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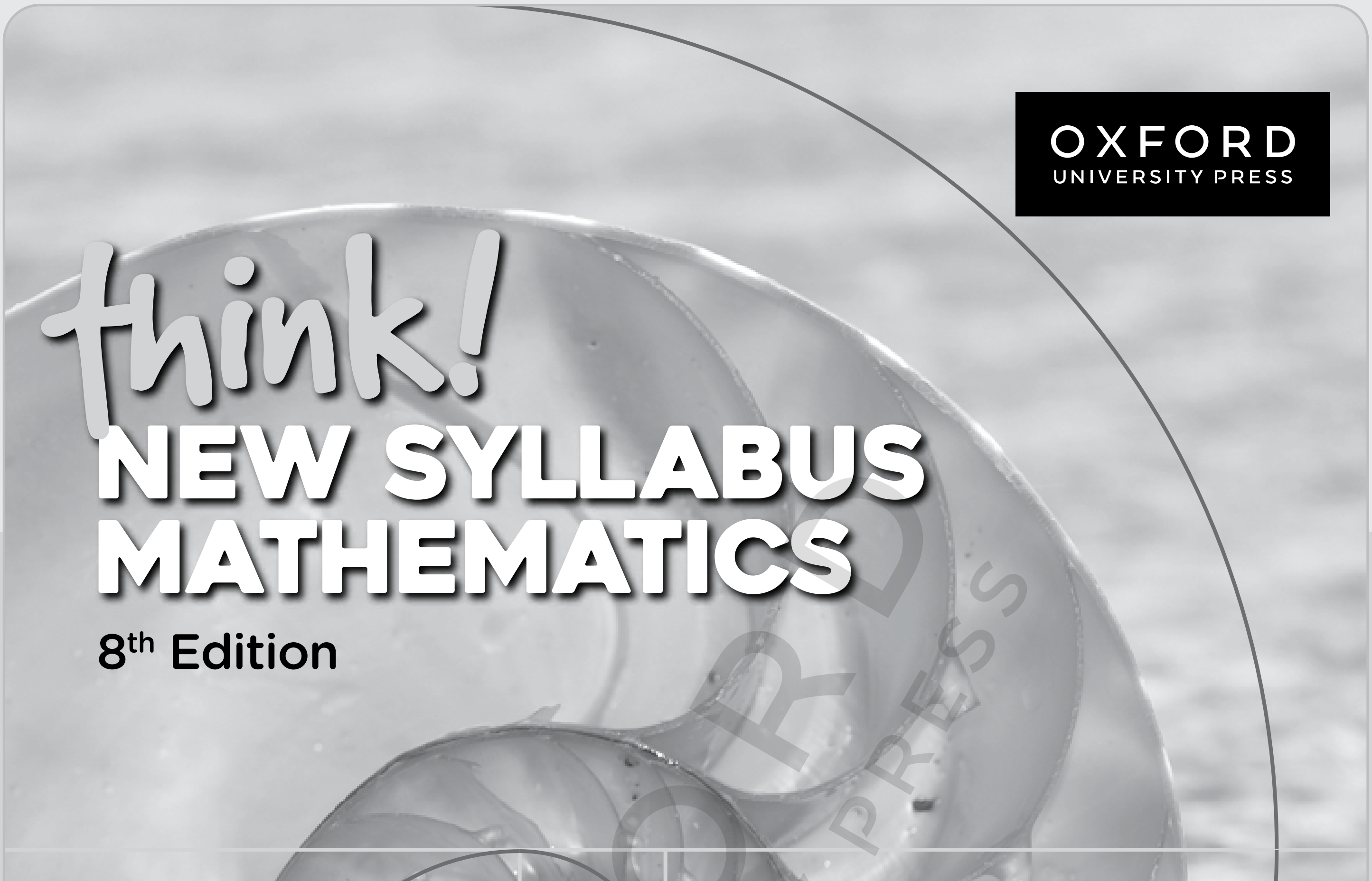
think!
NEW SYLLABUS MATHEMATICS

8th Edition



Workbook Full Solutions 4

1 Further Sets

Worksheet 1A Intersection and union of two sets

1. **(a)** $A \cap B$ = **{q, r}**
(b) n($A \cap B$) = **2**

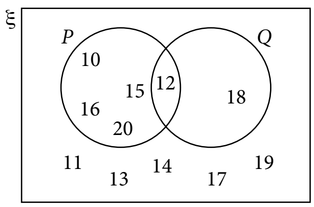
2. **(a)** $C \cap D$ = **{s, t}**
(b) n($C \cap D$) = **2**
(c) $\boldsymbol{C \cap D = D}$

3. **(a)** $P \cap Q$ = **{coffee, tea}**
(b) **No**. Although P and Q have the same number of elements, they do not have the same elements.

4. **(a)** B = **{3, 4, 5}**
(b) B = **{3, 4, 5, 7}**

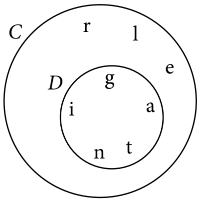
5. C = **{6, 16, 26, 36, 46}**

6. **(a)** P = {10, 12, 15, 16, 20}
Q = {12, 18}



(b) **(i)** n($P \cap Q$) = **1**
(ii) n($P \cap Q$)′ = **10**

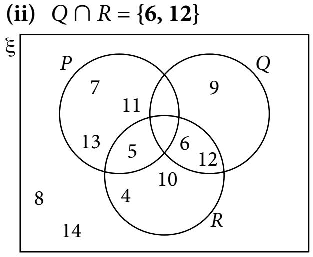
7. **(a)** $C \cap D$ = **{g, i, a, n, t}**
(b)



(c) **(i)** $\boldsymbol{(C \cap D) \subset C}$
(ii) $\boldsymbol{C \cap D = D}$

8. **(a)** **True.** An isosceles triangle has two equal sides, which is not scalene (three unequal sides).
(b) **True.** A scalene triangle can be right-angled, e.g. a triangle with sides 3 cm, 4 cm and 5 cm is scalene and right-angled.

9. **(a)** P = {5, 7, 11, 13}
Q = {6, 9, 12}
R = {4, 5, 6, 10, 12}
(i) $P \cap Q$ = { }
(ii) $Q \cap R$ = **{6, 12}**
(b)



10. **(a)** $A \cup B$ = **{p, q, r, s, t, u}**
(b) n($A \cup B$) = **6**

11. **(a)** $C \cup D$ = **{p, q, r, s, t}**
(b) n($C \cup D$) = **5**
(c) $\boldsymbol{C \cup D = C}$

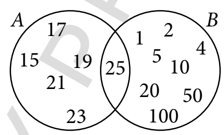
12. **(a)** **(i)** $P \cup R$ = **{a, b, c, d, e, f, g, i, j}**
(ii) $P \cup Q \cup R$ = **{a, b, c, d, e, f, g, h, i, j}**
(b) n(P) = 5, n(Q) = 5, n(R) = 5
n($P \cup Q \cup R$) = 10
$\therefore$ **n(P) + n(Q) + n(R) > n($P \cup Q \cup R$)**

13. **No.** $A \cup B \cup C$ = {butter, cocoa, egg, flour, salt, sugar}.

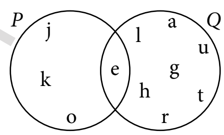
14. **(a)** $40 < 5x \leqslant 76$
$8 < x \leqslant 15.2$
D = {10, 12, 14}
E = {8, 9, 10, 12, 14, 15, 16}
$D \cup E$ = **{8, 9, 10, 12, 14, 15, 16}**
(b) Since n(D) = 3, n(E) = 7 and n($D \cup E$) = 7,
then n(D) + n(E) ≠ n($D \cup E$). (shown)

15. $\boldsymbol{P}$ **= {1, 3, 5}**
$\boldsymbol{Q}$ **= {2, 4, 6}**
$P \cup Q$ = {1, 2, 3, 4, 5, 6}
$\therefore$ n(P) + n(Q) = n($P \cup Q$) = 6

16. **(a)** A = {15, 17, 19, 21, 23, 25}
B = {1, 2, 4, 5, 10, 20, 25, 50, 100}
$A \cup B$ = **{1, 2, 4, 5, 10, 15, 17, 19, 20, 21, 23, 25, 50, 100}**
(b) n($A \cup B$) = **14**
(c)



17. **(a)** $P \cup Q$ = **{a, e, g, h, j, k, l, o, r, t, u}**
(b)



(c) **(i)** **True**
(ii) **True**

Challenge Myself!

18. **(a)** **(i)** When $x = -1$,
$y = -2(-1) + 3$
$= 5$
$\therefore$ (−1, 5) lies on $y = -2x + 3$, hence $(-1, 5) \in$ A. (shown)
(ii) **(0, 3)** $\in$ A
(b) **(i)** $x + hy = k$
$hy = -x + k$
$y = -\frac{1}{h}x + \frac{k}{h}$
$y = -2x + 3$ and $x + hy = k$ have the same gradient.
$-\frac{1}{h} = -2$
$h = \boldsymbol{\frac{1}{2}}$

(ii) Since $A \cap B = \varnothing$, then the lines are parallel but do not coincide.

$\frac{k}{h} \neq 3$

$k \neq 3h$

$k \neq 3\left(\frac{1}{2}\right)$

$k \neq 1.5$

$\therefore$ k can be any real number except 1.5.

(c) Maximum value of $n(A \cap C) = \mathbf{2}$

Worksheet 1B Applications of sets in real-world contexts

1. $A = \{2, 3, 5, 7\}$

$B = \{3, 6, 9\}$

(a) $A' = \mathbf{\{1, 4, 6, 8, 9, 10\}}$

(b) $A' \cap B = \mathbf{\{6, 9\}}$

(c) $A \cup B = \{2, 3, 5, 6, 7, 9\}$

$(A \cup B)' = \mathbf{\{1, 4, 8, 10\}}$

2. **(a)** $A = \{20, 25\}$

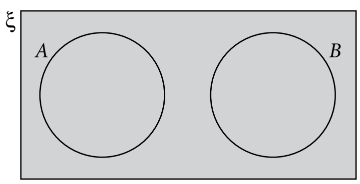
$A' = \{15, 16, 17, 18, 19, 21, 22, 23, 24\}$

$B = \{17, 19, 23\}$

$B' = \{15, 16, 18, 20, 21, 22, 24, 25\}$

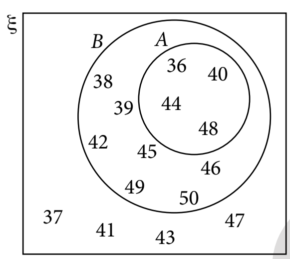
$A' \cup B' = \mathbf{\{15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25\}}$

(b)



3. **(a)** $A = \{36, 40, 44, 48\}$

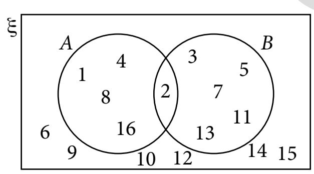
$B = \{36, 38, 39, 40, 42, 44, 45, 46, 48, 49, 50\}$



(b) $(A \cup B)' = \mathbf{\{37, 41, 43, 47\}}$

(c) $\mathbf{B'}$

4. **(a)**



(b) Set B contains all the prime numbers from 1 to 16.

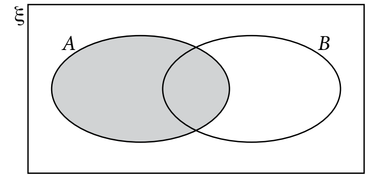
(c) $A' \cap B' = \mathbf{\{6, 9, 10, 12, 14, 15\}}$

(d) $A \cup B' = \{1, 2, 4, 6, 8, 9, 10, 12, 14, 15, 16\}$

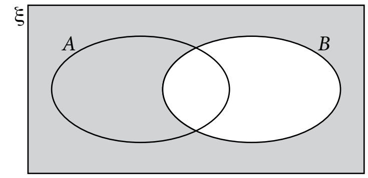
$A' \cap B = \{3, 5, 7, 11, 13\}$

$\therefore n((A \cup B') \cap (A' \cap B)) = \mathbf{0}$

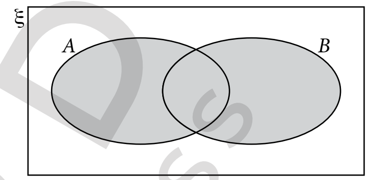
5. **(a)** A



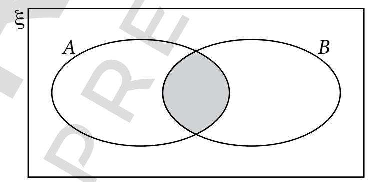
(b) B'



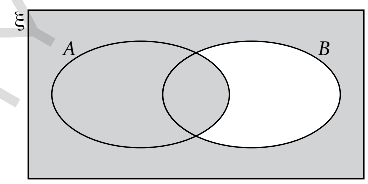
(c) $A \cup B$



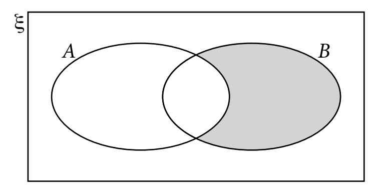
(d) $A \cap B$



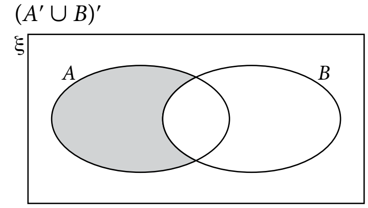
(e) $A \cup B'$



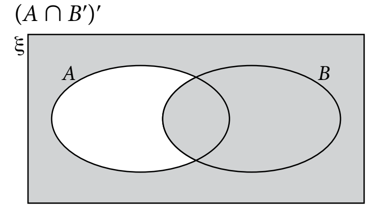
(f) $A' \cap B$



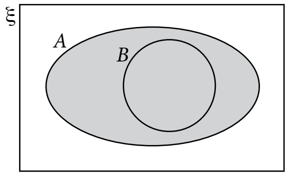
(g) $(A' \cup B)'$



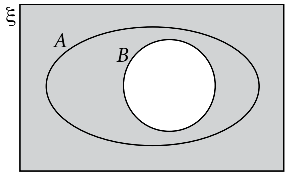
(h) $(A \cap B')'$



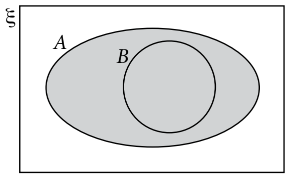
6. **(a)** A



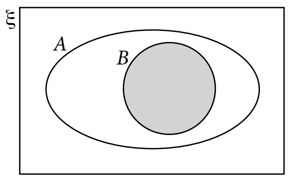
(b) B'



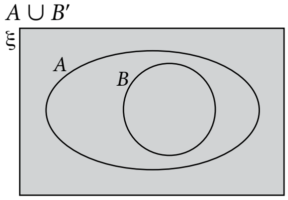
(c) $A \cup B$



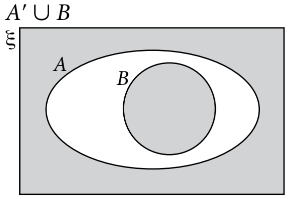
(d) $A \cap B$



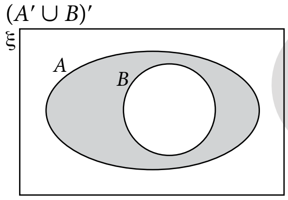
(e) $A \cup B'$



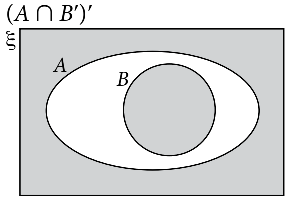
(f) $A' \cup B$



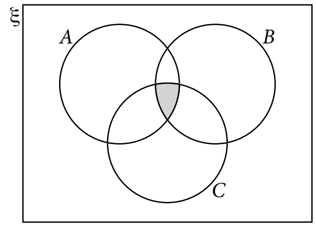
(g) $(A' \cup B)'$



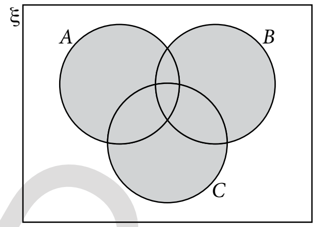
(h) $(A \cap B')'$



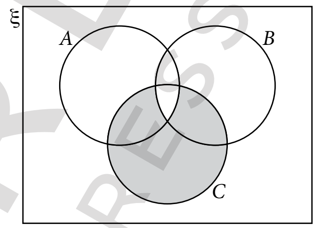
7. **(a)** $A \cap B \cap C$



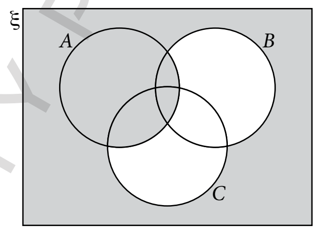
(b) $A \cup B \cup C$



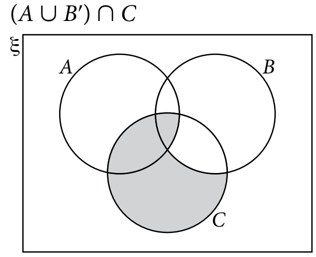
(c) $(A \cap B)' \cap C$



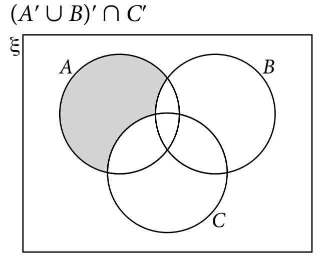
(d) $A \cup (B \cup C)'$



(e) $(A \cup B') \cap C$



(f) $(A' \cup B)' \cap C'$

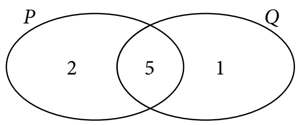


8. **(a)** $\boldsymbol{A \cap B}$
(b) $\boldsymbol{(A \cup B)'}$
(c) $\boldsymbol{A' \cup B}$
(d) $\boldsymbol{(A \cap B) \cup (A \cup B)'}$
(e) $\boldsymbol{(A \cap B) \cup (B \cap C) \cup (A \cap C)}$
(f) $\boldsymbol{(A \cup B \cup C)' \cup (A \cap B \cap C')}$

9. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$= 14 + 15 - 6$
$= \mathbf{23}$

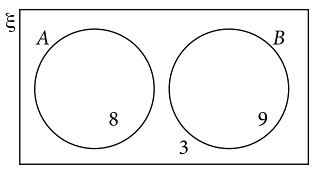
10. $n(C \cap D) = n(C) + n(D) - n(C \cup D)$
$= 8 + 3 - 10$
$= \mathbf{1}$

11. **(a)** Since $n(P \cap Q) = 5$, then $\mathbf{n(Q) \geqslant 5}$.
(b) OPEN Let $n(Q) = 6$.
$n(P \cup Q) = n(P) + n(Q) - n(P \cap Q)$
$= 7 + 6 - 5$
$= 8$
$\therefore$ An example of the values is $n(Q) = \mathbf{6}$ and $n(P \cup Q) = \mathbf{8}$.



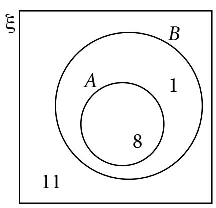
The numbers in the Venn diagram represent the number of elements in each set.

12. **(a)**



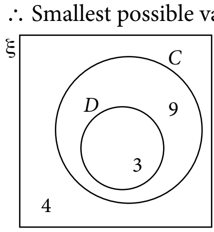
The numbers in the Venn diagram represent the number of elements in each set.
$\therefore$ Largest possible value of $n(A \cup B) = \mathbf{17}$

(b)



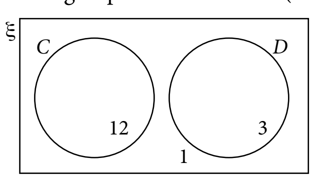
The numbers in the Venn diagram represent the number of elements in each set.
$\therefore$ Smallest possible value of $n(A \cup B) = \mathbf{9}$

13. **(a)**



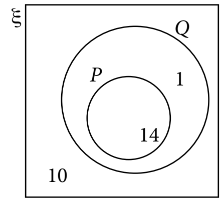
The numbers in the Venn diagram represent the number of elements in each set.
$\therefore$ Largest possible value of $n(C \cap D) = \mathbf{3}$

(b)



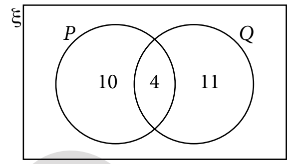
The numbers in the Venn diagram represent the number of elements in each set.
$\therefore$ Smallest possible value of $n(C \cap D) = \mathbf{0}$

14. **(a)**



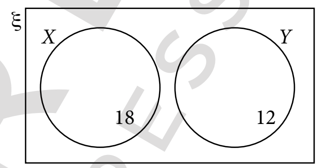
The numbers in the Venn diagram represent the number of elements in each set.
$\therefore$ Largest possible value of $n(P \cup Q)' = \mathbf{10}$

(b)



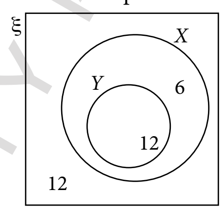
The numbers in the Venn diagram represent the number of elements in each set.
$\therefore$ Smallest possible value of $n(P \cup Q)' = \mathbf{0}$

15. **(a)**



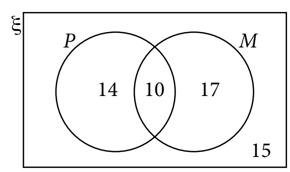
The numbers in the Venn diagram represent the number of elements in each set.
$\therefore$ Smallest possible value of $n(X \cap Y) = \mathbf{0}$

(b)



The numbers in the Venn diagram represent the number of elements in each set.
$\therefore$ Largest possible value of $n(X \cap Y)' = 12 + 6$
$= \mathbf{18}$

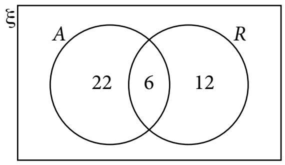
16. Let P and M represent the sets of people who use physical journals and mobile devices respectively.



The numbers in the Venn diagram represent the number of elements in each set.
Number of people who use neither physical journals nor mobile devices = **15**

17. Let A and R represent the sets of students who joined the class on animations and on robotics respectively.

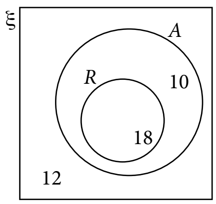
(a)



The numbers in the Venn diagram represent the number of elements in each set.

∴ Smallest possible number of students who joined both classes = **6**

(b)



The numbers in the Venn diagram represent the number of elements in each set.

∴ Smallest possible number of students who joined the class on animations only = **10**

Challenge Myself!

18. **(a)** None of the open-top cars have exactly two doors.

(b) Some cars with exactly two doors have rear dash cameras.

Review Exercise 1

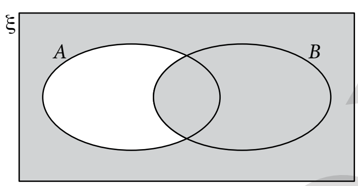
1. **(a)** **(i)** $C = \{\mathbf{6, 10, 12, 15, 20}\}$

(ii) $A = \{7, 11, 13, 17, 19, 23\}$

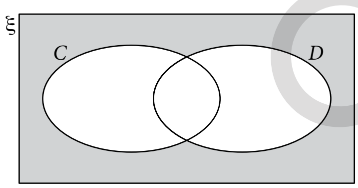
$A \cap C = \varnothing$

(b) $\boldsymbol{B \subset C, 8 \notin A}$

2. **(a)**



(b)



3. **(a)** **(i)** $A \cap B = \{\mathbf{a, e, r}\}$

(ii) $(A \cup B) = \{a, e, h, l, p, r, t, x, y\}$

$(A \cup B)' = \varnothing$

(b) $\boldsymbol{(P \cup Q) \cap (P \cap Q)'}$

4. **(a)** $\{2, 14\} \subset B$

(b) $5 \in C$

(c) $A \cap C = \varnothing$

2 Probability of Combined Events

Worksheet 2A Probability of single event

1. **{(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (3, 4), (4, 1), (4, 2), (4, 3), (4, 4)}**

2. **(a)** $\text{P(letter card)} = \frac{6}{8+6+7}$

$= \mathbf{\frac{2}{7}}$

(b) $\text{P(not a picture card)} = \frac{6+7}{21}$

$= \mathbf{\frac{13}{21}}$

(c) P(blank card) = **0**

3. **(a)** $\text{P(not a postcard of Italy)} = \frac{5+6}{5+9+6}$

$= \mathbf{\frac{11}{20}}$

(b) $\frac{9}{20-x} = 0.5$

$9 = 10 - 0.5x$

$0.5x = 1$

$x = \mathbf{2}$

4. **(a)** $\text{P(red counter)} = \frac{12}{12+17+1}$

$= \mathbf{\frac{2}{5}}$

(b) Let x be the number of blue counters that have to be added.

$\frac{17+x}{12+17+1+x} = \frac{4}{5}$

$\frac{17+x}{30+x} = \frac{4}{5}$

$85 + 5x = 120 + 4x$

$x = 35$

∴ **35** blue counters have to be added.

(c) Let y be the number of red counters that are removed.

$\frac{12-y}{12+17+1-y} = \frac{1}{5}$

$\frac{12-y}{30-y} = \frac{1}{5}$

$60 - 5y = 30 - y$

$4y = 30$

$y = 7.5$

Since y is not a positive integer, it is not possible for the probability to be $\frac{1}{5}$.

5. **(a)** $\text{Area of quadrant} = \frac{1}{4}\pi(4)^2$

$= 4\pi \text{ cm}^2$

$\text{Area of rhombus} = 2 \times \frac{1}{2}(x)(x)\sin 35°$

$= x^2 \sin 35°$

$\text{P(point lies within quadrant)} = \frac{4\pi}{x^2 \sin 35°}$

$\approx \frac{22}{x^2}$ (shown)

(b) If $x = 4$, then $\frac{22}{x^2} = 1.375 > 1$.

Since the probability cannot exceed 1, then $x \neq 4$.

Worksheet 2B Probability of combined events

1. **(a)**

Spinner 1	Spinner 2
2	7
2	8
2	9
3	7
3	8
3	9
4	7
4	8
4	9

(b) **(i)** P(total is 10) = $\mathbf{\frac{2}{9}}$

(ii) P(difference is 5) = $\frac{3}{9}$

= $\mathbf{\frac{1}{3}}$

2. **(a)**

Left earphone	Right earphone
white	white
white	blue
white	red
blue	white
blue	blue
blue	red
red	white
red	blue
red	red

(b) P(same colour) = $\frac{3}{9}$

= $\mathbf{\frac{1}{3}}$

3. **(a)**

×	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	**9**	**12**	**15**	**18**
4	**4**	**8**	**12**	**16**	**20**	**24**
5	**5**	**10**	**15**	**20**	**25**	**30**
6	**6**	**12**	**18**	**24**	**30**	**36**

(b) **(i)** P(product is 10) = $\frac{2}{36}$

= $\mathbf{\frac{1}{18}}$

(ii) P(product is a prime number) = $\frac{6}{36}$

= $\mathbf{\frac{1}{6}}$

(iii) P(product is less than 15) = $\mathbf{\frac{23}{36}}$

(iv) P(product is a factor of 60) = $\frac{26}{36}$

= $\mathbf{\frac{13}{18}}$

4. **(a)**

	3	4	5	7	8
3	(3, 3)	(3, 4)	(3, 5)	(3, 7)	(3, 8)
4	(4, 3)	(4, 4)	(4, 5)	(4, 7)	(4, 8)
5	(5, 3)	(5, 4)	(5, 5)	(5, 7)	(5, 8)
7	(7, 3)	(7, 4)	(7, 5)	(7, 7)	(7, 8)
8	(8, 3)	(8, 4)	(8, 5)	(8, 7)	(8, 8)

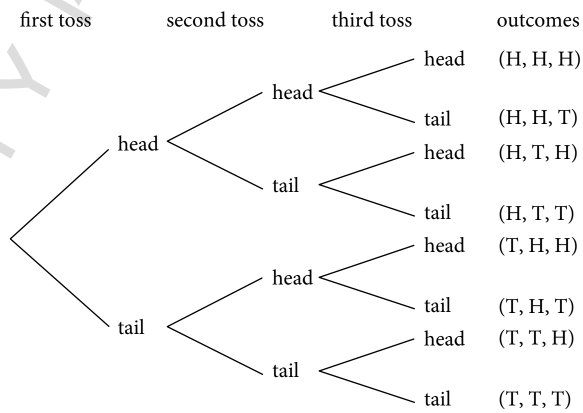
(b) **(i)** P(both counters have numbers less than 5) = $\mathbf{\frac{4}{25}}$

(ii) P(neither counter has an odd number) = $\mathbf{\frac{4}{25}}$

(iii) P(sum is 12) = $\mathbf{\frac{4}{25}}$

(iv) P(product is not more than 20) = $\mathbf{\frac{8}{25}}$

5. **(a)**



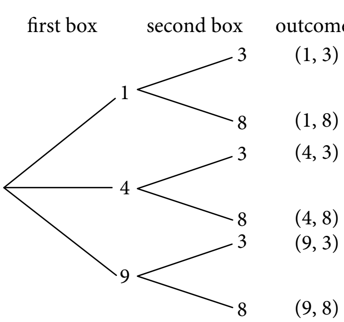
(b) **(i)** P(3 heads) = $\mathbf{\frac{1}{8}}$

(ii) P(exactly 2 heads) = $\mathbf{\frac{3}{8}}$

(iii) P(not more than 1 head) = $\frac{4}{8}$

= $\mathbf{\frac{1}{2}}$

6. **(a)**



(b) **(i)** P(2 prime numbers) = **0**

(ii) P(sum is 12) = $\frac{2}{6}$

$= \mathbf{\frac{1}{3}}$

(iii) P(2 factors of 72) = **1**

(iv) P(exactly 1 perfect square or perfect cube) = $\frac{3}{6}$

$= \mathbf{\frac{1}{2}}$

Worksheet 2C Addition Law of Probability and mutually exclusive events

1. $\mathbf{P(A \cup B) = P(A) + P(B)}$

2. **(a)** Even numbers: 2, 4, 6, 8, 10, 12, 14

Multiples of 3: 3, 6, 9, 12, 15

P(an even number or a multiple of 3) = $\frac{10}{15}$

$= \mathbf{\frac{2}{3}}$

(b) Perfect cubes: 1, 8

Perfect squares: 1, 4, 9

P(not a perfect cube or a perfect square) = $1 - \frac{4}{15}$

$= \mathbf{\frac{11}{15}}$

3. **(a)** P(a '7' or a spade) = $\frac{4+13-1}{52}$

$= \mathbf{\frac{4}{13}}$

(b) P(not an Ace or the King of Diamonds) = $1 - \frac{4}{52} - \frac{1}{52}$

$= \mathbf{\frac{47}{52}}$

4. **(a)** Total probability = 1

Since $\frac{1}{4} + \frac{3}{8} + \frac{5}{24} = \frac{5}{6} < 1$, the total probability of the other students getting selected is $\frac{1}{6}$.

(b) **(i)** P(none of them are selected) = $\mathbf{\frac{1}{6}}$

(ii) P(either Anna or Charles is selected) = $\frac{1}{4} + \frac{3}{8}$

$= \mathbf{\frac{5}{8}}$

(iii) P(neither Anna nor Edwin is selected) = $1 - \left(\frac{1}{4} + \frac{5}{24}\right)$

$= \mathbf{\frac{13}{24}}$

5. **(a)** Let A represent the event that the envelope contains the \$50 voucher, and B represent the event that the envelope contains the \$1000 voucher.

Possible outcomes: ***B*, *AB*, *AAB*, *AAA***

(b) **No.** Q includes the event that exactly three envelopes are opened. Since P and Q can occur at the same time, they are not mutually exclusive events.

Worksheet 2D Multiplication Law of Probability and independent events

1. $\mathbf{P(A \cap B) = P(A) \times P(B)}$

2. **(a)** P(stops on a consonant) = $\frac{4}{6}$

$= \mathbf{\frac{2}{3}}$

(b) P(stops on a consonant both times) = $\frac{4}{6} \times \frac{4}{6}$

$= \mathbf{\frac{4}{9}}$

(c) P(stops on a vowel exactly once) = $\frac{4}{6} \times \frac{2}{6} + \frac{2}{6} \times \frac{4}{6}$

$= \mathbf{\frac{4}{9}}$

3. **(a)** P(completes 2.4 km run within 14 min and 3.2 km run within 25 min) = $\frac{3}{8} \times \frac{5}{6}$

$= \mathbf{\frac{5}{16}}$

(b) P(does not complete 2.4 km run within 14 min or 3.2 km run within 25 min)

$= \frac{5}{8} \times \frac{1}{6}$

$= \mathbf{\frac{5}{48}}$

4. P(cycles on exactly 2 days)

= (0.6)(0.6)(0.4) + (0.6)(0.4)(0.6) + (0.4)(0.6)(0.6)

= **0.432**

5. **(a)** P(Ryan does not wake up on time on 2 consecutive days)

$= \frac{5}{7} \times \frac{5}{7}$

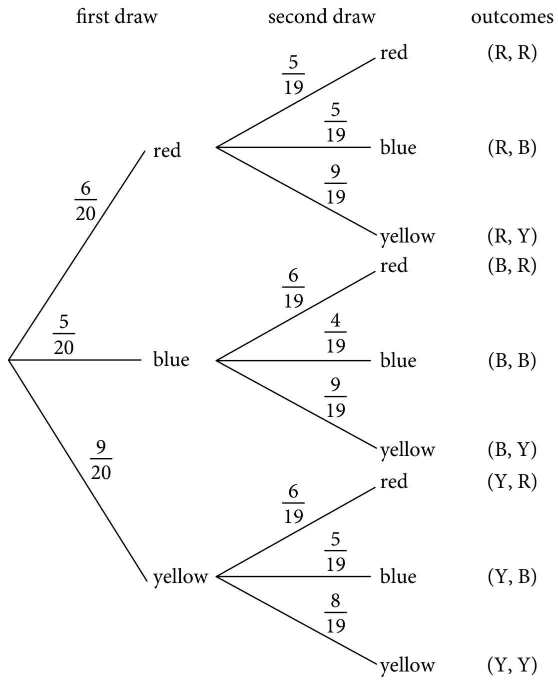
$= \mathbf{\frac{25}{49}}$

(b) P(Ryan forgets to take his vitamins on a particular day)

$= \frac{2}{7} \times \frac{1}{4} + \frac{5}{7} \times \frac{4}{5}$

$= \mathbf{\frac{9}{14}}$

6. **(a)**



(b) **(i)** P(same colour) $= \frac{6}{20}\times\frac{5}{19}+\frac{5}{20}\times\frac{4}{19}+\frac{9}{20}\times\frac{8}{19}$

$= \mathbf{\frac{61}{190}}$

(ii) P(at least 1 card is yellow) $= \frac{6}{20}\times\frac{9}{19}+\frac{5}{20}\times\frac{9}{19}+\frac{9}{20}$

$= \mathbf{\frac{27}{38}}$

(c) P(exactly 2 cards are blue)

= P(R, B, B) + P(B, B, B’) + P(B, B’, B) + P(Y, B, B)

$= \frac{6}{20}\times\frac{5}{19}\times\frac{4}{18}+\frac{5}{20}\times\frac{4}{19}\times\frac{15}{18}+\frac{5}{20}\times\frac{15}{19}\times\frac{4}{18}+\frac{9}{20}\times\frac{5}{19}\times\frac{4}{18}$

$= \mathbf{\frac{5}{38}}$

7. **(a)** **(i)** Total number of students = 40 + 35 + 12 + 3

= 90

P(full-time Typography student) $= \frac{3}{90}$

$= \mathbf{\frac{1}{30}}$

(ii) P(Branding student) $= \frac{40+35}{90}$

$= \mathbf{\frac{5}{6}}$

(b) **(i)** P(both are full-time students) $= \frac{38}{90}\times\frac{37}{89}$

$= \mathbf{\frac{703}{4005}}$

(ii) P(neither is a part-time Branding student) $= \frac{50}{90}\times\frac{49}{89}$

$= \mathbf{\frac{245}{801}}$

(c) P(all are full-time Typography students) $= \frac{3}{90}\times\frac{2}{89}\times\frac{1}{88}$

$= \mathbf{\frac{1}{117\ 480}}$

8. **(a)** $7 + 63 + 29 + m + 0 + 104 + 51 + n = 300$

$m + n = 46$ — (1)

$n - m = 10$ — (2)

(1) + (2): $2n = 56$

$n = 28$

Substitute $n = 28$ into (1):

$m + 28 = 46$

$m = 18$

$\therefore$ $\mathbf{m = 18, n = 28}$

(b) **(i)** P(Premium member aged > 40 years) $= \frac{28}{300}$

$= \mathbf{\frac{7}{75}}$

(ii) P(member aged ⩽ 30 years) $= \frac{7+63+104}{300}$

$= \mathbf{\frac{29}{50}}$

(c) **(i)** P(both are Classic members aged > 30 years)

$= \frac{47}{300}\times\frac{46}{299}$

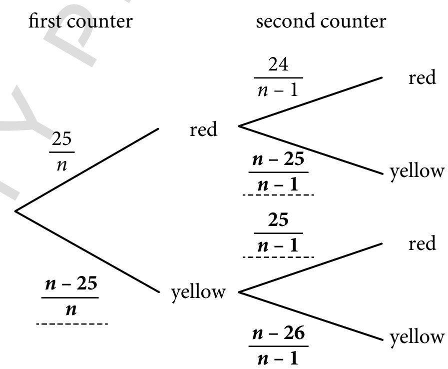
= **0.0241** (to 3 s.f.)

(ii) P(only one is a Premium member)

$= \frac{183}{300}\times\frac{117}{299}+\frac{117}{300}\times\frac{183}{299}$

= **0.477** (to 3 s.f.)

9. **(a)**



(b) $\left(\frac{n-25}{n}\right)\left(\frac{n-26}{n-1}\right) = \frac{7}{52}$

$52(n - 25)(n - 26) = 7n(n - 1)$

$52(n^2 - 51n + 650) = 7n^2 - 7n$

$52n^2 - 2652n + 33\ 800 = 7n^2 - 7n$

$45n^2 - 2645n + 33\ 800 = 0$

$9n^2 - 529n + 6760 = 0$ (shown)

(c) $9n^2 - 529n + 6760 = 0$

$(9n - 169)(n - 40) = 0$

$9n - 169 = 0$ or $n - 40 = 0$

$9n = 169$ $\quad$ $n = 40$

$n = \frac{169}{9}$

$\therefore$ $n = \mathbf{\frac{169}{9}}$ or $n = \mathbf{40}$

(d) Since n has to be a positive integer, we reject $n = \frac{169}{9}$.

(e) Probability $= \frac{25}{40}\times\frac{15}{39}+\frac{15}{40}\times\frac{25}{39}$

$= \mathbf{\frac{25}{52}}$

10. **(a)** Landing in A, B, C, then C would give Corey 18 points. Landing in B, B, B, then C would give Corey 19 points.

(b) **(i)** Area of $A = \pi(5)^2$
$= 25\pi \text{ cm}^2$
Area of $B = \pi(7)^2 - \pi(5)^2$
$= 24\pi \text{ cm}^2$
Total area $= 2 \times \frac{1}{2}(20)(20)\sin 75°$
$= 400 \sin 75° \text{ cm}^2$
Area of $C = (400 \sin 75° - 49\pi) \text{ cm}^2$
P(scores 12 points after 2 throws)
$= P(B, B)$
$= \frac{24\pi}{400\sin 75°} \times \frac{24\pi}{400\sin 75°}$
$= \mathbf{0.0381}$ (to 3 s.f.)

(ii) P(scores at least 10 points after 2 throws)
= 1 – P(scores less than 10 points after 2 throws)
= 1 – [P(lands in B, then C) + P(lands in C, then B) + P(lands in C, then C)]

$$= 1 - \left[\frac{24\pi}{400\sin 75°} \times \frac{400\sin 75° - 49\pi}{400\sin 75°} + \frac{400\sin 75° - 49\pi}{400\sin 75°} \times \frac{24\pi}{400\sin 75°} + \frac{400\sin 75° - 49\pi}{400\sin 75°} \times \frac{400\sin 75° - 49\pi}{400\sin 75°}\right]$$

$= \mathbf{0.403}$ (to 3 s.f.)

Challenge Myself!

11. **(a)** P(eliminated after 1 roll) $= \mathbf{\frac{1}{6}}$

(b) P(lands on "start" after 2 rolls) = **0**

(c) P(eliminated after 2 rolls)
= P(rolls a '1', then a '1') + P(rolls a '3', then a '5') + P(rolls a '4', then a '4') + P(rolls a '5', then a '3') + P(rolls a '5', then a '6')
$= \frac{1}{6} \times \frac{1}{6} \times 5$
$= \frac{5}{36}$ (shown)

(d) P(wins after 3 rolls)
= P(rolls a '1', then a '2', then a '3') + P(rolls a '1', then a '3', then a '2') + P(rolls a '1', then a '4', then a '1')
$= \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} \times 3$
$= \mathbf{\frac{1}{72}}$

Review Exercise 2

1. **(a)**

–	1	2	3	4	5	6
1	0	–1	–2	–3	–4	–5
2	–1	0	–1	–2	–3	–4
3	–2	–1	0	–1	–2	–3
4	–3	–2	–1	0	–1	–2
5	–4	–3	–2	–1	0	–1
6	–5	–4	–3	–2	–1	0

∴ Vera's possible scores are **–5, –4, –3, –2, –1 and 0**.

(b) P(score is less than –3) $= \frac{6}{36}$
$= \mathbf{\frac{1}{6}}$

2. **(i)** P(lands on 7 on both spins) = **0**

(ii) P(lands on 2 on both spins) $= \frac{3}{8} \times \frac{3}{8}$
$= \mathbf{\frac{9}{64}}$

(iii) P(does not land on 4 on either spin) $= \frac{7}{8} \times \frac{7}{8}$
$= \mathbf{\frac{49}{64}}$

(iv) P(lands on 8 on exactly one spin) $= \frac{2}{8} \times \frac{6}{8} \times 2$
$= \mathbf{\frac{3}{8}}$

3. **(a)** Maximum number of latecomers = **4**

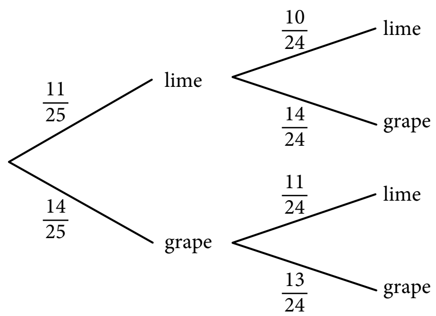
(b) Total number of classes = 11 + 6 + 7 + 5 + 3
= 32
P(no latecomers) $= \mathbf{\frac{11}{32}}$

(c) **(i)** P(both had 1 latecomer) $= \frac{6}{32} \times \frac{5}{31}$
$= \mathbf{\frac{15}{496}}$

(ii) P(one had > 3 latecomers, one had < 3 latecomers)
$= \frac{3}{32} \times \frac{24}{31} \times 2$
$= \mathbf{\frac{9}{62}}$

4. **(i)** Paul incorrectly assumed that the first jelly is replaced, which is not possible.
Paul's incorrect working: $\frac{14}{25} \times \frac{14}{25} = \frac{196}{625}$
The correct working should be: $\frac{14}{25} \times \frac{13}{24} = \frac{91}{300}$

(ii)



(iii) P(one of the jellies is a lime jelly) $= \frac{11}{25} \times \frac{14}{24} + \frac{14}{25} \times \frac{11}{24}$

$= \mathbf{\frac{77}{150}}$

3 Statistical Data Analysis

Worksheet 3A Cumulative frequency table and curve

1. **(a)**

Length (x cm)	$x < 45$	$x < 50$	$x < 55$	$x < 60$
Cumulative frequency	**6**	**21**	**33**	**36**

(b)

Force (y N)	**Cumulative frequency**
$y \leqslant 0.2$	**7**
$y \leqslant 0.4$	**15**
$y \leqslant 0.6$	**34**
$y \leqslant 0.8$	**58**
$y \leqslant 1.0$	**68**

2. **(a)**

Mass (x kg)	$0 < x \leqslant 10$	$10 < x \leqslant 20$	$20 < x \leqslant 30$	$30 < x \leqslant 40$	$40 < x \leqslant 50$
Frequency	5	**6**	**12**	**7**	**12**

(b)

Duration (y h)	**Frequency**
$0 \leqslant y < 3$	16
$3 \leqslant y < 6$	**17**
$6 \leqslant y < 9$	**19**
$9 \leqslant y < 12$	**45**
$12 \leqslant y < 15$	**3**

3. **(a)** **(i)** Estimated number of outlets = **18**

(ii) Estimated probability $= \frac{50-44}{50}$

$= \mathbf{\frac{3}{25}}$

(iii) Estimated percentage $= \frac{40-33}{50} \times 100\%$

$= \mathbf{14\%}$

(b) The new cumulative frequency curve will likely lie on the left side of the original curve, as a discount will probably increase the mass of tea leaves sold.

4. **(a)**

Amount spent (\$$x$)	**Cumulative frequency**
$x < 2$	5
$x < 4$	50
$x < 6$	77
$x < 8$	79
$x < 10$	80

(b) **(i)** Number of students who spend less than \$4 = **50**

(ii) Number of students who spend at least \$6 = 2 + 1

= **3**

(c) **(i)** P(student spends less than \$2) $= \frac{5}{80}$

$= \mathbf{\frac{1}{16}}$

(ii) P(student spends at least \$2, but less than \$8)

$= \frac{45+27+2}{80}$

$= \mathbf{\frac{37}{40}}$

5. **(a)** P(overestimated the height) $= \frac{40-8}{40}$

$= \mathbf{\frac{4}{5}}$

(b) 20% of actual height = 20% × 50 cm

= 10 cm

Number of students who estimated the height to be between 40 cm and 60 cm inclusive = 17 − 3

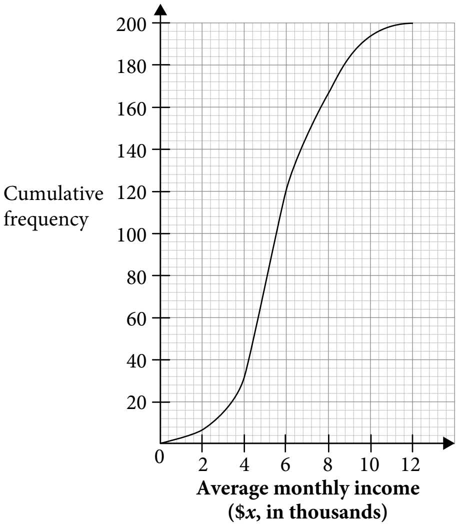
= **14**

(c) OPEN An example of the values is $\boldsymbol{p} = \mathbf{74}$ and $\boldsymbol{q} = \mathbf{84}$.

6. **(a)**

Average monthly income (\$$x$, in thousands)	**Frequency**
$x \leqslant 2$	**7**
$x \leqslant 4$	**32**
$x \leqslant 6$	**122**
$x \leqslant 8$	**165**
$x \leqslant 10$	**194**
$x \leqslant 12$	**200**

(b)



(c) Estimated number of households = **16**

(d) Number of households entitled to receive transport vouchers $= \frac{1}{5} \times 200$
$= 40$
Estimated monthly income limit = **$4200**

Worksheet 3B Median, quartiles, percentiles, range and interquartile range

1. (a) Range = 9 – 3 = **6**
Median = **6**
Lower quartile $= \frac{3+4}{2} = \mathbf{3.5}$
Upper quartile $= \frac{9+9}{2} = \mathbf{9}$
Interquartile range = 9 – 3.5 = **5.5**

(b) Range = 10 – 2 = **8**
Median = **5**
Lower quartile = **5**
Upper quartile = **8**
Interquartile range = 8 – 5 = **3**

(c) Range = 6 – 1 = **5**
Median $= \frac{3+4}{2} = \mathbf{3.5}$
Lower quartile = **2**
Upper quartile = **5**
Interquartile range = 5 – 2 = **3**

(d) Range = 16 – 4 = **12**
Median $= \frac{7+9}{2} = \mathbf{8}$
Lower quartile $= \frac{4+5}{2} = \mathbf{4.5}$
Upper quartile $= \frac{9+11}{2} = \mathbf{10}$
Interquartile range = 10 – 4.5 = **5.5**

(e) 2, 5, 7, 9, 10
Range = 10 – 2 = **8**
Median = **7**
Lower quartile $= \frac{2+5}{2} = \mathbf{3.5}$
Upper quartile $= \frac{9+10}{2} = \mathbf{9.5}$
Interquartile range = 9.5 – 3.5 = **6**

(f) 0, 0.3, 0.6, 0.9, 1.2, 1.5
Range = 1.5 – 0 = **1.5**
Median $= \frac{0.6+0.9}{2} = \mathbf{0.75}$
Lower quartile = **0.3**
Upper quartile = **1.2**
Interquartile range = 1.2 – 0.3 = **0.9**

(g) 2, 3, 7, 8, 14, 19, 26, 50, 50
Range = 50 – 2 = **48**
Median = **14**
Lower quartile $= \frac{3+7}{2} = \mathbf{5}$
Upper quartile $= \frac{26+50}{2} = \mathbf{38}$
Interquartile range = 38 – 5 = **33**

(h) 3, 3, 6, 7, 9, 9, 10, 10, 12, 18, 24, 31
Range = 31 – 3 = **28**
Median $= \frac{9+10}{2} = \mathbf{9.5}$
Lower quartile $= \frac{6+7}{2} = \mathbf{6.5}$
Upper quartile $= \frac{12+18}{2} = \mathbf{15}$
Interquartile range = 15 – 6.5 = **8.5**

(i) –12, –8, –4, –3, 0, 1, 8, 9
Range = 9 – (–12) = **21**
Median $= \frac{-3+0}{2} = \mathbf{-1.5}$
Lower quartile $= \frac{-8+(-4)}{2} = \mathbf{-6}$
Upper quartile $= \frac{1+8}{2} = \mathbf{4.5}$
Interquartile range = 4.5 – (–6) = **10.5**

(j) $\frac{1}{6}, \frac{1}{5}, \frac{3}{10}, \frac{3}{8}, \frac{1}{2}, \frac{3}{4}, \frac{9}{10}$
Range $= \frac{9}{10} - \frac{1}{6} = \mathbf{\frac{11}{15}}$
Median $= \mathbf{\frac{3}{8}}$
Lower quartile $= \mathbf{\frac{1}{5}}$
Upper quartile $= \mathbf{\frac{3}{4}}$
Interquartile range $= \frac{3}{4} - \frac{1}{5} = \mathbf{\frac{11}{20}}$

2. (a) Total frequency = 40
Range = 9 – 6 = **3 subjects**
Median = **8 subjects**
Lower quartile = **7 subjects**
Upper quartile = **8 subjects**
Interquartile range = 8 – 7 = **1 subject**

(b) Total frequency = 173
Range = 8 – 5 = **3 V**
Median = **7 V**
Lower quartile = **5 V**
Upper quartile = **8 V**
Interquartile range = 8 – 5 = **3 V**

3. **(a)** Range = 100 – 0 = **100 minutes**
Median = **40 minutes**
Lower quartile = **30 minutes**
Upper quartile = **52 minutes**
Interquartile range = 52 – 30 = **22 minutes**

(b) Range = 20 – 4 = **$16**
Median = **$9.20**
Lower quartile = **$8.40**
Upper quartile = **$12**
Interquartile range = 12 – 8.4 = **$3.60**

4. **(a)** 20^{th} percentile = **20 °C**
85^{th} percentile = **38 °C**

(b) 20^{th} percentile = **34 mm**
85^{th} percentile = **56 mm**

5. **(a)** Total number of students = **35**

(b) **(i)** Median = **168.5 cm**

(ii) Interquartile range = Upper quartile – lower quartile
= 175 – 163
= **12 cm**

(iii) $\frac{80}{100} \times 35 = 28$
80^{th} percentile = **176.5 cm**

6. **(a)** 1.5 hours = 90 minutes
Number of players unable to complete the task
= 200 – 184
= **16**

(b) Estimated median = **71 minutes**
Half of the players completed the task in less than 71 minutes.

(c) 60^{th} percentile = 73 minutes
30^{th} percentile = 64 minutes
∴ The 60^{th} percentile is not twice the 30^{th} percentile.

(d) Estimated probability = $\frac{12}{200} \times \frac{6}{199} + \frac{6}{200} \times \frac{12}{199}$
$= \mathbf{\frac{18}{4975}}$

(e) Number of players who took 60 to 80 minutes = 160 – 44
= 116

Percentage of players who took 60 to 80 minutes
$= \frac{116}{200} \times 100\%$
= 58%
≈ 60%
∴ I agree with the organiser.

(f) Since the median time taken by the players in the Senior category is lower, the curve representing the players in the Senior category will be on the left side of the curve representing the times taken by those in the Junior category.

7. **(a)** Total number of families = 800 + 800
= **1600**

(b) Estimated interquartile range = 500 – 410
= **90 kWh**

(c) Interquartile range for group A = 375 – 270
= 105 kWh
Since group B has a lower interquartile range than group A, **group B** has a more consistent electricity usage.

(d) Since a family of three is expected to use less electricity than a family of five, the curve corresponding to **group A** is more likely to represent the electricity usage of a family of three.

(e) **(i)** OPEN Possible median = **400 kWh**

(ii) The cumulative frequency curve corresponding to a family of four will likely lie between the two given curves.

8. **(a)** **(i)** Estimated median = **10 days**

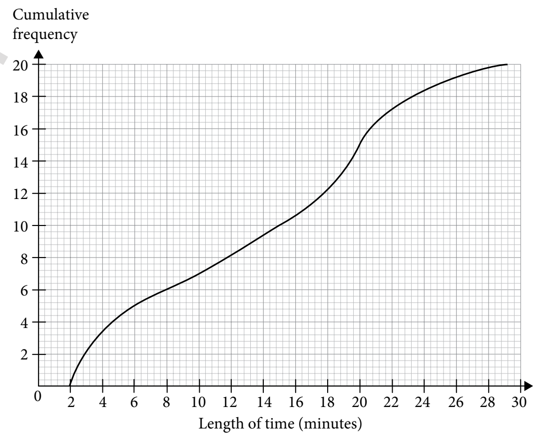
(ii) Estimated interquartile range = 14 – 7
= **7 days**

(b) Estimated value of $k = \frac{60-3}{60} \times 100$
= **95**

(c) OPEN An example is when the number of undelivered packages from a particular shipment is drawn against the number of days elapsed.

Challenge Myself!

9. OPEN **(a)**



(b) **(i)** Estimated probability = $\mathbf{\frac{7}{20}}$

(ii) Estimated 80^{th} percentile = **20.6 minutes**
80% of the customers waited not more than 20.6 minutes for their food.

(c) **(i)** The new cumulative frequency graph will lie on the left side of the original graph, but it might not be a translation of 5 minutes to the left, as customers within the first 20% currently have waiting times of not more than 5 minutes.

(ii) The manager could employ more staff to prepare the food.

1. (a) Mean = $\frac{29 + 26 + 28 + 28 + 24 + 25 + 26}{7}$ min
= **26.6 min** (to 3 s.f.)
Range = 29 min – 24 min
= **5 min**

(b) Mean = $\frac{27 + 28 + 28 + 26 + 22 + 24 + 26}{7}$ min
= **25.9 min** (to 3 s.f.)
Range = 28 min – 22 min
= **6 min**

(c) On average, Caleb takes longer than Tyler to run 5 rounds in the park.
However, there is a greater spread of the time taken by Tyler as compared to the time taken by Caleb.

2. (a) Mean = $\frac{192 + 194 + 196 + \ldots + 211}{10}$ g
= **202 g**
Range = 211 g – 192 g
= **19 g**

(b) On average, the red apples have a slightly greater mass than the green apples.
However, there is a greater spread of the masses of the red apples than the green apples.

3. (a) $p + 4 + 11 + 15 + 12 + q + 2 = 50$
$p + q = 6$ — (1)
$p \times 0 + 4 \times 1 + 11 \times 2 + 15 \times 3 + 12 \times 4 + q \times 5 + 2 \times 6$
$= 2.92 \times 50$
$5q + 131 = 146$
$5q = 15$
$q = 3$
Substitute $q = 3$ into (1):
$p + 3 = 6$
$p = 3$
$\therefore$ **$p = 3$, $q = 3$**

(b) Range = 6 – 0
= **6**

(c) On average, the students completed more Chemistry practice papers than English practice papers.
However, there is a greater spread in the number of English practice papers completed than the number of Chemistry practice papers.

4. (a) **No.** The estimated mean age is 15.1 years. There is insufficient information to conclude whether the mean age is 14 years.

(b) Although the category $16 \leqslant x < 20$ has the most participants, there is insufficient information to conclude whether at least 11 of these participants are 18 years old (if 10 participants are exactly 10 years old), or whether most of the participants in Group 2 are 18 years old.

(c) There is **insufficient information to conclude** whether Tiffy is correct. If the youngest and oldest participants in Group 1 are 9 years old and 19 years old respectively, whereas the youngest and oldest participants in Group 2 are 11 years old and 17 years old respectively, then the range of the ages would be 10 years and 6 years respectively, and Tiffy would not be correct. However, if the youngest and oldest participants in Group 1 are 11 years old and 17 years old respectively, whereas the youngest and oldest participants in Group 2 are 9 years old and 19 years old respectively, then the range of the ages would be 6 years and 10 years respectively, then Tiffy would be correct.

5. (a) (i) Estimated mean volume in Batch 1
$= \frac{0 \times 287.5 + 8 \times 292.5 + 23 \times 297.5 + 47 \times 302.5 + 2 \times 307.5}{80}$ ml
= **300 ml** (to 3 s.f.)
Estimated mean volume in Batch 2
$= \frac{1 \times 287.5 + 4 \times 292.5 + 39 \times 297.5 + 36 \times 302.5 + 0 \times 307.5}{80}$ ml
= **299 ml** (to 3 s.f.)

(ii) Estimated range in Batch 1 = 307.5 ml – 292.5 ml
= **15 ml**
Estimated range in Batch 2 = 302.5 ml – 287.5 ml
= **15 ml**

(iii) Class interval in Batch 1 that contains the median:
$300 \leqslant V < 305$
Class interval in Batch 2 that contains the median:
$295 \leqslant V < 300$

(iv) Probability in Batch 1 = $\mathbf{\frac{47}{80}}$
Probability in Batch 2 = $\frac{36}{80}$
= $\mathbf{\frac{9}{20}}$

(b) **Batch 1** contains more soy sauce as it has a higher mean.

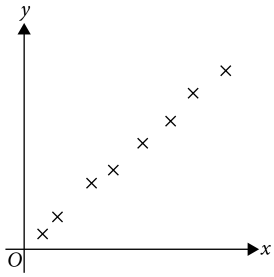
(c) Both batches have the **same consistency** as they have the same range.

(d) OPEN

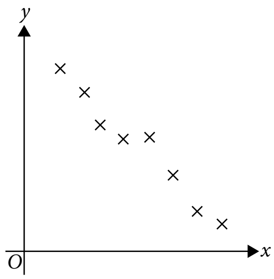
Volume (V millilitres)	Number of bottles in Batch 3
$285 \leqslant V < 290$	0
$290 \leqslant V < 295$	0
$295 \leqslant V < 300$	5
$300 \leqslant V < 305$	70
$305 \leqslant V < 310$	5

1.

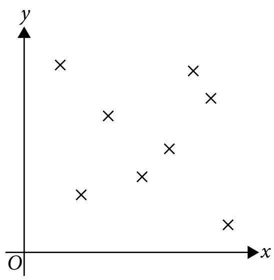
(a)



(b)

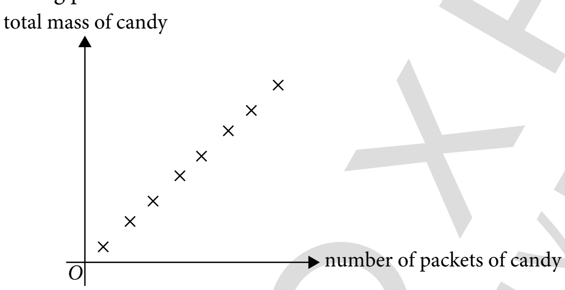


(c)

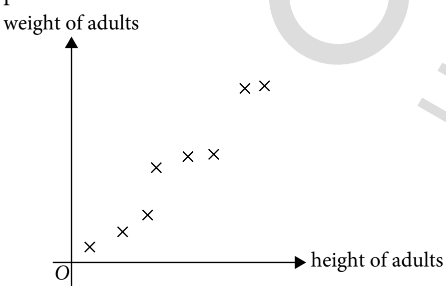


2.

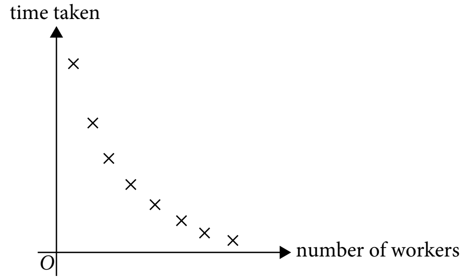
(a) strong positive correlation



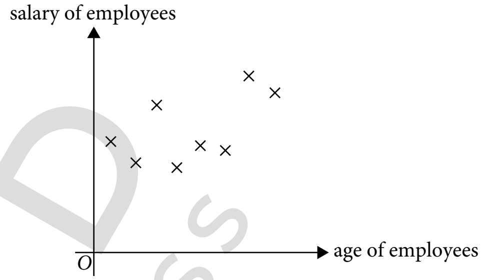
(b) positive correlation



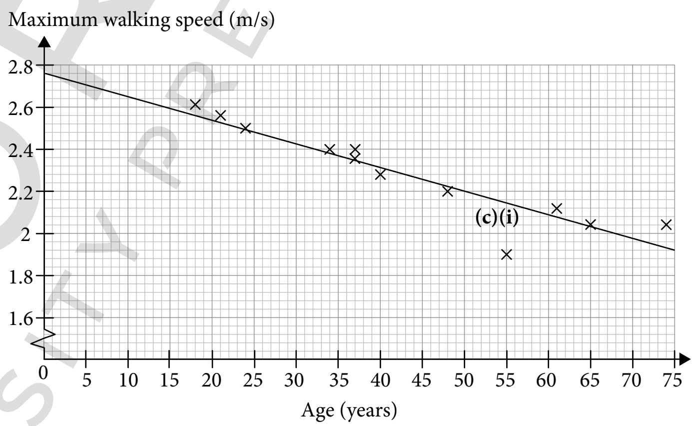
(c) negative correlation



(d) no correlation



3. **(a)**

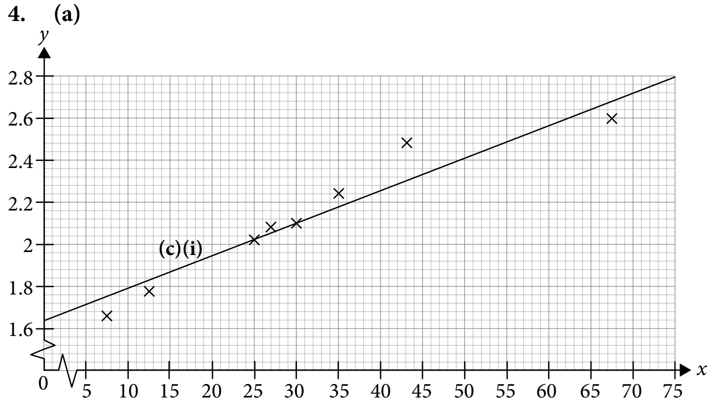


(b) **Negative correlation**

(c) **(ii)** From the line of best fit, estimated walking speed = **2.26 m/s**.

(iii) Although there is one data point which does not lie close to the line of best fit, the line of best fit drawn using the other data points show a strong negative correlation. Hence, this estimate is expected to be reliable.

4. **(a)**



(b) **Positive correlation**

(c) (ii) Gradient = $\frac{31.5 - 22}{13 - 10.8}$
= 4.32 (to 3 s.f.)
Substitute $x = 13$, $y = 31.5$ into $y = 4.32x + c$:
$31.5 = 4.32(13) + c$
$c = -24.7$
Equation of line of best fit: $y = \mathbf{4.32x - 24.7}$
(iii) When $x = 11.9$,
$y = 4.32(11.9) - 24.7$
= 26.7 (to 3 s.f.)
∴ The student takes about **26.7 min**.

(d) **No.** The line of best fit is drawn based on the given range of data, and using it to estimate the time taken by 1 student who runs 5 km in 46.5 min would require a significant extrapolation of the data, which might result in a poor estimate.

Review Exercise 3

1. (a) Mean = $\frac{3 + 4 + 4 + 5 + 8 + 10 + 12 + 12 + 12}{9}$
= **7.78** (to 3 s.f.)
Range = 12 − 3
= **9**
Median = **8**
Lower quartile = **4**
Upper quartile = **12**
Interquartile range = 12 − 4
= **8**

(b) 6, 6, 7, 9, 9, 10, 11, 11
Mean = $\frac{6 + 6 + 7 + 9 + 9 + 10 + 11 + 11}{8}$
= **8.625**
Range = 11 − 6
= **5**
Median = $\frac{9 + 9}{2}$
= **9**
Lower quartile = $\frac{6 + 7}{2}$
= **6.5**
Upper quartile = $\frac{10 + 11}{2}$
= **10.5**
Interquartile range = 10.5 − 6.5
= **4**

2. (a) Mean = $\frac{1 \times 1 + 6 \times 2 + 17 \times 5 + 24 \times 10 + 2 \times 20}{50}$ amperes
= **7.56 amperes**
Range = 20 amperes − 1 ampere
= **19 amperes**

(b) Estimated mean
= $\$\frac{4 \times 10 + 20 \times 30 + 39 \times 50 + 32 \times 70 + 17 \times 90}{112}$
= **\$56.79** (to 2 d.p.)
Estimated range = \$90 − \$10
= **\$80**

3. OPEN
(a) **23, 28, 32, 33, 34**
(b) Range = 34 − 23
= **11**

4. Correct mean = (264 + 10) g
= **274 g**
Correct median = (261 + 10) g
= **271 g**
Correct range = **38 g**

5. (a) $p + 4 + 8 + q + 27 + 12 + 4 = 80$
$p + q = 25$ —— (1)
Mean = $\frac{p(0)+4(1)+8(2)+q(3)+27(4)+12(5)+4(6)}{80}$
$3.55 = \frac{4+16+3q+108+60+24}{80}$
$284 = 3q + 212$
$3q = 72$
$q = 24$
Substitute $q = 24$ into (1):
$p + 24 = 25$
$p = 1$
∴ $\mathbf{p = 1, q = 24}$

(b) Range = 6 − 0
= **6**

(c) On average, the adults ate fewer servings of fruit and vegetables than the children.
However, there is a greater spread in the number of servings of fruit and vegetables eaten by the adults than by the children.

6. (a) Estimated median = **39 defects**
Estimated interquartile range = 46 − 33
= **13 defects**

(b) Number of apartments = 30% × 80 = 24
From the graph, k = **34**.

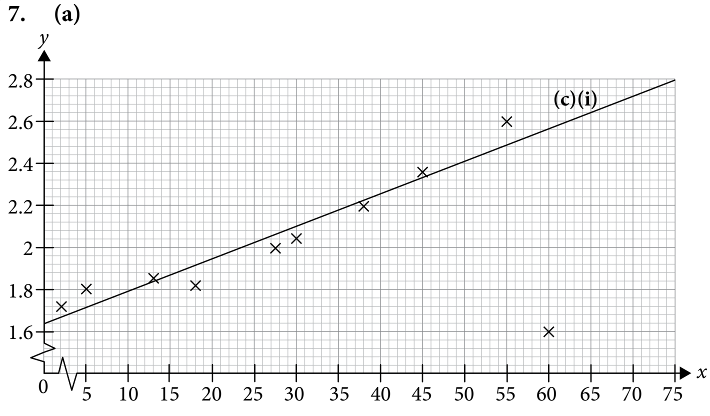
(c)

Number of defects, x	$0 \leqslant x < 20$	$20 \leqslant x < 40$	$40 \leqslant x < 60$	$60 \leqslant x < 80$	$80 \leqslant x < 100$
Frequency	5	40	25	6	4

(d) Required probability = $\frac{4}{80} \times \frac{3}{79}$
= $\mathbf{\frac{3}{1580}}$

(e) The number of defects spotted in Wind Apartments is more consistent than the number spotted in Gold Apartments.

7. (a)



(b) **Positive correlation**

(c) (ii) $\frac{3}{4}$ h = 45 min
From the line of best fit, estimated time taken = **82 min**.
(iii) Although there is one data point which does not lie close to the line of best fit, the line of best fit drawn using the other data points show a strong positive correlation. Hence, this estimate is expected to be reliable.

(d) **No.** Using the line of best fit, the student spends about 79 minutes on social media.

4 Vectors

Worksheet 4A Vectors in two dimensions

1. $\overrightarrow{AB} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

$|\overrightarrow{AB}| = \sqrt{3^2 + 2^2}$
$= \sqrt{13}$
$=$ **3.61 units** (to 3 s.f.)

$\overrightarrow{CD} = \begin{pmatrix} 6 \\ 0 \end{pmatrix}$

$|\overrightarrow{CD}| =$ **6 units**

$\overrightarrow{EF} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$

$|\overrightarrow{EF}| =$ **2 units**

$\overrightarrow{GH} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$

$|\overrightarrow{GH}| = \sqrt{2^2 + (-3)^2}$
$= \sqrt{13}$
$=$ **3.61 units** (to 3 s.f.)

$\mathbf{p} = \begin{pmatrix} -3 \\ -3 \end{pmatrix}$

$|\mathbf{p}| = \sqrt{(-3)^2 + (-3)^2}$
$= \sqrt{18}$
$=$ **4.24 units** (to 3 s.f.)

$\mathbf{q} = \begin{pmatrix} 0 \\ -6 \end{pmatrix}$

$|\mathbf{q}| =$ **6 units**

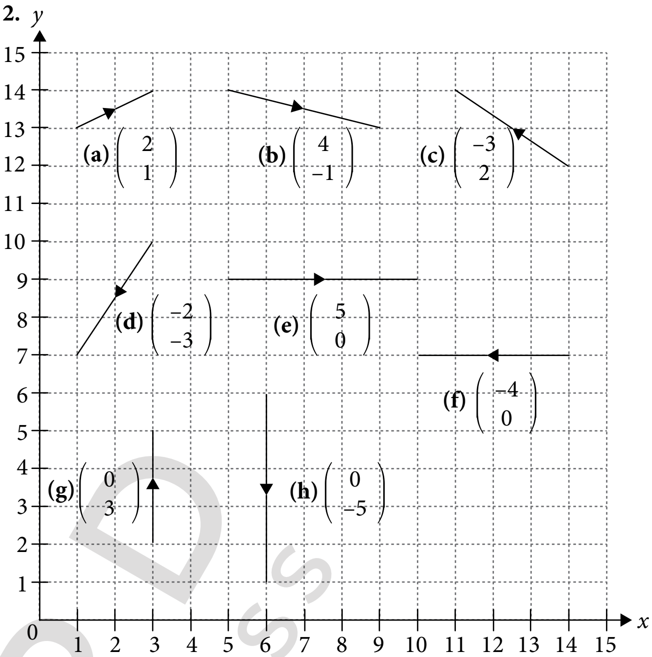
$\mathbf{r} = \begin{pmatrix} -6 \\ 1 \end{pmatrix}$

$|\mathbf{r}| = \sqrt{(-6)^2 + 1^2}$
$= \sqrt{37}$
$=$ **6.08 units** (to 3 s.f.)

$\mathbf{s} = \begin{pmatrix} -2 \\ 0 \end{pmatrix}$

$|\mathbf{s}| =$ **2 units**

2.



3. **(a)** $-\begin{pmatrix} 5 \\ -2 \end{pmatrix} = \begin{pmatrix} \mathbf{-5} \\ \mathbf{2} \end{pmatrix}$

(b) $-\begin{pmatrix} 0 \\ 4 \end{pmatrix} = \begin{pmatrix} \mathbf{0} \\ \mathbf{-4} \end{pmatrix}$

(c) $-\begin{pmatrix} -7.6 \\ 9 \end{pmatrix} = \begin{pmatrix} \mathbf{7.6} \\ \mathbf{-9} \end{pmatrix}$

(d) $-\begin{pmatrix} -8 \\ -1 \end{pmatrix} = \begin{pmatrix} \mathbf{8} \\ \mathbf{1} \end{pmatrix}$

4. **(a)** Magnitude of $\begin{pmatrix} 6 \\ 8 \end{pmatrix} = \sqrt{6^2 + 8^2}$
$= \sqrt{100}$
$=$ **10 units**

(b) Magnitude of $\begin{pmatrix} 10 \\ 0 \end{pmatrix} =$ **10 units**

(c) Magnitude of $\begin{pmatrix} -2.4 \\ 0.7 \end{pmatrix} = \sqrt{(-2.4)^2 + 0.7^2}$
$= \sqrt{6.25}$
$=$ **2.5 units**

(d) Magnitude of $\begin{pmatrix} -3.5 \\ -3.5 \end{pmatrix} = \sqrt{(-3.5)^2 + (-3.5)^2}$
$= \sqrt{24.5}$
$=$ **4.95 units** (to 3 s.f.)

5. **(a)** Since **p** = **q**,

$\begin{pmatrix} 4x \\ 7 \end{pmatrix} = \begin{pmatrix} 11 - y \\ x + y \end{pmatrix}$

$4x = 11 - y$
$4x + y = 11$ —— (1)
$7 = x + y$
$x + y = 7$ —— (2)
(1) − (2): $3x = 4$
$x = \frac{4}{3}$

Substitute $x = \frac{4}{3}$ into (2):

$$\frac{4}{3} + y = 7$$

$$y = \frac{17}{3}$$

$\therefore$ $\boldsymbol{x = 1\frac{1}{3}}$, $\boldsymbol{y = 5\frac{2}{3}}$

(b) $|\mathbf{q}| = |\mathbf{p}|$

$$= \sqrt{\left[4\left(\frac{4}{3}\right)\right]^2 + 7^2}$$

$$= \sqrt{\frac{697}{9}}$$

= **8.80 units** (to 3 s.f.)

6. Since $|\mathbf{r}| = 50$,

$$\sqrt{a^2 + (2b)^2} = 50$$

$$a^2 + 4b^2 = 2500$$

Let $a = 30$:

$$30^2 + 4b^2 = 2500$$

$$4b^2 = 1600$$

$$b^2 = 400$$

$$b = \pm 20$$

$\therefore$ An example is $\boldsymbol{a = 30, b = 20}$.

Worksheet 4B Addition of vectors

1. **(a)** $\begin{pmatrix} 3 \\ 1 \end{pmatrix} + \begin{pmatrix} 2 \\ 8 \end{pmatrix} = \begin{pmatrix} \mathbf{5} \\ \mathbf{9} \end{pmatrix}$

(b) $\begin{pmatrix} 4 \\ -7 \end{pmatrix} + \begin{pmatrix} -5 \\ 9 \end{pmatrix} = \begin{pmatrix} \mathbf{-1} \\ \mathbf{2} \end{pmatrix}$

(c) $\begin{pmatrix} 2.4 \\ -1.6 \end{pmatrix} + \begin{pmatrix} -2.4 \\ 1.6 \end{pmatrix} = \begin{pmatrix} \mathbf{0} \\ \mathbf{0} \end{pmatrix}$

(d) $\begin{pmatrix} 0 \\ 6 \end{pmatrix} + \begin{pmatrix} 2.7 \\ -4.5 \end{pmatrix} + \begin{pmatrix} -3.6 \\ 0 \end{pmatrix} = \begin{pmatrix} \mathbf{-0.9} \\ \mathbf{1.5} \end{pmatrix}$

2. **(a)** $\mathbf{a} + \mathbf{b} = \begin{pmatrix} -4 \\ 2 \end{pmatrix} + \begin{pmatrix} 8 \\ -5 \end{pmatrix}$

$$= \begin{pmatrix} \mathbf{4} \\ \mathbf{-3} \end{pmatrix}$$

(b) $\mathbf{b} + \mathbf{a} = \begin{pmatrix} 8 \\ -5 \end{pmatrix} + \begin{pmatrix} -4 \\ 2 \end{pmatrix}$

$$= \begin{pmatrix} \mathbf{4} \\ \mathbf{-3} \end{pmatrix}$$

(c) $\mathbf{b} + \mathbf{c} = \begin{pmatrix} 8 \\ -5 \end{pmatrix} + \begin{pmatrix} 7 \\ 6 \end{pmatrix}$

$$= \begin{pmatrix} \mathbf{15} \\ \mathbf{1} \end{pmatrix}$$

(d) $\mathbf{c} + \mathbf{b} = \begin{pmatrix} 7 \\ 6 \end{pmatrix} + \begin{pmatrix} 8 \\ -5 \end{pmatrix}$

$$= \begin{pmatrix} \mathbf{15} \\ \mathbf{1} \end{pmatrix}$$

(e) $(\mathbf{a} + \mathbf{b}) + \mathbf{c} = \begin{pmatrix} 4 \\ -3 \end{pmatrix} + \begin{pmatrix} 7 \\ 6 \end{pmatrix}$

$$= \begin{pmatrix} \mathbf{11} \\ \mathbf{3} \end{pmatrix}$$

(f) $\mathbf{a} + (\mathbf{b} + \mathbf{c}) = \begin{pmatrix} -4 \\ 2 \end{pmatrix} + \begin{pmatrix} 15 \\ 1 \end{pmatrix}$

$$= \begin{pmatrix} \mathbf{11} \\ \mathbf{3} \end{pmatrix}$$

3. **(a)** $\mathbf{p} + \mathbf{r} = \begin{pmatrix} 1 \\ -6 \end{pmatrix} + \begin{pmatrix} 2 \\ -4 \end{pmatrix}$

$$= \begin{pmatrix} 3 \\ -10 \end{pmatrix}$$

$$|\mathbf{p} + \mathbf{r}| = \sqrt{3^2 + (-10)^2}$$

$$= \sqrt{109}$$

= **10.4** (to 3 s.f.)

(b) $\mathbf{r} + \mathbf{q} + \mathbf{p} = \begin{pmatrix} 2 \\ -4 \end{pmatrix} + \begin{pmatrix} -3 \\ 7 \end{pmatrix} + \begin{pmatrix} 1 \\ -6 \end{pmatrix}$

$$= \begin{pmatrix} 0 \\ -3 \end{pmatrix}$$

$\boldsymbol{|\mathbf{r} + \mathbf{q} + \mathbf{p}| = 3}$

4. **(a)** $\begin{pmatrix} h \\ 6 \end{pmatrix} + \begin{pmatrix} -4 \\ 3k \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$$h - 4 = 0$$

$$h = 4$$

$$6 + 3k = 0$$

$$3k = -6$$

$$k = -2$$

$\therefore$ $\boldsymbol{h = 4, k = -2}$

(b) $\begin{pmatrix} 9 \\ -k \end{pmatrix} + \begin{pmatrix} 2h \\ 5 \end{pmatrix} = \mathbf{0}$

$$9 + 2h = 0$$

$$2h = -9$$

$$h = -4.5$$

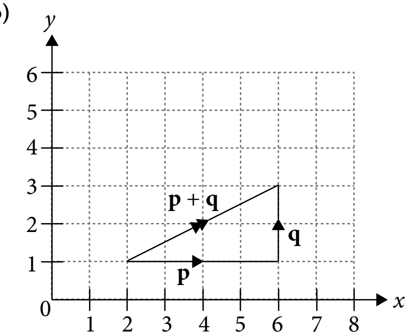
$$-k + 5 = 0$$

$$k = 5$$

$\therefore$ $\boldsymbol{h = -4.5, k = 5}$

5. **(a)** $\mathbf{p} = \begin{pmatrix} \mathbf{4} \\ \mathbf{0} \end{pmatrix}$, $\mathbf{q} = \begin{pmatrix} \mathbf{0} \\ \mathbf{2} \end{pmatrix}$

(b)



(c) $\mathbf{p} + \mathbf{q} = \begin{pmatrix} \mathbf{4} \\ \mathbf{2} \end{pmatrix}$

(d) Since $|\mathbf{p}| = 4$, $|\mathbf{q}| = 2$ and $|\mathbf{p} + \mathbf{q}| = \sqrt{4^2 + 2^2} = \sqrt{20}$,

$|\mathbf{p}| + |\mathbf{q}|$

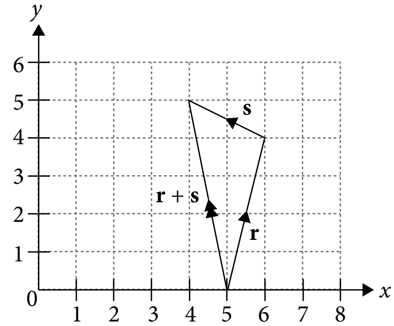
$= 4 + 2$

$= 6 \neq \sqrt{20}$

$\therefore$ $|\mathbf{p}| + |\mathbf{q}| \neq |\mathbf{p} + \mathbf{q}|$ (shown)

6. **(a)** $\mathbf{r} = \begin{pmatrix} \mathbf{1} \\ \mathbf{4} \end{pmatrix}$, $\mathbf{s} = \begin{pmatrix} \mathbf{-2} \\ \mathbf{1} \end{pmatrix}$

(b)



(c) $\mathbf{r}+\mathbf{s}=\begin{pmatrix}\mathbf{-1}\\ \mathbf{5}\end{pmatrix}$

(d) $|\mathbf{r}|=\sqrt{1^2+4^2}=\sqrt{17}$

$|\mathbf{s}|=\sqrt{(-2)^2+1^2}=\sqrt{5}$

$|\mathbf{r}|+|\mathbf{s}|=\sqrt{17}+\sqrt{5}$

$= 6.36$ (to 3 s.f.)

$|\mathbf{r}+\mathbf{s}|=\sqrt{(-1)^2+5^2}$

$=\sqrt{26}$

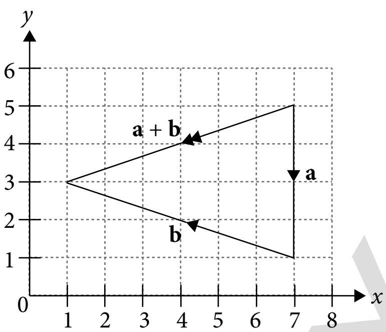
$= 5.10$ (to 3 s.f.)

$\therefore |\mathbf{r}|+|\mathbf{s}|>|\mathbf{r}+\mathbf{s}|$ (shown)

7. (a) **True**
(b) **True**
(c) **False**

8. OPEN

(a)



(b) $\begin{pmatrix}\mathbf{0}\\ \mathbf{-4}\end{pmatrix}+\begin{pmatrix}\mathbf{-6}\\ \mathbf{2}\end{pmatrix}=\begin{pmatrix}\mathbf{-6}\\ \mathbf{-2}\end{pmatrix}$

9. (a) $\overrightarrow{PS}+\overrightarrow{SR}=\overrightarrow{\mathbf{PR}}$

(b) $\overrightarrow{XS}+\overrightarrow{QX}=\overrightarrow{QX}+\overrightarrow{XS}$

$=\overrightarrow{\mathbf{QS}}$

(c) **Cannot be simplified.**

(d) $\overrightarrow{RS}+\overrightarrow{PX}+\overrightarrow{SP}=\overrightarrow{RS}+\overrightarrow{SP}+\overrightarrow{PX}$

$=\overrightarrow{\mathbf{RX}}$

Worksheet 4C Vector subtraction

1. (a) $\begin{pmatrix}7\\6\end{pmatrix}-\begin{pmatrix}1\\4\end{pmatrix}=\begin{pmatrix}\mathbf{6}\\ \mathbf{2}\end{pmatrix}$

(b) $\begin{pmatrix}9\\-2\end{pmatrix}-\begin{pmatrix}0\\-8\end{pmatrix}=\begin{pmatrix}\mathbf{9}\\ \mathbf{6}\end{pmatrix}$

(c) $\begin{pmatrix}-3.5\\5.3\end{pmatrix}-\begin{pmatrix}3.5\\5.3\end{pmatrix}=\begin{pmatrix}\mathbf{-7}\\ \mathbf{0}\end{pmatrix}$

(d) $\begin{pmatrix}0.6\\4.9\end{pmatrix}-\begin{pmatrix}5.4\\1.7\end{pmatrix}-\begin{pmatrix}-4.8\\3.2\end{pmatrix}=\begin{pmatrix}\mathbf{0}\\ \mathbf{0}\end{pmatrix}$

2. (a) $\mathbf{a}-\mathbf{b}=\begin{pmatrix}5\\4\end{pmatrix}-\begin{pmatrix}1\\-6\end{pmatrix}$

$=\begin{pmatrix}\mathbf{4}\\ \mathbf{10}\end{pmatrix}$

(b) $\mathbf{b}-\mathbf{a}=\begin{pmatrix}1\\-6\end{pmatrix}-\begin{pmatrix}5\\4\end{pmatrix}$

$=\begin{pmatrix}\mathbf{-4}\\ \mathbf{-10}\end{pmatrix}$

(c) $\mathbf{b}-\mathbf{c}=\begin{pmatrix}1\\-6\end{pmatrix}-\begin{pmatrix}2\\-9\end{pmatrix}$

$=\begin{pmatrix}\mathbf{-1}\\ \mathbf{3}\end{pmatrix}$

(d) $\mathbf{c}-\mathbf{b}=\begin{pmatrix}2\\-9\end{pmatrix}-\begin{pmatrix}1\\-6\end{pmatrix}$

$=\begin{pmatrix}\mathbf{1}\\ \mathbf{-3}\end{pmatrix}$

(e) $(\mathbf{a}-\mathbf{b})-\mathbf{c}=\begin{pmatrix}4\\10\end{pmatrix}-\begin{pmatrix}2\\-9\end{pmatrix}$

$=\begin{pmatrix}\mathbf{2}\\ \mathbf{19}\end{pmatrix}$

(f) $\mathbf{a}-(\mathbf{b}-\mathbf{c})=\begin{pmatrix}5\\4\end{pmatrix}-\begin{pmatrix}-1\\3\end{pmatrix}$

$=\begin{pmatrix}\mathbf{6}\\ \mathbf{1}\end{pmatrix}$

3. (a) $\mathbf{p}-\mathbf{r}=\begin{pmatrix}-3\\8\end{pmatrix}-\begin{pmatrix}4\\0\end{pmatrix}$

$=\begin{pmatrix}-7\\8\end{pmatrix}$

$|\mathbf{p}-\mathbf{r}|=\sqrt{(-7)^2+8^2}$

$=\sqrt{113}$

$=$ **10.6** (to 3 s.f.)

(b) $\mathbf{r}-\mathbf{p}=\begin{pmatrix}4\\0\end{pmatrix}-\begin{pmatrix}-3\\8\end{pmatrix}$

$=\begin{pmatrix}7\\-8\end{pmatrix}$

$|\mathbf{r}-\mathbf{p}|=\sqrt{7^2+(-8)^2}$

$=\sqrt{113}$

$=$ **10.6** (to 3 s.f.)

(c) $\mathbf{q}-\mathbf{r}=\begin{pmatrix}0\\3\end{pmatrix}-\begin{pmatrix}4\\0\end{pmatrix}$

$=\begin{pmatrix}-4\\3\end{pmatrix}$

$|\mathbf{q}-\mathbf{r}|=\sqrt{(-4)^2+3^2}$

$=\sqrt{25}$

$=$ **5**

(d) $\mathbf{r}-\mathbf{q}=\begin{pmatrix}4\\0\end{pmatrix}-\begin{pmatrix}0\\3\end{pmatrix}$

$=\begin{pmatrix}4\\-3\end{pmatrix}$

$|\mathbf{r} - \mathbf{q}| = \sqrt{4^2 + (-3)^2}$
$= \sqrt{25}$
$= \mathbf{5}$

4. **(a)** $\begin{pmatrix} 7h \\ 1 \end{pmatrix} - \begin{pmatrix} 5 \\ 2k \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$7h - 5 = 0$
$7h = 5$
$h = \frac{5}{7}$

$1 - 2k = 0$
$2k = 1$
$k = \frac{1}{2}$

$\therefore \boldsymbol{h = \frac{5}{7}, k = \frac{1}{2}}$

(b) $\begin{pmatrix} 10 \\ 4 - k \end{pmatrix} - \begin{pmatrix} 8h \\ 3 \end{pmatrix} = \mathbf{0}$

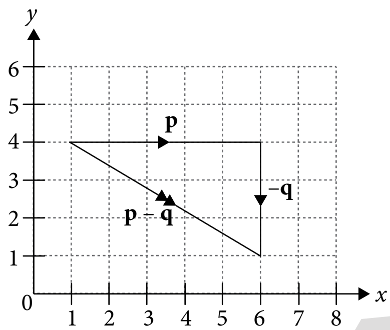
$10 - 8h = 0$
$8h = 10$
$h = 1.25$

$4 - k - 3 = 0$
$k = 1$

$\therefore \boldsymbol{h = 1.25, k = 1}$

5. **(a)** $\mathbf{p} = \begin{pmatrix} \mathbf{5} \\ \mathbf{0} \end{pmatrix}, \mathbf{q} = \begin{pmatrix} \mathbf{0} \\ \mathbf{3} \end{pmatrix}$

(b) $\mathbf{p} - \mathbf{q} = \mathbf{p} + (-\mathbf{q})$

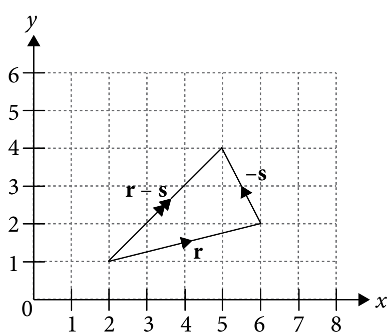


(c) $\mathbf{p} - \mathbf{q} = \begin{pmatrix} \mathbf{5} \\ \mathbf{-3} \end{pmatrix}$

(d) $|\mathbf{p} - \mathbf{q}| = \sqrt{5^2 + (-3)^2}$
$= \sqrt{34}$
$= \mathbf{5.83}$ (to 3 s.f.)

6. **(a)** $\mathbf{r} = \begin{pmatrix} \mathbf{4} \\ \mathbf{1} \end{pmatrix}, \mathbf{s} = \begin{pmatrix} \mathbf{1} \\ \mathbf{-2} \end{pmatrix}$

(b) $\mathbf{r} - \mathbf{s} = \mathbf{r} + (-\mathbf{s})$



(c) $\mathbf{r} - \mathbf{s} = \begin{pmatrix} \mathbf{3} \\ \mathbf{3} \end{pmatrix}$

(d) $|\mathbf{r}| = \sqrt{4^2 + 1^2} = \sqrt{17}$

$|\mathbf{s}| = \sqrt{1^2 + (-2)^2} = \sqrt{5}$

$|\mathbf{r}| - |\mathbf{s}| = \sqrt{17} - \sqrt{5}$
$= 1.89$ (to 3 s.f.)

$|\mathbf{r} - \mathbf{s}| = \sqrt{3^2 + 3^2}$
$= \sqrt{18}$
$= 4.24$ (to 3 s.f.)

$\therefore |\mathbf{r}| - |\mathbf{s}| \neq |\mathbf{r} - \mathbf{s}|$ (shown)

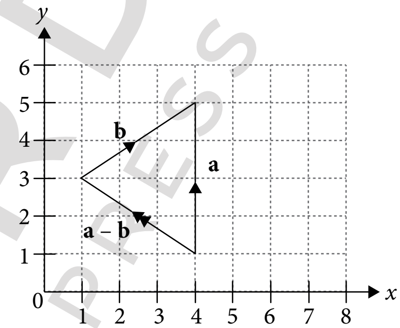
7. **(a)** **True**

(b) **False**

(c) **Insufficient information to conclude**

8. OPEN

(a) $\mathbf{a} - \mathbf{b} = \mathbf{a} + (-\mathbf{b})$



(b) $\begin{pmatrix} \mathbf{0} \\ \mathbf{4} \end{pmatrix} - \begin{pmatrix} \mathbf{3} \\ \mathbf{2} \end{pmatrix} = \begin{pmatrix} \mathbf{-3} \\ \mathbf{2} \end{pmatrix}$

Challenge Myself!

9. **(a)** $\overrightarrow{PX} - \overrightarrow{SX} = \overrightarrow{PX} + \overrightarrow{XS}$
$= \boldsymbol{\overrightarrow{PS}}$

(b) OPEN $\overrightarrow{QR} - \overrightarrow{UX} = \overrightarrow{QR} + \overrightarrow{XU}$
$= \overrightarrow{QR} + \overrightarrow{RX}$
$= \boldsymbol{\overrightarrow{QX}}$

(c) OPEN $\overrightarrow{UP} - \overrightarrow{TS} + \overrightarrow{PQ} = \overrightarrow{UP} + \overrightarrow{ST} + \overrightarrow{PQ}$
$= \boldsymbol{\overrightarrow{UP}}$

(d) OPEN $\overrightarrow{RX} - \overrightarrow{UP} - \overrightarrow{RT} = \overrightarrow{RX} + \overrightarrow{PU} + \overrightarrow{TR}$
$= \overrightarrow{TR} + \overrightarrow{RX} + \overrightarrow{PU}$
$= \overrightarrow{TX} + \overrightarrow{PU}$
$= \mathbf{0}$

Worksheet 4D Scalar multiples of a vector

1. **(a)** $4\begin{pmatrix} 1 \\ -2 \end{pmatrix} + \begin{pmatrix} 5 \\ -1 \end{pmatrix} = \begin{pmatrix} 4 \\ -8 \end{pmatrix} + \begin{pmatrix} 5 \\ -1 \end{pmatrix}$
$= \begin{pmatrix} \mathbf{9} \\ \mathbf{-9} \end{pmatrix}$

(b) $\begin{pmatrix} 3 \\ -9 \end{pmatrix} - 2\begin{pmatrix} -6 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ -9 \end{pmatrix} - \begin{pmatrix} -12 \\ 0 \end{pmatrix}$
$= \begin{pmatrix} \mathbf{15} \\ \mathbf{-9} \end{pmatrix}$

(c) $7\begin{pmatrix}0\\8\end{pmatrix}+3\begin{pmatrix}12\\-10\end{pmatrix}=\begin{pmatrix}0\\56\end{pmatrix}+\begin{pmatrix}36\\-30\end{pmatrix}$

$=\begin{pmatrix}\mathbf{36}\\\mathbf{26}\end{pmatrix}$

(d) $6\begin{pmatrix}4\\1\end{pmatrix}-0.5\begin{pmatrix}-8\\8\end{pmatrix}=\begin{pmatrix}24\\6\end{pmatrix}-\begin{pmatrix}-4\\4\end{pmatrix}$

$=\begin{pmatrix}\mathbf{28}\\\mathbf{2}\end{pmatrix}$

2. **(a)** $3\mathbf{a}+2\mathbf{b}=3\begin{pmatrix}3\\8\end{pmatrix}+2\begin{pmatrix}-4\\5\end{pmatrix}$

$=\begin{pmatrix}9\\24\end{pmatrix}+\begin{pmatrix}-8\\10\end{pmatrix}$

$=\begin{pmatrix}\mathbf{1}\\\mathbf{34}\end{pmatrix}$

(b) $4\mathbf{b}-5\mathbf{a}=4\begin{pmatrix}-4\\5\end{pmatrix}-5\begin{pmatrix}3\\8\end{pmatrix}$

$=\begin{pmatrix}-16\\20\end{pmatrix}-\begin{pmatrix}15\\40\end{pmatrix}$

$=\begin{pmatrix}\mathbf{-31}\\\mathbf{-20}\end{pmatrix}$

(c) $7\mathbf{b}+9\mathbf{c}=7\begin{pmatrix}-4\\5\end{pmatrix}+9\begin{pmatrix}7\\1\end{pmatrix}$

$=\begin{pmatrix}-28\\35\end{pmatrix}+\begin{pmatrix}63\\9\end{pmatrix}$

$=\begin{pmatrix}\mathbf{35}\\\mathbf{44}\end{pmatrix}$

(d) $6\mathbf{c}-2.5\mathbf{a}=6\begin{pmatrix}7\\1\end{pmatrix}-2.5\begin{pmatrix}3\\8\end{pmatrix}$

$=\begin{pmatrix}42\\6\end{pmatrix}-\begin{pmatrix}7.5\\20\end{pmatrix}$

$=\begin{pmatrix}\mathbf{34.5}\\\mathbf{-14}\end{pmatrix}$

(e) $8\mathbf{a}+\mathbf{b}-4\mathbf{c}=8\begin{pmatrix}3\\8\end{pmatrix}+\begin{pmatrix}-4\\5\end{pmatrix}-4\begin{pmatrix}7\\1\end{pmatrix}$

$=\begin{pmatrix}24\\64\end{pmatrix}+\begin{pmatrix}-4\\5\end{pmatrix}-\begin{pmatrix}28\\4\end{pmatrix}$

$=\begin{pmatrix}20\\69\end{pmatrix}-\begin{pmatrix}28\\4\end{pmatrix}$

$=\begin{pmatrix}\mathbf{-8}\\\mathbf{65}\end{pmatrix}$

(f) $7\mathbf{a}-2(\mathbf{b}+\mathbf{c})=7\begin{pmatrix}3\\8\end{pmatrix}-2\left[\begin{pmatrix}-4\\5\end{pmatrix}+\begin{pmatrix}7\\1\end{pmatrix}\right]$

$=\begin{pmatrix}21\\56\end{pmatrix}-2\begin{pmatrix}3\\6\end{pmatrix}$

$=\begin{pmatrix}21\\56\end{pmatrix}-\begin{pmatrix}6\\12\end{pmatrix}$

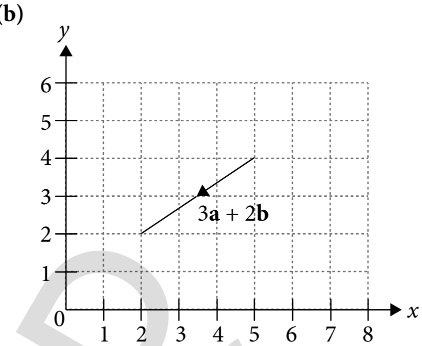
$=\begin{pmatrix}\mathbf{15}\\\mathbf{44}\end{pmatrix}$

3. **(a)** $3\mathbf{a}+2\mathbf{b}=3\begin{pmatrix}1\\-2\end{pmatrix}+2\begin{pmatrix}-3\\2\end{pmatrix}$

$=\begin{pmatrix}3\\-6\end{pmatrix}+\begin{pmatrix}-6\\4\end{pmatrix}$

$=\begin{pmatrix}\mathbf{-3}\\\mathbf{-2}\end{pmatrix}$

(b)



4. **(a)** $\begin{pmatrix}9\\6\end{pmatrix}=3\begin{pmatrix}3\\2\end{pmatrix}$

Since $\begin{pmatrix}9\\6\end{pmatrix}=3\begin{pmatrix}3\\2\end{pmatrix}$, then they are **parallel**, and in **the same direction**.

(b) $\begin{pmatrix}-2\\8\end{pmatrix}=2\begin{pmatrix}-1\\4\end{pmatrix}$

Since $\begin{pmatrix}-2\\8\end{pmatrix}\neq k\begin{pmatrix}1\\4\end{pmatrix}$, where k is a non-zero constant, then they are **not parallel**.

(c) $\begin{pmatrix}5\\0\end{pmatrix}=-5\begin{pmatrix}1\\0\end{pmatrix}$

Since $\begin{pmatrix}5\\0\end{pmatrix}=-5\begin{pmatrix}1\\0\end{pmatrix}$, then they are **parallel**, and **in opposite directions**.

(d) $\begin{pmatrix}-2.7\\-3.6\end{pmatrix}=-0.9\begin{pmatrix}3\\4\end{pmatrix}$

$\begin{pmatrix}3.6\\4.8\end{pmatrix}=1.2\begin{pmatrix}3\\4\end{pmatrix}$

Since $\begin{pmatrix}3.6\\4.8\end{pmatrix}=-\frac{4}{3}\begin{pmatrix}-2.7\\-3.6\end{pmatrix}$, then they are **parallel**, and **in opposite directions**.

5. **(a)** $|\mathbf{p}|=\sqrt{3^2+(-4)^2}$

= 5 units

(b) OPEN **(i)** An example is $\mathbf{q}=2\begin{pmatrix}3\\-4\end{pmatrix}=\begin{pmatrix}\mathbf{6}\\\mathbf{-8}\end{pmatrix}$.

(ii) $|\mathbf{q}|=\sqrt{6^2+(-8)^2}$

$=\sqrt{100}$

= 10 units

(c) OPEN **(i)** An example is $\mathbf{r}=-3\begin{pmatrix}3\\-4\end{pmatrix}=\begin{pmatrix}\mathbf{-9}\\\mathbf{12}\end{pmatrix}$.

(ii) $|\mathbf{r}|=\sqrt{(-9)^2+12^2}$

$=\sqrt{225}$

= 15 units

(d) $|a\mathbf{p}|=$ **$|5a|$ units**

6. Let $\begin{pmatrix} 8 \\ 15 \end{pmatrix} = a\begin{pmatrix} 12 \\ k \end{pmatrix}$.

$8 = 12a$

$a = \frac{2}{3}$

$15 = ak$

$15 = \frac{2}{3}k$

$k = 22.5$

$\therefore$ k = **22.5**

7. **(a)** Since $\begin{pmatrix} 3 \\ h \end{pmatrix}$ and $\begin{pmatrix} k \\ 10 \end{pmatrix}$ are parallel,

$\frac{h}{3} = \frac{10}{k}$

$\boldsymbol{hk = 30}$

(b) OPEN An example of the values is $\boldsymbol{h = 6}$ and $\boldsymbol{k = 5}$.

8. **(a)** $\mathbf{b} = k\mathbf{a} = k\begin{pmatrix} 5 \\ -12 \end{pmatrix} = \begin{pmatrix} 5k \\ -12k \end{pmatrix}$

Since $|\mathbf{b}| = 65$,

$\sqrt{(5k)^2 + (-12k)^2} = 65$

$25k^2 + 144k^2 = 4225$

$169k^2 = 4225$

$k^2 = 25$

$k = \pm 5$

$\therefore k = \boldsymbol{\pm 5}$

(b) When $k = 5$, $\mathbf{b} = \begin{pmatrix} \mathbf{25} \\ \mathbf{-60} \end{pmatrix}$.

When $k = -5$, $\mathbf{b} = \begin{pmatrix} \mathbf{-25} \\ \mathbf{60} \end{pmatrix}$.

9. **(a)** **(i)** $3\mathbf{p} - \mathbf{q} = 3\begin{pmatrix} 9 \\ 14 \end{pmatrix} - \begin{pmatrix} 7 \\ 17 \end{pmatrix}$

$= \begin{pmatrix} \mathbf{20} \\ \mathbf{25} \end{pmatrix}$

(ii) $\mathbf{q} + \frac{1}{2}\mathbf{r} = \begin{pmatrix} 7 \\ 17 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 18 \\ 6 \end{pmatrix}$

$= \begin{pmatrix} \mathbf{16} \\ \mathbf{20} \end{pmatrix}$

(b) $3\mathbf{p} - \mathbf{q} = \begin{pmatrix} 20 \\ 25 \end{pmatrix} = 5\begin{pmatrix} 4 \\ 5 \end{pmatrix}$

$\mathbf{q} + \frac{1}{2}\mathbf{r} = \begin{pmatrix} 16 \\ 20 \end{pmatrix} = 4\begin{pmatrix} 4 \\ 5 \end{pmatrix}$

$\therefore$ **3p – q** is parallel to $\mathbf{q} + \frac{1}{2}\mathbf{r}$, and $|3\mathbf{p} - \mathbf{q}| = \frac{5}{4}\left|\mathbf{q} + \frac{1}{2}\mathbf{r}\right|$.

Worksheet 4E Expression of a vector in terms of two other vectors

1. **(a)** $\overrightarrow{AB} = \mathbf{3p}$

(b) $\overrightarrow{CD} = \boldsymbol{-\frac{2}{3}\mathbf{q}}$

(c) $\overrightarrow{EF} = \mathbf{3p + 2q}$

(d) $\overrightarrow{GH} = \boldsymbol{\frac{1}{2}\mathbf{p} - \mathbf{q}}$

Challenge Myself!

2. **(a)** $\overrightarrow{OX} = \boldsymbol{\frac{1}{2}\overrightarrow{OP} - \overrightarrow{OQ}}$

(b) Let $\overrightarrow{OX} = m\overrightarrow{OP} + n\overrightarrow{OQ}$.

$\begin{pmatrix} -3 \\ 2 \end{pmatrix} = m\begin{pmatrix} 2 \\ 2 \end{pmatrix} + n\begin{pmatrix} 4 \\ -1 \end{pmatrix}$

$= \begin{pmatrix} 2m \\ 2m \end{pmatrix} + \begin{pmatrix} 4n \\ -n \end{pmatrix}$

$= \begin{pmatrix} 2m + 4n \\ 2m - n \end{pmatrix}$

$2m + 4n = -3$ —— (1)

$2m - n = 2$ —— (2)

(1) – (2): $5n = -5$

$n = -1$

Substitute $n = -1$ into (2):

$2m - (-1) = 2$

$2m = 1$

$m = \frac{1}{2}$

$\therefore \overrightarrow{OX} = \boldsymbol{\frac{1}{2}\overrightarrow{OP} - \overrightarrow{OQ}}$

Worksheet 4F Position vectors

1. **(a)** $\overrightarrow{OA} = \begin{pmatrix} \mathbf{3} \\ \mathbf{8} \end{pmatrix}$

(b) $\overrightarrow{OB} = \begin{pmatrix} \mathbf{7} \\ \mathbf{-2} \end{pmatrix}$

(c) $\overrightarrow{OC} = \begin{pmatrix} \mathbf{-4} \\ \mathbf{10} \end{pmatrix}$

(d) $\overrightarrow{OD} = \begin{pmatrix} \boldsymbol{\frac{1}{5}} \\ \boldsymbol{-\frac{1}{6}} \end{pmatrix}$

2. **(a)** ***E*****(2, 9)**

(b) ***F*****(6, 0)**

(c) ***G*****(4, –5)**

(d) ***H*****(–3.8, 7.2)**

3. $\overrightarrow{OA} = \begin{pmatrix} 1 \\ 6 \end{pmatrix}$, $\overrightarrow{OB} = \begin{pmatrix} 4 \\ -5 \end{pmatrix}$, $\overrightarrow{OC} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$

(a) $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$= \begin{pmatrix} 4 \\ -5 \end{pmatrix} - \begin{pmatrix} 1 \\ 6 \end{pmatrix}$

$= \begin{pmatrix} \mathbf{3} \\ \mathbf{-11} \end{pmatrix}$

(b) $\overrightarrow{BA} = \overrightarrow{OA} - \overrightarrow{OB}$

$= \begin{pmatrix} 1 \\ 6 \end{pmatrix} - \begin{pmatrix} 4 \\ -5 \end{pmatrix}$

$= \begin{pmatrix} \mathbf{-3} \\ \mathbf{11} \end{pmatrix}$

(c) $\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}$

$= \begin{pmatrix} -3 \\ 2 \end{pmatrix} - \begin{pmatrix} 4 \\ -5 \end{pmatrix}$

$= \begin{pmatrix} \mathbf{-7} \\ \mathbf{7} \end{pmatrix}$

(d) $\overrightarrow{CB} = \overrightarrow{OB} - \overrightarrow{OC}$

$= \begin{pmatrix} 4 \\ -5 \end{pmatrix} - \begin{pmatrix} -3 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} \mathbf{7} \\ \mathbf{-7} \end{pmatrix}$

4. $\overrightarrow{OP} + \overrightarrow{PQ} = \overrightarrow{OQ}$

$\begin{pmatrix} 7 \\ 8 \end{pmatrix} + \begin{pmatrix} -8 \\ 3 \end{pmatrix} = \overrightarrow{OQ}$

$\overrightarrow{OQ} = \begin{pmatrix} -1 \\ 11 \end{pmatrix}$

$\therefore$ **Q(−1, 11)**

5. $\overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OB}$

$\overrightarrow{OA} + \begin{pmatrix} 4 \\ -9 \end{pmatrix} = \begin{pmatrix} -6 \\ 2 \end{pmatrix}$

$\overrightarrow{OA} = \begin{pmatrix} -6 \\ 2 \end{pmatrix} - \begin{pmatrix} 4 \\ -9 \end{pmatrix}$

$= \begin{pmatrix} -10 \\ 11 \end{pmatrix}$

$\therefore$ **A(−10, 11)**

6. (a) $\overrightarrow{CD} = \overrightarrow{OD} - \overrightarrow{OC}$

$= \begin{pmatrix} 1 \\ k \end{pmatrix} - \begin{pmatrix} 5 \\ -3 \end{pmatrix}$

$= \begin{pmatrix} \mathbf{-4} \\ \mathbf{k+3} \end{pmatrix}$

(b) Since $\overrightarrow{CD}$ and $\begin{pmatrix} -7 \\ 6 \end{pmatrix}$ are parallel,

$\frac{k+3}{-4} = \frac{6}{-7}$

$k + 3 = \frac{24}{7}$

$k = \frac{3}{7}$

$\therefore k = \mathbf{\frac{3}{7}}$

7. (a) (i) $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$

$= \begin{pmatrix} -2 \\ 4 \end{pmatrix} - \begin{pmatrix} 6 \\ -5 \end{pmatrix}$

$= \begin{pmatrix} \mathbf{-8} \\ \mathbf{9} \end{pmatrix}$

(ii) $|\overrightarrow{QP}| = \sqrt{(-8)^2 + 9^2}$

$= \sqrt{145}$ units

$=$ **12.0 units** (to 3 s.f.)

(b) $\overrightarrow{QR} = \overrightarrow{OR} - \overrightarrow{OQ}$

$= \begin{pmatrix} -3 \\ k \end{pmatrix} - \begin{pmatrix} -2 \\ 4 \end{pmatrix}$

$= \begin{pmatrix} -1 \\ k-4 \end{pmatrix}$

Since $|\overrightarrow{QR}| = \sqrt{26}$,

$\sqrt{(-1)^2 + (k-4)^2} = (\sqrt{26})^2$

$1 + (k - 4)^2 = 26$

$1 + k^2 - 8k + 16 = 26$

$k^2 - 8k - 9 = 0$

$(k - 9)(k + 1) = 0$

$k = 9$ or $k = -1$

$\therefore k = \mathbf{9}$ or $k = \mathbf{-1}$

Worksheet 4G Applications of vectors

1. (a) $\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC}$

$= \mathbf{-p + q}$

(b) $\overrightarrow{CX} = \frac{3}{4}\overrightarrow{CB}$

$= \frac{3}{4}(\overrightarrow{CA} + \overrightarrow{AB})$

$= \frac{3}{4}(-\mathbf{q} + \mathbf{p})$

$= \mathbf{\frac{3}{4}p - \frac{3}{4}q}$

(c) $\overrightarrow{XA} = \overrightarrow{XC} + \overrightarrow{CA}$

$= -\frac{3}{4}(\mathbf{p} - \mathbf{q}) - \mathbf{q}$

$= -\frac{3}{4}\mathbf{p} + \frac{3}{4}\mathbf{q} - \mathbf{q}$

$= \mathbf{-\frac{3}{4}p - \frac{1}{4}q}$

2. (a) $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$= \mathbf{b - a}$

(b) $\overrightarrow{AQ} = \overrightarrow{AP} + \overrightarrow{PQ}$

$= \mathbf{2a + 2b}$

(c) $\overrightarrow{QB} = \overrightarrow{QR} + \overrightarrow{RB}$

$= \mathbf{-3a - b}$

3. (a) Since *ABCD* is a parallelogram,

$\overrightarrow{AB} = \overrightarrow{DC}$

$\overrightarrow{OB} - \overrightarrow{OA} = \overrightarrow{OC} - \overrightarrow{OD}$

$\begin{pmatrix} 1 \\ 5 \end{pmatrix} - \begin{pmatrix} 6 \\ 3 \end{pmatrix} = \begin{pmatrix} -4 \\ -4 \end{pmatrix} - \overrightarrow{OD}$

$\overrightarrow{OD} = \begin{pmatrix} -4 \\ -4 \end{pmatrix} - \begin{pmatrix} 1 \\ 5 \end{pmatrix} + \begin{pmatrix} 6 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} 1 \\ -6 \end{pmatrix}$

$\therefore$ **D(1, −6)**

(b) Since *ABDC* is a parallelogram,

$\overrightarrow{AB} = \overrightarrow{CD}$

$\overrightarrow{OB} - \overrightarrow{OA} = \overrightarrow{OD} - \overrightarrow{OC}$

$\begin{pmatrix} 1 \\ 5 \end{pmatrix} - \begin{pmatrix} 6 \\ 3 \end{pmatrix} = \overrightarrow{OD} - \begin{pmatrix} -4 \\ -4 \end{pmatrix}$

$\overrightarrow{OD} = \begin{pmatrix} 1 \\ 5 \end{pmatrix} - \begin{pmatrix} 6 \\ 3 \end{pmatrix} + \begin{pmatrix} -4 \\ -4 \end{pmatrix}$

$= \begin{pmatrix} -9 \\ -2 \end{pmatrix}$

$\therefore$ **D(−9, −2)**

4. **(a)** Since $|\overrightarrow{AB}| = \frac{7\sqrt{5}}{2}$,

$$\sqrt{(2h)^2 + h^2} = \frac{7\sqrt{5}}{2}$$
$$4h^2 + h^2 = \frac{245}{4}$$
$$5h^2 = \frac{245}{4}$$
$$h^2 = \frac{49}{4}$$
$$h = \pm 3.5$$
$\therefore$ **$h = \pm 3.5$**

(b) **(i)** $\overrightarrow{OA} = \begin{pmatrix} -9 \\ 3 \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} 1 \\ -12 \end{pmatrix}$

$$\therefore \overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \begin{pmatrix} 1 \\ -12 \end{pmatrix} - \begin{pmatrix} -9 \\ 3 \end{pmatrix} = \begin{pmatrix} \mathbf{10} \\ \mathbf{-15} \end{pmatrix}$$

(ii) $\overrightarrow{OC} = \begin{pmatrix} -7 \\ 0 \end{pmatrix}$

$$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = \begin{pmatrix} -7 \\ 0 \end{pmatrix} - \begin{pmatrix} -9 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$$

Since $\overrightarrow{AC} = \frac{1}{5}\overrightarrow{AB}$, then $C(-7, 0)$ lies on this line.

5. **(a)** $$\overrightarrow{BC} = \overrightarrow{AC} - \overrightarrow{AB} = \begin{pmatrix} 2 \\ -3 \end{pmatrix} - \begin{pmatrix} -6 \\ 3 \end{pmatrix} = \begin{pmatrix} 8 \\ -6 \end{pmatrix}$$

$\therefore BC = \sqrt{8^2 + (-6)^2}$
= **10 units**

(b) $AB = \sqrt{(-6)^2 + 3^2}$
$= \sqrt{45}$ units

$AC = \sqrt{2^2 + (-3)^2}$
$= \sqrt{13}$ units

Since $AB^2 + AC^2 \neq BC^2$, by the converse of Pythagoras' Theorem, $\triangle ABC$ is not a right-angled triangle.

6. **(a)** Gradient of AB × gradient of $BC = -1$

$$\frac{3-(-1)}{9-7} \times \frac{-1-3}{k-9} = -1$$
$$2 \times \frac{-4}{k-9} = -1$$
$$-8 = -k + 9$$
$$k = 17 \text{ (shown)}$$

(b) $\overrightarrow{OA} = \begin{pmatrix} 7 \\ -1 \end{pmatrix}$, $\overrightarrow{OB} = \begin{pmatrix} 9 \\ 3 \end{pmatrix}$ and $\overrightarrow{OC} = \begin{pmatrix} 17 \\ -1 \end{pmatrix}$

$$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB} = \begin{pmatrix} 17 \\ -1 \end{pmatrix} - \begin{pmatrix} 9 \\ 3 \end{pmatrix} = \begin{pmatrix} 8 \\ -4 \end{pmatrix}$$

$$\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AD} = \overrightarrow{OA} + \overrightarrow{BC} = \begin{pmatrix} 7 \\ -1 \end{pmatrix} + \begin{pmatrix} 8 \\ -4 \end{pmatrix} = \begin{pmatrix} 15 \\ -5 \end{pmatrix}$$

$\therefore$ ***D*(15, −5)**

(c) Area of rectangle $ABCD = 2 \times \frac{1}{2}(10)(4)$
= **40 units²**

7. **(a)** **(i)** $$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP} = \overrightarrow{OA} + \frac{3}{4}\overrightarrow{AB} = \overrightarrow{OA} + \frac{3}{4}(\overrightarrow{OB} - \overrightarrow{OA}) = 4\mathbf{a} + \frac{3}{4}(4\mathbf{b} - 4\mathbf{a})$$
$= 4\mathbf{a} + 3\mathbf{b} - 3\mathbf{a}$
$= \mathbf{a + 3b}$

(ii) $$\overrightarrow{PQ} = \overrightarrow{PA} + \overrightarrow{AQ} = -(3\mathbf{b} - 3\mathbf{a}) + \frac{1}{3}(13\mathbf{b} - 3\mathbf{a}) = -3\mathbf{b} + 3\mathbf{a} + \frac{13}{3}\mathbf{b} - \mathbf{a} = \mathbf{2a + \frac{4}{3}b}$$

(b) **(i)** $$\overrightarrow{PQ} = 2\mathbf{a} + \frac{4}{3}\mathbf{b} = \frac{2}{3}(3\mathbf{a} + 2\mathbf{b}) = \frac{2}{3}\overrightarrow{BC}$$

Since $\overrightarrow{PQ} = \frac{2}{3}\overrightarrow{BC}$, PQ is parallel to BC.

(ii) $PQ : BC = $ **2 : 3**

8. **(a)** $\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP}$
$= \mathbf{a} + k(\mathbf{b} - \mathbf{a})$
$= \mathbf{a} + k\mathbf{b} - k\mathbf{a}$
$= \mathbf{(1 - k)a + kb}$

(b) **(i)** $\overrightarrow{OC} = 3(1 - k)\mathbf{a} + 3k\mathbf{b}$
$= 3[(1 - k)\mathbf{a} + k\mathbf{b}]$
$= 3\overrightarrow{OP}$

Since $\overrightarrow{OC} = 3\overrightarrow{OP}$, then O, P and C lie on a straight line. (shown)

(ii) $\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC}$

$= \mathbf{a} + h\mathbf{b}$

Equating coefficients of **a**,

$3(1 - k) = 1$

$1 - k = \frac{1}{3}$

$k = \frac{2}{3}$

Equating coefficients of **b**,

$h = 3k$

$= 2$

$\therefore \boldsymbol{h = 2, k = \frac{2}{3}}$

(iii) $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$

$\overrightarrow{AP} = \frac{2}{3}(\mathbf{b} - \mathbf{a})$

$= \frac{2}{3}\overrightarrow{AB}$

$\therefore AP : PB = \mathbf{2 : 1}$

9. **(a)** **(i)** $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$

$= \mathbf{q} - \mathbf{p}$

(ii) $\overrightarrow{QB} = \overrightarrow{QR} + \overrightarrow{RB}$

$= (2\mathbf{p} - \mathbf{q}) - (2\mathbf{p} - 4\mathbf{q})$

$= 2\mathbf{p} - \mathbf{q} - 2\mathbf{p} + 4\mathbf{q}$

$= \mathbf{3q}$

(b) $\overrightarrow{PR} = \overrightarrow{PQ} + \overrightarrow{QR}$

$= (\mathbf{q} - \mathbf{p}) + (2\mathbf{p} - \mathbf{q})$

$= \mathbf{q} - \mathbf{p} + 2\mathbf{p} - \mathbf{q}$

$= \mathbf{p}$

$\overrightarrow{AB} = \overrightarrow{AR} + \overrightarrow{RB}$

$= 2(-\mathbf{p}) - (2\mathbf{p} - 4\mathbf{q})$

$= -2\mathbf{p} - 2\mathbf{p} + 4\mathbf{q}$

$= 4\mathbf{q} - 4\mathbf{p}$ (shown)

(c) Since $\overrightarrow{AB} = 4\overrightarrow{PQ}$, $AB \parallel PQ$.

$\angle OAB = \angle OPQ$ (corr. ∠s, $AB \parallel PQ$)

$\angle AOB = \angle POQ$ (common ∠)

$\therefore \triangle OPQ$ is similar to $\triangle OAB$. (shown)

10. **(a)** $\overrightarrow{OR} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$

$\overrightarrow{OQ} = \overrightarrow{OR} + \overrightarrow{RQ}$

$= \begin{pmatrix} 2 \\ -2 \end{pmatrix} - \begin{pmatrix} 7 \\ -3 \end{pmatrix}$

$= \begin{pmatrix} -5 \\ 1 \end{pmatrix}$

$\therefore$ **Q(–5, 1)**

(b) $\overrightarrow{OP} = \overrightarrow{OQ} + \overrightarrow{QP}$

$= \begin{pmatrix} -5 \\ 1 \end{pmatrix} - \begin{pmatrix} 4 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} -9 \\ 0 \end{pmatrix}$

$\therefore P(-9, 0)$

$\overrightarrow{SR} = \overrightarrow{PQ}$

$\overrightarrow{OR} - \overrightarrow{OS} = \overrightarrow{PQ}$

$\begin{pmatrix} 2 \\ -2 \end{pmatrix} - \overrightarrow{OS} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$

$\overrightarrow{OS} = \begin{pmatrix} 2 \\ -2 \end{pmatrix} - \begin{pmatrix} 4 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} -2 \\ -3 \end{pmatrix}$

$\therefore S(-2, -3)$

$\therefore$ ***P*(–9, 0), S(–2, –3)**

(c) Let x_1 be the angle between PQ and the x-axis.

$\tan x_1 = \frac{1}{4}$

$x_1 = \tan^{-1}\frac{1}{4}$

$= 14.036°$ (to 3 d.p.)

Let x_2 be the angle between PS and the x-axis.

$\tan x_2 = \frac{3}{7}$

$x_2 = \tan^{-1}\frac{3}{7}$

$= 23.199°$ (to 3 d.p.)

$\therefore \angle QPS = 14.036° + 23.199°$

$= 37.2°$ (to 1 d.p.) (shown)

(d) $QS = \sqrt{[-2-(-5)]^2 + (-3-1)^2}$

= **5 units**

Review Exercise 4

1. **(a)** $2\mathbf{p} - 5\mathbf{q} = 2\begin{pmatrix} 6 \\ 4 \end{pmatrix} - 5\begin{pmatrix} 1 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} 12 \\ 8 \end{pmatrix} - \begin{pmatrix} 5 \\ 15 \end{pmatrix}$

$= \begin{pmatrix} 7 \\ -7 \end{pmatrix}$

$= 7\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$\therefore k = \mathbf{7}$

(b) $|2\mathbf{p} - 5\mathbf{q}| = \sqrt{7^2 + (-7)^2}$

$= \sqrt{98}$

= **9.90 units** (to 3 s.f.)

2. **(a)** $\overrightarrow{OQ} = \begin{pmatrix} 4 \\ -6 \end{pmatrix}$

$\overrightarrow{OP} + \begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 \\ -6 \end{pmatrix}$

$\overrightarrow{OP} = \begin{pmatrix} 4 \\ -6 \end{pmatrix} - \begin{pmatrix} -1 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} 5 \\ -8 \end{pmatrix}$

$\therefore$ ***P*(5, –8)**

(b) Magnitude of $\begin{pmatrix} -1 \\ 2 \end{pmatrix} = \sqrt{(-1)^2 + 2^2}$

$= \sqrt{5}$

= **2.24 units** (to 3 s.f.)

3. **(a)** $\overrightarrow{OA} = \begin{pmatrix} 1 \\ -5 \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} -4 \\ 3 \end{pmatrix}$

$$\begin{aligned}\overrightarrow{AB} &= \overrightarrow{OB} - \overrightarrow{OA} \\ &= \begin{pmatrix} -4 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ -5 \end{pmatrix} \\ &= \begin{pmatrix} \mathbf{-5} \\ \mathbf{8} \end{pmatrix}\end{aligned}$$

(b) $\overrightarrow{OC} = \begin{pmatrix} 2 \\ h \end{pmatrix}$

$$\begin{aligned}\overrightarrow{AC} &= k\overrightarrow{AB} \\ \overrightarrow{OC} - \overrightarrow{OA} &= k\overrightarrow{AB} \\ \begin{pmatrix} 2 \\ h \end{pmatrix} - \begin{pmatrix} 1 \\ -5 \end{pmatrix} &= k\begin{pmatrix} -5 \\ 8 \end{pmatrix} \\ \begin{pmatrix} 1 \\ h+5 \end{pmatrix} &= \begin{pmatrix} -5k \\ 8k \end{pmatrix}\end{aligned}$$

Equating components,

x: $1 = -5k$

$k = -0.2$

y: $h + 5 = 8(-0.2)$

$h = -6.6$

$\therefore$ $\boldsymbol{h = -6.6, k = -0.2}$

4. **(a)** $\overrightarrow{OA} = \begin{pmatrix} 6 \\ 2 \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} -4 \\ -1 \end{pmatrix}$

$$\begin{aligned}\overrightarrow{BA} &= \overrightarrow{OA} - \overrightarrow{OB} \\ &= \begin{pmatrix} 6 \\ 2 \end{pmatrix} - \begin{pmatrix} -4 \\ -1 \end{pmatrix} \\ &= \begin{pmatrix} \mathbf{10} \\ \mathbf{3} \end{pmatrix}\end{aligned}$$

(b) $|\overrightarrow{BA}| = \sqrt{10^2 + 3^2}$

$= \sqrt{109}$

$=$ **10.4 units** (to 3 s.f.)

(c)

$$\begin{aligned}\overrightarrow{AB} &= 3\overrightarrow{AC} \\ -\overrightarrow{BA} &= 3(\overrightarrow{OC} - \overrightarrow{OA}) \\ \begin{pmatrix} -10 \\ -3 \end{pmatrix} &= 3\left[\overrightarrow{OC} - \begin{pmatrix} 6 \\ 2 \end{pmatrix}\right] \\ \begin{pmatrix} -\frac{10}{3} \\ -1 \end{pmatrix} &= \overrightarrow{OC} - \begin{pmatrix} 6 \\ 2 \end{pmatrix} \\ \overrightarrow{OC} &= \begin{pmatrix} -\frac{10}{3} \\ -1 \end{pmatrix} + \begin{pmatrix} 6 \\ 2 \end{pmatrix} \\ &= \begin{pmatrix} 2\frac{2}{3} \\ 1 \end{pmatrix}\end{aligned}$$

$\therefore$ $\boldsymbol{C\left(2\frac{2}{3}, 1\right)}$

5. **(a)** Gradient of $PQ = \frac{4}{-3} = -\frac{4}{3}$

Substitute $m = -\frac{4}{3}$, $x = 9$ and $y = 5$ into $y = mx + c$:

$5 = -\frac{4}{3}(9) + c$

$c = 17$

$\therefore$ Equation of PQ: $\boldsymbol{y = -\frac{4}{3}x + 17}$

(b) $y = -\frac{4}{3}x + 17$ —— (1)

$2y + 7x = 8$ —— (2)

Substitute (1) into (2):

$$\begin{aligned}2\left(-\frac{4}{3}x + 17\right) + 7x &= 8 \\ -\frac{8}{3}x + 34 + 7x &= 8 \\ \frac{13}{3}x &= -26 \\ x &= -6\end{aligned}$$

Substitute $x = -6$ into (1):

$$\begin{aligned}y &= -\frac{4}{3}(-6) + 17 \\ &= 25\end{aligned}$$

$\therefore$ Coordinates of point of intersection are **(–6, 25)**

(c) OPEN An example of l is $\boldsymbol{y = -\frac{4}{3}x + 2}$.

6. **(a)**

$$\begin{aligned}\overrightarrow{AP} &= \frac{2}{3}\overrightarrow{AB} \\ &= \frac{2}{3}(\overrightarrow{OB} - \overrightarrow{OA}) \\ &= \boldsymbol{\frac{2}{3}(\mathbf{b} - \mathbf{a})}\end{aligned}$$

(b) $\triangle OAB$ is similar to $\triangle QPB$.

Since $\frac{PB}{AB} = \frac{1}{3}$, then $\frac{QB}{OB} = \frac{1}{3}$ and $\frac{OQ}{OB} = \frac{2}{3}$.

$\therefore$ $\overrightarrow{OQ} = \frac{2}{3}\overrightarrow{OB} = \boldsymbol{\frac{2}{3}\mathbf{b}}$

7. **(a)**

$$\begin{aligned}\overrightarrow{QX} &= \overrightarrow{QR} + \overrightarrow{RX} \\ &= \overrightarrow{QR} + \frac{1}{5}\overrightarrow{RS} \\ &= \overrightarrow{PS} + \frac{1}{5}\overrightarrow{QP} \\ &= \mathbf{b} + \frac{1}{5}(-\mathbf{a}) \\ &= \boldsymbol{-\frac{1}{5}\mathbf{a} + \mathbf{b}}\end{aligned}$$

(b)

$$\begin{aligned}\overrightarrow{WX} &= \overrightarrow{WQ} + \overrightarrow{QX} \\ &= \overrightarrow{SX} + \overrightarrow{QX} \\ &= \frac{4}{5}\mathbf{a} + \left(-\frac{1}{5}\mathbf{a} + \mathbf{b}\right) \\ &= \frac{4}{5}\mathbf{a} - \frac{1}{5}\mathbf{a} + \mathbf{b} \\ &= \boldsymbol{\frac{3}{5}\mathbf{a} + \mathbf{b}}\end{aligned}$$

5 Relations and Functions

Worksheet 5A Functions

1. **(a)** **(i)** $f(2) = 10 - 3(2)$
$= \mathbf{4}$

(ii) $f(-4) = 10 - 3(-4)$
$= \mathbf{22}$

(iii) $f\left(-\frac{1}{3}\right) = 10 - 3\left(-\frac{1}{3}\right)$
$= \mathbf{11}$

(iv) $f(0) = 10 - 3(0)$
$= \mathbf{10}$

(b) When $f(x) = x$,
$10 - 3x = x$
$4x = 10$
$x = \mathbf{2.5}$

2. **(a)** **(i)** $f(-8) = \frac{3}{4}(-8) + 1$
$= \mathbf{-5}$

(ii) $g(3) = \frac{1}{5}(3)^2$
$= \mathbf{1\frac{4}{5}}$

(iii) $f\left(\frac{2}{3}\right) + g\left(\frac{2}{3}\right) = \frac{3}{4}\left(\frac{2}{3}\right) + 1 + \frac{1}{5}\left(\frac{2}{3}\right)^2$
$= \mathbf{1\frac{53}{90}}$

(iv) $f(-6) \times g(5) = \left[\frac{3}{4}(-6) + 1\right] \times \frac{1}{5}(5)^2$
$= \mathbf{-17\frac{1}{2}}$

(b) When $g(x) = 5$,
$\frac{1}{5}x^2 = 5$
$x^2 = 25$
$x = \mathbf{\pm 5}$

(c) OPEN
$g(-1) - f(-1) = \frac{1}{5}(-1)^2 - \left[\frac{3}{4}(-1) + 1\right]$
$= -\frac{1}{20}$

For $\frac{p}{-1+q} = -\frac{1}{20}$, a possible pair of values is $\boldsymbol{p = 1, q = -19}$.

3. **(a)** $f(-1) = 4(-1) + 1$
$= -3$
$g(-1) = 2 - (-1)$
$= 3$
$\therefore$ f(−1) does not give the same value as g(−1).

(b) $f(0.2) - g(0.2) = 4(0.2) + 1 - (2 - 0.2)$
$= \mathbf{0}$

(c) $3f(x) = 7g(x)$
$3(4x + 1) = 7(2 - x)$
$12x + 3 = 14 - 7x$
$19x = 11$
$x = \mathbf{\frac{11}{19}}$

(d) When $g(x) < f(x)$,
$2 - x < 4x + 1$
$5x > 1$
$\boldsymbol{x > \frac{1}{5}}$

4. **(a)** **(i)** $f(p - 1) = 5(p - 1) + 6$
$= 5p - 5 + 6$
$= \mathbf{5p + 1}$

(ii) $f(3p) = 5(3p) + 6$
$= \mathbf{15p + 6}$

(iii) $f(p^3) = \mathbf{5p^3 + 6}$

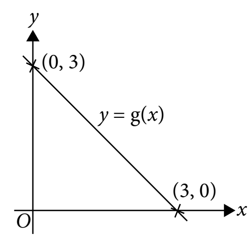
(iv) $f\left(\frac{1}{p}\right) - f(9) = \left[5\left(\frac{1}{p}\right) + 6\right] - [5(9) + 6]$
$= \frac{5}{p} + 6 - 51$
$= \mathbf{\frac{5}{p} - 45}$

(b) $f(6p) = 5(6p) + 6$
$= 30p + 6$
$2f(3p) = 2[5(3p) + 6]$
$= 2(15p + 6)$
$= 30p + 12$
$\therefore f(6p) \neq 2f(3p)$ (shown)

(c) $g(5x + 6) = 5(5x + 6)^2 + 6$
$= 5(25x^2 + 60x + 36) + 6$
$= 125x^2 + 300x + 180 + 6$
$= \mathbf{125x^2 + 300x + 186}$

5. **(a)** $g(5) = -2: 5a + b = -2$ —— (1)
$g(-2) = 5: -2a + b = 5$ —— (2)
(1) − (2): $7a = -7$
$a = -1$
Substitute $a = -1$ into (2):
$-2(-1) + b = 5$
$2 + b = 5$
$b = 3$
$\therefore \boldsymbol{a = -1, b = 3}$

(b)



6. **(a)** $f\left(-\frac{3}{4}\right) = -1: h\left(-\frac{3}{4}\right)^2 + 8 = -1$
$\frac{9}{16}h + 8 = -1$
$\frac{9}{16}h = -9$
$h = -16$

$g\left(-\frac{3}{4}\right) = -1: 2\left(-\frac{3}{4}\right) + k = -1$
$-\frac{3}{2} + k = -1$
$k = \frac{1}{2}$

$\therefore \boldsymbol{h = -16, k = \frac{1}{2}}$

(b) $f(x) = 8 - 16x^2$

$g(x) = 2x + \frac{1}{2}$

$f\left(\frac{3}{4}\right) \times g\left(\frac{3}{4}\right) = \left[8 - 16\left(\frac{3}{4}\right)^2\right] \times \left[2\left(\frac{3}{4}\right) + \frac{1}{2}\right]$

$= \mathbf{-2}$

Worksheet 5B Inverse functions

1. **(a)** $f(x) = 5x + 7$

Let $y = 5x + 7$.

$5x = y - 7$

$x = \frac{y-7}{5}$

$\therefore \mathbf{f^{-1} : x \mapsto \frac{x-7}{5}}$

(b) $f(2) + f^{-1}(2) = 5(2) + 7 + \frac{2-7}{5}$

$= \mathbf{16}$

2. **(a)** $g(x) = \frac{1}{3x}$

Let $y = \frac{1}{3x}$.

$x = \frac{1}{3y}$

$\therefore \mathbf{g^{-1} : x \mapsto \frac{1}{3x}, x < 0}$

(b) When $g^{-1}(x) = x$,

$\frac{1}{3x} = x$

$x^2 = \frac{1}{3}$

$x = -\frac{1}{\sqrt{3}}$ $(x < 0)$

3. **(a)** $h(x) = \frac{2x+9}{2}$

Let $y = \frac{2x+9}{2}$.

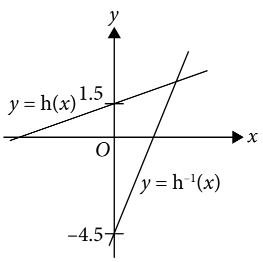
$6y = 2x + 9$

$2x = 6y - 9$

$x = \frac{6y-9}{2}$

$\therefore \mathbf{h^{-1} : x \mapsto \frac{6x-9}{2}}$

(b)



(c) When $h(x) = h^{-1}(x)$,

$\frac{2x+9}{6} = \frac{6x-9}{2}$

$2x + 9 = 18x - 27$

$16x = 36$

$x = \mathbf{2\frac{1}{4}}$

4. **(a)** $k = \mathbf{-9}$

(b) Let $y = \frac{4x-3}{x+9}$.

$xy + 9y = 4x - 3$

$4x - xy = 9y + 3$

$x(4 - y) = 9y + 3$

$x = \frac{9y+3}{4-y}$

$\therefore \mathbf{f^{-1} : x \mapsto \frac{9x+3}{4-x}, x \neq 4}$

(c) When $y = -2$,

$\frac{9x+3}{4-x} = -2$

$9x + 3 = -8 + 2x$

$7x = -11$

$x = -1\frac{4}{7}$

$\therefore \mathbf{a = -1\frac{4}{7}, b = -2}$

5. **(a)** $g(x) = a - bx$

Let $y = a - bx$.

$bx = a - y$

$x = \frac{a-y}{b}$

$\therefore g^{-1} : x \mapsto \frac{a-x}{b}$

$g^{-1}(-3) = 6$: $\frac{a+3}{b} = 6$

$a + 3 = 6b$ —— (1)

$g^{-1}(0) = 4$: $\frac{a}{b} = 4$

$a = 4b$ —— (2)

Substitute (2) into (1):

$4b + 3 = 6b$

$2b = 3$

$b = \frac{3}{2}$

Substitute $b = \frac{3}{2}$ into (2):

$a = 4\left(\frac{3}{2}\right)$

$= 6$

$\therefore \mathbf{a = 6, b = \frac{3}{2}}$

(b) When $g^{-1}(x) = x$,

$\frac{6-x}{\frac{3}{2}} = x$

$6 - x = \frac{3}{2}x$

$\frac{5}{2}x = 6$

$x = \mathbf{2\frac{2}{5}}$

Challenge Myself!

6. (a) $b = \mathbf{4}$

(b) $f(x) = \frac{ax}{2 - 4x}$

Let $y = \frac{ax}{2 - 4x}$.

$2y - 4xy = ax$

$4xy + ax = 2y$

$x(4y + a) = 2y$

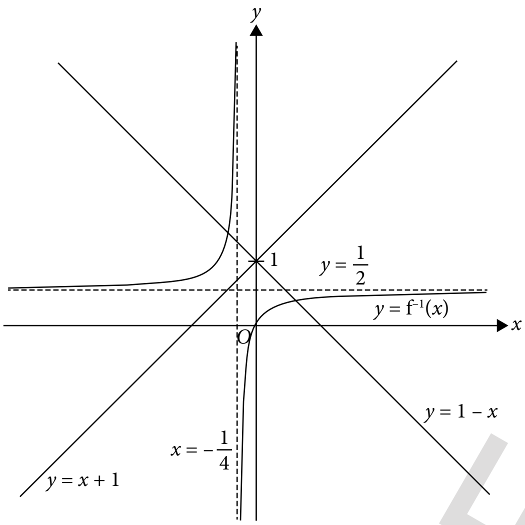
$x = \frac{2y}{4y + a}$

$\therefore f^{-1} : x \mapsto \frac{2x}{4x + a}, x \neq -\frac{\boldsymbol{a}}{\mathbf{4}}$

(c) $f^{-1}(x) = \frac{2x}{4x + 1}, x \neq -\frac{1}{4}$

Equations of asymptotes: $\boldsymbol{x = -\frac{1}{4}}, \boldsymbol{y = \frac{1}{2}}$

(d)



(e) When $p = 1$ and $q = 1$, $f^{-1}(x) = x + 1$ has 0 solutions.
When $p = -1$ and $q = 1$, $f^{-1}(x) = 1 - x$ has 2 solutions.
$\therefore f^{-1}(x) = px + q$ could have **0** or **2** solutions.

Worksheet 5C Composite functions

1. (a) $f(x) = \frac{1}{5}x - 2$

$g(x) = -x$

$fg(x) = f(-x)$

$= -\frac{1}{5}x - 2$

$gf(x) = g\left(\frac{1}{5}x - 2\right)$

$= 2 - \frac{1}{5}x$

$\therefore \mathbf{fg(x) = -\frac{1}{5}x - 2, gf(x) = 2 - \frac{1}{5}x}$

(b) (i) $fg(10) = -\frac{1}{5}(10) - 2$

$= \mathbf{-4}$

(ii) $gf(10) = 2 - \frac{1}{5}(10)$

$= \mathbf{0}$

(iii) $fg(-4) = -\frac{1}{5}(-4) - 2$

$= \mathbf{-1\frac{1}{5}}$

(iv) $gf(-4) = 2 - \frac{1}{5}(-4)$

$= \mathbf{2\frac{4}{5}}$

2. (a) $f(x) = 2x + 3$

$g(x) = 4 - 5x$

$fg(2) = f[4 - 5(2)]$

$= f(-6)$

$= 2(-6) + 3$

$= -9$ (shown)

(b) (i) $gf(x) = g(2x + 3)$

$= 4 - 5(2x + 3)$

$= 4 - 10x - 15$

$= \mathbf{-11 - 10x}$

(ii) $gf(2) = -11 - 10(2)$

$= \mathbf{-31}$

3. (a) $fg(x) = f(6x)$

$= 1 - 6(6x)$

$= \mathbf{1 - 36x}$

$gf(x) = f(1 - 6x)$

$= 6(1 - 6x)$

$= \mathbf{6 - 36x}$

(b) $fg(x) = f\left(\frac{1}{2}x\right)$

$= 4\left(\frac{1}{2}x\right)^2 + 7$

$= 4\left(\frac{1}{4}x^2\right) + 7$

$= \mathbf{x^2 + 7}$

$gf(x) = g(4x^2 + 7)$

$= \frac{1}{2}(4x^2 + 7)$

$= \mathbf{2x^2 + \frac{7}{2}}$

(c) $fg(x) = f\left(\frac{1}{x - 1}\right)$

$= \frac{1}{\frac{1}{x - 1}} - 1$

$= x - 1 - 1$

$= \mathbf{x - 2}$

$gf(x) = g\left(\frac{1}{x} - 1\right)$

$= \frac{1}{\frac{1}{x} - 1 - 1}$

$= \frac{1}{\frac{1}{x} - 2}$

$= \mathbf{\frac{x}{1 - 2x}, x \neq \frac{1}{2}}$

(d) $fg(x) = f\left(\frac{9x}{9-x}\right)$

$= 9\left(\frac{9x}{9-x} + 1\right)$

$= 9\left(\frac{9x + 9 - x}{9-x}\right)$

$= 9\left(\frac{8x+9}{9-x}\right)$

$= \mathbf{\frac{72x+81}{9-x}, x \neq 9}$

$gf(x) = g[9(x+1)]$

$= g(9x+9)$

$= \frac{9(9x+9)}{9-(9x+9)}$

$= \frac{9(9x+9)}{-9x}$

$= \mathbf{-\frac{9x+9}{x}, x \neq 0}$

4. (a) $f(x) = \sqrt{x}$

$g(x) = \frac{1}{4}(ax+1)^2$

$fg(-2) = 4.5$: $f\left[\frac{1}{4}(1-2a)^2\right] = 4.5$

$\sqrt{\frac{1}{4}(1-2a)^2} = 4.5$

$\frac{1}{2}(1-2a) = 4.5$

$1 - 2a = 9$

$2a = -8$

$a = \mathbf{-4}$

(b) $gf(9) = g\left(\sqrt{9}\right)$

$= g(3)$

$= \frac{1}{4}[-4(3)+1]^2$

$= \mathbf{30\frac{1}{4}}$

5. (a) $f(x) = hx - 6$

$g(x) = kx^2$

$fg(x) = f(kx^2)$

$= hkx^2 - 6$

$\therefore$ $\mathbf{fg : x \mapsto hkx^2 - 6}$

$gf(x) = g(hx-6)$

$= k(hx-6)^2$

$\therefore$ $\mathbf{gf : x \mapsto k(hx-6)^2}$

(b)

$fg(1) = hk - 6$

$gf(1) = k(h-6)^2$

Let $h = 2$: $2k - 6 = k(-4)^2$

$= 16k$

$14k = -6$

$k = -\frac{3}{7}$

$\therefore$ $\mathbf{h = 2, k = -\frac{3}{7}}$

6. (a) $f(x) = \frac{3}{x}$

$ff(x) = \frac{3}{\frac{3}{x}}$

$= \mathbf{x}$

(b) $f^{(3)}(x) = \frac{3}{x}$

$f^{(10)}(x) = \mathbf{x}$

$f^{(2025)}(x) = \mathbf{\frac{3}{x}, x \neq 0}$

Challenge Myself!

7. $g(x) = x + 1$

$fg(x) = x^2 + 5x + 6$

$= x^2 + 2x + 1 + 3x + 5$

$= (x+1)^2 + 3(x+1) + 2$

$\therefore$ $\mathbf{f : x \mapsto x^2 + 3x + 2}$

Review Exercise 5

1. (a) $f(x) = \frac{3x+1}{4}$

$g(x) = \frac{2}{5-x}$

(i) $f(7) = \frac{3(7)+1}{4}$

$= \mathbf{5\frac{1}{2}}$

(ii) $g(-3) = \frac{2}{5-(-3)}$

$= \mathbf{\frac{1}{4}}$

(b) When $f(x) = g(x)$,

$\frac{3x+1}{4} = \frac{2}{5-x}$

$(3x+1)(5-x) = 8$

$15x - 3x^2 + 5 - x = 8$

$3x^2 - 14x + 3 = 0$

$x = \frac{-(-14) \pm \sqrt{(-14)^2 - 4(3)(3)}}{2(3)}$

$= \frac{14 \pm \sqrt{160}}{6}$

$= 4.44$ or 0.225 (to 3 s.f.)

$\therefore$ $\mathbf{x = 4.44}$ or $\mathbf{x = 0.225}$

2. (a) $2x + 9 > 0$

$2x > -9$

$x > -4\frac{1}{2}$

$\therefore$ Smallest possible value of the integer $a = \mathbf{-4}$

(b) $h(x) = \frac{4}{2x+9}$

Let $y = \frac{4}{2x+9}$.

$2xy + 9y = 4$

$2xy = 4 - 9y$

$x = \frac{4-9y}{2y}$

$\therefore$ $\mathbf{h^{-1} : x \mapsto \frac{4-9x}{2x}, x > 0}$

(c) $h^{-1}h(2x) = h^{-1}\left(\dfrac{4}{2(2x)+9}\right)$

$= h^{-1}\left(\dfrac{4}{4x+9}\right)$

$= \dfrac{4 - 9\left(\dfrac{4}{4x+9}\right)}{2\left(\dfrac{4}{4x+9}\right)}$

$= \dfrac{4(4x+9) - 36}{8}$

$= \dfrac{16x + 36 - 36}{8}$

$= \mathbf{2x}$

3. **(a)** $f(x) = kx - 10$

$g(x) = 5x + k$

$fg(x) = f(5x + k)$

$= k(5x + k) - 10$

$= 5kx + (k^2 - 10)$

$\therefore$ $\mathbf{fg : x \mapsto 5kx + (k^2 - 10)}$

$gf(x) = g(kx - 10)$

$= 5(kx - 10) + k$

$= 5kx + (k - 50)$

$\therefore$ $\mathbf{gf : x \mapsto 5kx + (k - 50)}$

(b) When $fg(x) = gf(x)$,

$5kx + (k^2 - 10) = 5kx + (k - 50)$

$k^2 - 10 = k - 50$

$k^2 - k + 40 = 0$

$k = \dfrac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(40)}}{2(1)}$

$= \dfrac{1 \pm \sqrt{-159}}{2}$

Since k is undefined, there are no real values of x that satisfy the equation. (shown)

4. **(a)** $f(x) = 4x - 7$

Let $y = 4x - 7$.

$4x = y + 7$

$x = \dfrac{y+7}{4}$

$\therefore$ $\mathbf{f^{-1} : x \mapsto \dfrac{x+7}{4}}$

(b) $g(x) = \dfrac{x}{x-1}$

Let $y = \dfrac{x}{x-1}$.

$xy - y = x$

$xy - x = y$

$x(y - 1) = y$

$x = \dfrac{y}{y-1}$

$\therefore$ $\mathbf{g^{-1} : x \mapsto \dfrac{x}{x-1}, x \neq 1}$

(c) $gf(x) = g(4x - 7)$

$= \dfrac{4x-7}{4x-7-1}$

$= \dfrac{4x-7}{4x-8}$ (shown)

Let $y = \dfrac{4x-7}{4x-8}$.

$4xy - 8y = 4x - 7$

$4xy - 4x = 8y - 7$

$4x(y - 1) = 8y - 7$

$x = \dfrac{8y-7}{4(y-1)}$

$\therefore$ $\mathbf{(gf)^{-1}(x) = \dfrac{8x-7}{4(x-1)}, x \neq 1}$

(d) $f^{-1}g^{-1}(x) = f^{-1}\left(\dfrac{x}{x-1}\right)$

$= \dfrac{1}{4}\left(\dfrac{x}{x-1} + 7\right)$

$= \dfrac{1}{4}\left(\dfrac{x + 7x - 7}{x-1}\right)$

$= \dfrac{1}{4}\left(\dfrac{8x-7}{x-1}\right)$

$= \dfrac{8x-7}{4(x-1)}$

$\therefore$ $f^{-1}g^{-1}(x) = (gf)^{-1}(x)$

Mid-year Checkpoint A

Section A

1. $8 - (4 \times 3 + 6) - 10 = -20$ [1]

2. $5px - 4py - 10qx + 8qy = p(5x - 4y) - 2q(5x - 4y)$ [1]

$= \mathbf{(5x - 4y)(p - 2q)}$ [1]

3. **(a)** Size of remaining angle $= \dfrac{180° - 68°}{2}$

$= \mathbf{56°}$ [1]

(b) Sum of remaining angles $= 360° - 49°$

$= \mathbf{311°}$ [1]

4. Total $= 52.6 \times 5 = 263$

$\therefore$ The numbers are **49, 49, 51, 53 and 61**. [2]

5. **(a)** $f(x) = 3x$

$g(x) = \dfrac{1}{2}x^2$

$gf(4) = \dfrac{1}{2}(12)^2$

$= \mathbf{72}$ [1]

(b) Let $y = 3x$.

$x = \dfrac{y}{3}$

$\therefore$ $f^{-1} : x \mapsto \mathbf{\dfrac{x}{3}}$ [1]

6. **(a)** $\dfrac{6x+5}{6} \geqslant 3 - 7x$

$6x + 5 \geqslant 18 - 42x$

$48x \geqslant 13$ [1]

$\mathbf{x \geqslant \dfrac{13}{48}}$ [1]

(b) Smallest rational number $= \mathbf{\dfrac{13}{48}}$ [1]

7. **(a)** $5^5 + 5^5 + 5^5 + 5^5 + 5^5 = 5(5^5)$

$= 5^6$

$\therefore a = \mathbf{6}$ **[1]**

(b) $\left(\dfrac{x^9}{27y^{15}}\right)^{\frac{4}{3}} = \left(\dfrac{x^9}{3^3y^{15}}\right)^{\frac{4}{3}}$

$= \boldsymbol{\dfrac{x^{12}}{81y^{20}}}$ **[2]**

8. **(a)** When $k = 0.5$, $h = -1$,

$p = 4(0.5)[0.5 - 2(-1)]$

$= \mathbf{5}$ **[1]**

(b) $p = 4k(k - 2h)$

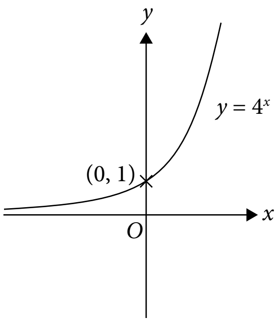
$\dfrac{p}{4k} = k - 2h$

$2h = k - \dfrac{p}{4k}$ **[1]**

$= \dfrac{4k^2 - p}{4k}$

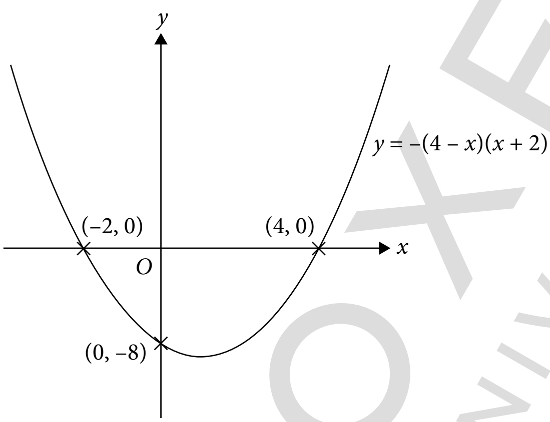
$\boldsymbol{h = \dfrac{4k^2 - p}{8k}}$ **[1]**

9. **(a)**



[1]

(b)



[2]

10. **(a)** Equation of line of symmetry: $\boldsymbol{x = 1.5}$ **[1]**

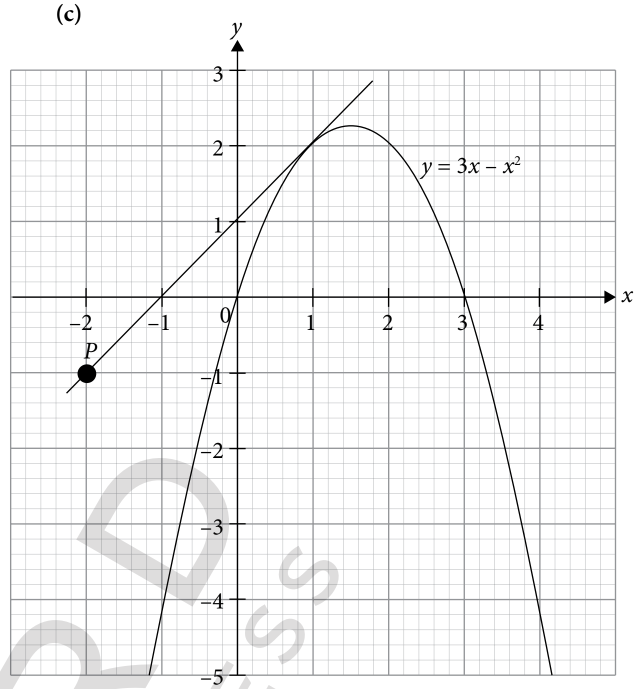
(b) $x^2 - 3x - 1 = 0$

$3x - x^2 = -1$

Draw $y = -1$.

From the graph, $x = \mathbf{-0.3}$ or $x = \mathbf{3.3}$. **[1]**

(c)



[1]

11. **(a)** $A = \{2, 3, 5, 7\}$

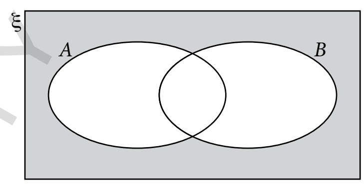
$B = \{1, 2, 5, 10\}$

$A' = \{1, 4, 6, 8, 9, 10\}$

$B' = \{3, 4, 6, 7, 8, 9\}$

$\therefore A' \cap B' = \mathbf{\{4, 6, 8, 9\}}$ **[1]**

(b)



[1]

(c) **(i)** $\{3, 5\} \subset A$ **[1]**

(ii) $2 \in B$ **[1]**

12. **(a)** $AB = \sqrt{[6-(-2)]^2 + (-3-9)^2}$

$= \sqrt{208}$

$=$ **14.4 units** (to 3 s.f.) **[1]**

(b) Gradient of $AB = \dfrac{-3-9}{6-(-2)} = -\dfrac{3}{2}$ **[1]**

Substitute $x = -2$, $y = 9$, $m = -\dfrac{3}{2}$ into $y = mx + c$:

$9 = -\dfrac{3}{2}(-2) + c$

$c = 6$

$\therefore$ Equation of AB: $\boldsymbol{y = -\dfrac{3}{2}x + 6}$ **[1]**

(c) $2x + 3y = 7$

$3y = -2x + 7$

$y = -\dfrac{2}{3}x + \dfrac{7}{3}$

Since the gradient of l is not $-\dfrac{3}{2}$, l is not parallel to AB.

$\therefore$ l intersects AB. **[1]**

13. **(a)** $\frac{5a^3b^2}{8} \times \frac{32a^3}{15b^5} = \mathbf{\frac{4a^6}{3b^3}}$ [2]

(b) $\frac{16x^2-4}{4x^2+1-4x} = \frac{4(4x^2-1)}{4x^2-4x+1}$

$= \frac{4(2x+1)(2x-1)}{(2x-1)^2}$ [1]

$= \mathbf{\frac{4(2x+1)}{2x-1}}$ [1]

14. **(a)** Total distance travelled = 330 m

$\frac{1}{2}(30)(v) = 330$ [1]

$v = 22$

∴ The greatest speed is **22 m/s**. [1]

(b) Gradient = $\frac{0-22}{30-25}$

$= -4.4$

∴ The deceleration is **4.4 m/s²**. [1]

(c) 8 m/s = $\frac{8 \div 1000}{1 \div 3600}$ km/h

= **28.8 km/h** [1]

Section B

15. **(a)** Breadth = $\mathbf{\frac{1}{2}x}$ **cm** [1]

Height = **(x – 2) cm** [1]

(b) $2\left[x\left(\frac{1}{2}x\right)+x(x-2)+\frac{1}{2}x(x-2)\right] = 108$ [1]

$2\left(\frac{1}{2}x^2+x^2-2x+\frac{1}{2}x^2-x\right) = 108$

$x^2 + 2x^2 - 4x + x^2 - 2x = 108$

$4x^2 - 6x - 108 = 0$

$2x^2 - 3x - 54 = 0$ (shown) [1]

(c) $2x^2 - 3x - 54 = 0$

$(2x + 9)(x - 6) = 0$ [1]

$x = -4.5$ or $x = 6$ [1]

∴ x = **–4.5** or x = **6**

(d) The length of the cuboid must be a positive value, so $x = -4.5$ must be rejected. [1]

(e) Volume of pyramid = (6)(3)(4)

= 72 cm³ [1]

Let the length of the square base be y cm.

$\frac{1}{3}y^2(7) = 72$ [1]

$y^2 = \frac{216}{7}$

$y = \sqrt{\frac{216}{7}}$

= 5.55 cm (to 3 s.f.)

∴ The length of each side of the square base is **5.55 cm**. [1]

16. **(a)** **(i)** **(a)** Estimated mean

$= \frac{2(7.5)+26(12.5)+48(17.5)+19(25)+5(45)}{100}$ [1]

= **18.8 minutes** [1]

(b) Estimated range

= 45 – 7.5

= **37.5 min** [1]

(ii) On average, the waiting time at clinic B was longer than at clinic A. [1]

However, there is a greater spread of the waiting time at clinic B. [1]

(iii) The correct median waiting time at clinic A is 1 minute less than the recorded waiting time. [1]

The correct range of waiting times remains unchanged. [1]

(b)

	Part-time staff	Full-time staff
Medically trained	**3**	4
Not medically trained	**12**	**1**

[3]

17. **(a)** **(i)** $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$= \begin{pmatrix} 1 \\ -4 \end{pmatrix} - \begin{pmatrix} -5 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} \mathbf{6} \\ \mathbf{-6} \end{pmatrix}$ [1]

(ii) $\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}$

$= \begin{pmatrix} 4 \\ k \end{pmatrix} - \begin{pmatrix} 1 \\ -4 \end{pmatrix}$

$= \begin{pmatrix} 3 \\ k+4 \end{pmatrix}$ [1]

Since $|\overrightarrow{BC}| = 5$,

$\sqrt{3^2 + (k+4)^2} = 5$ [1]

$9 + (k + 4)^2 = 25$

$9 + k^2 + 8k + 16 = 25$

$k^2 + 8k = 0$

$k(k + 8) = 0$

$k = 0$ or $k = -8$ [1]

∴ k = **0** or k = **–8**

(b) **(i)** $\overrightarrow{QP} = \overrightarrow{QX} + \overrightarrow{XP}$

$= \mathbf{\frac{1}{2}a - b}$ [2]

(ii) $\overrightarrow{PZ} = \overrightarrow{XY}$

$\overrightarrow{PS} + \overrightarrow{SZ} = \overrightarrow{XR} + \overrightarrow{RY}$ [1]

$\mathbf{a} + \overrightarrow{SZ} = \frac{1}{2}\mathbf{a} + \frac{1}{3}\left(\frac{1}{2}\mathbf{a} - \mathbf{b}\right)$ [2]

$\overrightarrow{SZ} = \frac{1}{2}\mathbf{a} + \frac{1}{6}\mathbf{a} - \frac{1}{3}\mathbf{b} - \mathbf{a}$

$= \mathbf{-\frac{1}{3}a - \frac{1}{3}b}$ [1]

18. **(a)** Volume of Type A = $\frac{1}{3}\pi(6)^2(5)$

= 60π cm³ [1]

Volume of Type B = $\frac{2}{3}\pi(4.5)^3$

= 60.75π cm³ [1]

∴ **Type B** has a greater capacity. [1]

(b) Let the radius of the circular surface of the mixture be p cm.

Using similar triangles,

$\frac{p}{6} = \frac{3.5}{5}$

$p = 4.2$

Volume of mixture $= \frac{1}{3}\pi(4.2)^2(3.5)$ **[1]**

$= 20.58\pi \text{ cm}^3$

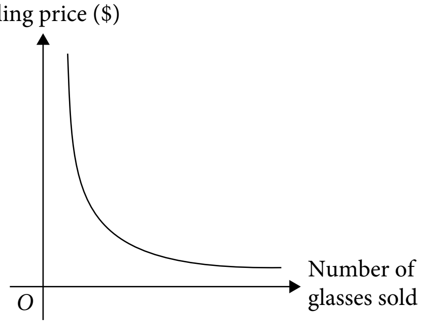
Percentage of glass filled $= \frac{20.58\pi}{60\pi} \times 100\%$ **[1]**

$= 34.3\%$

$\neq 70\%$ **[1]**

$\therefore$ Bryan is wrong.

(c) **(i)** Selling price ($)



[1]

(ii) Let $y = \frac{k}{x}$, where x represents the number of glasses sold and y represents the selling price of each glass. When $x = 80$, $y = 6$,

$6 = \frac{k}{80}$

$k = 480$

$\therefore y = \frac{480}{x}$

Let Bryan's estimated earnings after 30 days be $120 000.

Earnings per day $= \frac{\$120\ 000}{30} = \$4000$

Assume Bryan prices each glass at $5. **[1]**

Number of glasses sold per day $= \frac{480}{5} = 96$ **[1]**

Number of glasses sold in 30 days $= 96 \times 30 = 2880$ **[1]**

$\therefore$ If Bryan prices each glass at **$5**, and he would sell **2880 glasses** in a month and earn $120 000.

Mid-year Checkpoint B

Section A

1. $\frac{2479 \times \sqrt[3]{105}}{6^3 - 8^2} = \mathbf{80}$ (to 1 s.f.) **[1]**

2. **No.** There could be more accidents occurring at night than in the day, but fewer fatal accidents at night. **[1]**

3. $f(x) = \frac{2}{3x - 1}$

$g(x) = x^2$

When $gf(x) = 1$,

$g\left(\frac{2}{3x-1}\right) = 1$

$\left(\frac{2}{3x-1}\right)^2 = 1$ **[1]**

$\frac{2}{3x-1} = 1$ or $\frac{2}{3x-1} = -1$

$3x - 1 = 2$ $3x - 1 = -2$

$3x = 3$ $3x = -1$

$x = 1$ $x = -\frac{1}{3}$ **[1]**

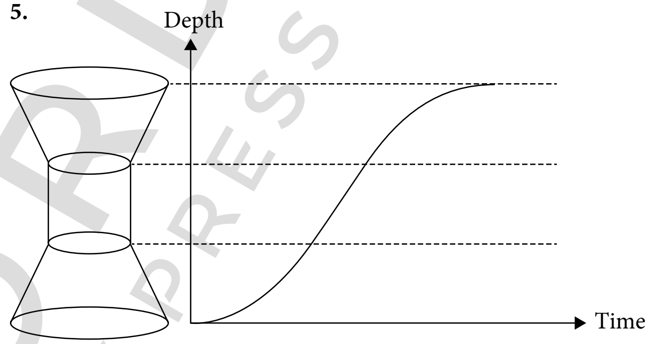
$\therefore x = \mathbf{1}$ or $x = -\mathbf{\frac{1}{3}}$

4. $14pq = (2 \times 7) \times (2^3 \times 5 \times 7^4) \times (2^2 \times 5 \times 7^3)$

$= 2^6 \times 5^2 \times 7^8$ **[1]**

Since the powers of the prime factors are multiples of 2, $14pq$ is a perfect square. **[1]**

5.



[2]

6. red : blue : yellow

9 : 12 : 13

18 : 24 : 26 **[1]**

5 units represent 20 counters. **[1]**

1 unit represents 4 counters.

24 units represent 96 counters.

$\therefore$ There are **96** blue counters in the box. **[1]**

7. **(a)** $8^7 = (2^3)^7$

$= \mathbf{2^{21}}$ **[1]**

(b) $\frac{9xy^2}{16} \div \frac{3x}{4y^3} = \frac{9xy^2}{16} \times \frac{4y^3}{3x}$ **[1]**

$= \mathbf{\frac{3y^5}{4}}$ **[1]**

8.

(a) $PQ = \sqrt{(h-4)^2 + [k-(-7)]^2}$ **[1]**

$9 = \sqrt{(h-4)^2 + (k+7)^2}$

$81 = (h - 4)^2 + (k + 7)^2$

Let $h = 4$:

$(k + 7)^2 = 81$

$k + 7 = -9$ (since $hk < 0$)

$k = -16$

$\therefore$ A possible pair of coordinates is **Q(4, –16)**. **[1]**

(b) PQ is a vertical line.

$\therefore$ The gradient of PQ is **undefined.** **[1]**

9. $$x^2 = \frac{y^2+4}{y^2-9}$$
$$x^2(y^2-9) = y^2+4$$
$$x^2y^2 - 9x^2 = y^2+4$$
$$x^2y^2 - y^2 = 9x^2+4 \quad [1]$$
$$y^2(x^2-1) = 9x^2+4$$
$$y^2 = \frac{9x^2+4}{x^2-1} \quad [1]$$
$$\boldsymbol{y = \pm\sqrt{\frac{9x^2+4}{x^2-1}}} \quad [1]$$

10. Consider plan A.

Interest $= \$\left[\frac{80\ 000(2.5)(20)}{100}\right]$

$= \$40\ 000$ **[1]**

Consider plan B.

Interest $= \$\left[80\ 000\left(1+\frac{1}{100}\right)^{40} - 80\ 000\right]$

$= \$39\ 109.0987$ (to 4 d.p.) **[1]**

Difference in interest = $40 000 – $39 109.0987

= $890.90 (to 2 d.p.)

∴ **Plan A** yields a higher interest of **$890.90**. **[1]**

11. (a) $4x^2 + 5x - 6 = \boldsymbol{(4x-3)(x+2)}$ **[2]**

(b) Replace x with $4y - 1$:

$4(4y-1)^2 + 5(4y-1) - 6 = [4(4y-1)-3][(4y-1)+2]$

$= (16y - 4 - 3)(4y - 1 + 2)$

$= \boldsymbol{(16y-7)(4y+1)}$ **[1]**

12. (a) 1 cm represents 400 cm = 4 m.

120 m is represented by $\frac{120}{4} = 30$ cm.

76 m is represented by $\frac{76}{4} = 19$ cm.

∴ Area of the plantation on the scale drawing

$= 30 \times 19$ **[1]**

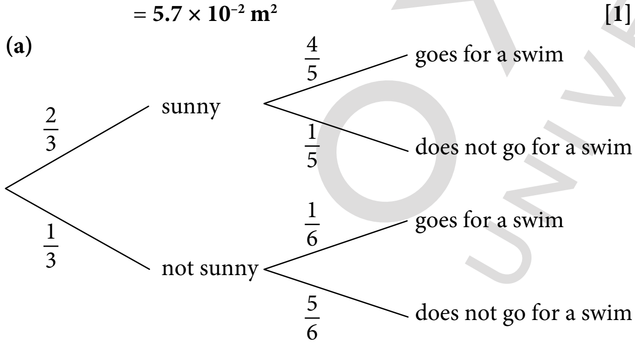
$= \boldsymbol{570\ \text{cm}^2}$ **[1]**

(b) $570\ \text{cm}^2 = 570 \times (10^{-2})^2\ \text{m}^2$ **[1]**

$= 570 \times 10^{-4}\ \text{m}^2$

$= \boldsymbol{5.7 \times 10^{-2}\ \text{m}^2}$ **[1]**

13. (a)

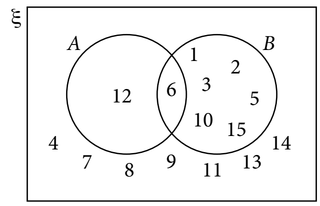


[3]

(b) P(Joe goes for a swim) $= \frac{2}{3}\times\frac{4}{5}+\frac{1}{3}\times\frac{1}{6}$ **[1]**

$= \boldsymbol{\frac{53}{90}}$ **[1]**

14. (a) (i)



[2]

(ii) $n(A \cup B) = \boldsymbol{8}$ **[1]**

(iii) $A' \cap B' = \boldsymbol{\{4, 7, 8, 9, 11, 13, 14\}}$ **[1]**

(b) $\boldsymbol{B \cup (A \cap C)}$ **[1]**

Section B

15. (a) $\frac{3}{4}(1-2x) \leqslant 5 - x$

$3 - 6x \leqslant 20 - 4x$

$-2x \leqslant 17$

$x \geqslant -8.5$ **[1]**

∴ Smallest possible value of $x^2 = 0^2 = \boldsymbol{0}$ **[1]**

(b) $16xy^2 - 36 - 24xy + 24y = 4(4xy^2 - 9 - 6xy + 6y)$

$= 4(4xy^2 - 6xy + 6y - 9)$

$= 4[2xy(2y-3) + 3(2y-3)]$ **[1]**

$= \boldsymbol{4(2y-3)(2xy+3)}$ **[1]**

(c) $5x - 10y = 8$ —— (1)

$2x + 4y = 7$ —— (2)

From (2),

$2x = 7 - 4y$

$x = 3.5 - 2y$ —— (3)

Substitute (3) into (1):

$5(3.5 - 2y) - 10y = 8$ **[1]**

$17.5 - 10y - 10y = 8$

$20y = 9.5$

$y = 0.475$ **[1]**

Substitute $y = 0.475$ into (3):

$x = 3.5 - 2(0.475)$

$= 2.55$ **[1]**

∴ $\boldsymbol{x = 2.55, y = 0.475}$

(d) $\frac{3}{2x+3} - \frac{x}{9-4x^2} = \frac{3}{2x+3} + \frac{x}{4x^2-9}$

$= \frac{3}{2x+3} + \frac{x}{(2x+3)(2x-3)}$ **[1]**

$= \frac{3(2x-3)+x}{(2x+3)(2x-3)}$ **[1]**

$= \frac{6x-9+x}{(2x+3)(2x-3)}$

$= \boldsymbol{\frac{7x-9}{(2x+3)(2x-3)}}$ **[1]**

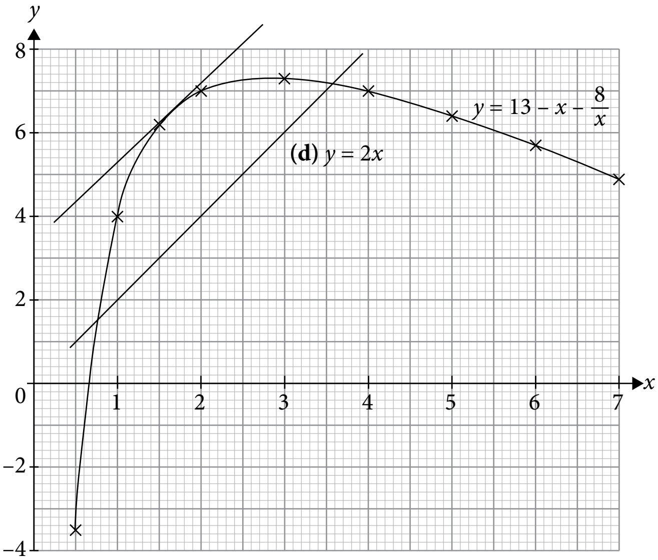
16. (a) When $x = 3$,

$y = 13 - 3 - \frac{8}{3}$

$= 7.3$ (to 1 d.p.) [1]

x	0.5	1	1.5	2	3	4	5	6	7
y	–3.5	4	6.2	7	**7.3**	7	6.4	5.7	4.9

(b)



[3]

(c) From the graph, the range of values is $\mathbf{1.15 < x < 6.85}$. **[1]**

(d) $13 - 3x - \frac{8}{x} = 0$

$13 - x - \frac{8}{x} = 2x$

Draw $y = 2x$. **[1]**

From the graph, $x = \mathbf{0.75}$ or $x = \mathbf{3.6}$. **[1]**

(e) From the graph, the coordinates are **(1.65, 6.5)**. **[2]**

(f) OPEN $x + \frac{8}{x} = k$

$-x - \frac{8}{x} = -k$

$13 - x - \frac{8}{x} = 13 - k$

For $x + \frac{8}{x} = k$ to have no solution, $y = 13 - k$ does not intersect $y = 13 - x - \frac{8}{x}$.

$\therefore$ A possible value is $k = \mathbf{2}$. **[1]**

17. (a) (i) Estimated median = **\$390 000** **[1]**

(ii) Estimated interquartile range = \$480 000 – \$330 000 **[1]**

= **\$150 000** **[1]**

(b) Percentage of companies that budgeted more than the average

$= \frac{80 - 45}{80} \times 100\%$ **[1]**

= **43.75%** **[1]**

(c) Estimated probability $= \frac{5}{80} \times \frac{9}{79} + \frac{9}{80} \times \frac{5}{79}$ **[1]**

$= \mathbf{\frac{9}{632}}$ **[1]**

(d) (i) Actual median = **\$690 000**

Actual lower quartile = **\$630 000**

Actual upper quartile = **\$780 000**

Actual minimum = **\$360 000**

Actual maximum = **\$1 100 000** **[2]**

(ii) The curve representing the actual advertising expenditure will be on the right side of the curve representing the budget. **[1]**

18. (a) (i) $\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}$

$= \frac{9}{5}\overrightarrow{OA} - \overrightarrow{OB}$ **[1]**

$= \mathbf{\frac{9}{5}a - b}$ **[1]**

(ii) $\overrightarrow{PA} = \frac{2}{5}\overrightarrow{BA}$ **[1]**

$= \frac{2}{5}(\overrightarrow{OA} - \overrightarrow{OB})$

$= \mathbf{\frac{2}{5}(a - b)}$ **[1]**

(b) For $BCQP$ to be a trapezium, PQ // BC and $\triangle APQ$ is similar to $\triangle ABC$.

$\frac{PQ}{BC} = \frac{AP}{AB} = \frac{2}{5}$ **[1]**

$\therefore \overrightarrow{PQ} = \frac{2}{5}\overrightarrow{BC}$ **[1]**

$= \frac{2}{5}\left(\frac{9}{5}\mathbf{a} - \mathbf{b}\right)$

$= \mathbf{\frac{18}{25}a - \frac{2}{5}b}$ **[1]**

(c) Area of $\triangle OBC = \frac{1}{2}(k)\left(\frac{9}{5}k\right)\sin(180° - 95° - 40°)$ **[2]**

$= \mathbf{0.64k^2}$ **units²** (to 2 d.p.) **[1]**

6 Further Trigonometry

Worksheet 6A Sine and cosine of obtuse angles

1. (a) sin 150° = **0.5**

(b) sin 149° = **0.515** (to 3 s.f.)

(c) cos 120° = **−0.5**

(d) cos 149° = **−0.857** (to 3 s.f.)

2. (a) sin 100° = sin (180° − 100°)

= **sin 80°**

(b) sin 165° = sin (180° − 165°)

= **sin 15°**

(c) cos 147° = −cos (180° − 147°)

= **−cos 33°**

(d) cos 98° = −cos (180° − 98°)

= **−cos 82°**

3. (a) Since sin 79° > 0 and cos 155° < 0,

then sin 79° × cos 155° < 0.

$\therefore$ It gives a **negative** value.

(b) Since sin 128° > 0 and cos 36° > 0,

then $\frac{\sin 128°}{\cos 36°} > 0$.

$\therefore$ It gives a **positive** value.

(c) Since sin 140° > 0 and $(\cos 93°)^2 > 0$,

then sin 140° + $(\cos 93°)^2 > 0$.

$\therefore$ It gives a **positive** value.

(d) Since cos 116° < 0 and sin 54° > 0,

then cos 116° − sin 54° < 0

and $(\cos 116° - \sin 54°)^3 < 0$.

$\therefore$ It gives a **negative** value.

4. $\sin 130° + \cos 56° = \sin (180° - 130°) + [-\cos (180° - 56°)]$
$= \sin 50° - \cos 124°$
$= 0.766 - (-0.559)$
$= 0.766 + 0.559$
$= \mathbf{1.325}$

5. $\dfrac{8\sin 42°}{3\cos 101°} = \dfrac{8\sin(180° - 42°)}{3[-\cos(180° - 101°)]}$
$= \dfrac{8\sin 138°}{-3\cos 79°}$
$= \dfrac{8p}{-3(-q)}$
$= \mathbf{\dfrac{8p}{3q}}$

6. **(a)** Using Pythagoras' Theorem,
$AC^2 = 9^2 + 12^2$
$= 225$
$AC = \sqrt{225}\ (AC > 0)$
$= 15$ cm (shown)

(b) **(i)** $\sin \angle ACD = \sin (180° - \angle ACB)$
$= \sin \angle ACB$
$= \dfrac{9}{15}$
$= \mathbf{\dfrac{3}{5}}$

(ii) $\cos \angle ACD = \cos (180° - \angle ACB)$
$= -\cos \angle ACB$
$= -\dfrac{12}{15}$
$= \mathbf{-\dfrac{4}{5}}$

7. **(a)** Let D be the point (5, −2).
Using Pythagoras' Theorem,
$AC^2 = 3^2 + 4^2$
$= 25$
$AC = \sqrt{25}\ (AC > 0)$
$= 5$ units
$\sin \angle ACB = \sin (180° - \angle ACD)$
$= \sin \angle ACD$
$= \mathbf{\dfrac{3}{5}}$

(b) $\cos \angle ACB = \cos (180° - \angle ACD)$
$= -\cos \angle ACD$
$= \mathbf{-\dfrac{4}{5}}$

(c) $\tan \angle ABC = \dfrac{3}{9}$
$= \mathbf{\dfrac{1}{3}}$

8. $\sin A° = 0.6$
$A° = 180° - \sin^{-1} 0.6$
$= 143.1°$ (to 1 d.p.)
$\therefore A = \mathbf{143.1}$

9. OPEN
$\sin \theta = 0.1357$
$\theta = \sin^{-1} 0.1357$ or $180° - \sin^{-1} 0.1357$
$= 7.8°$ or $172.2°$ (to 1 d.p.)
$\therefore$ The angle could be **7.8°** or **172.2°**.

10. **(a)** $\cos y° = -\cos \angle ABD$
$= \mathbf{-\dfrac{7}{x}}$

(b) Area of $ABCD = 4 \times$ area of $\triangle ABD$
$\dfrac{1}{2}(7 + DC)(AD) = 4 \times \dfrac{1}{2}(7)(AD)$
$7 + DC = 28$
$DC = \mathbf{21\ cm}$

Challenge Myself!

11. OPEN Let $A = 135°$.
$\sin 135° = \sin 45°$
$= \cos 45°$
$= -\cos 135°$
$\sin 135° + \cos 135° = 0$
$\therefore$ A possible pair of values is $\mathbf{A = 135°}$ and $\mathbf{B = 135°}$.

Worksheet 6B Area of triangle

1. **(a)** Area of triangle $= \dfrac{1}{2}(4)(6)\sin 55°$
$= \mathbf{10\ cm^2}$ (to the nearest integer)

(b) Area of triangle $= \dfrac{1}{2}(7)(7)\sin 103°$
$= \mathbf{24\ cm^2}$ (to the nearest integer)

(c) Area of triangle $= \dfrac{1}{2}(8.8)(8.8)\sin 60°$
$= \mathbf{34\ cm^2}$ (to the nearest integer)

(d) Area of triangle $= \dfrac{1}{2}(9.5)(7.1)\sin(180° - 40° - 120°)$
$= \mathbf{12\ cm^2}$ (to the nearest integer)

2. **(a)** Area of quadrilateral $= 2 \times \dfrac{1}{2}(3.9)(6.3)\sin 58°$
$= \mathbf{20.8\ cm^2}$ (to 3 s.f.)

(b) Area of quadrilateral $= 2 \times \dfrac{1}{2}(17)(17)\sin 104°$
$= \mathbf{280\ cm^2}$ (to 3 s.f.)

3. **(a)** $\angle BAC = 180° - 90° - 63°$ (∠ sum of a △)
$= 27°$
Area of $\triangle ABC = \dfrac{1}{2}(7.9)(8.9)\sin 27°$
$= 16\ cm^2$ (to 2 s.f.)

(b) Using Pythagoras' Theorem,
$7.9^2 + BC^2 = 8.9^2$
$BC^2 = 8.9^2 - 7.9^2$
$= 16.8$
$BC = \sqrt{16.8}\ (BC > 0)$
$= 4.0988$ cm (to 5 s.f.)
Area of $\triangle ABC = \dfrac{1}{2}(7.9)(4.0988)$
$= 16\ cm^2$ (to 2 s.f.)

4. **(i)** Area of $\triangle ABC = \dfrac{1}{2}(26)(28)\sin 43°$
$= \mathbf{248\ cm^2}$ (to 3 s.f.)

(ii) Let the shortest distance from A to BC be h cm.

$$\frac{1}{2}(28)h = 248.25$$
$$h = 17.7 \text{ (to 3 s.f.)}$$

$\therefore$ Shortest distance from A to BC = **17.7 cm**

(iii) Let the shortest distance from A to BC be h cm.

$$\sin 43° = \frac{h}{26}$$
$$h = 26 \sin 43°$$
$$= 17.7 \text{ (to 3 s.f.)}$$

$\therefore$ Shortest distance from A to BC = **17.7 cm**

5. **(i)** Using Pythagoras' Theorem,

$$AC^2 = 35^2 + 67^2$$
$$= 5714$$
$$AC = \sqrt{5714} \ (AC > 0)$$
$$= 75.591 \text{ cm (to 5 s.f.)}$$

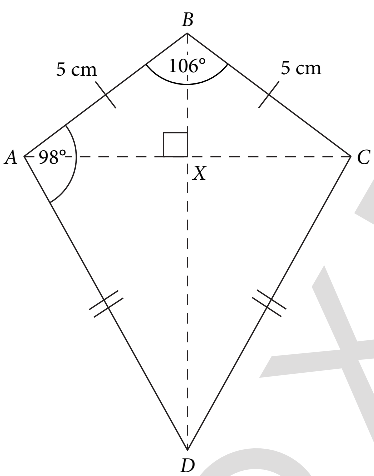
$$\text{Area of } ABCD = \frac{1}{2}(35)(67) + \frac{1}{2}(75.591)(90)\sin 41°$$
$$= \mathbf{3400 \text{ cm}^2} \text{ (to 3 s.f.)}$$

(ii) Let the perpendicular distance from C to AD be h cm.

$$\sin 41° = \frac{h}{75.591}$$
$$h = 75.591 \sin 41°$$
$$= 49.6 \text{ (to 3 s.f.)}$$

$\therefore$ Perpendicular distance from C to AD = **49.6 cm**

6.



(i) **Kite**

(ii) $\sin\frac{106°}{2} = \frac{AX}{5}$

$$AX = 5 \sin 53°$$
$$= 3.9932 \text{ cm (to 5 s.f.)}$$

$\therefore AC = 2(3.9932)$

$= 7.99$ cm (to 3 s.f.) (shown)

(iii) $\angle BAC = \frac{180° - 106°}{2}$ (base $\angle$s of isos. $\triangle$)

$= 37°$

$\angle DAX = 98° - 37°$

$= 61°$

$$\cos 61° = \frac{3.9932}{AD}$$
$$AD = \frac{3.9932}{\cos 61°}$$
$$= 8.2366 \text{ cm (to 5 s.f.)}$$

$$\therefore \text{Area of } ABCD = 2 \times \frac{1}{2}(5)(8.2366)\sin 98°$$
$$= \mathbf{40.8 \text{ cm}^2} \text{ (to 3 s.f.)}$$

7. Area of $\triangle PQR = 95.4$ cm²

$$\frac{1}{2}(17.6)(23.8)\sin\angle PQR = 95.4$$
$$\sin\angle PQR = 0.455\ 50 \text{ (to 5 s.f.)}$$
$$\angle PQR = 27.1° \text{ (to 1 d.p.) or } 152.9° \text{ (to 1 d.p.)}$$

$\therefore \angle PQR$ = **27.1°** or **152.9°**

8. **(a)** Area of $\triangle ABC = 117$ cm²

$$\frac{1}{2}(18)(x)\sin 60° = 117$$
$$x = \mathbf{15.0} \text{ (to 3 s.f.)}$$

(b) Using similar triangles,

$$\frac{PR}{AC} = \frac{PQ}{AB}$$
$$\frac{PR}{15.011} = \frac{12}{18}$$
$$PR = \mathbf{10 \text{ cm}} \text{ (to the nearest cm)}$$

9. $\angle AOB = \frac{360°}{5}$ ($\angle$s at a pt.)

$= 72°$

$$\text{Area of pentagon} = 5 \times \text{area of } \triangle OAB$$
$$= 5 \times \frac{1}{2}(x)^2 \sin 72°$$
$$= \mathbf{2.38x^2 \text{ cm}^2} \text{ (to 3 s.f.)}$$

10. $\angle BCD = \angle QRS = 83°$

Using similar triangles,

$$\frac{CD}{RS} = \frac{BC}{QR}$$
$$\frac{CD}{8} = \frac{8.4}{6}$$
$$CD = 11.2 \text{ cm}$$

$$\text{Area of } \triangle BCD = \frac{1}{2}(8.4)(11.2)\sin 83°$$
$$= \mathbf{46.7 \text{ cm}^2} \text{ (to 3 s.f.)}$$

11. $CD = \frac{3}{2}(4k) = 6k$ cm

$$\text{Area of } ABCD = 2 \times \text{area of } \triangle ACD$$
$$= 2 \times \frac{1}{2}(4k)(6k)\sin 110°$$
$$= 24k^2 \sin 110°$$
$$= \mathbf{23k^2 \text{ cm}^2} \text{ (to 2 s.f.)}$$

Challenge Myself!

12. Let the length of each stick be x units.

Perimeter of hexagon = 12 units,
i.e. length of each side of hexagon = 2 units

Perimeter of triangle = 18 units,
i.e. length of each side of triangle = 6 units

$$\text{Area of hexagon} = 6 \times \frac{1}{2}(2)(2)\sin 60°$$
$$= 12 \sin 60° \text{ units}^2$$

Area of triangle $= \frac{1}{2}(6)(6)\sin 60°$

$= 18 \sin 60°$ units^2

$\therefore$ Area of triangle : area of hexagon $= 18 \sin 60° : 12 \sin 60°$

$= \mathbf{3 : 2}$

Worksheet 6C Sine Rule

1. **(a)** Using Sine Rule,

$$\frac{x}{\sin 61°} = \frac{8}{\sin 41°}$$

$$x = \frac{8\sin 61°}{\sin 41°}$$

$= 10.7$ (to 3 s.f.)

$\therefore x = \mathbf{10.7}$

(b) Using Sine Rule,

$$\frac{x}{\sin(180° - 23° - 54°)} = \frac{7}{\sin 23°}$$

$$x = \frac{7\sin 103°}{\sin 23°}$$

$= 17.5$ (to 3 s.f.)

$\therefore x = \mathbf{17.5}$

(c) Using Sine Rule,

$$\frac{\sin y°}{21} = \frac{\sin 70°}{37}$$

$$\sin y° = \frac{21\sin 70°}{37}$$

$= 0.533\ 34$ (to 5 s.f.)

$y° = \sin^{-1} 0.533\ 34$

$= 32.2°$ (to 1 d.p.)

$\therefore y = \mathbf{32.2}$

(d) Using Sine Rule,

$$\frac{\sin \angle TSU}{32} = \frac{\sin 74°}{35}$$

$$\sin \angle TSU = \frac{32\sin 74°}{35}$$

$= 0.878\ 87$ (to 5 s.f.)

$\angle TSU = \sin^{-1} 0.878\ 87$

$= 61.506°$ (to 3 d.p.)

$y° = 180° - 74° - 61.506°$ ($\angle$ sum of a $\triangle$)

$= 44.5°$ (to 1 d.p.)

$\therefore y = \mathbf{44.5}$

2. **(i)** Using Sine Rule,

$$\frac{\sin \angle ACB}{10} = \frac{\sin 80°}{17.2}$$

$$\sin \angle ACB = \frac{10\sin 80°}{17.2}$$

$= 0.572\ 56$ (to 5 s.f.)

$\angle ACB = \sin^{-1} 0.572\ 56$

$= 34.929°$ (to 3 d.p.)

$\angle ABC = 180° - 80° - 34.929°$ ($\angle$ sum of a $\triangle$)

$= 65.071°$ (to 3 d.p.)

Using Sine Rule,

$$\frac{AC}{\sin 65.071°} = \frac{17.2}{\sin 80°}$$

$$AC = \frac{17.2\sin 65.071°}{\sin 80°}$$

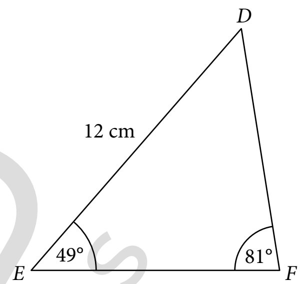
$= 15.8$ cm (to 3 s.f.)

$\therefore \angle ABC = \mathbf{65.1°}$, $\angle ACB = \mathbf{34.9°}$, $AC = \mathbf{15.8\ cm}$

(ii) Area of triangle $= \frac{1}{2}(10)(17.2)\sin 65.071°$

$= \mathbf{78.0\ cm^2}$ (to 3 s.f.)

3.



Using Sine Rule,

$$\frac{DF}{\sin \angle DEF} = \frac{DE}{\sin \angle DFE}$$

$$\frac{DF}{\sin 49°} = \frac{12}{\sin 81°}$$

$$DF = \frac{12\sin 49°}{\sin 81°}$$

$= 9.17$ cm (to 3 s.f.)

Using Sine Rule,

$$\frac{EF}{\sin \angle EDF} = \frac{DE}{\sin \angle DFE}$$

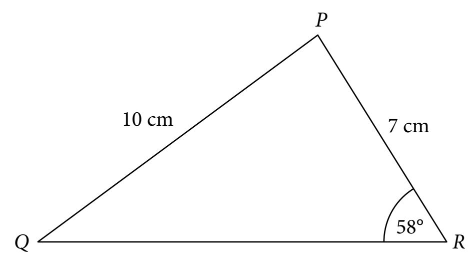
$$\frac{EF}{\sin(180° - 49° - 81°)} = \frac{12}{\sin 81°}$$

$$EF = \frac{12\sin 50°}{\sin 81°}$$

$= 9.31$ cm (to 3 s.f.)

$\therefore DF = \mathbf{9.17\ cm}$, $EF = \mathbf{9.31\ cm}$

4.



Using Sine Rule,

$$\frac{\sin \angle PQR}{PR} = \frac{\sin \angle PRQ}{PQ}$$

$$\frac{\sin \angle PQR}{7} = \frac{\sin 58°}{10}$$

$$\sin \angle PQR = \frac{7\sin 58°}{10}$$

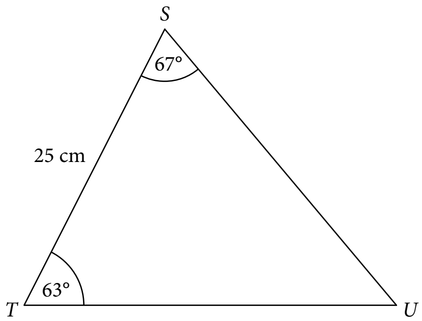
$= 0.593\ 63$ (to 5 s.f.)

$\angle PQR = \sin^{-1} 0.593\ 63$

$= 36.4°$ (to 1 d.p.)

$\therefore \angle PQR = \mathbf{36.4°}$

5.



(i) $\angle SUT = 180° - 63° - 67°$ ($\angle$ sum of a $\triangle$)

$= 50°$

The shortest side of a triangle is opposite the smallest angle. Since $\angle SUT$ is the smallest angle, then ST is the shortest side.

(ii) Since $\angle TSU$ is the largest angle, then TU is the longest side.

Using Sine Rule,

$$\frac{TU}{\sin \angle TSU} = \frac{ST}{\sin \angle SUT}$$

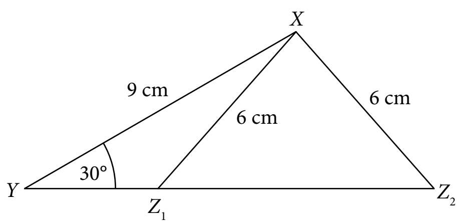
$$\frac{TU}{\sin 67°} = \frac{25}{\sin 50°}$$

$$TU = \frac{25 \sin 67°}{\sin 50°}$$

$$= 30.0 \text{ cm (to 3 s.f.)}$$

$\therefore$ Length of the longest side = **30.0 cm**

6.



Using Sine Rule,

$$\frac{\sin \angle XZY}{9} = \frac{\sin 30°}{6}$$

$$\sin \angle XZY = \frac{9 \sin 30°}{6}$$

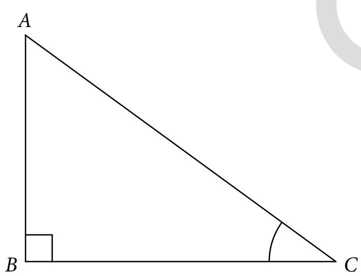
$$= 0.75$$

$$\angle XZY = \sin^{-1} 0.75 \text{ or } 180° - \sin^{-1} 0.75$$

$$= 48.6° \text{ or } 131.4° \text{ (to 1 d.p.)}$$

$\therefore$ $\angle XZY$ = **48.6°** or **131.4°**

7.



Consider $\triangle ABC$, where $\angle ABC = 90°$.

Assume we are given $\angle ACB$, then $\angle BAC = 180° - 90° - \angle ACB$.

If we are given the length of AB, then using the Sine Rule,

$$\frac{AC}{\sin 90°} = \frac{AB \text{ (known)}}{\sin \angle ACB \text{ (known)}} \text{ and}$$

$$\frac{BC}{\sin \angle BAC \text{ (known)}} = \frac{AB \text{ (known)}}{\sin \angle ACB \text{ (known)}}$$

If we are given the length of BC, then using the Sine Rule,

$$\frac{AC}{\sin 90°} = \frac{BC \text{ (known)}}{\sin \angle BAC \text{ (known)}} \text{ and}$$

$$\frac{AB}{\sin \angle ACB \text{ (known)}} = \frac{BC \text{ (known)}}{\sin \angle BAC \text{ (known)}}$$

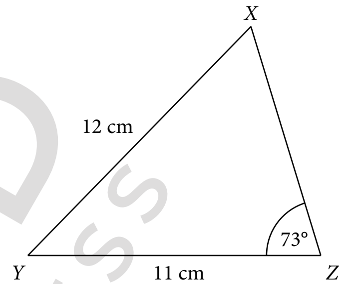
If we are given the length of AC, then using the Sine Rule,

$$\frac{AB}{\sin \angle ACB \text{ (known)}} = \frac{AC \text{ (known)}}{\sin 90°} \text{ and}$$

$$\frac{BC}{\sin \angle BAC \text{ (known)}} = \frac{AC \text{ (known)}}{\sin 90°}$$

$\therefore$ I agree with the student.

8. (i)



Using Sine Rule,

$$\frac{\sin \angle YXZ}{11} = \frac{\sin 73°}{12}$$

$$\sin \angle YXZ = \frac{11 \sin 73°}{12}$$

$$= 0.876\ 61 \text{ (to 5 s.f.)}$$

$$\angle YXZ = \sin^{-1} 0.876\ 61$$

$$= 61.236° \text{ (to 3 d.p.)}$$

Using Sine Rule,

$$\frac{XZ}{\sin(180° - 73° - 61.236°)} = \frac{12}{\sin 73°}$$

$$XZ = \frac{12 \sin 45.764°}{\sin 73°}$$

$$= 8.9904 \text{ cm (to 5 s.f.)}$$

$\therefore$ Perimeter of triangle $= 12 + 11 + 8.9904$

≈ 32 cm (shown)

(ii) Area of triangle $= \frac{1}{2}(12)(11)\sin 45.764°$

$\approx 47 \text{ cm}^2$ (shown)

9. (i) $\angle CAD = 180° - 32° - 56°$ (int. $\angle$s, AB // DC)

= **92°**

(ii) Using Sine Rule,

$$\frac{AC}{\sin 56°} = \frac{17.4}{\sin 92°}$$

$$AC = \frac{17.4 \sin 56°}{\sin 92°}$$

= **14.4 cm** (to 3 s.f.)

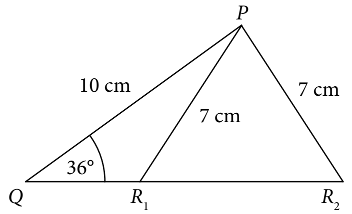
(iii) Area of $ABCD$

$$= \frac{1}{2}(14.434)(17.4)\sin 32° + \frac{1}{2}(14.434)(11.2)\sin 32°$$

= **109 cm²** (to 3 s.f.)

Challenge Myself!

10.



From the scale drawing, QR = **4.3 cm** or QR = **11.9 cm**.

Worksheet 6D Cosine Rule

1. **(a)** Using Cosine Rule,

$x^2 = 5.1^2 + 7.4^2 - 2(5.1)(7.4)\cos 60°$
$= 43.03$
$x = \sqrt{43.03}$ $(x > 0)$
$= 6.56$ (to 3 s.f.)
$\therefore x =$ **6.56**

(b) Using Cosine Rule,

$x^2 = 23^2 + 13^2 - 2(23)(13)\cos 108°$
$= 882.79$ (to 5 s.f.)
$x = \sqrt{882.79}$ $(x > 0)$
$= 29.7$ (to 3 s.f.)
$\therefore x =$ **29.7**

(c) Using Cosine Rule,

$25.5^2 = 28^2 + 12^2 - 2(28)(12)\cos y°$
$650.25 = 928 - 672\cos y°$
$672\cos y° = 277.75$
$\cos y° = 0.413\ 32$ (to 5 s.f.)
$y° = \cos^{-1} 0.413\ 32$
$= 65.6°$ (to 1 d.p.)
$\therefore y =$ **65.6**

(d) Using Cosine Rule,

$124^2 = 62^2 + 89^2 - 2(62)(89)\cos y°$
$15\ 376 = 11\ 765 - 11\ 036\cos y°$
$11\ 036\cos y° = -3611$
$\cos y° = -0.327\ 20$ (to 5 s.f.)
$y° = \cos^{-1}(-0.327\ 20)$
$= 109.1°$ (to 1 d.p.)
$\therefore y =$ **109.1**

2. **(i)** Using Cosine Rule,

$12.5^2 = 9.4^2 + 5.6^2 - 2(9.4)(5.6)\cos \angle BAC$
$156.25 = 119.72 - 105.28\cos \angle BAC$
$105.28\cos \angle BAC = -36.53$
$\cos \angle BAC = -0.346\ 98$ (to 5 s.f.)
$\angle BAC = \cos^{-1}(-0.346\ 98)$
$= 110.303°$ (to 3 d.p.)

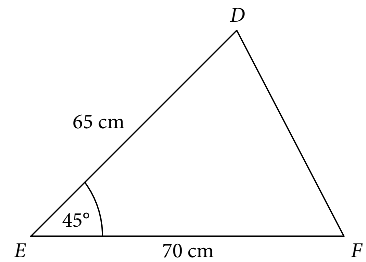
Using Sine Rule,

$\frac{\sin \angle ABC}{5.6} = \frac{\sin 110.303°}{12.5}$
$\sin \angle ABC = \frac{5.6 \sin 110.303°}{12.5}$
$= 0.420\ 17$ (to 5 s.f.)
$\angle ABC = \sin^{-1} 0.420\ 17$
$= 24.845°$ (to 3 d.p.)

$\angle ACB = 180° - 110.303° - 24.845°$ (∠ sum of a △)
$= 44.9°$ (to 1 d.p.)
$\therefore \angle ABC =$ **24.8°**, $\angle ACB =$ **44.9°**, $\angle BAC =$ **110.3°**

(ii) Area of triangle $= \frac{1}{2}(9.4)(12.5)\sin 24.845°$
$=$ **24.7 cm²** (to 3 s.f.)

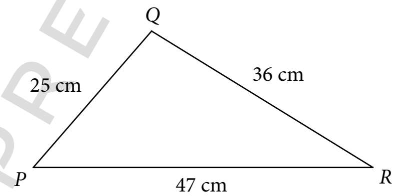
3.



Using Cosine Rule,

$DF^2 = 65^2 + 70^2 - 2(65)(70)\cos 45°$
$= 2690.3$ (to 5 s.f.)
$DF = \sqrt{2690.3}$ $(DF > 0)$
$= 51.9$ cm (to 3 s.f.)
$\therefore DF =$ **51.9 cm**

4.



Using Cosine Rule,

$25^2 = 36^2 + 47^2 - 2(36)(47)\cos \angle PRQ$
$625 = 3505 - 3384\cos \angle PRQ$
$3384\cos \angle PRQ = 2880$
$\cos \angle PRQ = \frac{40}{47}$
$\angle PRQ = \cos^{-1}\frac{40}{47}$
$= 31.7°$ (to 1 d.p.)
$\therefore \angle PRQ =$ **31.7°**

5. **(i)** Using Cosine Rule,

$93^2 = x^2 + 41^2 - 2(x)(41)\cos 85°$
$8649 = x^2 + 1681 - 7.1468x$
$x^2 - 7.1468x - 6968 = 0$
$x = \frac{-(-7.1468) \pm \sqrt{(-7.1468)^2 - 4(1)(-6968)}}{2(1)}$
$= \frac{7.1468 \pm \sqrt{27\ 923}}{2}$
$= 87.1$ or -80.0 (to 3 s.f.)
$\therefore x =$ **87.1**

(ii) Let the shortest distance from S to TU be h cm.

$\frac{1}{2}(93)(h) = \frac{1}{2}(87.124)(41)\sin 85°$
$h = 38.3$ (to 3 s.f.)
$\therefore$ Shortest distance from S to TU = **38.3 cm**

6. **(a)** **(i)** $\angle ABC = 180° - \angle DAP$ (int. ∠s, DA // CB)
$= 180° - 71°$
$=$ **109°**

(ii) $\angle PCD = 71° - 23°$
$=$ **48°**

(b) Area of $ABCD = 2 \times$ area of $\triangle BCD$

$$= 2 \times \frac{1}{2}(BC)(CD)\sin\angle BCD$$
$$= (5.6)(9.4)\sin 71^\circ$$
$$= \mathbf{49.8\ cm^2} \text{ (to 3 s.f.)}$$

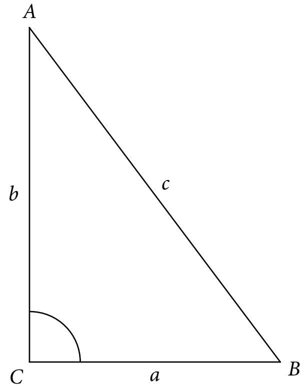
(c) Using Sine Rule,

$$\frac{CP}{\sin\angle PBC} = \frac{BC}{\sin\angle BPC}$$
$$\frac{CP}{\sin 109^\circ} = \frac{5.6}{\sin(180^\circ - 109^\circ - 23^\circ)}$$
$$CP = \frac{5.6\sin 109^\circ}{\sin 48^\circ}$$
$$= 7.125 \text{ cm (to 4 s.f.) (shown)}$$

(d) Using Cosine Rule,

$$DP^2 = CD^2 + CP^2 - 2(CD)(CP)\cos\angle PCD$$
$$= 9.4^2 + 7.1250^2 - 2(9.4)(7.1250)\cos 48^\circ$$
$$= 49.496 \text{ (to 5 s.f.)}$$
$$DP = \sqrt{49.496} \quad (DP > 0)$$
$$= \mathbf{7.04\ cm} \text{ (to 3 s.f.)}$$

7.



Consider $\triangle ABC$.

Using Cosine Rule,

$$c^2 = a^2 + b^2 - 2ab\cos\hat{C}$$

Let $\hat{C} = 90^\circ$.

$$c^2 = a^2 + b^2 - 2ab\cos 90^\circ$$
$$= a^2 + b^2 - 2ab(0)$$
$$= a^2 + b^2$$

$\therefore$ I agree with Owen.

8. Given that $AB = 20$ cm, $BC = 30$ cm and $AC = 40$ cm,

$$AC^2 = AB^2 + BC^2 - 2(AB)(BC)\cos\angle ABC$$
$$40^2 = 20^2 + 30^2 - 2(20)(30)\cos\angle ABC$$
$$1600 = 1300 - 1200\cos\angle ABC$$
$$1200\cos\angle ABC = -300$$
$$\cos\angle ABC = -\frac{1}{4}$$
$$\angle ABC = \cos^{-1}\left(-\frac{1}{4}\right)$$
$$= 104.5^\circ \text{ (to 1 d.p.)}$$

$\therefore \angle ABC = \mathbf{104.5^\circ}$

I do not agree with the student.

9. **(a)** **(i)** Using Cosine Rule,

$$5.7^2 = 7.8^2 + 4.5^2 - 2(7.8)(4.5)\cos\angle PRS$$
$$32.49 = 81.09 - 70.2\cos\angle PRS$$
$$70.2\cos\angle PRS = 48.6$$
$$\cos\angle PRS = \mathbf{\frac{9}{13}}$$

(ii) $\cos\angle PRQ = \cos(180^\circ - \angle PRS)$

$$= -\cos\angle PRS$$
$$= \mathbf{-\frac{9}{13}}$$

(b) $\cos\angle PRS = \frac{9}{13}$

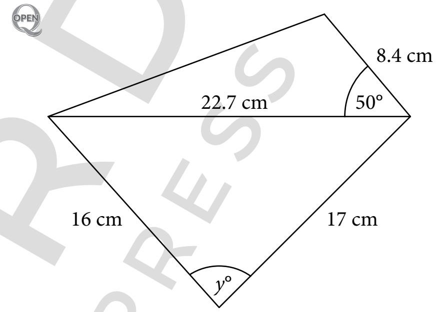
$$\angle PRS = \cos^{-1}\frac{9}{13}$$
$$= 46.187^\circ \text{ (to 3 d.p.)}$$

Let the perpendicular distance from P to QS produced be h m.

$$\sin 46.187^\circ = \frac{h}{7.8}$$
$$h = 7.8\sin 46.187^\circ$$
$$= 5.63 \text{ (to 3 s.f.)}$$

$\therefore$ Perpendicular distance from P to QS produced is **5.63 m**

10.



Using Cosine Rule,

$$22.7^2 = 16^2 + 17^2 - 2(16)(17)\cos y^\circ$$
$$515.29 = 545 - 544\cos y^\circ$$
$$544\cos y^\circ = 29.71$$
$$\cos y^\circ = 0.054\,614 \text{ (to 5 s.f.)}$$

Since $\cos y^\circ > 0$, then y° must be between 0° and 90°.

$$y^\circ = \cos^{-1} 0.054\,614$$
$$= 86.9^\circ \text{ (to 1 d.p.)}$$

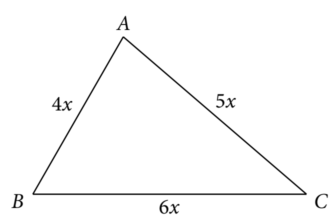
Elaine might have read off the outer scale of the protractor instead of the inner scale.

Replace the 93°-angle with an 87°-angle.

$$\text{Area of quadrilateral} = \frac{1}{2}(16)(17)\sin 87^\circ + \frac{1}{2}(22.7)(8.4)\sin 50^\circ$$
$$= \mathbf{209\ cm^2} \text{ (to 3 s.f.)}$$

Challenge Myself!

11.



Using Cosine Rule,

$$(6x)^2 = (4x)^2 + (5x)^2 - 2(4x)(5x)\cos\angle BAC$$
$$36x^2 = 16x^2 + 25x^2 - 40x^2\cos\angle BAC$$
$$40\cos\angle BAC = 5$$
$$\cos\angle BAC = \frac{1}{8}$$
$$\angle BAC = \cos^{-1}\frac{1}{8}$$

Using Cosine Rule,

$$(4x)^2 = (5x)^2 + (6x)^2 - 2(5x)(6x) \cos \angle ACB$$
$$16x^2 = 25x^2 + 36x^2 - 60x^2 \cos \angle ACB$$
$$60 \cos \angle ACB = 45$$
$$\cos \angle ACB = \frac{3}{4}$$
$$\angle ACB = \cos^{-1} \frac{3}{4}$$

Using Sine Rule,

$$\frac{\sin \angle ACB}{4x} = \frac{\sin\left(\cos^{-1}\frac{1}{8}\right)}{6x}$$
$$\sin \angle ACB = \frac{2}{3}\sin\left(\cos^{-1}\frac{1}{8}\right)$$
$$\sin\left(\cos^{-1}\frac{3}{4}\right) = \frac{2}{3}\sin\left(\cos^{-1}\frac{1}{8}\right)$$
$$3\sin\left(\cos^{-1}\frac{3}{4}\right) = 2\sin\left(\cos^{-1}\frac{1}{8}\right) \text{ (shown)}$$

Review Exercise 6

1. **(a)** $\sin x° = 0.3$

$x° = 180° - \sin^{-1} 0.3$
$= 162.5°$ (to 1 d.p.)
$\therefore x = \mathbf{162.5}$

(b) $\cos 154° = -\cos (180° - 154°)$
$= -\cos 26°$
$\therefore y = \mathbf{26}$

2. **(a)** $QR^2 + RS^2 = 12^2 + 9^2$
$= 225$
$QS^2 = 15^2$
$= 225$

Since $QR^2 + RS^2 = QS^2$, then by the converse of Pythagoras' Theorem, $\triangle QRS$ is a right-angled triangle. (shown)

(b) Using Pythagoras' Theorem,

$(x + 9)^2 + 12^2 = 37^2$
$(x + 9)^2 = 1225$
$x + 9 = 35$ (reject -35)
$x = 26$
$\therefore x = \mathbf{26}$

(c) **(i)** $\sin \angle PSQ = \sin (180° - \angle QSR)$
$= \sin \angle QSR$
$= \frac{12}{15}$
$= \mathbf{\frac{4}{5}}$

(ii) $\cos \angle PSQ = \cos (180° - \angle QSR)$
$= -\cos \angle QSR$
$= -\frac{9}{15}$
$= \mathbf{-\frac{3}{5}}$

(iii) $\tan \angle QPR = \frac{12}{26+9}$
$= \mathbf{\frac{12}{35}}$

3. **(a)** $\angle ABX = 360° - 348°$ ($\angle$s at a pt.)
$= 12°$

Using Cosine Rule,

$AX^2 = 11.5^2 + 8.4^2 - 2(11.5)(8.4) \cos 12°$
$= 13.832$ (to 5 s.f.)
$AX = \sqrt{13.832}$ $(AX > 0)$
$= 3.72$ cm (to 3 s.f.) (shown)

(b) Using Sine Rule,

$\frac{\sin \angle XCB}{8.4} = \frac{\sin 120°}{13.2}$
$\sin \angle XCB = \frac{8.4 \sin 120°}{13.2}$
$= 0.551\ 11$ (to 5 s.f.)
$\angle XCB = \sin^{-1} 0.551\ 11$
$= \mathbf{33.4°}$ (to 1 d.p.)

(c) $\angle XBC = 180° - 120° - 33.443°$
$= 26.557°$ (to 3 d.p.)

Area of $\triangle ABC = \frac{1}{2}(11.5)(13.2)\sin(12° + 26.557°)$
$= \mathbf{47.3\ cm^2}$ (to 3 s.f.)

(d) Let the shortest distance from X to BC be h cm.

$\sin 26.557° = \frac{h}{8.4}$
$h = 8.4 \sin 26.557°$
$= 3.76$ (to 3 s.f.)
$\therefore$ Shortest distance from X to BC = **3.76 cm**

7 Applications of Trigonometry

Worksheet 7A Angles of elevation and depression

1. **(a)** $\tan 37° = \frac{PB}{28}$

$PB = 28 \tan 37°$
$= 21.1$ cm (to 3 s.f.)
$\therefore$ Perpendicular height of P above B = **21.1 cm**

(b) $\sin 26° = \frac{PB}{45}$

$PB = 45 \sin 26°$
$= 19.7$ m (to 3 s.f.)
$\therefore$ Perpendicular height of P above B = **19.7 m**

2. **(a)** $\tan \angle PAB = \frac{6.8}{11.2}$

$\angle PAB = \tan^{-1} \frac{6.8}{11.2}$
$= 31.3°$ (to 1 d.p.)
$\therefore$ Angle of elevation of P from A = **31.3°**

(b) $\cos \angle PAB = \frac{2.5}{4.2}$

$\angle PAB = \cos^{-1} \frac{2.5}{4.2}$
$= 53.5°$ (to 1 d.p.)
$\therefore$ Angle of elevation of P from A = **53.5°**

3. Let the height of the building be h m.

$\tan 85.3° = \frac{h}{45}$

$h = 45 \tan 85.3°$

$= 550$ (to the nearest 10 metres)

$\therefore$ Height of the Lotte World Tower is **550 m**

4. Let the angle of elevation be α.

$\tan \alpha = \frac{632}{20}$

$\alpha = \tan^{-1} \frac{632}{20}$

$= 88.2°$ (to 1 d.p.)

$\therefore$ Angle of elevation of the top of the Shanghai Tower from P is **88.2°**

5. **(a)** Let the horizontal distance be x m.

$\tan 79° = \frac{414}{x}$

$x = \frac{414}{\tan 79°}$

$= 80.5$ (to 3 s.f.)

$\therefore$ Horizontal distance between P and the foot of the Princess Tower is **80.5 m**

(b) Let the angle of depression be β.

$\tan \beta = \frac{414 - 57.1}{86}$

$\beta = \tan^{-1} \frac{356.9}{86}$

$= 76.5°$ (to 1 d.p.)

$\therefore$ Angle of depression of the fire engine from Vicky is **76.5°**

6. Consider $\triangle ABC$.

$\tan 40° = \frac{5.6}{BC}$

$BC = \frac{5.6}{\tan 40°}$

$= 6.6738$ m (to 5 s.f.)

Consider $\triangle ABD$.

$\tan \angle ADB = \frac{5.6}{6.6738 + 3.9}$

$\angle ADB = 27.9°$ (to 1 d.p.)

$\therefore$ Angle of elevation is **27.9°**

7. **(i)** $AB = 12 \times 28$

$= 336$ m

Since $\angle BAC = 35°$ and $\angle ABC = 50°$,

$\angle ACB = 180° - 35° - 50°$ ($\angle$ sum of a $\triangle$)

$= 95°$

Using Sine Rule,

$$\frac{BC}{\sin \angle BAC} = \frac{AB}{\sin \angle ACB}$$

$$\frac{BC}{\sin 35°} = \frac{336}{\sin 95°}$$

$$BC = \frac{336 \sin 35°}{\sin 95°}$$

$= 193$ m (to 3 s.f.) (shown)

(ii) Let the height at which the drone is flying be h m.

$\sin 50° = \frac{h}{193.46}$

$h = 193.46 \sin 50°$

$= 148$ (to 3 s.f.)

$\therefore$ The drone is flying at a height of **148 m**.

8. **(i)** Using Cosine Rule,

$4^2 = 4.5^2 + 1^2 - 2(4.5)(1) \cos \angle APQ$

$16 = 21.25 - 9 \cos \angle APQ$

$9 \cos \angle APQ = 5.25$

$\cos \angle APQ = \frac{7}{12}$

$\angle APQ = \cos^{-1} \frac{7}{12}$

$= 54.315°$ (to 3 d.p.)

$\angle PAB = 180° - 90° - 54.315°$ ($\angle$ sum of a $\triangle$)

$= 35.7°$ (to 1 d.p.)

$\therefore$ Angle of elevation of P from A = **35.7°**

(ii) $\cos 54.315° = \frac{1 + QB}{4.5}$

$1 + QB = 4.5 \cos 54.315°$

$QB = 4.5 \cos 54.315° - 1$

$= 1.63$ m (to 3 s.f.)

$\therefore$ Vertical height of the pole = **1.63 m**

(iii) $\sin \angle QAB = \frac{1.625}{4}$

$\angle QAB = \sin^{-1} \frac{1.625}{4}$

$= 24.0°$ (to 1 d.p.)

$\therefore$ Angle of depression of A from Q = **24.0°** (alt. $\angle$s)

9. **(i)** Angle of depression of B from P = **49°**

(ii) $\tan 36° = \frac{PQ}{2.9 + BQ}$

$PQ = 2.9 \tan 36° + BQ \tan 36°$ — (1)

$\tan 49° = \frac{PQ}{BQ}$

$PQ = BQ \tan 49°$ — (2)

(1) = (2): $2.9 \tan 36° + BQ \tan 36° = BQ \tan 49°$

$BQ \tan 49° - BQ \tan 36° = 2.9 \tan 36°$

$BQ(\tan 49° - \tan 36°) = 2.9 \tan 36°$

$$BQ = \frac{2.9 \tan 36°}{\tan 49° - \tan 36°}$$

$= 4.9713$ m (to 5 s.f.)

$\tan 49° = \frac{PQ}{4.9713}$

$PQ = 4.9713 \tan 49°$

$=$ **5.72 m** (to 3 s.f.)

Worksheet 7B Bearings

1. **(a)** Bearing of A from O = **050°**

(b) $180° - 60° = 120°$

Bearing of B from O = **120°**

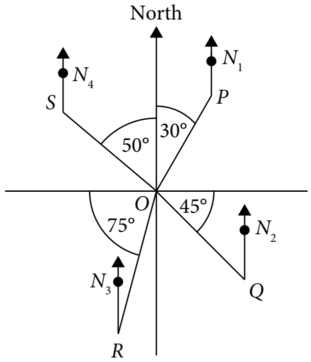
(c) $270° - 45° = 225°$

Bearing of C from O = **225°**

(d) $270° + 65° = 335°$

Bearing of D from O = **335°**

2.



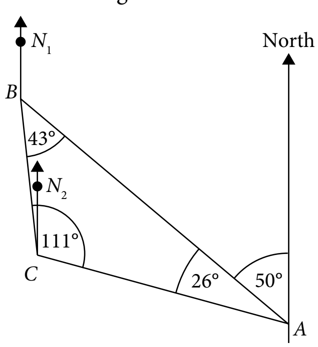
(a) Reflex $\angle OPN_1 = 180° + 30°$
$= 210°$
Bearing of O from P = **210°**

(b) $\angle OQN_2 = 180° - 90° - 45°$ ($\angle$ sum of a $\triangle$)
$= 45°$
$360° - 45° = 315°$ ($\angle$s at a pt.)
Bearing of O from Q = **315°**

(c) $\angle ORN_3 = 180° - 90° - 75°$ ($\angle$ sum of a $\triangle$)
$= 15°$
Bearing of O from R = **015°**

(d) $180° - 50° = 130°$ (int. $\angle$s, SN_4 // ON)
Bearing of O from S = **130°**

3.



(a) Reflex $\angle BAN = 360° - 50°$ ($\angle$s at a pt.)
$= 310°$
Bearing of B from A = **310°**

(b) $\angle ABN_1 = 180° - 50°$ (int. $\angle$s, BN_1 // AN)
$= 130°$
Bearing of A from B = **130°**

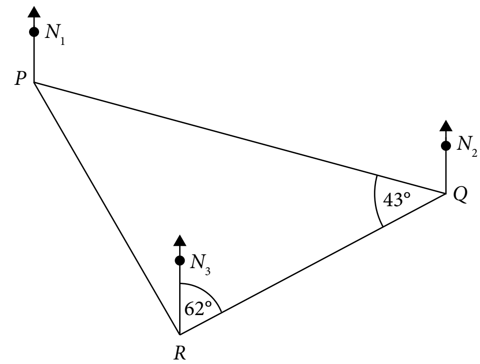
(c) Reflex $\angle CAN = 360° - 26° - 50°$ ($\angle$s at a pt.)
$= 284°$
Bearing of C from A = **284°**

(d) $\angle ACN_2 = 180° - 26° - 50°$ (int. $\angle$s, CN_2 // AN)
$= 104°$
Bearing of A from C = **104°**

(e) $\angle BCN_2 = 111° - 104°$
$= 7°$
Reflex $\angle BCN_2 = 360° - 7°$ ($\angle$s at a pt.)
$= 353°$
Bearing of B from C = **353°**

(f) $\angle CBN_1 = 130° + 43°$
$= 173°$
Bearing of C from B = **173°**

4. **(a)**



$\angle PRN_3 = 360° - 330°$ ($\angle$s at a pt.)
$= 30°$
$\angle N_1PR = 180° - 30°$ (int. $\angle$s, PN_1 // RN_3)
$= 150°$
$\therefore$ Bearing of R from P = **150°**

(b) $\angle N_3RQ = 62°$
$\angle PQN_2 = 180° - 62° - 43°$ (int. $\angle$s, RN_3 // QN_2)
$= 75°$
$360° - 75° = 285°$
$\therefore$ Bearing of P from Q = **285°**

5. Using Cosine Rule,
$$QR^2 = PQ^2 + PR^2 - 2(PQ)(PR)\cos\angle QPR$$
$$7^2 = 9.6^2 + 9.4^2 - 2(9.6)(9.4)\cos\angle QPR$$
$\cos\angle QPR = 0.728\ 72$ (to 5 s.f.)
$\angle QPR = 43.221°$ (to 3 d.p.)
$60° + 43.221° = 103.2°$ (to 1 d.p.)
$\therefore$ Bearing of R from P = **103.2°**

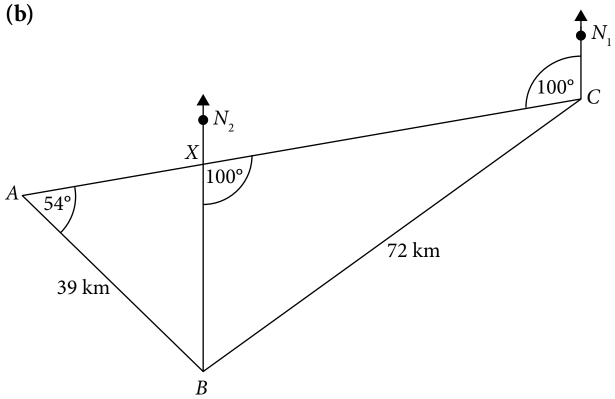
6. **(a)** $\angle ABC = 50° + (180° - 120°)$
$= 110°$ (shown)

(b) Using Cosine Rule,
$$AC^2 = AB^2 + BC^2 - 2(AB)(BC)\cos\angle ABC$$
$$= 65^2 + 50^2 - 2(65)(50)\cos 110°$$
$$= 8948.1 \text{ (to 5 s.f.)}$$
AC = **94.6 km** (to 3 s.f.)

(c) Using Sine Rule,
$$\frac{\sin\angle BCA}{AB} = \frac{\sin\angle ABC}{AC}$$
$$\frac{\sin\angle BCA}{65} = \frac{\sin 110°}{94.595}$$
$$\sin\angle BCA = \frac{65\sin 110°}{94.595}$$
$$= 0.645\ 70 \text{ (to 5 s.f.)}$$
$\angle BCA = 40.218°$ (to 3 d.p.)
$360° - 40.218° - (180° - 120°) = 259.8°$ (to 1 d.p.)
$\therefore$ Bearing of A from C = **259.8°**

7. **(a)** Using Sine Rule,
$$\frac{\sin\angle ACB}{AB} = \frac{\sin\angle CAB}{BC}$$
$$\frac{\sin\angle ACB}{39} = \frac{\sin 54°}{72}$$
$$\sin\angle ACB = \frac{39\sin 54°}{72}$$
$$= 0.438\ 22 \text{ (to 5 s.f.)}$$
$\angle ACB$ = **26.0°** (to 1 d.p.)

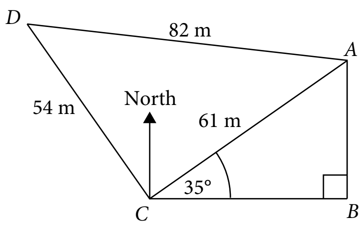
(b)



$\angle BXC = \angle N_1CX$ (alt. ∠s, $CN_1 // BN_2$)
$= 100°$
$\angle XBA = 100° - 54°$ (ext. ∠ of a △)
$= 46°$
$360° - 46° = 314°$
∴ Bearing of Alpha Town from Beta Ville = **314°**

8. **(a)** Using Cosine Rule,
$AD^2 = AC^2 + CD^2 - 2(AC)(CD) \cos \angle ACD$
$82^2 = 61^2 + 54^2 - 2(61)(54) \cos \angle ACD$
$\cos \angle ACD = -0.013\ 206$ (to 5 s.f.)
$\angle ACD = 90.8°$ (to 1 d.p.) (shown)

(b)



$\angle NCA = 90° - 35°$
$= 55°$
$\angle NCD = 90.757° - 55°$
$= 35.757°$ (to 3 d.p.)
$360° - 35.757° = 324.2°$ (to 1 d.p.)
∴ Bearing of D from C is **324.2°**

9. **(a)** Using Cosine Rule,
$AC^2 = BA^2 + BC^2 - 2(BA)(BC) \cos \angle ABC$
$= 960^2 + 710^2 - 2(960)(710) \cos 60°$
$= 744\ 100$
$AC = 863$ m (to 3 s.f.) (shown)

(b) Using Sine Rule,
$$\frac{\sin \angle ADC}{AC} = \frac{\sin \angle ACD}{AD}$$
$$\frac{\sin \angle ADC}{862.61} = \frac{\sin 34°}{500}$$
$$\sin \angle ADC = \frac{862.61 \sin 34°}{500}$$
$= 0.964\ 73$ (to 5 s.f.)
$\angle ADC = 74.738°$ (to 3 d.p.)
$180° + 74.738° = 254.7°$ (to 1 d.p.) (alt. ∠s)
∴ Bearing of D from C = **254.7°**

(c) Time taken $= \dfrac{960 + 710 + 862.61}{2.9}$
$= 873.31$ s (to 5 s.f.)
$= 14.555$ min (to 5 s.f.)
= **14 min 33 s** (to the nearest second)

Challenge Myself!

10. **(a)** Bearing of B from A = **255°**
(b) Distance within shaded area = 34 m
Length of time $= \dfrac{34}{5}$
= **6.8 s** (to the nearest second)

Worksheet 7C Three-dimensional problems

1. **(a)** Using Pythagoras' Theorem,
$AC^2 = 6^2 + 6^2$
$= 72$
$AC = \sqrt{72}$ ($AC > 0$)
$= 8.49$ cm (to 3 s.f.) (shown)

(b) $\tan \angle HBD = \dfrac{6}{\sqrt{72}}$
$\angle HBD = \tan^{-1} \dfrac{6}{\sqrt{72}}$
= **35.3°** (to 1 d.p.)

(c) Using Pythagoras' Theorem,
$BH^2 = 6^2 + 72$
$= 108$
$BH = \sqrt{108}$ ($BH > 0$)
= **10.4 cm** (to 3 s.f.)

(d) $\cos \angle ABH = \dfrac{6}{\sqrt{108}}$
$\angle ABH = \cos^{-1} \dfrac{6}{\sqrt{108}}$
= **54.7°** (to 1 d.p.)

2. Using Pythagoras' Theorem,
$BX^2 = 8^2 + 15^2$
$= 289$
$BX = \sqrt{289}$ ($BX > 0$)
$= 17$ cm
$\tan \angle AXB = \dfrac{6}{17}$
$\angle AXB = \tan^{-1} \dfrac{6}{17}$
= **19.4°** (to 1 d.p.)

3. **(a)** **(i)** $\tan \angle FAB = \dfrac{2}{4}$
$= \dfrac{1}{2}$
$\angle FAB = \tan^{-1} \dfrac{1}{2}$
$= 26.6°$ (to 1 d.p.)
∴ Angle of elevation of F from A is **26.6°**

(ii) Using Pythagoras' Theorem,
$BD^2 = 2^2 + 4^2$
$= 20$
$BD = \sqrt{20}$ cm ($BD > 0$)
$\tan \angle HBD = \dfrac{2}{\sqrt{20}}$
$\angle HBD = \tan^{-1} \dfrac{2}{\sqrt{20}}$
$= 24.1°$ (to 1 d.p.)
∴ Angle of elevation of H from B is **24.1°**

(b) Let the shortest distance travelled be x cm.
Using Pythagoras' Theorem,
$$x^2 = 4^2 + 4^2$$
$$= 32$$
$$x = \sqrt{32}\ (x > 0)$$
$$= 5.66 \text{ (to 3 s.f.)}$$
$\therefore$ Shortest distance = **5.66 cm**

(c) Let X and Y be the midpoints of AB and EF respectively.
Using Pythagoras' Theorem,
$$XC^2 = 2^2 + 2^2$$
$$= 8$$
$$XC = \sqrt{8} \text{ cm } (XC > 0)$$
$$\tan \angle YCX = \frac{2}{\sqrt{8}}$$
$$\angle YCX = \tan^{-1}\frac{2}{\sqrt{8}}$$
$$= 35.3° \text{ (to 1 d.p.)}$$
$\therefore$ Angle of depression of C from the midpoint of EF is **35.3°**

(d) OPEN P could be at the midpoint of EG.

4. (a) In $\triangle ABC$,
$$AB^2 + BC^2 = 3^2 + 4^2$$
$$= 25$$
$$AC^2 = 5^2$$
$$= 25$$
Since $AB^2 + BC^2 = AC^2$, by the converse of Pythagoras' Theorem, $\triangle ABC$ is a right-angled triangle. (shown)

(b) Total surface area $= 2 \times \frac{1}{2}(4)(3) + 10(5 + 4 + 3)$
$= \mathbf{132\ cm^2}$

(c) Let the shortest distance from B to AC be h cm.
$$\frac{1}{2}(5)(h) = \frac{1}{2}(3)(4)$$
$$h = 2.4$$
$\therefore$ Shortest distance from B to AC = **2.4 cm**

(d) Using Pythagoras' Theorem,
$$BD^2 = 10^2 + 4^2$$
$$= 116$$
$$BD = \sqrt{116} \text{ cm } (BD > 0)$$
Let the angle of elevation of A from D be α.
$$\tan \alpha = \frac{3}{\sqrt{116}}$$
$$\alpha = 15.6° \text{ (to 1 d.p.)}$$
$\therefore$ Angle of elevation of A from D is **15.6°**

5. (a) (i) Using Sine Rule,
$$\frac{\sin \angle BAC}{BC} = \frac{\sin \angle ACB}{AB}$$
$$\frac{\sin \angle BAC}{5.4} = \frac{\sin 85°}{7.1}$$
$$\sin \angle BAC = \frac{5.4 \sin 85°}{7.1}$$
$$= 0.757\ 67 \text{ (to 5 s.f.)}$$
$\angle BAC$ = **49.3°** (to 1 d.p.)

(ii) $\angle ABC = 180° - 49.259° - 85°$ ($\angle$ sum of a $\triangle$)
$= 45.741°$ (to 3 d.p.)
$\therefore$ Area of garden $= \frac{1}{2}(7.1)(5.4)\sin 45.741°$
$= \mathbf{13.7\ m^2}$ (to 3 s.f.)

(b) (i) $\tan 36° = \frac{PB}{5.4}$
$$PB = 5.4 \tan 36°$$
$$= 3.92 \text{ m (to 3 s.f.)}$$
$\therefore$ Height of pole = **3.92 m**

(ii) Using Pythagoras' Theorem,
$$PA^2 = PB^2 + AB^2$$
$$= 3.9233^2 + 7.1^2$$
$$= 65.803 \text{ (to 5 s.f.)}$$
PA = **8.11 m** (to 3 s.f.)

(iii) $\cos 36° = \frac{5.4}{PC}$
$$PC = \frac{5.4}{\cos 36°}$$
$$= 6.6748 \text{ m (to 5 s.f.)}$$
Using Sine Rule,
$$\frac{AC}{\sin 45.741°} = \frac{5.4}{\sin 49.259°}$$
$$AC = \frac{5.4 \sin 45.741°}{\sin 49.259°}$$
$$= 5.1044 \text{ m (to 5 s.f.)}$$
Using Cosine Rule,
$$AC^2 = PA^2 + PC^2 - 2(PA)(PC) \cos \angle APC$$
$$5.1044^2 = 8.1119^2 + 6.6748^2 - 2(8.1119)(6.6748) \cos \angle APC$$
$$\cos \angle APC = 0.778\ 47 \text{ (to 5 s.f.)}$$
$\angle APC$ = **38.9°** (to 1 d.p.)

6. (a) Length = x cm
Width = $\mathbf{0.5}\boldsymbol{x}$ **cm**
Height = $(\boldsymbol{x} - \mathbf{4})$ **cm**

(b) Using Pythagoras' Theorem,
$$FH^2 = x^2 + (0.5x)^2$$
$$= x^2 + 0.25x^2$$
$$= \mathbf{1.25x^2} \text{ (shown)}$$

(c) Using Pythagoras' Theorem,
$$BH^2 = BF^2 + FH^2$$
$$15^2 = (x - 4)^2 + 1.25x^2$$
$$225 = x^2 - 8x + 16 + 1.25x^2$$
$$2.25x^2 - 8x - 209 = 0$$
$$9x^2 - 32x - 836 = 0 \text{ (shown)}$$

(d) $9x^2 - 32x - 836 = 0$
$$x = \frac{-(-32) \pm \sqrt{(-32)^2 - 4(9)(-836)}}{2(9)}$$
$$= 11.578 \text{ (to 3 d.p.) or } -8.023 \text{ (to 3 d.p.)}$$
$\therefore x$ = **11.578** or x = **−8.023**

(e) Length = 11.578 cm (to 5 s.f.)
Width = 5.7891 cm (to 5 s.f.)
Height = 7.5783 cm (to 5 s.f.)
Total surface area $= 2[(11.578)(5.7891) + (11.578)(7.5783) + (5.7891)(7.5783)]$
$= \mathbf{397\ cm^2}$ (to 3 s.f.)

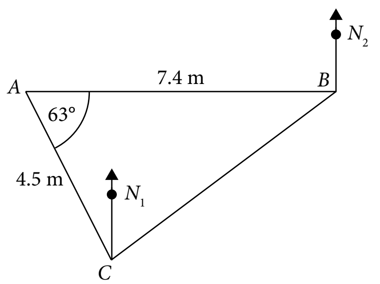
7. (a) (i) Using Cosine Rule,
$$BC^2 = AB^2 + AC^2 - 2(AB)(AC) \cos \angle BAC$$
$$= 7.4^2 + 4.5^2 - 2(7.4)(4.5) \cos 63°$$
$$= 44.774 \text{ (to 5 s.f.)}$$
BC = **6.69 m** (to 3 s.f.)

(ii) Area of $\triangle ABC = \frac{1}{2}(7.4)(4.5)\sin 63°$
$= \mathbf{14.8\ m^2}$ (to 3 s.f.)

(iii) Using Sine Rule,

$$\frac{\sin\angle ABC}{AC} = \frac{\sin\angle BAC}{BC}$$
$$\frac{\sin\angle ABC}{4.5} = \frac{\sin 63°}{6.6914}$$
$$\sin\angle ABC = \frac{4.5\sin 63°}{6.6914}$$
$$= 0.599\ 21 \text{ (to 5 s.f.)}$$
$$\angle ABC = \mathbf{36.8°} \text{ (to 1 d.p.)}$$

(iv)



$\angle N_1CB = 180° - 90° - 36.813°$ (int. ∠s, $CN_1 // BN_2$)
$= 53.187°$ (to 3 d.p.)
∴ Bearing of B from C = **053.2°** (to 1 d.p.)

(b) Let the height of the scarecrow be h m.

$$\tan 32° = \frac{h}{4.5}$$
$$h = 4.5\tan 32°$$
$$= 2.8119 \text{ (to 5 s.f.)}$$

Let the angle of elevation of T from B be $\alpha°$.

$$\tan\alpha° = \frac{2.8119}{6.6914}$$
$$\alpha° = 22.8° \text{ (to 1 d.p.)}$$

∴ Angle of elevation of top of scarecrow from B = **22.8°**

8. (a) $\angle ADB = \frac{180° - 80°}{2}$ (base ∠s of isos. △)
$= 50°$

$$\tan\angle BDC = \frac{92}{47}$$
$$\angle BDC = 62.939° \text{ (to 3 d.p.)}$$

∴ $\angle ADC = 50° + 62.939°$
$= 112.9°$ (to 1 d.p.) (shown)

(b) Using Pythagoras' Theorem,

$$BD^2 = 47^2 + 92^2$$
$$= 10\ 673$$
$$BD = \sqrt{10\ 673} \text{ m } (BD > 0)$$
$$\sin 40° = \frac{\frac{1}{2}BD}{AB}$$
$$AB = \frac{\frac{1}{2}\left(\sqrt{10\ 673}\right)}{\sin 40°}$$
$$= \mathbf{80.4\ m} \text{ (to 3 s.f.)}$$

(c) Area of land $= \frac{1}{2}(92)(47) + \frac{1}{2}(80.361)^2\sin 80°$
$= 5341.9\ \text{m}^2$ (to 5 s.f.)

Amount of money required = 5341.9 × 7000
= \$37 393 000 (to 5 s.f.)
> \$35 000 000

∴ He does not have enough money to purchase the land.

(d) Let the height of the drone be h m.

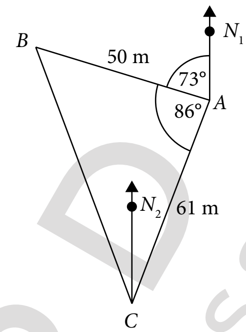
$$\tan 24° = \frac{h}{47}$$
$$h = 47\tan 24°$$
$$= 20.926 \text{ (to 5 s.f.)}$$

Let the angle of elevation of the drone from B be α.

$$\tan\alpha = \frac{20.926}{92}$$
$$\alpha = 12.8° \text{ (to 1 d.p.)}$$

∴ Angle of elevation of drone from B = **12.8°**

9.



(a) Using Cosine Rule,

$$BC^2 = AB^2 + AC^2 - 2(AB)(AC)\cos\angle BAC$$
$$= 50^2 + 61^2 - 2(50)(61)\cos 86°$$
$$= 5795.5 \text{ (to 5 s.f.)}$$
$$BC = \mathbf{76.1\ m} \text{ (to 3 s.f.)}$$

(b) Using Sine Rule,

$$\frac{\sin\angle ACB}{AB} = \frac{\sin\angle BAC}{BC}$$
$$\frac{\sin\angle ACB}{50} = \frac{\sin 86°}{76.128}$$
$$\sin\angle ACB = \frac{50\sin 86°}{76.128}$$
$$= 0.655\ 19 \text{ (to 5 s.f.)}$$
$$\angle ACB = 40.934° \text{ (to 3 d.p.)}$$

$\angle N_2CA = 180° - 86° - 73°$ (int. ∠s, $N_2C // N_1A$)
$= 21°$

$\angle BCN_2 = 40.934° - 21°$
$= 19.934°$ (to 3 d.p.)

$360° - 19.934° = 340.1°$ (to 1 d.p.)

∴ Bearing of B from C = **340.1°**

(c) Let the shortest distance between Jenny and B be x m.

$$\sin 86° = \frac{x}{50}$$
$$x = 50\sin 86°$$
$$= 49.9 \text{ (to 3 s.f.)}$$

∴ Shortest distance between Jenny and B = **49.9 m**

(d) $\tan 34° = \frac{TA}{61}$

$$TA = 61\tan 34°$$
$$= 41.1 \text{ m (to 3 s.f.)}$$

∴ Height of building = **41.1 m**

Review Exercise 7

1. Using Pythagoras' Theorem,

$DQ^2 = 6^2 + 21^2$
$= 477$
$DQ = \sqrt{477}$ cm $(DQ > 0)$

Let the angle of elevation of P from D be α.

$\tan \alpha = \frac{7}{\sqrt{477}}$
$\alpha = 17.8°$ (to 1 d.p.)

$\therefore$ Angle of elevation of P from D is **17.8°**

2. **(i)** Using Cosine Rule,

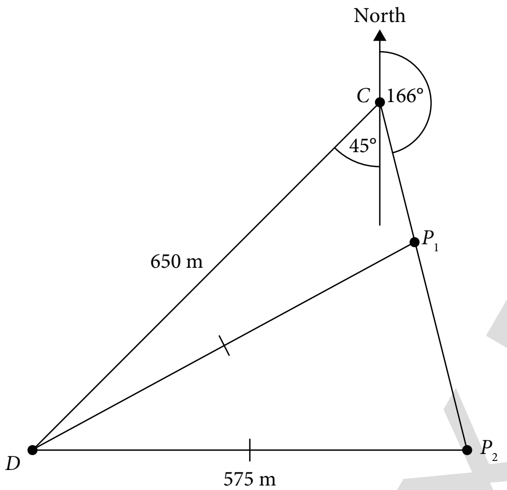
$AB^2 = BC^2 + AC^2 - 2(BC)(AC) \cos \angle ACB$
$32.9^2 = 24.8^2 + 12.3^2 - 2(24.8)(12.3) \cos \angle ACB$
$\cos \angle ACB = -0.518\ 10$ (to 5 s.f.)
$\angle ACB =$ **121.2°** (to 1 d.p.)

(ii) Let the angle of elevation be α.

$\tan \alpha = \frac{3.5}{12.3}$
$\alpha = 15.9°$ (to 1 d.p.)

$\therefore$ Angle of depression of A from the top of the post is **15.9°**

3.



$\angle DCP = 45° + (180° - 166°)$
$= 59°$

Using Sine Rule,

$\frac{\sin\angle CPD}{650} = \frac{\sin 59°}{575}$

$\sin \angle CPD = \frac{650 \sin 59°}{575}$
$= 0.968\ 97$ (to 5 s.f.)

$\angle CPD = \sin^{-1} 0.968\ 97$ or $180° - \sin^{-1} 0.968\ 97$
$= 75.690°$ or $104.310°$ (to 3 d.p.)

Consider $\angle CP_1D = 104.310°$.

Using Cosine Rule,

$CP_1^{\ 2} = 650^2 + 575^2 - 2(650)(575) \cos (180° - 104.310° - 59°)$
$= 37\ 115$ (to 5 s.f.)
$CP_1 = \sqrt{37\ 115}$ $(CP_1 > 0)$
$= 193$ m (to 3 s.f.)

Consider $\angle CP_2D = 75.690°$.

Using Cosine Rule,

$CP_2^{\ 2} = 650^2 + 575^2 - 2(650)(575) \cos (180° - 75.690° - 59°)$
$= 227\ 430$ (to 5 s.f.)
$CP_2 = \sqrt{227\ 430}$ $(CP_2 > 0)$
$= 477$ m (to 3 s.f.)

$\therefore$ The possible distances are **193 m** and **477 m**.

8 Arc Length and Sector Area

Worksheet 8A Length of arc

1. **(a)** Arc length $= \frac{1}{4} \times 2\pi(8)$
$= 4\pi$ cm
$=$ **12.6 cm** (to 3 s.f.)

(b) Arc length $= \frac{150°}{360°} \times 2\pi(24)$
$= 20\pi$ cm
$=$ **62.8 cm** (to 3 s.f.)

(c) Arc length $= \frac{225°}{360°} \times 2\pi(37)$
$= 46.25\pi$ cm
$=$ **145 cm** (to 3 s.f.)

(d) Arc length $= \frac{320°}{360°} \times 2\pi(19)$
$= \frac{304}{9}\pi$ cm
$=$ **106 cm** (to 3 s.f.)

(e) Arc length $= \frac{360° - 136°}{360°} \times 2\pi(5)$
$= \frac{56}{9}\pi$ cm
$=$ **19.5 cm** (to 3 s.f.)

(f) Arc length $= \frac{7}{8} \times 2\pi(10)$
$= 17.5\pi$ cm
$=$ **55.0 cm** (to 3 s.f.)

2. Perimeter $= 2(7) + \frac{105°}{360°} \times 2\pi(7)$
$=$ **26.8 cm** (to 3 s.f.)

3. Perimeter $= 2(24) + \frac{150°}{360°} \times 2\pi(24)$
$= (48 + 20\pi)$ cm

$\therefore$ $\boldsymbol{a = 48, b = 20}$

4. $\angle AOB = 60°$

Arc length $= \frac{60°}{360°} \times 2\pi(9)$
$= 3\pi$ cm

$\therefore$ Perimeter of shaded region $= 9 + 3\pi$
$=$ **18.4 cm** (to 3 s.f.)

5. **(a)** $\frac{70°}{360°} \times 2\pi(OA) = 45$
$OA = 36.8$ cm (to 3 s.f.)

$\therefore$ Radius of circle $=$ **36.8 cm**

(b) $\frac{360° - 322°}{360°} \times 2\pi(OA) = 45$

$OA = 67.9$ cm (to 3 s.f.)

$\therefore$ Radius of circle = **67.9 cm**

6. **(a)** Let the angle subtended at the centre be θ.

$\frac{\theta}{360°} \times 2\pi(4) = 3\pi$

$\theta = 135°$

$\therefore$ Angle subtended = **135°**

(b) Let the angle subtended at the centre be θ.

$\frac{\theta}{360°} \times 2\pi(5) = 21$

$\theta = 240.6°$ (to 1 d.p.)

$\therefore$ Angle subtended = **240.6°**

(c) Let the angle subtended at the centre be θ.

$2(6) + \frac{\theta}{360°} \times 2\pi(6) = 18$

$\frac{\theta}{360°} \times 2\pi(6) = 6$

$\theta = 57.3°$ (to 1 d.p.)

$\therefore$ Angle subtended = **57.3°**

(d) Radius $= \frac{51 - 37}{2}$

$= 7$ cm

Let the angle subtended at the centre be θ.

$\frac{\theta}{360°} \times 2\pi(7) = 37$

$\theta = 302.8°$ (to 1 d.p.)

$\therefore$ Angle subtended = **302.8°**

7. Length of arc $AB = \frac{40°}{360°} \times 2\pi(8)$

$= \frac{16}{9}\pi$ cm

Length of arc $PQ = \frac{270°}{360°} \times 2\pi(4)$

$= 6\pi$ cm

$\therefore$ Perimeter of shape $= \frac{16}{9}\pi + 8 + 4 + 6\pi + 4$

$= \mathbf{\left(16 + \frac{70}{9}\pi\right)}$ **cm**

8. $\frac{\angle AOB}{360°} \times 2\pi(27) = 68$

$\angle AOB = 144.300°$ (to 3 d.p.)

Using Cosine Rule,

$AB^2 = 27^2 + 27^2 - 2(27)(27) \cos 144.300°$

$= 2642.0$ (to 5 s.f.)

$AB = \sqrt{2642.0}$ $(AB > 0)$

$= 51.401$ cm (to 5 s.f.)

$\therefore$ Perimeter of shaded region $= 68 + 51.401$

$=$ **119 cm** (to 3 s.f.)

9. **(i)** $OA = 3x$ units and $OP = 4x$ units

$\frac{\text{reflex } \angle AOB}{360°} \times 2\pi(4x) = 96$

reflex $\angle AOB = \mathbf{\left(\frac{4320}{\pi x}\right)^{\circ}}$

(ii) Length of major arc $AB = \frac{\left(\frac{4320}{\pi x}\right)^{\circ}}{360°} \times 2\pi(3x)$

$= 72$ units

$\therefore$ Perimeter of shaded region $= 96 + 72 + 2x$

$=$ **$(2x + 168)$ units**

(iii) Since the larger sector is an enlargement of the smaller sector by a scale factor $\frac{4}{3}$, then length of major arc $AB = \frac{3}{4}(96) = 72$ units.

$\therefore$ Perimeter of shaded region $= 96 + 72 + 2x$

$=$ **$(2x + 168)$ units**

10. **(a)** Distance travelled $= 2 \times 2\pi(15)$

$=$ **60π cm**

(b) **(i)** Let the angle which the hour hand travels through be θ.

$\frac{\theta}{360°} \times 2\pi(15) = 8$

$\theta = 30.6°$ (to 1 d.p.)

$\therefore$ Angle which the hour hand travels through is **30.6°**

(ii) Since 360° corresponds to 12 h, 30° (≈ 8 cm) corresponds to 1 h and 0.6° corresponds to 0.02 h = 1.2 min.

Distance travelled in 1 h is less than 8 cm (30° < 30.6°)

Distance travelled in 2 h is less than 16 cm

The hour hand moves 16 cm in more than 2 h and in less than 2 h 15 min.

$\therefore$ I agree with Jeslyn.

Worksheet 8B Area of sector

1. **(a)** Area of sector $= \frac{1}{4} \times \pi(10)^2$

$=$ **78.5 cm²** (to 3 s.f.)

(b) Area of sector $= \frac{118°}{360°} \times \pi(9.4)^2$

$=$ **91.0 cm²** (to 3 s.f.)

(c) Area of sector $= \frac{234°}{360°} \times \pi(27)^2$

$=$ **1490 cm²** (to 3 s.f.)

(d) Area of sector $= \frac{305°}{360°} \times \pi(6)^2$

$=$ **95.8 cm²** (to 3 s.f.)

(e) Area of sector $= \frac{360° - 75°}{360°} \times \pi(16)^2$

$=$ **637 cm²** (to 3 s.f.)

(f) Area of sector $= \frac{5}{6} \times \pi(7)^2$

$=$ **128 cm²** (to 3 s.f.)

2. Area of sector $= \frac{63°}{360°} \times \pi(12)^2$

$=$ **79.2 cm²** (to 3 s.f.)

3. Total area $= (34 \times 17) + 2 \times \frac{60°}{360°} \times \pi(17)^2$

$=$ **881 cm²** (to 3 s.f.)

4. $\frac{\theta°}{360°} \times \pi r^2 = 924$

$\frac{\theta°}{360°} \times \frac{22}{7} r^2 = 924$

$\frac{\theta°}{360°} \times r^2 = 294$

Let $\theta = 240$ and $r = 21$.

Perimeter of sector $= \frac{240°}{360°} \times 2\pi(21) + 2(21)$

$= 130$ cm

$\therefore$ $\boldsymbol{\theta = 240}$, $\boldsymbol{r = 21}$; perimeter = **130 cm**

5. Area of shaded region $= \frac{126°}{360°} \times \pi(9.5)^2 - \frac{1}{2}(9.5)^2 \sin 126°$

$=$ **62.7 cm²** (to 3 s.f.)

6. Percentage of sector that is shaded

$= \dfrac{\frac{140°}{360°} \times \pi(7)^2 - \frac{1}{2}(7)^2 \sin 140°}{\frac{140°}{360°} \times \pi(7)^2} \times 100\%$

= **73.7%** (to 3 s.f.)

7. Volume of solid $= \frac{39°}{360°} \times \pi(11)^2(8)$

$=$ **329 cm³** (to 3 s.f.)

8. **(i)** $\frac{OP}{OA} = \frac{5}{3}$

$\frac{OP}{9} = \frac{5}{3}$

$OP = 15$ cm

$AP = 6$ cm

Perimeter of $ABQP$ $= \frac{75°}{360°} \times 2\pi(9) + \frac{75°}{360°} \times 2\pi(15) + 2(6)$

$=$ **43.4 cm** (to 3 s.f.)

(ii) Area of $ABQP$ $= \frac{75°}{360°} \times \pi(15)^2 - \frac{75°}{360°} \times \pi(9)^2$

$=$ **94.2 cm²** (to 3 s.f.)

9. **(a)** **(i)** $\cos 30° = \frac{45}{OA}$

$OA = \frac{45}{\cos 30°}$

$= 51.962$ cm (to 5 s.f.)

$\therefore$ Radius of sector = **52.0 cm**

(ii) $\angle AOP = 180° - 90° - 30°$ ($\angle$ sum of a $\triangle$)

$= 60°$

$\angle AOB = 180° - 2(60°)$ (adj. $\angle$s on a str. line)

$=$ **60°**

(iii) $\tan 30° = \frac{OP}{45}$

$OP = 45 \tan 30°$

$= 25.981$ cm (to 5 s.f.)

$\therefore$ Perimeter of figure

$= 2(25.981) + 2(45) + \frac{60°}{360°} \times 2\pi(51.962)$

$=$ **196 cm** (to 3 s.f.)

(b) Area of figure

$= 2 \times \frac{1}{2}(45)(51.962) \sin 30° + \frac{60°}{360°} \times \pi(51.962)^2$

$= 2582.9$ cm² (to 5 s.f.)

Cost of manufacturing $= \frac{2582.9}{1000} \times \$40$

= **$103** (to the nearest dollar)

10. $OB = 12 - 7 = 5$ cm

$\cos \angle AOB = \frac{5}{7}$

$\angle AOB = 44.415°$ (to 3 d.p.)

$\angle AOC = 2 \times 44.415°$

$= 88.831°$ (to 3 d.p.)

Area of segment $= \frac{1}{2}(7)^2 \sin 88.831° + \frac{360° - 88.831°}{360°} \times \pi(7)^2$

$=$ **140 cm²** (to 3 s.f.)

11. **(i)** $OP = 10$ cm

$BP = \frac{100}{40} \times 10$ cm $= 25$ cm

Using Cosine Rule,

$25^2 = 10^2 + 20^2 - 2(10)(20) \cos \angle BOP$

$625 = 500 - 400 \cos \angle BOP$

$400 \cos \angle BOP = -125$

$\cos \angle BOP = -\frac{5}{16}$

$\angle BOP = \cos^{-1}\left(-\frac{5}{16}\right)$

$= 108.210°$ (to 3 d.p.)

$\therefore$ Perimeter of shaded region

$= \frac{108.210°}{360°} \times 2\pi(20) + 10 + 25$

$=$ **72.8 cm** (to 3 s.f.)

(ii) Area of shaded region

$= \frac{108.210°}{360°} \times \pi(20)^2 - \frac{1}{2}(10)(20) \sin 108.210°$

$= 280$ cm² (to 2 s.f.) (shown)

12. **(a)** Arc length $= \frac{\theta}{360°} \times 2\pi r$, where θ is the angle subtended at the centre and r is the radius

Since r is a constant, then arc length $= k\theta$, where k is an arbitrary constant.

$\therefore$ The statement is **true**.

(b) Area of sector $= \frac{\theta}{360°} \times \pi r^2$, where θ is the angle subtended at the centre and r is the radius

Since θ is a constant, then area of sector $= kr^2$, where k is an arbitrary constant.

The area of a sector is directly proportional to the square of the radius.

$\therefore$ The statement is **false**.

(c) Since θ is a constant, then area of sector $= k_1 r^2$ and arc length $= k_2 r$, where k_1 and k_2 are arbitrary constants.

Rearranging the terms, r is directly proportional to the square root of the area, and to the arc length.

$\therefore$ The statement is **false**.

Challenge Myself!

13. (a) $\frac{130°}{360°} \times 2\pi(AP) = \frac{65}{6}\pi$

$AP = 15$ cm

$PQ = 15$ cm

$\angle POQ = 130° - 90°$ (ext. ∠ of a △)

$= 40°$

$\sin 40° = \frac{15}{OP}$

$OP = \frac{15}{\sin 40°}$

$= 23.336$ cm (to 5 s.f.)

Area of region Y $= \frac{1}{2}(15)(23.336)\sin 50° - \frac{50°}{360°} \times \pi(15)^2$

$= 35.898$ cm² (to 5 s.f.)

Area of region X $= \frac{40°}{360°} \times \pi(15 + 23.336)^2 - 35.898 - \frac{1}{2}\pi(15)^2$

$= 123.67$ cm² (to 5 s.f.)

∴ Required percentage $= \frac{123.67}{35.898} \times 100\%$

$=$ **345%** (to 3 s.f.)

(b) T lies on OQ such that $\angle OTC = 90°$.

Review Exercise 8

1. (a) Arc length $= \frac{65°}{360°} \times 2\pi(4)$

$=$ **4.54 cm** (to 3 s.f.)

Area of sector $= \frac{65°}{360°} \times \pi(4)^2$

$=$ **9.08 cm²** (to 3 s.f.)

(b) Arc length $= \frac{360° - 40°}{360°} \times 2\pi(6)$

$=$ **33.5 cm** (to 3 s.f.)

Area of sector $= \frac{360° - 40°}{360°} \times \pi(6)^2$

$=$ **101 cm²** (to 3 s.f.)

2. (i) $\frac{\angle POQ}{360°} \times 2\pi(10) + \frac{\angle POQ}{360°} \times 2\pi(25) + 2(25 - 10) = 107$

$\frac{\angle POQ}{360°} \times 70\pi + 30 = 107$

$\frac{\angle POQ}{360°} \times 70\pi = 77$

$\angle POQ =$ **126.1°** (to 1 d.p.)

(ii) Area of shaded region

$= \frac{126.051°}{360°} \times \pi(25)^2 - \frac{126.051°}{360°} \times \pi(10)^2$

$=$ **577.5 cm²**

3. (i) $\frac{\angle AOB}{360°} \times 2\pi(9) + 2(9) = 33.75$

$\frac{\angle AOB}{360°} \times 18\pi = 15.75$

$\angle AOB =$ **100.3°** (to 1 d.p.)

(ii) Total area

$= \frac{100.268°}{360°} \times \pi(9)^2 + \frac{360° - 100.268°}{360°} \times [\pi(12)^2 - \pi(9)^2]$

$=$ **214 cm²** (to 3 s.f.)

(iii) Required probability $= \frac{213.670}{\pi(12)^2}$

$=$ **0.472** (to 3 s.f.)

4. (a) Arc length $= \frac{240°}{360°} \times 2\pi(12)$

$= 16\pi$ cm

$=$ **50.3 cm** (to 3 s.f.)

(b) Let the base radius of the cone be r cm.

$2\pi r = 16\pi$

$r = 8$

∴ Base radius of the cone = **8 cm**

5. $\frac{\angle POQ}{360°} \times 2\pi r = 4.8$

$\frac{\angle POQ}{360°} \times \pi r = 2.4$ — (1)

$\frac{\angle POQ}{360°} \times \pi r^2 = 7.68$ — (2)

Substitute (1) into (2):

$r = 3.2$

Substitute $r = 3.2$ into (1):

$\frac{\angle POQ}{360°} \times 3.2\pi = 2.4$

$\angle POQ = 85.944°$ (to 3 d.p.)

Using Cosine Rule,

$PQ^2 = 3.2^2 + 3.2^2 - 2(3.2)(3.2)\cos 85.944°$

$= 19.031$

$PQ = 4.36$ units (to 3 s.f.) ($PQ > 0$) (shown)

9 Geometrical Properties of Circles

Worksheet 9A Symmetric properties of circles

1. (a) Let the midpoint of AB be M.

$AM = MB$ (⊥ bisector of a chord)

Using Pythagoras' Theorem,

$MB^2 + 6^2 = 10^2$

$MB^2 = 64$

$MB = \sqrt{64}$ ($MB > 0$)

$= 8$ cm

∴ $AB = 2(8)$

$=$ **16 cm**

(b) Let the midpoint of AB be M.

$\angle OMA = \angle OMB = 90°$ (⊥ bisector of a chord)

$\angle OAM = \angle OBM = 47°$ (base ∠s of isos. △)

$\cos 47° = \frac{AM}{5}$

$AM = 5\cos 47°$

∴ $AB = 2(5\cos 47°)$

$=$ **6.82 cm** (to 3 s.f.)

2. **(a)** Let the midpoint of AB be M.

$AM = MB = 4.5$ cm ($\perp$ bisector of a chord)

Using Pythagoras' Theorem,

$OA^2 = 4.5^2 + 7^2$

$= 69.25$

$OA = \sqrt{69.25}$ $(OA > 0)$

$= 8.32$ cm (to 3 s.f.)

$\therefore$ Radius = **8.32 cm**

(b) Let the midpoint of AB be M.

$\angle OMA = \angle OMB = 90°$ ($\perp$ bisector of a chord)

$AM = MB = 12$ cm

$\sin 58° = \dfrac{12}{OA}$

$OA = \dfrac{12}{\sin 58°}$

$= 14.2$ cm (to 3 s.f.)

$\therefore$ Radius = **14.2 cm**

3. **(i)** Let the midpoint of PQ be M.

$PM = MQ = 8.5$ cm ($\perp$ bisector of a chord)

Using Pythagoras' Theorem,

$OM^2 + 8.5^2 = 10^2$

$OM^2 = 27.75$

$OM = \sqrt{27.75}$ $(OM > 0)$

$= 5.27$ cm (to 3 s.f.) (shown)

(ii) Using Cosine Rule,

$17^2 = 10^2 + 10^2 - 2(10)(10)\cos \angle POQ$

$289 = 200 - 200 \cos \angle POQ$

$200 \cos \angle POQ = -89$

$\cos \angle POQ = -\dfrac{89}{200}$

$\angle POQ = \cos^{-1}\left(-\dfrac{89}{200}\right)$

$=$ **116.4°** (to 1 d.p.)

4. **(i)** $x =$ **5** (equal chords)

(ii) $\sin \angle OPQ = \dfrac{5}{7}$

$\angle OPQ = \sin^{-1}\dfrac{5}{7}$

$=$ **45.6°** (to 1 d.p.)

5. **(a)** $x° = 90°$ (tangent $\perp$ radius)

$\angle OAQ = 90°$ (tangent $\perp$ radius)

$y° = 180° - 90° - 60°$ ($\angle$ sum of a $\triangle$)

$= 30°$

$\therefore$ $\boldsymbol{x = 90, y = 30}$

(b) $\angle OAB = \angle OBA = 28°$ (base $\angle$s of isos. $\triangle$)

$\angle OAP = 90°$ (tangent $\perp$ radius)

$x° = 90° - 28°$

$= 62°$

$\angle AOB = 180° - 28° - 28°$ ($\angle$ sum of a $\triangle$)

$= 124°$

$y° = 360° - 124°$ ($\angle$s at a pt.)

$= 236°$

$\therefore$ $\boldsymbol{x = 62, y = 236}$

(c) $\angle PAB = \angle ABO = x°$ (alt. $\angle$s, $PQ // BO$)

$\angle OAB = \angle OBA = x°$ (base $\angle$s of isos. $\triangle$)

$\angle OAP = 90°$ (tangent $\perp$ radius)

$x° + x° = 90°$

$2x° = 90°$

$x° = 45°$

Using Pythagoras' Theorem,

$y^2 + y^2 = 4^2$

$2y^2 = 16$

$y^2 = 8$

$y = \sqrt{8}$ $(y > 0)$

$= 2.83$

$\therefore$ $\boldsymbol{x = 45, y = 2.83}$

(d) $\angle OAQ = 90°$ (tangent $\perp$ radius)

$x° = 180° - 90° - 32°$ ($\angle$ sum of a $\triangle$)

$= 58°$

$\tan 58° = \dfrac{y}{8}$

$y = 8 \tan 58°$

$= 12.8$ (to 3 s.f.)

$\therefore$ $\boldsymbol{x = 58, y = 12.8}$

6. **(a)** $x° = 90°$ (tangent $\perp$ radius)

$\angle OAT = \angle OBT = 90°$ (tangent $\perp$ radius)

$y° = 360° - 90° - 110° - 90°$

$= 70°$

$\therefore$ $\boldsymbol{x = 90, y = 70}$

(b) $\angle OAB = \angle OBA = 30°$ (base $\angle$s of isos. $\triangle$)

$\angle OAT = 90°$ (tangent $\perp$ radius)

$x° = 90° - 30°$

$= 60°$

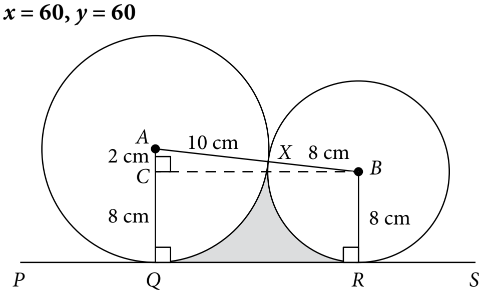
$\angle BAT = \angle ABT = 60°$ (tangents from an ext. pt. are equal)

$y° = 180° - 60° - 60°$ ($\angle$ sum of a $\triangle$)

$= 60°$

$\therefore$ $\boldsymbol{x = 60, y = 60}$

7.



$\cos \angle CAB = \dfrac{2}{18}$

$\angle CAB = \cos^{-1}\dfrac{2}{18}$

$= 83.621°$ (to 3 d.p.)

$\angle ABR = 180° - 83.621°$ (int. $\angle$s, $AQ // BR$)

$= 96.379°$ (to 3 d.p.)

Using Pythagoras' Theorem,

$BC^2 + 2^2 = 18^2$

$BC^2 = 320$

$BC = \sqrt{320}$ $(BC > 0)$

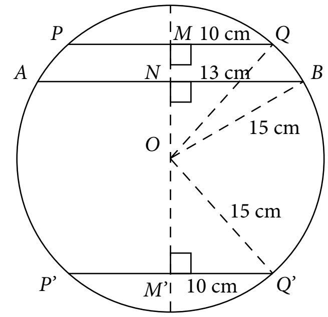
$= 17.889$ cm (to 5 s.f.)

$\therefore$ Area of shaded region

$= \dfrac{1}{2}(10+8)(17.889) - \dfrac{83.621°}{360°} \times \pi(10)^2 - \dfrac{96.379°}{360°} \times \pi(8)^2$

$=$ **34.2 cm²** (to 3 s.f.)

8.



Using Pythagoras' Theorem,

$ON^2 + 13^2 = 15^2$

$ON^2 = 56$

$ON = \sqrt{56}$ cm ($ON > 0$)

Using Pythagoras' Theorem,

$OM^2 + 10^2 = 15^2$

$OM^2 = 125$

$OM = \sqrt{125}$ cm ($OM > 0$)

If AB and PQ are on the same side of O,

Perpendicular distance $= \sqrt{125} - \sqrt{56}$

$= 3.70$ cm (to 3 s.f.)

If AB and PQ are on opposite sides of O,

Perpendicular distance $= \sqrt{125} + \sqrt{56}$

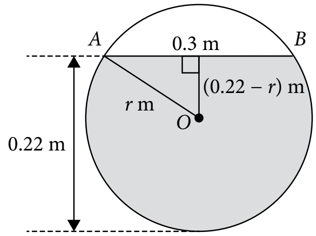
$= 18.7$ cm (to 3 s.f.)

∴ The two possible perpendicular distances are **3.70 cm** and **18.7 cm**.

9. OPEN

(a) A possible value of h is **0.22**.

(b)



(i) Shortest distance from O to AB = **(0.22 – *r*) m**

(ii) Using Pythagoras' Theorem,

$0.15^2 + (0.22 - r)^2 = r^2$

$0.0225 + 0.0484 - 0.44r + r^2 = r^2$

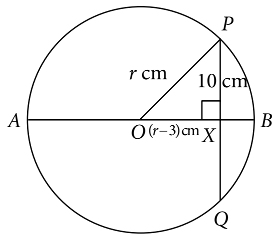
$0.44r = 0.0709$

$r = 0.161$ (to 3 s.f.)

∴ Radius of pipe = **0.161 m**

Challenge Myself!

10.



Let $OP = r$ cm, then $OX = (r - 3)$ cm.

$PX = XQ = 10$ cm (⊥ bisector of a chord)

Using Pythagoras' Theorem,

$(r - 3)^2 + 10^2 = r^2$

$r^2 - 6r + 9 + 100 = r^2$

$6r = 109$

$r = \frac{109}{6}$

∴ Area of circle $= \pi\left(\frac{109}{6}\right)^2$

$=$ **1040 cm²** (to 3 s.f.)

Worksheet 9B Angle properties of circles

1. **(a)** $x° = \frac{1}{2} \times 100°$ (∠ at centre = 2 ∠ at circumference)

$= 50°$

∴ $x =$ **50**

(b) $x° = 2 \times 30°$ (∠ at centre = 2 ∠ at circumference)

$= 60°$

∴ $x =$ **60**

(c) $x° = \frac{1}{2} \times 110°$ (∠ at centre = 2 ∠ at circumference)

$= 55°$

∴ $x =$ **55**

(d) Reflex $\angle AOB = 2 \times 120°$ (∠ at centre = 2 ∠ at circumference)

$= 240°$

$x° = 360° - 240°$ (∠s at a pt.)

$= 120°$

∴ $x =$ **120**

2. **(a)** $\angle APB = 90°$ (rt. ∠ in a semicircle)

$x° = 180° - 90° - 61°$ (∠ sum of a △)

$= 29°$

∴ $x =$ **29**

(b) $\angle OAB = \angle OBA = 28°$ (base ∠s of isos. △)

$\angle PAB = 90°$ (rt. ∠ in a semicircle)

$x° = 90° - 28°$

$= 62°$

∴ $x =$ **62**

3. $\angle ABQ = 180° - \angle BAQ - \angle AQB$ (∠ sum of a △)

$= 180° - 45° - 48°$

$= 87°$

Since $\angle ABQ \neq 90°$, by the converse of right angle in a semicircle, AB is not a diameter of the circle.

4. **(a)** $x° = 50°$ (∠s in the same segment)

$y° = 41°$ (∠s in the same segment)

∴ **$x = 50$, $y = 41$**

(b) $x° = y° = 52°$ (∠s in the same segment)

∴ **$x = 52$, $y = 52$**

5. $x° + 97° = 180°$ (∠s in opp. segments)

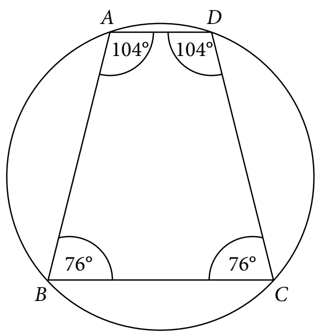
$x° = 83°$

$y° + 86° = 180°$ (∠s in opp. segments)

$y° = 94°$

∴ **$x = 83$, $y = 94$**

6. (a) **Yes.**
Since $\angle ABC + \angle CDA = 180°$, then $\angle BCD + \angle DAB = 180°$.
By the converse of angles in opposite segments, $ABCD$ is a cyclic quadrilateral and a circle can be drawn to pass through all four vertices.
(b) **No.**
In a cyclic quadrilateral, the sum of two opposite angles must be equal.
If one of its angles is a right angle, then the angle opposite it must also be a right angle.
It is not possible to draw a cyclic quadrilateral such that exactly one of its angles is a right angle.
(c) OPEN



7. (i) $\angle ABC = \frac{1}{2} \times \angle AOC$ ($\angle$ at centre = 2 $\angle$ at circumference)
$= \mathbf{2.5x°}$
(ii) $\angle ABC + \angle ADC = 180°$ ($\angle$s in opp. segments)
$2.5x + (3x - 7) = 180$
$5.5x = 187$
$x = 34$
$\therefore x = \mathbf{34}$

8. (i) $\angle DAC = \angle DBC$ ($\angle$s in the same segment)
$= \mathbf{71°}$
(ii) $\angle ADE = \angle DAC + \angle ACD$ (ext. $\angle$ of a $\triangle$)
$= 71° + 43°$
$= \mathbf{114°}$

9. (i) Reflex $\angle BOC = 360° - 160°$ ($\angle$s at a pt.)
$= 200°$
$\angle BDC = \frac{1}{2} \times 200°$ ($\angle$ at centre = 2 $\angle$ at circumference)
$= \mathbf{100°}$
(ii) $\angle BAC = \angle BDC$ ($\angle$s in same segment)
$= 100°$
$\angle CAT = 180° - 100°$ (adj. $\angle$s on a str. line)
$= \mathbf{80°}$

10. (a) $\angle TOA = 57°$ (alt. $\angle$s, TO // AB)
$\angle OAT = 90°$ (tangent $\perp$ radius)
$\therefore \angle OTA = 180 - 90° - 57°$ ($\angle$ sum of a $\triangle$)
$= \mathbf{33°}$
(b) $\cos 57° = \frac{p}{OT}$
$OT = \frac{p}{\cos 57°}$
$\therefore$ Radius of circle $= \frac{1}{2}\left(\frac{p}{\cos 57°}\right)$
$= \mathbf{0.918p}$ **cm** (to 3 s.f.)

11. (i) (a) $\angle OBT = \angle OAT = 90°$ (tangent $\perp$ radius)
$\angle BOT = 180° - \angle OBT - \angle OTB$ ($\angle$ sum of a $\triangle$)
$= 180° - 90° - 21°$
$= \mathbf{69°}$
(b) $\angle AOT = \angle BOT = 69°$
$\angle OAC = \frac{180° - 69°}{2}$ (base $\angle$s of isos. $\triangle$)
$= \mathbf{55.5°}$
(c) Reflex $\angle ACB = 360° - \angle ACO - \angle BCO$ ($\angle$s at a pt.)
$= 360° - 55.5° - 55.5°$
$= \mathbf{249°}$
(ii) Consider $\triangle OBT$.
$\tan 21° = \frac{5}{BT}$
$BT = \frac{5}{\tan 21°}$
$= 13.025$ cm (to 5 s.f.)
$\therefore$ Area of $AOBT = 2 \times \frac{1}{2}(13.025)(5)$
$= \mathbf{65.1}$ $\mathbf{cm^2}$ (to 3 s.f.)

12. (a) (i) $\angle AOB = 180° - \angle OAB - \angle OBA$ ($\angle$ sum of a $\triangle$)
$= 180° - 30° - 30°$
$= \mathbf{120°}$
(ii) $\angle ACB = \frac{1}{2} \times \angle AOB$ ($\angle$ at centre = 2 $\angle$ at circumference)
$= \frac{1}{2} \times 120°$
$= \mathbf{60°}$
(iii) $\angle OAT = \angle OBT = 90°$ (tangent $\perp$ radius)
$\angle ATB = 360° - \angle OAT - \angle OBT - \angle AOB$
$= 360° - 90° - 90° - 120°$
$= \mathbf{60°}$
(b) $\angle ATO = 30°$
$\tan 30° = \frac{OA}{9}$
$OA = 9 \tan 30°$
$= 5.1962$ cm (to 5 s.f.)
$\therefore$ Area of circle $= \pi(5.1962)^2$
$= 85$ cm^2 (to the nearest cm^2) (shown)

13. (a) (i) $\angle BCD = 180° - \angle DPC - \angle ADC$ ($\angle$ sum of a $\triangle$)
$= 180° - 30° - x°$
$= \mathbf{(150 - x)°}$
(ii) $\angle BAD = 180° - \angle AQD - \angle ADC$ ($\angle$ sum of a $\triangle$)
$= 180° - 32° - x°$
$= \mathbf{(148 - x)°}$
Alternatively,
$\angle BAD = 180° - \angle BCD$ ($\angle$s in opp. segments)
$= 180° - (150 - x)°$
$= 180° - 150° + x°$
$= \mathbf{(x + 30)°}$
(b) $\angle BAD + \angle BCD = 180°$ ($\angle$s in opp. segments)
$(148 - x)° + (150 - x)° = 180°$
$298 - 2x = 180$
$2x = 118$
$x = \mathbf{59}$

14. (a) $\angle OAB = 32°$ (base ∠s of isos. △)
$\angle OAD = 38°$ (base ∠s of isos. △)
$\therefore \angle BAD = 32° + 38°$
$= \mathbf{70°}$

(b) $\angle BOD = 2 \times \angle BAD$ (∠ at centre = 2 ∠ at circumference)
$= 2 \times 70°$
$= \mathbf{140°}$

(c) $\angle BCD = 180° - \angle BAD$ (∠s in opp. segments)
$= 180° - 70°$
$= 110°$
$\angle ODC = 180° - \angle BCD$ (int. ∠s, $DO \parallel CB$)
$= 180° - 110°$
$= \mathbf{70°}$

15. (i) $\angle OCE = \angle OEC = 24°$ (base ∠ of isos. △)
$\angle COE = 180° - 24° - 24°$ (∠ sum of a △)
$= 132°$
$\therefore \angle CAE = \frac{1}{2} \times \angle COE$ (∠ at centre = 2 ∠ at circumference)
$= \frac{1}{2} \times 132°$
$= \mathbf{66°}$

(ii) $\angle CDE = 180° - 66°$ (∠s in opp. segments)
$= 114°$
$\therefore \angle CED = 180° - 47° - 114°$ (∠ sum of a △)
$= \mathbf{19°}$

16. (i) $\angle APB = \angle AQB$ (∠s in the same segment)
$= \mathbf{65°}$

(ii) $\angle ACB = 180° - \angle AQB$ (∠s in opp. segments)
$= 180° - 65°$
$= \mathbf{115°}$

(iii) $\angle AOB = 2 \times \angle AQB$ (∠ at centre = 2 ∠ at circumference)
$= 2 \times 65°$
$= 130°$
$\angle OAT = \angle OBT = 90°$ (tangent ⊥ radius)
$\angle ATB = 360° - \angle OAT - \angle OBT - \angle AOB$
$= 360° - 90° - 90° - 130°$
$= \mathbf{50°}$

(iv) $\angle OAC = 90° - 34°$
$= 56°$
$\angle OBC = 360° - \angle OAC - \angle ACB - \angle AOB$
$= 360° - 56° - 115° - 130°$
$= \mathbf{59°}$

17. (i) OPEN △***OAT*** is congruent to △***OBT***.

(ii) (a) $\angle BTO = \angle ATO$
$= 24°$
$\angle OBT = 90°$ (tangent ⊥ radius)
$\therefore \angle BOT = 180° - \angle OBT - \angle BTO$ (∠ sum of a △)
$= 180° - 90° - 24°$
$= \mathbf{66°}$

(b) $\angle OXB = 90°$ (⊥ bisector of a chord)
$\angle OBA = 180° - \angle OXB - \angle BOT$ (∠ sum of a △)
$= 180° - 90° - 66°$
$= \mathbf{24°}$

(c) $\angle BDA = \frac{1}{2} \times \angle AOB$
$= \frac{1}{2} \times 2(66°)$
$= 66°$
$\therefore \angle BAD = 180° - \angle BDA - \angle ABD$ (∠ sum of a △)
$= 180° - 66° - (24° + 21°)$
$= \mathbf{69°}$

(d) $\angle BCD = 180° - \angle BAD$ (∠s in opp. segments)
$= 180° - 69°$
$= \mathbf{111°}$

(iii) Since $\angle OAT = 90°$, by the converse of right angle in a semicircle, OT is a diameter of the circle. Hence, a semicircle, with OT as diameter, passes through A.

18. (a) (i) $\angle ADB = \frac{1}{2} \times \angle AOB$ (∠ at centre = 2 ∠ at circumference)
$= \frac{1}{2} \times 108°$
$= \mathbf{54°}$

(ii) $\angle OAB = \frac{180° - 108°}{2}$ (base ∠s of isos. △)
$= 36°$
$\angle DCB = 180° - \angle DAB$ (∠s in opp. segments)
$= 180° - (36° + 38°)$
$= \mathbf{106°}$

(b) $\angle DBA = 180° - \angle DAB - \angle ADB$ (∠ sum of a △)
$= 180° - 74° - 54°$
$= 52°$
Using Sine Rule,
$$\frac{AB}{\sin \angle ADB} = \frac{AD}{\sin \angle DBA}$$
$$\frac{AB}{\sin 54°} = \frac{6}{\sin 52°}$$
$$AB = \frac{6 \sin 54°}{\sin 52°}$$
$= \mathbf{6.16\ cm}$ (to 3 s.f.)

19. (i) $\angle BCA = \angle ADY$ (∠s in the same segment)
$= \mathbf{45°}$

(ii) $\angle ADC = 90°$ (rt. ∠ in a semicircle)
$\angle BDC = 90° - 45°$
$= 45°$
$\angle BAC = \angle BDC$ (∠s in the same segment)
$= \mathbf{45°}$

(iii) $\angle ODC = \frac{45°}{2}$ (DX bisects $\angle BDC$)
$= 22.5°$
$\angle DOC = 180° - \angle ODC - \angle OCD$ (∠ sum of a △)
$= 180° - 22.5° - 22.5°$
$= \mathbf{135°}$

(iv) $\angle BYC = \angle BDC + \angle DCA$ (ext. ∠ of a △)
$= 45° + 22.5°$
$= \mathbf{67.5°}$

20. (a) (i) $\angle ABC = 180° - \angle ADC$ (∠s in opp. segments)
$= 180° - 120°$
$= 60°$

(ii) $\angle AOC = 2 \times \angle ABC$ (∠ at centre = 2 ∠ at circumference)
$= 2 \times 60°$
$= \mathbf{120°}$

(b) **(i)** Using Cosine Rule,

$AC^2 = AD^2 + CD^2 - 2(AD)(CD)\cos\angle ADC$
$= 3.2^2 + 5.9^2 - 2(3.2)(5.9)\cos 120°$
$= 63.93$
$AC =$ **8.00 cm** (to 3 s.f.)

(ii) Using Sine Rule,

$\frac{\sin\angle CAD}{CD} = \frac{\sin\angle ADC}{AD}$

$\frac{\sin\angle CAD}{5.9} = \frac{\sin 120°}{7.9956}$

$\sin\angle CAD = \frac{5.9\sin 120°}{7.9956}$

$\angle CAD =$ **39.7°** (to 1 d.p.)

Worksheet 9C Alternate Segment Theorem

1. **(i)** $\angle OAB = \frac{180° - 110°}{2}$ (base ∠s of isos. △)
$= 35°$
$\angle OAQ = 90°$ (tangent ⊥ radius)
$\angle CAQ = 90° - 35° - 20°$
$=$ **35°**

(ii) $\angle ABC = \angle CAQ$ (∠s in alt. segments)
$= 35°$
$\angle ACB = 180° - 20° - 35°$ (∠ sum of a △)
$=$ **25°**

2. **(a)** **(i)** $TA = TB$ (tangents from an ext. pt. are equal)

$\angle TAB = \frac{180° - x°}{2}$ (base ∠s of isos. △)

$= \left(\mathbf{90 - \frac{x}{2}}\right)°$

(ii) $\angle OAT = \angle OBT = 90°$ (tangent ⊥ radius)
$\angle AOB = 360° - 90° - 90° - x°$
$= 180° - x°$
Reflex $\angle AOB = 360° - (180° - x°)$ (∠s at a pt.)
$= 180° + x°$

$\angle ACB = \frac{1}{2} \times$ reflex $\angle AOB$

$= \frac{1}{2}(180° + x°)$

$= \left(\mathbf{90 + \frac{x}{2}}\right)°$

(b) **No**, Lily is not correct. By the Alternate Segment Theorem, angle TAC is equal to angle ABC.

Review Exercise 9

1. **(i)** $\angle OAT =$ **90°** because **the tangent *AT* is perpendicular to the radius *OA***.

(ii) $\angle AOB =$ **140°** because **△*OAT* ≡ △*OBT* and angle sum of *OATB* = 360°**.

(iii) $\angle ACB =$ **70°** because **angle at centre = 2 × angle at circumference**.

2. **(a)** $\angle BAC = 144° - 68°$ (ext. ∠ of a △)
$= 76°$
Since $\angle BAC \neq 90°$, by the converse of right angle in a semicircle, BC is not a diameter of the circle.

(b) OPEN An example of the values is $\boldsymbol{p = 70}$ and $\boldsymbol{q = 160}$.

3. **(i)** $\angle ECD = \angle EAD$ (∠s in the same segment)
$= 52°$
$\angle AED = 90°$ (rt. ∠ in a semicircle)
$\therefore \angle EDA = 180 - \angle AED - \angle EAD$ (∠ sum of a △)
$= 180° - 90° - 52°$
$= 38°$
$\angle AEC = 180° - \angle ABC$ (∠s in opp. segments)
$= 180° - 106°$
$= 74°$
$\therefore \angle ECD =$ **52°**, $\angle EDA =$ **38°**, $\angle AEC =$ **74°**

(ii) **(a)** $\angle AXC = \angle EAX + \angle AEX$ (ext. ∠ of a △)
$= 52° + 74°$
$=$ **126°**

(b) $\angle CED = 90° - 74°$
$= 16°$
$\angle CAD = \angle CED$ (∠s in the same segment)
$= 16°$
$\angle BAC = 180° - \angle ABC - \angle CAD$ (int. ∠s, $BC \parallel AD$)
$= 180° - 106° - 16°$
$=$ **58°**

10 Geometrical Transformations

Worksheet 10A Reflection

1. **(a)** **(i)** **(1, –4)**
(ii) **(5, 4)**
(iii) **(1, –1)**

(b) **(i)** Gradient of $BC = \frac{-1 - (-3)}{1 - (-1)}$
$= 1$
Equation of image: $\boldsymbol{y = -x + 2}$

(ii) Equation of image: $\boldsymbol{y = -x - 2}$

2. Gradient of line of reflection $= \frac{2 - 0}{2 - (-2)}$
$= \frac{1}{2}$
Equation of line of reflection: $\boldsymbol{y = \frac{1}{2}x + 1}$

3. **(a)** **(6, –2)** **(b)** **(–2, 6)**
(c) **(–5, 8)** **(d)** **(4, 6)**
(e) **(15, 0)** **(f)** **(–9, –16)**
(g) **(–1, –1)** **(h)** **(3, –5)**

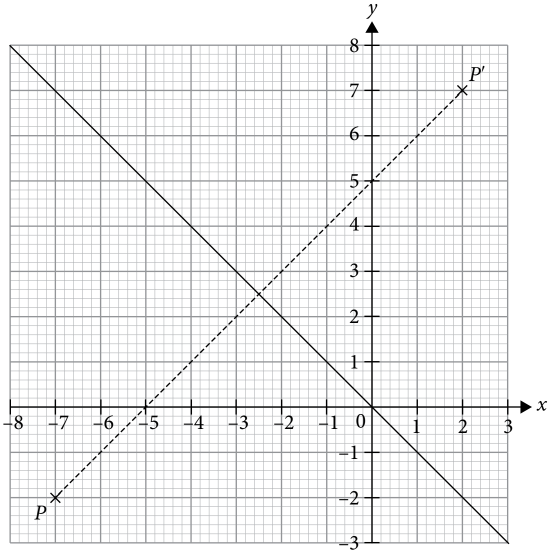
4. **(a)** The x-coordinates are the same.
Equation of l: $y = \frac{0 + (-3)}{2}$
$\boldsymbol{y = -1.5}$

(b) The y-coordinates are the same.

Equation of l: $x = \frac{-4 + 6}{2}$

$\boldsymbol{x = 1}$

(c)

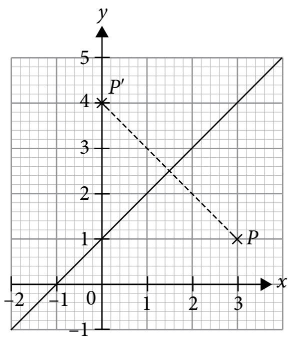


Gradient of $l = \frac{3 - 0}{-3 - 0}$

$= -1$

Equation of l: $\boldsymbol{y = -x}$

(d)

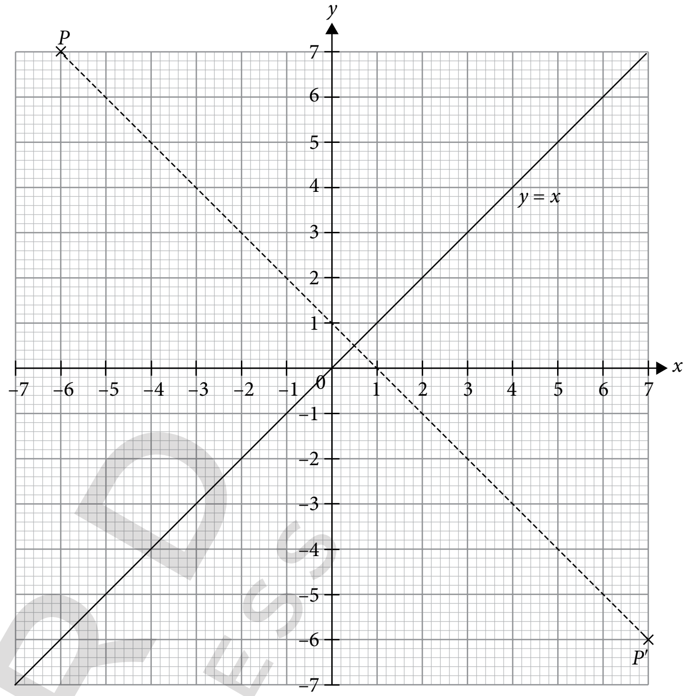


Gradient of $l = \frac{3 - 1}{2 - 0}$

$= 1$

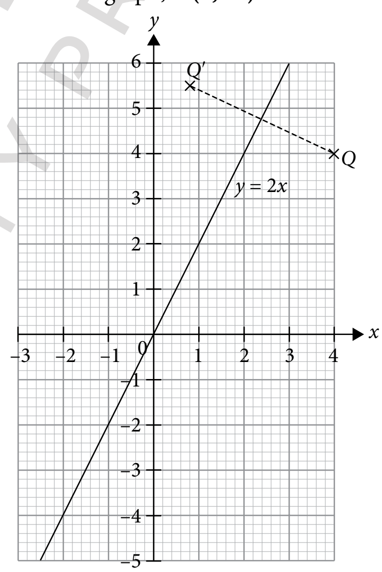
Equation of l: $\boldsymbol{y = x + 1}$

5. (a)



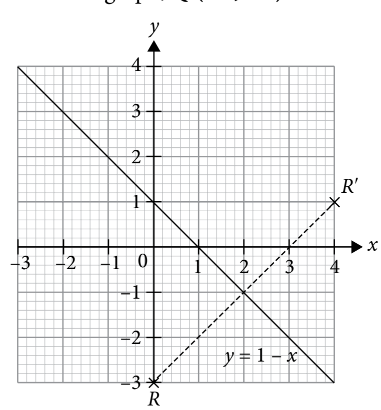
From the graph, $\boldsymbol{P'(7, -6)}$.

(b)



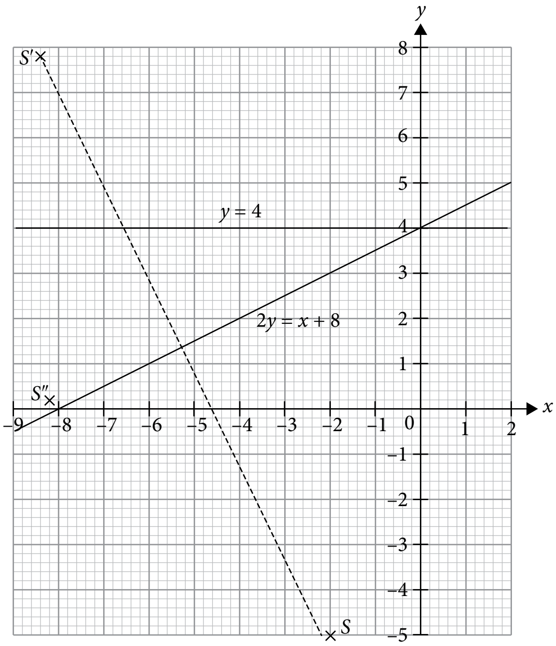
From the graph, $\boldsymbol{Q'(0.8, 5.6)}$.

(c)



From the graph, $\boldsymbol{R'(4, 1)}$.

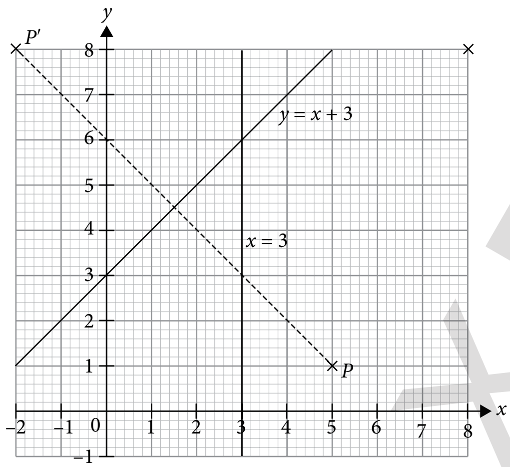
(d)



From the graph, ***S*″(−8.4, 0.2)**.

Challenge Myself!

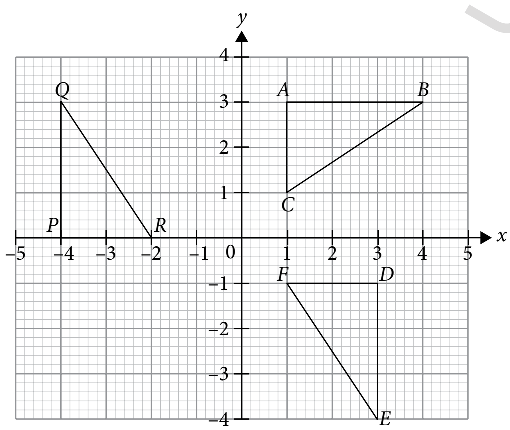
6.



From the graph, ***a* = 5**, ***b* = 1**.

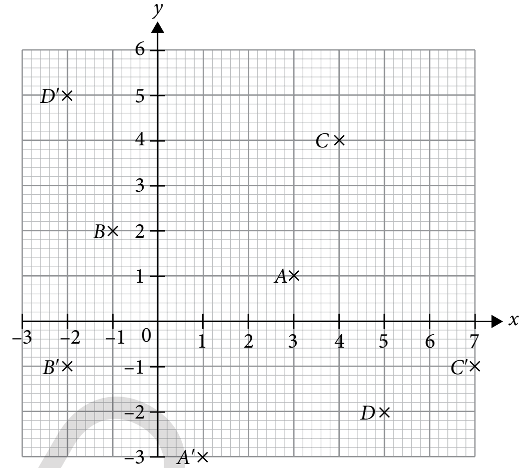
Worksheet 10B Rotation

1.



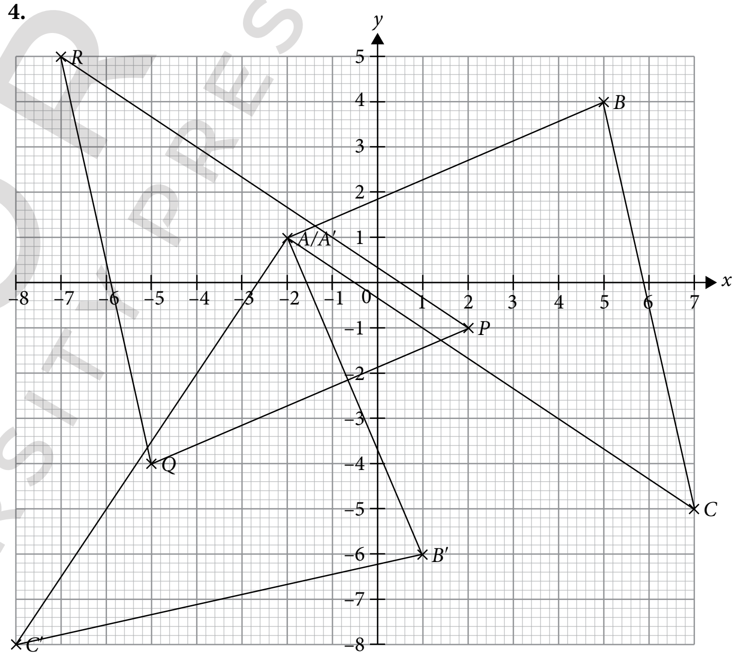
2. Centre of rotation: **(−1, 0)**
Angle and direction of rotation: **90° anticlockwise**

3.

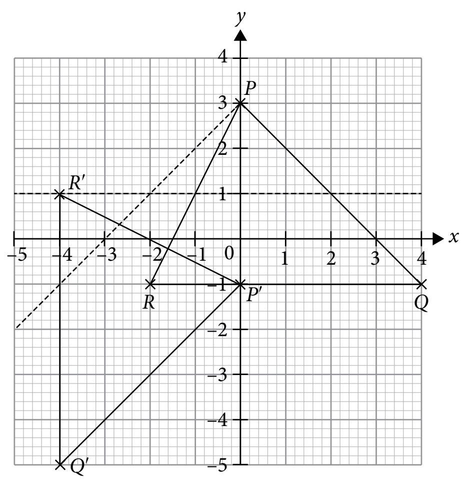


(a) ***A*′(1, −3)** (b) ***B*′(−2, −1)**
(c) ***C*′(7, −1)** (d) ***D*′(−2, 5)**

4.



5.



From the graph, ***W*(−2, 1)**.

Challenge Myself!

6. **(a)** Gradient of $l_1 = \frac{-2-3}{6-3}$

$= -\frac{5}{3}$

Equation of l_1: $y = -\frac{5}{3}x + c$ — (1)

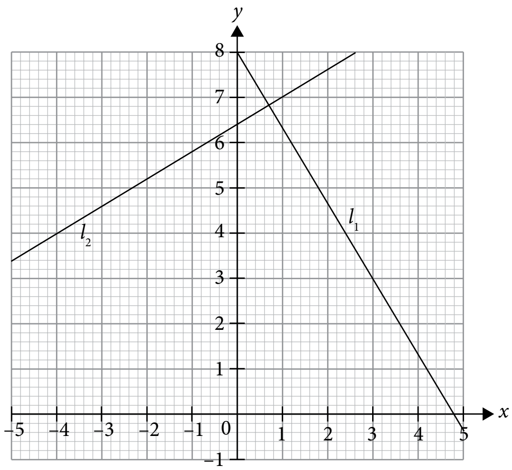
Substitute $x = 3$, $y = 3$ into (1):

$3 = -\frac{5}{3}(3) + c$

$c = 8$

$\therefore$ Equation of l_1: $\boldsymbol{y = -\frac{5}{3}x + 8}$

(b)



Gradient of $l_2 = \frac{7-4}{1-(-4)}$

$= \frac{3}{5}$

From $5y - 3x = 44$,

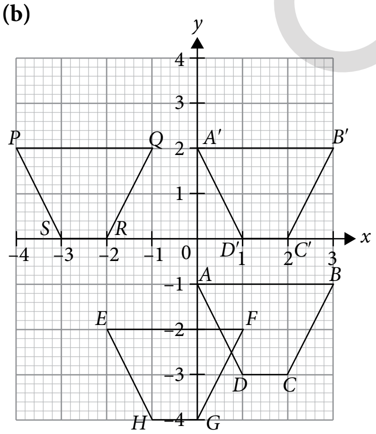
$y = \frac{3}{5}x + \frac{44}{5}$

Gradient $= \frac{3}{5}$

$\therefore$ l_2 is parallel to $5y - 3x = 44$. l_2 does not pass through the point $(-4, 5)$.

Worksheet 10C Translation

1. **(a), (b)**



(c) *ABCD* is mapped onto *PQRS* by a translation of 4 units in the negative *x*-axis, followed by a translation of 3 units in the positive *y*-axis.

2. **(a)** $\boldsymbol{A'(10, 1)}$ **(b)** $\boldsymbol{B'(-2, 9)}$

(c) $\boldsymbol{C'(4, -12)}$ **(d)** $\boldsymbol{D'(0, 10)}$

3. **(a)** $\begin{pmatrix} \mathbf{6} \\ \mathbf{-2} \end{pmatrix}$ **(b)** $\begin{pmatrix} \mathbf{0} \\ \mathbf{-6} \end{pmatrix}$

(c) $\begin{pmatrix} \mathbf{10} \\ \mathbf{0} \end{pmatrix}$ **(d)** $\begin{pmatrix} \mathbf{-5} \\ \mathbf{5} \end{pmatrix}$

4. **(a)** Coordinates of B: $(h - 3, k + 5)$

Coordinates of C: $(h + 5, k - 3)$

$h + 5 = 2k$ — (1)

$k - 3 = h$ — (2)

Substitute (2) into (1):

$k - 3 + 5 = 2k$

$k = 2$

Substitute $k = 2$ into (2):

$h = 2 - 3$

$= -1$

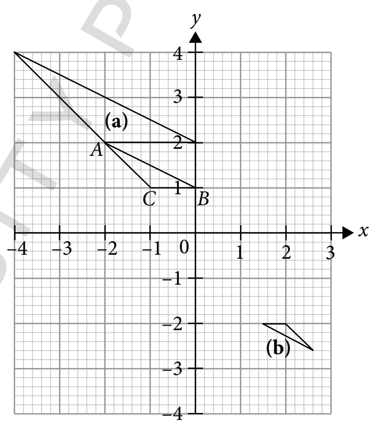
$\therefore$ $\boldsymbol{h = -1, k = 2}$

(b) $A(-1, 2)$ is mapped onto D under $\begin{pmatrix} -1 \\ -1 \end{pmatrix}$.

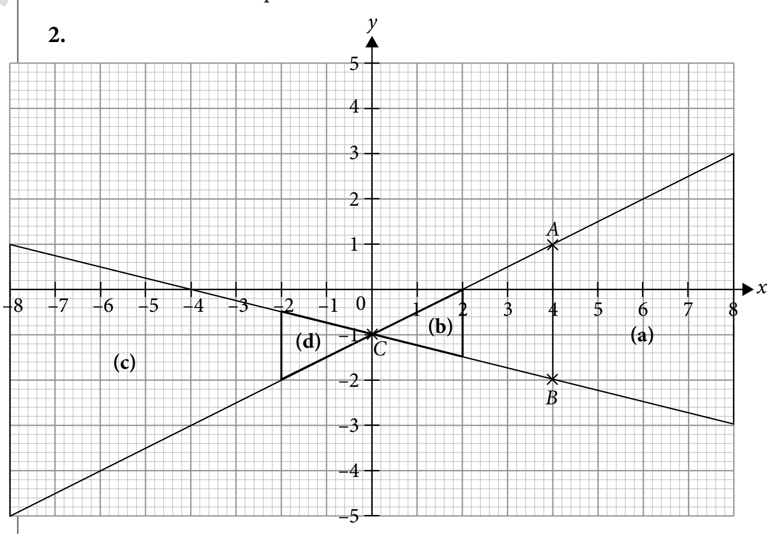
$\therefore$ $\boldsymbol{D(-2, 1)}$

Worksheet 10D Enlargement

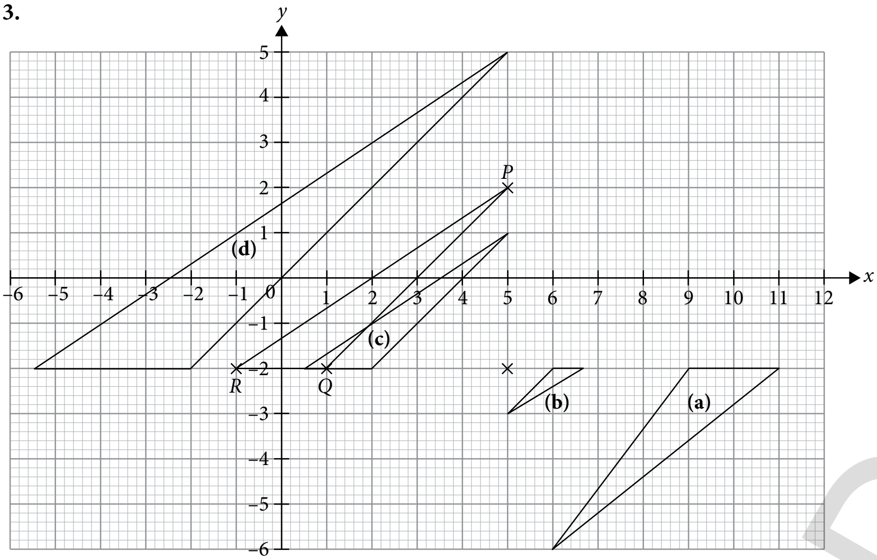
1.



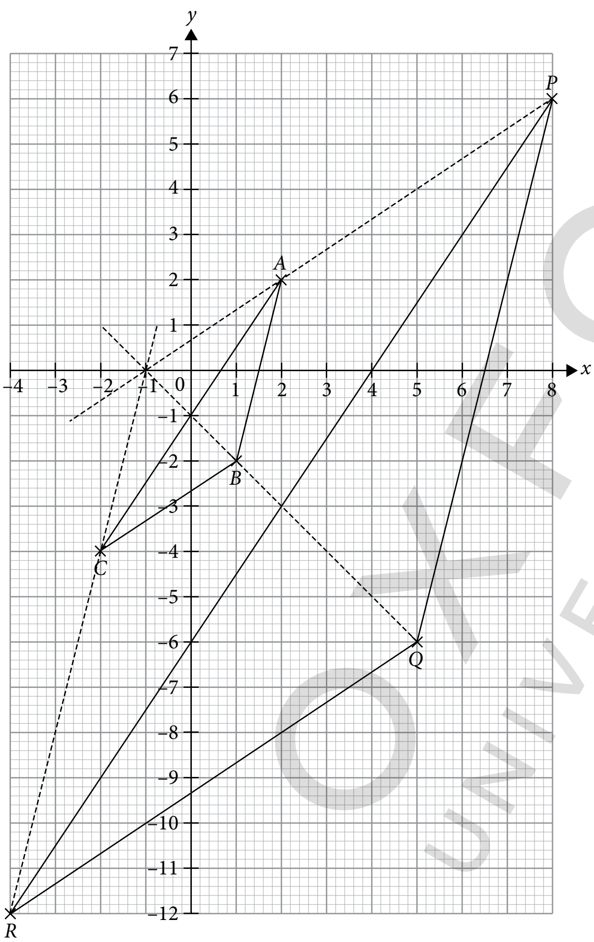
2.



3.



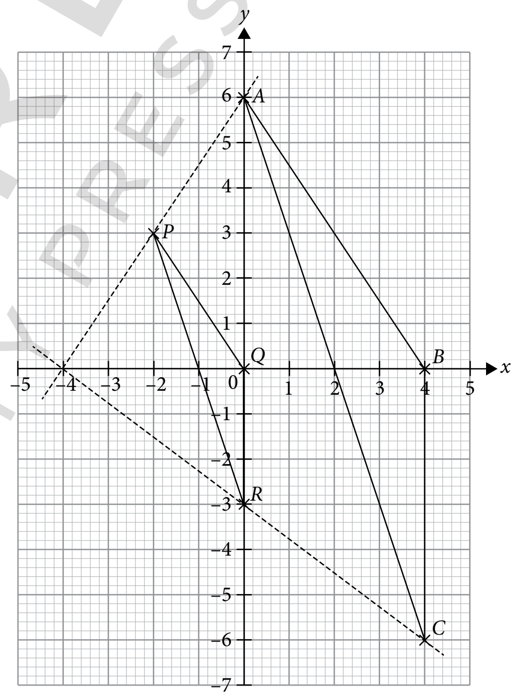
4. **(a)**



Centre of enlargement: **(–1, 0)**

Scale factor = **3**

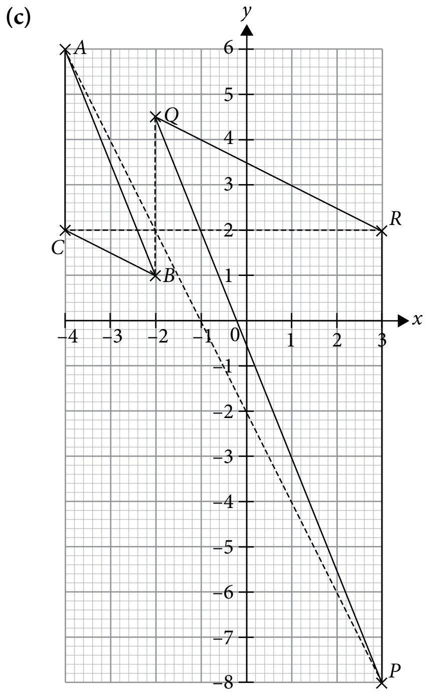
(b)



Centre of enlargement: **(–4, 0)**

Scale factor = $\mathbf{\frac{1}{2}}$

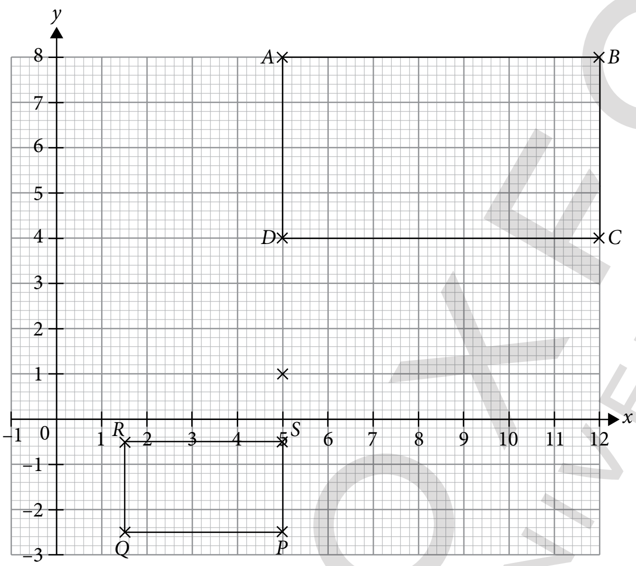
(c)



Centre of enlargement: **(–2, 2)**

Scale factor = $\mathbf{\frac{5}{2}}$

5.

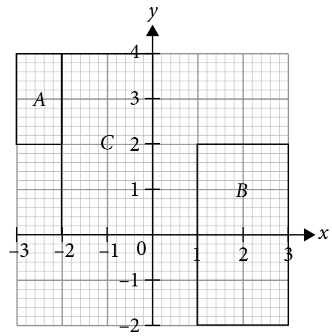


∴ ***P*(5, –2.5), *Q*(1.5, –2.5), *R*(1.5, –0.5), *S*(5, –0.5)**

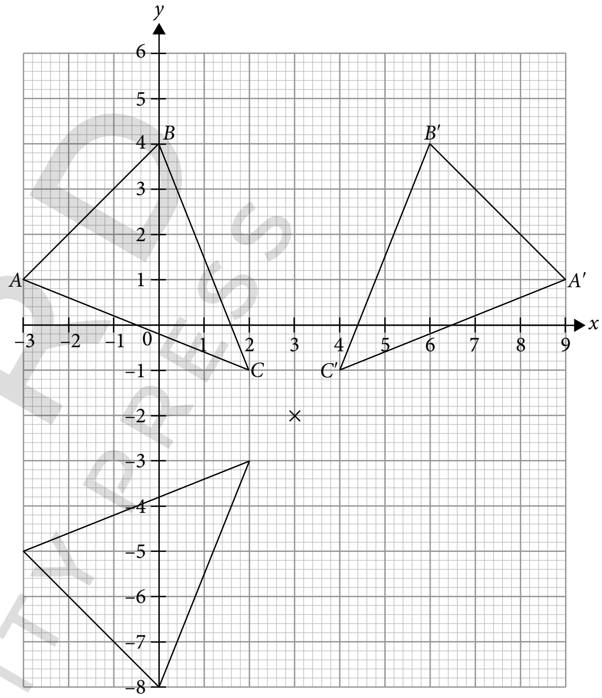
Worksheet 10E Combined transformations

1. (a) Coordinates of *B*: (–5, 2)
   Coordinates of *C*: (–5, –2)
   Coordinates of *D*: **(–1, –5)**
   (b) Coordinates of *Q*: (–5, 15)
   Coordinates of *R*: (11, 15)
   Coordinates of *S*: **(0, 25)**

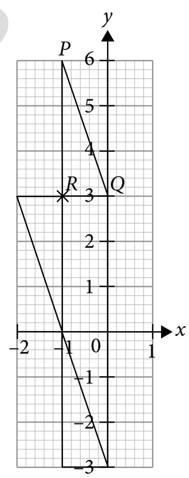
2.



3.



4.



∴ Coordinates of invariant point: **(–1, 3)**

5. (a) (i) An enlargement with scale factor $\frac{1}{2}$ and centre of enlargement at (1, 2), followed by a translation with vector $\begin{pmatrix} -4 \\ 2 \end{pmatrix}$.
   (ii) An enlargement with scale factor 2 and centre of enlargement at (–3, 4), followed by a translation with vector $\begin{pmatrix} 4 \\ -2 \end{pmatrix}$.

(iii) A 90° anticlockwise rotation about (–1, 3), followed by an enlargement with scale factor 3 and centre of enlargement at (–1, 3).

(iv) A 90° clockwise rotation about (–1, 3), followed by an enlargement with scale factor $\frac{1}{3}$ and centre of enlargement at (–1, 3).

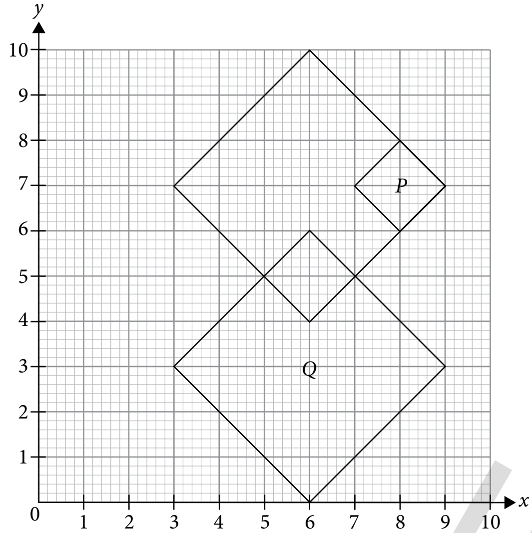
(b) An enlargement with scale factor $\frac{1}{2}$ and centre of enlargement at (–7, 6).

Challenge Myself!

6. OPEN

(a) **(7, 7), (8, 8), (9, 7), (8, 6)**

(b)



(c) An enlargement with scale factor 3 and centre of enlargement at (9, 7), followed by a reflection in the line $y = 5$.

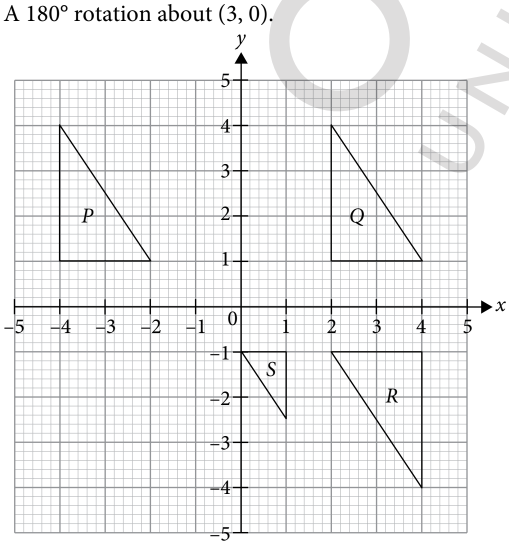
Review Exercise 10

1. **(a)** $\begin{pmatrix} \mathbf{6} \\ \mathbf{0} \end{pmatrix}$

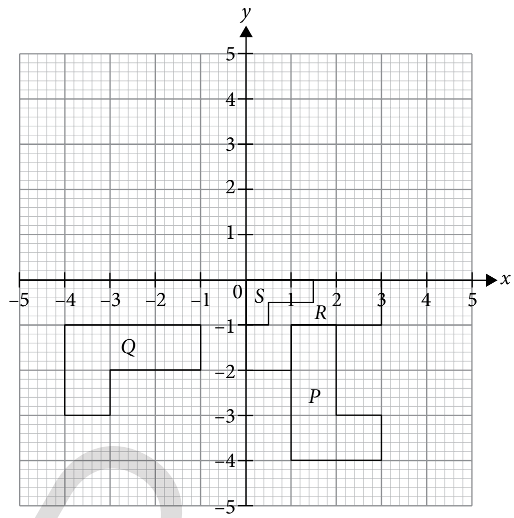
(b) A 180° rotation about (0, 0).

(c) A 180° rotation about (3, 0).

(d)



2.



3. An enlargement with scale factor 2 and centre of enlargement at (1, 0).

11 Area and Volume of Similar Figures and Solids

Worksheet 11A Area of similar figures

1. **(a)** $\frac{p}{12} = \left(\frac{6.8}{5}\right)^2$

$p = \left(\frac{6.8}{5}\right)^2 \times 12$

$= \mathbf{22.2}$ (to 3 s.f.)

(b) $\frac{q}{360} = \left(\frac{10}{12}\right)^2$

$q = \left(\frac{10}{12}\right)^2 \times 360$

$= \mathbf{250}$

(c) $\frac{w}{400} = \left(\frac{2h}{4h}\right)^2$

$w = \left(\frac{2}{4}\right)^2 \times 400$

$= \mathbf{100}$

(d) $\left(\frac{x}{23}\right)^2 = \frac{273}{125}$

$\frac{x}{23} = \sqrt{\frac{273}{125}}$

$x = \sqrt{\frac{273}{125}} \times 23$

$= \mathbf{34.0}$ (to 3 s.f.)

(e) $\left(\frac{y}{20}\right)^2 = \frac{90}{160}$

$\frac{y}{20} = \sqrt{\frac{90}{160}}$

$y = \sqrt{\frac{90}{160}} \times 20$

$= \mathbf{15}$

(f) $\left(\frac{z}{29.8}\right)^2 = \frac{75}{48}$

$\frac{z}{29.8} = \sqrt{\frac{75}{48}}$

$z = \sqrt{\frac{75}{48}} \times 29.8$

$= \mathbf{37.25}$

2. Let the area of $\triangle ADE$ be k cm^2.

$\frac{k}{k+35} = \left(\frac{7}{11}\right)^2$

$= \frac{49}{121}$

$121k = 49k + 1715$

$72k = 1715$

$k = 23.8$ (to 3 s.f.)

$\therefore$ Area of $\triangle ADE$ = **23.8 cm²**

3. **(a)** Using similar triangles,

$\frac{AB}{19} = \frac{6}{9}$

$AB = \frac{6}{9} \times 19$

$= \mathbf{12\frac{2}{3}}$ **cm**

(b) Using similar triangles,

$\frac{QX}{10} = \frac{9}{6}$

$QX = \frac{9}{6} \times 10$

$= 15$ cm

$\therefore AQ = 6 + 15$

$= \mathbf{21}$ **cm**

(c) $\frac{\text{Area of } \triangle PQX}{k} = \left(\frac{9}{6}\right)^2$

Area of $\triangle PQX = \left(\frac{9}{6}\right)^2 \times k$

$= \mathbf{2.25}\boldsymbol{k}$ **cm²**

4. **(a)** **True.** The ratio of the perimeters is equal to the ratio of two corresponding lengths.

(b) **False.** The ratio of the areas is $p^2 : q^2$.

5. **(a)** $\frac{\text{Area of } \triangle APX}{\text{Area of } \triangle ABR} = \left(\frac{5k}{7k}\right)^2$

$= \mathbf{\frac{25}{49}}$

(b) $\frac{\text{Area of } \triangle AXQ}{\text{Area of } \triangle ARC} = \left(\frac{AX}{AR}\right)^2$

$= \left(\frac{PX}{BR}\right)^2$

$= \left(\frac{5k}{7k}\right)^2$

$= \frac{25}{49}$

$\therefore$ Area of $\triangle AXQ$: area of $\triangle ARC$ = **25 : 49**

(c) $\frac{\text{Area of } \triangle ARC}{\text{Area of } \triangle ABC} = \frac{4k}{11k}$

$= \mathbf{\frac{4}{11}}$

(d) Area of $\triangle ARC$: area of $\triangle ABR = 4 : 7$

Area of $\triangle ABR$: area of $PBRX = 49 : 24$

$\therefore$ Area of $\triangle ARC$: area of $PBRX = 28 : 24$

$=$ **7 : 6**

6. OPEN

(a) Let the radii be 4 cm and 9 cm respectively.

Then the areas are $\pi(4)^2$ cm^2 = 16 cm^2 and $\pi(9)^2$ cm^2 = 81π cm^2 respectively.

$\therefore$ Radius = **4 cm**, area = **16π cm²**; radius = **9 cm**, area = **81π cm²**

(b) Ratio of lengths = 2 : 3

Let the dimensions of the smaller rectangle be 10 cm by 5 cm.

Then the dimensions of the larger rectangle are 15 cm by 7.5 cm.

$\therefore$ Dimensions are **10 cm by 5 cm** and **15 cm by 7.5 cm**

7. **(a)** $\angle CBX = \angle BXA$ (alt. ∠s, CB // XA)

$= \mathbf{80°}$

(b) $\angle AXB = \angle DXE$ (vert. opp. ∠s)

$\angle BAX = \angle EDX$ (alt. ∠s, AB // ED)

$\therefore \triangle ABX$ is similar to $\triangle DEX$. (shown)

(c) $\triangle$***CEB*** is similar to $\triangle ABX$ and $\triangle DEX$.

(d) **(i)** $ED : AB = 3 : 8$

$\therefore BC : AX$ = **11 : 8**

(ii) $\frac{\text{Area of } \triangle ABX}{2.25} = \left(\frac{8}{3}\right)^2$

Area of $\triangle ABX = \left(\frac{8}{3}\right)^2 \times 2.25$

$= 16$ cm^2

$\frac{\text{Area of } \triangle CEB}{2.25} = \left(\frac{11}{3}\right)^2$

Area of $\triangle CEB = \left(\frac{11}{3}\right)^2 \times 2.25$

$= 30.25$ cm^2

Area of $ABCEX = 16 + 30.25$

$=$ **46.25 cm²**

Worksheet 11B Volume of similar solids

1. (a) $\dfrac{p}{81} = \left(\dfrac{12}{9}\right)^3$

$p = \left(\dfrac{12}{9}\right)^3 \times 81$

$= \mathbf{192}$

(b) $\dfrac{q}{1280\pi} = \left(\dfrac{8}{16}\right)^3$

$q = \left(\dfrac{8}{16}\right)^3 \times 1280\pi$

$= 160\pi$

$= \mathbf{503}$ (to 3 s.f.)

(c) $\dfrac{w}{480} = \left(\dfrac{6}{9}\right)^3$

$w = \left(\dfrac{6}{9}\right)^3 \times 480$

$= \mathbf{142}$ (to 3 s.f.)

(d) $\left(\dfrac{x}{7}\right)^3 = \dfrac{200}{500}$

$\dfrac{x}{7} = \sqrt[3]{\dfrac{200}{500}}$

$x = \sqrt[3]{\dfrac{200}{500}} \times 7$

$= \mathbf{5.16}$ (to 3 s.f.)

(e) $\left(\dfrac{y}{4}\right)^3 = \dfrac{86}{14.7}$

$\dfrac{y}{4} = \sqrt[3]{\dfrac{86}{14.7}}$

$y = \sqrt[3]{\dfrac{86}{14.7}} \times 4$

$= \mathbf{7.21}$ (to 3 s.f.)

(f) $\left(\dfrac{z}{18}\right)^3 = \dfrac{760}{950}$

$\dfrac{z}{18} = \sqrt[3]{\dfrac{760}{950}}$

$z = \sqrt[3]{\dfrac{760}{950}} \times 18$

$= \mathbf{16.7}$ (to 3 s.f.)

2. $\dfrac{V_{\text{small}}}{V_{\text{large}}} = \left(\dfrac{r_{\text{small}}}{r_{\text{large}}}\right)^3$

$\dfrac{400}{800} = \left(\dfrac{r_{\text{small}}}{r_{\text{large}}}\right)^3$

$\dfrac{r_{\text{small}}}{r_{\text{large}}} = \sqrt[3]{\dfrac{400}{800}}$

$\therefore$ Required percentage $= \sqrt[3]{\dfrac{400}{800}} \times 100\%$

$= \mathbf{79.4\%}$ (to 3 s.f.)

3. $\dfrac{V_{\text{model}}}{V_{\text{actual}}} = \left(\dfrac{h_{\text{model}}}{h_{\text{actual}}}\right)^3$

$\dfrac{V_{\text{model}}}{2800} = \left(\dfrac{1}{20}\right)^3$

$V_{\text{model}} = \left(\dfrac{1}{20}\right)^3 \times 2800$

$= 0.35 \text{ m}^3$

$= \mathbf{(3.5 \times 10^{-1})\ m^3}$

4. (a) $\dfrac{m_{\text{large}}}{m_{\text{small}}} = \left(\dfrac{h_{\text{large}}}{h_{\text{small}}}\right)^3$

$\dfrac{m_{\text{large}}}{6} = \left(\dfrac{125}{75}\right)^3$

$m_{\text{large}} = \left(\dfrac{125}{75}\right)^3 \times 6$

$= 27.8$ (to 3 s.f.)

$\therefore$ Mass of larger figurine = **27.8 kg**

(b) Mass of larger figurine = 27.8×10^3 g

$= \mathbf{2.78 \times 10^4\ g}$

5. $\dfrac{V_{\text{small}}}{V_{\text{large}}} = \left(\dfrac{h_{\text{small}}}{h_{\text{large}}}\right)^3$

$\dfrac{1.25}{2.16} = \left(\dfrac{h_{\text{small}}}{h_{\text{large}}}\right)^3$

$\dfrac{h_{\text{small}}}{h_{\text{large}}} = \sqrt[3]{\dfrac{1.25}{2.16}}$

$= \dfrac{5}{6}$

$\dfrac{A_{\text{small}}}{A_{\text{large}}} = \left(\dfrac{5}{6}\right)^2$

$= \dfrac{25}{36}$

$\therefore$ Ratio of the total surface areas is **25 : 36**

6. (a) The volume of the ball will be $2^3 = 8$ times of its initial volume.

(b) The volume of the ball will then be $\left(\dfrac{1}{2}\right)^3 = \dfrac{1}{8}$ times of its initial volume.

Worksheet 11C Solving problems involving similar solids

1. (a) $\dfrac{V_{\text{large}}}{V_{\text{small}}} = \left(\dfrac{h_{\text{large}}}{h_{\text{small}}}\right)^3$

$\dfrac{3}{2.4} = \left(\dfrac{h_{\text{large}}}{21}\right)^3$

$\dfrac{h_{\text{large}}}{21} = \sqrt[3]{\dfrac{3}{2.4}}$

$h_{\text{large}} = \sqrt[3]{\dfrac{3}{2.4}} \times 21$

$= 22.6$ (to 3 s.f.)

$\therefore$ Height of larger container = **22.6 cm**

(b) $\dfrac{A_{small}}{A_{large}} = \left(\dfrac{h_{small}}{h_{large}}\right)^2$

$= \left(\sqrt[3]{\dfrac{2.4}{3}}\right)^2$

$= 0.86$ (to 2 d.p.)

$\therefore k = \mathbf{0.86}$

2. Daniel's method

Let the radii of the spheres be r_1 cm and r_2 cm respectively.

$\left(\dfrac{r_2}{r_1}\right)^2 = \dfrac{600}{450}$

$\dfrac{r_2}{r_1} = \sqrt{\dfrac{600}{450}}$

$= \sqrt{\dfrac{4}{3}}$

$\dfrac{V_2}{V_1} = \left(\dfrac{r_2}{r_1}\right)^3$

$= \left(\sqrt{\dfrac{4}{3}}\right)^3$

$V_2 = \left(\sqrt{\dfrac{4}{3}}\right)^3 \times V_1$

$\therefore$ If the value of V_1 is known, then the value of V_2 can be found.

Fiona's method

Let the radius of the larger sphere be r_2 cm.

$4\pi r_2^2 = 600$

$r_2^2 = \dfrac{600}{4\pi}$

$r_2 = \sqrt{\dfrac{600}{4\pi}}$

$= \sqrt{\dfrac{150}{\pi}}$

$V_2 = \dfrac{4}{3}\pi\left(\sqrt{\dfrac{150}{\pi}}\right)^3$

$= 1380$ (to 3 s.f.)

$\therefore$ The value of V_2 can be found without the value of V_1.

Both Daniel and Fiona are correct.

3. (a) Height of A : height of B : height of C = **1 : 1 : 1**

(b) $\dfrac{\text{Volume of C}}{\text{Volume of B+C}} = \left(\dfrac{1}{2}\right)^3$

$= \dfrac{1}{8}$

$\therefore$ Volume of B : volume of C = 7 : 1

$\dfrac{\text{Volume of C}}{\text{Volume of A+B+C}} = \left(\dfrac{1}{3}\right)^3$

$= \dfrac{1}{27}$

$\therefore$ Volume of A : volume of B : volume of C = **19 : 7 : 1**

(c) Required percentage $= \dfrac{7}{27} \times 100\%$

$= \mathbf{25.9\%}$ (to 3 s.f.)

4. (i) Using Pythagoras' Theorem,

$x^2 = \left(\dfrac{84-52}{2}\right)^2 + 33^2$

$= 1345$

$x = \sqrt{1345}$ $(x > 0)$

$= 36.7$ (to 3 s.f.) (shown)

(ii) Let the slant height of the cone be l cm.

Using similar triangles,

$\dfrac{l}{l-\sqrt{1345}} = \dfrac{93}{60}$

$60l = 93l - 93\sqrt{1345}$

$33l = 93\sqrt{1345}$

$l = \dfrac{93\sqrt{1345}}{33}$

$= 103.35$ (to 5 s.f.)

Total surface area $= \pi(42)^2 + \pi(26)^2 + \pi(42)(103.35) - \pi(26)(103.35 - 36.674)$

$= \mathbf{16\,000\ cm^2}$ (to 2 s.f.)

(iii) $\dfrac{V_{small}}{V_{large}} = \left(\dfrac{h_{small}}{h_{large}}\right)^3$

$\dfrac{1}{2} = \left(\dfrac{h_{small}}{33}\right)^3$

$\dfrac{h_{small}}{33} = \sqrt[3]{\dfrac{1}{2}}$

$h_{small} = \sqrt[3]{\dfrac{1}{2}} \times 33$

$= 26.2$ (to 3 s.f.)

$\therefore$ Height of smaller frustum = **26.2 cm**

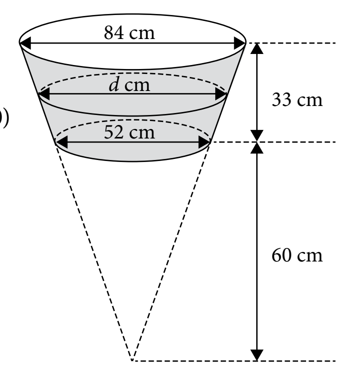
(iv) OPEN Let the diameter of the top surface of the smaller frustum be d cm.

$\dfrac{d}{84} = \dfrac{52}{d}$

$d^2 = 4368$

$d = \sqrt{4368}$ $(d > 0)$

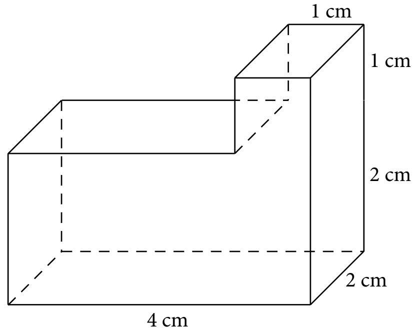
$= 66.1$ (to 3 s.f.)



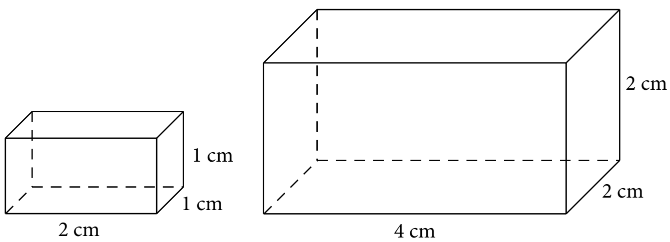
$\therefore$ One condition is that the diameter of the top surface of the smaller frustum is 66.1 cm.

Challenge Myself!

5. Original prism



Two similar solids



Review Exercise 11

1. $\left(\dfrac{\text{Length of side of }\triangle ABC}{\text{Length of corresponding side of }\triangle PQR}\right)^2 = \left(\dfrac{10}{15}\right)^2$

$= \dfrac{4}{9}$

$\neq \dfrac{2}{3}$

$\therefore$ Since $\dfrac{\text{area of }\triangle ABC}{\text{area of }\triangle PQR}$

$\neq \left(\dfrac{\text{length of side of }\triangle ABC}{\text{length of corresponding side of }\triangle PQR}\right)^2$,

$\triangle ABC$ is not similar to $\triangle PQR$. (shown)

2. $\dfrac{A_X}{A_Y} = \dfrac{72\pi}{18\pi}$

$= 4$

$= 2^2$

$\dfrac{V_X}{V_Y} = \dfrac{72k\pi}{9k\pi}$

$= 8$

$= 2^3$

Let the heights of cylinders X and Y be h_X and h_Y respectively.

Since $\dfrac{A_X}{A_Y} = \left(\dfrac{h_X}{h_Y}\right)^2$ and $\dfrac{V_X}{V_Y} = \left(\dfrac{h_X}{h_Y}\right)^3$, the two cylinders are similar solids.

3. **(a)** $\angle BFC = \angle BED$ (corr. $\angle$s, CF // DE)

$\angle BCF = \angle BDE$ (corr. $\angle$s, CF // DE)

$\therefore$ $\triangle BFC$ is similar to $\triangle BED$.

(b) $\triangle$***ABD*** is similar to $\triangle$***FCD***.

(c) **(i)** $\dfrac{\text{Area of }\triangle FCD}{\text{Area of }\triangle ABD} = \left(\dfrac{8}{17}\right)^2$

$= \mathbf{\dfrac{64}{289}}$

(ii) $\dfrac{\text{Area of }\triangle BFC}{\text{Area of }\triangle DFC} = \mathbf{\dfrac{9}{8}}$

4. $\dfrac{V_{\text{water}}}{V_{\text{cone}}} = \left(\dfrac{h_{\text{water}}}{h_{\text{cone}}}\right)^3$

$\dfrac{1}{2} = \left(\dfrac{h_{\text{water}}}{24}\right)^3$

$\dfrac{h_{\text{water}}}{24} = \sqrt[3]{\dfrac{1}{2}}$

$h_{\text{water}} = \sqrt[3]{\dfrac{1}{2}} \times 24$

$= 19.049$ (to 5 s.f.)

$\neq 12$

$\therefore$ The classmate is incorrect. (shown)

$\therefore$ $x = 24 - 19.049$

$=$ **4.95** (to 3 s.f.)

5. **(i)** Since $2AD = 5AX$, then $\dfrac{AX}{AD} = \dfrac{2}{5}$ and $\dfrac{AX}{XD} = \dfrac{2}{3}$.

$\dfrac{\text{Area of }\triangle ABX}{\text{Area of }\triangle DCX} = \left(\dfrac{2}{3}\right)^2$

$= \dfrac{4}{9}$

$\therefore$ Area of $\triangle ABX$: area of $\triangle DCX$ = **4 : 9**

(ii) Since the heights are the same,

Volume of prism with triangular base ABX : volume of prism with triangular base DCX = **4 : 9**

End-of-year Checkpoint A

Section A

1. Total value $= \$\left[8000 + \dfrac{8000(1.65)(5)}{100}\right]$ [1]

$=$ **\$8660** [1]

2. Sum = 5 × 15.4 = 77

___, ___, 14, 19, 19

Sum of remaining two numbers = 77 – (14 + 19 + 19)

= 25

$\therefore$ The numbers are **12, 13, 14, 19** and **19**. [2]

3. $-7 \leqslant 2x + 3 < 1$

$-10 \leqslant 2x < -2$

$\mathbf{-5 \leqslant x < -1}$ [2]

4. $f(x) = \sqrt{x}$

$g(x) = \dfrac{2}{3}x$

$gf(64) = g\left(\sqrt{64}\right)$

$= g(8)$ [1]

$= \dfrac{2}{3}(8)$

$= \mathbf{5\dfrac{1}{3}}$ [1]

5. $\left(\dfrac{27b^{15}}{64a^{12}}\right)^{-\frac{1}{3}} = \left(\dfrac{64a^{12}}{27b^{15}}\right)^{\frac{1}{3}}$ [1]

$= \mathbf{\dfrac{4a^4}{3b^5}}$ [1]

6. (a) $(A \cap B)'$ **[1]**

(b) (i) $P' = \{1, 4, 6, 8, 9\}$ **[1]**

(ii) $Q \cap R = \{\ \}$ **[1]**

7. (a) $PQ = \sqrt{[5-(-3)]^2 + (-2-7)^2}$ **[1]**

$= \sqrt{145}$

$\therefore m = \mathbf{145}$ **[1]**

(b) An example of the coordinates is ***R*(5, 0)**. **[1]**

8. $\frac{5}{x} + \frac{2}{x-4} = 3$

$5(x-4) + 2x = 3x(x-4)$

$5x - 20 + 2x = 3x^2 - 12x$

$3x^2 - 19x + 20 = 0$ **[1]**

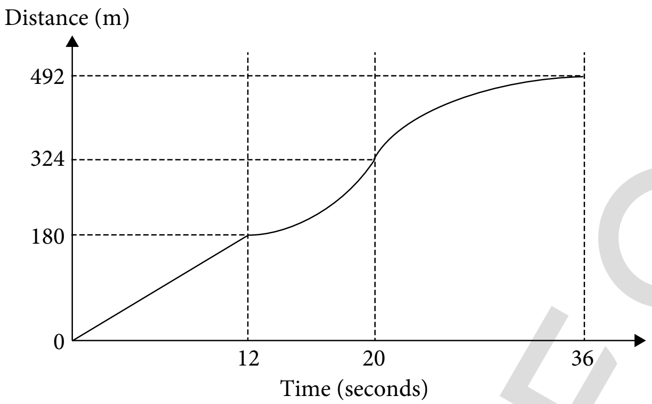
$(3x-4)(x-5) = 0$ **[1]**

$3x - 4 = 0$ or $x - 5 = 0$

$3x = 4$ $\quad x = 5$

$x = 1\frac{1}{3}$

$\therefore x = \mathbf{1\frac{1}{3}}$ or $x = \mathbf{5}$ **[1]**

9. 

[3]

10. $36^{4xy-2+y-8x} = 1$

$4xy - 2 + y - 8x = 0$ **[1]**

$4xy - 8x + y - 2 = 0$

$4x(y-2) + (y-2) = 0$

$(y-2)(4x+1) = 0$ **[1]**

$y - 2 = 0$ or $4x + 1 = 0$

$y = 2$ $\quad 4x = -1$

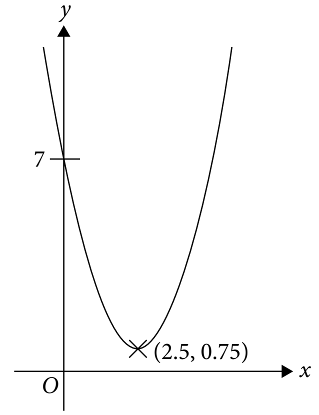
$x = -\frac{1}{4}$

$\therefore x = \mathbf{-\frac{1}{4}}, y = \mathbf{2}$ **[1]**

11. (a) $7 - 5x + x^2 = x^2 - 5x + 7$

$= (x-2.5)^2 - 2.5^2 + 7$

$= \mathbf{(x-2.5)^2 + 0.75}$ **[1]**

(b) 

[2]

(c) Equation of line of symmetry: $\mathbf{x = 2.5}$ **[1]**

12. $y = ax^2 + bx + 4$

When $x = 2$, $y = 1$,

$4a + 2b + 4 = 1$

$4a + 2b = -3$ —— (1) **[1]**

When $x = -6$, $y = -35$,

$36a - 6b + 4 = -35$

$36a - 6b = -39$

$12a - 2b = -13$ —— (2) **[1]**

(1) + (2): $16a = -16$

$a = -1$ **[1]**

Substitute $a = -1$ into (1):

$-4 + 2b = -3$

$2b = 1$

$b = \frac{1}{2}$ **[1]**

$\therefore \mathbf{a = -1, b = \frac{1}{2}}$

13. (a) $|\overrightarrow{AB}| = \sqrt{5^2 + (-12)^2}$

$= \mathbf{13}$ **[1]**

(b) $\overrightarrow{OP} = \begin{pmatrix} -3 \\ 3 \end{pmatrix}$

$\overrightarrow{QP} = 2\overrightarrow{AB}$

$\overrightarrow{OP} - \overrightarrow{OQ} = 2\overrightarrow{AB}$

$\begin{pmatrix} -3 \\ 3 \end{pmatrix} - \overrightarrow{OQ} = 2\begin{pmatrix} 5 \\ -12 \end{pmatrix}$

$\overrightarrow{OQ} = \begin{pmatrix} -3 \\ 3 \end{pmatrix} - 2\begin{pmatrix} 5 \\ -12 \end{pmatrix}$

$= \begin{pmatrix} -3 \\ 3 \end{pmatrix} - \begin{pmatrix} 10 \\ -24 \end{pmatrix}$

$= \begin{pmatrix} -13 \\ 27 \end{pmatrix}$ **[1]**

$\therefore$ **Q(−13, 27)** **[1]**

(c) **Trapezium** **[1]**

14. (a) Total cost = 2($69) + 2($59) + 2($70) **[1]**

= **$396** **[1]**

(b) 10% of $396 = $39.60 **[1]**

< $40

∴ Mr Chan should use the **$40 voucher** as he will save more. **[1]**

15. **(a)** In $\triangle ABC$,
$AB^2 + BC^2 = 12^2 + 16^2$
$= 400$
$AC^2 = 20^2$
$= 400$
Since $AB^2 + BC^2 = AC^2$, by the converse of Pythagoras' Theorem, $\triangle ABC$ is a right-angled triangle. **[1]**

(b) $\cos \angle ACD + \tan \angle BAC = -\cos \angle ACB + \tan \angle BAC$
$= -\frac{16}{20} + \frac{16}{12}$ **[1]**
$= \mathbf{\frac{8}{15}}$ **[1]**

(c) Area of $\triangle ACD = \frac{2}{3} \times \left(\frac{1}{2} \times 16 \times 12\right)$
$= \mathbf{64\ cm^2}$ **[1]**

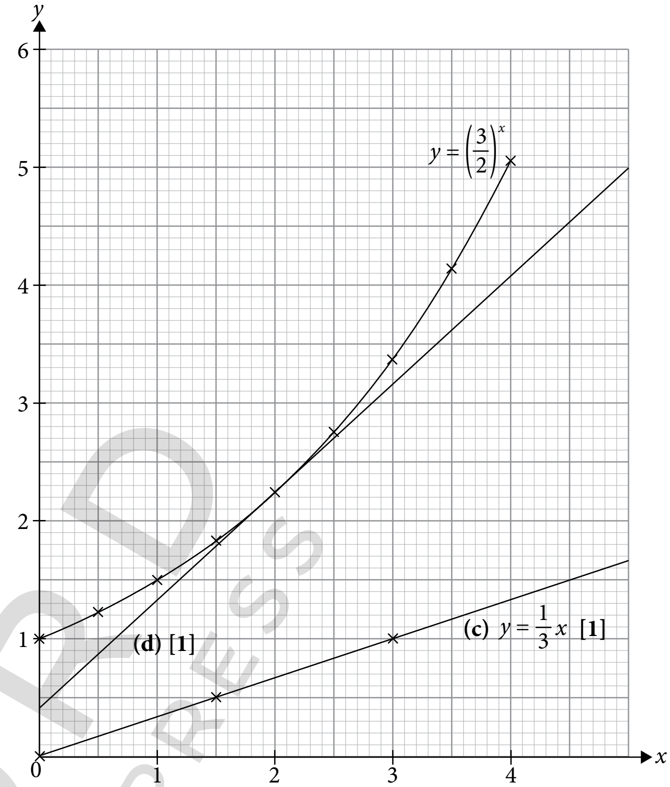
16. **(a)** Area of sector $OABC = \frac{108°}{360°} \times \pi(25)^2$ **[1]**
$= \mathbf{589\ cm^2}$ (to 3 s.f.) **[1]**

(b) Using Cosine Rule,
$AC^2 = OA^2 + OC^2 - 2(OA)(OC) \cos \angle AOC$
$= 25^2 + 25^2 - 2(25)(25) \cos 108°$
$= 1636.3$ (to 5 s.f.)
$AC = \sqrt{1636.3}$ $(AC > 0)$
$= 40.451$ cm (to 5 s.f.) **[1]**
$\therefore$ Perimeter of shaded region $= 40.451 + \frac{108°}{360°} \times 2\pi(25)$ **[1]**
$= \mathbf{87.6\ cm}$ (to 3 s.f.) **[1]**

Section B

17. **(a)** When $x = 2.5$,
$y = \left(\frac{3}{2}\right)^{2.5}$
$= 2.76$ (to 3 s.f.)
$\therefore \mathbf{p = 2.76}$ **[1]**

(b)



[3]

(c) $\left(\frac{3}{2}\right)^x - \frac{x}{3} = 0$
$\left(\frac{3}{2}\right)^x = \frac{1}{3}x$
The graph of $y = \frac{1}{3}x$ does not intersect the curve $y = \left(\frac{3}{2}\right)^x$. **[1]**
$\therefore$ The equation has no solutions.

(d) Gradient of tangent $= \frac{5 - 1.35}{5 - 1}$
$= \mathbf{\frac{73}{80}}$ **[1]**

(e) Substitute $m = \frac{73}{80}$, $x = 4$, $y = 0$ into $y = mx + c$:
$0 = \frac{73}{80}(4) + c$
$c = -\frac{73}{20}$ **[1]**
$\therefore$ Equation of line: $y = \frac{73}{80}x - \frac{73}{20}$
$80y = 73x - 292$
$\mathbf{73x - 80y = 292}$ **[1]**

18. **(a)** Volume of cone $= \frac{1}{2}(1500\pi)$
$= 750\pi\ \text{cm}^3$
$\frac{1}{3}\pi(10)^2 h = 750\pi$ **[1]**
$h = \frac{750\pi}{\frac{100}{3}\pi}$
$= 22.5$ **[1]**
$\therefore h = \mathbf{22.5}$

(b) Let the slant height of the cone be l cm.
Using Pythagoras' Theorem,
$l^2 = 10^2 + 22.5^2$
$= 606.25$
$l = \sqrt{606.25}$ $(l > 0)$
$= 24.622$ (to 5 s.f.) **[1]**
$\therefore$ Total surface area of remaining solid
$= \pi(10)^2 + 2\pi(10)(22.5) + \pi(10)(24.622)$ **[2]**
$= 2500$ cm^2 (to 3 s.f.) (shown) **[1]**

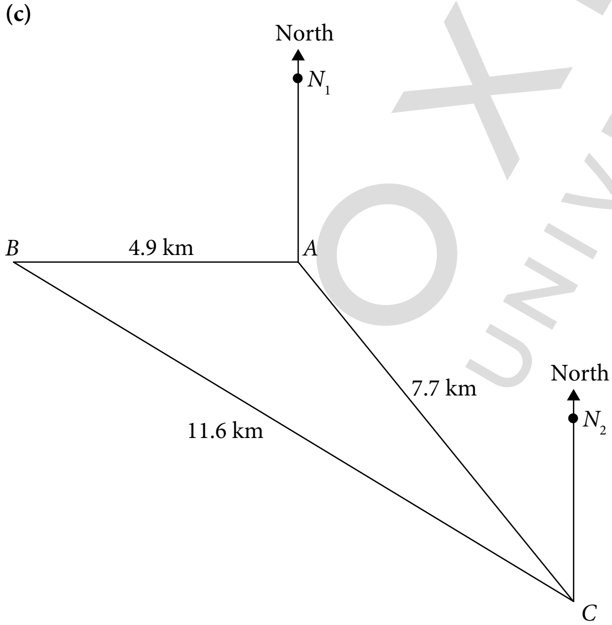
(c) Maximum number of ball bearings
$= \dfrac{750\pi}{\frac{4}{3}\pi(1.5)^3}$ **[1]**
$= 166\frac{2}{3}$
$= 166$ (round down to the nearest whole number) **[1]**
$\therefore$ A maximum of **166** ball bearings can be formed.

(d) Volume of pyramid $= \frac{1}{3}(20)^2(22.5)$
$= 3000$ cm^3
Difference in volume $= 3000 - 750\pi$
$= 644$ cm^3 (to 3 s.f.) **[1]**
The **pyramid** has a greater volume and the difference is **644 cm^3**. **[1]**

19. (a) Using Cosine Rule,
$BC^2 = AB^2 + AC^2 - 2(AB)(AC)\cos\angle BAC$
$11.6^2 = 4.9^2 + 7.7^2 - 2(4.9)(7.7)\cos\angle BAC$ **[1]**
$75.46\cos\angle BAC = -51.26$ **[1]**
$\angle BAC = \cos^{-1}\left(-\dfrac{51.26}{75.46}\right)$
$=$ **132.8°** (to 1 d.p.) **[1]**

(b) Area of $\triangle ABC = \frac{1}{2}(4.9)(7.7)\sin 132.789°$ **[1]**
$=$ **13.8 km^2** (to 3 s.f.) **[1]**

(c)



$\angle N_1AC = 360° - 90° - 132.789°$ ($\angle$s at a pt.)
$= 137.211°$ (to 3 d.p.) **[1]**
Reflex $\angle N_2CA = 180° + 137.211°$ (alt. $\angle$s, $AN_1 // CN_2$) **[1]**
$= 317.2°$ (to 1 d.p.)
$\therefore$ Bearing of A from C = **317.2°** **[1]**

(d) Let the angle of depression be $x°$.
$\tan x° = \dfrac{880}{11\ 600}$ **[1]**
$x° = \tan^{-1}\dfrac{880}{11\ 600}$
$= 4.3°$ (to 1 d.p.)
$\therefore$ Angle of depression = **4.3°** **[1]**

20. (a) (i)

Nursery	Lower quartile	Median	Upper quartile	Interquartile range
A	**107**	**114**	**121**	**14**
B	**105**	**120**	**124**	**19**

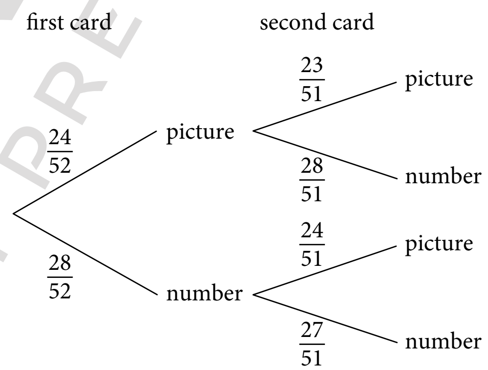
[3]

(ii) (a) **Agree.** The median height of the plants in nursery *B* is greater than that in nursery *A*. **[1]**

(b) **Disagree.** The spread of the heights of the first 20 plants in nursery *A* is greater than that in nursery *B*. **[1]**

(b) (i) George calculated the probability based on replacement of the first card. **[1]**

(ii)



[2]

(iii) P(only 1 is a picture card) $= \dfrac{24}{52}\times\dfrac{28}{51}+\dfrac{28}{52}\times\dfrac{24}{51}$ **[1]**
$= \mathbf{\dfrac{112}{221}}$ **[1]**

End-of-year Checkpoint B

Section A

1. 3.15 hours = 3 hours 9 minutes
3 hours 150 seconds = 3 hours 2 minutes 30 seconds
185 minutes = 3 hours 5 minutes
$\therefore$ **3 hours 15 minutes, 3.15 hours, 185 minutes, 3 hours 150 seconds** **[1]**

2. $f(x) = 3x - 5$
Let $y = 3x - 5$.
$3x = y + 5$
$x = \dfrac{y+5}{3}$
$\therefore$ $\mathbf{f^{-1} : x \mapsto \dfrac{x+5}{3}}$ **[1]**

3. (a) 32.964 million $= 32.964 \times 10^6$
$= 3.2964 \times 10^7$
$= \mathbf{3.30 \times 10^7}$ (to 3 s.f.) **[1]**

(b) P(not above the age of 75 years) $= 1 - \frac{700\ 000}{32.964 \times 10^6}$

$= \mathbf{0.979}$ (to 3 s.f.) [1]

4. $924 = 2^2 \times 3 \times 7 \times 11$

$1452 = 2^2 \times 3 \times 11^2$

HCF $= 66 = 2 \times 3 \times 11$

LCM $= 91\ 476 = 2^2 \times 3^3 \times 7 \times 11^2$

Largest possible value of $p = 2 \times 3^3 \times 7 \times 11^2$

$= 45\ 738$

Smallest possible value of $p = 2 \times 3^3 \times 11$

$= 594$

$\therefore$ Difference $= 45\ 738 - 594$ [1]

$= \mathbf{45\ 144}$ [1]

5. $11^{2023} + 11^{2025} = 11^{2023}(1 + 11^2)$

$= 11^{2023}(122)$

$= 2 \times 61 \times 11^{2023}$ [1]

Since 61 is a factor of $11^{2023} + 11^{2025}$, then $11^{2023} + 11^{2025}$ is exactly divisible by 61. (shown) [1]

6. (a) $\frac{5p^6}{14q} \div \frac{15p^2}{7q^3} = \frac{5p^6}{14q} \times \frac{7q^3}{15p^2}$

$= \mathbf{\frac{p^4q^2}{6}}$ [1]

(b) $6x^2 - 21x - 12 = 3(2x^2 - 7x - 4)$

$= \mathbf{3(2x + 1)(x - 4)}$ [1]

7. (a) Number of cubes $= 18 \times 12 \times 10$

$= \mathbf{2160}$ [1]

(b) Volume of empty space $= (75 \times 50 \times 40) - 2160 \times 4^3$ [1]

$= \mathbf{11\ 760\ cm^3}$ [1]

8. (a) **(0, 4), (−4, 2)** [1]

(b) $y = \frac{1}{2}x + 4$ —— (1)

$6y - x = 3$ —— (2)

Substitute (1) into (2):

$6\left(\frac{1}{2}x + 4\right) - x = 3$

$3x + 24 - x = 3$

$2x = -21$

$x = -10.5$

Substitute $x = -10.5$ into (1):

$y = \frac{1}{2}(-10.5) + 4$

$= -1.25$

$\therefore k$ is **(−10.5, −1.25)**. [1]

(c) From $6y - x = 3$, we have $y = \frac{1}{6}x + \frac{1}{2}$.

From $ax + by = 5$, we have $y = -\frac{a}{b}x + \frac{5}{b}$.

The lines must be parallel and have different y-intercepts.

Let $b = -1$: $-\frac{a}{-1} = \frac{1}{6}$

$a = \frac{1}{6}$

$\therefore$ An example of the values is $\mathbf{a = \frac{1}{6}, b = -1}$. [1]

9. $P = \frac{k}{\sqrt{Q}}$

$\frac{k}{\sqrt{9}} + \frac{k}{\sqrt{16}} = 21$ [1]

$\frac{k}{3} + \frac{k}{4} = 21$

$\frac{7}{12}k = 21$

$k = 36$

$\therefore P = \frac{36}{\sqrt{Q}}$ [1]

When $P = 100$,

$100 = \frac{36}{\sqrt{Q}}$

$\sqrt{Q} = \frac{36}{100}$

$= \frac{9}{25}$

$Q = \left(\frac{9}{25}\right)^2$

$= \mathbf{\frac{81}{625}}$ [1]

10. $V = \frac{1}{3}\pi r^2 h$

When $r = r_0$, $h = h_0$,

$V_0 = \frac{1}{3}\pi r_0^2 h_0$

When $r = 1.25r_0$, $h = 0.75h_0$,

$V = \frac{1}{3}\pi(1.25r_0)^2(0.75h_0)$

$= \frac{25}{64}\pi r_0^2 h_0$

Percentage change $= \frac{\frac{25}{64}\pi r_0^2 h_0 - \frac{1}{3}\pi r_0^2 h_0}{\frac{1}{3}\pi r_0^2 h_0} \times 100\%$ [1]

$= \frac{\frac{11}{192}\pi r_0^2 h_0}{\frac{1}{3}\pi r_0^2 h_0} \times 100\%$

$= 17.2\%$ (to 3 s.f.) [1]

$\therefore$ The volume of the cone **increases** by **17.2%**. [1]

11. (a) $16p^2 - r^2 - 8pq + q^2$

$= 16p^2 - 8pq + q^2 - r^2$

$= (4p - q)^2 - r^2$ [1]

$= \mathbf{(4p - q + r)(4p - q - r)}$ [1]

(b) $(3x + 2)^2 + (7y - 4)^4 = 0$

$3x + 2 = 0$ and $7y - 4 = 0$

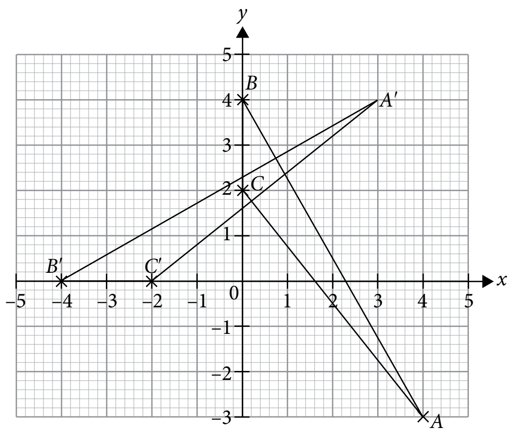
$3x = -2$ $7y = 4$

$x = -\frac{2}{3}$ $y = \frac{4}{7}$ [1]

$\therefore \frac{x+2}{y-4} = \frac{-\frac{2}{3} + 2}{\frac{4}{7} - 4}$

$= \mathbf{-\frac{7}{18}}$ [1]

12.



[4]

13. (i) P(passes on the 2nd attempt) = P(fail, pass)

$= (0.2)(0.8)$

$= \mathbf{0.16}$ [1]

(ii) P(fails the first 4 tests) = P(fail, fail, fail, fail)

$= 0.2^4$

$= \mathbf{0.0016}$ [1]

(iii) P(takes not more than 3 attempts)

= P(pass) + P(fail, pass) + P(fail, fail, pass)

$= 0.8 + (0.2)(0.8) + (0.2)(0.2)(0.8)$ [2]

$= \mathbf{0.992}$ [1]

14. (a) $3 \times 2^x = \dfrac{6}{\sqrt{128}}$

$2^x = \dfrac{2}{\sqrt{128}}$

$= \dfrac{2}{\sqrt{2^7}}$

$= \dfrac{2}{2^{\frac{7}{2}}}$

$= 2^{-\frac{5}{2}}$ [1]

$\therefore x = -\dfrac{5}{2} = \mathbf{-2.5}$ [1]

(b) $4^{y+1} + 4^y = 160$

$4(4^y) + 4^y = 160$

$5(4^y) = 160$ [1]

$4^y = 32$

$2^{2y} = 2^5$ [1]

$2y = 5$

$\therefore y = \mathbf{2.5}$ [1]

15. Area of $\triangle ABC = 174$ cm²

$\frac{1}{2}(32.8)(29.5) \sin \angle BAC = 174$ [1]

$\sin \angle BAC = 0.359\,65$ (to 5 s.f.)

$\angle BAC = \sin^{-1} 0.359\,65$ or $180° - \sin^{-1} 0.359\,65$

$= 21.079°$ or $158.921°$ (to 3 d.p.) [1]

Using Cosine Rule,

$BC^2 = AB^2 + AC^2 - 2(AB)(AC) \cos \angle BAC$

$= 32.8^2 + 29.5^2 - 2(32.8)(29.5) \cos 158.921°$

$= 3751.8$ (to 5 s.f.) [1]

$BC = \sqrt{3751.8}$ $(BC > 0)$

$= 61.252$ cm (to 5 s.f.) [1]

$\therefore$ Perimeter of $\triangle ABC = 32.8 + 29.5 + 61.252$

= **124 cm** (to 3 s.f.) [1]

16. (a) Sum of int. ∠s $= (8 - 2) \times 180°$

$= \mathbf{1080°}$ [1]

(b) (i) $\angle PRQ = \dfrac{1080°}{8} \div 2$

$= \mathbf{67.5°}$ [1]

(ii) $\angle RQC = \angle PRQ$ (alt. ∠s, PR // QC)

$= \mathbf{67.5°}$ [1]

(c) Let the length of the equal sides of each isosceles triangle be x cm.

Using Pythagoras' Theorem,

$x^2 + x^2 = 5^2$

$2x^2 = 25$

$x^2 = 12.5$

$x = \sqrt{12.5}$ $(x > 0)$ [1]

$\therefore$ Area of $ABCD = \left(2\sqrt{12.5} + 5\right)^2$

$= 146$ cm² (to 3 s.f.) (shown) [1]

Section B

17. (a) $\dfrac{3 - 8x}{2} \geqslant 5x + 1$

$3 - 8x \geqslant 10x + 2$

$-18x \geqslant -1$ [1]

$\boldsymbol{x \leqslant \dfrac{1}{18}}$ [1]

(b) OPEN From $5x + 2y = 7$, we have $\dfrac{5}{7}x + \dfrac{2}{7}y = 1$.

From $4x - y = 16$, we have $\dfrac{1}{4}x - \dfrac{1}{16}y = 1$.

$\therefore$ The equations are $\boldsymbol{\dfrac{5}{7}x + \dfrac{2}{7}y = 1}$ and $\boldsymbol{\dfrac{1}{4}x - \dfrac{1}{16}y = 1}$. [2]

(c) $\dfrac{7}{5 - 2x} - \dfrac{3(x-4)}{6x^2 - 11x - 10}$

$= \dfrac{7}{5 - 2x} - \dfrac{3(x-4)}{(3x+2)(2x-5)}$ [1]

$= \dfrac{7}{5 - 2x} + \dfrac{3(x-4)}{(3x+2)(5-2x)}$

$= \dfrac{7(3x+2) + 3(x-4)}{(3x+2)(5-2x)}$ [1]

$= \dfrac{21x + 14 + 3x - 12}{(3x+2)(5-2x)}$

$= \boldsymbol{\dfrac{24x + 2}{(3x+2)(5-2x)}}$ [1]

(d) $c = 4\sqrt{\dfrac{a^2 - 25}{b}}$

$\dfrac{c}{4} = \sqrt{\dfrac{a^2 - 25}{b}}$

$\dfrac{c^2}{16} = \dfrac{a^2 - 25}{b}$ [1]

$a^2 - 25 = \dfrac{bc^2}{16}$

$a^2 = \dfrac{bc^2}{16} + 25$ [1]

$\boldsymbol{a = \pm\sqrt{\dfrac{bc^2}{16} + 25}}$ [1]

18. **(a)** $AB^2 = 80^2$
$= 6400$
$AC^2 + BC^2 = 68^2 + 43^2$
$= 6473$ **[1]**
Since $AB^2 \neq AC^2 + BC^2$, by the converse of Pythagoras' Theorem, $\triangle ABC$ is not a right-angled triangle. (shown) **[1]**

(b) Using Cosine Rule,
$AB^2 = AC^2 + BC^2 - 2(AC)(BC) \cos \angle ACB$
$80^2 = 68^2 + 43^2 - 2(68)(43) \cos \angle ACB$ **[1]**
$5848 \cos \angle ACB = 73$
$\cos \angle ACB = \dfrac{73}{5848}$
$\angle ACB = \cos^{-1} \dfrac{73}{5848}$
$= 89.285°$ (to 3 d.p.) **[1]**
$360° - 89.285° = 270.7°$ (to 1 d.p.) ($\angle$s at a pt.) **[1]**
$\therefore$ Bearing of A from C is **270.7°** **[1]**

(c) Using Sine Rule,
$\dfrac{\sin \angle ABC}{AC} = \dfrac{\sin \angle ACB}{AB}$
$\dfrac{\sin \angle ABC}{68} = \dfrac{\sin 89.285°}{80}$
$\sin \angle ABC = \dfrac{68 \sin 89.285°}{80}$
$= 0.849\ 93$ (to 5 s.f.)
$\angle ABC = \sin^{-1} 0.849\ 93$
$= 58.204°$ (to 3 d.p.) **[1]**
$\therefore \angle CBD = 58.204°$
$\sin 58.204° = \dfrac{CD}{43}$
$CD = 43 \sin 58.204°$
$= 36.547$ m (to 5 s.f.) **[1]**
Let the height of the bird above D be h m.
$\tan 16° = \dfrac{h}{36.547}$ **[1]**
$h = 36.547 \tan 16°$
$= 10.5$ (to 3 s.f.)
$\therefore$ The bird is **10.5 m** above D. **[1]**

19. **(a)** $C(-3, 2)$
Gradient of $CE = \dfrac{8-2}{9-(-3)}$
$= \dfrac{1}{2}$ **[1]**
Substitute $m = \dfrac{1}{2}$, $x = 9$, $y = 8$ into $y = mx + c$:
$8 = \dfrac{1}{2}(9) + c$
$c = \dfrac{7}{2}$
$\therefore$ Equation of CE: $\boldsymbol{y = \frac{1}{2}x + \frac{7}{2}}$ **[1]**

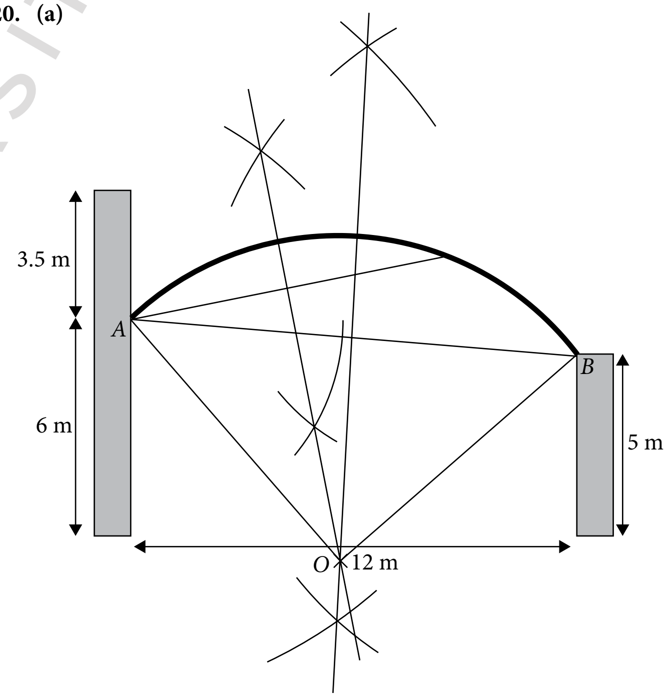
(b) Let the shortest distance from D to CE be h units.
$\dfrac{1}{2}(CD)(DE) = \dfrac{1}{2}(CE)h$ **[1]**
$\dfrac{1}{2}(12)(6) = \dfrac{1}{2}\left(\sqrt{[9-(-3)]^2 + (8-2)^2}\right)(h)$ **[1]**
$36 = \dfrac{1}{2}\left(\sqrt{180}\right)h$ **[1]**
$h = \dfrac{72}{\sqrt{180}}$
$= 5.37$ (to 3 s.f.)
$\therefore$ The shortest distance between D and CE is 5.37 units. (shown) **[1]**

(c) Using similar triangles,
$\dfrac{AB}{CD} = \dfrac{BC}{DE}$
$\dfrac{AB}{12} = \dfrac{4}{6}$
$AB = \dfrac{4}{6} \times 12$
$= 8$ units **[1]**
x-coordinate of $A = -3 - 4 = -7$
y-coordinate of $A = 2 + 8 = 10$
$\therefore$ ***A*(−7, 10)** **[1]**

(d) $\cos \angle BCE = -\cos \angle ECD$
$= \dfrac{-12}{\sqrt{180}}$
$= \mathbf{-0.894}$ (to 3 s.f.) **[1]**

(e) For $ACEP$ to be a parallelogram,
x-coordinate of $P = -7 + 12 = 5$
y-coordinate of $P = 10 + 6 = 16$
$\therefore$ ***P*(5, 16)** **[1]**

20. **(a)**



[2]
$OA =$ **8.5 m** **[1]**

(b) Since this is an outdoor theme park, the architect should choose **Type *R* tiles**, to cater for inclement weather conditions. **[2]**

$\angle AOB = 88°$

Length of bridge $= \frac{88°}{360°} \times 2\pi(8.5)$

$= \frac{187}{45}\pi$ m **[1]**

Area to be tiled $= \frac{187}{45}\pi \times 2.7$

$= 11.22\pi$ m^2 **[1]**

Assume a more conservative exchange rate of S\$1 = US\$0.732.

Assume that the architect orders 10% more tiles as reserves.

Price of tiles = $11.22\pi \times$ US\$50 $\times$ 110% **[1]**

= U\$1938.6768 (to 4 d.p.)

$= \text{S\$}\left(\frac{1938.6768}{0.732}\right)$ **[1]**

= S\$2648.47 (to 2 d.p.)

$\therefore$ The architect should set aside about **S\$2700**, rounded up to the nearest \$100. **[1]**