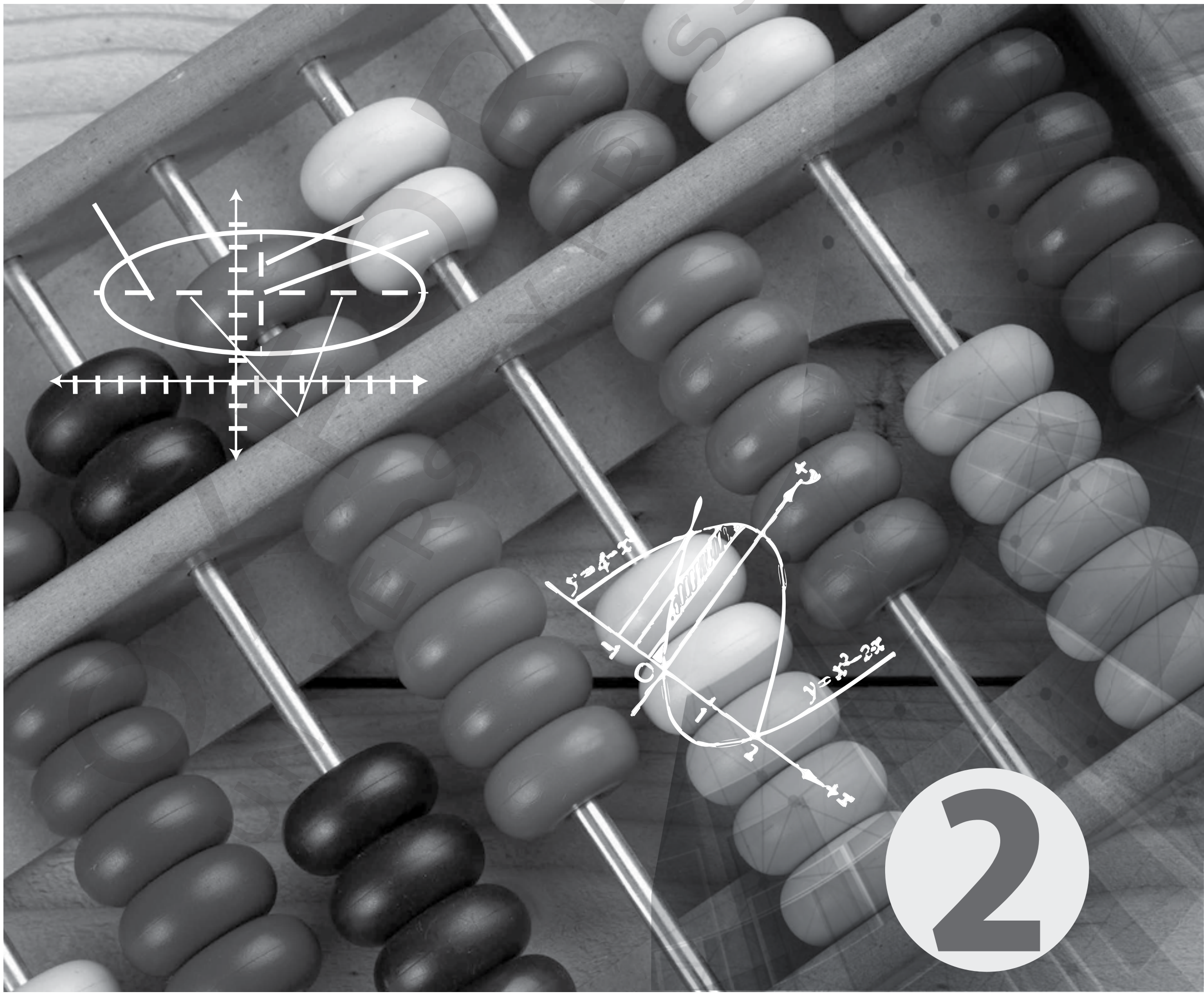
UPDATED 7th EDITION

NEW SYLLABUS MATHEMATICS
TEACHER'S RESOURCE BOOK

A Comprehensive Mathematics Programme for Grade 7



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Syllabus Matching Grid
National Curriculum of Pakistan 2022 with New Syllabus Mathematics 2 (Updated 7th Edition)

SLOs	Domain A: Numbers and Operations	Reference
M-07-A-01	With increasing degree of challenge, use the concept of place value for whole numbers, integers, rational numbers and decimal numbers.	Chapter 1
M-07-A-02	Round off whole numbers, integers, rational numbers, and decimal numbers to a required degree of accuracy, significance or decimal places (up to 3 decimal places).	Chapter 1
M-07-A-03	Use knowledge of rounding off to give an estimate to a calculation; to check the reasonableness of the solution.	Chapter 1
M-07-A-04	Recall H.C.F and L.C.M.	Chapter 1
M-07-A-05	Recall - Recognise, identify and represent integers (positive, negative and neutral integers) and their absolute or numerical value.	Chapter 1
M-07-A-06	Identify and represent (on a number line) rational numbers.	Chapter 1
M-07-A-07	Represent whole numbers, integers, and decimal numbers on a number line.	Chapter 1
M-07-A-08	Identify and convert between various types of fractions.	Chapter 1
M-07-A-09	Compare (using symbols $<$, $>$, $=$, $\leq$ and $\geq$) and arrange (in ascending or descending order) whole numbers, integers, rational numbers and decimal numbers.	Chapter 1
M-07-A-10	Verify associative and commutative properties of rational numbers.	Chapter 1
M-07-A-11	Verify associative, commutative, and distributive properties of rational numbers.	Chapter 1
M-07-A-12	Solve real-world word problems involving operations on rational numbers.	Chapter 1
M-07-A-13	Recognise the order of operations and use it to solve mathematical expressions involving whole numbers, decimals, fractions, and integers.	Chapter 1
M-07-A-14	Calculate rate and average rate of quantities.	Chapter 2
M-07-A-15	Calculate increase and decrease in a ratio based on change in quantities.	Chapter 2
M-07-A-16	Explain and calculate direct and inverse proportion and solve real-world word problems related to direct and inverse proportion.	Chapter 2
M-07-A-17	Identify and differentiate between selling price, cost price, loss, discount, profit percentage, and loss percentage.	Chapter 3
M-07-A-18	Explain income tax, property tax, general sales tax, value-added tax, zakat, and ushr.	Chapter 3
M-07-A-19	Solve real-world word problems involving profit, loss, discount, commission, tax, zakat, and ushr.	Chapter 3
M-07-A-20	Recognise and calculate squares of numbers up to 3-digits.	Chapter 1
M-07-A-21	Find the square roots of perfect squares of (up to 3-digits) natural numbers, fractions, and decimals.	Chapter 1
M-07-A-22	Solve real-world word problems involving squares and square roots.	Chapter 3
M-07-A-23	Use language, notation, and Venn Diagrams to represent different sets and their elements. (natural numbers, whole numbers, integers, even numbers, odd numbers, prime numbers)	Chapter 4
M-07-A-24	Identify and differentiate between: - subset and superset - proper and improper - equal and equivalent - disjoint and overlapping.	Chapter 4
M-07-A-25	Describe and perform operations on sets (union, intersection, difference and complement).	Chapter 4

M-07-A-26	Verify the following: $A \cap A' = \emptyset$ $A \cup A' = U$ $(A \cup B)' = A \cap B$ $(A \cap B)' = A \cup B$	Chapter 4
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M-07-B-02	Recall how to continue a given number sequence and find: - term to term rule - position to term rule	Chapter 5
M-07-B-03	Find terms of a sequence when the general term (n^{th} term) is given.	Chapter 5
M-07-B-04	Solve real-life problems involving number sequences and patterns.	Chapter 5
M-07-B-05	Students will know Muhammad bin Musa Al- Khwarizmi as the founding father of Algebra.	Book 1 Chapter 4
M-07-B-06	Recall variables as a quantity which can take various numerical values.	Chapter 6
M-07-B-07	Recognise open and close sentences, like and unlike terms, variable, constant, expression, equation, and inequality.	Chapter 6
M-07-B-08	Recognise polynomials as algebraic expressions in which the powers of variables are whole numbers.	Chapter 6
M-07-B-09	Identify a monomial, a binomial, and a trinomial as a polynomial.	Chapter 6
M-07-B-10	Add and subtract two or more polynomials.	Chapter 6
M-07-B-11	Find the product of: - monomial with monomial - monomial with binomial/trinomial - binomials with binomial/trinomial	Chapter 6
M-07-B-12	Simplify algebraic expressions (by expanding products of algebraic expressions by a number, a variable or an algebraic expression) involving addition, subtraction, and multiplication division.	Chapter 6 Chapter7
M-07-B-13	Explore the following algebraic identities and use them to expand expressions: $(a + b)^2 = a^2 + b^2 + 2ab$ $(a - b)^2 = a^2 + b^2 - 2ab$ $a^2 - b^2 = (a + b)(a - b)$	
M-07-B-14	Factorise algebraic expressions (by taking out common terms and by regrouping).	Chapter 7
M-07-B-15	Factorise quadratic expressions (by middle term breaking method).	Chapter 7
M-07-B-16	Construct linear equations in two variables such as; $ax + by = c$, where a and b are not zero.	Chapter 8
M-07-B-17	Recall solving linear equations in one variable.	Chapter 8
M-07-B-18	Introduction to Cartesian coordinate system.	Chapter 8
M-07-B-19	Plot the graph of the linear equation $ax + b = 0$ where $a \neq 0$ and of linear equations in two variables.	Chapter 8
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M-07-C-02	Convert 12-hour clock to 24-hour clock and vice versa.	Chapter 9
M-07-C-03	Convert between different units of time and speed.	Chapter 9

M-07-C-04	Calculate arrival time, departure time, and journey time in a given situation (on the previous day and the next day).	Chapter 9
M-07-C-05	Solve real-world word problems involving distance, time, and average speed.	Chapter 9
M-07-C-06	Differentiate between uniform and average speeds.	Chapter 9
M-07-C-07	Calculate the area and perimeter of the shaded/unshaded region in composite shapes.	Chapter 12
M-07-C-08	Calculate the circumference and area of a circle.	Chapter 12
M-07-C-09	Calculate the surface area and volume of any simple 3-D shape including right prisms and cylinders.	Chapter 12
M-07-C-10	Convert between standard units of area (m^2, cm^2, mm^2 and vice versa) and volume (m^3, cm^3 and mm^3 and vice versa).	Chapter 12
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SLOs	**Domain D: Geometry**	
M-07-D-01	Recognise quadrilaterals and their characteristics (parallel sides, equal sides, equal angles, right angles, lines of symmetry etc) Square, rectangle, parallelogram, rhombus trapezium, and kite.	Chapter 10
M-07-D-02	Differentiate between convex and concave polygons.	Chapter 10
M-07-D-03	Translate an object and give precise description of transformation.	Chapter 10
M-07-D-04	Know that the perpendicular distance from a point to a line is the shortest distance to the line.	Chapter 10
M-07-D-05	Describe the properties of a circle; centre, radius, diameter, chord, arcs, major and minor arc, semi-circle, and segment of a circle.	Chapter 10
M-07-D-06	Calculate unknown angles in quadrilaterals using the properties of quadrilaterals (square, rectangle, parallelogram, rhombus, trapezium, and kite).	Chapter 10
M-07-D-07	Understand the relationship between interior and exterior angles of polygons and between opposite interior and exterior angles in a triangle.	Chapter 10
M-07-D-08	Calculate the interior and exterior angles of a polygon and the sum of interior angles of a polygon.	Chapter 10
M-07-D-09	Recognise identity and draw lines of symmetry in 2-D shapes and rotate objects using rotational symmetry; and find the order of rotational symmetry.	Chapter 11
M-07-D-10	Calculate unknown angles in a triangle.	Chapter 10
M-07- D-11	Construct different types of triangles (equilateral, isosceles, scalene, acute- angled, right- angled, and obtuse-angled).	Chapter 10
SLOs	**Domain E: Statistics and Probability**	
M-07-E-01	- Recognise drawing and interpreting of bar graphs, line graphs, and pie charts. - Differentiate between a histogram and a bar graph. - Construct and compare histograms for both discrete and continuous data with equal interval range. - Select and justify the most appropriate graph(s) for a given data set and draw simple conclusions based on the shape of the graph.	Chapter 13
M-07-E-02	Recognise the difference between discrete, continuous, grouped and ungrouped data.	Chapter 13
M-07-E-03	Calculate the mean, median, and mode for ungrouped data and the mean for grouped data and solve related real-world problems; Compare, choose, and justify the appropriate measures of central tendency for a given set of data.	Chapter 13
M-07-E-04	Construct frequency distribution tables for given data (i.e., frequency, lower class limit, upper class limit, class interval and mid-point) and solve related real-world problems.	Chapter 13
M-07-E-05	Explain and compute the probability of: certain events, impossible events, and complement of an event (including real-world word problems).	Chapter 14

<u>Scheme of Work – New Syllabus Mathematics Book 2 (Updated 7th Edition)</u>

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
1, 2	**1** **Real numbers and Approximation**	**1.1** **Rational Numbers, Real Numbers, and Decimal Fractions (pp. 2 – 14)**	• Recall HCF and LCM • Use rational numbers and real numbers in a real- world context • Represent real numbers and rational numbers on a number line and order the numbers • Perform operations in real numbers, including using the calculator • Verify the properties of rational number	Use directed numbers in practical situations. Main Text – 'The number is Represent rational numbers on a number line marked out on the number line. Explain how the point on the number line is obtained. Order quantities by magnitude and demonstrate familiarity with the symbols =, ≠, <, >, ≤, ≥.	Thinking Time (p. 7) Investigation Terminating, Recurring, and Non-recurring Decimals (p. 9) Use of electronic calculator (p. 6) Investigation Some interesting facts about the Irrational Numbers (p. 10)	Investigation Interesting Facts about Irrational Number π (p.10)		Thinking Time (p. 7)
3			• Perform operations in real numbers, including using the calculator	Use the four operations for calculations ordering of operations and use of brackets.	Main Text (p. 11)	<u>Main Text – 'Alternatively, you may visit http://www. shinglee.com. sg/ Student Resources/</u>		

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
4		**1.2 Approximation (pp. 14 – 16)**	• With increasing degree of challenge, use the concept of place value for whole numbers, integers, rational numbers and decimal numbers. Use knowledge of rounding off to give an estimate to a calculation; to check the reasonableness of the solution.		Class discussion Actual and estimated values (p. 14)			Class Discussion – Actual and Approximated Value (p. 14) Practise Now 6 Q 2 (p. 15) Practise Now 7 Q 2 (p. 16) Ex 1B Q 5 – 7 (p. 17)
5		**1.3 Estimation**	• Use knowledge of rounding off to give an estimate to a calculation; to check the reasonableness of the solution.	Make estimates of numbers, quantities, and lengths	Use of smaller quantity to estimate Larger quantity (p. 20)	Investigation Use of smaller quantity to Estimate a Larger Quantity (p. 20) Performance task Estimation in Our daily Life (p. 21)	Review Exercise 1 Q 14 - 16 (p. 26)	
6	**2 Direct and Inverse Proportions**	**2.1 Rate and Ratio Rate and Average Rate Increasing and Decreasing Ratio (p. 28)**	• Distinguish between constant and average rate Solve Problems involving rate	Demonstrate an understanding of rate and average rate Use common measures of rate				

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
7		**2.2 Direct and Inverse Proportions (pp. 29 – 36)**	• Calculate increase and decrease in a ratio based on change in quantities • Solve real- life word problems related to direct and inverse proportions	Demonstrate and understanding of ratio and proportion Express increase and decrease in a ratio based on the change in quantities	Investigation Direct Proportion (p. 29) Investigation Direct Proportion (p.29) Investigation Inverse Proportion (p.32)			Investigation – Direct Proportion (p. 29) Class Discussions Real- Life examples of Quantities in Direct Proportion (p. 30) Investigation – Indirect Proportion (p. 32-33) Class Discussions Real- Life examples of Quantities in Inverse Proportion (p. 33)
8	**3 Application of Mathematics in Practical Situations**	**3.1 Profit and Loss (p. 39 – 40)**	• Identify and differentiate between selling price, cost price, loss, discount, profit percentage, and loss percentage	Solve problems involving profit and loss Use given data to solve problems on personal and small business finance, involving earning				
9		**3.2 Discount, Taxation, Commission, Zakat, and Ushr (pp. 41 – 50)**	• Explain commission, income tax, property tax, general sales tax, value added tax, zakat, and ushr	Solve problems involving commission, income tax, property tax, general sales tax, value added tax, zakat, and ushr	Investigation Discount, Service Charges, and GST (p. 43) Investigation Percentage Point			
9		**Miscellaneous**					Solutions for Challenge Yourself	

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
10	**4** **Sets**	**4.1 Venn Diagrams and Types of sets (pp. 54 – 59)**	• Use language notation, and Venn diagrams to represent different sets and their elements (natural numbers, whole numbers, integers, even numbers, odd numbers, and prime numbers)	Use set language, set notation and Venn diagrams to describe sets and represent relationships between sets Definition of sets: e.g. A = {x : x is a natural number}, $B = \{(x, y): y = mx + c\}$, $C = \{x : a \le x \le b\}$, D = {a, b, c, …}				Performance Task (p.67)
11		**4.2 Intersection of two sets (pp. 60 – 62)**	• Identify and differentiate between subsets and supersets proper and improper subsets equal and equivalent sets disjoint and overlapping set		Thinking Time (p.55) Class Discussion (p. 57)			
11		**4.3 Union of two sets (pp. 62 – 63)**	• Describe and perform operations on sets (union, intersection, difference and comparing					Thinking Time (p. 55)
12		**4.4 Combining Universal set, Subset, Intersection and Union of Sets (pp. 63 – 70)**	• Verify the following: $A \cap A' = \emptyset$ $A \cup A' = U$ $(A \cup B)' = A \cap B$ $(A \cap B)' = A \cup B$	e.g. $A = \{x : x$ is a natural number}, $B = \{(x, y): y = mx + c\}$, $C = \{x : a \le x \le b\}$, $D = \{a, b, c, \ldots\}$	Thinking Time (p. 67)			
12		**Miscellaneous**					Solutions for Challenge Yourself	

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
13	**5** **Number Patterns**	**5.1 General Term of a Number Sequency (pp. 74 – 76)**	• Recall recognising simple patterns from various number sequences • Recall how to continue a given number sequence and find: term to term rule position to term rule	Continue a given number sequence Recognise patterns sequences and relationship between different sequences Generalise sequence as simple algebraic statements	Class discussion (p. 75) Worked Example 2 (p. 76)			Worked Example 1 (p. 75) Class Discussion The Triangular Number Sequence (p. 77) Worked Example 3 (p. 76)
13		**5.2 Number Pattern in Real World Context (pp. 77 – 79)**	• Solve real-life problems involving number sequences and patterns		Investigation Fibonacci Sequence (p.77)			
14	**6** **Simplification and Expansion**	**6.1 Simplification of Linear Expressions with Fractional Coefficients (pp. 81 – 89)**	• Simplify linear expressions • Explore the following algebraic identities and use them to expand expressions $(a+b)^2 = a^2 + b^2 + 2ab$ $(a-b)^2 = a^2 + b^2 - 2ab$ $a^2 - b^2 = (a+b)(a-b)$	Expand product of algebraic expressions Addition of polynomials Product of polynomials Expand product of algebraic expressions	Main Text (p. 82 - 84) Worked Example 3 (p. 85) Worked Example 5 and 6 (p. 87) Class Discussion (p. 83) Worked Example 1 (P. 84)			Practise Now 3 (p. 85) Worked Example 7 and 6 (p. 87) Worked Example 2 (p. 84) Practise Now 1 Practise Now 2 (p. 84)
15	**7** **Expansion and Factorisation of Quadratic Expressions**	**7.1 Quadratic Expressions (pp. 93 – 98)**	• Recognise quadratic expressions				Practise Now (p. 95) Practise Now (p. 96) Practise Now (p. 97)	

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
15		**7.2 Expansion and Simplification of Quadratic Expressions (pp. 99 – 106)**	• Expand and simplify quadratic expressions		Class Discussion – Expansion of Quadratic Expressions of the Form $(a + b)(c + d)$ (p. 104)	Practise Now (p. 100) Class Discussion – Expansion of Quadratic Expressions of the Form $(a + b)(c + d)$ (p. 104)		
16		**7.3 Factorisation of Algebraic Expressions (pp. 107 – 115)**	• Use a multiplication frame to factorise quadratic expressions	Factorise where possible expressions of the form: $a^2 + 2ab + b^2$ $ax^2 + bx + c$	Main Text (pp. 107– 108)	Practise Now (p. 111)		
16		**Miscellaneous**					Solutions for Challenge Yourself	
17	**8 Linear Equations and Coordinate Geometry**	**8.1 Cartesian Coordinates (pp. 121 – 125)**	• State the coordinates of a point • Plot a point in a Cartesian plane	Demonstrate familiarity with Cartesian coordinates in two dimensions	Class Discussion – Battleship Game (Two Players) (p. 121) Internet Resources (p. 121) Class Discussion – Ordered Pairs (p. 122) Journal Writing (p. 123)	Internet Resources (p. 121) Story Time (p. 124)		Ordered Pairs (p. 122)

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
17		**8.2 Horizontal and vertical lines (pp. 126 – 131)**	• Recognise and state the equation of a horizontal and a vertical line	State the equation of a horizontal line and of a vertical line		Investigation – Equation of a Horizontal Line (pp. 126 - 127) Investigation – Equation of a Vertical Line (p128 - 129)		Investigation – Equation of a Horizontal Line (p. 130) Investigation – Equation of a Vertical Line (p.131)
18		**8.3 Graphs of Linear Equations (pp. 132 – 135)**	• Draw graphs of linear equations in the form $ax + by = k$	Draw graphs from given data				Challenge Yourself (p. 139)
19	**9 Time and Speed**	**9.1 Time (pp. 141 – 143)**	• Solve problems involving time	Calculate times in terms of the 24-hour and 12- hour clock Read clocks, dials and timetables	Main Text (pp. 141 - 142)			
		9.2 Speed (pp. 145 – 150)	• Discuss special types of rates such as speed and rate of rotation • Solve problems involving speed	Solve problems involving average speed	Main Text (pp. 144 - 146) Performance Task (p. 147) Thinking Time (p.147)	Internet Resource (p.145) Performance Task (p.147)		Just for Fun (p. 144) Thinking Time (p. 146)
		Miscellaneous					Solutions for Challenge Yourself	

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
20	**10 Triangles, Quadrilaterals and Polygon**	**10.1 Triangles (pp. 153 – 162)**	• Identify different types of triangles and state their properties • Solve problems involving the properties of triangles	Use and interpret the geometrical terms: interior and exterior angles Use and interpret vocabulary of triangles Calculate unknown angles and give simple explanations using angle properties of triangles	Thinking Time (p. 154)			Thinking Time (p. 154)
21		**10.2 Quadrilaterals (pp. 163 – 171)**	• Identify different types of special quadrilaterals and state their properties • Solve problems involving the properties of special quadrilaterals	Calculate unknown angles and give simple explanations using angle properties of quadrilaterals Use and interpret vocabulary of quadrilaterals	Investigation – Properties of Special Quadrilaterals (pp. 163) Thinking Time (p. 165)	Investigation –Properties of Special Quadrilaterals (p. 163)		Thinking Time (p. 165) Just for Fun (p. 166)

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
22		**10.3 Polygons (pp. 172 - 183)**	• Identify different types of polygons and state their properties • Solve problems involving the properties of polygons	Use and interpret vocabulary of polygons Calculate unknown angles and give simple explanations using angle properties of regular and irregular polygons	Investigation – Sum of Interior Angles of a Polygon (pp. 176 -177) Investigation – Sum of Exterior Angles of a Pentagon (p 179) Thinking Time (p. 179) Class Discussion – Naming of Polygons (p. 173) Thinking Time (p. 174) Investigation – Properties of a Regular Polygon (p. 175) Journal Writing (p. 175)	Class Discussion – Naming of Polygons (p. 173) Internet Resources (p. 173) Investigation – Sum of Exterior Angles of a Pentagon (p. 179)		Main Text – 'The shapes shown in Fig. 10.8 are *not* polygons. Why?' (p. 172) Thinking Time (p. 174) Journal Writing (p. 175) Investigation – Sum of Exterior Angles of a Pentagon (p..179) Thinking Time (p.179)
23		**10.4 Translation (pp. 183 – 185)**	• Translate an object				Practise Now (p.186)	
24		**10.5 Circles (pp. 186 – 190)**	• Elements of a circle • Properties of a circle	Identify centre, circumference, radius, chord, arc, and segment (p. 186- 187) Fundamental properties of a circle related to chord and diameter (p. 188- 189)				
24		**Miscellaneous**					Solutions for Challenge Yourself	

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
25	**11** **Symmetry**	**11.1 Symmetry in Triangles, Quadrilaterals and Polygons** **(pp. 196 – 202)**	• Make use of the symmetrical properties of triangles, quadrilaterals and regular polygons		Investigation – Symmetry in Triangles (pp. 196 _197)) Investigation – Symmetry in Regular Polygons (pp. 200 - 201)			Investigation – Symmetry in Triangles (pp. 196 - 197) Investigation – Symmetry in Regular Polygons (pp. 200 - 201)
26	**12** **Mensuration: Perimeter, Area, Surface Area, and Volume**	**12.1 Area and perimeter** **(p. 205)**	• Convert between cm^2 and m^2 Area and perimeter of a circle Area and perimeter of shaded region • Solve problems involving the perimeter and area of composite figures	Use current units of mass, length and area in practical situations and express quantities in terms of larger or smaller units Solve problems involving the perimeter and area of a rectangle and triangle, and the circumference and area of a circle	Practise Now (p. 206) Practise Now (p. 208) Practise Now (p. 209)			
27		**12.2 Volume and Surface Area of Prisms** **(pp. 348 – 353)**	• Find the volume and surface area of prisms	Solve problems involving the surface area and volume of a prism	Thinking Time (p. 211			Thinking Time (p. 211) Main Text – 'Can you find a relationship between the volume of a prism and the area of its cross section?' (p. 212)

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
28		**12.3 Volume and Surface Area of Cylinders (pp. 217 - 222)**	• Convert between cm^3 and m^3 • Find the volume and surface area of cylinders	Use current units of volume and capacity in practical situations and express quantities in terms of larger or smaller units Solve problems involving the surface area and volume of a cylinder	Thinking Time (p. 216) Investigation Compare between a cylinder and a prism (p. 217) Thinking Time (p. 219) Class Discussion – Total Surface Area of Other Types of Cylinders (p. 219)			Thinking Time (p.216) Thinking Time (p. 219) Class Discussion – Total Surface Area of Other Types of Cylinders (p. 219)
29	**13 Averages of Statistical Data**	**13.1 Statistical Diagrams (p. 226)**						
		13.2 Histogram (p. 227)	• Bar graphs and Histograms		Journal Writing (p. 231)			
29		**13.3 Histograms for Grouped Data (pp. 234 - 240)**	• Construct and interpret data from histograms • Evaluate the purposes and appropriateness of the use of different statistical diagrams • Explain why some statistical diagrams can lead to a mis-interpretation of data	Construct and interpret histograms with equal and unequal intervals Construct and interpret frequency polygons	Journal Writing (p. 231) Class Discussion Evaluation of Statistical Diagrams Performance Task (p. 236) Main Text (p. 477)			Journal Writing (p. 231) Performance Task (p. 236)

Week (5 classes × 45 min)	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Additional Resources	Reasoning, Communication and Connection
30		**13.4 Mean, Median, and Mode (pp. 241 – 248)**	• Find the mean of a set of data • Calculate and estimate for the mean • Find the median of a set of data • Find the class interval where the median lies • Find the mode of a set of data • State the modal class of a set of grouped data • Evaluate the purposes and appropriateness of the use of mean, median and mode	Calculate the mean for individual and discrete data Calculate an estimate of the mean for grouped and continuous data Calculate the median for individual and discrete data Calculate the mode for individual and discrete data Identify the modal class from a grouped frequency distribution Distinguish between the purposes for which the mean, median and mode are used				
31		**Miscellaneous**					Solutions for Challenge Yourself	
32	**14 Probability**	**14.1 Further probability of Single Events (pp. 252 - 257)**	• Solve problems involving the probability of single events					
		Miscellaneous					Challenge Yourself	

Chapter 1 Real Numbers and Approximation

TEACHING NOTES

Suggested Approach

Students have already learnt how to work with integers. In this chapter, students will work with real numbers and their real-life applications while also revising previous topics, such as HCF and LCM, and using rational number for estimation and approximation.

Teachers can give students a real-life example when an approximated or estimated value is used before getting them to discuss occasions when they use approximation and estimation in their daily lives. In this chapter, they will learn how to round off numbers to a specified numbers. Students will also learn how to carry out estimation through worked examples that involve situations in real-world contexts.

Section 1.1: Rational Numbers, Real Numbers, and Decimal Fractions

Traditionally, real numbers are classified as either rational or irrational numbers. Another way to classify real numbers is according to whether their decimal forms are terminating, recurring, or non-recurring. If teachers show students the first million digits of π (see page 10 of the textbook), many students may be surprised that π has so many digits! This suggests that students do not know that π has an infinite number of decimal places. Teachers may wish to celebrate Pi Day with students on March 14 by talking about π or singing the Pi song.

Section 1.2: Approximation

To make learning of mathematics relevant, students should know some reasons why they need to use approximations in their daily lives (see Class Discussion: Actual and Approximated Values).

Teachers should do a recap with students on what they have learnt in primary school, i.e. how to round off numbers to the nearest tenth, whole number and 10 etc.

Section 1.3: Estimation

Teachers can impress upon students that there are differences between approximation and estimation. Since students need to be aware when an answer is obviously wrong, estimation allows them to check the reasonableness of an answer obtained from a calculator (see Worked Example 10).

Students will also learn an important estimation strategy: use a smaller quantity to estimate a larger quantity (see Investigation: Use of a Smaller Quantity to Estimate a Larger Quantity).

Teachers should get students to work in groups to estimate quantities in a variety of contexts, compare their estimates and share their estimation strategies with one another. (see the performance task on page 21 of the textbook).

WORKED SOLUTIONS

Thinking Time (Page 07)

(a) Any integer m can be expressed in the form $\frac{m}{1}$, e.g. $2 = \frac{2}{1}$ and $-3 = \frac{-3}{1}$.

In particular, the integer 0 can be expressed in the form $\frac{0}{n}$, where n is any integer except 0.

(b) There is more than one way to express a decimal in the form $\frac{a}{b}$,

e.g. $0.5 = \frac{1}{2} = \frac{2}{4} = \frac{3}{6} \ldots$ and $0.333\ldots = \frac{1}{3} = \frac{2}{6} = \frac{3}{9} \ldots$

Investigation (Terminating, Recurring and Non-Recurring Decimals)

Group 1	Group 2	Group 3
$\frac{9}{4} = 2.25$	$\frac{1}{3} = 0.333\ 333\ 333\ 3$	$\frac{1}{\sqrt{2}} = 0.707\ 106\ 781\ 2$
$-3\frac{1}{8} = -3.125$	$-\frac{123}{99} = -1.242\ 424\ 242$	$-\sqrt[3]{5} = -1.709\ 975\ 947$
$\frac{63}{64} = 0.984\ 375$	$\frac{22}{7} = 3.142\ 857\ 143$	$\pi = 3.141\ 592\ 654$

Table 1.1

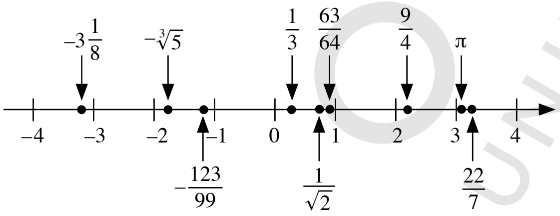
1. Based on the calculator values, π is not equal to $\frac{22}{7}$.
2. For each of the numbers in Group 2, some digits after the decimal point repeat themselves indefinitely. The numbers in Group 2 are rational numbers.
3. For each of the numbers in Group 1, the digits after the decimal point terminate. The numbers in Group 1 are rational numbers.
   For each of the numbers in Group 3, the digits after the decimal point do not repeat but they continue indefinitely. The numbers in Group 3 are irrational numbers.
4. 

Fig. 1.2

5. $\frac{22}{7}, \pi, \frac{9}{4}, \frac{63}{64}, \frac{1}{\sqrt{2}}, \frac{1}{3}, -\frac{123}{99}, -\sqrt[3]{5}, -3\frac{1}{8}$

Investigation (Some Interesting Facts about the Irrational Number π)

1. The 1 000 000th digit of π is 1.
2. The 5 000 000 000 000th digit of π is 2.
3. Lu Chao, a graduate student from China, took 24 hours and 4 minutes to recite π to 67 890 decimal places in 2005.

Class Discussion (Actual and Approximated Values)

1. The actual values indicated in the article include '7 267 582 passengers', and one terminal' while approximated values include 'over 25 airlines' and '12 million passengers'. Actual values are exact numbers while approximated values are values which are usually rounded off.
2. **(a)** It is not necessary to specify the actual number of airlines, as an approximation is sufficient to show that Jinnah International Airport carters many airlines.
   (b) A headline serves as a brief summary of the article to draw readers' attentions, thus it is more appropriate to use an approximated value instead of the actual value.

Investigation (Use of a Smaller Quantity to Estimate a Larger Quantity)

For this investigation, the smaller box used is of length 9.2 cm, width 5.6 cm and height 2.7 cm.

Three trials are carried out to find the average number of 5-rupee coin that can fill the box. The result of each trial is shown in the table.

Trial	Number of 5-rupee coin
1	294
2	280
3	284

Average number of 5-rupee coin that can fill the smaller box

$= \frac{294 + 280 + 284}{3}$

$= \frac{858}{3}$

$= 286$

Volume of smaller box $= 9.2 \times 5.6 \times 2.7$
$= 139.104\ \text{cm}^3$

Volume of tank $= 50 \times 23 \times 13$
$= 14\ 950\ \text{cm}^3$

Number of 5-rupee coin coins that can fill the tank $= \frac{286}{139.104} \times 14950$
$= 30\ 737$ (to the nearest whole number)

$\therefore$ Amount of money in the tank $= 30\ 737 \times 5$
= PKR 153 685
= PKR 150 000 (to the nearest whole number)

Performance Task (Estimation in Our Daily Lives)

1. Use surveys, questionnaires or verbal questioning to find out the number of hours spent surfing the Internet by each student in the class on *a weekday* and on *a Saturday or Sunday*. Ensure that students have a common understanding of the phrase 'surfing the Internet'.

Calculate the total number of hours spent surfing the Internet by *all* the students in the class on *a weekday* and on *a Saturday or Sunday*.

Total amount of time spent surfing the Internet by all the students in the class on a weekday = x hours

Total amount of time spent surfing the Internet by all the students in the class on a Saturday or Sunday = y hours

Estimate the total number of hours spent surfing the Internet by *all* the students in the class in *a month*. Assume that the average number of weekdays and the average number of Saturdays and Sundays in a month are 22 and 8 respectively.

Total amount of time spent surfing the Internet by all the students in the class in a month ≈ $(22x + 8y)$ hours

2. Assume that there are 8 slices in a large pizza. Use verbal questioning to find out the number of slices needed to feed *one class* (e.g. about 40 students) in the school when they go for an excursion.

Number of slices needed to feed one class in the school = x

Number of pizzas needed to feed one class in the school = $\frac{x}{8}$

Find out the number of classes in the school. Ensure that there is approximately the same number of students in each class, e.g. 40 students.

Number of classes in the school = y

Estimate the amount of pizza needed to feed all the students in the school during an excursion.

Total number of pizzas needed to feed all the students in the school $\approx \frac{xy}{8}$

3. Find out the opening hours of the drinks stall on a weekday and determine the durations of the peak (e.g. recess and lunchtime) and non-peak periods respectively.

Duration of peak period = x hours

Duration of non-peak period = y hours

Find out the amount of money collected by the drinks stall in *half an hour* during the peak period and *half an hour* during the non-peak period.

Amount of money collected by drinks stall in half an hour during peak period = PKR p

Amount of money collected by drinks stall in half an hour during non-peak period = PKR q

Estimate the total amount of money collected for both the peak and non-peak periods.

Total amount of money collected by drinks stall during peak period ≈ PKR $2px$

Total amount of money collected by drinks stall during non-peak period ≈ PKR $2qy$

Hence,

Total amount of money collected by drinks stall on a weekday ≈ PKR $(2px + 2qy)$

Practise Now 1

$$\begin{aligned}
-2\frac{3}{4} + \left(-\frac{5}{6}\right) - \left(-\frac{2}{3}\right) &= -2\frac{3}{4} - \frac{5}{6} + \frac{2}{3} \\
&= -\frac{11}{4} - \frac{5}{6} + \frac{2}{3} \\
&= -\frac{33}{12} - \frac{10}{12} + \frac{8}{12} \\
&= \frac{-33 - 10 + 8}{12} \\
&= \frac{-43 + 8}{12} \\
&= \frac{-35}{12} \\
&= -2\frac{11}{12}
\end{aligned}$$

Practise Now 2a

$$\begin{aligned}
1\frac{3}{4} \times \left[\frac{6}{5} + \left(-\frac{1}{2}\right)\right] &= \frac{7}{4} \times \left(\frac{6}{5} - \frac{1}{2}\right) \\
&= \frac{7}{4} \times \left(\frac{12}{10} - \frac{5}{10}\right) \\
&= \frac{7}{4} \times \frac{7}{10} \\
&= \frac{49}{40} \\
&= 1\frac{9}{40}
\end{aligned}$$

Practise Now 2b

$$1\frac{3}{4} \times \left[\frac{6}{5} + \left(-\frac{1}{2}\right)\right] = 1\frac{9}{40}$$

Practise Now 3

(a) $3.2 - (-1.6) = 3.2 + 1.6$

$= 4.8$

(b) $1.3 + (-3.5) = -2.2$

(c)
$$\begin{aligned}
\frac{0.12}{0.4} \times \frac{-0.23}{0.6} &= \frac{1.2}{4} \times \frac{-0.23}{0.6} \\
&= 0.3 \times \frac{-0.23}{0.6} \\
&= {}^{1}\cancel{3} \times \frac{-0.23}{\cancel{6}_{2}} \\
&= -0.115
\end{aligned}$$

(d) $-0.3^2 \times \dfrac{4.5}{-2.7} - 0.65 = -0.3^2 \times \dfrac{45}{-27} - 0.65$

$= \overset{0.03}{\cancel{-0.09}} \times \dfrac{\cancel{45}^{5}}{-\cancel{27}_{\cancel{3}\,1}} - 0.65$

$= 0.15 - 0.65$

$= -0.5$

Practise Now 4

(a) $-3.1 > -5.1$

(b) $-7.2 < -2.1$

(c) $0.4 > -0.4$

(d) $-0.06 > -1.00$

Practise Now 5

$\dfrac{\pi \times 0.7^2}{\sqrt[3]{2.4} + 1\frac{3}{10}} = 0.583$ (to 3 d.p.)

Practise Now 6

1. **(a)** 3 409 725 = 3 409 730 (to the nearest 10)
   (b) 3 409 725 = 3 409 700 (to the nearest 100)
   (c) 3 409 725 = 3 410 000 (to the nearest 1000)
   (d) 3 409 725 = 3 410 000 (to the nearest 10 000)
2. Largest possible number of overseas visitors = 11 649 999
   Smallest possible number of overseas visitors = 11 550 000

Practise Now 7

1. **(a)** 78.4695 = 78.5 (to 1 d.p.)
   (b) 78.4695 = 78 (to the nearest whole number)
   (c) 78.4695 = 78.47 (to the nearest hundredth)
   (d) 78.4695 = 78.470 (to the nearest 0.001)
2. No, I do not agree with Salman. 8.40 is rounded off to 2 decimal places which is more accurate than 8.4 which is rounded off to 1 decimal place.

Practise Now 8

Cost of 450 kWh of electricity = 450 × PKR 25.53
= PKR 10588.5

Cost of 38 m^3 of water = 38 × PKR 1.17
= PKR 44.46

Total amount of money the household has to pay
= PKR 10588.5 + PKR 44.46
= PKR 10 632.96
= PKR 10 000 (to the nearest 10 000)

Practise Now 9

1. $798 \times 195 \approx 800 \times 200$
   $= 160\ 000$
   ∴ Nadia's answer is not reasonable.
2. **(a)** $5712 \div 297 \approx 5700 \div 300$
   $= 19$
   Using a calculator, $5712 \div 297 = 19.2$ (to 3 s.f.).
   ∴ The estimated value is close to the actual value.
   (b) $\sqrt{63} \times \sqrt[3]{129} \approx \sqrt{64} \times \sqrt[3]{125}$
   $= 8 \times 5$
   $= 40$
   Using a calculator, $\sqrt{63} \times \sqrt[3]{129} = 40.1$ (to 3 s.f.).
   ∴ The estimated value is close to the actual value.
3. Time taken to drive from Lahore to Karachi $= \dfrac{1445}{80}$
   $\approx \dfrac{1450}{80}$ hours

Practise Now 10

PKR 1000 = S$5

PKR 1 = S$ $\dfrac{5}{1000}$

PKR 25000 = S$ $\dfrac{5}{1000} \times 25000$

=S$ 125

Practise Now 11

For option A, 300 m*l* costs PKR 580

Then, 150 *ml* will lost about PKR 290 and 50 *ml* will cost about PKR 96.3.

Since 300 *ml* + 50 *ml* will lost PKR 580-96.3 = PKR 676.3

Where as for option B, 300 *ml* + 50 *ml* cost PKR 1040
which is greater than PKR 676.3.

∴ So option A is better value for money.

Practise Now 12

Percentage of shaded region $= \dfrac{2}{3} \times 100\%$

$= 66\dfrac{2}{3}\ \%$

Practise Now 13

(a)

3	441
3	147
9	49
7	7
	1

$\sqrt{441} = \sqrt{3 \times 3 \times 7 \times 7}$

$\sqrt{441} = \sqrt{3^2 \times 7^2}$

$441 = 3 \times 7$

$= 21$

(b) $\sqrt{1.44} = \sqrt{\dfrac{144}{100}}$

2	144
2	72
2	36
2	18
3	9
3	3
	1

$= \dfrac{\sqrt{\overline{2\times 2}\times\overline{2\times 2}\times\overline{3\times 3}}}{\sqrt{10\times 10}}$

$= \dfrac{2\times 2\times 3}{10}$

$= \dfrac{12}{10}$

$= 1.2$

(c) $\sqrt{12.25} = \sqrt{\dfrac{1225}{100}}$

5	1225
5	245
7	49
7	7
	1

$= \dfrac{\sqrt{\overline{5\times 5}\times\overline{7\times 7}}}{\sqrt{10\times 10}}$

$= \dfrac{5\times 7}{10}$

$= \dfrac{35}{10}$

$= 3.5$

Exercise 1A

1. **(a)** $-\dfrac{8}{5} - \left(-2\dfrac{1}{4}\right) - \dfrac{1}{2} = -\dfrac{8}{5} + 2\dfrac{1}{4} - \dfrac{1}{2}$

$= -\dfrac{8}{5} + \dfrac{9}{4} - \dfrac{1}{2}$

$= -\dfrac{32}{20} + \dfrac{45}{20} - \dfrac{10}{20}$

$= \dfrac{-32 + 45 - 10}{20}$

$= \dfrac{3}{20}$

(b) $6\dfrac{1}{5} - \left(-\dfrac{3}{4}\right) + \left(-4\dfrac{1}{10}\right) = 6\dfrac{1}{5} + \dfrac{3}{4} - 4\dfrac{1}{10}$

$= \dfrac{31}{5} + \dfrac{3}{4} - \dfrac{41}{10}$

$= \dfrac{124}{20} + \dfrac{15}{20} - \dfrac{82}{20}$

$= \dfrac{124 + 15 - 82}{20}$

$= \dfrac{57}{20}$

$= 2\dfrac{17}{20}$

(c) $4\dfrac{2}{7} + \left(-6\dfrac{1}{3}\right) - \left(-\dfrac{4}{21}\right) = 4\dfrac{2}{7} - 6\dfrac{1}{3} + \dfrac{4}{21}$

$= \dfrac{30}{7} - \dfrac{19}{3} + \dfrac{4}{21}$

$= \dfrac{90}{21} - \dfrac{133}{21} + \dfrac{4}{21}$

$= \dfrac{90 - 133 + 4}{21}$

$= \dfrac{-39}{21}$

$= \dfrac{-13}{7}$

$= -1\dfrac{6}{7}$

(d) $-4 + \left(-3\dfrac{1}{8}\right) + \left(-\dfrac{4}{3}\right) = -4 - 3\dfrac{1}{8} - \dfrac{4}{3}$

$= -4 - 3\dfrac{1}{8} - 1\dfrac{1}{3}$

$= -4 - 3\dfrac{3}{24} - 1\dfrac{8}{24}$

$= -8\dfrac{11}{24}$

(e) $-\dfrac{1}{5} + 2\dfrac{1}{4} + \left(-\dfrac{7}{2}\right) = -\dfrac{1}{5} + 2\dfrac{1}{4} - \dfrac{7}{2}$

$= -\dfrac{1}{5} + \dfrac{9}{4} - \dfrac{7}{2}$

$= -\dfrac{4}{20} + \dfrac{45}{20} - \dfrac{70}{20}$

$= \dfrac{-4 + 45 - 70}{20}$

$= \dfrac{-29}{20}$

$= -1\dfrac{9}{20}$

2. **(a)** $-\dfrac{8}{5} - \left(-2\dfrac{1}{4}\right) - \dfrac{1}{2} = \dfrac{3}{20}$

(b) $6\dfrac{1}{5} - \left(-\dfrac{3}{4}\right) + \left(-4\dfrac{1}{10}\right) = 2\dfrac{17}{20}$

(c) $4\dfrac{2}{7} + \left(-6\dfrac{1}{3}\right) - \left(-\dfrac{4}{21}\right) = -1\dfrac{6}{7}$

(d) $-4 + \left(-3\dfrac{1}{8}\right) + \left(-\dfrac{4}{3}\right) = -8\dfrac{11}{24}$

(e) $-\frac{1}{5} + 2\frac{1}{4} + \left(-\frac{7}{2}\right) = -1\frac{9}{20}$

3. (a) $-\frac{5}{7} \times \left(-\frac{28}{15} + 1\frac{2}{3}\right) = -\frac{5}{7} \times \quad -\frac{28}{15} + \frac{5}{3}$

$= -\frac{5}{7} \times \quad -\frac{28}{15} + \frac{25}{15}$

$= -\frac{5}{7} \times \quad -\frac{3}{15}$

$= -\frac{{}^{1}\cancel{5}}{7} \times \quad -\frac{1}{\cancel{5}_{1}}$

$= \frac{1}{7}$

(b) $\left[-\frac{1}{4} - \left(-\frac{1}{3}\right)\right] \div \left(\frac{1}{4} - \frac{1}{3}\right) = \left(-\frac{1}{4} + \frac{1}{3}\right) \div \left(\frac{1}{4} - \frac{1}{3}\right)$

$= \left(-\frac{3}{12} + \frac{4}{12}\right) \div \left(\frac{3}{12} - \frac{4}{12}\right)$

$= \frac{1}{12} \div \left(-\frac{1}{12}\right)$

$= \frac{1}{{}_{1}\cancel{12}} \times (-\cancel{12}^{1})$

$= -1$

(c) $10 - \frac{15}{8} \times \left(\frac{3}{2} \div 4\frac{1}{2}\right) + \left(-\frac{1}{4}\right)$

$= 10 - \frac{15}{8} \times \left(\frac{3}{2} \div \frac{9}{2}\right) + \left(-\frac{1}{4}\right)$

$= 10 - \frac{15}{8} \times \left(\frac{{}^{1}\cancel{3}}{{}_{1}\cancel{2}} \times \frac{\cancel{2}^{1}}{\cancel{9}_{3}}\right) + \left(-\frac{1}{4}\right)$

$= 10 - \frac{{}^{5}\cancel{15}}{8} \times \frac{1}{\cancel{3}_{1}} + \left(-\frac{1}{4}\right)$

$= 10 - \frac{5}{8} + \left(-\frac{1}{4}\right)$

$= 10 - \frac{5}{8} - \frac{1}{4}$

$= 10 - \frac{5}{8} - \frac{2}{8}$

$= \left(9 + \frac{8}{8}\right) - \frac{5}{8} - \frac{2}{8}$

$= 9\frac{1}{8}$

(d) $\left(\frac{1}{2}\right)^{3} - \left(\frac{3}{4}\right)^{2} + \left(-\frac{3}{4}\right) = \frac{1}{8} - \frac{9}{16} + \left(-\frac{3}{4}\right)$

$= \frac{1}{8} - \frac{9}{16} - \frac{3}{4}$

$= \frac{2}{16} - \frac{9}{16} - \frac{12}{16}$

$= \frac{2 - 9 - 12}{16}$

$= \frac{-19}{16}$

$= -1\frac{3}{16}$

(e) $\frac{1}{3} + \frac{4}{9} \times \left(-\frac{1}{2}\right)^{2} = \frac{1}{3} + \frac{{}^{1}\cancel{4}}{9} \times \frac{1}{\cancel{4}_{1}}$

$= \frac{1}{3} + \frac{1}{9}$

$= \frac{3}{9} + \frac{1}{9}$

$= \frac{4}{9}$

(f) $\left(\frac{3}{2}\right)^{2} \times \left(\frac{1}{15} - 2\frac{1}{3}\right) = \left(\frac{3}{2}\right)^{2} \times \left(\frac{1}{15} - \frac{7}{3}\right)$

$= \left(\frac{3}{2}\right)^{2} \times \left(\frac{1}{15} - \frac{35}{15}\right)$

$= \left(\frac{3}{2}\right)^{2} \times \left(-\frac{34}{15}\right)$

$= \frac{{}^{3}\cancel{9}}{{}_{2}\cancel{4}} \times \left(-\frac{\cancel{34}^{17}}{\cancel{15}_{5}}\right)$

$= -\frac{51}{10}$

$= -5\frac{1}{10}$

4. (a) $-\frac{5}{7} \times \left(-\frac{28}{15} + 1\frac{2}{3}\right) = \frac{1}{7}$

(b) $\left[-\frac{1}{4} - \left(-\frac{1}{3}\right)\right] \div \left(\frac{1}{4} - \frac{1}{3}\right) = -1$

(c) $10 - \frac{15}{8} \times \left(\frac{3}{2} \div 4\frac{1}{2}\right) + \quad -\frac{1}{4} \quad = 9\frac{1}{8}$

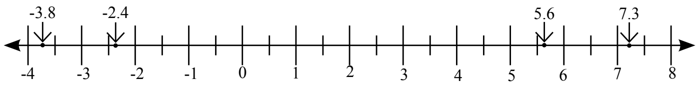
(d) $\left(\frac{1}{2}\right)^{3} - \left(\frac{3}{4}\right)^{2} + \left(-\frac{3}{4}\right) = -1\frac{3}{16}$

(e) $\frac{1}{3} + \frac{4}{9} \times \left(-\frac{1}{2}\right)^{2} = \frac{4}{9}$

(f) $\left(\frac{3}{2}\right)^{2} \times \left(\frac{1}{15} - 2\frac{1}{3}\right) = -5\frac{1}{10}$

5. (a) 5 (b) 6
(c) $\frac{2}{3}$ (d) 123
(e) $\frac{5}{6}$ (f) $\frac{8}{9}$

6.



7.

8. **(a)** Commutative property of addition slates that, a+b = b+a

$\frac{3}{5}+\frac{7}{3}=\frac{7}{3}+\frac{3}{5}$

L.H.S

$\frac{3}{5}+\frac{7}{3}=\frac{9}{15}+\frac{35}{15}$

$=\frac{44}{15}$

$=2\,\frac{14}{15}$

R.H.S

$\frac{7}{3}+\frac{3}{5}=\frac{35}{15}+\frac{9}{15}$

$=\frac{44}{15}$

$=2\frac{14}{15}$

Since, L.H.S = R.H.S

Hence, the commutative property of addition is verified.

Commutative property of multiplication states that,

$a\times b=b\times a$

$\frac{3}{5}\times\frac{7}{3}=\frac{7}{3}\times\frac{3}{5}$

L.H.S

$\frac{3}{5}\times\frac{7}{3}=1\frac{2}{5}$

R.H.S

$\frac{7}{3}\times\frac{3}{5}=\frac{7}{5}$

$=1\frac{2}{5}$

Since, L.H.S = R.H.S

Hince, the commutative property of multiplication is approved.

(b) $\frac{8}{5}+\frac{7}{5}=\frac{7}{5}+\frac{8}{5}$

L.H.S

$\frac{8}{5}+\frac{7}{5}=\frac{15}{5}=3$

R.H.S

$\frac{7}{5}+\frac{8}{5}=\frac{15}{5}=3$

Since, L.H.S = R.H.S

Hence, the commutative property is verified.

$\frac{8}{5}\times\frac{7}{5}=\frac{7}{5}\times\frac{8}{5}$

L.H.S

$\frac{8}{5}\times\frac{7}{5}=\frac{56}{25}$

$=2\,\frac{6}{25}$

R.H.S

$\frac{7}{5}\times\frac{8}{5}=\frac{56}{25}$

$=2\,\frac{6}{25}$

Since, L.H.S = R.H.S

Hence, the commutative property of multiplication is approved.

9. Associative property of addition states that

$a+(b+\mathrm{c})=(a+b)+\mathrm{c}$

$\frac{1}{2}+\left(\frac{3}{4}+\frac{5}{3}\right)=\left(\frac{1}{2}+\frac{3}{4}\right)+\frac{5}{3}$

L.H.S

$\frac{1}{2}+\left(\frac{3}{4}+\frac{5}{3}\right)$

$=\frac{1}{2}+\left(\frac{9}{12}+\frac{20}{12}\right)$

$=\frac{1}{2}+\frac{29}{12}$

$=\frac{6}{12}+\frac{29}{12}$

$=\frac{35}{12}$

$=2\,\frac{11}{12}$

R.H.S

$\left(\frac{1}{2}+\frac{3}{4}\right)+\frac{5}{3}$

$=\left(\frac{2}{4}+\frac{3}{4}\right)+\frac{5}{3}$

$=\frac{5}{4}+\frac{5}{3}$

$=\frac{15}{12}+\frac{20}{12}$

$=\frac{35}{12}$

Since, L.H.S = R.H.S

Hence, the association property of addition is approved.

Associative property of multiplication states that,

$a\times(b\times\mathrm{c})=(a\times b)\times\mathrm{c}$

$\frac{1}{2}\times\left(\frac{3}{4}\times\frac{5}{3}\right)=\left(\frac{1}{2}\times\frac{3}{4}\right)\times\frac{5}{3}$

L.H.S

$\frac{1}{2} \times \left(\frac{3}{4} \times \frac{5}{3}\right)$

$= \frac{1}{2} \times \frac{5}{4}$

$= \frac{5}{8}$

R.H.S

$\left(\frac{1}{2} \times \frac{3}{4}\right) \times \frac{5}{3}$

$= \frac{3}{8} \times \frac{5}{3}$

$= \frac{5}{8}$

Since, L.H.S = R.H.S

Hence, the associative property of multiplication is approved.

10. (a) Distribute property of multiplication over addition states that,

a × (b + c) = a×b + a×c

$\frac{2}{3} \times \left(\frac{6}{7} + \frac{9}{10}\right) = \frac{2}{3} \times \frac{6}{7} + \frac{2}{3} \times \frac{9}{10}$

L.H.S

$\frac{2}{3} \times \left(\frac{6}{7} + \frac{9}{10}\right)$

$= \frac{2}{3} \times \frac{60 + 63}{70}$

$= \frac{2}{3} \times \frac{123}{70}$

$= \frac{41}{35}$

$= 1\frac{6}{35}$

R.H.S

$\frac{2}{3} \times \frac{6}{7} + \frac{2}{3} \times \frac{9}{10}$

$= \frac{4}{7} + \frac{3}{5}$

$= \frac{20 + 21}{35}$

$= \frac{41}{35}$

$= 1\frac{6}{35}$

Since L.H.S = R.H.S,

Hence the property is verified.

(b) $\frac{8}{13} \times \left(\frac{5}{10} + \frac{2}{24}\right) = \frac{8}{13} \times \frac{5}{12} + \frac{8}{13} \times \frac{2}{24}$

L.H.S

$\frac{8}{13} \times \frac{10 + 2}{24}$

$= \frac{8}{13} \times \frac{12}{24}$

$= \frac{4}{13}$

R.H.S

$\frac{8}{13} \times \frac{5}{12} + \frac{8}{13} \times \frac{2}{24}$

$= \frac{10}{39} + \frac{2}{39}$

$= \frac{12}{39}$

$= \frac{4}{13}$

11. (a) $\frac{0.15}{0.5} \times \left(\frac{-0.16}{1.2}\right) = \frac{1.5}{5} \times \left(\frac{-0.16}{1.2}\right)$

$= 0.3 \times \left(\frac{-0.16}{1.2}\right)$

$= {}^{1}\cancel{3} \times \left(\frac{-0.16}{\cancel{12}_{4}}\right)$

$= -0.04$

(b) $\frac{0.027}{0.03} \times \left(\frac{1.4}{-0.18}\right) = \frac{2.7}{3} \times \left(\frac{1.4}{-0.18}\right)$

$= 0.9 \times \left(\frac{1.4}{-0.18}\right)$

$= {}^{1}\cancel{9} \times \left(\frac{14}{-\cancel{18}_{2}}\right)$

$= -7$

(c) $-0.4^2 \times \left(\frac{-1.3}{0.8}\right) - 0.62 = -0.4^2 \times \left(\frac{-13}{8}\right) - 0.62$

$= {}^{0.02}\cancel{-0.16} \times \left(\frac{-13}{\cancel{8}_{1}}\right) - 0.62$

$= 0.26 - 0.62$

$= -0.36$

(d) $(-0.2)^3 \times \frac{27}{1.6} + 0.105 = (-0.2)^3 \times \frac{270}{16} + 0.105$

$= {}^{0.001}\cancel{-0.008} \times \frac{\cancel{270}^{135}}{\cancel{16}\ \cancel{8}_{1}} + 0.105$

$= -0.135 + 0.105$

$= -0.03$

12. (a) $\left(\frac{\pi + 5\frac{1}{2}}{-2.1}\right)^2 = 16.934$ (to 3 d.p.)

(b) $-\frac{\pi^2 + \sqrt{2}}{7 - \sqrt[3]{4}} = -2.085$ (to 3 d.p.)

(c) $\frac{\sqrt[3]{14^2 + 19^2}}{\pi - 4.55} = -5.842$ (to 3 d.p.)

(d) $\sqrt{\frac{4.6^2 + 8.3^2 - \left(6\frac{1}{2}\right)^2}{2 \times 4.6 - 8.3}} = 7.288$ (to 3 d.p.)

13. Amount of time Nadia spent on visiting old folks' homes

$= \frac{4}{7} \times 8\frac{1}{16}$

$= \frac{^1\cancel{4}}{7} \times \frac{129}{\cancel{16}_4}$

$= \frac{129}{28}$

$= 4\frac{17}{28}$ hours

14. **(a)** $|(-9)| = 9$
(b) $|6 - 3| = 3$
(c) $|-5 - 7| = |-12| = 12$
(d) $|-11 - 3| = |-14| = 14$

15. $5\frac{3}{4} - 2\frac{5}{6} + \left(-\frac{23}{15}\right) - \left(-4\frac{7}{10}\right) = 5\frac{3}{4} - 2\frac{5}{6} - \frac{23}{15} + 4\frac{7}{10}$

$= \frac{23}{4} - \frac{17}{6} - \frac{23}{15} + \frac{47}{10}$

$= \frac{345}{60} - \frac{170}{60} - \frac{92}{60} + \frac{282}{60}$

$= \frac{345 - 170 - 92 + 282}{60}$

$= \frac{365}{60}$

$= \frac{73}{12}$

$= 6\frac{1}{12}$

16. Fraction of sum of money left after Farhan has taken his share

$= 1 - \frac{1}{5}$

$= \frac{4}{5}$

Fraction of sum of money left after Hussain has taken his share

$= \left(1 - \frac{1}{3}\right) \times \frac{4}{5}$

$= \frac{2}{3} \times \frac{4}{5}$

$= \frac{8}{15}$

Fraction of sum of money left after Anosha has taken her share

$= \left(1 - \frac{1}{4}\right) \times \frac{8}{15}$

$= \frac{^1\cancel{3}}{_1\cancel{4}} \times \frac{\cancel{8}^2}{\cancel{15}_5}$

$= \frac{2}{5}$

Fraction of sum of money taken by Sarah $= \left(1 - \frac{1}{7}\right) \times \frac{2}{5}$

$= \frac{6}{7} \times \frac{2}{5}$

Fraction of money Salman takes $= \frac{12}{35}$

Exercise 1B

1. **(a)** 698 352 = 698 400 (to the nearest 100)
(b) 698 352 = 698 000 (to the nearest 1000)
(c) 698 352 = 700 000 (to the nearest 10 000)

2. **(a)** 45.7395 = 45.7 (to 1 d.p.)
(b) 45.7395 = 46 (to the nearest whole number)
(c) 45.7395 = 45.740 (to 3 d.p.)

3. **(i)** Perimeter of land = 2(28.3 + 53.7)
= 2(82)
= 160 m (to the nearest 10 m)

(ii) Area of grass needed to fill up the entire plot of land
= 28.3 × 53.7
= 1519.71 m^2
= 1500 m^2 (to the nearest 100 m^2)

4. **(a)** 4.918 m = 4.9 m (to the nearest 0.1 m)
(b) 9.71 cm = 10 cm (to the nearest cm)
(c) PKR 10.982 = PKR 11.00 (to the nearest hundredth)
(d) 6.489 kg = 6.49 kg (to the nearest $\frac{1}{100}$ kg)

5. No, I do not agree with Kiran. She needs to put a '0' in the ones place as a place holder after dropping the digit '2', i.e. 5192.3 = 5190 (to the nearest 10).

6. Largest possible value of the country's population = 5 077 499
Smallest possible value of the country's population = 5 076 500

7. No, I do not agree with Farhan. 27.0 is rounded off to 1 decimal place which is more accurate than 27 which is rounded off to the nearest whole number.

Exercise 1C

1. $218 \div 31 \approx 210 \div 30$
$= 7$

∴ Seema's answer is not reasonable.
Using a calculator, 218 ÷ 31 = 7.03 (to 3 s.f.).
∴ The estimated value is close to the actual value.

2. **(a)** $2013 \times 39 \approx 2000 \times 40$
$= 80\,000$

Using a calculator, 2013 × 39 = 78 507.
∴ The estimated value is close to the actual value.

(b) $\sqrt{145.6} \div \sqrt[3]{65.4} \approx \sqrt{144} \div \sqrt[3]{64}$
$= 12 \div 4$
$= 3$

Using a calculator, $\sqrt{145.6} \div \sqrt[3]{65.4} = 2.99$ (to 3 s.f.).
∴ The estimated value is close to the actual value.

3. **(i)** 3.612 = 3.6 (to 2 s.f.)
29.87 = 30 (to 2 s.f.)
(ii) $3.612 \div 29.87 \approx 3.6 \div 30$
$= 0.12$ (to 2 s.f.)

4. Amount of petrol used $= \frac{274}{9.1}$

$\approx \frac{270}{9}\ l$

5.

2	6400
2	3200
2	1600
2	800
2	400
2	200
2	100
2	50
2	25
5	5
	1

$$\sqrt{6400} = \sqrt{\overline{2\times2} \times \overline{2\times2} \times \overline{2\times2} \times \overline{2\times2} \times \overline{5\times5}}$$

$= \quad 2 \times 2\times 2 \times 2 \times 5$

$= \quad 80$

Then the length of side of the table is 80 cm

6.

2	216
2	108
2	54
3	27
3	9
3	3
	1

$216 = \overline{2\times2} \times 2 \times \overline{3\times3} \times 3$

We can see that 2 and 3 have no pairs

There 216 must be divided by $2 \times 3 = 6$ to become a perfect square.

$216 \div 6 = 36$

36 is a perfect square.

$\sqrt{36} = 6$

7. $25 \times 25 =$ 625 trees

8. Ratio of area of shaded region to that of unshaded region = 1 : 2

9. (a) –14.12, -1.8, 1.5, 0.5, 7.25, 13.1

(b) –13.47, -0.134, 0.134, 1.34, 134.7, 1347

10. (a) 17.6, 1.76, 0, -7.8, -15.5, -18.2

(b) 25.4, 9.3, 2.54, 0.13, -0.55, -9.3

11. (a) **(i)** $23^2 = 23\times23 = 529$

(ii) $212^2 = 212\times212 = 44944$

(iii) $510^2 = 510\times510 = 260100$

(iv) $611^2 = 611\times611 = 373321$

(b)

19	367
19	19
	1

$= \sqrt{361} = \sqrt{19 \times 19}$

$= 19$

(c)

2	576
2	288
2	144
2	72
2	36
2	18
3	9
3	3
	1

$\sqrt{576} = \sqrt{\overline{2\times2} \times \overline{2\times2} \times \overline{2\times2} \times \overline{3\times3}}$

$= 2\times2\times2\times3$

$= 36$

(iii)

$$\sqrt{\frac{16.81}{22}} = \sqrt{\frac{1681}{22 \times 100}}$$

$$\frac{41}{\sqrt{2200}} = 0.87$$

(iv)

2	784
2	392
2	196
2	98
7	49
7	7
	1

$\sqrt{784} = \sqrt{\overline{2\times2} \times \overline{2\times2} \times \overline{7\times7}}$

$= 2 \times 2 \times 7$

$= 28$

(v) $\sqrt{5.29} = \sqrt{\dfrac{529}{100}}$

$$\begin{array}{r|l} 23 & 529 \\ \hline 23 & 23 \\ \hline & 1 \end{array}$$

$= \sqrt{\dfrac{23 \times 23}{10 \times 10}}$

$= \dfrac{23}{10}$

$= 2.3$

(vi) $\sqrt{2.89} = \sqrt{\dfrac{289}{100}}$

$= \sqrt{\dfrac{17 \times 17}{10 \times 10}}$

$= \dfrac{17}{10}$

$= 1.7$

(vii) $\sqrt{\dfrac{484}{256}} = \dfrac{\sqrt{484}}{\sqrt{256}}$

$$\begin{array}{r|l} 2 & 484 \\ \hline 2 & 242 \\ \hline 11 & 121 \\ \hline 11 & 11 \\ \hline & 1 \end{array}$$

$$\begin{array}{r|l} 2 & 256 \\ \hline 2 & 128 \\ \hline 2 & 64 \\ \hline 2 & 32 \\ \hline 2 & 16 \\ \hline 2 & 8 \\ \hline 2 & 4 \\ \hline 2 & 2 \\ \hline & 1 \end{array}$$

$= \dfrac{\sqrt{484}}{\sqrt{256}}$

$= \dfrac{\sqrt{2 \times 2 \times 11 \times 11}}{\sqrt{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}}$

$= \dfrac{2 \times 11}{2 \times 2 \times 2 \times 2}$

$= \dfrac{22}{16}$

$= 1\dfrac{6}{16}$

(viii) $\sqrt{65.61} = \sqrt{\dfrac{6561}{100}}$

$$\begin{array}{r|l} 3 & 6561 \\ \hline 3 & 2187 \\ \hline 3 & 729 \\ \hline 3 & 243 \\ \hline 3 & 81 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

$\sqrt{\dfrac{6561}{100}} = \sqrt{\dfrac{\overline{3 \times 3} \times \overline{3 \times 3} \times \overline{3 \times 3} \times \overline{3 \times 3}}{\overline{10 \times 10}}}$

$= \dfrac{3 \times 3 \times 3 \times 3}{10}$

$= 8.1$

(ix) $\sqrt{\dfrac{900}{441}} = \dfrac{\sqrt{900}}{\sqrt{441}}$

$= \dfrac{\sqrt{\overline{3 \times 3} \times \overline{10 \times 10}}}{\sqrt{\overline{3 \times 3} \times \overline{7 \times 7}}}$

$= \dfrac{3 \times 10}{3 \times 7}$

$= \dfrac{30}{21}$

$= 1\dfrac{9}{21}$

(x) $\sqrt{1.96} = \sqrt{\dfrac{196}{100}}$

$= \dfrac{\sqrt{\overline{2 \times 2} \times \overline{7 \times 7}}}{\sqrt{\overline{10 \times 10}}}$

$= \dfrac{2 \times 7}{10} = \dfrac{14}{10}$

$= 1.4$

12. Total amount of money that the shopkeeper has to pay
= 32 × PKR 18 + 18 × PKR 8 + 47 × PKR 26 + 63 × PKR 23 + 52 × PKR 9
≈ 30 × PKR 20 + 20 × PKR 10 + 50 × PKR 30 + 60 × PKR 20 + 50 × PKR 10
= PKR 600 + PKR 200 + PKR 1500 + PKR 1200 + PKR 500
= PKR 4000 (to the nearest hundred rupee)

13. RM 1 ≃ PKR 53
RM 25 ≃ PKR 53½25
≃ PKR 1325

14. For option A, 300 g costs about PKR 600.

Thus 100 g will cost about PKR 200.

$\therefore$ For option A, 500 g will cost about $5 \times$ PKR 200 = PKR 1000.

For option B, 500 g costs PKR 990 which is PKR 10 cheaper than option A.

However, for option A, 300 g actually costs PKR 580 which is PKR 20 less than PKR 600.

Thus for option A, 500 g will cost at least PKR 20 less than the estimated PKR 1000.

$\therefore$ Option A is better value for money.

15. Price of juice in Shop A after a 20% discount $= \frac{80}{100} \times$ PKR 79.50

$\approx \frac{80}{100} \times$ PKR 80

Price of juice in Shop B after a 10% discount $= \frac{90}{100} \times$ PKR 69.50

$\approx \frac{90}{100} \times$ PKR 70

16. The cost handbag is = PKR 0.18×26700

= PKR 4806

Review Exercise 1

1. $7\frac{1}{5} - \left(-3\frac{3}{10}\right) = 7\frac{1}{5} + 3\frac{3}{10}$

$= 7\frac{2}{10} + 3\frac{3}{10}$

$= 10\frac{5}{10}$

$= 10\frac{1}{2}$

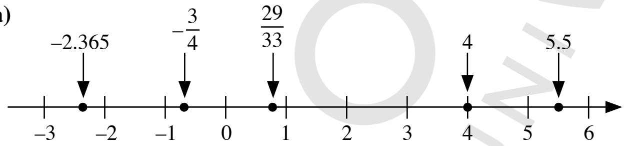
$19\frac{2}{5} + \left(-8\frac{1}{10}\right) = 19\frac{2}{5} - 8\frac{1}{10}$

$= 19\frac{4}{10} - 8\frac{1}{10}$

$= 11\frac{3}{10}$

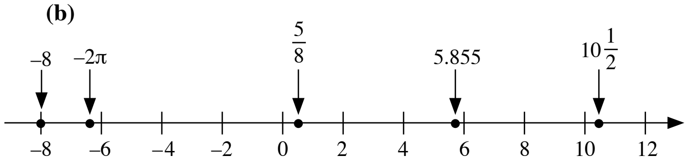
$\therefore 7\frac{1}{5} - \left(-3\frac{3}{10}\right) < 19\frac{2}{5} + \left(-8\frac{1}{10}\right)$

2. **(a)**



$\therefore 5.5, 4, \frac{29}{33}, -\frac{3}{4}, -2.365$

(b)



$\therefore 10\frac{1}{2}, 5.855, \frac{5}{8}, -2\pi, -8$

3. **(a)** 6479.952 = 6500 (to the nearest 100)

(b) 6479.952 = 6000 (to the nearest 1000)

(c) 6479.952 = 6480.0 (to the nearest tenth)

4. **(i)** 4.793 = 4.8 (to 2 s.f.)

39.51 = 40 (to 2 s.f.)

(ii) $4.793 \div 39.51 \approx 4.8 \div 40$

= 0.12 (to 2 s.f.)

5. $$\frac{-18 - \left[\sqrt[3]{-3375} - (-6)^2\right]}{\sqrt{4} + 9} = \frac{-18 - \left[\sqrt[3]{-(3^3 \times 5^3)} - (-6)^2\right]}{\sqrt{4} + 9}$$

$$= \frac{-18 - [-(3 \times 5) - 36]}{2 + 9}$$

$$= \frac{-18 - (-15 - 36)}{11}$$

$$= \frac{-18 - (-51)}{11}$$

$$= \frac{-18 + 51}{11}$$

$$= \frac{33}{11}$$

$$= 3$$

6. **(a)** $$3\frac{4}{7} + 1\frac{2}{5} - \left(-\frac{3}{7}\right) = 3\frac{4}{7} + 1\frac{2}{5} + \frac{3}{7}$$

$$= \frac{25}{7} + \frac{7}{5} + \frac{3}{7}$$

$$= \frac{125}{35} + \frac{49}{35} + \frac{15}{35}$$

$$= \frac{125 + 49 + 15}{35}$$

$$= \frac{189}{35}$$

$$= \frac{27}{5}$$

$$= 5\frac{2}{5}$$

(b) $$\frac{2}{3} - \left(-3\frac{3}{20}\right) + \left(-\frac{2}{5}\right) = \frac{2}{3} + 3\frac{3}{20} - \frac{2}{5}$$

$$= \frac{2}{3} + \frac{63}{20} - \frac{2}{5}$$

$$= \frac{40}{60} + \frac{189}{60} - \frac{24}{60}$$

$$= \frac{40 + 189 - 24}{60}$$

$$= \frac{205}{60}$$

$$= \frac{41}{12}$$

$$= 3\frac{5}{12}$$

(c) $-6\frac{4}{9} - 3\frac{3}{4} - 3\frac{5}{9} = -6\frac{16}{36} - 3\frac{27}{36} - 3\frac{20}{36}$

$= -12\frac{63}{36}$

$= -12\frac{7}{4}$

$= -13\frac{3}{4}$

(d) $\left(-\frac{1}{2} + \frac{1}{3}\right) + \left[\frac{1}{4} + \left(-\frac{1}{3}\right)\right] + \left(-\frac{1}{20}\right)$

$= -\frac{1}{2} + \frac{1}{3} + \frac{1}{4} - \frac{1}{3} - \frac{1}{20}$

$= -\frac{1}{2} + \frac{1}{3} - \frac{1}{3} + \frac{1}{4} - \frac{1}{20}$

$= -\frac{1}{2} + \frac{1}{4} - \frac{1}{20}$

$= -\frac{10}{20} + \frac{5}{20} - \frac{1}{20}$

$= \frac{-10 + 5 - 1}{20}$

$= \frac{-6}{20}$

$= -\frac{3}{10}$

(e) $-3\frac{1}{4} \times 1\frac{3}{5} \times \left(-1\frac{2}{13}\right) = -\frac{{}^{1}\cancel{13}}{{}_{1}\cancel{4}} \times \frac{\cancel{8}^{2}}{\cancel{5}_{1}} \times \left(-\frac{\cancel{15}^{3}}{\cancel{13}_{1}}\right)$

$= 6$

(f) $\frac{3}{5} \times \left(-\frac{1}{4} - \frac{1}{6}\right) \div \left(-2\frac{1}{3} + 1\frac{1}{4}\right)$

$= \frac{3}{5} \times \left(-\frac{3}{12} - \frac{2}{12}\right) \div \left(-2\frac{4}{12} + 1\frac{3}{12}\right)$

$= \frac{3}{5} \times \left(-\frac{5}{12}\right) \div \left(-1\frac{1}{12}\right)$

$= \frac{3}{5} \times \left(-\frac{5}{12}\right) \div \left(-\frac{13}{12}\right)$

$= \frac{3}{{}_{1}\cancel{5}} \times \left(-\frac{\cancel{5}^{1}}{\cancel{12}_{1}}\right) \times \left(-\frac{\cancel{12}^{1}}{13}\right)$

$= \frac{3}{13}$

(g) $-3\frac{9}{16} \div 1\frac{3}{16} - \frac{1}{3} \times \left(-1\frac{3}{4}\right) = -\frac{57}{16} \div \frac{19}{16} - \frac{1}{3} \times \left(-\frac{7}{4}\right)$

$= -\frac{{}^{3}\cancel{57}}{{}_{1}\cancel{16}} \times \frac{\cancel{16}^{1}}{\cancel{19}_{1}} - \frac{1}{3} \times \left(-\frac{7}{4}\right)$

$= -3 - \left(-\frac{7}{12}\right)$

$= -3 + \frac{7}{12}$

$= -2\frac{5}{12}$

(h) $-12\frac{1}{2} + 1\frac{2}{3} \div (-4) - \frac{5}{7} \times \left(-2\frac{4}{5}\right)$

$= -12\frac{1}{2} + \frac{5}{3} \div (-4) - \frac{5}{7} \times \left(-\frac{14}{5}\right)$

$= -12\frac{1}{2} + \frac{5}{3} \times \left(-\frac{1}{4}\right) - \frac{{}^{1}\cancel{5}}{{}_{1}\cancel{7}} \times \left(-\frac{\cancel{14}^{2}}{\cancel{5}_{1}}\right)$

$= -12\frac{1}{2} + \left(-\frac{5}{12}\right) - (-2)$

$= -12\frac{1}{2} - \frac{5}{12} + 2$

$= -12\frac{6}{12} - \frac{5}{12} + 2$

$= -10\frac{11}{12}$

7. $\dfrac{\left(-\frac{4}{7}\right)^{2} - \left(-\frac{2}{5}\right)^{3}}{-\sqrt{\frac{64}{625}} \div \sqrt[3]{-\frac{8}{125}}} = \frac{598}{1225}$

8. **(a)** $-12.8 - 88.2 = -101$

(b) $500.3 - (-200.2) - 210.1 = 500.3 + 200.2 - 210.1$

$= 700.5 - 210.1$

$= 490.4$

(c) $1.44 \div 1.2 \times (-0.4) = \frac{1.44}{1.2} \times (-0.4)$

$= \frac{14.4}{12} \times (-0.4)$

$= 1.2 \times (-0.4)$

$= -0.48$

(d) $(-0.3)^2 \div (-0.2) + (-2.56) = 0.09 \div (-0.2) + (-2.56)$

$= \frac{0.09}{-0.2} + (-2.56)$

$= \frac{0.9}{-2} + (-2.56)$

$= -0.45 + (-2.56)$

$= -0.45 - 2.56$

$= -3.01$

9. Smallest possible mass of chocolate truffle = 0.0245 kg

10. Indonesian rupiah, Rp 1 = PKR 0.018

Rp 35000 = PKR 0.018 × 35000

= PKR 630

11. Total mass = $3 \times 109 + 2 \times 148 + 5 \times 84$

$\approx (3 \times 110 + 2 \times 150 + 5 \times 80)$ g

12. Number of batteries required $= \frac{28.2}{4.03}$

$\approx \frac{28}{4}$

13. Price of hard disk in Store A after a 20% discount

$$= \frac{80}{100} \times \text{PKR } 85.05$$
$$\approx \frac{80}{100} \times \text{PKR } 85$$

Price of hard disk in Store B after a 10% discount

$$= \frac{90}{100} \times \text{PKR } 76.05$$
$$\approx \frac{90}{100} \times \text{PKR } 76$$

14. For option A, 250 ml costs about PKR 150.
Thus 50 ml will cost about PKR 30, and 100 ml will cost about PKR 60.
$\therefore$ For option A, 300 ml will cost about 3 × PKR 60 = PKR 180.
Furthermore, for option A, 250 ml actually costs PKR 152 which is PKR 2 more than PKR 150.
Thus for option A, 300 ml will cost at least PKR 2 more than the estimated PKR 180.
$\therefore$ Option B is better value for money.

15. $\sqrt{900} = 30$

30 rows can be planted.

16. $\frac{1352}{2} = 676$

$\sqrt{676} = 26$

He formed 26 rows for each group.

Challenge Yourself

1. 987 × 123 is more than 988 × 122 because 987 × 123 = 987 × (122 + 1), i.e. there is an additional 987 × 1; but 988 × 122 = (987 + 1) × 122, i.e. there is only an additional 1 × 122. In fact, 987 × 123 – 988 × 122 = 987 – 122 = 865.

2. This question tests students' sense of mass. The mass of an ordinary car is likely to be 2000 kg.
Teachers may wish to get students to give examples of objects with masses of 20 kg, 200 kg and 20 000 kg, e.g. 2 10-kg bags of rice have a total mass of 20 kg, 5 Secondary 1 students have a total mass of about 200 kg and a rocket has a mass of about 20 000 kg.

Chapter 2 Direct and Inverse Proportions

TEACHING NOTES

Suggested Approach

In grade 6, students have learnt rates such as PKR 0.25 per egg, or 13.5 km per litre of petrol etc. Teachers may wish to expand this further by asking what the prices of 2, 4 or 10 eggs are, or the distance that can be covered with 2, 4 or 10 litres of petrol, and leading to the introduction of direct proportion. After students are familiar with direct proportion, teachers can show the opposite scenario that is inverse proportions.

Section 2.1: Direct and Inverse Proportion

When introducing direct proportion, rates need not be stated explicitly. Rates can be used implicitly (see Investigation: Direct Proportion). By showing how one quantity increases proportionally with the other quantity, the concept should be easily relatable. More examples of direct proportion should be discussed and explored to test and enhance thinking and analysis skills (see Class Discussion: Real-Life Examples of Quantities in Direct Proportion). Teachers should discuss the linkages between direct proportion, algebra, rates and ratios to assess and improve students' understanding at this stage (see page 30 of the textbook). Teachers should also show the unitary method and proportion method in the worked example and advise students to adopt the method that is most comfortable for them.

The other form of proportion, inverse proportion, can be explored and studied by students (see Investigation: Inverse Proportion). When one variable increases, the other variable decreases proportionally. It is the main difference between direct and inverse proportion and must be emphasised clearly.

Students should be tasked with giving real-life examples of inverse proportion and explaining how they are inversely proportional (see Class Discussion: Real-Life Examples of Quantities in Inverse Proportion).

Teachers should present another difference between both kinds of proportions by reminding students that $\frac{y}{x}$ is a constant in direct proportion while xy is a constant in inverse proportion (see page 33 of the textbook).

WORKED SOLUTIONS

Investigation (Direct Proportion)

1. The fine will increase if the number of days a book is overdue increases.

2. $\frac{\text{Fine when a book is overdue for 6 days}}{\text{Fine when a book is overdue for 3 days}} = \frac{90}{45}$
$= 2$

The fine will be doubled if the number of days a book is overdue is doubled.

3. $\frac{\text{Fine when a book is overdue for 6 days}}{\text{Fine when a book is overdue for 2 days}} = \frac{90}{30}$
$= 3$

The fine will be tripled if the number of days a book is overdue is tripled.

4. $\frac{\text{Fine when a book is overdue for 5 days}}{\text{Fine when a book is overdue for 10 days}} = \frac{75}{150}$
$= \frac{1}{2}$

The fine will be halved if the number of days a book is overdue is halved.

5. $\frac{\text{Fine when a book is overdue for 3 days}}{\text{Fine when a book is overdue for 9 days}} = \frac{45}{135}$
$= \frac{1}{3}$

The fine will be reduced to $\frac{1}{3}$ of the original number if the number of days a book is overdue is reduced to $\frac{1}{3}$ of the original number.

Class Discussion (Real-Life Examples of Quantities in Direct Proportion)

The following are some real-life examples of quantities that are in direct proportion and why they are directly proportional to each other.

- In an hourly-rated job, one gets paid by the number of hours he worked. The longer one works, the more wages he will get.
  The wages one gets is directly proportional to the number of hours he worked.
- A coin weights approximately 6 g. As the number of coins increases, the total mass of the coins will increase proportionally. The total mass of the coins is directly proportional to the number of coins.
- The circumference of a circle is equivalent to the product of π and the diameter of the circle. As the diameter increases, the circumference increases proportionally. The circumference of the circle is directly proportional to the diameter of the circle.
- The speed of a moving object is the distance travelled by the object per unit time. If the object is moving at a constant speed, as the distance travelled increases, then the time spent in travelling increases proportionally. The distance travelled by the object is directly proportional to the time spent in travelling for an object moving at constant speed.
- The length of a spring can be compressed or extended depending on the force applied on it. The force required to compress or extend a spring is directly proportional to the change in the length of the spring. This is known as Hooke's Law, which has many practical applications in science and engineering.

Teachers may wish to note that the list is not exhaustive.

Investigation (Inverse Proportion)

1. The time taken decreases when the speed of the car increases.

2. $\frac{\text{Time taken when speed of the car is 40 km/h}}{\text{Time taken when speed of the car is 20 km/h}} = \frac{3}{6}$
$= \frac{1}{2}$

The time taken will be halved when the speed of the car is doubled.

3. $\frac{\text{Time taken when speed of the car is 60 km/h}}{\text{Time taken when speed of the car is 20 km/h}} = \frac{2}{6}$
$= \frac{1}{3}$

The time taken will be reduced to $\frac{1}{3}$ of the original number when the speed of the car is tripled.

4. $\frac{\text{Time taken when speed of the car is 30 km/h}}{\text{Time taken when speed of the car is 60 km/h}} = \frac{4}{2}$
$= 2$

The time taken will be doubled when the speed of the car is halved.

5. $\frac{\text{Time taken when speed of the car is 40 km/h}}{\text{Time taken when speed of the car is 120 km/h}} = \frac{3}{1}$
$= 3$

The time taken will be tripled when the speed of the car is reduced to $\frac{1}{3}$ of its original speed.

Class Discussion (Real-Life Examples of Quantities in Inverse Proportion)

The following are some real-life examples of quantities that are in inverse proportion and why they are inversely proportional to each other.

- Soldiers often dig trenches while serving in the army. The more soldiers there are digging the same trench, the faster it will take. The time to dig a trench is therefore inversely proportional to the number of soldiers.
- The area of a rectangle is the product of its length and breadth. Given a rectangle with a fixed area, if the length increases, then the breadth decreases proportionally. Therefore, the length of the rectangle is inversely proportional to the breadth of the rectangle.
- The density of a material is the mass of the material per unit volume. For an object of a material with a fixed mass, the density increases when the volume decreases proportionally. The density of the material is inversely proportional to the volume of the material.
- The speed of a moving object is the distance travelled by the object per unit time. For the same distance, when the speed of the object increases, the time to cover the distance is decreased

proportionally. The speed of the object is inversely proportional to the time to cover a fixed distance.

- For a fixed amount of force applied on it, the acceleration of the object is dependent on the mass of the object. When the mass of the object increases or decreases, the acceleration of the object decreases or increases proportionally. This is known as Newton's Second Law and has helped to explain many physical phenomena occurring around us.

Teachers may wish to note that the list is not exhaustive.

Practise Now 1

a) New quantity $= \frac{b}{a} \times$ original quantity

$$= \frac{4}{\cancel{3}} \times \cancel{75}^{25}$$

New quantity = 100 m

b) New quantity $= \frac{a}{b} \times$ original quantity

$$= \frac{2}{\cancel{5}} \times \cancel{250}^{50}$$

New quantity = 100 *ml*

Practise Now 2

(a) The cost of the sweets is directly proportional to the mass of the sweets.

Method 1: Unitary Method

50 g of sweets cost PKR 2.10.

1 g of sweets cost $\frac{\text{PKR } 2.10}{50}$.

380 g of sweets cost $\frac{\text{PKR } 2.10}{50} \times 380 =$ PKR 15.96.

PKR 15.96 = PKR 16 (to the nearest whole number)

Method 2: Proportion Method

Let the cost of 380 g of sweets be PKR x.

Then $\frac{x}{380} = \frac{2.1}{50}$. $\quad \frac{x_1}{y_1} = \frac{x_2}{y_2}$

$$x = \frac{2.1}{50} \times 380$$

$$= 15.96$$

$$= 16 \text{ (to the nearest whole number)}$$

Alternatively,

$\frac{x}{2.1} = \frac{380}{50} \quad \frac{x_1}{x_2} = \frac{y_1}{y_2}$

$$x = \frac{380}{50} \times 2.1$$

$$= 15.96$$

$$= 16 \text{ (to the nearest whole number)}$$

$\therefore$ 380 g of sweets cost PKR 16.

(b) The amount of metal is directly proportional to the mass of the metal.

Method 1: Unitary Method

$\frac{3}{4}$ of a piece of metal weighs 15 kg.

A whole piece of metal weighs $\frac{15}{\frac{3}{4}}$ kg.

$\frac{2}{5}$ of a piece of metal weighs $\frac{15}{\frac{3}{4}} \times \frac{2}{5} = 8$ kg.

Method 2: Proportion Method

Let the mass of $\frac{2}{5}$ of the piece of metal be x kg.

Then $\frac{x}{\frac{2}{5}} = \frac{15}{\frac{3}{4}}$. $\quad \frac{x_1}{y_1} = \frac{x_2}{y_2}$

$$x = \frac{15}{\frac{3}{4}} \times \frac{2}{5}$$

$$= 8$$

Alternatively,

$\frac{x}{15} = \frac{\frac{2}{5}}{\frac{3}{4}} \quad \frac{x_1}{x_2} = \frac{y_1}{y_2}$

$$x = \frac{\frac{2}{5}}{\frac{3}{4}} \times 15$$

$$= 8$$

$\therefore$ The mass of $\frac{2}{5}$ of the piece of metal is 8 kg.

Practise Now 3

The time taken to fill the tank is inversely proportional to the number of taps used.

Method 1: Unitary Method

4 taps can fill the tank in 70 minutes.

1 tap can fill the tank in (70 × 4) minutes.

7 taps can fill the tank in $\frac{70 \times 4}{7} = 40$ minutes.

Method 2: Proportion Method

Let the time taken for 7 taps to fill the tank by y minutes.

Then $7y = 4 \times 70$ $(x_1y_1 = x_2y_2)$

$$y = \frac{4 \times 70}{7}$$

$$= 40$$

$\therefore$ 7 taps can fill the tank in 40 minutes.

Practise Now 4

(a) The three variables are 'number of men', 'number of trenches' and 'number of hours'.

First, we keep the number of trenches constant.

Number of men	Number of trenches	Number of hours
3	2	5
1	2	5×3
5	2	$\frac{5 \times 3}{5} = 3$

Next, we keep the number of men constant.

Number of men	Number of trenches	Number of hours
5	2	3
5	1	$\frac{3}{2}$
5	7	$\frac{3}{2} \times 7 = 10.5$

$\therefore$ 5 men will take 10.5 hours to dig 7 trenches.

(b) The three variables are 'number of taps', 'number of tanks' and 'number of minutes'.

First, we keep the number of tanks constant.

Number of taps	Number of tanks	Number of minutes
7	3	45
1	3	45×7
5	3	$\frac{45 \times 7}{5} = 6.3$

Next, we keep the number of taps constant.

Number of taps	Number of tanks	Number of minutes
5	3	63
5	1	$\frac{63}{3} = 21$

$\therefore$ 5 taps will take 21 minutes to fill one tank.

Exercise 2A

1. **(i)** The number of books is directly proportional to the mass of books.

108 books have a mass of 30 kg.

1 book has a mass of $\frac{30}{108}$ kg.

150 books have a mass of $\frac{30}{108} \times 150 = 41\frac{2}{3}$ kg.

(ii) The mass of books is directly proportional to the number of books.

30 kg is the mass of 108 books.

1 kg is the mass of $\frac{108}{30}$ books.

20 kg is the mass of $\frac{108}{30} \times 20 = 72$ books.

2. **(i)** The number of books is directly proportional to the length occupied by the books.

60 books occupy a length of 1.5 m.

1 book occupies a length of $\frac{1.5}{60}$ m.

50 books occupy a length of $\frac{1.5}{60} \times 50 = 1.25$ m.

(ii) The length occupied by the books is directly proportional to the number of books.

1.5 m = (1.5 × 100) cm

= 150 m

150 cm is the length occupied by 60 books.

1 cm is the length occupied by $\frac{60}{150}$ books.

80 cm is the length occupied by $\frac{60}{150} \times 80 = 32$ books.

3. **(a)**

Original value	New value
30	x
1.5	26

$x = \frac{30 \times 26}{15}$

$x = 52$

(b)

Original value	New value
77	x
11	8

$x = \frac{77 \times 8}{11}$

$x = 56$

4. **(a)** The quantity of eraser is directly proportional to the cost of erasers.

3 kg of erasers cost PKR 18.

1 kg of erasers cost PKR $\frac{18}{3}$.

10 kg of erasers cost PKR $\frac{18}{3} \times 10$ = PKR 60.

(b) The mass of sugar is directly proportional to the cost.

b kg of sugar cost PKR 55.

1 kg of sugar cost PKR $\frac{55}{b}$.

a kg of sugar cost PKR $\frac{55}{b} \times a$ = PKR $\frac{55a}{b}$.

5. The amount of metal is directly proportional to the mass of the metal.

$\frac{5}{9}$ of a piece of metal has a mass of 7 kg.

A whole piece of metal has a mass of $\frac{7}{\frac{5}{9}}$ kg.

$\frac{2}{7}$ of a piece of metal has a mass of $\frac{7}{\frac{5}{9}} \times \frac{2}{7} = 3\frac{3}{5}$ kg.

6. Since z is directly proportional to x,

$$\frac{x_2}{z_2} = \frac{x_1}{z_1}$$
$$\frac{x}{18} = \frac{3}{12}$$
$$x = \frac{3}{12} \times 18$$
$$= 4.5$$

7. Since B is directly proportional to A,

$$\frac{B_2}{A_2} = \frac{B_1}{A_1}$$
$$\frac{B}{24} = \frac{3}{18}$$
$$B = \frac{3}{18} \times 24$$
$$= 4$$

Exercise 2B

1. **(a)** The number of pencils is directly proportional to the total cost of the pencils.
Assumption: All pencils are identical and cost the same each.

(b) The number of taps filling a tank is inversely proportional to the time taken to fill the tank.
Assumption: All taps are identical and each tap takes the same time to fill the tank.

(c) The number of men laying a road is inversely proportional to the taken to finish laying the road.
Assumption: All the men work at the same rate in laying the road.

(d) The number of cattle to be fed is directly proportional to the amount of fodder.
Assumption: All the cattle eat the same amount of fodder.

(e) The number of cattle to be fed is inversely proportional to the time taken to finish a certain amount of the fodder.
Assumption: All the cattle eat the fodder at the same rate.

$\therefore$ **(b)**, **(c)** and **(e)** are in inverse proportion.

2. The number of men to build a bridge is inversely proportional to the number of days to build the bridge.
8 men can build a bridge in 12 days.
1 man can build the bridge in (12 × 8) days.
6 men can build the bridge in $\frac{12 \times 8}{6} = 16$ days.
The assumption made is that all the men work at the same rate in building the bridge.

3. The number of days is inversely proportional to the number of workers employed.
16 days are needed for 35 workers to complete the projet.
1 day is needed for (35 × 16) workers to complete the project.
14 days are needed for $\frac{35 \times 16}{14} = 40$ workers to complete the project.
Number of additional workers to employ $= 40 - 35$
$= 5$

4. **(i)** The number of days is inversely proportional to the number of cattle to consume a consignment of fodder.
50 days are needed for 1260 cattle to consume a consignment of fodder.
1 day is needed for (1260 × 50) cattle to consume a consignment of fodder.
75 days are needed for $\frac{1260 \times 50}{75} = 840$ cattle to consume a consignment of fodder.

(ii) 1260 cattle consume a consignment of fodder in 50 days.
1 cattle consume a consignment of fodder in (50 × 1260) days.
1575 cattle consume a consignment of fodder in
$\frac{50 \times 1260}{1575} = 40$ days.

5. The number of athletes is inversely proportional to the number of days the food can last.
72 athletes take 6 days to consume the food.
1 athlete takes (6 × 72) days to consume the food.
$72 - 18 = 54$ athletes take $\frac{6 \times 72}{54} = 8$ days to consume the food.
Number of additional days the food can last $= 8 - 6$
$= 2$ days
The assumption made is that all athletes consume the same amount of food every day.

6. The three variables are 'number of glassblowers', 'number of vases' and 'number of minutes'.
First, we keep the number of vases constant.

Number of glassblowers	Number of vases	Number of minutes
12	12	9
1	12	9 × 12
8	12	$\frac{9 \times 12}{8} = 13.5$

Next, we keep the number of glassblowers constant.

Number of glassblowers	Number of vases	Number of minutes
8	12	13.5
8	1	$\frac{13.5}{12}$
8	32	$\frac{13.5}{12} \times 32 = 36$

$\therefore$ 8 glassblowers will take 36 minutes to make 32 vases.

7. The three variables are 'number of sheep', 'number of consignments' and 'number of days'.
First, we keep the number of consignments constant.

Number of sheep	Number of consignments	Number of days
100	1	20
1	1	20 × 1000
550	1	$\frac{20 \times 1000}{550} = 36\frac{4}{11}$

Next, we keep the number of sheep constant.

Number of sheep	Number of consignments	Number of days
550	1	$36\frac{4}{11}$
550	$400 \div 36\frac{4}{11} = 11$	400

$\therefore$ 11 consignments of fodder are needed.

8. In 1 minute, tap A alone fills up $\frac{1}{6}$ of the tank.

In 1 minute, tap B alone fills up $\frac{1}{9}$ of the tank.

In 1 minute, pipe C alone empties $\frac{1}{15}$ of the tank.

In 1 minute, when both taps and the pipe are turned on,

$\frac{1}{6} + \frac{1}{9} - \frac{1}{15} = \frac{19}{90}$ of the tank is filled up.

$$\text{Time to fill up the tank} = \frac{90}{19}$$
$$= 4\frac{14}{19} \text{ minutes}$$

9. Total number of hours worked on the road after 20 working days
$= 20 \times 50 \times 8$
$= 8000$ hours

The length of the road laid is directly proportional to the number of hours.

1200 m of road is laid in 8000 hours.

1 m of road is laid in $\frac{8000}{1200}$ hours.

$3000 - 1200 = 1800$ m of road is laid in $\frac{8000}{1200} \times 1800 = 12\,000$ hours.

Let the number of additional men to employ be x.

$$(30 - 20) \times (50 + x) \times 10 = 12\,000$$
$$100(50 + x) = 12\,000$$
$$50 + x = 120$$
$$x = 70$$

$\therefore$ 70 more men needs to be employed.

Review Exercise 2

1. Let the donations Kiran makes be PKR d,
the savings of Kiran be PKR s.

Since d is directly proportional to s^2,
then $d = ks^2$, where k is a constant.

When $s = 900$,
$d = k \times 900^2$
$= 810\,000k$

When $s = 1200$,
$d = k \times 1200^2$
$= 1\,440\,000k$

Since Kiran's donation increases by PKR 35,

$$1\,440\,000k - 810\,000k = 35$$
$$63\,000k = 35$$
$$k = \frac{35}{630\,000}$$
$$= \frac{1}{18\,000}$$

$$\text{Amount Kiran donates in January} = \frac{1}{18\,000} \times 900^2$$
$$= \text{PKR } 45$$

$$\text{Amount Kiran donates in February} = \frac{1}{18\,000} \times 1200^2 \text{ or } 45 + 35$$
$$= \text{PKR } 80$$

Challenge Yourself

1. Let the number of days for 5 men to complete the job be x.

The number of men is inversely proportional to the number of days to complete the job.

5 men take x days to complete the job.

1 man takes $x \times 5$ days to complete the job.

6 men take $\frac{x \times 5}{6}$ days to complete the job.

Since the job can be completed 8 days earlier when 1 more man is hired,

$$\frac{x \times 5}{6} = x - 8$$
$$\frac{5x}{6} = x - 8$$
$$5x = 6(x - 8)$$
$$= 6x - 48$$
$$x = 48$$

It takes (48×5) days for 1 man to complete the job.

It takes 1 day for $(1 \times 48 \times 5)$ men to complete the job.

It takes $48 - 28 = 20$ days for $\frac{1 \times 48 \times 5}{20} = 12$ men to complete the job.

$$\text{Additional number of men to hire} = 12 - 5$$
$$= 7$$

$\therefore$ 7 more men should be hired.

Chapter 3 Application of Mathematics in Practical Situations

TEACHING NOTES

Suggested Approach

Teachers can get students to discuss examples of percentages, which are used in everyday life. Although the concepts covered in this chapter are applicable to the real world, students might not have encountered the need to be familiar with them and hence might not identify with the situations easily. Teachers should prepare more relatable material, such as advertisements on discounted products, to allow students to appreciate the application of mathematics in practical situations.

Section 3.1: Profit and Loss

The definitions of profit and loss should be made clear to students, whereby:

$$\text{Profit} = \text{Selling price} - \text{Cost price}$$

$$\text{Loss} = \text{Cost price} - \text{Selling price}.$$

Teachers should also emphasise the difference between the expression of profit and loss as a percentage of the cost price and the calculation of percentage gain or loss in terms of the selling price, that may occur in some business transactions.

$$\frac{\text{Profit}}{\text{Cost price}} \times 100\% \qquad\qquad \frac{\text{Loss}}{\text{Cost price}} \times 100\%$$

$$\text{Percentage gain} = \frac{\text{Profit}}{\text{Selling price}} \times 100\% \qquad\qquad \text{Percentage loss} = \frac{\text{Loss}}{\text{Selling price}} \times 100\%$$

Thus, teachers should remind students to read the questions carefully in order to ascertain the correct percentage to report.

Section 3.2: Discount, Taxation and Commission

These real-world concepts would be useful for students when they start to work and plan their finances. However, teachers should note that students may not encounter terms such as discount, GST and commission often, and thus should explain the terms clearly before going through the topic.

The examples and figures used in the textbook are those applicable in Pakistan, but the theory involved would be relevant for use in other countries. Teachers may supplement the questions with current figures and tax brackets in Pakistan, for instance, and further discuss with students the implications and importance of taxes in a country.

WORKED SOLUTIONS

Investigation (Discount, Service Charge and GST)

1. GST as calculated by Kiran $= \frac{7}{100} \times$ PKR 14.40

   = PKR 1.01 (to the nearest paisa)

   GST as calculated by the market

   $= \frac{7}{100} \times \left[\text{PKR } 14.40 + \left(\frac{10}{100} \times \text{PKR } 14.40\right)\right]$

   $= \frac{7}{100} \times (\text{PKR } 14.40 + \text{PKR } 1.44)$

   $= \frac{7}{100} \times \text{PKR } 15.84$

   = PKR 1.11 (to the nearest paisa)
2. GST is an acronym for Goods and Services Tax, thus the tax is also imposed on the service charge, which is 10% of the subtotal.
3. If the discount is given before the service charge and GST are taken into account, the bill received will be as follows:

Z's Market	
Candies:	PKR 8.50
Jellies:	PKR 9.50
Subtotal:	PKR 18.00
Discount:	– PKR 3.60
Subtotal:	PKR 14.40
Service Charge 10%:	PKR 1.44
GST 7%:	PKR 1.1088
Total:	**PKR 16.9488**

If the discount is given after the service charge and GST have been taken into account, the bill received will be as follows:

Z's Market	
Candies:	PKR 8.50
Jellies:	PKR 9.50
Subtotal:	PKR 18.00
Service Charge 10%:	PKR 1.80
GST 7%:	PKR 1.386
Subtotal:	PKR 21.186
Discount:	– PKR 4.2372
Total:	**PKR 16.9488**

4. If the discount is given before the service charge and GST are taken into account, both the service charge and GST will be calculated based on a smaller amount, i.e. PKR 14.40, and thus the service charge and GST will have already been discounted.

   If the discount is given after the service charge and GST have been taken into account, both the service charge and GST will be calculated based on a greater amount, i.e. PKR 18, and thus the discount will be given on the service charge and GST as well.

   Hence, it makes no difference whether the discount is given before or after the service charge and GST are taken into account as the total bill will still be the same.

Investigation (Percentage Point)

1. Increase from 17% to 18% = 18% - 17%

   = 1%

   Percentage increase from 17% to 18% $= \frac{1}{17} \times 100\%$

   = 5.9%
2. Yes Salman is right in singing that the increase from 17% to 18% is an increase of 5.9% please see solution to Question no 1.
3. While the percentage increase see from 17% to 18% is 5.9%, the difference between 17% and 18% is 1%

   The term 'percentage point' is used to describe the difference between two percentage.

Practise Now 1

1. **(a)** Required percentage $= \frac{\text{PKR } 2400 - \text{PKR } 1800}{\text{PKR } 1800} \times 100\%$

   $= \frac{\text{PKR } 600}{\text{PKR } 1800} \times 100\%$

   $= 33\frac{1}{3}\%$

   (b) Required percentage $= \frac{\text{PKR } 6000 - \text{PKR } 5000}{\text{PKR } 5000} \times 100\%$

   $= \frac{\text{PKR } 1000}{\text{PKR } 5000} \times 100\%$

   = 20%
2. **(a)** Selling price of t-shirt$= \frac{127}{100} \times$ PKR 500

   = PKR 635

   (b) Selling price of t-shirt$= \frac{94}{100} \times$ PKR 78 400

   = PKR 73 696

Practise Now 2

1. 135% of cost price = PKR 1282.50

   1% of cost price $= \frac{\text{PKR } 1282.50}{135}$

   100% of cost price $= \frac{\text{PKR } 1282.50}{135} \times 100$

   = PKR 950

   The cost price of the stool is PKR 950.
2. 88% of cost price = PKR 16.50

   1% of cost price $= \frac{\text{PKR } 16.50}{88}$

   100% of cost price $= \frac{\text{PKR } 16.50}{88} \times 100$

   = PKR 18.75

   The cost price of the pen is PKR 18.75.

Practise Now 3

Cost price of 1800 eggs $= \frac{1800}{12} \times$ PKR 120

= PKR 18 000

Total selling price of eggs so as to earn a 33% profit on the cost price

$= \frac{133}{100} \times$ PKR 18 000

= PKR 23940

Number of eggs that the shopkeeper can sell $= \frac{95}{100} \times 1800$

$= 1710$

Selling price of each egg $= \frac{23\,940}{1710}$

= PKR 14

Practise Now 4

1. Percentage discount $= \frac{\text{PKR } 100 - \text{PKR } 88}{\text{PKR } 100} \times 100\%$

$= \frac{\text{PKR } 12}{\text{PKR } 100} \times 100\%$

$= 12\%$

2. Sale price of bucket $= \frac{94}{100} \times$ PKR 600

= PKR 564

Practise Now 5

(i) 91% of marked price = PKR 1274

1% of marked price $= \frac{\text{PKR } 1274}{91}$

100% of marked price $= \frac{\text{PKR } 1274}{91} \times 100$

= PKR 1400

The marked price of the carton of milk is PKR 1400.

(ii) Sale price of carton of milk after a 5% discount

$= \frac{95}{100} \times$ PKR 1400

= PKR 1330

Sale price of carton of milk after a further discount of 4%

$= \frac{96}{100} \times$ PKR 1330

= PKR 1276.80

No, the sale price would not be PKR 1274.

Practise Now 6

1. GST payable $= \frac{7}{100} \times$ PKR 85

= PKR 5.95

Total amount of money the man has to pay for article

= PKR 85 + PKR 5.95

= PKR 90.95

2. 107% of marked price = PKR 642

1% of marked price $= \frac{\text{PKR } 642}{107}$

100% of marked price $= \frac{\text{PKR } 642}{107} \times 100$

= PKR 600

The marked price of the printer is PKR 600.

Practise Now 7

1. Discount $= \frac{15}{100} \times$ PKR 69

= PKR 10.35

Service charge = 10% × (marked price – discount)

$= \frac{10}{100} \times$ (PKR 69 – PKR 10.35)

$= \frac{10}{100} \times$ PKR 58.65

= PKR 5.865

GST payable = 7% × (marked price – discount + service charge)

$= \frac{7}{100} \times$ (PKR 69 – PKR 10.35 + PKR 5.865)

$= \frac{7}{100} \times$ PKR 64.515

= PKR 4.51605

Total amount payable

= marked price – discount + service charge + GST payable

= PKR 69 – PKR 10.35 + PKR 5.865 + PKR 4.51605

= PKR 69 (to the whole number)

2. 117.7% of price after discount = PKR 235.4

1% of price after discount $= \frac{\text{PKR } 235.4}{117.7}$

100% of price after discount $= \frac{\text{PKR } 235.4}{117.7} \times 100$

= PKR 200

The price of the set meal after discount is PKR 20.

80% of marked price = PKR 200

1% of marked price $= \frac{\text{PKR } 200}{80}$

100% of marked price $= \frac{\text{PKR } 200}{80} \times 100$

= PKR 250

The marked price of the set meal is PKR 250.

Practise Now 8

Exceeding amount = PKR 1 364 000 – PKR 1 200 000 = PKR 164 000

12.5% of PKR 164 000 $= \frac{12.50}{100} \times 100 \times$ PKR 164 000

= PKR 20 500

Total tax = PKR 20 500 + PKR 15 000

= PKR 35 500

Practise Now 9

Total reliefs = PKR 3000 + 2(PKR 4000) + PKR 28 500 + PKR 3500
= PKR 43 000

Taxable income = PKR 284 000 – PKR 43 000
= PKR 241 000

Tax

PKR 241 000 → First PKR 200 000 : PKR 20 750

PKR 241 000 → Next PKR 41 000 at 18% : $\frac{18}{100}$ × PKR 41 000 = PKR 7380

∴ Income tax payable = PKR 20 750 + PKR 7380
= PKR 28 130

Practise Now 10

Property tax rate = 1.5%

Property tax = 30000

15% of two plots value = PKR 30 000

Value of two plots = $\frac{\text{PKR } 30000}{1.5} \times 100$
= PKR 2000000

Value of one plot = $\frac{\text{PKR } 2000000}{2}$

= PKR 1 000 000

Practice Now 11

(a) Amount of interest the man has to pay at the end of 1 year
= PKR 150 000 × $\frac{5.5}{100}$
= PKR 8250
Amount of interest the man has to pay at the end of 3 years
= PKR 8250 × 3
= PKR 24 750
Total amount he owes the bank
= PKR 150 000 + PKR 24 750
= PKR 174 750

(b) Total amount of interest Sarah earns
= PKR 6720 – PKR 6000
= PKR 720
Amount of interest Sarah earns per last year
= PKR 6000 × $\frac{3}{100}$
= PKR 180
Time taken for her investment to grow to PKR 6720
= $\frac{\text{PKR } 720}{\text{PKR } 180}$
= 4 years

Practise Now 12

Amount of Zakat = PKR 1125

2.5% of yearly savings = PKR 1125

yearly savings = $\frac{\text{PKR } 1125}{2.5} \times 100$
= PKR 45000

Exercise 3A

1. **(i)** 35% of cost price = PKR 280

1% of cost price = $\frac{\text{PKR } 2800}{35}$

100% of cost price = $\frac{\text{PKR } 2800}{35} \times 100$
= PKR 800

The cost price of the cup is PKR 800.

(ii) Selling price of cup = PKR 800 + PKR 280
= PKR 1080

3. Cost price of 5 kg of mixture = 2 × PKR 80 + 3 × PKR 60
= PKR 160 + PKR 180
= PKR 340

Selling price of 5 kg of mixture = 20 × PKR 25.5
= PKR 510

Required percentage = $\frac{\text{PKR } 510 - \text{PKR } 340}{\text{PKR } 510} \times 100\%$
= $\frac{\text{PKR } 170}{\text{PKR } 510} \times 100\%$
= $33\frac{1}{3}\%$

4. Selling price of one dozen of roses = 12 × PKR 1.20
= PKR 14.40

Required percentage = $\frac{\text{PKR } 18 - \text{PKR } 14.40}{\text{PKR } 14.40} \times 100\%$
= $\frac{\text{PKR } 3.60}{\text{PKR } 14.40} \times 100\%$
= 25%

5. 75% of price Bina buys from Sarah = PKR 360

1% of price Bina buys from Sarah = $\frac{\text{PKR } 360}{75}$

100% of price Bina buys from Sarah = $\frac{\text{PKR } 360}{75} \times 100$
= PKR 480

Bina buys the fax machine from Sarah at PKR 480.

125% of cost price = PKR 480

1% of cost price = $\frac{\text{PKR } 480}{125}$

100% of cost price = $\frac{\text{PKR } 480}{125} \times 100$
= PKR 384

Sarah paid PKR 384 for the fax machine.

6. Total number of apples Ali buys = 200×60

= 12 000

Cost price of 12 000 apples = $200 \times$ PKR 28

= PKR 5600

Total selling price of apples so as to earn a 80% profit on the cost price

$= \frac{180}{100} \times$ PKR 5600

= PKR 10 080

Number of apples that Ali can sell $= \frac{85}{100} \times 12\,000$

= 10 200

Selling price per apple $= \frac{\text{PKR } 10\,080}{10\,200}$

= PKR 0.99 (to the nearest cent)

7. Cost price of each article $= \frac{\text{PKR } 1500}{300}$

= PKR 5

Selling price of each of the 260 articles $= \frac{120}{100} \times$ PKR 5

= PKR 6

Selling price of each of the remaining 40 articles $= \frac{50}{100} \times$ PKR 6

= PKR 3

Selling price of articles = $260 \times$ PKR 6 + $40 \times$ PKR 3

= PKR 1560 + PKR 120

= PKR 1680

Required percentage $= \frac{\text{PKR } 1680 - \text{PKR } 1500}{\text{PKR } 1500} \times 100\%$

$= \frac{\text{PKR } 180}{\text{PKR } 1500} \times 100\%$

= 12%

Exercise 3B

1. Percentage discount $= \frac{\text{PKR } 580 - \text{PKR } 464}{\text{PKR } 580} \times 100\%$

$= \frac{\text{PKR } 116}{\text{PKR } 580} \times 100\%$

= 20%

2. Sale price of book $= \frac{88}{100} \times$ PKR 45

= PKR 39.60

3. **(i)** 7% of marked price = PKR 49

1% of marked price $= \frac{\text{PKR } 49}{7}$

100% of marked price $= \frac{\text{PKR } 49}{7} \times 100$

= PKR 700

The marked price of the pastry is PKR 700.

(ii) Sale price of pastry = PKR 700 – PKR 49

= PKR 651

4. GST payable $= \frac{7}{100} \times$ PKR 270

= PKR 18.90

Total amount of money Ethan has to pay for the pack of biscuit

= PKR 270 + PKR 18.90

= PKR 288.90

5. 107% of marked price = PKR 1391

1% of marked price $= \frac{\text{PKR } 1391}{107}$

100% of marked price $= \frac{\text{PKR } 1391}{107} \times 100$

= PKR 1300

The marked price of the electronic gadget is PKR 1300.

6. Total number of apples Saad buys = 200×60

= 12 000

Cost price of 12 000 apples = $200 \times$ PKR 300

= PKR 60 000

Total selling price of apples so as to earn a 80% profit on the cost price

$= \frac{180}{100} \times$ PKR 60 000

= PKR 108 000

Number of apples that Saad can sell

$= \frac{90}{100} \times 12000$

= 108 00

Selling price per apples = PKR $\frac{1080000}{110800}$

= PKR 10

7. **(a)** Amount of commission the agent receives

$= \frac{2.5}{100} \times$ PKR 650 000

= PKR 16 250

(b) 2.5% of selling price = PKR 12 000

1% of selling price $= \frac{\text{PKR } 12\,000}{2.5}$

100% of selling price $= \frac{\text{PKR } 12\,000}{2.5} \times 100$

= PKR 480 000

The selling price of the house is PKR 480 000.

8. Amount of zakat = 2.5% of yearly savings

$= \frac{2.5 \times \text{PKR } 400\,000}{100}$

= PKR 10000

9. Amount of zakat = 2.5% of PKR 73 800

$= \frac{2.5 \times \text{PKR } 93800}{100}$

= PKR 2345

10. 10% of agricultural output = PKR 6700

$$\text{agricultural output} = \frac{\text{PKR}6700 \times 100}{10}$$
$$= \text{PKR } 67000$$

11. **(i)** 87.5% of marked price = PKR 700

$$1\% \text{ of marked price} = \frac{\text{PKR } 700}{87.5}$$

$$100\% \text{ of marked price} = \frac{\text{PKR } 700}{87.5} \times 100$$
$$= \text{PKR } 800$$

The marked price of the umbrella is PKR 800.

(ii) Sale price of umbrella after a 10% discount

$$= \frac{90}{100} \times \text{PKR } 800$$
$$= \text{PKR } 720$$

Sale price of umbrella after a further discount of 2.5%

$$= \frac{97.5}{100} \times \text{PKR } 720$$
$$= \text{PKR } 702$$

No, the sale price would not be PKR 700.

12. Price of seafood fried rice after discount $= \frac{75}{100} \times \text{PKR } 95$
$= \text{PKR } 71.25$

Total amount payable $= \frac{117.7}{100} \times \text{PKR } 71.25$
$= \text{PKR } 83.86$

13. Gross annual income = PKR 1 850 000

The amount subjected to income tax = PKR 1 850 000 – PKR 200 000
= PKR 1 830 000

Income tax payable = 5% PKR 1 830 000
= PKR 91 500

14. Property tax payable yearly $= \text{PKR } 28\,800 \times \frac{10}{100}$
$= \text{PKR } 2880$

Property tax payable for 6 months $= \frac{\text{PKR } 2880}{2}$
$= \text{PKR } 1440$

15. Amount of commission Faiza receives = PKR 1220 – PKR 500
= PKR 720

4% of Faiza's sales = PKR 720

$$1\% \text{ of Faiza's sales} = \frac{\text{PKR } 720}{4}$$

$$100\% \text{ of Faiza's sales} = \frac{\text{PKR } 720}{4} \times 100$$
$$= \text{PKR } 18\,000$$

Faiza's sales for that month are PKR 18 000.

16. 117.7% of price after discount = PKR 101.3

$$1\% \text{ of price after discount} = \frac{\text{PKR } 101.3}{117.7}$$

$$100\% \text{ of price after discount} = \frac{\text{PKR } 101.3}{117.7} \times 100$$
$$= \text{PKR } 86.1 \text{ (to the nearest paisa)}$$

The price of the soup after discount is PKR 86.1.

82% of marked price = PKR 86.1

$$1\% \text{ of marked price} = \frac{\text{PKR } 86.1}{82}$$

$$100\% \text{ of marked price} = \frac{\text{PKR } 86.1}{82} \times 100$$
$$= \text{PKR } 105 \text{ (to the nearest paisa)}$$

The marked price of the soup is PKR 105.

Review Exercise 3

1. Profit the man earns = 12 × PKR 36
= PKR 432

Selling price of cameras = PKR 1800 + PKR 432
= PKR 2232

$$\text{Required percentage} = \frac{\text{PKR } 432}{\text{PKR } 2232} \times 100\%$$
$$= 19\frac{11}{31}\%$$

2. 125% of cost price of Item A = PKR 48

$$1\% \text{ of cost price of Item } A = \frac{\text{PKR } 48}{125}$$

$$100\% \text{ of cost price of Item } A = \frac{\text{PKR } 48}{125} \times 100$$
$$= \text{PKR } 38.40$$

The cost price of Item A is PKR 38.40.

80% of cost price of Item B = PKR 48

$$1\% \text{ of cost price of Item } B = \frac{\text{PKR } 48}{80}$$

$$100\% \text{ of cost price of Item } B = \frac{\text{PKR } 48}{80} \times 100$$
$$= \text{PKR } 60$$

The cost price of Item B is PKR 60.

Total cost price of the two items = PKR 38.40 + PKR 60
= PKR 98.40

Total selling price of the two items = 2 × PKR 48
= PKR 96

Net loss = PKR 98.40 – PKR 96
= PKR 2.40

$$\text{Required percentage} = \frac{\text{PKR } 2.40}{\text{PKR } 98.40} \times 100\%$$
$$= 2\frac{18}{41}\%$$

3. $$\text{Percentage discount} = \frac{\text{PKR } 60\,000 - \text{PKR } 57\,000}{\text{PKR } 60\,000} \times 100\%$$
$$= \frac{\text{PKR } 3000}{\text{PKR } 60\,000} \times 100\%$$
$$= 5\%$$

4. Marked price of 200 pencils = 200 × PKR 20
= PKR 4000

Amount of money the school has to pay $= \frac{93.5}{100} \times \text{PKR } 4000$
$= \text{PKR } 3740$

5. **(a)** Value-added tax payable $= \frac{15}{100} \times \text{PKR } 20$

$= \text{PKR } 3$

Amount of profit the retailer makes $= \text{PKR } 26 - (\text{PKR } 20 + \text{PKR } 3)$

$= \text{PKR } 26 - \text{PKR } 23$

$= \text{PKR } 3$

(b) **(i)** Price the retailer buys each bread toaster at from the manufacturer

$= \frac{120}{100} \times \text{PKR } 20$

$= \text{PKR } 24$

Value-added tax payable $= \frac{25}{100} \times \text{PKR } 24$

$= \text{PKR } 6$

Price a customer has to pay for bread

$= \text{PKR } 24 + \text{PKR } 6 + \text{PKR } 3$

$= \text{PKR } 33$

(ii) Selling price of bread $= \frac{130}{100} \times \text{PKR } 33$

$= \text{PKR } 42.90$

Amount of profit the retailer makes on a bread toaster

$= \text{PKR } 42.90 - \text{PKR } 24 - \text{PKR } 6$

$= \text{PKR } 12.90$

Amount of profit the retailer makes on 25 breads

$= 25 \times \text{PKR } 12.90$

$= \text{PKR } 322.50$

6. 117.7% of price after discount = PKR 79.4

1% of price after discount $= \frac{\text{PKR } 79.4}{117.7}$

100% of price after discount $= \frac{\text{PKR } 79.4}{117.7} \times 100$

$= \text{PKR } 67.5$ (to the nearest rupee)

The price of the noodles after discount is PKR 67.5.

90% of marked price = PKR 67.5

1% of marked price $= \frac{\text{PKR } 67.5}{90}$

100% of marked price $= \frac{\text{PKR } 67.5}{90} \times 100$

$= \text{PKR } 7.50$ (to the nearest cent)

The marked price of the noodles is PKR 75.0.

7. Total reliefs $= \text{PKR } 3000 + 2(\text{PKR } 5000) + \text{PKR } 16\,000 + \text{PKR } 750$

$= \text{PKR } 29\,750$

Taxable income $= \text{PKR } 80\,000 - \text{PKR } 29\,750$

$= \text{PKR } 50\,250$

Tax

PKR 80 000 → First PKR 40 000 : PKR 550

PKR 80 000 → Next PKR 10 250 at 7% : $\frac{7}{100} \times \text{PKR } 10\,250 = \text{PKR } 717.50$

$\therefore$ Income tax payable $= \text{PKR } 550 + \text{PKR } 717.50$

$= \text{PKR } 1267.50$

8. Amount of commission the agent receives

$= \frac{5}{100} \times \text{PKR } 50\,000 + \frac{2.25}{100} \times (\text{PKR } 240\,000 - \text{PKR } 50\,000)$

$= \frac{5}{100} \times \text{PKR } 50\,000 + \frac{2.25}{100} \times \text{PKR } 190\,000$

$= \text{PKR } 2500 + \text{PKR } 4275$

$= \text{PKR } 6775$

Challenge Yourself

(i) GST paid by Faiza $= \frac{7}{100} \times \text{PKR } 500$

$= \text{PKR } 35$

(ii) GST paid by Sarah $= \frac{7}{107} \times \text{PKR } 500$

$= \text{PKR } 32.71$ (to the nearest cent)

(iii) GST on PKR 500 is PKR 35 which is the same answer as in **(i)**. The shopkeeper is not complaining about it because he rather pays a GST of PKR 32.71 than a GST of PKR 35 to the government.

(iv) The amount paid by each customer at Shops *B* and *C* is PKR 500. As far as the government is concerned, this amount must be inclusive of GST. Another way of looking at this is to ask how the government can keep track of the shops which absorb GST and charge them a different GST amount. All the shops will tell the government that the final transacted amount is inclusive of GST because they can pay a *lower* amount for GST, so this agrees with why the final transacted amount is inclusive of GST regardless of whether the shops charge or absorb GST.

(v) Yes, it makes a difference. The difference is the *original* selling price of the handbag *before* the government announces that they will charge GST. Shop *C* has been selling the handbag for PKR 500 and decides to absorb GST after the announcement, so it still sells the handbag for PKR 500 (inclusive of GST). If Shop *C* decides not to absorb GST, they will sell the handbag for PKR 500 (before GST) or PKR 535 (inclusive of GST), just like what Shop *A* does. Since Shop *B* has been selling the handbag for *about* PKR 467.29 and decides to charge GST after the announcement, it sells the handbag for PKR 500 (inclusive of GST) *now*.

Chapter 4 Sets

TEACHING NOTES

Suggested Approach:

Teachers should not take an abstract approach when introducing the complement of a set, and the union and intersection of sets. Teachers should try to apply the set language to describe things in daily life to arouse students' interest to learn this topic.

Section 4.1: Venn Diagrams, and Types of Sets

Teachers may want to introduce Venn diagrams as a way to show the relationship between the set(s) that are under consideration. Teachers should go through some pointers (see Attention on page 54 of the textbook) when drawing the Venn diagram.

When introducing the complement of a set, it will be good if an example can be illustrated using a Venn diagram. By having the students to discuss whether the complement of a set will exist if the universal set is not defined (see Thinking Time on page 55), they can have a better understanding of the meaning of the complement of a set.

Students should be given more opportunities to discuss with each other on proper subsets (see Class Discussion: Understanding Subsets on page 57 of the textbook). It is crucial that the difference between a subset and a proper subset is discussed with the students so that they have a better understanding on proper subsets.

Section 4.2: Intersection of Two Sets

Teachers may wish to use the Chapter Opener to introduce the intersection of two sets and guide the students to think how they can represent the intersection of the two sets on the Venn diagram since all the elements in a set are distinct. Conclude that the intersection of two sets refers to the set of elements which are common to both sets and this is represented by the overlapping region of the two sets.

Teachers could use Practise Now 4, Question 1, to reinforce the meaning of subset and Practise Now 4, Question 2, to introduce disjoint sets.

Section 4.3: Union of Two Sets

Teachers should use Venn diagram to help students to visualise the meaning of the union of two sets, i.e. all the elements which are in either sets.

Section 4.4: Combining Universal Set, Complement of a Set, Subset, Intersection and Union of Sets

By recapping what was covered in the previous section, teachers may guide the students to solve problems involving universal set, intersection and union of sets in this section.

Teachers may use step-by-step approach as shown in Worked Example 6 to identify and shade the required region. It should be reinforced that for a union, shade all the regions with at least one tick; for an intersection, shade all the regions with exactly two ticks and for complement, shade all the regions without any tick.

Challenge Yourself

For Question 1, teachers may wish to use the inductive approach to lead the students to observe a pattern for the number of proper subsets when a set has *n* elements.

Question 2 involves the understanding of the properties of the different types of quadrilaterals and using one of the properties to classify them on a Venn diagram.

WORKED SOLUTIONS

Thinking Time (Page 55)

No, since A' is defined to be the set of all elements in the universal set but not in A.

Class Discussion (Understanding Subsets)

1. Yes, since a subset is a collection of well-defined and distinct objects.
2. No, since not every element of P is in Q and vice versa.

Thinking Time (Page 67)

1. $(X \cup Y)' \neq X' \cup Y'$
   $(X \cup Y)' = X' \cap Y'$
2. $(X \cap Y')' \neq X' \cap Y$
   $(X \cap Y')' = X' \cup Y$

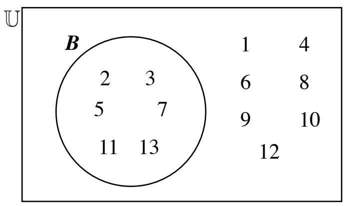
Performance Task

1. The universal set will be the students in my class.
2. Yes, G is an empty set. It should be included in the Venn diagram.
3. No, the sets will not be distinct.
4. Yes, the sets should be drawn such that there are overlapping regions between them.
5. Yes, since I am a student of the class which is the universal set $\mathbb{U}$.

Practise Now 1

(i) $\mathbb{U} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13\}$ and $B = \{2, 3, 5, 7, 11, 13\}$

(ii)

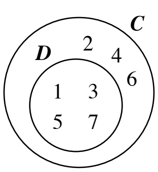


(iii) $B' = \{1, 4, 6, 8, 9, 10, 12\}$

(iv) B' is the set of all integers between 1 and 13 inclusive which are not prime numbers.

Practise Now 2

1. **(i)**



   (ii) Yes, D is a proper subset of C because every element of D is an element of C, and $D \neq C$.

2. **(i)** $P = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13\}$
   $Q = \{2, 3, 5, 7, 11\}$

   (ii) $Q \subset P$ because every element of Q is an element of P, and $Q \neq P$.

   (iii) $R = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13\}$

   (iv) R and P are equal sets because all the elements of R and P are the same, i.e. $R = P$.

Practise Now 3

(i) **(a)** {7}, {8}, {7, 8}
(b) {7}, {8}

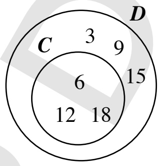
(ii) **(a)** {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, {a, b, c}
(b) { }, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}

Practise Now 4

1. **(i)** $C = \{6, 12, 18\}$ and $D = \{3, 6, 9, 12, 15, 18\}$

   (ii) $C \cap D = \{6, 12, 18\}$

   (iii)

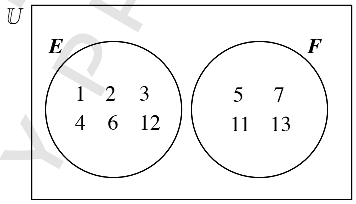


   (iv) Yes, since all of the elements of C are also in D.

2. **(i)** $E = \{1, 2, 3, 4, 6, 12\}$ and $F = \{5, 7, 11, 13\}$

   (ii) $E \cap F = \varnothing$ since E and F do not share any common elements.

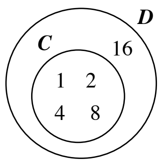
   (iii)



Practise Now 5

1. **(i)** $C = \{1, 2, 4, 8\}$ and $D = \{1, 2, 4, 8, 16\}$

   (ii)

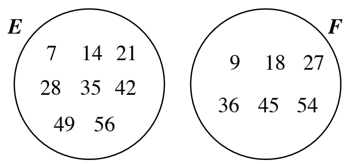


   (iii) $C \cup D = \{1, 2, 4, 8, 16\}$

   (iv) Yes, since all of the elements of C are also in D.

2. **(i)** $E = \{7, 14, 21, 28, 35, 42, 49, 56\}$ and $F = \{9, 18, 27, 36, 45, 54\}$

   (ii)

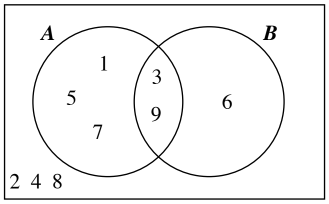


   (iii) $E \cup F = \{7, 9, 14, 18, 21, 27, 28, 35, 36, 42, 45, 49, 54, 56\}$

Practise Now 6

1. **(a)** $\mathbb{U} = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{1, 3, 5, 7, 9\}$ and $B = \{3, 6, 9\}$

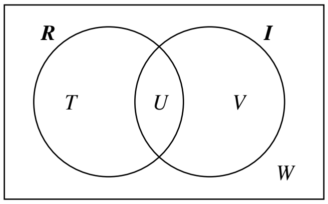
   (b) $\mathbb{U}$



   (c) **(i)** $(A \cup B)' = \{2, 4, 8\}$

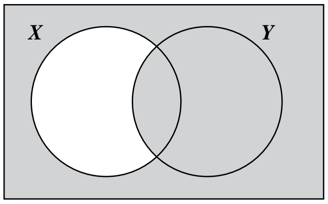
   (ii) $A \cap B' = \{1, 5, 7\}$

2. $\mathbb{U}$

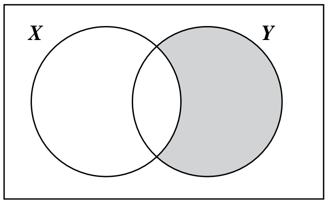


Practise Now 7

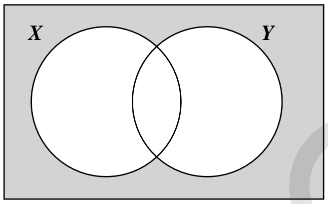
(i) $\mathbb{U}$



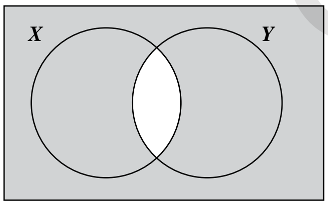
(ii) $\mathbb{U}$



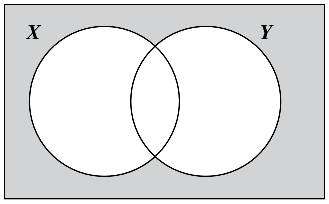
(iii) $\mathbb{U}$



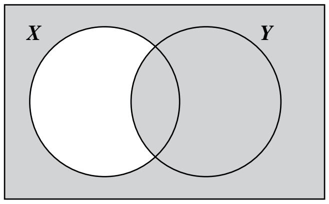
(iv) $\mathbb{U}$



(v) $\mathbb{U}$



(vi) $\mathbb{U}$



Practise Now 8

(i) $B \cap B' = \varnothing$

$B = \{c, e, g\}$, $B' = \{a, b, d, f\}$

$B \cap B' = \{\ \}$ or $\varnothing$

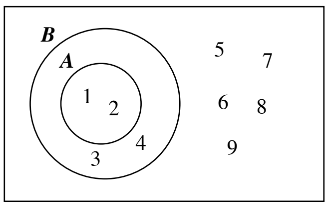
(ii) $B \cup B' = \mathbb{U}$

$\{c, e, g\} \cup \{a, b, d, f\} = \{a, b, c, d, e, f, g\}$

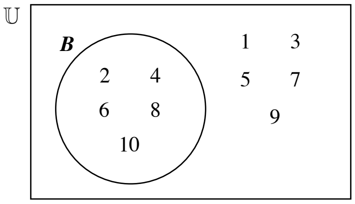
$= \mathbb{U}$

Exercise 4A

1. **(a)** {2, 4, 6, 8, 10, …, 20}
   (b) {4, 8, 12, 16, 20}
   (c) {3, 6, 9, 12, 15, 18}
   (d) {1, 3, 5, 7, 9, 11, 13, 15, 17, 19}
   (e) {1, 2, 3, 5, 6, 7, 9, 10, 11, 13, 14, 15, 17, 18, 19}
   (f) {1, 2, 4, 5, 7, 8, 10, 11, 13, 14, 16, 17, 19, 20}
2. **(a)** {30, 31, 32, 34, 35, 36, 38, 40, 41, 43, 45}
   (b) {35, 43, 44}
   (c) {31, 37, 41, 43}
   (d) {30, 31, 33, 34, 35, 37, 38, 39, 41, 42, 43, 45}
   (e) {31, 32, 34, 35, 37, 38, 40, 41, 43, 44}
3. **(a)** {2, 3, 5, 7}
   (b) {2, 4, 6, 8, 10}
   (c) {5, 10}
   (d) {1, 4, 6, 8, 9,10}
   (e) {1, 3, 5, 7, 9}
   (f) {1, 2, 3, 4, 6, 7, 8, 9}
4. **(i)** A = {cat, dog, mouse}
   $\mathbb{U}$ = {cat, dog, mouse, lion, tiger}
   (ii) A' = {tiger, lion}
5. $\mathbb{U}$



6. **(i)** $\mathbb{U} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
   $B = \{2, 4, 6, 8, 10\}$

(ii) 

(iii) $B' = \{1, 3, 5, 7, 9\}$

(iv) B' is the set of all integers between 1 and 10 inclusive which are odd numbers.

7. **(a)** {20, 40, 60, 80}

(b) {60}

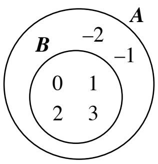
(c) {40, 80}

(d) $\varnothing$

8. **(i)** $A = \{s, t, u\}$

$B = \{s, t, u, v, w, x, y, z\}$

(ii) Yes, A is a proper subset of B because every element of A is an element of B, and $A \neq B$.

9. **(i)** 

(ii) Yes, B is a proper subset of A because every element of B is an element of A, and $B \neq A$.

10. **(i)** $\mathbb{U} = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$,

$C = \{1, 4, 6, 8, 9\}$,

$C' = \{2, 3, 5, 7\}$

(ii) C' is a set of all integers between 0 and 10 which are prime numbers.

11. **(i)** $\mathbb{U} = \{a, b, c, d, e, f, g, h, i, j\}$,

$D = \{b, c, d, f, g, h, j\}$,

$D' = \{a, e, i\}$

(ii) D' is a set of the first 10 letters of the English alphabet which are vowels.

12. **(i)** $E = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$

$F = \{4, 8, 12, 16\}$

(ii) $F \subset E$ because every element of F is an element of E, and $F \neq E$.

(iii) $G = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$

(iv) E and G are equal sets because all the elements of E and G are the same, i.e. $E = G$.

13. Yes, because every element of I is an element of H, and $I \neq H$.

14. **(a)** True

(b) True

(c) True

(d) False

(e) True

15. **(a)** { }, {1}, {2}, {1, 2}

(b) { }, {pen}, {ink}, {ruler}, {pen, ink}, {pen, ruler}, (ink, ruler}, {pen, ink, ruler}

(c) { }, {Thailand}, {Vietnam}, {Thailand, Vietnam}

(d) { }, {a}, {e}, {i}, {o}, {a, e}, {a, i}, {a, o}, {e, i}, {e, o}, {i, o}, {a, e, i}, {e, i, o}, {a, e, o}, {a, i, o}, {a, e, i, o}

16. **(a)** { }, {x}, {y}

(b) { }, {Singapore}, {Malaysia}

(c) { }, {3}, {4}, {5}, {3, 4}, {3, 5}, {4, 5}

(d) { }, {a}, {b}, {c}, {d}, {a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}

17. **(i)** $O' = \{1, 2, 4, 5, 7, 8, 10, 11, 13, 14, 16, 17, 19, 20\}$

(ii) $O' = \{x : x$ is a positive integer less than 21 which is not divisible by 3$\}$

18. **(a)** $\neq$

(b) $\supset$

(c) $=$

(d) $\neq$

(e) $\neq$

(f) $=$

(g) $\subset$

(h) $=$

(i) $\neq$

(j) $=$

(k) $\subset$

(l) $\neq$

19. **(a)** True

(b) True

(c) False

(d) False

(e) False

(f) True

(g) True

(h) False

(i) True

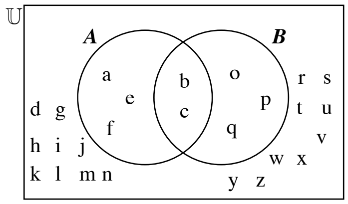
(j) True

(k) False

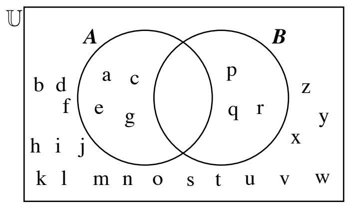
(l) True

(m) False

Exercise 4B

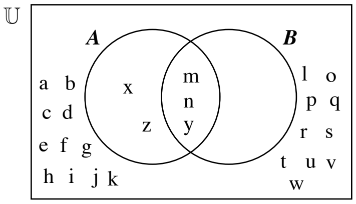
1. **(i)** 

$A \cap B = \{b, c\}$

(ii) 

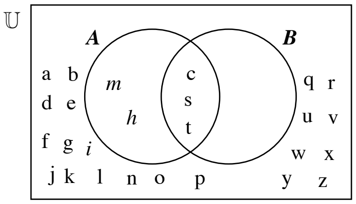
$A \cap B = \varnothing$

(iii)



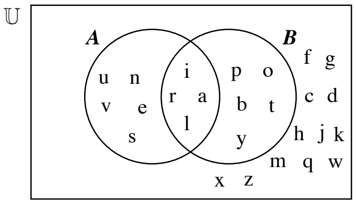
$A \cap B$ = {m, n, y}

(iv)



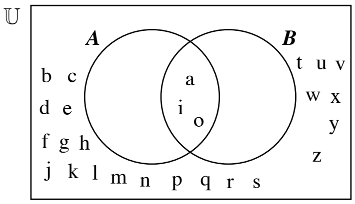
$A \cap B$ = {c, s, t}

(v)



$A \cap B$ = {a, i, r, l}

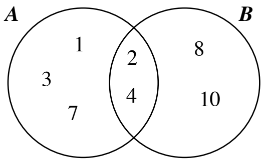
(vi)



$A \cap B$ = {a, i, o}

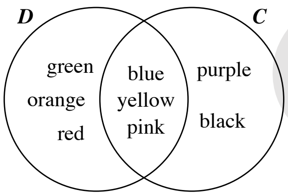
2. **(i)** $A \cap B$ = {2, 4}

(ii)

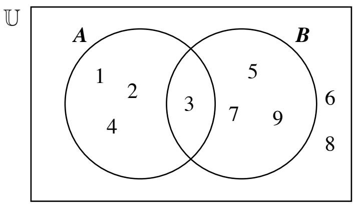


3. **(i)** $C \cap D$ = {blue, yellow, pink}

(ii)

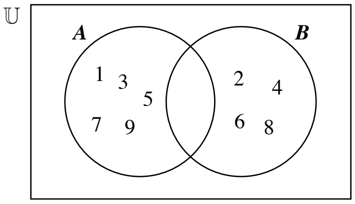


4. **(i)**



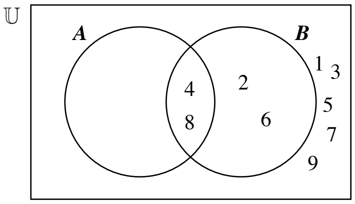
$A \cup B$ = {1, 2, 3, 4, 5, 7, 9}

(ii)



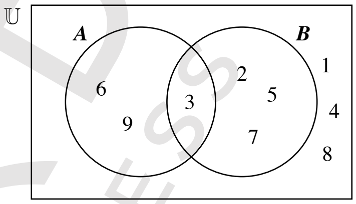
$A \cup B$ = {1, 2, 3, 4, 5, 6, 7, 8, 9}

(iii)



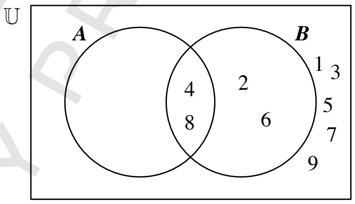
$A \cup B$ = {2, 4, 6, 8}

(iv)



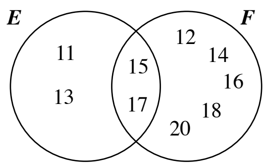
$A \cup B$ = {2, 3, 5, 6, 7, 9}

(v)



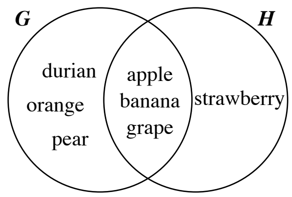
$A \cup B$ = {2, 4, 6, 8}

5. **(i)**



(ii) $E \cup F$ = {11, 12, 13, 14, 15, 16, 18, 20}

6. **(i)**



(ii) $G \cup H$ = {apple, banana, durian, grape, orange, pear, strawberry}

7. **(i)** $A \cup B$ = {durian, mango, pineapple, rambutan, soursop}

(ii) $A \cap B$ = {durian, mango}

8. **(a)** $A \cup B$ = {3, 6, 8, 9, 12}

$A \cap B$ = {6, 9}

(c) $C \cup D$ ={a, b, x, y, m, n, o, p}

$C \cap D = \varnothing$

(c) $E \cup F$ = {monkey, goat, lion, tiger}

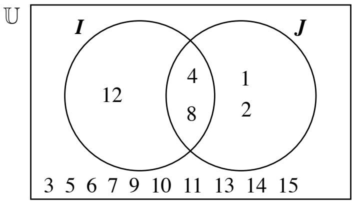
$E \cap F$ = {goat}

(d) $G \cup H$ = {a, m, k, y}

$G \cap H = \varnothing$

9. **(a)** $\mathbb{U} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$, $I = \{4, 8, 12\}$ and $J = \{1, 2, 4, 8\}$

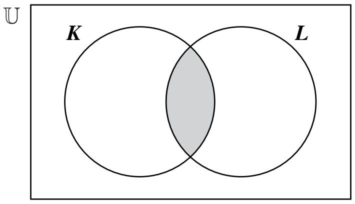
(b)



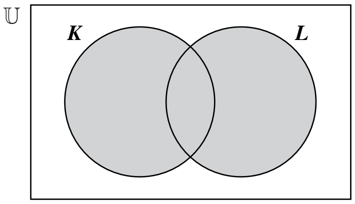
(c) **(i)** $(I \cup J)' = \{3, 5, 6, 7, 9, 10, 11, 13, 14\ 15\}$

(ii) $I \cap J' = \{12\}$

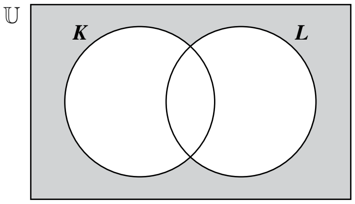
10. **(i)**



(ii)

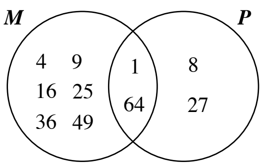


(iii)



11. **(i)** $M = \{1, 4, 9, 16, 25, 36, 49, 64\}$ and $P = \{1, 8, 27, 64\}$

(ii)

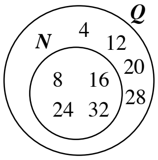


(iii) $M \cap P = \{1, 64\}$

12. **(i)** $N = \{8, 16, 24, 32\}$ and $Q = \{4, 8, 12, 16, 20, 24, 28, 32\}$

(ii) $N \cap Q = \{8, 16, 24, 32\}$

(iii)

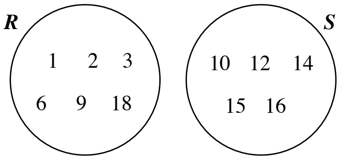


(iv) Yes, since all the elements of N are also in Q.

13. **(i)** $R = \{1, 2, 3, 6, 9, 18\}$ and $S = \{10, 12, 14, 15, 16\}$

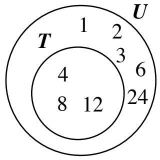
(ii) $R \cap S = \varnothing$ since R and S do not share any common elements.

(iii)



14. **(i)** $T = \{4, 8, 12\}$ and $U = \{1, 2, 3, 4, 6, 8, 12, 14\}$

(ii)

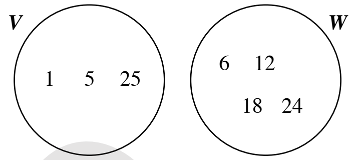


(iii) $T \cup U = \{1, 2, 3, 4, 6, 8, 12, 24\}$

(iv) Yes, since all of the elements of T are also in U.

15. **(i)** $V = \{1, 5, 25\}$ and $W = \{6, 12, 18, 24\}$

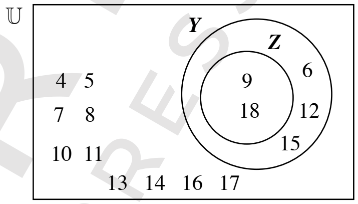
(ii)



(iii) $V \cup W = \{1, 5, 6, 12, 18, 24, 25\}$

16. **(a)** $\mathbb{U} = \{4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18\}$, $Y = \{6, 9, 12, 15, 18\}$ and $Z = \{9, 18\}$

(b)

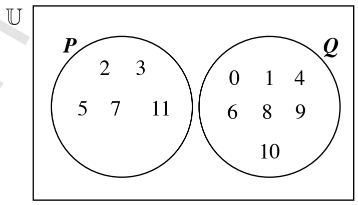


(c) **(i)** $(Y \cup Z)' = \{4, 5, 7, 8, 10, 11, 13, 14, 16, 17\}$

(ii) $Y \cap Z' = \{6, 12, 15\}$

17. **(a)** $\mathbb{U} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$, $P = \{2, 3, 5, 7, 11\}$ and $Q = \{0, 1, 4, 6, 8, 9, 10\}$

(b)

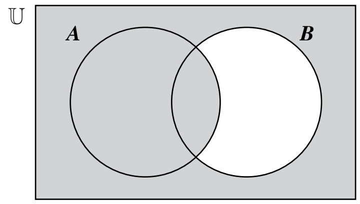


(c) **(i)** $P \cup Q = \ = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$

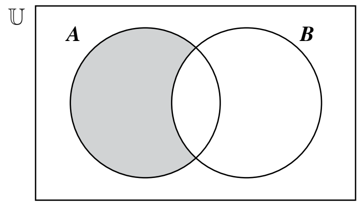
(ii) $(P \cup Q)' = \varnothing$

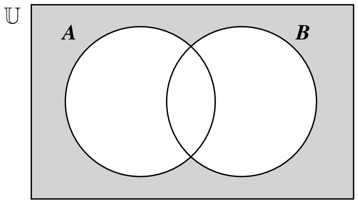
(iii) $P' \cap Q = Q = \{0, 1, 4, 6, 8, 9, 10\}$

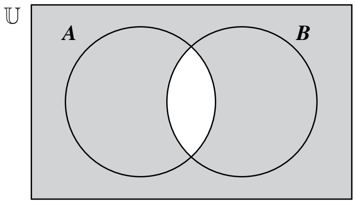
18. **(a)** **(i)**

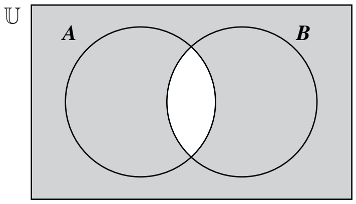


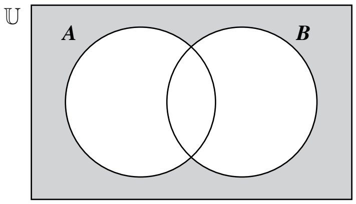
(ii)

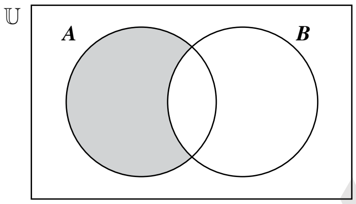


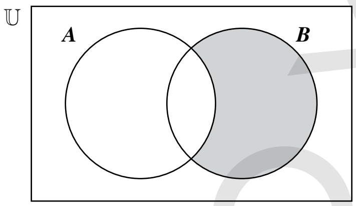
(iii) 

(iv) 

(v) 

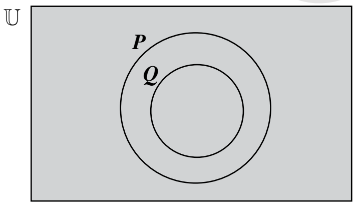
(vi) 

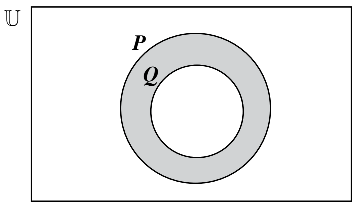
(vii) 

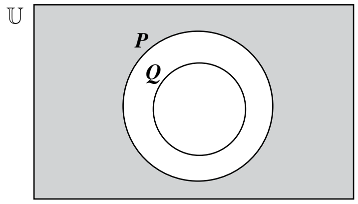
(viii) 

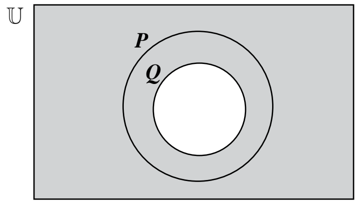
(b) **(i)** They are the same.

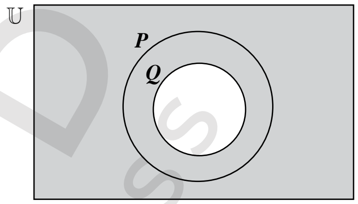
(ii) They are the same.

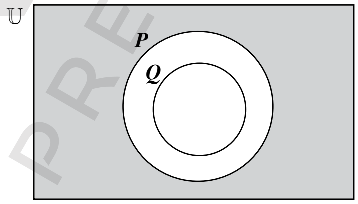
19. **(i)** 

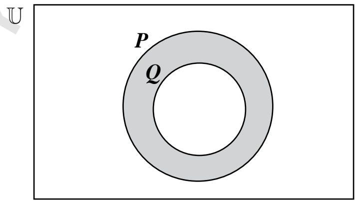
(ii) 

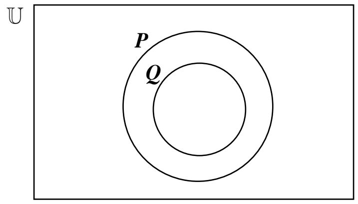
(iii) 

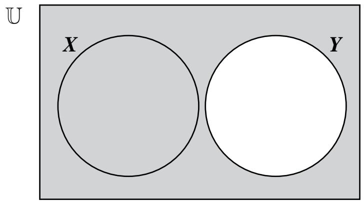
(iv) 

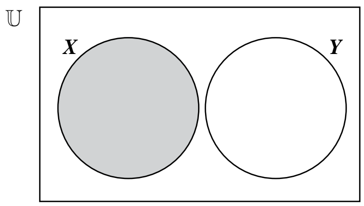
(v) 

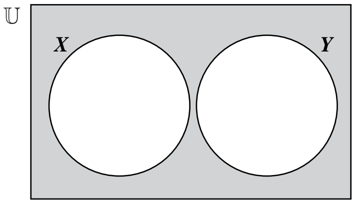
(vi) 

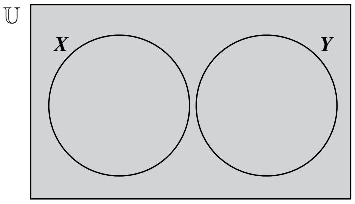
(vii) 

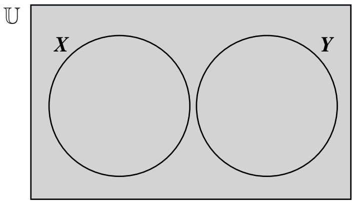
(viii) 

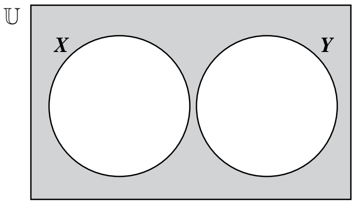
20. **(i)** 

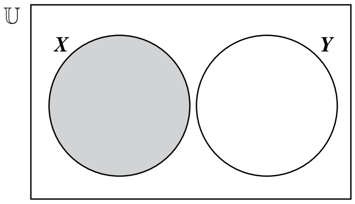
(ii) 

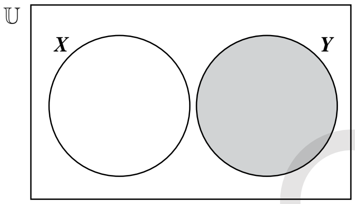
(iii) 

(iv) 

(v) 

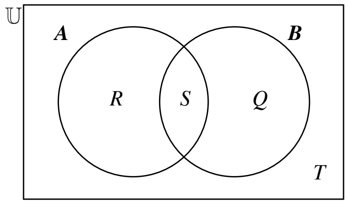
(vi) 

(vii) 

(viii) 

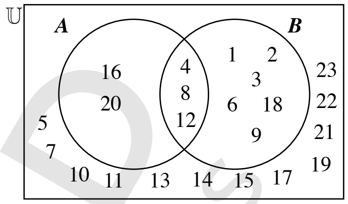
21. **(i)** $A \cap \mathbb{U} = A$
(ii) $A \cup \mathbb{U} = \mathbb{U}$
(iii) $A \cap \varnothing = \varnothing$
(iv) $A \cup \varnothing = A$

22. **(i)** $A \cap B = A$
(ii) $A \cup B = B$
(iii) Not possible to simplify further
(iv) Not possible to simplify further
(v) $(B \cup C) \cap A = A$
(vi) $(B \cap C) \cap A = \varnothing$
(vii) $(A \cup C) \cap B = A \cup C$
(viii) $(A \cap C) \cup B = B$

23. 

Review Exercise 4

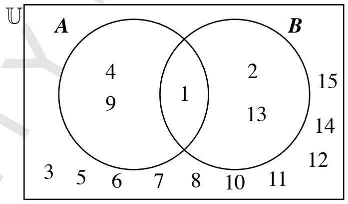
1. **(i)** $\mathbb{U}$ = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23}, A = {4, 8, 12, 16, 20} and B = {1, 2, 3, 4, 6, 8, 12, 18}



(ii) $A \cup B'$ = {4, 5, 7, 8, 10, 11, 12, 13, 14, 15, 16, 17, 19, 20, 21, 22, 23}

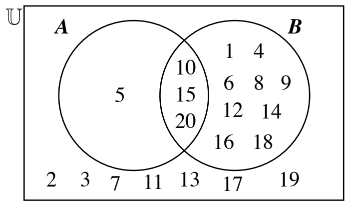
2. **(i)** A' = {–7, 7}
(ii) $A \cap B$ = {1, 2, 3, 4, 5, 6}
(iii) $A \cup B'$ = {–7, –6, –5, –4, –3, –2, –1, 0, 1, 2, 3, 4, 5, 6}

3. **(i)** $\mathbb{U}$ = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15}, A = {1, 4, 9} and B = {1, 2, 13}



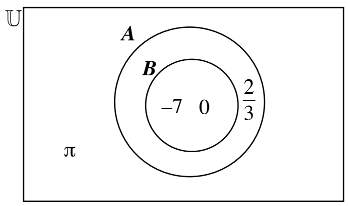
(ii) $A' \cap B'$ = {3, 5, 6, 7, 8, 10, 11, 12, 14, 15}

4. **(i)** $\mathbb{U}$ = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20}, A = {5, 10, 15, 20} and B = {1, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20}



(ii) $A \cap B'$ = {5}

5. **(i)** { }, {s}, {i}, {t}, {s, i}, {s, t}, {i, t}, {s, i, t}
(ii) { }, {s}, {i}, {t}, {s, i}, {s, t}, {i, t}

6. 

Challenge Yourself

1. If the set *S* has 2 elements, e.g. x and y then there are 4 subsets: ∅, {x}, {y} and {x, y}.

If the set *S* has 3 elements, e.g. 3, 4 and 5, then there are 8 subsets: ∅, {3}, {4}, {5}, {3, 4}, {3, 5}, {4, 5} and {3, 4, 5}.

If the set *S* has 4 elements, e.g. a, b, c and d, then there are 16 subsets: ∅, {a}, {b}, {c}, {d}, {a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d} and {a, b, c, d}.

No. of elements in a set S	No. of subsets
2	$4 = 2^2$
3	$8 = 2^3$
4	$16 = 2^4$

Note that the total number of subsets doubles each time. Hence if the set *S* has *n* elements, then there are 2^n subsets. So, if the set *S* has *n* elements, then there are $2^n - 1$ proper subsets, taking away the original set.

2. **(a)** A trapezium has at least two parallel sides.

(b) A kite has at least 2 equal adjacent sides.

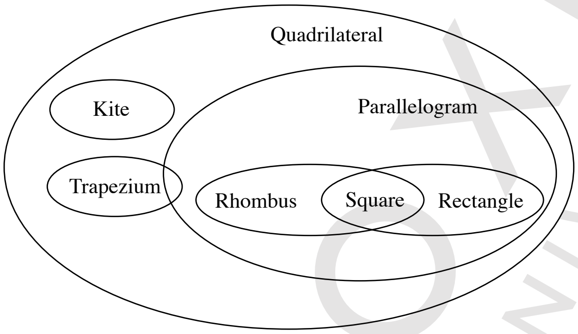
(c) A parallelogram has 2 pairs of parallel sides and 2 pairs of equal sides.

(d) A rectangle has 2 pairs of parallel sides, 2 pairs of equal sides and 4 equal angles.

(e) A rhombus has 2 pairs of parallel sides and 4 equal sides.

(f) A square has 2 pairs of parallel sides, 4 equal sides and 4 equal angles.

The Venn diagram based on the number of pairs of parallel sides a quadrilateral has is shown below.



Chapter 5 Number Patterns

TEACHING NOTES

Suggested Approach

Students have done problems involving number sequences and patterns in Grade 6. These problems required the students to recognise simple patterns from various number sequences and determine either the next few terms or a specific term. Teachers can arouse students' interest in this topic by bringing in real-life applications. (See Investigation: Fibonacci Sequence).

Section 5.1: General Term of a Number Sequence

Teachers can build upon what students have learnt in Grade 6 (Number Pattern and Algebraic and Manipulation) and teach students how to observe a number sequence and look for a pattern so that they can use algebra and find a formula for the general term, $T_n = n^{\text{th}}$ term.

Teachers can get students to work in pairs to find a formula for the general term and hence find a specific term for different number sequences (see Class Discussion: Generalising Simple Sequences). After the students have learnt how to generalise simple sequences, they should know that the aim is not to simply solve the problem but to represent it so that it becomes a general expression which can be used to find specific terms.

Section 5.2: Number Patterns in Real-World Contexts

Teachers may get students to discover number patterns in real-world contexts (e.g. shells, pine cones, rocks, wallpaper, floor tiles) and ask them to represent that number pattern into a general expression.

Challenge Yourself

Questions 1 and 2: Teachers have to get the students to think beyond just the four operations. The students have to consider more ways and observe carefully how each term in the sequence is obtained. Once they have figured this out, they are able to search on the Internet to find out the names for the sequences.

WORKED SOLUTIONS

Class Discussion (Generalising Simple Sequences)

(a) Hence, $T_n = 3n$.
100^{th} term, $T_{100} = 3 \times 100$
$= 300$

(b) Hence, $T_n = n^2$.
100^{th} term, $T_{100} = 100^2$
$= 10\ 000$

(c) Hence, $T_n = n^3$.
100^{th} term, $T_{100} = 100^3$
$= 1\ 000\ 000$

Class Discussion (The Triangular Number Sequence)

1.

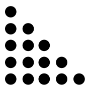


Figure 5

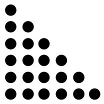


Figure 6

Investigation (Fibonacci Sequence)

1. 1; 5; 13; 21

2. 3, 5, 8, 13, 21, 34

Practise Now 1

1. **(a)** 2, 6, 10, 14, 18, ...
Add to each term to get the next term.
The next three terms are 22, 26, and 29.

(b) 5, 11, 17, 23, 29, ...
Add 6 to each term to get the next term.
The next three terms are 35, 41, and 47.

(c) 0, 3, -6, -9, -12, ...
Subract 3 from each term to get the next term .
The next three terms are -15, -18, and -21 .

(d) -1, -3, -5, -7, -9, ...
Subtract 2 from each term to get next term.
The next three terms are -11, -13, and -15.

2. 3, 7, 11, 15, 19, ...
Add 4 to each term to get the next term.
The next twi terms are 23 and 27.

Practise Now 2

(i) $T_4 = 4(4) + 7$
$= 16 + 7$
$= 23$

(ii) $T_7 = 4(7) + 7$
$= 28 + 7$
$= 35$

Sum of 4^{th} term and 7^{th} term of sequence $= T_4 + T_7$
$= 23 + 35$
$= 58$

Practise Now 3

1. **(i)**

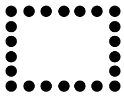


Figure 5

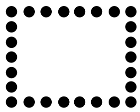


Figure 6

Exercise 5A

1. **(a)** Rule: Add 6 to each term.
The next four terms are 37, 43, 49, and 155.

(b) Rule: Add 3 to each term.
The next four terms are 11 14 17 and 20.

(c) Rule: Add 7 to each term
The nect four terms are 95, 102, 109, and 116

(d) Rule: Subtract 3 from each term.
The next four term are -1, -4, -7, and -10.

2. **(i)** $T_5 = 2(5) + 5$
$= 10 + 5$
$= 15$

(ii) $T_8 = 2(8) + 5$
$= 16 + 5$
$= 21$

(iii) $15 = 3 \times 5$
$21 = 3 \times 7$
LCM of 5^{th} term and 8^{th} term of sequence $= 3 \times 5 \times 7$
$= 105$

3. **(i)** The next two term are 18 and 21

(ii) $T_n = 3n$
$T_{105} = 3(105)$
$= 315$

4. **(i)**

Number of points	1	2	3	4	5	6
Number of segments	1 + 1 = 2	2 + 1 = 3	3 + 1 = 4	4 + 1 = 5	5 + 1 = 6	6 + 1 = 7

(ii) Let the number of points be n.
Number of segments $= n + 1$.
When $n = 49$, number of segments $= 49 + 1$
$= 50$

(iii) $101 = n + 1$
$\therefore n = 101 - 1$
$= 100$

5.

Bus	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}
Time	9.00 a.m	9.30 a.m	10.00 a.m	10.30 a.m	11.00 a.m

Asif should reach terminal before 11.00 a.m.

6. **(i)** Pattern 1 has perimeter 6cm.

Pattern 2 has the perimeter 8cm.

(ii)

pattern number	1	2	3	4	5
Perimeter	6 cm	8 cm	10 cm	12 cm	14 cm

(iii)

$T_n = 2n + 4$

$T_{18} = 2(18) + 4$

$= 36 + 4$

$= 40$

7. 6, 10, 14, ...

$T_n = 4n + 2$

$26 = 4n + 2$

$4_n + 2 = 26$

$4_n = 26 - 2$

$n = \frac{24}{4}$

$n = 6$

25 guest will sit on 6th table .

8. In 4th round :

$T_4 = 5(4) + 3$

$= 20 + 3$

$= 23$

In 5th round :

$T_5 = 5(5) + 3$

$= 52 + 3$

$= 28$

He scored 23 point in 4th round and 28 point in 5th round .

$5n + 3 = 53$

$5n = 53 - 3$

$5n = 50$

$n = \frac{50}{5}$

$n = 10$

He scored 53 point in 10th round .

Review Exercise 5

1. **(i)** 64, 81

(ii) General term of sequence, $T_n = (n + 2)^2$

(iii) $T_{25} = (25 + 2)^2$

$= 27^2$

$= 729$

Challenge Yourself

1. **(i)** 4, 9

(ii) The general term, T_n, of the sequence is obtained by continuously finding the sum of the digits of n^2 until a single-digit number is left, e.g. to obtain T_7,

$7^2 = 49 \rightarrow 4 + 9 = 13 \rightarrow 1 + 3 = 4, \therefore T_7 = 4.$

2. **(i)** 11, 18

(ii) For $n \geqslant 3$, $T_n = T_{n-1} + T_{n-2}$.

(iii) Lucas Numbers (which is different from Lucas Sequence)

Chapter 6 Simplification and Expansion

TEACHING NOTES

Suggested Approach

Since many Grade 6 students are still in the concrete operational stage (according to Piaget), the use of algebra discs can help them to learn the concepts more easily. However, there is still a need to guide students to move from the 'concrete' to the 'abstract', partly because they cannot use algebra discs in examinations, and partly because they cannot use algebra discs to manipulate algebraic expressions which consist of algebraic terms that have large or fractional coefficients (see Section 6.1).

Section 6.1: Simplification of Linear Expressions with Fractional Coefficients

After going through Worked Example 3 and 4, students should observe that the procedure for simplifying linear expressions with fractional coefficients is similar to that of simplifying ordinary numerical fractions.

WORKED SOLUTIONS

Class Discussion (Special Algebraic Identities)

1. $(a+b)^2 = (a+b)(a+b)$
$= a(a+b) + b(a+b)$
$= a^2 + ab + ab + b^2$
$= a^2 + 2ab + b^2$

2. $(a-b)^2 = (a-b)(a-b)$
$= a(a-b) - b(a-b)$
$= a^2 - ab - ab + b^2$
$= a^2 - 2ab + b^2$

3. $(a+b)(a-b) = a(a-b) + b(a-b)$
$= a^2 - ab + ab - b^2$
$= a^2 - b^2$

Class Discussion (Equivalent Expressions)

The five pairs of equivalent expressions are as follows:

1. **D** and **F**

$$\begin{aligned}3(x-2y)-2(3x-y) &= 3x-6y-6x+2y\\ &= 3x-6x-6y+2y\\ &= -3x-4y\end{aligned}$$

*Students may mistakenly match **D** and **O** due to an error in their working as shown:*

$$\begin{aligned}3(x-2y)-2(3x-y) &= 3x-6y-6x-2y\\ &= 3x-6x-6y-2y\\ &= -3x-8y\end{aligned}$$

2. **A** and **E**

$$\begin{aligned}\frac{x-3}{2}-\frac{2x-5}{3} &= \frac{3(x-3)-2(2x-5)}{6}\\ &= \frac{3x-9-4x+10}{6}\\ &= \frac{3x-4x-9+10}{6}\\ &= \frac{-x+1}{6}\\ &= \frac{1-x}{6}\end{aligned}$$

*Students may mistakenly match **E** and **H** due to an error in their working as shown:*

$$\begin{aligned}\frac{x-3}{2}-\frac{2x-5}{3} &= \frac{3(x-3)-2(2x-5)}{6}\\ &= \frac{3x-9-4x-10}{6}\\ &= \frac{3x-4x-9-10}{6}\\ &= \frac{-x-19}{6}\end{aligned}$$

3. **G** and **N**

$$\begin{aligned}\frac{3(x+3)}{4}-\frac{4(2x+3)}{3} &= \frac{9(x+3)-16(2x+3)}{12}\\ &= \frac{9x+27-32x-48}{12}\\ &= \frac{9x-32x+27-48}{12}\\ &= \frac{-23x-21}{12}\end{aligned}$$

*Students may mistakenly match **B** and **G** due to an error in their working as shown:*

$$\begin{aligned}\frac{3(x+3)}{4}-\frac{4(2x+3)}{3} &= \frac{9(x+3)-16(2x+3)}{12}\\ &= \frac{9x+27-32x+48}{12}\\ &= \frac{9x-32x+27+48}{12}\\ &= \frac{-23x+75}{12}\end{aligned}$$

4. **I** and **M**

$$\begin{aligned}2x-3[5x-y-2(7x-y)] &= 2x-3(5x-y-14x+2y)\\ &= 2x-3(5x-14x-y+2y)\\ &= 2x-3(-9x+y)\\ &= 2x+27x-3y\\ &= 29x-3y\end{aligned}$$

*Students may mistakenly match **L** and **M** due to errors in their working as shown:*

$$\begin{aligned}2x-3[5x-y-2(7x-y)] &= 2x-3(5x-y-14x-2y)\\ &= 2x-3(5x-14x-y-2y)\\ &= 2x-3(-9x-3y)\\ &= 2x-27x-9y\\ &= -25x-9y\end{aligned}$$

5. **C** and **J, C** and **K** or **J** and **K**

$7ay - 49y = 7(ay - 7y) = 7y(a - 7)$

Teachers may wish to get students to indicate the expression which is obtained when the expression 7ay – 49y is factorised completely.

Practise Now 1

1. **(a)** $(x+2)^2 = x^2 + 2(x)(2) + 2^2$
$= x^2 + 4x + 4$

(b) $(5x+3)^2 = (5x)^2 + 2(5x)(3) + 3^2$
$= 25x^2 + 30x + 9$

2. **(a)** $(1-3x)^2 = 1^2 - 2(1)(3x) + (3x)^2$
$= 1 - 6x + 9x^2$

(b) $(2x-3y)^2 = (2x)^2 - 2(2x)(3y) + (3y)^2$
$= 4x^2 - 12xy + 9y^2$

Practise Now 2

1. (a) $(5x+8)(5x-8) = (5x)^2 - 8^2$
$= 25x^2 - 64$

(b) $(-2x+7y)(-2x-7y) = (-2x)^2 - (7y)^2$
$= 4x^2 - 49y^2$

Practise Now 3

(a) $\frac{1}{2}x + \frac{1}{4}y - \frac{2}{5}y - \frac{1}{3}x = \frac{1}{2}x - \frac{1}{3}x + \frac{1}{4}y - \frac{2}{5}y$
$= \frac{3}{6}x - \frac{2}{6}x + \frac{5}{20}y - \frac{8}{20}y$
$= \frac{1}{6}x - \frac{3}{20}y$

(b) $\frac{1}{8}[-y - 3(16x - 3y)] = \frac{1}{8}(-y - 48x + 9y)$
$= \frac{1}{8}(-y + 9y - 48x)$
$= \frac{1}{8}(8y - 48x)$
$= y - 6x$

Practise Now 4

1. (a) $\frac{x-3}{2} + \frac{2x-5}{3} = \frac{3(x-3)}{6} + \frac{2(2x-5)}{6}$
$= \frac{3(x-3)+2(2x-5)}{6}$
$= \frac{3x-9+4x-10}{6}$
$= \frac{3x+4x-9-10}{6}$
$= \frac{7x-19}{6}$

(b) $\frac{x-2}{4} - \frac{2x-7}{3} = \frac{3(x-2)}{12} - \frac{4(2x-7)}{12}$
$= \frac{3(x-2)-4(2x-7)}{12}$
$= \frac{3x-6-8x+28}{12}$
$= \frac{3x-8x-6+28}{12}$
$= \frac{-5x+22}{12}$

2. (a) $\frac{x-1}{3} + \frac{1}{2} - \frac{2x-3}{4} = \frac{4(x-1)}{12} + \frac{6}{12} - \frac{3(2x-3)}{12}$
$= \frac{4(x-1)+6-3(2x-3)}{12}$
$= \frac{4x-4+6-6x+9}{12}$
$= \frac{4x-6x-4+6+9}{12}$
$= \frac{-2x+11}{12}$

(b) $2x + \frac{x-4}{9} - \frac{2x-5}{3} = \frac{9(2x)}{9} + \frac{x-4}{9} - \frac{3(2x-5)}{9}$
$= \frac{9(2x)+x-4-3(2x-5)}{9}$
$= \frac{18x+x-4-6x+15}{9}$
$= \frac{18x+x-6x-4+15}{9}$
$= \frac{13x+11}{9}$

Exercise 6A

1. (a) $\frac{1}{4}x + \frac{1}{5}y - \frac{1}{6}x - \frac{1}{10}y = \frac{1}{4}x - \frac{1}{6}x + \frac{1}{5}y - \frac{1}{10}y$
$= \frac{3}{12}x - \frac{2}{12}x + \frac{2}{10}y - \frac{1}{10}y$
$= \frac{1}{12}x + \frac{1}{10}y$

(b) $\frac{2}{3}a - \frac{1}{7}b + 2a - \frac{3}{5}b = \frac{2}{3}a + 2a - \frac{1}{7}b - \frac{3}{5}b$
$= \frac{2}{3}a + \frac{6}{3}a - \frac{5}{35}b - \frac{21}{35}b$
$= \frac{8}{3}a - \frac{26}{35}b$

(c) $\frac{5}{9}c + \frac{3}{4}d - \frac{7}{8}c - \frac{4}{3}d = \frac{5}{9}c - \frac{7}{8}c + \frac{3}{4}d - \frac{4}{3}d$
$= \frac{40}{72}c - \frac{63}{72}c + \frac{9}{12}d - \frac{16}{12}d$
$= -\frac{23}{72}c - \frac{7}{12}d$

(d) $2f - \frac{5}{3}h + \frac{9}{4}k - \frac{1}{2}f - \frac{28}{5}k + \frac{5}{4}h$
$= 2f - \frac{1}{2}f - \frac{5}{3}h + \frac{5}{4}h + \frac{9}{4}k - \frac{28}{5}k$
$= \frac{4}{2}f - \frac{1}{2}f - \frac{20}{12}h + \frac{15}{12}h + \frac{45}{20}k - \frac{112}{20}k$
$= \frac{3}{2}f - \frac{5}{12}h - \frac{67}{20}k$

2. (a) $5a + 4b - 3c - \left(2a - \frac{3}{2}b + \frac{3}{2}c\right)$
$= 5a + 4b - 3c - 2a + \frac{3}{2}b - \frac{3}{2}c$
$= 5a - 2a + 4b + \frac{3}{2}b - 3c - \frac{3}{2}c$
$= 3a + \frac{8}{2}b + \frac{3}{2}b - \frac{6}{2}c - \frac{3}{2}c$
$= 3a + \frac{11}{2}b - \frac{9}{2}c$

(b) $\frac{1}{2}[2x + 2(x-3)] = \frac{1}{2}(2x + 2x - 6)$
$= \frac{1}{2}(4x - 6)$
$= 2x - 3$

(c) $\frac{2}{5}[12p - (5 + 2p)] = \frac{2}{5}(12p - 5 - 2p)$

$= \frac{2}{5}(12p - 2p - 5)$

$= \frac{2}{5}(10p - 5)$

$= 4p - 2$

(d) $\frac{1}{2}[8x + 10 - 6(1 - 4x)] = \frac{1}{2}(8x + 10 - 6 + 24x)$

$= \frac{1}{2}(8x + 24x + 10 - 6)$

$= \frac{1}{2}(32x + 4)$

$= 16x + 2$

3. **(a)** $\frac{x}{2} + \frac{2x}{5} = \frac{5x}{10} + \frac{4x}{10}$

$= \frac{9}{10}x$

(b) $\frac{a}{3} - \frac{a}{4} = \frac{4a}{12} - \frac{3a}{12}$

$= \frac{1}{12}a$

(c) $\frac{2h}{7} + \frac{h+1}{5} = \frac{10h}{35} + \frac{7(h+1)}{35}$

$= \frac{10h + 7(h+1)}{35}$

$= \frac{10h + 7h + 7}{35}$

$= \frac{17h + 7}{35}$

(d) $\frac{3x}{8} - \frac{x+2}{4} = \frac{3x}{8} - \frac{2(x+2)}{8}$

$= \frac{3x - 2(x+2)}{8}$

$= \frac{3x - 2x - 4}{8}$

$= \frac{x - 4}{8}$

(e) $\frac{4x+1}{5} + \frac{3x-1}{2} = \frac{2(4x+1)}{10} + \frac{5(3x-1)}{10}$

$= \frac{2(4x+1) + 5(3x-1)}{10}$

$= \frac{8x + 2 + 15x - 5}{10}$

$= \frac{8x + 15x + 2 - 5}{10}$

$= \frac{23x - 3}{10}$

(f) $\frac{3y-1}{4} - \frac{2y-3}{6} = \frac{3(3y-1)}{12} - \frac{2(2y-3)}{12}$

$= \frac{3(3y-1) - 2(2y-3)}{12}$

$= \frac{9y - 3 - 4y + 6}{12}$

$= \frac{9y - 4y - 3 + 6}{12}$

$= \frac{5y + 3}{12}$

(g) $\frac{a-2}{4} - \frac{a+7}{8} = \frac{2(a-2)}{8} - \frac{a+7}{8}$

$= \frac{2(a-2) - (a+7)}{8}$

$= \frac{2a - 4 - a - 7}{8}$

$= \frac{2a - a - 4 - 7}{8}$

$= \frac{a - 11}{8}$

(h) $\frac{3p-2q}{3} - \frac{4p-5q}{4} = \frac{4(3p-2q)}{12} - \frac{3(4p-5q)}{12}$

$= \frac{4(3p-2q) - 3(4p-5q)}{12}$

$= \frac{12p - 8q - 12p + 15q}{12}$

$= \frac{12p - 12p - 8q + 15q}{12}$

$= \frac{7}{12}q$

4. **(a)** $y - \frac{2}{3}(9x - 3y) = y - 2(3x - y)$

$= y - 6x + 2y$

$= -6x + y + 2y$

$= -6x + 3y$

(b) $-\frac{1}{3}\{6(p+q) - 3[p - 2(p - 3q)]\}$

$= -\frac{1}{3}[6(p+q) - 3(p - 2p + 6q)]$

$= -\frac{1}{3}[6(p+q) - 3(-p + 6q)]$

$= -\frac{1}{3}(6p + 6q + 3p - 18q)$

$= -\frac{1}{3}(6p + 3p + 6q - 18q)$

$= -\frac{1}{3}(9p - 12q)$

$= -3p + 4q$

5. **(a)** $$\frac{7(x+3)}{2}+\frac{5(2x-5)}{3}=\frac{21(x+3)}{6}+\frac{10(2x-5)}{6}$$
$$=\frac{21(x+3)+10(2x-5)}{6}$$
$$=\frac{21x+63+20x-50}{6}$$
$$=\frac{21x+20x+63-50}{6}$$
$$=\frac{41x+13}{6}$$

(b) $$\frac{3x-4}{5}-\frac{3(x-1)}{2}=\frac{2(3x-4)}{10}-\frac{15(x-1)}{10}$$
$$=\frac{2(3x-4)-15(x-1)}{10}$$
$$=\frac{6x-8-15x+15}{10}$$
$$=\frac{6x-15x-8+15}{10}$$
$$=\frac{-9x+7}{10}$$

(c) $$\frac{3(z-2)}{4}-\frac{4(2z-3)}{5}=\frac{15(z-2)}{20}-\frac{16(2z-3)}{20}$$
$$=\frac{15(z-2)-16(2z-3)}{20}$$
$$=\frac{15z-30-32z+48}{20}$$
$$=\frac{15z-32z-30+48}{20}$$
$$=\frac{-17z+18}{20}$$

(d) $$\frac{2(p-4q)}{3}-\frac{3(2p+q)}{2}=\frac{4(p-4q)}{6}-\frac{9(2p+q)}{6}$$
$$=\frac{4(p-4q)-9(2p+q)}{6}$$
$$=\frac{4p-16q-18p-9q}{6}$$
$$=\frac{4p-18p-16q-9q}{6}$$
$$=\frac{-14p-25q}{6}$$

(e) $$-\frac{2b}{3}-\frac{3(a-2b)}{5}=-\frac{10b}{15}-\frac{9(a-2b)}{15}$$
$$=\frac{-10b-9(a-2b)}{15}$$
$$=\frac{-10b-9a+18b}{15}$$
$$=\frac{-9a-10b+18b}{15}$$
$$=\frac{-9a+8b}{15}$$

(f) $$\frac{2(x+3)}{5}-\frac{1}{2}+\frac{3x-4}{4}=\frac{8(x+3)}{20}-\frac{10}{20}+\frac{5(3x-4)}{20}$$
$$=\frac{8(x+3)-10+5(3x-4)}{20}$$
$$=\frac{8x+24-10+15x-20}{20}$$
$$=\frac{8x+15x+24-10-20}{20}$$
$$=\frac{23x-6}{20}$$

(g) $$\frac{a+1}{2}-\frac{a+3}{3}-\frac{5a-2}{4}$$
$$=\frac{6(a+1)}{12}-\frac{4(a+3)}{12}-\frac{3(5a-2)}{12}$$
$$=\frac{6(a+1)-4(a+3)-3(5a-2)}{12}$$
$$=\frac{6a+6-4a-12-15a+6}{12}$$
$$=\frac{6a-4a-15a+6-12+6}{12}$$
$$=-\frac{13}{12}a$$

(h) $$\frac{x+1}{2}+\frac{x+3}{3}-\frac{5x-1}{6}=\frac{3(x+1)}{6}+\frac{2(x+3)}{6}-\frac{5x-1}{6}$$
$$=\frac{3(x+1)+2(x+3)-(5x-1)}{6}$$
$$=\frac{3x+3+2x+6-5x+1}{6}$$
$$=\frac{3x+2x-5x+3+6+1}{6}$$
$$=\frac{10}{6}$$
$$=\frac{5}{3}$$
$$=1\frac{2}{3}$$

(i) $$\frac{2(a-b)}{7}-\frac{2a+3b}{14}+\frac{a+b}{2}$$
$$=\frac{4(a-b)}{14}-\frac{2a+3b}{14}+\frac{7(a+b)}{14}$$
$$=\frac{4(a-b)-(2a+3b)+7(a+b)}{14}$$
$$=\frac{4a-4b-2a-3b+7a+7b}{14}$$
$$=\frac{4a-2a+7a-4b-3b+7b}{14}$$
$$=\frac{9}{14}a$$

(j) $\dfrac{x+3}{3} + \dfrac{5(3x+4)}{6} + 1 = \dfrac{2(x+3)}{6} + \dfrac{5(3x+4)}{6} + \dfrac{6}{6}$

$= \dfrac{2(x+3) + 5(3x+4) + 6}{6}$

$= \dfrac{2x + 6 + 15x + 20 + 6}{6}$

$= \dfrac{2x + 15x + 6 + 20 + 6}{6}$

$= \dfrac{17x + 32}{6}$

6. (a) $\dfrac{5(p-q)}{2} - \dfrac{2q-p}{14} - \dfrac{2(p+q)}{7}$

$= \dfrac{35(p-q)}{14} - \dfrac{2q-p}{14} - \dfrac{4(p+q)}{14}$

$= \dfrac{35(p-q) - (2q-p) - 4(p+q)}{14}$

$= \dfrac{35p - 35q - 2q + p - 4p - 4q}{14}$

$= \dfrac{35p + p - 4p - 35q - 2q - 4q}{14}$

$= \dfrac{32p - 41q}{14}$

(b) $-\dfrac{2a+b}{3} - \dfrac{3(a-3b)}{2} - \dfrac{4(a+2b)}{5}$

$= -\dfrac{2a+b}{3} - \dfrac{3(a-3b)}{2} + \dfrac{4(a+2b)}{5}$

$= -\dfrac{10(2a+b)}{30} - \dfrac{45(a-3b)}{30} + \dfrac{24(a+2b)}{30}$

$= \dfrac{-10(2a+b) - 45(a-3b) + 24(a+2b)}{30}$

$= \dfrac{-20a - 10b - 45a + 135b + 24a + 48b}{30}$

$= \dfrac{-20a - 45a + 24a - 10b + 135b + 48b}{30}$

$= \dfrac{-41a + 173b}{30}$

(c) $\dfrac{3(f-h)}{4} - \dfrac{7(h+k)}{6} + \dfrac{5(k-f)}{2}$

$= \dfrac{9(f-h)}{12} - \dfrac{14(h+k)}{12} + \dfrac{30(k-f)}{12}$

$= \dfrac{9(f-h) - 14(h+k) + 30(k-f)}{12}$

$= \dfrac{9f - 9h - 14h - 14k + 30k - 30f}{12}$

$= \dfrac{9f - 30f - 9h - 14h - 14k + 30k}{12}$

$= \dfrac{-21f - 23h + 16k}{12}$

(d) $4 - \dfrac{x-y}{3} - \dfrac{3(y+4z)}{4} + \dfrac{5(x+3z)}{8}$

$= \dfrac{96}{24} - \dfrac{8(x-y)}{24} - \dfrac{18(y+z)}{24} + \dfrac{15(x+3z)}{24}$

$= \dfrac{96 - 8(x-y) - 18(y+z) + 15(x+3z)}{24}$

$= \dfrac{96 - 8x + 8y - 18y - 18z + 15x + 45z}{24}$

$= \dfrac{96 - 8x + 15x + 8y - 18y - 18z + 45z}{24}$

$= \dfrac{96 + 7x - 10y + 27z}{24}$

Review Exercise 6

1. (a) $\dfrac{2x}{3} + \dfrac{5-x}{4} = \dfrac{8x}{12} + \dfrac{3(5-x)}{12}$

$= \dfrac{8x + 3(5-x)}{12}$

$= \dfrac{8x + 15 - 3x}{12}$

$= \dfrac{8x - 3x + 15}{12}$

$= \dfrac{5x + 15}{12}$

(b) $\dfrac{x-y}{8} - \dfrac{3x-2y}{12} = \dfrac{3(x-y)}{24} - \dfrac{2(3x-2y)}{24}$

$= \dfrac{3(x-y) - 2(3x-2y)}{24}$

$= \dfrac{3x - 3y - 6x + 4y}{24}$

$= \dfrac{3x - 6x - 3y + 4y}{24}$

$= \dfrac{-3x + y}{24}$

(c) $\dfrac{4(2a-b)}{3} - \dfrac{2(3a+b)}{5} = \dfrac{20(2a-b)}{15} - \dfrac{6(3a+b)}{15}$

$= \dfrac{20(2a-b) - 6(3a+b)}{15}$

$= \dfrac{40a - 20b - 18a - 6b}{15}$

$= \dfrac{40a - 18a - 20b - 6b}{15}$

$= \dfrac{22a - 26b}{15}$

(d) $\dfrac{h+f}{3} - \dfrac{f+k}{2} + \dfrac{4h-k}{5}$

$= \dfrac{10(h+f)}{30} - \dfrac{15(f+k)}{30} + \dfrac{6(4h-k)}{30}$

$= \dfrac{10(h+f) - 15(f+k) + 6(4h-k)}{30}$

$= \dfrac{10h + 10f - 15f - 15k + 24h - 6k}{30}$

$= \dfrac{10f - 15f + 10h + 24h - 15k - 6k}{30}$

$= \dfrac{-5f + 34h - 21k}{30}$

(e) $3q - \dfrac{4p - 3q}{5} - \dfrac{q - 4p}{6}$

$= \dfrac{90q}{30} - \dfrac{6(4p - 3q)}{30} - \dfrac{5(q - 4p)}{30}$

$= \dfrac{90q - 6(4p - 3q) - 5(q - 4p)}{30}$

$= \dfrac{90q - 24p + 18q - 5q + 20p}{30}$

$= \dfrac{-24p + 20p + 90q + 18q - 5q}{30}$

$= \dfrac{-4p + 103q}{30}$

(f) $\dfrac{4(x - 5)}{7} - \dfrac{5(x - y)}{6} + \dfrac{7x - y}{21}$

$= \dfrac{4(x - 5)}{7} - \dfrac{5(x - y)}{6} - \dfrac{7x - y}{21}$

$= \dfrac{24(x - 5)}{42} - \dfrac{35(x - y)}{42} - \dfrac{2(7x - y)}{42}$

$= \dfrac{24(x - 5) - 35(x - y) - 2(7x - y)}{42}$

$= \dfrac{24x - 120 - 35x + 35y - 14x + 2y}{42}$

$= \dfrac{24x - 35x - 14x + 35y + 2y - 120}{42}$

$= \dfrac{-25x + 37y - 120}{42}$

2. **(a)** $21pq + 14q - 28qr = 7q(3p + 2 - 4r)$

(b) $4x - 8(y - 2z) = 4[x - 2(y - 2z)]$

$= 4(x - 2y + 4z)$

3. Distance Farhan can cycle in 1 minute $= \dfrac{x}{3 \times 60}$

$= \dfrac{x}{180}$ km

Distance Farhan can cycle in y minutes $= \dfrac{xy}{180}$ km

4. **(a)** Required difference $= 3y \times 60 - 25y$

$= 180y - 25y$

$= 155y$ seconds

(b) Required sum $= 50(3z - 2) \times 60 + 4(z + 1) \times 3600$

$= 3000(3z - 2) + 14\,400(z + 1)$

$= 9000z - 6000 + 14\,400z + 14\,400$

$= 9000z + 14\,400z - 6000 + 14\,400$

$= (23\,400z + 8400)$ seconds

5. **(i)** Total amount Sarah earned $=$ PKR $[(25 - 5) \times x + 5 \times 1.5x]$

$=$ PKR $(20 \times x + 5 \times 1.5x)$

$=$ PKR $(20x + 7.5x)$

$=$ PKR $27.5x$

Total amount Kiran earned $=$ PKR $[(18 - 4) \times y + 4 \times 1.5y]$

$=$ PKR $(14 \times y + 4 \times 1.5y)$

$=$ PKR $(14y + 6y)$

$=$ PKR $20y$

Total amount they earned $=$ PKR $(27.5x + 20y)$

(ii) Amount Kiran was paid per hour $=$ PKR 5.50 + PKR 0.50

$=$ PKR 6

Total amount they earned $=$ PKR $[27.5(5.5) + 20(6)]$

$=$ PKR $(151.25 + 120)$

$=$ PKR 271.25

6. **(i)** Total score obtained by Maaz in the first two papers

$= p - 3q + 13 + 3p + 5q - 4$

$= p + 3p - 3q + 5q + 13 - 4$

$= (4p + 2q + 9)$ marks

(ii) Score obtained by Maaz in the third paper

$= 10p + 5q - (4p + 2q + 9)$

$= 10p + 5q - 4p - 2q - 9$

$= 10p - 4p + 5q - 2q - 9$

$= (6p + 3q - 9)$ marks

(iii) $6p + 3q - 9 = 3(2p + q - 3)$

Challenge Yourself

1. **(a)** Total value of 5-rupee coins $=$ PKR $5x$

(b) Total value of 10-rupee coins $=$ PKR $(3x \times 10)$

$=$ PKR $30x$

(c) Number of 10-rupee coins $= \dfrac{3}{7}x$

Total value of coins $=$ PKR $5x + \dfrac{3}{7}x \times 10$

$=$ PKR $5x + \dfrac{30}{7}x$

$=$ PKR $\dfrac{35}{7}x + \dfrac{30}{7}x$

$=$ PKR $\dfrac{65}{7}x$

Chapter 7 Expansion and Factorisation of Quadratic Expressions

TEACHING NOTES

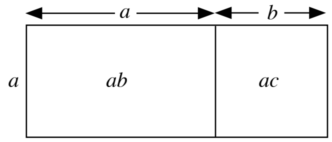
Suggested Approach

The teaching of the expansion and factorisation of algebraic expressions should focus primarily on the Concrete-Pictorial-Approach. Teachers may want to show the expansion of algebraic expressions using the area of rectangles.

E.g. Expand $a(b + c)$.

Area of rectangle $= a(b + c)$

$= ab + ac$



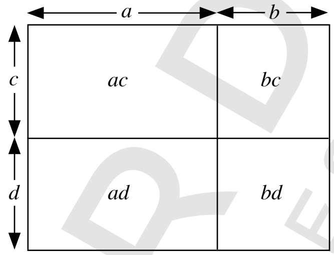
Teachers can further reinforce the concept of expanding quadratic expressions using the area of rectangles.

E.g. Expand $(a + b)(c + d)$.

Area of rectangle

$= (a + b)(c + d)$

$= ac + ad + bc + bd$



Section 7.1: Quadratic Expressions

Students have learnt how to simplify simple linear algebraic expressions in Grade 6 using algebra discs (E.g. 'x' disc, '$-x$' disc, '1' disc, '-1' disc). Teachers should further introduce another two more digital algebra discs (E.g. 'x^2' disc, '$-x^2$' disc) to help students to visualise and learn how to form and simplify simple quadratic expressions. Use the Practise Now examples in the textbook.

Section 7.2: Expansion and Simplification of Quadratic Expressions

Teachers can build upon prerequisites and move from expanding linear expressions to expanding simple quadratic expressions, of the form $a(b + c)$ using the algebra discs (see Practise Now on page 98 of the textbook).

To expand quadratic expressions of the form $(a + b)(c + d)$, teachers may use the algebra discs to illustrate how the 'expanded terms' can be arranged in the rectangular array. Teachers should also highlight to students how to 'fill up' the 'terms' in the multiplication frame after the expansion process (see Class Discussion: Expansion of Quadratic Expressions of the Form $(a + b)(c + d)$).

Section 7.3: Factorisation of Algebraic Expressions

Most students would find factorising quadratic expressions of the form $ax^2 + bx + c$ difficult. Students should be provided with ample practice questions and the factorisation process may need to be reiterated multiple times. Teachers should begin with simple quadratic expressions (E.g. those of the form $x^2 + bx + c$) to allow students to gain confidence in obtaining the 2 linear factors of the quadratic expressions.

Teachers should instruct students to explore the factorisation process of simple quadratic expressions using the algebra discs (see Practise Now on page 111 of the textbook).

Next, without using algebra discs, teacher should illustrate to students the steps to factorising quadratic expressions using a 'Multiplication Frame' (see Page 112).

Once students have acquired the technique in factorising simple quadratic expressions, teachers can then challenge the students with more difficult quadratic expressions.

WORKED SOLUTIONS

Class Discussion (Expansion of Quadratic Expressions of the Form $(a + b)(c + d)$)

1. **(a)** $(x + 3)(x + 6) = x(x + 6) + 3(x + 6)$
$= x^2 + 6x + 3x + 18$
$= x^2 + 9x + 18$

(b) $(x + 3)(x - 6) = x(x - 6) + 3(x - 6)$
$= x^2 - 6x + 3x - 18$
$= x^2 - 3x - 18$

(c) $(x - 3)(x + 6) = x(x + 6) - 3(x + 6)$
$= x^2 + 6x - 3x - 18$
$= x^2 + 3x - 18$

(d) $(x - 3)(x - 6) = x(x - 6) - 3(x - 6)$
$= x^2 - 6x - 3x + 18$
$= x^2 - 9x + 18$

(e) $(3x + 1)(2x + 3) = 3x(2x + 3) + 1(2x + 3)$
$= 6x^2 + 9x + 2x + 3$
$= 6x^2 + 11x + 3$

(f) $(3x + 1)(2x - 3) = 3x(2x - 3) + 1(2x - 3)$
$= 6x^2 - 9x + 2x - 3$
$= 6x^2 - 7x - 3$

(g) $(3x - 1)(2x + 3) = 3x(2x + 3) - 1(2x + 3)$
$= 6x^2 + 9x - 2x - 3$
$= 6x^2 + 7x - 3$

(h) $(3x - 1)(2x - 3) = 3x(2x - 3) - 1(2x - 3)$
$= 6x^2 - 9x - 2x + 3$
$= 6x^2 - 11x + 3$

2. $(a + b)(c + d) = a(c + d) + b(c + d)$
$= ac + ad + bc + bd$

Practise Now 1

(a) $-4x^2 + 2x^2 = -2x^2$

(b) $-4x^2 + (-2x^2) = -4x^2 - 2x^2$
$= -6x^2$

(c) $4x^2 - 2x^2 = 2x^2$

(d) $4x^2 - (-2x^2) = 4x^2 + 2x^2$
$= 6x^2$

(e) $2x^2 - 3 - x^2 + 1 = 2x^2 - x^2 - 3 + 1$
$= x^2 - 2$

(f) $5x^2 + (-x) + 2 - (-2x^2) - 3x - 4$
$= 5x^2 - x + 2 + 2x^2 - 3x - 4$
$= 5x^2 + 2x^2 - x - 3x + 2 - 4$
$= 7x^2 - 4x - 2$

Practise Now 2

(a) $-(2x^2 + x + 1) = -2x^2 - x - 1$

(b) $-(-2x^2 - x + 1) = 2x^2 + x - 1$

(c) $x^2 + 2x + 1 - (3x^2 + 5x - 2)$
$= x^2 + 2x + 1 - 3x^2 - 5x + 2$
$= x^2 - 3x^2 + 2x - 5x + 1 + 2$
$= -2x^2 - 3x + 3$

(d) $-(-x^2 - 4) + 2x^2 - 7x + 3$
$= x^2 + 4 + 2x^2 - 7x + 3$
$= x^2 + 2x^2 - 7x + 4 + 3$
$= 3x^2 - 7x + 7$

Practise Now 3

(a) $2(-2x^2 + x - 1) = -4x^2 + 2x - 2$

(b) $3(x^2 - 2x + 3) = 3x^2 - 6x + 9$

(c) $4x^2 + (-3x) + (-1) + 3(x^2 - 4)$
$= 4x^2 - 3x - 1 + 3x^2 - 12$
$= 4x^2 + 3x^2 - 3x - 1 - 12$
$= 7x^2 - 3x - 13$

(d) $2(x^2 + 4x - 5) - (6 + x^2)$
$= 2x^2 + 8x - 10 - 6 - x^2$
$= 2x^2 - x^2 + 8x - 10 - 6$
$= x^2 + 8x - 16$

Practise Now 4

(a) $7x^2 - 4x + 6x^2 - x = 7x^2 + 6x^2 - 4x - x$
$= 13x^2 - 5x$

(b) $-(-5x^2) + 3x + (-6) + 2(3x^2 - 8x + 4)$
$= 5x^2 + 3x - 6 + 6x^2 - 16x + 8$
$= 5x^2 + 6x^2 + 3x - 16x - 6 + 8$
$= 11x^2 - 13x + 2$

(c) $4x^2 - 1 - (7x^2 + 13x - 2)$
$= 4x^2 - 1 - 7x^2 - 13x + 2$
$= 4x^2 - 7x^2 - 13x - 1 + 2$
$= -3x^2 - 13x + 1$

(d) $-(3x^2 + 5x - 8) + x^2 + 6x + 5$
$= -3x^2 - 5x + 8 + x^2 + 6x + 5$
$= -3x^2 + x^2 - 5x + 6x + 8 + 5$
$= -2x^2 + x + 13$

Practise Now 5

(a) $3(2x + 1) = 3(2x) + 3(1)$
$= 6x + 3$

(b) $-3(2x - 1) = -3(2x) + (-3)(-1)$
$= -6x + 3$

(c) $x(-2x + 3) = x(-2x) + x(3)$
$= -2x^2 + 3x$

(d) $-2x(x - 3) = -2x(x) + (-2x)(-3)$
$= -2x^2 + 6x$

Practise Now 6

(a) $3(4x + 1) = 12x + 3$

(b) $7(5x - 2) = 35x - 14$

(c) $5x(2x - 3) = 10x^2 - 15x$

(d) $-2x(8x - 3) = -16x^2 + 6x$

Practise Now 7

(a) $5(x-4)-3(2x+4)$
$= 5x-20-6x-12$
$= 5x-6x-20-12$
$= -x-32$

(b) $2x(2x+3)-x(2-5x)$
$= 4x^2+6x-2x+5x^2$
$= 4x^2+5x^2+6x-2x$
$= 9x^2+4x$

Practise Now 8

(a) $(x+2)(x+4)$
$= x(x+4)+2(x+4)$
$= x^2+4x+2x+8$
$= x^2+6x+8$

(b) $(3x-4)(5x-6)$
$= 3x(5x-6)-4(5x-6)$
$= 15x^2-18x-20x+24$
$= 15x^2-38x+24$

(c) $(5+x)(2-3x)$
$= 5(2-3x)+x(2-3x)$
$= 10-15x+2x-3x^2$
$= 10-13x-3x^2$

(d) $(1-7x)(11x-4)$
$= (11x-4)-7x(11x-4)$
$= 11x-4-77x^2+28x$
$= -77x^2+11x+28x-4$
$= -77x^2+39x-4$

Practise Now 9

$(2x-1)(x+5)-5x(x-4)$
$= 2x(x+5)-(x+5)-5x(x-4)$
$= 2x^2+10x-x-5-5x^2+20x$
$= 2x^2-5x^2+10x-x+20x-5$
$= -3x^2+29x-5$

Practise Now 10

(a) $-10x+25 = -5(2x-5)$

(b) $18a+54ay+36az = 9a(2-6y+4z)$

Practise Now 11

(a) $2(x+1)+a(1+x) = (x+1)(2+a)$

(b) $9(x-2)-b(x+2) = (x+2)(9-b)$

(c) $3c(2x-3)-6d(2x-3) = 3[c(2x-3)-2d(2x-3)$
$= 3(2x-3)\,(c-2d)$

(d) $7h(4-x)-(x-4) = 7h(4-x)+(4-x)$
$= (4-x)\,(7h+1)$

Practise Now 12

(a)

$\times$	x	5
x	x^2	$5x$
1	x	5

$\therefore\ x^2+6x+5 = (x+1)(x+5)$

(b)

$\times$	x	-5
x	x^2	$-5x$
-1	$-x$	5

$\therefore\ x^2-6x+5 = (x-1)(x-5)$

(c)

$\times$	x	6
x	x^2	$6x$
2	$2x$	12

$\therefore\ x^2+8x+12 = (x+2)(x+6)$

(d)

$\times$	x	-6
x	x^2	$-6x$
-2	$-2x$	12

$\therefore\ x^2-8x+12 = (x-2)(x-6)$

Practise Now 13

(a) $x^2 = x \times x$
$7 = 1 \times 7$ or $(-1) \times (-7)$

$\times$	x	7
x	x^2	$7x$
1	x	7

$x+7x = 8x$
$\therefore\ x^2+8x+7 = (x+1)(x+7)$

(b) $x^2 = x \times x$
$28 = 1 \times 28$ or $(-1) \times (-28)$
$= 2 \times 14$ or $(-2) \times (-14)$
$= 4 \times 7$ or $(-4) \times (-7)$

$\times$	x	-7
x	x^2	$-7x$
-4	$-4x$	28

$(-4x)+(-7x) = -11x$
$\therefore\ x^2-11x+28 = (x-4)(x-7)$

(c) $x^2 = x \times x$
$-2 = 1 \times (-2)$ or $(-1) \times 2$

$\times$	x	2
x	x^2	$2x$
-1	$-x$	-2

$(-x)+2x = x$
$\therefore\ x^2+x-2 = (x-1)(x+2)$

(d) $x^2 = x \times x$

$-8 = 1 \times (-8)$ or $(-1) \times 8$

$= 2 \times (-4)$ or $(-2) \times 4$

$\times$	x	-8
x	x^2	$-8x$
1	x	-8

$x + (-8x) = -7x$

$\therefore x^2 - 7x - 8 = (x + 1)(x - 8)$

Practise Now 14

(a) $2x^2 = 2x \times x$

$12 = 1 \times 12$ or $(-1) \times (-12)$

$= 2 \times 6$ or $(-2) \times (-6)$

$= 3 \times 4$ or $(-3) \times (-4)$

$\times$	x	4
$2x$	$2x^2$	$8x$
3	$3x$	12

$3x + 8x = 11x$

$\therefore 2x^2 + 11x + 12 = (2x + 3)(x + 4)$

(b) $5x^2 = 5x \times x$

$6 = 1 \times 6$ or $(-1) \times (-6)$

$= 2 \times 3$ or $(-2) \times (-3)$

$\times$	x	-2
$5x$	$5x^2$	$-10x$
-3	$-3x$	6

$(-3x) + (-10x) = -13x$

$\therefore 5x^2 - 13x + 6 = (5x - 3)(x - 2)$

(c) $-2x^2 = -2x \times x$

$-9 = 1 \times (-9)$ or $(-1) \times 9$

$= 3 \times (-3)$ or $(-3) \times 3$

$\times$	x	-3
$-2x$	$-2x^2$	$6x$
3	$3x$	-9

$3x + 6x = 9x$

$\therefore -2x^2 + 9x - 9 = (-2x + 3)(x - 3)$

(d) $9x^2 - 33x + 24 = 3(3x^2 - 11x + 8)$

$3x^2 = 3x \times x$

$8 = 1 \times 8$ or $(-1) \times (-8)$

$= 2 \times 4$ or $(-2) \times (-4)$

$\times$	x	-1
$3x$	$3x^2$	$-3x$
-8	$-8x$	8

$(-8x) + (-3x) = -11x$

$\therefore 9x^2 - 33x + 24 = 3(3x - 8)(x - 1)$

Exercise 7A

1. **(a)** $6x^2 + 19 + 9x^2 - 8$
$= 6x^2 + 9x^2 + 19 - 8$
$= 15x^2 + 11$

(b) $x^2 + 2x - 7 - (-11x^2) - 5x - 1$
$= x^2 + 2x - 7 + 11x^2 - 5x - 1$
$= x^2 + 11x^2 + 2x - 5x - 7 - 1$
$= 12x^2 - 3x - 8$

(c) $y + (-3y^2) + 2(y^2 - 6y)$
$= y - 3y^2 + 2y^2 - 12y$
$= -3y^2 + 2y^2 + y - 12y$
$= -y^2 - 11y$

(d) $5x^2 - x - (x^2 - 10x)$
$= 5x^2 - x - x^2 + 10x$
$= 5x^2 - x^2 - x + 10x$
$= 4x^2 + 9x$

(e) $-(4x^2 + 9x + 2) + 3x^2 - 7x + 2$
$= -4x^2 - 9x - 2 + 3x^2 - 7x + 2$
$= -4x^2 + 3x^2 - 9x - 7x - 2 + 2$
$= -x^2 - 16x$

(f) $-(1 - 7y - 8y^2) + 2(y^2 - 3y - 1)$
$= -1 + 7y + 8y^2 + 2y^2 - 6y - 2$
$= 8y^2 + 2y^2 + 7y - 6y - 1 - 2$
$= 10y^2 + y - 3$

2. **(a)** $12 \times 5x = 12 \times 5 \times x$
$= 60x$

(b) $x \times 6x = x \times 6 \times x$
$= 6 \times x \times x$
$= 6x^2$

(c) $(-2x) \times 8x = (-2) \times x \times 8 \times x$
$= (-2) \times 8 \times x \times x$
$= -16x^2$

(d) $(-3x) \times (-10x)$
$= (-3) \times x \times (-10) \times x$
$= (-3) \times (-10) \times x \times x$
$= 30x^2$

3. **(a)** $4(3x + 4) = 12x + 16$

(b) $-6(-7x - 3) = 42x + 18$

(c) $8(-x - 3) = -8x - 24$

(d) $-2(5x - 1) = -10x + 2$

(e) $5x(3x - 4) = 15x^2 - 20x$

(f) $-8x(3x + 5) = -24x^2 - 40x$

(g) $-5x(2 - 3x) = -10x + 15x^2$

(h) $-x(-x - 1) = x^2 + x$

4. **(a)** $4(2a + 3) + 5(a + 3) = 8a + 12 + 5a + 15$
$= 8a + 5a + 12 + 15$
$= 13a + 27$

(b) $9(5 - 2b) + 3(6 - 5b) = 45 - 18b + 18 - 15b$
$= 45 + 18 - 18b - 15b$
$= 63 - 33b$

(c) $c(3c + 1) + 2c(c + 3) = 3c^2 + c + 2c^2 + 6c$
$= 3c^2 + 2c^2 + c + 6c$
$= 5c^2 + 7c$

(d) $6d(5d-4)+2d(3d-2) = 30d^2-24d+6d^2-4d$
$= 30d^2+6d^2-24d-4d$
$= 36d^2-28d$

5. **(a)** $(x+3)(x+7) = x(x+7)+3(x+7)$
$= x^2+7x+3x+21$
$= x^2+10x+21$

(b) $(4x+1)(3x+5) = 4x(3x+5)+(3x+5)$
$= 12x^2+20x+3x+5$
$= 12x^2+23x+5$

6. **(a)** $7(2a+1)-4(8a+3)$
$= 14a+7-32a-12$
$= 14a-32a+7-12$
$= -18a-5$

(b) $3(2b-1)-2(5b-3)$
$= 6b-3-10b+6$
$= 6b-10b-3+6$
$= -4b+3$

(c) $3c(5+c)-2c(3c-7)$
$= 15c+3c^2-6c^2+14c$
$= 15c+14c+3c^2-6c^2$
$= 29c-3c^2$

(d) $2d(3d-5)-d(2-d)$
$= 6d^2-10d-2d+d^2$
$= 6d^2+d^2-10d-2d$
$= 7d^2-12d$

(e) $-f(9-2f)+4f(f-8)$
$= -9f+2f^2+4f^2-32f$
$= 2f^2+4f^2-9f-32f$
$= 6f^2-41f$

(f) $-2h(3+4h)-5h(h-1)$
$= -6h-8h^2-5h^2+5h$
$= -8h^2-5h^2-6h+5h$
$= -13h^2-h$

7. **(a)** $(a+1)(a-9) = a(a-9)+(a-9)$
$= a^2-9a+a-9$
$= a^2-8a-9$

(b) $(b-2)(b+7) = b(b+7)-2(b+7)$
$= b^2+7b-2b-14$
$= b^2+5b-14$

(c) $(c-5)(c-6) = c(c-6)-5(c-6)$
$= c^2-6c-5c+30$
$= c^2-11c+30$

(d) $(3d+1)(5-2d) = 3d(5-2d)+(5-2d)$
$= 15d-6d^2+5-2d$
$= -6d^2+15d-2d+5$
$= -6d^2+13d+5$

(e) $(1-f)(7f+6) = (7f+6)-f(7f+6)$
$= 7f+6-7f^2-6f$
$= -7f^2+7f-6f+6$
$= -7f^2+f+6$

(f) $(4-3h)(10-9h) = 4(10-9h)-3h(10-9h)$
$= 40-36h-30h+27h^2$
$= 40-66h+27h^2$

8. **(a)** $5+(x+1)(x+3) = 5+x(x+3)+(x+3)$
$= 5+x^2+3x+x+3$
$= x^2+3x+x+5+3$
$= x^2+4x+8$

(b) $3x+(x+7)(2x-1) = 3x+x(2x-1)+7(2x-1)$
$= 3x+2x^2-x+14x-7$
$= 2x^2+3x-x+14x-7$
$= 2x^2+16x-7$

(c) $(3x+2)(x-9)+2x(4x+1)$
$= 3x(x-9)+2(x-9)+8x^2+2x$
$= 3x^2-27x+2x-18+8x^2+2x$
$= 3x^2+8x^2-27x+2x+2x-18$
$= 11x^2-23x-18$

(d) $(x-3)(x-8)+(x-4)(2x+9)$
$= x(x-8)-3(x-8)+x(2x+9)-4(2x+9)$
$= x^2-8x-3x+24+2x^2+9x-8x-36$
$= x^2+2x^2-8x-3x+9x-8x+24-36$
$= 3x^2-10x-12$

9. **(a)** $4x^2-(3x-4)(2x+1) = 4x^2-[3x(2x+1)-4(2x+1)]$
$= 4x^2-(6x^2+3x-8x-4)$
$= 4x^2-6x^2-3x+8x+4$
$= -2x^2+5x+4$

(b) $2x(x-6)-(2x+5)(7-x) = 2x^2-12x-[2x(7-x)+5(7-x)]$
$= 2x^2-12x-(14x-2x^2+35-5x)$
$= 2x^2-12x-14x+2x^2-35+5x$
$= 2x^2+2x^2-12x-14x+5x-35$
$= 4x^2-23x-35$

(c) $(4x-3)(x+2)-(3x-5)(-x-9)$
$= [4x(x+2)-3(x+2)]-[3x(-x-9)-5(-x-9)]$
$= (4x^2+8x-3x-6)-(-3x^2-27x+5x+45)$
$= 4x^2+5x-6+3x^2+22x-45$
$= 4x^2+3x^2+5x+22x-6-45$
$= 7x^2+27x-51$

(d) $(2x+3)(5x-2)-2(5x-3)(x+1)$
$= [2x(5x-2)+3(5x-2)]-2[5x(x+1)-3(x+1)]$
$= (10x^2-4x+15x-6)-2(5x^2+5x-3x-3)$
$= 10x^2+11x-6-2(5x^2+2x-3)$
$= 10x^2+11x-6-10x^2-4x+6$
$= 10x^2-10x^2+11x-4x-6+6$
$= 7x$

Exercise 7B

1. **(a)** $12x-9 = 3(4x-3)$

(b) $-125y-35 = -5\ (5y+7)$

(c) $27b-36by = 9b\ (3-4y)$

(d) $8ax+12a-4az = 4a\ (2x+3-z)$

(e) $4m-6my-18mz = 2m\ (2-3y-9z)$

2. **(a)** $9x + 27y$

$= 9(x + 3y)$

(b) $2ab^2 - ab^2c - 5ab$

$= ab\,(2b - bc - 5)$

(c) $12x^2y + 21x^2y^2$

$= 12xy\,(x + 2xy)$

(d) $9a^2 + 18b + 27$

$= 9(a^2 + 2b + 3)$

(e) $15a^3 - 5a^2 - a$

$a(15a^2 - 5a - 1)$

(f) $25x^2 + 15x^2y + 20x^2\,2y^3$

$5x2\,(5 + 3y + 4y^3)$

3. **(a)** $ax + by + ay + bz$

$= ax + ay + by + bz$

$= a(x+y) + b(y+z)$

(b) $x^2 - ax - bx + ab$

$= x(x - a) - b\,(x-a)$

$= (x - a)\,(x - b)$

(c) $l^2 - ln + lm - mn$

$= l(l - n) + m(l - n)$

$= (l - n)\,(l + m)$

(d) $4xy - y^2 + 12xz - 3yz$

$= y\,(4x - y) + 3z\,(4x - y)$

$= (4x - y)\,(y + 3z)$

(e) $a^3 - 5a^2 + a - 5$

$= a^2\,(a-5) + 1(a - 5)$

$= (a^2 + 1)\,(a - 5)$

(f) $mn^2 - np^2 - mn + p^2$

$= mn^2 - mn - np^2 + p^2$

$= mn\,(n - 1) - p^2(n - 1)$

$= (n - 1)\,(mn - p^2)$

4. **(a)** $a^2 = a \times a$

$8 = 1 \times 8$ or $(-1) \times (-8)$

$= 2 \times 4$ or $(-2) \times (-4)$

$\times$	a	8
a	a^2	$8a$
1	a	8

$a + 8a = 9a$

$\therefore\ a^2 + 9a + 8 = (a + 1)(a + 8)$

(b) $b^2 = b \times b$

$15 = 1 \times 15$ or $(-1) \times (-15)$

$= 3 \times 5$ or $(-3) \times (-5)$

$\times$	b	5
b	b^2	$5b$
3	$3b$	15

$3b + 5b = 8b$

$\therefore\ b^2 + 8b + 15 = (b + 3)(b + 5)$

(c) $c^2 = c \times c$

$20 = 1 \times 20$ or $(-1) \times (-20)$

$= 2 \times 10$ or $(-2) \times (-10)$

$= 4 \times 5$ or $(-4) \times (-5)$

$\times$	c	-5
c	c^2	$-5c$
-4	$-4c$	20

$(-4c) + (-5c) = -9c$

$\therefore\ c^2 - 9c + 20 = (c - 4)(c - 5)$

(d) $d^2 = d \times d$

$28 = 1 \times 28$ or $(-1) \times (-28)$

$= 2 \times 14$ or $(-2) \times (-14)$

$= 4 \times 7$ or $(-4) \times (-7)$

$\times$	d	-14
d	d^2	$-14d$
-2	$-2d$	28

$(-2d) + (-14d) = -16d$

$\therefore\ d^2 - 16d + 28 = (d - 2)(d - 14)$

(e) $f^2 = f \times f$

$-16 = 1 \times (-16)$ or $(-1) \times 16$

$= 2 \times (-8)$ or $(-2) \times 8$

$= 4 \times (-4)$ or $(-4) \times 4$

$\times$	f	8
f	f^2	$8f$
-2	$-2f$	-16

$(-2f) + 8f = 6f$

$\therefore\ f^2 + 6f - 16 = (f - 2)(f + 8)$

(f) $h^2 = h \times h$

$-120 = 1 \times (-120)$ or $(-1) \times 120$

$= 2 \times (-60)$ or $(-2) \times 60$

$= 3 \times (-40)$ or $(-3) \times 40$

$= 4 \times (-30)$ or $(-4) \times 30$

$= 5 \times (-24)$ or $(-5) \times 24$

$= 6 \times (-20)$ or $(-6) \times 20$

$= 8 \times (-15)$ or $(-8) \times 15$

$= 10 \times (-12)$ or $(-10) \times 12$

$\times$	h	12
h	h^2	$12h$
-10	$-10h$	-120

$(-10h) + 12h = 2h$

$\therefore\ h^2 + 2h - 120 = (h - 10)(h + 12)$

(g) $k^2 = k \times k$

$-12 = 1 \times (-12)$ or $(-1) \times 12$

$= 2 \times (-6)$ or $(-2) \times 6$

$= 3 \times (-4)$ or $(-3) \times 4$

$\times$	k	-6
k	k^2	$-6k$
2	$-2k$	-12

$2k + (-6k) = -4k$

$\therefore k^2 - 4k - 12 = (k + 2)(k - 6)$

(h) $m^2 = m \times m$

$-21 = 1 \times (-21)$ or $(-1) \times 21$

$= 3 \times (-7)$ or $(-3) \times 7$

$\times$	m	-21
m	m^2	$-21m$
1	m	21

$m + (-21m) = -20m$

$\therefore m^2 - 20m - 21 = (m + 1)(m - 21)$

5. (a) $3n^2 = 3n \times n$

$7 = 1 \times 7$ or $(-1) \times (-7)$

$\times$	n	1
$3n$	$3n^2$	$3n$
7	$7n$	7

$7n + 3n = 10n$

$\therefore 3n^2 + 10n + 7 = (3n + 7)(n + 1)$

(b) $4p^2 = 4p \times p$ or $2p \times 2p$

$3 = 1 \times 3$ or $(-1) \times (-3)$

$\times$	$2p$	3
$2p$	$4p^2$	$6p$
1	$2p$	3

$2p + 6p = 8p$

$\therefore 4p^2 + 8p + 3 = (2p + 1)(2p + 3)$

(c) $6q^2 = 6q \times q$ or $3q \times 2q$

$12 = 1 \times 12$ or $(-1) \times (-12)$

$= 2 \times 6$ or $(-2) \times (-6)$

$= 3 \times 4$ or $(-3) \times (-4)$

$\times$	$2q$	-3
$3p$	$6p^2$	$-9q$
-4	$-8q$	12

$(-8q) + (-9q) = -17q$

$\therefore 6q^2 - 17q + 12 = (3q - 4)(2q - 3)$

(d) $4r^2 = 4r \times r$ or $2r \times 2r$

$3 = 1 \times 3$ or $(-1) \times (-3)$

$\times$	r	-1
$4r$	$4r^2$	$-4r$
-3	$-3r$	3

$(-3r) + (-4r) = -7r$

$\therefore 4r^2 - 7r + 3 = (4r - 3)(r - 1)$

(e) $8s^2 = 8s \times s$ or $4s \times 2s$

$-15 = 1 \times (-15)$ or $(-1) \times 15$

$= 3 \times (-5)$ or $(-3) \times 5$

$\times$	$2s$	3
$4s$	$8s^2$	$-12s$
-5	$-10s$	-15

$(-10s) + 12s = 2s$

$\therefore 8s^2 + 2s - 15 = (4s - 5)(2s + 3)$

(f) $6t^2 = 6t \times t$ or $3t \times 2t$

$-20 = 1 \times (-20)$ or $(-1) \times 20$

$= 2 \times (-10)$ or $(-2) \times 10$

$= 4 \times (-5)$ or $(-4) \times 5$

$\times$	t	4
$6t$	$6t^2$	$24c$
-5	$-5t$	-20

$(-5t) + 24t = 19t$

$\therefore 6t^2 + 19t - 20 = (6t - 5)(t + 4)$

(g) $4u^2 = 4u \times u$ or $2u \times 2u$

$-21 = 1 \times (-21)$ or $(-1) \times 21$

$= 3 \times (-7)$ or $(-3) \times 7$

$\times$	$2u$	-7
$2u$	$4u^2$	$-14u$
3	$6u$	-21

$6u + (-14u) = -8u$

$\therefore 4u^2 - 8u - 21 = (2u + 3)(2u - 7)$

(h) $18w^2 = 18w \times w$ or $9w \times 2w$ or $6w \times 3w$

$-39 = 1 \times (-39)$ or $(-1) \times 39$

$= 3 \times (-13)$ or $(-3) \times 13$

$\times$	$2w$	-3
$9w$	$18w^2$	$-27w$
13	$26w$	-39

$26w + (-27w) = -w$

$\therefore 18w^2 - w - 39 = (9w + 13)(2w - 3)$

6. **(a)** $-a^2 = -a \times a$

$35 = 1 \times 35$ or $(-1) \times (-35)$

$= 5 \times 7$ or $(-5) \times (-7)$

×	a	5
$-a$	$-a^2$	$-5a$
7	$7a$	35

$7a + (-5a) = 2a$

$\therefore -a^2 + 2a + 35 = (-a + 7)(a + 5)$

(b) $-3b^2 = -3b \times b$

$-25 = 1 \times (-25)$ or $(-1) \times 25$

$= 5 \times (-5)$ or $(-5) \times 5$

×	b	-25
$-3b$	$-3b^2$	$75b$
1	b	-25

$b + 75b = 76b$

$\therefore -3b^2 + 76b - 25 = (-3b + 1)(b - 25)$

(c) $4c^2 + 10c + 4 = 2(2c^2 + 5c + 2)$

$2c^2 = 2c \times c$

$2 = 1 \times 2$ or $(-1) \times (-2)$

×	c	2
$2c$	$2c^2$	$4c$
1	c	2

$c + 4c = 5c$

$\therefore 4c^2 + 10c + 4 = 2(2c + 1)(c + 2)$

(d) $5d^2 - 145d + 600 = 5(d^2 - 29d + 120)$

$d^2 = d \times d$

$120 = 1 \times 120$ or $(-1) \times (-120)$

$= 2 \times 60$ or $(-2) \times (-60)$

$= 3 \times 40$ or $(-3) \times (-40)$

$= 4 \times 30$ or $(-4) \times (-30)$

$= 5 \times 24$ or $(-5) \times (-24)$

$= 6 \times 20$ or $(-6) \times (-20)$

$= 8 \times 15$ or $(-8) \times (-15)$

$= 10 \times 12$ or $(-10) \times (-12)$

×	d	-24
d	d^2	$-24d$
-5	$-5h$	120

$(-5h) + (-24h) = -29d$

$\therefore 5d^2 - 145d + 600 = 5(d - 5)(d - 24)$

(e) $8f^2 + 4f - 60 = 4(2f^2 + f - 15)$

$2f^2 = 2f \times f$

$-15 = 1 \times (-15)$ or $(-1) \times 15$

$= 3 \times (-5)$ or $(-3) \times 5$

×	f	3
$2f$	$2f^2$	$6f$
-5	$-5f$	-15

$(-5f) + 6f = f$

$\therefore 8f^2 + 4f - 60 = 4(2f - 5)(f + 3)$

(f) $24h^2 - 15h - 9 = 3(8h^2 - 5h - 3)$

$8h^2 = 8h \times h$ or $4h \times 2h$

$-3 = 1 \times (-3)$ or $(-1) \times 3$

×	h	-1
$8h$	$8h^2$	$-8h$
3	$3h$	-3

$3h + (-8h) = -5h$

$\therefore 24h^2 - 15h - 9 = 3(8h + 3)(h - 1)$

(g) $30 + 14k - 4k^2 = 2(15 + 7k - 2k^2)$

$-2k = -2k \times k$

$15 = 1 \times 15$ or $(-1) \times (-15)$

$= 3 \times 5$ or $(-3) \times (-5)$

×	$2k$	3
$-k$	$-2k^2$	$-3k$
5	$10k$	15

$10k + (-3k) = 7k$

$\therefore 30 + 14k - 4k^2 = 2(-k + 5)(2k + 3)$

(h) $35m^2n + 5mn - 30n = 5n(7m^2 + m - 6)$

$7m^2 = 7m \times m$

$-6 = 1 \times (-6)$ or $(-1) \times 6$

$= 2 \times (-3)$ or $(-2) \times 3$

×	m	1
$7m$	$7m^2$	$7m$
-6	$-6m$	-6

$(-6m) + 7m = m$

$\therefore 35m^2n + 5mn - 30n = 5n(7m - 6)(m + 1)$

7. $x^2 = x \times x$

$12 = 1 \times 12$ or $(-1) \times (-12)$

$= 2 \times 6$ or $(-2) \times (-6)$

$= 3 \times 4$ or $(-3) \times (-4)$

×	x	6
x	x^2	$6x$
2	$2x$	12

$2x + 6x = 8x$

$\therefore x^2 + 8x + 12 = (x + 2)(x + 6)$

Since the area is $(x^2 + 8x + 12)$ cm^2 and the length is $(x + 6)$ cm,

$\therefore$ the breadth is $(x + 2)$ cm.

8. **(a)** $\frac{4}{9}p^2 + p - 1 = \frac{1}{9}(4p^2 + 9p - 9)$

$4p^2 = 4p \times p$ or $2p \times 2p$

$-9 = 1 \times (-9)$ or $(-1) \times 9$

$= 3 \times (-3)$ or $(-3) \times 3$

×	p	3
$4p$	$4p^2$	$12p$
-3	$-3p$	-9

$(-3p) + 12p = 9p$

$\therefore \frac{4}{9}p^2 + p - 1 = \frac{1}{9}(4p - 3)(p + 3)$

(b) $0.6r - 0.8qr - 12.8q^2r = -0.2r(64q^2 + 4q - 3)$

$64q^2 = 64q \times q$ or $32q \times 2q$ or $16q \times 4q$ or $8q \times 8q$

$-3 = 1 \times (-3)$ or $(-1) \times 3$

$\times$	$4q$	1
$16q$	$64q^2$	$16q$
-3	$-12q$	-3

$(-12q) + 16q = 4q$

$\therefore 0.6r - 0.8qr - 12.8q^2r = -0.2r(16q - 3)(4q + 1)$

Review Exercise 7

1. **(a)** $10a(2a - 7) = 20a^2 - 70a$

(b) $-3b(7 - 4b) = -21b + 12b^2$

(c) $(c - 4)(c - 11) = c(c - 11) - 4(c - 11)$
$= c^2 - 11c - 4c + 44$
$= c^2 - 15c + 44$

(d) $(3d - 5)(4 - d) = 3d(4 - d) - 5(4 - d)$
$= 12d - 3d^2 - 20 + 5d$
$= -3d^2 + 12d + 5d - 20$
$= -3d^2 + 17d - 20$

2. **(a)** $7f(3f - 4) + 4f(3 - 2f) = 21f^2 - 28f + 12f - 8f^2$
$= 21f^2 - 8f^2 - 28f + 12f$
$= 13f^2 - 16f$

(b) $6h^2 + (2h + 3)(h - 1) = 6h^2 + 2h(h - 1) + 3(h - 1)$
$= 6h^2 + 2h^2 - 2h + 3h - 3$
$= 8h^2 + h - 3$

(c) $(2k - 1)(k - 4) - 3k(k - 7)$
$= 2k(k - 4) - (k - 4) - 3k^2 + 21k$
$= 2k^2 - 8k - k + 4 - 3k^2 + 21k$
$= 2k^2 - 3k^2 - 8k - k + 21k + 4$
$= -k^2 + 12k + 4$

(d) $(m + 2)(m + 1) - (3m + 5)(9 - 5m)$
$= m(m + 1) + 2(m + 1) - [3m(9 - 5m) + 5(9 - 5m)]$
$= m^2 + m + 2m + 2 - (27m - 15m^2 + 45 - 25m)$
$= m^2 + m + 2m + 2 - 27m + 15m^2 - 45 + 25m$
$= m^2 + 15m^2 + m + 2m - 27m + 25m + 2 - 45$
$= 16m^2 + m - 43$

3. **(a)** $a^2 = a \times a$

$36 = 1 \times 36$ or $(-1) \times (-36)$
$= 2 \times 18$ or $(-2) \times (-18)$
$= 3 \times 12$ or $(-3) \times (-12)$
$= 4 \times 9$ or $(-4) \times (-9)$
$= 6 \times 6$ or $(-6) \times (-6)$

$\times$	a	9
a	a^2	$9a$
4	$4a$	36

$4a + 9 = 13a$

$\therefore a^2 + 13a + 36 = (a + 4)(a + 9)$

(b) $b^2 = b \times b$

$56 = 1 \times 56$ or $(-1) \times (-56)$
$= 2 \times 28$ or $(-2) \times (-28)$
$= 4 \times 14$ or $(-4) \times (-14)$
$= 7 \times 8$ or $(-7) \times (-8)$

$\times$	b	-8
b	b^2	$-8b$
-7	$-7b$	56

$(-7b) + (-8b) = -15b$

$\therefore b^2 - 15b + 56 = (b - 7)(b - 8)$

(c) $c^2 = c \times c$

$-51 = 1 \times (-51)$ or $(-1) \times 51$
$= 3 \times (-17)$ or $(-3) \times 17$

$\times$	c	17
c	c^2	$17c$
-3	$-3c$	-51

$(-3c) + 17c = 14c$

$\therefore c^2 + 14c - 51 = (c - 3)(c + 17)$

(d) $d^2 = d \times d$

$-45 = 1 \times (-45)$ or $(-1) \times 45$
$= 3 \times (-15)$ or $(-3) \times 15$
$= 5 \times (-9)$ or $(-5) \times 9$

$\times$	d	-15
d	d^2	$-15d$
3	$3d$	-45

$3d + (-15d) = -15d$

$\therefore d^2 - 12d - 45 = (d + 3)(d - 15)$

4. **(a)** $9f^2 = 9f \times f$ or $3f \times 3f$

$-16 = 1 \times (-16)$ or $(-1) \times 16$
$= 2 \times (-8)$ or $(-2) \times 8$
$= 4 \times (-4)$ or $(-4) \times 4$

$\times$	$3f$	8
$3f$	$9f^2$	$24f$
-2	$-6f$	-16

$(-6f) + 24f = 18f$

$\therefore 9f^2 + 18f - 16 = (3f - 2)(3f + 8)$

(b) $3h^2 = 3h \times h$

$-14 = 1 \times (-14)$ or $(-1) \times 14$
$= 2 \times (-7)$ or $(-2) \times 7$

$\times$	h	-7
$3h$	$3h^2$	$-21h$
2	$2h$	-14

$2h + (-21h) = -19h$

$\therefore 3h^2 - 19h - 14 = (3h + 2)(h - 7)$

(c) $14k^2 + 49k + 21 = 7(2k^2 + 7k + 3)$

$2k^2 = 2k \times k$

$3 = 1 \times 3$ or $(-1) \times (-3)$

$\times$	k	3
$2k$	$2k^2$	$6k$
1	k	3

$k + 6k = 7k$

$\therefore\ 14k^2 + 49k + 21 = 7(2k + 1)(k + 3)$

(d) $18m^2 - 39m + 18 = 3(6m^2 - 13m + 6)$

$6m^2 = 6m \times m$ or $3m \times 2m$

$6 = 1 \times 6$ or $(-1) \times (-6)$

$= 2 \times 3$ or $(-2) \times (-3)$

$\times$	$2m$	-3
$3m$	$6m^2$	$-9m$
-2	$-4m$	6

$(-4m) + (-9m) = -13m$

$\therefore\ 18m^2 - 39m + 18 = 3(3m - 2)(2m - 3)$

5. $3x^2 - \dfrac{11}{2}x - 5 = \dfrac{1}{2}(6x^2 - 11x - 10)$

$6x^2 = 6x \times x$ or $3x \times 2x$

$-10 = 1 \times (-10)$ or $(-1) \times 10$

$= 2 \times (-5)$ or $(-2) \times 5$

$\times$	$2x$	-5
$3x$	$6x^2$	$-15x$
2	$4x$	-10

$4x + (-15x) = -11x$

$\therefore\ 3x^2 - \dfrac{11}{2}x - 5 = \dfrac{1}{2}(3x + 2)(2x - 5)$

6. **(a)** $-2a(a - 5b + 7) = -2a^2 + 10ab - 14a$

(b) $(2c + 3d)(3c + 4d) = 2c(3c + 4d) + 3d(3c + 4d)$

$= 6c^2 + 8cd + 9cd + 12d^2$

$= 6c^2 + 17cd + 12d^2$

(c) $(k + 3h)(5h - 4k) = k(5h - 4k) + 3h(5h - 4k)$

$= 5hk - 4k^2 + 15h^2 - 12hk$

$= -4k^2 + 5hk - 12hk + 15h^2$

$= -4k^2 - 7hk + 15h^2$

(d) $(2m + 1)(m^2 + 3m - 1) = 2m(m^2 + 3m - 1) + (m^2 + 3m - 1)$

$= 2m^3 + 6m^2 - 2m + m^2 + 3m - 1$

$= 2m^3 + 6m^2 + m^2 - 2m + 3m - 1$

$= 2m^3 + 7m^2 + m - 1$

7. **(a)** $2p(3p - 5q) - q(2q - 3p) = 6p^2 - 10pq - 2q^2 + 3pq$

$= 6p^2 - 10pq + 3pq - 2q^2$

$= 6p^2 - 7pq - 2q^2$

(b) $-4s(3s + 4r) - 2r(2r - 5s) = -12s^2 - 16sr - 4r^2 + 10sr$

$= -12s^2 - 16sr + 10sr - 4r^2$

$= -12s^2 - 6sr - 4r^2$

(c) $(8t - u)(t + 9u) - t(2u - 7t) = 8t(t + 9u) - u(t + 9u) - t(2u - 7t)$

$= 8t^2 + 72tu - tu - 9u^2 - 2tu + 7t^2$

$= 8t^2 + 7t^2 + 72tu - tu - 2tu - 9u^2$

$= 15t^2 + 69tu - 9u^2$

(d) $(2w + 3x)(w - 5x) - (3w + 7x)(w - 7x)$

$= 2w(w - 5x) + 3x(w - 5x) - [3w(w - 7x) + 7x(w - 7x)]$

$= 2w^2 - 10wx + 3wx - 15x^2 - (3w^2 - 21wx + 7wx - 49x^2)$

$= 2w^2 - 10wx + 3wx - 15x^2 - 3w^2 + 21wx - 7wx + 49x^2$

$= 2w^2 - 3w^2 - 10wx + 3wx + 21wx - 7wx - 15x^2 + 49x^2$

$= -w^2 + 7wx - 34x^2$

8. **(a)** $x^2 = x \times x$

$-63y^2 = y \times (-63y)$ or $(-y) \times 63y$

$= 3y \times (-21y)$ or $(-3y) \times 21y$

$= 7y \times (-9y)$ or $(-7y) \times 9y$

$\times$	x	$9y$
x	x^2	$9xy$
$-7y$	$-7xy$	$-63y^2$

$(-7xy) + 9xy = 2xy$

$\therefore\ x^2 + 2xy - 63y^2 = (x - 7y)(x + 9y)$

(b) $2x^2 = 2x \times x$

$3y^2 = y \times 3y$ or $(-y) \times (-3y)$

$\times$	x	y
$2x$	$2x^2$	$2xy$
$3y$	$3xy$	$3y^2$

$3xy + 2xy = 5xy$

$\therefore\ 2x^2 + 5xy + 3y^2 = (2x + 3y)(x + y)$

(c) $6x^2y^2 = 6xy \times xy$ or $3xy \times 2xy$

$-4 = 1 \times (-4)$ or $(-1) \times 4$

$= 2 \times (-2)$ or $(-2) \times 2$

$\times$	$2xy$	1
$3xy$	$6x^2y^2$	$3xy$
-4	$-8xy$	-4

$(-8xy) + 3xy = -5xy$

$\therefore\ 6x^2y^2 - 5xy - 4 = (3xy - 4)(2xy + 1)$

(d) $3z - 8xyz + 4x^2y^2z = z(3 - 8xy + 4x^2y^2)$

$3 = 3 \times 1$

$4x^2y^2 = xy \times 4xy$ or $(-xy) \times (4xy)$

$= 2xy \times 2xy$ or $(-2xy) \times (-2xy)$

$\times$	1	$-2xy$
3	3	$-6xy$
$-2xy$	$-2xy$	$4x^2y^2$

$(-2xy) + (-6xy) = -8xy$

$\therefore\ 3z - 8xyz + 4x^2y^2z = z(3 - 2xy)(1 - 2xy)$

9. **(a)** $(-x + 5y)^2 = (-x)^2 + 2(-x)(5y) + (5y)^2$

$= x^2 - 10xy + 25y^2$

(b) $(x^2 + y)(x^2 - y) = (x^2)^2 - y^2$

$= x^4 - y^2$

(c) $\left(3x + \frac{4}{5}y\right)^2 = (3x)^2 + 2(3x)\left(\frac{4}{5}y\right) + \left(\frac{4}{5}y\right)^2$

$= 9x^2 + \frac{24}{5}xy + \frac{16}{25}y^2$

(d) $\left(-\frac{1}{4}x - \frac{1}{6}y\right)^2 = \left(-\frac{1}{4}x\right)^2 + 2\left(-\frac{1}{4}x\right)\left(-\frac{1}{6}y\right) + \left(-\frac{1}{6}y\right)^2$

$= \frac{1}{16}x^2 + \frac{1}{12}xy + \frac{1}{36}y^2$

(e) $\left(5x - \frac{7}{4}y\right)\left(5x + \frac{7}{4}y\right) = (5x)^2 - \left(\frac{7}{4}y\right)^2$

$= 25x^2 - \frac{49}{16}y^2$

(f) $\left(\frac{3}{4}xy + \frac{1}{3}z\right)\left(\frac{3}{4}xy - \frac{1}{3}z\right) = \left(\frac{3}{4}xy\right)^2 - \left(\frac{1}{3}z\right)^2$

$= \frac{9}{16}x^2y^2 - \frac{1}{9}z^2$

Challenge Yourself

$n^2 = n \times n$

$45 = 1 \times 45$ or $(-1) \times (-45)$

$= 3 \times 15$ or $(-3) \times (-15)$

$= 5 \times 9$ or $(-5) \times (-9)$

$\times$	n	-15
n	n^2	$-15n$
-3	$-3n$	45

$(-3n) + (-15n) = -18n$

$\therefore n^2 - 18n + 45 = (n - 3)(n - 15)$ or $(3 - n)(15 - n)$

For $n^2 - 18n + 45$ to be a prime number,

$(n - 3)(n - 15)$ or $(3 - n)(15 - n)$ must be a prime number.

The factors of a prime number are 1 and itself.

$\therefore$ $n - 3 = 1$ or $n - 15 = 1$ or $3 - n = 1$ or $15 - n = 1$

$n = 4$ $n = 16$ $n = 2$ $n = 14$

When $n = 4$,

$4^2 - 18(4) + 45 = -11$

When $n = 16$,

$16^2 - 18(16) + 45 = 13$

When $n = 2$,

$2^2 - 18(2) + 45 = 13$

When $n = 14$,

$14^2 - 18(14) + 45 = -11$

$\therefore n = 2$ or 16

Chapter 8 Linear Equations and Coordinate Geometry

TEACHING NOTES

Suggested Approach

Since many Grade 7 students are still in the concrete operational stage (according to Piaget), teaching students how to solve linear equations in one variable with the use of algebra discs on a balance can help them to learn the concepts more easily. However, there is still a need to guide students to move from the 'concrete' to the 'abstract', partly because they cannot use this approach in examinations, and partly because they cannot use this approach to solve linear equations which consist of algebraic terms that have large or fractional coefficients.

Section 8.1: Cartesian Coordinates

Students have learnt how to complete mathematical sentences such as 7 + ___ = 13 in primary school. Teachers can introduce equations by telling students that when we replace with x, we have $7 + x = 13$, which is an equation. Teachers should illustrate the meaning of 'solving an equation' using appropriate examples. Students should know the difference between linear expressions and linear equations. Teachers can use the 'Balance Method' to show how to solve linear equations which do not involve any brackets before illustrating how to solve those which involve brackets.

Teachers can build upon prerequisites, namely number lines to introduce the horizontal axis (x-axis) and the vertical axis (y-axis). Teachers can introduce this concept by playing a game (see Class Discussion: Battleship Game (Two Players)) to arouse students' interest. Teachers should teach students not only on how to draw horizontal and vertical axes and plot the given points, but also to determine the position of points. Teachers can impress upon students that the first number in each ordered pair is with reference to the horizontal scale while the second number is with reference to the vertical scale. As such, students need to take note that the point (3, 4) has a different position compared to the point (4, 3).

Section 8.2: Horizontal and Vertical Lines

Teachers should bring students' attention to the relationship between the graphs of $y = mx + c$ where $m = 0$, i.e. c units up or down parallel to the x-axis depending on whether $c > 0$ or $c < 0$. Hence, teachers can lead students to the conclusion that the graphs of $y = mx + c$ for various values of c are parallel and cut the y-axis at different points corresponding to different values of c. Students also need to know that vertical lines parallel to the x-axis have the equation x = a and how this is related to the graphs of $y = mx + c$.

Section 8.3: Graphs of Linear Functions

Teachers should illustrate how a graph of a linear function is drawn on a sheet of graph paper. Teachers can impress upon students that when they draw a graph, the graph must follow the scale stated for both the x-axis and y-axis and the graph is only drawn for the values of x stated in the range.

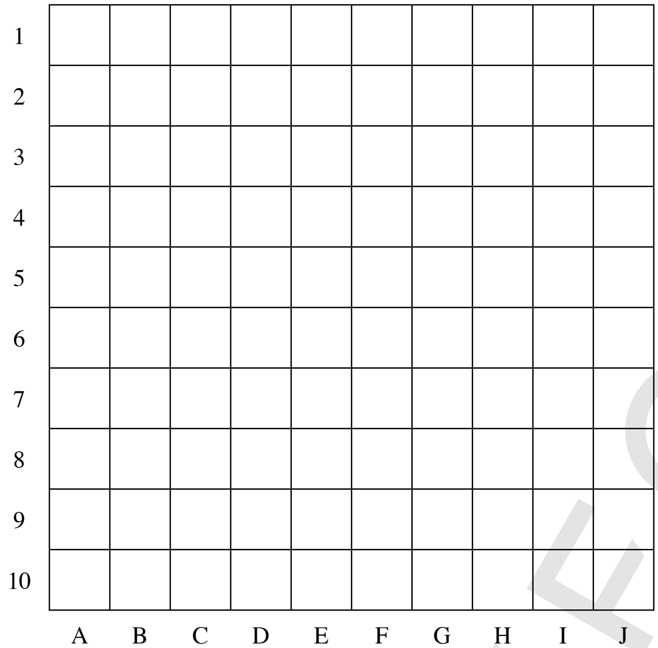
WORKED SOLUTIONS

Class Discussion (Battleship Game (Two Players))

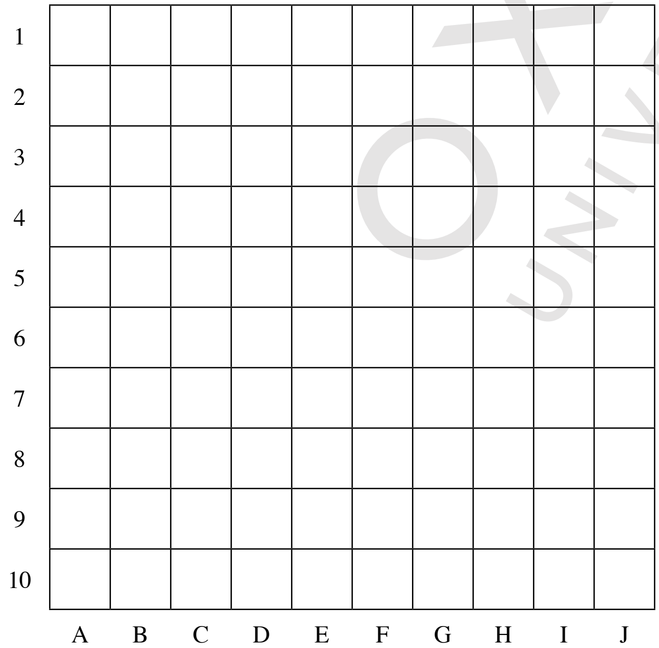
The purpose of this Battleship Game is to introduce students to the use of 2D Cartesian coordinates to specify points through an interesting and engaging activity.

Teachers may wish to emphasise to students that they should call out a location on the grid by calling the letter before calling the number, e.g. D7 instead of 7D.

Teachers may wish to use the grids provided (similar to that in Fig. 6.1) to conduct this activity.



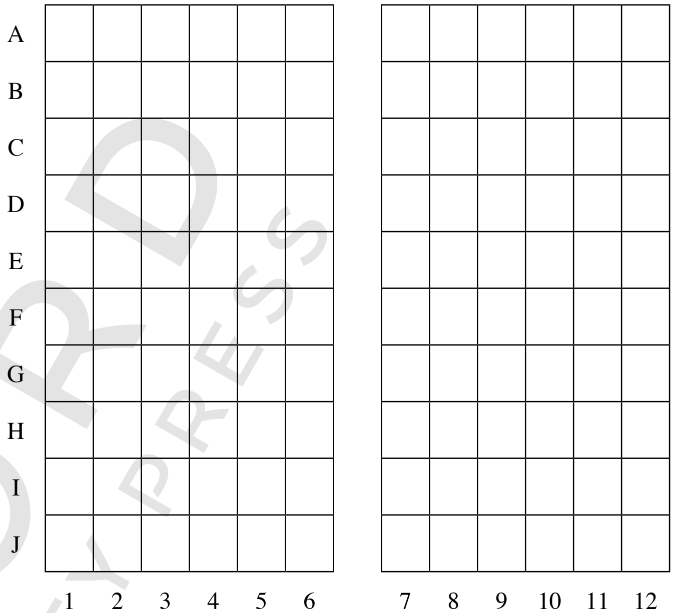
Self



Opponent

Class Discussion (Ordered Pairs)

1. A single number is not sufficient to describe the exact position of a student in the classroom seating plan. For example, when the number 1 is used to indicate the position of a student in the classroom, the student could be either in row 1 or column 1. From Fig. 6.2, we can see that there are 11 possible positions of the student.
   Similarly, the location of a seat in a cinema cannot be represented by a single number. An example of a seating plan of a cinema is as shown:



   From the seating plan shown, both the number and the letter are required to represent the location of a seat in the cinema.
2. The order in which two numbers are written are important, i.e. (5, 3) and (3, 5) do not indicate the same position.

Journal Writing (Page 123)

1. **Guiding Questions:**
   - How do you determine the locations of your house, a bus stop and a shopping mall in your neighbourhood on the map?
   - How can you obtain the approximate distances between your house, a bus stop and a shopping mall in your neighbourhood?
2. **Guiding Questions:**
   - What types of shops can normally be found on the ground floor of a shopping mall?
   - What is the size of each shop? How many spaces on the map should each shop occupy?
   - Are there any other considerations, e.g. walkways and washrooms, when designing the map?
3. **Guiding Questions:**
   - What types of horizontal and vertical scales are commonly used for the seating plan of a cinema in Pakistan?
   - What are the different types of seats, e.g. wheelchair berths and couple seats, which can be found in a cinema?

Investigation (Horizontal Line)

1. $B(-2, 3)$

2. $D(3, 3)$

Investigation (Vertical Line)

1. $Q(2, 1)$

2. $S(2, -4)$

Practise Now 1

1. **(a)** Let x years and y years by the ages of Fahad's two sons.

Sum of ages = $x + y$

Fahad's age = $3(x + y)$

(b) Let the number be x

Five times the number = $5x$

Let another number be y.

Difference = $y - 5$

Seven times the difference = $7(y - 5)$

$\therefore 5x = 7(y - 5)$

Practise Now 2

(a)
$$\begin{aligned} x + 3 &= 7 \\ x + 3 - 3 &= 7 - 3 \\ \therefore x &= 4 \end{aligned}$$

(b)
$$\begin{aligned} x - 7 &= 6 \\ x - 7 + 7 &= 6 + 7 \\ \therefore x &= 13 \end{aligned}$$

(c)
$$\begin{aligned} x + 3 &= -7 \\ x + 3 - 3 &= -7 - 3 \\ \therefore x &= -10 \end{aligned}$$

(d)
$$\begin{aligned} x - 2 &= -3 \\ x - 2 + 2 &= -3 + 2 \\ \therefore x &= -1 \end{aligned}$$

Practise Now 3

(a)
$$\begin{aligned} 2x - 5 &= 5 \\ 2x - 5 + 5 &= 5 + 5 \\ 2x &= 10 \\ \therefore x &= 5 \end{aligned}$$

(b)
$$\begin{aligned} 3x + 4 &= 7 \\ 3x + 4 - 4 &= 7 - 4 \\ 3x &= 3 \\ \therefore x &= 1 \end{aligned}$$

(c)
$$\begin{aligned} -3x + 3 &= 9 \\ -3x + 3 - 3 &= 9 - 3 \\ -3x &= 6 \\ 3x &= -6 \\ \therefore x &= -2 \end{aligned}$$

(d)
$$\begin{aligned} -5x - 2 &= 13 \\ -5x - 2 + 2 &= 13 + 2 \\ -5x &= 15 \\ 5x &= -15 \\ \therefore x &= -3 \end{aligned}$$

Practise Now 4

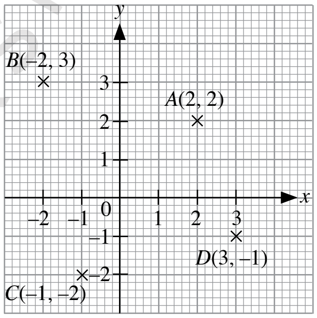
(a)
$$\begin{aligned} 3x + 4 &= x - 10 \\ 3x - x + 4 &= x - x - 10 \\ 2x + 4 &= -10 \\ 2x + 4 - 4 &= -10 - 4 \\ 2x &= -14 \\ \therefore x &= -7 \end{aligned}$$

(b)
$$\begin{aligned} 4x - 2 &= x + 7 \\ 4x - x - 2 &= x - x + 7 \\ 3x - 2 &= 7 \\ 3x - 2 + 2 &= 7 + 2 \\ 3x &= 9 \\ \therefore x &= 3 \end{aligned}$$

(c)
$$\begin{aligned} 3x - 2 &= -x + 14 \\ 3x + x - 2 &= -x + x + 14 \\ 4x - 2 &= 14 \\ 4x - 2 + 2 &= 14 + 2 \\ 4x &= 16 \\ \therefore x &= 4 \end{aligned}$$

(d)
$$\begin{aligned} -2x - 5 &= 5x - 12 \\ -2x - 5x - 5 &= 5x - 5x - 12 \\ -7x - 5 &= -12 \\ -7x - 5 + 5 &= -12 + 5 \\ -7x &= -7 \\ 7x &= 7 \\ x &= 1 \end{aligned}$$

Practise Now 5

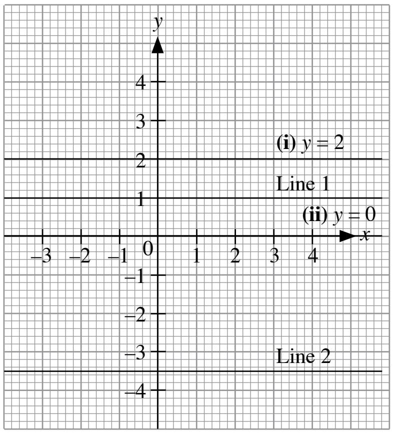


Practise Now 6

(a) Line 1: $y = 1$

Line 2: $y = -3.5$

(b)



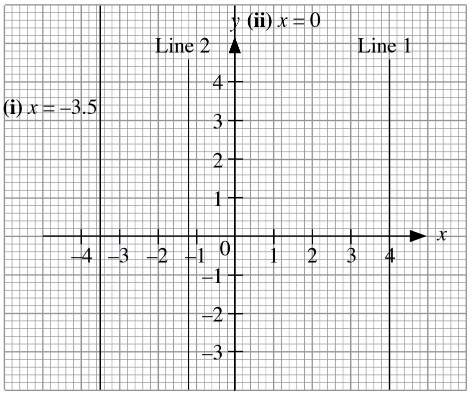
The lines are horizontal. The y-coordinates of all the points on the lines are a constant.

Practise Now 7

(a) Line 1: $x = 4$

Line 2: $x = -1.2$

(b)



The lines are vertical. The x-coordinates of all the points on the lines are a constant.

Practise Now 8

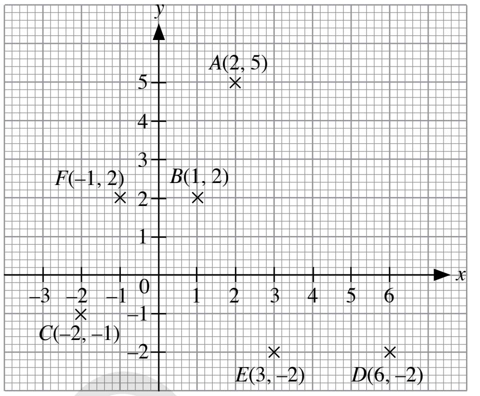
(a) P (3.5, 2)

(b) P (–2, –4)

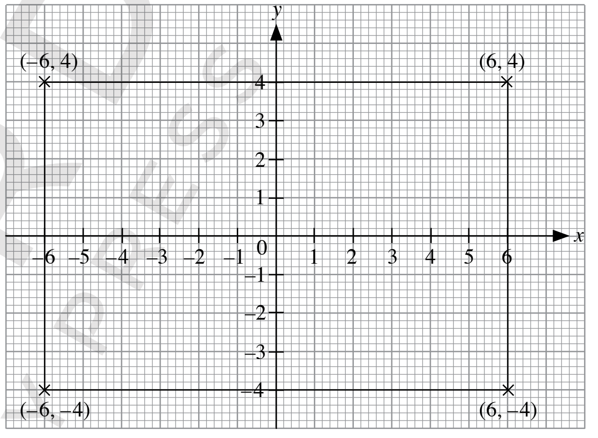
Exercise 8A

1. $A(-4, -3)$, $B(-2, 4)$, $C(3, 4)$, $D(4, 2)$, $E(1, 1)$, $F(3, -3)$

2.

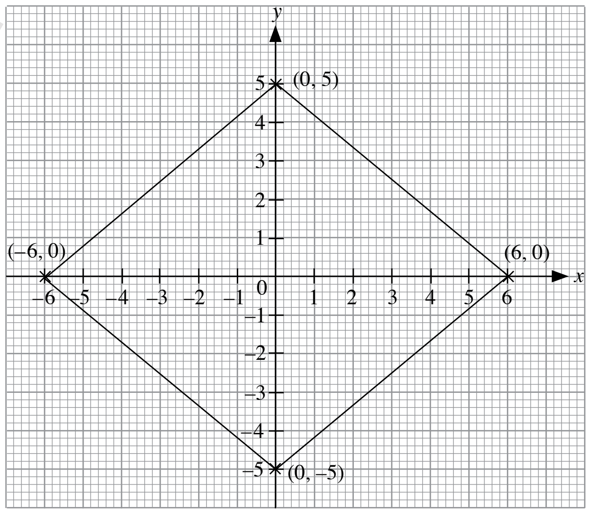


3. (a)



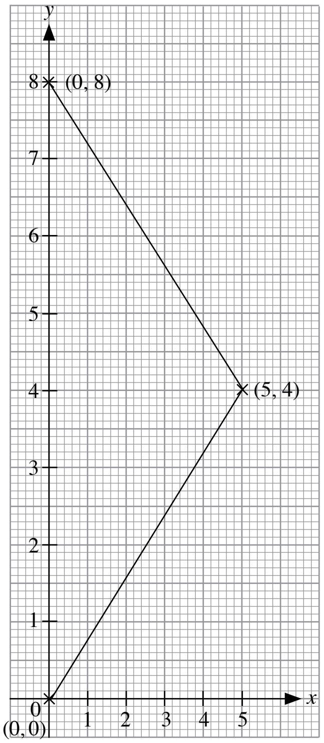
The figure shows a rectangle.

(b)



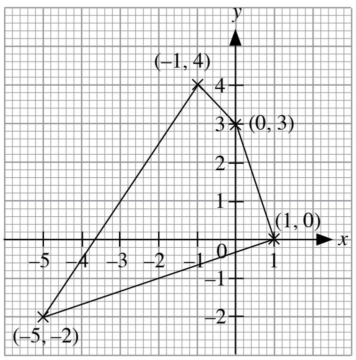
The figure shows a rhombus.

(c)



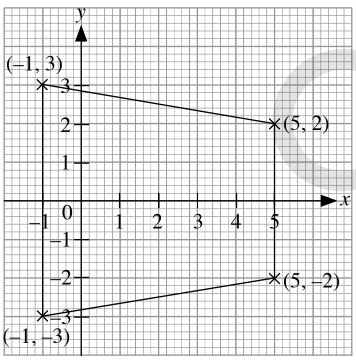
The figure shows an isosceles triangle.

(d)



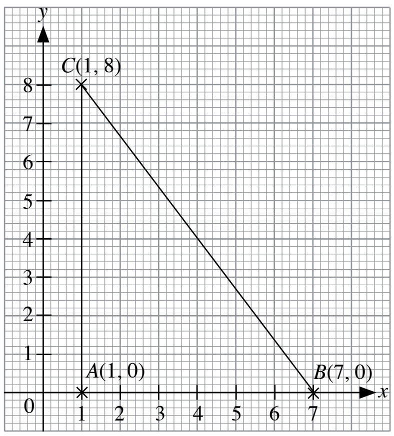
The figure shows a quadrilateral.

(e)



The figure shows a trapezium.

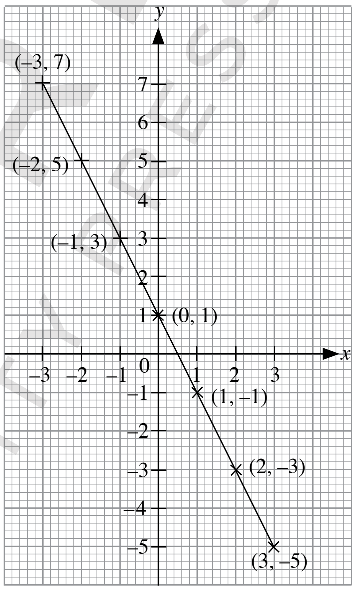
4.



Area of $\Delta ABC = \frac{1}{2} \times 6 \times 8$

$= 24 \text{ units}^2$

5.

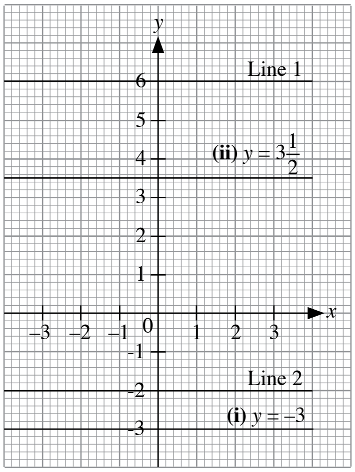


The points lie on a straight line.

Exercise 8B

1. **(a)** Line 1: $y = 6$
Line 2: $y = -2$

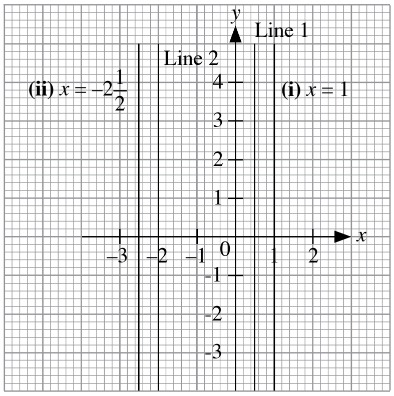
(b)



The lines are horizontal. The y-coordinates of all the points on the lines are a constant.

2. **(a)** Line 1: $x = 0.5$
Line 2: $x = -2$

(b)

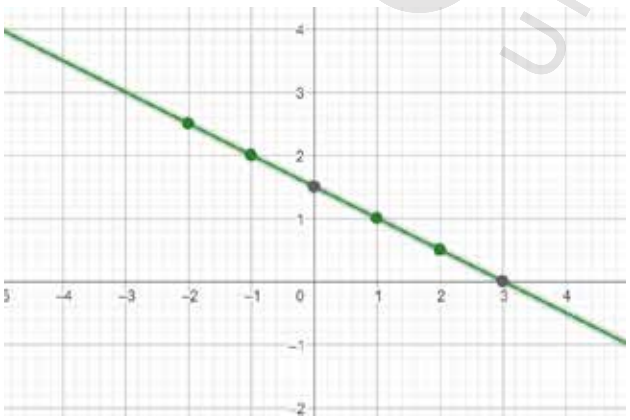


The lines are vertical. The x-coordinates of all the points on the lines are a constant.

3. 0, undefined

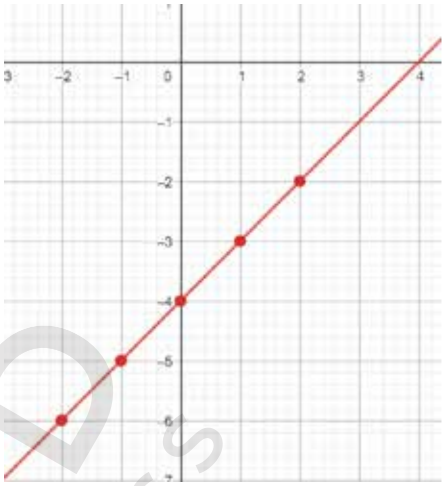
4. **(a)** $x + 2y = 3$

x	-2	-2	0	1	2
y	2.5	2	1.5	1	0.5



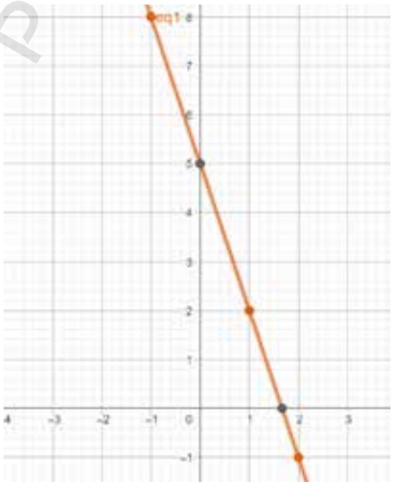
(b) $x - y = 4$

x	-2	-2	0	1	2
y	-6	-5	-4	-3	-2



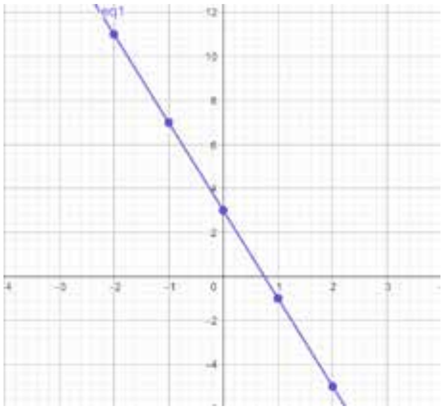
(c) $3x + y = 5$

x	-2	-2	0	1	2
y	11	8	5	2	-1



(d) $4x + y = -3s$

x	-2	-2	0	1	2
y	11	7	3	-1	-5



5. The coordinates of P are $(-2, -2)$

6. The coordinates of P, Q, and R are $(-2, -2)$, $(-2, 1)$, $(1, 4)$ respectively

7. The gradient of Line 1 = 0

The gradient of Line 2 = Gradient of Line 5

$= -3$

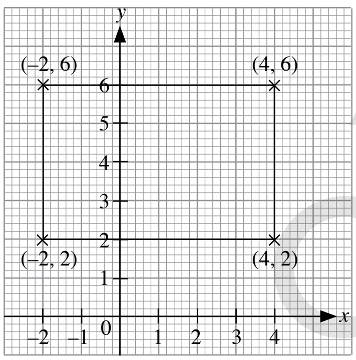
The slope of line 3 is undefined

Gradient of line 4 = Gradient of Line 6

$= \frac{1}{2}$

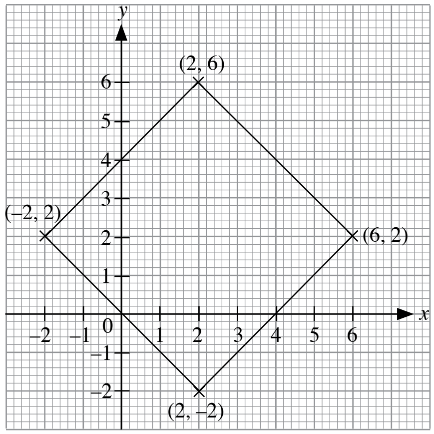
Review Exercise 8

1. **(a)**



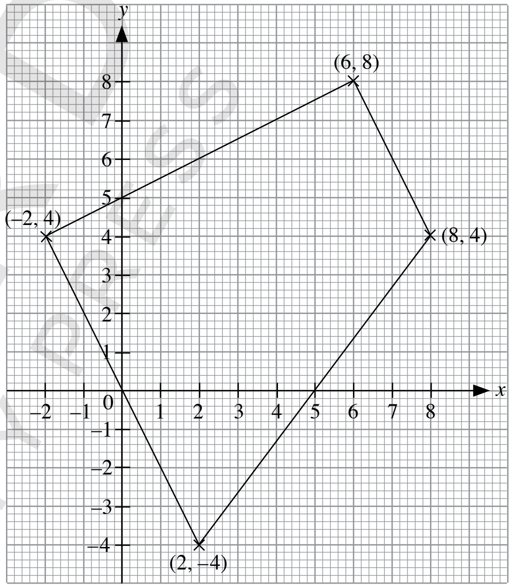
The figure shows a rectangle.

(b)



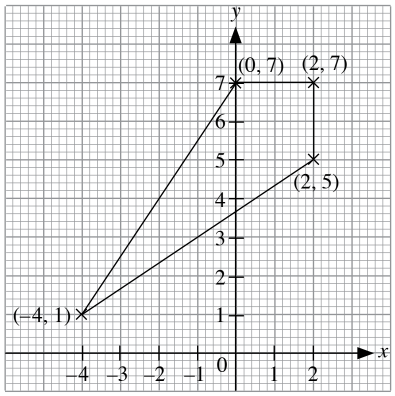
The figure shows a square.

(c)



The figure shows a trapezium.

(d)



The figure shows a kite.

2. **(a)** $A(-5, 0)$, $B(-4, 3)$, $C(-3, 4)$, $D(0, 5)$, $E(3, 4)$, $F(4, 3)$, $G(4, -3)$, $H(3, -4)$, $I(-3, -4)$, $J(-4, -3)$

(b) **(i)** H

(ii) G

Challenge Yourself

The coordinate of E are (–1.2, 1.6)

Chapter 9 Time and Speed

TEACHING NOTES

Suggested Approach

Teachers can bring in real-life examples for time and speed to arouse students' interest in this topic. Students will also learn how to solve problems involving time and speed through worked examples that involve situations in real-world contexts.

Section 9.1: Time

Teachers should emphasise that the addition and subtraction of times are not simply the same as adding and subtracting the numbers. For example, teachers can ask students why 30 + 40 = 70 = 110 (where 110 refers to 1 h 10 min). To prevent students from making careless mistakes, teachers should help students understand that:
6 hours 45 minutes is not the same as 6.45 hours, 1 hour is not the same as 100 minutes, 1 minute is not the same as 100 seconds.
Another important learning point would be dealing with time during the period before midnight and early morning. Teachers may also compare the time displayed on a digital clock with that on an analogue clock, and show students how the time is read.

Section 9.2: Speed

Teachers should inform students that speed is a special type of rate, i.e. speed is the distance covered per unit time. Teachers can get students to match appropriate speed to examples given (e.g. speed of a moving bicycle, lorry, car and aeroplane) to bring across the notion of speed.

Students need to know that average speed is defined as the total distance travelled by the object per unit time and not the average of the speeds of the object. Teachers should also impress upon students that there are differences between average speed and constant speed.

Teachers should teach students the conversion of units and highlight to them to use appropriate units when solving problems.

Challenge Yourself

Questions 1 and 2: Teachers can guide the students by getting them to use appropriate algebraic variables to represent the rates involved in the question. Students have to read the question carefully and form the linear equations which then can be solved to get the answers.

WORKED SOLUTIONS

Thinking Time (Page 147)

Do a recap on the definition of average speed which is defined as the total distance travelled by the total time taken. Average speed is different from the general meaning of 'average' in statistics. The word 'average' here does not refer to the sum of all individual speeds divided by the total number of individual speeds.

Performance Task (Page 147)

1. **(a)** *Teachers may wish to assign this activity as a pair work for the students to do.*
Students can help to record each other's walking speed.
Average walking speed of a human
= 5 km/h

(b) Minimum average speed

$$= \frac{2.4}{\frac{11 \times 60 + 30}{3600}}$$

$$= \frac{2.4}{\frac{690}{3600}}$$

= 12.5 km/h (to 3.s.f.)

(c) Average speed of a bicycle
= 22.5 km/h
Average speed of a sports car
= 280 km/h

(d) Average speed of an aeroplane
= 805 km/h

(e) Average speed of the spaceship
= 28 000 km/h

		Average speed (km/h)
(a)	Walking	5 km/h
(b)	Running	12.5 km/h
(c)	Bicycle	22.5 km/h
(d)	Sports car	280 km/h
(e)	Aeroplane	805 km/h
(f)	Spaceship	28 000 km/h

2. Seema cycles to school while Bina walks to school.

3. Number of times a spaceship is as fast as an aeroplane

$$= \frac{28\,000}{805}$$

= 34.8

4. Other examples of speeds which can be encountered in real life:
- Speed of a bus
- Speed of a cheetah

5. *Teachers may wish to ask the students to present their findings to the class.*

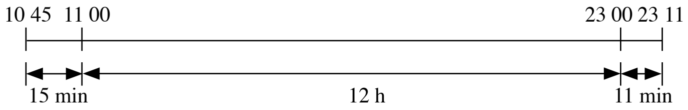
Practise Now 1

$7\frac{1}{4}$ h = 7 h 15 min

$$22\ 45 \xrightarrow{+7\text{ h}} \underset{(05\ 45)}{29\ 45} \xrightarrow{+15\text{ min}} 06\ 00$$

$\therefore$ The ship arrived at Port Y at 06 00 or 6 a.m. on Saturday.

Practise Now 2



15 min + 11 min = 26 min
$\therefore$ The bus journey was 12 h 26 min long.

Practise Now 3

1. **(i)** 25 minutes = $\frac{25}{60}$ hours

Speed of the train = $\dfrac{16.8}{\frac{25}{60}}$ = 40.32 km/h

(ii) 16.8 km = 16 800 m
25 minutes = 25 × 60 = 1500 seconds

Speed of the train = $\frac{16\,800}{1500}$ = 11.2 m/s

2. 55 km/h = $\frac{55\text{ km}}{1\text{ h}} = \frac{(55 \times 1000)\text{ m}}{3600\text{s}} = 15\frac{5}{18}$ m/s

12 minutes 30 seconds = (12 × 60) + 30 = 750 seconds

Distance travelled = $15\frac{5}{18} \times 750 = 11\,458\frac{1}{3}$ m

3. Let the speed of the bus be x km/h.

$$13\ 20\text{ hours} \xrightarrow{3\text{ hours}} 16\ 20\text{ hours}$$

Distance the car travelled in 3 hours = 90 × 3 = 270 km

$$270 + (3 \times x) = 510$$
$$3x = 510 - 270$$
$$3x = 240$$
$$x = 80$$

The speed of the bus is 80 km/h.

Practise Now 4

1. **(i)** Speed of the train
= 48.6 km/h

$$= \frac{48.6\text{ km}}{1\text{ h}}$$

$$= \frac{48\,600\text{ m}}{3600\text{ s}}$$ (convert 48.6 km to m and 1 h into s)

= 13.5 m/s

(ii) Speed of the train
= 48.6 km/h

$$= \frac{48.6\text{ km}}{1\text{ h}}$$

$= \dfrac{4\ 860\ 000 \text{ cm}}{60 \text{ min}}$ (convert 48.6 km to cm and 1 h into min)

$= 81\ 000$ cm/min

2. Speed of the fastest human sprinter

$= \dfrac{100 \text{ m}}{9.58 \text{ s}}$

$= \dfrac{(100 \surd 1000) \text{ km}}{(9.58 \surd 3600) \text{ h}}$

$= 37\dfrac{277}{479}$ km/h

No. of times a cheetah is as fast as the fastest human sprinter

$= \dfrac{110}{37\dfrac{277}{479}}$

$= 2.93$

Practise Now 5

Time taken for Farhan to swim a distance of 1.5 km

$= \dfrac{1.5}{2.5}$ h

$= \dfrac{3}{5}$ h

Total time taken

$= \dfrac{3}{5} + 1\dfrac{1}{2} + 1\dfrac{1}{9}$

$= \dfrac{3}{5} + \dfrac{3}{2} + \dfrac{10}{9}$

$= \dfrac{54}{90} + \dfrac{135}{90} + \dfrac{100}{90}$

$= \dfrac{289}{90}$ hours

Distance that Farhan runs

$= 9 \times 1\dfrac{1}{9}$

$= 9 \times \dfrac{10}{9}$

$= 10$ km

Total distance travelled

$= 1.5 + 40 + 10$

$= 51.5$ km

Average speed for the entire competition

$= \dfrac{\text{Total distance travelled}}{\text{Total time taken}}$

$= \dfrac{51.5}{\dfrac{289}{90}}$

$= 16\dfrac{11}{289}$ km/h

Practise Now 6

Let the distance for the car to travel from Town A to Town B to meet the truck $= x$ km.

Then the time taken for the car to travel from Town A to Town B to meet the truck at an average speed of 72 km/h

$= \dfrac{x}{72}$ hour, and

the time taken for the truck to travel from Town B to Town A to meet the car at an average speed of 38 km/h

$= \dfrac{550 - x}{38}$ hour

$\therefore \dfrac{x}{72} = \dfrac{550 - x}{38}$

$38x = 39\ 600 - 72x$

$38x + 72x = 39\ 600$

$110x = 39\ 600$

$x = 360$ km

Hence, the time taken for the two vehicles to meet

$= \dfrac{360}{72}$

$= 5$ hours

Practise Now 7

Radius of the wheel of the car

$= \dfrac{0.75}{2}$

$= 0.375$ m

Circumference of the wheel of the car

$= 2 \times \pi \times 0.375$

$= 2 \times 3.142 \times 0.375$

$= 2.3565$ m

Distance travelled by a car in 1 minute

$= 14 \times 60$

$= 840$ m

Number of revolutions made by the wheel per minute

$= \dfrac{840}{2.3565}$

$= 356$ (to the nearest whole number)

Exercise 9A

1. **(a)** 08:00
(b) 21:42
(c) 00:00
(d) 02:42

2. **(a)** 3.30 a.m.
(b) 11.12 p.m.
(c) 7.15 p.m.
(d) 12.00 a.m.

3.

	Departure Time	Journey Time	Arrival Time
(a)	02:40	55 minutes	03:35
(b)	22:35	8 hours	06:35 (next day)
(c)	15:45	$2\frac{1}{4}$ h or 2 h 15 min	17:50

(d)	09:48	$12\frac{7}{15}$ h or 12 h 28 min	22:16
(e)	20:35 (Tuesday)	$10\frac{2}{3}$ h or 10 h 40 min	07:15 (Wednesday)
(f)	22:35	$1\frac{1}{4}$ h	23:50

4. $21\ 55 \xrightarrow{+\,9\text{ h}} \underset{(06\ 55)}{30\ 55} \xrightarrow{+\,5\text{ min}} 07\ 00 \xrightarrow{+\,13\text{ min}} 07\ 13$

$\therefore$ The train arrived at its destination at 07 13 or 7.13 a.m. on Tuesday.

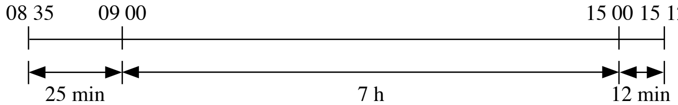
5. $? \xrightarrow{+\,4\text{ h}} ? \xrightarrow{+\,15\text{ min}} 15\ 06$

$10\ 51 \xleftarrow[-\,4\text{ h}]{} 14\ 51 \xleftarrow[-\,9\text{ min}]{} 15\ 00 \xleftarrow[-\,6\text{ min}]{} 15\ 06$

Working backwards, the car started the journey at 10 51.

6. 8.35 a.m. $\Rightarrow$ 08 35

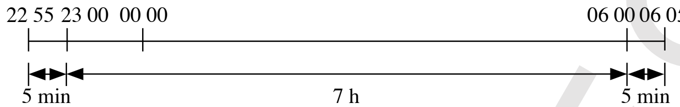
3.12 p.m. $\Rightarrow$ 15 12



25 min + 12 min = 37 min

$\therefore$ The journey took 6 h 37 min.

7. **(i)**



5 min + 5 min = 10 min

$\therefore$The journey took 7 h 10 min.

(ii) 35 min = 30 min + 5 min

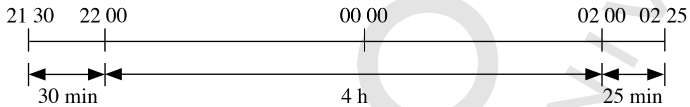
5 min before 06 05 is 06 00

30 min before 06 00 is 05 30

$\therefore$ The coach reached its destination at 05 30.

8. Assume time taken includes breaks in between stations.

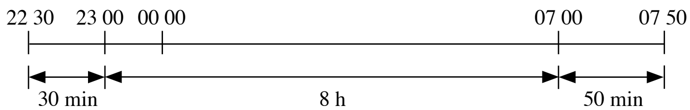
(a) Depart from A: 21 30; Arrive at C: 02 25



30 min + 25 min = 55 min

$\therefore$ The time taken from A to C was 4 h 55 min.

(b) Depart from B: 22 30; Arrive at E: 07 50

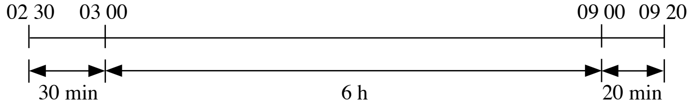


30 min + 50 min = 80 min

= 1 h 20 min

$\therefore$ The time taken from B to E was 9 h 20 min.

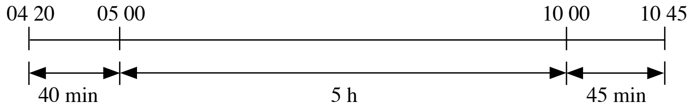
(c) Depart from C: 02 30; Arrive at F: 09 20



30 min + 20 min = 50 min

$\therefore$ The time taken from C to F was 6 h 50 min.

(d) Depart from D: 04 20; Arrive at G: 10 45

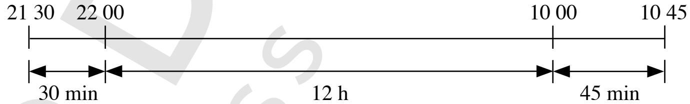


40 min + 45 min = 85 min

= 1 h 25 min

$\therefore$ The time taken from D to G was 6 h 25 min.

(e) Depart from A: 21 30; Arrive at G: 10 45



30 min + 45 min = 75 min

= 1 h 15 min

$\therefore$ The time taken from A to G was 13 h 15 min.

Exercise 9B

1. **(i)** 30 minutes = $\frac{30}{60} = \frac{1}{2}$ hour

Speed of the particle

$= \dfrac{24.6\text{ km}}{\frac{1}{2}\text{ hour}}$

= 49.2 km/h

(ii) 24.6 km = 24.6 × 1000 = 24 600 m

30 minutes = 30 × 60 = 1800 s

Speed of the particle

$= \dfrac{24\ 600\text{ m}}{1800\text{ s}}$

$= 13\frac{2}{3}$ m/s

2. 12 24 hours $\xrightarrow{\text{1 hour 48 minutes}}$ 14 12 hours

1 hour 48 minutes = $1\frac{48}{60} = 1\frac{4}{5}$ hours

Distance between the two stations

$= 200 \times 1\frac{4}{5}$

= 360 km

= 360 × 1000 m

= 360 000 m

3. **(a)** 8.4 km/min

$= \dfrac{8.4\text{ km}}{1\text{ min}}$

$= \dfrac{8.4\text{ km}}{\frac{1}{60}\text{ h}}$

= 504 km/h

(b) 315 m/s

$= \dfrac{315\text{ m}}{1\text{ s}}$

$= \dfrac{\frac{315}{1000}\text{ km}}{\frac{1}{3600}\text{ h}}$

$= 1134$ km/h

(c) 242 m/min

$= \dfrac{242\text{ m}}{1\text{ min}}$

$= \dfrac{\frac{242}{1000}\text{ km}}{\frac{1}{60}\text{ h}}$

$= 14\frac{13}{25}$ km/h

(d) 125 cm/s

$= \dfrac{125\text{ cm}}{1\text{ s}}$

$= \dfrac{\frac{125}{100\,000}\text{ km}}{\frac{1}{3600}\text{ h}}$

$= 4.5$ km/h

4. **(a)** 65 cm/s

$= \dfrac{65\text{ cm}}{1\text{ s}}$

$= \dfrac{\frac{65}{100}\text{ km}}{1\text{ s}}$

$= \frac{13}{20}$ m/s

(b) 367 km/h

$= \dfrac{367\text{ km}}{1\text{ h}}$

$= \dfrac{367 \times 1000\text{ m}}{3600\text{ s}}$

$= \dfrac{367\,000\text{ m}}{3600\text{ s}}$

$= 107\frac{17}{18}$ m/s

(c) 1000 cm/min

$= \dfrac{1000\text{ cm}}{1\text{ min}}$

$= \dfrac{\frac{1000}{100}\text{ m}}{60\text{ s}}$

$= \frac{1}{6}$ m/s

(d) 86 km/min

$= \dfrac{86\text{ km}}{1\text{ min}}$

$= \dfrac{86 \times 1000\text{ m}}{60\text{ s}}$

$= 1433\frac{1}{3}$ m/s

5. Speed of the fastest national sprinter

$= \dfrac{100\text{ m}}{10.37\text{ s}}$

$= \dfrac{\frac{100}{1000}\text{ km}}{\frac{10.37}{3600}\text{ h}}$

$= 34\frac{742}{1037}$ km/h

Number of times the bullet train is as fast as the fastest national sprinter

$= \dfrac{365}{34\frac{742}{1037}}$

$= 10\frac{3701}{7200}$

6. Time taken to travel the first part of the journey

$= \frac{19}{57}$

$= \frac{1}{3}$ h

Time taken to travel the remaining part of the journey

$= \frac{55}{110}$

$= \frac{1}{2}$ h

Average speed of the car for its entire journey

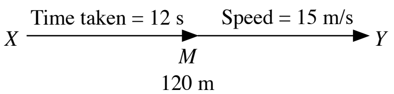
$= \dfrac{\text{Total distance travelled}}{\text{Total time taken}}$

$= \dfrac{19 + 55}{\frac{1}{3} + \frac{1}{2}}$

$= \dfrac{74}{\frac{5}{6}}$

$= 88\frac{4}{5}$ km/h

7.



(i) Time taken to travel from M to Y

$= \frac{60}{15}$

$= 4$ s

(ii) Average speed of the object for its entire journey from X to Y

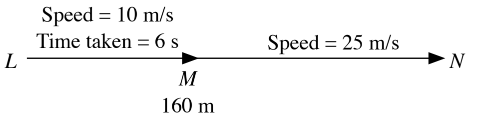
$= \dfrac{\text{Total distance travelled}}{\text{Total time taken}}$

$= \dfrac{120}{12 + 4}$

$= \dfrac{120}{16}$

$= 7\frac{1}{2}$ m/s

8.



Distance travelled from L to M

$= 10 \times 6$

$= 60$ m

Thus, distance travelled from M to N

$= 160 - 60$

$= 100$ m

Time taken to travel from M to N

$= \frac{100}{25}$

$= 4$ s

Average speed of the object for its entire journey from L to N

$= \frac{\text{Total distance travelled}}{\text{Total time taken}}$

$= \frac{160}{6 + 4}$

$= \frac{160}{10}$

$= 16$ m/s

9. Time taken to travel the first 50 km of its journey

$= \frac{50 \times 1000 \text{ m}}{25 \text{ m/s}}$

$= 2000$ s

$= \frac{2000}{3600}$ h

$= \frac{5}{9}$ h

Time taken to travel the next 120 km of its journey

$= \frac{120}{80}$

$= 1\frac{1}{2}$ h

Distance travelled for the last part of its journey

$= 90 \times \frac{35}{60}$

$= 52.5$ km

Average speed of the object for its entire journey

$= \frac{\text{Total distance travelled}}{\text{Total time taken}}$

$= \frac{50 + 120 + 52.5}{\frac{5}{9} + 1\frac{1}{2} + \frac{35}{60}}$

$= 84\frac{6}{19}$ km/h

10. Radius of the wheel of the car

$= \frac{60}{2}$

$= 30$ cm

$= 0.3$ m

Circumference of the wheel of the car

$= 2 \times \pi \times 0.3$

$= 2 \times 3.142 \times 0.3$

$= 1.8852$ m

Distance travelled by the car in 1 hour

$= 13.2 \times 60 \times 60$

$= 47\,520$ m

Number of revolutions made by the wheel per minute

$= \frac{47\,520}{1.8852}$

$= 25\,207$ (to the nearest whole number)

11. Length of the goods train

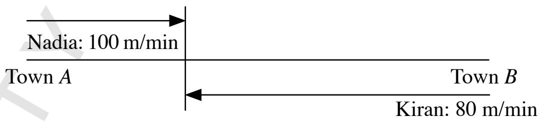
$= 72 \times \frac{8}{3600} + 54 \times \frac{8}{3600}$

$= \frac{7}{25}$ km

$= \frac{7}{25} \times 1000$

$= 280$ m

12.

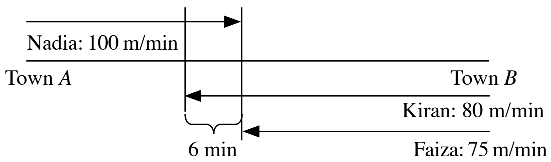


Let the time taken for Nadia to meet Kiran be x min.

So distance between Town A and Town B

$= 100 \times x + 80 \times x$

$= 180x$ m



Then the time taken for Nadia to meet Faiza will be $(x + 6)$ min.

Distance between Town A and Town B

$= 100 \times (x + 6) + 75 \times (x + 6)$

$= 100x + 600 + 75x + 450$

$= (175x + 1050)$ m

Thus,

$$180x = 175x + 1050$$
$$10x - 175x = 1050$$
$$5x = 1050$$
$$x = 210$$

Distance between Town A and Town B

$= 180 \times 210$

$= 37\,800$ m

Review Exercise 9

1. (i)

$$? \xrightarrow{+2\text{ h}} ? \xrightarrow{+15\text{ min}} 12\ 06$$

$$09\ 51 \xleftarrow[-2\text{ h}]{} 11\ 51 \xleftarrow[-9\text{ min}]{} 12\ 00 \xleftarrow[-6\text{ min}]{} 12\ 06$$

Working backwards, the time the journey started was 09 51.

(ii) Total distance = 198 km

Total time = 2 h 15 min = $2\frac{1}{4}$ h = $\frac{9}{4}$ h

Average speed = $\dfrac{198}{\frac{9}{4}}$

= 88 km/h

2. (i) Time taken = $\dfrac{195}{52}$

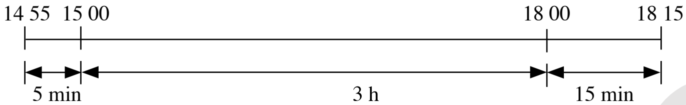
= $3\frac{3}{4}$ h

= 3 h 45 min

$$08\ 45 \xrightarrow{+3\text{ h}} 11\ 45 \xrightarrow{+15\text{ min}} 12\ 00 \xrightarrow{+30\text{ min}} 12\ 30$$

∴ The time at which the lorry arrives at its destination is 12 30.

(ii)



5 min + 15 min = 20 min

∴ The time taken was 3 h 20 min.

3 h 20 min = $3\frac{1}{3}$ h

= $\frac{10}{3}$ h

Average speed = $\dfrac{195}{\frac{10}{3}}$

= 58.5 km/h

3. Distance that the athlete cycles

= $40 \times \frac{30}{60}$

= 20 km

Time taken for the athlete to run

= $\dfrac{5 \times 1000}{3}$

= $1666\frac{2}{3}$ s

= $1666\frac{2}{3} \div 3600$

= $\frac{25}{54}$ h

His average speed for the entire competition

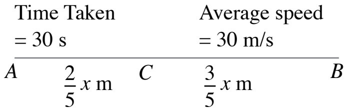
= $\dfrac{\text{Total distance travelled}}{\text{Total time taken}}$

= $\dfrac{\frac{750}{1000} + 20 + 5}{\frac{15}{60} + \frac{30}{60} + \frac{25}{54}}$

= $21\frac{30}{131}$ km/h

4. Let the distance from A to C be $\frac{2}{5}x$ m.

Then the distance from C to B be $\frac{3}{5}x$ m.



(i) Time taken for the object to travel from C to B

= $\dfrac{\frac{3}{5}x}{30}$

= $\frac{3}{5}x \div 30$

= $\frac{3}{5}x \times \frac{1}{30}$

= $\frac{1}{50}x$ s

(ii) Average speed of the object for its entire journey from A to B

= $\dfrac{x \text{ m}}{30 + \frac{1}{50}x \text{ s}}$

= $\dfrac{50x}{1500 + x}$ m/s

5. Let the first part of the journey be x km.

Then the remaining part of the journey be $(150 - x)$ km.

Time taken for the entire journey = 4.5 h

$$\therefore \frac{x}{35} + \frac{150 - x}{5} = 4.5$$

$$35 \times \frac{x}{35} + 35 \times \frac{150 - x}{5} = 35 \times 4.5$$

$$\begin{aligned} x + 7(150 - x) &= 157.5 \\ x + 1050 - 7x &= 157.5 \\ x - 7x &= 157.5 - 1050 \\ -6x &= -892.5 \\ x &= 148.75 \end{aligned}$$

∴ Distance = 148.75 km

6. Radius of the wheel of the car

= $\frac{48}{2}$

= 24 cm

Circumference of the wheel of the car

= $2 \times \pi \times 24$

= $2 \times 3.142 \times 24$

= 150.816 cm

Distance travelled by a car in 1 minute

= $3.5 \div 60$

$= \frac{7}{120}$ km

$= \frac{7}{120} \times 100\,000$ cm

$= 5833\frac{1}{3}$ cm

Number of revolutions made by the wheel per minute

$= \frac{5833\frac{1}{3}}{150.816}$

= 39 (to the nearest whole number)

Challenge Yourself

1. Let the rate of the moving escalator be x steps per second.

When she is walking down at a rate of 2 steps per second, then the total steps (including the steps covered by the moving escalator) covered in 1 second is $(x + 2)$. Since she use 18 steps to reach the bottom from the top, therefore, the time taken is $(18 \div 2) = 9$ seconds. When she is exhausted, then the total steps (including the steps covered by the moving escalator) covered in 1 second is $(x + 1)$. Since she use 12 steps to reach the bottom from the top, therefore, the time taken is $(12 \div 1) = 12$ seconds.

Hence,

$9(x + 2) = 12(x + 1)$

$9x + 18 = 12x + 12$

$9x - 12x = 12 - 18$

$-3x = -6$

$x = 2$

$\therefore$ Total steps covered by the moving escalator $= 9(2 + 2) = 36$.

Hence the time taken for her to reach the bottom from the top if she stands on the escalator

$= \frac{36}{2}$

= 18 s

2. Let Bilal's speed be x m/s and Salman's speed be y m/s.

Then in the first race, when Bilal ran pass the end point 100 m, Salman is only at 90 m of the race. Hence, at the same time,

$\frac{100}{x} = \frac{90}{y}$

$100y = 90x$

$x = \frac{100y}{90}$

For the second race,

Let the time for the first person to pass the end point be t s.

Time taken for Bilal to finish the 100 m race

$= \frac{110}{x}$

$= \frac{110}{\frac{100y}{90}}$ (Substitute $x = \frac{100y}{90}$)

$= \frac{99}{y}$ s

Time taken for Salman to finish the 100 m race

$= \frac{100}{y}$ s

At time t s, distance that Bilal covered

$t = \frac{99}{y}$

$ty = 99$ m

At time t s, distance that Salman covered

$t = \frac{100}{y}$

$ty = 100$ m

$\therefore$ Bilal win the race by $100 - 99 = 1$ m.

Chapter 10 Triangles, Quadrilaterals and Polygons

TEACHING NOTES

Suggested Approach

Students have learnt about triangles, and quadrilaterals such as parallelograms, rhombuses and trapeziums in primary school. They would have learnt the properties. In this chapter, students begin from 3-sided triangles, to 4-sided quadrilaterals and finally n-sided polygons. The incremental approach is to ensure that students have a good understanding before they move on to a higher level. Teachers may want to dedicate more time and attention to the section on polygons in the last section of this chapter.

Section 10.1: Triangles

Students have learnt about isosceles triangles, equilateral triangles and right-angled triangles in primary school. In this chapter, students should be aware that triangles can be classified by the number of equal sides or the types of angles. Teachers may want to check students' understanding on the classification of triangles (see Thinking Time on page 154). Teachers should highlight to the students that equilateral triangles are a special type of isosceles triangles while scalene triangles are triangles that are not isosceles, and are definitely not equilateral triangles.

Students should explore and discover that the longest side of a triangle is opposite the largest angle, and the sum of two sides is always larger than the third side.

Teachers should ensure students are clear what exterior angles are before stating the relation between exterior angles and its interior opposite angles. Some may think that the exterior angle of a triangle is the same as the reflex angle at a vertex of a triangle.

Section 10.2: Quadrilaterals

Teachers may want to first recap students' knowledge of parallelograms, rhombuses and trapeziums based on what they have learnt in primary school. Teachers can use what students have learnt in previous class, reintroduce and build up their understanding of the different types of quadrilaterals and their properties. For further understanding, teachers may wish to show the taxonomy of quadrilaterals to demonstrate their relations.

Before proceeding onto the next section, teachers may want to go through with the students the angle properties of triangles and quadrilaterals. This reinforces the students' knowledge as well as prepares them for the section on polygons.

Section 10.3: Polygons

Teachers should emphasise to the students that triangles and quadrilaterals are polygons so that they are aware that all the concepts which they have learnt so far remains applicable in this topic. Students should learn the different terms with regards to polygons. In this section, most polygons studied will be simple, convex polygons.

Students need to know the names of polygons with 10 sides or less and the general naming convention of polygons (see Class Discussion: Naming of Polygons). Through the class discussion, students should be able to develop a good understanding on polygons and be able to name them. They should also know and appreciate the properties of regular polygons (see Investigation: Properties of a Regular Polygon).

Teachers can ask students to recall the properties of triangles and quadrilaterals during the investigation of the sum of interior angles and sum of exterior angles of a polygon. Students should see a pattern in how the sum of interior angles differs as the number of sides increases and understand its formula, (see Investigation: Sum of Interior Angles of a Polygon) as well as discover that the sum of exterior angles is always equal to 360° regardless of the number of sides of the polygon (see Investigation: Sum of Exterior Angles of a Pentagon).

Challenge Yourself

Some of the questions (e.g. Questions 1 and 2) may be challenging for most students while the rest of the questions can be done with guidance from teachers.

Question 1: Two new points need to be added. The first point (say, E) is the midpoint of BC and the second point (say, F) lies on the line AE such that $\triangle BCF$ is equilateral. Draw the lines AE, CF and DF. Begin by finding $A\hat{B}C$ and continue from there.

Question 2: Draw DG such that $BC \parallel DG$, and mark E at the point where DG cuts CD. Join E and F. Begin by finding $A\hat{C}B$ and continue from there.

WORKED SOLUTIONS

Thinking Time (Page 154)

A represents isosceles triangles.
B represents scalene triangles.
C represents acute-angled triangles.
D represents right-angled triangles.

Investigation (Properties of Special Quadrilaterals)

1. $AB = 2.8$ cm, $BC = 1.8$ cm, $DC = 2.8$ cm, $AD = 1.8$ cm
$AB = DC$ and $BC = AD$ (Opposite sides are equal in length.)

2. $B\hat{A}D = 90°$, $A\hat{B}C = 90°$, $B\hat{C}D = 90°$, $A\hat{D}C = 90°$
$B\hat{A}D = A\hat{B}C = B\hat{C}D = A\hat{D}C = 90°$ (All four interior angles are right angles.)

3. $AE = 1.7$ cm, $BE = 1.7$ cm, $CE = 1.7$ cm, $DE = 1.7$ cm
$AE = BE = CE = DE = 1.7$ cm (Diagonals bisect each other.)

4. $AE + CE = 1.7 + 1.7 = 3.4$ cm,
$BE + DE = 1.7 + 1.7 = 3.4$ cm
Both of the sums are equal. (The two diagonals are equal in length.)

5. The following properties hold:
- Opposite sides are equal in length.
- All four interior angles are right angles.
- Diagonals bisect each other.
- The two diagonals are equal in length.

6. **(a)** Square : All sides are equal in length.
Parallelogram : Opposite sides are equal in length.
Rhombus : All sides are equal in length.
Trapezium : All sides are not equal in length.
Kite : There are two pairs of equal adjacent sides.

(b) Square : All four interior angles are right angles.
Parallelogram : Opposite interior angles are equal.
Rhombus : Opposite interior angles are equal.
Trapezium : All four interior angles are not equal.
Kite : One pair of opposite interior angles is equal.

(c) Square : The two diagonals are equal in length.
Parallelogram : The two diagonals are not equal in length.
Rhombus : The two diagonals are not equal in length.
Trapezium : The two diagonals are not equal in length.
Kite : The two diagonals are not equal in length.

(d) Square : The diagonals bisect each other.
Parallelogram : The diagonals bisect each other.
Rhombus : The diagonals bisect each other.
Trapezium : The diagonals do not bisect each other.
Kite : The diagonals do not bisect each other.

(e) Square : The diagonals are perpendicular to each other.
Parallelogram : The diagonals are not perpendicular to each other.
Rhombus : The diagonals are perpendicular to each other.
Trapezium : The diagonals are not perpendicular to each other.
Kite : The diagonals are perpendicular to each other.

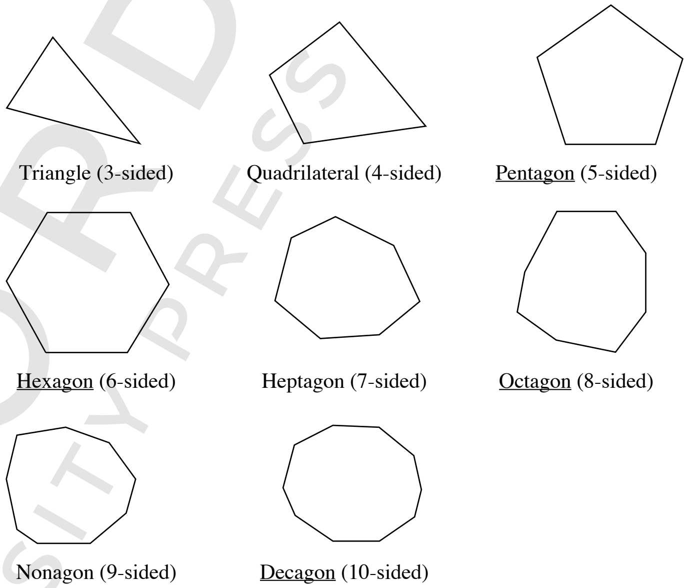
(f) Square : The diagonals bisect the interior angles.
Parallelogram : The diagonals do not bisect the interior angles.
Rhombus : The diagonals bisect the interior angles.
Trapezium : The diagonals do not bisect the interior angles.
Kite : One diagonal bisects the interior angles.

Thinking Time (Page 165)

(a) Yes **(b)** Yes **(c)** Yes
(d) Yes **(e)** Yes

A represents kites.
B represents parallelograms.
C represents rhombus.
D represents squares.

Class Discussion (Naming of Polygons)

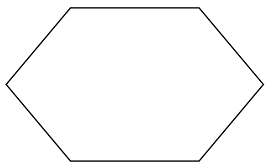


Thinking Time (Page 174)

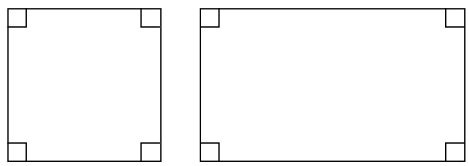
The name of a regular triangle is an equilateral triangle and the name of a regular quadrilateral is a square.

Investigation (Properties of a Regular Polygon)

1. Yes.
(a) Rhombus
(b)



2. Yes.
(a) Square and Rectangle
(b)



Journal Writing (Page 175)

Since a regular polygon is a polygon with all sides equal and all angles equal, the statement made by Devi is correct as she stated one of the two *properties* of a regular polygon.

On the other hand, the statement made by Michael is wrong as he stated an incomplete *definition* of a regular polygon, i.e. the *conditions* of a regular polygon. A polygon with all sides equal may not be regular, e.g. a square is a regular polygon (see Fig. (a)) but a rhombus is not a regular polygon (see Fig. (b)). This is because even though a rhombus is a polygon with all sides equal, not all its angles are equal. The hexagon shown in Fig. (d) is a regular polygon but the hexagon shown in Fig. (e) is not a regular polygon because even though all its sides are equal, not all its angles are equal. Hence, it does not mean that a polygon with all sides equal is regular.

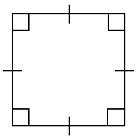


Fig. (a)

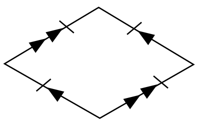


Fig. (b)

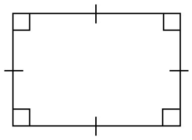


Fig. (c)

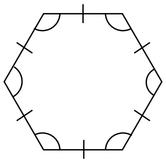


Fig. (d)

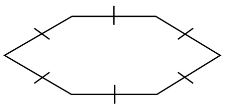


Fig. (e)

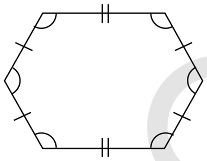


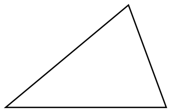
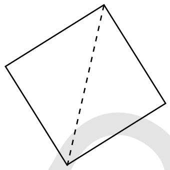
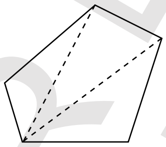
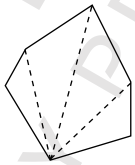
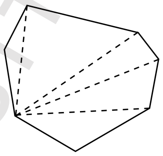
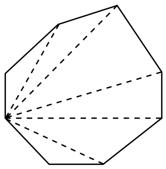
Fig. (f)

In addition, a polygon with all angles equal may not be regular. For example, a rectangle is a polygon (see Fig. (c)) but it is not regular because not all its sides are equal although all its angles are equal. Another example is the hexagon as shown in Fig. (f). It is not a regular polygon because even though all its angles are equal, not all its sides are equal. Hence, it does not mean that a polygon with all angles equal is regular.

In conclusion, a regular polygon is a polygon with all sides equal **and** all angles equal.

Investigation (Sum of Interior Angles of a Polygon)

1.

Polygon	Number of sides	Number of Triangle(s) formed	Sum of Interior Angles
 Triangle	3	1	$1 \times 180° = (3 - 2) \times 180°$
 Quadrilateral	4	2	$2 \times 180° = (4 - 2) \times 180°$
 Pentagon	5	3	$3 \times 180° = (5 - 2) \times 180°$
 Hexagon	6	4	$4 \times 180° = (6 - 2) \times 180°$
 Heptagon	7	5	$5 \times 180° = (7 - 2) \times 180°$
 Octagon	8	6	$6 \times 180° = (7 - 2) \times 180°$
n-gon	n	$(n - 2)$	$(n - 2) \times 180°$

2. If a polygon has n sides, then it will form $(n - 2)$ triangles.

Investigation (Sum of Exterior Angles of a Pentagon)

1. –

2. The sum of exterior angles of a pentagon is 360° as all the exterior angles will meet at a vertex.
From the investigation, we observe that the sum of exterior angles of a pentagon is 360°.
A proof of the above result is given as follows:
Consider the pentagon in Fig. 11.24.

We have $\angle a + \angle p = 180°$, $\angle b + \angle q = 180°$,

$\angle c + \angle r = \underline{180°}$, $\angle d + \angle s = \underline{180°}$ and $\angle e + \angle t = \underline{180°}$.

$\therefore \angle a + \angle p + \angle b + \angle q + \angle c + \angle r + \angle d + \angle s + \angle e + \angle t$

$= \underline{5} \times 180°$

$(\angle a + \angle b + \angle c + \angle d + \angle e) + (\angle p + \angle q + \angle r + \angle s + \angle t) = 900°$

Since the sum of interior angles of a pentagon

$= \angle a + \angle b + \angle c + \angle d + \angle e$

$= (5 - 2) \times 180° = 540°$,

$540° + (\angle p + \angle q + \angle r + \angle s + \angle t) = 900°$.

$\therefore \angle p + \angle q + \angle r + \angle s + \angle t = 900° - \underline{540°} = \underline{360°}$

By using this method, we can show that the sum of exterior angles of a hexagon, of a heptagon and of an octagon is also 360°.

Thinking Time (Page 179)

1. **(i)** No. Since 70° is not an exact divisor of 360°, hence a regular polygon to have an exterior angle of 70° is not possible.

(ii) Since

$360° = 3 \times 120°$,
$360° = 4 \times 90°$,
$360° = 6 \times 60°$,
$360° = 8 \times 45°$,
$360° = 9 \times 40°$,
$360° = 10 \times 36°$,
$360° = 12 \times 30°$,
$360° = 15 \times 24°$,
$360° = 18 \times 20°$,
$360° = 20 \times 18°$,
$360° = 25 \times 15°$,
$360° = 30 \times 12°$,
$360° = 40 \times 9°$,
$360° = 45 \times 8°$,
$360° = 60 \times 6°$,
$360° = 90 \times 4°$,
$360° = 120 \times 3°$,
$360° = 180 \times 2°$,

All the possible values of the angle are 2°, 3°, 4°, 6°, 8°, 9°, 12°, 15°, 18°, 20°, 24°, 30°, 36°, 40°, 45°, 60°, 90° and 120°.

2. No, it is not possible as a concave polygon has one or more interior angles that are greater than 180° while as a regular polygons has all interior angles that are less than 180°.

Practise Now 1

1. $90° + 65° + a° = 180°$ ($\angle$ sum of $\triangle$)

$a° = 180° - 90° - 65°$
$= 25°$
$\therefore a = 25$

2. Since $AC = BC$, $\therefore C\hat{A}B = C\hat{B}A = b°$

$b° + 52° + b° = 180°$ ($\angle$ sum of $\triangle$)
$2b° = 180° - 52°$
$= 128°$
$b° = \dfrac{128°}{2}$
$= 64°$
$\therefore b = 64$

Practise Now 2

(a) $a° = 53° + 48°$ (ext. $\angle$ of $\triangle$)
$= 101°$
$\therefore a = 101$

(b) $F\hat{D}E = 93°$ (vert. opp. $\angle$s)
$b° + 33° + 93° = 180°$ ($\angle$ sum of $\triangle$)
$b° = 180° - 33° - 93°$
$= 54°$
$\therefore b = 54$
$c° = 41° + 93°$ (ext. $\angle$ of $\triangle ABD$)
$= 134°$
$\therefore c = 134$

Practise Now 3

1. **(i)** $D\hat{A}E = 90°$ (right angle)
$51° + 90° + A\hat{E}D = 180°$ ($\angle$ sum of $\triangle AED$)
$A\hat{E}D = 180° - 51° - 90°$
$= 39°$

(ii) $C\hat{D}E + 51° = 90°$ ($\angle ADC$ is a right angle)
$C\hat{D}E = 90° - 51°$
$= 39°$
$68° + 39° + C\hat{E}D = 180°$ ($\angle$ sum of $\triangle CDE$)
$C\hat{E}D = 180° - 68° - 39°$
$= 73°$

2. **(i)** Since $EB = EC$ (diagonals bisect each other), $\therefore E\hat{B}C = 63°$
$63° + B\hat{E}C + 63° = 180°$ ($\angle$ sum of $\triangle BEC$)
$B\hat{E}C = 180° - 63° - 63°$
$= 54°$

(ii) $D\hat{E}C + 54° = 180°$ (adj. $\angle$s on a str. line)
$D\hat{E}C = 180° - 54°$
$= 126°$
Since $ED = EC$ (diagonals bisect each other),
$\therefore C\hat{D}E = D\hat{C}E = x°$.
$x° + 126° + x° = 180°$ ($\angle$ sum of $\triangle CDE$)
$2x° = 180° - 126°$
$= 54°$
$x° = \dfrac{54°}{2}$
$= 27°$
$\therefore C\hat{D}E = 27°$

Practise Now 4

1. **(i)** $A\hat{B}C = 108°$ (opp. $\angle$s of // gram)
$9x° = 108°$
$x = \dfrac{108°}{9}$
$= 12°$
$\therefore x = 12$

(ii) $(D\hat{C}E + 38°) + 108° = 180°$ (int. ∠s, $AD \parallel BC$)

$D\hat{C}E = 180° - 38° - 108°$

$= 34°$

2. $(5x + 6)° + (2x + 13)° = 180°$ (int. ∠s, $AB \parallel DC$)

$7x° + 19° = 180°$

$7x° = 180° - 19°$

$= 161°$

$x° = \frac{161°}{7}$

$= 23°$

$\therefore x = 23$

$[5(23) + 6]° + (y + 17°) = 180°$ (int. ∠s, $AB \parallel DC$)

$y° = 180° - 121° - 17°$

$= 42°$

$\therefore y = 42$

Practise Now 5

1. **(i)** $C\hat{A}B = 32°$ (alt. ∠s, $AB \parallel DC$)

Since $BA = BC$, $\therefore A\hat{C}B = C\hat{A}B = 32°$

$32° + A\hat{B}C + 32° = 180°$ (∠sum of $\triangle ABC$)

$A\hat{B}C = 180° - 32° - 32°$

$= 116°$

(ii) Since $AC = CE$, $\therefore C\hat{E}A = C\hat{A}E = 32°$

$32° + (32° + B\hat{C}E) + 32° = 180°$ (∠ sum of $\triangle ABC$)

$B\hat{C}E = 180° - 32° - 32° - 32°$

$= 84°$

2. $B\hat{D}C = (3x + 13)°$ (diagonals bisect interior angles of a rhombus)

$D\hat{A}C = (x + 45)°$ (diagonals bisect interior angles of a rhombus)

$2(3x + 13)° + 2(x + 45)° = 180°$ (int. ∠ s, $AB \parallel DC$)

$6x° + 26° + 2x° + 90° = 180°$

$8x° = 180° - 26° - 90°$

$8x° = 64°$

$x° = \frac{64°}{8}$

$= 8°$

$\therefore x = 8$

Practise Now 6

1. Sum of interior angles of a pentagon

$= (n - 2) \times 180°$

$= (5 - 2) \times 180°$

$= 540°$

$a° + 121° + a° + a° + 107° = 540°$

$3a° = 540° - 121° - 107°$

$3a° = 312°$

$a° = \frac{312°}{3}$

$= 104°$

$\therefore a = 104$

2. Sum of interior angles of a hexagon

$= (n - 2) \times 180°$

$= (6 - 2) \times 180°$

$= 720°$

$3b° + 4b° + 104° + 114° + 128° + 122° = 720°$

$7b° = 720° - 104° - 114° - 128° - 122°$

$7b° = 252°$

$b° = \frac{252°}{7}$

$= 36°$

$\therefore b = 36$

Practise Now 7

(i) Sum of interior angles of a regular polygon with 24 sides

$= (n - 2) \times 180°$

$= (24 - 2) \times 180°$

$= 3960°$

(ii) Size of each interior angle of a regular polygon with 24 sides

$= \frac{3960°}{24}$

$= 165°$

Practise Now 8

1. **(a)** The sum of exterior angles of the regular polygon is 360°.

$\therefore$ Number of sides of the polygon

$= \frac{360°}{40°}$

$= 9$

(b) Size of each exterior angle of a regular polygon

$= 180° - 178°$

$= 2°$

The sum of exterior angles of the regular polygon is 360°.

$\therefore$ Number of sides of the polygon

$= \frac{360°}{2°}$

$= 180$

2. The sum of exterior angles of the regular decagon is 360°.

$\therefore$ Size of each exterior angle of the regular decagon

$= \frac{360°}{10}$

$= 36°$

$\therefore$ Size of each interior angle of the regular decagon

$= 180° - 36°$

$= 144°$

3. The sum of exterior angles of an n-sided polygon is 360°.

$25° + 26° + 3(180° - 161°) + (n - 5)(180° - 159°) = 360°$

$25° + 26° + 3(19°) + (n - 5)(21°) = 360°$

$25° + 26° + 57° + n(21°) - 105° = 360°$

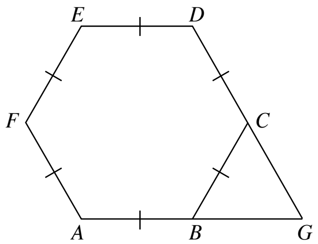
$n(21°) = 360° - 25° - 26° - 57° + 105°$

$= 357°$

$n = \frac{357°}{21°}$

$= 17$

Practise Now 9



Size of each exterior angle of the hexagon

$= \dfrac{360°}{6}$

$= 60°$

$C\hat{B}G = B\hat{C}G = 60°$

$B\hat{G}C + 60° + 60° = 180°$ (∠ sum of △BCG)

$B\hat{G}C = 180° - 60° - 60°$

$= 60°$

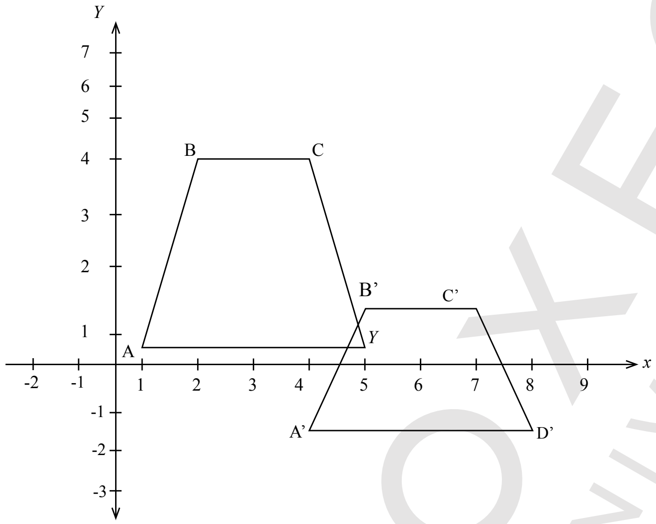
Practise Now 10

A = (1, 1)   A' = (1+3 , 1-2) = (4, -1)

B = (2, 4)   B' = (2+3 , 4 - 2) = (5 - 2)

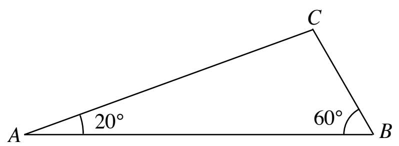
C = (4, 4)   C' = (4+2 , 4 - 2) = (7 , 2)

D = (5, 1)   D' = (5 + 3 , 10 - 2) = (8 , -1)



Exercise 10A

1. (a)

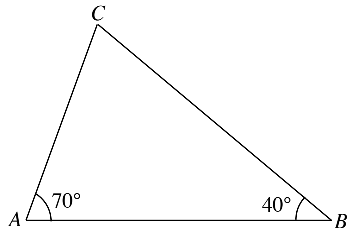


$\angle C = 180° - 20° - 60°$ (∠ sum of △)

$= 100°$

It is a scalene triangle and an obtuse-angled triangle.

(b)

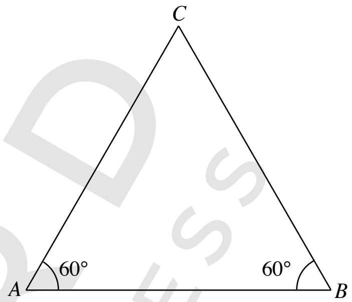


$\angle C = 180° - 70° - 40°$ (∠ sum of △)

$= 70°$

It is an isosceles triangle and acute-angled triangle.

(c)

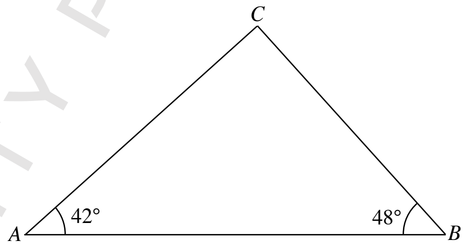


$\angle C = 180° - 60° - 60°$ (∠ sum of △)

$= 60°$

It is an equilateral triangle and acute-angled triangle.

(d)



$\angle C = 180° - 42° - 48°$ (∠ sum of △)

$= 90°$

It is a scalene triangle and right-angled triangle.

2. (a) Third angle of the triangle

$= 180° - 40° - 40°$ (∠ sum of △)

$= 100°$

(b) Third angle of the triangle

$= 180° - 87° - 87°$ (∠ sum of △)

$= 6°$

(c) Third angle of the triangle

$= 180° - 15° - 15°$ (∠ sum of △)

$= 150°$

(d) Third angle of the triangle

$= 180° - 79° - 79°$ (∠ sum of △)

$= 22°$

3. (a) $39° + 90° + a° = 180°$ (∠ sum of △)

$a° = 180° - 39° - 90°$

$= 51°$

$\therefore a = 51$

(b) $68° + 2b° + 64° = 180°$ (∠ sum of △)

$$2b° = 180° - 68° - 64°$$
$$= 48°$$
$$b° = \frac{48°}{2}$$
$$= 24°$$
$$\therefore b = 24$$

(c) $4c° + 3c° + 40° = 180°$ (∠ sum of △)

$$4c° + 3c° = 180° - 40°$$
$$7c° = 140°$$
$$c = \frac{140°}{7}$$
$$= 20°$$
$$\therefore c = 20$$

(d) $3d° + 4d° + d° = 180°$ (∠ sum of △)

$$8d° = 180°$$
$$d° = \frac{180°}{8}$$
$$= 22.5°$$
$$\therefore d = 22.5$$

(e) Since $BA = BC$, $\therefore B\hat{C}A = B\hat{A}C = 62°$

$62° + e° + 62° = 180°$ (∠ sum of △)

$$e° = 180° - 62° - 62°$$
$$= 56°$$
$$\therefore e = 56$$

(f) Since $AC = BC = AB$, $\therefore C\hat{A}B = C\hat{B}A = A\hat{C}B = f°$

$f° + f° + f° = 180°$ (∠ sum of △)

$$3f° = 180°$$
$$f° = \frac{180°}{3}$$
$$= 60°$$
$$\therefore f = 60$$

4. **(a)** $a° = 47° + 55°$ (ext. ∠ of △)

$$= 102°$$
$$\therefore a = 102$$

(b) $90° + b° + 50° = 180°$ (∠ sum of △)

$$b° = 180° - 90° - 50°$$
$$= 40°$$
$$\therefore b = 40$$

$90° + c° + 35° = 180°$ (∠ sum of △)

$$c° = 180° - 90° - 35°$$
$$= 55°$$
$$\therefore c = 55$$

(c) $d° + 110° = 180°$ (adj. ∠s on a str. line)

$$d° = 180° - 110°$$
$$= 70°$$
$$\therefore d = 70$$

$2e° + 3e° = 110°$ (ext. ∠ of △)

$$5e° = 110°$$
$$e° = \frac{110°}{5}$$
$$= 22°$$
$$\therefore e = 22$$

5. $3x° + 4x° + 5x° = 180°$ (∠ sum of △)

$$12x° = 180°$$
$$x° = \frac{180°}{12}$$
$$= 15°$$
$$\therefore x = 15$$

Smallest angle of the triangle

$= 3(15°)$

$= 45°$

6. **(i)** Let $A\hat{D}B = B\hat{D}C = x°$

$90° + 20° + 2x° = 180°$ (∠ sum of △)

$$2x° = 180° - 90° - 20°$$
$$= 70°$$
$$x° = \frac{70°}{2}$$
$$= 35°$$
$$\therefore B\hat{D}C = 35°$$

(ii) $C\hat{B}D + 20° + 35° = 180°$ (∠ sum of △)

$$C\hat{B}D = 180° - 20° - 35°$$
$$= 125°$$

7. **(a)** $a° + (180° + 120°) = 110°$ (ext < of Δ)

$$a° + 60° = 110°$$
$$a° = 110° - 60°$$
$$a° = 50°$$

(b) $a° + a° + 52° = 180°$ (isoseeles Δ)

$$2a° = 180° - 52°$$
$$a° = \frac{180°}{2}$$
$$a° = 64°$$

(C) $a° + 2a° + (360° - 270°) = 180°$

$$3a° + 90° = 180°$$
$$3a° = 180° - 90°$$
$$3a° = 90°$$
$$a° = 30°$$

(d) $a° + 100° (360° - 310°) = 180°$

$$a° = 180° - 100° - 50°$$
$$a° = 30°$$

8. **(a)** $a° + 90° = 115°$ (ext. ∠ of $\triangle BCE$)

$$a° = 115° - 90°$$
$$= 25°$$
$$\therefore a = 25$$

$b° = 90° + 32°$ (ext. ∠ of $\triangle EFG$)

$$= 122°$$
$$\therefore b = 122$$

(b) $A\hat{B}E = A\hat{B}D = 89° + 27°$ (ext. ∠ of $\triangle BCD$)

$$= 116°$$

$c° = 116° + 22°$ (ext. ∠ of $\triangle ABE$)

$$= 138°$$
$$\therefore c = 138$$

9. $(x - 35)^\circ + (x - 25)^\circ + \frac{1}{2}x - 10^{\circ} = 180^\circ$ ($\angle$ sum of $\triangle$)

$$\frac{5}{2}x^\circ - 70^\circ = 180^\circ$$
$$\frac{5}{2}x^\circ = 180^\circ + 70^\circ$$
$$\frac{5}{2}x^\circ = 250^\circ$$
$$x^\circ = \frac{250^\circ}{\frac{5}{2}}$$
$$= 100^\circ$$
$$\therefore x = 100$$

10. **(i)** $D\hat{C}E + 61^\circ + 41^\circ = 180^\circ$ ($\angle$ sum of $\triangle$)

$D\hat{C}E = 180^\circ - 61^\circ - 41^\circ$

$= 78^\circ$

$A\hat{C}B = 78^\circ$ (vert. opp. $\angle$s)

(ii) $A\hat{B}C + 78^\circ + 50^\circ = 180^\circ$ ($\angle$ sum of $\triangle$)

$A\hat{B}C = 180^\circ - 78^\circ - 50^\circ$

$= 52^\circ$

11. $a^\circ = (360^\circ - 310^\circ) + (180^\circ - 100^\circ)$

$a^\circ = 50^\circ + 80^\circ$

$a^\circ = 130^\circ$

12. **(a)** $82^\circ + 40^\circ + a^\circ = 180^\circ$ ($\angle$ sum of $\triangle$)

$a^\circ = 180^\circ - 82^\circ - 40^\circ$

$= 58^\circ$

$\therefore a = 58$

$A\hat{D}B = 82^\circ$ (vert. opp. $\angle$s)

$b^\circ = 45^\circ + 82^\circ$ (ext. $\angle$ of $\triangle ABD$)

$= 127^\circ$

$\therefore b = 127$

(b) $E\hat{D}F + 44^\circ + 57^\circ = 180^\circ$ ($\angle$ sum of $\triangle$)

$E\hat{D}F = 180^\circ - 44^\circ - 57^\circ$

$= 79^\circ$

$A\hat{D}B = 79^\circ$ (vert. opp. $\angle$s)

$c^\circ = 51^\circ + 79^\circ$ (ext. $\angle$ of $\triangle ABD$)

$= 130^\circ$

$\therefore c = 130$

13. **(i)** $A\hat{B}C + 50^\circ + 26^\circ = 180^\circ$ ($\angle$ sum of $\triangle$)

$A\hat{B}C = 180^\circ - 50^\circ - 26^\circ$

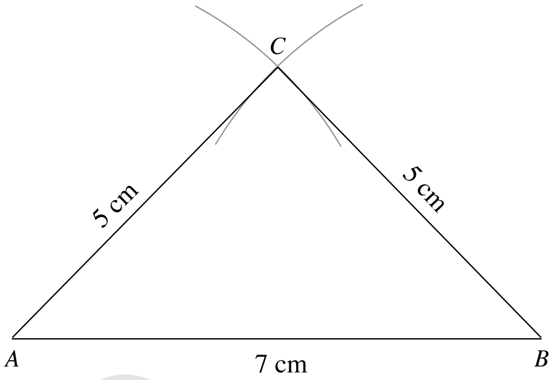
$= 104^\circ$

(ii) $C\hat{B}D = 50^\circ + 26^\circ$ (ext. $\angle$ of $\triangle$)

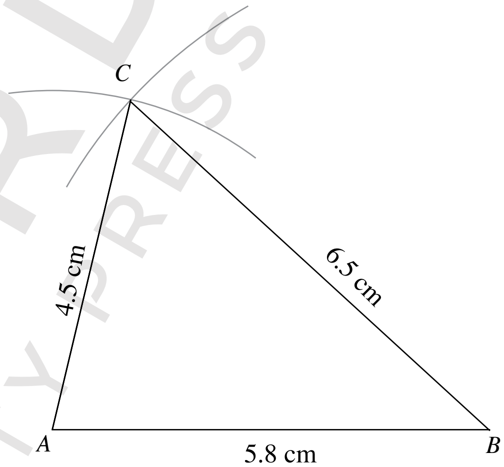
$= 76^\circ$

Exercise 10B

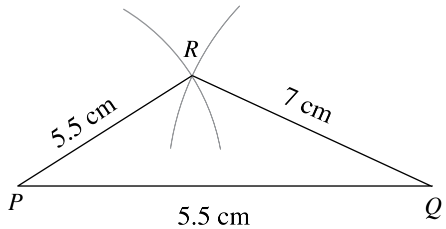
1. **(a)**



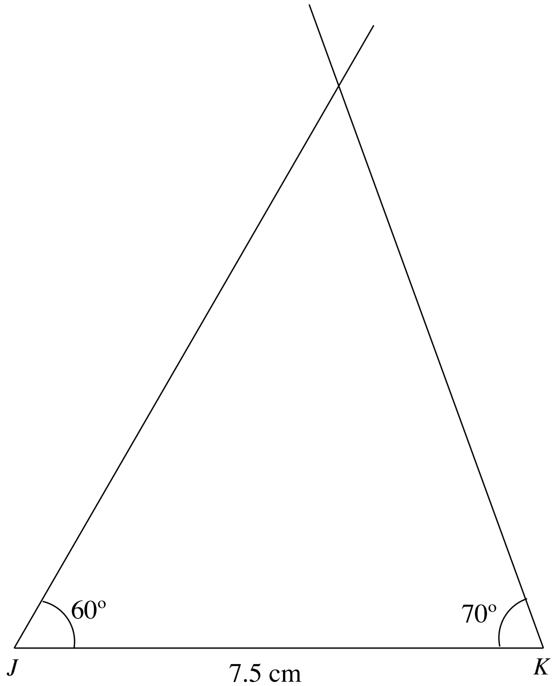
(b)



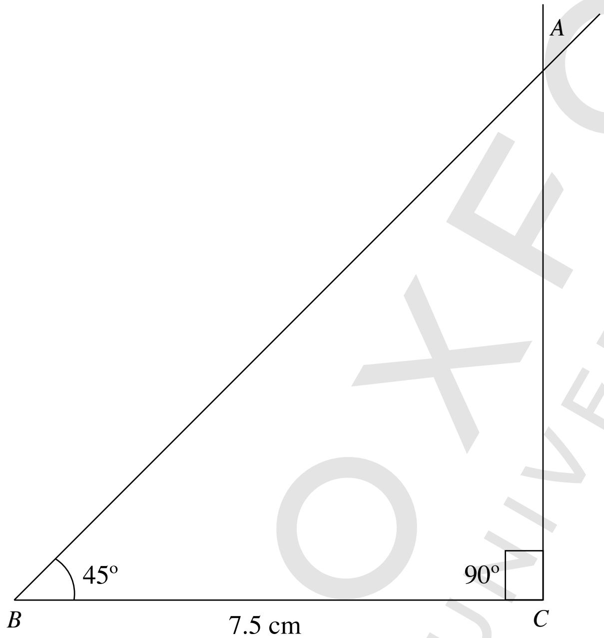
(c)



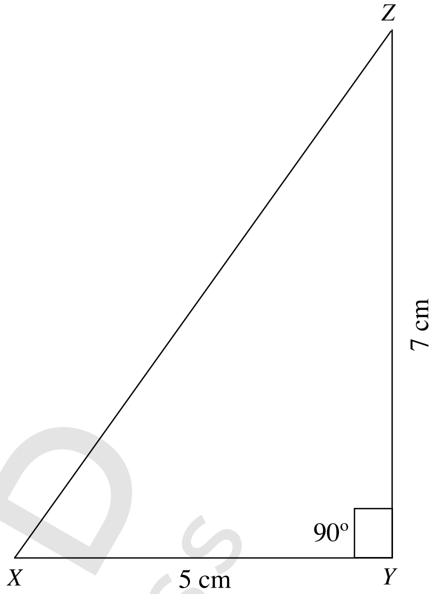
(d)



(e)



(f)



2. **(a)** $a° + 54° = 90°$ ($B\hat{C}D$ is a right angle)

$a° = 90° - 54°$

$= 36°$

$\therefore a = 36$

$b° = 36°$ (alt. $\angle$s, $AB \,//\, DC$)

$\therefore b = 36$

(b) $E\hat{B}C = 90°$ (right angle)

$90° + 39° + c° = 180°$ ($\angle$ sum of $\triangle BCE$)

$c° = 180° - 90° - 39°$

$= 51°$

$\therefore c = 51$

$D\hat{C}E + 39° = 90°$ ($B\hat{C}D$ is a right angle)

$D\hat{C}E = 90° - 39°$

$= 51°$

$51° + d° + 78° = 180°$ ($\angle$ sum of $\triangle CDE$)

$d° = 180° - 51° - 78°$

$= 51°$

$\therefore d = 51$

3. **(a)** $a° = 106°$ (opp. $\angle$s of // gram)

$\therefore a = 106$

$b° = 48°$ (alt. $\angle$s, $AD \,//\, BC$)

$\therefore b = 48$

(b) $4c° + 5c° = 180°$ (int. $\angle$s, $AB \,//\, DC$)

$9c° = 180°$

$c° = \dfrac{180°}{9}$

$= 20°$

$\therefore c = 20$

$2d° = 4(20°)$ (opp. $\angle$ s of // gram)

$d° = \dfrac{80°}{2}$

$= 40°$

$\therefore d = 40$

4. (a) Since $ABCD$ is a kite, $\therefore AD = CD$ and so $A\hat{C}D = C\hat{A}D = a°$

$a° + 100° + a° = 180°$ ($\angle$ sum of $\triangle ACD$)

$2a° = 180° - 100°$

$= 80°$

$a° = \frac{80°}{2}$

$= 40°$

$\therefore a = 40$

Since $ABCD$ is a kite, $\therefore AB = CB$ and so $C\hat{A}B = A\hat{C}B = 61°$.

$61° + b° + 61° = 180°$ ($\angle$sum of $\triangle ABC$)

$b° = 180° - 61° - 61°$

$= 58°$

$\therefore b = 58$

(b) Since $ABCD$ is a kite, $\therefore D\hat{A}C = B\hat{A}C = 40°$.

(One diagonal bisects the interior angles)

$40° + 26° + c° = 180°$ ($\angle$ sum of $\triangle ACD$)

$c° = 180° - 40° - 26°$

$= 114°$

$\therefore c = 114$

5. (a) Since $ABCD$ is a square, $\therefore D\hat{A}C = B\hat{A}C = 45°$ and hence $D\hat{A}E = 45°$.

(Diagonals bisect the interior angles)

$A\hat{E}D + 82° = 180°$ (adj. $\angle$s on a str. line)

$A\hat{E}D = 180° - 82°$

$= 98°$

$45° + 98° + a° = 180°$ ($\angle$ sum of $\triangle ADE$)

$a° = 180° - 45° - 98°$

$= 37°$

$\therefore a = 37$

Since $ABCD$ is a square, $\therefore B\hat{A}C = D\hat{A}C = 45°$ and hence $E\hat{A}F = 45°$.

(Diagonals bisect the interior angles)

$A\hat{E}F = 82°$ (vert. opp. $\angle$)

$b° = 45° + 82°$ (ext. $\angle$ of $\triangle AEF$)

$= 127°$

$\therefore b = 127$

(b) Since $ABCD$ is a square, $\therefore B\hat{C}A = D\hat{C}A = 45°$ and hence $E\hat{C}F = 45°$.

(Diagonals bisect the interior angles)

$c° + 45° + c° = 180°$ ($\angle$ sum of $\triangle CEF$)

$2c° = 180° - 45°$

$= 135°$

$c° = \frac{135°}{2}$

$= 67.5°$

$\therefore c = 67.5$

Since $ABCD$ is a square, $\therefore C\hat{E}D = 90°$.

(Diagonals bisect each other at right angles)

Hence,

$d° + 67.5° = 90°$

$d° = 90° - 67.5°$

$= 22.5°$

$\therefore d = 22.5$

6. (a) Since $ABCD$ is a rhombus, $\therefore A\hat{C}B = A\hat{D}C = 114°$ (Opposite angles are equal) and hence $a = 114$.

Since $ABCD$ is a rhombus, $\therefore AB = CB$ and hence $A\hat{C}B = C\hat{A}B = b°$.

$b° + 114° + b° = 180°$ ($\angle$ sum of $\triangle ABC$)

$2b° = 180° - 114°$

$= 66°$

$b° = \frac{66°}{2}$

$= 33°$

$\therefore b = 33$

(b) $C\hat{B}D = B\hat{D}A = 38°$ (alt. $\angle$s, $AD // BC$)

$c° = 38°$

$\therefore c = 38$

Since $ABCD$ is a rhombus, $\therefore AB = AD$ and hence $B\hat{D}A = D\hat{B}A = 38°$.

$38° + d° + 38° = 180°$ ($\angle$ sum of $\triangle ABD$)

$d° = 180° - 38° - 38°$

$= 104°$

$\therefore d = 104$

(c) $D\hat{C}A = C\hat{A}B = 42°$ (alt, $\angle$s, $AB // DC$)

$e° = 42°$

$\therefore e = 42$

Since $ABCD$ is a rhombus, $\therefore A\hat{D}B = C\hat{D}B = f°$.

(Diagonals bisect the interior angles)

Also, $AD = CD$ and hence $C\hat{A}D = A\hat{C}D = 42°$

$42° + 2f° + 42° = 180°$ ($\angle$ sum of $\triangle ACD$)

$2f° = 180° - 42° - 42°$

$= 96°$

$f° = \frac{96°}{2}$

$= 48°$

$\therefore f = 48$

7. (i) $A\hat{E}D = 52°$ (vert. opp. $\angle$s)

Since $AE = DE$, $\therefore A\hat{D}E = D\hat{A}E = x°$.

$x° + 52° + x° = 180°$ ($\angle$ sum of $\triangle ADE$)

$2x° = 180° - 52°$

$= 128°$

$x° = \frac{128°}{2}$

$= 64°$

$\therefore A\hat{D}B = A\hat{D}E = 64°$

(ii) $A\hat{D}C = 90°$ (right angle of a rectangle)

$64° + 90° + A\hat{C}D = 180°$ ($\angle$ sum of $\triangle ACD$)

$A\hat{C}D = 180° - 64° - 90°$

$= 26°$

8. (i) $A\hat{D}E + 65° = 180°$ (int. $\angle$s, $AB // DC$)

$A\hat{D}E = 180° - 65°$

$= 115°$

(ii) $B\hat{C}D = 65°$ (opp. $\angle$s of // gram)

$C\hat{B}E + 65° = 125°$ (ext. $\angle$ of $\triangle BCE$)

$C\hat{B}E = 125° - 65°$

$= 60°$

9. $A\hat{D}B = (3x + 7)°$ (diagonals bisect interior angles of a rhombus)

$D\hat{A}C = (2x + 53)°$ (diagonals bisect interior angles of a rhombus)

$2(3x + 7)° + 2(2x + 53)° = 180°$ (int. $\angle$s, $AB \parallel DC$)

$$6x° + 14° + 4x° + 106° = 180°$$
$$10x° = 180° - 14° - 106°$$
$$10x° = 60°$$
$$x° = \frac{60°}{10}$$
$$= 6°$$
$$\therefore x = 6$$

10. $5x° + x° = 180°$ (int. $\angle$s, $AB \parallel DC$)

$$6x° = 180°$$
$$x° = \frac{180°}{6}$$
$$= 30°$$
$$\therefore x = 30$$

$2.2(30°) + y° = 180°$ (int. $\angle$s, $AB \parallel DC$)

$$y° = 180° - 66°$$
$$= 114°$$
$$\therefore y = 114$$

11. (i) Since $ABCD$ is a kite, $\therefore B\hat{A}C = D\hat{A}C = 25°$
(One diagonal bisects the interior angles) and
since $AB = AD$, $\therefore B\hat{D}A = D\hat{B}A = x°$

$x° + 2(25°) + x° = 180°$ ($\angle$ sum of $\triangle ABD$)

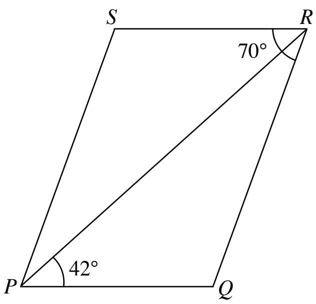
$$2x° = 180° - 50°$$
$$= 130°$$
$$x° = \frac{130°}{2}$$
$$= 65°$$
$$\therefore A\hat{B}D = 65°$$

(ii) Since $ABCD$ is a kite, $\therefore B\hat{C}A = D\hat{C}A = 44°$
One diagonal bisects the interior angles) and
since $CB = CD$, $\therefore B\hat{D}C = D\hat{B}C = y°$

$y° + 2(44°) + y° = 180°$ ($\angle$ sum of $\triangle BCD$)

$$2y° = 180° - 88°$$
$$= 92°$$
$$y° = \frac{92°}{2}$$
$$= 46°$$
$$\therefore C\hat{B}D = 46°$$

12.



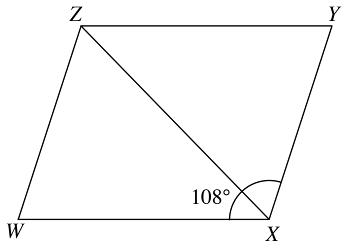
(i) $P\hat{Q}R + 70° = 180°$ (int. $\angle$s, $PQ \parallel SR$)

$$P\hat{Q}R = 180° - 70°$$
$$= 110°$$

(ii) $42° + 110° + P\hat{R}Q = 180°$ ($\angle$ sum of $\triangle PQR$)

$$P\hat{R}Q = 180° - 42° - 110°$$
$$= 28°$$

13.



(i) Since $WXYZ$ is a rhombus, $W\hat{Z}Y = W\hat{X}Y = 108°$ (opp. $\angle$s of a // gram) and $X\hat{Z}Y = X\hat{Z}W = x°$ (Diagonal bisect the interior angles), hence $W\hat{Z}Y = 2x°$

$$2x° = 108°$$
$$x° = \frac{108°}{2}$$
$$= 54°$$
$$\therefore X\hat{Z}Y = 54°$$

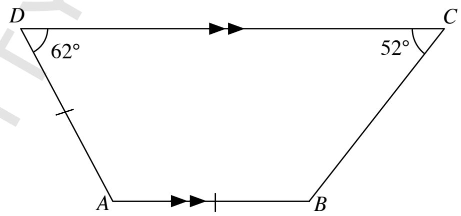
(ii) $X\hat{Y}Z + 108° = 180°$ (int. $\angle$s, $WX \parallel ZY$)

$$X\hat{Y}Z = 180° - 108°$$
$$= 72°$$

(iii) Since $WXYZ$ is a rhombus, $X\hat{W}Z = X\hat{Y}Z = 72°$ (opp. $\angle$s of a // gram) and $X\hat{W}Y = Z\hat{W}Y = y°$ (Diagonals bisect the interior angles), hence $X\hat{W}Z = 2y°$

$$2y° = 72°$$
$$y° = \frac{72°}{2}$$
$$= 36°$$
$$\therefore X\hat{W}Y = 36°$$

14.



(i) $B\hat{A}D + 62° = 180°$ (int. $\angle$s, $AB \parallel DC$)

$$B\hat{A}D = 180° - 62°$$
$$= 118°$$

Since $AB = AD$, $\therefore A\hat{B}D = A\hat{D}B = x°$

$x° + 118° + x° = 180°$ ($\angle$ sum of $\triangle ABD$)

$$2x° = 180° - 118°$$
$$= 62°$$
$$x° = \frac{62°}{2}$$
$$= 31°$$
$$\therefore A\hat{B}D = 31°$$

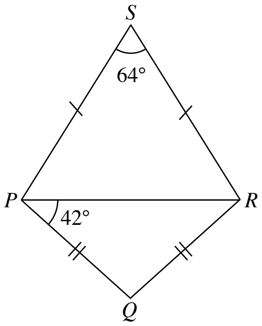
(ii) $A\hat{B}C + 52° = 180°$ (int. $\angle$s, $AB \parallel DC$)

$$A\hat{B}C = 180° - 52°$$
$$= 128°$$

From (i), $A\hat{B}D = 31°$.

$$36° + C\hat{B}D = 128°$$
$$C\hat{B}D = 128° - 31°$$
$$= 97°$$

15.



(i) Since $PS = RS$, $\therefore R\hat{P}S = P\hat{R}S = x°$.

$x° + 64° + x° = 180°$ ($\angle$ sum of $\triangle PRS$)

$2x° = 180° - 64°$

$= 116°$

$x° = \frac{116°}{2}$

$= 58°$

$\therefore P\hat{R}S = 58°$

(ii) Since $PQ = QR$, $\therefore Q\hat{P}R = Q\hat{R}P = 42°$.

$42° + P\hat{Q}R + 42° = 180°$ ($\angle$ sum of $\triangle PQR$)

$P\hat{Q}R = 180° - 42° - 42°$

$= 96°$

Exercise 10C

1. **(a)** Sum of interior angles of a 11-gon

$= (n - 2) \times 180°$

$= (11 - 2) \times 180°$

$= 1620°$

(b) Sum of interior angles of a 12-gon

$= (n - 2) \times 180°$

$= (12 - 2) \times 180°$

$= 1800°$

(c) Sum of interior angles of a 15-gon

$= (n - 2) \times 180°$

$= (15 - 2) \times 180°$

$= 2340°$

(d) Sum of interior angles of a 20-gon

$= (n - 2) \times 180°$

$= (20 - 2) \times 180°$

$= 3240°$

2. **(a)** Sum of interior angles of a quadrilateral

$= (n - 2) \times 180°$

$= (4 - 2) \times 180°$

$= 360°$

$78° + 62° + a° + 110° = 360°$

$a° = 360° - 78° - 62° - 110°$

$= 110°$

$\therefore a = 110$

(b) Sum of interior angles of a quadrilateral

$= (n - 2) \times 180°$

$= (4 - 2) \times 180°$

$= 360°$

$b° + 78° + 2b° + 84° = 360°$

$3b° = 360° - 78° - 84°$

$= 198°$

$b° = \frac{198°}{3}$

$= 66°$

$\therefore b = 66$

(c) Sum of interior angles of a pentagon

$= (n - 2) \times 180°$

$= (5 - 2) \times 180°$

$= 540°$

$c° + 152° + 38° + 2c° + 101° = 540°$

$3c° = 540° - 152° - 38° - 101°$

$3c° = 249°$

$c° = \frac{249°}{3}$

$= 83°$

$\therefore c = 83$

(d) Sum of interior angles of a hexagon

$= (n - 2) \times 180°$

$= (6 - 2) \times 180°$

$= 720°$

$102° + 5d° + 4d° + 4d° + 108° + 4d° = 720°$

$17d° = 720° - 102° - 108°$

$= 510°$

$d° = \frac{510°}{17}$

$= 30°$

$\therefore d = 30$

3. **(a)** **(i)** Sum of interior angles of a hexagon

$= (n - 2) \times 180°$

$= (6 - 2) \times 180°$

$= 720°$

(ii) Hence, size of each interior angle of a hexagon

$= \frac{720°}{6}$

$= 120°$

(b) **(i)** Sum of interior angles of a regular polygon with 18 sides

$= (n - 2) \times 180°$

$= (18 - 2) \times 180°$

$= 2880°$

(ii) Hence, size of each interior angle of a regular polygon with 18 sides

$= \frac{2880°}{18}$

$= 160°$

4. **(a)** The sum of exterior angles of the regular polygon is 360°.

$\therefore$ Size of each exterior angle of the regular polygon

$= \frac{360°}{24}$

$= 15°$

$\therefore$ Size of each interior angle of a regular polygon with 24 sides

$= 180° - 15°$

$= 165°$

(b) The sum of exterior angles of the regular polygon is 360°.

$\therefore$ Size of each exterior angle of the regular polygon

$= \frac{360°}{36}$

$= 10°$

$\therefore$ Size of each interior angle of a regular polygon with 36 sides

$= 180° - 10°$

$= 170°$

5. **(a)** The sum of exterior angles of the regular polygon is 360°.

$\therefore$ Number of sides of the polygon

$= \frac{360°}{90°}$

$= 4$

(b) The sum of exterior angles of the regular polygon is 360°.

$\therefore$ Number of sides of the polygon

$= \frac{360°}{45°}$

$= 8$

(c) The sum of exterior angles of the regular polygon is 360°.

$\therefore$ Number of sides of the polygon

$= \frac{360°}{12°}$

$= 30$

(d) The sum of exterior angles of the regular polygon is 360°.

$\therefore$ Number of sides of the polygon

$= \frac{360°}{4°}$

$= 90$

6. **(a)** Size of each interior angle of a regular polygon

$= 180° - 140°$

$= 40°$

The sum of exterior angles of the regular polygon is 360°.

$\therefore$ Number of sides of the polygon

$= \frac{360°}{40°}$

$= 9$

(b) Size of each interior angle of a regular polygon

$= 180° - 162°$

$= 18°$

The sum of exterior angles of the regular polygon is 360°

$\therefore$ Number of sides of the polygon

$= \frac{360°}{18°}$

$= 20$

(c) Size of each interior angle of a regular polygon

$= 180° - 172°$

$= 8°$

The sum of exterior angles of the regular polygon is 360°.

$\therefore$ Number of sides of the polygon

$= \frac{360°}{8°}$

$= 45$

(d) Size of each interior angle of a regular polygon

$= 180° - 175°$

$= 5°$

The sum of exterior angles of the regular polygon is 360°

$\therefore$ Number of sides of the polygon

$= \frac{360°}{5°}$

$= 72$

7. Sum of interior angles of a pentagon

$= (n - 2) \times 180°$

$= (5 - 2) \times 180°$

$= 540°$

$$\begin{aligned} 2x° + 3x° + 4x° + 5x° + 6x° &= 540° \\ 20x° &= 540° \\ x° &= \frac{540°}{20} \\ &= 27° \end{aligned}$$

Hence, the largest interior angle of the pentagon

$= 6(27°)$

$= 162°$

8. **(i)** The sum of exterior angles of the triangle is 360°.

$$\begin{aligned} 3y° + 4y° + 5y° &= 360° \\ 12y° &= 360° \\ y° &= \frac{360°}{12} \\ &= 30° \\ \therefore y &= 30 \end{aligned}$$

(ii) Smallest interior angle of the triangle

$= 180° - 5(30°)$

$= 180° - 150°$

$= 30°$

9. The sum of exterior angles of an n-sided polygon is 360°.

$$\begin{aligned} 15° + 25° + 70° + (n - 3) \times 50° &= 360° \\ 15° + 25° + 70° + n(50°) - 150° &= 360° \\ n(50°) &= 360° - 15° - 25° - 70° + 150° \\ n(50°) &= 400° \\ n &= \frac{400°}{50°} \\ &= 8 \end{aligned}$$

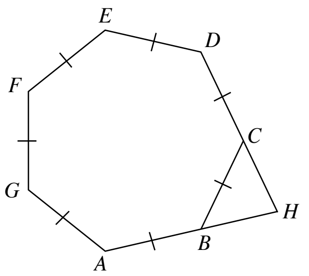
10. The sum of exterior angles of a n-sided polygon is 360°.

$3(50°) + (180° - 127°) + (180° - 135°) + (n - 5)(180° - 173°)$

$= 360°$

$$\begin{aligned} 150° + 53° + 45° + (n - 5)(7°) &= 360° \\ 150° + 53° + 45° + n(7°) - 35° &= 360° \\ n(7°) &= 360° - 150° - 53° - 45° + 35° \\ &= 147° \\ n &= \frac{147°}{7°} \\ &= 21 \end{aligned}$$

11.



Size of each exterior angle of the heptagon

$= \dfrac{360°}{7}$

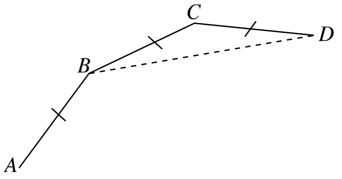
$= 51.43°$

$B\hat{H}C + 51.43° + 51.43° = 180°$ ($\angle$ sum of $\triangle BCH$)

$B\hat{H}C = 180° - 51.43° - 51.43°$

$= 77.1°$ (to 1 d.p.)

12.



(i) Sum of interior angles of a regular polygon with 20 sides

$= (n - 2) \times 180°$

$= (20 - 2) \times 180°$

$= 3240°$

Hence, size of each interior angles of a regular polygon with 20 sides

$= \dfrac{3240°}{20}$

$= 162°$

$\therefore A\hat{B}C = 162°$

(ii) Since size of each interior angle of a regular polygon with 20 sides = 162°,

$\therefore B\hat{C}D = 162°$

Let $C\hat{B}D = C\hat{D}B = x°$ (base $\angle$s of isos. $\triangle BCD$)

$x° + x° + 162° = 180°$ ($\angle$ sum of $\triangle BCD$)

$2x° = 180° - 162°$

$2x° = 18°$

$x° = \dfrac{18°}{2}$

$= 9°$

$\therefore x = 9$

Hence,

$A\hat{B}D = A\hat{B}C - C\hat{B}D$

$= 162° - 9°$

$= 153°$

13. (i) Sum of interior angles of a hexagon

$= (n - 2) \times 180°$

$= (6 - 2) \times 180°$

$= 720°$

$\therefore$ Size of each interior angle of a hexagon

$= \dfrac{720°}{6}$

$= 120°$

Since $A\hat{B}P$ is an interior angle of a hexagon,

$\therefore A\hat{B}P = 120°$.

(ii) Since $P\hat{Q}R$ is an interior angle of a hexagon,

$\therefore P\hat{Q}R = 120°$.

$P\hat{Q}X = \dfrac{120°}{2}$ (QA is a line of symmetry)

$= 60°$

(iii) $A\hat{X}B = \dfrac{360°}{6}$ ($\angle$s at a point)

$= 60°$

(iv) Sum of interior angles of a pentagon

$= (n - 2) \times 180°$

$= (5 - 2) \times 180°$

$= 540°$

$\therefore$ Size of each interior angle of a pentagon

$= \dfrac{540°}{5}$

$= 108°$

Since $A\hat{B}C$ is an interior angle of a pentagon,

$\therefore A\hat{B}C = 108°$.

(v) Since size of each interior angle of a pentagon = 108°,

$\therefore B\hat{C}D = 108°$

Let $B\hat{A}C = B\hat{C}A = x°$ (base $\angle$s of isos. $\triangle ABC$)

$x° + x° + 108° = 180°$ ($\angle$ sum of $\triangle ABC$)

$2x° = 180° - 108°$

$2x° = 72°$

$x° = \dfrac{72°}{2}$

$= 36°$

$\therefore x = 36$

Hence,

$A\hat{C}D = B\hat{C}D - B\hat{C}A$

$= 108° - 36°$

$= 72°$

(vi) Since size of each interior angle of a hexagon = 120°,

$\therefore B\hat{A}S = 120°$

Since size of each interior angle of a pentagon = 108°,

$\therefore B\hat{A}E = 108°$

$120° + 108° + S\hat{A}E = 360°$ ($\angle$s at a point)

$S\hat{A}E = 360° - 120° - 108°$

$= 132°$

Let $A\hat{S}E = A\hat{E}S = x°$ (base ∠ of isos. △AES)

$x° + x° + 132° = 180°$ (∠ sum of △AES)

$2x° = 180° - 132°$

$2x° = 48°$

$x° = \dfrac{48°}{2}$

$= 24°$

$\therefore A\hat{S}E = 24°$

14. (i) Let the interior angle be $5x°$ and the exterior angle be $x°$.

$5x° + x° = 180°$ (adj. ∠s on a str. line)

$6x° = 180°$

$x° = \dfrac{180°}{6}$

$= 30°$

Since sum of exterior angles of a n-sided polygon is 360°,

$\therefore n = \dfrac{360°}{30°} = 12$

(ii) $A\hat{B}C = 5(30°) = 150°$ (int. ∠ of a 12-sided polygon)

Let $B\hat{A}C = B\hat{C}A = x°$ (base ∠s of isos. △ABC)

$x° + x° + 150° = 180°$ (∠ sum of △ABC)

$2x° = 180° - 150°$

$2x° = 30°$

$x° = \dfrac{30°}{2}$

$= 15°$

Hence,

$A\hat{C}D = B\hat{C}D - B\hat{C}A$

$= 150° - 15°$

$= 135°$

(iii) $A\hat{B}C = B\hat{C}D = 150°$ (int. ∠ of a 12-sided polygon)

$B\hat{A}D = A\hat{D}C = y°$ (base ∠s of isos. quadrilateral, $BA = CD$)

$y° + y° + 150° + 150° = 360°$ (∠ sum of quadrilateral)

$2y° = 360° - 150° - 150°$

$2y° = 60°$

$y° = \dfrac{60°}{2}$

$= 30°$

$\therefore A\hat{D}C = 30°$

$C\hat{D}E = 150°$ (int. ∠ of a 12-sided polygon)

Hence,

$A\hat{D}E = C\hat{D}E - A\hat{D}C$

$= 150° - 30°$

$= 120°$

15. (i) Since sum of exterior angles of a n-sided polygon is 360°,

$\therefore n = \dfrac{360°}{36°} = 10$

(ii) Size of an interior angle of the n-sided polygon

$= 180° - 36°$ (adj. ∠s on a str. line)

$= 144°$

Let $C\hat{B}D = C\hat{D}B = x°$ (base ∠s of isos. △BCD)

$x° + x° + 144° = 180°$ (∠ sum of △BCD)

$2x° = 180° - 144°$

$2x° = 36°$

$x° = \dfrac{36°}{2}$

$= 18°$

$\therefore C\hat{D}B = 18°$

$C\hat{D}E = 144°$ (int. ∠ of a 10-sided polygon)

Hence,

$B\hat{D}E = C\hat{D}E - C\hat{D}B$

$= 144° - 18°$

$= 126°$

(iii) Let $X\hat{C}D = X\hat{D}C = 18°$ (base ∠s of isos. △CDX, $CX = DX$)

$18° + 18° + C\hat{X}D = 180°$ (∠ sum of △CDX)

$C\hat{X}D = 180° - 18° - 18°$

$= 144°$

16. $A\hat{C}E = \angle a + \angle b$ (ext. ∠ of △ABC)

$J\hat{C}E + \angle a + \angle b = 180°$ (adj. ∠s on a str. line)

$J\hat{C}E = 180° - \angle a - \angle b$

$D\hat{E}C = \angle c + \angle d$ (ext. ∠ of △DEF)

$G\hat{E}C + \angle c + \angle d = 180°$ (adj. ∠s on a str. line)

$G\hat{E}C = 180° - \angle c - \angle d$

$H\hat{G}J = \angle e + \angle f$ (ext. ∠ of △GHI)

$E\hat{G}J + \angle e + \angle f = 180°$ (adj. ∠s on a str. line)

$E\hat{G}J = 180° - \angle e - \angle f$

$C\hat{J}G = \angle g + \angle h$ (ext. ∠ of △JKL)

$C\hat{G}J + \angle g + \angle h = 180°$ (adj. ∠s on a str. line)

$C\hat{J}G = 180° - \angle g - \angle h$

Sum of interior angles of quadrilateral = $(4 - 2) \times 180° = 360°$

$\therefore J\hat{C}E + G\hat{E}C + E\hat{G}J + C\hat{J}G = 360°$

$(180° - \angle a - \angle b) + (180° - \angle c - \angle d) + (180° - \angle e - \angle f) + (180° - \angle g - \angle h) = 360°$

$-\angle a - \angle b - \angle c - \angle d - \angle e - \angle f - \angle g - \angle h$

$= 360° - 180° - 180° - 180° - 180°$

$-\angle a - \angle b - \angle c - \angle d - \angle e - \angle f - \angle g - \angle h = -360°$

Hence,

$\angle a + \angle b + \angle c + \angle d + \angle e + \angle f + \angle g + \angle h = 360°$

17. Sum of interior angles of a pentagon = 540°

Let the exterior angle of the pentagon be $x°$.

$5(180° - x°) = 540°$

$900° - 5x° = 540°$

$-5x° = 540° - 900°$

$-5x° = -360°$

$x° = \dfrac{360°}{5}$

$= 72°$

$\angle a + 72° + 72° = 180°$

$\angle a = 180° - 72° - 72°$

$= 36°$

Hence, $\angle a + \angle b + \angle c + \angle d + \angle e = 5 \times 36° = 180°$

18. $a_1 + x_1 = 180°$ (adj. $\angle$s on a str. line)

$a_2 + x_2 = 180°$ (adj. $\angle$s on a str. line)

$a_3 + x_3 = 180°$ (adj. $\angle$s on a str. line)

$a_4 + x_4 = 180°$ (adj. $\angle$s on a str. line)

$a_n + x_n = 180°$ (adj. $\angle$s on a str. line)

Hence,

$a_1 + x_1 + a_2 + x_2 + a_3 + x_3 + a_4 + a_4 + \cdots + a_n + x_n = n \times 180°$

$\underbrace{a_1 + a_2 + a_3 + a_4 + \cdots + a_n}_{} + \underbrace{x_1 + x_2 + x_3 + x_4 + \cdots + x_n}_{} = n \times 180°$

$(n-2) \times 180° + x_1 + x_2 + x_3 + x_4 + \cdots + x_n = n \times 180°$

$x_1 + x_2 + x_3 + x_4 + \cdots + x_n = n \times 180° - (n-2) + 360°$

$x_1 + x_2 + x_3 + x_4 + \cdots + x_n = 180°n - 180°n + 360°$

$\therefore x_1 + x_2 + x_3 + x_4 + \cdots + x_n = 360°$

19. **(i)** Two regular polygons are equilateral triangles and squares.

(ii) The interior angles of the polygons meeting at a vertex must add to 360°.

(iii)

Shape	Interior Angle in degrees
Triangle	60
Square	90
Pentagon	108
Hexagon	120
More than six sides	More than 120 degrees

Since the interior angles of the polygon meeting at a vertex must add to 360°, hence the interior angle must be an exact divisor of 360°. This will work only for triangles, squares and hexagons as the interior angle are all divisor of 360°.

(iv) The reason is that the hexagon has the smallest perimeter for a given area as compared to the square and the triangle. This will allow the bees to make more honey using less wax and less work.

Exercise 10D

1. Given:

In a circle with centre O, $\overline{PQ}$ is a chord and R is the mid–point of $\overline{PQ}$.

$\overline{PR} = \overline{RQ}$, $\overline{OR}$ is perpendicular to $\overline{PQ}$.

$m \angle ROP = 42°$

In Δ OPR

$m \angle ORP + m \angle POR + m \angle ORP = 180°$

$90° + 42° + m \angle OPR = 180°$

$132° + m \angle OPR = 180°$

$m \angle OPR = 180° - 132° = 48°$

2. Given:

$\overline{AB}$ is a chord of the circle with centre O.

$\overline{OC} \perp \overline{AB}$

$m\overline{AC} = 4$ cm

We know that a perpendicular, from the centre f a circle to a chord, bisects the chord (Property 2)

$\therefore \quad m\overline{AC} = m\overline{CB} = 4$ cm

$m\overline{AC} = m\overline{AC} + m\overline{CB}$

$= 4 + 4$

$= 8$ cm

3. Given:

In the given circle with centre O, $\overline{AB}$ and $\overline{CD}$ are chords to the circle.

$\overline{OM} \perp \overline{AB}$, $\overline{ON} \perp \overline{CD}$

$m\overline{OM} = m\overline{ON} = 3$ cm

$m\overline{AM} = 3.2$ cm

Now, $m\overline{AM} = m\overline{BM} = 3.2$ cm

($\therefore$ OM AB, it bisects $\overline{AB}$)

$m\overline{AB} = m\overline{AM} + m\overline{MB}$

$= 3.2$ cm $+ 3.2$ cm $= 6.4$ cm

It is given that $m\overline{OM} = m\overline{ON}$

$\therefore \quad \overline{AB}$ and $\overline{CD}$ are equidistant from the circle.

Hence in $\overline{AB} = m\overline{CD} = 6.4$ cm

4. Given:

$m \angle AOB = m \angle BOC = 60°$

$m\overline{AB} = 4$ cm

Now,

$m\overline{OA} = m\overline{OB} = m\overline{OC}$ (radii of the circle)

Since the angles subtended by $\overline{AB}$ and $\overline{BC}$ at the centre are equal, then the chords are equal.

$\therefore \quad m\overline{AB} = m\overline{BC} = 4$ cm

Δ AOB is an isosceles triangle with vertex angle = 60°

$\therefore \quad \angle OAB = \angle OBA$

$2 \angle OAB = 180° - 60° = 120°$

$\angle OAB = 60°$

$\therefore \quad \Delta$ OAB is an equilateral triangle

Thus $m\overline{AB} = m\overline{OA} = 4$ cm

$\therefore \quad m\overline{OC} = 4$ cm

$\angle ABC = \angle ABO + \angle OBC = 60° + 60° = 120°$

5. $P\,(1,3) \longrightarrow P'\,(1+3, 3-2) = P'\,(4, 1)$

$Q\,(7,5) \longrightarrow Q'\,(7+3, 5-2) = Q'\,(10, 3)$

$R\,(2,0) \longrightarrow R'\,(2+3, 0-2) = R'\,(5, -2)$

7. $A\,(51\ -1) \longrightarrow A'\,(2, 3)$

$T_1 \longrightarrow$ Transformation is 3 units left and 4 unit up .

B (-2,5) $\longrightarrow$ B′ (4,-5)

T_2 Transformation is 3 units right and 10 units up.

(a) P (7,6) $\longrightarrow$ P′ (7-3 , 6+3) = P′ (4,9)

(b) P (7,6) $\longrightarrow$ P′ (7+6 , 6-10) = P′ (13 , -4)

(c) $P(7,6) \longrightarrow P'(4, 9) \longrightarrow P''(4 + 6, 9 - 10)$
$= P''(10, -1)$

(d) $P(7,6) \longrightarrow P'(13, 4) \longrightarrow P''(13 - 3, -4 + 3)$
$= P''(10, -1)$

Review Exercise 10

1. **(a)** Since $AB = AC$, $\therefore A\hat{C}B = A\hat{B}C = 3a°$.

$3a° + 2a° + 3a° = 180°$ ($\angle$ sum of $\triangle ABC$)
$8a° = 180°$
$a° = \frac{180°}{8}$
$= 22.5°$
$\therefore a = 22.5$

(b) Since $DA = DB$, $\therefore D\hat{B}A = D\hat{A}B = 32°$.

$32° + A\hat{D}B + 32° = 180°$ ($\angle$ sum of $\triangle ABD$)
$A\hat{D}B = 180° - 32° - 32°$
$= 116°$
$116° + b° = 360°$ ($\angle$s at a point)
$b° = 360° - 116°$
$= 244°$
$\therefore b = 244$

Since $CA = CB$, $\therefore C\hat{A}B = C\hat{B}A = x°$.

$x° + 64° + x° = 180°$ ($\angle$ sum of $\triangle ABC$)
$2x° = 180° - 64°$
$= 116°$
$x° = \frac{116°}{2}$
$= 58°$
$c° + 32° = 58°$
$c° = 58° - 32°$
$= 26°$
$\therefore c = 26$

2. **(a)** Since $BA = BD$, $\therefore B\hat{D}A = B\hat{A}D = a°$.

$a° + 40° + a° = 180°$ ($\angle$ sum of $\triangle ABD$)
$2a° = 180° - 40°$
$= 140°$
$a° = \frac{140°}{2}$
$= 70°$
$\therefore a = 70$
$C\hat{B}D + 40° = 180°$ (adj. $\angle$ s on a str. line)
$C\hat{B}D = 180° - 40°$
$= 140°$

Since $BC = BD$, $\therefore B\hat{C}D = B\hat{D}C = b°$.

$b° + 140° + b° = 180°$ ($\angle$ sum of $\triangle BCD$)
$2b° = 180° - 140°$
$= 40°$
$b° = \frac{40°}{2}$
$= 20°$
$\therefore b = 20$

(b) Since $BA = BD$, $\therefore B\hat{D}A = B\hat{A}D = c°$.

$c° + c° = 78°$ (ext. $\angle$ of $\triangle ABD$)
$2c° = 78°$
$c° = \frac{78°}{2}$
$= 39°$
$\therefore c = 39$

Since $DA = DC$, $\therefore D\hat{C}A = D\hat{A}C = 39°$.

$39° + A\hat{B}C + 39° = 180°$ ($\angle$ sum of $\triangle ACD$)
$A\hat{D}C = 180° - 39° - 39°$
$= 102°$
$39° + d° = 102°$
$d° = 102° - 39°$
$= 63°$
$\therefore d = 63$

(c) $e° + 62° + 52° = 180°$ ($\angle$ sum of $\triangle BCD$)
$e° = 180° - 62° - 52°$
$= 66°$
$\therefore e = 66$
$48° + f° + 66° = 180°$ (adj. $\angle$s on a str. line)
$f° = 180° - 48° - 66°$
$= 66°$
$\therefore f = 66$

(d) $110° + D\hat{B}C = 180°$ (adj. $\angle$s on a str. line)
$D\hat{B}C = 180° - 110°$
$= 70°$

Since $DB = DC$, $\therefore D\hat{C}B = D\hat{B}C = 70°$.

Hence, $g° = 70°$
$\therefore g = 70$
$70° + h° = 110°$ (ext. $\angle$ of $\triangle BCD$)
$h° = 110° - 70°$
$= 40°$
$\therefore h = 40$

(e) Since $DB = DC$, $\therefore D\hat{B}C = D\hat{C}B = 3i°$.

$(5i + 4)° + 3i° = 180°$ (adj. $\angle$s on a str. line)
$8i° = 180° - 4°$
$= 176°$
$i° = \frac{176°}{8}$
$= 22°$
$\therefore i = 22$
$3(22°) + 2j° = [5(22) + 4]°$ (ext. $\angle$ of $\triangle BCD$)
$2j° = 114° - 66°$
$= 48°$
$j° = \frac{48°}{2}$
$= 24°$
$\therefore j = 24$

(f) $k° + 78° = 3k°$ (ext. $\angle$ of $\triangle ABD$)

$3k° - k° = 78°$

$2k° = 78°$

$k° = \frac{78°}{2}$

$= 39°$

$\therefore k = 39$

$39° + l° + 78° = 180°$ ($\angle$ sum of $\triangle ABD$)

$l° = 180° - 39° - 78°$

$= 63°$

$\therefore l = 63$

3. **(a)** Since $AB = AC$, $A\hat{C}B = A\hat{B}C = a°$.

$a° + B\hat{A}C + a° = 180°$ ($\angle$ sum of $\triangle ABC$)

$B\hat{A}C = 180° - 2a°$

$D\hat{C}A = 180° - 2a°$ (alt. $\angle$s, $AB \parallel DC$)

Since $AC = AD = CD$, $D\hat{C}A = C\hat{D}A = C\hat{A}D = 60°$

$180° - 2a° = 60°$

$-2a° = 60° - 180°$

$= -120°$

$a° = \frac{-120°}{-2}$

$= 60°$

$\therefore a = 60$

(b) $b° + b° + 76° = 180°$ (int. $\angle$s, $AB \parallel DC$)

$2b° = 180° - 76°$

$= 104°$

$b° = \frac{104°}{2}$

$= 52°$

$\therefore b = 52$

$c° + c° + 118° = 180°$ (int. $\angle$s, $AB \parallel DC$)

$2c° = 180° - 118°$

$= 62°$

$c° = \frac{62°}{2}$

$= 31°$

$\therefore c = 31$

$52° + 31° + d° = 180°$ ($\angle$ sum of $\triangle ABE$)

$d° = 180° - 52° - 31°$

$= 97°$

$\therefore d = 97$

(c) Since $EA = EB$, $E\hat{A}B = E\hat{B}A = 58°$.

$58° + e° = 180°$ (int. $\angle$s, $AB \parallel DC$)

$e° = 180° - 58°$

$= 122°$

$\therefore e = 122$

$f° = 58°$ (corr. $\angle$s, $AB \parallel DC$)

$\therefore f = 58$

Since $ED = EC$, $E\hat{D}C = E\hat{C}D = 58°$.

$58° + g° + 58° = 180°$ ($\angle$ sum of $\triangle CDE$).

$g° = 180° - 58° - 58°$

$= 64°$

$\therefore g = 64$

4. **(a)** $112° + A\hat{B}C = 180°$ (adj. $\angle$s on a str. line)

$A\hat{B}C = 180° - 112°$

$= 68°$

$62° + H\hat{E}D = 180°$ (adj. $\angle$s on a str. line)

$H\hat{E}D = 180° - 62°$

$= 118°$

$a° + B\hat{C}D = 180°$ (adj. $\angle$s on a str. line)

$B\hat{C}D = 180° - a°$

Sum of the interior angles of a pentagon $= (5 - 2) \times 180° = 540°$.

$\therefore 114° + 68° + 180° - a° + 95° + 118° = 540°$

$-a° = 540° - 114° - 68° - 180° - 95° - 118°$

$= -35°$

$\therefore a = 35$

(b) Sum of exterior angles of a hexagon = 360°

$\therefore 2b° + 4b° + 3b° + b° + b° + b° = 360°$

$12b° = 360°$

$b° = \frac{360°}{12}$

$= 30°$

$\therefore b = 30$

$c° + 3(30°) = 180°$ (adj. $\angle$s on a str. line)

$c° = 180° - 90°$

$= 90°$

$\therefore c = 90$

5. **(i)** $A\hat{C}D = 40°$ (alt. $\angle$s, $AB \parallel DC$)

(ii) $C\hat{A}D + 108° + 40° = 180°$ (int. $\angle$s, $AD \parallel BC$)

$C\hat{A}D = 180° - 108° - 40°$

$= 32°$

6. **(i)** Since $AB = AD$, $\therefore A\hat{D}B = A\hat{B}D = 62°$.

$62° + B\hat{A}D + 62° = 180°$ ($\angle$ sum of $\triangle ABD$)

$B\hat{A}D = 180° - 62° - 62°$

$= 56°$

(ii) Since $CB = CD$, $\therefore B\hat{D}C = D\hat{B}C = x°$.

$x° + 118° + x° = 180°$ ($\angle$ sum of $\triangle BCD$)

$2x° = 180° - 118°$

$= 62°$

$x° = \frac{62°}{2}$

$= 31°$

$\therefore B\hat{D}C = 31°$

7. Since $\triangle ABE$ is an equilateral triangle, $AB = AE = BE$ and $E\hat{A}B = E\hat{B}A = A\hat{E}B = 60°$.

$D\hat{A}E + 60° = 90°$ (right angle of a square)

$D\hat{A}E = 90° - 60°$

$= 30°$

Since $AD = AB$, $\therefore AE = AD$ and $A\hat{E}D = A\hat{D}E = x°$.

$x° + 30° + x° = 180°$ ($\angle$ sum of $\triangle ADE$)

$2x° = 180° - 30°$

$= 150°$

$x° = \frac{150°}{2}$

$= 75°$

$C\hat{B}E + 60° = 90°$ (right angle of a square)

$C\hat{B}E = 90° - 60°$

$= 30°$

Since $BC = AB$, $\therefore$ $BE = BC$ and $B\hat{E}C = B\hat{C}E = y°$.

$y° + 30° + y° = 180°$ ($\angle$ sum of $\triangle BEC$)

$2y° = 180° - 30°$

$= 150°$

$y° = \dfrac{150°}{2}$

$= 75°$

$75° + 60° + 75° + C\hat{E}D = 360°$ ($\angle$s at a point)

$C\hat{E}D = 360° - 75° - 60° - 75°$

$= 150°$

8. Sum of interior angles of a $(2n - 3)$-sided polygon

$= [(2n - 3) - 2] \times 180°$

Hence,

$[(2n - 3) - 2] \times 180° = 62 \times 90°$

$(2n - 5) \times 180° = 5580°$

$360°n - 900° = 5580°$

$360°n = 5580° + 900°$

$360°n = 6480°$

$n = \dfrac{6480°}{360°}$

$= 18$

9. Sum of interior angles of a n-sided polygon

$= (n - 2) \times 180°$

$126° + (n - 1) \times 162° = (n - 2) \times 180°$

$126° + 162°n - 162° = 180°n - 360°$

$180°n - 162°n = 360° + 126° - 162°$

$18°n = 324°$

$n = \dfrac{324°}{18°}$

$= 18$

10. Sum of interior angles of a pentagon

$= (5 - 2) \times 180°$

$= 3 \times 180°$

$= 540°$

Let the 5 interior angles be $3x°$, $4x°$, $5x°$, $5x°$ and $7x°$.

$3x° + 4x° + 5x° + 5x° + 7x° = 540°$

$24x° = 540°$

$x° = \dfrac{540\Upsilon}{24}$

$= 22.5°$

(i) Largest interior angle $= 7 \times 22.5°$

$= 157.5°$

(ii) Largest exterior angle $= 180° - 3 \times 22.5°$

$= 112.5°$

11. Sum of exterior angles of a n-sided polygon $= 360°$

$35° + 72° + (n - 2) \times 23° = 360°$

$23°n = 360° - 35° - 72° + 46°$

$= 299°$

$n = \dfrac{299°}{23°}$

$= 13$

12. Let the interior angle be $13x°$ and the exterior angle be $2x°$.

$13x° + 2x° = 180°$ (adj. $\angle$s on a str. line)

$15x° = 180°$

$x° = \dfrac{180\Upsilon}{15}$

$= 12°$

Sum of exterior angles of a n-sided polygon $= 360°$

Hence,

$n = \dfrac{360°}{2(12°)}$

$= 15$

13. Sum of the interior angles of a n-sided polygon $= (n - 2) \times 180°$

Sum of the exterior angles of a n-sided polygon $= 360°$

$(n - 2) \times 180° = 4 \times 360°$

$180°n = 1440° + 360°$

$= 1800°$

$n = \dfrac{1800°}{180°}$

$= 10$

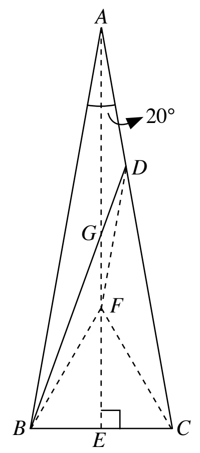
14.

Challenge Yourself

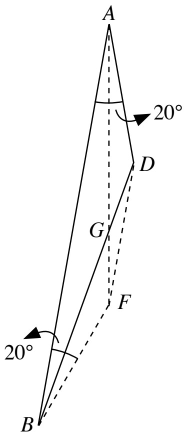
1. $A\hat{B}C = \dfrac{180° - 20°}{2}$ (base $\angle$s of isos. $\triangle ABC$)

$= \dfrac{160°}{2}$

$= 80°$



Draw E on BC such that $AE \perp BC$.
Draw F on AE such that $\triangle BCF$ is an equilateral triangle.
Then $A\hat{B}F = 80° - 60° = 20°$ and $BF = BC = AD$.
Consider the quadrilateral $ABFD$.



Since $A\hat{B}F = B\hat{A}D = B\hat{A}C = 20°$ and $BF = AD$,
then by symmetry, AB // DF and $ABFD$ is an isosceles trapezium.
In the isosceles trapezium $ABFD$, by symmetry, $AG = BG$, so $\triangle ABG$ is an isosceles triangle.

Since $B\hat{A}G = B\hat{A}E = \dfrac{20°}{2} = 10°$ (AE bisects $B\hat{A}C$),
then $A\hat{B}G = B\hat{A}G = 10°$ (base $\angle$s of isos. $\triangle ABG$).

$$\begin{aligned}
\therefore A\hat{D}B + A\hat{B}D + B\hat{A}D &= 180° \quad (\angle \text{ sum of } \triangle ABD)\\
A\hat{D}B + A\hat{B}G + 20° &= 180°\\
A\hat{D}B + 10° + 20° &= 180°\\
A\hat{D}B &= 180° - 10° - 20°\\
&= 150°
\end{aligned}$$

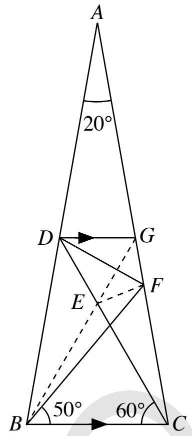
Teachers may wish to note the usefulness of the symmetric properties of an isosceles trapezium. Otherwise, formal proofs using congruent triangles are beyond the scope of Secondary 1 syllabus.

2. $$\begin{aligned}
A\hat{C}B &= \frac{180° - 20°}{2} \quad (\text{base } \angle\text{s of isos. } \triangle ABC)\\
&= \frac{160°}{2}\\
&= 80°
\end{aligned}$$

$$\begin{aligned}
\therefore D\hat{C}F &= D\hat{C}A\\
&= A\hat{C}B - 60°\\
&= 80° - 60°\\
&= 20°
\end{aligned}$$

$$\begin{aligned}
B\hat{F}C + F\hat{C}B + 50° &= 180° \quad (\angle \text{ sum of } \triangle BCF)\\
B\hat{F}C + A\hat{C}B + 50° &= 180°\\
B\hat{F}C + 80° + 50° &= 180°\\
B\hat{F}C &= 180° - 80° - 50°\\
&= 50°
\end{aligned}$$

Since $C\hat{B}F = B\hat{F}C = 50°$, i.e. $CB = CF$,
then $\triangle BCF$ is an isosceles triangle.



Draw G on AG such that DG // BC.
Draw BG to cut CD at E.
Draw EF.
By symmetry, $BE = CE$, so $\triangle BCE$ is an isosceles triangle.
Since the base angle of $\triangle BCE$ is 60°,
then $\triangle BCE$ is an equilateral triangle,
i.e. $B\hat{E}C = 60°$ and $E\hat{B}F = 60° - 50° = 10°$.

$$\begin{aligned}
\therefore CE &= CB \quad (\text{sides of equilateral } \triangle BCE)\\
&= CF \quad (\text{sides of isosceles } \triangle BCF)
\end{aligned}$$

Since $CE = CF$,
then $\triangle CEF$ is an isosceles triangle.

$$\begin{aligned}
C\hat{F}E &= \frac{180° - E\hat{C}F}{2} \quad (\text{base } \angle\text{s of isos. } \triangle CEF)\\
&= \frac{180° - D\hat{C}F}{2}\\
&= \frac{180° - 20°}{2}\\
&= \frac{160°}{2}\\
&= 80°
\end{aligned}$$

$$\begin{aligned}
\therefore B\hat{F}E &= C\hat{F}E - B\hat{F}C\\
&= 80° - 50°\\
&= 30°
\end{aligned}$$

$$\begin{aligned}
F\hat{E}G &= E\hat{B}F + B\hat{F}E \quad (\text{ext. } \angle \text{ of } \triangle BEF)\\
&= 10° + 30°\\
&= 40°
\end{aligned}$$

$$\begin{aligned}
D\hat{E}G &= B\hat{E}C \quad (\text{vert. opp. } \angle\text{s})\\
&= 60°
\end{aligned}$$

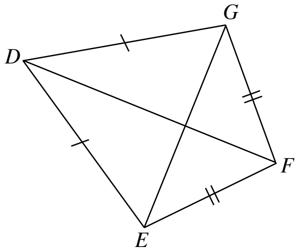
$$\begin{aligned}
D\hat{G}E &= C\hat{B}E \quad (\text{alt. } \angle\text{s, } DG \text{ // } BC)\\
&= 60°
\end{aligned}$$

Since the base angle of $\triangle DEG$ is 60°,
then $\triangle DEG$ is an equilateral triangle,
i.e. $E\hat{D}G = 60°$ and $DE = DG$.

$$\begin{aligned}
A\hat{G}D &= A\hat{C}B \quad (\text{corr. } \angle\text{s, } DG \text{ // } BC)\\
&= 80°
\end{aligned}$$

$$\begin{aligned}
\therefore F\hat{G}E + D\hat{G}E + A\hat{G}D &= 180° \quad (\text{adj. } \angle\text{s on a str. line})\\
F\hat{G}E + 60° + 80° &= 180°\\
F\hat{G}E &= 180° - 60° - 80°\\
&= 40°
\end{aligned}$$

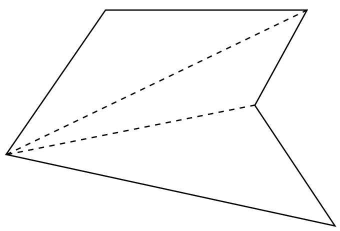
Since $F\hat{E}G = F\hat{G}E = 40°$,
then $\triangle EFG$ is an isosceles triangle,
i.e. $FE = FG$.
Consider the quadrilateral $DEFG$.



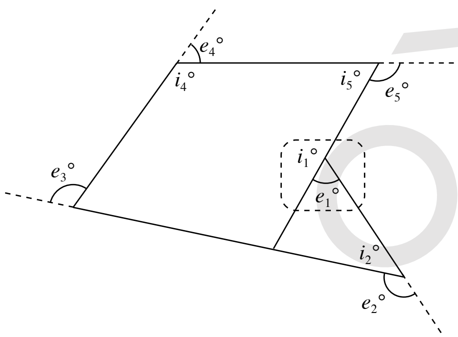
Since $DE = DG$ and $FE = FG$,
then $DEFG$ is a kite.
In the kite $DEFG$, the longer diagonal DF bisects $E\hat{D}G$.

$$\therefore C\hat{D}F = E\hat{D}F = \frac{60°}{2} = 30°$$

3. Yes. For any n-sided concave polygon, it can still form $(n - 2)$ triangles in the polygon.
Hence the sum of the interior angles is still the same.
E.g.



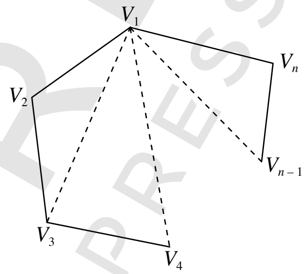
4. **(i)** An exterior angle of a concave polygon has a negative measure and is inside the polygon as shown in the diagram below.
E.g.



(ii) Yes. Exterior angle of the vertex which is "pushed in" will flip over into the inside of the polygon and becomes negative. Adding all the exterior angles as before, they will still add to 360°.

E.g. $i_1° + (-e_1°) + i_2° + e_2° + i_3° + e_3° + i_4° + e_4° + i_5° + e_5° +$
$= 5 \times 180°$
$[i_1° + i_2° + i_3° + i_4° + i_5°] + (-e_1°) + e_2° + e_3° + e_4° + e_5°$
$= 900°$
$(-e_1°) + e_2° + e_3° + e_4° + e_5°$
$= 900 - [i_1° + i_2° + i_3° + i_4° + i_5°]$
$(-e_1°) + e_2° + e_3° + e_4° + e_5° = 900° - (5 - 2) \times 180°$
$(-e_1°) + e_2° + e_3° + e_4° + e_5° = 900° - 540°$
$(-e_1°) + e_2° + e_3° + e_4° + e_5° = 360°$
The above proof holds for any n-sided polygon.

5. In a n-sided polygon, each diagonal connects one vertex to another vertex which is not its next-door neighbour. Since there are n vertices in an n-sided polygon, therefore there are n starting points for the diagonals. For each diagonal, it (e.g. V_1) can join to other $(n - 3)$ vertices since it cannot join itself (V_1) or either of the two neighbouring vertices (V_2 and V_n). So the total number of diagonals formed is $n \times (n - 3)$. However, in this way, each diagonal would be formed twice (to and from each vertex), so the product $n(n - 3)$ must be divided by 2. Hence the formula is $\dfrac{n(n-3)}{2}$.
E.g.



Chapter 11 Symmetry

TEACHING NOTES

Suggested Approach:

Many buildings and objects in our surroundings are symmetrical in shape. Teachers can make use of these real life examples to allow students to appreciate the significance of symmetry in the way things are designed (see Chapter Opener on Page 355). Teachers can also highlight that this is not the first contact students have with symmetry, as most animals and insects, and even our faces, are symmetrical in shape. Students can be encouraged to think about symmetry around them and if there are any animals that are asymmetrical.

Section 11.1: Symmetry in Triangles, Quadrilaterals and Polygons

Students can visualise the symmetrical properties of triangles, quadrilaterals and polygons through hands-on interaction by making their own paper cut-outs (see Investigation on Pages 196 to 201).

WORKED SOLUTIONS

Investigation (Symmetry in Triangles)

2. The triangle has one line of symmetry, shown by line *AD*.

3. Yes. Since *ABC* is an isosceles triangle with line of symmetry *AD*, the two triangles *ABD* and *ACD* are congruent, and thus the sides *AB* and *AC* and angles *B* and *C* can be deduced to be equal.

4. Yes. The order of rotational symmetry is 1 as it needs to be rotated one full round for the shape to be the same.

5. An equilateral triangle has 3 lines of symmetry and an order of rotational symmetry of order 3. A scalene triangle has no line of symmetry and rotational symmetry of order 1.

Investigation (Symmetry in Special Quadrilaterals)

(a) *The answers are provided in Table 11.2 on Page 199 and 200 of the textbook.*

(b) *The answers are provided in Table 11.2 on Page 199 and 200 of the textbook.*

(c) The angles directly across each other from the line of symmetry are equal.

Investigation (Symmetry in Regular Polygons)

3.

Types of quadrilateral	Number of lines of symmetry	Order of rotational symmetry
Equilateral triangle	3	$\frac{360^\circ}{120^\circ} = 3$
Square	4	$\frac{360^\circ}{90^\circ} = 4$
Regular pentagon	5	$\frac{360^\circ}{72^\circ} = 5$
Regular hexagon	6	$\frac{360^\circ}{60^\circ} = 6$
Regular heptagon	7	$\frac{360^\circ}{51\frac{3}{7}^\circ} = 7$
Regular octagon	8	$\frac{360^\circ}{45^\circ} = 8$
Regular nonagon	9	$\frac{360^\circ}{40^\circ} = 9$
Regular decagon	10	$\frac{360^\circ}{36^\circ} = 10$

Table 11.2

4. **(a)** Number of lines of symmetry = n

(b) Order of rotational symmetry = n

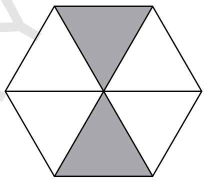
5. Smallest angle of rotational symmetry = $\frac{360^\circ}{n}$

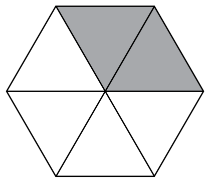
Exercise 11A

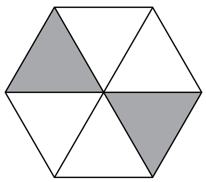
1. **(a)** True
(b) False
(c) True
(d) True
(e) True
(f) True
(g) False
(h) True
(i) False
(j) False

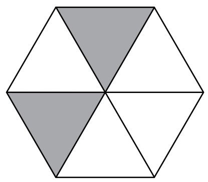
2.

	(i) Lines of symmetry	(ii) Order of rotational symmetry
(a)	2	2
(b)	0	4
(c)	8	8
(d)	3	3
(e)	1	1
(f)	0	2
(g)	2	4

3. **(a)**


(b)


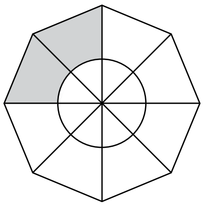
(c)


(d)


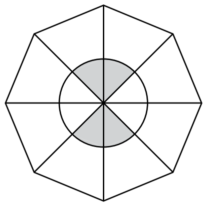
Teachers should note that there are other ways to shade the triangles, and these are not the only answers. However, students should bear in mind that the question states that "exactly two triangles" are to be shaded.

Review Exercise

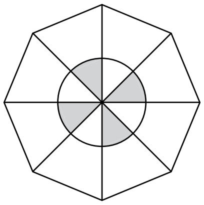
1. **(a)**



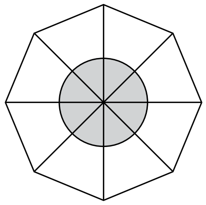
(b)



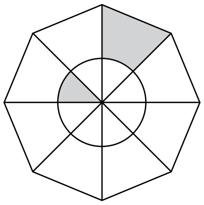
(c)



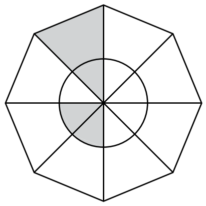
(d)



(e)



(f)



Teachers should note that there are other ways to shade the figure.

Challenge Yourself

1. **(a)** 1
   (b) 2
   (c) 1
   (d) 2
   (e) 2
   (f) 1
   (g) 2
   (h) 2

Chapter 12 Mensuration: Perimeter, Area, Surface Area, and Volume

TEACHING NOTES

Suggested Approach

Students have learnt the conversion of unit area and perimeter and area of plane figures. This chapter will be dealing with the area and perimeter of circle and conversion of unit volumes and the volume and surface area of solids, which is a natural transition from two-dimensional to three-dimensional. To assist in the students' understanding, teachers should continually remind students to be aware of the linkages between both topics, as well as introducing real-life applications that can reinforce learning.

Section 12.1: Area and Perimeter

Teachers can impress upon the students that the value of in calculators are used when the value is not stated in the question. Unless specified, all answers that are not exact shoved be rounded off to 3 significant figures.

Section 12.2: Volume and Surface Area of Prisms

Teachers should recap the unit conversion of lengths and areas, proceed to introduce of volume by stating actual applications (see Class Discussion: Measurement in Daily Lives), and then stating the different units associated with volume (e.g. m, cm^3 and m^3).

Students should recognise how the number of dimensions and the unit representation for lengths, areas and volumes are related (e.g. cm, cm^2 and cm^3). Students should recall calculations such as 1 cm^3 = 1 cm × 1 cm × 1 cm and solve problems involving conversion of unit volumes.

Teachers can introduce prisms to the students by stacking a few cubes to form a prism and show them how a prism looks like. Students should know terms like lateral faces and cross-sections, and learn that prisms are solids with uniform polygonal cross-sections. Teachers can ask the students to name some real-life examples of prisms and use this opportunity to get them to explain why certain objects are not prisms so that they can get a better understanding about prisms.

Observant students should realise that cuboids are prisms. Teachers can highlight to the students that prisms do not necessarily have square bases and challenge students to think of bases of other possible shapes (see Fig. 12.2 on page 210).

Teachers should illustrate and derive the formulas for the volume and total surface area. Students need to understand the definitions of volume and total surface area rather than memorise the formulas.

Section 12.3: Volume and Surface Area of Cylinders

Similar to the last section, teachers can introduce cylinders by stacking coins or showing students real-life examples of cylindrical objects. Only right circular cylinders are covered in this syllabus.

Some students may think that cylinders are also prisms since both have uniform cross-sections. Teachers need to impress upon students that this is not the case even though cylinders and prisms share similarities (see Investigation: Comparison between a Cylinder and a Prism).

Teachers should also cover the formulas for the volume and total surface area of cylinders. Again, students need to understand the definitions of volume and total surface area rather than memorise formulas.

WORKED SOLUTIONS

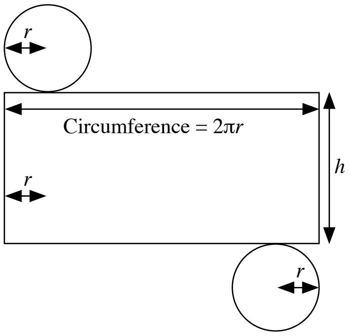
Thinking Time (Page 216)

Examples are can drinks, toilet rolls and iron rods. They are shaped as cylinders as they have a uniform circular cross-section.

Investigation (Comparison between a Cylinder and a Prism)

1. The polygon will become a circle.
2. The prism will become like a cylinder.

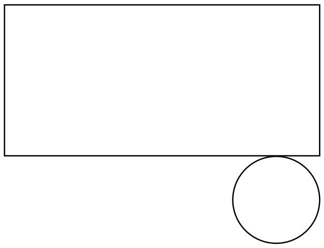
Thinking Time (Page 219)



Total outer surface of a closed cylinder
$= \pi r^2 + 2\pi rh + \pi r^2$
$= 2\pi r^2 + 2\pi rh$

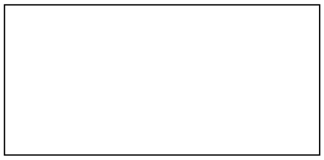
Class Discussion (Total Surface Area of Other Types of Cylinders)

(a) an open cylinder



Total outer surface of an open cylinder $= 2\pi rh + \pi r^2$

(b) a pipe of negligible thinkness



Total outer surface of a pipe of negligible thickness $= 2\pi rh$

Practise Now 1

radius = 5.5 cm
area of the circle $= \pi r^2$
$= 3.142 \times 5.5^2\,\text{cm}^2$
$= 95.05\,\text{cm}^2$

Circumference of the circle $= 2\pi r$
$= 2 \times 3.142 \times 5.5$ cm
$= 34.\ 56$ cm

Practise Now 2

(i) Perimeter of unshaded area $= \frac{3}{4} \times 2\pi(14) + 2(14)$
$= 21\pi + 28$
$= 94.0$ cm. (to 31 s.f)

(ii) Area of unshaded region $= \frac{3}{4}\pi(14)^2$
$= 147\pi$
$= 462\,\text{cm}^2$ (to 3 s.f)

(iii) Area of shaded region = area of square - area of unshadedregion
$= (2 \times 14)^2 - 147\pi$
$= 28^2 - 147\pi$
$= 784 - 147\pi$
$= 322\,\text{cm}^2$ (1 to 3 s.f)

Practise Now 3

1. Total area of shaded region

= area of parallelogran *ABJK* + area of parallelogran *CBIJ* + area of parallelogran *DEGH*

$= 4 \times 12 + (2 \times 4) \times 12 + 4 \times 12$

$= 48 + 8 \times 12 + 48$

$= 48 + 96 + 48$

$= 192\,\text{m}^2$

Practise Now 4

Area of trapezium $= \frac{1}{2} \times$ (Sum of paralell sides) $\times$ height

$= \frac{1}{2} \times (16 + 48) \times 20\,\text{m}^2$

$= 64 \times 10\,\text{m}^2$

$= 640\,\text{m}^2$

Practise Now 5

1. Base area = area of square
$= 4 \times 4$
$= 16\,\text{m}^2$
Volume of the prism = base area × height
$= 16 \times 10$
$= 160\,\text{m}^3$

2. Base area = area of triangle
$= \frac{1}{2} \times 5.6 \times x$
$= 2.8x\,\text{cm}^2$
Volume of the prism = base area × height $= 2.8x \times 12 = 151.2$
$33.6x = 151.2$
$x = 4.5$

Practise Now 6

(i) Volume of the prism = base area × height

$$= \frac{1}{2} \times 3 \times 4 \quad + (6 \times 5) \quad \times 4.5$$

$= 36 \times 4.5$

$= 162 \text{ cm}^3$

(ii) Total surface area of the prism

= perimeter of the base × height + 2 × base area

$= (3 + 4 + 6 + 5 + 6) \times 4.5 + 2 \times 36$

$= 180 \text{ cm}^2$

Practise Now 7

1. Base radius = 18 ÷ 2 = 9 cm

Height of the cylinder = 2.5 × 9 = 22.5 cm

Volume of the cylinder $= \pi r^2 h$

$= \pi(9)^2(22.5)$

$= 5730 \text{ cm}^3$ (to 3 s.f.)

2. Base radius = 12 ÷ 2 = 6 cm

Volume of the cylinder $= \pi(6)^2 h = 1000$

$$h = \frac{1000}{\pi(6)^2}$$

$h = 8.84$ cm (to 3 s.f.)

Practise Now 8

1. (i) Total surface area of the can

$= 2\pi r^2 + 2\pi rh$

$= 2\pi(3.5)^2 + 2\pi(3.5)(10)$

$= 24.5\pi + 70\pi$

$= 94.5\pi$

$= 297 \text{ cm}^2$ (to 3 s.f.)

(ii) Area of the can that is painted

$= \pi r^2 + 2\pi rh$ (An open cylinder has only one

$= \pi(3.5)^2 + 2\pi(3.5)(10)$ base and a curved surface)

$= 12.25\pi + 70\pi$

$= 82.25\pi$

Ratio of the area of the can that is painted, to the total surface area found in **(i)**.

$= 94.5\pi : 82.25\pi$

$= 94.5 : 82.25$

$= 54 : 47$

2. (i) Area of the cross section of the pipe

$= \pi(2.5)^2 - \pi(2.1)^2$

$= 6.25\pi - 4.41\pi$

$= 1.84\pi \text{ cm}^2$

(ii) Internal curved surface area of the pipe

$= 2\pi(2.1)(12)$

$= 50.4\pi$

$= 158 \text{ cm}^2$ (to 3 s.f.)

(iii) Total surface area of the pipe

$= 2(1.84\pi) + 50.4\pi + 2\pi(2.5)(12)$

$= 3.68\pi + 50.4\pi + 60\pi$

$= 114.08\pi$

$= 358 \text{ cm}^2$ (to 3 s.f.)

Exercise 12A

1. (a) volume of cuboid $= l \times b \times h$

$= 120 \times 59 \times 46 \text{ cm}^3$

$= 325680 \text{ cm}^3$

(b) volume of cube $= l^3$

$= 18^3 \text{ cm}^3$

$= 5832 \text{ cm}^3$

(c) volume of cuboid $= 8 \times 20 \times 14 \text{ cm}^3$

$= 2240 \text{ cm}^3$

(d) volume of cuboid $= 62 \times 20 \times 14 \text{ cm}^3$

$= 53816 \text{ cm}^3$

(e) voulme of cuboid $= 12 \times 3 \times 1 \text{ cm}^3$

$= 36 \text{ cm}^3$

2.

	AB	***BC***	***BC***	**Area of $\triangle ABC$**	**Volume of prism**
(a)	3 cm	4 cm	7 cm	6 cm^2	42 cm^3
(b)	9 cm	14 cm	11 cm	63 cm^2	693 cm^3
(c)	32 cm	15 cm	300 cm	240 cm^2	72 000 cm^3
(d)	24.6 cm	7.8 cm	400 cm	95.94 cm^2	38 376 cm^3

(a) Area of $\triangle ABC$

$= \frac{1}{2} \times 4 \times 3$

$= 6 \text{ cm}^2$

Volume of prism

$= 6 \times 7$

$= 42 \text{ cm}^3$

(b) Area of $\triangle ABC$

$= \frac{1}{2} \times BC \times 9 = 63$

$4.5BC = 63$

$BC = 14$ cm

Volume of prism

$= 63 \times 11$

$= 693 \text{ cm}^3$

(c) Volume of prism

= Area of $\triangle ABC \times 300 = 72\,000$

Area of $\triangle ABC = 240 \text{ cm}^2$

Area of $\triangle ABC$

$= \frac{1}{2} \times 15 \times AB = 240$

$7.5AB = 240$

$AB = 32$ cm

(d) Area of $\triangle ABC$

$= \frac{1}{2} \times 7.8 \times 24.6$

$= 95.94 \text{ cm}^2$

Volume of prism

$= 95.94 \times CD = 38\,376$

$CD = 400 \text{ cm}$

3. Air space in the hall = Volume of the prism

$= \text{base area} \times \text{height}$

$= \frac{1}{2} \times 42 \times (38 - 23) + 42 \times 23 \quad \times 80$

$= 1281 \times 80$

$= 102\,480 \text{ m}^3$

4. (a) (i) Volume of the prism = base area × height

$= \frac{1}{2} \times 6 \times 4 \quad \times 15$

$= 12 \times 15$

$= 180 \text{ cm}^3$

(ii) Total surface area of the prism

= perimeter of the base × height + 2 × base area

$= (5 + 5 + 6) \times 15 + 2 \times 12$

$= 264 \text{ cm}^2$

Exercise 12B

1. (a) (i) Volume of the closed cylinder

$= \pi r^2 h$

$= \pi(7)^2(12)$

$= 1850 \text{ cm}^3$ (to 3 s.f.)

(ii) Total surface area of the closed cylinder

$= 2\pi r^2 + 2\pi rh$

$= 2\pi(7)^2 + 2\pi(7)(12)$

$= 98\pi + 168\pi$

$= 266\pi$

$= 836 \text{ cm}^2$ (to 3 s.f.)

(b) Base radius = 1.2 ÷ 2 = 0.6 m

(i) Volume of the closed cylinder

$= \pi r^2 h$

$= \pi(0.6)^2(4)$

$= 4.52 \text{ m}^3$ (to 3 s.f.)

(ii) Total surface area of the closed cylinder

$= 2\pi r^2 + 2\pi rh$

$= 2\pi(0.6)^2 + 2\pi(0.6)(4)$

$= 0.72\pi + 4.8\pi$

$= 5.52\pi$

$= 17.3 \text{ m}^2$ (to 3 s.f.)

(c) (i) Volume of the closed cylinder

$= \pi r^2 h$

$= \pi(15)^2(63)$

$= 44\,500 \text{ mm}^3$ (to 3 s.f.)

(ii) Total surface area of the closed cylinder

$= 2\pi r^2 + 2\pi rh$

$= 2\pi(15)^2 + 2\pi(15)(63)$

$= 450\pi + 1890\pi$

$= 2340\pi$

$= 7350 \text{ mm}^2$ (to 3 s.f.)

2.

	Diameter	**Radius**	**Height**	**Volume**	**Total surface area**
(a)	8.00 cm	4.00 cm	14 cm	704 cm^3	453 cm^2
(b)	28.0 cm	14.0 cm	20 cm	12 320 cm^3	2990 cm^2
(c)	4 cm	2 cm	42.0 cm	528 cm^3	553 cm^2
(d)	8 m	4 m	21.0 m	1056 m^3	629 m^2

(a) Volume = 704 cm^3

$\pi r^2(14) = 704$

$r^2 = \frac{704}{14\pi}$

$r = \sqrt{\frac{704}{14\pi}}$

$= 4.00 \text{ cm}$ (to 3 s.f.)

$\therefore\ d = 2 \times 4.00 = 8.00 \text{ cm}$ (to 3 s.f.)

Total surface area

$= 2\pi r^2 + 2\pi rh$

$= 2\pi(4.001)^2 + 2\pi(4.001)(14)$

$= 453 \text{ cm}^2$

(b) Volume = 12 320 cm^3

$\pi r^2(20) = 12\,320$

$r^2 = \frac{12\,320}{20\pi}$

$r = \sqrt{\frac{12\,320}{20\pi}}$

$= 14.0 \text{ cm}$ (to 3 s.f.)

$\therefore\ d = 2 \times 14.0 = 28.0 \text{ cm}$ (to 3 s.f.)

Total surface area

$= 2\pi r^2 + 2\pi rh$

$= 2\pi(14.00)^2 + 2\pi(14.00)(20)$

$= 2990 \text{ cm}^2$ (to 3 s.f.)

(c) $r = 4 \div 2 = 2 \text{ cm}$

Volume = 528 cm^3

$\pi(2)^2 h = 528$

$h = \frac{528}{4\pi}$

$= 42.0 \text{ cm}$ (to 3 s.f.)

Total surface area

$= 2\pi r^2 + 2\pi rh$

$= 2\pi(2)^2 + 2\pi(2)(42.02)$

$= 553 \text{ cm}^2$ (to 3 s.f.)

(d) $d = 4 \times 2 = 8$ m

Volume = 1056 m^3

$\pi(4)^2 h = 1056$

$h = \dfrac{1056}{16\pi}$

$= 21.0$ m (to 3 s.f.)

Total surface area

$= 2\pi r^2 + 2\pi rh$

$= 2\pi(4)^2 + 2\pi(4)(21.01)$

$= 629$ m^2 (to 3 s.f.)

3. Base radius = $0.4 \div 2 = 0.2$ m

Height of the cylinder = $\dfrac{3}{4} \times 0.2 = 0.15$ m

Volume of the cylinder = $\pi r^2 h$

$= \pi(0.2)^2(0.15)$

$= 0.006\pi$ m^3

$= 6000\pi$ cm^3

$= \dfrac{6000\pi}{1000}$ l

$= 18.8$ l

4. Let the depth of water in the drum be d cm.

Base radius = $48 \div 2 = 24$ cm

150 l = $150\ 000$ ml = $150\ 000$ cm^3

Volume of water in the drum = $\pi r^2 d = 150\ 000$ cm^3

$\pi(24)^2 d = 150\ 000$

$d = \dfrac{150\ 000}{\pi(24)^2}$

$= 82.9$

$\therefore$ Depth of water = 82.9 cm

5. Base radius = $15 \div 2 = 7.5$ cm

Capacity of the drink trough

$= \dfrac{1}{2} \times \pi \times (7.5)^2 \times 84$

$= 7420$ cm^3 (to 3 s.f.)

$= 7.42$ l

6. 35 mm = $35 \div 10 = 3.5$ cm

Base radius = $3.5 \div 2 = 1.75$ cm

Total surface area that need to be painted for 1 wooden closed cylinder

$= 2\pi r^2 + 2\pi rh$

$= 2\pi(1.75)^2 + 2\pi(1.75)(7)$

$= 6.125\pi + 24.5\pi$

$= 30.625\pi$

Total surface area that need to be painted for 200 wooden closed cylinders

$= 200 \times 30.625\pi$

$= 19\ 200$ cm^2 (to 3 s.f.)

7. Base radius = $2.4 \div 2 = 1.2$ m

Volume of the tank = $\pi r^2 h$

$= \pi(1.2)^2(6.4)$

$= 9.216\pi$ m^3

$= 9\ 216\ 000\pi$ cm^3

Volume of the cylinder container = $\pi r^2 h$

$= \pi(8.2)^2(28)$

$= 1882.72\pi$ cm^3

Number of completed cylindrical container which can be filled by the oil in the tank

$= \dfrac{9\ 216\ 000\pi}{1882.72\pi}$

$= 4895$ (to the nearest whole number)

8. External base radius = $28 \div 2 = 14$ mm = 1.4 cm

Internal base radius = $20 \div 2 = 10$ mm = 1 cm

Volume of the metal used in making the pipe

$= \pi(1.4)^2(35) - \pi(1)^2(35)$

$= 68.6\pi - 35\pi$

$= 33.6\pi$

$= 106$ cm^3 (to 3 s.f.)

9. Base radius of the copper cylindrical rod = $14 \div 2 = 7$ cm

Volume of the copper cylindrical rod

$= \pi r^2 h$

$= \pi(7)^2(47)$

$= 2303\pi$

Let the length of the wire be l.

Base radius of the wire = $8 \div 2 = 4$ mm = 0.4 cm

Volume of the wire = $\pi(0.4)^2\, l = 2303\pi$

$(0.4)^2\, l = 2303$

$l = \dfrac{2303}{0.16}$

$= 14\ 400$ cm (to 3 s.f.)

$= 144$ m

10. Base radius = $2.4 \div 2 = 1.2$ cm

Since water is discharged through the pipe at a rate of 2.8 m/s, i.e. 280 cm/s, in 1 second, the volume of water discharged is the volume of water that fills the pipe to a length of 280 cm.

In 1 second, volume of water discharged

= volume of pipe of length 280 cm

$= \pi r^2 h$

$= \pi(1.2)^2(280)$

$= 403.2\pi$ cm^3

Half an hour = 30 minutes

In 30 minutes, volume of water discharged

$= 403.2\pi \times 30 \times 60$

$= 725\ 760\pi$ cm^3

$= 2\ 280\ 000$ cm^3 (to 3 s.f.)

$= 2280$ l

11. Base radius of the pipe = 64 ÷ 2 = 32 mm

Since water is discharged through the pipe at a rate of 2.05 mm/s, i.e. in 1 second, the volume of water discharged is the volume of water that fills the pipe to a length of 2.05 mm.

In 1 second, volume of water discharged

= volume of pipe of length 2.05 mm

$= \pi r^2 h$

$= \pi(32)^2(2.05)$

$= 2099.2\pi$ mm^3

Base radius of the cylindrical tank = 7.6 ÷ 2 = 3.8 cm = 38 mm

2.3 m = 230 cm = 2300 mm

Volume of the cylindrical tank

$= \pi r^2 h$

$= \pi(38)^2(2300)$

$= 3\ 321\ 200\pi$ cm^3

Time required to fill the tank

$= \frac{3\ 321\ 200\pi}{2099.2\pi}$

$= 1582\frac{83}{656}$ s

= 26 min (to the nearest minute)

12. **(i)** Volume of water in the tank = 18 × 16 × 13

$= 3744$ cm^3

(ii) Let the height of water in the cylindrical container be h.

Base radius of the cylindrical container = 17 ÷ 2 = 8.5 cm

Volume of water in the cylindrical container $= \pi(8.5)^2h$

$= 3744$

$h = \frac{3744}{\pi(8.5)^2}$

$= 16.5$

∴ Height of water = 16.5 cm

(iii) Surface area of the cylindrical container that is in contact with the water

$= \pi r^2 + 2\pi rh$

$= \pi(8.5)^2 + 2\pi(8.5)(16.49)$

$= 72.25\pi + 280.33\pi$

$= 352.58\pi$

$= 1110$ cm^2 (to 3 s.f.)

13. **(i)** Base radius = 186 ÷ 2 = 93 mm = 9.3 cm

Height $= \frac{1}{3} \times 93 = 31$ mm = 3.1 cm

Total surface area of the container

$= 2\pi r^2 + 2\pi rh$

$= 2\pi(9.3)^2 + 2\pi(9.3)(3.1)$

$= 172.98\pi + 57.66\pi$

$= 230.64\pi$

$= 725$ cm^2 (to 3 s.f.)

(ii) Area of the container that is painted

$= \pi r^2 + 2\pi rh$ (An open cylinder has only one base and a curved surface)

$= \pi(9.3)^2 + 2\pi r(9.3)(3.1)$

$= 86.49\pi + 57.66\pi$

$= 144.15\pi$

Fraction

$= \frac{144.15\pi}{230.64\pi}$

$= \frac{5}{8}$

14. **(i)** Base radius = 23 ÷ 2 = 11.5 mm = 1.15 cm

Height = 4 mm = 0.4 cm

Volume of water and metal discs in the tank

$= (32 \times 28 \times 19) + 2580[\pi \times (1.15)^2 \times 0.4]$

$= (17\ 024 + 1364.82\pi)$ cm^3

Let the new height in the tank be h.

Volume in the tank $= 32 \times 28 \times h$

$= 17\ 024 + 1364.82\pi$

$896h = 17\ 024 + 1364.82\pi$

$h = \frac{17\ 024 + 1364.82\pi}{896}$

$= 23.8$ (to 3 s.f.)

∴ New height of water in the tank = 23.8 cm

(ii) Surface area of the tank that is in contact with the water after the discs have been added

$= 2(32 \times 23.79 + 28 \times 23.79) + 32 \times 28$

$= 3750$ cm^2 (to 3 s.f.)

15. Total surface area of the pipe

$= 2[\pi(3.8 + 0.8)^2 - \pi(3.8)^2] + 2\pi(3.8 + 0.8)(15) + 2\pi(3.8)(15)$

$= 13.44\pi + 138\pi + 114\pi$

$= 265.44\pi$

$= 834$ cm^2 (to 3 s.f.)

16. 124 mm = 12.4 cm = 0.124 m

28 km^2 = 28 000 000 m^2

Volume of the rain

$= 28\ 000\ 000 \times 0.124$

$= 3\ 472\ 000$ m^3

Volume of each channel

$= 18 \times 26.4$

$= 475.2$ m^3

Time required for the channels to drain off the rain

$= \frac{3\ 472\ 000}{475.2 \times 2}$

$= 3653\frac{59}{297}$ s

= 61 minutes (to the nearest minute)

Review Exercise 12

1. **(a)** (i) voulme of cuboid = 2 × 12×3 cm^3

= 72 cm^3

(ii) total surface area = 2 (l×b + b×h + hl)

= 2 (12×3 + 3×2 + 2×12)

= 2 (36 + 6 + 24)

= 2 (66)

= 123 cm^2

(b) (i) Volume of the shape = $(3\times2\times4 + 3\times5\times2)$ cm^3

$= (24 + 30)$ cm^3

$= 54$ cm^3

(ii) total surface area = $2(3\times4 + 3\times2 + 5\times2) + 2\times4 + 3\times2 + 2\times1$

$= 2(12 + 6 + 10) + 8 + 6 + 2$

$= 56 + 8 + 6 + 2$

$= 72$ cm^2

(c) **(i)** Volume of the solid

$= 4 \times 5 \times 1 - 2(1 \times 3)$

$= 20 - 6$

$= 14$ cm^3

(ii) Total surface area of the solid

$= 2(1 \times 4) + 8(1 \times 1) + 2(1 \times 3) + 2[4 \times 5 - 2(1 \times 3)]$

$= 8 + 8 + 6 + 40 - 12$

$= 50$ cm^2

(d) **(i)** Volume of the solid

$= 1 \times 1 \times 5 + 2 \times 4 \times 1 + 1 \times 1 \times 3$

$= 5 + 8 + 3$

$= 16$ cm^3

(ii) Total surface area of the solid

$= 2(1 \times 5) + 2(1 \times 1) + 2(1 \times 3) + 2(1 \times 4) + 2[1 \times 5 + 2 \times 3 + 1 \times 5]$

$= 10 + 2 + 6 + 8 + 32$

$= 58$ cm^2

2. 4.5 m = 450 cm, 3.6 m = 360 cm

Number of bricks required

$= \frac{450}{18} \times \frac{18}{9} \times \frac{360}{6}$

$= 3000$

3. Volume of the rectangular block of metal

$= 256 \times 152 \times 81$

$= 3\ 151\ 872$ mm^3

Let the length of the cube be l mm.

$l^3 = 3\ 151\ 872$

$l = \sqrt[3]{3\,151\,872}$

$= 147$ (to 3 s.f.)

$\therefore$ Length of each side = 147 mm

4. Let the length of the cube be l cm.

$l^3 = 343$

$l = \sqrt[3]{343}$

$= 7$

Total surface area of a cube

$= 6l^2$

$= 6(7)^2$

$= 294$ cm^2

5. **(i)** Its volume

$= \frac{1}{2} \times (20 + 22.5) \times 17 \times 45.5$

$= 16\ 436.875$ cm^3

(ii) Volume of a gold bar with a mass of 200 g

$= \frac{16\ 436.875}{250\ 000} \times 200$

$= 13.1495$ cm^3

$= 13.1495 \times 1000$ mm^3

$= 13\ 149.5$ mm^3

(iii) Volume of the gold bar weighing 200 g = 13 149.5 mm^3

$\frac{1}{2} \times (20 + x) \times 15 \times 50 = 13\ 149.5$

$375(20 + x) = 13\ 149.5$

$20 + x = \frac{13\,149.5}{375}$

$x = \frac{13\,149.5}{375} - 20$

$\therefore x = 15\frac{49}{750}$

6. Base radius of the cylindrical barrel = $70 \div 2 = 35$ cm

Volume of water in the cylindrical barrel that is drained away

$= \pi(35)^2(6)$

$= 7350\pi$ cm^3

$0.2\ l = 200$ ml = 200 cm^3

Time taken for the water level in the barrel to drop by 6 cm

$= \frac{7350\neq}{200}$

= 115 minutes (to 3 s.f.)

7. **(i)** Volume of water in the pail

$= \pi(32)^2(25)$

$= 25\ 600\pi$

$= 80\ 400$ cm^3 (to 3 s.f.)

(ii) Volume of water in the pail after 2000 metal cubes are added to it

$= 25\ 600\pi + 2000(2 \times 2 \times 2)$

$= 25\ 600\pi + 16\ 000$

Let the new height of water in the pail be h cm.

$\pi(32)^2h = 25\ 600\pi + 16\ 000$

$h = \frac{25600\pi + 16000}{\pi(32)^2}$

$= 30.0$ (to 3 s.f.)

$\therefore$ New height = 30.0 cm

8. **(i)** Internal radius = $4.2 \div 2 = 2.1$ cm

External radius = $5 \div 2 = 2.5$ cm

Volume of metal used in making the pipe

$= [\pi(2.5)^2 - \pi(2.1)^2] \times 8.9$

$= 16.376\pi$

$= 51.4$ cm^3 (to 3 s.f.)

(ii) 51.45 cm^3 = 0.00 005 145 m^3

Cost of the pipe = $0.00\ 005\ 145 \times 2700 \times \$8$

= \$1.11

9. **(i)** Volume of the solid

$= \pi(6)^2(14) + 22 \times 18 \times 8$

$= 504\pi + 3168$

$= 4750$ cm^3 (to 3 s.f.)

(ii) Total surface area of the solid

$= 2(22 \times 18) + 2\pi(6)(14) + 2(8 \times 22) + 2(8 \times 18)$

$= 792 + 168\pi + 352 + 288$

$= 1432 + 168\pi$

$= 1960 \text{ cm}^2$ (to 3 s.f.)

Challenge Yourself

(i) Volume of the remaining solid

$= 15 \times 24 \times 16 - \pi(4)^2(7)$

$= 5760 - 112\pi$

$= 5410 \text{ cm}^3$ (to 3 s.f.)

(ii) Area that will be covered in paint

$= 2\pi(4)(7) + 2(15 \times 24) + 2(16 \times 24) + 2(16 \times 15)$

$= 56\pi + 720 + 768 + 480$

$= 56\pi + 1968$

$= 2140 \text{ cm}^2$ (to 3 s.f.)

Chapter 13 Averages of Statistical Data

TEACHING NOTES

Suggested Approach

In Grade 6, students would have explored the usage of some statistical diagrams like pictograms, bar graphs, pie charts and line graphs.

In this chapter, students will learn about other statistical diagrams, namely histograms. Other than illustrating how is used, teachers may either provide more examples than shown in the textbook, or instruct students to develop their own examples. Students are expected to be familiar with the similarities and differences, advantages and disadvantages among various statistical diagrams and decide the most suitable statistical diagram to use.

Section 13.1: Statistical Diagrams

Students will recap what are pictograms, bar graphs, pie charts and line graphs here. Teachers may revise examples from Book 1 to allow recollection and application of the diagrams.

Section 13.2: Histograms for Ungrouped Data

Teachers may want to introduce histograms by revising the explaining the difference between grouped and ungrouped data. It is important that students realise that histograms and bar graphs, though they look similar, are different representations of sets of data.

Teachers may want to group students, discuss and present the similarities, differences, advantages and disadvantages between a bar graph and a histogram (see Journal Writing on page 231). The same may be done for the Class Discussion (see Class Discussion: Evaluation of Statistical Diagrams)

Section 13.3: Histograms for Grouped Data

It is crucial that the differences between ungrouped data and grouped data are stated at the beginning. The use of class intervals for grouped data is what differentiates both types of data, as the interval 'groups' similar data together. Students are also required to make frequency tables in this section.

Similar to the previous section, students can be grouped together to discuss and present the similarities, differences, advantages and disadvantages between a stem-and-leaf diagram and a histogram for grouped data.

To assess and reinforce students' understanding, teachers can get the entire class to measure the length of their right shoe and present the results in the form of a histogram (see Performance Task on page 236).

The last activity in this section is to expose students to the usage of histograms for grouped data with unequal class intervals.

Section 13.4: Mean, Median, and Mode

Questions involving all three numerical averages will be covered in this section. Students may need to recall the algebraic skills they have picked up at the first half of the textbook.
In this section, teachers should use the activities that compare the mean, median mode and question students on the most suitable numerical average depending on the set of data provided.

WORKED SOLUTIONS

Journal Writing (Page 231)

1. Both bar charts and histograms are an important element of statistics. Both graphs make use of vertical bars to represent information on the *x* and *y* axes. The histogram is used to show a graphical presentation that represents the data in the form of frequency, whereas a bar chart is also a graphical representation of data and the information that is used for the comparison of two or more categories.
   The primary difference is that, in a histogram, the bars are closely spaced without forming gaps, whereas in a bar graph, there will be fixed gaps between bars. In simple words, bars are connected and continuous in a histogram, unlike a bar graph. It also indicates that a histogram represents continuous data using bars and intervals formed by grouping it. In contrast, a bar graph represents categorical data using bars so that each bar represents different items.
2. **(a)** A bar graph is more appropriate than a histogram when there are extreme values and gaps in between values. Also, a bar graph is more suitable when the number of data and the data set is small.
   (b) A histogram is more appropriate than a bar graph when there are no gaps in between values and the data set is large.

Class Discussion (Evaluation of Statistical Diagrams)

1. The favourite ice-cream flavours of Nadia's classmates is a form of categorical data. Pictograms, bar charts or pie charts are suitable for showing categorical data.
   Dot diagrams are more suited for numerical data, where the horizontal axis is the number line.
   Therefore, the choice of a dot diagram to present Nadia's survey data is not suitable.
2. Salman's first statement, that there are three times as many households which own 2 smartphones as that which own 0 smartphones and that which own 1 smartphone, is incorrect.

   The numbers of households who own 0, 1 and 2 smartphones are 100, 100 and 200 respectively. Hence, the number of households who own 2 smartphones is actually twice, and not three times that of households which own 0 smartphones and that which own 1 smartphone.

   His second statement, that the number of households which own 3 smartphones, is half of that which own 2 smartphones is incorrect as well.

   The numbers of households who own 2 and 3 smartphones are 200 and 125 respectively. Hence, the number of households who own 3 smartphones is more than half that of households which own 2 smartphones.

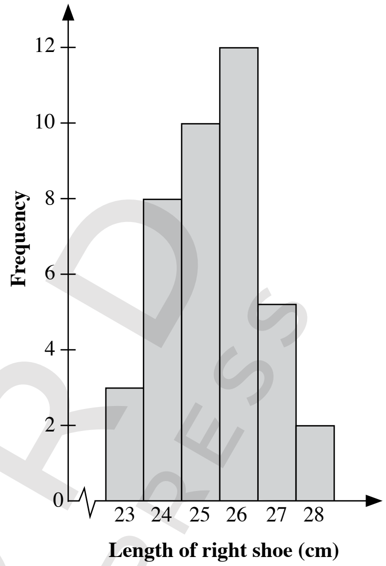
   The most probable reason why Salman was mistaken with his statements was that he took the height of the rectangle to be proportional to the frequency. However, the vertical axis starts from 50, and not 0, in this histogram, which contributed to his error.

Performance Task (Page 236)

1. The length of the right shoe for most students, correct to the nearest cm, should range from 23 cm to 28 cm.
2. Sample frequency table for a class of 40 students:

Length of right shoe (cm)	23	24	25	26	27	28
Frequency	3	8	10	12	5	2

3.



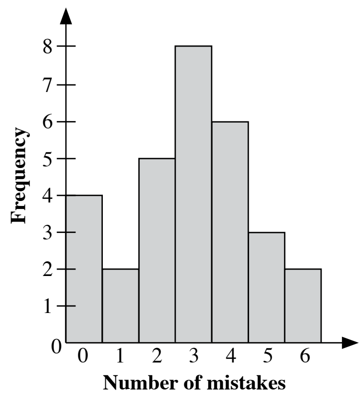
4. The length of the right shoe, correct to the nearest cm, for a class of 40 students ranges from 23 cm to 28 cm. The length is clustered around 24 cm to 28 cm.

Practise Now 1

(i)

Number of mistakes	0	1	2	3	4	5	6
Frequency	4	2	5	8	6	3	2

(ii)



(iii) The most common number of mistakes made is 3.

(iv) Fraction of students who made at most 3 mistakes in the test

$$= \frac{4+2+5+8}{30}$$

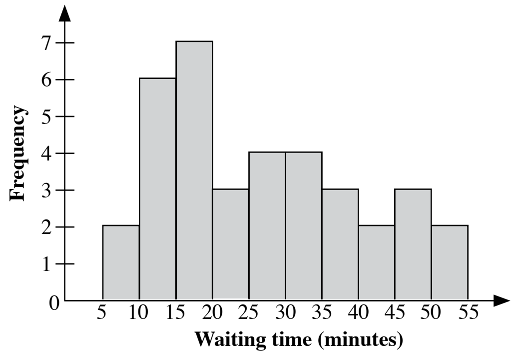
$$= \frac{19}{30}$$

Practise Now 2

(i)

Waiting time (x minutes)	Tally	Frequency
$5 \leqslant x < 10$	//	2
$10 \leqslant x < 15$	### /	6
$15 \leqslant x < 20$	### //	7
$20 \leqslant x < 25$	///	3
$25 \leqslant x < 30$	////	4
$30 \leqslant x < 35$	////	4
$35 \leqslant x < 40$	///	3
$40 \leqslant x < 45$	//	2
$45 \leqslant x < 50$	///	3
$50 \leqslant x < 55$	//	2
Total frequency		36

(ii)



Practise Now 3

1. Mean time taken by the group of students

$= \dfrac{\Sigma fx}{\Sigma f}$

$= \dfrac{5 \times 20 + 3 \times 30 + 10 \times 40 + 1 \times 50 + 1 \times 60}{5 + 3 + 10 + 1 + 1}$

$= \dfrac{700}{20}$

= 35 minutes

Practise Now 4

Age (x years)	Frequency (f)	Mid-value (x)	fx
$20 < x \leqslant 30$	12	25	300
$30 < x \leqslant 40$	10	35	350
$40 < x \leqslant 50$	20	45	900
$50 < x \leqslant 60$	15	55	825
$60 < x \leqslant 70$	18	65	1170
	$\Sigma f = 75$		$\Sigma fx = 3545$

Estimated mean age of the employees $= \dfrac{3545}{75}$

= 47.3 years (to 3 s.f.)

Practise Now 5

1. Total number of data = 16

Middle position $= \dfrac{16 + 1}{2}$

$= 8.5^{\text{th}}$ position

$\therefore$ Median = mean of the data in the 8^{th} and the 9^{th} position

$= \dfrac{3 + 3}{2}$

= 3

Practise Now 6

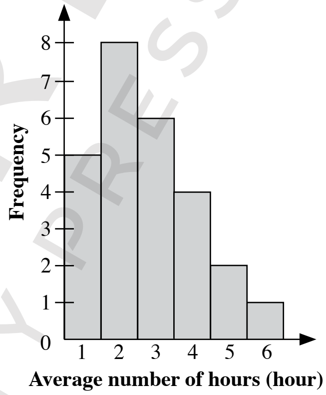
Mode = 1

Exercise 13A

1. (i)

Average number of hours (hour)	1	2	3	4	5	6
Frequency	5	8	6	4	2	1

(ii)



(iii) The most common average number of hours is 2 hours.

(iv) Number of teachers who participated in the survey

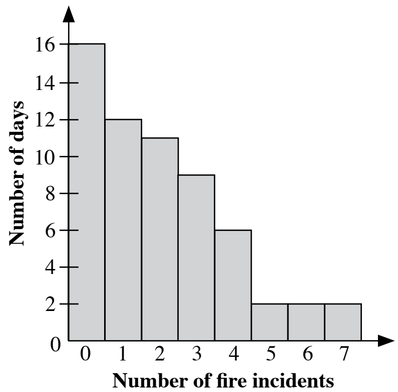
= 5 + 8 + 6 + 4 + 2 + 1

= 26

(v) Fraction of teachers who spent an average of 3 hours $= \dfrac{6}{26}$

$= \dfrac{3}{13}$

2. (i)



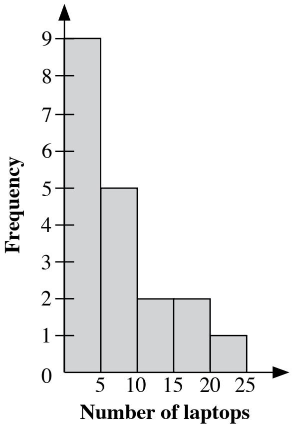
(ii) Required ratio = 16 : (2 + 2)

= 16 : 4

= 4 : 1

(iii) No. The most common number of fire incidents is 0, of which 16 days out of 60 days had 0 fire incidents.

3. **(i)**



(ii) Number of days number of laptops sold are more than or equal to 15

= 2 + 1

= 3 days

4. **(i)**

Number of hour (x hours)	Frequency
$0 \leqslant x < 2$	0
$2 \leqslant x < 4$	30
$4 \leqslant x < 6$	40
$6 \leqslant x < 8$	60
$8 \leqslant x < 10$	20
Total frequency	150

(ii) Number of students who spent more than or equal to 4, but less than 6 hours

= 40

(iii) Total number of Secondary Two students

= 30 + 40 + 60 + 20

= 150

(iv) Percentage of students who spent less than 6 hours

$= \dfrac{70}{150} \times 100\%$

$= 46\dfrac{2}{3}\ \%$

5. **(a)**

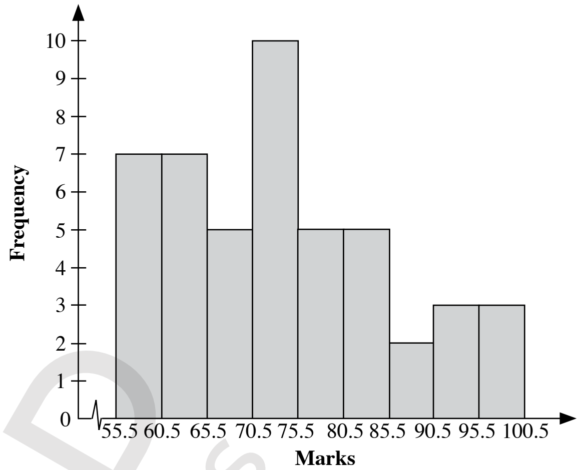
Marks	Tally	Lower class boundary	Upper class boundary	Frequency
56 – 60	### //	55.5	60.5	7
61 – 65	### //	60.5	65.5	7
66 – 70	###	65.5	70.5	5
71 – 75	### ###	70.5	75.5	10
76 – 80	###	75.5	80.5	5
81 – 85	###	80.5	85.5	5
86 – 90	//	85.5	90.5	2
91 – 95	///	90.5	95.5	3
96 – 100	///	95.5	100.5	3

(b) 90.9: 91 – 95

66.2: 66 – 70

81.5: 81 – 85

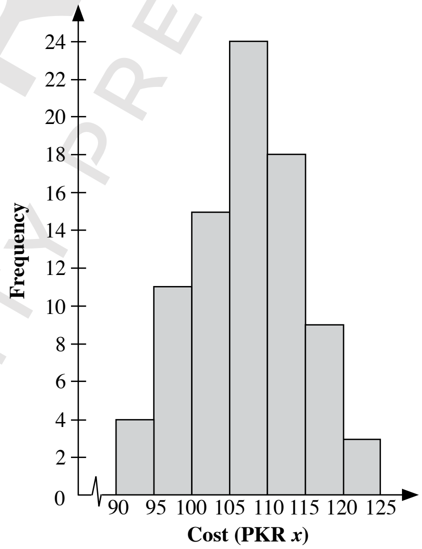
(c)



6. **(a)** Total number of shops = 4 + 11 + 15 + 24 + 18 + 9 + 3

= 84

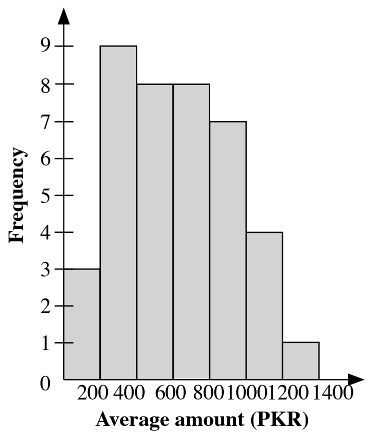
(b)



7. **(i)**

Average amount (PKR x)	Tally	Frequency
$0 \leqslant x < 200$	///	3
$200 \leqslant x < 400$	### ////	9
$400 \leqslant x < 600$	### ///	8
$600 \leqslant x < 800$	### ///	8
$800 \leqslant x < 1000$	### //	7
$1000 \leqslant x < 1200$	////	4
$1200 \leqslant x < 1400$	/	1
Total frequency		40

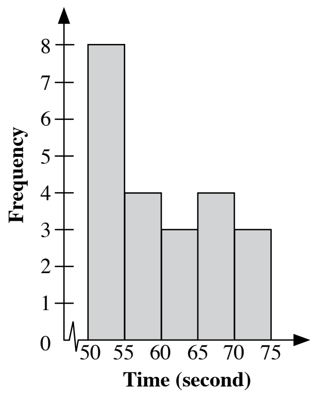
(ii)



8. **(i)**

Time (x seconds)	**Frequency**
$50 \leqslant x < 55$	8
$55 \leqslant x < 60$	4
$60 \leqslant x < 65$	3
$65 \leqslant x < 70$	4
$70 \leqslant x < 75$	3

(ii)



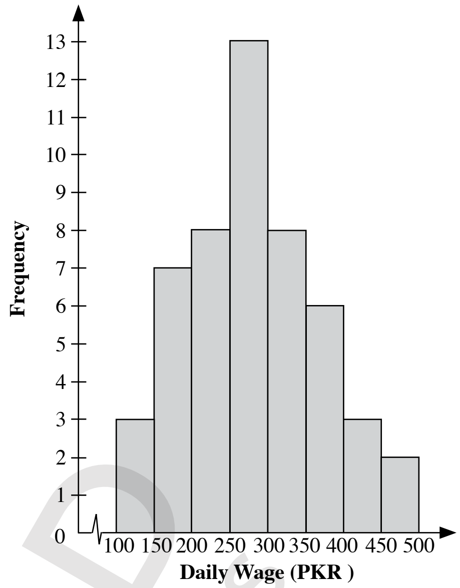
(iii) Required percentage $= \dfrac{8}{7} \times 100\%$

$= 114\dfrac{2}{7}\,\%$

(iv) Advancements in nutrition, fitness and training regiments lead to fitter and better swimmers over the years, thus resulting in the faster times from 1920 to 2012.

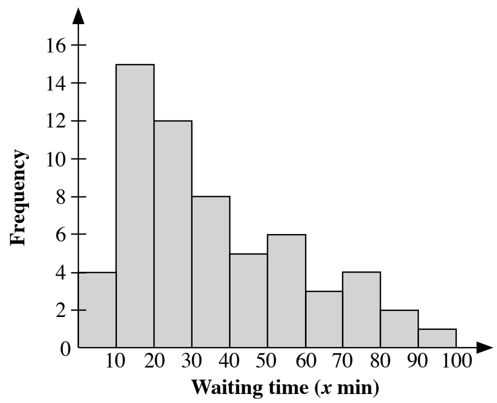
9.

Daily wage (PKR)	**Frequency**
10 – 14	3
15 – 19	7
20 – 24	8
25 – 29	13
30 – 34	8
35 – 39	6
40 – 44	3
45 – 49	2

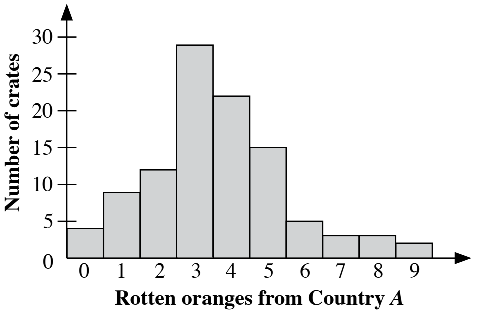


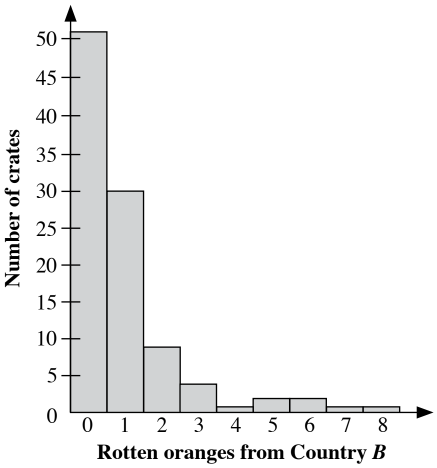
10.

Waiting time (x minutes)	**Frequency**
$0 < x \leqslant 10$	4
$10 < x \leqslant 20$	15
$20 < x \leqslant 30$	12
$30 < x \leqslant 40$	8
$40 < x \leqslant 50$	5
$50 < x \leqslant 60$	6
$60 < x \leqslant 70$	3
$70 < x \leqslant 80$	4
$80 < x \leqslant 90$	2
$90 < x \leqslant 100$	1



11. (i)





(ii) Largest number of rotten oranges found in a crate from country $A = 9$

Largest number of rotten oranges found in a crate from country $B = 8$

(iii) Total number of rotten oranges from country A

$= (4 \times 0) + (9 \times 1) + (12 \times 2) + (28 \times 3) + (22 \times 4) + (15 \times 5) + (5 \times 6) + (2 \times 7) + (2 \times 8) + (1 \times 9)$

$= 0 + 9 + 24 + 84 + 88 + 75 + 30 + 14 + 16 + 9$

$= 349$

Total number of rotten oranges from country B

$= (51 \times 0) + (30 \times 1) + (8 \times 2) + (4 \times 3) + (1 \times 4) + (2 \times 5) + (2 \times 6) + (1 \times 7) + (1 \times 8)$

$= 0 + 30 + 16 + 12 + 4 + 10 + 12 + 7 + 16$

$= 99$

(iv) P(crate contains no fewer than p rotten oranges) $= \dfrac{3}{4}$

Number of crates with no fewer than p rotten oranges

$= \dfrac{3}{4} \times 100$

$= 75$

Since $1 + 2 + 2 + 5 + 15 + 22 + 28 = 75$,

$\therefore p = 3$

Exercise 13B

1. Mean $= \dfrac{\Sigma fx}{\Sigma f}$

$= \dfrac{1 \times 6 + 2 \times 7 + 1 \times 8 + 4 \times 9 + 3 \times 10 + 1 \times 11 + 1 \times 12}{1 + 2 + 1 + 4 + 3 + 1 + 1}$

$= \dfrac{117}{13}$

$= 9$

Mean $= \dfrac{\Sigma fx}{\Sigma f}$

$= \dfrac{3 \times 3 + 5 \times 4 + 6 \times 5 + 4 \times 6 + 2 \times 7}{3 + 5 + 6 + 4 + 2}$

$= \dfrac{97}{20}$

$= 4.85$ years

2. Total number of data $= 21$

Middle position $= \dfrac{21 + 1}{2}$

$= 11^{\text{th}}$ position

$\therefore$ Median = mean of the data in the 11^{th} position

$= 70$

3. Mode $= 30$

4. (i)

Height (x cm)	Frequency (f)	Mid-value (x)	fx
$0 < x \leqslant 10$	4	5	20
$10 < x \leqslant 20$	6	15	90
$20 < x \leqslant 30$	14	25	350
$30 < x \leqslant 40$	6	35	210
$40 < x \leqslant 50$	10	45	450
	$\Sigma f = 40$		$\Sigma fx = 1120$

Estimate for the mean height of the plants $= \dfrac{1120}{40}$

$= 28$ cm

(ii) Number of plants not taller than 40 cm $= 4 + 6 + 14 + 6$

$= 30$

P(plant not taller than 40 cm) $= \dfrac{30}{40}$

$= \dfrac{3}{4}$

5. **(i)**

Time taken (t minutes)	Frequency (f)	Mid-value (x)	fx
$116 \leqslant t < 118$	1	117	117
$118 \leqslant t < 120$	6	119	714
$120 \leqslant t < 122$	23	121	2783
$122 \leqslant t < 124$	28	123	3444
$124 \leqslant t < 126$	27	125	3375
$126 \leqslant t < 128$	9	127	1143
$128 \leqslant t < 130$	5	129	645
$130 \leqslant t < 132$	1	131	131
	$\Sigma f = 100$		$\Sigma fx = 12\ 352$

Estimate for the mean travelling time of the lorries

$= \frac{12\ 352}{100}$

$= 123.52$ minutes

(ii) Number of lorries which took less than 124 minutes

$= 1 + 6 + 23 + 28$

$= 58$

Fraction of lorries which took less than 124 minutes $= \frac{58}{100}$

$= \frac{29}{50}$

6. **(i)**

Speed (x km/h)	Frequency (f)	Mid-value (x)	fx
$30 < x \leqslant 40$	16	35	560
$40 < x \leqslant 50$	25	45	1125
$50 < x \leqslant 60$	35	55	1925
$60 < x \leqslant 70$	14	65	910
$70 < x \leqslant 80$	10	75	750
	$\Sigma f = 100$		$\Sigma fx = 5270$

Estimate for the mean speed of the vehicles $= \frac{5270}{100}$

$= 52.7$ km/h

(ii) Required ratio $= 16 : (14 + 10)$

$= 16 : 24$

$= 2 : 3$

7. **(i)**

Mean distance (d million km)	Frequency (f)
$21.0 \leqslant d < 21.5$	7
$21.5 \leqslant d < 22.0$	0
$22.0 \leqslant d < 22.5$	1
$22.5 \leqslant d < 23.0$	1
$23.0 \leqslant d < 23.5$	8
$23.5 \leqslant d < 24.0$	3

(ii)

Mean distance (d million km)	Frequency (f)	Mid-value (x)	fx
$21.0 \leqslant d < 21.5$	7	21.25	148.75
$21.5 \leqslant d < 22.0$	0	21.75	0
$22.0 \leqslant d < 22.5$	1	22.25	22.25
$22.5 \leqslant d < 23.0$	1	22.75	22.75
$23.0 \leqslant d < 23.5$	8	23.25	186
$23.5 \leqslant d < 24.0$	3	23.75	71.25
	$\Sigma f = 20$		$\Sigma fx = 451$

Estimate for the mean of the mean distances of the moons from Jupiter

$= \frac{451}{20}$

$= 22.55$ million km

8. **a)** **(i)** Mean allowance

$$= \frac{4\times300 + 5\times310 + 9\times210 + 7\times330 + 4\times340 + 1\times350}{4 + 5 + 9 + 7 + 4 + 1}$$

$$= \frac{9650}{30}$$

$$= 321.7$$

(ii) Total number of data = 30

Middle position $= \frac{30 + 1}{2}$

$= 15.5^{th}$ position.

$\therefore$ Median allowance = mean of data in the 15^{th} and 16^{th} position

$= \frac{320 + 320}{2}$

= PKR 320

(iii) Modal allowance = PKR 320

b) Fraction of student who receive more than PKR 320 a week:

$= \frac{4 + 5 + 9}{30}$

$= \frac{18}{30}$

$= \frac{3}{5}$.

9. **(i)** The number of students who did less than or equal to 5 pull-ups, or more than or equal to 10 pull-ups are grouped together.

(ii) Total number of data = 21

Middle position = $\dfrac{21 + 1}{2}$

= 11^{th} position

$\therefore$ Median number by secondary 2A = data in the 11^{th} position = 7

$\therefore$ Median number by secondary 2B = data in the 11^{th} position = 7

(iii) Modal number by secondary 2A = 6

Modal number by secondary 2B = 8

(iv) The mode gives a better comparison. The mode shows the most common number of pull-ups by students in both classes, thus giving a better comparison.

10. **(i)** The number of SMS messages sent by students in June ranges from 65 to 95 messages. Most students sent between 75 to 80 messages in June.

The number of SMS messages sent by students in July ranges from 60 to 95 messages. Most students sent between 80 to 85 messages in July.

(ii) If a datum of 25 is added into June, the mean will be affected more than the median.

Most of the values range from 65 to 95, thus a value of 25 is an extreme value.

Extreme values affect the mean more than the median.

11. **(i)** Mean

$= \dfrac{\text{Sum of the lifespans of 30 light bulbs}}{30}$

$= \dfrac{167 + 171 + 179 + 167 + 171 + 165 + 175 + 179 + 169 + 168 + 171 + 177 + 169 + 171 + 177 + 173 + 165 + 175 + 167 + 174 + 177 + 172 + 164 + 175 + 179 + 179 + 174 + 174 + 168 + 171}{30}$

$= \dfrac{5163}{30}$

= 172.1 hours

(ii)

Lifespan (x hour)	Frequency
$164 \leqslant x < 167$	3
$167 \leqslant x < 170$	7
$170 \leqslant x < 173$	6
$173 \leqslant x < 176$	7
$176 \leqslant x < 179$	3
$179 \leqslant x < 182$	4

(iii)

Lifespan (x hour)	Frequency (f)	Mid-value (x)	fx
$164 \leqslant x < 167$	3	165.5	496.5
$167 \leqslant x < 170$	7	168.5	1179.5
$170 \leqslant x < 173$	6	171.5	1029
$173 \leqslant x < 176$	7	174.5	1221.5
$176 \leqslant x < 179$	3	177.5	532.5
$179 \leqslant x < 182$	4	180.5	722
	$\Sigma f = 30$		$\Sigma fx = 5181$

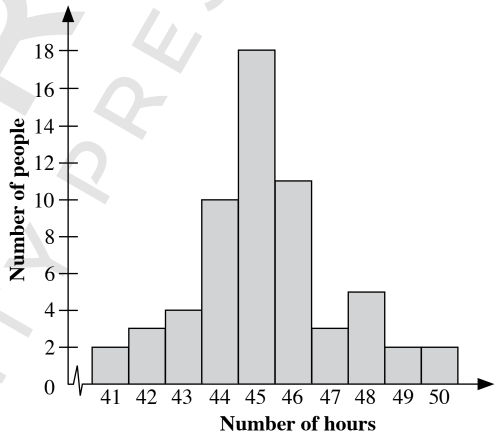
Estimate for the mean lifespan of the lightbulbs = $\dfrac{5181}{30}$

= 172.7 hours

(iv) The two values are different. The value in **(iii)** is an estimate of the actual value **(i)**.

Review Exercise 13

1. **(i)**



(ii) Fraction of workers who worked no more than 45 hours in that week = $\dfrac{37}{60}$

(iii) Total salary to workers who worked 49 hours in that week

= 2 × [(42 × PKR 600) + (7 × PKR 900)]

= (PKR 25200 + PKR 6300)

= 2(PKR 31500)

= PKR 63000

Total salary to workers who worked 50 hours in that week

= 2 × [(42 × PKR 600) + (8 × PKR 900)

= 2(PKR 25200 + PKR 7200)

= 2(PKR 32400)

= PKR 64800

Total salary paid to workers who worked more than 48 hours in that week

= PKR 63000 + PKR 64800

= PKR 127800

2. (a)

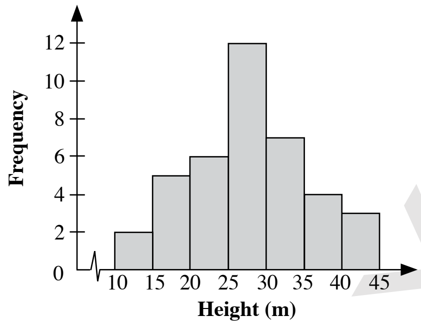
Height (x m)	Number of flats
$10 < x \leqslant 15$	2
$15 < x \leqslant 20$	5
$20 < x \leqslant 25$	6
$25 < x \leqslant 30$	12
$30 < x \leqslant 35$	7
$35 < x \leqslant 40$	4
$40 < x \leqslant 45$	3

(b) Total number of flats = 2 + 5 + 6 + 12 + 7 + 4 + 3
= 39

(c)

Height (x m)	Mid-value	Frequency
$10 < x \leqslant 15$	12.5	2
$15 < x \leqslant 20$	17.5	5
$20 < x \leqslant 25$	22.5	6
$25 < x \leqslant 30$	25.5	12
$30 < x \leqslant 35$	32.5	7
$35 < x \leqslant 40$	35.5	4
$40 < x \leqslant 45$	42.5	3

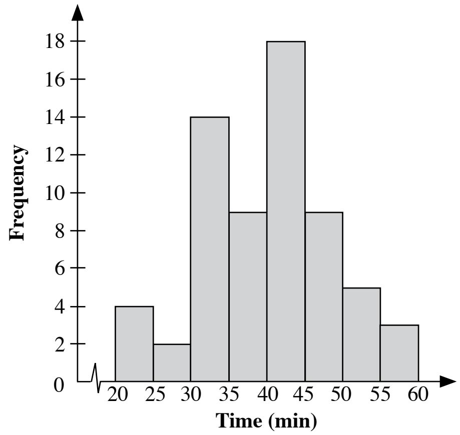
(d)



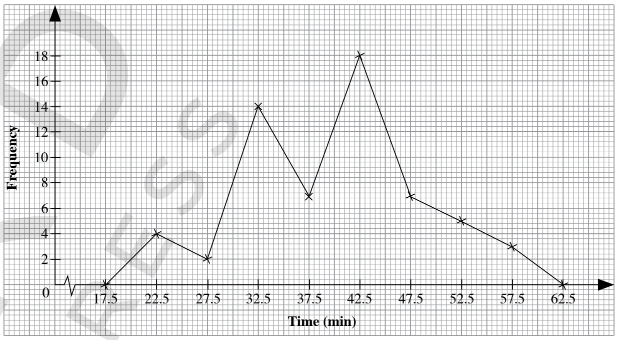
3. (a)

Time (x minutes)	Tally	Frequency
$20 < x \leqslant 25$	////	4
$25 < x \leqslant 30$	//	2
$30 < x \leqslant 35$	~~////~~ ~~////~~ ////	14
$35 < x \leqslant 40$	~~////~~ //	7
$40 < x \leqslant 45$	~~////~~ ~~////~~ ~~////~~ ///	18
$45 < x \leqslant 50$	~~////~~ //	7
$50 < x \leqslant 55$	~~////~~	5
$55 < x \leqslant 60$	///	3

(b)



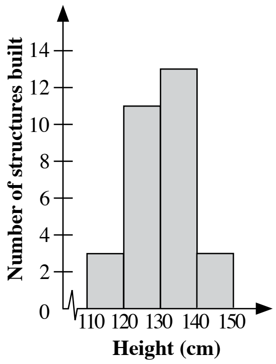
(c)



4. (a)

Height (x cm)	Frequency
$110 \leqslant x < 120$	3
$120 \leqslant x < 130$	11
$130 \leqslant x < 140$	13
$140 \leqslant x < 150$	3

(b) (i)



(ii)

Height (x cm)	Frequency (f)	Mid-value (x)	fx
$110 \leqslant x < 120$	3	115	345
$120 \leqslant x < 130$	11	125	1375
$130 \leqslant x < 140$	13	135	1755
$140 \leqslant x < 150$	3	145	435
	$\Sigma f = 30$		$\Sigma fx = 3910$

Estimate for the mean height of the structures $= \frac{3910}{30}$

$= 130\frac{1}{3}$ cm

(iii) Percentage of structures shorter than 140 cm

$= \frac{3 + 11 + 13}{30} \times 100\%$

$= \frac{27}{30} \times 100\%$

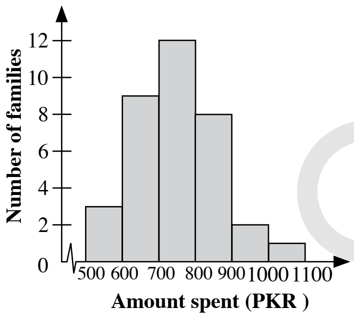
$= 90\%$

Challenge Yourself

1. **(i)**

Amount spent (PKR x)	Tally	Frequency
$5000 \leqslant x < 6000$	///	3
$6000 \leqslant x < 7000$	~~////~~ ////	9
$7000 \leqslant x < 8000$	~~////~~ ~~////~~ //	12
$8000 \leqslant x < 9000$	~~////~~ ///	8
$9000 \leqslant x < 10\,000$	//	2
$10\,000 \leqslant x < 11\,000$	/	1
Total frequency		35

(ii)



(iii) P(family selected is eligible for draw) $= \frac{6}{35}$

Chapter 14 Probability of Single Events

TEACHING NOTES

Suggested Approach

Students would not have learnt probability in grade 6 so the concept will not be new for them. Teachers may begin the lesson by first arousing the students' interest in this topic.

Section 14.1 Further Examples on Probability of Single Events

In this section, there are more calculations of probability using real-life examples. Teachers should use this section to reinforce the concept of probability.

Challenge Yourself

By listing the possible outcomes in the question, students should be able to answer all three parts.

WORKED SOLUTIONS

Practise Now 1

(i) Number of girls = 40 – 24
= 16

P(student chosen is a girl) = $\frac{16}{40}$

$= \frac{2}{5}$

(ii) Number of boys who are not short-sighted = 16

Number of girls who are not short-sighted = 16 – 4
= 12

Number of students who are not short-sighted = 16 + 12
= 28

P(student is not short-sighted) = $\frac{28}{40}$

$= \frac{7}{10}$

Practise Now 2

(i) P(point selected lies in the red sector)

$= \frac{\text{Area of the red sector}}{\text{Area of the circle}}$

$= \frac{\text{Area of the red sector}}{360^\circ}$

$= \frac{135^\circ}{360^\circ}$

$= \frac{3}{8}$

(ii) Angle of the blue sector = 360° – 90° – 135° – 45° – 30°
= 60°

P(point selected lies in the blue sector)

$= \frac{\text{Area of the blue sector}}{\text{Area of the circle}}$

$= \frac{\text{Area of the blue sector}}{360^\circ}$

$= \frac{60^\circ}{360^\circ}$

$= \frac{1}{6}$

(iii) P(point selected lies in the purple sector)

$= \frac{\text{Area of the purple sector}}{\text{Area of the circle}}$

$= \frac{0}{\text{Area of the circle}}$

= 0

(iv) P(point selected lies in the green or white sector)

$= \frac{\text{Area of the green sector + Area of the white sector}}{\text{Area of the circle}}$

$= \frac{\text{Area of the green sector + Area of the white sector}}{360^\circ}$

$= \frac{45^\circ + 30^\circ}{360^\circ}$

$= \frac{75^\circ}{360^\circ}$

$= \frac{5}{24}$

Practise Now 3

1. P(primary colour) = $\frac{\text{No. of primary coloured balls}}{\text{Total no of balls}}$

$= \frac{6}{10}$

$= \frac{3}{5}$

P(not a primary colour) = $1 - \frac{3}{5}$

$= \frac{5-3}{5}$

$= \frac{2}{5}$

2. P(Shan winning the game) = 1- P(Rashid winning the game)

$= 1 - \frac{9}{25}$

$= \frac{25-9}{25}$

$= \frac{16}{25}$

Exercise 14A

1. **(i)** Total number of girls = 8 + 3 + 1
= 12

P(student is a girl) = $\frac{12}{30}$

$= \frac{2}{5}$

(ii) Total number of non-Chinese students = 3 + 1 + 4 + 3
= 11

P(student is not a Chinese) = $\frac{11}{30}$

(iii) P(student is an Indian boy) = $\frac{3}{30}$

$= \frac{1}{10}$

P(student is not an Indian boy) = 1 – P(student is an Indian boy)

$= 1 - \frac{1}{10}$

$= \frac{9}{10}$

(iv) Number of Eurasian students = 0

P(student is a Eurasian) = $\frac{0}{30}$

= 0

2. **(i)** Total number of books = 5 + 5
= 10

P(book chosen is in Japanese) = $\frac{3}{10}$

(ii) Number of novels in Japanese $= 3 - 2$
$= 1$

Number of novels in English $= 10 - 5 - 1$
$= 4$

P(book chosen is a novel in English) $= \dfrac{4}{10}$

$= \dfrac{2}{5}$

3. **(i)** P(student prefers apple)

$= \dfrac{\text{Area of the sector of denoting apple}}{\text{Area of the circle}}$

$= \dfrac{\text{Area of the sector of denoting apple}}{360°}$

$= \dfrac{150°}{360°}$

$= \dfrac{5}{12}$

(ii) Angle of the sector denoting mango

$= 360° - 150° - 90° - 45°$

$= 75°$

P(student prefers mango)

$= \dfrac{\text{Area of the sector denoting mango}}{\text{Area of the circle}}$

$= \dfrac{\text{Area of the sector denoting mango}}{360°}$

$= \dfrac{75°}{360°}$

$= \dfrac{5}{24}$

(iii) Angle of sector denoting papaya or guava

$= 90° + 45°$

$= 135°$

P(student prefers papaya or guava)

$= \dfrac{\text{Area of the sector denoting papaya or guava}}{\text{Area of the circle}}$

$= \dfrac{\text{Area of the sector denoting papaya or guava}}{360°}$

$= \dfrac{135°}{360°}$

$= \dfrac{3}{8}$

4. **(i)** Total number of sides in an octagon = 8

P(point lies in region R) $= \dfrac{1}{8}$

(ii) P(points lies in region S) $= \dfrac{3}{8}$

(iii) P(point lies in region P or Q) $= \dfrac{2 + 2}{8}$

$= \dfrac{4}{8}$

$= \dfrac{1}{2}$

5. **(i)** Total number of students $= 15 + x$
$= x + 15$

(ii) P(student is a girl) $= \dfrac{15}{x + 15}$

(iii) Given that $\dfrac{15}{x + 15} = \dfrac{1}{5}$,

$5(15) = x + 15$

$75 = x + 15$

$\therefore x = 60$

6. P(not green) $= \dfrac{\text{No. of other colour balls}}{\text{Total no. of balls}}$

$= \dfrac{18}{20}$

$= \dfrac{9}{10}$

7. P(not a ring) $= \dfrac{\text{No. of item other than ring}}{\text{Total no. of items}}$

$= \dfrac{16}{28}$

$= \dfrac{4}{7}$

8. **(i)** P(student is a girl who did not check in her luggage) $= \dfrac{8}{38}$

$= \dfrac{4}{19}$

(ii) Total number of girls $= 38 - 18$
$= 20$

Total number of girls who checked in their luggage $= 20 - 8$
$= 12$

Total number of students who checked in their luggage

$= 12 + 6$

$= 18$

P(student checked in his/her luggage) $= \dfrac{18}{38}$

$= \dfrac{9}{19}$

9. **(a)** **(i)** Total number of students $= 16 + 24$
$= 40$

P(student is a boy) $= \dfrac{16}{40}$

$= \dfrac{2}{5}$

(ii) Total number of left-handed students $= 3 + 2$
$= 5$

P(student is left-handed) $= \dfrac{5}{40}$

$= \dfrac{1}{8}$

(b) **(i)** Total number of students who can borrow the visualiser

$= 40 - 1$

$= 39$

P(students is a boy who is left-handed) $= \dfrac{3}{39}$

$= \dfrac{1}{13}$

(ii) Number of girls who are not left-handed $= 24 - 2 - 1$
$= 21$

$$\text{P(student is a girl who is not left-handed)} = \frac{21}{39} = \frac{7}{13}$$

10. Total number of presents $= (3h + 11) + (h + 5)$
$= 4h + 16$

$$\text{P(Ethan obtains a red present)} = \frac{3h + 11}{4h + 16}$$

Given that $\frac{3h + 11}{4h + 16} = \frac{19}{26}$,

$$26(3h + 11) = 19(4h + 16)$$
$$78h + 286 = 76h + 304$$
$$2h = 18$$
$$h = 9$$

11. $\frac{7}{13} + \frac{1}{k} + \frac{1}{2k} = 1$

$$14k + 26 + 13 = 26k$$
$$12k = 39$$
$$\therefore k = \frac{39}{12} = 3\frac{1}{4}$$

The value of k is $3\frac{1}{4}$.

12. Total number of toothbrushes $= 15 + 5$
$= 20$

$$\text{P(draw a toothbrush with soft bristles)} = \frac{p + 5}{20}$$

Given that $\frac{p + 5}{20} = \frac{3}{4}$,

$$4(p + 5) = 20(3)$$
$$4p + 20 = 60$$
$$4p = 40$$
$$p = 10$$

13. Total number of boys and girls remaining after graduation
$= 23 + 35 - q - (q + 4)$
$= 54 - 2q$
Total number of boys remaining after graduation
$= 23 - q$

$$\text{P(boy is selected for event)} = \frac{23 - q}{54 - 2q}$$

Given that $\frac{23 - q}{54 - 2q} = \frac{2}{5}$,

$$5(23 - q) = 2(54 - 2q)$$
$$115 - 5q = 108 - 4q$$
$$q = 7$$

14. (i) P(drawing a black ball)
$= 1 - \text{P(drawing a red ball)} - \text{P(drawing a yellow ball)}$
$= 1 - \frac{1}{4} - \frac{2}{5}$
$= \frac{7}{20}$

(ii) Total number of balls in the bag now
$= 40 + (2x + 1) + (x + 2) - (x - 3)$
$= 40 + 2x + 1 + x + 2 - x + 3$
$= 2x + 46$

(iii) Before the balls were added,

$$\text{P(drawing a yellow ball)} = \frac{2}{5}$$

$$\frac{\text{Number of yellow balls}}{\text{Total number of balls}} = \frac{2}{5}$$

$$\therefore \text{Number of yellow balls} = \frac{2}{5} \times 40 = 16$$

After the balls were added,
Total number of yellow balls $= 16 + x + 2$
$= x + 18$

$$\text{P(drawing a yellow ball)} = \frac{x + 18}{2x + 46}$$

Given that $\frac{x + 18}{2x + 46} = \frac{3}{7}$

$$7(x + 18) = 3(2x + 46)$$
$$7x + 126 = 6x + 138$$
$$x = 12$$

Number of yellow balls in the bag now $= 12 + 18$
$= 30$

15. Total number of students $= 50$
$2x + y = 50$ — (1)
After some boys left and some girls entered the auditorium,
Total number of students $= 50 - (y - 6) + (2x - 5)$
$= 50 - y + 6 + 2x - 5$
$= 2x - y + 51$
Total number of girls $= y + 2x - 5$
$= 2x + y - 5$

$$\text{P(girl is selected at random)} = \frac{2x + y - 5}{2x - y + 51}$$

Given that $\frac{2x + y - 5}{2x - y + 51} = \frac{9}{13}$,

$$13(2x + y - 5) = 9(2x - y + 51)$$
$$26x + 13y - 65 = 18x - 9y + 459$$
$$8x + 22y = 524$$
$4x + 11y = 262$ — (2)

From (1),
$y = 50 - 2x$ — (3)
Substitute (3) into (2):

$$4x + 11(50 - 2x) = 262$$
$$4x + 550 - 22x = 262$$
$$18x = 288$$
$$\therefore x = 16$$

Substitute $x = 16$ into (3):

$$\therefore y = 50 - 2(16) = 18$$

The value of x and of y are 16 and 18 respectively.

Review Exercise 14

1. **(i)** There are 7 'orange' symbols.

P(shows the symbol 'orange') = $\frac{7}{22}$

(ii) Number of 'grape' symbols $= 22 - 4 - 7 - 9$

$= 2$

P(shows the symbol 'grape') = $\frac{2}{22}$

$= \frac{1}{11}$

(iii) There are no 'pineapple' symbol.

P(shows the symbol 'pineapple') = $\frac{0}{22}$

$= 0$

(iv) There are $4 + 9 = 13$ symbols that are either 'cherry' or 'peach'.

P(shows either the symbol 'cherry' or the symbol 'peach') = $\frac{13}{22}$

2. **(i)** P(sweet is a mint wrapped in red paper) = $\frac{4}{20}$

$= \frac{1}{5}$

(ii) There are $7 + 3 = 10$ toffees in the bag.

P(sweet is a toffee) = $\frac{10}{20}$

$= \frac{1}{2}$

(iii) There are $7 + 6 = 13$ sweets wrapped in green paper.

P(sweet is wrapped in green paper) = $\frac{13}{20}$

3. **(a)** **(i)** Total number of staplers $= 7 + 11$

$= 18$

There are no green staplers in the bag.

P(stapler is green) = $\frac{0}{18}$

$= 0$

(ii) There are $7 + 11 = 18$ staplers that are either white or orange in the bag.

P(stapler is either white or orange) = $\frac{18}{18}$

$= 1$

(b) **(i)** Total number of staplers $= 18 + 12$

$= 30$

There are 12 red staplers in the bag.

P(stapler is red) = $\frac{12}{30}$

$= \frac{2}{5}$

(ii) There are 11 orange staplers in the bag.

P(stapler is orange) = $\frac{11}{30}$

P(stapler is not orange) = $1 - \frac{11}{30}$

$= \frac{19}{30}$

4. **(i)** Angle of the sector denoting travel by car

$= 360° - 90° - 110° - 60°$

$= 100°$

P(student travels to school by car)

$= \frac{\text{Area of the sector denoting travel by car}}{\text{Area of the circle}}$

$= \frac{\text{Area of the sector denoting travel by car}}{360\Upsilon}$

$= \frac{100\Upsilon}{360\Upsilon}$

$= \frac{5}{18}$

(ii) Angle of the sector denoting travel by train or on foot

$= 110° + 60°$

$= 170°$

P(students travels to school by train or on foot)

$= \frac{\text{Area of the sector denoting travel by train or on foot}}{\text{Area of the circle}}$

$= \frac{\text{Area of the sector denoting travel by train or on foot}}{360\Upsilon}$

$= \frac{170\Upsilon}{360\Upsilon}$

$= \frac{17}{36}$

(iii) Angle of sector denoting travel by bicycle = 0°

P(student travels to school by bicycle)

$= \frac{\text{Area of the sector denoting travel by bicycle}}{\text{Area of the circle}}$

$= \frac{\text{Area of the sector denoting travel by bicycle}}{360\Upsilon}$

$= \frac{0\Upsilon}{360\Upsilon}$

$= 0$

5. **(i)** Number of tulips $= 100 - 20 - h$

$= 80 - h$

P(picking a stalk of tulip) = $\frac{80 - h}{100}$

Given that $\frac{80 - h}{100} = \frac{1}{4}$,

$4(80 - h) = 100$

$320 - 4h = 100$

$4h = 220$

$h = 55$

(ii) Number of flowers remaining $= 100 - 10$

$= 90$

Number of roses = 55

P(picking a stalk of rose) = $\frac{55}{90}$

$= \frac{11}{18}$

6. **(i)** Total number of vehicles $= 125 + 3p + 2q + 20$
$= 3p + 2q + 145$

P(vehicle is a motorcycle) $= \dfrac{3p}{3p + 2q + 145}$

Given that $\dfrac{3p}{3p + 2q + 145} = \dfrac{3}{40}$,

$$40(3p) = 3(3p + 2q + 145)$$
$$120p = 9p + 6q + 435$$
$$111p - 6q - 435 = 0$$
$$37p - 2q - 145 = 0$$

(ii) P(vehicle is a bus) $= \dfrac{20}{3p + 2q + 145}$

Given that $\dfrac{20}{3p + 2q + 145} = \dfrac{1}{10}$,

$$10(20) = 3p + 2q + 145$$
$$200 = 3p + 2q + 145$$
$$3p + 2q - 55 = 0$$

(iii) $37p - 2q - 145 = 0$ — (1)

$3p + 2q - 55 = 0$ — (2)

(1) + (2):

$$(37p - 2q - 145) + (3p + 2q - 55) = 0 + 0$$
$$40p - 200 = 0$$
$$40p = 200$$
$$\therefore p = 5$$

Substitute $p = 5$ into (2):

$$3(5) + 2q - 55 = 0$$
$$15 + 2q - 55 = 0$$
$$2q = 40$$
$$\therefore q = 20$$

The value of p and of q are 5 and 20 respectively.

Challenge Yourself

1. **(i)** Let the 3 friends Farhan, Bilal and Maaz be A, B and C, and the letters be 1, 2 and 3.
i.e. Farhan should receive letter 1, Bilal should receive letter 2 and Maaz should receive letter 3. $A \leftrightarrow 1$, $B \leftrightarrow 2$, $C \leftrightarrow 3$.
The sample space consists of $\{A1, B2, C3\}$, $\{A1, B3, C2\}$, $\{A2, B1, C3\}$, $\{A2, B3, C1\}$, $\{A3, B1, C2\}$, $\{A3, B2, C1\}$.
Total number of possible outcomes = 6
There are 3 ways where exactly one of his friends receive the correct letter,
i.e. $\{A1, B3, C2\}$, $\{A2, B1, C3\}$ and $\{A3, B2, C1\}$.

P(exactly one of his friends receive the correct letter) $= \dfrac{3}{6}$
$= \dfrac{1}{2}$

(ii) There are no ways where exactly two of his friends receive the correct letters.

P(exactly two of his friends receive the correct letter) $= \dfrac{0}{6}$
$= 0$

(iii) There is only one way where all three of his friends receive the correct letters, i.e. $\{A1, B2, C3\}$.

P(all three of his friends receive the correct letters) $= \dfrac{1}{6}$