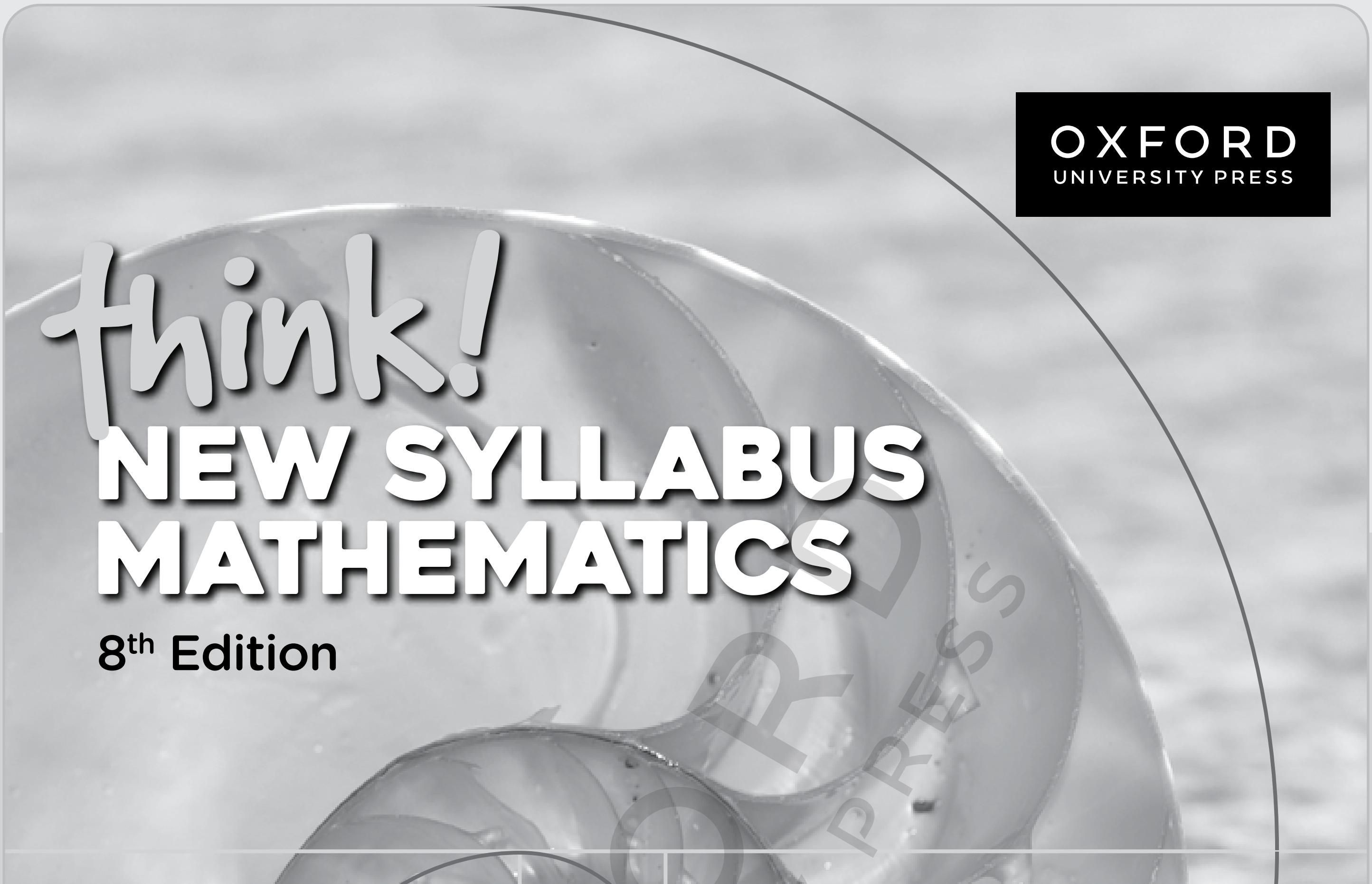

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# think! NEW SYLLABUS MATHEMATICS

8th Edition

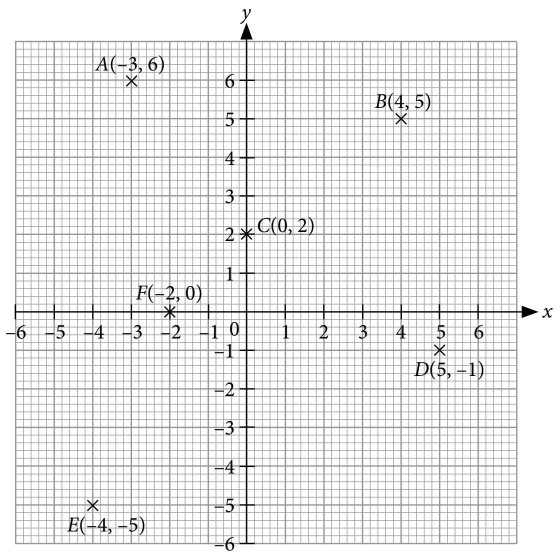
## Workbook Full Solutions 2

# 1 Linear Functions and Graphs

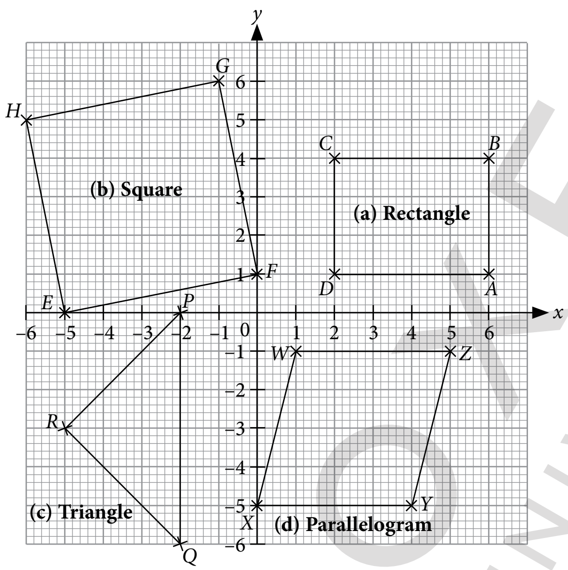
## Worksheet 1A Cartesian coordinates

1. $A(5, 2)$, $B(4, -2)$, $C(-3, -5)$, $D(-5, 4)$, $E(0, 6)$, $F(1, 0)$

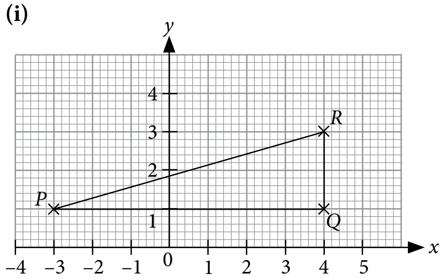
2.




3.




4. (i)

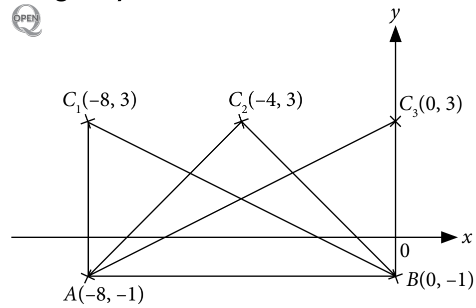



(ii) Area of $\triangle PQR = \frac{1}{2} \times 7 \times 2$

$= \mathbf{7}$ **units**$^2$

## Challenge Myself!

5.




∴ Three possible pairs of coordinates are **(–8, 3), (–4, 3) and (0, 3)**.

## Worksheet 1B Functions

1. (i) When $x = 2$,
$y = 4(2) + 1$
$= \mathbf{9}$

(ii) When $y = -3$,
$4x + 1 = 3$
$4x = -4$
$x = \mathbf{-1}$

2. (i) When $x = -\frac{1}{2}$,
$y = 7 - 2\left(-\frac{1}{2}\right)$
$= \mathbf{8}$

(ii) When $y = 5$,
$7 - 2x = 5$
$-2x = -2$
$x = \mathbf{1}$

3. (i) When $x = -6$,
$y = \frac{1}{6}(-6) - 3$
$= \mathbf{-4}$

(ii) When $y = 1\frac{4}{5}$,
$1\frac{4}{5} = \frac{1}{6}x - 3$
$4\frac{4}{5} = \frac{1}{6}x$
$x = \mathbf{28\frac{4}{5}}$

4. (i) When $x = 8.5$,
$y = 10 - 0.9(8.5)$
$= \mathbf{2.35}$

(ii) When $y = 0$,
$10 - 0.9x = 0$
$-0.9x = -10$
$x = \mathbf{11\frac{1}{9}}$

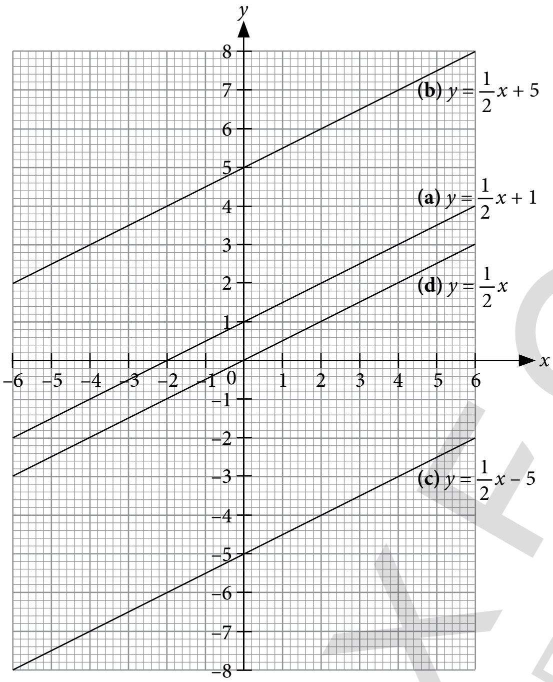
5. When $x = 4$ and $y = -5$, $4a + b = -5$
∴ An example of the values is $\boldsymbol{a = 2}$ **and** $\boldsymbol{b = -13}$.

## Worksheet 1C Linear Functions

1. (a) **3, –1** (b) **–1, 8**
   (c) **1, 7** (d) **–4, 10**
   (e) $\mathbf{\frac{1}{4}, -5}$ (f) **–1.2, 9**
   (g) $\mathbf{y = 2x + \frac{3}{4}}$ (h) $\mathbf{y = -6x}$
   (i) $\mathbf{y = -\frac{1}{3}x + 6}$ (j) $\mathbf{y = -2.7}$

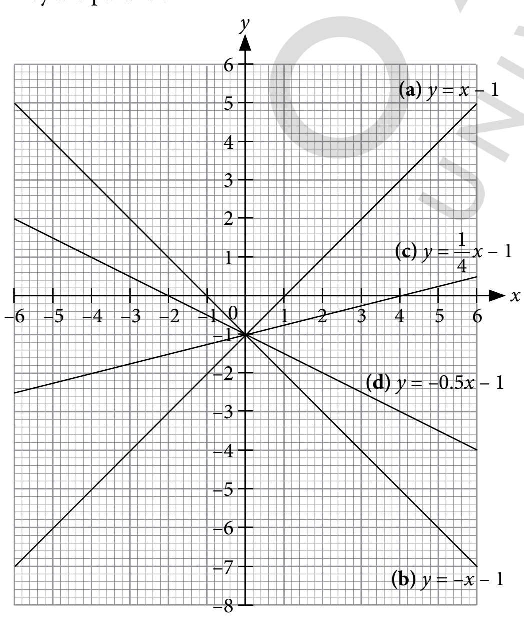
2. (i) When $x = 1$,
   $y = 5(1) - 2$
   $= 5 - 2$
   $= 3$
   $\neq 7$
   $\therefore$ (1, 7) does not lie on the line. (shown)
   (ii) OPEN A possible pair of coordinates is ***P*(1, 3)**.

3. (i)



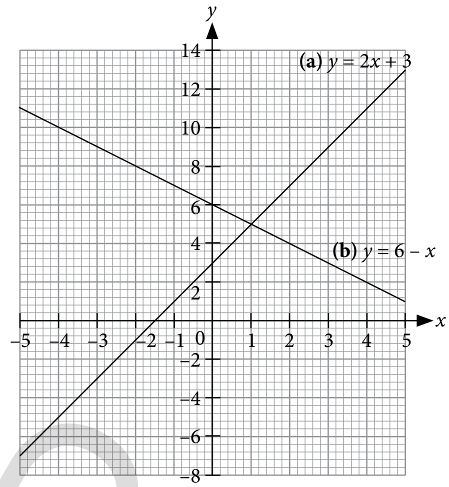

   (ii) They are parallel.

4. (i)




   (ii) They have the same $y$-intercept.

5. (i)




   (ii) **–1.5** (iii) **2**
   (iv) $\mathbf{h = 1, k = 5}$

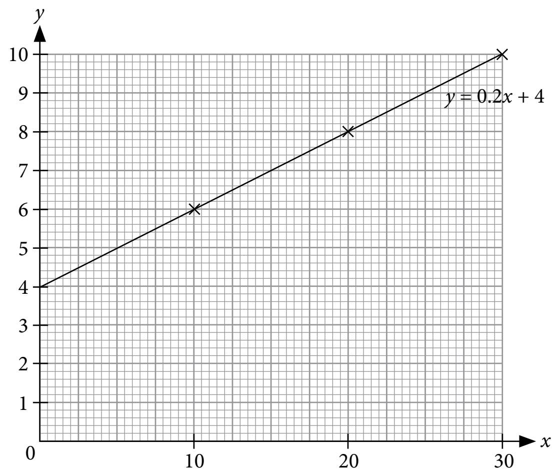
6. (a) Consider the points (0, 1) and (2, 5).
   Vertical change = 5 – 1 = 4
   Horizontal change = 2 – 0 = 2
   $\therefore$ Gradient $= \frac{4}{2}$
   $= \mathbf{2}$
   $y$-intercept = **1**
   (b) Consider the points (0, 4) and (4, 0).
   Vertical change = 0 – 4 = –4
   Horizontal change = 4 – 0 = 4
   $\therefore$ Gradient $= \frac{-4}{4}$
   $= \mathbf{-1}$
   $y$-intercept = **4**
   (c) The line is horizontal, i.e. gradient = **0**.
   $y$-intercept = **–3**
   (d) Consider the points (–2, 0) and (0, –1).
   Vertical change = –1 – 0 = –1
   Horizontal change = 0 – (–2) = 2
   $\therefore$ Gradient $= \frac{-1}{2}$
   $= \mathbf{-\frac{1}{2}}$
   $y$-intercept = **–1**

## Worksheet 1D Applications of linear graphs in real-world contexts

1. (i) (a) **\$140** (b) **\$230**
   (ii) (a) **4 kg** (b) **88 kg**
2. (i) (a) **£100** (b) **£30**
   (ii) (a) **\$100** (b) **\$40**
3. (i)

| | | | |
|---|---|---|---|
| $x$ | 10 | 20 | 30 |
| $y$ | **6** | **8** | **10** |

**(ii)**




**(iii)** **(a)** **9 cm** **(b)** **7.6 cm**

**(iv)** It refers to the original length of the spring (4 cm) when no mass is attached to it.

**4.** **(i)** Jimmy waited from 08 33 to 08 40, i.e. **7 minutes**.

**(ii)** **2.6 km**

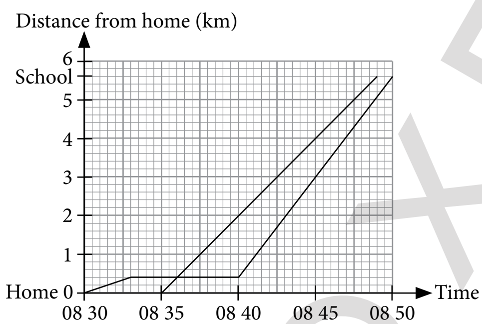
**(iii)** Vertical change = 5.6 km – 0.4 km
= 5.2 km

Horizontal change = 10 min
$= \frac{1}{6}$ h

$\therefore$ Gradient $= \dfrac{5.2 \text{ km}}{\frac{1}{6} \text{ h}}$
= **31.2 km/h**

It refers to the speed of the bus.

**(iv)**




## Review Exercise 1

**1.** **(i)** ***A*(4, 2), *B*(5, –1), *C*(3, –5), *D*(–4, –4), *E*(–3, 3), *F*(0, 4)**

**(ii)** **(a)** **(–4, 2)**

**(b)** **(3, 3)**

**(iii)** OPEN Two possible points are **(–5, –1) and (0, –6)**.

**2.** **(i)** When $x = -8$,

$y = 5 - \frac{3}{4}(-8)$

= **11**

**(ii)** When $y = \frac{1}{2}$,

$5 - \frac{3}{4}x = \frac{1}{2}$

$-\frac{3}{4}x = -4\frac{1}{2}$

$x =$ **6**

**3.** When $x = -1$,

$y = 7(-1) + \frac{23}{3}$

$= \frac{2}{3}$

$\therefore$ The line **passes through** $\left(-1, \frac{2}{3}\right)$.

**4.** **(a)** Consider the points (0, –30) and (40, 0).

Vertical change = 0 – (–30) = 30

Horizontal change = 40 – 0 = 40

$\therefore$ Gradient $= \frac{30}{40} = \mathbf{\frac{3}{4}}$

$y$-intercept = **–30**

**(b)** Consider the points (2, 9) and (4, 6).

Vertical change = 6 – 9 = –3

Horizontal change = 4 – 2 = 2

$\therefore$ Gradient $= \frac{-3}{2}$

$= \mathbf{-1\frac{1}{2}}$

$y$-intercept = **12**

**5.** **(i)**

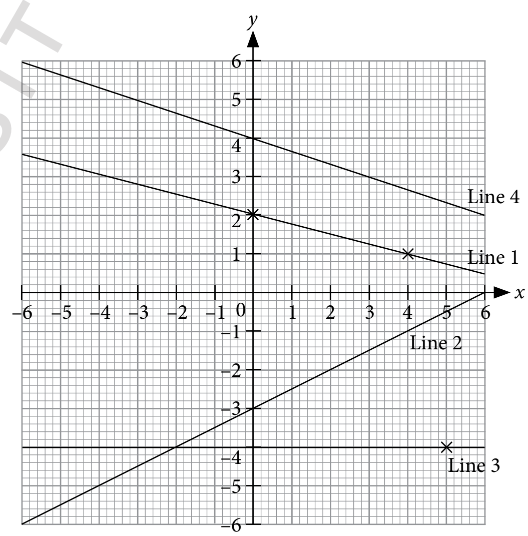
| ***n*** | 10 | 20 | 40 |
|---|---|---|---|
| ***C*** | **200** | **320** | **560** |

**(ii)** **No.**

**(iii)** The fixed cost of \$80 could be the minimum amount chargeable, or it could be the transport cost.

**(iv)** The fixed cost of \$80 is divided among the number of people. When there are more people, the cost for each person decreases.

**6.**



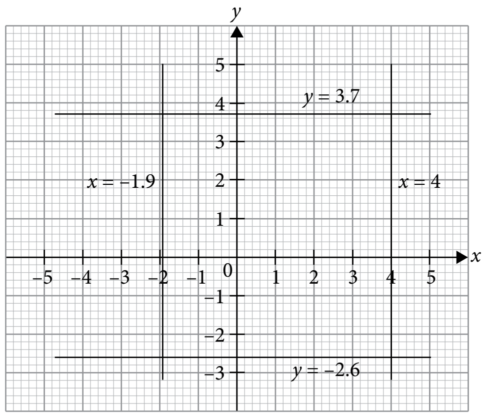
# 2 Linear Graphs and Simultaneous Linear Equations

## Worksheet 2A Equations of straight lines

**1.** **(a)** Line 1: $y$ = **4**
Line 2: $x$ = **3.8**
Line 3: $y$ = **−1.4**
Line 4: $x$ = **−3.5**

**(b)** Area of rectangle = [4 − (−1.4)] × [3.8 − (−3.5)]
= **39.42 units²**

**2.**

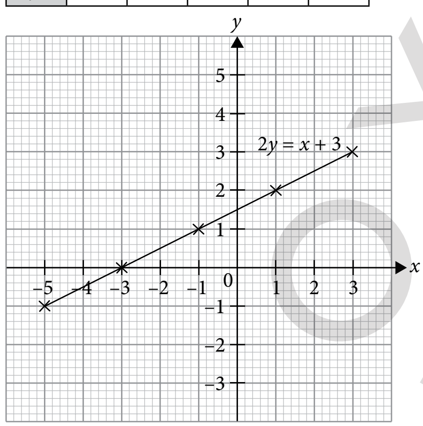



## Worksheet 2B Graphs of linear equations in the form $ax + by = k$

**1.** **(a)**

| $x$ | −5 | −3 | −1 | 1 | 3 |
|---|---|---|---|---|---|
| $y$ | −1 | **0** | **1** | 2 | **3** |

**(b)**

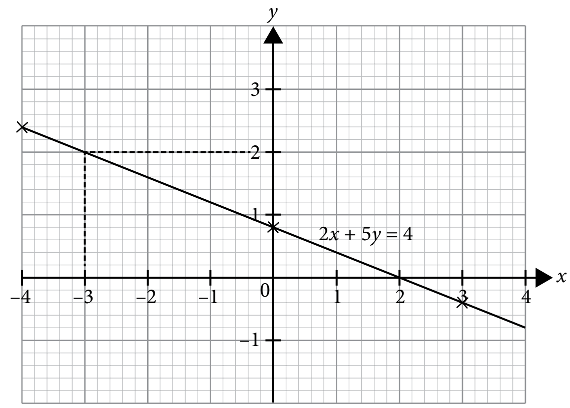



**(c)** **(−3, 0)**

**(d)** $x$ = **−2**

**2.** **(a)** When $x = -4$, $y = p$,
$2(-4) + 5p = 4$
$5p - 8 = 4$
$5p = 12$
$p = 2.4$
$\therefore p$ = **2.4**

**(b)**

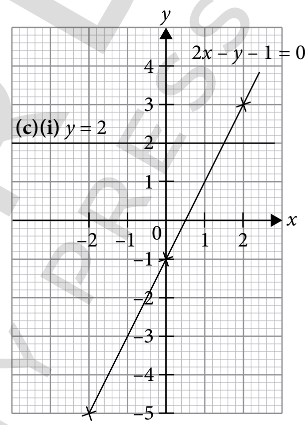



**(c)** From the graph, $q$ = **2** and $r$ = **2.**

**3.** **(a)**

| $x$ | −2 | 0 | 2 |
|---|---|---|---|
| $y$ | **−5** | **−1** | **3** |

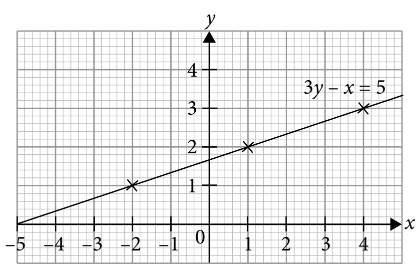
**(b)**




**(c)** **(ii)** Area of trapezium = $\frac{1}{2}(1.5 + 0.5)(2)$
= **2 units²**

**4.** **(a)** A possible pair of values is $a$ = **1** and $b$ = **2.**

**(b)**




**(c)** **$P$(−5, 0), $Q$(0, 1.65)**

**(d)** **$R$(−5, 1.65)** or **$R$(0, 3.3)**

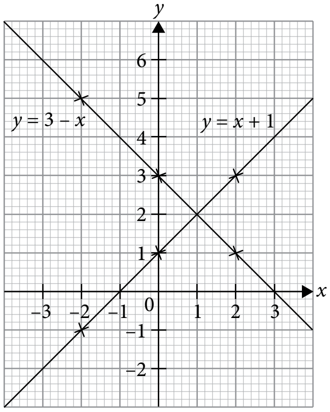
## Worksheet 2C Solving simultaneous linear equations using graphical method

**1.** **(a)**

| $x$ | –2 | 0 | 2 |
|---|---|---|---|
| $y = 3 - x$ | **5** | **3** | **1** |

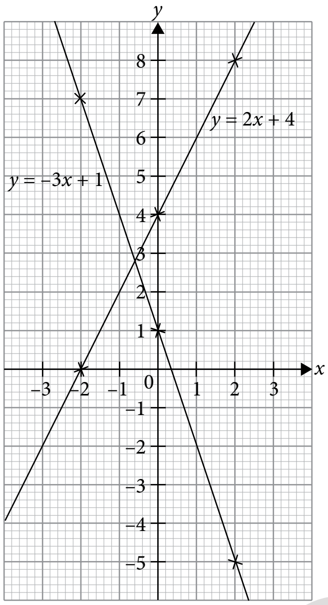
| $x$ | –2 | 0 | 2 |
|---|---|---|---|
| $y = x + 1$ | **–1** | **1** | **3** |

**(b)**



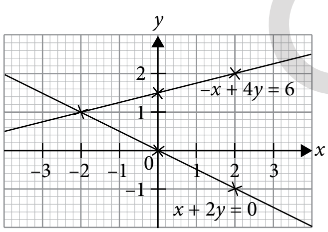

**(c)** From the graph, $\boldsymbol{x = 1}$ and $\boldsymbol{y = 2}$.

**2.** **(a)**



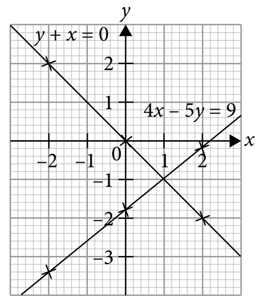

From the graph, $\boldsymbol{x = -0.6}$ and $\boldsymbol{y = 2.8}$.

**(b)**



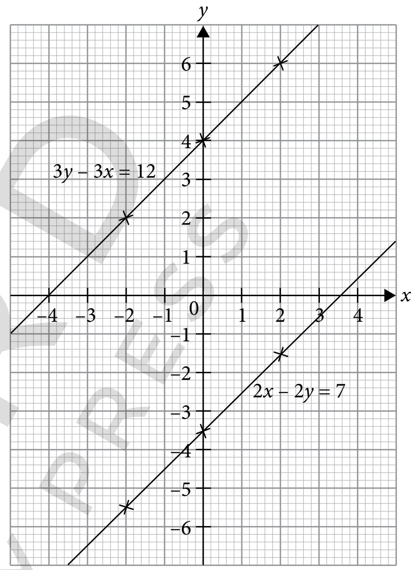

From the graph, $\boldsymbol{x = -2}$ and $\boldsymbol{y = 1}$.

**3.** **(a)**




From the graph, $\boldsymbol{x = 1}$ and $\boldsymbol{y = -1}$.

**(b)**



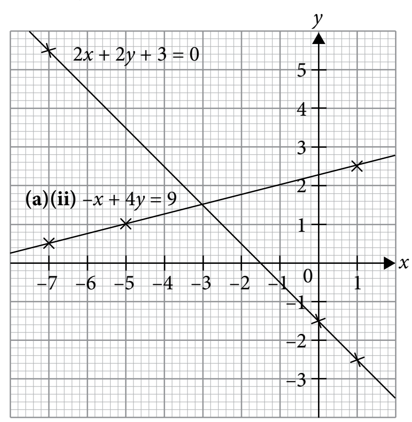

From the graph, there are **no solutions**.

**4.** **(a)** **(i)** When $x = -5$, $y = p$,

$$-(-5) + 4p = 9$$
$$5 + 4p = 9$$
$$4p = 4$$
$$p = 1$$

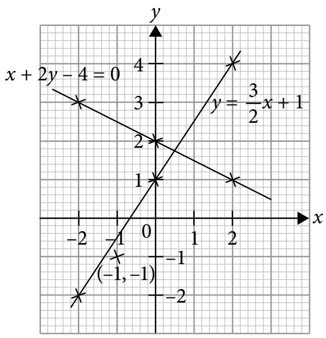
$\therefore$ $p = \mathbf{1}$

**(b)**




**(c)** From the graph, $\boldsymbol{x = -3}$ and $\boldsymbol{y = 1.5}$.

**5.** **(i)**



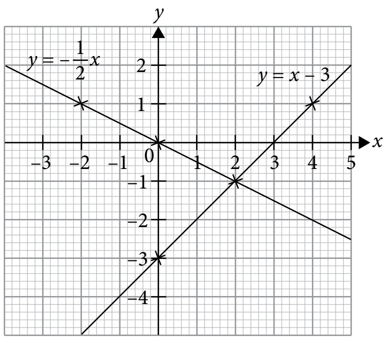

**(ii)** (−1, −1) is not the point of intersection between the two lines.

**(iii)** From the graph, $\boldsymbol{x = 0.5}$ and $\boldsymbol{y = 1.75}$.

## Challenge Myself!

**6.** **(i)** A possible pair of equations is $\boldsymbol{y = x - 3}$ and $\boldsymbol{y = -\frac{1}{2}x}$.

**(ii)**




## Worksheet 2D Solving simultaneous linear equations using algebraic methods

*If the question does not specify the method, students may use either the method of elimination or the method of substitution.*

**1.** **(a)** $2x + y = 5$ — (1)
$x + y = 1$ — (2)
(1) − (2): $x = 4$
Substitute $x = 4$ into (2):
$4 + y = 1$
$y = -3$
$\therefore \boldsymbol{x = 4, y = -3}$

**(b)** $2x + y = 5$ — (1)
$x - y = 1$ — (2)
(1) + (2): $3x = 6$
$x = 2$
Substitute $x = 2$ into (2):
$2 - y = 1$
$y = 1$
$\therefore \boldsymbol{x = 2, y = 1}$

**(c)** $2x - y = 5$ — (1)
$x - y = 1$ — (2)
(1) − (2): $x = 4$
Substitute $x = 4$ into (2):
$4 - y = 1$
$y = 3$
$\therefore \boldsymbol{x = 4, y = 3}$

**(d)** $2x - y = 5$ — (1)
$x + y = 1$ — (2)
(1) + (2): $3x = 6$
$x = 2$
Substitute $x = 2$ into (2):
$2 + y = 1$
$y = -1$
$\therefore \boldsymbol{x = 2, y = -1}$

**(e)** $5x + y = 6$ — (1)
$-5x + y = 4$ — (2)
(1) + (2): $2y = 10$
$y = 5$
Substitute $y = 5$ into (1):
$5x + 5 = 6$
$5x = 1$
$x = \frac{1}{5}$
$\therefore \boldsymbol{x = \frac{1}{5}, y = 5}$

**(f)** $4x + 9y = -1$ — (1)
$9y + 10x = 11$ — (2)
(2) − (1): $6x = 12$
$x = 2$
Substitute $x = 2$ into (1):
$4(2) + 9y = -1$
$8 + 9y = -1$
$9y = -9$
$y = -1$
$\therefore \boldsymbol{x = 2, y = -1}$

**(g)** $6x - 3y = 2$ — (1)
$3y - 7x = 10$ — (2)
(1) + (2): $-x = 12$
$x = -12$
Substitute $x = -12$ into (2):
$3y - 7(-12) = 10$
$3y + 84 = 10$
$3y = -74$
$y = -24\frac{2}{3}$
$\therefore \boldsymbol{x = -12, y = -24\frac{2}{3}}$

**(h)** $-8y + 3x = 7$ — (1)
$9x - 8y = -3$ — (2)
(2) − (1): $6x = -10$
$x = -1\frac{2}{3}$
Substitute $x = -1\frac{2}{3}$ into (1):
$-8y + 3\left(-1\frac{2}{3}\right) = 7$
$-8y - 5 = 7$
$-8y = 12$
$y = -1\frac{1}{2}$
$\therefore \boldsymbol{x = -1\frac{2}{3}, y = -1\frac{1}{2}}$

**(i)** $y = 5x - 9$ — (1)
$x = y + 1$ — (2)
Substitute (2) into (1):
$y = 5(y + 1) - 9$
$= 5y + 5 - 9$
$-4y = -4$
$y = 1$
Substitute $y = 1$ into (2):
$x = 1 + 1$
$= 2$
$\therefore$ **$x = 2, y = 1$**

**(j)** $x + 5y = 16$ — (1)
$y - 3x - 8 = 0$ — (2)
From (2), $y = 3x + 8$ — (3)
Substitute (3) into (1):
$x + 5(3x + 8) = 16$
$x + 15x + 40 = 16$
$16x = -24$
$x = -1\frac{1}{2}$
Substitute $x = -1\frac{1}{2}$ into (3):
$y = 3\left(-1\frac{1}{2}\right) + 8$
$= 3\frac{1}{2}$
$\therefore$ **$x = -1\frac{1}{2}, y = 3\frac{1}{2}$**

**(k)** $x + 6y = 5$ — (1)
$4x + 3y = 13$ — (2)
From (1), $x = 5 - 6y$ — (3)
Substitute (3) into (2):
$4(5 - 6y) + 3y = 13$
$20 - 24y + 3y = 13$
$-21y = -7$
$y = \frac{1}{3}$
Substitute $y = \frac{1}{3}$ into (3):
$x = 5 - 6\left(\frac{1}{3}\right)$
$= 3$
$\therefore$ **$x = 3, y = \frac{1}{3}$**

**(l)** $8x + 3y = 8$ — (1)
$4x - y = 14$ — (2)
(2) × 2: $8x - 2y = 28$ — (3)
(1) – (3): $5y = -20$
$y = -4$
Substitute $y = -4$ into (2):
$4x - (-4) = 14$
$4x + 4 = 14$
$4x = 10$
$x = 2\frac{1}{2}$
$\therefore$ **$x = 2\frac{1}{2}, y = -4$**

**(m)** $8x - 2y = 11$ — (1)
$3x - 4y = -4$ — (2)
(1) × 2: $16x - 4y = 22$ — (3)
(3) – (2): $13x = 26$
$x = 2$
Substitute $x = 2$ into (1):
$8(2) - 2y = 11$
$16 - 2y = 11$
$-2y = -5$
$y = 2\frac{1}{2}$
$\therefore$ **$x = 2, y = 2\frac{1}{2}$**

**(n)** $9x + 3y + 8 = 0$ — (1)
$6x - 6y = 0$ — (2)
From (2), $x - y = 0$, i.e. $y = x$ — (3)
Substitute (3) into (1):
$9x + 3x + 8 = 0$
$12x = -8$
$x = -\frac{2}{3}$
$\therefore$ **$x = -\frac{2}{3}, y = -\frac{2}{3}$**

**(o)** $4x - 8y = 19$ — (1)
$2y = 5x - 1$ — (2)
Substitute (2) into (1):
$4x - 4(5x - 1) = 19$
$4x - 20x + 4 = 19$
$-16x = 15$
$x = -\frac{15}{16}$
Substitute $x = -\frac{15}{16}$ into (2):
$2y = 5\left(-\frac{15}{16}\right) - 1$
$= -\frac{91}{16}$
$y = -\frac{91}{32}$
$= -2\frac{27}{32}$
$\therefore$ **$x = -\frac{15}{16}, y = -2\frac{27}{32}$**

**(p)** $7x + 4y = 2$ — (1)
$8x + 5y = 1$ — (2)
(1) × 5: $35x + 20y = 10$ — (3)
(2) × 4: $32x + 20y = 4$ — (4)
(3) – (4): $3x = 6$
$x = 2$
Substitute $x = 2$ into (1):
$7(2) + 4y = 2$
$14 + 4y = 2$
$4y = -12$
$y = -3$
$\therefore$ **$x = 2, y = -3$**

**(q)** $2x - 9y = 3$ — (1)
$5x - 10y = 4$ — (2)
(1) × 5: $10x - 45y = 15$ — (3)
(2) × 2: $10x - 20y = 8$ — (4)
(4) – (3): $25y = -7$
$y = -\frac{7}{25}$

Substitute $y = -\frac{7}{25}$ into (1):

$2x - 9\left(-\frac{7}{25}\right) = 3$

$2x + \frac{63}{25} = 3$

$2x = \frac{12}{25}$

$x = \frac{6}{25}$

$\therefore$ **$x = \frac{6}{25}$, $y = -\frac{7}{25}$**

**(r)** $7x + 10y = 6$ — (1)

$7y - 8x = 24$ — (2)

(1) × 8: $56x + 80y = 48$ — (3)

(2) × 7: $49y - 56x = 168$ — (4)

(3) + (4): $129y = 216$

$y = 1\frac{29}{43}$

Substitute $y = 1\frac{29}{43}$ into (1):

$7x + 10\left(1\frac{29}{43}\right) = 6$

$7x + \frac{720}{43} = 6$

$7x = -\frac{462}{43}$

$x = -1\frac{23}{43}$

$\therefore$ **$x = -1\frac{23}{43}$, $y = 1\frac{29}{43}$**

**2.** **(a)** $\frac{x}{3} - \frac{y}{2} = 4$ — (1)

$2x + 3y = 0$ — (2)

(1) × 6: $2x - 3y = 24$ — (3)

(2) + (3): $4x = 24$

$x = 6$

Substitute $x = 6$ into (2):

$2(6) + 3y = 0$

$12 + 3y = 0$

$3y = -12$

$y = -4$

$\therefore$ **$x = 6$, $y = -4$**

**(b)** $\frac{1}{3}x + 4y = 3$ — (1)

$4x + \frac{37}{3} = \frac{1}{3}y$ — (2)

(1) × 3: $x + 12y = 9$ — (3)

(2) × 3: $12x + 37 = y$ — (4)

Substitute (4) into (3):

$x + 12(12x + 37) = 9$

$x + 144x + 444 = 9$

$145x = -435$

$x = -3$

Substitute $x = -3$ into (4):

$y = 12(-3) + 37$

$= 1$

$\therefore$ **$x = -3$, $y = 1$**

**(c)** $y + 5x = -21$ — (1)

$\frac{10}{9x - 7y} = \frac{2}{y + x}$ — (2)

From (2),

$10(y + x) = 2(9x - 7y)$

$10x + 10y = 18x - 14y$

$8x = 24y$

$x = 3y$ — (3)

Substitute (3) into (1):

$y + 5(3y) = -21$

$y + 15y = -21$

$16y = -21$

$y = -1\frac{5}{16}$

Substitute $y = -1\frac{5}{16}$ into (3):

$x = 3\left(-1\frac{5}{16}\right)$

$= -3\frac{15}{16}$

$\therefore$ **$x = -3\frac{15}{16}$, $y = -1\frac{5}{16}$**

**(d)** $\frac{x + 5}{y + 9} = \frac{4}{7}$ — (1)

$\frac{x - 5}{y - 9} = \frac{6}{11}$ — (2)

From (1),

$7(x + 5) = 4(y + 9)$

$7x + 35 = 4y + 36$

$7x - 4y = 1$ — (3)

From (2),

$11(x - 5) = 6(y - 9)$

$11x - 55 = 6y - 54$

$11x - 6y = 1$ — (4)

(3) × 3: $21x - 12y = 3$ — (5)

(4) × 2: $22x - 12y = 2$ — (6)

(6) – (5): $x = -1$

Substitute $x = -1$ into (3):

$7(-1) - 4y = 1$

$-7 - 4y = 1$

$4y = -8$

$y = -2$

$\therefore$ **$x = -1$, $y = -2$**

**(e)** $\frac{3}{4}(7x - 3y) = y + 1$ — (1)

$\frac{4}{9}(x + 8y) - 2 = x - y$ — (2)

From (1),

$3(7x - 3y) = 4(y + 1)$

$21x - 9y = 4y + 4$

$21x - 13y = 4$ — (3)

From (2),

$4(x + 8y) - 18 = 9(x - y)$

$4x + 32y - 18 = 9x - 9y$

$5x = 41y - 18$

$x = \frac{41y - 18}{5}$ — (4)

Substitute (4) into (3):

$21\left(\frac{41y-18}{5}\right)-13y=4$

$21(41y - 18) - 65y = 20$

$861y - 378 - 65y = 20$

$796y = 398$

$y = \frac{1}{2}$

Substitute $y = \frac{1}{2}$ into (4):

$x = \frac{41\left(\frac{1}{2}\right)-18}{5}$

$= \frac{1}{2}$

$\therefore$ $\mathbf{x = \frac{1}{2}, y = \frac{1}{2}}$

**(f)** $2x + 9y = 8$ — (1)

$0.5y - x = 6$ — (2)

(2) × 2: $y - 2x = 12$ — (3)

(1) + (3): $10y = 20$

$y = 2$

Substitute $y = 2$ into (2):

$0.5(2) - x = 6$

$1 - x = 6$

$x = -5$

$\therefore$ $\mathbf{x = -5, y = 2}$

**(g)** $2.4x - 6.8y = 11.6$ — (1)

$7x - 15 = y$ — (2)

Substitute (2) into (1):

$2.4x - 6.8(7x - 15) = 11.6$

$2.4x - 47.6x + 102 = 11.6$

$45.2x = 90.4$

$x = 2$

Substitute $x = 2$ into (2):

$7(2) - 15 = y$

$y = -1$

$\therefore$ $\mathbf{x = 2, y = -1}$

**(h)** $3.5x - 1.5y = 0.2$ — (1)

$1.5x - 3.5y = -2.2$ — (2)

(1) × 10: $35x - 15y = 2$ — (3)

(2) × 10: $15x - 35y = -22$ — (4)

From (4),

$15x = 35y - 22$

$x = \frac{35y-22}{15}$ — (5)

Substitute (5) into (3):

$35\left(\frac{35y-22}{15}\right)-15y=2$

$\frac{7}{3}(35y-22)-15y=2$

$245y - 154 - 45y = 6$

$200y = 160$

$y = 0.8$

Substitute $y = 0.8$ into (5):

$x = \frac{35(0.8)-22}{15}$

$= 0.4$

$\therefore$ $\mathbf{x = 0.4, y = 0.8}$

**3.** $3x + 5y = 12$ — (1)

$9x - y = 12$ — (2)

From (2), $y = 9x - 12$ — (3)

Substitute (3) into (1):

$3x + 5(9x - 12) = 12$

$3x + 45x - 60 = 12$

$48x = 72$

$x = 1\frac{1}{2}$

Substitute $x = 1\frac{1}{2}$ into (3):

$y = 9\left(1\frac{1}{2}\right)-12$

$= 1\frac{1}{2}$

$\therefore$ $\mathbf{x = 1\frac{1}{2}, y = 1\frac{1}{2}}$

**4.** $ax + by = 2$ — (1)

$ax - by = 22$ — (2)

Substitute $x = 4$ and $y = -2$ into (1) and (2):

$4a - 2b = 2$

$2a - b = 1$ — (3)

$4a + 2b = 22$

$2a + b = 11$ — (4)

(3) + (4): $4a = 12$

$a = 3$

Substitute $a = 3$ into (4):

$2(3) + b = 11$

$6 + b = 11$

$b = 5$

$\therefore$ $\mathbf{a = 3, b = 5}$

## Challenge Myself!

**5.** $2x + 3y + 4z = 12$ — (1)

$4x - 3y + 8z = 6$ — (2)

$x + y + z = 7$ — (3)

(3) × 2: $2x + 2y + 2z = 14$ — (4)

(1) − (4): $y + 2z = -2$ — (5)

(3) × 4: $4x + 4y + 4z = 28$ — (6)

(6) − (2): $7y - 4z = 22$ — (7)

From (5), $y = -2z - 2$ — (8)

Substitute (8) into (7):

$7(-2z - 2) - 4z = 22$

$-14z - 14 - 4z = 22$

$-18z = 36$

$z = -2$

Substitute $z = -2$ into (8):

$y = -2(-2) - 2$

$= 2$

Substitute $y = 2$ and $z = -2$ into (3):

$x + 2 + (-2) = 7$

$x = 7$

$\therefore$ $\mathbf{x = 7, y = 2, z = -2}$

## Worksheet 2E Applications of simultaneous equations in real-world contexts

1. Let the numbers be $x$ and $y$.
$x + y = 121$ — (1)
$x - y = 5$ — (2)
(1) + (2): $2x = 126$
$x = 63$
Substitute $x = 63$ into (1):
$63 + y = 121$
$y = 58$
∴ The numbers are **63** and **58**.

2. Let the original fraction be $\frac{x}{y}$.
$\frac{x+2}{y+3} = \frac{3}{5}$ — (1)
$\frac{x-2}{y-3} = \frac{4}{7}$ — (2)
From (1),
$5(x + 2) = 3(y + 3)$
$5x + 10 = 3y + 9$
$5x - 3y = -1$ — (3)
From (2),
$7(x - 2) = 4(y - 3)$
$7x - 14 = 4y - 12$
$7x = 4y + 2$
$x = \frac{4y+2}{7}$ — (4)
Substitute (4) into (3):
$5\left(\frac{4y+2}{7}\right) - 3y = -1$
$5(4y + 2) - 21y = -7$
$20y + 10 - 21y = -7$
$y = 17$
Substitute $y = 17$ into (4):
$x = \frac{4(17)+2}{7}$
$= 10$
∴ The original fraction is $\mathbf{\frac{10}{17}}$.

3. Let $10x + y$ represent the original number.
$x + y = 12$ — (1)
$(10x + y) - (10y + x) = 54$ — (2)
From (2),
$10x + y - 10y - x = 54$
$9x - 9y = 54$
$x - y = 6$ — (3)
(1) + (2): $2x = 18$
$x = 9$
Substitute $x = 9$ into (1):
$9 + y = 12$
$y = 3$
∴ The original number is **93**.

4. Let the costs of one bottle of juice and one box of cookies be $\$x$ and $\$y$ respectively.
$4x + 3y = 29.6$ — (1)
$4y + 5x = 38.4$ — (2)
From (2),
$4y = 38.4 - 5x$
$y = 9.6 - 1.25x$ — (3)
Substitute (3) into (1):
$4x + 3(9.6 - 1.25x) = 29.6$
$4x + 28.8 - 3.75x = 29.6$
$0.25x = 0.8$
$x = 3.2$
Substitute $x = 3.2$ into (3):
$y = 9.6 - 1.25(3.2)$
$= 9.6 - 4$
$= 5.6$
∴ Total cost of one bottle of juice and one box of cookies
= \$3.20 + \$5.60
= **\$8.80**

5. Let the prices of one granola bar and one box of cereal be $\$x$ and $\$y$ respectively.
$6x + 2y = 29.6$ — (1)
$4x + 3y = 30.4$ — (2)
From (1),
$2y = 29.6 - 6x$
$y = 14.8 - 3x$ — (3)
Substitute (3) into (2):
$4x + 3(14.8 - 3x) = 30.4$
$4x + 44.4 - 9x = 30.4$
$5x = 14$
$x = 2.8$
Substitute $x = 2.8$ into (3):
$y = 14.8 - 3(2.8)$
$= 6.4$
∴ One granola bar costs **\$2.80** and one box of cereal costs **\$6.40**.

6. 600 g of assam black tea leaves cost \$7.20.
1 kg of assam black tea leaves costs $\$\left(\frac{7.20}{600} \times 1000\right) = \$12$.
Let the amounts of green tea leaves and assam black tea leaves be $x$ kg and $y$ kg respectively.
$x + y = 100$ — (1)
$8x + 12y = 1000$ — (2)
From (1), $y = 100 - x$ — (3)
Substitute (3) into (2):
$8x + 12(100 - x) = 1000$
$8x + 1200 - 12x = 1000$
$4x = 200$
$x = 50$
Substitute $x = 50$ into (3):
$y = 100 - 50$
$= 50$
∴ He should order **50 kg** of green tea leaves and **50 kg** of assam black tea leaves.

**7.** **(a)** $3x - 22 = y + 23$ (base ∠s of isos. △) — (1)

$5x - 18 + 3x - 22 + y + 23 = 180$ (∠ sum of a △) — (2)

From (1),

$3x - y = 45$ — (3)

From (2),

$8x + y = 197$ — (4)

∴ The equations are $\mathbf{3x - y = 45}$ and $\mathbf{8x + y = 197}$.

**(b)** (3) + (4): $11x = 242$

$x = 22$

Substitute $x = 22$ into (3):

$3(22) - y = 45$

$66 - y = 45$

$y = 21$

∴ $\angle PQR = \mathbf{44°}$, $\angle PRQ = \mathbf{44°}$ and $\angle QPR = \mathbf{92°}$

**8.** **(a)** $4h - 64 = 2k$

$2h - 32 = k$ (shown)

**(b)** $h + 5k = 115$ — (1)

$2h - 32 = k$ — (2)

Substitute (2) into (1):

$h + 5(2h - 32) = 115$

$h + 10h - 160 = 115$

$11h = 275$

$h = 25$

Substitute $h = 25$ into (2):

$k = 2(25) - 32$

$= 18$

∴ $\mathbf{h = 25, k = 18}$

**9.** **(a)** $25x + 15y = 680$

$5x + 3y = 136$ (shown)

**(b)** $16x + 18y = 536$

$8x + 9y = 268$ (shown)

**(c)** $5x + 3y = 136$ — (1)

$8x + 9y = 268$ — (2)

(1) × 3: $15x + 9y = 408$ — (3)

(3) − (2): $7x = 140$

$x = 20$

Substitute $x = 20$ into (1):

$5(20) + 3y = 136$

$100 + 3y = 136$

$3y = 36$

$y = 12$

∴ $\mathbf{x = 20, y = 12}$

**(d)** Difference in price = \$20 − \$12

= **\$8**

## Challenge Myself!

**10.** Let $A = \dfrac{1}{x}$ and $B = \dfrac{1}{y}$.

Then the equations are:

$9A + 2B = 4$ — (1)

$\dfrac{7}{2}B - 3A = 6$ — (2)

From (1),

$2B = 4 - 9A$

$B = 2 - \dfrac{9}{2}A$ — (3)

Substitute (3) into (2):

$\dfrac{7}{2}\left(2 - \dfrac{9}{2}A\right) - 3A = 6$

$7 - \dfrac{63}{4}A - 3A = 6$

$28 - 63A - 12A = 24$

$75A = 4$

$A = \dfrac{4}{75}$

Substitute $A = \dfrac{4}{75}$ into (3):

$B = 2 - \dfrac{9}{2}\left(\dfrac{4}{75}\right)$

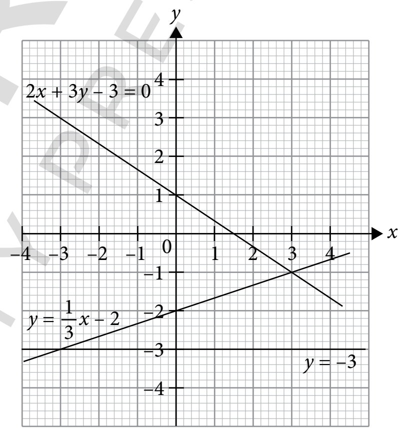
$= \dfrac{132}{75}$

$= \dfrac{44}{25}$

∴ $\mathbf{x = \dfrac{75}{4}, y = \dfrac{25}{44}}$

## Review Exercise 2

**1.** **(a)**




**(b)** From the graph, $\mathbf{x = 3}$ and $\mathbf{y = -1}$.

**2.** $7x - y = 3$ — (1)

$2x + 2y = 15$ — (2)

From (1),

$y = 7x - 3$ — (3)

Substitute (3) into (2):

$2x + 2(7x - 3) = 15$

$2x + 14x - 6 = 15$

$16x = 21$

$x = 1\dfrac{5}{16}$

Substitute $x = 1\dfrac{5}{16}$ into (3):

$y = 7\left(1\dfrac{5}{16}\right) - 3$

$= 6\dfrac{3}{16}$

∴ $\mathbf{x = 1\dfrac{5}{16}, y = 6\dfrac{3}{16}}$

3. Let $10x + y$ represent the original number.

$x + y = \frac{1}{4}(10x + y)$ — (1)

$(10x + y) - (10y + x) = -27$ — (2)

From (1),

$4(x + y) = 10x + y$

$4x + 4y = 10x + y$

$3y = 6x$

$y = 2x$ — (3)

From (2),

$10x + y - 10y - x = -27$

$9x - 9y = -27$

$x - y = -3$ — (4)

Substitute (3) into (4):

$x - 2x = -3$

$-x = -3$

$x = 3$

Substitute $x = 3$ into (3):

$y = 6$

$\therefore$ The original number is **36**.

4. **(i)** Consider Jimmy's usage:

$p + 5(q + 5) = 62$

$p + 5q + 25 = 62$

$p + 5q = 37$ — (1)

Consider Carol's usage:

$(3p - 4) + 12q = 92$

$3p - 4 + 12q = 92$

$3p + 12q = 96$

$p + 4q = 32$ — (2)

(1) – (2): $q = 5$

Substitute $q = 5$ into (2):

$p + 4(5) = 32$

$p + 20 = 32$

$p = 12$

$\therefore$ $\boldsymbol{p = 12, q = 5}$

**(ii)** Since \$2$p$ = \$24 and \$(3$p$ – 4) = \$32, Henry should choose **two 7-day tourist SIM cards**.

# 3 Linear Inequalities

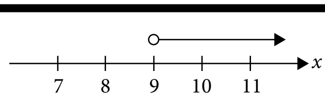
## Worksheet 3A Simple inequalities

1. $x = \boldsymbol{-3, -1, 0, 1, 2, 3}$
2. **2, 3, 5, 7, 11, 13, 17, 19**

## Worksheet 3B Solving simple linear inequalities

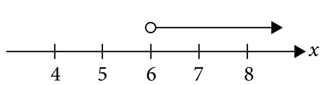
1. **(a)** $8x > 72$

$\boldsymbol{x > 9}$



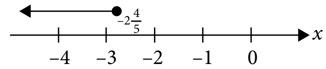

**(b)** $-x < -6$

$\boldsymbol{x > 6}$




**(c)** $-5x \geqslant 14$

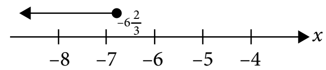
$\boldsymbol{x \leqslant -2\frac{4}{5}}$




**(d)** $20 \leqslant -3x$

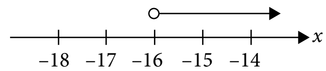
$3x \leqslant -20$

$\boldsymbol{x \leqslant -6\frac{2}{3}}$



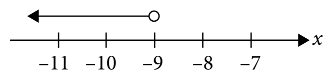

**(e)** $\frac{1}{4}y > -4$

$\boldsymbol{y > -16}$




**(f)** $0.9y < -8.1$

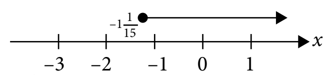
$\boldsymbol{y < -9}$




**(g)** $\frac{2}{5} \geqslant -\frac{3}{8}y$

$\frac{3}{8}y \geqslant -\frac{2}{5}$

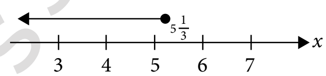
$\boldsymbol{y \geqslant -1\frac{1}{15}}$




**(h)** $-6.4 \leqslant -1.2y$

$1.2y \leqslant 6.4$

$\boldsymbol{y \leqslant 5\frac{1}{3}}$




2. **(a)** $a + 6 > 15$

$\boldsymbol{a > 9}$

**(b)** $a - 10 < 2$

$\boldsymbol{a < 12}$

**(c)** $7 - b \geqslant 3$

$-b \geqslant -4$

$\boldsymbol{b \leqslant 4}$

**(d)** $0 \leqslant 8 + b$

$-b \leqslant 8$

$\boldsymbol{b \geqslant -8}$

**(e)** $4h + 1 > 9$

$4h > 8$

$\boldsymbol{h > 2}$

**(f)** $6 - 2h < -11$

$-2h < -17$

$\boldsymbol{h > 8\frac{1}{2}}$

**(g)** $\frac{1}{10}k \geqslant 2$

$\boldsymbol{k \geqslant 20}$

**(h)** $\frac{5}{6}k \leqslant 25$

$\boldsymbol{k \leqslant 30}$

**(i)** $-\frac{3}{8}p > 1$

$\boldsymbol{p < -2\frac{2}{3}}$

**(j)** $-\frac{2}{7}p < -\frac{4}{5}$

$\boldsymbol{p > 2\frac{4}{5}}$

**(k)** $8 + \frac{2}{3}q \geqslant -6$

$\frac{2}{3}q \geqslant -14$

$\boldsymbol{q \geqslant -21}$

**(l)** $-4 \leqslant 3(1 + 7q)$

$3(1 + 7q) \geqslant -4$

$1 + 7q \geqslant -\frac{4}{3}$

$7q \geqslant -\frac{7}{3}$

$\boldsymbol{q \geqslant -\frac{1}{3}}$

**(m)** $\frac{1}{3}(6x - 1) > -\frac{3}{4}$

$6x - 1 > -\frac{9}{4}$

$6x > -\frac{5}{4}$

$\boldsymbol{x > -\frac{5}{24}}$

**(n)** $\frac{5(8 - 3x)}{2} < \frac{9}{10}$

$50(8 - 3x) < 18$

$400 - 150x < 18$

$-150x < -382$

$-x < -\frac{191}{75}$

$\boldsymbol{x > 2\frac{41}{75}}$

**(o)** $-1.5(9 - 0.2y) \geqslant 0$

$9 - 0.2y \leqslant 0$

$-0.2y \leqslant -9$

$\boldsymbol{y \geqslant 45}$

**(p)** $2.4(-5y) + 1.6y \leqslant -5.2$

$-12y + 1.6y \leqslant -5.2$

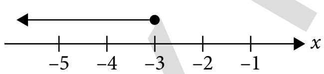
$-10.4y \leqslant -5.2$

$\boldsymbol{y \geqslant \frac{1}{2}}$

**3.** **(a)** $9x + 1 \leqslant 6x - 8$

$3x \leqslant -9$

$\boldsymbol{x \leqslant -3}$

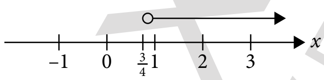



**(b)** $3(1 - x) < x$

$3 - 3x < x$

$-4x < -3$

$\boldsymbol{x > \frac{3}{4}}$



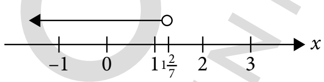

**(c)** $\frac{7 - 3x}{2} > \frac{x + 5}{4}$

$4(7 - 3x) > 2(x + 5)$

$28 - 12x > 2x + 10$

$-14x > -18$

$\boldsymbol{x < 1\frac{2}{7}}$

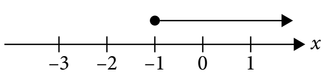



**(d)** $2.5(2x + 3) \geqslant 1.5 - x$

$5x + 7.5 \geqslant 1.5 - x$

$6x \geqslant -6$

$\boldsymbol{x \geqslant -1}$




**4.** **(a)** $4(2x + 15) \leqslant \frac{1}{2}x$

$8x + 60 \leqslant \frac{1}{2}x$

$16x + 120 \leqslant x$

$15x \leqslant -120$

$\boldsymbol{x \leqslant -8}$

**(b)** $\frac{3}{8}(3x - 2) < 2x + 1$

$9x - 6 < 16x + 8$

$-7x < 14$

$\boldsymbol{x > -2}$

**(c)** $\frac{5x - 2}{5} \geqslant \frac{8 - 3x}{2}$

$10x - 4 \geqslant 40 - 15x$

$25x \geqslant 44$

$\boldsymbol{x \geqslant 1\frac{19}{25}}$

**(d)** $\frac{6x + 11}{9} > \frac{2(1 - 2x)}{10} - 4$

$60x + 110 > 18 - 36x - 360$

$96x > -452$

$\boldsymbol{x > -4\frac{17}{24}}$

**(e)** $3.2(x - 1.5) > 2.8x - 4.8$

$3.2x - 4.8 > 2.8x - 4.8$

$0.4x > 0$

$\boldsymbol{x > 0}$

**(f)** $0.4(10 - 7x) \leqslant 0.1(6x - 5) + x$

$4 - 2.8x \leqslant 0.6x - 0.5 + x$

$-4.4x \leqslant -4.5$

$\boldsymbol{x \geqslant 1\frac{1}{44}}$

**5.** **(a)** **32**

**(b)** **37**

**(c)** **36**

**(d)** **64**

**6.** **(i)** $8m > 57$

$\boldsymbol{m > 7\frac{1}{8}}$

**(ii)** **8**

**7.** **(i)** $6n \leqslant -101$

$\boldsymbol{n \leqslant -16\frac{5}{6}}$

**(ii)** **−17**

**8.** **(i)** $\frac{p + 3}{27} \geqslant \frac{1}{2}$

$2p + 6 \geqslant 27$

$2p \geqslant 21$

$\boldsymbol{p \geqslant 10\frac{1}{2}}$

**(ii)** **27**

**9.** **(i)** $25 - 3(1 + 4q) < -2$

$25 - 3 - 12q < -2$

$22 - 12q < -2$

$-12q < -24$

$\boldsymbol{q > 2}$

**(ii)** **4**

**10.** $\dfrac{2(3-4x)}{5} \geqslant -7$

$6 - 8x \geqslant -35$

$-8x \geqslant -41$

$x \leqslant 5\dfrac{1}{8}$

$\therefore$ Greatest rational value of $x$ is $\mathbf{5\dfrac{1}{8}}$.

**11. (i)** $\dfrac{4(16-3y)}{5} \leqslant 2\dfrac{1}{5}$

$\dfrac{64-12y}{5} \leqslant \dfrac{11}{5}$

$64 - 12y \leqslant 11$

$-12y \leqslant -53$

$\mathbf{y \geqslant 4\dfrac{5}{12}}$

**(ii)** **9**

**12.** $-\dfrac{4}{9}(x-2) < -35$

$-4(x-2) < -315$

$4x - 8 > 315$

$4x > 323$

$x > 80\dfrac{3}{4}$

$\therefore$ A possible value of $x$ is **84**.

**13. (a) (i)** $\dfrac{x+3}{7} \geqslant -4$

$x + 3 \geqslant -28$

$\mathbf{x \geqslant -31}$

**(ii)** $\dfrac{10-2x}{5} > 6$

$10 - 2x > 30$

$-2x > 20$

$\mathbf{x < -10}$

**(b)** $\mathbf{-\sqrt{125}}$

**14. (i)** $7(2x+9) > 3(3x-4) + 45$

$14x + 63 > 9x - 12 + 45$

$5x > -30$

$\mathbf{x > -6}$

**(ii)** **–5, –4, –3, –2, –1**

**15. (i)** $\dfrac{4(7-3x)}{5} \leqslant \dfrac{2x+1}{6}$

$24(7-3x) \leqslant 5(2x+1)$

$168 - 72x \leqslant 10x + 5$

$-82x \leqslant -163$

$\mathbf{x \geqslant \dfrac{163}{82}}$

**(ii)** Least possible value of $x = \mathbf{\dfrac{163}{82}}$

**16.** $\dfrac{8x-45}{2} > 1 - \dfrac{6-7x}{4}$

$16x - 90 > 4 - 6 + 7x$

$9x > 88$

$\mathbf{x > 9\dfrac{7}{9}}$

$\therefore$ A possible value of $x$ is **60**.

**17.** Let $a = 2$, $b = 4$, $c = 5$ and $d = 3$.

$2x + 4 \leqslant 5x + 3$

$-3x \leqslant -1$

$x \geqslant \dfrac{1}{3}$

$\therefore$ The inequality is $\mathbf{2x + 4 \leqslant 5x + 3}$.

## Challenge Myself!

**18. (a)** Let $x = 3$ and $y = 4$. Then $\dfrac{1}{x} = \dfrac{1}{3}$ and $\dfrac{1}{y} = \dfrac{1}{4}$, but $\dfrac{1}{x}$ is greater than $\dfrac{1}{y}$.

Let $x = -3$ and $y = -4$. Then $\dfrac{1}{x} = -\dfrac{1}{3}$ and $\dfrac{1}{y} = -\dfrac{1}{4}$, so $\dfrac{1}{x}$ is less than $\dfrac{1}{y}$.

$\therefore$ The statement is **sometimes true**.

**(b)** Let $x = 5$ and $y = 2$. Then $x^2 = 25$ and $y^2 = 4$, so $x^2$ is greater than $y^2$.

Let $x = -2$ and $y = -5$. Then $x^2 = 4$ and $y^2 = 25$, but $x^2$ is less than $y^2$.

$\therefore$ The statement is **sometimes true**.

## Worksheet 3C Solving problems involving linear inequalities

**1.** Let $x$ be the number of cups of bubble tea.

$4.2x \leqslant 35$

$x \leqslant 8\dfrac{1}{3}$

$\therefore$ Maximum number of cups of bubble tea = **8**

**2.** Greatest possible length $= \sqrt{256}$ cm

$= 16$ cm

$\therefore$ Greatest possible perimeter = 4(16) cm

= **64 cm**

**3.** Let $x$ be the integer.

$12x > \dfrac{1}{2}\sqrt[3]{343}$

$12x > \dfrac{7}{2}$

$x > \dfrac{7}{24}$

$\therefore$ Smallest possible integer = **1**

**4.** Let the largest of the three numbers be $x$.

$x + (x-2) + (x-4) < 140$

$3x - 6 < 140$

$3x < 146$

$x < 48\dfrac{2}{3}$

$\therefore$ Largest possible number = $\mathbf{48 = 2^4 \times 3}$

**5.** Let $x$ be the number of \$10-notes.

$10x + 50(20 - x) > 650$

$10x + 1000 - 50x > 650$

$-40x > -350$

$x < 8\dfrac{3}{4}$

$\therefore$ Maximum number of \$10-notes = **8**

6. Four cups of coffee cost less than \$25. What is the maximum cost of each cup of coffee?

7. Number of hours spent sewing plain aprons = $\frac{1}{2}x$

Number of hours spent sewing aprons with designs = $\frac{3}{4}x$

$\frac{1}{2}x + \frac{3}{4}x \leqslant 36$

$\frac{5}{4}x \leqslant 36$

$x \leqslant 28\frac{4}{5}$

∴ She can sew a maximum of **28** plain aprons.

8. (a) Let $n$ be the number of additional outfits.

$30 + 7n \leqslant 55$

$7n \leqslant 25$

$n \leqslant 3\frac{4}{7}$

∴ She can rent a maximum of 3 + 2 = **5** outfits in that month.

(b) Let $p$ be the number of sets of 3 additional outfits.

$30 + 12p \leqslant 55$

$12p \leqslant 25$

$p \leqslant 2\frac{1}{12}$

∴ She can rent a maximum of 2(3) + 2 = **8** outfits in that month.

## Challenge Myself!

9. (i) Let $x$ be the number of hours she spends tutoring students.

$\mathbf{25x + 12(8 - x) \geqslant 150}$

(ii) $25x + 96 - 12x \geqslant 150$

$13x \geqslant 54$

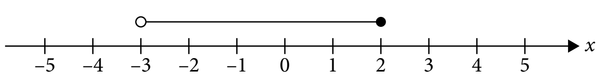
$x \geqslant 4\frac{2}{13}$

The combinations are:

**5 h of tutoring and 3 h of working at the library,**
**6 h of tutoring,**
**6 h of tutoring and 1 h of working at the library,**
**6 h of tutoring and 2 h of working at the library,**
**7 h of tutoring,**
**7 h of tutoring and 1 h of working at the library,**
**8 h of tutoring.**

## Worksheet 3D Simultaneous linear inequalities

1.




2. (i) $\mathbf{-4 \leqslant x < 3}$

(ii) **–4, –3, –2, –1, 0, 1, 2**

3. (a) $x + 4 < 6$ and $7x \geqslant 3 - 3x$

$x < 2$ $\qquad 10x \geqslant 3$

$x \geqslant \frac{3}{10}$

∴ $\mathbf{\frac{3}{10} \leqslant x < 2}$

(b) $9x - 5 \leqslant 2 + x$ and $x - 8 > 4x + 10$

$8x \leqslant 7$ $\qquad -3x > 18$

$x \leqslant \frac{7}{8}$ $\qquad x < -6$

∴ $\mathbf{x < -6}$

(c) $\frac{2x+7}{3} > 8$ and $4(1 - x) < 5x$

$2x + 7 > 24$ $\qquad 4 - 4x < 5x$

$2x > 17$ $\qquad -9x < -4$

$x > 8\frac{1}{2}$ $\qquad x > \frac{4}{9}$

∴ $\mathbf{x > 8\frac{1}{2}}$

(d) $0.6(3x - 10) \geqslant 1$ and $2.5x \leqslant x + 3$

$1.8x - 6 \geqslant 1$ $\qquad 1.5x \leqslant 3$

$1.8x \geqslant 7$ $\qquad x \leqslant 2$

$x \geqslant 3\frac{8}{9}$

∴ There are **no solutions**.

4. (a) $8x - 3 < 12 < 6x + 4$

$8x - 3 < 12$ and $12 < 6x + 4$

$8x < 15$ $\qquad -6x < -8$

$x < 1\frac{7}{8}$ $\qquad x > 1\frac{1}{3}$

∴ $\mathbf{1\frac{1}{3} < x < 1\frac{7}{8}}$

(b) $-7 \leqslant 2x - 5 \leqslant 7$

$-7 \leqslant 2x - 5$ and $2x - 5 \leqslant 7$

$-2x \leqslant 2$ $\qquad 2x \leqslant 12$

$x \geqslant -1$ $\qquad x \leqslant 6$

∴ $\mathbf{-1 \leqslant x \leqslant 6}$

(c) $-2 < \frac{10x-7}{6} \leqslant 3x$

$-2 < \frac{10x-7}{6}$ and $\frac{10x-7}{6} \leqslant 3x$

$-12 < 10x - 7$ $\qquad 10x - 7 \leqslant 18x$

$-10x < 5$ $\qquad -8x \leqslant 7$

$x > -\frac{1}{2}$ $\qquad x \geqslant -\frac{7}{8}$

∴ $\mathbf{x > -\frac{1}{2}}$

(d) $\frac{x}{5} \leqslant \frac{4x+9}{2} < \frac{7x-1}{3}$

$\frac{x}{5} \leqslant \frac{4x+9}{2}$ and $\frac{4x+9}{2} < \frac{7x-1}{3}$

$2x \leqslant 20x + 45$ $\qquad 12x + 27 < 14x - 2$

$-18x \leqslant 45$ $\qquad -2x < -29$

$x \geqslant -2\frac{1}{2}$ $\qquad x > 14\frac{1}{2}$

∴ $\mathbf{x > 14\frac{1}{2}}$

(e) $9x - 10 > 1 - 2x \geqslant 4(3x - 11)$

$9x - 10 > 1 - 2x$ and $1 - 2x \geqslant 4(3x - 11)$

$11x > 11$ $\qquad 1 - 2x \geqslant 12x - 44$

$x > 1$ $\qquad -14x \geqslant -45$

$x \leqslant 3\frac{3}{14}$

∴ $\mathbf{1 < x \leqslant 3\frac{3}{14}}$

**(f)** $\frac{3(8x+3)}{4} \geqslant \frac{2(x-7)}{5} > \frac{9+x}{6}$

$\frac{3(8x+3)}{4} \geqslant \frac{2(x-7)}{5}$ and $\frac{2(x-7)}{5} > \frac{9+x}{6}$

$15(8x + 3) \geqslant 8(x - 7)$ $\quad$ $12(x - 7) > 5(9 + x)$

$120x + 45 \geqslant 8x - 56$ $\quad$ $12x - 84 > 45 + 5x$

$112x \geqslant -101$ $\quad$ $7x > 129$

$x \geqslant -\frac{101}{112}$ $\quad$ $x > 18\frac{3}{7}$

$\therefore$ $\boldsymbol{x > 18\frac{3}{7}}$

**5.** $-6 < 3(x + 4) \leqslant 10$

$-2 < x + 4 \leqslant \frac{10}{3}$

$-6 < x \leqslant -\frac{2}{3}$

**6.** **(i)** $\frac{x}{8} \leqslant \frac{7x-2}{4} < \frac{5x+19}{5}$

$\frac{x}{8} \leqslant \frac{7x-2}{4}$ and $\frac{7x-2}{4} < \frac{5x+19}{5}$

$x \leqslant 14x - 4$ $\quad$ $35x - 10 < 20x + 76$

$-13x \leqslant -4$ $\quad$ $15x < 86$

$x \geqslant \frac{4}{13}$ $\quad$ $x < 5\frac{11}{15}$

$\therefore$ $\boldsymbol{\frac{4}{13} \leqslant x < 5\frac{11}{15}}$

**(ii)** A possible value of $x$ is **5**.

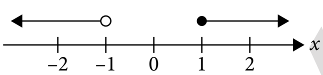
**7.** Let $a = 10$, $b = 9$, $c = 2$ and $d = -1$.

$10x + 9 < x \leqslant 2x - 1$

$10x + 9 < x$ and $x \leqslant 2x - 1$

$9x < -9$ $\quad$ $-x \leqslant -1$

$x < -1$ $\quad$ $x \geqslant 1$



Since there is no overlapping region, there is no real value of $x$ such that $x < -1$ and $x \geqslant 1$.

$\therefore$ The inequality $\boldsymbol{10x + 9 < x \leqslant 2x - 1}$ has no solutions.

**8.** **(i)** $\frac{1}{2}x \leqslant 5x - 9 \leqslant 4x + 11$

$\frac{1}{2}x \leqslant 5x - 9$ and $5x - 9 \leqslant 4x + 11$

$-4.5x \leqslant -9$ $\quad$ $x \leqslant 20$

$x \geqslant 2$

$\therefore$ $\boldsymbol{2 \leqslant x \leqslant 20}$

**(ii)** Smallest possible value of $y = 2^2 - 9 = -5$

Greatest possible value of $y = 20^2 - 9 = 391$

$\therefore$ Range of values of $y$ is $\boldsymbol{-5 \leqslant y \leqslant 391}$

**9.** **(a)** If $p \leqslant q$, then $-\frac{p}{2} \boldsymbol{\geqslant} -\frac{q}{2}$.

**(b)** If $p^2 \leqslant q^2$, then $4 - \frac{1}{p^2} \boldsymbol{\leqslant} 4 - \frac{1}{q^2}$.

**10.** **(i)** $\frac{x+h}{4} < \frac{3x-1}{2} \leqslant kx$

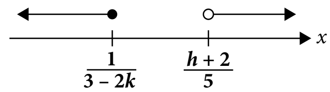
$\frac{x+h}{4} < \frac{3x-1}{2}$ and $\frac{3x-1}{2} \leqslant kx$

$x + h < 6x - 2$ $\quad$ $3x - 1 \leqslant 2kx$

$5x > h + 2$ $\quad$ $(3 - 2k)x \leqslant 1$

$x > \frac{h+2}{5}$ $\quad$ $x \leqslant \frac{1}{3-2k}$

For the inequalities to have no solution,



$\therefore$ $\frac{h+2}{5} > \frac{1}{3-2k}$ (shown)

**(ii)** A possible pair of values is $\boldsymbol{h = \sqrt{7}}$ and $\boldsymbol{k = \sqrt{3}}$.

## Challenge Myself!

**11.** **(a)** **Yes.**

$a < b$ — (1)

$c < d$ — (2)

(1) + (2): $a + c < b + d$

**(b)** **No.**

If both $a$ and $b$ are negative numbers (e.g. $a = -10$ and $b = -1$), then $a^2 > b^2$, so $a^2 + c > b^2 + c$.

**(c)** **No.**

E.g. if $a = -10$, $b = -1$, $c = 2$ and $d = 20$, then $c - a = 12$ and $d - b = 21$, so $c - a < d - b$.

**(d)** **No.**

E.g. if $a = -10$, $b = -1$, $c = 2$ and $d = 50$, then $ac = -20$ and $bd = -50$, so $ac > bd$.

## Worksheet 3E Solving problems involving simultaneous linear inequalities

**1.** Chris replaces $\frac{x-3}{2}$ tyres in $25\left(\frac{x-3}{2}\right)$ min.

Jim replaces $\frac{x+3}{2}$ tyres in $20\left(\frac{x+3}{2}\right)$ min.

$20\left(\frac{x+3}{2}\right) \leqslant 25\left(\frac{x-3}{2}\right)$

$10x + 30 \leqslant 12.5x - 37.5$

$-2.5x \leqslant -67.5$

$x \geqslant 27$

$\therefore$ Minimum number of tyres to be replaced = **27**

**2.** Let the time taken to wash a car and a van be $x$ min and $(x + 10)$ min respectively.

$175 \leqslant 5x + 4(x + 10) < 3(60) + 10$

$175 \leqslant 5x + 4x + 40 < 190$

$175 \leqslant 9x + 40 < 190$

$135 \leqslant 9x < 150$

$\boldsymbol{15 \leqslant x < 16\frac{2}{3}}$

**3.** **(a)** $\boldsymbol{65 \leqslant x < 80}$

**(b)** $65 \leqslant \frac{88+y+64}{3} < 80$

$195 \leqslant y + 152 < 240$

$\boldsymbol{43 \leqslant y < 88}$

4. **(i)** $3(5.8) + 2(6.4) \leqslant 3x + 2y \leqslant 3(9.6) + 2(10.2)$
$\mathbf{\$30.20 \leqslant \$(3x + 2y) \leqslant \$49.20}$

**(ii)** Maximum price difference = \$10.20 – \$5.80
= **\$4.40**

**(iii)** OPEN She could have paid **\$9.60** for the ham sandwich set and **\$8.90** for the bacon roll set.

## Challenge Myself!

5. For $\triangle ABC$ to exist, the sum of the lengths of the two shorter sides must be greater than the length of the third side.

$BC + CA > AB$

$x - 8 + x - 10 > x - 6$

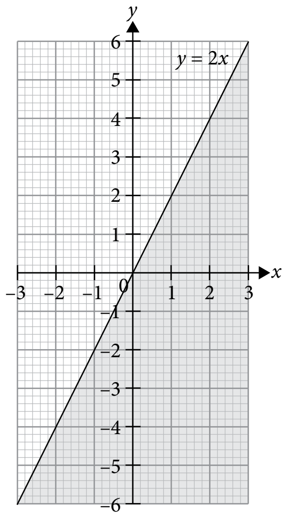
$\mathbf{x > 12}$

## Worksheet 3F Linear inequalities in two variables

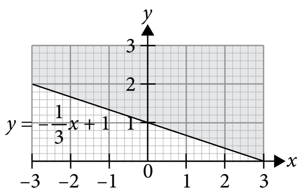
1. $\mathbf{y > x + 3}$ **and** $\mathbf{y < 3}$

2. $\mathbf{y \leqslant \frac{1}{4}x}$ **and** $\mathbf{y < 2x}$

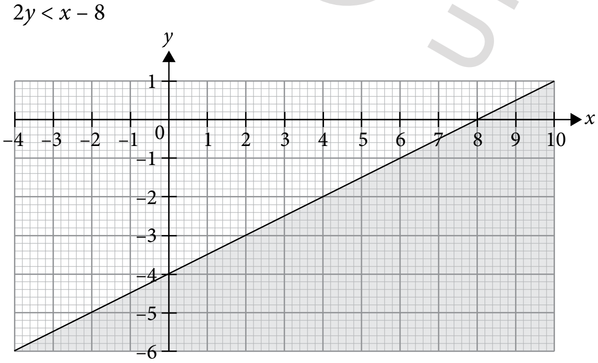
3.



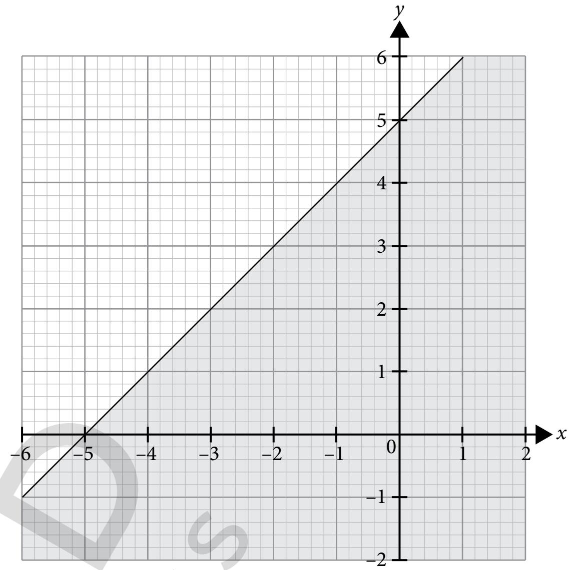

4.



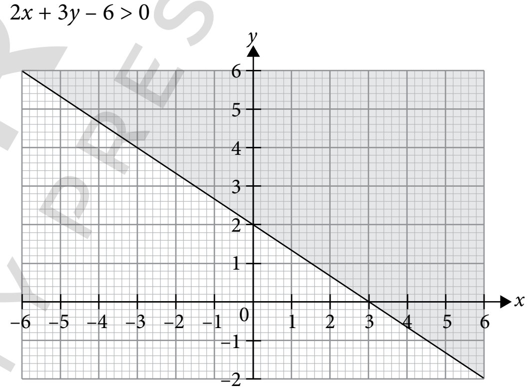

5. **(a)** $2y < x - 8$



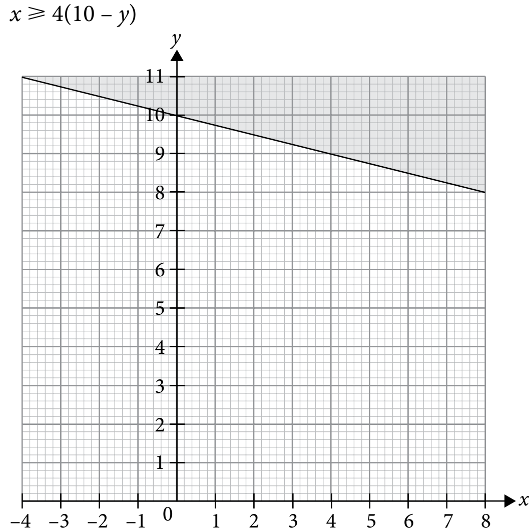
**(b)** $y - x \leqslant 5$



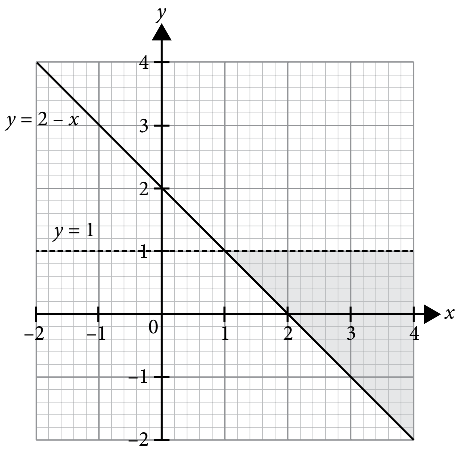
**(c)** $2x + 3y - 6 > 0$



**(d)** $x \geqslant 4(10 - y)$



6.

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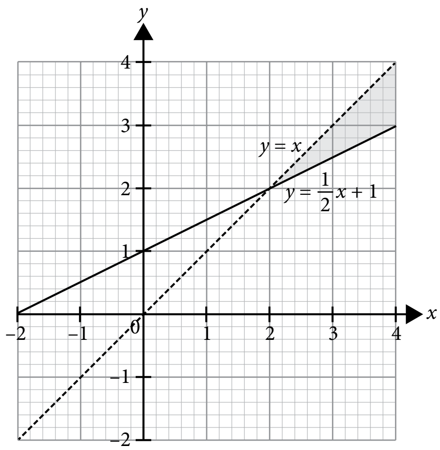
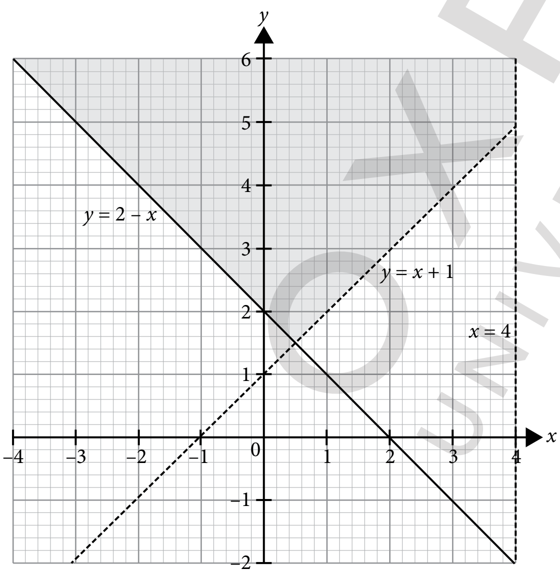
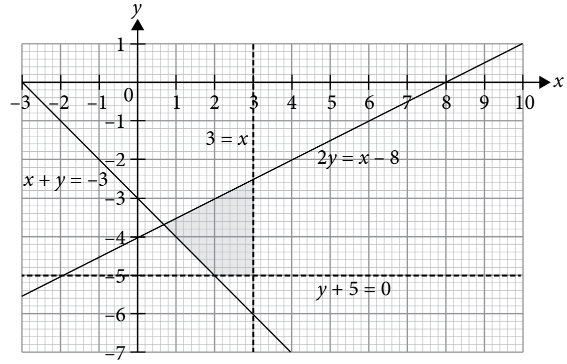



8. (a)




(b)



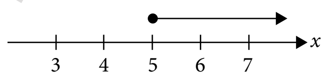

9. $y \geqslant -\frac{4}{5}x - 1$, $y \leqslant \frac{3}{4}x + 2$ **and** $y < -\frac{5}{3}x - 1$

## Review Exercise 3

1. $x = -4, -2, -1, 0, 1$
2. (a) (i) **28**
   (ii) **29**
   (iii) **64**
   (b) OPEN **45**
3. $5 - \frac{1}{5}x \leqslant 4$

   $-\frac{1}{5}x \leqslant -1$

   $\boldsymbol{x \geqslant 5}$



4. (i) $3x \geqslant 28$

   $\boldsymbol{x \geqslant 9\frac{1}{3}}$

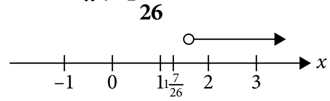
   (ii) $\boldsymbol{9\frac{1}{3}}$
5. (a) $3(6x - 5) > \frac{2}{3}x + 7$

   $9(6x - 5) > 2x + 21$

   $54x - 45 > 2x + 21$

   $52x > 66$

   $\boldsymbol{x > 1\frac{7}{26}}$

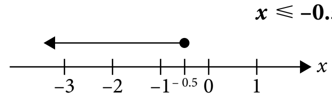


   (b) $3.5(2x - 1) + 0.4(8x + 21.5) \leqslant 0$

   $7x - 3.5 + 3.2x + 8.6 \leqslant 0$

   $10.2x \leqslant -5.1$

   $\boldsymbol{x \leqslant -0.5}$



6. (a) $7x \leqslant 5 - x$ and $\frac{3x}{4} + 1 > 0$

   $8x \leqslant 5$ $\quad$ $3x + 4 > 0$

   $x \leqslant \frac{5}{8}$ $\quad$ $3x > -4$

   $x > -1\frac{1}{3}$

   $\therefore\ \boldsymbol{-1\frac{1}{3} < x \leqslant \frac{5}{8}}$

**(b)** $0.1x + 8 < 0.5x$ and $3(2x - 7) \geqslant 4$

$-0.4x < -8$ $\qquad$ $6x - 21 \geqslant 4$

$x > 20$ $\qquad$ $6x \geqslant 25$

$x \geqslant \frac{25}{6}$

$\therefore$ $\boldsymbol{x > 20}$

**(c)** $6x - 9 < \frac{7x+12}{2} < 4(5x - 11)$

$6x - 9 < \frac{7x+12}{2}$ and $\frac{7x+12}{2} < 4(5x - 11)$

$12x - 18 < 7x + 12$ $\qquad$ $7x + 12 < 8(5x - 11)$

$5x < 30$ $\qquad$ $7x + 12 < 40x - 88$

$x < 6$ $\qquad$ $-33x < -100$

$x > 3\frac{1}{33}$

$\therefore$ $\boldsymbol{3\frac{1}{33} < x < 6}$

**(d)** $4x + 5 \geqslant \frac{10}{3}x > 9x + 50$

$4x + 5 \geqslant \frac{10}{3}x$ and $\frac{10}{3}x > 9x + 50$

$12x + 15 \geqslant 10x$ $\qquad$ $10x > 27x + 150$

$2x \geqslant -15$ $\qquad$ $-17x > 150$

$x \geqslant -7\frac{1}{2}$ $\qquad$ $x < -8\frac{14}{17}$

$\therefore$ There are **no solutions**.

**7.** **(i)** $3x - 11 < -14 \leqslant 2x + 65$

$3x - 11 < -14$ and $-14 \leqslant 2x + 65$

$3x < -3$ $\qquad$ $-2x \leqslant 79$

$x < -1$ $\qquad$ $x \geqslant -39\frac{1}{2}$

$\therefore$ $\boldsymbol{-39\frac{1}{2} \leqslant x < -1}$

**(ii)** When $x = -4$,

$x^2 = 16$

$\therefore$ Greatest possible value of $x$ is $\boldsymbol{-4}$

**8.** (OPEN) Let $a = \frac{1}{2}$, $b = -1$, $c = 9$ and $d = -1$.

$\frac{1}{2}x - 1 \leqslant 5x < 9x - 1$

$\frac{1}{2}x - 1 \leqslant 5x$ and $5x < 9x - 1$

$x - 2 \leqslant 10x$ $\qquad$ $-4x < -1$

$-9x \leqslant 2$ $\qquad$ $x > \frac{1}{4}$

$x \geqslant -\frac{2}{9}$

$\therefore$ A possible inequality is $\boldsymbol{\frac{1}{2}x - 1 \leqslant 5x < 9x - 1}$.

**9.** Let the smallest of the three numbers be $x$.

$x + (x + 2) + (x + 4) > 111$

$3x + 6 > 111$

$3x > 105$

$x > 35$

$\therefore$ Smallest possible product $= (37)(39)(41)$

$= \mathbf{59\ 163}$

**10.** Let $x$ be the number of laptops.

$2250x \leqslant 40\ 000$

$x \leqslant 17\frac{7}{9}$

$\therefore$ The company can buy a maximum of **17** laptops.

**11.** Let the length of the cube be $x$ cm.

$x^3 \geqslant 95.5$

$x \geqslant 4.57$ (to 2 d.p.)

$\therefore$ Least possible length = **4.57 cm**

**12.** $\frac{90}{60} < x < \frac{120}{60}$

$\therefore$ $\boldsymbol{1.5 < x < 2}$

**13.** **(a)** (OPEN) **30 wins, 5 draws and 3 losses**

Number of points $= 30(3) + 5(1) + 3(0)$

$= 95$

**(b)** Let $x$ be the number of wins.

$3x + 1(38 - x) \geqslant 88$

$3x + 38 - x \geqslant 88$

$2x \geqslant 50$

$x \geqslant 25$

$\therefore$ Maximum number of matches his team could afford to draw $= 38 - 25$

$= \mathbf{13}$

**14.** Amount charged by Spick and Span Cleaners $= \$[22 + 18(x - 1)]$

$= \$(22 + 18x - 18)$

$= \$(18x + 4)$

Amount charged by The Cleaning Professional $= \$[60 + p(x - 3)]$

$= \$(60 + px - 3p)$

$18x + 4 > 60 + px - 3p$

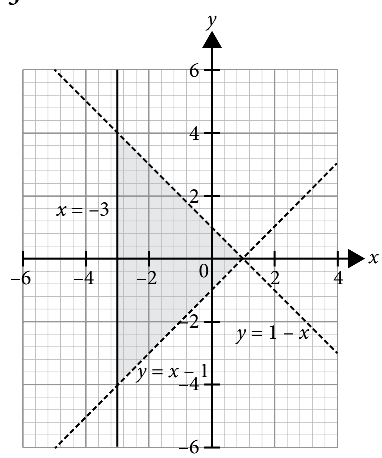
$18x - px > 56 - 3p$

$(18 - p)x > 56 - 3p$

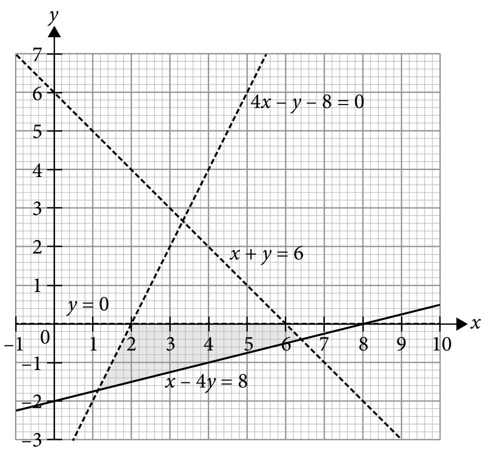
$\boldsymbol{x > \frac{56 - 3p}{18 - p}}$

**15.** $\boldsymbol{y \leqslant \frac{2}{3}x + 4,\ x + y + 1 \geqslant 0}$ **and** $\boldsymbol{x < -1}$

**16.** **(a)**



(b)




## 4 Expansion and Factorisation of Algebraic Expressions

### Worksheet 4A Addition and subtraction of quadratic expressions

**1.** (a) $12x^2 + 4x^2 = \mathbf{16x^2}$

(b) $5y^2 - 2y^2 = \mathbf{3y^2}$

(c) $8x^2 + (-5x^2) = 8x^2 - 5x^2$
$= \mathbf{3x^2}$

(d) $-y^2 + 10y^2 = \mathbf{9y^2}$

(e) $-6x^2 + x^2 = \mathbf{-5x^2}$

(f) $-3y^2 - 9y^2 = \mathbf{-12y^2}$

(g) $-7x^2 - (-11x^2) = -7x^2 + 11x^2$
$= \mathbf{4x^2}$

(h) $-4y^2 - (-4y^2) = -4y^2 + 4y^2$
$= \mathbf{0}$

**2.** (a) $3x^2 + 7 + 2x^2 + 5 = 3x^2 + 2x^2 + 7 + 5$
$= \mathbf{5x^2 + 12}$

(b) $8y^2 - 1 - 6y^2 + 4 = 8y^2 - 6y^2 + 4 - 1$
$= \mathbf{2y^2 + 3}$

(c) $9x^2 + 10 + (-x^2) - 3 = 9x^2 + 10 - x^2 - 3$
$= 9x^2 - x^2 + 10 - 3$
$= \mathbf{8x^2 + 7}$

(d) $4 - 2y^2 - 11 + 7y^2 = 7y^2 - 2y^2 + 4 - 11$
$= \mathbf{5y^2 - 7}$

(e) $5x^2 + 6x + 2 + 9x^2 + 4x + 1 = 5x^2 + 9x^2 + 6x + 4x + 2 + 1$
$= \mathbf{14x^2 + 10x + 3}$

(f) $3y^2 - 10y + 8 - (-3y^2) + 10y + 8$
$= 3y^2 - 10y + 8 + 3y^2 + 10y + 8$
$= 3y^2 + 3y^2 - 10y + 10y + 8 + 8$
$= \mathbf{6y^2 + 16}$

(g) $-x^2 + 12 - 7x + 4x^2 - 3x - 8 = 4x^2 - x^2 - 7x - 3x + 12 - 8$
$= \mathbf{3x^2 - 10x + 4}$

(h) $9 - 6y^2 + 9y - (-2y^2) + 9xy = 9 - 6y^2 + 9y + 2y^2 + 9xy$
$= 2y^2 - 6y^2 + 9y + 9xy + 9$
$= \mathbf{-4y^2 + 9y + 9xy + 9}$

**3.** (a) $(20x + 5x^2) + (x^2 - 7x) = 20x + 5x^2 + x^2 - 7x$
$= 5x^2 + x^2 + 20x - 7x$
$= \mathbf{6x^2 + 13x}$

(b) $(-8y^2 + 3y) + (4y - 11y^2) = -8y^2 + 3y + 4y - 11y^2$
$= 3y + 4y - 8y^2 - 11y^2$
$= \mathbf{7y - 19y^2}$

(c) $(1 - 6x^2) + (3x + 8 - 9x^2) = 1 - 6x^2 + 3x + 8 - 9x^2$
$= 1 + 8 + 3x - 6x^2 - 9x^2$
$= \mathbf{9 + 3x - 15x^2}$

(d) $[10y + (-y^2)] + [10 - (-7y^2)] = (10y - y^2) + (10 + 7y^2)$
$= 10y - y^2 + 10 + 7y^2$
$= 7y^2 - y^2 + 10y + 10$
$= \mathbf{6y^2 + 10y + 10}$

(e) $\left(-\frac{1}{2}y^2 + x\right) + \left(\frac{1}{2}y^2 - x\right) = -\frac{1}{2}y^2 + x + \frac{1}{2}y^2 - x$
$= x - x + \frac{1}{2}y^2 - \frac{1}{2}y^2$
$= \mathbf{0}$

(f) $(2.5x^2 - 3y^2 + xy) + (8.4y^2 - xy - 1.2x^2)$
$= 2.5x^2 - 3y^2 + xy + 8.4y^2 - xy - 1.2x^2$
$= 2.5x^2 - 1.2x^2 + xy - xy + 8.4y^2 - 3y^2$
$= \mathbf{1.3x^2 + 5.4y^2}$

**4.** (a) $(6x^2 + 24x) - (4x^2 + 15x) = 6x^2 + 24x - 4x^2 - 15x$
$= 6x^2 - 4x^2 + 24x - 15x$
$= \mathbf{2x^2 + 9x}$

(b) $(3y^2 - x + 10) - (8y^2 - 7) = 3y^2 - x + 10 - 8y^2 + 7$
$= 10 + 7 - x + 3y^2 - 8y^2$
$= \mathbf{17 - x - 5y^2}$

(c) $\left(-\frac{5}{8}x^2 + y^2\right) - \left(y^2 - \frac{3}{4}x^2\right) = -\frac{5}{8}x^2 + y^2 - y^2 + \frac{3}{4}x^2$
$= \frac{3}{4}x^2 - \frac{5}{8}x^2 + y^2 - y^2$
$= \mathbf{\frac{1}{8}x^2}$

(d) $(9x - 0.8y^2) - (x + y - 3.3y^2) = 9x - 0.8y^2 - x - y + 3.3y^2$
$= 9x - x - y + 3.3y^2 - 0.8y^2$
$= \mathbf{8x - y + 2.5y^2}$

### Challenge Myself!

**5.** (a) $ax^2 + bx + c - 5x^2 + 9x - 1 = 4x^2 + 7x - 3$
$ax^2 - 5x^2 + bx + 9x + c - 1 = 4x^2 + 7x - 3$
$(a - 5)x^2 + (b + 9)x + (c - 1) = 4x^2 + 7x - 3$
Comparing coefficients of $x^2$,
$a - 5 = 4$
$a = 9$
Comparing coefficients of $x$,
$b + 9 = 7$
$b = -2$
Comparing constants,
$c - 1 = -3$
$c = -2$
$\therefore$ **$a = 9, b = -2, c = -2$**

(b) $hx^2 + kx + m + 2x^2 - 8x + 6 = 10x^2$
$hx^2 + 2x^2 + kx - 8x + m + 6 = 10x^2$
$(h + 2)x^2 + (k - 8)x + (m + 6) = 10x^2$
Comparing coefficients of $x^2$,
$h + 2 = 10$
$h = 8$

Comparing coefficients of $x$,
$k - 8 = 0$
$k = 8$
Comparing constants,
$m + 6 = 0$
$m = -6$
$\therefore$ **$h = 8, k = 8, m = -6$**

(c) $px - qx^2 - n + (-3x^2) - (-x) + 12 = 0$
$px - qx^2 - n - 3x^2 + x + 12 = 0$
$qx^2 - px + n = -3x^2 + x + 12$
Comparing coefficients of $x^2$,
$q = -3$
Comparing coefficients of $x$,
$-p = 1$
$p = -1$
Comparing constants,
$n = 12$
$\therefore$ **$p = -1, q = -3, n = 12$**

## Worksheet 4B Expansion of algebraic expressions of the form $(a + b)(c + d)$

**1.** (a) $7(5x + 4) =$ **$35x + 28$**
(b) $3(9x - 8) =$ **$27x - 24$**
(c) $-2(6x + 1) =$ **$-12x - 2$**
(d) $-(10 - 4x) = -10 + 4x$
$=$ **$4x - 10$**
(e) $6a(2x + 7y) =$ **$12ax + 42ay$**
(f) $4a(3y - 20x) =$ **$12ay - 80ax$**
(g) $-3a(y + 8x) =$ **$-3ay - 24ax$**
(h) $-5a(2x - 9y) = -10ax + 45ay$
$=$ **$45ay - 10ax$**

**2.** (a) $4(10x + y) + 20x = 40x + 4y + 20x$
$=$ **$60x + 4y$**
(b) $8(x + 3y) - 25y = 8x + 24y - 25y$
$=$ **$8x - y$**
(c) $7x + 4(6x - 5y) = 7x + 24x - 20y$
$=$ **$31x - 20y$**
(d) $-2x - 9(3x - 7y) = -2x - 27x + 63y$
$=$ **$63y - 29x$**
(e) $12(2x - y) + 2(12x + y) = 24x - 12y + 24x + 2y$
$=$ **$48x - 10y$**
(f) $-3(9x + y) + 4(y - 10x) = -27x - 3y + 4y - 40x$
$=$ **$y - 67x$**
(g) $5a(9x + 4y) + 3a(5x + 6y) = 45ax + 20ay + 15ax + 18ay$
$=$ **$60ax + 38ay$**
(h) $2a(3y - 10x) - 7a(x - 8y) = 6ay - 20ax - 7ax + 56ay$
$=$ **$62ay - 27ax$**

**3.** (a) $(a + b)(x + y) =$ **$ax + ay + bx + by$**
(b) $(a + b)(x - y) =$ **$ax - ay + bx - by$**
(c) $(a - b)(x + y) =$ **$ax + ay - bx - by$**
(d) $(a - b)(x - y) =$ **$ax - ay - bx + by$**
(e) $(2h + k)(7x + y) =$ **$14hx + 2hy + 7kx + ky$**
(f) $(3h + 4k)(9x - 2y) =$ **$27hx - 6hy + 36kx - 8ky$**
(g) $(h - 6k)(4x + 5y) =$ **$4hx + 5hy - 24kx - 30ky$**
(h) $(5h - k)(x - 10y) =$ **$5hx - 50hy - kx + 10ky$**
(i) $(m + n)(3x + 8y + z)$
$=$ **$3mx + 8my + mz + 3nx + 8ny + nz$**
(j) $(2m - n)(4x - 5y - 6z)$
$=$ **$8mx - 10my - 12mz - 4nx + 5ny + 6nz$**

**4.** (a) $(4a + b)(8x - 9y + 10)$
$=$ **$32ax - 36ay + 40a + 8bx - 9by + 10b$**
(b) $(6b - 5a)(3 - 2x + 7y)$
$=$ **$18b - 12bx + 42by - 15a + 10ax - 35ay$**

**5.** $(5k - 3h)(8x + 10y + 12) - (7x - 4y)(6h - k - 9)$
$= (40kx + 50ky + 60k - 24hx - 30hy - 36h) -$
$(42hx - 7kx - 63x - 24hy + 4ky + 36y)$
$= 40kx + 50ky + 60k - 24hx - 30hy - 36h - 42hx + 7kx + 63x$
$+ 24hy - 4ky - 36y$
$=$ **$47kx + 46ky + 60k - 66hx - 6hy - 36h + 63x - 36y$**

## Worksheet 4C Expansion of quadratic and complex expressions

**1.** (a) $5x(2x + 2) =$ **$10x^2 + 10x$**
(b) $6x(10 - x) =$ **$60x - 6x^2$**
(c) $-4x(8x + 3) =$ **$-32x^2 - 12x$**
(d) $-x(7 - 9x) = -7x + 9x^2$
$=$ **$9x^2 - 7x$**

**2.** (a) $5x^2 + 2x(1 + 6x) = 5x^2 + 2x + 12x^2$
$=$ **$17x^2 + 2x$**
(b) $x(3x - 1) + 7x(x + 1) = 3x^2 - x + 7x^2 + 7x$
$=$ **$10x^2 + 6x$**
(c) $4x(x + 9) - x(4x + 9) = 4x^2 + 36x - 4x^2 - 9x$
$=$ **$27x$**
(d) $-2x(8x - 3) + 5x(x - 10) = -16x^2 + 6x + 5x^2 - 50x$
$=$ **$-11x^2 - 44x$**

**3.** (a) $3x(6x - 11y) =$ **$18x^2 - 33xy$**
(b) $-8x(5y + 9x^2) =$ **$-40xy - 72x^3$**
(c) $2xy(7x^2 + y - 4) =$ **$14x^3y + 2xy^2 - 8xy$**
(d) $-4xy(xz - 10y^2 + z) =$ **$-4x^2yz + 40xy^3 - 4xyz$**

**4.** (a) $8x(3x - 4y) + 12x^2 = 24x^2 - 32xy + 12x^2$
$=$ **$36x^2 - 32xy$**
(b) $5x(x - 6y) - x(9y + x) = 5x^2 - 30xy - 9xy - x^2$
$=$ **$4x^2 - 39xy$**
(c) $-4y(5y - 8x) + 3y(10x + y) = -20y^2 + 32xy + 30xy + 3y^2$
$=$ **$62xy - 17y^2$**
(d) $-y(7y + 2x) - 2x(7x - 2y) = -7y^2 - 2xy - 14x^2 + 4xy$
$=$ **$-14x^2 + 2xy - 7y^2$**

**5.** (a) $(x + 8)(x + 2) = x^2 + 2x + 8x + 16$
$=$ **$x^2 + 10x + 16$**
(b) $(x + 9)(x - 3) = x^2 - 3x + 9x - 27$
$=$ **$x^2 + 6x - 27$**
(c) $(2x - 7)(x + 4) = 2x^2 + 8x - 7x - 28$
$=$ **$2x^2 + x - 28$**
(d) $(x - 5)(11x - 6) = 11x^2 - 6x - 55x + 30$
$=$ **$11x^2 - 61x + 30$**
(e) $(4x + 5)(2x + 9) = 8x^2 + 36x + 10x + 45$
$=$ **$8x^2 + 46x + 45$**
(f) $(7x + 2)(3x - 1) = 21x^2 - 7x + 6x - 2$
$=$ **$21x^2 - x - 2$**
(g) $(9x - 2)(x + 6) = 9x^2 + 54x - 2x - 12$
$=$ **$9x^2 + 52x - 12$**

**(h)** $(3x - 8)(2x - 8) = 6x^2 - 24x - 16x + 64$
$= \mathbf{6x^2 - 40x + 64}$

**(i)** $(8 + 3x)(6 + 7x) = 48 + 56x + 18x + 21x^2$
$= \mathbf{48 + 74x + 21x^2}$

**(j)** $(10 + x)(5 - 10x) = 50 - 100x + 5x - 10x^2$
$= \mathbf{50 - 95x - 10x^2}$

**(k)** $(7 - 5x)(4 + 5x) = 28 + 35x - 20x - 25x^2$
$= \mathbf{28 + 15x - 25x^2}$

**(l)** $(12 - 7x)(9 - 2x) = 108 - 24x - 63x + 14x^2$
$= \mathbf{108 - 87x + 14x^2}$

**6.** **(a)** $(6x + 1)(x - 5) + 3x(8x - 3)$
$= (6x^2 - 30x + x - 5) + (24x^2 - 9x)$
$= (6x^2 - 29x - 5) + (24x^2 - 9x)$
$= 6x^2 - 29x - 5 + 24x^2 - 9x$
$= \mathbf{30x^2 - 38x - 5}$

**(b)** $(7x - 3)(4x - 9) - 2(4x + 5)(2x + 5)$
$= (28x^2 - 63x - 12x + 27) - 2(8x^2 + 20x + 10x + 25)$
$= (28x^2 - 75x + 27) - 2(8x^2 + 30x + 25)$
$= 28x^2 - 75x + 27 - 16x^2 - 60x - 50$
$= \mathbf{12x^2 - 135x - 23}$

**7.** **(a)** $(x + 2y)(x + y) = x^2 + xy + 2xy + 2y^2$
$= \mathbf{x^2 + 3xy + 2y^2}$

**(b)** $(x + 5y)(x - y) = x^2 - xy + 5xy - 5y^2$
$= \mathbf{x^2 + 4xy - 5y^2}$

**(c)** $(x - y)(8x + y) = 8x^2 + xy - 8xy - y^2$
$= \mathbf{8x^2 - 7xy - y^2}$

**(d)** $(x - 4y)(x - 5y) = x^2 - 5xy - 4xy + 20y^2$
$= \mathbf{x^2 - 9xy + 20y^2}$

**(e)** $(4x + 7y)(10x + 9y) = 40x^2 + 36xy + 70xy + 63y^2$
$= \mathbf{40x^2 + 106xy + 63y^2}$

**(f)** $(3x + 7y)(3x - 8y) = 9x^2 - 24xy + 21xy - 56y^2$
$= \mathbf{9x^2 - 3xy - 56y^2}$

**(g)** $(8x - 5y)(6x + 5y) = 48x^2 + 40xy - 30xy - 25y^2$
$= \mathbf{48x^2 + 10xy - 25y^2}$

**(h)** $(11x - 6y)(12x - y) = 132x^2 - 11xy - 72xy + 6y^2$
$= \mathbf{132x^2 - 83xy + 6y^2}$

**(i)** $(3y + 8x)(4y + 5x) = 12y^2 + 15xy + 32xy + 40x^2$
$= \mathbf{12y^2 + 47xy + 40x^2}$

**(j)** $(2y + 9x)(5y - 6x) = 10y^2 - 12xy + 45xy - 54x^2$
$= \mathbf{10y^2 + 33xy - 54x^2}$

**(k)** $(6y - 7x)(12y + x) = 72y^2 + 6xy - 84xy - 7x^2$
$= \mathbf{72y^2 - 78xy - 7x^2}$

**(l)** $(8y - 5x)(8y - 3x) = 64y^2 - 24xy - 40xy + 15x^2$
$= \mathbf{64y^2 - 64xy + 15x^2}$

**8.** **(a)** $(9x + 4y)(x - 4y) + (8x - 3y)(x + y)$
$= (9x^2 - 36xy + 4xy - 16y^2) + (8x^2 + 8xy - 3xy - 3y^2)$
$= (9x^2 - 32xy - 16y^2) + (8x^2 + 5xy - 3y^2)$
$= 9x^2 - 32xy - 16y^2 + 8x^2 + 5xy - 3y^2$
$= \mathbf{17x^2 - 27xy - 19y^2}$

**(b)** $4x(10y - 3x) - 3(5x - y)(7y - 2x)$
$= (40xy - 12x^2) - 3(35xy - 10x^2 - 7y^2 + 2xy)$
$= (40xy - 12x^2) - 3(-10x^2 + 37xy - 7y^2)$
$= 40xy - 12x^2 + 30x^2 - 111xy + 21y^2$
$= \mathbf{18x^2 - 71xy + 21y^2}$

**9.** **(a)** $(x + 4)(x^2 + 8) = x^3 + 8x + 4x^2 + 32$
$= \mathbf{x^3 + 4x^2 + 8x + 32}$

**(b)** $(x^2 + 4)(x - 8) = \mathbf{x^3 - 8x^2 + 4x - 32}$

**(c)** $(2x + 7y)(7x - 2y - 6)$
$= 14x^2 - 4xy - 12x + 49xy - 14y^2 - 42y$
$= \mathbf{14x^2 - 12x + 45xy - 42y - 14y^2}$

**(d)** $(10x - y)(9 - 5x + 4y)$
$= 90x - 50x^2 + 40xy - 9y + 5xy - 4y^2$
$= \mathbf{-50x^2 + 90x + 45xy - 9y - 4y^2}$

**(e)** $(9x + 5)(x^2 + x + 7) = 9x^3 + 9x^2 + 63x + 5x^2 + 5x + 35$
$= \mathbf{9x^3 + 14x^2 + 68x + 35}$

**(f)** $(6y^2 - y + 3)(2y - 1) = 12y^3 - 6y^2 - 2y^2 + y + 6y - 3$
$= \mathbf{12y^3 - 8y^2 + 7y - 3}$

**10.** **(a)** $(5x + 3)(2x - h) = 10x^2 - 5hx + 6x - 3h$
$= 10x^2 + (6 - 5h)x - 3h$
Comparing coefficients of $x$,
$k = 6 - 5h$ — (1)
Comparing constants,
$k = -3h$ — (2)
$\therefore$ The equations are $\mathbf{k = 6 - 5h}$ and $\mathbf{k = -3h}$.

**(b)** Substitute (1) into (2):
$6 - 5h = -3h$
$2h = 6$
$h = 3$
Substitute $h = 3$ into (2):
$k = -3(3)$
$= -9$
$\therefore$ $\mathbf{h = 3, k = -9}$

## Challenge Myself!

**11.** Let $a = 1$, $b = 2$, $c = 3$ and $d = -4$.
$(ax + b)(cx + d) = (x + 2)(3x - 4)$
$= 3x^2 - 4x + 6x - 8$
$= 3x^2 + 2x - 8$
Comparing coefficients of $x^2$,
$p = 3$
Comparing coefficients of $x$,
$q = 2$
Comparing constants,
$r = -8$
$\therefore$ A possible set of values is $\mathbf{a = 1, b = 2, c = 3}$ $\mathbf{d = -4, p = 3,}$ $\mathbf{q = 2}$ **and** $\mathbf{r = -8}$.

## Worksheet 4D Factorisation of quadratic expressions

**1.** **(a)** $5x + 20 = \mathbf{5(x + 4)}$
**(b)** $14x - 7 = \mathbf{7(2x - 1)}$
**(c)** $16 - 12x = \mathbf{4(4 - 3x)}$
**(d)** $-9x - 3 = \mathbf{-3(3x + 1)}$
**(e)** $10x^2 + 11x = \mathbf{x(10x + 11)}$
**(f)** $25x^2 - 40x = \mathbf{5x(5x - 8)}$
**(g)** $axy - 6ax^2 = \mathbf{ax(y - 6x)}$
**(h)** $-28x^2y - 32xy^2 = \mathbf{-4xy(7x + 8y)}$

**2.** **(a)** $x^2 + 5x + 4 = \mathbf{(x + 1)(x + 4)}$
**(b)** $x^2 + 8x + 15 = \mathbf{(x + 3)(x + 5)}$
**(c)** $x^2 - 9x + 8 = \mathbf{(x - 1)(x - 8)}$
**(d)** $x^2 - 7x + 12 = \mathbf{(x - 3)(x - 4)}$
**(e)** $x^2 + 10x - 11 = \mathbf{(x - 1)(x + 11)}$
**(f)** $x^2 + 6x - 16 = \mathbf{(x - 2)(x + 8)}$
**(g)** $x^2 - x - 6 = \mathbf{(x + 2)(x - 3)}$

(h) $x^2 - 2x - 48 = \mathbf{(x + 6)(x - 8)}$

3. (a) $5x^2 + 8x + 3 = \mathbf{(5x + 3)(x + 1)}$
(b) $14x^2 + 51x + 27 = \mathbf{(14x + 9)(x + 3)}$
(c) $11x^2 - 23x + 12 = \mathbf{(11x - 12)(x - 1)}$
(d) $4x^2 - 29x + 30 = \mathbf{(4x - 5)(x - 6)}$
(e) $10x^2 + 11x - 6 = \mathbf{(5x - 2)(2x + 3)}$
(f) $8x^2 + 7x - 51 = \mathbf{(8x - 17)(x + 3)}$
(g) $3x^2 - 10x - 8 = \mathbf{(3x + 2)(x - 4)}$
(h) $24x^2 - 98x - 45 = \mathbf{(12x + 5)(2x - 9)}$

4. (a) $3x^2 + 21x + 30 = 3(x^2 + 7x + 10)$
$= \mathbf{3(x + 2)(x + 5)}$
(b) $8x^2 + 2x - 28 = 2(4x^2 + x - 14)$
$= \mathbf{2(4x - 7)(x + 2)}$
(c) $15x^2 - 70x + 80 = 5(3x^2 - 14x + 16)$
$= \mathbf{5(3x - 8)(x - 2)}$
(d) $-x^2 + x + 56 = -(x^2 - x - 56)$
$= \mathbf{-(x - 8)(x + 7)}$
(e) $-9x^2 - 3x + 72 = -3(3x^2 + x - 24)$
$= \mathbf{-3(3x - 8)(x + 3)}$
(f) $84 + 16x - 4x^2 = -4(x^2 - 4x - 21)$
$= \mathbf{-4(x - 7)(x + 3)}$
(g) $2ax^2 - 21ax + 10a = a(2x^2 - 21x + 10)$
$= \mathbf{a(2x - 1)(x - 10)}$
(h) $3x^3 + 7x^2 - 20x = x(3x^2 + 7x - 20)$
$= \mathbf{x(3x - 5)(x + 4)}$

5. (a) $x^2 + 5xy + 6y^2 = \mathbf{(x + 2y)(x + 3y)}$
(b) $x^2 - 5xy + 6y^2 = \mathbf{(x - 2y)(x - 3y)}$
(c) $x^2 + 5xy - 6y^2 = \mathbf{(x - y)(x + 6y)}$
(d) $x^2 - 5xy - 6y^2 = \mathbf{(x + y)(x - 6y)}$
(e) $8x^2 + 6xy + y^2 = \mathbf{(4x + y)(2x + y)}$
(f) $x^2 - 14xy + 33y^2 = \mathbf{(x - 3y)(x - 11y)}$
(g) $7x^2 + 27xy - 4y^2 = \mathbf{(7x - y)(x + 4y)}$
(h) $6x^2 - 7xy - 20y^2 = \mathbf{(3x + 4y)(2x - 5y)}$
(i) $24x^2 + 112xy + 18y^2 = 2(12x^2 + 56xy + 9y^2)$
$= \mathbf{2(6x + y)(2x + 9y)}$
(j) $60x^2 - 148xy + 56y^2 = 4(15x^2 - 37xy + 14y^2)$
$= \mathbf{4(15x - 7y)(x - 2y)}$
(k) $-x^2 - 13xy + 68y^2 = -(x^2 + 13xy - 68y^2)$
$= \mathbf{-(x - 4y)(x + 17y)}$
(l) $45x^2 - 27xy - 18y^2 = 9(5x^2 - 3xy - 2y^2)$
$= \mathbf{9(5x + 2y)(x - y)}$
(m) $16x^2y^2 - 2xy - 3 = \mathbf{(8xy + 3)(2xy - 1)}$
(n) $21 - 51xy + 30x^2y^2 = 3(7 - 17xy + 10x^2y^2)$
$= \mathbf{3(7 - 10xy)(1 - xy)}$
(o) $30ax^2 - 35axy + 10ay^2 = 5a(6x^2 - 7xy + 2y^2)$
$= \mathbf{5a(3x - 2y)(2x - y)}$
(p) $-44x^2y^3 - 137xy^2 - 44y = -y(44x^2y^2 + 137xy + 44)$
$= \mathbf{-y(4xy + 11)(11xy + 4)}$

6. (a) $3x^2 - 16x - 12 = \mathbf{(3x + 2)(x - 6)}$
(b) $3(2y - 1)^2 - 16(2y - 1) - 12$
$= [3(2y - 1) + 2][(2y - 1) - 6]$
$= (6y - 3 + 2)(2y - 1 - 6)$
$= \mathbf{(6y - 1)(2y - 7)}$

7. (a) $8x^2 - 50x + 63 = \mathbf{(4x - 7)(2x - 9)}$
(b) $8(3y^2 + 2)^2 - 150y^2 - 37$
$= 8(3y^2 + 2)^2 - 50(3y^2 + 2) + 100 - 37$
$= 8(3y^2 + 2)^2 - 50(3y^2 + 2) + 63$
$= [4(3y^2 + 2) - 7][2(3y^2 + 2) - 9]$
$= (12y^2 + 8 - 7)(6y^2 + 4 - 9)$
$= \mathbf{(12y^2 + 1)(6y^2 - 5)}$

8. (a) $2a^2 + 3ab - 20b^2 = \mathbf{(2a - 5b)(a + 4b)}$
(b) $2(7x + 2y)^2 + 21xz + 6yz - 20z^2$
$= 2(7x + 2y)^2 + 3(7x + 2y)(z) - 20z^2$
$= [2(7x + 2y) - 5z][(7x + 2y) + 4z]$
$= \mathbf{(14x + 4y - 5z)(7x + 2y + 4z)}$

9. The product of the base and height of the triangle should be an expression with $50x^2$, not one with $100x^2$.

10. (i) Total surface area of a cube with sides of length $(ax + 9)$ cm will have a constant term $6(9)^2 = 486$, not 54.
(ii) Total surface area $= (24x^2 - 72x + 54)$ cm²
$= 6(4x^2 - 12x + 9)$ cm²
$= 6(2x - 3)^2$ cm²
$\therefore$ Length of each side = **(2x – 3) cm**

## Worksheet 4E Factorisation of algebraic expressions in the form $(a + b)(c + d)$

1. (a) $ab + 2a + 4b + 8 = a(b + 2) + 4(b + 2)$
$= \mathbf{(b + 2)(a + 4)}$
(b) $2ab + ac + 6b + 3c = a(2b + c) + 3(2b + c)$
$= \mathbf{(2b + c)(a + 3)}$
(c) $5ac + 5ad + 8bc + 8bd = 5a(c + d) + 8b(c + d)$
$= \mathbf{(c + d)(5a + 8b)}$
(d) $7ab + 21ac + 2bd + 6cd = 7a(b + 3c) + 2d(b + 3c)$
$= \mathbf{(b + 3c)(7a + 2d)}$
(e) $ab - 2a + 3b - 6 = a(b - 2) + 3(b - 2)$
$= \mathbf{(b - 2)(a + 3)}$
(f) $36ab + 9bc - 4a - c = 9b(4a + c) - (4a + c)$
$= \mathbf{(4a + c)(9b - 1)}$
(g) $7bc - 28ac - 3b + 12a = 7c(b - 4a) - 3(b - 4a)$
$= \mathbf{(b - 4a)(7c - 3)}$
(h) $20ac - 6ad - 10bc + 3bd = 2a(10c - 3d) - b(10c - 3d)$
$= \mathbf{(10c - 3d)(2a - b)}$
(i) $8abc + 2 + ac + 16b = 8abc + ac + 16b + 2$
$= ac(8b + 1) + 2(8b + 1)$
$= \mathbf{(8b + 1)(ac + 2)}$
(j) $15ab + cd + 5ac + 3bd = 15ab + 5ac + 3bd + cd$
$= 5a(3b + c) + d(3b + c)$
$= \mathbf{(3b + c)(5a + d)}$
(k) $32ad - 15bc - 24cd + 20ab = 32ad - 24cd + 20ab - 15bc$
$= 8d(4a - 3c) + 5b(4a - 3c)$
$= \mathbf{(4a - 3c)(8d + 5b)}$
(l) $-21ab - 20cd + 6ac + 70bd$
$= 70bd - 21ab - 20cd + 6ac$
$= 7b(10d - 3a) - 2c(10d - 3a)$
$= \mathbf{(10d - 3a)(7b - 2c)}$

2. (a) $xy + x + 8y^2 + 8y = x(y + 1) + 8y(y + 1)$
$= \mathbf{(y + 1)(x + 8y)}$
(b) $2x^2 + 8x - 3xy - 12y = 2x(x + 4) - 3y(x + 4)$
$= \mathbf{(x + 4)(2x - 3y)}$

(c) $12x^2y - 3xy + 4x - 1 = 3xy(4x - 1) + (4x - 1)$
$= \mathbf{(4x - 1)(3xy + 1)}$

(d) $6z^2 + 5xy - 30z - xyz = 6z^2 - xyz - 30z + 5xy$
$= z(6z - xy) - 5(6z - xy)$
$= \mathbf{(6z - xy)(z - 5)}$

(e) $30xy - 24xz + 105y^2 - 84yz$
$= 3(10xy - 8xz + 35y^2 - 28yz)$
$= 3[2x(5y - 4z) + 7y(5y - 4z)]$
$= \mathbf{3(5y - 4z)(2x + 7y)}$

(f) $84xz + 7y^3 - 21xy^2 - 28yz = 84xz - 21xy^2 - 28yz + 7y^3$
$= 7(12xz - 3xy^2 - 4yz + y^3)$
$= 7[3x(4z - y^2) - y(4z - y^2)]$
$= \mathbf{7(4z - y^2)(3x - y)}$

(g) $x^2y^3z - y^3z^2 - x^2yz^2 + yz^3 = yz(x^2y^2 - y^2z - x^2z + z^2)$
$= yz[y^2(x^2 - z) - z(x^2 - z)]$
$= \mathbf{yz(x^2 - z)(y^2 - z)}$

(h) $6xy^3z - 18x^2y^2z - 54x^2y^3 + 2xy^2z^2$
$= 6xy^3z - 54x^2y^3 + 2xy^2z^2 - 18x^2y^2z$
$= 2xy^2(3yz - 27xy + z^2 - 9xz)$
$= 2xy^2[3y(z - 9x) + z(z - 9x)]$
$= \mathbf{2xy^2(z - 9x)(3y + z)}$

3. $4hxy - \frac{2}{3}hyz - 18hxz + 3hz^2 = \frac{1}{3}h(12xy - 2yz - 54xz + 9z^2)$
$= \frac{1}{3}h(6x - z)(2y - 9z)$

Since $6x - z = -(z - 6x)$ and $2y - 9z = -(9z - 2y)$, then

$4hxy - \frac{2}{3}hyz - 18hxz + 3hz^2 = \frac{1}{3}h[-(z - 6x)][-(9z - 2y)]$
$= \frac{1}{3}h(z - 6x)(9z - 2y)$

Alvin's and Zach's expressions are equivalent.
∴ Both Alvin and Zach are correct.

## Challenge Myself!

4. (i) $(3p + 2)(3p - 2) = 9p^2 - 6p + 6p - 4$
$= \mathbf{9p^2 - 4}$

(ii) $126a^2kn^2 + 54ahn^2 - 24ah - 56a^2k$
$= 54ahn^2 + 126a^2kn^2 - 24ah - 56a^2k$
$= 2a(27hn^2 + 63akn^2 - 12h - 28ak)$
$= 2a[9n^2(3h + 7ak) - 4(3h + 7ak)]$
$= 2a(3h + 7ak)(9n^2 - 4)$
$= 2a(3h + 7ak)(3n + 2)(3n - 2)$
∴ The factors are **2, *a*, 3*h* + 7*ak*, 3*n* + 2 and 3*n* – 2**.

## Worksheet 4F Expansion using special algebraic identities

1. (a) $(a + 5)^2 = a^2 + 2(a)(5) + 5^2$
$= \mathbf{a^2 + 10a + 25}$

(b) $(9 + b)^2 = 9^2 + 2(9)(b) + b^2$
$= \mathbf{81 + 18b + b^2}$

(c) $(3c + 1)^2 = (3c)^2 + 2(3c)(1) + 1^2$
$= \mathbf{9c^2 + 6c + 1}$

(d) $(7d + 4)^2 = (7d)^2 + 2(7d)(4) + 4^2$
$= \mathbf{49d^2 + 56d + 16}$

(e) $(h + 6k)^2 = h^2 + 2h(6k) + (6k)^2$
$= \mathbf{h^2 + 12hk + 36k^2}$

(f) $(8h + k)^2 = (8h)^2 + 2(8h)(k) + k^2$
$= \mathbf{64h^2 + 16hk + k^2}$

(g) $(5m + 2n)^2 = (5m)^2 + 2(5m)(2n) + (2n)^2$
$= \mathbf{25m^2 + 20mn + 4n^2}$

(h) $(3m + 10n)(10n + 3m) = (3m + 10n)^2$
$= (3m)^2 + 2(3m)(10n) + (10n)^2$
$= \mathbf{9m^2 + 60mn + 100n^2}$

(i) $\left(2p + \frac{1}{4}\right)^2 = (2p)^2 + 2(2p)\left(\frac{1}{4}\right) + \left(\frac{1}{4}\right)^2$
$= \mathbf{4p^2 + p + \frac{1}{16}}$

(j) $\left(\frac{1}{4}q + 2\right)^2 = \left(\frac{1}{4}q\right)^2 + 2\left(\frac{1}{4}q\right)(2) + 2^2$
$= \mathbf{\frac{1}{16}q^2 + q + 4}$

(k) $\left(8p + \frac{3}{2}q\right)^2 = (8p)^2 + 2(8p)\left(\frac{3}{2}q\right) + \left(\frac{3}{2}q\right)^2$
$= \mathbf{64p^2 + 24pq + \frac{9}{4}q^2}$

(l) $\left(\frac{2}{5}p + 10q\right)^2 = \left(\frac{2}{5}p\right)^2 + 2\left(\frac{2}{5}p\right)(10q) + (10q)^2$
$= \mathbf{\frac{4}{25}p^2 + 8pq + 100q^2}$

(m) $(xy + 4)^2 = (xy)^2 + 2(xy)(4) + 4^2$
$= \mathbf{x^2y^2 + 8xy + 16}$

(n) $(6 + 5xy)^2 = 6^2 + 2(6)(5xy) + (5xy)^2$
$= \mathbf{36 + 60xy + 25x^2y^2}$

(o) $\left(\frac{1}{6}xy + 3z\right)^2 = \left(\frac{1}{6}xy\right)^2 + 2\left(\frac{1}{6}xy\right)(3z) + (3z)^2$
$= \mathbf{\frac{1}{36}x^2y^2 + xyz + 9z^2}$

(p) $\left(\frac{3}{8}x + \frac{4}{5}yz\right)^2 = \left(\frac{3}{8}x\right)^2 + 2\left(\frac{3}{8}x\right)\left(\frac{4}{5}yz\right) + \left(\frac{4}{5}yz\right)^2$
$= \mathbf{\frac{9}{64}x^2 + \frac{3}{5}xyz + \frac{16}{25}y^2z^2}$

2. (a) $(a - 8)^2 = a^2 - 2(a)(8) + 8^2$
$= \mathbf{a^2 - 16a + 64}$

(b) $(11 - b)^2 = 11^2 - 2(11)(b) + b^2$
$= \mathbf{121 - 22b + b^2}$

(c) $(4c - 1)^2 = (4c)^2 - 2(4c)(1) + 1^2$
$= \mathbf{16c^2 - 8c + 1}$

(d) $(7 - 2d)^2 = 7^2 - 2(7)(2d) + (2d)^2$
$= \mathbf{49 - 28d + 4d^2}$

(e) $(h - 5k)^2 = h^2 - 2(h)(5k) + (5k)^2$
$= \mathbf{h^2 - 10hk + 25k^2}$

(f) $(9h - k)^2 = (9h)^2 - 2(9h)(k) + k^2$
$= \mathbf{81h^2 - 18hk + k^2}$

(g) $(3m - 10n)^2 = (3m)^2 - 2(3m)(10n) + (10n)^2$
$= \mathbf{9m^2 - 60mn + 100n^2}$

(h) $(6m - 5n)(5n - 6m) = -(6m - 5n)^2$
$= -[(6m)^2 - 2(6m)(5n) + (5n)^2]$
$= -(36m^2 - 60mn + 25n^2)$
$= \mathbf{-36m^2 + 60mn - 25n^2}$

(i) $\left(2p - \frac{1}{3}\right)^2 = (2p)^2 - 2(2p)\left(\frac{1}{3}\right) + \left(\frac{1}{3}\right)^2$
$= \mathbf{4p^2 - \frac{4}{3}p + \frac{1}{9}}$

(j) $\left(\frac{1}{3}p - 2\right)^2 = \left(\frac{1}{3}p\right)^2 - 2\left(\frac{1}{3}p\right)(2) + 2^2$

$= \mathbf{\frac{1}{9}p^2 - \frac{4}{3}p + 4}$

(k) $\left(7p - \frac{5}{6}q\right)^2 = (7p)^2 - 2(7p)\left(\frac{5}{6}q\right) + \left(\frac{5}{6}q\right)^2$

$= \mathbf{49p^2 - \frac{35}{3}pq + \frac{25}{36}q^2}$

(l) $\left(\frac{1}{10}p - 10q\right)^2 = \left(\frac{1}{10}p\right)^2 - 2\left(\frac{1}{10}p\right)(10q) + (10q)^2$

$= \mathbf{\frac{1}{100}p^2 - 2pq + 100q^2}$

(m) $(xy - 9)^2 = (xy)^2 - 2(xy)(9) + 9^2$

$= \mathbf{x^2y^2 - 18xy + 81}$

(n) $(3 - 8xy)^2 = 3^2 - 2(3)(8xy) + (8xy)^2$

$= \mathbf{9 - 48xy + 64x^2y^2}$

(o) $\left(\frac{1}{2}xy - 4z\right)^2 = \left(\frac{1}{2}xy\right)^2 - 2\left(\frac{1}{2}xy\right)(4z) + (4z)^2$

$= \mathbf{\frac{1}{4}x^2y^2 - 4xyz + 16z^2}$

(p) $\left(\frac{4}{5}x - \frac{5}{6}yz\right)^2 = \left(\frac{4}{5}x\right)^2 - 2\left(\frac{4}{5}x\right)\left(\frac{5}{6}yz\right) + \left(\frac{5}{6}yz\right)^2$

$= \mathbf{\frac{16}{25}x^2 - \frac{4}{3}xyz + \frac{25}{36}y^2z^2}$

3. (a) $(a + 9)(a - 9) = a^2 - 9^2$

$= \mathbf{a^2 - 81}$

(b) $(5 + b)(5 - b) = 5^2 - b^2$

$= \mathbf{25 - b^2}$

(c) $(6c + 1)(6c - 1) = (6c)^2 - 1^2$

$= \mathbf{36c^2 - 1}$

(d) $(8 + 3d)(8 - 3d) = 8^2 - (3d)^2$

$= \mathbf{64 - 9d^2}$

(e) $(h + 4k)(h - 4k) = h^2 - (4k)^2$

$= \mathbf{h^2 - 16k^2}$

(f) $(9h - k)(9h + k) = (9h)^2 - k^2$

$= \mathbf{81h^2 - k^2}$

(g) $(7m + 2n)(7m - 2n) = (7m)^2 - (2n)^2$

$= \mathbf{49m^2 - 4n^2}$

(h) $(4n - 11m)(11m + 4n) = (4n - 11m)(4n + 11m)$

$= (4n)^2 - (11m)^2$

$= \mathbf{16n^2 - 121m^2}$

(i) $\left(10p + \frac{1}{2}\right)\left(10p - \frac{1}{2}\right) = (10p)^2 - \left(\frac{1}{2}\right)^2$

$= \mathbf{100p^2 - \frac{1}{4}}$

(j) $\left(\frac{1}{5}p + 3\right)\left(3 - \frac{1}{5}p\right) = 3^2 - \left(\frac{1}{5}p\right)^2$

$= \mathbf{9 - \frac{1}{25}p^2}$

(k) $\left(9p + \frac{2}{3}q\right)\left(9p - \frac{2}{3}q\right) = (9p)^2 - \left(\frac{2}{3}q\right)^2$

$= \mathbf{81p^2 - \frac{4}{9}q^2}$

(l) $\left(\frac{3}{4}p - 4q\right)\left(4q + \frac{3}{4}p\right) = \left(\frac{3}{4}p - 4q\right)\left(\frac{3}{4}p + 4q\right)$

$= \left(\frac{3}{4}p\right)^2 - (4q)^2$

$= \mathbf{\frac{9}{16}p^2 - 16q^2}$

(m) $(xy + 12)(xy - 12) = (xy)^2 - 12^2$

$= \mathbf{x^2y^2 - 144}$

(n) $(7 - 5xy)(5xy + 7) = (7 - 5xy)(7 + 5xy)$

$= 7^2 - (5xy)^2$

$= \mathbf{49 - 25x^2y^2}$

(o) $\left(\frac{1}{4}xy + z\right)\left(\frac{1}{4}xy - z\right) = \left(\frac{1}{4}xy\right)^2 - z^2$

$= \mathbf{\frac{1}{16}x^2y^2 - z^2}$

(p) $\left(\frac{3}{5}x - \frac{2}{3}yz\right)\left(\frac{2}{3}yz + \frac{3}{5}x\right) = \left(\frac{3}{5}x - \frac{2}{3}yz\right)\left(\frac{3}{5}x + \frac{2}{3}yz\right)$

$= \left(\frac{3}{5}x\right)^2 - \left(\frac{2}{3}yz\right)^2$

$= \mathbf{\frac{9}{25}x^2 - \frac{4}{9}y^2z^2}$

4. (a) $6x^2 + (x + 2)^2 = 6x^2 + (x^2 + 4x + 4)$

$= \mathbf{7x^2 + 4x + 4}$

(b) $(3x - 7)^2 + 49 = (9x^2 - 42x + 49) + 49$

$= \mathbf{9x^2 - 42x + 98}$

(c) $(4 + 9x)^2 - (-4 - 9x)^2 = (4 + 9x)^2 - (4 + 9x)^2$

$= \mathbf{0}$

(d) $(x - y + z)(x - y - z) = [(x - y) + z][(x - y) - z]$

$= (x - y)^2 - z^2$

$= \mathbf{x^2 - 2xy + y^2 - z^2}$

5. Paul seems to be unsure as to whether $b$ is $q$ or $-q$, and substituted $b$ as $-q$ in the second term of the expansion of $(px - q)^2$.

$(px - q)^2 = (px)^2 - 2(px)(q) + q^2$

$= \mathbf{p^2x^2 - 2pqx + q^2}$

6. (a) $(a + b)^2 = a^2 + 2ab + b^2$

$= (a^2 + b^2) + 2ab$

$= 6 + 2(3)$

$= \mathbf{12}$

(b) $(a - b)^2 = a^2 - 2ab + b^2$

$= (a^2 + b^2) - 2ab$

$= 6 - 2(3)$

$= \mathbf{0}$

7. (a) $4(h + k)^2 = 4(h^2 + 2hk + k^2)$

$= 4[(h^2 + k^2) + 2hk]$

$= 4[20 + 2(11)]$

$= \mathbf{168}$

(b) $(4h - 4k)^2 = 16(h - k)^2$

$= 16(h^2 - 2hk + k^2)$

$= 16[(h^2 + k^2) - 2hk]$

$= 16[20 - 2(11)]$

$= \mathbf{-32}$

8. Since $2m^2 + 2n^2 = 14$ and $5mn = 12$,

then $m^2 + n^2 = 7$ and $mn = \frac{12}{5}$.

**(a)** $(m + n)^2 = m^2 + 2mn + n^2$
$= (m^2 + n^2) + 2mn$
$= 7 + 2\left(\frac{12}{5}\right)$
$= \mathbf{11\frac{4}{5}}$

**(b)** $\left(\frac{1}{2}n - \frac{1}{2}m\right)^2 = \frac{1}{4}(m - n)^2$
$= \frac{1}{4}(m^2 - 2mn + n^2)$
$= \frac{1}{4}[(m^2 + n^2) - 2mn]$
$= \frac{1}{4}\left[7 - 2\left(\frac{12}{5}\right)\right]$
$= \mathbf{\frac{11}{20}}$

**9.** $\left(p + \frac{1}{p}\right)^2 = 43$
$p^2 + 2 + \frac{1}{p^2} = 43$
$p^2 + \frac{1}{p^2} = \mathbf{41}$

**10.** $(x + y)(x - y) = (6.5)(8)$
$x^2 - y^2 = 52$
$\frac{1}{3}x^2 - \frac{1}{3}y^2 = \mathbf{17\frac{1}{3}}$

**11.** **(i)** $(m + 1)(m - 1) = \boldsymbol{m^2 - 1}$
**(ii)** Replace $m$ with $5x + 4y$:
$(5x + 4y + 1)(5x + 4y - 1) = (5x + 4y)^2 - 1$
$= \boldsymbol{25x^2 + 40xy + 16y^2 - 1}$

**12.** $(9n + 2)^2 - (9n - 2)^2 = (81n^2 + 36n + 4) - (81n^2 - 36n + 4)$
$= 81n^2 + 36n + 4 - 81n^2 + 36n - 4$
$= 72n$
$= 24(3n)$, which is a multiple of 24 (shown)

**13.** **(i)** $x^2 - (x + a)(x - a) = x^2 - (x^2 - a^2)$
$= x^2 - x^2 + a^2$
$= \boldsymbol{a^2}$
**(ii)** $89\,013^2 - 89\,017 \times 89\,009$
$= 89\,013^2 - (89\,013 + 4)(89\,013 - 4)$
$= 4^2$
$= \mathbf{16}$

**14.** **(i)** $59.8^2 \approx 60^2 = 3600$
$\therefore$ His answer is not reasonable.
**(ii)** $59.8^2 = (60 - 0.2)^2$
$= 60^2 - 2(60)(0.2) + 0.2^2$
$= 3600 - 24 + 0.04$
$= 3576.04$

**15.** **(a)** $801^2 = (800 + 1)^2$
$= 800^2 + 2(800)(1) + 1^2$
$= 640\,000 + 1600 + 1$
$= \mathbf{641\,601}$
**(b)** $502^2 = (500 + 2)^2$
$= 500^2 + 2(500)(2) + 2^2$
$= 250\,000 + 2000 + 4$
$= \mathbf{252\,004}$
**(c)** $199^2 = (200 - 1)^2$
$= 200^2 - 2(200)(1) + 1^2$
$= 40\,000 - 400 + 1$
$= \mathbf{39\,601}$
**(d)** $497^2 = (500 - 3)^2$
$= 500^2 - 2(500)(3) + 3^2$
$= 250\,000 - 3000 + 9$
$= \mathbf{247\,009}$
**(e)** $305 \times 295 = (300 + 5)(300 - 5)$
$= 300^2 - 5^2$
$= 90\,000 - 25$
$= \mathbf{89\,975}$
**(f)** $1996 \times 2004 = (2000 - 4)(2000 + 4)$
$= 2000^2 - 4^2$
$= 4\,000\,000 - 16$
$= \mathbf{3\,999\,984}$

**16.** **(a)** $\frac{2021}{99^2 - 100 \times 98} = \frac{2021}{99^2 - (99 + 1)(99 - 1)}$
$= \frac{2021}{99^2 - (99^2 - 1)}$
$= \frac{2021}{99^2 - 99^2 + 1}$
$= \mathbf{2021}$
**(b)** $\frac{675^2 - 680 \times 670}{50} = \frac{675^2 - (675 + 5)(675 - 5)}{50}$
$= \frac{675^2 - (675^2 - 5^2)}{50}$
$= \frac{675^2 - 675^2 + 5^2}{50}$
$= \frac{5^2}{50}$
$= \frac{25}{50}$
$= \mathbf{\frac{1}{2}}$

## Challenge Myself!

**17.** **(a)** Given that $x + y = 18$ and $xy = 77$,
$(x + y)^2 = 18^2$
$x^2 + 2xy + y^2 = 324$
$x^2 + 2(77) + y^2 = 324$
$x^2 + y^2 = 170$
$(x - y)^2 = x^2 - 2xy + y^2$
$= 170 - 2(77)$
$= 16$
$x - y = \mathbf{\pm 4}$

(b) $x^2 - 6x + 1 = 0$

$x^2 + 1 = 6x$

$x + \frac{1}{x} = 6$

$\left(x + \frac{1}{x}\right)^2 = 36$

$x^2 + 2x\left(\frac{1}{x}\right) + \frac{1}{x^2} = 36$

$x^2 + 2 + \frac{1}{x^2} = 36$

$x^2 + \frac{1}{x^2} = 34$

$\left(x^2 + \frac{1}{x^2}\right)^2 = 1156$

$x^4 + 2x^2\left(\frac{1}{x^2}\right) + \frac{1}{x^4} = 1156$

$x^4 + 2 + \frac{1}{x^4} = 1156$

$x^4 + \frac{1}{x^4} = \mathbf{1154}$

## Worksheet 4G Factorisation using special algebraic identities

1. (a) $a^2 + 10a + 25 = a^2 + 2(a)(5) + 5^2$
$= \mathbf{(a + 5)^2}$

(b) $4 + 4b + b^2 = 2^2 + 2(2)(b) + b^2$
$= \mathbf{(2 + b)^2}$

(c) $9c^2 + 6c + 1 = (3c)^2 + 2(3c)(1) + 1^2$
$= \mathbf{(3c + 1)^2}$

(d) $49d^2 + 42d + 9 = (7d)^2 + 2(7d)(3) + 3^2$
$= \mathbf{(7d + 3)^2}$

(e) $h^2 + 8hk + 16k^2 = h^2 + 2(h)(4k) + (4k)^2$
$= \mathbf{(h + 4k)^2}$

(f) $81h^2 + 18hk + k^2 = (9h)^2 + 2(9h)(k) + k^2$
$= \mathbf{(9h + k)^2}$

(g) $25m^2 + 80mn + 64n^2 = (5m)^2 + 2(5m)(8n) + (8n)^2$
$= \mathbf{(5m + 8n)^2}$

(h) $121m^2 + 100n^2 + 220mn$
$= 121m^2 + 220mn + 100n^2$
$= (11m)^2 + 2(11m)(10n) + (10n)^2$
$= \mathbf{(11m + 10n)^2}$

(i) $4p^2q^2 + 4pq + 1 = (2pq)^2 + 2(2pq)(1) + 1^2$
$= \mathbf{(2pq + 1)^2}$

(j) $49 + 56pq + 16p^2q^2 = 7^2 + 2(7)(4pq) + (4pq)^2$
$= \mathbf{(7 + 4pq)^2}$

(k) $96xyz + 256x^2y^2z^2 + 9 = 256x^2y^2z^2 + 96xyz + 9$
$= (16xyz)^2 + 2(16xyz)(3) + 3^2$
$= \mathbf{(16xyz + 3)^2}$

(l) $25x^2z^2 + 10xy^2z + y^4 = (5xz)^2 + 2(5xz)(y^2) + (y^2)^2$
$= \mathbf{(5xz + y^2)^2}$

2. (a) $a^2 - 12a + 36 = a^2 - 2(a)(6) + 6^2$
$= \mathbf{(a - 6)^2}$

(b) $4 - 4b + b^2 = 2^2 - 2(2)(b) + b^2$
$= \mathbf{(2 - b)^2}$

(c) $100c^2 - 20c + 1 = (10c)^2 - 2(10c)(1) + 1^2$
$= \mathbf{(10c - 1)^2}$

(d) $25d^2 - 70d + 49 = (5d)^2 - 2(5d)(7) + 7^2$
$= \mathbf{(5d - 7)^2}$

(e) $h^2 - 16hk + 64k^2 = h^2 - 2(h)(8k) + (8k)^2$
$= \mathbf{(h - 8k)^2}$

(f) $900h^2 - 60hk + k^2 = (30h)^2 - 2(30h)(k) + k^2$
$= \mathbf{(30h - k)^2}$

(g) $81m^2 - 36mn + 4n^2 = (9m)^2 - 2(9m)(2n) + (2n)^2$
$= \mathbf{(9m - 2n)^2}$

(h) $196m^2 + 9n^2 - 84mn = 196m^2 - 84mn + 9n^2$
$= (14m)^2 - 2(14m)(3n) + (3n)^2$
$= \mathbf{(14m - 3n)^2}$

(i) $121p^2q^2 - 22pq + 1 = (11pq)^2 - 2(11pq)(1) + 1^2$
$= \mathbf{(11pq - 1)^2}$

(j) $25 - 60pq + 36p^2q^2 = 5^2 - 2(5)(6pq) + (6pq)^2$
$= \mathbf{(5 - 6pq)^2}$

(k) $4 + 225x^2y^2z^2 - 60xyz = 4 - 60xyz + 225x^2y^2z^2$
$= 2^2 - 2(2)(15xyz) + (15xyz)^2$
$= \mathbf{(2 - 15xyz)^2}$

(l) $16x^4z^2 - 24x^2y^2z + 9y^4 = (4x^2z)^2 - 2(4x^2z)(3y^2) + (3y^2)^2$
$= \mathbf{(4x^2z - 3y^2)^2}$

3. (a) $a^2 - 100 = a^2 - 10^2$
$= \mathbf{(a + 10)(a - 10)}$

(b) $81 - b^2 = 9^2 - b^2$
$= \mathbf{(9 + b)(9 - b)}$

(c) $49h^2 - 9 = (7h)^2 - 3^2$
$= \mathbf{(7h + 3)(7h - 3)}$

(d) $1 - 64k^2 = 1^2 - (8k)^2$
$= \mathbf{(1 + 8k)(1 - 8k)}$

(e) $16m^2 - 25n^2 = (4m)^2 - (5n)^2$
$= \mathbf{(4m + 5n)(4m - 5n)}$

(f) $121n^2 - 36m^2 = (11n)^2 - (6m)^2$
$= \mathbf{(11n + 6m)(11n - 6m)}$

(g) $p^2q^2 - 144 = (pq)^2 - 12^2$
$= \mathbf{(pq + 12)(pq - 12)}$

(h) $4 - 81p^2q^2 = 2^2 - (9pq)^2$
$= \mathbf{(2 + 9pq)(2 - 9pq)}$

(i) $9x^2 - 100y^2z^2 = (3x)^2 - (10yz)^2$
$= \mathbf{(3x + 10yz)(3x - 10yz)}$

(j) $225x^2y^4 - 49z^4 = (15xy^2)^2 - (7z^2)^2$
$= \mathbf{(15xy^2 + 7z^2)(15xy^2 - 7z^2)}$

4. $58^2 - 46^2 = (58 + 46)(58 - 46)$
$= 104(12)$
$\therefore k = \mathbf{104}$

5. (i) $x^2 - 441 = x^2 - 21^2$
$= \mathbf{(x + 21)(x - 21)}$

(ii) Since $5959 + 441 = 6400 = 80^2$,
let $x = 80$:
$5959 = 80^2 - 441$
$= (80 + 21)(80 - 21)$
$= 101 \times 59$
$\therefore$ Two factors of 5959 are **101** and **59**.

6. (a) $2x^2 + 20x + 50 = 2(x^2 + 10x + 25)$
$= 2[x^2 + 2(x)(5) + 5^2]$
$= \mathbf{2(x + 5)^2}$

(b) $160x^2 + 80xy + 10y^2 = 10(16x^2 + 8xy + y^2)$
$= 10[(4x)^2 + 2(4x)(y) + y^2]$
$= \mathbf{10(4x + y)^2}$

(c) $54ax^2 + 72ax + 24a = 6a(9x^2 + 12x + 4)$
$= 6a[(3x)^2 + 2(3x)(2) + 2^2]$
$= \mathbf{6a(3x + 2)^2}$

(d) $12bx^2 + 84bxy + 147by^2 = 3b(4x^2 + 28xy + 49y^2)$
$= 3b[(2x)^2 + 2(2x)(7y) + (7y)^2]$
$= \mathbf{3b(2x + 7y)^2}$

(e) $8x^2 - 16x + 8 = 8(x^2 - 2x + 1)$
$= 8[x^2 - 2(x)(1) + 1^2]$
$= \mathbf{8(x - 1)^2}$

(f) $45x^2 - 300xy + 500y^2 = 5(9x^2 - 60xy + 100y^2)$
$= 5[(3x)^2 - 2(3x)(10y) + (10y)^2]$
$= \mathbf{5(3x - 10y)^2}$

(g) $175hx^2 - 280hx + 112h = 7h(25x^2 - 40x + 16)$
$= 7h[(5x)^2 - 2(5x)(4) + 4^2]$
$= \mathbf{7h(5x - 4)^2}$

(h) $16kx^2y^2 - 72kxy + 81k = k(16x^2y^2 - 72xy + 81)$
$= k[(4xy)^2 - 2(4xy)(9) + 9^2]$
$= \mathbf{k(4xy - 9)^2}$

(i) $128x^2 - 98 = 2(64x^2 - 49)$
$= 2[(8x)^2 - 7^2]$
$= \mathbf{2(8x + 7)(8x - 7)}$

(j) $3x^2 - 108y^2 = 3(x^2 - 36y^2)$
$= 3[x^2 - (6y)^2]$
$= \mathbf{3(x + 6y)(x - 6y)}$

(k) $125px^2 - 20p = 5p(25x^2 - 4)$
$= 5p[(5x)^2 - 2^2]$
$= \mathbf{5p(5x + 2)(5x - 2)}$

(l) $81qx - 144qx^3y^2 = 9qx(9 - 16x^2y^2)$
$= 9qx[3^2 - (4xy)^2]$
$= \mathbf{9qx(3 + 4xy)(3 - 4xy)}$

7. (a) $(a + 1)^2 - 16 = (a + 1)^2 - 4^2$
$= (a + 1 + 4)(a + 1 - 4)$
$= \mathbf{(a + 5)(a - 3)}$

(b) $(5b + 3)^2 - b^2 = (5b + 3 + b)(5b + 3 - b)$
$= (6b + 3)(4b + 3)$
$= \mathbf{3(4b + 3)(2b + 1)}$

(c) $9 - (h + 2)^2 = 3^2 - (h + 2)^2$
$= [3 + (h + 2)][3 - (h + 2)]$
$= (3 + h + 2)(3 - h - 2)$
$= \mathbf{(5 + h)(1 - h)}$

(d) $4k^2 - (3k - 7)^2 = (2k)^2 - (3k - 7)^2$
$= [2k + (3k - 7)][2k - (3k - 7)]$
$= (2k + 3k - 7)(2k - 3k + 7)$
$= \mathbf{(5k - 7)(7 - k)}$

(e) $(5p + 8)^2 - (2p + 7)^2$
$= [(5p + 8) + (2p + 7)][(5p + 8) - (2p + 7)]$
$= (5p + 8 + 2p + 7)(5p + 8 - 2p - 7)$
$= \mathbf{(7p + 15)(3p + 1)}$

(f) $(6q - 1)^2 - 16(2q - 9)^2$
$= (6q - 1)^2 - (8q - 36)^2$
$= [(6q - 1) + (8q - 36)][(6q - 1) - (8q - 36)]$
$= (6q - 1 + 8q - 36)(6q - 1 - 8q + 36)$
$= \mathbf{(14q - 37)(35 - 2q)}$

(g) $10x^4 - 10 = 10(x^4 - 1)$
$= 10[(x^2)^2 - 1^2]$
$= 10(x^2 + 1)(x^2 - 1)$
$= \mathbf{10(x^2 + 1)(x + 1)(x - 1)}$

(h) $x^2 - 8x - 16y^2 + 16 = (x^2 - 8x + 16) - 16y^2$
$= [x^2 - 2(x)(4) + 4^2] - 16y^2$
$= (x - 4)^2 - (4y)^2$
$= (x - 4 + 4y)(x - 4 - 4y)$
$= \mathbf{(x + 4y - 4)(x - 4y - 4)}$

8. (a) $79^2 + 158 + 1 = 79^2 + 2(79)(1) + 1^2$
$= (79 + 1)^2$
$= 80^2$
$= \mathbf{6400}$

(b) $397^2 + 2382 + 9 = 397^2 + 2(397)(3) + 3^2$
$= (397 + 3)^2$
$= 400^2$
$= \mathbf{160\ 000}$

(c) $42^2 - 168 + 4 = 42^2 - 2(42)(2) + 2^2$
$= (42 - 2)^2$
$= 40^2$
$= \mathbf{1600}$

(d) $605^2 - 6050 + 25 = 605^2 - 2(605)(5) + 5^2$
$= (605 - 5)^2$
$= 600^2$
$= \mathbf{360\ 000}$

(e) $81^2 - 19^2 = (81 + 19)(81 - 19)$
$= (100)(62)$
$= \mathbf{6200}$

(f) $9.4^2 - 8.4^2 = (9.4 + 8.4)(9.4 - 8.4)$
$= (17.8)(1.0)$
$= \mathbf{17.8}$

(g) $9999^2 - 1 = 9999^2 - 1^2$
$= (9999 + 1)(9999 - 1)$
$= (10\ 000)(9998)$
$= \mathbf{99\ 980\ 000}$

(h) $0.23^2 - 1.77^2 = (0.23 + 1.77)(0.23 - 1.77)$
$= (2.00)(-1.54)$
$= \mathbf{-3.08}$

9. Total surface area $= (24x^2 - 120x + 150)\ \text{cm}^2$
$= 6(4x^2 - 20x + 25)\ \text{cm}^2$
$= 6[(2x)^2 - 2(2x)(5) + 5^2]\ \text{cm}^2$
$= 6(2x - 5)^2\ \text{cm}^2$

Area of one face $= (2x - 5)^2\ \text{cm}^2$
Length of side $= (2x - 5)$ cm
Volume of cube $= \mathbf{(2x - 5)^3\ cm^3}$

10. (a)
$$y^2 + \frac{1}{y^2} = 14$$
$$y^2 + 2 + \frac{1}{y^2} = 14 + 2$$
$$y^2 + 2(y)\left(\frac{1}{y}\right) + \left(\frac{1}{y}\right)^2 = 16$$
$$\left(y + \frac{1}{y}\right)^2 = 16$$
$$y + \frac{1}{y} = \pm 4$$

Since $y > 0$, then $y + \frac{1}{y} = \mathbf{4}$.

**(b)** $\left(y - \frac{1}{y}\right)^2 = y^2 - 2(y)\left(\frac{1}{y}\right) + \frac{1}{y^2}$
$= y^2 + \frac{1}{y^2} - 2$
$= 14 - 2$
$= 12$ (shown)

**11.** **(i)** $48^2 + 192 + 4 = 48^2 + 2(48)(2) + 2^2$
$= (48 + 2)^2$
$= 50^2$
$= 2500$

$53^2 - 318 + 9 = 53^2 - 2(53)(3) + 3^2$
$= (53 - 3)^2$
$= 50^2$
$= 2500$

$\therefore$ The values are the same.

**(ii)** $2500 = (54 - 4)^2$
$= 54^2 - 2(54)(4) + 4^2$
$= 54^2 - 432 + 16$

$\therefore$ A possible set of values is $\boldsymbol{h = 54}$ and $\boldsymbol{k = -432}$.

## Challenge Myself!

**12.** $x^3 - 8x^2y - 36xy + 16xy^2 + 144y^2$
$= x^3 - 8x^2y + 16xy^2 - 36xy + 144y^2$
$= x(x^2 - 8xy + 16y^2) - 36y(x - 4y)$
$= x(x - 4y)^2 - 36y(x - 4y)$
$= x(9)^2 - 36y(9)$
$= 81x - 324y$
$= 81(x - 4y)$
$= 81(9)$
$= \mathbf{729}$

## Review Exercise 4

**1.** **(a)** $(9a + 2b)^2 = (9a)^2 + 2(9a)(2b) + (2b)^2$
$= \boldsymbol{81a^2 + 36ab + 4b^2}$

**(b)** $\left(\frac{3}{4} - \frac{4}{3}hk\right)^2 = \left(\frac{3}{4}\right)^2 - 2\left(\frac{3}{4}\right)\left(\frac{4}{3}hk\right) + \left(\frac{4}{3}hk\right)^2$
$= \boldsymbol{\frac{9}{16} - 2hk + \frac{16}{9}h^2k^2}$

**(c)** $(5p + q)^2 - (5p - q)^2 + (5p + q)(5p - q)$
$= (25p^2 + 10pq + q^2) - (25p^2 - 10pq + q^2) + (25p^2 - q^2)$
$= 25p^2 + 10pq + q^2 - 25p^2 + 10pq - q^2 + 25p^2 - q^2$
$= \boldsymbol{25p^2 + 20pq - q^2}$

**(d)** $(x + y + z)^2 = [(x + y) + z]^2$
$= (x + y)^2 + 2(x + y)(z) + z^2$
$= x^2 + 2xy + y^2 + 2xz + 2yz + z^2$
$= \boldsymbol{x^2 + y^2 + z^2 + 2xy + 2yz + 2xz}$

**2.** Polynomial $= (7x^2 - 3x + 1)(5x - 4) + 2$
$= (35x^3 - 28x^2 - 15x^2 + 12x + 5x - 4) + 2$
$= \boldsymbol{35x^3 - 43x^2 + 17x - 2}$

**3.** **(a)** $6x^2 + 36ax = \boldsymbol{6x(x + 6a)}$

**(b)** $21bc^2 - 14b^3c = \boldsymbol{7bc(3c - 2b^2)}$

**(c)** $72m^2 + 110mn - 28n^2 = 2(36m^2 + 55mn - 14n^2)$
$= \boldsymbol{2(9m - 2n)(4m + 7n)}$

**(d)** $-48p^2q^2 - 106pq - 33 = -(48p^2q^2 + 106pq + 33)$
$= \boldsymbol{-(6pq + 11)(8pq + 3)}$

**(e)** $5hx + 6ky - 15hy - 2kx = 5hx - 15hy - 2kx + 6ky$
$= 5h(x - 3y) - 2k(x - 3y)$
$= \boldsymbol{(x - 3y)(5h - 2k)}$

**(f)** $16x(xy^2 - 5z) + 8z^2(xy^2 - 5z)$
$= 16x^2y^2 + 8xy^2z^2 - 80xz - 40z^3$
$= 8(2x^2y^2 + xy^2z^2 - 10xz - 5z^3)$
$= 8[xy^2(2x + z^2) - 5z(2x + z^2)]$
$= \boldsymbol{8(2x + z^2)(xy^2 - 5z)}$

**4.** **(a)** $98a^2 + 84ab + 18b^2 = 2(49a^2 + 42ab + 9b^2)$
$= 2[(7a)^2 + 2(7a)(3b) + (3b)^2]$
$= \boldsymbol{2(7a + 3b)^2}$

**(b)** $-100h^2n + 20hkn - k^2n = -n(100h^2 - 20kh + k^2)$
$= -n[(10h)^2 - 2(10h)(k) + k^2]$
$= \boldsymbol{-n(10h - k)^2}$

**(c)** $121(p - 1)^2 - 36p^2 = (11p - 11)^2 - (6p)^2$
$= (11p - 11 + 6p)(11p - 11 - 6p)$
$= \boldsymbol{(17p - 11)(5p - 11)}$

**(d)** $240xy + 225 - 400y^2 - 36x^2$
$= 225 - (36x^2 - 240xy + 400y^2)$
$= 225 - 4(9x^2 - 60xy + 100y^2)$
$= 225 - 4[(3x)^2 - 2(3x)(10y) + (10y)^2]$
$= 15^2 - 4(3x - 10y)^2$
$= 15^2 - (6x - 20y)^2$
$= [15 + (6x - 20y)][15 - (6x - 20y)]$
$= \boldsymbol{(15 + 6x - 20y)(15 - 6x + 20y)}$

**5.** **(a)** In $4(2x + 3)(x + 2)$, the coefficient of the term in $x^2$ is 8 ($\neq 6$) and the constant term is 24 ($\neq -18$).

**(b)** $6x^2 + 3x - 18 = 3(2x^2 + x - 6)$
$= \boldsymbol{3(2x - 3)(x + 2)}$

**6.** **(a)** $5x^2 + 3x - 8 = \boldsymbol{(5x + 8)(x - 1)}$

**(b)** $5(4y - 1)^2 + 3(4y - 1) - 8 = [5(4y - 1) + 8][(4y - 1) - 1]$
$= (20y - 5 + 8)(4y - 1 - 1)$
$= (20y + 3)(4y - 2)$
$= \boldsymbol{2(20y + 3)(2y - 1)}$

**7.** $12x^2 + 16xy - 35y^2 = 12x^2 + 30xy - 14xy - 35y^2$
$= 6x(2x + 5y) - 7y(2x + 5y)$
$= \boldsymbol{(2x + 5y)(6x - 7y)}$

**8.** **(i)** Area of rectangle $= (7x - 4y)(3y - x)$ cm$^2$
$= (21xy - 7x^2 - 12y^2 + 4xy)$ cm$^2$
$= \boldsymbol{(25xy - 7x^2 - 12y^2)}$ **cm$^2$**

**(ii)** Let $x = 4$ and $y = 5$.
Length of one side $= [7(4) - 4(5)]$ cm $= 8$ cm
Length of the other side $= [3(5) - 4]$ cm $= 11$ cm
Area of rectangle $= 8$ cm $\times$ 11 cm
$= 88$ cm$^2$
Substitute $x = 4$ and $y = 5$ into the expression in part **(i)**:
Area of rectangle $= (25xy - 7x^2 - 12y^2)$ cm$^2$
$= [25(4)(5) - 7(4)^2 - 12(5)^2]$ cm$^2$
$= 88$ cm$^2$
Since the values are equal, the expression obtained in part **(i)** is correct. (shown)

**9.** **(i)** $\left(\frac{1}{3}x + \frac{1}{4}y\right)(hx + ky) = \boldsymbol{\frac{h}{3}x^2 + \left(\frac{h}{4} + \frac{k}{3}\right)xy + \frac{k}{4}y^2}$

**(ii)** Comparing coefficients of $y^2$,
$\frac{k}{4} = 1$
$k = \mathbf{4}$

**(iii)** Comparing coefficients of $xy$,

$\frac{h}{4} + \frac{k}{3} = -1$

$\frac{h}{4} + \frac{4}{3} = -1$

$\frac{h}{4} = -\frac{7}{3}$

$h = -\frac{28}{3}$

Comparing coefficients of $x^2$,

$a = \frac{h}{3}$

$= \frac{-\frac{28}{3}}{3}$

$= -\frac{28}{9}$

$\therefore$ $\boldsymbol{a = -\frac{28}{9}, h = -\frac{28}{3}}$

**10.** $48x^2 - 60xy + \frac{75}{4}y^2 = \frac{3}{4}(64x^2 - 80xy + 25y^2)$

$= \frac{3}{4}[(8x)^2 - 2(8x)(5y) + (5y)^2]$

$= \boldsymbol{\frac{3}{4}(8x - 5y)^2}$

**11.** $x^2 - y^2 = 30$

$(x + y)(x - y) = 30$

$5(x + y) = 30$

$x + y = 6$

$(x + y)^2 = \mathbf{36}$

**12.** $(7n + 1)^2 + 6 = (49n^2 + 14n + 1) + 6$

$= 49n^2 + 14n + 7$

$= 7(7n^2 + 2n + 1)$, which is a multiple of 7. (shown)

**13.** **(i)** $x^2 - y^2 = \boldsymbol{(x + y)(x - y)}$

**(ii)** $123\,321^2 - 123\,322^2$

$= (123\,321 + 123\,322)(123\,321 - 123\,322)$

$= (246\,643)(-1)$

$= \mathbf{-246\,643}$

**14.** **(i)** $x^2 - 10\,201 = x^2 - 101^2$

$= \boldsymbol{(x + 101)(x - 101)}$

**(ii)** Since $4199 + 10\,201 = 14\,400 = 120^2$,

let $x = 120$:

$4199 = 120^2 - 10\,201$

$= (120 + 101)(120 - 101)$

$= 221 \times 19$

$\therefore$ Two factors of 4199 are **221** and **19**.

**15.** $\frac{394^2 - 401 \times 387}{49} = \frac{394^2 - (394 + 7)(394 - 7)}{49}$

$= \frac{394^2 - (394^2 - 7^2)}{49}$

$= \frac{394^2 - 394^2 + 7^2}{49}$

$= \frac{7^2}{49}$

$= \mathbf{1}$

**16.** Area of square $= (196p^2 - 168pq + 36q^2)$ cm²

$= 4(49p^2 - 42pq + 9q^2)$ cm²

$= 4(7p - 3q)^2$ cm²

$= (14p - 6q)^2$ cm²

$\therefore$ Length of square $= (14p - 6q)$ cm

$= \mathbf{2(7p - 3q)}$ **cm**

**17.** Consider $(-a + b)^2$.

$(-a + b)^2 = (b - a)^2$

$= (-1)^2(a - b)^2$

$= (a - b)^2$

Consider $(-a - b)^2$.

$(-a - b)^2 = (-1)^2(a + b)^2$

$= (a + b)^2$

$\therefore$ $\boldsymbol{(a + b)^2 = (-a - b)^2; (a - b)^2 = (b - a)^2 = (-a + b)^2}$

# 5 Number Patterns

## Worksheet 5A Number sequences

**1.** **(a)** Start with 15, then add 7 to each term to get the next term.
The next two terms are **50 and 57**.

**(b)** Start with 50, then subtract 4 from each term to get the next term.
The next two terms are **30 and 26**.

**(c)** Start with 8, then multiply each term by 2 to get the next term.
The next two terms are **256 and 512**.

**(d)** Start with –2400, then divide each term by (–2) to get the next term.
The next two terms are **75 and** $\boldsymbol{-37\frac{1}{2}}$.

**2.** **(a)** $19 + 4 = 23$

$23 + 4 = 27$

$\therefore$ The next two terms are **23 and 27**.

**(b)** $162 \times 3 = 486$

$486 \times 3 = 1458$

$\therefore$ The next two terms are **486 and 1458**.

**(c)** $25 - 6 = 19$

$19 - 6 = 13$

$\therefore$ The next two terms are **19 and 13**.

**(d)** $18 \div 2 = 9$

$9 \div 2 = 4\frac{1}{2}$

$\therefore$ The next two terms are **9 and** $\boldsymbol{4\frac{1}{2}}$.

**(e)** $4 \div (-2) = -2$

$(-2) \div (-2) = 1$

$\therefore$ The next two terms are **–2 and 1**.

**(f)** $\frac{1}{15} \div 3 = \frac{1}{45}$

$\frac{1}{45} \div 3 = \frac{1}{135}$

$\therefore$ The next two terms are $\boldsymbol{\frac{1}{45}}$ **and** $\boldsymbol{\frac{1}{135}}$.

**3.** **(a)** $14 - 1 = 13$

$13 + 3 = 16$

$\therefore$ The next two terms are **13 and 16**.

**(b)** $14 + 23 = 37$

$23 + 37 = 60$

∴ The next two terms are **37 and 60**.

**(c)** $50 \times 500 = 25\ 000$

$500 \times 25\ 000 = 12\ 500\ 000$

∴ The next two terms are **25 000 and 12 500 000**.

**(d)** $18 + 1 = 19$

$19 \times 2 = 38$

∴ The next two terms are **19 and 38**.

**(e)** $-9 - 16 = -25$

$-25 - 32 = -57$

∴ The next two terms are **–25 and –57**.

**(f)** $21.9 + 25 = 46.9$

$46.9 + 36 = 82.9$

∴ The next two terms are **46.9 and 82.9**.

**4.** **(a)** $T_1 = 3(1) + 8 = \mathbf{11}$

$T_2 = 3(2) + 8 = \mathbf{14}$

$T_3 = 3(3) + 8 = \mathbf{17}$

**(b)** $T_1 = 8(1) - 9 = \mathbf{-1}$

$T_2 = 8(2) - 9 = \mathbf{7}$

$T_3 = 8(3) - 9 = \mathbf{15}$

**(c)** $T_1 = 1^2 + 2 = \mathbf{3}$

$T_2 = 2^2 + 2 = \mathbf{6}$

$T_3 = 3^2 + 2 = \mathbf{11}$

**(d)** $T_1 = \frac{1}{2}[6(1) - 1] = \mathbf{2\frac{1}{2}}$

$T_2 = \frac{1}{2}[6(2) - 1] = \mathbf{5\frac{1}{2}}$

$T_3 = \frac{1}{2}[6(3) - 1] = \mathbf{8\frac{1}{2}}$

**(e)** $T_1 = \frac{10}{1^3} = \mathbf{10}$

$T_2 = \frac{10}{2^3} = \mathbf{1\frac{1}{4}}$

$T_3 = \frac{10}{3^3} = \mathbf{\frac{10}{27}}$

**(f)** $T_1 = \frac{4(1)}{1+5} = \mathbf{\frac{2}{3}}$

$T_2 = \frac{4(2)}{2+5} = \mathbf{1\frac{1}{7}}$

$T_3 = \frac{4(3)}{3+5} = \mathbf{1\frac{1}{2}}$

**5.** **(i)** $T_2 = 48 - 9 = \mathbf{39}$

$T_3 = 39 - 9 = \mathbf{30}$

**(ii)** $T_n = \mathbf{57 - 9n}$

**6.** **(i)** $T_n = \mathbf{7n - 3}$

**(ii)** Let $7n - 3 = 333$.

$7n = 336$

$n = 48$

∴ 333 is the 48th term in the sequence.

**7.** **(i)** $T_5 = -13 - 6 = \mathbf{-19}$

$T_6 = -19 - 6 = \mathbf{-25}$

**(ii)** $T_n = \mathbf{11 - 6n}$

**(iii)** Given that $T_k = -163$,

$11 - 6k = -163$

$-6k = -174$

$k = \mathbf{29}$

**8.** **(i)** $T_8 = 15 + 4 + 4 + 4 + 4 = \mathbf{31}$

**(ii)** $T_n = \mathbf{4n - 1}$

**(iii)** Given that $T_n = 231$,

$4n - 1 = 231$

$4n = 232$

$n = \mathbf{58}$

**9.** **(i)** Common difference $= \frac{36 - 15}{3} = 7$

$a = 15 - 7 = 8$

$b = 15 + 7 = 22$

$c = 22 + 7 = 29$

∴ $\mathbf{a = 8, b = 22, c = 29}$

**(ii)** $T_n = \mathbf{1 + 7n}$

**(iii)** Let $1 + 7n = 112$.

$7n = 111$

$n = 15\frac{6}{7}$

Since $n$ is not a positive integer, 112 is **not a term** of this sequence.

**10.** **(i)** $\mathbf{2n - 1, 2n + 1}$

**(ii)** **(a)** Sum of numbers $= (2n - 3) + (2n - 1) + (2n + 1)$

$= \mathbf{6n - 3}$

**(b)** Since $6n - 3 = 3(2n - 1)$, the sum is a multiple of 3.

**11.** **(i)** $k = \mathbf{-12}$ **(ii)** $T_n = \mathbf{2 - 7n}$

**12.** **(a)** $T_1 = 3(1)^2 - 1 = \mathbf{2}$

$T_2 = 3(2)^2 - 1 = \mathbf{11}$

$T_3 = 3(3)^2 - 1 = \mathbf{26}$

$T_4 = 3(4)^2 - 1 = \mathbf{47}$

**(b)** **(i)** $U_n = T_n + 2$

$= 3n^2 - 1 + 2$

$= \mathbf{3n^2 + 1}$

**(ii)** $U_{25} = 3(25)^2 + 1$

$= \mathbf{1876}$

## Challenge Myself!

**13.**

| $T_1$ | $T_2$ | $T_3$ | $T_4$ | $T_5$ | |
|---|---|---|---|---|---|
| 2, | 6, | 46, | 446, | 4446, | … |

+4, +40 = 4(10), +400 = $4(10^2)$, +4000 = $4(10^3)$

$T_5 = T_4 + 4(10^3)$

$\mathbf{T_{n+1} = T_n + 4(10^{n-1})}$

## Worksheet 5B Number sequences and patterns

**1.** **(i)**

| Figure number | Length of side of large square | Number of grey squares | Number of white squares | Total number of squares |
|---|---|---|---|---|
| 1 | 2 | 1 | 3 | 4 |
| 2 | 3 | 4 | 5 | 9 |
| 3 | **4** | **9** | **7** | **16** |
| 4 | **5** | **16** | **9** | **25** |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| $n$ | $\mathbf{n + 1}$ | $\mathbf{n^2}$ | $\mathbf{1 + 2n}$ | $\mathbf{n^2 + 2n + 1}$ **or** $\mathbf{(n + 1)^2}$ |

**(ii)** Since $n + 1 = 65$,

$n = 64$.

$\therefore$ Number of white squares $= 1 + 2(64)$

$= \mathbf{129}$

**(iii)** **No.** 920 is not a perfect square.

**2.** **(i)**

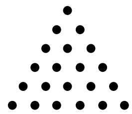


Figure 5

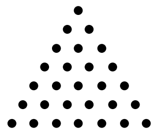


Figure 6

**(ii)**

| Figure number | Number of dots |
|---|---|
| 1 | 3 |
| 2 | 6 |
| 3 | **10** |
| 4 | **15** |
| ⋮ | ⋮ |
| 10 | **66** |

**(iii)** As the figure number increases from $n$ to $n + 1$, the number of dots increases by $n + 2$.

**(iv)** Using the formula $n^2 + 3n + 2$, the number of dots in Figures 1 to 4 are 6, 12, 20 and 30 respectively. Belle is not correct. The formula should be $\mathbf{\frac{1}{2}(n^2 + 3n + 2)}$.

**3.** **(i)** $T_4 = 4^2 + 16 = \mathbf{32}$

**(ii)** $T_n = \mathbf{n^2 + 4n}$

**(iii)** $T_{50} = 50^2 + 4(50)$

$= \mathbf{2700}$

**4.** **(i)** $\mathbf{\frac{1}{6} + \frac{1}{7} = \frac{7+6}{6 \times 7} = \frac{13}{42}}$

**(ii)** $\mathbf{k = 20, p = 41, q = 420}$

**(iii)** $\frac{5}{80} + \frac{5}{81} = 5\left(\frac{1}{80} + \frac{1}{81}\right)$

$= 5\left(\frac{81+80}{80 \times 81}\right)$

$= 5\left(\frac{161}{6480}\right)$

$= \mathbf{\frac{161}{1296}}$

**5.** **(i)** $\mathbf{100 = 10^2 = 1^3 + 2^3 + 3^3 + 4^3}$

**(ii)** $a = \sqrt{3025}$

$= 55$

$1 + 2 + 3 + \ldots + b = 55$

$b = 10$

$\therefore \mathbf{a = 55, b = 10}$

**6.** **(i)** $\mathbf{45 = 5 \times 9 = 5 \times (5 + 4)}$

**(ii)** $396 = 18 \times 22$

$\therefore \mathbf{a = 18, b = 22}$

**(iii)** $\mathbf{480 = 20 \times 24 = 20 \times (20 + 4)}$

**(iv)** **No.** The last digit of the number on the left-hand side will not be 4.

## Review Exercise 5

**1.** $T_1 = \frac{1}{4}(1)(1 + 1)(1 + 2) = \mathbf{1\frac{1}{2}}$

$T_2 = \frac{1}{4}(2)(2 + 1)(2 + 2) = \mathbf{6}$

$T_3 = \frac{1}{4}(3)(3 + 1)(3 + 2) = \mathbf{15}$

**2.**

| $n$ | 1 | 4 | **9** | 16 |
|---|---|---|---|---|
| $T_n$ | 97 | **52** | −143 | **−668** |

**3.** **(i)** $T_1 = 1$

$T_2 = (1 + 3) \times 2 = \mathbf{8}$

$T_3 = (8 + 3) \times 2 = \mathbf{22}$

**(ii)** $U_3 = 54$

$U_2 = (54 \div 2) - 3 = 24$

$U_1 = (24 \div 2) - 3 = \mathbf{9}$

**4.** **(i)** $T_7 = -1\frac{1}{2} - 1\frac{1}{3} - 1\frac{1}{3} - 1\frac{1}{3} = \mathbf{-5\frac{1}{2}}$

**(ii)** $T_n = \mathbf{\frac{23}{6} - \frac{4}{3}n}$

**(iii)** Let $\frac{23}{6} - \frac{4}{3}n = -18\frac{5}{6}$.

$-\frac{4}{3}n = -\frac{68}{3}$

$n = \mathbf{17}$

**5.** **(i)** $\mathbf{2n + 2, 2n + 4}$

**(ii)** **(a)** Sum of numbers $= 2n + (2n + 2) + (2n + 4)$

$= \mathbf{6n + 6}$

**(b)** $6n + 6 = 4(1.5n + 1.5)$

When $n = 7$,

Sum of numbers = 48, which is a multiple of 4

When $n = 8$,

Sum of numbers = 54, which is not a multiple of 4

$\therefore$ The sum is **not always a multiple of 4**.

**6.** **(i)** $T_4 = 4^3 - 5 = \mathbf{59}$

**(ii)** $T_n = n^3 - (n + 1)$

$= \mathbf{n^3 - n - 1}$

**(iii)** $T_{18} = 18^3 - 18 - 1$

$= \mathbf{5813}$

**7.** **(a)** $T_1 = 2(1)^2 + 5 = \mathbf{7}$

$T_2 = 2(2)^2 + 5 = \mathbf{13}$

$T_3 = 2(3)^2 + 5 = \mathbf{23}$

$T_4 = 2(4)^2 + 5 = \mathbf{37}$

**(b)** **(i)** $U_n = T_n + n$

$= \mathbf{2n^2 + n + 5}$

**(ii)** $U_{36} = 2(36)^2 + 36 + 5$

$= \mathbf{2633}$

**8.** **(i)** $\mathbf{6^2 + 6 \times 5 + 5^2 = 91 = 6^3 - 5^3}$

**(ii)** $\mathbf{(n + 1)^2 + (n + 1) \times n + n^2 = (n + 1)^3 - n^3}$

**(iii)** When $n = 23$,

$24^2 + 24 \times 23 + 23^2 = 24^3 - 23^3$

$= \mathbf{1657}$

9. (i)

| Figure number | Number of white hexagons | Number of grey hexagons | Total number of hexagons |
|---|---|---|---|
| 1 | 1 | 0 | 1 |
| 2 | 1 | 1 | 2 |
| 3 | **2** | **1** | **3** |
| 4 | **2** | **2** | **4** |
| ⋮ | ⋮ | ⋮ | ⋮ |
| 12 | **6** | **6** | **12** |

(ii) $\mathbf{\dfrac{n-1}{2}}$ (iii) **Figure 155**

# 6 Financial Transactions

## Worksheet 6A Percentage, ratio and rate

1. (a) 50% of \$2000 $= \dfrac{50}{100} \times \$2000$
$= \mathbf{\$1000}$

(b) 12% of 3 m $= \dfrac{12}{100} \times 3$ m
$= \mathbf{0.36\ m}$

(c) 47% of 900 g $= \dfrac{47}{100} \times 900$ g
$= \mathbf{423\ g}$

(d) 200% of 650 ml $= \dfrac{200}{100} \times 650$ ml
$= \mathbf{1300\ ml}$

2. (a) Required percentage $= \dfrac{20\text{ m}}{1\text{ km}} \times 100\%$
$= \dfrac{20\text{ m}}{1000\text{ m}} \times 100\%$
$= \mathbf{2\%}$

(b) Required percentage $= \dfrac{90°}{270°} \times 100\%$
$= \mathbf{33\tfrac{1}{3}\ \%}$

(c) Required percentage $= \dfrac{300\text{ s}}{0.5\text{ h}} \times 100\%$
$= \dfrac{300\text{ s}}{0.5 \times 60 \times 60\text{ s}} \times 100\%$
$= \mathbf{16\tfrac{2}{3}\ \%}$

(d) Required percentage $= \dfrac{6800\text{ cm}^3}{4\text{ m}^3} \times 100\%$
$= \dfrac{6800\text{ cm}^3}{4 \times 100^3\text{ m}^3} \times 100\%$
$= \mathbf{0.17\%}$

3. Percentage decrease $= \dfrac{2\ 100\ 000 - 1\ 488\ 000}{1\ 488\ 000} \times 100\%$
$= \mathbf{41.1\%}$ (to 3 s.f.)

4. (a) 900 : 600 = **3 : 2** (b) 0.2 : 1.4 = **1 : 7**

(c) $\dfrac{1}{5} : \dfrac{7}{8} = \mathbf{8 : 35}$ (d) 21 : 36 : 15 = **7 : 12 : 5**

5. **18 : 8, 45 : 20**

6. Volume of apple concentrate needed $= \dfrac{5600}{7} \times 2$ ml
$= 1600$ ml

Cost of apple concentrate = 2 × \$14
= \$28

Volume of pear juice needed $= \dfrac{5600}{7} \times 3$ ml
$= 2400$ ml

Cost of pear juice = 4 × \$3.50
= \$14

Minimum amount of money = \$28 + \$14
= **\$42**

7. (a) Bisma's rate $= \dfrac{4\text{ puzzles}}{1\text{ min }20\text{ s}}$
$= \dfrac{4\text{ puzzles}}{1\frac{1}{3}\text{ min}}$
$= \mathbf{3\ puzzles/min}$

(b) Faiq's rate $= \dfrac{6\text{ puzzles}}{1.9\text{ min}}$
$= \mathbf{3.16\ puzzles/min}$ (to 3 s.f.)

(c) Laiba's rate $= \dfrac{7\text{ puzzles}}{160\text{ s}}$
$= \dfrac{7\text{ puzzles}}{\frac{160}{60}\text{ min}}$
$= \mathbf{2.625\ puzzles/min}$

(d) Usman's rate $= \dfrac{10\text{ puzzles}}{3\text{ min}}$
$= \mathbf{3.33\ puzzles/min}$ (to 3 s.f.)

8. Peggy's rate $= \dfrac{5\text{ km}}{33\text{ min}}$
$= 0.152$ km/min (to 3 s.f.)

Fraser's rate $= \dfrac{8\text{ km}}{55\text{ min}}$
$= 0.145$ km/min (to 3 s.f.)

∴ **Peggy** ran at a faster rate.

## Worksheet 6B Profit, loss, discount, General Sales Tax and commission

1. (a) **Profit** as a percentage of cost price
$= \dfrac{\$75 - \$60}{\$60} \times 100\%$
$= \mathbf{25\%}$

(b) **Loss** as a percentage of cost price
$= \dfrac{\$250 - \$190}{\$250} \times 100\%$
$= \mathbf{24\%}$

(c) **Profit** as a percentage of cost price
$= \dfrac{\text{PKR } 19\ 620 - \text{PKR } 18\ 000}{\text{PKR } 18\ 000} \times 100\%$
$= \mathbf{9\%}$

(d) **Loss** as a percentage of cost price

$= \frac{\text{PKR } 73\,900 - \text{PKR } 67\,249}{\text{PKR } 67\,249} \times 100\%$

$=$ **9%**

(e) Selling price $= \frac{116}{100} \times \$4000$

$=$ **\$4640**

(f) Selling price $= \frac{92}{100} \times \$3120$

$=$ **\$2870.40**

(g) Selling price $= \frac{111}{100} \times \text{PKR } 25\,000$

$=$ **PKR 27 750**

(h) Selling price $= \frac{97}{100} \times \text{PKR } 48\,000$

$=$ **PKR 46 560**

(i) Cost price $= \frac{100}{102.5} \times \$164$

$=$ **\$160**

(j) Cost price $= \frac{100}{92.6} \times \$444.48$

$=$ **\$480**

(k) Cost price $= \frac{100}{112.5} \times \text{PKR } 64\,125$

$=$ **PKR 57 000**

(l) Cost price $= \frac{100}{90} \times \text{PKR } 63\,000$

$=$ **PKR 70 000**

2. (a) Profit as a percentage of the cost price

$= \frac{\$1049 - \$999}{\$999} \times 100\%$

$=$ **5.01%** (to 3 s.f.)

(b) Loss as a percentage of the selling price

$= \frac{\$999 - \$499}{\$499} \times 100\%$

$=$ **100%** (to 3 s.f.)

3. Selling price $= \frac{83}{100} \times \$898$

$=$ **\$745.34**

4. Cost price $= \frac{100}{112} \times \$1344$

$=$ **\$1200**

5. Cost price $= \frac{100}{74} \times \$95$

$=$ **\$128.38** (to 2 d.p.)

6. Number of apples that are not rotten $= \frac{90}{100} \times 300$

$= 270$

Cost price of 300 apples $= \frac{\$1.80}{12} \times 300$

$= \$45$

Total selling price of 270 apples $= \frac{600}{100} \times \$45$

$= \$270$

Selling price of each apple $= \frac{\$270}{270}$

$=$ **\$1**

7. Cost price = \$8000

Loss $= \frac{10}{100} \times \$8000$

$= \$800$

Selling price = \$8000 – \$800

= \$7200

∴ The cost price and selling price could be **\$8000** and **\$7200** respectively.

8. Discount as a percentage of the selling price

$= \frac{\$428 - \$360}{\$360} \times 100\%$

$=$ **18.9%** (to 3 s.f.)

9. Marked price $= \frac{100}{15} \times \$384$

$=$ **\$2560**

10. (i) Sale price at Retailer A $= \frac{90}{100} \times \$126$

$=$ **\$113.40**

(ii) Sale price at Retailer B $= \frac{98}{100} \times \left(\frac{92}{100} \times \$126\right)$

$= \$113.6016$

Difference in price = \$113.6016 – \$113.40

= \$0.20 (to 2 d.p.)

∴ **Yes.** Grandma Lucy is correct.

11. (a) Amount of GST $= \frac{9}{100} \times \$100$

$=$ **\$9**

Marked price inclusive of GST = \$100 + \$9

= **\$109**

(b) Amount of GST $= \frac{9}{100} \times \$29.90$

$=$ **\$2.69** (to 2 d.p.)

Marked price inclusive of GST = \$29.90 + \$2.691

= **\$32.59** (to 2 d.p.)

(c) Amount of GST $= \frac{18}{100} \times \text{PKR } 10\,000$

$=$ **PKR 1800**

Marked price inclusive of GST = PKR 10 000 + PKR 1800

= **PKR 11 800**

(d) Amount of GST $= \frac{18}{100} \times \text{PKR } 45\,000$

$=$ **PKR 8100**

Marked price inclusive of GST = PKR 45 000 + PKR 8100

= **PKR 53 100**

(e) Price before GST $= \frac{100}{109} \times \$8720$

$=$ **\$8000**

Amount of GST = \$8720 – \$8000

= **\$720**

(f) Price before GST $= \frac{100}{109} \times \$381.50$

$=$ **\$350**

Amount of GST = \$381.50 – \$350

= **\$31.50**

(g) Price before GST $= \frac{100}{118} \times \text{PKR } 90\,860$

$=$ **PKR 77 000**

Amount of GST = PKR 90 860 – PKR 77 000

= **PKR 13 860**

**(h)** Price before GST = $\frac{100}{118}$ × PKR 60 000
= **PKR 55 046** (to the nearest PKR)
Amount of GST = PKR 60 000 – PKR 55 046
= **PKR 4954**

**(i)** Price before GST = $\frac{100}{9}$ × $4.50
= **$50**
Marked price inclusive of GST = $50 + $4.50
= **$54.50**

**(j)** Price before GST = $\frac{100}{9}$ × $71
= **$788.89** (to 2 d.p.)
Marked price inclusive of GST = $788.8889 + $71
= **$859.89** (to 2 d.p.)

**(k)** Price before GST = $\frac{100}{18}$ × PKR 3600
= **PKR 20 000**
Marked price inclusive of GST = PKR 20 000 + PKR 3600
= **PKR 23 600**

**(l)** Price before GST = $\frac{100}{18}$ × PKR 92 000
= **PKR 511 111**
Marked price inclusive of GST = PKR 511 111 + PKR 92 000
= **PKR 603 111**

**12.** **(i)** Amount of GST = $\frac{18}{100}$ × $380
= **$68.40**

**(ii)** Total amount = $380 + $68.40
= **$448.40**

**13.** Price before GST = $\frac{100}{118}$ × $812.05
= **$688.18** (to 2 d.p.)

**14.** Total amount = $\frac{118}{100} \times \left(\frac{110}{100} \times \$24.90\right) \times 3$
= **$96.96** (to 2 d.p.)

**15.** **(i)** Amount Ying received = $\frac{99}{100}$ × $825 000
= **$816 750**

**(ii)** Amount the property agent received
= $\frac{3}{100}$ × $825 000
= **$24 750**

**16.** **(i)** Selling price = $\frac{120}{100}$ × $3 600 000
= **$4 320 000**

**(ii)** Correct amount = $\frac{109}{100} \times \left(\frac{2}{100} \times \$4\,320\,000\right)$
= **$94 176**

**17.** Total commission = $\frac{2}{100}$ × ($800 000 + $1 200 000)
= $40 000
∴ The selling prices could be **$800 000** and **$1 200 000** respectively.

**18.** **(a)** **(i)** Price after 20% discount = $\frac{80}{100}$ × 2 × $99.90
= $159.84
Price after $18 discount = $159.84 – $18
= $141.84

Amount Ronald has to pay if he uses a credit card
= $\frac{102}{100}$ × $141.84
= **$144.68** (to 2 d.p.)

**(ii)** Amount payable in cash = $\frac{98}{100}$ × $$141.84
= $139.00 (to 2 d.p.)
Difference in payment amounts
= $\frac{\$144.68 - \$139.00}{\$139.00} \times 100\%$
= **4.09%** (to 3 s.f.)

**(b)** Banks may charge the retailers a fee for each transaction, so the retailers may translate this cost to the customers.

**19.** Commission = $\frac{2}{100}$ × $1 980 000
= **$39 600**

**20.** Total commission = 12 × 2 × $\frac{1.5}{100}$ × $700 000
= $252 000 > $250 000
∴ Joseph is correct.

**21.** Commission on each vacuum cleaner = $\frac{5}{100}$ × $490
= $24.50
If Wang sells 10 vacuum cleaners,
Total commission and bonus = 10 × $24.50 + $120
= $365
If Wang sells 20 vacuum cleaners,
Total commission and bonus = 2 × $365
= $730
Total number of vacuum cleaners sold = 20 + $\frac{\$926 - \$730}{\$24.50}$
= **28**

## Worksheet 6C Insurance, hire purchase and interest

1. Insurance premium rate = $\frac{11\,250}{500\,000}$ × 100%
= **2.25%**

2. $\frac{3}{100}$ × PKR $x$ = PKR 27 000
$x$ = **900 000**

3. Total amount paid = 20 × $\frac{2.8}{100}$ × $200 000
= **$112 000**

4. Total premiums = $\frac{3}{100}$ × PKR 1 000 000 + 12 × $\frac{1}{100}$ × PKR 800 000
= PKR 30 000 + PKR 96 000
= **PKR 126 000**

5. **(a)** Premiums payable = $\frac{2.5}{100}$ × PKR 3 899 000
= **PKR 97 475**

**(b)** Premiums payable = $\frac{2.5}{100}$ × PKR 29 000 000
= **PKR 725 000**

6. **(a)** Annual premium = $\frac{\text{PKR } 360\,000}{5}$
= **PKR 72 000**
∴ Aafaq purchased the **Gold Plan for self, spouse and up to 3 children**.

(b) Total annual premium = 12 × PKR 12 000

= PKR 144 000

Additional amount needed = PKR 152 000 – PKR 144 000

= PKR 8000

∴ **No**, Hisham cannot afford to purchase the Pro Plan. Hisham needs another **PKR 8000**.

7. (a) Difference in premiums = 12 × \$87 – \$1000

= **\$44**

(b) Total premiums paid = 6 × \$1000 + 2 × \$1200

= \$8400

Difference = \$9600 – \$8400

= **\$1200**

8. (i) Total amount she pays = $\frac{1}{3}$ × \$999 + 18 × \$44.40

= **\$1132.20**

(ii) Extra cost as a percentage of the cash price

$= \frac{\$1132.20 - \$999}{\$999} \times 100\%$

= **13.3%** (to 3 s.f.)

9. Amount she pays = $\frac{20}{100}$ × \$470 + 24 × \$22

= \$622

Extra amount she pays as a percentage of the cash price

$= \frac{\$622 - \$470}{\$470} \times 100\%$

= **32.3%** (to 3 s.f.)

10. (a) Interest = $\$10\,000 \times \frac{2}{100} \times 5$

= **\$1000**

(b) Interest = $\$36\,000 \times \frac{1.4}{100} \times 8$

= **\$4032**

(c) Interest = $\$36\,000 \times \frac{2.8}{100} \times 8$

= **\$8064**

(d) Interest = $\$75\,000 \times \frac{3.5}{100} \times \frac{75}{12}$

= **\$16 406.25**

11. (a) Interest = $\$10\,000\left(1 + \frac{2}{100}\right)^5 - \$10\,000$

= **\$1040.81** (to 2 d.p.)

(b) Interest = $\$36\,000\left(1 + \frac{1.4}{100}\right)^8 - \$36\,000$

= **\$4235.20** (to 2 d.p.)

(c) Interest = $\$36\,000\left(1 + \frac{2.8}{100}\right)^8 - \$36\,000$

= **\$8900.11** (to 2 d.p.)

(d) Interest = $\$75\,000\left(1 + \frac{2.8}{100}\right)^{\frac{7.5}{12}} - \$75\,000$

= **\$17 990.47** (to 2 d.p.)

12. Amount of interest = $\$25\,000 \times \frac{3.6}{100} \times 5$

= **\$4500**

13. OPEN Let the simple interest rate be 1.85%.

Amount he could receive

$= \$30\,000 \times \frac{1.85}{100} \times 4 + \$30\,000$

= **\$32 220**

14. Total amount $= 50\,000\left(1+\frac{1.02}{100}\right)^4$

= **\$52 071.42** (to the nearest cent)

15. Let the principal sum be \$$P$.

$P + 1493 = P\left(1+\frac{2.4}{100}\right)^4$

$= P(1.024)^4$

$P(1.024)^4 - P = 1493$

$P(1.024^4 - 1) = 1493$

$P = \frac{1493}{1.024^4 - 1}$

= 15 000 (to the nearest ten dollars)

∴ Leslie invested **\$15 000**.

16. Total amount = \$60 000

$48\,000\left(1+\frac{r}{100}\right)^{15} = 60\,000$

$\left(1+\frac{r}{100}\right)^{15} = 1.25$

$1+\frac{r}{100} = \sqrt[15]{1.25}$

$\frac{r}{100} = \sqrt[15]{1.25} - 1$

$r = 100\left(\sqrt[15]{1.25} - 1\right)$

= 1.50 (to 3 s.f.)

∴ $r$ = **1.50**

17. Let the principal sum and the number of years be \$$P$ and $n$ respectively.

$20\,000\left(1+\frac{3}{100}\right)^n - 20\,000 = 1854.54$

$20\,000(1.03)^n = 21\,854.54$

$1.03^n = 1.092\,727$

$= 1.03^3$

$n = 3$

∴ $x$ = 3 × 12 = **36**

18. $48\left(1+\frac{r}{100}\right)^{12} = 348$

$\left(1+\frac{r}{100}\right)^{12} = 7.25$

$1+\frac{r}{100} = \sqrt[12]{7.25}$

$\frac{r}{100} = \sqrt[12]{7.25} - 1$

$r = 100\left(\sqrt[12]{7.25} - 1\right)$

= 17.9 (to 3 s.f.)

∴ $r$ = **17.9**

19. (i) $P\left(1+\frac{10}{100}\right)^3 - P - \frac{P(11)(2)}{100} = 3330$

$P(1.1)^3 - P - 0.22P = 3330$

$0.111P = 3330$

$P$ = **30 000**

(ii) Total amount at the end of the 1st year

$= \$\left(5\,000\,000 + \frac{10}{100} \times 5\,000\,000\right)$

= \$5 500 000

Interest received at the end of the 2nd year

$= \$\left(\frac{4}{100} \times 5\,500\,000\right)$

= **\$220 000**

(iii) Required percentage $= \dfrac{220\ 000}{5\ 000\ 000} \times 100\%$
$= \mathbf{4.4\%}$

**20.** **(a)** Total interest $= \$\left[40\ 000\left(1+\dfrac{3}{100}\right)^{6} - 40\ 000\right]$
$= \mathbf{\$7762.09}$ (to 2 d.p.)

**(b)** Total interest $= \$\left[40\ 000\left(1+\dfrac{3 \div 2}{100}\right)^{6\times2} - 40\ 000\right]$
$= \mathbf{\$7824.73}$ (to 2 d.p.)

**(c)** Total interest $= \$\left[40\ 000\left(1+\dfrac{3 \div 12}{100}\right)^{6\times12} - 40\ 000\right]$
$= \mathbf{\$7877.94}$ (to 2 d.p.)

**21.** Jessica could propose to charge interest based on a daily (or even hourly) computation.
The higher the frequency of interest compounded, the higher the interest received.

**22.** **(a)** Amount owed $= \$80\ 000\left(1+\dfrac{2.7 \div 2}{100}\right)^{5\times2}$
$= \mathbf{\$91\ 480.30}$ (to the nearest 10 cents)

**(b)** Interest owed in the first 3 years
$= \$\left[80\ 000\left(1+\dfrac{2.7 \div 2}{100}\right)^{3\times2} - 80\ 000\right]$
$= \$6702.6767$ (to 4 d.p.)

Interest owed in the next 2 years
$= \$\left[80\ 000\left(1+\dfrac{2.5 \div 12}{100}\right)^{12\times2} - 80\ 000\right]$
$= \$4097.3136$ (to 4 d.p.)
$\therefore$ Total interest owed $= \$6702.6767 + \$4097.3136$
$= \mathbf{\$10\ 800}$ (to the nearest hundred dollars)

**23.** Let the principal sum be $\$P$.
Amount received from Annuity A $= \$P\left(1+\dfrac{2.76 \div 4}{100}\right)^{20\times4}$
$= \$P(1.0069)^{80}$
$= \$1.7334P$ (to 5 s.f.)
Amount received from Annuity B $= \$P\left(1+\dfrac{2.88}{100}\right)^{20}$
$= \$P(1.0288)^{20}$
$= \$1.7645P$ (to 5 s.f.)
$\therefore$ I do not agree with Mona. The amount received from Annuity B will be higher, so Mona should invest in Annuity B instead.

## Challenge Myself!

**24.** **(a)** Interest from Product A $= \$\dfrac{60\ 000(3.5)(8)}{100}$
$= \$16\ 800$
Interest from Product B
$= \$\left[60\ 000\left(1+\dfrac{2.99 \div 12}{100}\right)^{8\times12} - 60\ 000\right]$
$= \$16\ 191.28$ (to 2 d.p.)
Since the interest from Product A is more than that from Product B, Sharon should invest in **Product A**.

**(b)** Let $p = 5.2$:
Total amount from Product C $= \$\left[60\ 000\left(1+\dfrac{5.2 \div 2}{100}\right)^{8\times2}\right]$
$= \$90\ 470.92$ (to 2 d.p.)
Mike is correct in his calculation.
However, I do not agree with his suggestion as the interest rate is not guaranteed to be 5.2%, and it could potentially yield less than if she were to invest her money in Product A or B.

## Worksheet 6D Zakat, ushr and income tax

**1.** Amount of zakat $= \dfrac{2.5}{100} \times$ PKR 120 000
= **PKR 3000**

**2.** Amount of zakat $= 5 \times \dfrac{2.5}{100} \times$ PKR 95 000
= **PKR 11 875**

**3.** Amount saved $= \dfrac{100}{2.5} \times$ PKR 4650
= **PKR 186 000**

**4.** Amount of ushr $= \dfrac{10}{100} \times$ PKR 1 000 000
= **PKR 100 000**

**5.** Sale price $= \dfrac{100}{5} \times$ PKR 738 000
= **PKR 14.76 million**

**6.** **(a)** **(i)** Income tax
= PKR 15 000 + $\dfrac{2.5}{100} \times$ (PKR 1 500 000 − PKR 1 200 000)
= **PKR 22 500**

**(ii)** Income tax
= PKR 1 095 000
+ $\dfrac{35}{100} \times$ (PKR 7 280 000 − PKR 6 000 000)
= **PKR 1 543 000**

**(b)** **(i)** Chargeable income
= (PKR 232 500 − PKR 165 000) $\times \dfrac{100}{22.5}$ + PKR 2 400 000
= **PKR 2 700 000**

**(ii)** Chargeable income
= (PKR 2 845 000 − PKR 1 095 000) $\times \dfrac{100}{35}$
+ PKR 6 000 000
= **PKR 11 000 000**

**7.** **(a)** **(i)** Tax for the first \$40 000 = \$550
Tax for the next \$2000 $= \dfrac{7}{100} \times \$2000$
$= \$140$
$\therefore$ Income tax payable = \$550 + \$140
= **\$690**

**(ii)** Tax for the first \$120 000 = \$7950
Tax for the next \$35 500 $= \dfrac{15}{100} \times \$35\ 500$
$= \$5325$
$\therefore$ Income tax payable = \$7950 + \$5325
= **\$13 275**

(b) (i) Tax for the first \$30 000 = \$200

Tax for the next $\$x$ = \$280

$$\$x = \frac{100}{3.5} \times \$280$$

$$= \$8000$$

∴ Chargeable income = \$30 000 + \$8000

= **\$38 000**

(ii) Tax for the first \$80 000 = \$3350

Tax for the next $\$x$ = \$1250

$$\$x = \frac{100}{11.5} \times \$1250$$

= \$10 869.57 (to 2 d.p.)

∴ Chargeable income = \$80 000 + \$10 869.57

= **\$90 869.57** (to 2 d.p.)

**8.** Total reliefs = \$1000 + 2 × \$9000 + \$19 200 + \$7000

= \$45 200

Chargeable income = \$96 000 – \$45 200

= \$50 800

Tax for the first \$40 000 = \$550

Tax for the next \$10 800 = $\frac{7}{100} \times \$10\,800$

= \$756

∴ Income tax payable = \$550 + \$756

= **\$1306**

## Worksheet 6E Inheritance and partnership

**1.** Amount 1st heir receives = $\frac{3}{10}$ × PKR 4 900 000

= **PKR 1 470 000**

Amount 2nd heir receives = $\frac{2}{10}$ × PKR 4 900 000

= **PKR 980 000**

Amount 3rd heir receives = $\frac{5}{10}$ × PKR 4 900 000

= **PKR 2 450 000**

**2.** Amount property was worth = $\frac{10}{3}$ × PKR 372 000

= **PKR 1 240 000**

**3.** Amount son received = $\frac{2}{3}$ × PKR 390 000

= **PKR 260 000**

Amount daughter received = $\frac{1}{3}$ × PKR 390 000

= **PKR 130 000**

**4.** Amount wife received = $\frac{1}{8}$ × PKR 885 000

= **PKR 110 625**

Amount son received = $\frac{2}{3}$ × (PKR 885 000 – PKR 110 625)

= **PKR 516 250**

Amount daughter received = $\frac{1}{3}$ × (PKR 885 000 – PKR 110 625)

= **PKR 258 125**

**5.** Amount wife received = $\frac{1}{8}$ × (PKR 1 360 000 – PKR 9000)

= **PKR 168 875**

Amount each son received = $\frac{2}{10} \times \frac{7}{8}$ × (PKR 1 360 000 – PKR 9000)

= **PKR 236 425**

Amount each daughter received

= $\frac{1}{10} \times \frac{7}{8}$ × (PKR 1 360 000 – PKR 9000)

= **PKR 118 212** (round down to the nearest integer)

**6.** Amount of assets

= PKR 697 000 – PKR 8600 – PKR 250 000 + PKR 140 000

= PKR 578 400

Amount wife received = $\frac{1}{4}$ × PKR 578 400

= **PKR 144 600**

Amount son received = $\frac{2}{4} \times \frac{3}{4}$ × PKR 578 400

= **PKR 216 900**

Amount daughter received = $\frac{1}{2}$ × PKR 216 900

= **PKR 108 450**

**7.** (a) Amount of assets = 8 × PKR 180 000

= **PKR 1 440 000**

(b) Amount of assets = 7 × PKR 227 000 × $\frac{8}{7}$

= **PKR 1 816 000**

(c) Amount of assets = $\frac{7}{2}$ × PKR 335 000 × $\frac{8}{7}$

= **PKR 1 340 000**

**8.** Amount 1st friend received = $\frac{4}{15}$ × (\$4 200 000 – \$3 000 000)

= **\$320 000**

Amount 2nd friend received = $\frac{5}{15}$ × (\$4 200 000 – \$3 000 000)

= **\$400 000**

Amount 3rd friend received = $\frac{6}{15}$ × (\$4 200 000 – \$3 000 000)

= **\$480 000**

**9.** Leo's share : Molly's share : Nick's share

= 25 000 : 37 500 : 87 500

= 2 : 3 : 7

Amount Leo received = $\frac{2}{2+3+7}$ × \$240 000 – \$25 000

= \$15 000

Amount Molly received = $\frac{3}{2+3+7}$ × \$240 000 – \$37 500

= \$22 500

Amount Nick received = $\frac{7}{2+3+7}$ × \$240 000 – \$87 500

= \$52 500

∴ They received **\$15 000**, **\$22 500** and **\$52 500** respectively.

**10.** (a) Manaal invested 40% of the total amount.

Difference = $\frac{5}{40}$ × PKR 240 000

= **PKR 30 000**

(b) Amount Manaal receives this year = 2 × PKR 240 000

= **PKR 480 000**

Amount Sidra receives this year = $\frac{60}{40}$ × PKR 480 000

= **PKR 720 000**

## Review Exercise 6

1. Let $\$x$ be the original price of a T-shirt.

$$x + \frac{85}{100}x = 51.8$$
$$1.85x = 51.8$$
$$x = 28$$

$\therefore$ The non-discounted price of each T-shirt is **$28**.

2. OPEN

(a) $5\%$ of $\$1600 = \frac{5}{100} \times \$1600$
$= \$80$

$\therefore$ The minimum purchase amount could be **$1600** so that the amount of discount is the same.

(b) $5\%$ of $\$2880 = \frac{5}{100} \times \$2800$
$= \$144$

$\therefore$ The customer should choose to get a **5% discount on the marked price** as he will save more.

3. Marked price of the buffet dinner for a child

$= \frac{50}{100} \times \$85$

$= \$42.50$

Amount payable before service charge and GST

$= 4 \times \$85 + 3 \times \$42.50 + 5 \times \$7$

$= \$502.50$

Total amount $= \frac{109}{100} \times \left(\frac{110}{100} \times \$502.50\right)$

$=$ **$602.50** (to 2 d.p.)

4. (i) Amount of commission $= \frac{2}{100} \times \$1\,000\,000$
$=$ **$20 000**

(ii) Amount of commission for first $1 000 000
$= \$20\,000$

Amount of commission for next $1 000 000
$= \frac{1.5}{100} \times \$1\,000\,000$
$= \$15\,000$

$\therefore$ Total amount of commission
$= \$20\,000 + \$15\,000$
$=$ **$35 000**

5. Commission rate for house $= \frac{\$67\,500}{\$3\,750\,000} \times 100\%$
$= 1.8\%$

Commission rate for apartment $= \frac{\$15\,688}{\$848\,000} \times 100\%$
$= 1.85\%$

$\therefore$ The property sold the **apartment** at a higher rate of commission.

6. (a) (i) Total amount received based on 2% per annum simple interest

$= \$\left(10\,000 \times \frac{2}{100} \times 4 + 10\,000\right)$

$= \$10\,800$

$\therefore$ Eric assumes that he would receive 2% per annum simple interest, but the advertisement states "up to 2%".

(ii) Interest paid $= \$10\,640 - \$10\,000$
$= \$640$

Let the interest rate used be $x\%$.

$$10\,000 \times \frac{x}{100} \times 4 = 640$$
$$x = 1.6$$

$\therefore$ The bank used an interest rate of **1.6%**.

(b) OPEN

Total sum $= \$\left(8000 \times \frac{1}{100} \times 4 + 8000\right)$
$= \$8320$

$\therefore$ Rico might have borrowed **$8000** for **4 years**.

7. (a) $80\,000\left(1+\frac{5}{100}\right)^9 = \frac{80\,000(r)(9)}{100} + 80\,000$

$$1.05^9 = 0.09r + 1$$
$$0.09r = 1.05^9 - 1$$
$$r = \frac{1.05^9 - 1}{0.09}$$
$$= 6.13 \text{ (to 3 s.f.)}$$

$\therefore r =$ **6.13**

(b) Amount of money $= \$\left[80\,000(1.05)^9\right]\left[1+\frac{5.5 \div 4}{100}\right]^{6\times4}$

$=$ **$172 240** (to the nearest dollar)

8. Amount of zakat $= \frac{2.5}{100} \times$ PKR 336 000

$=$ **PKR 8400**

9. Amount Dawood sold his crop of mangoes for

$= \frac{100}{5} \times$ PKR 46 250

$=$ **PKR 925 000**

10. (a) Income tax

$=$ PKR 435 000 $+ \frac{27.5}{100} \times$ (PKR 4 800 000 − PKR 3 600 000)

$=$ PKR 765 000 (shown)

(b) Chargeable income

$=$ (PKR 21 000 − PKR 15 000) $\times \frac{100}{2.5} +$ PKR 1 200 000

$=$ **PKR 1 440 000**

11. (a) Tax for the first $20 000 = $0

Tax for the next \$10 000 $= \frac{2}{100} \times \$10\,000$
$= \$200$

Tax for the next \$10 000 $= \frac{3.5}{100} \times \$10\,000$
$= \$350$

Tax for the next \$40 000 $= \frac{7}{100} \times \$40\,000$
$= \$2800$

Tax for the next \$10 000 $= \frac{11.5}{100} \times \$10\,000$
$= \$1150$

$\therefore$ Income tax payable

$= \$200 + \$350 + \$2800 + \$1150$

$= \$4500$ (shown)

(b) Tax for the first $20 000 = $0

Tax for the next $10 000 (at 2%) = $200

Tax for the next $10 000 (at 3.5%) = $350

Tax for the next $40 000 (at 7%) = $2800

Let the income taxed at 7% be $\$x$.

$200 + 350 + \frac{7}{100}x = 3210$

$\frac{7}{100}x = 2660$

$x = 38\ 000$

$\therefore$ Chargeable income

= \$20 000 + \$10 000 + \$10 000 + \$38 000

= **\$78 000**

**12.** Amount of assets = PKR 868 000 − PKR 7400 + PKR 49 000

= PKR 909 600

Amount wife received = $\frac{1}{8}$ × PKR 909 600

= **PKR 113 700**

Amount son received = $\frac{2}{5} \times \frac{7}{8}$ × PKR 909 600

= **PKR 318 360**

Amount daughter received = $\frac{1}{2}$ × PKR 318 360

= **PKR 159 680**

**13.** Total profit

= \$1 288 000 − (\$112 000 + \$80 000 + \$64 000 + \$112 000)

= \$920 000

Ratio of investments: 7 : 5 : 4 : 7

Amount Alan received = $\frac{7}{23}$ × \$920 000

= **\$280 000**

Amount Brian received = $\frac{5}{23}$ × \$920 000

= **\$200 000**

Amount Cooper received = $\frac{4}{23}$ × \$920 000

= **\$160 000**

Amount Dennis received = **\$280 000**

## Mid-year Checkpoint A

**Section A**

**1.** Amount of ushr = $\frac{5}{100}$ × PKR 480 000

= **PKR 24 000** [1]

**2.** Vertical change = −4 − 0 = −4

Horizontal change = 5 − (−3) = 8

$\therefore$ Gradient = $\frac{-4}{8}$ [1]

$= -\mathbf{\frac{1}{2}}$ [1]

**3.** Selling price = $\frac{85}{100} \times \$4600$ [1]

= **\$3910** [1]

**4.** **(a)** $\angle DBC = \angle BDC$ (base $\angle$s of isos. $\triangle$)

= **70°** [1]

**(b)** $\angle DMC = 90°$

$\angle BAC = \angle ACD$ (alt. $\angle$s, $AB$ // $DC$)

= 180° − 90° − 70° ($\angle$ sum of $\triangle$)

= **20°** [1]

**5.** $5x - y = 3$ — (1)

$2x + 2y = 7$ — (2)

(2) ÷ 2: $x + y = \frac{7}{2}$ — (3)

(1) + (3): $6x = \frac{7}{2}$ [1]

$x = 1\frac{1}{12}$ [1]

Substitute $x = 1\frac{1}{12}$ into (3):

$1\frac{1}{12} + y = \frac{7}{2}$

$y = 2\frac{5}{12}$ [1]

$\therefore$ $\mathbf{x = 1\frac{1}{12}, y = 2\frac{5}{12}}$

**6.** $3ay - ab - 4b + 12y = 3ay - ab + 12y - 4b$

$= a(3y - b) + 4(3y - b)$

$= (3y - b)(a + 4)$ [1]

$\therefore$ The factors are $\mathbf{3y - b}$ **and** $\mathbf{a + 4}$. [1]

**7.** $98^2 - 95^2 = (98 + 95)(98 - 95)$ [1]

$= (193)(3)$

$\therefore k =$ **193** [1]

**8.** Amount wife received = $\frac{1}{8}$ × PKR 840 000

= **PKR 105 000**

Amount son received = $\frac{2}{3} \times \frac{7}{8}$ × PKR 840 000

= **PKR 490 000** [1]

Amount daughter received = $\frac{1}{3} \times \frac{7}{8}$ × PKR 840 000

= **PKR 245 000** [1]

**9.** $\frac{7(4-5x)}{3} \geqslant -8$

$28 - 35x \geqslant -24$

$-35x \geqslant -52$ [1]

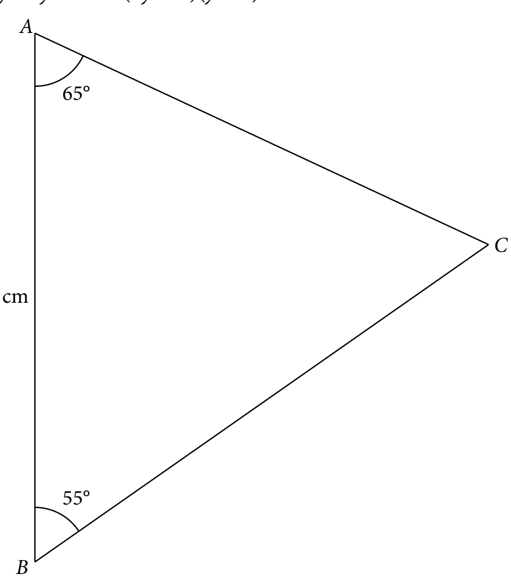
$x \leqslant 1\frac{17}{35}$ [1]

$\therefore$ Greatest rational value of $x$ is $\mathbf{1\frac{17}{35}}$ [1]

**10.** **(a)** $(x + 5)(x^2 - 6) = \mathbf{x^3 + 5x^2 - 6x - 30}$ [2]

**(b)** $4y^2 + y - 18 = (4y + 9)(y - 2)$ [2]

**11.** **(a)**

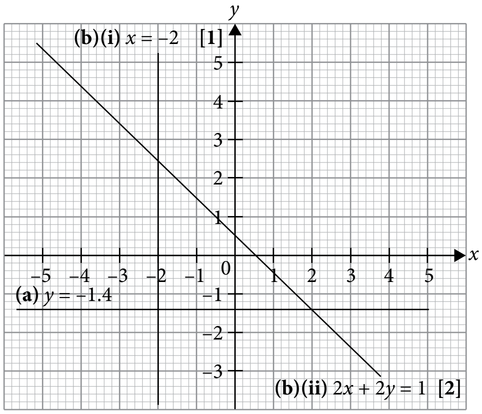



[2]

**(b)** $AC$ = **6.6 cm** [1]

**12.** **(a)** $\boldsymbol{y = -1.4}$ **[1]**

**(b)**




**Section B**

**13.** Since $AB = DC$,

$11x + 6y - 12 = 18y - 5x$

$16x - 12y = 12$

$4x - 3y = 3$ — (1) **[1]**

Since $AD = BC$,

$10y - 2 = 3x + 7y + 7$

$3y = 3x + 9$

$y = x + 3$ — (2) **[1]**

Substitute (2) into (1):

$4x - 3(x + 3) = 3$

$4x - 3x - 9 = 3$

$x = 12$

Substitute $x = 12$ into (2):

$y = 12 + 3$

$= 15$ **[1]**

$AD = 10(15) - 2$

$= 148$ mm

$DC = 18(15) - 5(12)$

$= 210$ mm

Perimeter of rectangle = 2[148 + 210]

= **716 mm** **[1]**

Area of rectangle = (148)(210) $\text{mm}^2$

= **31 080 $\text{mm}^2$** **[1]**

**14.** **(a)** Common difference = $\dfrac{13 - 41}{4}$

$= -7$

$p = 41 - 7 = 34$

$q = 34 - 7 = 27$

$r = 27 - 7 = 20$

$\therefore$ $\boldsymbol{p = 34, q = 27, r = 20}$ **[2]**

**(b)** $T_n = \boldsymbol{48 - 7n}$ **[2]**

**(c)** Let $48 - 7n = -201$.

$-7n = 249$

$n = 35\dfrac{4}{7}$

Since $n$ is not a positive integer, –201 is not a term of this sequence. **[1]**

**(d)** $T_{20} = 48 - 7(20)$

$= \boldsymbol{-92}$ **[1]**

**(e)** $U_n = \dfrac{1}{48 - 7n}$

$U_8 = \dfrac{1}{48 - 7(8)} = -\dfrac{1}{8}$

$U_9 = \dfrac{1}{48 - 7(9)} = -\dfrac{1}{15}$

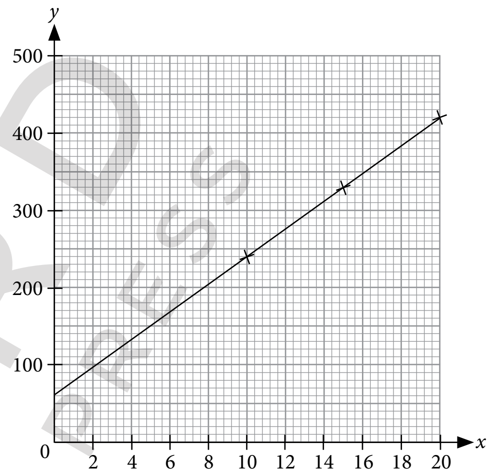
$\therefore$ Sum of the 8th and 9th terms

$= -\dfrac{1}{8} + \left(-\dfrac{1}{15}\right)$ **[1]**

$= -\dfrac{\mathbf{23}}{\mathbf{120}}$ **[1]**

**15.** **(i)** **(10, 240), (15, 330), (20, 420)** **[1]**

**(ii)**




**[3]**

**(iii)** The fixed cost of \$60 could be the minimum amount chargeable before the printing firm takes on the job. **[1]**

**(iv)** Price of each journal if 10 journals are printed

$= \dfrac{\$240}{10}$

$= \$24$

Price of each journal if 15 journals are printed

$= \dfrac{\$330}{15}$

$= \$22$

Price of each journal if 20 journals are printed

$= \dfrac{\$420}{20}$

$= \$21$ **[1]**

The caption is more accurate if it states “The more you order, the less you pay for each journal!” **[1]**

## Mid-year Checkpoint B

**Section A**

**1.** $\boldsymbol{0.95^{\frac{3}{4}}, \sqrt{0.81}, \dfrac{36}{49}, 0.706}$ **[1]**

**2.** $3 - \dfrac{1}{4}x \leqslant \dfrac{4}{5}$

$60 - 5x \leqslant 16$

$-5x \leqslant -44$ **[1]**

$x \geqslant \boldsymbol{8\dfrac{4}{5}}$ **[1]**

**3.** $12ax + 21a - 8bx - 14b = 3a(4x + 7) - 2b(4x + 7)$ **[1]**

$= \boldsymbol{(4x + 7)(3a - 2b)}$ **[1]**

**4.** $y = mx + c$

When $x = \frac{5}{6}$, $y = -\frac{2}{3}$,

$$-\frac{2}{3} = \frac{5}{6}m + c$$

$5m + 6c = -4$

A possible pair of values is $\boldsymbol{m = -\frac{1}{4}}$ and $\boldsymbol{c = -\frac{11}{24}}$. [2]

**5.** $(x - y)^2 = x^2 - 2xy + y^2$

$= 9 - 2(5)$ [1]

$= \mathbf{-1}$ [1]

**6.** $(7n + 3)^2 - (7n - 3)^2 = (49n^2 + 42n + 9) - (49n^2 - 42n + 9)$ [1]

$= 49n^2 + 42n + 9 - 49n^2 + 42n - 9$

$= 84n$, which is a multiple of 84 (shown) [1]

**7.** $3 - x < 15 < 2x + 1$

$3 - x < 15$ and $15 < 2x + 1$

$-x < 12$    $14 < 2x$

$x > -12$ [1]    $x > 7$ [1]

$\therefore \boldsymbol{x > 7}$ [1]

**8.** $y = 5 - \frac{1}{3}x$ — (1)

$\frac{x}{6} - \frac{y}{4} = 1$ — (2)

(1) × 3: $3y = 15 - x$ — (3)

(2) × 12: $2x - 3y = 12$ — (4)

Substitute (3) into (4):

$2x - (15 - x) = 12$ [1]

$2x - 15 + x = 12$

$3x = 27$

$x = 9$

Substitute $x = 9$ into (3):

$3y = 15 - 9$

$= 6$

$y = 2$ [1]

$\therefore \boldsymbol{p = 9, q = 2}$ [1]

**9.** **(a)** Number of sticks in Diagram $n = \mathbf{4n + 1}$ [2]

**(b)** Let $4n + 1 = 203$.

$4n = 202$

$n = 50.5$

Since $n$ is not a positive integer, it is not possible to have a diagram with 203 sticks. [1]

**10.** **(a)** $6x^2y^2 - 8x^3y = \mathbf{2x^2y(3y - 4x)}$ [1]

**(b)** $48p^3 - 3pq^4 = 3p(16p^2 - q^4)$ [1]

$= \mathbf{3p(4p + q^2)(4p - q^2)}$ [1]

**11.** **(a)** Chargeable income

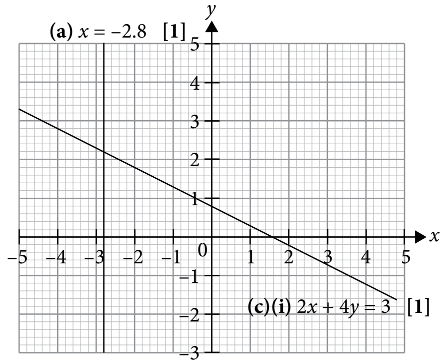
$= (\text{PKR } 29\,250 - \text{PKR } 15\,000) \times \frac{100}{2.5} + \text{PKR } 1\,200\,000$ [1]

= **PKR 1 770 000** [1]

**(b)** Amount of zakat $= \frac{2.5}{100} \times \text{PKR } 86\,000$

= **PKR 2150** [1]

**12.** **(a)**




**(b)** $\boldsymbol{y = 0}$ [1]

**(c)** **(ii)** Gradient $= \mathbf{-\frac{1}{2}}$ [1]

**Section B**

**13.** **(a)** Total amount paid $= \$\left(\frac{1}{5} \times 87\,500 + 48 \times 1960\right)$ [1]

= **$111 580** [1]

**(b)** Total amount paid $= \frac{105}{100} \times \$776$ [1]

= $814.80 [1]

Amount of each monthly payment $= \frac{\$814.80}{12}$

= **$67.90** [1]

**14.** **(i)** $T_5 = \frac{4(5)+1}{195-5(5)}$

$= \mathbf{\frac{21}{170}}$ [1]

**(ii)** $\frac{4k+1}{195-5k} = \frac{5}{33}$

$33(4k + 1) = 5(195 - 5k)$ [1]

$132k + 33 = 975 - 25k$ [1]

$157k = 942$

$k = \mathbf{6}$ [1]

**(iii)** Let $4n + 1 = 195 - 5n$. [1]

$9n = 194$

$n = 21\frac{5}{9}$ [1]

For $T_n$ to be greater than 1, least value of $n$ is **22**. [1]

**15.** **(i)** Aaron waited from 17 35 to 17 48, i.e. **13 minutes**. [1]

**(ii)** **4.6 km** [1]

**(iii)** Consider the journey between 17 48 and 18 10.

Vertical change = 10 km

Horizontal change = 22 min

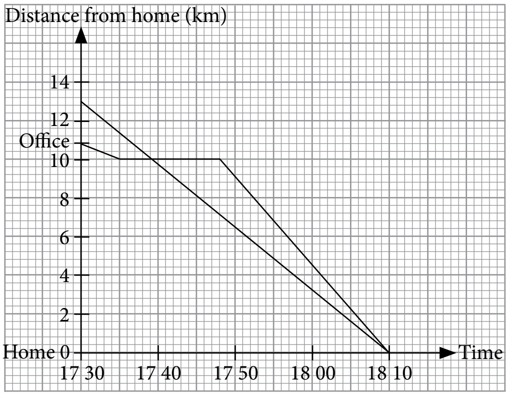
$= \frac{11}{30}$ h

$\therefore$ Gradient $= \frac{10 \text{ km}}{\frac{11}{30} \text{ h}}$

= **27.3 km/h** (to 3 s.f.) [1]

It refers to the speed of the bus. [1]

**(iv) (a)**




[2]

**(b)** OPEN At the point of intersection, they were both 10 km away from home. This occurred at about 17 39. [2]

# 7 Direct and Inverse Proportions

## Worksheet 7A Direct proportion

**1.** **(i)** Height of 1 textbook $= \frac{48}{30}$ cm

$= \mathbf{1.6\ cm}$

**(ii)** Height of 35 textbooks = 35(1.6) cm

$= \mathbf{56\ cm}$

**2.** Number of dumplings $= \frac{15}{2} \times 20$

$= \mathbf{150}$

**3.** Cost of 0.24 kg $= \$\left(\frac{4}{100} \times 240\right)$

$= \mathbf{\$9.60}$

**4.** Amount it charges $= \$\left(\frac{C}{x} \times y\right)$

$= \mathbf{\$\frac{Cy}{x}}$

**5.** Cost of fish food $= \$\left(\frac{53.5}{5} \times 8\right)$

$= \$85.60$

Amount of change = 2($50) – $85.60

$= \mathbf{\$14.40}$

**6.** Amount of fuel $= \frac{1}{2}(65)$ litres

= 32.5 litres

Maximum distance = 32.5(15) km

$= \mathbf{487.5\ km}$

**7.** Amount of carbohydrate $= \frac{7.9}{15} \times 100$ g

$= \mathbf{52.7\ g}$ (to 3 s.f.)

Amount of protein $= \frac{1.0}{15} \times 100$ g

$= \mathbf{6.67\ g}$ (to 3 s.f.)

Amount of fat $= \frac{37.4}{100} \times 15$ g

$= \mathbf{5.61\ g}$

Amount of sodium $= \frac{1083}{100} \times 15$ mg

$= \mathbf{162.45\ mg}$

**8.** Amount based on daily charge = $84 × 10

= $840

Amount based on weekly charge = $550 × 2

= $1100

Amount based on a combination = $550 + 3($84)

= $802

∴ Minimum amount of rental = **$802**

**9.** **(i)** Time taken $= \frac{12\ 000}{24}$ min

= 500 min

> 60 min

∴ Alicia cannot finish typing the manuscript in 1 hour.

**(ii)** Amount author pays $= \$\left(\frac{12\ 000}{360} \times 6\right)$

$= \mathbf{\$200}$

## Worksheet 7B Algebraic and graphical representations of direct proportion

**1.** **(a)**

| $x$ | 2 | 4 | 7 |
|---|---|---|---|
| $y$ | 18 | 36 | 54 |
| $\frac{y}{x}$ | 9 | 9 | 7.71 |

Since $\frac{y}{x} \neq$ constant, $x$ and $y$ are not in direct proportion.

**(b)**

| $x$ | 3 | 5 | 11 |
|---|---|---|---|
| $y$ | 5.1 | 8.5 | 18.7 |
| $\frac{y}{x}$ | 1.7 | 1.7 | 1.7 |

Since $\frac{y}{x}$ = constant, $x$ and $y$ are in direct proportion.

**2.** **(a)** Since the graph of $y$ against $x$ is not a straight line passing through the origin, $x$ and $y$ are not in direct proportion.

**(b)** Since the graph of $y$ against $x$ is not a straight line passing through the origin, $x$ and $y$ are not in direct proportion.

**3.** **(a)**

| $x$ | 8 | 9 | 12 |
|---|---|---|---|
| $y$ | 56 | 63 | 84 |
| $\frac{y}{x}$ | 7 | 7 | 7 |

Since $\frac{y}{x}$ = constant, $x$ and $y$ are in direct proportion. (shown)

**(b)**

| $x$ | 0.3 | 2.5 | 3.9 |
|---|---|---|---|
| $y$ | 5.4 | 45 | 70.2 |
| $\frac{y}{x}$ | 18 | 18 | 18 |

Since $\frac{y}{x}$ = constant, $x$ and $y$ are in direct proportion. (shown)

**4.** **(a)** **Yes**

**(b)** **No**

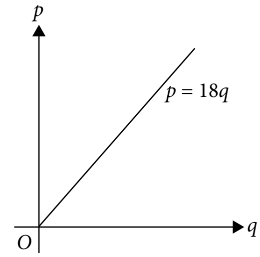
**(c)** **No**

**(d)** **Yes**

5. $y = kx$
When $x = 5$, $y = 4$,
$4 = k(5)$
$k = \frac{4}{5}$
$\therefore y = \frac{4}{5}x$
When $x = 8$, $y = p$,
$p = \frac{4}{5}(8)$
$= 6.4$
When $x = q$, $y = 9.6$,
$\frac{4}{5}q = 9.6$
$q = 12$
$\therefore$ $\boldsymbol{p = 6.4, q = 12}$

6. (a) $y = kx$
When $x = 2$, $y = 14$,
$14 = k(2)$
$k = 7$
$\therefore$ $\boldsymbol{y = 7x}$
(b) When $x = 3$,
$y = 7(3)$
$= \mathbf{21}$
(c) When $y = 20$,
$7x = 20$
$x = \mathbf{2\frac{6}{7}}$

7. $y = kx$
When $x = 12$, $y = 8$,
$8 = 12k$
$k = \frac{2}{3}$
$\therefore y = \frac{2}{3}x$
When $x = 4$,
$y = \frac{2}{3}(4)$
$= \mathbf{2\frac{2}{3}}$

8. (a) $p = kq$
When $p = 6$, $q = \frac{1}{3}$,
$6 = k\left(\frac{1}{3}\right)$
$k = 18$
$\therefore p = 18q$
(i) When $q = 4.5$,
$p = 18(4.5)$
$= \mathbf{81}$
(ii) When $p = 0.9$,
$18q = 0.9$
$q = \mathbf{0.05}$
(b)



9. Since $x$ and $y$ are in direct proportion, when $x$ is halved, $y$ is halved.
$\therefore$ $\boldsymbol{y = 2}$

10. (a) $x = kf$
When $f = 3$, $x = 4.5$,
$4.5 = 3k$
$k = 1.5$
$\therefore$ $\boldsymbol{x = 1.5f}$
(b) When $f = 5$,
$x = 1.5(5)$
$= 7.5$
$\therefore$ Original length of spring = (10 – 7.5) cm
= **2.5 cm**

11. $I = kP$
When $I = 950$, $P = 25\,000$,
$950 = k(25\,000)$
$k = 0.038$
$\therefore I = 0.038P$
When $P = 30\,000$,
$I = 0.038(30\,000)$
$= 1140$
$\therefore$ Interest charged = **$1140**

12.

| Volume (ml) | 88 | 100 | 250 |
|---|---|---|---|
| Cost ($) | 4.95 | 5.55 | 12.80 |
| $\frac{\text{Cost (\$)}}{\text{Volume (ml)}}$ | 0.056 25 | 0.0555 | 0.0512 |

Since $\frac{\text{cost (\$)}}{\text{volume (ml)}} \neq$ constant, the cost of the sunscreen is not directly proportional to the volume of the sunscreen. (shown)

13. (a)

| $n$ | 10 | 25 | 40 |
|---|---|---|---|
| $C$ | 248 | 620 | 992 |
| $\frac{C}{n}$ | 24.8 | 24.8 | 24.8 |

Since $\frac{C}{n}$ = constant, $C$ is directly proportional to $n$. (shown)
(b) $\boldsymbol{C = 24.8n}$
(c) Amount of commission = $[24.8(36)]
= **$892.80**

14. Since $V$ and $T$ are in direct proportion, when $T$ is doubled, $V$ is also doubled.
Lydia is correct.
The proportion between $V$ and $T$ holds only when either quantity is multiplied (or divided) by a constant factor. It does not apply to addition (or subtraction) of a value.
Howard is incorrect.

## Worksheet 7C Other forms of direct proportion

**1.**

| $x^3$ | 8 | 64 | 216 |
|---|---|---|---|
| $y$ | 2 | 16 | 48 |
| $\frac{y}{x^3}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{2}{9}$ |

Since $\frac{y}{x^3} \neq$ constant, $x^3$ and $y$ are not in direct proportion.

**2.**

| $x$ | 3 | 12 | 27 |
|---|---|---|---|
| $y$ | 9 | 36 | 81 |
| $\frac{y^2}{x}$ | 3 | 3 | 3 |

Since $\frac{y^2}{x}$ = constant, $x$ and $y^2$ are in direct proportion. (shown)

**3.** **(a)** $\boldsymbol{x^3}$ **and** $\boldsymbol{y}$

**(b)** $\boldsymbol{x}$ **and** $\boldsymbol{y}$

**(c)** $\boldsymbol{\sqrt{x}}$ **and** $\boldsymbol{y^3}$

**(d)** $\boldsymbol{\frac{1}{x}}$ **and** $\boldsymbol{\sqrt[3]{y}}$

**4.** $y = kx^2$

When $x = 10$, $y = 50$,

$50 = k(10)^2$

$k = \frac{1}{2}$

$\therefore y = \frac{1}{2}x^2$

When $x = 4$, $y = a$,

$a = \frac{1}{2}(4)^2$

$= 8$

When $x = b$, $y = 128$,

$128 = \frac{1}{2}b^2$

$b^2 = 256$

$b = \pm 16$

$\therefore \boldsymbol{a = 8, b = \pm 16}$

**5.** $y = k\sqrt{x+1}$

When $x = 3$, $y = 8$,

$8 = k\sqrt{3+1}$

$k = 4$

$\therefore y = 4\sqrt{x+1}$

When $x = 15$, $y = p$,

$p = 4\sqrt{15+1}$

$= 16$

When $x = q$, $y = 20$,

$20 = 4\sqrt{q+1}$

$\sqrt{q+1} = 5$

$q + 1 = 25$

$q = 24$

$\therefore \boldsymbol{k = 4, p = 16, q = 24}$

**6.** **(a)** $y = kx^3$

When $x = 8$, $y = 64$,

$64 = k(8)^3$

$k = \frac{1}{8}$

$\therefore \boldsymbol{y = \frac{1}{8}x^3}$

**(b)** When $x = \frac{1}{2}$,

$y = \frac{1}{8}\left(\frac{1}{2}\right)^3$

$= \boldsymbol{\frac{1}{64}}$

**7.** $y = k\sqrt{x}$

When $x = 9$, $y = 5$,

$5 = k\sqrt{9}$

$k = \frac{5}{3}$

$\therefore y = \frac{5}{3}\sqrt{x}$

When $x = 36$,

$y = \frac{5}{3}\sqrt{36}$

$= \boldsymbol{10}$

**8.** **(a)** **(i)** $y = k\sqrt[3]{x}$

When $x = 27$, $y = 9$,

$9 = k\sqrt[3]{27}$

$k = 3$

$\therefore \boldsymbol{y = 3\sqrt[3]{x}}$

**(ii)** When $x = 1000$,

$y = 3\sqrt[3]{1000}$

$= \boldsymbol{30}$

**(iii)** When $y = 2$,

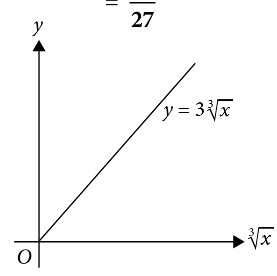
$2 = 3\sqrt[3]{x}$

$\sqrt[3]{x} = \frac{2}{3}$

$x = \left(\frac{2}{3}\right)^3$

$= \boldsymbol{\frac{8}{27}}$

**(b)**




**9.** **(a)** **(i)** $y = k(2x + 1)^3$

When $x = 3$, $y = 49$,

$49 = k[2(3) + 1]^3$

$k = \frac{1}{7}$

$\therefore \boldsymbol{y = \frac{1}{7}(2x + 1)^3}$

**(ii)** When $x = \frac{1}{2}$,

$y = \frac{1}{7}\left[2\left(\frac{1}{2}\right)+1\right]^3$

$= \mathbf{1\frac{1}{7}}$

**(iii)** When $y = 16\ 807$,

$16\ 807 = \frac{1}{7}(2x+1)^3$

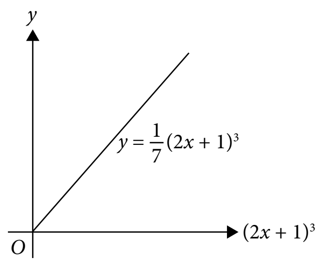
$(2x + 1)^3 = 117\ 649$

$2x + 1 = 49$

$2x = 48$

$x = \mathbf{24}$

**(b)**




**10.** $y^3 = kx^2$

When $x = 4$, $y = 2$,

$2^3 = k(4)^2$

$k = \frac{1}{2}$

$\therefore y^3 = \frac{1}{2}x^2$

When $y = 8$,

$8^3 = \frac{1}{2}x^2$

$x^2 = 1024$

$x = \mathbf{\pm 32}$

**11.** $y = kx^2$

When $x = a$, $y = 7$,

$7 = ka^2$

When $x = 2a$,

$y = k(2a)^2$

$= 4ka^2$

$= \mathbf{28}$

**12.** $y = kx^2$

When $x = 3$,

$y = k(3)^2$

$= 9k$

When $x = 6$,

$y = k(6)^2$

$= 36k$

$36k - 9k = 9$

$27k = 9$

$k = \frac{1}{3}$

$\therefore y = \frac{1}{3}x^2$

When $x = 9$,

$y = \frac{1}{3}(9)^2$

$= \mathbf{27}$

**13.** **(a)** $m = kr^3$

When $r = 0.7$, $m = 10.78$,

$10.78 = k(0.7)^3$

$k = \frac{220}{7}$

$\therefore \boldsymbol{m = \frac{220}{7}r^3}$

**(b)** When $r = 0.8$,

$m = \frac{220}{7}(0.8)^3$

$= 16.1$ (to 3 s.f.)

$\therefore$ Mass of ball bearing is **16.1 g**

**14.** **(a)** $s = kt^2$

When $t = 3$, $s = 45$,

$45 = k(3)^2$

$k = 5$

$\therefore \boldsymbol{s = 5t^2}$

**(b)** $s = 5t^2$

When $t = 4$, $s = 5(4)^2 = \mathbf{80}$.

$\therefore$ The particle hits the ground when $t = 4$.

**15.** **(a)** $T = k\sqrt{L}$

When $L = 0.36$, $T = 1.2$,

$1.2 = k\sqrt{0.36}$

$k = 2$

$\therefore \boldsymbol{T = 2\sqrt{L}}$

**(b)** When $T = 1.1$,

$1.1 = 2\sqrt{L}$

$\sqrt{L} = 0.55$

$L = 0.3025$

$\therefore$ Length of pendulum = **0.3025 m**

**(c)** Since $\frac{T_1}{T_2} = \frac{3}{2}$,

$\frac{k\sqrt{L_1}}{k\sqrt{L_2}} = \frac{3}{2}$

$\sqrt{\frac{L_1}{L_2}} = \frac{3}{2}$

$\frac{L_1}{L_2} = \frac{9}{4}$

$\therefore$ The ratio of the lengths is **9 : 4**.

## Challenge Myself!

**16.** **(a)** **False.**

$y$ is directly proportional to $x^2$ if and only if the equation is in the form $y = kx^2$.

**(b)** **True.**

The equation can be written as $x^2 = y - 1$, which is in the form $x^2 = k(y - 1)$.

**(c)** **False.**

$y$ is linearly related to $x^2$.

**(d)** **False.**

$x$ is linearly related to $\sqrt{y-1}$ ; in fact, $x$ is directly proportional to $\sqrt{y-1}$ .

## Worksheet 7D Inverse proportion

1. Number of robots = $5 \times 2$
$= \mathbf{10}$

2. 20 workers take 4 days.
1 worker takes $20 \times 4 = 80$ days.
25 workers take $\frac{80}{25}$ = **3.2 days**.

3. 4 painters take 3.5 hours.
1 painter takes $4 \times 3.5 = 14$ hours.
7 painters take $\frac{14}{7}$ = **2 hours**.

4. **(i)** Time taken = $50 \times 20$ min
= **1000 min**
**(ii)** Time taken = $\frac{1000}{8}$ min
= 125 min
= **2 h 5 min**

5. 8 pipes take 30 minutes.
1 pipe takes $8 \times 30 = 240$ minutes.
6 pipes take $\frac{240}{6}$ = 40 minutes.
∴ The remaining pipes can empty the tank in 40 minutes.

6. 3 seamstresses sew 6 blouses in 2 hours.
3 seamstresses sew 15 blouses in $\frac{15}{6} \times 2$ = 5 hours.
2 seamstresses sew 15 blouses in $\frac{3}{2} \times 5$ = **7.5 hours**.

7. **(i)** 12 workers take 10 days.
$\frac{12}{8} \times 10$ = 15 workers take 8 days.
∴ An additional **3** workers are needed.
**(ii)** Assume that each worker works at the same rate and that each of them works for 6 hours a day.

### Challenge Myself!

8. Assume the half-hour break is from 2.30 p.m. to 3 p.m.
Total man-hours required to complete the work = $6 \times 3$
= 18
Man-hours put in by 2.30 p.m. = $2 \times 3$
= 6
Man-hours left to complete the work = 18 – 6
= 12

| Time | Man-hours | Cost |
|---|---|---|
| 12.30 p.m. to 2.30 p.m. | 6 | 6 × \$45 = \$270 |
| 3 p.m. to 5 p.m. | 4 | 4 × \$45 = \$180 |
| 5 p.m. to 9 p.m. | 8 | 8 × 2 × \$30 = \$480 |
| Total | 18 | \$930 |

∴ The owner has to pay **\$930**.

## Worksheet 7E Algebraic and graphical representations of inverse proportion

1. (a)

| $x$ | 5 | 10 | 15 |
|---|---|---|---|
| $y$ | 18 | 9 | 6 |
| $xy$ | 90 | 90 | 90 |

Since $xy$ = constant, $x$ and $y$ are in inverse proportion.

(b)

| $x$ | 6 | 9 | 12 |
|---|---|---|---|
| $y$ | 1.2 | 0.9 | 0.6 |
| $xy$ | 7.2 | 8.1 | 7.2 |

Since $xy \neq$ constant, $x$ and $y$ are not in inverse proportion.

2. (a)

| $x$ | 0.5 | 1 | 2 | 4 | 5 |
|---|---|---|---|---|---|
| $y$ | 4 | 2 | 1 | 0.5 | 0.4 |
| $xy$ | 2 | 2 | 2 | 2 | 2 |

Since $xy$ = constant, $x$ and $y$ are in inverse proportion.

(b) Since the graph of $y$ against $\frac{1}{x}$ is not a straight line passing through the origin, $x$ and $y$ are not in inverse proportion.

3. (a)

| $x$ | 2 | 4 | 8 |
|---|---|---|---|
| $y$ | 12 | 6 | 3 |
| $xy$ | 24 | 24 | 24 |

Since $xy$ = constant, $x$ and $y$ are in inverse proportion. (shown)

(b)

| $x$ | 5 | 8 | 11 |
|---|---|---|---|
| $y$ | $7\frac{1}{5}$ | $4\frac{1}{2}$ | $3\frac{3}{11}$ |
| $xy$ | 36 | 36 | 36 |

Since $xy$ = constant, $x$ and $y$ are in inverse proportion. (shown)

4. (a) **Yes**
(b) **No**
(c) **Yes**
(d) **No**

5. $y = \frac{k}{x}$
When $x = 3$, $y = 8$,
$8 = \frac{k}{3}$
$k = 24$
$\therefore y = \frac{24}{x}$
When $x = 4.8$, $y = p$,
$p = \frac{24}{4.8}$
$= 5$
When $x = q$, $y = 1.5$,
$1.5 = \frac{24}{q}$
$q = 16$
$\therefore \mathbf{p = 5, q = 16}$

**6.** **(a)** $y = \dfrac{k}{x}$

When $x = 10$, $y = 5$,

$5 = \dfrac{k}{10}$

$k = 50$

$\therefore \boldsymbol{y = \dfrac{50}{x}}$

**(b)** When $x = 4$,

$y = \dfrac{50}{4}$

$= \mathbf{12.5}$

**(c)** When $y = 20$,

$20 = \dfrac{50}{x}$

$x = \mathbf{2.5}$

**7.** $y = \dfrac{k}{x}$

When $x = 12$, $y = 8$,

$8 = \dfrac{k}{12}$

$k = 96$

$\therefore y = \dfrac{96}{x}$

When $x = 4$,

$y = \dfrac{96}{4}$

$= \mathbf{24}$

**8.** **(a)** $p = \dfrac{k}{q}$

When $p = \dfrac{1}{2}$, $q = \dfrac{3}{4}$,

$\dfrac{1}{2} = \dfrac{k}{\frac{3}{4}}$

$k = \dfrac{3}{8}$

$\therefore p = \dfrac{3}{8q}$

**(i)** When $q = \dfrac{5}{6}$,

$p = \dfrac{3}{8\left(\frac{5}{6}\right)}$

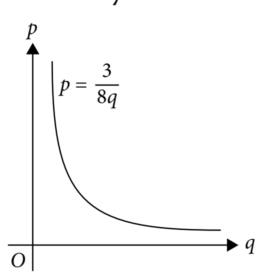
$= \mathbf{\dfrac{9}{20}}$

**(ii)** When $p = \dfrac{7}{8}$,

$\dfrac{7}{8} = \dfrac{3}{8q}$

$q = \mathbf{\dfrac{3}{7}}$

**(b)**




**9.** Since $x$ and $y$ are in inverse proportion, when $x$ is tripled, $y$ is reduced to one-third of its original value.

$\therefore y = \dfrac{1}{3}\left(\dfrac{4}{5}\right) = \mathbf{\dfrac{4}{15}}$

**10.** **(a)** $N = \dfrac{k}{x}$

When $N = 50$, $x = 12$,

$50 = \dfrac{k}{12}$

$k = 600$

$\therefore \boldsymbol{N = \dfrac{600}{x}}$

**(b)** When $N = 120$,

$120 = \dfrac{600}{x}$

$x = 5$

$\therefore$ A sack of corn can feed 120 chickens for **5 days**.

**(c)** When $x = 7$,

$N = \dfrac{600}{7}$

$= 85$ (round down to the nearest integer)

$\therefore$ There are **85** chickens on the farm.

**11.** $V = \dfrac{k}{P}$

When $V = 2.5$, $P = 240$,

$2.5 = \dfrac{k}{240}$

$k = 600$

$\therefore V = \dfrac{600}{P}$

**(a)** When $P = 180$,

$V = \dfrac{600}{180}$

$= 3.33$ (to 3 s.f.)

$\therefore$ Volume = **3.33 m**$^3$

**(b)** When $V = 3.5$,

$3.5 = \dfrac{600}{P}$

$P = 171$ (to 3 s.f.)

$\therefore$ Pressure = **171 pascals**

**12.** **(a)** $Fd = k$

$F = \dfrac{k}{d}$

When $d = 1.8$, $F = 6$,

$6 = \dfrac{k}{1.8}$

$k = \dfrac{54}{5}$

$\therefore \boldsymbol{F = \dfrac{54}{5d}}$

**(b)** When $d = 3.2$,

$F = \dfrac{54}{5(3.2)}$

$= 3.375$

$\therefore$ Force needed = **3.375 newtons**

**(c)** When $F = 5$,

$5 = \dfrac{54}{5d}$

$d = 2.16$

$\therefore$ Distance moved = **2.16 m**

13. (a)

| $x$ | 2 | 5 | 10 |
|---|---|---|---|
| $T$ | 3.75 | 1.5 | 0.75 |
| $Tx$ | 7.5 | 7.5 | 7.5 |

Since $Tx$ = constant, $T$ is inversely proportional to $x$. (shown)

(b) $\boldsymbol{T = \frac{15}{2x}}$

(c) When $x = 6$,
$T = \frac{15}{2(6)}$
$= 1.25$
$\therefore$ 6 workers take **1.25 h**.

(d) Amount of time taken $= \frac{500}{20} \times 1.25$ h
$=$ **31.25 h**

## Challenge Myself!

14. In 1 hour,

Patricia completes $\frac{1}{3} = \frac{8}{24}$ of the work,

Queenie completes $\frac{1}{8} = \frac{3}{24}$ of the work,

Rosa completes $\frac{1}{4} = \frac{6}{24}$ of the work.

They should receive payment in the ratio 8 : 3 : 6.

Amount Patricia should receive $= \frac{8}{17} \times \$340$ = **$160**

Amount Queenie should receive $= \frac{3}{17} \times \$340$ = **$60**

Amount Rosa should receive $= \frac{4}{17} \times \$340$ = **$80**

## Worksheet 7F Other forms of inverse proportion

1.

| $x$ | 4 | 25 | 50 |
|---|---|---|---|
| $y^2$ | 25 | 4 | 1 |
| $xy^2$ | 100 | 100 | 50 |

Since $xy^2 \neq$ constant, $x$ and $y^2$ are not in inverse proportion.

2.

| $\sqrt{x}$ | 1 | 4 | 7 |
|---|---|---|---|
| $y$ | 4 | 1 | $\frac{4}{7}$ |
| $y\sqrt{x}$ | 4 | 4 | 4 |

Since $y\sqrt{x}$ = constant, $\sqrt{x}$ and $y$ are in inverse proportion. (shown)

3. (a) $\boldsymbol{x^3}$ **and** $\boldsymbol{y}$
(b) $\boldsymbol{\sqrt{x}}$ **and** $\boldsymbol{y}$
(c) $\boldsymbol{x}$ **and** $\boldsymbol{y^3}$
(d) $\boldsymbol{\frac{1}{x}}$ **and** $\boldsymbol{\sqrt[3]{y}}$

4. $y = \frac{k}{\sqrt[3]{x}}$
When $x = 27$, $y = 1.5$,
$1.5 = \frac{k}{\sqrt[3]{27}}$
$k = \frac{9}{2}$
$\therefore y = \frac{9}{2\sqrt[3]{x}}$
When $x = 64$, $y = a$,
$a = \frac{9}{2\sqrt[3]{64}}$
$= 1.125$
When $x = b$, $y = 0.45$,
$0.45 = \frac{9}{2\sqrt[3]{b}}$
$\sqrt[3]{b} = 10$
$b = 1000$
$\therefore$ $\boldsymbol{a = 1.125, b = 1000}$

5. $y = \frac{k}{(2x-1)^2}$
When $x = 2$, $y = \frac{2}{3}$,
$\frac{2}{3} = \frac{k}{[2(2)-1]^2}$
$k = 6$
$\therefore y = \frac{6}{(2x-1)^2}$
When $x = 3$, $y = p$,
$p = \frac{6}{[2(3)-1]^2}$
$= \frac{6}{25}$
When $x = q$, $y = 600$,
$600 = \frac{6}{(2q-1)^2}$
$(2q-1)^2 = \frac{1}{100}$
$2q - 1 = \frac{1}{10}$ or $2q - 1 = -\frac{1}{10}$
$2q = \frac{11}{10}$ $\quad$ $2q = \frac{9}{10}$
$q = \frac{11}{20}$ $\quad$ $q = \frac{9}{20}$
$\therefore$ $\boldsymbol{k = 6, p = \frac{6}{25}, q = \frac{11}{20}}$ **or** $\boldsymbol{\frac{9}{20}}$

6. $y = \frac{k}{x^3}$
When $x = 2$, $y = 5$,
$5 = \frac{k}{2^3}$
$k = 40$
$\therefore y = \frac{40}{x^3}$
When $x = 3$,
$y = \frac{40}{3^3}$
$= \boldsymbol{1\frac{13}{27}}$

7. **(a)** $y = \frac{k}{x^2}$

When $x = 3$, $y = 9$,

$9 = \frac{k}{3^2}$

$k = 81$

$\therefore \boldsymbol{y = \frac{81}{x^2}}$

**(b)** When $x = 6$,

$y = \frac{81}{6^2}$

$= \boldsymbol{2\frac{1}{4}}$

**(c)** When $y = 100$,

$100 = \frac{81}{x^2}$

$x^2 = \frac{81}{100}$

$x = \boldsymbol{\pm\frac{9}{10}}$

8. **(a)** **(i)** $y = \frac{k}{\sqrt{x}}$

When $x = 64$, $y = \frac{1}{4}$,

$\frac{1}{4} = \frac{k}{\sqrt{64}}$

$k = 2$

$\therefore \boldsymbol{y = \frac{2}{\sqrt{x}}}$

**(ii)** When $x = 256$,

$y = \frac{2}{\sqrt{256}}$

$= \boldsymbol{\frac{1}{8}}$

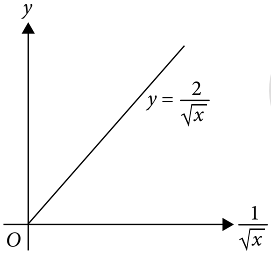
**(iii)** When $y = 9$,

$9 = \frac{2}{\sqrt{x}}$

$\sqrt{x} = \frac{2}{9}$

$x = \boldsymbol{\frac{4}{81}}$

**(b)**




9. **(a)** **(i)** $p^2 = \frac{k}{q+1}$

When $q = 8$, $p = 2$,

$2^2 = \frac{k}{8+1}$

$k = 36$

$\therefore \boldsymbol{p^2 = \frac{36}{q+1}}$

**(ii)** When $q = 3$,

$p^2 = \frac{36}{3+1}$

$= 9$

$p = \boldsymbol{\pm 3}$

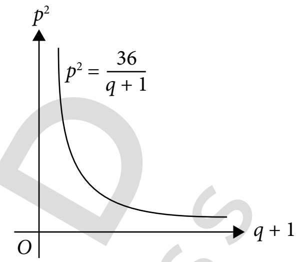
**(iii)** When $p = \frac{1}{5}$,

$\left(\frac{1}{5}\right)^2 = \frac{36}{q+1}$

$q + 1 = 900$

$q = \mathbf{899}$

**(b)**




10. $y - 4 = \frac{k}{x-4}$

When $x = 10$, $y = 28$,

$28 - 4 = \frac{k}{10-4}$

$k = 144$

$\therefore y - 4 = \frac{144}{x-4}$

**(a)** **(i)** When $x = 12$,

$y - 4 = \frac{144}{12-4}$

$y = \mathbf{22}$

**(ii)** When $y = 4.5$,

$4.5 - 4 = \frac{144}{x-4}$

$x - 4 = 288$

$x = \mathbf{292}$

**(b)** **No**

11. $x = \frac{h}{y^2}$

When $y = a$, $x = k$,

$k = \frac{h}{a^2}$

When $y = 4a$,

$x = \frac{h}{(4a)^2}$

$= \frac{h}{16a^2}$

$= \boldsymbol{\frac{1}{16}k}$

12. $p = \frac{k}{5q-1}$

When $q = 2$,

$p = \frac{k}{5(2)-1}$

$= \frac{k}{9}$

When $q = 5$,

$$p = \frac{k}{5(5)-1}$$
$$= \frac{k}{24}$$
$$\frac{k}{9} - \frac{k}{24} = \frac{5}{27}$$
$$\frac{5k}{72} = \frac{5}{27}$$
$$k = \frac{8}{3}$$
$$\therefore p = \frac{8}{3(5q-1)}$$

When $q = 1$,

$$p = \frac{8}{3[5(1)-1]}$$
$$= \mathbf{\frac{2}{3}}$$

**13.** $y = \dfrac{k}{x^3}$

When $x = 2$,

$$y = \frac{k}{2^3}$$
$$= \frac{k}{8}$$

When $x = 4$,

$$y = \frac{k}{4^3}$$
$$= \frac{k}{64}$$
$$\frac{k}{8} + \frac{k}{64} = 13.5$$
$$\frac{9k}{64} = 13.5$$
$$k = 96$$
$$y = \frac{96}{x^3}$$

When $x = 6$,

$$y = \frac{96}{6^3}$$
$$= \mathbf{\frac{4}{9}}$$

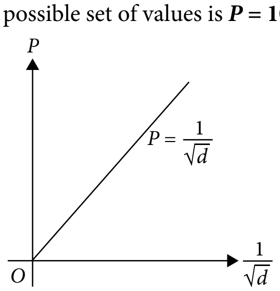
**14.** OPEN **(a)** $P\sqrt{d} = k$

Let $P = 100$ and $d = 4$.

$$100\sqrt{4} = k$$
$$k = 200$$

$\therefore$ A possible set of values is $\boldsymbol{P = 100}$, $\boldsymbol{d = 4}$ and $\boldsymbol{k = 200}$.

**(b) (i)**




**(ii)** As the quantity demanded, $d$, increases, the value of $\dfrac{1}{\sqrt{d}}$ decreases, so the price $P$ decreases.

**15.** $F = \dfrac{k}{d^2}$

When $d = a$,

$$F = \frac{k}{a^2}$$

When $d = \dfrac{5}{4}a$,

$$F = \frac{k}{\left(\frac{5}{4}a\right)^2}$$
$$= \frac{k}{\frac{25}{16}a^2}$$
$$= \frac{16k}{25a^2}$$

$$\text{Percentage change} = \frac{\frac{16k}{25a^2} - \frac{k}{a^2}}{\frac{k}{a^2}} \times 100\%$$
$$= \frac{-\frac{9k}{25a^2}}{\frac{k}{a^2}} \times 100\%$$
$$= -36\%$$

$\therefore$ The force **decreases** by **36%**.

## Challenge Myself!

**16.** $y^2 = \dfrac{k}{x^m}$

When $x = 1$, $y = 2\frac{1}{2}$,

$$\left(2\frac{1}{2}\right)^2 = \frac{k}{1^m}$$
$$k = \frac{25}{4}$$
$$y^2 = \frac{25}{4x^m}$$

When $x = 0.25$, $y = 5$,

$$5^2 = \frac{25}{4(0.25^m)}$$
$$0.25^m = \frac{1}{4}$$
$$\left(\frac{1}{4}\right)^m = \frac{1}{4}$$
$$m = 1$$
$$y^2 = \frac{25}{4x}$$

When $x = 0.64$, $y = p$,

$$p^2 = \frac{25}{4(0.64)}$$
$$= \frac{625}{64}$$
$$p = 3\frac{1}{8}$$

When $x = q$, $y = 1\frac{2}{3}$,

$\left(1\frac{2}{3}\right)^2 = \frac{25}{4q}$

$q = 2\frac{1}{4}$

$\therefore \boldsymbol{p = 3\frac{1}{8}, q = 2\frac{1}{4}}$

## Review Exercise 7

**1.** **(i)** Amount of time $= \frac{45}{2} \times 24$

$= 540$ min

$= \mathbf{9\ h}$

**(ii)** Assume that she takes the same amount of time to design each poster.

**2.** **(a)** $p = k\sqrt[3]{q}$

When $q = 125$, $p = 3$,

$3 = k\sqrt[3]{125}$

$k = \frac{3}{5}$

$\therefore \boldsymbol{p = \frac{3}{5}\sqrt[3]{q}}$

**(b)** When $p = 0.2$,

$0.2 = \frac{3}{5}\sqrt[3]{q}$

$\sqrt[3]{q} = \frac{1}{3}$

$q = \boldsymbol{\frac{1}{27}}$

**3.** $x = \frac{y}{k}$

When $y = 17$,

$x = \frac{17}{k}$

When $y = 20$,

$x = \frac{20}{k}$

$\frac{20}{k} - \frac{17}{k} = 4$

$\frac{3}{k} = 4$

$k = \frac{3}{4}$

$\frac{y}{x} = \frac{3}{4}$

When $x = 68$,

$y = \frac{3}{4} \times 68$

$= \mathbf{51}$

**4.** **(a)**

| $x$ | 2 | 3 | 5 | 7 |
|---|---|---|---|---|
| $m$ | 12 | 40.5 | 187.5 | 514.5 |
| $\frac{m}{x}$ | 6 | 13.5 | 37.5 | 73.5 |

Since $\frac{m}{x} \neq$ constant, $m$ is not directly proportional to $x$. (shown)

**(b)**

| $x^3$ | 8 | 27 | 125 | 343 |
|---|---|---|---|---|
| $m$ | 12 | 40.5 | 187.5 | 514.5 |
| $\frac{m}{x^3}$ | 1.5 | 1.5 | 1.5 | 1.5 |

Since $\frac{m}{x^3}$ = constant, $m$ is directly proportional to $x^3$.

**(c)** $\frac{m}{x^3} = 1.5$

When $x = 6$,

$\frac{m}{6^3} = 1.5$

$m = 324$

$\therefore$ Mass of object is **324 g**

**5.** OPEN **(a)** $\boldsymbol{y = 4x}$

**(b)** $\boldsymbol{y = 5\sqrt{x}}$

**(c)** $\boldsymbol{y = \frac{3}{\sqrt[3]{x}}}$

**(d)** $\boldsymbol{y = 6x}$

**6.** 6 workers take 1.5 h.

8 workers take $\frac{1.5}{8} \times 6 = \mathbf{1.125\ h}$.

**7.** **(a)** **(i)** $y = \frac{k}{\sqrt[3]{x}}$

When $x = 64$, $y = 3$,

$3 = \frac{k}{\sqrt[3]{64}}$

$k = 12$

$\boldsymbol{y = \frac{12}{\sqrt[3]{x}}}$

**(ii)** When $x = 343$,

$y = \frac{12}{\sqrt[3]{343}}$

$= \boldsymbol{1\frac{5}{7}}$

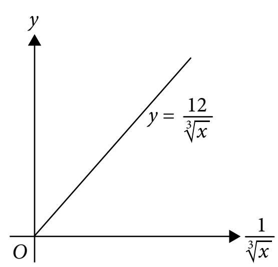
**(iii)** When $y = 5$,

$5 = \frac{12}{\sqrt[3]{x}}$

$\sqrt[3]{x} = \frac{12}{5}$

$x = \boldsymbol{13\frac{103}{125}}$

**(b)**




**8.** **(i)** $I = \frac{k}{d^2}$

As the value of $d$ increases, the value of $I$ **decreases**.

**(ii)** When the distance is $0.75d$,

$$I = \frac{k}{(0.75d)^2}$$
$$= \frac{16k}{9d^2}$$

$$\text{Percentage change} = \frac{\frac{16k}{9d^2} - \frac{k}{d^2}}{\frac{k}{d^2}} \times 100\%$$
$$= \frac{\frac{7k}{9d^2}}{\frac{k}{d^2}} \times 100\%$$
$$= 77.8\% \text{ (to 3 s.f.)}$$

When the distance between the object and the light source decreases by 25%, the intensity of light increases by 77.8%.

# 8 Congruence and Similarity

## Worksheet 8A Congruent figures

**1.** **(a)** **Yes**
**(b)** **No**
**(c)** **Yes**
**(d)** **Yes**

**2.** **(a)** $AB = \boldsymbol{PQ} = \mathbf{44\ cm}$
**(b)** $BC = \boldsymbol{QR} = \mathbf{41\ cm}$
**(c)** $AC = \boldsymbol{PR} = \mathbf{27\ cm}$
**(d)** $\angle ABC = \angle \boldsymbol{PQR} = \mathbf{37°}$
**(e)** $\angle BCA = \angle \boldsymbol{QRP} = \mathbf{78°}$
**(f)** $\angle CAB = \angle \boldsymbol{RPQ} = \mathbf{65°}$

**3.** **(a)** $BC = EF = \mathbf{10\ cm}$
**(b)** $\angle DFE = \angle ACB = \mathbf{30°}$
**(c)** $\angle EDF = \angle BAC = \mathbf{60°}$

**4.** **(a)** $\angle PQR = \angle ABC$
$y° = 32°$
$\therefore y = \mathbf{32}$
**(b)** $\angle ACB = \angle PRQ$
$x° = 180° - 32° - 51°$ (∠ sum of a △)
$= 97°$
$\therefore x = \mathbf{97}$

**5.** **(a)** $AB = \boldsymbol{RS} = \mathbf{25\ cm}$
**(b)** $BC = \boldsymbol{SP} = \mathbf{28\ cm}$
**(c)** $CD = \boldsymbol{PQ} = \mathbf{44\ cm}$
**(d)** $DA = \boldsymbol{QR} = \mathbf{27\ cm}$
**(e)** Quadrilateral *ABCD* is congruent to quadrilateral ***RSPQ***.

**6.** **(a)** $BC = EH = \mathbf{4.1\ cm}$
**(b)** $\angle EFG = \angle BAD = \mathbf{93°}$

**7.** **(a)** $PS = P'S' = 41$ cm
$P'Q' = PQ = 47$ cm
$\therefore PS + P'Q' = (41 + 47)$ cm
$= \mathbf{88\ cm}$
**(b)** $\angle QRS = \angle Q'R'S'$
$= 360° - 131°$ (∠s at a pt.)
$= 229°$
$\angle PQR = 360° - 65° - 229° - 36°$ (∠ sum of a quad.)
$= \mathbf{30°}$

**8.** **(a)** $AB = PR = 5.6$ cm
$BC = RQ = 3.3$ cm
$AC = PQ = 6.5$ cm
$\therefore \triangle \boldsymbol{ABC} \equiv \triangle \boldsymbol{PRQ}$
**(b)** In $\triangle PQR$, the side opposite the 84° angle is 37 cm, not 42 cm.
∴ The triangles are not congruent.

**9.** **(a)** $AB = CB = ED = 49$ cm
$\therefore AC = (49 + 49)$ cm
$= \mathbf{98\ cm}$
**(b)** $\angle ECD = 180° - 90° - 31°$ (∠ sum of a △)
$= 59°$
$\angle AEB = \angle CEB = \angle ECD = 59°$
$\therefore \angle AEC = 59° + 59°$
$= \mathbf{118°}$

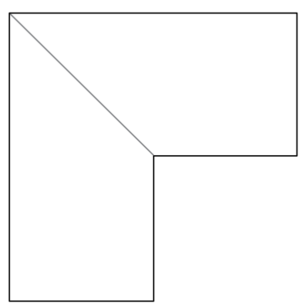
**10.** $RS = PQ$
$x = 15.6$
$\angle SQR = \angle QSP$
$y° = 180° - 63° - 23°$ (∠ sum of a △)
$= 94°$
$\therefore \boldsymbol{x = 15.6, y = 94}$

**11.** $AB = BE = EC = p$ cm
$DE = CB = 2p$ cm
Area of $ADEB = \frac{1}{2}(p + 2p)(p)$ cm²
$= \boldsymbol{\frac{3}{2}p^2}$ **cm²**

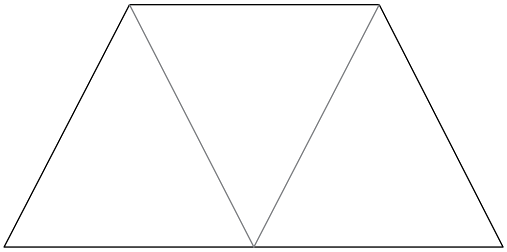
**12.** **(a)** **(i)** $DC = AB = \mathbf{6.4\ cm}$
**(ii)** $AX = DX = \mathbf{3.5\ cm}$
**(b)** $\angle XDC = \angle XAB = 68°$
$\angle DXC = 180° - 68° - 33°$ (∠ sum of a △)
$= 79°$

**13.** **(a)** **No.** A parallelogram can only be divided into four congruent triangles if all four sides are equal, i.e. it is a rhombus.
**(b)** **Yes.** All 7 sides of a regular heptagon are equal, and the length from the centre to the each of the 7 vertices is equal.

**14.** **(a)**



**(b)**



## Challenge Myself!

**15.** An isosceles triangle $ABC$ with $AB = AC$ can be divided into two congruent triangles $ABX$ and $ACX$ such that $AX$ is perpendicular to $BC$ and $BX = CX$.
Consider an isosceles triangle $PQR$ in which $\angle QPR = 120°$ and $\angle PQR = \angle PRQ = 30°$. When three of these triangles are joined at the point $P$, an equilateral triangle is formed.
$\therefore$ The student is correct.

## Worksheet 8B Similar figures

**1.** **(a)** $\frac{BC}{QR} = \frac{5}{5} = 1$

$\frac{AC}{PQ} = \frac{10}{15} = \frac{2}{3}$

Since not all the ratios of the corresponding sides are equal, the triangles are not similar.

**(b)** $\frac{AB}{PQ} = \frac{\sqrt{5^2+6^2}}{\sqrt{6^2+9^2}} = \sqrt{\frac{61}{117}}$

$\frac{BC}{QR} = \frac{8}{12} = \frac{2}{3}$

Since not all the ratios of the corresponding sides are equal, the triangles are not similar.

**2.** **(a)** $\frac{AB}{PQ} = \frac{30}{20} = \frac{3}{2}$

$\frac{BC}{QR} = \frac{63}{43}$

Since not all the ratios of the corresponding sides are equal, the triangles are not similar.

**(b)** $\angle ABC = \frac{180° - 42°}{2}$ (base $\angle$s of an isos. $\triangle$)

$= 69°$

$\angle ABC = \angle QPR = 69°$

$\angle ACB = \angle QRP = 69°$

$\angle BAC = \angle PQR = 42°$

Since all the corresponding angles are equal, the triangles are similar.

**3.** **(a)** $\angle BAC = \angle \boldsymbol{QPR}$

**(b)** $\frac{BC}{QR} = \frac{AC}{PR}$

$\frac{x}{3} = \frac{8}{4}$

$x = \frac{8}{4} \times 3$

$= \mathbf{6}$

**(c)** $\frac{PQ}{AB} = \frac{PR}{AC}$

$\frac{y}{10} = \frac{4}{8}$

$y = \frac{4}{8} \times 10$

$= \mathbf{5}$

**4.** **(a)** $\angle DEF = \angle ABC$

$= \mathbf{75°}$

**(b)** $\frac{DF}{AC} = \frac{DE}{AB}$

$\frac{DF}{7} = \frac{3}{4}$

$DF = \frac{3}{4} \times 7$ cm

$= \mathbf{5\frac{1}{4}}$ **cm**

**(c)** $\frac{BC}{EF} = \frac{AB}{DE}$

$\frac{BC}{5} = \frac{4}{3}$

$BC = \frac{4}{3} \times 5$ cm

$= \mathbf{6\frac{2}{3}}$ **cm**

**5.** **(a)** $\frac{x}{48} = \frac{50}{75}$

$x = \frac{50}{75} \times 48$

$= \mathbf{32}$

**(b)** $\frac{y}{30} = \frac{75}{50}$

$y = \frac{75}{50} \times 30$

$= \mathbf{45}$

**6.** $\frac{x}{3.9} = \frac{6.8}{8.4}$

$x = \frac{6.8}{8.4} \times 3.9$

$= 3\frac{11}{70}$

$\frac{y}{1.4} = \frac{8.4}{6.8}$

$y = \frac{8.4}{6.8} \times 1.4$

$= 1\frac{62}{85}$

$\therefore \boldsymbol{x = 3\frac{11}{70}, y = 1\frac{62}{85}}$

**7.** $\frac{h}{21} = \frac{2(21)}{35}$

$h = \frac{2(21)}{35} \times 21$

$= \mathbf{25.2}$

**8.** **(a)** Length of side of larger cube $= \sqrt[3]{8p^3} = 2p$ cm

Length of side of smaller cube $= \sqrt[3]{p^3} = p$ cm

The ratio of the lengths of the cubes is 2 : 1, which is a constant.
Hence, the cubes are similar.

(b) **Yes.** The ratio of the lengths of any two cubes is a constant.

**9.** $\angle DBC = \angle EAC$

$p° = 83°$

$\frac{CD}{CE} = \frac{BD}{AE}$

$\frac{43}{q+43} = \frac{16}{32}$

$= \frac{1}{2}$

$86 = q + 43$

$q = 43$

$\therefore \boldsymbol{p = 83, q = 43}$

**10.** $\frac{PQ}{BC} = \frac{AP}{AB}$

$\frac{x}{21} = \frac{28+14}{28}$

$= \frac{3}{2}$

$x = \frac{3}{2} \times 21$

$= 31.5$

$\frac{AC}{AQ} = \frac{AB}{AP}$

$\frac{y}{y+21} = \frac{28}{28+14}$

$= \frac{2}{3}$

$3y = 2y + 42$

$y = 42$

$\therefore \boldsymbol{x = 31.5, y = 42}$

**11.** **(a)** $\frac{AQ}{AC} = \frac{PQ}{BC}$

$\frac{6.3}{6.3+QC} = \frac{7}{16}$

$100.8 = 44.1 + 7QC$

$7QC = 56.7$

$QC = \mathbf{8.1\ cm}$

**(b)** $\frac{AP}{AB} = \frac{PQ}{BC}$

$\frac{AP}{AP+10.9} = \frac{7}{16}$

$16AP = 7AP + 76.3$

$9AP = 76.3$

$AP = \mathbf{8.48\ cm}$ (to 3 s.f.)

**12.** **(a)** $\frac{CD}{CA} = \frac{DE}{AB}$

$\frac{h}{8} = \frac{13}{17}$

$h = \frac{13}{17} \times 8$

$= \boldsymbol{6\frac{2}{17}}$

**(b)** $\frac{EC}{BC} = \frac{DE}{AB}$

$\frac{k+8}{15} = \frac{13}{17}$

$k + 8 = \frac{195}{17}$

$k = \boldsymbol{3\frac{8}{17}}$

**13.** $\frac{AP}{AB} = \frac{PQ}{BC}$

$\frac{x}{10} = \frac{9}{12}$

$x = \frac{9}{12} \times 10$

$= 7\frac{1}{2}$

$\frac{AC}{AQ} = \frac{BC}{PQ}$

$\frac{y}{7} = \frac{12}{9}$

$y = \frac{12}{9} \times 7$

$= 9\frac{1}{3}$

$\therefore \boldsymbol{x = 7\frac{1}{2}, y = 9\frac{1}{3}}$

**14.** **(a)** $CE = \frac{2}{3}(4p)$ cm

$= \boldsymbol{\frac{8}{3}p}$ **cm**

**(b)** $\frac{AB}{ED} = \frac{CA}{CE}$

$\frac{CD}{q} = \frac{4p}{\frac{8}{3}p}$

$= \frac{3}{2}$

$CD = \boldsymbol{\frac{3}{2}q}$ **cm**

**15.** **(a)** $\frac{AQ}{AB} = \frac{AP}{AC}$

$\frac{AQ}{32+24} = \frac{32}{AQ+36}$

$AQ^2 + 36AQ = 1792$

$AQ^2 + 36AQ - 1792 = 0$

$(AQ - 28)(AQ + 64) = 0$

$AQ - 28 = 0$ or $AQ + 64 = 0$

$AQ = 28$ cm $\quad AQ = -64$ cm (rejected)

$\therefore AQ = \mathbf{28\ cm}$

**(b)** $\frac{BC}{PQ} = \frac{AB}{AQ} = \frac{56}{28} = \frac{2}{1}$

$\therefore BC : PQ = \mathbf{2 : 1}$

**16.** **(a)** $\angle ABF = \angle \mathbf{DCA}$

**(b)**
$$\frac{AC}{FB} = \frac{CD}{BA}$$
$$\frac{40 + BC}{22 + 42} = \frac{28}{40}$$
$$\frac{40 + BC}{64} = \frac{7}{10}$$
$$40 + BC = \frac{7}{10} \times 64$$
$$= 44.8$$
$$BC = \mathbf{4.8\ cm}$$

**17.** **(a)** Interior angle of a regular hexagon $= \frac{(6-2) \times 180°}{6}$
$= 120°$

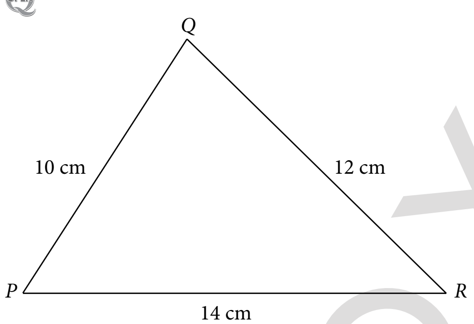
$\therefore \angle CDE = \angle QRS$

**(b)** Let $PB = TF = 2$ cm and $AT = 6$ cm.
Perimeter of $ABCDEF = 6(8\text{ cm})$
$= \mathbf{48\ cm}$
Perimeter of $APQRST = 6(6\text{ cm})$
$= \mathbf{36\ cm}$

**18.** **(a)**
$$\frac{BY}{MX} = \frac{AY}{AX}$$
$$\frac{BY}{1.4} = \frac{2AX}{AX}$$
$$= 2$$
$$BY = 2(1.4)\text{ m}$$
$$= \mathbf{2.8\ m}$$

**(b)** When light from a lamppost shines on a boy, similar triangles can be applied to find the height of the lamppost, provided the height of the boy, the length of his shadow and the distance between the boy and the foot of the lamppost are known.

**19.**




## Challenge Myself!

**20.** **(a)** The two triangles are congruent if and only if the angle opposite the side measuring 8 cm is the same. Hence, the statement is **sometimes true**.

**(b)** When one side of each triangle is 8 cm long, the two triangles are similar if and only if all the ratios of the corresponding sides are equal. Hence, the statement is **sometimes true**.

## Worksheet 8C Similarity and enlargement

**1.** **(a)** Scale factor $= \frac{PS}{AD}$
$= \frac{9.5}{7.6}$
$= \mathbf{1.25}$

**(b)** $AB = \frac{9.6}{1.25}$
$= \mathbf{7.68\ cm}$

**(c)** Obtuse $\angle QRS$ = obtuse $\angle BCD$
$= 360° - 228°$ ($\angle$s at a pt.)
$= \mathbf{132°}$

**2.** 22.4 cm represents 56 km.
22.4 cm represents (56 × 100 000) cm = 5 600 000 cm.
1 cm represents $\frac{5\ 600\ 000}{22.4}$ cm = 250 000 cm.
$\therefore n = \mathbf{250\ 000}$

**3.** **(a)** 1 cm represents 4000 cm $= \frac{4000}{100\ 000}$ km = 0.04 km.
7.5 cm represents (7.5 × 0.04) km = 0.3 km.
∴ Actual distance between tourist attractions = **0.3 km**

**(b)** 0.04 km is represented by 1 cm.
5.6 km is represented by $\frac{5.6}{0.04}$ cm = 140 cm
∴ Distance on the map = **140 cm**

**4.** **(a)** 4 cm represents 10 km.
1 cm represents $\frac{10}{4}$ km = 2.5 km.
1 cm represents (2.5 × 100 000) cm = 250 000 cm.
∴ Scale is **1 : 250 000**

**(b)** 1 cm represents 2.5 km.
1 $cm^2$ represents $2.5^2$ $km^2$ = 6.25 $km^2$.
56 $cm^2$ represents (56 × 6.25) $km^2$ = 350 $km^2$.
∴ Actual area occupied by educational institution = **350 $km^2$**

**5.** **(a)** 1 cm represents 20 cm = 0.2 m.
96 cm represents (96 × 0.2) m = 19.2 m.
∴ Actual length of office = **19.2 m**

**(b)** 1 cm represents 0.2 m.
1 $cm^2$ represents $0.2^2$ $m^2$ = 0.04 $m^2$.
0.04 $m^2$ is represented by 1 $cm^2$.
4.8 $m^2$ is represented by $\frac{4.8}{0.04}$ $cm^2$ = 120 $cm^2$.
∴ Area of pantry on the plan = **120 $cm^2$**

**6.** **(a)** 1 cm represents 25 cm = 0.25 m.
0.25 m is represented by 1 cm.
12 m is represented by $\frac{12}{0.25}$ cm = 48 cm.
∴ Width of library on the plan = **48 cm**

**(b)** 1 cm represents 0.25 m.
1 $cm^2$ represents $0.25^2$ $m^2$ = 0.0625 $m^2$.
4800 $cm^2$ represents (4800 × 0.0625) $m^2$ = 300 $m^2$.
∴ Floor area of library = **300 $m^2$**

**7.** **(a)** 1 cm represents 250 000 cm $= \frac{250\ 000}{100\ 000}$ km = 2.5 km.
∴ Scale is **1 cm to 2.5 km**

**(b)** 1 cm represents 2.5 km.
24 cm represents (24 × 2.5) km = 60 km.
∴ Actual length of river = **60 km**

**(c)** 2.5 km is represented by 1 cm.
$2.5^2$ $km^2$ is represented by 1 $cm^2$.
60 $km^2$ is represented by $\frac{60}{2.5^2}$ $cm^2$ = 9.6 $cm^2$.
∴ Area of river on the map = **9.6 $cm^2$**

**8.** **(a)** 1 cm represents 12 500 000 cm = $\frac{12\ 500\ 000}{100\ 000}$ km = 125 km.

28 cm represents (28 × 125) km = 3500 km.

∴ Actual length of the Great Dividing Range = **3500 km**

**(b)** 125 km is represented by 1 cm.

$125^2$ km$^2$ is represented by 1 cm$^2$.

68 400 km$^2$ is represented by $\frac{68\ 400}{125^2}$ cm$^2$ = 4.38 cm$^2$ (to 3 s.f.)

∴ Area of Tasmania on the map = **4.38 cm²**

**9.** **(a)** 1 cm represents 40 000 cm = $\frac{40\ 000}{100\ 000}$ km = 0.4 km.

∴ $n$ = **0.4**

**(b)** 1 cm represents 0.4 km.

22.5 cm represents (22.5 × 0.4) km = 9 km.

∴ Actual distance between the two petrol stations = **9 km**

**(c)** 0.4 km is represented by 1 cm.

$0.4^2$ km$^2$ is represented by 1 cm$^2$.

0.8 km$^2$ is represented by $\frac{0.8}{0.4^2}$ cm$^2$ = 5 cm$^2$.

∴ Area of golf course on the map = **5 cm²**

**10.** **(a)** 5 cm$^2$ represents 32 000 m$^2$.

1 cm$^2$ represents **6400 m²**.

**(b)** 1 cm$^2$ represents 6400 m$^2$.

1 cm represents 80 m = 0.08 km.

∴ $p$ = **0.08**

**(c)** 1 cm represents 80 m = 8000 cm.

∴ Linear scale is **1 : 8000**

**11.** 2.8 cm$^2$ represents 70 km$^2$.

1 cm$^2$ represents $\frac{70}{2.8}$ km$^2$ = 25 km$^2$.

1 cm represents 5 km = (5 × 100 000) cm = 500 000 cm.

∴ The representative fraction is $\mathbf{\frac{1}{500\ 000}}$.

**12.** **(a)** 1 cm represents 25 000 cm = $\frac{25\ 000}{100\ 000}$ km = 0.25 km.

4.8 cm represents (4.8 × 0.25) km = 1.2 km.

∴ Actual distance between $P$ and $Q$ = **1.2 km**

**(b)** 0.25 km is represented by 1 cm.

$0.25^2$ km$^2$ is represented by 1 cm$^2$.

16 km$^2$ is represented by $\frac{16}{0.25^2}$ cm$^2$ = 256 cm$^2$.

∴ Area of town on the map = **256 cm²**

**13.** **(a)** 5 cm represents 2.5 m.

1 cm represents $\frac{2.5}{5}$ m = 0.5 m.

2 cm represents (2 × 0.5) m = 1 m.

∴ $p$ = **1**

**(b)** Actual area of floor = [(3.8)(2.6) – (1.3)(0.5)] m$^2$

= **9.23 m²**

**14.** **(a)** 3.5 cm$^2$ represents 14 km$^2$.

1 cm$^2$ represents $\frac{14}{3.5}$ cm$^2$ = 4 km$^2$.

1 cm represents 2 km = (2 × 100 000) cm = 200 000 cm.

∴ Linear scale is **1 : 200 000**

**(b)** **(i)** Let the actual dimensions of the plot of land be 7 km by 2 km.

Actual perimeter = 2(7 + 2) km

= 18 km

∴ $p$ = **18**

**(ii)** 2 km is represented by 1 cm.

18 km is represented by $\frac{18}{2}$ cm = 9 cm.

∴ Perimeter of the plot of land on the map = **9 cm**

**15.** **(a)** 5 cm represents 2 km.

1 cm represents 0.4 km = (0.4 × 100 000) cm = 40 000 cm.

∴ Scale of the map is **1 : 40 000**

**(b)** 1 cm represents 0.4 km.

1 cm$^2$ represents $0.4^2$ km$^2$ = 0.16 km$^2$.

50 cm$^2$ represents (50 × 0.16) km$^2$ = 8 km$^2$.

∴ Actual area of the green zone = **8 km²**

**(c)** 1 cm represents 25 000 cm = $\frac{25\ 000}{100\ 000}$ km = 0.25 km.

1 cm$^2$ represents $0.25^2$ km$^2$ = 0.0625 km$^2$.

0.0625 km$^2$ is represented by 1 cm$^2$.

8 km$^2$ is represented by $\frac{8}{0.0625}$ cm$^2$ = 128 cm$^2$.

∴ Area of the green zone on the second map = **128 cm²**

**16.** **(a)** 1 cm represents 200 000 cm = $\frac{200\ 000}{100\ 000}$ km = 2 km.

14.1 cm represents (14.1 × 2) km = 28.2 km.

∴ Actual road distance = **28.2 km**

**(b)** 2 km is represented by 1 cm.

4 km$^2$ is represented by 1 cm$^2$.

1.01 km$^2$ is represented by $\frac{1.01}{4}$ cm$^2$ = 0.2525 cm$^2$.

∴ Area covered by Gardens by the Bay on the map = **0.2525 cm²**

**(c)** Area covered by Gardens by the Bay on Lee's map = 4(0.2525) cm$^2$ = 1.01 cm$^2$

1.01 cm$^2$ represents 1.01 km$^2$.

1 cm$^2$ represents 1 km$^2$.

1 cm represents 1 km = 100 000 cm.

∴ Linear scale of Lee's map is **1 : 100 000**

## Review Exercise 8

**1.** **(a)** Scale factor = $\frac{PQ}{AB}$

$= \frac{40}{55}$

$= \mathbf{\frac{8}{11}}$

**(b)** $k = \frac{8}{11} \times 60$

$= \mathbf{43\frac{7}{11}}$

**2.** **(a)** $\frac{AB}{PQ} = \frac{OB}{AB}$

**(b)** **No.** To maintain a pair of similar triangles $OAB$ and $OPQ$, the image of $PQ$ will be shifted more than $k$ mm towards the point $O$.

**3.** The student is incorrect.

4 cm$^2$ represents 100 m$^2$.

1 cm$^2$ represents 25 m$^2$.

1 cm represents 5 m = 500 cm.

∴ The linear scale of the map is **1 : 500**.

4. **(a)** 1 cm represents 10 000 000 cm = $\frac{10\ 000\ 000}{100\ 000}$ km = 100 km.
17.4 cm represents (17.4 × 100) km = 1740 km.
∴ Actual length of the river Mackenzie = **1740 km**

**(b)** 1 cm represents 100 km.
1 $cm^2$ represents $100^2$ $km^2$ = 10 000 $km^2$.
10 000 $km^2$ is represented by 1 $cm^2$.
55 284 $km^2$ is represented by $\frac{55\ 284}{10\ 000}$ $cm^2$ = 5.5284 $cm^2$.
∴ Area of Nova Scotia on the map = **5.5284 cm²**

## 9 Pythagoras' Theorem

### Worksheet 9A Pythagoras' Theorem

1. **(a)** Using Pythagoras' Theorem,
$a^2 = 6^2 + 8^2$
$= 100$
$a = \sqrt{100}$ (since $a > 0$)
$= 10$
∴ $a$ = **10**

**(b)** Using Pythagoras' Theorem,
$b^2 = 24^2 + 10^2$
$= 676$
$b = \sqrt{676}$ (since $b > 0$)
$= 26$
∴ $b$ = **26**

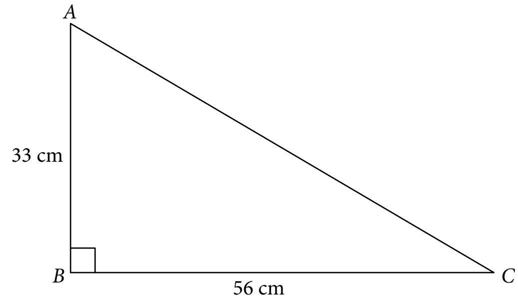
**(c)** Using Pythagoras' Theorem,
$h^2 = 8^2 + 15^2$
$= 289$
$h = \sqrt{289}$ (since $h > 0$)
$= 17$
∴ $h$ = **17**

**(d)** Using Pythagoras' Theorem,
$k^2 + 21^2 = 29^2$
$k^2 = 29^2 - 21^2$
$= 400$
$k = \sqrt{400}$ (since $k > 0$)
$= 20$
∴ $k$ = **20**

**(e)** Using Pythagoras' Theorem,
$p^2 + 28^2 = 53^2$
$p^2 = 53^2 - 28^2$
$= 2025$
$p = \sqrt{2025}$ (since $p > 0$)
$= 45$
∴ $p$ = **45**

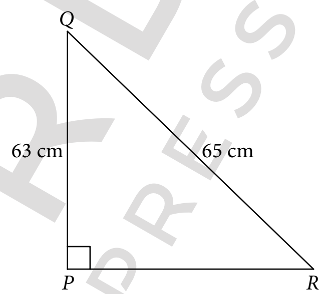
**(f)** Using Pythagoras' Theorem,
$q^2 + 8.4^2 = 8.5^2$
$q^2 = 8.5^2 - 8.4^2$
$= 1.69$
$q = \sqrt{1.69}$ (since $q > 0$)
$= 1.3$
∴ $q$ = **1.3**

2.



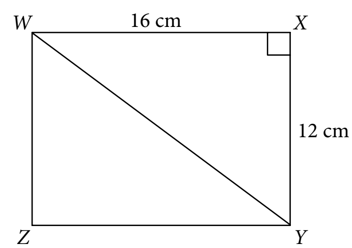

Using Pythagoras' Theorem,
$AC^2 = AB^2 + BC^2$
$= 33^2 + 56^2$
$= 4225$
$AC = \sqrt{4225}$ cm (since $AC > 0$)
$= 65$ cm
∴ $AC$ = **65 cm**

3.




Using Pythagoras' Theorem,
$PQ^2 + PR^2 = QR^2$
$63^2 + PR^2 = 65^2$
$PR^2 = 65^2 - 63^2$
$= 256$
$PR = \sqrt{256}$ cm (since $PR > 0$)
$= 16$ cm
∴ $PR$ = **16 cm**

4.




Using Pythagoras' Theorem,
$WY^2 = WX^2 + XY^2$
$= 16^2 + 12^2$
$= 400$
$WY = \sqrt{400}$ cm (since $WY > 0$)
$= 20$ cm
∴ $WY$ = **20 cm**

5. **(i)** Let the length of the side of the square be $x$ cm.
Using Pythagoras' Theorem,
$x^2 + x^2 = 20^2$
$2x^2 = 400$
$x^2 = 200$
$x = \sqrt{200}$ (since $x > 0$)
$= 14.1$ (to 3 s.f.)
∴ Length of a side = **14.1 cm**

(ii) Area of square = $\left(\sqrt{200}\right)^2$
$= 200 \text{ cm}^2$ (shown)

**6.** **(i)** Consider $\triangle ATC$.
Using Pythagoras' Theorem,
$AT^2 + TC^2 = AC^2$
$AT^2 + 5^2 = 6^2$
$AT^2 = 6^2 - 5^2$
$= 11$
$AT = \sqrt{11}$ cm (since $AT > 0$)
$= 3.32$ cm (to 3 s.f.)
$\therefore AT = \mathbf{3.32\ cm}$

**(ii)** Consider $\triangle ABT$.
Using Pythagoras' Theorem,
$AT^2 + BT^2 = AB^2$
$(\sqrt{11})^2 + BT^2 = 4^2$
$11 + BT^2 = 16$
$BT^2 = 5$
$BT = \sqrt{5}$ cm (since $BT > 0$)
$= 2.24$ cm (to 3 s.f.)
$\therefore BT = \mathbf{2.24\ cm}$

**7.** **(i)** Consider $\triangle PQR$.
Using Pythagoras' Theorem,
$PQ^2 + QR^2 = PR^2$
$PQ^2 + 30^2 = 47^2$
$PQ^2 = 47^2 - 30^2$
$= 1309$
$PQ = \sqrt{1309}$ cm (since $PQ > 0$)
$= 36.2$ cm (to 3 s.f.)
$\therefore PQ = \mathbf{36.2\ cm}$

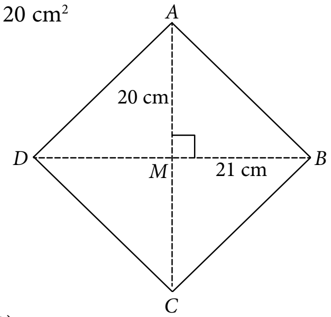
**(ii)** Consider $\triangle PQS$.
Using Pythagoras' Theorem,
$PS^2 = PQ^2 + QS^2$
$= \left(\sqrt{1309}\right)^2 + (30 + 42)^2$
$= 6493$
$PS = \sqrt{6493}$ cm (since $PS > 0$)
$= 80.6$ cm (to 3 s.f.)
$\therefore PS = 80.6$ cm (shown)

**8.** Consider $\triangle BCD$.
Using Pythagoras' Theorem,
$BD^2 = BC^2 + CD^2$
$= 32^2 + 56^2$
$= 4160$
$BD = \sqrt{4160}$ cm
Consider $\triangle ABD$.
Using Pythagoras' Theorem,
$AD^2 = AB^2 + BD^2$
$= 41^2 + \left(\sqrt{4160}\right)^2$
$= 5841$
$AD = \sqrt{5841}$ cm (since $AD > 0$)
$= 76.4$ cm (to 3 s.f.)
$\therefore AD = \mathbf{76.4\ cm}$

**9.** **(i)** Consider $\triangle AMB$.
Using Pythagoras' Theorem,
$AM^2 + MB^2 = AB^2$
$AM^2 + 30^2 = 35^2$
$AM^2 = 35^2 - 30^2$
$= 325$
$AM = \sqrt{325}$ cm (since $AM > 0$)
$= 18.0$ cm (to 3 s.f.)
$\therefore AM = \mathbf{18.0\ cm}$

**(ii)** Area of rhombus $= 4 \times \frac{1}{2} \times 30 \times \sqrt{325}$ cm$^2$
$= \mathbf{1080\ cm^2}$ (to 3 s.f.)

**10.** Area of rhombus $= 4 \times \frac{1}{2} \times 21 \times 20$ cm$^2$
$= \mathbf{840\ cm^2}$




Consider $\triangle AMB$.
Using Pythagoras' Theorem,
$AB^2 = AM^2 + MB^2$
$= 20^2 + 21^2$
$= 841$
$AB = \sqrt{841}$ cm (since $AB > 0$)
$= 29$ cm
$\therefore$ Perimeter of rhombus = 4(29) cm
$= \mathbf{116\ cm}$

**11.** $DP = \frac{8}{19} \times DC$
$= \frac{8}{19} \times 57$ cm
$= 24$ cm
Area of parallelogram = 1938 cm$^2$
$57 \times AP = 1938$
$AP = 34$ cm
Consider $\triangle ADP$.
Using Pythagoras' Theorem,
$AD^2 = DP^2 + AP^2$
$= 24^2 + 34^2$
$= 1732$
$AD = \sqrt{1732}$ cm (since $AD > 0$)
$= 41.6$ cm (to 3 s.f.)
$\therefore BC = \mathbf{41.6\ cm}$

**12.** Consider $\triangle ABD$.
Using Pythagoras' Theorem,
$BD^2 + AD^2 = AB^2$
$BD^2 + 60^2 = 70^2$
$BD^2 = 70^2 - 60^2$
$= 1300$
$BD = \sqrt{1300}$ cm (since $BD > 0$)
Consider $\triangle BCD$.
Using Pythagoras' Theorem,
$BC^2 + CD^2 = BD^2$
$x^2 + x^2 = \left(\sqrt{1300}\right)^2$
$2x^2 = 1300$
$x^2 = 650$
$x = \sqrt{650}$ (since $x > 0$)
$= 25.5$ (to 3 s.f.)
$\therefore x = \mathbf{25.5}$

**13. (a)** Area of $\triangle ABC = 96$ cm²

$\frac{1}{2} \times 24 \times AP = 96$

$AP = 8$ cm

Consider $\triangle ABP$.

Using Pythagoras' Theorem,

$AB^2 = BP^2 + AP^2$

$= 8^2 + 8^2$

$= 128$

$AB = \sqrt{128}$ cm (since $AB > 0$)

$= 11.3$ cm (to 3 s.f.)

$\therefore AB =$ **11.3 cm**

**(b)** Area of $\triangle APC = \frac{1}{2} \times (24 - 8) \times 8$ cm²

$=$ **64 cm²**

**14.** Consider $\triangle EAD$.

Using Pythagoras' Theorem,

$AD^2 + AE^2 = DE^2$

$AD^2 + 18^2 = 21^2$

$AD^2 = 21^2 - 18^2$

$= 117$

$AD = \sqrt{117}$ cm (since $AD > 0$)

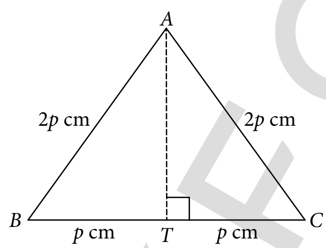
$\therefore DC = AC - AD$

$= \left(21 - \sqrt{117}\right)$ cm

$=$ **10.2 cm** (to 3 s.f.)

**15.** Consider $\triangle ABT$.

Using Pythagoras' Theorem,




$AT^2 + BT^2 = AB^2$

$AT^2 + p^2 = (2p)^2$

$= 4p^2$

$AT^2 = 3p^2$

$AT = \sqrt{3p^2}$ cm

$= \sqrt{3}p$ cm

Area of $\triangle ABC = \frac{1}{2} \times 2p \times \sqrt{3}p$ cm²

$= \sqrt{3}p^2$ cm²

$\therefore$ I agree with Ryan.

**16. (i)** Consider $\triangle ABC$.

Using Pythagoras' Theorem,

$AB^2 + BC^2 = AC^2$

$(7x - 20)^2 + (2x + 1)^2 = [5(x + 9)]^2$

$49x^2 - 280x + 400 + 4x^2 + 4x + 1 = 25(x^2 + 18x + 81)$

$53x^2 - 276x + 401 = 25x^2 + 450x + 2025$

$28x^2 - 726x - 1624 = 0$

$14x^2 - 363x - 812 = 0$ (shown)

**(ii)** $14x^2 - 363x - 812 = 0$

$(14x + 29)(x - 28) = 0$

$14x + 29 = 0$ or $x - 28 = 0$

$14x = -29$ $\quad$ $x = 28$

$x = -2\frac{1}{14}$

$\therefore x =$ $\mathbf{-2\frac{1}{14}}$ or $x =$ **28**

$AB = [7(28) - 20] = 176$ cm

$BC = [2(28) + 1] = 57$ cm

$\therefore$ Perimeter of $ABCD = 2(176 + 57)$ cm

$=$ **466 cm**

**17. (a)** Consider $\triangle ABC$.

Using Pythagoras' Theorem,

$AB^2 + AC^2 = BC^2$

$(3x - 2 + x)^2 + (2x - 1 + 5)^2 = (7x - 4)^2$

$(4x - 2)^2 + (2x + 4)^2 = (7x - 4)^2$

$16x^2 - 16x + 4 + 4x^2 + 16x + 16 = 49x^2 - 56x + 16$

$29x^2 - 56x - 4 = 0$ (shown)

**(b)** $29x^2 - 56x - 4 = 0$

$(29x + 2)(x - 2) = 0$

$29x + 2 = 0$ or $x - 2 = 0$

$29x = -2$ $\quad$ $x = 2$

$x = -\frac{2}{29}$

$\therefore x =$ $\mathbf{-\frac{2}{29}}$ or $x =$ **2**

**(c)** $AB = [4(2) - 2]$ cm

$= 6$ cm

$AC = [2(2) + 4]$ cm

$= 8$ cm

$BC = [7(2) - 4]$ cm

$= 10$ cm

$AP = [3(2) - 2]$ cm

$= 4$ cm

$AQ = [2(2) - 1]$ cm

$= 3$ cm

Consider $\triangle APQ$.

Using Pythagoras' Theorem,

$PQ^2 = AP^2 + AQ^2$

$= 4^2 + 3^2$

$= 25$

$PQ = \sqrt{25}$ (since $PQ > 0$)

$= 5$ cm

Area of shaded region = Area of $\triangle ABC$ – area of $\triangle APQ$

$= \left(\frac{1}{2} \times 6 \times 8 - \frac{1}{2} \times 3 \times 4\right)$ cm²

$=$ **18 cm²**

Perimeter of shaded region $= PQ + QC + BC + PB$

$= (5 + 5 + 10 + 2)$ cm

$=$ **22 cm**

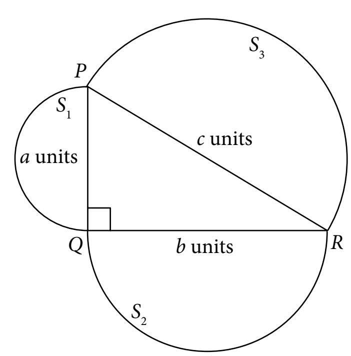
## Challenge Myself!

**18.** Let the diameters of the semicircles $S_1$, $S_2$ and $S_3$ be $a$ units, $b$ units and $c$ units respectively.

Area of $S_1 = \frac{1}{2}\pi\left(\frac{a}{2}\right)^2$ units²

$= \frac{1}{2}\pi\left(\frac{a^2}{4}\right)$ units²

$= \frac{1}{8}\pi a^2$ units²




Area of $S_2 = \frac{1}{2}\pi\left(\frac{b}{2}\right)^2$ units²

$= \frac{1}{2}\pi\left(\frac{b^2}{4}\right)$ units²

$= \frac{1}{8}\pi b^2$ units²

Area of $S_3 = \frac{1}{2}\pi\left(\frac{c}{2}\right)^2$ units$^2$

$= \frac{1}{2}\pi\left(\frac{c^2}{4}\right)$ units$^2$

$= \frac{1}{8}\pi c^2$ units$^2$

Using Pythagoras' Theorem,

$a^2 + b^2 = c^2$ — (1)

Multiplying (1) throughout by $\frac{1}{8}\pi$,

$\frac{1}{8}\pi a^2 + \frac{1}{8}\pi b^2 = \frac{1}{8}\pi c^2$

Area of $S_1$ + area of $S_2$ = Area of $S_3$

∴ Sum of areas of the two smaller semicircles = Area of the largest semicircle

## Worksheet 9B Applications of Pythagoras' Theorem in real-world contexts

**1.** Consider △$ABC$.

Using Pythagoras' Theorem,

$AC^2 = AB^2 + BC^2$

$= 1.2^2 + 1^2$

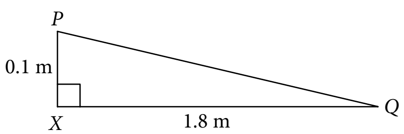
$= 2.44$

$AC = \sqrt{2.44}$ m (since $AC > 0$)

$= 1.56$ m (to 3 s.f.)

∴ Distance covered = **1.56 m**

**2.**




Consider △$PQX$.

Using Pythagoras' Theorem,

$PQ^2 = PX^2 + QX^2$

$= 0.1^2 + 1.8^2$

$= 3.25$

$PQ = \sqrt{3.25}$ m (since $PQ > 0$)

$= 1.8028$ m (to 5 s.f.)

∴ Minimum length of tape = (1.8028 + 0.15 + 0.15) m

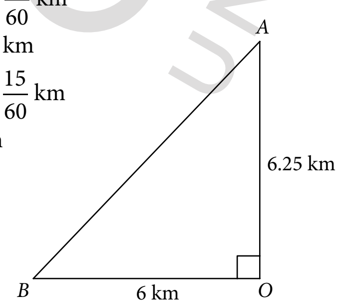
= **2.10 m** (to 3 s.f.)

**3.** Distance travelled by $A = 25 \times \frac{15}{60}$ km

$= 6.25$ km

Distance travelled by $B = 24 \times \frac{15}{60}$ km

$= 6$ km




Consider △$OAB$.

Using Pythagoras' Theorem,

$AB^2 = OA^2 + OB^2$

$= 6.25^2 + 6^2$

$= 75.0625$

$AB = \sqrt{75.0625}$ km (since $AB > 0$)

$= 8.66$ km (to 3 s.f.)

∴ Distance between the ships = **8.66 km**

**4.** **(i)** Consider △$OPQ$.

Using Pythagoras' Theorem,

$PQ^2 = OP^2 + OQ^2$

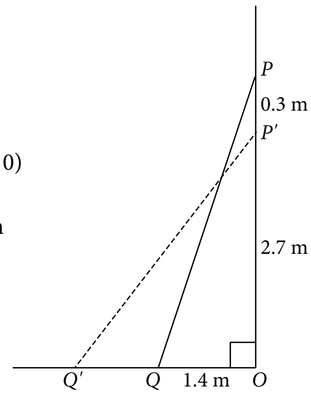
$= 3^2 + 1.4^2$

$= 10.96$

$PQ = \sqrt{10.96}$ m (since $PQ > 0$)

$= 3.31$ m (to 3 s.f.)

∴ Length of ladder = **3.31 m**




**(ii)** Consider △$OP'Q'$.

Using Pythagoras' Theorem,

$(OP')^2 + (OQ')^2 = (P'Q')^2$

$2.7^2 + (OQ')^2 = \left(\sqrt{10.96}\right)^2$

$(OQ')^2 = 10.96 - 2.7^2$

$= 3.67$

$OQ' = \sqrt{3.67}$ m (since $OQ' > 0$)

∴ $Q'Q = OQ' - OQ$

$= \left(\sqrt{3.67} - 1.4\right)$ m

= **0.516 m** (to 3 s.f.)

**5.** △$BAP$ is congruent to △$BQP$.

∴ $BQ = BA = 297$ mm

Consider △$BQC$.

Using Pythagoras' Theorem,

$BC^2 + QC^2 = BQ^2$

$210^2 + QC^2 = 297^2$

$QC^2 = 297^2 - 210^2$

$= 44\ 109$

$QC = \sqrt{44\ 109}$ mm

$= 210$ mm (to 3 s.f.)

∴ $QC$ = **210 mm**

**6.** Consider △$OAB$.

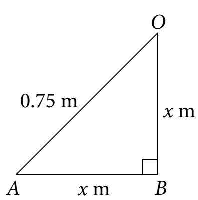
Using Pythagoras' Theorem,

$x^2 + x^2 = 0.75^2$

$2x^2 = 0.5625$

$x^2 = 0.281\ 25$

$x = \sqrt{0.281\ 25}$ (since $x > 0$)

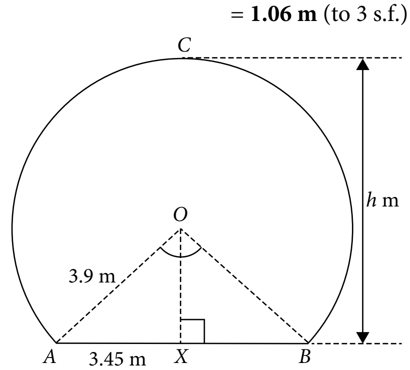



∴ Length of table mat = $2x$ m

$= 2\sqrt{0.281\ 25}$ m

= **1.06 m** (to 3 s.f.)

**7.**




Consider △$OAX$.

Using Pythagoras' Theorem,

$OX^2 + AX^2 = OA^2$

$OX^2 + 3.45^2 = 3.9^2$

$OX^2 = 3.9^2 - 3.45^2$

$= 3.3075$

$OX = \sqrt{3.3075}$ m (since $OX > 0$)

$= 1.8187$ m (to 5 s.f.)

$\therefore h = 1.8187 + 3.9$

$= 5.72$ (to 3 s.f.)

$\therefore$ Height of entrance = **5.72 m**

## Challenge Myself!

**8.** $y = 0.1x(x - 3)$

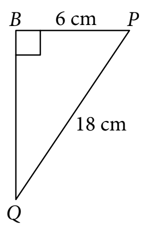
When $x = 1.5$,

$y = 0.1(1.5)(1.5 - 3)$

$= -0.225$

$B$ lies 22.5 cm from the top of the wall, i.e. 197.5 cm above the ground.

Consider $\triangle BPQ$.




Using Pythagoras' Theorem,

$BP^2 + BQ^2 = PQ^2$

$6^2 + BQ^2 = 18^2$

$BQ^2 = 18^2 - 6^2$

$= 288$

$BQ = \sqrt{288}$ cm (since $BQ > 0$)

$\therefore$ Height of lowest point above the ground $= \left(197.5 - \sqrt{288}\right)$cm

$= 181$ cm (to 3 s.f.)

$> 1.6$ m

$\therefore$ Donna does not have to adjust the bunting.

## Worksheet 9C Converse of Pythagoras' Theorem

**1. (a)** $AB^2 + BC^2 = 3.9^2 + 7.9^2$

$= 77.62$

$AC^2 = 8.9^2$

$= 79.21$

Since $AB^2 + BC^2 \neq AC^2$, then by the converse of Pythagoras' Theorem, $\triangle ABC$ is not a right-angled triangle.

**(b)** $DE^2 + EF^2 = 65^2 + 72^2$

$= 9409$

$DF^2 = 97^2$

$= 9409$

Since $DE^2 + EF^2 = DF^2$, then by the converse of Pythagoras' Theorem, $\triangle DEF$ is a right-angled triangle with $\angle$***DEF*** = 90°.

**(c)** $PQ^2 + PR^2 = 1.7^2 + 1.4^2$

$= 4.85$

$QR^2 = 2.3^2$

$= 5.29$

Since $PQ^2 + PR^2 \neq QR^2$, then by the converse of Pythagoras' Theorem, $\triangle PQR$ is not a right-angled triangle.

**(d)** $XZ^2 + YZ^2 = 5.2^2 + 16.5^2$

$= 299.29$

$XY^2 = 17.3^2$

$= 299.29$

Since $XZ^2 + YZ^2 = XY^2$, then by the converse of Pythagoras' Theorem, $\triangle XYZ$ is a right-angled triangle with $\angle$***XZY*** = 90°.

**2. (a)** $AC^2 + BC^2 = 20^2 + 21^2$

$= 841$

$AB^2 = 29^2$

$= 841$

Since $AC^2 + BC^2 = AB^2$, then by the converse of Pythagoras' Theorem, $\triangle ABC$ is a right-angled triangle with $\angle$***ACB*** = 90°.

**(b)** $PQ^2 + QR^2 = 4.4^2 + 11.7^2$

$= 156.25$

$PR^2 = 12.5^2$

$= 156.25$

Since $PQ^2 + QR^2 = PR^2$, then by the converse of Pythagoras' Theorem, $\triangle PQR$ is a right-angled triangle with $\angle$***PQR*** = 90°.

**3. (a)** $AC^2 + BC^2 = 35^2 + 12^2$

$= 1369$

$AB^2 = 37^2$

$= 1369$

Since $AC^2 + BC^2 = AB^2$, then by the converse of Pythagoras' Theorem, $\triangle ABC$ is a right-angled triangle with $\angle$***ACB*** = 90°. (shown)

**(b)** $PQ^2 + PR^2 = 16.8^2 + 9.5^2$

$= 372.49$

$QR^2 = 19.3^2$

$= 372.49$

Since $PQ^2 + PR^2 = QR^2$, then by the converse of Pythagoras' Theorem, $\triangle PQR$ is a right-angled triangle with $\angle$***QPR*** = 90°. (shown)

**4. (i)** Consider $\triangle ACD$.

Using Pythagoras' Theorem,

$AC^2 + CD^2 = AD^2$

$AC^2 + 9.1^2 = 10.9^2$

$AC^2 = 10.9^2 - 9.1^2$

$= 36$

$AC = \sqrt{36}$ cm (since $AC > 0$)

$= 6$ cm

$\therefore AC =$ **6 cm**

**(ii)** Consider $\triangle ABC$.

$AC^2 + BC^2 = 6^2 + 4.8^2$

$= 59.04$

$AB^2 = 6.2^2$

$= 38.44$

Since $AC^2 + BC^2 \neq AB^2$, then by the converse of Pythagoras' Theorem, $\triangle ABC$ is not a right-angled triangle, i.e. $\angle ACB \neq 90°$.

$\therefore BCD$ is not a straight line.

**5. (i)** $PB = (74 - 18)$ cm

$= 56$ cm

Consider $\triangle PBQ$.

Using Pythagoras' Theorem,

$PQ^2 = PB^2 + BQ^2$

$= 56^2 + 23^2$

$= 3665$

$PQ = \sqrt{3665}$ cm

$= 60.5$ cm (to 3 s.f.)

$\therefore PQ =$ **60.5 cm**

**(ii)** Consider $\triangle APD$.
Using Pythagoras' Theorem,
$DP^2 = AP^2 + AD^2$
$= 18^2 + 42^2$
$= 2088$
Consider $\triangle QDC$.
$QC = (42 - 23)$ cm
$= 19$ cm
Using Pythagoras' Theorem,
$DQ^2 = DC^2 + QC^2$
$= 74^2 + 19^2$
$= 5837$
Consider $\triangle DPQ$.
$DP^2 + PQ^2 = 2088 + 3665$
$= 5753$
$DQ^2 = 5837$
Since $DP^2 + PQ^2 \neq DQ^2$, then by the converse of Pythagoras' Theorem, $\triangle DPQ$ is not a right-angled triangle. (shown)

**6.** **(a)** Let the perpendicular height be $h$ cm.
Area of $\triangle PQR = 10.71$ cm$^2$
$\frac{1}{2} \times QR \times h = 10.71$
$\frac{1}{2} \times 6.3 \times h = 10.71$
$h = 3.4$
$\therefore$ Perpendicular height is **3.4 cm**

**(b)** $QR^2 + h^2 = 6.3^2 + 3.4^2$
$= 51.25$
$PQ^2 = 7.4^2$
$= 54.76$
Since $QR^2 + h^2 \neq PQ^2$, then by the converse of Pythagoras' Theorem, $\triangle PQR$ is not a right-angled triangle. (shown)

**7.** Consider $\triangle PDC$.
$DP^2 + PC^2 = 55^2 + 48^2$
$= 5329$
$DC^2 = 73^2$
$= 5329$
Since $DP^2 + PC^2 = DC^2$, then by the converse of Pythagoras' Theorem, $\triangle PDC$ is a right-angled triangle with $\angle DPC = 90°$.
Area of $\triangle PDC = \frac{1}{2} \times 55 \times 48$ cm$^2$
$= 1320$ cm$^2$
$\therefore$ Sum of areas of $\triangle ADP$ and $\triangle BCP$ = area of $\triangle PDC$
= **1320 cm²**

**8.** **(a)** $6^2 + 8^2 = 10^2$
$\therefore$ The statement is **true**.

**(b)** The square of an odd number is an odd number.
The sum of the squares of two odd numbers is an even number.
Hence, a Pythagorean Triple cannot be made up of three odd numbers.
$\therefore$ The statement is **true**.

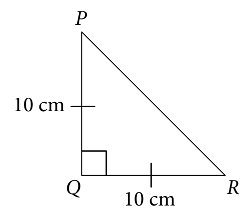
## Challenge Myself!

**9.** Attach one end of the piece of twine to a point $A$ on the post at one of the marked intervals, and the other end to the point $P$.
If the twine is not taut, then adjust it such that $AP$ is a straight line, and use the post to measure the length of $AP$.
Let the foot of the post be $O$.
Find the value of $OA^2 + OP^2$ and of $AP^2$.
If the values are equal, then by the converse of Pythagoras' Theorem, $\triangle OAP$ is a right-angled triangle with $\angle AOP = 90°$ and the post is vertical.

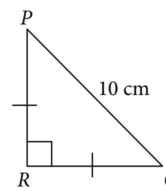
## Review Exercise 9

**1.** Consider $\triangle ADC$.
Using Pythagoras' Theorem,
$AC^2 = AD^2 + CD^2$
$= 6.8^2 + 3.6^2$
$= 59.2$
Consider $\triangle ABC$.
Using Pythagoras' Theorem,
$AB^2 + BC^2 = AC^2$
$x^2 + 5.9^2 = 59.2$
$x^2 = 59.2 - 5.9^2$
$= 24.39$
$x = \sqrt{24.39}$ (since $x > 0$)
$= 4.94$ (to 3 s.f.)
$\therefore x =$ **4.94**

**2.** Consider $\triangle PQR$ in which $PQ = QR$.
$\therefore QR = 10$ cm

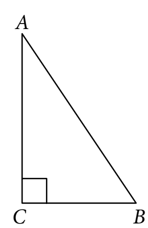



Consider $\triangle PQR$ in which $PR = QR$.
Using Pythagoras' Theorem,
$PR^2 + QR^2 = PQ^2$
$QR^2 + QR^2 = PQ^2$
$2QR^2 = 10^2$
$= 100$
$QR^2 = 50$
$QR = \sqrt{50}$ (since $QR > 0$)
$= 7.07$ cm (to 3 s.f.)




$\therefore QR =$ **10 cm** or $QR =$ **7.07 cm**

**3.** **(a)** Let $AB = 26$ cm and $BC = 24$ cm.
$AC^2 + BC^2 = 10^2 + 24^2$
$= 676$
$AB^2 = 26^2$
$= 676$
$\therefore$ When $AB =$ **26 cm**, $BC =$ **24 cm** and $AC = 10$ cm, $\triangle ABC$ is right-angled at $C$.

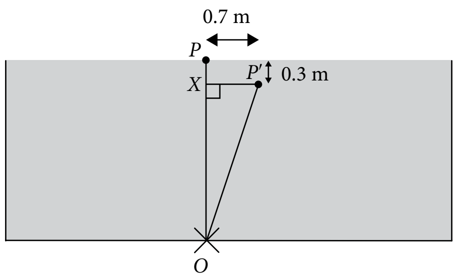


(b) Let $AC = 40$ cm and $BC = 75$ cm.

$AC^2 + BC^2 = 40^2 + 75^2$
$= 7225$
$AB^2 = 85^2$
$= 7225$

∴ When $AC =$ **40 cm**, $BC =$ **75 cm** and $AB = 85$ cm, $\triangle ABC$ is right-angled at $C$.

4.




Consider $\triangle OP'X$.

Using Pythagoras' Theorem,

$$OX^2 + (XP')^2 = (OP')^2$$
$$(OP - 0.3)^2 + 0.7^2 = OP^2$$
$$OP^2 - 0.6OP + 0.09 + 0.49 = OP^2$$
$$0.6OP = 0.58$$
$$OP = 0.967 \text{ m (to 3 s.f.)}$$

∴ Depth of swimming pool = **0.967 m**

5. (a) $a = m^2 - n^2$, $b = 2mn$ and $c = m^2 + n^2$

Let $m = 3$ and $n = 2$: $a = 5$, $b = 12$ and $c = 13$
Let $m = 4$ and $n = 3$: $a = 7$, $b = 24$ and $c = 25$
Let $m = 5$ and $n = 4$: $a = 9$, $b = 40$ and $c = 41$
Let $m = 6$ and $n = 5$: $a = 11$, $b = 60$ and $c = 61$

∴ Four more Pythagorean Triples are **(5, 12, 13)**, **(7, 24, 25)**, **(9, 40, 41)** and **(11, 60, 61)**.

(b) $a^2 = (m^2 - n^2)^2$
$= m^4 - 2m^2n^2 + n^4$
$b^2 = (2mn)^2$
$= 4m^2n^2$
$c^2 = (m^2 + n^2)^2$
$= m^4 + 2m^2n^2 + n^4$

Since $a^2 + b^2 = c^2$, then Pythagoras' Theorem holds true. (shown)

## 10 Trigonometric Ratios

### Worksheet 10A Trigonometric ratios

1. (a) ***AB***
(b) ***BC***
(c) ***AC***
(d) ***AC***
(e) ***BC***

2. (a) $\sin A = \mathbf{\frac{8}{17}}$
(b) $\cos A = \mathbf{\frac{15}{17}}$
(c) $\tan A = \mathbf{\frac{8}{15}}$
(d) $\sin B = \mathbf{\frac{15}{17}}$
(e) $\cos B = \mathbf{\frac{8}{17}}$
(f) $\tan B = \mathbf{\frac{15}{8}}$

3. (a) $\sin A = \boldsymbol{\frac{p}{r}}$
(b) $\cos A = \boldsymbol{\frac{q}{r}}$
(c) $\tan A = \boldsymbol{\frac{p}{q}}$
(d) $\sin B = \boldsymbol{\frac{q}{r}}$
(e) $\cos B = \boldsymbol{\frac{p}{r}}$
(f) $\tan B = \boldsymbol{\frac{q}{p}}$

4. (a) sin 30° = **0.5**
(b) sin 60° = **0.866** (to 3 s.f.)
(c) sin 21° = **0.358** (to 3 s.f.)
(d) sin 54.8° = **0.817** (to 3 s.f.)
(e) cos 30° = **0.866** (to 3 s.f.)
(f) cos 60° = **0.5**
(g) cos 69° = **0.358** (to 3 s.f.)
(h) cos 35.2° = **0.817** (to 3 s.f.)

5. (i) Complementary angle of $x°$ = **(90 – *x*)°**
(ii) The sine of an angle is equal to the cosine of its complementary angle.

6. (a) tan 15° = **0.268** (to 3 s.f.)
(b) tan 30° = **0.577** (to 3 s.f.)
(c) tan 60° = **1.73** (to 3 s.f.)
(d) tan 75° = **3.73** (to 3 s.f.)

7. Since $\frac{1}{\tan 60°} = 0.577$ (to 3 s.f.) and $\frac{1}{\tan 75°} = 0.268$ (to 3 s.f.), then the tangent of an angle is equal to the reciprocal of the tangent of its complementary angle.

8. (a) $\frac{1}{2}\sin 90° =$ **0.5**
(b) 5 cos 15° – 2 tan 8° = **4.55** (to 3 s.f.)
(c) $3(\cos 45°)^2 =$ **1.5**
(d) $\frac{\sin 23°}{\sin 46°} =$ **0.543** (to 3 s.f.)

### Challenge Myself!

9. (a) For $\sin A = \cos B$, angles $A$ and $B$ are complementary angles, i.e. $A + B = 90°$.
∴ A possible pair of values is ***A* = 56°** and ***B* = 34°**.

(b) For $\tan A = \frac{1}{\tan B}$, angles $A$ and $B$ are complementary angles, i.e. $A + B = 90°$.
∴ A possible pair of values is ***A* = 29°** and ***B* = 61°**.

## Worksheet 10B Applications of trigonometric ratios to find unknown sides of right-angled triangles

**1.** **(a)** $\sin 31° = \frac{x}{47}$

$x = 47 \sin 31°$

$= \mathbf{24.2}$ (to 3 s.f.)

**(b)** $\sin 48° = \frac{6}{x}$

$x = \frac{6}{\sin 48°}$

$= \mathbf{8.07}$ (to 3 s.f.)

**(c)** $\cos 35° = \frac{y}{20}$

$y = 20 \cos 35°$

$= \mathbf{16.4}$ (to 3 s.f.)

**(d)** $\cos 46° = \frac{19}{y}$

$y = \frac{19}{\cos 46°}$

$= \mathbf{27.4}$ (to 3 s.f.)

**(e)** $\tan 51° = \frac{z}{85}$

$z = 85 \tan 51°$

$= \mathbf{105}$ (to 3 s.f.)

**(f)** $\tan 58° = \frac{33}{z}$

$z = \frac{33}{\tan 58°}$

$= \mathbf{20.6}$ (to 3 s.f.)

**2.** **(a)** $\sin 22° = \frac{x}{5}$

$x = 5 \sin 22°$

$= 1.87$ (to 3 s.f.)

$\cos 22° = \frac{y}{5}$

$y = 5 \cos 22°$

$= 4.64$ (to 3 s.f.)

$\therefore \mathbf{x = 1.87, y = 4.64}$

**(b)** $\cos 57° = \frac{x}{16}$

$x = 16 \cos 57°$

$= 8.71$ (to 3 s.f.)

$\sin 57° = \frac{y}{16}$

$y = 16 \sin 57°$

$= 13.4$ (to 3 s.f.)

$\therefore \mathbf{x = 8.71, y = 13.4}$

**(c)** $\tan 31° = \frac{x}{8.4}$

$x = 8.4 \tan 31°$

$= 5.05$ (to 3 s.f.)

$\cos 31° = \frac{8.4}{y}$

$y = \frac{8.4}{\cos 31°}$

$= 9.80$ (to 3 s.f.)

$\therefore \mathbf{x = 5.05, y = 9.80}$

**(d)** $\cos 52° = \frac{33}{x}$

$x = \frac{33}{\cos 52°}$

$= 53.6$ (to 3 s.f.)

$\tan 52° = \frac{y}{33}$

$y = 33 \tan 52°$

$= 42.2$ (to 3 s.f.)

$\therefore \mathbf{x = 53.6, y = 42.2}$

**(e)** $\tan 58° = \frac{7.9}{x}$

$x = \frac{7.9}{\tan 58°}$

$= 4.94$ (to 3 s.f.)

$\sin 58° = \frac{7.9}{y}$

$y = \frac{7.9}{\sin 58°}$

$= 9.32$ (to 3 s.f.)

$\therefore \mathbf{x = 4.94, y = 9.32}$

**(f)** $\sin 37° = \frac{21}{x}$

$x = \frac{21}{\sin 37°}$

$= 34.9$ (to 3 s.f.)

$\tan 37° = \frac{21}{y}$

$y = \frac{21}{\tan 37°}$

$= 27.9$ (to 3 s.f.)

$\therefore \mathbf{x = 34.9, y = 27.9}$

**3.** **(a)** Consider $\triangle ABM$.

$\angle BAM = \frac{1}{2}(105°) = 52.5°$

$\cos 52.5° = \frac{AM}{AB}$

$= \frac{AM}{42}$

$AM = 42 \cos 52.5°$

$= \mathbf{25.6\ cm}$ (to 3 s.f.)

**(b)** Consider $\triangle ABM$.

$\sin 52.5° = \frac{BM}{AB}$

$= \frac{BM}{42}$

$BM = 42 \sin 52.5°$

$BD = 2(42 \sin 52.5°)$

$= \mathbf{66.6\ cm}$ (to 3 s.f.)

**4.** **(a)** **(i)** Consider $\triangle PTR$.

Using Pythagoras' Theorem,

$PT^2 + TR^2 = PR^2$

$PT^2 + 6.2^2 = 7.8^2$

$PT^2 = 7.8^2 - 6.2^2$

$= 22.4$

$PT = \sqrt{22.4}$ cm (since $PT > 0$)

$= \mathbf{4.73\ cm}$ (to 3 s.f.)

**(ii)** Consider $\triangle PQT$.

$$\tan 71° = \frac{PT}{QT}$$
$$= \frac{4.7329}{QT}$$
$$QT = \frac{4.7329}{\tan 71°}$$
$$= \mathbf{1.63\text{ cm}} \text{ (to 3 s.f.)}$$

**(b)** Area of $\triangle PQR = \frac{1}{2} \times (1.6297 + 6.2) \times 4.7329\ \text{cm}^2$

$$= \mathbf{18.5\text{ cm}^2} \text{ (to 3 s.f.)}$$

**5.** Consider $\triangle ACD$.

$$\tan 30° = \frac{AC}{CD}$$
$$= \frac{AC}{24}$$
$$AC = 24 \tan 30°$$
$$= 13.856 \text{ cm (to 5 s.f.)}$$

Consider $\triangle BCD$.

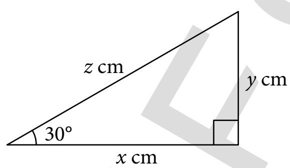
$$\tan 20° = \frac{BC}{CD}$$
$$= \frac{BC}{24}$$
$$BC = 24 \tan 20°$$
$$= 8.7353 \text{ cm (to 5 s.f.)}$$

$\therefore AB = AC - BC$

$= (13.856 - 8.7353)$ cm

$= \mathbf{5.12\text{ cm}}$ (to 3 s.f.)

**6.** $\tan 30° = \frac{y}{x}$




$y = x \tan 30°$

Area of triangle = 80 cm$^2$

$$\frac{1}{2}xy = 80$$
$$\frac{1}{2}x(x \tan 30) = 80$$
$$x^2 = \frac{160}{\tan 30}$$
$$x = \sqrt{\frac{160}{\tan 30}}$$
$$= 16.647 \text{ (to 5 s.f.)}$$

$y = 16.647 \tan 30°$

$= 9.6112$ (to 5 s.f.)

Using Pythagoras' Theorem,

$z^2 = x^2 + y^2$

$= 16.647^2 + 9.6112^2$

$= 369.50$ (to 5 s.f.)

$z = \sqrt{369.50}$ (since $z > 0$)

$= 19.222$ (to 5 s.f.)

$\therefore$ Perimeter of triangle = (16.647 + 9.6112 + 19.222) cm

$= \mathbf{45.5\text{ cm}}$ (to 3 s.f.)

**7.** **(a)** Consider $\triangle ABC$.

$$\tan x° = \frac{p}{q}$$
$$\tan 40° = \frac{10}{q}$$
$$q = \frac{10}{\tan 40°}$$
$$= 11.918 \text{ (to 5 s.f.)}$$

Consider $\triangle ABD$.

$$\tan y° = \frac{p}{q+r}$$
$$\tan 30° = \frac{10}{11.918 + r}$$
$$11.918 + r = \frac{10}{\tan 30°}$$
$$r = \frac{10}{\tan 30°} - 11.918$$
$$= 5.40 \text{ (to 3 s.f.)}$$

$\therefore$ **$q$ = 11.9, $r$ = 5.40**

**(b)** Consider $\triangle ABC$.

$$\tan x° = \frac{p}{q}$$
$$\tan 42° = \frac{p}{15}$$
$$p = 15 \tan 42°$$
$$= 13.506 \text{ (to 5 s.f.)}$$

Consider $\triangle ABD$.

$$\tan y° = \frac{p}{q+r}$$
$$\tan 35° = \frac{13.506}{15+r}$$
$$15 + r = \frac{13.506}{\tan 35°}$$
$$r = \frac{13.506}{\tan 35°} - 15$$
$$= 4.29 \text{ (to 3 s.f.)}$$

$\therefore$ **$p$ = 13.5, $r$ = 4.29**

**(c)** Consider $\triangle ABC$.

$$\tan x° = \frac{p}{q}$$
$$\tan 36° = \frac{p}{q}$$
$$p = q \tan 36° \quad \text{— (1)}$$

Consider $\triangle ABD$.

$$\tan y° = \frac{p}{q+r}$$
$$\tan 27° = \frac{p}{q+8}$$
$$p = (q+8) \tan 27° \quad \text{— (2)}$$

Substitute (1) into (2):

$$q \tan 36° = (q+8) \tan 27°$$
$$= q \tan 27° + 8 \tan 27°$$
$$q \tan 36° - q \tan 27° = 8 \tan 27°$$
$$q(\tan 36° - \tan 27°) = 8 \tan 27°$$
$$q = \frac{8 \tan 27°}{\tan 36° - \tan 27°}$$
$$= 18.783 \text{ (to 5 s.f.)}$$

Substitute $q$ = 18.783 into (1):

$p = 18.783 \tan 36°$

$= 13.6$ (to 3 s.f.)

$\therefore$ **$p$ = 13.6, $q$ = 18.8**

## Worksheet 10C Applications of trigonometric ratios to find unknown angles in right-angled triangles

**1.** **(a)** $\sin A = 0.3$

$A = \sin^{-1} 0.3$

$= \mathbf{17.5°}$ (to 1 d.p.)

**(b)** $\cos B = 0.18$

$B = \cos^{-1} 0.18$

$= \mathbf{79.6°}$ (to 1 d.p.)

**(c)** $\tan C = \frac{5}{6}$

$C = \tan^{-1}\frac{5}{6}$

$= \mathbf{39.8°}$ (to 1 d.p.)

**(d)** $9 \sin D = 7$

$\sin D = \frac{7}{9}$

$D = \sin^{-1}\frac{7}{9}$

$= \mathbf{51.1°}$ (to 1 d.p.)

**2.** **(a)** $\sin x° = \frac{4}{5}$

$x° = \sin^{-1}\frac{4}{5}$

$= \mathbf{53.1°}$ (to 1 d.p.)

**(b)** $\sin x° = \frac{33}{36}$

$x° = \sin^{-1}\frac{33}{36}$

$= \mathbf{66.4°}$ (to 1 d.p.)

**(c)** $\cos y° = \frac{12}{18}$

$y° = \cos^{-1}\frac{12}{18}$

$= \mathbf{48.2°}$ (to 1 d.p.)

**(d)** $\cos y° = \frac{22}{42}$

$y° = \cos^{-1}\frac{22}{42}$

$= \mathbf{58.4°}$ (to 1 d.p.)

**(e)** $\tan z° = \frac{51}{27}$

$z° = \tan^{-1}\frac{51}{27}$

$= \mathbf{62.1°}$ (to 1 d.p.)

**(f)** $\tan z° = \frac{5.6}{8.8}$

$z° = \tan^{-1}\frac{5.6}{8.8}$

$= \mathbf{32.5°}$ (to 1 d.p.)

**3.** $\sin y° = \frac{3.5}{6.3}$

$y° = \sin^{-1}\frac{3.5}{6.3}$

$= 33.7°$ (to 1 d.p.)

Using Pythagoras' Theorem,

$x^2 + 3.5^2 = 6.3^2$

$x^2 = 6.3^2 - 3.5^2$

$= 27.44$

$x = \sqrt{27.44}$ (since $x > 0$)

$= 5.24$ (to 3 s.f.)

$\therefore x = 5.24, y = 33.7$

**4.** **(a)** Consider $\triangle PRT$.

$\tan \angle PRT = \frac{9.6}{15.5}$

$\angle PRT = \tan^{-1}\frac{9.6}{15.5}$

$= 31.8°$ (to 1 d.p.)

$\therefore \angle PRQ = \mathbf{31.8°}$

**(b)** Consider $\triangle PQT$.

$\sin 56° = \frac{9.6}{PQ}$

$PQ = \frac{9.6}{\sin 56°}$

$= \mathbf{11.6\ cm}$ (to 3 s.f.)

**5.** **(a)** Consider $\triangle ABC$.

$\tan \angle BAC = \frac{28}{50}$

$\angle BAC = \tan^{-1}\frac{28}{50}$

$= \mathbf{29.2°}$ (to 1 d.p.)

**(b)** $\angle AMB = 180° - 2(29.249°)$ ($\angle$ sum of a $\triangle$)

$= 121.5°$ (to 1 d.p.)

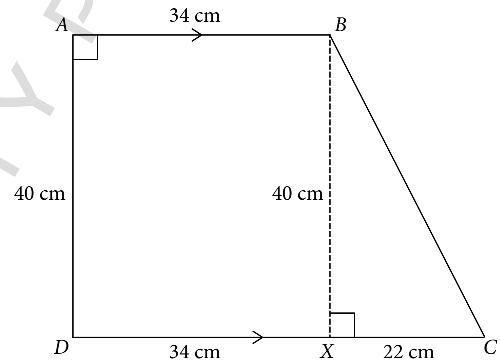
$\therefore \angle DMC = \mathbf{121.5°}$ (vert. opp. $\angle$s)

**6.** **(a)** Area of trapezium $= 1800\ cm^2$

$\frac{1}{2} \times (34 + DC) \times 40 = 1800$

$34 + DC = 90$

$DC = 56$ cm




Consider $\triangle BCX$.

Using Pythagoras' Theorem,

$BC^2 = BX^2 + XC^2$

$= 40^2 + 22^2$

$= 2084$

$BC = \sqrt{2084}$ (since $BC > 0$)

$= \mathbf{45.7\ cm}$ (to 3 s.f.)

**(b)** Consider $\triangle BCX$.

$\tan \angle XBC = \frac{22}{40}$

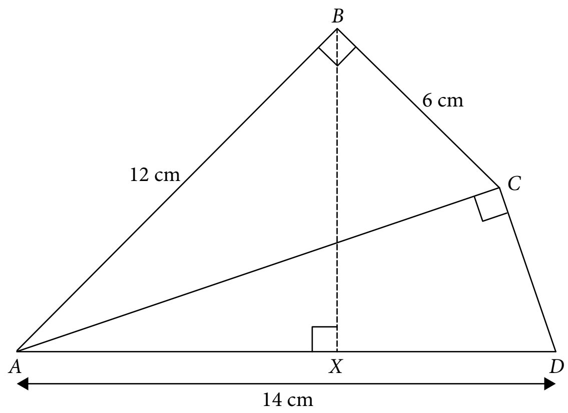
$\angle XBC = \tan^{-1}\frac{22}{40}$

$= 28.811°$ (to 3 d.p.)

$\therefore \angle ABC = 90° + 28.811°$

$= \mathbf{118.8°}$ (to 1 d.p.)

**7.** **(a)**




Consider $\triangle ABC$.

Using Pythagoras' Theorem,

$AC^2 = AB^2 + BC^2$

$= 12^2 + 6^2$

$= 180$

$AC = \sqrt{180}$ (since $AC > 0$)

$= 13.416$ cm (to 5 s.f.)

$\tan \angle BAC = \frac{6}{12}$

$\angle BAC = \tan^{-1} \frac{6}{12}$

$= 26.565°$ (to 3 d.p.)

Consider $\triangle ACD$.

$\cos \angle CAD = \frac{13.416}{14}$

$\angle CAD = \cos^{-1} \frac{13.416}{14}$

$= 16.602°$ (to 3 d.p.)

$\therefore \angle BAD = 26.565° + 16.602°$

$=$ **43.2°** (to 1 d.p.)

**(b)** Consider $\triangle BAX$.

$\sin 43.167° = \frac{BX}{12}$

$BX = 12 \sin 43.167°$

$= 8.21$ cm (to 3 s.f.)

$\therefore$ Shortest distance from $B$ to $AD = 8.21$ cm (shown)

**(c)** Consider $\triangle ACD$.

$\tan 16.602° = \frac{CD}{13.416}$

$CD = 13.416 \tan 16.602°$

$= 4$ cm

$\therefore$ Area of $ABCD$ = Area of $\triangle ABC$ + area of $\triangle ACD$

$= \left[\frac{1}{2}(12)(6) + \frac{1}{2}(13.416)(4)\right]$ cm²

$=$ **62.8 cm²** (to 3 s.f.)

**8.** Let $PQ = 6$ cm and $QB = BA = 5$ cm.

Consider $\triangle PQB$.

$\tan x° = \frac{6}{5}$

$x° = \tan^{-1} \frac{6}{5}$

$= 50.194°$ (to 3 d.p.)

Consider $\triangle PQA$.

$\tan y° = \frac{6}{10}$

$y° = \tan^{-1} \frac{6}{10}$

$= 30.964°$ (to 3 d.p.)

Since $2y° = 61.928°$ (to 3 d.p.), then $x°$ is not twice of $y°$.

**9.** **(a)** **(i)** Consider $\triangle ABC$.

Using Pythagoras' Theorem,

$AB^2 + AC^2 = BC^2$

$AB^2 + 5^2 = 10^2$

$AB^2 = 10^2 - 5^2$

$= 75$

$AB = \sqrt{75}$ (since $AB > 0$)

$=$ **8.66 cm** (to 3 s.f.)

**(ii)** Area of $\triangle ABC = \frac{1}{2}(8.6603)(5)$ cm²

$=$ **21.7 cm²** (to 3 s.f.)

**(iii)** Area of $\triangle ABC = 21.651$ cm²

$\frac{1}{2} \times BC \times AN = 21.651$

$AN =$ **4.33 cm** (to 3 s.f.)

**(b)** **(i)** Consider $\triangle ABC$.

$\sin \angle ABC = \frac{5}{10}$

$\angle ABC = \sin^{-1} \frac{5}{10}$

$=$ **30°**

**(ii)** Consider $\triangle ABN$.

$\sin 30° = \frac{AN}{8.6603}$

$AN = 8.6603 \sin 30°$

$=$ **4.33 cm** (to 3 s.f.)

**(c)** The answers are the same.

## Worksheet 10D Applications of trigonometric ratios in real-world contexts

**1.** **(a)** $\cos \angle PRQ = \frac{4}{8}$

$\angle PRQ = \cos^{-1} \frac{4}{8}$

$=$ **60°**

**(b)** Using Pythagoras' Theorem,

$PQ^2 + QR^2 = PR^2$

$PQ^2 + 4^2 = 8^2$

$PQ^2 = 8^2 - 4^2$

$= 48$

$PQ = \sqrt{48}$ (since $PQ > 0$)

$= 6.93$ m (to 3 s.f.)

$\therefore$ Height of pole = **6.93 m**

**2.** $\cos x° = \frac{0.93}{3.6}$

$x° = \cos^{-1} \frac{0.93}{3.6}$

$= 75.0°$ (to 1 d.p.)

$\therefore$ The ladder makes an angle of **75.0°** with the horizontal ground.

**3.** Let $x$ be 54.5 and the vertical height of the kite above her head be $h$ m.

$\sin 54.5° = \dfrac{h}{80}$

$h = 80 \sin 54.5°$

$= 65.129$ (to 5 s.f.)

$\therefore$ Vertical height of kite from ground = (65.129 + 1.69) m

= **66.8 m** (to 3 s.f.)

**4.** **(a)** $\tan 75° = 3.73 \approx 4$ (to the nearest integer)

$\dfrac{\text{vertical change}}{\text{horizontal change}} = \dfrac{4}{1}$

The "1 in 4" rule is obtained by ensuring that the horizontal distance from the foot of the ladder to the wall and the vertical height of the topmost point of the ladder against the wall are in the ratio 1 : 4.

**(b)** Let the angle between the ladder and the ground be $x°$.

$\cos x° = \dfrac{1.1}{3.2}$

$x° = \cos^{-1}\dfrac{1.1}{3.2}$

$= 69.9°$ (to 1 d.p.)

Since $x°$ does not fall within 74° and 76°, it is not in a safe position to be used. (shown)

**5.** **(a)** $PA' = PA = 32$ cm

Let the horizontal distance between $A$ and $A'$ be $x$ cm.

$\sin 20° = \dfrac{x}{32}$

$x = 32 \sin 20°$

$= 10.9$ cm (to 3 s.f.)(shown)

**(b)** **(i)** $\sin \angle APA' = \dfrac{15}{32}$

$\angle APA' = \sin^{-1}\dfrac{15}{32}$

= **28.0°** (to 1 d.p.)

**(ii)** $\angle PA'Q = \angle APA' = 27.953°$ (alt. $\angle$s, $AP$ // $BQ$)

$\cos 27.953° = \dfrac{A'Q}{32}$

$A'Q = 32 \cos 27.953°$

$= 28.267$ cm (to 5 s.f.)

$\therefore$ Distance between $A'$ and $B$ = (32 – 28.267) cm

= **3.73 cm** (to 3 s.f.)

**6.** Consider $\triangle OAP$.

$\tan 40° = \dfrac{OP}{OA}$

$OP = OA \tan 40°$ — (1)

Consider $\triangle OBP$.

$\tan 30° = \dfrac{OP}{OA + 6}$

$OP = OA \tan 30° + 6 \tan 30°$ — (2)

Substitute (1) into (2):

$OA \tan 40° = OA \tan 30° + 6 \tan 30°$

$OA \tan 40° - OA \tan 30° = 6 \tan 30°$

$OA(\tan 40° - \tan 30°) = 6 \tan 30°$

$OA = \dfrac{6 \tan 30°}{\tan 40° - \tan 30°}$

$= 13.234$ m (to 5 s.f.)

Substitute $OA = 13.234$ into (1):

$OP = 13.234 \tan 40°$

$= 11.105$ m (to 5 s.f.)

Floor the actor is on $= \dfrac{11.105 - 3.8}{3.6} + 1$

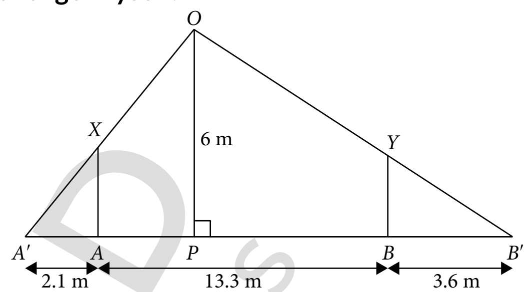
$= 3.44$ (to 3 s.f.)

$\approx 4$ (round up to the nearest integer)

$\therefore$ The actor is on the **4th floor**.

## Challenge Myself!

**7.**




**(i)** Let Sam's height be $h$.

Consider $\triangle OA'P$ and $\triangle XA'A$.

Using similar triangles,

$\dfrac{A'A}{A'P} = \dfrac{XA}{OP}$

$\dfrac{2.1}{A'P} = \dfrac{h}{6}$ — (1)

Consider $\triangle OB'P$ and $\triangle YB'B$.

Using similar triangles,

$\dfrac{BB'}{PB'} = \dfrac{YB}{OP}$

$\dfrac{3.6}{PB'} = \dfrac{h}{6}$ — (2)

(1) = (2): $\dfrac{2.1}{A'P} = \dfrac{3.6}{PB'}$

$2.1PB' = 3.6A'P$

$2.1(PB + 3.6) = 3.6(2.1 + AP)$

$2.1(13.3 - AP + 3.6) = 3.6(2.1 + AP)$

$35.49 - 2.1AP = 7.56 + 3.6AP$

$5.7AP = 27.93$

$AP = 4.9$ m

Substitute $A'P = 2.1 + 4.9$ into (1):

$\dfrac{2.1}{7} = \dfrac{h}{6}$

$h = 1.8$

$\therefore$ Sam is **1.8 m** tall.

**(ii)** $\tan \angle OA'P = \dfrac{1.8}{2.1}$

$\angle OA'P = \tan^{-1}\dfrac{1.8}{2.1}$

$= 40.601°$ (to 3 d.p.)

$\tan \angle OB'P = \dfrac{1.8}{3.6}$

$\angle OA'P = \tan^{-1}\dfrac{1.8}{3.6}$

$= 26.565°$ (to 3 d.p.)

$\therefore \angle A'OB' = 180° - 40.601° - 26.565°$ ($\angle$ sum of a $\triangle$)

= **112.8°** (to 1 d.p.)

## Review Exercise 10

**1.** Let the angle be $x$.

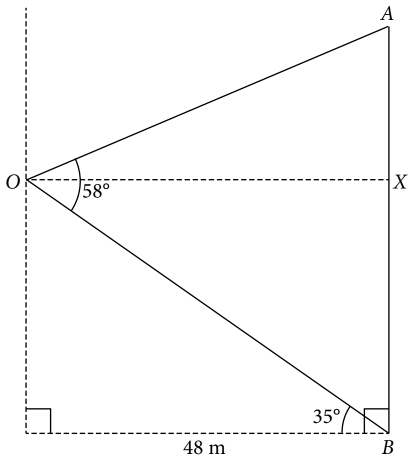
$\sin x = 0.2390$

$x = \sin^{-1} 0.2390$

$= 13.8°$ (to 1 d.p.)

$\therefore$ A possible angle is **13.8°**.

**2.**




Consider $\triangle OBX$.

$\tan 35° = \dfrac{XB}{48}$

$XB = 48 \tan 35°$

$= 33.610$ m (to 5 s.f.)

Consider $\triangle OAX$.

$\tan 23° = \dfrac{AX}{48}$

$AX = 48 \tan 23°$

$= 20.375$ m (to 5 s.f.)

$\therefore$ Distance between $A$ and $B$ = (33.610 + 20.375) m

= **54.0 m** (to 3 s.f.)

**3.** **(a)** Consider $\triangle OAT$.

Using Pythagoras' Theorem,

$AT^2 + OA^2 = OT^2$

$AT^2 + 10^2 = 17^2$

$AT^2 = 17^2 - 10^2$

$= 189$

$AT = \sqrt{189}$ (since $AT > 0$)

= **13.7 cm** (to 3 s.f.)

**(b)** Consider $\triangle OAT$.

$\sin \angle OTA = \dfrac{10}{17}$

$\angle OTA = \sin^{-1} \dfrac{10}{17}$

$= 36.0°$ (to 1 d.p.)

$\therefore \angle BTA =$ **36.0°**

**4.** **(a)** $\tan \theta° = \dfrac{0.5}{3}$

$\theta° = \tan^{-1} \dfrac{0.5}{3}$

= **9.5°** (to 1 d.p.)

**(b)** $\boldsymbol{h = 0.8, \theta = 30}$

**(c)** $\tan 30° = \dfrac{0.8}{x}$

$x = \dfrac{0.8}{\tan 30°}$

= **1.39** (to 3 s.f.)

# 11 Volume and Surface Area of Prisms and Cylinders

## Worksheet 11A Conversion of units

**1.** **(a)** $7 \text{ m}^3 = 7 \times 100^3 \text{ cm}^3$

= **7 000 000 cm³**

**(b)** $0.2 \text{ m}^3 = 0.2 \times 100^3 \text{ cm}^3$

= **200 000 cm³**

**2.** **(a)** $4 \text{ m}^3 = 4 \times 100^3 \text{ cm}^3$

= 4 000 000 cm³

= **4 000 000 ml**

**(b)** $2.4 \text{ m}^3 = 2.4 \times 100^3 \text{ cm}^3$

= 2 400 000 cm³

= **2 400 000 ml**

**3.** **(a)** $9 \text{ m}^3 = 9 \times 1000\ l$

= **9000 *l***

**(b)** $5.01 \text{ m}^3 = 5.01 \times 1000\ l$

= **5010 *l***

**4.** **(a)** $300\,000 \text{ cm}^3 = \dfrac{300\,000}{1000}\ l$

= **300 *l***

**(b)** $88\,000 \text{ cm}^3 = \dfrac{88\,000}{1000}\ l$

= **88 *l***

**5.** **(a)** $600\,000 \text{ cm}^3 = \dfrac{600\,000}{100^3} \text{ m}^3$

= **0.6 m³**

**(b)** $92\,000 \text{ cm}^3 = \dfrac{920\,000}{100^3} \text{ m}^3$

= **0.092 m³**

**6.** $0.4 \text{ m}^3 = 0.4 \times 1000\ l$

$= 400\ l$

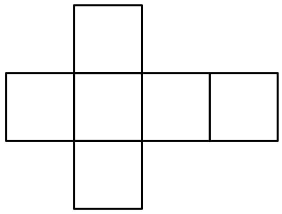
Difference $= 400\ l - 37.5\ l$

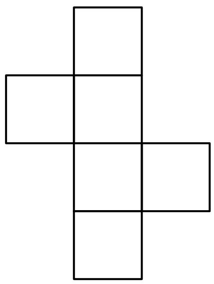
$= 362.5\ l$

$\therefore$ **Tank *Q*** contains **362.5 *l*** more water.

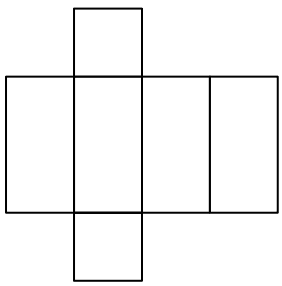
## Worksheet 11B Three-dimensional solids

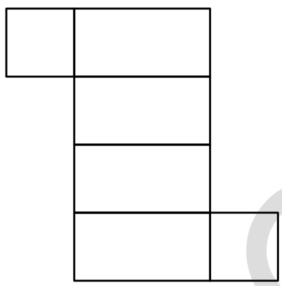
**1.**



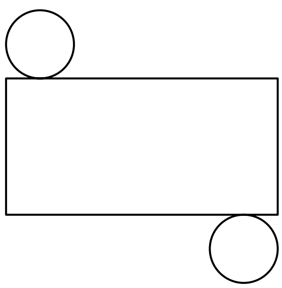


**2.**





**3.**



## Worksheet 11C Volume and surface area of cubes and cuboids

**1.** **(a)** Volume of cube = 18 cm × 18 cm × 18 cm
= **5832 cm³**
Total surface area of cube = (6 × 18 cm × 18 cm)
= **1944 cm²**

**(b)** Volume of cube = 0.4 m × 0.4 m × 0.4 m
= **0.064 m³**
Total surface area of cube = 6 × 0.4 m × 0.4 m
= **0.96 m²**

**2.** **(a)** Length of cube = $\sqrt[3]{125}$ cm
= 5 cm
Total surface area of cube = 6 × 5 cm × 5 cm
= **150 cm²**

**(b)** Length of cube = $\sqrt{\frac{294}{6}}$ cm
= 7 cm
Volume of cube = 7 cm × 7 cm × 7 cm
= **343 cm³**

**3.** **(a)** Volume of cuboid = 40 cm × 10 cm × 30 cm
= **12 000 cm³**
Surface area of cuboid
= 2[40 × 10 + 10 × 30 + 40 × 30] cm²
= **3800 cm²**

**(b)** Volume of cuboid = 0.5 m × 0.5 m × 4.2 m
= **1.05 m³**
Surface area of cuboid = $[2(0.5^2) + 4(0.5)(4.2)]$ m²
= **8.9 m²**

**4.** **(a)** Volume of cuboid = 25 cm × 5 cm × $h$ cm
$1000 = 125h$
$125h = 1000$
$h =$ **8**

**(b)** Volume of cuboid = 4 m × $l$ m × 4 m
$288 = 16l$
$16l = 288$
$l =$ **18**

**(c)** Volume of cuboid = $p$ cm × 12 cm × 26 cm
$17\ 160 = 312p$
$312p = 17\ 160$
$p =$ **55**

**(d)** Volume of cuboid = $x$ m × $x$ m × 3 m
$5.88 = 3x^2$
$3x^2 = 5.88$
$x^2 = 1.96$
$x =$ **1.4**

**5.** **(a)** **(i)** Number of cubes she can get
$= \frac{24}{2} \times \frac{14}{2} \times \frac{20}{2}$
= **840**

**(ii)** Volume of leftover chocolate
= 24 cm × 1 cm × 20 cm
= **480 cm³**

**(b)** She can melt the leftover chocolate and place it in a mould with dimensions that are multiples of 2 cm to obtain an additional $\frac{480}{2^3}$ = 60 cubes of chocolate.

**6.** 1 m = 100 cm
1 m² = 10 000 cm²
0.0948 m² = 948 cm²
Total surface area = $2(x \times 6 + x \times 4 + 6 \times 4)$ cm²
$948 = 2(6x + 4x + 24)$
$2(10x + 24) = 948$
$20x + 48 = 948$
$20x = 900$
$x =$ **45**

**7.**

**(i)** Let the length of its base be 5 cm and the height be 8 cm.
Total surface area = (2 × 5 × 5 + 4 × 5 × 8) cm²
= 210 cm²
∴ The length of its base and the height could be **5 cm** and **8 cm** respectively.

**(ii)** Volume of cuboid = 5 cm × 5 cm × 8 cm
= **200 cm³**

8. **(i)** Height of tank = $\frac{100}{90} \times 45$ cm
= **50 cm**

**(ii)** Surface area of tank that is in contact with the water
= (80 × 60 + 2 × 80 × 45 + 2 × 60 × 45) cm²
= **17 400 cm²**

**(iii)** 100 cm = 1 m
10 000 cm² = 1 m²
17 400 cm² = **1.74 m²**

9. Volume of wood
= [(0.35 × 2.4 × 0.2) – (0.31 × 2.36 × 0.16)] m³
= **0.051 m³** (to 2 s.f.)

## Challenge Myself!

10. OPEN

**(a)** **(i)** Volume of wood
= [(1.5 × 1.2 × 0.6) – (1.4 × 1.1 × 0.55)] m³
= **0.233 m³**

**(ii)** 1 m = 100 cm
1 m³ = 1 000 000 cm³
0.233 m³ = 233 000 cm³
∴ Volume of wood = **233 000 cm³**

**(b)** Total surface area of box
= (2 × 1.5 × 1.2 + 2 × 1.5 × 0.6 + 2 × 1.2 × 0.6 + 2 × 1.4 × 0.55 + 2 × 1.1 × 0.55) m²
= **9.59 m²**

## Worksheet 11D Volume and surface area of prisms

1. **(a)** Volume of prism = $\left(\frac{1}{2}\times 6\times 8\times 17\right)$ cm³
= **408 cm³**

**(b)** Total surface area of prism
= $\left(2\times\frac{1}{2}\times 6\times 8+8\times 17+10\times 17+6\times 17\right)$ cm²
= **456 cm²**

2. Volume of prism = (380 × 42) cm³
= **15 960 cm³**

3. **(a)** Base area of prism = (16 × 12 – 8 × 8) cm²
= 128 cm²
Volume of prism = (128 × 4) cm³
= **512 cm³**
Total surface area of prism
= (2 × 128 + 2 × 16 × 4 + 2 × 12 × 4 + 2 × 8 × 4) cm²
= **544 cm²**

**(b)** Base area of prism = $\left(8\times 3+\frac{1}{2}\times 8\times 3\right)$ m²
= 36 m²
Volume of prism = (36 × 5) m³
= **180 m³**
Total surface area of prism
= [2 × 36 + 2 × 5 × 3 + 2 × 5 × 5 + 8 × 5] m²
= **192 m²**

**(c)** Base area of prism = $\frac{1}{2}$ × (41 + 23) × 25 cm²
= 800 cm²
Volume of prism = (800 × 20) cm³
= **16 000 cm³**
Total surface area of prism
= [2 × 800 + 20 × (23 + 31 + 41 + 25)] cm²
= **4000 cm²**

**(d)** Base area of prism = (5.2 × 1.9) m²
= 9.88 m²
Volume of prism = (9.88 × 3.6) m³
= **35.568 m³**
Total surface area of prism
= (2 × 9.88 + 2 × 3.6 × 5.2 + 2 × 3.6 × 2.1) m²
= **72.32 m²**

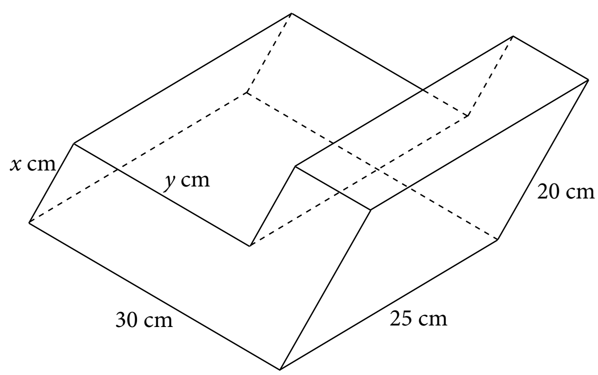
4. OPEN Volume of prism = $\left(\frac{1}{2}\times 12\times 10\times 58\right)$ cm³
= 3480 cm³
∴ A possible set of values is ***x* = 12 and *y* = 10**.

5. Volume of prism = $\left[\frac{1}{2}\times(6+11)\times 9\times h\right]$ cm³
1224 = 76.5$h$
76.5$h$ = 1224
$h$ = **16**

6. **(i)** Area of cross section
= $\left(50\times 1.8-2\times\frac{1}{2}\times 20\times 0.6\right)$ m²
= 78 m²
Volume of water = (78 × 25) m³
= **1950 m³**

**(ii)** Area in contact with water
= [2 × 78 + 25 × (1.2 + $x$ + 10 + $x$ + 1.2)] m²
= [156 + 25(2$x$ + 12.4)] m²
= **(50$x$ + 466) m²**

7. **No.** Dimensions such as the values of $x$ and $y$ shown in the diagram are needed.




## Worksheet 11E Volume and surface area of cylinders

1. **(a)** Volume of cylinder = π(7)²(10) cm³
= **1540 cm³** (to 3 s.f.)
Total surface area of cylinder
= [2π(7)² + 2π(7)(10)] cm²
= **748 cm²** (to 3 s.f.)

**(b)** Base radius = $\frac{1.4}{2}$ m

$= 0.7$ m

Volume of cylinder = $\pi(0.7)^2(3.2)$ m³

$=$ **4.93 m³** (to 3 s.f.)

Total surface area of cylinder

$= [2\pi(0.7)^2 + 2\pi(0.7)(3.2)]$ m²

$=$ **17.2 m²** (to 3 s.f.)

**2.** Volume of cylinder = $\pi(12)^2(27)$ cm³

$=$ **12 200 cm³** (to 3 s.f.)

**3.** **(i)** Let the height of the cylinder be $h$ cm.

Volume of cylinder = $\pi(10)^2h$ cm³

$200\pi = 100\pi h$

$100\pi h = 200\pi$

$h = 2$

∴ Its height is **2 cm**.

**(ii)** Total surface area of cylinder

$= [2\pi(10)^2 + 2\pi(10)(2)]$ cm²

$=$ **754 cm²** (to 3 s.f.)

**4.** **(i)** Let the base radius of the cylinder be $r$ m.

Volume of cylinder = $\pi r^2(0.8)$ m³

$6.4 = 0.8\pi r^2$

$0.8\pi r^2 = 6.4$

$r^2 = \frac{6.4}{0.8\pi}$

$r = \sqrt{\frac{6.4}{0.8\pi}}$

$= 1.60$ (to 3 s.f.)

∴ Its base radius is **1.60 m**.

**(ii)** Total surface area of cylinder

$= [2\pi(1.5957)^2 + 2\pi(1.5957)(0.8)]$ m²

$=$ **24.0 m²** (to 3 s.f.)

**5.** **(i)** Let the height of the cylinder be $h$ m.

Total surface area of cylinder

$= [2\pi(2.9)^2 + 2\pi(2.9)h]$ m²

$270 = 16.82\pi + 5.8\pi h$

$5.8\pi h = 270 - 16.82\pi$

$h = \frac{270 - 16.82\pi}{5.8\pi}$

$= 11.9$ (to 3 s.f.)

∴ Its height is **11.9 m**.

**(ii)** Volume of cylinder = $\pi(2.9)^2(11.917)$

$=$ **315 m³** (to 3 s.f.)

**6.** 1.6 litres = $(1.6 \times 1000)$ cm³

$= 1600$ cm³

Let the height of the container be $h$ cm.

Capacity of container = $\pi(7.8)^2h$ cm³

$1600 = \pi(7.8)^2h$

$\pi(7.8)^2h = 1600$

$h = \frac{1600}{\pi(7.8)^2}$

$= 8.37$ (to 3 s.f.)

∴ The height of the container is **8.37 cm**.

**7.** **(i)** Volume of water = $\frac{1}{2} \times \pi\left(\frac{d}{2}\right)^2 (1)$ m³

$= \mathbf{\frac{1}{8}\pi d^2}$ **m³**

**(ii)** Since the container is half-filled, the height of the water is **0.5 m**.

**8.** **(i)** Capacity of tank = $\pi(3.6)^2(1.4)$ m³

$=$ **57.0 m³** (to 3 s.f.)

**(ii)** Surface area of tank in contact with the liquid

$= \left[\pi(3.6)^2 + 2\pi(3.6)\left(\frac{3}{4}\times 1.4\right)\right]$ m²

$=$ **64.5 m²** (to 3 s.f.)

**(iii)** 1 m = 100 cm

1 m² = 10 000 cm²

64.5 m² = **645 000 cm²**

**9.** Volume of disc

= Volume of larger cylinder – volume of smaller cylinder

$= [\pi(2.5)^2h - \pi(1.5)^2h]$ cm³

$= [6.25\pi h - 2.25\pi h]$ cm³

$=$ **4π*h* cm³**

**10.** **(i)** Volume of large cylinder = $\pi(10.5)^2(33)$ cm³

$= 3638.25\pi$ cm³

12 mm = 1.2 cm and 24 mm = 2.4 cm

Volume of small cylinder = $\pi(1.2)^2(2.4)$ cm³

$= 3.456\pi$ cm³

Number of small cylinders formed

$= \frac{3628.25\pi}{3.456\pi}$

$=$ **1052** (round down to the nearest whole number)

**(ii)** Total surface area

$= 1052 \times [2\pi(1.2)^2 + 2\pi(1.2)(2.4)]$ cm²

$=$ **28 600 cm²** (to 3 s.f.)

**11.** **(i)** Volume of water in cylindrical tank

$= \pi(6.8)^2\left(\frac{3}{4}\times 17.5\right)$ cm³

$= 606.9\pi$ cm³

$=$ **1910 cm³** (to 3 s.f.)

**(ii)** Capacity of cylindrical tank

$= \pi(2 \times 6.8)^2\left(\frac{1}{2}\times 17.5\right)$ cm³

$= 1618.4\pi$ cm³

Fraction of cylindrical tank filled with water

$= \frac{606.9\pi}{618.4\pi}$

$= \frac{3}{8}$

∴ **No.** The cylindrical tank is not $\frac{3}{4}$-filled with water.

**(iii)** Surface area of cylindrical tank in contact with water

$= \left[2\pi(2\times 6.8)^2 + 2\pi(2\times 6.8)\left(\frac{3}{8}\times\frac{1}{2}\times 17.5\right)\right]$ cm²

$=$ **1440 cm²** (to 3 s.f.)

**12.** Volume of coins = $[200 \times 150 \times (81 - 80)]$ cm³

$= 30\,000$ cm³

Volume of each coin = $\pi(1.1)^2(0.4)$ cm³

$= 0.484\pi$ cm³

Number of coins added = $\frac{30\,000}{0.484\pi}$

$=$ **19 730** (to the nearest integer)

**13.** Capacity of tank = $\pi(50)^2(110)$ cm$^3$
= $275\,000\pi$ cm$^3$
Rate of water flow = 24 $l$/min
= 24 000 cm$^3$/min
Time taken to fill the tank = $\frac{275\,000\pi}{24\,000}$ min
= **36.0 min** (to 3 s.f.)

**14.** **(i)** Volume of water discharged per minute
= $60 \times \pi(3.6)^2(180)$ cm$^3$
= $139\,968\pi$ cm$^3$
= $\frac{139\,968\pi}{1000}$ $l$
= **440 *l*** (to 3 s.f.)

**(ii)** Volume of water discharged in 11 min
= $(11 \times 139\,968\pi)$ cm$^3$
= $1\,539\,648\pi$ cm$^3$
Capacity of tank = $\pi(140)^2(120)$
= $2\,352\,000\pi$ cm$^3$
∴ The tank is **not yet completely filled** with water.

**15.** **(i)** Diameter of each piece of pencil lead = $\frac{9}{15}$ mm
= 0.6 mm
Radius of each piece of pencil lead = 0.3 mm
Volume of each piece of pencil lead
= $\pi(0.3)^2(60)$ mm$^3$
= **17.0 mm³** (to 3 s.f.)

**(ii)** Price per mm$^3$ of pencil lead = $\frac{\$2.60}{40 \times 16.964}$
= **\$0.004** (to 3 d.p.)

## Worksheet 11F Volume and surface area of composite solids

**1.** **(i)** Volume of toy = $[(24 \times 10 \times 10) + \pi(3)^2(20)]$ cm$^3$
= **2970 cm³** (to 3 s.f.)

**(ii)** Total surface area covered by yellow paint
= $[\pi(3)^2 + 2\pi(3)(20)]$ cm$^2$
= **405 cm²** (to 3 s.f.)

**(iii)** Total surface area covered by red paint
= $[2 \times 10 \times 10 + 4 \times 24 \times 10 - \pi(3)^2]$ cm$^2$
= **1130 cm²** (to 3 s.f.)

**2.** **(i)** Volume of figure
= $\left[(18\times10\times35)+\frac{1}{4}\times\pi(10)^2(35)\right]$ cm$^3$
= $(6300 + 875\pi)$ cm$^3$ (shown)

**(ii)** Area of cross section = $\left[(18\times10)+\frac{1}{4}\times\pi(10)^2\right]$ cm$^2$
= 258.54 cm$^2$ (to 5 s.f.)
Total surface area of figure
= $\left\{2\times258.54+\left[\frac{1}{4}\times2\pi(10)+18+10+18+10\right]\times35\right\}$ cm$^2$
= **3030 cm²** (to 3 s.f.)

**3.** **(i)** Assume that the loaf of bread is made up of a cuboid and a half-cylinder.
The radius and height of the half-cylinder are 5.5 cm and 15 cm respectively
The dimensions of the cuboid are 11 cm × 11.5 cm × 15 cm.
Volume of the loaf of bread
= $\left[\frac{1}{2}\times\pi(5.5)^2(15)+11\times11.5\times15\right]$ cm$^3$
= **2610 cm³** (to 3 s.f.)

**(ii)** Mass of the loaf of bread = $(0.2 \times 2610.2)$ g
= **522 g** (to 3 s.f.)

**4.** **(a)** OPEN $x$ could be **16 cm**. It refers to the diameter of the roll of kitchen paper towel.

**(b)** $\boldsymbol{y = x + 4}$

**(c)** **(i)** Area of cross section
= $\left[(x+4)\times4-\frac{1}{2}\pi\left(\frac{x}{2}\right)^2\right]$ cm$^2$
= $\left(4x+16-\frac{1}{8}\pi x^2\right)$ cm$^2$
Volume of kitchen paper towel holder
= $26 \times \left(4x+16-\frac{1}{8}\pi x^2\right)$ cm$^3$
= $\boldsymbol{\left(104x+416-\frac{13}{4}\pi x^2\right)}$ **cm³**

**(ii)** Total surface area of kitchen paper towel holder
= $\left\{2\times\left(104x+416-\frac{1}{8}\pi x^2\right)\right.$
$\left.+\ 26\left[\frac{1}{2}\times2\pi\left(\frac{x}{2}\right)+2+4+x+4+4+2\right]\right\}$ cm$^2$
= $\left[8x+32-\frac{1}{4}\pi x^2+26\left(\frac{1}{2}\pi x+16+x\right)\right]$ cm$^2$
= $\left(8x+32-\frac{1}{4}\pi x^2+13\pi x+416+26x\right)$ cm$^2$
= $\boldsymbol{\left(34x+13\pi x+448-\frac{1}{4}\pi x^2\right)}$ **cm²**

**5.** **(i)** Volume of prism before the hole was drilled
= $\left(\frac{1}{2}\times62\times62\times85\right)$ cm$^3$
= 163 370 cm$^3$
Volume of cylinder removed = $(\pi r^2 \times 85)$ cm$^3$
= $85\pi r^2$ cm$^3$
$\frac{\text{Volume of solid removed}}{\text{Volume of solid}} = \frac{85\pi r^2}{163\,370-85\pi r^2}$
$\frac{1}{6} = \frac{85\pi r^2}{163\,370-85\pi r^2}$
$163\,370 - 85\pi r^2 = 510\pi r^2$
$163\,370 = 595\pi r^2$
$595\pi r^2 = 163\,370$
$r^2 = \frac{163\,370}{595\pi}$
$r = \sqrt{\frac{163\,370}{595\pi}}$
= **9.35** (to 3 s.f.)

**(ii)** Area of cross section

$= \left[\frac{1}{2} \times 62 \times 62 - \pi(9.3487)^2\right]$ cm$^2$

= 1647.4 cm$^2$

Total surface area of solid

= [2 × 1647.4 + (62 + 62 + 88) × 85 + 2π(9.3487)(85)] cm$^2$

= **26 300 cm$^2$** (to 3 s.f.)

**6.** Area of cross section $= \left[6 \times \frac{1}{2} \times 1.2 \times 1.04 - \pi(0.85)^2\right]$ m$^2$

= 1.4742 m$^2$ (to 5 s.f.)

Volume of frame = (1.4742 × 4) m$^3$

= 5.8968 m$^3$ (to 5 s.f.)

Volume of concrete used $= \frac{70}{100} \times 5.8968$ m$^3$

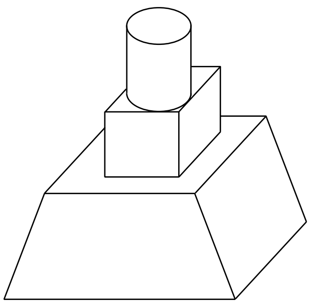
= 4.1278 m$^3$ (to 5 s.f.)

Total cost of concrete = **\$4.13*x*** (to 3 s.f.)

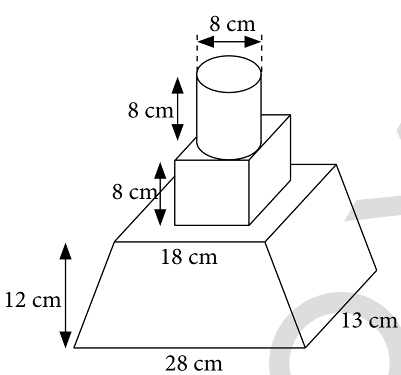
## Challenge Myself!

**7.** OPEN

**(i)**



**(ii)**



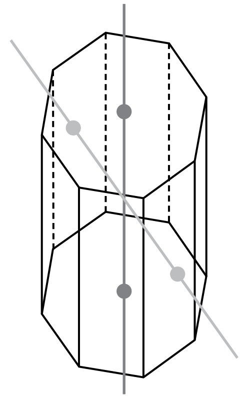

## Worksheet 11G Symmetry in right prisms and cylinders

**1.** **(a)** **infinite number**

**(b)** **No**, *PQ* does not divide the cylinder into two identical parts.

**2.** **(a)** **9**

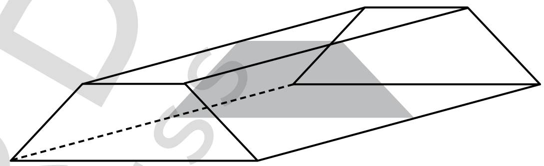
**(b)**



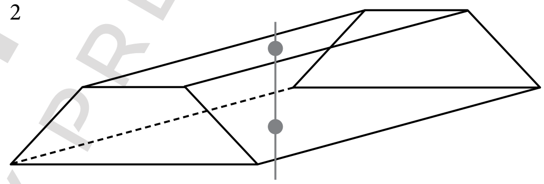
**3.** **(a)** 9

**(b)** 4

**4.** **(a)**



**(b)** 2



## Review Exercise 11

**1.** **(i)** Let the height of the cuboid be *h* cm.

Volume of cuboid = (4.5 × 4.5 × *h*) cm$^3$

162 = 20.25*h*

20.25*h* = 162

*h* = 8

∴ The height of the cuboid is **8 cm**.

**(ii)** Total surface area of cuboid

= [2 × 4.5 × 4.5 + (4 × 4.5 × 8)] cm$^2$

= **184.5 cm$^2$**

**2.** **(i)** Area of the trapezium $= \left[\frac{1}{2} \times (24 + 32) \times 15\right]$ cm$^2$

= **420 cm$^2$**

**(ii)** Volume of prism = 420 × *l*

11 970 = 420*l*

420*l* = 11 970

*l* = 28.5

∴ The length of the prism is **28.5 cm**.

**3.** **(a)** **(i)** Volume of water discharged per minute

$= 60 \times \pi\left(\frac{116}{2}\right)^2 (300)$ cm$^3$

= 605 520π cm$^3$

Volume of water discharged in 15 minutes

= (15 × 605 520π) cm$^3$

= 9 082 800π cm$^3$

= **28 500 000 cm$^3$** (to 3 s.f.)

**(ii)** $28\,500\,000 \text{ cm}^3 = \frac{28\,500\,000}{1000}\ l$

$= \mathbf{28\,500\ \boldsymbol{l}}$

**(b)** Volume discharged by 2 pipes per second

$= 2 \times \pi\left(\frac{11.6}{2}\right)^2 (300) \text{ cm}^3$

$= 63\,410 \text{ cm}^3$ (to 5 s.f.)

∴ **Yes.** The pipes are able to discharge the water as quickly as it enters the area.

**4.** **(i)** Volume of one candle $= \pi(8)^2(75) \text{ mm}^3$

$= 4800\pi \text{ mm}^3$

$= \mathbf{15\,100\ mm^3}$ (to 3 s.f.)

**(ii)** Maximum number of candles

$= \frac{400}{16} \times \frac{240}{16} \times \frac{150}{75}$

$= \mathbf{750}$

**(iii)** Volume of empty space

$= [400 \times 240 \times 150 - 750 \times \pi(8)^2(75)] \text{ cm}^3$

$= \mathbf{3\,090\,000\ mm^3}$ (to 3 s.f.)

**5.** **(i)** Volume of each slice of pineapple

$= \left[\pi\left(\frac{15}{2}\right)^2(2) - \pi\left(\frac{8}{2}\right)^2(2)\right] \text{ cm}^3$

$= 80.5\pi \text{ cm}^3$

$= \mathbf{253\ cm^3}$ (to 3 s.f.)

**(ii)** Mass of each slice of pineapple $= (80.5\pi \times 1.4)$ g

$= 112.7\pi$ g

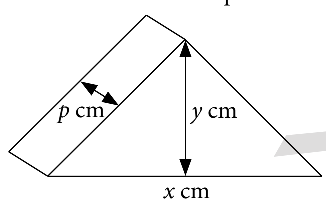
$= 354.06$ g (to 5 s.f.)

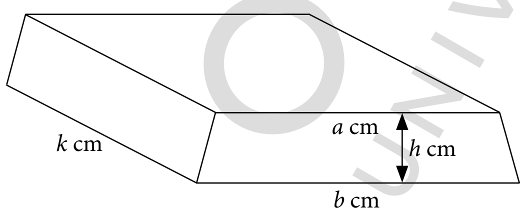
Amount of sugar in each slice of pineapple

$= \frac{112.7\pi}{112}$ g

$= \mathbf{28.5\ g}$ (to 3 s.f.)

**6.** **(i)** Let the dimensions of the two parts be as shown.




Estimated volume of each prism with a triangular cross section $= \frac{1}{2}xyp \text{ cm}^3$, where $p$ cm is the width of the prism along the middle

Estimated volume of the prism with a trapezoidal cross section $= \frac{1}{2}(a + b)hk \text{ cm}^3$

∴ Estimated volume of the bar of chocolate

$= \left[\frac{9}{2}pxy + \frac{1}{2}(a+b)hk\right] \text{ cm}^3$

**(ii)** The second part is assumed to have 9 identical prisms, but they do not have a uniform width.

# 12 Introduction to Sets and Probability

## Worksheet 12A Introduction to sets and set notations

**1.** **(a)** **{125, 216, 343, 512, 729}**

**(b)** **{2}**

**2.** **(i)** $A$ = **{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11}**

**(ii)** n($A$) = **11**

**(iii)** **(a)** **True**

**(b)** **True**

**3.** **(a)** **Yes**, $A = B$.

**(b)** **No**, $C \neq D$.

**4.** **(a)** **No.** $A$ and $B$ are not equal sets because they do not have the same elements.

**(b)** **No.** $C$ is not a null set because it contains the element 0.

**5.** (OPEN) **(a)** $P$ = **{1, 2, 3, 4}**

$Q$ = **{1, 2, 3, 4, 5}**

**(b)** $P$ = **{1, 2, 3, 4}**

$Q$ = **{5, 6, 7, 8}**

**6.** **(a)** (OPEN) **(0, 7)**

**(b)** Substitute $x = 1$ and $y = 3$:

$x + 2y = 1 + 2(3)$

$= 7$

$\neq 5$

∴ (1, 3) is not an element of $B$.

**(c)** When $y = 7$,

$x + 2(7) = 5$

$x = -9$

∴ The common element is **(–9, 7)**.

## Challenge Myself!

**7.** (OPEN) $P$ = **{0, 4, 6, 7, 11}**

## Worksheet 12B Venn diagrams, universal sets, complements of sets and subsets

**1.** **(a)** **{p}, {q}, {p, q}, { }**

**(b)** **{car}, {bus}, {train}, {car, bus}, {car, train}, {bus, train}, {car, bus, train}, { }**

**2.** **(a)** **{4}, {5}, { }**

**(b)** **{red}, {blue}, {yellow}, {red, blue}, {red, yellow}, {blue, yellow}, { }**

**3.** **(a)** **(i)** $P$ = **{1, 8}**

**(ii)** $Q'$ = **{2, 4, 6, 8, 10}**

**(b)** $Q$ = {1, 3, 5, 7, 9}

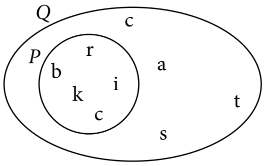
n($Q$) = **5**

**4.** **(a)** $A$ = **{1, 2, 3, 4, 5, 6, 7, 8, 9}**

**(b)** $B$ = **{1, 2, 3, 6}**

**(c)** $B$ is a proper subset of $A$ because every element in $B$ is an element of $A$.

**5.** **(a)**




**(b)** **Yes**, $P \subseteq Q$ because every element in $P$ is an element of $Q$ and $n(P) \leqslant n(Q)$.

**(c)** **No**, $Q \not\subseteq P$ because $n(Q) > n(P)$ and there are elements in $Q$ that are not elements of $P$.

**6.** **(a)** **(i)** $A'$ is the set of perfect squares between 2 and 17 inclusive.

**(ii)** $B'$ is the set of composite numbers between 2 and 17 inclusive.

**(b)** $\boldsymbol{B \subset A}$

**7.** **(a)**

$2x + 3 \leqslant 27$

$2x \leqslant 24$

$x \leqslant 12$

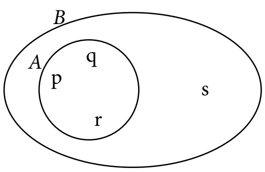
$\xi = \{1, 3, 5, 7, 9, 11\}$

$M = \{3, 9\}$

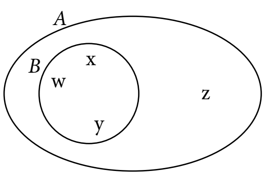
$\therefore M' = \mathbf{\{1, 5, 7, 11\}}$

**(b)** $M'$ is the set of odd integers between 1 and 11 inclusive which are not multiples of 3.

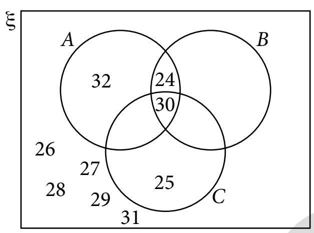
**8.** **(a)**




**(b)**




**9.** **(a)**

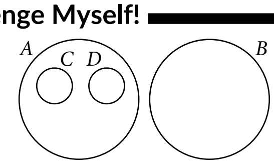



**(b)** **(i)** $\{24, 30\} \not\subseteq \boldsymbol{C}$

**(ii)** $25 \in \boldsymbol{C}$

## Challenge Myself!

**10.** **(a)**




**(b)** $\mathbf{0.\dot{2}} \in C$; $\mathbf{0.1} \in D$

## Worksheet 12C Probability experiment and sample space

**1.** **(i)** **{head, tail}**

**(ii)** Total number of possible outcomes = **2**

**2.** **(i)** **{1, 2, 3, 4, 5, 6}**

**(ii)** Total number of possible outcomes = **6**

**3.** **(i)** **{\$20 voucher, Free salad, Free ginseng chicken, \$10 voucher, Free ice cream, Free seafood pancake}**

**(ii)** Total number of possible outcomes = **6**

**4.** **(i)** **{January, February, March, April, May, June, July, August, September, October, November, December}**

**(ii)** Total number of possible outcomes = **12**

**5.** **(i)** $\mathbf{\{A_1, A_2, E_1, E_2, I_1, I_2, O_1, O_2, U_1, U_2\}}$

**(ii)** Total number of possible outcomes = **10**

**6.** **(i)** **{1086, 1087, 1088, …, 1103}**

**(ii)** Total number of possible outcomes = **18**

**7.** **(i)** **{00, 01, 02, …, 59}**

**(ii)** Total number of possible outcomes = **60**

**8.** **(i)** **{24, 29, 42, 49, 92, 94}**

**(ii)** Total number of possible outcomes = **6**

## Worksheet 12D Probability of single events

**1.** **(a)** P(a head) = $\mathbf{\frac{1}{2}}$

**(b)** P(a tail) = $\mathbf{\frac{1}{2}}$

**2.** **(a)** P(a ‘4’) = $\mathbf{\frac{1}{6}}$

**(b)** P(a ‘1’ or a ‘6’) = $\frac{2}{6}$

$= \mathbf{\frac{1}{3}}$

**(c)** P(a prime number) = P(a ‘2’, a ‘3’ or a ‘5’)

$= \frac{3}{6}$

$= \mathbf{\frac{1}{2}}$

**(d)** P(a multiple of 10) = **0**

**3.** **(a)** **{1, 2, 3, 5, 7}**

**(b)** **(i)** P(a number not more than 5) = $\mathbf{\frac{4}{5}}$

**(ii)** P(a prime number) = $\mathbf{\frac{4}{5}}$

**4.** **(a)** P(a ‘P’) = $\mathbf{\frac{2}{9}}$

**(b)** P(a ‘T’) = **0**

**5.** **(a)** P(a red card) = $\frac{7}{6+8+7}$

$= \mathbf{\frac{1}{3}}$

**(b)** P(not a green card) = $\frac{6+7}{21}$

$= \mathbf{\frac{13}{21}}$

**(c)** P(a yellow card) = **0**

6. (i) P(passenger sits in the Premium economy section)

$= \frac{28}{4+48+28+184}$

$= \mathbf{\frac{7}{66}}$

(ii) P(passenger does not sit in Business class) $= \frac{4+28+184}{264}$

$= \mathbf{\frac{9}{11}}$

7. (i) P(an 'I') $= \mathbf{\frac{2}{9}}$

(ii) P(a consonant) $= \mathbf{\frac{4}{9}}$

(iii) P(a vowel) $= \mathbf{\frac{5}{9}}$

(iv) P(a 'P') = **0**

8. (a) P(a '30') $= \mathbf{\frac{1}{20}}$

(b) P(an even number) $= \mathbf{\frac{1}{2}}$

(c) P(a multiple of 64) = **0**

(d) P(a perfect square or a perfect cube)

= P(a '25', a '27' or a '36')

$= \mathbf{\frac{3}{20}}$

9. Sample space: {30, 36, 37, 60, 63, 67, 70, 73, 76}

(i) P(number is 60) $= \mathbf{\frac{1}{9}}$

(ii) P(number formed is not 49) = **1**

10. Sample space: {10, 11, 14, 19, 40, 41, 44, 49, 90, 91, 94, 99}

(i) P(an odd number) $= \frac{6}{12}$

$= \mathbf{\frac{1}{2}}$

(ii) P(a factor of 266) = P(a '14' or a '19')

$= \frac{2}{12}$

$= \mathbf{\frac{1}{6}}$

11. (a) **{100, 101, 102, …, 999}**

(b) (i) P(an even number) $= \mathbf{\frac{1}{2}}$

(ii) P(a perfect cube)

= P(a '125', a '216', a '343', a '512' or a '729')

$= \frac{5}{900}$

$= \mathbf{\frac{1}{180}}$

12. Number of cans which are not chilli tuna flakes $= \frac{5}{8} \times 40$

= **25**

13. (a) P(a black or white ball) = 0.36 + 0.4

= **0.76**

(b) P(a purple ball) = 1 – 0.76

= **0.24**

14. P(rain) = 0.15

Let P(cloudy) = P(sunny) = 0.1.

Then P(thunderstorm) = 1 – 0.15 – 2(0.1)

= 0.65.

∴ P(thunderstorm) = **0.65**, P(sunny) = **0.1**

15. (a) P(a girl with perfect vision) $= \frac{24}{30+24+21+35}$

$= \mathbf{\frac{12}{55}}$

(b) P(a student who is myopic) $= \frac{21+35}{110}$

$= \mathbf{\frac{28}{55}}$

16. (a) Number of hours on an analogue clock = 12

Number of hours between 4:45 and 6:15 = 1.5

P(clock shows a time between 4:45 and 6:15) $= \frac{1.5}{12}$

$= \frac{1}{8}$ (shown)

(b) P(hour hand points to an area between '2' and '3') $= \mathbf{\frac{1}{12}}$

17. Let P(a '1') = P(a '3') = P(a '5') = $x$.

Then P(a '4') $= \frac{9}{5}x$, P(a '6') $= \frac{12}{5}x$ and P(a '2') $= \frac{4}{5}x$.

$x + x + x + \frac{9}{5}x + \frac{12}{5}x + \frac{4}{5}x = 1$

$8x = 1$

$x = \frac{1}{8}$

∴ P(a '4') $= \frac{9}{5}\left(\frac{1}{8}\right)$

$= \mathbf{\frac{9}{40}}$

18. **No.** The sum of the probabilities of a team winning or losing or ending in a draw is 1, but the individual probabilities might not necessarily be $\frac{1}{3}$.

## Challenge Myself!

19. Based on Clue 1, there are 3 composite numbers and 1 prime number.

Based on Clue 2, one of the numbers can be 1 or 64.

Based on Clue 3, one of the numbers is a 2, and none of the numbers is a 1; hence 2 and 64 are two of the numbers, with the remaining two being composite numbers, one of which is perfect square.

Based on Clue 4, the remaining two numbers are odd numbers.

Based on Clue 5, one of the remaining two numbers has two of 3, 5 and 7 as its factors.

∴ A set of possible numbers is **2, 35, 64** and **81**.

## Worksheet 12E Further examples of probability of single events

1. P(point lies on $PQ$) $= \frac{4}{4+1}$

$= \mathbf{\frac{4}{5}}$

2. (a) P(point lies in sector $V$)

$= \frac{360° - 90° - 45° - 105° - 45°}{360°}$ (∠s at a pt.)

$= \mathbf{\frac{5}{24}}$

(b) P(point lies in sector $X$ or $Z$) $= \frac{105° + 90°}{360°}$

$= \mathbf{\frac{13}{24}}$

3. (a) $4.5p° + 90° + 2p° + (2p + 15)° = 360°$ (∠s at a pt.)

$$8.5p° + 105° = 360°$$
$$8.5p° = 255°$$
$$p° = 30°$$

$\therefore p = \mathbf{30}$

(b) (i) P(student will travel to Europe) $= \frac{4.5(30°)}{360°} = \mathbf{\frac{3}{8}}$

(ii) P(student will travel to Asia or North America)

$$= \frac{2(30°)+15°+90°}{360°} = \mathbf{\frac{11}{24}}$$

(iii) P(student will travel to South America) = **0**

(c) Number of students whose exchange programme will take place in Asia

$$= \frac{2(30°)+15°}{360°}\times 144 = \mathbf{30}$$

4. P(a white cabbage) = 0.56

$$\frac{14}{14+x} = 0.56$$
$$14 = 7.84 + 0.56x$$
$$0.56x = 6.16$$
$$x = \mathbf{11}$$

5. P(a green cushion) $= 1-\frac{4}{11} = \frac{7}{11}$

Number of red cushions $= \frac{280}{7}\times 4 = \mathbf{160}$

6. P(a red apple) $= \frac{3}{7}$

$$\frac{45-p}{(5p+3)+(45-p)} = \frac{3}{7}$$
$$\frac{45-p}{4p+48} = \frac{3}{7}$$
$$315 - 7p = 12p + 144$$
$$19p = 171$$
$$p = \mathbf{9}$$

7. (a) $p = 50 - 35 = \mathbf{15}$

(b)

| | Brown rice | White rice |
|---|---|---|
| Vegetarian | 5 | 10 |
| Seafood | 20 | 15 |

(i) P(a vegetarian rice roll made from white rice) $= \frac{10}{50} = \mathbf{\frac{1}{5}}$

(ii) P(a rice roll made from brown rice) $= \frac{5+20}{50} = \mathbf{\frac{1}{2}}$

8. (a) P(not a toffee) $= \frac{12+11}{12+9+11} = \mathbf{\frac{23}{32}}$

(b) Let $x$ be the number of fruit pastilles eaten.

P(a mint) = 0.5

$$\frac{12}{32-x} = 0.5$$
$$12 = 16 - 0.5x$$
$$0.5x = 4$$
$$x = 8$$

∴ **8** fruit pastilles were eaten.

9. (a) P(a picture card) $= \frac{6}{8+10+6} = \mathbf{\frac{1}{4}}$

(b) Let $x$ be the number of alphabet cards removed.

P(an alphabet card) $= \frac{1}{3}$

$$\frac{10-x}{24-x} = \frac{1}{3}$$
$$30 - 3x = 24 - x$$
$$2x = 6$$
$$x = 3$$

∴ **3** alphabet cards must be removed.

(c) Assume that $y$ number cards are added.

P(a number card) $= \frac{3}{7}$

$$\frac{8+y}{24+y} = \frac{3}{7}$$
$$56 + 7y = 72 + 3y$$
$$4y = 16$$
$$y = 4$$

∴ **4** number cards must be **added**.

10. Area of each triangle $= \frac{1}{2}(4)(4)$ cm$^2$ $= 8$ cm$^2$

Area of rectangle $= [(12 + 4) \times (4 + 4)]$ cm$^2$ $= 128$ cm$^2$

P(the point lies inside one of the triangles) $= \frac{8+8}{128} = \mathbf{\frac{1}{8}}$

11. (i) Arc length of semicircle $C = 3\pi$ cm

$$\frac{1}{2}(2\pi r_C) = 3\pi$$
$$r_C = 3$$

∴ Radius of semicircle $C$ = **3 cm**

(ii) Area of semicircle $A = \frac{1}{2}\left[\pi(2)^2\right]$ cm$^2$ $= 2\pi$ cm$^2$

Area of semicircle $B = \frac{1}{2}\left[\pi(4)^2\right]$ cm$^2$ $= 8\pi$ cm$^2$

Area of semicircle $C = \frac{1}{2}\left[\pi(3)^2\right] = \frac{9}{2}\pi$ cm$^2$

P(the point lies inside semicircle $B$) $= \frac{8\pi}{2\pi+8\pi+\frac{9}{2}\pi} = \mathbf{\frac{16}{29}}$

## Challenge Myself!

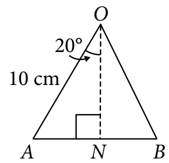
**12.** Circumference of circle = $20\pi$ cm

$$2\pi(OA) = 20\pi$$
$$OA = 10 \text{ cm}$$

$\angle AOB = \frac{1}{9} \times 360°$

$= 40°$

Consider $\triangle OAB$.




$\sin 20° = \frac{AN}{10}$

$AN = 10 \sin 20°$

$\cos 20° = \frac{ON}{10}$

$ON = 10 \cos 20°$

Area of $\triangle OAB = \frac{1}{2}(2 \times 10 \sin 20°)(10 \cos 20°)$ cm$^2$

$= 32.139$ cm$^2$ (to 5 s.f.)

P(the point lies outside the triangle) $= \frac{\pi(10)^2 - 32.139}{\pi(10)^2}$

$=$ **0.90** (to 2 d.p.)

## Review Exercise 12

**1.** **(a)** **{1, 2, 4, 8}**

**(b)** **{3, 5, 6, 7, 9}**

**(c)** $\boldsymbol{P \subset Q}$

**2.** **(a)** **(i)** $A'$ is the set of perfect cubes between 1 and 12 inclusive.

**(ii)** $B'$ is the set of prime numbers between 1 and 12 inclusive.

**(b)** $C' = \{1, 3, 5, 7, 9, 11\}$

**No**, $B \not\subset C'$ because there is an element in $B$ (i.e. 2) that is not an element of $C'$.

**3.** **(a)** **{2, 3, 5, 7, 11, 13, 17, 19}**

**(b)** **(i)** P(an odd number) $= \boldsymbol{\frac{7}{8}}$

**(ii)** P(a factor of 24) = P(a '2' or a '3')

$= \frac{2}{8}$

$= \boldsymbol{\frac{1}{4}}$

**4.** Number of devices that are not faulty = $(1 - 0.065) \times 2000$

$=$ **1870**

**5.** **(a)** The probability of obtaining each number is not the same, i.e. $\frac{1}{6}$.

**(b)** **(i)** $x = 150 - (23 + 18 + 42 + 30 + 17)$

$=$ **20**

**(ii)** P(a '3' or a '5') $= \frac{42 + 30}{150}$

$= \boldsymbol{\frac{12}{25}}$

**6.** Sample space: {508, 518, 528, 538, 548, 558, 568, 578, 588, 598}

**(i)** P(number formed is 548) $= \boldsymbol{\frac{1}{10}}$

**(ii)** P(number formed is greater than 590) $= \boldsymbol{\frac{1}{10}}$

**(iii)** P(number formed is an even number) = **1**

**(iv)** P(number formed is a multiple of 25) = **0**

**7.** **(a)** P(not a red card) $= \frac{9 + 10}{9 + 6 + 10}$

$= \boldsymbol{\frac{19}{25}}$

**(b)** P(a blue card) $= \frac{1}{2}$

$$\frac{9}{25 - x} = \frac{1}{2}$$
$$18 = 25 - x$$
$$x = \mathbf{7}$$

**8.** **(i)** P(student's favourite snack is dried fruit)

$= \frac{7}{3 + 9 + 7 + 10}$

$= \boldsymbol{\frac{7}{29}}$

**(ii)** P(student's favourite snack is not biscuits) $= 1 - \frac{3}{29}$

$= \boldsymbol{\frac{26}{29}}$

**9.** Area of largest circle $= \pi(3x)^2$ cm$^2$

$= 9\pi x^2$ cm$^2$

Area of region $C = \pi x^2$ cm$^2$

Area of region $B = [\pi(2x)^2 - \pi x^2]$ cm$^2$

$= (4\pi x^2 - \pi x^2)$ cm$^2$

$= 3\pi x^2$ cm$^2$

**(a)** P(the point lies inside region $C$) $= \frac{\pi x^2}{9\pi x^2}$

$= \boldsymbol{\frac{1}{9}}$

**(b)** P(the point lies inside region $B$) $= \frac{3\pi x^2}{9\pi x^2}$

$= \boldsymbol{\frac{1}{3}}$

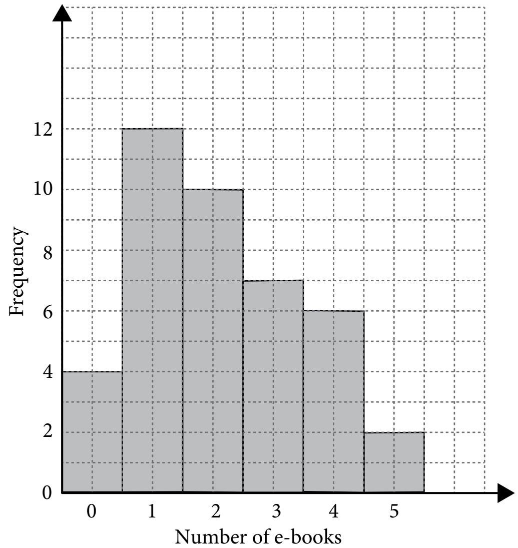
# 13 Histograms

## Worksheet 13A Histograms for ungrouped data

**1.** **(a)**

| Number of e-books | Tally | Frequency |
|---|---|---|
| 0 | \|\|\|\| | **4** |
| 1 | 𝍸 𝍸 \|\| | **12** |
| 2 | 𝍸 𝍸 | **10** |
| 3 | 𝍸 \|\| | **7** |
| 4 | 𝍸 \| | **6** |
| 5 | \|\| | **2** |

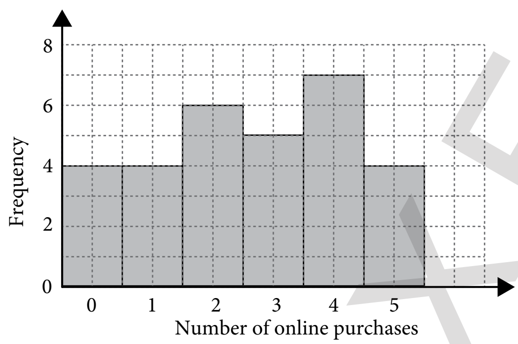
(b)




2. (a)

| Number of online purchases | Frequency |
|---|---|
| 0 | 4 |
| 1 | 4 |
| 2 | 6 |
| 3 | 5 |
| 4 | 7 |
| 5 | 4 |

(b)




(c) Most common number of online purchases = **4**

## Worksheet 13B Histograms for grouped data

1. (a) Number of bottles of chemical solution that had a pH value of between 7 and 8 = **6**

(b) P(a bottle contains chemical solution that is not acidic)

$= \dfrac{6+7}{30}$

$= \mathbf{\dfrac{13}{30}}$

2. (a) Number of players who caught between 10 and 25 monsters = 20 + 36 + 41

= **97**

(b) Percentage of players who caught between 25 and 30 monsters

$= \dfrac{48}{15+28+20+36+41+48} \times 100\%$

= 25.5%

Marcus incorrectly deduced that of the 48 players who lie in the interval 25–30, all of them caught the maximum of 30 monsters.

3. (i)

| Earnings ($\$x$) | Frequency |
|---|---|
| $0 \leqslant x < 20$ | 2 |
| $20 \leqslant x < 40$ | 10 |
| $40 \leqslant x < 60$ | 16 |
| $60 \leqslant x < 80$ | 19 |
| $80 \leqslant x < 100$ | 3 |

(ii) Interval that contains the most food delivery drivers is $\mathbf{60 \leqslant x < 80}$

(iii) Number of drivers who earned at least \$20 on that day

= 10 + 16 + 19 + 3

= **48**

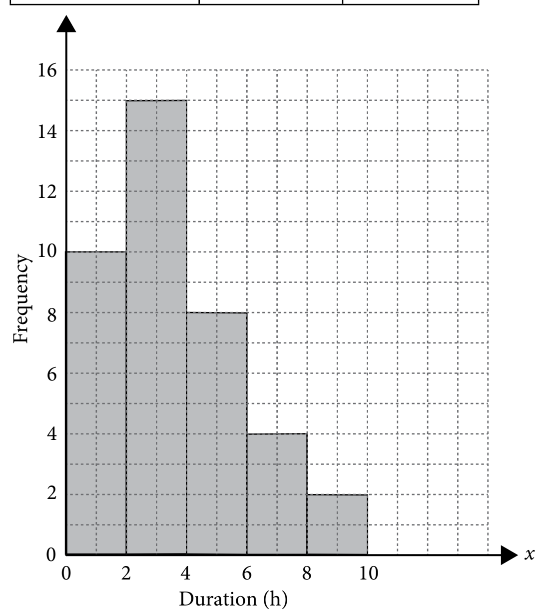
(iv) Required percentage $= \dfrac{2+10+16}{2+10+16+19+3} \times 100\%$

= **56%**

4. (a)

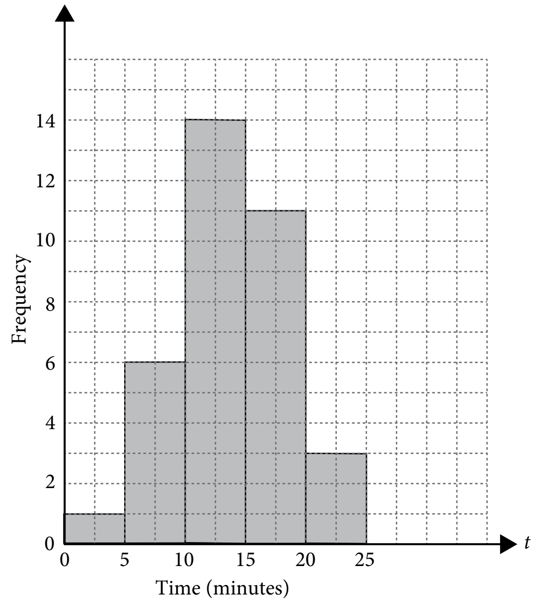
| Duration ($x$ h) | Tally | Frequency |
|---|---|---|
| $0 \leqslant x < 2$ | ~~||||~~ ~~||||~~ | **10** |
| $2 \leqslant x < 4$ | ~~||||~~ ~~||||~~ ~~||||~~ | **15** |
| $4 \leqslant x < 6$ | ~~||||~~ ||| | **8** |
| $6 \leqslant x < 8$ | |||| | **4** |
| $8 \leqslant x < 10$ | || | **2** |

(b)




5. (a) The values of $t = 5$, $t = 10$ and so on would appear in two intervals.

**(b)**




**(c)** P(student took more than 15 minutes) $= \dfrac{11+3}{1+6+14+11+3}$

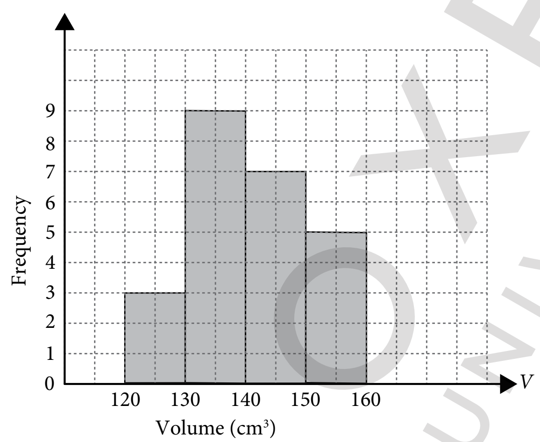
$= \mathbf{\dfrac{2}{5}}$

**(d)** It would be placed in $\mathbf{5 < t \leqslant 10}$.

**6.** **(a)**

| Volume ($V$ cm³) | Frequency |
|---|---|
| $120 \leqslant V < 130$ | 3 |
| $130 \leqslant V < 140$ | 9 |
| $140 \leqslant V < 150$ | 7 |
| $150 \leqslant V < 160$ | 5 |

**(b)**



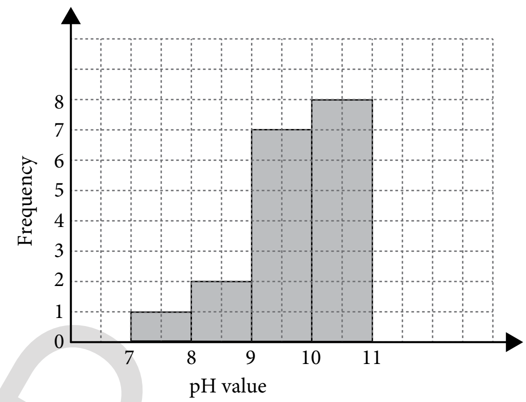

**(c)** OPEN At least **3** sets of data were added, e.g. **142 cm³**, **143 cm³** and **144 cm³**.

## Review Exercise 13

**1.** **(a)** Lowest pH value recorded = **7.5**

**(b)** Class interval with the highest frequency is **10.0–10.9**

**(c)**




## End-of-year Checkpoint A

**Section A**

**1.** $\dfrac{8}{9} > 88\%$ **[1]**

**2.** **(a)** P(plain paper) $= \dfrac{14}{9+17+14}$

$= \mathbf{\dfrac{7}{20}}$ **[1]**

**(b)** Let $x$ be the number of notebooks with lined paper that must be added.

$\dfrac{17+x}{9+17+14+x} = \dfrac{1}{2}$ **[1]**

$\dfrac{17+x}{40+x} = \dfrac{1}{2}$

$2(17+x) = 40 + x$

$34 + 2x = 40 + x$

$x = 6$

∴ **6** notebooks with lined paper must be added. **[1]**

**3.** $\dfrac{5(1-3x)}{6} \geqslant -4$

$5 - 15x \geqslant -24$

$-15x \geqslant -29$

$x \leqslant 1\dfrac{14}{15}$ **[1]**

∴ Greatest rational value of $x$ is $\mathbf{1\dfrac{14}{15}}$ **[1]**

**4.** $9ax^2 - 30axy + 25ay^2 = a(9x^2 - 30xy + 25y^2)$ **[1]**

$= a[(3x)^2 - 2(3x)(5y) + (5y)^2]$

$= \mathbf{a(3x - 5y)^2}$ **[1]**

5. $y = \frac{k}{x^3}$

When $x = \frac{1}{4}$, $y = 4$,

$4 = \frac{k}{\left(\frac{1}{4}\right)^3}$

$k = \frac{1}{16}$

$y = \frac{1}{16x^3}$ [1]

When $x = \frac{1}{3}$,

$y = \frac{1}{16\left(\frac{1}{3}\right)^3}$

$= 1\frac{11}{16}$ [1]

$\therefore$ $y = \mathbf{1\frac{11}{16}}$

6. $\frac{PQ}{BC} = \frac{AP}{AB}$

$\frac{x}{14} = \frac{24+6}{24}$

$x = \frac{30}{24} \times 14$

$= 17.5$ [1]

$\frac{AC}{AQ} = \frac{AB}{AP}$

$\frac{y}{y+5} = \frac{24}{24+6}$

$= \frac{4}{5}$

$5y = 4(y + 5)$

$= 4y + 20$

$y = 20$ [1]

$\therefore$ $\mathbf{x = 17.5, y = 20}$

7. $12ax - 5by - 6bx + 10ay = 12ax - 6bx + 10ay - 5by$

$= 6x(2a - b) + 5y(2a - b)$ [1]

$= \mathbf{(2a - b)(6x + 5y)}$ [1]

8. Consider $\triangle PRS$.

$\tan 35° = \frac{PR}{16}$

$PR = 16 \tan 35°$ cm

Consider $\triangle QRS$.

$\tan 24° = \frac{QR}{16}$

$QR = 16 \tan 24°$ cm [1]

$\therefore PQ = PR - QR$

$= 16 \tan 35° - 16 \tan 24°$

= **4.08 cm** (to 3 s.f.) [1]

9. **(i)** $x^2 - 9801 = x^2 - 99^2$

$= \mathbf{(x + 99)(x - 99)}$ [1]

**(ii)** Since $31\ 003 + 9801 = 40\ 804 = 202^2$,

let $x = 202$:

$31\ 003 = 202^2 - 9801$

$= (202 + 99)(202 - 99)$

$= 301 \times 103$

$\therefore$ Two factors of 31 003 are **301** and **103**. [1]

10. **(a)** 1 cm represents 5 000 000 cm = $\frac{5\ 000\ 000}{100 \times 1000}$ km = 50 km.

80.6 cm represents (80.6 × 50) km = 4030 km.

$\therefore$ Actual distance = **4030 km** [1]

**(b)** 1 cm represents 50 km.

1 cm$^2$ represents $50^2$ km$^2$ = 2500 km$^2$. [1]

2500 km$^2$ is represented by 1 cm$^2$.

7 692 000 km$^2$ is represented by $\frac{7\ 692\ 000}{2500}$ cm$^2$ = 3076.8 cm$^2$.

$\therefore$ Area of Australia on the map = **3076.8 cm$^2$** [1]

11. **(a)** $S_n = an - bn^2$

When $n = 2$,

$S_2 = a(2) - b(2)^2 = 12 + 5$

$2a - 4b = 17$ (shown) [1]

**(b)** $a - b = 12$ — (1)

$2a - 4b = 17$ — (2)

(1) × 2: $2a - 2b = 24$ — (3)

(3) – (2): $2b = 7$ [1]

$b = 3.5$ [1]

Substitute $b = 3.5$ into (1): $a - 3.5 = 12$, i.e. $a = 15.5$ [1]

$\therefore$ $\mathbf{a = 15.5, b = 3.5}$

12. Consider the small bottle.

$\frac{\text{cost}}{\text{volume}} = \frac{8.5}{50}$

$= 0.17$ [1]

Consider the large bottle.

$\frac{\text{cost}}{\text{volume}} = \frac{37.4}{220}$

$= 0.17$ [1]

Since $\frac{\text{cost}}{\text{volume}}$ is the same, the cost is directly proportional to its volume. (shown) [1]

13. (OPEN) **A histogram for grouped data** could be used. [1]

It is suitable to display a data set containing many different values as it is unlikely that a few students walk an identical number of steps each day. [1]

The number of steps could be grouped into equal class intervals such as 1–1000, 1001–2000, 2001–3000, and so on. [1]

14. Let the length of the diagonal be $x$ cm.

Consider a rectangle with dimensions 10 units × 1 unit.

Width = 6 cm

Length = 60 cm

Using Pythagoras' Theorem,

$x^2 = 60^2 + 6^2$ [1]

$= 3636$

$x = \sqrt{3636}$ (since $x > 0$)

$= 60.3$ (to 3 s.f.) [1]

Consider a rectangle with dimensions 5 units × 2 units.

Width = 6 cm

Length = 15 cm

Using Pythagoras' Theorem,

$x^2 = 15^2 + 6^2$ [1]

$= 261$

$x = \sqrt{261}$ (since $x > 0$)

$= 16.2$ (to 3 s.f.) [1]

$\therefore$ The possible lengths are **60.3 cm** and **16.2 cm**.

**Section B**

**15.** **(a)** $y = k\sqrt{x}$

When $x = 64$, $y = 6$,

$6 = k\sqrt{64}$

$k = \frac{3}{4}$ **[1]**

$\therefore \boldsymbol{y = \frac{3}{4}\sqrt{x}}$ **[1]**

**(b)** When $x = 100$,

$y = \frac{3}{4}\sqrt{100}$

$= \mathbf{7.5}$ **[1]**

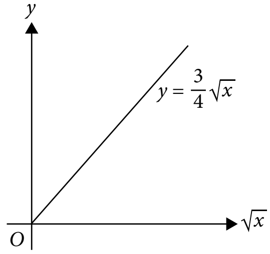
**(c)** When $y = 9$,

$9 = \frac{3}{4}\sqrt{x}$

$\sqrt{x} = 12$

$x = \mathbf{144}$ **[1]**

**(d)**




**[1]**

**16.** **(a)** **(i)** $\tan \angle RPS = \frac{37}{65}$ **[1]**

$\angle RPS = \tan^{-1}\frac{37}{65}$

$= \mathbf{29.6°}$ (to 1 d.p.) **[1]**

**(ii)** $PQ^2 + QR^2 = 33^2 + 56^2$

$= 4225$

$PR^2 = 65^2$

$= 4225$ **[1]**

Since $PQ^2 + QR^2 = PR^2$, then by the converse of Pythagoras' Theorem, $\triangle PQR$ is a right-angled triangle with $\angle PQR = 90°$. **[1]**

**(iii)** Area of $PQRS$ = Area of $\triangle PQR$ + area of $\triangle PRS$

$= \left(\frac{1}{2} \times 56 \times 33 + \frac{1}{2} \times 37 \times 65\right)$ cm$^2$ **[1]**

$= \mathbf{2126.5\ cm^2}$ **[1]**

**(b)** **(i)** (6 cm × 3 cm) represents 72 m$^2$.

18 cm$^2$ represents 72 m$^2$.

1 cm$^2$ represents 4 m$^2$. **[1]**

1 cm represents 2 m.

$\therefore n = \mathbf{2}$ **[1]**

**(ii)** Amount of paint needed $= \frac{72}{10}$ litres

$= 7.2$ litres

The interior designer could buy one 5-litre can, two 1-litre cans and one 400-ml can. **[1]**

Amount spent = \$72 + 2(\$23) + \$9.90

= **\$127.90** **[1]**

**17.** **(i)** Amount of material used

= Volume of large cylinder – volume of small cylinder

$= \left[\pi\left(\frac{2.7}{2}\right)^2(13.8) - \pi\left(\frac{2.7}{2} - 0.24\right)^2(13.8 - 0.12)\right]$ cm$^3$ **[3]**

$= \mathbf{26.1\ cm^3}$ (to 3 s.f.) **[1]**

**(ii)** Estimated capacity of the tube

$= \left[\pi\left(\frac{2.7}{2} - 0.24\right)^2(13.8 - 0.12 - 1.5)\right]$ cm$^3$ **[1]**

$= \mathbf{47.1\ cm^3}$ (to 3 s.f.) **[1]**

**(iii)** Maximum volume of each tablet

$= \frac{47.146}{15}$ cm$^3$ **[1]**

$= \mathbf{3.1\ cm^3}$ (to 2 s.f.) **[1]**

Assume that each tablet is in the shape of a cylinder and that there is minimum empty space between the tablets or between each tablet and the side of the container. **[1]**

**(iv)** Recommended daily amount of Vitamin C

$= \frac{100}{300} \times 240$ mg

$= \mathbf{80\ mg}$ **[1]**

## End-of-year Checkpoint B

**Section A**

**1.** Amount of zakat $= \frac{2.5}{100} \times$ PKR 300 000

= **PKR 7500** **[1]**

**2.** $24x^2y^2 - 111xy - 45 = 3(8x^2y^2 - 37xy - 15)$ **[1]**

$= \mathbf{3(8xy + 3)(xy - 5)}$ **[1]**

**3.** **(a)** P(ball is blue) $= 1 - 0.4 - \frac{1}{3}$

$= \mathbf{\frac{4}{15}}$ **[1]**

**(b)** P(ball is red) $= \frac{2}{5}$

P( ball is yellow) $= \frac{1}{3}$

LCM of 5 and 3 = 15

$\therefore$ There could be **15** or **30** balls in the sack. **[1]**

**4.**

| $x^2$ | 9 | 36 | 100 |
|---|---|---|---|
| $y$ | 40 | 10 | 4 |
| $x^2y$ | 360 | 360 | 400 |

Since $x^2y \neq$ constant, $x^2$ and $y$ are not in inverse proportion. **[2]**

**5.** $298^2 = (300 - 2)^2$

$= 300^2 - 2(300)(2) + 2^2$ **[1]**

$= 90\ 000 - 1200 + 4$

$= \mathbf{88\ 804}$ **[1]**

**6.** $(5n + 2)^2 + 1 = 25n^2 + 20n + 4 + 1$ **[1]**

$= 25n^2 + 20n + 5$

$= 5(5n^2 + 4n + 1)$, which is a multiple of 5 (shown) **[1]**

**7.** **(a)** $T_n = \mathbf{19 + 4n}$ **[1]**

**(b)** Since $4n$ is a multiple of 2, then $19 + 4n$ will be an odd number. **[1]**

$\therefore$ A term in the sequence cannot be a multiple of 8.

**8.** Using similar triangles,

$$\frac{AC}{DC} = \frac{CD}{CB}$$

$$\frac{AB+10}{15} = \frac{15}{10} \quad \text{[1]}$$

$$AB + 10 = 22.5$$

$$AB = 12.5 \text{ cm} \quad \text{[1]}$$

$\therefore AB =$ **12.5 cm**

**9.** Original length of picture = [29.7 – 2(2.5)] cm
= 24.7 cm

Original breadth of picture = [(21 – 2(2.5)] cm
= 16 cm

Area of enlarged picture = [2(24.7) × 2(16)] $cm^2$ [1]
= **1600 $cm^2$** (to 2 s.f.) [1]

**10.** **(a)** $9x^4 - y^2 = (3x^2)^2 - y^2$
$= \mathbf{(3x^2 + y)(3x^2 - y)}$ [1]

**(b)** $20pq - 2 - 8q + 5p = 20pq + 5p - 8q - 2$
$= 5p(4q + 1) - 2(4q + 1)$ [1]
$= \mathbf{(5p - 2)(4q + 1)}$ [1]

**11.** $8x + 2y = 1$ — (1)
$6x - 5y = 4$ — (2)
(1) × 3: $24x + 6y = 3$ — (3)
(2) × 4: $24x - 20y = 16$ — (4)
(3) – (4): $26y = -13$ [1]

$$y = -\frac{1}{2} \quad \text{[1]}$$

Substitute $y = -\frac{1}{2}$ into (1):

$$8x + 2\left(-\frac{1}{2}\right) = 1$$

$$8x - 1 = 1$$

$$8x = 2$$

$$x = \frac{1}{4} \quad \text{[1]}$$

$\therefore \mathbf{x = \frac{1}{4}, y = -\frac{1}{2}}$

**12.** **(a)** Macy's speed $= \frac{360 \text{ m}}{5 \text{ min}}$

$= \frac{360 \text{ m}}{(5 \times 60)\text{s}}$

= **1.2 m/s** [1]

**(b)** **(i)** They pass each other for the first time at about **9.07 a.m.** [1]

**(ii)** Distance away from the dance school = 660 m – 360 m
= **300 m** [1]

**(c)** 3.6 km/h $= \frac{(3.6 \times 1000) \text{ m}}{(1 \times 3600)\text{s}}$
= **1 m/s** [1]

**13.** Area of cross section

$= \left[2 \times \frac{1}{2} \times 1.8 \times 1.2 - \pi(0.45)^2\right] \text{cm}^2$ [1]

$= (2.16 - 0.2025\pi) \text{ cm}^2$

Volume of pendant = [0.2 × (2.16 – 0.2025π)] $cm^3$
= 0.304 77 $cm^3$ (to 5 s.f.) [1]

Mass of silver = (10.5 × 0.304 77) g
= 3.2000 g (to 5 s.f.) [1]

Value of silver = $(0.49 × 3.2000)
= **$1.57** (to the nearest cent) [1]

**14.** Since $\triangle ABC$ is congruent to $\triangle CDE$,
$CD = AB = x$, $DE = BC = y$ and $CE = AC = z$.
Area of trapezium $ABDE$ = Area of $\triangle ABC$ + area of $\triangle ACE$ + area of $\triangle CDE$ [1]

$$\frac{1}{2}(x+y)(x+y) = \frac{1}{2}xy + \frac{1}{2}z^2 + \frac{1}{2}xy \quad \text{[1]}$$

$$(x + y)^2 = xy + z^2 + xy$$

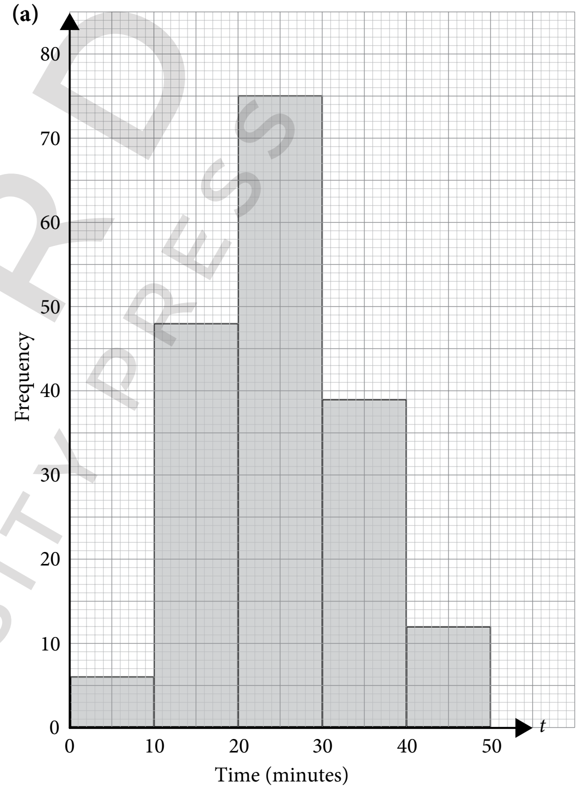
$$x^2 + 2xy + y^2 = 2xy + z^2$$

$$x^2 + y^2 = z^2 \quad \text{[1]}$$

Since the three sides of right-angled $\triangle ABC$ are $x$, $y$ and $z$, where $z$ is the hypotenuse, then $x^2 + y^2 = z^2$ proves Pythagoras' Theorem. (shown) [1]

## Section B

**15.** **(a)**




[3]

**(b)** P(customer spent more than half an hour)

$= \frac{39 + 12}{6 + 48 + 75 + 39 + 12}$ [1]

$= \mathbf{\frac{17}{60}}$ [1]

**(c)** Maximum number of customers = 6 + 48 + 75 + 39 + 12
= **180** [1]

It is possible that all the customers were inside the supermarket at the same time in that particular morning. [1]

16. **(a)** $A = k\sqrt{B}$

Given that $B = \frac{1}{4}$ and $A = 3$,

$3 = k\sqrt{\frac{1}{4}}$

$= \frac{1}{2}k$

$k = 6$

$\therefore A = 6\sqrt{B}$ **[1]**

When $B = \frac{1}{9}$,

$A = 6\sqrt{\frac{1}{9}}$

$= 6\left(\frac{1}{3}\right)$

$=$ **2** **[1]**

**(b)** $p = \sqrt{64}$ **[1]**

$=$ **8** **[1]**

17. **(a)** 1 cm represents 50 000 cm $= \frac{50\ 000}{100 \times 1000}$ km = 0.5 km.

$\therefore n =$ **0.5** **[1]**

**(b)** 1 cm represents 0.5 km.

2.8 cm represents (2.8 × 0.5) km = 1.4 km.

∴ Actual distance = **1.4 km** **[1]**

**(c)** 1 cm represents 0.5 km.

1 $cm^2$ represents $0.5^2$ $km^2$ = 0.25 $km^2$. **[1]**

0.25 $km^2$ is represented by 1 $cm^2$.

4.5 $km^2$ is represented by $\frac{4.5}{0.25}$ $cm^2$ = 18 $cm^2$.

∴ Area of the park on the map = **18 $cm^2$** **[1]**

18. **(a)** **(i)** $AD^2 + DC^2 = 21^2 + 28^2$

$= 1225$

$AC^2 = 35^2$

$= 1225$

Since $AD^2 + DC^2 = AC^2$, then by the converse of Pythagoras' Theorem, $\triangle ADC$ is a right-angled triangle with $\angle ADC = 90°$. (shown) **[1]**

**(ii)** $\tan \angle CAD = \frac{28}{21}$ **[1]**

$\angle CAD = \tan^{-1} \frac{28}{21}$

$=$ **53.1°** (to 1 d.p.) **[1]**

**(iii)** Area of $ABCD$ = 525 $cm^2$

$\frac{1}{2}(28 + AB)(21) = 525$ **[2]**

$28 + AB = 50$

$AB =$ **22 cm** **[1]**

**(b)** **(i)** Let $\theta$ be the angle of the ramp.

$\tan \theta = \frac{1}{12}$

$\theta = \tan^{-1} \frac{1}{12}$

$= 4.76°$ (to 2 d.p.) (shown) **[1]**

**(ii)** Let $x$ m be the length of one sloping edge.

$\sin 4.7636° = \frac{0.19}{x}$ **[1]**

$x = \frac{0.19}{\sin 4.7636°}$

$= 2.2879$ (to 5 s.f.)

∴ Total length = 2 × 2.2879 m **[1]**

= **4.58 m** (to 3 s.f.) **[1]**