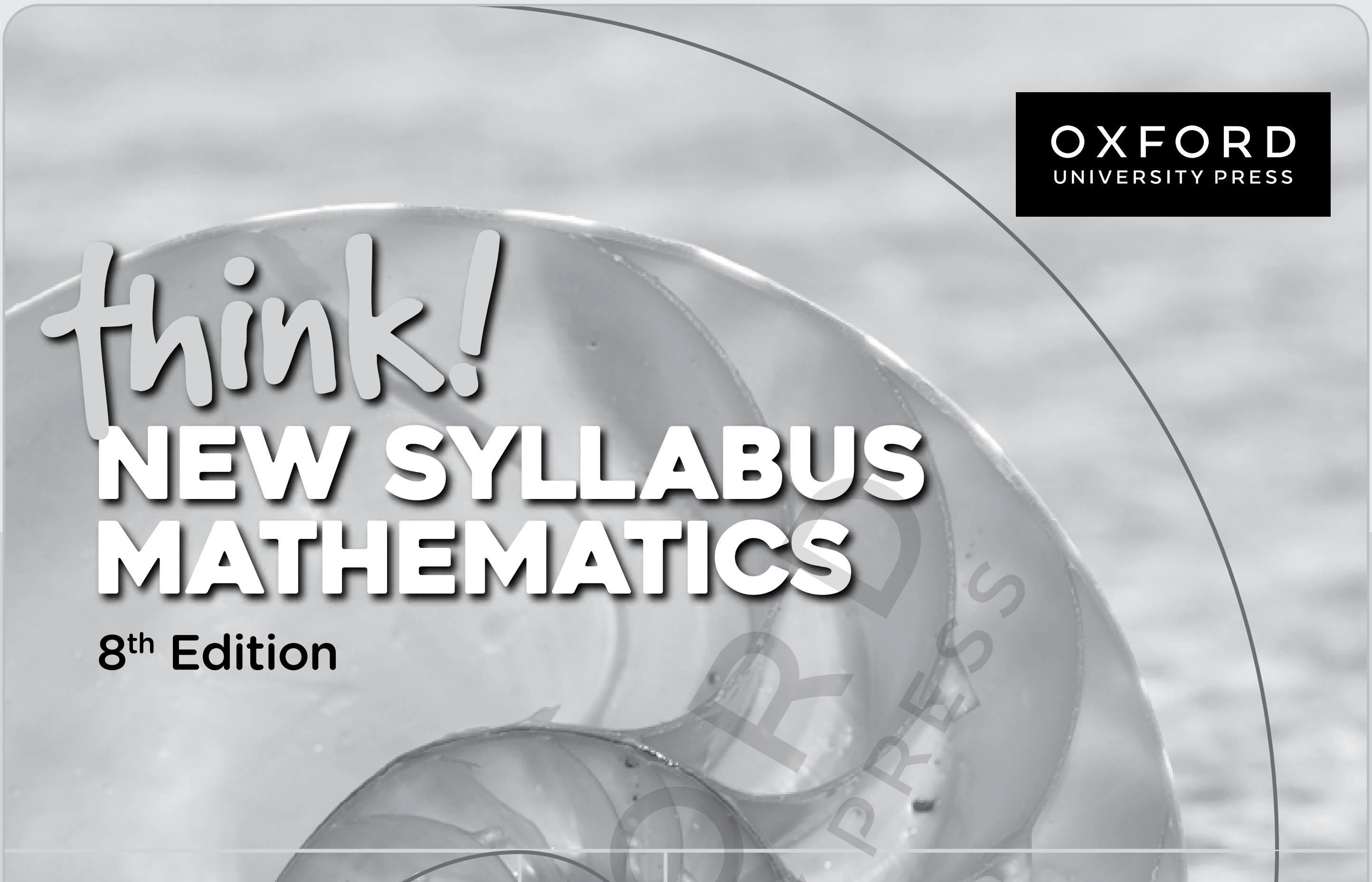

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8th Edition

Workbook Full Solutions 3

1 Algebraic Fractions and Formulae

Worksheet 1A Algebraic fractions

1. (a) $\frac{10ab}{100bc} = \mathbf{\frac{a}{10c}}$

(b) $\frac{7d^5}{21d^6} = \mathbf{\frac{1}{3d}}$

(c) $\frac{8h^2k}{2hk^2} = \mathbf{\frac{4h}{k}}$

(d) $\frac{6hj^3k^2}{9hj^4k} = \mathbf{\frac{2k}{3j}}$

(e) $\frac{p^3(p+4q)}{3p(p+4q)^2} = \mathbf{\frac{p^2}{3(p+4q)}}$

(f) $\frac{30p(2q+r)}{24p^2(2r+q)} = \mathbf{\frac{5(2q+r)}{4p(2r+q)}}$

(g) $\frac{15(x-y)^4}{35(x-y)^3} = \mathbf{\frac{3(x-y)}{7}}$

(h) $\frac{49x^3z(x^2-y)}{(7yz)^3(y-x^2)} = \frac{49x^3z(x^2-y)}{343y^3z^3(y-x^2)}$

$= \mathbf{-\frac{x^3}{7y^3z^2}}$

2. (a) $\frac{a}{5a^2+a} = \frac{a}{a(5a+1)}$

$= \mathbf{\frac{1}{5a+1}}$

(b) $\frac{b-b^2}{b} = \frac{b(1-b)}{b}$

$= \mathbf{1-b}$

(c) $\frac{16h^2+2hk}{6h^2} = \frac{2h(8h+k)}{6h^2}$

$= \mathbf{\frac{8h+k}{3h}}$

(d) $\frac{3hj-3hk}{9hj-9hk} = \frac{3h(j-k)}{9h(j-k)}$

$= \mathbf{\frac{1}{3}}$

(e) $\frac{4p+4q}{(p+q)^2} = \frac{4(p+q)}{(p+q)^2}$

$= \mathbf{\frac{4}{p+q}}$

(f) $\frac{7p-q}{8qr-56pr} = \frac{7p-q}{8r(q-7p)}$

$= \mathbf{-\frac{1}{8r}}$

(g) $\frac{22xy+33y^2}{99xy+66x^2} = \frac{11y(2x+3y)}{33x(3y+2x)}$

$= \mathbf{\frac{y}{3x}}$

(h) $\frac{14y^3z-14xy^3}{7y(x-z)^2} = \frac{14y^3(z-x)}{7y(x-z)^2}$

$= -\frac{2y^2}{x-z}$

$= \mathbf{\frac{2y^2}{z-x}}$

3. (a) $\frac{a^2-16b^2}{ab+4b^2} = \frac{(a+4b)(a-4b)}{b(a+4b)}$

$= \mathbf{\frac{a-4b}{b}}$

(b) $\frac{(6a-6b)^2}{6a^2-6b^2} = \frac{[6(a-b)]^2}{6(a^2-b^2)}$

$= \frac{36(a-b)^2}{6(a+b)(a-b)}$

$= \mathbf{\frac{6(a-b)}{a+b}}$

(c) $\frac{5c+1}{5c^2+11c+2} = \frac{5c+1}{(5c+1)(c+2)}$

$= \mathbf{\frac{1}{c+2}}$

(d) $\frac{d^2+3d-28}{d-4} = \frac{(d+7)(d-4)}{d-4}$

$= \mathbf{d+7}$

(e) $\frac{h^2-9h+18}{h^2-6h} = \frac{(h-3)(h-6)}{h(h-6)}$

$= \mathbf{\frac{h-3}{h}}$

(f) $\frac{h^2+8hk+16k^2}{5h^2+18hk-8k^2} = \frac{(h+4k)^2}{(5h-2k)(h+4k)}$

$= \mathbf{\frac{h+4k}{5h-2k}}$

(g) $\frac{12m^2-10m-12}{24m^2-72m+54} = \frac{2(6m^2-5m-6)}{6(4m^2-12m+9)}$

$= \frac{2(3m+2)(2m-3)}{6(2m-3)^2}$

$= \mathbf{\frac{3m+2}{3(2m-3)}}$

(h) $\frac{15am-4n-5m+12an}{8n^2+6mn-5m^2} = \frac{15am-5m+12an-4n}{(4n+5m)(2n-m)}$

$= \frac{5m(3a-1)+4n(3a-1)}{(4n+5m)(2n-m)}$

$= \frac{(5m+4n)(3a-1)}{(4n+5m)(2n-m)}$

$= \mathbf{\frac{3a-1}{2n-m}}$

(i) $\frac{49p^2-28pq+4q^2}{8q^2-98p^2} = \frac{(7p-2q)^2}{2(4q^2-49p^2)}$

$= \frac{(7p-2q)^2}{2(2q+7p)(2q-7p)}$

$= \frac{(2q-7p)^2}{2(2q+7p)(2q-7p)}$

$= \mathbf{\frac{2q-7p}{2(2q+7p)}}$

(j) $\dfrac{9p^2-25q^2}{(3p+q)^2-16q^2} = \dfrac{(3p+5q)(3p-5q)}{(3p+q+4q)(3p+q-4q)}$

$= \dfrac{(3p+5q)(3p-5q)}{(3p+5q)(3p-3q)}$

$= \mathbf{\dfrac{3p-5q}{3(p-q)}}$

(k) $\dfrac{7x-xy+35-5y}{7x+5y-35-xy} = \dfrac{7x-xy+35-5y}{7x-35-xy+5y}$

$= \dfrac{x(7-y)+5(7-y)}{7(x-5)-y(x-5)}$

$= \dfrac{(x+5)(7-y)}{(7-y)(x-5)}$

$= \mathbf{\dfrac{x+5}{x-5}}$

(l) $\dfrac{wxy-wyz+xyz-yz^2}{wx^2-wxz-x^2z+xz^2} = \dfrac{y\left(wx-wz+xz-z^2\right)}{x\left(wx-wz-xz+z^2\right)}$

$= \dfrac{y[w(x-z)+z(x-z)]}{x[w(x-z)-z(x-z)]}$

$= \dfrac{y(w+z)(x-z)}{x(w-z)(x-z)}$

$= \mathbf{\dfrac{y(w+z)}{x(w-z)}}$

4. (i) $50x^2-2 = 2(25x^2-1)$

$= \mathbf{2(5x+1)(5x-1)}$

(ii) $\dfrac{50x^2-2}{10x^2-8x-2} = \dfrac{2(5x+1)(5x-1)}{2\left(5x^2-4x-1\right)}$

$= \dfrac{2(5x+1)(5x-1)}{2(5x+1)(x-1)}$

$= \mathbf{\dfrac{5x-1}{x-1}}$

5. **No**, I do not agree with the student.

$\dfrac{3x^2-x}{9ax^3-6ax^2+ax} = \dfrac{x(3x-1)}{ax\left(9x^2-6x+1\right)}$

$= \dfrac{x(3x-1)}{ax(3x-1)^2}$

$= \dfrac{1}{a(3x-1)}$

Challenge Myself!

6. (a) $\dfrac{4}{x-1} = \dfrac{4(x+3)}{(x-1)(x+3)}$

$= \dfrac{4x+12}{x^2+2x-3}$

$\therefore$ A possible fraction is $\mathbf{\dfrac{4x+12}{x^2+2x-3}}$.

(b) $\dfrac{4}{x-1} = \dfrac{4(x+1)}{(x-1)(x+1)}$

$= \dfrac{4x+4}{x^2-1}$

$\therefore$ A possible fraction is $\mathbf{\dfrac{4x+4}{x^2-1}}$.

(c) $\dfrac{x-2y}{x+3} = \dfrac{(x-2y)(x-5)}{(x+3)(x-5)}$

$= \dfrac{x^2-5x-2xy+10y}{x^2-2x-15}$

$\therefore$ A possible fraction is $\mathbf{\dfrac{x^2-5x-2xy+10y}{x^2-2x-15}}$.

Worksheet 1B Multiplication and division of algebraic fractions

1. (a) $\dfrac{6a}{7}\times\dfrac{14}{15a} = \mathbf{\dfrac{4}{5}}$

(b) $\dfrac{6a^3}{b^2}\times\dfrac{ab^2}{8} = \mathbf{\dfrac{3a^4}{4}}$

(c) $\left(-\dfrac{5c}{2d}\right)\times\sqrt{100d^2} = \left(-\dfrac{5c}{2d}\right)\times 10d$

$= \mathbf{-25c}$

(d) $\dfrac{14d}{81c}\times\dfrac{(-3cd)^2}{8} = \dfrac{14d}{81c}\times\dfrac{9c^2d^2}{8}$

$= \mathbf{\dfrac{7cd^3}{36}}$

(e) $\dfrac{12h}{25}\div\dfrac{10h^3}{9} = \dfrac{12h}{25}\times\dfrac{9}{10h^3}$

$= \mathbf{\dfrac{54}{125h^2}}$

(f) $\dfrac{7j}{4h}\div\dfrac{21j^2}{8} = \dfrac{7j}{4h}\times\dfrac{8}{21j^2}$

$= \mathbf{\dfrac{2}{3hj}}$

(g) $\dfrac{27mn}{15}\div\dfrac{15m^2}{20n} = \dfrac{27mn}{15}\times\dfrac{20n}{15m^2}$

$= \dfrac{9mn}{5}\times\dfrac{4n}{3m^2}$

$= \mathbf{\dfrac{12n^2}{5m}}$

(h) $\dfrac{28mn^3}{5k}\div\dfrac{7m^3n}{25k} = \dfrac{28mn^3}{5k}\times\dfrac{25k}{7m^3n}$

$= \mathbf{\dfrac{20n^2}{m^2}}$

(i) $\dfrac{4q^2}{27p^2}\times\dfrac{9q}{16p}\times\dfrac{20}{pq^2} = \mathbf{\dfrac{5q}{3p^4}}$

(j) $\dfrac{(pr)^2}{45q}\times\dfrac{5}{pqr}\div\dfrac{12}{q^2r^2} = \dfrac{p^2r^2}{45q}\times\dfrac{5}{pqr}\times\dfrac{q^2r^2}{12}$

$= \mathbf{\dfrac{pr^3}{108}}$

(k) $\dfrac{6y}{x}\div\dfrac{3xy^2}{2}\div\dfrac{y}{x^2} = \dfrac{6y}{x}\times\dfrac{2}{3xy^2}\times\dfrac{x^2}{y}$

$= \mathbf{\dfrac{4}{y^2}}$

(l) $\dfrac{4}{y}\div\left(\dfrac{3xz^2}{y^2}\times\dfrac{9y}{\sqrt[3]{-216x^3}}\right) = \dfrac{4}{y}\div\left(\dfrac{3xz^2}{y^2}\times\dfrac{9y}{-6x}\right)$

$= \dfrac{4}{y}\div\left(-\dfrac{9z^2}{2y}\right)$

$= \dfrac{4}{y}\times\left(-\dfrac{2y}{9z^2}\right)$

$= \mathbf{-\dfrac{8}{9z^2}}$

2. (a) $\frac{a+5}{3}\times\frac{9}{(a+5)^2} = \mathbf{\frac{3}{a+5}}$

(b) $\frac{12b}{49(4a-3b)}\div\frac{16b}{35(4a-3b)^3} = \frac{12b}{49(4a-3b)}\times\frac{35(4a-3b)^3}{16b}$

$= \mathbf{\frac{15(4a-3b)^2}{28}}$

(c) $\frac{3c+1}{6c-1}\times\frac{2(6c-1)^2}{7(3c+1)} = \mathbf{\frac{2(6c-1)}{7}}$

(d) $\frac{(5c-2d)^3}{(9c+d)}\div\frac{(9c+d)(5c-2d)}{5c}$

$= \frac{(5c-2d)^3}{(9c+d)}\times\frac{5c}{(9c+d)(5c-2d)}$

$= \mathbf{\frac{5c(5c-2d)^2}{(9c+d)^2}}$

(e) $\frac{6}{2h-8}\times\frac{h-4}{8} = \frac{6}{2(h-4)}\times\frac{h-4}{8}$

$= \mathbf{\frac{3}{8}}$

(f) $\frac{3k-9}{jk^2}\div\frac{15-3k}{j^2k} = \frac{3k-9}{jk^2}\times\frac{j^2k}{15-3k}$

$= \frac{3(k-3)}{jk^2}\times\frac{j^2k}{3(5-k)}$

$= \mathbf{\frac{j(k-3)}{k(5-k)}}$

(g) $\frac{14m^2}{8m+4}\times\frac{(2m+1)^2}{35m^3} = \frac{14m^2}{4(2m+1)}\times\frac{(2m+1)^2}{35m^3}$

$= \mathbf{\frac{2m+1}{10m}}$

(h) $\frac{mn+3m}{m^2n+3mn}\div\frac{n^2+3n}{mn+3n} = \frac{mn+3m}{m^2n+3mn}\times\frac{mn+3n}{n^2+3n}$

$= \frac{m(n+3)}{mn(m+3)}\times\frac{n(m+3)}{n(n+3)}$

$= \mathbf{\frac{1}{n}}$

(i) $\frac{3p^2}{(3p-1)^2}\times\frac{1}{p(3p-1)}\times\frac{(3p-1)^3}{6p} = \mathbf{\frac{1}{2}}$

(j) $\frac{pq+4q}{p}\div\frac{p-4}{p^4}\div\frac{p^2+4p}{p-4} = \frac{pq+4q}{p}\times\frac{p^4}{p-4}\times\frac{p-4}{p^2+4p}$

$= \frac{q(p+4)}{p}\times\frac{p^4}{p-4}\times\frac{p-4}{p(p+4)}$

$= \mathbf{p^2q}$

(k) $\frac{2y^2(x-y)}{5x^2}\times\frac{1}{xy(8y-8x)} = \frac{2y^2(x-y)}{5x^2}\times\frac{1}{8xy(y-x)}$

$= \mathbf{-\frac{y}{20x^3}}$

(l) $\frac{xy(x-7y)}{(-x)^3}\div\frac{x(7y-x)^2}{y} = \frac{xy(x-7y)}{(-x)^3}\times\frac{y}{x(7y-x)^2}$

$= \frac{xy(x-7y)}{-x^3}\times\frac{y}{x(x-7y)^2}$

$= -\frac{y^2}{x^3(x-7y)}$

$= \mathbf{\frac{y^2}{x^3(7y-x)}}$

3. (a) $\frac{a+4}{4a^2-5a+1}\times\frac{4a-1}{a^2+4a} = \frac{a+4}{(4a-1)(a-1)}\times\frac{4a-1}{a(a+4)}$

$= \mathbf{\frac{1}{a(a-1)}}$

(b) $\frac{b^2-25}{2b+10}\div\frac{(b-5)^2}{5b^2} = \frac{b^2-25}{2b+10}\times\frac{5b^2}{(b-5)^2}$

$= \frac{(b+5)(b-5)}{2(b+5)}\times\frac{5b^2}{(b-5)^2}$

$= \mathbf{\frac{5b^2}{2(b-5)}}$

(c) $\frac{h^2-7h+12}{h^2-9}\times\frac{1}{(2h-8)^2} = \frac{(h-3)(h-4)}{(h+3)(h-3)}\times\frac{1}{4(h-4)^2}$

$= \mathbf{\frac{1}{4(h+3)(h-4)}}$

(d) $(7k-h)^2\div\frac{4h-28k}{7hk} = \frac{(7k-h)^2}{1}\times\frac{7hk}{4h-28k}$

$= \frac{(h-7k)^2}{1}\times\frac{7hk}{4(h-7k)}$

$= \mathbf{\frac{7hk(h-7k)}{4}}$

(e) $\frac{am+an-mn-n^2}{n+a}\times\frac{a^2-n^2}{3m+3n}$

$= \frac{a(m+n)-n(m+n)}{n+a}\times\frac{(a+n)(a-n)}{3(m+n)}$

$= \frac{(m+n)(a-n)}{n+a}\times\frac{(a+n)(a-n)}{3(m+n)}$

$= \mathbf{\frac{(a-n)^2}{3}}$

(f) $\frac{20m^2-25mn}{5mn+4n^2}\div\frac{16m^2-25n^2}{5m^2+4mn}$

$= \frac{20m^2-25mn}{5mn+4n^2}\times\frac{5m^2+4mn}{16m^2-25n^2}$

$= \frac{5m(4m-5n)}{n(5m+4n)}\times\frac{m(5m+4n)}{(4m+5n)(4m-5n)}$

$= \mathbf{\frac{5m^2}{n(4m+5n)}}$

(g) $\frac{p^2-7p+10}{p^2+3p-40}\times\frac{p^2+9p+18}{p^2+4p-12}$

$= \frac{(p-2)(p-5)}{(p+8)(p-5)}\times\frac{(p+3)(p+6)}{(p+6)(p-2)}$

$= \mathbf{\frac{p+3}{p+8}}$

(h) $\frac{15p^2+13pq+2q^2}{18p^2-8q^2}\div\frac{(5p+q)^2}{9p-4q}$

$= \frac{15p^2+13pq+2q^2}{18p^2-8q^2}\times\frac{9p-4q}{(5p+q)^2}$

$= \frac{(5p+q)(3p+2q)}{2(9p^2-4q^2)}\times\frac{9p-4q}{(5p+q)^2}$

$= \frac{(5p+q)(3p+2q)}{2(3p+2q)(3p-2q)}\times\frac{9p-4q}{(5p+q)^2}$

$= \mathbf{\frac{9p-4q}{2(5p+q)(3p-2q)}}$

(i) $\dfrac{2x-2y}{ax^2+axy+xy+y^2}\times\dfrac{2(x+y)^2}{x^2-y^2}$

$=\dfrac{2(x-y)}{ax(x+y)+y(x+y)}\times\dfrac{2(x+y)^2}{(x+y)(x-y)}$

$=\dfrac{2(x-y)}{(x+y)(ax+y)}\times\dfrac{2(x+y)^2}{(x+y)(x-y)}$

$=\boldsymbol{\dfrac{4}{ax+y}}$

(j) $\dfrac{x^2y^2+3xy-10}{x^2y^2-4xy+4}\div\dfrac{4xy+20}{3x^2y^2-7xy+2}$

$=\dfrac{x^2y^2+3xy-10}{x^2y^2-4xy+4}\times\dfrac{3x^2y^2-7xy+2}{4xy+20}$

$=\dfrac{(xy+5)(xy-2)}{(xy-2)^2}\times\dfrac{(3xy-1)(xy-2)}{4(xy+5)}$

$=\boldsymbol{\dfrac{3xy-1}{4}}$

4. **(a)** The first to fourth tiers in $\dfrac{\frac{a}{b}}{\frac{c}{d}}$ are *a*, *b*, *c* and *d* respectively.

$\dfrac{\frac{a}{b}}{\frac{c}{d}}$ can be simplified into $\dfrac{ad}{bc}$, in which the numerator is the product of the first and fourth tiers, and the denominator is the product of the second and third tiers.

(b) **(i)** $\dfrac{2}{\frac{1}{6}}=\dfrac{\frac{2}{1}}{\frac{1}{6}}$

$=\mathbf{12}$

(ii) $\dfrac{\frac{4}{5}}{\frac{2}{3}}=\boldsymbol{\dfrac{6}{5}}$

(iii) $\dfrac{\frac{m-4}{m}}{m-4}=\dfrac{\frac{m-4}{m}}{\frac{m-4}{1}}$

$=\boldsymbol{\dfrac{1}{m}}$

(iv) $\dfrac{\frac{n^3}{n+1}}{\frac{n^2}{a(n+1)}}=\boldsymbol{an}$

5. **(a)** $\dfrac{\frac{(2p+5)^2-16}{2p+9}}{\frac{(2p+1)^2}{2p}}=\dfrac{(2p+5)^2-16}{2p+9}\div\dfrac{(2p+1)^2}{2p}$

$=\dfrac{(2p+5+4)(2p+5-4)}{2p+9}\times\dfrac{2p}{(2p+1)^2}$

$=\dfrac{(2p+9)(2p+1)}{2p+9}\times\dfrac{2p}{(2p+1)^2}$

$=\boldsymbol{\dfrac{2p}{2p+1}}$

(b) $\dfrac{ax^2y^2-z-axy+xyz}{4x^2y^2-8xy+4}\div\dfrac{162a-32ax^2y^2}{4x^2y^2+5xy-9}$

$=\dfrac{ax^2y^2-z-axy+xyz}{4x^2y^2-8xy+4}\times\dfrac{4x^2y^2+5xy-9}{162a-32ax^2y^2}$

$=\dfrac{ax^2y^2-axy+xyz-z}{4(x^2y^2-2xy+1)}\times\dfrac{4x^2y^2+5xy-9}{2a(81-16x^2y^2)}$

$=\dfrac{axy(xy-1)+z(xy-1)}{4(xy-1)^2}\times\dfrac{(4xy+9)(xy-1)}{2a(9+4xy)(9-4xy)}$

$=\dfrac{(axy+z)(xy-1)}{4(xy-1)^2}\times\dfrac{(4xy+9)(xy-1)}{2a(9+4xy)(9-4xy)}$

$=\boldsymbol{\dfrac{axy+z}{8a(9-4xy)}}$

Worksheet 1C Addition and subtraction of algebraic fractions

1. **(a)** $\dfrac{5}{a}+\dfrac{1}{6a}=\dfrac{30+1}{6a}$

$=\boldsymbol{\dfrac{31}{6a}}$

(b) $\dfrac{10}{3b}-\dfrac{2}{b}=\dfrac{10-6}{3b}$

$=\boldsymbol{\dfrac{4}{3b}}$

(c) $\dfrac{7}{2c}+\dfrac{3}{4d}=\dfrac{14d+3c}{4cd}$

$=\boldsymbol{\dfrac{3c+14d}{4cd}}$

(d) $\dfrac{1}{6f}-\dfrac{1}{9g}=\boldsymbol{\dfrac{3g-2f}{18fg}}$

(e) $\dfrac{9}{2h}+\dfrac{8}{3h}-\dfrac{7}{4h}=\dfrac{54+32-21}{12h}$

$=\boldsymbol{\dfrac{65}{12h}}$

(f) $\dfrac{1}{8k}-\dfrac{9}{k}+\dfrac{3}{5k}=\dfrac{5-360+24}{40k}$

$=\boldsymbol{-\dfrac{331}{40k}}$

(g) $\dfrac{4}{3m+9}+\dfrac{1}{m+3}=\dfrac{4}{3(m+3)}+\dfrac{1}{m+3}$

$=\dfrac{4+3}{3(m+3)}$

$=\boldsymbol{\dfrac{7}{3(m+3)}}$

(h) $\dfrac{5}{8n-4}-\dfrac{7}{12n-6}=\dfrac{5}{4(2n-1)}-\dfrac{7}{6(2n-1)}$

$=\dfrac{15-14}{12(2n-1)}$

$=\boldsymbol{\dfrac{1}{12(2n-1)}}$

(i) $\frac{2}{2p-5}+\frac{7}{15-6p} = \frac{2}{2p-5}+\frac{7}{3(5-2p)}$

$= \frac{2}{2p-5}-\frac{7}{3(2p-5)}$

$= \frac{6-7}{3(2p-5)}$

$= -\frac{1}{3(2p-5)}$

$= \mathbf{\frac{1}{3(5-2p)}}$

(j) $\frac{3}{20r-45q}-\frac{8}{9q-4r} = \frac{3}{5(4r-9q)}+\frac{8}{4r-9q}$

$= \frac{3+40}{5(4r-9q)}$

$= \mathbf{\frac{43}{5(4r-9q)}}$

(k) $6+\frac{2}{5x-1} = \frac{6(5x-1)+2}{5x-1}$

$= \frac{30x-6+2}{5x-1}$

$= \mathbf{\frac{30x-4}{5x-1}}$

(l) $\frac{8}{3y+4}-y = \frac{8-y(3y+4)}{3y+4}$

$= \frac{8-3y^2-4y}{3y+4}$

$= \mathbf{\frac{8-4y-3y^2}{3y+4}}$

2. (a) $\frac{6}{a}+\frac{6}{a+2} = \frac{6(a+2)+6a}{a(a+2)}$

$= \frac{6a+12+6a}{a(a+2)}$

$= \mathbf{\frac{12a+12}{a(a+2)}}$

(b) $\frac{4}{b+7}-\frac{1}{7b} = \frac{4(7b)-(b+7)}{7b(b+7)}$

$= \frac{28b-b-7}{7b(b+7)}$

$= \mathbf{\frac{27b-7}{7b(b+7)}}$

(c) $\frac{5}{c-6}+\frac{3}{3c+1} = \frac{5(3c+1)+3(c-6)}{(3c+1)(c-6)}$

$= \frac{15c+5+3c-18}{(3c+1)(c-6)}$

$= \mathbf{\frac{18c-13}{(3c+1)(c-6)}}$

(d) $\frac{8}{2d+1}-\frac{1}{d-2} = \frac{8(d-2)-(2d+1)}{(2d+1)(d-2)}$

$= \frac{8d-16-2d-1}{(2d+1)(d-2)}$

$= \mathbf{\frac{6d-17}{(2d+1)(d-2)}}$

(e) $\frac{2f}{7f-2}+\frac{4}{7f+4} = \frac{2f(7f+4)+4(7f-2)}{(7f-2)(7f+4)}$

$= \frac{14f^2+8f+28f-8}{(7f-2)(7f+4)}$

$= \mathbf{\frac{14f^2+36f-8}{(7f-2)(7f+4)}}$

(f) $\frac{9}{4h-6}-\frac{5h}{3-4h} = \frac{9(3-4h)-5h(4h-6)}{(4h-6)(3-4h)}$

$= \frac{27-36h-20h^2+30h}{2(2h-3)(3-4h)}$

$= \frac{-20h^2-6h+27}{2(2h-3)(3-4h)}$

$= \mathbf{\frac{20h^2+6h-27}{2(4h-3)(2h-3)}}$

(g) $\frac{3}{9k-1}+\frac{8}{3(2k+5)} = \frac{3(3)(2k+5)+8(9k-1)}{3(9k-1)(2k+5)}$

$= \frac{18k+45+72k-8}{3(9k-1)(2k+5)}$

$= \mathbf{\frac{90k+37}{3(9k-1)(2k+5)}}$

(h) $\frac{10}{4n-3m}-\frac{9}{2(3n-m)} = \frac{10(2)(3n-m)-9(4n-3m)}{2(4n-3m)(3n-m)}$

$= \frac{60n-20m-36n+27m}{2(4n-3m)(3n-m)}$

$= \mathbf{\frac{7m+24n}{2(4n-3m)(3n-m)}}$

(i) $-\frac{7}{4+5p}+\frac{2}{7p-3} = \frac{2}{7p-3}-\frac{7}{5p+4}$

$= \frac{2(5p+4)-7(7p-3)}{(7p-3)(5p+4)}$

$= \frac{10p+8-49p+21}{(7p-3)(5p+4)}$

$= \mathbf{\frac{29-39p}{(7p-3)(5p+4)}}$

(j) $-\frac{1}{5(15r-3q)}-\frac{q}{2q-r} = \frac{1}{15(q-5r)}-\frac{q}{2q-r}$

$= \frac{(2q-r)-15q(q-5r)}{15(2q-r)(q-5r)}$

$= \frac{2q-r-15q^2+75qr}{15(2q-r)(q-5r)}$

$= \mathbf{\frac{2q-r+75qr-15q^2}{15(2q-r)(q-5r)}}$

(k) $\frac{1}{2x-1}+\frac{1}{2x}+\frac{1}{2x+1}$

$= \frac{2x(2x+1)+(2x+1)(2x-1)+2x(2x-1)}{2x(2x+1)(2x-1)}$

$= \frac{4x^2+2x+4x^2-1+4x^2-2x}{2x(2x+1)(2x-1)}$

$= \mathbf{\frac{12x^2-1}{2x(2x+1)(2x-1)}}$

(l) $\frac{2}{z+5y}-\frac{8}{3(z-6y)}-\frac{4}{z+7y}$

$=\frac{2(3)(z-6y)(z+7y)-8(z+5y)(z+7y)-4(3)(z+5y)(z-6y)}{3(z+5y)(z-6y)(z+7y)}$

$=\frac{6(z^2+yz-42y^2)-8(z^2+12yz+35y^2)-12(z^2-yz-30y^2)}{3(z+5y)(z-6y)(z+7y)}$

$=\frac{6z^2+6yz-252y^2-8z^2-96yz-280y^2-12z^2+12yz+360y^2}{3(z+5y)(z-6y)(z+7y)}$

$=\frac{-14z^2-78yz-172y^2}{3(z+5y)(z-6y)(z+7y)}$

$=-\boldsymbol{\frac{14z^2+78yz+172y^2}{3(z+5y)(z-6y)(z+7y)}}$

3. (a) $\frac{10}{a}+\frac{20}{a^2}=\boldsymbol{\frac{10a+20}{a^2}}$

(b) $\frac{4}{b-3}+\frac{5}{(b-3)^2}=\frac{4(b-3)+5}{(b-3)^2}$

$=\frac{4b-12+5}{(b-3)^2}$

$=\boldsymbol{\frac{4b-7}{(b-3)^2}}$

(c) $\frac{6}{(c+4)^2}-\frac{2}{c+4}=\frac{6-2(c+4)}{(c+4)^2}$

$=\frac{6-2c-8}{(c+4)^2}$

$=\frac{-2c-2}{(c+4)^2}$

$=-\boldsymbol{\frac{2c+2}{(c+4)^2}}$

(d) $\frac{4d}{7-4d}-\frac{5d}{(7-4d)^2}=\frac{4d(7-4d)-5d}{(7-4d)^2}$

$=\frac{28d-16d^2-5d}{(7-4d)^2}$

$=\boldsymbol{\frac{23d-16d^2}{(7-4d)^2}}$

(e) $\frac{5f}{4f^2+4f+1}-\frac{9}{2f+1}=\frac{5f}{(2f+1)^2}-\frac{9}{2f+1}$

$=\frac{5f-9(2f+1)}{(2f+1)^2}$

$=\frac{5f-18f-9}{(2f+1)^2}$

$=\frac{-13f-9}{(2f+1)^2}$

$=-\boldsymbol{\frac{13f+9}{(2f+1)^2}}$

(f) $\frac{1}{h-8}+\frac{3h}{h^2-16h+64}=\frac{1}{h-8}+\frac{3h}{(h-8)^2}$

$=\frac{(h-8)+3h}{(h-8)^2}$

$=\boldsymbol{\frac{4h-8}{(h-8)^2}}$

(g) $\frac{1}{k^2-25}+\frac{3}{k+5}=\frac{1}{(k+5)(k-5)}+\frac{3}{k+5}$

$=\frac{1+3(k-5)}{(k+5)(k-5)}$

$=\frac{1+3k-15}{(k+5)(k-5)}$

$=\boldsymbol{\frac{3k-14}{(k+5)(k-5)}}$

(h) $\frac{6}{(m-9n)^2}-\frac{1}{9n-m}=\frac{6}{(m-9n)^2}+\frac{1}{m-9n}$

$=\frac{6+(m-9n)}{(m-9n)^2}$

$=\boldsymbol{\frac{m-9n+6}{(m-9n)^2}}$

(i) $\frac{3}{7p^2+11p+4}+\frac{2}{7p+4}=\frac{3}{(7p+4)(p+1)}+\frac{2}{7p+4}$

$=\frac{3+2(p+1)}{(7p+4)(p+1)}$

$=\frac{3+2p+2}{(7p+4)(p+1)}$

$=\boldsymbol{\frac{2p+5}{(7p+4)(p+1)}}$

(j) $\frac{4}{3q-8r}-\frac{q}{6q^2-qr-40r^2}=\frac{4}{3q-8r}-\frac{q}{(3q-8r)(2q+5r)}$

$=\frac{4(2q+5r)-q}{(3q-8r)(2q+5r)}$

$=\frac{8q+20r-q}{(3q-8r)(2q+5r)}$

$=\boldsymbol{\frac{7q+20r}{(3q-8r)(2q+5r)}}$

(k) $\frac{9}{(x+1)(x+2)}-\frac{10}{(x+2)(x+3)}=\frac{9(x+3)-10(x+1)}{(x+1)(x+2)(x+3)}$

$=\frac{9x+27-10x-10}{(x+1)(x+2)(x+3)}$

$=\boldsymbol{\frac{17-x}{(x+1)(x+2)(x+3)}}$

(l) $\frac{y}{2y^2+2y-24}+\frac{5}{y^2-y-20}$

$=\frac{y}{2(y^2+y-12)}+\frac{5}{y^2-y-20}$

$=\frac{y}{2(y+4)(y-3)}+\frac{5}{(y-5)(y+4)}$

$=\frac{y(y-5)+5(2)(y-3)}{2(y-3)(y+4)(y-5)}$

$=\frac{y^2-5y+10y-30}{2(y-3)(y+4)(y-5)}$

$=\boldsymbol{\frac{y^2+5y-30}{2(y-3)(y+4)(y-5)}}$

4. $2yz - \frac{5}{y-z} + \frac{y}{yz-z^2} = 2yz - \frac{5}{y-z} + \frac{y}{z(y-z)}$

$= \frac{2yz^2(y-z) - 5z + y}{z(y-z)}$

$= \mathbf{\frac{2y^2z^2 - 2yz^3 + y - 5z}{z(y-z)}}$

5. $\frac{7}{4x-3} + \frac{5}{4x+3} + \frac{hx+k}{9-16x^2} = \frac{5}{3+4x} - \frac{7}{3-4x} + \frac{hx+k}{(3+4x)(3-4x)}$

$= \frac{5(3-4x) - 7(3+4x) + (hx+k)}{(3+4x)(3-4x)}$

$= \frac{15 - 20x - 21 - 28x + hx + k}{(3+4x)(3-4x)}$

$= \frac{(h-48)x + (k-6)}{(3+4x)(3-4x)}$

Since $\frac{(h-48)x+(k-6)}{(3+4x)(3-4x)} = \frac{Ax+B}{9-16x^2}$,

then $\boldsymbol{A = h - 48}$ and $\boldsymbol{B = k - 6}$.

6. $\frac{\frac{1}{x}+\frac{1}{y}}{\frac{4}{y^2}-\frac{4}{x^2}} = \frac{\frac{y+x}{xy}}{\frac{4x^2-4y^2}{x^2y^2}}$

$= \frac{\frac{x+y}{xy}}{\frac{4(x+y)(x-y)}{x^2y^2}}$

$= \mathbf{\frac{xy}{4(x-y)}}$

Challenge Myself!

7. $\frac{A}{1+5x} + \frac{B}{4x+1} + \frac{C}{1-5x}$

$= \frac{A}{5x+1} + \frac{B}{4x+1} - \frac{C}{5x-1}$

$= \frac{A(5x-1)(4x+1) + B(5x+1)(5x-1) - C(5x+1)(4x+1)}{(5x+1)(5x-1)(4x+1)}$

$= \frac{A(20x^2+x-1) + B(25x^2-1) - C(20x^2+9x+1)}{(5x+1)(5x-1)(4x+1)}$

$= \frac{20Ax^2 + Ax - A + 25Bx^2 - B - 20Cx^2 - 9Cx - C}{(5x+1)(5x-1)(4x+1)}$

$= \frac{(20A+25B-20C)x^2 + (A-9C)x - (A+B+C)}{(5x+1)(5x-1)(4x+1)}$

Comparing coefficients of x^2,

$20A + 25B - 20C = 235$

$4A + 5B - 4C = 47$ — (1)

Comparing coefficients of x,

$A - 9C = 24$ — (2)

Comparing constants,

$-(A + B + C) = -7$

$A + B + C = 7$ — (3)

From (2), $A = 9C + 24$ — (4)

Substitute (4) into (1):

$4(9C + 24) + 5B - 4C = 47$

$36C + 96 + 5B - 4C = 47$

$5B + 32C = -49$ — (5)

Substitute (4) into (3):

$(9C + 24) + B + C = 7$

$9C + C + 24 + B = 7$

$B = -10C - 17$ — (6)

Substitute (6) into (5):

$5(-10C - 17) + 32C = -49$

$-50C - 85 + 32C = -49$

$18C = -36$

$C = -2$

Substitute $C = -2$ into (6):

$B = -10(-2) - 17$

$= 3$

Substitute $C = -2$ into (4):

$A = 9(-2) + 24$

$= 6$

$\therefore$ **$A = 6, B = 3, C = -2$**

Worksheet 1D Solving equations involving algebraic fractions

1. (a) $\frac{3a-4}{8} + \frac{a+3}{4} = \frac{7}{6}$

$3(3a - 4) + 6(a + 3) = 28$

$9a - 12 + 6a + 18 = 28$

$15a = 22$

$a = \mathbf{1\frac{7}{15}}$

(b) $\frac{10}{b} = \frac{1}{4}$

$b = \mathbf{40}$

(c) $\frac{9}{c-2} = 5$

$5(c - 2) = 9$

$c - 2 = \frac{9}{5}$

$c = \mathbf{3\frac{4}{5}}$

(d) $10 - \frac{4}{6d+1} = 8$

$\frac{4}{6d+1} = 2$

$2(6d + 1) = 4$

$6d + 1 = 2$

$6d = 1$

$d = \mathbf{\frac{1}{6}}$

(e) $\frac{7h-1}{4h+5} = 3$

$7h - 1 = 3(4h + 5)$

$7h - 1 = 12h + 15$

$5h = -16$

$h = \mathbf{-3\frac{1}{5}}$

(f) $\frac{6k}{k+1} = \frac{5}{12}$

$12(6k) = 5(k + 1)$

$72k = 5k + 5$

$67k = 5$

$k = \mathbf{\frac{5}{67}}$

(g) $\frac{3}{2m+7} = \frac{4}{8-m}$

$3(8 - m) = 4(2m + 7)$

$24 - 3m = 8m + 28$

$11m = -4$

$m = \mathbf{-\frac{4}{11}}$

(h) $\frac{7}{n} - \frac{2}{n+3} = 0$

$\frac{7}{n} = \frac{2}{n+3}$

$7(n + 3) = 2n$

$7n + 21 = 2n$

$5n = -21$

$n = \mathbf{-4\frac{1}{5}}$

(i) $\frac{8}{4p-5} + \frac{6}{5p-4} = 0$

$\frac{8}{4p-5} = -\frac{6}{5p-4}$

$= \frac{6}{4-5p}$

$8(4 - 5p) = 6(4p - 5)$

$32 - 40p = 24p - 30$

$64p = 62$

$p = \mathbf{\frac{31}{32}}$

(j) $\frac{9}{q} + \frac{7}{2q} - \frac{5}{3q} = 1$

$\frac{54+21-10}{6q} = 1$

$\frac{65}{6q} = 1$

$6q = 65$

$q = \mathbf{10\frac{5}{6}}$

(k) $\frac{4}{3x+5} + \frac{1}{6x+10} = 6$

$\frac{4}{3x+5} + \frac{1}{2(3x+5)} = 6$

$\frac{8+1}{2(3x+5)} = 6$

$\frac{9}{2(3x+5)} = 6$

$12(3x + 5) = 9$

$36x + 60 = 9$

$36x = -51$

$x = \mathbf{-1\frac{5}{12}}$

(l) $\frac{12}{9-2y} - \frac{11}{2y-9} = 23$

$\frac{12}{9-2y} + \frac{11}{9-2y} = 23$

$\frac{23}{9-2y} = 23$

$9 - 2y = 1$

$2y = 8$

$y = \mathbf{4}$

2. (a) $\frac{3a+2}{20} = \frac{5}{3a+2}$

$(3a + 2)^2 = 100$

$3a + 2 = 10$ or $3a + 2 = -10$

$3a = 8$ $\quad$ $3a = -12$

$a = 2\frac{2}{3}$ $\quad$ $a = -4$

$\therefore a = \mathbf{2\frac{2}{3}}$ or $a = \mathbf{-4}$

(b) $b = \frac{12}{b+1}$

$b(b + 1) = 12$

$b^2 + b - 12 = 0$

$(b + 4)(b - 3) = 0$

$b = -4$ or $b = 3$

$\therefore b = \mathbf{-4}$ or $b = \mathbf{3}$

(c) $\frac{18}{h-8} = h - 1$

$(h - 8)(h - 1) = 18$

$h^2 - 9h + 8 = 18$

$h^2 - 9h - 10 = 0$

$(h - 10)(h + 1) = 0$

$h = 10$ or $h = -1$

$\therefore h = \mathbf{10}$ or $h = \mathbf{-1}$

(d) $4k - 1 = \frac{11}{4-k}$

$(4k - 1)(4 - k) = 11$

$-4k^2 + 17k - 4 = 11$

$4k^2 - 17k + 15 = 0$

$(4k - 5)(k - 3) = 0$

$k = 1\frac{1}{4}$ or $k = 3$

$\therefore k = \mathbf{1\frac{1}{4}}$ or $k = \mathbf{3}$

(e) $7m + 2 = \frac{5}{7m-2}$

$(7m + 2)(7m - 2) = 5$

$49m^2 - 4 = 5$

$49m^2 = 9$

$m^2 = \frac{9}{49}$

$m = \pm\frac{3}{7}$

$\therefore m = \pm\mathbf{\frac{3}{7}}$

(f) $25 + 20n + 4n^2 = \dfrac{8}{2n+5}$

$$(2n+5)^2 = \frac{8}{2n+5}$$
$$(2n+5)^3 = 8$$
$$2n + 5 = 2$$
$$2n = -3$$
$$n = -1\frac{1}{2}$$

$\therefore n = \mathbf{-1\frac{1}{2}}$

(g) $\dfrac{9-p}{p+5} = 2p - 3$

$$(2p-3)(p+5) = 9 - p$$
$$2p^2 + 7p - 15 = 9 - p$$
$$2p^2 + 8p - 24 = 0$$
$$p^2 + 4p - 12 = 0$$
$$(p+6)(p-2) = 0$$
$$p = -6 \quad \text{or} \quad p = 2$$

$\therefore p = \mathbf{-6}$ or $p = \mathbf{2}$

(h) $\dfrac{7q-1}{7q+3} = \dfrac{7q+2}{7q+5}$

$$(7q+5)(7q-1) = (7q+2)(7q+3)$$
$$49q^2 + 28q - 5 = 49q^2 + 35q + 6$$
$$7q = -11$$
$$q = -1\frac{4}{7}$$

$\therefore q = \mathbf{-1\frac{4}{7}}$

(i) $\dfrac{36}{x} - x = 5$

$$36 - x^2 = 5x$$
$$x^2 + 5x - 36 = 0$$
$$(x+9)(x-4) = 0$$
$$x = -9 \quad \text{or} \quad x = 4$$

$\therefore x = \mathbf{-9}$ or $x = \mathbf{4}$

(j) $4y^2 = 65 - \dfrac{16}{y^2}$

$$4(y^2)^2 = 65y^2 - 16$$
$$4(y^2)^2 - 65y^2 + 16 = 0$$
$$(4y^2 - 1)(y^2 - 16) = 0$$
$$y^2 = \frac{1}{4} \quad \text{or} \quad y^2 = 16$$
$$y = \pm\frac{1}{2} \qquad y = \pm 4$$

$\therefore y = \mathbf{\pm\frac{1}{2}}$ or $y = \mathbf{\pm 4}$

3. (a) $\dfrac{10}{(5a+2)(3a-1)} + \dfrac{2}{3a-1} = 0$

$$10 + 2(5a+2) = 0$$
$$10 + 10a + 4 = 0$$
$$10a = -14$$
$$a = -1\frac{2}{5}$$

$\therefore a = \mathbf{-1\frac{2}{5}}$

(b) $\dfrac{4}{8b-3} - \dfrac{b}{8b^2 - 3b} = 0$

$$\frac{4}{8b-3} - \frac{b}{b(8b-3)} = 0$$
$$4b - b = 0$$
$$3b = 0$$
$$b = 0$$

$\therefore$ The equation has **no real solutions**.

(c) $\dfrac{7}{2h+1} + \dfrac{5}{4h^2 - 1} = 0$

$$\frac{7}{2h+1} + \frac{5}{(2h+1)(2h-1)} = 0$$
$$7(2h-1) + 5 = 0$$
$$14h - 7 + 5 = 0$$
$$14h = 2$$
$$h = \frac{1}{7}$$

$\therefore h = \mathbf{\frac{1}{7}}$

(d) $\dfrac{2}{3k-1} - \dfrac{17}{6k+5} = \dfrac{6k-3}{18k^2 + 9k - 5}$

$$= \frac{6k-3}{(6k+5)(3k-1)}$$
$$2(6k+5) - 17(3k-1) = 6k - 3$$
$$12k + 10 - 51k + 17 = 6k - 3$$
$$45k = 30$$
$$k = \frac{2}{3}$$

$\therefore k = \mathbf{\frac{2}{3}}$

(e) $\dfrac{3}{m+3} + \dfrac{2}{m-3} = 1$

$$3(m-3) + 2(m+3) = (m+3)(m-3)$$
$$3m - 9 + 2m + 6 = m^2 - 9$$
$$m^2 - 5m - 6 = 0$$
$$(m-6)(m+1) = 0$$
$$m = 6 \quad \text{or} \quad m = -1$$

$\therefore m = \mathbf{6}$ or $m = \mathbf{-1}$

(f) $\dfrac{6}{n-6} = 2 - \dfrac{1}{n-8}$

$$6(n-8) = 2(n-6)(n-8) - (n-6)$$
$$6n - 48 = 2(n^2 - 14n + 48) - n + 6$$
$$= 2n^2 - 28n + 96 - n + 6$$
$$2n^2 - 35n + 150 = 0$$
$$(2n-15)(n-10) = 0$$
$$n = 7\frac{1}{2} \quad \text{or} \quad n = 10$$

$\therefore n = \mathbf{7\frac{1}{2}}$ or $n = \mathbf{10}$

(g) $\dfrac{16}{3p+12} + \dfrac{12}{3p-8} = 1$

$$16(3p-8) + 12(3p+12) = (3p+12)(3p-8)$$
$$48p - 128 + 36p + 144 = 9p^2 + 12p - 96$$
$$9p^2 - 72p - 112 = 0$$
$$(3p-28)(3p+4) = 0$$
$$p = 9\frac{1}{3} \quad \text{or} \quad p = -1\frac{1}{3}$$

$\therefore p = \mathbf{9\frac{1}{3}}$ or $p = \mathbf{-1\frac{1}{3}}$

(h) $\frac{q-3}{q} - \frac{7}{q-5} = 3$

$(q-3)(q-5) - 7q = 3q(q-5)$

$q^2 - 8q + 15 - 7q = 3q^2 - 15q$

$2q^2 = 15$

$q^2 = 7.5$

$q = \pm 2.74$ (to 3 s.f.)

$\therefore$ $q = \mathbf{\pm 2.74}$

(i) $\frac{5}{x} = 2 + \frac{10}{5-3x}$

$5(5-3x) = 2x(5-3x) + 10x$

$25 - 15x = 10x - 6x^2 + 10x$

$6x^2 - 35x + 25 = 0$

$(6x-5)(x-5) = 0$

$x = \frac{5}{6}$ or $x = 5$

$\therefore$ $x = \mathbf{\frac{5}{6}}$ or $x = \mathbf{5}$

(j) $\frac{4y-12}{y+2} - \frac{1}{y-2} = 4$

$(4y-12)(y-2) - (y+2) = 4(y+2)(y-2)$

$4y^2 - 20y + 24 - y - 2 = 4(y^2 - 4)$

$4y^2 - 21y + 22 = 4y^2 - 16$

$21y = 38$

$y = 1\frac{17}{21}$

$\therefore$ $y = \mathbf{1\frac{17}{21}}$

4. **(i)** $3x^2 - 2x - 8 = \mathbf{(3x+4)(x-2)}$

(ii) $\frac{4x}{3x^2-2x-8} + \frac{7}{2-x} = 0$

$\frac{4x}{(3x+4)(x-2)} - \frac{7}{x-2} = 0$

$4x - 7(3x+4) = 0$

$4x - 21x - 28 = 0$

$17x = -28$

$x = -1\frac{11}{17}$

$\therefore$ $x = \mathbf{-1\frac{11}{17}}$

5. Substitute $x = 5$ into $\frac{A}{8x-4} - \frac{B}{1-2x} = 6$:

$\frac{1}{36}A + \frac{1}{9}B = 6$

$A + 4B = 216$

$\therefore$ A possible pair of values is $\boldsymbol{A} = \mathbf{200}$ and $\boldsymbol{B} = \mathbf{4}$.

6. Let the fraction be $\frac{x+2}{x}$.

$\frac{x+2}{x} + \frac{x}{x+2} = \frac{34}{15}$

$15(x+2)^2 + 15x^2 = 34x(x+2)$

$15(x^2 + 4x + 4) + 15x^2 = 34x^2 + 68x$

$15x^2 + 60x + 60 + 15x^2 = 34x^2 + 68x$

$4x^2 + 8x - 60 = 0$

$x^2 + 2x - 15 = 0$

$(x+5)(x-3) = 0$

$x = -5$ or $x = 3$

$\therefore$ The fractions are $\mathbf{\frac{5}{3}}$ and $\mathbf{\frac{3}{5}}$.

7. **(i)** Volume of tank = $(200 \times 150 \times 120)$ cm^3

$= 3\,600\,000$ cm^3

Time taken by tap $P = \frac{3\,600\,000}{x}$ s $= \frac{1000}{x}$ h

Time taken by taps P and $Q = \frac{3\,600\,000}{x+x+40}$ s $= \frac{1000}{2x+40}$ h

$= \frac{500}{x+20}$ h

$\frac{1000}{x} - \frac{500}{x+20} = 5\frac{5}{6}$

$= \frac{35}{6}$

$6000(x+20) - 3000x = 35x(x+20)$

$6000x + 120\,000 - 3000x = 35x^2 + 700x$

$35x^2 - 2300x - 120\,000 = 0$

$7x^2 - 460x - 24\,000 = 0$

$(7x + 240)(x - 100) = 0$

$x = -34\frac{2}{7}$ or $x = 100$

$\therefore$ $x = \mathbf{100}$

(ii) Time taken by tap $Q = \frac{1000}{x+40}$ h

$= \frac{1000}{140}$ h

$= 7\frac{1}{7}$ h

= **7 h 9 min** (to the nearest minute)

Worksheet 1E Manipulating algebraic formulae

1. **(a)** $y = 7p + 4q$

$7p = y - 4q$

$\boldsymbol{p} = \mathbf{\frac{y - 4q}{7}}$

(b) $ax + by = c$

$by = c - ax$

$\boldsymbol{y} = \mathbf{\frac{c - ax}{b}}$

(c) $hk = abc$

$\boldsymbol{b} = \mathbf{\frac{hk}{ac}}$

(d) $pq^3 = \frac{3}{4}mn$

$3mn = 4pq^3$

$\boldsymbol{m} = \mathbf{\frac{4pq^3}{3n}}$

(e) $x = \frac{4p - 5q}{8}$

$8x = 4p - 5q$

$4p = 5q + 8x$

$\boldsymbol{p} = \mathbf{\frac{5q + 8x}{4}}$

(f) $y = \frac{ab}{x-c}$

$x - c = \frac{ab}{y}$

$\boldsymbol{x} = \boldsymbol{c} + \mathbf{\frac{ab}{y}}$

(g) $S = \frac{n}{2}(2a + l)$

$2a + l = \frac{2S}{n}$

$\boldsymbol{l = \frac{2S}{n} - 2a}$

(h) $D = b^2 - 4ac$

$4ac = b^2 - D$

$\boldsymbol{a = \frac{b^2 - D}{4c}}$

(i) $v^2 = u^2 + 2gs$

$u^2 = v^2 - 2gs$

$\boldsymbol{u = \pm\sqrt{v^2 - 2gs}}$

(j) $L = \frac{gT^2}{4\pi^2}$

$4\pi^2 L = gT^2$

$T^2 = \frac{4\pi^2 L}{g}$

$\boldsymbol{T = \pm\sqrt{\frac{4\pi^2 L}{g}}}$

(k) $y = \sqrt{ax^2 + b}$

$y^2 = ax^2 + b$

$ax^2 = y^2 - b$

$\boldsymbol{a = \frac{y^2 - b}{x^2}}$

(l) $x + y = \sqrt[3]{6p + q}$

$(x + y)^3 = 6p + q$

$6p = (x + y)^3 - q$

$\boldsymbol{p = \frac{(x + y)^3 - q}{6}}$

(m) $hx = kx + c$

$hx - kx = c$

$x(h - k) = c$

$\boldsymbol{x = \frac{c}{h - k}}$

(n) $V - r^3 = \frac{4}{3}\pi r^3$

$3V - 3r^3 = 4\pi r^3$

$4\pi r^3 + 3r^3 = 3V$

$r^3(4\pi + 3) = 3V$

$r^3 = \frac{3V}{4\pi + 3}$

$\boldsymbol{r = \sqrt[3]{\frac{3V}{4\pi + 3}}}$

(o) $xy = a(x - 6)$

$= ax - 6a$

$ax - xy = 6a$

$x(a - y) = 6a$

$\boldsymbol{x = \frac{6a}{a - y}}$

(p) $\pi(a + b) = \frac{b}{kc}$

$\pi ck(a + b) = b$

$\pi ack + \pi bck = b$

$b - \pi bck = \pi ack$

$b(1 - \pi ck) = \pi ack$

$\boldsymbol{b = \frac{\pi ack}{1 - \pi ck}}$

(q) $x = \frac{a(b - 3)}{3 + b}$

$3x + bx = ab - 3a$

$ab - bx = 3a + 3x$

$b(a - x) = 3a + 3x$

$\boldsymbol{b = \frac{3a + 3x}{a - x}}$

(r) $h = \frac{2k^2}{h - k^2}$

$h(h - k^2) = 2k^2$

$h^2 - hk^2 = 2k^2$

$hk^2 + 2k^2 = h^2$

$k^2(h + 2) = h^2$

$k^2 = \frac{h^2}{h + 2}$

$\boldsymbol{k = \pm\sqrt{\frac{h^2}{h + 2}}}$

(s) $\frac{m}{n} = \frac{an + bn}{m}$

$m^2 = n(an + bn)$

$= n^2(a + b)$

$n^2 = \frac{m^2}{a + b}$

$\boldsymbol{n = \pm\sqrt{\frac{m^2}{a + b}}}$

(t) $\frac{1}{4p} + \frac{2}{5q} = \frac{3}{r}$

$\frac{2}{5q} = \frac{3}{r} - \frac{1}{4p}$

$= \frac{12p - r}{4pr}$

$\frac{5q}{2} = \frac{4pr}{12p - r}$

$\boldsymbol{q = \frac{8pr}{5(12p - r)}}$

(u) $a\sqrt{x} + b = p\sqrt{x} + q$

$a\sqrt{x} - p\sqrt{x} = q - b$

$\sqrt{x}(a - p) = q - b$

$\sqrt{x} = \frac{q - b}{a - p}$

$\boldsymbol{x = \left(\frac{q - b}{a - p}\right)^2}$

(v) $\sqrt[3]{\frac{m}{n} - \frac{an^3}{m^3}} = b$

$\frac{m}{n} - \frac{an^3}{m^3} = b^3$

$\frac{an^3}{m^3} = \frac{m}{n} - b^3$

$= \frac{m - b^3 n}{n}$

$a = \frac{m^3}{n^3}\left(\frac{m - b^3 n}{n}\right)$

$\boldsymbol{a = \frac{m^3(m - b^3 n)}{n^4}}$

(w) $x^2 + 2xy = 1 - y^2$

$x^2 + 2xy + y^2 = 1$

$(x + y)^2 = 1$

$x + y = \pm 1$

$\boldsymbol{y = -x \pm 1}$

(x) $\frac{c}{x} - \frac{b^2y^2}{ax} = \frac{1}{9}ax\left(a^2x^2 - 6by\right)$

$9ac - 9b^2y^2 = a^2x^2(a^2x^2 - 6by)$

$= a^4x^4 - 6a^2bx^2y$

$a^4x^4 - 6a^2bx^2y + 9b^2y^2 = 9ac$

$(a^2x^2 - 3by)^2 = 9ac$

$a^2x^2 - 3by = \pm\sqrt{9ac}$

$3by = a^2x^2 \pm \sqrt{9ac}$

$\boldsymbol{b = \frac{a^2x^2 \pm \sqrt{9ac}}{3y}}$

2. $A = 3\pi a(6a + b)$

$6a + b = \frac{A}{3\pi a}$

$\boldsymbol{b = \frac{A}{3\pi a} - 6a}$

3. (i) $h = \frac{1}{4}k\left(a^2 - b^2\right)$

When $a = 8$, $b = -6$ and $k = 5$,

$h = \frac{1}{4}(5)\left[8^2 - (-6)^2\right]$

$= \mathbf{35}$

(ii) $h = \frac{1}{4}k\left(a^2 - b^2\right)$

$\frac{4h}{k} = a^2 - b^2$

$b^2 = a^2 - \frac{4h}{k}$

$\boldsymbol{b = \pm\sqrt{a^2 - \frac{4h}{k}}}$

4. (i) $s = ut - \frac{1}{2}gt^2$

When $u = 4$, $t = 1$ and $g = 10$,

$s = (4)(1) - \frac{1}{2}(10)(1)^2$

$= \mathbf{-1}$

(ii) $s = ut - \frac{1}{2}gt^2$

$2s = 2ut - gt^2$

$gt^2 = 2ut - 2s$

$\boldsymbol{g = \frac{2tu - 2s}{t^2}}$

5. $y = \frac{x^2 + a}{x^2 - b}$

$y(x^2 - b) = x^2 + a$

$x^2y - by = x^2 + a$

$x^2y - x^2 = a + by$

$x^2(y - 1) = a + by$

$x^2 = \frac{a + by}{y - 1}$

$\boldsymbol{x = \pm\sqrt{\frac{a + by}{y - 1}}}$

6. $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2}$

$\frac{1}{b^2} = \frac{1}{c^2} - \frac{1}{a^2}$

$= \frac{a^2 - c^2}{a^2c^2}$

$b^2 = \frac{a^2c^2}{a^2 - c^2}$

$\boldsymbol{b = \pm\sqrt{\frac{a^2c^2}{a^2 - c^2}}}$

7. $T = 2\pi\sqrt{\frac{L + r}{g}}$

$T^2 = 4\pi^2\left(\frac{L + r}{g}\right)$

$\frac{gT^2}{4\pi^2} = L + r$

$\boldsymbol{L = \frac{gT^2}{4\pi^2} - r}$

8. (i) $(x - a)^2 + (y - b)^2 = r^2$

When $a = b = 0$ and $r = 6$,

Equation of circle: $\boldsymbol{x^2 + y^2 = 36}$

(ii) $(x - a)^2 + (y - b)^2 = r^2$

$(y - b)^2 = r^2 - (x - a)^2$

$y - b = \pm\sqrt{r^2 - (x - a)^2}$

$\boldsymbol{y = b \pm \sqrt{r^2 - (x - a)^2}}$

9. (i) $I = P\left(1 + \frac{R}{100}\right)^n - P$

When $P = 30\ 000$, $R = 4$ and $n = 5$,

$I = \$\left[30\ 000\left(1 + \frac{4}{100}\right)^5 - 30\ 000\right]$

$= \$6499.59$ (to 2 d.p.)

∴ Interest charged is **$6499.59**

(ii) $I = P\left(1 + \frac{R}{100}\right)^n - P$

$= P\left[\left(1 + \frac{R}{100}\right)^n - 1\right]$

$\boldsymbol{P = \frac{I}{\left(1 + \frac{R}{100}\right)^n - 1}}$

(iii) $P = \frac{I}{\left(1 + \frac{R}{100}\right)^n - 1}$

When $I = 1460$, $R = 2.5$ and $n = 2$,

$P = \frac{1460}{\left(1 + \frac{2.5}{100}\right)^2 - 1} = 28\ 800$ (to the nearest hundred)

∴ Amount borrowed is **$28 800**

Challenge Myself!

10. (i) $y = mx + c$

$mx = y - c$

$m = \boldsymbol{\frac{y - c}{x}}$

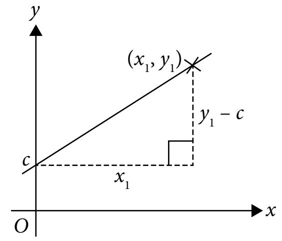
(ii) Let (x_1, y_1) be the coordinates of a point that lies on the line.

Then gradient of line $= \dfrac{\text{vertical change}}{\text{horizontal change}}$

$= \dfrac{y_1 - c}{x_1}$

Replacing (x_1, y_1) with (x, y) to represent any point that lies on the line,

we have $m = \dfrac{y-c}{x}$.



Review Exercise 1

1. **(a)** $\dfrac{6ab^2}{5} \div \dfrac{4b^3}{a} \div \left(\dfrac{b}{a}\right)^2 = \dfrac{6ab^2}{5} \times \dfrac{a}{4b^3} \times \left(\dfrac{a}{b}\right)^2$

$= \dfrac{6ab^2}{5} \times \dfrac{a}{4b^3} \times \dfrac{a^2}{b^2}$

$= \mathbf{\dfrac{3a^4}{10b^3}}$

(b) $\dfrac{18y^2 + 12xy}{4x^2 - 9y^2} = \dfrac{6y(3y+2x)}{(2x+3y)(2x-3y)}$

$= \mathbf{\dfrac{6y}{2x - 3y}}$

2. **(a)** $2x^2 - 9x + 4 = \mathbf{(2x - 1)(x - 4)}$

(b) **(i)** $2x^2 - 9x + 4 = 0$

$(2x - 1)(x - 4) = 0$

$x = \dfrac{1}{2}$ or $x = 4$

$\therefore x = \mathbf{\dfrac{1}{2}}$ or $x = \mathbf{4}$

(ii) $\dfrac{2x^2 - 9x + 4}{x^2 - 16} = \dfrac{(2x-1)(x-4)}{(x+4)(x-4)}$

$= \mathbf{\dfrac{2x - 1}{x + 4}}$

3. **(a)** $\dfrac{4}{x+4} + \dfrac{5}{x-5} = \dfrac{4(x-5)+5(x+4)}{(x+4)(x-5)}$

$= \dfrac{4x - 20 + 5x + 20}{(x+4)(x-5)}$

$= \mathbf{\dfrac{9x}{(x+4)(x-5)}}$

(b) $\dfrac{2}{x-4} + \dfrac{3}{x+2} = 1$

$2(x + 2) + 3(x - 4) = (x - 4)(x + 2)$

$2x + 4 + 3x - 12 = x^2 - 2x - 8$

$x^2 - 7x = 0$

$x(x - 7) = 0$

$x = 0$ or $x = 7$

$\therefore x = \mathbf{0}$ or $x = \mathbf{7}$

4. $\dfrac{A}{B} = \dfrac{20x - 15xy - 8 + 6y}{24x - 3y - 18xy + 4}$

$= \dfrac{20x - 15xy - 8 + 6y}{24x + 4 - 18xy - 3y}$

$= \dfrac{5x(4-3y) - 2(4-3y)}{4(6x+1) - 3y(6x+1)}$

$= \dfrac{(4-3y)(5x-2)}{(6x+1)(4-3y)}$

$= \mathbf{\dfrac{5x - 2}{6x + 1}}$

5. $\left(1 - \dfrac{1}{x^2}\right) \div \left(1 - \dfrac{1}{x}\right) = \dfrac{x^2-1}{x^2} \div \dfrac{x-1}{x}$

$= \dfrac{x^2-1}{x^2} \times \dfrac{x}{x-1}$

$= \dfrac{(x+1)(x-1)}{x^2} \times \dfrac{x}{x-1}$

$= \mathbf{\dfrac{x + 1}{x}}$

6. **(i)** $V = \dfrac{\pi}{3}\left(R^3 - r^3\right)$

When $R = 7.2$ and $r = 6.8$,

$V = \dfrac{\pi}{3}\left(7.2^3 - 6.8^3\right)$

$= \mathbf{61.6}$ (to 3 s.f.)

(ii) $V = \dfrac{\pi}{3}\left(R^3 - r^3\right)$

$\dfrac{3V}{\pi} = R^3 - r^3$

$r^3 = R^3 - \dfrac{3V}{\pi}$

$\mathbf{r = \sqrt[3]{R^3 - \dfrac{3V}{\pi}}}$

7. $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$2ax = -b \pm \sqrt{b^2 - 4ac}$

$2ax + b = \pm\sqrt{b^2 - 4ac}$

$(2ax + b)^2 = b^2 - 4ac$

$4a^2x^2 + 4abx + b^2 = b^2 - 4ac$

$4a^2x^2 + 4abx + 4ac = 0$

$4a(ax^2 + bx + c) = 0$

Since $a \neq 0$, then $ax^2 + bx + c = 0$. (shown)

8. $\dfrac{4320}{v} + \dfrac{720}{v-1} = 1800$

$4320(v - 1) + 720v = 1800v(v - 1)$

$4320v - 4320 + 720v = 1800v^2 - 1800v$

$1800v^2 - 6840v + 4320 = 0$

$5v^2 - 19v + 12 = 0$

$(5v - 4)(v - 3) = 0$

$v = \dfrac{4}{5}$ or $v = 3$

$\therefore v = \mathbf{3}$

2 Quadratic Equations and Graphs

Worksheet 2A Solving quadratic equations by factorisation

1. **(a)** $x(x-7)=0$

$x=0$ or $x-7=0$

$x=7$

$\therefore x=\mathbf{0}$ or $x=\mathbf{7}$

(b) $8x(x+9)=0$

$8x=0$ or $x+9=0$

$x=0$ $\quad x=-9$

$\therefore x=\mathbf{0}$ or $x=\mathbf{-9}$

(c) $(x-6)(5x+1)=0$

$x-6=0$ or $5x+1=0$

$x=6$ $\quad 5x=-1$

$x=-\frac{1}{5}$

$\therefore x=\mathbf{6}$ or $x=\mathbf{-\frac{1}{5}}$

(d) $(4x+1)(x-2)=0$

$4x+1=0$ or $x-2=0$

$4x=-1$ $\quad x=2$

$x=-\frac{1}{4}$

$\therefore x=\mathbf{-\frac{1}{4}}$ or $x=\mathbf{2}$

(e) $7(8-3x)(x-4)=0$

$8-3x=0$ or $x-4=0$

$-3x=-8$ $\quad x=4$

$x=2\frac{2}{3}$

$\therefore x=\mathbf{2\frac{2}{3}}$ or $x=\mathbf{4}$

(f) $\frac{2}{5}(6x+1)(5+9x)=0$

$(6x+1)(5+9x)=0$

$6x+1=0$ or $5+9x=0$

$6x=-1$ $\quad 9x=-5$

$x=-\frac{1}{6}$ $\quad x=-\frac{5}{9}$

$\therefore x=\mathbf{-\frac{1}{6}}$ or $x=\mathbf{-\frac{5}{9}}$

(g) $(2x-7)^2=0$

$2x-7=0$

$2x=7$

$x=3\frac{1}{2}$

$\therefore x=\mathbf{3\frac{1}{2}}$

(h) $\frac{5}{8}(11-4x)^2=0$

$11-4x=0$

$-4x=-11$

$x=2\frac{3}{4}$

$\therefore x=\mathbf{2\frac{3}{4}}$

(i) $x^2-5x=0$

$x(x-5)=0$

$x=0$ or $x-5=0$

$x=5$

$\therefore x=\mathbf{0}$ or $x=\mathbf{5}$

(j) $3x^2+12x=0$

$3x(x+4)=0$

$3x=0$ or $x+4=0$

$x=0$ $\quad x=-4$

$\therefore x=\mathbf{0}$ or $x=\mathbf{-4}$

(k) $-x-4x^2=0$

$-x(1+4x)=0$

$-x=0$ or $1+4x=0$

$x=0$ $\quad 4x=-1$

$x=-\frac{1}{4}$

$\therefore x=\mathbf{0}$ or $x=\mathbf{-\frac{1}{4}}$

(l) $10x-\frac{1}{2}x^2=0$

$20x-x^2=0$

$x(20-x)=0$

$x=0$ or $20-x=0$

$x=20$

$\therefore x=\mathbf{0}$ or $x=\mathbf{20}$

(m) $7x^2=21x$

$7x^2-21x=0$

$7x(x-3)=0$

$7x=0$ or $x-3=0$

$x=0$ $\quad x=3$

$\therefore x=\mathbf{0}$ or $x=\mathbf{3}$

(n) $x=\frac{1}{4}x^2$

$4x=x^2$

$4x-x^2=0$

$x(4-x)=0$

$x=0$ or $4-x=0$

$x=4$

$\therefore x=\mathbf{0}$ or $x=\mathbf{4}$

2. **(a)** $x^2+7x+12=0$

$(x+3)(x+4)=0$

$x+3=0$ or $x+4=0$

$x=-3$ $\quad x=-4$

$\therefore x=\mathbf{-3}$ or $x=\mathbf{-4}$

(b) $x^2+12x+36=0$

$(x+6)^2=0$

$x+6=0$

$x=-6$

$\therefore x=\mathbf{-6}$

(c) $x^2-9x+20=0$

$(x-4)(x-5)=0$

$x-4=0$ or $x-5=0$

$x=4$ $\quad x=5$

$\therefore x=\mathbf{4}$ or $x=\mathbf{5}$

(d) $x^2-16x+64=0$

$(x-8)^2=0$

$x-8=0$

$x=8$

$\therefore x=\mathbf{8}$

(e) $x^2 + 11x + 30 = 0$
$(x + 5)(x + 6) = 0$
$x + 5 = 0$ or $x + 6 = 0$
$x = -5$ $\quad$ $x = -6$
$\therefore x = \mathbf{-5}$ or $x = \mathbf{-6}$

(f) $x^2 - 14x + 24 = 0$
$(x - 2)(x - 12) = 0$
$x - 2 = 0$ or $x - 12 = 0$
$x = 2$ $\quad$ $x = 12$
$\therefore x = \mathbf{2}$ or $x = \mathbf{12}$

(g) $x^2 + 4x - 21 = 0$
$(x + 7)(x - 3) = 0$
$x + 7 = 0$ or $x - 3 = 0$
$x = -7$ $\quad$ $x = 3$
$\therefore x = \mathbf{-7}$ or $x = \mathbf{3}$

(h) $x^2 - 5x - 36 = 0$
$(x - 9)(x + 4) = 0$
$x - 9 = 0$ or $x + 4 = 0$
$x = 9$ $\quad$ $x = -4$
$\therefore x = \mathbf{9}$ or $x = \mathbf{-4}$

(i) $5x^2 + 12x + 4 = 0$
$(5x + 2)(x + 2) = 0$
$5x + 2 = 0$ or $x + 2 = 0$
$5x = -2$ $\quad$ $x = -2$
$x = -\frac{2}{5}$
$\therefore x = \mathbf{-\frac{2}{5}}$ or $x = \mathbf{-2}$

(j) $3x^2 - 11x + 8 = 0$
$(3x - 8)(x - 1) = 0$
$3x - 8 = 0$ or $x - 1 = 0$
$3x = 8$ $\quad$ $x = 1$
$x = 2\frac{2}{3}$
$\therefore x = \mathbf{2\frac{2}{3}}$ or $x = \mathbf{1}$

(k) $8x^2 + 10x - 7 = 0$
$(4x + 7)(2x - 1) = 0$
$4x + 7 = 0$ or $2x - 1 = 0$
$4x = -7$ $\quad$ $2x = 1$
$x = -1\frac{3}{4}$ $\quad$ $x = \frac{1}{2}$
$\therefore x = \mathbf{-1\frac{3}{4}}$ or $x = \mathbf{\frac{1}{2}}$

(l) $9x^2 - 15x - 50 = 0$
$(3x - 10)(3x + 5) = 0$
$3x - 10 = 0$ or $3x + 5 = 0$
$3x = 10$ $\quad$ $3x = -5$
$x = 3\frac{1}{3}$ $\quad$ $x = -1\frac{2}{3}$
$\therefore x = \mathbf{3\frac{1}{3}}$ or $x = \mathbf{-1\frac{2}{3}}$

(m) $3x^2 + 27x + 24 = 0$
$x^2 + 9x + 8 = 0$
$(x + 1)(x + 8) = 0$
$x + 1 = 0$ or $x + 8 = 0$
$x = -1$ $\quad$ $x = -8$
$\therefore x = \mathbf{-1}$ or $x = \mathbf{-8}$

(n) $5x^2 + 50x + 125 = 0$
$x^2 + 10x + 25 = 0$
$(x + 5)^2 = 0$
$x + 5 = 0$
$x = -5$
$\therefore x = \mathbf{-5}$

(o) $2x^2 - 12x + 16 = 0$
$x^2 - 6x + 8 = 0$
$(x - 2)(x - 4) = 0$
$x - 2 = 0$ or $x - 4 = 0$
$x = 2$ $\quad$ $x = 4$
$\therefore x = \mathbf{2}$ or $x = \mathbf{4}$

(p) $24x^2 - 72x + 54 = 0$
$4x^2 - 12x + 9 = 0$
$(2x - 3)^2 = 0$
$2x - 3 = 0$
$2x = 3$
$x = 1\frac{1}{2}$
$\therefore x = \mathbf{1\frac{1}{2}}$

(q) $x^2 - 100 = 0$
$x^2 = 100$
$x = \pm 10$
$\therefore x = \mathbf{\pm 10}$

(r) $25 - 16x^2 = 0$
$-16x^2 = -25$
$x^2 = \frac{25}{16}$
$x = \pm\frac{5}{4}$
$\therefore x = \mathbf{\pm 1\frac{1}{4}}$

(s) $12x^2 - 243 = 0$
$12x^2 = 243$
$x^2 = \frac{81}{4}$
$x = \pm\frac{9}{2}$
$\therefore x = \mathbf{\pm 4\frac{1}{2}}$

(t) $36x^3 - 4x = 0$
$4x(9x^2 - 1) = 0$
$4x(3x + 1)(3x - 1) = 0$
$4x = 0$ or $3x + 1 = 0$ or $3x - 1 = 0$
$x = 0$ $\quad$ $3x = -1$ $\quad$ $3x = 1$
$x = -\frac{1}{3}$ $\quad$ $x = \frac{1}{3}$
$\therefore x = \mathbf{0}$, $x = \mathbf{-\frac{1}{3}}$ or $x = \mathbf{\frac{1}{3}}$

3. **(a)** $(x + 4)^2 = 1$
$x + 4 = 1$ or $x + 4 = -1$
$x = -3$ $\quad$ $x = -5$
$\therefore x = \mathbf{-3}$ or $x = \mathbf{-5}$

(b) $(10 - 3x)^2 - 25 = 0$

$(10 - 3x)^2 = 25$

$10 - 3x = 5$ or $10 - 3x = -5$

$-3x = -5$ $\quad$ $-3x = -15$

$x = 1\frac{2}{3}$ $\quad$ $x = 5$

$\therefore x = \mathbf{1\frac{2}{3}}$ or $x = \mathbf{5}$

(c) $3x^2 + 2 = \frac{25}{2}x$

$6x^2 + 4 = 25x$

$6x^2 - 25x + 4 = 0$

$(6x - 1)(x - 4) = 0$

$6x - 1 = 0$ or $x - 4 = 0$

$6x = 1$ $\quad$ $x = 4$

$x = \frac{1}{6}$

$\therefore x = \mathbf{\frac{1}{6}}$ or $x = \mathbf{4}$

(d) $x(5x - 2) = 16$

$5x^2 - 2x = 16$

$5x^2 - 2x - 16 = 0$

$(5x + 8)(x - 2) = 0$

$5x + 8 = 0$ or $x - 2 = 0$

$5x = -8$ $\quad$ $x = 2$

$x = -1\frac{3}{5}$

$\therefore x = \mathbf{-1\frac{3}{5}}$ or $x = \mathbf{2}$

(e) $(x - 2)(x - 6) = 96$

$x^2 - 8x + 12 = 96$

$x^2 - 8x - 84 = 0$

$(x - 14)(x + 6) = 0$

$x - 14 = 0$ or $x + 6 = 0$

$x = 14$ $\quad$ $x = -6$

$\therefore x = \mathbf{14}$ or $x = \mathbf{-6}$

(f) $(4x - 1)(x - 1) = 8 - 2x$

$4x^2 - 5x + 1 = 8 - 2x$

$4x^2 - 3x - 7 = 0$

$(4x - 7)(x + 1) = 0$

$4x - 7 = 0$ or $x + 1 = 0$

$4x = 7$ $\quad$ $x = -1$

$x = 1\frac{3}{4}$

$\therefore x = \mathbf{1\frac{3}{4}}$ or $x = \mathbf{-1}$

(g) $\frac{2}{9}(9x + 17) = (x + 2)(x + 3)$

$2(9x + 17) = 9(x + 2)(x + 3)$

$18x + 34 = 9(x^2 + 5x + 6)$

$= 9x^2 + 45x + 54$

$9x^2 + 27x + 20 = 0$

$(3x + 5)(3x + 4) = 0$

$3x + 5 = 0$ or $3x + 4 = 0$

$3x = -5$ $\quad$ $3x = -4$

$x = -1\frac{2}{3}$ $\quad$ $x = -1\frac{1}{3}$

$\therefore x = \mathbf{-1\frac{2}{3}}$ or $x = \mathbf{-1\frac{1}{3}}$

(h) $5(7x^2 - 14x - 4) = (6x - 5)^2 + 4x$

$35x^2 - 70x - 20 = 36x^2 - 60x + 25 + 4x$

$x^2 + 14x + 45 = 0$

$(x + 9)(x + 5) = 0$

$x + 9 = 0$ or $x + 5 = 0$

$x = -9$ $\quad$ $x = -5$

$\therefore x = \mathbf{-9}$ or $x = \mathbf{-5}$

4. **(i)** $x^2 - 3x - 40 = \mathbf{(x - 8)(x + 5)}$

(ii) $x^2 - 3x - 40 = 0$

$(x - 8)(x + 5) = 0$

$x - 8 = 0$ or $x + 5 = 0$

$x = 8$ $\quad$ $x = -5$

$\therefore x = \mathbf{8}$ or $x = \mathbf{-5}$

5. **(i)** $10x^2 + 53x + 36 = 0$

$(5x + 4)(2x + 9) = 0$

$5x + 4 = 0$ or $2x + 9 = 0$

$5x = -4$ $\quad$ $2x = -9$

$x = -\frac{4}{5}$ $\quad$ $x = -4\frac{1}{2}$

$\therefore x = \mathbf{-\frac{4}{5}}$ or $x = \mathbf{-4\frac{1}{2}}$

(ii) $10(x - 1)^2 + 53(x - 1) + 36 = 0$

$x - 1 = -\frac{4}{5}$ or $x - 1 = -4\frac{1}{2}$

$x = \frac{1}{5}$ $\quad$ $x = -3\frac{1}{2}$

$\therefore x = \mathbf{\frac{1}{5}}$ or $x = \mathbf{-3\frac{1}{2}}$

6. **(a)** $(7x - 2)^2 = 9$

$7x - 2 = \pm 3$

$7x - 2 = 3$ or $7x - 2 = -3$

$7x = 5$ $\quad$ $7x = -1$

$x = \frac{5}{7}$ $\quad$ $x = -\frac{1}{7}$

$\therefore x = \mathbf{\frac{5}{7}}$ or $x = \mathbf{-\frac{1}{7}}$

(b) $(7x - 2)^2 = 9$

$(7x - 2)^2 - 3^2 = 0$

$(7x - 2 + 3)(7x - 2 - 3) = 0$

$(7x + 1)(7x - 5) = 0$

$7x + 1 = 0$ or $7x - 5 = 0$

$7x = -1$ $\quad$ $7x = 5$

$x = -\frac{1}{7}$ $\quad$ $x = \frac{5}{7}$

$\therefore x = \mathbf{-\frac{1}{7}}$ or $x = \mathbf{\frac{5}{7}}$

7. **(a)** $kx^2 + (3k + 1)x - 8 = 0$

When $x = -4$,

$k(-4)^2 + (3k + 1)(-4) - 8 = 0$

$16k - 12k - 4 - 8 = 0$

$4k = 12$

$k = 3$

$\therefore k = \mathbf{3}$

(b) $3x^2 + 10x - 8 = 0$

$(3x - 2)(x + 4) = 0$

$3x - 2 = 0$ or $x + 4 = 0$

$3x = 2$ $\quad$ $x = -4$

$x = \frac{2}{3}$

$\therefore$ The second possible value of x is $\mathbf{\frac{2}{3}}$.

8. $42(x^2 + y^2) = 85xy$

$42x^2 - 85xy + 42y^2 = 0$

$(7x - 6y)(6x - 7y) = 0$

$7x = 6y$ or $6x = 7y$

$\frac{y}{x} = \frac{7}{6}$ $\quad\quad \frac{y}{x} = \frac{6}{7}$

$\therefore \frac{y}{x} = \mathbf{\frac{7}{6}}$ or $\frac{y}{x} = \mathbf{\frac{6}{7}}$

9. Let x be the smaller number.

$x^2 + (x + 2)^2 = 802$

$x^2 + x^2 + 4x + 4 = 802$

$2x^2 + 4x - 798 = 0$

$x^2 + 2x - 399 = 0$

$(x - 19)(x + 21) = 0$

$x - 19 = 0$ or $x + 21 = 0$

$x = 19$ $\quad\quad x = -21$

$\therefore$ Square of the sum $= (19 + 21)^2$

$= \mathbf{1600}$

10. Let the numbers be x and $x + 1$.

$x(x + 1) = 6 + 6(x + x + 1)$

$x^2 + x = 6 + 6(2x + 1)$

$= 6 + 12x + 6$

$= 12x + 12$

$x^2 - 11x - 12 = 0$

$(x - 12)(x + 1) = 0$

$x - 12 = 0$ or $x + 1 = 0$

$x = 12$ $\quad\quad x = -1$

$\therefore$ The integers are **12** and **13**.

11. Let $10x + y$ represent the number.

$y = x + 6$ — (1)

$10x + y = 12 + xy$ — (2)

Substitute (1) into (2):

$10x + x + 6 = 12 + x(x + 6)$

$11x + 6 = 12 + x^2 + 6x$

$x^2 - 5x + 6 = 0$

$(x - 2)(x - 3) = 0$

$x - 2 = 0$ or $x - 3 = 0$

$x = 2$ $\quad\quad x = 3$

$\therefore$ The original number is **28** or **39**.

12. Let the length of the rectangle be x cm.

Since the perimeter is 92 cm,

Breadth of rectangle $= (46 - x)$ cm

$x(46 - x) = 480$

$46x - x^2 = 480$

$x^2 - 46x + 480 = 0$

$(x - 16)(x - 30) = 0$

$x - 16 = 0$ or $x - 30 = 0$

$x = 16$ $\quad\quad x = 30$

$\therefore$ The length and breadth are **30 cm** and **16 cm** respectively.

13. Let the length and breadth of the rectangle be $4x$ cm and $3x$ cm respectively.

$(4x)(3x) = 192$

$12x^2 = 192$

$x^2 = 16$

$x = \pm 4$

Length of rectangle = 4(4) = 16 cm

Breadth of rectangle = 3(4) = 12 cm

$\therefore$ Perimeter of rectangle = 2(16 + 12)

= **56 cm**

14. **(a)** $6(2x - 3) + (x + 3)^2 = 79$

$12x - 18 + x^2 + 6x + 9 = 79$

$x^2 + 18x - 88 = 0$ (shown)

(b) $x^2 + 18x - 88 = 0$

$(x + 22)(x - 4) = 0$

$x + 22 = 0$ or $x - 4 = 0$

$x = -22$ $\quad\quad x = 4$

$\therefore$ Total perimeter $= 2[6 + 2(4) - 3] + 4(4 + 3)$

= **50 m**

15. Let the width of the border be x cm.

$(91 - 2x)(61 - 2x) = 4399$

$5551 - 182x - 122x + 4x^2 = 4399$

$4x^2 - 304x + 1152 = 0$

$x^2 - 76x + 288 = 0$

$(x - 4)(x - 72) = 0$

$x - 4 = 0$ or $x - 72 = 0$

$x = 4$ $\quad\quad x = 72$

$\therefore$ The width of the border is **4 cm**.

16. Let the width of the deck be x m.

$(25 + 2x)(18.3 + 2x) = (25)(18.3) + 100.1$

$457.5 + 50x + 36.6x + 4x^2 = 557.6$

$4x^2 + 86.6x - 100.1 = 0$

$40x^2 + 866x - 1001 = 0$

$(10x - 11)(4x + 91) = 0$

$10x - 11 = 0$ or $4x + 91 = 0$

$10x = 11$ $\quad\quad 4x = -91$

$x = 1.1$ $\quad\quad x = -22.75$

$\therefore$ The width of the deck is **1.1 m**.

17. **(a)** Area of $ABCD = (2x + 1)(3x - 2)$

$= (6x^2 - x - 2)$ cm^2

Area of $PQRS = \frac{1}{2}(x + 2 + 5x - 8)(4x - 11)$ cm^2

$= \frac{1}{2}(6x - 6)(4x - 11)$ cm^2

$= (3x - 3)(4x - 11)$ cm^2

$= (12x^2 - 45x + 33)$ cm^2

$6x^2 - x - 2 = 2(12x^2 - 45x + 33)$

$= 24x^2 - 90x + 66$

$18x^2 - 89x + 68 = 0$ (shown)

(b) $18x^2 - 89x + 68 = 0$

$(18x - 17)(x - 4) = 0$

$18x - 17 = 0$ or $x - 4 = 0$

$18x = 17$ $\quad\quad x = 4$

$x = \frac{17}{18}$

$\therefore x = \mathbf{\frac{17}{18}}$ or $x = \mathbf{4}$

(c) $x = \frac{17}{18}$ must be rejected as the length cannot be a negative value.

(d) Perimeter of $ABCD = 2[(2x + 1) + (3x - 2)]$

$= 2(2x + 1 + 3x - 2)$

$= 2(5x - 1)$

$= 2[5(4) - 1]$

$= \mathbf{38\ cm}$

Challenge Myself!

18. (i) $(x + a)^2 + b = x^2 + 2ax + a^2 + b$

Comparing coefficients of x,

$2a = -7$

$a = -3.5$

Comparing constants,

$a^2 + b = 8$

$(-3.5)^2 + b = 8$

$b = 8 - (-3.5)^2$

$= -4.25$

$\therefore x^2 - 7x + 8 = \mathbf{(x - 3.5)^2 - 4.25}$

(ii) $x^2 - 7x + 8 = 0$

$(x - 3.5)^2 - 4.25 = 0$

$(x - 3.5)^2 = 4.25$

$x - 3.5 = \pm\sqrt{4.25}$

$x = 3.5 \pm \sqrt{4.25}$

$= 5.56$ or 1.44 (to 2 d.p.)

$\therefore x = \mathbf{5.56}$ or $x = \mathbf{1.44}$

Worksheet 2B Quadratic functions and graphs

1. (i) When $y = 0$,

$-(x + 3)(x - 1) = 0$

$x + 3 = 0$ or $x - 1 = 0$

$x = -3$ $\quad x = 1$

$\therefore A(-3, 0), B(1, 0)$

When $x = 0$,

$y = 3$

$\therefore C(0, 3)$

$\therefore \mathbf{A(-3, 0), B(1, 0), C(0, 3)}$

(ii) When $x = -1$,

$y = 4$

$\therefore$ Coordinates of maximum point are **(−1, 4)**

2. (i) When $y = 0$,

$(x - 4)(x + 2) = 0$

$x - 4 = 0$ or $x + 2 = 0$

$x = 4$ $\quad x = -2$

$\therefore A(-2, 0), B(4, 0)$

When $x = 0$,

$y = -8$

$\therefore C(0, -8)$

$\therefore \mathbf{A(-2, 0), B(4, 0), C(0, -8)}$

(ii) When $x = -3$, $y = k$,

$k = (-3 - 4)(-3 + 2)$

$= 7$

$\therefore k = \mathbf{7}$

(iii) When $x = 3.5$,

$y = (3.5 - 4)(3.5 + 2)$

$= -2.75$

$\neq -2.5$

$\therefore Q$ does not lie on the curve.

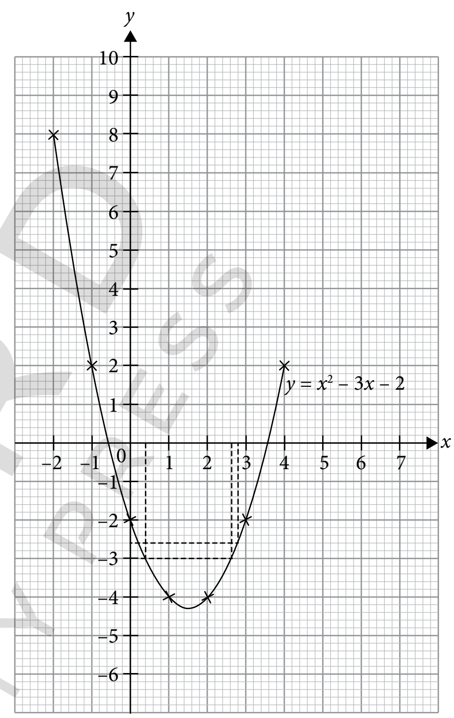
3. (a) When $x = -2$, $y = p$,

$p = (-2)^2 - 3(-2) - 2$

$= 8$

$\therefore p = \mathbf{8}$

(b)



(c) (i) When $x = 2.8$,

$y = \mathbf{-2.6}$.

(ii) When $y = -3$,

$x = \mathbf{0.4}$ or $x = \mathbf{2.6}$.

(iii) The minimum value of y is **−4.3** and this occurs when $x = \mathbf{1.5}$.

(d) $\mathbf{x = 1.5}$

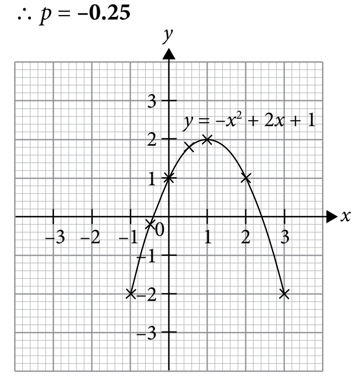
4. (a) When $x = -0.5$, $y = p$,

$p = -(-0.5)^2 + 2(-0.5) + 1$

$= -0.25$

$\therefore p = \mathbf{-0.25}$

(b)



(c) (i) When $x = -0.7$,

$y = \mathbf{-0.95}$.

(ii) When $y = 0.5$,

$x = \mathbf{-0.2}$ or $x = \mathbf{2.2}$.

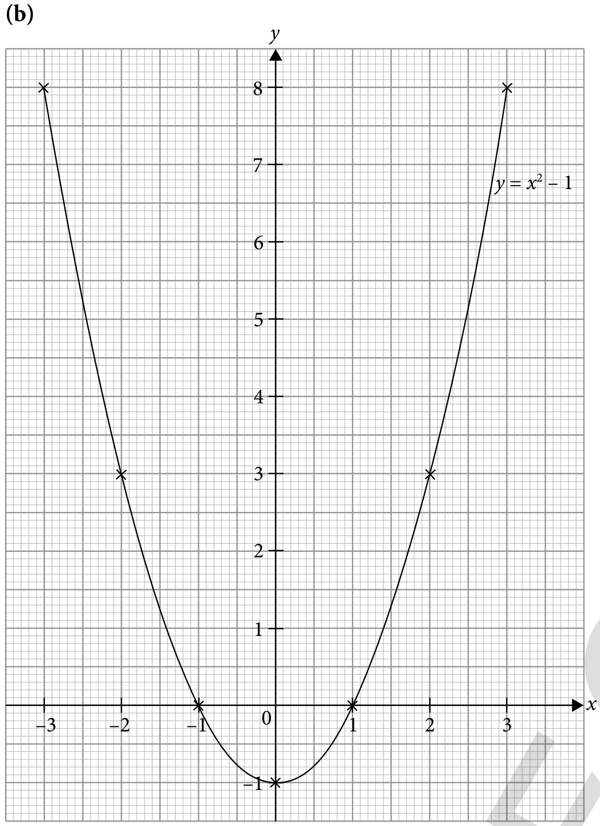
(iii) The maximum value of y is **2** and this occurs when $x = \mathbf{1}$.

(d) $x = 1$

5. (a)

x	–3	–2	–1	0	1	2	3
$y = x^2 - 1$	**8**	**3**	**0**	**–1**	**0**	**3**	**8**

(b)



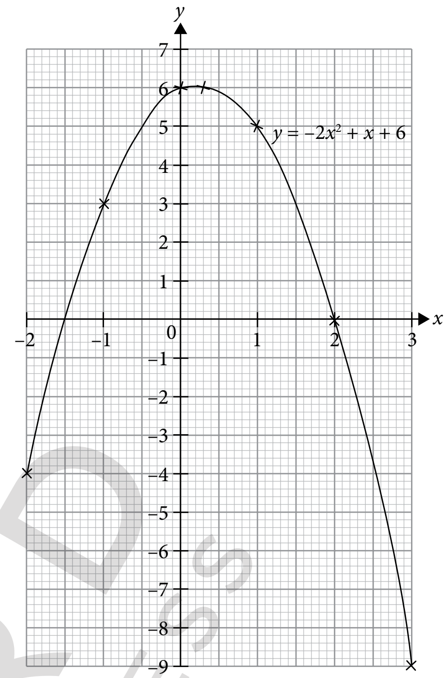
(c) (i) **(0, –1)**

(ii) $x = \mathbf{0}$

6. (a)

x	–2	–1	0	0.5	1	2	3
$y = -2x^2 + x + 6$	**–4**	**3**	**6**	**6**	**5**	**0**	**–9**

(b)



(c) (i) **(0.25, 6.1), maximum point**

(ii) $x = \mathbf{0.25}$

7. (a) Since the perimeter is 70 m,

Breadth of enclosure = $(35 - x)$ m

$\therefore A = x(35 - x)$ (shown)

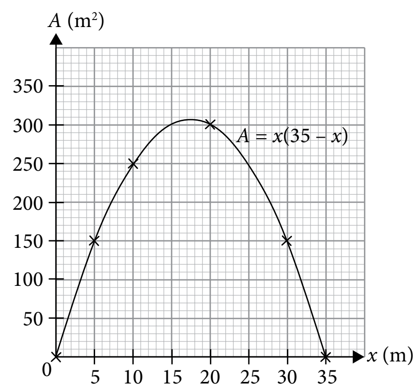
(b) (i) When $x = 20$, $A = p$,

$p = 20(35 - 20)$

$= 300$

$\therefore p = \mathbf{300}$

(ii)



(c) (i) $x = \mathbf{17.5}$

(ii) Maximum value of A = **304**

8. (a) (i) $-4t^2 + 12t = 0$

$4t(-t + 3) = 0$

$4t = 0$ or $-t + 3 = 0$

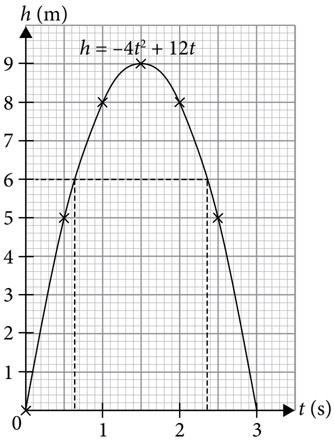
$t = 0$ $\quad$ $t = 3$

$\therefore t = \mathbf{0}$ or $t = \mathbf{3}$

(ii) Duration of flight = **3 s**

(b)

t	0	0.5	1	1.5	2	2.5	3
h	0	5	8	9	8	5	0

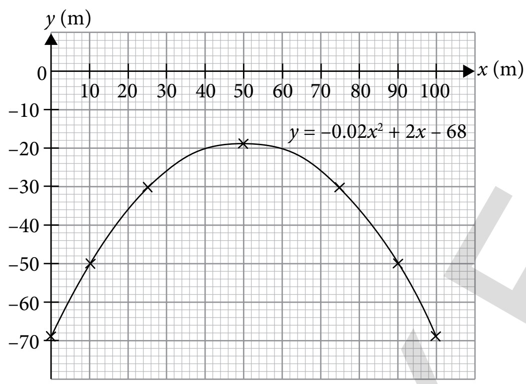


(c) (i) **9 m**

(ii) When $h = 6$,
$t =$ **0.65** or $t =$ **2.35**.

9\. (a)

x	0	10	25	50	75	90	100
y	−68	−50	−30.5	−18	−30.5	−50	−68



(b) (i) When $y = -68$, $x = 0$ or $x = 100$.
$\therefore$ They are located at the same depth.

(ii) Minimum height difference = **18 m**

Review Exercise 2

1\. (a) $y = x^2 + bx + c$ — (1)
Substitute $x = -1$ and $y = 8$ into (1):
$8 = (-1)^2 + b(-1) + c$
$b - c = -7$ — (2)
Substitute $x = 3$ and $y = -4$ into (1):
$-4 = 3^2 + b(3) + c$
$3b + c = -13$ — (3)
(2) + (3): $4b = -20$
$b = -5$
Substitute $b = -5$ into (2):
$-5 - c = -7$
$c = 2$
$\therefore$ $\boldsymbol{b = -5, c = 2}$

(b) When $x = 0$,
$y = 2$
$\therefore$ **(0, 2)**

2\. (a) $86x^2 = 0$
$x^2 = 0$
$x = 0$
$\therefore x = \mathbf{0}$

(b) $3 - \frac{1}{27}x^2 = 0$
$81 - x^2 = 0$
$x^2 = 81$
$x = \pm 9$
$\therefore x = \mathbf{\pm 9}$

(c) $12(x + 1)^2 = 17 - 4x$
$12(x^2 + 2x + 1) = 17 - 4x$
$12x^2 + 24x + 12 = 17 - 4x$
$12x^2 + 28x - 5 = 0$
$(6x - 1)(2x + 5) = 0$
$6x - 1 = 0$ or $2x + 5 = 0$
$6x = 1$ $\quad$ $2x = -5$
$x = \frac{1}{6}$ $\quad$ $x = -2\frac{1}{2}$
$\therefore x = \mathbf{\frac{1}{6}}$ or $x = \mathbf{-2\frac{1}{2}}$

(d) $x + 32 = \frac{1}{7}(x+7)(x-4)$
$7(x + 32) = (x + 7)(x - 4)$
$7x + 224 = x^2 + 3x - 28$
$x^2 - 4x - 252 = 0$
$(x - 18)(x + 14) = 0$
$x - 18 = 0$ or $x + 14 = 0$
$x = 18$ $\quad$ $x = -14$
$\therefore x =$ **18** or $x =$ **−14**

3\. (a) $7(x^2 + 3kx + 5) = (1 - k)x - 1$
When $x = 9$,
$7[9^2 + 3k(9) + 5] = (1 - k)(9) - 1$
$7(27k + 86) = 9 - 9k - 1$
$189k + 602 = 8 - 9k$
$198k = -594$
$k = -3$
$\therefore k =$ **−3**

(b) $7(x^2 - 9x + 5) = 4x - 1$
$7x^2 - 63x + 35 = 4x - 1$
$7x^2 - 67x + 36 = 0$
$(7x - 4)(x - 9) = 0$
$7x - 4 = 0$ or $x - 9 = 0$
$7x = 4$ $\quad$ $x = 9$
$x = \frac{4}{7}$
$\therefore$ The other solution is $x = \mathbf{\frac{4}{7}}$.

4\. (i) $8x^2 - 35x + 12 = 0$
$(8x - 3)(x - 4) = 0$
$8x - 3 = 0$ or $x - 4 = 0$
$8x = 3$ $\quad$ $x = 4$
$x = \frac{3}{8}$
$\therefore x = \mathbf{\frac{3}{8}}$ or $x =$ **4**

(ii) $8(x + 4)^2 - 35(x + 4) + 12 = 0$

$$x + 4 = \frac{3}{8} \quad \text{or} \quad x + 4 = 4$$

$$x = -3\frac{5}{8} \quad\quad x = 0$$

$\therefore x = \mathbf{-3\frac{5}{8}}$ or $x = \mathbf{0}$

5. Let x be the smallest number.

$$x^2 + (x + 2)^2 + (x + 4)^2 = 1208$$
$$x^2 + x^2 + 4x + 4 + x^2 + 8x + 16 = 1208$$
$$3x^2 + 12x - 1188 = 0$$
$$x^2 + 4x - 396 = 0$$
$$(x - 18)(x + 22) = 0$$
$$x - 18 = 0 \quad \text{or} \quad x + 22 = 0$$
$$x = 18 \quad\quad x = -22$$

$\therefore$ The three numbers are **18**, **20** and **22**.

6. Let $10x + y$ represent the number.

$y = x - 4$ — (1)

$(10x + y) + xy = 56$ — (2)

Substitute (1) into (2):

$$10x + x - 4 + x(x - 4) = 56$$
$$11x - 4 + x^2 - 4x = 56$$
$$x^2 + 7x - 60 = 0$$
$$(x + 12)(x - 5) = 0$$
$$x + 12 = 0 \quad \text{or} \quad x - 5 = 0$$
$$x = -12 \quad\quad x = 5$$

$\therefore$ The original number is **51**.

7. (a) Length = x cm

Height = $\mathbf{(x - 22)}$ **cm**

Width = $\mathbf{2(x - 22)}$ **cm**

(b) $x[2(x - 22)] + 2[x + 2(x - 22)](x - 22) = 1806$

$$2x(x - 22) + 2(x - 22)(3x - 44) = 1806$$
$$2x^2 - 44x + 2(3x^2 - 110x + 968) = 1806$$
$$2x^2 - 44x + 6x^2 - 220x + 1936 = 1806$$
$$8x^2 - 264x + 130 = 0$$
$$4x^2 - 132x + 65 = 0 \text{ (shown)}$$

(c) $4x^2 - 132x + 65 = 0$

$$(2x - 65)(2x - 1) = 0$$
$$2x - 65 = 0 \quad \text{or} \quad 2x - 1 = 0$$
$$2x = 65 \quad\quad 2x = 1$$
$$x = 32.5 \quad\quad x = 0.5$$

$\therefore x = \mathbf{32.5}$ or $x = \mathbf{0.5}$

(d) $x = 0.5$ has to be rejected as the height and the width of the container cannot be a negative value.

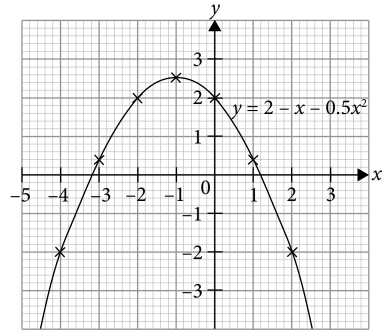
(e) Volume of container $= (32.5 \times 10.5 \times 21)$ cm^3

$= \mathbf{7166.25\ cm^3}$

8. (a)

x	−4	−3	−2	−1	0	1	2
$y = 2 - x - 0.5x^2$	**−2**	**0.5**	**2**	**2.5**	**2**	**0.5**	**−2**

(b)



(c) (i) **(−1, 2.5), maximum point**

(ii) $\mathbf{x = -1}$

3 Quadratic and Fractional Equations

Worksheet 3A Solving quadratic equations by completing the square

1. (a) $x^2 + 8x = (x + 4)^2 - 4^2$

$= \mathbf{(x + 4)^2 - 16}$

(b) $x^2 - 6x = (x - 3)^2 - 3^2$

$= \mathbf{(x - 3)^2 - 9}$

(c) $x^2 + 4x - 3 = (x + 2)^2 - 2^2 - 3$

$= \mathbf{(x + 2)^2 - 7}$

(d) $x^2 - 5x + 7 = \left(x - \frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2 + 7$

$= \mathbf{\left(x - \frac{5}{2}\right)^2 + \frac{3}{4}}$

(e) $x^2 + \frac{2}{9}x + 1 = \left(x + \frac{1}{9}\right)^2 - \left(\frac{1}{9}\right)^2 + 1$

$= \mathbf{\left(x + \frac{1}{9}\right)^2 + \frac{80}{81}}$

(f) $x^2 - 0.3x - 0.25 = (x - 0.15)^2 - 0.15^2 - 0.25$

$= \mathbf{(x - 0.15)^2 - 0.2725}$

2. (a) $-x^2 + 10x = -(x^2 - 10x)$

$= -[(x - 5)^2 - 5^2]$

$= -[(x - 5)^2 - 25]$

$= \mathbf{-(x - 5)^2 + 25}$

(b) $-x^2 - 4x = -(x^2 + 4x)$

$= -[(x + 2)^2 - 2^2]$

$= -[(x + 2)^2 - 4]$

$= \mathbf{-(x + 2)^2 + 4}$

(c) $-x - x^2 = -(x^2 + x)$

$= -\left[\left(x + \frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2\right]$

$= -\left[\left(x + \frac{1}{2}\right)^2 - \frac{1}{4}\right]$

$= \mathbf{-\left(x + \frac{1}{2}\right)^2 + \frac{1}{4}}$

(d) $9x - x^2 + 1 = -(x^2 - 9x) + 1$
$$= -\left[\left(x - \frac{9}{2}\right)^2 - \left(\frac{9}{2}\right)^2\right] + 1$$
$$= -\left[\left(x - \frac{9}{2}\right)^2 - \frac{81}{4}\right] + 1$$
$$= -\left(x - \frac{9}{2}\right)^2 + \frac{81}{4} + 1$$
$$= -\left(\boldsymbol{x} - \frac{\mathbf{9}}{\mathbf{2}}\right)^2 + \frac{\mathbf{85}}{\mathbf{4}}$$

3. (a) $(x + 2)^2 = 10$
$x + 2 = \pm\sqrt{10}$
$x = -2 \pm \sqrt{10}$
$= 1.16$ or -5.16 (to 2 d.p.)
$\therefore x = \mathbf{1.16}$ or $x = \mathbf{-5.16}$

(b) $(3x - 1)^2 = 16$
$3x - 1 = \pm 4$
$3x = 1 \pm 4$
$3x = 5$ or $3x = -3$
$x = 1\frac{2}{3}$ $\quad x = -1$
$\therefore x = \mathbf{1\frac{2}{3}}$ or $x = \mathbf{-1}$

4. (a) $x^2 + 6x = 7$
$(x + 3)^2 - 3^2 = 7$
$(x + 3)^2 = 16$
$x + 3 = \pm 4$
$x = -3 \pm 4$
$= 1$ or -7
$\therefore x = \mathbf{1}$ or $x = \mathbf{-7}$

(b) $x^2 - 10x - 9 = 0$
$(x - 5)^2 - 5^2 - 9 = 0$
$(x - 5)^2 = 34$
$x - 5 = \pm\sqrt{34}$
$x = 5 \pm \sqrt{34}$
$= 10.83$ or -0.83 (to 2 d.p.)
$\therefore x = \mathbf{10.83}$ or $x = \mathbf{-0.83}$

(c) $x^2 + \frac{1}{5}x + \frac{1}{200} = 0$
$$\left(x + \frac{1}{10}\right)^2 - \left(\frac{1}{10}\right)^2 + \frac{1}{200} = 0$$
$$\left(x + \frac{1}{10}\right)^2 = \frac{1}{200}$$
$$x + \frac{1}{10} = \pm\sqrt{\frac{1}{200}}$$
$$x = -\frac{1}{10} \pm \sqrt{\frac{1}{200}}$$
$= -0.03$ or -0.17 (to 2 d.p.)
$\therefore x = \mathbf{-0.03}$ or $x = \mathbf{-0.17}$

(d) $x^2 - 0.4x = 0.08$
$(x - 0.2)^2 - 0.2^2 = 0.08$
$(x - 0.2)^2 = 0.12$
$x - 0.2 = \pm\sqrt{0.12}$
$x = 0.2 \pm \sqrt{0.12}$
$= 0.55$ or -0.15 (to 2 d.p.)
$\therefore x = \mathbf{0.55}$ or $x = \mathbf{-0.15}$

(e) $3x(x + 3) = 25$
$$x^2 + 3x = \frac{25}{3}$$
$$\left(x + \frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 = \frac{25}{3}$$
$$\left(x + \frac{3}{2}\right)^2 = \frac{127}{12}$$
$$x + \frac{3}{2} = \pm\sqrt{\frac{127}{12}}$$
$$x = -\frac{3}{2} \pm \sqrt{\frac{127}{12}}$$
$= 1.75$ or -4.75 (to 2 d.p.)
$\therefore x = \mathbf{1.75}$ or $x = \mathbf{-4.75}$

(f) $\frac{1}{2}x^2 + 5 = (1 - x)^2$
$\frac{1}{2}x^2 + 5 = 1 - 2x + x^2$
$x^2 + 10 = 2 - 4x + 2x^2$
$x^2 - 4x = 8$
$(x - 2)^2 - 2^2 = 8$
$(x - 2)^2 = 12$
$x - 2 = \pm\sqrt{12}$
$x = 2 \pm \sqrt{12}$
$= 5.46$ or -1.46 (to 2 d.p.)
$\therefore x = \mathbf{5.46}$ or $x = \mathbf{-1.46}$

5. (a) $x^2 - 12x + 25 = (x - 6)^2 - 6^2 + 25$
$= (x - 6)^2 - 11$
$\therefore \boldsymbol{p} = \mathbf{-6}, \boldsymbol{q} = \mathbf{-11}$

(b) $x^2 - 12x + 25 = 0$
$(x - 6)^2 - 11 = 0$
$(x - 6)^2 = 11$
$x - 6 = \pm\sqrt{11}$
$x = 6 \pm \sqrt{11}$
$= 9.32$ or 2.68 (to 2 d.p.)
$\therefore x = \mathbf{9.32}$ or $x = \mathbf{2.68}$

6. (a) $x^2 + 8x - 7 = (x + 4)^2 - 4^2 - 7$
$= (x + 4)^2 - 23$
$\therefore \boldsymbol{a} = \mathbf{4}, \boldsymbol{b} = \mathbf{-23}$

(b) $x^2 + 8x - 7 = 0$
$(x + 4)^2 - 23 = 0$
$(x + 4)^2 = 23$
$x + 4 = \pm\sqrt{23}$
$x = -4 \pm \sqrt{23}$
$= 0.80$ or -8.80 (to 2 d.p.)
$\therefore x = \mathbf{0.80}$ or $x = \mathbf{-8.80}$

7. (a) $x^2 - 6x - 9 = (x - 3)^2 - 3^2 - 9$
$= (x - 3)^2 - 18$
$\therefore \boldsymbol{a} = \mathbf{-3}, \boldsymbol{b} = \mathbf{-18}$

(b) $x^2 - 6x - 9 = 0$
$(x - 3)^2 - 18 = 0$
$(x - 3)^2 = 18$
$x - 3 = \pm\sqrt{18}$
$x = 3 \pm \sqrt{18}$
$= 7.24$ or -1.24 (to 3 s.f.)
$\therefore x = \mathbf{7.24}$ or $x = \mathbf{-1.24}$

8. (i) $x^2 + 20x - 3 = 0$

$(x + 10)^2 - 10^2 - 3 = 0$

$(x + 10)^2 = 103$

$x + 10 = \pm\sqrt{103}$

$x = -10 \pm \sqrt{103}$

$= 0.149$ or -20.1 (to 3 s.f.)

$\therefore$ $x =$ **0.149** or $x =$ **−20.1**

(ii) $x^4 = 3 - 20x^2$

$x^4 + 20x^2 - 3 = 0$

From part **(i)**,

$x^2 = 0.148\ 89$ or $x^2 = -20.149$ (no solution)

$x = \pm 0.386$ (to 3 s.f.)

$\therefore$ There are **2** solutions.

9. $ax^2 + bx + c = 0$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

$$\left(x + \frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2 = -\frac{c}{a}$$

$$\left(x + \frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}$$

$$= \frac{b^2}{4a^2} - \frac{c}{a}$$

$$= \frac{b^2 - 4ac}{4a^2}$$

$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$= \pm\frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$\therefore$ The solutions are $\frac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $\frac{-b - \sqrt{b^2 - 4ac}}{2a}$. (shown)

Worksheet 3B Solving quadratic equations using formula

1. (a) $x^2 + 5x + 3 = 0$

$$x = \frac{-5 \pm \sqrt{5^2 - 4(1)(3)}}{2(1)}$$

$$= \frac{-5 \pm \sqrt{13}}{2}$$

$= -0.697$ or -4.30 (to 3 s.f.)

$\therefore$ $x =$ **−0.697** or $x =$ **−4.30**

(b) $6x^2 - 8x - 1 = 0$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(6)(-1)}}{2(6)}$$

$$= \frac{8 \pm \sqrt{88}}{12}$$

$= 1.45$ or -0.115 (to 3 s.f.)

$\therefore$ $x =$ **1.45** or $x =$ **−0.115**

(c) $2x^2 = 4x - 9$

$2x^2 - 4x + 9 = 0$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(9)}}{2(2)}$$

$$= \frac{4 \pm \sqrt{-56}}{4}$$

$\therefore$ There are **no real solutions**.

(d) $7 - x^2 = 12x$

$x^2 + 12x - 7 = 0$

$$x = \frac{-12 \pm \sqrt{12^2 - 4(1)(-7)}}{2(1)}$$

$$= \frac{-12 \pm \sqrt{172}}{2}$$

$= 0.557$ or -12.6 (to 3 s.f.)

$\therefore$ $x =$ **0.557** or $x =$ **−12.6**

(e) $8x + 10 = \frac{2}{5}x^2$

$40x + 50 = 2x^2$

$2x^2 - 40x - 50 = 0$

$x^2 - 20x - 25 = 0$

$$x = \frac{-(-20) \pm \sqrt{(-20)^2 - 4(1)(-25)}}{2(1)}$$

$$= \frac{20 \pm \sqrt{500}}{2}$$

$= 21.2$ or -1.18 (to 3 s.f.)

$\therefore$ $x =$ **21.2** or $x =$ **−1.18**

(f) $0.4x(9x + 5) = 3x - 1$

$3.6x^2 + 2x = 3x - 1$

$3.6x^2 - x + 1 = 0$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(3.6)(1)}}{2(3.6)}$$

$$= \frac{1 \pm \sqrt{-13.4}}{7.2}$$

$\therefore$ There are **no real solutions**.

(g) $3x(2 - x) = 10 - \frac{1}{2}x^2$

$6x - 3x^2 = 10 - \frac{1}{2}x^2$

$12x - 6x^2 = 20 - x^2$

$5x^2 - 12x + 20 = 0$

$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(5)(20)}}{2(5)}$$

$$= \frac{12 \pm \sqrt{-256}}{10}$$

$\therefore$ There are **no real solutions**.

(h) $(6x + 5)(5 - 6x) = (5x + 6)^2 - 15$

$25 - 36x^2 = 25x^2 + 60x + 36 - 15$

$61x^2 + 60x - 4 = 0$

$$x = \frac{-60 \pm \sqrt{60^2 - 4(61)(-4)}}{2(61)}$$

$$= \frac{-60 \pm \sqrt{4576}}{122}$$

$= 0.0627$ or -1.05 (to 3 s.f.)

$\therefore$ $x =$ **0.0627** or $x =$ **−1.05**

2. **(a)**
$$\begin{aligned} x^2 - 6x + 5 &= 0 \\ (x-3)^2 - 3^2 + 5 &= 0 \\ (x-3)^2 - 4 &= 0 \\ (x-3)^2 &= 4 \\ x - 3 &= \pm 2 \\ x = 3 + 2 \quad &\text{or} \quad x = 3 - 2 \\ = 5 \quad &\quad\quad\; = 1 \end{aligned}$$
$\therefore$ $x =$ **5** or $x =$ **1**

(b) $x^2 - 6x + 5 = 0$
$$\begin{aligned} x &= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(5)}}{2(1)} \\ &= \frac{6 \pm \sqrt{16}}{2} \\ &= 5 \text{ or } 1 \end{aligned}$$
$\therefore$ $x =$ **5** or $x =$ **1**

3. I do not agree with the student.

(i)
$$\begin{aligned} 7x - \frac{2}{3}x^2 &= 4 \\ 21x - 2x^2 &= 12 \\ 2x^2 - 21x + 12 &= 0 \\ x &= \frac{-(-21) \pm \sqrt{(-21)^2 - 4(2)(12)}}{2(2)} \\ &= \frac{21 \pm \sqrt{345}}{4} \\ &= 9.89 \text{ or } 0.606 \text{ (to 3 s.f.)} \end{aligned}$$
$\therefore$ $x =$ **9.89** or $x =$ **0.606**

(ii)
$$14x - \frac{8}{3}x^2 = 4$$
$$7(2x) - \frac{2}{3}(2x)^2 = 4$$
From part **(i)**,
$$\begin{aligned} 2x = 9.8935 \quad &\text{or} \quad 2x = 0.606\,46 \\ x = 4.95 \text{ (to 3 s.f.)} \quad &\quad\;\; x = 0.303 \text{ (to 3 s.f.)} \end{aligned}$$
$\therefore$ $x =$ **4.95** or $x =$ **0.303**

4.
$$\begin{aligned} 4x^2 - 3x &= 8 \\ 4x^2 - 3x - 8 &= 0 \\ x &= \frac{-(-3) \pm \sqrt{(-3)^2 - 4(4)(-8)}}{2(4)} \\ &= \frac{3 \pm \sqrt{137}}{8} \\ &= 1.84 \text{ or } -1.09 \text{ (to 3 s.f.)} \end{aligned}$$
$\therefore$ $x =$ **1.84** or $x =$ **−1.09**

5.

(a) $\mathbf{2x^2 + 10x - 1 = 0}$ has two real solutions.
(b) $\mathbf{x^2 + 6x + 9 = 0}$ has one real solution.
(c) $\mathbf{5x^2 - x + 4 = 0}$ has no real solutions.

6.

(a) A possible equation is $\mathbf{x^2 - 5x + 4 = 0}$.
$$\begin{aligned} x^2 - 5x + 4 &= 0 \\ (x-1)(x-4) &= 0 \\ x = 1 \quad &\text{or} \quad x = 4 \end{aligned}$$
$\therefore$ $x =$ **1** or $x =$ **4**

(b) A possible equation is $\mathbf{x^2 - 8x = 10}$.
$$\begin{aligned} x^2 - 8x &= 10 \\ (x-4)^2 - 16 &= 10 \\ (x-4)^2 &= 26 \\ x - 4 &= \pm\sqrt{26} \\ x &= 4 \pm \sqrt{26} \\ &= 9.10 \text{ or } -1.10 \text{ (to 3 s.f.)} \end{aligned}$$
$\therefore$ $x =$ **9.10** or $x =$ **−1.10**

(c) A possible equation is $\mathbf{10x^2 + 3x - 2 = 0}$.
$$\begin{aligned} 10x^2 + 3x - 2 &= 0 \\ x &= \frac{-3 \pm \sqrt{3^2 - 4(10)(-2)}}{2(10)} \\ &= \frac{-3 \pm \sqrt{89}}{20} \\ &= 0.322 \text{ or } -0.622 \text{ (to 3 s.f.)} \end{aligned}$$
$\therefore$ $x =$ **0.322** or $x =$ **−0.622**

7. Using Pythagoras' Theorem,
$$\begin{aligned} (5x-1)^2 + (x+1)^2 &= (4x+5)^2 \\ 25x^2 - 10x + 1 + x^2 + 2x + 1 &= 16x^2 + 40x + 25 \\ 10x^2 - 48x - 23 &= 0 \\ x &= \frac{-(-48) \pm \sqrt{(-48)^2 - 4(10)(-23)}}{2(10)} \\ &= \frac{48 \pm \sqrt{3224}}{20} \\ &= 5.24 \text{ or } -0.439 \text{ (to 3 s.f.)} \end{aligned}$$
$\therefore$ $x =$ **5.24**

Worksheet 3C Solving fractional equations reducible to quadratic equations

1. **(a)**
$$\begin{aligned} \frac{4x-1}{4} &= \frac{16}{4x-1} \\ (4x-1)^2 &= 64 \\ 4x - 1 &= \pm 8 \\ 4x &= 1 \pm 8 \\ 4x = 9 \quad &\text{or} \quad 4x = -7 \\ x = 2\frac{1}{4} \quad &\quad\quad\; x = -1\frac{3}{4} \end{aligned}$$
$\therefore$ $x = \mathbf{2\frac{1}{4}}$ or $x = \mathbf{-1\frac{3}{4}}$

(b)
$$\begin{aligned} x + \frac{10}{x} &= 11 \\ x^2 + 10 &= 11x \\ x^2 - 11x + 10 &= 0 \\ (x-1)(x-10) &= 0 \\ x = 1 \quad &\text{or} \quad x = 10 \end{aligned}$$
$\therefore$ $x =$ **1** or $x =$ **10**

(c)
$$\frac{20}{x-1} = 2x+3$$
$$(2x+3)(x-1) = 20$$
$$2x^2 + x - 3 = 20$$
$$2x^2 + x - 23 = 0$$
$$x = \frac{-1 \pm \sqrt{1^2 - 4(2)(-23)}}{2(2)}$$
$$= \frac{-1 \pm \sqrt{185}}{4}$$
$$= 3.15 \text{ or } -3.65 \text{ (to 3 s.f.)}$$
$\therefore x =$ **3.15** or $x =$ **−3.65**

(d)
$$6x = \frac{10x}{2-x} - 1$$
$$= \frac{10x - 2 + x}{2-x}$$
$$= \frac{11x-2}{2-x}$$
$$6x(2-x) = 11x - 2$$
$$12x - 6x^2 = 11x - 2$$
$$6x^2 - x - 2 = 0$$
$$(3x-2)(2x+1) = 0$$
$$x = \frac{2}{3} \quad \text{or} \quad x = -\frac{1}{2}$$
$\therefore x = \mathbf{\frac{2}{3}}$ or $x = \mathbf{-\frac{1}{2}}$

(e)
$$\frac{6}{x-5} + \frac{1}{x-3} = 7$$
$$6(x-3) + x - 5 = 7(x-5)(x-3)$$
$$6x - 18 + x - 5 = 7(x^2 - 8x + 15)$$
$$7x - 23 = 7x^2 - 56x + 105$$
$$7x^2 - 63x + 128 = 0$$
$$x = \frac{-(-63) \pm \sqrt{(-63)^2 - 4(7)(128)}}{2(7)}$$
$$= \frac{63 \pm \sqrt{385}}{14}$$
$$= 5.90 \text{ or } 3.10 \text{ (to 3 s.f.)}$$
$\therefore x =$ **5.90** or $x =$ **3.10**

(f)
$$\frac{4}{2x-1} - \frac{8}{2x+1} = \frac{5}{9}$$
$$36(2x+1) - 72(2x-1) = 5(2x+1)(2x-1)$$
$$72x + 36 - 144x + 72 = 5(4x^2 - 1)$$
$$108 - 72x = 20x^2 - 5$$
$$20x^2 + 72x - 113 = 0$$
$$x = \frac{-72 \pm \sqrt{72^2 - 4(20)(-113)}}{2(20)}$$
$$= \frac{-72 \pm \sqrt{14\ 224}}{40}$$
$$= 1.18 \text{ or } -4.78 \text{ (to 3 s.f.)}$$
$\therefore x =$ **1.18** or $x =$ **−4.78**

(g)
$$\frac{12}{4x^2 + 28x + 49} = \frac{9}{2x+7} - \frac{23}{27}$$
$$\frac{12}{(2x+7)^2} = \frac{9}{2x+7} - \frac{23}{27}$$
$$324 = 243(2x+7) - 23(2x+7)^2$$
$$= 486x + 1701 - 23(4x^2 + 28x + 49)$$
$$= 486x + 1701 - 92x^2 - 644x - 1127$$
$$92x^2 + 158x - 250 = 0$$
$$46x^2 + 79x - 125 = 0$$
$$x = \frac{-79 \pm \sqrt{79^2 - 4(46)(-125)}}{2(46)}$$
$$= \frac{-79 \pm \sqrt{29\ 241}}{92}$$
$$= 1 \text{ or } -2\frac{33}{46}$$
$\therefore x =$ **1** or $x = \mathbf{-2\frac{33}{46}}$

(h)
$$\frac{2}{3x^2 - 48} + \frac{5}{4-x} = 10$$
$$\frac{2}{3(x^2-16)} + \frac{5}{4-x} = 10$$
$$\frac{2}{3(x+4)(x-4)} - \frac{5}{x-4} = 10$$
$$2 - 15(x+4) = 30(x+4)(x-4)$$
$$2 - 15x - 60 = 30(x^2 - 16)$$
$$-15x - 58 = 30x^2 - 480$$
$$30x^2 + 15x - 422 = 0$$
$$x = \frac{-15 \pm \sqrt{15^2 - 4(30)(-422)}}{2(30)}$$
$$= \frac{-15 \pm \sqrt{50\ 865}}{60}$$
$$= 3.51 \text{ or } -4.01 \text{ (to 3 s.f.)}$$
$\therefore x =$ **3.51** or $x =$ **−4.01**

(i)
$$\frac{8}{2x+5} = \frac{11}{2x^2 + 9x + 10} - 1$$
$$= \frac{11 - 2x^2 - 9x - 10}{2x^2 + 9x + 10}$$
$$= \frac{1 - 9x - 2x^2}{(2x+5)(x+2)}$$
$$8(x+2) = 1 - 9x - 2x^2$$
$$8x + 16 = 1 - 9x - 2x^2$$
$$2x^2 + 17x + 15 = 0$$
$$(2x+15)(x+1) = 0$$
$$x = -7\frac{1}{2} \quad \text{or} \quad x = -1$$
$\therefore x = \mathbf{-7\frac{1}{2}}$ or $x =$ **−1**

(j)
$$\frac{4}{x+1} + \frac{5}{x} - \frac{9}{3x-1} = 0$$
$$4x(3x-1) + 5(x+1)(3x-1) - 9x(x+1) = 0$$
$$12x^2 - 4x + 5(3x^2 + 2x - 1) - 9x^2 - 9x = 0$$
$$12x^2 - 4x + 15x^2 + 10x - 5 - 9x^2 - 9x = 0$$
$$18x^2 - 3x - 5 = 0$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(18)(-5)}}{2(18)}$$

$$= \frac{3 \pm \sqrt{369}}{36}$$

$= 0.617$ or -0.450 (to 3 s.f.)

$\therefore x = \mathbf{0.617}$ or $x = \mathbf{-0.450}$

2. **(i)** $$\frac{8}{x-5} = x$$

$$8 = x^2 - 5x$$

$$x^2 - 5x - 8 = 0$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-8)}}{2(1)}$$

$$= \frac{5 \pm \sqrt{57}}{2}$$

$= 6.27$ or -1.27 (to 2 d.p.)

$\therefore x = \mathbf{6.27}$ or $x = \mathbf{-1.27}$

(ii) $$\frac{8}{\sqrt{x}-5} - \sqrt{x} = 0$$

$$\frac{8}{\sqrt{x}-5} = \sqrt{x}$$

From part **(i)**,

$\sqrt{x} = 6.2749$ or $\sqrt{x} = -1.2749$ (no solution)

$x = 39.4$ (to 3 s.f.)

$\therefore \mathbf{x = 39.4}$

3. Let the number be x.

$$x + \frac{1}{x} = \frac{85}{18}$$

$$18x^2 + 18 = 85x$$

$$18x^2 - 85x + 18 = 0$$

$$(9x - 2)(2x - 9) = 0$$

$$x = \frac{9}{2} \quad \text{or} \quad x = \frac{2}{9}$$

$\therefore$ The number and its reciprocal are $\frac{9}{2}$ and $\frac{2}{9}$ (in either order).

$$\text{Difference} = \frac{9}{2} - \frac{2}{9}$$

$$= \mathbf{\frac{77}{18}}$$

4. $$\frac{100x}{(9-5x)^2} = x$$

$$100x = x(9 - 5x)^2$$

$$100x - x(9 - 5x)^2 = 0$$

$$x[100 - (9 - 5x)^2] = 0$$

$x = 0$ or $(9 - 5x)^2 = 100$

$9 - 5x = \pm 10$

$5x = 9 \pm 10$

$5x = 19$ or $5x = -1$

$x = 3\frac{4}{5}$ $\quad x = -\frac{1}{5}$

$\therefore$ There are three values of x that satisfy the equation. Both Terry and Veron are incorrect.

Challenge Myself!

5. OPEN

(a) $$\frac{3}{x} + \frac{2}{ax+b} = 1 \quad \text{— (1)}$$

Substitute $x = 5$ into (1):

$$\frac{3}{5} + \frac{2}{5a+b} = 1$$

$$\frac{2}{5a+b} = \frac{2}{5}$$

$$5a + b = 5$$

Let $a = -\frac{1}{5}$: $5\left(-\frac{1}{5}\right) + b = 5$

$-1 + b = 5$

$b = 6$

$\therefore$ A possible pair of numbers is $\boldsymbol{a = -\frac{1}{5}}$ and $\boldsymbol{b = 6}$.

(b) Substitute $a = -\frac{1}{5}$, $b = 6$ into (1):

$$\frac{3}{x} + \frac{2}{6 - \frac{1}{5}x} = 1$$

$$3\left(6 - \frac{1}{5}x\right) + 2x = x\left(6 - \frac{1}{5}x\right)$$

$$18 - \frac{3}{5}x + 2x = 6x - \frac{1}{5}x^2$$

$$90 - 3x + 10x = 30x - x^2$$

$$x^2 - 23x + 90 = 0$$

$$(x - 5)(x - 18) = 0$$

$$x = 5 \quad \text{or} \quad x = 18$$

$\therefore$ The other solution is $x = \mathbf{18}$.

Worksheet 3D Solving quadratic equations by graphical method

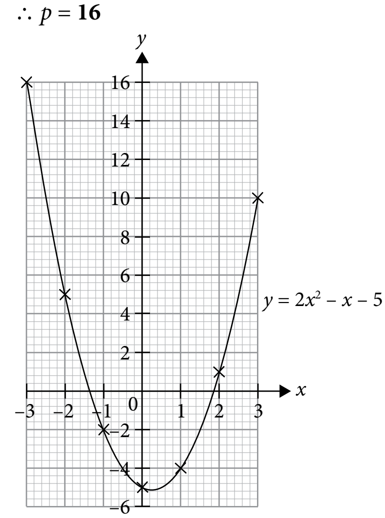
1. **(a)** When $x = -3$,

$y = 2(-3)^2 - (-3) - 5$

$= 16$

$\therefore p = \mathbf{16}$

(b)

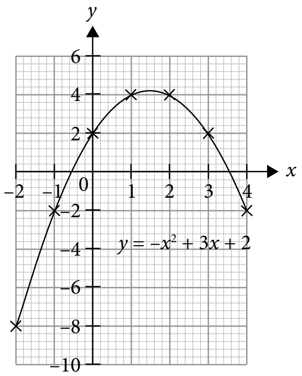


(c) From the graph, $x = \mathbf{-1.4}$ or $x = \mathbf{1.9}$.

2. **(a)**

x	−2	−1	0	1	2	3	4
$y = -x^2 + 3x + 2$	**−8**	**−2**	**2**	**4**	**4**	**2**	**−2**

(b)



(c) From the graph, $x = \mathbf{-0.6}$ or $x = \mathbf{3.6}$.

(d) $-x^2 + 3x + 3 = 0$

$-x^2 + 3x + 2 = -1$

Draw the line $y = -1$.

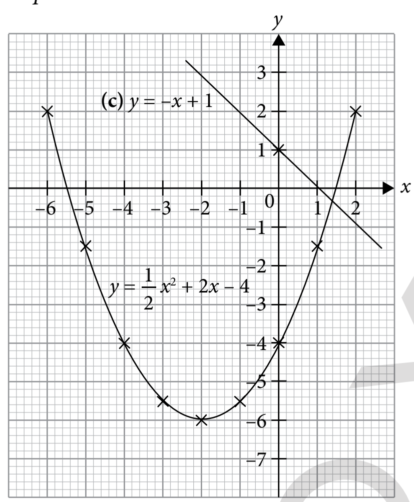
The solutions of the equation correspond to the x-coordinates of the points of intersection of the curve $y = -x^2 + 3x + 2$ and $y = -1$.

3. **(a)** When $x = -5$,

$$y = \frac{1}{2}(-5)^2 + 2(-5) - 4$$
$$= -1.5$$

$\therefore p = \mathbf{-1.5}$

(b)



(c) $x^2 + 6x - 10 = 0$

$$\frac{1}{2}x^2 + 3x - 5 = 0$$

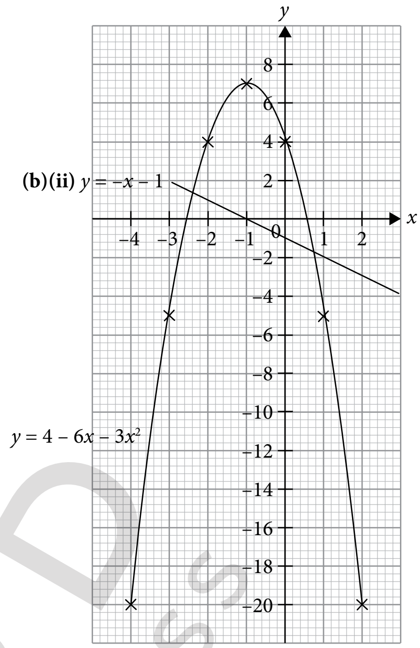
$$\frac{1}{2}x^2 + 2x - 4 = -x + 1$$

Draw $y = -x + 1$.

From the graph, $x = \mathbf{1.4}$.

4. **(a)**

x	−4	−3	−2	−1	0	1	2
y	−20	−5	4	7	4	−5	−20



(b) **(i)** From the graph, $x = \mathbf{-2.5}$ or $x = \mathbf{0.50}$.

(ii) $3x^2 = 5 - 5x$

$5 - 5x - 3x^2 = 0$

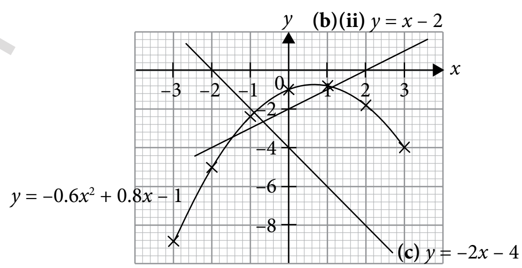
$4 - 6x - 3x^2 = -1 - x$

Draw $y = -x - 1$.

From the graph, $x = \mathbf{-2.4}$ or $x = \mathbf{0.70}$.

5. **(a)**

x	−3	−2	−1	0	1	2	3
y	−8.8	−5	−2.4	−1	−0.8	−1.8	−4



(b) **(i)** Since the graph of $y = -0.6x^2 + 0.8x - 1$ does not intersect the x-axis, the equation has no solution.

(ii) $-0.6x^2 = 0.2x - 1$

$-0.6x^2 + 0.8x - 1 = x - 2$

Draw $y = x - 2$.

From the graph, $x = \mathbf{-1.4}$ or $\mathbf{1.1}$.

(c) OPEN $-0.6x^2 + 0.8x - 1 = mx - 4$

Let $m = \mathbf{-2}$.

Draw $y = -2x - 4$.

From the graph, the solution is $x = \mathbf{-0.9}$.

Challenge Myself!

6. **(a)** **No**. The x-coordinates of the points where the graph of $y = 5x - x^2 + 4$ meets the x-axis give the solutions of the equation.

(b) $5x - x^2 + 4 = 0$

$x^2 - 5x = 4$

$\therefore k = \mathbf{4}$

(c) $5x - x^2 + 4 = 0$

$x^2 - 5x - 4 = 0$

$x^2 - 4x - 5 = x - 1$

Draw $\mathbf{y = x^2 - 4x - 5}$ and $\mathbf{y = x - 1}$.

$5x - x^2 + 4 = 0$

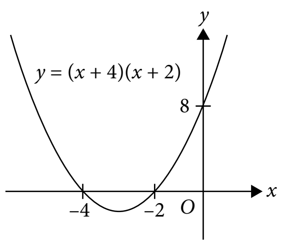
$x^2 - 5x - 4 = 0$

$x^2 - x - 7 = 4x - 3$

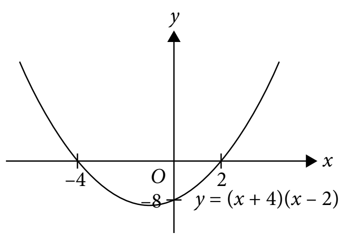
Draw $\mathbf{y = x^2 - x - 7}$ and $\mathbf{y = 4x - 3}$.

Worksheet 3E Sketching graphs of quadratic functions

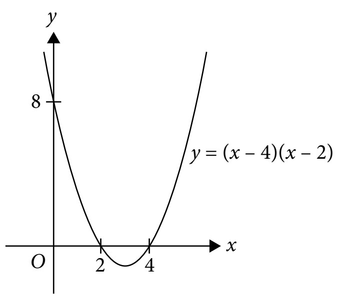
1. **(a)**



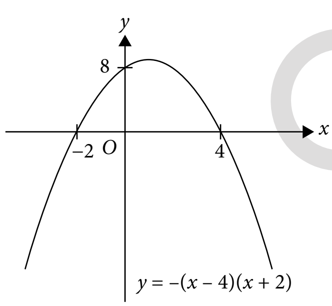
(b)



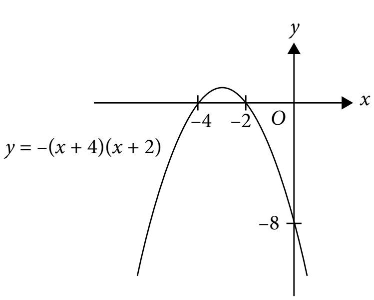
(c)



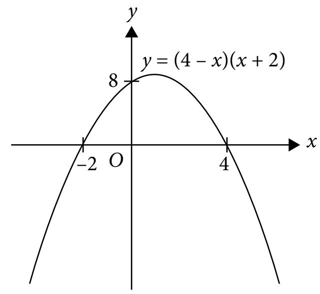
(d)



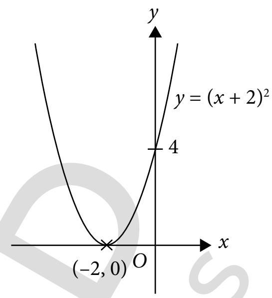
(e)



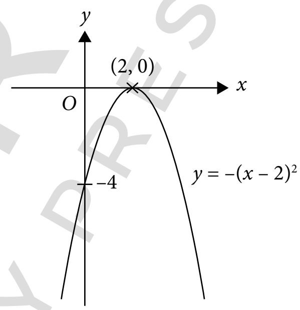
(f)



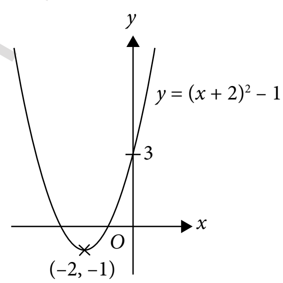
2. **(a)**



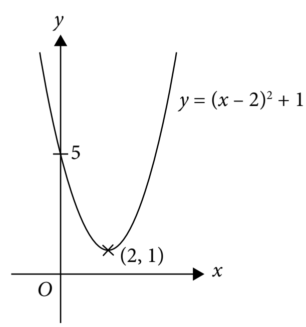
(b)



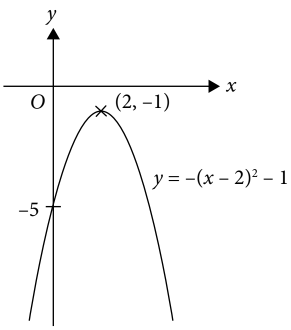
(c)



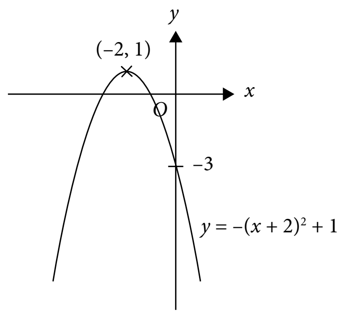
(d)



(e)

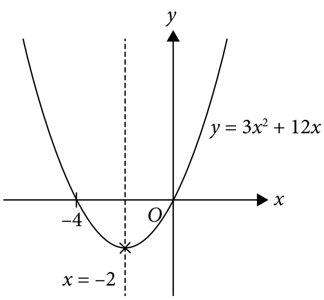


(f)



3. **(i)** $3x^2 + 12x = \mathbf{3x(x + 4)}$

(ii)



4. **(a)** Equation of line of symmetry: $x = \dfrac{-1+9}{2}$

$\mathbf{x = 4}$

(b) $y = (x + 1)(x - 9)$ — (1)

Substitute $x = 4$ into (1):

$y = (4 + 1)(4 - 9)$

$= -25$

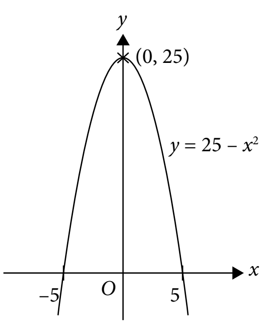
∴ Coordinates of the minimum point are **(4, –25)**

5. **(i)** **Maximum point**

(ii) **(0, 25)**

(iii) Equation of line of symmetry: $\mathbf{x = 0}$

(iv)



6.

(a) $\mathbf{y = (x + 3)^2 + 2}$

(b) Let the curve pass through (–4, 0) and (–2, 0).

$y = a(x + 4)(x + 2)$ — (1)

Substitute $x = -3$, $y = 2$ into (1):

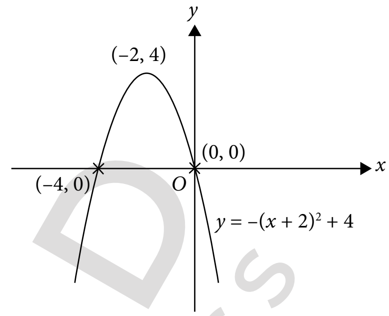
$2 = a(-3 + 4)(-3 + 2)$

$= -a$

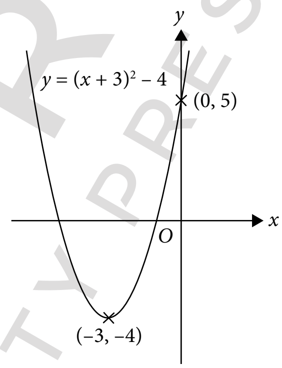
$a = -2$

∴ A possible equation is $\mathbf{y = -2(x + 4)(x + 2)}$.

7.



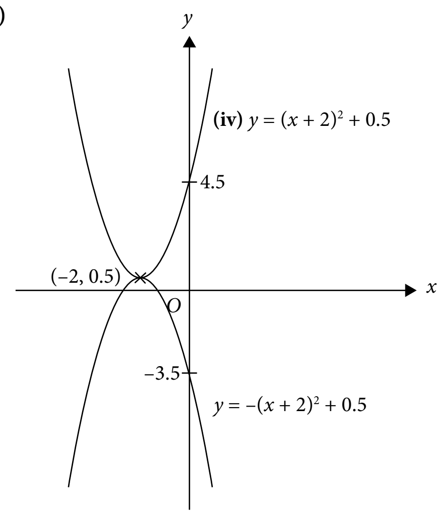
8.



9. **(i)** **Maximum point**

(ii) $\mathbf{h = 2, k = 0.5}$

(iii)-(iv)



10. **(i)** $11 - 6x + x^2 = x^2 - 6x + 11$

$= (x - 3)^2 - 3^2 + 11$

$= \mathbf{2 + (x - 3)^2}$

(ii) Coordinates of minimum point are **(3, 2)**

11. **(i)** $x^2 - 5x + 5 = (x - 2.5)^2 - 2.5^2 + 5$

$= \mathbf{(x - 2.5)^2 - 1.25}$

(ii) Minimum value = **–1.25**

(iii) Equation of line of symmetry: $\mathbf{x = 2.5}$

(iv) Substitute $x = -1$ into $y = x^2 - 5x + 5$:

$y = (-1)^2 - 5(-1) + 5$
$= 11$
$\neq 9$

$\therefore$ (–1, 9) does not lie on the graph.

Challenge Myself!

12. (a) Substitute $x = -1$ into $y = x^2 + x - 4$:

$y = (-1)^2 + (-1) - 4$
$= -4$

$\therefore$ (–1, 4) does not lie on $y = x^2 + x - 4$.

Substitute $x = -1$ into $y = -x^2 - x + 4$:

$y = -(-1)^2 - (-1) + 4$
$= 4$

$\therefore$ (–1, 4) lies on $y = -x^2 - x + 4$.

$\therefore$ Lauren is not correct.

(b) Let (–1, 4) be the coordinates of the maximum point.

The equation of the line of symmetry is $x = -1$.

By symmetry, **(–4, 0)** lies on the curve.

Let the equation of the curve be $y = a(x + 1)^2 + 4$.

Substitute $x = 2$, $y = 0$ into $y = a(x + 1)^2 + 4$:

$0 = 9a + 4$

$a = -\frac{4}{9}$

$\therefore$ A possible equation is $\mathbf{y = -\frac{4}{9}(x+1)^2 + 4}$.

Worksheet 3F Applications of quadratic equations and functions in real-world contexts

1. Let the initial number of children be x.

$$\frac{360}{x-6} - \frac{360}{x} = 2$$
$$360x - 360(x - 6) = 2x(x - 6)$$
$$360x - 360x + 2160 = 2x^2 - 12x$$
$$2x^2 - 12x - 2160 = 0$$
$$x^2 - 6x - 1080 = 0$$
$$(x - 36)(x + 30) = 0$$
$$x = 36 \quad \text{or} \quad x = -30$$

$\therefore$ There are **36** children initially.

2. Let the average speed of the Nozomi train be x km/h.

$$\frac{400}{x-30} - \frac{400}{x} = \frac{30}{60} = \frac{1}{2}$$
$$800x - 800(x - 30) = x(x - 30)$$
$$800x - 800x + 24\ 000 = x^2 - 30x$$
$$x^2 - 30x - 24\ 000 = 0$$
$$x = \frac{-(-30) \pm \sqrt{(-30)^2 - 4(1)(-24\ 000)}}{2(1)}$$
$$= \frac{30 \pm \sqrt{96\ 900}}{2}$$
$$= 171 \text{ or } -141 \text{ (to 3 s.f.)}$$

$\therefore$ The average speed of the Nozomi train is **171 km/h**.

3. (i) $320 \text{ cm}^2 = (320 \times 10^2) \text{ mm}^2$
$\mathbf{= 32\ 000\ mm^2}$

(ii) Let the width of the border be x mm.

$$(297 - 2x)(210 - 2x) = 32\ 000$$
$$62\ 370 - 594x - 420x + 4x^2 = 32\ 000$$
$$4x^2 - 1014x + 30\ 370 = 0$$
$$2x^2 - 507x + 15\ 185 = 0$$
$$x = \frac{-(-507) \pm \sqrt{(-507)^2 - 4(2)(15\ 185)}}{2(2)}$$
$$= \frac{507 \pm \sqrt{135\ 569}}{4}$$
$$= 219 \text{ (rejected) or } 34.7 \text{ (to 3 s.f.)}$$

$\therefore$ The width of the border is **34.7 mm**.

4. (a) Width = $\mathbf{(x - 4)}$ **cm**

Height = $\mathbf{(x + 1)}$ **cm**

(b) Total surface area = 480 cm²

$$2[x(x - 4) + x(x + 1) + (x - 4)(x + 1)] = 480$$
$$x^2 - 4x + x^2 + x + x^2 - 3x - 4 = 240$$
$$3x^2 - 6x - 4 = 240$$
$$3x^2 - 6x - 244 = 0 \text{ (shown)}$$

(c) $3x^2 - 6x - 244 = 0$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(3)(-244)}}{2(3)}$$
$$= 10.07 \text{ (to 2 d.p.) or } -8.07 \text{ (to 2 d.p.)}$$

$\therefore$ x = **10.07** or x = **–8.07**

(d) The length of the box cannot be a negative value, so $x = -8.07$ must be rejected.

(e) Volume of box $= x(x - 4)(x + 1)$
$= 10.074(10.074 - 4)(10.074 + 1)$
= **678 cm³** (to 3 s.f.)

5. (a) Speed for first 30 km $= \frac{30}{x \div 60}$
$= \mathbf{\frac{1800}{x}}$ **km/h**

(b) Speed for next 30 km $= \frac{30}{(x-5) \div 60}$
$= \mathbf{\frac{1800}{x-5}}$ **km/h**

(c)
$$\frac{1800}{x-5} - \frac{1800}{x} = 8$$
$$1800x - 1800(x - 5) = 8x(x - 5)$$
$$1800x - 1800x + 9000 = 8x^2 - 40x$$
$$8x^2 - 40x - 9000 = 0$$
$$x^2 - 5x - 1125 = 0 \text{ (shown)}$$

(d) $x^2 - 5x - 1125 = 0$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-1125)}}{2(1)}$$
$$= 36.13 \text{ (to 2 d.p.) or } -31.13 \text{ (to 2 d.p.)}$$

$\therefore$ x = **36.13** or x = **–31.13**

(e) Total time taken = $(2x - 5)$ min

Average speed $= \frac{75}{(2x-5) \div 60}$
$= \frac{4500}{2(36.13) - 5}$
= **66.9 km/h** (to 3 s.f.)

6. (a) Amount of water $= \frac{80}{100} \times 4000$
$= 3200$ litres

Number of minutes $= \mathbf{\frac{3200}{x}}$

(b) Number of minutes $= \mathbf{\frac{3200}{x-8}}$

(c) $$\frac{3200}{x-8} - \frac{3200}{x} = 18$$
$$3200x - 3200(x-8) = 18x(x-8)$$
$$3200x - 3200x + 25\ 600 = 18x^2 - 144x$$
$$18x^2 - 144x - 25\ 600 = 0$$
$$9x^2 - 72x - 12\ 800 = 0 \text{ (shown)}$$

(d) $9x^2 - 72x - 12\ 800 = 0$
$$x = \frac{-(-72) \pm \sqrt{(-72)^2 - 4(9)(-12\ 800)}}{2(9)}$$
$= 41.92$ (to 2 d.p.) or -33.92 (to 2 d.p.)

$\therefore x = \mathbf{41.92}$ or $x = \mathbf{-33.92}$

(e) Rate of both pumps $= (2x - 8)$ litres per minute

Time taken $= \frac{3200}{2(41.9239) - 8}$
$= 42.190$ min (to 5 s.f.)
$=$ **42 min 10 s** (to the nearest 10 seconds)

Review Exercise 3

1. (a) B (b) C
(c) A (d) E
(e) F (f) D

2. (a) $x^2 + 9x = \mathbf{(x + 4.5)^2 - 20.25}$

(b) $x^2 - 8x + 1 = (x - 4)^2 - 16 + 1$
$= \mathbf{(x - 4)^2 - 15}$

3. (a) $$x^2 + 6x = 50$$
$$(x + 3)^2 - 9 = 50$$
$$(x + 3)^2 = 59$$
$$x + 3 = \pm\sqrt{59}$$
$$x = -3 \pm \sqrt{59}$$
$= 4.68$ or -10.68 (to 2 d.p.)

$\therefore x = \mathbf{4.68}$ or $x = \mathbf{-10.68}$

(b) $$2 - 0.5x^2 = x$$
$$4 - x^2 = 2x$$
$$x^2 + 2x = 4$$
$$(x + 1)^2 - 1 = 4$$
$$(x + 1)^2 = 5$$
$$x + 1 = \pm\sqrt{5}$$
$$x = -1 \pm \sqrt{5}$$
$= 1.24$ or -3.24 (to 2 d.p.)

$\therefore x = \mathbf{1.24}$ or $x = \mathbf{-3.24}$

(c) $$4x - 7 = \frac{2}{3}x^2$$
$$12x - 21 = 2x^2$$
$$2x^2 - 12x = -21$$
$$x^2 - 6x = -10.5$$
$$(x - 3)^2 - 9 = -10.5$$
$$(x - 3)^2 = -1.5$$
$\therefore$ There are **no real solutions**.

(d) $$3x(6x + 1) = 7$$
$$18x^2 + 3x = 7$$
$$x^2 + \frac{1}{6}x = \frac{7}{18}$$
$$\left(x + \frac{1}{2}\right)^2 - \frac{1}{144} = \frac{7}{18}$$
$$\left(x + \frac{1}{2}\right)^2 = \frac{19}{48}$$
$$x + \frac{1}{12} = \pm\sqrt{\frac{19}{48}}$$
$$x = -\frac{1}{12} \pm \sqrt{\frac{19}{48}}$$
$= 0.55$ or -0.71 (to 2 d.p.)

$\therefore x = \mathbf{0.55}$ or $x = \mathbf{-0.71}$

4. (a) $$x^2 + 6x = 50$$
$$x^2 + 6x - 50 = 0$$
$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(-50)}}{2(1)}$$
$$= \frac{-1 \pm \sqrt{5}}{2}$$
$= 4.68$ or -10.68 (to 2 d.p.)

$\therefore x = \mathbf{4.68}$ or $x = \mathbf{-10.68}$

(b) $$2 - 0.5x^2 = x$$
$$0.5x^2 + x - 2 = 0$$
$$x = \frac{-1 \pm \sqrt{1^2 - 4(0.5)(-2)}}{2(0.5)}$$
$$= \frac{-1 \pm \sqrt{5}}{1}$$
$= 1.24$ or -3.24 (to 2 d.p.)

$\therefore x = \mathbf{1.24}$ or $x = \mathbf{-3.24}$

(c) $$4x - 7 = \frac{2}{3}x^2$$
$$\frac{2}{3}x^2 - 4x + 7 = 0$$
$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4\left(\frac{2}{3}\right)(7)}}{2\left(\frac{2}{3}\right)}$$
$$= \frac{4 \pm \sqrt{-\frac{2}{3}}}{\frac{2}{3}}$$
$\therefore$ There are **no real solutions**.

(d) $$3x(6x + 1) = 7$$
$$18x^2 + 3x = 7$$
$$18x^2 + 3x - 7 = 0$$
$$x = \frac{-3 \pm \sqrt{3^2 - 4(18)(-7)}}{2(18)}$$
$$= \frac{-3 \pm \sqrt{513}}{36}$$
$= 0.55$ or -0.71 (to 2 d.p.)

$\therefore x = \mathbf{0.55}$ or $x = \mathbf{-0.71}$

5. **(i)** $$\frac{8}{x}+4=\frac{9}{2x-1}$$
$$8(2x-1)+4x(2x-1)=9x$$
$$16x-8+8x^2-4x=9x$$
$$8x^2+11x-8=0$$
$$x=\frac{-11\pm\sqrt{11^2-4(8)(-8)}}{2(8)}$$
$$=\frac{-11\pm\sqrt{377}}{16}$$
$$=0.526 \text{ or } -1.901 \text{ (to 3 s.f.)}$$
$\therefore x = \mathbf{0.526}$ or $x = \mathbf{-1.901}$

(ii) $\frac{8}{x^2}+4=\frac{9}{2x^2-1}$

From part **(i)**,

$x^2 = 0.526\ 03$ (to 5 s.f.)

$x = \pm\sqrt{0.526\ 03}$

$= \pm 0.725$ (to 3 s.f.)

$\therefore x = \pm\ \mathbf{0.725}$

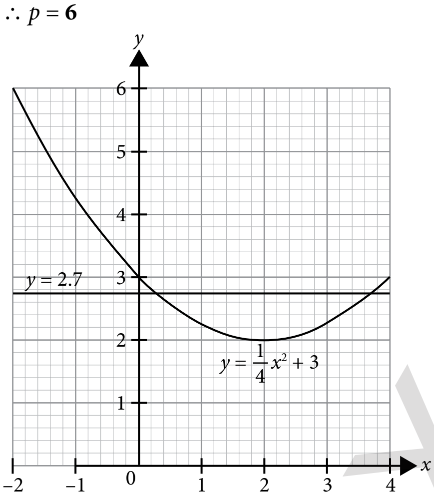
6. **(a)** When $x = -2$,

$y=\frac{1}{4}(-2)^2-(-2)+3$

$=6$

$\therefore p = \mathbf{6}$

(b)



(c) $-\frac{\pi}{4}$

(d) $x^2-4x+1.2=0$

$\frac{1}{4}x^2-x+0.3=0$

$\frac{1}{4}x^2-x+3=2.7$

Draw $y = 2.7$.

From the graph, $x = \mathbf{0.325}$ or $\mathbf{3.65}$.

7. Let the numbers be x, $x + 2$ and $x + 4$.
$$2[x^2+(x+2)^2+(x+4)^2]+1067=(x+x+2+x+4)^2$$
$$2(x^2+x^2+4x+4+x^2+8x+16)+1067=(3x+6)^2$$
$$2(3x^2+12x+20)+1067=9x^2+36x+36$$
$$6x^2+24x+40+1067=9x^2+36x+36$$
$$3x^2+12x-1071=0$$
$$x^2+4x-357=0$$
$$(x-17)(x+21)=0$$
$$x=17 \quad \text{or} \quad x=-21$$
$\therefore$ The numbers are −21, −19 and −17.

$\therefore$ Required number $= (-17)^3$

$= \mathbf{-4913}$

8. **(a)** $1.2\text{ m}^3 = 1.2 \times 100^3\text{ cm}^3$

$= 1\ 200\ 000\text{ cm}^3$

Number of minutes $= \mathbf{\frac{1\ 200\ 000}{x}}$

(b) Number of minutes $= \mathbf{\frac{1\ 200\ 000}{x+20\ 000}}$

(c) $$\frac{1\ 200\ 000}{x}-\frac{1\ 200\ 000}{x+20\ 000}=15\times 60$$
$$\frac{4000}{x}-\frac{4000}{x+20\ 000}=3$$
$$4000(x+20\ 000)-4000x=3x(x+20\ 000)$$
$$4000x+80\ 000\ 000-4000x=3x^2+60\ 000x$$
$$3x^2+60\ 000x-80\ 000\ 000=0 \text{ (shown)}$$

(d) $3x^2+60\ 000x-80\ 000\ 000=0$
$$x=\frac{-60\ 000\pm\sqrt{60\ 000^2-4(3)(-80\ 000\ 000)}}{2(3)}$$
$$=\frac{-60\ 000\pm\sqrt{4\ 560\ 000\ 000}}{6}$$
$= 1255$ or $-21\ 255$ (to the nearest integer)

$\therefore x = \mathbf{1255}$ or $x = \mathbf{-21\ 255}$

(e) Amount of liquid $= (1254.6 + 20\ 000) \times 30 \div 100^3$

$= \mathbf{0.638\ m^3}$ (to 3 s.f.)

4 Indices, Surds, Exponential Growth and Decay, and Standard Form

Worksheet 4A Laws of Indices

1. **(a)** $5^6 \times 5^3 = 5^{6+3}$

$= \mathbf{5^9}$

(b) $11^2 \times 11^8 = 11^{2+8}$

$= \mathbf{11^{10}}$

(c) $x^4 \times x^{10} = x^{4+10}$

$= \boldsymbol{x^{14}}$

(d) $8x^7 \times x = 8x^{7+1}$

$= \mathbf{8}\boldsymbol{x^8}$

(e) $7x^h \times 3x^k = (7 \times 3)x^{h+k}$

$= \mathbf{21}\boldsymbol{x^{h+k}}$

(f) $ax^p \times bx^q = (a \times b)x^{p+q}$

$= \boldsymbol{abx^{p+q}}$

2. **(a)** $7^9 \div 7^3 = 7^{9-3}$

$= \mathbf{7^6}$

(b) $3^{10} \div 3^4 = 3^{10-4}$

$= \mathbf{3^6}$

(c) $x^8 \div x = x^{8-1}$

$= \boldsymbol{x^7}$

(d) $10x^5 \div 5x^2 = (10 \div 5)x^{5-2}$

$= \mathbf{2}\boldsymbol{x^3}$

(e) $8x^h \div 12x^k = (8 \div 12)x^{h-k}$

$= \boldsymbol{\frac{2}{3}x^{h-k}}$

(f) $ax^p \div bx^q = (a \div b)x^{p-q}$

$= \boldsymbol{\frac{a}{b}x^{p-q}}$

3. **(a)** $(11^6)^2 = 11^{6\times 2}$
$= \mathbf{11^{12}}$

(b) $(7^4)^4 = 7^{4\times 4}$
$= \mathbf{7^{16}}$

(c) $(5x^8)^3 = 5^3x^{8\times 3}$
$= \mathbf{125x^{24}}$

(d) $(ax^p)^q = a^qx^{p\times q}$
$= \boldsymbol{a^qx^{pq}}$

4. **(a)** $7^6 \times 2^6 = (7\times 2)^6$
$= \mathbf{14^6}$

(b) $3^{10}\times 19^{10} = (3\times 19)^{10}$
$= \mathbf{57^{10}}$

(c) $x^9 \times y^9 = \boldsymbol{x^9y^9}$

(d) $4x^5 \times 5y^5 = (4\times 5)x^5y^5$
$= \boldsymbol{20x^5y^5}$

(e) $11x^k \times 8y^k = (11\times 8)x^ky^k$
$= \boldsymbol{88x^ky^k}$

(f) $ax^p \times by^p = \boldsymbol{abx^py^p}$

5. **(a)** $20^7 \div 4^7 = \left(\frac{20}{4}\right)^7$
$= \mathbf{5^7}$

(b) $99^{11}\div 3^{11} = \left(\frac{99}{3}\right)^{11}$
$= \mathbf{33^{11}}$

(c) $x^8 \div y^8 = \boldsymbol{\frac{x^8}{y^8}}$

(d) $36x^3 \div 6y^3 = \left(\frac{36}{6}\right)\frac{x^3}{y^3}$
$= \boldsymbol{\frac{6x^3}{y^3}}$

(e) $5x^k \div 50y^k = \left(\frac{5}{50}\right)\frac{x^k}{y^k}$
$= \boldsymbol{\frac{x^k}{10y^k}}$

(f) $ax^p \div by^p = \left(\frac{a}{b}\right)\frac{x^p}{y^p}$
$= \boldsymbol{\frac{ax^p}{by^p}}$

6. **(a)** $\frac{3^4\times 3^9}{3^3} = 3^{4+9-3}$
$= \mathbf{3^{10}}$

(b) $\frac{p^{10}}{(p^2)^3} = \frac{p^{10}}{p^6}$
$= \boldsymbol{p^4}$

(c) $\frac{(q^k)^5}{q^{2k}\times q} = \frac{q^{5k}}{q^{2k+1}}$
$= \boldsymbol{q^{3k-1}}$

7. $(32^4)^7 \div 16^3 = [(2^5)^4]^7 \div (2^4)^3$
$= 2^{140}\div 2^{12}$
$= \mathbf{2^{128}}$

8. **(a)** $a^4b \times ab^5 = \boldsymbol{a^5b^6}$

(b) $(-3cd^3)^2 \times c^6d^7 = 9c^2d^6 \times c^6d^7$
$= \boldsymbol{9c^8d^{13}}$

(c) $h^9k^{18} \div (hk^4)^3 = h^9k^{18} \div h^3k^{12}$
$= \boldsymbol{h^6k^6}$

(d) $24m^{11}n^{25} \div 10(-m^2n)^5 = 24m^{11}n^{25} \div (-10m^{10}n^5)$
$= \boldsymbol{-\frac{12mn^{20}}{5}}$

(e) $(4x^8yz^2)^2 \times 3xy^6z^4 = 16x^{16}y^2z^4 \times 3xy^6z^4$
$= \boldsymbol{48x^{17}y^8z^8}$

(f) $15x^9y^3z^4 \div (5x^3yz)^2 = 15x^9y^3z^4 \div 25x^6y^2z^2$
$= \boldsymbol{\frac{3x^3yz^2}{5}}$

9. **(a)** $\frac{14a}{b^6}\times\frac{a^3}{8b^2} = \boldsymbol{\frac{7a^4}{4b^8}}$

(b) $\frac{5c^5}{d^2}\div\frac{10c}{d^4} = \frac{5c^5}{d^2}\times\frac{d^4}{10c}$
$= \boldsymbol{\frac{c^4d^2}{2}}$

(c) $\left(\frac{h^7}{k^3}\right)^2\times\left(\frac{2}{hk^4}\right)^3 = \frac{h^{14}}{k^6}\times\frac{8}{h^3k^{12}}$
$= \boldsymbol{\frac{8h^{11}}{k^{18}}}$

(d) $9\left(m^4n\right)^3 \div \frac{6n^2}{m} = 9m^{12}n^3\times\frac{m}{6n^2}$
$= \boldsymbol{\frac{3m^{13}n}{2}}$

(e) $\frac{q^5}{3p^4}\times\frac{9p^8}{2q}\times\frac{1}{6pq^2} = \boldsymbol{\frac{p^3q^2}{4}}$

(f) $\frac{5x^{17}z^6}{y^2}\div\left(\frac{10x^7y^3}{z^4}\div\frac{2y^2z}{5x^8}\right) = \frac{5x^{17}z^6}{y^2}\div\left(\frac{10x^7y^3}{z^4}\times\frac{5x^8}{2y^2z}\right)$
$= \frac{5x^{17}z^6}{y^2}\div\frac{25x^{15}y}{z^5}$
$= \frac{5x^{17}z^6}{y^2}\times\frac{z^5}{25x^{15}y}$
$= \boldsymbol{\frac{x^2z^{11}}{5y^3}}$

10. $5^8+5^8+5^8+5^8+5^8 = 5(5^8)$
$= 5^9$
$\therefore k = \mathbf{9}$

11. **(i)** $\frac{\left(-4x^8\right)^2}{18y^6}\div\left(\frac{2x^5}{3y^3}\right)^3 = \frac{16x^{16}}{18y^6}\div\frac{8x^{15}}{27y^9}$
$= \frac{16x^{16}}{18y^6}\times\frac{27y^9}{8x^{15}}$
$= 3xy^3$
$\therefore a = \mathbf{3}$
Equating the indices of x,
$h - k = 1$ — (1) (shown)

(ii) Equating the indices of y,
$\mathbf{4k - h = 3}$ — (2)

(iii) (1) + (2): $3k = 4$
$k = \frac{4}{3}$
Substitute $k = \frac{4}{3}$ into (1):
$h - \frac{4}{3} = 1$
$h = \frac{7}{3}$
$\therefore \boldsymbol{h = \frac{7}{3}, k = \frac{4}{3}}$

Challenge Myself!

12. $111^{12k} = (111^2)^{6k}$
$= 12\,321^{6k}$
$> 9999^{6k}$
$\therefore$ $\mathbf{111^{12k}}$ is greater.

1. (a) $8^0 = \mathbf{1}$
   (b) $(-3)^0 = \mathbf{1}$
   (c) $\left(\frac{2}{7}\right)^0 = \mathbf{1}$
   (d) $0.94^0 = \mathbf{1}$
   (e) $5^2 + 5^0 = 25 + 1$
   $= \mathbf{26}$
   (f) $7^0 - 12 = 1 - 12$
   $= \mathbf{-11}$
   (g) $\frac{45}{2+99^0} = \frac{45}{2+1}$
   $= \mathbf{15}$
   (h) $\sqrt[3]{8\left(-\frac{1}{6}\right)^0} = \sqrt[3]{8}$
   $= \mathbf{2}$

2. (a) $7^{-1} = \mathbf{\frac{1}{7}}$
   (b) $8^{-2} = \frac{1}{8^2}$
   $= \mathbf{\frac{1}{64}}$
   (c) $(-5)^{-1} = \frac{1}{-5}$
   $= \mathbf{-\frac{1}{5}}$
   (d) $(-3)^{-4} = \frac{1}{(-3)^4}$
   $= \mathbf{\frac{1}{81}}$
   (e) $\left(\frac{1}{4}\right)^{-1} = \left(\frac{4}{1}\right)^1$
   $= \mathbf{4}$
   (f) $\left(\frac{2}{9}\right)^{-2} = \left(\frac{9}{2}\right)^2$
   $= \mathbf{\frac{81}{4}}$
   (g) $3^2 + 3^{-2} = 3^2 + \frac{1}{3^2}$
   $= 9 + \frac{1}{9}$
   $= \mathbf{9\frac{1}{9}}$
   (h) $\left(\frac{4}{5}\right)^{-3} \times \left(\frac{3}{2}\right)^{-4} = \left(\frac{5}{4}\right)^3 \times \left(\frac{2}{3}\right)^4$
   $= \frac{125}{64} \times \frac{16}{81}$
   $= \mathbf{\frac{125}{324}}$

3. (a) $(a^3b^{-3})^8 = a^{24}b^{-24}$
   $= \mathbf{\frac{a^{24}}{b^{24}}}$
   (b) $(c^7d^2)^{-4} = c^{-28}d^{-8}$
   $= \mathbf{\frac{1}{c^{28}d^8}}$
   (c) $(3h^{-2}k^4)^{-1} = 3^{-1}h^2k^{-4}$
   $= \mathbf{\frac{h^2}{3k^4}}$
   (d) $\left(\frac{n}{2m^5}\right)^{-3} = \left(\frac{2m^5}{n}\right)^3$
   $= \mathbf{\frac{8m^{15}}{n^3}}$
   (e) $\frac{10p}{5p^3q^{-6}} = \mathbf{\frac{2q^6}{p^2}}$
   (f) $\frac{8x^2y^{-1}}{12x^{-9}y^2} = \mathbf{\frac{2x^{11}}{3y^3}}$

4. (a) $a^7 \times a^{-2} \div a^{15} = a^{-10}$
   $= \mathbf{\frac{1}{a^{10}}}$
   (b) $6b^{-4} \div (2b \times 9b^{-8}) = \frac{6b^{-4}}{2b \times 9b^{-8}}$
   $= \frac{6b^{-4}}{18b^{-7}}$
   $= \mathbf{\frac{b^3}{3}}$
   (c) $(h^{-5}j^{-9}k)^2 \times (3hj^3k^{-4})^4 = h^{-10}j^{-18}k^2 \times 81h^4j^{12}k^{-16}$
   $= 81h^{-6}j^{-6}k^{-14}$
   $= \mathbf{\frac{81}{h^6j^6k^{14}}}$
   (d) $\left(\frac{m^{-1}}{n^7}\right)^{-3} \div \left(\frac{n^{-6}}{m^5}\right)^2 = \frac{m^3}{n^{-21}} \div \frac{n^{-12}}{m^{10}}$
   $= \frac{m^3}{n^{-21}} \times \frac{m^{10}}{n^{-12}}$
   $= \frac{m^{13}}{n^{-33}}$
   $= \mathbf{m^{13}n^{33}}$
   (e) $3p \times \frac{4p}{q} - \frac{8q^{-1}}{p^{-2}} + \frac{(5pq)^2}{q^3} = \frac{12p^2}{q} - \frac{8p^2}{q} + \frac{25p^2q^2}{q^3}$
   $= \frac{4p^2}{q} + \frac{25p^2}{q}$
   $= \mathbf{\frac{29p^2}{q}}$
   (f) $\frac{4(xy^8)^2}{x^{-3}y} \times \left(\frac{y^7}{x}\right)^0 \times \frac{(x^2y^5)^{-2}}{x^2y^5} = \frac{4x^2y^{16}}{x^{-3}y} \times 1 \times (x^2y^5)^{-3}$
   $= 4x^5y^{15} \times x^{-6}y^{-15}$
   $= 4x^{-1}$
   $= \mathbf{\frac{4}{x}}$

5. $\frac{5^7 \times 5^0}{5^{-2}} = 5^{7+0-(-2)}$
   $= 5^9$
   $\therefore a = \mathbf{9}$

6. $\frac{2^2 \times 2^6}{2^{-3}} = 2^{2+6-(-3)}$
   $= \mathbf{2^{11}}$

7. (a) $\frac{1}{125} = \frac{1}{5^3}$
   $= 5^{-3}$
   $\therefore k = \mathbf{-3}$

(b) $64 \times 8^n = 1$
$2^6 \times 2^{3n} = 1$
$2^{6+3n} = 1$
$6 + 3n = 0$
$3n = -6$
$n = -2$
$\therefore n = \mathbf{-2}$

(c) $9^{p+7} = \frac{1}{3}$
$(3^2)^{p+7} = 3^{-1}$
$3^{2p+14} = 3^{-1}$
$2p + 14 = -1$
$2p = -15$
$p = -\frac{15}{2}$
$\therefore p = \mathbf{-\frac{15}{2}}$

(d) $2^{q^2-100} - 1 = 0$
$2^{q^2-100} = 1$
$= 2^0$
$q^2 - 100 = 0$
$q^2 = 100$
$q = \pm 10$
$\therefore q = \mathbf{\pm 10}$

8. $27^{p+2q} = \frac{1}{81}$
$3^{3p+6q} = 3^{-4}$
$3p + 6q = -4$
$p + 2q = -\frac{4}{3}$
$p = -\frac{4}{3} - 2q$
Let $q = -\frac{1}{2}$: $p = -\frac{4}{3} - 2\left(-\frac{1}{2}\right)$
$= -\frac{1}{3}$
$\therefore$ An example of the values is $\mathbf{p = -\frac{1}{3}}$, $\mathbf{q = -\frac{1}{2}}$.

9. $\frac{x^6z^{-4}}{x^3y^2z} \times \left(\frac{y^5z^{-1}}{x^8}\right)^{-5} = \frac{x^3}{y^2z^5} \times \frac{y^{-25}z^5}{x^{-40}}$
$= \mathbf{\frac{x^{43}}{y^{27}}}$

10. $\frac{x^{-2}y^7z^{-3}}{\left(x^{-1}y\right)^4} \div \frac{1}{\left(xy^{-4}z^2\right)^{-3}} = \frac{x^{-2}y^7z^{-3}}{x^{-4}y^4} \times \left(xy^{-4}z^2\right)^{-3}$
$= x^2y^3z^{-3} \times x^{-3}y^{12}z^{-6}$
$= x^{-1}y^{15}z^{-9}$
$= \mathbf{\frac{x^{-1}z^{-9}}{y^{-15}}}$

Challenge Myself!

11. $8x^{-1} = \frac{8}{x}$, not $\frac{1}{8x}$ and $3x^0 = 3$, not 1.
$\therefore$ The student is not correct.
$8x^{-1} + 3x^0 = 4$
$\frac{8}{x} + 3 = 4$
$\frac{8}{x} = 1$
$x = \mathbf{8}$

Worksheet 4C Rational indices

1. (a) $\sqrt{225} = \sqrt{15^2}$
$= \mathbf{15}$

(b) $\sqrt[3]{512} = \sqrt[3]{8^3}$
$= \mathbf{8}$

(c) $\sqrt[4]{\frac{16}{81}} = \sqrt[4]{\left(\frac{2}{3}\right)^4}$
$= \mathbf{\frac{2}{3}}$

(d) $\sqrt[5]{-\frac{1}{1024}} = \sqrt[5]{\left(-\frac{1}{4}\right)^5}$
$= \mathbf{-\frac{1}{4}}$

(e) $\left(\sqrt{100}\right)^2 = \mathbf{100}$

(f) $\left(\sqrt[3]{-27}\right)^4 = \left(\sqrt[3]{(-3)^3}\right)^4$
$= (-3)^4$
$= \mathbf{81}$

3. (a) $121^{\frac{1}{2}} = \left(11^2\right)^{\frac{1}{2}}$
$= \mathbf{11}$

(b) $1000^{\frac{1}{3}} = \left(10^3\right)^{\frac{1}{3}}$
$= \mathbf{10}$

(c) $256^{\frac{1}{4}} = \left(4^4\right)^{\frac{1}{4}}$
$= \mathbf{4}$

(d) $(-243)^{\frac{1}{5}} = \left[(-3)^5\right]^{\frac{1}{5}}$
$= \mathbf{-3}$

(e) $81^{\frac{3}{2}} = \left(9^2\right)^{\frac{3}{2}}$
$= 9^3$
$= \mathbf{729}$

(f) $16^{\frac{7}{4}} = \left(2^4\right)^{\frac{7}{4}}$
$= 2^7$
$= \mathbf{128}$

(g) $0.49^{1.5} = (0.7^2)^{1.5}$
$= 0.7^3$
$= \mathbf{0.343}$

(h) $400^{-0.5} = (20^2)^{-0.5}$
$= 20^{-1}$
$= \mathbf{\frac{1}{20}}$

(i) $\left(\frac{25}{144}\right)^{-\frac{1}{2}} = \left(\frac{144}{25}\right)^{\frac{1}{2}}$

$= \left[\left(\frac{12}{5}\right)^2\right]^{\frac{1}{2}}$

$= \mathbf{\frac{12}{5}}$

(j) $\left(\frac{1}{3125}\right)^{-\frac{3}{5}} = 3125^{\frac{3}{5}}$

$= \left(5^5\right)^{\frac{3}{5}}$

$= 5^3$

$= \mathbf{125}$

5. **(a)** $x^{\frac{3}{4}} = \sqrt[4]{\boldsymbol{x^3}}$

(b) $x^{\frac{9}{8}} = \sqrt[8]{\boldsymbol{x^9}}$

6. $\sqrt[10]{x^5} = x^{\frac{5}{10}}$

$= x^{\frac{1}{2}}$

$= \sqrt{x}$

$\therefore \sqrt[10]{x^5} = \sqrt{x}$

7. **(a)** $\sqrt{x} = \boldsymbol{x^{\frac{1}{2}}}$

(b) $\sqrt[3]{x} = \boldsymbol{x^{\frac{1}{3}}}$

(c) $\sqrt[7]{x^6} = \boldsymbol{x^{\frac{6}{7}}}$

(d) $\left(\sqrt[8]{x}\right)^9 = \boldsymbol{x^{\frac{9}{8}}}$

(e) $\frac{1}{\sqrt[4]{x}} = \frac{1}{x^{\frac{1}{4}}}$

$= \boldsymbol{x^{-\frac{1}{4}}}$

(f) $\frac{1}{\sqrt[5]{x^2}} = \frac{1}{x^{\frac{2}{5}}}$

$= \boldsymbol{x^{-\frac{2}{5}}}$

(g) $\frac{1}{\left(\sqrt[3]{x}\right)^4} = \frac{1}{x^{\frac{4}{3}}}$

$= \boldsymbol{x^{-\frac{4}{3}}}$

(h) $\frac{1}{\left(\sqrt[6]{x^8}\right)^2} = \frac{1}{\left(x^{\frac{8}{6}}\right)^2}$

$= \frac{1}{x^{\frac{8}{3}}}$

$= \boldsymbol{x^{-\frac{8}{3}}}$

(i) $\left(x^{\frac{3}{4}}y^{-\frac{1}{3}}\right)^{\frac{1}{2}} = \boldsymbol{x^{\frac{3}{8}}y^{-\frac{1}{6}}}$

(j) $\sqrt[4]{\left(16xy^{-\frac{5}{6}}\right)^3} = \left(16xy^{-\frac{5}{6}}\right)^{\frac{3}{4}}$

$= \boldsymbol{8x^{\frac{3}{4}}y^{-\frac{5}{8}}}$

(k) $\frac{1}{\left(x^{\frac{1}{5}}y^{-\frac{2}{3}}\right)^2} = \left(x^{\frac{1}{5}}y^{-\frac{2}{3}}\right)^{-2}$

$= \boldsymbol{x^{-\frac{2}{5}}y^{\frac{4}{3}}}$

(l) $\frac{1}{\sqrt[9]{x^{-\frac{7}{2}}y^{\frac{4}{3}}}} = \frac{1}{\left(x^{-\frac{7}{2}}y^{\frac{4}{3}}\right)^{\frac{1}{9}}}$

$= \left(x^{-\frac{7}{2}}y^{\frac{4}{3}}\right)^{-\frac{1}{9}}$

$= \boldsymbol{x^{\frac{7}{18}}y^{-\frac{4}{27}}}$

8. **(a)** $x^{\frac{1}{3}} \times x^{\frac{1}{6}} = \boldsymbol{x^{\frac{1}{2}}}$

(b) $x^2 \div x^{\frac{5}{9}} = \boldsymbol{x^{\frac{13}{9}}}$

(c) $x^{\frac{7}{8}} \times x^{-2} = x^{-\frac{9}{8}}$

$= \mathbf{\frac{1}{\boldsymbol{x^{\frac{9}{8}}}}}$

(d) $x^{-\frac{1}{4}} \div x^{-\frac{1}{5}} = x^{-\frac{1}{20}}$

$= \mathbf{\frac{1}{\boldsymbol{x^{\frac{1}{20}}}}}$

(e) $\sqrt{x} \times \sqrt[6]{x} = x^{\frac{1}{2}} \times x^{\frac{1}{6}}$

$= \boldsymbol{x^{\frac{2}{3}}}$

(f) $\sqrt[7]{x^4} \div \sqrt[3]{x^5} = x^{\frac{4}{7}} \div x^{\frac{5}{3}}$

$= x^{-\frac{23}{21}}$

$= \mathbf{\frac{1}{\boldsymbol{x^{\frac{23}{21}}}}}$

(g) $x^{\frac{4}{5}} \times x^{\frac{1}{2}} \div x^{\frac{7}{10}} = \boldsymbol{x^{\frac{3}{5}}}$

(h) $x^{-\frac{5}{6}} \div \left(x^{\frac{1}{3}} \div x\right) = x^{-\frac{5}{6}} \div x^{-\frac{2}{3}}$

$= x^{-\frac{1}{6}}$

$= \mathbf{\frac{1}{\boldsymbol{x^{\frac{1}{6}}}}}$

(i) $\left(x^{\frac{3}{8}} \div x^2\right)^{\frac{2}{5}} = \left(x^{-\frac{13}{8}}\right)^{\frac{2}{5}}$

$= x^{-\frac{13}{20}}$

$= \mathbf{\frac{1}{\boldsymbol{x^{\frac{13}{20}}}}}$

(j) $\left(x^{-99^2} \times x^{\frac{1}{99}}\right)^0 \times \frac{1}{x} = \mathbf{\frac{1}{\boldsymbol{x}}}$

(k) $\left(8\sqrt{x^9 \times x^{-3}}\right)^{\frac{7}{3}} = \left(8\sqrt{x^6}\right)^{\frac{7}{3}}$

$= \left(2^3x^3\right)^{\frac{7}{3}}$

$= 2^7x^7$

$= \mathbf{128}\boldsymbol{x^7}$

(l) $\frac{5}{\sqrt[4]{81x^{-1} \div x^{-10}}} = \frac{5}{\sqrt[4]{81x^{9}}}$

$= \frac{5}{\left(3^4 x^9\right)^{\frac{1}{4}}}$

$= \mathbf{\frac{5}{3x^{\frac{9}{4}}}}$

9.

(a) Product $= ax^p \times (bx)^q$

$= ax^p \times b^q x^q$

$= (ab^q) \times x^{p+q}$

Let $a = 1$, $b = 1$, $p = 4$ and $q = 8$:

Product $= (1 \times 1^8) \times x^{4+8}$

$= x^{12}$

∴ A possible set of values is $\boldsymbol{a = 1}$, $\boldsymbol{b = 1}$, $\boldsymbol{p = 4}$ and $\boldsymbol{q = 8}$.

(b) Let $a = 1$, $b = 1$, $p = \frac{19}{2}$ and $q = \frac{5}{2}$:

Product $= \left(1 \times 1^{\frac{5}{2}}\right) \times x^{\frac{19}{2}+\frac{5}{2}}$

$= x^{12}$

∴ A possible set of values is $\boldsymbol{a = 1}$, $\boldsymbol{b = 1}$, $\boldsymbol{p = \frac{19}{2}}$ and $\boldsymbol{q = \frac{5}{2}}$.

10. (a) $\left(a^5 b^{-2}\right)^{\frac{1}{3}} \times \left(a^2 b^8\right)^{\frac{1}{4}} = a^{\frac{5}{3}} b^{-\frac{2}{3}} \times a^{\frac{1}{2}} b^2$

$= \boldsymbol{a^{\frac{13}{6}} b^{\frac{4}{3}}}$

(b) $\left(\frac{c}{d^{-6}}\right)^{\frac{5}{6}} \times \left(\frac{d^4}{c^{-9}}\right)^{-\frac{2}{3}} = \frac{c^{\frac{5}{6}}}{d^{-5}} \times \frac{d^{-\frac{8}{3}}}{c^6}$

$= c^{-\frac{31}{6}} d^{\frac{7}{3}}$

$= \mathbf{\frac{d^{\frac{7}{3}}}{c^{\frac{31}{6}}}}$

(c) $\frac{h^{\frac{1}{6}} k^{-\frac{3}{5}}}{\left(h^{-\frac{1}{3}} k^2\right)^2} = \frac{h^{\frac{1}{6}} k^{-\frac{3}{5}}}{h^{-\frac{2}{3}} k^4}$

$= h^{\frac{5}{6}} k^{-\frac{23}{5}}$

$= \mathbf{\frac{h^{\frac{5}{6}}}{k^{\frac{23}{5}}}}$

(d) $m^{\frac{8}{9}} n \div \left(\frac{m^{-\frac{2}{7}}}{n^3}\right)^4 = m^{\frac{8}{9}} n \div \frac{m^{-\frac{8}{7}}}{n^{12}}$

$= m^{\frac{8}{9}} n \times \frac{n^{12}}{m^{-\frac{8}{7}}}$

$= m^{\frac{8}{9}} n \times m^{\frac{8}{7}} n^{12}$

$= \boldsymbol{m^{\frac{128}{63}} n^{13}}$

(e) $\left(\frac{49p^8}{q^{-4}}\right)^{-\frac{1}{2}} \times \left(pq^{\frac{6}{5}}\right)^{-2} = \left(\frac{q^{-4}}{49p^8}\right)^{\frac{1}{2}} \times \left(pq^{\frac{6}{5}}\right)^{-2}$

$= \frac{q^{-2}}{7p^4} \times p^{-2} q^{-\frac{12}{5}}$

$= \frac{p^{-6} q^{-\frac{22}{5}}}{7}$

$= \mathbf{\frac{1}{7p^6 q^{\frac{22}{5}}}}$

(f) $\sqrt{64x^{\frac{1}{2}} y^{-\frac{1}{4}}} \div \frac{\sqrt[5]{32x^{\frac{1}{4}}}}{y^3} = \left(64x^{\frac{1}{2}} y^{-\frac{1}{4}}\right)^{\frac{1}{2}} \div \frac{\left(32x^{\frac{1}{4}}\right)^{\frac{1}{5}}}{y^3}$

$= 8x^{\frac{1}{4}} y^{-\frac{1}{8}} \div \frac{2x^{\frac{1}{20}}}{y^3}$

$= 8x^{\frac{1}{4}} y^{-\frac{1}{8}} \times \frac{y^3}{2x^{\frac{1}{20}}}$

$= \boldsymbol{4x^{\frac{1}{5}} y^{\frac{23}{8}}}$

11. $243^{\frac{1}{2}} = \left(3^5\right)^{\frac{1}{2}}$

$= 3^{\frac{5}{2}}$

∴ $n = \mathbf{\frac{5}{2}}$

12. $27^{\frac{5}{3}} = \left(3^3\right)^{\frac{5}{3}}$

$= 3^5$

∴ $n = \mathbf{5}$

13. (a) $4 \times 32^{\frac{1}{5}} = 2^2 \times \left(2^5\right)^{\frac{1}{5}}$

$= 2^2 \times 2^1$

$= 2^3$

∴ $n = \mathbf{3}$

(b) $n^{(n+1)(n-1)} = 3^{4 \times 2}$

$= 3^8$

$= 6561$ (shown)

14. (i) $x^{\frac{1}{3}} = 27$

$x = 27^3$

$= \mathbf{19\,683}$

(ii) $19\,683 = 27^3$

$= (3^3)^3$

$= 3^9$

$= 3^{2 \times \frac{9}{2}}$

$= \mathbf{9^{\frac{9}{2}}}$

15. (i) $\sqrt{2^{18}} = \mathbf{2^9}$

(ii) $x^{0.75} = \sqrt{2^{18}}$

$x^{\frac{3}{4}} = 2^9$

$x = \left(2^9\right)^{\frac{4}{3}}$

$= \mathbf{2^{12}}$

16. (i) $(a + b)(a^2 - ab + b^2) = a^3 - a^2b + ab^2 + a^2b - ab^2 + b^3$

$= \boldsymbol{a^3 + b^3}$

(ii) $\left(\sqrt[3]{x}+\sqrt[3]{y}\right)\left(\sqrt[3]{x^2}-\sqrt[3]{xy}+\sqrt[3]{y^2}\right)$
$= \left(x^{\frac{1}{3}}+y^{\frac{1}{3}}\right)\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}y^{\frac{1}{3}}+y^{\frac{2}{3}}\right)$
$= x - x^{\frac{2}{3}}y^{\frac{1}{3}} + x^{\frac{1}{3}}y^{\frac{2}{3}} + x^{\frac{2}{3}}y^{\frac{1}{3}} - x^{\frac{1}{3}}y^{\frac{2}{3}} + y$
$= \boldsymbol{x+y}$

(iii) Let $a = \sqrt[3]{x}$ and $b = \sqrt[3]{y}$:
$\left(\sqrt[3]{x}+\sqrt[3]{y}\right)\left(\sqrt[3]{x^2}-\sqrt[3]{xy}+\sqrt[3]{y^2}\right) = \left(\sqrt[3]{x}\right)^3+\left(\sqrt[3]{y}\right)^3$
$= \boldsymbol{x+y}$

17. (a) $x^3 = 50$
$x = 50^{\frac{1}{3}}$
$= \mathbf{3.68}$ (to 3 s.f.)

(b) $x^7 = 4321$
$x = 4321^{\frac{1}{7}}$
$= \mathbf{3.31}$ (to 3 s.f.)

(c) $x^{\frac{4}{3}} = 9$
$x = 9^{\frac{3}{4}}$
$= \mathbf{5.20}$ (to 3 s.f.)

(d) $x^{0.2} = 6.8$
$x = 6.8^5$
$= \mathbf{14\,500}$ (to 3 s.f.)

(e) $\sqrt[6]{x} = 1.7$
$x^{\frac{1}{6}} = 1.7$
$x = 1.7^6$
$= \mathbf{24.1}$ (to 3 s.f.)

(f) $\sqrt[8]{x^5} = 0.305$
$x^{\frac{5}{8}} = 0.305$
$x = 0.305^{\frac{8}{5}}$
$= \mathbf{0.150}$ (to 3 s.f.)

18. (a) $x^5 = 243$
$= 3^5$
$x = \mathbf{3}$

(b) (i) $y^5 = 244$
$y = \mathbf{244^{\frac{1}{5}}}$

(ii) $y = 244^{\frac{1}{5}}$
$= \mathbf{3.00}$ (to 3 s.f.)

19. $\sqrt[p]{x^n} = x^{\frac{n}{p}}$

Since $\frac{n}{p} = 24$, possible pairs of values are $\boldsymbol{n = 48, p = 2}$ or $\boldsymbol{n = 72, p = 3}$.

Worksheet 4D Surds

1. (a) $\sqrt{75}+\sqrt{27}-\sqrt{12} = \sqrt{25\times3}+\sqrt{9\times3}-\sqrt{4\times3}$
$= 5\sqrt{3}+3\sqrt{3}-2\sqrt{3}$
$= \mathbf{6\sqrt{3}}$

(b) $3\sqrt{8}-\sqrt{50}+\frac{1}{2}\sqrt{32} = 3\sqrt{4\times2}-\sqrt{25\times2}+\frac{1}{2}\sqrt{16\times2}$
$= 3\left(2\sqrt{2}\right)-5\sqrt{2}+\frac{1}{2}\left(4\sqrt{2}\right)$
$= 6\sqrt{2}-5\sqrt{2}+2\sqrt{2}$
$= \mathbf{3\sqrt{2}}$

(c) $6\sqrt{6}\times\sqrt{12}\div4\sqrt{2} = \frac{6}{4}\times\sqrt{\frac{6\times12}{2}}$
$= \frac{3}{2}\sqrt{36}$
$= \frac{3}{2}(6)$
$= \mathbf{9}$

(d) $\left(10-\sqrt{5}\right)\left(9+\sqrt{5}\right) = 90+10\sqrt{5}-9\sqrt{5}-5$
$= \mathbf{85+\sqrt{5}}$

(e) $4\sqrt{117}-\frac{1}{2}\sqrt{208}+\frac{6}{\sqrt{52}}$
$= 4\sqrt{9\times13}-\frac{1}{2}\sqrt{16\times13}+\frac{6}{\sqrt{4\times13}}$
$= 4\left(3\sqrt{13}\right)-\frac{1}{2}\left(4\sqrt{13}\right)+\frac{6}{2\sqrt{13}}$
$= 12\sqrt{13}-2\sqrt{13}+\frac{3}{\sqrt{13}}$
$= 10\sqrt{13}+\frac{3\sqrt{13}}{13}$
$= \mathbf{\frac{133}{13}\sqrt{13}}$

(f) $\frac{\left(4-\sqrt{10}\right)\left(\sqrt{10}+5\right)}{3+\sqrt{10}} = \frac{4\sqrt{10}+20-10-5\sqrt{10}}{3+\sqrt{10}}$
$= \frac{10-\sqrt{10}}{\sqrt{10}+3}$
$= \frac{10-\sqrt{10}}{\sqrt{10}+3}\times\frac{\sqrt{10}-3}{\sqrt{10}-3}$
$= \frac{10\sqrt{10}-30-10+3\sqrt{10}}{10-9}$
$= \mathbf{13\sqrt{10}-40}$

2. $\left(5+\sqrt{3}\right)^2+\left(4-\sqrt{3}\right)^2 = 25+10\sqrt{3}+3+16-8\sqrt{3}+3$
$= 47+2\sqrt{3}$
$\therefore \boldsymbol{a = 47, b = 2}$

3. $\frac{1}{6-5\sqrt{2}} = \frac{1}{6-5\sqrt{2}} \times \frac{6+5\sqrt{2}}{6+5\sqrt{2}}$

$= \frac{6+5\sqrt{2}}{36-50}$

$= \frac{6+5\sqrt{2}}{-14}$

$= -\frac{3}{7} - \frac{5}{14}\sqrt{2}$

$\therefore$ $\boldsymbol{a = -\frac{3}{7}, b = -\frac{5}{14}}$

4. $\frac{p^2-2}{p+1} = \frac{(8-\sqrt{5})^2 - 2}{(8-\sqrt{5})+1}$

$= \frac{64-16\sqrt{5}+5-2}{9-\sqrt{5}}$

$= \frac{67-16\sqrt{5}}{9-\sqrt{5}}$

$= \frac{67-16\sqrt{5}}{9-\sqrt{5}} \times \frac{9+\sqrt{5}}{9+\sqrt{5}}$

$= \frac{603+67\sqrt{5}-144\sqrt{5}-80}{81-5}$

$= \frac{523-77\sqrt{5}}{76}$

$= \boldsymbol{\frac{523}{76} - \frac{77}{76}\sqrt{5}}$

5. $\frac{6}{\sqrt{3}+1} - \frac{5}{\sqrt{3}} = \frac{6}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} - \frac{5}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{6\sqrt{3}-6}{3-1} - \frac{5\sqrt{3}}{3}$

$= \frac{6\sqrt{3}-6}{2} - \frac{5\sqrt{3}}{3}$

$= 3\sqrt{3} - 3 - \frac{5}{3}\sqrt{3}$

$= \frac{4}{3}\sqrt{3} - 3$

$\therefore$ $\boldsymbol{a = \frac{4}{3}, b = -3}$

6. (a) $\frac{2-\sqrt{3}}{\sqrt{7}-\sqrt{5}} + \frac{2+\sqrt{3}}{\sqrt{7}+\sqrt{5}}$

$= \frac{2-\sqrt{3}}{\sqrt{7}-\sqrt{5}} \times \frac{\sqrt{7}+\sqrt{5}}{\sqrt{7}+\sqrt{5}} + \frac{2+\sqrt{3}}{\sqrt{7}+\sqrt{5}} \times \frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}-\sqrt{5}}$

$= \frac{2\sqrt{7}+2\sqrt{5}-\sqrt{21}-\sqrt{15}}{7-5} + \frac{2\sqrt{7}-2\sqrt{5}+\sqrt{21}-\sqrt{15}}{7-5}$

$= \frac{2\sqrt{7}+2\sqrt{5}-\sqrt{21}-\sqrt{15}}{2} + \frac{2\sqrt{7}-2\sqrt{5}+\sqrt{21}-\sqrt{15}}{2}$

$= \frac{4\sqrt{7}-2\sqrt{15}}{2}$

$= 2\sqrt{7} - \sqrt{15}$

$\therefore$ $\boldsymbol{a = 2, b = -1}$

(b) $\frac{2+\sqrt{3}}{\sqrt{7}-\sqrt{5}} - \frac{2-\sqrt{3}}{\sqrt{7}+\sqrt{5}}$

$= \frac{2+\sqrt{3}}{\sqrt{7}-\sqrt{5}} \times \frac{\sqrt{7}+\sqrt{5}}{\sqrt{7}+\sqrt{5}} - \frac{2-\sqrt{3}}{\sqrt{7}+\sqrt{5}} \times \frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}-\sqrt{5}}$

$= \frac{2\sqrt{7}+2\sqrt{5}+\sqrt{21}+\sqrt{15}}{7-5} - \frac{2\sqrt{7}-2\sqrt{5}-\sqrt{21}+\sqrt{15}}{7-5}$

$= \frac{2\sqrt{7}+2\sqrt{5}+\sqrt{21}+\sqrt{15}}{2} - \frac{2\sqrt{7}-2\sqrt{5}-\sqrt{21}+\sqrt{15}}{2}$

$= \frac{4\sqrt{5}+2\sqrt{21}}{2}$

$= 2\sqrt{5} + \sqrt{21}$

$= \sqrt{4\times5} + \sqrt{21}$

$= 2\sqrt{20} + \sqrt{21}$

$\therefore$ $\boldsymbol{c = 20, d = 21}$

7. (a) $\frac{1}{\sqrt{5}} = \frac{1}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$

$= \frac{\sqrt{5}}{5}$

$= \frac{2.236}{5}$

$=$ **0.447** (to 3 d.p.)

(b) $\sqrt{32} = \sqrt{16\times2}$

$= 4\sqrt{2}$

$= 4(1.414)$

$=$ **5.656**

(c) $\frac{4}{\sqrt{20}} = \frac{4}{\sqrt{4\times5}}$

$= \frac{4}{2\sqrt{5}}$

$= \frac{2}{\sqrt{5}}$

$= \frac{2}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$

$= \frac{2\sqrt{5}}{5}$

$= \frac{2(2.236)}{5}$

$=$ **0.894** (to 3 d.p.)

(d) $\frac{1}{\sqrt{5}+\sqrt{2}} = \frac{1}{\sqrt{5}+\sqrt{2}} \times \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}}$

$= \frac{\sqrt{5}-\sqrt{2}}{5-2}$

$= \frac{2.236-1.414}{3}$

$=$ **0.274**

Worksheet 4E Exponential growth and decay

1. (a) $V = 68\,000 \times 1.07^t$
When $t = 0$,
$V = 68\,000$
$\therefore$ Initial value = **$68 000**

(b) When $t = 10$,
$V = 68\,000 \times 1.07^{10}$
$= 134\,000$ (to the nearest thousand)
$\therefore$ Value after 10 years = **$134 000**

2. (a) Height of P after 1 week $= \frac{118}{100} \times 30$ cm
$= \mathbf{35.4\text{ cm}}$
Height of Q after 1 week $= \frac{115}{100} \times 36$ cm
$= \mathbf{41.4\text{ cm}}$
Height of R after 1 week $= \frac{111}{100} \times 40$ cm
$= \mathbf{44.4\text{ cm}}$

(b) Height of P after 8 weeks $= \left(\frac{118}{100}\right)^8 \times 30$ cm
$= 112.77$ cm (to 5 s.f.)
Height of Q after 8 weeks $= \left(\frac{115}{100}\right)^8 \times 36$ cm
$= 110.12$ cm (to 5 s.f.)
Height of R after 8 weeks $= \left(\frac{111}{100}\right)^8 \times 40$ cm
$= 92.182$ cm (to 5 s.f.)
$\therefore$ Plant **P** is the tallest.

3. (a) Population in December 2027 $= \left(\frac{105}{100}\right)^3 \times 80\,000$
$= \mathbf{92\,610}$

(b) Population in December 2030 $= \left(\frac{105}{100}\right)^6 \times 80\,000$
$= 107\,208$ (to the nearest whole number)
Population in December 2032 $= \left(\frac{105}{100}\right)^8 \times 80\,000$
$= 118\,196$ (to the nearest whole number)
Population in December 2033 $= \left(\frac{105}{100}\right)^9 \times 80\,000$
$= 124\,106$ (to the nearest whole number)
$\therefore$ The population will exceed 120 000 in **2033**.

4. (a) Initial volume of water $= \frac{1}{2} \times 1.4 \times 0.8 \times 1.2$ m^3
$= 0.672$ m^3
$= 672\ l$
Volume of water left after 24 h $= \left(\frac{98}{100}\right)^{24} \times 672\ l$
$= \mathbf{414\ \mathit{l}}$ (to 3 s.f.)

(b) Volume of water leaked $= 413.80 - \left(\frac{97}{100}\right)^6 \times 413.80$
$= \mathbf{69.1\ \mathit{l}}$ (to 3 s.f.)

5. (a) Number of times the mass was halved $= \frac{5 \times 60}{75}$
$= 4$
Initial mass $= 90 \times 2^4$ g
$= 1440$ g
$\therefore$ Mass of substance after t hours, $\mathbf{m = 1440 \times 0.5^{0.8t}}$

(b) 720 min = 12 h
When $t = 12$,
$m = 1440 \times 0.5^{0.8(12)}$
$= 0.567$ g (to 3 s.f.) < 2 g (shown)

Worksheet 4F Standard form

1. (a) $7.16 \times 10^3 = \mathbf{7160}$
(b) $4.80 \times 10^5 = \mathbf{480\,000}$

2. (a) $3.92 \times 10^{-2} = \mathbf{0.0392}$
(b) $5.70 \times 10^{-6} = \mathbf{0.000\,005\,70}$

3. (a) $4.195 \times 10^{-7} + 8.3 \times 10^{-6} = \mathbf{8.72 \times 10^{-6}}$ (to 3 s.f.)
(b) $7.02 \times 10^8 - 3.4 \times 10^7 = \mathbf{6.68 \times 10^8}$
(c) $(6.52 \times 10^4) \times (1.8 \times 10^{-9}) = \mathbf{1.17 \times 10^{-4}}$ (to 3 s.f.)
(d) $(9.6 \times 10^{-3}) \div (8 \times 10^{-5}) = \mathbf{1.2 \times 10^2}$

4. OPEN $4.65 \times 10^7 = 4 \times 10^7 + 0.65 \times 10^7$
$= 4 \times 10^7 + 6.5 \times 10^6$
$= 6.5 \times 10^6 + 4 \times 10^7$
$\therefore$ An example of the values is $\mathbf{n = 6}$, $\mathbf{p = 6.5}$ and $\mathbf{q = 4}$.

5. (i) 0.000 090 45 kg = **0.000 0905 kg** (to 3 s.f.)
(ii) 0.000 0905 kg = $\mathbf{9.05 \times 10^{-5}}$ **kg**

6. (i) 300 000 000 m/s = $\mathbf{3 \times 10^8}$ **m/s**
(ii) Distance travelled $= \frac{(3 \times 10^8) \times (60 \times 60)}{1000}$
$= \mathbf{1.08 \times 10^9}$ **km**

7. (i) 328 nanoseconds = **330 ns** (to 2 s.f.)
(ii) 330 ns $= 330 \times 10^{-9}$ s
$= \mathbf{3.3 \times 10^{-7}}$ **s**

8. Total population $= (145.0 + 50.18 + 18.46 + 4.220) \times 10^6$
$= 217.86 \times 10^6$
$= \mathbf{2.18 \times 10^8}$ (to 3 s.f.)

9. (i) $0.000\,06 = \mathbf{6 \times 10^{-5}}$
(ii) Average number of people $= \frac{9.65 \times 10^7}{3.31 \times 10^5}$
$= \mathbf{2.92 \times 10^2}$ (to 3 s.f.)

10. Speed of light ray $= \frac{10\text{ km}}{33.3\ \mu\text{s}}$
$= \frac{10 \times 10^3\text{ m}}{33.3 \times 10^{-6}\text{ s}}$
$= 300\,000\,000$ m/s (to 3 s.f.)
$= \mathbf{3.00 \times 10^8}$ **m/s**

11. (a) Population of India = $\mathbf{1.38 \times 10^9}$
(b) Number of people $= (6.05 \times 10^7) - (1.67 \times 10^7)$
$= 4.38 \times 10^7$
$= 43.8 \times 10^6$
$=$ **43.8 million**
(c) Population of Namibia $\approx \frac{1}{50}(1.29 \times 10^8)$
$= 2.58 \times 10^6$
$=$ **2.6 million** (to 2 s.f.)

12. **(a)** Mass of 1 CO_2 molecule $= 1.99 \times 10^{-23} + 2(2.66 \times 10^{-23})$
$= 7.31 \times 10^{-23}$ g
$= \mathbf{7.31 \times 10^{-20}}$ **mg**

(b) Mass of 1 H_2O molecule $= 2(1.66 \times 10^{-24}) + 2.66 \times 10^{-23}$
$= 2.992 \times 10^{-23}$ g

Number of water molecules $= \dfrac{500}{2.992 \times 10^{-23}}$
$= \mathbf{1.67 \times 10^{25}}$ (to 3 s.f.)

13. **(a)** $2870 = \mathbf{2.87 \times 10^3}$

(b) **(i)** Percentage decrease $= \dfrac{1.34 \times 10^3 - 9.86 \times 10^2}{1.34 \times 10^3} \times 100\%$
$= \mathbf{26.4\%}$ (to 3 s.f.)

(ii) Decrease in volume $= (1.34 \times 10^3 - 9.86 \times 10^2) \times 60$
$= 21\ 240$ litres
$= 2.124 \times 10^4$ litres
$= \mathbf{2.1 \times 10^4}$ **litres** (to 2 s.f.)

Review Exercise 4

1. **(a)** $(-5)^0 > -5^0$

(b) $\left(\frac{7}{6}\right)^{-4} > \left(\frac{1}{8}\right)^{\frac{2}{3}}$

2. **(a)** $5^2 + 5^0 - 5^{-2} = 25 + 1 - \frac{1}{25}$
$= \mathbf{25\frac{24}{25}}$

(b) $\sqrt[7]{\left(-\frac{3}{5}\right)^2 + \left(\frac{5}{4}\right)^{-2}} = \sqrt[7]{\left(-\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2}$
$= \sqrt[7]{\frac{9}{25} + \frac{16}{25}}$
$= \sqrt[7]{1}$
$= \mathbf{1}$

3. **(a)** $\left(\frac{27x^6}{64y^{12}}\right)^{-\frac{2}{3}} = \left(\frac{64y^{12}}{27x^6}\right)^{\frac{2}{3}}$
$= \left(\frac{4^3 y^{12}}{3^3 x^6}\right)^{\frac{2}{3}}$
$= \mathbf{\frac{16y^8}{9x^4}}$

(b) $\sqrt[3]{x} \times \frac{9}{\sqrt{x}} \div \sqrt[4]{x^{17}} = x^{\frac{1}{3}} \times 9x^{-\frac{1}{2}} \div x^{\frac{17}{4}}$
$= 9x^{-\frac{53}{12}}$
$= \mathbf{\frac{9}{\sqrt[12]{x^{53}}}}$

4. **(a)** $8^{x-2} = 1$
$= 8^0$
$x - 2 = 0$
$x = 2$
$\therefore x = \mathbf{2}$

(b) $49^x = 343^{1+3x}$
$(7^2)^x = (7^3)^{1+3x}$
$7^{2x} = 7^{3+9x}$
$2x = 3 + 9x$
$7x = -3$
$x = -\frac{3}{7}$
$\therefore x = \mathbf{-\frac{3}{7}}$

(c) $625^x \times 5 = \frac{1}{25}$
$(5^4)^x \times 5 = \frac{1}{5^2}$
$5^{4x+1} = 5^{-2}$
$4x + 1 = -2$
$4x = -3$
$x = -\frac{3}{4}$
$\therefore x = \mathbf{-\frac{3}{4}}$

(d) $3^{x+1} + 3^x = 4$
$3(3^x) + 3^x = 4$
$4(3^x) = 4$
$3^x = 1$
$= 3^0$
$x = 0$
$\therefore x = \mathbf{0}$

5. $36^{x+y} = 216$ — (1)
$\sqrt{9^y} = \left(\sqrt[3]{27^x}\right)^4$ — (2)

From (1),
$(6^2)^{x+y} = 6^3$
$6^{2x+2y} = 6^3$
$2x + 2y = 3$ — (3)

From (2),
$\sqrt{3^{2y}} = (27^x)^{\frac{4}{3}}$
$(3^{2y})^{\frac{1}{2}} = [(3)^{3x}]^{\frac{4}{3}}$
$3^y = 3^{4x}$
$y = 4x$ — (4)

Substitute (4) into (3):
$2x + 2(4x) = 3$
$2x + 8x = 3$
$10x = 3$
$x = \frac{3}{10}$

Substitute $x = \frac{3}{10}$ into (4):
$y = 4\left(\frac{3}{10}\right)$
$= \frac{6}{5}$
$\therefore x = \mathbf{\frac{3}{10}}, y = \mathbf{\frac{6}{5}}$

6. (a) $6.5 \times 10^{-12} = 6.5 \times 10^{-5} \times 10^{-7}$
$= 0.000\,065 \times 10^{-7}$
$\therefore a = \mathbf{0.000\,065}$
(b) (i) 2.3 billion = $\mathbf{2.3 \times 10^9}$
(ii) Population $= (2.3 \times 10^9) \times 2^5$
$= 73.6 \times 10^9$
$= \mathbf{7.36 \times 10^{10}}$

7. (a) $\left(\dfrac{\sqrt{3} \times \sqrt{375}}{\sqrt{5}}\right)^2 = \dfrac{3 \times 375}{5}$
$= \mathbf{225}$
(b) $\dfrac{7}{\sqrt{2}}\left(\dfrac{\sqrt{128}}{6} - \dfrac{6}{\sqrt{2}}\right) = \dfrac{7}{\sqrt{2}}\left(\dfrac{\sqrt{64 \times 2}}{6} - \dfrac{6}{\sqrt{2}} \times \dfrac{\sqrt{2}}{\sqrt{2}}\right)$
$= \dfrac{7}{\sqrt{2}}\left(\dfrac{8\sqrt{2}}{6} - \dfrac{6\sqrt{2}}{2}\right)$
$= \dfrac{7}{\sqrt{2}}\left(-\dfrac{5\sqrt{2}}{3}\right)$
$= -\mathbf{\dfrac{35}{3}}$

8. Using Pythagoras' Theorem,
$PR^2 = PQ^2 + QR^2$
$= (7\sqrt{3} - 4\sqrt{5})^2 + (10\sqrt{5} - 6\sqrt{3})^2$
$= 147 - 56\sqrt{15} + 80 + 500 - 120\sqrt{15} + 108$
$= \mathbf{835 - 176\sqrt{15}}$

9. $\sqrt{a + b\sqrt{2}} = \dfrac{6}{(\sqrt{2} - 1)^2}$
$\left(\sqrt{a + b\sqrt{2}}\right)^2 = \left[\dfrac{6}{(\sqrt{2} - 1)^2}\right]^2$
$a + b\sqrt{2} = \left(\dfrac{6}{2 - 2\sqrt{2} + 1}\right)^2$
$= \left(\dfrac{6}{3 - 2\sqrt{2}}\right)^2$
$= \dfrac{36}{9 - 12\sqrt{2} + 8}$
$= \dfrac{36}{17 - 12\sqrt{2}}$
$= \dfrac{36}{17 - 12\sqrt{2}} \times \dfrac{17 + 12\sqrt{2}}{17 + 12\sqrt{2}}$
$= \dfrac{612 + 432\sqrt{2}}{289 - 288}$
$= 612 + 432\sqrt{2}$
$\therefore \mathbf{a = 612, b = 432}$

10. $a + b = (\sqrt{a} + \sqrt{b})^2 - 2 \times \sqrt{a} \times \sqrt{b}$
$= (\sqrt{11} + \sqrt{15})^2 - 2 \times (\sqrt{165} - \sqrt{11})$
$= 11 + 2\sqrt{165} + 15 - 2\sqrt{165} + 2\sqrt{11}$
$= 26 + 2\sqrt{11}$
$\therefore \mathbf{p = 26, q = 11}$

11. (i) $\dfrac{\sqrt{7} + 1}{\sqrt{7} - 2} = \dfrac{\sqrt{7} + 1}{\sqrt{7} - 2} \times \dfrac{\sqrt{7} + 2}{\sqrt{7} + 2}$
$= \dfrac{7 + 2\sqrt{7} + \sqrt{7} + 2}{7 - 4}$
$= \dfrac{9 + 3\sqrt{7}}{3}$
$= \mathbf{3 + \sqrt{7}}$
(ii) Area of triangle $= \left(\dfrac{1}{2} \times 2\sqrt{7} \times h\right)$ cm^2
$\dfrac{\sqrt{7} + 1}{\sqrt{7} - 2} = h\sqrt{7}$
$h\sqrt{7} = \dfrac{\sqrt{7} + 1}{\sqrt{7} - 2}$
$h = \dfrac{\sqrt{7} + 1}{7 - 2\sqrt{7}}$
$= \dfrac{\sqrt{7} + 1}{7 - 2\sqrt{7}} \times \dfrac{7 + 2\sqrt{7}}{7 + 2\sqrt{7}}$
$= \dfrac{7\sqrt{7} + 14 + 7 + 2\sqrt{7}}{49 - 28}$
$= \dfrac{9\sqrt{7} + 21}{21}$
$= \dfrac{3\sqrt{7} + 7}{7}$
$= \dfrac{1}{7}(3\sqrt{7} + 7)$ (shown)

Mid-year Checkpoint A

Section A

1. $\dfrac{1}{125} = 5^k$
$\dfrac{1}{5^3} = 5^k$
$5^{-3} = 5^k$
$k = \mathbf{-3}$ [1]

2. Diagram **1** [1]

3. $\dfrac{5x}{7} - \dfrac{4x - 9}{2} = \dfrac{2(5x) - 7(4x - 9)}{14}$
$= \dfrac{10x - 28x + 63}{14}$ [1]
$= \mathbf{\dfrac{63 - 18x}{14}}$ [1]

4. $-9 \leqslant 2x + 5 < 3$
$-14 \leqslant 2x < -2$ [1]
$\mathbf{-7 \leqslant x < -1}$ [1]

5. Gradient $= \frac{-2-8}{1-(-3)}$

$= -\frac{5}{2}$ [1]

Substitute $m = -\frac{5}{2}$, $x = 1$, $y = -2$ into $y = mx + c$:

$-2 = -\frac{5}{2}(1)+c$

$= -\frac{5}{2}+c$

$c = \frac{1}{2}$

∴ Equation of line: $\boldsymbol{y = -\frac{5}{2}x+\frac{1}{2}}$ [1]

6. $8x - 4y = 5$ — (1)

$6x + 3y = 1$ — (2)

(1) ÷ 4: $2x - y = \frac{5}{4}$ — (3)

(2) ÷ 3: $2x + y = \frac{1}{3}$ — (4)

(3) + (4): $4x = \frac{19}{12}$ [1]

$x = \frac{19}{48}$ [1]

Substitute $x = \frac{19}{48}$ into (4):

$2\left(\frac{19}{48}\right)+y = \frac{1}{3}$

$\frac{19}{24}+y = \frac{1}{3}$

$y = -\frac{11}{24}$ [1]

∴ $\boldsymbol{x = \frac{19}{48}, y = -\frac{11}{24}}$

7. (a) $16x^2 + kx - 9 = 0$

When $x = -\frac{3}{8}$,

$16\left(-\frac{3}{8}\right)^2+k\left(-\frac{3}{8}\right)-9 = 0$

$-\frac{3}{8}k-\frac{27}{4} = 0$

$\frac{3}{8}k = -\frac{27}{4}$

$k = -18$ [1]

∴ $k = \mathbf{-18}$

(b) $16x^2 - 18x - 9 = 0$

$(8x + 3)(2x - 3) = 0$ [1]

$8x + 3 = 0$ or $2x - 3 = 0$

$8x = -3$ $\quad$ $2x = 3$

$x = -\frac{3}{8}$ $\quad$ $x = 1\frac{1}{2}$

∴ The second possible value of x is $\mathbf{1\frac{1}{2}}$. [1]

8. $\frac{4x^2-12x+9}{4x^2-9} = \frac{(2x-3)^2}{(2x+3)(2x-3)}$ [2]

$= \boldsymbol{\frac{2x-3}{2x+3}}$ [1]

9. $(3x + 4)(2x - 1) = 8$

$6x^2 + 5x - 4 = 8$

$6x^2 + 5x - 12 = 0$ [1]

$x = \frac{-5\pm\sqrt{5^2-4(6)(-12)}}{2(6)}$ [1]

$= \frac{-5\pm\sqrt{313}}{12}$

$= 1.06$ or -1.89 (to 3 s.f.) [1]

∴ $x = \mathbf{1.06}$ or $x = \mathbf{-1.89}$

10. (a) $9^7 = (3^2)^7$

$= \mathbf{3^{14}}$ [1]

(b) $\frac{33a^5}{10b} \div \frac{44ab}{5} = \frac{33a^5}{10b} \times \frac{5}{44ab}$

$= \boldsymbol{\frac{3a^4}{8b^2}}$ [2]

11. Let there be x members in the choir.

$x(x - 1) = 702$ [1]

$x^2 - x - 702 = 0$

$(x - 27)(x + 26) = 0$ [1]

$x - 27 = 0$ or $x + 26 = 0$

$x = 27$ or $x = -26$

∴ There are **27** members in the choir. [1]

12. (i) $y = (x + 4)(x - 5)$

When $y = 0$,

$(x + 4)(x - 5) = 0$

$x + 4 = 0$ or $x - 5 = 0$

$x = -4$ $\quad$ $x = 5$

∴ $A(-4, 0)$, $B(5, 0)$ [2]

When $x = 0$,

$y = -20$

∴ $C(0, -20)$ [1]

∴ $\boldsymbol{A(-4, 0), B(5, 0), C(0, -20)}$

(ii) When $x = 0.5$,

$y = (0.5 + 4)(0.5 - 5)$

$= -20.25$

∴ Coordinates of minimum point are **(0.5, –20.25)** [1]

Section B

13. (a) $s = ut+\frac{1}{2}at^2$

$2s = 2ut + at^2$ [1]

$at^2 = 2s - 2ut$ [1]

$\boldsymbol{a = \frac{2s-2ut}{t^2}}$ [1]

(b) $\frac{3}{x-5}-\frac{6}{2x+1} = \frac{3(2x+1)-6(x-5)}{(2x+1)(x-5)}$ [1]

$= \frac{6x+3-6x+30}{(2x+1)(x-5)}$ [1]

$= \boldsymbol{\frac{33}{(2x+1)(x-5)}}$ [1]

14. Volume of cylinder = $(6+5\sqrt{2})\pi$ cm³

Let the height of the cylinder be h cm.

$$\pi(2+\sqrt{2})^2 h = (6+5\sqrt{2})\pi \quad \textbf{[1]}$$
$$(4+4\sqrt{2}+2)h = 6+5\sqrt{2}$$
$$(6+4\sqrt{2})h = 6+5\sqrt{2}$$
$$h = \frac{6+5\sqrt{2}}{6+4\sqrt{2}}$$
$$= \frac{6+5\sqrt{2}}{6+4\sqrt{2}} \times \frac{6-4\sqrt{2}}{6-4\sqrt{2}} \quad \textbf{[1]}$$
$$= \frac{(6+5\sqrt{2})(6-4\sqrt{2})}{6^2-(4\sqrt{2})^2}$$
$$= \frac{36-24\sqrt{2}+30\sqrt{2}-40}{36-32} \quad \textbf{[1]}$$
$$= \frac{6\sqrt{2}-4}{4} \quad \textbf{[1]}$$
$$= \frac{3}{2}\sqrt{2}-1$$

∴ The height of the cylinder is $\left(\frac{3}{2}\sqrt{2}-1\right)$ **cm**. **[1]**

15. **(a)** **70 m** **[1]**

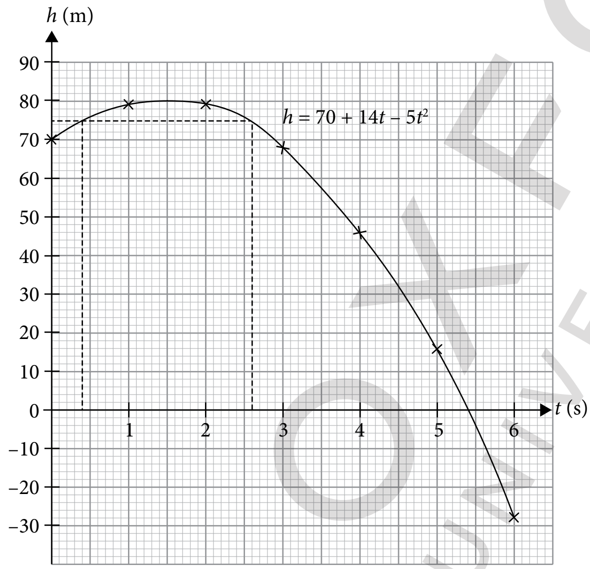
(b) When $t = 6$, $h = p$,

$p = 70 + 14(6) - 5(6)^2$

$= -26$ **[1]**

∴ $p = \mathbf{-26}$

(c)



[2]

(d) **(i)** **80 m** **[1]**

(ii) Length of time = (2.6 – 0.4) s **[1]**

= **2.2 s** **[1]**

(e) **(i)** $t = \mathbf{5.4}$ **[1]**

(ii) The pebble hits the sea 5.4 s after leaving Don's hand. **[1]**

Mid-year Checkpoint B

Section A

1. 2.59×10^{-4} milliseconds $= 2.59 \times 10^{-4} \times 10^{-3}$ seconds

$= 2.59 \times 10^{-7}$ seconds **[1]**

$= \dfrac{2.59 \times 10^{-7}}{10^{-9}}$ nanoseconds

$= 2.59 \times 10^{2}$ nanoseconds

∴ $k = \mathbf{259}$ **[1]**

2. $\dfrac{9a^3b}{c^4} \div \dfrac{27ab}{2c^3} = \dfrac{9a^3b}{c^4} \times \dfrac{2c^3}{27ab}$

$= \mathbf{\dfrac{2a^2}{3c}}$ **[2]**

3. Prime numbers: 101, 103, 107, 109

P(prime number) $= \dfrac{4}{10}$ **[1]**

$= \mathbf{\dfrac{2}{5}}$ **[1]**

4. $9x^4 - 144 = 9(x^4 - 16)$

$= 9(x^2 + 4)(x^2 - 4)$ **[1]**

$= \mathbf{9(x^2 + 4)(x + 2)(x - 2)}$ **[1]**

5. $\left(\sqrt[3]{p} - \sqrt[3]{q}\right)\left(\sqrt[3]{p^2} + \sqrt[3]{pq} + \sqrt[3]{q^2}\right)$

$= \left(p^{\frac{1}{3}} - q^{\frac{1}{3}}\right)\left(p^{\frac{2}{3}} + p^{\frac{1}{3}}q^{\frac{1}{3}} + q^{\frac{2}{3}}\right)$

$= p + p^{\frac{2}{3}}q^{\frac{1}{3}} + p^{\frac{1}{3}}q^{\frac{2}{3}} - p^{\frac{2}{3}}q^{\frac{1}{3}} - p^{\frac{1}{3}}q^{\frac{2}{3}} - q$ **[1]**

$= \mathbf{p - q}$ **[1]**

6. $2^9 + 2^9 + 2^9 + 2^9 = 4(2^9)$

$= 4[(2^2)^{4.5}]$

$= 4(4^{4.5})$ **[1]**

$= 4^{5.5}$

∴ $a = \mathbf{5.5}$ **[1]**

7. **(i)** $y = -(2x + 5)(x - 1)$

When $y = 0$,

$-(2x + 5)(x - 1) = 0$

$2x + 5 = 0$ or $x - 1 = 0$

$2x = -5$ $\quad x = 1$

$x = -2\frac{1}{2}$

∴ x-coordinate of A is $\mathbf{-2\frac{1}{2}}$ **[1]**

(ii) When $x = 0$,

$y = 5$

∴ $C(0, 5)$

Area of △$ABC = \dfrac{1}{2}\left[1-\left(-2\dfrac{1}{2}\right)\right](5)$ units² **[1]**

= **8.75 units²** **[1]**

8. $\dfrac{400}{v} - \dfrac{400}{v+30} = 3$ **[1]**

$400(v + 30) - 400v = 3v(v + 30)$

$400v + 12\,000 - 400v = 3v^2 + 90v$

$3v^2 + 90v - 12\,000 = 0$

$v^2 + 30v - 4000 = 0$

$(v + 80)(v - 50) = 0$ **[1]**

$v + 80 = 0$ or $v - 50 = 0$

$v = -80$ $\quad v = 50$

∴ $v = \mathbf{50}$ **[1]**

9. Using similar triangles,

$$\frac{BD}{BA} = \frac{BA}{BC}$$

$$\frac{q+CD}{p} = \frac{p}{q} \quad [1]$$

$$q + CD = \frac{p^2}{q}$$

$$CD = \left(\boldsymbol{\frac{p^2}{q} - q}\right) \textbf{ cm} \quad [1]$$

10. (i) $A = 3 \times 2 \times 7^2$

$B = 3 \times 2^3 \times 7$

$\therefore$ $\boldsymbol{p = 2, q = 7, r = 1}$ [2]

(ii) HCF $= 2 \times 3 \times 7$

$= \mathbf{42}$ [1]

11. $P = kQ^3$

Percentage change $= 2^3 \times 100\%$ [1]

$= 800\%$ [1]

$\therefore$ Percentage change $= 800\% - 100\%$

$= \mathbf{700\%}$ [1]

12. (i) $\frac{36}{c} = \frac{c}{1}$

$c^2 = 36$

$c = \pm 6$

$\therefore c = 6$ [1]

$b = \frac{1}{6}$

$a = \frac{1}{6} \div 6$

$= \frac{1}{36}$ [1]

$\therefore \boldsymbol{a = \frac{1}{36}, b = \frac{1}{6}, c = 6}$

(ii) $T_n = \boldsymbol{\frac{1}{216}(6^n)}$ [1]

(iii) Since $36^9 = 6^{18}$ is a multiple of 6, then $36^9 + 1$ has a remainder of 1 when divided by 6. There is no integer value of n for which $36^n + 1$ is a multiple of 6. Hence $36^9 + 1$ is not a multiple of 6 and is not a term of this sequence. [1]

Section B

13. (a) $x^2 + 7x - 5 = x^2 + 7x + \left(\frac{7}{2}\right)^2 - \left(\frac{7}{2}\right)^2 - 5$ [1]

$= \boldsymbol{\left(x + \frac{7}{2}\right)^2 - \frac{69}{4}}$ [1]

(b) Coordinates of minimum point: $\boldsymbol{\left(-3\frac{1}{2}, -17\frac{1}{4}\right)}$ [1]

(c) $y = x^2 + 7x - 5$

$= \left(x + \frac{7}{2}\right)^2 - \frac{69}{4}$

$\left(x + \frac{7}{2}\right)^2 = y + \frac{69}{4}$ [1]

$x + \frac{7}{2} = \pm\sqrt{y + \frac{69}{4}}$

$x = \boldsymbol{-\frac{7}{2} \pm \sqrt{y + \frac{69}{4}}}$ [1]

14. (a) $\frac{4}{5-2x} - \frac{7}{x+3} = \frac{4(x+3) - 7(5-2x)}{(5-2x)(x+3)}$

$= \frac{4x + 12 - 35 + 14x}{(5-2x)(x+3)}$ [1]

$= \boldsymbol{\frac{18x - 23}{(5-2x)(x+3)}}$ [1]

(b) $\frac{42}{5-x} = 4x + 9$

$42 = (4x + 9)(5 - x)$

$= 20x - 4x^2 + 45 - 9x$

$= -4x^2 + 11x + 45$

$4x^2 - 11x - 3 = 0$

$(4x + 1)(x - 3) = 0$ [1]

$4x + 1 = 0$ or $x - 3 = 0$

$4x = -1$ $\quad$ $x = 3$ [1]

$x = -\frac{1}{4}$ [1]

$\therefore x = \boldsymbol{-\frac{1}{4}}$ **or** $\boldsymbol{x = 3}$

15. (a) Number of minutes $= \boldsymbol{\frac{4500}{x}}$ [1]

(b) Number of minutes $= \boldsymbol{\frac{4500}{x-8}}$ [1]

(c) $\frac{4500}{x-8} - \frac{4500}{x} = \frac{30}{60}$ [1]

$\frac{4500x - 4500(x-8)}{x(x-8)} = \frac{1}{2}$

$\frac{4500x - 4500x + 36\ 000}{x(x-8)} = \frac{1}{2}$

$\frac{36\ 000}{x(x-8)} = \frac{1}{2}$ [1]

$72\ 000 = x^2 - 8x$

$x^2 - 8x - 72\ 000 = 0$ (shown) [1]

(d) $x^2 - 8x - 72\ 000 = 0$

$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(-72\ 000)}}{2(1)}$ [1]

$= \frac{8 \pm \sqrt{288\ 064}}{2}$

$= 272.36$ or -264.36 (to 2 d.p.) [2]

$\therefore x = \mathbf{272.36}$ or $x = \mathbf{-264.36}$

(e) Time taken $= \frac{4500}{2(272.36) - 8}$

$= 8.3843$ min (to 5 s.f.) [1]

$=$ **8 min 20 s** (to the nearest 10 s) [1]

5 Coordinate Geometry

Worksheet 5A Length of a line segment

1. **(a)** Length of $AB = \sqrt{[3-(-5)]^2+(1-7)^2}$

$= \sqrt{100}$

$= \mathbf{10\ units}$

(b) $C(0, -4)$

Length of $CD = \sqrt{(6-0)^2+[-2-(-4)]^2}$

$= \sqrt{40}$

$=$ **6.32 units** (to 3 s.f.)

(c) Length of $EF = \sqrt{(2-9)^2+(-8-0)^2}$

$= \sqrt{113}$

$=$ **10.6 units** (to 3 s.f.)

(d) Length of $GH = \sqrt{[0-(-1)]^2+[5-(-5)]^2}$

$= \sqrt{101}$

$=$ **10.0 units** (to 3 s.f.)

(e) PQ is a horizontal line.

Length of $PQ = 11 - 4$

$=$ **7 units**

(f) RS is a vertical line.

Length of $RS = 7 - (-3)$

$=$ **10 units**

2. Since distance $= \sqrt{52}$ units,

$\sqrt{(k-3)^2+[-5-(-1)]^2} = \sqrt{52}$

$\sqrt{(k-3)^2+16} = \sqrt{52}$

$(k-3)^2 + 16 = 52$

$(k-3)^2 = 36$

$k - 3 = 6$ or $k - 3 = -6$

$k = 9$ $\quad$ $k = -3$

$\therefore$ $k =$ **9** or $k =$ **−3**

3. **(a)** When $y = 0$,

$4x + 21 = 0$

$4x = -21$

$x = -5.25$

$\therefore P(-5.25, 0)$

When $x = 0$,

$-3y + 21 = 0$

$3y = 21$

$y = 7$

$\therefore Q(0, 7)$

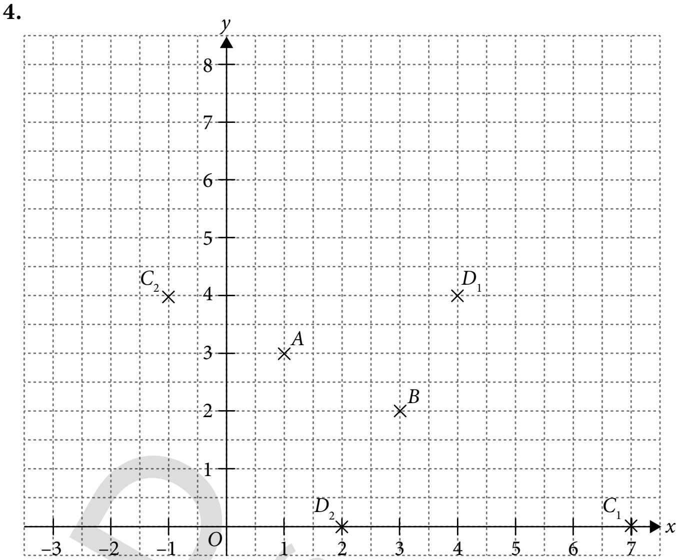
$\therefore$ **$P(-5.25, 0)$, $Q(0, 7)$**

(b) Length of $PQ = \sqrt{(-5.25-0)^2+(0-7)^2}$

$= \sqrt{76.5625}$

$=$ **8.75 units**

4.



5. **(a)** **(i)** Let the coordinates of R be $(x_R, 0)$.

$PQ = QR$

$\sqrt{(-2-2)^2+(4-1)^2} = \sqrt{[x_R-(-2)]^2+(0-4)^2}$

$25 = (x_R + 2)^2 + 16$

$(x_R + 2)^2 = 9$

$x_R + 2 = 3$ or $x_R + 2 = -3$

$x_R = 1$ $\quad$ $x_R = -5$

$\therefore$ **$R(1, 0)$** or **$R(-5, 0)$**

(ii) Let the coordinates of R be $(0, y_R)$.

$PR = QR$

$\sqrt{(0-2)^2+(y_R-1)^2} = \sqrt{[0-(-2)]^2+(y_R-4)^2}$

$4 + y_R^2 - 2y_R + 1 = 4 + y_R^2 - 8y_R + 16$

$6y_R = 15$

$y_R = 2.5$

$\therefore$ **$R(0, 2.5)$**

(b) $PS = QS$

$\sqrt{(a-2)^2+(b-1)^2} = \sqrt{[a-(-2)]^2+(b-4)^2}$

$(a-2)^2 + (b-1)^2 = (a+2)^2 + (b-4)^2$

$a^2 - 4a + 4 + b^2 - 2b + 1 = a^2 + 4a + 4 + b^2 - 8b + 16$

$\mathbf{8a - 6b + 15 = 0}$

6. **(i)** $BC = \sqrt{[1-(-4)]^2+[3-(-2)]^2}$

$= \sqrt{50}$

$=$ **7.07 units** (to 3 s.f.)

(ii) $AB = 1 - (-5)$

$= 6$ units

$AC = \sqrt{[-4-(-5)]^2+(-2-3)^2}$

$= \sqrt{26}$ units

Since no two sides are of the same length, $\triangle ABC$ is not an isosceles triangle.

(iii) **$D(2, -2)$**

(iv) OPEN A possible point is **$P(-2, -2)$**.

(v) Let the shortest distance from A to BC be h units.

Area of $\triangle ABC = \frac{1}{2}(6)(5)$

$$\frac{1}{2}\left(\sqrt{50}\right)h = \frac{1}{2}(6)(5)$$

$$h = \frac{30}{\sqrt{50}}$$

$= 4.24$ units (to 3 s.f.)

$\therefore$ Shortest distance from A to BC = **4.24 units**

Worksheet 5B Gradient of a straight line

1. (a) Gradient of $AB = \frac{9-2}{5-(-4)}$

$= \mathbf{\frac{7}{9}}$

(b) $D(0, -7)$

Gradient of $CD = \frac{-7-(-3)}{0-(-8)}$

$= \mathbf{-\frac{1}{2}}$

(c) Gradient of $EF = \frac{12-6}{3-0}$

$= \mathbf{2}$

(d) Gradient of $GH = \frac{0-(-10)}{4-7}$

$= \mathbf{-\frac{10}{3}}$

(e) PQ is a horizontal line.
Gradient of PQ = **0**

(f) RS is a vertical line.
Gradient of PQ is **undefined**

2. (a) Gradient of $AB = \frac{-7-5}{2-(-4)}$

$= \mathbf{-2}$

(b) Length of $AB = \sqrt{(-4-2)^2 + [5-(-7)]^2}$

$= \sqrt{180}$

= **13.4 units** (to 3 s.f.)

3. (i) Gradient of $AB = \frac{-1-4}{6-0}$

$= \mathbf{-\frac{5}{6}}$

(ii) Length of $BC = \sqrt{(7-6)^2 + [2-(-1)]^2}$

$= \sqrt{10}$

= **3.16 units** (to 3 s.f.)

(iii) x-coordinate of $D = 0 + 1$

$= 1$

y-coordinate of $D = 4 + 3$

$= 7$

$\therefore$ ***D*(1, 7)**

4. Gradient $= -\frac{1}{4}$

$$\frac{-3-(-2)}{k-8} = -\frac{1}{4}$$

$$-\frac{1}{k-8} = -\frac{1}{4}$$

$$k - 8 = 4$$

$$k = 12$$

$\therefore k = \mathbf{12}$

5. Let the coordinates of B be (h, k).

Gradient of $AB = \frac{2}{3}$

$$\frac{k-5}{h-6} = \frac{2}{3}$$

$$3k - 15 = 2h - 12$$

$$3k - 2h = 3$$

Let $h = 0$: $3k = 3$

$k = 1$

Let $h = -1$: $3k - 2(-1) = 3$

$3k + 2 = 3$

$3k = 1$

$k = \frac{1}{3}$

$\therefore$ Two possible pairs of coordinates are $\mathbf{B(0, 1)}$ and $\mathbf{B\left(-1, \frac{1}{3}\right)}$.

6. Gradient of AB = Gradient of AC

$$\frac{4-9}{k-1} = \frac{-3-9}{6-1}$$

$$-\frac{5}{k-1} = -\frac{12}{5}$$

$$k - 1 = \frac{25}{12}$$

$$k = \frac{37}{12}$$

$\therefore k = \mathbf{\frac{37}{12}}$

7. (i) Gradient of AB = Gradient of BC

$$\frac{-7-(-1)}{2-(-4)} = \frac{k-(-7)}{h-2}$$

$$-1 = \frac{k+7}{h-2}$$

$$2 - h = k + 7$$

$h + k + 5 = 0$ (shown)

(ii) Let $h = 1$: $1 + k + 5 = 0$

$k = -6$

$\therefore$ A possible point is $\mathbf{C(1, -6)}$.

Challenge Myself!

8. (a) Gradient of $PR = \frac{8-6}{7-2}$

$= \frac{2}{5}$

Gradient of $PQ = \frac{3-6}{5-2}$

$= -1$

Since gradient of PR × gradient of $PQ = -\frac{2}{5} \neq -1$, then PR is not perpendicular to PQ. (shown)

(b) Let the coordinates of S be $(x_S, 1)$.

Gradient of PQ × gradient of $RS = -1$

$$-1 \times \frac{1-8}{x_S - 7} = -1$$
$$-\frac{7}{x_S - 7} = 1$$
$$x_S - 7 = -7$$
$$x_S = 0$$

∴ x-coordinate of S = **0**

(c) Let the acute angle that RS makes with the x-axis be $\theta°$.

$\tan\theta$ = Gradient of RS

$$= \frac{1-8}{0-7}$$
$$= 1$$
$$\theta = 45°$$

∴ The acute angle that RS makes with the x-axis is **45°**.

Worksheet 5C Equation of a straight line

1. (a) $\boldsymbol{y = 2x + 9}$

(b) $\boldsymbol{y = \frac{3}{8}x - 7}$

(c) $\boldsymbol{y = -\frac{1}{6}x + 3}$

(d) $\boldsymbol{y = -1.5x - 4}$

(e) $\boldsymbol{y = -8}$

(f) $\boldsymbol{x = 5}$

2. (a) Gradient $= \frac{5-3}{5-(-1)}$

$= \frac{1}{3}$

Substitute $m = \frac{1}{3}$, $x = 5$, $y = 5$ into $y = mx + c$:

$$5 = \frac{1}{3}(5) + c$$
$$c = \frac{10}{3}$$

Equation of line: $\boldsymbol{y = \frac{1}{3}x + \frac{10}{3}}$

(b) Gradient $= \frac{6-0}{-4-0}$

$= -\frac{3}{2}$

Equation of line: $\boldsymbol{y = -\frac{3}{2}x}$

(c) Gradient $= \frac{0-4}{7-0}$

$= -\frac{4}{7}$

Equation of line: $\boldsymbol{y = -\frac{4}{7}x + 4}$

(d) Gradient $= \frac{0-(-8)}{6-(-3)}$

$= \frac{8}{9}$

Substitute $m = \frac{8}{9}$, $x = 6$, $y = 0$ into $y = mx + c$:

$$0 = \frac{8}{9}(6) + c$$
$$c = -\frac{16}{3}$$

Equation of line: $\boldsymbol{y = \frac{8}{9}x - \frac{16}{3}}$

3. (a) Gradient of $AB = \frac{12-0}{7-3}$

$= 3$

Substitute $m = 3$, $x = 3$, $y = 0$ into $y = mx + c$:

$$0 = 3(3) + c$$
$$c = -9$$

Equation of line: $\boldsymbol{y = 3x - 9}$

(b) Gradient $= \frac{-9-1}{0-(-4)}$

$= -\frac{5}{2}$

Equation of line: $\boldsymbol{y = -\frac{5}{2}x - 9}$

(c) Equation of line: $\boldsymbol{y = -5}$

(d) Equation of line: $\boldsymbol{x = 2}$

4. (a) Substitute $m = \frac{2}{3}$, $x = 1$, $y = -3$ into $y = mx + c$:

$$-3 = \frac{2}{3}(1) + c$$
$$= \frac{2}{3} + c$$
$$c = -\frac{11}{3}$$

Equation of line: $\boldsymbol{y = \frac{2}{3}x - \frac{11}{3}}$

(b) Substitute $m = -4$, $x = 5$, $y = 0$ into $y = mx + c$:

$$0 = -4(5) + c$$
$$= -20 + c$$
$$c = 20$$

Equation of line: $\boldsymbol{y = -4x + 20}$

5. From $3y = 7 - x$, we have $y = -\frac{1}{3}x + \frac{7}{3}$.

∴ $g = \boldsymbol{-\frac{1}{3}}$, $(a, b) = \boldsymbol{\left(0, 2\frac{1}{3}\right)}$

6. (a) (i) From $2x - 5y = k$, we have $y = \frac{2}{5}x - \frac{k}{5}$.

∴ Gradient = $\boldsymbol{\frac{2}{5}}$

(ii) y-intercept = 14

$$-\frac{k}{5} = 14$$
$$k = -70$$

∴ k = **−70**

(b) $2x - 5y = -70$

Let $x = 10$: $2(10) - 5y = -70$

$20 - 5y = -70$

$5y = 90$

$y = 18$

∴ The point **(10, 18)** lies on the line.

(c) Gradient = $\frac{2}{5}$

∴ A possible equation is $\boldsymbol{y = \frac{2}{5}x + 1}$.

7. **(i)** Gradient of $AB = \frac{1-(-2)}{5-(-3)}$

$= \frac{3}{8}$

(gradient of AB) × (gradient of PT) = –1

$\frac{3}{8} \times (\text{gradient of } PT) = -1$

gradient of $PT = -\frac{8}{3}$

Substitute $m = -\frac{8}{3}$, $x = 2$, $y = 4$ into $y = mx + c$:

$4 = -\frac{8}{3}(2) + c$

$c = \frac{28}{3}$

Equation of PT: $\boldsymbol{y = -\frac{8}{3}x + \frac{28}{3}}$

(ii) Let $x = 3$: $y = -\frac{8}{3}(3) + \frac{28}{3}$

$= \frac{4}{3}$

∴ An example is $\boldsymbol{Q\left(3, 1\frac{1}{3}\right)}$.

8. **(a)** Consider $y - x = 4$.

When $y = 0$,

$-x = 4$

$x = -4$

∴ $A(-4, 0)$

Consider $2x + 3y + 5 = 0$.

When $x = 0$,

$3y + 5 = 0$

$3y = -5$

$y = -1\frac{2}{3}$

∴ $B\left(0, -1\frac{2}{3}\right)$

∴ $\boldsymbol{A(-4, 0), B\left(0, -1\frac{2}{3}\right)}$

(b) Equation of vertical line: $\boldsymbol{x = -4}$

(c) $2x + 3y + 5 = 0$

$3y = -2x - 5$

$y = -\frac{2}{3}x - \frac{5}{3}$

∴ Gradient of line = $\boldsymbol{-\frac{2}{3}}$

(d) $h = \frac{-4+0}{2}$

$= -2$

$k = \frac{0 + \left(-1\frac{2}{3}\right)}{2}$

$= -\frac{5}{6}$

∴ $\boldsymbol{h = -2, k = -\frac{5}{6}}$

9. **(a)** $y = ka^{-x}$ — (1)

Substitute $x = 0$, $y = 3$ into (1):

$k = 3$

Substitute $k = 3$, $x = -3$, $y = 24$ into (1):

$3a^3 = 24$

$a^3 = 8$

$a = 2$

∴ $\boldsymbol{k = 3, a = 2}$

(b) Gradient of $AB = \frac{24-3}{-3-0}$

$= -7$

∴ Equation of AB: $\boldsymbol{y = -7x + 3}$

10. **(i)** $\boldsymbol{y = -1}$

(ii) Area of $ABCD$ = 2 × area of △ABC

$= 2 \times \frac{1}{2}(7)(2)$

= **14 units²**

(iii) $AB = \sqrt{[-1-(-3)]^2 + [1-(-1)]^2}$

$= \sqrt{8}$

= **2.83 units** (to 2 d.p.)

(iv) Gradient of $BC = \frac{-1-1}{4-(-1)}$

$= -\frac{2}{5}$

Substitute $m = -\frac{2}{5}$, $x = 4$, $y = -1$ into $y = mx + c$:

$-1 = -\frac{2}{5}(4) + c$

$c = \frac{3}{5}$

Equation of BC: $\boldsymbol{y = -\frac{2}{5}x + \frac{3}{5}}$

11. **(a)** x-coordinate of $B = 2 - 5$

$= -3$

∴ **B(–3, 9)**

(b) **(i)** The line crosses the y-axis at **(0, 9)**.

(ii) Gradient of line = $\boldsymbol{-\frac{3}{4}}$

(iii) $y = -\frac{3}{4}x + 9$ — (1)

Let the coordinates of C be $(2k, k)$.

Substitute $x = 2k$, $y = k$ into (1):

$$k = -\frac{3}{4}(2k) + 9$$
$$= -\frac{3}{2}k + 9$$
$$\frac{5}{2}k = 9$$
$$k = 3.6$$

$\therefore$ **$C(7.2, 3.6)$**

12. (a) x-coordinate of $D = -4$

y-coordinate of $D = 1 - 5 = -4$

$\therefore$ **$D(-4, -4)$**

(b) (i) Equation of BC: $\mathbf{x = 2}$

(ii) Gradient of $AB = \frac{4-1}{2-(-4)}$

$= \frac{1}{2}$

Substitute $x = 2$, $y = 4$ and $m = \frac{1}{2}$ into $y = mx + c$:

$$4 = \frac{1}{2}(2) + c$$
$$c = 3$$

$\therefore$ Equation of AB: $\mathbf{y = \frac{1}{2}x + 3}$

13. (a) x-coordinate of $D = 1 - 4$

$= -3$

y-coordinate of $D = 1$

$\therefore$ **$D(-3, 1)$**

(b) (i) Equation of AB: $\mathbf{y = 7}$

(ii) Gradient of $AD = \frac{7-1}{-6-(-3)}$

$= -2$

Substitute $m = -2$, $x = -3$ and $y = 1$ into $y = mx + c$:

$$1 = -2(-3) + c$$
$$c = -5$$

$\therefore$ Equation of AD: $\mathbf{y = -2x - 5}$ — (1)

(c) Substitute $y = 0$ into (1):

$$-2x - 5 = 0$$
$$x = -2\frac{1}{2}$$

$\therefore$ AD cuts the x-axis at $\left(\mathbf{-2\frac{1}{2}, 0}\right)$.

14. (a) Gradient $= \frac{8-2}{0-3}$

$= -2$

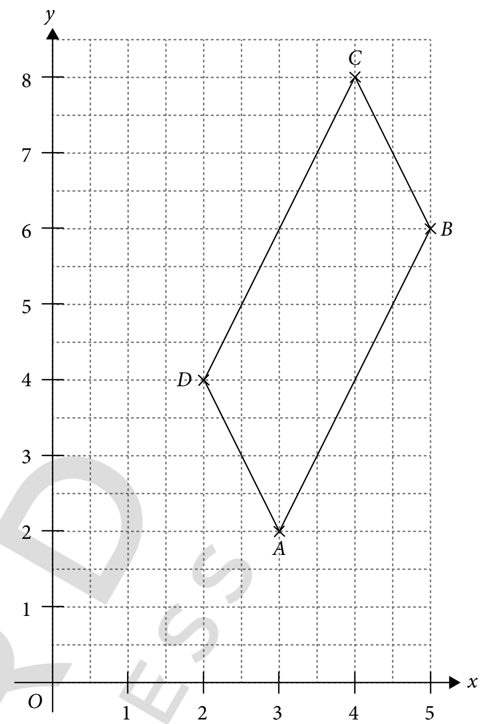
$\therefore$ Equation of line: $\mathbf{y = -2x + 8}$

(b) Length of $AB = \sqrt{(5-3)^2 + (6-2)^2}$

$= \sqrt{20}$

$=$ **4.47 units** (to 3 s.f.)

(c)



From the graph, **$C(4, 8)$**.

(d) **No,** there are no two equal adjacent sides.

Worksheet 5D Midpoint of a line segment

1. (a) Midpoint of A and $B = \left(\frac{6+1}{2}, \frac{-2+8}{2}\right)$

$= \left(\mathbf{3\frac{1}{2}, 3}\right)$

(b) Midpoint of A and $B = \left(7, \frac{4+(-3)}{2}\right)$

$= \left(\mathbf{7, \frac{1}{2}}\right)$

(c) Midpoint of A and $B = \left(\frac{0+9}{2}, \frac{1}{2}\right)$

$= \left(\mathbf{4\frac{1}{2}, \frac{1}{2}}\right)$

(d) Midpoint of A and $B = \left(\frac{5p+(-p)}{2}, \frac{2+(4p-2)}{2}\right)$

$=$ **$(2p, 2p)$**

2. (a) $\left(\frac{5+x_B}{2}, \frac{2+y_B}{2}\right) = (3, 8)$

$\frac{5+x_B}{2} = 3$ and $\frac{2+y_B}{2} = 8$

$5 + x_B = 6$ $\quad$ $2 + y_B = 16$

$x_B = 1$ $\quad$ $y_B = 14$

$\therefore$ **$B(1, 14)$**

(b) $\left(\frac{-1+x_B}{2}, \frac{9+y_B}{2}\right) = (0, 0)$

$$\frac{-1+x_B}{2} = 0 \quad \text{and} \quad \frac{9+y_B}{2} = 0$$
$$-1 + x_B = 0 \qquad 9 + y_B = 0$$
$$x_B = 1 \qquad y_B = -9$$

$\therefore$ ***B*(1, –9)**

(c) $\left(\frac{0+x_B}{2}, \frac{0+y_B}{2}\right) = (-5, 0.75)$

$$\frac{x_B}{2} = -5 \quad \text{and} \quad \frac{y_B}{2} = 0.75$$
$$x_B = -10 \qquad y_B = 1.5$$

$\therefore$ ***B*(–10, 1.5)**

(d) $\left(\frac{2q+3+x_B}{2}, \frac{-q+y_B}{2}\right) = (q + 1, 4)$

$$\frac{2q+3+x_B}{2} = q + 1 \quad \text{and} \quad \frac{-q+y_B}{2} = 4$$
$$2q + 3 + x_B = 2q + 2 \qquad -q + y_B = 8$$
$$x_B = -1 \qquad y_B = q + 8$$

$\therefore$ ***B*(–1, *q* + 8)**

3. **(a)** Since *ABCD* is a parallelogram,
Midpoint of *AC* = Midpoint of *BD*

$$\left(\frac{1+(-2)}{2}, \frac{3+1}{2}\right) = \left(\frac{0+x_D}{2}, \frac{5+y_D}{2}\right)$$
$$\left(-\frac{1}{2}, 2\right) = \left(\frac{x_D}{2}, \frac{5+y_D}{2}\right)$$
$$-\frac{1}{2} = \frac{x_D}{2} \quad \text{and} \quad 2 = \frac{5+y_D}{2}$$
$$x_D = -1 \qquad 5 + y_D = 4$$
$$y_D = -1$$

$\therefore$ ***D*(–1, –1)**

(b) Since *ABDC* is a parallelogram,
Midpoint of *AD* = Midpoint of *BC*

$$\left(\frac{1+x_D}{2}, \frac{3+y_D}{2}\right) = \left(\frac{0+(-2)}{2}, \frac{5+1}{2}\right)$$
$$= (-1, 3)$$
$$\frac{1+x_D}{2} = -1 \quad \text{and} \quad \frac{3+y_D}{2} = 3$$
$$1 + x_D = -2 \qquad 3 + y_D = 6$$
$$x_D = -3 \qquad y_D = 3$$

$\therefore$ ***D*(–3, 3)**

4. **(i)** Since *PQRS* is a parallelogram,
Midpoint of *PR* = Midpoint of *QS*

$$\left(\frac{2+5}{2}, \frac{6+3}{2}\right) = \left(\frac{7+x_S}{2}, \frac{8+y_S}{2}\right)$$
$$\left(\frac{7}{2}, \frac{9}{2}\right) = \left(\frac{7+x_S}{2}, \frac{8+y_S}{2}\right)$$
$$\frac{7}{2} = \frac{7+x_S}{2} \quad \text{and} \quad \frac{9}{2} = \frac{8+y_S}{2}$$
$$7 = 7 + x_S \qquad 9 = 8 + y_S$$
$$x_S = 0 \qquad y_S = 1$$

$\therefore$ ***S*(0, 1)**

(ii) $QS = \sqrt{(0-7)^2 + (1-8)^2}$
$= \sqrt{98}$
$= 7\sqrt{2}$ **units**

(iii) $m_{PR} = \frac{3-6}{5-2} = -1$

$m_{QS} = \frac{1-8}{0-7} = 1$

Since $m_{PR}m_{QS} = -1$, the diagonals of the parallelogram are perpendicular to each other, i.e. *PQRS* is a rhombus. (shown)

5. **(i)** $m_{RP} = \frac{2-4}{-6-4}$
$= \frac{1}{5}$

Let θ be the acute angle that *RP* produced makes with the *x*-axis.

$\tan\theta = \frac{1}{5}$

$\theta = 11.3°$ (to 1 d.p.)

$\therefore$ *RP* produced makes an angle of **11.3°** with the *x*-axis.

(ii) Midpoint of $PR = \left(\frac{-6+4}{2}, \frac{2+4}{2}\right)$
$= (-1, 3)$

$\therefore M(-1, 3)$

Since $MS \perp PR$, then $m_{MS} = -5$.

$$\frac{0-3}{x_S-(-1)} = -5$$
$$\frac{3}{5} = x_S + 1$$
$$x_S = -\frac{2}{5}$$

$\therefore$ $\boldsymbol{S\left(-\frac{2}{5}, 0\right)}$

(iii) Since *PQRS* is a parallelogram,
Midpoint of *PR* = Midpoint of *QS*

$$(-1, 3) = \left(\frac{x_Q+\left(-\frac{2}{5}\right)}{2}, \frac{y_Q+0}{2}\right)$$
$$(-1, 3) = \left(\frac{x_Q-\frac{2}{5}}{2}, \frac{y_Q}{2}\right)$$
$$-1 = \frac{x_Q-\frac{2}{5}}{2} \quad \text{and} \quad 3 = \frac{y_Q}{2}$$
$$x_Q - \frac{2}{5} = -2 \qquad y_Q = 6$$
$$x_Q = -\frac{8}{5} \qquad y_Q = 6$$

$\therefore$ $\boldsymbol{Q\left(-1\frac{3}{5}, 6\right)}$

Worksheet 5E Parallel and perpendicular lines

1. **(a)** Since $m_{AB} = m_{CD}$,

$$\frac{13-5}{7-3} = \frac{k-4}{8-5}$$
$$2 = \frac{k-4}{3}$$
$$k - 4 = 6$$
$$k = 10$$

$\therefore k = \mathbf{10}$

(b) Since $m_{AB} = m_{CD}$,

$$\frac{-3-2}{0-k} = \frac{9-8}{4-1}$$
$$\frac{5}{k} = \frac{1}{3}$$
$$k = 15$$

$\therefore k = \mathbf{15}$

(c) Since $m_{AB} = m_{CD}$,

$$\frac{k-(-1)}{6-2} = \frac{6-(-2k)}{7-3}$$
$$\frac{k+1}{4} = \frac{2k+6}{4}$$
$$k + 1 = 2k + 6$$
$$k = -5$$

$\therefore k = \mathbf{-5}$

(d) Since $m_{AB} = m_{CD}$,

$$\frac{3-4k}{3-0} = \frac{10-\frac{1}{3}}{k-1-8}$$
$$\frac{3-4k}{3} = \frac{\frac{29}{3}}{k-9}$$
$$(3 - 4k)(k - 9) = 29$$
$$3k - 27 - 4k^2 + 36k = 29$$
$$4k^2 - 39k + 56 = 0$$
$$(4k - 7)(k - 8) = 0$$
$$k = \frac{7}{4} \quad \text{or} \quad k = 8$$

$\therefore k = \mathbf{\frac{7}{4}}$ or $k = \mathbf{8}$

2. **(a)** Since $m_{AB} \times m_{CD} = -1$,

$$\frac{0-9}{5-2} \times \frac{8-k}{4-1} = -1$$
$$-3 \times \frac{8-k}{3} = -1$$
$$k - 8 = -1$$
$$k = 7$$

$\therefore k = \mathbf{7}$

(b) Since $m_{AB} \times m_{CD} = -1$,

$$\frac{k-6}{7-(-3)} \times \frac{1-(-4)}{5-2} = -1$$
$$\frac{k-6}{10} \times \frac{5}{3} = -1$$
$$k - 6 = -6$$
$$k = 0$$

$\therefore k = \mathbf{0}$

(c) Since $m_{AB} \times m_{CD} = -1$,

$$\frac{0-2k}{-1-6} \times \frac{-18-8}{8k+1-3} = -1$$
$$\frac{2k}{7} \times \frac{-26}{8k-2} = -1$$
$$-52k = -56k + 14$$
$$4k = 14$$
$$k = \frac{7}{2}$$

$\therefore k = \mathbf{\frac{7}{2}}$

(d) Since $m_{AB} \times m_{CD} = -1$,

$$\frac{k-(-k)}{3k+1-(k+1)} \times \frac{4k-2k}{0-(k-1)} = -1$$
$$\frac{2k}{2k} \times \frac{2k}{-(k-1)} = -1$$
$$2k = k - 1$$
$$k = -1$$

$\therefore k = \mathbf{-1}$

3. **(a)** l_1: $y = 2x - 5$

l_2: $2y + x = 5$, i.e. $y = -\frac{1}{2}x + \frac{5}{2}$

Since $m_1 m_2 = 2 \times \left(-\frac{1}{2}\right) = -1$, the lines are **perpendicular**.

(b) l_1: $y = 8 - \frac{3}{4}x$, i.e. $y = -\frac{3}{4}x + 8$

l_2: $3x + 4y = 10$, i.e. $y = -\frac{3}{4}x + \frac{5}{2}$

Since $m_1 = m_2$, the lines are **parallel**.

4. **(i)** $8y = 6 - 7x$ — (1)

$x^2 + 8xy + 12 = 0$ — (2)

Substitute (1) into (2):

$$x^2 + x(6 - 7x) + 12 = 0$$
$$x^2 + 6x - 7x^2 + 12 = 0$$
$$6x^2 - 6x - 12 = 0$$
$$x^2 - x - 2 = 0$$
$$(x - 2)(x + 1) = 0$$
$$x = 2 \quad \text{or} \quad x = -1$$

Substitute $x = 2$ into (1): $8y = 6 - 7(2) = -8$, i.e. $y = -1$

Substitute $x = -1$ into (1): $8y = 6 - 7(-1) = 13$, i.e. $y = \frac{13}{8}$

$\therefore P(2, -1), Q\left(-1, \frac{13}{8}\right)$

$$\text{Midpoint of } PQ = \left(\frac{2+(-1)}{2}, \frac{-1+\frac{13}{8}}{2}\right)$$
$$= \left(\mathbf{\frac{1}{2}}, \mathbf{\frac{5}{16}}\right)$$

(ii) $m_{PQ} = \dfrac{\frac{13}{8}-(-1)}{-1-2} = -\dfrac{7}{8}$

$m_{PR} = \dfrac{\frac{1}{2}-(-1)}{0-2} = -\dfrac{3}{4}$

Since $m_{PQ} \neq m_{PR}$, the points are not collinear.

5. $m_{AC} = \dfrac{2-8}{5-8} = 2$

$m_{BC} = \dfrac{2-3.5}{5-2} = -\dfrac{1}{2}$

Since $m_{AC} m_{BC} = -1$, then $AC \perp BC$.

$\therefore \triangle ABC$ is a right-angled triangle with **angle ACB** = 90°.

Challenge Myself!

6. **(a)** **No.** There is insufficient information to conclude that they are collinear.

For example, consider the points $A(1, 4)$, $B(3, 6)$, $C(5, 3)$ and $D(7, 5)$.

Although $m_{AB} = \frac{6-4}{3-1} = 2$ and $m_{CD} = \frac{5-3}{7-5} = 2$, but $m_{BC} = \frac{3-6}{5-3} = -\frac{3}{2}$.

$\therefore$ The points do not lie on the same straight line.

(b) **No.** There is insufficient information to conclude that P is the midpoint of AB.

For example, consider the points $A(-2, 0)$ and $B(4, 0)$.

The midpoint $(1, 0)$ is equidistant from A and B.

However, the points $(1, 4)$ and $(1, -8)$, which lie on the perpendicular bisector of AB, are also equidistant from A and B.

$\therefore$ P is not necessarily the midpoint of AB.

Worksheet 5F Equation of a straight line involving parallel and perpendicular lines

1. **(i)** Equation of l_1:
$$\begin{aligned} y - 3 &= 4[x - (-1)] \\ &= 4(x + 1) \\ &= 4x + 4 \\ \mathbf{y} &= \mathbf{4x + 7} \end{aligned}$$

(ii) A possible equation of l_2 is $\mathbf{y = 4x - 3}$.

(iii) A possible equation of l_3 is $\mathbf{y = -\frac{1}{4}x}$.

2. $7y + x = 0$, i.e. $y = -\frac{1}{7}x$

Equation of line:
$$\begin{aligned} y - \frac{1}{4} &= -\frac{1}{7}\left(x - \frac{1}{2}\right) \\ &= -\frac{1}{7}x + \frac{1}{14} \\ \mathbf{y} &= \mathbf{-\frac{1}{7}x + \frac{9}{28}} \end{aligned}$$

3. Midpoint of $(-5, 1)$ and $(2, -4)$
$$\begin{aligned} &= \left(\frac{-5+2}{2}, \frac{1+(-4)}{2}\right) \\ &= \left(-\frac{3}{2}, -\frac{3}{2}\right) \end{aligned}$$

Gradient of line segment
$$\begin{aligned} &= \frac{-4-1}{2-(-5)} \\ &= -\frac{5}{7} \end{aligned}$$

Gradient of perpendicular bisector $= \frac{7}{5}$

$\therefore$ Equation of perpendicular bisector:
$$\begin{aligned} y - \left(-\frac{3}{2}\right) &= \frac{7}{5}\left[x - \left(-\frac{3}{2}\right)\right] \\ y + \frac{3}{2} &= \frac{7}{5}\left(x + \frac{3}{2}\right) \\ &= \frac{7}{5}x + \frac{21}{10} \\ \mathbf{y} &= \mathbf{\frac{7}{5}x + \frac{3}{5}} \end{aligned}$$

4. $4x + 3y = \sqrt{2}$, i.e. $y = -\frac{4}{3}x + \frac{\sqrt{2}}{3}$

Midpoint of $(9, -1)$ and $(3, 5)$
$$\begin{aligned} &= \left(\frac{9+3}{2}, \frac{-1+5}{2}\right) \\ &= (6, 2) \end{aligned}$$

Equation of line:
$$\begin{aligned} y - 2 &= -\frac{4}{3}(x - 6) \\ &= -\frac{4}{3}x + 8 \\ \mathbf{y} &= \mathbf{-\frac{4}{3}x + 10} \end{aligned}$$

5. **(i)** Since $AB = BC$,
$$\begin{aligned} \sqrt{[0-(-10)]^2 + (y_B - 8)^2} &= \sqrt{(2-0)^2 + (2 - y_B)^2} \\ 10^2 + (y_B - 8)^2 &= 2^2 + (2 - y_B)^2 \\ 100 + y_B^{\,2} - 16y_B + 64 &= 4 + 4 - 4y_B + y_B^{\,2} \\ 12y_B &= 156 \\ y_B &= 13 \end{aligned}$$

$\therefore$ $B(0, 13)$

Since $ABCD$ is a rhombus,

Midpoint of AC = Midpoint of BD
$$\begin{aligned} \left(\frac{-10+2}{2}, \frac{8+2}{2}\right) &= \left(\frac{0+x_D}{2}, \frac{13+y_D}{2}\right) \\ (-4, 5) &= \left(\frac{x_D}{2}, \frac{13+y_D}{2}\right) \end{aligned}$$

$-4 = \frac{x_D}{2}$ and $5 = \frac{13+y_D}{2}$

$x_D = -8$ $\quad$ $13 + y_D = 10$

$y_D = -3$

$\therefore$ $D(-8, -3)$

$\therefore$ $\mathbf{B(0, 13), D(-8, -3)}$

(ii)
$$\begin{aligned} m_{CD} &= \frac{-3-2}{-8-2} \\ &= \frac{1}{2} \end{aligned}$$

Let θ be the acute angle that CD makes with the x-axis.
$$\begin{aligned} \tan\theta &= \frac{1}{2} \\ \theta &= 26.6° \text{ (to 1 d.p.)} \end{aligned}$$

$\therefore$ CD makes an angle of **26.6°** with the x-axis.

6. **(i)**
$$\begin{aligned} m_{PR} &= \frac{8-0}{0-4} \\ &= -2 \end{aligned}$$

Since $OQ \perp PR$, then $m_{OQ} = \frac{1}{2}$.

$\therefore$ Equation of OQ: $\mathbf{y = \frac{1}{2}x}$ — (1)

(ii) Equation of PR: $y = -2x + 8$ — (2)

Substitute (1) into (2):

$$\frac{1}{2}x = -2x + 8$$

$$\frac{5}{2}x = 8$$

$$x = \frac{16}{5}$$

Substitute $x = \frac{16}{5}$ into (1): $y = \frac{1}{2}\left(\frac{16}{5}\right) = \frac{8}{5}$

$\therefore$ Coordinates of midpoint of OQ are

$$\left(\frac{0+x_Q}{2}, \frac{0+y_Q}{2}\right) = \left(\frac{16}{5}, \frac{8}{5}\right)$$

$$\frac{x_Q}{2} = \frac{16}{5} \quad \text{and} \quad \frac{y_Q}{2} = \frac{8}{5}$$

$$x_Q = \frac{32}{5} \qquad y_Q = \frac{16}{5}$$

$\therefore$ $\boldsymbol{Q\left(6\frac{2}{5}, 3\frac{1}{5}\right)}$

7. (i) $m_{AD} = m_{BC}$

$$= \frac{3-1}{7-3}$$

$$= \frac{1}{2}$$

Equation of AD: $y - (-5) = \frac{1}{2}(x - 1)$

$$y + 5 = \frac{1}{2}x - \frac{1}{2}$$

$$\boldsymbol{y = \frac{1}{2}x - \frac{11}{2}} \quad \text{— (1)}$$

(ii) Since $CD \perp BC$, then $m_{CD} = -2$.

Equation of CD: $y - 3 = -2(x - 7)$

$$= -2x + 14$$

$$\boldsymbol{y = -2x + 17} \quad \text{— (2)}$$

(iii) Substitute (1) into (2):

$$\frac{1}{2}x - \frac{11}{2} = -2x + 17$$

$$\frac{5}{2}x = \frac{45}{2}$$

$$x = 9$$

Substitute $x = 9$ into (2): $y = -2(9) + 17 = -1$

$\therefore$ **D(9, –1)**

8. (i) Midpoint of $AC = \left(\frac{-1+5}{2}, \frac{-4+8}{2}\right)$

$$= (2, 2)$$

$\therefore$ **M(2, 2)**

(ii) $m_{AC} = \frac{8-(-4)}{5-(-1)}$

$$= 2$$

Since $BD \perp AC$, then $m_{BD} = -\frac{1}{2}$.

$$\frac{0-2}{x_D - 2} = -\frac{1}{2}$$

$$x_D - 2 = 4$$

$$x_D = 6$$

$\therefore D(6, 0)$

Since $m_{BD} = -\frac{1}{2}$,

$$\frac{y_B - 0}{-1-6} = -\frac{1}{2}$$

$$y_B = \frac{7}{2}$$

$\therefore B\left(-1, 3\frac{1}{2}\right)$

$\therefore$ $\boldsymbol{B\left(-1, 3\frac{1}{2}\right), D(6, 0)}$

(iii) $m_{CD} = \frac{0-8}{6-5}$

$$= -8$$

Gradient of perpendicular bisector of $CD = \frac{1}{8}$

Midpoint of $CD = \left(\frac{5+6}{2}, \frac{8+0}{2}\right)$

$$= \left(\frac{11}{2}, 4\right)$$

Equation of perpendicular bisector of CD:

$$y - 4 = \frac{1}{8}\left(x - \frac{11}{2}\right)$$

$$= \frac{1}{8}x - \frac{11}{16}$$

$$\boldsymbol{y = \frac{1}{8}x + \frac{53}{16}} \quad \text{— (1)}$$

Substitute $x = 2$ into (1): $y = \frac{1}{8}(2) + \frac{53}{16} = \frac{57}{16} \neq 2$

$\therefore$ The perpendicular bisector of CD does not pass through M.

9. (i) l_{QR}: $2y + 5x = 64$, i.e. $y = -\frac{5}{2}x + 32$ — (1)

Since $PQ \perp QR$, then $m_{PQ} = \frac{2}{5}$.

Equation of PQ: $y - 8 = \frac{2}{5}[x - (-2)]$

$$= \frac{2}{5}(x + 2)$$

$$= \frac{2}{5}x + \frac{4}{5}$$

$$y = \frac{2}{5}x + \frac{44}{5} \quad \text{— (2)}$$

Substitute (2) into (1):

$$\frac{2}{5}x + \frac{44}{5} = -\frac{5}{2}x + 32$$

$$\frac{29}{10}x = \frac{116}{5}$$

$$x = 8$$

Substitute $x = 8$ into (1): $y = -\frac{5}{2}(8) + 32 = 12$

$\therefore$ **Q(8, 12)**

(ii) Substitute $y = 0$ into (1): $-\frac{5}{2}x + 32 = 0$, i.e. $x = \frac{64}{5}$

$\therefore R\left(12\frac{4}{5}, 0\right)$

Midpoint of $PR = \left(\frac{-2+\frac{64}{5}}{2}, \frac{8+0}{2}\right)$

$= \left(\frac{27}{5}, 4\right)$

$\therefore M\left(5\frac{2}{5}, 4\right)$

Since $PQRS$ is a rectangle,
Midpoint of PR = Midpoint of QS

$\left(\frac{27}{5}, 4\right) = \left(\frac{8+x_S}{2}, \frac{12+y_S}{2}\right)$

$\frac{27}{5} = \frac{8+x_S}{2}$ and $4 = \frac{12+y_S}{2}$

$8 + x_S = \frac{54}{5}$ $\quad$ $12 + y_S = 8$

$x_S = \frac{14}{5}$ $\quad$ $y_S = -4$

$\therefore S\left(2\frac{4}{5}, -4\right)$

$\therefore \boldsymbol{M\left(5\frac{2}{5}, 4\right), S\left(2\frac{4}{5}, -4\right)}$

Challenge Myself!

10. (i) Midpoint of $AB = \left(\frac{-2+1}{2}, \frac{6+2}{2}\right)$

$= \left(-\frac{1}{2}, 4\right)$

$m_{AB} = \frac{2-6}{1-(-2)}$

$= -\frac{4}{3}$

Since $CD \perp AB$, then $m_{CD} = \frac{3}{4}$.

Equation of CD: $y - 4 = \frac{3}{4}\left[x - \left(-\frac{1}{2}\right)\right]$

$= \frac{3}{4}\left(x + \frac{1}{2}\right)$

$= \frac{3}{4}x + \frac{3}{8}$

$y = \frac{3}{4}x + \frac{35}{8}$ — (1)

i.e. coordinates of C and D are in the form $\left(k, \frac{3}{4}k + \frac{35}{8}\right)$

Since $AC \perp BC$, $m_{AC}m_{BC} = -1$.

$$\frac{\frac{3}{4}k + \frac{35}{8} - 6}{k-(-2)} \times \frac{\frac{3}{4}k + \frac{35}{8} - 2}{k-1} = -1$$

$$\frac{\frac{3}{4}k - \frac{13}{8}}{k+2} \times \frac{\frac{3}{4}k + \frac{19}{8}}{k-1} = -1$$

$$\left(\frac{3}{4}k - \frac{13}{8}\right)\left(\frac{3}{4}k + \frac{19}{8}\right) = -(k+2)(k-1)$$

$$\frac{9}{16}k^2 + \frac{9}{16}k - \frac{247}{64} = -k^2 - k + 2$$

$$36k^2 + 36k - 247 = -64k^2 - 64k + 128$$

$$100k^2 + 100k - 375 = 0$$

$$4k^2 + 4k - 15 = 0$$

$$(2k+5)(2k-3) = 0$$

$$k = -\frac{5}{2} \quad \text{or} \quad k = \frac{3}{2}$$

Substitute $x = -\frac{5}{2}$ into (1): $y = \frac{3}{4}\left(-\frac{5}{2}\right) + \frac{35}{8} = \frac{5}{2}$

Substitute $x = \frac{3}{2}$ into (1): $y = \frac{3}{4}\left(\frac{3}{2}\right) + \frac{35}{8} = \frac{11}{2}$

$\therefore \boldsymbol{C\left(-2\frac{1}{2}, 2\frac{1}{2}\right), D\left(1\frac{1}{2}, 5\frac{1}{2}\right)}$ **or**

$\boldsymbol{C\left(1\frac{1}{2}, 5\frac{1}{2}\right), D\left(-2\frac{1}{2}, 2\frac{1}{2}\right)}$

(ii) OPEN C and D lie on $y = \frac{3}{4}x + \frac{35}{8}$.

Since $\triangle ACB$ is an acute angle, $x_C < -\frac{5}{2}$.

Let $x_C = -4$: $y = \frac{3}{4}(-4) + \frac{35}{8} = \frac{11}{8}$

$\therefore C\left(-4, 1\frac{3}{8}\right)$

Midpoint of $CD = \left(-\frac{1}{2}, 4\right)$

$$\left(\frac{-4+x_D}{2}, \frac{\frac{11}{8}+y_D}{2}\right) = \left(-\frac{1}{2}, 4\right)$$

$\frac{-4+x_D}{2} = -\frac{1}{2}$ and $\frac{\frac{11}{8}+y_D}{2} = 4$

$-4 + x_D = -1$ $\quad$ $\frac{11}{8} + y_D = 8$

$x_D = 3$ $\quad$ $y_D = \frac{53}{8}$

$\therefore D\left(3, 6\frac{5}{8}\right)$

$\therefore$ An example is $\boldsymbol{C\left(-4, 1\frac{3}{8}\right)}$ and $\boldsymbol{D\left(3, 6\frac{5}{8}\right)}$.

Review Exercise 5

1. **(a)** **(i)** Gradient of $PQ = \dfrac{4-(-2)}{7-5}$

$= \mathbf{3}$

(ii) Length of $PQ = \sqrt{(7-5)^2 + [4-(-2)]^2}$

$= \sqrt{40}$

$= \mathbf{6.32\ units}$ (to 3 s.f.)

(b) Substitute $m = 3$, $x = -3$, $y = 6$ into $y = mx + c$:

$$\begin{aligned} 6 &= 3(-3) + c \\ &= -9 + c \\ c &= 15 \end{aligned}$$

$\therefore$ Equation of line: $\boldsymbol{y = 3x + 15}$

2. **(a)** Gradient = **0**

(b) **(0, 8)** and **(4, 8)**

(c) $\boldsymbol{x = 1}$

3. Using similar triangles,

x-coordinate of $C = 1 + 2(6)$

$= 13$

y-coordinate of $C = 8 + 2(8)$

$= 24$

$\therefore$ $\boldsymbol{h = 13, k = 24}$

4. $x = y + 1$ — (1)

$2x^2 + y^2 = 9$ — (2)

Substitute (1) into (2):

$$\begin{aligned} 2(y+1)^2 + y^2 &= 9 \\ 2(y^2 + 2y + 1) + y^2 &= 9 \\ 2y^2 + 4y + 2 + y^2 &= 9 \\ 3y^2 + 4y - 7 &= 0 \\ (3y+7)(y-1) &= 0 \end{aligned}$$

$y = -\dfrac{7}{3}$ or $y = 1$

Substitute $y = -\dfrac{7}{3}$ into (1): $x = -\dfrac{7}{3} + 1 = -\dfrac{4}{3}$

Substitute $y = 1$ into (1): $x = 1 + 1 = 2$

$\therefore P\left(-\dfrac{4}{3}, -\dfrac{7}{3}\right)$, $Q(2, 1)$

Midpoint of $PQ = \left(\dfrac{-\frac{4}{3}+2}{2}, \dfrac{-\frac{7}{3}+1}{2}\right)$

$= \left(\dfrac{1}{3}, -\dfrac{2}{3}\right)$

$6y = 3x + 10$ — (3)

Substitute $y = -\dfrac{2}{3}$ into the LHS of (3):

$\text{LHS} = 6\left(-\dfrac{2}{3}\right) = -4$

Substitute $x = \dfrac{1}{3}$ into the RHS of (3):

$\text{RHS} = 3\left(\dfrac{1}{3}\right) + 10 = 11$

Since LHS ≠ RHS, the midpoint of PQ does not lie on $6y = 3x + 10$.

5. Since $m_{CD} = m_{AB}$,

$$\begin{aligned} \frac{0-3}{x_D - (-1)} &= \frac{2-7}{11-5} \\ -\frac{3}{x_D + 1} &= -\frac{5}{6} \\ 5x_D + 5 &= 18 \\ 5x_D &= 13 \\ x_D &= \frac{13}{5} \end{aligned}$$

$\therefore \boldsymbol{D\left(2\frac{3}{5}, 0\right)}$

6. **(i)** Equation of AB: $\boldsymbol{x = -4}$

(ii) Gradient of $BC = \dfrac{3-(-1)}{2-(-4)}$

$= \dfrac{2}{3}$

Substitute $m = \dfrac{2}{3}$, $x = 2$, $y = 3$ into $y = mx + c$:

$$\begin{aligned} 3 &= \frac{2}{3}(2) + c \\ &= \frac{4}{3} + c \\ c &= \frac{5}{3} \end{aligned}$$

Equation of BC: $y = \dfrac{2}{3}x + \dfrac{5}{3}$

Substitute $y = 0$ into $y = \dfrac{2}{3}x + \dfrac{5}{3}$:

$x = -\dfrac{5}{2}$

$\therefore$ $\boldsymbol{P\left(-2\frac{1}{2}, 0\right)}$

(iii) $BC = \sqrt{[2-(-4)]^2 + [3-(-1)]^2}$

$= \sqrt{52}$ units

Let the shortest distance from A to BC be h units.

Area of $\triangle ABC = \dfrac{1}{2}(5)(6)$

$$\begin{aligned} \frac{1}{2}\left(\sqrt{52}\right)h &= \frac{1}{2}(5)(6) \\ h &= \frac{30}{\sqrt{52}} \\ &= 4.16 \text{ units (to 3 s.f.)} \end{aligned}$$

$\therefore$ Shortest distance from A to BC = 4.16 units (shown)

(iv) $\boldsymbol{D(2, 0)}$ or $\boldsymbol{D(2, 9)}$

7. **(i)** l_1: $5y + x = 48$, i.e. $x = 48 - 5y$ — (1)

l_2: $4y = 3x - 30$ — (2)

Substitute (1) into (2):

$$\begin{aligned} 4y &= 3(48 - 5y) - 30 \\ &= 144 - 15y - 30 \\ 19y &= 114 \\ y &= 6 \end{aligned}$$

Substitute $y = 6$ into (1): $x = 48 - 5(6) = 18$

$\therefore P(18, 6)$

Midpoint of $OP = \left(\dfrac{0+18}{2}, \dfrac{0+6}{2}\right)$

$= (9, 3)$

$\therefore M(9, 3)$

Substitute $y = 0$ into (2): $3x - 30 = 0$, i.e. $x = 10$

$\therefore R(10, 0)$

$$m_{OP} = \frac{6-0}{18-0} = \frac{1}{3}$$

$$m_{MR} = \frac{0-3}{10-9} = -3$$

Since $m_{OP}m_{MR} = -1$, then $OM \perp MR$, i.e. $\angle OMR = 90°$. (shown)

(ii) Substitute $x = 0$ into (1): $5y = 48$, i.e. $y = 9.6$

$\therefore Q(0, 9.6)$

$$\text{Area of } \triangle OPQ = \frac{1}{2}(9.6)(18) = 86.4 \text{ units}^2$$

$$\text{Area of } \triangle OPR = \frac{1}{2}(10)(6) = 30 \text{ units}^2$$

$\therefore$ Area of $\triangle OPQ$: Area of quadrilateral $OPQR$ = 86.4 : (86.4 + 30) = **72 : 97**

8. (i) Midpoint of $AB = \left(\frac{5+(-p)}{2}, \frac{p+7}{2}\right) = \left(\frac{5-p}{2}, \frac{p+7}{2}\right)$

$$m_{AB} = \frac{7-p}{-p-5} = \frac{p-7}{p+5}$$

Gradient of perpendicular bisector $= \frac{p+5}{7-p}$

$$\frac{\frac{p+7}{2} - 0}{\frac{5-p}{2} - (-2)} = \frac{p+5}{7-p}$$

$$\frac{\frac{p+7}{2}}{\frac{5-p+4}{2}} = \frac{p+5}{7-p}$$

$$\frac{p+7}{9-p} = \frac{p+5}{7-p}$$

$$(7+p)(7-p) = (p+5)(9-p)$$

$$49 - p^2 = -p^2 + 4p + 45$$

$$4p = 4$$

$$p = 1$$

$\therefore p = \mathbf{1}$

(ii) Since D lies on the perpendicular bisector of AB, $AD = BD$.

$\therefore \triangle ABD$ is **isosceles**.

(iii) $m_{AB} = \frac{1-7}{1+5} = -1$

Let θ be the acute angle that BA produced makes with the x-axis.

$\tan \theta = 1$

$\theta = 45°$

$\therefore \angle BXC = \mathbf{45°}$

(iv) Since $BYAC$ is a kite, Y lies on the perpendicular bisector of AB.

Equation of perpendicular bisector: $y - 0 = 1[x - (-2)]$

$$y = x + 2$$

$\therefore$ An example of a pair of coordinates is $\boldsymbol{Y(10, 12)}$.

9. (i) $m_{PQ} = \frac{15-3}{11-5} = 2$

Equation of line: $y - 1 = 2[x - (-4)]$

$$= 2(x + 4)$$

$$= 2x + 8$$

$$\mathbf{2x - y + 9 = 0}$$

(ii) Gradient of perpendicular bisector of $PQ = -\frac{1}{2}$

Midpoint of $PQ = \left(\frac{5+11}{2}, \frac{3+15}{2}\right) = (8, 9)$

Equation of perpendicular bisector of PQ: $y - 9 = -\frac{1}{2}(x - 8)$

$$= -\frac{1}{2}x + 4$$

$$\boldsymbol{y = -\frac{1}{2}x + 13}$$

(iii) Let the coordinates of R be $\left(h, 13 - \frac{1}{2}h\right)$.

Given that distance between R and $PQ = \sqrt{50}$,

$$\sqrt{(h-8)^2 + \left(13 - \frac{1}{2}h - 9\right)^2} = \sqrt{50}$$

$$(h-8)^2 + \left(4 - \frac{1}{2}h\right)^2 = 50$$

$$h^2 - 16h + 64 + 16 - 4h + \frac{1}{4}h^2 = 50$$

$$\frac{5}{4}h^2 - 20h + 30 = 0$$

$$h^2 - 16h + 24 = 0$$

$$h = \frac{-(-16) \pm \sqrt{(-16)^2 - 4(1)(24)}}{2(1)} = \frac{16 \pm \sqrt{160}}{2} = \frac{16 \pm 4\sqrt{10}}{2} = 8 \pm 2\sqrt{10}$$

Let the coordinates of R be $\left(8+2\sqrt{10},\ 9-\sqrt{10}\right)$.
Area of $\triangle PQR$

$$= \frac{1}{2}\begin{vmatrix} 11 & 5 & 8+2\sqrt{10} & 11 \\ 15 & 3 & 9-\sqrt{10} & 15 \end{vmatrix}$$

$$= \frac{1}{2}\left\{\left[(11)(3)+(5)\left(9-\sqrt{10}\right)+\left(8+2\sqrt{10}\right)(15)\right] - \left[(15)(5)+(3)\left(8+2\sqrt{10}\right)+\left(9-\sqrt{10}\right)(11)\right]\right\}$$

$$= \frac{1}{2}\left[\left(33+45-5\sqrt{10}+120+30\sqrt{10}\right) - \left(75+24+6\sqrt{10}+99-11\sqrt{10}\right)\right]$$

$$= \frac{1}{2}\left[\left(198+25\sqrt{10}\right)-\left(198-5\sqrt{10}\right)\right]$$

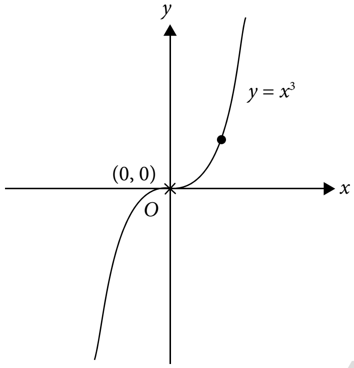
$$= \frac{1}{2}\left(30\sqrt{10}\right)$$

$= 15\sqrt{10}$ units^2 (shown)

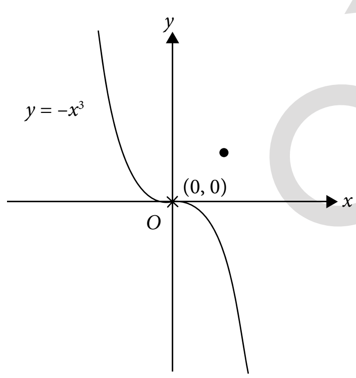
6 Graphs of Functions and Graphical Solution

Worksheet 6A Graphs of cubic functions

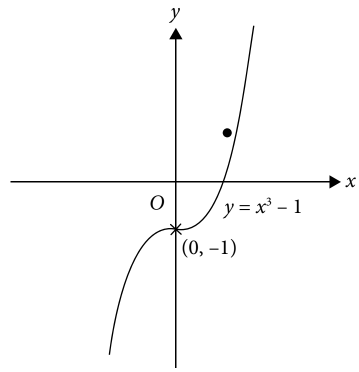
1. (a)



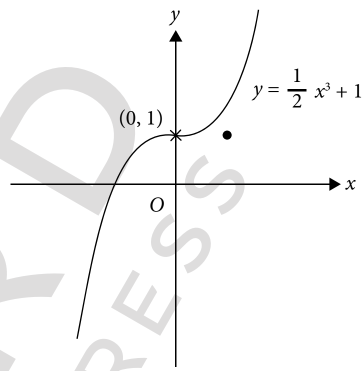
(b)



(c)



(d)



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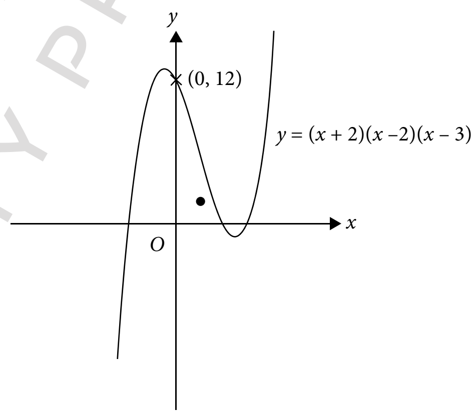

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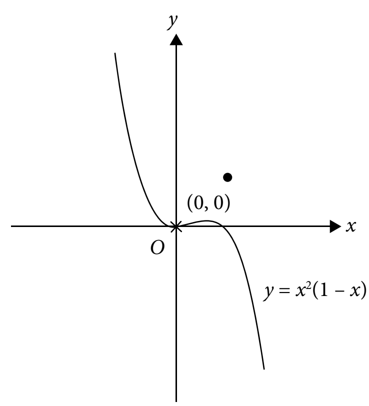
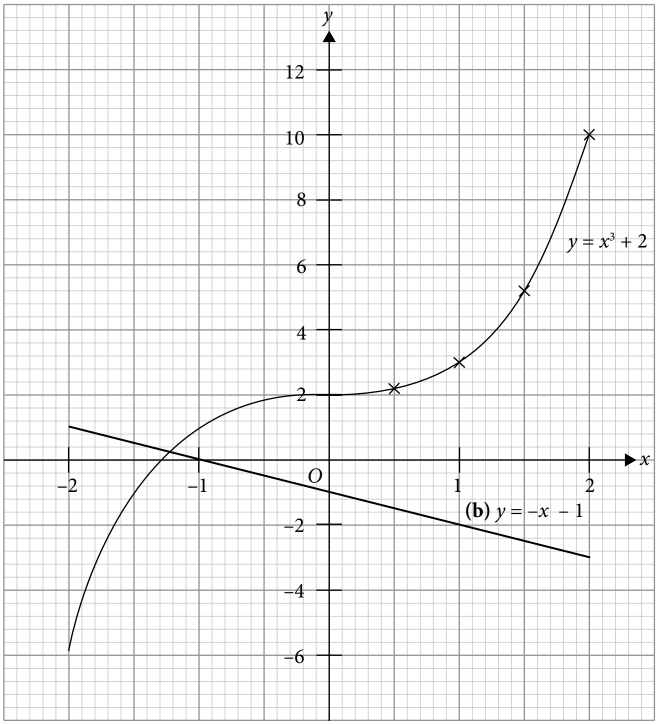


2. **(a)**



(c) **(−1.2, 0.2)**

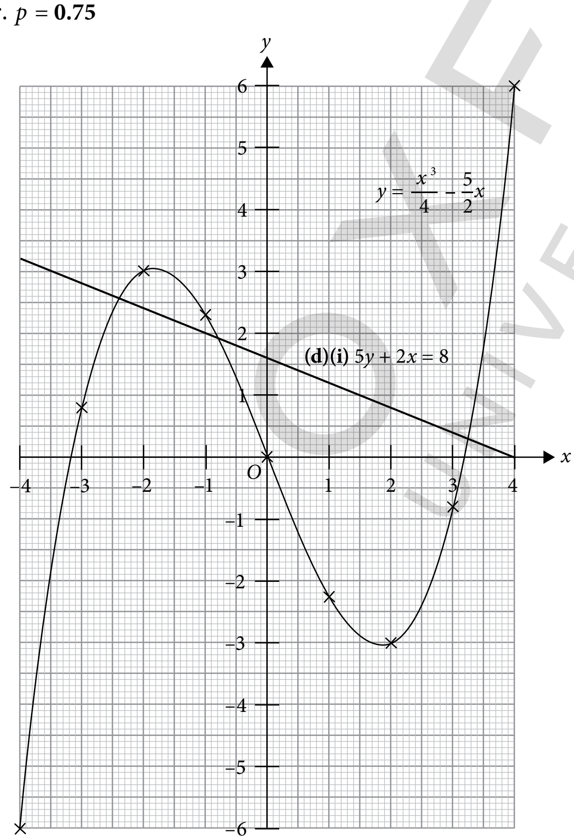
3. **(a)** When $x = -3$,

$$y = \frac{(-3)^3}{4} - \frac{5}{2}(-3)$$

$= 0.75$

$\therefore p = \mathbf{0.75}$

(b)



(c) $\boldsymbol{x < -3.8}$

(d) **(ii)** $y = \frac{x^3}{4} - \frac{5}{2}x$ — (1)

$5y + 2x = 8$ — (2)

Substitute (1) into (2):

$$5\left(\frac{x^3}{4} - \frac{5}{2}x\right) + 2x = 8$$

$$\frac{5x^3}{4} - \frac{25}{2}x + 2x = 8$$

$$5x^3 - 50x + 8x = 32$$

$$5x^3 - 42x - 32 = 0 \text{ (shown)}$$

(iii) From the graph, $x = \mathbf{-2.4}$ or $x = \mathbf{-0.8}$ or $x = \mathbf{3.25}$.

4. **(a)** $\boldsymbol{p = 1, q = 2}$

(b) $y = x(x + 1)(x - 2)$ — (1)

$4y = 1 - x$ — (2)

Substitute (1) into (2):

$$4x(x + 1)(x - 2) = 1 - x$$

$$4x(x^2 - x - 2) = 1 - x$$

$$4x^3 - 4x^2 - 8x = 1 - x$$

$$\boldsymbol{4x^3 - 4x^2 - 7x - 1 = 0}$$

(c) **(i)** $4x^3 - 4x^2 - 8x + 6 = 0$

$$4x^3 - 4x^2 - 8x = -6$$

$$x^3 - x^2 - 2x = -1.5$$

$\therefore$ Equation of line: $\boldsymbol{y = -1.5}$

(ii) From the graph, when $y = -1.5$,

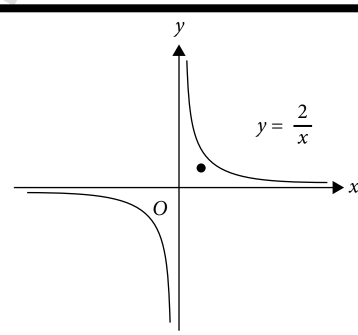
$x = -1.35$ or $x = 0.7$ or $x = 1.75$

$\therefore x = \mathbf{-1.35}$ or $x = \mathbf{0.7}$ or $x = \mathbf{1.75}$

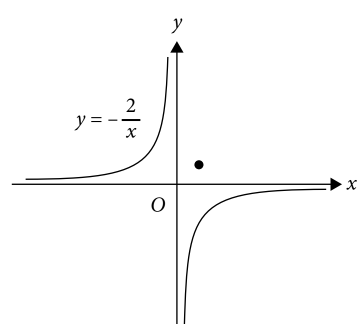
(d) OPEN $\boldsymbol{x = 1.2}$

Worksheet 6B Graphs of reciprocal functions

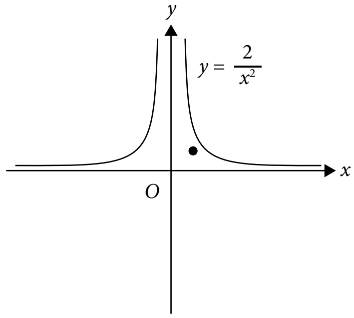
1. **(a)**



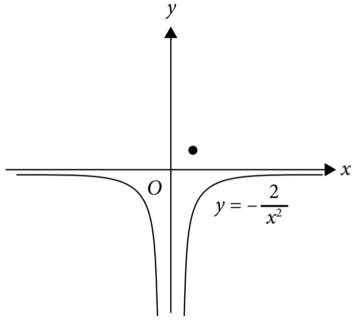
(b)



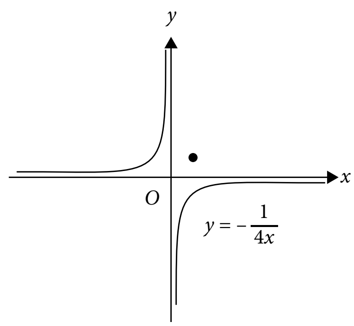
(c)



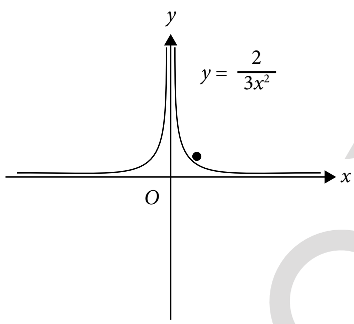
(d)



(e)



(f)

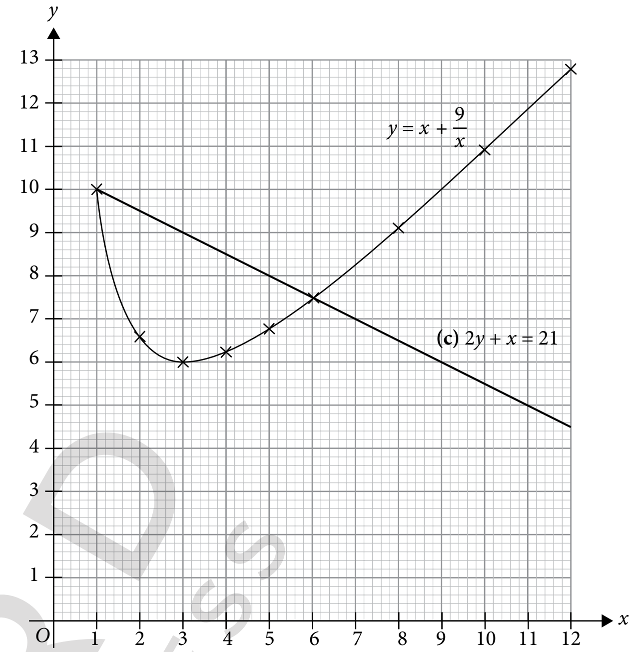


2. **(a)** When $x = 8$,

$y = 8 + \frac{9}{8}$

$= \mathbf{9.125}$

(b)



(d) From the graph, $x = \mathbf{1}$ or $x = \mathbf{6}$.

3. **(a)** When $x = 1$,

$y = 1 + \frac{8}{1} - 9$

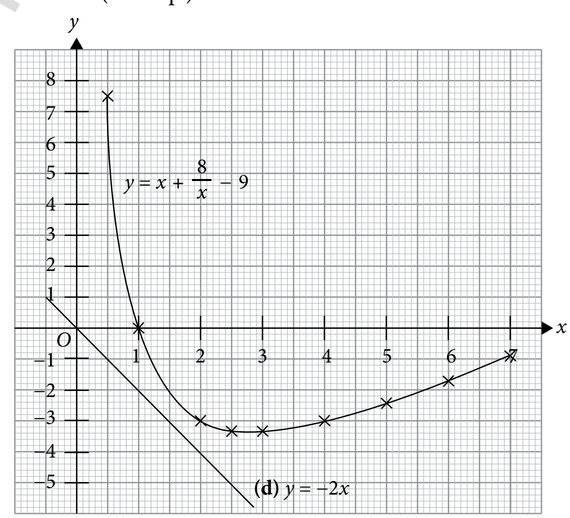
$= \mathbf{0}$

When $x = 3$,

$y = 3 + \frac{8}{3} - 9$

$= \mathbf{-3.3}$ (to 1 d.p.)

(b)



(c) $x + \frac{8}{x} = 7$

$x + \frac{8}{x} - 9 = -2$

From the graph, when $y = -2$,

$x = 1.45$ or $x = 5.6$

$\therefore\ x = \mathbf{1.45}$ or $x = \mathbf{5.6}$

(d) $(m-1)x^2 + 9x - 8 = 0$

$$mx^2 - x^2 + 9x - 8 = 0$$
$$x^2 + 8 - 9x = mx^2$$
$$x + \frac{8}{x} - 9 = mx$$

Let $m = \mathbf{-2}$:

From the graph, $y = -2x$ does not intersect the curve $y = x + \frac{8}{x} - 9$ in the interval $0.5 \leqslant x \leqslant 7$.

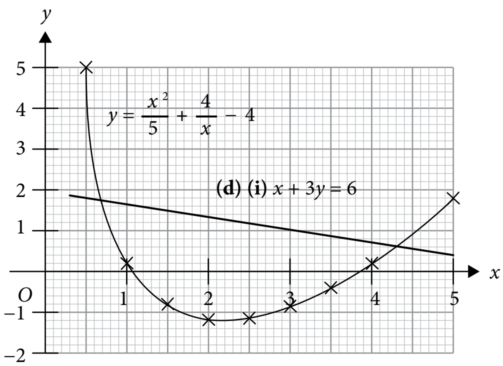
$\therefore$ The equation has no solution in the given interval. (shown)

4. **(a)** When $x = 3$,

$$y = \frac{3^2}{5} + \frac{4}{3} - 4$$
$$= -0.9 \text{ (to 1 d.p.)}$$

$\therefore p = \mathbf{-0.9}$

(b)



(c) **(1.05, 0)** and **(3.85, 0)**

(d) **(ii)** $y = \frac{x^2}{5} + \frac{4}{x} - 4$ — (1)

$x + 3y = 6$ — (2)

Substitute (1) into (2):

$$x + 3\left(\frac{x^2}{5} + \frac{4}{x} - 4\right) = 6$$
$$x + \frac{3x^2}{5} + \frac{12}{x} - 12 = 6$$
$$5x^2 + 3x^3 + 60 - 60x = 30x$$
$$3x^3 + 5x^2 - 90x + 60 = 0$$

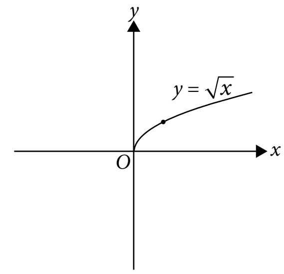
$\therefore$ ***b* = 5, *c* = −90, *d* = 60**

(iii) From the graph, $x = \mathbf{0.7}$ or $x = \mathbf{4.3}$.

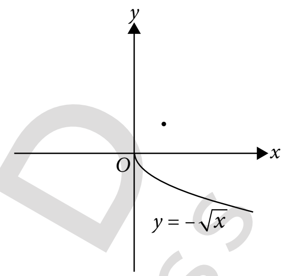
(e) $k = \mathbf{-2}$

Worksheet 6C Graphs of functions involving $\sqrt{x}$

1. **(a)**



(b)



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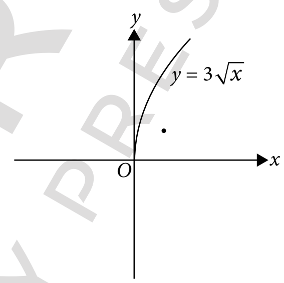
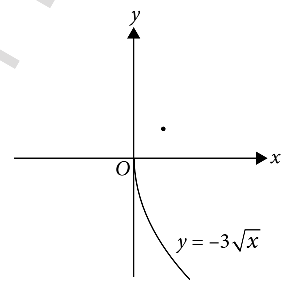


(d)



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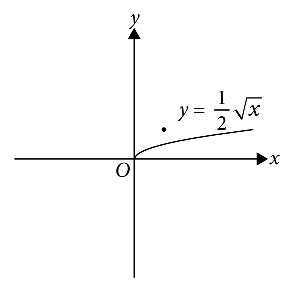
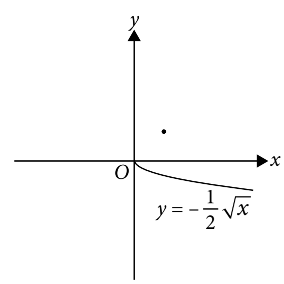
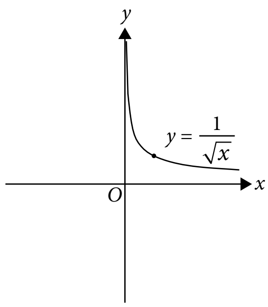


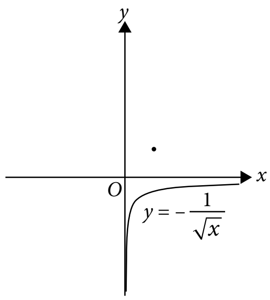
(f)



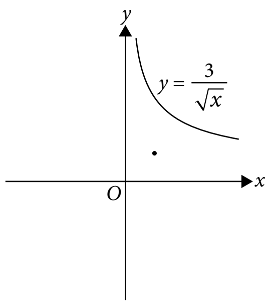
2. **(a)**



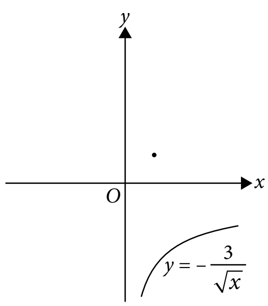
(b)



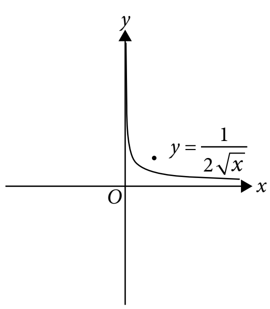
(c)



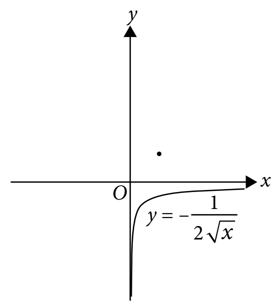
(d)



(e)



(f)



3. **(a)** When $x = 2$,

$y = \frac{4}{\sqrt{2}}$

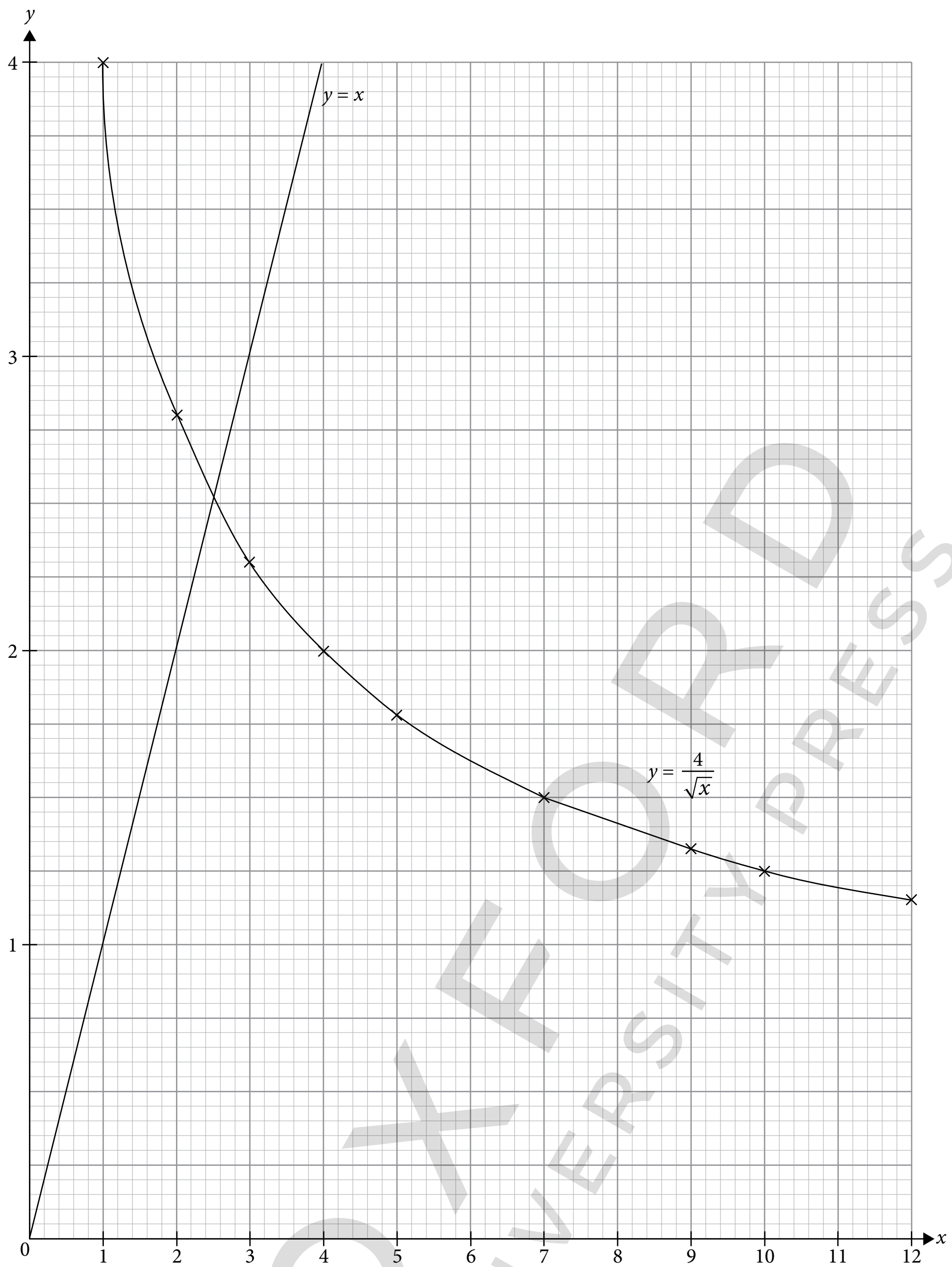
$= \mathbf{2.83}$ (to 2 d.p.)

When $x = 7$,

$y = \frac{4}{\sqrt{7}}$

$= \mathbf{1.51}$ (to 2 d.p.)

(b)



(c) From the graph, $x =$ **2.5**.

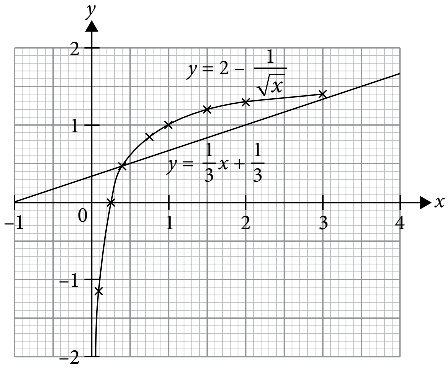
4. **(a)** When $x = 0.25$,

$$y = 2 - \frac{1}{\sqrt{0.25}}$$

$$= 0$$

$\therefore p =$ **0**

(b)



(c) From the graph, the coordinates are **(0.25, 0)**.

(d) **(i)** $\sqrt{x}\,(5 - x) = 3$

$$5 - x = \frac{3}{\sqrt{x}}$$

$$\frac{1}{\sqrt{x}} = \frac{5}{3} - \frac{1}{3}x$$

$$-\frac{1}{\sqrt{x}} = \frac{1}{3}x - \frac{5}{3}$$

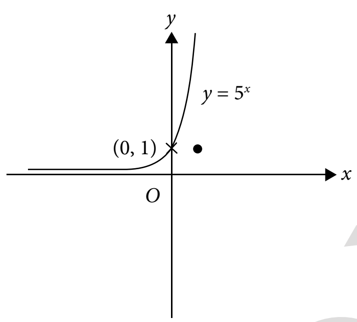
$$2 - \frac{1}{\sqrt{x}} = \frac{1}{3}x + \frac{1}{3}$$

$\therefore$ Equation of line: $\boldsymbol{y = \frac{1}{3}x + \frac{1}{3}}$

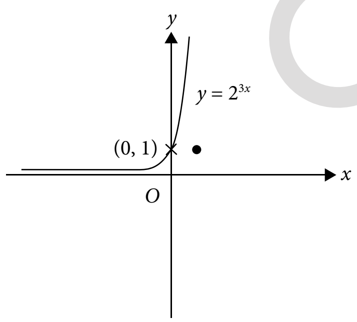
(ii) From the graph, $x =$ **0.40**.

Worksheet 6D Graphs of exponential functions

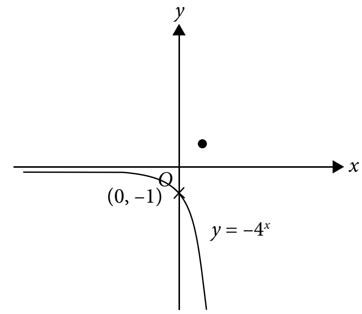
1. **(a)**



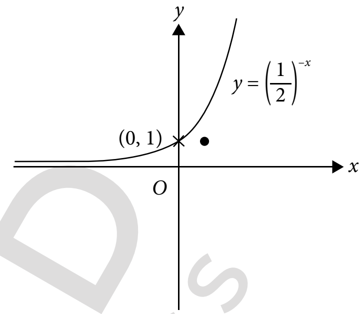
(b)



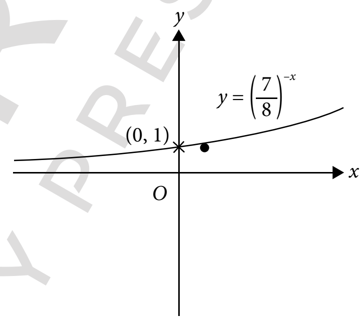
(c)



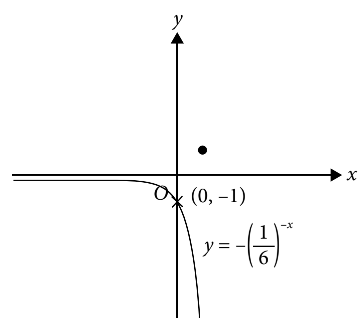
(d)



(e)



(f)



2. **(i)** When $x = 0$,

$y = 2 \times 3^0$

$= 2$

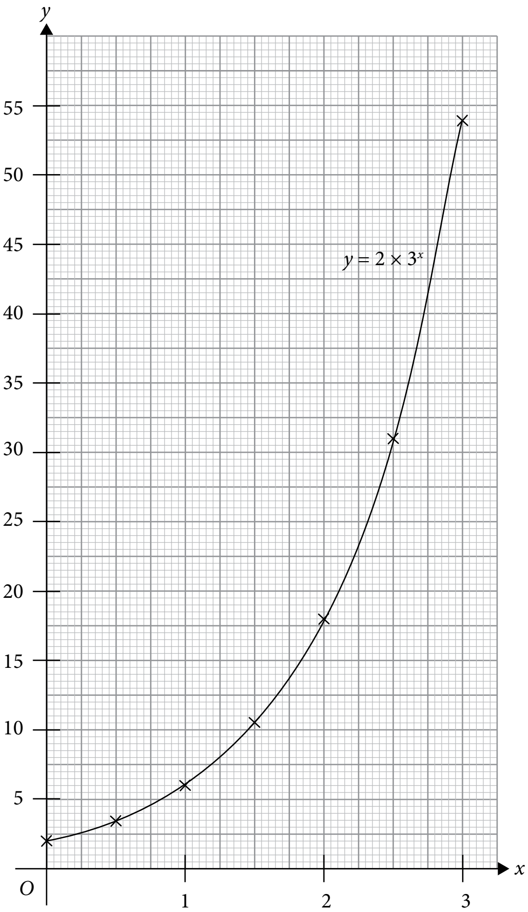
When $x = 2.5$,

$y = 2 \times 3^{2.5}$

$= 31.2$ (to 3 s.f.)

$\therefore$ $\boldsymbol{p = 2, q = 31.2}$

(ii)



(iii) From the graph, when $y = 36$,

$x = 2.625$.

$\therefore x = \mathbf{2.625}$

3. **(a)** Substitute $x = 0$, $y = 0$ into $y = 2^x + k$:

$0 = 2^0 + k$

$= 1 + k$

$k = -1$

Since the y-intercept of $y = mx + c$ is 2,

$c = 2$

$\text{Gradient} = \dfrac{2-0}{0-(-2)}$

$m = 1$

$\therefore \mathbf{k = -1, m = 1, c = 2}$

(b) $y = 2^x - 1$ — (1)

$y = x + 2$ — (2)

Substitute (1) into (2):

$2^x - 1 = x + 2$

$2^x - x = 3$ (shown)

Worksheet 6E Graphs of rational functions

1. **(a)** $y = \dfrac{x+4}{3x}$

Equations of asymptotes: $\mathbf{x = 0, y = \dfrac{1}{3}}$

(b) $y = \dfrac{x^2+4}{3x}$

Equation of asymptote: $\mathbf{x = 0}$

(c) $y = \dfrac{x+4}{3x^2}$

Equations of asymptotes: $\mathbf{x = 0, y = 0}$

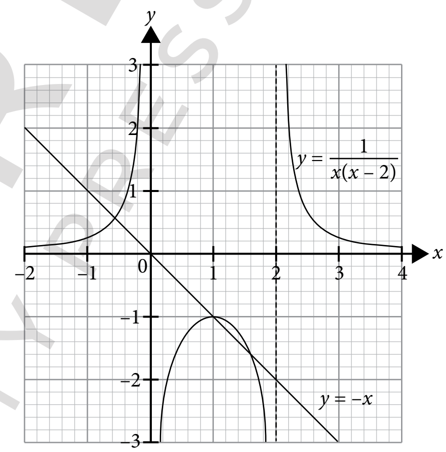
(d) $y = \dfrac{x^2+4}{3x^2}$

Equations of asymptotes: $\mathbf{x = 0, y = \dfrac{1}{3}}$

2. **(a)** $y = \dfrac{1}{x(x-2)}$

Equations of asymptotes: $\mathbf{x = 0, x = 2, y = 0}$

(b)



(c) From the graph, $x = \mathbf{-0.6}$, $x = \mathbf{1}$ or $x = \mathbf{1.6}$.

3. **(a)** $y = \dfrac{3x^2-1}{x^2+4}$

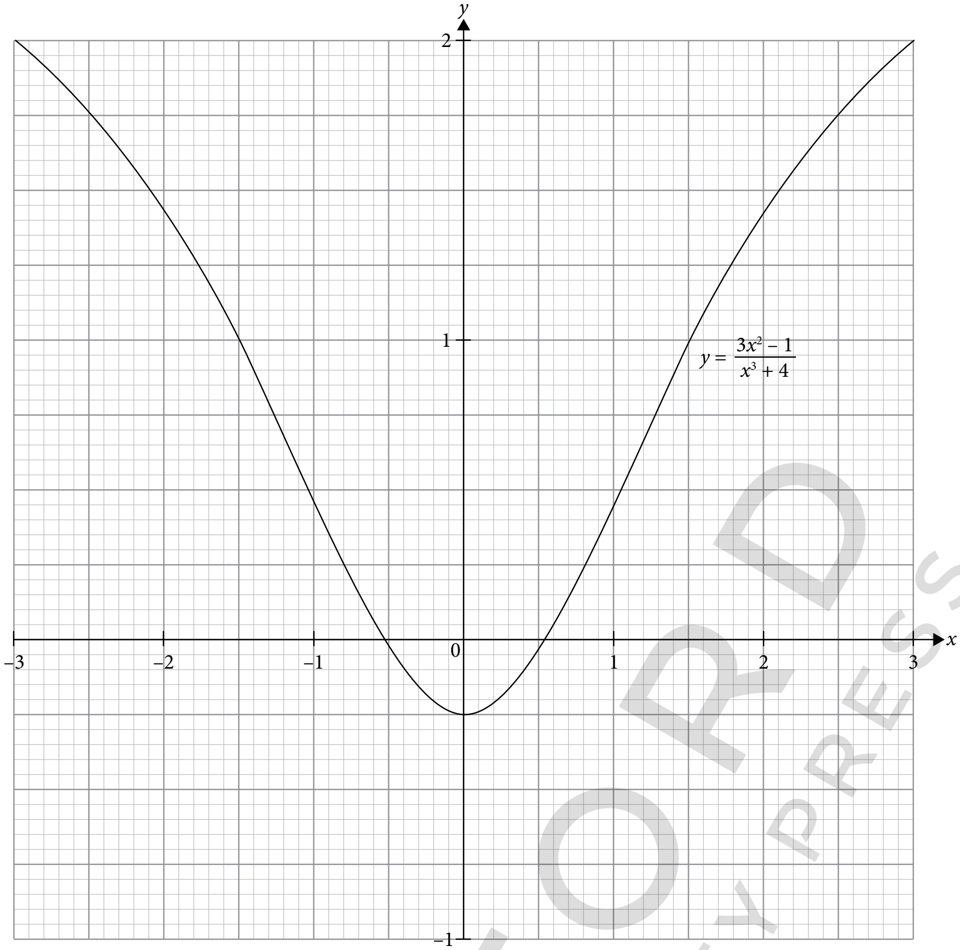
Equation of asymptote: $\mathbf{y = 3}$

(b) When $x = 0$,

$y = -0.25$

$\therefore p = \mathbf{-0.25}$

(c)



(d) From the graph, $x = -1.4$ or $x = 1.4$.

(e) $y = -0.25$, $x = 0$

Worksheet 6F Gradient of a curve

1. **(a)** The line $y = k$ does not intersect the curve $y = \frac{1}{2}x^2 - x$ for $k < -0.5$.

(b) Equation of line of symmetry: $\boldsymbol{x = 1}$

(c) $x^2 - 2x = 1.6$

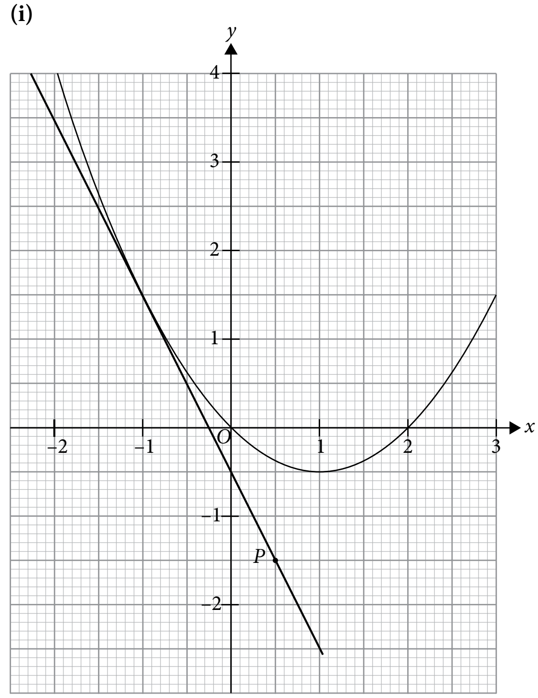
$\frac{1}{2}x^2 - x = 0.8$

From the graph, when $y = 0.8$,

$x = -0.6$ or $x = 2.6$

$\therefore x = \mathbf{-0.6}$ or $x = \mathbf{2.6}$

(d) **(i)**



(ii) Using (0.5, –1.5) and (–1, 1.5),

$$\text{Gradient of tangent} = \frac{1.5-(-1.5)}{-1-0.5} = -2$$

Substitute $m = -2$, $x = -1$ and $y = 1.5$ into $y = mx + c$:

$$1.5 = -2(-1) + c$$
$$= 2 + c$$
$$c = -0.5$$

$\therefore$ Equation of tangent: $\mathbf{y = -2x - 0.5}$

2. **(a)** When $x = -2$,

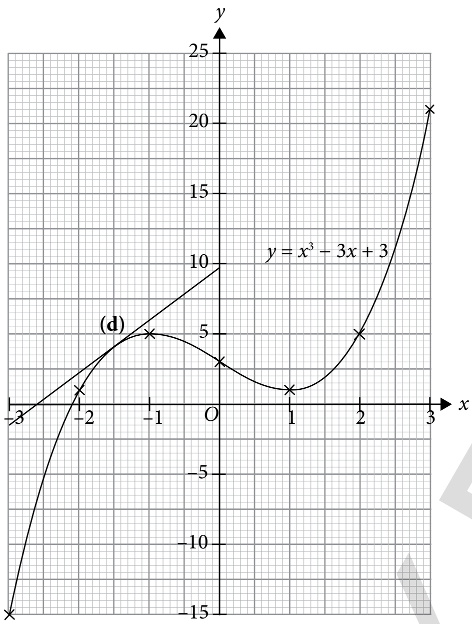
$y = (-2)^3 - 3(-2) + 3$
$= 1$

When $x = 3$,

$y = 3^3 - 3(3) + 3$
$= 21$

$\therefore$ $\mathbf{p = 1, q = 21}$

(b)



(c) From the graph, when $y = 2$,
Largest value of x = **1.5**

(d) Using (0, 9.75) and (–2.6, 0)

$$\text{Gradient} = \frac{0-9.75}{-2.6-0} = \mathbf{3.75}$$

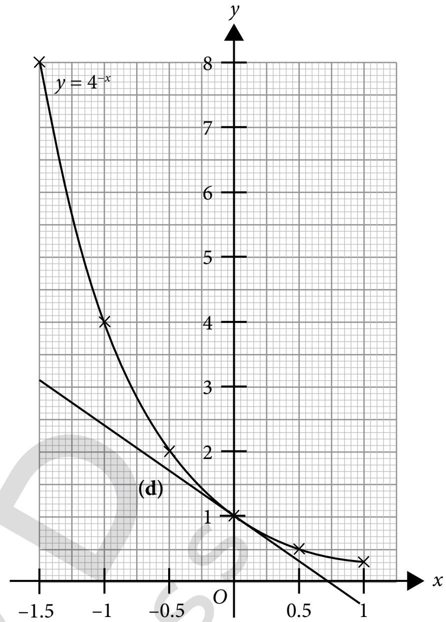
3. **(a)** When $x = -1$,

$y = 4^{-(-1)}$
$= \mathbf{4}$

When $x = 0.5$,

$y = 4^{-0.5}$
$= \mathbf{0.5}$

(b)



(c) From the graph, when $y = 3$,
$x = \mathbf{-0.8}$.

(d) Using (0, 1) and (–1, 2.4)

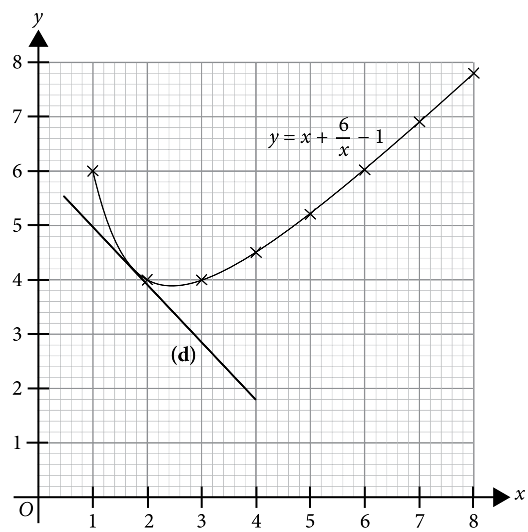
$$\text{Gradient} = \frac{2.4-1}{-1-0} = \mathbf{-1.4}$$

4. **(a)** When $x = 5$,

$$y = 5 + \frac{6}{5} - 1 = 5.2$$

$\therefore$ $p = \mathbf{5.2}$

(b)



(c) Coordinates of minimum point: **(2.5, 3.9)**

(d) From the graph, when $y = 5.5$,
$x = 1.1$ or $x = 5.4$.
$\therefore$ $x = \mathbf{1.1}$ or $x = \mathbf{5.4}$

(e) Using (0.6, 5.4) and (3.4, 2.4),

$$\text{Gradient} = \frac{2.4 - 5.4}{3.4 - 0.6}$$
$$= \mathbf{-1.07} \text{ (to 3 s.f.)}$$

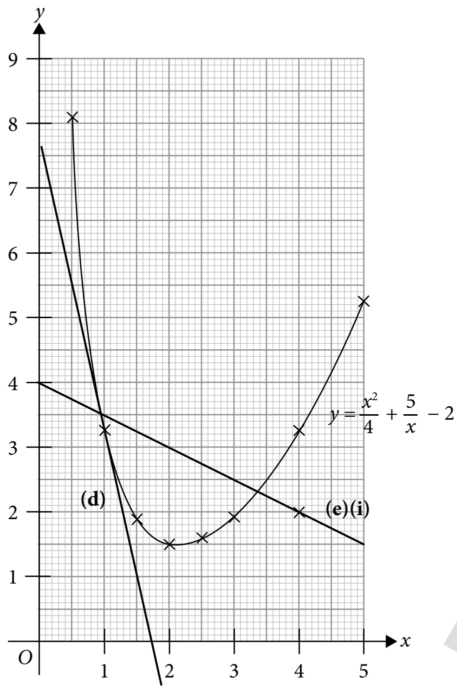
5. **(a)** $y = \frac{x^2}{4} + \frac{5}{x} - 2$

When $x = 0.5$,

$$y = \frac{0.5^2}{4} + \frac{5}{0.5} - 2$$
$$= 8.06 \text{ (to 2 d.p.)}$$

$\therefore p = \mathbf{8.06}$

(b)



(c)
$$\frac{x^2}{4} + \frac{5}{x} = 7$$
$$\frac{x^2}{4} + \frac{5}{x} - 2 = 5$$

From the graph, when $y = 5$,

$x = 0.7$ or $x = 4.9$

$\therefore x = \mathbf{0.7}$ or $x = \mathbf{4.9}$

(d) Using (0, 7.6) and (1.5, 1),

$$\text{Gradient} = \frac{1 - 7.6}{1.5 - 0}$$
$$= \mathbf{-4.4}$$

(e) **(ii)** Equation of line: $\mathbf{y = -0.5x + 4}$

(iii) Coordinates of intersection points: **(0.9, 3.5)** and **(3.4, 2.3)**

Worksheet 6G Applications of graphs in real-world contexts

1. **(a)** Initial speed = **0 m/s**

(b)
$$\text{Gradient} = \frac{0 - 60}{35 - 10}$$
$$= -2.4$$

$\therefore$ Speed = **2.4 m/s**

2. **(a)**
$$\text{Gradient} = \frac{0 - 21}{75 - 60}$$
$$= -1.4$$

$\therefore$ Deceleration = **1.4 m/s²**

(b)
$$\text{Total distance} = \frac{1}{2}(45 + 75)(21)$$
$$= \mathbf{1260 \text{ m}}$$

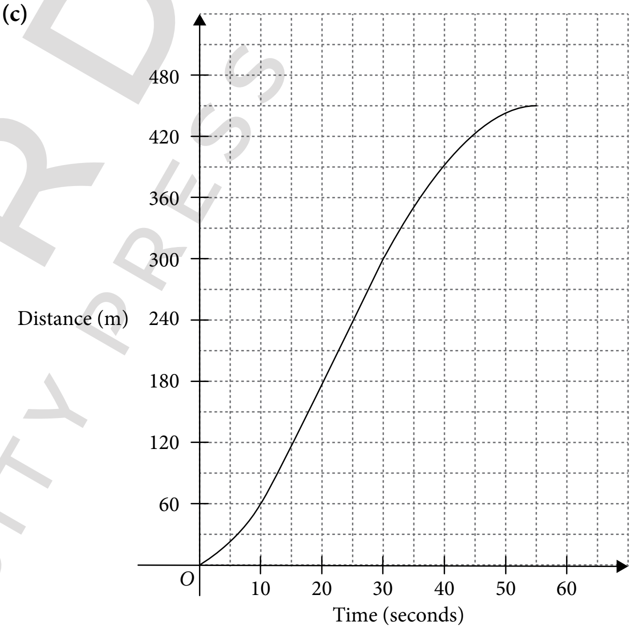
3. **(a)** Total distance = 450 m

$$\frac{1}{2}(55 + 20)v = 450$$
$$37.5v = 450$$
$$v = 12$$

$\therefore$ Greatest speed = **12 m/s**

(b)
$$\text{Acceleration} = \frac{12 - 0}{10 - 0}$$
$$= \mathbf{1.2 \text{ m/s}^2}$$

(c)

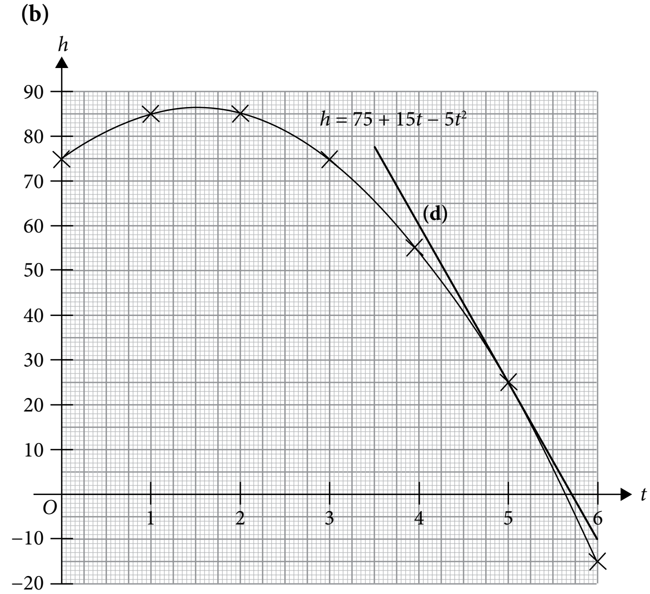


4. **(a)** When $t = 6$,

$$h = 75 + 15(6) - 5(6)^2$$
$$= -15$$

$\therefore p = \mathbf{-15}$

(b)



(c) **(i)** From the graph,
Maximum height = **86.5 m**

(ii) From the graph,
Length of time $= 2.6 - 0.4$
$= \mathbf{2.2\ s}$

(iii) From the graph,
Time taken = **5.65 s**

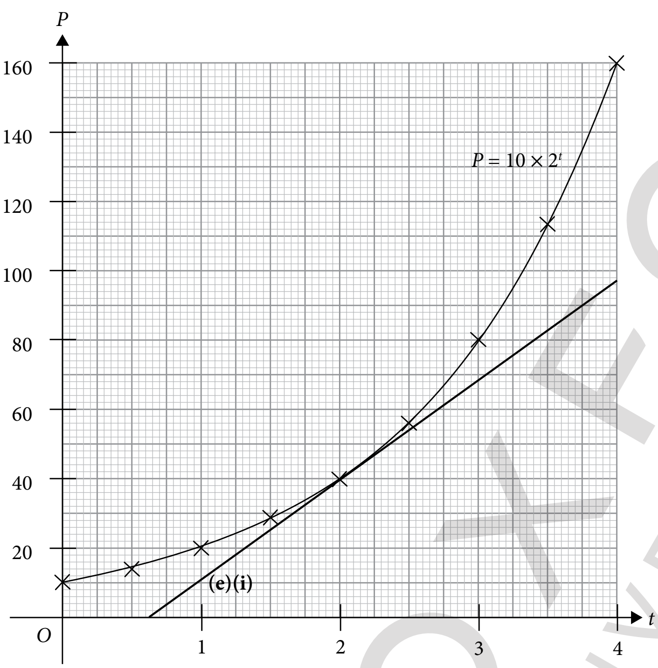
(d) Using (5, 25) and (6, –10),

$$\text{Gradient} = \frac{-10-25}{6-5}$$
$$= \mathbf{-35\ m/s}$$

5. **(a)** When $t = 0$,
$P = 10 \times 2^0$
$= 10$
$\therefore$ Launch price = **\$10**

(b) When $t = 0.5$,
$P = 10 \times 2^{0.5}$
$= 14.1$ (to 3 s.f.)
$\therefore x = \mathbf{14.1}$

(c)



(d) From the graph, when $P = 100$,
$t = 3.3$
$\therefore$ It takes about **3.3 months**.

(e) **(i)** Using (2, 40) and (3.4, 80),

$$\text{Gradient} = \frac{80-40}{3.4-2}$$
$$= \mathbf{28.6} \text{ (to 3 s.f.)}$$

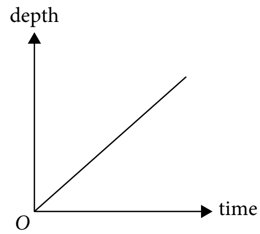
(ii) It represents the rate of change of the share price, i.e. at the instant when $t = 2$, the share price is increasing at a rate of \$28.60 per month.

6. **(a)** **(i)** From the graph, it costs **\$600**.

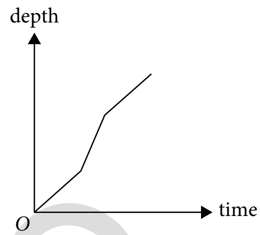
(ii) 30 min = 0.5 h
From the graph, it costs **\$300**.

(b) \$460 + \$680 = \$1140
Ella could have spent 1.25 h = **1 h 15 min**.
Darren could have spent 3.5 h = **3 h 30 min**.

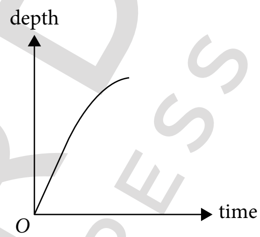
7. **(a)**



(b)



(c)



Review Exercise 6

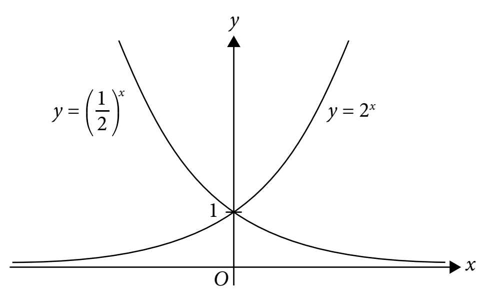
1. **(a)** The equation could be $y = x^3 - 2$.
$\therefore$ $\boldsymbol{a = 1, b = -2, n = 3}$

(b) The equation could be $y = -x + 2$.
$\therefore$ $\boldsymbol{a = -1, b = 2, n = 1}$

(c) The equation could be $y = \frac{1}{x^2} - 1$.
$\therefore$ $\boldsymbol{a = 1, b = -1, n = -2}$

(d) The equation could be $y = -x^2 + 2$.
$\therefore$ $\boldsymbol{a = -1, b = 2, n = 2}$

2. **(i)**



(ii) Substitute $x = -3$ into $y = 2^x$:
$y = 2^{-3}$
$= \frac{1}{8}$

Substitute $x = 3$ into $y = \left(\frac{1}{2}\right)^x$:
$y = \left(\frac{1}{2}\right)^3$
$= \frac{1}{8}$

Estimated area = 2 × area of trapezium

$= 2\times\frac{1}{2}\left(\frac{1}{8}+1\right)(3)$

$= 3.375$ units^2

Due to the curvature of $y = 2^x$ and $y = \left(\frac{1}{2}\right)^x$, the area calculated is an **overestimate**.

3. **(a)** $y = x+\frac{2}{x^2}-3$

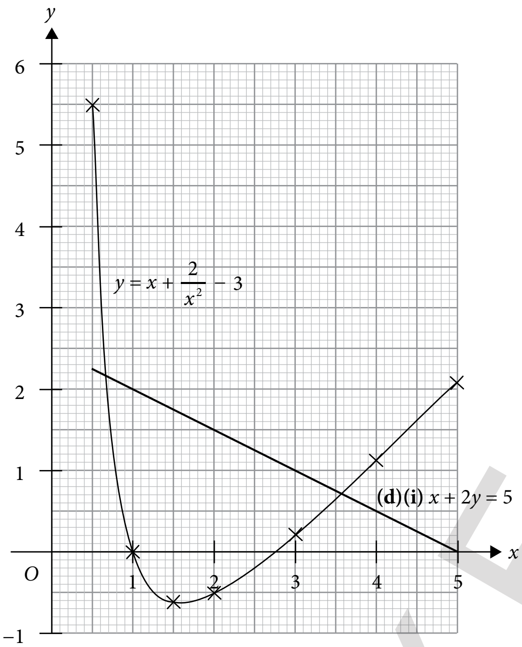
When $x = 1.5$,

$y = 1.5+\frac{2}{1.5^2}-3$

$= -0.61$ (to 2 d.p.)

$\therefore p = \mathbf{-0.61}$

(b)



(c) $x+\frac{2}{x^2} = 4$

$x+\frac{2}{x^2}-3 = 1$

From the graph, when $y = 1$,

$x = 0.8$ or $x = 3.85$

$\therefore x = \mathbf{0.8}$ or $x = \mathbf{3.85}$

(d) **(ii)** From the graph,

$x = \mathbf{0.65}$ or $x = \mathbf{3.55}$.

(iii) $y = x+\frac{2}{x^2}-3$ — (1)

$x + 2y = 5$ — (2)

Substitute (1) into (2):

$x+2\left(x+\frac{2}{x^2}-3\right) = 5$

$x+2x+\frac{4}{x^2}-6 = 5$

$3x+\frac{4}{x^2}-11 = 0$

$3x^3 - 11x^2 + 4 = 0$

$\therefore \boldsymbol{h = -11, k = 4}$

7 Volume, Surface Area, and Symmetry of Pyramids, Cones and Spheres

Worksheet 7A Volume, surface area and symmetry of pyramids

1. **(a)** Volume of pyramid $= \frac{1}{3}\times\left(\frac{1}{2}\times 8\times 5\right)\times 4$ cm^3

$= \mathbf{26\frac{2}{3}\ cm^3}$

(b) Volume of pyramid $= \frac{1}{3}\times(12\times 12)\times 15$ cm^3

$= \mathbf{720\ cm^3}$

(c) Volume of pyramid $= \frac{1}{3}\times(27\times 24)\times 30$ cm^3

$= \mathbf{6480\ cm^3}$

(d) Volume of pyramid $= \frac{1}{3}\times 400\times 20$ cm^3

$= \mathbf{2666\frac{2}{3}\ cm^3}$

2. Volume of pyramid $= \frac{1}{3}\times 58\times 12$ cm^3

$= \mathbf{232\ cm^3}$

3. Let the height of the pyramid be h cm.

Volume of pyramid = 388 cm^3

$\frac{1}{3}\times(5\times 5)\times h = 388$

$h = 46.56$

$\therefore$ Height of pyramid = **46.56 cm**

4. Let the base area of the pyramid be A cm^2.

Volume of pyramid = 1280 cm^3

$\frac{1}{3}\times A\times 40 = 1280$

$A = 96$

$\therefore$ Base area of pyramid = **96 cm²**

5. (OPEN) Volume of tetrahedron = 300 cm^3

$\frac{1}{3}\times p\times q = 300$

$pq = 900$

$\therefore$ A possible pair of values is $\boldsymbol{p = 300}$ and $\boldsymbol{q = 3}$.

6. **(i)** Let the length of the square base be x cm.

Using Pythagoras' Theorem,

$x^2 + x^2 = 96^2$

$2x^2 = 9216$

$x^2 = 4608$

$x = \sqrt{4608}$ (since $x > 0$)

$= 67.9$ (to 3 s.f.)

$\therefore$ Length of square base = **67.9 cm**

(ii) Let the height of the pyramid be h cm.

Volume of pyramid = 57 000 cm^3

$\frac{1}{3}\times\left(\sqrt{4608}\right)^2\times h = 57\ 000$

$h = 37.1$ (to 3 s.f.)

$\therefore$ Height of pyramid = **37.1 cm**

7. **(a)** **2**

(b) **1**

(c) **4**

(d) Volume of pyramid $= \frac{1}{3} \times (5 \times 5) \times 6$ cm^3

$= \mathbf{50\ cm^3}$

(e) Let the height of each triangular face be h cm.

Using Pythagoras' Theorem,

$6^2 + 2.5^2 = h^2$

$h^2 = 42.25$

$h = \sqrt{42.25}$ (since $h > 0$)

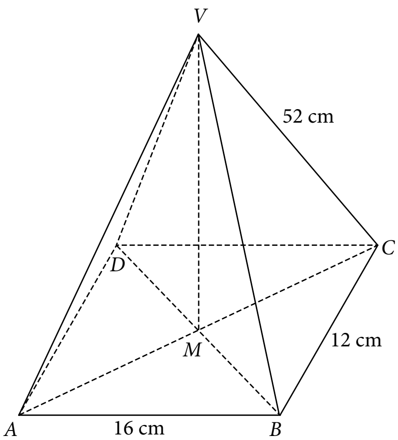
$= 6.5$

$\therefore$ Total surface area of pyramid

$= \left[(5 \times 5) + 4 \times \left(\frac{1}{2} \times 5 \times 6.5\right)\right]$ cm^2

$= \mathbf{90\ cm^2}$

8. **(a)**



Consider $\triangle ABC$.

Using Pythagoras' Theorem,

$AC^2 = 16^2 + 12^2$

$= 400$

$AC = \sqrt{400}$ (since $AC > 0$)

$= 20$ cm

$MC = 10$ cm

Consider $\triangle VMC$.

Using Pythagoras' Theorem,

$VM^2 + 10^2 = 52^2$

$VM^2 = 52^2 - 10^2$

$= 2604$

$VM = \sqrt{2604}$ (since VM > 0)

$= 51.029$ cm (to 5 s.f.)

$\therefore$ Volume of pyramid $= \frac{1}{3} \times (16 \times 12) \times 51.029$ cm^3

$= \mathbf{3270\ cm^3}$ (to 3 s.f.)

(b) Let the height of $\triangle VBC$ be h cm.

Using Pythagoras' Theorem,

$h^2 + 6^2 = 52^2$

$h^2 = 52^2 - 6^2$

$= 2668$

$h = \sqrt{2668}$ (since $h > 0$)

$= 51.653$ (to 5 s.f.)

Let the height of $\triangle VAB$ be k cm.

Using Pythagoras' Theorem,

$k^2 + 8^2 = 52^2$

$k^2 = 52^2 - 8^2$

$= 2640$

$k = \sqrt{2640}$ (since $k > 0$)

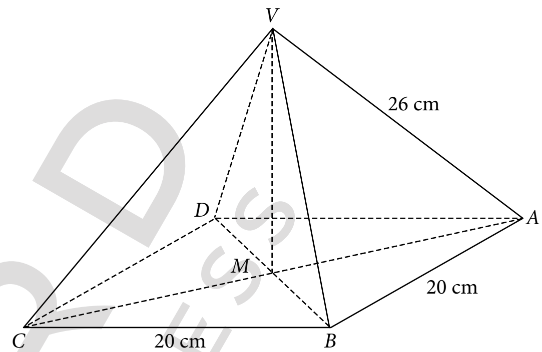
$= 51.381$ (to 5 s.f.)

$\therefore$ Total surface area of pyramid

$= \left[(16 \times 12) + 2 \times \left(\frac{1}{2} \times 12 \times 51.653\right) + 2 \times \left(\frac{1}{2} \times 16 \times 51.381\right)\right]$ cm^2

$= \mathbf{1630\ cm^2}$ (to 3 s.f.)

9.



Consider $\triangle ABC$.

Using Pythagoras' Theorem,

$AC^2 = 20^2 + 20^2$

$= 800$

$AC = \sqrt{800}$ (since $AC > 0$)

$= 28.284$ cm (to 5 s.f.)

$MC = 14.142$ cm (to 5 s.f.)

Consider $\triangle VMC$.

Using Pythagoras' Theorem,

$VM^2 + 14.142^2 = 26^2$

$VM^2 = 26^2 - 14.142^2$

$= 476$

$VM = \sqrt{476}$ (since $VM > 0$)

$= 21.817$ cm (to 5 s.f.)

$\therefore$ Volume of pyramid $= \frac{1}{3} \times (20 \times 20) \times 21.817$ cm^3

$= \mathbf{2910\ cm^3}$ (to 3 s.f.)

Let the height of $\triangle VBC$ be h cm.

Using Pythagoras' Theorem,

$h^2 + 10^2 = 26^2$

$h^2 = 26^2 - 10^2$

$= 576$

$h = \sqrt{576}$ (since $h > 0$)

$= 24$

$\therefore$ Total surface area of pyramid

$= \left[(20 \times 20) + 4 \times \left(\frac{1}{2} \times 20 \times 24\right)\right]$ cm^2

$= \mathbf{1360\ cm^2}$

10. **(a)** $VN^2 + AN^2 = 8^2 + 6^2$
$= 100$
$VA^2 = 10^2$
$= 100$
Since $VN^2 + AN^2 = VA^2$, then by the converse of Pythagoras' Theorem, $\triangle VAN$ is a right-angled triangle where $\angle VNA = 90°$.
$\angle VNB = 180° - \angle VNA$ (adj. $\angle$s on a str. line)
$= 90°$

(b) **(i)** Total surface area of pyramid
$= \left[(12 \times 12) + 4 \times \left(\frac{1}{2} \times 12 \times 8\right)\right]$ cm^2
$=$ **336 cm²**

(ii) Consider $\triangle VMN$.
Using Pythagoras' Theorem,
$VM^2 + 6^2 = 8^2$
$VM^2 = 8^2 - 6^2$
$= 28$
$VM = \sqrt{28}$ (since $VM > 0$)
$=$ **5.29 cm** (to 3 s.f.)

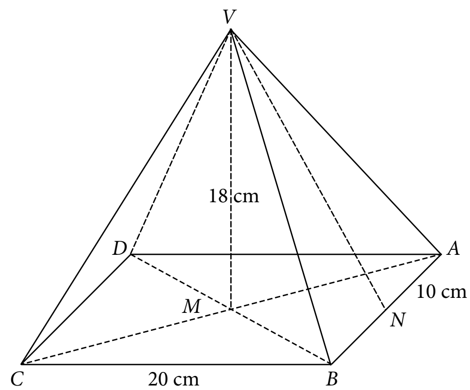
(iii) Volume of pyramid $= \frac{1}{3} \times (12 \times 12) \times 5.2915$ cm^3
$=$ **254 cm³** (to 3 s.f.)

11. **(a)** Interior angle of a regular hexagon $= \frac{(6-2) \times 180°}{6}$
$= 120°$
$\angle ODN = \frac{120°}{2} = 60°$
Consider $\triangle ODN$.
$\tan 60° = \frac{ON}{4}$
$ON = 4 \tan 60°$
$= 6.9282$ cm (to 5 s.f.)
$\therefore$ Area of $ABCDEF = 6 \times \left(\frac{1}{2} \times 8 \times 6.9282\right)$ cm^2
$=$ **166 cm²** (to 3 s.f.)

(b) Consider $\triangle VON$.
Using Pythagoras' Theorem,
$VO^2 + (4 \tan 60°)^2 = 20^2$
$VO^2 = 20^2 - (4 \tan 60°)^2$
$= 352$
$VO = \sqrt{352}$ (since $VO > 0$)
$= 18.762$ cm (to 5 s.f.)
$\therefore$ Volume of pyramid $= \frac{1}{3} \times 166.28 \times 18.762$ cm^3
$=$ **1040 cm³** (to 3 s.f.)

Challenge Myself!

12. **(a)**



Consider $\triangle ABC$.
Using Pythagoras' Theorem,
$AC^2 = 20^2 + 20^2$
$= 800$
$AC = \sqrt{800}$ (since $AC > 0$)
$= 28.284$ cm (to 5 s.f.)
$AM = 14.142$ cm (to 5 s.f.)
Consider $\triangle VAM$.
Using Pythagoras' Theorem,
$AV^2 = 18^2 + 14.142^2$
$= 524$
$AV = \sqrt{524}$ (since $AV > 0$)
$= 22.9$ cm (to 3 s.f.) (shown)

(b) Consider $\triangle VAN$.
$\cos \angle BAV = \frac{10}{22.891}$
$\angle BAV = \cos^{-1} \frac{10}{22.891}$
$=$ **64.1°** (to 1 d.p.)

(c) Consider $\triangle VAN$.
Using Pythagoras' Theorem,
$VN^2 + 10^2 = \left(\sqrt{524}\right)^2$
$VN^2 = 524 - 10^2$
$= 424$
$VN = \sqrt{424}$ (since $VN > 0$)
$= 20.591$ cm (to 5 s.f.)
$\therefore$ Total surface area of pyramid
$= \left[(20 \times 20) + 4 \times \left(\frac{1}{2} \times 20 \times 20.591\right)\right]$ cm^2
$=$ **1220 cm²** (to 3 s.f.)

(d) Consider the pair of similar triangles $\triangle VYX$ and $\triangle VCB$.
$\frac{YX}{20} = \frac{20.591 - 3}{20.591}$
$YX = \frac{20.591 - 3}{20.591} \times 20$ cm
$= 17.086$ cm (to 5 s.f.)
$\therefore$ Area of $WXYZ = 17.086^2$ cm^2
$=$ **292 cm²** (to 3 s.f.)

Worksheet 7B Volume, surface area and symmetry of cones

1. (a) Volume of cone = $\frac{1}{3}\pi(4)^2(8)$ cm^3

= **134 cm³** (to 3 s.f.)

Consider $\triangle VOB$.

Using Pythagoras' Theorem,

$VB^2 = 8^2 + 4^2$

$= 80$

$VB = \sqrt{80}$ (since $VB > 0$)

$= 8.9443$ cm (to 5 s.f.)

Total surface area of cone = $[\pi(4)^2 + \pi(4)(8.9443)]$ cm^2

= **163 cm²** (to 3 s.f.)

(b) Volume of cone = $\frac{1}{3}\pi(7)^2(15)$ cm^3

= **770 cm³** (to 3 s.f.)

Consider $\triangle VOA$.

Using Pythagoras' Theorem,

$VA^2 = 15^2 + 7^2$

$= 274$

$VA = \sqrt{274}$ (since $VA > 0$)

$= 16.553$ cm (to 5 s.f.)

Total surface area of cone = $[\pi(7)^2 + \pi(7)(16.553)]$ cm^2

= **518 cm²** (to 3 s.f.)

(c) Volume of cone = $\frac{1}{3}\pi(10)^2(20)$ cm^3

= **2090 cm³** (to 3 s.f.)

Consider $\triangle VOA$.

Using Pythagoras' Theorem,

$VA^2 = 10^2 + 20^2$

$= 500$

$VA = \sqrt{500}$ (since $VA > 0$)

$= 22.361$ cm (to 5 s.f.)

Total surface area of cone = $[\pi(10)^2 + \pi(10)(22.361)]$ cm^2

= **1020 cm²** (to 3 s.f.)

(d) Consider $\triangle VOA$.

Using Pythagoras' Theorem,

$VO^2 + 15^2 = 39^2$

$VO^2 = 39^2 - 15^2$

$= 1296$

$VO = \sqrt{1296}$ (since $VO > 0$)

$= 36$ cm

Volume of cone = $\frac{1}{3}\pi(15)^2(36)$ cm^3

= **8480 cm³** (to 3 s.f.)

Total surface area of cone = $[\pi(15)^2 + \pi(15)(39)]$ cm^2

= **2540 cm²** (to 3 s.f.)

2. (a) (i) **Infinite number**

(ii) **Infinite number**

(b) Total surface area of cone = $[\pi(5)^2 + \pi(5)(8)]$ cm^2

= **65π cm²**

3. Volume of cone = $\frac{1}{3}\pi(6)^2(6)$ cm^3

= **226 cm³** (to 3 s.f.)

Let the slant height of the cone be l cm.

Using Pythagoras' Theorem,

$l^2 = 6^2 + 6^2$

$= 72$

$l = \sqrt{72}$ (since $l > 0$)

$= 8.4853$ (to 5 s.f.)

Total surface area of cone = $[\pi(6)^2 + \pi(6)(8.4853)]$ cm^2

= **273 cm²** (to 3 s.f.)

4. (i) Let the base radius of the cone be r cm.

Volume of cone = 250π cm^3

$\frac{1}{3}\pi r^2(10\pi) = 250\pi$

$r^2 = \frac{75}{\pi}$

$r = \sqrt{\frac{75}{\pi}}$ (since $r > 0$)

$= 4.89$ (to 3 s.f.)

∴ Base radius of cone = **4.89 cm**

(ii) Let the slant height of the cone be l cm.

Using Pythagoras' Theorem,

$l^2 = \left(\sqrt{\frac{75}{\pi}}\right)^2 + (10\pi)^2$

$= 1010.8$ (to 5 s.f.)

$l = \sqrt{1010.8}$ (since $l > 0$)

$= 31.794$ (to 5 s.f.)

Curved surface area of cone = $\pi(4.8860)(31.794)$ cm^2

= **488 cm²** (to 3 s.f.)

5. Let the height of the cone be h cm.

Volume of cone = 3888π cm^3

$\frac{1}{3}(324\pi)h = 3888\pi$

$h = 36$

Let the base radius of the cone be r cm.

Base area of cone = 324π cm^2

$\pi r^2 = 324\pi$

$r^2 = 324$

$r = \sqrt{324}$ (since $r > 0$)

$= 18$

Let the slant height of the cone be l cm.

Using Pythagoras' Theorem,

$l^2 = 18^2 + 36^2$

$= 1620$

$l = \sqrt{1620}$ (since $l > 0$)

$= 40.249$ (to 5 s.f.)

Total surface area of cone = $[\pi(18)^2 + \pi(18)(40.249)]$ cm^2

= **3290 cm²** (to 3 s.f.)

6. OPEN Let base radius = 6 cm and height = 12 cm.

Volume of cone = $\frac{1}{3}\pi(6)^2(12)$ cm^3

$= 144\pi$ cm^3

∴ A possible base radius is **6 cm** and a possible height is **12 cm**.

7. Let the slant height of the cone be l cm.

Total surface area = 720 cm^2

$$\pi(9)^2 + \pi(9)(l) = 720$$
$$81\pi + 9\pi l = 720$$
$$9\pi l = 720 - 81\pi$$
$$l = \frac{720 - 81\pi}{9\pi}$$
$$= 16.465 \text{ (to 5 s.f.)}$$

Let the height of the cone be h cm.

Using Pythagoras' Theorem,

$$h^2 + 9^2 = 16.465^2$$
$$h^2 = 16.465^2 - 9^2$$
$$= 190.09 \text{ (to 5 s.f.)}$$
$$h = \sqrt{190.09} \text{ (since } h > 0)$$
$$= 13.787 \text{ (to 5 s.f.)}$$

$\therefore$ Volume of cone $= \frac{1}{3}\pi(9)^2(13.787)$ cm^3

$= \mathbf{1170\ cm^3}$ (to 3 s.f.)

8. $$\frac{\text{Volume of smaller cone}}{\text{Volume of larger cone}} = \frac{\frac{1}{3}\pi(R)^2 H}{\frac{1}{3}\pi(kR)^2(kH)}$$
$$= \frac{\frac{1}{3}\pi R^2 H}{\frac{1}{3}\pi k^3 R^2 H}$$
$$= \frac{1}{k^3}$$

$\therefore$ The ratio of their volumes is $\mathbf{1 : k^3}$.

Using similar triangles, their slant heights are L and kL respectively.

$$\frac{\text{Total surface area of smaller cone}}{\text{Total surface area of larger cone}} = \frac{\pi R^2 + \pi RL}{\pi(kR)^2 + \pi(kR)(kL)}$$
$$= \frac{\pi R^2 + \pi RL}{\pi k^2 R^2 + \pi k^2 RL}$$
$$= \frac{\pi R^2 + \pi RL}{k^2(\pi R^2 + \pi RL)}$$
$$= \frac{1}{k^2}$$

$\therefore$ The ratio of their total surface areas is $\mathbf{1 : k^2}$.

9. (i) Let the base radius of the cone be r cm.

Using Pythagoras' Theorem,

$$r^2 + 7.5^2 = 9^2$$
$$r^2 = 9^2 - 7.5^2$$
$$= 24.75$$
$$r = \sqrt{24.75} \text{ (since } r > 0)$$
$$= 4.97 \text{ (to 3 s.f.)}$$

$\therefore$ Base radius of cone = **4.97 cm**

(ii) Volume of cone $= \frac{1}{3}\pi\left(\sqrt{24.75}\right)^2(7.5)$

$= \frac{495}{8}\pi$ cm^3

Time taken for the solid to completely melt

$$= \frac{\frac{495}{8}\pi}{4.2} \text{ min}$$
$$= 46.282 \text{ min (to 5 s.f.)}$$

$\therefore$ The solid has completely melted at **3.43 p.m.**

(iii) Let the height of the mould be H cm.

Volume of mould $= \frac{495}{8}\pi$ cm^3

$$\frac{1}{3}\pi(4)^2 H = \frac{495}{8}\pi$$
$$H = 11.602 \text{ (to 5 s.f.)}$$

Let the slant height of the mould be l cm.

Using Pythagoras' Theorem,

$$l^2 = 4^2 + 11.602^2$$
$$= 150.60 \text{ (to 5 s.f.)}$$
$$l = \sqrt{150.60} \text{ (since } l > 0)$$
$$= 12.272 \text{ (to 5 s.f.)}$$

$\therefore$ Curved surface area of mould = $\pi(4)(12.272)$ cm^2

$= \mathbf{154\ cm^2}$ (to 3 s.f.)

10. (i) Let the base radius of the cone be r cm.

Base circumference = 6 cm

$$2\pi r = 6$$
$$r = \frac{6}{2\pi} = 0.954\,93 \text{ (to 5 s.f.)}$$

Let the height of the cone be h cm.

Using Pythagoras' Theorem,

$$h^2 + 0.954\,93^2 = 3^2$$
$$h^2 = 3^2 - 0.954\,93^2$$
$$= 8.0881 \text{ (to 5 s.f.)}$$
$$h = \sqrt{8.0881} \text{ (since } h > 0)$$
$$= 2.8440 \text{ (to 5 s.f.)}$$

Estimated volume $= \frac{1}{3}\pi(0.954\,93)^2(2.8840)$ cm^3

$= \mathbf{2.72\ cm^3}$ (to 3 s.f.)

(ii) Estimated mass of each piece of snack = (2.7158×0.14) g

$= 0.380\,21$ g (to 5 s.f.)

Estimated number of pieces of snack $= \frac{60}{0.380\,21}$

= **158** (to the nearest whole number)

11. (i) Length of arc $AB = 28\pi$ cm

$$\pi(OA) = 28\pi$$
$$OA = \mathbf{28\ cm}$$

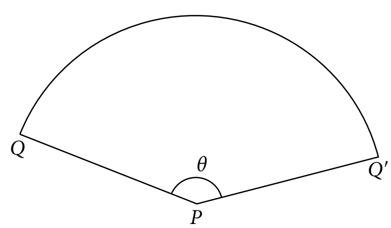
(ii) Let the base radius of the cone be r cm.

Base circumference = 28π cm

$$2\pi r = 28\pi$$
$$r = 14$$

$\therefore$ Base radius of cone = **14 cm**

12. (i)



Base circumference of cone = $2\pi(6)$ cm

$= 12\pi$ cm

Length of arc $QQ' = 12\pi$ cm

$$\frac{\theta}{360°} \times 2\pi(15) = 12\pi$$
$$\theta = 144°$$

$\therefore$ Angle at centre = **144°**

(ii) Perimeter of cardboard = $(12\pi + 15 + 15)$ cm

= **67.7 cm** (to 3 s.f.)

13. (i) Let the height of the smaller cone be h cm.
Since $\triangle VAB$ is similar to $\triangle VPQ$,

$$\frac{h}{h+6} = \frac{5}{7}$$
$$7h = 5h + 30$$
$$2h = 30$$
$$h = 15$$

$\therefore$ Height of smaller cone = **15 cm**

(ii) Volume of frustum
= Volume of larger cone – volume of smaller cone

$$= \left[\frac{1}{3}\pi(7)^2(15+6) - \frac{1}{3}\pi(5)^2(15)\right] \text{ cm}^3$$

= **218π cm³**

14. (a) Let the radius of the cone be r cm.
Using Pythagoras' Theorem,

$$r^2 + 24^2 = 26^2$$
$$r^2 = 26^2 - 24^2$$
$$= 100$$
$$r = \sqrt{100} \text{ (since } r > 0)$$
$$= 10$$

$\therefore$ Radius of cone = **10 cm**

(b) (i) Consider $\triangle VXA$ and $\triangle VYB$.

$$\frac{24}{24+XY} = \frac{26}{58.5}$$
$$= \frac{4}{9}$$
$$216 = 96 + 4XY$$
$$4XY = 120$$
$$XY = 30 \text{ cm}$$
$$\frac{YB}{10} = \frac{58.5}{26}$$
$$YB = \frac{58.5}{26} \times 10$$
$$= 22.5 \text{ cm}$$

$\therefore$ ***XY* = 30 cm, *YB* = 22.5 cm**

(ii) Volume of frustum
= Volume of larger cone – volume of smaller cone

$$= \left[\frac{1}{3}\pi(22.5)^2(24+30) - \frac{1}{3}\pi(10)^2(24)\right] \text{ cm}^3$$

= **26 100 cm³** (to 3 s.f.)

Challenge Myself!

15. Let the radius of the surface of water in the inverted container be r cm.
Using similar triangles,

$$\frac{r}{12} = \frac{20}{30}$$
$$r = \frac{20}{30} \times 12$$
$$= 8$$

Volume of water $= \frac{1}{3}\pi(8)^2(20)$ cm³

$= \frac{1280}{3}\pi$ cm³

Total volume $= \frac{1}{3}\pi(12)^2(30)$ cm³

$= 1440\pi$ cm³

Volume of water : total volume $= \frac{1280}{3}\pi : 1440\pi$

$= 8 : 27$

Consider the case when the container is inverted.
Since the triangles are similar and the ratio of the volumes of the cones is 19 : 27, then the ratio of the heights is 1 : 1.1243.

$$\therefore h = \frac{1.1243 - 1}{1.1243} \times 30$$
$$= 3.32 \text{ (to 3 s.f)}$$

$\therefore$ Depth of water is now **3.32 cm**

Worksheet 7C Volume and surface area of spheres

1. (a) Volume of sphere $= \frac{4}{3}\pi(7)^3$ cm³
= **1440 cm³** (to 3 s.f.)
Total surface area of sphere = $4\pi(7)^2$ cm²
= **616 cm²** (to 3 s.f.)

(b) Volume of sphere $= \frac{4}{3}\pi(8.4)^3$ cm³
= **2480 cm³** (to 3 s.f.)
Total surface area of sphere = $4\pi(8.4)^2$ cm²
= **887 cm²** (to 3 s.f.)

(c) Volume of sphere $= \frac{4}{3}\pi(5)^3$ cm³
= **524 cm³** (to 3 s.f.)
Total surface area of sphere = $4\pi(5)^2$ cm²
= **314 cm²** (to 3 s.f.)

(d) Volume of sphere $= \frac{4}{3}\pi(3.15)^3$ cm³
= **131 cm³** (to 3 s.f.)
Total surface area of sphere = $4\pi(3.15)^2$ cm²
= **125 cm²** (to 3 s.f.)

2. (a) Volume of hemisphere $= \frac{2}{3}\pi(6)^3$ cm³
= **452 cm³** (to 3 s.f.)
Total surface area of hemisphere = $[2\pi(6)^2 + \pi(6)^2]$ cm²
= **339 cm²** (to 3 s.f.)

(b) Volume of hemisphere $= \frac{2}{3}\pi(2.8)^3$ cm³
= **46.0 cm³** (to 3 s.f.)
Total surface area of hemisphere = $[2\pi(2.8)^2 + \pi(2.8)^2]$ cm²
= **73.9 cm²** (to 3 s.f.)

(c) Volume of hemisphere $= \frac{2}{3}\pi(11)^3$ cm³
= **2790 cm³** (to 3 s.f.)
Total surface area of hemisphere = $[2\pi(11)^2 + \pi(11)^2]$ cm²
= **1140 cm²** (to 3 s.f.)

(d) Volume of hemisphere $= \frac{2}{3}\pi(4.75)^3$ cm³
= **224 cm³** (to 3 s.f.)
Total surface area of hemisphere = $[2\pi(4.75)^2 + \pi(4.75)^2]$ cm²
= **213 cm²** (to 3 s.f.)

3. **(a)** Let the radius of the sphere be r cm.
Volume = 930 cm³

$$\frac{4}{3}\pi r^3 = 930$$
$$r^3 = \frac{1395}{2\pi}$$
$$r = \sqrt[3]{\frac{1395}{2\pi}}$$
$$= 6.06 \text{ (to 3 s.f.)}$$

∴ Radius of sphere = **6.06 cm**

Total surface area = $4\pi(6.0552)^2$ cm²
= **461 cm²** (to 3 s.f.)

(b) Let the radius of the sphere be r cm.
Volume = 125π cm³

$$\frac{4}{3}\pi r^3 = 125\pi$$
$$r^3 = \frac{375}{4}$$
$$r = \sqrt[3]{\frac{375}{4}}$$
$$= 4.54 \text{ (to 3 s.f.)}$$

∴ Radius of sphere = **4.54 cm**

Total surface area = $4\pi(4.5428)^2$ cm²
= **259 cm²** (to 3 s.f.)

4. **(a)** Let the radius of the sphere be r cm.
Total surface area = 67 cm²

$$4\pi r^2 = 67$$
$$r^2 = \frac{67}{4\pi}$$
$$r = \sqrt{\frac{67}{4\pi}} \text{ (since } r > 0\text{)}$$
$$= 2.31 \text{ (to 3 s.f.)}$$

∴ Radius of sphere = **2.31 cm**

Volume of sphere = $\frac{4}{3}\pi(2.3090)^3$ cm³
= **51.6 cm³** (to 3 s.f.)

(b) Let the radius of the sphere be r cm.
Total surface area = 484π cm²

$$4\pi r^2 = 484\pi$$
$$r^2 = 121$$
$$r = \sqrt{121} \text{ (since } r > 0\text{)}$$
$$= 11$$

∴ Radius of sphere = **11 cm**

Volume of sphere = $\frac{4}{3}\pi(11)^3$ cm³
= **5580 cm³** (to 3 s.f.)

5. **(i)** Let the radius of the hemisphere be r cm.

Total surface area = $\frac{75}{4}\pi$ cm²

$$2\pi r^2 + \pi r^2 = \frac{75}{4}\pi$$
$$3\pi r^2 = \frac{75}{4}\pi$$
$$r^2 = \frac{25}{4}$$
$$r = \sqrt{\frac{25}{4}} \text{ (since } r > 0\text{)}$$
$$= 2.5$$

∴ Radius of hemisphere = **2.5 cm**

(ii) Volume of hemisphere = $\frac{2}{3}\pi(2.5)^3$ cm³
= **32.7 cm³** (to 3 s.f.)

6. Let the radius of the hemisphere be r cm.
Volume of hemisphere = 812 cm³

$$\frac{2}{3}\pi r^3 = 812$$
$$r^3 = \frac{1218}{\pi}$$
$$r = \sqrt[3]{\frac{1218}{\pi}}$$
$$= 7.2918 \text{ (to 5 s.f.)}$$

Total surface area of hemisphere = $[2\pi(7.2918)^2 + \pi(7.2918)^2]$ cm²
= **501 cm²** (to 3 s.f.)

7. Volume of sphere = 100 cm³

$$\frac{4}{3}\pi(4x)^3 = 100$$
$$\frac{256}{3}\pi x^3 = 100$$
$$x^3 = \frac{75}{64\pi}$$
$$\boldsymbol{x = \sqrt[3]{\frac{75}{64\pi}}}$$

8. **(a)** Let the radii of the spheres be r units and kr units respectively.

$$\frac{V_1}{V_2} = \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi(kr)^3}$$
$$= \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi k^3 r^3}$$
$$= \frac{1}{k^3}$$

∴ The ratio of the volumes is **1 : k^3**.

(b)
$$\frac{A_1}{A_2} = \frac{4\pi r^2}{4\pi(kr)^2}$$
$$= \frac{4\pi r^2}{4\pi k^2 r^2}$$
$$= \frac{1}{k^2}$$

∴ The ratio of the total surface areas is **1 : k^2**.

9. **(i)** Total surface area = 912 cm²

$$2\pi(3p)^2 + \pi(3p)^2 = 912$$
$$27\pi p^2 = 912$$
$$p^2 = \frac{304}{9\pi}$$
$$p = \sqrt{\frac{304}{9\pi}} \text{ (since } p > 0\text{)}$$
$$= 3.28 \text{ (to 3 s.f.)}$$

∴ p = **3.28**

(ii) Volume of 24 hemispheres = $24 \times \frac{2}{3}\pi(3.2790)^3$ cm^3

$= 1772.1$ cm^3 (to 5 s.f.)

Let the radius of the larger hemisphere be R cm.

Volume of larger hemisphere = 1772.1 cm^3

$\frac{2}{3}\pi R^3 = 1772.1$

$R^3 = 846.12$ (to 5 s.f.)

$R = \sqrt[3]{846.12}$

$= 9.4583$ (to 5 s.f.)

$24 \times 3p = 24 \times 3(3.2790)$

$= 236.09$ (to 5 s.f.)

$\neq R$

$\therefore$ Carl is wrong, and the correct radius is **9.46 cm** (to 3 s.f.).

10. Estimated volume of ice-cream = $3 \times \frac{4}{3}\pi(2.8)^3$ cm^3

= **276 cm^3** (to 3 s.f.)

11. Volume of glass

= Volume of larger sphere – volume of smaller sphere

$= \left[\frac{4}{3}\pi(4.6)^3 - \frac{4}{3}\pi(3.4)^3\right]$ cm^3

= **243 cm^3** (to 3 s.f.)

12. Let the radius of each small sphere be r units.

Volume of each small sphere = $\frac{4}{3}\pi r^3$

Volume of larger sphere = $\frac{4}{3}\pi(4r)^3$ cm^3

$= \frac{256}{3}\pi r^3$ cm^3

$\therefore$ Required percentage = $\dfrac{\frac{256}{3}\pi r^3 - 4\left(\frac{4}{3}\pi r^3\right)}{\frac{256}{3}\pi r^3} \times 100\%$

$= \dfrac{\frac{256}{3} - \frac{16}{3}}{\frac{256}{3}} \times 100\%$

$= 93.75\%$ (to 3 s.f.)

Worksheet 7D Volume and surface area of composite solids

1. **(a)** Volume of solid = $\left[\pi(6)^2(25) + \frac{1}{3}\pi(6)^2(15)\right]$ cm^2

= **3390 cm^3** (to 3 s.f.)

(b) Let the slant height of the cone be l cm.

Using Pythagoras' Theorem,

$l^2 = 6^2 + 15^2$

$= 261$

$l = \sqrt{261}$ (since $l > 0$)

$= 16.2$ (to 3 s.f.)

$\therefore$ Slant height of cone = **16.2 cm**

(c) Total surface area of solid

$= [\pi(6)^2 + 2\pi(6)(25) + \pi(6)(16.155)]$ cm^2

= **1360 cm^2** (to 3 s.f.)

2. Volume of chemical solution

$= \frac{1}{2}\left[\frac{2}{3}\pi(0.7)^3 + \pi(0.7)^2(8.5 - 0.7)\right]$ cm^3

= **6.36 cm^3** (to 3 s.f.)

3. Volume of empty space

= Volume of cylinder – volume of sphere

$= \left[\pi(7)^2(14) - \frac{4}{3}\pi(7)^3\right]$ cm^3

= **$718\frac{2}{3}$ cm^3**

4. **(a)** Using Pythagoras' Theorem,

$\boldsymbol{p^2 + q^2 = r^2}$ — (1)

(b) Substitute $p = 10$ and $r = 26$ into (1):

$10^2 + q^2 = 26^2$

$q^2 = 26^2 - 10^2$

$= 576$

$q = \sqrt{576}$ (since $q > 0$)

$= 24$

Volume of solid = $\left[\frac{2}{3}\pi(10)^3 + \frac{1}{3}\pi(10)^2(24)\right]$ cm^3

= **4610 cm^3** (to 3 s.f.)

5. **(i)** Capacity of bowl = $\frac{2}{3}\pi(15)^3$ cm^3

$= 2250\pi$ cm^3 (shown)

(ii) Volume of each can of green tea = $\pi(3.3)^2(11.5)$cm^3

$= 125.235\pi$ cm^3

Maximum number of cans of green tea

$= \dfrac{2250\pi}{125.235\pi}$

= **17** (round down to the nearest integer)

(iii) Total volume of green tea = $(17 \times 125.235\pi)$ cm^3

$= 2128.995\pi$ cm^3

Volume of each cup = $\frac{1}{3}\pi(2.7)^2(7)$ cm^3

$= 17.01\pi$ cm^3

Maximum number of cups

$= \dfrac{2128.995\pi}{17.01\pi}$

= **125** (round down to the nearest integer)

6. Radius of hemisphere = $3a$ cm

Height of cylinder = $4a$ cm

Total surface area of toy = $[2\pi(3a)^2 + 2\pi(a)(4a) + \pi(3a)^2]$ cm^2

= **$35\pi a^2$ cm^2**

7. Let the slant height of the cone be l cm.

Using Pythagoras' Theorem,

$l^2 = (3r)^2 + (4r)^2$

$= 25r^2$

$l = 5r$

Total surface area of solid = $[2 \times \pi(3r)(5r) + 2\pi(3r)(8r)]$ cm^2

$= 78\pi r^2$ cm^2

Total surface area of hemisphere = $(2\pi R^2 + \pi R^2)$ cm^2

$= 3\pi R^2$ cm^2

Since $78\pi r^2 = \frac{1}{2}(3\pi R^2)$,

$R^2 = 52r^2$

$\boldsymbol{R = \sqrt{52}\, r}$

8. **(a)** Volume of solid $= \left[\pi(10)^2(24) - 2 \times \frac{1}{3}\pi(10)^2(10)\right]$ cm³

$= \frac{5200}{3}\pi$ cm³

$=$ **5450 cm³** (to 3 s.f.)

(b) Mass of solid $= \left(\frac{5200}{3}\pi \times 1.15\right)$ g

$= 6262.2$ g (to 5 s.f.)

$=$ **6.26 kg** (to 3 s.f.)

9. **(a)** Consider $\triangle ABC$.

Using Pythagoras' Theorem,

$AC^2 = 14^2 + 14^2$

$= 392$

$AC = \sqrt{392}$ (since $AC > 0$)

$= 19.799$ cm (to 5 s.f.)

Radius of hemisphere $= \frac{19.799}{2}$ cm

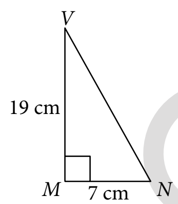
$= 9.8995$ cm (to 5 s.f.)

Volume of combined object

$= \left[\frac{1}{3} \times (14 \times 14) \times 19 + \frac{2}{3}\pi(9.8995)^3\right]$ cm³

$=$ **3270 cm³** (to 3 s.f.)

(b) Consider $\triangle VMN$.



Using Pythagoras' Theorem,

$VN^2 = 19^2 + 7^2$

$= 410$

$VN = \sqrt{410}$ (since $VN > 0$)

$= 20.248$ cm (to 5 s.f.)

Total surface area of combined object

$= \left[4 \times \frac{1}{2}(14)(20.248) + \pi(9.8995)^2 - 14^2 + 2\pi(9.8995)^2\right]$ cm²

$=$ **1290 cm²** (to 3 s.f.)

Challenge Myself!

10. Let the radius of the smaller cone be r cm.

Using similar triangles,

$\frac{r}{12.5} = \frac{12.5}{48}$

$r = \frac{12.5}{48} \times 12.5$

$= 3.2552$ (to 5 s.f.)

Volume of solid

= Volume of hemisphere + volume of larger cone – volume of smaller cone

$= \left[\frac{2}{3}\pi(12.5)^3 + \frac{1}{3}\pi(12.5)^2(48) - \frac{1}{3}\pi(3.2552)^2(12.5)\right]$ cm³

$=$ **11 800 cm³** (to 3 s.f.)

Review Exercise 7

1. Volume of cone = Volume of hemisphere

$\frac{1}{3}\pi r^2 h = \frac{2}{3}\pi r^3$

$r = \frac{h}{2}$

Total surface area of hemisphere $= 2\pi r^2 + \pi r^2$

$= 3\pi r^2$

$= 3\pi\left(\frac{h}{2}\right)^2$

$= \mathbf{\frac{3}{4}\pi h^2}$ **cm²**

2. **(a)** Height of cone $= (68 - 22 - 22)$ cm

$= 24$ cm

Volume of toy

$= \left[\pi(11)^2(22) + \frac{1}{3}\pi(11)^2(24) + \frac{4}{3}\pi(11)^3\right]$ cm³

$=$ **17 000 cm³** (to 3 s.f.)

(b) Let the slant height of the cone be l cm.

Using Pythagoras' Theorem,

$l^2 = 11^2 + 24^2$

$= 697$

$l = \sqrt{697}$ (since $l > 0$)

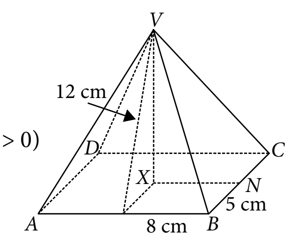
$= 26.401$ (to 5 s.f.)

Total surface area

$= [\pi(11)^2 + 2\pi(11)(22) + \pi(11)(26.401) + 4\pi(11)^2]$ cm²

$=$ **4330 cm²** (to 3 s.f.)

3. **(a)** Consider $\triangle VXM$.



Using Pythagoras' Theorem,

$VX^2 + 5^2 = 12^2$

$VX^2 = 12^2 - 5^2$

$= 119$

$VX = \sqrt{119}$ cm (since $VX > 0$)

Consider $\triangle VXN$.

Using Pythagoras' Theorem,

$VN^2 = \left(\sqrt{119}\right)^2 + 8^2$

$= 183$

$VN = \sqrt{183}$ cm (since $VN > 0$)

Total surface area

$= \left[2 \times \frac{1}{2}(16)(12) + 2 \times \frac{1}{2}(10)\left(\sqrt{183}\right) + (16)(10)\right]$ cm²

$= 487$ cm² (to 3 s.f.) (shown)

(b) Volume of material $= \frac{1}{3} \times (16 \times 10) \times \sqrt{119}$ cm³

$= 581.80$ cm³ (to 5 s.f.)

Mass of ornament $= (0.94 \times 581.80)$ g

$=$ **547 g** (to 3 s.f.)

8 Averages of Statistical Data

Worksheet 8A Mean

1. (a) Mean $= \frac{17+19+25}{3}$
$= \mathbf{20\frac{1}{3}}$

(b) Mean $= \frac{38+36+34+32}{4}$
$= \mathbf{35}$

(c) Mean $= \frac{8.3+2.6+7.4+1.8+9.5}{5}$
$= \mathbf{5.92}$

(d) Mean $= \frac{-5+2+(-1)+0+(-9)+4}{6}$
$= \mathbf{-1.5}$

2. (a) Mean $= \frac{11(2)+17(3)+24(4)+\ldots+15(7)}{11+17+24+\ldots+15}$
$= \mathbf{4.60}$ (to 3 s.f.)

(b) Mean $= \frac{35(0)+76(1)+48(2)+22(3)}{35+76+48+22}$
$= \mathbf{1.31}$ (to 3 s.f.)

(c) Mean $= \frac{7(8)+10(9)+3(10)}{7+10+3}$
$= \mathbf{8.8}$

3. Estimated mean $= \frac{6(5)+8(15)+12(25)+4(35)+10(45)}{6+8+12+4+10}$ books
$=$ **26 books**

4. (a) Estimated mean $= \frac{5(15)+12(17)+23(19)+10(21)}{5+12+23+10}$
$= \mathbf{18.52}$

(b) Estimated mean $= \frac{26(5)+35(15)+18(25)+7(35)+4(45)}{26+35+18+7+4}$
$= \mathbf{17}$

5. Given that mean = 39,
$\frac{14+30+27+58+x}{5} = 39$
$129 + x = 195$
$x = \mathbf{66}$

6. Mean $= \frac{20+22+24+26+28}{5}$
$= 24$
∴ A possible set of data is **20, 22, 24, 26** and **28**.

7. Let the numbers be x, $x + 2$, $x + 4$, … and $x + 10$.
Given that mean = 96,
$\frac{x+x+2+x+4+x+6+x+8+x+10}{6} = 96$
$6x + 30 = 576$
$6x = 546$
$x = 91$
∴ The smallest number is **91**.

8. Total mass of 6 girls = (45 × 6) kg
= 270 kg
Total mass of 7 girls = (44.8 × 7) kg
= 313.6 kg
∴ Zoe's mass = (313.6 – 270) kg
= **43.6 kg**

9. Total shirt size of 12 men = 16 × 12
= 192
Total shirt size of 9 men = (18 × 9)
= 162
Total shirt size of 3 men = 192 – 162
= 30
Mean shirt size of 3 men $= \frac{30}{3}$
= **10**

10. Total mass of 5 lemons = (72 × 5) g
= 360 g
Total mass of 6 lemons = (360 – 69 + 2 × 84) g
= 459 g
Mean mass of 6 lemons $= \frac{459}{6}$ g
= **76.5 g**

11. Let there be x girls in the group.
Total mass of boys + total mass of girls = Total mass of students
$55(x + 3) + 44x = 50(x + x + 3)$
$55x + 165 + 44x = 100x + 150$
$x = 15$
∴ Total number of students = 2(15) + 3
= **33**

12. Total amount of solvent used by the boys = (5 × 26.7) cm^3
= 133.5 cm^3
Total amount of solvent used by the students = (11 × 26.3) cm^3
= 289.3 cm^3
Total amount of solvent used by the girls = (289.3 – 133.5) cm^3
= 155.8 cm^3
∴ Mean amount of solvent used by the girls $= \frac{155.8}{6}$ cm^3
= **26.0 cm^3**
(to 3 s.f.)

13. (a) Let his mean time taken from Monday to Saturday be t min.
Total time taken from Monday to Saturday = $6t$ min
Total time taken from Monday to Sunday = $7(t + 0.2)$ min
$7(t + 0.2) = 6t + 11$
$7t + 1.4 = 6t + 11$
$t = 9.6$
∴ His mean time taken from Monday to Saturday was **9.6 min.**

(b) Total time taken = (9.5 × 7) min
= 66.5 min
Time taken on Sunday = (66.5 – 6 × 9.6) min
= 8.9 min
= **8 min 54 s**

14. (a) Total number of members = 5 + 18 + 24 + 32 + 11 + 10
= 100
Required fraction $= \frac{11+10}{100}$
$= \mathbf{\frac{21}{100}}$

(b) Estimated mean
$= \$\left[\frac{5(25)+18(75)+24(125)+\ldots+10(275)}{100}\right]$
= **$153**

15. $a + 4 = -2c \quad \text{— (1)}$

$c + 16 = b + d \quad \text{— (2)}$

$(1) - (2): (a + 4) - (c + 16) = -2c - (b + d)$

$a + 4 - c - 16 = -2c - b - d$

$a + b + c + d = 12$

$\therefore$ Mean of a, b, c and $d = \dfrac{12}{4}$

$= \mathbf{3}$

Challenge Myself!

16. **(a)** Given that $\dfrac{p+q+r}{3} = 12$,

Mean of $p - 2$, $q - 2$ and $r - 2 = \dfrac{(p-2)+(q-2)+(r-2)}{3}$

$= \dfrac{(p+q+r)-6}{3}$

$= \dfrac{p+q+r}{3} - 2$

$= 12 - 2$

$= \mathbf{10}$

(b) Mean of $\dfrac{3}{4}p+1$, $\dfrac{3}{4}q-7$ and $\dfrac{3}{4}r$

$= \dfrac{\left(\frac{3}{4}p+1\right)+\left(\frac{3}{4}q-7\right)+\frac{3}{4}r}{3}$

$= \dfrac{\left(\frac{3}{4}p+\frac{3}{4}q+\frac{3}{4}r\right)-6}{3}$

$= \dfrac{1}{4}(p+q+r)-2$

$= \dfrac{1}{4}(36)-2$

$= \mathbf{7}$

Worksheet 8B Median

1. **(a)** 3, 5, 7, 9, 100

Median = **7**

(b) 27, 41, 50, 83, 96

Median = **50**

(c) 1, 6, 16, 45

Median $= \dfrac{6+16}{2}$

$= \mathbf{11}$

(d) 2, 2, 4, 8, 12, 33

Median $= \dfrac{4+8}{2}$

$= \mathbf{6}$

2. **(a)** Total frequency = 7 + 10 + 12 + 5

= 34

Median positions: 17th and 18th

Median $= \dfrac{1+2}{2}$ siblings

= **1.5 siblings**

(b) Total frequency = 8 + 11 + 9 + 24 + 3

= 55

Median position: 28th

Median = **7 hours**

(c) Total frequency = 9 + 15 + 11 + 9 + 11 + 4

= 59

Median position: 30th

Median = **3 club memberships**

(d) Total frequency = 7 + 5 + 2 + 6 + 4

= 24

Median positions: 12th and 13th

Median $= \dfrac{27+28}{2}$

= **27.5 calls**

3. 28, 39, x, 51, 64, 70

Given that median = 48,

$\dfrac{x+51}{2} = 48$

$x + 51 = 96$

$x = \mathbf{45}$

4. Largest possible value of $p = 2 + 8 + 5$

$= \mathbf{15}$

5. Smallest possible value of $q = 9 + 10 - 6 + 1$

$= \mathbf{14}$

6. **(a)** 53, 54, 58, 59, 60, 61, 62, 67

Median $= \dfrac{59+60}{2}$

= **59.5 s**

(b) The median time will also be reduced by x seconds, i.e. **(59.5 – x) s**.

7. The smallest number is 27.

The largest number is 36 + 1 = 37.

27, ____, ____, ____, ____, 37

The other prime numbers in the interval are 29 and 31.

Since the median is 29, then the numbers are **27, 28, 28, 30, 31** and **37**.

8. **(a)** A possible set of data is **11, 14, 17, 20** and **23**.

(b) ____, ____, 18, 22, ____, ____

3, ____, 18, 22, ____, 37

3, 9, 18, 22, 26, 37

A possible set of data is **3, 9, 18, 22, 26** and **37**.

Challenge Myself!

9. Since the median = 6,

let $2k + 11 + 1 = 6 + 3k + 1 + 4k$

$2k + 12 = 7k + 7$

$5k = 5$

$k = \mathbf{1}$

Worksheet 8C Mode

1. **(a)** Mode = **2**

(b) Mode = **84**

(c) Bimodal, modes = **18** and **90**

(d) Mode = **2.5**

2. **(a)** Mode = **8 V**

(b) Mode = **8 subjects**

3. (a) Modal class interval: **60 to 80 questions**
(b) **No.** There are 15 students lying in the interval 80 to 100 questions, but it is possible that all of them practised 90 questions or fewer, for example.
4. (a) Mode = **12 flights of stairs**
(b) He could have climbed **6**, **8** or **15 flights of stairs.**

Worksheet 8D Measures of central tendency

1. (a) $12 + 8 + 6 + x + 4 = 36$
$x + 30 = 36$
$x = \mathbf{6}$
(b) (i) Mean number of surgeries per day
$= \frac{12(0)+8(1)+6(2)+6(3)+4(4)}{36}$
$= \mathbf{1.5}$
(ii) Median positions: 18th and 19th
Median number of surgeries per day $= \frac{1+1}{2}$
$= \mathbf{1}$
(iii) Modal number of surgeries per day = **0**
2. Median $= \frac{7+8}{2}$
$= 7.5$
Mean $= 7.5 + 2.5$
$= 10$
$\frac{4+6+4(7)+2(8)+9+x+19+20}{12} = 10$
$x + 102 = 120$
$x = \mathbf{18}$
3. (i) Five possible four-digit numbers are **1036, 1234, 4321, 5320** and **7120**.
(ii) Median = **4321**
4. (a) New mean = $\mathbf{53.4 + x}$
(b) New median = $\mathbf{58.1 + x}$
(c) New mode = $\mathbf{62 + x}$
(d) New range of marks = **70**
5. (a) The numbers are $p - 4$, $p - 2$, p, $p + 2$ and $p + 4$.
The mean of the five integers is p.
∴ Ken is correct.
(b) A counterexample is: 1, 3, 21, 23 and 27.
The mean is 15 but the median is 21.
∴ Leslie is incorrect.
6. The shoe sizes are: 5, 5, 6, 8, 8, 8, 9, 9, 10, 10, 12, 12
∴ Largest possible shoe size = **12**
7. (a) Total frequency = 3 + 7 + 9 + 18 + 2 + 1
= 40
Estimated mean waiting time
$= \frac{3(2.5)+7(7.5)+9(12.5)+18(17.5)+2(22.5)+1(27.5)}{40}$ min
= **14 min**
(b) Since 19 customers waited for 15 minutes or less, I do not agree with Sarah that half (20) of the customers waited for longer than 17.5 minutes.
(c) Percentage of customers who waited for 20 minutes or less
$= \frac{3+7+9+18}{40} \times 100\%$
= 92.5%
∴ I agree with Mr Chean.
8. (a) (i) Mean amount of PM2.5 in the environment from 2012 to 2018
$= \frac{19+20+18+24+15+14+15}{7}$
= **17.9 micrograms per cubic metre** (to 3 s.f.)
(ii) Mean amount of nitrogen dioxide in the environment from 2012 to 2015
$= \frac{25+25+24+22}{4}$
= **24 micrograms per cubic metre**
(b) (i) Median amount of PM2.5 in the environment from 2012 to 2018
= **18 micrograms per cubic metre**
(ii) Median amount of nitrogen dioxide in the environment from 2012 to 2015
$= \frac{25+24}{2}$ micrograms per cubic metre
= **24.5 micrograms per cubic metre**
(c) **No.** Between 2014 and 2015, there was an increase in the amount of PM2.5 in the environment, but the amount of nitrogen in the environment decreased.
9. (a) (i) Mean height attained by Leslie
$= \frac{1.77+1.81+1.78+1.80+1.81+1.78}{6}$ m
= **1.79 m** (to 3 s.f.)
(ii) Mean height attained by Mark
$= \frac{1.83+1.90+1.74+1.62+1.85+1.67}{6}$ m
= **1.77 m** (to 3 s.f.)
(b) The coach could select Leslie as his mean height attained is higher than that of Mark. However, the coach could also select Mark as there is a chance of him attaining a height greater than that of Leslie.
10. (a) Total number of sit-ups by 25 students = 39.6 × 25 = 990
Total number of sit-ups by 26 students = 40 × 26 = 1040
Number of sit-ups the extra student did = 1040 − 990
= **50**
(b) The new median **might be greater than 39 but might still be 39**.

Challenge Myself!

11. ___, ___, 11, ___, ___
Two possible sets are **5, 7, 11, 11, 11** and **3, 4, 11, 11, 16**.

Review Exercise 8

1. (a) Mode = **1 item**
(b) Mean number of items
$= \frac{11(0)+12(1)+6(2)+7(3)+11(4)+9(5)}{56}$
= **2.39** (to 3 s.f.)

(c) $\frac{1}{4}$ of the shoppers corresponds to 14 people.
Since more than 14 shoppers had more than 3 items in their shopping carts at that particular time, the statement is correct.

2. (a) Total heights of all 7 players = (1.95×7) m
$= 13.65$ m
Total height of the other 5 players = $(13.65 - 1.96 - 1.87)$ m
$= 9.82$ m
$\therefore$ Mean height of the other 5 players = $\frac{9.82}{5}$ m
$= \mathbf{1.964\ m}$

(b) OPEN Let the modes be 1.93 m and 2.01 m.
A possible set of data is **1.87 m, 1.93 m, 1.93 m, 1.94 m, 1.96 m, 2.01 m** and **2.01 m**.

3. The numbers are **7, 7, 8, 9, 10, 10** and **12.**

End-of-year Checkpoint A

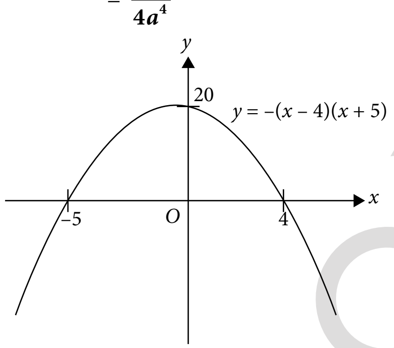
Section A

1. $\frac{76\ 538}{19^3 - 42^2} = \mathbf{20}$ (to 1 s.f.) **[1]**

2. $7^{-5} \times 7^k = \frac{1}{7}$
$7^{k-5} = 7^{-1}$
$k - 5 = -1$
$k = \mathbf{4}$ **[1]**

3. $\left(\frac{64a^{12}}{b^{18}}\right)^{-\frac{1}{3}} = \left(\frac{b^{18}}{64a^{12}}\right)^{\frac{1}{3}}$ **[1]**
$= \mathbf{\frac{b^6}{4a^4}}$ **[1]**

4.



[2]

5. Let $y = k(\sqrt{x} - 1)$.
When $x = 9$, $y = 9$,
$9 = k(\sqrt{9} - 1)$
$= 2k$
$k = \frac{9}{2}$
$\therefore y = \frac{9}{2}(\sqrt{x} - 1)$ **[1]**
When $x = 64$,
$y = \frac{9}{2}(\sqrt{64} - 1)$
$= \mathbf{\frac{63}{2}}$ **[1]**

6. $\frac{9}{8-4x} - \frac{1}{3x-6} = \frac{9}{4(2-x)} - \frac{1}{3(x-2)}$
$= \frac{9}{4(2-x)} + \frac{1}{3(2-x)}$ **[1]**
$= \frac{27+4}{12(2-x)}$
$= \mathbf{\frac{31}{12(2-x)}}$ **[1]**

7. $7(7^7 + 7^7 + 7^7 + 7^7 + 7^7 + 7^7 + 7^7)$
$= 7[7(7^7)]$
$= 7^{1+1+7}$ **[1]**
$= 7^9$
$\therefore p = \mathbf{9}$ **[1]**

8. $\frac{8}{2x+1} = 5 - x$
$8 = (5 - x)(2x + 1)$
$= 10x + 5 - 2x^2 - x$
$2x^2 - 9x + 3 = 0$ **[1]**
$x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(2)(3)}}{2(2)}$
$= \frac{9 \pm \sqrt{57}}{2}$
$= 4.14$ or 0.363 (to 3 s.f.) **[2]**
$\therefore x = \mathbf{4.14}$ or $x = \mathbf{0.363}$

9. Consider $\triangle PRS$.
$\tan 35° = \frac{PR}{16}$
$PR = 16 \tan 35°$ cm **[1]**
Consider $\triangle QRS$.
$\tan 24° = \frac{QR}{16}$
$QR = 16 \tan 24°$ cm **[1]**
$\therefore PQ = PR - QR$
$= 16 \tan 35° - 16 \tan 24°$
$= \mathbf{4.08\ cm}$ (to 3 s.f.) **[1]**

10. (a) $S_n = an - bn^2$
When $n = 2$,
$S_2 = a(2) - b(2)^2 = 12 + 5$
$2a - 4b = 17$ (shown) **[1]**

(b) $a - b = 12$ — (1)
$2a - 4b = 17$ — (2)
(1) × 2: $2a - 2b = 24$ — (3)
(3) – (2): $2b = 7$
$b = 3.5$ **[1]**
Substitute $b = 3.5$ into (1): $a - 3.5 = 12$, i.e. $a = 15.5$ **[1]**
$\therefore$ **$a = 15.5$, $b = 3.5$**

11. (i) $p = \frac{8r - q}{r + 1}$
When $q = 4$ and $r = -3$,
$p = \frac{8(-3) - 4}{-3 + 1}$
$= \mathbf{14}$ **[1]**

(ii) $p = \dfrac{8r - q}{r + 1}$

$p(r + 1) = 8r - q$

$pr + p = 8r - q$

$8r - pr = p + q$ **[1]**

$r(8 - p) = p + q$

$\boldsymbol{r = \dfrac{p + q}{8 - p}}$ **[1]**

12. **(i)** Median position $= \dfrac{63+1}{2} = 32^{\text{nd}}$

$\therefore$ The interval **6000 to 7000 steps** contains the median number of steps. **[1]**

(ii) Mean number of steps $= \dfrac{3 \times 3500 + 9 \times 4500 + \ldots + 8 \times 9500}{63}$ **[1]**

$=$ **6750** (to 3 s.f.) **[1]**

13. **(a)** $\dfrac{3x^2 + 4(x-1)}{4 + 9x^2 - 12x} = \dfrac{3x^2 + 4x - 4}{9x^2 - 12x + 4}$

$= \dfrac{(3x-2)(x+2)}{(3x-2)^2}$ **[1]**

$= \boldsymbol{\dfrac{x+2}{3x-2}}$ **[1]**

(b) $x^3 + 8 - 4x - 2x^2 = x^3 - 2x^2 - 4x + 8$

$= x^2(x - 2) - 4(x - 2)$ **[1]**

$= (x - 2)(x^2 - 4)$

$= (x - 2)(x + 2)(x - 2)$

$= \boldsymbol{(x + 2)(x - 2)^2}$ **[1]**

14. Let the length of the diagonal be x cm.

Consider a rectangle with dimensions 10 units × 1 unit.

Width = 6 cm

Length = 60 cm

Using Pythagoras' Theorem,

$x^2 = 60^2 + 6^2$ **[1]**

$= 3636$

$x = \sqrt{3636}$ (since $x > 0$)

$= 60.3$ (to 3 s.f.) **[1]**

Consider a rectangle with dimensions 5 units × 2 units.

Width = 6 cm

Length = 15 cm

Using Pythagoras' Theorem,

$x^2 = 15^2 + 6^2$ **[1]**

$= 261$

$x = \sqrt{261}$ (since $x > 0$)

$= 16.2$ (to 3 s.f.) **[1]**

$\therefore$ The possible lengths are **60.3 cm** and **16.2 cm**.

Section B

15. **(i)** $m_{AC} = \dfrac{5-0}{0-10}$

$= -\dfrac{1}{2}$ **[1]**

Since $OB \perp AC$, then $m_{OB} = 2$. **[1]**

$\therefore$ Equation of OB: $\boldsymbol{y = 2x}$ — (1) **[1]**

(ii) Equation of AC: $y = -\dfrac{1}{2}x + 5$ — (2) **[1]**

Substitute (1) into (2):

$2x = -\dfrac{1}{2}x + 5$

$\dfrac{5}{2}x = 5$

$x = 2$

Substitute $x = 2$ into (1): $y = 2(2) = 4$

$\therefore$ Coordinates of midpoint of OQ are (2, 4) **[1]**

$\left(\dfrac{0 + x_B}{2}, \dfrac{0 + y_B}{2}\right) = (2, 4)$ **[1]**

$\dfrac{x_B}{2} = 2$ and $\dfrac{y_B}{2} = 4$

$x_B = 4$ $\quad$ $y_B = 8$

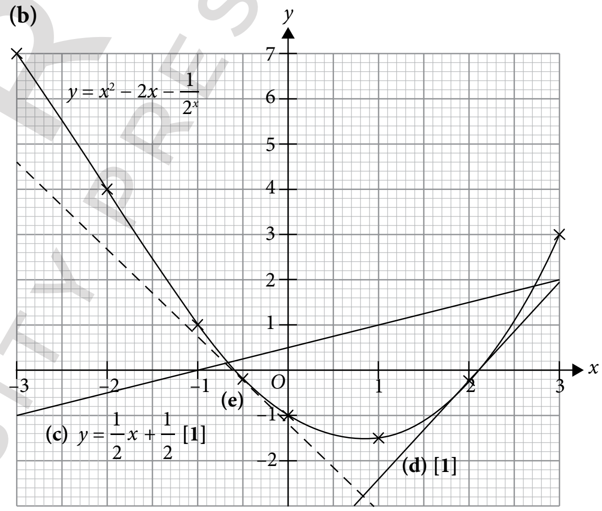
$\therefore$ **B(4, 8)** **[1]**

16. **(a)** When $x = 3$,

$y = 3^2 - 2(3) - \dfrac{1}{2^3}$

$=$ **2.9** (to 1 d.p.) **[1]**

(b)



[2]

(c) $2x^2 - 5x = 2^{1-x} + 1$

$x^2 - \dfrac{5}{2}x = 2^{-x} + \dfrac{1}{2}$

$x^2 - 2x = 2^{-x} + \dfrac{1}{2}x + \dfrac{1}{2}$

$x^2 - 2x - \dfrac{1}{2^x} = \dfrac{1}{2}x + \dfrac{1}{2}$

Draw $y = \dfrac{1}{2}x + \dfrac{1}{2}$.

From the graph, $x =$ **−0.65** or $x =$ **2.75**. **[1]**

(d) Using (1, −2.4) and (3, 1.9),

Gradient $= \dfrac{1.9 - (-2.4)}{3 - 1}$

$=$ **2.15** **[1]**

(e) From the graph, $x \approx$ **−0.5**. **[1]**

17. **(a)** $\pi rl = \pi r^2$

$\boldsymbol{l = r}$ **[1]**

(b) **(i)** Let the height of the cone be h cm.

Using Pythagoras' Theorem,

$h^2 + 0.9^2 = 2.1^2$

$h^2 = 3.6$

$h = \sqrt{3.6}$ (since $h > 0$) **[1]**

Volume of each cone = $\frac{1}{3}\pi(0.9)^2\left(\sqrt{3.6}\right)$ cm³ **[1]**

= **1.61 cm³** (to 3 s.f.) **[1]**

(ii) Volume of each shell = $\left[\frac{2}{3}\pi(1)^3 - \frac{2}{3}\pi(0.8)^3\right]$ cm³ **[1]**

= **1.02 cm³** (to 3 s.f.) **[1]**

(iii) Let the height of the pyramid be H cm.
Using Pythagoras' Theorem,

$H^2 + 3^2 = 5^2$

$H^2 = 5^2 - 3^2$

$= 16$

$H = 4$ (since $H > 0$) **[1]**

Volume of pyramid = $\frac{1}{3}(6)^2(4)$ cm³

= 48 cm³ **[1]**

Volume of chocolate = [80(1.6094) + 60(1.0221)] cm³

= 190.08 cm³ (to 5 s.f.) **[1]**

Maximum number of pyramids = $\frac{190.08}{48}$

= 3 (round down to the nearest integer)

∴ Lynn could have made a maximum of **3** chocolate pyramids. **[1]**

End-of-year Checkpoint B

Section A

1. $\frac{5a^3 \times a^2}{a^{-6}} = \frac{5a^5}{a^{-6}}$

$= \mathbf{5a^{11}}$ **[1]**

2. 2.5×10^6 km $= 2.5 \times 10^6 \times 10^3 \times 10^2$ cm

$= 2.5 \times 10^{11}$ cm

∴ p = **11** **[1]**

3. $m = \left(\frac{1}{3}\right)^2$

$= \mathbf{\frac{1}{9}}$ **[1]**

4. Length of rectangle = 36 − 11

= 25 units

∴ x-coordinate of B = 5 + 25

= 30 **[1]**

Width of rectangle = 40 − 30

= 10 units

∴ y-coordinate of B = 11 + 10

= 21

∴ $\mathbf{B(30, 21)}$ **[1]**

5. $\frac{1}{4x-9} - \frac{5}{3x-5} = \frac{3x-5-5(4x-9)}{(4x-9)(3x-5)}$

$= \frac{3x-5-20x+45}{(4x-9)(3x-5)}$ **[1]**

$= \mathbf{\frac{40-17x}{(4x-9)(3x-5)}}$ **[1]**

6. **(a)** By symmetry,

$\frac{x_A + (-5)}{2} = -1.5$

$x_A - 5 = -3$

$x_A = 2$

∴ $\mathbf{A(2, 0)}$ **[1]**

(b) Coordinates of maximum point are **(−1.5, 8)** **[1]**

7. $\frac{9x^2-16}{27x^2-33x-4} = \frac{(3x+4)(3x-4)}{(9x+1)(3x-4)}$ **[2]**

$= \mathbf{\frac{3x+4}{9x+1}}$ **[1]**

8. Gradient of line joining (5, 2) and (−3, −9) = $\frac{-9-2}{-3-5}$

$= \frac{11}{8}$

Gradient of perpendicular bisector = $-\frac{8}{11}$ **[1]**

Midpoint = $\left(\frac{5+(-3)}{2}, \frac{2+(-9)}{2}\right)$

$= \left(1, -\frac{7}{2}\right)$ **[1]**

Equation of perpendicular bisector: $y = -\frac{8}{11}x + c$ — (1)

Substitute $x = 1$, $y = -\frac{7}{2}$ into (1):

$-\frac{7}{2} = -\frac{8}{11}(1) + c$

$c = -\frac{61}{22}$

∴ Equation of perpendicular bisector: $y = -\frac{8}{11}x - \frac{61}{22}$

$22y = -16x - 61$

$\mathbf{22y + 16x + 61 = 0}$ **[1]**

9. **(i)** $4 < \frac{2}{5}x - 3 \leqslant 89 - x$

$4 < \frac{2}{5}x - 3$ and $\frac{2}{5}x - 3 \leqslant 89 - x$

$-\frac{2}{5}x < -7$ $\quad\quad$ $\frac{7}{5}x \leqslant 92$

$x > 17\frac{1}{2}$ $\quad\quad$ $x \leqslant 65\frac{5}{7}$ **[1]**

∴ $\mathbf{17\frac{1}{2} < x \leqslant 65\frac{5}{7}}$ **[1]**

(ii) An example of a prime number in the interval is **23**. **[1]**

10. **(a)** $480 = \mathbf{2^5 \times 3 \times 5}$ **[1]**

(b) Let $p = 5^2$ and $q = 2^2 \times 3$.

$\frac{480p}{q} = \frac{(2^5 \times 3 \times 5) \times 5^2}{2^2 \times 3}$

$= 2^3 \times 5^3$ **[1]**

∴ $\mathbf{p = 25, q = 12}$ **[1]**

11. $p = \frac{a(h^3+k)}{k-2h^3}$

$kp - 2h^3p = ah^3 + ak$

$ah^3 + 2h^3p = kp - ak$ **[1]**

$h^3(a + 2p) = kp - ak$

$h^3 = \dfrac{kp - ak}{a + 2p}$ **[1]**

$h = \sqrt[3]{\dfrac{\boldsymbol{kp - ak}}{\boldsymbol{a + 2p}}}$ **[1]**

12. **(a)** $x^2 + 7x - 6 = x^2 + 7x + \left(\dfrac{7}{2}\right)^2 - \left(\dfrac{7}{2}\right)^2 - 6$

$= \left(\boldsymbol{x + \dfrac{7}{2}}\right)^2 - \dfrac{\boldsymbol{73}}{\boldsymbol{4}}$ **[1]**

(b) Coordinates of **minimum point** are $\left(\boldsymbol{-3\frac{1}{2}, -18\frac{1}{4}}\right)$ **[2]**

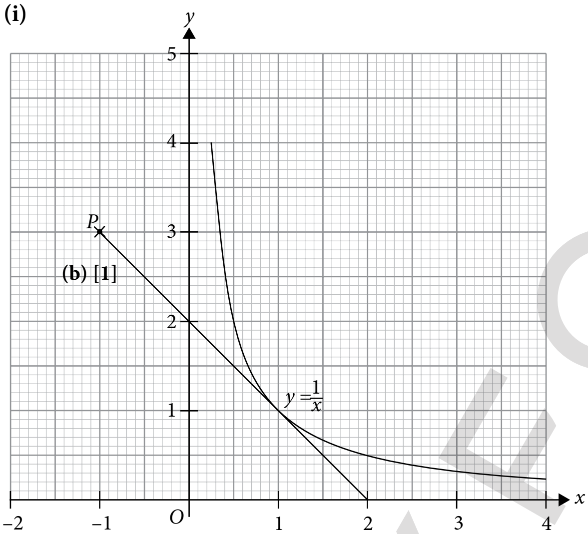
13. **(a)** $\dfrac{7}{5} - \dfrac{1}{x} = 0$

$\dfrac{1}{x} = 1.4$

From the graph, when $y = 1.4$,

$x =$ **0.7**. **[1]**

(b) **(i)**



(ii) Using (−1, 3) and (2, 0),

Gradient $= \dfrac{0 - 3}{2 - (-1)}$

$= -1$ **[1]**

Equation of tangent: $\boldsymbol{y = -x + 2}$ **[1]**

14. **(a)** $m_X > m_Y$

$m_0\left(1 + \dfrac{r}{100}\right)^n > \dfrac{m_0(r)(n)}{100} + m_0$ **[1]**

$\left(1 + \dfrac{r}{100}\right)^{365} > 3.65r + 1$ (shown) **[1]**

(b) **(i)** Graph A: $\boldsymbol{y = \left(1 + \dfrac{x}{100}\right)^{365}}$

Graph B: $\boldsymbol{y = 3.65x + 1}$ **[1]**

(ii) $\boldsymbol{P(0, 1)}$ **[1]**

Section B

15. **(i)** It is likely that the people coming out of the cyber café have an interest in online gaming. **[1]**

(ii) The question does not indicate a time frame, such as one week. **[1]**

(iii) **(a)** Mean time

$= \dfrac{14 \times 1 + 18 \times 3 + 36 \times 5 + 17 \times 7 + 15 \times 9}{100}$ **[1]**

= **5.02 h** **[1]**

(b) Number of respondents who spend at least x h

$= \dfrac{8}{25} \times 100$

$= 32$

$\therefore$ $\boldsymbol{x = 6}$ **[1]**

16. **(a)** Total volume $= \dfrac{3}{4} \times \pi(36)^2(45)$ cm³ **[1]**

= **43 740π cm³** **[1]**

(b) Volume of each bath bomb $= \dfrac{43\ 740\pi}{120}$ cm³

$= \dfrac{729\pi}{2}$ cm³

Let the radius of the bath bomb be r cm.

$\dfrac{4}{3}\pi r^3 = \dfrac{729\pi}{2}$ **[1]**

$r^3 = \dfrac{2187}{8}$

$r = 6.49$ (to 3 s.f.) **[1]**

$\therefore$ Radius of the bath bomb = 6.49 cm (shown)

(c) Dimensions of box: $6r$ cm by $4r$ cm by $2r$ cm

Volume of empty space

$= \left[(6r)(4r)(2r) - 6 \times \dfrac{4}{3}\pi r^3\right]$ cm³ **[1]**

$= \left[48r^3 - 6 \times \dfrac{4}{3}\pi r^3\right]$ cm³

$= \left[48\left(\dfrac{2187}{8}\right) - 6 \times \dfrac{4}{3}\pi\left(\dfrac{2187}{8}\right)\right]$ cm³ **[1]**

= **6250 cm³** (to 3 s.f.) **[1]**

(d) Number of bath bombs that can fit in Type A box

= 2 × 2 × 2

= 8

Number of bath bombs that can fit in Type B box

= 3 × 1 × 1

= 3

Number of bath bombs that can fit in Type C box

= 3 × 2 × 2

= 12

Number of bath bombs that can fit in Type D box

= 2 × 2 × 4

= 16 **[1]**

Number of Type A boxes she should order $= \dfrac{120}{8}$

$= 15$

Amount of money she spends on Type A boxes

= 15 × $0.60

= $9

Number of Type C boxes she should order $= \dfrac{120}{12}$

$= 10$

Amount of money she spends on Type C boxes
$= 10 \times \$0.80$
$= \$8$ [1]
∴ Jen should order **10 Type C boxes**. [1]

17. **(a)** Speed for the first part $= \dfrac{30 \text{ km}}{\frac{x}{60} \text{ h}}$

$$= \frac{1800}{x} \text{ km/h}$$

Speed for the second part $= \dfrac{90 \text{ km}}{\frac{x+55}{60} \text{ h}}$

$$= \frac{5400}{x+55} \text{ km/h} \quad [1]$$

$$\frac{1800}{x} - 10 = \frac{5400}{x+55} \quad [1]$$

$$1800(x + 55) - 10x(x + 55) = 5400x$$
$$1800x + 99\,000 - 10x^2 - 550x = 5400x$$
$$10x^2 + 4150x - 99\,000 = 0$$
$$x^2 + 415x - 9900 = 0 \text{ (shown)} \quad [1]$$

(b) $x^2 + 415x - 9900 = 0$

$$x = \frac{-415 \pm \sqrt{415^2 - 4(1)(-9900)}}{2(1)} \quad [1]$$

$$= \frac{-415 \pm \sqrt{211\,825}}{2}$$

$= 22.62$ or -437.62 (to 2 d.p.) [2]

∴ $x =$ **22.62** or $x =$ **−437.62**

(c) Average speed $= \dfrac{\text{Total distance}}{\text{Total time}}$

$$= \frac{120 \text{ km}}{\left[\frac{2(22.622)+55}{60} + \frac{1}{2}\right] \text{h}} \quad [1]$$

$=$ **55.3 km/h** (to 3 s.f.) [1]

(d) $55.281 \text{ km/h} = \dfrac{(55.281 \times 1000) \text{ m}}{3600 \text{ s}}$ [1]

$=$ **15.4 m/s** (to 3 s.f.) [1]