = 6 cm

Practise Now 10



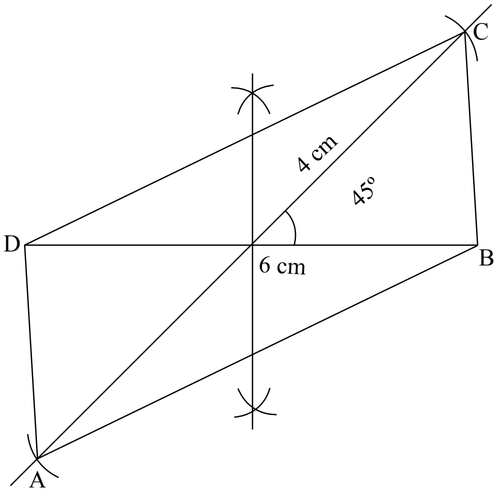
$m\overline{RS}$ =5 cm

Practise Now 12



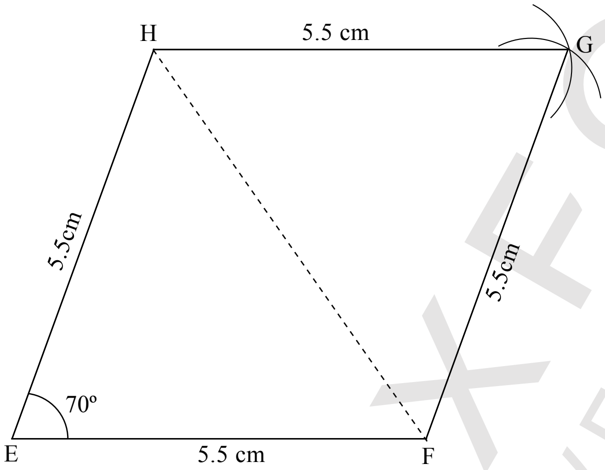
Length of the rectangle is 6.9 cm

Practise Now 13



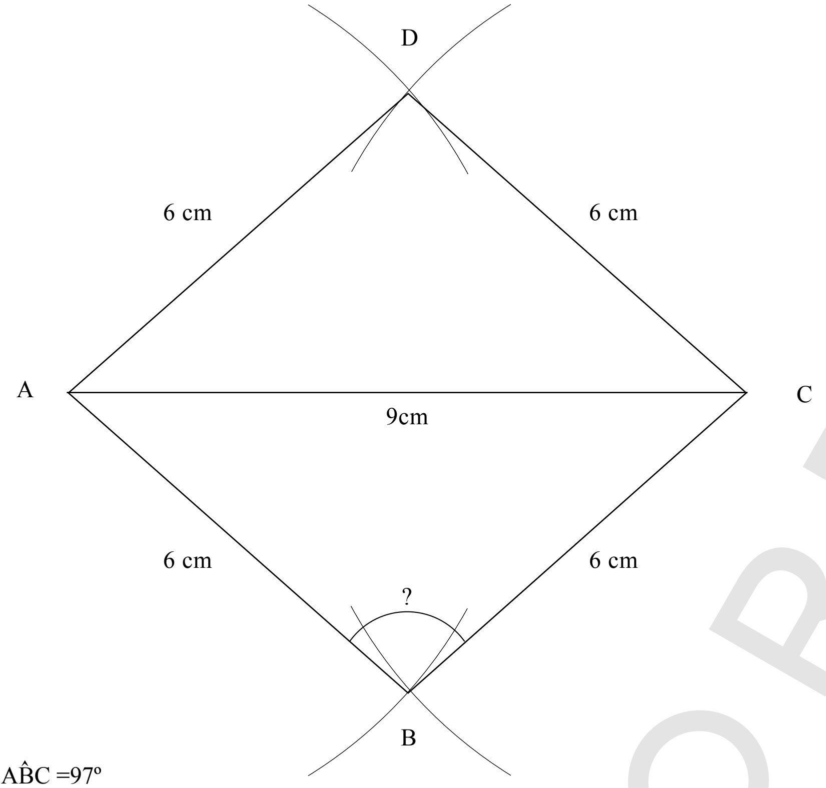
Length of the longest side is 2.8 cm

Practise Now 14



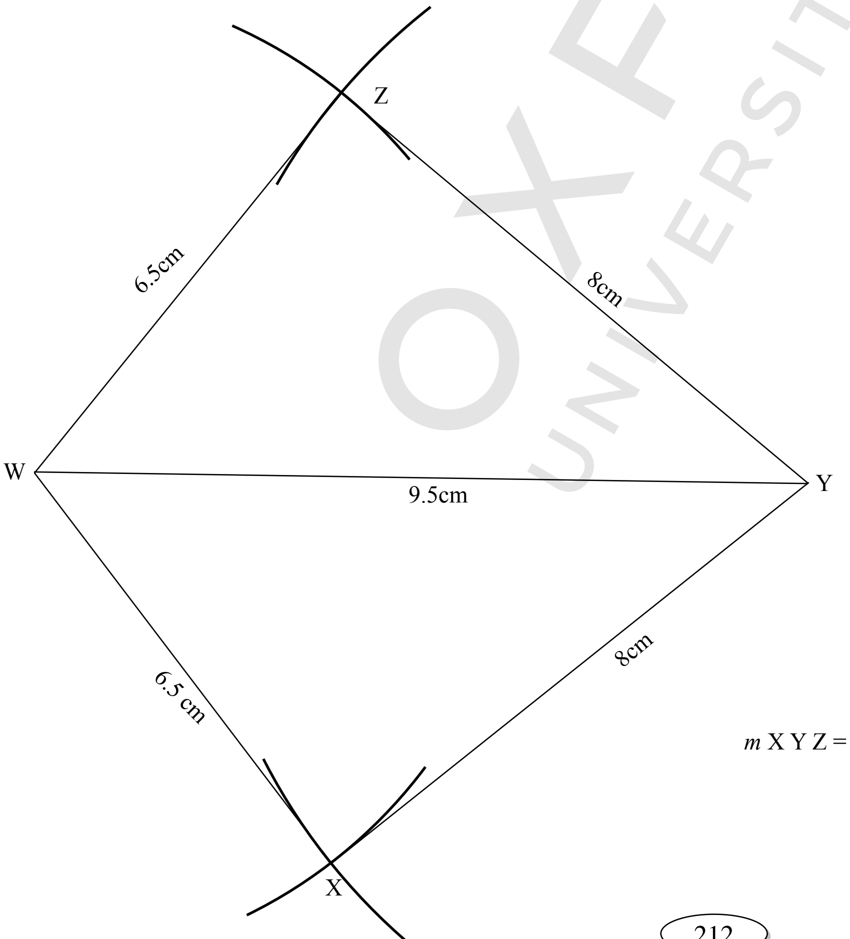
$m\overline{FH} = 6.3$

Practise Now 15



$mA\hat{B}C = 97^{\circ}$

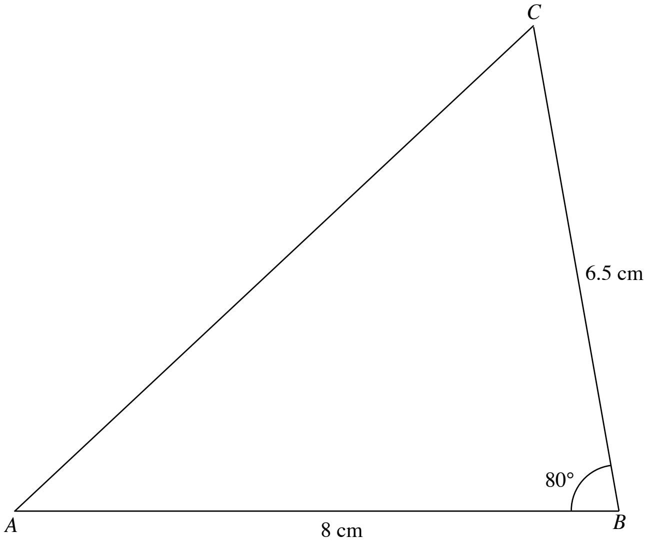
Practise Now 16



m X Y Z = 78 ˆ

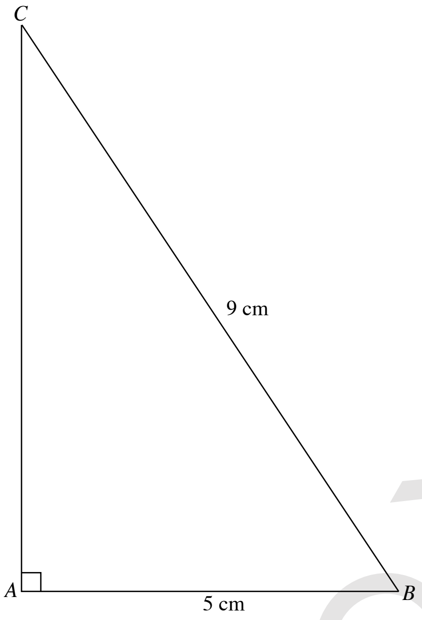
Exercise 11A

1.



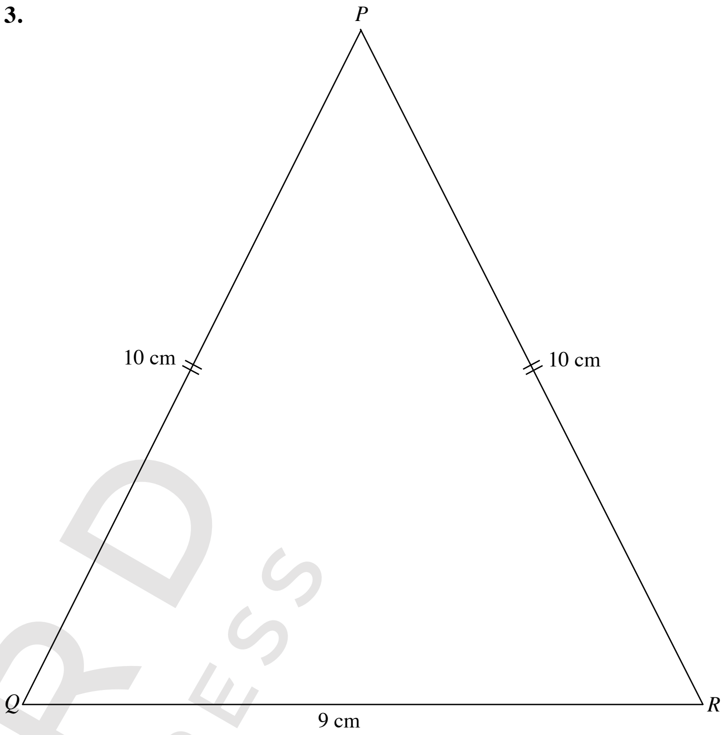
Length of *AC* = 9.4 cm

2.



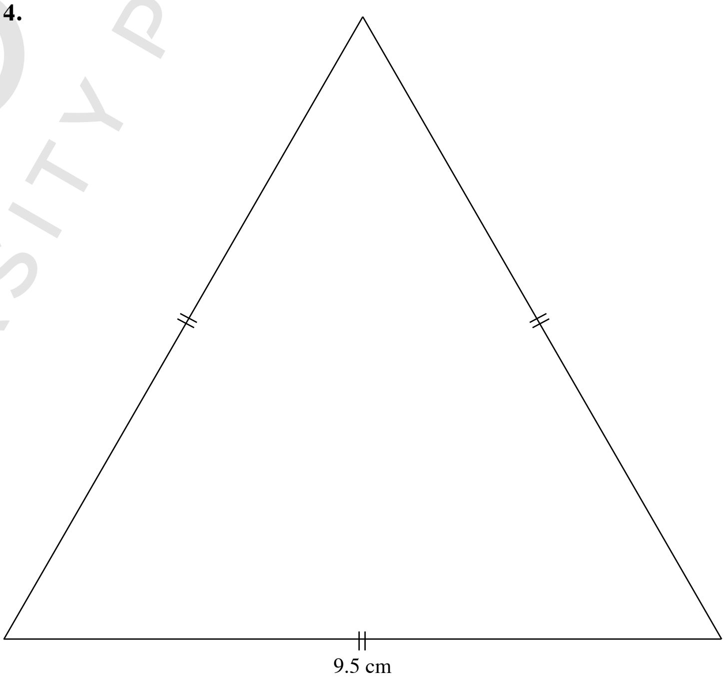
Length of *AC* = 7.5 cm

3.

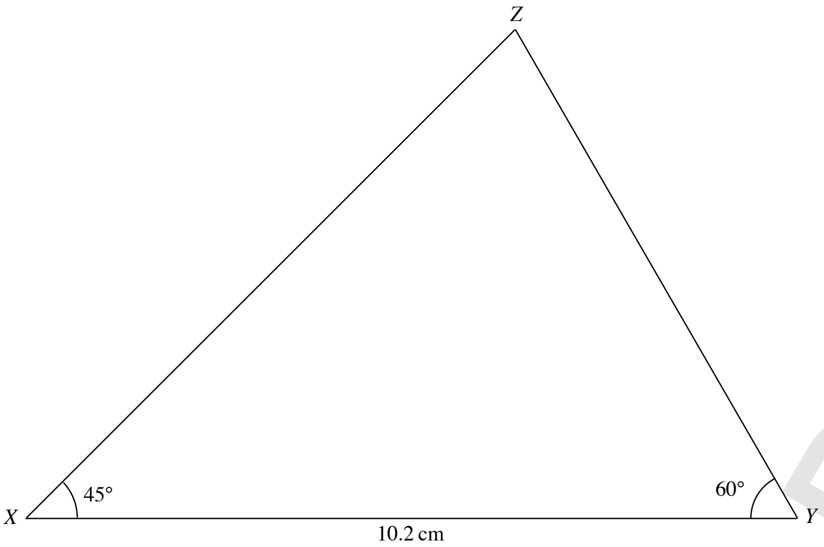


$Q\hat{P}R = 53°$

4.

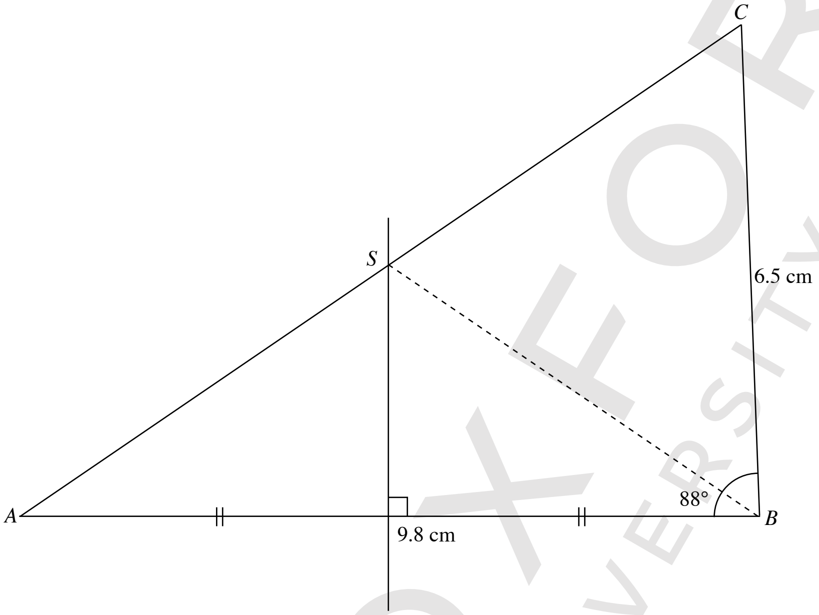


5.



Length of $XZ = 9.1$ cm

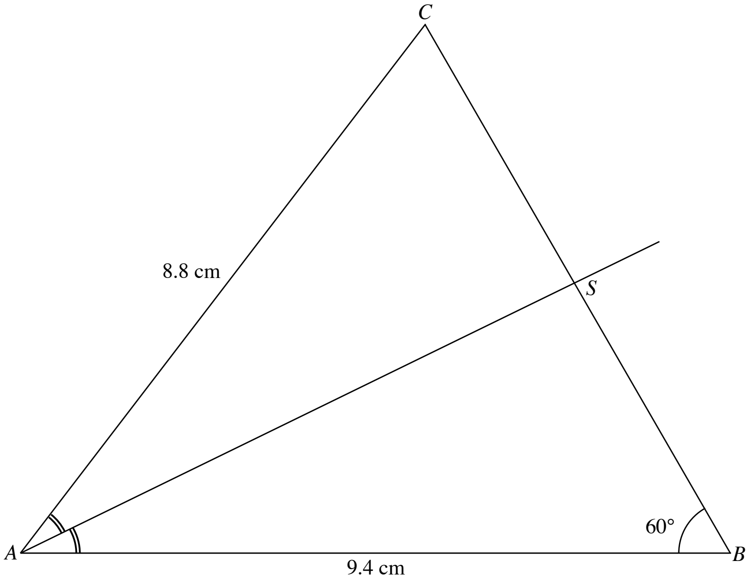
6.



(i) Length of $AC = 11.6$ cm

(ii) Length of $BS = 5.9$ cm

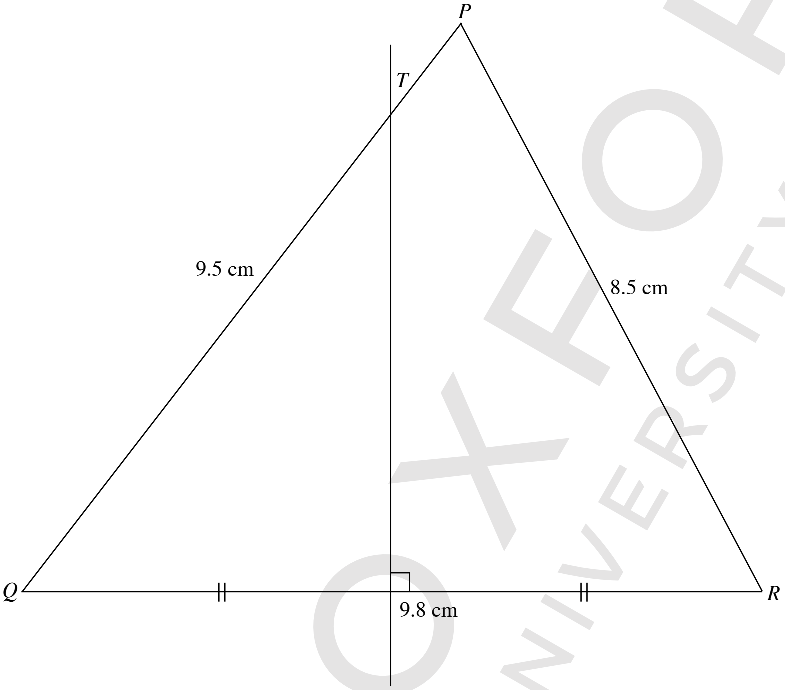
7.



(i) Required angle, $B\hat{A}C = 52°$

(ii) Length of CS = 3.9 cm

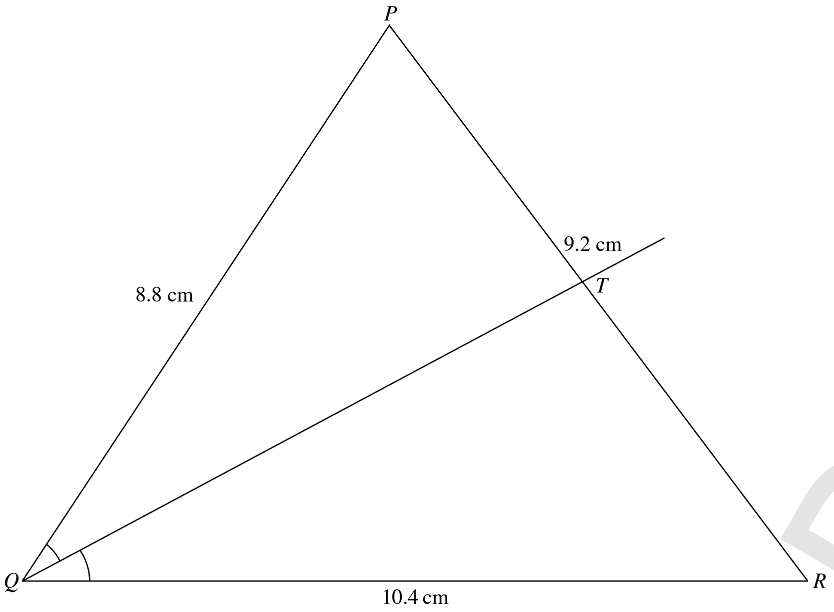
8.



(i) Required angle, $P\hat{Q}R = 52°$

(ii) Length of QT = 8.0 cm

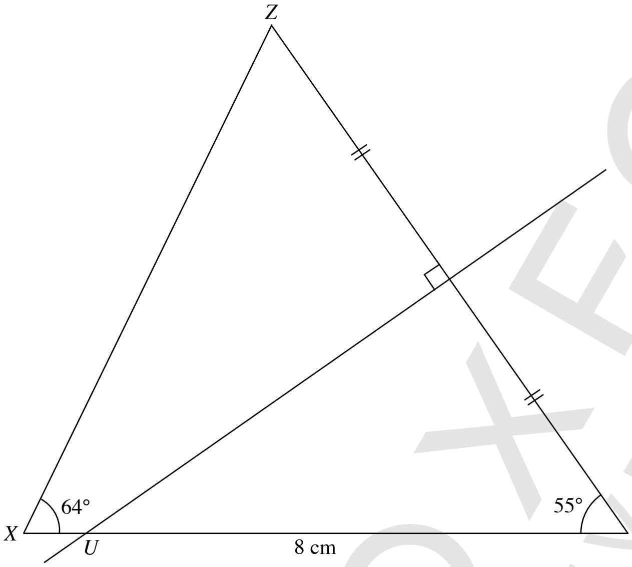
9.



(i) Required angle, $Q\hat{P}R = 71°$

(ii) $PT = 4.2$ cm

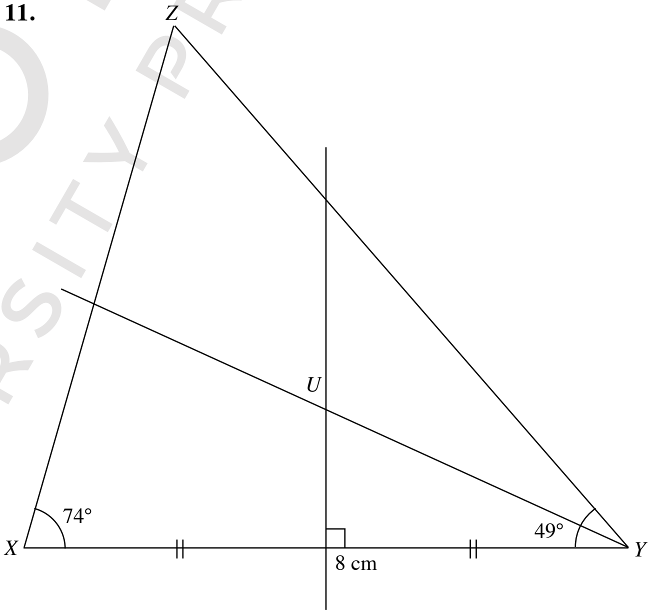
10.



(i) Length of $XZ = 7.5$ cm

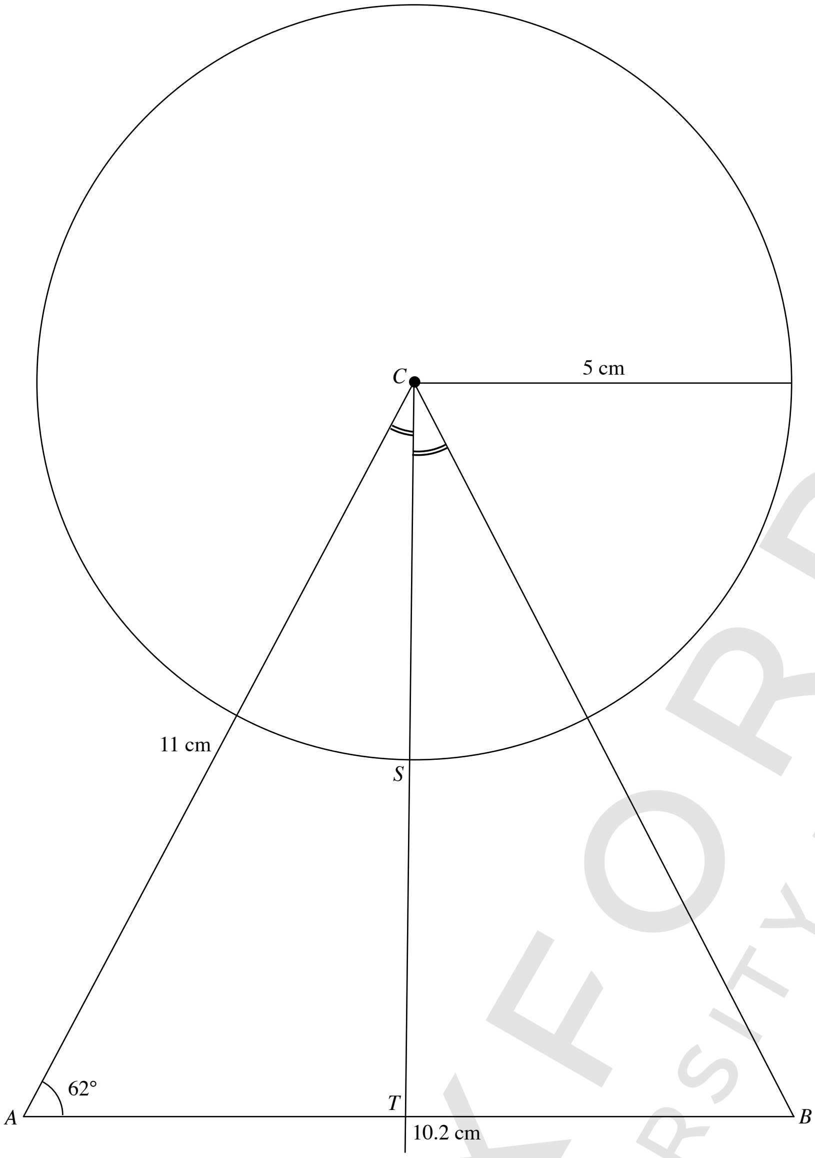
(ii) Length of $UY = 7.2$ cm

11.



(iii) The point U is equidistant from the points X and Y, and equidistant from the lines XY and YZ.

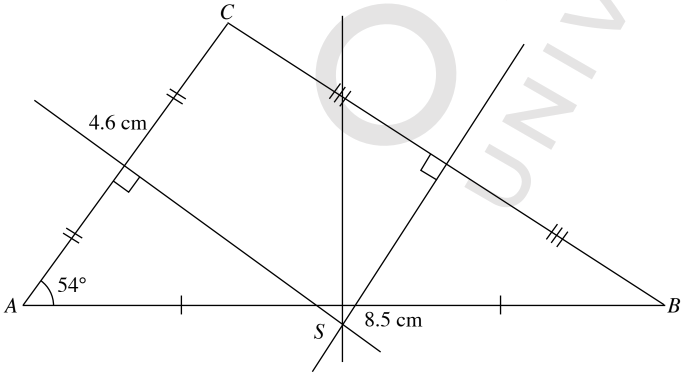
12.



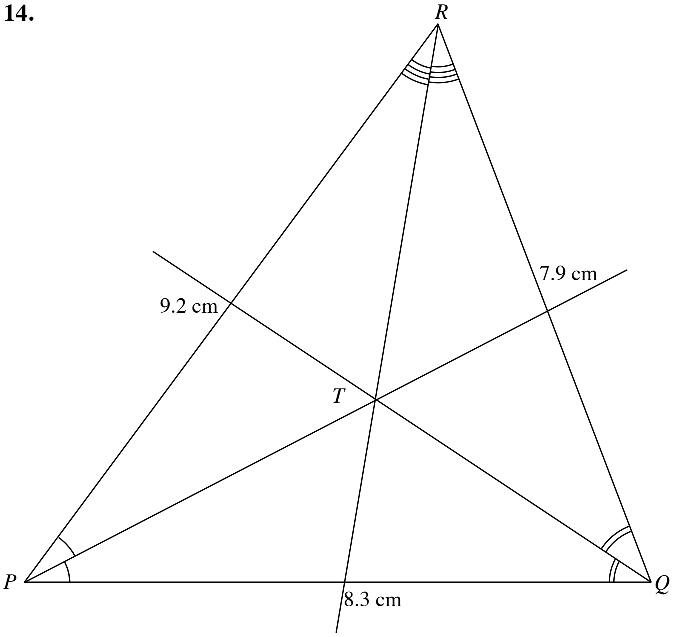
(i) Length of BC = 10.9 cm

(iii) Length of ST = 4.7 cm

13.

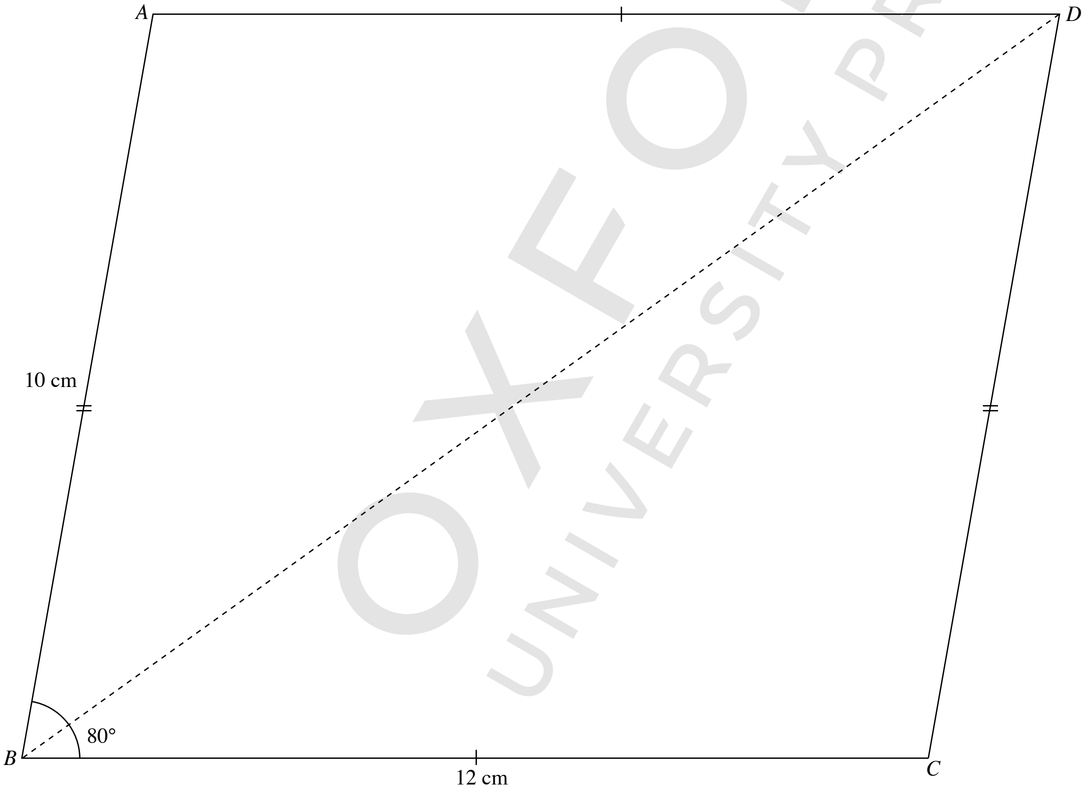


14.



Exercise 11B

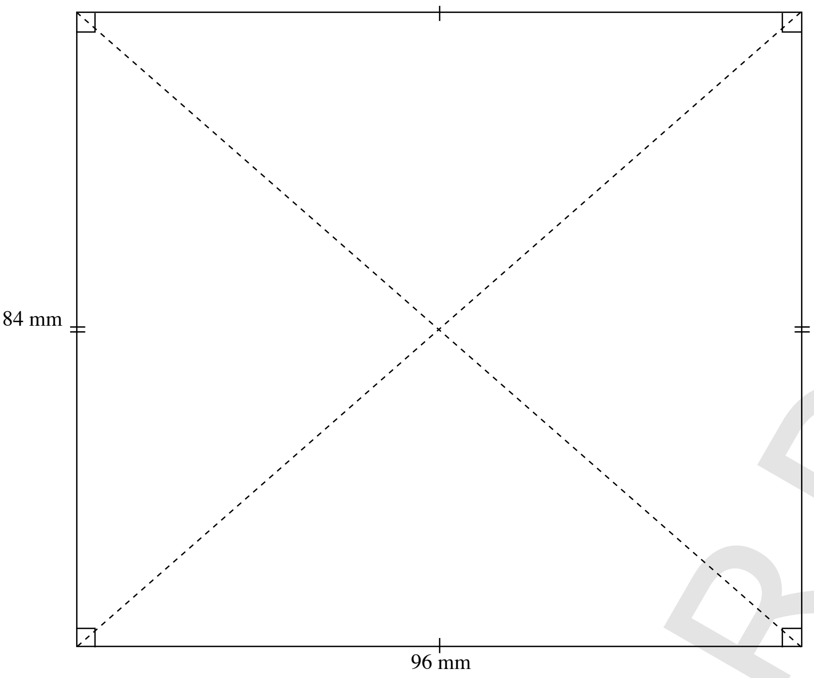
1.



Length of diagonal *BD* = 16.9 cm

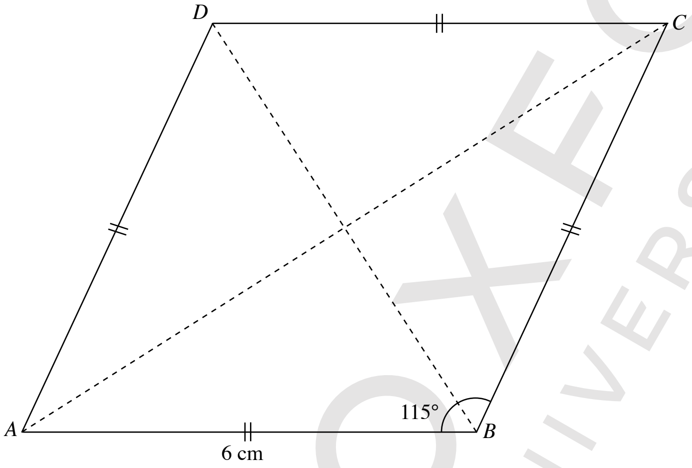
2. 96 mm = 9.6 cm

84 mm = 8.4 cm



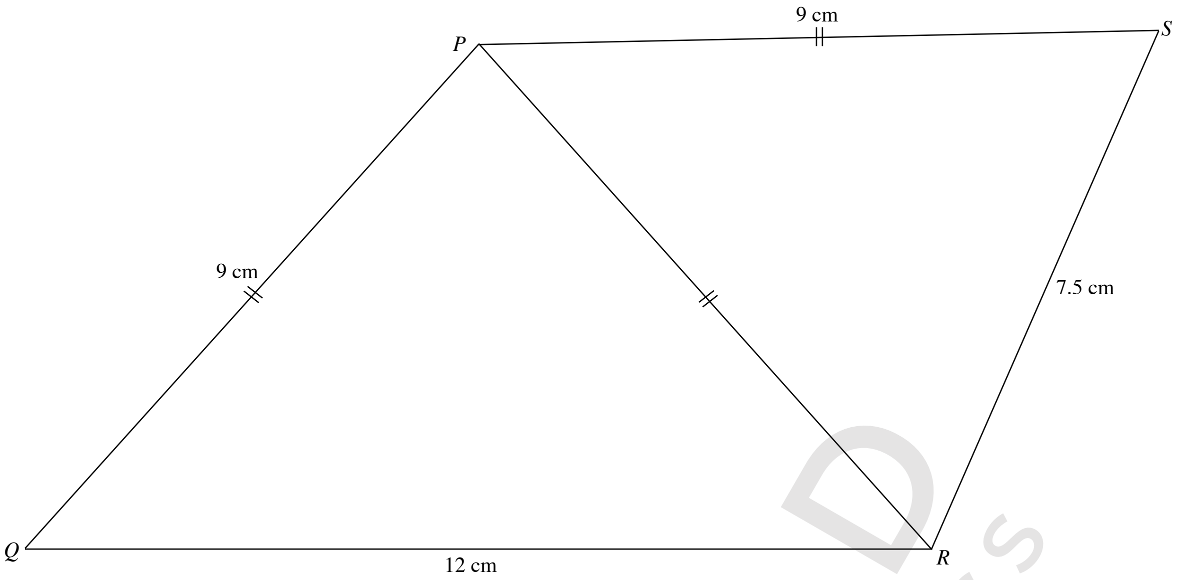
Length of each of the two diagonals = 12.8 cm

3.



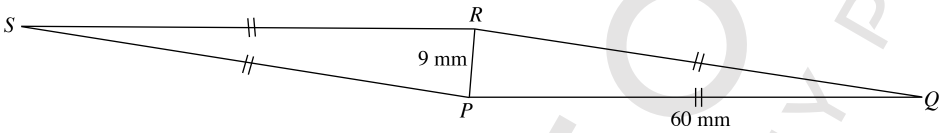
Length of each of the two diagonals = 10.1 cm, 6.5 cm

4.



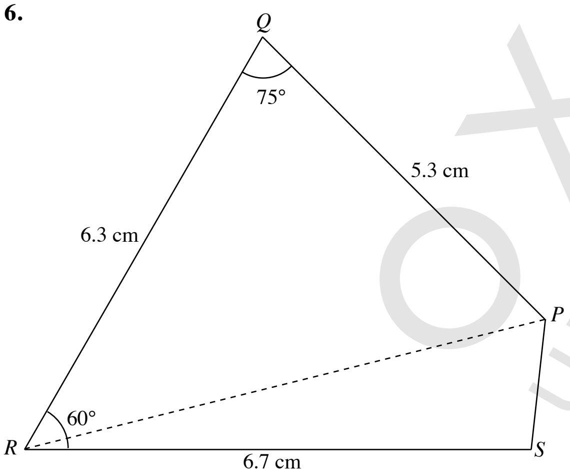
$Q\hat{P}S = 133°$

5. 60 mm = 6 cm
9 mm = 0.9 cm



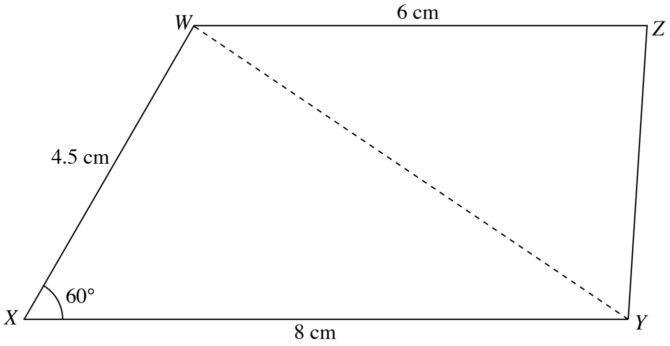
$Q\hat{P}S = 171°$

6.



(i) Length of $PR = 7.1$ cm
(ii) $R\hat{P}S = 70°$

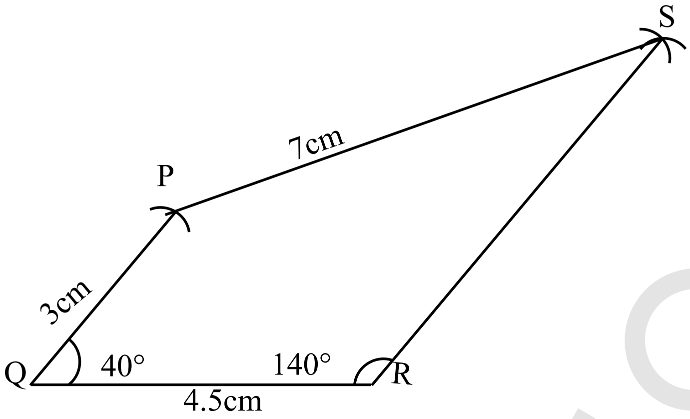
7.



Length of *YZ* = 3.9 cm

Length of *WY* = 6.9 cm

8. Quadrilateral

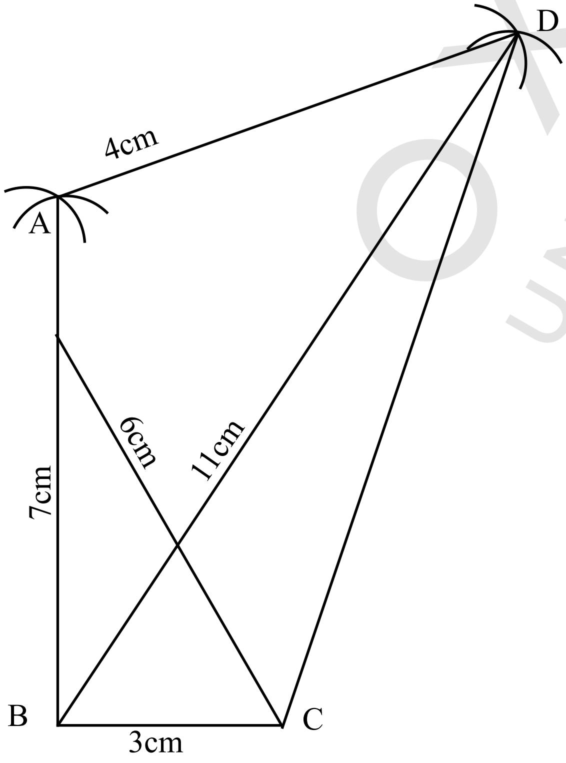


Length of *YZ* = 3.9 cm

m<QRS = 140°

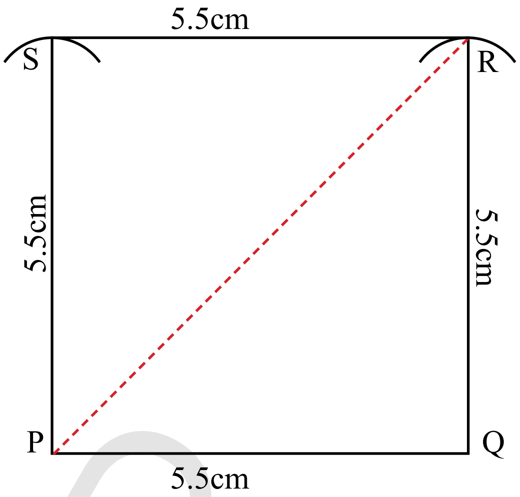
measure & write the size of sides & angles marked wired.

9. Quadrilateral



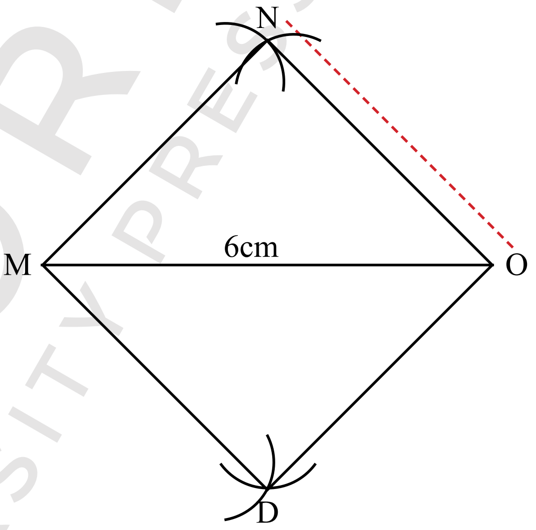
In each case Don't draw the redline it is to be measured only

10.



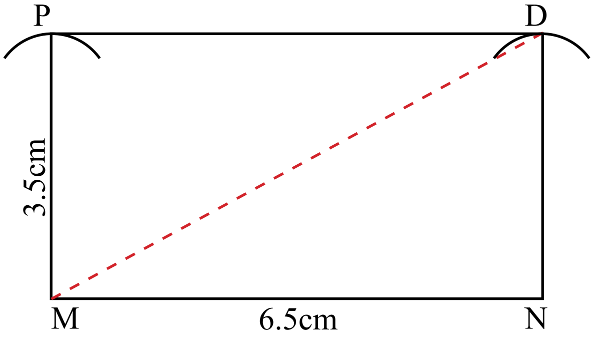
Diagonal PR = 7.5 cm

11.



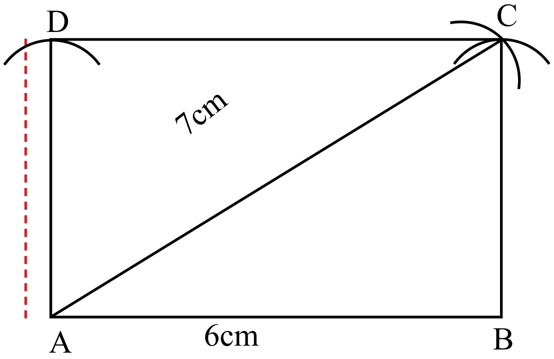
mNO = 4.2 cm

12.



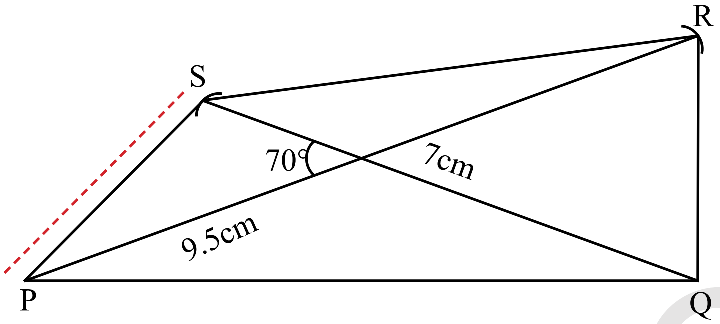
Length of diagonal MO = 7.2 cm

13.



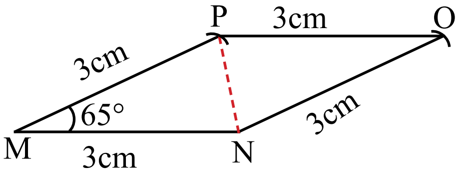
Breadth of the rectangle ABCD = 3.5 cm

14.



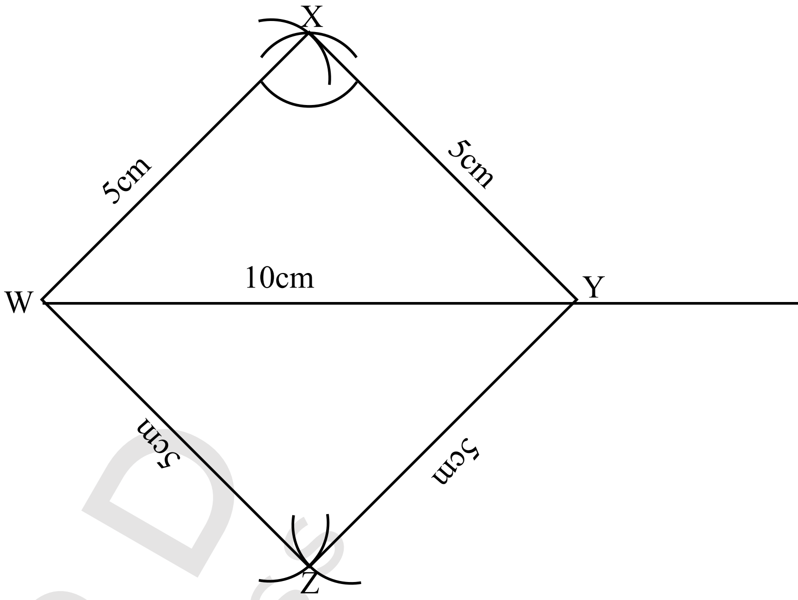
Length of the shorter side Ps = 3.4 cm

15.



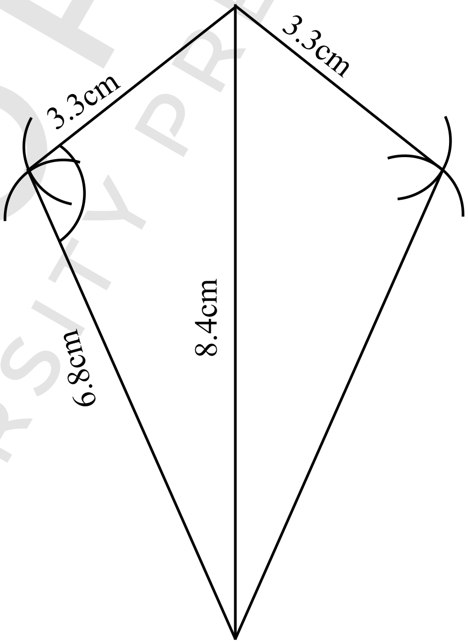
mNP = 1.3

16.



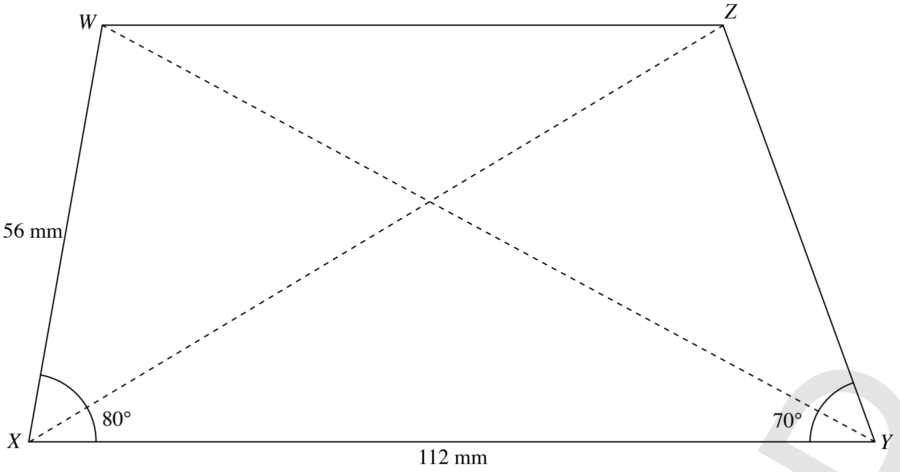
m<WXY= 90°

17.



18. 56 mm = 5.6 cm

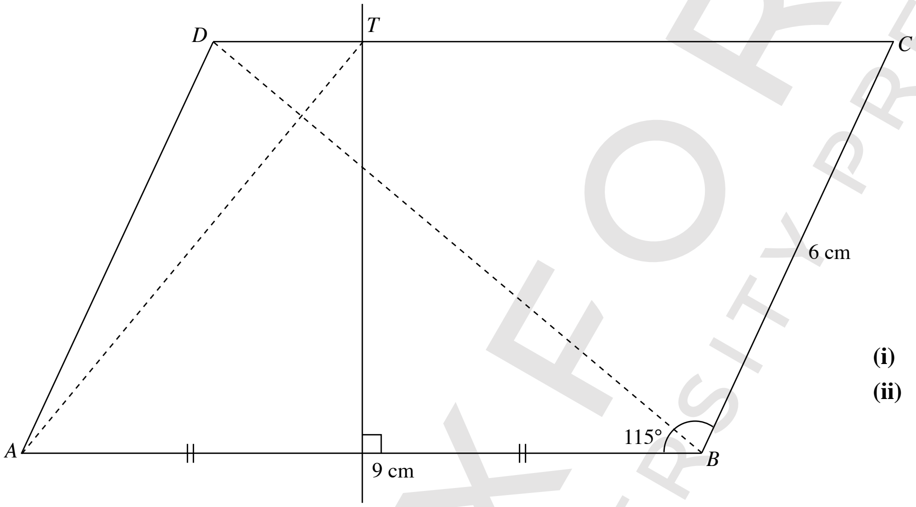
112 mm = 11.2 cm



Length of *WY* = 11.6 cm

Length of *XZ* = 10.7 cm

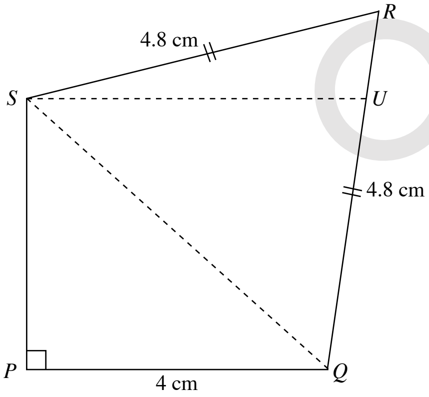
19.



(i) Length of diagonal *BD* = 8.4 cm

(ii) Length of *AT* = 7.1 cm

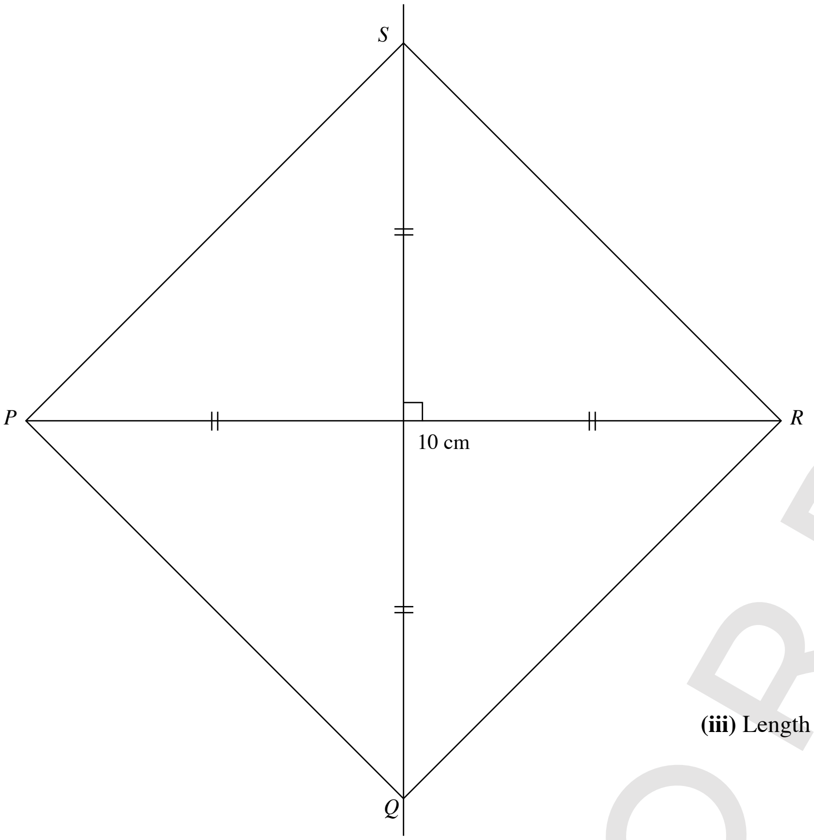
20.



(i) Length of *QS* = 5.4 cm

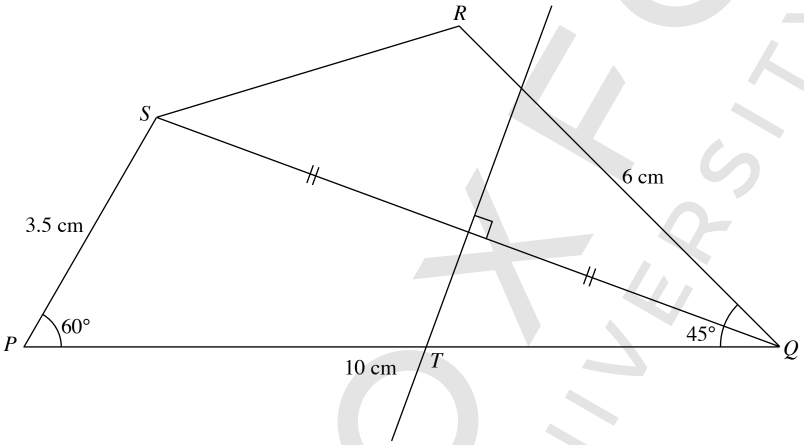
(ii) Length of *SU* = 4.5 cm

21.



(iii) Length of $PQ = 7.0$ cm

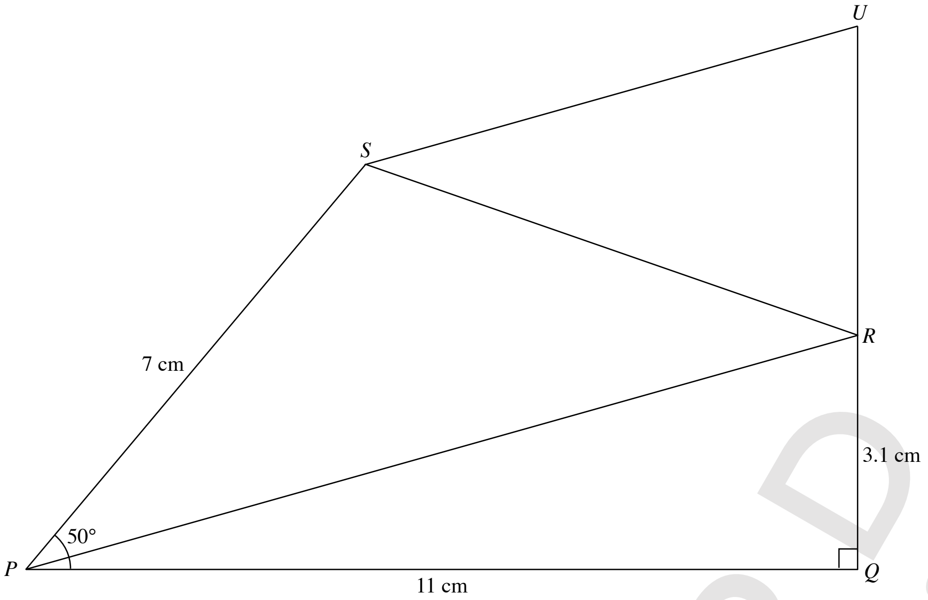
22.



(i) $Q\hat{R}S = 119°$

(ii) Length of $PT = 5.4$ cm

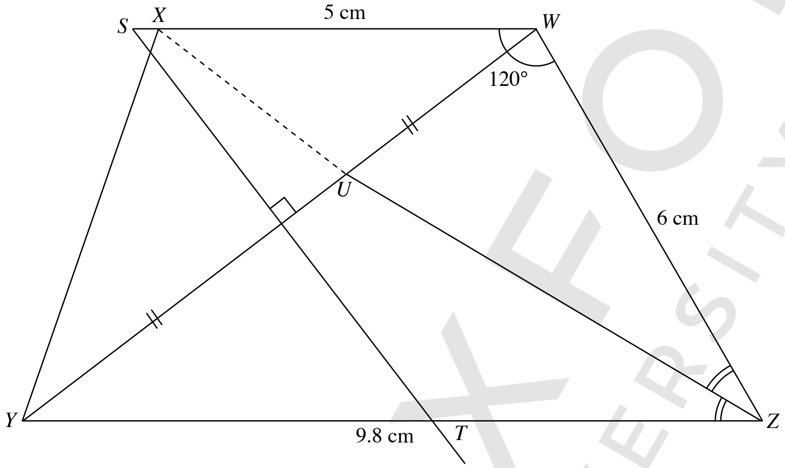
23.



(i) $Q\hat{R}S = 109°$

(ii) Length of RU = 4.1 cm

24.

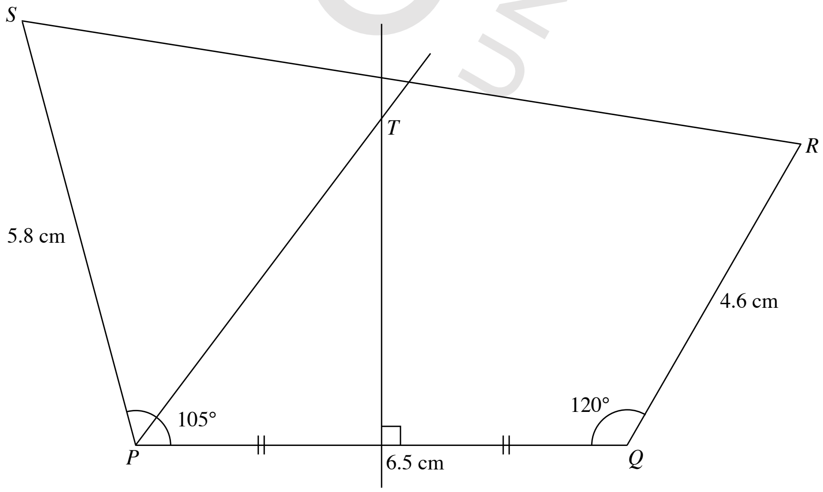


(i) Length of WY = 8.6 cm

(ii) Length of ST = 6.5 cm

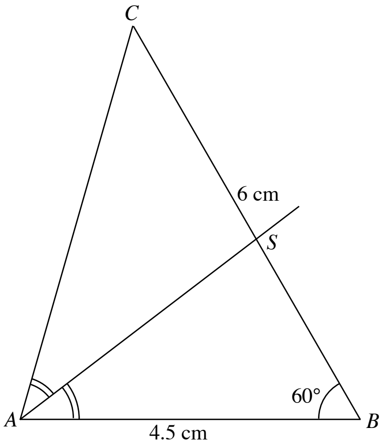
(iii) $W\hat{U}X = 105°$

25.



Review Exercise 11

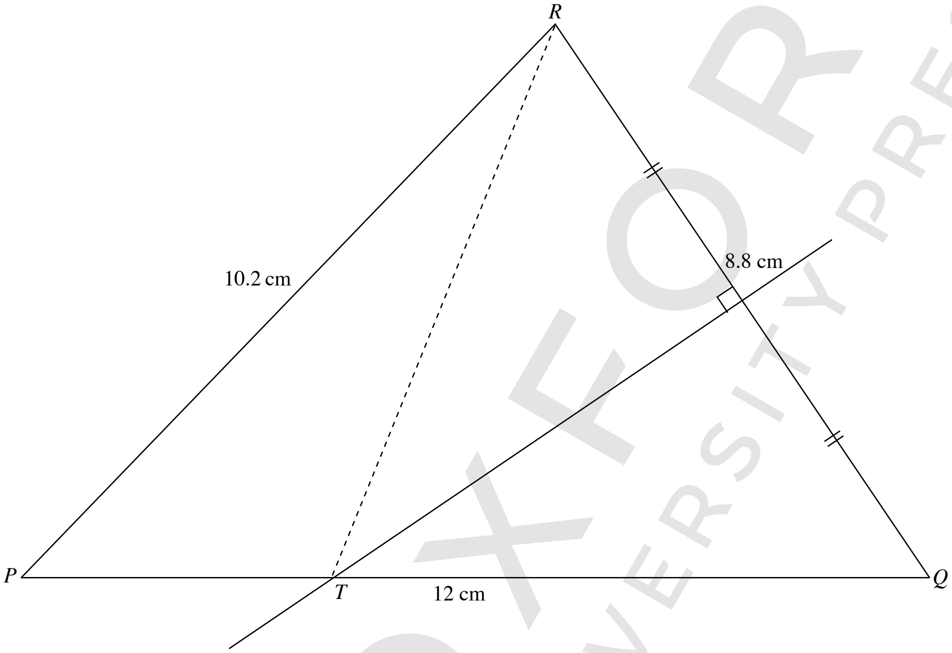
1.



(i) Length of $AC = 5.4$ cm

(ii) Length of $CS = 3.3$ cm

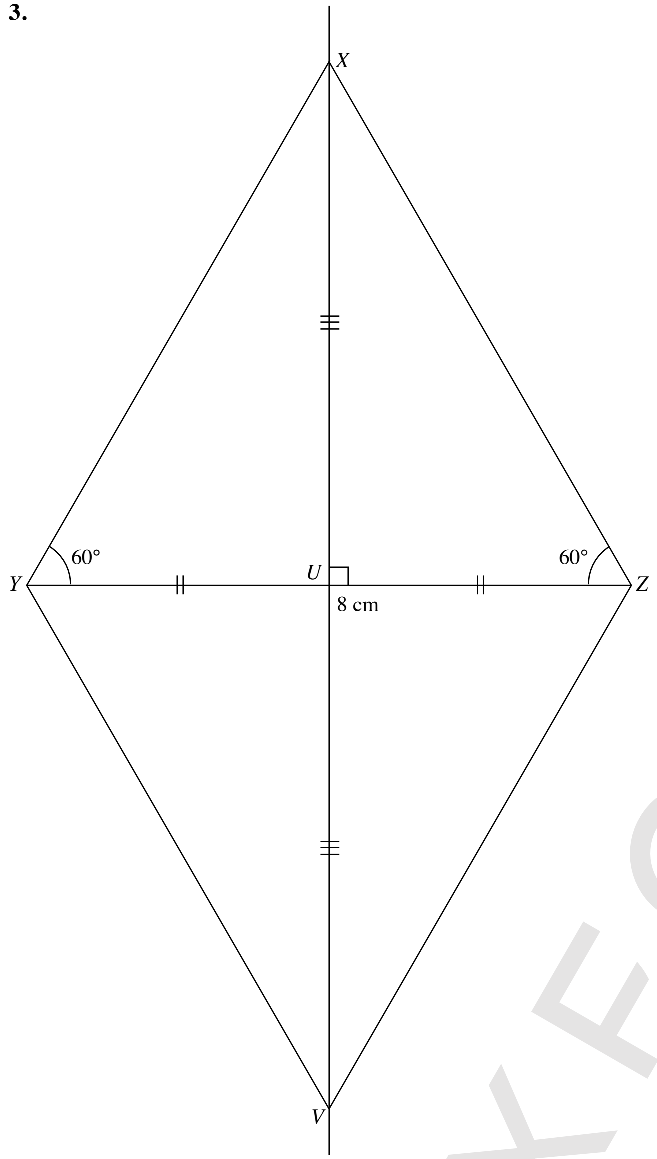
2.



(i) Required angle, $Q\hat{P}R = 46°$

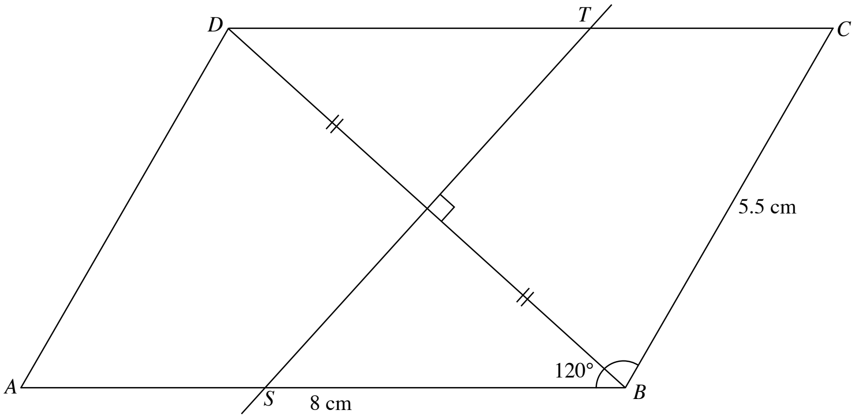
(ii) $RT = 7.9$ cm

3.



(iii) Rhombus

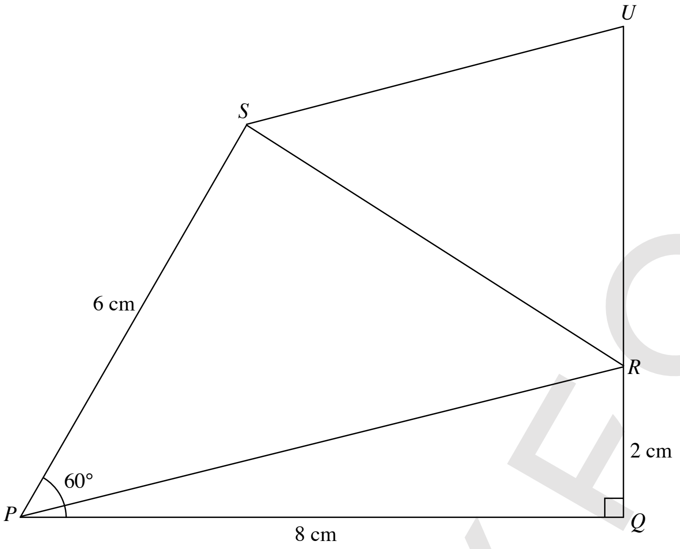
4.



(i) Length of *BD* = 7.1 cm

(ii) Length of *ST* = 6.5 cm

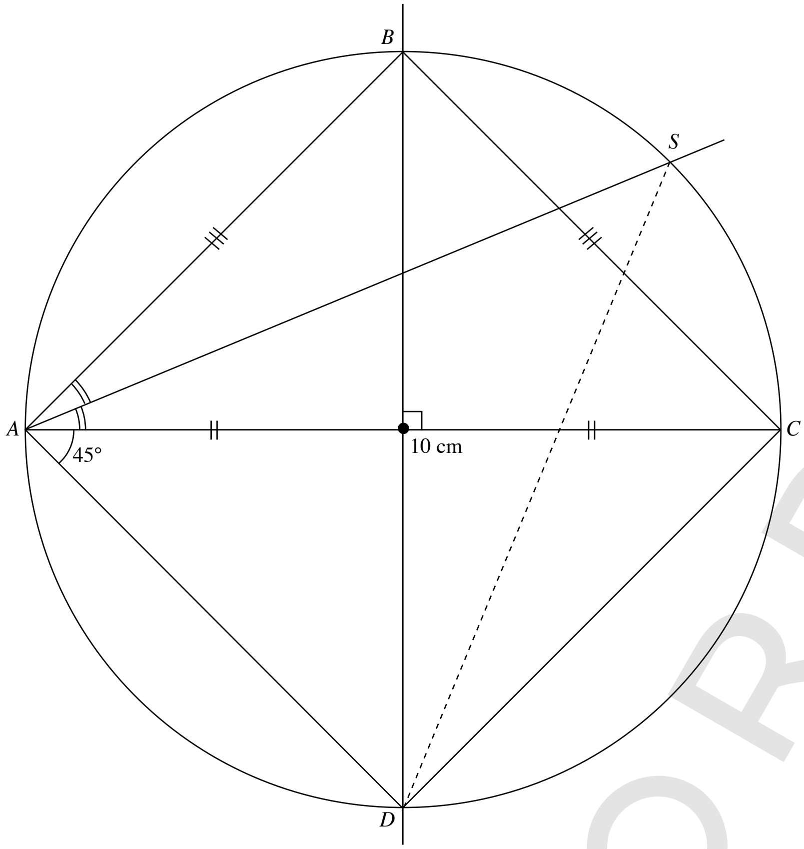
5.



(i) Required angle, $Q\hat{R}S = 123°$

(ii) Length of *QU* = 6.5 cm

6.



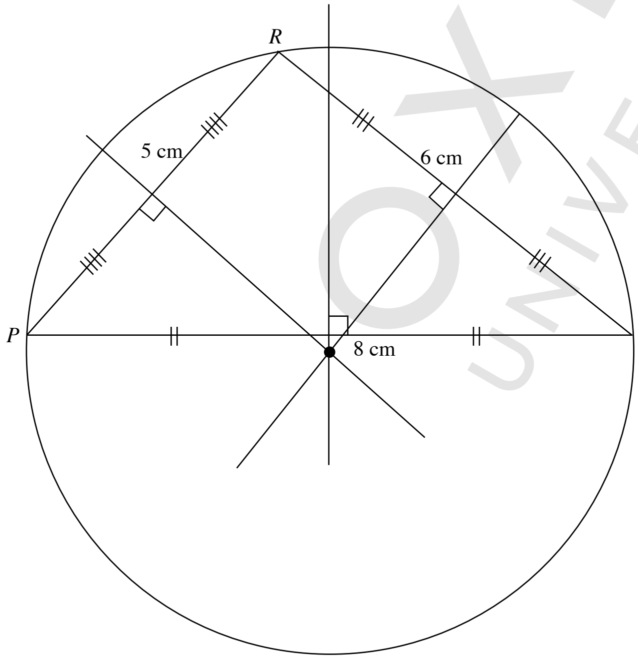
(iii) $AB = BC = AD = CD = 7.1$ cm

ABCD is a square.

(iv) Length of $DS = 9.3$ cm

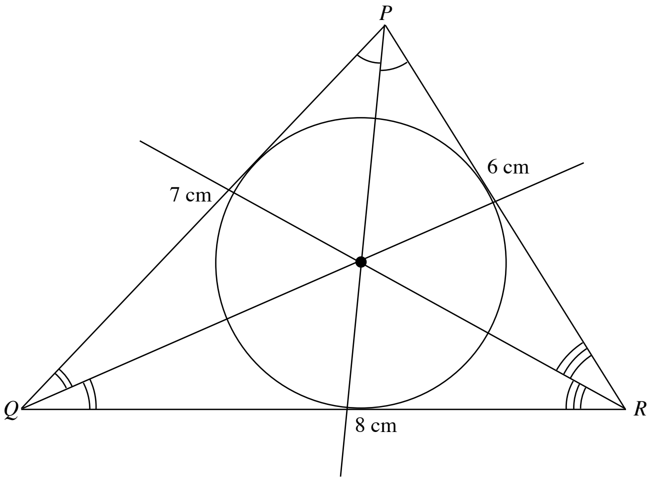
Challenge Yourself

1.



Circumcircle

2.



Incircle

Chapter 12 Further Geometrical Transformations

TEACHING NOTES

Suggested Approach:

This topic deals with spatial visualisation and teachers would be able to find many examples in the surroundings. Teachers should make use of these everyday examples to help students understand transformations from a three-dimensional point of view, in order for them to apply the concepts to the drawing of graphs.

Section 12.1: Rotation

Teachers should highlight the importance of providing exact specifications for transformations. In the case of a rotation, the centre of rotation needs to be specified. The importance of specifying the centre can be illustrated by calling up students to stretch out an arm each, and rotate it 90 clockwise. A few possibilities would arise as some might rotate their arms such that the centre of rotation is the shoulder, or at the elbow joints, or with their wrists at the centre of rotation.

Section 12.2: Enlargement

Teachers can revise the construction steps needed for the enlargement of a given figure with a positive scale factor. Teachers should then proceed to illustrate the construction steps for negative scale factors, and the construction steps involved in finding the centre of enlargement as well as the scale factor if given the original figure and its image.

WORKED SOLUTIONS

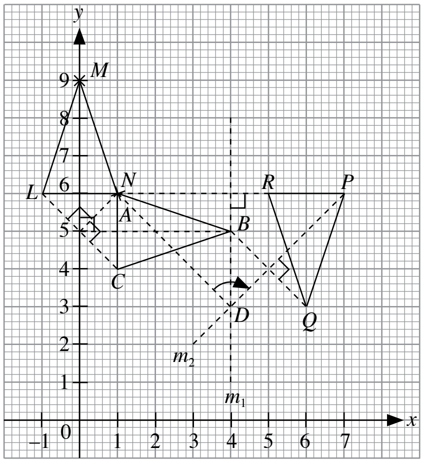
Thinking Time (Page 350)

The distance between A and A' and that between C and C' from the centre of rotation will be the same. Points along the perpendicular bisectors of AA' and CC' indicate where the respective distances between A and A', and C and C' are equal. Hence, the point of intersection of the perpendicular bisectors would mean that this point is equidistant from both A to A' and C to C', thus the centre of rotation lies here.

Class Discussion (Enlargement in our surroundings)

Teachers can come up with an example of enlargement in the classroom, for example an A5 notebook compared to an A4 one, before getting students to build on this and discuss with each other.

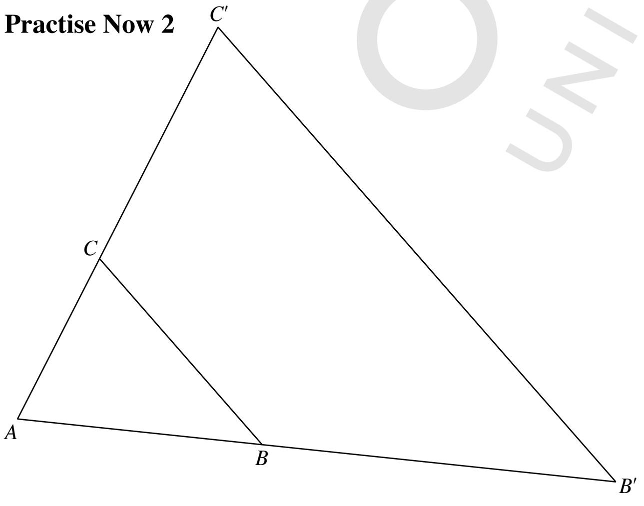
Practise Now 1



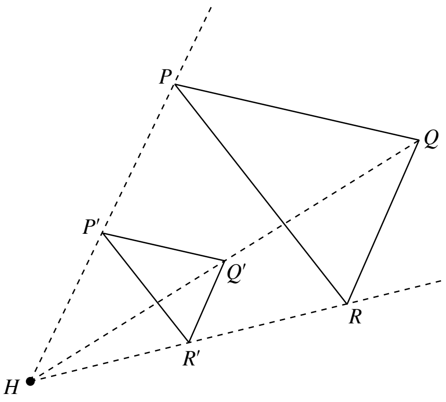
(a) From graph, vertices of $\triangle LMN$ are $L(-1, 6)$, $M(0, 9)$ and $N(1, 6)$.

(b) From graph, line m_1 is the perpendicular bisector of AP while line m_2 is the perpendicular bisector of BQ. The point of intersection of these two perpendicular bisectors $D(4, 3)$ gives the centre of rotation. Joining AD and PD gives the angle of rotation which is 90 clockwise.

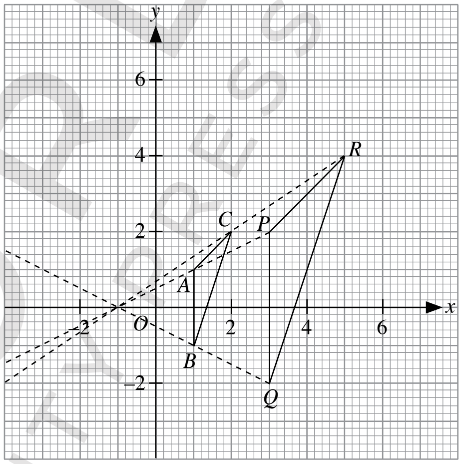
Practise Now 2



Practise Now 3



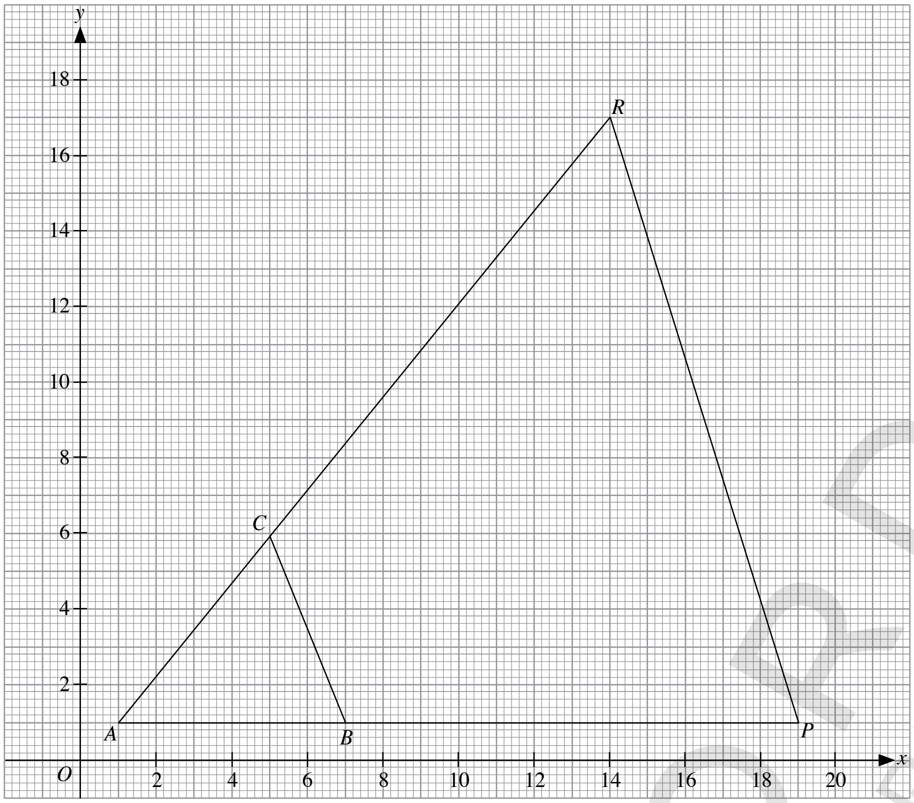
Practise Now 4



(a) From the graph plotted, coordinates of centre of enlargement are (–1, 0).

(b) Scale factor = $\frac{PQ}{AB} = \frac{4}{2} = 2$

Practise Now 5



(a) From the graph plotted, *A* is the invariant point and hence, centre of enlargement is (1, 1).

(b) Length of *AB* = 6 units

Height of $\triangle ABC$ = 5 units

$$\therefore \text{Area of } \triangle ABC = \frac{1}{2}(6)(5)$$

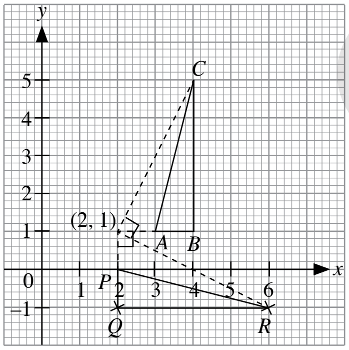
$$= 15 \text{ units}^2$$

$$\text{Area of } \triangle APR = 3^2 \times 15$$

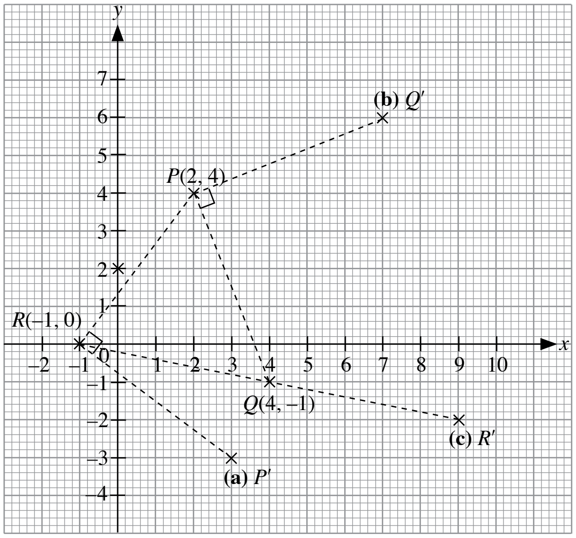
$$= 135 \text{ units}^2$$

Exercise 12A

1.



2.



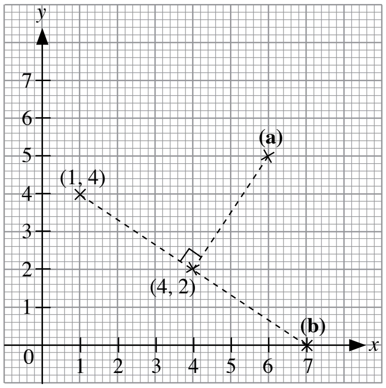
From graph,

(a) Image of P under clockwise rotation of 90 about R is $P'(3, -3)$

(b) Image of Q under anticlockwise rotation of 90 about P is $Q'(7, 6)$

(c) Image of R under 180 rotation about Q is $R'(9, -2)$

3.

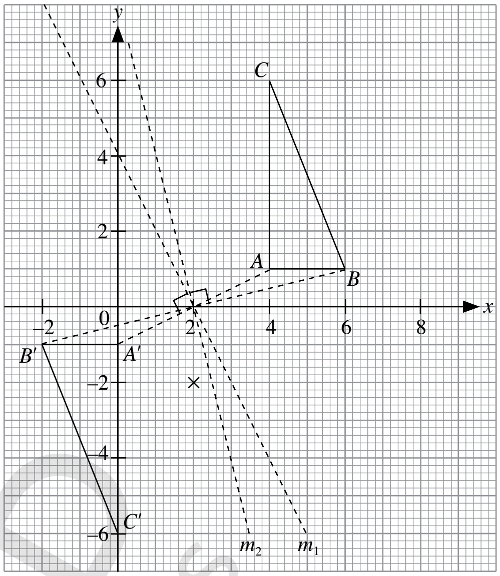


(a) From graph, coordinates of the image are (6, 5).

(b) From graph, coordinates of the image are (7, 0).

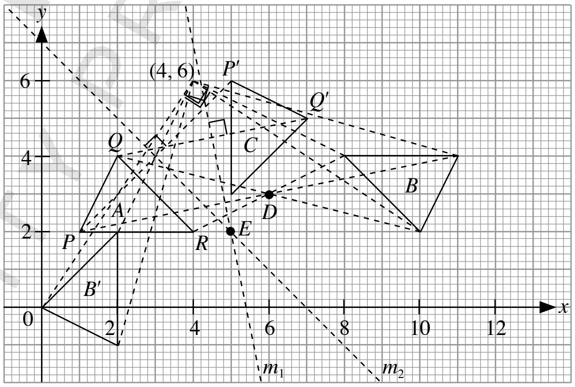
4. Since R represents an anticlockwise rotation of 240° about the origin, R^2 will be $(240° \times 2) - 360° = 480° - 360° = 120°$ anticlockwise rotation about the origin. R^4 will then be $120° \times 2 = 240°$ anticlockwise rotation about the origin.

5. (i)



(ii) From graph, line m_1 is the perpendicular bisector of AA' while line m_2 is the perpendicular bisector of BB'. The point of intersection of these two perpendicular bisectors (2, 0) gives the centre of rotation. The angle of rotation is 180°.

6.



(a) From graph, centre of rotation is given by point of intersection D.

  (i) Coordinates are (6, 3)

  (ii) Angle of rotation = 180°

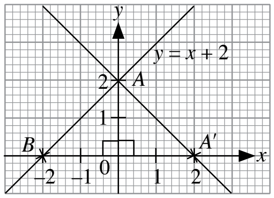
(b) From graph, line m_1 is the perpendicular bisector of QQ' while line m_2 is the perpendicular bisector of PP'.

  (i) The point of intersection of these two perpendicular bisectors at E gives the centre of rotation, with coordinates (5, 2).

  (ii) The angle of rotation obtained by joining PE and PE' is 90°.

(c) From graph, the coordinates of the vertices are (2, 2), (0, 0) and (2, –1).

7.



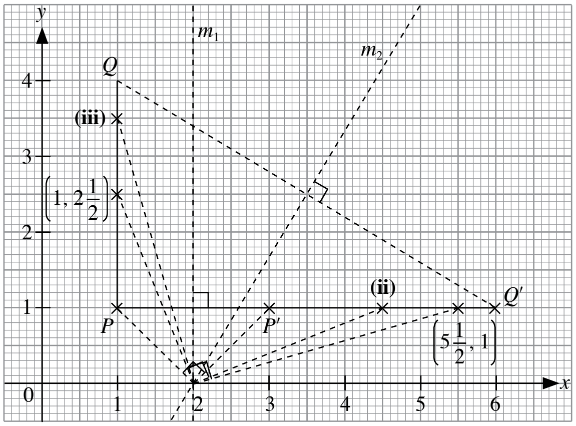
Points $A(0, 2)$ and $B(-2, 0)$ lie on the line $y = x + 2$. Under a clockwise rotation of 90 about the origin, point A becomes A', and B becomes A.

Gradient $= \dfrac{\text{rise}}{\text{run}} = -1$

y-intercept = 2

$\therefore$ Equation of the line is $y = -x + 2$ i.e. $x + y = 2$.

8.



(i) From graph, coordinates of Q' are (6, 1). $\therefore k = 6$.

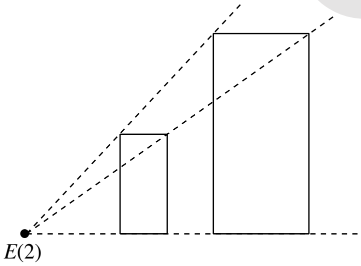
(ii) From graph, line m_1 is the perpendicular bisector of PP' while line m_2 is the perpendicular bisector of QQ'. The point of intersection of these two perpendicular bisectors (2, 0) gives the centre of rotation. The angle of rotation is 90° clockwise.

$\therefore$ The image of the point $\left(1, 2\dfrac{1}{2}\right)$ on line PQ is $\left(4\dfrac{1}{2}, 1\right)$ on line $P'Q'$.

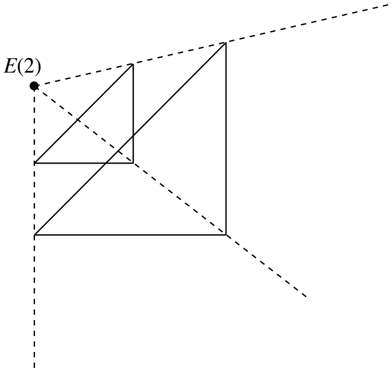
(iii) The coordinates of the point on line PQ whose image is $\left(5\dfrac{1}{2}, 1\right)$ on line $P'Q'$ are $\left(1, 3\dfrac{1}{2}\right)$.

Exercise 12B

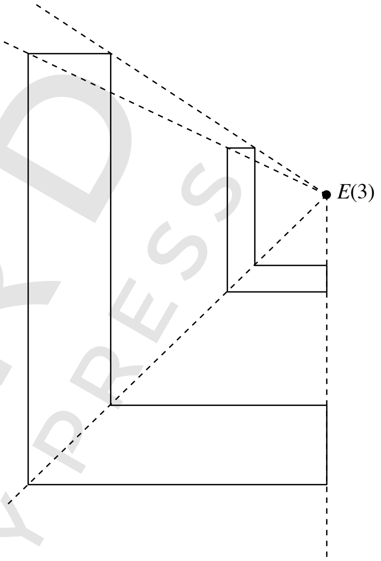
1. (a)



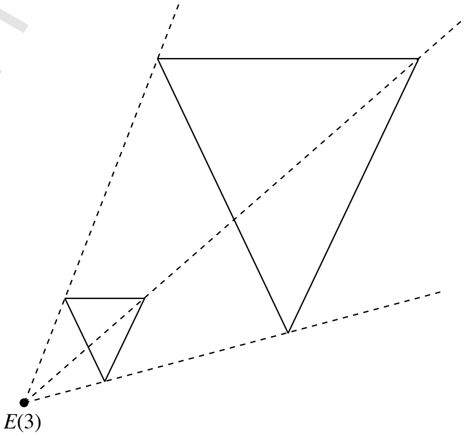
(b)



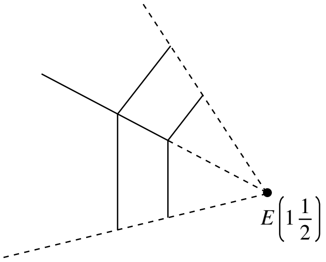
(c)



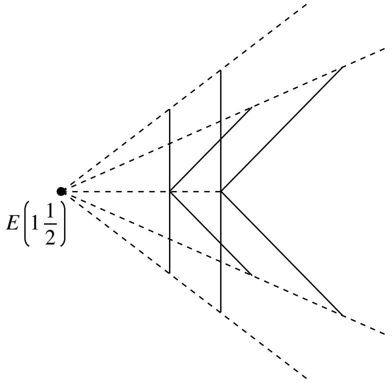
(d)



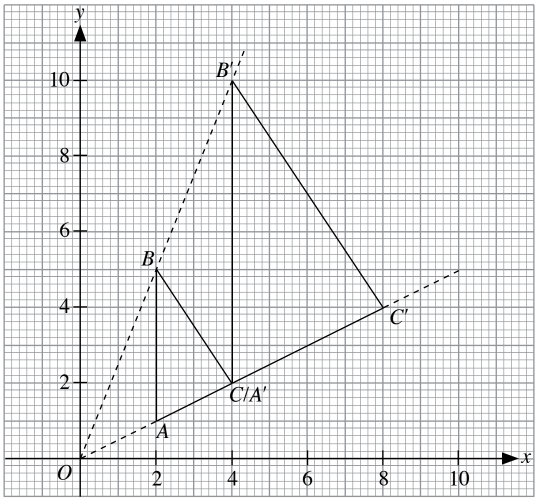
(e)



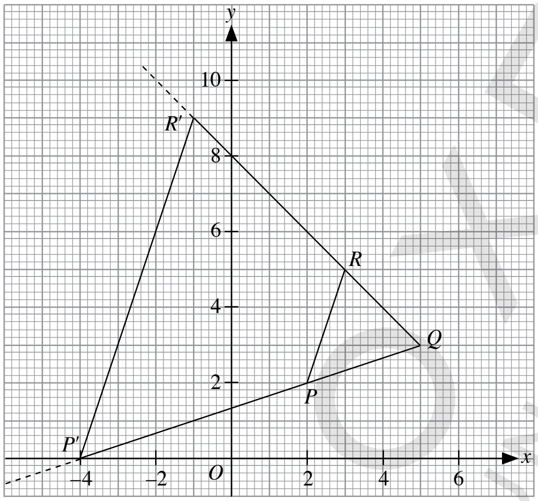
(f)



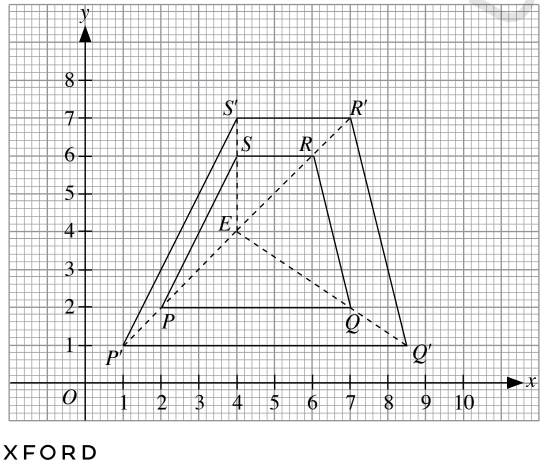
2.



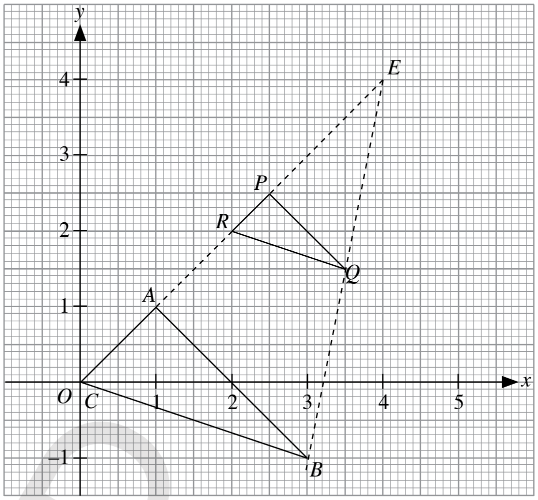
3.



4.

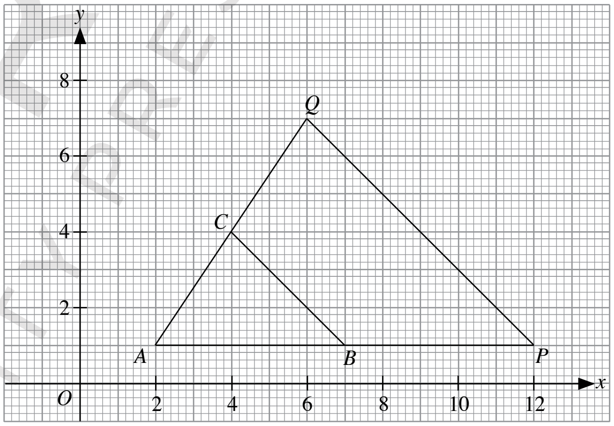


5.



From the graph, the coordinates are $P\left(2\frac{1}{2}, 2\frac{1}{2}\right)$, $Q\left(3\frac{1}{2}, 1\frac{1}{2}\right)$ and $R(2, 2)$.

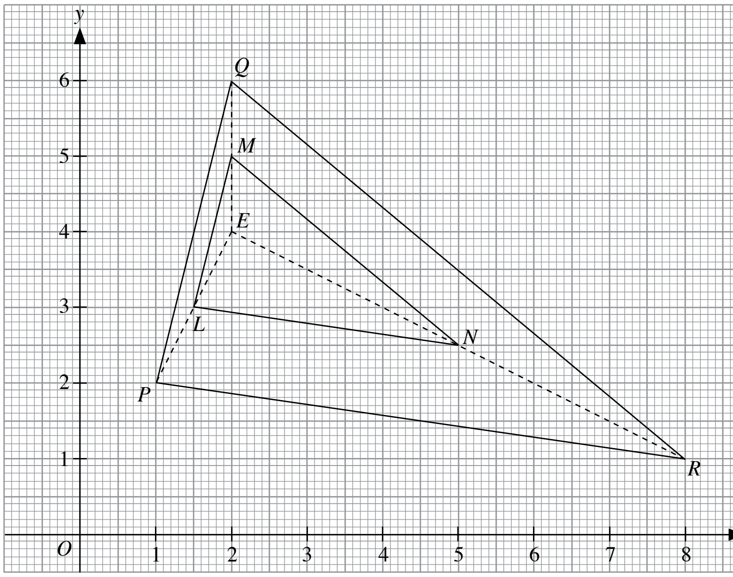
6.



(a) From the graph plotted, A is the invariant point and hence, centre of enlargement is $(2, 1)$.

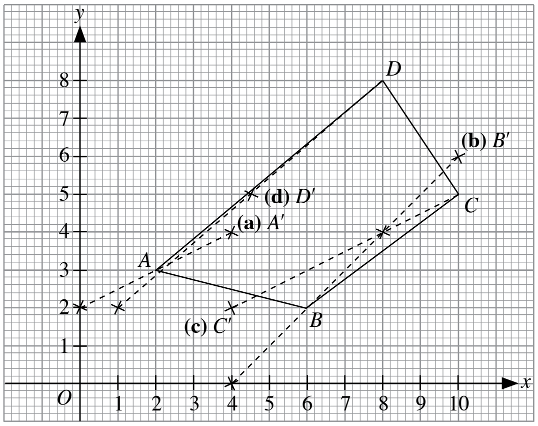
(b) From the graph, coordinates are $P(12, 1)$ and $Q(6, 7)$.

7.



From the graph plotted, the coordinates are $L\left(1\frac{1}{2}, 3\right)$, $M(2, 5)$ and $N\left(5, 2\frac{1}{2}\right)$.

8.



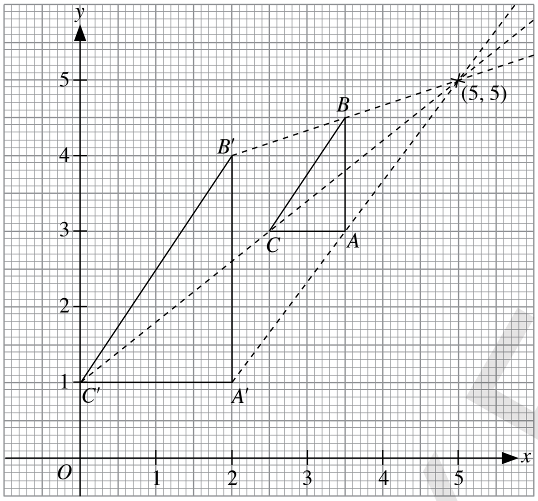
(a) Image of A is $A'(4, 4)$

(b) Image of B is $B'(10, 6)$

(c) Image of C is $C'(4, 2)$

(d) Image of D is $D'\left(4\frac{1}{2}, 5\right)$

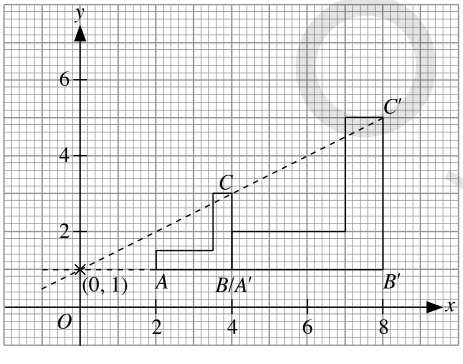
9. **(a)**



From graph, coordinates of centre of enlargement are (5, 5).

Scale factor = $\frac{A'B'}{AB} = \frac{3}{1\frac{1}{2}} = 2$

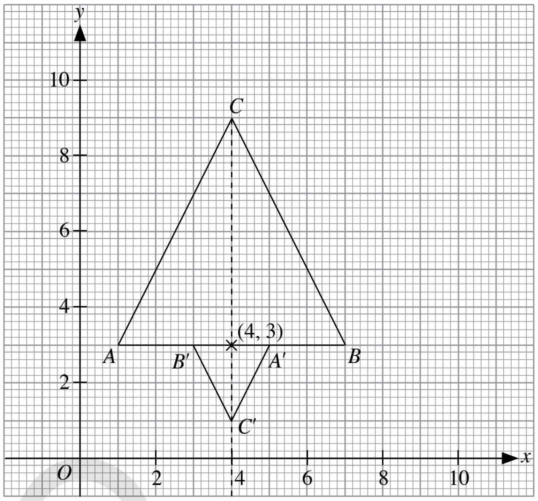
(b)



From graph, coordinates of centre of enlargement are (0, 1).

Scale factor = $\frac{C'B'}{CB} = \frac{4}{2} = 2$

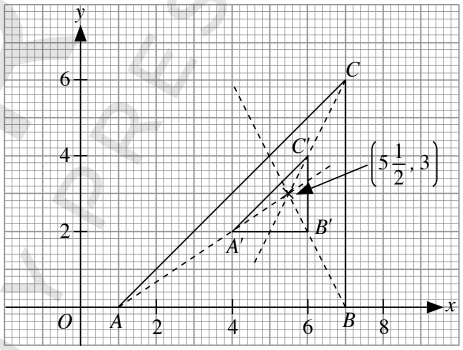
(c)



From graph, coordinates of centre of enlargement are (4, 3).

Scale factor = $-\frac{A'B'}{AB} = -\frac{2}{6} = -\frac{1}{3}$

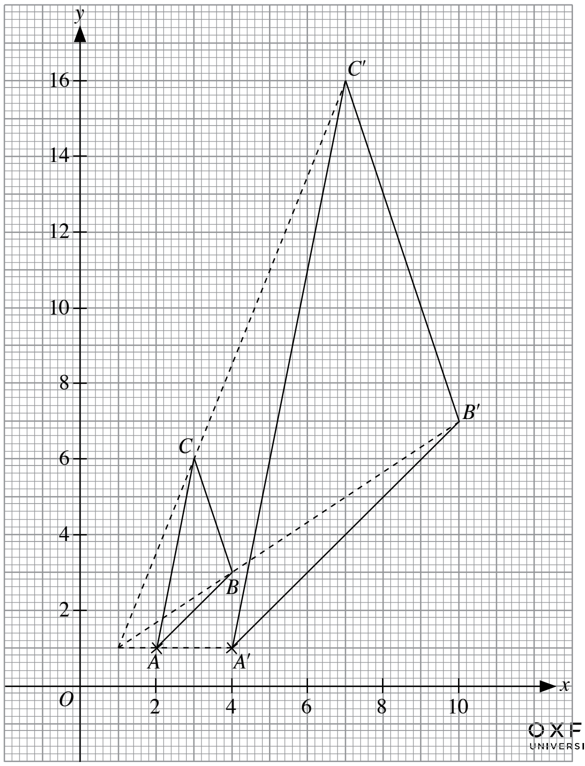
(d)



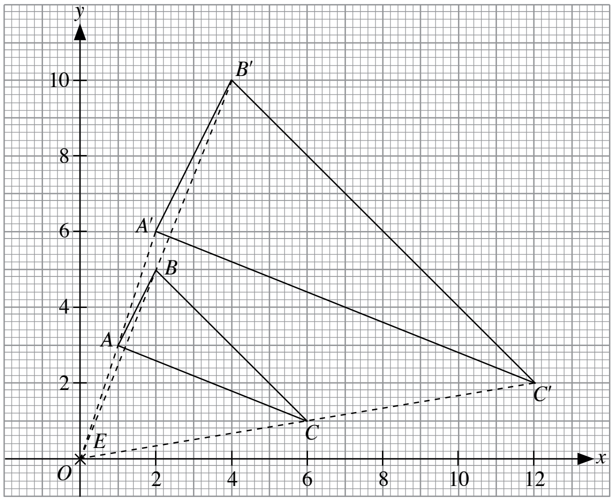
From graph, coordinates of centre of enlargement are $\left(5\frac{1}{2}, 3\right)$.

Scale factor = $\frac{A'B'}{AB} = \frac{2}{6} = \frac{1}{3}$

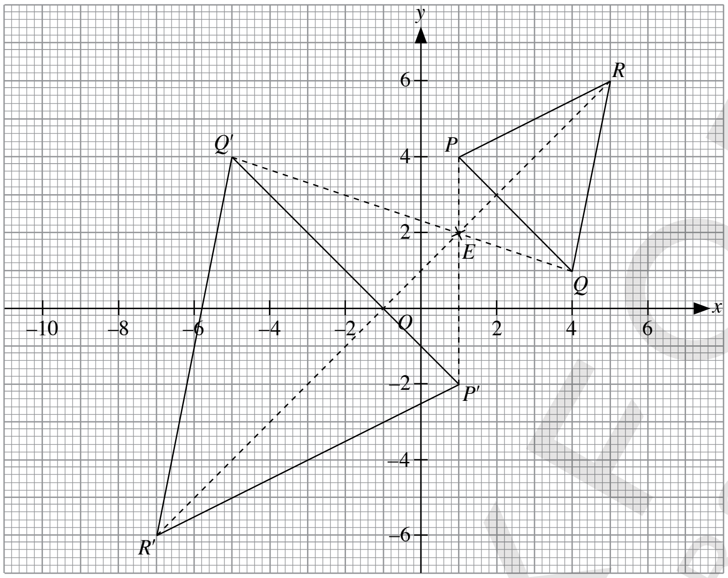
10.



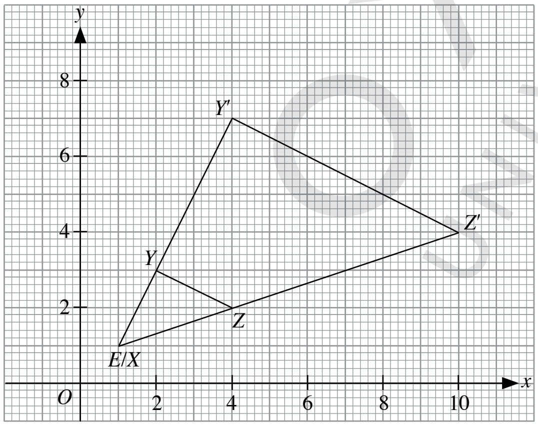
11. (a)



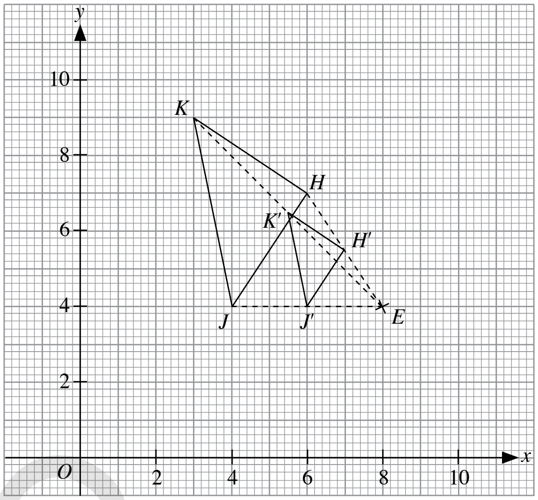
(b)



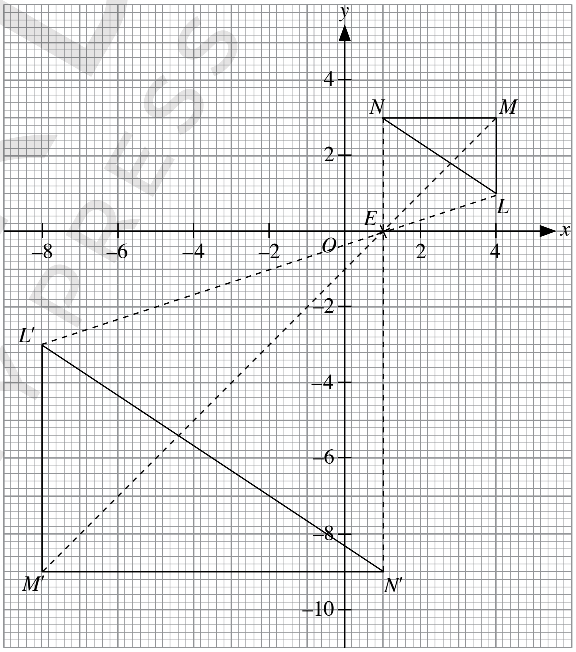
(c)



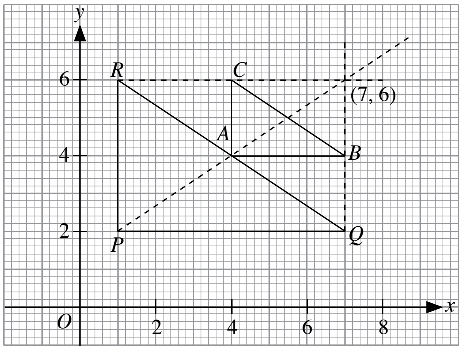
(d)



(e)



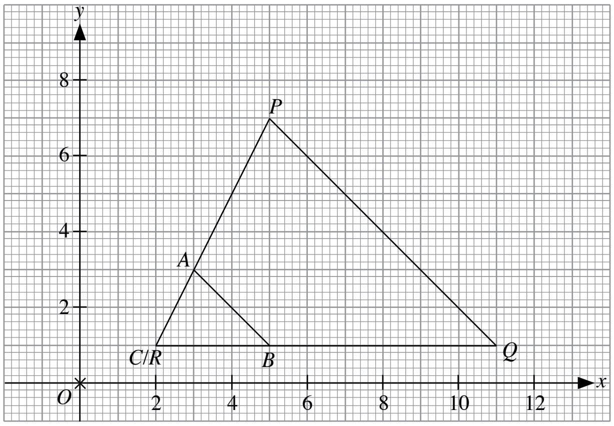
12. (a)



From graph, coordinates of centre of enlargement are (7, 6).

Scale factor = $\frac{PQ}{AB} = \frac{6}{3} = 2$

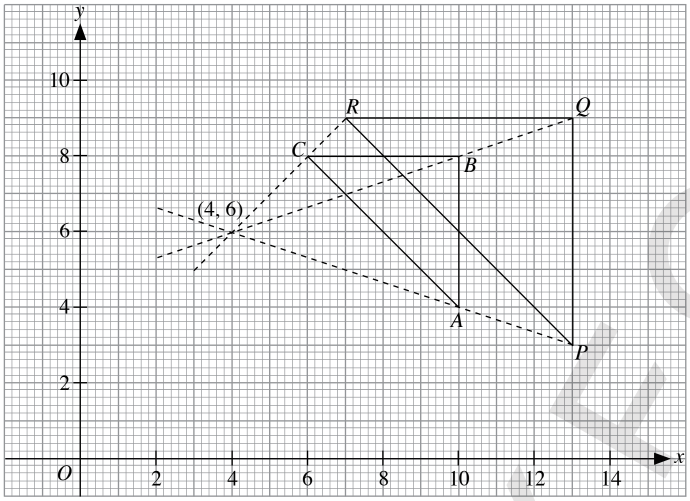
(b)



From graph, coordinates of centre of enlargement are (2, 1).

Scale factor = $\frac{RQ}{CB} = \frac{9}{3} = 3$

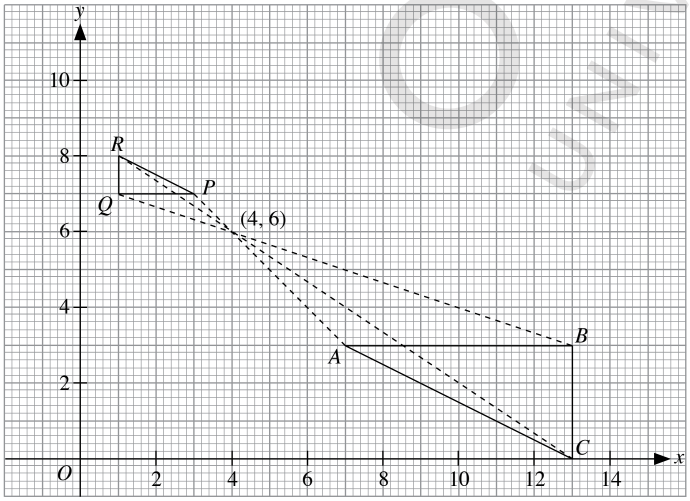
(c)



From graph, coordinates of centre of enlargement are (4, 6).

Scale factor = $\frac{RQ}{CB} = \frac{6}{4} = 1\frac{1}{2}$

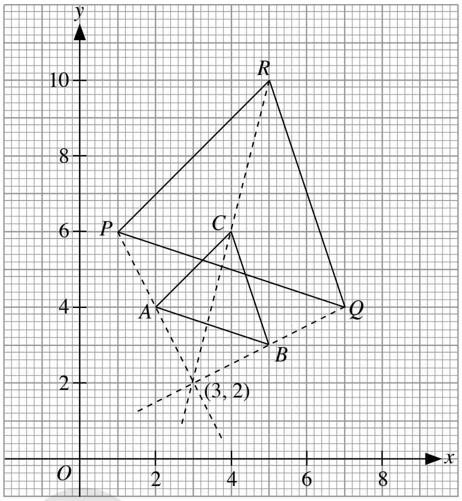
(d)



From graph, coordinates of centre of enlargement are (4, 6).

Scale factor = $-\frac{PQ}{AB} = -\frac{2}{6} = -\frac{1}{3}$

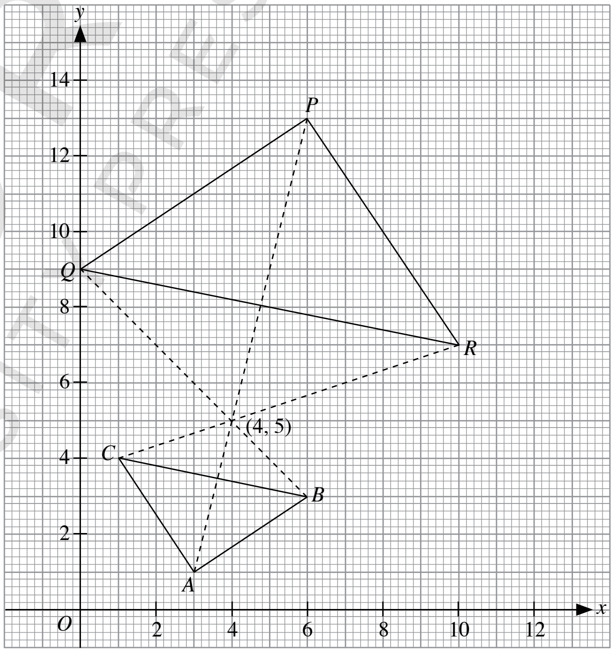
(e)



From graph, coordinates of centre of enlargement are (3, 2).

Scale factor = $\frac{PQ}{AB} = \frac{3}{1\frac{1}{2}} = 2$

(f)



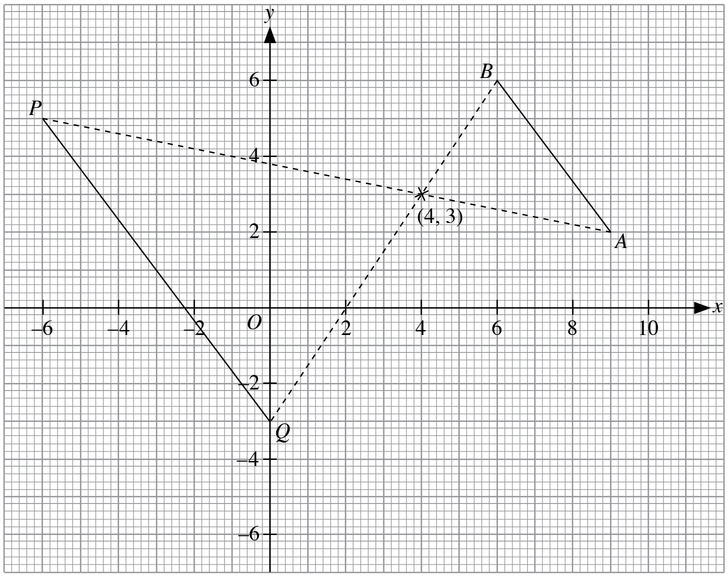
From graph, coordinates of centre of enlargement are (4, 5).

Scale factor = $-\frac{PQ}{AB} = -\frac{3.6}{1.8} = -2$

13. **(a)** The image figure is $\triangle ABC$.

(b) The image figure is rectangle *PBQR*.

14.

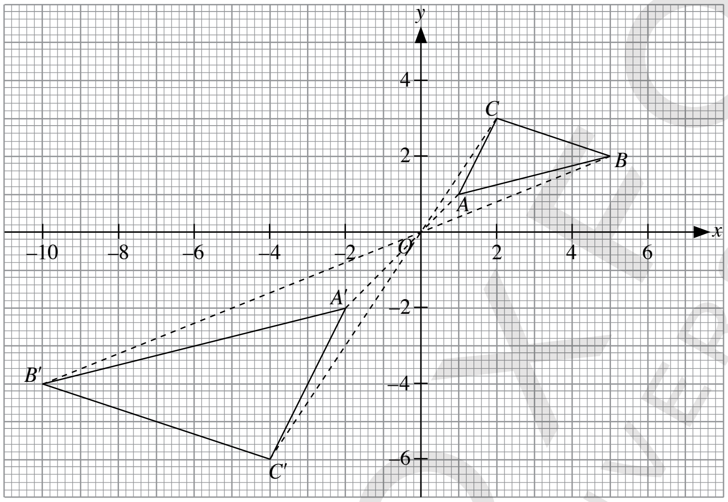


(a) From the graph plotted, the coordinates are $P(-6, 5)$ and $Q(0, -3)$.

(b) Length of $PQ = 5 \times 2$ units

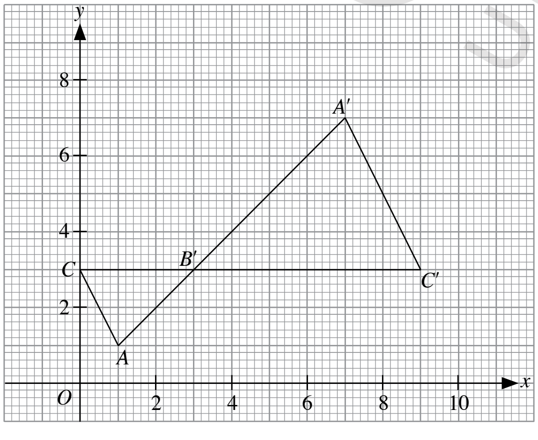
$= 10$ units

15.



From the graph plotted, the coordinates of $\triangle ABC$ are $A(1, 1)$, $B(5, 2)$ and $C(2, 3)$.

16.



From the graph plotted, the coordinates are $A(1, 1)$ and $C(0, 3)$.

17. Since $PQRS$ is a square, $PQ = RQ = 20$ cm

Scale factor $= \dfrac{PQ}{AB} = \dfrac{20}{10} = 2$

Let the distance of centre of enlargement from point A be x.

Distance of centre of enlargement from point P will be twice that of point A., i.e.

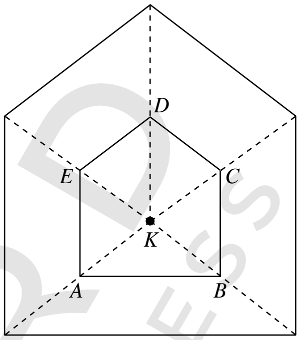
$2x = x + 20$

$x = 20$

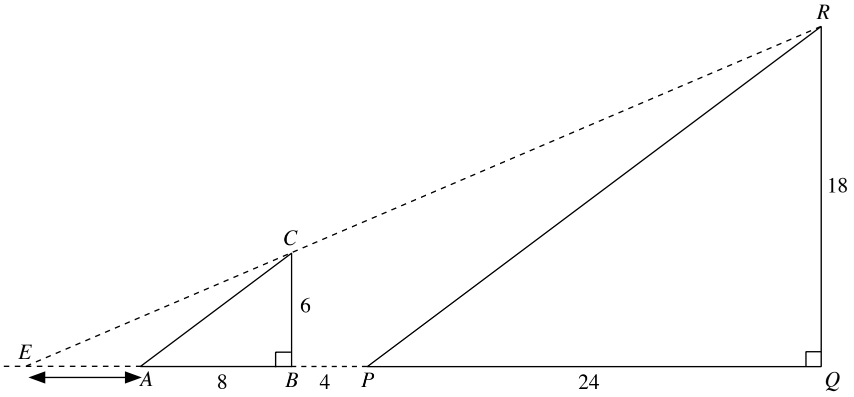
$\therefore$ The distance of the centre of enlargement from point A is 20 cm.

18. Area of image $= k^2 \times$ (area of original figure)

When $k^2 = 4$, $k = 2$



19.

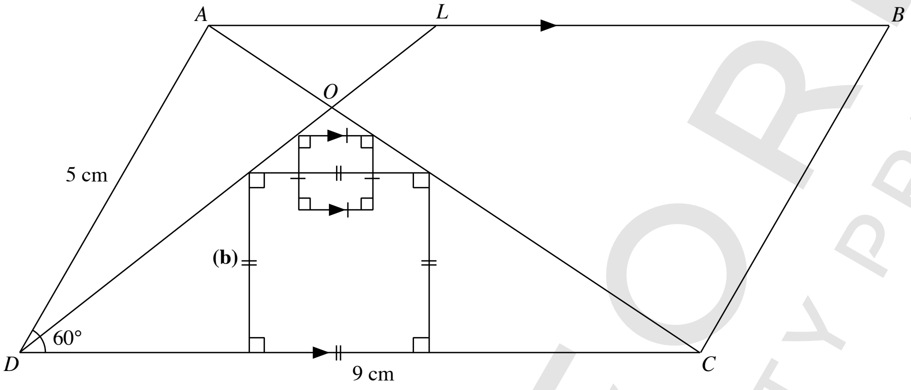


From drawn diagram, $EA = 1.5$ cm

$\therefore$ Length of $EA = 1.5 \times 4$

$= 6$ cm

20.



(a) $\triangle ALO$ is enlarged by a scale factor of -3 at centre O.

Review Exercise 12

1. **(a)** Under a reflection in the y-axis, $(a, b) \rightarrow (-a, b)$

$\therefore$ Coordinates of image of P are $(-2, -1)$.

(b) Under a 90° anticlockwise rotation about the origin, $(a, b) \rightarrow (-b, a)$

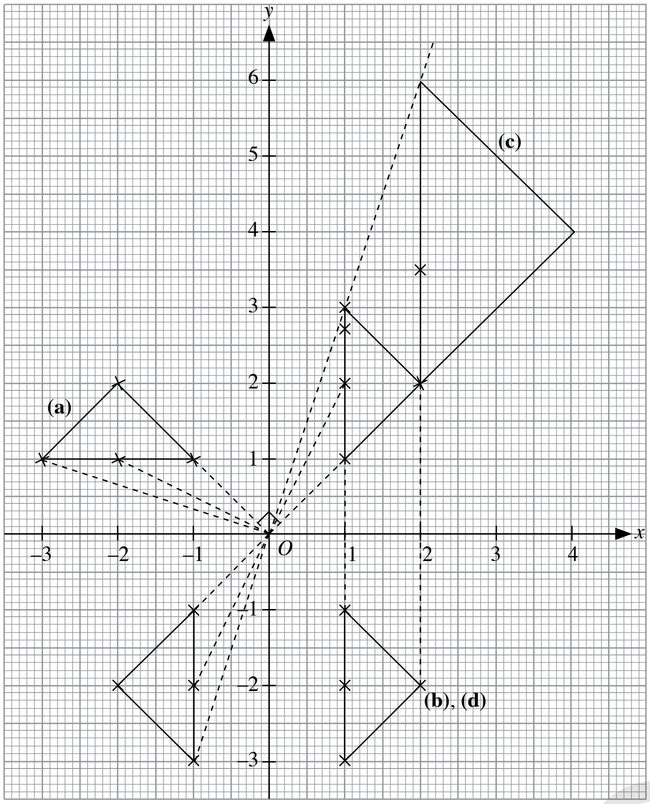
$\therefore$ Coordinates of image of P are $(1, 2)$.

(c) The translation is represented by $\begin{pmatrix} 1 \\ 5 \end{pmatrix}$.

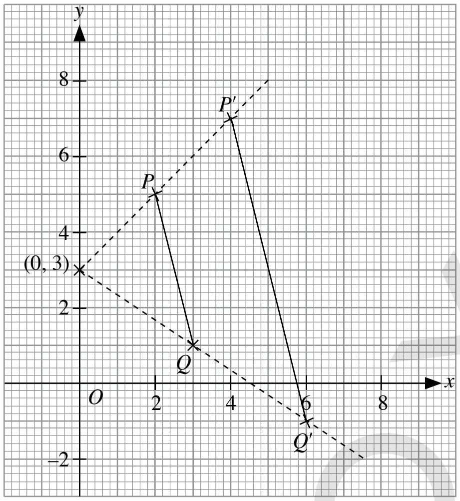
$$\begin{pmatrix} 2 \\ -1 \end{pmatrix} + \begin{pmatrix} 1 \\ 5 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

$\therefore$ Coordinates of image of P are $(3, 4)$.

3.



4.



From the graph plotted, from enlargement centre (0, 3), the image of P at $x = 4$ will give $P'(4, 7)$. Hence, scale factor of enlargement = 2.

Coordinates of the image of Q, Q' are (6, –1).

$\therefore p = 7, m = 6$ and $n = -1$

5. Scale factor, $k = 4$

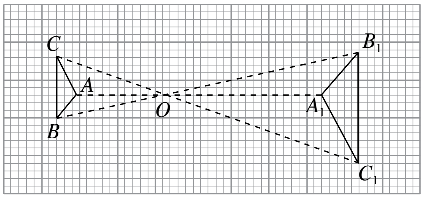
(a) $\dfrac{B'C'}{BC} = k = 4$

(b) Size of $AB'C'$ = size of ABC

$\therefore \dfrac{AB'C'}{ABC} = 1$

(c) $\dfrac{\text{area of } AB'C'}{\text{area of } ABC} = k^2 = 4^2 = 16$

6.



(a) $AB : A_1B_1 = 1 : 2$

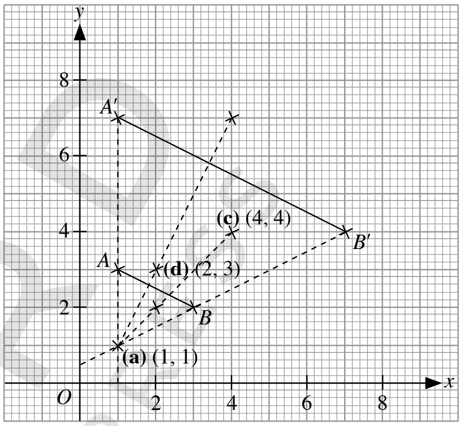
(b) Scale factor of enlargement = –2

Area of $\triangle A_1B_1C_1$: area of $\triangle ABC$

$= (-2)^2 : 1$

$= 4 : 1$

7.



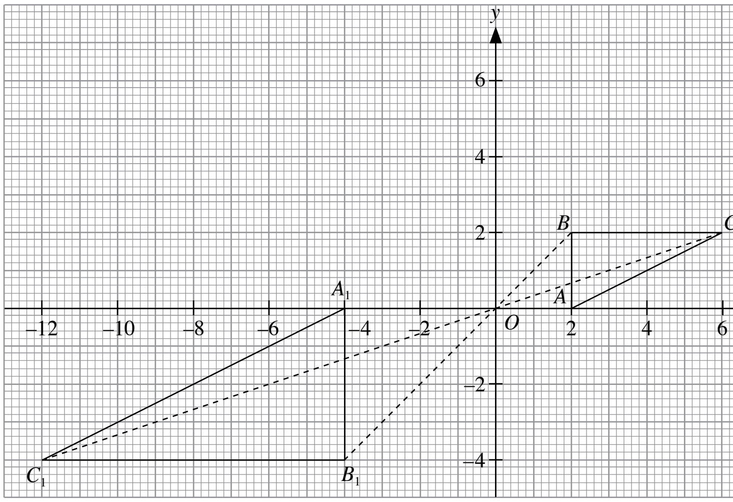
(a) From the graph plotted, coordinates of centre of enlargement are (1, 1).

(b) Scale factor = $\dfrac{A'B'}{AB} = 3$

(c) From graph, coordinates of the image are (4, 4).

(d) From graph, coordinates of the point are (2, 3).

8.



(a) From the graph plotted, the coordinates are $A_1(-4, 0)$, $B_1(-4, -4)$ and $C_1(-12, -4)$.

(b) $\dfrac{A_1B_1}{AB} = \dfrac{4}{2} = 2$

(c) $\dfrac{\text{area of } \triangle ABC}{\text{area of } \triangle A_1B_1C_1} = \dfrac{1}{2^2} = \dfrac{1}{4}$

9. **(a)** Enlargement at centre A with scale factor 4

(b) Translation parallel to AP with length AP

(c) 180° rotation about point P

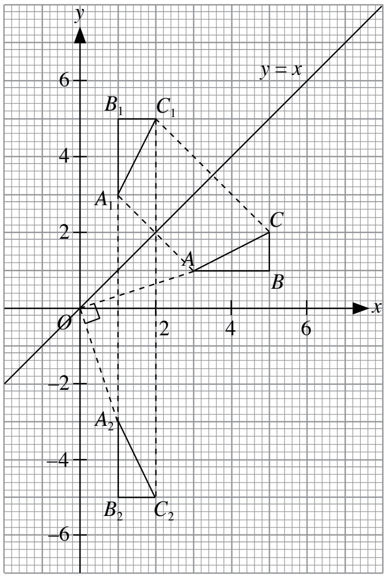
(d) Enlargement at centre A with scale factor 2

(e) Enlargement at centre point X, where $2HX = XK$, with scale factor 3

(f) Enlargement at centre B with scale factor 2

Challenge Yourself

1.



From the graph plotted, $\triangle ABC$ will be mapped onto $\triangle A_2B_2C_2$ under a 90° clockwise rotation about the origin.

2. **(a)** PQ would be a reflection in the x-axis followed by a 90° clockwise rotation about the origin. Taking the point (1, 1):
After Q, the image would be (1, –1); followed by P, the image would be (–1, –1).
∴ A single transformation equivalent to PQ would be a reflection in the line $y + x = 0$.

(b) QP would be a 90° clockwise rotation about the origin followed by a reflection in the x-axis.
Taking the point (2, 1):
After P, the image would be (1, –2); followed by Q, the image would be (1, 2)
∴ A single transformation equivalent to QP would be a reflection in the line $y = x$.

Chapter 13 Statistics

TEACHING NOTES

Suggested Approach

Students are already familiarised by histograms. In this chapter, they will learn about frequency polygon using histogram of grouped data with unequal class intervals. They will further learn about standard variance and its applications in real-life.

Section 13.1 Frequency Polygons

It is crucial that the differences between ungrouped data and grouped data as well as discrete and continuous data are stated at the beginning. The use of class intervals for grouped data is what differentiates both types of data, as the interval 'groups' similar data together. Students are also required to make frequency tables in this section.

Students can be grouped together to discuss and present the similarities, differences, advantages and disadvantages between a stem-and-leaf diagram and a histogram for grouped data.

Students are to be exposed to the usage of histograms for grouped data with unequal class intervals.

Section 13.2: Standard Deviation

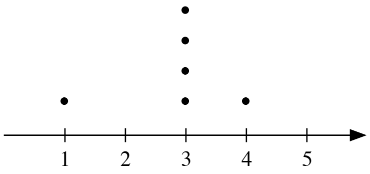
Teachers can use use the Investigation activity on page 375 to introduce the need to have a statistical measure — standard deviation — to describe the distribution of a set of data. Then, guide the students to go through the Investigation activity on page 376 to obtain the formula for a new measure of spread. With this, there should be more opportunities for the students to compare the means and standard deviations of two sets of data by referring to the context of the questions.

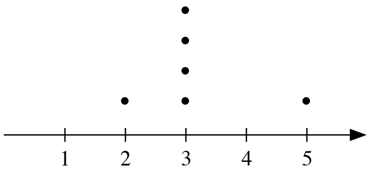
Teachers should take time to go through the use of calculator to find the mean and standard deviation for a set of data. Also, teachers should engage the class in the Class Discussion on page 385 to allow the students to discuss about examples of inappropriate representations of data from newspapers and other sources, e.g. whether certain representations are misleading.

WORKED SOLUTIONS

Investigation (Are Averages Adequate for Comparing Distributions?)

1.





2. No, as these three averages are not able to describe the distribution of the set of data.

Investigation (Obtaining a Formula for a New Measure of Spread)

Part 1: Mean Temperatures

1. Mean temperature of City *A*

$$= \frac{25 + 24 + 26 + 33 + 31 + 29}{6}$$

= 28 °C

Mean temperature of City *B*

$$= \frac{21 + 15 + 23 + 36 + 41 + 32}{6}$$

= 28 °C

2. Yes

3. The spread of the temperatures of City *A* is less wide as compared to the spread of the temperatures in City *B*.

Part 2: Spread of the Temperatures

4.

x	$x - \overline{x}$
25	$25 - 28 = -3$
24	$24 - 28 = -4$
26	$26 - 28 = -2$
33	$33 - 28 = 5$
31	$31 - 28 = 3$
29	$29 - 28 = 1$
Sum	$\Sigma(x - \overline{x}) = 0$

5. The data of City *B* is more spread out.

6.

x	$x - \overline{x}$
21	$21 - 28 = -7$
15	$15 - 28 = -13$
23	$23 - 28 = -5$
36	$36 - 28 = 8$
41	$41 - 28 = 13$
32	$32 - 28 = 4$
Sum	$\Sigma(x - \overline{x}) = 0$

The value of $\Sigma(x - \overline{x})$ from City *B* is the same as the value of $\Sigma(x - \overline{x})$ from City *A*.

This is not a good measure of spread since it does not show that the spread of the temperatures in City *B* is wider than that in City *A*.

7. For City *A*,

x	$(x - \overline{x})^2$
25	$(25 - 28)^2 = 9$
24	$(24 - 28)^2 = 16$
26	$(26 - 28)^2 = 4$
33	$(33 - 28)^2 = 25$
31	$(31 - 28)^2 = 9$
29	$(29 - 28)^2 = 1$
Sum	$\Sigma(x - \overline{x})^2 = 64$

For City *B*,

x	$(x - \overline{x})^2$
21	$(21 - 28)^2 = 49$
15	$(15 - 28)^2 = 169$
23	$(23 - 28)^2 = 25$
36	$(36 - 28)^2 = 64$
41	$(41 - 28)^2 = 169$
32	$(32 - 28)^2 = 16$
Sum	$\Sigma(x - \overline{x})^2 = 492$

The value of $\Sigma(x - \overline{x})^2$ from City *B* is greater than the value of $\Sigma(x - \overline{x})^2$ from City *A*.

This is a good measure of spread since this will remove the negative value of the difference between each data and the mean and hence show that the spread of the temperatures in City *B* is wider than that in City *A*.

8. $\Sigma(x - \overline{x})^2$ will increase when there are more data values.

No, it does not mean that the spread will increase when there are more data values.

9. For City *A*,

$$\frac{\Sigma\left(x-\bar{x}\right)^2}{6} = \frac{64}{6} = 10.7 \text{ (to 3 s.f.)}$$

For City *B*,

$$\frac{\Sigma\left(x-\bar{x}\right)^2}{6} = \frac{492}{6} = 82$$

This will provide a good indication on how the data are spread about the mean since it takes into account the number of data.

10. For City *A*,

$$\sqrt{\frac{\Sigma\left(x-\bar{x}\right)^2}{n}} = \sqrt{\frac{64}{6}} = 3.26 \text{ (to 3 s.f.)}$$

11. For City *B*,

$$\sqrt{\frac{\Sigma\left(x-\bar{x}\right)^2}{n}} = \sqrt{\frac{492}{6}} = 9.06 \text{ (to 3 s.f.)}$$

12. The standard deviation for City *B* is larger. This means that the temperatures of City *B* are more widely spread than those of City *A*.

Thinking Time (Page 384)

The measured mean is 1.5 °C higher than the correct mean since the total error of the measurements is divided by the number of measurements.

There will be no change in the standard deviation since the difference between each data and the mean cancels the 1.5 °C error.

Class Discussion (Matching Histograms with Data Sets)

Histogram *A* represents data set V since the distribution is skewed to the right so its median should match the bar on the left.

Histogram *B* represents data set III since the distribution is skewed to the left so its median should match the bar on the right.

Histogram *C* represents data set VI since its mean and median are close to each other so its distribution is more symmetrical.

Histogram *D* represents data set II since its mean and median are the same so its distribution is more symmetrical. But the data is more widely spread so standard deviation is higher.

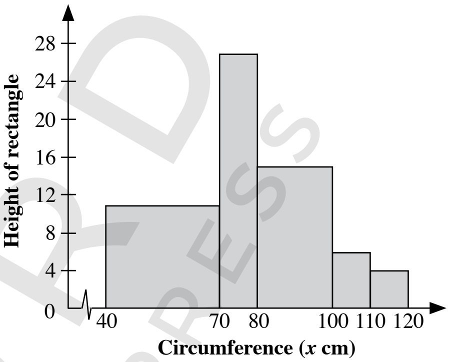
Histogram *E* represents data set I since the distribution is skewed to the left so its median should match the bar on the right. Also, its mean and median are close to each other.

Histogram *F* represents data set IV since the distribution is skewed to the right so its median should match the bar on the left.

Practise Now 1

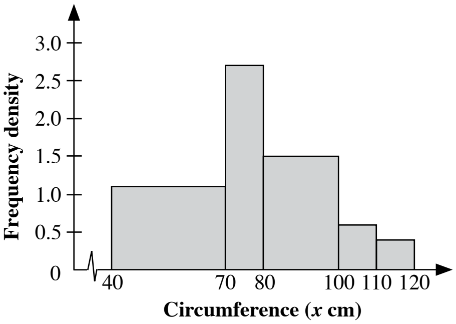
Method 1 (Using the heights of rectangles)

Circumference (x cm)	Class width		Frequency	Rectangle's height
$40 < x \leqslant 70$	30	3 × standard	33	33 ÷ 3 = 11
$70 < x \leqslant 80$	10	1 × standard	27	27 ÷ 1 = 27
$80 < x \leqslant 100$	20	2 × standard	30	30 ÷ 2 = 15
$100 < x \leqslant 110$	10	1 × standard	6	6 ÷ 1 = 6
$110 < x \leqslant 120$	10	1 × standard	4	4 ÷ 1 = 4



Method 2 (Using frequency densities)

Circumference (x cm)	Frequency	Class width	Frequency density $= \dfrac{\text{Frequency}}{\text{Class width}}$
$40 < x \leqslant 70$	33	30	33 ÷ 30 = 1.1
$70 < x \leqslant 80$	27	10	27 ÷ 10 = 2.7
$80 < x \leqslant 100$	30	20	30 ÷ 20 = 1.5
$100 < x \leqslant 110$	6	10	6 ÷ 10 = 0.6
$110 < x \leqslant 120$	4	10	4 ÷ 10 = 0.4

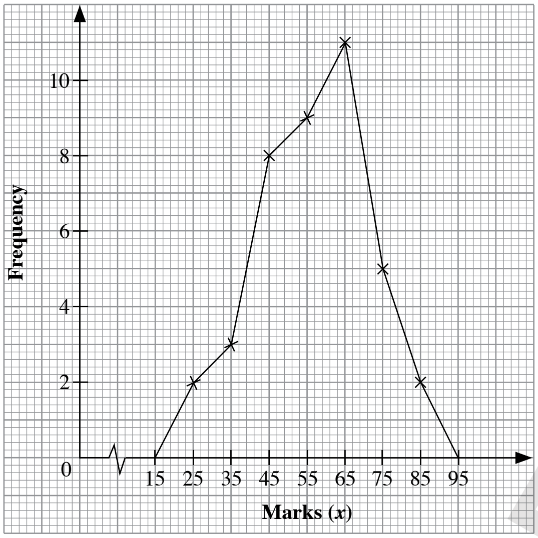


Practise Now 2

(a) Mid-value = 45

(b)

Marks (x)	Mid-value	Number of students
$20 < x \leqslant 30$	25	2
$30 < x \leqslant 40$	35	3
$40 < x \leqslant 50$	45	8
$50 < x \leqslant 60$	55	9
$60 < x \leqslant 70$	65	11
$70 < x \leqslant 80$	75	5
$80 < x \leqslant 90$	85	2



Practise Now 3

x	x^2
6	36
9	81
15	225
26	676
10	100
14	196
21	441
3	9
$\Sigma x = 104$	$\Sigma x^2 = 1764$

$$\text{Mean, } \bar{x} = \frac{\Sigma x}{n}$$
$$= \frac{104}{8}$$
$$= 13$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2}$$
$$= \sqrt{\frac{1764}{8} - 13^2}$$
$$= 7.18 \text{ (to 3 s.f.)}$$

Practise Now 4

Standard deviation of their ages = 3.74 years

Practise Now 5

(a)

Marks	Frequency	Mid-value (x)	fx	fx^2
$0 < x \leqslant 4$	3	2	6	12
$4 < x \leqslant 8$	8	6	48	288
$8 < x \leqslant 12$	14	10	140	1400
$12 < x \leqslant 16$	2	14	28	392
$16 < x \leqslant 20$	3	18	54	972
Sum	$\Sigma f = 30$		$\Sigma fx = 276$	$\Sigma fx^2 = 3064$

(i) Mean, $\bar{x} = \dfrac{\Sigma fx}{\Sigma f}$

$$= \frac{276}{30}$$
$$= 9.2$$

(ii) Standard deviation $= \sqrt{\dfrac{\Sigma fx^2}{\Sigma f} - \bar{x}^2}$

$$= \sqrt{\frac{3064}{30} - 9.2^2}$$
$$= 4.18 \text{ (to 3 s.f.)}$$

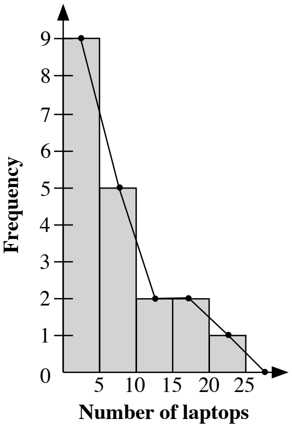
(b) Since the mean mark for Class *A* is lower than that of Class *B*, the students of Class *A* did not perform as well overall in comparison to the students of Class *B*.
Since the standard deviation of Class *A* is higher than that of Class *B*, this indicates that there is a greater spread of marks in Class *A*, i.e. some students scored very high marks while some scored very low marks.

Practise Now 6

Standard deviation = 12.1 g

Exercise 13A

1. **(i)**



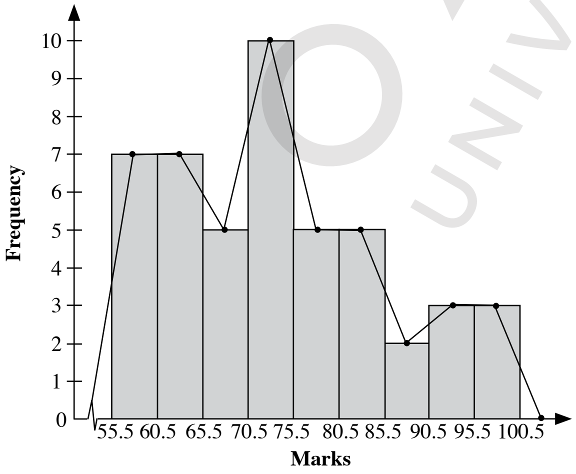
(ii) The line on the histogram shows the required frequency polygon.

2. **(a)**

Marks	Tally	Lower class boundary	Upper class boundary	Frequency
56 – 60	𝍸 //	55.5	60.5	7
61 – 65	𝍸 //	60.5	65.5	7
66 – 70	𝍸	65.5	70.5	5
71 – 75	𝍸 𝍸	70.5	75.5	10
76 – 80	𝍸	75.5	80.5	5
81 – 85	𝍸	80.5	85.5	5
86 – 90	//	85.5	90.5	2
91 – 95	///	90.5	95.5	3
96 – 100	///	95.5	100.5	3

(b) 90.9: 91 – 95
66.2: 66 – 70
81.5: 81 – 85

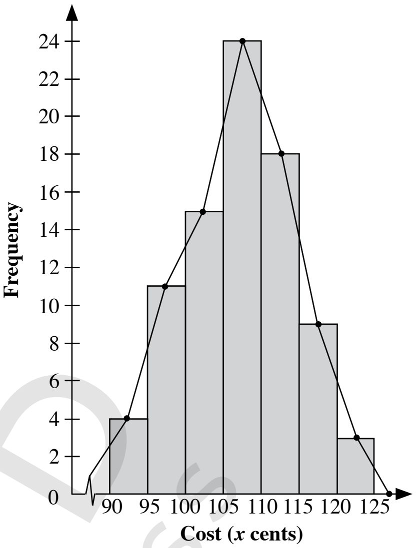
(c)



(d) The line on the histogram shows the required frequency polygram.

3. **(a)** Total number of shops = 4 + 11 + 15 + 24 + 18 + 9 + 3
= 84

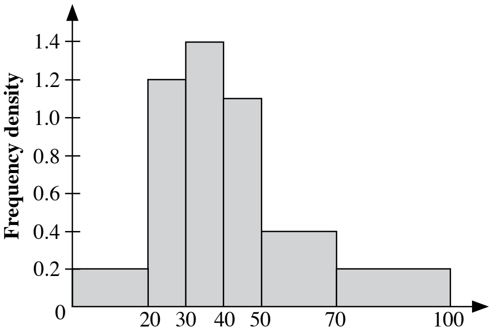
(b)



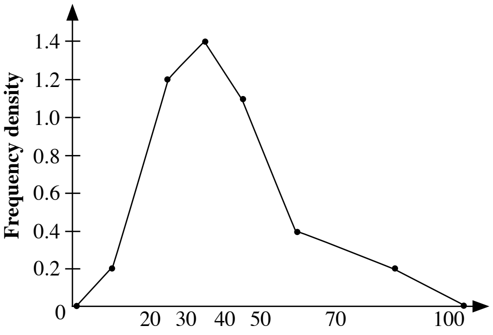
4. **(a)**

Class interval	Class width	Frequency	Frequency density $= \frac{\text{Frequency}}{\text{Class width}}$
$0 < x \leqslant 20$	20	4	$4 \div 20 = 0.2$
$20 < x \leqslant 30$	10	12	$12 \div 10 = 1.2$
$30 < x \leqslant 40$	10	14	$14 \div 10 = 1.4$
$40 < x \leqslant 50$	10	11	$11 \div 10 = 1.1$
$50 < x \leqslant 70$	20	8	$8 \div 20 = 0.4$
$70 < x \leqslant 100$	30	6	$6 \div 30 = 0.2$

(b)



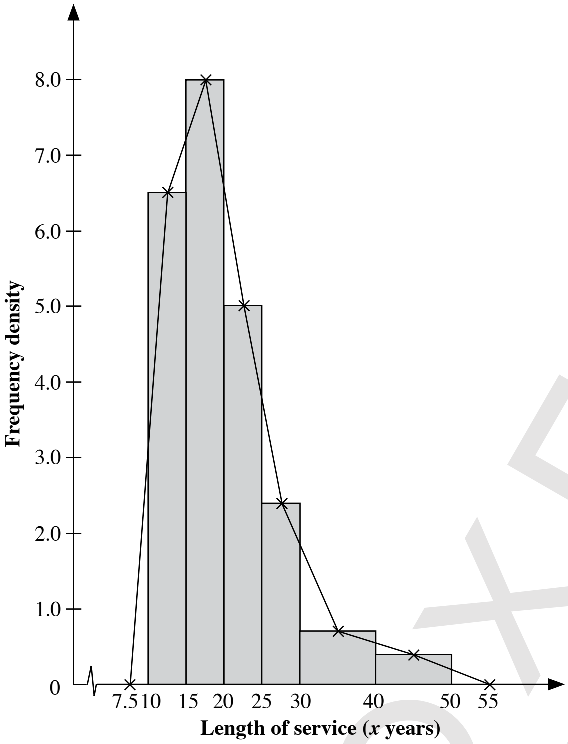
5.



6. It is an open ended question. Students may give different answers.

7. Since the class intervals are unequal, the histogram is to be drawn using either height of rectangle or frequency density.

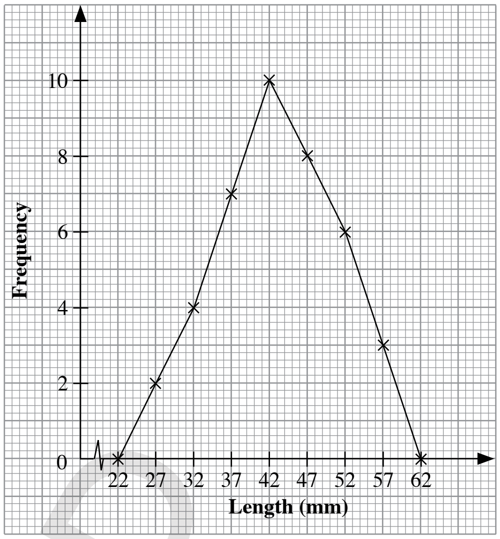
Class interval	Class width	Frequency	Frequency density $= \frac{\text{Frequency}}{\text{Class width}}$
$10 \leqslant x < 15$	5	32	$32 \div 5 = 6.4$
$15 \leqslant x < 20$	5	40	$40 \div 5 = 8$
$20 \leqslant x < 25$	5	25	$25 \div 5 = 5$
$25 \leqslant x < 30$	5	12	$12 \div 5 = 2.4$
$30 \leqslant x < 40$	10	7	$7 \div 10 = 0.7$
$40 \leqslant x < 50$	10	4	$4 \div 10 = 0.4$



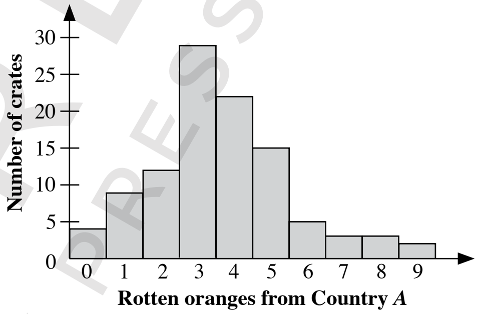
8.

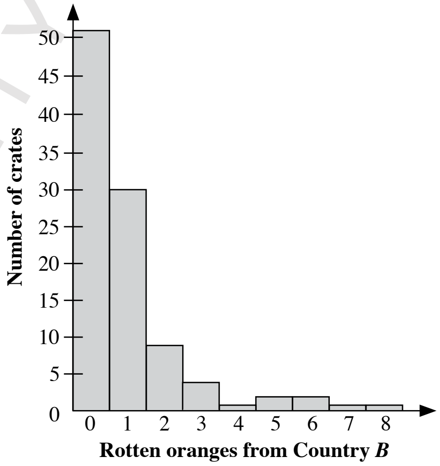
Length (mm)	Mid-value	Frequency
25 – 29	27	2
30 – 34	32	4
35 – 39	37	7
40 – 44	42	10
45 – 49	47	8
50 – 54	52	6
55 – 59	57	3

The points to be plotted are (22, 0), (27, 2), (32, 4), (37, 7), (42, 10), (47, 8), (52, 6), (57, 3) and (62, 0).



9. **(i)**





(ii) Largest number of rotten oranges found in a crate from country $A = 9$

Largest number of rotten oranges found in a crate from country $B = 8$

(iii) Total number of rotten oranges from country A

$= (4 \times 0) + (9 \times 1) + (12 \times 2) + (28 \times 3) + (22 \times 4) + (15 \times 5) + (5 \times 6) + (2 \times 7) + (2 \times 8) + (1 \times 9)$

$= 0 + 9 + 24 + 84 + 88 + 75 + 30 + 14 + 16 + 9$

$= 349$

Total number of rotten oranges from country B

$= (51 \times 0) + (30 \times 1) + (8 \times 2) + (4 \times 3) + (1 \times 4) + (2 \times 5) + (2 \times 6) + (1 \times 7) + (1 \times 8)$

$= 0 + 30 + 16 + 12 + 4 + 10 + 12 + 7 + 16$

$= 99$

(iv) P(crate contains no fewer than p rotten oranges) = $\frac{3}{4}$

Number of crates with no fewer than p rotten oranges

$= \frac{3}{4} \times 100$

$= 75$

Since $1 + 2 + 2 + 5 + 15 + 22 + 28 = 75$,

$\therefore p = 3$

Exercise 13B

1. **(a)**

x	x^2
3	9
4	16
5	25
7	49
8	64
10	100
13	169
$\Sigma x = 50$	$\Sigma x^2 = 432$

Mean, $\bar{x} = \frac{\Sigma x}{n}$

$= \frac{50}{7}$

$= 7.1429$ (to 5 s.f.)

Standard deviation $= \sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2}$

$= \sqrt{\frac{432}{7} - 7.1429^2}$

$= 3.27$ (to 3 s.f.)

(b)

x	x^2
28	784
25	625
32	1024
20	400
30	900
19	361
22	484
24	576
27	729
23	529
$\Sigma x = 250$	$\Sigma x^2 = 6412$

Mean, $\bar{x} = \frac{\Sigma x}{n}$

$= \frac{250}{10}$

$= 25$

Standard deviation $= \sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2}$

$= \sqrt{\frac{6412}{10} - 25^2}$

$= 4.02$ (to 3 s.f.)

(c)

x	x^2
–5	25
–4	16
0	0
1	1
4	16
–2	4
$\Sigma x = -6$	$\Sigma x^2 = 62$

Mean, $\bar{x} = \frac{\Sigma x}{n}$

$= \frac{-6}{6}$

$= -1$

Standard deviation $= \sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2}$

$= \sqrt{\frac{62}{6} - (-1)^2}$

$= 3.06$ (to 3 s.f.)

2. **(a)** Standard deviation = 11.1 (to 3 s.f.)

(b) Standard deviation = 9.35 (to 3 s.f.)

(c) Standard deviation = 11.9 (to 3 s.f.)

3.

Marks (x)	Frequency	fx	fx^2
2	5	10	20
3	7	21	63
4	6	24	96
5	4	20	100
6	9	54	324
7	3	21	147
8	6	48	384
Sum	$\Sigma f = 40$	$\Sigma fx = 198$	$\Sigma fx^2 = 1134$

Mean, $\bar{x} = \frac{\Sigma fx}{\Sigma f}$

$= \frac{198}{40}$

$= 4.95$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \overline{x}^2}$$
$$= \sqrt{\frac{1134}{40} - 4.95^2}$$
$$= 1.96 \text{ (to 3 s.f.)}$$

4.

Number of goals scored per match (x)	**Frequency**	**fx**	**fx^2**
0	10	0	0
1	8	8	8
2	7	14	28
3	6	18	54
4	2	8	32
5	3	15	75
6	1	6	36
Sum	$\Sigma f = 37$	$\Sigma fx = 69$	$\Sigma fx^2 = 233$

$$\text{Mean, } \overline{x} = \frac{\Sigma fx}{\Sigma f}$$
$$= \frac{69}{37}$$
$$= 1.86 \text{ (to 3 s.f.)}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \overline{x}^2}$$
$$= \sqrt{\frac{233}{37} - \left(\frac{69}{37}\right)^2}$$
$$= 1.68 \text{ (to 3 s.f.)}$$

5.

x	**Frequency**	**Mid-value (x)**	**fx**	**fx^2**
$0 < x \leqslant 5$	4	2.5	10	25
$5 < x \leqslant 10$	12	7.5	90	675
$10 < x \leqslant 15$	20	12.5	250	3125
$15 < x \leqslant 20$	24	17.5	420	7350
$20 < x \leqslant 25$	16	22.5	360	8100
$25 < x \leqslant 30$	4	27.5	110	3025
Sum	$\Sigma f = 80$		$\Sigma fx = 1240$	$\Sigma fx^2 = 22\,300$

$$\text{Mean, } \overline{x} = \frac{\Sigma fx}{\Sigma f}$$
$$= \frac{1240}{80}$$
$$= 15.5$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \overline{x}^2}$$
$$= \sqrt{\frac{22\,300}{80} - 15.5^2}$$
$$= 6.20 \text{ (to 3 s.f.)}$$

6.

Salary (PKR)	**Frequency**	**Mid-value (x)**	**fx**	**fx^2**
$200 < x \leqslant 220$	8	210	1680	352 800
$220 < x \leqslant 240$	23	230	5290	1 216 700
$240 < x \leqslant 260$	16	250	4000	1 000 000
$260 < x \leqslant 280$	3	270	810	218 700
$280 < x \leqslant 300$	10	290	2900	841 000
Sum	$\Sigma f = 60$		$\Sigma fx = 14\,680$	$\Sigma fx^2 = 3\,629\,200$

$$\text{Mean, } \overline{x} = \frac{\Sigma fx}{\Sigma f}$$
$$= \frac{14\,680}{60}$$
$$= 245 \text{ (to 3 s.f.)}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \overline{x}^2}$$
$$= \sqrt{\frac{3\,629\,200}{60} - \left(244\frac{2}{3}\right)^2}$$
$$= 25.0 \text{ (to 3 s.f.)}$$

7. **(a)** Standard deviation = 11.7 (to 3 s.f.)

(b) Standard deviation = 7.23 (to 3 s.f.)

8. **(i)** For Class *A*,

x	**x^2**
4	16
6	36
6	36
7	49
8	64
10	100
11	121
12	144
$\Sigma x = 64$	$\Sigma x^2 = 566$

$$\text{Mean, } \overline{x} = \frac{\Sigma x}{n}$$
$$= \frac{64}{8}$$
$$= 8$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2}$$

$$= \sqrt{\frac{566}{8} - 8^2}$$

$$= 2.60 \text{ (to 3 s.f.)}$$

For Class *B*,

x	x^2
0	0
1	1
1	1
2	4
3	9
14	196
17	289
25	625
$\Sigma x = 63$	$\Sigma x^2 = 1125$

$$\text{Mean, } \bar{x} = \frac{\Sigma x}{n}$$

$$= \frac{63}{8}$$

$$= 7.875$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2}$$

$$= \sqrt{\frac{1125}{8} - 7.875^2}$$

$$= 8.87 \text{ (to 3 s.f.)}$$

(ii) The mean scores of the students from Class *A* and Class *B* are approximately the same. This means that the students from each class in general did not perform better or worse than the students from the other class. However, the scores of Class *B* have a higher standard deviation than those of Class *A*, which indicates that there is a greater spread in the scores of Class *B*.

9. (i) Mean mark = 10

$$\frac{x + 5 + 16 + 6 + 10 + 4}{6} = 10$$

$$x + 41 = 60$$

$$x = 19$$

(ii)

x	x^2
4	16
5	25
6	36
10	100
16	256
19	361
$\Sigma x = 60$	$\Sigma x^2 = 794$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2}$$

$$= \sqrt{\frac{794}{6} - 10^2}$$

$$= 5.69 \text{ (to 3 s.f.)}$$

(iii) Seema had the highest score of 19 marks, and performed much better than her friends, given that her score was almost twice that of the mean mark.

10. (i)

x	x^2
23	529
15	225
8	64
13	169
28	784
6	36
15	225
$\Sigma x = 108$	$\Sigma x^2 = 2032$

$$\text{Mean, } \bar{x} = \frac{\Sigma x}{n}$$

$$= \frac{108}{7}$$

$$= 15.4 \text{ minutes}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2}$$

$$= \sqrt{\frac{2032}{7} - \left(\frac{108}{7}\right)^2}$$

$$= 7.23 \text{ minutes (to 3 s.f.)}$$

(ii)

x	x^2
20	400
12	144
5	25
10	100
25	625
3	9
12	144
$\Sigma x = 87$	$\Sigma x^2 = 1447$

$$\text{Mean, } \overline{x} = \frac{\Sigma x}{n}$$
$$= \frac{87}{7}$$
$$= 12.4 \text{ minutes (to 3 s.f.)}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma x^2}{n} - \overline{x}^2}$$
$$= \sqrt{\frac{1447}{7} - \left(\frac{87}{7}\right)^2}$$
$$= 7.23 \text{ minutes (to 3 s.f.)}$$

(iii) Since the mean time taken for Kiran to fall asleep in Nathia Gali is lower than that in Home, therefore the time taken for Kiran to fall asleep in Nathia Gali is shorter than the time taken for her to fall asleep when she is in Home.

Both standard deviations are approximately the same which indicates that the spread of the time taken for Kiran to fall asleep in Nathia Gali and Home is the same.

11. **(i)** For Train *A*,

Time (minutes, *x*)	**Frequency**	***fx***	***fx*²**
2	3	6	12
3	2	6	18
4	5	20	80
5	12	60	300
6	10	60	360
7	6	42	294
8	1	8	64
9	1	9	81
Sum	$\Sigma f = 40$	$\Sigma fx = 211$	$\Sigma fx^2 = 1209$

$$\text{Mean, } \overline{x} = \frac{\Sigma fx}{\Sigma f}$$
$$= \frac{211}{40}$$
$$= 5.28 \text{ (to 3 s.f.)}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \overline{x}^2}$$
$$= \sqrt{\frac{1209}{40} - 5.275^2}$$
$$= 1.55 \text{ (to 3 s.f.)}$$

For Train *B*,

Time (minutes, *x*)	**Frequency**	***fx***	***fx*²**
2	4	8	16
3	3	9	27
4	9	36	144
5	9	45	225
6	7	42	252
7	5	35	245
8	3	24	192
9	0	0	0
Sum	$\Sigma f = 40$	$\Sigma fx = 199$	$\Sigma fx^2 = 1101$

$$\text{Mean, } \overline{x} = \frac{\Sigma fx}{\Sigma f}$$
$$= \frac{199}{40}$$
$$= 4.98 \text{ (to 3 s.f.)}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \overline{x}^2}$$
$$= \sqrt{\frac{1101}{40} - 4.975^2}$$
$$= 1.67 \text{ (to 3 s.f.)}$$

(ii) Train *A* arrives late more consistently than Train *B* since its standard deviation of the time of arriving after the scheduled time is lower than that of Train *B*.

(iii) Train *B* is more punctual on the whole than Train *A* since its mean time of arriving after the scheduled time is shorter than that of Train *A*.

12. **(a)**

Time (minutes)	**Frequency**	**Mid-value (*x*)**	***fx***	***fx*²**
$20 < t \leqslant 22$	5	21	1680	352 800
$22 < t \leqslant 24$	11	23	5290	1 216 700
$24 < t \leqslant 26$	27	25	4000	1 000 000
$26 < t \leqslant 28$	13	27	810	218 700
$28 < t \leqslant 30$	4	29	2900	841 000
Sum	$\Sigma f = 60$		$\Sigma fx = 1500$	$\Sigma fx^2 = 37\ 740$

(i) Mean, $\bar{x} = \dfrac{\Sigma fx}{\Sigma f}$

$$= \frac{1500}{60}$$

$$= 25$$

(ii) Standard deviation $= \sqrt{\dfrac{\Sigma fx^2}{\Sigma f} - \bar{x}^2}$

$$= \sqrt{\frac{37\,400}{60} - 25^2}$$

$$= 2$$

(b) The patients in both hospitals have the same waiting time on the whole since their mean waiting time is the same. However, Hillview Hospital has a higher standard deviation, which indicates that there is a greater spread in the waiting time, i.e. some patients have a much longer waiting time than other patients.

13. (a) For City *A*,

Temperature (°C)	**Frequency**	**Mid-value (x)**	**fx**	**fx^2**
$35 \leqslant x < 40$	1	37.5	37.5	1406.25
$40 \leqslant x < 45$	4	42.5	170	7225
$45 \leqslant x < 50$	12	47.5	570	27 075
$50 \leqslant x < 55$	23	52.5	1207.5	63 393.75
$55 \leqslant x < 60$	7	57.5	402.5	23 143.75
$60 \leqslant x < 65$	3	62.5	187.5	11 718.75
Sum	$\Sigma f = 50$		$\Sigma fx = 2575$	$\Sigma fx^2 = 133\,962.5$

(i) Mean, $\bar{x} = \dfrac{\Sigma fx}{\Sigma f}$

$$= \frac{2575}{50}$$

$$= 51.5\ °\text{C}$$

(ii) Standard deviation $= \sqrt{\dfrac{\Sigma fx^2}{\Sigma f} - \bar{x}^2}$

$$= \sqrt{\frac{133\,962.5}{50} - 51.5^2}$$

$$= 5.20\ °\text{C (to 3 s.f.)}$$

For City *B*,

Temperature (°C)	**Frequency**	**Mid-value (x)**	**fx**	**fx^2**
$35 \leqslant x < 40$	2	37.5	37.5	1406.25
$40 \leqslant x < 45$	14	42.5	170	7225
$45 \leqslant x < 50$	16	47.5	570	27 075
$50 \leqslant x < 55$	10	52.5	1207.5	63 393.75
$55 \leqslant x < 60$	5	57.5	402.5	23 143.75
$60 \leqslant x < 65$	3	62.5	187.5	11 718.75
Sum	$\Sigma f = 50$		$\Sigma fx = 2430$	$\Sigma fx^2 = 120\,012.5$

(i) Mean, $\bar{x} = \dfrac{\Sigma fx}{\Sigma f}$

$$= \frac{2430}{50}$$

$$= 48.6°\text{C}$$

(ii) Standard deviation $= \sqrt{\dfrac{\Sigma fx^2}{\Sigma f} - \bar{x}^2}$

$$= \sqrt{\frac{120\,012.5}{50} - 48.6^2}$$

$$= 6.19°\text{C (to 3 s.f.)}$$

(b) City *A* is warmer on the whole than City *B* because its mean temperature is higher than that of City *B*.

(c) City *A*'s daily temperature is more consistent as its standard deviation is lower.

14. Mean = 9

$$\begin{aligned}
\frac{10 + 6 + 18 + x + 15 + y}{6} &= 9 \\
10 + 6 + 18 + x + 15 + y &= 54 \\
x + y &= 5 \\
y &= 5 - x \quad \text{— (1)}
\end{aligned}$$

$$10^2 + 6^2 + 18^2 + x^2 + 15^2 + y^2 = 685 + x^2 + y^2$$

Standard deviation = 6

$$\begin{aligned}
\sqrt{\frac{685 + x^2 + y^2}{6} - 9^2} &= 6 \\
\frac{685 + x^2 + y^2}{6} - 9^2 &= 36 \\
\frac{685 + x^2 + y^2}{6} &= 117 \\
685 + x^2 + y^2 &= 702 \\
x^2 + y^2 &= 17 \quad \text{— (2)}
\end{aligned}$$

Substitute (1) into (2):

$$\begin{aligned}
x^2 + (5 - x)^2 &= 17 \\
x^2 + 25 - 10x + x^2 &= 17 \\
2x^2 - 10x + 8 &= 0 \\
2(x^2 - 5x + 4) &= 0 \\
2(x - 1)(x - 4) &= 0 \\
x = 1 \text{ or } x &= 4
\end{aligned}$$

When $x = 1$, $y = 4$.
When $x = 4$, $y = 1$.
$\therefore x = 1, y = 4$ or $x = 4, y = 1$

15. **(i)** Sets A and C since the mean of each set is 5.

(ii) Set C since the numbers in the set are the closest to each other compared to the numbers in the other two sets.

16. **(i)** Yes, we can use $\frac{\overline{x}+\overline{y}}{2}$ to find the combined mean since the number of students from each school is the same.

$$\frac{\overline{x}+\overline{y}}{2} = \frac{1}{2}\left(\frac{\Sigma fx}{100}+\frac{\Sigma fy}{100}\right)$$

$$= \frac{\Sigma fx + \Sigma fy}{200}$$

$$= \frac{\Sigma(fx + fy)}{200}$$

$$= \overline{(x+y)}$$

which is the combined mean of x and y.

(ii) No, not possible since the sum of the standard deviations of the masses of both schools $\sqrt{\frac{\Sigma fx^2}{100}-\overline{x}^2} + \sqrt{\frac{\Sigma fy^2}{100}-\overline{y}^2}$

$\neq \sqrt{\frac{\Sigma fx^2+\Sigma fy^2}{200}-\overline{(x+y)}^2}$ which is the combined standard deviation.

(iii) For School A and School B combined,

Mass (x kg)	Frequency	fx	fx^2
40	7	280	11 200
45	26	1170	52 650
50	60	3000	150 000
55	34	1870	102 850
60	25	1500	90 000
65	33	2145	139 425
70	10	700	49 000
75	1	75	5625
80	4	320	25 600
Sum	$\Sigma f = 200$	$\Sigma fx = 11\,060$	$\Sigma fx^2 = 626\,350$

Combined mean mass

$$= \frac{11\,060}{200}$$

$= 55.3$ kg

Combined standard deviation

$$= \sqrt{\frac{626\,350}{200}-55.3^2}$$

$= 8.58$ kg (to 3 s.f.)

Review Exercise 13

1. **(a)** For Mishal's shots,

x	x^2
47	2209
16	256
32	1024
1	1
19	361
35	1225
$\Sigma x = 150$	$\Sigma x^2 = 5076$

(i) Mean distance, $\overline{x} = \frac{\Sigma x}{n}$

$$= \frac{150}{6}$$

$= 25$ mm

(ii) Standard deviation $= \sqrt{\frac{\Sigma x^2}{n}-\overline{x}^2}$

$$= \sqrt{\frac{5076}{6}-25^2}$$

$= 14.9$ mm (to 3 s.f.)

For Jamil's shots,

x	x^2
20	400
9	81
16	256
43	1849
13	169
4	16
$\Sigma x = 105$	$\Sigma x^2 = 2771$

(i) Mean distance, $\overline{x} = \frac{\Sigma x}{n}$

$$= \frac{105}{6}$$

$= 17.5$ mm

(ii) Standard deviation $= \sqrt{\frac{\Sigma x^2}{n}-\overline{x}^2}$

$$= \sqrt{\frac{2771}{6}-17.5^2}$$

$= 12.5$ mm (to 3 s.f.)

(b) Mishal's shots are less accurate than Jamil's shots since the mean distance of Mishal's shots from the centre of the target is higher. Also, Mishal's shots have a higher standard deviation which indicates the greater spread of the distance of each shot from the centre of the target.

2.

For Company A,

Lifespans (hours)	**Frequency**	**Mid-value (x)**	fx	fx^2
$600 \leqslant t < 700$	2	650	1300	845 000
$700 \leqslant t < 800$	9	750	6750	5 062 500
$800 \leqslant t < 900$	16	850	12 750	10 837 500
$900 \leqslant t < 1000$	21	950	19 950	18 952 500
$1000 \leqslant t < 1100$	29	1050	15750	16 537 500
$1100 \leqslant t < 1200$	18	1150	20 700	23 805 000
$1200 \leqslant t < 1300$	5	1250	6250	7 812 500
Sum	$\Sigma f = 100$		Σfx $= 99\,000$	Σfx^2 $= 100\,010\,000$

(i) Mean, $\overline{x} = \dfrac{\Sigma fx}{\Sigma f}$

$= \dfrac{99\,000}{100}$

$= 990$ hours

$\therefore p = 990$

Standard deviation $= \sqrt{\dfrac{\Sigma fx^2}{\Sigma f} - \overline{x}^2}$

$= \sqrt{\dfrac{100\,010\,000}{100} - 990^2}$

$= 141$ hours (to 3 s.f.)

$\therefore q = 150$

$r = 100 - 8 - 10 - 12 - 16 - 18 - 12$

$= 24$

For Company B,

Lifespans (hours)	**Frequency**	**Mid-value (x)**	fx	fx^2
$600 \leqslant t < 700$	8	650	5200	3 380 000
$700 \leqslant t < 800$	10	750	7500	5 625 000
$800 \leqslant t < 900$	12	850	10 200	8 670 000
$900 \leqslant t < 1000$	16	950	15 200	14 400 000
$1000 \leqslant t < 1100$	24	1050	25 200	26 460 000
$1100 \leqslant t < 1200$	18	1150	20 700	23 805 000
$1200 \leqslant t < 1300$	12	1250	15 000	18 750 000
Sum	$\Sigma f = 100$		Σfx $= 99\,000$	Σfx^2 $= 101\,130\,000$

Standard deviation $= \sqrt{\dfrac{\Sigma fx^2}{\Sigma f} - \overline{x}^2}$

$= \sqrt{\dfrac{101\,130\,000}{100} - 989.5^2}$

$= 179$ hours (to 3 s.f.)

$\therefore t = 179$

(ii) The light bulbs produced by both companies have the same amount of lifespans in general as the median lifespans are almost the same. However, the light bulbs produced by Company B have a higher standard deviation which indicates that there is a greater spread of the lifespans of their light bulbs. Hence Company B is less consistent than Company A in producing light bulbs with the same lifespan.

Challenge Yourself

1. Sets W and X since the spread between the data in each of these sets is the same as Set A.

2. $M = \{-2, -1, 0, 1, 2\}$

$N = \{-\sqrt{5}\ , 0, 0, 0, \sqrt{5}\ \}$

$n = 5$

Mean $= 0$

Standard deviation $= \sqrt{2}$

Chapter 14 Probability of Combined Events

TEACHING NOTES

Suggested Approach

As the pupils are already familiar with the concept of probability that they learnt in Grade 7, it would be easier to approach this topic. A quick revision is suggested with the summary list on the first page of this chapter.

Following the revision, teachers can get students to think about how games in sports such as tennis, football or hockey are started. Teachers can prompt students to notice that generally, a coin or something else with two sides is used, and a player from each team will choose either face, determining who has the first advantage based on the outcome of the toss. Why is the coin the norm in most cases? Why not use a die or any other objects?

Teachers can then get the whole class to throw a coin 20 times each and record the number of occurrences of heads and tails. Students can then tally the number of heads and tails to draw a distinction between the theoretical and actual probabilities occurring in an event. Teachers can urge students to think about whether the outcome of one toss affects the outcome of the next toss. Teachers may then discuss some cases where probability is useful in making real-life predictions, and demonstrate why learning about the probability of combined events, and not just single events, is important in the real-world context.

Section 14.1: Probability of Single Events

As the pupils are already familiar with set notation, teachers can introduce the concept of sample space and events using set notation.

Section 14.2: Simple Combined Events, Possibility Diagrams and Tree Diagrams

In this section, we introduce the possibility diagram and tree diagram when two events are taking place. Possibility diagrams and tree diagrams are very useful for solving problems involving two events. Teachers can go through the Worked Examples or work out the Practise Now questions on the board for the class and let students read the Worked Examples themselves.

Section 14.3: Addition Law of Probability and Mutually Exclusive Events

Go through the Investigation on Mutually Exclusive and Non-Mutually Exclusive Events on page 406 so that pupils can distinguish the difference between them. The main concept is that Mutually Exclusive Events cannot occur at the same time and P(A or B) or $P(A \cup B) = P(A) + P(B)$.

Section 14.4: Multiplication Law of Probability and Independent Events

Discuss the concept of independent events and dependent events using simple everyday life examples such as the following:

(i) Throwing a coin followed by tossing a die. Will the first event affect the result of the second event?

(ii) Tossing a white die followed by tossing a red die. Will the first event affect the result of the second event?

(iii) A bag has 5 red marbles and 7 blue marbles. All the marbles are identical except for their colour. A marble is selected, its colour is noted and it is put back into the bag. A second marble is then picked and its colour noted. Will the first event affect the result of the second event?

(iv) A bag has 8 red marbles and 9 blue marbles. All the marbles are identical except for their colour. A marble is selected, its colour is noted and it is put aside. A second marble is then picked and its colour is noted. Will the first event affect the result of the second event?

Teachers can work through the Investigation on Dependent Events on page 415 for a better understanding of dependent events. Teachers can also use some of the questions in Practise Now 9 and 10 to show how problems involving independent and dependent events can solved, and teachers can get students to work with tree diagrams as well as possibility diagrams.

If time permits, ask the pupils to work on the Performance Task on page 417 for enrichment.

Challenge Yourself

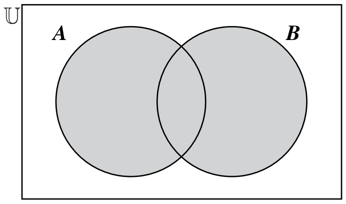
For question 2, teachers may advise the students to search on the Internet by keying in ‘Monte Hall problem’ to understand more on this question.

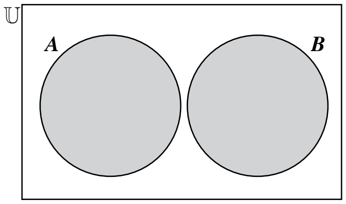
Question 3 involves the concept of the probability of the complement of any event, i.e. $P(E') = 1 - P(E)$, to find out the probability of at least two students having their birthday falling on the same day of the year.

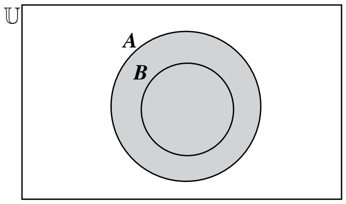
WORKED SOLUTIONS

Thinking Time (Page 393)

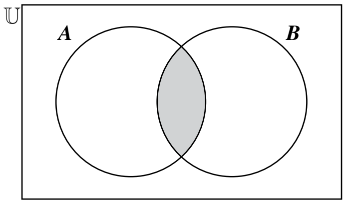
P($A \cup B$) refers to the probability of an event landing in *A* or in *B* or in both *A* and *B*.

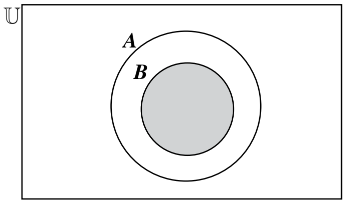






P($A \cap B$) refers to the probability of an event landing in *A* and *B*.





Investigation (Mutually Exclusive and Non-Mutually Exclusive Events)

1. The sample space is {1, 2, 3, 4, 5, 6, 7, 8}.

Part 1: Mutually Exclusive Events

2. $A = \{2, 3, 5, 7\}$ and $P(A) = \frac{4}{8} = \frac{1}{2}$

3. $B = \{4, 8\}$ and $P(B) = \frac{2}{8} = \frac{1}{4}$

4. No

5. $A \cup B = \{2, 3, 4, 5, 7, 8\}$ and $P(A \cup B) = \frac{6}{8} = \frac{3}{4}$

6. Yes, since the two events are mutually exclusive.

Part 2: Non-Mutually Exclusive Events

7. $C = \{1, 3, 5, 7\}$ and $P(C) = \frac{4}{8} = \frac{1}{2}$

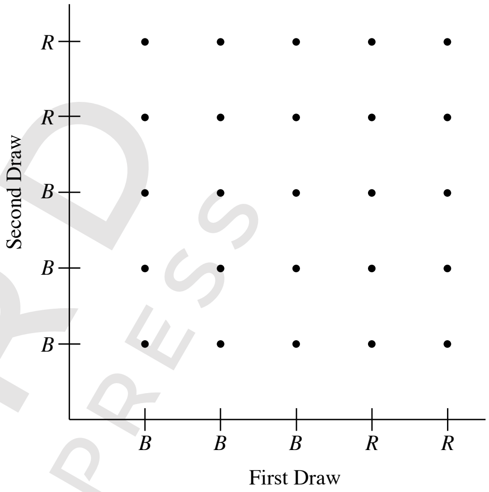
8. Yes

9. $A \cup C = \{1, 2, 3, 5, 7\}$ and $P(A \cup C) = \frac{5}{8}$

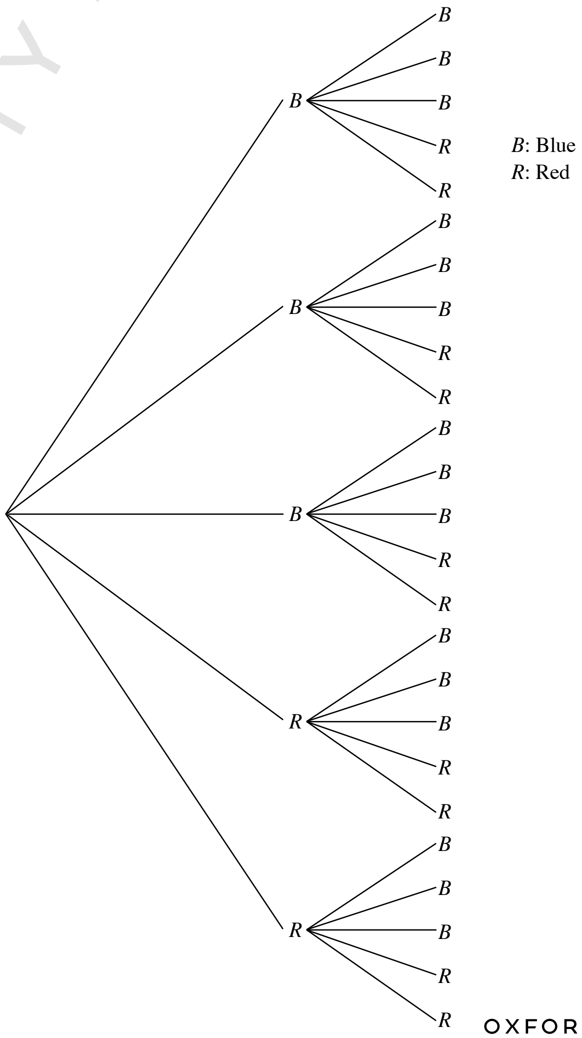
10. No, since the two events are not mutually exclusive.

Class Discussion (Choosing a Diagram to Represent the Sample Space)

1. **(a)**



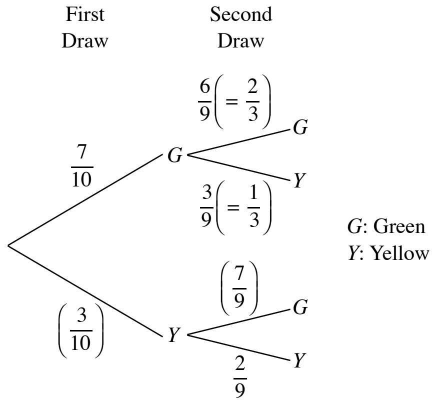
(b)



2. It is easier to represent the sample space on the possibility diagram than the tree diagram.

Investigation (Dependent Events)

1.



2. **(i)** Probability = $\frac{1}{3}$

(ii) Probability = $\frac{2}{9}$

3. No

Yes. If the first marble drawn is green, then there will still be 3 yellow marbles in the bag. However, if the first marble drawn is yellow, then there will only be 2 yellow marbles left.

4. P(*GY* or *YY*)

$= \left(\frac{7}{10}\times\frac{3}{9}\right) + \left(\frac{3}{10}\times\frac{2}{9}\right)$

$= \frac{3}{10}$

No

5. **(i)** Event *B* is dependent on event *A* since the probability of *B* happening depends on the outcome of *A*.

(ii) No

Performance Task (Page 417)

(a) Area of the unit circle

$= \pi(1)^2$

= 3.14 square units (to 3 s.f.)

(b) The number of points within the unit circle

(c) =D2/E2*4

(d) The mean area of the unit circle is 3.20 square units (to 3 s.f.).

Yes, it is close enough to the value of π.

Practise Now 1

(a) Let *S* represent the sample space.

S = {22, 23, 25, 32, 33, 35, 52, 53, 55}

(b) **(i)** Let *A* be the event that the two-digit number formed is prime.

A = {23, 53}

$P(A) = \frac{2}{9}$

(ii) Let *B* be the event that the two-digit number contains the digit '2'.

B = {22, 23, 25, 32, 52}

$P(B) = \frac{5}{9}$

(iii) Let *C* be the event that the two-digit number is divisible by 4.

C = {32, 52}

$P(C) = \frac{2}{9}$

(iv) Let *D* be the event that the two-digit number is divisible by 13.

D = {52}

$P(D) = \frac{1}{9}$

(v) Let *E* be the event that the two-digit number is not divisible by 13.

E = {22, 23, 25, 32, 33, 35, 53, 55}

$P(E) = \frac{8}{9}$

Alternatively,

$P(D') = 1 - P(D)$

$= 1 - \frac{1}{9}$

$= \frac{8}{9}$

Practise Now 2

Let *S* represent the sample space.

$S = \{C, L, E_1, V, E_2, R\}$

(i) Let *A* be the event that the letter chosen is an 'E'.

$A = \{E_1, E_2\}$

$P(A) = \frac{2}{6}$

$= \frac{1}{3}$

(ii) Let *B* be the event that the letter chosen is a 'C' or a 'R'.

B = {C, R}

$P(B) = \frac{2}{6}$

$= \frac{1}{3}$

(iii) Let *C* be the event that the letter chosen is a 'K'.

C = { }

$P(C) = \frac{0}{6}$

= 0

(iv) Let *D* be the event that the letter chosen is a consonant.

D = {C, L, V, R}

$P(D) = \frac{4}{6}$

$= \frac{2}{3}$

Practise Now 3

1. **(a)**

6-sided Die

Tetrahedral Die

	1	**2**	**3**	**4**	**5**	**6**
1	1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
2	2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
5	5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
6	6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

(b) **(i)** P(both dice show the same number) $= \frac{4}{24}$

$= \frac{1}{6}$

(ii) P(the number shown on the tetrahedral die is greater than the number shown on the 6-sided die)

$= \frac{10}{24}$

$= \frac{5}{12}$

(iii) P(the numbers shown on both dice are prime numbers)

$= \frac{6}{24}$

$= \frac{1}{4}$

2.

Second Card

First Card

	1	**2**	**3**	**4**	**5**
1	1, 1	1, 2	1, 3	1, 4	1, 5
2	2, 1	2, 2	2, 3	2, 4	2, 5
3	3, 1	3, 2	3, 3	3, 4	3, 5
4	4, 1	4, 2	4, 3	4, 4	4, 5
5	5, 1	5, 2	5, 3	5, 4	5, 5

(i) P(number shown on the second card is greater than the number shown on the first card)

$= \frac{10}{25}$

$= \frac{2}{5}$

(ii) P(sum of the two numbers shown is greater than 7)

$= \frac{6}{25}$

(iii) P(product of the two numbers shown is greater than 10)

$= \frac{8}{25}$

Practise Now 4

1. **(a)**

Tetrahedral Die

6-sided Die

+	**1**	**2**	**5**	**6**
1	2	3	6	7
2	3	4	7	8
3	4	5	8	9
4	5	6	9	10
5	6	7	10	11
6	7	8	11	12

Tetrahedral Die

6-sided Die

×	**1**	**2**	**5**	**6**
1	1	2	5	6
2	2	4	10	12
3	3	6	15	18
4	4	8	20	24
5	5	10	25	30
6	6	12	30	36

(b) **(i)** P(sum of the scores is even)

$= \frac{12}{24}$

$= \frac{1}{2}$

(ii) P(sum of the scores is divisible by 3)

$= \frac{8}{24}$

$= \frac{1}{3}$

(iii) P(sum of the scores is a perfect square)

$= \frac{4}{24}$

$= \frac{1}{6}$

(iv) P(sum of the scores is less than 2)

$= 0$

(c) **(i)** P(product of the scores is odd)

$= \frac{6}{24}$

$= \frac{1}{4}$

(ii) P(product of the scores is larger than 12)

$= \frac{8}{24}$

$= \frac{1}{3}$

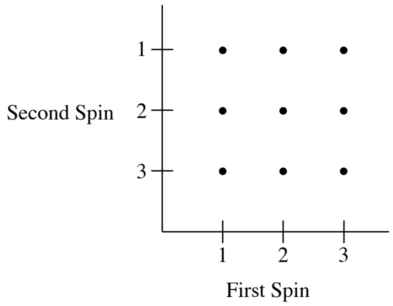
(iii) P(product of the scores is a prime number)

$= \frac{5}{24}$

(iv) P(product of the scores is less than 37)

$= 1$

2. **(a)**



(i) P(each score is a '1')

$= \frac{1}{9}$

(ii) P(at least one of the scores is a '3')

$= \frac{5}{9}$

(b) **(i)**

	1	**2**	**3**
1	1	2	3
2	2	2	3
3	3	3	3

(ii) P(final score is even)

$= \frac{3}{9}$

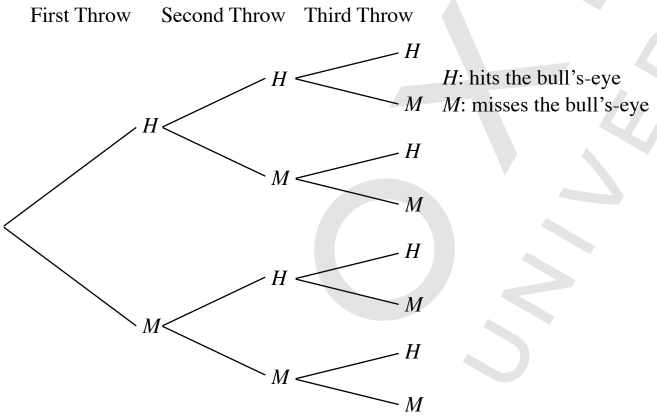
$= \frac{1}{3}$

(iii) P(final score is a prime number)

$= \frac{8}{9}$

Practise Now 5

1.



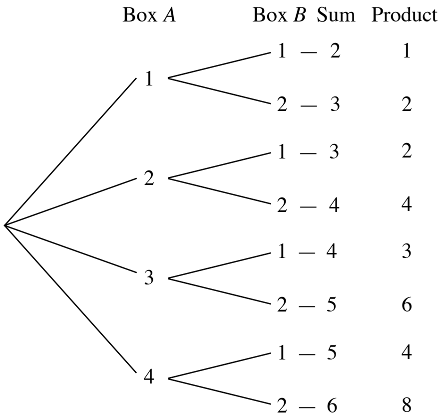
(i) P(misses the bull's-eye once)

$= \frac{3}{8}$

(ii) P(hits the bull's-eye at least once)

$= \frac{7}{8}$

2. **(a)**



(b) **(i)** P(at least one '1' is obtained)

$= \frac{5}{8}$

(ii) P(the sum of the two numbers is 3)

$= \frac{2}{8}$

$= \frac{1}{4}$

(iii) P(the product of two numbers is at least 4)

$= \frac{4}{8}$

$= \frac{1}{2}$

(iv) P(the sum is equal to the product)

$= \frac{1}{8}$

Practise Now 6

1. **(i)** P(picture card or Ace)

= P(picture card) + P(Ace)

$= \frac{12}{52} + \frac{4}{52}$

$= \frac{16}{52}$

$= \frac{4}{13}$

(ii) P(Ace or card bearing a number which is divisible by 3)

= P(Ace) + P(card bearing a number which is divisible by 3)

$= \frac{4}{52} + \frac{12}{52}$

$= \frac{4}{52} + \frac{12}{52}$

$= \frac{16}{52}$

$= \frac{4}{13}$

(iii) P(King or Queen)

= P(King) + P(Queen)

$= \frac{4}{52} + \frac{4}{52}$

$= \frac{8}{52}$

$= \frac{2}{13}$

(iv) P(Jack or Ace)

= P(Jack) + P(Ace)

$= \frac{4}{52} + \frac{4}{52}$

$= \frac{8}{52}$

$= \frac{2}{13}$

P(neither Jack or Ace)

$= 1 - \frac{2}{13}$

$= \frac{11}{13}$

Practise Now 7

1. **(i)** P(P or Q wins)

= P(P wins) + P(Q wins)

$= \frac{1}{5} + \frac{1}{6}$

$= \frac{11}{30}$

(ii) P(Q or R or S wins)

= P(Q wins) + P(P wins) + P(S wins)

$= \frac{1}{6} + \frac{1}{7} + \frac{1}{8}$

$= \frac{73}{168}$

(iii) P(P or Q or R or S wins)

= P(P wins) + P(Q wins) + P(P wins) + P(S wins)

$= \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8}$

$= \frac{533}{840}$

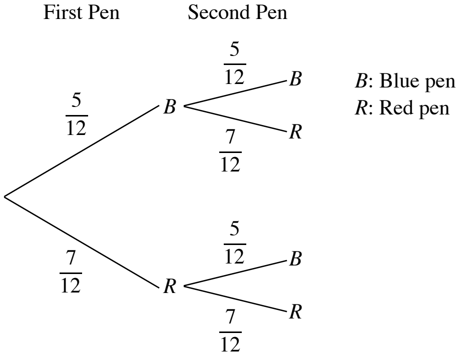
P(none wins)

$= 1 - \frac{533}{840}$

$= \frac{307}{840}$

Practise Now 8

1.



(i) P(first pen selected is R)

$= \frac{7}{12}$

(ii) P(second pen selected is B, given that the first pen selected is B)

$= \frac{5}{12}$

(iii) P(first pen selected is B and the second pen selected is B)

$= \frac{5}{12} \times \frac{5}{12}$

$= \frac{25}{144}$

(iv) P(second pen selected is B)

= P(B, B) + P(R, B)

$= \frac{5}{12} \times \frac{5}{12} + \frac{7}{12} \times \frac{5}{12}$

$= \frac{5}{12}$

(v) P(no blue pen was selected)

$= \frac{7}{12} \times \frac{7}{12}$

$= \frac{49}{144}$

Practise Now 9

1. In the ‘Administrative’ Department, there are 6 men and 8 women and in the Technical Department, there are 12 men and 4 women.

(i) P(both the chairman and chairwoman are from the ‘Technical’ Department)

$= \frac{12}{18} \times \frac{4}{12}$

$= \frac{2}{9}$

(ii) P(the chairman is from the ‘Administrative’ Department and the chairwoman is from the ‘Technical’ Department)

$= \frac{6}{18} \times \frac{4}{12}$

$= \frac{1}{9}$

2. **(i)** P(both laptops break down)

$= 0.1 \times 0.35$

$= 0.035$

(ii) P(Laptop X breaks down but Laptop Y does not)

$= 0.1 \times (1 - 0.35)$

$= 0.065$

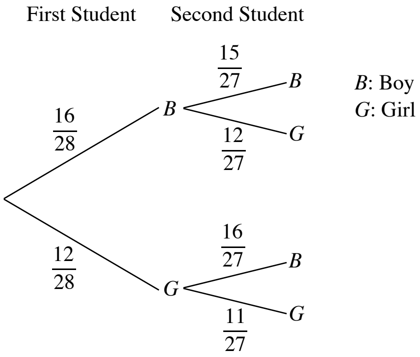
(iii) P(exactly one of the laptops breaks down)

$= [0.1 \times (1 - 0.35)] + [(1 - 0.1) \times 0.35]$

$= 0.38$

Practise Now 10

1.



(i) P(first student is B and the second student is G)

$= \text{P}(B, G)$

$= \frac{16}{28} \times \frac{12}{27}$

$= \frac{16}{63}$

(ii) P(one student is B while the other student is G)

$= \text{P}(B, G) + \text{P}(G, B)$

$= \frac{16}{28} \times \frac{12}{27} + \frac{12}{28} \times \frac{16}{27}$

$= \frac{32}{63}$

(iii) P(at least one of the students is G)

$= \text{P}(B, G) + \text{P}(G, B) + \text{P}(G, G)$

$= \frac{16}{28} \times \frac{12}{27} + \frac{12}{28} \times \frac{16}{27} + \frac{12}{28} \times \frac{11}{27}$

$= \frac{43}{63}$

Alternatively,

P(at least one of the students is G)

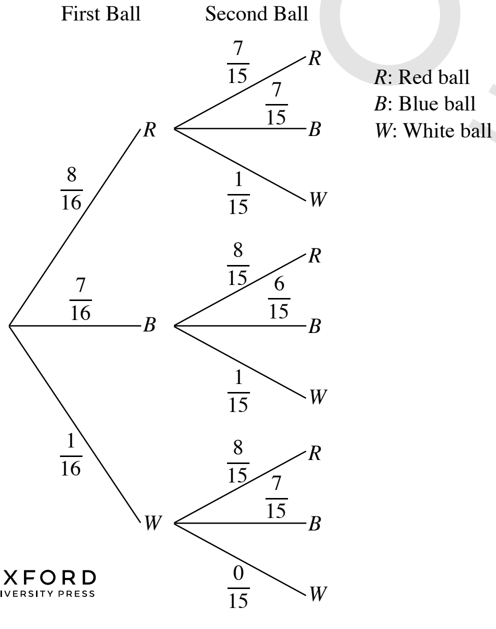
$= 1 - \text{P(both students are } B)$

$= 1 - (B, B)$

$= 1 - \frac{16}{28} \times \frac{15}{27}$

$= \frac{43}{63}$

2.



(i) P(first ball is R and the second ball is B)

$= \text{P}(R, B)$

$= \frac{8}{16} \times \frac{7}{15}$

$= \frac{7}{30}$

(ii) P(one ball is R while the other ball is B)

$= \text{P}(R, B) + \text{P}(B, R)$

$= \frac{8}{16} \times \frac{7}{15} + \frac{7}{16} \times \frac{8}{15}$

$= \frac{7}{15}$

(iii) P(two balls are of the same colour)

$= \text{P}(R, R) + \text{P}(B, B) + \text{P}(W, W)$

$= \frac{8}{16} \times \frac{7}{15} + \frac{7}{16} \times \frac{6}{15} + \frac{1}{16} \times \frac{0}{15}$

$= \frac{49}{120}$

Exercise 14A

1. Let S represent the sample space.

$S = \{(H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6), (T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6)\}$

2. Let S represent the sample space.

Let F be the event that the pen drawn is faulty.

Let N be the event that the pen drawn is not faulty.

$S = \{F_1, F_2, F_3, N_1, N_2, N_3, N_4\}$

Probability that the pen drawn is not faulty $= \frac{4}{7}$

After drawing the first pen, $S = \{F_1, F_2, F_3, N_1, N_2, N_3\}$

Probability that the second pen drawn is faulty $= \frac{3}{6}$

$= \frac{1}{2}$

3. Let S represent the sample space.

$S = \{\text{B, I}_1, \text{I}_2, \text{I}_3, \text{L, O, P, S}_1, \text{S}_2, \text{T, Y}\}$

(i) P(the letter on the chosen card is a ‘S’)

$= \frac{2}{11}$

(ii) P(the letter on the chosen card is a ‘P’ or an ‘I’)

$= \frac{4}{11}$

(iii) P(the letter on the chosen card is a vowel)

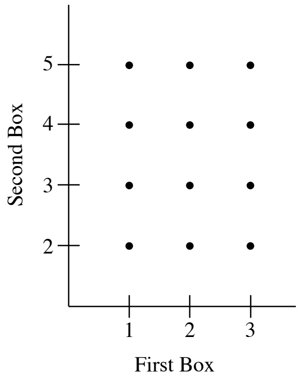
$= \frac{4}{11}$

(iv) P(the letter on the chosen card is a consonant)

$= 1 - \frac{4}{11}$

$= \frac{7}{11}$

4. **(a)**



(b) **(i)** P(cards bear the same number)

$= \frac{2}{12}$

$= \frac{1}{6}$

(ii) P(numbers on the cards are different)

$= \frac{10}{12}$

$= \frac{5}{6}$

Alternatively,

P(numbers on the cards are different)

= 1 – P(numbers on the cards are the same)

$= 1 - \frac{1}{6}$

$= \frac{5}{6}$

(iii) P(larger of the two numbers on the card is 3)

$= \frac{3}{12}$

$= \frac{1}{4}$

5. **(a)**

First Number

Second Number

+	**0**	**1**	**2**	**3**	**4**	**5**
0	0	1	2	3	4	5
1	1	2	3	4	5	6
2	2	3	4	5	6	7
3	3	4	5	6	7	8
4	4	5	6	7	8	9
5	5	6	7	8	9	10

(b) 36 possible outcomes

(c) **(i)** P(sum of the two numbers is 7)

$= \frac{4}{36}$

$= \frac{1}{9}$

(ii) P(sum of the two numbers is a prime number)

$= \frac{17}{36}$

(iii) P(sum of the two numbers is not a prime number)

$= 1 - \frac{17}{36}$

$= \frac{19}{36}$

(iv) P(sum of the two numbers is even)

$= \frac{18}{36}$

$= \frac{1}{2}$

(v) P(sum of the two numbers is not even)

$= 1 - \frac{1}{2}$

$= \frac{1}{2}$

(d) The sum of 7 is more likely to occur.

6. **(a)**

x

y

+	**4**	**5**	**6**
7	11	12	13
8	12	13	14
9	13	14	15

x

y

×	**4**	**5**	**6**
7	28	35	42
8	32	40	48
9	36	45	54

(b) **(i)** P(sum $x + y$ is prime)

$= \frac{4}{9}$

(ii) P(sum $x + y$ is greater than 12)

$= \frac{6}{9}$

$= \frac{2}{3}$

(iii) P(sum $x + y$ is at most 14)

$= \frac{8}{9}$

(c) **(i)** P(product xy is odd)

$= \frac{2}{9}$

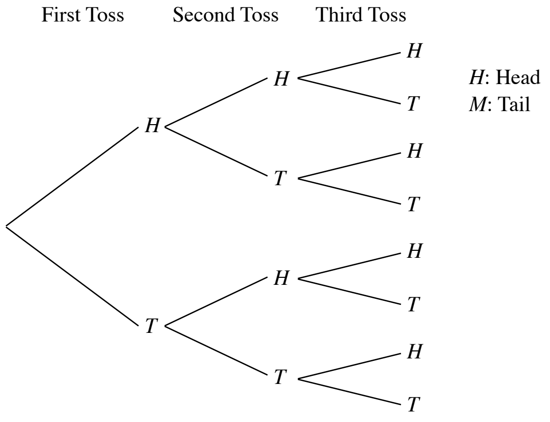
(ii) P(product xy is even)

$= \frac{7}{9}$

(iii) P(product xy is at most 40)

$= \frac{5}{9}$

7.



(i) P(three heads)

$= \frac{1}{8}$

(ii) P(exactly two heads)

$= \frac{3}{8}$

(iii) P(at least two heads)

$= \frac{4}{8}$

$= \frac{1}{2}$

Alternatively,

P(at least two heads)

= P(exactly two heads) + P(three heads)

$= \frac{3}{8} + \frac{1}{8}$

$= \frac{4}{8}$

$= \frac{1}{2}$

8. Let S represent the sample space.

$S = \{RB, BB, WB, RR, BR, WR\}$

(i) P(marbles selected are of the same colour)

$= \frac{2}{6}$

$= \frac{1}{3}$

(ii) P(marbles selected are blue and red)

$= \frac{2}{6}$

$= \frac{1}{3}$

(iii) P(marbles selected are of different colours)

$= 1 - \frac{1}{3}$

$= \frac{2}{3}$

9. **(a)** Let S represent the sample space.

$S = \{11, 12, 13, 21, 22, 23, 31, 32, 33\}$

(b) **(i)** P(number formed is divisible by 3)

$= \frac{3}{9}$

$= \frac{1}{3}$

(ii) P(number formed is a perfect square)

= 0

(iii) P(number formed is a prime number)

$= \frac{4}{9}$

(iv) P(number formed is a composite number)

$= \frac{5}{9}$

10. Let S represent the sample space.

$S = \{BBB, BGB, BBG, GBB, BGG, GBG, GGB, GGG\}$

(i) P(three grandsons)

$= \frac{1}{8}$

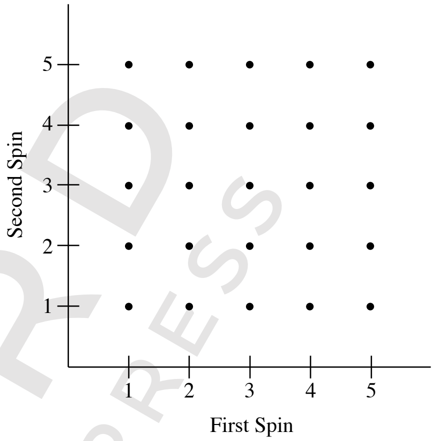
(ii) P(two grandsons and one granddaughter)

$= \frac{3}{8}$

(iii) P(one grandson and two granddaughters)

$= \frac{3}{8}$

11.



(a) **(i)** P(numbers on the spinners whose sum is 6)

$= \frac{5}{25}$

$= \frac{1}{5}$

(ii) P(the same numbers on both spinners)

$= \frac{5}{25}$

$= \frac{1}{5}$

(iii) P(different numbers on both spinners)

$= 1 - \frac{1}{5}$

$= \frac{4}{5}$

(iv) P(two different prime numbers)

$= \frac{6}{25}$

(b) P(first number less than second number)

$= \frac{10}{25}$

$= \frac{2}{5}$

12. **(a)**

Die

		1	**2**	**3**	**4**	**5**	**6**
Coin	***H***	1	2	3	4	5	6
	T	2	4	6	8	10	12

(b) **(i)** P(player's score is odd)

$= \frac{3}{12}$

$= \frac{1}{4}$

(ii) P(player's score is even)

$= 1 - \frac{1}{4}$

$= \frac{3}{4}$

(iii) P(player's score is a prime number)

$= \frac{3}{12}$

$= \frac{1}{4}$

(iv) P(player's score is less than or equal to 8)

$= \frac{10}{12}$

$= \frac{5}{6}$

(v) P(player's score is a multiple of 3)

$= \frac{4}{12}$

$= \frac{1}{3}$

13. **(a)**

First Die

Second Die

–	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	0	1	2	3	4
2	2	1	0	1	2	3
3	3	2	1	0	1	2
4	4	3	2	1	0	1
5	5	4	3	2	1	0

(b) **(i)** P(difference of the two numbers is 1)

$= \frac{10}{36}$

$= \frac{5}{18}$

(ii) P(difference of the two numbers is non-zero)

$= \frac{30}{36}$

$= \frac{5}{6}$

(iii) P(difference of the two numbers is odd)

$= \frac{18}{36}$

$= \frac{1}{2}$

(iv) P(difference of the two numbers is a prime number)

$= \frac{16}{36}$

$= \frac{4}{9}$

(v) P(difference of the two numbers is more than 2)

$= \frac{12}{36}$

$= \frac{1}{3}$

14.

Second Ball

7	1, 7	2, 7	4, 7	5, 7	
5	1, 5	2, 5	4, 5		7, 5
4	1, 4	2, 4		5, 4	7, 4
2	1, 2		4, 2	5, 3	7, 2
1		2, 1	4, 1	5, 1	7, 1
	1	**2**	**4**	**5**	**7**

First Ball

(i) P(numbers obtained on both balls are prime)

$= \frac{6}{20}$

$= \frac{3}{10}$

(ii) P(sum of the numbers obtained is odd)

$= \frac{11}{20}$

(iii) P(product of the numbers obtained is greater than 20)

$= \frac{4}{20}$

$= \frac{1}{5}$

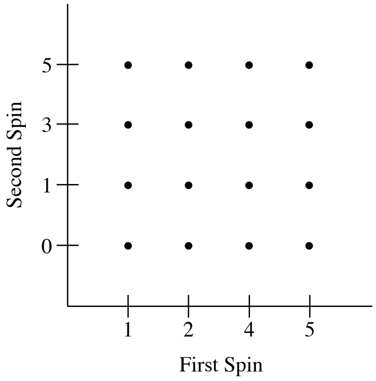
(iv) P(difference in the numbers obtained is less than 7)

$= 1$

(v) P(product of the numbers obtained is divisible by 9)

$= 0$

15.



(i) P(scores on both cards are the same)

$= \frac{2}{16}$

$= \frac{1}{8}$

(ii) P(scores on both cards are prime)

$= \frac{4}{16}$

$= \frac{1}{4}$

(iii) P(sum of the scores is odd)

$= \frac{8}{16}$

$= \frac{1}{2}$

(iv) P(sum of the scores is divisible by 5)

$= \frac{4}{16}$

$= \frac{1}{4}$

(v) P(sum of the scores is 6 or less)

$= \frac{11}{16}$

(vi) P(product of the scores is not 0)

$= \frac{12}{16}$

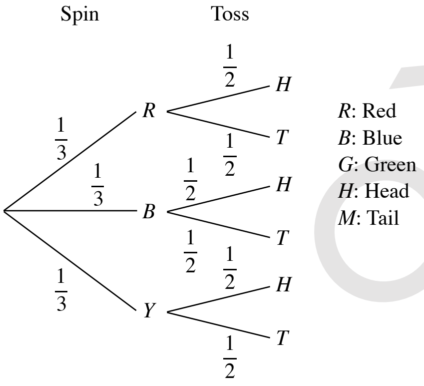
$= \frac{3}{4}$

(vii) P(product of the scores is greater than 11)

$= \frac{4}{16}$

$= \frac{1}{4}$

16.



(i) P(red on the spinner and tail on the coin)

$= \frac{1}{3} \times \frac{1}{2}$

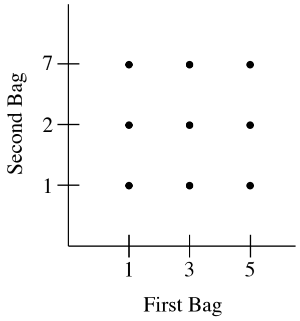
$= \frac{1}{6}$

(ii) P(blue or yellow on the spinner and head on the coin)

$= \left(\frac{1}{3} \times \frac{1}{2}\right) + \left(\frac{1}{3} \times \frac{1}{2}\right)$

$= \frac{1}{3}$

17.



(i) P(two numbers obtained are both odd)

$= \frac{6}{9}$

$= \frac{2}{3}$

(ii) P(two numbers obtained are prime)

$= \frac{4}{9}$

(iii) P(sum of the numbers greater than 4)

$= \frac{6}{9}$

$= \frac{2}{3}$

(iv) P(sum of the numbers is even)

$= \frac{6}{9}$

$= \frac{2}{3}$

(v) P(product is prime)

$= \frac{4}{9}$

(vi) P(product is greater than 20)

$= \frac{2}{9}$

(vii) P(product is divisible by 7)

$= \frac{3}{9}$

$= \frac{1}{3}$

18. (a) (i) P(land with the face printed '4' down)

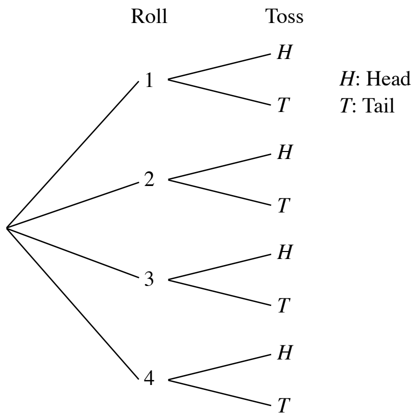
$= \frac{1}{4}$

(ii) P(land such that the sum of the three upper faces is odd)

$= \frac{2}{4}$

$= \frac{1}{2}$

(b)



19. P(first component tested is defective) = $\frac{1}{7}$

20.

Kiran's Room

Nora's Room

	1A	1B	1C	2A	2B	2C
1A		1	2	3	4	5
1B	1		3	4	5	6
1C	2	3		5	6	7
2A	3	4	5		7	8
2B	4	5	6	7		9
2C	5	6	7	8	9	

(a) **(i)** P(stay next to each other)

$= \frac{8}{30}$

$= \frac{4}{15}$

(ii) P(stay on different storeys)

$= \frac{18}{30}$

$= \frac{3}{5}$

(iii) P(do not stay next to each other)

$= 1 - \frac{4}{15}$

$= \frac{11}{15}$

(b) P(Kiran stays on the second floor and next to Nora)

$= \frac{4}{15}$

21. There are 4 × 4 × 6 = 96 outcomes.

Let the event of the score on the 6-sided die greater than the sum of the scores of the two tetrahedral dice be A.

20 outcomes for event A:

{611, 612, 613, 614, 621, 622, 623, 631, 632, 641, 511, 512, 513, 521, 522, 531, 411, 412, 421, 311}

$\therefore$ P(A) $= \frac{20}{96}$

$= \frac{5}{24}$

Exercise 14B

1. Let S represent the sample space.

$S = \{11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21\}$

(i) P(number is even)

$= \frac{5}{11}$

(ii) P(number is prime)

$= \frac{4}{11}$

(iii) P(number is either even or prime)

= P(number is even) + P(number is prime)

$= \frac{5}{11} + \frac{4}{11}$

$= \frac{9}{11}$

(iv) P(number is divisible by 3)

$= \frac{4}{11}$

(v) P(number is neither even nor prime)

= 1 − P(number is either even or prime)

$= 1 - \frac{9}{11}$

$= \frac{2}{11}$

2. Let S represent the sample space.

$S = \{R_1, R_2, R_3, R_4, R_5, R_6, R_7, G_1, G_2, G_3, G_4, G_5, B_1, B_2, B_3\}$

(i) P(red marble)

$= \frac{7}{15}$

(ii) P(green marble)

$= \frac{5}{15}$

$= \frac{1}{3}$

(iii) P(either red or green marble)

= P(red marble) + P(green marble)

$= \frac{7}{15} + \frac{5}{15}$

$= \frac{12}{15}$

$= \frac{4}{5}$

(iv) P(neither red nor green marble)

= 1 − P(either red or green marble)

$= 1 - \frac{4}{5}$

$= \frac{1}{5}$

3. Let S represent the sample space.

$S = \{A, C, E_1, E_2, I, L_1, L_2, L_3\ M, S, T, U_1, U_2, U_3, V, X, Y\}$

(i) P(letter ‘U’)

$= \frac{3}{17}$

(ii) P(letter ‘E’)

$= \frac{2}{17}$

(iii) P(letter ‘U’ or ‘E’)

= P(letter ‘U’) + P(letter ‘E’)

$= \frac{3}{17} + \frac{2}{17}$

$= \frac{5}{17}$

(iv) P(consonant)

$= \frac{10}{17}$

(v) P(letter ‘U’ or consonant)

= P(letter ‘U’) + P(consonant)

$= \frac{3}{17} + \frac{10}{17}$

$= \frac{13}{17}$

(vi) P(letter ‘U’ or ‘E’ or ‘L’)

= P(letter ‘U’) + P(letter ‘E’) + P(letter ‘L’)

$= \frac{13}{17} + \frac{2}{17} + \frac{13}{17}$

$= \frac{8}{17}$

4. **(i)** P(team wins or loses a particular match)

= P(team wins) + P(team loses)

$= \frac{7}{10} + \frac{2}{15}$

$= \frac{5}{6}$

(ii) P(team neither wins nor loses a particular match)

= P(match ends in a draw)

$= 1 - \frac{5}{6}$

$= \frac{1}{6}$

5. **(i)** P(King or Jack)

= P(King) + P(Jack)

$= \frac{4}{52} + \frac{4}{52}$

$= \frac{8}{52}$

$= \frac{2}{13}$

(ii) P(Queen or card bearing a prime number)

= P(Queen) + P(card bearing a prime number)

$= \frac{4}{52} + \frac{16}{52}$

$= \frac{20}{52}$

$= \frac{5}{13}$

(iii) P(card bearing a number that is divisible by 3 or 5)

$= \frac{20}{52}$

$= \frac{5}{13}$

(iv) P(neither King nor Jack)

= 1 – P(King or Jack)

$= 1 - \frac{2}{13}$

$= \frac{11}{13}$

6. **(i)** P(4 or 5 strokes)

= P(4 strokes) + P(5 strokes)

$= \frac{1}{14} + \frac{2}{7}$

$= \frac{5}{14}$

(ii) P(4, 5 or 6 strokes)

= P(4 strokes) + P(5 strokes) + P(6 strokes)

$= \frac{1}{14} + \frac{2}{7} + \frac{3}{7}$

$= \frac{11}{14}$

(iii) P(more than 6 strokes)

$= 1 - \frac{11}{14}$

$= \frac{3}{14}$

7. **(i)** P(Alpha or Gamma wins)

= P(Alpha wins) + P(Gamma wins)

$= \frac{4}{15} + \frac{1}{5}$

$= \frac{7}{15}$

(ii) P(Alpha, Beta or Gamma wins)

= P(Alpha wins) + P(Beta wins) + P(Gamma wins)

$= \frac{4}{15} + \frac{1}{10} + \frac{1}{5}$

$= \frac{17}{30}$

(iii) P(neither Alpha nor Gamma wins)

= 1 – P(Alpha or Gamma wins)

$= 1 - \frac{7}{15}$

$= \frac{8}{15}$

(iv) P(none wins)

$= 1 - \text{P(Alpha, Beta or Gamma wins)}$

$= 1 - \frac{17}{30}$

$= \frac{13}{30}$

8. **(i)** P(one of them wins the award)

$= \text{P(Seema wins or Rizwan wins or Amirah wins)}$

$= \text{P(Seema wins)} + \text{P(Rizwan wins)} + \text{P(Amirah wins)}$

$= \frac{1}{3} + \frac{1}{8} + \frac{1}{20}$

$= \frac{61}{120}$

(ii) P(none of them wins the award)

$= 1 - \text{P(one of them wins the award)}$

$= 1 - \frac{61}{120}$

$= \frac{59}{120}$

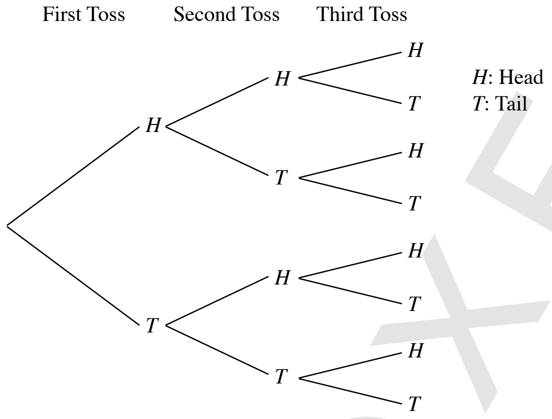
(iii) P(Seema and Rizwan will not win the award)

$= 1 - \text{P(Seema wins)} - \text{P(Rizwan wins)}$

$= 1 - \frac{1}{3} - \frac{1}{8}$

$= \frac{13}{24}$

9. **(a)**



(b) **(i)** Mutually exclusive

(ii) Not mutually exclusive

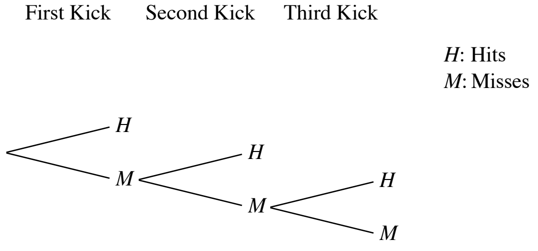
(iii) Not mutually exclusive

(iv) Mutually exclusive

(v) Not mutually exclusive

(vi) Not mutually exclusive

10. **(a)**

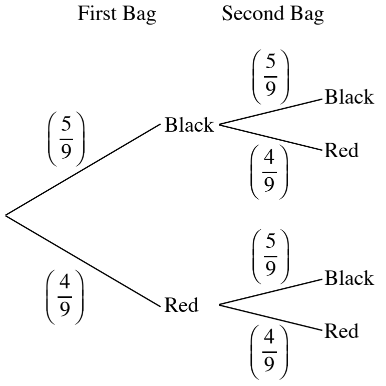


4 outcomes

(b) Not mutually exclusive as Event *A* and *B* can happen at the same time.

Exercise 14C

1. **(a)**



(b) **(i)** P(black marble from the first bag)

$= \frac{5}{9}$

(ii) P(red marble from the second bag, given that a black marble is drawn from the first bag)

$= \frac{4}{9}$

(iii) P(black marble from the first bag, red marble from the second bag)

$= \frac{5}{9} \times \frac{4}{9}$

$= \frac{20}{81}$

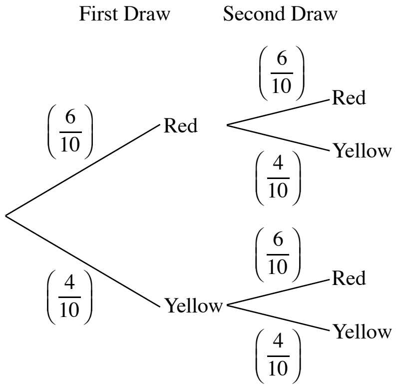
(iv) P(red marble from the second bag)

$= \text{P(red, red)} + \text{P(black, red)}$

$= \left(\frac{4}{9} \times \frac{4}{9}\right) + \left(\frac{5}{9} \times \frac{4}{9}\right)$

$= \frac{4}{9}$

2. **(a)**



(b) **(i)** P(two red balls)

$= \frac{6}{10} \times \frac{6}{10}$

$= \frac{9}{25}$

(ii) P(one ball of each colour)

= P(red, yellow) + P(yellow, red)

$= \left(\frac{6}{10} \times \frac{4}{10}\right) + \left(\frac{4}{10} \times \frac{6}{10}\right)$

$= \frac{12}{25}$

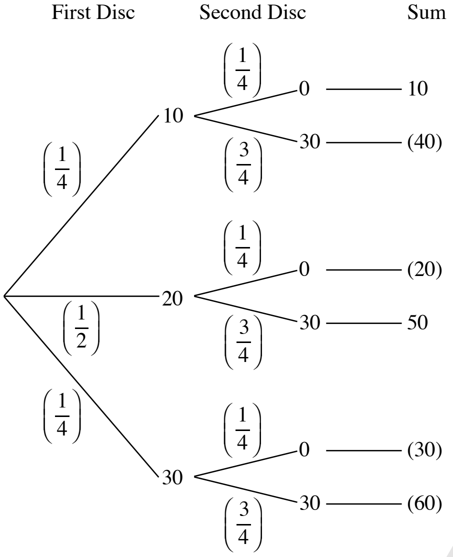
(iii) P(yellow ball on the second draw)

= P(red, yellow) + P(yellow, yellow)

$= \left(\frac{6}{10} \times \frac{4}{10}\right) + \left(\frac{4}{10} \times \frac{4}{10}\right)$

$= \frac{2}{5}$

3. **(a)**



(b) **(i)** P(first number ⩽ second number)

$= \left(\frac{1}{4} \times \frac{3}{4}\right) + \left(\frac{1}{2} \times \frac{3}{4}\right) + \left(\frac{1}{4} \times \frac{3}{4}\right)$

$= \frac{3}{4}$

(ii) P(second number is zero)

$= \left(\frac{1}{4} \times \frac{1}{4}\right) + \left(\frac{1}{2} \times \frac{1}{4}\right) + \left(\frac{1}{4} \times \frac{1}{4}\right)$

$= \frac{1}{4}$

(c) **(i)** P(gets PKR 2)

$= \left(\frac{1}{4} \times \frac{3}{4}\right) + \left(\frac{1}{2} \times \frac{1}{4}\right) + \left(\frac{1}{4} \times \frac{1}{4}\right)$

$= \frac{3}{8}$

(ii) P(gets PKR 5)

$= \left(\frac{1}{2} \times \frac{3}{4}\right) + \left(\frac{1}{4} \times \frac{3}{4}\right)$

$= \frac{9}{16}$

(iii) P(gets PKR 2 or PKR 5)

= P(gets PKR 2) + P(gets PKR 5)

$= \frac{3}{8} + \frac{9}{16}$

$= \frac{15}{16}$

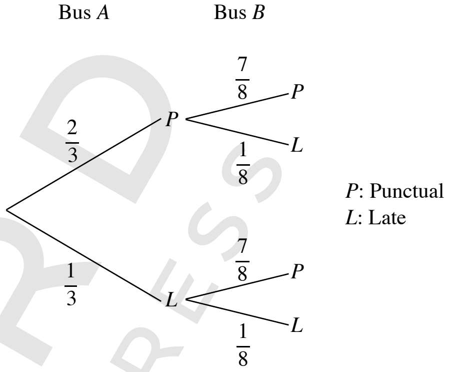
(iv) P(gets nothing)

= 1 – P(gets PKR 2 or PKR 5)

$= 1 - \frac{15}{16}$

$= \frac{1}{16}$

4.



(i) P(both buses are punctual)

$= \frac{2}{3} \times \frac{7}{8}$

$= \frac{7}{12}$

(ii) P(Bus *A* is late, Bus *B* is punctual)

$= \frac{1}{3} \times \frac{7}{8}$

$= \frac{7}{24}$

(iii) P(exactly one bus is late)

$= \left(\frac{1}{3} \times \frac{7}{8}\right) + \left(\frac{2}{3} \times \frac{1}{8}\right)$

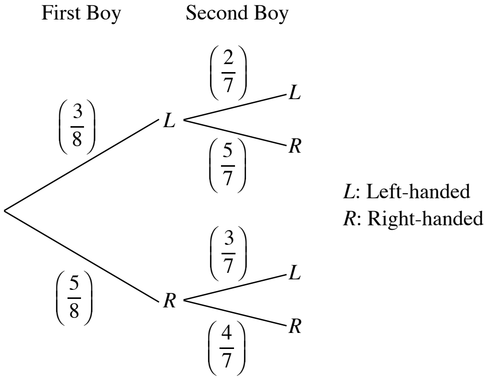
$= \frac{3}{8}$

5. P(boy is left-handed) $= \frac{3}{8}$

(a) P(second boy is left-handed, given that the first boy is left-handed)

$= \frac{2}{7}$

(b)



(c) **(i)** P(first boy is right-handed, second boy is left-handed)

$= P(R, L)$

$= \frac{5}{8} \times \frac{3}{7}$

$= \frac{15}{56}$

(ii) P(both boys are left-handed)

$= P(L, L)$

$= \frac{3}{8} \times \frac{2}{7}$

$= \frac{3}{28}$

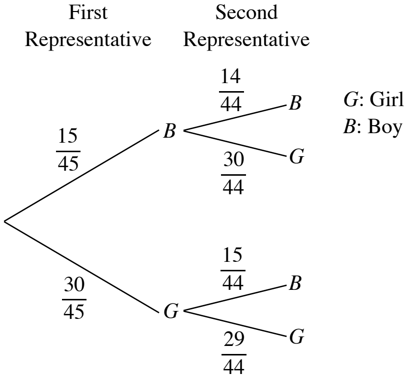
(iii) P(second boy chosen is left-handed)

$= P(R, L) + P(L, L)$

$= \frac{15}{56} + \frac{3}{28}$

$= \frac{3}{8}$

6.



(i) P(first representative is a girl)

$= \frac{30}{45}$

$= \frac{2}{3}$

(ii) P(second representative is a girl, given that the first representative is a boy)

$= \frac{30}{44}$

$= \frac{15}{22}$

(iii) P(first representative is a boy and second representative is a girl)

$= \frac{15}{45} \times \frac{30}{44}$

$= \frac{5}{22}$

(iv) P(one boy and one girl)

$= \left(\frac{15}{45} \times \frac{30}{44}\right) + \left(\frac{30}{45} \times \frac{15}{44}\right)$

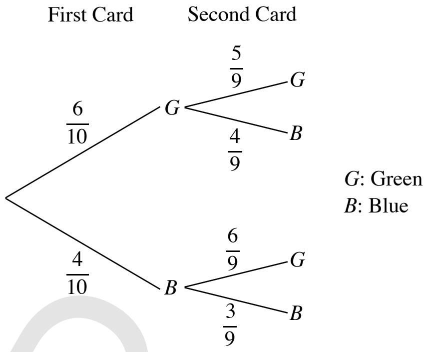
$= \frac{5}{11}$

7. **(a)** P(green card)

$= \frac{6}{10}$

$= \frac{3}{5}$

(b)



(i) P(two green cards)

$= P(G, G)$

$= \frac{6}{10} \times \frac{5}{9}$

$= \frac{1}{3}$

(ii) P(one card of each colour)

$= P(G, B) + P(B, G)$

$= \left(\frac{6}{10} \times \frac{4}{9}\right) + \left(\frac{4}{10} \times \frac{6}{9}\right)$

$= \frac{8}{15}$

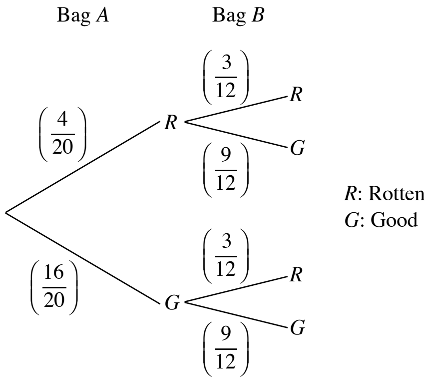
(iii) P(at least one blue card)

$= 1 - P(\text{two green cards})$

$= 1 - \frac{1}{3}$

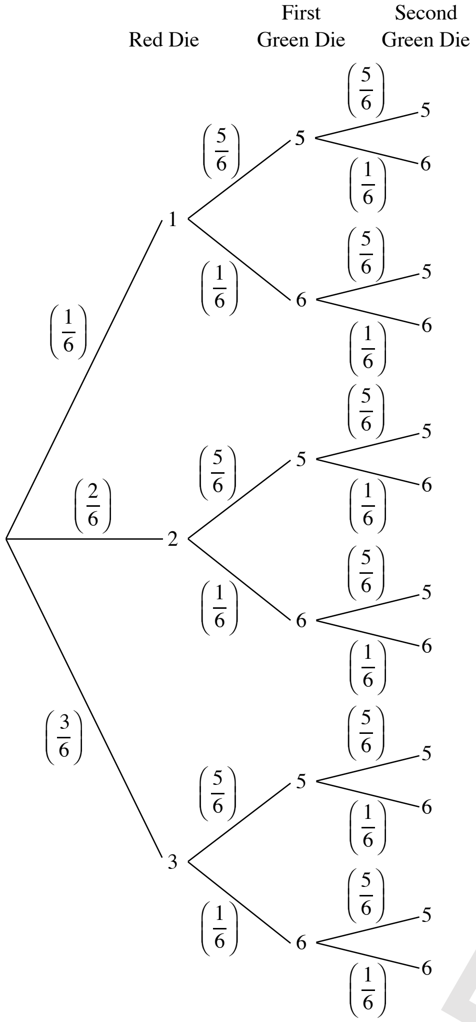
$= \frac{2}{3}$

8. **(a)**



(b) Yes. Since the selections of the rotten potatoes from the two bags are independent events, hence the Multiplication Law of Probability applies.

9. (a)



(b) (i) P(2, 5, 6)

$= \frac{2}{6} \times \frac{5}{6} \times \frac{1}{6}$

$= \frac{5}{108}$

(ii) P(3, 6, 6)

$= \frac{3}{6} \times \frac{1}{6} \times \frac{1}{6}$

$= \frac{1}{72}$

(iii) P(exactly two sixes)

= P(1, 6, 6) + P(2, 6, 6) + P(3, 6, 6)

$= 1 \times \frac{1}{6} \times \frac{1}{6}$

$= \frac{1}{36}$

(iv) P(a sum of 12)

= P(1, 5, 6) + P(1, 6, 5) + P(2, 5, 5)

$= \left(\frac{1}{6} \times \frac{5}{6} \times \frac{1}{6}\right) + \left(\frac{1}{6} \times \frac{1}{6} \times \frac{5}{6}\right) + \left(\frac{2}{6} \times \frac{5}{6} \times \frac{5}{6}\right)$

$= \frac{5}{18}$

(v) P(a sum which is divisible by 3)

= P(a sum of 12 or a sum of 15)

= P(a sum of 12) + P(3, 6, 6)

$= \frac{5}{18} + \left(\frac{3}{6} \times \frac{1}{6} \times \frac{1}{6}\right)$

$= \frac{7}{24}$

10. (i) P(will not buy a sack of rice in a particular month)

$= 1 - \frac{4}{9}$

$= \frac{5}{9}$

(ii) P(will not buy a sack of rice in two consecutive months)

$= \frac{5}{9} \times \frac{5}{9}$

$= \frac{25}{81}$

(iii) P(buys a sack of rice in one of two particular months)

$= \left(\frac{4}{9} \times \frac{5}{9}\right) + \left(\frac{5}{9} \times \frac{4}{9}\right)$

$= \frac{40}{81}$

11. (i) P(three representatives are females)

= P(F, F, F)

$= \frac{36}{76} \times \frac{35}{90} \times \frac{52}{90}$

$= \frac{91}{855}$

(ii) P(representative from the front office is male while the other two representatives are females)

= P(M, F, F)

$= \frac{40}{76} \times \frac{35}{90} \times \frac{52}{90}$

$= \frac{182}{1539}$

(iii) P(exactly one representative is male)

= P(M, F, F) + P(F, M, F) + P(F, F, M)

$= \left(\frac{40}{76} \times \frac{35}{90} \times \frac{52}{90}\right) + \left(\frac{36}{76} \times \frac{55}{90} \times \frac{52}{90}\right) + \left(\frac{36}{76} \times \frac{35}{90} \times \frac{38}{90}\right)$

$= \frac{5591}{15\,390}$

12. (a) (i) P(two black shirts)

$= \frac{8}{16} \times \frac{7}{15}$

$= \frac{7}{30}$

(ii) P(one shirt is black and one shirt is white)

$= \left(\frac{8}{16} \times \frac{6}{15}\right) + \left(\frac{6}{16} \times \frac{8}{15}\right)$

$= \frac{2}{5}$

(iii) P(two shirts are of the same colour)

$$= \left(\frac{8}{16}\times\frac{7}{15}\right)+\left(\frac{6}{16}\times\frac{5}{15}\right)+\left(\frac{2}{16}\times\frac{1}{15}\right)$$

$$= \frac{11}{30}$$

(b) P(all three shirts are black)

$$= \frac{8}{16} \times \frac{7}{15} \times \frac{6}{14}$$

$$= \frac{1}{10}$$

(c) No, since selections of the three black shirts are dependent events.

13. (i) P(first card bears the letter 'O')

= P(O)

$$= \frac{3}{10}$$

(ii) P(two cards bear the letter 'P' and 'O' in that order)

= P(P, O)

$$= \frac{2}{10} \times \frac{3}{9}$$

$$= \frac{1}{15}$$

(iii) P(two cards bear the letter 'P' and 'O' in any order)

= P(P, O) + P(O, P)

$$= \left(\frac{2}{10}\times\frac{3}{9}\right)+\left(\frac{3}{10}\times\frac{2}{9}\right)$$

$$= \frac{2}{15}$$

(iv) P(two cards bear the same letter)

= P(R, R) + P(O, O) + P(P, P)

$$= \left(\frac{2}{10}\times\frac{1}{9}\right)+\left(\frac{3}{10}\times\frac{2}{9}\right)+\left(\frac{2}{10}\times\frac{1}{9}\right)$$

$$= \frac{1}{9}$$

14. (a) P(ball numbered '8')

$$= \frac{1}{5}$$

(b) (i) P(number on each ball is even)

= P(2, 8) + P(8, 2)

$$= \left(\frac{1}{5}\times\frac{1}{4}\right)+\left(\frac{1}{5}\times\frac{1}{4}\right)$$

$$= \frac{1}{10}$$

(ii) P(sum of the numbers on the balls is more than 10)

= P(2, 9) + P(9, 2) + P(5, 8) + P(8, 5) + P(5, 9) + P(9, 5) + P(8, 9) + P(9, 8)

$$= 8\left(\frac{1}{5}\times\frac{1}{4}\right)$$

$$= \frac{2}{5}$$

(iii) P(number on each ball is not a prime number)

= P(1, 8) + P(8, 1) + P(1, 9) + P(9, 1) + P(8, 9) + P(9, 8)

$$= 6\left(\frac{1}{5}\times\frac{1}{4}\right)$$

$$= \frac{3}{10}$$

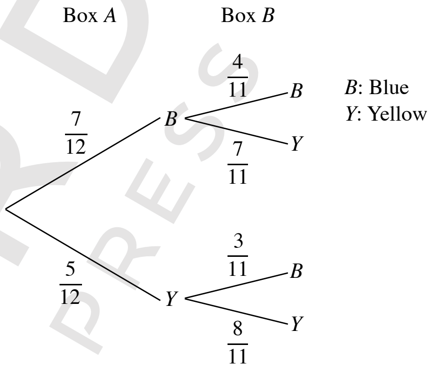
(iv) P(only one ball is odd number)

= P(1, 2) + P(2, 1) + P(1, 8) + P(8, 1) + P(2, 5) + P(5, 2) + P(2, 9) + P(9, 2) + P(5, 8) + P(8, 5) + P(8, 9) + P(9, 8)

$$= 12\left(\frac{1}{5}\times\frac{1}{4}\right)$$

$$= \frac{3}{5}$$

15. (a)



(b) (i) P(Box *A* has more yellow balls than blue balls)

= 0

(ii) P(Box *A* has exactly 7 blue and 5 yellow balls)

= P(*B*, *B*) + P(*Y*, *Y*)

$$= \left(\frac{7}{12}\times\frac{4}{11}\right)+\left(\frac{5}{12}\times\frac{8}{11}\right)$$

$$= \frac{17}{33}$$

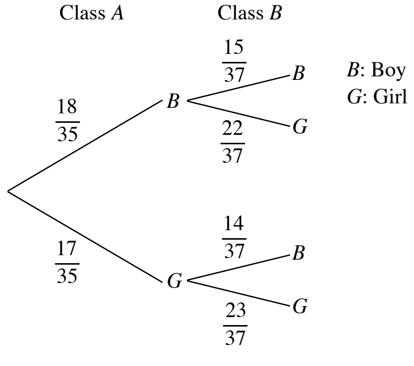
(iii) P(Box *A* has twice as many blue balls as yellow balls)

= P(*Y*, *B*)

$$= \frac{5}{12} \times \frac{3}{11}$$

$$= \frac{5}{44}$$

16.



(i) P(student was initially from Class A)

$= \frac{1}{37}$

(ii) P(student is a boy)

$= \text{P}(B, B) + \text{P}(G, B)$

$= \left(\frac{18}{35} \times \frac{15}{37}\right) + \left(\frac{17}{35} \times \frac{14}{37}\right)$

$= \frac{508}{1295}$

17. **(i)** P(volcanic eruptions in one of the three countries)

$= 0.03 \times 0.12 \times 0.3$

$= 0.001\ 08$

(ii) P(no volcanic eruptions)

$= (1 - 0.03) \times (1 - 0.12) \times (1 - 0.3)$

$= 0.598$ (to 3 s.f.)

(iii) P(at least one volcanic eruptions)

= 1 − P(no volcanic eruptions)

$= 1 - 0.598$

$= 0.402$ (to 3 s.f.)

(iv) P(exactly two volcanic eruptions)

$= (0.97 \times 0.12 \times 0.3) + (0.03 \times 0.88 \times 0.3) + (0.03 \times 0.12 \times 0.7)$

$= 0.0454$ (to 3 s.f.)

18. **(i)** P(a red and two blue balls in that order)

$= \frac{10}{26} \times \frac{8}{24} \times \frac{3}{65}$

$= \frac{3}{65}$

(ii) P(a red, a yellow and a blue ball in that order)

$= \frac{10}{26} \times \frac{7}{25} \times \frac{9}{24}$

$= \frac{21}{520}$

(iii) P(three balls of different colours)

= P(red, yellow, blue) + P(red, blue, yellow)
+ P(yellow, red, blue) + P(yellow, blue, red)
+ P(blue, yellow, red) + P(blue, red, yellow)

$= 6 \times \left(\frac{10}{26} \times \frac{7}{25} \times \frac{9}{24}\right)$

$= \frac{63}{260}$

19. **(a)** **(i)** P(game ends on the third roll)

$= \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6}$

$= \frac{25}{216}$

(ii) P(game ends on the fourth roll)

$= \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6}$

$= \frac{125}{1296}$

(iii) P(game ends by the fourth roll)

$= \frac{1}{6} + \left(\frac{5}{6} \times \frac{1}{6}\right) + \left(\frac{5}{6} \times \frac{5}{6} \times \frac{1}{6}\right) + \left(\frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6}\right)$

$= \frac{671}{1296}$

(b) **(i)** P(game ends on the third roll)

$= \left(\frac{5}{6} \times \frac{1}{6} \times \frac{1}{6}\right) + \left(\frac{1}{6} \times \frac{5}{6} \times \frac{1}{6}\right)$

$= \frac{5}{108}$

(ii) P(game ends on the third roll and the sum of the scores is odd)

$= \left(\frac{1}{2} \times \frac{1}{6} \times \frac{1}{6}\right) + \left(\frac{1}{6} \times \frac{1}{2} \times \frac{1}{6}\right)$

$= \frac{1}{36}$

Review Exercise 14

1. **(i)** P(number ‘3’ followed by a head)

$= \frac{1}{6} \times \frac{1}{2}$

$= \frac{1}{12}$

(ii) P(even number followed by a tail)

$= \frac{3}{6} \times \frac{1}{2}$

$= \frac{1}{4}$

2. **(i)** P(same number)

$= 1 \times \frac{1}{6}$

$= \frac{1}{6}$

(ii) P(two even numbers)

$= \frac{3}{6} \times \frac{3}{6}$

$= \frac{1}{4}$

(iii) P(two odd numbers)

$= \frac{3}{6} \times \frac{3}{6}$

$= \frac{1}{4}$

(iv) P(one odd and one even number)

$= \left(\frac{3}{6} \times \frac{3}{6}\right) + \left(\frac{3}{6} \times \frac{3}{6}\right)$

$= \frac{1}{2}$

3. **(i)** P(number is greater than 28)

$= \frac{22}{50}$

$= \frac{11}{25}$

(ii) P(number includes the digit ‘3’)

$= \frac{14}{50}$

$= \frac{7}{25}$

(iii) P(number is prime)

$= \frac{15}{50}$

$= \frac{3}{10}$

(iv) P(number is divisible by 4)

$= \frac{12}{50}$

$= \frac{6}{25}$

4. **(i)** P(two people born in the same month)

$= 1 \times \frac{1}{12}$

$= \frac{1}{12}$

(ii) P(three people born in the same month)

$= 1 \times \frac{1}{12} \times \frac{1}{12}$

$= \frac{1}{144}$

P(three people not born in the same month)

= 1 – P(three people born in the same month)

$= 1 - \frac{1}{144}$

$= \frac{143}{144}$

(iii) P(four people born in the same month)

$= 1 \times \frac{1}{12} \times \frac{1}{12} \times \frac{1}{12}$

$= \frac{1}{1728}$

5. **(i)** P(Anosha will catch her bus on a particular day)

$= 1 - \frac{1}{7}$

$= \frac{6}{7}$

(ii) P(Anosha will miss her bus on two particular consecutive days)

$= \frac{1}{7} \times \frac{1}{7}$

$= \frac{1}{49}$

(iii) P(Anosha will miss her bus on one of two particular consecutive days)

$= \left(\frac{1}{7} \times \frac{6}{7}\right) + \left(\frac{6}{7} \times \frac{1}{7}\right)$

$= \frac{12}{49}$

(iv) P(Anosha will catch her bus on three particular consecutive days)

$= \frac{6}{7} \times \frac{6}{7} \times \frac{6}{7}$

$= \frac{216}{343}$

6. **(i)** P(one of them wins the gold medal)

= P(Rizwan wins) + P(Maaz wins) + P(Hussain wins)

$= \frac{1}{2} + \frac{1}{6} + \frac{1}{8}$

$= \frac{19}{24}$

(ii) P(none of them wins the gold medal)

$= 1 - \frac{19}{24}$

$= \frac{5}{24}$

(iii) P(Rizwan fails to win the gold medal)

$= 1 - \frac{1}{2}$

$= \frac{1}{2}$

7.

Second Disc

6	16	26	36	46	56	
5	15	25	35	45		65
4	14	24	34		54	64
3	13	23		43	53	63
2	12		32	42	52	62
1		21	31	41	51	61
	1	**2**	**3**	**4**	**5**	**6**

First Disc

(i) P(number formed is divisible by 2)

$= \frac{15}{30}$

$= \frac{1}{2}$

(ii) P(number formed is divisible by 5)

$= \frac{5}{30}$

$= \frac{1}{6}$

(iii) P(number formed is a prime number)

$= \frac{7}{30}$

(iv) P(number formed is a perfect square)

$= \frac{4}{30}$

$= \frac{2}{15}$

8. **(i)** P(first two cards are letter 'O')

$$= \left(\frac{1}{6} \times \frac{1}{5}\right) + \left(\frac{1}{5} \times \frac{1}{6}\right)$$

$$= \frac{1}{15}$$

(ii) P(second card is letter 'F')

$$= \frac{5}{6} \times \frac{1}{5}$$

$$= \frac{1}{6}$$

(iii) P(word 'FOLLOW' is obtained, in that order)

$$= \frac{1}{6} \times \frac{2}{5} \times \frac{2}{4} \times \frac{1}{3} \times \frac{1}{2} \times 1$$

$$= \frac{1}{180}$$

9. **(i)** P(Hussain selects a dark chocolate)

$$= \frac{y}{x + y}$$

(ii) P(Hussain selects a white chocolate, Seema selects a dark chocolate)

$$= \frac{x}{x + y} \times \frac{y}{x + y - 1}$$

$$= \frac{xy}{(x + y)(x + y - 1)}$$

(iii) P(chocolate selected by them are of different types)

$$= \left(\frac{x}{x + y} + \frac{y}{x + y - 1}\right) + \left(\frac{y}{x + y} \times \frac{x}{x + y - 1}\right)$$

$$= \frac{2xy}{(x + y)(x + y - 1)}$$

10. P(traffic jam)

$$= \left(\frac{1}{4} \times \frac{2}{5}\right) + \left(\frac{3}{4} \times \frac{1}{5}\right)$$

$$= \frac{1}{4}$$

11. **(a)** **(i)** P(one girl, one boy)

$$= \left(\frac{10}{30} \times \frac{20}{29}\right) + \left(\frac{20}{30} \times \frac{10}{29}\right)$$

$$= \frac{40}{87}$$

(ii) P(no girls)

$$= \frac{20}{30} \times \frac{19}{29}$$

$$= \frac{38}{87}$$

(b) **(i)** P(both travel to school by bus)

$$= \frac{6}{10} \times \frac{5}{9}$$

$$= \frac{1}{3}$$

(ii) P(both travel to school by different means of transportation)

$$= \left(\frac{6}{10} \times \frac{4}{9}\right) + \left(\frac{4}{10} \times \frac{6}{9}\right)$$

$$= \frac{8}{15}$$

(iii) P(at least one travels to school by bus)

$$= \left(\frac{6}{10} \times \frac{4}{9}\right) + \left(\frac{4}{10} \times \frac{6}{9}\right) + \left(\frac{6}{10} \times \frac{5}{9}\right)$$

$$= \frac{13}{15}$$

12. **(i)** P(next two days also wet)

$= 0.6 \times 0.6$

$= 0.36$

(ii) P(Tuesday is wet, Wednesday is fine)

$= 0.6 \times 0.4$

$= 0.24$

(iii) P(one day fine, one day wet)

$= (0.6 \times 0.4) + (0.4 \times 0.2)$

$= 0.32$

(iv) P(two of the three days are wet)

$= (0.6 \times 0.6 \times 0.4) + (0.6 \times 0.4 \times 0.2) + (0.4 \times 0.2 \times 0.6)$

$= 0.24$

13. **(i)** P(first two sweets are different)

$$= \left(\frac{2}{10} \times \frac{3}{9}\right) + \left(\frac{3}{10} \times \frac{2}{9}\right) + \left(\frac{2}{10} \times \frac{5}{9}\right) + \left(\frac{5}{10} \times \frac{2}{9}\right) + \left(\frac{3}{10} \times \frac{5}{9}\right) + \left(\frac{5}{10} \times \frac{3}{9}\right)$$

$$= \frac{31}{45}$$

(ii) P(three sweets are the same)

$$= \left(\frac{3}{10} \times \frac{2}{9} \times \frac{1}{8}\right) + \left(\frac{5}{10} \times \frac{4}{9} \times \frac{3}{8}\right)$$

$$= \frac{11}{120}$$

(iii) P(first two sweets are the same, third sweet is a toffee)

$$= \left(\frac{2}{10} \times \frac{1}{9} \times \frac{5}{8}\right) + \left(\frac{3}{10} \times \frac{2}{9} \times \frac{5}{8}\right) + \left(\frac{3}{10} \times \frac{4}{9} \times \frac{3}{8}\right)$$

$$= \frac{5}{36}$$

14. **(i)** P(all three airplanes land at Terminal 2)

$$= \frac{1}{4} \times \frac{1}{3} \times \frac{1}{6}$$

$$= \frac{1}{72}$$

(ii) P(exactly two airplanes land at Terminal 1)

$$= \left(\frac{3}{4} \times \frac{2}{3} \times \frac{1}{6}\right) + \left(\frac{3}{4} \times \frac{1}{3} \times \frac{5}{6}\right) + \left(\frac{1}{4} \times \frac{2}{3} \times \frac{5}{6}\right)$$

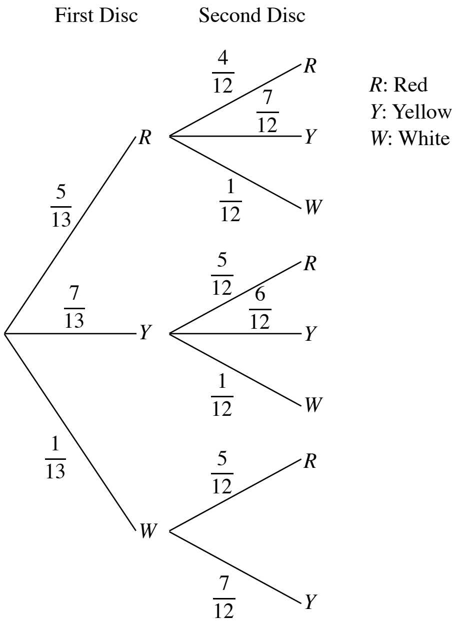
$$= \frac{31}{72}$$

(iii) P(exactly one airplane lands at Terminal 1)

$$= \left(\frac{3}{4} \times \frac{1}{3} \times \frac{1}{6}\right) + \left(\frac{1}{4} \times \frac{1}{3} \times \frac{5}{6}\right) + \left(\frac{1}{4} \times \frac{2}{3} \times \frac{1}{6}\right)$$

$$= \frac{5}{36}$$

15.



(i) P(two red discs)

$= \frac{5}{13} \times \frac{4}{12}$

$= \frac{5}{39}$

(ii) P(a red and a yellow disc in that order)

$= \frac{5}{13} \times \frac{7}{12}$

$= \frac{35}{136}$

(iii) P(two white discs)

$= 0$

(iv) P(two discs of different colours)

= 1 – P(two discs of the same colour)

$= 1 - \left[\left(\frac{5}{13} \times \frac{4}{12}\right) + \left(\frac{7}{13} \times \frac{6}{12}\right)\right]$

$= \frac{47}{78}$

16. (a) (i) P(all three men hit the target)

$= \frac{2}{3} \times \frac{3}{5} \times \frac{4}{7}$

$= \frac{8}{35}$

(ii) P(all three men miss the target)

$= \frac{1}{3} \times \frac{2}{5} \times \frac{3}{7}$

$= \frac{2}{35}$

(iii) P(exactly two of them hit the target)

$= \left(\frac{2}{3} \times \frac{3}{5} \times \frac{3}{7}\right) + \left(\frac{2}{3} \times \frac{2}{5} \times \frac{4}{7}\right) + \left(\frac{1}{3} \times \frac{3}{5} \times \frac{4}{7}\right)$

$= \frac{46}{105}$

(iv) P(at least one of them hits the target)

= 1 – P(none of them hits the target)

$= 1 - \left(\frac{1}{3} \times \frac{2}{5} \times \frac{3}{7}\right)$

$= \frac{33}{35}$

(b) (i) P(game ends after two shots)

$= \frac{1}{3} \times \frac{3}{5}$

$= \frac{1}{5}$

(ii) P(game ends after three shots)

$= \frac{1}{3} \times \frac{2}{5} \times \frac{4}{7}$

$= \frac{8}{105}$

(iii) P(game ends by the third shot)

= P(game ends after the first shot)
+ P(game ends after the second shot)
+ P(game ends after the third shot)

$= \frac{2}{3} + \frac{1}{5} + \frac{8}{105}$

$= \frac{33}{35}$

Challenge Yourself

1.

Second Die

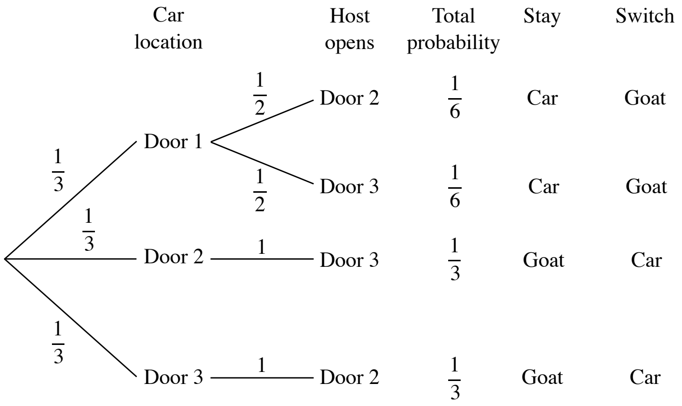
6	1, 6	2, 6	3, 6	4, 6	5, 6	6, 6
5	1, 5	2, 5	3, 5	4, 5	5, 5	6, 5
4	1, 4	2, 4	3, 4	4, 4	5, 4	6, 4
3	1, 3	2, 3	3, 3	4, 3	5, 3	6, 3
2	1, 2	2, 2	3, 2	4, 2	5, 2	6, 2
1	1, 1	2, 1	3, 1	4, 1	5, 1	6, 1
	1	**2**	**3**	**4**	**5**	**6**

First Die

There are at least 11 outcomes with at least a '3' and the probability that both of the scores are '3's is $\frac{1}{11}$.

2. Contestants who switch doors have a $\frac{2}{3}$ chance of winning the car, while contestants who stick to their choice have only a $\frac{1}{3}$ chance. By definition, the conditional probability of winning by switching, given the contestant initially picks door 1 and the host opens door 3, is the probability for the event "car is behind door 2 and host opens door 3" divided by the probability for "host opens door 3". These probabilities can be determined by referring to the probability tree as shown. The conditional probability of winning by switching is $\frac{\frac{1}{3}}{\frac{1}{3} + \frac{1}{6}} = \frac{2}{3}$.

The tree diagram showing the probability of every possible outcome if the player initially picks Door 1 is shown below.



3. **(a)** 1

(b) $1 - \frac{364}{365} \times \frac{363}{365} \times \frac{362}{365} \times \ldots \times \frac{326}{365} = 0.8912$

Yes

(c) $1 - \frac{364}{365} \times \frac{363}{365} \times \frac{362}{365} \times \ldots \times \frac{324 - x + 1}{365} > \frac{1}{2}$

$\frac{364}{365} \times \frac{363}{365} \times \frac{362}{365} \times \ldots \times \frac{364 - x + 1}{365} < \frac{1}{2}$

Least number of students is 22.

Chapter 15 Sets

TEACHING NOTES

Suggested Approach

Students have learnt, in their previous class, how to write sets in three different notations. They have also learnt about the different types of sets, and how to use union and interaction.

In this chapter, students will learn how to use different laws on sets.

Section 15.1: Sets

The section is a recap of all that the students have studied in previous grades. The teacher should use Worked Examples 1, 2, and 3 to recap their prior knowledge.

Section 15.2: Commutative, Associative, and Distributive Laws

Students shall use Venn Diagram to represent and verify the commutative, associative, and distributive laws. Practice Now 4, 5, and 6 can be carried out in class with the students to help them grasp the concept.

Section 15.3: De Morgan's Laws

The teacher may should define De Morgan's Laws using Worked Example 7 and illustrate it using Venn Diagram.

Exercise 15A

1. (i) $\{\{\}, \{a\}\}$
(ii) $\{\{\}. \{x\}, \{y\}, \{x,y\}\}$
(iii) $\{\{\}, \{1\}, \{2\}, \{3\}, \{1,2\}, \{2,3\}, \{1,3\}, \{1,2,3\}$
(iv) $\{\{\}\}$
(v) $\{\{\} ,\{a\}, \{b\}, \{c\}, \{d\}, \{a,b\}, \{a,c\}, \{a,d\},$
$\{b,c\}, \{b,d\}, \{c,d\}, \{a,b,c\}, \{a,b,d\},$
$\{\{b,c,d\}, \{a,c,d\},\{a,b,c,d\}\}$
(vi) $\{\{\}, \{0\}\}$

2. (i) $A \cup C = \{0,2,4,6,8\} \cup \{1,2,3,8\}$
$= \{0,1,2,4,5,6,,8\}$
(ii) $A \cup D = \{0,2,4,6,8\} \cup \{4,5,6,7\}$
$= \{0,2,4,5,6,7,8\}$
(iii) $C \cup D = \{1,2,3,8\} \cup \{4,5,6,7\}$
$= \{1,2,3,4,5,6,7,8\}$
(iv) $C \cup \mathbb{U} = \{1,2,3,8\} \cup \{0,1,2,....,9\}$
$= \{0,1,2,....,9\}$
(v) $A \cup \varnothing = \{0,2,4,6,8\} \cup \varnothing$
$= \{0,2,4,6,8\}$
(vi) $= A \cap B = \{0,2,4,6,8\} \cap \{1,3,5,7,9\}$
$= \{\}$
(vii) $= B \cap C = \{1,3,5,7,9\} \cap \{1,2,3,8\}$
$= \{1,3\}$
(viii) $= C \cap D = \{1,2,3,8\} \cap \{4,5,6,7\}$
$= \{\}$
(ix) $= B \cap \varnothing =\{1,3,5,7,9\} \cap \varnothing = \varnothing$
(x) $= B \cap \mathbb{U} = \{1,3,5,7,9\} \cap \{0,1,2,3.....,9\}$
$= \{1,3,5,7,9\}$

3. (i) $A' = \mathbb{U} - A$
$= \{1,2,3,4,5,6,7\} - \{1,2,5,7\}$
$= \{3,4,6\}$
(ii) $B' = \mathbb{U} - B$
$=\{1,2,3,4,5,6,7,\} - \{1,3,6,7\}$
$= \{2,4,5\}$
(iii) $(A \cap B)'$
$A \cap B = \{1,7\}$
$(A \cap B)' = \{2,3,4,5,6,\}$
(iv) $(A \cup B)'$
$A \cup B = \{1,2,3,5,6,7\}$
$(A \cup B)'= \{4\}$
(v) $A' \cup B' = \{3,4,6\} \cup \{2,4,5\}$
$= \{2,3,4,5,6\}$
(vi) $A' \cap B' = \{3,4,6\} \cup \{2,4,5\}$
$= \{4\}$

4. **(a) (i)** $A \cup (B \cup C)$
$B \cup C = \{2,4,5\}$
$A \cup (B \cup C) = \{1,2,3,4,5\}$
(ii) $(A \cup B) \cap C$
$A \cup B = \{1,2,3,4\}$
$(A \cup B) \cap C =\{1,2,3,4,\} \cap \{5\}$
$= \{\}$
(iii) $(A \cap B) \cup C$
$A \cap B = \{ \}$
$(A \cap B) \cup C = \{ \} \cup \{5\}$
$\{5\}$
(iv) $A \cap B \cap C$
$A \cap B = \{ \}$
$(A \cap B) \cap C = \{ \} \cap \{ \}$
$= \{ \}$
(v) $A \cup (B \cap C)$
$B \cap C = \{ \}$
$A \cup (B \cup C) = \{1,3\} \cup \{ \}$
$= \{1,3\}$
(vi) $A \cap (B \cup C)$
$B \cup C = \{2,4,5\}$
$A \cap (B \cup C) = \{1,3\} \cap \{2,4,5\}$
$= \{ \}$

(c) (i) $A' \cup B' = (A \cap B)'$
L.H.S
$A' \cup B' = \{2,4,5,\} \cup \{1,3,5\}$
$= \{1,2,3,4,5\}$
R.H.S
$(A \cap B)' = \mathbb{U} -\{ \}$
$= \{1,2,3,4,5\}$
L.H.S = R.H.S

(ii) $A' \cap B' = (A \cup B)$`
L.H.S
$A' \cap B' = \{2,4,5,\} \cap \{1,3,5\}$
$= \{5\}$
R.H.S
$(A \cup B)' = \mathbb{U} - \{1,2,3,4\}$
$= \{5\}$
L.H.S = R.H.S

5. (i) $B = [(A \cup B) - A] \cup (A \cap B)$
$= [\{2,4,5,6,7,8\} - \{2,4,6,8\}] \cup \{6,8\}$
$= \{5,7\} \cup \{6,8\}$
$= \{5,6,7,8\}$
(ii) $B = [(A \cup B) - A] \cup (A \cap B$
$= [\{1,2,3,4\} - \{3,4\}] \cup \{3\}$
$= \{1,2\} - \{3\}$
$= \{1,2,3\}$

6. (i) $A \cup B = B \cup A$
(ii) $A \cup \varnothing = A$
(iii) $\varnothing \cup A = \varnothing$
(iv) $A \cup A = A$
(v) $A = B$

7. Associative Property Of Union state that:

$A \cup (B \cup C) = (A \cup B) \cup C$

Lets take left hand side

L.H.S

$A \cup (B \cup C) = \{2,3,4\} \cup \{3,5,6,7,9\}$

$= \{2,3,4,5,6,7,9\}$

R.H.S

$(A \cup B) \cup C = \{2,3,4,5,6\} \cup \{5,7,9\}$

$= \{2,3,4,5,6,7,9\}$

Since **L.H.S = R.H.S**

Hence, the associative property of union proved .

8. Associative property of intersection states that:

$A \cap (B \cap C) = (A \cap B) \cap C$

L.H.S

$A \cap (B \cap C) = \{m,n,o\} \cap \{p,q\}$

$\{\ \}$

R.H.S

$(A \cap B) \cap C = \{\ \} \cap \{n,o,p,q\}$

$\{\ \}$

Since **L.H.S= R.H.S**

Hence, the associative property of intersection is proved.

9. De Morgan's first law states that

$(A \cup B)` = A` \cap B`$

L.H.S

$(A \cap B)` = \mathbb{U} - (A \cup B)$

$= \{1,2,34,5,6,\} - \{1,2,3,4,5,6\}$

$= \{\ \}$

R.H.S

$A` \cap B` = \{5,6,\} \cap \{1,2\}$

$= \{\ \}$

Since **L.H.S = R.H.S**

Hence , De Morgan's first law is proved

10. (a) Swimming = {B,E,F,H}

Coding = {A,C,F,G}

Painting {B,C,D,F}

(b) B and F are taking swimming and painting classes both.

(c) F is taking all the classes.

Review Exercise 15

1. (i) Commutative law of union states that $A \cup B = B \cup A$

L.H.S

$A \cup B = \{4, 8, 12, 16, 20\} \cup \{1, 2, 3, 4, 6, 9, 12, 18\}$

$= \{1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 20\}$

R.H.S

$B \cup A = \{1, 2, 3, 4, 6, 9, 12, 18\} \cup \{4, 8, 12, 16, 20\}$

$= \{1, 2, 3, 4, 6, 8, 9, 12, 16,18, 20\}$

Since **L.H.S = R.H.S**

Hence, commutative law of union is proved.

(ii) Commutative law of intersection states that $A \cap B = B \cap A$

L.H.S

$A \cap B = \{4, 8, 12, 16, 20\} \cap \{1, 2, 3, 4, 6, 9, 12, 18\}$

$= \{4, 12\}$

R.H.S

$B \cap A = \{1, 2, 3, 4, 6, 9, 12, 18\} \cap \{4, 8, 12, 16, 20\}$

$= \{4, 12\}$

Since **L.H.S = R.H.S**

Hence, commutative law of union is proved.

2. (i) Associative law of union states that:

$A \cup (B \cup C) = (A \cup B) \cup C$

L.H.S

$A \cup (B \cup C) = \{-6, -5, -4, ..., 5, 6\} \cup \{1, ..., 7\}$

$= \{-6, -5, ..., 7\}$

R.H.S

$(A \cup B) \cup C = \{-6, -5, ..., 6, 7\} \cup \{4, 5, 6\}$

$= \{-6, -5, ..., 7\}$

Since **L.H.S = R.H.S**, Hence, The associative law of union is proved.

(ii) Associative law of intersection states that,

$A \cap (B \cap C) = (A \cap B) \cap C$

L.H.S

$A \cap (B \cap C) = \{-6, -5, ..., 5, 6\} \cap \{4, 5, 6\}$

$= \{4, 5, 6\}$

R.H.S

$(A \cap B) \cap C = \{1, 2, ..., 7\} \cap \{4, 5, 6\}$

$= \{4, 5, 6\}$

Since **L.H.S = R.H.S**

Hence,The associative law of intersection of proved.

3. (i) De Morgan's first law

$(A \cup B)' = A' \cap B'$

L.H.S

$(A \cup B)' = \{-7, -6, ... 6, 7\} - \{-6, -5, ..., 7\}$

$= \{-7\}$

R.H.S

$A' \cap B' = \{-7, 7\} \cap \{-7, -6, ..., 0\}$

$= \{-7\}$

Since **L.H.S = R.H.S**

Hence, De Morgan's first law is proved.

(ii) De Morgan's second law states

$(A \cap B)' = A' \cup B'$

L.H.S

$(A \cap B)' = \{-7, -6, ... 6, 7\} - \{1, 2, ..., 6\}$

$= \{-7, -6, -5, -4, -3, -2, -1, 0, 7\}$

R.H.S

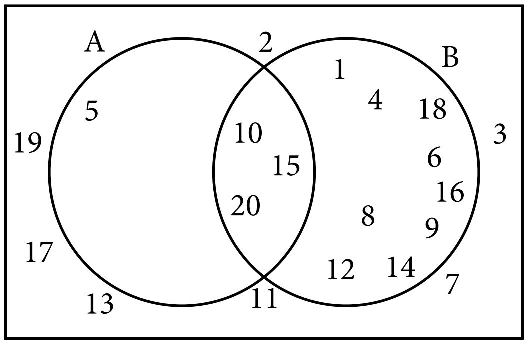
$A' \cup B' = \{-7, 7\} \cup \{-7, -6, -5, -4, -3, -2, -1, 0\}$

$= \{-7, -6, -5, -4, -3, -2, -1, 0, 7\}$

Since **L.H.S = R.H.S**

Hence, De Morgan's second law is proved.

4. **(i)** $\mathbb{U}$



(ii) A ∩ B′ = { 5,10,15,20} ∩ {2,3,5,7,11,13,17,19}
= {5}

5. S = {s,i,t}

(i) Subset; { }, {s}, {i},{t}, {s,i}, {s,t}, {i,t}, {s,i,t}

(ii) Proper subset; { }, {s}, {i},{t}, {s,i}, {s,t}, {i,t}