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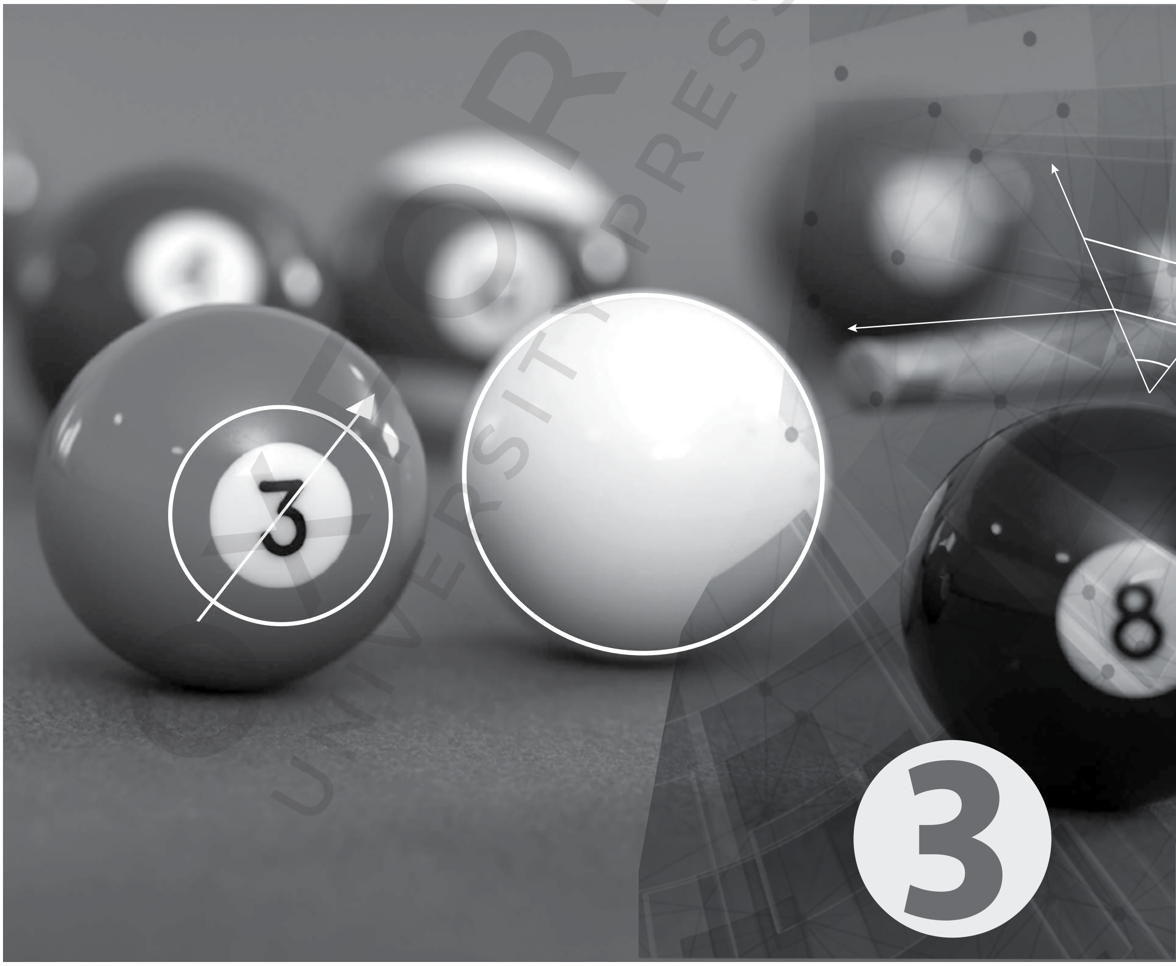
UPDATED 7th EDITION

NEW SYLLABUS MATHEMATICS

WORKBOOK FULL SOLUTIONS

A Comprehensive Mathematics Programme for Grade 8



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ANSWERS

Chapter 1 Number Operations and Direct and Inverse Proportions

Basic

1. **(a)** We observe that 48 is close to 49 which is a perfect square. Thus $\sqrt{48} \approx \sqrt{49} = 7$.
   (b) We observe that 626 is close to 625 which is a perfect square. Thus $\sqrt{626} \approx \sqrt{625} = 25$.
   (c) 65 is close to 64 which is a perfect cube. Thus $\sqrt[3]{65} \approx \sqrt[3]{64} = 4$.
   (d) 998 is close to 1000 which is a perfect cube. Thus $\sqrt[3]{998} \approx \sqrt[3]{1000} = 10$.
   (e) We observe that 99 is close to 100 which is a perfect square and 28 is close to 27 which is a perfect cube. Thus $\sqrt{99} - \sqrt[3]{28} \approx \sqrt{100} - \sqrt[3]{27} = 7$.
   (f) We observe that 19 is close to 20 and 10 004 is close to 10 000 which is a perfect square. Thus $19^2 \times \sqrt{10\,004} \approx 20^2 \times \sqrt{10\,000} = 400 \times 100$
   $= 40\,000$.
   (g) We observe that 11 is close to 10 and 7999 is close to 8000 which is a perfect cube. Thus $11^3 + \sqrt[3]{7999} \approx 10^3 + \sqrt[3]{8000} = 1000 + 20 = 1020$.
2. **(a)** $693 + 1262 - \sqrt{71\,289} \times \sqrt[3]{912\,673} = 318\,486$
   (b) $\dfrac{\sqrt[3]{12\,167} \times 57^2 - 56^3}{\sqrt{153\,664}} = -257.3699$ (to 4 d.p.)
   (c) $\dfrac{\left(\sqrt{576} + \sqrt{961} - \sqrt[3]{12\,167}\right)}{\sqrt[3]{4096}} = 2$
   (d) $\dfrac{18^3}{\sqrt{5184}} + \dfrac{16^2 - \sqrt[3]{75\,357}}{22^3 - 103^2 - \sqrt[3]{753\,571}}$
3. **(a)** All zeros between non-zero digits are significant.
   5 significant figures
   (b) In a decimal, all zeros before a non-zero digit are not significant.
   4 significant figures
   (c) 5 significant figures
   (d) 9 or 10 significant figures.
4. **(a)** 3.9 (to 2 s.f.)
   (b) 20 (to 2 s.f.)
   (c) 38 (to 2 s.f.)
   (d) 4.07 (to 3 s.f.)
   (e) 18.1 (to 3 s.f.)
   (f) 0.0326 (to 3 s.f.)
   (g) 0.0770 (to 3 s.f.)
   (h) 0.008 17 (to 3 s.f.)
   (i) 18.14 (to 4 s.f.)
   (j) 240.0 (to 4 s.f.)
   (k) 5004 (to 4 s.f.)
   (l) 0.054 45 (to 4 s.f.)
5. **(a)** 20 (to 1 s.f.)
   (b) 19 (to 2 s.f.)
6. **(a)** 0.007 (to 1 s.f.)
   (b) 0.007 20 (to 3 s.f.)
7. **(a)** 984.6 (to 4 s.f.)
   (b) 984.61 (to the nearest hundredth)
8. **(a)** 0.000 143 (to 3 s.f.)
   (b) 1000 (to 2 s.f.)
9. **(a)** 0.3403 (to 4 s.f.)
   (b) 10.255 (to 5 s.f.)
   (c) 64 704 800 (to 6 s.f.)
10. **(a)** 428.2 (to 4 s.f.)
   The number of decimal places in the answer is 1.
   (b) 0.000 90 (to 5 d.p.)
   The number of significant figures is 1 or 2, depending on whether the last zero is included or otherwise.
11. **(a)** 61.994 06 – 29.980 78
   = 32.013 28
   = 30 (to 1 s.f.)
   (b) 64.967 02 – 36.230 87
   = 28.736 15
   = 30 (to 1 s.f.)
   (c) 4987 × 91.2
   = 454 814.4
   = 500 000 (to 1 s.f.)
   (d) 30.9 × 98.6
   = 3046.74
   = 3000 (to 1 s.f.)
   (e) 0.0079 × 21.7
   = 0.171 43
   = 0.2 (to 1 s.f.)
   (f) 1793 × 0.000 97
   = 1.739 21
   = 2 (to 1 s.f.)
   (g) 9801 × 0.0613
   = 600.8013
   = 600 (to 1 s.f.)
   (h) $(8.907)^2$
   = 79.334 649
   = 80 (to 1 s.f.)

(i) $(398)^2 \times 0.062$

$= 9821.048$

$= 10\,000$ (to 1 s.f.)

(j) $81.09 \div 1.592$

$= 50.935\ldots$

$= 50$ (to 1 s.f.)

(k) $\frac{49.82}{9.784}$

$= 5.091\,98\ldots$

$= 5$ (to 1 s.f.)

(l) $\frac{163.4}{0.0818}$

$= 1997.555\,012\ldots$

$= 2000$ (to 1 s.f.)

(m) $15.002 \div 0.019\,99 - 68.12$

$= 682.355\,237\,6\ldots$

$= 700$ (to 1 s.f.)

(n) $\frac{59.26 \times 5.109}{3.817}$

$= \frac{302.759\,34}{3.817}$

$= 79.318\,663\,87\ldots$

$= 80$ (to 1 s.f.)

(o) $\frac{4.18 \times 0.0309}{0.0212}$

$= \frac{0.129\,162}{0.0212}$

$= 6.092\,547\,17$

$= 6$ (to 1 s.f.)

(p) $\frac{16.02 \times 0.0341}{0.079\,21}$

$= \frac{0.546\,282}{0.079\,21}$

$= 6.896\,629\,213\ldots$

$= 7$ (to 1 s.f.)

(q) $\sqrt{\frac{35.807}{101.09}}$

$= \sqrt{0.354\,209\,12}$

$= 0.595\,154\,703\ldots$

$= 0.6$ (to 1 s.f.)

(r) $\sqrt{\frac{18.01 \times 36.01}{1.989}}$

$= \sqrt{\frac{648.5401}{1.989}}$

$= \sqrt{326.063\,398\,7}$

$= 18.057\,225\,66\ldots$

$= 20$ (to 1 s.f.)

12. $340 \div 21$

$\approx 340 \div 20$

$= 34 \div 2$

$= 17$

$\therefore$ Rizwan's answer is wrong.

Using a calculator, the actual answer is 16.190 476 19.

Hence, his estimated value 15 is close to actual value 16.190 476 19.

He has underestimated the value by using the estimation $300 \div 20$.

13. (i) **(a)** $45.3125 = 45$ (to 2 s.f.)

(b) $3.9568 = 4.0$ (to 2 s.f.)

(ii) $45.3125 \div 3.9568$

$\approx 45 \div 4.0$

$= 11.25$

(iii) Using a calculator, the actual value is 11.451 804 49. The estimated value is close to the actual value. The estimated value is approximately 0.20 less than the actual value.

14. (a) $0.052\,639\,81 = 0.052\,640$ (to 5 s.f.)

(b) $1793 \times 0.000\,979$

$= 1.755\,347$

$= 1.8$ (to 1 d.p.)

(c) $\frac{31.205 \times 4.97}{1.925}$

$= \frac{155.088\,85}{1.925}$

$= 80.565\,636\,36\ldots$

$= 80$ (to 1 s.f.)

15. The calculation is $297 \div 19.91$.

$297 \div 19.91$

$\approx 300 \div 20$

$= 15$ (to 2 s.f.)

15 litres of petrol is used to travel 1 km.

16. Total cost of set meals = PKR 6.90×9

= PKR 7×9

= PKR 63

Ahsan should pay less than PKR 63 for the set meals. Therefore, he has paid the wrong amount.

17. Cost of 15 l of petrol = $\frac{\text{PKR } 14.70}{7} \times 15$

= PKR 31.50

18. (i) $y = kx$

When $x = 200$, $y = 40$,

$40 = k(200)$

$k = \frac{40}{200}$

$= \frac{1}{5}$

$\therefore y = \frac{1}{5}x$

(ii) When $x = 15$,

$y = \frac{1}{5}(15)$

$= 3$

(iii) When $y = 8$,

$8 = \frac{1}{5}x$

$x = 40$

19. (i) $y = k(4x + 1)$

When $x = 2$, $y = 3$,

$3 = k(8 + 1)$

$k = \frac{3}{9}$

$= \frac{1}{3}$

$\therefore y = \frac{1}{3}(4x + 1)$

(ii) When $x = 5$,

$y = \frac{1}{3}(20 + 1)$

$= 7$

(iii) When $y = 11$,

$11 = \frac{1}{3}(4x + 1)$

$33 = 4x + 1$

$4x = 32$

$x = 8$

20. Time taken for 1 tap to fill the bath tub $= 15 \times 2$

$= 30$ minutes

Time taken for 3 taps to fill the bath tub $= \frac{30}{3}$

$= 10$ minutes

21. (i) When $x = 5$,

$y = 100 \times 2$

$= 200$

(ii) $y = \frac{k}{x}$

When $x = 10$, $y = 100$,

$100 = \frac{k}{10}$

$k = 1000$

$\therefore y = \frac{1000}{x}$

(iii) When $y = 80$,

$80 = \frac{1000}{x}$

$x = \frac{1000}{80}$

$= 12.5$

Intermediate

22. (a) (i) Divide 216 000 by the smallest prime number until we get 1.

2	216 000
2	108 000
2	54 000
2	27 000
2	13 500
2	6750
3	3375
3	1125
3	375
5	125
5	25
5	5
	1

$216\,000 = 2^6 \times 3^3 \times 5^3$

(ii) Divide 518 400 by the smallest prime number until we get 1.

2	518 400
2	259 200
2	129 600
2	64 800
2	32 400
2	16 200
2	8100
2	4050
3	2025
3	675
3	225
3	75
5	25
5	5
	1

$518\,400 = 2^8 \times 3^4 \times 5^2$

(b) (i) $216\,000 = 2^6 \times 3^3 \times 5^3 = (2^2 \times 3 \times 5)^3$

$\sqrt[3]{216\,000} = 2^2 \times 3 \times 5 = 60$

(ii) $518\,400 = 2^8 \times 3^4 \times 5^2 = (2^4 \times 3^2 \times 5)^2$

$\sqrt{518\,400} = 2^4 \times 3^2 \times 5 = 720$

23. (i) Divide 13 824 by the smallest prime number until we get 1.

2	13 824
2	6912
2	3456
2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

$13\ 824 = 2^9 \times 3^3$

5	42 875
5	8575
5	1715
7	343
7	49
7	7
	1

$42\ 875 = 5^3 \times 7^3$

$13\ 824 \times 42\ 875 = 2^9 \times 3^3 \times 5^3 \times 7^3$

(ii) $13\ 824 \times 42\ 875 = 2^9 \times 3^3 \times 5^3 \times 7^3$

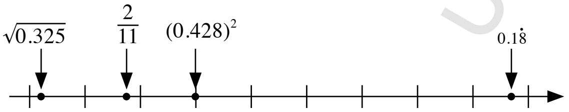
$= (2^3 \times 3 \times 5 \times 7)^3$

$\sqrt[3]{13\ 824 \times 42\ 875} = 2^3 \times 3 \times 5 \times 7 = 840$

24. (a) $\frac{2}{11} = 0.1818$

$\sqrt{0.325} = 0.1803$

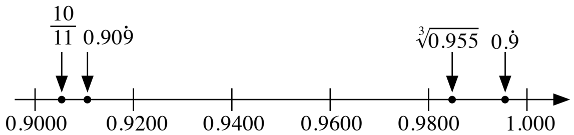
$(0.428)^2 = 0.1830$



$\therefore 0.1\dot{8}, (0.428)^2, \frac{2}{11}, \sqrt{0.325}$

(b) $\frac{10}{11} = 0.9090$

$\sqrt[3]{0.955} = 0.984$



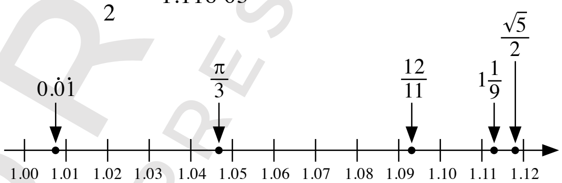
$\therefore 0.\dot{9}, \sqrt[3]{0.955}, 0.90\dot{9}, \frac{10}{11}$

(c) $\frac{\pi}{3} = 1.047\ 20$

$1\frac{1}{9} = 1.1111$

$\frac{12}{11} = 1.0909$

$\frac{\sqrt{5}}{2} = 1.118\ 03$



$\therefore \frac{\sqrt{5}}{2}, 1\frac{1}{9}, \frac{12}{11}, \frac{\pi}{3}, 1.\dot{0}\dot{1}$

25. (a) $[109 - (-19)] \div (-2)^3 \times (-5)$

$= (109 + 19) \div (-2)^3 \times (-5)$

$= 128 \div (-8) \times (-5)$

$= \left(\frac{128}{-8}\right) \times (-5)$

$= -16 \times (-5)$

$= -(-80)$

$= 80$

(b) $(13 - 9)^2 - 5^2 - (28 - 31)^3$

$= 4^2 - 5^2 - (-3)^3$

$= 16 - 25 - (-27)$

$= 16 - 25 + 27$

$= 18$

(c) $[(-5) \times (-8)^2 - (-2)^3 \times 7] \div (-11)$

$= [(-5) \times 64 - (-8) \times 7] \div (-11)$

$= [-(5 \times 64) - [-(8 \times 7)]] \div (-11)$

$= [-320 - (-56)] \div (-11)$

$= (-320 + 56) \div (-11)$

$= (-264) \div (-11)$

$= 24$

(d) $\{[(-23) - (-11)] \div 6 - 7 \div (-7)\} \times 1997$
$= [(-23 + 11) \div 6 - 7 \div (-7) \times 1997$
$= [(-12) \div 6 - 7 \div (-7)] \times 1997$
$= \left[\left(\frac{-12}{6}\right) - \left(\frac{7}{-7}\right)\right] \times 1997$
$= [(-2) - (-1)] \times 1997$
$= (-2 + 1) \times 1997$
$= (-1) \times 1997$
$= -1997$

(e) $(-7)^3 + (-2)^3 - [(-21) + 35 - \sqrt[3]{125} \times (-8)]$
$= -343 + (-8) - [(-21) + 35 - 5 \times (-8)]$
$= -343 + (-8) - [(-21) + 35 - (-40)]$
$= -343 + (-8) - [(-21) + 35 + 40]$
$= -343 + (-8) - (14 + 40)$
$= -343 - 8 - 54$
$= -(343 + 8 + 54)$
$= -405$

26. (a) $[109 - (-19)] \div (-2)^3 \times (-5) = 80$
(b) $(13 - 9)^2 - 5^2 - (28 - 31)^3 = 18$
(c) $[(-5) \times (-8)^2 - (-2)^3 \times 7] \div (-11) = 24$
(d) $\{[(-23) - (-11)] \div 6 - 7 \div (-7)\} \times 1997 = -1997$
(e) $(-7)^3 + (-2)^3 - [(-21) + 35 - \sqrt[3]{125} \times (-8)] = -405$

27. (a) $(-0.3)^2 \times \left(\frac{-1.4}{0.07}\right) - 0.78$
$= \left(-\frac{3}{10}\right)^2 \times \left(\frac{-140}{7}\right) - 0.78$
$= \left(\frac{9}{100}\right) \times (-20) - 0.78$
$= -1.8 - 0.78$
$= -2.58$

(b) $(-0.4)^3 \times \left(\frac{-3.3}{0.11}\right) + 0.123$
$= \left(-\frac{4}{10}\right)^3 \times \left(\frac{-3.3}{0.11}\right) + 0.123$
$= \left(-\frac{64}{1000}\right) \times \left(\frac{-330}{11}\right) + 0.123$
$= \left(-\frac{64}{1000}\right) \times (-30) + 0.123$
$= 1.92 + 0.123$
$= 2.043$

28. (a) $\dfrac{1\frac{8}{13} \times \frac{13}{42} + 5\frac{1}{5} \div \frac{7}{45}}{\left(\frac{7}{9} + \frac{7}{18}\right) \div \frac{1}{18} \times \frac{1}{7}} = 11\frac{13}{42}$

(b) $\dfrac{\sqrt[3]{13} - \sqrt{7}}{\sqrt{48} - \sqrt[3]{101}} = -0.130$ (to 3 d.p.)

(c) $\dfrac{\sqrt[3]{42.7863} \times (41.567)^2}{94\ 536.721} = 0.064$ (to 3 d.p.)

(d) $\sqrt[3]{\dfrac{9206 \times (29.5)^3}{(11.86)^2}} = 118.884$ (to 3 d.p.)

(e) $\sqrt{\dfrac{46.3^2 + 85.9^2 - 70.7^2}{2 \times 46.3 \times 85.9}} = 0.754$ (to 3 d.p.)

(f) $\sqrt{\dfrac{18 \times (4.359)^2 + 10 \times (3.465)^2}{(4.359)^3 + 3 \times (3.465)^3}} = 1.492$ (to 3 d.p.)

29. (a) $(16.245 - 5.001)^3 \times \sqrt{122.05}$
$= 15\ 704.76\ldots$
$= 20\ 000$ (to 1 s.f.)

(b) $\dfrac{6.01 \times 0.0312}{0.0622}$
$= 3.014\ 66\ldots$
$= 3$ (to 1 s.f.)

(c) $\dfrac{29.12 \times 5.167}{1.895}$
$= 79.400\ldots$
$= 80$ (to 1 s.f.)

(d) $\dfrac{41.41}{10.02 \times 0.018\ 65}$
$= 221.594\ 344\ 8$
$= 200$ (to 1 s.f.)

(e) $\dfrac{\pi(8.5^2 - 7.5^2) \times 26}{169.8}$
$= 7.6967\ldots$
$= 8$ (to 1 s.f.)

(f) $\dfrac{\sqrt{24.997} \times 28.0349}{19.897}$
$= 7.044\ 58\ldots$
$= 7$ (to 1 s.f.)

(g) $\dfrac{2905 \times (0.512)^3}{0.004\ 987}$
$= 78\ 183.77\ldots$
$= 80\ 000$ (to 1 s.f.)

(h) $\dfrac{59.701 + 41.098}{\sqrt[3]{998.07}}$
$= 10.086\ 393\ 09\ldots$
$= 10$ (to 1 s.f.)

(i) $\dfrac{4.311 - 2.9016}{\sqrt[3]{981} \times 0.0231}$
$= 6.140\ 437\ 069\ldots$
$= 6$ (1 s.f.)

(j) $\dfrac{(20.315)^3 - \sqrt{82.0548}}{\sqrt[3]{85.002 - 21.997}}$
$= 2104.695\ 751\ldots$
$= 2000$ (to 1 s.f.)

30. (i) $\dfrac{12.01 \times 4.8}{2.99}$

$\approx \dfrac{12 \times 4.8}{3.0}$

$= 19.2$

$= 20$ (to 1 s.f.)

(ii) $\dfrac{12.01 \times 0.048}{0.299}$

$\approx \dfrac{12 \times 4.8 \div 100}{3.0 \div 10}$

$= 20 \div 10$

$= 2$

31. (a) (i) 24.988 = 25 (to 2 s.f.)

(ii) 39.6817 = 40 (to 2 s.f.)

(iii) 198.97 = 200 (to 2 s.f.)

(b) $\dfrac{\sqrt{24.988} \times 39.6817}{198.97}$

$\approx \dfrac{\sqrt{25} \times 40}{200}$

$= \dfrac{5 \times 40}{200}$

$= 1$ (to 1 s.f.)

32. (a) $\dfrac{17.47 \times 6.87}{5.61 - 3.52}$

$= 57.425\ 311$

$= 57.425$ (to 5 s.f.)

(b) $\dfrac{1.743 \times 5.3 \times 2.9454}{(11.71)^2}$

$= 0.198\ 428\ 362\ldots$

$= 0.198\ 43$ (to 5 s.f.)

(c) $7.593 - 6.219 \times \dfrac{1.47}{(1.4987)^3}$

$= 4.877\ 225\ 103\ldots$

$= 4.8772$ (to 5 s.f.)

(d) $\dfrac{119.73 - 13.27 \times 4.711}{88.77 \div 66.158}$

$= 42.640\ 891\ 68\ldots$

$= 42.641$ (to 5 s.f.)

(e) $\left(\dfrac{32.41 - 10.479}{7.218}\right) \times \left(\dfrac{4.7103 \times 21.483}{8.4691}\right)$

$= 36.303\ 441\ 14\ldots$

$= 36.303$ (to 5 s.f.)

(f) $\dfrac{(0.629)^2 - \sqrt{7.318}}{2.873}$

$= -0.803\ 877\ 207\ldots$

$= -0.803\ 88$ (to 5 s.f.)

(g) $\sqrt[3]{\dfrac{11.84 \times 0.871}{0.9542}}$

$= 2.210\ 939\ 278\ldots$

$= 2.2109$ (to 5 s.f.)

(h) $\dfrac{7.295 - \sqrt{7.295}}{(7.295)^2} + \dfrac{(6.98)^3 - 6.98}{\sqrt[3]{6.98}}$

$= 0.086\ 327\ 152 + 174.290\ 757\ 4$

$= 174.377\ 084\ 6\ldots$

$= 174.38$ (to 5 s.f.)

33. (a) (i) 271.569 = 270 (to 2 s.f.)

(ii) 9.9068 = 10 (to the nearest whole number)

(iii) 3.0198 = 3.0 (to 1 d.p.)

(b) $\dfrac{271.569 \times (9.9068)^2}{(3.0198)^3}$

$\approx \dfrac{270 \times (10)^2}{(3.0)^3}$

$= \dfrac{270 \times 100}{27}$

$= 1000$ (to 1 s.f.)

(c) $\dfrac{271.569 \times (9.9068)^2}{(3.0198)^3}$

$= 967.859\ 777\ 4\ldots$

$= 970$ (to 2 s.f.)

(d) No, the answers are close but not the same. The estimated value is 30 more than the actual value.

34. (a) Perimeter of the rectangular sheet of metal

$= 2(9.96 + 5.08)$

$= 2(15.04)$

$= 30.08$

$= 30$ m (to 1 s.f.)

(b) Area of rectangular sheet of metal

$= 9.96 \times 5.08$

$= 50.5968$

$= 50.6\ m^2$

35. (a) Smallest possible number of customers = 250

(b) Largest possible number of customers = 349

36. Total number of students that the school can accommodate

$= 33 \times 37$

$= 1221$

$= 1200$ (to 2 s.f.)

The school can accommodate approximately 1200 students.

37. Number of pens bought
$= 815 \div 85$
$= 9.588\ldots$
$= 9$ (to 1 s.f.)
The greatest number of pens that he can buy is 9.

38. (i) Thickness of each piece of paper
$= \dfrac{60 \div 10}{500}$
$= \dfrac{6}{500}$
$= 0.012$
$= 0.01$ cm (to 1 d.p.)

(ii) Thickness of a piece of paper
$= 0.012$ cm
$= 0.000\ 12$ m
$= 0.0001$ m (to 1 s.f.)

39. (i) Length of the carpet
$= \dfrac{11.9089}{4.04}$
$= 2.947\ 747\ 525\ldots$
$= 2.95$ m (to 3 s.f.)

(ii) Perimeter of the carpet
$\approx 2(2.9477 + 4.04)$
$= 2(6.9877)$
$= 13.9754$
$= 13.98$ m (to 4 s.f.)

40. (i) $18\ 905 = 19\ 000$ (to 2 s.f.)

(ii) Cost of each ticket
$= \dfrac{7\,000\,000}{19\,000}$
$= \dfrac{7\,000}{19}$
$\approx 368.421\ 052\ 6$
= PKR 368 (to the nearest Rupees)

41. (a) (i) Radius
$= 497$
$= 500$ mm (to 2 s.f)
Circumference of circle
$= 2\pi(500)$
$= 1000\pi$
$= 3141.59\ldots$
$= 3000$ mm (to 1 s.f.)

(ii) Radius
$= 5.12$
$= 5.1$ m (to 2 s.f.)
Circumference of circle
$= 2\pi(5.1)$
$= 10.2\pi$
$= 32.044\ldots$
$= 30$ m (to 1 s.f.)

(b) (i) Radius
$= 10.09$
$= 10$ m (to 2 s.f.)
Area of circle
$= \pi(10)^2$
$= 100\pi$
$= 314.159\ldots$
$= 300\ m^2$ (to 1 s.f.)

(ii) Radius
$= 98.4$
$= 98$ mm (to 2 s.f.)
Area of circle
$= \pi(98)^2$
$= 9604\pi$
$= 30\ 171.855\ \ldots$
$= 30\ 000\ mm^2$ (to 1 s.f.)

42. Total cost of 20-paisa coins
$= 31 \times 0.2$
= PKR 6.20
Total cost of 5-paisa coins
= PKR 7.35 – PKR 6.20
= PKR 1.15
Number of 5-paisa coins
$= \dfrac{1.15}{0.05}$
$= \dfrac{1.2}{0.05}$ (to 2 s.f.)
$= \dfrac{120}{5}$
$= 24$
There are about 24 5-paisa coins in the box.

43. Total amount that Lixin has to pay
$= 18 \times (0.99 \div 3) + 1.2 \times 1.5 + 2 \times 0.81 + 2.2 \times 3.4$
$= 18 \times 0.33 + 1.2 \times 1.5 + 2 \times 0.8 + 2.2 \times 3.4$
$= 5.94 + 1.8 + 1.6 + 7.48$
= PKR 16.84
The total amount she has to pay, to the nearest Rupees, is PKR 17.

44. For option *A*,
700 m*l* costs about PKR 4.00.
For option *B*,
1400 m*l* costs PKR 8.90.
Thus 700 m*l* will cost about $(8.90 \div 2)$ = PKR 4.45
For option *C*,
950 m*l* costs PKR 9.90.
Thus 700 m*l* will cost about $(9.90 \div 950) \times 700$
$\approx$ PKR 7.00
$\therefore$ Option *A* is better value for money.

Advanced

45. (i) $a = kb$

When $b = 15$, $a = 75$,

$75 = k(15)$

$k = \frac{75}{15}$

$= 5$

$\therefore a = 5b$

When $b = 37.5$,

$a = 5(37.5)$

$= 187.5$

(ii) When $a = 195$,

$195 = 5b$

$b = \frac{195}{5}$

$= 39$

46. $h = kl$

When $l = 36$, $h = 30$,

$30 = k(36)$

$k = \frac{30}{36}$

$= \frac{5}{6}$

$\therefore h = \frac{5}{6} l$

When $h = 15$,

$15 = \frac{5}{6} l$

$l = \frac{6}{5} \times 15$

$= 18$

When $l = 72$,

$h = \frac{5}{6} (72)$

$= 60$

When $h = 75$,

$75 = \frac{5}{6} l$

$l = \frac{6}{5} \times 75$

$= 90$

h	15	30	60	75
l	18	36	72	90

47. (i) $w = kt$

When $t = 0.3$, $w = 1.8$,

$1.8 = k(0.3)$

$k = \frac{1.8}{0.3}$

$= 6$

$\therefore w = 6t$

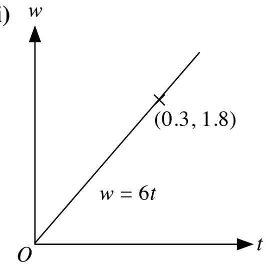
(ii) When $t = 2.5$,

$w = 6(2.5)$

$= 15$

$\therefore$ 15 g of silver will be deposited.

(iii)



48. (i) $F = km$

When $m = 250$, $F = 60$,

$60 = k(250)$

$k = \frac{60}{250}$

$= \frac{6}{25}$

$\therefore F = \frac{6}{25} m$

(ii) When $m = 300$,

$F = \frac{6}{25} (300)$

$= 72$

$\therefore$ The net force required is 72 newtons.

(iii) When $F = 102$,

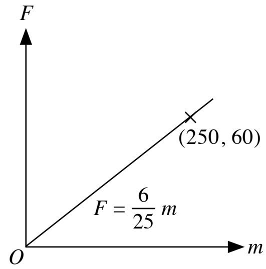
$102 = \frac{6}{25} m$

$m = \frac{25}{6} \times 102$

$= 425$

$\therefore$ The mass of the box is 425 kg.

(iv)



49. (i) $C = an + b$

When $n = 200$, $C = 55\ 000$,

$55\ 000 = 200a + b$ —(1)

When $n = 500$, $C = 62\ 500$,

$62\ 500 = 500a + b$ —(2)

(2) – (1): $300a = 7500$

$a = \frac{7500}{300}$

$= 25$

Substitute $a = 25$ into (1):

$200(25) + b = 55\ 000$

$5000 + b = 55\ 000$

$b = 55\ 000 - 5000$

$= 50\ 000$

$\therefore\ a = 25,\ b = 50\ 000$

(ii) $C = 25n + 50\ 000$

When $n = 420$,

$C = 25(420) + 50\ 000$

$= 60\ 500$

$\therefore$ The total cost is PKR 60 500.

(iii) When $C = 70\ 000$,

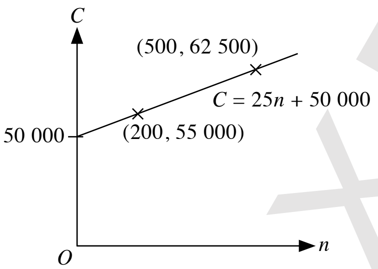
$70\ 000 = 25n + 50\ 000$

$25n = 20\ 000$

$n = \frac{20\ 000}{25}$

$= 800$

(iv)



No, C is not directly proportional to n since the graph of C against n does not pass through the origin.

50. (i) Annual premium payable

$= \text{PKR } 25 + \frac{\text{PKR } 20\ 000}{\text{PKR } 1000} \times \text{PKR } 2$

$= \text{PKR } 65$

(ii) Face value $= (\text{PKR } 155 - \text{PKR } 25) \times \frac{\text{PKR } 1000}{\text{PKR } 2}$

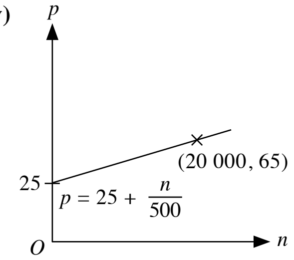
$= \text{PKR } 65\ 000$

(iii) $p = 25 + \frac{n}{1000} \times 2$

$= 25 + \frac{2n}{1000}$

$= 25 + \frac{n}{500}$

(iv)



No, p is not directly proportional to n since the graph of p against n does not pass through the origin.

51. (i) $n = km^3$

When $m = 1\frac{1}{2}$, $n = 27$,

$27 = k\left(\frac{3}{2}\right)^3$

$= \frac{27}{8}k$

$k = 8$

$\therefore\ n = 8m^3$

When $m = 2$,

$n = 8(2)^3$

$= 64$

(ii) When $n = 125$,

$125 = 8m^3$

$m^3 = \frac{125}{8}$

$m = \frac{5}{2}$

$= 2\frac{1}{2}$

52. Number of workers to complete in 1 day $= 6 \times 8$

$= 48$

Number of workers to complete in 12 days $= \frac{48}{12}$

$= 4$

53.

	Number of girls	Number of paper cranes		Number of minutes	
	8	5		6	
			$\times 24$		$\times 24$
	8	120		144	
$\div 8$					$\times 8$
	1	120		1152	
$\times 36$					$\div 36$
	36	120		32	

$\therefore$ 36 girls take 32 minutes to fold 120 paper cranes. Assume that all the girls have the same rate of folding paper cranes.

54. $P = \frac{k}{V}$

When $V = 2$, $P = 500$,

$500 = \frac{k}{2}$

$k = 500 \times 2$

$= 1000$

$\therefore P = \frac{1000}{V}$

When $V = 5$,

$P = \frac{1000}{5}$

$= 200$

$\therefore$ The pressure of the gas is 200 pascals.

55. (a) 406 A45 when correct to 3 significant figures is 406 000, so $A < 5$.

$\therefore$ The maximum prime value of A is 3.

(b) 398 200 is the estimated value for

398 150 to 398 199, if corrected to 4 significant figures;

398 195 to 398 204, if corrected to 5 significant figures;

398 200.1 to 398 200.4, if corrected to 6 significant figures.

$\therefore m = 4, 5$ or 6

56. 2000 is the estimated value for 1999 to 2004, if corrected to 1, 2 and 3 significant figures.

$\therefore$ The smallest number is 1999 and the largest number is 2004.

57. Rp 7872.5300 = S\$1

Rp 8000 $\approx$ S\$1

Price of cup noodle in Rp

= Rp 27 800

$\approx$ Rp 28 000

Price of cup noodle in S\$ = S\$ $\frac{28\,000}{8000}$

= S\$3.50

The cup noodle costs S\$3.50.

58. $\dfrac{\sqrt{16\,500.07 \times 39.59 - \left(119\,999.999 + \dfrac{485\,200.023}{(2.6)^2}\right)}}{\sqrt[3]{1.02 \times (13.5874 + 19.0007)^2 - 99.998}}$

$\approx \dfrac{\sqrt{17\,000 \times 40 - \left(120\,000 + \dfrac{490\,000}{(2.6)^2}\right)}}{\sqrt[3]{1.0 \times (14 + 20)^2 - 100}}$

$= \dfrac{\sqrt{17\,000 \times 40 - \left(120\,000 + \dfrac{490\,000}{6.76}\right)}}{\sqrt[3]{989}}$

$\approx \dfrac{\sqrt{680\,000 - \left(120\,000 + \dfrac{490\,000}{7}\right)}}{\sqrt[3]{1000}}$

(Note: 6.76 and 989 are estimated so that the division and cube root can be carried out, without the use of calculator)

$= \dfrac{\sqrt{680\,000 - 190\,000}}{10}$

$= \dfrac{\sqrt{490\,000}}{10}$

$= \dfrac{700}{10}$

= 70 (to 1 s.f.)

New Trend

59. Arranging in ascending order,

$0.85^{\frac{3}{2}}, \frac{\pi}{4}, \sqrt{0.64}, 0.801$

60. **(a)** $\frac{16.8^5}{3(7.1) - 1.55} \approx 67\ 760$

(b) 67 760 = 67 800 (to 3 s.f.)

61. **(a)** $\frac{(0.984\ 52)^3 \times \sqrt{2525}}{102.016}$

$\approx \frac{(1.0)^3 \times \sqrt{2500}}{100}$

= 0.5 (to 1 s.f.)

(b) $\frac{(0.984\ 52)^3 \times \sqrt{2525}}{102.016}$

= 0.470 041 311

= 0.47 (to 2 s.f.)

62. $\sqrt[3]{\frac{(1.92)^2}{(4.3)^3 - \sqrt{4.788}}}$

= 0.362 609 371

= 0.362 61 (to 5 s.f.)

63. **(a)** 8.5 kg

(b) Greatest possible mass of 1 m^3 of wood

$= \frac{9.5}{2.5}$

= 3.8 kg

Chapter 2 Financial Transactions

Basic

1. (a) (i) Simple interest $= 6\%$ of PKR 700

$= \frac{6}{100} \times 700$

$=$ PKR 42

Simple interest for 5 years

$= 5 \times$ PKR 42 = PKR 210

(ii) Total amount of money loaned after 5 years

= PKR 700 + PKR 210

= PKR 910

(b) (i) Simple interest $= 8\%$ of PKR 360

$= \frac{8}{100} \times 360$

= PKR 28.80

(ii) Simple interest for 3.5 years $= 3.5 \times$ PKR 28.80

= PKR 100.80

Total amount of money loaned after 3.5 years

= PKR 360 + PKR 100.80

= PKR 460.80

(c) (i) Simple interest $= 4\frac{1}{4}\%$ of PKR 480

$= \frac{4\frac{1}{4}}{100} \times 480$

= PKR 20.40

Convert 4 years and 8 months to years.

4 years and 8 months $= 4 + \frac{8}{12}$

$= 4\frac{2}{3}$ years

Simple interest for $4\frac{2}{3}$ years

$= 4\frac{2}{3} \times$ PKR 20.40

= PKR 95.20

(ii) Total amount of money loaned after 5 years

= PKR 480 + PKR 95.20

= PKR 575.20

(d) (i) Simple interest $= 9\frac{3}{8}\%$ of PKR 1600

$= \frac{9\frac{3}{8}}{100} \times 1600$

= PKR 150

Convert 18 months to years.

18 months $= \frac{18}{12}$

$= 1\frac{1}{2}$ years

Simple interest for $1\frac{1}{2}$ years

$= 1\frac{1}{2} \times$ PKR 150

= PKR 225

(ii) Total amount of money loaned after 18 months

= PKR 1600 + PKR 225

= PKR 1825

2. Amount of interest given to Ahsan

= PKR 5355 – PKR 4500

= PKR 855

Let T years denote the time taken for the investment to grow to PKR 5355.

$$855 = \frac{4500 \times 4\frac{3}{4} \times T}{100}$$

$$855 = 213.75 \times T$$

3. (a) $A = 2500\left(1 + \frac{3}{100}\right)^2$

= PKR 2652.25

I = PKR 2652.25 – PKR 2500

= PKR 152.25

(b) $A = 2500\left(1 + \frac{\frac{3}{12}}{100}\right)^{24}$

= PKR 2654.39 (to 2 d.p.)

I = PKR 2654.39 – PKR 2500

= PKR 154.39 (to 2 d.p.)

4. Let the initial invested amount be PKR P.

$$I = \frac{PRT}{100}$$

$$25.20 = \frac{P \times 4 \times \frac{9}{12}}{100}$$

$$25.20 = 0.03P$$

$$P = 25.2 \div 0.03$$

$$= 840$$

For the new interest rate,

$$44.80 + 25.20 = \frac{840 \times x \times \frac{20}{12}}{100}$$

$$70 = 14x$$

$$x = 5$$

5. $A = 20\,000\left(1 + \frac{\frac{3.2}{12}}{100}\right)^{48}$

= PKR 22 727.19 (to 2 d.p.)

6. $A = 6050\left(1+\frac{\frac{4}{4}}{100}\right)^8$

= PKR 6551 (to the nearest dollar)

7. $28\ 121.60 = 25\ 000\left(1+\frac{r}{100}\right)^3$

$\left(1+\frac{r}{100}\right)^3 = 1.124\ 864$

$1 + \frac{r}{100} = \sqrt[3]{1.124\ 864}$

$\frac{r}{100} = \sqrt[3]{1.124\ 864} - 1$

$r = 4$

8. $P + 11\ 798.38 = P\left(1+\frac{\frac{6}{2}}{100}\right)^6$

$11\ 798.38 = P(1.03)^6 - P$

$= P(1.03^6 - 1)$

$P = \frac{11\ 798.38}{1.03^6 - 1}$

= PKR 60 800 (to the nearest rupee)

9. **(i)** Deposit = 25% of PKR 1300

$= \frac{25}{100} \times$ PKR 1300

= PKR 325

Remaining amount = PKR 1300 – PKR 325

= PKR 975

Amount of interest the man owes at the end of 1 year

= PKR 975 × $\frac{18}{100}$

= PKR 175.50

Amount of interest the man has to pay at the end of 2 years

= PKR 175.50 × 2

= PKR 351

Total amount to be paid in monthly instalments

= PKR 975 + PKR 351

= PKR 1326

Monthly instalment

$= \frac{\text{PKR } 1326}{24}$

= PKR 55.25

(ii) Total amount the man has to pay for the TV set

= PKR 325 + PKR 1326

= PKR 1651

(iii) Difference in the amount paid with hire purchase

= PKR 1651 – PKR 1300

= PKR 351

11. **(a)** Number of packets = $\frac{24\ 000}{4}$

= 6000

Total selling price = 6000 × PKR 1.20

= PKR 7200

(b) Costs of labour and materials = PKR 0.17 × 24 000

= PKR 4080

Total cost of production

= cost of administration

+ cost of labour and materials

= 1545 + 4080

= PKR 5625

Profit = 7200 – 5625

= PKR 1575

Percentage profit made = $\frac{1575}{5625} \times 100\%$

= 28%

(c) Number of packets = $\frac{212\ 000}{4}$

= 53 000

Total selling price = 53 000 × PKR 1.20

= PKR 63 600

$132\frac{1}{2}\%$ of the cost of production = PKR 63 600

Cost of production = $63\ 600 \div 132\frac{1}{2}\%$

$= 63\ 600 \div \frac{265}{2}\%$

$= 63\ 600 \times \frac{2}{265} \times 100$

= PKR 48 000

The cost of producing 212 000 rubber pieces is PKR 48 000.

12. **(i)** Selling price of the condominium

= 90% of PKR 950 000

$= \frac{90}{100} \times 950\ 000$

= PKR 855 000

(ii) Amount Mei Shan received after paying the agent

= 98% of PKR 855 000

$= \frac{98}{100} \times 855\ 000$

= PKR 837 900

(iii) Amount agent received from seller

= PKR 855 000 – PKR 837 900

= PKR 17 100

Amount agent received from buyer

= 5% of PKR 855 000

$= \frac{5}{100} \times 855\,000$

= PKR 42 750

Total amount received by the agent

= 42 750 + 17 100

= PKR 59 850

13. (i) Number of litres used $= \frac{\text{PKR } 3600}{\text{PKR } 2.00} = 1800$ litres

(ii) Total distance travelled = 1800 × 16

= 28 800 km

(iii) Total cost in 2011

= PKR 3600 + PKR 2000 + PKR 850 + PKR 880

= PKR 7330

(iv) Total cost in 2012

$= \text{PKR } 880 + \left(\text{PKR } 3600 \times \frac{100+5}{100}\right)$

$+ \left(\text{PKR } 850 \times \frac{100+15}{100}\right) + \left(\text{PKR } 2000 \times \frac{100-10}{100}\right)$

= PKR 880 + PKR 3780 + PKR 977.50 + PKR 1800

= PKR 7437.50

Increase = PKR 7437.50 – PKR 7330 = PKR 107.50

Percentage increase $= \frac{\text{PKR } 107.50}{\text{PKR } 7330} \times 100\%$

= 1.5% (to 2 s.f.)

14. (i) Total cash price

= PKR 580 + PKR 380 + PKR 140 + PKR 480 + PKR 240

= PKR 1820

(ii) (a) Deposit = 20% of PKR 1820

$= \frac{12}{100} \times \text{PKR } 1820$

= PKR 364

Remaining amount = PKR 1820 – PKR 364

= PKR 1456

Credit charge = 12% of PKR 1456

$= \frac{12}{100} \times \text{PKR } 1456$

= PKR 174.72

Total amount to be paid in instalments

= PKR 1456 + PKR 174.72

= PKR 1630.72

Monthly instalment

$= \frac{\text{PKR } 1630.72}{12}$

= PKR 135.893

= PKR 135.89 (to the nearest paisa)

(b) Total hire purchase = PKR 364 + PKR 1630.72

= PKR 1994.72

(iii) Total cash price after reduction

$= \left(\text{PKR } 580 \times \frac{100-10}{100}\right) + \left(\text{PKR } 380 \times \frac{100-5}{100}\right)$

$+ \left(\text{PKR } 480 \times \frac{100-3}{100}\right) + \text{PKR } 140 + \text{PKR } 240$

= PKR 522 + PKR 361 + PKR 465.60 + PKR 140 + PKR 240

= PKR 1728.60

15. (i) Number of litres of petrol required to drive around France

$= \frac{1920}{12}$

= 160 litres

(ii) Total cost of the petrol used in Euros

= €1.47 × 160

= €235.20

(iii) Total cost of the petrol in Singapore dollars

= €235.20 × S\$1.5599

= S\$366.888

= S\$367 (to the nearest dollar)

(iv) Cost of each adult ferry ticket in Singapore dollars

= £100 × S\$1.9399

= S\$193.99

16. $9004.07 = P\left(1 + \frac{3}{100}\right)^4$

$P = \frac{9004.07}{(1.03)^4}$

= PKR 8000 (to the nearest rupee)

17. Deposit = 20% of PKR 1299

$= \frac{20}{100} \times \text{PKR } 1299$

= PKR 259.80

Let PKR x be one monthly payment.

$1348.80 = 259.80 + 18x$

$18x = 1089$

$x = 60.5$

One monthly payment is PKR 60.50.

18. Extra charge for making monthly payments

= 8% of PKR 1280

= PKR 102.40

Monthly payment

$= \frac{\text{PKR } (1280 + 102.40)}{12}$

= PKR 115.20

19. If he changes in Singapore,

S$1 = £0.51

S$2400 = 2400 × 0.51

= £1224

If he changes in London,

S$2.02 = £1

S$2400 = $\frac{2400}{2.02}$

= £1188.1188 (to 4 d.p.)

Difference in the amount exchanged

= 1224 – 1188.1188

= £35.88 (to the nearest pound)

20. **(a)** PKR 1 = S$1.38

PKR 500 = S$(500 × 1.38)

= S$690

(b) S$1.38 = PKR 1

S$800 = $\frac{\text{S\$800}}{\text{S\$1.38}}$ × PKR 1

= PKR 579 $\frac{49}{69}$

PKR 1.12 = €1

PKR 579 $\frac{49}{69}$ = $\frac{\text{US\$579}\frac{49}{69}}{\text{US\$1.12}}$ × €1

= €518 (to the nearest euro)

21. **(a)** $A = 1668\left(1+\frac{2.6}{100}\right)^3$

= PKR 1801.52 (to 2 d.p.)

I = 1801.52 – 1668

= PKR 133.52

(b) Amount to be paid in euros = 799 + $\left(\frac{0.8}{100} \times 799\right)$

= €805.392

€0.65 = S$1

€805.632 = S$ $\frac{805.632}{0.65}$

= S$1239.43 (to the nearest cent)

Chapter 3 Expansion and Factorisation of Algebraic Expressions

Basic

1. **(a)** Since the common difference is 5, $T_n = 5n + ?$.
The term before T_1 is $c = T_0 = 12 - 5 = 7$.
$\therefore$ General term of the sequence, $T_n = 5n + 7$

(b) Since the common difference is -6, $T_n = -6n + ?$.
The term before T_1 is $c = T_0 = 83 + 6 = 89$.
$\therefore$ General term of the sequence, $T_n = -6n + 89$.

(c) Since the common difference is 7, $T_n = 7n + ?$.
The term before T_1 is $c = T_0 = 2 - 7 = -5$.
$\therefore$ General term of the sequence, $T_n = 7n - 5$.

(d) Since the common difference is 6, $T_n = 6n + ?$.
The term before T_1 is $c = T_0 = 7 - 6 = 1$.
$\therefore$ General term of the sequence, $T_n = 6n + 1$.

(e) Since the common difference is -4, $T_n = -4n + ?$.
The term before T_1 is $c = T_0 = 39 + 4 = 43$.
$\therefore$ General term of the sequence, $T_n = -4n + 43$.

(f) To find the formula, consider the following:
1, 2, 4, 8, 16, …
as $2^0, 2^1, 2^2, 2^3, 2^4, \ldots$
$\therefore$ General term of the sequence, $T_n = 2^{n-1}$,
$n = 1, 2, 3, \ldots$

(g) To find the formula, consider the following:
2, 6, 18, 54, 162, …
as $2 \times 3^0, 2 \times 3^1, 2 \times 3^2, 2 \times 3^3, 2 \times 3^4, \ldots$
$\therefore$ General term of the sequence, $T_n = 2 \times 3^{n-1}$,
$n = 1, 2, 3, \ldots$

(h) To find the formula, consider the following:
12, 36, 108, 324, 972, …
as $4 \times 3, 4 \times 3^2, 4 \times 3^3, 4 \times 3^4, 4 \times 3^5, \ldots$
$\therefore$ General term of the sequence, $T_n = 4 \times 3^n$,
$n = 1, 2, 3, \ldots$

(i) To find the formula, consider the following:
2000, 1000, 500, 250, 125, …
as $\frac{4000}{2}, \frac{4000}{2^2}, \frac{4000}{2^3}, \frac{4000}{2^4}, \frac{4000}{2^5}, \ldots$
$\therefore$ General term of the sequence, $T_n = \frac{4000}{2^n}$,
$n = 1, 2, 3, \ldots$

2. **(i)** The next two terms of the sequence are 96 and 192.

(ii) To find the formula, consider the following:
$3, 3 \times 2, 6 \times 2, 12 \times 2, 24 \times 2, \ldots$
$3 \times 2^0, 3 \times 2, 3 \times 2^2, 3 \times 2^3, 3 \times 2^4, \ldots$
$\therefore$ General term of the sequence, $T_n = 3 \times 2^{n-1}$

(iii) Let $3 \times 2^{m-1} = 1536$
$$2^{m-1} = \frac{1536}{3} = 512$$
By trial and error, $2^9 = 512$
$\therefore m - 1 = 9$
$m = 9 + 1 = 10$

4. **(i)**

```
  E
D E
D E
D E
D E
D E
D E
D E
  E
```

(ii)

Letter	Number of Letters
A	$2(1) - 1 = 1$
B	$2(2) - 1 = 3$
C	$2(3) - 1 = 5$
D	$2(4) - 1 = 7$
E	$2(5) - 1 = 9$
⋮	⋮
n^{th} letter	Tn

(iii) For the letter J, $2(10) - 1 = 19$.

(iv) Since the common difference is 2, $T_n = 2n + ?$.
The term before T_1 is $c = T_0 = 1 - 2 = -1$.
$\therefore$ General term of the sequence, $T_n = 2n - 1$.
$2n - 1 = 29$
$2n = 29 + 1$
$2n = 30$
$n = 15$
When $n = 15$, it is the letter O.

5. **(a)** $(a + 5)^2$
$= a^2 + 10a + 25$

(b) $(2b + 3)^2$
$= 4b^2 + 12b + 9$

(c) $(c + 6d)^2$
$= c^2 + 12cd + 36d^2$

(d) $(7e + 4f)^2$
$= 49e^2 + 56ef + 16f^2$

6. **(a)** $(a - 8)^2$
$= a^2 - 16a + 64$

(b) $(4b - 1)^2$
$= 16b^2 - 8b + 1$

(c) $(c - 3d)^2$
$= c^2 - 6cd + 9d^2$

(d) $(9e - 2f)^2$
$= 81e^2 - 36ef + 4f^2$

7. **(a)** $(a + 6)(a - 6)$
$= a^2 - 36$

(b) $(4b + 3)(4b - 3)$
$= 16b^2 - 9$

(c) $(9 + 4c)(9 - 4c)$
$= 81 - 16c^2$

(d) $(5d + e)(5d - e)$
$= 25d^2 - e^2$

8. (a) 904^2
$= (900 + 4)^2$
$= 900^2 + 2(900)(4) + 4^2$
$= 810\,000 + 7200 + 16$
$= 817\,216$

(b) 791^2
$= (800 - 9)^2$
$= 800^2 - 2(800)(9) + 9^2$
$= 640\,000 - 14\,400 + 81$
$= 625\,681$

(c) 603×597
$= (600 + 3)(600 - 3)$
$= 600^2 - 3^2$
$= 360\,000 - 9$
$= 359\,991$

(d) 99×101
$= (100 - 1)(100 + 1)$
$= 100^2 - 1^2$
$= 10\,000 - 1$
$= 9999$

9. $(a + b)^2 = a^2 + 2ab + b^2$
$73 = a^2 + b^2 + 2(65)$
$= a^2 + b^2 + 130$
$a^2 + b^2 = 73 - 130$
$= -57$

10. (a) $a^2 + 12a + 36$
$= (a + 6)^2$

(b) $9b^2 + 12b + 4$
$= (3b + 2)^2$

(c) $4c^2 + 4cd + d^2$
$= (2c + d)^2$

(d) $16e^2 + 40ef + 25f^2$
$= (4e + 5f)^2$

11. (a) $a^2 - 18a + 81$
$= (a - 9)^2$

(b) $25b^2 - 20b + 4$
$= (5b - 2)^2$

(c) $9c^2 - 6cd + d^2$
$= (3c - d)^2$

(d) $49e^2 - 28ef + 4f^2$
$= (7e - 2f)^2$

12. (a) $a^2 - 196$
$= a^2 - 14^2$
$= (a + 14)(a - 14)$

(b) $4b^2 - 81$
$= (2b)^2 - 9^2$
$= (2b + 9)(2b - 9)$

(c) $289 - 36c^2$
$= 17^2 - (6c)^2$
$= (17 + 6c)(17 - 6c)$

(d) $9d^2 - e^2$
$= (3d)^2 - e^2$
$= (3d + e)(3d - e)$

Intermediate

13. (i) The next two terms of the sequence are 642 and 621.

(ii) Since the common difference is –21,
$T_n = -21n + ?$.
The term before T_1 is $c = T_0 = 747 + 21 = 768$.
$\therefore$ General term of the sequence, $T_n = 768 - 21n$.

(iii) $768 - 21r = 390$
$21r = 768 - 390$
$= 378$
$r = 18$

14. (a) When $n = 1$, $2(1)^2 - 3(1) + 5 = 4$
When $n = 2$, $2(2)^2 - 3(2) + 5 = 7$
When $n = 3$, $2(3)^2 - 3(3) + 5 = 14$
When $n = 4$, $2(4)^2 - 3(4) + 5 = 25$
The first four terms of the sequence are 4, 7, 14 and 25.

(b) (i) Comparing the two sequences, the common difference between two sequences is –3.
Since the formula for the sequence in part **(a)** is $2n^2 - 3n + 5$, then the formula for the sequence is $2n^2 - 3n + 5 - 3 = 2n^2 - 3n + 2$.

(ii) When $n = 385$,
$2(385)^2 - 3(385) + 2$
$= 295\,297$.

15. (i) 5^{th} line: $n = 5$, $6 \times 5 - 10 = 20$

(ii) Note that the product is the value of n and the value of 1 more than n.
$\therefore a = 29$
The value of b is an even number and it is the product of n and 2.
$\therefore b = 28 \times 2 = 56$
The value of c is $29 \times 28 - 56 = 756$.

(iii) When $n = 50$,
$51 \times 50 - 50 \times 2 = 2450$

16. (i) 7^{th} line: $7^3 - 7 = 336 = (7 - 1) \times 7 \times (7 + 1)$

(ii) 1320 is divisible by 10. Thus the factors of 1320 are 10, 11 and 12.

$1320 = (11 - 1) \times 11 \times (11 + 1)$

$\therefore n = 11$

(iii) $19^3 - 19 = (19 - 1) \times 19 \times (19 + 1) = 6840$

17. (a)

Figure Number	Number of Dots	Number of Small Right-Angled Triangles
1	4	2
2	9	8
3	16	18
4	25	32
⋮	⋮	⋮
10	121	200
⋮	⋮	⋮
19	400	722
⋮	⋮	⋮
n	x	y

(b) (i) $x = (n + 1)^2$

(ii) $y = 2n^2$

18. (i)

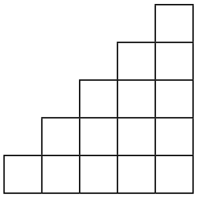


Figure 5

(ii) When $n = 5$,

Height of figure = 5

Number of squares $= 5 + 4 + 3 + 2 + 1$

$= \dfrac{5(5 + 1)}{2} = 15$

When $n = 6$,

Height of figure = 6

Number of squares $= 6 + 5 + 4 + 3 + 2 + 1$

$= \dfrac{6(6 + 1)}{2} = 21$

When $n = n$,

Number of squares $= n + (n - 1) + (n - 2) + \ldots + 3 + 2 + 1$

$= \dfrac{n(n + 1)}{2}$

19. (a) $\left(a + \dfrac{b}{3}\right)^2$

$= a^2 + \dfrac{2ab}{3} + \dfrac{b^2}{9}$

(b) $(0.5c + d)^2$

$= 0.25c^2 + cd + d^2$

(c) $(ef + 2)^2$

$= e^2f^2 + 4ef + 4$

(d) $\left(g + \dfrac{2}{g}\right)^2$

$= g^2 + 4 + \dfrac{4}{g^2}$

(e) $(h^2 + 3)^2$

$= h^4 + 6h^2 + 9$

(f) $(k^3 + 4)^2$

$= k^6 + 8k^3 + 16$

(g) $\left(\dfrac{2}{p} + \dfrac{3}{q}\right)^2$

$= \dfrac{4}{p^2} + \dfrac{12}{pq} + \dfrac{9}{q^2}$

(h) $\left(\dfrac{x}{y} + 3y\right)^2$

$= \dfrac{x^2}{y^2} + 6x + 9y^2$

20. (a) $\left(3a - \dfrac{1}{4}b\right)^2$

$= 9a^2 - \dfrac{3}{2}ab + \dfrac{1}{16}b^2$

(b) $(10c - 0.1d)^2$

$= 100c^2 - 2cd + 0.01d^2$

(c) $(2ef - 1)^2$

$= 4e^2f^2 - 4ef + 1$

(d) $\left(2h - \dfrac{1}{h}\right)^2$

$= 4h^2 - 4 + \dfrac{1}{h^2}$

(e) $(p^4 - 2)^2$

$= p^8 - 4p^4 + 4$

(f) $\left(\dfrac{x}{y} - \dfrac{y}{x}\right)^2$

$= \dfrac{x^2}{y^2} - 2 + \dfrac{y^2}{x^2}$

21. (a) $\left(\dfrac{1}{2}a + b\right)\left(\dfrac{1}{2}a - b\right)$

$= \dfrac{1}{4}a^2 - b^2$

(b) $(0.2c + d)(d - 0.2c)$

$= (d + 0.2c)(d - 0.2c)$

$= d^2 - 0.04c^2$

(c) $(3ef + 4)(3ef - 4)$

$= 9e^2f^2 - 16$

(d) $\left(\frac{g}{2}-\frac{h}{4}\right)\left(\frac{h}{4}+\frac{g}{2}\right)$

$= \left(\frac{g}{2}+\frac{h}{4}\right)\left(\frac{g}{2}-\frac{h}{4}\right)$

$= \frac{g^2}{4} - \frac{h^2}{16}$

22. $x^2 - y^2 = 6$

$(x + y)(x - y) = 6$

$2(x + y) = 6$

$x + y = 3$

$\therefore (x + y)^2 = 9$

23. (i) $(x + y)^2 = x^2 + 2xy + y^2$

$= 43 + 24$

$= 67$

(ii) $(2x - 2y)^2 = 4x^2 - 8xy + 4y^2$

$= 4(43) - 2(48)$

$= 76$

24. (i) $x^2 - 4y^2 = (x + 2y)(x - 2y)$

$= (-2)(18)$

$= -36$

(ii) $x + 2y = -2$ —(1)

$x - 2y = 18$ —(2)

(1) + (2): $2x = 16$

$x = 8$

(1) – (2): $4y = -20$

$y = -5$

$\therefore x^2 + 4y^2 = 8^2 + 4(-5)^2$

$= 164$

25. (i) $a^2 - b^2 = (a + b)(a - b)$

(ii) $2030^2 - 2029^2 + 2028^2 - 2027^2$

$= (2030 + 2029)(2030 - 2029)$

$+ (2028 + 2027)(2028 - 2027)$

$= 2030 + 2029 + 2028 + 2027$

$= 8114$

26. (a) $4a^2 + 32a + 64$

$= 4(a^2 + 8a + 16)$

$= 4(a + 4)^2$

(b) $\frac{1}{4}b^2 + 4bc + 16c^2$

$= \left(\frac{1}{2}b+4c\right)^2$

(c) $\frac{1}{9}d^2 + \frac{4}{15}de + \frac{4}{25}e^2$

$= \left(\frac{1}{3}d+\frac{2}{5}e\right)^2$

(d) $f^4 + 8f^2 + 16$

$= (f^2 + 4)^2$

27. (a) $3a^2 - 36a + 108$

$= 3(a^2 - 12a + 36)$

$= 3(a - 6)^2$

(b) $64b^2 - 4bc + \frac{1}{16}c^2$

$= \left(8b-\frac{1}{4}c\right)^2$

(c) $e^2f^2 - 10ef + 25$

$= (ef - 5)^2$

(d) $\frac{1}{4}g^2 - \frac{1}{4}gh + \frac{1}{16}h^2$

$= \left(\frac{1}{2}g-\frac{1}{4}h\right)^2$

28. (a) $\frac{1}{4}a^2 - b^2$

$= \left(\frac{1}{2}a+b\right)\left(\frac{1}{2}a-b\right)$

(b) $4c^3 - 49c$

$= c(4c^2 - 49)$

$= c(2c + 7)(2c - 7)$

(c) $81ef^2 - 4eg^2$

$= e(81f^2 - 4g^2)$

$= e(9f + 2g)(9f - 2g)$

(d) $18h^3 - 8hk^2$

$= 2h(9h^2 - 4k^2)$

$= 2h(3h + 2k)(3h - 2k)$

(e) $81m^5n^3 - 121m^3n^5$

$= m^3n^3(81m^2 - 121n^2)$

$= m^3n^3(9m + 11n)(9m - 11n)$

(f) $p^4 - 81q^4$

$= (p^2 + 9q^2)(p^2 - 9q^2)$

$= (p^2 + 9q^2)(p + 3q)(p - 3q)$

(g) $(t^2 - 1)^2 - 9$

$= (t^2 - 1 + 3)(t^2 - 1 - 3)$

$= (t^2 + 2)(t^2 - 4)$

$= (t^2 + 2)(t + 2)(t - 2)$

(h) $9 - (a - b)^2$

$= (3 + a - b)(3 - a + b)$

(i) $(d + 2c)^2 - c^2$

$= (d + 2c + c)(d + 2c - c)$

$= (d + 3c)(d + c)$

(j) $(e - 3)^2 - 16f^2$

$= (e - 3 + 4f)(e - 3 - 4f)$

(k) $(3g - h)^2 - g^2$

$= (3g - h + g)(3g - h - g)$

$= (4g - h)(2g - h)$

(l) $4j^2 - (k - 2)^2$

$= (2j + k - 2)(2j - k + 2)$

(m) $9m^2 - (3m - 2n)^2$

$= (3m + 3n - 2n)(3m - 3m + 2n)$

$= (6m - 2n)(2n)$

$= 4n(3m - n)$

(n) $9p^2 - 4(p - 2q)^2$
$= (3p)^2 - (2p - 4q)^2$
$= (3p + 2p - 4q)(3p - 2p + 4q)$
$= (5p - 4q)(p + 4q)$

(o) $(3x - 2y)^2 - (2x - 3y)^2$
$= (3x - 2y + 2x - 3y)(3x - 2y - 2x + 3y)$
$= (5x - 5y)(x + y)$
$= 5(x + y)(x - y)$

29. (a) $41^2 + 738 + 81$
$= 41^2 + 2(41)(9) + 9^2$
$= (41 + 9)^2$
$= 50^2$
$= 2500$

(b) $65^2 + 650 + 25$
$= 65^2 + 2(65)(5) + 5^2$
$= (65 + 5)^2$
$= 70^2$
$= 4900$

(c) $92^2 - 368 + 4$
$= 92^2 - 2(92)(2) + 2^2$
$= (92 - 2)^2$
$= 90^2$
$= 8100$

(d) $201^2 - 402 + 1$
$= 201^2 - 2(201)(1) + 1^2$
$= (201 - 1)^2$
$= 200^2$
$= 40\ 000$

(e) $201^2 - 99^2$
$= (201 + 99)(201 - 99)$
$= (300)(102)$
$= 30\ 600$

(f) $1.013^2 - 0.013^2$
$= (1.013 + 0.013)(1.013 - 0.013)$
$= 1.026$

Advanced

30. (a) $2a^2b^2 + 4ab - 48 = 2(a^2b^2 + 2ab - 24)$

$\times$	ab	6
ab	a^2b^2	$6ab$
-4	$-4ab$	-24

$\therefore\ 2a^2b^2 + 4ab - 48 = 2(ab + 6)(ab - 4)$

(b) $15c^2d^2e - 77cde + 10e = e(15c^2d^2 - 77cd + 10)$

$\times$	$15cd$	-2
cd	$15c^2d^2$	$-2cd$
-5	$-75cd$	10

$\therefore\ 15c^2d^2e - 77cde + 10e = e(15cd - 2)(cd - 5)$

(c) $12p^2q^2r - 34pqr - 28r = 2r(6p^2q^2 - 17pq - 14)$

$\times$	$3pq$	2
$2pq$	$6p^2q^2$	$4pq$
-7	$-21pq$	-14

$\therefore\ 12p^2q^2r - 34pqr - 28r = 2r(3pq + 2)(2pq - 7)$

(d) $3x^2 + 7xy + \frac{15}{4}y^2 = \frac{1}{4}(12x^2 + 28xy + 15y^2)$

$\times$	$6x$	$5y$
$2x$	$12x^2$	$10xy$
$3y$	$18xy$	$15y^2$

$\therefore\ 3x^2 + 7xy + \frac{15}{4}y^2 = \frac{1}{4}(6x + 5y)(2x + 3y)$

31. $(x^2 - y)(x^2 + y)(x^4 + y^2)$
$= (x^4 - y^2)(x^4 + y^2)$
$= x^8 - y^4$

32. (a) $10^2 - 9^2 + 8^2 - 7^2 + 6^2 - 5^2 + 4^2 - 3^2 + 2^2 - 1^2$
$= (10 + 9)(10 - 9) + (8 + 7)(8 - 7) + (6 + 5)(6 - 5) + (4 + 3)(4 - 3) + (2 + 1)(2 - 1)$
$= 19 + 15 + 11 + 7 + 3$
$= 55$

(b) $2008^2 - 2007^2 + 2006^2 - 2005^2 + 2004^2 - 2003^2$
$= (2008 + 2007)(2008 - 2007)$
$+ (2006 + 2005)(2006 - 2005)$
$+ (2004 + 2003)(2004 - 2003)$
$= 2008 + 2007 + 2006 + 2005 + 2004 + 2003$
$= 12\ 033$

33. (a) $a(b - c) + bc - a^2$
$= ab - ac + bc - a^2$
$= ab + bc - a^2 - ac$
$= b(a + c) - a(a + c)$
$= (b - a)(a + c)$

(b) $25x^4 + \frac{9}{4}y^2z^2 - x^2z^2 - \frac{225}{4}x^2y^2$

$= \frac{1}{4}[100x^4 + 9y^2z^2 - 4x^2z^2 - 225x^2y^2]$

$= \frac{1}{4}[100x^4 - 4x^2z^2 + 9y^2z^2 - 225x^2y^2]$

$= \frac{1}{4}[4x^2(25x^2 - z^2) + 9y^2(z^2 - 25x^2)]$

$= \frac{1}{4}[4x^2(25x^2 - z^2) - 9y^2(25x^2 - z^2)]$

$= \frac{1}{4}(4x^2 - 9y^2)(25x^2 - z^2)$

$= \frac{1}{4}(2x + 3y)(2x - 3y)(5x + z)(5x - z)$

34. (i) $\frac{1}{3}xy + \frac{1}{4}x^2y - y^2 - \frac{1}{12}x^3$

$= \frac{1}{12}[4xy + 3x^2y - 12y^2 - x^3]$

$= \frac{1}{12}[4xy - 12y^2 + 3x^2y - x^3]$

$= \frac{1}{12}[4y(x - 3y) + x^2(3y - x)]$

$= \frac{1}{12}[4y(x - 3y) - x^2(x - 3y)]$

$= \frac{1}{12}(4y - x^2)(x - 3y)$

(ii) Let $x = 22$ and $y = 9$:

$\frac{1}{3} \times 22 \times 9 + \frac{1}{4} \times 484 \times 9 - 81 - \frac{1}{12} \times 10\,648$

$= \frac{1}{12}[4(9) - 22^2][22 - 3(9)]$

$= 186\frac{2}{3}$

New Trend

35. (a) (i) Next line is the 6^{th} line: $6^2 - 6 = 30$.

(ii) 8^{th} line: $8^2 - 8 = 56$

(iii) From the number pattern, we observe that

$1^2 - 1 = 1(1 - 1)$

$2^2 - 2 = 2(2 - 1)$

$3^2 - 3 = 3(3 - 1)$

$4^2 - 4 = 4(4 - 1)$

$5^2 - 5 = 5(5 - 1)$

⋮

n^{th} line: $n^2 - n = n(n - 1)$

(b) $139^2 - 139 = 139(139 - 1) = 19\,182$

36. (a) $27d^3 - 48d$

$= 3d(9d^2 - 16)$

$= 3d(3d + 4)(3d - 4)$

(b) $3x^2 - 75y^2$

$= 3(x^2 - 25y^2)$

$= 3(x + 5y)(x - 5y)$

Chapter 4 Graphs of Linear Equations and Simultaneous Linear Equations

Basic

1. **(a)** Take two points (0, 2) and (7, 2).
Vertical change (or rise) = 2 – 2 = 0
Horizontal change (or run) = 7 – 0 = 7
$\therefore$ Gradient $= \frac{\text{rise}}{\text{run}}$
$= \frac{0}{7} = 0$

(b) Take two points (7, 0) and (7, 7).
Vertical change (or rise) = 7 – 0 = 7
Horizontal change (or run) = 7 – 7 = 0
$\therefore$ Gradient $= \frac{\text{rise}}{\text{run}}$
$= \frac{7}{0}$ = undefined

(c) Take two points (0, 2) and (4, 6).
Vertical change (or rise) = 6 – 2 = 4
Horizontal change (or run) = 4 – 0 = 4
Since the line slopes upwards from the left to the right, its gradient is positive.
$\therefore$ Gradient $= \frac{\text{rise}}{\text{run}}$
$= \frac{4}{4} = 1$

(d) Take two points (4, 6) and (7, 0).
Vertical change (or rise) = 6 – 0 = 6
Horizontal change (or run) = 7 – 4 = 3
Since the line slopes downwards from the left to the right, its gradient is negative.
$\therefore$ Gradient $= \frac{\text{rise}}{\text{run}}$
$= -\frac{6}{3} = -2$

2. **(a)** Take two points (–3, 4) and (4, 4).
Vertical change (or rise) = 4 – 4 = 0
Horizontal change (or run) = 4 – (–3) = 7
$\therefore$ Gradient $= \frac{\text{rise}}{\text{run}}$
$= \frac{0}{7} = 0$

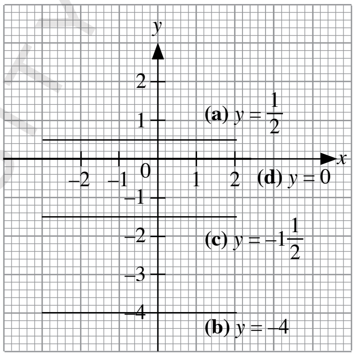
(b) Take two points (–3, –3) and (4, –3).
Vertical change (or rise) = –3 – (–3) = 0
Horizontal change (or run) = 4 – (–3) = 7
$\therefore$ Gradient $= \frac{\text{rise}}{\text{run}}$
$= \frac{0}{7} = 0$

(c) Take two points (–3, 4) and (–3, –3).
Vertical change (or rise) = 4 – (–3) = 7
Horizontal change (or run) = –3 – (–3) = 0
$\therefore$ Gradient $= \frac{\text{rise}}{\text{run}}$
$= \frac{7}{0}$ = undefined

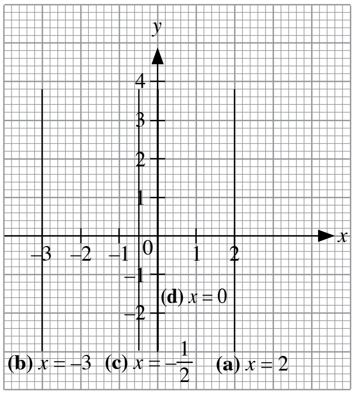
(d) Take two points (–4, 4) and (0, –3).
Vertical change (or rise) = 4 – (–3) = 7
Horizontal change (or run) = 0 – (–4) = 4
Since the line slopes downwards from the left to the right, its gradient is negative.
$\therefore$ Gradient $= \frac{\text{rise}}{\text{run}}$
$= -\frac{7}{4}$

(e) Take two points (0, –3) and (4, 4).
Vertical change (or rise) = 4 – (–3) = 7
Horizontal change (or run) = 4 – 0 = 4
Since the line slopes upwards from the left to the right, its gradient is positive.
$\therefore$ Gradient $= \frac{\text{rise}}{\text{run}}$
$= \frac{7}{4}$

3.



4.



5. **(a)** Line 1: $x = 1$
Line 2: $x = -1.2$
Line 3: $y = 2$
Line 4: $y = -2.6$

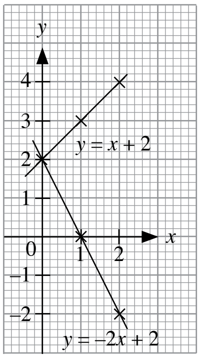
(b) Area enclosed = $(2.2)(4.6)$
$= 10.12$ units^2

6. **(a)** $y = x + 2$

x	0	1	2
y	2	3	4

$y = -2x + 2$

x	0	1	2
y	2	0	–2



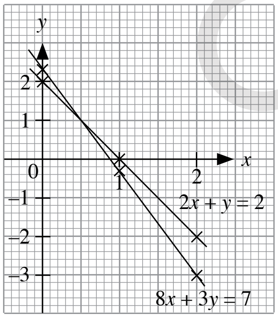
From the graph,
$x = 0$ and $y = 2$

(b) $8x + 3y = 7$

x	0	1	2
y	2.3	–0.3	–3

$2x + y = 2$

x	0	1	2
y	2	0	–2



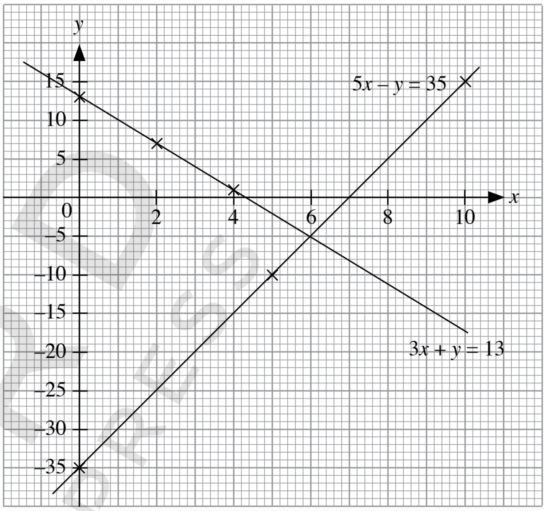
From the graph,
$x = \frac{1}{2}$ and $y = 1$.

(c) $3x + y = 13$

x	0	2	4
y	13	7	1

$5x - y = 35$

x	0	5	10
y	–35	–10	15



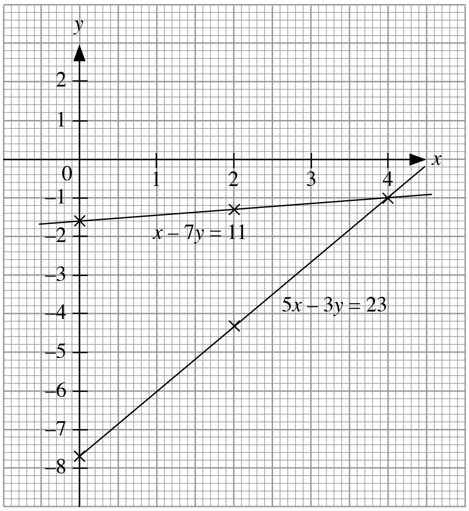
From the graph,
$x = 6$ and $y = -5$.

(d) $5x - 3y = 23$

x	0	2	4
y	–7.7	–4.3	–1

$x - 7y = 11$

x	0	2	4
y	–1.6	–1.3	–1



From the graph,
$x = 4$ and $y = -1$.

7. **(a)** $x + y = 7$ —(1)

$x - y = 3$ —(2)

(1) + (2): $2x = 10$

$x = 5$

Substitute $x = 5$ into (1):

$5 + y = 7$

$y = 2$

$\therefore x = 5, y = 2$

(b) $5x - 4y = 18$ —(1)

$3x + 2y = 13$ —(2)

(2) × 2: $6x + 4y = 26$ —(3)

(1) + (3): $11x = 44$

$x = 4$

Substitute $x = 4$ into (1):

$5(4) - 4y = 18$

$20 - 4y = 18$

$4y = 2$

$y = \frac{1}{2}$

$\therefore x = 4, y = \frac{1}{2}$

(c) $x + 3y = 7$ —(1)

$x + y = 3$ —(2)

(1) – (2): $2y = 4$

$y = 2$

Substitute $y = 2$ into (2):

$x + 2 = 3$

$x = 1$

$\therefore x = 1, y = 2$

(d) $3x - 5y = 19$ —(1)

$5x + 2y = 11$ —(2)

(1) × 2: $6x - 10y = 38$ —(3)

(2) × 5: $25x + 10y = 55$ —(4)

(3) + (4): $31x = 93$

$x = 3$

Substitute $x = 3$ into (2):

$5(3) + 2y = 11$

$15 + 2y = 11$

$2y = -4$

$y = -2$

$\therefore x = 3, y = -2$

(e) $3x - 4y = 30$ —(1)

$2x - 7y = 33$ —(2)

(1) × 2: $6x - 8y = 60$ —(3)

(2) × 3: $6x - 21y = 99$ —(4)

(3) – (4): $13y = -39$

$y = -3$

Substitute $y = -3$ into (2):

$2x - 7(-3) = 33$

$2x + 21 = 33$

$2x = 12$

$x = 6$

$\therefore x = 6, y = -3$

8. **(a)** $3x + y = 17$ —(1)

$3x - y = 19$ —(2)

From (1),

$y = 17 - 3x$ —(3)

Substitute (3) into (2):

$3x - (17 - 3x) = 19$

$3x - 17 + 3x = 19$

$6x = 36$

$x = 6$

Substitute $x = 6$ into (3):

$y = 17 - 3(6)$

$= -1$

$\therefore x = 6, y = -1$

(b) $2x - y = 3$ —(1)

$x + y = 0$ —(2)

From (1),

$y = 2x - 3$ —(3)

Substitute (3) into (2):

$x + (2x - 3) = 0$

$x + 2x - 3 = 0$

$3x = 3$

$x = 1$

Substitute $x = 1$ into (3):

$y = 2(1) - 3$

$= 2 - 3$

$= -1$

$\therefore x = 1, y = -1$

(c) $3x + 3 = 6y$ —(1)

$x - y = 1$ —(2)

From (2),

$y = x - 1$ —(3)

Substitute (3) into (1):

$3x + 3 = 6(x - 1)$

$= 6x - 6$

$3x = 9$

$x = 3$

Substitute $x = 3$ into (3):

$y = 3 - 1$

$= 2$

$\therefore x = 3, y = 2$

(d) $6x + 2y = -3$ —(1)

$4x - 7y = 23$ —(2)

From (1),

$y = \left(\frac{-3 - 6x}{2}\right)$ —(3)

Substitute (3) into (2):

$4x - 7\left(\frac{-3 - 6x}{2}\right) = 23$

$8x + 21 + 42x = 46$

$50x = 25$

$x = \frac{1}{2}$

Substitute $x = \frac{1}{2}$ into (3):

$y = \frac{-3 - 6\left(\frac{1}{2}\right)}{2}$

$= -3$

$\therefore x = \frac{1}{2}, y = -3$

(e) $5x + y = 7$ —(1)

$3x - 5y = 13$ —(2)

From (1),

$y = 7 - 5x$ —(3)

Substitute (3) into (2):

$3x - 5(7 - 5x) = 13$

$3x - 35 + 25x = 13$

$28x = 48$

$x = 1\frac{5}{7}$

Substitute $x = 1\frac{5}{7}$ into (3):

$y = 7 - 5\left(1\frac{5}{7}\right)$

$= -1\frac{4}{7}$

$\therefore x = 1\frac{5}{7}, y = -1\frac{4}{7}$

9. **(a)** $3x - y = -1$ —(1)

$x + y = -3$ —(2)

(1) + (2): $4x = -4$

$x = -1$

Substitute $x = -1$ into (2):

$-1 + y = -3$

$y = -2$

$\therefore x = -1, y = -2$

(b) $2x - 3y = 13$ —(1)

$3x - 12y = 42$ —(2)

From (2),

$x - 4y = 14$

$x = 4y + 14$ —(3)

Substitute (3) into (1):

$2(4y + 14) - 3y = 13$

$8y + 28 - 3y = 13$

$5y = -15$

$y = -3$

Substitute $y = -3$ into (3):

$x = 4(-3) + 14$

$= -12 + 14$

$= 2$

$\therefore x = 2, y = -3$

(c) $14x + 6y = 9$ —(1)

$6x - 15y = -2$ —(2)

(1) × 5: $70x + 30y = 45$ —(3)

(2) × 2: $12x - 30y = -4$ —(4)

(3) + (4): $82x = 41$

$x = \frac{1}{2}$

Substitute $x = \frac{1}{2}$ into (2):

$6\left(\frac{1}{2}\right) - 15y = -2$

$3 - 15y = -2$

$15y = 5$

$y = \frac{1}{3}$

$\therefore x = \frac{1}{2}, y = \frac{1}{3}$

(d) $8x + y = 24$ —(1)

$4x - y = 6$ —(2)

(1) + (2): $12x = 30$

$x = 2\frac{1}{2}$

Substitute $x = 2\frac{1}{2}$ into (2):

$4\left(2\frac{1}{2}\right) - y = 6$

$10 - y = 6$

$y = 4$

$\therefore x = 2\frac{1}{2}, y = 4$

(e) $3x + 7y = 17$ —(1)

$3x - 6y = 4$ —(2)

(1) – (2): $13y = 13$

$y = 1$

Substitute $y = 1$ into (1):

$3x + 7(1) = 17$

$3x + 7 = 17$

$3x = 10$

$x = 3\frac{1}{3}$

$\therefore x = 3\frac{1}{3}, y = 1$

(f) $7x - 3y = 6$ —(1)

$7x - 4y = 8$ —(2)

(1) – (2): $y = -2$

Substitute $y = -2$ into (1):

$7x - 3(-2) = 6$

$7x + 6 = 6$

$7x = 0$

$x = 0$

$\therefore x = 0, y = -2$

Intermediate

10. For L_1:

Vertical change (or rise) = 6 – 2 = 4

Horizontal change (or run) = 4 – 0 = 4

Since the line slopes upwards from the left to the right, its gradient is positive.

m = gradient of line

$= \frac{4}{4}$

$= 1$

c = y-intercept

$= 2$

For L_2:

Vertical change (or rise) = 6 – (–2) = 8

Horizontal change (or run) = 4 – 0 = 4

Since the line slopes upwards from the left to the right, its gradient is positive.

m = gradient of line

$= \frac{8}{4}$

$= 2$

c = y-intercept

$= -2$

For L_3:

Vertical change (or rise) = 4 – 0 = 4

Horizontal change (or run) = 4 – 0 = 4

Since the line slopes downwards from the left to the right, its gradient is negative.

m = gradient of line

$= -\frac{4}{4}$

$= -1$

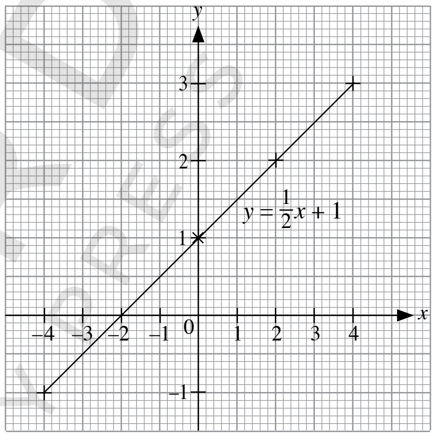
c = y-intercept

$= 4$

11. (a)

x	–4	0	2	4
$y = \frac{1}{2}x + 1$	$y = \frac{1}{2}(-4) + 1$ $= -1$	$y = \frac{1}{2}(0) + 1$ $= 1$	$y = \frac{1}{2}(2) + 1$ $= 2$	$y = \frac{1}{2}(4) + 1$ $= 3$

(b)



(c) From the graph, the point (3, 2.5) lies on the line but the point $\left(-1, -\frac{1}{2}\right)$ does not lie on the line.

(d) From the graph, the line cuts the x-axis at $x = -2$. The coordinates are (–2, 0).

(e) Vertical change (or rise) = 3 – (–1) = 4

Horizontal change (or run) = 4 – (–4) = 8

Since the line slopes upwards from the left to the right, its gradient is positive.

m = gradient of line

$= \frac{4}{8}$

$= \frac{1}{2}$

12. The equation of a straight line is in the form of $y = mx + c$, where m is the gradient. So, to find the gradient of the lines, express the equation of the given lines to be in the form of the equation of a straight line.

(a) $y + x = 5$

$y = -x + 5$

From the equation, the value of the gradient m is –1.

(b) $3y + x = 6$

$3y = -x + 6$

$\frac{3y}{3} = \frac{-x + 6}{3}$

$y = \frac{-x}{3} + 2$

$= -\frac{1}{3}x + 2$

From the equation, the value of m is $-\frac{1}{3}$.

(c) $2y + 3x = 7$

$2y = -3x + 7$

$\frac{2y}{2} = \frac{-3x + 7}{2}$

$y = \frac{-3x}{2} + \frac{7}{2}$

From the equation, the value of m is $-\frac{3}{2}$.

(d) $2x - 5y = 9$

$2x = 9 + 5y$

$2x - 9 = 5y$

$5y = 2x - 9$

$\frac{5y}{5} = \frac{2x - 9}{5}$

$y = \frac{2x}{5} - \frac{9}{5}$

From the equation, the value of m is $\frac{2}{5}$.

(e) $4x - 6y + 1 = 0$

$4x + 1 = 6y$

$6y = 4x + 1$

$\frac{6y}{6} = \frac{4x + 1}{6}$

$y = \frac{4x}{6} - \frac{1}{6}$

$y = \frac{2x}{3} - \frac{1}{6}$

From the equation, the value of m is $\frac{2}{3}$.

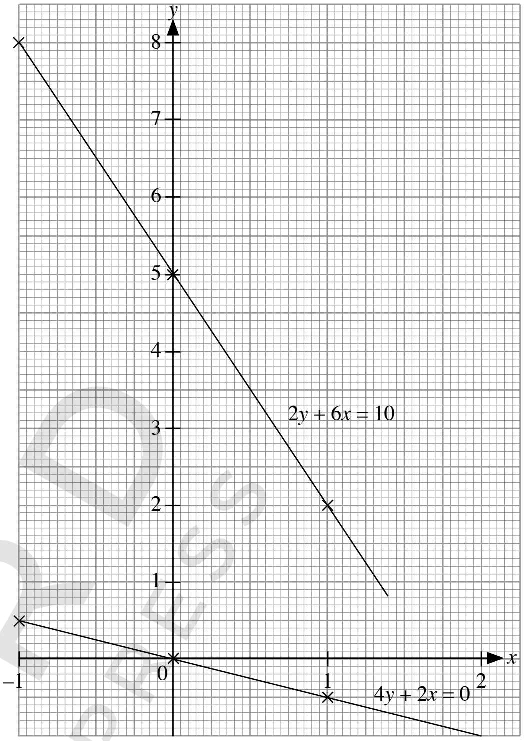
(f) $\frac{1}{2}x - \frac{2}{3}y - 5 = 0$

$\frac{2}{3}y = \frac{1}{2}x - 5$

$y = \frac{3}{4}x - 7\frac{1}{2}$

From the equation, the value of m is $\frac{3}{4}$.

13. (a)



For $4y + 2x = 0$,

Vertical change (or rise) $= \frac{1}{2} - 0$

$= \frac{1}{2}$

Horizontal change (or run) $= 0 - (-1)$

$= 1$

Since the line slopes downwards from the left to the right, its gradient is negative.

m = gradient of line

$= \frac{\frac{1}{2}}{1}$

$= -\frac{1}{2}$

For $2y + 6x = 10$,

Vertical change (or rise) $= 5 - 2$

$= 3$

Horizontal change (or run) $= 1 - 0$

$= 1$

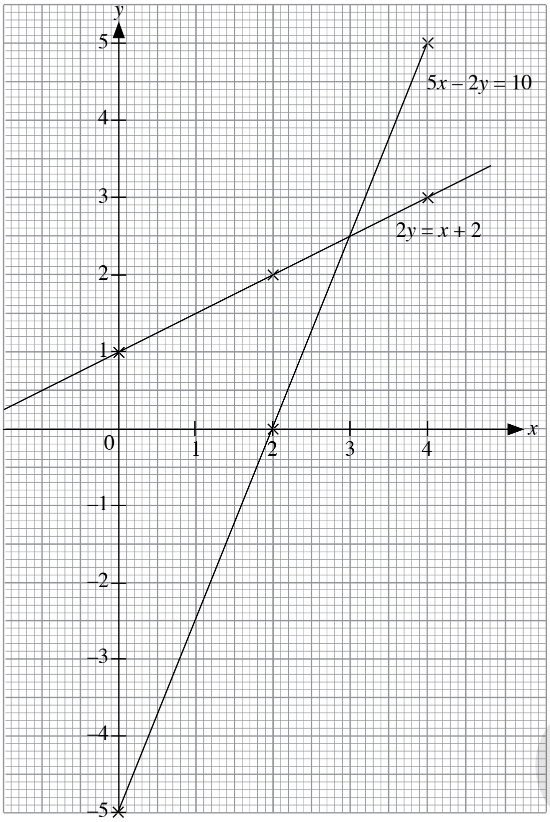
Since the line slopes downwards from the left to the right, its gradient is negative.

m = gradient of line

$= -\frac{3}{1}$

$= -3$

(b)



For $2y = x + 2$,

Vertical change (or rise) $= 2\frac{1}{2} - 1\frac{1}{2}$

$= -1$

Horizontal change (or run) $= 3 - 1$

$= -2$

Since the line slopes upwards from the left to the right, its gradient is positive.

m = gradient of line

$= \frac{1}{2}$

For $5x - 2y = 10$,

Vertical change (or rise) $= 2\frac{1}{2} - (-2\frac{1}{2})$

$= 5$

Horizontal change (or run) $= 3 - 1$

$= 2$

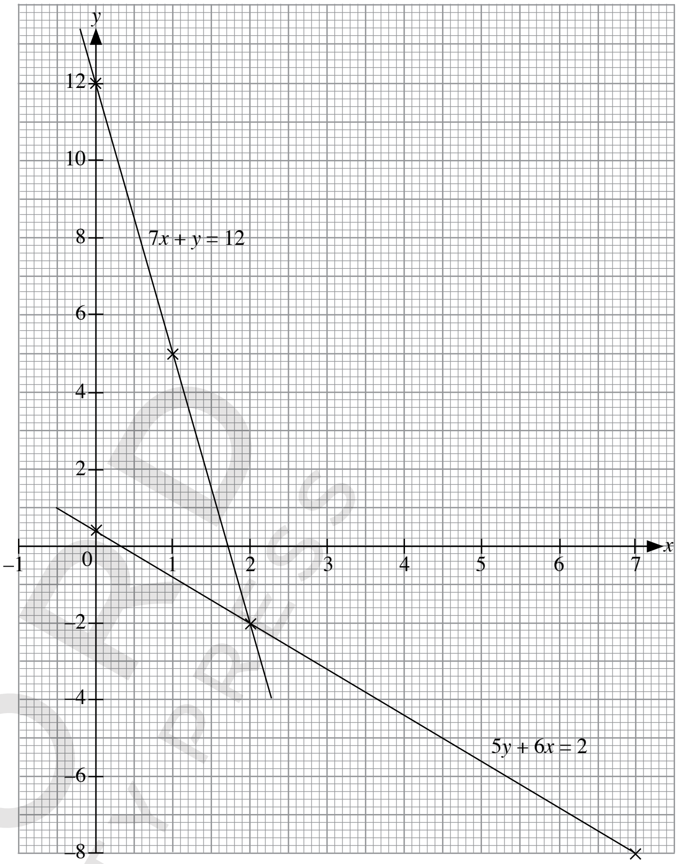
Since the line slopes upwards from the left to the right, its gradient is positive.

m = gradient of line

$= \frac{5}{2}$

$= 2\frac{1}{2}$

(c)



For $7x + y = 12$,

Vertical change (or rise) $= 12 - 5$

$= 7$

Horizontal change (or run) $= 1 - 0$

$= 1$

Since the line slopes downwards from the left to the right, its gradient is negative.

m = gradient of line

$= \frac{7}{1}$

$= -7$

For $5y + 6x = 2$,

Vertical change (or rise) $= -2 - (-8)$

$= 6$

Horizontal change (or run) $= 7 - 2$

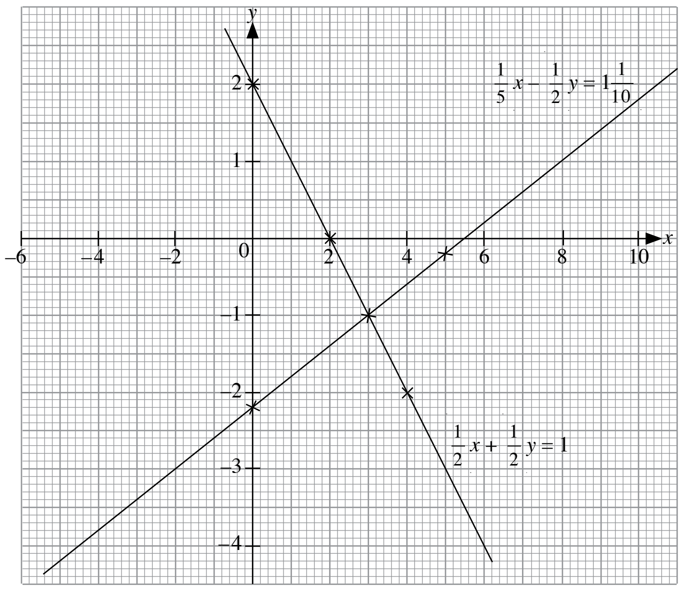
$= 5$

Since the line slopes downwards from the left to the right, its gradient is negative.

m = gradient of line

$= -\frac{6}{5}$

(d)



For $\frac{1}{2}x + \frac{1}{2}y = 1$,

Vertical change (or rise) $= 0 - (-4)$
$= 4$

Horizontal change (or run) $= 6 - 2$
$= 4$

Since the line slopes downwards from the left to the right, its gradient is negative.

m = gradient of line
$= -\frac{4}{4}$
$= -1$

For $\frac{1}{5}x - \frac{1}{2}y = 1\frac{1}{10}$,

Vertical change (or rise) $= 1 - (-1)$
$= 2$

Horizontal change (or run) $= 8 - 3$
$= 5$

Since the line slopes upwards from the left to the right, its gradient is positive.

m = gradient of line
$= \frac{2}{5}$

14. (i) From the graph, the value of x can be obtained by taking the value of the y-intercept, i.e. when the number of units used is zero.

$\therefore x = 14$

The value of y can be obtained by find the gradient of the line since the gradient, in this case, represents the cost for every unit of electricity used.

Vertical change (or rise) $= 54 - 14 = 40$

Horizontal change (or run) $= 400 - 0 = 400$

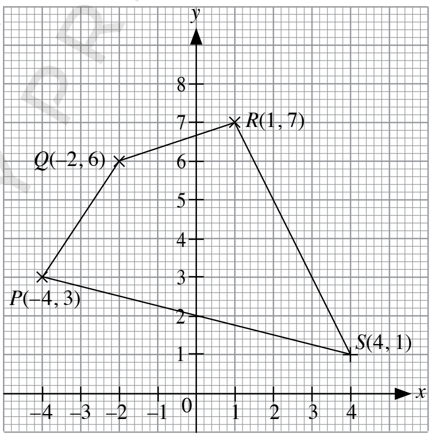
Since the line slopes upwards from the left to the right, its gradient is positive.

$y = m$ = gradient of line
$= \frac{40}{400}$
$= \frac{1}{10}$

(ii) From the graph, the cost of using 300 units of electricity is PKR 44.

(iii) From the graph, the number of units of electricity used if the cost is PKR 32 is 180.

15. (a)



(b) (i) Vertical change (or rise) $= 6 - 3 = 3$

Horizontal change (or run) $= -2 - (-4) = 2$

Since the line slopes upwards from the left to the right, its gradient is positive.

Gradient of line $= \frac{3}{2}$

(ii) Vertical change (or rise) $= 7 - 6 = 1$

Horizontal change (or run) $= 1 - (-2) = 3$

Since the line slopes upwards from the left to the right, its gradient is positive.

Gradient of line $= \frac{1}{3}$

(iii) Vertical change (or rise) $= 7 - 1 = 6$

Horizontal change (or run) $= 4 - 1 = 3$

Since the line slopes downwards from the left to the right, its gradient is negative.

Gradient of line $= -\frac{6}{3} = -2$

(iv) Vertical change (or rise) = 3 – 1 = 2

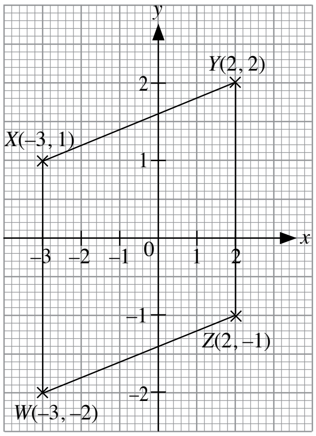
Horizontal change (or run) = 4 – (–4) = 8

Since the line slopes downwards from the left to the right, its gradient is negative.

Gradient of line = $-\frac{2}{8} = -\frac{1}{4}$

(c) From the graph, the coordinates of the point is (0, 2).

16. (a)



(b) (i) Vertical change (or rise) = 1 – (–2) = 3

Horizontal change (or run) = –3 – (–3) = 0

Gradient of line = $\frac{3}{0}$ = undefined

(ii) Vertical change (or rise) = 2 – 1 = 1

Horizontal change (or run) = 2 – (–3) = 5

Since the line slopes upwards from the left to the right, its gradient is positive.

Gradient of line = $\frac{1}{5}$

(iii) Vertical change (or rise) = 2 – (–1) = 3

Horizontal change (or run) = 2 – 2 = 0

Gradient of line = $\frac{3}{0}$ = undefined

(iv) Vertical change (or rise) = –1 – (–2)

Horizontal change (or run) = 2 – (–3) = 5

Since the line slopes upwards from the left to the right, its gradient is positive.

Gradient of line = $\frac{1}{5}$

(c) The quadrilateral *WXYZ* is a parallelogram.

17. (a) $4x - 6y = 12$ —(1)

$2x + 4y = -4.5$ —(2)

(1) ÷ 2: $2x - 3y = 6$ —(3)

(2) – (3): $7y = -10.5$

$y = -1.5$

Substitute $y = -1.5$ into (3):

$2x - 3(-1.5) = 6$

$2x + 4.5 = 6$

$2x = 1.5$

$x = 0.75$

$\therefore x = 0.75, y = -1.5$

(b) $3x - 5y = 2$ —(1)

$x - 2y = \frac{4}{15}$ —(2)

(2) × 3: $3x - 6y = \frac{4}{5}$ —(3)

(1) – (3): $y = \frac{6}{5}$

$= 1\frac{1}{5}$

Substitute $y = 1\frac{1}{5}$ into (2):

$x - 2\left(1\frac{1}{5}\right) = \frac{4}{15}$

$x = \frac{8}{3}$

$= 2\frac{2}{3}$

$\therefore x = 2\frac{2}{3}, y = 1\frac{1}{5}$

(c) $5x - 8y = 23\frac{1}{2}$ —(1)

$4x + y = 22\frac{1}{2}$ —(2)

(2) × 8: $32x + 8y = 180$ —(3)

(1) + (3): $37x = 203\frac{1}{2}$

$x = 5\frac{1}{2}$

Substitute $x = 5\frac{1}{2}$ into (2):

$4\left(5\frac{1}{2}\right) + y = 22\frac{1}{2}$

$22 + y = 22\frac{1}{2}$

$y = \frac{1}{2}$

$\therefore x = 5\frac{1}{2}, y = \frac{1}{2}$

(d) $5x - 3y = 1.4 \quad —(1)$

$2x + 5y = 14.2 \quad —(2)$

$(1) \times 2: 10x - 6y = 2.8 \quad —(3)$

$(2) \times 5: 10x + 25y = 71 \quad —(4)$

$(4) - (3): 31y = 68.2$

$y = 2.2$

Substitute $y = 2.2$ into (2):

$2x + 5(2.2) = 14.2$

$2x + 11 = 14.2$

$2x = 3.2$

$x = 1.6$

$\therefore x = 1.6, y = 2.2$

18. (a) $15x - 7y = 14\frac{1}{4} \quad —(1)$

$5x - y = 3\frac{3}{4} \quad —(2)$

From (2),

$y = 5x - 3\frac{3}{4} \quad —(3)$

Substitute (3) into (1):

$15x - 7\left(5x - 3\frac{3}{4}\right) = 14\frac{1}{4}$

$15x - 35x + \frac{105}{4} = \frac{57}{4}$

$20x = 12$

$x = \frac{3}{5}$

Substitute $x = \frac{3}{5}$ into (3):

$y = 5\left(\frac{3}{5}\right) - 3\frac{3}{4}$

$= 3 - 3\frac{3}{4}$

$= -\frac{3}{4}$

$\therefore x = \frac{3}{5}, y = -\frac{3}{4}$

(b) $3x + 1.4y = 0.1 \quad —(1)$

$x - 3.6y = 10.2 \quad —(2)$

From (2),

$x = 3.6y + 10.2 \quad —(3)$

Substitute (3) into (1):

$3(3.6y + 10.2) + 1.4y = 0.1$

$10.8y + 30.6 + 1.4y = 0.1$

$12.2y = -30.5$

$y = -2.5$

Substitute $y = -2.5$ into (3):

$x = 3.6(-2.5) + 10.2$

$= 1.2$

$\therefore x = 1.2, y = -2.5$

(c) $\frac{1}{2}x - \frac{1}{3}y - 1 = 0 \quad —(1)$

$x + 6y + 8 = 0 \quad —(2)$

From (2),

$x = -6y - 8 \quad —(3)$

Substitute (3) into (1):

$\frac{1}{2}(-6y - 8) - \frac{1}{3}y - 1 = 0$

$-3y - 4 - \frac{1}{3}y - 1 = 0$

$-\frac{10}{3}y = 5$

$y = -\frac{3}{2}$

$= -1\frac{1}{2}$

Substitute $y = -1\frac{1}{2}$ into (3):

$x = -6\left(-1\frac{1}{2}\right) - 8$

$= 9 - 8$

$= 1$

$\therefore x = 1, y = -1\frac{1}{2}$

(d) $3x - 2y = 8 \quad —(1)$

$\frac{1}{8}x + \frac{1}{2}y = 1.25 \quad —(2)$

From (2),

$\frac{1}{2}y = 1.25 - \frac{1}{8}x$

$y = 2.5 - \frac{1}{4}x \quad —(3)$

Substitute (3) into (1):

$3x - 2\left(2.5 - \frac{1}{4}x\right) = 8$

$3x - 5 + \frac{1}{2}x = 8$

$\frac{7}{2}x = 13$

$x = \frac{26}{7}$

$= 3\frac{5}{7}$

Substitute $x = 3\frac{5}{7}$ into (3):

$y = 2\frac{1}{2} - \frac{1}{4}\left(3\frac{5}{7}\right)$

$= 1\frac{4}{7}$

$\therefore x = 3\frac{5}{7}, y = 1\frac{4}{7}$

19. (a) $3x + 2y + 7 = 0$ —(1)

$5x - 2y + 1 = 0$ —(2)

$(1) + (2)$: $8x + 8 = 0$

$8x = -8$

$x = -1$

Substitute $x = -1$ into (1):

$3(-1) + 2y + 7 = 0$

$-3 + 2y + 7 = 0$

$2y = -4$

$y = -2$

$\therefore x = -1, y = -2$

(b) $2y - 7x + 69 = 0$ —(1)

$4x - 3y - 45 = 0$ —(2)

$(1) \times 3$: $6y - 21x + 207 = 0$ —(3)

$(2) \times 2$: $8x - 6y - 90 = 0$ —(4)

$(3) + (4)$: $-13x + 117 = 0$

$13x = 117$

$x = 9$

Substitute $x = 9$ into (1):

$2y - 7(9) + 69 = 0$

$2y - 63 + 69 = 0$

$2y = -6$

$y = -3$

$\therefore x = 9, y = -3$

(c) $0.5x - 0.2y = 2$ —(1)

$2.5x + 0.6y = 2$ —(2)

$(1) \times 3$: $1.5x - 0.6y = 6$ —(3)

$(2) + (3)$: $4x = 8$

$x = 2$

Substitute $x = 2$ into (1):

$0.5(2) - 0.2y = 2$

$1 - 0.2y = 2$

$0.2y = -1$

$y = -5$

$\therefore x = 2, y = -5$

(d) $x + \frac{1}{2}y = 9$ —(1)

$3x - 2y = 13$ —(2)

$(1) \times 4$: $4x + 2y = 36$ —(3)

$(2) + (3)$: $7x = 49$

$x = 7$

Substitute $x = 7$ into (1):

$7 + \frac{1}{2}y = 9$

$\frac{1}{2}y = 2$

$y = 4$

$\therefore x = 7, y = 4$

(e) $\frac{1}{3}(x + 1) + y - 8 = 0$ —(1)

$x + 4 = \frac{y + 1}{3}$ —(2)

From (1),

$x + 1 + 3y - 24 = 0$

$x = 23 - 3y$ —(3)

Substitute (3) into (2):

$23 - 3y + 4 = \frac{y + 1}{3}$

$27 - 3y = \frac{y + 1}{3}$

$81 - 9y = y + 1$

$10y = 80$

$y = 8$

Substitute $y = 8$ into (3):

$x = 23 - 3(8)$

$= 23 - 24$

$= -1$

$\therefore x = -1, y = 8$

(f) $\frac{1}{5}x + \frac{3}{4}y = -1\frac{1}{2}$ —(1)

$\frac{5}{6}x - \frac{1}{8}y = 13\frac{1}{4}$ —(2)

$(1) \times 20$: $4x + 15y = -30$ —(3)

$(2) \times 24$: $20x - 3y = 318$ —(4)

From (4),

$3y = 20x - 318$ —(5)

Substitute (5) into (3):

$4x + 5(20x - 318) = -30$

$4x + 100x - 1590 = -30$

$104x = 1560$

$x = 15$

Substitute $x = 15$ into (5):

$3y = 20(15) - 318$

$= -18$

$y = -6$

$\therefore x = 15, y = -6$

(g) $\frac{1}{3}x - \frac{2}{3}y + 5 = 0$ —(1)

$\frac{1}{2}x + \frac{1}{3}y - \frac{1}{2} = 0$ —(2)

$(1) \times 3$: $x - 2y + 15 = 0$ —(3)

$(2) \times 6$: $3x + 2y - 3 = 0$ —(4)

$(3) + (4)$: $4x + 12 = 0$

$4x = -12$

$x = -3$

Substitute $x = -3$ into (3):
$-3 - 2y + 15 = 0$
$2y = 12$
$y = 6$
$\therefore x = -3, y = 6$

(h) $\dfrac{x + y}{13 - 7y} = \dfrac{1}{3}$ —(1)
$\dfrac{4x - 4y - 3}{6y - 3x + 2} = \dfrac{4}{3}$ —(2)
From (1),
$3x + 3y = 13 - 7y$
$3x + 10y = 13$ —(3)
From (2),
$12x - 12y - 9 = 24y - 12x + 8$
$24x - 36y = 17$ —(4)
From (3),
$3x = 13 - 10y$ —(5)
Substitute (5) into (4):
$8(13 - 10y) - 36y = 17$
$104 - 80y - 36y = 17$
$116y = 87$
$y = \dfrac{3}{4}$
Substitute $y = \dfrac{3}{4}$ into (5):
$3x = 13 - 10\left(\dfrac{3}{4}\right)$
$= \dfrac{11}{2}$
$x = \dfrac{11}{6}$
$= 1\dfrac{5}{6}$
$\therefore x = 1\dfrac{5}{6}, y = \dfrac{3}{4}$

20. (a) $4x + 4 = 5x = 60y - 100$
$4x + 4 = 5x$ —(1)
$5x = 60y - 100$ —(2)
From (1),
$x = 4$
Substitute $x = 4$ into (2):
$5(4) = 60y - 100$
$20 = 60y - 100$
$60y = 120$
$y = 2$
$\therefore x = 4, y = 2$

(b) $2x - 2 + 12y = 9 = 4x - 2y$
$2x - 2 + 12y = 9$ —(1)
$4x - 2y = 9$ —(2)
From (1),
$2x + 12y = 11$
$x = \dfrac{11 - 12y}{2}$ —(3)
Substitute (3) into (2):
$4\left(\dfrac{11 - 12y}{2}\right) - 2y = 9$
$22 - 24y - 2y = 9$
$26y = 13$
$y = \dfrac{1}{2}$
Substitute $y = \dfrac{1}{2}$ into (3):
$x = \dfrac{11 - 12\left(\dfrac{1}{2}\right)}{2}$
$= \dfrac{5}{2}$
$= 2\dfrac{1}{2}$
$\therefore x = 2\dfrac{1}{2}, y = \dfrac{1}{2}$

(c) $5x + 3y = 2x + 7y = 29$
$5x + 3y = 29$ —(1)
$2x + 7y = 29$ —(2)
$(1) \times 2$: $10x + 6y = 58$ —(3)
$(2) \times 5$: $10x + 35y = 145$ —(4)
$(4) - (3)$: $29y = 87$
$y = 3$
Substitute $y = 3$ into (2):
$2x + 7(3) = 29$
$2x + 21 = 29$
$2x = 8$
$x = 4$
$\therefore x = 4, y = 3$

(d) $10x - 15y = 12x - 8y = 150$
$10x - 15y = 150$ —(1)
$12x - 8y = 150$ —(2)
$(1) \div 5$: $2x - 3y = 30$ —(3)
$(2) \div 2$: $6x - 4y = 75$ —(4)
From (3),
$2x = 3y + 30$ —(5)
Substitute (5) into (4):
$3(3y + 30) - 4y = 75$
$9y + 90 - 4y = 75$
$5y = -15$
$y = -3$

Substitute $y = -3$ into (5):

$2x = 3(-3) + 30$

$= -9 + 30$

$= 21$

$x = \frac{21}{2}$

$= 10\frac{1}{2}$

$\therefore x = 10\frac{1}{2}, y = -3$

(e) $x + y + 3 = 3y - 2 = 2x + y$

$x + y + 3 = 3y - 2$ —(1)

$x + y + 3 = 2x + y$ —(2)

From (2),

$x = 3$

Substitute $x = 3$ into (1):

$3 + y + 3 = 3y - 2$

$2y = 8$

$y = 4$

$\therefore x = 3, y = 4$

(f) $5x - 8y = 3y - x + 8 = 2x - y + 1$

$5x - 8y = 3y - x + 8$ —(1)

$5x - 8y = 2x - y + 1$ —(2)

From (1),

$6x - 11y = 8$ —(3)

From (2),

$3x - 7y = 1$

$3x = 7y + 1$ —(4)

Substitute (4) into (3):

$2(7y + 1) - 11y = 8$

$14y + 2 - 11y = 8$

$3y = 6$

$y = 2$

Substitute $y = 2$ into (4):

$3x = 7(2) + 1$

$= 15$

$x = 5$

$\therefore x = 5, y = 2$

(g) $4x + 2y = x - 3y + 1 = 2x + y + 3$

$4x + 2y = x - 3y + 1$ —(1)

$4x + 2y = 2x + y + 3$ —(2)

From (1),

$3x + 5y = 1$ —(3)

From (2),

$2x + y = 3$

$y = 3 - 2x$ —(4)

Substitute (4) into (3):

$3x + 5(3 - 2x) = 1$

$3x + 15 - 10x = 1$

$7x = 14$

$x = 2$

Substitute $x = 2$ into (4):

$y = 3 - 2(2)$

$= 3 - 4$

$= -1$

$\therefore x = 2, y = -1$

(h) $3x - 4y - 7 = y + 10x - 10 = 4x - 7y$

$3x - 4y - 7 = y + 10x - 10$ —(1)

$3x - 4y - 7 = 4x - 7y$ —(2)

From (1),

$7x + 5y = 3$ —(3)

From (2),

$x - 3y = -7$

$x = 3y - 7$ —(4)

Substitute (4) into (3):

$7(3y - 7) + 5y = 3$

$21y - 49 + 5y = 3$

$26y = 52$

$y = 2$

Substitute $y = 2$ into (4):

$x = 3(2) - 7$

$= 6 - 7$

$= -1$

$\therefore x = -1, y = 2$

21. $6x - 3y = 4$ —(1)

$y = 2x + 5$ —(2)

Substitute (2) into (1):

$6x - 3(2x + 5) = 4$

$6x - 6x - 15 = 4$

$-15 = 4$ (N.A.)

From (1),

$3y = 6x - 4$

$y = 2x - \frac{4}{3}$

Since the gradients of the lines are equal, the lines are parallel and have no solution.

22. $6y + 3x = 15$ —(1)

$y = -\frac{1}{2}x + \frac{5}{2}$ —(2)

From (1),

$6y = -3x + 15$

$y = -\frac{1}{2}x + \frac{5}{2}$

Since the lines are identical, they overlap each other and have an infinite number of solutions.

23. **(a)** $x + y + 2 = 3y + 1 = 2x$

$x + y + 2 = 3y + 1$ —(1)

$3y + 1 = 2x$ —(2)

From (1),

$x = 2y - 1$ —(3)

Substitute (3) into (2):

$3y + 1 = 2(y - 1)$

$= 4y - 2$

$y = 3$

$\therefore$ Perimeter $= 3[3(3) + 1]$

$= 30$ cm

(b) $x + 5y + 9 = 2x + 3y - 3 = x + y + 1$

$x + 5y + 9 = 2x + 3y - 3$ —(1)

$x + 5y + 9 = x + y + 1$ —(2)

From (2),

$4y = -8$

$y = -2$

Substitute $y = -2$ into (1):

$x + 5(-2) + 9 = 2x + 3(-2) - 3$

$x - 1 = 2x - 9$

$x = 8$

$\therefore$ Perimeter $= 3[8 + (-2) + 1]$

$= 21$ cm

24. **(a)** $2x + y + 1 = 12$ —(1)

$4x + y + 2 = 3x + 3y$ —(2)

From (1),

$y = 11 - 2x$ —(3)

Substitute (3) into (2):

$4x + 11 - 2x + 2 = 3x + 3(11 - 2x)$

$2x + 13 = 3x + 33 - 6x$

$= 33 - 3x$

$5x = 20$

$x = 4$

Substitute $x = 4$ into (3):

$y = 11 - 2(4)$

$= 11 - 8$

$= 3$

$\therefore$ Perimeter $= 2[3(4) + 3(3) + 12]$

$= 66$ cm

Area $= 12[3(4) + 3(3)]$

$= 252\ \text{cm}^2$

(b) $3x + y + 6 = 4x - y$ —(1)

$5x - 2y + 1 = 6x + y$ —(2)

From (1),

$x = 2y + 6$ —(3)

Substitute (3) into (2):

$5(2y + 6) - 2y + 1 = 6(2y + 6) + y$

$10y + 30 - 2y + 1 = 12y + 36 + y$

$8y + 31 = 13y + 36$

$5y = -5$

$y = -1$

Substitute $y = -1$ into (3):

$x = 2(-1) + 6$

$= -2 + 6$

$= 4$

$\therefore$ Perimeter $= 2[6(4) + (-1) + 4(4) - (-1)]$

$= 80$ cm

Area $= [6(4) + (-1)][4(4) - (-1)]$

$= 391\ \text{cm}^2$

25. $y - 1 = x + 5$ —(1)

$2x + y + 1 = 3x - y + 18$ —(2)

From (1),

$y = x + 6$ —(3)

Substitute (3) into (2):

$2x + x + 6 + 1 = 3x - (x + 6) + 18$

$3x + 7 = 3x - x - 6 + 18$

$= 2x + 12$

$x = 5$

Substitute $x = 5$ into (3):

$y = 5 + 6$

$= 11$

$\therefore$ Perimeter $= 2[2(5) + 11 + 1 + 11 - 1]$

$= 64$ cm

26. $y + 2 = x - 1$ —(1)

$2x + y = 3x - y + 12$ —(2)

From (1),

$y = x - 3$ —(3)

Substitute (3) into (2):

$2x + x - 3 = 3x - (x - 3) + 12$

$3x - 3 = 3x - x + 3 + 12$

$= 2x + 15$

$x = 18$

Substitute $x = 18$ into (3):

$y = 18 - 3$

$= 15$

$y + 2 = 15 + 2$

$= 17$

$2x + y = 2(18) + 15$

$= 51$

Since the lengths of the sides are not equal, the quadrilateral is not a rhombus.

27. $0.3x + 0.4y = 7$ —(1)

$1.1x - 0.3y = 8$ —(2)

(1) × 30: $9x + 12y = 210$ —(3)

(2) × 40: $44x - 12y = 320$ —(4)

(3) + (4): $53x = 530$

$x = 10$

Substitute $x = 10$ into (1):

$0.3(10) + 0.4y = 7$

$3 + 0.4y = 7$

$0.4y = 4$

$y = 10$

$\therefore p = 10, q = 10$

28. $3x - y = 7$ —(1)

$2x + 5y = -1$ —(2)

From (1),

$y = 3x - 7$ —(3)

Substitute (3) into (2):

$2x + 5(3x - 7) = -1$

$2x + 15x - 35 = -1$

$17x = 34$

$x = 2$

Substitute $x = 2$ into (3):

$y = 3(2) - 7$

$= 6 - 7$

$= -1$

$\therefore$ Coordinates of point of intersection are $(2, -1)$.

29. $x^2 + ax + b = 0$ —(1)

Substitute $x = 3$ into (1):

$3^2 + a(3) + b = 0$

$3a + b = -9$ —(2)

Substitute $x = -4$ into (1):

$(-4)^2 + a(-4) + b = 0$

$4a - b = 16$ —(3)

(2) + (3): $7a = 7$

$a = 1$

Substitute $a = 1$ into (2):

$3(1) + b = -9$

$b = -9 - 3$

$= -12$

$\therefore a = 1, b = -12$

30. $ax - by = 1$ —(1)

$ay + bx = -7$ —(2)

Substitute $x = -1, y = 2$ into (1):

$a(-1) - b(2) = 1$

$-a - 2b = 1$ —(3)

Substitute $x = -1, y = 2$ into (2):

$a(2) + b(-1) = -7$

$2a - b = -7$

$b = 2a + 7$ —(4)

Substitute (4) into (3):

$-a - 2(2a + 7) = 1$

$-a - 4a - 14 = 1$

$5a = -15$

$a = -3$

Substitute $a = -3$ into (4):

$b = 2(-3) + 7$

$= 1$

$\therefore a = -3, b = 1$

31. Using the same method,

$4x - 3y = 48x + 8y$

$44x = -11y$

$4x = -y$

$\therefore$ This method cannot be used as we have one equation with two unknowns at the end.

32. Let Hussain's age be x years and his aunt's age be y years.

$y = 4x$ —(1)

$y + 8 = \frac{5}{2}(x + 8)$ —(2)

Substitute (1) into (2):

$4x + 8 = \frac{5}{2}(x + 8)$

$8x + 16 = 5x + 40$

$3x = 24$

$x = 8$

Substitute $x = 8$ into (1):

$y = 4(8)$

$= 32$

$\therefore$ His aunt's present age is 32 years.

33. **(i)** Let Jamil's age be x years and his mother's age be y years.

$x + y = 61$ —(1)

$y - x = 29$ —(2)

(1) – (2): $2x = 32$

$x = 16$

$\therefore$ Jamil's present age is 16 years.

(ii) Substitute $x = 16$ into (2):

$y - 16 = 29$

$y = 45$

$y + 5 = 45 + 5$

$= 50$

$\therefore$ Jamil's mother will be 50 years old.

34. Let the numbers be x and y.

$y + 7 = 4x$ —(1)

$x + 28 = 2y$ —(2)

From (1),

$y = 4x - 7$ —(3)

Substitute (3) into (2):

$x + 28 = 2(4x - 7)$

$= 8x - 14$

$7x = 42$

$x = 6$

Substitute $x = 6$ into (3):

$y = 4(6) - 7$

$= 17$

$\therefore$ The numbers are 17 and 6.

35. Let the original fraction be $\frac{x}{y}$.

$\frac{x-1}{y-1} = \frac{3}{4}$ —(1)

$\frac{x+1}{y+1} = \frac{4}{5}$ —(2)

From (1),

$4x - 4 = 3y - 3$

$4x - 3y = 1$ —(3)

From (2),

$5x + 5 = 4y + 4$

$4y = 5x + 1$

$y = \frac{1}{4}(5x + 1)$ —(4)

Substitute (4) into (3):

$4x - \frac{3}{4}(5x + 1) = 1$

$16x - 15x - 3 = 4$

$x = 7$

Substitute $x = 7$ into (4):

$y = \frac{1}{4}(35 + 1)$

$= 9$

$\therefore$ The fraction is $\frac{7}{9}$.

36. Let the fractions be represented by x and y.

$x + y = 3(y - x)$ —(1)

$6x - y = \frac{3}{2}$ —(2)

From (2),

$y = 6x - \frac{3}{2}$ —(3)

Substitute (3) into (1):

$x + 6x - \frac{3}{2} = 3\left(6x - \frac{3}{2} - x\right)$

$7x - \frac{3}{2} = 15x - \frac{9}{2}$

$8x = 3$

$x = \frac{3}{8}$

Substitute $x = \frac{3}{8}$ into (3):

$y = 6\left(\frac{3}{8}\right) - \frac{3}{2}$

$= \frac{3}{4}$

$\therefore$ The fractions are $\frac{3}{4}$ and $\frac{3}{8}$.

37. Let the price of a chicken be PKR x and that of a duck be PKR y.

$5x + 5y = 100$ —(1)

$10x + 17y = 287.5$ —(2)

From (1),

$x + y = 20$

$y = 20 - x$ —(3)

Substitute (3) into (2):

$10x + 17(20 - x) = 287.5$

$10x + 340 - 17x = 287.5$

$7x = 52.5$

$x = 7.5$

Substitute $x = 7.5$ into (3):

$y = 20 - 7.5$

$= 12.5$

$3x + 2y = 3(7.5) + 2(12.5)$

$= 47.5$

$\therefore$ He will receive PKR 47.50.

38. Let the number of chickens and goats be x and y respectively.

$x + y = 45$ —(1)

$2x + 4y = 150$ —(2)

From (2),

$x + 2y = 75$ —(3)

(2) – (1): $y = 30$

Substitute $y = 30$ into (1):

$x + 30 = 45$

$x = 15$

$y - x = 30 - 15$

$= 15$

$\therefore$ There are 15 more goats than chickens.

39. Let the cost of 1 can of condensed milk and 1 jar of instant coffee be PKR x and PKR y respectively.

$5x + 3y = 27$ —(1)

$12x + 5y = 49.4$ —(2)

From (1),

$3y = 27 - 5x$

$y = 9 - \frac{5}{3}x$ —(3)

Substitute (3) into (2):

$$12x + 5\left(9 - \frac{5}{3}x\right) = 49.4$$

$$12x + 45 - \frac{25}{3}x = 49.4$$

$$\frac{11}{3}x = 4.4$$

$$x = 1.2$$

Substitute $x = 1.2$ into (3):

$$y = 9 - \frac{5}{3}(1.2)$$

$$= 7$$

$$7x + 2y = 7(1.2) + 2(7)$$

$$= 22.4$$

∴ The total cost is PKR 22.40.

40. Let the cost of 1 kiwi fruit and 1 pear be PKR x and PKR y respectively.

$8x + 7y = 4.1$ —(1)

$4x + 9y = 3.7$ —(2)

(2) × 2: $8x + 18y = 7.4$ —(3)

(3) – (1): $11y = 3.3$

$y = 0.3$

Substitute $y = 0.3$ into (1):

$8x + 7(0.3) = 4.1$

$8x = 2.0$

$x = 0.25$

$2x + 2y = 2(0.25) + 2(0.3)$

$= 1.1$

∴ The cost is PKR 1.10.

41. Let the number of research staff and laboratory assistants be x and y respectively.

$x + y = 540$ —(1)

$240x + 200y = 120\,000$ —(2)

From (2),

$6x + 5y = 3000$ —(3)

(1) × 5: $5x + 5y = 2700$ —(4)

(3) – (4): $x = 300$

Substitute $x = 300$ into (1):

$300 + y = 540$

$y = 240$

∴ The facility employs 300 research staff and 240 laboratory assistants.

42. Let the time taken to travel at 90 km/h and 80 km/h be x hours and y hours respectively.

$x + y = 8$ —(1)

$90x + 80y = 690$ —(2)

From (2),

$9x + 8y = 69$ —(3)

(1) × 9: $9x + 9y = 72$ —(4)

(4) – (3): $y = 3$

$80y = 80(3)$

$= 240$

∴ The distance he covered was 240 km.

Advanced

43. (a) $$\frac{2}{3}x - \frac{3}{5}y - 4 = \frac{1}{20}x - y + \frac{17}{30} = 2x - y - 18\frac{14}{15}$$

$$\frac{2}{3}x - \frac{3}{5}y - 4 = \frac{1}{20}x - y + \frac{17}{30} \quad \text{—(1)}$$

$$\frac{1}{20}x - y + \frac{17}{30} = 2x - y - 18\frac{14}{15} \quad \text{—(2)}$$

From (1),

$40x - 36y - 240 = 3x - 60y + 34$

$37x + 24y = 274$ —(3)

From (2),

$3x - 60y + 34 = 120x - 60y - 1136$

$117x = 1170$

$x = 10$

Substitute $x = 10$ into (3):

$37(10) + 24y = 274$

$24y = –96$

$y = –4$

∴ $x = 10$, $y = –4$

(b) $$\frac{2}{7}x + \frac{3}{4}y - 4 = \frac{3}{5}x - \frac{2}{7}y - 44 = \frac{7}{15}x + y - 3\frac{1}{3}$$

$$\frac{2}{7}x + \frac{3}{4}y - 4 = \frac{3}{5}x - \frac{2}{7}y - 44 \quad \text{—(1)}$$

$$\frac{3}{5}x - \frac{2}{7}y - 44 = \frac{7}{15}x + y - 3\frac{1}{3} \quad \text{—(2)}$$

From (1),

$40x + 105y - 560 = 84x - 40y - 6160$

$44x - 145y = 5600$ —(3)

From (2),

$63x - 30y - 4620 = 49x + 105y - 350$

$14x = 135y + 4270$

$$x = \frac{135}{14}y + 305 \quad \text{—(4)}$$

Substitute (4) into (3):

$44\left(\frac{135}{14}y + 305\right) - 145y = 5600$

$\frac{2970}{7}y + 13\,420 - 145y = 5600$

$\frac{1955}{7}y = -7820$

$y = -28$

Substitute $y = -28$ into (4):

$x = \frac{135}{14}(-28) + 305$

$= 35$

$\therefore x = 35, y = -28$

44 . Let the number be represented by $10x + y$.

$10x + y = 4(x + y)$ —(1)

$(10y + x) - (10x + y) = 27$ —(2)

From (1),

$10x + y = 4x + 4y$

$6x = 3y$

$y = 2x$ —(3)

From (2),

$9y - 9x = 27$

$y - x = 3$ —(4)

Substitute (3) into (4):

$2x - x = 3$

$x = 3$

Substitute $x = 3$ into (3):

$y = 2(3)$

$= 6$

$\therefore$ The original number is 36.

45. Let the digit in the tens place be x and the digit in the ones place be y.

$x = \frac{1}{2}y$ —(1)

$(10y + x) - (10x + y) = 36$ —(2)

From (2),

$9y - 9x = 36$

$y - x = 4$ —(3)

Substitute (1) into (3):

$y - \frac{1}{2}y = 4$

$\frac{1}{2}y = 4$

$y = 8$

Substitute $y = 8$ into (1):

$x = \frac{1}{2}(8)$

$= 4$

$\therefore$ The original number is 48.

46. Let the larger number be x and the smaller number be y.

$x + y = 55$ —(1)

$x = 2y + 7$ —(2)

Substitute (2) into (1):

$2y + 7 + y = 55$

$3y = 48$

$y = 16$

Substitute $y = 16$ into (2):

$x = 2(16) + 7$

$= 39$

Difference in the reciprocals $= \frac{1}{16} - \frac{1}{39}$

$= \frac{23}{624}$

47. Let the walking speed of Ahsan and Maaz be x m/s and y m/s respectively.

$8x + 8y = 64$ —(1)

$32x - 64 = 32y$ —(2)

From (1),

$x + y = 8$ —(3)

From (2),

$32x - 32y = 64$

$x - y = 2$ —(4)

(3) + (4): $2x = 10$

$x = 5$

Substitute $x = 5$ into (4):

$5 - y = 2$

$y = 3$

$\therefore$ Ahsan's walking speed is 5 m/s and Maaz's walking speed is 3 m/s.

The assumption is that when they are walking in the same direction, Ahsan starts off 64 m behind Maaz.

New Trend

48. $3x = y + 1$ —(1)

$y - x = 3$ —(2)

From (1),

$y = 3x - 1$ —(3)

Substitute (3) into (2):

$3x - 1 - x = 3$

$2x = 4$

$x = 2$

Substitute $x = 2$ into (3):

$y = 3(2) - 1$

$= 5$

$\therefore x = 2, y = 5$

49. **(a)** Let the speed of the faster ship and slower ship be x km/h and y km/h respectively.

$x = y + 8$ —(1)

$60x + 60y = 4320$ —(2)

From (2),

$x + y = 72$ —(3)

Substitute (1) into (3):

$y + 8 + y = 72$

$2y = 64$

$y = 32$

Substitute $y = 32$ into (1):

$x = 32 + 8$

$= 40$

$\therefore$ The speeds of the faster ship and slower ship are 40 km/h and 32 km/h respectively.

(b) $\frac{1780}{32} - \frac{1780}{40}$

$= 55.625 - 44.5$

$= 11.125$ h

$= 11$h 8 min (nearest min)

50. At x-axis, $y = 0$

$3x = 30$

$x = 10$

At y-axis, $x = 0$

$-5y = 30$

$y = -6$

$\therefore$ The coordinates of P are $(10, 0)$ and of Q are $(0, -6)$.

51. **(a)** $4x - 6 = 5y - 7$ (isos. trapezium)

$4x - 5y = -1$ —(1)

$(4x - 6) + (5x + 6y + 33) = 180$ (int. $\angle$s)

$9x + 6y = 153$

$3x + 2y = 51$ —(2)

(b) (1) × 3: $12x - 15y = -3$ —(3)

(2) × 4: $12x + 8y = 204$ —(4)

(4) – (3): $23y = 207$

$y = 9$

$\hat{B} = \hat{C}$

$= [5(9) - 7]°$

$= 38°$

$\hat{A} = 180° - \hat{B}$

$= 180° - 38°$

$= 142°$

$\therefore \hat{A} = 142°$ and $\hat{B} = 38°$

52. **(a)** $4x - 2y - 5 = 0$

$2y = 4x - 5$

$y = 2x - 2\frac{1}{2}$

(i) Gradient of line $l = 2$

(ii) y-intercept of line $l = -2\frac{1}{2}$

(b) $2x + 3y = -5$ —(1)

$4x - 2y = 5$ —(2)

(1) × 2: $4x + 6y = -10$ —(3)

(3) – (2): $8y = -15$

$y = -1\frac{7}{8}$

Substitute $y = -1\frac{7}{8}$ into (1):

$2x + 3(-1\frac{7}{8}) = -5$

$2x - 5\frac{5}{8} = -5$

$2x = \frac{5}{8}$

$x = \frac{5}{16}$

$\therefore$ The coordinates of C are $\left(\frac{5}{16}, -1\frac{7}{8}\right)$.

53. **(a)** $y = 7 - 2x$ —(1)

$y = x + 10$ —(2)

Substitute $x = -9$ into (1):

$y = 7 - 2(-9)$

$= 7 + 18$

$= 25$

Substitute $x = -9$ into (2):

$y = -9 + 10$

$= 1$

$\therefore$ The coordinates of A are $(-9, 25)$ and of B are $(-9, 1)$.

(b) $y = 7 - 2x$

From the equation, gradient of the line $= -2$.

(c) $(0, k)$ lies on the perpendicular bisector of AB.

$\therefore k = \frac{1+25}{2}$

$= 13$

Chapter 5 Indices and Standard Form

Basic

1. **(a)** $a^4 \div a^{-2} \times a^7$
$= a^{4-(-2)+7}$
$= a^{13}$

(b) $2b^7 \times 4b^{-3}$
$= 8b^{7+(-3)}$
$= 8b^4$

(c) $c^{-2} \times (c^{\frac{1}{2}})^6 \times c^{-1}$
$= c^{-2} \times c^3 \times c^{-1}$
$= c^{-2+3+(-1)}$
$= c^0$
$= 1$

(d) $\sqrt[3]{d^2} \times \sqrt{d^3} \div d^2$
$= d^{\frac{2}{3}} \times d^{\frac{3}{2}} \div d^2$
$= d^{\frac{2}{3}+\frac{3}{2}-2}$
$= d^{\frac{1}{6}}$

$\dfrac{e^{-5} \times e^9}{e}$
$= e^{-5+9-1}$
$= e^3$

(e) $\dfrac{e^{-5} \times e^9}{e}$
$= e^{-5+9-11}$
$= e^3$

(f) $\dfrac{f^{-\frac{1}{2}} \times f^4}{f^0 \times \sqrt{f} \div f^{-2}}$
$= \dfrac{f^{-\frac{1}{2}+4}}{f^{\frac{1}{2}-(-2)}}$
$= \dfrac{f^{3\frac{1}{2}}}{f^{2\frac{1}{2}}}$
$= f$

2. **(a)** $\left(\dfrac{3w}{5}\right)^{-2}$
$= \left(\dfrac{5}{3w}\right)^2$
$= \dfrac{25}{9w^2}$

(b) $\left(\dfrac{3}{7x}\right)^{-2}$
$= \left(\dfrac{7x}{3}\right)^2$
$= \dfrac{49x^2}{9}$

(c) $3 \div 9y^2$
$= 3 \div \dfrac{9}{y^2}$
$= 3 \times \dfrac{y^2}{9}$
$= \dfrac{y^2}{3}$

(d) $(5z)^0 \div 8z^{-4}$
$= 1 \div \dfrac{8}{z^4}$
$= 1 \times \dfrac{z^4}{8}$
$= \dfrac{z^4}{8}$

3. **(a)** $(-27)^{\frac{2}{3}}$
$= (\sqrt[3]{-27})^2$
$= (-3)^2$
$= 9$

(b) $8^{-\frac{2}{3}}$
$= \dfrac{1}{8^{\frac{2}{3}}}$
$= \dfrac{1}{(\sqrt[3]{8})^2}$
$= \dfrac{1}{2^2}$
$= \dfrac{1}{4}$

(c) $\sqrt[3]{0.027}$
$= \sqrt[3]{\dfrac{27}{1000}}$
$= \sqrt[3]{\left(\dfrac{3}{10}\right)^3}$
$= \dfrac{3}{10}$

(d) $3^4 - 3^3$
$= 81 - 27$
$= 54$

4. **(a)** $2^{2a-1} = 128$
$= 2^7$
$2a - 1 = 7$
$2a = 8$
$a = 4$

(b) $6^{3b} = 216$
$= 6^3$
$3b = 3$
$b = 1$

(c) $3^{c+1} = 27^{-1}$
$= (3^3)^{-1}$
$= 3^{-3}$
$c + 1 = -3$
$c = -4$

(d) $8^{3d-1} = 1$
$3d - 1 = 0$
$3d = 1$
$d = \frac{1}{3}$

5. (a) $0.0231 = 2.31 \times 10^{-2}$
(b) $62\ 500 = 6.25 \times 10^4$
(c) $5\ 390\ 000 = 5.39 \times 10^6$
(d) $0.000\ 005\ 345 = 5.345 \times 10^{-6}$

6. (a) $9.43 \times 10^{-4} = 0.000\ 943$
(b) $6.1 \times 10^4 = 61\ 000$
(c) $2.795 \times 10^6 = 2795\ 000$
(d) $7 \times 10^{-7} = 0.000\ 0007$

7. (a) $(8.59 \times 10^{-7}) \times (0.392 \times 10^5)$
$= 3.37 \times 10^{-2}$ (to 3 s.f.)
(b) $(8.05 \times 10^6) \div (7 \times 10^{-2})$
$= 1.15 \times 10^8$
(c) $3.2 \times 10^6 + 1.8 \times 10^4$
$= 3.22 \times 10^6$ (to 3 s.f.)
(d) $1.97 \times 10^7 - 2.02 \times 10^5$
$= 1.95 \times 10^7$ (to 3 s.f.)

8. 750 gigabytes $= 750 \times 10^9$ bytes
$= 7.5 \times 10^{11}$ bytes

9. 0.5 MHz $= 0.5 \times 10^6$ hertz
$= 5 \times 10^5$ hertz

10. 76 μg $= 76 \times 10^{-6}$ g
$= 7.6 \times 10^{-5}$ g

11. (i) 273 picograms $= 273 \times 10^{-12}$ g
$= 2.73 \times 10^{-10}$ g
(ii) Total mass $= (0.3 \times 10^9) \times (2.73 \times 10^{-10})$
$= 8.19 \times 10^{-2}$

12. (a) $q : p = 1.2 \times 10^6 : 9.6 \times 10^5$
$= 12 : 9.6$
$= 5 : 4$
(b) Distance between Beijing and Tokyo
$= 9.6 \times 10^5 + 1.2 \times 10^6$
$= 2.16 \times 10^6$ m

13. (a) Difference in mass $= 2.66 \times 10^{-23} - 1.99 \times 10^{-23}$
$= 0.67 \times 10^{-23}$
$= 6.7 \times 10^{-24}$ g
(b) Mass of one molecule $= 1.99 \times 10^{-23} + 2(2.66 \times 10^{-23})$
$= 1.99 \times 10^{-23} + 5.32 \times 10^{-23}$
$= 7.31 \times 10^{-23}$ g

Intermediate

14. (a) $(5a^2b^3)^3$
$= 125a^{2\times3}b^{3\times3}$
$= 125a^6b^9$
(b) $5a^4b^8 \times 9a^2b^3$
$= 45a^{4+2}b^{8+3}$
$= 45a^6b^{11}$
(c) $\frac{a^4 \times (ab^2)^2}{(a^8b)^2}$
$= \frac{a^4 \times a^{1\times2}b^{2\times2}}{a^{8\times2}b^{1\times2}}$
$= \frac{a^{4+2}b^4}{a^{16}b^2}$
$= \frac{a^6b^4}{a^{16}b^2}$
$= \frac{b^{4-2}}{a^{16-6}}$
$= \frac{b^2}{a^{10}}$
(d) $-(4a^3b)^2 \times \frac{3a^5}{8b^4}$
$= -16a^{3\times2}b^{1\times2} \times \frac{3a^5}{8b^4}$
$= -16a^6b^2 \times \frac{3a^5}{8b^4}$
$= -\frac{6a^{6+5}}{b^{4-2}}$
$= -\frac{6a^{11}}{b^2}$

15. (a) $3^0 + \left(\frac{1}{3}\right)^{-4}$
$= 1 + 3^4$
$= 1 + 81$
$= 82$
(b) $8^{-2} + 8^0 - 8^1$
$= \frac{1}{8^2} + 1 - 8$
$= \frac{1}{64} - 7$
$= -6\frac{63}{64}$
(c) $9^{-1} + 9^0 + 9^{\frac{1}{2}}$
$= \frac{1}{9} + 1 + 3$
$= 4\frac{1}{9}$

(d) $16^{-\frac{3}{4}} \times 8^2 \div 2^{-1}$

$= (2^4)^{-\frac{3}{4}} \times (2^3)^2 \div 2^{-1}$

$= 2^{-3} \times 2^6 \div 2^{-1}$

$= 2^{-3+6-(-1)}$

$= 2^4$

$= 16$

(e) $\left(\frac{3}{4}\right)^{-2} + 3^{-1} - 3$

$= \left(\frac{4}{3}\right)^{2} + \frac{1}{3} - 3$

$= \frac{16}{9} - 2\frac{2}{3}$

$= -\frac{8}{9}$

(f) $(81^{\frac{1}{2}} - 4^0) \times 3^{-2}$

$= (9 - 1) \times \frac{1}{3^2}$

$= 8 \times \frac{1}{9}$

$= \frac{8}{9}$

(g) $\left(\frac{1}{27}\right)^{0} \times \left(\frac{27}{8}\right)^{-\frac{2}{3}} \div \frac{1}{3^2}$

$= 1 \times \left(\frac{27}{8}\right)^{-\frac{2}{3}} \times 3^2$

$= \left[\left(\frac{2}{3}\right)^{3}\right]^{\frac{2}{3}} \times 9$

$= \left(\frac{2}{3}\right)^{2} \times 9$

$= \frac{4}{9} \times 9$

$= 4$

(h) $\left(\frac{2}{3}\right)^{2} \div 125^{\frac{1}{3}}$

$= \left(\frac{2}{5}\right)^{-2} \div (5^3)^{\frac{1}{3}}$

$= \frac{25}{4} \div 5$

$= \frac{25}{4} \times \frac{1}{5}$

$= \frac{5}{4}$

$= 1\frac{1}{4}$

16. (a) $\frac{(-2x^2y)^3}{4x^{-1}(y^2)^3}$

$= \frac{-8x^{2\times3}y^{1\times3}}{4x^{-1}y^{2\times3}}$

$= -\frac{2x^6y^3}{x^{-1}y^6}$

$= -\frac{2x^{6-(-1)}}{y^{6-3}}$

$= -\frac{2x^7}{y^3}$

(b) $\frac{(2x^2y)^3 \times \sqrt{x^8}}{x^{-2}y^5}$

$= \frac{8x^{2\times3}y^{1\times3} \times x^4}{x^{-2}y^5}$

$= \frac{8x^{6+4}y^3}{x^{-2}y^5}$

$= \frac{8x^{10-(-2)}}{y^{5-3}}$

$= \frac{8x^{12}}{y^2}$

(c) $\frac{(2xy)^2}{35xy^7} \div \left(\frac{x^{-1}y^{-2}}{4}\right)^{-2}$

$= \frac{4x^{1\times2}y^{1\times2}}{35xy^7} \div \left(\frac{4}{x^{-1}y^{-2}}\right)^{2}$

$= \frac{4x^2y^2}{35xy^7} \div (4xy^2)^2$

$= \frac{4x^{2-1}}{35y^{7-2}} \div 16x^{1\times2}y^{2\times2}$

$= \frac{4x}{35y^5} \times \frac{1}{16x^2y^4}$

$= \frac{1}{140xy^9}$

(d) $\left(\frac{2x}{y^{-1}}\right)^{2} \div \left(\frac{2}{x^{-2}y}\right)^{-2}$

$= \frac{4x^{1\times2}}{y^{-1\times2}} \div \left(\frac{x^{-2}y}{2}\right)^{2}$

$= \frac{4x^2}{y^{-2}} \div \left(\frac{y}{2x^2}\right)^{2}$

$= 4x^2y^2 \div \frac{y^{1\times2}}{4x^{2\times2}}$

$= 4x^2y^2 \div \frac{y^2}{4x^4}$

$= 4x^2y^2 \times \frac{4x^4}{y^2}$

$= 16x^6$

17. $\frac{5^p}{\sqrt{5}} = 5^{-p}$

$\frac{5^p}{5^{\frac{1}{2}}} = 5^{-p}$

$5^{p-\frac{1}{2}} = 5^{-p}$

$p - \frac{1}{2} = -p$

$2p = \frac{1}{2}$

$p = \frac{1}{4}$

18. $\frac{a^3 \times \sqrt[3]{a}}{\sqrt{a^5}} = a^w$

$\frac{a^3 \times a^{\frac{1}{3}}}{a^{\frac{5}{2}}} = a^w$

$a^{3+\frac{1}{3}-\frac{5}{2}} = a^w$

$w = \frac{5}{6}$

19. $10^{3q+2q-r}$

$= \frac{(10^{3p})(10^{2q})}{10^r}$

$= \frac{(10^p)^3(10^q)^2}{10^r}$

$= \frac{(2)^3(3)^2}{1250}$

$= 5.76 \times 10^{-2}$

20. (a) $10^{-4} - 3.12 \times 10^{-5}$

$= 6.88 \times 10^{-5}$

(b) $\frac{0.26 \times 10^{-4}}{2.31 \times 23 \times 10^{-2}}$

$= 4.89 \times 10^{-5}$ (to 3 s.f.)

(c) $1.2 \times 10^8 + 2(3.5 \times 10^7)$

$= 1.9 \times 10^8$

(d) $\sqrt[4]{1600 \times 10^{-4}}$

$= 6.32 \times 10^{-1}$ (to 3 s.f.)

(e) $\frac{7.5 \times 10^6}{1.5 \times 10^3} + 4.1 \times 10^4$

$= 4.6 \times 10^4$

(f) $\frac{(4 \times 10^2)^5 - (5 \times 10^6)}{\sqrt{16 \times 10^{-4}}}$

$= 2.56 \times 10^{14}$ (to 3 s.f.)

21. (a) $\frac{2b}{a} = \frac{2(2 \times 10^2)}{5 \times 10^{-3}}$

$= 8 \times 10^4$

(b) $\frac{3}{a} - b = \frac{3}{5 \times 10^{-3}} - 2 \times 10^2$

$= 4 \times 10^2$

22. (a) $p \times 2q = 4 \times 10^9 \times 2 \times 3 \times 10^5$

$= 2.4 \times 10^{15}$

(b) $\frac{q^2}{p} = \frac{(3 \times 10^3)^2}{4 \times 10^9}$

$= 2.25 \times 10^1$

23. 3.3 nanoseconds = 3.3×10^{-9} seconds

4.2 billion km = 4.2×10^9 km

$= 4.2 \times 10^{12}$ m

Time taken $= \frac{4.2 \times 10^{12}}{1 \div (3.3 \times 10^{-9})}$

$= 1.386 \times 10^4$ seconds

24. (a) Difference in population $= 50 \times 10^6 - 5.18 \times 10^6$

$= 4.482 \times 10^7$

(b) $5.18 \times 10^6 : 6.97 \times 10^9$

1 : 1350 (to 3 s.f.)

25. (i) 0.000 001 654 cm = 1.654×10^{-6} cm

(ii) Volume $= \frac{4}{3}\pi\left(\frac{1.654 \times 10^{-6}}{2}\right)^3 \times 10^6$

$= 2.37 \times 10^{-12}$ cm^3 (to 3 s.f.)

26. $x = 1, y = -2$

Advanced

27. (i) Number of daughter cells at the end of 1 hour = 2^3

(ii) Number of daughter cells at the end of 1 day = 2^{72}

(iii) Number of daughter cells at the end of 1 week = 2^{504}

28. $\frac{8(9^{3x}) - 27^{2x}}{3^{2x+1} \times 81^{x-1}} = \frac{8(3^2)^{3x} - (3^3)^{2x}}{3(3^{2x}) \times (3^4)^{x-1}}$

$= \frac{8(3^{6x}) - 3^{6x}}{3(3^{2x}) \times 3^{4x} \times 3^{-4}}$

$= \frac{7(3^{6x})}{3^{-3}(3^{6x})}$

$= 189$

29. (a) $\frac{2^{15}}{8^5} = \frac{(2^3)^5}{8^5}$

$= \frac{8^5}{8^5}$

$= 1$

(b) $2^8 \times 5^4 = (2^2)^4 \times 5^4$

$= 4^4 \times 5^4$

$= 20^4$

$= 160\ 000$

30. $9^n + 9^n + 9^n = 243$

$3(9^n) = 243$

$9^n = 81$

$= 9^2$

$n = 2$

New Trend

31. $16 \times 64^n = 1$

$4^2 \times (4^3)^n = 4^0$

$4^{2+3n} = 4^0$

$2 + 3n = 0$

$n = -\frac{2}{3}$

32. (a) $2^n \times 2^{-2} = \frac{1}{32}$

$2^{n-2} = \frac{1}{2^5}$

$= 2^{-5}$

$n - 2 = -5$

$n = -3$

(b) $\frac{1}{36} = 6^k$

$\frac{1}{6^2} = 6^k$

$6^k = 6^{-2}$

$k = -2$

33. $\left(\frac{2x}{y^{-1}}\right)^2 \div \frac{1}{3x^{-3}y^{-3}}$

$= \frac{4x^2}{y^{-1\times2}} \div \frac{x^3y^3}{3}$

$= \frac{4x^2}{y^{-2}} \times \frac{3}{x^3y^3}$

$= \frac{12}{xy}$

34. (a) $(x^9y^{-3})^{\frac{1}{3}} \times (x^8y^{-2})^{\frac{3}{2}}$

$= x^{9\times\frac{1}{3}}y^{-3\times\frac{1}{3}} \times x^{8\times\frac{3}{2}}y^{-2\times\frac{3}{2}}$

$= x^3y^{-1} \times x^{12}y^{-3}$

$= x^{3+12}y^{-1+(-3)}$

$= x^{15}y^{-4}$

$= \frac{x^{15}}{y^4}$

(b) $\left(\frac{125}{x^{27}}\right)^{-\frac{1}{3}} = \left(\frac{x^{27}}{125}\right)^{\frac{1}{3}}$

$= \frac{x^9}{5}$

35. (a) (i) $11^{20} \div 11^5 = 11^{20-5}$

$= 11^{15}$

(ii) $\frac{1}{121} = \frac{1}{11^2}$

$= 11^{-2}$

(iii) $\sqrt[6]{11} = 11^{\frac{1}{6}}$

36. (i) $46\ \mu\text{m} = 46 \times 10^{-6}$ m

$= 4.6 \times 10^{-5}$ m

(ii) Area $= \pi(4.6 \times 10^{-5})^2$

$= 6.65 \times 10^{-9}\ \text{m}^2$ (to 3 s.f.)

37. (a) $12\,000 = 1.2 \times 10^4$

(b) Percentage increase in speed

$= \frac{1.14 \times 10^7 - 9.7 \times 10^6}{9.7 \times 10^6} \times 100\%$

$= \frac{10^6(1.14 \times 10 - 9.7)}{9.7 \times 10^6} \times 100\%$

$= \frac{1.7}{9.7} \times 100\%$

$= 17.5\%$ (to 3 s.f.)

(c) $29\ \text{m/s} = \frac{29\ \text{m}}{1\ \text{s}}$

$= \frac{(29 \div 1000)\ \text{km}}{(1 \div 3600)\ \text{h}}$

$= 104.4$ km/h

$= 1.044 \times 10^2$ km/h

38. (a) Difference in population $= 6.64 \times 10^7 - 5.077 \times 10^6$

$= 6.64 \times 10^7 - 0.5077 \times 10^7$

$= 6.1323 \times 10^7$

(b) 100% represent the population of Thailand in 1950.

338% represent the population of Thailand in 2010

$= 6.64 \times 10^7$

Population of Thailand in 1950 $= \frac{6.64 \times 10^7}{338} \times 100$

$= 1.96 \times 10^7$ (to 3 s.f.)

39. (a) $50\,197.4 \times 10^9$ Wh $= 50\,197.4 \times 10^6$ kWh

$= 5.019\,74 \times 10^{10}$ kWh

(b) Mean domestic electricity consumed per person

$= \frac{4716.1 \times 10^9}{3.111 \times 10^6}$

$= 1516$ kWh (to the nearest kWh)

(c) 100% represent electricity consumption in 2000.

Electricity consumption in 2015 is represented by $100 - 41.6 = 58.4\%$

Electricity consumption in 2000

$= \frac{5471.2}{58.4} \times 100$

$= 9368$ GWh (to the nearest GWh)

40. (i) When $t = 0$,

$V = 20\,000 \times 1.1^0$

$= 20\,000$

∴ The value of the flat when it was first built was PKR 20 000.

(ii) When $t = 2$,

PKR $V = 20\,000 \times 1.1^2$

$= 24\,200$

Percentage increase $= \frac{24\,200 - 20\,000}{20\,000} \times 100\%$

$= 21\%$

∴ The value of the flat increased by 21% after two years.

41. **(a)** $P = 35\,480 \times 1.0125^5$

$= \text{PKR } 37\,753.63$ (to the nearest paisa)

(b) Percentage increase in the balance

$= \frac{37\,753.63 - 35\,480}{35\,480} \times 100\%$

$= 6.41\%$ (to 3 s.f.)

42. $200 \text{ ha} = 200\,000 \text{ m}^2$

Number of trees on $200\,000 \text{ m}^2 = \frac{200\,000}{10} \times 4$

$= 80\,000$

Total number of fruits on trees $= 60 \times 80\,000$

$= 4\,800\,000$

$= 4.8 \times 10^6$

Average number of seeds produced

by these fruits $= \frac{1.44 \times 10^7}{4.8 \times 10^6}$

$= 3$

43. **(a)** 8.48 light years $= 8.48 \times 9.46 \times 10^{15}$ m

$= 80.2208 \times 10^{15}$ m

$= 8.02208 \times 10^{13}$ km

(b) 4.35 light years $= 4.35 \times 9.46 \times 10^{15}$ m

$= 41.151 \times 10^{15}$ m

$= 4.1151 \times 10^{16}$ m

$= 4.1151 \times 10^{13}$ km

Time taken $= \frac{4.1151 \times 10^{13}}{50\,000}$

$= 0.823\,02 \times 10^9$ h

$= \frac{0.823\,02 \times 10^9 \text{ h}}{(365 \times 24) \text{ h}}$

$= 0.000\,093\,952\,05 \times 10^9$ years

$= 94\,000$ years (to 2 s.f.)

Chapter 6 Linear Inequalities

Basic

1. **(a)** $15 < 30$

(b) $-2 > -5$

(c) $(-3)^2 > -9$

(d) $-2^4 = -16$

(e) $\left(-\frac{1}{3}\right)^{11} < \left(-\frac{1}{3}\right)^{4}$

(f) $\sqrt{16} < \sqrt{10}$

(g) $h - 3 > h - 4$

(h) $k + 10 > k + 7$

(i) $12 - p < 14 - p$

(j) $16 - 4q < 2(8 - q)$

2. **(a)** $a < b$

(b) $d > -3$

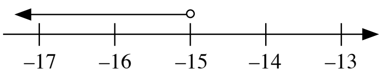
(c) $-\frac{h}{2} < -\frac{k}{2}$

(d) $3m \geqslant 3n$

(e) $-6p \geqslant -6q$

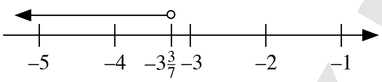
3. **(a)** $-5x > 75$

$x > 15$



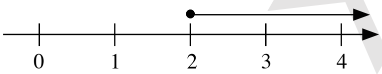
(b) $-7x \geqslant 24$

$x > -3\frac{3}{7}$



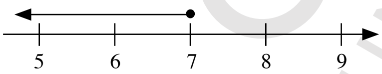
(c) $a + 1 \geqslant 3$

$a \geqslant 2$



(d) $b - 2 \leqslant 5$

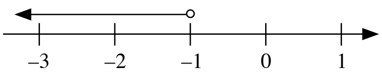
$b \leqslant 7$



(e) $-c + 1 > 2$

$-c > 1$

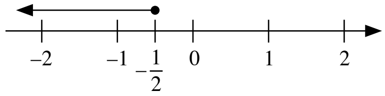
$c < -1$



(f) $-6d - 3 \geqslant 0$

$-6d \geqslant 3$

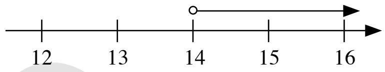
$d \leqslant -\frac{1}{2}$



(g) $12 - e < -2$

$-e < -14$

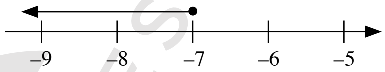
$e > 14$



(h) $20 + 4f \leqslant f - 1$

$3f \leqslant -21$

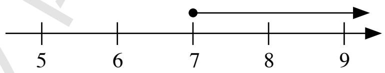
$f \leqslant -7$



(i) $3 - 2g \leqslant -4 - g$

$-g \leqslant -7$

$g \geqslant 7$

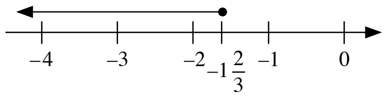


(j) $2(1 - 5h) \geqslant 4(3 - h)$

$2 - 10h \geqslant 12 - 4h$

$-6h \geqslant 10$

$h \leqslant -1\frac{2}{3}$

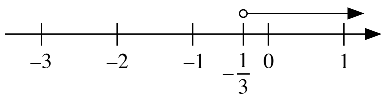


(k) $2(i + 3) > 4(1 - i)$

$2i + 6 > 4 - 4i$

$6i > -2$

$i > -\frac{1}{3}$

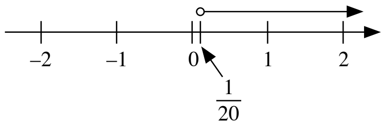


(l) $8j - 1 > 4(-3j)$

$8j - 1 > -12j$

$20j > 1$

$j > \frac{1}{20}$

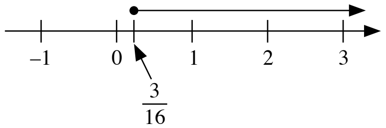


(m) $9(1 - 2k) \leqslant 2(3 - k)$

$9 - 18k \leqslant 6 - 2k$

$-16k \leqslant -3$

$k \geqslant \frac{3}{16}$

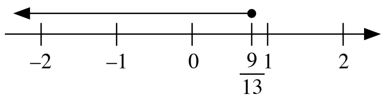


(n) $2(5 - 4l) \geqslant 5l + 1$

$10 - 8l \geqslant 5l + 1$

$-13l \geqslant -9$

$l \leqslant \frac{9}{13}$



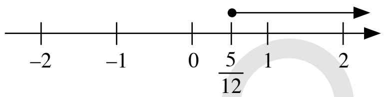
(o) $4(2m + 3) \geqslant 2(m + 7) - 3(2m - 1)$

$8m + 12 \geqslant 2m + 14 - 6m + 3$

$8m + 12 \geqslant 17 - 4m$

$12m \geqslant 5$

$m \geqslant \frac{5}{12}$



4. **(b)** $2x + 1 > 16$

$2x > 15$

$x > 7\frac{1}{2}$

$\therefore$ Smallest integer value of x is 8.

(c) $9x + 12 > 30$

$9x > 18$

$x > 2$

$\therefore$ Smallest integer value of x is 3.

(d) $10x + 2 \geqslant 20$

$10x \geqslant 18$

$x \geqslant 1.8$

$\therefore$ Smallest integer value of x is 2.

5. **(b)** $3y - 2 < 13$

$3y < 15$

$y < 5$

$\therefore$ Largest integer value of y is 4.

(c) $16y + 1 \leqslant 31$

$16y \leqslant 30$

$y \leqslant 1\frac{7}{8}$

$\therefore$ Largest integer value of y is 1.

() $4(2y + 3) < 24$

$2y + 3 < 6$

$2y < 3$

$y < 1\frac{1}{2}$

$\therefore$ Largest integer value of y is 1.

6. $\frac{1}{2}h + \frac{1}{3}(h - 6) \geqslant 3$

$\frac{1}{2}h + \frac{1}{3}h - 2 \geqslant 13$

$\frac{5}{6}h \geqslant 15$

$h \geqslant 18$

(a) Least integer value of h is 18.

(b) Least prime number h is 19.

7. $3(x + 2) \geqslant 5(x - 1)$

$3x + 6 \geqslant 5x - 5$

$-2x \geqslant -11$

$x \leqslant 5\frac{1}{2}$

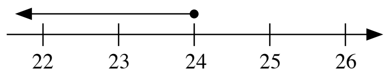
(a) $5\frac{1}{2}$

(b) 5

(c) 5

8. $6 + x \leqslant 30$

$x \leqslant 24$



(a) 2, 3, 5, 7, 11, 13, 17, 19, 23

(b) 16

9. Let x be the number of PKR 2 notes.

$2x + 10(21 - x) < 110$

$2x + 210 - 10x < 110$

$-8x < -100$

$x > 12.5$

$\therefore$ Minimum number of PKR 2 notes is 13.

10. Let x be the mark Sarah scores for her third History test.

$\frac{72 + 58 + x}{3} \geqslant 70$

$130 + x \geqslant 210$

$x \geqslant 80$

$\therefore$ Minimum mark is 80.

11. Let PKR x be the amount that Nasir pays.

$x + 50 + x \leqslant 220$

$2x + 50 \leqslant 220$

$2x \leqslant 170$

$x \leqslant 85$

$\therefore$ Greatest amount that Mishal pays is PKR 135.

12. Let x be the number of kiwi fruits he sells.

$0.55x - 66.50 \geqslant 20$

$0.55x \geqslant 86.5$

$x \geqslant 157\frac{3}{11}$

$\therefore$ Least number of kiwi fruits is 158.

13. (i) Maximum amount $=$ PKR 1.50×12

$=$ PKR 18

Minimum amount $=$ PKR 1.20×12

$=$ PKR 14.40

(ii) Let x be the number of cups of ice-cream.

$(1.50)x + (1.20)(2) + (1.20)(10 - x) \leqslant 16$

$1.5x + 2.4 + 12 - 1.2x \leqslant 16$

$0.3x \leqslant 1.6$

$x \leqslant 5\frac{1}{3}$

$\therefore$ Maximum number of cups of ice-cream is 5.

14. Let the length of the square be x cm.

$4x \leqslant 50$

$x \leqslant 12.5$

Largest possible area $= 12.5^2$

$= 156.3$ cm^2 (to 4 s.f.)

15. (a) $x + 1 \leqslant 5$ and $2x > -8$

$x \leqslant 4$ $\quad$ $x > -4$

$\therefore -4 < x \leqslant 4$

(b) $4x + 2 < 10$ and $3x - 1 \geqslant 11$

$4x < 8$ $\quad$ $3x \geqslant 12$

$x < 2$ $\quad$ $x \geqslant 4$

$\therefore$ No solution

(c) $x + 1 < 14$ and $2x + 3 > 12$

$x < 13$ $\quad$ $2x > 9$

$x > 4\frac{1}{2}$

$\therefore 4\frac{1}{2} < x < 13$

(d) $6 + 2x > 0$ and $20 - 4x > 1 - 2x$

$2x > -6$ $\quad$ $-2x > -19$

$x > -3$ $\quad$ $x < 9\frac{1}{2}$

$\therefore -3 < x < 9\frac{1}{2}$

(e) $x + 3 < 22$ and $14 \leqslant 5x - 2$

$x < 19$ $\quad$ $-5x \leqslant -16$

$x \geqslant 3\frac{1}{5}$

$\therefore 3\frac{1}{5} \leqslant x < 19$

(f) $x - 1 < 10$ and $4x + 1 > 7$

$x < 11$ $\quad$ $4x > 6$

$x > 1\frac{1}{2}$

$\therefore 1\frac{1}{2} < x < 11$

(g) $2x - 3 \leqslant 5$ and $7 - 6x \leqslant -3$

$2x \leqslant 8$ $\quad$ $-6x \leqslant -10$

$x \leqslant 4$ $\quad$ $x \geqslant 1\frac{2}{3}$

$\therefore 1\frac{2}{3} \leqslant x \leqslant 4$

(h) $10x - 7 < 11$ and $5x - 2 > -4$

$10x < 18$ $\quad$ $5x > -2$

$x < 1\frac{4}{5}$ $\quad$ $x > -\frac{2}{5}$

$\therefore -\frac{2}{5} < x < 1\frac{4}{5}$

(i) $2x - 9 < 14$ and $3x - 8 > 11$

$2x < 23$ $\quad$ $3x > 19$

$x < 11\frac{1}{2}$ $\quad$ $x > 6\frac{1}{3}$

$\therefore 6\frac{1}{3} < x < 11\frac{1}{2}$

(j) $14 - x > 3$ and $1 - 2x < 10$

$-x > -11$ $\quad$ $-2x < 9$

$x < 11$ $\quad$ $x > -4\frac{1}{2}$

$\therefore -4\frac{1}{2} < x < 11$

Intermediate

21. (a) $\frac{3x}{6} \leqslant -8$

$x \leqslant -16$

(b) $\frac{x+1}{4} \geqslant \frac{x}{3}$

$3x + 3 \geqslant 4x$

$-x \geqslant -3$

$x \leqslant 3$

(c) $\frac{1}{4} + \frac{1}{3}x > 3x - \frac{1}{2}$

$\frac{3+4x}{12} > \frac{6x-1}{2}$

$6 + 8x > 72x - 12$

$-64x > -18$

$x < \frac{9}{32}$

(d) $\frac{x-1}{2} - \frac{x+1}{3} < 1\frac{1}{6}$

$\frac{3(x-1)-2(x+1)}{6} < \frac{7}{6}$

$3x - 3 - 2x - 2 < 7$

$x - 5 < 7$

$x < 12$

(e) $\frac{2x+1}{3} < \frac{3x-4}{5} + \frac{2}{3}$

$\frac{2x+1}{3} < \frac{3(3x-4)+2(5)}{15}$

$\frac{2x+1}{3} < \frac{9x-12+10}{15}$

$\frac{2x+1}{3} < \frac{9x-2}{15}$

$30x + 15 < 27x - 6$

$3x < -21$

$x < -7$

(f) $\frac{2x-1}{4} - \frac{2x-7}{3} < \frac{5}{7}$

$\frac{3(2x-1)-4(2x-7)}{12} < \frac{5}{7}$

$\frac{6x-3-8x+28}{12} < \frac{5}{7}$

$\frac{25-2x}{12} < \frac{5}{7}$

$175 - 14x < 60$

$-14x < -115$

$x > 8\frac{3}{14}$

(g) $\frac{5x}{6} - \frac{7}{9} \leqslant 2x - 4\frac{1}{2}$

$\frac{15x-14}{18} \leqslant \frac{4x-9}{2}$

$30x - 28 \leqslant 72x - 162$

$-42x \leqslant -134$

$x \geqslant 3\frac{4}{21}$

(h) $\frac{2-4x}{5} \geqslant 2\frac{1}{2} - 6x$

$\frac{2-4x}{5} \geqslant \frac{5-12x}{2}$

$4 - 8x \geqslant 25 - 60x$

$52x \geqslant 21$

$x \geqslant \frac{21}{52}$

(i) $\frac{2x-7}{8} + \frac{x-3}{4} \leqslant \frac{2x+3}{6} + 1$

$\frac{2x-7+2(x-3)}{8} \leqslant \frac{2x+3+6}{6}$

$\frac{2x-7+2x-6}{8} \leqslant \frac{2x+9}{6}$

$\frac{4x-13}{8} \leqslant \frac{2x+9}{6}$

$24x - 78 \leqslant 16x + 72$

$8x \leqslant 150$

$x \leqslant 18\frac{3}{4}$

(j) $\frac{x}{5} - 4 < 3 - \frac{5}{4}x$

$\frac{x}{5} + \frac{5}{4}x < 7$

$\frac{4x+25x}{20} < 7$

$\frac{29x}{20} < 7$

$x < 4\frac{24}{29}$

(k) $\frac{1}{3}(4x-3) > \frac{1}{2}(x+5)$

$8x - 6 > 3x + 15$

$5x > 21$

$x > 4\frac{1}{5}$

22. (a) $2 - x < 2x + 3 \leqslant x + 6$

$2 - x < 2x + 3$ and $2x + 3 \leqslant x + 6$

$-3x < 1$ $\quad$ $x \leqslant 3$

$x > -\frac{1}{3}$

$\therefore -\frac{1}{3} < x \leqslant 3$

(b) $x + 2 < 14 < 3x + 1$

$x + 2 < 14$ and $14 < 3x + 1$

$x > 12$ $\quad$ $-3x < -13$

$x > 4\frac{1}{3}$

$\therefore 4\frac{1}{3} < x < 12$

(c) $8x + 1 \leqslant 2x + 1 \leqslant 3x + 2$

$$8x + 1 \leqslant 2x + 2 \qquad \text{and} \qquad 2x + 1 \leqslant 3x + 2$$
$$6x \leqslant 0 \qquad\qquad -x \leqslant 1$$
$$x \leqslant 0 \qquad\qquad x \geqslant -1$$

$\therefore -1 \leqslant x \leqslant 0$

(d) $3x - 3 \leqslant 5x + 9 \leqslant x + 35$

$$3x - 3 \leqslant 5x + 9 \qquad \text{and} \qquad 5x + 9 \leqslant x + 35$$
$$-2x \leqslant 12 \qquad\qquad 4x \leqslant 26$$
$$x \geqslant -6 \qquad\qquad x \leqslant 6\frac{1}{2}$$

$\therefore -6 \leqslant x \leqslant 6\frac{1}{2}$

(e) $6x + 4 < 4x - 2 \leqslant 2x + 1$

$$6x + 4 < 4x - 2 \qquad \text{and} \qquad 4x - 2 \leqslant 2x + 1$$
$$2x < -6 \qquad\qquad 2x \leqslant 3$$
$$x < -3 \qquad\qquad x \leqslant 1\frac{1}{2}$$

$\therefore x < -3$

(f) $x + 2 \geqslant 1 - 3x > x - 11$

$$x + 2 \geqslant 1 - 3x \qquad \text{and} \qquad 1 - 3x > x - 11$$
$$4x \geqslant -1 \qquad\qquad -4x > -12$$
$$x \geqslant -\frac{1}{4} \qquad\qquad x < 3$$

$\therefore -\frac{1}{4} \leqslant x < 3$

(g) $3x - 5 < 26 \leqslant 4x - 6$

$$3x - 5 < 26 \qquad \text{and} \qquad 26 \leqslant 4x - 6$$
$$3x < 31 \qquad\qquad -4x \leqslant -32$$
$$x < 10\frac{1}{3} \qquad\qquad x \geqslant 8$$

$\therefore 8 \leqslant x < 10\frac{1}{3}$

23. (a) $x - \frac{3}{2} < \frac{5 - 6x}{4} < x + \frac{1}{2}$

$$x - \frac{3}{2} < \frac{5 - 6x}{4} \qquad \text{and} \qquad \frac{5 - 6x}{4} < x + \frac{1}{2}$$
$$\frac{2x - 3}{2} < \frac{5 - 6x}{4} \qquad\qquad \frac{5 - 6x}{4} < \frac{2x + 1}{2}$$
$$8x - 12 < 10 - 12x \qquad\qquad 10 - 12x < 8x + 4$$
$$20x < 22 \qquad\qquad -20x < -6$$
$$x < 1\frac{1}{10} \qquad\qquad x > \frac{3}{10}$$

$\therefore \frac{3}{10} < x < 1\frac{1}{10}$

(b) $2 + \frac{3x}{2} \leqslant \frac{5x + 1}{3} \leqslant \frac{3x + 11}{2}$

$$2 + \frac{3x}{2} \leqslant \frac{5x + 1}{3} \qquad \text{and} \qquad \frac{5x + 1}{3} \leqslant \frac{3x + 11}{2}$$
$$\frac{4 + 3x}{2} \leqslant \frac{5x + 1}{3} \qquad\qquad 10x + 2 \leqslant 9x + 33$$
$$12 + 9x \leqslant 10x + 2 \qquad\qquad x \leqslant 31$$
$$-x \leqslant -10$$
$$x \geqslant 10$$

$\therefore 10 \leqslant x \leqslant 31$

(c) $2x + 3 > \frac{7x + 6}{4} \geqslant 3x + 2$

$$2x + 3 > \frac{7x + 6}{4} \qquad \text{and} \qquad \frac{7x + 6}{4} \geqslant 3x + 2$$
$$8x + 12 > 7x + 6 \qquad\qquad 7x + 6 \geqslant 12x + 8$$
$$x > -6 \qquad\qquad -5x \geqslant 2$$
$$x \leqslant -\frac{2}{5}$$

$\therefore -6 < x \leqslant -\frac{2}{5}$

(d) $2x - 15\frac{1}{2} > x + \frac{1}{2} \geqslant 2x - 25\frac{1}{2}$

$$2x - 15\frac{1}{2} > x + \frac{1}{2} \qquad \text{and} \qquad x + \frac{1}{2} \geqslant 2x - 25\frac{1}{2}$$
$$x > 16 \qquad\qquad -x \geqslant -26$$
$$x \leqslant 26$$

$\therefore 16 < x \leqslant 26$

(e) $\frac{x}{2} + \frac{1}{5} \geqslant \frac{2x}{5} > x - 5$

$$\frac{x}{2} + \frac{1}{5} \geqslant \frac{2x}{5} \qquad \text{and} \qquad \frac{2x}{5} > x - 5$$
$$\frac{5x + 2}{10} \geqslant \frac{2x}{5} \qquad\qquad 2x > 5x - 25$$
$$25x + 10 \geqslant 20x \qquad\qquad -3x > -25$$
$$5x \geqslant -10 \qquad\qquad x < 8\frac{1}{3}$$
$$x \geqslant -2$$

$\therefore -2 \leqslant x < 8\frac{1}{3}$

(f) $\frac{1}{2}x + 6 < \frac{1}{4}x + 10 < x + 5$

$$\frac{1}{2}x + 6 < \frac{1}{4}x + 10 \qquad \text{and} \qquad \frac{1}{4}x + 10 < x + 5$$
$$\frac{1}{4}x < 4 \qquad\qquad -\frac{3}{4}x < -5$$
$$x < 16 \qquad\qquad x > 6\frac{2}{3}$$

$\therefore 6\frac{2}{3} < x < 16$

(g) $-2x + 4 \leqslant \frac{3x - 5}{3} \leqslant 5x - 6$

$$-2x + 4 \leqslant \frac{3x - 5}{3} \qquad \text{and} \qquad \frac{3x - 5}{3} \leqslant 5x - 6$$
$$-6x + 12 \leqslant 3x - 5 \qquad\qquad 3x - 5 \leqslant 15x - 18$$
$$-9x \leqslant -17 \qquad\qquad -12x \leqslant -13$$
$$x \geqslant 1\frac{8}{9} \qquad\qquad x \geqslant 1\frac{1}{12}$$

$\therefore x \geqslant 1\frac{8}{9}$

(h) $\frac{2}{5}x < 2x - 1 \leqslant \frac{10 + 2x}{15}$

$\frac{2}{5}x < 2x - 1$ and $2x - 1 \leqslant \frac{10 + 2x}{15}$

$2x < 10x - 5$ $\quad$ $30x - 15 \leqslant 10 + 2x$

$-8x < -5$ $\quad$ $28x \leqslant 25$

$x > \frac{5}{8}$ $\quad$ $x \leqslant \frac{25}{28}$

$\therefore \frac{5}{8} < x \leqslant \frac{25}{28}$

24. $\frac{1}{2}(y - 4) > \frac{2y}{3}$

$3y - 12 > 4y$

$-y > 12$

$y < -12$

$\therefore$ Largest integer value of y is -13.

25. $3 - 3x \leqslant 2 + 2x < 5x + 1$

$3 - 3x \leqslant 2 + 2x$ and $2 + 2x < 5x + 1$

$-5x \leqslant -1$ $\quad$ $-3x < -1$

$x \geqslant \frac{1}{5}$ $\quad$ $x > \frac{1}{3}$

$\therefore x > \frac{1}{3}$

(a) 1

(b) 2

26. $3x + 5 < 4x - 2 \leqslant 3x + 7$

$3x + 5 < 4x - 2$ and $4x - 2 \leqslant 3x + 7$

$-x < -7$ $\quad$ $x \leqslant 9$

$x > 7$

$\therefore 7 < x \leqslant 9$

Integer values of x are 8 and 9.

27. $\frac{q + 8}{3} \leqslant \frac{4q}{3} - 4$

$\frac{q + 8}{3} \leqslant \frac{4q - 12}{3}$

$q + 8 \leqslant 4q - 12$

$-3q \leqslant -20$

$q \geqslant 6\frac{2}{3}$

(a) 7

(b) 7

28. $\frac{1}{4}x - \frac{3}{5}\left(x + \frac{1}{3}\right) \leqslant \frac{1}{2}(x - 9)$

$\frac{1}{4}x - \frac{3}{5}x - \frac{1}{5} \leqslant \frac{1}{2}x - \frac{9}{2}$

$-\frac{7}{20}x - \frac{1}{5} \leqslant \frac{1}{2}x - \frac{9}{2}$

$-\frac{17}{20}x \leqslant -\frac{43}{10}$

$x \geqslant 5\frac{1}{17}$

(a) $5\frac{1}{17}$

(b) 6

29. $\frac{y + 8}{3} \leqslant \frac{4y}{5} - 1$

$\frac{y + 8}{3} \leqslant \frac{4y - 5}{5}$

$5y + 40 \leqslant 12y - 15$

$-7y \leqslant -55$

$y \geqslant 7\frac{6}{7}$

(a) 8

(b) 11

30. $40 < 60 - 50t < 50$

$40 < 60 - 50t$ and $60 - 50t < 50$

$50t < 20$ $\quad$ $-50t < -10$

$t < \frac{2}{5}$ $\quad$ $t > \frac{1}{5}$

$\therefore \frac{1}{5} < t < \frac{2}{5}$

31. $5 < x - 1 < 9$ and $9\frac{1}{2} < 2x + 1\frac{1}{2} < 18$

$6 < x < 10$ $\quad$ $8 < 2x < 16\frac{1}{2}$

$4 < x < 8\frac{1}{4}$

$\therefore 6 < x < 8\frac{1}{4}$

Integers are 7 and 8.

32. $x < 3 + 8$

$\therefore x < 11$

33. Let the integers be x, $x + 1$ and $x + 2$.

$x + x + 1 + x + 2 \leqslant 370$

$3x + 3 \leqslant 370$

$3x \leqslant 367$

$x \leqslant 122\frac{1}{3}$

(a) 123

(b) $\sqrt{124} = 11.1$ (to 3 s.f.)

34. Let x m be the breadth of the plot.

$2(4x + x) \leqslant 220$

$10x \leqslant 220$

$x \leqslant 22$

Largest possible area = (88)(22)

$= 1936\ m^2$

3 5. Let Farhan's age be x years.

$x + 2x \geqslant 53$

$3x \geqslant 53$

$x \geqslant 17\frac{2}{3}$

$\therefore$ Minimum age of Farhan is 18 years.

36. Let the number of questions he answered correctly be x.

$$2x - (18 - x) > 30$$
$$2x - 18 + x > 30$$
$$3x > 48$$
$$x > 16$$

$\therefore$ Minimum number of questions he answered correctly is 17.

37. Let x be the number of strawberries.

$$x + \frac{2}{3}x \leqslant 65$$
$$\frac{5}{3}x \leqslant 65$$
$$x \leqslant 39$$

$\therefore$ Maximum number of strawberries is 39.

38. Let the number of 50-paisa coins be x.

$$3(50) + 20(2) + x(0.5) \leqslant 200$$
$$150 + 40 + 0.5x \leqslant 200$$
$$0.5x \leqslant 10$$
$$x \leqslant 20$$

$\therefore$ Maximum number of 50-paisa coins is 20.

39. **(a)** Greatest possible value of $a + b = 3 + (-2)$ $= 1$

(b) Least possible value of $a - b = -5 - (-2)$ $= -3$

(c) Largest possible value of $ab = (-5)(-8)$ $= 40$

(d) Smallest possible value of $\frac{a}{b} = \frac{3}{-2}$ $= -1\frac{1}{2}$

(e) Greatest possible value of $a^2 = (-5)^2$ $= 25$

Least possible value of $a^2 = 0^2$ $= 0$

Advanced

43. **(a)** Greatest possible value of $(x - y)^2 = [8 - (-5)]^2$ $= 169$

(b) Least possible value of $(x + y)^2 = [5 + (-5)]^2$ $= 0$

(c) Largest possible value of $\frac{2y}{x} = \frac{2(2)}{2}$ $= 2$

(d) Largest possible value of $\frac{y^2}{x} = \frac{(-5)^2}{2}$ $= 12\frac{1}{2}$

(e) Greatest possible value of $x^3 - y^3 = 8^3 - (-5)^3$ $= 637$

Least possible value of $x^3 - y^3 = 2^3 - 2^3$ $= 0$

44. **(a)** Least possible value of $p^2 - q^2 = \left(-\frac{1}{2}\right)^2 - 6^2$ $= -35\frac{3}{4}$

(b) Least possible value of $p^2 + q^2 = \left(-\frac{1}{2}\right)^2 + 0^2$ $= \frac{1}{4}$

(c) Largest possible value of $pq = (-2)(-1)$ $= 2$

(d) Smallest possible value of $\frac{q}{p} = \frac{6}{-\frac{1}{2}}$ $= -12$

(e) Greatest possible value of $p^3 + q^3 = \left(-\frac{1}{2}\right)^2 + 6^3$ $= 215\frac{7}{8}$

Least possible value of $p^3 + q^3 = (-2)^3 + (-1)^3$ $= -9$

New Trend

45. **(i)** $-10 < 7 - 2x \leqslant -1$

$-10 < 7 - 2x$ and $7 - 2x \leqslant -1$

$2x < 17$ $\quad$ $-2x \leqslant -8$

$x < 8\frac{1}{2}$ $\quad$ $x \geqslant 4$

$\therefore 4 \leqslant x < 8\frac{1}{2}$

(ii) Integers are 4, 5, 6, 7 and 8.

46. $2(x + 1) > \frac{3}{5}(x - 4)$

$$10(x + 1) > 3(x - 4)$$
$$10x + 10 > 3x - 12$$
$$7x > -22$$
$$x > -3\frac{1}{7}$$

47. **(a)** $-5 < x \leqslant 3$

Integers are $-4, -3, -2, -1, 0, 1, 2$ and 3.

(b) $x - 3 < 2x - 1 < 5 + x$

$x - 3 < 2x - 1$ and $2x - 1 < 5 + x$

$-x < 2$ $\quad$ $x < 6$

$x > -2$

$\therefore -2 < x < 6$

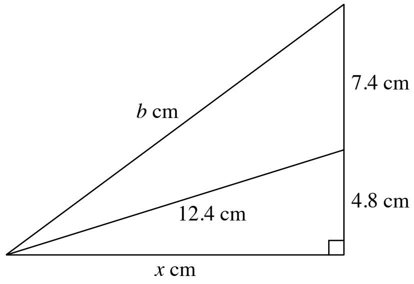
Chapter 7 Pythagoras' Theorem

Basic

1. (a) Using Pythagoras' Theorem,

$a^2 = 11.9^2 + 6.8^2$
$= 187.85$
$a = \sqrt{187.85}$
$= 13.7$ (to 3 s.f.)

(b)



Using Pythagoras' Theorem,

$x^2 + 4.8^2 = 12.4^2$
$x^2 = 130.72$
$x = \sqrt{130.72}$

Using Pythagoras' Theorem,

$b^2 = 130.72 + (7.4 + 4.8)^2$
$= 279.56$
$b = \sqrt{279.56}$
$= 16.7$ (to 3 s.f.)

2. (a) Using Pythagoras' Theorem,

$(3a)^2 + (2a)^2 = 18.9^2$
$9a^2 + 4a^2 = 357.21$
$13a^2 = 357.21$
$a^2 = 27.47$ (to 4 s.f.)
$a = \sqrt{27.47}$
$= 5.24$ (to 3 s.f.)

(b) Using Pythagoras' Theorem,

$(3b + 4b + 3b)^2 + 16.3^2 = 29.6^2$
$(10b)^2 = 29.6^2 - 16.3^2$
$100b^2 = 610.47$
$b^2 = 6.1047$
$b = \sqrt{6.1047}$
$= 2.47$ (to 3 s.f.)

3. Using Pythagoras' Theorem,

$a^2 = 5^2 + 12^2$
$= 169$
$a = \sqrt{169}$
$= 13$

Using Pythagoras' Theorem,

$b^2 + 12^2 = 21^2$
$b^2 = 21^2 - 12^2$
$= 297$
$b = \sqrt{297}$
$= 17.2$ (to 3 s.f.)

$\therefore a = 13, b = 17.2$

4. Using Pythagoras' Theorem,

$(x + 1)^2 + (4x)^2 = (4x + 1)^2$
$x^2 + 2x + 1 + 16x^2 = 16x^2 + 8x + 1$
$x^2 - 6x = 0$
$x(x - 6) = 0$
$x = 0$ (rejected) or $x = 6$

5. Let the length of the ladder be x m.

Using Pythagoras' Theorem,

$x^2 = 3.2^2 + 0.8^2$
$= \sqrt{10.88}$
$x = 3.30$ (to 3 s.f.)

$\therefore$ The length of the ladder is 3.30 m.

6. Let the vertical height of the cone be h cm.

Using Pythagoras' Theorem,

$h^2 + 8^2 = 12^2$
$h^2 = 12^2 - 8^2$
$= 80$
$h = \sqrt{80}$
$= 8.94$ (to 3 s.f.)

$\therefore$ The vertical height of the cone is 8.94 cm.

7. Let the length of the diagonal be x m.

Using Pythagoras' Theorem,

$x^2 = 14^2 + 12^2$
$= 340$
$x = \sqrt{340}$
$= 18.4$ (to 3 s.f.)

$\therefore$ The length of the fence is 18.4 m.

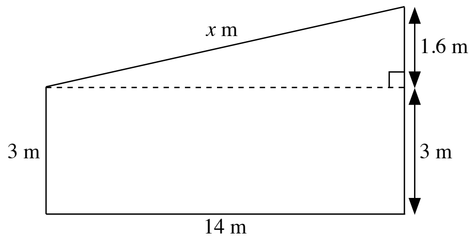
8. Let the distance between the tips of the hands be x m.

Using Pythagoras' Theorem,

$x^2 = 3.05^2 + 3.85^2$
$= 24.125$
$x = \sqrt{24.125}$
$= 4.91$ (to 3 s.f.)

$\therefore$ The distance between the tips of the hands is 4.91 m.

9.



Using Pythagoras' Theorem,

$x^2 = 14^2 + 1.6^2$

$= 198.56$

$x = \sqrt{198.56}$

$= 14.1$ (to 3 s.f.)

$\therefore$ The distance between the top of the two posts is 14.1 m.

10. Using Pythagoras' Theorem,

$\left(\frac{d}{2}\right)^2 + 9^2 = 18^2$

$\left(\frac{d}{2}\right)^2 = 18^2 - 9^2$

$= 243$

$\frac{d}{2} = \sqrt{243}$

$d = 2\sqrt{243}$

$= 31.2$ (to 3 s.f.)

11. **(a)** $AC^2 = 32^2$

$= 1024$

$AB^2 + BC^2 = 24^2 + 28^2$

$= 1360$

Since $AC^2 \neq AB^2 + BC^2$,

$\therefore \triangle ABC$ is not a right-angled triangle.

(b) $DF^2 = 85^2$

$= 7225$

$DE^2 + EF^2 = 13^2 + 84^2$

$= 7225$

Since $DF^2 = DE^2 + EF^2$,

$\therefore \triangle DEF$ is a right-angled triangle with $\angle DEF = 90°$.

(c) $HI^2 = 6.5^2$

$= 42.25^2$

$GH^2 + GI^2 = 3.3^2 + 5.6^2$

$= 42.25$

Since $HI^2 = GH^2 + GI^2$,

$\therefore \triangle GHI$ is a right-angled triangle with $\angle HGI = 90°$.

(d) $KL^2 = \left(2\frac{3}{17}\right)^2$

$= 4\frac{213}{289}$

$JK^2 + JL^2 = \left(\frac{12}{17}\right)^2 + 2^2$

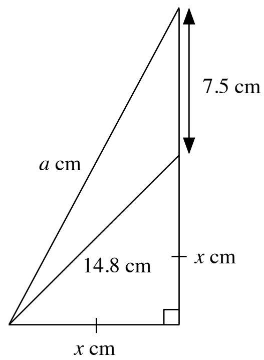
$= 4\frac{144}{289}$

Since $KL^2 \neq JK^2 + JL^2$,

$\therefore \triangle JKL$ is not a right-angled triangle.

Intermediate

12. **(a)**



Using Pythagoras' Theorem,

$x^2 + x^2 = 14.8^2$

$2x^2 = 219.04$

$x^2 = 109.52$

$x = \sqrt{109.52}$

$= 10.47$ (to 4 s.f.)

Using Pythagoras' Theorem,

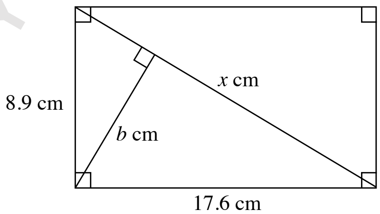
$a^2 = 10.47^2 + (7.5 + 10.47)^2$

$= 432.2$ (to 4 s.f.)

$a = \sqrt{432.2}$

$= 20.8$ (to 3 s.f.)

(b)



Using Pythagoras' Theorem,

$x^2 = 8.9^2 + 17.6^2$

$= 388.97$

$x = \sqrt{388.97}$

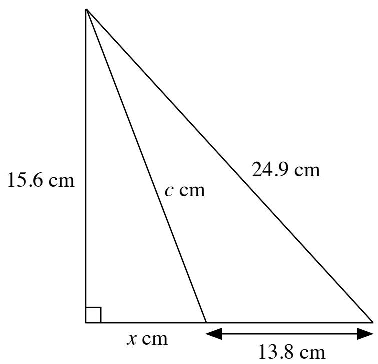
Area of triangle $= \frac{1}{2} \times$ base $\times$ height

$\frac{1}{2} \times \sqrt{388.97} \times b = \frac{1}{2} \times 17.6 \times 8.9$

$b = \frac{17.6 \times 8.9}{\sqrt{388.97}}$

$= 7.94$ (to 3 s.f.)

(c)



Using Pythagoras' Theorem,

$(x + 13.8)^2 + 15.6^2 = 24.9^2$

$(x + 13.8)^2 = 376.65$

$x + 13.8 = \sqrt{376.65}$

$x = \sqrt{376.65} - 13.8$

$= 5.607$ (to 4 s.f.)

Using Pythagoras' Theorem,

$c^2 = 15.6^2 + 5.607^2$

$= 274.8$ (to 4 s.f.)

$c = \sqrt{274.8}$

$= 16.6$ (to 3 s.f.)

13. Using Pythagoras' Theorem,

$a^2 = 8^2 + 9^2$

$= 145$

$a = \sqrt{145}$

$= 12.0$ (to 3 s.f.)

Using Pythagoras' Theorem,

$b^2 = 16^2 + 9^2$

$= 337$

$b = \sqrt{337}$

$= 18.4$ (to 3 s.f.)

$\therefore a = 12.0, b = 18.4$

14. (i) Using Pythagoras' Theorem,

$QR^2 + 8.5^2 = 12.3^2$

$QR^2 = 12.3^2 - 8.5^2$

$= 79.04$

$QR = \sqrt{79.04}$

$= 8.89$ cm (to 3 s.f.)

(ii) Using Pythagoras' Theorem,

$PS^2 + 12.3^2 = 17.8^2$

$PS^2 = 17.8^2 - 12.3^2$

$= 165.55$

$PS = \sqrt{165.55}$

$= 12.9$ cm (to 3 s.f.)

(iii) Area of trapezium $PQRS = \frac{1}{2}(8.5 + 17.8)\sqrt{79.04}$

$= 117 \text{ cm}^2$ (to 3 s.f.)

15. Area of $\triangle ABC = \frac{1}{2} \times AB \times 14$

$180 = 7AB$

$AB = \frac{180}{7}$ cm

Using Pythagoras' Theorem,

$AC^2 = \left(\frac{180}{7}\right)^2 + 14^2$

$= 857.2$ (to 4 s.f.)

$AC = \sqrt{857.2}$

$= 29.3$ cm (to 3 s.f.)

16. Using Pythagoras' Theorem,

$BK^2 + 7^2 = 12^2$

$BK^2 = 12^2 - 7^2$

$= 95$

$BK = \sqrt{95}$

$= 9.746$ cm (to 4 s.f.)

$BC = 2(9.746)$

$= 19.49$ cm (to 4 s.f.)

Using Pythagoras' Theorem,

$(2x + 3)^2 = 19.49^2 + 8^2$

$= 444$

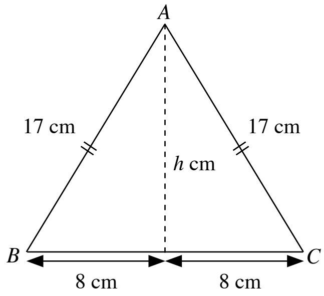
$2x + 3 = \sqrt{444}$

$= 21.07$ (to 4 s.f.)

$2x = 18.07$

$x = 9.04$ (to 3 s.f.)

17.



Using Pythagoras' Theorem,

$h^2 + 8^2 = 17^2$

$h^2 = 17^2 - 8^2$

$= 225$

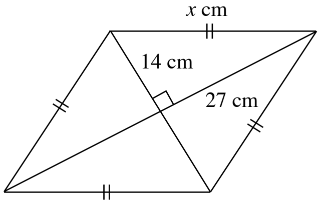
$h = \sqrt{225}$

$= 15$

Area of $\triangle ABC = \frac{1}{2}(16)(15)$

$= 120 \text{ cm}^2$

18.



Using Pythagoras' Theorem,

$x^2 = 14^2 + 27^2$

$= 925$

$x = \sqrt{925}$

$= 30.41$ (to 4 s.f.)

$\therefore$ Perimeter $= 4(30.41)$

$= 122$ cm (to 3 s.f.)

19. Using Pythagoras' Theorem,

$PQ^2 = (28 - 11)^2 + (28 - 9)^2$

$= 17^2 + 19^2$

$= 650$

Area of $PQRS = PQ^2$

$= 650 \text{ cm}^2$

20. Using Pythagoras' Theorem,

$XB^2 + 1.3^2 = 5^2$

$XB^2 = 5^2 - 1.3^2$

$= 23.31$

$XB = \sqrt{23.31}$

$= 4.828$ cm (to 4 s.f.)

$\therefore XY = 2(4.828)$

$= 9.66$ cm (to 3 s.f.)

21. $P(-2, -1)$, $T(6, 5)$

Using Pythagoras' Theorem,

$PT^2 = 8^2 + 6^2$

$= 100$

$PT = 10$

$\therefore$ The player has to run 10 units.

22. Let the height of the LCD screen be h inches.

Using Pythagoras' Theorem,

$h^2 + 48.5^2 = 55^2$

$h^2 = 55^2 - 48.5^2$

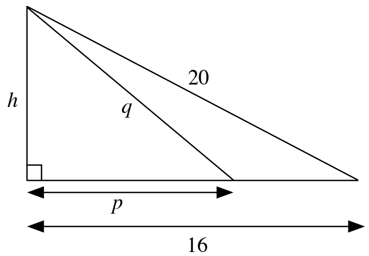
$= 672.75$

$h = \sqrt{672.75}$

$= 25.9$ (to 3 s.f.)

Since $h > 24$, the box will not fit the LCD screen.

23.



Using Pythagoras' Theorem,

$h^2 + 16^2 = 20^2$

$h^2 = 20^2 - 16^2$

$= 144$

Using Pythagoras' Theorem,

$p^2 + 144 = q^2$

24. Using Pythagoras' Theorem,

$(x + 2)^2 + x^2 = (x + 4)^2$

$x^2 + 4x + 4 + x^2 = x^2 + 8x + 16$

$x^2 - 4x - 12 = 0$

$(x - 6)(x + 2) = 0$

$x = 6$ or $x = -2$ (rejected)

Perimeter $= 2(x + 2 + x)$

$= 4x + 4$

$= 4(6) + 4$

$= 28$ m

25. Using Pythagoras' Theorem,

$(2x)^2 + (4x - 1)^2 = (4x + 1)^2$

$4x^2 + 16x^2 - 8x + 1 = 16x^2 + 8x + 1$

$4x^2 - 16x = 0$

$4x(x - 4) = 0$

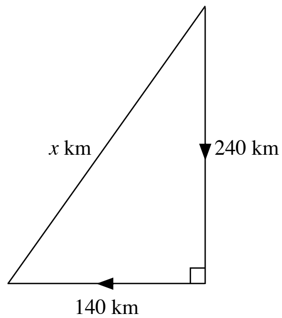
$x = 0$ or $x = 4$

(rejected)

Cross-sectional area of sandwich $= \frac{1}{2}(8)(15)$

$= 60 \text{ cm}^2$

26.



Using Pythagoras' Theorem,

$x^2 = 140^2 + 240^2$

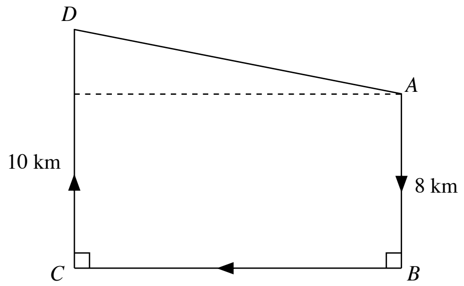
$= 77\ 200$

$x = \sqrt{77\ 200}$

$= 278$ (to 3 s.f.)

$\therefore$ The distance from the starting point is 278 km.

27.



$AB = 40 \times \frac{12}{60}$

$= 8$ km

$BC = 15$ km

$CD = 60 \times \frac{10}{60}$

$= 10$ km

Using Pythagoras' Theorem,

$DA^2 = 15^2 + 2^2$

$= 229$

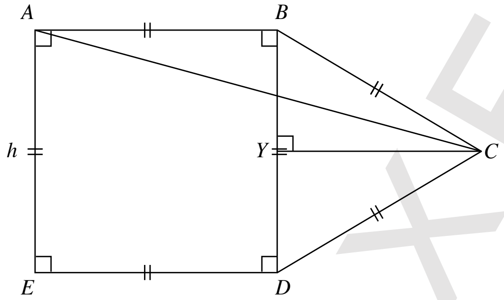
$DA = \sqrt{229}$

$= 15.1$ km (to 3 s.f.)

$\therefore$ The shortest distance is 15.1 km.

Advanced

28.



Using Pythagoras' Theorem,

$YC^2 + \left(\frac{h}{2}\right)^2 = h^2$

$YC^2 + \frac{h^2}{4} = h^2$

$YC^2 = h^2 - \frac{h^2}{4}$

$= \frac{3h^2}{4}$

$YC = \sqrt{\frac{3}{4}}\,h$

$= \frac{\sqrt{3}}{2}\,h$

Using Pythagoras' Theorem,

$AC^2 = \left(\frac{h}{2}\right)^2 + \left(h + \frac{\sqrt{3}}{2}\,h\right)^2$

$= 0.25h^2 + 3.482h^2$

$= 3.732h^2$

$AC = \sqrt{3.732h^2}$

$= 1.93h$ units (to 3 s.f.)

29. (a) At x-axis, $y = 0$

$3x + 15 = 0$

$x = -5$

At y-axis, $x = 0$

$y + 15 = 0$

$y = -15$

$\therefore$ The coordinates of A are $(-5, 0)$ and B are $(0, -15)$.

(b) Using Pythagoras' Theorem,

$AB^2 = 5^2 + 15^2$

$= 250$

$AB = \sqrt{250}$

$= 15.8$ units (to 3 s.f.)

$\therefore$ The length of the line joining A to B is 15.8 units.

30. (a) $BC = 23x - 2 - (3x - 2) - (5x + 1) - (6x - 7)$

$= 23x - 3x - 5x - 6x - 2 + 2 - 1 + 7$

$= (9x + 6)$ cm

(b) Since $BC = 2AD$,

$9x + 6 = 2(5x + 1)$

$9x + 6 = 10x + 2$

$x = 4$

Perimeter of trapezium $= 23x - 2$

$= 23(4) - 2$

$= 90$ cm

(c) $BX + CY = BC - AD$

$= 9(4) + 6 - [5(4) + 1]$

$= 42 - 21$

$= 21$

Since $5BX = 2CY$, $\frac{BX}{CY} = \frac{2}{5}$

$BX = \frac{21}{7} \times 2$

$= 6$

$AB = 3(4) - 2$

$= 10$

Using Pythagoras' Theorem,

$AX^2 = 10^2 - 6^2$

$= 64$

$AX = \sqrt{64}$

$= 8$ cm

Area of trapezium $= \frac{1}{2} \times 8 \times (21 + 42)$

$= 252$ cm^2

Chapter 8 Arc Length, Area of Sector, and Radian Measure

Basic

1. (a) (i) Perimeter $= \frac{70°}{360°}(2\pi)(6) + 2(6)$

$= 19\frac{1}{3}$ cm

(ii) Area $= \frac{70°}{360°}(\pi)(6)^2$

$= 22$ cm^2

(b) (i) Perimeter $= \frac{280°}{360°}(2\pi)(9) + 2(9)$

$= 62$ cm

(ii) Area $= \frac{280°}{360°}(\pi)(9)^2$

$= 198$ cm^2

(c) (i) Perimeter $= \frac{360° - 36°}{360°}(2\pi)(35) + 2(35)$

$= 268$ cm

(ii) Area $= \frac{360° - 36°}{360°}(\pi)(35)^2$

$= 3465$ cm^2

2. (a) Area of sector = 25.5 m^2

$\frac{\theta}{360}(\pi)(8)^2 = 25.5$

$\theta = 45.7$ (to 1 d.p.)

(b) Area of sector = 6.6 m^2

$\frac{\theta}{360}(\pi)(8)^2 = 6.6$

$\theta = 11.8$ (to 1 d.p.)

(c) Area of sector = 8 m^2

$\frac{\theta}{360}(\pi)(8)^2 = 8$

$\theta = 14.3$ (to 1 d.p.)

3. (a) Perimeter $= \frac{30°}{360°}(2\pi)(20) + \frac{30°}{360°}(2\pi)(30) + 2(10)$

$= 46.2$ cm (to 3 s.f.)

Area $= \frac{30°}{360°}(\pi)(30)^2 - \frac{30°}{360°}(\pi)(20)^2$

$= 131$ cm^2 (to 3 s.f.)

(b) Perimeter $= \frac{120°}{360°}(2\pi)(21)^2 + \frac{120°}{360°}(2\pi)(11) + 2(10)$

$= 87.0$ cm (to 3 s.f.)

Area $= \frac{120°}{360°}(\pi)(21)^2 - \frac{120°}{360°}(\pi)(11)^2$

$= 335$ cm^2 (to 3 s.f.)

Intermediate

4. (i) Perimeter $= \frac{50°}{360°}(2\pi)(20) + \frac{50°}{360°}(2\pi)(36) + 2(16)$

$= 80.9$ m (to 3 s.f.)

(ii) Using Cosine Rule,

$AC^2 = 20^2 + 36^2 - 2(20)(36)\cos 50°$

$AC = 27.8$ m (to 3 s.f.)

5. (i) Circumference of circle = 35.2 + 52.8

= 88 cm

Let the radius of the circle be r cm.

$2\pi r = 88$

$r = 14.0$ (to 3 s.f.)

∴ Radius of circle is 14.0 cm.

(ii) Let the angle subtended at the centre of the circle be θ rad.

$\theta = \frac{35.2}{14.00}$

$= 2.51$ (to 3 s.f.)

∴ Angle subtended is 2.51 rad.

6. (a) Time taken $= \frac{156°}{360°} \times 60$

= 26 minutes

(b) (i) Distance moved $= \frac{12}{60}(\pi)(42)$

= 26.4 cm

(ii) Distance moved $= \frac{45}{60}(\pi)(42)$

= 99 cm

7. Total area $= \frac{120°}{360°}(\pi)(10)^2 - \frac{120°}{360°}(\pi)(6)^2 + \frac{360° - 120°}{360°}(\pi)(6)^2$

$= 142$ cm^2 (to the nearest cm^2)

Advanced

8. (i) $r\theta + 2r = 4$

$r\theta = 4 - 2r$

$\theta = \left(\frac{4}{r} - 2\right)$

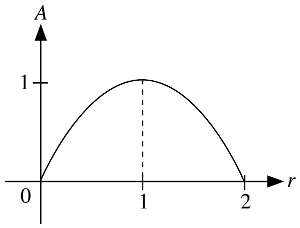
(ii) Area $= \frac{1}{2}r^2\theta$

$= \frac{1}{2}r^2\left(\frac{4}{r} - 2\right)$

$= 2r - r^2$

Let $A = 2r - r^2$

$= r(2 - r)$



When the area is a maximum,

$r = 1$.

(iii) When $r = 1$,

Area $= 1(2 - 1)$

$= 1 \text{ m}^2$

$\theta = \frac{4}{1} - 2$

$= 2$ rad

New Trend

9. **(i)** $(2d)\theta = 20$

$\theta = \frac{10}{d}$

(ii) Area of $R_1 = \frac{1}{2}(2d)^2\theta$

$= 2d^2\theta \text{ cm}^2$

Area of $R_2 = 6d^2\theta \text{ cm}^2$

$\frac{1}{2}(OD)^2\theta = 6d^2\theta + 2d^2\theta$

$OD^2 = 16d^2$

$OD = 4d$ cm

10. (a) Volume of sphere = $\frac{4}{3}\pi r^3$

$$34 = \frac{4}{3}\pi r^3$$

$$r^3 = \frac{51}{2\pi}$$

$$r = \sqrt[3]{\frac{51}{2\pi}}$$

$= 2.009$ (to 4 s.f.)

$= 2.01$ cm (to 3 s.f.)

Surface area of sphere $= 4\pi(2.009)^2$

$= 50.8$ cm^2 (to 3 s.f.)

(b) Volume of sphere = $\frac{4}{3}\pi r^3$

$$68.2 = \frac{4}{3}\pi r^3$$

$$r^3 = \frac{51.15}{\pi}$$

$$r = \sqrt[3]{\frac{51.15}{\pi}}$$

$= 2.534$ (to 4 s.f.)

$= 2.53$ m (to 3 s.f.)

Surface area of sphere $= 4\pi(2.534)^2$

$= 80.7$ m^2 (to 3 s.f.)

11. Surface area of sphere $= 4\pi(8)^2$

$= 256\pi$ m^2

Cost of painting $= \frac{256\pi}{8} \times 8.5$

$=$ PKR 854.51 (to 2 d.p.)

Intermediate

12. Let the height and the slant height of the pyramid be h cm and l cm respectively.

Total surface area of pyramid $= 8^2 + 4 \times \frac{1}{2}(8)l$

$$144 = 64 + 16l$$

$$16l = 80$$

$$l = 5$$

Using Pythagoras' Theorem,

$4^2 + h^2 = 5^2$

$16 + h^2 = 25$

$h^2 = 9$

$h = 3$

$\therefore$ Volume of pyramid $= \frac{1}{3} \times 8^2 \times 3$

$= 64$ cm^3

13. (i) Let the radius of the base be r m.

$2\pi r = 8.5$

$r = \frac{4.25}{\pi}$

$= 1.352$ (to 4 s.f)

Volume of rice $= \frac{1}{3}\pi(1.352)^2(1.2)$

$= 2.29$ (to 3 s.f.)

$= 2.3$ m^3 (to 2 s.f.)

(ii) Number of bags $= \frac{2.29}{0.5}$

$= 4.59$ (to 3 s.f.)

≈ 5

Assume that the space between the grains of rice is negligible.

14. Volume of crew cabin

$= \frac{1}{3}\pi\left(\frac{75}{2}\right)^2(92) - \frac{1}{3}\pi\left(\frac{27}{2}\right)^2(92 - 59)$

$= 129\ 000$ cm^3 (to 3 s.f.)

15. (i) Let the radius of the base be r cm.

$2\pi r = 88$

$r = \frac{44}{\pi}$

$= 14.00$ (to 4 s.f.)

Curved surface area of cone $= \pi\left(\frac{44}{\pi}\right)(15)$

$= 660$ cm^2

(ii) Total surface area of cone

$= 660 + \pi(14.00)^2$

$= 1276$ cm^2 (to the nearest integer)

16. (i) Curved surface area of cone $= \pi(x - 5)(x + 5)$

$75\pi = \pi(x^2 - 25)$

$75 = x^2 - 25$

$x^2 = 100$

$x = 10$

(ii) Base radius = 5 cm

Slant height = 15 cm

Height $= \sqrt{15^2 - 5^2}$

$= \sqrt{200}$

$\therefore$ Volume of cone $= \frac{1}{3}\pi(5)^2(\sqrt{200})$

$= 370$ cm^3 (to 3 s.f.)

Chapter 9 Volume and Surface Area of Pyramids, Cones and Spheres

Basic

1. (a) Volume of pyramid $= \frac{1}{3} \times 16^2 \times 27$
$= 2304 \text{ cm}^3$

(b) Volume of pyramid $= \frac{1}{3} \times \left(\frac{1}{2} \times 12 \times 9\right) \times 20$
$= 360 \text{ cm}^3$

(c) Volume of pyramid $= \frac{1}{3} \times 9 \times 5 \times 3$
$= 45 \text{ m}^3$

2. Volume of pyramid $= \frac{1}{3} \times 8 \times h$

$42 = \frac{8}{3}h$

$h = 15.75$

$\therefore$ The height of the figurine is 15.75 cm.

3. Volume of pyramid $= \frac{1}{3} \times 8 \times 3 \times h$

$86 = 8h$

$h = 10.75$

$\therefore$ The height of the pyramid is 10.75 m.

4. Volume of pyramid $= \frac{1}{3} \times \left(\frac{1}{2} \times 12 \times 5\right) \times h$

$160 = 10h$

$h = 16$

$\therefore$ The height of the pyramid is 16 m.

5. Total surface area $= 16^2 + 4 \times \frac{1}{2} \times 16 \times 17$
$= 800 \text{ m}^2$

6. $V = \frac{1}{3}\pi r^2 h$

(a) When $r = 8$ and $V = 320$,

$320 = \frac{1}{3}\pi(8)^2 h$

$h = \frac{960}{64\pi}$

$= 4.77$ (to 3 s.f.)

(b) When $r = 10.6$ and $V = 342.8$,

$342.8 = \frac{1}{3}\pi(10.6)^2 h$

$h = \frac{1028.4}{112.36\pi}$

$= 2.91$ (to 3 s.f.)

(c) When $h = 6$ and $V = 254$,

$254 = \frac{1}{3}\pi r^2(6)$

$r^2 = \frac{762}{6\pi}$

$r = \sqrt{\frac{762}{6\pi}}$

$= 6.36$ (to 3 s.f.)

(d) When $h = 11$ and $V = 695$,

$695 = \frac{1}{3}\pi r^2(11)$

$r^2 = \frac{2085}{11\pi}$

$r = \sqrt{\frac{2085}{11\pi}}$

$= 7.77$ (to 3 s.f.)

	Radius, r cm	Height, h cm	Volume, V cm^3
(a)	8	4.77	320
(b)	10.6	2.91	342.8
(c)	6.36	6	254
(d)	7.77	11	695

7. (a) Volume of cone $= \frac{1}{3}\pi(6)^2(8)$
$= 302 \text{ cm}^3$ (to 3 s.f.)

Total surface area of cone $= \pi(6)^2 + \pi(6)(10)$
$= 302 \text{ cm}^2$ (to 3 s.f.)

(b) Volume of cone $= \frac{1}{3}\pi(12)^2(28.8)$
$= 4340 \text{ cm}^3$ (to 3 s.f.)

Total surface area of cone $= \pi(12)^2 + \pi(12)(31.2)$
$= 1630 \text{ cm}^2$ (to 3 s.f.)

8. (a) Volume of sphere $= \frac{4}{3}\pi(5.8)^3$
$= 817 \text{ cm}^3$ (to 3 s.f.)

(b) Volume of sphere $= \frac{4}{3}\pi(12.6)^3$
$= 8380 \text{ m}^3$ (to 3 s.f.)

9. (a) Volume of sphere $= \frac{4}{3}\pi\left(\frac{24.2}{2}\right)^3$
$= 7420 \text{ cm}^3$ (to 3 s.f.)

(b) Volume of sphere $= \frac{4}{3}\pi\left(\frac{6.25}{2}\right)^3$
$= 128 \text{ mm}^3$ (to 3 s.f.)

17. **(i)** Volume of solid $= \frac{2}{3}\pi h^3 - \frac{2}{3}\pi\left(\frac{h}{2}\right)^3$

$= \frac{2}{3}\pi h^3 - \frac{1}{12}\pi h^3$

$= \frac{7}{12}\pi h^3$

(ii) Total surface area of solid

$= 2\pi h^2 + \left[\pi h^2 - \pi\left(\frac{h}{2}\right)^2\right] + 2\pi\left(\frac{h}{2}\right)^2$

$= 2\pi h^2 + \pi h^2 - \frac{1}{4}\pi h^2 + \frac{1}{2}\pi h^2$

$= \frac{13}{4}\pi h^2$

18. Volume of plastic $= \frac{4}{3}\pi(4)^3 - \frac{4}{3}\pi(3.6)^3$

$= 72.7 \text{ cm}^3$ (to 3 s.f.)

19. Volume of steel

$= 100 \times \left[\frac{4}{3}\pi\left(\frac{16}{2}\right)^3 - \frac{4}{3}\pi\left(\frac{16}{2} - 0.8\right)^3\right]$

$= 58\ 100 \text{ cm}^3$ (to 3 s.f.)

20. Amount of space $= 6^3 - \frac{4}{3}\pi\left(\frac{6}{2}\right)^3$

$= 103 \text{ cm}^3$ (to 3 s.f.)

21. **(a)** Total surface area of hemisphere $= 2\pi r^2 + \pi r^2$

$374 = 3\pi r^2$

$r^2 = \frac{374}{3\pi}$

$r = \sqrt{\frac{374}{3\pi}}$

$= 6.3$ cm (to 1 d.p.)

Volume of hemisphere $= \frac{2}{3}\pi\left(\sqrt{\frac{374}{3\pi}}\right)^3$

$= 523.6 \text{ cm}^3$ (to 1 d.p.)

(b) Total surface area of hemisphere $= 2\pi r^2 + \pi r^2$

$1058.4 = 3\pi r^2$

$r^2 = \frac{352.8}{\pi}$

$r = \sqrt{\frac{352.8}{\pi}}$

$= 10.6$ m (to 1 d.p.)

Volume of hemisphere $= \frac{2}{3}\pi\left(\sqrt{\frac{352.8}{\pi}}\right)^3$

$= 2492.5 \text{ m}^3$ (to 1 d.p.)

22. **(i)** Volume of sphere $= \frac{4}{3}\pi\left(\frac{x+2}{2}\right)^3$

$972\pi = \frac{4}{3}\pi\left(\frac{x+2}{2}\right)^3$

$\left(\frac{x+2}{2}\right)^3 = 729$

$\frac{x+2}{2} = 9$

$x + 2 = 18$

$x = 16$

(ii) Surface area of sphere $= 4\pi\left(\frac{18}{2}\right)^2$

$= 1020 \text{ cm}^2$ (to 3 s.f.)

23. Volume of glass

= volume of prism + volume of pyramid

$= \left(\frac{1}{2} \times 3.6 \times 4.8\right)(6) + \frac{1}{3}\left(\frac{1}{2} \times 3.6 \times 4.8\right)(12)$

$= 86.4 \text{ m}^3$

24. Volume of hemisphere $= \frac{2}{3}\pi(4)^3$

$= \frac{128}{3}\pi \text{ cm}^3$

$\therefore$ Volume of model $= \frac{37}{4} \times \frac{128}{3}\pi$

$= 1240 \text{ cm}^3$ (to 3 s.f.)

25. **(i)** Capacity of container $= \frac{1}{3}\pi(21)^2(21)$

$= 9698 \text{ cm}^3$ (to 4 s.f.)

$= 9.70\ l$ (to 3 s.f.)

(ii) Mass of container $= 9698 \times 1.5$

$= 14\ 540$ g (to 4 s.f.)

$= 15$ kg (to the nearest kg)

Advanced

26. **(i)** Volume of iron $= \frac{1}{3}\pi(1)^2(0.5)$

$= \frac{\pi}{6}$

$= 0.524 \text{ m}^3$ (to 3 s.f.)

Volume of lead $= \pi(2)^2(3) - \frac{\pi}{6}$

$= 12\pi - \frac{\pi}{6}$

$= \frac{71\pi}{6}$

$= 37.2 \text{ m}^3$ (to 3 s.f.)

(ii) Let the denisty of lead be ρ g/m^3.

Original mass of cylinder $= \pi(2)^2(3)\rho$

$= 12\pi\rho$ g

New mass of cylinder $= \frac{\pi}{6}\left(\frac{2}{3}\rho\right) + \frac{71\pi}{6}(\rho)$

$= \frac{\pi}{9}\rho + \frac{71\pi}{6}\rho$

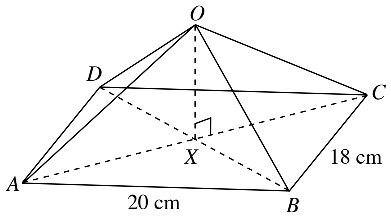
$= \frac{215\pi}{18}\rho$ g

$\therefore$ Percentage reduction in mass

$= \frac{12\pi\rho - \frac{215\pi}{18}\rho}{12\pi\rho} \times 100\%$

$= \frac{25}{54}\%$

27.



Using Pythagoras' Theorem,

$AC^2 = 20^2 + 18^2$

$= 724$

$AC = \sqrt{724}$ cm

$\tan 50° = \frac{AX}{OX}$

$OX = \frac{AX}{\tan 50°}$

$= \frac{\frac{1}{2}\sqrt{724}}{\tan 50°}$

$\therefore$ Volume of pyramid $= \frac{1}{3}(20 \times 18)\left(\frac{\frac{1}{2}\sqrt{724}}{\tan 50°}\right)$

$= 1350$ cm^3 (to 3 s.f.)

New Trend

28. **(a)** Using Pythagoras' Theorem,

$h^2 + 8^2 = 17^2$

$h^2 = 225$

$h = \sqrt{225}$

$= 15$

$\therefore$ The height of the cone is 15 cm. (shown)

(b) Volume of solid

= volume of cone + volume of hemisphere

$= \frac{1}{3}\pi(8)^2(15) + \frac{1}{2}\left[\frac{4}{3}\pi(8)^3\right]$

$= 2080$ cm^3 (to 3 s.f.)

29. Total surface area of solid $= \frac{1}{2}(4\pi x)^2 + 2\pi x(3x) + \pi x^2$

$= 2\pi x^2 + 6\pi x^2 + \pi x^2$

$= 9\pi x^2$

Total surface area of solid = 2 × surface area of cone

$9\pi x^2 = 2(\pi xl + \pi x^2)$

$7\pi x^2 = 2\pi xl$

$l = \frac{7\pi x^2}{2\pi x}$

$= \frac{7x}{2}$

30. **(a)** Let the height of the pyramid be h cm.

Using Pythagoras' Theorem,

$h^2 + 15^2 = 39^2$

$h^2 = 1296$

$h = \sqrt{1296}$

$= 36$

Volume of solid $= (30)(30)(70) + \frac{1}{3}(30)^2(36)$

$= 73\ 800$ cm^3

(b) Volume of spherical candle $= \frac{1}{10} \times 73\ 800$

$\frac{4}{3}\pi r^3 = 7380$

$r^3 = \frac{7380 \times 3}{4\pi}$

$r = \sqrt[3]{\frac{7380 \times 3}{4\pi}}$

$= 12.078$ cm (to 5 s.f.)

$= 12.1$ cm (to 3 s.f.)

(shown)

(c) Volume of cuboid

$= 4(12.078) \times 2(12.078) \times 2(12.078)$

$= 28\ 191$ cm^3 (to 5 s.f.)

Volume of empty space $= 28\ 191 - 2(7380)$

$= 13\ 400$ cm^3 (to 3 s.f.)

31. Total surface area $= \pi(4r)^2 + 2(2\pi r)(3r) + \frac{1}{2}[4\pi(4r)^2]$

$= 16\pi r^2 + 12\pi r^2 + 32\pi r^2$

$= 60\pi r^2$ cm^2

32. **(i)** Using Pythagoras' Theorem,

$x^2 = (15 - 9)^2 + 16^2$

$x^2 = 292$

$x = \sqrt{292}$

$= 17.088$ (to 5 s.f.)

$= 17.09$ cm (to 4 s.f.) (shown)

(ii) Let the slant height of the cone with radius 9 cm be l cm.

Using Pythagoras' Theorem,

$l^2 = (40 - 16)^2 + 9^2$

$l^2 = 657$

$l = \sqrt{657}$

$= 25.63$ cm (to 2 d.p.)

Total surface area of vase

$= \pi(15)(17.088 + 25.63) - \pi(9)(25.63) + \pi(15)^2$

$= 1995$ cm^2 (to the nearest whole number)

Chapter 9 Congruence and Similarity Tests

Basic

1. **(a)** $AB = ZY$

$BC = YX$

$AC = ZX$

$\therefore \triangle ABC \equiv \triangle ZYX$ (SSS)

(b) $PQ = LM$

$\angle QPR = \angle MLN$

$\angle PRQ = \angle LNM$

$\therefore \triangle PQR \equiv \triangle LMN$ (AAS)

(c) $AB = XY$

$AC = XZ$

$\angle BAC = \angle YXZ$

$\therefore \triangle ABC \equiv \triangle XYZ$ (SAS)

(d) $TP = SR$

$TQ = SQ$

$PQ = RQ$

$\therefore \triangle TPQ \equiv \triangle SRQ$ (SSS)

(e) $\angle CAB = \angle CBA$ (base $\angle$s of isos. $\triangle$)

$\angle FED = \angle FDE$ (base $\angle$s of isos. $\triangle$)

$\angle CAB = \angle FED$

$\angle CBA = \angle FDE$

$CA = FE$

$\therefore \triangle CAB \equiv \triangle FED$ (AAS)

(f) $ML = PQ$

$MO = PO$

$\angle LMO = \angle QPO$

$\therefore \triangle MLO \equiv \triangle PQO$ (SAS)

(g) $AB = ED$

$AC = EC$

$\angle ACB = \angle ECD$, which is not the included angle.

$\therefore$ The triangles may not be congruent.

(h) $PQ = PS$

$QR = SR$

PR is a common side.

$\therefore \triangle PQR \equiv \triangle PSR$ (SSS)

(i) $OL = OP$

$\angle OLM = \angle OPQ$

$\angle LOM = \angle POQ$

$\therefore \triangle OLM \equiv \triangle OPQ$ (AAS)

(j) $AB = CB$

BD is a common side.

$\angle BAD = \angle BCD = 90°$

$\therefore \triangle ABD \equiv \triangle CBD$ (RHS)

(k) $PQ = AB$

$\angle OPQ = \angle OAB$ (alt. $\angle$s, QP // AB)

$\angle POQ = \angle AOB$ (vert. opp. $\angle$s)

$\therefore \triangle OPQ \equiv \triangle OAB$ (AAS)

(l) $BC = EF$

$\angle BAC = \angle EDF$

$\angle BCA = \angle EFD$

$\therefore \triangle ABC \equiv \triangle DEF$ (AAS)

2. **(a)** $\angle BAC = \angle ZXY$

$\angle ACB = \angle XYZ$

$\therefore \triangle ABC$ is similar to $\triangle XZY$

(2 pairs of corr. $\angle$s equal).

(b) $\angle ABC = 180° - 90° - 30°$ ($\angle$ sum of a $\triangle$)

$= 60°$

$\angle ABC = \angle YZX$

$\angle CAB = \angle XYZ$

$\therefore \triangle ABC$ is similar to $\triangle YZX$

(2 pairs of corr. $\angle$s equal).

(c) $\dfrac{AC}{ZX} = \dfrac{13}{13} = 1$

$\dfrac{AB}{ZY} = \dfrac{13}{12}$

$\dfrac{BC}{YX} = \dfrac{5}{10} = \dfrac{1}{2}$

Since the ratios of the corresponding sides are not equal, the triangles are not similar.

(d) $\dfrac{AB}{XY} = \dfrac{14}{7} = 2$

$\dfrac{BC}{YZ} = \dfrac{6}{2} = 3$

Since the ratios of the corresponding sides are not equal, the triangles are not similar.

3. Let the height of the lamp post be h m.

Using similar triangles,

$$\frac{h}{1.7} = \frac{2.3 + 1.7}{1.7} = \frac{4.0}{1.7}$$

$h = 4.0$

$\therefore$ Height of lamp post is 4.0 m.

Intermediate

4. **(a)** $\triangle APD \equiv \triangle DSC \equiv \triangle BQA \equiv \triangle CRB$

(b) $\triangle AQP \equiv \triangle BSR$

$\triangle AQR \equiv \triangle BSP$

$\triangle ABP \equiv \triangle ABR$

(c) $\triangle RSX \equiv \triangle RQX$

$\triangle PSX \equiv \triangle PQX$

$\triangle PSR \equiv \triangle PQR$

(d) $\triangle PQT \equiv \triangle SRT$

5. $\angle EAB = \angle EDC$ (base $\angle$s of isos. $\triangle$)

$\angle EBA = \angle ECD$ (adj. $\angle$s. on a str. line)

$EA = ED$

$\therefore \triangle EAB \equiv \triangle EDC$ (AAS)

6. $\angle ABE + \angle EBD = \angle EBD + \angle DBC$

i.e. $\angle ABD = \angle CBE$

$\angle ADB = \angle CEB$

$AB = CB$

$\therefore \triangle ABD \equiv \triangle CBE$ (AAS)

7. **(a)** $\triangle AQR$

(b) $\triangle ASP$

8. $\triangle QZS$ and $\triangle YZX$

9. **(a)** $\triangle CAX$

(b) $\triangle EYZ$

10. **(a)** **(i)** $\triangle DXC$

(ii) $\triangle CDB$

(b) $\triangle DXA$

11. **(a)** $\triangle TSR$

Using similar triangles,

$$\frac{x}{18} = \frac{5}{9}$$

$$x = \frac{5}{9} \times 18$$

$$= 10$$

$$\frac{y}{6} = \frac{9}{5}$$

$$y = \frac{9}{5} \times 6$$

$$= 10.8$$

(b) $\triangle ABR$

Using similar triangles,

$$\frac{x+5}{5} = \frac{6}{2}$$

$$x + 5 = \frac{6}{2} \times 5$$

$$= 15$$

$$x = 10$$

$$\frac{y+4}{4} = \frac{6}{2}$$

$$y + 4 = \frac{6}{2} \times 4$$

$$= 12$$

$$y = 8$$

(c) $\triangle QAR$

Using similar triangles,

$$\frac{x+12}{15} = \frac{15}{12}$$

$$x + 12 = \frac{15}{12} \times 15$$

$$= 18\frac{3}{4}$$

$$x = 6\frac{3}{4}$$

$$\frac{y}{9} = \frac{15}{12}$$

$$y = \frac{15}{12} \times 9$$

$$= 11\frac{1}{4}$$

(d) $\triangle PXQ$

Using similar triangles,

$$\frac{x}{12} = \frac{12}{18}$$

$$x = \frac{12}{18} \times 12$$

$$= 8$$

$$\frac{y}{10} = \frac{18}{12}$$

$$y = \frac{18}{12} \times 10$$

$$= 15$$

(e) $\triangle ARB$

Using similar triangles,

$$\frac{x}{6} = \frac{15}{12}$$

$$x = \frac{15}{12} \times 6$$

$$= 7\frac{1}{2}$$

$$\frac{y}{10} = \frac{12}{15}$$

$$y = \frac{12}{15} \times 10$$

$$= 8$$

(f) $\triangle MLR$

Using similar triangles,

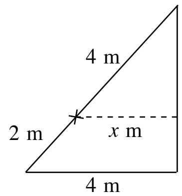
$$\frac{x}{12 - x} = \frac{6}{9}$$

$$9x = 72 - 6x$$

$$15x = 72$$

$$x = 4\frac{4}{5}$$

12.



Let the horizontal distance between the lizard and the wall be x m.

Using similar triangles,

$$\frac{x}{4} = \frac{4}{6}$$

$$x = \frac{4}{6} \times 4$$

$$= 2\frac{2}{3}$$

$\therefore$ Horizontal distance is $2\frac{2}{3}$ m.

13. **(i)** $\angle PQR = \angle PXZ$ (corr. $\angle$s, QR // XZ)

$\angle PQR = \angle XQY$ (common $\angle$)

$\angle QPR = \angle QXY$ (corr. $\angle$s. PR // XY)

$\angle QPR = \angle XPZ$ (common $\angle$)

$\therefore$ $\triangle PQR$, $\triangle PXZ$ and $\triangle XQY$ are similar (2 pairs of corr. $\angle$s equal).

(ii) Using similar triangles,

$$\frac{XY}{PR} = \frac{QY}{QR}$$

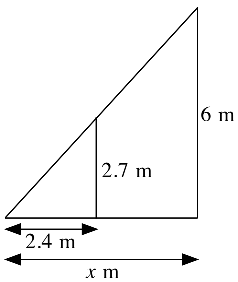
$$\frac{XY}{8.5} = \frac{QR - XZ}{6.75}$$

$$\frac{XY}{8.5} = \frac{6.75 - 3}{6.75}$$

$XY = 4.72$ (to 3 s.f.)

$\therefore$ The length of XY is 4.72 cm.

14.



Using similar triangles,

$$\frac{x}{2.4} = \frac{6}{2.7}$$

$$x = \frac{6}{2.7} \times 2.4$$

$$= 5\frac{1}{3}$$

15. $7OP = 5PQ$

$$\frac{OP}{PQ} = \frac{5}{7}$$

$$\frac{OP}{OQ} = \frac{5}{12}$$

$$\frac{QB}{PA} = \frac{12}{5}$$

$\therefore$ $QB : PA = 12 : 5$

16. $\triangle ABC$ is similar to $\triangle CDE$.

$$\frac{BC}{12 - BC} = \frac{5}{7}$$

$7BC = 60 - 5BC$

$12BC = 60$

$BC = 5$ cm

17. $\triangle LMN$ is similar to $\triangle LCB$.

$$\frac{BC}{6} = \frac{10}{4}$$

$$BC = \frac{10}{4} \times 6$$

$= 15$ cm

$\triangle AMN$ is similar to $\triangle ABC$.

$$\frac{AM}{AM + 10} = \frac{6}{15}$$

$15AM = 6AM + 60$

$9AM = 60$

$AM = 6\frac{2}{3}$ cm

18. **(i)** $\angle SQT = \angle RPT$ (given)

$\angle STQ = \angle RTP$ (common $\angle$)

$\therefore$ $\triangle SQT$ is similar to $\triangle RPT$ (2 pairs of corr. $\angle$s equal).

(ii) Using similar triangles,

$$\frac{QS}{9} = \frac{7}{6}$$

$$QS = \frac{7}{6} \times 9$$

$= 10.5$ cm

19. **(a)** $\angle BAC = \angle CBD$ (given)

$\angle ABC = \angle BCD$ (alt. $\angle$s, AB // CD)

$\therefore$ $\triangle ABC$ is similar to $\triangle BCD$ (2 pairs of corr. $\angle$s equal).

(b) **(i)** Using similar triangles,

$$\frac{BC}{9} = \frac{16}{BC}$$

$BC^2 = 144$

$BC = 12$ cm

(ii)
$$\frac{AC}{BD} = \frac{9}{12}$$

$$= \frac{3}{4}$$

20. (i) $\angle PSQ = \angle PQR$ (given)

$\angle QPS = \angle RPQ$ (common $\angle$)

$\therefore$ $\triangle PSQ$ is similar to $\triangle PQR$ (2 pairs of corr. $\angle$s equal).

(ii) Using similar triangles,

$$\frac{PQ}{9+16} = \frac{9}{PQ}$$
$$PQ^2 = 225$$
$$PQ = 15 \text{ cm}$$

21. $\triangle AXZ$ is similar to $\triangle BYZ$.

$$\frac{YZ}{YZ+10} = \frac{4}{6}$$
$$6YZ = 4YZ + 40$$
$$2YZ = 40$$
$$YZ = 20 \text{ cm}$$

22. (a) $\triangle ZAB$ is similar to $\triangle ZYX$.

Using similar triangles,

$$\frac{BZ}{3\frac{1}{2}} = \frac{6}{3}$$
$$BZ = \frac{6}{3} \times 3\frac{1}{2}$$
$$= 7 \text{ cm}$$

(b) $\triangle ZXY$ is similar to $\triangle ZQR$.

Using similar triangles,

$$\frac{YZ}{YZ+16} = \frac{3}{11}$$
$$11YZ = 3YZ + 48$$
$$8YZ = 48$$
$$YZ = 6 \text{ cm}$$

23. $\triangle ZXY$ is similar to $\triangle ZCB$.

Using similar triangles,

$$\frac{BC}{2.8} = \frac{2}{1.4}$$
$$BC = \frac{2}{1.4} \times 2.8$$
$$= 4 \text{ m}$$

$\triangle AXY$ is similar to $\triangle ABC$.

$$\frac{YC + 3.2}{3.2} = \frac{4}{2.8}$$
$$YC + 3.2 = \frac{4}{2.8} \times 3.2$$
$$YC = 1.37 \text{ m (to 3 s.f.)}$$
$$\frac{CZ}{1.2} = \frac{2}{1.4}$$
$$CZ = \frac{2}{1.4} \times 1.2$$
$$= 1.71 \text{ m (to 3 s.f.)}$$

Advanced

24. $\triangle MQP$ is similar to $\triangle MRS$.

$$\frac{QP}{RS} = \frac{6}{10}$$
$$= \frac{3}{5}$$

$\triangle PML$ is similar to $\triangle PSR$.

$PM : MS$

$3 : 5$

$\therefore PM : PS$

$3 : 8$

$$\frac{PM}{PS} = \frac{LM}{RS}$$
$$\frac{3}{8} = \frac{LM}{10}$$
$$LM = \frac{3 \times 10}{8}$$
$$= 3.75 \text{ cm}$$

25. $\triangle PQR$ is similar to $\triangle YXR$.

$$\frac{2x-y}{7} = \frac{2x+3y}{9}$$
$$18x - 9y = 14x + 21y$$
$$4x = 30y$$
$$\frac{x}{y} = \frac{15}{2}$$

$\therefore x : y = 15 : 2$

New Trend

26. (a) $\angle DPQ = \angle APB$ (common $\angle$)

$\angle PDQ = \angle PAB$ (corr. $\angle$s, DC // AB)

$\therefore$ $\triangle PDQ$ is similar to $\triangle PAB$ (2 pairs of corr. $\angle$s equal).

(b) $\triangle BCQ$

(c) Using similar triangles,

$$\frac{DQ}{AB} = \frac{PD}{PA}$$
$$= \frac{1}{3}$$

$\therefore DQ : AB = 1 : 3$

(d) $$\frac{10}{10 + 8 + RB} = \frac{1}{3}$$
$$30 = 18 + RB$$
$$RB = 12 \text{ cm}$$

27. (a) $\angle CAB = \angle NCB$ (given)

$\angle ABC = \angle CBN$ (common $\angle$)

$\therefore$ $\triangle ABC$ is similar to $\triangle CBN$ (2 pairs of corr. $\angle$s equal).

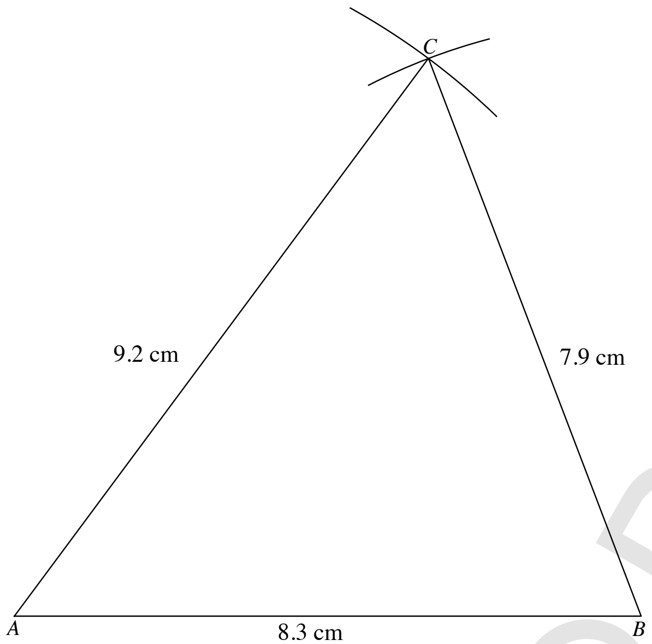
(b) Using similar triangles,

$$\frac{BC}{25} = \frac{13}{BC}$$
$$BC^2 = 325$$
$$BC = 18 \text{ cm}$$

Chapter 11 Geometrical Constructions

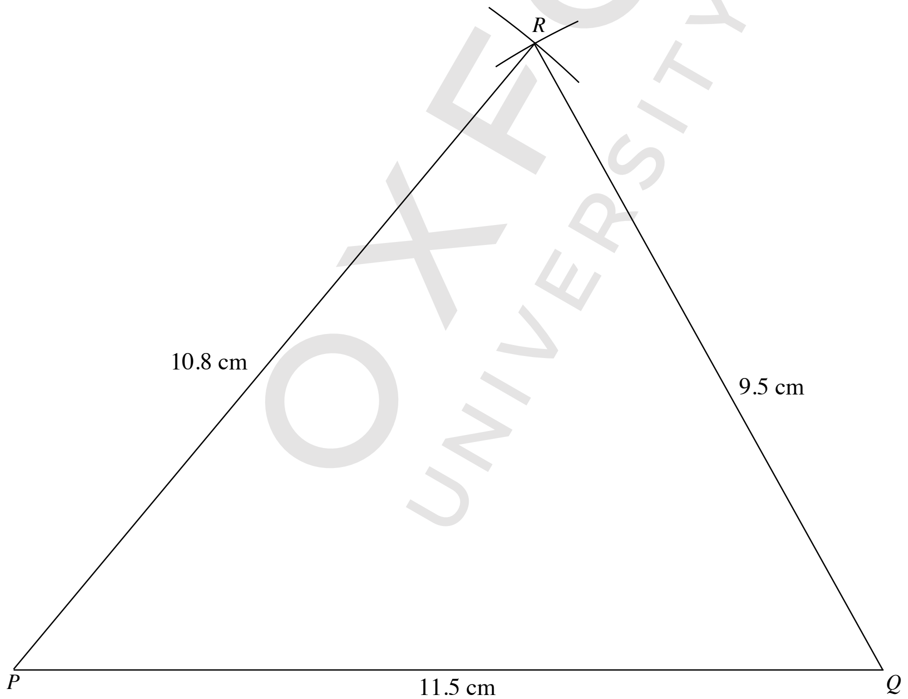
Basic

1.



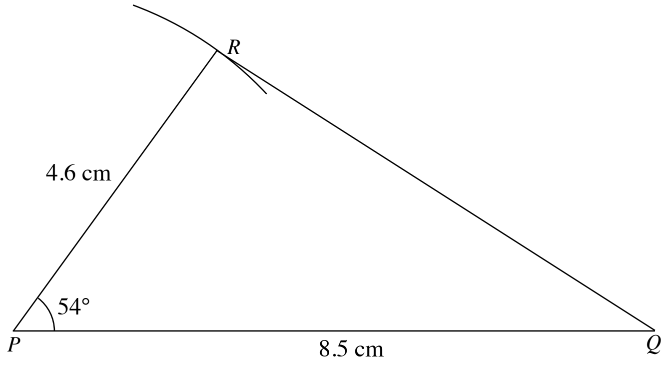
$\angle ABC = 69°$

2.



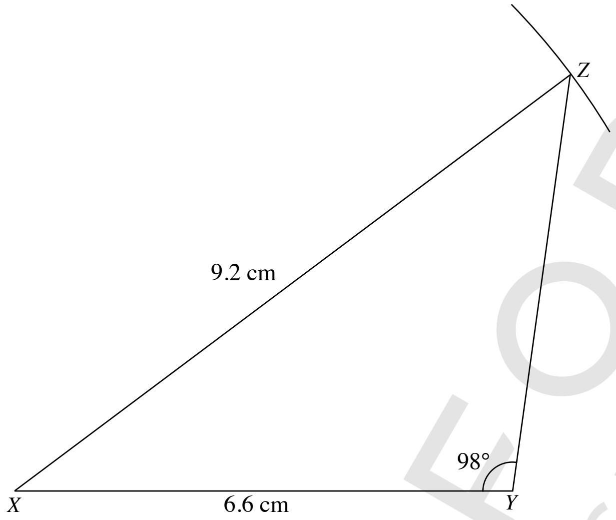
$\angle QPR = 50°$
$\angle PQR = 61°$

3.



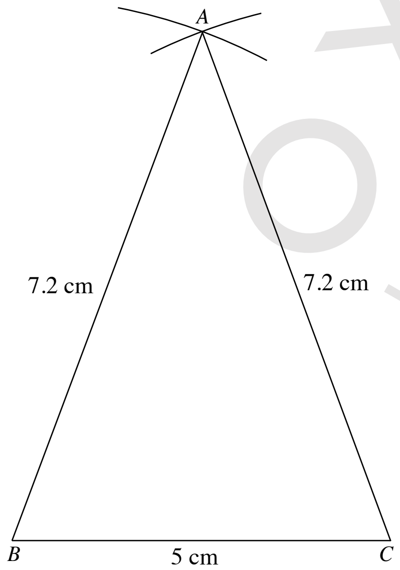
Length of $QR = 6.9$ cm.

4.



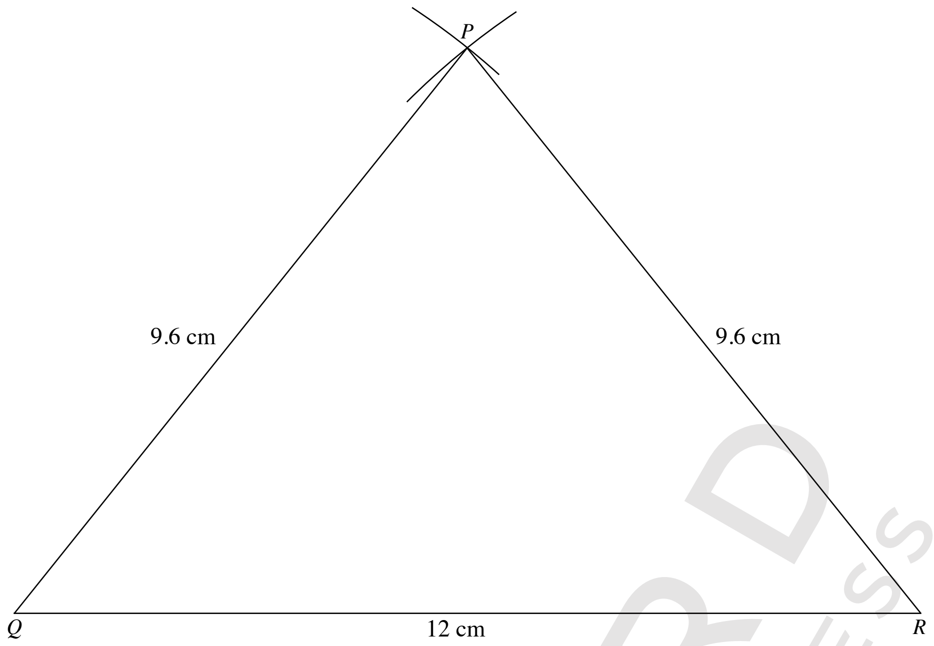
$\angle YXZ = 37°$

5.



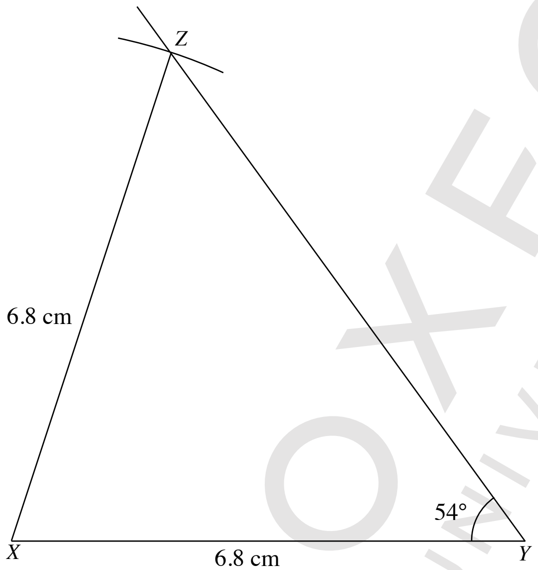
$\angle ABC = 70°$

6.



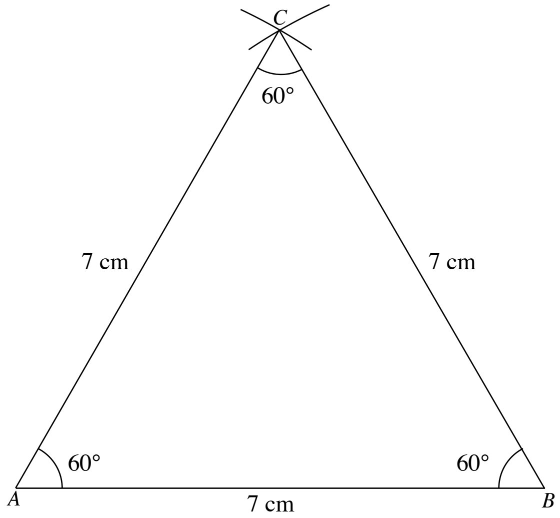
$\angle PQR = 51°$

7.

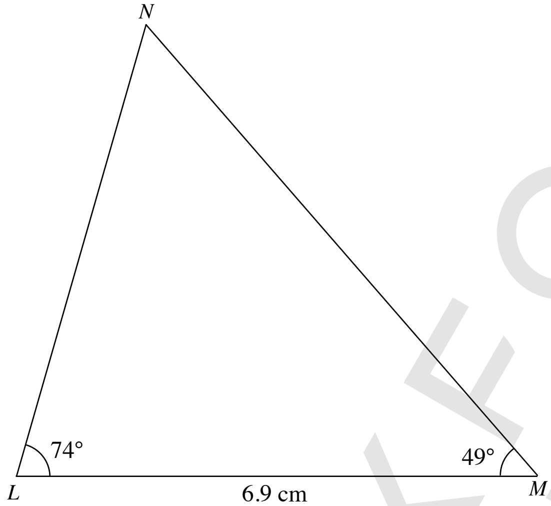


Length of YZ = 8.0 cm

8.

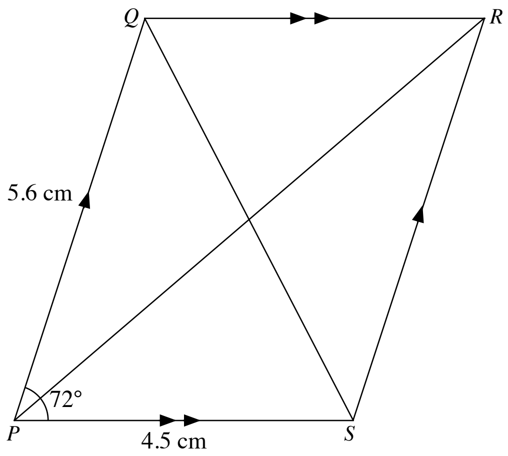


9.



Length of $LN = 6.2$ cm

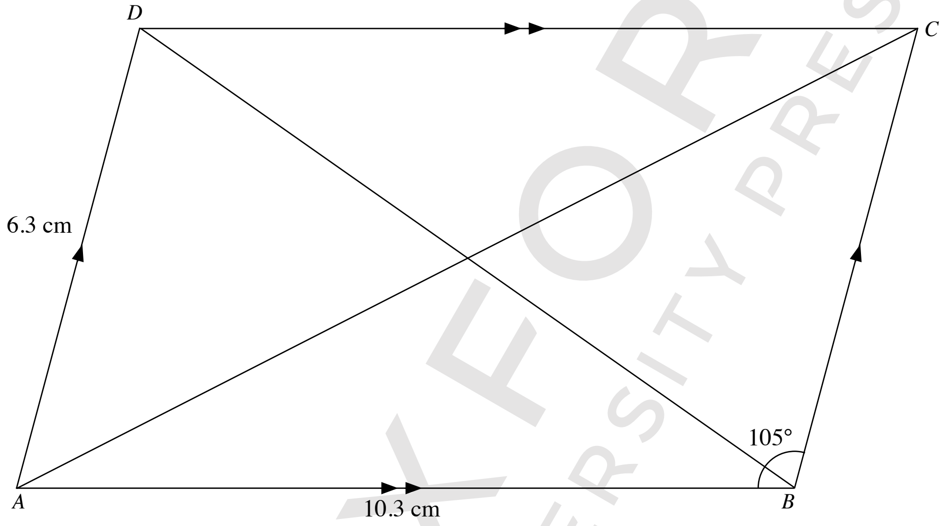
10.



Length of diagonal *PR* = 8.2 cm

Length of diagonal *QS* = 6 cm

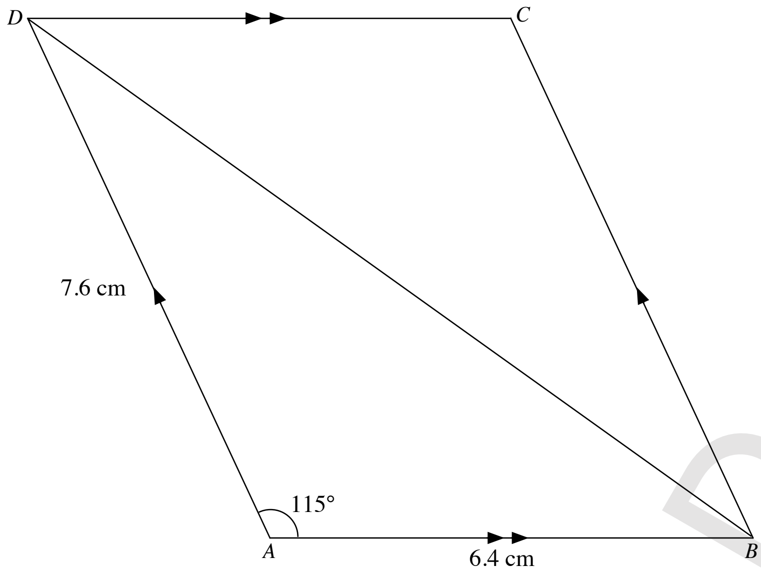
11.



Length of diagonal *AC* = 13.4 cm

Length of diagonal *BD* = 10.6 cm

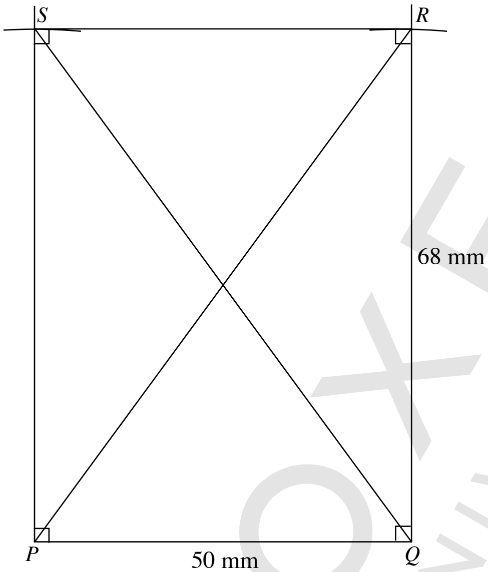
12.



Length of BD = 11.8 cm

$\angle BDA = 29°$

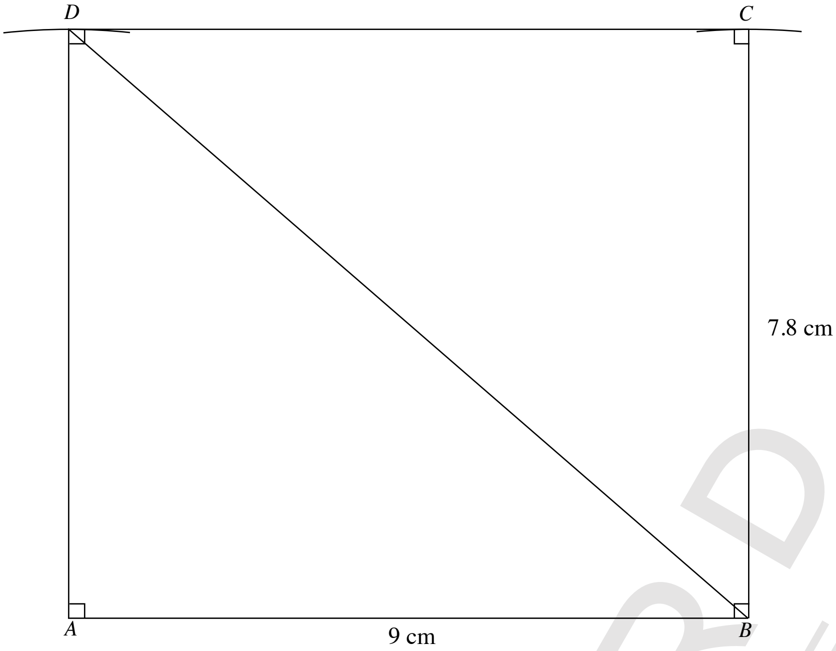
13.



Length of diagonal PR = 84 mm

Length of diagonal QS = 84 mm

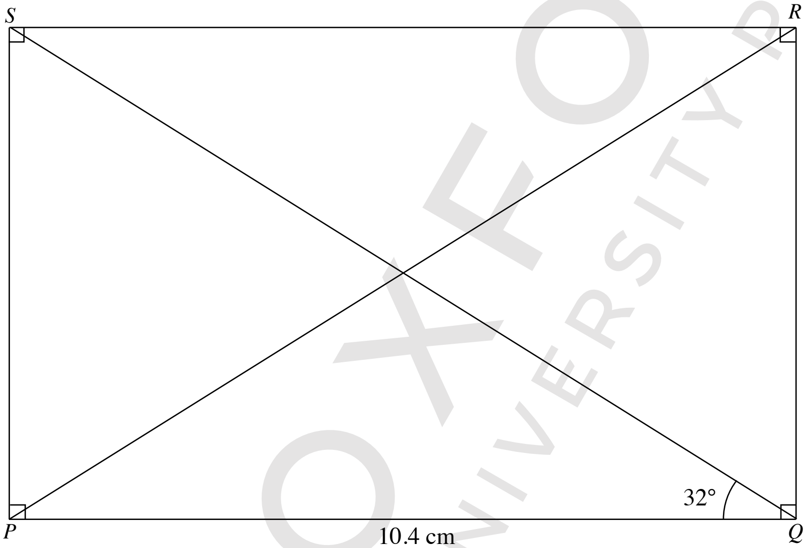
14.



Length of BD = 11.9 cm

$\angle ABD$ = 41°

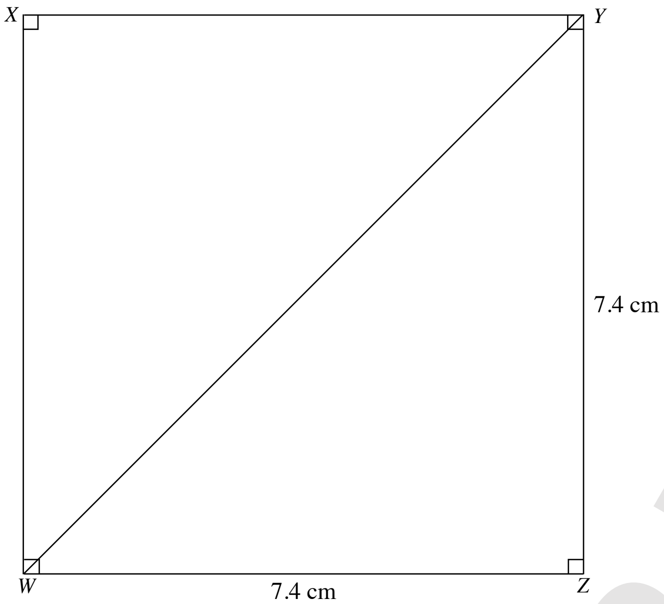
15.



Length of PS = 6.5 cm

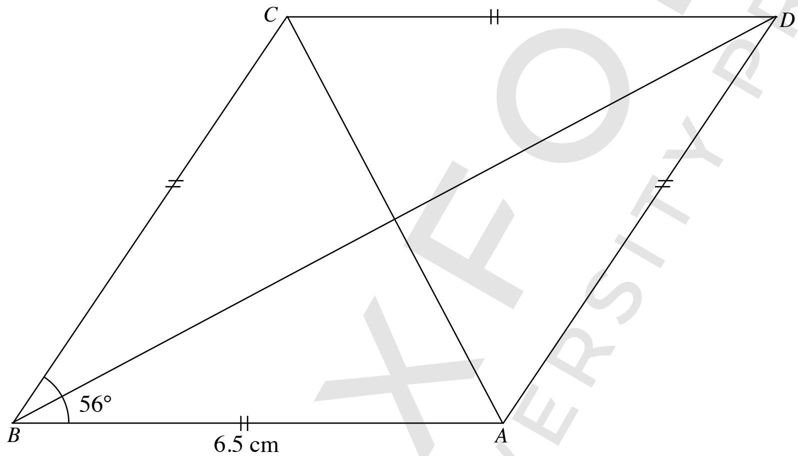
Length of PR = 12.3 cm

16.



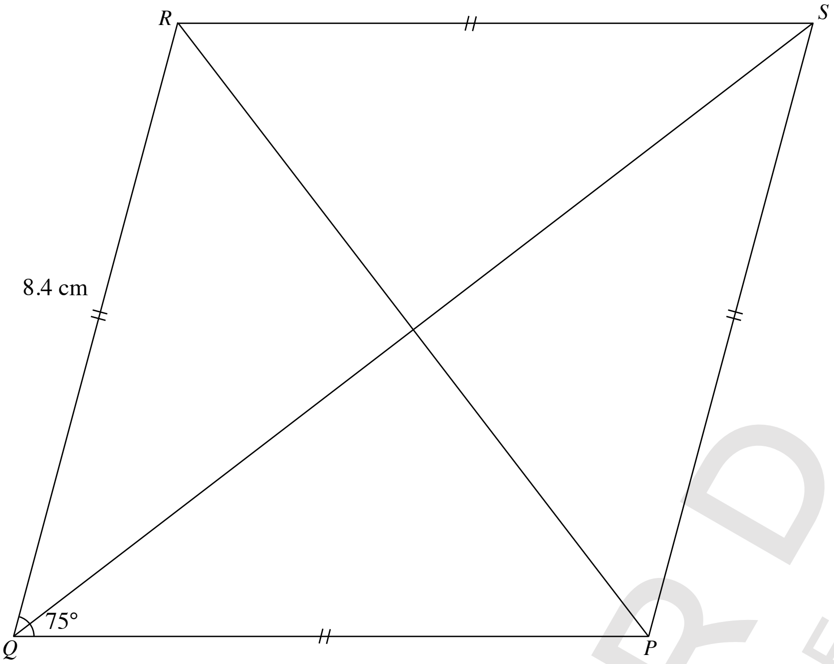
Length of diagonal $WY = 10.5$ cm

17.



Length of diagonal $BD = 11.5$ cm
Length of diagonal $AC = 6.1$ cm

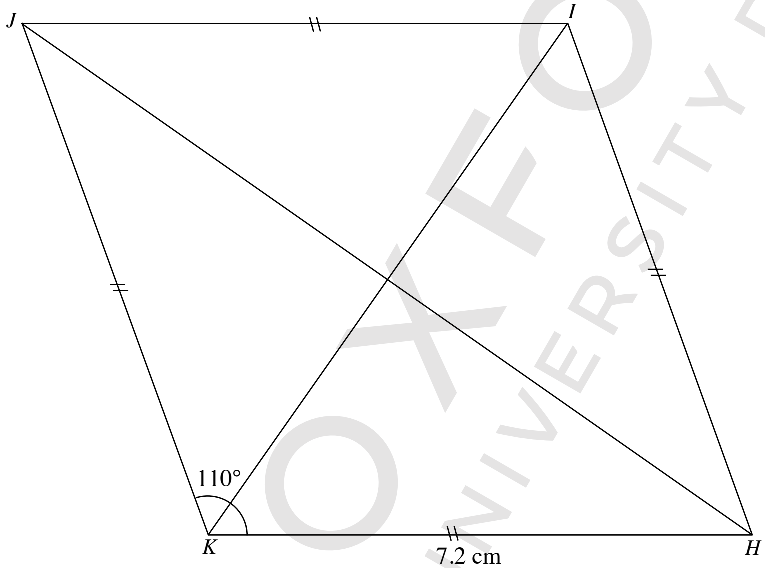
18.



Length of diagonal *PR* = 10.2 cm

Length of diagonal *QS* = 13.3 cm

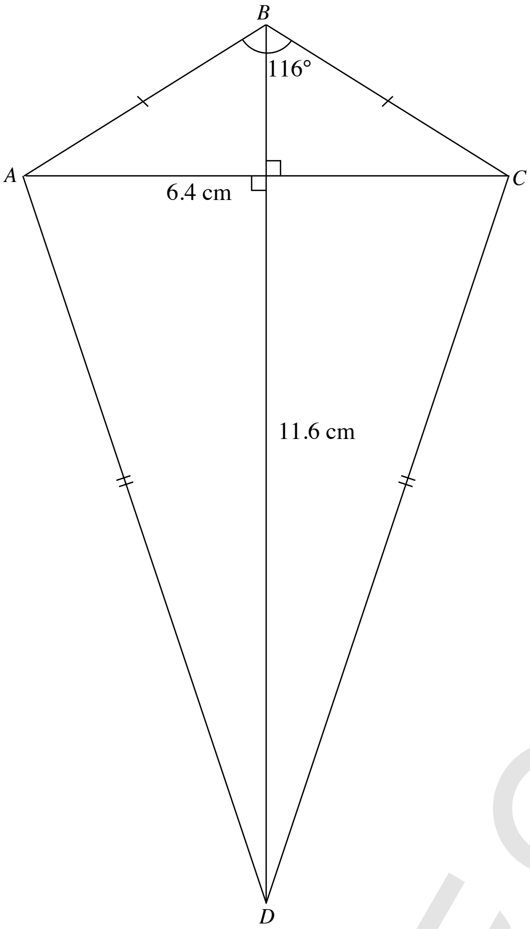
19.



Length of diagonal *HJ* = 11.8 cm

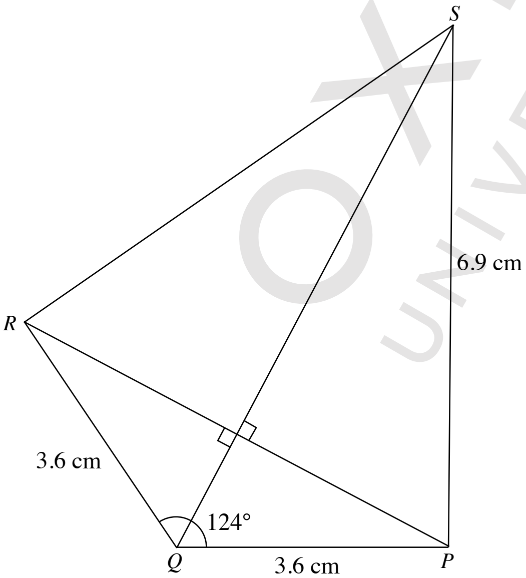
Length of diagonal *KI* = 8.4 cm

20.



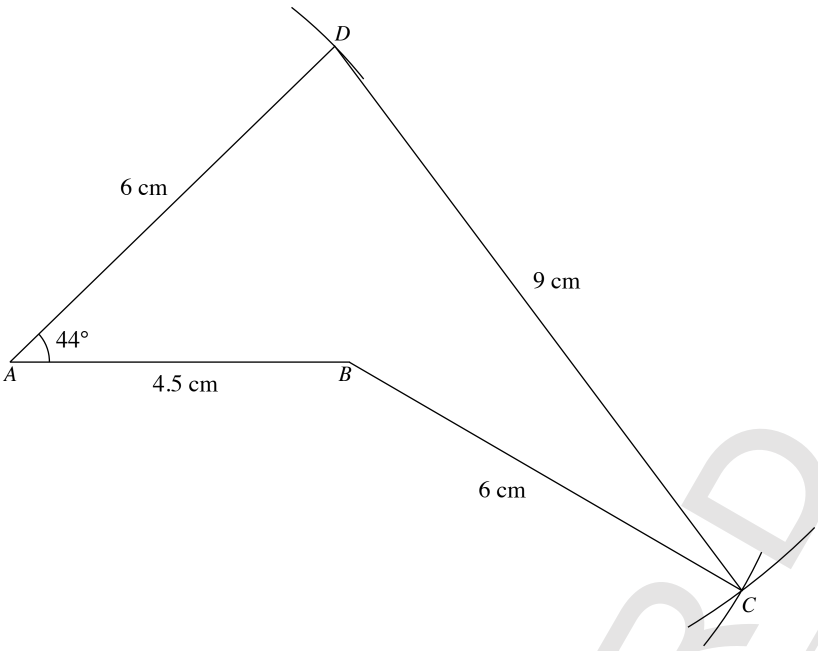
Length of AB = 3.8 cm
Length of AD = 10.1 cm

21.



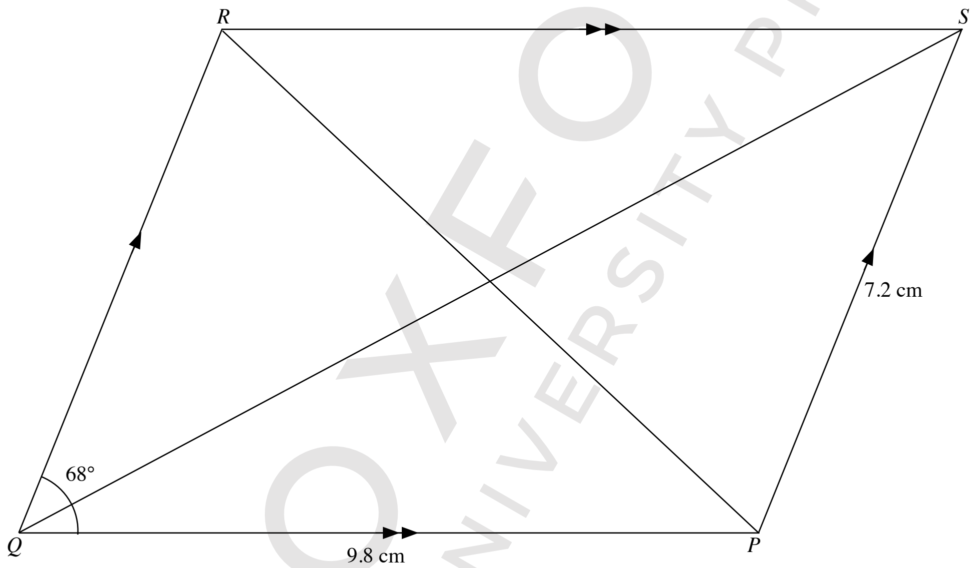
Length of diagonal PR = 6.4 cm
Length of diagonal QS = 7.8 cm

22.



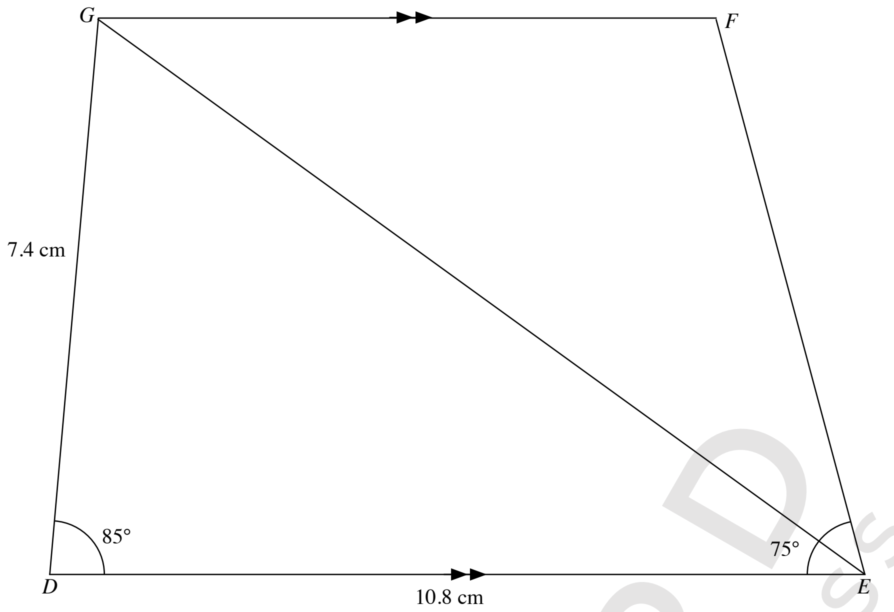
$\angle ADC = 82°$

23.



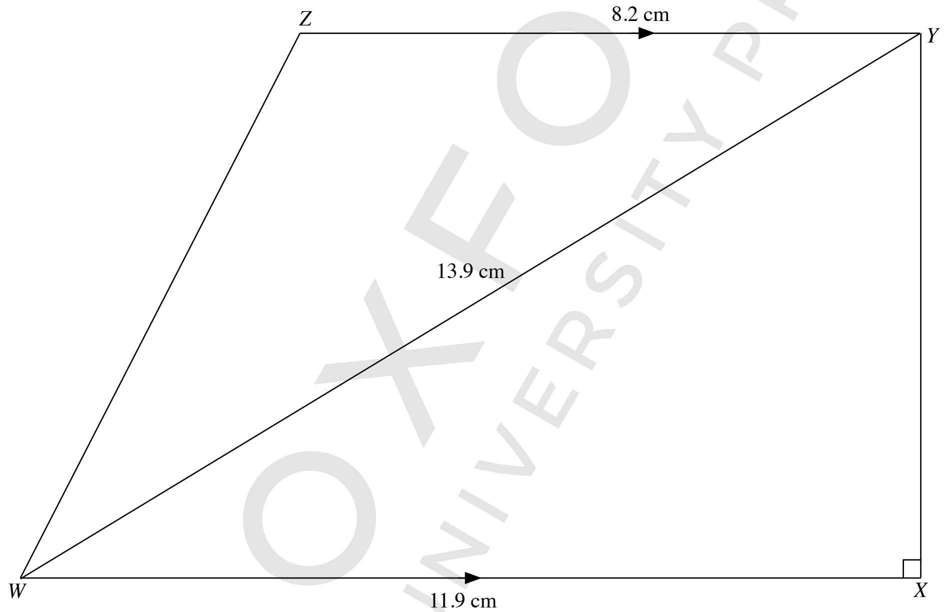
Length of diagonal PR = 9.7 cm
Length of diagonal QS = 14.2 cm

24.



Length of GE = 12.5 cm
Length of GF = 8.2 cm

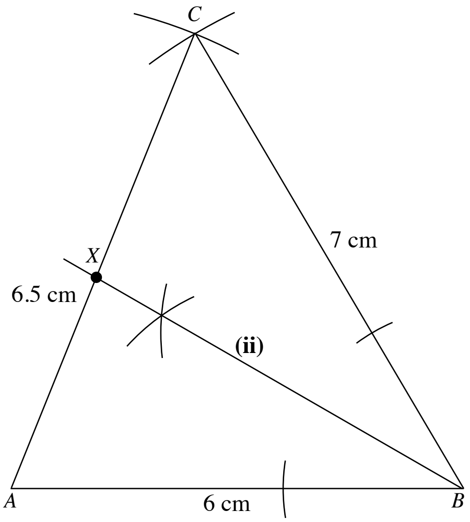
25.



Length of WZ = 8.1 cm
$\angle XWZ = 63°$

Intermediate

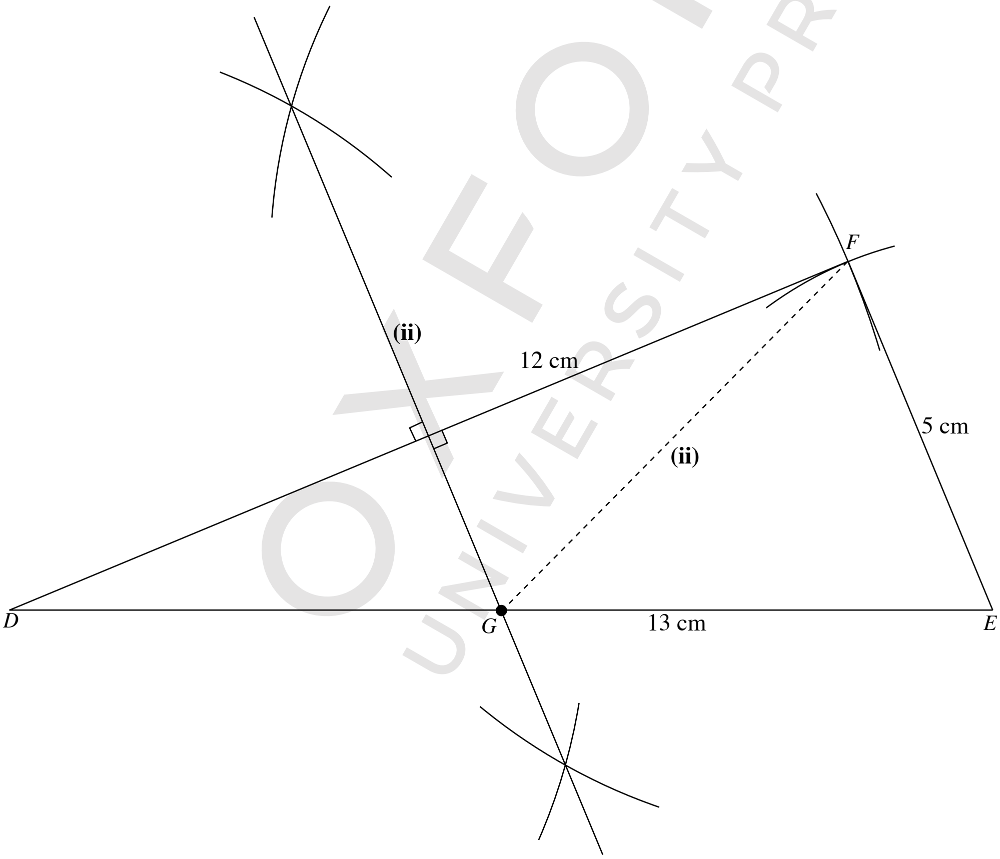
26.



(i) $\angle ABC = 58°$

(ii) Length of BX = 5.6 cm

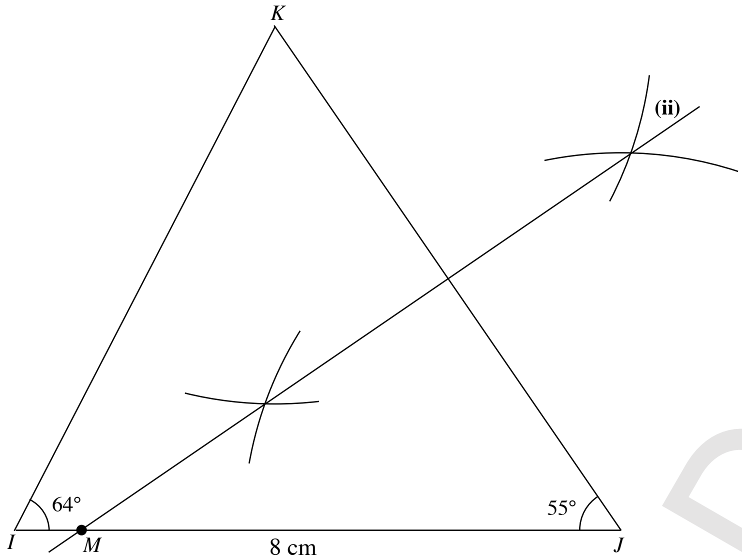
27.



(i) $\angle DEF = 67°$

(ii) Length of GF = 6.5 cm

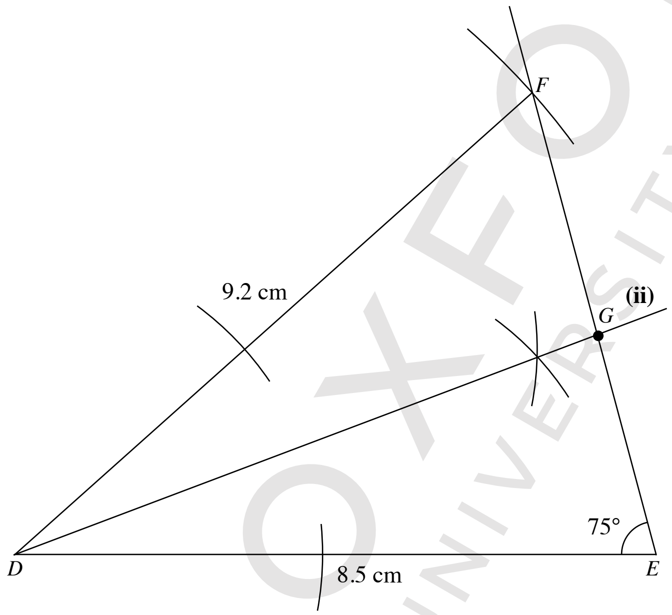
28.



(i) Length of $IK = 7.5$ cm
Length of $JK = 8.1$ cm

(ii) Length of $IM = 0.9$ cm

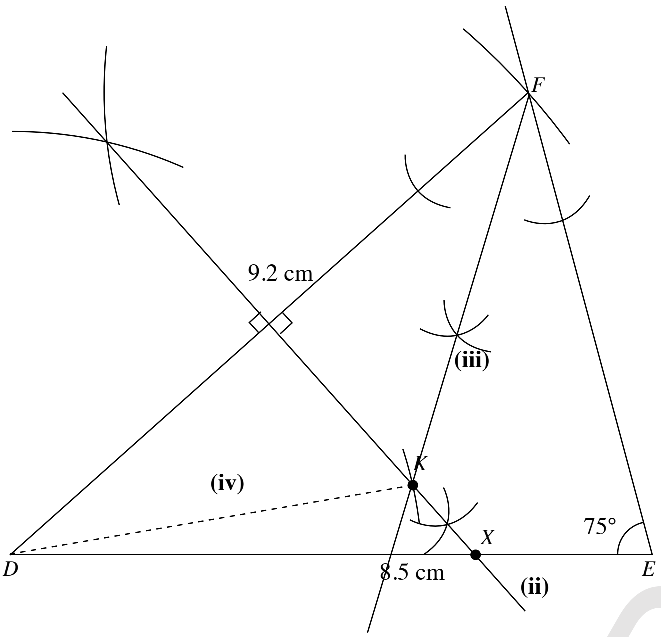
29.



(i) The angle that is facing the longest side is $\angle DEF$.
The size of $\angle DEF = 75°$.

(ii) Length of $DG = 8.2$ cm

30.

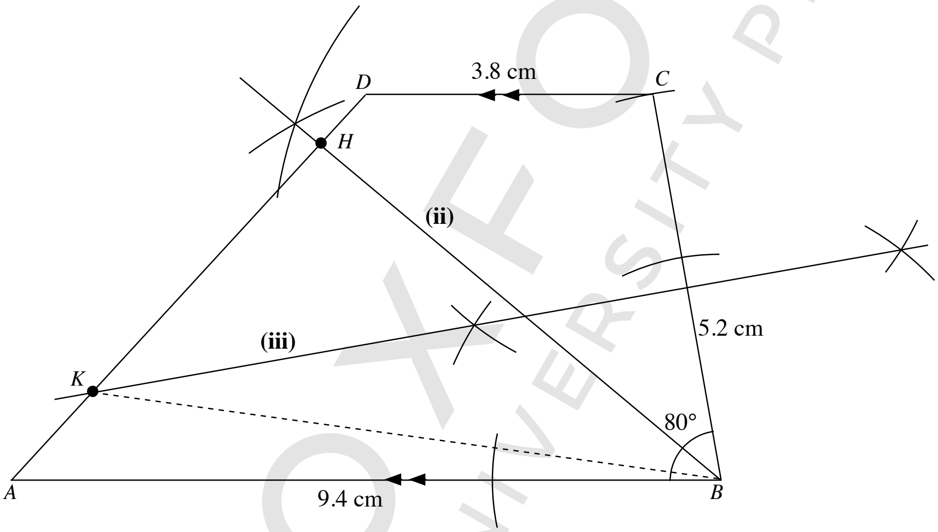


(i) Length of $EF = 6.3$ cm

(ii) Length of $DX = 6.2$ cm

(iv) Length of $DK = 5.4$ cm

31.



(i) Length of $AD = 7$ cm

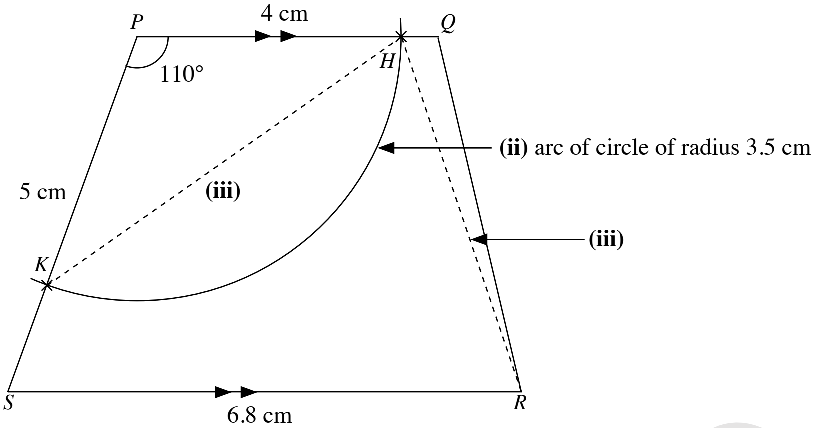
$\angle BAD = 47°$

(ii) Length of $HB = 7$ cm

(iii) Length of $KB = 8.4$ cm

(iv) Length of $HK = 4.5$ cm

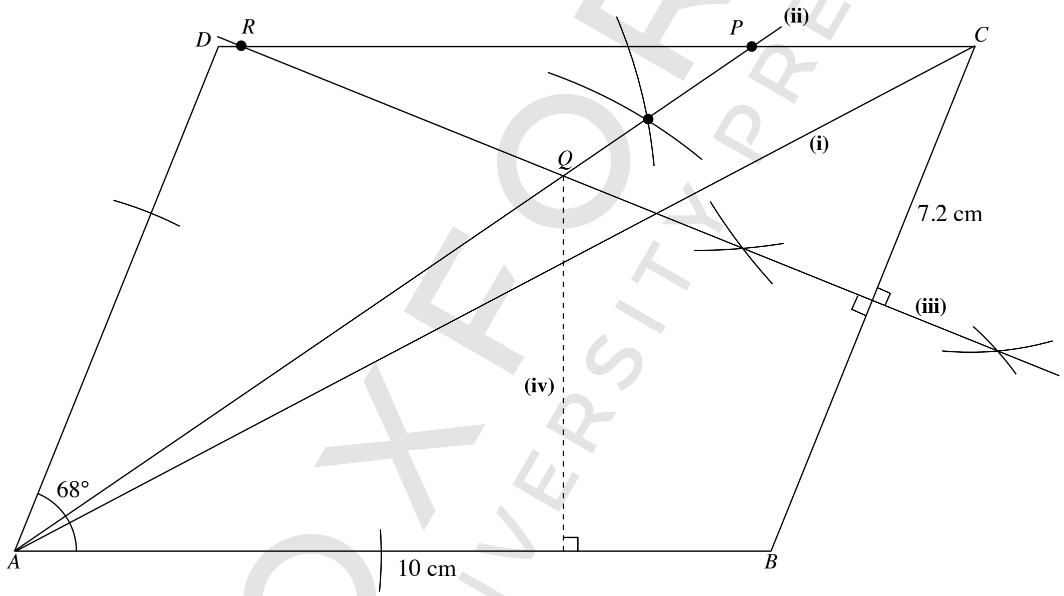
32.



(i) Length of QR = 4.8 cm
$\angle PQR$ = 104°

(iii) Length of RH = 4.9 cm
Length of HK = 5.8 cm

33.



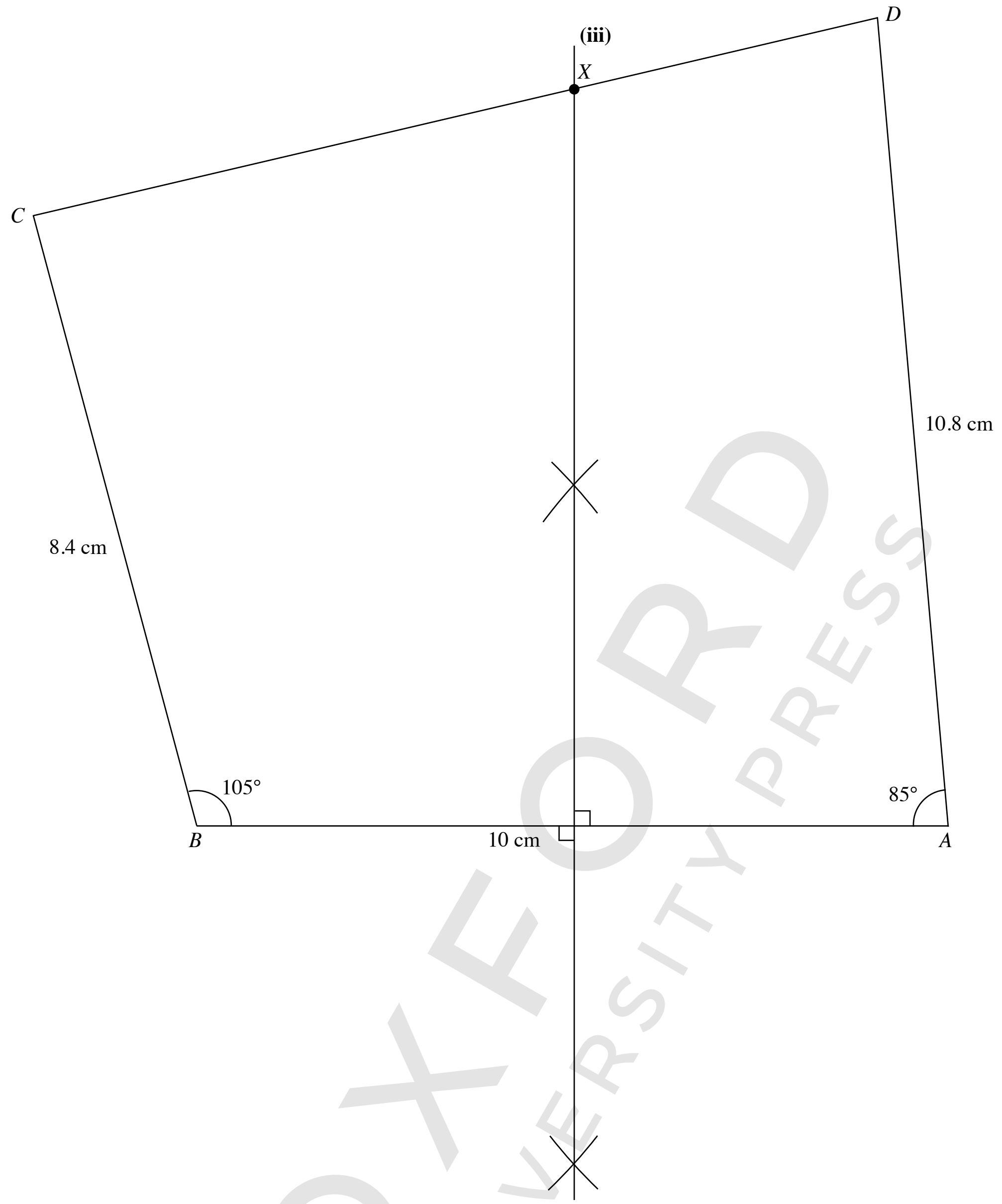
(i) Length of diagonal AC = 14.3 cm

(ii) Length of PC = 3.0 cm

(iii) Length of DR = 0.3 cm

(iv) Perpendicular height of Q to the base of the parallelogram
= 5.0 cm

34.

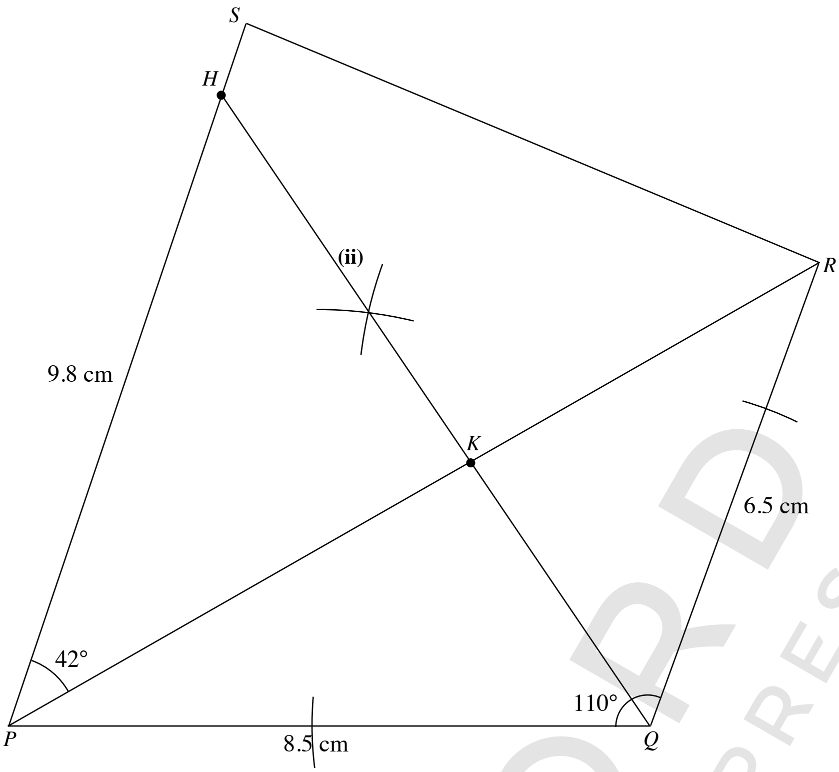


(i) Length of CD = 11.5 cm

(ii) $\angle ADC$ = 82°

(iii) Length of CX = 7.4 cm

35.



(i) Length of *RS* = 8.3 cm

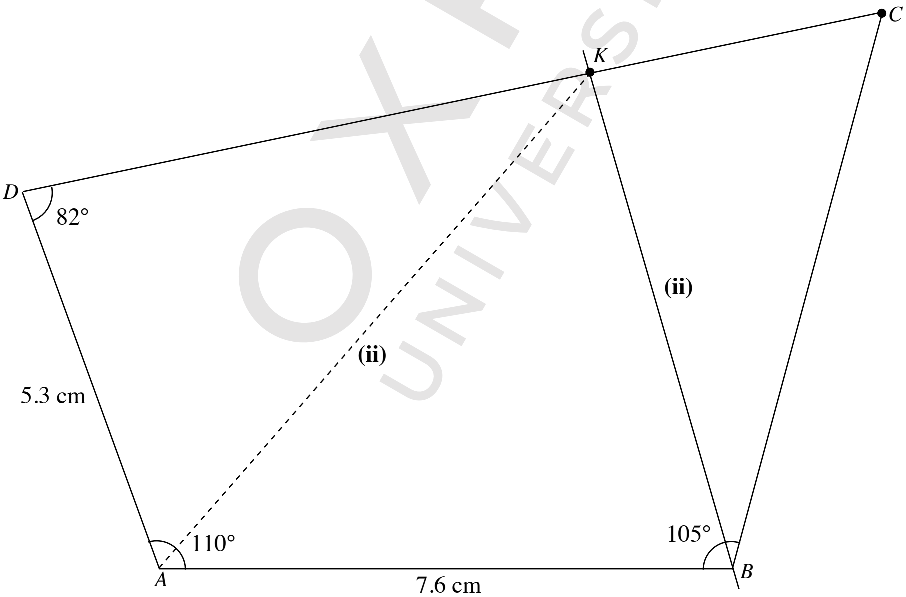
(ii) Length of *QK* = 4.2 cm

Length of *HK* = 6 cm

(iii) Ratio of *QK*: *KH* = 4.2 : 6

= 7 : 10

36.



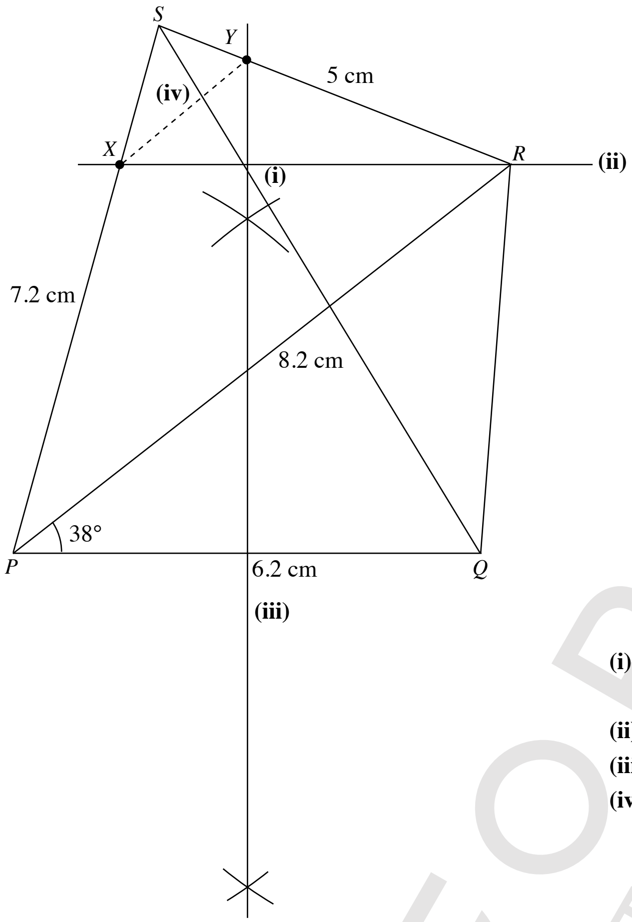
(i) Length of *CD* = 11.6 cm

Length of *BC* = 7.6 cm

(ii) Length of *AK* = 8.7 cm

Length of *BK* = 6.8 cm

37.



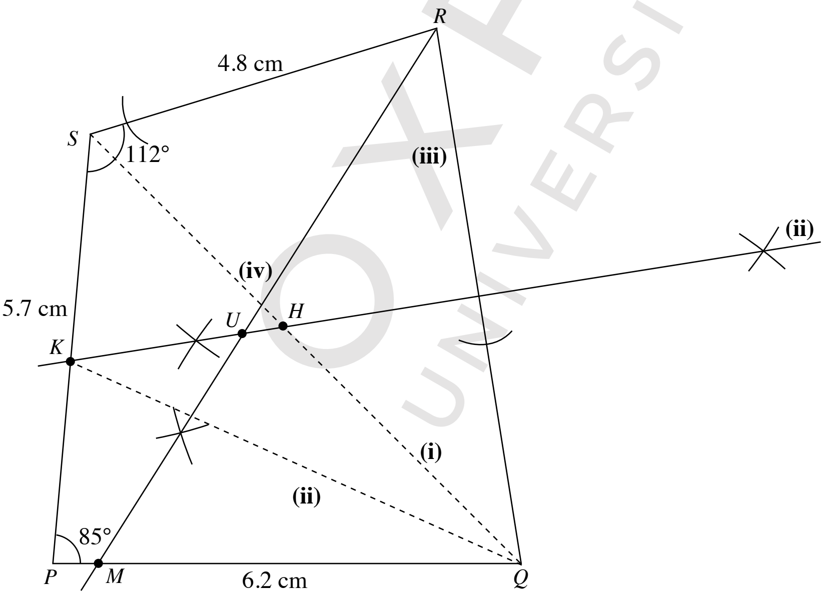
(i) Length of *SQ* = 8.2 cm
$\angle PRS = 60°$

(ii) Length of *PX* = 5.3 cm

(iii) Length of *RY* = 3.7 cm

(iv) Length of *XY* = 2.3 cm

38.



(i) Length of *QR* = 7.2 cm
Length of *QS* = 8.1 cm

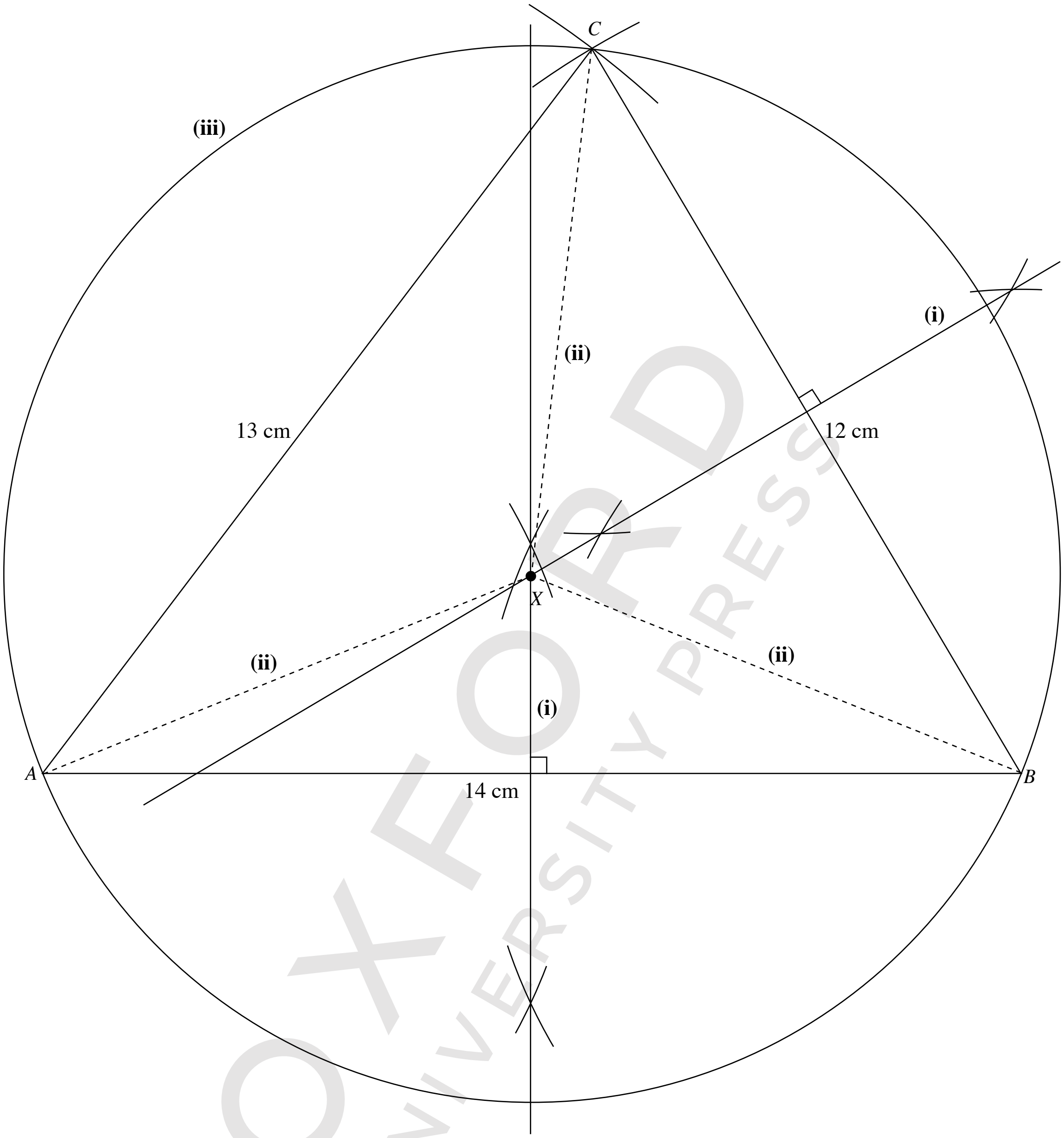
(ii) Length of *SH* = 3.6 cm
Length of *KQ* = 6.6 cm

(iii) Length of *RM* = 8.4 cm

(iv) Length of *UM* = 3.7 cm

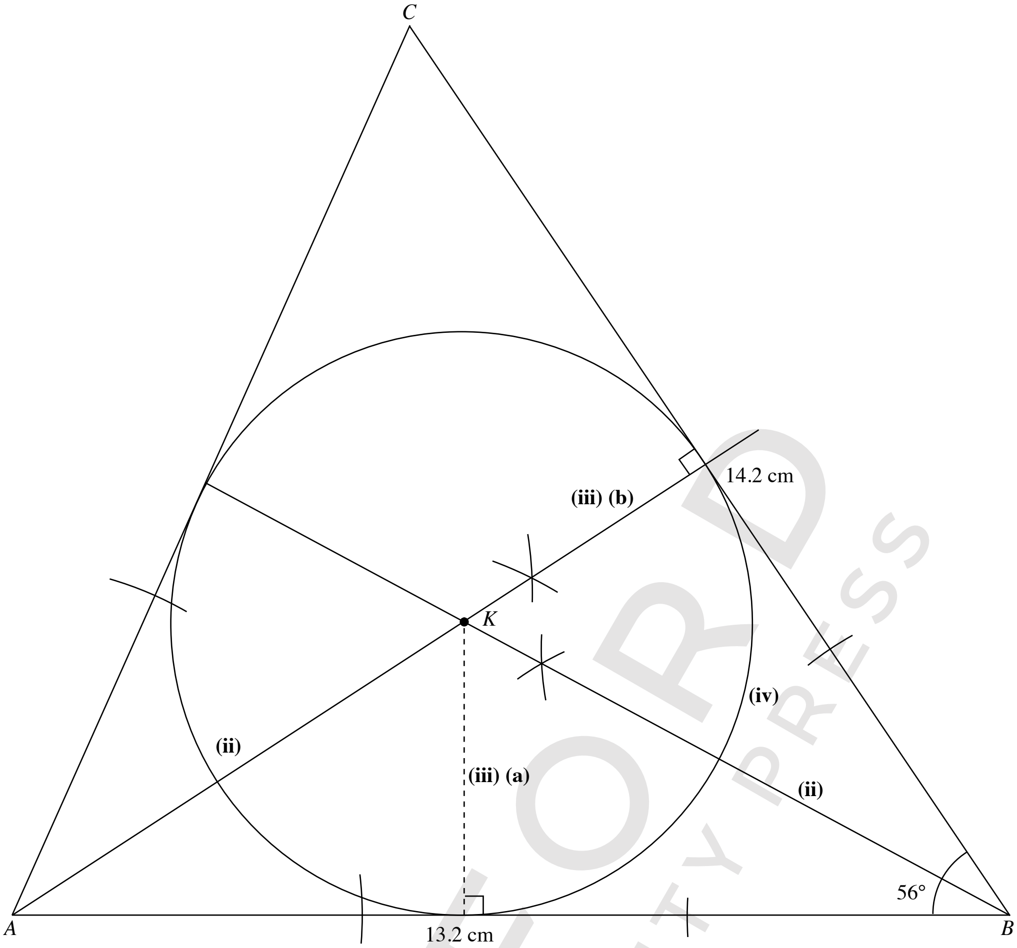
Advanced

39.



(ii) The length of AX, of BX and of CX = 7.6 cm

40.



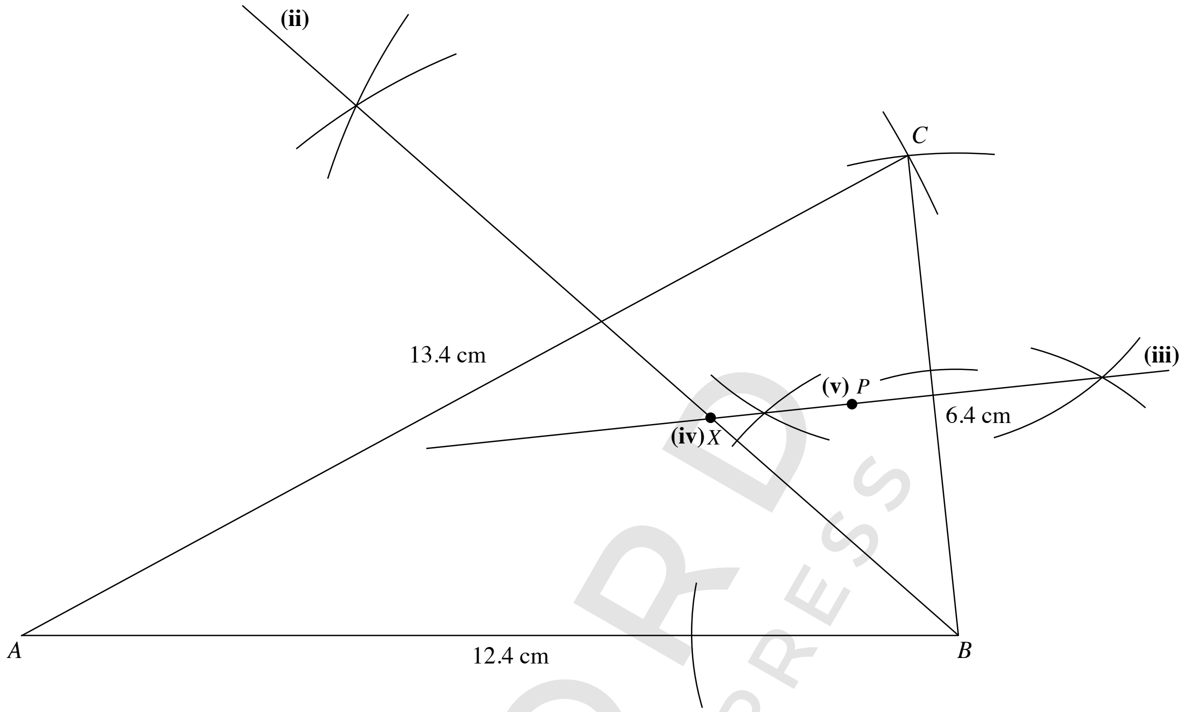
(i) Length of AC = 12.9 cm

(iii) (a) Shortest distance of K from AB = 4.0 cm

(b) Shortest distance of K from BC = 4.0 cm

New Trend

41.



(i) The angle that is facing the longest side is $\angle ABC$.
$\angle ABC = 84°$

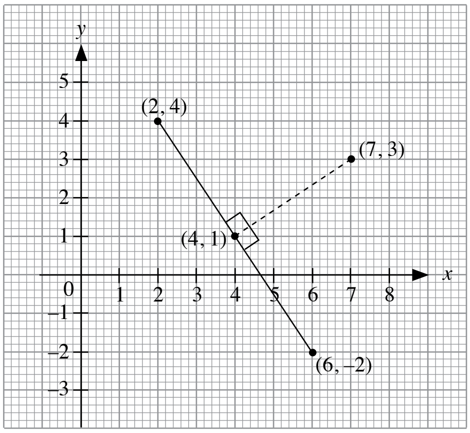
(iv) The point X is equidistant from the points $\underline{B}$ and $\underline{C}$, and equidistant from the lines $\underline{AB}$ and $\underline{BC}$.

(v) Point P is on the perpendicular bisector to the right of the angle bisector, closer to BC than BA.

Chapter 12 Geometrical Transformation

Basic

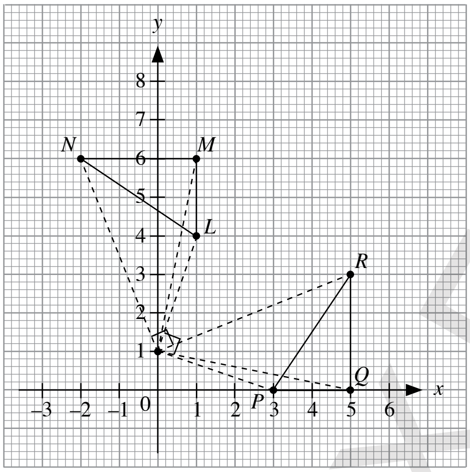
1.



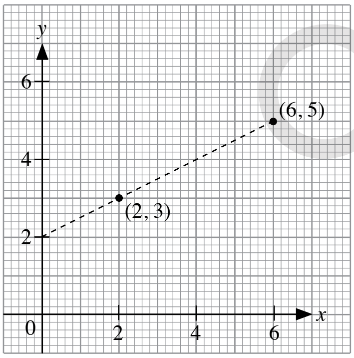
(a) (7, 3)

(b) (6, –2)

2.

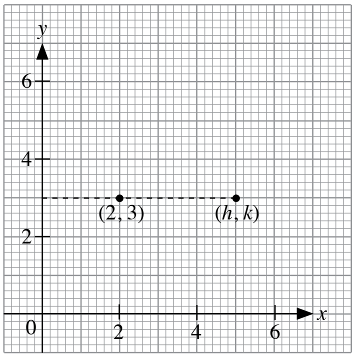


3.



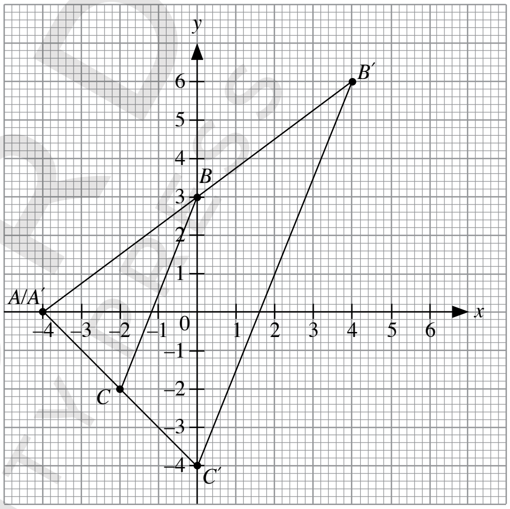
$\therefore k = 5$

4.



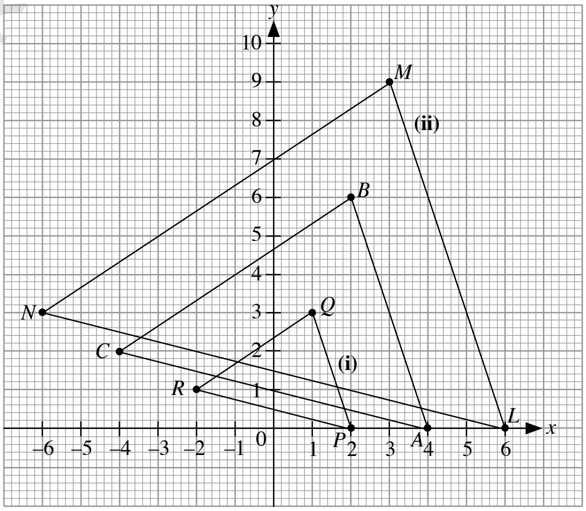
$\therefore h = 5, k = 3$

5.

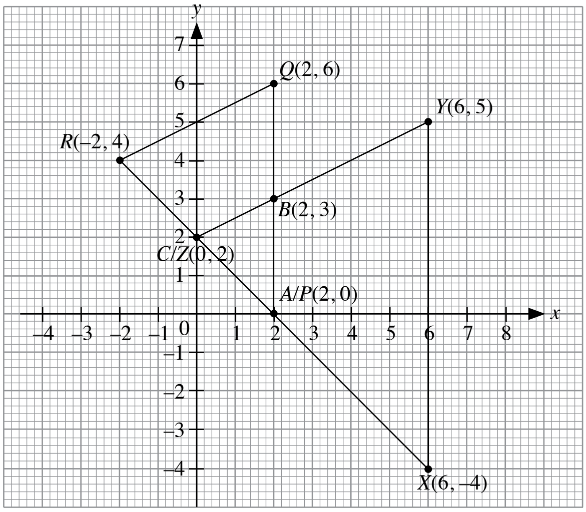


$\therefore A'(-4, 0), B'(4, 6)$ and $C'(0, -4)$

6.



7.



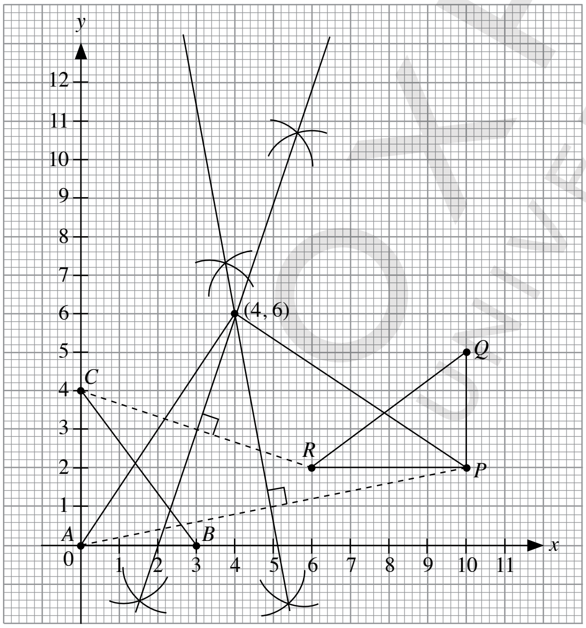
8. **(i)** $R^2\begin{pmatrix}3\\2\end{pmatrix} = R\begin{pmatrix}2\\-3\end{pmatrix} = \begin{pmatrix}-3\\-2\end{pmatrix}$ i.e. (–3, –2)

(ii) $E^2\begin{pmatrix}3\\2\end{pmatrix} = E\begin{pmatrix}5\\6\end{pmatrix} = \begin{pmatrix}8\\12\end{pmatrix}$ i.e. (8, 12)

(iii) $ER\begin{pmatrix}3\\2\end{pmatrix} = E\begin{pmatrix}2\\-3\end{pmatrix} = \begin{pmatrix}2\\-9\end{pmatrix}$ i.e. (2, –9)

(iv) $RE\begin{pmatrix}3\\2\end{pmatrix} = R\begin{pmatrix}5\\6\end{pmatrix} = \begin{pmatrix}6\\-5\end{pmatrix}$ i.e. (6, –5)

9.



(ii) The centre of rotation is (4, 6). The angle of rotation is 90° clockwise or 270° anticlockwise.

10. R^4 represents $(4 \times 160°) - 360°$

$= 280°$ anticlockwise rotation about the origin.

R^5 represents $(5 \times 160°) - 720°$

$= 80°$ anticlockwise rotation about the origin.

Advanced

11. **(i)** $Z\hat{X}X' = 20°$

$XZ = X'Z$

$\therefore Z\hat{X}X' = \dfrac{180° - 20°}{2}$

$= 80°$ (base ∠ of isos. △)

(ii) $Y\hat{Z}Y' = 20°$

$\tan X'\hat{Z}Y = \dfrac{7}{4}$

$X'\hat{Z}Y' = \tan^{-1}\dfrac{7}{4}$

$= 60.3°$ (to 1 d.p.)

$\therefore Y\hat{Z}X' = 60.3° - 20°$

$= 40.3°$

12. $\dfrac{x}{x+3} = \dfrac{3}{4}$

$4x = 3x + 9$

$x = 9$

$\dfrac{y}{y+2.8} = \dfrac{3}{4}$

$4y = 3y + 8.4$

$y = 8.4$

$\therefore x = 9, y = 8.4$

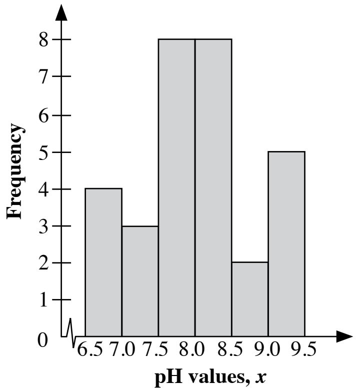
Chapter 13 Statistics

Basic

1. (i)

pH values, x	Tally	Frequency
$6.5 \leqslant x < 7.0$	////	4
$7.0 \leqslant x < 7.5$	///	3
$7.5 \leqslant x < 8.0$	~~////~~ ///	8
$8.0 \leqslant x < 8.5$	~~////~~ ///	8
$8.5 \leqslant x < 9.0$	//	2
$9.0 \leqslant x < 9.5$	~~////~~	5
Total frequency		30

(ii)



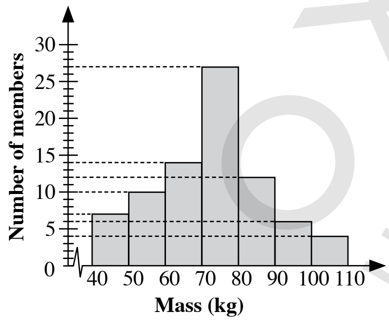
(iii) There are many distinct values in the set of data. Using a histogram for grouped data would be more suitable.

(iv) Percentage of the types which are alkaline

$= \frac{26}{30} \times 100\%$

$= 86.7\%$ (to 3 s.f.)

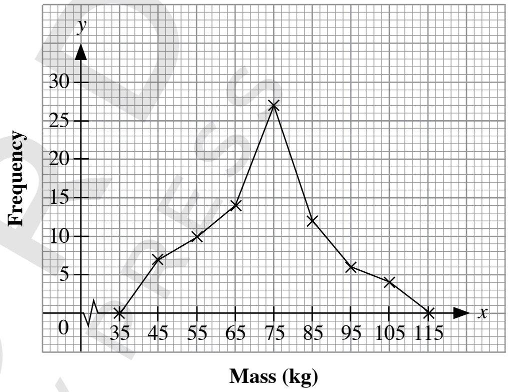
2. (a)



(b)

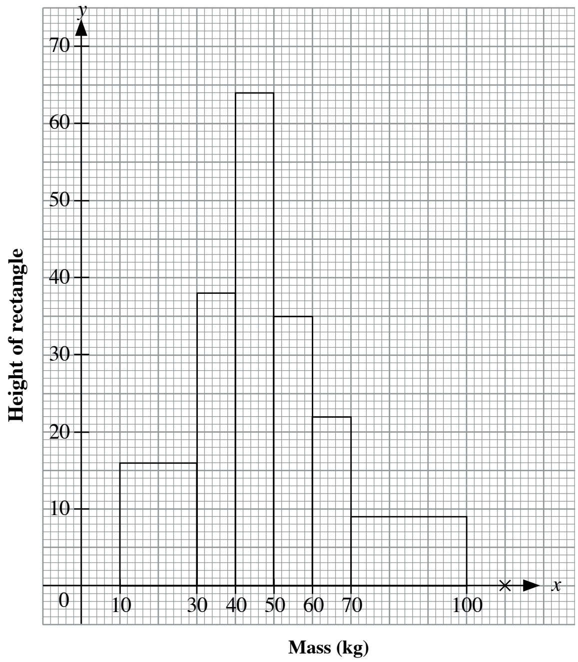
Mass, (x kg)	Mid-value	Frequency
$40 < x \leqslant 50$	45	7
$50 < x \leqslant 60$	55	10
$60 < x \leqslant 70$	65	14
$70 < x \leqslant 80$	75	27
$80 < x \leqslant 90$	85	12
$90 < x \leqslant 100$	95	6
$100 < x \leqslant 110$	105	4

The points to be plotted are (35, 0), (45, 7), (55, 10), (65, 14), (75, 27), (85, 12), (95, 6), (105, 4) and (115, 0).



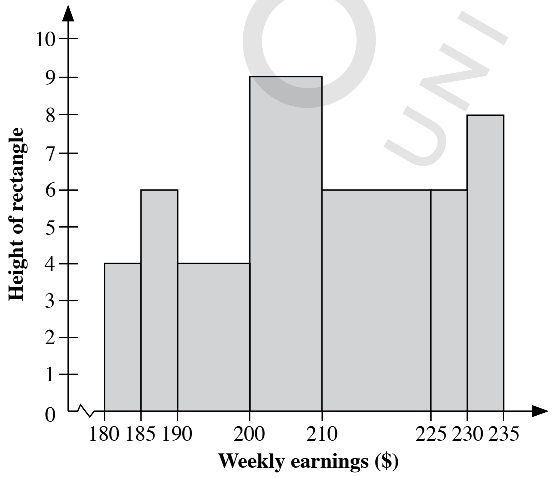
3. (a) Since the class intervals are unequal, the histogram is to be drawn using either height of rectangle or frequency density.

Class interval	Class width		Frequency	Rectangle's height
10 – 29	20	2 × standard	32	32 ÷ 2 = 16
30 – 39	10	1 × standard	38	38 ÷ 1 = 38
40 – 49	10	2 × standard	64	64 ÷ 1 = 64
50 – 59	10	2 × standard	35	35 ÷ 1 = 35
60 – 69	10	1 × standard	22	22 ÷ 1 = 22
70 – 99	30	3 × standard	9	9 ÷ 3 = 3



4. Since the class intervals are unequal, the histogram is to be drawn using either height of rectangle or frequency density.

Weekly earnings ($)	Class width		Frequency	Rectangle's height
$180 \leqslant x < 185$	5	1 × standard	4	4 ÷ 1 = 4
$185 \leqslant x < 190$	5	1 × standard	6	6 ÷ 1 = 6
$190 \leqslant x < 200$	10	2 × standard	8	8 ÷ 2 = 4
$200 \leqslant x < 210$	10	2 × standard	18	18 ÷ 2 = 9
$210 \leqslant x < 225$	15	3 × standard	18	18 ÷ 3 = 6
$225 \leqslant x < 230$	5	1 × standard	6	6 ÷ 1 = 6
$230 \leqslant x < 235$	5	1 × standard	8	8 ÷ 1 = 8



5. **(a)** For Class B,

Marks	Mid-value (x)	f	fx	fx^2
$10 < x \leqslant 30$	20	4	80	1600
$30 < x \leqslant 50$	40	9	360	14 400
$50 < x \leqslant 70$	60	12	720	43 200
$70 < x \leqslant 90$	80	5	400	32 000
Sum		$\Sigma f = 30$	$\Sigma fx = 1560$	$\Sigma fx^2 = 91\,200$

(i) Mean, $\bar{x} = \dfrac{\Sigma fx}{\Sigma f} = \dfrac{1560}{30} = 52$ marks

(ii) Standard deviation $= \sqrt{\dfrac{\Sigma fx^2}{\Sigma f} - \bar{x}^2}$

$= \sqrt{\dfrac{91\,200}{30} - 52^2}$

$= 18.3$ marks (to 3 s.f.)

(b) Class A performed better since its mean mark is higher than that of Class B.

6.

Time (min)	Mid-value (x)	f	fx	fx^2
$30 < x \leqslant 35$	32.5	4	130	4225
$35 < x \leqslant 40$	37.5	2	75	2812.5
$40 < x \leqslant 45$	42.5	4	170	7225
$45 < x \leqslant 50$	47.5	5	237.5	11 281.25
$50 < x \leqslant 55$	52.5	3	157.5	8268.75
$55 < x \leqslant 60$	57.5	3	172.5	9918.75
$60 < x \leqslant 65$	62.5	4	250	15 625
$65 < x \leqslant 70$	67.5	5	337.5	22 781.25
Sum		$\Sigma f = 30$	$\Sigma fx = 1530$	$\Sigma fx^2 = 82\,137.5$

(i) Mean, $\bar{x} = \dfrac{\Sigma fx}{\Sigma f} = \dfrac{1530}{30} = 51$ min

(ii) Standard deviation $= \sqrt{\dfrac{\Sigma fx^2}{\Sigma f} - \bar{x}^2}$

$= \sqrt{\dfrac{82\,137.5}{30} - 51^2}$

$= 11.7$ min (to 3 s.f.)

7. $15 + 6 + 18 + 9 + 2 + x = 9 \times 6$

$$50 + x = 54$$

$$x = 4$$

Standard deviation

$$= \sqrt{\frac{(15-9)^2 + (6-9)^2 + (18-9)^2 + (9-9)^2 + (2-9)^2 + (4-9)^2}{6}}$$

= 5.77 (to 3 s.f.)

8. $145 + 126 + 137 + 150 + x + 2x = 130 \times 6$

$$558 + 3x = 780$$

$$x = 74$$

Standard deviation

$$= \sqrt{\frac{(145-130)^2 + (126-130)^2 + (137-130)^2 + (150-130)^2 + (74-130)^2 + (148-130)^2}{6}}$$

= 26.3 (to 3 s.f.)

9. (a) For Latif,

(i) mean distance $= \frac{52 + 21 + 37 + 6 + 24 + 40}{6}$

$= 30$

(ii) standard deviation

$$= \sqrt{\frac{(52-30)^2 + (21-30)^2 + (37-30)^2 + (6-30)^2 + (24-30)^2 + (40-30)^2}{6}}$$

$= 14.9$

For Tariq,

(i) mean distance $= \frac{25 + 14 + 21 + 48 + 18 + 9}{6}$

$= 22.5$

(ii) standard deviation

$$= \sqrt{\frac{(25-22.5)^2 + (14-22.5)^2 + (21-22.5)^2 + (48-22.5)^2 + (18-22.5)^2 + (9-22.5)^2}{6}}$$

$= 12.5$

(b) Tariq's performance was more consistent since his standard deviation is smaller which means a smaller spread in data.

(c) Tariq was a better shooter since the mean distance from the centre of target each shot hit is smaller.

10. (a) (i) For Class X,

x	f	fx	fx^2
2	2	4	8
3	3	9	27
4	6	24	96
5	11	55	275
6	10	60	360
7	7	49	343
8	1	8	64
Sum	$\Sigma f = 40$	$\Sigma fx = 209$	$\Sigma fx^2 = 1173$

mean, $\bar{x} = \frac{\Sigma fx}{\Sigma f}$

$= \frac{209}{40}$

$= 5.225$

$= 5.23$ hours (to 3 s.f.)

standard deviation $= \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \bar{x}^2}$

$= \sqrt{\frac{1173}{40} - 5.225^2}$

$= 1.42$ hours (to 3 s.f.)

(ii) For Class Y,

x	f	fx	fx^2
2	4	8	16
3	4	12	36
4	9	36	144
5	8	40	200
6	7	42	252
7	5	35	245
8	3	24	192
Sum	$\Sigma f = 40$	$\Sigma fx = 197$	$\Sigma fx^2 = 1085$

mean, $\bar{x} = \frac{\Sigma fx}{\Sigma f}$

$= \frac{197}{40}$

$= 4.925$

$= 4.93$ hours (to 3 s.f.)

standard deviation $= \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \bar{x}^2}$

$= \sqrt{\frac{1085}{40} - 4.925^2}$

$= 1.69$ hours (to 3 s.f.)

(b) Class Y spends less time on surfing the Internet since the mean time spent by pupils on the Internet is lesser as compared to Class X.

11. (a) Mean of Nadeem = 20

$$\frac{21 + 43 + x + 8 + 34 + 24 + 12 + 2}{8} = 20$$

$144 + x = 160$

$x = 16$

Mean of Nasir = y

$$\frac{6 + 9 + 15 + 26 + 10 + 14 + 21 + 3}{8} = y$$

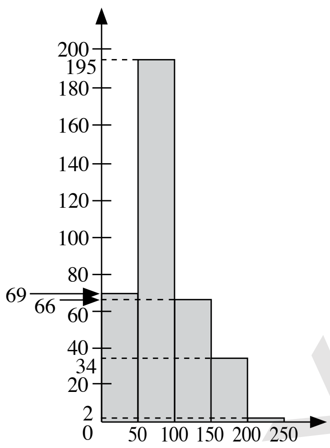
$y = 13$

$\therefore x = 16, y = 13$

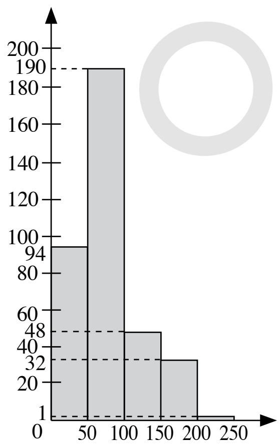
(b) Nadeem was more careless because the mean number of mistakes she made is higher than Nasir's.

(c) Nasir was more consistent because her standard deviation is smaller than Nadeem's, i.e. the number of mistakes is not as widely spread as Nadeem's.

12. (i)



PSI value, x, in 2012



PSI value, x, in 2013

(ii) The measures taken have been effective in improving the air quality as the PSI values in 2013 are generally lower than those in 2012.

New Trend

13. (a)

Marks	Mid-value (x)	City G			City P		
		f	fx	fx^2	f	fx	fx^2
$15.0 < x \leqslant 15.5$	15.25	3	45.75	697.69	22	335.5	5116.385
$15.5 < x \leqslant 16.0$	15.75	14	220.5	3472.88	27	425.25	6697.69
$16.0 < x \leqslant 16.5$	16.25	26	422.5	6865.63	19	308.75	5017.19
$16.5 < x \leqslant 17.0$	16.75	33	552.75	9258.56	20	335	5611.25
$17.0 < x \leqslant 17.5$	17.25	21	362.25	6248.81	16	276	4761
$17.5 < x \leqslant 18.0$	17.75	10	177.5	3150.63	5	88.75	1575.31
$18.0 < x \leqslant 18.5$	18.25	3	54.75	999.19	1	18.25	333.06
Sum		$\Sigma f = 110$	$\Sigma fx = 1836$	$\Sigma fx^2 = 30\,693.39$	$\Sigma f = 110$	$\Sigma fx = 1787.5$	$\Sigma fx^2 = 29\,111.88$

For City G,

(i) mean, $\bar{x} = \dfrac{\Sigma fx}{\Sigma f}$

$= \dfrac{1836}{110}$

$= 16.7$°C (to 3 s.f.)

(ii) standard deviation $= \sqrt{\dfrac{\Sigma fx^2}{\Sigma f} - \bar{x}^2}$

$= \sqrt{\dfrac{30\,693.39}{110} - 16.69^2}$

$= 0.689$°C (to 3 s.f.)

For City P,

(i) mean, $\bar{x} = \dfrac{\Sigma fx}{\Sigma f}$

$= \dfrac{1787.5}{110}$

$= 16.25$°C

(ii) standard deviation $= \sqrt{\dfrac{\Sigma fx^2}{\Sigma f} - \bar{x}^2}$

$= \sqrt{\dfrac{29\,111.88}{110} - 16.25^2}$

$= 0.769$°C (to 3 s.f.)

(b) City G is warmer because its mean temperature is higher.

(c) City G's temperature is more consistent because its standard deviation is smaller.

(d) For City G, mean = 19.7°C
standard deviation = 0.689°C
For City P, mean = 19.25°C
standard deviation = 0.769°C

14.

Blood pressure (mm Hg)	Mid-value (x)	f	fx	fx^2
$55 < x \leqslant 60$	57.5	1	57.5	3306.25
$60 < x \leqslant 65$	62.5	4	250	15 625
$65 < x \leqslant 70$	67.5	10	675	45 562.5
$70 < x \leqslant 75$	72.5	21	1522.5	110 381.25
$75 < x \leqslant 80$	77.5	35	2712.5	210 218.75
$80 < x \leqslant 85$	82.5	29	2392.5	197 381.25
$85 < x \leqslant 90$	87.5	13	1137.5	99 531.25
$90 < x \leqslant 95$	92.5	7	647.5	59 893.75
Sum		$\Sigma f = 120$	$\Sigma fx = 9395$	$\Sigma fx^2 = 741\,900$

Mean, $\bar{x} = \dfrac{\Sigma fx}{\Sigma f} = \dfrac{9395}{120} = 78.3$ mm Hg (to 3 s.f.)

Standard deviation $= \sqrt{\dfrac{\Sigma fx^2}{\Sigma f} - \bar{x}^2}$

$= \sqrt{\dfrac{741\,900}{120} - 78.29^2}$

$= 7.29$ mm Hg (to 3 s.f.)

Chapter 14 Probability of Combined Events

Basic

1. The fifteen cards are labelled 16, 17, 18, …, 30.

(a) P(contains 7) = $\frac{2}{15}$

(b) P(contains at least a 2) = $\frac{10}{15} = \frac{2}{3}$

(c) P(multiple of 3) = $\frac{5}{15} = \frac{1}{3}$

(d) P(prime) = $\frac{4}{15}$

(e) P(divisible by 5) = $\frac{3}{15} = \frac{1}{5}$

2. There are 5 red balls, 6 white balls and 9 green balls.

(a) P(green) = $\frac{9}{20}$

(b) P(red and white) = $\frac{11}{20}$

(c) There are no yellow balls.
P(yellow) = 0

(d) P(red, green or white) = 1

3. There are x white marbles (W), y blue marbles (B) and 8 red marbles (R).

$P(B) = \frac{y}{x + y + 8} = \frac{8}{15}$

$8x + 8y + 64 = 15y$

$8x + 64 = 7y$ — (1)

$P(W) = \frac{x}{x + y + 8} = \frac{1}{5}$

$5x = x + y + 8$

$4x - 8 = y$ — (2)

Substitute (2) into (1):

$8x + 64 = 7(4x - 8)$

$8x + 64 = 28x - 56$

$20x = 120$

$x = 6$

$\therefore x = 6$

$y = 4(6) - 8 = 16$

$\therefore$ Total number of marbles = 6 + 16 + 8 = 30

4. **(a)** P(a '5') = $\frac{1}{12}$

(b) P(a heart) = $\frac{2}{12} = \frac{1}{6}$

(c) P(a spade) = $\frac{6}{12} = \frac{1}{2}$

(d) P(a picture card) = $\frac{6}{12} = \frac{1}{2}$

(e) P(the ace of diamond) = 0

5. **(a)**

		y					
		1	2	3	4	5	6
x	1	0	–1	–2	–3	–4	–5
	2	1	0	–1	–2	–3	–4
	3	2	1	0	–1	–2	–3
	4	3	2	1	0	–1	–2
	5	4	3	2	1	0	–1
	6	5	4	3	2	1	0

(b) **(i)** P(negative) = $\frac{15}{36} = \frac{5}{12}$

(ii) P(positive and even) = $\frac{6}{36} = \frac{1}{6}$

(iii) P(non-zero) = $\frac{30}{36} = \frac{5}{6}$

(iv) P(≥ 2) = $\frac{10}{36} = \frac{5}{18}$

(v) P(not a multiple of 3) = $\frac{24}{36} = \frac{2}{3}$

6. There are x red balls and $(35 - x)$ blue balls.

(a) P(red) = $\frac{x}{35}$

(b) After 5 red balls are removed, there are $(x - 5)$ red balls and $(30 - x)$ blue balls.

P(red) = $\frac{x - 5}{30} = \frac{x}{35} - \frac{1}{14}$

$\frac{x - 5}{30} = \frac{2x - 5}{70}$

$70x - 350 = 60x - 150$

$10x = 200$

$\therefore x = 20$

Intermediate

7. **(a)** **(i)** P(< 4) = P(1, 2 or 3) = $\frac{3}{8}$

(ii) P(a prime number) = P(2, 3, 5, 7) = $\frac{4}{8} = \frac{1}{2}$

(iii) P(6 or 8) = $\frac{2}{8} = \frac{1}{4}$

(b)

×	1	2	3	4	5	6	7	8
1	1	2	3	4	5	6	7	8
2	2	4	6	8	10	12	14	16
3	3	6	9	12	15	18	21	24
4	4	8	12	16	20	24	28	32
5	5	10	15	20	25	30	35	40
6	6	12	18	24	30	36	42	48
7	7	14	21	28	35	42	49	56
8	8	16	24	32	40	48	56	64

(i) P(odd) = $\frac{16}{64} = \frac{1}{4}$

(ii) P(even) = 1 – P(odd) = $1 - \frac{1}{4} = \frac{3}{4}$

(iii) P(a perfect square) = $\frac{12}{64} = \frac{3}{16}$

(iv) P(not a perfect cube) = 1 – P(a perfect cube)

$= 1 - \frac{6}{64}$

$= \frac{29}{32}$

(v) P(a prime number) = $\frac{8}{64} = \frac{1}{8}$

(vi) P(a multiple of 6) = $\frac{21}{64}$

(vii) P(≤ 20) = $\frac{38}{64} = \frac{19}{32}$

(viii) P(divisible by 3 or 5) = $\frac{39}{64}$

(ix) P(divisible by 3 and 4) = $\frac{11}{64}$

8. (a)

+	1	2	3	4	5	6	8
3	4	5	6	7	8	9	11
5	6	7	8	9	10	11	13
7	8	9	10	11	12	13	15
9	10	11	12	13	14	15	17

×	1	2	3	4	5	6	8
3	3	6	9	12	15	18	24
5	5	10	15	20	25	30	40
7	7	14	21	28	35	42	56
9	9	18	27	36	45	54	72

(b) (i) P(sum > 5) = $\frac{26}{28} = \frac{13}{14}$

(ii) P(sum ⩽ 9) = $\frac{12}{28} = \frac{3}{7}$

(iii) P(sum is prime) = $\frac{11}{28}$

(iv) P(sum is a multiple of 5) = $\frac{6}{28} = \frac{3}{14}$

(v) P(product is odd) = $\frac{12}{28} = \frac{3}{7}$

(vi) P(product is even) = $\frac{16}{28} = \frac{4}{7}$

(vii) P(product consists of two digits) = $\frac{22}{28} = \frac{11}{14}$

(viii) P(product is divisible by 4) = $\frac{8}{28} = \frac{2}{7}$

(ix) P(product ⩾ 20) = $\frac{15}{28}$

(x) P(product is a perfect square) = $\frac{3}{28}$

9. $A = \{2, 3\}, B = \{1, 3, 9\}, C = \{2, 4, 6, 8, 10\}$

(a) $A \cap B = \{3\}$

(b) P(number is in C) = $\frac{5}{8}$

(c) P(number is in B) = $\frac{1}{2}$

10. $\mathbb{U} = \{41, 42, 43, \ldots, 59, 60\}$

(a) P(an even number) = $\frac{10}{20} = \frac{1}{2}$

(b) P(a perfect square) = $\frac{1}{20}$

(c) P(a multiple of 7) = $\frac{3}{20}$

(d) P(product of its two digits is odd)

= P(51, 53, 55, 57, 59)

$= \frac{5}{20}$

$= \frac{1}{4}$

(e) (i) P(sum > 10) = P(47, 48, 49, 56, 57, 58, 59)

$= \frac{7}{20}$

(ii) P(sum > 4) = 1

(iii) P(sum > 15) = 0

11. (a) P(Maaz does not proceed to JC or Poly)

$= 1 - \frac{3}{8} - \frac{1}{3}$

$= \frac{7}{24}$

(b) P(Maaz proceeds to JC while Sarah proceeds to neither JC nor Poly)

$= \frac{3}{8} \times \left[1 - \left(\frac{5}{8} + \frac{1}{4}\right)\right]$

$= \frac{3}{64}$

(c) P(only one proceeds to JC)

= P(Maaz proceeds to JC and Sarah does not) or P(Sarah proceeds to JC and Maaz does not)

$= \frac{3}{8} \times \left(1 - \frac{5}{8}\right) + \left(1 - \frac{3}{8}\right) \times \frac{5}{8}$

$= \frac{17}{32}$

12. There are 8 white discs (W), 12 green discs (G) and x yellow discs (Y).

(a) $P(Y) = \frac{x}{8 + 12 + x} = \frac{2}{7}$

$7x = 40 + 2x$

$5x = 40$

$x = 8$

(b) (i) $P(WW) = \frac{8}{28} \times \frac{7}{27} = \frac{2}{27}$

(ii) $P(GG) = \frac{12}{28} \times \frac{11}{27} = \frac{11}{63}$

(iii) $P(WY) = P(WY \text{ or } YW)$

$= \frac{8}{28} \times \frac{8}{27} + \frac{8}{28} \times \frac{8}{27}$

$= \frac{32}{189}$

(iv) P(G and black) = 0

13. (a) {(5C, 0W), (4C, 1W), (3C, 2W), (2C, 3W), (1C, 4W), (0C, 5W)}

= {20, 15, 13, 5, 0, –5}

(b) (i) P(20 marks) = $\frac{1}{6}$

(ii) P(0 marks) = $\frac{1}{6}$

(iii) P(> 6 marks) = $\frac{3}{6} = \frac{1}{2}$

(iv) P(< –3 marks) = $\frac{1}{6}$

14. (a)

+	1	2	3	4	5	6	7	8
1	2	3	4	5	6	7	8	9
2	3	4	5	6	7	8	9	10
3	4	5	6	7	8	9	10	11
4	5	6	7	8	9	10	11	12
5	6	7	8	9	10	11	12	13
6	7	8	9	10	11	12	13	14
7	8	9	10	11	12	13	14	15
8	9	10	11	12	13	14	15	16

(b) (i) P(even) = $\frac{32}{64} = \frac{1}{2}$

(ii) P(odd) = $\frac{32}{64} = \frac{1}{2}$

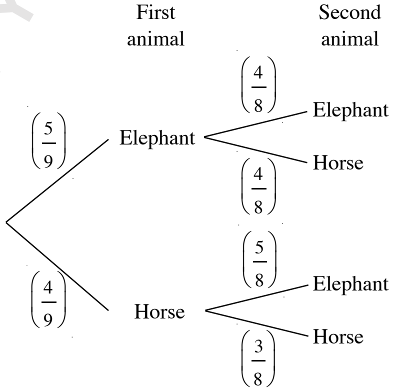
(iii) P(prime) = $\frac{23}{64}$

(iv) P(≤ 10) = $\frac{43}{64}$

(v) P(> 5) = $\frac{54}{64} = \frac{27}{32}$

(vi) P(multiple of 3) = $\frac{22}{64} = \frac{11}{32}$

15. (a)



(b) (i) P(first animal is horse and second is elephant)

$= \frac{4}{9} \times \frac{5}{8}$

$= \frac{5}{18}$

(ii) P(at least one of the animals is an elephant)

= 1 – P(both horses)

$= 1 - \frac{4}{9} \times \frac{3}{8}$

$= \frac{5}{6}$

Alternatively,

P(at least one of the animals is an elephant)

= P(Elephant, Elephant) or P(Elephant, Horse) or P(Horse, Elephant)

$= \frac{5}{9} \times \frac{4}{8} + \frac{5}{9} \times \frac{4}{8} + \frac{4}{9} \times \frac{5}{8}$

$= \frac{5}{6}$

(iii) P(second animal chosen is a horse)

= P(Elephant, Horse) or P(Horse, Horse)

$= \frac{5}{9} \times \frac{4}{8} + \frac{4}{9} \times \frac{3}{8}$

$= \frac{4}{9}$

16. There are x red marbles (R), y yellow marbles (Y) and 55 blue marbles (B).

(a) $P(R) = \frac{1}{8} = \frac{x}{x+y+55}$

$8x = x + y + 55$

$y = 7x - 55$ — (1)

$P(Y) = \frac{5}{12} = \frac{x}{x+y+55}$

$12y = 5x + 5y + 275$

$7y = 5x + 275$ — (2)

(b) Substitute (1) into (2) :

$7(7x - 55) = 5x + 275$

$49x - 385 = 5x + 275$

$44x = 660$

$x = 15$

Substitute $x = 15$ into (1) :

$y = 7(15) - 55$

$= 50$

(c) Now, there are 15 R, 50 Y and 55 B.

(i) $P(RR) = \frac{15}{120} \times \frac{14}{119} = \frac{1}{68}$

(ii) P(one R and one B)

= P(RB or BR)

$= \frac{15}{120} \times \frac{55}{119} + \frac{55}{120} \times \frac{15}{119}$

$= \frac{55}{476}$

(iii) P(2 marbles of different colours)

= P(RB , RY, YB, BR, YR, BY)

$= \left(\frac{15}{120} \times \frac{55}{119} + \frac{15}{120} \times \frac{50}{110} + \frac{50}{120} \times \frac{55}{119}\right) \times 2$

$= \frac{865}{1428}$

17. There are x red balls (R) and $(15 - x)$ white balls (W).

(a) $P(R) = \frac{x}{15}$

(b) $P(RR) = \frac{x}{15} \times \frac{x-1}{14} = \frac{x(x-1)}{210}$

(c) $\frac{x}{15} \times \frac{x-1}{14} = \frac{12}{35}$

$35x(x - 1) = 12 \times 210$

$x(x - 1) = 72$

$x^2 - x = 72$

(d) $x^2 - x - 72 = 0$

$(x + 8)(x - 9) = 0$

$\therefore x = -8$ (NA) or $x = 9$

$\therefore$ There are 6 white balls in the bag.

18. (a) $P(Y) = \frac{60°}{360°} = \frac{1}{6}$

(b) (i) $P(RB) = \frac{120}{360} \times \frac{120}{360} = \frac{1}{9}$

(ii) P(G at second spin)

= P(GG, RG, BG, YG)

$= \frac{60}{360} \times \frac{60}{360} + \frac{120}{360} \times \frac{60}{360} + \frac{120}{360} \times \frac{60}{360} + \frac{60}{360} \times \frac{60}{360}$

$= \frac{1}{6}$

(iii) P(Y or R)

= P(YY, YR, RY, RR)

$= \frac{1}{6} \times \frac{1}{6} + \frac{1}{6} \times \frac{120}{360} + \frac{120}{360} \times \frac{1}{6} + \frac{120}{360} \times \frac{120}{360}$

$= \frac{1}{4}$

(iv) P(different colours at both spins)

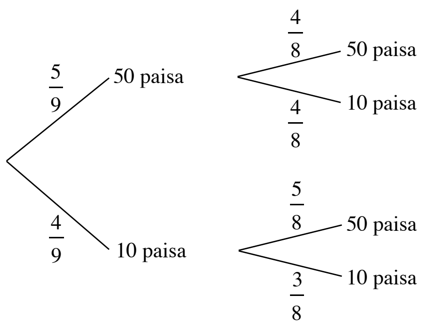
= 1 – P(same colour at both spins)

= 1 – P(RR or YY or BB or GG)

$= 1 - \left(\frac{1}{3} \times \frac{1}{3} + \frac{1}{6} \times \frac{1}{6} + \frac{1}{3} \times \frac{1}{3} + \frac{1}{6} \times \frac{1}{6}\right)$

$= \frac{13}{18}$

19. (a)



(b) (i) P(20 paisas in total) = P(10 paisas, 10 paisas)

$= \frac{4}{9} \times \frac{3}{8}$

$= \frac{1}{6}$

(ii) P(60 paisas in total)

= P(10 paisas, 50 paisas) or P(50 paisas, 10 paisas)

$= \frac{4}{9} \times \frac{5}{8} + \frac{5}{9} \times \frac{4}{8}$

$= \frac{5}{9}$

(c) (i) P(70 paisas in total)

= P(50 paisas, 10 paisas, 10 paisas) or P(10 paisas, 50 paisas, 10 paisas) or P(10 paisas, 10 paisas, 50 paisas)

$= \frac{5}{9} \times \frac{4}{8} \times \frac{3}{7} + \frac{4}{9} \times \frac{5}{8} \times \frac{3}{7} + \frac{4}{9} \times \frac{3}{8} \times \frac{5}{7}$

$= \frac{5}{14}$

(ii) P(at least PKR 1.10)

= P(50 paisas, 50 paisas, 50 paisas) or P(50 paisas, 50 paisas, 10 paisas) or P(10 paisas, 50 paisas, 50 paisas) or P(50 paisas, 10 paisas, 50 paisas)

$= \frac{5}{9} \times \frac{4}{8} \times \frac{3}{7} + \frac{5}{9} \times \frac{4}{8} \times \frac{4}{7} + \frac{4}{9} \times \frac{5}{8} \times \frac{4}{7} + \frac{5}{9} \times \frac{4}{8} \times \frac{4}{7}$

$= \frac{25}{42}$

20. (a) P(box B is chosen) $= \frac{1}{2}$

(b) P(even number on ball) = P(A even or B even)

$= \frac{1}{2} \times \frac{4}{9} + \frac{1}{2} \times \frac{3}{6}$

$= \frac{17}{36}$

(c) P(box A is chosen and even number on ball)

$= \frac{1}{2} \times \frac{4}{9}$

$= \frac{2}{9}$

(d) P(box B is chosen and prime number on ball)

$= \frac{1}{2} \times \frac{3}{6}$

$= \frac{1}{4}$

21. (a) P(both alive) $= 0.45 \times 0.5 = \frac{9}{40}$

(b) P(only wife alive) = P(man dies and wife survives)

$= (1 - 0.45) \times 0.5$

$= \frac{11}{40}$

(c) P(at least one of them survives)

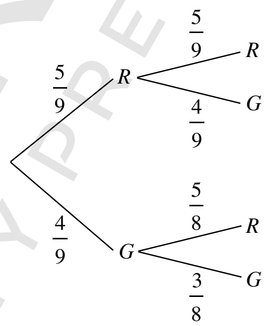
= 1 – P(both do not survive)

$= 1 - (1 - 0.45) \times (1 - 0.5)$

$= 1 - \frac{11}{40}$

$= \frac{29}{40}$

22. (a)



R: Red
G: Green

(b) (i) P(RR) $= \frac{5}{9} \times \frac{5}{9} = \frac{25}{81}$

(ii) P(different colours) = P(RG or GR)

$= \frac{5}{9} \times \frac{4}{9} + \frac{4}{9} \times \frac{5}{8}$

$= \frac{85}{162}$

(iii) P(at least three green balls are left)

= 1 – P(RR) – P(RG) – P(GR)

$= 1 - \frac{5}{9} \times \frac{5}{9} - \frac{5}{9} \times \frac{4}{9} - \frac{4}{9} \times \frac{5}{8}$

$= \frac{1}{6}$

23. (a) P(only *Laila* solves)

= P(*Laila* solves and *Leena* does not solve)

$= \frac{1}{2} \times \left(1 - \frac{2}{5}\right)$

$= \frac{3}{10}$

(b) P(at least one of them solves)

= 1 – P(both do not solve)

$= 1 - \frac{3}{5} \times \frac{1}{2}$

$= \frac{7}{10}$

24. **(a)** P(two diamonds) = $\frac{13}{52} \times \frac{12}{51} = \frac{1}{17}$

(b) P(two Queens) = $\frac{4}{52} \times \frac{3}{51} = \frac{1}{221}$

(c) P(one heart and one spade)

= P(heart, spade or spade, heart)

$= \frac{13}{52} \times \frac{13}{51} + \frac{13}{52} \times \frac{13}{51}$

$= \frac{13}{102}$

25. There are 7 toffees in green paper (*TG*), 4 barley sugar in red paper (*BR*), 3 toffees in red paper (*TR*) and 6 barley sugar in green paper (*BG*).

(a) P(*T* and *BR*) = $\frac{10}{20} \times \frac{4}{19} = \frac{2}{19}$

(b) P(*TT*) = $\frac{10}{20} \times \frac{9}{19} = \frac{9}{38}$

(c) P(*BG*, *BG*) = $\frac{6}{20} \times \frac{5}{19} = \frac{3}{38}$

(d) P(same flavour) = P(*TT* or *BB*)

$= \frac{10}{20} \times \frac{9}{19} + \frac{10}{20} \times \frac{9}{19}$

$= \frac{9}{19}$

(e) P(different colour) = P(*GR* or *RG*)

$= \frac{13}{20} \times \frac{7}{19} + \frac{7}{20} \times \frac{13}{19}$

$= \frac{91}{190}$

26. There are 6 yellow marbles (*Y*) and 3 green marbles (*G*).

(a) P(*YY* with replacement) = $\frac{6}{9} \times \frac{6}{9} = \frac{4}{9}$

(b) P(*YY* without replacement) = $\frac{6}{9} \times \frac{5}{8} = \frac{5}{12}$

Advanced

27. **(a)** P(to *Q*) = $\frac{1}{3}$

(b) P(to *T*) = P(straight and right)

$= \frac{1}{2} \times \frac{1}{6}$

$= \frac{1}{12}$

(c) P(to *U*) = P(straight and straight)

$= \frac{1}{2} \times \frac{1}{2}$

$= \frac{1}{4}$

28. There are 3 red socks (*R*) and 5 green socks (*G*) in the first bag and 6 red socks (*R*) and 4 green socks (*G*) in the second bag.

(a) P(both *R*) = P(*RR*)

$= \frac{3}{8} \times \frac{6}{5}$

$= \frac{9}{40}$

(b) P(at least one is *G*) = 1 – P(*RR*)

$= 1 - \frac{9}{40}$

$= \frac{31}{40}$

(c) P(different colours) = P(*RG* or *GR*)

$= \frac{3}{8} \times \frac{4}{10} + \frac{5}{8} \times \frac{6}{10}$

$= \frac{21}{40}$

29. P(getting distinction in English) = P(*E*) = $\frac{5}{7}$

P(getting distinction in Maths) = P(*M*) = $\frac{3}{4}$

P(getting distinction in Science) = P(*S*) = $\frac{5}{6}$

(a) P(no distinction) = $\frac{2}{7} \times \frac{1}{4} \times \frac{1}{6} = \frac{1}{84}$

(b) P(exactly one distinction)

= P(*EM'S'* or *E'MS'* or *E'M'S*)

$= \frac{5}{7} \times \frac{1}{4} \times \frac{1}{6} + \frac{2}{7} \times \frac{3}{4} \times \frac{1}{6} + \frac{2}{7} \times \frac{1}{4} \times \frac{5}{6}$

$= \frac{1}{8}$

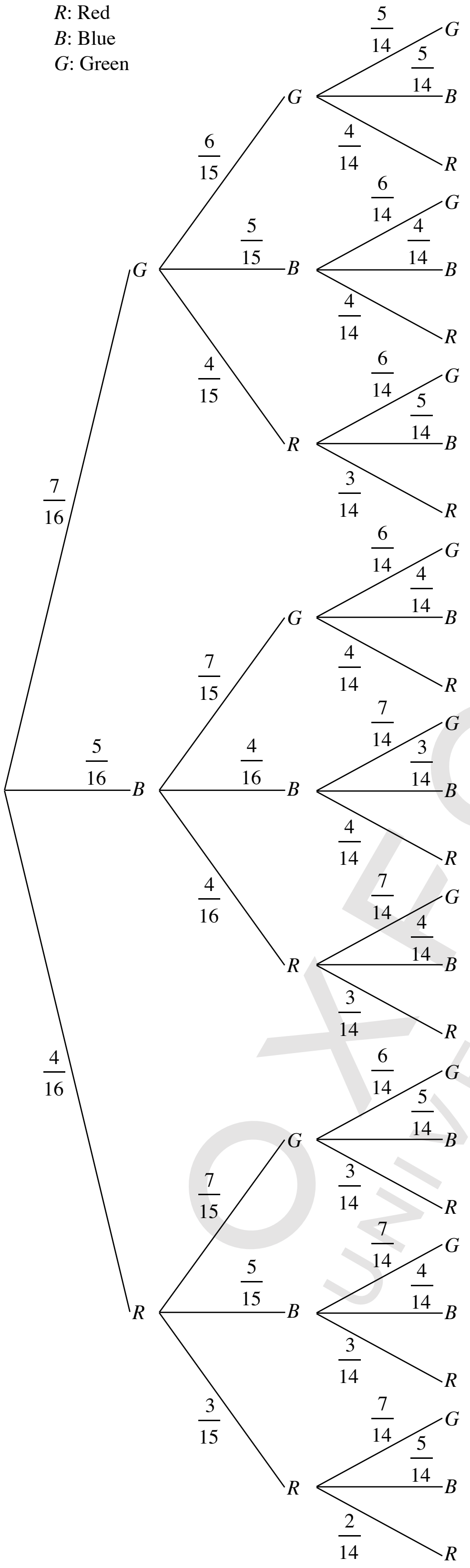
(c) P(qualify for entry)

= 1 – P(no distinction or exactly one distinction)

$= 1 - \frac{1}{84} - \frac{1}{8}$

$= \frac{145}{168}$

30. (a) R: Red
B: Blue
G: Green



(b) (i) P(one ball of each colour)

$= \text{P}(GBR \text{ or } GRB \text{ or } BGR \text{ or } BRG \text{ or } RGB \text{ or } RBG)$

$= \left(\frac{7}{16} \times \frac{5}{15} \times \frac{4}{14}\right) \times 6$

$= \frac{1}{4}$

(ii) P(exactly one is blue)

$= \text{P}(BB'B' \text{ or } B'BB' \text{ or } B'B'B)$

$= \frac{5}{16} \times \frac{11}{15} \times \frac{10}{14} + \frac{11}{16} \times \frac{5}{15} \times \frac{10}{14} + \frac{11}{16} \times \frac{10}{15} \times \frac{5}{14}$

$= \frac{55}{112}$

(iii) P(no red balls) $= \text{P}(R'R'R')$

$= \frac{12}{16} \times \frac{11}{15} \times \frac{10}{14}$

$= \frac{11}{28}$

(iv) P(second ball is G)

$= \text{P}(GG \text{ any}, BG \text{ any}, RG \text{ any})$

$= \frac{7}{16} \times \frac{6}{15} \times 1 + \frac{5}{16} \times \frac{7}{15} \times 1 + \frac{4}{16} \times \frac{7}{15} \times 1$

$= \frac{7}{16}$

31. There are 4 white counters (W) and 3 black counters (B).

P(two counters of each colour are left)

$= \text{P}(WWB \text{ or } WBW \text{ or } BWW)$

$= \frac{4}{7} \times \frac{3}{6} \times \frac{3}{5} + \frac{4}{7} \times \frac{3}{6} \times \frac{3}{5} + \frac{3}{7} \times \frac{4}{6} \times \frac{3}{5}$

$= \frac{18}{35}$

New Trend

32. (a) P(both balls are black) $= \frac{15-n}{15}\left(\frac{14-n}{14}\right)$

$= \frac{210 - 29n + n^2}{210}$

(b) $\frac{210 - 29n + n^2}{210} = \frac{2}{35}$

$210 - 29n + n^2 = 12$

$n^2 - 29n + 198 = 0$ (shown)

(c) $n^2 - 29n + 198 = 0$

$(n - 11)(n - 18) = 0$

$n = 11$ or $n = 18$

$\therefore$ There are $15 - 11 = 4$ black balls.

33. (a) (i) P(student from School A who obtains > 30 marks)

$= \frac{23 + 19}{160}$

$= \frac{21}{80}$

(ii) P(student gets a score $\leqslant$ 20 marks)

$= \frac{17 + 9}{160}$

$= \frac{13}{80}$

(b) P(both students from School B who obtain > 40 marks)

$= \frac{22}{160} \times \frac{21}{159}$

$= 0.0182$ (to 3 s.f.)

34. (a) P(prime) $= \frac{5}{10}$

$= \frac{1}{2}$

(b) P(both even) $= \frac{1}{2} \times \frac{1}{2}$

$= \frac{1}{4}$

(c) P(sum is 3) = P(1, 2) + P(2, 1)

$= \frac{2}{10} \times \frac{2}{10} + \frac{2}{10} \times \frac{2}{10}$

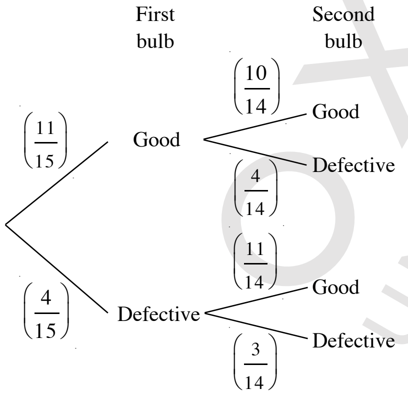
$= \frac{2}{25}$

P(sum is not 3) = 1 – P(sum is 3)

$= 1 - \frac{2}{25}$

$= \frac{23}{25}$

35. (a)



(b) (i) P(first bulb is good and second bulb is defective)

$= \frac{11}{15} \times \frac{4}{14}$

$= \frac{22}{105}$

(ii) P(both bulbs are good) $= \frac{11}{15} \times \frac{10}{14}$

$= \frac{11}{21}$

(iii) P(neither bulb is good) $= \frac{4}{15} \times \frac{3}{14}$

$= \frac{2}{35}$

(iv) P(one bulb is defective)

= P(first is good and second is defective)

+ P(first is defective and second is good)

$= \frac{11}{15} \times \frac{4}{14} + \frac{4}{15} \times \frac{11}{14}$

$= \frac{44}{105}$

36. (a)

		First Outcome					
		1	2	3	4	5	6
Second Outcome	6	(1, 6)	(2, 6)	(3, 6)	(4, 6)	(5, 6)	
	5	(1, 5)	(2, 5)	(3, 5)	(4, 5)		(6, 5)
	4	(1, 4)	(2, 4)	(3, 4)		(5, 4)	(6, 4)
	3	(1, 3)	(2, 3)		(4, 3)	(5, 3)	(6, 3)
	2	(1, 2)		(3, 2)	(4, 2)	(5, 2)	(6, 2)
	1		(2, 1)	(3, 1)	(4, 1)	(5, 1)	(6, 1)

(b) Total number of outcomes = 30

(i) P(both numbers more than 4) $= \frac{2}{30}$

$= \frac{1}{15}$

(ii) P(sum of numbers is 12) = 0

(iii) P(product is less than 6) $= \frac{8}{30}$

$= \frac{4}{15}$

(iv) P(neither counter has an odd number)

= P(both counters have even numbers)

$= \frac{6}{30}$

$= \frac{1}{5}$

37. (i) (a) P(girl who comes to school by public transport)

$= \frac{8}{40}$

$= \frac{1}{5}$

(b) P(boy who comes to school by private transport)

$= \frac{7}{40}$

(c) P(pupil who comes to school by public transport)

$= \frac{20}{40}$

$= \frac{1}{2}$

(d) P(pupil is a boy) $= \frac{19}{40}$

(ii) (a) P(both female) $= \frac{21}{40} \times \frac{20}{39}$

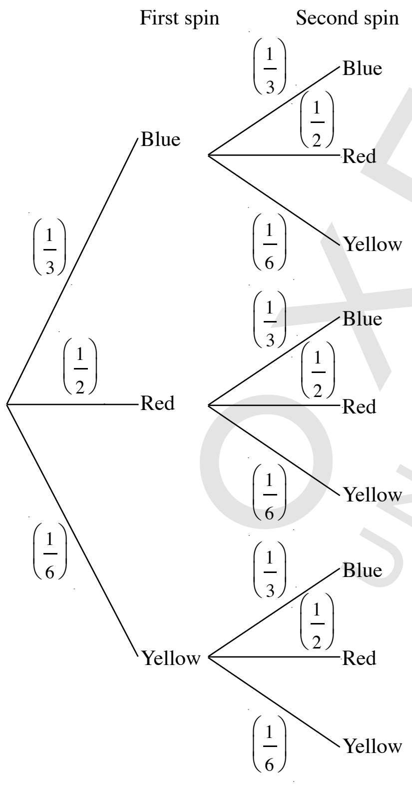
$= \frac{7}{26}$

(b) P(neither are boys taking public transport)

$= \frac{28}{40} \times \frac{27}{39}$

$= \frac{63}{130}$

38. (a)



(b) (i) P(blue, red) $= \frac{1}{3} \times \frac{1}{2}$

$= \frac{1}{6}$

(ii) P(same colour at both spins)

= P(blue, blue) or P(red, red) or P(yellow, yellow)

$= \frac{1}{3} \times \frac{1}{3} + \frac{1}{2} \times \frac{1}{2} + \frac{1}{6} \times \frac{1}{6}$

$= \frac{7}{18}$

(iii) P(different colours at both spins)

= 1 – P(same colour at both spins)

$= 1 - \frac{7}{18}$

$= \frac{11}{18}$