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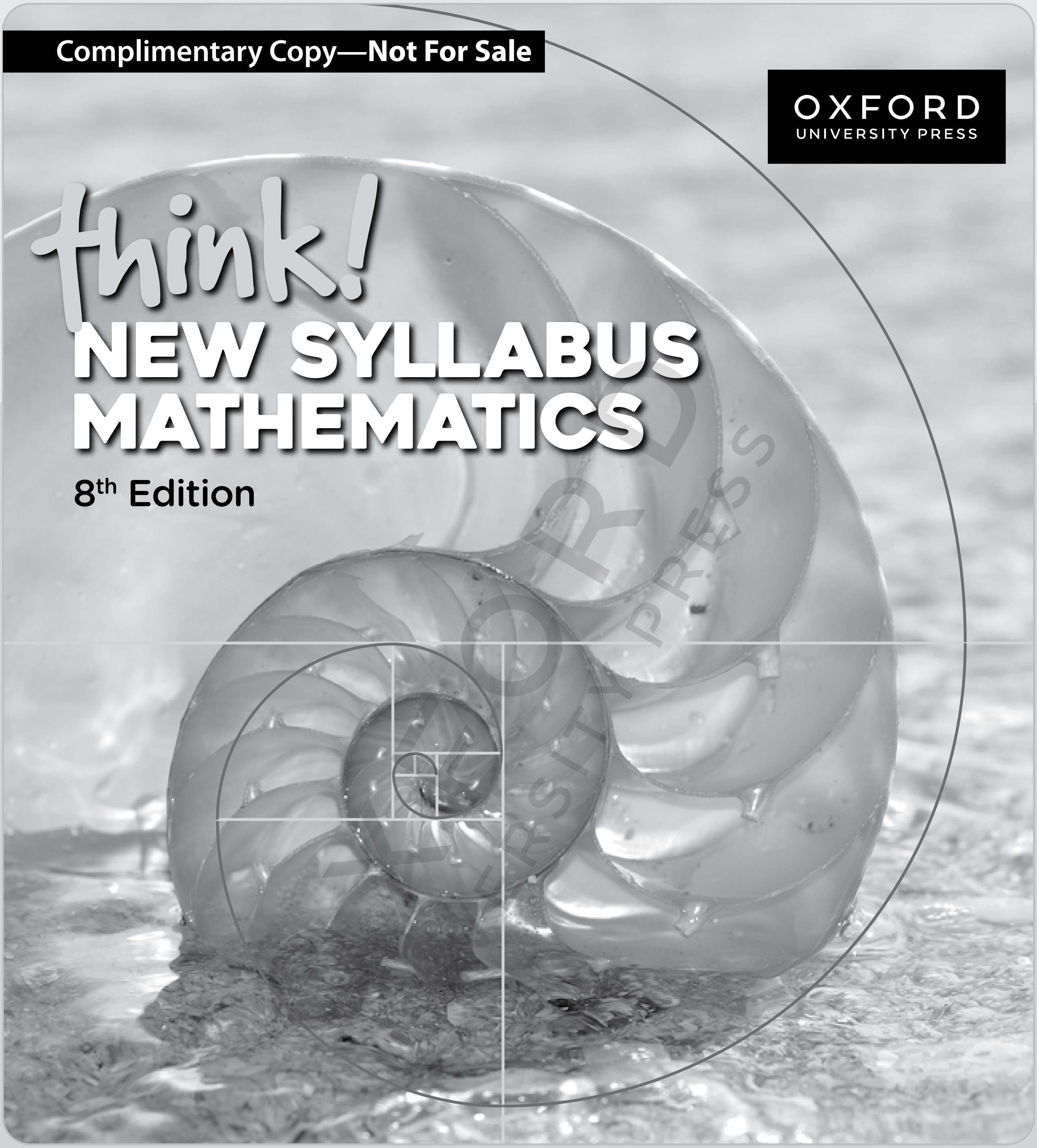
think! NEW SYLLABUS MATHEMATICS

8th Edition



Teacher's Resource Book 2

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CONTENTS

Syllabus Matching Grid

Cambridge O Level Mathematics (Syllabus D) 4024. Syllabus for examination in 2025, 2026 and 2027.

Theme or Topic	Subject Content	Reference
1. Number	Identify and use: • Natural numbers • Integers (positive, negative and zero) • Prime numbers • Square numbers • Cube numbers • Common factors and common multiples • Rational and irrational numbers (e.g. π, $\sqrt{2}$) • Reciprocals.	Book 1: Chapter 1 Chapter 2 Chapter 4
2. Sets	• Use set language, set notation and Venn diagrams to describe sets and represent relationships between sets	Book 2: Chapter 12 Book 4: Chapter 1
2. Powers and roots	Calculate with the following: • Squares • Square roots • Cubes and cube roots of numbers • Cube roots • other powers and roots of numbers	Book 1: Chapter 1 Chapter 2 Chapter 3 Chapter 4
4. Fractions, decimals and percentages	• Use the language and notation of the following in appropriate contexts: (a) proper fractions (b) improper fractions (c) mixed numbers (d) decimals (e) percentages. • Recognise equivalence and convert between these forms.	Book 1: Chapter 2 Chapter 3 Chapter 8
5. Ordering	• Order quantities by magnitude and demonstrate familiarity with the symbols $=, \neq, <, >, \leqslant, \geqslant$.	Book 1: Chapter 2 Chapter 3 Chapter 4 Book 2: Chapter 3
6. The four operations	• Use the four operations for calculations with integers, fractions and decimals, including correct ordering of operations and use of brackets.	Book 1: Chapter 2 Chapter 3 Chapter 4
7. Indices I	• Understand and use indices (positive, zero, negative and fractional). • Understand and use the rules of indices.	Book 3: Chapter 4
8. Standard form	• Use the standard form $A \times 10^n$, where n is a positive or negative integer, and $1 \leqslant A < 10$. • Convert numbers into and out of standard form. • Calculate with values in standard form.	Book 3: Chapter 4

9. Estimation	• Round values to a specified degree of accuracy. • Make estimates for calculations involving numbers, quantities and measurements. • Round answers to a reasonable degree of accuracy in the context of a given problem.	Book 1: Chapter 5
10. Limits of accuracy	• Give upper and lower bounds for data rounded to a specified accuracy. • Find upper and lower bounds of the results of calculations which have used data rounded to a specified accuracy.	Book 1: Chapter 5
11. Ratio and proportion	Understand and use ratio and proportion to: • give ratios in their simplest form • divide a quantity in a given ratio • use proportional reasoning and ratios in context.	Book 1: Chapter 9
12. Rates	• Use common measures of rate. • Apply other measures of rate. • Solve problems involving average speed.	Book 1: Chapter 9
13. Percentages	• Calculate a given percentage of a quantity. • Express one quantity as a percentage of another. • Calculate percentage increase or decrease. • Calculate with simple and compound interest. • Calculate using reverse percentages.	Book 1: Chapter 8
14. Using a calculator	• Use a calculator efficiently. • Enter values appropriately on a calculator. • Interpret the calculator display appropriately.	Book 1: Chapter 1 Chapter 4 Chapter 12
15. Time	• Calculate with time: seconds (s), minutes (min), hours (h), days, weeks, months, years, including the relationship between units. • Calculate times in terms of the 24-hour and 12-hour clock. • Read clocks and timetables.	Book 1: Chapter 9
16. Money	• Calculate with money. • Convert from one currency to another.	Book 1: Chapter 9 Book 2: Chapter 6
17. Exponential growth and decay	• Use exponential growth and decay.	Book 3: Chapter 4
18. Surds	• Understand and use surds, including simplifying expressions. • Rationalise the denominator.	Book 3: Chapter 4
19. Introduction to algebra	• Know that letters can be used to represent generalised numbers. • Substitute numbers into expressions and formulas.	Book 1: Chapter 6
20. Algebraic manipulation	• Simplify expressions by collecting like terms. • Expand products of algebraic expressions. • Factorise by extracting common factors. • Factorise expressions of the form: (a) $ax + bx + kay + kby$ (b) $a^2x^2 - b^2y^2$ (c) $a^2 + 2ab + b^2$ (d) $ax^2 + bx + c$ (e) $ax^2 + bx^2 + cx$ • Complete the square for expressions in the form $ax^2 + bx + c$.	Book 1: Chapter 6 Book 2: Chapter 4

21. Algebraic fractions	• Manipulate algebraic fractions. • Factorise and simplify rational expressions.	Book 3: Chapter 1
22. Indices II	• Understand and use indices (positive, zero, negative and fractional). • Understand and use the rules of indices.	Book 3: Chapter 4
23. Equations	• Construct expressions, equations and formulas. • Solve linear equations in one unknown. • Solve fractional equations with numerical and linear algebraic denominators. • Solve simultaneous linear equations in two unknowns. • Solve quadratic equations by factorisation, completing the square and by use of the quadratic formula. • Change the subject of formulas.	Book 1: Chapter 7 Book 2: Chapter 2 Book 3: Chapter 1 Chapter 2 Chapter 3
24. Inequalities	• Represent and interpret inequalities, including on a number line. • Construct, solve and interpret linear inequalities. • Represent and interpret linear inequalities in two variables graphically. • List inequalities that define a given region.	Book 2: Chapter 3
25. Sequences	• Continue a given number sequence or pattern. • Recognise patterns in sequences, including the term-to-term rule, and relationships between different sequences. • Find and use the nth term of sequences.	Book 2: Chapter 5
26. Proportion	• Express direct and inverse proportion in algebraic terms and use this form of expression to find unknown quantities.	Book 2: Chapter 7
27. Graphs in practical situations	• Use and interpret graphs in practical situations including travel graphs and conversion graphs. • Draw graphs from given data. • Apply the idea of rate of change to simple kinematics involving distance-time and speed-time graphs, acceleration and deceleration. • Calculate distance travelled as area under a speed-time graph.	Book 2: Chapter 1 Book 3: Chapter 6
28. Graphs of functions	• Construct tables of values, and draw, recognise and interpret graphs for functions of the following forms: (a) ax^n (includes sums of no more than three of these) (b) $ab^x + c$ where $n = -2, -1, -\frac{1}{2}, 0, \frac{1}{2}, 1, 2, 3$; a and c are rational numbers; and b is a positive integer. • Solve associated equations graphically, including finding and interpreting roots by graphical methods. • Draw and interpret graphs representing exponential growth and decay problems. • Estimate gradients of curves by drawing tangents.	Book 3: Chapter 6
29. Sketching curves	• Recognise, sketch and interpret graphs of the following functions: (a) linear (b) quadratic (c) cubic (d) reciprocal (e) exponential.	Book 2: Chapter 1 Book 3: Chapter 2 Chapter 3 Chapter 6
30. Functions	• Understand functions, domain and range, and use function notation. • Understand and find inverse functions $f^{-1}(x)$. • Form composite functions as defined by $gf(x) = g(f(x))$.	Book 4: Chapter 5

31.	Coordinates	• Use and interpret Cartesian coordinates in two dimensions.	Book 2: Chapter 1
32.	Drawing linear graphs	• Draw straight-line graphs for linear equations.	Book 2: Chapter 1
33.	Gradient of linear graphs	• Find the gradient of a straight line. • Calculate the gradient of a straight line from the coordinates of two points on it.	Book 2: Chapter 1 Book 3: Chapter 5
34.	Length and midpoint	• Calculate the length of a line segment. • Find the coordinates of the midpoint of a line segment.	Book 3: Chapter 5
35.	Equations of linear graphs	• Interpret and obtain the equation of a straight-line graph.	Book 2: Chapter 1 Book 3: Chapter 5
36.	Parallel lines	• Find the gradient and equation of a straight line parallel to a given line.	Book 3: Chapter 5
37.	Perpendicular lines	• Find the gradient and equation of a straight line perpendicular to a given line.	Book 3: Chapter 5
38.	Geometrical terms	• Use and interpret the following geometrical terms: (a) point (b) vertex (c) line (d) plane (e) parallel (f) perpendicular (g) perpendicular bisector (h) bearing (i) right angle (j) acute, obtuse and reflex angles (k) interior and exterior angles (l) similar (m) congruent (n) scale factor. • Use and interpret the vocabulary of: (a) triangles (b) special quadrilaterals (c) polygons (d) nets (e) solids. • Use and interpret the vocabulary of a circle.	Book 1: Chapter 10 Chapter 11 Book 2: Chapter 8 Chapter 11
39.	Geometrical constructions	• Measure and draw lines and angles. • Construct a triangle, given the lengths of all sides, using a ruler and pair of compasses only. • Draw, use and interpret nets.	Book 1: Chapter 11

40.	Scale drawings	• Draw and interpret scale drawings. • Use and interpret three-figure bearings.	Book 2: Chapter 8 Book 4: Chapter 7
41.	Similarity	• Calculate lengths of similar shapes. • Use the relationships between lengths and areas of similar shapes and lengths, surface areas and volumes of similar solids. • Solve problems and give simple explanations involving similarity.	Book 2: Chapter 8 Book 4: Chapter 11
42.	Symmetry	• Recognise line symmetry and order of rotational symmetry in two dimensions. • Recognise symmetry properties of prisms, cylinders, pyramids and cones.	Book 1: Chapter 11 Book 2: Chapter 11 Book 3: Chapter 7
43.	Angles	• Calculate unknown angles and give simple explanations using the following geometrical properties: (a) sum of angles at a point = 360° (b) sum of angles at a point on a straight line = 180° (c) vertically opposite angles are equal (d) angle sum of a triangle = 180° and angle sum of a quadrilateral = 360°. • Calculate unknown angles and give geometric explanations for angles formed within parallel lines: (a) corresponding angles are equal (b) alternate angles are equal (c) co-interior (supplementary) angles sum to 180°. • Know and use angle properties of regular and irregular polygons.	Book 1: Chapter 10 Chapter 11
44.	Circle theorems I	• Calculate unknown angles and give explanations using the following geometrical properties of circles: • angle in a semicircle = 90° • angle between tangent and radius = 90° • angle at the centre is twice the angle at the circumference • angles in the same segment are equal • opposite angles of a cyclic quadrilateral sum to 180° (supplementary) • alternate segment theorem.	Book 4: Chapter 9
45.	Circle theorems II	• Use the following symmetry properties of circles: • equal chords are equidistant from the centre • the perpendicular bisector of a chord passes through the centre • tangents from an external point are equal in length.	Book 4: Chapter 9
46.	Units of measure	• Use metric units of mass, length, area, volume and capacity in practical situations and convert quantities into larger or smaller units.	Book 1: Chapter 3 Chapter 12 Book 2: Chapter 11
47.	Area and perimeter	• Carry out calculations involving the perimeter and area of a rectangle, triangle, parallelogram and trapezium.	Book 1: Chapter 12

48.	Circles, arcs and sectors	• Carry out calculations involving the circumference and area of a circle. • Carry out calculations involving arc length and sector area as fractions of the circumference and area of a circle.	Book 1: Chapter 12 Book 4: Chapter 8
49.	Surface area and volume	Carry out calculations and solve problems involving the surface area and volume of a: • cuboid • prism • cylinder • sphere • pyramid • cone.	Book 2: Chapter 11 Book 3: Chapter 7
50.	Compound shapes and parts of shapes	• Carry out calculations and solve problems involving perimeters and areas of: (a) compound shapes (b) parts of shapes. • Carry out calculations and solve problems involving surface areas and volumes of: (a) compound solids (b) parts of solids.	Book 1: Chapter 12 Book 2: Chapter 11 Book 3: Chapter 7
51.	Pythagoras' theorem	• Know and use Pythagoras' theorem.	Book 2: Chapter 9
52.	Right-angled triangles	• Know and use the sine, cosine and tangent ratios for acute angles in calculations involving sides and angles of a right-angled triangle. • Solve problems in two dimensions using Pythagoras' theorem and trigonometry. • Know that the perpendicular distance from a point to a line is the shortest distance to the line. • Carry out calculations involving angles of elevation and depression.	Book 2: Chapter 9 Chapter 10 Book 4: Chapter 6 Chapter 7
53.	Non-right-angled triangles	• Use the sine and cosine rules in calculations involving lengths and angles for any triangle. • Use the formula area of triangle = $\frac{1}{2}ab\sin C$	Book 4: Chapter 6
54.	Pythagoras' theorem and trigonometry in 3D	• Carry out calculations and solve problems in three dimensions using Pythagoras' theorem and trigonometry, including calculating the angle between a line and a plane.	Book 4: Chapter 7
55.	Transformations	Recognise, describe and draw the following transformations: • Reflection of a shape in a straight line. • Rotation of a shape about a centre through multiples of 90°. • Enlargement of a shape from a centre by a scale factor. • Translation of a shape by a vector $\begin{pmatrix} x \\ y \end{pmatrix}$.	Book 4: Chapter 10
56.	Vectors in two dimensions	• Describe a translation using a vector represented by $\begin{pmatrix} x \\ y \end{pmatrix}$, $\overrightarrow{AB}$ or $\mathbf{a}$ • Add and subtract vectors. • Multiply a vector by a scalar.	Book 4: Chapter 7
57.	Magnitude of a vector	• Calculate the magnitude of a vectorvector $\begin{pmatrix} x \\ y \end{pmatrix}$ as $\sqrt{x^2 + y^2}$	Book 4: Chapter 4

58.	Vector geometry	• Represent vectors by directed line segments. • Use position vectors. • Use the sum and difference of two or more vectors to express given vectors in terms of two coplanar vectors. • Use vectors to reason and to solve geometric problems.	Book 4: Chapter 4
59.	Introduction to probability	• Understand and use the probability scale from 0 to 1. • Understand and use probability notation. • Calculate the probability of a single event. • Understand that the probability of an event not occurring = 1 – the probability of the event occurring.	Book 2: Chapter 12
60.	Relative and expected frequencies	• Understand relative frequency as an estimate of probability. • Calculate expected frequencies.	Book 2: Chapter 12
61.	Probability of combined events	Calculate the probability of combined events using, where appropriate: • sample space diagrams • Venn diagrams • tree diagrams.	Book 4: Chapter 2
62.	Classifying statistical data	• Classify and tabulate statistical data.	Book 1: Chapter 13
63.	Interpreting statistical data	• Read, interpret and draw inferences from tables and statistical diagrams. • Compare sets of data using tables, graphs and statistical measures. • Appreciate restrictions on drawing conclusions from given data.	Book 1: Chapter 13
64.	Averages and measures of spread	• Calculate the mean, median, mode and range for individual data and distinguish between the purposes for which these are used. • Calculate an estimate of the mean for grouped discrete or grouped continuous data. • Identify the modal class from a grouped frequency distribution.	Book 3: Chapter 8
65.	Statistical charts and diagrams	• Draw and interpret: (a) bar charts (b) pie charts (c) pictograms (d) simple frequency distributions.	Book 1: Chapter 13
66.	Scatter diagrams	• Draw and interpret scatter diagrams. • Understand what is meant by positive, negative and zero correlation. • Draw by eye, interpret and use a straight line of best fit.	Book 4: Chapter 3
67.	Cumulative frequency diagrams	• Draw and interpret cumulative frequency tables and diagrams. • Estimate and interpret the median, percentiles, quartiles and interquartile range from cumulative frequency diagrams.	Book 4: Chapter 3
68.	Histograms	• Draw and interpret histograms. • Calculate with frequency density.	Book 2: Chapter 13

think! Mathematics Secondary 2 Mathematics Scheme of Work

Week	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Reasoning, Communication and Connection
1	**1 Linear Functions and Graphs**	**1.1 Cartesian coordinates** (pp. 3–5)	• State the coordinates of a point • Plot a point in a Cartesian plane	Use and interpret Cartesian coordinates in two dimensions	Introductory Problem (p. 2) Class Discussion – Cartesian coordinate system (p. 3) Journal Writing (p. 5)		Introductory Problem (p. 2) Class Discussion – Cartesian coordinate system (p. 3) Journal Writing (p. 5) Ex 1A Q7 (p. 13)
1, 2		**1.2 Functions** (pp. 6–13)	• Find the value of an unknown in the equation of a function	Substitute numbers into expressions and formulas	Investigation – Function machine (pp. 6–8) Thinking Time (pp. 9–10)		Investigation – Function machine (pp. 6–8) Thinking Time (pp. 9–10)
2		**1.3 Linear functions** (pp. 14–28)	• Draw the graph of a linear function	Draw straight–line graphs for linear equations	Class Discussion – Graphs of linear functions (p. 15)		Class Discussion – Graphs of linear functions (p. 15)
			• Calculate the gradient of a straight line • State the y–intercept of a straight line • Find the equation of a straight–line graph	Find the gradient of a straight line Interpret and obtain the equation of a straight–line graph	Investigation – Equation of straight line (pp. 15–16) Class Discussion – Gradient of straight line (p. 18) Thinking Time (p. 22) Class Discussion – Sense of magnitude of steepness (p. 23) Investigation – Gradient of horizontal line (pp. 23–24) Investigation – Gradient of vertical line (p. 24)	Investigation – Equation of straight line (pp. 15–16)	Investigation – Equation of straight line (pp. 15–16) Class Discussion – Gradient of straight line (p. 18) Big Idea (p. 19) Thinking Time (p. 22) Class Discussion – Sense of magnitude of steepness (p. 23) Investigation – Gradient of horizontal line (pp. 23–24) Investigation – Gradient of vertical line (p. 24) Ex 1B Q1–3 (p. 25)

Week	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Reasoning, Communication and Connection
2, 3	**1** **Linear Functions and Graphs**	**1.4 Applications of linear graphs in real–world contexts** (pp. 28–41)	• Solve problems involving linear graphs in real–world contexts	Use and interpret graphs in practical situations including travel graphs and conversion graphs Draw graphs from given data Apply the idea of rate of change to simple kinematics involving distance–time graphs	Class Discussion – Graphical representation of height of flag (p. 28)		Class Discussion – Graphical representation of height of flag (p. 28) Reflection (p. 30) Big Idea (p. 34) Ex 1C Q5, 6 (pp. 38 – 39)
3	**2** **Linear Graphs and Simultaneous Linear Equations**	**2.1 Equations of straight lines** (pp. 44–48)	• State the equation of a horizontal line and of a vertical line	Interpret and obtain the equation of a straight-line graph	Investigation – Equation of a horizontal line (pp. 45 – 46) Investigation – Equation of a vertical line (p. 47)		Investigation – Equation of a horizontal line (pp. 45–46) Investigation – Equation of a vertical line(p. 47) Information (p. 47) Ex 2A Q1(b), 2(b) (p. 52)
3		**2.2 Graphs of linear equations in the form $ax + by = k$** (pp. 48–53)	• Plot graphs of linear equations in the form $ax + by = k$	Draw straight-line graphs for linear equations	Investigation – Graphs of $ax + by = k$ (p. 49)		Investigation – Graphs of $ax + by = k$ (p. 49)
			• Sketch graphs of linear equations in the form $ax + by = k$	Recognise, sketch and interpret graphs of linear functions			
4		**2.3 Solving simultaneous linear equations using graphical method** (pp. 54–57)	• Solve simultaneous linear equations in two variables using the graphical method	Solve simultaneous linear equations in two unknowns	Investigation – Solving simultaneous linear equation graphically (p. 54) Class Discussion – Choice of appropriate scales for graphs and accuracy of graphs (p. 55) Class Discussion (p. 56) Thinking Time (p. 56)		Investigation – Solving simultaneous linear equation graphically (p. 54) Class Discussion – Choice of appropriate scales for graphs and accuracy of graphs (p. 55) Class Discussion (p. 56) Thinking Time (p. 56)

Week	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Reasoning, Communication and Connection
4, 5	**2 Linear Graphs and Simultaneous Linear Equations**	**2.4 Solving simultaneous linear equations using algebraic methods** (pp. 58–66)	• Solve simultaneous linear equations in two variables using the elimination method		Thinking Time (p. 60)		Big Idea (p. 58) Thinking Time (p. 60)
			• Solve simultaneous linear equations intwo variables using the substitution method		Thinking Time (p. 63)		Reflection (p. 62) Thinking Time (p. 63)
5		**2.5 Applications of simultaneous equations in real–world contexts** (pp. 67–73)	• Formulate a pair of linear equations in two variables to solve mathematical and real-life problems		Thinking Time (p. 68) Introductory Problem Revisited (p. 71)		Thinking Time (p. 68) Introductory Problem Revisited (p. 71)
5	**3 Linear Inequalities**	**3.1 Simple inequalities** (pp. 76–79)	• Construct inequalities, based on real-world contexts	Represent and interpret inequalities, including on a number line Construct, solve and interpret linear inequalities	Class Discussion – Real-life examples of inequalities (p. 77) Investigation – Properties of inequalities (pp. 77 – 78)		Class Discussion – Real-life examples of inequalities (p. 77) Investigation – Properties of inequalities (pp. 77–78) Ex 1A Q9(a)–(f) (p. 84)
5		**3.2 Solving simple linear inequalities** (pp. 79–84)	• Represent and interpret inequalities, including on a number line • Solve linear inequalities in one variable and represent the solution on a number line		Main text (pp. 79–80) Thinking time (p. 81)		Thinking time (p. 81)
5, 6		**3.3 Solving problems involving linear inequalities** (pp. 85–87)	• Apply linear inequalities to solve word problems		Introductory Problem Revisited (p. 86)		Introductory Problem Revisited (p. 86) Ex 3B Q7 (p. 87)
6		**3.4 Simultaneous linear inequalities** (pp. 87–89)	• Solve simultaneous linear inequalities in one variable and represent the solution on a number line				Reflection (p. 89)
6		**3.5 Solving problems involving simultaneous linear inequalities** (p. 90)	• Apply simultaneous linear inequalities in one variable to solve word problems				Ex 3C Q16 (p. 97)

Week	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Reasoning, Communication and Connection
6	**3 Linear Inequalities**	**3.6 Linear inequalities in two variables** (pp. 91–97)	• Represent linear inequalities in two variables graphically • Represent a system of linear inequalities in two variables graphically • Identify the linear inequalities from a given region in a graph • Solve problems involving linear inequalities in two variables	Represent and interpret linear inequalities in two variables graphically List inequalities that define a given region			Reflection (p. 95)
7	**4 Expansion and Factorisation of Algebraic Expressions**	**4.1 Addition and subtraction of quadratic expressions** (pp. 102–104)	• Recognise quadratic expressions • Add and subtract quadratic expressions	Simplify expressions by collecting like terms	Class Discussion – Recap of algebraic expressions (p. 102) Class Discussion – Recap of linear expressions (p. 102)		Class Discussion – Recap of algebraic expressions (p. 102) Class Discussion – Recap of linear expressions (p. 102)
		4.2 Expansion of algebraic expressions of the form $(a+b)(c+d)$ (pp. 105–108)	• Expand and simplify $(a+b)(c+d)$	Expand products of algebraic expressions			Problem-solving Tip (p. 107) Ex 4A Q11 (p. 108)
7		**4.3 Expansion of quadratic and complex expressions** (pp. 109–116)	• Expand and simplify quadratic expressions • Expand and simplify expressions of the form $(px+q)(rx+s)(tx+u)$		Investigation – Expansion of expressions of the form $p(qx+r)$ and $px(qx+r)$ (p. 110) Investigation – Expansion of expressions of the form $(px+q)(rx+s)$ (p. 112)		Investigation – Expansion of expressions of the form $p(qx+r)$ and $px(qz+r)$ (p. 110) Investigation – Expansion of expressions of the form $(px+q)(rx+s)$ (p. 112) Reflection (p. 113) Reflection (p. 114)

Week	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Reasoning, Communication and Connection
8	**4** **Expansion and Factorisation of Algebraic Expressions**	**4.4 Factorisation of quadratic expressions** (pp. 116–129)	• Factorise quadratic expressions by extracting common factors • Factorise quadratic expressions of the form $ax^2 + bx + c$ • Factorise algebraic expressions of the form $ax^3 + bx^2 + cx$	Factorise by extracting common factors Factorise expressions of the form: • $ax + bx + kay + kby$ • $a^2x^2 - b^2y^2$ • $a^2 + 2ab + b^2$ • $ax^2 + bx + c$ • $ax^3 + bx^2 + cx$	Thinking Time (p. 117) Investigation – Factorisation of quadratic expressions of the form $x^2 + bx + c$ where c > 0 (and $b > 0$) (p. 120) Investigation – Factorisation of quadratic expressions of the form $x^2 + bx + c$ where $c > 0$ (and $b < 0$) (p. 121) Investigation – Factorisation of quadratic expressions of the form $x^2 + bx + c$ where $c < 0$ (p. 122) Class Discussion – Factorisation of quadratic expressions (p. 123)		Thinking Time (p. 117) Practise Now 1 Q2 (pp. 107) Investigation – Factorisation of quadratic expressions of the form $x^2 + bx + c$ where $c > 0$ (and $b > 0$) (p. 120) Investigation – Factorisation of quadratic expressions of the form $x^2 + bx + c$ where $c > 0$ (and $b < 0$) (p. 121) Investigation – Factorisation of quadratic expressions of the form $x^2 + bx + c$ where $c < 0$ (p. 122) Class Discussion – Factorisation of quadratic expressions (p. 123) Introductory Problem Revisited (p. 123) Worked Example 16 (p. 126) Practise Now 16 (p. 126) Ex 4C Q9, 14 (p. 129)
9		**4.5 Factorisation of algebraic expressions into the form $(a + b)(c + d)$** (pp. 129–134)	• Factorise algebraic expressions into the form $(a + b)(c + d)$ by multiplication frame		Class Discussion – Arrangement of terms for factorisation using multiplication frame (p. 130)		Class Discussion – Arrangement of terms for factorisation using multiplication frame (p. 130)
			• Factorise algebraic expressions by grouping		Class Discussion – Arrangement of terms for factorisation grouping (p. 130) Thinking Time (p. 133)		Class Discussion – Arrangement of terms for factorisation grouping (p. 130) Reflection (p. 131) Thinking Time (p. 133) Ex 4D Q3 (p. 134)

Week	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Reasoning, Communication and Connection
9	**4** **Expansion and Factorisation of Algebraic Expressions**	**4.6 Expansion using special algebraic identities** (pp. 134–143)	• Recognise and apply the three special algebraic identities to expand algebraic expressions	Expand products of algebraic expressions	Investigation – First special algebraic identity (pp. 134–135) Investigation – Second special algebraic identity (p. 137) Investigation – Third special algebraic identity (pp. 138–139)		Investigation – First special algebraic identity (pp. 134–135) Investigation – Second special algebraic identity (p. 137) Investigation – Third special algebraic identity (pp. 138–139) Problem-solving Tip (p. 140) Worked Example 27 (p. 141) Practise Now 27 (p. 141) Ex 4E Q16, 17 (p. 143)
9		**4.7 Factorisation using special algebraic identities** (pp. 143–148)	• Recognise and apply the three special algebraic identities to factorise algebraic expressions	Factorise expressions of the form: • $ax + bx + kay + kby$ • $a^2x^2 - b^2y^2$ • $a^2 + 2ab + b^2$ • $ax^2 + bx + c$ • $ax^3 + bx^2 + cx$ Complete the square for expressions of the form $ax^2 + bx + c$	Class Discussion – Equivalent expressions (p. 147)		Class Discussion – Equivalent expressions (p. 147) Reflection (p. 147)
10, 11	**5** **Number Patterns**	**5.1 Number sequences** (pp. 152–166)	• Recognise simple patterns from various number sequences and determine the next few terms	Continue a given number sequence or pattern Recognise patterns in sequences, including the term-to-term rule, and relationships between different sequences	Class Discussion – Number sequences (p. 153)		Class Discussion – Number sequences (p. 153)
			• Find a formula for the general term of simple and of linear number sequences	Find and use the *n*th term of sequences	Class Discussion – Number sequences (p. 153) Investigation – Finding general term of simple sequences (p. 155)		Class Discussion – Number sequences (p. 153) Investigation – Finding general term of simple sequences (p. 155) Reflection (p. 158) Ex 5A Q11(ii), 13(b) (p. 166)

Week	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Reasoning, Communication and Connection
10, 11	**5** **Number Patterns**		• Find a formula for the general term of a quadratic sequence • Find a formula for the general term of a cubic sequence • Find a formula for the general term of an exponential sequence • Solve problems involving number sequences		Investigation ¬– Identifying cubic sequence (p. 161) Investigation – Finding general term of sequences with exponential patterns (pp. 162–163)		Investigation – Identifying cubic sequence (p. 161) Investigation – Finding general term of sequences with exponential patterns (pp. 162–163) Ex 5A Q8(i) (p. 165)
11		**5.2 Number sequences and patterns** (pp. 167–177)	• Solve problems involving number patterns in real-world contexts		Investigation – Fibonacci sequence (pp. 171–172) Introductory Problem Revisited (p. 173)		Investigation – Fibonacci sequence (pp. 171–172) Ex 5B Q2–5, 7 (p. 176)
12	**6** **Financial Transactions**	**6.1 Percentage, ratio and rate** (pp. 180–181)	• Solve problems involving percentages, ratio and rate	Calculate a given percentage of a quantity Express one quantity as a percentage of another Calculate percentage increase or decrease Calculate using reverse percentages Understand and use ratio and proportion to: • give ratios in their simplest form • divide a quantity in a given ratio • use proportional reasoning and ratios in context Apply common measures of rate			

Week	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Reasoning, Communication and Connection
12	**6 Financial Transactions**	**6.2 Profit, loss, discount, General Sales Tax, and commission** (pp. 182–190)	• Solve problems involving profit, loss, discount, General Sales Tax and commission by applying concept of percentage	Calculate a given percentage of a quantity Express one quantity as a percentage of another Calculate percentage increase or decrease Calculate using reverse percentages	Thinking Time (p. 183) Investigation – Discount, service charge and GST (p. 186)		Attention (p. 182) Information (p. 182) Thinking Time (p. 183) Reflection (p.186) Investigation – Discount, service charge and GST (p. 186) Reflection (p.187) Ex 6A Q20 (p. 190)
12, 13		**6.3 Insurance, hire purchase and interest** (pp. 190–198)	• Solve problems involving insurance, hire purchase and interest by applying measures of rate	Apply common measures of rate Calculate with simple and compound interest	Introductory Problem (p. 180) Investigation – Exploring simple interest and compound interest (p. 194)		Introductory Problem (p. 180) Investigation – Exploring simple interest and compound interest (p. 194) Reflection (p. 196) Ex6B Q11 (p. 197)
13		**6.4 Zakat, ushr and income tax** (pp. 198–200)	• Solve problems involving zakat, ushr and income tax by applying concept of percentage	Calculate a given percentage of a quantity Calculate using reverse percentages	Class Discussion – What is a reasonable way to tax income? (p. 199)		Class Discussion – What is a reasonable way to tax income? (p. 199)
13		**6.5 Inheritance and partnership** (pp. 201–204)	• Solve problems involving inheritance and partnership by applying concept of ratio and proportion	Understand and use ratio and proportion to: • give ratios in their simplest form • divide a quantity in a given ratio • use proportional reasoning and ratios in context			
14	**7 Direct and Inverse Proportions**	**7.1 Direct proportion** (pp. 208–210)	• Explain the concept of direct proportion • Solve problems involving direct proportion	Express direct and inverse proportion in algebraic terms and use this form of expression to find unknown quantities	Investigation – Direct proportion (p. 208)		Investigation – Direct proportion (p. 208) Reflection (p. 209)

Week	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Reasoning, Communication and Connection
14	**7 Direct and Inverse Proportions**	**7.2 Algebraic and graphical representations of direct proportion** (pp. 210–216)	• Explain the concept of direct proportion using tables, equations and graphs • Solve problems involving direct proportion	Express direct and inverse proportion in algebraic terms and use this form of expression to find unknown quantities	Investigation – Graphical representation of direct proportion (p. 210) Thinking Time (p. 211)		Investigation – Graphical representation of direct proportion (p. 210) Thinking Time (p. 211) Reflection (p. 212) Worked Example 4 (pp. 213–214) Practise Now 4 (p. 214) Ex 7A Q16, 17 (p. 216)
14		**7.3 Other forms of direct proportion** (pp. 216–222)	• Explain the concept of direct proportion using tables, equations and graphs • Solve problems involving direct proportion		Investigation – Other forms of direct proportion (pp. 216–218)		Investigation – Other forms of direct proportion (pp. 216–218) Ex 7B Q13 (p. 222)
14		**7.4 Inverse proportion** (pp. 222–225)	• Explain the concept of inverse proportion • Solve problems involving inverse proportion		Investigation – Inverse proportion (p. 222) Class Discussion – Real-life examples of quantities in inverse proportion (p. 223) Introductory Problem Revisited (p. 225)		Investigation – Inverse proportion (p. 222) Class Discussion –Real-life examples of quantities in inverse proportion (p. 223) Attention (p. 223)
15		**7.5 Algebraic and graphical representations of inverse proportion** (pp. 225–233)	• Explain the concept of inverse proportion using tables, equations and graphs • Solve problems involving inverse proportion		Thinking Time (p. 225) Investigation – Graphical representation of inverse proportion (pp. 226–227) Thinking Time (p. 228)		Thinking Time (p. 225) Investigation – Graphical representation of inverse proportion (pp. 226–227) Thinking Time (p. 228) Ex 7C Q1(a)–(e), 2, 7 (pp. 231–232)

Week	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Reasoning, Communication and Connection
15	**7** **Direct and Inverse Proportions**	**7.6 Other forms of inverse proportion** (pp. 233–236)	• Explain the concept of inverse proportion using tables, equations and graphs • Solve problems involving inverse proportion				Ex 7D Q12 (p.236)
16	**8** **Congruence and Similarity**	**8.1 Congruent figures** (pp. 240–249)	• Identify and explain if two figures are congruent • Solve simple problems involving congruence	Use and interpret the geometrical term, congruent	Investigation – Properties of congruent figures (p. 241) Thinking Time (p. 241)	Main text (p. 241)	Investigation – Properties of congruent figures (p. 241) Thinking Time (p. 241) Worked Example 3 (p. 245) Practise Now 3 (p. 246) Worked Example 4 (pp. 246–247) Practise Now 4 (p. 247) Ex 8A Q4(a)–(c) (p. 248)
16, 17		**8.2 Similar figures** (pp. 249–259)	• Identify and explain if two figures are similar • State the properties of similar triangles and polygons • Solve simple problems involving similarity	Use and interpret the geometrical term, similar Calculate lengths of similar shapes Solve problems and give simple explanations involving similarity	Investigation – Similar polygons (p. 250) Thinking Time (p. 251) Class Discussion – Identifying similar triangles (p. 252)		Investigation – Similar polygons (p. 250) Thinking Time (p. 251) Class Discussion – Identifying similar triangles (p. 252) Worked Example 5 (pp. 252–253) Practise Now 5 (p. 253) Introductory Problem Revisited (p. 257) Ex 8B Q2(a), (b), 12 (pp. 257–259)
17		**8.3 Similarity and enlargement** (pp. 260–269)	• Construct simple scale drawing with a given scale factor	Use and interpret the geometrical term, scale factor			
			• Interpret maps, floor plans and other scale drawings	Draw and interpret scale drawings			Ex 8C Q13(iii) (p. 268)
18	**9** **Pythagoras' Theorem**	**9.1 Pythagoras' Theorem** (pp. 272–282)	• Solve problems using Pythagoras' Theorem	Know and use Pythagoras' Theorem	Investigation – Pythagoras' Theorem (pp. 273–274) Thinking Time (p. 275)		Investigation – Pythagoras' Theorem (pp. 273–274) Thinking Time (p. 275) Worked Example 3 (p. 278) Practise Now 3 Q1(b) (p. 279) Ex 9A Q9(b) (p. 280)

Week	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Reasoning, Communication and Connection
18	**9** **Pythagoras' Theorem**	**9.2 Applications of Pythagoras' Theorem in real-world contexts** (pp. 283–291)	• Solve problems using Pythagoras' Theorem in real-world contexts	Know and use Pythagoras' Theorem	Class Discussion – Modelling real-world phenomena (p. 285) Introductory Problem Revisited (p. 288)		Practise Now 4 Q2 (p. 284) Class Discussion – Modelling real-world phenomena (p. 285) Introductory Problem Revisited (p. 288) Ex 9B Q15(d) (p. 291)
19		**9.3 Converse of Pythagoras' Theorem** (pp. 291–293)	• Determine whether a triangle is right-angled given the lengths of three sides				
19	**10** **Trigonometric Ratios**	**10.1 Trigonometric ratios** (pp. 296–302)	• State the trigonometric ratios for acute angles in a right-angled triangle	Known and use the sine, cosine and tangent ratios for acute angles in calculations involving sides and angles of a right-angled triangle Use a calculator efficiently Enter values appropriately on a calculator	Investigation – Trigonometric ratios (pp. 296–297) Thinking Time (p. 298) Thinking Time (p. 301)	Investigation – Trigonometric ratios (pp. 296–297)	Investigation – Trigonometric ratios (pp. 296–297) Thinking Time (p. 298) Thinking Time (p. 301)
19		**10.2 Applications of trigonometric ratios to find unknown sides of right-angled triangles** (pp. 302–306)	• Find the unknown sides of right-angled triangles given an acute angle				
20		**10.3 Applications of trigonometric ratios to find unknown angles in right-angled triangles** (pp. 307–311)	• Find the unknown angles in right-angled triangles given two sides		Journal Writing (p. 309)		Journal Writing (p. 309)
20		**10.4 Applications of trigonometric ratios in real-world contexts** (pp. 312–319)	• Apply trigonometric ratios to solve problems in real-world contexts	Solve problems in two dimensions using Pythagoras' Theorem and trigonometry	Introductory Problem Revisited (p. 316)		Introductory Problem Revisited (p. 316)

Week	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Reasoning, Communication and Connection
21	**11 Volume, Surface Area and Symmetry of Prisms and Cylinders**	**11.1 Conversion of units** (pp. 322–324)	• Convert between cm^3 and m^3	Use metric units of volume in practical situations and convert quantities into larger or smaller units	Class Discussion – Measurements in daily lives (p. 322)	Class Discussion – Measurements in daily lives (p. 322)	Class Discussion – Measurements in daily lives (p. 322)
21		**11.2 Three-dimensional solids** (pp. 324–326)	• Draw the net of a solid • Identify right angles, and parallel and perpendicular line segments of a simple solid	Draw, use and interpret nets	Investigation – Visualising 3D solids (pp. 324–325) Investigation – Cubes, cuboids, prisms and cylinders (pp. 325–326)		Investigation – Visualising 3D solids (pp. 324–325) Investigation – Cubes, cuboids, prisms and cylinders (pp. 325–326) Reflection (p. 326)
21		**11.3 Volume and surface area of cubes and cuboids** (pp. 327–332)	• Find the volume and surface area of cubes and cuboids	Carry out calculations and solve problems involving the surface area and volume of a: • cuboid • prism • cylinder	Class Discussion – Surface area of cubes and cuboids (p. 329)		Class Discussion – Surface area of cubes and cuboids (p. 329) Ex 11A Q17, 18 (p. 332)
21, 22		**11.4 Volume and surface area of prisms** (pp. 333–339)	• Find the volume and surface area of prisms		Thinking Time (p. 334) Investigation – Volume of prism (p. 334)		Thinking Time (p. 334) Investigation – Volume of prism (p. 334)
22		**11.5 Volume and surface area of cylinders** (pp. 339–346)	• Find the volume and surface area of cylinders		Thinking Time (p. 340) Investigation – Comparison between cylinder and prism (p. 340) Journal Writing (p. 342) Thinking Time (p. 343) Class Discussion – Total surface area of other types of cylinders (p. 343)		Thinking Time (p. 340) Investigation – Comparison between cylinder and prism (p. 340) Journal Writing (p. 342) Thinking Time (p. 343) Class Discussion – Total surface area of other types of cylinders (p. 343) Just For Fun (p. 341)

Week	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Reasoning, Communication and Connection
22	**11** **Volume, Surface Area and Symmetry of Prisms and Cylinders**	**11.6 Volume and surface area of composite solids** (pp. 346–347)	• Solve problems involving the volume and surface area of composite solids	Carry out calculations and solve problems involving surface areas and volumes of: • compound solids • parts of solids			
22		**11.7 Symmetry in right prisms and cylinders** (pp. 347–353)	• Identify the planes and axes of symmetry of a solid • State the order of rotational symmetry of a solid about a given axis of symmetry	Recognise symmetry properties of prisms and cylinders	Investigation – Plane and rotational symmetries of a cylinder (p. 351) Thinking Time (p. 351)		Investigation – Plane and rotational symmetries of a cylinder (p. 351) Thinking Time (p. 351)
23	**12** **Introduction to Set Notation and Probability**	**12.1 Sets and set notations** (pp. 357–361)	• Use set language and notation to describe a set • Identify well-defined and distinct elements in a set	Understand and use set language, notation and Venn diagrams to describe sets and represent relationships between sets	Class Discussion – Well-defined and distinct objects in a set (p. 358) Thinking Time (p. 359)		Class Discussion – Well-defined and distinct objects in a set (p. 358) Thinking Time (p. 359) Worked Example 2 (p. 360) Practise Now 2 (p. 360) Ex 12A Q8–11 (p. 361)
23		**12.2 Venn diagrams, universal sets, complements of set and subsets** (pp. 362–368)	• Represent a set using a Venn diagram • Identify the elements of a subset of a universal set • Identify the elements of the complement of a set		Thinking Time (p. 363) Class Discussion – Understanding subset (pp. 365–366)		Thinking Time (p.363) Class Discussion – Understanding subset (pp. 365 –366) Reflection (p. 367) Ex 12B Q3, 5(a)-(d), 8, 9, 12 (pp. 367–368)

Week	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Reasoning, Communication and Connection
23		**12.3 Probability experiment and sample space** (pp. 368–371)	• Define probability as a measure of chance • List the sample space of a probability experiment	Understand and use the probability scale from 0 to 1 Understand and use probability notation			
24	**12 Introduction to Set Notation and Probability**	**12.4 Probability of single events** (pp. 371–376)	• Find the probability of a single event	Calculate the probability of a single event Understand that the probability of an event not occurring = 1 – probability of the event occurring			Attention (p. 371)
24		**12.5 Further examples of probability of single events** (pp. 377–379)	• Solve problems involving the probability of single events				
24		**12.6 Experimental approach to finding probability** (pp. 380–383)	• State that the relative frequency of an event occurring in a number of trials is the ratio of the number of occurrences to the total number of trials • Calculate the expected frequencies of an event occurring using the theoretical probability of the event	Understand relative frequency as an estimate of probability Calculate expected frequencies	Investigation – Tossing a coin (pp. 380–381)		Investigation – Tossing a coin (pp.380–381)

Week	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Reasoning, Communication and Connection
24, 25	**13 Histograms**	**13.1 Histograms for ungrouped data** (pp. 389–393)	• Construct and interpret data from histograms for ungrouped data • Evaluate the purposes and appropriateness of the use of different statistical diagrams • Explain why some statistical diagrams can lead to a misinterpretation of data	Draw and interpret histograms	Introductory Problem (p. 388) Class Discussion – Purposes and appropriateness of histogram for ungrouped data (p. 390)		Introductory Problem (p. 388) Class Discussion – Purposes and appropriateness of histogram for ungrouped data (p. 390) Ex 13A Q2 (p. 392)
25		**13.2 Histograms for grouped data** (pp. 393–405)	• Calculate the frequency density from the frequency and size of the interval of a given class • Construct and interpret data from histograms for grouped data with equal class intervals and with unequal class intervals • Evaluate the purposes and appropriateness of the use of different statis-tical diagrams • Explain why some statistical diagrams can lead to a misinterpretation of data	Calculate frequency density Draw and interpret histograms	Thinking Time (pp. 394–395) Class Discussion – Histogram for grouped data with unequal class intervals (pp. 397–400)		Thinking Time (pp. 394–395) Class Discussion – Histogram for grouped data with unequal class intervals (pp. 397–400)

Chapter 1 Linear Functions and Graphs

TEACHING NOTES

Suggested Approach

Although the topic on functions and linear graphs is new to most students, they do encounter examples of their applications in their daily lives, e.g. maps show the usage of Cartesian coordinates; escalators and moving walkways illustrate the concept of steepness. Teachers can get students to discuss about in detail these real-life examples. When students are able to appreciate their uses, teachers can proceed to introduce the concept of functions and linear graphs.

Section 1.1: Cartesian coordinates

Teachers can build upon prerequisites, namely number lines to introduce the horizontal axis (x-axis) and the vertical axis (y-axis). Teachers can introduce this concept by playing a game (see Introductory Problem on page 2 and Class Discussion: Cartesian coordinate system) to arouse students' interest.

Teachers should teach students not only on how to draw horizontal and vertical axes and plot the given points, but also to determine the position of points. Teachers can impress upon students that the first number in each ordered pair is with reference to the horizontal scale while the second number is with reference to the vertical scale. As such, students need to take note that the point (3, 4) has a different position compared to the point (4, 3).

Section 1.2: Functions

Teachers can use the Function Machine (see Investigation: Function machine) to explore the concept of a function with the students and show that when a function is applied to any input x, it will produce exactly one output y. Once the students have understood the relationship between the input x and the output y, they are then able to represent the function using an equation, a table and a graph.

Section 1.3: Linear functions

Teachers should illustrate how a graph of a linear function is drawn on a sheet of graph paper. Teachers can impress upon students that when they draw a graph, the graph has to follow the scale stated for both the x-axis and y-axis and the graph is only drawn for the values of x stated in the range.

Teachers should teach students how to take two points on the line and use it to calculate the vertical change (rise) and horizontal change (run), and then the gradient of the straight line.

To make learning more interactive, students can explore how the graph of a straight line in the form $y = mx + c$ changes when either m or c varies (see Investigation: Equation of a straight line). Through this investigation, students should be able to observe what happens to the line when m varies. Students should also learn how to differentiate between lines with a positive value of m, a negative value of m and when the value of m is 0. When calculating the gradient, teachers should impress upon the students the importance of including the negative sign when the line slopes downwards from left to right.

Section 1.4 Applications of linear graphs in real-world contexts

Through the class discussion on page 28, students should learnt that a graph is an abstract representation of the relationship between two variables.

Teachers can give further examples of linear graphs used in many daily situations and explain what each of the graphs is used for. Through Worked Examples 4 to 6, students will learn how functions and linear graphs are applied in real-world contexts and solve similar problems

Introductory Problem

1. The grids have to be labelled in order to call out a specific square.
2. The squares may each be given a number.
3. Another way would be to assign a value to each row and column, such that each square has a specific set of two values that corresponds to its position. This way is better because less numbers are used and labelling every single square with one number will be tedious for a big grid.

1.1 Cartesian coordinates

Class Discussion (Cartesian coordinate system)

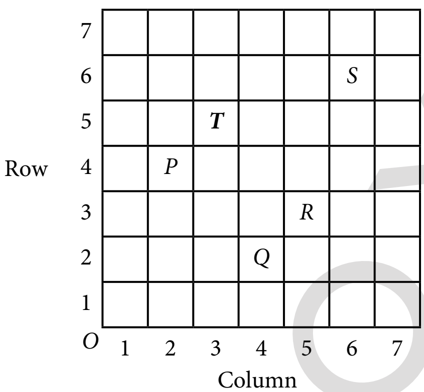
1. **No**, a single number is not sufficient to describe the exact position of the square *P*. It will be unclear if the number refers to the row or the column, and square *P* will have many possible positions.
2. **Yes**, the order is important since the first number represents the column number and the second number represents the row number. (2, 4), where square *P* is, and (4, 2), where square *Q* is, do not indicate the same position.
3. In an ordered pair, the two numbers are written in a certain order. For example in (2, 4), the first number '2' corresponds to Column 2 and the second number '4' corresponds to Row 4.
4. Square *R*: **(5, 3)**
   Square *S*: **(6, 6)**
5. 

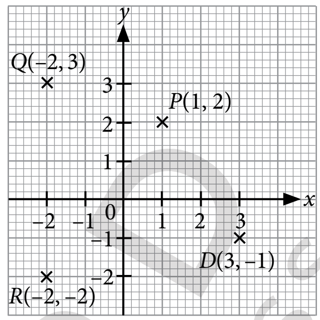
Fig. 1.2

6. Ordered pairs are useful to locate a place on a map and to plan the positions of buildings, for example.

Journal Writing (Page 5)

1. The point *C* is 1 unit to the left of the *y*-axis and 2 units above the *x*-axis. Hence, its coordinates are (–1, 2).
2. The point *D* is 1 unit to the right of the *y*-axis and 3 units below the *x*-axis. Hence, its coordinates are **(1, –3)**.
3. The coordinates of the origin *O* are **(0, 0)**. The origin is the point where the *x*-axis and *y*-axis intersect. The values of *x* and *y* at the origin are both 0.
4. An ordered pair refers to two numbers for which the order they are given in is important. The coordinates of a point refer specifically to the position of a point on a Cartesian plane. Coordinates are examples of ordered pairs, but an ordered pair might not always be used in the context of a Cartesian plane.

Practise Now 1A



1.2 Functions

Investigation (Function machine)

1. $y = \boldsymbol{x + 3}$
2. **(a)** Input $x = 4 \rightarrow$ Output $y = 4 + 3 = \mathbf{7}$
   (b) Input $x = -7 \rightarrow$ Output $y = -7 + 3 = \mathbf{-4}$
3. **(a)** Input $x = 9 - 3 = \mathbf{6} \rightarrow$ Output $y = 9$
   (b) Input $x = 0 - 3 = \mathbf{-3} \rightarrow$ Output $y = 0$
4.

x	–7	**–3**	2	4	**6**
y	**–4**	0	5	**7**	9

Table 1.1

5.

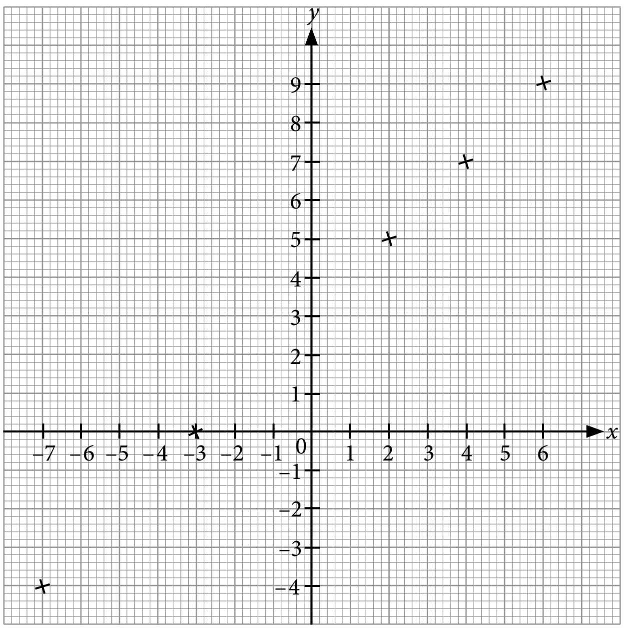


Fig. 1.6

6. Every input x produces exactly **one** output y.
7. **No**, the new function is not the same. The inputs are not restricted to be only integers, which will result in different outputs as well. For example, we can input $x = 1.5$ and the output will be $y = 4.5$.
8. **Yes**, the equation will be same. Even though the input can be any real number, the function is still represented by $y = x + 3$.
9. The graph of this function will include the points in the graph in Question 5, with the points connected by a straight line.
10. $y = \mathbf{-2x - 1}$
11.

x	−1	**−0.5**	**0**	2	3
y	**1**	0	−1	**−5**	−7

Table 1.2

12.

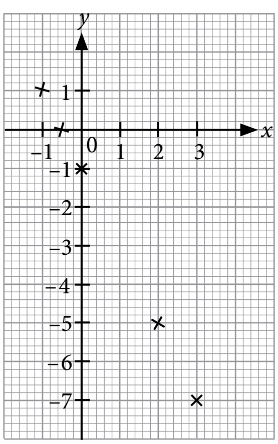


Fig. 1.8

13. Every input x produces exactly **one** output y.

Thinking Time (Page 9)

1. **No**. The relationship between two variables x and y is a function only if exactly one output of y is produced for every input x.
2. **Yes**, the relationship between x and y may still be a function. It is possible that for this function, the inputs are specified to be less than 0 for example, and $x = 4$ will not have any output value.
3. **Yes**. It is possible for a function to have two input values x with the same output value y, as long as each input value x only gives one output value y.
4. Graph A: Tea leaves \$2 per kg
   Graph B: Tea bags \$2 per box

Practise Now 1B

1. **(i)** When $x = 4$,

$$\begin{aligned} y &= 2(4) - 3 \\ &= 8 - 3 \\ &= \mathbf{5} \end{aligned}$$

(ii) When $y = -5$,

$$\begin{aligned} -5 &= 2x - 3 \\ -5 + 3 &= 2x \\ -2 &= 2x \\ x &= \mathbf{-1} \end{aligned}$$

2. **(i)** When $x = 0$,

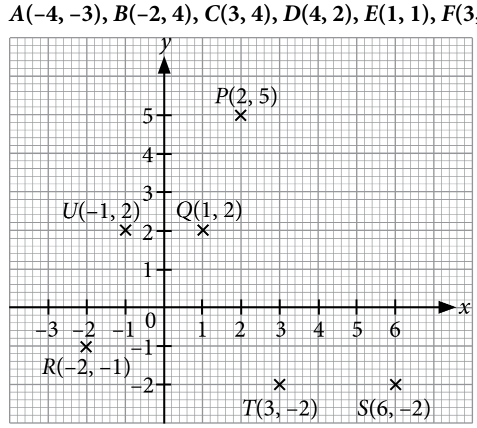
$$\begin{aligned} y &= -\frac{1}{3}(0) - \frac{2}{5} \\ &= 0 - \frac{2}{5} \\ &= \mathbf{-\frac{2}{5}} \end{aligned}$$

(ii) When $y = -\frac{2}{3}$,

$$\begin{aligned} -\frac{2}{3} &= -\frac{1}{3}x - \frac{2}{5} \\ -\frac{2}{3} + \frac{2}{5} &= -\frac{1}{3}x \\ -\frac{4}{15} &= -\frac{1}{3}x \\ \therefore x &= \mathbf{\frac{4}{5}} \end{aligned}$$

Exercise 1A

1. **A(−4, −3), B(−2, 4), C(3, 4), D(4, 2), E(1, 1), F(3, −3)**
2.



3. **(i)** When $x = 3$,

$y = 4(3) + 5$

$= 12 + 5$

$= \mathbf{17}$

(ii) When $x = -2$,

$y = 4(-2) + 5$

$= -8 + 5$

$= \mathbf{-3}$

4. **(i)** When $y = 34$,

$34 = 25 - 3x$

$3x = 25 - 34$

$3x = -9$

$x = \mathbf{-3}$

(ii) When $y = -5$,

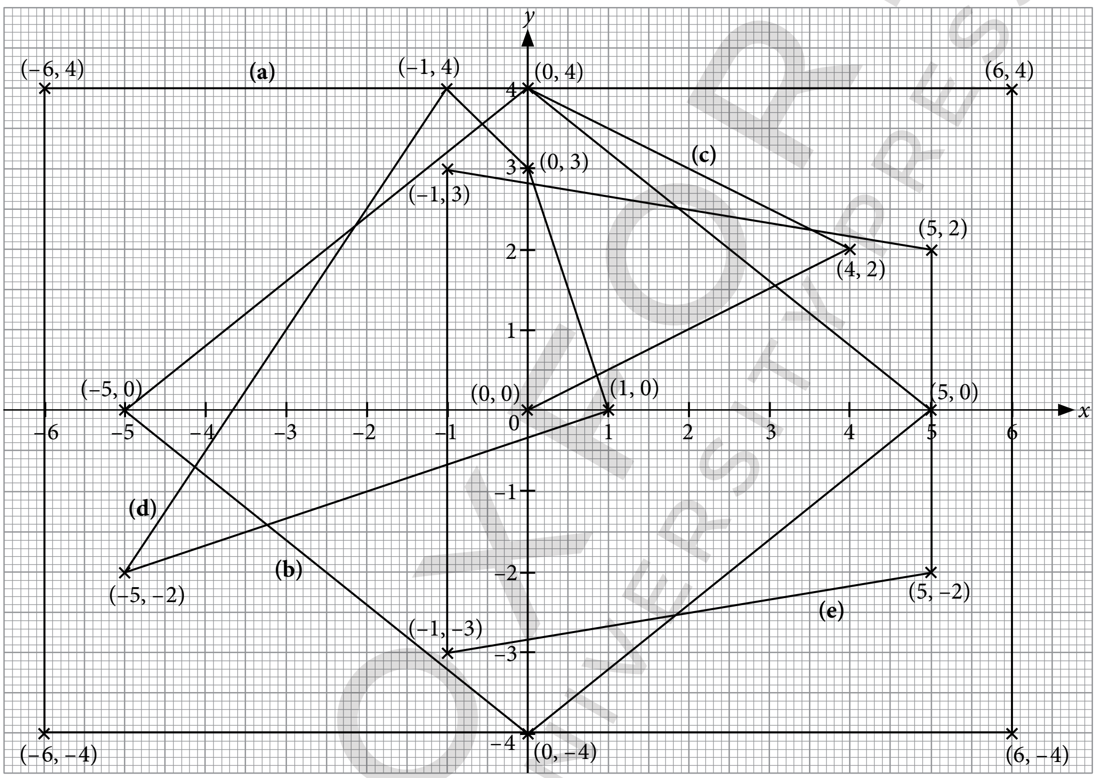
$-5 = 25 - 3x$

$3x = 25 + 5$

$3x = 30$

$x = \mathbf{10}$

5.



(a) **Rectangle**

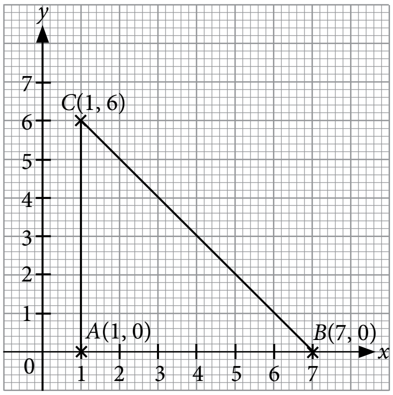
(b) **Rhombus**

(c) **Isosceles triangle**

(d) **Quadrilateral**

(e) **Trapezium**

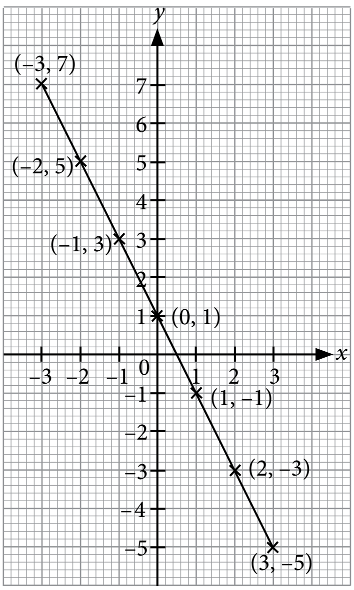
6.



Area of $\triangle ABC = \frac{1}{2} \times 6 \times 6$

$= \mathbf{18\ units^2}$

7.



The points lie on a straight line.

8. (a) (i) When $x = -3$,

$$y = \frac{2}{3}(-3) + \frac{1}{3}$$
$$= -2 + \frac{1}{3}$$
$$= \mathbf{-1\frac{2}{3}}$$

(ii) When $x = 1\frac{1}{2}$,

$$y = \frac{2}{3}\left(1\frac{1}{2}\right) + \frac{1}{3}$$
$$= 1 + \frac{1}{3}$$
$$= \mathbf{1\frac{1}{3}}$$

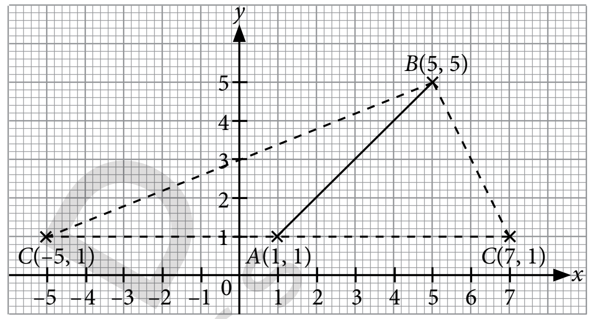
(b) (i) When $y = 1$,

$$1 = \frac{2}{3}x + \frac{1}{3}$$
$$1 - \frac{1}{3} = \frac{2}{3}x$$
$$\frac{2}{3} = \frac{2}{3}x$$
$$x = \mathbf{1}$$

(ii) When $y = -\frac{1}{6}$,

$$-\frac{1}{6} = \frac{2}{3}x + \frac{1}{3}$$
$$-\frac{1}{6} - \frac{1}{3} = \frac{2}{3}x$$
$$-\frac{1}{2} = \frac{2}{3}x$$
$$x = \mathbf{-\frac{3}{4}}$$

9.



Since the y-coordinate of C is 1, when the base of $\triangle ABC$ is taken to be AC, the height of $\triangle ABC$ is 4 units.

$\therefore \frac{1}{2} \times AC \times 4 = 12$

$AC = 6$ units

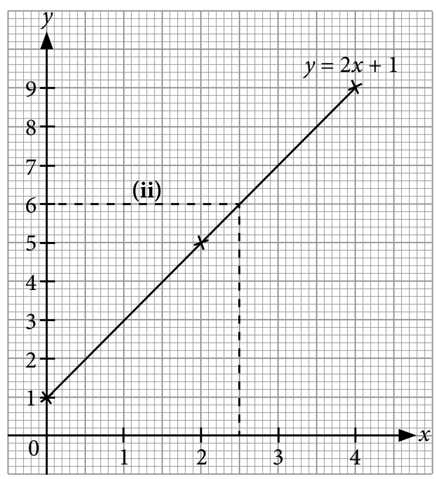
$\therefore$ the possible x-coordinates of C are **–5 and 7**.

1.3 Linear functions

Practise Now 1C

1. (i)

x	0	2	4
$y = 2x + 1$	1	5	9

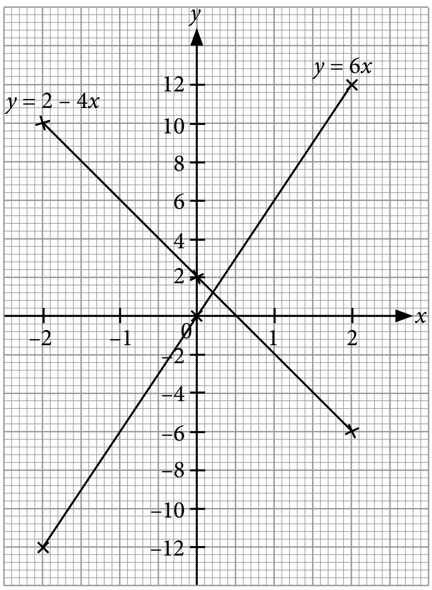


(ii) From the graph in (i),
when $y = 6$,
$q = x = \mathbf{2\frac{1}{2}}$

2.

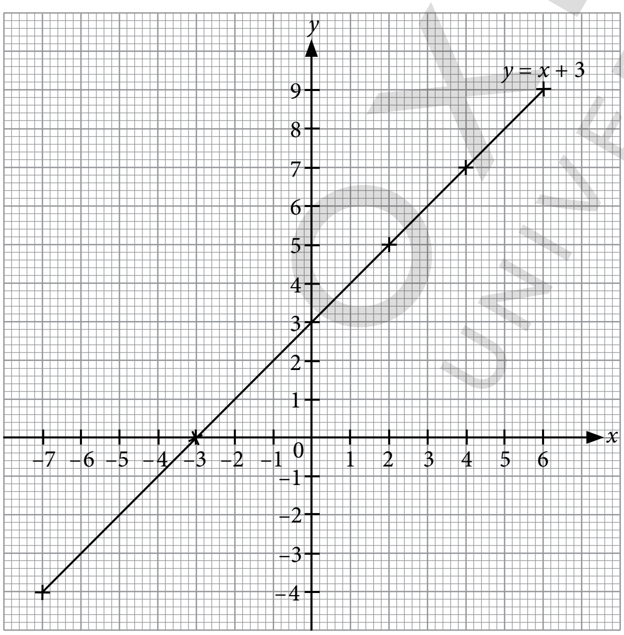
x	-2	0	2
$y = 6x$	-12	0	12

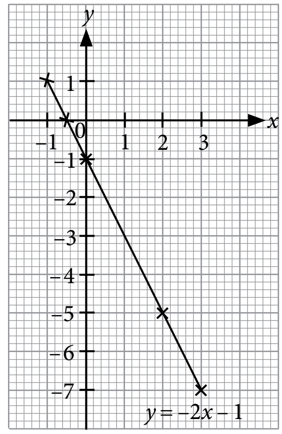
x	-2	0	2
$y = 2 - 4x$	10	2	-6



Class Discussion (Graphs of linear functions)

1. **No**. A straight line contains infinitely many points. The 3 points are specific points plotted, sufficient to draw a straight line accurately.
2. Since the point A lies on the graph of the function $y = 2x$, its coordinates satisfy the equation of the function $y = 2x$. Since the point B does not lie on the graph of the function $y = 2x$, its coordinates do not satisfy the equation of the function $y = 2x$.
3.





Yes, I agree with Nadia. As shown above, the graphs of the functions $y = x + 3$ and $y = -2x - 1$ are straight lines. Hence, they are linear.

Investigation (Equation of straight line)

1. As the value of c changes, the y-coordinate of the point of intersection of the line with the y-axis changes. The coordinates of the point where the line cuts the y-axis are **$(0, c)$**.
2. As the value of m increases from 0 to 4, the steepness of the line increases and the line slopes upwards from left to right.
3. As the value of m decreases from 0 to -4, the steepness of the line increases and the line slopes downwards from left to right.
4. A line with a positive value for m slopes upwards from the left to the right while a line with a negative value for m slopes downwards from the left to the right.

Practise Now 2

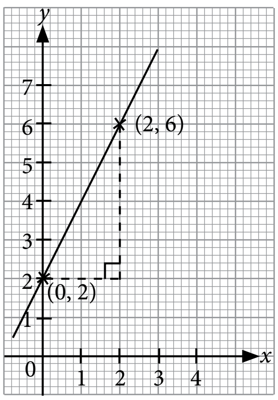
1. **(a)** Gradient = **4**, y-intercept = **-3**
   (b) Gradient = **-1**, y-intercept = **7**
   (c) Gradient = **1**, y-intercept = $\mathbf{\frac{5}{2}}$
   (d) Gradient = **-0.5**, y-intercept = **7.16**
   (e) Gradient = **6**, y-intercept = **0**
   (f) Gradient = **0**, y-intercept = **6**
2. **(a)** $\mathbf{y = 3x + 5}$
   (b) $\mathbf{y = -7x - 2}$
   (c) $\mathbf{y = x - \frac{2}{3}}$
   (d) $\mathbf{y = 7.69 - x}$
   (e) $\mathbf{y = -\frac{1}{2}x}$
   (f) $\mathbf{y = -\frac{1}{2}}$

Class Discussion (Gradient of straight line)

1. The scales used are different.
2. Graph A appears to be steeper but both lines have the same gradient of 2.
3. A gradient of 2 means that in both graphs, the slope of the line as measured by the vertical change and the horizontal change has a value of 2.

Practise Now 3

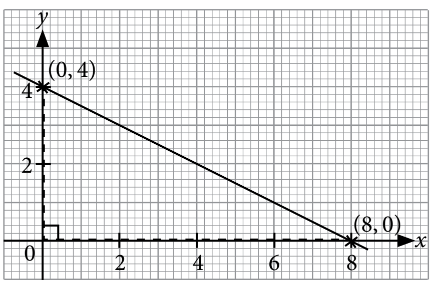
(a)



Gradient = $\frac{4}{2}$

= **2**

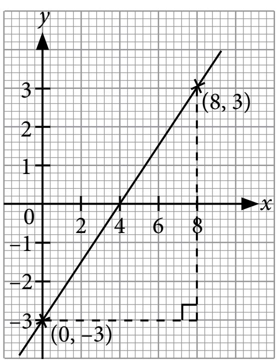
(b)



Gradient = $-\frac{4}{8}$

= $-\mathbf{\frac{1}{2}}$

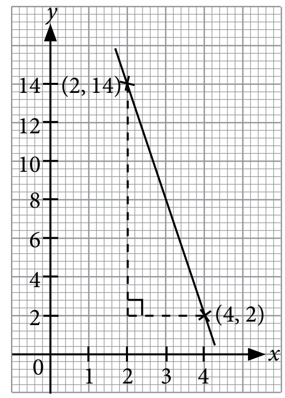
(c)



Gradient = $\frac{6}{8}$

= $\mathbf{\frac{3}{4}}$

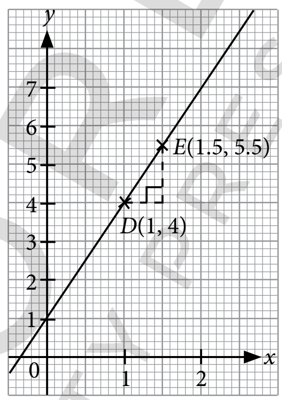
(d)



Gradient = $-\frac{12}{2}$

= **–6**

Thinking Time (Page 22)



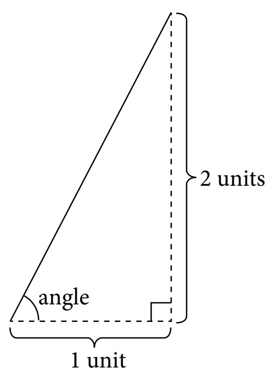
1. Gradient of DE = $\frac{1.5}{0.5}$

   = **3**
2. **Yes**, gradient of DE = gradient of AB.
3. **Yes**, we can choose *any* two points on a line to find its gradient because the gradient of a straight line is constant.

Class Discussion (Sense of magnitude of steepness)

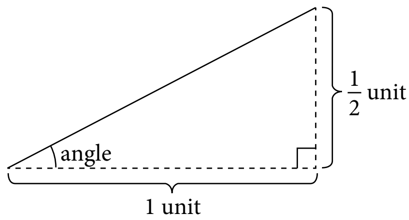
1. **Yes**, the gradient of a road can be negative, depending on the direction it is viewed from.
2. **(i)** Angle of inclination = **45°**

   (ii)



Angle of inclination = **63°**

(iii)



Angle of inclination = **27°**

3. **(i)** A road with a gradient of 1 is generally considered to be steep. *Teachers may wish to get students to name some roads in Pakistan which they think may have an approximate gradient of 1 and to ask students how they can determine the gradients of the roads they have named.*

   (ii) A road with a gradient of $\frac{1}{2}$ is generally considered to be steep.
   Since most roads are generally gentle, the gradients of most roads should be less than $\frac{1}{2}$.

4. 0.35 is less than $\frac{1}{2}$. Since the gradient of the steepest road in the world is less than $\frac{1}{2}$, the gradients of all roads in Pakistan are **less than $\frac{1}{2}$**.

Investigation (Gradient of horizontal line)

1. ***B*(−1, 2), *D*(4, 2)**
2. **(i)** In the line segment *AC*,
   vertical change (rise) = **0** and horizontal change (run) = **3**.
   (ii) In the line segment *BD*,
   vertical change (rise) = **0** and horizontal change (run) = **5**.
3. Gradient of $AC = \frac{\text{rise}}{\text{run}}$
   $= \frac{0}{3}$
   $= 0$
   Gradient of $BD = \frac{\text{rise}}{\text{run}}$
   $= \frac{0}{5}$
   $= 0$
   $\therefore$ the gradient of a horizontal line is **0**.

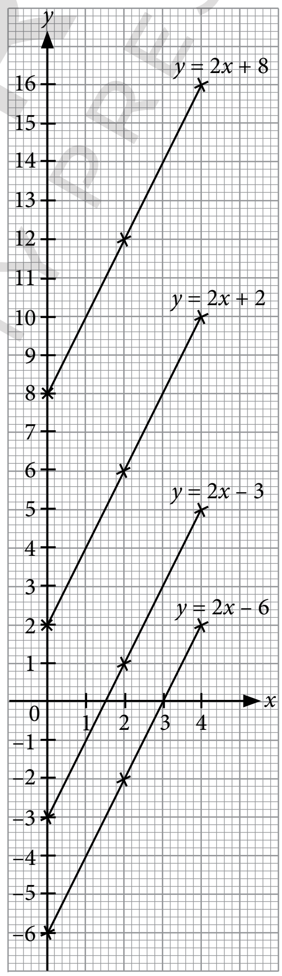
Investigation (Gradient of vertical line)

1. ***Q*(3, 2), *S*(3, −3)**
2. **(i)** In the line segment *PR*,
   vertical change (rise) = **4** and horizontal change (run) = **0**
   (ii) In the line segment *QS*,
   vertical change (rise) = **5** and horizontal change (run) = **0**
3. Gradient of $PR = \frac{\text{rise}}{\text{run}}$
   $= \frac{4}{0}$
   $\therefore$ the gradient of *PR* is undefined.
   Gradient of $QS = \frac{\text{rise}}{\text{run}}$
   $= \frac{5}{0}$
   $\therefore$ the gradient of *QS* is undefined.
   $\therefore$ the gradient of a vertical line is **undefined**.

Exercise 1B

1. **(i)**

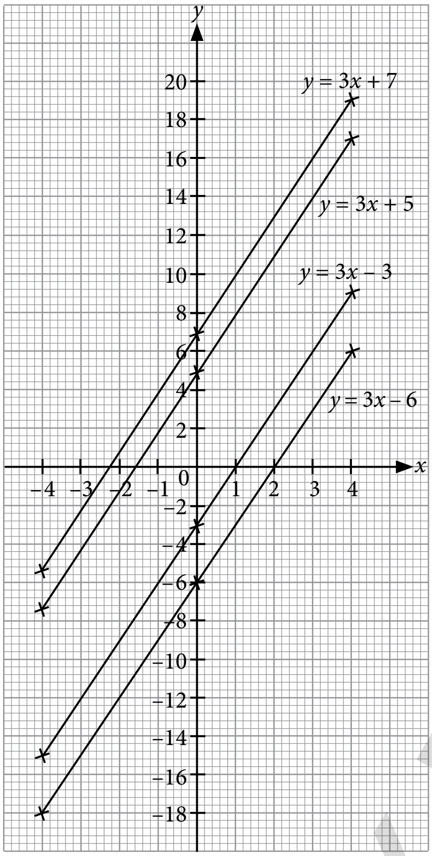
x	0	2	4
$y = 2x + 8$	8	12	16
$y = 2x + 2$	2	6	10
$y = 2x - 3$	−3	1	5
$y = 2x - 6$	−6	−2	2



(ii) **They are parallel lines.**

2. **(i)**

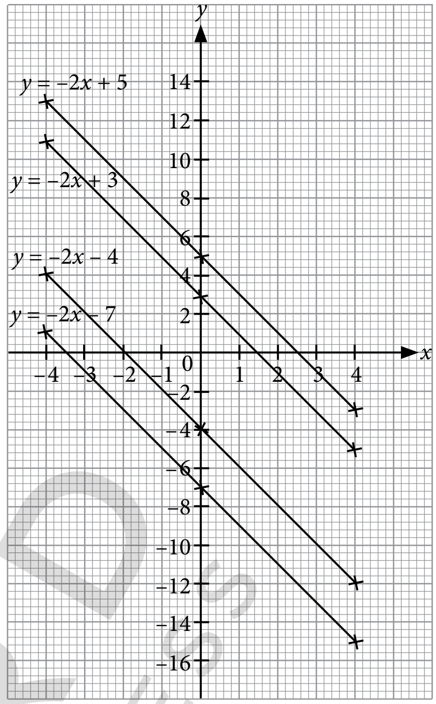
x	−4	0	4
$y = 3x + 7$	−5	7	19
$y = 3x + 5$	−7	5	17
$y = 3x - 3$	−15	−3	9
$y = 3x - 6$	−18	−6	6



(ii) **They are parallel lines.**

3. **(i)**

x	−4	0	4
$y = -2x + 5$	13	5	−3
$y = -2x + 3$	11	3	−5
$y = -2x - 4$	4	−4	−12
$y = -2x - 7$	1	−7	−15



(ii) **They are parallel lines.**

4. **(a)** Gradient = **3**, y-intercept = **7**

(b) Gradient = **−1**, y-intercept = **−1**

(c) Gradient = **6**, y-intercept = **6**

(d) Gradient = **−4**, y-intercept = $\mathbf{\frac{10}{3}}$

(e) Gradient = **0.2**, y-intercept = **0**

(f) Gradient = **0**, y-intercept = **−11**

5. **(a)** $\mathbf{y = 2x + 4}$

(b) $\mathbf{y = -2x - 4}$

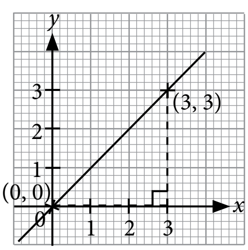
(c) $\mathbf{y = x - \frac{1}{5}}$

(d) $\mathbf{y = 3.78 - x}$

(e) $\mathbf{y = -\frac{2}{3}x}$

(f) $\mathbf{y = -\frac{2}{3}}$

6. **(a)**

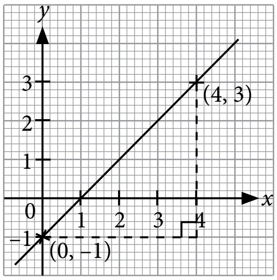


$m = \frac{3}{3}$

$= \mathbf{1}$

$c = \mathbf{0}$

(b)

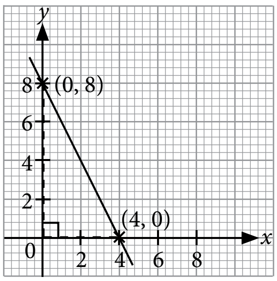


$m = \frac{4}{4}$

$= \mathbf{1}$

$c = \mathbf{-1}$

(c)

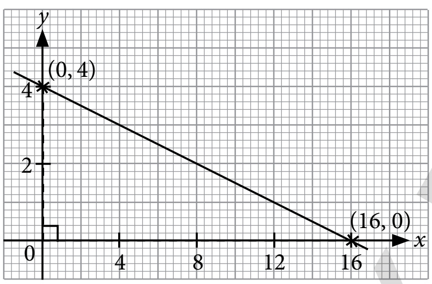


$m = \frac{8}{4}$

$= \mathbf{-2}$

$c = \mathbf{8}$

(d)

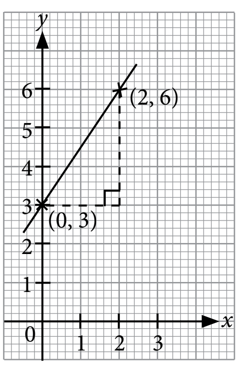


$m = -\frac{4}{16}$

$= \mathbf{-\frac{1}{4}}$

$c = \mathbf{4}$

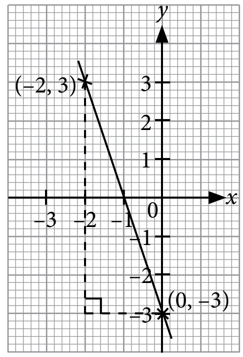
(e)



$m = \mathbf{\frac{3}{2}}$

$c = \mathbf{3}$

(f)

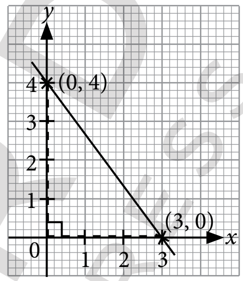


$m = -\frac{6}{2}$

$= \mathbf{-3}$

$c = \mathbf{-3}$

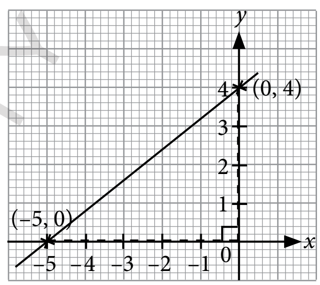
(g)



$m = -\mathbf{\frac{4}{3}}$

$c = \mathbf{4}$

(h)



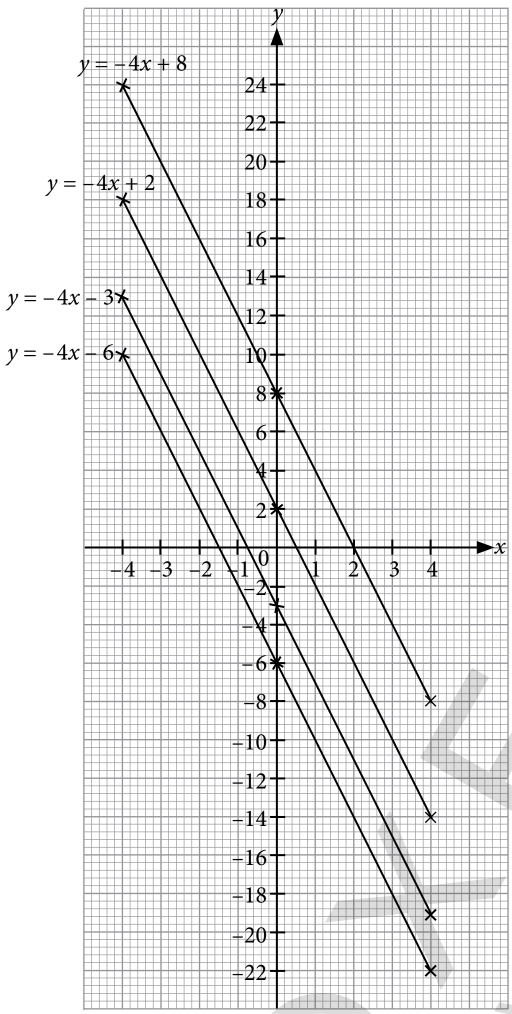
$m = \mathbf{\frac{4}{5}}$

$c = \mathbf{4}$

7. Gradient of Line 1 = **0**
Gradient of Line 2 is **undefined**.

8. **(i)**

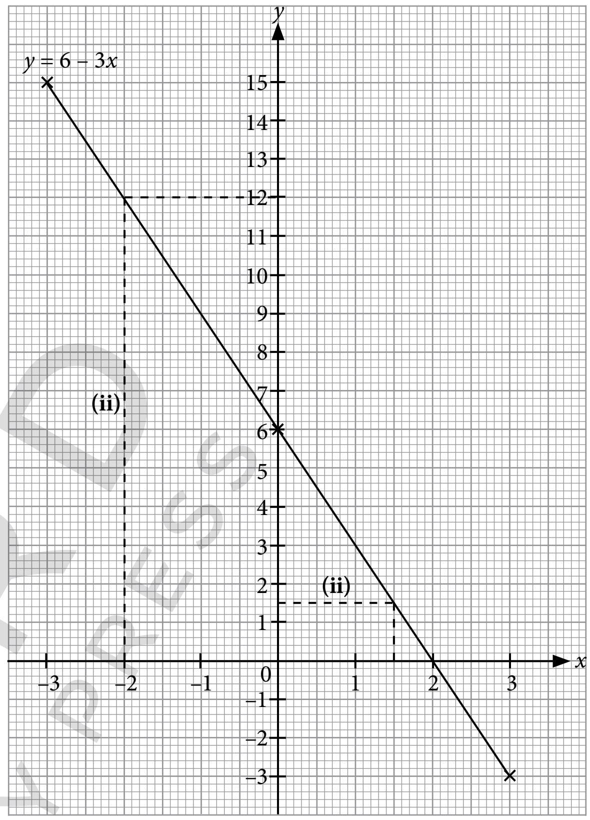
x	–4	0	4
$y = -4x + 8$	24	8	–8
$y = -4x + 2$	18	2	–14
$y = -4x - 3$	13	–3	–19
$y = -4x - 6$	10	–6	–22



(ii) $\boldsymbol{y = mx + c_1, y = mx + c_2, y = mx + c_3, y = mx + c_4}$, where m, c_1, c_2, c_3 and c_4 are constants

9. **(i)**

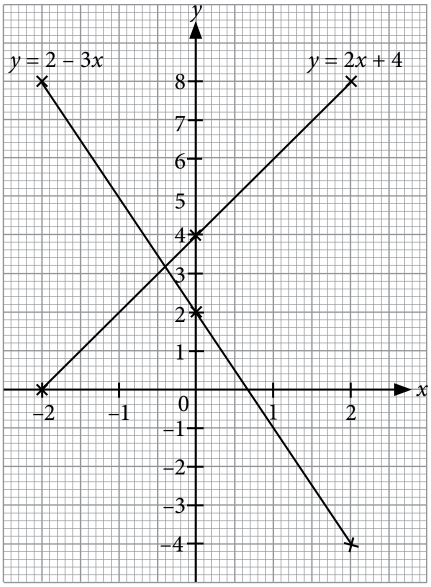
x	–3	0	3
$y = 6 - 3x$	15	6	–3



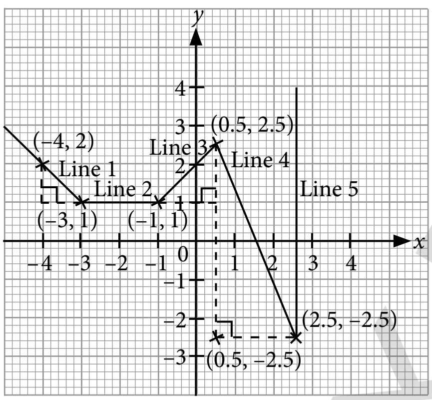
(ii) From the graph in **(i)**,
when $y = 0$,
$a = x = \mathbf{2}$
From the graph in **(i)**,
when $x = -2$,
$b = y = \mathbf{12}$
From the graph in **(i)**,
when $y = 1.5$,
$c = x = \mathbf{1.5}$

10.

x	-2	0	2
$y = 2x + 4$	0	4	8
$y = 2 - 3x$	8	2	-4



11. (i)



Gradient of Line 1 $= -\dfrac{2}{2}$

$= \mathbf{-1}$

Gradient of Line 2 = **0**

Gradient of Line 3 $= \dfrac{1.5}{1.5}$

$= \mathbf{1}$

Gradient of Line 4 $= -\dfrac{5}{2}$

$= \mathbf{-2\frac{1}{2}}$

The gradient of Line 5 is **undefined**.

(ii) y-intercept of Line 1 = –2

$\therefore$ the equation of Line 1 is $\boldsymbol{y = -x - 2}$.

y-intercept of Line 3 = 2

$\therefore$ the equation of Line 3 is $\boldsymbol{y = x + 2}$.

y-intercept of Line 4 $= 3\dfrac{3}{4}$

$\therefore$ the equation of Line 4 is $\boldsymbol{y = -2\frac{1}{2}x + 3\frac{3}{4}}$.

12. Gradient of Line 1 = **0**

Gradient of Line 2 = gradient of Line 5

$= \mathbf{-3}$

The gradient of Line 3 is **undefined**.

Gradient of Line 4 = gradient of Line 6

$= \mathbf{\frac{1}{2}}$

1.4 Applications of linear graphs in real-world contexts

Class Discussion (Graphical representation of height of flag)

Graph B represents the height of a school flag during the flag raising ceremony. The height of the flag does not start at '0' since the flag does not start from the ground. The height of the flag also increases slowly and would reach a maximum height, and not an indefinite height.

Practise Now 4

(i) (a) Amount charged = \$30 + \$12

= **\$42**

(b) Amount charged = \$30 + (5 – 2) × \$12

= **\$66**

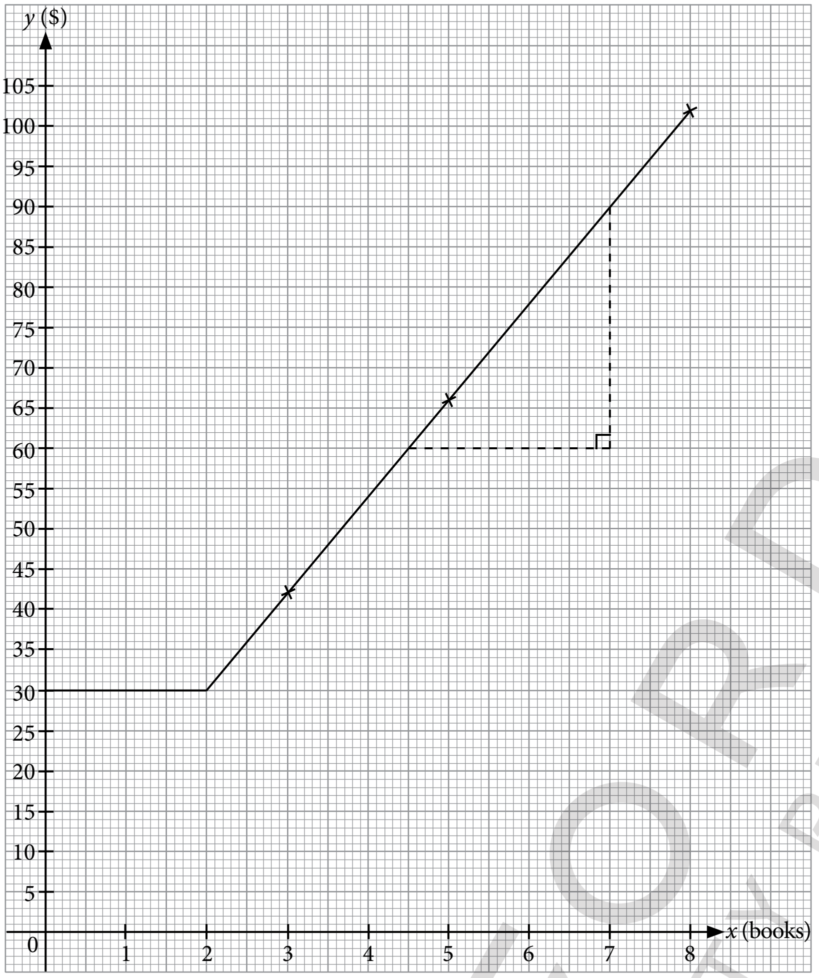
(c) Amount charged = \$30 + (8 – 2) × \$12

= **\$102**

(ii)

x	3	5	8
y	**42**	**66**	**102**

(iii)



(iv) From the graph, gradient = $\frac{90 - 60}{7 - 4.5}$

$= 12$

y-intercept = 6

$\therefore$ the equation of the straight line is $\boldsymbol{y = 12x + 6}$.

(v) **(a)** Substitute $x = 7$ into $y = 12x + 6$:

$y = 12(7) + 6$

$= 90$

$\therefore$ the customer has to pay **$90**.

(b) Substitute $x = 13$ into $y = 12x + 6$:

$y = 12(13) + 6$

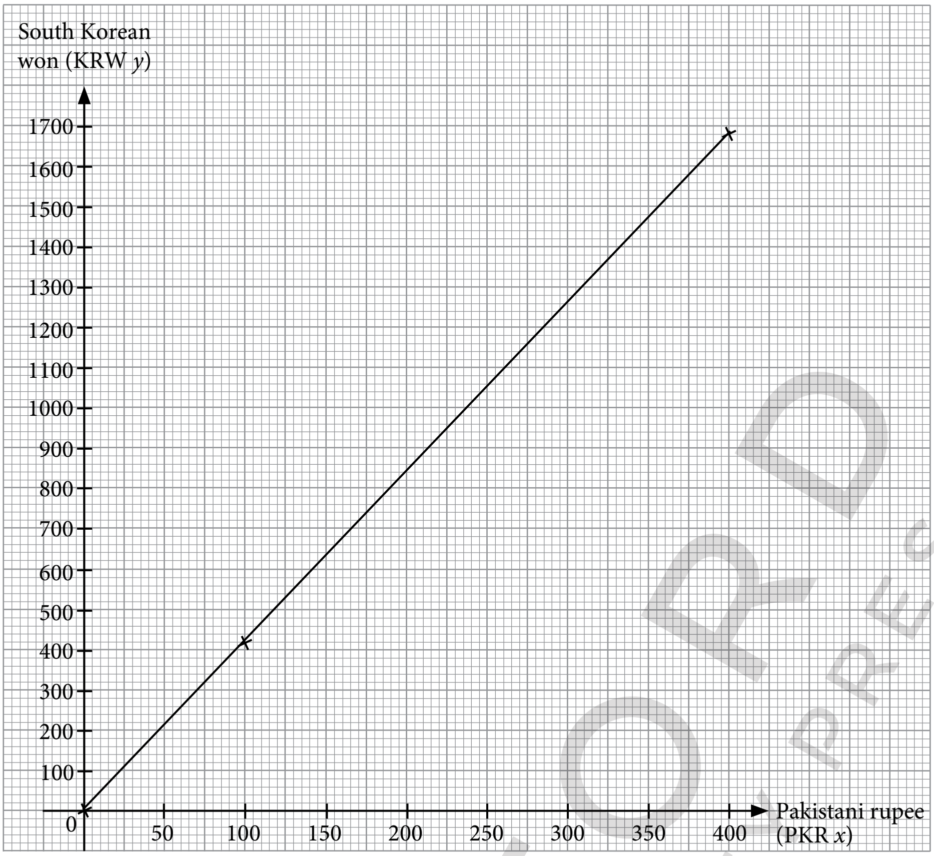
$= 162$

$\therefore$ the customer has to pay **$162**.

Practise Now 5

(i) Missing value is **0**.

(ii)



(iii) From the graph,

$$\text{gradient} = \frac{\text{rise}}{\text{run}} = \frac{1680}{400} = 4.2$$

y–intercept $= 0$

The equation of the line is $\boldsymbol{y = 4.2x}$.

$\therefore$ currency exchange rate on that day is KRW 1 = PKR 4.2.

(iv) **(a)** Substituting $x = 300$ into $y = 4.2x$,

$y = 4.2(300)$
$= 1260$

$\therefore$ amount of South Korean won received = **KRW 1260**.

Alternatively,

From the graph in (ii), when $x = 300$, $y = 1260$.

$\therefore$ amount of South Korean won received = **KRW 1260**.

(b) Substituting $y = 50$ into $y = 4.2x$,

$50 = 4.2x$
$x = 50 \div 4.2$
$= 12$ (to the nearest whole number)

$\therefore$ amount of Pakistani rupee needed = **PKR 12**.

(c) Substituting $x = 270$ into $y = 4.2x$,

$y = 4.2(270)$
$= 1134$

$\therefore$ amount of South Korean won received = **KRW 1134**.

Practise Now 6

(i) Time taken for the technician to repair each computer = **20 minutes**

(ii) Distance between the technician's workshop and his first customer = **9 km**

(iii) **(a)** Gradient of $OA = \boldsymbol{\frac{9}{10}}$

The technician was travelling away from his workshop at an average speed of $\frac{9}{10}$ km/min.

(b) Gradient of $AB =$ **0**

The technician stopped to repair a computer and hence had an average speed of 0 km/min.

(c) Gradient of $BC = \boldsymbol{-\frac{4}{5}}$

The technician was travelling back to his workshop at an average speed of $\frac{4}{5}$ km/min.

(d) Gradient of $CD =$ **0**

The technician stopped to repair a computer and hence had an average speed of 0 km/min.

(e) Gradient of $DE = \boldsymbol{-\frac{5}{7}}$

The technician was travelling back to his workshop at an average speed of $\frac{5}{7}$ km/min.

Exercise 1C

1. (i) (a) Amount payable if the package weighs 0.5 kg
= $3 + 0.5 × $2
= **$4**

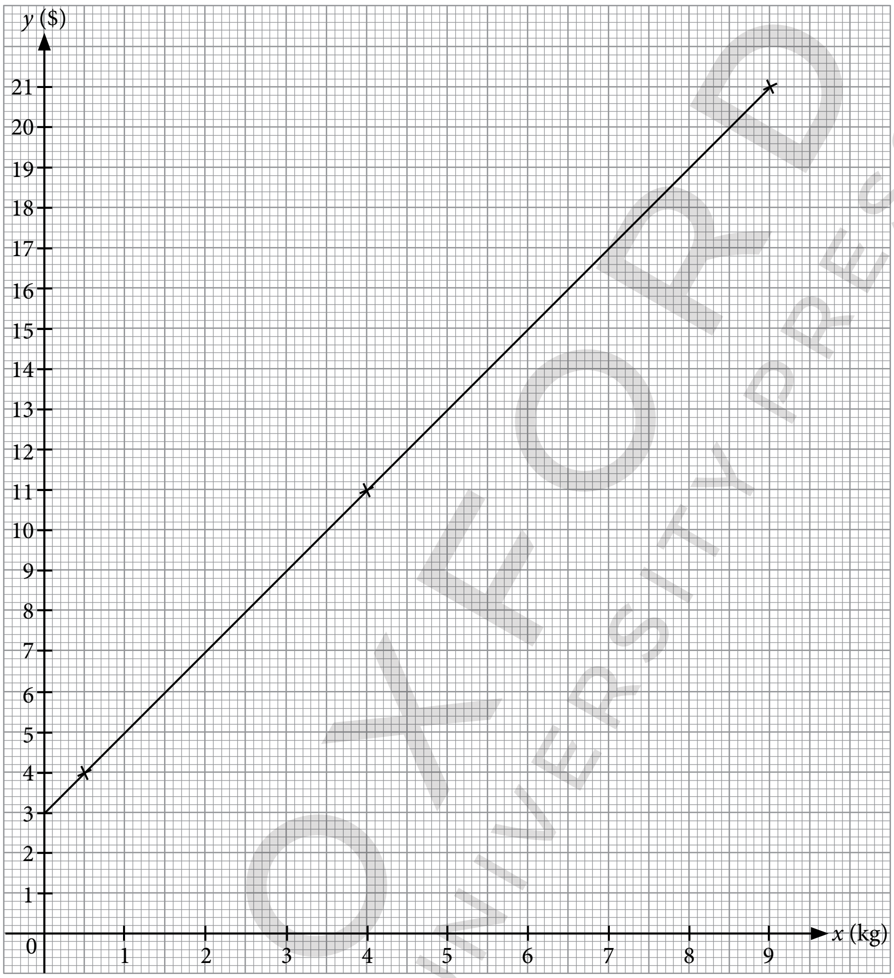
(b) Amount payable if the package weighs 4 kg
= $3 + 4 × $2
= **$11**

(c) Amount payable if the package weighs 9 kg
= $3 + 9 × $2
= **$21**

(ii)

x	0.5	4	9
y	**4**	**11**	**21**

(iii)



(iv) From the graph, gradient $= \dfrac{11 - 4}{4 - 0.5}$
$= 2$

y-intercept = 3

∴ the equation of the straight line is $\boldsymbol{y = 2x + 3}$.

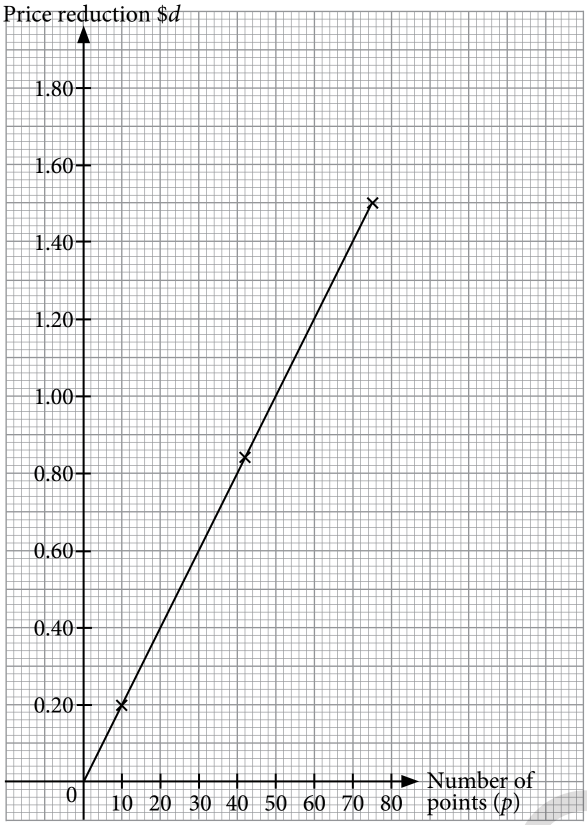
(v) (a) Substitute $x = 7.5$ into $y = 2x + 3$:
$y = 2(7.5) + 3$
$= 18$
∴ the amount payable if the package weighs 7.5 kg is **$18**.

(b) Substitute $x = 18.5$ into $y = 2x + 3$:
$y = 2(18.5) + 3$
$= 40$
∴ the amount payable if the package weighs 18.5 kg is **$40**.

2. (i) (a) Distance car can travel if it has 2 l of petrol = **15 km**
(b) Distance car can travel if it has 5.2 l of petrol = **39 km**

(ii) Amount of petrol required to travel 30 km = 4 l
∴ cost of petrol required to travel 30 km = 4 × $1.40
= **$5.60**

3. (i)



(ii) From the graph,

$$\text{gradient} = \frac{\text{rise}}{\text{run}} = \frac{1.30}{65} = 0.02$$

y–intercept = 0

The equation of the line is $\boldsymbol{d = 0.02p}$.

(iii) Substituting $p = 210$ into $d = 0.02p$,

$d = 0.02(210)$
$= 4.2$

Amount of price reduction = **$4.20**.

∴ Amount paid in cash
= $8.90 – $4.20
= **$4.70**

4. (i)

N	10	30	50	70
C	**100**	**200**	**300**	**400**

(ii) **There is a fixed overhead cost of $50.**

(iii) Amount of money David has to pay for 68 T-shirts = **$390**

(iv) Number of T-shirts David can order with $410 = **72**

5. (i) Price Company A charges for 10 minutes of talk time = **$0.80**

(ii) Price Company B charges for 25 minutes of talk time = **$3.80**

(iii) For less than 15 minutes of talk time, Company B charges a lower price than Company A, thus **Company B** would be able to offer Raju a better price.

(iv) m_A = gradient of A

$$= \frac{4}{50} = \frac{2}{25}$$

m_B = gradient of B

$$= \frac{5}{40} = \frac{1}{8}$$

Since $m_B > m_A$, **Company B** has a greater rate of increase in charges.

(v) At $4 a month,
duration of talk time offered by Company A = 60 minutes and
duration of talk time offered by Company B = 52 minutes.
Since Company A offers more talk time for $4 per month, Albert should choose **Company A**.

6. (i) From the graph,

x	5	10	20
y	**18**	**36**	**72**

(ii) From the graph,

$$\text{gradient} = \frac{\text{rise}}{\text{run}} = \frac{54}{15} = 3.6$$

y–intercept = 0

The equation of the line is $\boldsymbol{y = 3.6x}$.

A value x in m/s can be converted to y km/h by **multiplying *x* by 3.6**.

Since $y = 3.6x$, $x = \frac{y}{3.6}$. Thus, a value y in km/h can be converted to x m/s by **dividing *y* by 3.6**.

(iii) Substituting $x = 25$ into $y = 3.6x$,

$y = 3.6(25)$
$= 90$

The speed of the car = 90 km/h, which is more than the speed limit of 80 km/h. Thus, the driver cannot drive at a speed of 25 m/s on these roads.

Alternatively,
Substituting $y = 80$ into $y = 3.6x$,
$80 = 3.6x$
$x = 80 \div 3.6$
$= 22.2$ m/s (to 3 s.f.)

The speed limit on these roads = 22.2 m/s, which is less than the speed of the car. Thus, the driver cannot drive at a speed of 25 m/s on these roads.

7. **(i)** Ken left home at **1000 hours**.

(ii) Distance Ken travelled before he reached the cafeteria
= **50 km**

(iii) (a) Gradient of $OA = \frac{50}{1}$
$= \mathbf{50}$
Ken's average speed was 50 km/h.

(b) Gradient of $AB = \mathbf{0}$
Ken's average speed was 0 km/h, i.e. he was stationary.

(c) Gradient of $BC = \dfrac{30}{\frac{1}{2}}$
$= \mathbf{60}$
Ken's average speed was 60 km/h.

8. **(i)** Distance between Waseem's home and the post office
= **40 km**

(ii) Total time Waseem stayed at the post office and at the hawker centre $= 1 + \frac{1}{2}$
$= \mathbf{1\frac{1}{2}}$ **hours**

(iii) (a) Gradient of $OA = \frac{40}{2}$
$= \mathbf{20}$
Waseem was travelling away from his home to the post office at an average speed of 20 km/h.

(b) Gradient of $BC = -\dfrac{20}{1\frac{1}{2}}$
$= \mathbf{-13\frac{1}{3}}$
Waseem was travelling away from the post office back to his home at an average speed of $13\frac{1}{3}$ km/h.

(c) Gradient of $DE = -\frac{20}{1}$
$= \mathbf{-20}$
Waseem was travelling away from the post office back to his home at an average speed of 20 km/h.

(iv) (a) Waseem's speed at 0700 hours was **20 km/h**.

(b) Gradient of $AB = 0$
Waseem's speed at 0830 hours was **0 km/h**

(c) Waseem's speed at 1015 hours was $\mathbf{13\frac{1}{3}}$ **km/h**.

Chapter 2 Linear Graphs and Simultaneous Linear Equations

TEACHING NOTES

Suggested Approach

Students have learnt the graphs of straight lines in the form $y = mx + c$ in Chapter 1. In this chapter, this will be expanded to cover linear equations in the form $ax + by = k$.

They have also learnt how to solve simple linear equations in Secondary One. Here, they will be learning how to solve simultaneous linear equations to obtain a pair of values of x and of y that satisfies two linear equations at the same time. Students are expected to know how to solve them graphically and algebraically and apply this to real-life scenarios by the end of the chapter.

Teachers can build up on past knowledge learnt by students when covering this chapter.

Section 2.1 Equations of straight lines

Teachers can build upon students' knowledge of the equation of straight line of the form $y = mx + c$ to derive the equation $y = c$ for horizontal lines (see Investigation: Equation of a horizontal line). Teachers should explain to the students that lines of the form $y = c$ are parallel to the x-axis and cut the y-axis at c. These lines are above or beneath the x-axis depending on whether $c > 0$ or $c < 0$. Teachers can use this observation to guide students in deducing the equation of vertical lines (see Investigation: Equation of a vertical line). Students should know that lines of the form $x = a$ are parallel to the y-axis and cut the x-axis at a.

Section 2.2 Graphs of linear equations in the form $ax + by = k$

Before students start plotting the graphs of the functions in this section, they should revise the choice of scales and labelling of scales on both axes. Students are often weak in some of these areas. Many errors in students' work arise from their choice of scales. Teachers should spend some time to ensure that students are able to choose appropriate scales, even though the choice of scales are specified in most of the questions at this stage.

In the conclusion that an equation of the form $ax + by = k$ can be rewritten in the form $y = mx + c$ (see Investigation: Graphs of $ax + by = k$), teachers may highlight that the two equations are equivalent, and link the idea of equivalence to the solving of linear equations (Book 1 Chapter 7) where an equation is manipulated to other equivalent forms to solve for an unknown. Here each pair of coordinates that lies on the line $ax + by = k$ will also lie on the equivalent line in the form $y = mx + c$.

Teachers should ensure that students are confident in plotting the graphs of the form $ax + by = k$ before progressing to sketching these graphs (see Worked Example 2). For a sketch, important characteristics of the graph should be indicated. For linear graphs, these are the x- and y-intercept. Teachers should emphasise that the equation of the line should be written next to the sketched line.

Section 2.3: Solving simultaneous linear equations using graphical method

It is important that teachers state the concept clearly that the point(s) of intersection of two graphs gives the solution to a pair of simultaneous equations and this can be illustrated by solving a pair of linear simultaneous equations and then plotting the graphs of these two linear equations to verify the results (see Investigation: Solving simultaneous linear equations graphically)

Teachers should show clearly that a pair of simultaneous linear equations may have an infinite number of solutions or no solution (see Class Discussion: Coincident lines and parallel lines, and Thinking Time on page 50).

Section 2.4: Solving simultaneous linear equations using algebraic methods

The ability to solve equations is crucial to the study of mathematics. The concept of solving simultaneous linear equations by adding or subtracting both sides of equations can be illustrated using physical examples. An example is drawing a balance and adding or removing coins from both sides of the balance.

Some students make common errors when they are careless in the multiplication or division of both sides of an equation and they may forget that all terms must be multiplied or divided by the same number throughout.

The following are some examples.

- $x + 3y = 5$ is taken to imply $2x + 6y = 5$
- $5x + 15y = 14$ is taken to imply $x + 3y = 14$, and then $x = 14 - 3y$

Section 2.5: Applications of simultaneous equations in real-world contexts

Struggling learners may face challenges formulating a pair of simultaneous equations from a given problem. Teachers may wish to show more examples and allow more practice for students. Teachers may also want to group students of varying ability together, so that the better students can help the weaker students.

Introductory Problem

The solutions to this problem can be found in Introductory Problem Revisited (after Practise Now 12).

2.1 Equations of straight lines

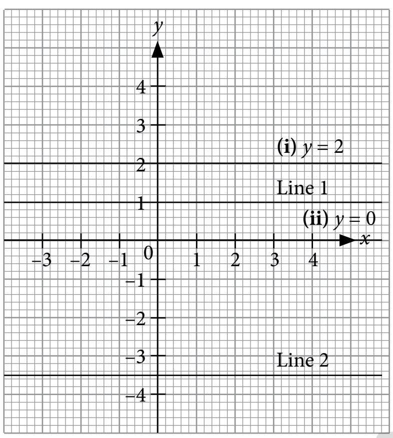
Investigation (Equation of a horizontal line)

1. The gradient of the horizontal line is **0**.
2. $\boldsymbol{B(-2, 3), D(3, 3)}$
3. The y-coordinates of all the four points are equal to **3**.
4. A straight line can be drawn through A, B, C, D and the point $(k, 3)$.
5. $\boldsymbol{y = 3}$

Practise Now 1A

(a) Line 1: $\boldsymbol{y = 1}$
Line 2: $\boldsymbol{y = -3.5}$

(b)



The lines are horizontal. The y-coordinates of all the points on the lines are a constant.

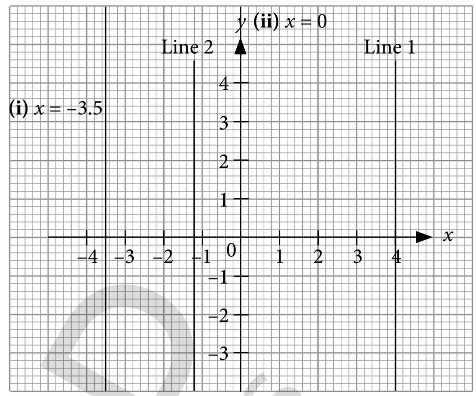
Investigation (Equation of a vertical line)

1. The gradient of the vertical line is **undefined**.
2. $\boldsymbol{Q(2, 1), S(2, -4)}$
3. The x-coordinates of all the four points are equal to **2**.
4. A straight line can be drawn through P, Q, R, S and the point $(2, k)$.
5. $\boldsymbol{x = 2}$

Practise Now 1B

(a) Line 1: $\boldsymbol{x = 4}$
Line 2: $\boldsymbol{x = -1.2}$

(b)

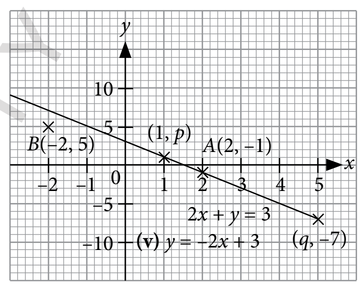


The lines are vertical. The x-coordinates of all the points on the lines are a constant.

2.2 Graphs of linear equations in the form $ax + by = k$

Investigation (Graphs of $ax + by = k$)

1. (i)



(ii) The point $A(2, -1)$ lies on the graph. The point $B(-2, 5)$ does not lie on the graph.

When $x = 2$, $2(2) + y = 3$
$4 + y = 3$
$y = -1$

When $x = -2$, $2(-2) + y = 3$
$-4 + y = 3$
$y = 7 \neq 5$

$A(2, -1)$ satisfies the equation $2x + y = 3$. $B(-2, 5)$ does not satisfy the equation $2x + y = 3$.

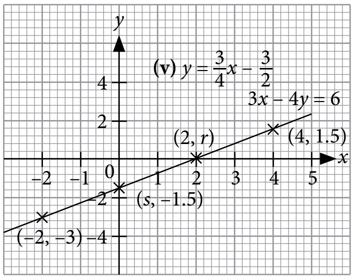
(iii) When $x = 1$, $y = p = \mathbf{1}$.

(iv) When $y = -7$, $x = q = \mathbf{5}$.

(v) The graph of $y = -2x + 3$ coincides with the graph of $2x + y = 3$.

$$2x + y = 3$$
$$2x - 2x + y = -2x + 3$$
$$y = -2x + 3 \quad \text{(shown)}$$

2. **(i)**



(ii) When $x = 2$, $y = r = \mathbf{0}$

(iii) When $y = -1.5$, $x = s = \mathbf{0}$

(iv) The coordinates of two other points are **(–2, –3)** and **(4, 1.5)**.

Other points can be used, as long as they lie on the line.

(v) The graph of $y = \frac{3}{4}x - \frac{3}{2}$ coincides with the graph of $3x - 4y = 6$.

$$3x - 4y = 6$$
$$3x - 3x - 4y = -3x + 6$$
$$-4y = -3x + 6$$
$$\frac{-4y}{-4} = \frac{-3x+6}{-4}$$
$$y = \frac{3}{4}x - \frac{3}{2} \quad \text{(shown)}$$

Practise Now 1C

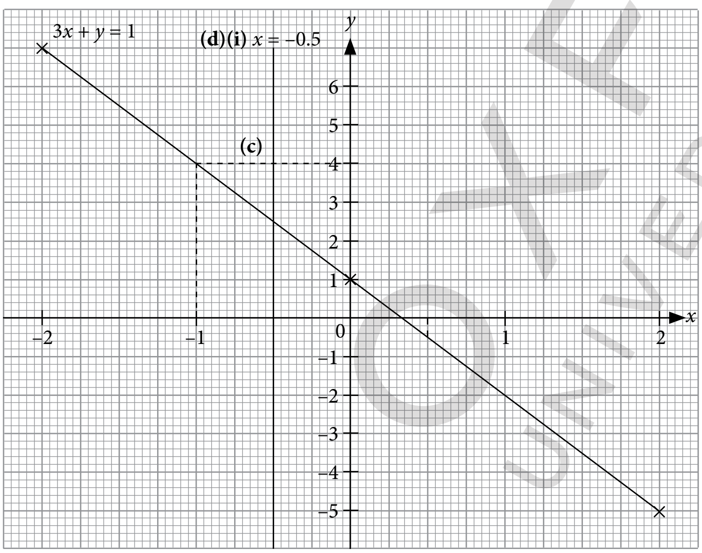
(a) When $x = -2$, $y = p$,

$3(-2) + p = 1$

$-6 + p = 1$

$\therefore p = \mathbf{7}$

(b)



(c) From the graph in **(b)**,

When $x = -1$,

$q = y = \mathbf{4}$

(d) **(ii)** y-coordinate = **2.5**

Practise Now 2

(a) $3x - y = 1$

Substitute $x = 0$ into $3x - y = 1$:

$3(0) - y = 1$

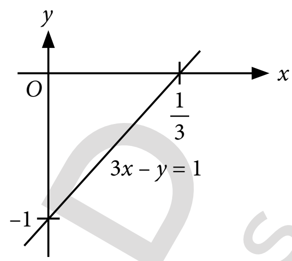
$y = -1$

Substitute $y = 0$ into $3x - y = 1$:

$3x - 0 = 1$

$x = \frac{1}{3}$

$\therefore$ the graph cuts the x- and y-axes at $\mathbf{(\frac{1}{3}, 0)}$ **and** **(0, –1)**.



(b) $4y + \frac{1}{2}x = -2$

Substitute $x = 0$ into $4y + \frac{1}{2}x = -2$:

$4y + \frac{1}{2}(0) = -2$

$y = \frac{-2}{4}$

$= -\frac{1}{2}$

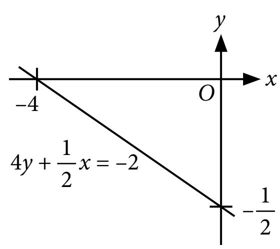
Substitute y = 0 into $4y + \frac{1}{2}x = -2$:

$4(0) + \frac{1}{2}x = -2$

$x = -2 \div \frac{1}{2}$

$= -4$

$\therefore$ the graph cuts the x- and y-axes at **(–4, 0)** and $\mathbf{(0, -\frac{1}{2})}$.



(c) $2y = 4x + 3$

Substitute $x = 0$ into $2y = 4x + 3$:

$2y = 4(0) + 3$

$2y = 3$

$y = \frac{3}{2}$

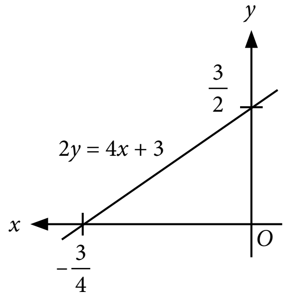
Substitute $y = 0$ into $2y = 4x + 3$:

$2(0) = 4x + 3$

$4x = -3$

$x = -\frac{3}{4}$

$\therefore$ the graph cuts the x- and y-axes at $\mathbf{(-\frac{3}{4}, 0)}$ and $\mathbf{(0, \frac{3}{2})}$.



(d) $x + y = 0$

Substitute $x = 0$ into $x + y = 0$:

$(0) + y = 0$

$y = 0$

Substitute $y = 0$ into $x + y = 0$:

$x + (0) = 0$

$x = 0$

Teachers should highlight that since the y-intercept of the line is 0, it is necessary determine another point through which the line passes. This is because any line of the form $ax + by = 0$ (or $y = mx$) have the same y-intercept.

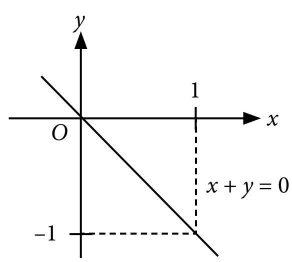
Substitute $x = 1$ into $x + y = 0$:

$1 + y = 0$

$y = -1$

Choice of x-value is arbitrary.

$\therefore$ the graph passes through **(0, 0)** and **(1, –1)**

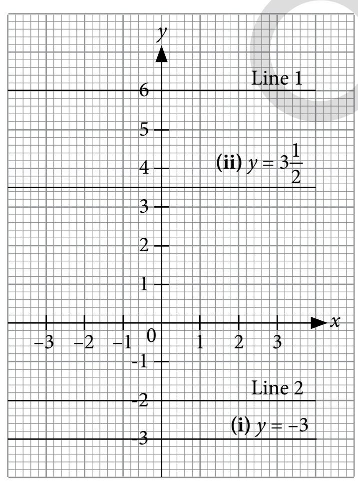


Exercise 2A

1. **(a)** Line 1: $\boldsymbol{y = 6}$

Line 2: $\boldsymbol{y = -2}$

(b)

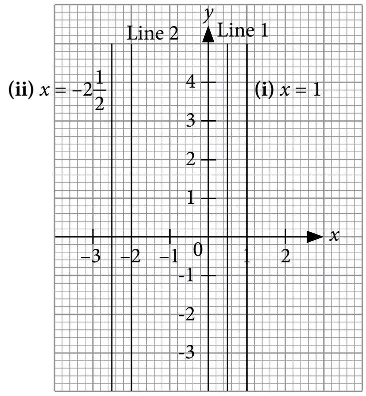


The lines are horizontal. The y-coordinates of all the points on the lines are a constant.

2. **(a)** Line 1: $\boldsymbol{x = 0.5}$

Line 2: $\boldsymbol{x = -2}$

(b)



The lines are vertical. The x-coordinates of all the points on the lines are a constant.

3. **(a)** $y - 2x = -1$

Substitute $x = 0$ into $y - 2x = -1$:

$y - 2(0) = -1$

$y = -1$

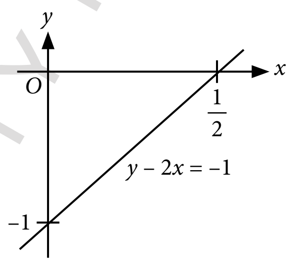
Substitute y = 0 into $y - 2x = -1$:

$0 - 2x = -1$

$-2x = -1$

$x = \frac{1}{2}$

$\therefore$ the graph cuts the x- and y-axes at $\boldsymbol{(\frac{1}{2}, 0)}$ and **(0, –1)**.



(b) $3x + 2y = 3$

Substitute $x = 0$ into $3x + 2y = 3$:

$3(0) + 2y = 3$

$2y = 3$

$y = \frac{3}{2}$

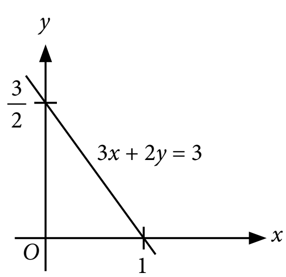
Substitute $y = 0$ into $3x + 2y = 3$:

$3x + 2(0) = 3$

$3x = 3$

$x = 1$

$\therefore$ the graph cuts the x- and y-axes at **(1, 0)** and $\boldsymbol{(0, \frac{3}{2})}$.



(c) $2x - 5y = -10$

Substitute $x = 0$ into $2x - 5y = -10$:

$2(0) - 5y = -10$

$-5y = -10$

$y = 2$

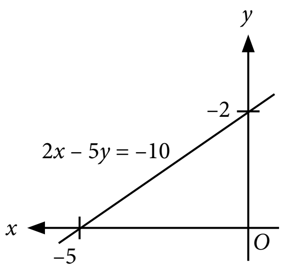
Substitute $y = 0$ into $2x - 5y = -10$:

$2x - 5(0) = -10$

$2x = -10$

$x = -5$

$\therefore$ the graph cuts the x- and y-axes at **(–5, 0)** and **(0, 2)**.



(d) $2.5x - 3y = -4$

Substitute $x = 0$ into $2.5x - 3y = -4$:

$2.5(0) - 3y = -4$

$-3y = -4$

$y = \frac{4}{3}$

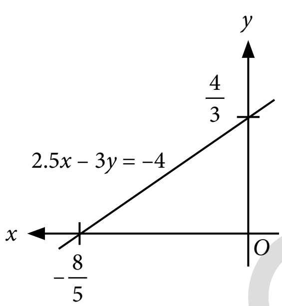
Substitute $y = 0$ into $2.5x - 3y = -4$:

$2.5x - 3(0) = -4$

$2.5x = -4$

$x = -\frac{8}{5}$

$\therefore$ the graph cuts the x- and y-axes at $\mathbf{(-\frac{8}{5}, 0)}$ and $\mathbf{(0, \frac{4}{3})}$.



(e) $\frac{1}{2}y + x = -2$

Substitute x = 0 into $\frac{1}{2}y + x = -2$:

$\frac{1}{2}y + 0 = -2$

$\frac{1}{2}y = -2$

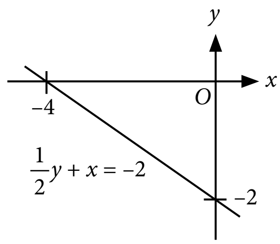
$y = -4$

Substitute $y = 0$ into $\frac{1}{2}y + x = -2$:

$\frac{1}{2}(0) + x = -2$

$x = -2$

$\therefore$ the graph cuts the x- and y-axes at **(–2, 0)** and **(0,–4)**.



(f) $5x - 4y = 0$

Substitute $x = 0$ into $5x - 4y = 0$:

$5(0) - 4y = 0$

$y = 0$

Substitute $y = 0$ into $5x - 4y = 0$:

$5x - 4(0) = 0$

$x = 0$

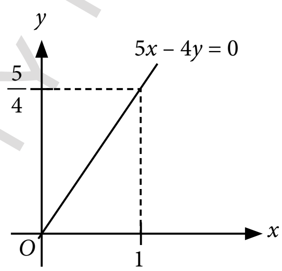
Substitute $x = 1$ into $5x - 4y = 0$:

$5(1) - 4y = 0$

$-4y = -5$

$y = \frac{5}{4}$

$\therefore$ the graph cuts the x- and y-axes at **(0, 0)** and $\mathbf{(0, \frac{5}{4})}$.



4. **(a)** When $x = -5$, $y = p$,

$-(-5) + 2p = 4$

$5 + 2p = 4$

$2p = -1$

$p = -0.5$

When $x = 5$, $y = q$,

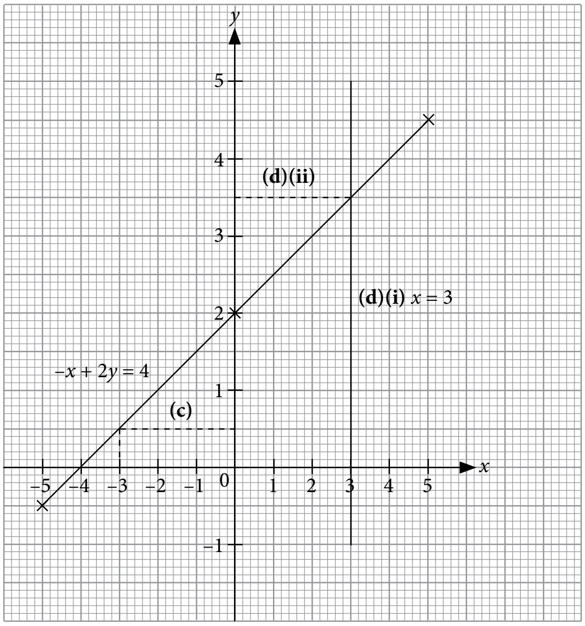
$-5 + 2q = 4$

$2q = 9$

$q = 4.5$

$\therefore$ $p = \mathbf{-0.5}$, $q = \mathbf{4.5}$

(b)



(c) From the graph in **(b)**,
When $y = 0.5$,
$r = x = \mathbf{-3}$

(d) **(ii)** y-coordinate = **3.5**

5. **(i)** From the sketch,

$$\text{gradient} = \frac{\text{rise}}{\text{run}} = \frac{4}{2} = 2$$

y-intercept $= -4$

$\therefore$ equation of the function is $\mathbf{y = 2x - 4}$.

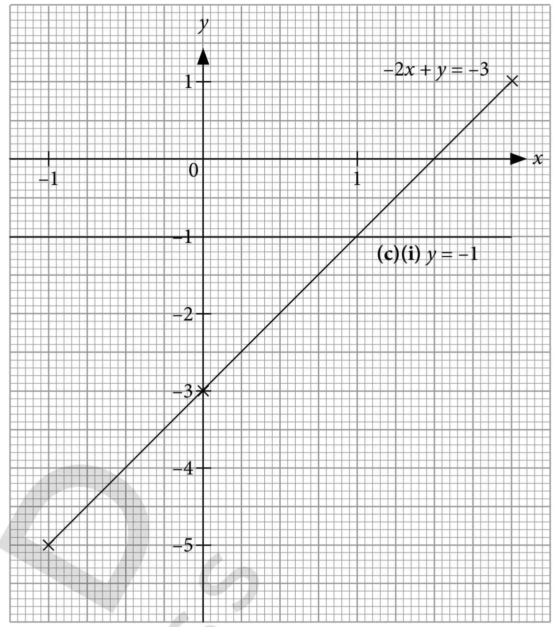
(ii) Substitute $x = 1$ and $y = p$ into $y = 2x - 4$:
$p = 2(1) - 4$
$= \mathbf{-2}$
Substitute $x = q$ and $y = 3$ into $y = 2x - 4$:
$3 = 2q - 4$
$2q = 3 + 4$
$q = 7 \div 2 = \mathbf{3.5}$

(iii) Substitute $x = 4$ and $y = r$ into $y = 2x - 4$:
$r = 2(4) - 4$
$r = 8 - 4$
$= \mathbf{4}$
The point (4, 4) lies on the graph. Two possible values of r are **–2, 1**.

6. **(a)** $-2x + y = -3$

x	–1	0	2
y	**–5**	**–3**	**1**

(b)

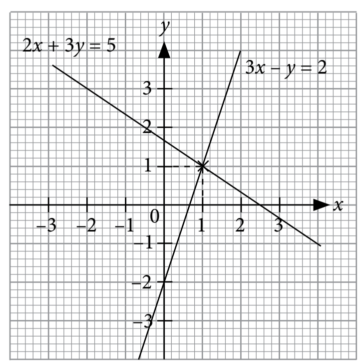


(c) **(ii)** Area of trapezium
$= \frac{1}{2} \times (1.5 + 1) \times 1$
$= \mathbf{1.25\ units^2}$

2.3 Solving simultaneous linear equations using graphical method

Investigation (Solving simultaneous linear equations graphically)

1. **(i)**



(ii) The coordinates of the point of intersection of the two graphs are **(1, 1)**.

(iii) For $2x + 3y = 5$
When $x = -2$, $2(-2) + 3y = 5$
$y = 3$
For $3x - y = 2$
When $x = -2$, $3(-2) - y = 2$
$y = -8$
$(-2, 3)$ satisfies equation $2x + 3y = 5$ but not equation $3x - y = 2$.

(iv) For $2x + 3y = 5$

When $x = 0$, $2(0) + 3y = 5$

$y = \frac{5}{3}$

When $x = 1$, $2(1) + 3y = 5$

$y = 1$

When $x = 2$, $2(2) + 3y = 5$

$y = \frac{1}{3}$

When $x = 4$, $2(4) + 3y = 5$

$y = -1$

For $3x - y = 2$

When $x = 0$, $3(0) - y = 2$

$y = -2$

When $x = 1$, $3(1) - y = 2$

$y = 1$

When $x = 2$, $3(2) - y = 2$

$y = 4$

When $x = 4$, $3(4) - y = 2$

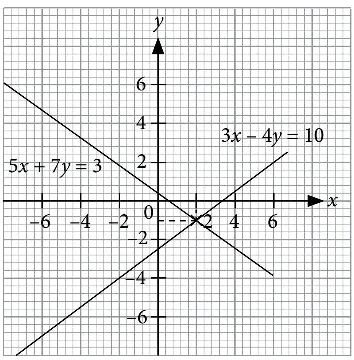
$y = 10$

$(0, -2)$ and $(2, 4)$ satisfies equation $3x - y = 2$ but not $2x + 3y = 5$.

$(1, 1)$ satisfies both equations $2x + 3y = 5$ and $3x - y = 2$.

$(4, -1)$ satisfies equation $2x + 3y = 5$ but not equation $3x - y = 2$.

2. **(i)**



(ii) The coordinates of the point of intersection of the two graphs are **(2, –1)**.

(iii) The pair of values of x and y that satisfies both equations are $\boldsymbol{x = 2}$ and $\boldsymbol{y = -1}$.

3. The coordinates of the point of intersection of the two graphs is the pair of values of x and y that satisfies both the equations. A point that lies on one line will satisfy the equation of that line. The same applies to the second line. Hence, the coordinates of the point of intersection is the same as the point that lies on both lines and that satisfy both equations.

Class Discussion (Choice of appropriate scales for graphs and accuracy of graphs)

1. The graphs should look different to students who have used different scales in both axes.
2. **(i)** $y = \mathbf{2.9}$
   (ii) $x = \mathbf{-0.6}$
3. By substituting the given value into the linear equation, one can check for the accuracy of the answers.
4. Use a larger scale and redraw the graph to improve its accuracy.

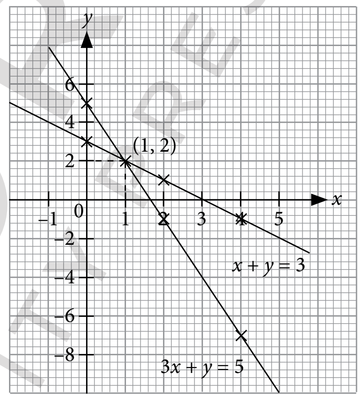
Practise Now 3

1. $x + y = 3$

x	0	2	4
y	3	1	–1

$3x + y = 5$

x	0	2	4
y	5	–1	–7



The graphs intersect at the point (1, 2).

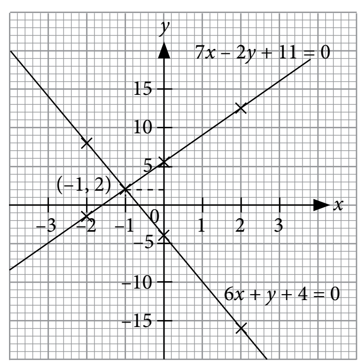
$\therefore$ the solution is $\boldsymbol{x = 1}$ and $\boldsymbol{y = 2}$.

2. $7x - 2y + 11 = 0$

x	–2	0	2
y	–1.5	5.5	12.5

$6x + y + 4 = 0$

x	–2	0	2
y	8	–4	–16

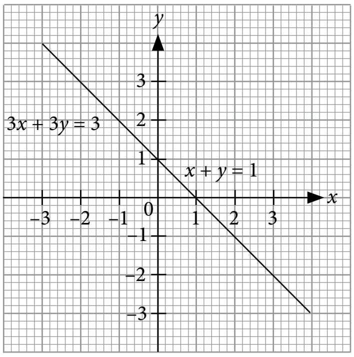


The graphs intersect at the point (–1, 2).

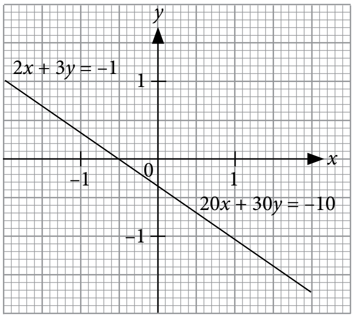
$\therefore$ the solution is $\boldsymbol{x = -1}$ and $\boldsymbol{y = 2}$.

Class Discussion (Coincident lines and parallel lines)

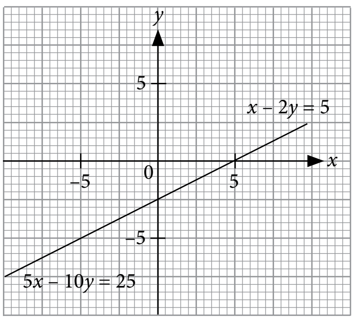
1. (a) (i)



(ii)



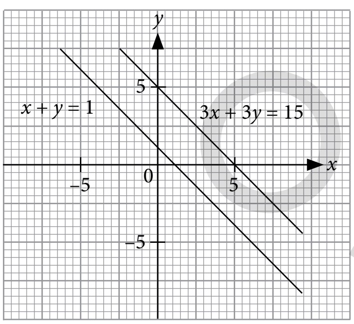
(iii)



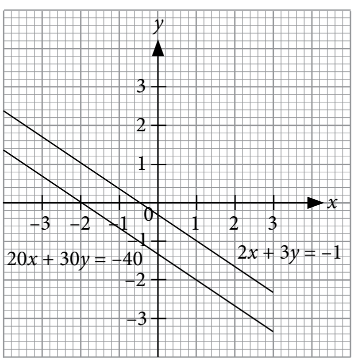
(b) The graphs of each pair of simultaneous equations are a pair of lines that coincide.

(c) Yes, each pair of simultaneous equations has solutions. The solutions are all the points that lie on the line.

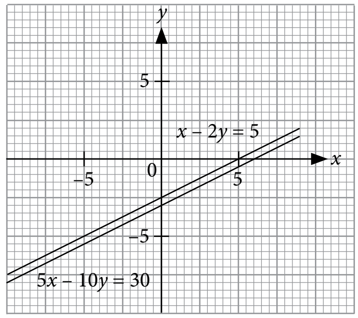
2. (a) (i)



(ii)



(iii)



(b) The graphs of each pair of simultaneous equations are a pair of parallel lines.

(c) No, each pair of simultaneous equations does not have any solution since they do not have any point of intersection.

Thinking Time (Page 56)

(a) Plot the pair of simultaneous equations on the same axes. If there is only one point of intersection, the pair of simultaneous equations has only one solution.

(b) Plot the pair of simultaneous equations on the same axes. If the two plots coincide, the pair of simultaneous equations has infinitely many solutions.

(c) Plot the pair of simultaneous equations on the same axes. If the two plots are parallel to each other with no intersection point, the pair of simultaneous equations has no solution.

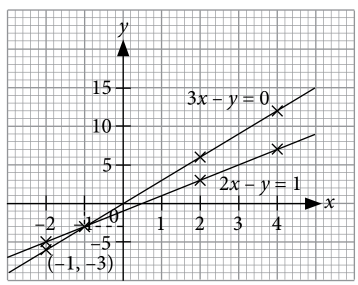
Exercise 2B

1. (a) $3x - y = 0$

x	–2	2	4
y	–6	6	12

$2x - y = 1$

x	–2	2	4
y	–5	3	7



The graphs intersect at the point (–1, –3).

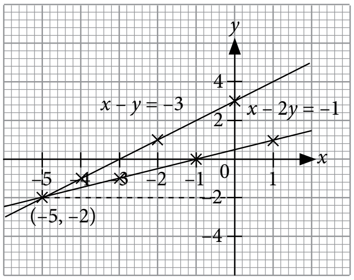
∴ the solution is $\boldsymbol{x = -1}$ and $\boldsymbol{y = -3}$.

(b) $x - y = -3$

x	-4	-2	0
y	-1	1	3

$x - 2y = -1$

x	-3	-1	1
y	-1	0	1



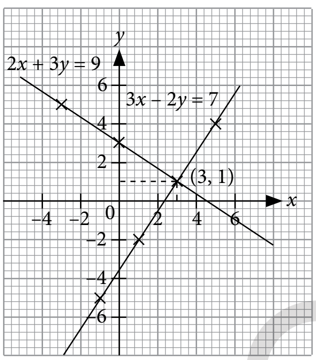
The graphs intersect at the point $(-5, -2)$.
$\therefore$ the solution is $\boldsymbol{x = -5}$ and $\boldsymbol{y = -2}$.

(c) $3x - 2y = 7$

x	-1	1	5
y	-5	-2	4

$2x + 3y = 9$

x	-3	0	3
y	5	3	1



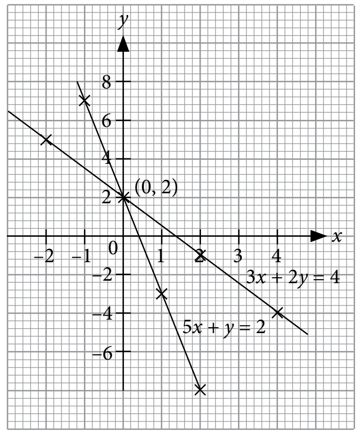
The graphs intersect at the point $(3, 1)$.
$\therefore$ the solution is $\boldsymbol{x = 3}$ and $\boldsymbol{y = 1}$.

(d) $3x + 2y = 4$

x	-2	2	4
y	5	-1	-4

$5x + y = 2$

x	-1	1	2
y	7	-3	-8



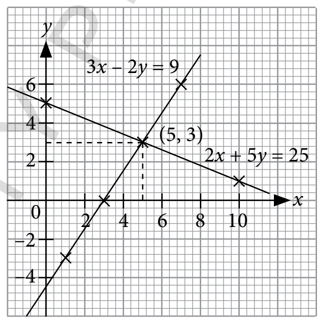
The graphs intersect at the point $(0, 2)$.
$\therefore$ the solution is $\boldsymbol{x = 0}$ and $\boldsymbol{y = 2}$.

(e) $2x + 5y = 25$

x	0	5	10
y	5	3	1

$3x - 2y = 9$

x	1	3	7
y	-3	0	6



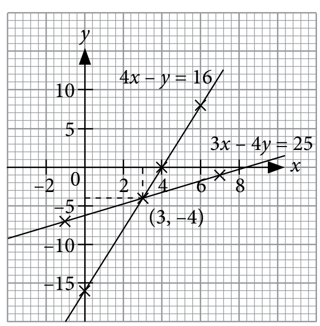
The graphs intersect at the point $(5, 3)$.
$\therefore$ the solution is $\boldsymbol{x = 5}$ and $\boldsymbol{y = 3}$.

(f) $3x - 4y = 25$

x	-1	3	7
y	-7	-4	-1

$4x - y = 16$

x	0	4	6
y	-16	0	8



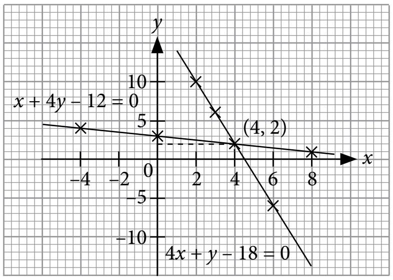
The graphs intersect at the point $(3, -4)$.
$\therefore$ the solution is $\boldsymbol{x = 3}$ and $\boldsymbol{y = -4}$.

2. **(a)** $x + 4y - 12 = 0$

x	−4	0	8
y	4	3	1

$4x + y - 18 = 0$

x	2	3	6
y	10	6	−6



The graphs intersect at the point (4, 2).

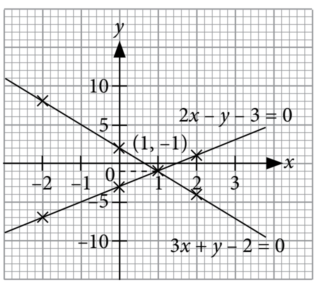
$\therefore$ the solution is $\boldsymbol{x = 4}$ and $\boldsymbol{y = 2}$.

(b) $3x + y - 2 = 0$

x	−2	0	2
y	8	2	−4

$2x - y - 3 = 0$

x	−2	0	2
y	−7	−3	1



The graphs intersect at the point (1, −1).

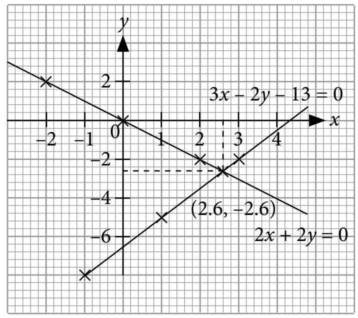
$\therefore$ the solution is $\boldsymbol{x = 1}$ and $\boldsymbol{y = -1}$.

(c) $3x - 2y - 13 = 0$

x	−1	1	3
y	−8	−5	−2

$2x + 2y = 0$

x	−2	0	2
y	2	0	−2



The graphs intersect at the point (2.6, −2.6).

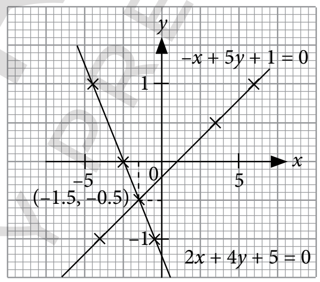
$\therefore$ the solution is $\boldsymbol{x = 2.6}$ and $\boldsymbol{y = -2.6}$.

(d) $2x + 4y + 5 = 0$

x	−4.5	−2.5	−0.5
y	1	0	−1

$-x + 5y + 1 = 0$

x	−4	3.5	6
y	−1	0.5	1



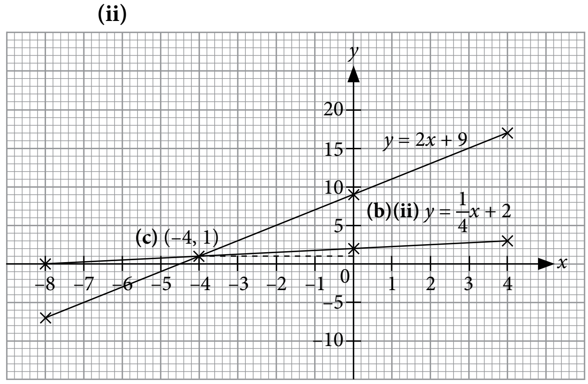
The graphs intersect at the point (−1.5, −0.5).

$\therefore$ the solution is $\boldsymbol{x = -1.5}$ and $\boldsymbol{y = -0.5}$.

3. **(a)** **(i)** $y = 2x + 9$

x	−8	0	4
y	**−7**	**9**	**17**

(ii)



(b) **(i)** $y = \frac{1}{4}x + 2$

x	−8	0	4
y	**0**	**2**	**3**

(c) $2x - y = -9$ — (1)
$x - 4y = -8$ — (2)
From (1), $y = 2x + 9$
From (2), $4y = x + 8$

$$y = \frac{1}{4}x + 2$$

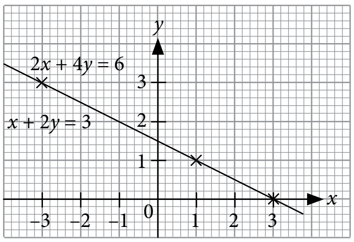
From **(a)(ii)**, the graphs intersect at the point $(-4, 1)$.
$\therefore$ the solution is $\boldsymbol{x = -4}$ and $\boldsymbol{y = 1}$.

4. **(a)** $x + 2y = 3$

x	-3	1	3
y	3	1	0

$2x + 4y = 6$

x	-3	1	3
y	3	1	0



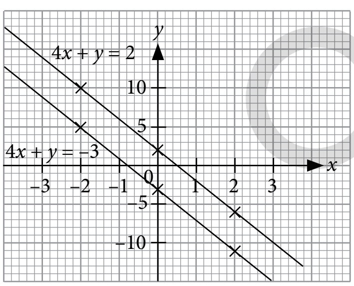
The graphs of each pair of simultaneous equations are identical.
The simultaneous equations have an **infinite number of solutions**.

(b) $4x + y = 2$

x	-2	0	2
y	10	2	-6

$4x + y = -3$

x	-2	0	2
y	5	-3	-11



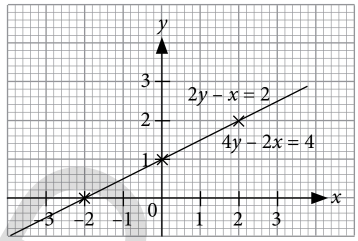
The graphs of each pair of simultaneous equations are parallel and have no intersection point.
The simultaneous equations have **no solution**.

(c) $2y - x = 2$

x	-2	0	2
y	0	1	2

$4y - 2x = 4$

x	-2	0	2
y	0	1	2



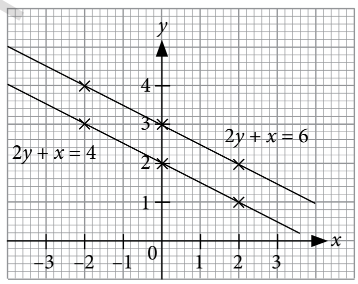
The graphs of each pair of simultaneous equations are identical.
The simultaneous equations have an **infinite number of solutions**.

(d) $2y + x = 4$

x	-2	0	2
y	3	2	1

$2y + x = 6$

x	-2	0	2
y	4	3	2



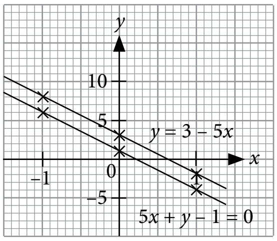
The graphs of each pair of simultaneous equations are parallel and have no intersection point.
The simultaneous equations have **no solution**.

5. **(a)** $y = 3 - 5x$

x	-1	0	1
y	8	3	-2

$5x + y - 1 = 0$

x	-1	0	1
y	6	1	-4



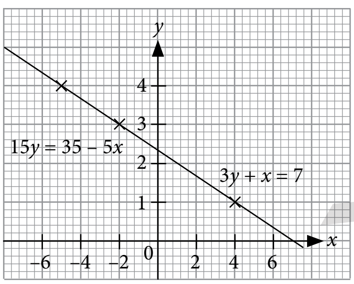
The graphs of each pair of simultaneous equations are parallel and have no intersection point.
The simultaneous equations have **no solution**.

(b) $3y + x = 7$

x	-5	-2	4
y	4	3	1

$15y = 35 - 5x$

x	-5	-2	4
y	4	3	1



The graphs of each pair of simultaneous equations are identical.
The simultaneous equations an **infinite number of solutions**.

2.4 Solving simultaneous linear equations using algebraic methods

Practise Now 4

1. **(a)** $x - y = 3 \quad \text{— (1)}$
$4x + y = 17 \quad \text{— (2)}$
(2) + (1):
$(4x + y) + (x - y) = 17 + 3$
$4x + y + x - y = 20$
$5x = 20$
$x = 4$

Substitute $x = 4$ into (2):
$4(4) + y = 17$
$16 + y = 17$
$y = 1$
$\therefore$ the solution is $\boldsymbol{x = 4}$ and $\boldsymbol{y = 1}$.

(b) $7x + 2y = 19 \quad \text{— (1)}$
$7x + 8y = 13 \quad \text{— (2)}$
(2) − (1):
$(7x + 8y) - (7x + 2y) = 13 - 19$
$7x + 8y - 7x - 2y = -6$
$6y = -6$
$y = -1$
Substitute $y = -1$ into (1):
$7x + 2(-1) = 19$
$7x - 2 = 19$
$7x = 21$
$x = 3$
$\therefore$ the solution is $\boldsymbol{x = 3}$ and $\boldsymbol{y = -1}$.

(c) $43x + 9y = 4 \quad \text{— (1)}$
$17x - 9y = 26 \quad \text{— (2)}$
(1) + (2):
$(43x + 9y) + (17x - 9y) = 4 + 26$
$43x + 9y + 17x - 9y = 30$
$60x = 30$
$x = \frac{1}{2}$

Substitute $x = \frac{1}{2}$ into (1):

$43\left(\frac{1}{2}\right) + 9y = 4$

$\frac{43}{2} + 9y = 4$

$9y = -\frac{35}{2}$

$y = -\frac{35}{18}$

$\therefore$ the solution is $\boldsymbol{x = \frac{1}{2}}$ and $\boldsymbol{y = -\frac{35}{18}}$.

(d) $4x - 5y = 17 \quad \text{— (1)}$
$2x - 5y = 8 \quad \text{— (2)}$
(1) − (2):
$(4x - 5y) - (2x - 5y) = 17 - 8$
$4x - 5y - 2x + 5y = 9$
$2x = 9$
$x = \frac{9}{2}$

Substitute $x = \frac{9}{2}$ into (2):

$2\left(\frac{9}{2}\right) - 5y = 8$
$9 - 5y = 8$
$5y = 1$
$y = \frac{1}{5}$

$\therefore$ the solution is $\boldsymbol{x = \frac{9}{2}}$ and $\boldsymbol{y = \frac{1}{5}}$.

2. $3x - y + 14 = 0$ — (1)
$2x + y + 1 = 0$ — (2)
(1) + (2):
$(3x - y + 14) + (2x + y + 1) = 0 + 0$
$3x - y + 14 + 2x + y + 1 = 0$
$5x + 15 = 0$
$5x = -15$
$x = -3$
Substitute $x = -3$ into (2):
$2(-3) + y + 1 = 0$
$y - 5 = 0$
$y = 5$
$\therefore$ the solution is $\boldsymbol{x = -3}$ and $\boldsymbol{y = 5}$.

Practise Now 5

1. (a) $2x + 3y = 18$ — (1)
$3x - y = 5$ — (2)
$3 \times (2)$: $9x - 3y = 15$ — (3)
(1) + (3):
$(2x + 3y) + (9x - 3y) = 18 + 15$
$2x + 3y + 9x - 3y = 33$
$11x = 33$
$x = 3$
Substitute $x = 3$ into (2):
$3(3) - y = 5$
$9 - y = 5$
$y = 4$
$\therefore$ the solution is $\boldsymbol{x = 3}$ and $\boldsymbol{y = 4}$.

(b) $x + 4y = 11$ — (1)
$2x + 3y = 7$ — (2)
$2 \times (1)$: $2x + 8y = 22$ — (3)
(3) – (2):
$(2x + 8y) - (2x + 3y) = 22 - 7$
$2x + 8y - 2x - 3y = 15$
$5y = 15$
$y = 3$
Substitute $y = 3$ into (1):
$x + 4(3) = 11$
$x + 12 = 11$
$x = -1$
$\therefore$ the solution is $\boldsymbol{x = -1}$ and $\boldsymbol{y = 3}$.

2. (a) $9x + 2y = 5$ — (1)
$7x - 3y = 13$ — (2)
$3 \times (1)$: $27x + 6y = 15$ — (3)
$2 \times (2)$: $14x - 6y = 26$ — (4)
(3) + (4):
$(27x + 6y) + (14x - 6y) = 15 + 26$
$27x + 6y + 14x - 6y = 41$
$41x = 41$
$x = 1$
Substitute $x = 1$ into (1):
$9(1) + 2y = 5$
$9 + 2y = 5$
$2y = -4$
$y = -2$
$\therefore$ the solution is $\boldsymbol{x = 1}$ and $\boldsymbol{y = -2}$.

(b) $5x - 4y = 17$ — (1)
$2x - 3y = 11$ — (2)
$2 \times (1)$: $10x - 8y = 34$ — (3)
$5 \times (2)$: $10x - 15y = 55$ — (4)
(3) – (4):
$(10x - 8y) - (10x - 15y) = 34 - 55$
$10x - 8y - 10x + 15y = -21$
$7y = -21$
$y = -3$
Substitute $y = -3$ into (2):
$2x - 3(-3) = 11$
$2x + 9 = 11$
$2x = 2$
$x = 1$
$\therefore$ the solution is $\boldsymbol{x = 1}$ and $\boldsymbol{y = -3}$.

Thinking Time (Page 60)

$13x - 6y = 20$ — (1)
$7x + 4y = 18$ — (2)
$7 \times (1)$: $91x - 42y = 140$ — (3)
$13 \times (2)$: $91x + 52y = 234$ — (4)
(4) – (3):
$(91x + 52y) - (91x - 42y) = 234 - 140$
$91x + 52y - 91x + 42y = 94$
$94y = 94$
$y = 1$
Substitute $y = 1$ into (1):
$13x - 6(1) = 20$
$13x - 6 = 20$
$13x = 26$
$x = 2$
$\therefore$ the solution is $x = 2$ and $y = 1$.
No, it is not easier to eliminate x first as the LCM of 13 and 7 is larger than 12.

Practise Now 6

Method 1:

$\frac{x}{2} - \frac{y}{3} = 4$ — (1)

$\frac{2}{5}x - \frac{y}{6} = 3\frac{1}{2}$ — (2)

$\frac{1}{2} \times (1)$: $\frac{x}{4} - \frac{y}{6} = 2$ — (3)

(2) – (3):

$\left(\frac{2}{5}x - \frac{y}{6}\right) - \left(\frac{x}{4} - \frac{y}{6}\right) = 3\frac{1}{2} - 2$

$\frac{2}{5}x - \frac{y}{6} - \frac{x}{4} + \frac{y}{6} = 1\frac{1}{2}$

$\frac{3}{20}x = 1\frac{1}{2}$

$x = 10$

Substitute $x = 10$ into (1):

$\frac{10}{2} - \frac{y}{3} = 4$

$5 - \frac{y}{3} = 4$

$\frac{y}{3} = 1$

$y = 3$

$\therefore$ the solution is $\boldsymbol{x = 10}$ and $\boldsymbol{y = 3}$.

Method 2:

$\frac{x}{2} - \frac{y}{3} = 4$ — (1)

$\frac{2}{5}x - \frac{y}{6} = 3\frac{1}{2}$ — (2)

$30 \times (1)$: $15x - 10y = 120$ — (3)

$60 \times (2)$: $24x - 10y = 210$ — (4)

$(4) - (3)$:

$$\begin{aligned}(24x - 10y) - (15x - 10y) &= 210 - 120\\ 24x - 10y - 15x + 10y &= 90\\ 9x &= 90\\ x &= 10\end{aligned}$$

Substitute $x = 10$ into (3):

$$\begin{aligned}15(10) - 10y &= 120\\ 150 - 10y &= 120\\ -10y &= -30\\ y &= 3\end{aligned}$$

$\therefore$ the solution is $\boldsymbol{x = 10}$ and $\boldsymbol{y = 3}$.

Practise Now 7

1. $3y - x = 7$ — (1)

$2x + 3y = 4$ — (2)

From (1), $x = 3y - 7$ — (3)

Substitute (3) into (2):

$$\begin{aligned}2(3y - 7) + 3y &= 4\\ 6y - 14 + 3y &= 4\\ 9y &= 18\\ y &= 2\end{aligned}$$

Substitute $y = 2$ into (3):

$$\begin{aligned}x &= 3(2) - 7\\ &= -1\end{aligned}$$

$\therefore$ the solution is $\boldsymbol{x = -1}$ and $\boldsymbol{y = 2}$.

2. $3x - 2y = 8$ — (1)

$4x + 3y = 5$ — (2)

From (1), $3x = 2y + 8$

$x = \frac{2y+8}{3}$ — (3)

Substitute (3) into (2):

$$\begin{aligned}4\left(\frac{2y+8}{3}\right) + 3y &= 5\\ 4(2y + 8) + 9y &= 15\\ 8y + 32 + 9y &= 15\\ 17y + 32 &= 15\\ 17y &= -17\\ y &= -1\end{aligned}$$

Substitute $y = -1$ into (3):

$$\begin{aligned}x &= \frac{2(-1)+8}{3}\\ &= 2\end{aligned}$$

$\therefore$ the solution is $\boldsymbol{x = 2}$ and $\boldsymbol{y = -1}$.

Teacher may use to the following to guide students in their reflection on page 62.

1. $7x - 2y = 21$ — (1)

$4x + y = 57$ — (2)

From (2), $x = \frac{57 - y}{4}$ — (3)

Substitute (3) into (1):

$$\begin{aligned}7\left(\frac{57 - y}{4}\right) - 2y &= 21\\ 7(57 - y) - 8y &= 84\\ 399 - 7y - 8y &= 84\\ 15y &= 315\\ y &= 21\end{aligned}$$

Substitute $y = 21$ into (3): $x = \frac{57 - 21}{4}$

$= 9$

$\therefore$ the solution is $x = 9$ and $y = 21$.

If x is made the subject of equation (1) or (2), we will get the same solution. Making y as the subject of equation is easier since algebraic fractions will not be introduced then.

2. $7x - 2y = 21$ — (1)

$4x + y = 57$ — (2)

$2 \times (8 :(2x + 2y = 114$ — (3)

$(1) + (3)$:

$$\begin{aligned}(7x - 2y) + (8x + 2y) &= 21 + 114\\ 7x - 2y + 8x + 2y &= 135\\ 15x &= 135\\ x &= 9\end{aligned}$$

Substitute $x = 9$ into (2):

$$\begin{aligned}4(9) + y &= 57\\ 36 + y &= 57\\ y &= 21\end{aligned}$$

$\therefore$ the solution is $x = \mathbf{9}$ and $y = \mathbf{21}$.

Yes, we can use the method of elimination to solve Worked Example 7.

3. The difference in the substitution and elimination methods lies in how they isolate and solve for one variable when tackling a pair of simultaneous equations involving two variables.

The substitution method first expresses one variable in terms of the other using one of the equations. This expression is then substituted into the remaining equation to derive the possible value(s) of the variable.

The elimination method works by multiplying the equations by a constant, which may differ for each equation, such that the coefficient of one of the variables is the same in the resultant equations. The resultant equations are then added or subtracted to eliminate one of the variables and thereby deriving the possible value(s) of the remaining variable.

Once the possible value(s) of one variable is/are obtained, both methods solve for the possible value(s) of the other variable in a similar fashion by substituting the known values of one variable into either one of the pair of simultaneous equations or their equivalent form.

Thinking Time (Page 63)

$2x + y = 6$ — (1)

$x = 1 - \frac{1}{2}y$ — (2)

$2 \times (2)$: $2x = 2 - y$

$2x + y = 2$ — (3)

Comparing (1) and (3), we notice that the gradients of the 2 equations are the same but with different constants; i.e. they are parallel lines with no solution.

Practise Now 8

(a) $\frac{x-1}{y-3} = \frac{2}{3}$ — (1)

$\frac{x-2}{y-1} = \frac{1}{2}$ — (2)

From (1),

$3(x - 1) = 2(y - 3)$

$3x - 3 = 2y - 6$

$3x - 2y = -3$ — (3)

From (2),

$2(x - 2) = y - 1$

$2x - 4 = y - 1$

$y = 2x - 3$ — (4)

Substitute (4) into (3):

$3x - 2(2x - 3) = -3$

$3x - 4x + 6 = -3$

$-x + 6 = -3$

$x = 9$

Substitute $x = 9$ into (4):

$y = 2(9) - 3$

$= 15$

$\therefore$ the solution is $\boldsymbol{x = 9}$ and $\boldsymbol{y = 15}$.

(b) $3x + 2y = 3$ — (1)

$\frac{1}{x+y} = \frac{3}{x+2y}$ — (2)

From (2),

$x + 2y = 3(x + y)$

$= 3x + 3y$

$y = -2x$ — (3)

Substitute (3) into (1):

$3x + 2(-2x) = 3$

$3x - 4x = 3$

$x = -3$

Substitute $x = -3$ into (3):

$y = -2(-3)$

$= 6$

$\therefore$ the solution is $\boldsymbol{x = -3}$ and $\boldsymbol{y = 6}$.

Exercise 2C

1. **(a)** $x + y = 16$ — (1)

$x - y = 0$ — (2)

(1) + (2):

$(x + y) + (x - y) = 16 + 0$

$x + y + x - y = 16$

$2x = 16$

$x = 8$

Substitute $x = 8$ into (1):

$8 + y = 16$

$y = 8$

$\therefore$ the solution is $\boldsymbol{x = 8}$ and $\boldsymbol{y = 8}$.

(b) $x - y = 5$ — (1)

$x + y = 19$ — (2)

(2) + (1):

$(x + y) + (x - y) = 19 + 5$

$x + y + x - y = 24$

$2x = 24$

$x = 12$

Substitute $x = 12$ into (2):

$12 + y = 19$

$y = 7$

$\therefore$ the solution is $\boldsymbol{x = 12}$ and $\boldsymbol{y = 7}$.

(c) $11x + 4y = 12$ — (1)

$9x - 4y = 8$ — (2)

(1) + (2):

$(11x + 4y) + (9x - 4y) = 12 + 8$

$11x + 4y + 9x - 4y = 20$

$20x = 20$

$x = 1$

Substitute $x = 1$ into (1):

$11(1) + 4y = 12$

$11 + 4y = 12$

$4y = 1$

$y = \frac{1}{4}$

$\therefore$ the solution is $\boldsymbol{x = 1}$ and $\boldsymbol{y = \frac{1}{4}}$.

(d) $4y + x = 11$ — (1)

$3y - x = 3$ — (2)

(1) + (2):

$(4y + x) + (3y - x) = 11 + 3$

$4y + x + 3y - x = 14$

$7y = 14$

$y = 2$

Substitute $y = 2$ into (1):

$4(2) + x = 11$

$8 + x = 11$

$x = 3$

$\therefore$ the solution is $\boldsymbol{x = 3}$ and $\boldsymbol{y = 2}$.

(e) $3x + y = 5$ — (1)

$x + y = 3$ — (2)

(1) – (2):

$(3x + y) - (x + y) = 5 - 3$

$3x + y - x - y = 2$

$2x = 2$

$x = 1$

Substitute $x = 1$ into (2):

$1 + y = 3$

$y = 2$

$\therefore$ the solution is $\boldsymbol{x = 1}$ and $\boldsymbol{y = 2}$.

(f) $2x + 3y = 5$ — (1)
$2x + 7y = 9$ — (2)
(2) – (1):
$(2x + 7y) - (2x + 3y) = 9 - 5$
$2x + 7y - 2x - 3y = 4$
$4y = 4$
$y = 1$
Substitute $y = 1$ into (1):
$2x + 3(1) = 5$
$2x + 3 = 5$
$2x = 2$
$x = 1$
$\therefore$ the solution is $\boldsymbol{x = 1}$ and $\boldsymbol{y = 1}$.

(g) $7x - 3y = 15$ — (1)
$11x - 3y = 21$ — (2)
(2) – (1):
$(11x - 3y) - (7x - 3y) = 21 - 15$
$11x - 3y - 7x + 3y = 6$
$4x = 6$
$x = \frac{3}{2}$
Substitute $x = \frac{3}{2}$ into (1):
$7\left(\frac{3}{2}\right) - 3y = 15$
$\frac{21}{2} - 3y = 15$
$3y = -\frac{9}{2}$
$y = -\frac{3}{2}$
$\therefore$ the solution is $\boldsymbol{x = \frac{3}{2}}$ and $\boldsymbol{y = -\frac{3}{2}}$.

(h) $3y - 2x = 9$ — (1)
$2y - 2x = 7$ — (2)
(1) – (2):
$(3y - 2x) - (2y - 2x) = 9 - 7$
$3y - 2x - 2y + 2x = 2$
$y = 2$
Substitute $y = 2$ into (1):
$3(2) - 2x = 9$
$6 - 2x = 9$
$2x = -3$
$x = -\frac{3}{2}$
$\therefore$ the solution is $\boldsymbol{x = -\frac{3}{2}}$ and $\boldsymbol{y = 2}$.

(i) $3a - 2b = 5$ — (1)
$2b - 5a = 9$ — (2)
(1) + (2):
$(3a - 2b) + (2b - 5a) = 5 + 9$
$3a - 2b + 2b - 5a = 14$
$-2a = 14$
$a = -7$
Substitute $a = -7$ into (2):
$2b - 5(-7) = 9$
$2b + 35 = 9$
$2b = -26$
$b = -13$
$\therefore$ the solution is $\boldsymbol{a = -7}$ and $\boldsymbol{b = -13}$.

(j) $5c - 2d = 9$ — (1)
$3c + 2d = 7$ — (2)
(1) + (2):
$(5c - 2d) + (3c + 2d) = 9 + 7$
$5c - 2d + 3c + 2d = 16$
$8c = 16$
$c = 2$
Substitute $c = 2$ into (2):
$3(2) + 2d = 7$
$6 + 2d = 7$
$2d = 1$
$d = \frac{1}{2}$
$\therefore$ the solution is $\boldsymbol{c = 2}$ and $\boldsymbol{d = \frac{1}{2}}$.

(k) $3f + 4h = 1$ — (1)
$5f - 4h = 7$ — (2)
(1) + (2):
$(3f + 4h) + (5f - 4h) = 1 + 7$
$3f + 4h + 5f - 4h = 8$
$8f = 8$
$f = 1$
Substitute $f = 1$ into (1):
$3(1) + 4h = 1$
$3 + 4h = 1$
$4h = -2$
$h = -\frac{1}{2}$
$\therefore$ the solution is $\boldsymbol{f = 1}$ and $\boldsymbol{h = -\frac{1}{2}}$.

(l) $6j - k = 23$ — (1)
$3k + 6j = 11$ — (2)
(2) – (1):
$(3k + 6j) - (6j - k) = 11 - 23$
$3k + 6j - 6j + k = -12$
$4k = -12$
$k = -3$
Substitute $k = -3$ into (2):
$3(-3) + 6j = 11$
$-9 + 6j = 11$
$6j = 20$
$j = \frac{10}{3}$
$\therefore$ the solution is $\boldsymbol{j = \frac{10}{3}}$ and $\boldsymbol{k = -3}$.

2. **(a)** $7x - 2y = 17$ — (1)
$3x + 4y = 17$ — (2)
$2 \times$ (1): $14x - 4y = 34$ — (3)
(3) + (2):
$(14x - 4y) + (3x + 4y) = 34 + 17$
$14x - 4y + 3x + 4y = 51$
$17x = 51$
$x = 3$
Substitute $x = 3$ into (2):
$3(3) + 4y = 17$
$9 + 4y = 17$
$4y = 8$
$y = 2$
$\therefore$ the solution is $\boldsymbol{x = 3}$ and $\boldsymbol{y = 2}$.

(b) $16x + 5y = 39$ — (1)
$4x - 3y = 31$ — (2)
$4 \times (2)$: $16x - 12y = 124$ — (3)
$(1) - (3)$:
$(16x + 5y) - (16x - 12y) = 39 - 124$
$16x + 5y - 16x + 12y = -85$
$17y = -85$
$y = -5$
Substitute $y = -5$ into (2):
$4x - 3(-5) = 31$
$4x + 15 = 31$
$4x = 16$
$x = 4$
$\therefore$ the solution is $\boldsymbol{x = 4}$ and $\boldsymbol{y = -5}$.

(c) $x + 2y = 3$ — (1)
$3x + 5y = 7$ — (2)
$3 \times (1)$: $3x + 6y = 9$ — (3)
$(3) - (2)$:
$(3x + 6y) - (3x + 5y) = 9 - 7$
$3x + 6y - 3x - 5y = 2$
$y = 2$
Substitute $y = 2$ into (1):
$x + 2(2) = 3$
$x + 4 = 3$
$x = -1$
$\therefore$ the solution is $\boldsymbol{x = -1}$ and $\boldsymbol{y = 2}$.

(d) $3x + y = -5$ — (1)
$7x + 3y = 1$ — (2)
$3 \times (1)$: $9x + 3y = -15$ — (3)
$(3) - (2)$:
$(9x + 3y) - (7x + 3y) = -15 - 1$
$9x + 3y - 7x - 3y = -16$
$2x = -16$
$x = -8$
Substitute $x = -8$ into (1):
$3(-8) + y = -5$
$-24 + y = -5$
$y = 19$
$\therefore$ the solution is $\boldsymbol{x = -8}$ and $\boldsymbol{y = 19}$.

(e) $7x - 3y = 13$ — (1)
$2x - y = 3$ — (2)
$3 \times (2)$: $6x - 3y = 9$ — (3)
$(1) - (3)$:
$(7x - 3y) - (6x - 3y) = 13 - 9$
$7x - 3y - 6x + 3y = 4$
$x = 4$
Substitute $x = 4$ into (2):
$2(4) - y = 3$
$8 - y = 3$
$y = 5$
$\therefore$ the solution is $\boldsymbol{x = 4}$ and $\boldsymbol{y = 5}$.

(f) $9x - 5y = 2$ — (1)
$3x - 4y = 10$ — (2)
$3 \times (2)$: $9x - 12y = 30$ — (3)
$(1) - (3)$:
$(9x - 5y) - (9x - 12y) = 2 - 30$
$9x - 5y - 9x + 12y = -28$
$7y = -28$
$y = -4$
Substitute $y = -4$ into (2):
$3x - 4(-4) = 10$
$3x + 16 = 10$
$3x = -6$
$x = -2$
$\therefore$ the solution is $\boldsymbol{x = -2}$ and $\boldsymbol{y = -4}$.

3. (a) $7x - 3y = 18$ — (1)
$6x + 7y = 25$ — (2)
$7 \times (1)$: $49x - 21y = 126$ — (3)
$3 \times (2)$: $18x + 21y = 75$ — (4)
$(3) + (4)$:
$(49x - 21y) + (18x + 21y) = 126 + 75$
$49x - 21y + 18x + 21y = 201$
$67x = 201$
$x = 3$
Substitute $x = 3$ into (2):
$6(3) + 7y = 25$
$18 + 7y = 25$
$7y = 7$
$y = 1$
$\therefore$ the solution is $\boldsymbol{x = 3}$ and $\boldsymbol{y = 1}$.

(b) $4x + 3y = -5$ — (1)
$3x - 2y = 43$ — (2)
$2 \times (1)$: $8x + 6y = -10$ — (3)
$3 \times (2)$: $9x - 6y = 129$ — (4)
$(3) + (4)$:
$(8x + 6y) + (9x - 6y) = -10 + 129$
$8x + 6y + 9x - 6y = 119$
$17x = 119$
$x = 7$
Substitute $x = 7$ into (1):
$4(7) + 3y = -5$
$28 + 3y = -5$
$3y = -33$
$y = -11$
$\therefore$ the solution is $\boldsymbol{x = 7}$ and $\boldsymbol{y = -11}$.

(c) $2x + 3y = 8$ — (1)
$5x + 2y = 9$ — (2)
$2 \times (1)$: $4x + 6y = 16$ — (3)
$3 \times (2)$: $15x + 6y = 27$ — (4)
$(4) - (3)$:
$(15x + 6y) - (4x + 6y) = 27 - 16$
$15x + 6y - 4x - 6y = 11$
$11x = 11$
$x = 1$
Substitute $x = 1$ into (2):
$5(1) + 2y = 9$
$5 + 2y = 9$
$2y = 4$
$y = 2$
$\therefore$ the solution is $\boldsymbol{x = 1}$ and $\boldsymbol{y = 2}$.

(d) $5x + 4y = 11$ — (1)
$3x + 5y = 4$ — (2)
$3 \times (1)$: $15x + 12y = 33$ — (3)
$5 \times (2)$: $15x + 25y = 20$ — (4)
(4) – (3):
$(15x + 25y) - (15x + 12y) = 20 - 33$
$15x + 25y - 15x - 12y = -13$
$13y = -13$
$y = -1$
Substitute $y = -1$ into (1):
$5x + 4(-1) = 11$
$5x - 4 = 11$
$5x = 15$
$x = 3$
$\therefore$ the solution is $\boldsymbol{x = 3}$ and $\boldsymbol{y = -1}$.

(e) $4x - 3y = -1$ — (1)
$5x - 2y = 4$ — (2)
$2 \times (1)$: $8x - 6y = -2$ — (3)
$3 \times (2)$: $15x - 6y = 12$ — (4)
(4) – (3):
$(15x - 6y) - (8x - 6y) = 12 - (-2)$
$15x - 6y - 8x + 6x = 14$
$7x = 14$
$x = 2$
Substitute $x = 2$ into (2):
$5(2) - 2y = 4$
$10 - 2y = 4$
$2y = 6$
$y = 3$
$\therefore$ the solution is $\boldsymbol{x = 2}$ and $\boldsymbol{y = 3}$.

(f) $5x - 4y = 23$ — (1)
$2x - 7y = 11$ — (2)
$2 \times (1)$: $10x - 8y = 46$ — (3)
$5 \times (2)$: $10x - 35y = 55$ — (4)
(3) – (4):
$(10x - 8y) - (10x - 35y) = 46 - 55$
$10x - 8y - 10x + 35y = -9$
$27y = -9$
$y = -\frac{1}{3}$
Substitute $y = -\frac{1}{3}$ into (1):
$5x - 4\left(-\frac{1}{3}\right) = 23$
$5x + \frac{4}{3} = 23$
$5x = \frac{65}{3}$
$x = \frac{13}{3}$
$\therefore$ the solution is $\boldsymbol{x = \frac{13}{3}}$ and $\boldsymbol{y = -\frac{1}{3}}$.

4. **(a)** $x + y = 7$ — (1)
$x - y = 5$ — (2)
From (1), $y = 7 - x$ — (3)
Substitute (3) into (2):
$x - (7 - x) = 5$
$x - 7 + x = 5$
$2x = 12$
$x = 6$
Substitute $x = 6$ into (3):
$y = 7 - 6$
$= 1$
$\therefore$ the solution is $\boldsymbol{x = 6}$ and $\boldsymbol{y = 1}$.

(b) $3x - y = 0$ — (1)
$2x + y = 5$ — (2)
From (2), $y = 5 - 2x$ — (3)
Substitute (3) into (1):
$3x - (5 - 2x) = 0$
$3x - 5 + 2x = 0$
$5x = 5$
$x = 1$
Substitute $x = 1$ into (3):
$y = 5 - 2(1)$
$= 3$
$\therefore$ the solution is $\boldsymbol{x = 1}$ and $\boldsymbol{y = 3}$.

(c) $2x - 7y = 5$ — (1)
$3x + y = -4$ — (2)
From (2), $y = -4 - 3x$ — (3)
Substitute (3) into (1):
$2x - 7(-4 - 3x) = 5$
$2x + 28 + 21x = 5$
$23x = -23$
$x = -1$
Substitute $x = -1$ into (3):
$y = -4 - 3(-1)$
$= -1$
$\therefore$ the solution is $\boldsymbol{x = -1}$ and $\boldsymbol{y = -1}$.

(d) $5x - y = 5$ — (1)
$3x + 2y = 29$ — (2)
From (1), $y = 5x - 5$ — (3)
Substitute (3) into (2):
$3x + 2(5x - 5) = 29$
$3x + 10x - 10 = 29$
$13x = 39$
$x = 3$
Substitute $x = 3$ into (3):
$y = 5(3) - 5$
$= 10$
$\therefore$ the solution is $\boldsymbol{x = 3}$ and $\boldsymbol{y = 10}$.

(e) $5x + 3y = 11$ — (1)
$4x - y = 2$ — (2)
From (2), $y = 4x - 2$ — (3)
Substitute (3) into (1):
$5x + 3(4x - 2) = 11$
$5x + 12x - 6 = 11$
$17x = 17$
$x = 1$
Substitute $x = 1$ into (3):
$y = 4(1) - 2$
$= 2$
$\therefore$ the solution is $\boldsymbol{x = 1}$ and $\boldsymbol{y = 2}$.

(f) $3x + 5y = 10$ — (1)
$x - 2y = 7$ — (2)
From (2), $x = 2y + 7$ — (3)
Substitute (3) into (1):
$3(2y + 7) + 5y = 10$
$6y + 21 + 5y = 10$
$11y = -11$
$y = -1$
Substitute $y = -1$ into (3):
$x = 2(-1) + 7$
$= 5$
$\therefore$ the solution is $\boldsymbol{x = 5}$ and $\boldsymbol{y = -1}$.

(g) $x + y = 9$ — (1)
$5x - 2y = 4$ — (2)
From (1), $y = 9 - x$ — (3)
Substitute (3) into (2):
$5x - 2(9 - x) = 4$
$5x - 18 + 2x = 4$
$7x = 22$
$x = \frac{22}{7}$
Substitute $x = \frac{22}{7}$ into (3):
$y = 9 - \frac{22}{7}$
$= \frac{41}{7}$
$\therefore$ the solution is $\boldsymbol{x = \frac{22}{7}}$ and $\boldsymbol{y = \frac{41}{7}}$.

(h) $5x + 2y = 3$ — (1)
$x - 4y = -6$ — (2)
From (2), $x = 4y - 6$ — (3)
Substitute (3) into (1):
$5(4y - 6) + 2y = 3$
$20y - 30 + 2y = 3$
$22y = 33$
$y = \frac{3}{2}$
Substitute $y = \frac{3}{2}$ into (3):
$x = 4\left(\frac{3}{2}\right) - 6$
$= 0$
$\therefore$ the solution is $\boldsymbol{x = 0}$ and $\boldsymbol{y = \frac{3}{2}}$.

5. **(a)** $x + y = 0.5$ — (1)
$x - y = 1$ — (2)
(1) + (2):
$(x + y) + (x - y) = 0.5 + 1$
$x + y + x - y = 1.5$
$2x = 1.5$
$x = 0.75$
Substitute $x = 0.75$ into (1):
$0.75 + y = 0.5$
$y = -0.25$
$\therefore$ the solution is $\boldsymbol{x = 0.75}$ and $\boldsymbol{y = -0.25}$.

(b) $2x + 0.4y = 8$ — (1)
$5x - 1.2y = 9$ — (2)
$3 \times (1)$: $6x + 1.2y = 24$ — (3)
(3) + (2):
$(6x + 1.2y) + (5x - 1.2y) = 24 + 9$
$6x + 1.2y + 5x - 1.2y = 33$
$11x = 33$
$x = 3$
Substitute $x = 3$ into (1):
$2(3) + 0.4y = 8$
$6 + 0.4y = 8$
$0.4y = 2$
$y = 5$
$\therefore$ the solution is $\boldsymbol{x = 3}$ and $\boldsymbol{y = 5}$.

(c) $10x - 3y = 24.5$ — (1)
$3x - 5y = 13.5$ — (2)
$5 \times (1)$: $50x - 15y = 122.5$ — (3)
$3 \times (2)$: $9x - 15y = 40.5$ — (4)
(3) – (4):
$(50x - 15y) - (9x - 15y) = 122.5 - 40.5$
$50x - 15y - 9x + 15y = 82$
$41x = 82$
$x = 2$
Substitute $x = 2$ into (1):
$10(2) - 3y = 24.5$
$20 - 3y = 24.5$
$3y = -4.5$
$y = -1.5$
$\therefore$ the solution is $\boldsymbol{x = 2}$ and $\boldsymbol{y = -1.5}$.

(d) $6x + 5y = 10.5$ — (1)
$5x - 3y = -2$ — (2)
$3 \times (1)$: $18x + 15y = 31.5$ — (3)
$5 \times (2)$: $25x - 15y = -10$ — (4)
(4) + (3):
$(25x - 15y) + (18x + 15y) = -10 + 31.5$
$25x - 15y + 18x + 15y = 21.5$
$43x = 21.5$
$x = 0.5$
Substitute $x = 0.5$ into (1):
$6(0.5) + 5y = 10.5$
$3 + 5y = 10.5$
$5y = 7.5$
$y = 1.5$
$\therefore$ the solution is $\boldsymbol{x = 0.5}$ and $\boldsymbol{y = 1.5}$.

6. **(a)** $4x - y - 7 = 0$ — (1)
$4x + 3y - 11 = 0$ — (2)
(2) – (1):
$(4x + 3y - 11) - (4x - y - 7) = 0 - 0$
$4x + 3y - 11 - 4x + y + 7 = 0$
$4y = 4$
$y = 1$
Substitute $y = 1$ into (1):
$4x - 1 - 7 = 0$
$4x = 8$
$x = 2$
$\therefore$ the solution is $\boldsymbol{x = 2}$ and $\boldsymbol{y = 1}$.

(b) $7x + 2y - 33 = 0$ — (1)
$3y - 7x - 17 = 0$ — (2)
(1) + (2):
$(7x + 2y - 33) + (3y - 7x - 17) = 0 + 0$
$7x + 2y - 33 + 3y - 7x - 17 = 0$
$5y = 50$
$y = 10$
Substitute $y = 10$ into (1):
$7x + 2(10) - 33 = 0$
$7x + 20 - 33 = 0$
$7x = 13$
$x = \frac{13}{7}$
∴ the solution is $\boldsymbol{x = \frac{13}{7}}$ and $\boldsymbol{y = 10}$.

(c) $5x - 3y - 2 = 0$ — (1)
$x + 5y - 6 = 0$ — (2)
$5 \times (2)$: $5x + 25y - 30 = 0$ — (3)
(3) – (1):
$(5x + 25y - 30) - (5x - 3y - 2) = 0 - 0$
$5x + 25y - 30 - 5x + 3y + 2 = 0$
$28y = 28$
$y = 1$
Substitute $y = 1$ into (2):
$x + 5(1) - 6 = 0$
$x + 5 - 6 = 0$
$x = 1$
∴ the solution is $\boldsymbol{x = 1}$ and $\boldsymbol{y = 1}$.

(d) $5x - 3y - 13 = 0$ — (1)
$7x - 6y - 20 = 0$ — (2)
$2 \times (1)$: $10x - 6y - 26 = 0$ — (3)
(3) – (2):
$(10x - 6y - 26) - (7x - 6y - 20) = 0 - 0$
$10x - 6y - 26 - 7x + 6y + 20 = 0$
$3x = 6$
$x = 2$
Substitute $x = 2$ into (1):
$5(2) - 3y - 13 = 0$
$10 - 3y - 13 = 0$
$3y = -3$
$y = -1$
∴ the solution is $\boldsymbol{x = 2}$ and $\boldsymbol{y = -1}$.

(e) $7x + 3y - 8 = 0$ — (1)
$3x - 4y - 14 = 0$ — (2)
$4 \times (1)$: $28x + 12y - 32 = 0$ — (3)
$3 \times (2)$: $9x - 12y - 42 = 0$ — (4)
(3) + (4):
$(28x + 12y - 32) + (9x - 12y - 42) = 0 + 0$
$28x + 12y - 32 + 9x - 12y - 42 = 0$
$37x = 74$
$x = 2$
Substitute $x = 2$ into (1):
$7(2) + 3y - 8 = 0$
$14 + 3y - 8 = 0$
$3y = -6$
$y = -2$
∴ the solution is $\boldsymbol{x = 2}$ and $\boldsymbol{y = -2}$.

(f) $3x + 5y + 8 = 0$ — (1)
$4x + 13y - 2 = 0$ — (2)
$4 \times (1)$: $12x + 20y + 32 = 0$ — (3)
$3 \times (2)$: $12x + 39y - 6 = 0$ — (4)
(3) – (4):
$(12x + 20y + 32) - (12x + 39y - 6) = 0 - 0$
$12x + 20y + 32 - 12x - 39y + 6 = 0$
$19y = 38$
$y = 2$
Substitute $y = 2$ into (1):
$3x + 5(2) + 8 = 0$
$3x + 10 + 8 = 0$
$3x = -18$
$x = -6$
∴ the solution is $\boldsymbol{x = -6}$ and $\boldsymbol{y = 2}$.

7. (a) $\frac{x+1}{y+2} = \frac{3}{4}$ — (1)
$\frac{x-2}{y-1} = \frac{3}{5}$ — (2)
From (1),
$4(x + 1) = 3(y + 2)$
$4x + 4 = 3y + 6$
$4x - 3y = 2$ — (3)
From (2),
$5(x - 2) = 3(y - 1)$
$5x - 10 = 3y - 3$
$5x - 3y = 7$ — (4)
(4) – (3):
$(5x - 3y) - (4x - 3y) = 7 - 2$
$5x - 3y - 4x + 3y = 5$
$x = 5$
Substitute $x = 5$ into (3):
$4(5) - 3y = 2$
$20 - 3y = 2$
$3y = 18$
$y = 6$
∴ the solution is $\boldsymbol{x = 5}$ and $\boldsymbol{y = 6}$.

(b) $\frac{x}{3} - \frac{y}{2} = \frac{5}{6}$ — (1)
$3x - \frac{2}{5}y = 3\frac{2}{5}$ — (2)
$9 \times (1)$: $3x - \frac{9y}{2} = 7\frac{1}{2}$ — (3)
(2) – (3):
$\left(3x - \frac{2}{5}y\right) - \left(3x - \frac{9y}{2}\right) = 3\frac{2}{5} - 7\frac{1}{2}$
$3x - \frac{2}{5}y - 3x + \frac{9y}{2} = -4\frac{1}{10}$
$4\frac{1}{10}y = -4\frac{1}{10}$
$y = -1$
Substitute $y = -1$ into (2):
$3x - \frac{2}{5}(-1) = 3\frac{2}{5}$
$3x + \frac{2}{5} = 3\frac{2}{5}$
$3x = 3$
$x = 1$
∴ the solution is $\boldsymbol{x = 1}$ and $\boldsymbol{y = -1}$.

(c) $\frac{x}{4} - \frac{3}{8}y = 3$ — (1)

$\frac{5}{3}x - \frac{y}{2} = 12$ — (2)

$8 \times (1)$: $2x - 3y = 24$ — (3)

$6 \times (2)$: $10x - 3y = 72$ — (4)

(4) – (3):

$(10x - 3y) - (2x - 3y) = 72 - 24$

$10x - 3y - 2x + 3y = 48$

$8x = 48$

$x = 6$

Substitute $x = 6$ into (3):

$2(6) - 3y = 24$

$12 - 3y = 24$

$3y = -12$

$y = -4$

$\therefore$ the solution is $\boldsymbol{x = 6}$ and $\boldsymbol{y = -4}$.

(d) $\frac{x-3}{5} = \frac{y-7}{2}$ — (1)

$11x = 13y$ — (2)

$26 \times (1)$: $\frac{26}{5}(x - 3) = 13(y - 7)$

$\frac{26}{5}x - \frac{78}{5} = 13y - 91$ — (3)

(2) – (3):

$11x - \left(\frac{26}{5}x - \frac{78}{5}\right) = 13y - (13y - 91)$

$11x - \frac{26}{5}x + \frac{78}{5} = 13y - 13y + 91$

$5\frac{4}{5}x = 75\frac{2}{5}$

$x = 13$

Substitute $x = 13$ into (2):

$11(13) = 13y$

$y = 11$

$\therefore$ the solution is $\boldsymbol{x = 13}$ and $\boldsymbol{y = 11}$.

8. **(a)** $2x + 5y = 12$ — (1)

$4x + 3y = -4$ — (2)

From (1), $2x = 12 - 5y$

$x = \frac{12-5y}{2}$ — (3)

Substitute (3) into (2):

$4\left(\frac{12-5y}{2}\right) + 3y = -4$

$24 - 10y + 3y = -4$

$7y = 28$

$y = 4$

Substitute $y = 4$ into (3):

$x = \frac{12-5(4)}{2}$

$= -4$

$\therefore$ the solution is $\boldsymbol{x = -4}$ and $\boldsymbol{y = 4}$.

(b) $4x - 3y = 25$ — (1)

$6x + 5y = 9$ — (2)

From (1), $4x = 3y + 25$

$x = \frac{3y+25}{4}$ — (3)

Substitute (3) into (2):

$6\left(\frac{3y+25}{4}\right) + 5y = 9$

$\frac{9y}{2} + \frac{75}{2} + 5y = 9$

$9\frac{1}{2}y = -28\frac{1}{2}$

$y = -3$

Substitute $y = -3$ into (3):

$x = \frac{3(-3)+25}{4}$

$= 4$

$\therefore$ the solution is $\boldsymbol{x = 4}$ and $\boldsymbol{y = -3}$.

(c) $3x + 7y = 2$ — (1)

$6x - 5y = 4$ — (2)

From (1), $3x = 2 - 7y$

$x = \frac{2-7y}{3}$ — (3)

Substitute (3) into (2):

$6\left(\frac{2-7y}{3}\right) - 5y = 4$

$4 - 14y - 5y = 4$

$19y = 0$

$y = 0$

Substitute $y = 0$ into (3):

$x = \frac{2-7(0)}{3}$

$= \frac{2}{3}$

$\therefore$ the solution is $\boldsymbol{x = \frac{2}{3}}$ and $\boldsymbol{y = 0}$.

(d) $9x + 2y = 5$ — (1)

$7x - 3y = 13$ — (2)

From (1), $9x = 5 - 2y$

$x = \frac{5-2y}{9}$ — (3)

Substitute (3) into (2):

$7\left(\frac{5-2y}{9}\right) - 3y = 13$

$\frac{35}{9} - \frac{14}{9}y - 3y = 13$

$4\frac{5}{9}y = -9\frac{1}{9}$

$y = -2$

Substitute $y = -2$ into (3):

$x = \frac{5-2(-2)}{9}$

$= 1$

$\therefore$ the solution is $\boldsymbol{x = 1}$ and $\boldsymbol{y = -2}$.

(e) $2y - 5x = 25$ — (1)

$4x + 3y = 3$ — (2)

From (1), $2y = 5x + 25$

$y = \frac{5x+25}{2}$ — (3)

Substitute (3) into (2):

$4x + 3\left(\frac{5x+25}{2}\right) = 3$

$4x + \frac{15}{2}x + \frac{75}{2} = 3$

$11\frac{1}{2}x = -34\frac{1}{2}$

$x = -3$

Substitute $x = -3$ into (3):

$y = \frac{5(-3)+25}{2}$

$= 5$

$\therefore$ the solution is $\boldsymbol{x = -3}$ and $\boldsymbol{y = 5}$.

(f) $3x - 5y = 7$ — (1)

$4x - 3y = 3$ — (2)

From (1), $3x = 5y + 7$

$x = \frac{5y+7}{3}$ — (3)

Substitute (3) into (2):

$4\left(\frac{5y+7}{3}\right) - 3y = 3$

$\frac{20}{3}y + \frac{28}{3} - 3y = 3$

$3\frac{2}{3}y = -6\frac{1}{3}$

$y = -\frac{19}{11}$

Substitute $y = -\frac{19}{11}$ into (3):

$x = \frac{5\left(-\frac{19}{11}\right)+7}{3}$

$= -\frac{6}{11}$

$\therefore$ the solution is $\boldsymbol{x = -\frac{6}{11}}$ and $\boldsymbol{y = -\frac{19}{11}}$.

9. (a) $\frac{x}{5} + y + 2 = 0$ — (1)

$\frac{x}{3} - y - 10 = 0$ — (2)

From (1), $y = -\frac{x}{5} - 2$ — (3)

Substitute (3) into (2):

$\frac{x}{3} - \left(-\frac{x}{5} - 2\right) - 10 = 0$

$\frac{x}{3} + \frac{x}{5} + 2 - 10 = 0$

$\frac{8}{15}x = 8$

$x = 15$

Substitute $x = 15$ into (3):

$y = -\frac{15}{5} - 2$

$= -5$

$\therefore$ the solution is $\boldsymbol{x = 15}$ and $\boldsymbol{y = -5}$.

(b) $\frac{x+y}{3} = 3$ — (1)

$\frac{3x+y}{5} = 1$ — (2)

From (1), $x + y = 9$

$x = 9 - y$ — (3)

Substitute (3) into (2):

$\frac{3(9-y)+y}{5} = 1$

$27 - 3y + y = 5$

$2y = 22$

$y = 11$

Substitute $y = 11$ into (3):

$x = 9 - 11$

$= -2$

$\therefore$ the solution is $\boldsymbol{x = -2}$ and $\boldsymbol{y = 11}$.

(c) $3x - y = 23$ — (1)

$\frac{x}{3} + \frac{y}{4} = 4$ — (2)

From (1), $y = 3x - 23$ — (3)

Substitute (3) into (2):

$\frac{x}{3} + \frac{3x-23}{4} = 4$

$4x + 9x - 69 = 48$

$13x = 117$

$x = 9$

Substitute $x = 9$ into (3):

$y = 3(9) - 23$

$= 4$

$\therefore$ the solution is $\boldsymbol{x = 9}$ and $\boldsymbol{y = 4}$.

(d) $\frac{x}{3} + \frac{y}{2} = 4$ — (1)

$\frac{2}{3}x - \frac{y}{6} = 1$ — (2)

From (1), $2x + 3y = 24$

$2x = 24 - 3y$

$x = \frac{24-3y}{2}$ — (3)

Substitute (3) into (2):

$\frac{2}{3}\left(\frac{24-3y}{2}\right) - \frac{y}{6} = 1$

$48 - 6y - y = 6$

$7y = 42$

$y = 6$

Substitute $y = 6$ into (3):

$x = \frac{24-3(6)}{2}$

$= 3$

$\therefore$ the solution is $\boldsymbol{x = 3}$ and $\boldsymbol{y = 6}$.

10. When $x = 3$, $y = -1$,

$3p(3) + q(-1) = 11$

$9p - q = 11$ — (1)

$-q(3) + 5(-1) = p$

$p = -3q - 5$ — (2)

Substitute (2) into (1):

$9(-3q - 5) - q = 11$

$-27q - 45 - q = 11$

$28q = -56$

$q = -2$

Substitute $q = -2$ into (2):

$p = -3(-2) - 5$

$= 1$

$\therefore$ the values of p and of q are **1** and **–2** respectively.

11. When $x = -11$, $y = 5$,

$p(-11) + 5(5) = q$

$-11p + 25 = q$ — (1)

$q(-11) + 7(5) = p$

$-11q + 35 = p$ — (2)

Substitute (2) into (1):

$-11(-11q + 35) + 25 = q$

$121q - 385 + 25 = q$

$120q = 360$

$q = 3$

Substitute $q = 3$ into (2):

$-11(3) + 35 = p$

$p = 2$

$\therefore$ the values of p and of q are **2** and **3** respectively.

12. $8s - 3h = -9$ — (1)

$-29s + 10h = 16$ — (2)

$10 \times (1)$: $80s - 30h = -90$ — (3)

$3 \times (2)$: $-87s + 30h = 48$ — (4)

(3) + (4):

$(80s - 30h) + (-87s + 30h) = -90 + 48$

$7s = 42$

$s = 6$

Substitute $s = 6$ into (1):

$8(6) - 3h = -9$

$48 - 3h = -9$

$3h = 57$

$h = 19$

$\therefore$ the cat meets the mouse **6 s** after it starts moving at **19 m** above the ground.

13. **(a)** $\dfrac{2}{x+y} = \dfrac{1}{2x+y}$ — (1)

$3x + 4y = 9$ — (2)

From (1),

$2(2x + y) = x + y$

$4x + 2y = x + y$

$y = -3x$ — (3)

Substitute (3) into (2):

$3x + 4(-3x) = 9$

$3x - 12x = 9$

$-9x = 9$

$x = -1$

Substitute $x = -1$ into (3):

$y = -3(-1)$

$= 3$

$\therefore$ the solution is $\boldsymbol{x = -1}$ and $\boldsymbol{y = 3}$.

(b) $\dfrac{1}{5}(x - 2) = \dfrac{1}{4}(1 - y)$ — (1)

$\dfrac{1}{7}\left(x + 2\dfrac{2}{3}\right) = \dfrac{1}{3}(3 - y)$ — (2)

$20 \times (1)$:

$4(x - 2) = 5(1 - y)$

$4x - 8 = 5 - 5y$

$4x + 5y = 13$ — (3)

$21 \times (2)$:

$3\left(x + 2\dfrac{2}{3}\right) = 7(3 - y)$

$3x + 8 = 21 - 7y$

$3x = 13 - 7y$

$x = \dfrac{13 - 7y}{3}$ — (4)

Substitute (4) into (3):

$4\left(\dfrac{13 - 7y}{3}\right) + 5y = 13$

$\dfrac{52}{3} - \dfrac{28}{3}y + 5y = 13$

$4\dfrac{1}{3}y = 4\dfrac{1}{3}$

$y = 1$

Substitute $y = 1$ into (4):

$x = \dfrac{13 - 7(1)}{3}$

$= 2$

$\therefore$ the solution is $\boldsymbol{x = 2}$ and $\boldsymbol{y = 1}$.

(c) $\dfrac{5x+y}{9} = 2 - \dfrac{x+y}{5}$ — (1)

$\dfrac{7x-3}{2} = 1 + \dfrac{y-x}{3}$ — (2)

$45 \times (1)$:

$5(5x + y) = 90 - 9(x + y)$

$25x + 5y = 90 - 9x - 9y$

$34x + 14y = 90$

$17x + 7y = 45$ — (3)

$6 \times (2)$:

$3(7x - 3) = 6 + 2(y - x)$

$21x - 9 = 6 + 2y - 2x$

$2y = 23x - 15$

$y = \dfrac{23x - 15}{2}$ — (4)

Substitute (4) into (3):

$17x + 7\left(\dfrac{23x - 15}{2}\right) = 45$

$17x + \dfrac{161}{2}x - \dfrac{105}{2} = 45$

$97\dfrac{1}{2}x = 97\dfrac{1}{2}$

$x = 1$

Substitute $x = 1$ into (4):

$y = \dfrac{23(1) - 15}{2}$

$= 4$

$\therefore$ the solution is $\boldsymbol{x = 1}$ and $\boldsymbol{y = 4}$.

(d) $\dfrac{x+y}{3} = \dfrac{x-y}{5}$ — (1)

$\dfrac{x-y}{5} = 2x - 3y + 5$ — (2)

From (1), $5(x + y) = 3(x - y)$

$5x + 5y = 3x - 3y$

$2x = -8y$

$x = -4y$ — (3)

Substitute (3) into (2):

$\dfrac{-4y - y}{5} = 2(-4y) - 3y + 5$

$-y = -8y - 3y + 5$

$10y = 5$

$y = \dfrac{1}{2}$

Substitute $y = \dfrac{1}{2}$ into (3):

$x = -4\left(\dfrac{1}{2}\right)$

$= -2$

$\therefore$ the solution is $\boldsymbol{x = -2}$ and $\boldsymbol{y = \dfrac{1}{2}}$.

2.5 Applications of simultaneous equations in real-world contexts

Thinking Time (Page 68)

Let the smaller number be x. Then the greater number is $67 - x$.

$\therefore (67 - x) - x = 3$

$67 - 2x = 3$

$2x = 64$

$\therefore x = 32$

Greater number $= 67 - 32$

$= 35$

The two numbers are **32** and **35**.

Practise Now 9

1. Let the smaller number be x and the greater number be y.

   $x + y = 36$ — (1)

   $y - x = 9$ — (2)

   (1) + (2):

   $2y = 45$

   $y = 22.5$

   Substitute $y = 22.5$ into (1):

   $x + 22.5 = 36$

   $x = 13.5$

   $\therefore$ the two numbers are **13.5** and **22.5**.

2. Let the smaller angle be x and the greater angle be y.

   $\frac{1}{3}(x + y) = 60°$ — (1)

   $\frac{1}{4}(y - x) = 28°$ — (2)

   $3 \times (1)$: $x + y = 180°$ — (3)

   $4 \times (2)$: $y - x = 112°$ — (4)

   (3) + (4):

   $2y = 292°$

   $y = 146°$

   Substitute $y = 146°$ into (3):

   $x + 146° = 180°$

   $x = 34°$

   $\therefore$ the two angles are **34°** and **146°**.

3. $x + y + 2 = 2x + 1$ — (1)

   $2y = x + 2$ — (2)

   From (1),

   $y = x - 1$ — (3)

   Substitute (3) into (2):

   $2(x - 1) = x + 2$

   $2x - 2 = x + 2$

   $x = 4$

   Substitute $x = 4$ into (3):

   $y = 4 - 1$

   $= 3$

   Length of rectangle $= 2(4) + 1$

   $= 9$ cm

   Breadth of rectangle $= 2(3)$

   $= 6$ cm

   Perimeter of rectangle $= 2(9 + 6)$

   $= 30$ cm

   $\therefore$ the perimeter of the rectangle is **30 cm**.

Practise Now 10

1. Let the present age of Li Ting be x years and that of Li Ting's father be y years. Then in 5 years' time, Li Ting's father will be $(y + 5)$ years old and Li Ting will be $(x + 5)$ years old.

   4 years ago, Li Ting's father was $(y - 4)$ years old and Li Ting was $(x - 4)$ years old.

   $y + 5 = 3(x + 5)$ — (1)

   $y - 4 = 6(x - 4)$ — (2)

   From (1),

   $y + 5 = 3x + 15$

   $y = 3x + 10$ — (3)

   Substitute (3) into (2):

   $3x + 10 - 4 = 6(x - 4)$

   $= 6x - 24$

   $3x = 30$

   $x = 10$

   Substitute $x = 10$ into (3):

   $y = 3(10) + 10$

   $= 40$

   $\therefore$ Li Ting's present age is **10 years** and Li Ting's father's present age is **40 years**.

2. Let the amount an adult has to pay be \$$x$ and the amount a child has to pay be \$$y$.

   $11x + 5y = 280$ — (1)

   $14x + 9y = 388$ — (2)

   $9 \times (1)$: $99x + 45y = 2520$ — (3)

   $5 \times (2)$: $70x + 45y = 1940$ — (4)

   (3) – (4):

   $(99x + 45y) - (70x + 45y) = 2520 - 1940$

   $29x = 580$

   $x = 20$

   Substitute $x = 20$ into (1):

   $11(20) + 5y = 280$

   $220 + 5y = 280$

   $5y = 60$

   $y = 12$

   Total amount a family of 2 adults and 3 children has to pay

   $= \$(2x + 3y)$

   $= \$[2(20) + 3(12)]$

   $= \$76$

   $\therefore$ the family has to pay **\$76**.

Practise Now 11

Let the numerator of the fraction be x and its denominator be y, i.e. let the fraction be $\frac{x}{y}$.

$\frac{x+1}{y+1} = \frac{4}{5}$ — (1)

$\frac{x-5}{y-5} = \frac{1}{2}$ — (2)

From (1),

$5(x + 1) = 4(y + 1)$

$5x + 5 = 4y + 4$

$5x - 4y = -1$ — (3)

From (2),

$2(x - 5) = y - 5$

$2x - 10 = y - 5$

$y = 2x - 5$ — (4)

Substitute (4) into (3):
$5x - 4(2x - 5) = -1$
$5x - 8x + 20 = -1$
$-3x = -21$
$x = 7$
Substitute $x = 7$ into (4):
$y = 2(7) - 5$
$= 9$
$\therefore$ the fraction is $\mathbf{\frac{7}{9}}$.

Practise Now 12

Let the tens digit of the original number be x and its ones digit be y. Then the original number is $10x + y$, the number obtained when the digits of the original number are reversed is $10y + x$.
$x + y = 11$ — (1)
$10x + y - (10y + x) = 9$ — (2)
From (2),
$10x + y - 10y - x = 9$
$9x - 9y = 9$
$x - y = 1$ — (3)
(1) + (3):
$2x = 12$
$x = 6$
Substitute $x = 6$ into (1):
$6 + y = 11$
$y = 5$
$\therefore$ the original number is **65**.

Introductory Problem Revisited

Let the amount of money that Cheryl has be $\$c$.
Let the amount of money that Albert has be $\$d$.
$c - 40 = d + 40$
$c - d = 80$ — (1)
$c + 80 = 4(d - 80)$
$c + 80 = 4d - 320$
$c - 4d = -400$ — (2)
(1) – (2):
$3d = 480$
$d = 160$
Substitute $d = 160$ into (1):
$c - 160 = 80$
$c = 240$
$\therefore$ Cheryl has **\$240** and Albert has **\$160**.

Exercise 2D

1. Let the smaller number be x and the greater number be y.
$x + y = 138$ — (1)
$y - x = 88$ — (2)
(1) + (2):
$(x + y) + (y - x) = 138 + 88$
$x + y + y - x = 226$
$2y = 226$
$y = 113$
Substitute $y = 113$ into (1):
$x + 113 = 138$
$x = 25$
$\therefore$ the two numbers are **25** and **113**.

2. Let the smaller number be x and the greater number be y.
$y - x = 10$ — (1)
$x + y = 4x$ — (2)
From (2), $y = 3x$ — (3)
Substitute (3) into (1):
$3x - x = 10$
$2x = 10$
$x = 5$
Substitute $x = 5$ into (3):
$y = 3(5)$
$= 15$
$\therefore$ the two numbers are **5** and **15**.

3. Let the smaller number be x and the greater number be y.
$x + y = 48$ — (1)
$x = \frac{1}{5}y$ — (2)
Substitute (2) into (1):
$\frac{1}{5}y + y = 48$
$\frac{6}{5}y = 48$
$y = 40$
Substitute $y = 40$ into (2):
$x = \frac{1}{5}(40)$
$= 8$
$\therefore$ the two numbers are **8** and **40**.

4. Let the cost of a belt be $\$x$ and the cost of a wallet be $\$y$.
$x + y = 42$ — (1)
$7x + 4y = 213$ — (2)
From (1), $y = 42 - x$ — (3)
Substitute (3) into (2):
$7x + 4(42 - x) = 213$
$7x + 168 - 4x = 213$
$3x = 45$
$x = 15$
Substitute $x = 15$ into (3):
$y = 42 - 15$
$= 27$
$\therefore$ the cost of a belt is **\$15** and the cost of a wallet is **\$27**.

5. Let the cost of 1 kg of potatoes be $\$x$ and the cost of 1 kg of carrots be $\$y$.
$8x + 5y = 28$ — (1)
$2x + 3y = 11.2$ — (2)
$4 \times$ (2): $8x + 12y = 44.8$ — (3)
(3) – (1):
$(8x + 12y) - (8x + 5y) = 44.8 - 28$
$8x + 12y - 8x - 5y = 16.8$
$7y = 16.8$
$y = 2.4$
Substitute $y = 2.4$ into (2):
$2x + 3(2.4) = 11.2$
$2x + 7.2 = 11.2$
$2x = 4$
$x = 2$
$\therefore$ 1 kg of potatoes cost **\$2** and 1 kg of carrots cost **\$2.40**.

6. Let the first number be x and the second number be y.

$x + 7 = 2y$ — (1)

$y + 20 = 4x$ — (2)

From (1), $x = 2y - 7$ — (3)

Substitute (3) into (2):

$$\begin{aligned} y + 20 &= 4(2y - 7) \\ &= 8y - 28 \\ 7y &= 48 \\ y &= \frac{48}{7} \end{aligned}$$

Substitute $y = \frac{48}{7}$ into (3):

$$\begin{aligned} x &= 2\left(\frac{48}{7}\right) - 7 \\ &= \frac{47}{7} \end{aligned}$$

$\therefore$ the two numbers are $\mathbf{\frac{47}{7}}$ and $\mathbf{\frac{48}{7}}$.

7. Let the smaller angle be x and the greater angle be y.

$\frac{1}{5}(x + y) = 24°$ — (1)

$\frac{1}{2}(y - x) = 14°$ — (2)

$5 \times (1)$: $x + y = 120°$ — (3)

$2 \times (2)$: $y - x = 28°$ — (4)

(3) + (4):

$$\begin{aligned} (x + y) + (y - x) &= 120° + 28° \\ x + y + y - x &= 148° \\ 2y &= 148° \\ y &= 74° \end{aligned}$$

Substitute $y = 74°$ into (3):

$$\begin{aligned} x + 74° &= 120° \\ x &= 46° \end{aligned}$$

$\therefore$ the two angles are **46°** and **74°**.

8. The sides of an equilateral triangle are equal.

$$\begin{aligned} x + y - 9 &= y + 5 \\ x &= 14 \end{aligned}$$

$$\begin{aligned} \text{Length of each side} &= 2(14) - 7 \\ &= 21 \text{ cm} \end{aligned}$$

$\therefore$ the length of each side of the triangle is **21 cm**.

9. $3x - y = 2x + y$ — (1)

$3x - y + 2x + y + 2(2x - 3) = 120$ — (2)

From (2),

$$\begin{aligned} 3x - y + 2x + y + 4x - 6 &= 120 \\ 9x &= 126 \\ x &= 14 \end{aligned}$$

Substitute $x = 14$ into (1):

$$\begin{aligned} 3(14) - y &= 2(14) + y \\ 42 - y &= 28 + y \\ 2y &= 14 \\ y &= 7 \end{aligned}$$

$$\begin{aligned} \text{Area of rectangle} &= [3(14) - 7] \times [2(14) - 3] \\ &= 35 \times 25 \\ &= 875 \text{ cm}^2 \end{aligned}$$

$\therefore$ the area of the rectangle is $\mathbf{875\ cm^2}$.

10. The sides of a rhombus are equal.

$2x + y + 1 = \frac{3x - y - 2}{2}$ — (1)

$2x + y + 1 = x - y$ — (2)

From (2), $x = -2y - 1$ — (3)

Substitute (3) into (1):

$$\begin{aligned} 2(-2y - 1) + y + 1 &= \frac{3(-2y - 1) - y - 2}{2} \\ -4y - 2 + y + 1 &= \frac{-7y - 5}{2} \\ -6y - 2 &= -7y - 5 \\ y &= -3 \end{aligned}$$

Substitute $y = -3$ into (3):

$$\begin{aligned} x &= -2(-3) - 1 \\ &= 5 \end{aligned}$$

$$\begin{aligned} \text{Perimeter of the figure} &= 4[5 - (-3)] \\ &= 32 \text{ cm} \end{aligned}$$

$\therefore$ the perimeter of the figure is **32 cm**.

11. Let the age of the polar bear in 2013 be x years old and the age of the panda in 2013 be y years old.

$x + y = 11$ — (1)

$x + 9 = 3y$ — (2)

(1) – (2):

$$\begin{aligned} (x + y) - (x + 9) &= 11 - 3y \\ x + y - x - 9 &= 11 - 3y \\ y - 9 &= 11 - 3y \\ 4y &= 20 \\ y &= 5 \end{aligned}$$

Substitute $y = 5$ into (1):

$$\begin{aligned} x + 5 &= 11 \\ x &= 6 \end{aligned}$$

In 2014,

$$\begin{aligned} \text{Age of Kai Kai} &= 6 + 1 \\ &= 7 \end{aligned}$$

$$\begin{aligned} \text{Age of Jia Jia} &= 5 + 1 \\ &= 6 \end{aligned}$$

$\therefore$ in 2014, the ages of the polar bear and the panda are **7 years** and **6 years** respectively.

12. Let the amount an adult has to pay be $\$x$ and the amount a senior citizen has to pay be $\$y$.

$6x + 4y = 228$ — (1)

$13x + 7y = 459$ — (2)

From (1), $3x + 2y = 114$ — (3)

$2 \times (2)$: $26x + 14y = 918$ — (4)

$7 \times (3)$: $21x + 14y = 798$ — (5)

(4) – (5):

$$\begin{aligned} (26x + 14y) - (21x + 14y) &= 918 - 798 \\ 26x + 14y - 21x - 14y &= 120 \\ 5x &= 120 \\ x &= 24 \end{aligned}$$

Substitute $x = 24$ into (3):

$$\begin{aligned} 3(24) + 2y &= 114 \\ 72 + 2y &= 114 \\ 2y &= 42 \\ y &= 21 \end{aligned}$$

Total amount 2 adults and a senior citizen have to pay

$= 2(\$24) + \$21$

$= \$69$

$\therefore$ the total amount is **\$69**.

13. Let the number of Gifts A to buy be x and the number of Gifts B to buy be y.

$10x + 8y = 230$ — (1)

$x + y = 2 + 2 + 13 + 10$
$= 27$ — (2)

From (2), $y = 27 - x$ — (3)

Substitute (3) into (1):

$10x + 8(27 - x) = 230$
$10x + 216 - 8x = 230$
$2x = 14$
$x = 7$

Substitute $x = 7$ into (3):

$y = 27 - 7$
$= 20$

∴ Nadia should buy **7** Gifts A and **20** Gifts B.

14. Let the number of chickens be x and the number of goats be y.

$x + y = 50$ — (1)
$2x + 4y = 140$ — (2)

From (1), $y = 50 - x$ — (3)

Substitute (3) into (2):

$2x + 4(50 - x) = 140$
$2x + 200 - 4x = 140$
$2x = 60$
$x = 30$

Substitute $x = 30$ into (3):

$y = 50 - 30$
$= 20$

Difference between number of chickens and number of goats
$= 30 - 20$
$= 10$

∴ there are **10** more chickens than goats.

15. Let the amount Bernard has be $\$x$ and the amount Cheryl has be $\$y$.

$x + y = 80$ — (1)

$\frac{1}{4}x = \frac{1}{6}y$ — (2)

From (1), $y = 80 - x$ — (3)

Substitute (3) into (2):

$\frac{1}{4}x = \frac{1}{6}(80 - x)$
$3x = 160 - 2x$
$5x = 160$
$x = 32$

Substitute $x = 32$ into (1):

$32 + y = 80$
$y = 48$

∴ Bernard receives **\$32** and Cheryl receives **\$48**.

16. Let the amount deposited in Bank A be $\$x$ and the amount deposited in Bank B be $\$y$.

$x + y = 25\,000$ — (1)

$\frac{0.6}{100}x = \frac{0.65}{100}y$ — (2)

From (2), $y = \frac{12}{13}x$ — (3)

Substitute (3) into (1):

$x + \frac{12}{13}x = 25\,000$

$\frac{25}{13}x = 25\,000$

$x = 13\,000$

Substitute $x = 13\,000$ into (3):

$y = \frac{12}{13}(13\,000)$
$= 12\,000$

∴ Waseem deposited **\$13 000** in Bank A and **\$12 000** in Bank B.

17. Let the numerator of the fraction be x and its denominator be y, i.e. let the fraction be $\frac{x}{y}$.

$\frac{x-1}{y-1} = \frac{1}{2}$ — (1)

$\frac{x+1}{y+1} = \frac{2}{3}$ — (2)

From (1),

$2(x - 1) = y - 1$
$2x - 2 = y - 1$
$y = 2x - 1$ — (3)

Substitute (3) into (2):

$\frac{x+1}{2x-1+1} = \frac{2}{3}$
$3(x + 1) = 4x$
$3x + 3 = 4x$
$x = 3$

Substitute $x = 3$ into (3):

$y = 2(3) - 1$
$= 5$

∴ the fraction is $\mathbf{\frac{3}{5}}$.

18. Let the smaller number be x and the greater number be y.

$\frac{y-2}{x} = 2$ — (1)

$\frac{5x-2}{y} = 2$ — (2)

From (1), $y - 2 = 2x$

$x = \frac{y-2}{2}$ — (3)

Substitute (3) into (2):

$$\frac{5\left(\frac{y-2}{2}\right)-2}{y} = 2$$

$5\left(\frac{y-2}{2}\right) - 2 = 2y$

$\frac{5}{2}y - 5 - 2 = 2y$

$\frac{1}{2}y = 7$

$y = 14$

Substitute $y = 14$ into (3):

$x = \frac{14-2}{2}$
$= 6$

∴ the two numbers are **6** and **14**.

19. Let the cost of 1 pear be $\$x$ and the cost of 1 mango be $\$y$.

$8x + 5y = 10 + 1.1$ — (1)
$5x + 4y = 10 - 1.75$ — (2)
$4 \times (1)$: $32x + 20y = 44.4$ — (3)
$5 \times (2)$: $25x + 20y = 41.25$ — (4)
(3) – (4):
$(32x + 20y) - (25x + 20y) = 44.4 - 41.25$
$32x + 20y - 25x - 20y = 3.15$
$7x = 3.15$
$x = 0.45$

Substitute $x = 0.45$ into (2):

$$5(0.45) + 4y = 8.25$$
$$2.25 + 4y = 8.25$$
$$4y = 6$$
$$y = 1.5$$

$\therefore$ 1 pear costs **\$0.45** and 1 mango costs **\$1.50**.

20. **(i)** Let the number of shares of Company A Joyce's mother has be x and the share price of company B on Day 7 be \$$y$.

$$4.6x - 2000y = 7400 \quad \text{— (1)}$$
$$4.8x - 5000(y - 0.5) = -5800 \quad \text{— (2)}$$

From (1),

$$2000y = 4.6x - 7400$$
$$y = \frac{4.6x - 7400}{2000} \quad \text{— (3)}$$

Substitute (3) into (2):

$$4.8x - 5000\left(\frac{4.6x - 7400}{2000} - 0.5\right) = -5800$$
$$4.8x - 11.5x + 18\,500 + 2500 = -5800$$
$$6.7x = 26\,800$$
$$x = 4000$$

$\therefore$ Joyce's mother has **4000** shares of company A.

(ii) From **(i)**,

Substitute $x = 4000$ into (3):

$$y = \frac{4.6(4000) - 7400}{2000}$$
$$= 5.5$$

Share price of company B on Day 12 $= 5.5 - 0.5$
$= \$5$

$\therefore$ the share price of company B on Day 12 is **\$5**.

21. Let the tens digit of the original number be x and its ones digit be y.

Then the original number is $10x + y$, the number obtained when the digits of the original number are reversed is $10y + x$.

$$x + y = \frac{1}{8}(10x + y) \quad \text{— (1)}$$
$$(10x + y) - (10y + x) = 45 \quad \text{— (2)}$$

From (1),

$$8(x + y) = 10x + y$$
$$8x + 8y = 10x + y$$
$$2x = 7y$$
$$x = \frac{7}{2}y \quad \text{— (3)}$$

From (2),

$$10x + y - 10y - x = 45$$
$$9x - 9y = 45$$
$$x - y = 5 \quad \text{— (4)}$$

Substitute (3) into (4):

$$\frac{7}{2}y - y = 5$$
$$\frac{5}{2}y = 5$$
$$y = 2$$

Substitute $y = 2$ into (3):

$$x = \frac{7}{2}(2)$$
$$= 7$$

$\therefore$ the original number is **72**.

Chapter 3 Linear Inequalities

TEACHING NOTES

Suggested Approach

In secondary one, students have learnt to solve linear equations. They have also learnt to plot the graphs of linear functions in Chapter 1 of this book. In this chapter, they will learn to solve linear inequalities. Teachers can link students' prior knowledge of solving linear equations. By replacing the equality sign with inequality signs, teachers can emphasise that the variable can take more than one value.

The chapter can be classified into three parts. In the first part (see Sections 3.1 to 3.3), students will begin solving simple inequalities. Teachers can stress upon the students that "solving an inequality" involves finding all the solutions that satisfy the inequality, which is akin to solving a simple linear equation. Once students are confident in solving simple inequalities, teachers can progress to the second part (Sections 3.4 and 3.5) on solving simultaneous linear inequalities. Sections 3.1 to 3.5 involve linear inequalities in one variable. The last part (see Section 3.6) involves solving linear inequalities in two variables. Teachers may start the section by building on students' knowledge on functions and graph plotting.

Section 3.1 Simple inequalities

In the investigation on page 77 of the textbook, students are required to work with numerical examples before generalising the conclusions for some properties of inequalities. It is recommended that teachers get students to formulate inequalities based on real-world contexts (see Class Discussion: Real-life examples of inequalities).

Section 3.2 Solving simple linear inequalities

The section begins with the introduction to the use of a number line to represent an inequality. The use of number lines will help students to visualise and understand the meanings of $<$, $>$, $\leqslant$ and $\geqslant$. Teachers should guide students when solving linear inequalities that involve reversing the inequality signs when multiplying or dividing the inequalities by a negative number as this may be confusing to them. Teachers can use actual numbers to explain how the signs will change when multiplying and dividing by a negative number.

Section 3.3 Solving problems involving linear inequalities

When problem solving involving inequalities, students must work on their mathematical process of interpretation and thinking skills. Teachers can guide students to understand terms such as 'at most', 'at least', 'not more than' and 'not lesser than' and how to form an inequality and solve it to find the answer to the problem.

Section 3.4 Simultaneous linear inequalities

When solving simultaneous linear inequalities, teachers should guide students on how to solve two linear inequalities separately and to consider only the common solutions of the inequalities after representing both inequalities on a number line. The use of number lines in this section is useful to represent the solutions(s) of a pair of simultaneous linear inequalities. Teachers should highlight to students that there may not always be a solution to the simultaneous linear inequalities (see Worked Example 10 on page 89 of the textbook).

Section 3.5 Solving problems involving simultaneous linear inequalities

When solving word problems involving simultaneous linear inequalities, struggling learners may need guidance with formulating the inequalities based on the information given in the word problem. Teachers may use the five steps described in the Problem-solving Tip in Worked Example 11 when working through the worked examples and questions with their students.

Section 3.6 Linear inequalities in two variables

Teachers may begin this section by reminding the students that coordinate pairs which lie on the graph of a function satisfy the equation of the function. Teachers can then build on this knowledge by highlighting that coordinate pairs that satisfy a linear inequality with two variables lie in a region. Numerical examples can be used to illustrate this. Teachers should highlight the difference between the graphs of $ax + by = c$, $ax + by < c$ and $ax + by > c$.

Students should be given ample practice on drawing graphs of linear inequalities in two variables and writing linear inequalities in two variables from graphs. Teachers should bring to the students' attention that in a graph, the region containing solutions satisfying a system of linear inequalities is generally (but not always) not shaded.

Introductory Problem

The solutions to this problem can be found in Introductory Problem Revisited (after Practise Now 7).

3.1 Simple inequalities

Class Discussion (Real-life examples of inequalities)

1. $\boldsymbol{x \leqslant 40}$
2. $\boldsymbol{y \geqslant 16}$
   $\boldsymbol{z < 16}$

Investigation (Properties of inequalities)

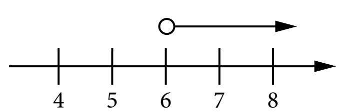
1. (a) (i) $<$ (ii) $<$
   (b) $<$; $<$
   (c) $>$; $>$
2. (a) (i) $<$ (ii) $<$
   (b) $<$; $<$
   (c) $>$; $>$
3. (a) (i) $>$ (ii) $>$
   (b) If $6 > -12$ and a is a real number, then $6 + a > -12 + a$ and $6 - a > -12 - a$.
   If $-12 < 6$ and a is a real number, then $-12 + a < -6 + a$ and $-12 - a < 6 - a$.
4. If $x < y$ and a is a real number, then $x + a < y + a$ and $x - a < y - a$.
   If $x > y$ and a is a real number, then $x + a > y + a$ and $x - a > y - a$.
   If $x \leqslant y$ and a is a real number, then $x + a \leqslant y + a$ and $x - a \leqslant y - a$.
   If $x \geqslant y$ and a is a real number, then $x + a \geqslant y + a$ and $x - a \geqslant y - a$.
5. (a) (i) $<$ (ii) $>$
   (b) $<$; $>$
   (c) $>$; $<$
6. (a) (i) $<$ (ii) $>$
   (b) $<$; $>$
   (c) $>$; $<$
7. (a) (i) $>$ (ii) $<$
   (b) If $6 > -12$ and a is a positive real number, then $6 \times a > -12 \times a$.
   If $6 > -12$ and a is a negative real number, then $6 \times a < -12 \times a$.
8. (a) Consider the inequality $6 < 12$.
   $$\frac{6}{2} < \frac{12}{2}$$
   Consider the inequality $-6 < 12$.
   $$\frac{-6}{2} < \frac{12}{2}$$
   Consider the inequality $6 > -12$.
   $$\frac{6}{2} > \frac{-12}{2}$$
   The inequality sign does not reverse when both sides of the inequality are divided by a positive real number.

   (b) Consider the inequality $6 < 12$.
   $$\frac{6}{-4} > \frac{12}{-4}$$
   Consider the inequality $-6 < 12$.
   $$\frac{-6}{-4} > \frac{12}{-4}$$
   Consider the inequality $6 > -12$.
   $$\frac{6}{-4} < \frac{-12}{-4}$$
   The inequality sign reverses when both sides of the inequality are divided by a negative real number.
9. If $x < y$ and a is a positive real number, then $x \times a < y \times a$ and $\frac{x}{a} < \frac{y}{a}$.
   If $x < y$ and a is a negative real number, then $x \times a > y \times a$ and $\frac{x}{a} > \frac{y}{a}$.
   If $x > y$ and a is a positive real number, then $x \times a > y \times a$ and $\frac{x}{a} > \frac{y}{a}$.
   If $x > y$ and a is a negative real number, then $x \times a < y \times a$ and $\frac{x}{a} < \frac{y}{a}$.
   If $x \leqslant y$ and a is a positive real number, then $x \times a \leqslant y \times a$ and $\frac{x}{a} \leqslant \frac{y}{a}$.
   If $x \leqslant y$ and a is a negative real number, then $x \times a \geqslant y \times a$ and $\frac{x}{a} \geqslant \frac{y}{a}$.
   If $x \geqslant y$ and a is a positive real number, then $x \times a \geqslant y \times a$ and $\frac{x}{a} \geqslant \frac{y}{a}$.
   If $x \geqslant y$ and a is a negative real number, then $x \times a \leqslant y \times a$ and $\frac{x}{a} \leqslant \frac{y}{a}$.

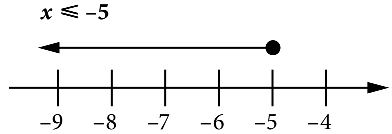
3.2 Solving simple linear inequalities

Practise Now 1

(a) $5x > 30$
$$\frac{5x}{5} > \frac{30}{5}$$
$$\boldsymbol{x > 6}$$



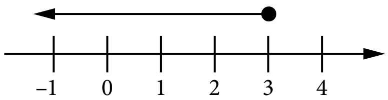
(b) $-4x \geqslant 20$
$$\frac{-4x}{-4} \leqslant \frac{20}{-4}$$
$$\boldsymbol{x \leqslant -5}$$



(c) $15x \leqslant 45$

$\frac{15x}{15} \leqslant \frac{45}{15}$

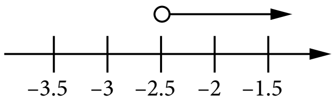
$\boldsymbol{x \leqslant 3}$



(d) $-6x < 15$

$\frac{-6x}{-6} > \frac{15}{-6}$

$\boldsymbol{x > -2.5}$

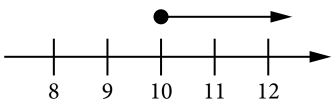


Practise Now 2

(a) $x - 3 \geqslant 7$

$x - 3 + 3 \geqslant 7 + 3$

$\boldsymbol{x \geqslant 10}$



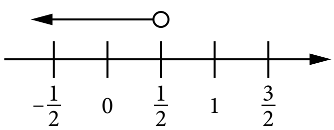
(b) $-2y + 4 > 3$

$-2y + 4 - 4 > 3 - 4$

$-2y > -1$

$\frac{-2y}{-2} < \frac{-1}{-2}$

$\boldsymbol{y < \frac{1}{2}}$



Thinking Time (Page 81)

1. For $a > 0$:

Step 1: $ax + b = c$

Step 2: $ax + b - b = c - b$

Step 3: $ax = c - b$

Step 4: $\frac{ax}{a} = \frac{c-b}{a}$

Step 5: $x = \frac{c-b}{a}$

The steps are **the same** for $a < 0$.

2. For $a > 0$:

Step 1: $ax + b > c$

Step 2: $ax + b - b > c - b$

Step 3: $ax > c - b$

Step 4: $\frac{ax}{a} > \frac{c-b}{a}$

Step 5: $x > \frac{c-b}{a}$

For $a < 0$, steps 4 and 5 will be different since the **inequality sign reverses** when the inequality is divided by a negative real number on both sides of the inequality.

For $a < 0$:

Step 4: $\frac{ax}{a} < \frac{c-b}{a}$

Step 5: $x < \boldsymbol{\frac{c-b}{a}}$

3. From Question 1, the solution of the linear equation $ax + b = c$ is $x = \frac{c-b}{a}$.

Inferring from Question 2,

For $a > 0$:

The solution of the linear inequality $ax + b < c$ is $x < \frac{c-b}{a}$.

The solution of the linear inequality $ax + b \leqslant c$ is $x \leqslant \frac{c-b}{a}$.

The solution of the linear inequality $ax + b > c$ is $x > \frac{c-b}{a}$.

The solution of the linear inequality $ax + b \geqslant c$ is $x \geqslant \frac{c-b}{a}$.

For $a < 0$:

The solution of the linear inequality $ax + b < c$ is $x > \frac{c-b}{a}$.

The solution of the linear inequality $ax + b \leqslant c$ is $x \geqslant \frac{c-b}{a}$.

The solution of the linear inequality $ax + b > c$ is $x < \frac{c-b}{a}$.

The solution of the linear inequality $ax + b \geqslant c$ is $x \leqslant \frac{c-b}{a}$.

Practise Now 3

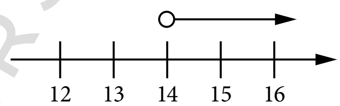
$5 - x < -9$

$5 - x - 5 < -9 - 5$

$-x < -14$

$-1 \times (-x) > -1 \times (-14)$

$\boldsymbol{x > 14}$



(i) $x = \mathbf{17}$

(ii) $x = \mathbf{27}$

Practise Now 4

1. **(a)** $5(3 + x) \geqslant 9$

$15 + 5x \geqslant 9$

$15 + 5x - 15 \geqslant 9 - 15$

$5x \geqslant -6$

$\frac{5x}{5} \geqslant \frac{-6}{5}$

$\boldsymbol{x \geqslant -\frac{6}{5}}$

(b) $\frac{1}{3} > \frac{y+1}{2}$

$2 \times 3 \times \frac{1}{3} > 2 \times 3 \times \frac{y+1}{2}$

$2 > 3(y + 1)$

$2 > 3y + 3$

$2 - 3 > 3y + 3 - 3$

$-1 > 3y$

$3y < -1$

$\frac{3y}{3} < \frac{-1}{3}$

$\boldsymbol{y < -\frac{1}{3}}$

(c) $\frac{1}{3}(z+1)+2 \leqslant \frac{1}{2}$

$3 \times 2 \times \left[\frac{1}{3}(z+1)+2\right] \leqslant 3 \times 2 \times \frac{1}{2}$

$2(z + 1) + 12 \leqslant 3$

$2z + 2 + 12 \leqslant 3$

$2z + 14 \leqslant 3$

$2z + 14 - 14 \leqslant 3 - 14$

$2z \leqslant -11$

$\frac{2z}{2} \leqslant \frac{-11}{2}$

$\boldsymbol{z \leqslant -\frac{11}{2}}$

2. $\frac{1}{2} - \frac{3}{4}(p-3) > -1$

$2 \times 4 \times \left[\frac{1}{2} - \frac{3}{4}(p-3)\right] > 2 \times 4 \times (-1)$

$4 - 6(p - 3) > -8$

$4 - 6p + 18 > -8$

$-6p + 22 > -8$

$-6p + 22 - 22 > -8 - 22$

$-6p > -30$

$\frac{-6p}{-6} < \frac{-30}{-6}$

$p < 5$

If p is a perfect square, the largest possible value of p = **4**.

Practise Now 5

(a) $4x < 2x + 3$

$4x - 2x < 2x + 3 - 2x$

$2x < 3$

$\frac{2x}{2} < \frac{3}{2}$

$\boldsymbol{x < \frac{3}{2}}$

(b) $3x > 7x + 10$

$3x - 7x > 7x + 10 - 7x$

$-4x > 10$

$\frac{-4x}{-4} < \frac{10}{-4}$

$\boldsymbol{x < -\frac{5}{2}}$

(c) $5x \geqslant 2(2x + 3)$

$5x \geqslant 4x + 6$

$5x - 4x \geqslant 4x + 6 - 4x$

$\boldsymbol{x \geqslant 6}$

(d) $2x + 5 \leqslant 6x - 13$

$2x + 5 - 6x \leqslant 6x - 13 - 6x$

$-4x + 5 \leqslant -13$

$-4x + 5 - 5 \leqslant -13 - 5$

$-4x \leqslant -18$

$\frac{-4x}{-4} \geqslant \frac{-18}{-4}$

$\boldsymbol{x \geqslant \frac{9}{2}}$

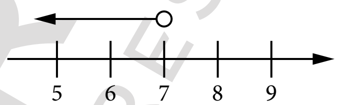
Exercise 3A

1. **(a)** $<$
(b) $<$
(c) $>$
(d) $>$
(e) $\leqslant$
(f) $\leqslant$

2. **(a)** $4x < 28$

$\frac{4x}{4} < \frac{28}{4}$

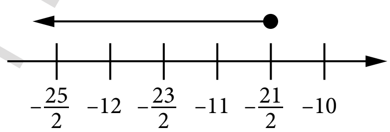
$\boldsymbol{x < 7}$



(b) $-12x \geqslant 126$

$\frac{-12x}{-12} \leqslant \frac{126}{-12}$

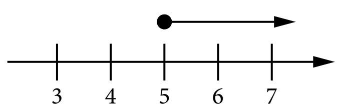
$\boldsymbol{x \leqslant -\frac{21}{2}}$



(c) $-y \leqslant -5$

$\frac{-y}{-1} \geqslant \frac{-5}{-1}$

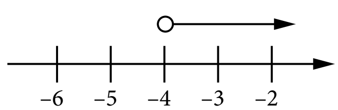
$\boldsymbol{y \geqslant 5}$



(d) $\frac{1}{2}y > -2$

$2 \times \frac{1}{2}y > 2 \times (-2)$

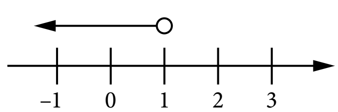
$\boldsymbol{y > -4}$



(e) $a + 2 < 3$

$a + 2 - 2 < 3 - 2$

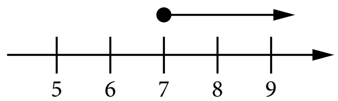
$\boldsymbol{a < 1}$



(f) $b - 3 \geqslant 4$

$b - 3 + 3 \geqslant 4 + 3$

$\boldsymbol{b \geqslant 7}$



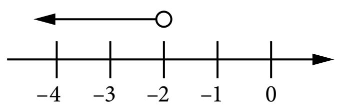
(g) $-c + 3 > 5$

$-c + 3 - 3 > 5 - 3$

$-c > 2$

$\frac{-c}{-1} < \frac{2}{-1}$

$\boldsymbol{c < -2}$



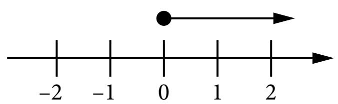
(h) $4 - d \leqslant 4$

$4 - d - 4 \leqslant 4 - 4$

$-d \leqslant 0$

$\frac{-d}{-1} \geqslant \frac{0}{-1}$

$\boldsymbol{d \geqslant 0}$



3. **(a)** $8 \leqslant 7y$

$7y \geqslant 8$

$\frac{7y}{7} \geqslant \frac{8}{7}$

$y \geqslant \frac{8}{7}$

The smallest rational value that satisfies the inequality is $\boldsymbol{\frac{8}{7}}$.

(b) $20x > 33$

$\frac{20x}{20} > \frac{33}{20}$

$x > \frac{33}{20}$

The smallest prime number that satisfies the inequality is **2**.

(c) $-3x > -105$

$\frac{-3x}{-3} < \frac{-105}{-3}$

$x < 35$

The greatest odd integer value that satisfies the inequality is **33**.

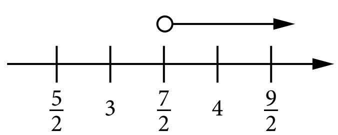
4. **(a)** $2a - 5 > 2$

$2a - 5 + 5 > 2 + 5$

$2a > 7$

$\frac{2a}{2} > \frac{7}{2}$

$\boldsymbol{a > \frac{7}{2}}$



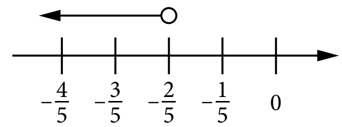
(b) $2 + 5b < 0$

$2 + 5b - 2 < 0 - 2$

$5b < -2$

$\frac{5b}{5} < \frac{-2}{5}$

$\boldsymbol{b < -\frac{2}{5}}$



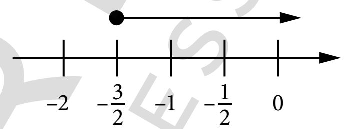
(c) $-2c - 1 \leqslant 2$

$-2c - 1 + 1 \leqslant 2 + 1$

$-2c \leqslant 3$

$\frac{-2c}{-2} \geqslant \frac{3}{-2}$

$\boldsymbol{c \geqslant -\frac{3}{2}}$



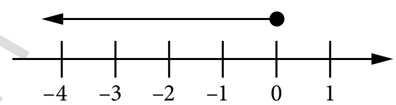
(d) $1 - 4d \geqslant 1$

$1 - 4d - 1 \geqslant 1 - 1$

$-4d \geqslant 0$

$\frac{-4d}{-4} \leqslant \frac{0}{-4}$

$\boldsymbol{d \leqslant 0}$

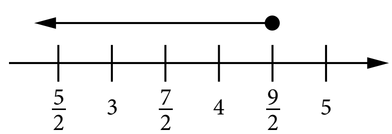


5. $7 + 2x \leqslant 16$

$7 + 2x - 7 \leqslant 16 - 7$

$2x \leqslant 9$

$\boldsymbol{x \leqslant \frac{9}{2}}$



(i) The largest possible integer that satisfies the inequality is **4**.

(ii) If x is a perfect square, the largest possible value of x is **4**.

6. **(a)** $2(e - 5) \leqslant 2$

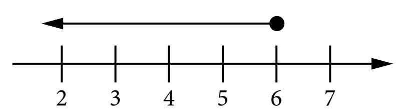
$2e - 10 \leqslant 2$

$2e - 10 + 10 \leqslant 2 + 10$

$2e \leqslant 12$

$\frac{2e}{2} \leqslant \frac{12}{2}$

$\boldsymbol{e \leqslant 6}$



(b) $4(f+1) > 5$

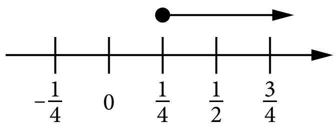
$4f + 4 > 5$

$4f + 4 - 4 > 5 - 4$

$4f > 1$

$\frac{4f}{4} > \frac{1}{4}$

$\boldsymbol{f > \frac{1}{4}}$



(c) $8 < 2(7 - g)$

$8 < 14 - 2g$

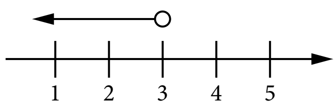
$8 - 14 < 14 - 2g - 14$

$-6 < -2g$

$-2g > -6$

$\frac{-2g}{-2} < \frac{-6}{-2}$

$\boldsymbol{g < 3}$



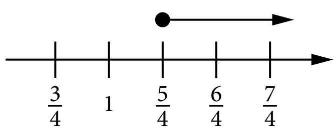
(d) $-5 \geqslant 2(-2h)$

$-5 \geqslant -4h$

$-4h \leqslant -5$

$\frac{-4h}{-4} \geqslant \frac{-5}{-4}$

$\boldsymbol{h \geqslant \frac{5}{4}}$



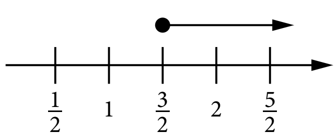
(e) $\frac{4a}{3} \geqslant 2$

$3 \times \frac{4a}{3} \geqslant 3 \times 2$

$4a \geqslant 6$

$\frac{4a}{4} \geqslant \frac{6}{4}$

$\boldsymbol{a \geqslant \frac{3}{2}}$



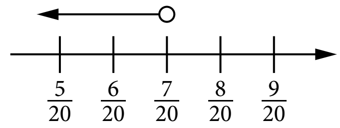
(f) $\frac{5b}{7} < \frac{1}{4}$

$28 \times \frac{5b}{7} < 28 \times \frac{1}{4}$

$20b < 7$

$\frac{20b}{20} < \frac{7}{20}$

$\boldsymbol{b < \frac{7}{20}}$



7. (a) $\frac{2p-1}{3} > \frac{7}{11}$

$33 \times \frac{2p-1}{3} > 33 \times \frac{7}{11}$

$11(2p - 1) > 21$

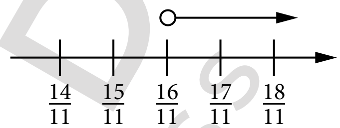
$22p - 11 > 21$

$22p - 11 + 11 > 21 + 11$

$22p > 32$

$\frac{22p}{22} > \frac{32}{22}$

$\boldsymbol{p > \frac{16}{11}}$



(b) $\frac{10q+9}{2} \leqslant \frac{3}{8}$

$8 \times \frac{10q+9}{2} \leqslant 8 \times \frac{3}{8}$

$4(10q + 9) \leqslant 3$

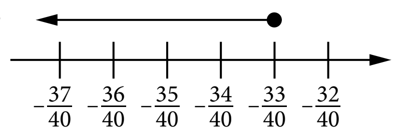
$40q + 36 \leqslant 3$

$40q + 36 - 36 \leqslant 3 - 36$

$40q \leqslant -33$

$\frac{40q}{40} \leqslant \frac{-33}{40}$

$\boldsymbol{q \leqslant -\frac{33}{40}}$



(c) $\frac{3(9-2r)}{4} \geqslant \frac{2}{5}$

$20 \times \frac{27-6r}{4} \geqslant 20 \times \frac{2}{5}$

$5(27 - 6r) \geqslant 8$

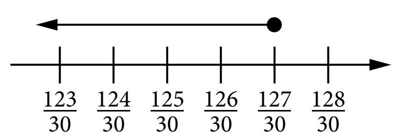
$135 - 30r \geqslant 8$

$135 - 30r - 135 \geqslant 8 - 135$

$-30r \geqslant -127$

$\frac{-30r}{-30} \leqslant \frac{-127}{-30}$

$\boldsymbol{r \leqslant \frac{127}{30}}$



(d) $$\frac{5(-5s-1)}{6} > -\frac{9}{5}$$

$$30\times\frac{5(-5s-1)}{6} > 30\times\left(-\frac{9}{5}\right)$$

$$25(-5s-1) > -54$$

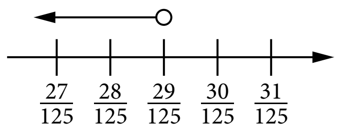
$$-125s - 25 > -54$$

$$-125s - 25 + 25 > -54 + 25$$

$$-125s > -29$$

$$\frac{-125s}{-125} < \frac{-29}{-125}$$

$$\boldsymbol{s < \frac{29}{125}}$$



8. **(a)** $$3x < x - 7$$

$$3x - x < x - 7 - x$$

$$2x < -7$$

$$\frac{2x}{2} < \frac{-7}{2}$$

$$\boldsymbol{x < -\frac{7}{2}}$$

(b) $$2x + 8 > 9x$$

$$2x + 8 - 9x > 9x - 9x$$

$$-7x + 8 > 0$$

$$-7x + 8 - 8 > 0 - 8$$

$$-7x > -8$$

$$\frac{-7x}{-7} < \frac{-8}{-7}$$

$$\boldsymbol{x < \frac{8}{7}}$$

(c) $$5x \geqslant 3(x - 4)$$

$$5x \geqslant 3x - 12$$

$$5x - 3x \geqslant 3x - 12 - 3x$$

$$2x \geqslant -12$$

$$\frac{2x}{2} \geqslant \frac{-12}{2}$$

$$\boldsymbol{x \geqslant -6}$$

(d) $$4x - 2 \leqslant 7x - 5$$

$$4x - 2 - 7x \leqslant 7x - 5 - 7x$$

$$-3x - 2 \leqslant -5$$

$$-3x - 2 + 2 \leqslant -5 + 2$$

$$-3x \leqslant -3$$

$$\frac{-3x}{-3} \geqslant \frac{-3}{-3}$$

$$\boldsymbol{x \geqslant 1}$$

9. **(a)** **Always true**

(b) **Always true**

(c) **Always true**

(d) **Sometimes true**

True only when $b - a > d - c$

(e) **Sometimes true**

True only when $b - a > d - c$

(f) **Never true**

Let $a = 1$, $b = 2$ and $c = 3$.

$c - a = 3 - 1$

$\quad = 2$

$c - b = 3 - 2$

$\quad = 1$

$\therefore c - a > c - b$

Alternatively,

if $\quad c - a < c - b$,

$c - a - c < c - b - c$

$-a < -b$

$\frac{-a}{-1} > \frac{-b}{-1}$

$a > b$

Since $a < b$, this inequality is never true.

3.3 Solving problems involving linear inequalities

Practise Now 6

Let the number of buses that are needed to ferry 520 students be x.

Then $\quad 45x \geqslant 520$

$$\frac{45x}{45} \geqslant \frac{520}{45}$$

$$x \geqslant 11.6 \text{ (to 3 s.f.)}$$

Since the number of buses must be an integer, the minimum number of buses is **12**.

Practise Now 7

Let the mark she scored in her first quiz be x.

$$\frac{x+76+89}{3} \geqslant 75$$

$$3\times\frac{x+76+89}{3} \geqslant 3 \times 75$$

$$x + 165 \geqslant 225$$

$$x + 165 - 165 \geqslant 225 - 165$$

$$x \geqslant 60$$

The minimum mark she scored in her first quiz is **60**.

Introductory Problem Revisited

(a) Let the marks Yasir scores for the presentation be x.

$$\frac{65}{100}\times 15+\frac{8}{10}\times 10+\frac{75}{100}\times 35+\frac{x}{100}\times 40 \geqslant 70$$

$$9.75 + 8 + 26.25 + 0.4x \geqslant 70$$

$$0.4x \geqslant 70 - 9.75 - 8 - 26.25$$

$$\frac{0.4x}{0.4} \geqslant \frac{26}{0.4}$$

$$\boldsymbol{x \geqslant 65}$$

Yasir must score at least 65 marks for the presentation to qualify for an award.

(b) Let the marks Yasir scores for the presentation be x.

$$\frac{65}{100}\times 15+\frac{8}{10}\times 10+\frac{75}{100}\times 35+\frac{x}{100}\times 40 \geqslant 85$$

$$9.75 + 8 + 26.25 + 0.4x \geqslant 85$$

$$0.4x \geqslant 85 - 9.75 - 8 - 26.25$$

$$\frac{0.4x}{0.4} \geqslant \frac{41}{0.4}$$

$$x \geqslant 102.5$$

Since the maximum mark for the presentation is 100, Yasir cannot receive funding for the project.

Practise Now 8

Let the number of correct answers he obtained be x.

$$\begin{aligned} 3x - (19 - x) &> 32 \\ 3x - 19 + x &> 32 \\ 4x - 19 &> 32 \\ 4x - 19 + 19 &> 32 + 19 \\ 4x &> 51 \\ \frac{4x}{4} &> \frac{51}{4} \\ x &> 12.75 \end{aligned}$$

Since the number of correct answers he obtained must be an integer, the minimum number of correct answers he obtained is **13**.

Exercise 3B

1. Let the number of tickets Vani can buy be x.

$$\begin{aligned} 13.5x &\leqslant 265 \\ \frac{13.5x}{13.5} &\leqslant \frac{265}{13.5} \\ x &\leqslant 19.6 \text{ (to 3 s.f.)} \end{aligned}$$

Since the number of tickets must be an integer, the maximum number of tickets Vani can buy with \$265 is **19**.

2. Let x be the number of months that Cheryl keeps the money in the account.

$$\begin{aligned} 200 + 15x &> 2 \times 200 \\ 200 + 15x &> 400 \\ 200 + 15x - 200 &> 400 - 200 \\ 15x &> 200 \\ \frac{15x}{15} &> \frac{200}{15} \\ x &> 13.3 \text{ (to 3 s.f.)} \end{aligned}$$

Since the number of months must be an integer, Cheryl has to keep the money in the account for at least **14** months.

3. Let the mark he scored for his third class test be x.

$$\begin{aligned} \frac{66+72+x}{3} &\geqslant 75 \\ 3\times\frac{66+72+x}{3} &\geqslant 3 \times 75 \\ 138 + x &\geqslant 225 \\ 138 + x - 138 &\geqslant 225 - 138 \\ x &\geqslant 87 \end{aligned}$$

David has to score at least **87** marks for his third class test.

4. Let the largest integer be x.

$$\begin{aligned} x - 2 + x - 1 + x &< 75 \\ 3x - 3 &< 75 \\ 3x - 3 + 3 &< 75 + 3 \\ 3x &< 78 \\ \frac{3x}{3} &< \frac{78}{3} \\ x &< 26 \end{aligned}$$

The largest possible integer is 25.

The cube of the largest possible integer $= 25^3$

$= \mathbf{15\,625}$

5. Let the number of correct answers Bernard obtained be x.

$$\begin{aligned} 5x - 2(30 - x) &\leqslant 66 \\ 5x - 60 + 2x &\leqslant 66 \\ 7x - 60 &\leqslant 66 \\ 7x - 60 + 60 &\leqslant 66 + 60 \\ 7x &\leqslant 126 \\ \frac{7x}{7} &\leqslant \frac{126}{7} \\ x &\leqslant 18 \end{aligned}$$

The maximum number of correct answers he obtained is **18**.

6. Let the number of \$5-notes Siti has be x.

$$\begin{aligned} 5x + 2(50 - x) &> 132 \\ 5x + 100 - 2x &> 132 \\ 3x + 100 &> 132 \\ 3x + 100 - 100 &> 132 - 100 \\ 3x &> 32 \\ \frac{3x}{3} &> \frac{32}{3} \\ x &> 10.7 \text{ (to 3 s.f.)} \end{aligned}$$

Since the number of \$5-notes must be an integer, the minimum number of \$5-notes that Siti has is **11**.

7. Let the number of days Devi rents a car for be x.

Devi should choose Company A if:

$$\begin{aligned} 45x &< 38x + 75 \\ 45x - 38x &< 38x + 75 - 38x \\ 7x &< 75 \\ \frac{7x}{7} &< \frac{75}{7} \\ x &< 10.7 \text{ (to 3 s.f.)} \end{aligned}$$

Since the number of days Devi rents a car must be an integer, Devi should choose **Company A** if she rents a car for **10 days or less** and **Company B** if she rents a car for **more than 10 days**.

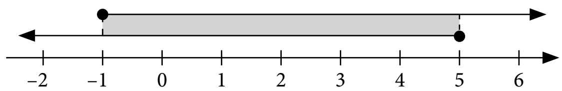
3.4 Solving simultaneous linear inequalities

Practise Now 9

Solving the two linear inequalities separately,

$$\begin{aligned} 2x - 3 &\leqslant 7 \\ 2x - 3 + 3 &\leqslant 7 + 3 \\ 2x &\leqslant 10 \\ x &\leqslant \frac{10}{2} \\ x &\leqslant 5 \end{aligned} \qquad \text{and} \qquad \begin{aligned} 2x + 1 &\geqslant -3x - 4 \\ 2x + 1 - 1 &\geqslant -3x - 4 - 1 \\ 2x &\geqslant -3x - 5 \\ 2x + 3x &\geqslant -3x - 5 + 3x \\ 5x &\geqslant -5 \\ x &\geqslant \frac{-5}{5} \\ x &\geqslant -1 \end{aligned}$$

Representing $x \leqslant 5$ and $x \geqslant -1$ on a number line,



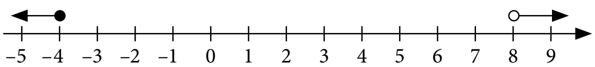
$\therefore$ the solutions satisfying both inequalities lie in the overlapping shaded region, i.e. $\mathbf{-1 \leqslant x \leqslant 5}$.

Practise Now 10

1. Solving the two linear inequalities separately,

$$\begin{aligned} 8x + 13 &\leqslant 4x - 3 \\ 8x + 13 - 4x &\leqslant 4x - 3 - 4x \\ 4x + 13 &\leqslant -3 \\ 4x + 13 - 13 &\leqslant -3 - 13 \\ 4x &\leqslant -16 \\ x &\leqslant \frac{-16}{4} \\ x &\leqslant -4 \end{aligned} \qquad \text{and} \qquad \begin{aligned} 4x - 3 &< 5x - 11 \\ 4x - 3 - 5x &< 5x - 11 - 5x \\ -x - 3 &< -11 \\ -x - 3 + 3 &< -11 + 3 \\ -x &< -8 \\ x &> 8 \end{aligned}$$

Representing $x \leqslant -4$ and $x > 8$ on a number line,

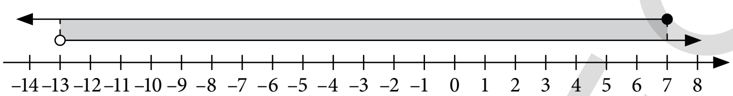


$\therefore$ these two simultaneous linear inequalities have **no solution**.

2. Solving the two linear inequalities separately,

$$\begin{aligned} \frac{y-2}{3} &< \frac{2y+1}{5} \\ \frac{y-2}{3} \times 3 \times 5 &< \frac{2y+1}{5} \times 3 \times 5 \\ 5(y-2) &< 3(2y+1) \\ 5y - 10 &< 6y + 3 \\ 5y - 10 - 6y &< 6y + 3 - 6y \\ -y - 10 &< 3 \\ -y - 10 + 10 &< 3 + 10 \\ -y &< 13 \\ y &> -13 \end{aligned} \qquad \text{and} \qquad \begin{aligned} \frac{2y+1}{5} &\leqslant 3 \\ \frac{2y+1}{5} \times 5 &\leqslant 3 \times 5 \\ 2y + 1 &\leqslant 15 \\ 2y + 1 - 1 &\leqslant 15 - 1 \\ 2y &\leqslant 14 \\ y &\leqslant \frac{14}{2} \\ y &\leqslant 7 \end{aligned}$$

Representing $y > -13$ and $y \leqslant 7$ on a number line,



$\therefore$ the solutions satisfying both inequalities lie in the overlapping shaded region, i.e. $\mathbf{-13 < y \leqslant 7}$.

3.5 Solving problems involving simultaneous linear inequalities

Practise Now 11

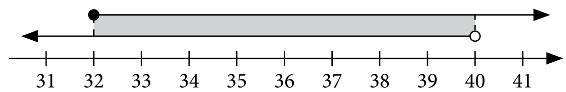
OPEN Let the number of 20-cent coins be x.
Then the number of 50-cent coins is $(48 - x)$.
Since the total value of all the coins is more than \$12,

$$\begin{aligned} 0.2x + 0.5(48 - x) &> 12 \\ 0.2x + 24 - 0.5x &> 12 \\ -0.3x + 24 &> 12 \\ -0.3x + 24 - 24 &> 12 - 24 \\ -0.3x &> -12 \\ x &< \frac{-12}{-0.3} \\ x &< 40 \end{aligned}$$

Since the number of 20-cent coins is at least twice the number of 50-cent coins,

$$\begin{aligned} x &\geqslant 2(48 - x) \\ x &\geqslant 96 - 2x \\ x + 2x &\geqslant 96 - 2x + 2x \\ 3x &\geqslant 96 \\ x &\geqslant \frac{96}{3} \\ x &\geqslant 32 \end{aligned}$$

Representing $x < 40$ and $x \geqslant 32$ on a number line,

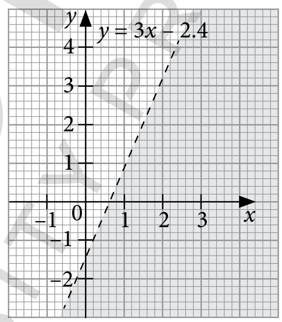


the solutions satisfying both inequalities is $32 \leqslant x < 40$.
$\therefore$ the possible number of 20-cent coins is 32, 33, 34, 35, 36, 37, 38 or 39.
Then the possible number of 50-cent coins is 16, 15, 14, 13, 12, 11, 10 or 9.
$\therefore$ one possible solution is **38 20-cent coins** and **10 50-cent coins**.

3.6 Linear inequalities in two variables

Practise Now 12

(a) From the graph, the point (2, 2) is in the shaded region.
Substituting $x = 2$ and $y = 2$ into $y < 3x - 2.4$, we have
$2 < 3(2) - 2.4$
$2 < 3.6$.
The shaded region represents $y < 3x - 2.4$.



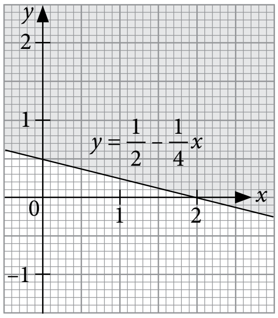
(b) $\frac{1}{2}x + 2y \geqslant 1$

$$\begin{aligned} 2y &\geqslant 1 - \frac{1}{2}x \\ 2y \times \frac{1}{2} &\geqslant \left(1 - \frac{1}{2}x\right) \times \frac{1}{2} \\ y &\geqslant \frac{1}{2} - \frac{1}{4}x \end{aligned}$$

From the graph, the point (1, 2) is in the shaded region.
Substituting $x = 1$ and $y = 2$ into $y \geqslant \frac{1}{2} - \frac{1}{4}x$, we have

$$\begin{aligned} y &\geqslant \frac{1}{2} - \frac{1}{4}x \\ 2 &\geqslant \frac{1}{2} - \frac{1}{4}(1) \\ 2 &\geqslant \frac{1}{2} \end{aligned}$$

The shaded region represents $y \geqslant \frac{1}{2} - \frac{1}{4}x$.



(c) $\frac{3}{2} - 2x \geqslant \frac{1}{2}y$

$\frac{1}{2}y \times 2 \leqslant \left(\frac{3}{2} - 2x\right) \times 2$

$y \leqslant 3 - 4x$

From the graph, the point (0, 0) is in the shaded region.

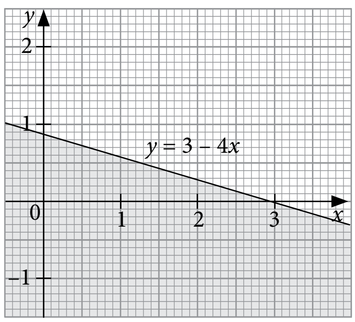
Substituting $x = 0$ and $y = 0$ into $y \leqslant 3 - 4x$, we have

$y \leqslant 3 - 4x$

$0 \leqslant 3 - 4(0)$

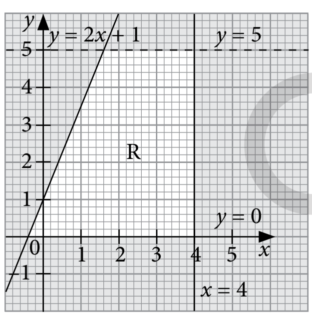
$0 \leqslant 3$

The shaded region represents $y \leqslant 3 - 4x$.

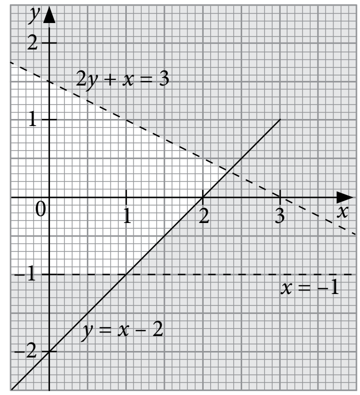


Practise Now 13

1. **(a)** The unshaded region labelled R represents $y < 2x + 1$, $y < 5$, $y \geqslant 0$ and $x \leqslant 4$.



(b) The unshaded region labelled R represents $2y + x < 3$, $y \geqslant x - 2$ and $x > -1$.



2. From the graph, (1, –1) falls in the shaded region.

Substituting $x = 1$ and y = 1 into $2y = -x + 4$,

LHS = $2(-1) = -2$

RHS = $-1 + 4 = 3$

Shaded region falls in the region representing $\boldsymbol{2y \leqslant -x + 4}$.

Substituting $x = 1$ and $y = 1$ into $y + 2x + 1 = 0$,

LHS = $-1 + 2(1) + 1 = 2$

RHS = 0

The shaded region falls in the region representing $\boldsymbol{y + 2x + 1 \geqslant 0}$.

Substituting $x = 1$ and $y = 1$ into $y = -x - 4$,

LHS = –1

RHS = $1 + 4 = 5$

Shaded region falls in the region representing $y > -x - 4$.

$\therefore$ the shaded region represents the inequalities $\boldsymbol{2y \leqslant -x + 4}$, $\boldsymbol{y + 2x + 1 \geqslant 0}$ and $\boldsymbol{y > x - 4}$.

3. **(i)** $p > 2q$;

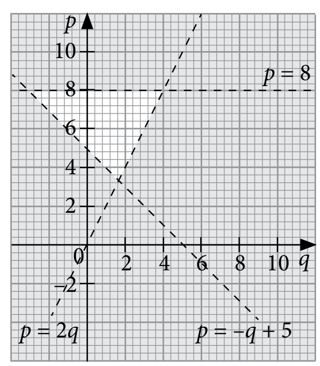
$p < 8$;

$p + q > 5$

(ii) $p + q > 5$

$\Rightarrow p > -q + 5$

The unshaded region labelled R represents $p > 2q$, $p < 8$ and $p + q > 5$.



(iii) All (p, q) satisfying the three inequalities fall in region R.

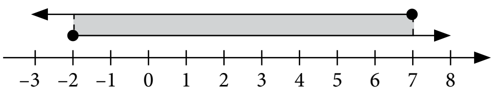
Since (0, 6) falls in R, $\boldsymbol{p = 6}$, and $\boldsymbol{q = 0}$.

Exercise 3C

1. **(a)** Solving the two linear inequalities separately,

$$\begin{array}{lcl} x - 4 \leqslant 3 & \text{and} & 3x \geqslant -6 \\ x - 4 + 4 \leqslant 3 + 4 & & x \geqslant \dfrac{-6}{3} \\ x \leqslant 7 & & x \geqslant -2 \end{array}$$

Representing $x \leqslant 7$ and $x \geqslant -2$ on a number line,

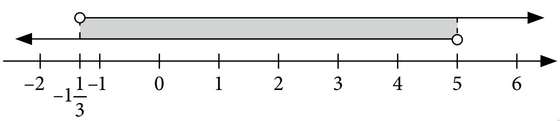


$\therefore$ the solutions satisfying both inequalities lie in the overlapping shaded region, i.e. $\boldsymbol{-2 \leqslant x \leqslant 7}$.

(b) Solving the two linear inequalities separately,

$$\begin{array}{lcl} 2x + 5 < 15 & \text{and} & 3x - 2 > -6 \\ 2x + 5 - 5 < 15 - 5 & & 3x - 2 + 2 > -6 + 2 \\ 2x < 10 & & 3x > -4 \\ x < \dfrac{10}{2} & & x > \dfrac{-4}{3} \\ x < 5 & & x > -1\dfrac{1}{3} \end{array}$$

Representing $x < 5$ and $x > -1\frac{1}{3}$ on a number line,

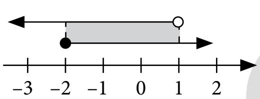


$\therefore$ the solutions satisfying both inequalities lie in the overlapping shaded region, i.e. $\boldsymbol{-1\frac{1}{3} < x < 5}$.

3. **(a)** Solving the two linear inequalities separately,

$$\begin{array}{lcl} 5x - 1 < 4 & \text{and} & 3x + 5 \geqslant x + 1 \\ 5x - 1 + 1 < 4 + 1 & & 3x + 5 - x \geqslant x + 1 - x \\ 5x < 5 & & 2x + 5 \geqslant 1 \\ x < \dfrac{5}{5} & & 2x + 5 - 5 \geqslant 1 - 5 \\ x < 1 & & 2x \geqslant -4 \\ & & x \geqslant \dfrac{-4}{2} \\ & & x \geqslant -2 \end{array}$$

Representing $x < 1$ and $x \geqslant -2$ on a number line,



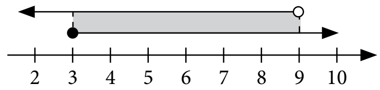
$\therefore$ the solutions satisfying both inequalities lie in the overlapping shaded region, i.e. $-2 \leqslant x < 1$.

$\therefore$ the integer values of x which satisfy both inequalities are **–2**, **–1** and **0**.

(b) Solving the two linear inequalities separately,

$$\begin{array}{lcl} 2x - 5 \geqslant 1 & \text{and} & 3x - 1 < 26 \\ 2x - 5 + 5 \geqslant 1 + 5 & & 3x - 1 + 1 < 26 + 1 \\ 2x \geqslant 6 & & 3x < 27 \\ x \geqslant \dfrac{6}{2} & & x < \dfrac{27}{3} \\ x \geqslant 3 & & x < 9 \end{array}$$

Representing $x \geqslant 3$ and $x < 9$ on a number line,



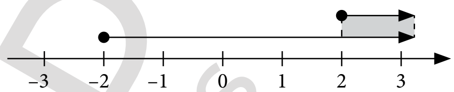
$\therefore$ the solutions satisfying both inequalities lie in the overlapping shaded region, i.e. $3 \leqslant x < 9$.

$\therefore$ the integer values of x which satisfy both inequalities are **3**, **4**, **5**, **6**, **7** and **8**.

3. **(a)** Solving the two linear inequalities separately,

$$\begin{array}{lcl} -4 \leqslant 2x & \text{and} & 2x \leqslant 3x - 2 \\ 2x \geqslant -4 & & 2x - 3x \leqslant 3x - 2 - 3x \\ x \geqslant \dfrac{-4}{2} & & -x \leqslant -2 \\ x \geqslant -2 & & x \geqslant 2 \end{array}$$

Representing $x \geqslant -2$ and $x \geqslant 2$ on a number line,

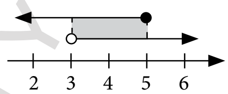


$\therefore$ the solutions satisfying both inequalities lie in the overlapping shaded region, i.e. $\boldsymbol{x \geqslant 2}$.

(b) Solving the two linear inequalities separately,

$$\begin{array}{lcl} 1 - x < -2 & \text{and} & -2 \leqslant 3 - x \\ 1 - x - 1 < -2 - 1 & & -2 + x \leqslant 3 - x + x \\ -x < -3 & & -2 + x \leqslant 3 \\ x > 3 & & -2 + x + 2 \leqslant 3 + 2 \\ & & x \leqslant 5 \end{array}$$

Representing $x > 3$ and $x \leqslant 5$ on a number line,

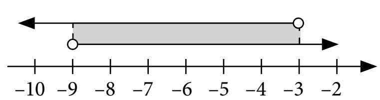


$\therefore$ the solutions satisfying both inequalities lie in the overlapping shaded region, i.e. $\boldsymbol{3 < x \leqslant 5}$.

(c) Solving the two linear inequalities separately,

$$\begin{array}{lcl} 3x - 3 < x - 9 & \text{and} & x - 9 < 2x \\ 3x - 3 - x < x - 9 - x & & x - 9 - 2x < 2x - 2x \\ 2x - 3 < -9 & & -x - 9 < 0 \\ 2x - 3 + 3 < -9 + 3 & & -x - 9 + 9 < 0 + 9 \\ 2x < -6 & & -x < 9 \\ x < \dfrac{-6}{2} & & x > -9 \\ x < -3 & & \end{array}$$

Representing $x < -3$ and $x > -9$ on a number line,



$\therefore$ the solutions satisfying both inequalities lie in the overlapping shaded region, i.e. $\boldsymbol{-9 < x < -3}$.

4. Let the number of apples sold be x.

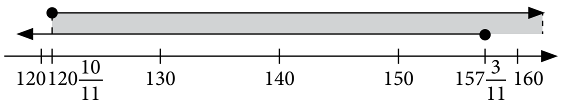
Revenue made from selling x apples = \$0.55x

Since the fruit seller was able to cover his cost but did not make a profit of more than \$20, $0 \leqslant 0.55x - 66.50 \leqslant 20$.

Solving the two linear inequalities separately,

$0 \leqslant 0.55x - 66.50$	and	$0.55x - 66.50 \leqslant 20$
$0 + 66.50 \leqslant 0.55x - 66.50 + 66.50$		$0.55x - 66.50 + 66.50 \leqslant 20 + 66.50$
$66.50 \leqslant 0.55x$		$0.55x \leqslant 86.50$
$0.55x \geqslant 66.50$		$x \leqslant \frac{86.50}{0.55}$
$x \geqslant \frac{66.50}{0.55}$		$x \leqslant 157\frac{3}{11}$
$x \geqslant 120\frac{10}{11}$		

Representing $x \geqslant 120\frac{10}{11}$ and $x \leqslant 157\frac{3}{11}$ on a number line,



$\therefore$ the solutions satisfying both inequalities lie in the overlapping shaded region, i.e. $120\frac{10}{11} \leqslant x \leqslant 157\frac{3}{11}$.

$\therefore$ the possible number of apples sold is **any integer between 121 and 157 inclusive**.

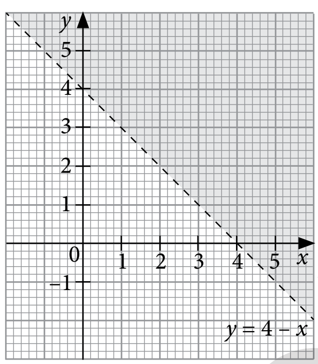
5. **(a)** From the graph, the point (3, 5) is in the shaded region.

Substituting $x = 3$ and $y = 5$ into $y > 4 - x$, we have

$5 > 4 - 3$

$5 > 1$.

The shaded region represents $y > 4 - x$.



(b) $y + 2x \leqslant \frac{2}{5}$

$y \leqslant -2x + \frac{2}{5}$

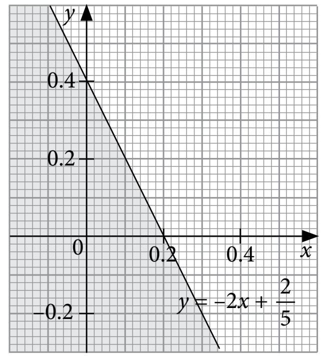
From the graph, the point (0, 0.2) is in the shaded region.

Substituting $x = 0$ and $y = 0.2$ into $y \leqslant -2x + \frac{2}{5}$, we have

$0.2 \leqslant -2(0) + \frac{2}{5}$

$0.2 \leqslant \frac{2}{5}$.

The shaded region represents $y \leqslant -2x + \frac{2}{5}$.



(c) $3x - 4y \geqslant 8$

$3x \geqslant 8 + 4y$

$3x - 8 \geqslant 4y$

$(3x - 8) \times \frac{1}{4} \geqslant 4y \times \frac{1}{4}$

$y \leqslant \frac{3}{4}x - 2$

From the graph, the point (2, –2) is in the shaded region.

Substituting $x = 2$ and $y = -2$ into

$y \leqslant \frac{3}{4}x - 2$, we have

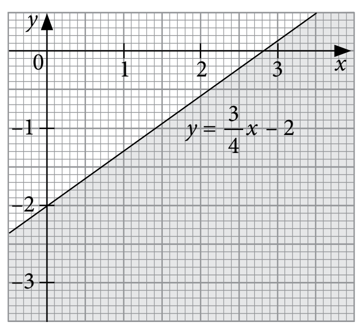
$-2 \leqslant \frac{3}{4}(2) - 2$

$-2 \leqslant \frac{3}{2} - 2$

$-2 \leqslant -\frac{1}{2}$.

The shaded region represents

$y \leqslant \frac{3}{4}x - 2$.

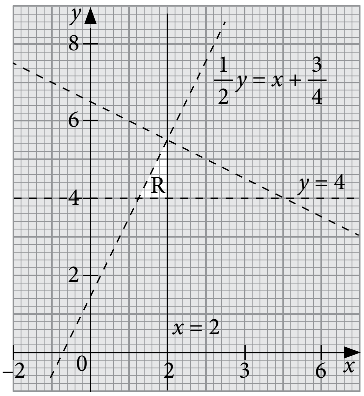


6. **(a)** $\frac{1}{2}y < x + \frac{3}{4}$

$\frac{1}{2}y \times 2 < \left(x + \frac{3}{4}\right) \times 2$

$y < 2x + \frac{3}{2}$

The unshaded region labelled R represents $\frac{1}{2}y < x + \frac{3}{4}$, $x \leqslant 2$ and $y > 4$.

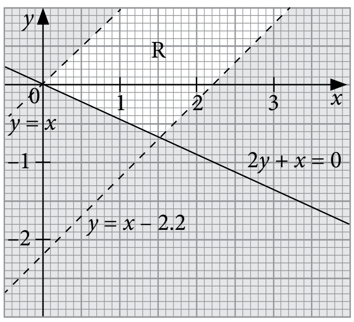


(b) $2y + x \geqslant 0$

$y \geqslant -x$

$y \geqslant -\frac{1}{2}x$

The unshaded region labelled R represents $y > x - 2.2$, $2y + x \geqslant 0$and $y < x$.



7. **(a)** From the graph, (2, –4) falls in the shaded region.

Substituting $x = 2$ and $y =-4$ into $y = -x - 2$,

LHS = –4

RHS = 2 – 2 = 0

Shaded region falls in the region representing $y < x - 2$.

Substituting $x = 2$ and $y = -4$ into $y = -x + 1$,

LHS = –4

RHS = –2 + 1 = –1

The shaded region falls in the region representing $y \leqslant -x + 1$.

Substituting $x = 2$ and $y = -4$ into $y = -2$,

LHS = –4

RHS = –2

Shaded region falls in the region representing $y \leqslant -2$.

$\therefore$ the shaded region represents the inequalities $y < x - 2$, $\boldsymbol{y \leqslant -x + 1}$ **and** $\boldsymbol{y \leqslant -2}$.

(b) From the graph, (0.5, –3) falls in the shaded region.

Substituting $x = 0.5$ and $y = -3$ into $y - 3x + 3 = 0$,

LHS = –3 – 3(0.5) + 3 = –1.5

RHS = 0

Shaded region falls in the region representing $y - 3x + 3 < 0$.

Substituting x = 0.5 and $y = -3$ into $y = -6$,

LHS = –3

RHS = –6

The shaded region falls in the region representing $y > -6$.

Substituting $x = 0.5$ and $y = -3$ into $x = 1$,

LHS = 0.5

RHS = 1

The shaded region falls in the region representing $x \leqslant 1$.

$\therefore$ the shaded region represents the inequalities $\boldsymbol{y - 3x + 3 < 0, y > -6}$ **and** $\boldsymbol{x \leqslant 1}$.

8. Solving the two linear inequalities separately,

$$\frac{1}{2}x - 4 > \frac{1}{3}x \quad \text{and} \quad \frac{1}{6}x + 1 < \frac{1}{8}x + 3$$

$$\frac{1}{2}x - 4 - \frac{1}{3}x > \frac{1}{3}x - \frac{1}{3}x \qquad \frac{1}{6}x + 1 - \frac{1}{8}x < \frac{1}{8}x + 3 - \frac{1}{8}x$$

$$\frac{1}{6}x - 4 > 0 \qquad \frac{1}{24}x + 1 < 3$$

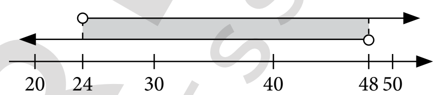
$$\frac{1}{6}x - 4 + 4 > 0 + 4 \qquad \frac{1}{24}x + 1 - 1 < 3 - 1$$

$$\frac{1}{6}x > 4 \qquad \frac{1}{24}x < 2$$

$$x > 4 \times 6 \qquad x < 24 \times 2$$

$$x > 24 \qquad x < 48$$

Representing $x > 24$ and $x < 48$ on a number line,



$\therefore$ the solutions satisfying both inequalities lie in the overlapping shaded region, i.e. $24 < x < 48$.

$\therefore$ the prime values of x which satisfy both inequalities are **29**, **31**, **37**, **41**, **43** and **47**.

9. Solving the two linear inequalities separately,

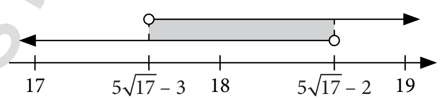
$$x + 2 < 5\sqrt{17} \quad \text{and} \quad 5\sqrt{17} < x + 3$$

$$x + 2 - 2 < 5\sqrt{17} - 2 \qquad x + 3 > 5\sqrt{17}$$

$$x < 5\sqrt{17} - 2 \qquad x + 3 - 3 > 5\sqrt{17} - 3$$

$$x > 5\sqrt{17} - 3$$

Representing $x < 5\sqrt{17} - 2$ and $x > 5\sqrt{17} - 3$ on a number line,



$\therefore$ the solutions satisfying both inequalities lie in the overlapping shaded region, i.e. $5\sqrt{17} - 3 < x < 5\sqrt{17} - 2$.

$\therefore$ the integer value of x is **18**.

10. **(a)** Solving the two linear inequalities separately,

$$3 - a \leqslant a - 4 \quad \text{and} \quad a - 4 \leqslant 9 - 2a$$

$$3 - a - a \leqslant a - 4 - a \qquad a - 4 + 2a \leqslant 9 - 2a + 2a$$

$$3 - 2a \leqslant -4 \qquad 3a - 4 \leqslant 9$$

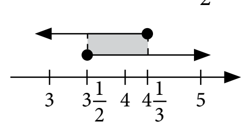
$$3 - 2a - 3 \leqslant -4 - 3 \qquad 3a - 4 + 4 \leqslant 9 + 4$$

$$-2a \leqslant -7 \qquad 3a \leqslant 13$$

$$a \geqslant \frac{-7}{-2} \qquad a \leqslant \frac{13}{3}$$

$$a \geqslant 3\frac{1}{2} \qquad a \leqslant 4\frac{1}{3}$$

Representing $a \geqslant 3\frac{1}{2}$ and $a \leqslant 4\frac{1}{3}$ on a number line,

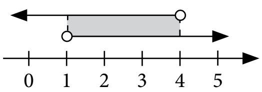


$\therefore$ the solutions satisfying both inequalities lie in the overlapping shaded region, i.e. $\boldsymbol{3\frac{1}{2} \leqslant a \leqslant 4\frac{1}{3}}$.

(b) Solving the two linear inequalities separately,

$1 - b < b - 1$	and	$b - 1 < 11 - 2b$
$1 - b - b < b - 1 - b$		$b - 1 + 2b < 11 - 2b + 2b$
$1 - 2b < -1$		$3b - 1 < 11$
$1 - 2b - 1 < -1 - 1$		$3b - 1 + 1 < 11 + 1$
$-2b < -2$		$3b < 12$
$b > \frac{-2}{-2}$		$b < \frac{12}{3}$
$b > 1$		$b < 4$

Representing $b > 1$ and $b < 4$ on a number line,

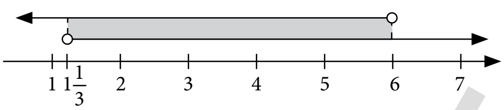


$\therefore$ the solutions satisfying both inequalities lie in the overlapping shaded region, i.e. $\mathbf{1 < b < 4}$.

(c) Solving the two linear inequalities separately,

$3 - c < 2c - 1$	and	$2c - 1 < 5 + c$
$3 - c - 2c < 2c - 1 - 2c$		$2c - 1 - c < 5 + c - c$
$3 - 3c < -1$		$c - 1 < 5$
$3 - 3c - 3 < -1 - 3$		$c - 1 + 1 < 5 + 1$
$-3c < -4$		$c < 6$
$c > \frac{-4}{-3}$		
$c > 1\frac{1}{3}$		

Representing $c > 1\frac{1}{3}$ and $c < 6$ on a number line,

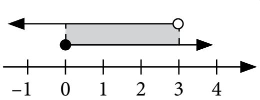


$\therefore$ the solutions satisfying both inequalities lie in the overlapping shaded region, i.e. $\mathbf{1\frac{1}{3} < c < 6}$.

(d) Solving the two linear inequalities separately,

$3d - 5 < d + 1$	and	$d + 1 \leqslant 2d + 1$
$3d - 5 - d < d + 1 - d$		$d + 1 - 2d \leqslant 2d + 1 - 2d$
$2d - 5 < 1$		$-d + 1 \leqslant 1$
$2d - 5 + 5 < 1 + 5$		$-d + 1 - 1 \leqslant 1 - 1$
$2d < 6$		$-d \leqslant 0$
$d < \frac{6}{2}$		$d \geqslant 0$
$d < 3$		

Representing $d < 3$ and $d \geqslant 0$ on a number line,

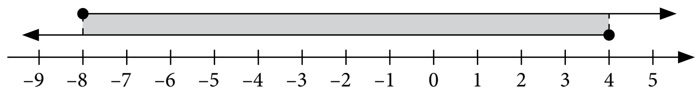


$\therefore$ the solutions satisfying both inequalities lie in the overlapping shaded region, i.e. $\mathbf{0 \leqslant d < 3}$.

11. (a) Solving the two linear inequalities separately,

$\frac{a}{4} + 3 \leqslant 4$	and	$4 \leqslant \frac{a}{4} + 6$
$\frac{a}{4} + 3 - 3 \leqslant 4 - 3$		$\frac{a}{4} + 6 \geqslant 4$
$\frac{a}{4} \leqslant 1$		$\frac{a}{4} + 6 - 6 \geqslant 4 - 6$
$a \leqslant 4 \times 1$		$\frac{a}{4} \geqslant -2$
$a \leqslant 4$		$a \geqslant -2 \times 4$
		$a \geqslant -8$

Representing $a \leqslant 4$ and $a \geqslant -8$ on a number line,

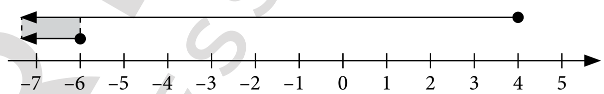


$\therefore$ the solutions satisfying both inequalities lie in the overlapping shaded region, i.e. $\mathbf{-8 \leqslant a \leqslant 4}$.

(b) Solving the two linear inequalities separately,

$\frac{b}{3} \geqslant \frac{b}{2} + 1$	and	$\frac{b}{2} + 1 \geqslant b - 1$
$\frac{b}{3} - \frac{b}{2} \geqslant \frac{b}{2} + 1 - \frac{b}{2}$		$\frac{b}{2} + 1 - b \geqslant b - 1 - b$
$-\frac{b}{6} \geqslant 1$		$-\frac{b}{2} + 1 \geqslant -1$
$b \leqslant 6-) \times 1)$		$-\frac{b}{2} + 1 - 1 \geqslant -1 - 1$
$b \leqslant -6$		$-\frac{b}{2} \geqslant -2$
		$b \leqslant -2 \times (-2)$
		$b \leqslant 4$

Representing $b \leqslant -6$ and $b \leqslant 4$ on a number line,

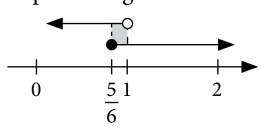


$\therefore$ the solutions satisfying both inequalities lie in the overlapping shaded region, i.e. $\mathbf{b \leqslant -6}$.

(c) Solving the two linear inequalities separately,

$2(1 - c) > c - 1$	and	$c - 1 \geqslant \frac{c - 2}{7}$
$2 - 2c > c - 1$		$(c - 1) \times 7 \geqslant \frac{c - 2}{7} \times 7$
$2 - 2c - c > c - 1 - c$		$7(c - 1) \geqslant c - 2$
$2 - 3c > -1$		$7c - 7 \geqslant c - 2$
$2 - 3c - 2 > -1 - 2$		$7c - 7 - c \geqslant c - 2 - c$
$-3c > -3$		$6c - 7 \geqslant -2$
$c < \frac{-3}{-3}$		$6c - 7 + 7 \geqslant -2 + 7$
$c < 1$		$6c \geqslant 5$
		$c \geqslant \frac{5}{6}$

Representing $c < 1$ and $c \geqslant \frac{5}{6}$ on a number line,

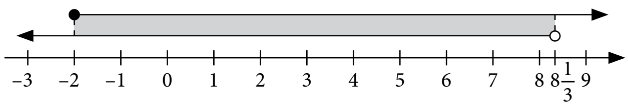


$\therefore$ the solutions satisfying both inequalities lie in the overlapping shaded region, i.e. $\mathbf{\frac{5}{6} \leqslant c < 1}$.

(d) Solving the two linear inequalities separately,

$d - 5 < \frac{2d}{5}$	and	$\frac{2d}{5} \leqslant \frac{d}{2} + \frac{1}{5}$
$d - 5 - \frac{2d}{5} < \frac{2d}{5} - \frac{2d}{5}$		$\frac{2d}{5} - \frac{d}{2} \leqslant \frac{d}{2} + \frac{1}{5} - \frac{d}{2}$
$\frac{3d}{5} - 5 < 0$		$-\frac{d}{10} \leqslant \frac{1}{5}$
$\frac{3d}{5} - 5 + 5 < 0 + 5$		$d \geqslant \frac{1}{5} \times (-10)$
$\frac{3d}{5} < 5$		$d \geqslant -2$
$d < \frac{5}{\frac{3}{5}}$		
$d < 8\frac{1}{3}$		

Representing $d < 8\frac{1}{3}$ and $d \geqslant -2$ on a number line,

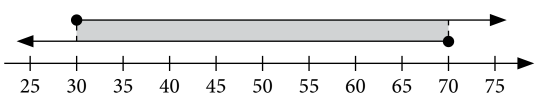


$\therefore$ the solutions satisfying both inequalities lie in the overlapping shaded region, i.e. $\mathbf{-2 \leqslant d < 8\frac{1}{3}}$.

12. Let the amount that Joyce has to pay be $\$x$.
Since the present costs no more than \$210,
Amount Waseem has to pay $\leqslant \$(210 - x)$
Since Waseem agrees to pay at least twice as much as but at most \$150 more than Joyce does, $2x \leqslant 210 - x \leqslant 150 + x$.
Solving the two linear inequalities separately,

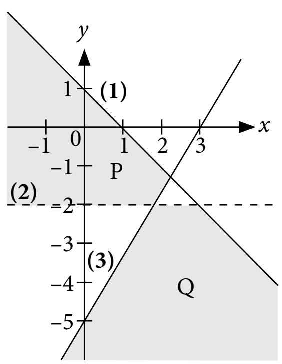
$$2x \leqslant 210 - x \quad \text{and} \quad 210 - x \leqslant 150 + x$$
$$2x + x \leqslant 210 - x + x \qquad 210 - x - x \leqslant 150 + x - x$$
$$3x \leqslant 210 \qquad 210 - 2x \leqslant 150$$
$$x \leqslant \frac{210}{3} \qquad 210 - 2x - 210 \leqslant 150 - 210$$
$$x \leqslant 70 \qquad -2x \leqslant -60$$
$$x \geqslant \frac{-60}{-2}$$
$$x \geqslant 30$$

Representing $x \leqslant 70$ and $x \geqslant 30$ on a number line,



$\therefore$ the solutions satisfying both inequalities lie in the overlapping shaded region, i.e. $30 \leqslant x \leqslant 70$.
Since the amount Waseem has to pay $\leqslant \$(210 - x)$, Waseem has to pay the greatest amount when Joyce pays the least.
$\therefore$ greatest amount Waseem has to pay $= 210 - 30$
$= \mathbf{\$180}$

13.



Line (1):
(0, 1) and (1, 0) lie on line (1).

$$\text{Gradient} = \frac{\text{rise}}{\text{run}} = \frac{1 - 0}{0 - 1} = -1$$

$\therefore$ equation of line (1) is $y = -x + 1$.

Equation of line (2) is $y = -2$.
Line (3):
(0, −5) and (3, 0) lie on line (3).

$$\text{Gradient} = \frac{\text{rise}}{\text{run}} = \frac{-5 - 0}{0 - 3} = \frac{5}{3}$$

$\therefore$ equation of line (1) is $y = \frac{5}{3}x - 5$.

(a) From the sketch, (−1, 1) lies in region P.
Substitute $x = -1$ and $y = 1$ into $y = -x + 1$:
LHS = 1
RHS = −(−1) + 1 = 2
P falls in the region representing $\mathbf{y \leqslant -x + 1}$.
Substitute $x = -1$ and $y = 1$ into $y = -2$:
LHS = 1
RHS = −2
P falls in the region representing $\mathbf{y > -2}$.
Substitute $x = -1$ and $y = 1$ into $y = \frac{5}{3}x - 5$:
LHS = 1
RHS $= \frac{5}{3}(-1) - 5 = -6\frac{2}{3}$
P falls in the region representing $\mathbf{y \geqslant \frac{5}{3}x - 5}$.
$\therefore$ P represents $\mathbf{y \leqslant -x + 1, y > -2}$ and $\mathbf{y \geqslant \frac{5}{3}x - 5}$.

(b) From the sketch, (3, −4) lies in region Q.
Substitute $x = 3$ and $y = -4$ into $y = -x + 1$:
LHS = −4
RHS = −(3) + 1 = −2
Q falls in the region representing $\mathbf{y \leqslant -x + 1}$.
Substitute $x = 3$ and $y = -4$ into $y = -2$:
LHS = −4
RHS = −2
Q falls in the region representing $\mathbf{y < -2}$.
Substitute $x = 3$ and $y = -4$ into $y = \frac{5}{3}x - 5$:
LHS = −4
RHS $= \frac{5}{3}(3) - 5 = 0$
Q falls in the region representing $\mathbf{y \leqslant \frac{5}{3}x - 5}$.
$\therefore$ Q represents $\mathbf{y \leqslant -x + 1, y < -2}$ and $\mathbf{y \leqslant \frac{5}{3}x - 5}$.

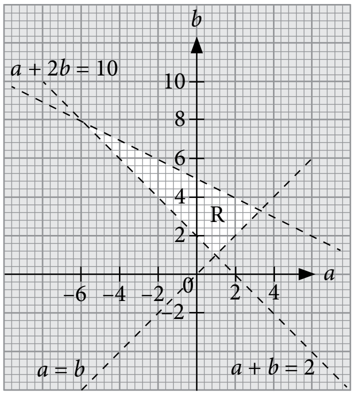
14. The inequalities are:

$a + 2b < 10$;

$a + b > 2$;

$b > a$.

Let vertical axis represent b and the horizontal axis represent a.



All pairs of coordinates (a, b) satisfying the three inequalities fall in the unshaded region in the graph.

Two possible pairs of integer values are

- $\boldsymbol{a = 1, b = 4}$
- $\boldsymbol{a = -2, b = 5}$

15. (a) Solving the two linear inequalities separately,

$$\begin{aligned} ax + 3 &> 2x + b \\ ax + 3 - 2x &> 2x + b - 2x \\ (a - 2)x + 3 &> b \\ (a - 2)x + 3 - 3 &> b - 3 \\ (a - 2)x &> b - 3 \end{aligned} \qquad \text{and} \qquad \begin{aligned} cx - 1 &> 5x - 4 \\ cx - 1 - 5x &> 5x - 4 - 5x \\ (c - 5)x - 1 &> -4 \\ (c - 5)x - 1 + 1 &> -4 + 1 \\ (c - 5)x &> -3 \end{aligned}$$

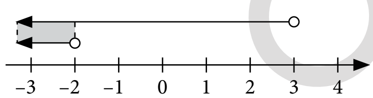
Assuming that the solution to $(a - 2)x > b - 3$ is $x < -2$, $(a - 2)$ must be negative and $(b - 3)$ must be positive. Take $a = 1$ and $b = 5$.

For the solution to both inequalities to be $x < -2$, the solution to $(c - 5)x > -3$ must be of the form $x < p$, where p must be at least -2, and $(c - 5)$ must be negative. Take $c = 4$.

The two linear inequalities then become

$$\begin{aligned} (1 - 2)x &> 5 - 3 \\ -x &> 2 \\ x &< -2 \end{aligned} \qquad \text{and} \qquad \begin{aligned} (4 - 5)x &> -3 \\ -x &> -3 \\ x &< 3 \end{aligned}$$

Representing $x < -2$ and $x < 3$ on a number line,



$\therefore$ the solutions satisfying both inequalities lie in the overlapping shaded region, i.e. $x < -2$.

$\therefore$ a set of possible values is $\boldsymbol{a = 1}$, $\boldsymbol{b = 5}$ and $\boldsymbol{c = 4}$.

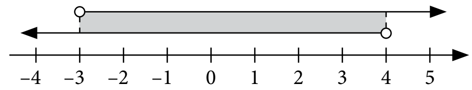
(b) Assuming that the solution to $(a - 2)x > b - 3$ is $x < 4$, both $(a - 2)$ and $(b - 3)$ must be negative. Take $a = 1$ and $b = -1$.

For the solution to both inequalities to be $-3 < x < 4$, the solution to $(c - 5)x > -3$ must be $x > -3$ and $c - 5$ must be positive. Take $c = 6$.

The two linear inequalities then become

$$\begin{aligned} (1 - 2)x &> -1 - 3 \\ -x &> -4 \\ x &< 4 \end{aligned} \qquad \text{and} \qquad \begin{aligned} (6 - 5)x &> -3 \\ x &> -3 \end{aligned}$$

Representing $x < 4$ and $x > -3$ on a number line,



$\therefore$ the solutions satisfying both inequalities lie in the overlapping shaded region, i.e. $-3 < x < 4$.

$\therefore$ a set of possible values is $\boldsymbol{a = 1}$, $\boldsymbol{b = -1}$ and $\boldsymbol{c = 6}$.

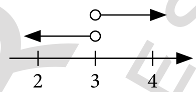
(c) For there to be no solution that satisfies both inequalities, the solution to both inequalities must not overlap. Let the solutions to $(a - 2)x > b - 3$ and $(c - 5)x > -3$ be $x > 3$ and $x < 3$ respectively.

For the solution to $(a - 2)x > b - 3$ to be $x > 3$, both $(a - 2)$ and $(b - 3)$ must be positive. Take $a = 3$ and $b = 6$.

For the solution to $(c - 5)x > -3$ to be $x < 3$, $(c - 5)$ must be negative. Take $c = 4$.

$$\begin{aligned} (3 - 2)x &> 6 - 3 \\ x &> 3 \end{aligned} \qquad \text{and} \qquad \begin{aligned} (4 - 5)x &> -3 \\ -x &> -3 \\ x &< 3 \end{aligned}$$

Representing $x > 3$ and $x < 3$ on a number line,



$\therefore$ these two simultaneous linear inequalities have no solution.

$\therefore$ a set of possible values is $\boldsymbol{a = 3}$, $\boldsymbol{b = 6}$ and $\boldsymbol{c = 4}$.

16. Let the number of friends taking bubble tea be x.

Then the number of friends taking ice-cream cones is $(12 - x)$.

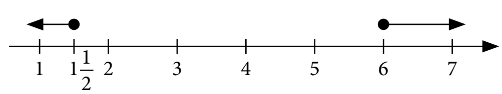
Since Cheryl intends to spend no more than \$30,

$$\begin{aligned} 3.20x + 2.40(12 - x) &\leqslant 30 \\ 3.20x + 28.80 - 2.40x &\leqslant 30 \\ 0.80x + 28.80 &\leqslant 30 \\ 0.80x + 28.80 - 28.80 &\leqslant 30 - 28.80 \\ 0.80x &\leqslant 1.20 \\ x &\leqslant \frac{1.20}{0.80} \\ x &\leqslant 1\frac{1}{2} \end{aligned}$$

Since more of Cheryl's friends want to have bubble tea than ice-cream cones,

$$\begin{aligned} x &\geqslant 12 - x \\ x + x &\geqslant 12 - x + x \\ 2x &\geqslant 12 \\ x &\geqslant \frac{12}{2} \\ x &\geqslant 6 \end{aligned}$$

Representing $x \leqslant 1\frac{1}{2}$ and $x \geqslant 6$ on a number line,



$\therefore$ these two simultaneous linear inequalities have no solution.

$\therefore$ **no**, Cheryl will not be able to give the treat.

Chapter 4 Expansion and Factorisation of Algebraic Expressions

TEACHING NOTES

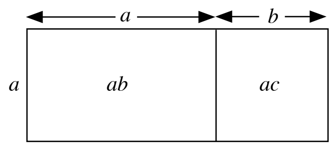
Suggested Approach

The teaching of the expansion and factorisation of algebraic expressions should focus primarily on the Concrete-Pictorial-Approach. In secondary one, students have learnt how to expand simple linear expressions using the Distributive Law. Teachers may want to show the expansion of algebraic expressions using the area of rectangles.

E.g. Expand $a(b + c)$.

Area of rectangle $= a(b + c)$

$= ab + ac$



Teachers can further reinforce the concept of expanding quadratic expressions using the area of rectangles.

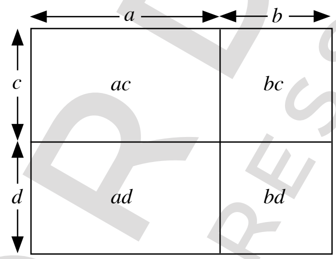
E.g. Expand $(a + b)(c + d)$.

Area of rectangle

$= (a + b)(c + d)$

$= ac + ad + bc + bd$

$= ac + ad + bc + bd$



Teachers can use the Concrete-Pictorial-Approach with algebra discs in a multiplication frame to illustrate the process of expansion and factorisation of quadratic expressions. However, teachers should guide students to progress from the 'concrete' to the 'abstract', by showing the algebraic notations next to the algebraic discs.

Section 4.1: Addition and subtraction of quadratic expressions

Students have learnt how to simplify simple linear algebraic expressions in secondary one using algebra discs (E.g. 'x' disc, '$-x$' disc, '1' disc, '-1' disc). Teachers should further introduce another two more digital algebra discs (E.g. 'x^2' disc, '$-x^2$' disc) to help students to visualise and learn how to form and simplify simple quadratic expressions. Use the Practise Now examples in the textbook.

Section 4.2 Expansion of algebraic expressions of the form $(a + b)(c + d)$

Teachers can progress from students' knowledge of expanding linear expressions like $a(b + c)$ by Distributive Law to more complex forms such as $(a + b)(c + d)$ (see Suggested Approach above). This allows students to transit to the use of multiplication frame when expanding and factorising quadratic and complex expressions in Sections 4.3 and 4.4.

Section 4.3 Expansion of quadratic and complex expressions

Teachers can begin with the expansion of simple quadratic expressions of the form $px(qx + r)$ using the algebra discs. Teachers may use the algebra discs to illustrate how the 'expanded terms' can be arranged in the rectangular array, similar to the result of the expansion of $(a + b)(c + d)$ using Distributive Law. Teachers should also highlight to students how to 'fill up' the 'terms' in the multiplication frame after the expansion in this section.

Teachers can remind students of the use of Distributive Law when expanding algebraic expressions of the form $(px + q)(rx + s)(tx + u)$.

Section 4.4 Factorisation of quadratic expressions

Most students would find factorising quadratic expressions of the form $ax^2 + bx + c$ difficult. Students should be provided with ample practice questions and the factorisation process may need to be reiterated multiple times. Teachers should begin with simple quadratic expressions (E.g. those of the form $x^2 + bx + c$) to allow students to gain confidence in obtaining the two linear factors of the quadratic expressions.

Teachers should instruct students to explore the factorisation process of simple quadratic expressions using the algebra discs (see pages 118 and 119 of the textbook). Next, without using algebra discs, teacher should illustrate to students the steps to factorising quadratic expressions using a 'Multiplication Frame' (see Page 120).

Once students have acquired the technique in factorising simple quadratic expressions, teachers can then challenge the students with more difficult quadratic expressions.

Section 4.5 Factorisation of algebraic expressions into the form $(a + b)(c + d)$

Teachers may highlight that the algebraic expressions in this section consists of two or more variables. Teachers can continue using the multiplication frame to factorise these algebraic expressions into the form $(a + b)(c + d)$.

Factorisation using the grouping method should be introduced here. This method involves rearranging the terms of the algebraic expression to identify the common factor(s) in the first two terms and another common factor(s) in the last 2 terms. For example, to factorise by grouping, we have

$ax + bx + kay + kby$
$= x(a + b) + ky(a + b)$
$= (a + b)(x + ky)$

Section 4.6: Expansion using special algebraic identities

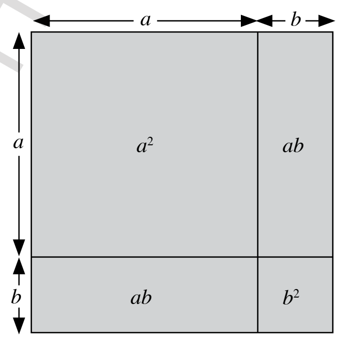
The area of squares and rectangles can be used to show the expansion of the three special algebraic identities. Teachers can also guide students to complete the investigations on pages 134, 137 and 138.

From the investigations, students should conclude that these algebraic identities known as **perfect squares**, $(a + b)^2$ and $(a - b)^2$ and the **difference of two squares** $(a + b)(a - b)$, are useful for expanding algebraic expressions.

Special Algebraic Identity 1

Expand $(a + b)^2$.

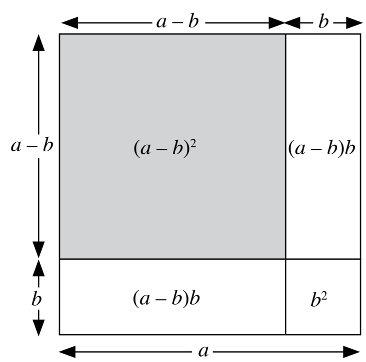
Area of square $= (a + b)^2$
$= a^2 + ab + ab + b^2$
$= a^2 + 2ab + b^2$



Special Algebraic Identity 2

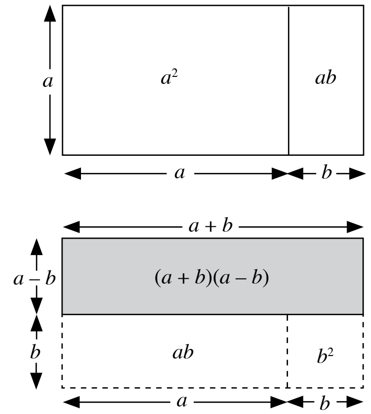
Expand $(a - b)^2$.

Area of small square
$= (a - b)^2$
$= a^2 - b^2 - 2(a - b)b$
$= a^2 - b^2 - 2ab + 2b^2$
$= a^2 - 2ab + b^2$



Special Algebraic Identity 3

Area of rectangle
$= (a + b)(a - b)$
$= (a^2 - ab) + (ab - b^2)$
$= a^2 - ab + ab - b^2$
$= a^2 - b^2$



As an additional activity, teachers may want to ask students the following:
Is $(a + b)^2 = a^2 + b^2$ and $(a - b)^2 = a^2 - b^2$? Explain your answer.

Below are some common misconceptions regarding expansion that teachers may want to remind students of.
- $(x + 2)^2 = x^2 + 4$ instead of $(x + 2)^2 = x^2 + 4x + 4$
- $(2x - 1)^2 = 4x^2 - 1$ instead of $(2x - 1)^2 = 4x^2 - 4x + 1$

Section 4.7 Factorisation using special algebraic identities

Since factorisation is the reverse of expansion, when we factorise the quadratic expression using the special algebraic identities, we have
- $a^2 + 2ab + b^2 = (a + b)^2$
- $a^2 - 2ab + b^2 = (a - b)^2$
- $a^2 - b^2 = (a + b)(a - b)$

Teachers should provide ample practice for students to check if the given quadratic expression can be factorised using the special algebraic identities. Get students to learn to identify the 'a' and 'b' terms in any given expression.

Introductory Problem

The solutions to this problem can be found in Introductory Problem Revisited (after Practise Now 15B).

4.1 Addition and subtraction of quadratic expressions

Class Discussion (Recap of algebraic expressions)

1. There are **2 variables** in the expression. The variables are $\boldsymbol{x}$ and $\boldsymbol{y}$.
2. There are **4 terms** in the above expression. The terms are $\boldsymbol{x}$, $\boldsymbol{-3xy}$, $\boldsymbol{\frac{4y}{3}}$ and $\boldsymbol{-7}$.
3. The coefficient of the variable in x is **1**.
   The coefficient of the variables in $-3xy$ is $\boldsymbol{-3}$.
   The coefficient of the variable in $\frac{4y}{3}$ is $\boldsymbol{\frac{4}{3}}$.
   The coefficient of the variable in -7 is $\boldsymbol{-7}$.
   Teachers should note that at this stage, it is still acceptable if students exclude the constant term -7 in this answer (refer to Teaching notes on page 2).
4. The last term in the expression is a **constant term**.
5. **Yes**. Examples of algebraic expressions that consist of only one term are: x, y and z.

Class Discussion (Recap of linear expressions)

(a) **Linear expression.** Not counting the constant term, it contains only one term in x.
(b) **Linear expression.** Not counting the constant term, it contains only one term in y.
(c) **Linear expression.** Not counting the constant term, it contains only one term in x.
(d) **Linear expression.** Not counting the constant term, it contains one term in x and one term in y. It is a linear expression in two variables.
(e) **Not a linear expression.** It only has a constant term.
(f) **Not a linear expression.** It contains a term in xy.
(g) **Not a linear expression.** It contains a term in x^2, which has a degree of more than 1.

Practise Now 1

(a) $x^2 + (-6x^2) = x^2 - 6x^2$
$= \boldsymbol{-5x^2}$
(b) $10x^2 + (-19x^2) = 10x^2 - 19x^2$
$= \boldsymbol{-9x^2}$
(c) $-13y^2 + 3y^2 = \boldsymbol{-10y^2}$
(d) $-28y^2 + 15y^2 = \boldsymbol{-13y^2}$
(e) $-8w^2 - 4w^2 = \boldsymbol{-12w^2}$
(f) $-11w^2 - 12w^2 = \boldsymbol{-23w^2}$
(g) $-x^2 - (-30x^2) = -x^2 + 30x^2$
$= \boldsymbol{29x^2}$
(h) $-25x^2 - (-15x^2) = -25x^2 + 15x^2$
$= \boldsymbol{-10x^2}$

Practise Now 2

(a) $-5x^2 + (-2x^2) + 3 - 7 = -5x^2 - 2x^2 + 3 - 7$
$= \boldsymbol{-7x^2 - 4}$
(b) $8x^2 + (-6x^2) + 4x - 9x = 8x^2 - 6x^2 + 4x - 9x$
$= \boldsymbol{2x^2 - 5x}$
(c) $-4y^2 - yx + 3y^2 - (-5xy) = -4y^2 + 3y^2 - xy - (-5xy)$
$= -y^2 - xy + 5xy$
$= \boldsymbol{-y^2 + 4xy}$
(d) $-7y^2 - 9y - 5y^2 - (-3xy) = -7y^2 - 5y^2 - 9y - (-3xy)$
$= \boldsymbol{-12y^2 - 9y + 3xy}$
(e) $10a^2 + (-12b^2) - 9 - (-3b^2) + 5 + (-6a^2)$
$= 10a^2 + (-6a^2) + (-12b^2) - (-3b^2) - 9 + 5$
$= 10a^2 - 6a^2 - 12b^2 + 3b^2 - 4$
$= \boldsymbol{4a^2 - 9b^2 - 4}$
(f) $16h^2 - (-5k^2) - 18hk + 3h^2 + (-k^2) + 15kh$
$= 16h^2 + 3h^2 - (-5k^2) + (-k^2) - 18hk + 15hk$
$= 19h^2 + 5k^2 - k^2 - 3hk$
$= \boldsymbol{19h^2 + 4k^2 - 3hk}$

4.2 Expansion of algebraic expressions of the form $(a + b)(c + d)$

Practise Now 3

(a) $2(x + 5) = \boldsymbol{2x + 10}$
(b) $-3(6x - y) = \boldsymbol{-18x + 3y}$
(c) $5 - a(-2b + 3c) = \boldsymbol{5 + 2ab - 3ac}$
(d) $-4 - 2a(-7x - 6y) = \boldsymbol{-4 + 14ax + 12ay}$

Practise Now 4

(a) $-3x(y + 4z) - 5x(2y - z) = -3xy - 12xz - 10xy + 5xz$
$= -3xy - 10xy - 12xz + 5xz$
$= \boldsymbol{-13xy - 7xz}$
(b) $2p(-4q - 3r) - 6q(3p + 2r) = -8pq - 6pr - 18pq - 12qr$
$= -8pq - 18pq - 6pr - 12qr$
$= \boldsymbol{-26pq - 6pr - 12qr}$

Practise Now 5

(a) $(a + b)(8x + 7y) = \boldsymbol{8ax + 7ay + 8bx + 7by}$
(b) $(2c + d)(5x + 9y) = \boldsymbol{10cx + 18cy + 5dx + 9dy}$
(c) $(5a + 2)(x - 2y) = \boldsymbol{5ax - 10ay + 2x - 4y}$
(d) $(6a + 5b)(3c - d) = \boldsymbol{18ac - 6ad + 15bc - 5bd}$
(e) $(x - 4y)(2c + 3d) = \boldsymbol{2cx + 3dx - 8cy - 12dy}$
(f) $(7x - 1)(3a + 2b) = \boldsymbol{21ax + 14bx - 3a - 2b}$
(g) $(6p - 5q)(3r - 4s) = \boldsymbol{18pr - 24ps - 15qr + 20qs}$
(h) $(2p - 9q)(7x - 3y) = \boldsymbol{14px - 6py - 63qx + 27qy}$
(i) $(-3a - 5b)(-7c + 3d) = \boldsymbol{21ac - 9ad + 35bc - 15bd}$
(j) $(-4r - 3s)(3 - 2t - 5u) = \boldsymbol{-12r + 8rt + 20ru - 9s + 6st + 15su}$

Practise Now 6

(a) $2ac - (3a + b)(c - 4d) = 2ac - (3ac - 12ad + bc - 4bd)$
$= 2ac - 3ac + 12ad - bc + 4bd$
$= -ac + 12ad - bc + 4bd$
$= \boldsymbol{12ad - ac - bc + 4bd}$

(b) $2x(3y - 4z) - (3x + y)(y - 3z) = 6xy - 8xz - (3xy - 9xz + y^2 - 3yz)$
$= 6xy - 8xz - 3xy + 9xz - y^2 + 3yz$
$= -y^2 + 6xy - 3xy - 8xz + 9xz + 3yz$
$= \mathbf{-y^2 + 3xy + xz + 3yz}$

(c) $(3p - q)(2r + s) - (p - 2q)(5r - 4s)$
$= 6pr + 3ps - 2qr - qs - (5pr - 4ps - 10qr + 8qs)$
$= 6pr + 3ps - 2qr - qs - 5pr + 4ps + 10qr - 8qs$
$= 6pr - 5pr + 3ps + 4ps - 2qr + 10qr - qs - 8qs$
$= \mathbf{pr + 7ps + 8qr - 9qs}$

(d) $(h + 6k)(2m - h) + (3h - 2m)(2k + h)$
$= 2hm - h^2 + 12km - 6hk + 6hk + 3h^2 - 4km - 2hm$
$= -h^2 + 3h^2 - 6hk + 6hk + 2hm - 2hm + 12km - 4km$
$= \mathbf{2h^2 + 8km}$

Exercise 4A

1. **(a)** $2x^2 + (-11x^2) = 2x^2 - 11x^2$
$= \mathbf{-9x^2}$

(b) $5x^2 - 17x^2 = \mathbf{-12x^2}$

(c) $-6y^2 + 15y^2 = \mathbf{9y^2}$

(d) $-30y^2 + 14y^2 = \mathbf{-16y^2}$

(e) $-3e^2 - 10e^2 = \mathbf{-13e^2}$

(f) $-12f^2 - 19f^2 = \mathbf{-31f^2}$

(g) $-20g^2 - (-21g^2) = -20g^2 + 21g^2$
$= \mathbf{g^2}$

(h) $-18h^2 - (-5h^2) = -18h^2 + 5h^2$
$= \mathbf{-13h^2}$

2. **(a)** $-3x^2 + (-7x^2) + 9 - 18 = -3x^2 - 7x^2 + 9 - 18$
$= \mathbf{-10x^2 - 9}$

(b) $14x^2 - 15x^2 + 8x - 10x = \mathbf{-x^2 - 2x}$

(c) $6y^2 + 19z + 9y^2 - 8yz = 6y^2 + 9y^2 + 19z - 8yz$
$= \mathbf{15y^2 + 19z - 8yz}$

(d) $5y^2 - xy - y^2 - (-10yx) = 5y^2 - y^2 - xy - (-10xy)$
$= 5y^2 - y^2 - xy + 10xy$
$= \mathbf{4y^2 + 9xy}$

(e) $w^2 + 2w - 7 - (-11w^2) - 5w - 1$
$= w^2 - (-11w^2) + 2w - 5w - 7 - 1$
$= w^2 + 11w^2 + 2w - 5w - 7 - 1$
$= \mathbf{12w^2 - 3w - 8}$

(f) $-4h^2 - 9k^2 - (-2hk) + 3h^2 - 7k^2 + 2kh$
$= -4h^2 + 3h^2 - 9k^2 - 7k^2 - (-2hk) + 2hk$
$= -4h^2 + 3h^2 - 9k^2 - 7k^2 + 2hk + 2hk$
$= \mathbf{-h^2 - 16k^2 + 4hk}$

3. **(a)** $10(x + 1) = \mathbf{10x + 10}$

(b) $-4(3x - 2y) = \mathbf{-12x + 8y}$

(c) $8x(y - 1) = \mathbf{8xy - 8x}$

(d) $-9x(3y - 2z) = \mathbf{-27xy + 18xz}$

(e) $2 + 3a(5 - 11b) = \mathbf{2 + 15a - 33ab}$

(f) $-5 - 3c(2d + 3e) = \mathbf{-5 - 6cd - 9ce}$

(g) $7 - 6p(7q - 3r) = \mathbf{7 - 42pq + 18pr}$

(h) $11 - 8s(-12t - 7u) = \mathbf{11 + 96st + 56su}$

4. **(a)** $5x(y + 6z) - 2x(2y + 10z) = 5xy + 30xz - 4xy - 20xz$
$= 5xy - 4xy + 30xz - 20xz$
$= \mathbf{xy + 10xz}$

(b) $4a(b - 5c) + 2a(3b - 7c) = 4ab - 20ac + 6ab - 14ac$
$= 4ab + 6ab - 20ac - 14ac$
$= \mathbf{10ab - 34ac}$

(c) $7d(3e - 4f) - 4d(3e - 2f) = 21de - 28df - 12de + 8df$
$= 21de - 12de - 28df + 8df$
$= \mathbf{9de - 20df}$

(d) $-3h(3k - 4m) - 8h(2k + 3m) = -9hk + 12hm - 16hk - 24hm$
$= -9hk - 16hk + 12hm - 24hm$
$= \mathbf{-25hk - 12hm}$

5. **(a)** $(a + b)(4x + 9y) = \mathbf{4ax + 9ay + 4bx + 9by}$

(b) $(5c + d)(5e + 2f) = \mathbf{25ce + 10cf + 5de + 2df}$

(c) $(7m + 3)(n - 3p) = \mathbf{7mn - 21mp + 3n - 9p}$

(d) $(3t - 7u)(7v + 4w) = \mathbf{21tv + 12tw - 49uv - 28uw}$

(e) $(2a - b)(x - 6y) = \mathbf{2ax - 12ay - bx + 6by}$

(f) $(3h - 5k)(-q - 7r) = \mathbf{-3hq - 21hr + 5kq + 35kr}$

6. **(a)** $3ac - (2a + b)(c + 3d) = 3ac - (2ac + 6ad + bc + 3bd)$
$= 3ac - 2ac - 6ad - bc - 3bd$
$= \mathbf{ac - 6ad - bc - 3bd}$

(b) $2xy + (x - 5a)(6y + 7b) = 2xy + 6xy + 7bx - 30ay - 35ab$
$= \mathbf{8xy + 7bx - 30ay - 35ab}$

(c) $9ps + (2p - 3r)(4q - 5s) = 9ps + 8pq - 10ps - 12qr + 15rs$
$= 8pq + 9ps - 10ps - 12qr + 15rs$
$= \mathbf{8pq - ps - 12qr + 15rs}$

(d) $10hk - (-3m - h)(8k - 3n) = 10hk - (-24km + 9mn - 8hk + 3hn)$
$= 10hk + 8hk + 24km - 9mn - 3hn$
$= \mathbf{18hk + 24km - 9mn - 3hn}$

7. **(a)** $3a(5b + c) - 2b(3c + a) = 15ab + 3ac - 6bc - 2ab$
$= 15ab - 2ab + 3ac - 6bc$
$= \mathbf{13ab + 3ac - 6bc}$

(b) $-2d(4f - 5h) - 8f(3d + 7h) = -8df + 10dh - 24df - 56hf$
$= 10dh - 8df - 24df - 56hf$
$= \mathbf{10dh - 32df - 56hf}$

(c) $4k(13m - 5n) - 13m(4k - 5n) = 52km - 20kn - 52km + 65mn$
$= 65mn + 52km - 52km - 20kn$
$= \mathbf{65mn - 20kn}$

(d) $-6w(7x - 12y) - 4y(11w - 9x) = -42wx + 72wy - 44wy + 36xy$
$= 72wy - 44wy - 42wx + 36xy$
$= \mathbf{28wy - 42wx + 36xy}$

8. **(a)** $(x + 9y)(a + 3b + 1) = \mathbf{ax + 3bx + x + 9ay + 27by + 9y}$

(b) $(2p + 5q)(7 - r + 5s) = \mathbf{14p - 2pr + 10ps + 35q - 5qr + 25qs}$

(c) $(11m - 12n)(4t - 3u - 10)$
$= \mathbf{44mt - 33mu - 110m - 48nt + 36nu + 120n}$

(d) $(-5w - 14y)(-2v - 9x - 6z)$
$= \mathbf{10vw + 45wx + 30wz + 28vy + 126xy + 84yz}$

9. **(a)** $2a(5b + 4c) - (2a + c)(3b - 5c)$
$= 10ab + 8ac - (6ab - 10ac + 3bc - 5c^2)$
$= 10ab + 8ac - 6ab + 10ac - 3bc + 5c^2$
$= 5c^2 + 10ab - 6ab + 8ac + 10ac - 3bc$
$= \mathbf{5c^2 + 4ab + 18ac - 3bc}$

(b) $(7x - 3y)(w - 4z) + (z - 2w)(5x - 9y)$
$= 7wx - 28xz - 3wy + 12yz + 5xz - 9yz - 10wx + 18wy$
$= -3wy + 18wy + 7wx - 10wx - 28xz + 5xz + 12yz - 9yz$
$= \mathbf{15wy - 3wx - 23xz + 3yz}$

(c) $(10p + q)(3r + 2q) - (5p - 4q)(-r - 6q)$
$= 30pr + 20pq + 3qr + 2q^2 - (-5pr - 30pq + 4qr + 24q^2)$
$= 30pr + 20pq + 3qr + 2q^2 + 5pr + 30pq - 4qr - 24q^2$
$= 2q^2 - 24q^2 + 30pr + 5pr + 20pq + 30pq + 3qr - 4qr$
$= \mathbf{-22q^2 + 35pr + 50pq - qr}$

(d) $(4h - 11k)(2m - 13h) + (-13h - 12m)(8k + 9h)$
$= 8hm - 52h^2 - 22km + 143hk - 104hk - 117h^2 - 96km - 108hm$
$= -52h^2 - 117h^2 + 8hm - 108hm - 22km - 96km + 143hk - 104hk$
$= \mathbf{-169h^2 - 100hm - 118km + 39hk}$

10. $(-13b - 3c)(20 - 11a - 7d) - (11a - 2b)(4c + 6d + 15)$
$= -260b + 143ab + 91bd - 60c + 33ac + 21cd - (44ac + 66ad + 165a - 8bc - 12bd - 30b)$
$= -260b + 143ab + 91bd - 60c + 33ac + 21cd - 44ac - 66ad - 165a + 8bc + 12bd + 30b$
$= 143ab - 260b + 30b + 91bd + 12bd + 33ac - 44ac - 60c + 21cd - 66ad - 165a + 8bc$
$= \mathbf{143ab - 230b + 103bd - 11ac - 60c + 21cd - 66ad - 165a + 8bc}$

11. Actual land used for planting crops
$= (2x + 3y)(5w - 8z) + (3z - 10w)(-4x - 9y)$
$= 10wx - 16xz + 15wy - 24yz - 12xz - 27yz + 40wx + 90wy$
$= 10wx + 40wx + 15wy + 90wy - 16xz - 12xz - 24yz - 27yz$
$= 50wx + 105wy - 28xz - 51yz$
Albert says that the total land used to plant crops
$= 10xw - 24yz - 12xz + 90wy$
$= 10wx + 90wy - 12xz - 24yz$
I **do not agree** with Albert since
$50wx + 105wy - 28xz - 51yz \neq 10wx + 90wy - 12xz - 24yz$.

4.3 Expansion of quadratic expressions

Investigation (Expansion of expressions of the form $p(qx + r)$ and $px(qx + r)$)

(a)

×	x	1	1	1	1
1	x	1	1	1	1
1	x	1	1	1	1
1	x	1	1	1	1

$\therefore 3(x + 4) = \mathbf{3x + 12}$

(b)

×	x	x	x	-1
-1	$-x$	$-x$	$-x$	1
-1	$-x$	$-x$	$-x$	1

$\therefore -2(3x - 1) = \mathbf{-6x + 2}$

(c)

×	1	1	1	$-x$	$-x$
-1	-1	-1	-1	x	x
-1	-1	-1	-1	x	x
-1	-1	-1	-1	x	x
-1	-1	-1	-1	x	x

$\therefore -4(3 - 2x) = -12 + 8x$
$= \mathbf{8x - 12}$

(d)

×	x	1	1	1
x	x^2	x	x	x
x	x^2	x	x	x

$\therefore 2x(x + 3) = \mathbf{2x^2 + 6x}$

(e)

×	x	x	1	1	1	1	1
$-x$	$-x^2$	$-x^2$	$-x$	$-x$	$-x$	$-x$	$-x$
$-x$	$-x^2$	$-x^2$	$-x$	$-x$	$-x$	$-x$	$-x$
$-x$	$-x^2$	$-x^2$	$-x$	$-x$	$-x$	$-x$	$-x$

$\therefore -3x(2x + 5) = \mathbf{-6x^2 - 15x}$

(f)

×	1	$-x$	$-x$	$-x$
$-x$	$-x$	x^2	x^2	x^2
$-x$	$-x$	x^2	x^2	x^2
$-x$	$-x$	x^2	x^2	x^2
$-x$	$-x$	x^2	x^2	x^2

$\therefore -4x(1 - 3x) = -4x + 12x^2$
$= \mathbf{12x^2 - 4x}$

Practise Now 7

(a) $4x(2x + 3) = \mathbf{8x^2 + 12x}$
(b) $11a(4 - a) = 44a - 11a^2$
$= \mathbf{-11a^2 + 44a}$
(c) $-5x(3x + 4) = \mathbf{-15x^2 - 20x}$
(d) $-n(12n - 29) = \mathbf{-12n^2 + 29n}$

Practise Now 8

(a) $x(7x - 4) - 3(x + 2) = 7x^2 - 4x - 3x - 6$
$= \mathbf{7x^2 - 7x - 6}$
(b) $-2x(x - 8) - 5(3x - 4) = -2x^2 + 16x - 15x + 20$
$= \mathbf{-2x^2 + x + 20}$
(c) $-(5y + 8) - 3y(4 - 9y) = -5y - 8 - 12y + 27y^2$
$= 27y^2 - 5y - 12y - 8$
$= \mathbf{27y^2 - 17y - 8}$
(d) $-6k(7 - k) + 5k(-2k - 3) = -42k + 6k^2 - 10k^2 - 15k$
$= 6k^2 - 10k^2 - 42k - 15k$
$= \mathbf{-4k^2 - 57k}$

Practise Now 9

(a) $xy(yz + x^2 - xy) = \mathbf{xy^2z + x^3y - x^2y^2}$
(b) $h^2(km + m) - m(h^2m - h^2k) = h^2km + h^2m - h^2m^2 + h^2km$
$= h^2km + h^2km + h^2m - h^2m^2$
$= \mathbf{2h^2km + h^2m - h^2m^2}$

Investigation (Expansion of expressions of the form $(px + q)(rx + s)$)

(a)

$\times$	x	1	1	1	1
x	x^2	x	x	x	x
1	x	1	1	1	1
1	x	1	1	1	1
1	x	1	1	1	1

$\therefore (x + 3)(x + 4) = x^2 + 4x + 3x + 12$
$= \mathbf{x^2 + 7x + 12}$

(b)

$\times$	x	-1	-1	-1
x	x^2	$-x$	$-x$	$-x$
x	x^2	$-x$	$-x$	$-x$
1	x	-1	-1	-1

$\therefore (2x + 1)(x - 3) = 2x^2 - 6x + x - 3$
$= \mathbf{2x^2 - 5x - 3}$

(c)

$\times$	y	y	-1
y	y^2	y^2	$-y$
y	y^2	y^2	$-y$
y	y^2	y^2	$-y$
-1	$-y$	$-y$	1
-1	$-y$	$-y$	1
-1	$-y$	$-y$	1
-1	$-y$	$-y$	1

$\therefore (3y - 4)(2y - 1) = 6y^2 - 3y - 8y + 4$
$= \mathbf{6y^2 - 11y + 4}$

(d) $(3 - 2x)(3x + 1) = (-2x + 3)(3x + 1)$

$\times$	x	x	x	1
$-x$	$-x^2$	$-x^2$	$-x^2$	$-x$
$-x$	$-x^2$	$-x^2$	$-x^2$	$-x$
1	x	x	x	1
1	x	x	x	1
1	x	x	x	1

$\therefore (3 - 2x)(3x + 1) = (-2x + 3)(3x + 1)$
$= -6x^2 - 2x + 9x + 3$
$= \mathbf{-6x^2 + 7x + 3}$

(e) $(3 - 2p)(4 - 3p) = (-2p + 3)(-3p + 4)$

$\times$	$-p$	$-p$	$-p$	1	1	1	1
$-p$	p^2	p^2	p^2	$-p$	$-p$	$-p$	$-p$
$-p$	p^2	p^2	p^2	$-p$	$-p$	$-p$	$-p$
1	$-p$	$-p$	$-p$	1	1	1	1
1	$-p$	$-p$	$-p$	1	1	1	1
1	$-p$	$-p$	$-p$	1	1	1	1

$\therefore (3 - 2p)(4 - 3p) = (-2p + 3)(-3p + 4)$
$= 6p^2 - 8p - 9p + 12$
$= \mathbf{6p^2 - 17p + 12}$

(f)

$\times$	x	x	1	1	1
$-x$	$-x^2$	$-x^2$	$-x$	$-x$	$-x$
$-x$	$-x^2$	$-x^2$	$-x$	$-x$	$-x$
$-x$	$-x^2$	$-x^2$	$-x$	$-x$	$-x$
-1	$-x$	$-x$	-1	-1	-1
-1	$-x$	$-x$	-1	-1	-1

$\therefore (-3x - 2)(2x + 3) = -6x^2 - 9x - 4x - 6$
$= \mathbf{-6x^2 - 13x - 6}$

Practise Now 10

(a)

$\times$	$4x$	$+7$
$3x$	$12x^2$	$+21x$
$+5$	$+20x$	$+35$

$(4x + 7)(3x + 5) = 12x^2 + 21x + 20x + 35$
$= \mathbf{12x^2 + 41x + 35}$

(b)

$\times$	x	$+6$
$9x$	$9x^2$	$+54x$
-4	$-4x$	-24

$(9x - 4)(x + 6) = 9x^2 + 54x - 4x - 24$
$= \mathbf{9x^2 + 50x - 24}$

(c)

$\times$	2	$-7y$
$3y$	$6y$	$-21y^2$
$+11$	$+22$	$-77y$

$(3y + 11)(2 - 7y) = 6y - 21y^2 + 22 - 77y$
$= -21y^2 + 6y - 77y + 22$
$= \mathbf{-21y^2 - 71y + 22}$

(d)

$\times$	4	$-7k$
5	20	$-35k$
$-3k$	$-12k$	$+21k^2$

$(5 - 3k)(4 - 7k) = 20 - 35k - 12k + 21k^2$
$= 21k^2 - 35k - 12k + 20$
$= \mathbf{21k^2 - 47k + 20}$

Practise Now 11

(a) $(3x - 2)(x + 4) - 5x(x - 3)$
$= 3x^2 + 12x - 2x - 8 - 5x^2 + 15x$
$= 3x^2 - 5x^2 + 12x + 15x - 2x - 8$
$= \mathbf{-2x^2 + 25x - 8}$

(b) $(5y - 1)(y + 6) + 3(4y - 5)(9 - 2y)$
$= 5y^2 + 30y - y - 6 + 3(36y - 8y^2 - 45 + 10y)$
$= 5y^2 + 30y - y - 6 + 108y - 24y^2 - 135 + 30y$
$= 5y^2 - 24y^2 + 30y - y + 108y + 30y - 6 - 135$
$= \mathbf{-19y^2 + 167y - 141}$

Practise Now 12

1. $(2x - 7y)(5x + y) = 10x^2 + 2xy - 35yx - 7y^2$
$= \mathbf{10x^2 - 33xy - 7y^2}$

2. $(3w + 5v)(2v - 5w) - 6w(w - 2v)$
$= 6wv - 15w^2 + 10v^2 - 25vw - 6w^2 + 12wv$
$= 10v^2 + 6vw - 25vw + 12vw - 15w^2 - 6w^2$
$= \mathbf{10v^2 - 7vw - 21w^2}$

Practise Now 13

(a) $(x + 1)(2 + 3x)(2 + x)$
$= (2x + 3x^2 + 2 + 3x)(2 + x)$
$= (3x^2 + 5x + 2)(2 + x)$
$= 6x^2 + 3x^3 + 10x + 5x^2 + 4 + 2x$
$= \mathbf{3x^3 + 11x^2 + 12x + 4}$

(b) $(p + 2q)(p + q)(2p - 3q)$
$= (p^2 + pq + 2qp + 2q^2)(2p - 3q)$
$= (p^2 + pq + 2pq + 2q^2)(2p - 3q)$
$= (p^2 + 3pq + 2q^2)(2p - 3q)$
$= 2p^3 - 3p^2q + 6p^2q - 9pq^2 + 4pq^2 - 6q^3$
$= \mathbf{2p^3 + 3p^2q - 5pq^2 - 6q^3}$

Exercise 4B

1. **(a)** $5a(3a - 4) = \mathbf{15a^2 - 20a}$
(b) $-8b(3b + 5) = \mathbf{-24b^2 - 40b}$
(c) $-5n(2 - 3n) = -10n + 15n^2$
$= \mathbf{15n^2 - 10n}$
(d) $-m(-m - 1) = \mathbf{m^2 + m}$

2. **(a)** $4(2a + 3) + 5a(a + 3)$
$= 8a + 12 + 5a^2 + 15a$
$= 5a^2 + 8a + 15a + 12$
$= \mathbf{5a^2 + 23a + 12}$
(b) $9b(5 - 2b) + 3(6 - 5b)$
$= 45b - 18b^2 + 18 - 15b$
$= -18b^2 + 45b - 15b + 18$
$= \mathbf{-18b^2 + 30b + 18}$
(c) $c(3c + 1) - 2c(c + 3)$
$= 3c^2 + c - 2c^2 - 6c$
$= 3c^2 - 2c^2 - 6c + c$
$= \mathbf{c^2 - 5c}$
(d) $-6d(5d - 4) + 2d(3d - 2)$
$= -30d^2 + 24d + 6d^2 - 4d$
$= -30d^2 + 6d^2 - 4d + 24d$
$= \mathbf{-24d^2 + 20d}$

3. **(a)** $-3a(2a + 3b^2) = \mathbf{-6a^2 - 9ab^2}$
(b) $-4c(2c^2 - 5cd) = \mathbf{-8c^3 + 20c^2d}$
(c) $-hk(7k - 3h) = \mathbf{-7hk^2 + 3h^2k}$
(d) $5xy(x - 4yz) = \mathbf{5x^2y - 20xy^2z}$

4. **(a)** $4k(3k + m) - 3k(2k - 5m)$
$= 12k^2 + 4km - 6k^2 + 15km$
$= 12k^2 - 6k^2 + 4km + 15km$
$= \mathbf{6k^2 + 19km}$
(b) $n(p - 2n) - 4n(n - 2p)$
$= np - 2n^2 - 4n^2 + 8np$
$= -2n^2 - 4n^2 + np + 8np$
$= \mathbf{-6n^2 + 9np}$
(c) $3w(wt - 2t^2) + t(3wt - 4w^2)$
$= 3w^2t - 6wt^2 + 3wt^2 - 4w^2t$
$= -6t^2w + 3t^2w + 3tw^2 - 4tw^2$
$= \mathbf{-3t^2w - tw^2}$
(d) $2x(-y - xy^2) - y(-2x + 3x^2y)$
$= -2xy - 2x^2y^2 + 2xy - 3x^2y^2$
$= -2x^2y^2 - 3x^2y^2 - 2xy + 2xy$
$= \mathbf{-5x^2y^2}$

5. **(a)** $(x + 3)(x + 7) = x^2 + 7x + 3x + 21$
$= \mathbf{x^2 + 10x + 21}$
(b) $(4y + 1)(3y + 5) = 12y^2 + 20y + 3y + 5$
$= \mathbf{12y^2 + 23y + 5}$
(c) $(t + 1)(t - 8) = t^2 - 8t + t - 8$
$= \mathbf{t^2 - 7t - 8}$
(d) $(5 - v)(7 - v) = 35 - 5v - 7v + v^2$
$= v^2 - 5v - 7v + 35$
$= \mathbf{v^2 - 12v + 35}$

6. **(a)** $(x + y)(x + 6y) = x^2 + 6xy + yx + 6y^2$
$= \mathbf{x^2 + 7xy + 6y^2}$
(b) $(x + 3y)(x - 5y) = x^2 - 5xy + 3yx - 15y^2$
$= \mathbf{x^2 - 2xy - 15y^2}$
(c) $(3c + 7d)(c - 2d) = 3c^2 - 6cd + 7dc - 14d^2$
$= \mathbf{3c^2 + cd - 14d^2}$
(d) $(3k - 5h)(-h - 7k) = -3kh - 21k^2 + 5h^2 + 35hk$
$= 5h^2 - 3hk + 35hk - 21k^2$
$= \mathbf{5h^2 + 32hk - 21k^2}$

7. **(a)** $(a + 1)(a + 2)(a + 3)$
$= (a^2 + 2a + a + 2)(a + 3)$
$= (a^2 + 3a + 2)(a + 3)$
$= a^3 + 3a^2 + 3a^2 + 9a + 2a + 6$
$= \mathbf{a^3 + 6a^2 + 11a + 6}$
(b) $(1 + b)(b - 4)(5 + b)$
$= (b - 4 + b^2 - 4b)(5 + b)$
$= (b^2 - 3b - 4)(5 + b)$
$= 5b^2 + b^3 - 15b - 3b^2 - 20 - 4b$
$= \mathbf{b^3 + 2b^2 - 19b - 20}$
(c) $(m - n)(3m + 2n)(2m - 3n)$
$= (3m^2 + 2mn - 3mn - 2n^2)(2m - 3n)$
$= (3m^2 - mn - 2n^2)(2m - 3n)$
$= 6m^3 - 9m^2n - 2m^2n + 3mn^2 - 4mn^2 + 6n^3$
$= \mathbf{6m^3 - 11m^2n - mn^2 + 6n^3}$
(d) $(x - 6y)(4x - y)(3x - 4y)$
$= (4x^2 - xy - 24xy + 6y^2)(3x - 4y)$
$= (4x^2 - 25xy + 6y^2)(3x - 4y)$
$= 12x^3 - 16x^2y - 75x^2y + 100xy^2 + 18xy^2 - 24y^3$
$= \mathbf{12x^3 - 91x^2y + 118xy^2 - 24y^3}$

8. **(a)** $7a(2a + 1) - 4(8a + 3) = 14a^2 + 7a - 32a - 12$
$= \mathbf{14a^2 - 25a - 12}$
(b) $3(2b - 1) - 2b(5b - 3) = 6b - 3 - 10b^2 + 6b$
$= -10b^2 + 6b + 6b - 3$
$= \mathbf{-10b^2 + 12b - 3}$
(c) $3c(5 + c) - 2c(3c - 7) = 15c + 3c^2 - 6c^2 + 14c$
$= 3c^2 - 6c^2 + 15c + 14c$
$= \mathbf{-3c^2 + 29c}$

(d) $2d(3d - 5) - d(2 - d) = 6d^2 - 10d - 2d + d^2$
$= 6d^2 + d^2 - 10d - 2d$
$= \mathbf{7d^2 - 12d}$

(e) $-f(9 - 2f) + 4f(f - 8) = -9f + 2f^2 + 4f^2 - 32f$
$= 2f^2 + 4f^2 - 9f - 32f$
$= \mathbf{6f^2 - 41f}$

(f) $-2h(3 + 4h) - 5h(h - 1) = -6h - 8h^2 - 5h^2 + 5h$
$= -8h^2 - 5h^2 - 6h + 5h$
$= \mathbf{-13h^2 - h}$

9. (a) $13x^2y(3xy - y) = \mathbf{39x^3y^2 - 13x^2y^2}$

(b) $-8mn(-12m + nw - 7n^2) = \mathbf{96m^2n - 8mn^2w + 56mn^3}$

(c) $2p(3p + q^2p^2 + 7qr^3) = 6p^2 + 2q^2p^3 + 14pqr^3$
$= \mathbf{6p^2 + 2p^3q^2 + 14pqr^3}$

(d) $-7s^2t(s - 4t^2 - 3su^3) = \mathbf{-7s^3t + 28s^2t^3 + 21s^3tu^3}$

10. (a) $2x^2(z - yz - xz) + 3z(xz - x^2y + 2x^3)$
$= 2x^2z - 2x^2yz - 2x^3z + 3xz^2 - 3zx^2y + 6zx^3$
$= 2x^2z - 2x^2yz - 3x^2yz - 2x^3z + 6x^3z + 3xz^2$
$= \mathbf{2x^2z - 5x^2yz + 4x^3z + 3xz^2}$

(b) $ab(ac + b^2 - c^2) - bc(a^2 - 2ac - 3ab^2)$
$= a^2bc + ab^3 - abc^2 - bca^2 + 2bac^2 + 3cab^3$
$= a^2bc - a^2bc + ab^3 - abc^2 + 2abc^2 + 3ab^3c$
$= \mathbf{ab^3 + abc^2 + 3ab^3c}$

11. (a) $(2a + 1)(3a - 9) = 6a^2 - 18a + 3a - 9$
$= \mathbf{6a^2 - 15a - 9}$

(b) $(5b - 2)(5b + 7) = 25b^2 + 35b - 10b - 14$
$= \mathbf{25b^2 + 25b - 14}$

(c) $(4c - 5)(7c - 10) = 28c^2 - 40c - 35c + 50$
$= \mathbf{28c^2 - 75c + 50}$

(d) $(3d + 14)(5 - 2d) = 15d - 6d^2 + 70 - 28d$
$= -6d^2 + 15d - 28d + 70$
$= \mathbf{-6d^2 - 13d + 70}$

(e) $(1 - f)(17f + 16) = 17f + 16 - 17f^2 - 16f$
$= -17f^2 + 17f - 16f + 16$
$= \mathbf{-17f^2 + f + 16}$

(f) $(19 - 3h)(10 - 9h) = 190 - 171h - 30h + 27h^2$
$= 27h^2 - 171h - 30h + 190$
$= \mathbf{27h^2 - 201h + 190}$

12. (a) $5 + (x + 1)(x + 3) = 5 + x^2 + 3x + x + 3$
$= x^2 + 3x + x + 5 + 3$
$= \mathbf{x^2 + 4x + 8}$

(b) $3y + (y + 7)(2y - 1) = 3y + 2y^2 - y + 14y - 7$
$= 2y^2 + 3y - y + 14y - 7$
$= \mathbf{2y^2 + 16y - 7}$

(c) $(3t + 2)(t - 9) + 2t(4t + 1)$
$= 3t^2 - 27t + 2t - 18 + 8t^2 + 2t$
$= 3t^2 + 8t^2 - 27t + 2t + 2t - 18$
$= \mathbf{11t^2 - 23t - 18}$

(d) $(w - 3)(w - 8) + (w - 4)(2w + 9)$
$= w^2 - 8w - 3w + 24 + 2w^2 + 9w - 8w - 36$
$= w^2 + 2w^2 - 8w - 3w + 9w - 8w + 24 - 36$
$= \mathbf{3w^2 - 10w - 12}$

13. (a) $4a^2 - (3a - 4)(2a + 1) = 4a^2 - (6a^2 + 3a - 8a - 4)$
$= 4a^2 - 6a^2 - 3a + 8a + 4$
$= \mathbf{-2a^2 + 5a + 4}$

(b) $2b(b - 6) - (2b + 5)(7 - b)$
$= 2b^2 - 12b - (14b - 2b^2 + 35 - 5b)$
$= 2b^2 - 12b - 14b + 2b^2 - 35 + 5b$
$= 2b^2 + 2b^2 - 12b - 14b + 5b - 35$
$= \mathbf{4b^2 - 21b - 35}$

(c) $(4c - 3)(c + 2) - (3c - 5)(-c - 9)$
$= 4c^2 + 8c - 3c - 6 - (-3c^2 - 27c + 5c + 45)$
$= 4c^2 + 8c - 3c - 6 + 3c^2 + 27c - 5c - 45$
$= 4c^2 + 3c^2 + 8c - 3c + 27c - 5c - 6 - 45$
$= \mathbf{7c^2 + 27c - 51}$

(d) $(2d + 3)(5d - 2) - 2(5d - 3)(d + 1)$
$= 10d^2 - 4d + 15d - 6 - 2(5d^2 + 5d - 3d - 3)$
$= 10d^2 - 4d + 15d - 6 - 10d^2 - 10d + 6d + 6$
$= 10d^2 - 10d^2 - 4d + 15d - 10d + 6d - 6 + 6$
$= \mathbf{7d}$

14. (a) $(x^2 + 2)(x + 5) = \mathbf{x^3 + 5x^2 + 2x + 10}$

(b) $(2x - 3y)(x + 5y - 2) = 2x^2 + 10xy - 4x - 3yx - 15y^2 + 6y$
$= 2x^2 + 10xy - 3xy - 4x - 15y^2 + 6y$
$= \mathbf{2x^2 + 7xy - 4x - 15y^2 + 6y}$

(c) $(x + 2)(x^2 + x + 1) = x^3 + x^2 + x + 2x^2 + 2x + 2$
$= x^3 + x^2 + 2x^2 + x + 2x + 2$
$= \mathbf{x^3 + 3x^2 + 3x + 2}$

(d) $(3x^2 - 3x + 4)(3 - x) = 9x^2 - 3x^3 - 9x + 3x^2 + 12 - 4x$
$= -3x^3 + 9x^2 + 3x^2 - 9x - 4x + 12$
$= \mathbf{-3x^3 + 12x^2 - 13x + 12}$

15. (a) $5x(x - 6y) + (x + 3y)(3x - 4y)$
$= 5x^2 - 30xy + 3x^2 - 4xy + 9yx - 12y^2$
$= 5x^2 + 3x^2 - 30xy - 4xy + 9xy - 12y^2$
$= \mathbf{8x^2 - 25xy - 12y^2}$

(b) $(7x - 3y)(x - 4y) + (5x - 9y)(y - 2x)$
$= 7x^2 - 28xy - 3yx + 12y^2 + 5xy - 10x^2 - 9y^2 + 18yx$
$= 7x^2 - 10x^2 - 28xy - 3xy + 5xy + 18xy + 12y^2 - 9y^2$
$= \mathbf{-3x^2 - 8xy + 3y^2}$

(c) $(8x - y)(x + 3y) - (4x + y)(9y - 2x)$
$= 8x^2 + 24xy - yx - 3y^2 - (36xy - 8x^2 + 9y^2 - 2yx)$
$= 8x^2 + 24xy - yx - 3y^2 - 36xy + 8x^2 - 9y^2 + 2yx$
$= 8x^2 + 8x^2 + 24xy - xy - 36xy + 2xy - 3y^2 - 9y^2$
$= \mathbf{16x^2 - 11xy - 12y^2}$

(d) $(10x + y)(3x + 2y) - (5x - 4y)(-x - 6y)$
$= 30x^2 + 20xy + 3yx + 2y^2 - (-5x^2 - 30xy + 4yx + 24y^2)$
$= 30x^2 + 20xy + 3yx + 2y^2 + 5x^2 + 30xy - 4yx - 24y^2$
$= 30x^2 + 5x^2 + 20xy + 3xy + 30xy - 4xy + 2y^2 - 24y^2$
$= \mathbf{35x^2 + 49xy - 22y^2}$

16. (a) $-2x(x + 3) + (x + 2)(3x + 1)(x + 5)$
$= -2x(x + 3) + (3x^2 + x + 6x + 2)(x + 5)$
$= -2x^2 - 6x + (3x^2 + 7x + 2)(x + 5)$
$= -2x^2 - 6x + 3x^3 + 15x^2 + 7x^2 + 35x + 2x + 10$
$= -2x^2 - 6x + 3x^3 + 22x^2 + 37x + 10$
$= \mathbf{3x^3 + 20x^2 + 31x + 10}$

(b) $(-2x + 1)(x - 3)(4x + 1) - (2x + 5)(13x - 1)$
$= (-2x^2 + 6x + x - 3)(4x + 1) - (26x^2 - 2x + 65x - 5)$
$= (-2x^2 + 7x - 3)(4x + 1) - (26x^2 + 63x - 5)$
$= -8x^3 - 2x^2 + 28x^2 + 7x - 12x - 3 - 26x^2 - 63x + 5$
$= -8x^3 + 26x^2 - 5x - 3 - 26x^2 - 63x + 5$
$= \mathbf{-8x^3 - 68x + 2}$

(c) $m(m + 2n)(-2m + n) + (4m + n)(m + n)(3m - n)$
$= (m^2 + 2mn)(-2m + n) + (4m^2 + 4mn + mn + n^2)(3m - n)$
$= (m^2 + 2mn)(-2m + n) + (4m^2 + 5mn + n^2)(3m - n)$
$= (-2m^3 + m^2n - 4m^2n + 2mn^2) + (12m^3 - 4m^2n + 15m^2n -$
$5mn^2 + 3mn^2 - n^3)$
$= -2m^3 - 3m^2n + 2mn^2 + 12m^3 + 11m^2n - 2mn^2 - n^3$
$= \mathbf{10m^3 + 8m^2n - n^3}$

(d) $(x-y)(2x+3y)(4x-6y) - 2x(x+y)(x-y)$
$= (2x^2 + 3xy - 2xy - 3y^2)(4x - 6y) - (2x^2 + 2xy)(x - y)$
$= (2x^2 + xy - 3y^2)(4x - 6y) - (2x^2 + 2xy)(x - y)$
$= (8x^3 - 12x^2y + 4x^2y - 6xy^2 - 12xy^2 + 18y^3) - (2x^3 - 2x^2y + 2x^2y - 2xy^2)$
$= 8x^3 - 8x^2y - 18xy^2 + 18y^3 - (2x^3 - 2xy^2)$
$= 8x^3 - 8x^2y - 18xy^2 + 18y^3 - 2x^3 + 2xy^2$
$= \mathbf{6x^3 - 8x^2y - 16xy^2 + 18y^3}$

17. $(7x-5)(9+2x) = 63x + 14x^2 - 45 - 10x$
$= 14x^2 + 63x - 10x - 45$
$= 14x^2 + 53x - 45$

$14x^2 + 53x - 45 = (10x^2 + 3x - 45) + (4x^2 + 50x)$
$= (-x^2 + 40x + 10) + (15x^2 + 13x - 55)$

$\therefore$ two possible pairs of quadratic expressions for the mass of the watermelon and the corresponding mass of the pack of lemons are:
mass of watermelon = ($\mathbf{10x^2 + 3x - 45}$) kg,
mass of pack of lemons = ($\mathbf{4x^2 + 50x}$) kg and
mass of watermelon = ($\mathbf{-x^2 + 40x + 10}$) kg,
mass of pack of lemons = ($\mathbf{15x^2 + 13x - 55}$) kg.

18. Total cost of five such pairs of shoes and four of the bags
$= 5(4x^2 - 9) + 4(2x^2 - 3x + 5)$
$= 20x^2 - 45 + 8x^2 - 12x + 20$
$= 20x^2 + 8x^2 - 12x - 45 + 20$
$= \mathbf{\$(28x^2 - 12x - 25)}$

4.4 Factorisation of quadratic expressions

Practise Now 14

(a) $12x + 8 = \mathbf{4(3x + 2)}$
(b) $21 + 35a = \mathbf{7(3 + 5a)}$
(c) $-15x - 25 = \mathbf{-5(3x + 5)}$
(d) $-8 - 20p = \mathbf{-4(2 + 5p)}$
(e) $-27ax + 12ay = 12ay - 27ax$
$= \mathbf{3a(4y - 9x)}$
(f) $-42xy - 12xz = \mathbf{-6x(7y + 2z)}$
(g) $36p - 54pq + 18pr = \mathbf{18p(2 - 3q + r)}$
(h) $-9z - 24bz - 15cz = \mathbf{-3z(3 + 8b + 5c)}$

Practise Now 15A

(a) $10x^2 + 8x = \mathbf{2x(5x + 4)}$
(b) $10a^2 - 15a = \mathbf{5a(2a - 3)}$
(c) $-49b - 28b^2 = \mathbf{-7b(7 + 4b)}$
(d) $2\pi r^2 + 2\pi rh = \mathbf{2\pi r(r + h)}$
(e) $x^2yz^3 - yz^2 = \mathbf{yz^2(x^2z - 1)}$
(f) $c^2d^3 + c^3d^2 - c^2d^2 = \mathbf{c^2d^2(d + c - 1)}$

Thinking Time (Page 117)

1. **Yes**, we can factorise quadratic expressions of the form $ax^2 + c$ using the method of extracting common factors when a and c have at least one common factor other than 1 or −1.
A possible example would be:
$4x^2 + 2 = 2(2x^2 + 1)$

2. **No**, we cannot use the method of extracting common factors to factorise $ax^2 + bx + c$ completely, where a, b and c have no common factors other than 1 or −1. Since we can use a multiplication frame to expand expressions of the form $(px + q)(rx + s)$ to obtain expressions of the form $ax^2 + bx + c$, then we will be able to use a multiplication frame to do the reverse and factorise expressions of the form $ax^2 + bx + c$.

Investigation (Factorisation of quadratic expressions of the form $x^2 + bx + c$, where $c > 0$ (and $b > 0$))

(a) **(i)** $x^2 = x \times x$
$6 = 1 \times 6$ or $(-1) \times (-6)$
$= 2 \times 3$ or $(-2) \times (-3)$

$\times$	x	1	1	1	1	1	1
x	x^2	x	x	x	x	x	x
1	x	1	1	1	1	1	1

$\therefore x^2 + 7x + 6 = \mathbf{(x + 1)(x + 6)}$

(ii) $x^2 = x \times x$
$6 = 1 \times 6$ or $(-1) \times (-6)$
$= 2 \times 3$ or $(-2) \times (-3)$

$\times$	x	$+6$
x	x^2	$+6x$
$+1$	$+x$	$+6$

$x + 6x = 7x$
$\therefore x^2 + 7x + 6 = \mathbf{(x + 1)(x + 6)}$

(b) **(i)** $x^2 = x \times x$
$8 = 1 \times 8$ or $(-1) \times (-8)$
$= 2 \times 4$ or $(-2) \times (-4)$

$\times$	x	1	1	1	1
x	x^2	x	x	x	x
1	x	1	1	1	1
1	x	1	1	1	1

$\therefore x^2 + 6x + 8 = \mathbf{(x + 2)(x + 4)}$

(ii) $x^2 = x \times x$
$8 = 1 \times 8$ or $(-1) \times (-8)$
$= 2 \times 4$ or $(-2) \times (-4)$

$\times$	x	$+4$
x	x^2	$+4x$
$+2$	$+2x$	$+8$

$2x + 4x = 6x$
$\therefore x^2 + 6x + 8 = \mathbf{(x + 2)(x + 4)}$

(c) **(i)** $x^2 = x \times x$

$8 = 1 \times 8$ or $(-1) \times (-8)$

$= 2 \times 4$ or $(-2) \times (-4)$

$\times$	x	1	1	1	1	1	1	1	1
x	x^2	x	x	x	x	x	x	x	x
1	x	1	1	1	1	1	1	1	1

$\therefore x^2 + 9x + 8 = \mathbf{(x + 1)(x + 8)}$

(ii) $x^2 = x \times x$

$8 = 1 \times 8$ or $(-1) \times (-8)$

$= 2 \times 4$ or $(-2) \times (-4)$

$\times$	x	$+8$
x	x^2	$+8x$
$+1$	$+x$	$+8$

$x + 8x = 9x$

$\therefore x^2 + 9x + 8 = \mathbf{(x + 1)(x + 8)}$

(d) **(i)** $x^2 = x \times x$

$12 = 1 \times 12$ or $(-1) \times (-12)$

$= 2 \times 6$ or $(-2) \times (-6)$

$= 3 \times 4$ or $(-3) \times (-4)$

$\times$	x	1	1	1	1	1	1
x	x^2	x	x	x	x	x	x
1	x	1	1	1	1	1	1
1	x	1	1	1	1	1	1

$\therefore x^2 + 8x + 12 = \mathbf{(x + 2)(x + 6)}$

(ii) $x^2 = x \times x$

$12 = 1 \times 12$ or $(-1) \times (-12)$

$= 2 \times 6$ or $(-2) \times (-6)$

$= 3 \times 4$ or $(-3) \times (-4)$

$\times$	x	$+6$
x	x^2	$+6x$
$+2$	$+2x$	$+12$

$2x + 6x = 8x$

$\therefore x^2 + 8x + 12 = \mathbf{(x + 2)(x + 6)}$

(e) **(i)** $x^2 = x \times x$

$12 = 1 \times 12$ or $(-1) \times (-12)$

$= 2 \times 6$ or $(-2) \times (-6)$

$= 3 \times 4$ or $(-3) \times (-4)$

$\times$	x	1	1	1	1
x	x^2	x	x	x	x
1	x	1	1	1	1
1	x	1	1	1	1
1	x	1	1	1	1

$\therefore x^2 + 7x + 12 = \mathbf{(x + 3)(x + 4)}$

(ii) $x^2 = x \times x$

$12 = 1 \times 12$ or $(-1) \times (-12)$

$= 2 \times 6$ or $(-2) \times (-6)$

$= 3 \times 4$ or $(-3) \times (-4)$

$\times$	x	$+4$
x	x^2	$+4x$
$+3$	$+3x$	$+12$

$3x + 4x = 7x$

$\therefore x^2 + 7x + 12 = \mathbf{(x + 3)(x + 4)}$

(f) **(i)** $x^2 = x \times x$

$12 = 1 \times 12$ or $(-1) \times (-12)$

$= 2 \times 6$ or $(-2) \times (-6)$

$= 3 \times 4$ or $(-3) \times (-4)$

$\times$	x	1	1	1	1	1	1	1	1	1	1	1	1
x	x^2	x	x	x	x	x	x	x	x	x	x	x	x
1	x	1	1	1	1	1	1	1	1	1	1	1	1

$\therefore x^2 + 13x + 12 = \mathbf{(x + 1)(x + 12)}$

(ii) $x^2 = x \times x$

$12 = 1 \times 12$ or $(-1) \times (-12)$

$= 2 \times 6$ or $(-2) \times (-6)$

$= 3 \times 4$ or $(-3) \times (-4)$

$\times$	x	$+12$
x	x^2	$+12x$
$+1$	$+x$	$+12$

$x + 12x = 13x$

$\therefore x^2 + 13x + 12 = \mathbf{(x + 1)(x + 12)}$

Investigation (Factorisation of quadratic expressions of the form $x^2 + bx + c$, where $c > 0$ (and $b < 0$))

(a) $x^2 = x \times x$

$6 = 1 \times 6$ or $(-1) \times (-6)$

$= 2 \times 3$ or $(-2) \times (-3)$

$\times$	x	-6
x	x^2	$-6x$
-1	$-x$	$+6$

$-x - 6x = -7x$

$\therefore x^2 - 7x + 6 = (x - 1)(x - 6)$

The two factors are $\mathbf{(x - 1)}$ and $\mathbf{(x - 6)}$.

(b) $x^2 = x \times x$

$8 = 1 \times 8$ or $(-1) \times (-8)$

$\quad = 2 \times 4$ or $(-2) \times (-4)$

$\times$	x	-4
x	x^2	$-4x$
-2	$-2x$	$+8$

$-2x - 4x = -6x$

$\therefore\ x^2 - 6x + 8 = (x - 2)(x - 4)$

The two factors are $\boldsymbol{(x - 2)}$ and $\boldsymbol{(x - 4)}$.

(c) $x^2 = x \times x$

$8 = 1 \times 8$ or $(-1) \times (-8)$

$\quad = 2 \times 4$ or $(-2) \times (-4)$

$\times$	x	-8
x	x^2	$-8x$
-1	$-x$	$+8$

$-x - 8x = -9x$

$\therefore\ x^2 - 9x + 8 = (x - 1)(x - 8)$

The two factors are $\boldsymbol{(x - 1)}$ and $\boldsymbol{(x - 8)}$.

(d) $x^2 = x \times x$

$12 = 1 \times 12$ or $(-1) \times (-12)$

$\quad = 2 \times 6$ or $(-2) \times (-6)$

$\quad = 3 \times 4$ or $(-3) \times (-4)$

$\times$	x	-6
x	x^2	$-6x$
-2	$-2x$	$+12$

$-2x - 6x = -8x$

$\therefore\ x^2 - 8x + 12 = (x - 2)(x - 6)$

The two factors are $\boldsymbol{(x - 2)}$ and $\boldsymbol{(x - 6)}$.

(e) $x^2 = x \times x$

$12 = 1 \times 12$ or $(-1) \times (-12)$

$\quad = 2 \times 6$ or $(-2) \times (-6)$

$\quad = 3 \times 4$ or $(-3) \times (-4)$

$\times$	x	-4
x	x^2	$-4x$
-3	$-3x$	$+12$

$-3x - 4x = -7x$

$\therefore\ x^2 - 7x + 12 = (x - 3)(x - 4)$

The two factors are $\boldsymbol{(x - 3)}$ and $\boldsymbol{(x - 4)}$.

(f) $x^2 = x \times x$

$12 = 1 \times 12$ or $(-1) \times (-12)$

$\quad = 2 \times 6$ or $(-2) \times (-6)$

$\quad = 3 \times 4$ or $(-3) \times (-4)$

$\times$	x	-12
x	x^2	$-12x$
-1	$-x$	$+12$

$-x - 12x = -13x$

$\therefore\ x^2 - 13x + 12 = (x - 1)(x - 12)$

The two factors are $\boldsymbol{(x - 1)}$ and $\boldsymbol{(x - 12)}$.

Investigation (Factorisation of quadratic expressions of the form $x^2 + bx + c$, where $c < 0$)

(a) $x^2 = x \times x$

$-6 = 1 \times (-6)$ or $(-1) \times 6$

$\quad = 2 \times (-3)$ or $(-2) \times 3$

$\times$	x	$+3$
x	x^2	$+3x$
-2	$-2x$	-6

$-2x + 3x = x$

$\therefore\ x^2 + x - 6 = \boldsymbol{(x - 2)(x + 3)}$

(b) $x^2 = x \times x$

$-6 = 1 \times (-6)$ or $(-1) \times 6$

$\quad = 2 \times (-3)$ or $(-2) \times 3$

$\times$	x	-3
x	x^2	$-3x$
$+2$	$+2x$	-6

$2x - 3x = -x$

$\therefore\ x^2 - x - 6 = \boldsymbol{(x + 2)(x - 3)}$

(c) $x^2 = x \times x$

$-8 = 1 \times (-8)$ or $(-1) \times 8$

$\quad = 2 \times (-4)$ or $(-2) \times 4$

$\times$	x	$+4$
x	x^2	$+4x$
-2	$-2x$	-8

$-2x + 4x = 2x$

$\therefore\ x^2 + 2x - 8 = \boldsymbol{(x - 2)(x + 4)}$

(d) $x^2 = x \times x$

$-8 = 1 \times (-8)$ or $(-1) \times 8$

$= 2 \times (-4)$ or $(-2) \times 4$

$\times$	x	-4
x	x^2	$-4x$
$+2$	$+2x$	-8

$2x - 4x = -2x$

$\therefore x^2 - 2x - 8 = \mathbf{(x + 2)(x - 4)}$

(e) $x^2 = x \times x$

$-8 = 1 \times (-8)$ or $(-1) \times 8$

$= 2 \times (-4)$ or $(-2) \times 4$

$\times$	x	-8
x	x^2	$-8x$
$+1$	$+x$	-8

$x - 8x = -7x$

$\therefore x^2 - 7x - 8 = \mathbf{(x + 1)(x - 8)}$

(f) $x^2 = x \times x$

$-8 = 1 \times (-8)$ or $(-1) \times 8$

$= 2 \times (-4)$ or $(-2) \times 4$

$\times$	x	$+8$
x	x^2	$+8x$
-1	$-x$	-8

$-x + 8x = 7x$

$\therefore x^2 + 7x - 8 = \mathbf{(x - 1)(x + 8)}$

Class Discussion (Factorisation of quadratic expressions)

1. Since the constant term is negative for Fig. 4.14**(c)** and **(d)** (in this case, –6), one of the two corresponding factors of –6 must be negative while the other positive,

i.e. $-6 = 1 \times (-6)$ or $(-1) \times 6$

$= 2 \times (-3)$ or $(-2) \times 3$.

From Fig. 4.14**(c)**, since the coefficient of x is positive (in this case, +5), then the absolute value of the positive factor of –6 must be larger than that of the negative factor of –6 and the two factors have a sum of 5. In this case, $-1 + 6 = 5$, so $x^2 + 5x - 6 = (x - 1)(x + 6)$.

From Fig. 4.14**(d)**, since the coefficient of x is negative (in this case, –5), then the absolute value of the positive factor of –6 must be smaller than that of the negative factor of –6 and the two factors have a sum of –5. In this case, $1 - 6 = -5$, so $x^2 - 5x - 6 = (x + 1)(x - 6)$.

2. By comparing similar structures shown in Fig. 3.14, we can deduce a systematic approach to factorise quadratic equations of the form $x^2 + bx + c$.

If the constant term, c, is positive (e.g. +6), the two factors of 6 must either be both positive (i.e. $6 = 1 \times 6$ or 2×3) or both negative (i.e. 6–) × (1–) = 6) or (–3–) × (2)).

If the constant term, c, is negative (e.g. –6), one of the two corresponding factors of –6 must be negative while the other positive,

i.e. $-6 = 1 \times (-6)$ or $(-1) \times 6$

$= 2 \times (-3)$ or $(-2) \times 3$.

Once we narrow down possible pairs of factors of the constant, we can then identify the pair of factors of the constant that will add up to the coefficient of x, i.e. b.

Practise Now 15B

(a) $x^2 = x \times x$

$5 = 1 \times 5$ or $(-1) \times (-5)$

$\times$	x	$+5$
x	x^2	$+5x$
$+1$	$+x$	$+5$

$x + 5x = 6x$

$\therefore x^2 + 6x + 5 = \mathbf{(x + 1)(x + 5)}$

(b) $x^2 = x \times x$

$-15 = (-1) \times 15$ or $1 \times (-15)$

$= (-3) \times 5$ or $3 \times (-5)$

$\times$	x	$+5$
x	x^2	$+5x$
-3	$-3x$	-15

$-3x + 5x = 2x$

$\therefore x^2 + 2x - 15 = \mathbf{(x + 5)(x - 3)}$

(c) $x^2 = x \times x$

$-12 = (-1) \times 12$ or $1 \times (-12)$

$= (-2) \times 6$ or $2 \times (-6)$

$= (-3) \times 4$ or $3 \times (-4)$

$\times$	x	-4
x	x^2	$-4x$
$+3$	$+3x$	-12

$3x - 4x = -x$

$\therefore x^2 - x - 12 = \mathbf{(x + 3)(x - 4)}$

(d) $x^2 = x \times x$

$-14 = (-1) \times 14$ or $1 \times (-14)$

$= (-2) \times 7$ or $2 \times (-7)$

$\times$	x	$+7$
x	x^2	$+7x$
-2	$-2x$	-14

$-2x + 7x = 5x$

$\therefore x^2 + 5x - 14 = \mathbf{(x + 7)(x - 2)}$

(e) $y^2 = y \times y$

$12 = 1 \times 12$ or $(-1) \times (-12)$

$= 2 \times 6$ or $(-2) \times (-6)$

$= 3 \times 4$ or $(-3) \times (-4)$

$\times$	y	-6
y	y^2	$-6y$
-2	$-2y$	12

$-2y - 6y = -8y$

$\therefore y^2 - 8y + 12 = \mathbf{(y - 2)(y - 6)}$

(f) $y^2 = y \times y$

$5 = 1 \times 5$ or $(-1) \times (-5)$

$\times$	y	-5
y	y^2	$-5y$
-1	$-y$	5

$-y - 5y = -6y$

$\therefore y^2 - 6y + 5 = \mathbf{(y - 1)(y - 5)}$

(g) $z^2 = z \times z$

$12 = 1 \times 12$ or $(-1) \times (-12)$

$= 2 \times 6$ or $(-2) \times (-6)$

$= 3 \times 4$ or $(-3) \times (-4)$

$\times$	z	$+6$
z	z^2	$+6z$
$+2$	$+2z$	$+12$

$2z + 6z = 8z$

$\therefore z^2 + 8z + 12 = \mathbf{(z + 2)(z + 6)}$

(h) $z^2 = z \times z$

$-8 = (-1) \times 8$ or $1 \times (-8)$

$= (-2) \times 4$ or $2 \times (-4)$

$\times$	z	-8
z	z^2	$-8z$
$+1$	$+z$	-8

$z - 8z = -7z$

$\therefore z^2 - 7z - 8 = \mathbf{(z + 1)(z - 8)}$

Introductory Problem Revisited

$x^2 = x \times x$

$24 = 1 \times 24$ or $(-1) \times (-24)$

$= 2 \times 12$ or $(-2) \times (-12)$

$= 3 \times 8$ or $(-3) \times (-8)$

$= 4 \times 6$ or $(-4) \times (-6)$

$\times$	x	$+8$
x	x^2	$+8x$
$+3$	$+3x$	$+24$

$3x + 8x = 11x$

$\therefore x^2 + 11x + 24 = \mathbf{(x + 3)(x + 8)}$

The length of the rectangle is $\mathbf{(x + 8)}$ **cm**.

Practise Now 15C

(a) $2x^2 = 2x \times x$

$6 = 1 \times 6$ or $(-1) \times (-6)$

$= 2 \times 3$ or $(-2) \times (-3)$

$\times$	x	$+2$
$2x$	$2x^2$	$+4x$
$+3$	$+3x$	$+6$

$3x + 4x = 7x$

$\therefore 2x^2 + 7x + 6 = \mathbf{(2x + 3)(x + 2)}$

(b) $3x^2 = 3x \times x$

$-8 = (-1) \times 8$ or $1 \times (-8)$

$= (-2) \times 4$ or $2 \times (-4)$

$\times$	x	$+4$
$3x$	$3x^2$	$+12x$
-2	$-2x$	-8

$-2x + 12x = 10x$

$\therefore 3x^2 + 10x - 8 = \mathbf{(3x - 2)(x + 4)}$

(c) $6y^2 = 6y \times y$

$= 3y \times 2y$

$4 = 1 \times 4$ or $(-1) \times (-4)$

$= 2 \times 2$ or $(-2) \times (-2)$

$\times$	$2y$	-1
$3y$	$6y^2$	$-3y$
-4	$-8y$	$+4$

$-8y - 3y = -11y$

$\therefore 6y^2 - 11y + 4 = \mathbf{(3y - 4)(2y - 1)}$

(d) $7 - 13x - 2x^2 = -(2x^2 + 13x - 7)$
$2x^2 = 2x \times x$
$-7 = (-1) \times 7$ or $1 \times (-7)$

$\times$	x	$+7$
$2x$	$2x^2$	$+14x$
-1	$-x$	-7

$-x + 14x = 13x$
$\therefore\ 7 - 13x - 2x^2 = \mathbf{-(2x - 1)(x + 7)}$
$= \mathbf{(1 - 2x)(x + 7)}$

(e) $-x^2 + 6x - 9 = -(x^2 - 6x + 9)$
$x^2 = x \times x$
$9 = 1 \times 9$ or $(-1) \times (-9)$
$= 3 \times 3$ or $(-3) \times (-3)$

$\times$	x	-3
x	x^2	$-3x$
-3	$-3x$	$+9$

$-3x - 3x = -6x$
$\therefore\ -x^2 + 6x - 9 = -(x - 3)(x - 3)$
$= \mathbf{(x - 3)(3 - x)}$
$= \mathbf{-(3 - x)^2}$

(f) $-6x^2 + 23x - 15 = -(6x^2 - 23x + 15)$
$6x^2 = 6x \times x$
$= 3x \times 2x$
$15 = 1 \times 15$ or $(-1) \times (-15)$
$= 3 \times 5$ or $(-3) \times (-5)$

$\times$	x	-3
$6x$	$6x^2$	$-18x$
-5	$-5x$	$+15$

$-5x - 18x = -23x$
$\therefore\ -6x^2 + 23x - 15 = \mathbf{-(6x - 5)(x - 3)}$
$= \mathbf{(6x - 5)(3 - x)}$
$= \mathbf{(5 - 6x)(x - 3)}$

(g) $4x^2 = 4x \times x$
$= 2x \times 2x$
$-4 = (-1) \times 4$ or $1 \times (-4)$
$= (-2) \times 2$

$\times$	$2x$	-4
$2x$	$4x^2$	$-8x$
$+1$	$+2x$	-4

$2x - 8x = -6x$
$\therefore\ 4x^2 - 6x - 4 = (2x + 1)(2x - 4)$
$= \mathbf{2(2x + 1)(x - 2)}$

(h) $-5a^2 - 17a - 6 = -(5a^2 + 17a + 6)$
$5a^2 = 5a \times a$
$6 = 1 \times 6$ or $(-1) \times (-6)$
$= 2 \times 3$ or $(-2) \times (-3)$

$\times$	a	$+3$
$5a$	$5a^2$	$+15a$
$+2$	$+2a$	$+6$

$2a + 15a = 17a$
$\therefore\ -5a^2 - 17a - 6 = \mathbf{-(5a + 2)(a + 3)}$

Practise Now 16

1. The constant term in $(2x + 3)(x - 5)$ is -15.
But the constant term in $3x^2 - 18x + 15$ is 15.
$\therefore$ the two expressions are not equivalent.

2. The coefficient of y^2 in $(y - 2)(2y + 1)$ is 2.
But the coefficient of y^2 in $4y^2 + 7y - 2$ is 4.
$\therefore$ the two expressions are not equivalent.

Practise Now 17

(a) $x^2 = x \times x$
$-8y^2 = (-y) \times 8y$ or $y \times (-8y)$
$= (-2y) \times 4y$ or $2y \times (-4y)$

$\times$	x	$+4y$
x	x^2	$+4xy$
$-2y$	$-2xy$	$-8y^2$

$-2xy + 4xy = 2xy$
$\therefore\ x^2 + 2xy - 8y^2 = \mathbf{(x - 2y)(x + 4y)}$

(b) $x^2 = x \times x$
$-15y^2 = (-y) \times 15y$ or $y \times (-15y)$
$= (-3y) \times 5y$ or $3y \times (-5y)$

$\times$	x	$-5y$
x	x^2	$-5xy$
$+3y$	$+3xy$	$-15y^2$

$3xy - 5xy = -2xy$
$\therefore\ x^2 - 2xy - 15y^2 = \mathbf{(x + 3y)(x - 5y)}$

(c) $6x^2 = 6x \times x$
$= 3x \times 2x$
$5y^2 = y \times 5y$ or $(-y) \times (-5y)$

$\times$	x	$+y$
$6x$	$6x^2$	$+6xy$
$+5y$	$+5xy$	$+5y^2$

$5xy + 6xy = 11xy$
$\therefore\ 6x^2 + 11xy + 5y^2 = \mathbf{(6x + 5y)(x + y)}$

(d) $6x^2 = 6x \times x$
$= 3x \times 2x$
$18y^2 = y \times 18y$ or $(-y) \times (-18y)$
$= 2y \times 9y$ or $(-2y) \times (-9y)$
$= 3y \times 6y$ or $(-3y) \times (-6y)$

$\times$	$3x$	$-6y$
$2x$	$6x^2$	$-12xy$
$-3y$	$-9xy$	$+18y^2$

$-9xy - 12xy = -21xy$
$\therefore\ 6x^2 - 21xy + 18y^2 = (2x - 3y)(3x - 6y)$
$= \mathbf{3(2x - 3y)(x - 2y)}$

(e) $-a^2 + 5ab - 6b^2 = -(a^2 - 5ab + 6b^2)$
$a^2 = a \times a$
$6b^2 = b \times 6b$ or $(-b) \times (-6b)$
$= 2b \times 3b$ or $(-2b) \times (-3b)$

$\times$	a	$-2b$
a	a^2	$-2ab$
$-3b$	$-3ab$	$+6b^2$

$-3ab - 2ab = -5ab$
$\therefore\ -a^2 + 5ab - 6b^2 = \mathbf{-(a - 3b)(a - 2b)}$
$= \mathbf{(a - 3b)(2b - a)}$
$= \mathbf{(3b - a)(a - 2b)}$

(f) $-2c^2 + 12cd - 18d^2 = -2(c^2 - 6cd + 9d^2)$
$c^2 = 2c \times c$
$9d^2 = d \times 9d$ or $(-d) \times (-9d)$
$= 3d \times 3d$ or $(-3d) \times (-3d)$

$\times$	c	$-3d$
c	c^2	$-3cd$
$-3d$	$-3cd$	$+9d^2$

$-3cd - 3cd = -6cd$
$\therefore\ -2c^2 + 12cd - 18d^2 = \mathbf{-2(c - 3d)^2}$
$= \mathbf{2(c - 3d)(3d - c)}$

(g) $6pq^2 - 57pqr + 105pr^2 = 3p(2q^2 - 19qr + 35r^2)$
$2q^2 = 2q \times q$
$35r^2 = r \times 35r$ or $(-r) \times (-35r)$
$= 5r \times 7r$ or $(-5r) \times (-7r)$

$\times$	q	$-7r$
$2q$	$2q^2$	$-14qr$
$-5r$	$-5qr$	$+35r^2$

$-5qr - 14qr = -19qr$
$\therefore\ 6pq^2 - 57pqr + 105pr^2 = \mathbf{3p(2q - 5r)(q - 7r)}$

(h) $3x^2y^2 = 3xy \times xy$
$-16 = (-1) \times 16$ or $1 \times (-16)$
$= (-2) \times 8$ or $2 \times (-8)$
$= (-4) \times 4$

$\times$	xy	$+2$
$3xy$	$3x^2y^2$	$+6xy$
-8	$-8xy$	-16

$-8xy + 6xy = -2xy$
$\therefore\ 3x^2y^2 - 2xy - 16 = \mathbf{(3xy - 8)(xy + 2)}$

Practise Now 18

(a) $a^3 + 5a^2 + 6a$
$= a(a^2 + 5a + 6)$
$= \mathbf{a(a + 3)(a + 2)}$

$\times$	a	$+3$
a	a^2	$+3a$
$+2$	$+2a$	$+6$

$a^2 + 5a + 6 = (a + 3)(a + 2)$

(b) $b^3 + 4b^2 - 5b$
$= b(b^2 + 4b - 5)$
$= \mathbf{b(b + 5)(b - 1)}$

$\times$	b	$+5$
b	b^2	$+5b$
-1	$-b$	-5

$b^2 + 4b - 5 = (b + 5)(b - 1)$

(c) $2c^3 + 7c^2 - 4c$
$= c(2c^2 + 7c - 4)$
$= \mathbf{c(2c - 1)(c + 4)}$

$\times$	c	-1
c	$2c^2$	$-c$
$+4$	$+8c$	-4

$2c^2 + 7c - 4 = (2c - 1)(c + 4)$

(d) $2d^3 - 9d^2 - 18d$
$= d(2d^2 - 9d - 18)$
$= \mathbf{d(2d + 3)(d - 6)}$

$\times$	$2d$	$+3$
d	$2d^2$	$+3d$
-6	$-12d$	-18

$2d^2 - 9d - 18 = (2d + 3)(d - 6)$

(e) $3e^3 - 5e^2 + 2e$
$= e(3e^2 - 5e + 2)$
$= \boldsymbol{e(3e - 2)(e - 1)}$

$\times$	$3e$	-2
e	$3e^2$	$-2e$
-1	$-3e$	2

$3e^2 - 5e + 2 = (3e - 2)(e - 1)$

(f) $4f^3 - 17f^2 + 4f$
$= f(4f^2 - 17f + 4)$
$= \boldsymbol{f(4f - 1)(f - 4)}$

$\times$	$4f$	-1
f	$4f^2$	$-f$
-4	$-16f$	$+4$

$4f^2 - 17f + 4 = (4f - 1)(f - 4)$

(g) $4g^3 - 4g^2 - 3g$
$= g(4g^2 - 4g - 3)$
$= \boldsymbol{g(2g - 3)(2g + 1)}$

$\times$	$2g$	-3
$2g$	$4g^2$	$-6g$
$+1$	$2g$	-3

$4g^2 - 4g - 3 = (2g - 3)(2g + 1)$

(h) $6h^3 - 5h^2 + h$
$= h(6h^2 - 5h + 1)$
$= \boldsymbol{h(3h - 1)(2h - 1)}$

$\times$	$3h$	-1
$2h$	$6h^2$	$-2h$
-1	$-3h$	$+1$

$6h^2 - 5h + 1 = (3h - 1)(2h - 1)$

Exercise 4C

1. **(a)** $8x + 64 = \boldsymbol{8(x + 8)}$
(b) $-12p - 27q = \boldsymbol{-3(4p + 9q)}$
(c) $16aw + 20av = \boldsymbol{4a(4w + 5v)}$
(d) $-36bc + 4bd = \boldsymbol{4b(d - 9c)}$
(e) $14xy - 7x + 21xz = \boldsymbol{7x(2y - 1 + 3z)}$
(f) $-8tu - 4u - 11su = \boldsymbol{-u(8t + 4 + 11s)}$

2. **(a)** $4x^2 + 16x = \boldsymbol{4x(x + 4)}$
(b) $18y^2 - 6y = \boldsymbol{6y(3y - 1)}$
(c) $39xy - 15x^2z = \boldsymbol{3x(13y - 5xz)}$
(d) $-8\pi xy^3 - 10\pi y^3 = \boldsymbol{-2\pi y^3(4x + 5)}$

3. **(a)** $a^2 = a \times a$
$8 = 1 \times 8$ or $(-1) \times (-8)$
$= 2 \times 4$ or $(-2) \times (-4)$

$\times$	a	$+8$
a	a^2	$+8a$
$+1$	$+a$	$+8$

$a + 8a = 9a$
$\therefore a^2 + 9a + 8 = \boldsymbol{(a + 1)(a + 8)}$

(b) $b^2 = b \times b$
$15 = 1 \times 15$ or $(-1) \times (-15)$
$= 3 \times 5$ or $(-3) \times (-5)$

$\times$	b	$+5$
b	b^2	$+5b$
$+3$	$+3b$	$+15$

$3b + 5b = 8b$
$\therefore b^2 + 8b + 15 = \boldsymbol{(b + 3)(b + 5)}$

(c) $c^2 = c \times c$
$20 = 1 \times 20$ or $(-1) \times (-20)$
$= 2 \times 10$ or $(-2) \times (-10)$
$= 4 \times 5$ or $(-4) \times (-5)$

$\times$	c	-5
c	c^2	$-5c$
-4	$-4c$	$+20$

$-4c - 5c = -9c$
$\therefore c^2 - 9c + 20 = \boldsymbol{(c - 4)(c - 5)}$

(d) $d^2 = d \times d$
$28 = 1 \times 28$ or $(-1) \times (-28)$
$= 2 \times 14$ or $(-2) \times (-14)$
$= 4 \times 7$ or $(-4) \times (-7)$

$\times$	d	-14
d	d^2	$-14d$
-2	$-2d$	$+28$

$-2d - 14d = -16d$
$\therefore d^2 - 16d + 28 = \boldsymbol{(d - 2)(d - 14)}$

(e) $f^2 = f \times f$
$-16 = (-1) \times 16$ or $1 \times (-16)$
$= (-2) \times 8$ or $2 \times (-8)$
$= (-4) \times 4$

$\times$	f	$+8$
f	f^2	$+8f$
-2	$-2f$	-16

$-2f + 8f = 6f$
$\therefore f^2 + 6f - 16 = \boldsymbol{(f - 2)(f + 8)}$

(f) $h^2 = h \times h$

$-120 = (-1) \times 120$ or $1 \times (-120)$
$= (-2) \times 60$ or $2 \times (-60)$
$= (-3) \times 40$ or $3 \times (-40)$
$= (-4) \times 30$ or $4 \times (-30)$
$= (-5) \times 24$ or $5 \times (-24)$
$= (-6) \times 20$ or $6 \times (-20)$
$= (-8) \times 15$ or $8 \times (-15)$
$= (-10) \times 12$ or $10 \times (-12)$

$\times$	h	$+12$
h	h^2	$+12h$
-10	$-10h$	-120

$-10h + 12h = 2h$

$\therefore h^2 + 2h - 120 = \boldsymbol{(h - 10)(h + 12)}$

(g) $k^2 = k \times k$

$-12 = (-1) \times 12$ or $1 \times (-12)$
$= (-2) \times 6$ or $2 \times (-6)$
$= (-3) \times 4$ or $3 \times (-4)$

$\times$	k	-6
k	k^2	$-6k$
$+2$	$+2k$	-12

$2k - 6k = -4k$

$\therefore k^2 - 4k - 12 = \boldsymbol{(k + 2)(k - 6)}$

(h) $m^2 = m \times m$

$-21 = (-1) \times 21$ or $1 \times (-21)$
$= (-3) \times 7$ or $3 \times (-7)$

$\times$	m	-21
m	m^2	$-21m$
$+1$	$+m$	-21

$m - 21m = -20m$

$\therefore m^2 - 20m - 21 = \boldsymbol{(m + 1)(m - 21)}$

4. (a) $3n^2 = 3n \times n$

$7 = 1 \times 7$ or $(-1) \times (-7)$

$\times$	n	$+1$
$3n$	$3n^2$	$+3n$
$+7$	$+7n$	$+7$

$7n + 3n = 10n$

$\therefore 3n^2 + 10n + 7 = \boldsymbol{(3n + 7)(n + 1)}$

(b) $4p^2 = 4p \times p$
$= 2p \times 2p$

$3 = 1 \times 3$ or $(-1) \times (-3)$

$\times$	$2p$	$+3$
$2p$	$4p^2$	$+6p$
$+1$	$+2p$	$+3$

$2p + 6p = 8p$

$\therefore 4p^2 + 8p + 3 = \boldsymbol{(2p + 1)(2p + 3)}$

(c) $6q^2 = 6q \times q$
$= 3q \times 2q$

$12 = 1 \times 12$ or $(-1) \times (-12)$
$= 2 \times 6$ or $(-2) \times (-6)$
$= 3 \times 4$ or $(-3) \times (-4)$

$\times$	$2q$	-3
$3q$	$6q^2$	$-9q$
-4	$-8q$	$+12$

$-8q - 9q = -17q$

$\therefore 6q^2 - 17q + 12 = \boldsymbol{(3q - 4)(2q - 3)}$

(d) $4r^2 = 4r \times r$
$= 2r \times 2r$

$3 = 1 \times 3$ or $(-1) \times (-3)$

$\times$	r	-1
$4r$	$4r^2$	$-4r$
-3	$-3r$	$+3$

$-3r - 4r = -7r$

$\therefore 4r^2 - 7r + 3 = \boldsymbol{(4r - 3)(r - 1)}$

(e) $8s^2 = 8s \times s$
$= 4s \times 2s$

$-15 = (-1) \times 15$ or $1 \times (-15)$
$= (-3) \times 5$ or $3 \times (-5)$

$\times$	$2s$	$+3$
$4s$	$8s^2$	$+12s$
-5	$-10s$	-15

$-10s + 12s = +2s$

$\therefore 8s^2 + 2s - 15 = \boldsymbol{(4s - 5)(2s + 3)}$

(f) $6t^2 = 6t \times t$
$= 3t \times 2t$

$-20 = (-1) \times 20$ or $1 \times (-20)$
$= (-2) \times 10$ or $2 \times (-10)$
$= (-4) \times 5$ or $4 \times (-5)$

$\times$	t	$+4$
$6t$	$6t^2$	$+24t$
-5	$-5t$	-20

$-5t + 24t = 19t$

$\therefore 6t^2 + 19t - 20 = \boldsymbol{(6t - 5)(t + 4)}$

(g) $4u^2 = 4u \times u$
$= 2u \times 2u$
$-21 = (-1) \times 21$ or $1 \times (-21)$
$= (-3) \times 7$ or $3 \times (-7)$

$\times$	$2u$	-7
$2u$	$4u^2$	$-14u$
$+3$	$+6u$	-21

$6u - 14u = -8u$
$\therefore 4u^2 - 8u - 21 = \mathbf{(2u + 3)(2u - 7)}$

(h) $18w^2 = 18w \times w$
$= 9w \times 2w$
$= 6w \times 3w$
$-39 = (-1) \times 39$ or $1 \times (-39)$
$= (-3) \times 13$ or $3 \times (-13)$

$\times$	$2w$	-3
$9w$	$18w^2$	$-27w$
$+13$	$+26w$	-39

$26w - 27w = -w$
$\therefore 18w^2 - w - 39 = \mathbf{(9w + 13)(2w - 3)}$

5. **(a)** $a^2 = a \times a$
$-4b^2 = (-b) \times 4b$ or $b \times (-4b)$
$= (-2b) \times 2b$

$\times$	a	$+4b$
a	a^2	$+4ab$
$-b$	$-ab$	$-4b^2$

$-ab + 4ab = 3ab$
$\therefore a^2 + 3ab - 4b^2 = \mathbf{(a - b)(a + 4b)}$

(b) $c^2 = c \times c$
$-21d^2 = (-d) \times 21d$ or $d \times (-21d)$
$= (-3d) \times 7d$ or $3d \times (-7d)$

$\times$	c	$-7d$
c	c^2	$-7cd$
$+3d$	$+3cd$	$-21d^2$

$3cd - 7cd = -4cd$
$\therefore c^2 - 4cd - 21d^2 = \mathbf{(c + 3d)(c - 7d)}$

(c) $2h^2 = 2h \times h$
$-15k^2 = (-k) \times 15k$ or $k \times (-15k)$
$= (-3k) \times 5k$ or $3k \times (-5k)$

$\times$	h	$+5k$
$2h$	$2h^2$	$+10hk$
$-3k$	$-3hk$	$-15k^2$

$-3hk + 10hk = 7hk$
$\therefore 2h^2 + 7hk - 15k^2 = \mathbf{(2h - 3k)(h + 5k)}$

(d) $3m^2 = 3m \times m$
$-12n^2 = (-n) \times 12n$ or $n \times (-12n)$
$= (-2n) \times 6n$ or $2n \times (-6n)$
$= (-3n) \times 4n$ or $3n \times (-4n)$

$\times$	m	$-6n$
$3m$	$3m^2$	$-18mn$
$+2n$	$+2mn$	$-12n^2$

$2mn - 18mn = -16mn$
$\therefore 3m^2 - 16mn - 12n^2 = \mathbf{(3m + 2n)(m - 6n)}$

6. **(a)** $a^3 + 5a^2 + 4a$
$= a(a^2 + 5a + 4)$
$= \mathbf{a(a + 1)(a + 4)}$

$\times$	a	$+1$
a	a^2	$+a$
$+4$	$+4a$	$+4$

$a^2 + 5a + 4 = (a + 1)(a + 4)$

(b) $3b^3 - 8b^2 - 3b$
$= b(3b^2 - 8b - 3)$
$= b(3b + 1)(b - 3)$

$\times$	$3b$	$+1$
b	$3b^2$	$+b$
-3	$-9b$	-3

$3b^2 - 8b - 3 = (3b + 1)(b - 3)$

(c) $6c^3 - 11c^2 + 5c$
$= c(6c^2 - 11c + 5)$
$= c(6c - 5)(c - 1)$

$\times$	$6b$	-5
c	$6c^2$	$-5c$
-1	$-6c$	$+5$

$6c^2 - 11c + 5 = (6c - 5)(c - 1)$

(d) $6d^3 - 13d^2 + 6d$
$= d(6d^2 - 13d + 6)$
$= \mathbf{d(3d - 2)(2d - 3)}$

$\times$	$3d$	-2
d	$6d^2$	$-4d$
-3	$-9d$	$+6$

$6d^2 - 13d + 6 = (3d - 2)(2d - 3)$

7. **(a)** $-xy^2z^2 - x^2y^3 = \mathbf{-xy^2(z^2 + xy)}$

(b) $12a^2b^3 + 6a^3b^2 - 2a^2b^2 = \mathbf{2a^2b^2(6b + 3a - 1)}$

(c) $10\pi p^2r - 20\pi p^2q - 14\pi pqr^3 = \mathbf{2\pi p(5pr - 10pq - 7qr^3)}$

(d) $3v^3w^2 - 18tv^2w^3 + \frac{1}{3}tv^2w = \mathbf{\frac{1}{3}v^2w\left(9vw - 54tw^2 + t\right)}$

8. **(a)** $-a^2 + 2a + 35 = -(a^2 - 2a - 35)$

$a^2 = a \times a$

$-35 = (-1) \times 35$ or $1 \times (-35)$

$= (-5) \times 7$ or $5 \times (-7)$

$\times$	a	$+5$
a	a^2	$+5a$
-7	$-7a$	-35

$-7a + 5a = -2a$

$\therefore -a^2 + 2a + 35 = \mathbf{-(a - 7)(a + 5)}$

$= \mathbf{(7 - a)(a + 5)}$

(b) $-3b^2 + 76b - 25 = -(3b^2 - 76b + 25)$

$3b^2 = 3b \times b$

$25 = 1 \times 25$ or $(-1) \times (-25)$

$= 5 \times 5$ or $(-5) \times (-5)$

$\times$	b	-25
$3b$	b^2	$-75b$
-1	$-b$	$+25$

$-b - 75b = -76b$

$\therefore -3b^2 + 76b - 25 = \mathbf{-(3b - 1)(b - 25)}$

$= \mathbf{(1 - 3b)(b - 25)}$

$= \mathbf{(3b - 1)(25 - b)}$

(c) $4c^2 + 10c + 4 = 2(2c^2 + 5c + 2)$

$2c^2 = 2c \times c$

$2 = 1 \times 2$ or $(-1) \times (-2)$

$\times$	c	$+2$
$2c$	$2c^2$	$+4c$
$+1$	$+c$	$+2$

$c + 4c = 5c$

$\therefore 4c^2 + 10c + 4 = \mathbf{2(2c + 1)(c + 2)}$

(d) $5d^2 - 145d + 600 = 5(d^2 - 29d + 120)$

$d^2 = d \times d$

$120 = 1 \times 120$ or $(-1) \times (-120)$

$= 2 \times 60$ or $(-2) \times (-60)$

$= 3 \times 40$ or $(-3) \times (-40)$

$= 4 \times 30$ or $(-4) \times (-30)$

$= 5 \times 24$ or $(-5) \times (-24)$

$= 6 \times 20$ or $(-6) \times (-20)$

$= 8 \times 15$ or $(-8) \times (-15)$

$= 10 \times 12$ or $(-10) \times (-12)$

$\times$	d	-24
d	d^2	$-24d$
-5	$-5d$	$+120$

$-5d - 24d = -29d$

$\therefore 5d^2 - 145d + 600 = \mathbf{5(d - 5)(d - 24)}$

(e) $8f^2 + 4f - 60 = 4(2f^2 + f - 15)$

$2f^2 = 2f \times f$

$-15 = (-1) \times 15$ or $1 \times (-15)$

$= (-3) \times 5$ or $3 \times (-5)$

$\times$	f	$+3$
$2f$	$2f^2$	$+6f$
-5	$-5f$	-15

$-5f + 6f = f$

$\therefore 8f^2 + 4f - 60 = \mathbf{4(2f - 5)(f + 3)}$

(f) $24h^2 - 15h - 9 = 3(8h^2 - 5h - 3)$

$8h^2 = 8h \times h$

$= 4h \times 2h$

$-3 = (-1) \times 3$ or $1 \times (-3)$

$\times$	h	-1
$8h$	$8h^2$	$-8h$
$+3$	$+3h$	-3

$3h - 8h = -5h$

$\therefore 24h^2 - 15h - 9 = \mathbf{3(8h + 3)(h - 1)}$

(g) $30 + 14k - 4k^2 = -2(2k^2 - 7k - 15)$

$2k^2 = 2k \times k$

$-15 = (-1) \times 15$ or $1 \times (-15)$

$= (-3) \times 5$ or $3 \times (-5)$

$\times$	$2k$	$+3$
k	$2k^2$	$+3k$
-5	$-10k$	-15

$-10k + 3k = -7k$

$\therefore 30 + 14k - 4k^2 = \mathbf{-2(k - 5)(2k + 3)}$

$= \mathbf{2(5 - k)(2k + 3)}$

(h) $35m^2 + 5m - 30 = 5(7m^2 + m - 6)$

$7m^2 = 7m \times m$

$-6 = (-1) \times 6$ or $1 \times (-6)$

$= (-2) \times 3$ or $2 \times (-3)$

$\times$	m	$+1$
$7m$	$7m^2$	$+7m$
-6	$-6m$	-6

$-6m + 7m = m$

$\therefore 35m^2 + 5m - 30 = \mathbf{5(7m - 6)(m + 1)}$

9. The constant term in $(2x - 3)(x + 5)$ is -15.
But the constant term in $4x^2 + 8x + 15$ is 15.
$\therefore$ the two expressions are not equivalent and hence the breadth of the rectangle is not $(x + 5)$ cm.

10. **(a)** $3p^2 + 15pq + 18q^2 = 3(p^2 + 5pq + 6q^2)$

$p^2 = p \times p$

$6q^2 = q \times 6q$ or $(-q) \times (-6q)$

$= 2q \times 3q$ or $(-2q) \times (-3q)$

$\times$	p	$+3q$
p	p^2	$+3pq$
$+2q$	$+2pq$	$+6q^2$

$2pq + 3pq = 5pq$

$\therefore\ 3p^2 + 15pq + 18q^2 = \mathbf{3(p + 2q)(p + 3q)}$

(b) $2r^2t - 9rst + 10s^2t = t(2r^2 - 9rs + 10s^2)$

$2r^2 = 2r \times r$

$10s^2 = s \times 10s$ or $(-s) \times (-10s)$

$= 2s \times 5s$ or $(-2s) \times (-5s)$

$\times$	r	$-2s$
$2r$	$2r^2$	$-4rs$
$-5s$	$-5rs$	$+10s^2$

$-5rs - 4rs = -9rs$

$\therefore\ 2r^2t - 9rst + 10s^2t = \mathbf{t(2r - 5s)(r - 2s)}$

(c) $x^2y^2 = xy \times xy$

$-15 = (-1) \times 15$ or $1 \times (-15)$

$= (-3) \times 5$ or $3 \times (-5)$

$\times$	xy	$+5$
xy	x^2y^2	$+5xy$
-3	$-3xy$	-15

$-3xy + 5xy = 2xy$

$\therefore\ x^2y^2 + 2xy - 15 = \mathbf{(xy - 3)(xy + 5)}$

(d) $12x^2y^2 = 12xy \times xy$

$= 6xy \times 2xy$

$= 4xy \times 3xy$

$-40 = (-1) \times 40$ or $1 \times (-40)$

$= (-2) \times 20$ or $2 \times (-20)$

$= (-4) \times 10$ or $4 \times (-10)$

$= (-5) \times 8$ or $5 \times (-8)$

$\times$	$3xy$	-8
$4xy$	$12x^2y^2$	$-32xy$
$+5$	$+15xy$	-40

$15xy - 32xy = -17xy$

$\therefore\ 12x^2y^2 - 17xy - 40 = \mathbf{(4xy + 5)(3xy - 8)}$

(e) $4x^2y^2z - 22xyz + 24z = 2z(2x^2y^2 - 11xy + 12)$

$2x^2y^2 = 2xy \times xy$

$12 = 1 \times 12$ or $(-1) \times (-12)$

$= 2 \times 6$ or $(-2) \times (-6)$

$= 3 \times 4$ or $(-3) \times (-4)$

$\times$	xy	-4
$2xy$	$2x^2y^2$	$-8xy$
-3	$-3xy$	$+12$

$-3xy - 8xy = -11xy$

$\therefore\ 4x^2y^2z - 22xyz + 24z = \mathbf{2z(2xy - 3)(xy - 4)}$

11. (a) $2x^3 + 6x^2 + 4x$

$= 2x(x^2 + 3x + 2)$

$= \mathbf{2x(x + 2)(x + 1)}$

$\times$	x	$+2$
x	x^2	$+2x$
$+1$	$+x$	$+2$

$x^2 + 3x + 2 = (x + 2)(x + 1)$

(b) $-3p^3 - 3p^2 + 18p$

$= -3p(p^2 + p - 6)$

$= \mathbf{-3p(p + 3)(p - 2)}$

$\times$	p	$+3$
p	p^2	$+3p$
-2	$-2p$	-6

$p^2 + p - 6 = (p + 3)(p - 2)$

12. (a) $(x + y)(a + b) - (y + z)(a + b)$

$= (a + b)[(x + y) - (y + z)]$

$= (a + b)(x + y - y - z)$

$= \mathbf{(a + b)(x - z)}$

(b) $(c + 2d)(c + 2d) - (c + 2d)(3c - 7d)$

$= (c + 2d)[(c + 2d) - (3c - 7d)]$

$= (c + 2d)(c + 2d - 3c + 7d)$

$= (c + 2d)(2d + 7d + c - 3c)$

$= \mathbf{(c + 2d)(9d - 2c)}$

13. (a) $\frac{4}{9}p^2 + p - 1 = \frac{1}{9}\left(4p^2 + 9p - 9\right)$

$4p^2 = 4p \times p$

$= 2p \times 2p$

$-9 = (-1) \times 9$ or $1 \times (-9)$

$= (-3) \times 3$

$\times$	p	$+3$
$4p$	$4p^2$	$+12p$
-3	$-3p$	-9

$-3p + 12p = 9p$

$\therefore\ \frac{4}{9}p^2 + p - 1 = \mathbf{\frac{1}{9}\left(4p - 3\right)\left(p + 3\right)}$

(b) $0.6r - 0.8qr - 12.8q^2r = -0.2r(64q^2 + 4q - 3)$

$64q^2 = 64q \times q$

$= 32q \times 2q$

$= 16q \times 4q$

$= 8q \times 8q$

$-3 = (-1) \times 3$ or $1 \times (-3)$

$\times$	$4q$	$+1$
$16q$	$64q^2$	$+16q$
-3	$-12q$	-3

$-12q + 16q = 4q$

$\therefore\ 0.6r - 0.8qr - 12.8q^2r = \mathbf{-0.2r(16q - 3)(4q + 1)}$

$= \mathbf{0.2r(3 - 16q)(4q + 1)}$

14. The constant term in $6(2x + 11)(2x + 11)$ is 726.
But the constant term in $4x^2 + 22x + 100$ is 100.
$\therefore$ the two expressions are not equivalent and hence the total surface area of the cube is not $(4x^2 + 22x + 100)$ cm^2.

15. **(i)** $7x^2 - \frac{99}{2}x - 85 = \frac{1}{2}(14x^2 - 99x - 170)$

$14x^2 = 14x \times x$
$= 7x \times 2x$
$-170 = (-1) \times 170$ or $1 \times (-170)$
$= (-2) \times 85$ or $2 \times (-85)$
$= (-5) \times 34$ or $5 \times (-34)$
$= (-10) \times 17$ or $10 \times (-17)$

$\times$	$2x$	-17
$7x$	$14x^2$	$-119x$
$+10$	$+20x$	-170

$20x - 119x = -99x$

$\therefore 7x^2 - \frac{99}{2}x - 85 = \mathbf{\frac{1}{2}(7x+10)(2x-17)}$

(ii) Total distance travelled $= \frac{1}{2}(7x + 10)(2x - 17)$
$=$ speed $\times$ time

Let speed $= 7x + 10$ and time $= \frac{1}{2}(2x - 17)$.

Substituting $x = 10$, we get:
speed $= 7(10) + 10$
$=$ **80 km/h**;
time taken $= \frac{1}{2}[2(10) - 17]$
$=$ **1.5 h**

4.5 Factorisation of algebraic expressions into the form $(a + b)(c + d)$

Class Discussion (Arrangement of terms for factorisation using multiplication frame)

Method 1 works if the terms in each row and column of the multiplication frame have common factors.

(i)

$\times$	a	$+b$
c	ac	$+bc$
$+d$	$+ad$	$+bd$

$ac + bc + ad + bc = (c + d)(a + b)$
$= (a + b)(c + d)$

Since the terms in each row and column of the multiplication frame have common factors, this arrangement works.

(ii)

$\times$	a	$+b$
	ac	$+bd$
	$+ad$	$+bc$

Since the terms in each row do not have a common factor, this arrangement does not work.

(iii)

$\times$	c	$+d$
	ac	$+bd$
	$+bc$	$+ad$

Since the terms in each row do not have a common factor, this arrangement does not work.

Class Discussion (Arrangement of terms for factorisation by grouping)

Method 2 works if
- the terms grouped together have common factors, and
- there is a common factor for the groups.

(i) $ac + bc + ad + bd = (ac + bc) + (ad + bd)$
$= c(a + b) + d(a + b)$
$= (a + b)(c + d)$
$\therefore$ this arrangement works.

(ii) $ac + bd + ad + bc = (ac + bd) + (ad + bc)$
Since the terms in $(ac + bd)$ do not have a common factor and the terms in $(ad + bc)$ do not have a common factor, this arrangement does not work.

(iii) $ac + bd + bc + ad = (ac + bd) + (bc + ad)$
Since the terms in $(ac + bd)$ do not have a common factor and the terms in $(bc + ad)$ do not have a common factor, this arrangement does not work.

Practise Now 19

(a) $ab + ac + 2bd + 2cd = (ab + ac) + (2bd + 2cd)$
$= a(b + c) + 2d(b + c)$
$= \mathbf{(a + 2d)(b + c)}$

(b) $3pq + 7rs + 3pr + 7qs = (3pq + 3pr) + (7qs + 7rs)$
$= 3p(q + r) + 7s(q + r)$
$= \mathbf{(3q + 7s)(q + r)}$

(c) $6ax - 20by - 8bx + 15ay = (6ax - 8bx) + (15ay - 20by)$
$= 2x(3a - 4b) + 5y(3a - 4b)$
$= \mathbf{(3a - 4b)(2x + 5y)}$

(d) $3hp - 12kq + 18kp - 2hq = (3hp + 18kp) - (2hq + 12kq)$
$= 3p(h + 6k) - 2q(h + 6k)$
$= \mathbf{(h + 6k)(3p - 2q)}$

Practise Now 20

(a) $6xy - 15x + 20 - 8y = (6xy - 8y) - (15x - 20)$
$= 2y(3x - 4) - 5(3x - 4)$
$= (3x - 4)(2y - 5)$
$\therefore$ the two factors are $\mathbf{(3x - 4)}$ and $\mathbf{(2y - 5)}$.

(b) $6ab - 9ac + 21c - 14b = (6ab - 14b) - (9ac - 21c)$
$= 2b(3a - 7) - 3c(3a - 7)$
$= (3a - 7)(2b - 3c)$
$\therefore$ the two factors are $\mathbf{(3a - 7)}$ and $\mathbf{(2b - 3c)}$.

Practise Now 21

(a) $x^2 + xy - 3x - 3y = (x^2 + xy) - (3x + 3y)$
$= x(x + y) - 3(x + y)$
$= \mathbf{(x + y)(x - 3)}$

(b) $15w^2 - 20w - 6wz + 8z = (15w^2 - 20w) - (6wz - 8z)$
$= 5w(3w - 4) - 2z(3w - 4)$
$= \mathbf{(3w - 4)(5w - 2z)}$

Thinking Time (Page 133)

1. This method of grouping makes use of the presence of common factors of the terms in each group. This is similar to the method of using a multiplication frame, where the terms in each row and column have common factors.
2. **(a)** $2x^2 + 7x + 6 = 2x^2 + 4x + 3x + 6$
$= (2x^2 + 4x) + (3x + 6)$
$= 2x(x + 2) + 3(x + 2)$
$= \mathbf{(2x + 3)(x + 2)}$

   (b) $3x^2 + 10x - 8 = 3x^2 + 12x - 2x - 8$
$= (3x^2 + 12x) - (2x + 8)$
$= 3x(x + 4) - 2(x + 4)$
$= \mathbf{(3x - 2)(x + 4)}$

Exercise 4D

1. **(a)** $xy + 4x + 3y + 12 = (xy + 4x) + (3y + 12)$
$= x(y + 4) + 3(y + 4)$
$= \mathbf{(x + 3)(y + 4)}$

   (b) $ax - 5a + 4x - 20 = (ax - 5a) + (4x - 20)$
$= a(x - 5) + 4(x - 5)$
$= \mathbf{(a + 4)(x - 5)}$

   (c) $12cy + 20c - 15 - 9y = (12cy + 20c) - (9y + 15)$
$= 4c(3y + 5) - 3(3y + 5)$
$= \mathbf{(4c - 3)(3y + 5)}$

   (d) $3by + 4ax + 12ay + bx = (4ax + 12ay) + (bx + 3by)$
$= 4a(x + 3y) + b(x + 3y)$
$= \mathbf{(4a + b)(x + 3y)}$

   (e) $6xy - 4x - 2z + 3yz = (6xy + 3yz) - (4x + 2z)$
$= 3y(2x + z) - 2(2x + z)$
$= \mathbf{(2x + z)(3y - 2)}$

   (f) $dy + fy - fz - dz = (dy + fy) - (dz + fz)$
$= y(d + f) - z(d + f)$
$= \mathbf{(d + f)(y - z)}$

2. **(a)** $2xy - 8x + 12 - 3y = (2xy - 8x) - (3y - 12)$
$= 2x(y - 4) - 3(y - 4)$
$= \mathbf{(2x - 3)(y - 4)}$

   (b) $6xy - 15y + 10 - 4x = (6xy - 15y) - (4x - 10)$
$= 3y(2x - 5) - 2(2x - 5)$
$= \mathbf{(2x - 5)(3y - 2)}$

   (c) $10 - 14p + 7pq - 5q = (10 - 14p) - (5q - 7pq)$
$= 2(5 - 7p) - q(5 - 7p)$
$= \mathbf{(5 - 7p)(2 - q)}$

   (d) $kx + hy - hx - ky = (kx - hx) - (ky - hy)$
$= x(k - h) - y(k - h)$
$= \mathbf{(k - h)(x - y)}$

   (e) $2ab - 6ad - bc + 3cd = (2ab - 6ad) - (bc - 3cd)$
$= 2a(b - 3d) - c(b - 3d)$
$= \mathbf{(2a - c)(b - 3d)}$

   (f) $24mx + 8my - 6nx - 2ny$
$= (24mx + 8my) - (6nx + 2ny)$
$= 8m(3x + y) - 2n(3x + y)$
$= (8m - 2n)(3x + y)$
$= \mathbf{2(4m - n)(3x + y)}$

3. The coefficient of km in $(2h - 3m)(k + n)$ is -3.
But the coefficient of km in $2hk + 2hn + 3km - 3mn$ is 3.
Since the two expressions are not equivalent, her answer is wrong.
4. **(a)** $x + xy + 2y + 2y^2 = (x + xy) + (2y + 2y^2)$
$= x(1 + y) + 2y(1 + y)$
$= \mathbf{(x + 2y)(1 + y)}$

   (b) $x^2 - 3x + 2xy - 6y = (x^2 - 3x) + (2xy - 6y)$
$= x(x - 3) + 2y(x - 3)$
$= \mathbf{(x + 2y)(x - 3)}$

   (c) $3x^2 + 6xy - 4xz - 8yz = (3x^2 + 6xy) - (4xz + 8yz)$
$= 3x(x + 2y) - 4z(x + 2y)$
$= \mathbf{(x + 2y)(3x - 4z)}$

   (d) $x^2y^2 - 5x^2y - 5xy^2 + xy^3 = (x^2y^2 - 5x^2y) + (xy^3 - 5xy^2)$
$= x^2y(y - 5) + xy^2(y - 5)$
$= (x^2y + xy^2)(y - 5)$
$= \mathbf{xy(y - 5)(x + y)}$

5. **(a)** $144p(y - 5x^2) - 12q(10x^2 - 2y)$
$= 144p(y - 5x^2) - 12q[-2(y - 5x^2)]$
$= 144p(y - 5x^2) + 24q(y^2 - 5x^2)$
$= (144p + 24q)(y - 5x^2)$
$= \mathbf{24(y - 5x^2)(6p + q)}$

   (b) $2(5x + 10y)(2y - x)^2 - 4(6y + 3x)(x - 2y)$
$= 2(5x + 10y)(2y - x)^2 + 4(3x + 6y)(2y - x)$
$= 10(x + 2y)(2y - x)^2 + 12(x + 2y)(2y - x)$
$= 2(x + 2y)(2y - x)[5(2y - x) + 6]$
$= \mathbf{2(x + 2y)(2y - x)(10y - 5x + 6)}$

6. **(i)** $\frac{1}{5}\left(5xy - 25x^2 + 50x - 10y\right) = xy - 5x^2 + 10x - 2y$
$= (xy - 5x^2) - (2y - 10x)$
$= x(y - 5x) - 2(y - 5x)$
$= (y - 5x)(x - 2)$

   The base area of the tank can be expressed as $(y - 5x)(x - 2)$ m^2.
   Since $(y - 5x)(x - 2) = (ay - bx)(ax - c)$, where a, b and c are integers, then $\mathbf{a = 1}$, $\mathbf{b = 5}$ and $\mathbf{c = 2}$.

   (ii) When $x = 6$ and $y = 40$,
$y - 5x = [40 - 5(6)]$
$= 40 - 30$
$= 10$
$x - 2 = 6 - 2$
$= 4$
$\therefore$ the dimensions of the tank are **10 m by 4 m by 5 m**.

4.6 Expansion using special algebraic identities

Investigation (First special algebraic identity)

1. $(a + b)^2$ means $(a + b) \times (a + b)$.
2. $(a + b)^2 = (a + b)(a + b)$
   $= \boldsymbol{a^2 + ab + ba + b^2}$
   $= \boldsymbol{a^2 + 2ab + b^2}$
3.

$\times$	a	$+b$
a	$\boldsymbol{a^2}$	$\boldsymbol{+ab}$
$+b$	$\boldsymbol{+ab}$	$\boldsymbol{+b^2}$

   $ab + ab = 2ab$
   $\therefore (a + b)^2 = \boldsymbol{a^2 + 2ab + b^2}$
4. **(i)** **Yes.**
   (ii) $(a + b)^2 = a^2 + 2ab + b^2$
   $= (a^2 + b^2) + 2ab$
   $\therefore (a + b)^2 \neq a^2 + b^2$ when a and/or b is not 0.
5. **(i)** Area of square $PQRS$ = $\boldsymbol{(a + b)^2}$
   (ii) Area of square $PQRS$
   = Area of square with length a + 2 × area of rectangle with length a and breadth b + area of square with length b
   $= a^2 + ab + ab + b^2$
   $= \boldsymbol{a^2 + 2ab + b^2}$
   (iii) The two expressions in Question 5 parts **(i)** and **(ii)** are equal since the area of the square $PQRS$ must be the same regardless of the method used.
   (iv) $(a + b)^2 = \boldsymbol{a^2 + 2ab + b^2}$
   (v) **Yes.**
   (vi) With reference to Fig. 4.1, $a^2 + b^2$ is the sum of the areas of the two smaller squares. Since $(a + b)^2$ is the area of the square $PQRS$, $(a + b)^2 \neq a^2 + b^2$ since $a^2 + b^2$ does not include the sum of the areas of the two rectangles, $2ab$.
6. **(i)** a^2 is called a perfect square as it can be represented by the area of a square with length a as shown in Fig. 4.1.
   (ii) b^2 is a perfect square since it can be represented by the area of a square with length b as shown in Fig. 4.1.
   (iii) $(a + b)^2$ is a perfect square since it can be represented by the area of a square with length $(a + b)$ as shown in Fig. 4.1.
7. $(a + b)^2 = a^2 + 2ab + b^2$ is an identity since it is true for all values of a and b.

Practise Now 22

(a) $(x + 6)^2 = x^2 + 2(x)(6) + 6^2$
$= \boldsymbol{x^2 + 12x + 36}$

(b) $(4y + 3)^2 = (4y)^2 + 2(4y)(3) + 3^2$
$= \boldsymbol{16y^2 + 24y + 9}$

(c) $(7 + 3a)^2 = 7^2 + 2(7)(3a) + (3a)^2$
$= \boldsymbol{49 + 42a + 9a^2}$

(d) $\left(\frac{1}{2}x+8\right)^2 = \left(\frac{1}{2}x\right)^2 + 2\left(\frac{1}{2}x\right)(8)+8^2$
$= \boldsymbol{\frac{1}{4}x^2 + 8x + 64}$

(e) $(2x + 3y)^2 = (2x)^2 + 2(2x)(3y) + (3y)^2$
$= \boldsymbol{4x^2 + 12xy + 9y^2}$

(f) $(5a + 2b)^2 = (5a)^2 + 2(5a)(2b) + (2b)^2$
$= \boldsymbol{25a^2 + 20ab + 4b^2}$

Investigation (Second special algebraic identity)

1. $(a - b)^2 = (a - b)(a - b)$
   $= \boldsymbol{a^2 - ab - ba + b^2}$
   $= \boldsymbol{a^2 - 2ab + b^2}$
2.

$\times$	a	$-b$
a	$\boldsymbol{a^2}$	$\boldsymbol{-ab}$
$-b$	$\boldsymbol{-ab}$	$\boldsymbol{+b^2}$

   $-ab - ab = -2ab$
   $\therefore (a - b)^2 = \boldsymbol{a^2 - 2ab + b^2}$
3. $(a - b)^2 = \boldsymbol{a^2 + 2a(-b) + (-b)^2}$
   $= \boldsymbol{a^2 - 2ab + b^2}$
4. **(i)** **Yes.**
   (ii) From Questions 1 to 3, $(a - b)^2 = a^2 - 2ab + b^2$.
   $\therefore (a - b)^2 \neq a^2 - b^2$ when b is not 0 and/or a is not equal to b.
5. **(i)** Length of the square $PWXY$ is $\boldsymbol{(a - b)}$.
   (ii) $(a - b)^2$ is a perfect square as it can be represented by the area of a square with length $(a - b)$ as shown in Fig. 4.2.
   (iii) With reference to Fig. 4.2, $a^2 - b^2$ is the difference between the areas of square $PTUV$ and smaller square $GUHX$, which is equivalent to the sum of the area of square $PWXY$ and the areas of the two rectangles $WTGX$ and $YXHV$, i.e. $(a - b)^2 + 2(a - b)(b) = (a - b)^2 + 2ab - 2b^2$.
   However, $(a - b)^2$ only represents the area of the square $PWXY$.
6. $(a - b)^2 = a^2 - 2ab + b^2$ is an identity since it is true for all values of a and b.

Practise Now 23

(a) $(x - 4)^2 = x^2 - 2(x)(4) + 4^2$
$= \boldsymbol{x^2 - 8x + 16}$

(b) $(5y - 3)^2 = (5y)^2 - 2(5y)(3) + 3^2$
$= \boldsymbol{25y^2 - 30y + 9}$

(c) $(8 - 2a)^2 = 8^2 - 2(8)(2a) + (2a)^2$
$= \boldsymbol{64 - 32a + 4a^2}$

(d) $\left(\frac{2}{3}x-6\right)^2 = \left(\frac{2}{3}x\right)^2 - 2\left(\frac{2}{3}x\right)(6)+6^2$
$= \boldsymbol{\frac{4}{9}x^2 - 8x + 36}$

(e) $(b - 3a)^2 = b^2 - 2(b)(3a) + (3a)^2$
$= \boldsymbol{b^2 - 6ab + 9a^2}$

(f) $(3a - 4b)^2 = (3a)^2 - 2(3a)(4b) + (4b)^2$
$= \boldsymbol{9a^2 - 24ab + 16b^2}$

Investigation (Third special algebraic identity)

1. $(a + b)(a - b) = \boldsymbol{a^2 - ab + ba - b^2}$
$\qquad\qquad\qquad = \boldsymbol{a^2 - b^2}$

2.

$\times$	a	$-b$
a	$\boldsymbol{a^2}$	$\boldsymbol{-ab}$
$+b$	$\boldsymbol{+ab}$	$\boldsymbol{-b^2}$

$ab - ab = 0$
$\therefore (a + b)(a - b) = \boldsymbol{a^2 - b^2}$

3. **Yes.**

4. **(i)** Area of polygon $PTGXHV$
= Area of square $PTUV$ – area of square $XGUH$
$= \boldsymbol{a^2 - b^2}$
(ii) With reference to Fig. 4.3**(a)**, $a^2 - b^2$ is a difference of two squares.

5. The dimensions of rectangle $YXHV$ is **$(a - b)$ by b**.

6. **(i)** **Yes**. The area of rectangle $PKJY$ is equal to the sum of the areas of rectangles $PTGY$ and $TKJG$, which is also equal to the area of polygon $PTGXHV$.
(ii) The dimensions of rectangle $PKJY$ is **$(a + b)$ by $(a - b)$**.
(iii) Area of rectangle $PKJY = \boldsymbol{(a + b)(a - b)}$
(iv) $(a + b)(a - b) = \boldsymbol{a^2 - b^2}$

Practise Now 24

(a) $(x + 3)(x - 3) = x^2 - 3^2$
$\qquad = \boldsymbol{x^2 - 9}$

(b) $(5y - 4)(5y + 4) = (5y)^2 - 4^2$
$\qquad = \boldsymbol{25y^2 - 16}$

(c) $(-3 + 2a)(-3 - 2a) = (-3)^2 - (2a)^2$
$\qquad = \boldsymbol{9 - 4a^2}$

(d) $\left(\frac{1}{4}x+8\right)\left(8-\frac{1}{4}x\right) = \left(8+\frac{1}{4}x\right)\left(8-\frac{1}{4}x\right)$
$\qquad = 8^2-\left(\frac{1}{4}x\right)^2$
$\qquad = \boldsymbol{64-\frac{1}{16}x^2}$

(e) $(2x + 7y)(2x - 7y) = (2x)^2 - (7y)^2$
$\qquad = \boldsymbol{4x^2 - 49y^2}$

(f) $(6b - a)(a + 6b) = (6b + a)(6b - a)$
$\qquad = (6b)^2 - a^2$
$\qquad = \boldsymbol{36b^2 - a^2}$

Practise Now 25

(a) $103^2 = (100 + 3)^2$
$\qquad = 100^2 + 2(100)(3) + 3^2$
$\qquad = 10\,000 + 600 + 9$
$\qquad = \mathbf{10\,609}$

(b) $1001^2 = (1000 + 1)^2$
$\qquad = 1000^2 + 2(1000)(1) + 1^2$
$\qquad = 1\,000\,000 + 2000 + 1$
$\qquad = \mathbf{1\,002\,001}$

(c) $49^2 = (50 - 1)^2$
$\qquad = 50^2 - 2(50)(1) + 1^2$
$\qquad = 2500 - 100 + 1$
$\qquad = \mathbf{2401}$

(d) $197^2 = (200 - 3)^2$
$\qquad = 200^2 - 2(200)(3) + 3^2$
$\qquad = 40\,000 - 1200 + 9$
$\qquad = \mathbf{38\,809}$

(e) $205 \times 195 = (200 + 5)(200 - 5)$
$\qquad = 200^2 - 5^2$
$\qquad = 40\,000 - 25$
$\qquad = \mathbf{39\,975}$

(f) $798 \times 802 = (800 - 2)(800 + 2)$
$\qquad = (800 + 2)(800 - 2)$
$\qquad = 800^2 - 2^2$
$\qquad = 640\,000 - 4$
$\qquad = \mathbf{639\,996}$

Practise Now 26

1. $(x + y)^2 = 38$
$x^2 + 2xy + y^2 = 38$
Since $xy = -24$, then $x^2 + 2(-24) + y^2 = 38$
$x^2 - 48 + y^2 = 38$
$x^2 + y^2 = \mathbf{86}$

2. $(a - b)^2 = 296$
$a^2 - 2ab + b^2 = 296$
Since $ab = -51$, then $a^2 - 2(-51) + b^2 = 296$
$a^2 + 102 + b^2 = 296$
$a^2 + b^2 = \mathbf{194}$

Practise Now 27

(i) Since $(2n + 1) \div 2 = n+\frac{1}{2}$ (which is not an integer), then 2 is not a factor of $(2n + 1)$.
$\therefore (2n + 1)$ is an odd number.

(ii) $\boldsymbol{2n + 3}$

(iii) $(2n + 1)^2 = (2n)^2 + 2(2n)(1) + 1^2$
$\qquad = \boldsymbol{4n^2 + 4n + 1}$
$(2n + 3)^2 = (2n)^2 + 2(2n)(3) + 3^2$
$\qquad = \boldsymbol{4n^2 + 12n + 9}$

(iv) $(2n + 3)^2 - (2n + 1)^2 = (4n^2 + 12n + 9) - (4n^2 + 4n + 1)$
$\qquad = 8n + 8$
$\qquad = 8(n + 1)$, which is a multiple of 8
$\therefore$ the difference between the squares of two consecutive odd numbers is always a multiple of 8.

Exercise 4E

1. **(a)** $(a + 4)^2 = a^2 + 2(a)(4) + 4^2$
$\qquad = \boldsymbol{a^2 + 8a + 16}$
(b) $(3b + 2)^2 = (3b)^2 + 2(3b)(2) + 2^2$
$\qquad = \boldsymbol{9b^2 + 12b + 4}$
(c) $(c + 4d)^2 = c^2 + 2(c)(4d) + (4d)^2$
$\qquad = \boldsymbol{c^2 + 8cd + 16d^2}$
(d) $(9h + 2k)^2 = (9h)^2 + 2(9h)(2k) + (2k)^2$
$\qquad = \boldsymbol{81h^2 + 36hk + 4k^2}$
(e) $(3a + 4b)^2 = (3a)^2 + 2(3a)(4b) + (4b)^2$
$\qquad = \boldsymbol{9a^2 + 24ab + 16b^2}$
(f) $(2b + 3a)^2 = (2b)^2 + 2(2b)(3a) + (3a)^2$
$\qquad = \boldsymbol{4b^2 + 12ab + 9a^2}$

2. (a) $(m-9)^2 = m^2 - 2(m)(9) + 9^2$
$= \mathbf{m^2 - 18m + 81}$
(b) $(5n-4)^2 = (5n)^2 - 2(5n)(4) + 4^2$
$= \mathbf{25n^2 - 40n + 16}$
(c) $(9-5p)^2 = 9^2 - 2(9)(5p) + (5p)^2$
$= \mathbf{81 - 90p + 25p^2}$
(d) $(3q-8r)^2 = (3q)^2 - 2(3q)(8r) + (8r)^2$
$= \mathbf{9q^2 - 48qr + 64r^2}$
(e) $(3a-4b)^2 = (3a)^2 - 2(3a)(4b) + (4b)^2$
$= \mathbf{9a^2 - 24ab + 16b^2}$
(f) $(5b-3a)^2 = (5b)^2 - 2(5b)(3a) + (3a)^2$
$= \mathbf{25b^2 - 30ab + 9a^2}$

3. (a) $(s+5)(s-5) = s^2 - 5^2$
$= \mathbf{s^2 - 25}$
(b) $(w-10x)(w+10x) = (w+10x)(w-10x)$
$= w^2 - (10x)^2$
$= \mathbf{w^2 - 100x^2}$
(c) $(2t+11)(2t-11) = (2t)^2 - 11^2$
$= \mathbf{4t^2 - 121}$
(d) $(7-2u)(7+2u) = (7+2u)(7-2u)$
$= 7^2 - (2u)^2$
$= \mathbf{49 - 4u^2}$

4. (a) $1203^2 = (1200+3)^2$
$= 1200^2 + 2(1200)(3) + 3^2$
$= 1\,440\,000 + 7200 + 9$
$= \mathbf{1\,447\,209}$
(b) $892^2 = (900-8)^2$
$= 900^2 - 2(900)(8) + 8^2$
$= 810\,000 - 14\,400 + 64$
$= \mathbf{795\,664}$
(c) $403 \times 397 = (400+3)(400-3)$
$= 400^2 - 3^2$
$= 160\,000 - 9$
$= \mathbf{159\,991}$
(d) $1998 \times 2002 = (2000-2)(2000+2)$
$= (2000+2)(2000-2)$
$= 2000^2 - 2^2$
$= 4\,000\,000 - 4$
$= \mathbf{3\,999\,996}$

5. $(x-y)^2 = x^2 - 2xy + y^2$
$= (x^2+y^2) - 2(xy)$
$= 80 - 2(12)$
$= \mathbf{56}$

6. $x^2 - y^2 = (x+y)(x-y)$
$= (10)(4)$
$= \mathbf{40}$

7. (a) $\left(\frac{1}{5}a+3b\right)^2 = \left(\frac{1}{5}a\right)^2 + 2\left(\frac{1}{5}a\right)(3b) + (3b)^2$
$= \mathbf{\frac{1}{25}a^2 + \frac{6}{5}ab + 9b^2}$
(b) $\left(\frac{1}{2}c+\frac{2}{3}d\right)^2 = \left(\frac{1}{2}c\right)^2 + 2\left(\frac{1}{2}c\right)\left(\frac{2}{3}d\right) + \left(\frac{2}{3}d\right)^2$
$= \mathbf{\frac{1}{4}c^2 + \frac{2}{3}cd + \frac{4}{9}d^2}$

8. (a) $\left(\frac{3}{2}h-5k\right)^2 = \left(\frac{3}{2}h\right)^2 - 2\left(\frac{3}{2}h\right)(5k) + (5k)^2$
$= \mathbf{\frac{9}{4}h^2 - 15hk + 25k^2}$
(b) $\left(-\frac{6}{5}m-3n\right)^2 = \left(-\frac{6}{5}m\right)^2 - 2\left(-\frac{6}{5}m\right)(3n) + (3n)^2$
$= \mathbf{\frac{36}{25}m^2 + \frac{36}{5}mn + 9n^2}$

9. (a) $(6p+5)(5-6p) = (5+6p)(5-6p)$
$= 5^2 - (6p)^2$
$= \mathbf{25 - 36p^2}$
(b) $\left(9r-\frac{4}{5}q\right)\left(9r+\frac{4}{5}q\right) = \left(9r+\frac{4}{5}q\right)\left(9r-\frac{4}{5}q\right)$
$= (9r)^2 - \left(\frac{4}{5}q\right)^2$
$= \mathbf{81r^2 - \frac{16}{25}q^2}$
(c) $\left(\frac{s}{2}+\frac{t}{3}\right)\left(\frac{t}{3}-\frac{s}{2}\right) = \left(\frac{t}{3}+\frac{s}{2}\right)\left(\frac{t}{3}-\frac{s}{2}\right)$
$= \left(\frac{t}{3}\right)^2 - \left(\frac{s}{2}\right)^2$
$= \mathbf{\frac{t^2}{9} - \frac{s^2}{4}}$
(d) $(u+2)(u-2)(u^2+4) = (u^2-2^2)(u^2+4)$
$= (u^2-4)(u^2+4)$
$= (u^2+4)(u^2-4)$
$= (u^2)^2 - 4^2$
$= \mathbf{u^4 - 16}$

10. (a) $4(x+3)^2 - 3(x+4)(x-4)$
$= 4[x^2 + 2(x)(3) + 3^2] - 3(x^2 - 4^2)$
$= 4(x^2 + 6x + 9) - 3(x^2 - 16)$
$= 4x^2 + 24x + 36 - 3x^2 + 48$
$= \mathbf{x^2 + 24x + 84}$
(b) $(5x-7y)(5x+7y) - 2(x-2y)^2$
$= (5x+7y)(5x-7y) - 2[x^2 - 2(x)(2y) + (2y)^2]$
$= (5x)^2 - (7y)^2 - 2(x^2 - 4xy + 4y^2)$
$= 25x^2 - 49y^2 - 2x^2 + 8xy - 8y^2$
$= \mathbf{23x^2 + 8xy - 57y^2}$

11. $\left(\frac{1}{2}x+\frac{1}{2}y\right)^2 = \left(\frac{1}{2}x\right)^2 + 2\left(\frac{1}{2}x\right)\left(\frac{1}{2}y\right) + \left(\frac{1}{2}y\right)^2$
$= \frac{1}{4}x^2 + \frac{1}{2}xy + \frac{1}{4}y^2$
$= \frac{1}{4}(x^2+y^2) + \frac{1}{2}(xy)$
$= \frac{1}{4}(14) + \frac{1}{2}(5)$
$= \mathbf{6}$

12. $2x^2 - 2y^2 = 125$
$2(x^2 - y^2) = 125$
$2[(x+y)(x-y)] = 125$
Since $(x-y) = 2.5$, then $2[(x+y)(2.5)] = 125$
$x + y = \frac{125}{2(2.5)}$
$x + y = \mathbf{25}$

13. $\left(\frac{1}{16}x^2+\frac{1}{25}y^2\right)\left(\frac{1}{4}x+\frac{1}{5}y\right)\left(\frac{1}{4}x-\frac{1}{5}y\right)$
$= \left(\frac{1}{16}x^2+\frac{1}{25}y^2\right)\left[\left(\frac{1}{4}x\right)^2 - \left(\frac{1}{5}y\right)^2\right]$
$= \left(\frac{1}{16}x^2+\frac{1}{25}y^2\right)\left(\frac{1}{16}x^2-\frac{1}{25}y^2\right)$
$= \left[\left(\frac{1}{16}x^2\right)^2 - \left(\frac{1}{25}y^2\right)^2\right]$
$= \mathbf{\frac{1}{256}x^4 - \frac{1}{625}y^4}$

14. (i) $(p - 2q)^2 - p(p - 4q) = p^2 - 2(p)(2q) + (2q)^2 - p^2 + 4pq$
$= p^2 - 4pq + 4q^2 - p^2 + 4pq$
$= p^2 - p^2 - 4pq + 4pq + 4q^2$
$= \mathbf{4q^2}$

(ii) Let p be 5330.
Let $(p - 2q)$ be 5310.
$p - 2q = 5310$
$2q = p - 5310$
$2q = 5330 - 5310$
$2q = 20$
$q = 10$
$5310^2 - 5330 \times 5290 = [5330 - 2(10)]^2 - 5330[5330 - 4(10)]$
$= 4(10)^2$
$= \mathbf{400}$

15. (i) $n^2 - (n - a)(n + a) = n^2 - (n^2 - a^2)$
$= n^2 - n^2 + a^2$
$= \mathbf{a^2}$

(ii) Let n be 16 947.
Let $(n - a)$ be 16 944.
$n - a = 16\,944$
$a = n - 16\,944$
$= 16\,947 - 16\,944$
$= 3$
$16\,947^2 - 16\,944 \times 16\,950 = 16\,947^2 - (16\,947 - 3) \times (16\,947 + 3)$
$= 3^2$
$= \mathbf{9}$

16. (i) Since $2m \div 2 = m$ (which is an integer), then $2m$ is divisible by 2.
$\therefore$ $2m$ is an even number.

(ii) $\mathbf{2m + 2}$

(iii) $(2m)^2 = 4m^2$
$(2m + 2)^2 = (2m)^2 + 2(2m)(2) + 2^2$
$= \mathbf{4m^2 + 8m + 4}$

(iv) $(2m + 2)^2 + (2m)^2 = 4m^2 + 8m + 4 + 4m^2$
$= 4m^2 + 4m^2 + 8m + 4$
$= 8m^2 + 8m + 4$
$= 4(2m^2 + 2m + 1)$, which is a multiple of 4
$\therefore$ the sum of the squares of two consecutive even numbers is always a multiple of 4.

17. (i) Since $(2m + 1) \div 2 = m + \frac{1}{2}$ (which is not an integer), then 2 is not a factor of $(2m + 1)$.
$\therefore$ $(2m + 1)$ is an odd number.

(ii) $\mathbf{2m + 3}$

(iii) $(2m + 1)^2 = (2m)^2 + 2(2m)(1) + 1^2$
$= \mathbf{4m^2 + 4m + 1}$
$(2m + 3)^2 = (2m)^2 + 2(2m)(3) + 3^2$
$= \mathbf{4m^2 + 12m + 9}$

(iv) $(2m + 3)^2 + (2m + 1)^2 = 4m^2 + 12m + 9 + 4m^2 + 4m + 1$
$= 4m^2 + 4m^2 + 12m + 4m + 9 + 1$
$= 8m^2 + 16m + 10$
$= 2(4m^2 + 8m + 5)$
Since $2(4m^2 + 8m + 5) \div 2 = 4m^2 + 8m + 5$ (which is an integer as m is an integer), then $2(4m^2 + 8m + 5)$ is divisible by 2.
$\therefore$ $2(4m^2 + 8m + 5)$ is an even number.

4.7 Factorisation using special algebraic identities

Practise Now 28

(a) $x^2 + 10x + 25 = x^2 + 2(x)(5) + 5^2$
$= \mathbf{(x + 5)^2}$

(b) **N.A.**

(c) $9y^2 + 24y + 16 = (3y)^2 + 2(3y)(4) + 4^2$
$= \mathbf{(3y + 4)^2}$

(d) $36a^2 + 8a + \frac{4}{9} = (6a)^2 + 2(6a)\left(\frac{2}{3}\right) + \left(\frac{2}{3}\right)^2$
$= \left(\mathbf{6a + \frac{2}{3}}\right)^2$

(e) $25a^2 + 40ab + 16b^2 = (5a)^2 + 2(5a)(4b) + (4b)^2$
$= \mathbf{(5a + 4b)^2}$

(f) **N.A.**

Practise Now 29

(a) $8x^2 - 56x + 98 = 2(4x^2 - 28x + 49)$
$= 2[(2x)^2 - 2(2x)(7) + 7^2]$
$= \mathbf{2(2x - 7)^2}$

(b) $\frac{4}{3}t^2 - 4t + 3 = \frac{1}{3}(4t^2 - 12t + 9)$
$= \frac{1}{3}\left[(2t)^2 - 2(2t)(3) + 3^2\right]$
$= \mathbf{\frac{1}{3}(2t - 3)^2}$

(c) $1 - \frac{2}{3}q + \frac{1}{9}q^2 = 1 - 2(1)\left(\frac{1}{3}q\right) + \left(\frac{1}{3}q\right)^2$
$= \left(\mathbf{1 - \frac{1}{3}q}\right)^2$

(d) $\frac{16}{25} - \frac{24}{5}n + 9n^2 = \left(\frac{4}{5}\right)^2 - 2\left(\frac{4}{5}\right)(3n) + (3n)^2$
$= \left(\mathbf{\frac{4}{5} - 3n}\right)^2$

(e) $25x^2 - 10xy + y^2 = (5x)^2 - 2(5x)(y) + y^2$
$= \mathbf{(5x - y)^2}$

(f) **N.A.**

Practise Now 30

(a) $81x^2 - 16 = (9x)^2 - 4^2$
$= \mathbf{(9x + 4)(9x - 4)}$

(b) $-25y^2 + 9 = 9 - 25y^2$
$= 3^2 - (5y)^2$
$= \mathbf{(3 + 5y)(3 - 5y)}$

(c) **N.A.**

(d) $4a^2 - 64b^2 = 4(a^2 - 16b^2)$
$= 4[a^2 - (4b)^2]$
$= \mathbf{4(a + 4b)(a - 4b)}$

(e) $\frac{8}{25}b^2 - 18a^2 = 2\left(\frac{4}{25}b^2 - 9a^2\right)$
$= 2\left[\left(\frac{2}{5}b\right)^2 - (3a)^2\right]$
$= \mathbf{2\left(\frac{2}{5}b + 3a\right)\left(\frac{2}{5}b - 3a\right)}$

(f) $(4x + 1)^2 - 49 = (4x + 1)^2 - 7^2$
$= (4x + 1 + 7)(4x + 1 - 7)$
$= (4x + 8)(4x - 6)$
$= (4)(x + 2)(2)(2x - 3)$
$= \mathbf{8(x + 2)(2x - 3)}$

Practise Now 31

(a) $103^2 - 9 = 103^2 - 3^2$
$= (103 + 3)(103 - 3)$
$= 106(100)$
$= \mathbf{10\ 600}$

(b) $211^2 - 121 = 211^2 - 11^2$
$= (211 + 11)(211 - 11)$
$= 222(200)$
$= \mathbf{44\ 400}$

(c) $49 - 107^2 = 7^2 - 107^2$
$= (7 + 107)(7 - 107)$
$= 114(-100)$
$= \mathbf{-11\ 400}$

(d) $247^2 - 147^2 = (247 + 147)(247 - 147)$
$= 394(100)$
$= \mathbf{39\ 400}$

Practise Now 32

1. **(i)** $x^2 - 4y^2 = x^2 - (2y)^2$
$= \mathbf{(x + 2y)(x - 2y)}$

(ii) $x^2 - 4y^2 = 5$
$(x + 2y)(x - 2y) = 5$
Since 5 is a prime number, it has exactly two factors: 1 and 5.
Since x and y are positive integers, $x - 2y$ is smaller than $x + 2y$.
$x - 2y = 1$ — (1)
$x + 2y = 5$ — (2)
(1) + (2): $2x = 6$
$x = 3$
Substitute $x = 3$ into (2): $3 + 2y = 5$
$2y = 2$
$y = 1$
$\therefore$ $\mathbf{x = 3}$ and $\mathbf{y = 1}$.

2. **(a)** $x^2 - 9 = x^2 - 3^2$
$= \mathbf{(x + 3)(x - 3)}$

(b) Let $(x^2 - 9)$ be 2491.
$x^2 - 9 = 2491$
$x^2 = 2500$
$x = \pm 50$
Let x be 50.
$2491 = (50 + 3)(50 - 3)$
$= 53 \times 47$
The two possible factors of 2491 are **47** and **53**.

Class Discussion (Equivalent expressions)

Consider the expressions A, I and M:
Substitute the same value of x and of y into both expressions, e.g. $x = 1$ and $y = 2$.
$(x - y)^2 = (1 - 2)^2$
$= (-1)^2$
$= 1$
$(x - y)(x - y) = (1 - 2)(1 - 2)$
$= (-1)(-1)$
$= 1$
$x^2 - 2xy + y^2 = 1^2 - 2(1)(2) + 2^2$
$= 1 - 4 + 4$
$= 1$
Since $(x - y)^2 = (x - y)(x - y) = x^2 - 2xy + y^2$, expressions **A, I and M** are equivalent.
Consider the expressions B, G and O:
Substitute the same value of x and of y into both expressions, e.g. $x = 1$ and $y = 2$.
$(x + y)(x + y) = (1 + 2)(1 + 2)$
$= 3^2$
$= 9$
$(x + y)^2 = (1 + 2)^2$
$= 3^2$
$= 9$
$x^2 + 2xy + y^2 = 1^2 + 2(1)(2) + 2^2$
$= 1 + 4 + 4$
$= 9$
Since $(x + y)(x + y) = (x + y)^2 = x^2 + 2xy + y^2$, expressions **B, G and O** are equivalent.
Consider the pair of expressions D and F:
Substitute the same values of w, x, y and z into both expressions, e.g. $w = 1$, $x = 2$, $y = 3$ and $z = 4$.
$(2w - x)(z - 3y) = [2(1) - 2][4 - 3(3)]$
$= (2 - 2)(4 - 9)$
$= 0(-5)$
$= 0$
$2wz - 6wy + 3xy - xz = 2(1)(4) - 6(1)(3) + 3(2)(3) - (2)(4)$
$= 8 - 18 + 18 - 8$
$= 0$
Since $(2w - x)(z - 3y) = 2wz - 6wy + 3xy - xz$, expressions **D and F** are equivalent.
Consider the pair of expressions E and L:
Substitute the same value of x into both expressions, e.g. $x = 1$.
$-5x^2 + 28x - 24 = -5(1)^2 + 28(1) - 24$
$= -5 + 28 - 24$
$= -1$
$2x - (x - 4)(5x - 6) = 2(1) - (1 - 4)[5(1) - 6]$
$= 2 - (-3)(-1)$
$= 2 - 3$
$= -1$
Since $-5x^2 + 28x - 24 = 2x - (x - 4)(5x - 6)$, expressions **E and L** are equivalent.

Consider the pair of expressions J and K:
Substitute the same value of x and of y into both expressions, e.g. $x = 1$ and $y = 2$.

$x^2 - y^2 = 1^2 - 2^2$
$= 1 - 4$
$= -3$

$(x + y)(x - y) = (1 + 2)(1 - 2)$
$= 3(-1)$
$= -3$

Since $x^2 - y^2 = (x + y)(x - y)$, expressions **J and K** are equivalent.

Exercise 4F

1. **(a)** $a^2 + 14a + 49 = a^2 + 2(a)(7) + 7^2$
$= \boldsymbol{(a + 7)^2}$

(b) $4b^2 + 4b + 1 = (2b)^2 + 2(2b)(1) + 1^2$
$= \boldsymbol{(2b + 1)^2}$

(c) $c^2 + 2cd + d^2 = \boldsymbol{(c + d)^2}$

(d) $4h^2 + 20hk + 25k^2 = (2h)^2 + 2(2h)(5k) + (5k)^2$
$= \boldsymbol{(2h + 5k)^2}$

(e) $9a^2 + 30ab + 25b^2 = (3a)^2 + 2(3a)(5b) + (5b)^2$
$= \boldsymbol{(3a + 5b)^2}$

2. **(a)** $m^2 - 10m + 25 = m^2 - 2(m)(5) + 5^2$
$= \boldsymbol{(m - 5)^2}$

(b) $169n^2 - 52n + 4 = (13n)^2 - 2(13n)(2) + 2^2$
$= \boldsymbol{(13n - 2)^2}$

(c) $81 - 180p + 100p^2 = 9^2 - 2(9)(10p) + (10p)^2$
$= \boldsymbol{(9 - 10p)^2}$

(d) $49q^2 - 42qr + 9r^2 = (7q)^2 - 2(7q)(3r) + (3r)^2$
$= \boldsymbol{(7q - 3r)^2}$

3. **(a)** $s^2 - 144 = s^2 - 12^2$
$= \boldsymbol{(s + 12)(s - 12)}$

(b) $36t^2 - 25 = (6t)^2 - 5^2$
$= \boldsymbol{(6t + 5)(6t - 5)}$

(c) $225 - 49u^2 = 15^2 - (7u)^2$
$= \boldsymbol{(15 + 7u)(15 - 7u)}$

(d) $49w^2 - 81x^2 = (7w)^2 - (9x)^2$
$= \boldsymbol{(7w + 9x)(7w - 9x)}$

4. **(a)** $59^2 - 41^2 = (59 + 41)(59 - 41)$
$= 100(18)$
$= \mathbf{1800}$

(b) $29^2 - 39^2 = (29 + 39)(29 - 39)$
$= 68(-10)$
$= \mathbf{-680}$

(c) $7.7^2 - 2.3^2 = (7.7 + 2.3)(7.7 - 2.3)$
$= 10(5.4)$
$= \mathbf{54}$

(d) $81 - 91^2 = 9^2 - 91^2$
$= (9 + 91)(9 - 91)$
$= 100(-82)$
$= \mathbf{-8200}$

5. **(a)** $3a^2 + 12a + 12 = 3(a^2 + 4a + 4)$
$= 3[a^2 + 2(a)(2) + 2^2]$
$= \boldsymbol{3(a + 2)^2}$

(b) $25b^2 + 5bc + \frac{1}{4}c^2 = (5b)^2 + 2(5b)\left(\frac{1}{2}c\right) + \left(\frac{1}{2}c\right)^2$
$= \boldsymbol{\left(5b + \frac{1}{2}c\right)^2}$

(c) **N.A.**

(d) $\frac{16}{49}w^2 + \frac{8}{35}wv + \frac{1}{25}v^2 = \left(\frac{4}{7}w\right)^2 + 2\left(\frac{4}{7}w\right)\left(\frac{1}{5}v\right) + \left(\frac{1}{5}v\right)^2$
$= \boldsymbol{\left(\frac{4}{7}w + \frac{1}{5}v\right)^2}$

(e) $h^4 + 2h^2k + k^2 = (h^2)^2 + 2(h^2)(k) + k^2$
$= \boldsymbol{(h^2 + k)^2}$

(f) **N.A.**

6. **(i)** $x^2 + 4x + 4 = x^2 + 2(x)(2) + 2^2$
$= (x + 2)^2$
The length of the cube is $\boldsymbol{(x + 2)}$ **cm**.

(ii) $(x^2 + 4x + 4)(x + 2) = x^3 + 2x^2 + 4x^2 + 8x + 4x + 8$
$= x^3 + 6x^2 + 12x + 8$
The volume of the cube is $\boldsymbol{(x^3 + 6x^2 + 12x + 8)}$ **cm³**.

7. **(a)** $36m^2 - 48mn + 16n^2 = 4(9m^2 - 12mn + 4n^2)$
$= 4[(3m)^2 - 2(3m)(2n) + (2n)^2]$
$= \boldsymbol{4(3m - 2n)^2}$

(b) **N.A.**

(c) $\frac{1}{3}p^2 - \frac{2}{3}pq + \frac{1}{3}q^2 = \frac{1}{3}(p^2 - 2pq + q^2)$
$= \boldsymbol{\frac{1}{3}(p - q)^2}$

(d) $16r^2 - rs + \frac{1}{64}s^2 = (4r)^2 - 2(4r)\left(\frac{1}{8}s\right) + \left(\frac{1}{8}s\right)^2$
$= \boldsymbol{\left(4r - \frac{1}{8}s\right)^2}$

(e) $25 - 10tu + t^2u^2 = 5^2 - 2(5)(tu) + (tu)^2$
$= \boldsymbol{(5 - tu)^2}$

(f) $75w^2 - \frac{15}{2}wz + \frac{3}{16}z^2 = 3\left(25w^2 - \frac{5}{2}wz + \frac{1}{16}z^2\right)$
$= 3\left[(5w)^2 - 2(5w)\left(\frac{1}{4}z\right) + \left(\frac{1}{4}z\right)^2\right]$
$= \boldsymbol{3\left(5w - \frac{1}{4}z\right)^2}$

8. **(a)** $32a^2 - 98b^2 = 2(16a^2 - 49b^2)$
$= 2[(4a)^2 - (7b)^2]$
$= \boldsymbol{2(4a + 7b)(4a - 7b)}$

(b) $c^2 - \frac{1}{4}d^2 = c^2 - \left(\frac{1}{2}d\right)^2$
$= \boldsymbol{\left(c + \frac{1}{2}d\right)\left(c - \frac{1}{2}d\right)}$

(c) $m^2 - 64n^4 = m^2 - (8n^2)^2$
$= \boldsymbol{(m + 8n^2)(m - 8n^2)}$

(d) **N.A.**

(e) $\frac{9h^2}{100} - 16k^2 = \left(\frac{3h}{10}\right)^2 - (4k)^2$
$= \boldsymbol{\left(\frac{3h}{10} + 4k\right)\left(\frac{3h}{10} - 4k\right)}$

(f) $60p^2 - \frac{15}{64}q^2 = 15\left(4p^2 - \frac{1}{64}q^2\right)$
$= 15\left[(2p)^2 - \left(\frac{1}{8}q\right)^2\right]$
$= \boldsymbol{15\left(2p + \frac{1}{8}q\right)\left(2p - \frac{1}{8}q\right)}$

9. **(a)** $(a+3)^2 - 9 = (a+3)^2 - 3^2$
$= (a+3+3)(a+3-3)$
$= (a+6)(a)$
$= \boldsymbol{a(a+6)}$

(b) $16 - 25(b+3)^2 = 4^2 - [5(b+3)]^2$
$= 4^2 - [5b+15)]^2$
$= (4+5b+15)[4-(5b+15)]$
$= (5b+19)(4-5b-15)$
$= (5b+19)(-11-5b)$
$= \boldsymbol{-(5b+19)(5b+11)}$

(c) $c^2 - (d+2)^2 = (c+d+2)[c-(d+2)]$
$= \boldsymbol{(c+d+2)(c-d-2)}$

(d) $(2h-1)^2 - 4k^2 = (2h-1)^2 - (2k)^2$
$= \boldsymbol{(2h-1+2k)(2h-1-2k)}$

(e) $(3x-5)^2 - 169 = (3x-5)^2 - 13^2$
$= (3x-5+13)(3x-5-13)$
$= (3x+8)(3x-18)$
$= \boldsymbol{3(x-6)(3x+8)}$

(f) $(p+1)^2 - (p-1)^2 = (p+1+p-1)[p+1-(p-1)]$
$= 2p(p+1-p+1)$
$= 2p(2)$
$= \boldsymbol{4p}$

10. $5 \times 88^2 - 720 = 5(88^2 - 144)$
$= 5(88^2 - 12^2)$
$= 5(88+12)(88-12)$
$= 5(100)(76)$
$= \mathbf{38\ 000}$

11. **(i)** $x^2 - 4y^2 = x^2 - (2y)^2$
$= \boldsymbol{(x+2y)(x-2y)}$

(ii) $x^2 - 4y^2 = 13$
$(x+2y)(x-2y) = 13$
Since 13 is a prime number, it has exactly two factors: 1 and 13.
Since x and y are positive integers, $x - 2y$ is smaller than $x + 2y$.
$x - 2y = 1$ — (1)
$x + 2y = 13$ — (2)
(1) + (2): $2x = 14$
$\boldsymbol{x = 7}$
Substitute $x = 7$ into (2): $7 + 2y = 13$
$2y = 6$
$\boldsymbol{y = 3}$

12. **(a)** $4(x-1)^2 - 81(x+1)^2 = [2(x-1)]^2 - [9(x+1)]^2$
$= (2x-2)^2 - (9x+9)^2$
$= (2x-2+9x+9)[2x-2-(9x+9)]$
$= (11x+7)(2x-2-9x-9)$
$= (11x+7)(-7x-11)$
$= \boldsymbol{-(11x+7)(7x+11)}$

(b) $16x^2 + 8x + 1 - 9y^2 = (4x)^2 + 2(4x)(1) + 1^2 - (3y)^2$
$= (4x+1)^2 - (3y)^2$
$= \boldsymbol{(4x+1+3y)(4x+1-3y)}$

(c) $4x^2 - y^2 + 4y - 4 = (2x)^2 - (y^2 - 4y + 4)$
$= (2x)^2 - [y^2 - 2(y)(2) + 2^2]$
$= (2x)^2 - [y-2]^2$
$= (2x+y-2)[2x-(y-2)]$
$= \boldsymbol{(2x+y-2)(2x-y+2)}$

(d) $13x^2 + 26xy + 13y^2 - 13 = 13(x^2 + 2xy + y^2 - 1)$
$= 13[(x+y)^2 - 1^2]$
$= \boldsymbol{13(x+y+1)(x+y-1)}$

13. **(a)** $x^2 - 121 = x^2 - 11^2$
$= \boldsymbol{(x+11)(x-11)}$

(b) Let $(x^2 - 121)$ be 7979.
$x^2 - 121 = 7979$
$x^2 = 8100$
$x = \pm 90$
Let x be 90.
$7979 = (90+11)(90-11)$
$= 101 \times 79$
The two possible factors of 7979 are **79** and **101**.

Chapter 5 Number Patterns

TEACHING NOTES

Suggested Approach

Students have done word problems involving number sequences and patterns in primary school. These word problems required the students to recognise simple patterns from various number sequences and determine either the next few terms or a specific term. However, they were not taught to use algebra to solve problems involving number patterns. Teachers can arouse students' interest in this topic by bringing in real-life applications (see chapter opener on page 151 and Investigation: Fibonacci sequence).

Section 5.1 Number sequences

In primary school, students were only asked how to find the next few terms and a specific term of number sequences but they have not been taught how to state the rule. Teachers can build on this by getting students to work in pairs to state the rules of number sequences and then write down the next few terms (see Class Discussion: Number sequences). Students should learn that they can add, subtract, multiply or divide or use a combination of arithmetic operations to get the next term of a number sequence.

Teachers can build upon what students have learnt in Chapter 6 of Book 1 (Basic Algebra and Algebraic Manipulation) and teach students how to observe a number sequence and look for a pattern so that they can use algebra and find a formula for the general term, $T_n = n^{\text{th}}$ term.

Teachers can get students to work in pairs to find a formula for the general term and hence find a specific term for different number sequences (see Class Discussion: Finding general term of simple sequences). After the students have learnt how to find the general term for simple sequences, they should know that the aim is not to simply solve the problem but to represent it so that it becomes a general expression which can be used to find specific terms.

Section 5.2 Number sequences and patterns

Through Worked Example 8, students will learn that in the real world, which in this case in Chemistry, the general term of a number sequence is important and advantageous in finding specific terms. In this worked example, finding the general term of the number of hydrogen atoms allowed one to find the member number, number of carbon atom(s) and number of hydrogen atoms easily without going through tedious workings, especially if the value of the specific term is large. For other figures, students should consider drawing the next figure in the sequence so as to identify the pattern.

Introductory Problem

Teachers may get students to list down the numbers systematically. Alternatively, the observations that can be made in this problem will be highlighted in Introductory Problem Revisited on Page 173 of the textbook.

5.1 Number sequences

Class Discussion (Number sequences)

1.

	Sequence	Rule
Positive even numbers	2, 4, 6, 8, 10, **12**, **14**, … +2 +2 +2 +2 **+2** **+2**	Start with **2**, then add **2** to each term to get the next term.
Positive odd numbers	1, 3, 5, 7, 9, **11**, **13**, … +2 +2 +2 +2 **+2** **+2**	Start with **1**, then add **2** to each term to get the next term.
Multiples of 3	3, 6, 9, 12, 15, **18**, **21**, … +3 +3 +3 +3 **+3** **+3**	Start with **3**, then add **3** to each term to get the next term.
Powers of 2	1, 2, 4, 8, 16, **32**, **64**, … ×2 ×2 ×2 ×2 **×2** **×2**	Start with **1**, then multiply each term by **2** to get the next term.
Powers of 3	1, 3, 9, 27, 81, **243**, **729**, … ×3 ×3 ×3 ×3 **×3** **×3**	Start with **1**, then multiply each term by **3** to get the next term.

Table 7.1

2. The sequence of positive odd numbers can be obtained by **subtracting 1 from each term of the sequence 2, 4, 6, 8, 10, ….**

   Teachers may wish to note that there are other possible answers to this question.

Practise Now 1

1. **(a)** Rule: Add 5 to each term to get the next term. The next two terms are **28** and **33**.
   (b) Rule: Subtract 6 from each term to get the next term. The next two terms are **–50** and **–56**.
   (c) Rule: Multiply each term by 3 to get the next term. The next two terms are **1215** and **3645**.
   (d) Rule: Divide each term by –3 to get the next term. The next two terms are **–18** and **6**.
2. **(a)** **22, 29**
   (b) **15, 11**

Investigation (Finding general term of simple sequences)

(a) Therefore, $T_n = \mathbf{3n}$.
100th term, $T_{100} = 3 \times 100$
$= \mathbf{300}$

(b) Therefore, $T_n = \mathbf{n^2}$.
100th term, $T_{100} = 100^2$
$= \mathbf{10\ 000}$

(c) Therefore, $T_n = \mathbf{n^3}$.
100th term, $T_{100} = 100^3$
$= \mathbf{1\ 000\ 000}$

Practise Now 2

(i) $T_4 = 4(4) + 7$
$= 16 + 7$
$= \mathbf{23}$

(ii) $T_7 = 4(7) + 7$
$= 28 + 7$
$= 35$

Sum of 4th term and 7th term of sequence $= T_4 + T_7$
$= 23 + 35$
$= \mathbf{58}$

Practise Now 3

1. **(a)** Since the common difference is 4, $T_n = 4n + ?$.
   The term before T_1 is $c = T_0$
   $= 5 - 4$
   $= 1$.
   $\therefore$ general term of sequence, $\mathbf{T_n = 4n + 1}$

   (b) Since the common difference is 5, $T_n = 5n + ?$.
   The term before T_1 is $c = T_0$
   $= 7 - 5$
   $= 2$.
   $\therefore$ general term of sequence, $\mathbf{T_n = 5n + 2}$

   (c) Since the common difference is 6, $T_n = 6n + ?$.
   The term before T_1 is $c = T_0$
   $= 2 - 6$
   $= -4$.
   $\therefore$ general term of sequence, $\mathbf{T_n = 6n - 4}$

   (d) Since the common difference is 3, $T_n = 3n + ?$.
   The term before T_1 is $c = T_0$
   $= 1 - 3$
   $= -2$.
   $\therefore$ general term of sequence, $\mathbf{T_n = 3n - 2}$

2. $$\begin{array}{cccccc} T_0 & T_1 & T_2 & T_3 & T_4 & T_5 \\ ?, & -10, & -2, & 6, & 14, & 22, \ldots \\ & -8 & +8 & +8 & +8 & +8 \end{array}$$

   (i) The next two terms are **30** and **38**.
   (ii) Since the common difference is 8, $T_n = 8n + ?$.
   The term before T_1 is $c = T_0$
   $= -10 - 8$
   $= -18$.
   $\therefore$ general term of the sequence, $\mathbf{T_n = 8n - 18}$
   (iii) $T_{50} = 8(50) - 18$
   $= \mathbf{382}$

Practise Now 4

1. **(a)**

T_1 T_1 T_3 T_4 T_5

3, 7, 13, 21, 31,

+ 4 + 6 + 8 + 10

+ 2 + 2 + 2

Let the general term $T_n = an^2 + bn + c$.

Second difference = 2

$2a = 2$

$a = 1$

$T_2 - T_1 = 4$

$3a + b = 4$

$3(1) + b = 4$

$b = 4 - 3$

$= 1$

$T_1 = 3$

$a + b + c = 3$

$1 + 1 + c = 4$

$c = 1$

$\therefore$ the general term of the sequence T_n

$= an^2 + bn + c$

$= (1)n^2 + (1)n + 1$

$= \boldsymbol{n^2 + n + 1}$

(b)

T_1 T_1 T_3 T_4 T_5

$1\frac{1}{2}$, 5, $9\frac{1}{2}$, 15, $21\frac{1}{2}$,

$+ 3\frac{1}{2}$ $+ 4\frac{1}{2}$ $+ 5\frac{1}{2}$ $+ 6\frac{1}{2}$

+ 1 + 1 + 1

Let the general term $T_n = an^2 + bn + c$.

Second difference = 1

$2a = 1$

$a = \frac{1}{2}$

$T_2 - T_1 = 3\frac{1}{2}$

$3a + b = 3\frac{1}{2}$

$3\left(\frac{1}{2}\right) + b = 3\frac{1}{2}$

$b = 3\frac{1}{2} - 1\frac{1}{2}$

$= 2$

$T_1 = 1\frac{1}{2}$

$a + b + c = 1\frac{1}{2}$

$\frac{1}{2} + 2 + c = 1\frac{1}{2}$

$c = -1$

$\therefore$ the general term of the sequence T_n

$= an^2 + bn + c$

$= \boldsymbol{\frac{1}{2}n^2 + 2n - 1}$

(c)

T_1 T_1 T_3 T_4 T_5

– 6, – 16, – 30, – 48, – 70,

– 10 – 14 – 18 – 22

– 4 – 4 – 4

Let the general term $T_n = an^2 + bn + c$.

Second difference = –4

$2a = -4$

$a = -2$

$T_2 - T_1 = -10$

$3a + b = -10$

$3(-2) + b = -10$

$b = -10 - 3(-2)$

$= -4$

$T_1 = -6$

$a + b + c = -6$

$-2 - 4 + c = -6$

$c = 0$

$\therefore$ the general term of the sequence T_n

$= an^2 + bn + c$

$= \boldsymbol{-2n^2 - 4n}$

(d)

T_1 T_1 T_3 T_4 T_5

– 0.25, – 1, – 2.25, – 4, – 6.25,

– 0.75 – 1.25 – 1.75 – 2.25

– 0.5 – 0.5 – 0.5

Let the general term $T_n = an^2 + bn + c$.

Second difference = –0.5

$2a = -0.5$

$a = -0.25$

$T_2 - T_1 = -0.75$

$3a + b = -0.75$

$3(-0.25) + b = -0.75$

$b = -0.75 - 3(-0.25)$

$= 0$

$T_1 = -0.25$

$a + b + c = -0.25$

$-0.25 + 0 + c = -0.25$

$c = 0$

$\therefore$ the general term of the sequence $T_n = an^2 + bn + c$

$= \boldsymbol{-0.25n^2}$

2. Let the general term $T_n = an^2 + bn + c$.

$T_3 = a(3)^2 + b(3) + c$
$25 = 9a + 3b + c$
Second difference = 4
$2a = 4$
$a = 2$

$T_2 - T_1 = 2$
$3a + b = 2$
$3(2) + b = 2$
$b = 2 - 3(2)$
$= -4$

Substitute $a = 2$ and $b = -4$ into $25 = 9a + 3b + c$,
$25 = 9a + 3b + c$
$25 = 9(2) + 3(-4) + c$
$c = 25 - 9(2) - 3(-4)$
$= 25 - 18 + 12$
$= 19$

(i) $T_1 = a + b + c$
$= 2 - 4 + 19$
$= 17$

(ii) The general term of the sequence $T_n = an^2 + bn + c$
$= \mathbf{2n^2 - 4n + 19}$

Practise Now 5

1. **(a)**

T_1	T_1	T_3	T_4	T_5
10,	26,	58,	112,	194,

+ 16   + 32   + 54   + 82

+ 16   + 22   + 28

+ 6   + 6

Let the general term $T_n = an^3 + bn^2 + cn + d$.
Third difference = 6
$6a = 6$
$a = 1$
$(T_3 - T_2) - (T_2 - T_1) = 32 - 16$
$12a + 2b = 16$
$12(1) + 2b = 16$
$2b = 16 - 12$
$= 4$
$b = 2$

$T_2 - T_1 = 16$
$7a + 3b + c = 16$
$7(1) + 3(2) + c = 16$
$c = 16 - 7 - 6$
$= 3$

$T_1 = 10$
$a + b + c + d = 10$
$1 + 2 + 3 + d = 10$
$d = 10 - 6$
$= 4$

$\therefore$ the general term of the sequence T_n
$= an^3 + bn^2 + cn + d$
$= (1)n^3 + (2)n^2 + (3)n + 4$
$= \mathbf{n^3 + 2n^2 + 3n + 4}$

(b)

T_1	T_1	T_3	T_4	T_5
2,	0,	−18,	−64,	−150,

− 2   − 18   − 46   − 86

− 16   − 28   − 40

− 12   − 12

Let the general term $T_n = an^3 + bn^2 + cn + d$.
Third difference = −12
$6a = -12$
$a = -2$

$(T_3 - T_2) - (T_2 - T_1) = -18 - (-2)$
$12a + 2b = -16$
$12(-2) + 2b = -16$
$2b = -16 + 24$
$= 8$
$b = 4$

$T_2 - T_1 = -2$
$7a + 3b + c = -2$
$7(-2) + 3(4) + c = -2$
$c = -2 + 14 - 12$
$= 0$

$T_1 = 2$
$a + b + c + d = 2$
$-2 + 4 + 0 + d = 2$
$d = 2 + 2 - 4$
$= 0$

$\therefore$ the general term of the sequence T_n
$= an^3 + bn^2 + cn + d$
$= (-2)n^3 + (4)n^2 + (0)n + 0$
$= \mathbf{-2n^3 + 4n^2}$

(c)

T_1	T_1	T_3	T_4	T_5
7,	6,	−7,	−38,	−93,

− 1   − 13   − 31   − 55

− 12   − 18   − 24

− 6   − 6

Let the general term $T_n = an^3 + bn^2 + cn + d$.
Third difference = −6
$6a = -6$
$a = -1$

$$(T_3 - T_2) - (T_2 - T_1) = -13 - (-1)$$
$$12a + 2b = -12$$
$$12(-1) + 2b = -12$$
$$2b = 0$$
$$b = 0$$

$$T_2 - T_1 = -1$$
$$7a + 3b + c = -1$$
$$7(-1) + 3(0) + c = -1$$
$$c = -1 + 7$$
$$= 6$$

$$T_1 = 7$$
$$a + b + c + d = 7$$
$$-1 + 0 + 6 + d = 7$$
$$d = 7 + 1 - 6$$
$$= 2$$

$\therefore$ the general term of the sequence T_n

$= an^3 + bn^2 + cn + d$

$= (-1)n^3 + (0)n^2 + (6)n + 2$

$= \mathbf{-n^3 + 6n + 2}$

(d)

T_1	T_1	T_3	T_4	T_5
4.5,	20,	48.5,	96,	168.5,

+ 15.5 + 28.5 + 47.5 + 72.5

+ 13 + 19 + 25

+ 6 + 6

Let the general term $T_n = an^3 + bn^2 + cn + d$.

Third difference = 6

$$6a = 6$$
$$a = 1$$
$$(T_3 - T_2) - (T_2 - T_1) = 28.5 - 15.5$$
$$12a + 2b = 13$$
$$12(1) + 2b = 13$$
$$2b = 1$$
$$b = 0.5$$

$$T_2 - T_1 = 15.5$$
$$7a + 3b + c = 15.5$$
$$7(1) + 3(0.5) + c = 15.5$$
$$c = 15.5 - 7 - 1.5$$
$$= 7$$

$$T_1 = 4.5$$
$$a + b + c + d = 4.5$$
$$1 + 0.5 + 7 + d = 4.5$$
$$d = 4.5 - 1 - 0.5 - 7$$
$$= -4$$

$\therefore$ the general term of the sequence T_n

$= an^3 + bn^2 + cn + d$

$= (1)n^3 + (0.5)n^2 + (7)n - 4$

$= \mathbf{n^3 + 0.5n^2 + 7n - 4}$

2. (i)

T_1	T_1	T_3	T_4	T_5
0,	– 7,	– 31,	– 81,	– 166,

– 7 – 24 – 50 – 85

– 17 – 26 – 35

– 9 – 9

The sequence has a **common (non-zero) third difference**. Thus, it is a cubic sequence.

(ii) Let the general term $T_n = an^3 + bn^2 + cn + d$.

Third difference = –9

$$6a = -9$$
$$a = -\frac{9}{6} = -\frac{3}{2}$$

$$(T_3 - T_2) - (T_2 - T_1) = -24 - (-7)$$
$$12a + 2b = -17$$
$$12\left(-\frac{3}{2}\right) + 2b = -17$$
$$2b = -17 + 18$$
$$= 1$$
$$b = \frac{1}{2}$$

$$T_2 - T_1 = -7$$
$$7a + 3b + c = -7$$
$$7\left(-\frac{3}{2}\right) + 3\left(\frac{1}{2}\right) + c = -7$$
$$c = -7 + \frac{21}{2} - \frac{3}{2}$$
$$= 2$$

$$T_1 = 0$$
$$a + b + c + d = 0$$
$$-\frac{3}{2} + \frac{1}{2} + 2 + d = 0$$
$$d = \frac{3}{2} - \frac{1}{2} - 2$$
$$= -1$$

$\therefore$ the general term of the sequence T_n

$= an^3 + bn^2 + cn + d$

$= \left(-\frac{3}{2}\right)n^3 + \left(\frac{1}{2}\right)n^2 + (2)n - 1$

$= \mathbf{-1.5n^3 + 0.5n^2 + 2n - 1}$

(iii) $T_{15} = -1.5(15)^3 + 0.5(15)^2 + 2(15) - 1 = \mathbf{-4921}$

Practise Now 6

1. **(a)**

$$\begin{array}{ccccccccc} T_1 & & T_1 & & T_3 & & T_4 & & T_5 \\ 1, & \xrightarrow{\times 2} & 2, & \xrightarrow{\times 2} & 4, & \xrightarrow{\times 2} & 8, & \xrightarrow{\times 2} & 16, \end{array}$$

$r = 2$

$a = 1$

$\therefore$ the general term of the sequence T_n

$= ar^{(n-1)}$

$= (1)(2^{(n-1)})$

$= \mathbf{2^{(n-1)}}$

Alternatively, by observation,

$T_1 = 1 \qquad = 2^0 \qquad = 2^{(1-1)}$

$T_2 = 2 \qquad = 2^1 \qquad = 2^{(2-1)}$

$T_3 = 4 \qquad = 2^2 \qquad = 2^{(3-1)}$

$T_4 = 8 \qquad = 2^3 \qquad = 2^{(4-1)}$

$T_5 = 16 \qquad = 2^4 \qquad = 2^{(5-1)}$

$T_n = \qquad = \mathbf{2^{(n-1)}}$

(b)

$$\begin{array}{ccccccccc} T_1 & & T_1 & & T_3 & & T_4 & & T_5 \\ 1, & \xrightarrow{\times \frac{1}{2}} & \frac{1}{2}, & \xrightarrow{\times \frac{1}{2}} & \frac{1}{4}, & \xrightarrow{\times \frac{1}{2}} & \frac{1}{8}, & \xrightarrow{\times \frac{1}{2}} & \frac{1}{16}, \end{array}$$

$r = \frac{1}{2}$

$a = 1$

$\therefore$ the general term of the sequence T_n

$= ar^{(n-1)}$

$= (1)\left[\left(\frac{1}{2}\right)^{(n-1)}\right]$

$= \boldsymbol{\left(\frac{1}{2}\right)^{(n-1)}}$

(c)

$$\begin{array}{ccccccccc} T_1 & & T_1 & & T_3 & & T_4 & & T_5 \\ -3, & \xrightarrow{\times 1.5} & -4.5, & \xrightarrow{\times 1.5} & -6.75, & \xrightarrow{\times 1.5} & -10.1025, & \xrightarrow{\times 1.5} & -15.1875 \end{array}$$

$r = 1.5$

$a = -3$

$\therefore$ the general term of the sequence T_n

$= ar^{(n-1)}$

$= \mathbf{-3(1.5^{(n-1)})}$

(d)

$$\begin{array}{ccccccccc} T_1 & & T_1 & & T_3 & & T_4 & & T_5 \\ -1, & \xrightarrow{\times 0.2} & -0.2, & \xrightarrow{\times 0.2} & -0.04, & \xrightarrow{\times 0.2} & -0.008, & \xrightarrow{\times 0.2} & -0.0016 \end{array}$$

$r = 0.2$

$a = -1$

$\therefore$ the general term of the sequence T_n

$= ar^{(n-1)}$

$= -1(0.2^{(n-1)})$

$= \mathbf{-0.2^{(n-1)}}$

2. **(i)**

$$\begin{array}{ccccccccc} T_1 & & T_1 & & T_3 & & T_4 & & T_5 \\ ?, & \xrightarrow{\div 2} & ?, & \xrightarrow{\div 2} & 3, & \xrightarrow{\div 2} & 6, & \xrightarrow{\div 2} & 12, \end{array}$$

$T_3 = T_2 \times 2$

$T_2 = T_3 \div 2$

$= 3 \div 2$

$= \mathbf{\frac{3}{2}}$

$T_2 = T_1 \times 2$

$T_1 = T_2 \div 2$

$= \frac{3}{2} \div 2$

$= \mathbf{\frac{3}{4}}$

(ii) $r = 2$

$a = \frac{3}{4}$

$\therefore$ the general term of the sequence T_n

$= ar^{(n-1)}$

$= \mathbf{\frac{3}{4}\,(2^{(n-1)})}$

(iii) $T_{10} = \frac{3}{4}(2^{(10-1)})$

$= \frac{3}{4} \times 2^9$

$= \mathbf{384}$

Exercise 5A

1. **(a)** Rule: Add 5 to each term to get the next term. The next two terms are **39** and **44**.

(b) Rule: Subtract 8 from each term to get the next term. The next two terms are **40** and **32**.

(c) Rule: Multiply each term by 2 to get the next term. The next two terms are **384** and **768**.

(d) Rule: Divide each term by 2 to get the next term. The next two terms are **50** and **25**.

(e) Rule: Divide each term by –4 to get the next term. The next two terms are **16** and **–4**.

(f) Rule: Multiply each term by –2 to get the next term. The next two terms are **–288** and **576**.

(g) Rule: Subtract 7 from each term to get the next term. The next two terms are **–87** and **–94**.

(h) Rule: Add 10 to each term to get the next term. The next two terms are **–50** and **–40**.

2. **(i)** $T_5 = 2(5) + 5$

$= 10 + 5$

$= \mathbf{15}$

(ii) $T_8 = 2(8) + 5$

$= 16 + 5$

$= \mathbf{21}$

(iii) $15 = 3 \times 5$

$21 = 3 \times 7$

LCM of 5th term and 8th term of sequence $= 3 \times 5 \times 7$

$= \mathbf{105}$

3. **(a)** Since the common difference is 6, $T_n = 6n + ?$.

The term before T_1 is $c = T_0$

$= 7 - 6$

$= 1$.

$\therefore$ general term of sequence, $\boldsymbol{T_n = 6n + 1}$

(b) Since the common difference is 3, $T_n = 3n + ?$.

The term before T_1 is $c = T_0$

$= -4 - 3$

$= -7$.

$\therefore$ general term of sequence, $\boldsymbol{T_n = 3n - 7}$

(c) Since the common difference is 7, $T_n = 7n + ?$.

The term before T_1 is $c = T_0$

$= 60 - 7$

$= 53$.

$\therefore$ general term of sequence, $\boldsymbol{T_n = 7n + 53}$

(d) Since the common difference is -3, $T_n = -3n + ?$.

The term before T_1 is $c = T_0$

$= 14 + 3$

$= 17$.

$\therefore$ general term of sequence, $\boldsymbol{T_n = -3n + 17}$

4. **(i)** **42**, **49**

(ii) Since the common difference is 7, $T_n = 7n + ?$.

The term before T_1 is $c = T_0$

$= 7 - 7$

$= 0$

$\therefore$ general term of sequence, $\boldsymbol{T_n = 7n}$

(iii) $T_{105} = 7(105)$

$= \mathbf{735}$

5. **(i)** **30**, **34**

(ii) Since the common difference is 4, $T_n = 4n + ?$.

The term before T_1 is $c = T_0$

$= 10 - 4$

$= 6$.

$\therefore$ general term of sequence, $\boldsymbol{T_n = 4n + 6}$

(iii) $T_{200} = 4(200) + 6$

$= 800 + 6$

$= \mathbf{806}$

6. **(a)** $T_1 = 12 - 5(1) = \mathbf{7}$

$T_2 = 12 - 5(2) = \mathbf{2}$

$T_3 = 12 - 5(3) = \mathbf{-3}$

(b) **(i)** Since the common difference is 7, $T_n = 7n + ?$.

The term before T_1 is $c = T_0$

$= 5 - 7$

$= -2$

$\therefore$ general term of sequence, $T_n = \mathbf{7n - 2}$

(ii) $T_{15} = 7(15) - 2$

$= \mathbf{103}$

(iii) When $n = k$, $T_n = 222$

$7k - 2 = 222$

$7k = 224$

$k = \mathbf{32}$

7. **(i)**

$$\begin{array}{ccccccccc} T_1 & & T_1 & & T_3 & & T_4 & & T_5 \\ 0, & & 3, & & 10, & & 21, & & 36, \\ & +3 & & +7 & & +11 & & +15 & \\ & & +4 & & +4 & & +4 & & \end{array}$$

The sequence has a **common (non-zero) second difference**. Thus, it is a quadratic sequence.

(ii) Let the general term $T_n = an^2 + bn + c$.

Second difference $= 4$

$2a = 4$

$a = 2$

$T_2 - T_1 = 3$

$3a + b = 3$

$3(2) + b = 3$

$b = 3 - 6$

$= -3$

$T_1 = 0$

$a + b + c = 0$

$2 - 3 + c = 0$

$c = 1$

$\therefore$ the general term of the sequence T_n

$= an^2 + bn + c$

$= \mathbf{2n^2 - 3n + 1}$

(iii) $T_{80} = 2(80)^2 - 3(80) + 1$

$= 12\,561$

$T_{81} = 2(81)^2 - 3(81) + 1$

$= 12\,880$

$T_{81} - T_{80} = 12\,880 - 12\,561$

$= \mathbf{319}$

8. **(i)**

$$\begin{array}{ccccccccc} T_1 & & T_1 & & T_3 & & T_4 & & T_5 \\ 4, & & 6, & & 16, & & 40, & & 84, \\ & +2 & & +10 & & +24 & & +44 & \\ & & +8 & & +14 & & +20 & & \\ & & & +6 & & +6 & & & \end{array}$$

The sequence has a **common (non-zero) third difference**. Thus, it is a **cubic sequence**.

(ii) Let the general term $T_n = an^3 + bn^2 + cn + d$.

Third difference $= 6$

$6a = 6$

$a = 1$

$(T_3 - T_2) - (T_2 - T_1) = 10 - 2$

$12a + 2b = 8$

$12(1) + 2b = 8$

$2b = 8 - 12$

$= -4$

$b = -2$

$T_2 - T_1 = 2$
$7a + 3b + c = 2$
$7(1) + 3(-2) + c = 2$
$c = 2 - 7 + 6$
$= 1$

$T_1 = 4$
$a + b + c + d = 4$
$1 - 2 + 1 + d = 4$
$d = 4$
$\therefore$ the general term of the sequence T_n
$= an^3 + bn^2 + cn + d$
$= (1)n^3 + (-2)n^2 + (1)n + 4$
$= \mathbf{n^3 - 2n^2 + n + 4}$

(iii) $T_{21} = (21)^3 - 2(21)^2 + 21 + 4$
$= \mathbf{8404}$

9. **(i)** $a = \mathbf{3}$
$r = \left(\frac{T_n}{T_{n-1}}\right)$
$= \frac{48}{3}$
$= \mathbf{16}$

(ii) The general term of the sequence T_n
$= ar^{(n-1)}$
$= 3(16^{(n-1)})$

$T_3 = 3(16^{(3-1)})$
$= 3 \times 16^2$
$= \mathbf{768}$

$T_4 = 3(16^{(4-1)})$
$= 3 \times 16^3$
$= \mathbf{12\ 288}$

$T_5 = 3(16^{(5-1)})$
$= 3 \times 16^4$
$= \mathbf{196\ 608}$

10. **(a)** **9, 15**
(b) **12, 8**
(c) **–33, –32**
(d) **88, 85**

11. **(i)** $T_n = 8n + 3$
$T_{57} = 8(57) + 3$
$= \mathbf{459}$

(ii) **Yes**. Given any value of n, $8n$ is always even, and adding an odd number 3 will give an odd number. $\therefore$ $8n + 3$ is always odd.

12. **(i)** **216, 343**
(ii) $T_n = \mathbf{n^3}$
(iii) When $n = p$, $T_n = \mathbf{3375}$
$p^3 = 3375$
$p = \sqrt[3]{3375}$
$= \mathbf{15}$

13. **(a)** **(i)** **–38, –45**
(ii) Since the common difference is -7, $T_n = -7n + ?$.
The term before T_1 is $c = T_0$
$= -3 + 7$
$= 4$
$\therefore$ general term of sequence, $\mathbf{T_n = -7n + 4}$

(b) **(i)** $T_n = -7n + 4 - 90$
$\mathbf{T_n = -7n - 86}$
(ii) $-7n - 86 = -268$
$-7n = -182$
$n = 26$
Since n is a positive integer, -268 is a term in the sequence.

14. **(a)** When $n = 1$,
$2n^2 + 1 = 2(1)^2 + 1$
$= 2 + 1$
$= 3$
When $n = 2$,
$2n^2 + 1 = 2(2)^2 + 1$
$= 8 + 1$
$= 9$
When $n = 3$,
$2n^2 + 1 = 2(3)^2 + 1$
$= 18 + 1$
$= 19$
When $n = 4$,
$2n^2 + 1 = 2(4)^2 + 1$
$= 32 + 1$
$= 33$
The first four terms of the sequence are **3**, **9**, **19** and **33**.

(b) **(i)** General term of sequence, $T_n = 2n^2 + 1 - 2$
$\mathbf{T_n = 2n^2 - 1}$
(ii) $T_{38} = 2(38)^2 - 1$
$= 2888 - 1$
$= \mathbf{2887}$

15. **(i)** $T_n = an^2 + bn + c$
$T_1 = 1$
$T_2 = 1$
Second difference $= 8$
$2a = 8$
$a = \mathbf{4}$

$T_2 - T_1 = 0$
$3a + b = 0$
$3(4) + b = 0$
$b = 0 - 12$
$= \mathbf{-12}$

$T_1 = 1$
$a + b + c = 1$
$4 - 12 + c = 1$
$c = 1 - 4 + 12$
$= \mathbf{9}$

(ii) The general term of the sequence $T_n = an^2 + bn + c$
$= \mathbf{4n^2 - 12n + 9}$

$T_3 = 4(3)^2 - 12(3) + 9$
$= 4(9) - 36 + 9$
$= \mathbf{9}$

$T_4 = 4(4)^2 - 12(4) + 9$
$= 4(16) - 48 + 9$
$= \mathbf{25}$

$T_5 = 4(5)^2 - 12(5) + 9$
$= 4(25) - 60 + 9$
$= \mathbf{49}$

16. $T_n = an^3 - 2n^2 + n + d$

(a) (i) Third difference = 12
$6a = 12$
$a = \mathbf{2}$

From the given general term, $b = -2$ and $c = 1$.
$T_1 = 4$
$a + b + c + d = 4$
$2 - 2 + 1 + d = 4$
$d = \mathbf{3}$

(ii) The general term of the sequence T_n
$= an^3 - 2n^2 + n + d$
$= 2n^3 - 2n^2 + n + 3$

$T_2 = 2(2)^3 - 2(2)^2 + 2 + 3$
$= \mathbf{13}$

$T_3 = 2(3)^3 - 2(3)^2 + 3 + 3$
$= \mathbf{42}$

$T_4 = 2(4)^3 - 2(4)^2 + 4 + 3$
$= \mathbf{103}$

$T_5 = 2(5)^3 - 2(5)^2 + 5 + 3$
$= \mathbf{208}$

(b) (i)

T_1	T_2	T_3	T_4	T_5
4,	13,	42,	103,	208,
↓ −4	↓ −4	↓ −4	↓ −4	↓ −4
0,	9,	38,	99,	204,

∴ the general term of the new sequence is T_n
$= 2n^3 - 2n^2 + n + 3 - 4$
$= \mathbf{2n^3 - 2n^2 + n - 1}$

(ii) $T_{75} = 2(75)^3 - 2(75)^2 + 75 - 1$
$= \mathbf{832\ 574}$

17. (a)

$$\begin{array}{ccccccccc} T_1 & & T_1 & & T_3 & & T_4 & & T_5 \\ 3, & \xrightarrow{\times 4} & 12, & \xrightarrow{\times 4} & 48, & \xrightarrow{\times 4} & 192, & \xrightarrow{\times 4} & 768, \end{array}$$

$r = 4$
$a = 3$
∴ the general term of the sequence T_n
$= ar^{(n-1)}$
$= \mathbf{3(4^{(n-1)})}$

(b) (i)

T_1	T_2	T_3	T_4	T_5	
3,	12,	48,	192,	768,	…
↓ +1 (=1^2)	↓ +4 (=2^2)	↓ +9 (=3^2)	↓ +16 (=4^2)	↓ +25(=5^2)	
4,	16,	57,	208,	793,	…

∴ the general term of the new sequence is T_n
$= \mathbf{3(4^{(n-1)}) + n^2}$

(ii) $T_{10} = 3(4^{(10-1)}) + 10^2$
$= 3(4^9) + 100$
$= \mathbf{786\ 532}$

18. (a) **−67, −131**
(b) **8, 13**
(c) **144, 196**
(d) **−216, 343**

5.2 Number sequences and patterns

Practise Now 7

1. (i)

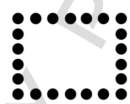


Figure 5

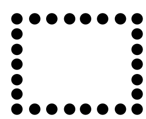


Figure 6

(ii)

Figure number	Number of dots
1	$2 + 1 \times 4 = 6$
2	$2 + 2 \times 4 = 10$
3	$2 + 3 \times 4 = 14$
4	$2 + 4 \times 4 = 18$
5	$\mathbf{2 + 5 \times 4 = 22}$
6	$\mathbf{2 + 6 \times 4 = 26}$
⋮	⋮
n	$\mathbf{2 + n \times 4 = 4n + 2}$

(iii) When $n = 2020$,
$4n + 2 = 4(2020) + 2$
$= 8082$
Number of dots in 2020th figure = **8082**

(iv) $4n + 2 = 80\ 000$
$4n = 80\ 000 - 2$
$n = 79\ 998 \div 4$
$= 19\ 999\frac{1}{2}$

No. Since n is not a positive integer, no figure in this sequence has 80 000 dots.

2. **(i)** 8th line: $\mathbf{72 = 8 \times 9}$
(ii) Since $110 = 10 \times 11 = 10(10 + 1)$,
$k = \mathbf{10}$.
(iii) $\sqrt{342} \approx 18$
$18(18 + 1) = 18 \times 19$
$= 342$
Yes. Since there exists a value of n such that $n(n + 1) = 342$, there will be a line with 342 on the left-hand side.

3. **(i)**

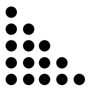


Figure 5

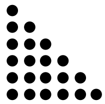


Figure 6

(ii)

Figure number, n	Number of dots at the base of the triangle, n	Total number of dots, T_n
1	1	$1 = 1 = \frac{1 \times 2}{2}$
2	2	$1 + 2 = 3 = \frac{2 \times 3}{2}$
3	3	$1 + 2 + 3 = 6 = \frac{3 \times 4}{2}$
4	4	$1 + 2 + 3 + 4 = 10 = \frac{4 \times 5}{2}$
5	**5**	$\mathbf{1 + 2 + 3 + 4 + 5 = 15 = \frac{5 \times 6}{2}}$
6	**6**	$\mathbf{1 + 2 + 3 + 4 + 5 + 6 = 21 = \frac{6 \times 7}{2}}$
⋮	⋮	⋮
n	$\boldsymbol{n}$	$\mathbf{1 + 2 + 3 + 4 + \cdots + \boldsymbol{n} = \frac{1}{2}\boldsymbol{n}(\boldsymbol{n} + 1)}$

(iii) When $n = 100$,
$$\frac{1}{2}n(n + 1) = \frac{1}{2} \times 100 \times (100 + 1)$$
$$= \frac{1}{2} \times 100 \times 101$$
$$= 5050$$
Total number of dots needed to form a triangle with a base that has 100 dots = **5050**

Investigation (Fibonacci sequence)

1. **1; 5; 13; 21**
2. **3, 5, 8, 13, 21, 34**
Each term is obtained by adding the previous two terms, i.e. in the sequence 1, 1, 2, 3, 5, 8, 13, 21, 34, the third term is 1 + 1 = 2, the fourth term is 1 + 2 = 3, the fifth term is 2 + 3 = 5, and so on.
3. Michaelmas Daisy has 21 + 34 = **55** petals.
4. **4, 6; 7, 10**

Practise Now 8

(i)

Member number	Number of carbon atom(s)	Number of hydrogen atoms
1	2	4
2	3	6
3	4	8
4	5	10
5	**6**	**12**
6	**7**	**14**
⋮	⋮	⋮
n	$\mathbf{n + 1}$	$\mathbf{2n + 2}$

(ii) Let $h + 1 = 55$.
$h = 55 - 1$
$= \mathbf{54}$
When $n = h = 54$,
$2n + 2 = 2(54) + 2$
$= 110$
Number of hydrogen atoms the member has = **110**

(iii) Let $2k + 2 = 120$.
$2k = 120 - 2$
$= 118$
$k = \mathbf{59}$
When $n = k = 59$,
$n + 1 = 59 + 1$
$= 60$
Number of carbon atoms the member has = **60**

(iv) When $2n + 2 = 72$,
$2n = 72 - 2$
$= 70$
$\therefore n = 35$
Number of carbon atoms $= 35 + 1$
$= 36$
$\therefore 3(a + b) = 36$
$a + b = 12$
Possible values for $a \geqslant 0$ and $a < b$:

a	b
1	11
2	10
3	9
4	8
5	7

Exercise 5B

1. **(i)**

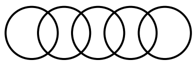


Figure 5

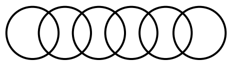


Figure 6

(ii)

Figure number	Number of intersection(s) between the circles
1	0
2	1
3	**2**
4	**3**
5	**4**
6	**5**
⋮	⋮
n	$\boldsymbol{n - 1}$

(iii) Let $n - 1 = 28$.

$n = 28 + 1$

$= \mathbf{29}$

2. **(i)**

Figure number	Number of small triangles
1	4
2	9
3	**16**
4	**25**
5	**36**
6	**49**
:	:
n	$\boldsymbol{(n + 1)^2}$

(ii) When $n = 20$,

$(n + 1)^2 = (20 + 1)^2$

$= 21^2$

$= 441$

Number of small triangles in the 20th figure = **441**

(iii) Let $(n + 1)^2 = 121$.

$n + 1 = 11$ or $n + 1 = -11$

$n = 11 - 1$ or $n = -11 - 1$

$= \mathbf{10}$ or $= -12$ (N.A. since $n > 0$)

(iv) $(n + 1)^2 = 2400$

$n + 1 = \sqrt{2400}$

No. Since $\sqrt{2400}$ is not a positive integer, no figure in this sequence has 2400 small triangles.

3. **(i)**

Pattern number	1	2	3	4	5
Number of purple tiles	1	2	3	4	**5**
Number of white tiles	2	3	4	5	**6**
Total number of tiles	3	5	7	9	**11**

(ii) **(a)** $\boldsymbol{n + 1}$

(b) $\mathbf{2}\boldsymbol{n}\mathbf{ + 1}$

(iii) When $n = 32$,

$2(32) + 1 = \mathbf{65}$

(iv) $2n + 1 = 501$

$2n = 500$

$n = 250$

Yes. Since n is a positive integer, the 250th pattern will have a total of 501 tiles.

4. **(i)** 6th line: $\mathbf{54 = 6 \times 9}$

(ii) Since $208 = 13 \times 16 = 13(13 + 3)$,

$k = \mathbf{13}$.

(iii) **No.** All numbers on the left-hand side must be even.

If n is even, $n + 3$ is odd.

If n is odd, $n + 3$ is even.

The product of an even number and an odd number is always even.

$\therefore$ $n(n + 3)$ will be an even number for all positive integers n, and there will not be a line with 1777 on the left-hand side.

5. **(i)** 5th line: $\mathbf{1 + 3 + 5 + 7 + 9 + 11 = 36 = 6^2 = (5 + 1)^2}$

(ii) $c = \sqrt{169}$

$= \mathbf{13}$

$d + 1 = 13$

$d = 13 - 1$

$= \mathbf{12}$

$a = 13 + 12$

$= \mathbf{25}$

(iii) **No**. The left-hand side of each line of the pattern is made up of odd numbers.

Since 86 868 is an even number, no line in the pattern will have a = 86 868.

6. **(i)**

Year	Amount of money ($)
1	**89 760**
2	**91 520**
3	93 280
4	95 040
5	96 800
6	98 560
⋮	⋮
n	**1760$\boldsymbol{n}$ + 88 000**

(ii) $1760n + 88\,000 = 120\,000$

$1760n = 32\,000$

$n = 18\frac{2}{11}$

He would close the account after **19 years**.

7. **(i)** **1 5 10 10 5 1**

(ii)

Row	Sum
1	$1 = 1 = 2^0$
2	$1 + 1 = 2 = 2^1$
3	$1 + 2 + 1 = 4 = 2^2$
4	$1 + 3 + 3 + 1 = 8 = 2^3$
5	$1 + 4 + 6 + 4 + 1 = 16 = 2^4$
6	$\mathbf{1 + 5 + 10 + 10 + 5 + 1 = 32 = 2^5}$
⋮	⋮
n	$1 + (n - 1) + \cdots + (n - 1) + 1 = \mathbf{2^{n-1}}$

(iii) From the pattern, the value of the sum can only have one prime factor, i.e. 2.

2	3072
2	1536
2	768
2	384
2	192
2	96
2	48
2	24
2	12
2	6
3	3
	1

No. Since $3072 = 2^{10} \times 3$, there will not be a line in the pattern with a sum of 3072.

8. **(i)**

Member number	Number of carbon atoms	Number of hydrogen atoms
1	3	4
2	4	6
3	5	8
4	6	10
5	**7**	**12**
6	**8**	**14**
⋮	⋮	⋮
n	$\mathbf{n + 2}$	$\mathbf{2n + 2}$

(ii) Let $h + 2 = 25$.

$h = 25 - 2$

$= \mathbf{23}$

When $n = h = 23$,

$2n + 2 = 2(23) + 2$

$= 48$

Number of hydrogen atoms the member has = **48**

(iii) When $2n + 2 = 64$,

$2n = 64 - 2$

$= 62$

$\therefore n = 31$

$j + k = 31$

Possible values of j and k:

j	k
3	28
5	26
7	24
11	20
13	18
17	14
19	12
23	8

Since $n = 31$, number of carbon atoms the member has = 33.

Chapter 6 Financial Transactions

TEACHING NOTES

Suggested Approach

In this chapter, students will learn how to apply the concepts of percentage, ratio, and rate to real-world situations involving financial transactions. Students should already have a good understanding of discounts and General Sales Tax (GST), and this will help reinforce their understanding of percentages. Some students may find it challenging to work with transactions involving rates, as these are usually expressed as a percentage. Teachers can explain what "percent per annum" means and demonstrate the relevant calculations step-by-step. For example, they can show how to calculate the value of the percentage first, and then find the total value over the number of years.

Section 6.1 Percentage, ratio and rate

Students will review the concepts of percentage, ratio and rate in this section. These will be applied in this chapter. Teachers are advised to make sure that students can solve the questions in Practise Now independently before moving on to the next section. Students who have trouble with solving problems involving reverse percentage, particularly Question 2 in Practise Now 1A, may need extra support. In such cases, teachers should consider revisiting some exercise questions in Book 1 or trying other questions.

Section 6.2 Profit, loss, discount, General Sales Tax, and commission

In this section, students will learn about financial transactions involving percentages, such as profit, loss, discount, and the General Sales Tax (GST). Teachers can explain the purpose and necessity of taxes in different It is recommended that students complete the activity on calculating discounts and GST to reinforce the concept of multiplying percentages (see Investigation: Discount, service charge, and GST). Some students may use the commutative property of multiplication to show that the order in which the discount and service charge are applied does not affect the total amount on the bill. Teachers may demonstrate how to calculate the discount, GST, and service charge in a single step (see Method 2 in Worked Example 6 on page 187).

Section 6.3 Insurance, hire purchase and interest

In this section, students will apply the concept of rates to solve problems involving insurance premiums, hire purchase and interest. Teachers may opt to go through simple interest together with insurance due to the relative simplicity and similarity in calculations. Some students may struggle here because rate is expressed as a percentage. To overcome this, teachers may explain what percent per annum means.

At the end of Worked Example 10, teachers may revisit the Introductory Problem and encourage the students to discuss the considerations for each option. It's important to emphasise to the students the importance of being prudent in financial matters, especially when taking out loans.

The difference between the simple and compound interests should be highlighted to the students in an activity (see Investigation: Exploring simple interest and compound interest). Teachers can get students to discuss the type of interest offered by banks or provide some examples where simple interest is used.

Section 6.4 Zakat, ushr and income tax

In this section, personal taxes and donations are discussed. By the end of the class discussion on page 199, students should understand that a progressive tax system is more beneficial to those with lower incomes. Teachers can use Worked Example 14 to introduce the tax slabs used in progressive tax systems.

Section 6.5 Inheritance and partnership

In this last section, students will apply the concept of ratio in distribution of inheritance and profits. Teachers can use the Worked Examples, Practise Now questions and exercise questions to demonstrate how each portion is calculated.

Introductory Problem

Teachers may revisit this problem after Practise Now 10, or at the end of Section 6.3.

1. **(i)** Downpayment for the laptop

$= \frac{20}{100} \times$ PKR 460 000

= **PKR 92 000**

Remaining price of laptop

= PKR 460 000 – PKR 92 000

= **PKR 368 000**

(ii) Total interest to be paid at the end of 12 months

= PKR 368 000 $\times$ 1

= **PKR 29 440**

Teachers may highlight that 'per annum' means per year.

(iii) Total amount to be paid in monthly instalments

= PKR 368 000 + PKR 29 440

= PKR 397 440

Monthly instalment

$= \frac{\text{PKR } 397\ 440}{12}$

= **PKR 33 120**

2. Total amount Imran will pay under the hire purchase option

= PKR 460 000 + PKR 29 440

= PKR 489 440

Some considerations for recommending hire purchase:

- Imran might need cashflow for unexpected expenses, but he expects a steady income over the next 12 months that will enable him to keep up with the monthly instalments.

Some considerations for not recommending hire purchase:

- Imran will be paying more in total. If Imran has enough money to cover unexpected expenses after paying for the laptop in full, he should consider making a one-time payment instead of opting for hire purchase.

6.1 Percentage, ratio and rate

Practise Now 1A

1. **(i)** Amount of money set aside as savings

$= \frac{32}{100} \times$ PKR 720 000

= **PKR 230 400**

Amount of money set aside for transportation

= $\times$ PKR 720 000

= **PKR 108 000**

(ii) Percentage of salary spent on food

$= \frac{\text{PKR } 180\ 000}{\text{PKR } 720\ 000} \times 100\%$

= **25%**

2. **(i)** 24% of the men = 60

1% of the men $= \frac{60}{100}$

100% of the men $= \frac{60}{24} \times 100$

$= 250$

There are **250** men in the audience.

(ii) 40% of the audience = 250

1% of the audience $= \frac{250}{40}$

100% of the audience $= \frac{250}{40} \times 100$

$= 625$

There are **625** people in the audience.

Practise Now 1B

1. **(i)** Number of pens

$= \frac{5}{5+3+6} \times 126$

= **45**

Number of pencils

$= \frac{3}{5+3+6} \times 126$

= **27**

Number of notebooks

$= \frac{6}{5+3+6} \times 126$

= **54**

Alternatively,

number of notebooks

= 126 – 45 – 27

= **54**

2. **(i)** Number of toys produced in 1 hour = 35

Number of toys produced in 6 hours $= 35 \times 6$

= **210**

(ii) Number of toys produced in 4.6 hours by first machine

$= 35 \times 4.6$

$= 161$

Number of toys produced in 4.6 hours by second machine

$= 161 + 92$

$= 253$

In 4.6 hours, the second machine produces 253 toys.

In 1 hour, the second machine produces $\frac{253}{4.6} = 55$ toys.

$\therefore$ the rate of toy production by second machine is **55 toys per hour**.

6.2 Profit, loss, discount, General Sales Tax, and commission

Practise Now 1C

1. (a) Required percentage $= \frac{\text{PKR } 24\,000 - \text{PKR } 18\,000}{\text{PKR } 18\,000} \times 100\%$

$= \frac{\text{PKR } 6000}{\text{PKR } 18\,000} \times 100\%$

$= \mathbf{33\frac{1}{3}\%}$ or **33.3%** (to 3 s.f.)

(b) Required percentage $= \frac{\$6000 - \$5000}{\$5000} \times 100\%$

$= \frac{\$1000}{\$5000} \times 100\%$

= **20%**

2. (a) Selling price of gold chain $= \frac{127}{100} \times \text{PKR } 112\,000$

= **PKR 142 240**

(b) Selling price of car $= \frac{94}{100} \times \$78\,400$

= **$73 696**

Practise Now 2

1. 135% of cost price = PKR 287 550

1% of cost price $= \frac{\text{PKR } 287\,550}{135}$

100% of cost price $= \frac{\text{PKR } 287\,550}{135} \times 100$

= PKR 213 000

The cost price of the smartphone is **PKR 213 000**.

2. Cost price of 1800 eggs $= \frac{1800}{12} \times \$1.20$

= $180

Total selling price of eggs so as to earn a 33% profit on the cost price

$= \frac{133}{100} \times \$180$

= $239.40

Number of eggs that the shopkeeper can sell $= \frac{95}{100} \times 1800$

= 1710

Selling price of each egg $= \frac{\$239.40}{1710}$

= **$0.14**

Thinking Time (Page 183)

There is insufficient information provided to find the answer. We will need to know either the cost price of the camera or the profit/loss made.

Practise Now 3

1. Percentage discount $= \frac{\text{PKR } 1000 - \text{PKR } 880}{\text{PKR } 1000} \times 100\%$

$= \frac{\text{PKR } 120}{\text{PKR } 1000} \times 100\%$

= **12%**

2. Sale price of washing machine $= \frac{94}{100} \times \$600$

= **$564**

Practise Now 4

1. (i) 91% of marked price = PKR 111 020

1% of marked price $= \frac{\text{PKR } 111\,020}{91}$

100% of marked price $= \frac{\text{PKR } 111\,020}{91} \times 100$

= PKR 122 000

The marked price of the laptop is **PKR 122 000**.

(ii) Sale price of laptop after a 5% discount $= \frac{95}{100} \times \text{PKR } 122\,000$

= PKR 115 900

Sale price of laptop after a further discount of 4%

$= \frac{96}{100} \times \text{PKR } 115\,900$

= PKR 111 264

No, the sale price would not be PKR 111 020.

2. (i) 90% of marked price = $180

1% of marked price $= \frac{\$180}{90}$

100% of marked price $= \frac{\$180}{90} \times 100$

= $200

The marked price of the handbag is **$200**.

(ii) Sale price of handbag after a 10% discount = $180

Sale price of handbag after a further 15% discount

$= \frac{85}{100} \times \$180$

= **$153**

Practise Now 5

1. GST payable $= \frac{18}{100} \times \text{PKR } 19\,000$

= PKR 3420

Total amount of money the man has to pay for article

= PKR 19 000 + PKR 3420

= **PKR 22 420**

2. 118% of marked price = $642

1% of marked price $= \frac{\$642}{118}$

100% of marked price $= \frac{\$642}{118} \times 100$

= $544.07 (to the nearest cent)

The marked price of the printer is **$544.07**.

Investigation (Discount, service charge and GST)

1. GST as calculated by the restaurant

$= \frac{18}{100} \times \left[\text{PKR } 1040 + \left(\frac{10}{100} \times \text{PKR } 1040\right)\right]$

$= \frac{18}{100} \times (\text{PKR } 1040 + \text{PKR } 104)$

$= \frac{18}{100} \times \text{PKR } 1144$

= PKR 206 (to the nearest PKR 1)

2. **No**, I do not agree. GST is an acronym for General Sales Tax, thus the tax is also imposed on the service charge, which is 10% of the subtotal.

3. If the discount is given before the service charge is taken into account, the bill received will be as follows:

Yummy Restaurant		
Fish and Chips:	PKR	600
Chicken Chop:	PKR	700
Subtotal:	PKR	1300
Discount 20%:	– PKR	260
Subtotal:	PKR	1040
Service Charge 10%:	PKR	104
GST 18%:	PKR	206
Total:	**PKR**	**1350**

If the discount is given after the service charge has been taken into account, the bill received will be as follows:

Yummy Restaurant		
Fish and Chips:	PKR	600
Chicken Chop:	PKR	700
Subtotal:	PKR	1300
Service Charge 10%:	PKR	130
Subtotal:	PKR	257
Subtotal:	PKR	1687
GST 9%:	PKR	337
Total:	PKR	**1350**

4. If the discount is given before the service charge is taken into account, the service charge will be calculated based on a smaller amount, i.e. PKR 1040, and thus the service charge will have already been discounted.
If the discount is given after the service charge has been taken into account, the service charge will be calculated based on a greater amount, i.e. PKR 1300, and thus the discount will be given on the service charge as well.
Hence, it makes no difference whether the discount is given before or after the service charge is taken into account as the total bill will still be the same.

Practise Now 6

1. Discount $= \frac{15}{100} \times \text{PKR } 640$

= PKR 96

Service charge = 10% × (marked price – discount)

$= \frac{10}{100} \times (\text{PKR } 640 - \text{PKR } 96)$

$= \frac{10}{100} \times \text{PKR } 544$

= PKR 54.4

GST payable = 18% × (marked price – discount + service charge)

$= \frac{18}{100} \times (\text{PKR } 640 - \text{PKR } 96 + \text{PKR } 54.4)$

$= \frac{18}{100} \times \text{PKR } 598.4$

= PKR 107.712

Total amount payable

= marked price – discount + service charge + GST payable

= PKR 640 - PKR 96 +PKR 54.4 + PKR 107.712

= **PKR 706** (to the nearest PKR 1)

2. 129.8% of price after discount = PKR 2600

1% of price after discount $= \frac{\text{PKR } 2600}{129.8}$

100% of price after discount $= \frac{\text{PKR } 2600}{129.8} \times 100$

= PKR 2003 (to the nearest PKR 1)

The price of the set meal after discount is PKR 2003.

80% of marked price = PKR 2003

1% of marked price $= \frac{\text{PKR } 2003}{80}$

100% of marked price $= \frac{\text{PKR } 2003}{80} \times 100$

= PKR 2504 (to the nearest PKR 1)

The marked price of the set meal is **PKR 2504**.

Practise Now 7

1. Amount of commission the agent receives

$= \frac{2}{100} \times \text{PKR } 14\,000\,000$

= **PKR 280 000**

2. 3.5% of selling price = \$25 375

1% of selling price $= \frac{\$25\,375}{3.5}$

100% of selling price $= \frac{\$25\,375}{3.5} \times 100$

= \$725 000

The selling price of the piece of property is **\$725 000**.

Exercise 6A

1. (a) Profit = \$45 – \$40

= **\$5**

Profit as a percentage of cost price $= \frac{5}{40} \times 100\%$

= **12.5%**

Profit as a percentage of selling price

$= \frac{5}{45} \times 100\%$

$= \mathbf{11\frac{1}{9}\%}$ or **11.1%** (to 3 s.f.)

(b) Loss = \$600 – \$480
= **\$120**

Loss as a percentage of cost price $= \frac{120}{600} \times 100\%$
= **20%**

Loss as a percentage of selling price $= \frac{120}{480} \times 100\%$
= **25%**

(c) Selling price $= \frac{104}{100} \times$ PKR 88 000
= **PKR 91 520**

Profit = PKR 91 520 – PKR88 000
= **PKR 3520**

Profit as a percentage of selling price
$= \frac{3520}{91\ 520} \times 100\%$
$= \mathbf{3\frac{11}{13}}$ **%** or **3.85%** (to 3 s.f.)

(d) Selling price $= \frac{77.5}{100} \times$ PKR 5680
= **PKR 4402**

Loss = PKR 5680 – PKR 4402
= **PKR 1278**

Loss as a percentage of selling price
$= \frac{1278}{4402} \times 100\%$
$= \mathbf{29\frac{1}{31}}$ **%** or **29.0%** (to 3 s.f.)

(e) $117\frac{1}{4}$ % of cost price = \$28.14

1% of cost price $= \frac{\$28.14}{117\frac{1}{4}}$

100% of cost price $= \frac{\$28.14}{117\frac{1}{4}} \times 100$
= \$24

Cost price = **\$24**

Profit = \$28.14 – \$24
= **\$4.14**

Profit as a percentage of selling price $= \frac{4.14}{28.14} \times 100\%$
= **14.7%** (to 3 s.f.)

(f) 93% of cost price = \$506.85

1% of cost price $= \frac{\$506.85}{93}$

100% of cost price $= \frac{\$506.85}{93} \times 100$
= \$545

Cost price = **\$545**

Loss = \$545 – \$506.85
= **\$38.15**

Loss as a percentage of selling price $= \frac{38.15}{506.85} \times 100\%$
= **7.53%** (to 3 s.f.)

2. Selling price of one dozen of roses = 12 × \$1.20
= \$14.40

Required percentage $= \frac{\$18 - \$14.40}{\$14.40} \times 100\%$
$= \frac{\$3.60}{\$14.40} \times 100\%$
= **25%**

3. **(i)** 35% of cost price = PKR 217 000

1% of cost price $= \frac{\text{PKR } 217\ 000}{35}$

100% of cost price $= \frac{\text{PKR } 217\ 000}{35} \times 100$
= PKR 620 000

The cost price of the refrigerator is **PKR 620 000**.

(ii) Selling price of refrigerator = PKR 620 000 + PKR 217 000
= **PKR 837 000**

4. 88% of cost price = \$16.50

1% of cost price $= \frac{\$16.50}{88}$

100% of cost price $= \frac{\$16.50}{88} \times 100$
= \$18.75

The cost price of the book is **\$18.75**.

5. Percentage discount $= \frac{\text{PKR } 129\ 800 - \text{PKR } 103\ 840}{\text{PKR } 129\ 800} \times 100\%$
$= \frac{\text{PKR } 25\ 960}{\text{PKR } 129\ 800} \times 100\%$
= **20%**

6. Sale price of folding table $= \frac{88}{100} \times$ PKR 3200
=PKR 2816

7. **(i)** 7% of marked price = \$49

1% of marked price $= \frac{\$49}{7}$

100% of marked price $= \frac{\$49}{7} \times 100$
= \$700

The marked price of the television set is **\$700**.

(ii) Sale price of television set = \$700 – \$49
= **\$651**

8. GST payable $= \frac{18}{100} \times$ PKR 20 000
= PKR 3600

Total amount of money Ali has to pay for microwave oven
= PKR 20 000 + PKR 3600
= **PKR 23 600**

9. 109% of marked price = \$1417

1% of marked price $= \frac{\$1417}{109}$

100% of marked price $= \frac{\$1417}{109} \times 100$
= \$1300

The marked price of the electronic gadget is **\$1300**.

10. **(a)** Amount of commission the agent receives $= \frac{2.5}{100} \times \$650\,000$
$= \mathbf{\$16\,250}$

(b) 2.5% of selling price = \$12 000

1% of selling price $= \frac{\$12\,000}{2.5}$

100% of selling price $= \frac{\$12\,000}{2.5} \times 100$
$= \$480\,000$

The selling price of the house is **\$480 000**.

11. Cost price of 5 kg of mixture $= 2 \times \$8 + 3 \times \$6$
$= \$16 + \$18$
$= \$34$

Selling price of 5 kg of mixture $= 20 \times \$2.55$
$= \$51$

Required percentage $= \frac{\$51 - \$34}{\$51} \times 100\%$
$= \frac{\$17}{\$51} \times 100\%$
$= \mathbf{33\frac{1}{3}\%}$ or **33.3%** (to 3 s.f.)

12. 75% of price Nadia buys from Li Ting = PKR 36 000

1% of price Nadia buys from Li Ting $= \frac{\text{PKR } 36\,000}{75}$

100% of price Nadia buys from Li Ting $= \frac{\text{PKR } 36\,000}{75} \times 100$
$= \text{PKR } 48\,000$

Nadia buys the fax machine from Li Ting at PKR 48 000.

125% of price Sara paid = PKR 48 000

1% of price Sara paid $= \frac{\text{PKR } 48\,000}{75}$

100% of price Sara paid $= \frac{\text{PKR } 48\,000}{75} \times 100$
$= \text{PKR } 38\,400$

Sara paid **PKR 38 400** for the fax machine.

13. Total number of apples Raju buys $= 200 \times 60$
$= 12\,000$

Cost price of 12 000 apples $= 200 \times \$28$
$= \$5600$

Total selling price of apples so as to earn a 80% profit on the cost price $= \frac{180}{100} \times \$5600$
$= \$10\,080$

Number of apples that Raju can sell $= \frac{85}{100} \times 12\,000$
$= 10\,200$

Selling price per apple $= \frac{\$10\,080}{10\,200}$
$= \mathbf{\$0.99}$ (to the nearest cent)

14. Cost price of each article $= \frac{\$1500}{300}$
$= \$5$

Selling price of each of the 260 articles $= \frac{120}{100} \times \$5$
$= \$6$

Selling price of each of the remaining 40 articles $= \frac{50}{100} \times \$6$
$= \$3$

Selling price of articles $= 260 \times \$6 + 40 \times \$3$
$= \$1560 + \$120$
$= \$1680$

Required percentage $= \frac{\$1680 - \$1500}{\$1500} \times 100\%$
$= \frac{\$180}{\$1500} \times 100\%$
$= \mathbf{12\%}$

15. A possible selling price of a book is **\$13**.

Let x be the cost price.

$\frac{13 - x}{x} \times 100\% = 30\%$

$\frac{13 - x}{x} = \frac{3}{10}$

$130 - 10x = 3x$

$13x = 130$

$x = 10$

$\therefore$ the cost price will be **\$10**.

16. **(i)** 87.5% of marked price = PKR 423 500

1% of marked price $= \frac{\text{PKR } 423\,500}{87.5}$

100% of marked price $= \frac{\text{PKR } 423\,500}{87.5} \times 100$
$= \text{PKR } 484\,000$

The marked price of the air conditioner is **PKR 484 000**.

(ii) Sale price of air conditioner after a 10% discount
$= \frac{90}{100} \times \text{PKR } 484\,000$
$= \text{PKR } 435\,600$

Sale price of air conditioner after a further discount of 2.5%
$= \frac{97.5}{100} \times \text{PKR } 435\,600$
$= \text{PKR } 424\,710$

No, the sale price would not be PKR 423 500.

17. **(i)** 84% of marked price = \$420

1% of marked price $= \frac{\$420}{84}$

100% of marked price $= \frac{\$420}{84} \times 100$
$= \$500$

The marked price of the item is **\$500**.

(ii) Sale price of item after a 16% discount = \$420

Sale price of item after a further discount of 14%
$= \frac{86}{100} \times \$420$
$= \$361.20$

The sale price of the same item if a member buys it is **\$361.20**.

18. Price of seafood fried rice after discount $= \frac{75}{100} \times \text{PKR } 900$
$= \text{PKR } 675$

Total amount payable $= \frac{129.8}{100} \times \text{PKR } 675$
$=$ **PKR 876** (to the nearest PKR 1)

19. Amount of commission Joyce received that month $= \$1220 - \$500$
$= \$720$

Amount of sales for that month $= \frac{\$720}{4} \times 100$
$= \mathbf{\$18\,000}$

20. Let the marked price of the sofa set be $\$x$.

Sale price of sofa set $= 90\% \times 80\% \times 75\% \times \$x$
$= 90\% \times 75\% \times 80\% \times \$x$
$= 80\% \times 90\% \times 75\% \times \$x$
$= 80\% \times 75\% \times 90\% \times \$x$
$= 75\% \times 90\% \times 80\% \times \$x$
$= 75\% \times 80\% \times 90\% \times \$x$
$= \$0.54x$

Thus **there is no difference** as the sale price of the sofa set is the same regardless of the order Kumar chooses to arrange the 3 discounts.

21. 129.8% of price after discount= PKR 2395

$$1\% \text{ of price after discount} = \frac{\text{PKR } 2395}{129.8}$$

$$100\% \text{ of price after discount} = \frac{\text{PKR } 2395}{129.8} \times 100$$

= PKR 1845 (to the nearesst PKR 1)

The price of the ramen after discount is PKR 1845.

82% of marked price = PKR 1845

$$1\% \text{ of marked price} = \frac{\text{PKR } 1845}{82}$$

$$100\% \text{ of marked price} = \frac{\text{PKR } 1845}{82} \times 100$$

= PKR 2250

The marked price of the ramen is **PKR 2250**.

6.3 Insurance, hire purchase and interest

Practise Now 8

1. Yearly premium
= 4% × insurance amount
$= \frac{3}{100} \times$ PKR 400 000
= **PKR 12 000**

2. Yearly premium = premium rate × insurance amount
PKR 21 000 = premium rate × PKR 600 000
Premium rate
$= \frac{\text{PKR } 21\,000}{\text{PKR } 600\,000} \times 100\%$
= **3.5%**

Practise Now 9

1. Annual premium
= 2.6% × value of car
= 2.6% × PKR 860 000
= **PKR 22 360**

2. Premium payable in the first year
= 2% × value of car in the first year
= 2% × PKR 650 000
= PKR 13 000
Value of car in the second year
= 96% × PKR 650 000
= PKR 624 000
Premium payable in the second year
= 2% × value of car in the second year
= 2% × PKR 624 000
= PKR 12 480
Total premium payable in the first two years
= PKR 13 000 + PKR 12 480
= **PKR 25 480**

Practise Now 10

(i) Down payment
$= \frac{20}{100} \times$ PKR 484 500
= PKR 96 900
Remaining amount
= PKR 484 500 – PKR 96 900
= PKR 387 600
Amount of interest Nadia owes at the end of 4 years
= PKR 387 600 $\times \frac{10}{100} \times 4$
= PKR 542 640
Monthly instalment
$= \frac{\text{PKR } 542\,640}{48}$
= **PKR 11 305**
(4 years = 48 months)

(ii) Total amount Nadia pays for the air conditioner
= PKR 542 640 + PKR96 900
= **PKR 637 540**

(iii) She has to pay **PKR 155 040** more for buying the air conditioner on hire purchase.

Practise Now 11

1. Amount of interest Bernard has to pay at the end of 3 years
$= \$150\,000 \times \frac{5.5}{100} \times 3$
= **$24 750**
Total amount he owes the bank
= $150 000 + $24 750
= **$174 750**

2. Amount of interest Joyce earns per year
= PKR 60 000 $\times \frac{3}{100}$
= PKR 1800
Time taken for her investment to grow to PKR 67 200
$= \frac{\text{PKR } 7200}{\text{PKR } 1800}$
= **4 years**

Investigation (Exploring simple interest and compound interest)

1. Interest $= \frac{PRT}{100}$
$= \frac{1000 \times 2 \times 3}{100}$
= **$60**

Total amount she would have after 3 years = $1000 + $60
= **$1060**

2. 1st year:

Principal $P_1 = \$1000$

Interest $I_1 = \$1000 \times 2\%$

$= \$\mathbf{20}$

Total amount at the end of the 1st year, $A_1 = P_1 + I_1$

$= \$1000 + \$\mathbf{20}$

$= \$1020$

2nd year:

Principal $P_2 = A_1 = \$1020$

Interest $I_2 = \$\mathbf{1020} \times 2\%$

$= \$\mathbf{20.40}$

Total amount at the end of the 2nd year, $A_2 = P_2 + I_2$

$= \$1020 + \$\mathbf{20.40}$

$= \$\mathbf{1040.40}$

3rd year:

Principal $P_3 = A_2 = \$\mathbf{1040.40}$

Interest $I_3 = \$\mathbf{1040.40} \times 2\%$

$= \$\mathbf{20.808}$

Total amount at the end of the 3rd year, A_3

$= P_3 + I_3$

$= \$\mathbf{1040.40} + \$\mathbf{20.808}$

$= \$\mathbf{1061.21}$ (to the nearest cent)

3. Interest offered by Bank B = \$1061.21 – \$1000

= \$61.21

Difference in amount of interest offered by Bank A and Bank B

= \$61.21 – \$60

= \$1.21

∴ Bank **B** offers a higher interest by **\$1.21**.

Practise Now 12

1. $A = P\left(1 + \frac{r}{100}\right)^n$

$= \text{PKR } 30\,000\left(1 + \frac{5}{100}\right)^4$

= PKR 36 465 (to the nearest PKR 1)

Compound interest $I = A - P$

= PKR 36 465 – PKR 30 000

= **PKR 6465**

2. **(a)** $A = P\left(1 + \frac{r}{100}\right)^n$

$= \$1500\left(1 + \frac{2}{100}\right)^2$

$= \$1560.60$

Compound interest $I = A - P$

= \$1560.60 – \$1500

= **\$60.60**

(b) Since interest is compounded monthly,

$r = \frac{2}{12} = \frac{1}{6}, n = 2 \times 12 = 24$

$A = P\left(1 + \frac{r}{100}\right)^n$

$= \$1500\left(1 + \frac{\frac{1}{6}}{100}\right)^{24}$

= \$1561.16 (to the nearest cent)

Compound interest $I = A - P$

= \$1561.16 – \$1500

= **\$61.16**

3. $A = P\left(1 + \frac{r}{100}\right)^n$

$\text{PKR } 42\,436 = \text{PKR } 40\,000\left(1 + \frac{r}{100}\right)^2$

$\left(1 + \frac{r}{100}\right)^2 = \frac{\text{PKR } 42436}{\text{PKR } 40\,000}$

$= 1.0609$

$1 + \frac{r}{100} = \sqrt{1.0609}$

$= 1.0300$ (to 5 s.f.)

$\frac{r}{100} = 0.0300$

$r = 3.00$ (to 3 s.f.)

∴ the interest rate is **3.00%**.

Exercise 6B

1. Yearly premium

= 2.8% × insurance amount

$= \frac{2.8}{100} \times \text{PKR } 620\,000$

= **PKR 17 630**

2. Annual premium

= 6.5% × value of car

= 6.5% × PKR 750 000

= **PKR 48 750**

3. **(i)** Remaining amount = \$3200 – \$480

= \$2720

Amount of interest the man has to pay at the end of 2 years

$= \$2720 \times \frac{9.5}{100} \times 2$

= \$516.80

Total amount to be paid in monthly instalments

= \$2720 + \$516.80

= \$3236.80

Monthly instalment $= \frac{\$3236.80}{24}$

= **\$134.87** (to the nearest cent)

(ii) Total amount the man pays for the computer system

= \$480 + \$3236.80

= **\$3716.80**

(iii) Difference = \$3716.80 – \$3200

= **\$516.80**

4. Amount of interest Albert has to pay at the end of 2 years

$= \text{PKR } 48\,000 \times \frac{6}{100} \times 2$

= PKR 5760

Total amount of money he has to pay at the end of 2 years

= PKR 48 000 + PKR 5760

= **PKR 53 760**

5. Total amount of interest the man earns = \$18 900 – \$16 800

= \$2100

Amount of interest the man earns per year

$= \$16\,800 \times \frac{5}{100}$

= \$840

Time taken for his investment to grow to \$16 800

$= \frac{\$2100}{\$840}$

$= \mathbf{2\frac{1}{2}}$ **years**

6. $A = P\left(1 + \frac{r}{100}\right)^n$

$= \text{PKR } 50\,000\left(1 + \frac{8}{100}\right)^3$

$= \text{PKR } 62\,986$ (to the nearest PKR 1)

Compound interest $I = A - P$

$= \text{PKR } 62\,986 - \text{PKR } 50\,000$

$= \text{PKR } 12\,986$

$\therefore$ the total interest in his account at the end of 3 years is **PKR 12 986**.

7. Total premium payable = premium rate × insurance value × number of years

PKR 67 500 = 3% × insurance value × 5

Insurance value

$= \dfrac{\text{PKR } 67\,500}{\frac{3}{100} \times 5}$

= **PKR 450 000**

8. Premium payable in the first year

= 3.5% × value of car in the first year

$= \frac{3.5}{100} \times 5 \times \text{PKR } 1\,000\,000$

= PKR 35 000

Premium payable in the second year

= 3.5% × value of car in the second year

$= \frac{3.5}{100} \times \left(\frac{95}{100} \times \text{PKR } 1\,000\,000\right)$

$= \frac{3.5}{100} \times \text{PKR } 950\,000$

= PKR 33 250

Premium payable in the third year

= 3.5% × value of car in the third year

$= \frac{3.5}{100} \times \left(\frac{95}{100} \times \text{PKR } 950\,000\right)$

$= \frac{3.5}{100} \times \text{PKR } 902\,500$

= PKR 31 588 (to the nearest PKR 1)

Total premium paid in the first three years

= PKR 35 000 + PKR 33 250 + PKR 31 588

= **PKR 99 838**

9. Down payment $= \frac{25}{100} \times \$x$

$= \$0.25x$

Remaining amount $= \$x - \$0.25x$

$= \$0.75x$

Amount of interest the man owes at the end of 30 months

(= 2.5 years) $= \$0.75x \times \frac{12}{100} \times 2.5$

$= \$0.225x$

Remaining amount including the interest that the man needs to pay $= \$0.75x + \$0.225x$

$= \$0.975x$

Total amount to be paid in monthly instalments = \$52 × 30

= \$1560

Hence $0.975x = 1560$

$x =$ **1600**

10. Amount of interest received before the interest rate decreases

$= \frac{2.75}{100} \times \$20\,000$

= \$550

Amount of interest received after the interest rate decreases

= \$550 − \$50

= \$500

$\frac{x}{100} \times \$20\,000 = \$500$

$200x = 500$

$x = \frac{500}{200}$

$\therefore x =$ **2.5**

11. **(a)** Interest $= \text{PKR } 400\,000 \times \frac{2.35}{100} \times 1$

= PKR 9400

Cheryl had expected to receive **PKR 9400** after one year.

(b) She misinterpreted the interest rate to be 2.35% per annum because she failed to notice the information – "10 Year High-Yield Account", which means that the 2.35% in interest will only be accumulated after 10 years.

12. **(a)** Since interest is compounded monthly,

$r = \frac{5.68}{12} = \frac{71}{150}$, $n = 6 \times 12 = 72$

$A = P\left(1 + \frac{r}{100}\right)^n$

$= \text{PKR } 15\,000\left(1 + \frac{\frac{71}{150}}{100}\right)^{72}$

= **PKR 21 074** (to the nearest PKR 1)

(b) Since interest is compounded half-yearly,

$r = \frac{5.68}{2} = 2.84$, $n = 6 \times 2 = 12$

$A = P\left(1 + \frac{r}{100}\right)^n$

$= \text{PKR } 15\,000\left(1 + \frac{2.84}{100}\right)^{12}$

= **PKR 20 991** (to the nearest PKR 1)

13. $A = P\left(1 + \frac{r}{100}\right)^n$

$\text{PKR } 58\,000 = \text{PKR } 50\,000\left(1 + \frac{r}{100}\right)^2$

$\left(1 + \frac{r}{100}\right)^n = \frac{\text{PKR } 58\,000}{\text{PKR } 50\,000}$

$= 1.16$

$1 + \frac{r}{100} = \sqrt[5]{1.16}$

$\frac{r}{100} = \sqrt[5]{1.16} - 1$

$r = 3.01$ (to 3 s.f.)

$\therefore$ the interest rate is **3.01%**.

14. Since interest is compounded quarterly,

$r = \frac{4.2}{4} = 1.05$, $n = 1 \times 4 = 4$

$$A = P\left(1 + \frac{r}{100}\right)^n$$

$$96.60 + P = P\left(1 + \frac{1.05}{100}\right)^4$$

$$= 1.0105^4P$$

$$1.0105^4P = P + 96.60$$

$$1.0105^4P - P = 96.60$$

$$P(1.0105^4 - 1) = 96.60$$

$$P = \frac{96.60}{1.0105^4 - 1}$$

$= \mathbf{\$2264}$ (to the nearest dollar)

15. $$A = P\left(1 + \frac{r}{100}\right)^n$$

$$\$36\ 757.94 = \$x\left(1 + \frac{2}{100}\right)^7$$

$$36\ 757.94 = x(1.02)^7$$

$$x = \frac{36\ 757.94}{1.02^7}$$

$= \mathbf{32\ 000.00}$ (to 2 d.p.)

6.4 Zakat, ushr and income tax

Practise Now 13

1. Amount of ushr payable
$= 5\% \times$ value of agricultural output
$= \frac{5}{100} \times$ PKR 74 200
= PKR 3710

2. Zakat payable = 2.5% × yearly savings
PKR 1255 = 2.5% × yearly savings
Yearly savings
$= \dfrac{\text{PKR } 1255}{\frac{2.5}{100}}$
= PKR 50 200

Class Discussion (What is a reasonable way to tax income?)

1. Income tax that Cheryl has to pay $= \frac{5}{100} \times$ PKR 200 000
= PKR 10 000
Income tax that Joyce has to pay $= \frac{5}{100} \times$ PKR 1 600 000
= PKR 80 000
∴ **Joyce** pays more income tax.

2. Amount Cheryl has left to spend = PKR 200 000 – PKR 10 000
= PKR 190 000
Amount Joyce has left to spend = PKR 1 600 000 – PKR 80 000
= PKR 1 520 000
∴ **Joyce** has more money left to spend.

3. Since Cheryl earns less than PKR 400 000, she does not have to pay any income tax i.e. **PKR 0**.
Amount Joyce has to pay $= \frac{10}{100}$ (PKR 1 600 000 – PKR 400 000)
$= \frac{10}{100}$ (PKR 1 200 000)
= PKR 120 000
∴ **Joyce** pays more income tax.

4. Amount Cheryl has left to spend = **PKR 200 000**
Amount Joyce has left to spend = PKR 1 600 000 – PKR 120 000
= PKR 1 480 000
∴ **Joyce** has more money left to spend.

5. **Tax option B** is more favourable for people with lower incomes. They will have relatively more money left to spend than if income tax was at a flat rate (e.g. PKR 200 000 vs PKR 190 000).

6. Pakistan follows a **progressive** tax rate, where higher income earners pay a proportionately higher tax. This serves as a more equitable system, which seeks to narrow the income gap between people who earn higher incomes and those who earn lower incomes.

Practise Now 14

Taxable income
= \$75 600 – \$1000 – (\$4000 × 2) – \$4500 – \$1500
= \$60 600

Income tax payable
= (\$60 600 – \$20 000) × 18%
= \$7308

6.5 Inheritance and partnership

Practise Now 15

1. Amount inherited by widow
$= \frac{1}{8} \times$ PKR 288 000
= PKR 36 000
Amount to be shared among children
= PKR 288 000 – PKR 36 000
= PKR 252 000
sons' share : daughters' share
= 2 × 2 : 1 × 2
= 2 : 1
The two sons will inherit $\frac{2}{3}$ of remaining amount.
Amount inherited by two sons
$= \frac{2}{3} \times$ PKR 252 000
= PKR 168 000
Amount inherited by each son
= PKR 168 000 ÷ 2
= PKR 84 000

Amount inherited by two daughters

$= \frac{1}{3} \times$ PKR 252 000

= PKR 84 000

Amount inherited by each daughter

= PKR 84 000 ÷ 2

= PKR 42 000

2. Sons' share : daughter's share

$= 3 \times 2 : 1$

$= 6 : 1$

The three sons inherited $\frac{6}{7}$ of the inheritance.

Total amount three sons received

= PKR 220 000 × 3

= PKR 660 000

$\frac{6}{7}$ of the inheritance = PKR 660 000

$\frac{1}{7}$ of the inheritance = PKR 660 000 ÷ 6

= PKR 110 000

∴ Amount obtained from sale of property

= PKR 110 000 × 7

= PKR 770 000

Practise Now 16

1. **(i)** Investments by Imran : Cheryl : Joyce

= 44 100 : 14 700 : 88 200

= 3 : 1 : 6

(ii) Profit received by Imran

$= \frac{3}{3+1+6} \times$ PKR 80 100

$= \frac{3}{10} \times$ PKR 80 100

= PKR 24 030

Profit received by Cheryl

$= \frac{1}{10} \times$ PKR 80 100

= PKR 8010

Profit received by Joyce

$= \frac{6}{10} \times$ PKR 80 100

= PKR 48 060

2. Nadia's share forms $\frac{5}{4+3+5} = \frac{5}{12}$ of the total profit.

$\frac{5}{12}$ of total profit = \$850

$\frac{1}{12}$ of total profit $= \frac{\$850}{5}$

∴ total profit $= \frac{\$850}{5} \times 12$

= \$2040

Exercise 6C

1. Amount of zakat

= 2.5% × yearly savings

$= \frac{2.5}{100} \times$ PKR 400 000

= PKR 10 000

2. Amount of ushr

= 5% × value of agricultural output

$= \frac{5}{100} \times$ PKR 71 000

= PKR 3550

3. **(a)** Income tax payable

= 2.5% × (PKR 685 000 – PKR 600 000)

= PKR 2125

(b) Income tax payable

= PKR 165 000 + (22.5% × (PKR 3 050 000 – PKR 2 400 000))

= PKR 165 000 + (22.5% × PKR 650 000)

= PKR 311 250

(c) Income tax payable

= PKR 435 000 + (27.5% × (PKR 6 000 000 – PKR 3 600 000))

= PKR 435 000 + (27.5% × PKR 2 400 000)

= PKR 1 095 000

(d) PKR 12 150 000

Income tax payable

= PKR 2 955 000 + (35% × (PKR 12 150 000 – PKR 12 000 000))

= PKR 2 955 000 + (35% × PKR 150 000)

= PKR 3007 500

4. Property tax payable yearly

$=$ PKR 938 952 $\times \frac{25}{100}$

= PKR 234 738

Property tax payable for 6 months

$= \frac{\text{PKR } 234\ 738}{2}$

= PKR 117 369

5. Amount inherited by widow $= \frac{1}{8} \times$ PKR 249 600

= PKR 31 200

Amount remaining = PKR 249 600 – PKR 31 200

= PKR 218 400

Son's share : daughters' share = 2 : 1 × 2

= 1 : 1

The son will inherit $\frac{1}{2}$ of the remaining amount.

Amount inherited by son $= \frac{1}{2} \times$ PKR 218 400

= PKR 109 200

Amount inherited by each daughter = PKR 109 200 ÷ 2

= PKR 54 600

6. Amount of ushr = 10% × value of agricultural output

PKR 6700 $= \frac{10}{100} \times$ value of agricultural output

Value of agricultural output $= \frac{\text{PKR } 6700}{\frac{10}{100}}$

= PKR 67 000

7. Taxable income
$= \$80\,000 - \$3000 - (\$5000 \times 2) - \$16\,000 - \$750$
$= \$50\,250$
Income tax payable
$= \$550 + 7\% \times (\$50\,250 - \$40\,000)$
$= \$550 + \frac{7}{100} \times \$10\,250$
= $1267.50

8. Amount inherited by 3 sons
$= \frac{7}{8}$ of total inheritance
Amount inherited by each son
$= (\frac{7}{8} \div 3)$ of total inheritance
$= \frac{7}{24}$ of total inheritance
$\frac{7}{24}$ of total inheritance = PKR 84 630
$\frac{1}{24}$ of total inheritance $= \frac{\text{PKR } 84\,630}{7}$
Total inheritance $= \frac{\text{PKR } 84\,630}{7} \times 24$
= PKR 290 160

9. Amount received by heir 1
$= \frac{4}{4+3+3} \times$ PKR 12 850 000
= PKR 5 140 000
Amount received by heir 2
$= \frac{3}{4+3+3} \times$ PKR 12 850 000
= PKR 3 855 000
= amount received by heir 3

10. Ratio of Albert, Imran and Sara's property investment
= 427 000 : 671 000 : 305 000
= 427 : 671 : 305
= 7 : 11 : 5
Total amount of profit earned
= \$1 897 500 − (\$427 000 + \$671 00 + \$305 000)
= \$494 500
Amount of profit Albert received
$= \frac{7}{7+11+5} \times \$494\,500$
= $150 500
Amount of profit Imran received
$= \frac{7}{7+11+5} \times \$494\,500$
= $236 500
Amount of profit Sara received
$= \frac{7}{7+11+5} \times \$494\,500$
= $107 500

11. Let the amount of money be taxable at 12.5% be PKR x.
Amount of tax payable for PKR x at 12.5%
= PKR 24 375 − PKR 15 000
= PKR 9375
Hence, $\frac{12.5}{100} \times$ PKR x = PKR 9375
$0.125x$ = PKR 9375
x = PKR 75 000

Total reliefs = PKR 60 000 + PKR 40 000 + (4 × PKR 80 000) + (2 × PKR 100 000) + 10% of gross income + PKR 4000
= PKR 624 000 + 10% of gross income
Gross income = PKR 1 200 000 + PKR 75 000 + PKR 624 000 + 10% of gross income
= PKR 1 899 000 + 10% of gross income
90% of gross income = PKR 1 899 000
Gross income = **PKR 2 110 000**

12. Ratio of Nadia's, Joyce's and Waseem's profits
= PKR 29 680 : PKR 44 520 : PKR 37 100
= 4 : 6 : 5
Amount invested by Nadia $= \frac{4}{4+6+5} \times$ PKR 978 000
= PKR 260 800
Amount invested by Joyce $= \frac{6}{4+6+5} \times$ PKR 978 000
= PKR 391 000
Amount invested by Waseem $= \frac{5}{4+6+5} \times$ PKR 978 000
= PKR 326 000

13. **(i)** Original ratio of Ken's, Shaha's and David's investment
= 3 : 5 : 4
= 15 : 25 : 20
New ratio of Ken's, Shaha's and David's investment
= 4 : 6 : 5
= 16 : 24 : 20
$\frac{1}{15+25+20} = \frac{1}{60}$ of the profit was given by Shaha to Ken.
$\frac{1}{60}$ of the profit = PKR 85 000
Amount of profit = PKR 85 000 × 60
= PKR 5 100 000

(ii) Original amount received by Ken
$= \frac{3}{3+5+4} \times$ PKR 5 100 000
= PKR 1 275 000
Original amount received by Shaha
$= \frac{5}{3+5+4} \times$ PKR 5 100 000
= PKR 2 125 000
Original amount received by David
$= \frac{4}{3+5+4} \times$ PKR 5 100 000
= PKR 1 700 000

Chapter 7 Direct and Inverse Proportion

TEACHING NOTES

Suggested Approach

In Secondary One, students have learnt rates such as \$0.25 per egg, or 13.5 km per litre of petrol etc. Teachers may wish to expand this further by asking what the prices of 2, 4 or 10 eggs are, or the distance that can be covered with 2, 4 or 10 litres of petrol, and leading to the introduction of direct proportion. After students are familiar with direct proportion, teachers can show the opposite scenario that is inverse proportions.

Section 7.1 Direct proportion

When introducing direct proportion, rates need not be stated explicitly. Rates can be used implicitly (see Investigation: Direct proportion). By showing how one quantity increases proportionally with the other quantity, the concept should be easily relatable. Teachers should discuss the linkages between direct proportion, algebra, rates and ratios to assess and improve students' understanding at this stage (see page 209 of the textbook). Teachers should also show the unitary method and proportion method in the worked example and advise students to adopt the method that is most comfortable for them.

Section 7.2 Algebraic and graphical representations of direct proportion

By recapping what was covered in the previous section, teachers should easily state the direct proportion formula between two quantities and the constant k. It is important to highlight the condition k $\neq$ 0 as the relation would not hold if $k = 0$.

Through studying how direct proportion means graphically (see Investigation: Graphical representation of direct proportion), students will gain an understanding on how direct proportion and linear functions are related, particularly the positive gradient of the straight line and the graph passing through the origin. The graphical representation will act as a test to determine if two variables are directly proportional. It is important to highlight the features of a graph that indicates direct proportion between two variables (see Thinking Time on page 211).

Section 7.3 Other forms of direct proportion

Direct proportion does not always involve two linear variables. If one variable divided by another gives a constant, then the two variables are directly proportional (see Investigation: Other forms of direct proportion). In this case, although the graph of y against x is a parabola, the graph of y against x^2 will be a straight line passing through the origin. Teachers may wish to illustrate the direct proportionality clearly by replacing variables with Y and X and showing $Y = kX$, which is in the form students learnt in the previous section.

Section 7.4 Inverse proportion

The other form of proportion, inverse proportion, can be explored and studied by students (see Investigation: Inverse Proportion). When one variable increases, the other variable decreases proportionally. It is the main difference between direct and inverse proportion and must be emphasised clearly.

Students should be tasked with giving real-life examples of inverse proportion and explaining how they are inversely proportional (see Class Discussion: Real-life examples of quantities in inverse proportion). Teachers should present another difference between both kinds of proportions by reminding students that $\frac{y}{x}$ is a constant in direct proportion while xy is a constant in inverse proportion (see page 223 of the textbook).

Section 7.5 Algebraic and graphical representations of inverse proportion

constant k. It is important to highlight the condition $k \neq 0$ as the relation would not hold if $k = 0$ (see Thinking Time on page 225).

Although plotting y against x gives a hyperbola, and does not provide any useful information, teachers can show by plotting y against $\frac{1}{x}$ and showing direct proportionality between the two variables (see Investigation: graphical representation of inverse proportion).

Section 7.6 Other forms of inverse proportion

Inverse proportion, just like direct proportion, may not involve two linear variables all the time. Again, teachers can replace the variables with Y and X and show the inverse proportionality relation $Y = \frac{k}{X}$.

Introductory Problem

The solutions to this problem can be found in Introductory Problem Revisited (after Practise Now 9).

7.1 Direct proportion

Investigation (Direct proportion)

1. The fine will **increase** if the number of days a book is overdue increases.

2. $\dfrac{\text{Fine when a book is overdue for 6 days}}{\text{Fine when a book is overdue for 3 days}} = \dfrac{90}{45}$

$= 2$

The fine will be **doubled** if the number of days a book is overdue is doubled.

3. $\dfrac{\text{Fine when a book is overdue for 6 days}}{\text{Fine when a book is overdue for 2 days}} = \dfrac{90}{30}$

$= 3$

The fine will be **tripled** if the number of days a book is overdue is tripled.

4. $\dfrac{\text{Fine when a book is overdue for 5 days}}{\text{Fine when a book is overdue for 10 days}} = \dfrac{75}{150}$

$= \dfrac{1}{2}$

The fine will be **halved** if the number of days a book is overdue is halved.

5. $\dfrac{\text{Fine when a book is overdue for 3 days}}{\text{Fine when a book is overdue for 9 days}} = \dfrac{45}{135}$

$= \dfrac{1}{3}$

The fine will be **reduced to $\frac{1}{3}$** of the original amount if the number of days a book is overdue is reduced to $\frac{1}{3}$ of the original number.

Practise Now 1

(a) Method 1: Unitary method

50 g of sweets cost \$2.10.

1 g of sweets costs $\dfrac{\$2.10}{50}$.

380 g of sweets cost $\dfrac{\$2.10}{50} \times 380 = \$15.96$.

$\therefore$ 380 g of sweets cost **\$15.95** (to the nearest 5 cents).

Method 2: Proportion method

Let the cost of 380 g of sweets be \$$x$.

Method 2a:

$\dfrac{x}{380} = \dfrac{2.10}{50}$

$x = \dfrac{2.10}{50} \times 380$

$= \$15.96$

Method 2b:

$\dfrac{x}{2.10} = \dfrac{380}{50}$

$x = \dfrac{380}{50} \times 2.10$

$= \$15.96$

$\therefore$ 380 g of sweets cost **\$15.95** (to the nearest 5 cents).

(b) Method 1: Unitary method

$\dfrac{3}{4}$ of a piece of metal weighs 15 kg.

The piece of metal weighs $\dfrac{15}{\frac{3}{4}} = 20$ kg.

$\dfrac{2}{5}$ of the piece of metal weighs $\dfrac{2}{5} \times 20 =$ **8 kg**.

Method 2: Proportion method

Let the mass of $\dfrac{2}{5}$ of the piece of metal be x kg.

Method 2a:

$\dfrac{x}{\frac{2}{5}} = \dfrac{15}{\frac{3}{4}}$

$x = \dfrac{15}{\frac{3}{4}} \times \dfrac{2}{5}$

$= 8$

Method 2b:

$\dfrac{x}{15} = \dfrac{\frac{2}{5}}{\frac{3}{4}}$

$x = \dfrac{\frac{2}{5}}{\frac{3}{4}} \times 15$

$= 8$

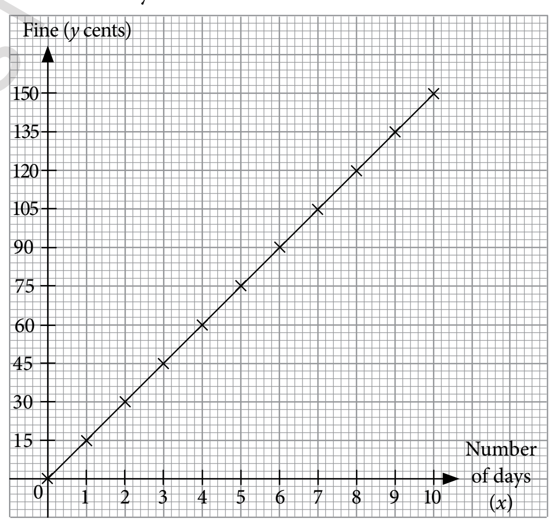
$\therefore$ the mass of $\dfrac{2}{5}$ of the piece of metal is **8 kg**.

7.2 Algebraic and graphical representations of direct proportion

Investigation (Graphical representation of direct proportion)

$y = 15x$ in this context means that the fine increases by 15 cents for each additional day that a book is overdue.

1.



2. The graph is a **straight line**.

3. The y-intercept is **(0, 0)**.

4. The gradient of the graph represents the **increase in the fine (y cents) for each additional day (x) that a book is overdue**, i.e. 15 cents/day.

Thinking Time (Page 211)

1. Since y is directly proportional to x, $y = kx$

$$x = \left(\frac{1}{k}\right)y$$

Since $k \neq 0$, then we can rename $\frac{1}{k} = k_1$ where k_1 is another constant.

Hence, $x = k_1 y$, where $k_1 \neq 0$ and x **is directly proportional** to y.

2. $x = k_1 y$ is the equation of a straight line. When $y = 0$, $x = 0$.
We will get a straight line of x against y that passes through the origin.

3. If the graph of y against x does not pass through the origin, then $y \neq 0$ when $x = 0$. Hence, y is not directly proportional to x.
Note that even if the graph passes through the origin, this does **not necessarily** conclude that y is directly proportional to x, e.g. $y = x^2$.

4. As x increases, y also increases. This does **not necessarily** conclude that y is directly proportional to x. It is important that when x increases, y increases proportionally. Also, when $x = 0$, $y = 0$. $y = kx + c$ is an example of how x increases and y increases, but y is not directly proportional to x.

Practise Now 2

1. (i) Since y is directly proportional to x, then $y = kx$, where k is a constant.
When $x = 2$, $y = 10$,
$10 = k \times 2$
$k = 5$
$\therefore$ $\boldsymbol{y = 5x}$

(ii) **Method 1:**
Substitute $x = 10$ into $y = 5x$:
$y = 5 \times 10$
$= \mathbf{50}$

Method 2:
When $x = 2$, $y = 10$.
When $x = 10$,
$y = 5 \times 10$
$= \mathbf{50}$

We can also use $\frac{y_2}{y_1} = \frac{x_2}{x_1}$,
i.e. $\frac{y}{10} = \frac{10}{2}$
$y = 5 \times 10$
$= \mathbf{50}$

(iii) Substitute $y = 60$ into $y = 5x$:
$60 = 5x$
$\therefore x = \frac{60}{5}$
$= \mathbf{12}$

2. Since y is directly proportional to x,
$\frac{y_2}{y_1} = \frac{x_2}{x_1}$
$\frac{y}{5} = \frac{7}{2}$
$y = \frac{7}{2} \times 5$
$= \mathbf{17.5}$

3. Since q is directly proportional to p, then $q = kp$, where k is a constant.
When $p = 5$, $q = 30$,
$30 = k \times 5$
$\therefore k = 6$
$\therefore q = 6p$
When $q = 48$,
$48 = 6 \times p$
$p = \frac{48}{6}$
$= 8$
When $q = 57$,
$57 = 6 \times p$
$p = \frac{57}{6}$
$= 9.5$
When $p = 4$,
$q = 6 \times 4$
$= 24$
When $p = 7$,
$q = 6 \times 7$
$= 42$

p	4	5	7	**8**	**9.5**
q	**24**	30	**42**	48	57

Practise Now 3

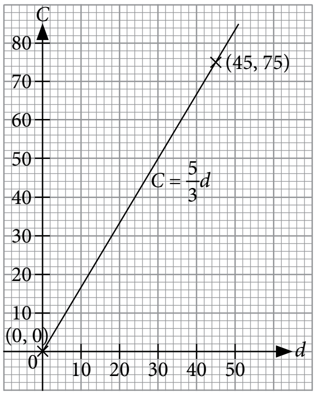
(i) Since C is directly proportional to d, then $C = kd$, where k is a constant.
When $d = 60$, $C = 100$,
$100 = k \times 60$
$\therefore k = \frac{5}{3}$
$\therefore \boldsymbol{C = \frac{5}{3}d}$

(ii) When $d = 45$,
$C = \frac{5}{3} \times 45$
$= 75$
$\therefore$ the cost of transporting goods is **$75**.

(iii) When $C = 120$,
$120 = \frac{5}{3} \times d$
$d = 120 \times \frac{3}{5}$
$= 72$
$\therefore$ the distance covered is **72 km**.

(iv) $C = \frac{5}{3}d$
Since C is directly proportional to d, then the graph passes through the origin.

From part **(ii)**, when $d = 45$, $C = 75$.



Practise Now 4

(i) Total monthly cost = \$5000 + 80 × 41
$\quad$ = **\$8280**

(ii) Variable amount = \$7378 – \$5000
$\quad$ = \$2378

Number of children enrolled = $\frac{\$2378}{\$41}$
$\quad$ = **58**

(iii) Variable amount = $n \times \$41$
$\quad$ = $\$41n$

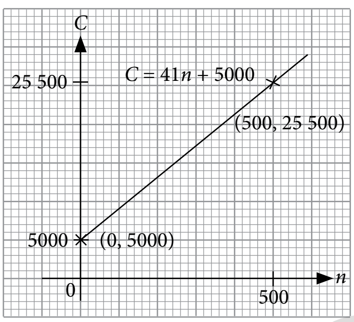
Total monthly cost = variable amount + fixed amount

$\therefore$ $\boldsymbol{C = 41n + 5000}$

(iv) $C = 41n + 5000$

When $n = 0$, $C = 5000$.

When $n = 500$, $C = 25\,500$.



C is **not** directly proportional to n because the line does not pass through the origin.

Exercise 7A

1. **(i)** The number of books is directly proportional to the mass of books.

108 books have a mass of 30 kg.

1 book has a mass of $\frac{30}{108}$ kg.

150 books have a mass of $\frac{30}{108} \times 150 = \mathbf{41\frac{2}{3}}$ **kg** or **41.7 kg** (to 3 s.f.)

(ii) The mass of books is directly proportional to the number of books.

30 kg is the mass of 108 books.

1 kg is the mass of $\frac{108}{30}$ books.

20 kg is the mass of $\frac{108}{30} \times 20 = \mathbf{72}$ books.

2. **(i)** The number of books is directly proportional to the length occupied by the books.

60 books occupy a length of 1.5 m.

1 book occupies a length of $\frac{1.5}{60}$ m.

50 books occupy a length of $\frac{1.5}{60} \times 50 = \mathbf{1.25\ m}$.

(ii) The length occupied by the books is directly proportional to the number of books.

1.5 m = (1.5 × 100) cm
$\quad$ = 150 cm

150 cm is the length occupied by 60 books.

1 cm is the length occupied by $\frac{60}{150}$ books.

80 cm is the length occupied by $\frac{60}{150} \times 80 = \mathbf{32}$ books.

3. **(i)** Since x is directly proportional to y, then $x = ky$, where k is a constant.

When $x = 4.5$, $y = 3$,

$4.5 = k \times 3$

$k = 1.5$

$\therefore$ $\boldsymbol{x = 1.5y}$

(ii) **Method 1:**

Substitute $y = 6$ into $x = 1.5y$:

$x = 1.5 \times 6$

$\quad = \mathbf{9}$

Method 2:

When $y = 6$,

$x = 2 \times 4.5$

$\quad = \mathbf{9}$

We can also use $\frac{x_2}{x_1} = \frac{y_2}{y_1}$,

i.e. $\frac{x}{4.5} = \frac{6}{3}$

$y = 2 \times 4.5$

$\quad = \mathbf{9}$

(iii) Substitute $x = 12$ into $x = 1.5y$:

$12 = 1.5y$

$\therefore y = \frac{12}{1.5}$

$\quad = \mathbf{8}$

4. **(i)** Since Q is directly proportional to P, then $Q = kP$, where k is a constant.

When $P = 4$, $Q = 28$,

$28 = k \times 4$

$\therefore k = 7$

$\therefore$ $\boldsymbol{Q = 7P}$

(ii) When $P = 5$,

$Q = 7 \times 5$

$\quad = \mathbf{35}$

(iii) When $Q = 42$,

$42 = 7 \times P$

$P = \mathbf{6}$

5. **(a)** The mass of tea leaves is directly proportional to the cost of tea leaves.

3 kg of tea leaves cost \$18.

1 kg of tea leaves costs $\$\left(\frac{18}{3}\right)$.

10 kg of tea leaves cost $\$\left(\frac{18}{3} \times 10\right) = \mathbf{\$60}$.

(b) The mass of sugar is directly proportional to the cost.

b kg of sugar cost \$$c$.

1 kg of sugar costs \$$\frac{c}{b}$.

a kg of sugar cost \$$\left(\frac{c}{b}\times a\right)$ = **\$$\boldsymbol{\frac{ac}{b}}$**.

6. The amount of metal is directly proportional to the mass of the metal.

$\frac{5}{9}$ of a piece of metal has a mass of 7 kg.

A whole piece of metal has a mass of $\frac{7}{\frac{5}{9}}$ kg.

$\frac{2}{7}$ of a piece of metal has a mass of $\frac{7}{\frac{5}{9}} \times \frac{2}{7} = \mathbf{3\frac{3}{5}}$ **kg** or **3.6 kg**.

7. Since z is directly proportional to x,

$\frac{x_2}{z_2} = \frac{x_1}{z_1}$

$\frac{x}{18} = \frac{3}{12}$

$x = \frac{3}{12} \times 18$

$= \mathbf{4.5}$

8. Since B is directly proportional to A,

$\frac{B_2}{A_2} = \frac{B_1}{A_1}$

$\frac{B}{24} = \frac{3}{18}$

$B = \frac{3}{18} \times 24$

$= \mathbf{4}$

9. **(a)** Since y is directly proportional to x, then $y = kx$, where k is a constant.

When $x = 24$, $y = 6$,

$6 = k \times 24$

$\therefore k = \frac{1}{4}$

$\therefore y = \frac{1}{4}x$

When $y = 9$,

$9 = \frac{1}{4} \times x$

$x = 9 \times 4$

$= 36$

When $y = 11$,

$11 = \frac{1}{4} \times x$

$x = 11 \times 4$

$= 44$

When $x = 4$,

$y = \frac{1}{4} \times 4$

$= 1$

When $x = 20$,

$y = \frac{1}{4} \times 20$

$= 5$

x	4	20	24	**36**	**44**
y	**1**	**5**	6	9	11

(b) Since y is directly proportional to x, then $y = kx$, where k is a constant.

When $x = 3$, $y = 3.6$,

$3.6 = k \times 3$

$\therefore k = 1.2$

$\therefore y = 1.2x$

When $y = 9.6$,

$9.6 = 1.2 \times x$

$x = \frac{9.6}{1.2}$

$= 8$

When $y = 11.4$,

$11.4 = 1.2 \times x$

$x = \frac{11.4}{1.2}$

$= 9.5$

When $x = 2$,

$y = 1.2 \times 2$

$= 2.4$

When $x = 5.5$,

$y = 1.2 \times 5.5$

$= 6.6$

x	2	3	5.5	**8**	**9.5**
y	**2.4**	3.6	**6.6**	9.6	11.4

10. **(i)** Since y is directly proportional to x, then $y = kx$, where k is a constant.

When $x = 5$, $y = 20$,

$20 = k \times 5$

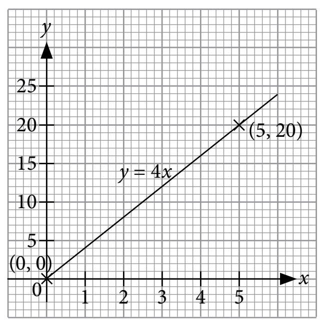
$k = 4$

$\therefore \boldsymbol{y = 4x}$

(ii) $y = 4x$

Since y is directly proportional to x, then the graph passes through the origin.

It was given that when $x = 5$, $y = 20$.



11. **(i)** Since z is directly proportional to y, then $z = ky$, where k is a constant.

When $y = 6$, $z = 48$,

$48 = k \times 6$

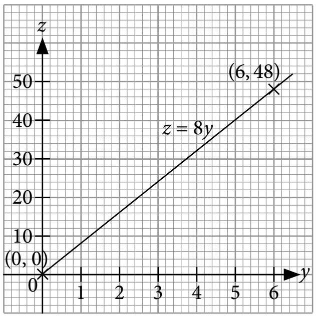
$k = 8$

$\therefore \boldsymbol{z = 8y}$

(ii) $z = 8y$

Since z is directly proportional to y, then the graph passes through the origin.

It was given that when $y = 6$, $z = 48$.



12. (i) Since F is directly proportional to m,
then $F = km$, where k is a constant.
When $m = 5$, $F = 49$,
$49 = k \times 5$
$\therefore k = 9.8$
$\therefore$ $\boldsymbol{F = 9.8m}$

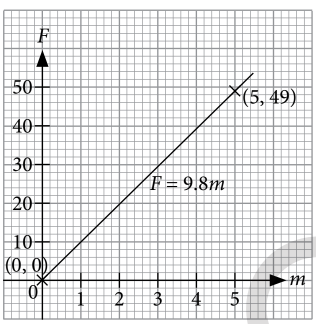
(ii) When $m = 14$,
$F = 9.8 \times 14$
$= \mathbf{137.2}$

(iii) When $F = 215.6$,
$215.6 = 9.8 \times m$
$m = \dfrac{215.6}{9.8}$
$= \mathbf{22}$

(iv) $F = 9.8m$

Since F is directly proportional to m, then the graph passes through the origin.

It was given that when $m = 5$, $F = 49$.



13. (i) Since P is directly proportional to T,
then $P = kT$, where k is a constant.
When $T = 10$, $P = 25$,
$25 = k \times 10$
$\therefore k = 2.5$
$\therefore$ $\boldsymbol{P = 2.5T}$

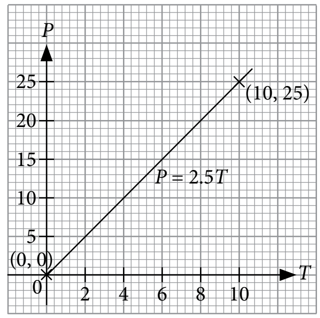
(ii) When $T = 24$,
$P = 2.5 \times 24$
$= \mathbf{60}$

(iii) When $P = 12$,
$12 = 2.5 \times T$
$T = \dfrac{12}{2.5}$
$= \mathbf{4.8}$

(iv) $P = 2.5T$

Since P is directly proportional to T, then the graph passes through the origin.

It was given that when $T = 10$, $P = 25$.



14. (i) Since V is directly proportional to R,
then $V = kR$, where k is a constant.
When $R = 6$, $V = 9$,
$9 = k \times 6$
$\therefore k = 1.5$
$\therefore$ $\boldsymbol{V = 1.5R}$

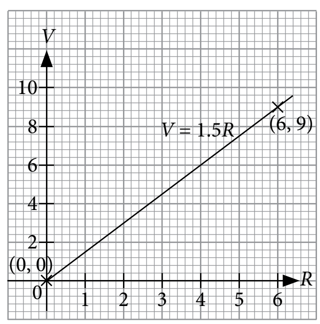
(ii) When $R = 15$,
$V = 1.5 \times 15$
$= \mathbf{22.5}$

(iii) When $V = 15$,
$15 = 1.5 \times R$
$R = \dfrac{15}{1.5}$
$= \mathbf{10}$

(iv) $V = 1.5R$

Since V is directly proportional to R, then the graph passes through the origin.

It was given that when $R = 6$, $V = 9$.



15. Let the mass of ice produced be m tonnes and the number of hours of production be T hours.
Since m is directly proportional to T,
then $m = kT$, where k is a constant.

When $T = \dfrac{30}{60} - \dfrac{10}{60} = \dfrac{1}{3}$, $m = 20$,

$20 = k \times \dfrac{1}{3}$

$\therefore k = 60$

$\therefore m = 60T$

When $T = 1.75 - \dfrac{10}{60}$,

$m = 60\left(1.75 - \dfrac{10}{60}\right)$
$= 95$

$\therefore$ the mass of ice manufactured is **95 tonnes**.

15. (i) Total income for that month
$= \$600 + \$8 \times 95$
$= \mathbf{\$1360}$

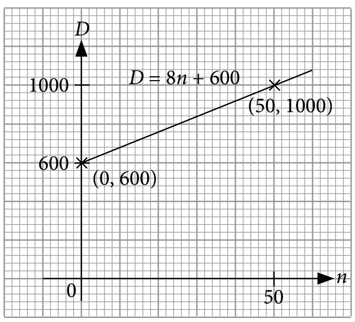
(ii) Variable amount $= \$1680 - \$600$
$= \$1080$

Number of tyres he sold in that month $= \dfrac{1080}{8}$
$= \mathbf{135}$

(iii) Variable amount $= n \times \$8$
$= \$8n$

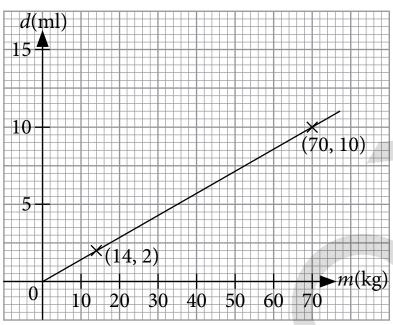
Total income = variable amount + fixed amount
$\therefore\ \boldsymbol{D = 8n + 600}$

(iv) $D = 8n + 600$
When $n = 0$, $D = 600$.
When $n = 50$, $D = 1000$.



D is **not** directly proportional to n because the line does not pass through the origin.

17. A real-world example is medication dosage given in proportion to the mass of the person. For example, if an adult with a mass, m, of 70 kg receives a medication dose, d, of 10 ml, then a child with a mass of 14 kg should receive a dose of $(14 \div 70) \times 10 = 2$ ml.
$\therefore$ the graph passes through the points (14, 2) and (70, 10).

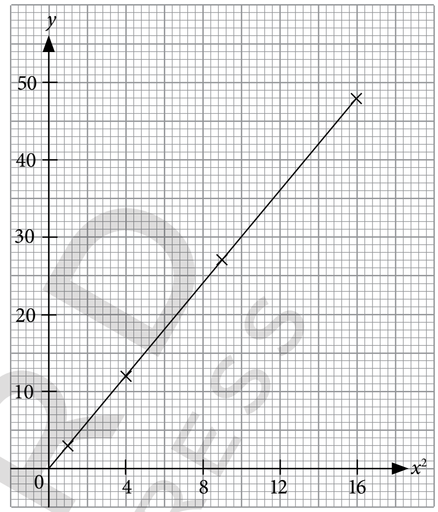


7.3 Other forms of direct proportion

Investigation (Other forms of direct proportion)

1. y is **not directly proportional** to x. The graph of y against x is not a straight line that passes through the origin.

2.



y is **directly proportional** to x^2. The graph of y against x^2 is a straight line that passes through the origin.

Practise Now 5

(a) Since $y = 6x^2$, i.e. $\dfrac{y}{x^2} = 6$ is a constant, then $\boldsymbol{y}$ and $\boldsymbol{x^2}$ are directly proportional to each other.

(b) Since $\sqrt{y} = x^3$, i.e. $\dfrac{\sqrt{y}}{x^3} = 1$ is a constant, then $\boldsymbol{\sqrt{y}}$ and $\boldsymbol{x^3}$ are directly proportional to each other.

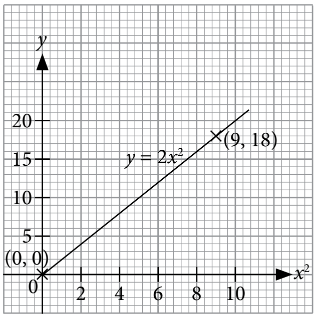
Practise Now 6

1. (i) Since y is directly proportional to x^2,
then $y = kx^2$, where k is a constant.
When $x = 3$, $y = 18$,
$18 = k \times 3^2$
$18 = 9k$
$\therefore\ k = 2$
$\therefore\ \boldsymbol{y = 2x^2}$

(ii) When $x = 5$,
$y = 2 \times 5^2$
$= \mathbf{50}$

(iii) When $y = 32$,
$32 = 2x^2$
$x^2 = 16$
$\therefore\ x = \pm\sqrt{16}$
$= \mathbf{\pm 4}$

(iv) Since y is directly proportional to x^2, then the graph of y against x^2 is a straight line that passes through the origin. It was given that when $x = 3$, $y = 18$. When $x = 3$, $x^2 = 9$.
$\therefore$ the graph will pass through the point (9, 18).



2. Since y is directly proportional to x^2,
then $y = kx^2$, where k is a constant.
When $x = 2$, $y = 21$,
$21 = k \times 2^2$
$21 = 4k$
$\therefore k = \frac{21}{4}$
$\therefore y = \frac{21}{4}x^2$
When $x = 4$,
$y = \frac{21}{4} \times 4^2$
$= \mathbf{84}$

3. Since k is directly proportional to h^2,
then $k = ah^2$, where a is a constant.
When $h = 3$, $k = 81$,
$81 = a \times 3^2$
$\therefore a = 9$
$\therefore k = 9h^2$
When $k = 56.25$,
$56.25 = 9 \times h^2$
$h^2 = 6.25$
$h = \sqrt{6.25}$ $(h > 0)$
$= 2.5$
When $k = 441$,
$441 = 9 \times h^2$
$h^2 = 49$
$h = \sqrt{49}$ $(h > 0)$
$= 7$
When $h = 2$,
$k = 9 \times 2^2$
$= 36$
When $h = 5$,
$k = 9 \times 5^2$
$= 225$

h	2	**2.5**	3	5	**7**
k	**36**	56.25	81	**225**	441

Practise Now 7

(i) Since l is directly proportional to T^2, then $l = kT^2$, where k is a constant.
When $l = 55.8$, $T = 1.5$,
$55.8 = k(1.5)^2$
$\therefore k = \frac{55.8}{2.25}$
$= 24.8$
$\therefore \boldsymbol{l = 24.8T^2}$

(ii) When $T = 0.8$,
$l = 24.8(0.8)^2$
$= 15.9$ (to 3 s.f.)
$\therefore$ the length of the pendulum is **15.9 cm**.

(iii) When $l = 0.36$ m $= 36$ cm,
$36 = 24.8T^2$
$T^2 = \frac{36}{24.8}$
$\therefore T = \sqrt{\frac{36}{24.8}}$ $(T > 0)$
$= 1.20$ (to 3 s.f.)
$\therefore$ the period of the pendulum is **1.20 seconds**.

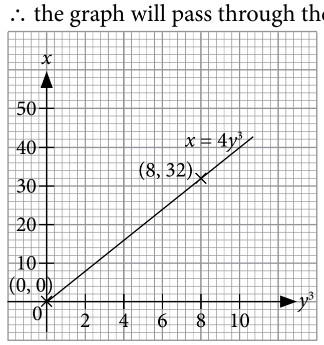
Exercise 7B

1. (i) Since x is directly proportional to y^3,
then $x = ky^3$, where k is a constant.
When $y = 2$, $x = 32$,
$32 = k \times 2^3$
$32 = 8k$
$\therefore k = 4$
$\therefore \boldsymbol{x = 4y^3}$

(ii) When $y = 6$,
$x = 4 \times 6^3$
$= \mathbf{864}$

(iii) When $x = 108$,
$108 = 4 \times y^3$
$y^3 = \frac{108}{4}$
$= 27$
$y = \mathbf{3}$

(iv) Since x is directly proportional to y^3, then the graph of x against y^3 is a straight line that passes through the origin. It was given that when $x = 32$, $y = 2$. When $y = 2$, $y^3 = 8$.
$\therefore$ the graph will pass through the point (8, 32).

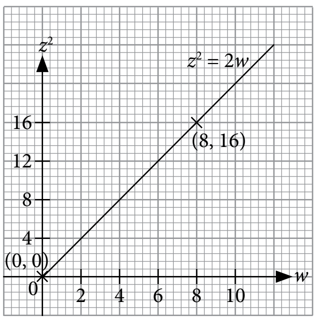


2. **(i)** Since z^2 is directly proportional to w,
then $z^2 = kw$, where k is a constant.
When $w = 8$, $z = 4$,
$4^2 = k \times 8$
$16 = 8k$
$\therefore k = 2$
$\therefore \boldsymbol{z^2 = 2w}$

(ii) When $w = 18$,
$z^2 = 2 \times 18$
$= 36$
$z = \pm\sqrt{36}$
$= \boldsymbol{\pm 6}$

(iii) When $z = 5$,
$5^2 = 2 \times w$
$w = \dfrac{25}{2}$
$= \mathbf{12.5}$

(iv) Since z^2 is directly proportional to w, then the graph of z^2 against w is a straight line that passes through the origin.
It was given that when $z = 4$, $w = 8$. When $z = 4$, $z^2 = 16$.
$\therefore$ the graph will pass through the point (8, 16).



3. **(i)** Since y is directly proportional to x^n,
then $y = kx^n$, where k is a constant.
Since y m^2 is the area of a square of length x m, then $y = x^2$.
$kx^n = x^2$
$\therefore n = \mathbf{2}$

(ii) Since y is directly proportional to x^n,
then $y = kx^n$, where k is a constant.
Since y cm^3 is the volume of a cube of length x cm, then $y = x^3$.
$kx^n = x^3$
$\therefore n = \mathbf{3}$

4. **(a)** Since $y = 4x^2$, i.e. $\dfrac{y}{x^2} = 4$ is a constant, then $\boldsymbol{y}$ and $\boldsymbol{x^2}$ are directly proportional to each other.

(b) Since $y = 3\sqrt{x}$, i.e. $\dfrac{y}{\sqrt{x}} = 3$ is a constant, then $\boldsymbol{y}$ and $\boldsymbol{\sqrt{x}}$ are directly proportional to each other.

(c) Since $y^2 = 5x^3$, i.e. $\dfrac{y^2}{x^3} = 5$ is a constant, then $\boldsymbol{y^2}$ and $\boldsymbol{x^3}$ are directly proportional to each other.

(d) Since $p^3 = q^2$, i.e. $\dfrac{p^3}{q^2} = 1$ is a constant, then $\boldsymbol{p^3}$ and $\boldsymbol{q^2}$ are directly proportional to each other.

5. Since z^2 is directly proportional to x^3,
$\dfrac{z_2^2}{x_2^3} = \dfrac{z_1^2}{x_1^3}$
$\dfrac{z^2}{9^3} = \dfrac{8^2}{4^3}$
$z^2 = \dfrac{8^2}{4^3} \times 9^3$
$= 729$
$z = \pm\sqrt{729}$
$= \boldsymbol{\pm 27}$

6. Since q is directly proportional to $(p - 1)^2$,
$\dfrac{(p_2 - 1)^2}{q_2} = \dfrac{(p_1 - 1)^2}{q_1}$
$\dfrac{(p - 1)^2}{80} = \dfrac{(3 - 1)^2}{20}$
$(p - 1)^2 = \dfrac{(3 - 1)^2}{20} \times 80$
$= 16$
$p - 1 = -4$ or $p - 1 = 4$
$p = -3$ $\quad$ $p = 5$
$\therefore p = \mathbf{-3}$ or $\mathbf{5}$

7. Since y is directly proportional to x^3,
then $y = kx^3$, where k is a constant.
When $x = 6$, $y = 648$,
$648 = k \times 6^3$
$\therefore k = 3$
$\therefore y = 3x^3$
When $y = 375$,
$375 = 3 \times x^3$
$x^3 = 125$
$x = \sqrt[3]{125}$
$= 5$
When $y = 1029$,
$1029 = 3 \times x^3$
$x^3 = 343$
$x = \sqrt[3]{343}$
$= 7$
When $x = 3$,
$y = 3 \times 3^3$
$= 81$
When $x = 4$,
$y = 3 \times 4^3$
$= 192$

$\boldsymbol{x}$	3	4	**5**	6	**7**
$\boldsymbol{y}$	**81**	**192**	375	648	1029

8. Since m is directly proportional to r^3,
then $m = kr^3$, where k is a constant.
When $r = 1.5$, $m = 6.75$,
$6.75 = k \times 1.5^3$
$\therefore k = 2$
$\therefore m = 2r^3$
When $m = 0.25$,
$0.25 = 2 \times r^3$
$r^3 = 0.125$
$r = \sqrt[3]{0.125}$
$= 0.5$

When $m = 11.664$,

$11.664 = 2 \times r^3$

$r^3 = 5.832$

$r = \sqrt[3]{5.832}$

$= 1.8$

When $r = 0.2$,

$m = 2 \times 0.2^3$

$= 0.016$

When $r = 0.7$,

$m = 2 \times 0.7^3$

$= 0.686$

r	0.2	**0.5**	0.7	1.5	**1.8**
m	**0.016**	0.25	**0.686**	6.75	11.664

9. **(i)** Since L is directly proportional to $\sqrt{N}$,

then $L = k\sqrt{N}$, where k is a constant.

When $N = 1$, $L = 2.5$,

$2.5 = k\sqrt{1}$

$\therefore k = 2.5$

$\therefore \boldsymbol{L = 2.5\sqrt{N}}$

(ii) When $N = 4$,

$L = 2.5 \times \sqrt{4}$

$= 5$

$\therefore$ the length of the earthworm 4 hours after its birth is **5 cm**.

(iii) When $L = 15$,

$15 = 2.5 \times \sqrt{N}$

$\sqrt{N} = 6$

$N = 6^2$

$= 36$

$\therefore$ it will take **36 hours** for the earthworm to grow to a length of 15 cm.

10. Since y is directly proportional to x^2,

then $y = kx^2$, where k is a constant.

When $x = 1$,

$y = k \times 1^2$

$= k$

When $x = 3$,

$y = k \times 3^2$

$= 9k$

Since the difference in the values of y is 32,

$9k - k = 32$

$8k = 32$

$\therefore k = 4$

$\therefore y = 4x^2$

When $x = -2$,

$y = 4 \times (-2)^2$

$= \mathbf{16}$

11. Since y is directly proportional to x^2,

$\dfrac{y_2}{x_2^2} = \dfrac{y_1}{x_1^2}$

$\dfrac{y}{(2x)^2} = \dfrac{a}{x^2}$

$y = \dfrac{a}{x^2} \times (2x)^2$

$= \dfrac{a}{x^2} \times 4x^2$

$= \boldsymbol{4a}$

12. Let the braking distance of a vehicle be D m and the speed of the vehicle be B m/s.

Since D is directly proportional to B^2, then $D = kB^2$, where k is a constant.

When $B = b$, $D = d$,

$d = k \times b^2$

$\therefore k = \dfrac{d}{b^2}$

$\therefore D = \dfrac{d}{b^2} B^2$

When the speed of the vehicle is increased by 200%,

$B = (100\% + 200\%) \times b$

$= \dfrac{100+200}{100} \times b$

$= 3b$

When $B = 3b$,

$D = \dfrac{d}{b^2}(3b)^2$

$= \dfrac{d}{b^2}(9b^2)$

$= 9d$

Percentage increase in its braking distance

$= \dfrac{9d - d}{d} \times 100\%$

$= \mathbf{800\%}$

13. **(i)**

x	2	4	5	8	10
y	5.2	41.6	81.25	332.8	650
$\dfrac{y}{x}$	$\dfrac{5.2}{2} = \dfrac{13}{5}$	$\dfrac{41.6}{4} = \dfrac{52}{5}$	$\dfrac{81.25}{5} = \dfrac{65}{4}$	$\dfrac{332.8}{8} = \dfrac{208}{5}$	$\dfrac{650}{10} = 65$
$\dfrac{y}{x^3}$	$\dfrac{5.2}{2^3} = \dfrac{13}{20}$	$\dfrac{41.6}{4^3} = \dfrac{13}{20}$	$\dfrac{81.25}{5^3} = \dfrac{13}{20}$	$\dfrac{332.8}{8^3} = \dfrac{13}{20}$	$\dfrac{650}{10^3} = \dfrac{13}{20}$

Since $\dfrac{y}{x}$ is not a constant but $\dfrac{y}{x^3} = \dfrac{13}{20}$ is a constant, then y is proportional to $\boldsymbol{x^3}$.

OPEN **(ii)** An example is that y could be the volume of a cube while x is the length of the side of the cube.

7.4 Inverse proportion

Investigation (Inverse proportion)

1. The time taken **decreases** when the speed of the car increases.

2. $\dfrac{\text{Time taken when speed of the car is 40 km/h}}{\text{Time taken when speed of the car is 20 km/h}} = \dfrac{3}{6}$

$= \dfrac{1}{2}$

The time taken will be **halved** when the speed of the car is doubled.

3. $\dfrac{\text{Time taken when speed of the car is 60 km/h}}{\text{Time taken when speed of the car is 20 km/h}} = \dfrac{2}{6}$

$= \dfrac{1}{3}$

The time taken will be **reduced to $\mathbf{\frac{1}{3}}$** of the original time taken when the speed of the car is tripled.

4. $\dfrac{\text{Time taken when speed of the car is 30 km/h}}{\text{Time taken when speed of the car is 60 km/h}} = \dfrac{4}{2}$

$= 2$

The time taken will be **doubled** when the speed of the car is halved.

5. $\dfrac{\text{Time taken when speed of the car is 40 km/h}}{\text{Time taken when speed of the car is 120 km/h}} = \dfrac{3}{1}$

$= 3$

The time taken will be **tripled** when the speed of the car is reduced to $\dfrac{1}{3}$ of its original speed.

Class Discussion (Real-life examples of quantities in inverse proportion)

The following are some real-life examples of quantities that are in inverse
proportion and why they are inversely proportional to each other.

- Soldiers often dig trenches while serving in the army. The more soldiers there are digging the same trench, the faster it will take. Assuming that each soldier digs at the same rate, then the time taken to dig a trench is inversely proportional to the number of soldiers.
- The area of a rectangle is the product of its length and breadth. Given a rectangle with a fixed area, if the length increases, then the breadth decreases proportionally. Therefore for a rectangle with a fixed area, its length and breadth are inversely proportional.
- The density of a material is the mass of the material per unit volume.
  For an object made using a material with a fixed mass, the density increases when the volume decreases proportionally. Therefore for an object made using a material with a fixed mass, its density is inversely proportional to its volume.
- The speed of a moving object is the distance travelled by the object per unit time. For the same distance, when the speed of the object increases, the time to cover the distance is decreased proportionally. The speed of a moving object is thus inversely proportional to the time taken by the object to cover a fixed distance.
- For a fixed amount of resultant force acting on an object, the acceleration of the object in the direction of the resultant force is dependent on the mass of the object. When the mass of the object increases or decreases, the acceleration of the object decreases or increases proportionally. This is known as Newton's Second Law and has helped to explain many physical phenomena occurring around us.

Teachers may wish to note that the list is not exhaustive.

Practise Now 8

The time taken to fill the tank is inversely proportional to the number of taps used.

Method 1: Unitary method

4 taps can fill the tank in 70 minutes.

1 tap can fill the tank in (70 × 4) minutes.

7 taps can fill the tank in $\dfrac{70\times 4}{7}$ = **40 minutes**.

Method 2: Proportion method

Let the time taken for 7 taps to fill the tank be y minutes.

Then $7y = 4 \times 70$

$y = \dfrac{4\times 70}{7}$

$= 40$

∴ 7 taps can fill the tank in **40 minutes**.

Practise Now 9

(a) 3 men can dig 2 trenches in 5 hours.

1 man can dig 2 trenches in $5 \times 3 = 15$ hours.

5 men can dig 2 trenches in $\dfrac{15}{5} = 3$ hours.

5 men can dig 1 trench in $\dfrac{3}{2} = 1.5$ hours.

5 men can dig 7 trenches in $1.5 \times 7 = 10.5$ hours.

∴ 5 men will take **10.5 hours** to dig 7 such trenches.

(b) 7 taps can fill 3 tanks in 45 minutes.

1 tap can fill 3 tanks in $45 \times 7 = 315$ minutes.

5 taps can fill 3 tanks in $\dfrac{315}{5} = 63$ minutes.

5 taps can fill 1 tank in $\dfrac{63}{3} = 21$ minutes.

∴ 5 taps will take **21 minutes** to fill 1 such tank.

Introductory Problem Revisited

In 20 days, 1000 sheep consume	1 consignment of fodder.	
In 400 days, 1000 sheep consume	1 × 20	= 20 consignments of fodder.
In 400 days, 1 sheep consumes	$\dfrac{20}{1000}$	= 0.02 consignments of fodder.
In 400 days, 550 sheep consume	0.02 × 550	= 11 consignments of fodder.

∴ **11** consignments of fodder are needed to feed 550 sheep for 400 days.

7.5 Algebraic and graphical representations of inverse proportion

Thinking Time (Page 225)

If we substitute $k = 0$ into $y = \dfrac{k}{x}$, then $y = 0$. This implies that for all values of
x, $y = 0$. y **cannot be inversely proportional** to x in this case.

Investigation (Graphical representation of inverse proportion)

1. The graph is **hyperbola** in shape. It is **not a straight line**.
2. Based on the points (20, 6) and (40, 3),

   Change in value of $y = \dfrac{3}{6}$

   $= \dfrac{1}{2}$

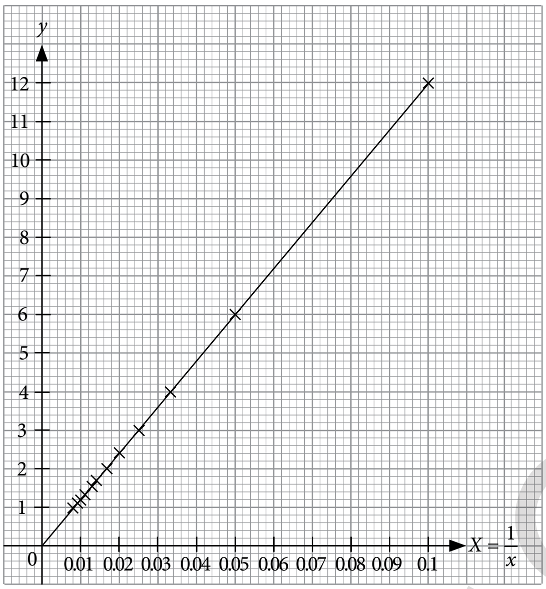
   ∴ the value of y will be **halved** when the value of x is doubled.

3. (i)

Speed (x km/h)	10	20	30	40	50	60
$X = \frac{1}{x}$	0.1	0.05	0.033	**0.025**	0.02	0.017
Time taken (y hours)	12	6	4	3	2.4	2

Speed (x km/h)	70	80	90	100	110	120
$X = \frac{1}{x}$	0.014	0.013	0.011	**0.01**	0.009	0.008
Time taken (y hours)	1.7	1.5	1.3	1.2	1.1	1

(ii)



4. The graph is a straight line. $\frac{y}{X}$ is a constant.

5. A linear relationship exists between y and X.

6. $y = kX$, where k is a constant. $\frac{y}{X}$ is a constant and y is linearly related to X.

Thinking Time (Page 228)

Since y is inversely proportional to x, $y = \frac{k}{x}$

$x = \frac{k}{y}$

Hence, $x = \frac{k}{y}$, where $k \neq 0$ and x **is inversely proportional** to y.

Practise Now 10

1. (i) When $x = 8$,

$y = \mathbf{\frac{5}{4}}$

Alternatively,

$x_2y_2 = x_1y_1$

$8 \times y = 2 \times 5$

$y = \frac{10}{8}$

$= \mathbf{\frac{5}{4}}$

(ii) Since y is inversely proportional to x, then $y = \frac{k}{x}$, where k is a constant.

When $x = 2$, $y = 5$,

$5 = \frac{k}{2}$

$\therefore k = 10$

$\therefore \mathbf{y = \frac{10}{x}}$

(iii) When $y = 10$,

$10 = \frac{10}{x}$

$\therefore x = \frac{10}{10}$

$= \mathbf{1}$

2. Since y is inversely proportional to x,

$x_2y_2 = x_1y_1$

$3 \times y = 2 \times 9$

$y = \frac{18}{3}$

$= \mathbf{6}$

3. Since n is inversely proportional to m,

then $n = \frac{k}{m}$, where k is a constant.

When $m = 2$, $n = 2$,

$2 = \frac{k}{2}$

$\therefore k = 4$

$\therefore n = \frac{4}{m}$

When $n = 4$,

$4 = \frac{4}{m}$

$m = \frac{4}{4}$

$= 1$

When $n = 0.8$,

$0.8 = \frac{4}{m}$

$m = \frac{4}{0.8}$

$= 5$

When $m = 0.5$,

$n = \frac{4}{0.5}$

$= 8$

When $m = 3$,

$n = \frac{4}{3}$

m	0.5	**1**	2	3	**5**
n	**8**	4	2	$\mathbf{\frac{4}{3}}$	0.8

Practise Now 11

(i) Method 1:

Since I is inversely proportional to R, then $I = \frac{k}{R}$, where k is a constant.

$k = IR$

When $I = 12$, $R = 0.5$,

$k = 12 \times 0.5$
$= 6$

$\therefore I = \frac{6}{R}$

When $R = 3$,

$I = \frac{6}{3}$
$= 2$

$\therefore$ the current flowing through the wire is **2 A**.

Method 2:

$I_2R_2 = I_1R_1$

$I_2 \times 3 = 0.5 \times 12$
$= 6$

$I_2 = \frac{6}{3}$
$= 2$

$\therefore$ the current flowing through the wire is **2 A**.

(ii) Method 1:

From part **(i)**, $I = \frac{6}{R}$.

When $I = 3$,

$3 = \frac{6}{R}$

$R = \frac{6}{3}$
$= 2\ \Omega$

$\therefore$ the resistance of the wire is **2 Ω**.

Method 2:

$I_2R_2 = I_1R_1$

$3 \times R_2 = 6$

$R_2 = \frac{6}{3}$
$= 2$

$\therefore$ the resistance of the wire is **2 Ω**.

Exercise 7C

1. **(a)** The number of pencils is directly proportional to the total cost of the pencils.
Assumption: All pencils are identical and cost the same.
(b) The number of taps filling a tank is inversely proportional to the time taken to fill the tank.
Assumption: All taps are identical and each tap takes the same time to fill the tank.
(c) The number of men laying a road is inversely proportional to the time taken to finish laying the road.
Assumption: All the men work at the same rate in laying the road.
(d) The number of cattle to be fed is directly proportional to the amount of fodder.
Assumption: All the cattle eat the same amount of fodder.
(e) The number of cattle to be fed is inversely proportional to the time taken to finish a certain amount of the fodder.
Assumption: All the cattle eat the fodder at the same rate.
$\therefore$ **(b)**, **(c)** and **(e)** are in inverse proportion.

2. The number of men is inversely proportional to the number of days to build the bridge.
8 men can build a bridge in 12 days.
1 man can build the bridge in (12×8) days.
6 men can build the bridge in $\frac{12 \times 8}{6}$ = **16 days**.
The assumption made is that all the men work at the same rate in building the bridge.

3. **(i)** Since x is inversely proportional to y,
$y_2x_2 = y_1x_1$
$25 \times x = 5 \times 40$
$x = \frac{5 \times 40}{25}$
$= \mathbf{8}$
(ii) Since x is inversely proportional to y,
then $x = \frac{k}{y}$, where k is a constant.
When $y = 5$, $x = 40$,
$40 = \frac{k}{5}$
$\therefore k = 200$
$\therefore \boldsymbol{x = \frac{200}{y}}$
(iii) When $x = 400$,
$400 = \frac{200}{y}$
$y = \frac{200}{400}$
$= \mathbf{0.5}$

4. **(i)** Since Q is inversely proportional to P,
then $Q = \frac{k}{P}$, where k is a constant.
When $P = 2$, $Q = 0.25$,
$0.25 = \frac{k}{2}$
$\therefore k = 0.5$
$Q = \frac{0.5}{P}$
$\therefore \boldsymbol{Q = \frac{1}{2P}}$
(ii) When $P = 5$,
$Q = \frac{1}{2(5)}$
$= \mathbf{0.1}$
(iii) When $Q = 0.2$,
$0.2 = \frac{1}{2P}$
$2P = \frac{1}{0.2}$
$= 5$
$P = \mathbf{2.5}$

5. The number of days is inversely proportional to the number of workers employed.
16 days are needed for 35 workers to complete the project.
1 day is needed for (35×16) workers to complete the project.
14 days are needed for $\frac{35 \times 16}{14}$ = 40 workers to complete the project.
Number of additional workers to employ = 40 – 35
= **5**

6. **(i)** The number of days is inversely proportional to the number of cattle to consume a consignment of fodder.
50 days are needed for 1260 cattle to consume a consignment of fodder.
1 day is needed for (1260 × 50) cattle to consume a consignment of fodder.
75 days are needed for $\frac{1260\times50}{75}$ = **840** cattle to consume a consignment of fodder.
(ii) 1260 cattle consume a consignment of fodder in 50 days.
1 cattle consumes a consignment of fodder in (50 × 1260) days.
1575 cattle consume a consignment of fodder in
$\frac{50\times1260}{1575}$ = **40 days**.

7. The number of athletes is inversely proportional to the number of days the food can last.
72 athletes take 6 days to consume the food.
1 athlete takes (6 × 72) days to consume the food.
72 – 18 = 54 athletes take $\frac{6\times72}{54}$ = 8 days to consume the food.
Number of additional days the food can last = 8 – 6
= **2 days**
The assumption made is that all athletes consume the same amount of food every day.

8. Since z is inversely proportional to x,
$x_2z_2 = x_1z_1$
$x \times 70 = 7 \times 5$
$x = \frac{7\times5}{70}$
$= \mathbf{0.5}$

9. Since B is inversely proportional to A,
$A_2B_2 = A_1B_1$
$1.4 \times B = 2 \times 3.5$
$B = \frac{2\times3.5}{1.4}$
$= \mathbf{5}$

10. **(a)** Since y is inversely proportional to x,
then $y = \frac{k}{x}$, where k is a constant.
When $x = 3$, $y = 4$,
$4 = \frac{k}{3}$
$\therefore k = 12$
$\therefore y = \frac{12}{x}$
When $y = 24$,
$24 = \frac{12}{x}$
$x = \frac{12}{24}$
$= 0.5$
When $y = 1.5$,
$1.5 = \frac{12}{x}$
$x = \frac{12}{1.5}$
$= 8$
When $x = 2$,
$y = \frac{12}{2}$
$= 6$
When $x = 2.5$,
$y = \frac{12}{2.5}$
$= 4.8$

x	**0.5**	2	2.5	3	**8**
y	24	**6**	**4.8**	4	1.5

(b) Since y is inversely proportional to x,
then $y = \frac{k}{x}$, where k is a constant.
When $x = 4$, $y = 9$,
$9 = \frac{k}{4}$
$\therefore k = 36$
$\therefore y = \frac{36}{x}$
When $y = 8$,
$8 = \frac{36}{x}$
$x = \frac{36}{8}$
$= 4.5$
When $y = 2.5$,
$2.5 = \frac{36}{x}$
$x = \frac{36}{2.5}$
$= 14.4$
When $x = 3$,
$y = \frac{36}{3}$
$= 12$
When $x = 25$,
$y = \frac{36}{25}$
$= 1.44$

x	3	4	**4.5**	**14.4**	25
y	**12**	9	8	2.5	**1.44**

11. **(i)** Since f is inversely proportional to λ,
then $f = \frac{k}{\lambda}$, where k is a constant.
When $\lambda = 3000$, $f = 100$,
$100 = \frac{k}{3000}$
$\therefore k = 300\,000$
$\therefore f = \frac{300\,000}{\lambda}$
When $\lambda = 500$,
$f = \frac{300\,000}{500}$
$= 600$
$\therefore$ the frequency of the radio wave is **600 kHz**.

(ii) When $f = 800$,

$800 = \frac{300\,000}{\lambda}$

$\lambda = \frac{300\,000}{800}$

$= 375$

$\therefore$ the wavelength of the radio wave is **375 m**.

12. **(i)** Since t is inversely proportional to N,

then $t = \frac{k}{N}$, where k is a constant.

When $N = 3$, $t = 8$,

$8 = \frac{k}{3}$

$\therefore k = 24$

$\therefore \boldsymbol{t = \frac{24}{N}}$

(ii) When $N = 6$,

$t = \frac{24}{6}$

$= 4$

$\therefore$ the number of hours needed by 6 men is **4 hours**.

(iii) When $t = \frac{3}{4}$,

$\frac{3}{4} = \frac{24}{N}$

$N = 24 \times \frac{4}{3}$

$= 32$

$\therefore$ **32** men need to be employed.

13. In 1 minute, Tap A alone fills up $\frac{1}{6}$ of the tank.

In 1 minute, Tap B alone fills up $\frac{1}{9}$ of the tank.

In 1 minute, Pipe C alone empties $\frac{1}{15}$ of the tank.

In 1 minute, when both taps and the pipe are turned on,

$\frac{1}{6} + \frac{1}{9} - \frac{1}{15} = \frac{19}{90}$ of the tank is filled up.

Time taken to fill up the tank $= \frac{90}{19}$

$= 4\frac{14}{19}$ **minutes** or **4.74 minutes** (to 3 s.f.)

14. 12 glassblowers can make 12 vases in 9 minutes.

1 glassblower can make 12 vases in $9 \times 12 = 108$ minutes.

8 glassblowers can make 12 vases in $\frac{108}{8} = 13.5$ minutes.

8 glassblowers can make 32 vases in $13.5 \times \frac{32}{12} = 36$ minutes.

$\therefore$ 8 glassblowers will take **36 minutes** to make 32 such vases.

15. Total number of hours worked on the road after 20 working days

$= 20 \times 50 \times 8$

$= 8000$ hours

The length of the road laid is directly proportional to the number of hours.

1200 m of road is laid in 8000 hours.

1 m of road is laid in $\frac{8000}{1200}$ hours.

$3000 - 1200 = 1800$ m of road is laid in $\frac{8000}{1200} \times 1800 = 12\,000$ hours.

Let the number of additional men to employ be x.

$(30 - 20) \times (50 + x) \times 10 = 12\,000$

$100(50 + x) = 12\,000$

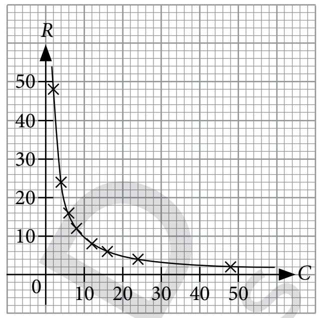
$50 + x = 120$

$x = 70$

$\therefore$ **70** more men need to be employed.

16.

Number of columns, C	48	24	16	12	8	6	4	2
Number of rows, R	2	4	6	8	12	16	24	48



The number of rows, R, is inversely proportional to the number of columns, C.

Since R is inversely proportional to C, then $R = \frac{k}{C}$, where k is a constant.

When $C = 48$, $R = 2$,

$2 = \frac{k}{48}$

$k = 96$

$\therefore \boldsymbol{R = \frac{96}{C}}$

7.6 Other forms of inverse proportion

Practise Now 12

(a) Since $y = \frac{4}{x^2}$, i.e. $x^2y = 4$ is a constant, then $\boldsymbol{y}$ and $\boldsymbol{x^2}$ are inversely proportional to each other.

(b) Since $y^2 = \frac{1}{\sqrt[3]{x}}$, i.e. $\sqrt[3]{x}\,y^2 = 1$ is a constant, then $\boldsymbol{y^2}$ and $\boldsymbol{\sqrt[3]{x}}$ are inversely proportional to each other.

(c) Since $y = \frac{5}{x+2}$, i.e. $(x + 2)y = 5$ is a constant, then $\boldsymbol{y}$ and $\boldsymbol{x + 2}$ are inversely proportional to each other.

Practise Now 13

1. **(i)** Since y is inversely proportional to x^2,

then yx^2 is a constant.

$\therefore y(8)^2 = 2(4)^2$

$64y = 32$

$y = \boldsymbol{\frac{1}{2}}$

(ii) Since y is inversely proportional to x^2,

then $y = \frac{k}{x^2}$, where k is a constant.

When $x = 4$, $y = 2$,

$2 = \frac{k}{4^2}$

$\therefore k = 32$

$\therefore \boldsymbol{y = \frac{32}{x^2}}$

(iii) When $y = 8$,

$8 = \frac{32}{x^2}$

$x^2 = \frac{32}{8}$

$= 4$

$x = \pm\sqrt{4}$

$= \mathbf{\pm 2}$

2. Since y is inversely proportional to $\sqrt{x}$,

then $y = \frac{k}{\sqrt{x}}$, where k is a constant.

When $x = 9$, $y = 6$,

$6 = \frac{k}{\sqrt{9}}$

$\therefore k = 18$

$\therefore y = \frac{18}{\sqrt{x}}$

When $x = 25$,

$y = \frac{18}{\sqrt{25}}$

$= \mathbf{3.6}$

3. Since b is inversely proportional to $\sqrt{a}$,

then $b = \frac{k}{\sqrt{a}}$, where k is a constant.

When $a = 1$, $b = 8$,

$8 = \frac{k}{\sqrt{1}}$

$\therefore k = 8$

$\therefore b = \frac{8}{\sqrt{a}}$

When $b = 16$,

$16 = \frac{8}{\sqrt{a}}$

$\sqrt{a} = \frac{8}{16}$

$= \frac{1}{2}$

$a = \left(\frac{1}{2}\right)^2$

$= \frac{1}{4}$

When $b = \frac{4}{3}$,

$\frac{4}{3} = \frac{8}{\sqrt{a}}$

$\sqrt{a} = \frac{8}{\frac{4}{3}}$

$= 6$

$a = 6^2$

$= 36$

When $a = 4$,

$b = \frac{8}{\sqrt{4}}$

$= 4$

When $a = 16$,

$b = \frac{8}{\sqrt{16}}$

$= 2$

a	$\mathbf{\frac{1}{4}}$	1	4	16	**36**
b	16	8	**4**	**2**	$\mathbf{\frac{4}{3}}$

Practise Now 14

(i) Since F is inversely proportional to d^2, then Fd^2 is a constant.

$\therefore F(5^2) = 10(2^2)$

$25F = 40$

$F = \frac{40}{25}$

$= 1.6$

$\therefore$ the force between the particles when they are 5 m apart is **1.6 N**.

(ii) Since F is inversely proportional to d^2, then Fd^2 is a constant.

$\therefore 25(d^2) = 10(2^2)$

$25d^2 = 40$

$d^2 = \frac{40}{25}$

$= 1.6$

$d = \sqrt{1.6}$ (since $d > 0$)

$= 1.26$ (to 3 s.f.)

$\therefore$ the distance between the particles when the force between them is 25 N is **1.26 m**.

Exercise 7D

1. (i) Since x is inversely proportional to y^3,

$y_2^3 x_2 = y_1^3 x_1$

$4^3 \times x = 2^3 \times 50$

$x = \frac{2^3 \times 50}{4^3}$

$= \mathbf{6.25}$

(ii) Since x is inversely proportional to y^3,

then $x = \frac{k}{y^3}$, where k is a constant.

When $y = 2$, $x = 50$,

$50 = \frac{k}{2^3}$

$\therefore k = 400$

$\therefore \boldsymbol{x = \frac{400}{y^3}}$

(iii) When $x = 3.2$,

$3.2 = \frac{400}{y^3}$

$y^3 = \frac{400}{3.2}$

$= 125$

$y = \sqrt[3]{125}$

$= \mathbf{5}$

2. **(i)** Since z is inversely proportional to $\sqrt{w}$,

then $z = \frac{k}{\sqrt{w}}$, where k is a constant.

When $w = 9$, $z = 9$,

$9 = \frac{k}{\sqrt{9}}$

$\therefore k = 27$

$\therefore \boldsymbol{z = \frac{27}{\sqrt{w}}}$

(ii) When $w = 16$,

$z = \frac{27}{\sqrt{16}}$

$= \mathbf{6.75}$

(iii) When $z = 3$,

$3 = \frac{27}{\sqrt{w}}$

$\sqrt{w} = \frac{27}{3}$

$= 9$

$w = 9^2$

$= \mathbf{81}$

3. **(a)** Since $y = \frac{3}{x^2}$, i.e. $yx^2 = 3$ is a constant, then $\boldsymbol{y}$ and $\boldsymbol{x^2}$ are inversely proportional to each other.

(b) Since $y = \frac{1}{\sqrt{x}}$, i.e. $y\sqrt{x} = 1$ is a constant, then $\boldsymbol{y}$ and $\boldsymbol{\sqrt{x}}$ are inversely proportional to each other.

(c) Since $y^2 = \frac{5}{x^3}$, i.e. $y^2x^3 = 5$ is a constant, then $\boldsymbol{y^2}$ and $\boldsymbol{x^3}$ are inversely proportional to each other.

(d) Since $n = \frac{7}{m-1}$, i.e. $n(m - 1) = 7$ is a constant, then $\boldsymbol{n}$ and $\boldsymbol{m - 1}$ are inversely proportional to each other.

(e) Since $q = \frac{4}{(p+1)^2}$, i.e. $q(p + 1)^2 = 4$ is a constant, then $\boldsymbol{q}$ and $\boldsymbol{(p + 1)^2}$ are inversely proportional to each other.

4. Since z is inversely proportional to $\sqrt[3]{x}$,

$\sqrt[3]{x_2}z_2 = \sqrt[3]{x_1}z_1$

$\sqrt[3]{216} \times z = \sqrt[3]{64} \times 5$

$z = \frac{\sqrt[3]{64} \times 5}{\sqrt[3]{216}}$

$= \mathbf{\frac{10}{3}}$

5. Since q^2 is inversely proportional to $p + 3$,

$(p_2 + 3)q_2^2 = (p_1 + 3)q_1^2$

$(17 + 3) \times q^2 = (2 + 3) \times 5^2$

$20q^2 = 125$

$q^2 = \frac{125}{20}$

$= 6.25$

$q = \pm\sqrt{6.25}$

$= \mathbf{\pm 2.5}$

6. Since t is inversely proportional to s^3,

then $t = \frac{k}{s^3}$, where k is a constant.

When $s = 1$, $t = 80$,

$80 = \frac{k}{1^3}$

$\therefore k = 80$

$\therefore t = \frac{80}{s^3}$

When $t = 0.08$,

$0.08 = \frac{80}{s^3}$

$s^3 = \frac{80}{0.08}$

$= 1000$

$s = \sqrt[3]{1000}$

$= 10$

When $t = 0.01$,

$0.01 = \frac{80}{s^3}$

$s^3 = \frac{80}{0.01}$

$= 8000$

$s = \sqrt[3]{8000}$

$= 20$

When $s = 2$,

$t = \frac{80}{2^3}$

$= 10$

When $s = 4$,

$t = \frac{80}{4^3}$

$= 1.25$

s	1	2	4	**10**	**20**
t	80	**10**	**1.25**	0.08	0.01

7. **(i)** Since F is inversely proportional to d^2, then $\boldsymbol{F = \frac{k}{d^2}}$**, where** $\boldsymbol{k}$ **is a constant.**

(ii) Since $F = \frac{k}{d^2}$, where k is a constant, Fd^2 is a constant.

$\therefore 20(d^2) = F\left(\frac{d}{2}\right)^2$

$20d^2 = \frac{Fd^2}{4}$

$\frac{F}{4} = 20$

$F = 80$

$\therefore$ the force when the distance is halved is **80 N**.

8. **(i)** Since h is inversely proportional to r^2, then hr^2 is a constant.

$\therefore h(3^2) = 5(6^2)$

$9h = 180$

$h = \frac{180}{9}$

$= 20$

$\therefore$ the height of Cone B is **20 cm**.

(ii) Since h is inversely proportional to r^2, then hr^2 is a constant.

$\therefore\ 1.25(r^2) = 5(6)^2$

$1.25r^2 = 180$

$r^2 = \dfrac{180}{1.25}$

$= 144$

$r = \sqrt{144}$ (since $r > 0$)

$= 12$

$\therefore$ the base radius of Cone C is **12 cm**.

9. Since y is inversely proportional to $2x + 1$,

then $y = \dfrac{k}{2x+1}$, where k is a constant.

When $x = 0.5$,

$y = \dfrac{k}{2(0.5)+1}$

$= \dfrac{k}{2}$

When $x = 2$,

$y = \dfrac{k}{2(2)+1}$

$= \dfrac{k}{5}$

Since the difference in the values of y is 0.9,

$\dfrac{k}{2} - \dfrac{k}{5} = 0.9$

$0.3k = 0.9$

$\therefore\ k = 3$

$\therefore\ y = \dfrac{3}{2x+1}$

When $x = -0.25$,

$y = \dfrac{3}{2(-0.25)+1}$

$= \mathbf{6}$

10. Since y is inversely proportional to x^2,

$x_2^2 y_2 = x_1^2 y_1$

$(3x)^2 y = x^2 b$

$9x^2 y = bx^2$

$y = \dfrac{bx^2}{9x^2}$

$= \boldsymbol{\dfrac{b}{9}}$

11. When the distance r cm is increased by 400%,

the new distance $= \dfrac{100+400}{100} \times r = 5r$ cm.

Since F is inversely proportional to r^2,

$\therefore\ F_1 r_1^{\,2} = F_2 r_2^{\,2}$

$F(r^2) = cF(5r)^2$

$Fr^2 = 25cFr^2$

$25c = 1$

$\therefore\ c = \mathbf{0.04}$

12. **No**, I do not agree.

Let the radius of the cylinder be r, the diameter of the cylinder be d and the height of the water level be h.

Since water takes the shape of the cylindrical container,

Volume of water $= \pi r^2 h$

$= \pi\left(\dfrac{d}{2}\right)^2 h$

$= \dfrac{1}{4}\pi d^2 h$

Given the same volume of water in the cylindrical containers, d^2h is a constant.

$\therefore$ the square of the diameter of the container and the height of the water level are inversely proportional to each other.

Chapter 8 Congruence and Similarity

TEACHING NOTES

Suggested Approach

In this chapter, students will be introduced to the concepts of congruence and similarity which are properties of geometrical figures. The definitions of both terms must be clearly stated, with their similarities and differences explored and discussed to minimise any confusion. A recap on angle properties and geometrical construction may be required in this chapter.

Section 8.1: Congruent figures

Teachers may wish to show the properties of congruent figures (see Investigation: Properties of congruent figures) before stating the definition. Students should recognise that congruence is a property of geometrical figures; two geometrical figures of the same size and shape are congruent (see Thinking Time on page 241).

In stating the congruence relation, it is crucial the order of vertices reflect the equal corresponding angles and sides in both congruent figures. A wrong order will indicate an incorrect relation.

The worked examples aim to allow students to understand and apply the properties of congruence, as well as test whether two figures are congruent. Teachers should provide guidance to students who require explanations and assistance.

Section 8.2 Similar figures

Students, after knowing the definition of similarity, should be able to realise that congruence is a special case of similarity. The Investigation on page 250 allows students to derive the properties that corresponding angles are equal and the ratios of corresponding sides are equal for two similar geometrical figures.

Students should explore the concept of similarity for different figures (see Thinking Time on page 251) as the results shows that both conditions are needed for polygons with four sides or more.

Teachers should also go through the activity on page 252 (see Class Discussion: Identifying similar triangles). Students should discover that right-angled triangles and isosceles triangles need not be similar but all equilateral triangles are definitely similar.

Section 8.3 Similarity and enlargement

From the previous section, when two figures are similar, one will be 'larger' than the other. The concept of a scale factor should then be a natural result. Teachers and students should note (see Information on page 260) that enlargement does not always mean the resultant figure is larger than the original figure. The resultant figure can be smaller than the original figure, and the scale factor will be less than 1, but it is still known as an enlargement. If the scale factor is 1, then the resultant figure is congruent to the original figure.

Students are required to recall their lessons on geometrical construction while learning about and making scale drawings. Observant students may note that scale drawing is actually an application of ratios, and the concepts of linear scales and area scales of maps/models further illustrate this.

Introductory Problem

The solutions to this problem can be found in Introductory Problem Revisited (after Practise Now 8).

8.1 Congruent figures

Investigation (Properties of congruent figures)

1. The shape and size of the pairs of scissors are the same, whereas the orientation and position of the pairs of scissors are different.
2. The pairs of scissors will stack on top of one another exactly.
3. The scissors in **(a)** can be moved to the position in **(c)** by a reflection $A_1 \rightarrow A_3$ about a vertical line.
   The scissors in **(a)** can be moved to the position in **(d)** by a rotation of $A_1 \rightarrow A_4$ about 135° in an anticlockwise direction.
   The scissors in **(a)** has the same orientation and position as **(e)**.

Thinking Time (Page 241)

Yes, the two pairs of scissors are congruent as they have the same shape and size.

Practise Now 1

A is congruent to ***H***.
B is congruent to ***E***.
C is congruent to ***F***.
D is congruent to ***G*** and ***I***.

Practise Now 2

(i) $PQ = AB = \mathbf{5}$ cm
(ii) $SR = \boldsymbol{DC} = 6$ cm
(iii) $PS = \boldsymbol{AD} = \mathbf{2}$ cm
(iv) $QR = \boldsymbol{BC} = \mathbf{5.3}$ cm
(v) $\angle PQR = \angle \boldsymbol{ABC} = \mathbf{90°}$

Practise Now 3

(a) In $\triangle ABC$,
$AB = 4$ cm, $BC = 5.4$ cm and $AC = 6.1$ cm.
In $\triangle PQR$,
$PQ = 8$ cm, $QR = 12.2$ cm and $PR = 10.8$ cm.
$\triangle ABC$ and $\triangle PQR$ do not have equal corresponding sides.
$\therefore$ $\triangle ABC$ is **not congruent** to $\triangle PQR$.

(b) $\angle TSU = 180° - 40° - 80°$ ($\angle$ sum of $\triangle TSU$)
$= 60°$
$\angle DFE = 180° - 60° - 80°$ ($\angle$ sum of $\triangle DEF$)
$= 40°$
$\angle EDF = \angle STU = °80$
$\angle DEF = \angle TSU = °60$
$\angle DFE = \angle TUS = °40$
$DE = TS = 4$ cm
$EF = SU = 6.13$ cm
$DF = TU = 5.39$ cm
$\therefore$ $\triangle \boldsymbol{DEF} \equiv \triangle \boldsymbol{TSU}$.

(c) $\angle MNL = 180° - 70° - 60°$ ($\angle$ sum of $\triangle MLN$)
$= 50°$
$\angle YXZ = 180° - 70° - 40°$ ($\angle$ sum of $\triangle XYZ$)
$= 70°$
$\triangle MLN$ and $\triangle XYZ$ do not have three pairs of corresponding equal angles.
$\therefore$ $\triangle MLN$ is **not congruent** to $\triangle XYZ$.

Practise Now 4

(a) **(i)** $\angle CDE = \angle ABC$
$= \mathbf{38°}$
(ii) $\angle CED = 180° - 114° - 38°$ ($\angle$ sum of $\triangle CDE$)
$= \mathbf{28°}$
(iii) $\angle ACB = \angle CED$
$= \mathbf{28°}$
(iv) Length of BC = length of DE
= **27 cm**
(v) Length of CE = length of AC
= 18 cm
$\therefore$ length of BE = length of BC – length of CE
$= 27 - 18$
= **9 cm**

(b) $\angle ACB + \angle DCE + \angle CDE$
$= 28° + 114° + 38°$
$= 180°$
Since the sum of $\angle CDE$ and $\angle ACD$ is 180°, then ***AC // ED*** (converse of int. $\angle$s).

Exercise 8A

1. ***A*** is congruent to ***F***.
   B is congruent to ***J***.
   C is congruent to ***E***.
   D is congruent to ***G***.
   I is congruent to ***K***.
2. **(i)** $PQ = VW = \mathbf{3.5}$ cm
   (ii) $PT = \boldsymbol{VZ} = 2$ cm
   (iii) $QR = \boldsymbol{WX} = \mathbf{3.5}$ cm
   (iv) $TS = \boldsymbol{ZY} = \mathbf{2.1}$ cm
   (v) $SR = \boldsymbol{YX} = \mathbf{2}$ cm
   (vi) $\angle PQR = \angle \boldsymbol{VWX} = \mathbf{90°}$
3. $\boldsymbol{EF} = LM = \mathbf{3.4}$ **cm**
   $\boldsymbol{GH} = NO = \mathbf{2.4}$ **cm**
   $\angle \boldsymbol{FEH} = \angle MLO = \mathbf{100°}$
   $\angle \boldsymbol{FGH} = \angle MNO = \mathbf{75°}$
   $\boldsymbol{MN} = FG = \mathbf{5}$ **cm**
   $\boldsymbol{OL} = HE = \mathbf{3}$ **cm**
   $\angle \boldsymbol{LMN} = \angle EFG = \mathbf{65°}$
   $\angle \boldsymbol{NOL} = \angle GHE = \mathbf{120°}$
4. **(a)** $\angle ACB = 180° - 90° - 36.9°$ ($\angle$ sum of $\triangle ABC$)
   $= 53.1°$
   $\angle PRQ = 180° - 90° - 36.9°$ ($\angle$ sum of $\triangle PQR$)
   $= 53.1°$
   $\angle BAC = \angle QPR = 36.9°$
   $\angle ABC = \angle PQR = °90$
   $\angle ACB = \angle PRQ = 53.1°$

$AB = PQ = 4$ cm
$BC = QR = 3$ cm
$AC = PR = 5$ cm
$\therefore \triangle \boldsymbol{ABC} \equiv \triangle \boldsymbol{PQR}$.

(b) $\angle EDF = 180° - 80° - 70°$ ($\angle$ sum of $\triangle DEF$)
$= 30°$
$\angle TUS = 180° - 80° - 30°$ ($\angle$ sum of $\triangle STU$)
$= 70°$
$\angle EDF = \angle STU = °30$
$\angle DEF = \angle TSU = °80$
$\angle DFE = \angle TUS = °70$
$DE = TS = 18.8$ cm
$EF = SU = 10$ cm
$DF = TU = 19.7$ cm
$\therefore \triangle \boldsymbol{DEF} \equiv \triangle \boldsymbol{TSU}$.

(c) Based on the corresponding angles, MN will be equal to YZ if the triangles are congruent. However, $MN = 4$ cm $\neq 5.13$ cm $= YZ$.
$\therefore \triangle LMN$ is **not congruent** to $\triangle XYZ$.

5. **(i)** $\angle ABK = \angle ACK$
$= 62°$
$\angle BAK = 180° - 90° - 62°$ ($\angle$ sum of $\triangle ABK$)
$= 28°$
$\angle CAK = \angle BAK$
$= 28°$
$\therefore \angle BAC = \angle BAK + \angle CAK$
$= 28° + 28°$
$= \mathbf{56°}$

(ii) Length of CK = length of BK
$= 8$ cm
$\therefore$ length of BC = length of BK + length of CK
$= 8 + 8$
$=$ **16 cm**

6. **(i)** $\angle BAC = \angle EDC$
$= 34°$
$\therefore \angle ABC = 180° - 71° - 34°$ ($\angle$ sum of $\triangle ABC$)
$= \mathbf{75°}$

(ii) Length of CD = length of CA
$= 6.9$ cm
$\therefore$ length of BD = length of BC + length of CD
$= 4 + 6.9$
$=$ **10.9 cm**

7. **(i)** $\angle AHC = \angle AKB$
$= 180° - 90°$ (adj. $\angle$s on a str. line)
$= 90°$
Length of AH = length of AK
$\therefore \triangle AHK$ is an isosceles triangle.
Let $\angle AHK$ be $x°$.
$\angle AKH = \angle AHK$ (base $\angle$s of isos. $\triangle AHK$)
$= x°$
$\angle CHK = 90° - x°$
$\angle CKH = 90° - x°$
$\therefore \triangle CHK$ is an isosceles triangle.
Let the length of CH be n cm.
Length of CK = length of $CH = n$ cm
Length of BK = length of $CH = n$ cm
$2n = 12$
$n = 6$
$\therefore$ the length of CH is **6 cm**.

(ii) $\angle BAC = 180° - 58° - 58°$ ($\angle$ sum of isos. $\triangle ABC$)
$= 64°$
$\angle ACH = \angle ABK$
$= 58°$
$\angle CAH = 180° - 90° - 58°$ ($\angle$ sum of $\triangle ACH$)
$= 32°$
$\therefore \angle BAH = \angle BAC + \angle CAH$
$= 64° + 32°$
$= \mathbf{96°}$

8.2 Similar figures

Investigation (Similar polygons)

1. **(a)** $\angle A = \mathbf{44°}$, $\angle A' = \mathbf{44°}$
(b) $\angle B = \mathbf{102°}$, $\angle B' = \mathbf{102°}$
(c) $\angle C = \mathbf{34°}$, $\angle C' = \mathbf{34°}$
The size of each pair of corresponding angles is the same.

2. **(a)** $\dfrac{A'B'}{AB} = \mathbf{2}$
(b) $\dfrac{B'C'}{BC} = \mathbf{2}$
(c) $\dfrac{A'C'}{AC} = \mathbf{2}$
The ratios of the lengths of the corresponding sides are all equal to 2.

Thinking Time (Page 251)

1. **(i)** $AB = CD = 3.3$ cm
$A'B' = C'D' = 5$ cm
$BC = DA = B'C' = D'A' = 1.65$ cm
$\dfrac{A'B'}{AB} = \dfrac{C'D'}{CD} = \dfrac{5}{3.3} = \dfrac{50}{33}$
$\dfrac{B'C'}{BC} = \dfrac{D'A'}{DA} = \dfrac{1.65}{1.65} = 1$
No, the ratios of the corresponding sides are not equal.
(ii) **Yes**, all the corresponding angles are equal to 90°.
(iii) **No**, the two rectangles are not similar.

2. **(i)** Length of square = 1.55 cm
Length of rhombus = 3.1 cm
$\dfrac{P'Q'}{PQ} = \dfrac{Q'R'}{QR} = \dfrac{R'S'}{RS} = \dfrac{S'P'}{SP} = \dfrac{3.1}{1.55} = 2$
Yes, all the ratios of the corresponding sides are equal.
(ii) **No**, the corresponding angles are not equal. The angles in the square are all equal to 90° but none of the angles in the rhombus are 90°.
(iii) **No**, the two quadrilaterals are not similar.

Class Discussion (Identifying similar triangles)

2. $\triangle \boldsymbol{A}$ **is similar to** $\triangle \boldsymbol{B}$ as $\triangle A$ can fit inside $\triangle B$ with an equivalent width around it.
$\triangle \boldsymbol{D}$ **is similar to** $\triangle \boldsymbol{E}$ and $\triangle \boldsymbol{F}$ as $\triangle F$ can fit inside $\triangle D$, which can fit inside $\triangle E$ with an equivalent width around it.
$\triangle \boldsymbol{G}$ **is similar to** $\triangle \boldsymbol{H}$ as $\triangle G$ can fit inside $\triangle H$ with an equivalent width around it.
$\triangle \boldsymbol{I}$ **is similar to** $\triangle \boldsymbol{J}$ as $\triangle I$ overlaps with $\triangle J$. $\triangle I \equiv \triangle J$.
$\triangle \boldsymbol{K}$ **is similar to** $\triangle \boldsymbol{L}$ **and** $\triangle \boldsymbol{M}$ as $\triangle L$ can fit inside $\triangle K$, which can fit inside $\triangle M$ with an equivalent width around it.

3. **(a)** **No.** The corresponding angles may not be equal.
(b) **No.** The corresponding angles may not be equal, e.g. $\triangle G$ and $\triangle I$.
(c) **Yes.** All angles are equal to 60° so the corresponding angles are the same. Since the sides of an equilateral triangle are the same, the ratios of the corresponding sides of two equilateral triangles are the same.

Practise Now 5

(a) $\angle C = 180° - 90° - 58°$ ($\angle$ sum of $\triangle ABC$)
$= 32°$
$\angle P = 180° - 90° - 35°$ ($\angle$ sum of $\triangle PQR$)
$= 55°$
$\angle B = \angle Q =$ °90
$\angle C = 35° \neq$ °32 $= \angle R$
$\angle A = 58° \neq 55° = \angle P$
Since not all the corresponding angles are equal, then $\triangle ABC$ is **not similar** to $\triangle PQR$.

(b) $\angle E = 180° - 100° - 44°$ ($\angle$ sum of $\triangle DEF$)
$= 36°$
$\angle U = 180° - 44° - 36°$ ($\angle$ sum of $\triangle STU$)
$= 100°$
$\angle E = \angle T =$ °36
$\angle F = \angle U = 100°$
$\angle D = \angle S = 44°$
$$\frac{DE}{ST} = \frac{12}{10} = 1.2$$
$$\frac{EF}{TU} = \frac{8.4}{7} = 1.2$$
$$\frac{DF}{SU} = \frac{7.2}{6} = 1.2$$
Since all the corresponding angles are equal and all the ratios of the lengths of the corresponding sides are equal, then $\triangle DEF$ is **similar** to $\triangle STU$.

Practise Now 6

1. Since $\triangle ABC$ is similar to $\triangle PRQ$, then all the corresponding angles are equal.
$\therefore x° = \angle QPR$
$= \angle CAB$
$= 30°$
Since $\triangle ABC$ is similar to $\triangle PRQ$, then all the ratios of the corresponding sides are equal.
$$\therefore \frac{BC}{RQ} = \frac{AC}{PQ}$$
$$\frac{y}{2.8} = \frac{6}{4}$$
$$y = \frac{6}{4} \times 2.8$$
$$= 4.2$$
$\therefore$ $\boldsymbol{x = 30, y = 4.2}$.

2. Since $ABCD$ is similar to $PQRS$, then all the corresponding angles are equal.
$\therefore w° = \angle BCD$
$= \angle QRS$
$= 60°$
$\therefore x° = \angle QPS$
$= \angle BAD$
$= 100°$
Since $ABCD$ is similar to $PQRS$, then all the ratios of the corresponding sides are equal.
$$\therefore \frac{BC}{QR} = \frac{AB}{PQ}$$
$$\frac{y}{5.4} = \frac{4}{3}$$
$$y = \frac{4}{3} \times 5.4$$
$$= 7.2$$
$$\therefore \frac{PS}{AD} = \frac{PQ}{AB}$$
$$\frac{z}{6} = \frac{3}{4}$$
$$z = \frac{3}{4} \times 6$$
$$= 4.5$$
$\therefore$ $\boldsymbol{w = 60, x = 100, y = 7.2, z = 4.5}$

Practise Now 7

1. Since $\triangle XYZ$ is similar to $\triangle XRS$, then all the corresponding angles are equal.
$\therefore a° = \angle XSR$
$= \angle XZY$
$= 30°$
Since $\triangle XYZ$ is similar to $\triangle XRS$, then all the ratios of the corresponding sides are equal.
$$\therefore \frac{XS}{XZ} = \frac{XR}{XY}$$
$$\frac{5+b}{5} = \frac{4+6}{4}$$
$$5 + b = \frac{10}{4} \times 5$$
$$= 12.5$$
$$b = 12.5 - 5$$
$$= 7.5$$
$\therefore$ $\boldsymbol{a = 30, b = 7.5}$

2. Since $\triangle ABC$ is similar to $\triangle DEC$, then all the corresponding angles are equal.
$\therefore \angle CED = \angle CBA$
$=$ °48
$x° = \angle CDE$
$= 180° - 60° - 48°$ ($\angle$ sum of $\triangle DEC$)
$= 72°$
Since $\triangle ABC$ is similar to $\triangle DEC$, then all the ratios of the corresponding sides are equal.
$$\therefore \frac{DE}{AB} = \frac{CE}{CB}$$
$$\frac{y}{7.3} = \frac{10}{8}$$
$$y = \frac{10}{8} \times 7.3$$
$$= 9.125$$
$\therefore$ $\boldsymbol{x = 72, y = 9.125}$

Practise Now 8

Let the height of the lamp post, AB, be x cm.

Since $\triangle ABD$ and $\triangle CED$ are similar, then all the ratios of the corresponding sides are equal.

$$\frac{AB}{CE} = \frac{AD}{CD}$$

$$\frac{x}{180} = \frac{256+144}{144}$$

$$x = \frac{180(400)}{144}$$

$$= 500$$

$\therefore$ the height of the lamp post is 500 cm = **5 m**.

Introductory Problem Revisited

$\triangle ABC$ and $\triangle AED$ are similar. Hence, the ratios of the corresponding sides are equal.

We can measure the lengths of AB, AC and AE.

Then we can determine the length of the bridge AD using $\dfrac{AD}{AC} = \dfrac{AE}{AB}$.

Exercise 8B

1. **(a)** Since $\triangle ABC$ is similar to $\triangle PQR$, then all the corresponding angles are equal.

$x° = \angle PQR$
$= \angle ABC$
$= 90°$

$y° = \angle ACB$
$= \angle PRQ$
$= 35°$

$z° = \angle QPR$
$= 180° - 90° - 35°$ ($\angle$ sum of $\triangle PQR$)
$= 55°$

$\therefore$ $\boldsymbol{x = 90, y = 35, z = 55}$

(b) Since $\triangle ABC$ is similar to $\triangle PQR$, then all the corresponding angles are equal.

$x° = \angle PRQ$
$= \angle ACB$
$= 28°$

$y° = \angle BAC$
$= \angle QPR$
$= °28 - °118 - °180$ ($\angle$ sum of $\triangle PQR$)
$= 34°$

$\therefore$ $\boldsymbol{x = 28, y = 34}$

(c) Since $\triangle ABC$ is similar to $\triangle PQR$, then all the ratios of the corresponding sides are equal.

$$\frac{QR}{BC} = \frac{PQ}{AB}$$

$$\frac{x}{12} = \frac{6}{10}$$

$$x = \frac{6}{10}\times 12$$

$$= 7.2$$

$$\frac{PR}{AC} = \frac{PQ}{AB}$$

$$\frac{y}{18} = \frac{6}{10}$$

$$y = \frac{6}{10}\times 18$$

$$= 10.8$$

$\therefore$ $\boldsymbol{x = 7.2, y = 10.8}$

(d) Since $\triangle ABC$ is similar to $\triangle PQR$, then all the ratios of the corresponding sides are equal.

$$\frac{AC}{PR} = \frac{AB}{PQ}$$

$$\frac{x}{8} = \frac{12}{10}$$

$$x = \frac{12}{10}\times 8$$

$$= 9.6$$

$$\frac{QR}{BC} = \frac{PQ}{AB}$$

$$\frac{y}{7} = \frac{10}{12}$$

$$y = \frac{10}{12}\times 7$$

$$= 5\frac{5}{6} \text{ or } 5.83 \text{ (to 3 s.f.)}$$

$\therefore$ $\boldsymbol{x = 9.6, y = 5\frac{5}{6}}$ **or 5.83**

2. **(a)** $\angle B = \angle C$ (base $\angle$s of isos. $\triangle ABC$)
$= \dfrac{180° - 40°}{2}$ ($\angle$ sum of isos. $\triangle ABC$)
$= 70°$

$\angle R = \angle Q$ (base $\angle$s of isos. $\triangle PQR$)
$= 50°$

$\angle P = 180° - 50° - 50°$ ($\angle$ sum of isos. $\triangle PQR$)
$= 80°$

$\angle A = °80 \neq °40 = \angle P$
$\angle B = °50 \neq °70 = \angle Q$
$\angle C = °50 \neq °70 = \angle R$

Since all the corresponding angles are not equal, then $\triangle ABC$ is **not similar** to $\triangle PQR$.

(b) $\angle E = 180° - 84° - 32°$ ($\angle$ sum of $\triangle DEF$)
$= 64°$

$\angle S = 180° - 64° - 32°$ ($\angle$ sum of $\triangle STU$)
$= 84°$

$\angle D = \angle S = 84°$
$\angle E = \angle T = 64°$
$\angle F = \angle U = 32°$

$$\frac{DE}{ST} = \frac{3.2}{1.6} = 2$$

$$\frac{EF}{TU} = \frac{6}{3} = 2$$

$$\frac{DF}{SU} = \frac{5.4}{2.7} = 2$$

Since all the corresponding angles are equal and all the ratios of the lengths of the corresponding sides are equal, then $\triangle DEF$ is **similar** to $\triangle STU$.

3. **(a)** Since $ABCD$ is similar to $PQRS$, then all the corresponding angles are equal.

$x° = \angle QPS$
$= \angle BAD$
$= 95°$

$y° = \angle QRS$
$= \angle BCD$
$= 360° - °108 - °105 - °95$ ($\angle$ sum of quad. $ABCD$)
$= 52°$

Since $ABCD$ is similar to $PQRS$, then all the ratios of the corresponding sides are equal.

$$\frac{PQ}{AB} = \frac{QR}{BC}$$
$$\frac{z}{8} = \frac{7.2}{12}$$
$$z = \frac{7.2}{12} \times 8$$
$$= 4.8$$

$\therefore$ $\boldsymbol{x = 95, y = 52, z = 4.8}$

(b) Since $ABCD$ is similar to $PQRS$, then all the corresponding angles are equal.

$x° = \angle ADC$
$= \angle PSR$
$= 180° - 100°$ (int. $\angle$s, PQ // SR)
$= 80°$

Since $ABCD$ is similar to $PQRS$, then all the ratios of the corresponding sides are equal.

$$\frac{PS}{AD} = \frac{RS}{CD}$$
$$\frac{y}{14} = \frac{9}{12}$$
$$y = \frac{9}{12} \times 14$$
$$= 10.5$$

$\therefore$ $\boldsymbol{x = 80, y = 10.5}$

4. Since the two water bottles are similar, then all the ratios of the corresponding sides are equal.

$$\frac{x}{10} = \frac{8}{5}$$
$$x = \frac{8}{5} \times 10$$
$$= 16$$
$$\frac{y}{3} = \frac{5}{8}$$
$$y = \frac{5}{8} \times 3$$
$$= 1.875$$

$\therefore$ $\boldsymbol{x = 16, y = 1.875}$

5. Since the two toy houses are similar, then all the corresponding angles are equal and all the ratios of the corresponding sides are equal.

$x° = 100°$

$$\frac{y}{180} = \frac{180}{120}$$
$$y = \frac{180}{120} \times 180$$
$$= 270$$
$$\frac{z}{150} = \frac{120}{180}$$
$$z = \frac{120}{180} \times 150$$
$$= 100$$

$\therefore$ $\boldsymbol{x = 100, y = 270, z = 100}$

6. Since $\triangle ABC$ is similar to $\triangle ADE$, then all the corresponding angles are equal.

$x° = \angle ADE$
$= \angle ABC$
$= 56°$

Since $\triangle ABC$ is similar to $\triangle ADE$, then all the ratios of the corresponding sides are equal.

$$\frac{AD}{AB} = \frac{AE}{AC}$$
$$\frac{4+y}{4} = \frac{6+9}{6}$$
$$4 + y = \frac{4(15)}{6}$$
$$= 10$$
$$y = 6$$

$\therefore$ $\boldsymbol{x = 56, y = 6}$

7. Since $\triangle PQR$ is similar to $\triangle BAR$, then all the corresponding angles are equal.

$\angle ABR = \angle QPR$
$= 61°$

$x° = \angle BAR$
$= 180° - 52° - 61°$ ($\angle$ sum of $\triangle BAR$)
$= 67°$

Since $\triangle PQR$ is similar to $\triangle BAR$, then all the ratios of the corresponding sides are equal.

$$\frac{BR}{PR} = \frac{AB}{QP}$$
$$\frac{y}{14} = \frac{9}{12}$$
$$y = \frac{9}{12} \times 14$$
$$= 10.5$$

$\therefore$ $\boldsymbol{x = 67, y = 10.5}$

8. Since $\triangle AQP$ is similar to $\triangle ABC$, then all the ratios of the corresponding sides are equal.

$$\frac{AQ}{AB} = \frac{AP}{AC}$$
$$\frac{AQ}{150} = \frac{75}{125}$$
$$AQ = \frac{75}{125} \times 150$$
$$= \mathbf{90\ cm}$$

9. Since $\triangle ABE$ and $\triangle DCE$ are similar, then all the ratios of the corresponding sides are equal.

$$\frac{AB}{DC} = \frac{BE}{CE}$$
$$\frac{AB}{3} = \frac{10+6}{6}$$
$$AB = \frac{3(16)}{6}$$
$$= 8 \text{ m}$$

$\therefore$ the height of the lamp AB is **8 m**.

10. $GC = AE = 4$ cm

$$\frac{BG}{BC} = \frac{3}{5}$$
$$\frac{BG}{BG+GC} = \frac{3}{5}$$
$$\frac{BG}{BG+4} = \frac{3}{5}$$
$$5BG = 3BG + 12$$
$$2BG = 12$$
$$BG = 6$$

Since $\triangle AEF$ and $\triangle BFG$ are similar, then all the ratios of the corresponding sides are equal.

$$\frac{BF}{AE} = \frac{BG}{AF}$$

$$\frac{BF}{4} = \frac{6}{3}$$

$$BF = \frac{6}{3}\times 4$$
$$= 8$$

Since $ABCD$ is a rectangle, $AB = CD$ and $AD = BC$.

Since $AF = CH$, then $BF = DH$.

Since $AE = GC$, then $DE = BG$.

Since $EFGH$ is a rectangle, $FG = EH$.

$\therefore \triangle BFG \equiv \triangle DHE$.

Since $EFGH$ is a rectangle, $FE = GH$.

$AE = CG$ and $AF = CH$.

$\therefore \triangle AEF \equiv \triangle CGH$.

Area of $EFGH$

= area of $ABCD$ – (area of $\triangle AEF + \triangle CGH + \triangle BFG + \triangle DHE$)

$= (3 + 8)(4 + 6) - \left(2\times\frac{1}{2}\times 4\times 3 + 2\times\frac{1}{2}\times 8\times 6\right)$

$= 110 - 60$

$= \mathbf{50\ cm^2}$

11. **(a)** **(i)** Since $\triangle TBP$ is similar to $\triangle TAQ$, then all the ratios of the corresponding sides are equal.

$$\frac{x}{y} = \frac{AQ}{BP}$$

$$\frac{x}{y} = \frac{6}{2}$$

$$x = 3y$$

$\therefore$ length of $PA = x + y$
$= 3y + y$
$= \mathbf{4\textit{y}\ m}$

(ii) Since $\triangle PTM$ is similar to $\triangle PQA$, then all the ratios of the corresponding sides are equal.

$$\frac{TM}{QA} = \frac{PM}{PA}$$

$$\frac{TM}{6} = \frac{y}{4y}$$

$$TM = \frac{y}{4y}\times 6$$
$$= \mathbf{1.5\ m}$$

(b) $BP = h$ and $AQ = k$

$$\frac{x}{y} = \frac{MA}{PM}$$

From **(a)(i)**, $\frac{x}{y} = \frac{AQ}{BP}$.

$$\therefore \frac{MA}{PM} = \frac{AQ}{BP}$$

$$MA = \frac{k}{h}\times PM$$

From **(a)(ii)**, $\frac{TM}{QA} = \frac{PM}{PA}$.

$$\frac{TM}{k} = \frac{PM}{PA}$$
$$= \frac{PM}{PM + MA}$$
$$= \frac{PM}{PM + \left(\frac{k}{h}\times PM\right)}$$
$$= \frac{PM}{PM\left(1+\frac{k}{h}\right)}$$
$$= \frac{1}{1+\frac{k}{h}}$$
$$= \frac{h}{h+k}$$

$$\boldsymbol{TM = \frac{hk}{h+k}}$$

12. **(i)** $AC = 8$ m

$BE = \frac{30}{2}$ cm = 15 cm = 0.15 m

$AB = 60$ cm = 0.6 m

Since $\triangle ABE$ and $\triangle ACD$ are similar, then all the ratios of the corresponding sides are equal.

$$\frac{CD}{BE} = \frac{AC}{AB}$$

$$\frac{CD}{0.15} = \frac{8}{0.6}$$

$$CD = \frac{8}{0.6}\times 0.15$$
$$= 2 \text{ m}$$

$$DF = 2 \times CD$$
$$= 4 \text{ m}$$

$\therefore$ the estimated height of the tree is **4 m**.

(ii) The person estimating the height of a very tall building using this method would have to stand a great distance away from the building. It will then be difficult to measure the distance between the person and the building accurately, which will affect the estimated height obtained.

8.3 Similarity and enlargement

Practise Now 9

1. $\triangle ABC$ is similar to $\triangle A'B'C'$ under enlargement.

$$\frac{A'B'}{AB} = \frac{A'C'}{AC} = 3$$

$$\frac{A'B'}{6} = 3 \quad \text{and} \quad \frac{A'C'}{10} = 3$$

$\therefore A'B'$ = **18 cm** and $A'C'$ = **30 cm**.

2. $\triangle XYZ$ is similar to $\triangle XY'Z'$ under enlargement.

$$\frac{XY'}{XY} = \frac{Y'Z'}{YZ} = 1.5$$

$$\frac{XY'}{5} = 1.5 \quad \text{and} \quad \frac{12}{YZ} = 1.5$$

$\therefore XY'$ = **7.5 cm** and YZ = **8 cm**.

3. Let the actual height of the house be x m.

$\frac{x}{0.225} = \frac{1.8}{0.09}$

$x = \frac{1.8}{0.09} \times 0.225$

$= 4.5$

$\therefore$ the actual height of the house is **4.5 m**.

Practise Now 10A

1. (i)

Plan		Actual
1 cm	represents	2.5 m
1.25 cm	represents	(1.25×2.5) m = 3.125 m

$\therefore$ the actual length of the dining room is **3.125 m**.

(ii)

Actual		Plan
2.5 m	is represented by	1 cm
1 m	is represented by	$(1 \div 2.5)$ cm = 0.4 cm
3.4 m	is represented by	(3.4×0.4) = 1.36 cm

$\therefore$ the length on the plan is **1.36 cm**.

2. (i)

Model		Actual
1 cm	represents	4 m
67 cm	represents	(67×4) m = 268 m

$\therefore$ the actual length of the cruise liner is **268 m**.

(ii)

Actual		Model
10 m	is represented by	1 cm
1 m	is represented by	$(1 \div 10)$ cm = 0.1 cm
268 m	is represented by	(268×0.1) = 26.8 cm

$\therefore$ the length of the model cruise liner will be **26.8 cm**.

Practise Now 10B

(i) By measuring the length of the apartment that represents 825 cm, we obtain 5.5 cm.

Plan		Actual
5.5 cm	represents	825 cm
1 cm	represents	$(825 \div 5.5)$ cm = 150 cm

$\therefore$ the scale is **1 : 150**.

(ii)

Plan		Actual
1 cm	represents	150 cm
8 cm	represents	(8×150) cm = 1200 cm
		= 12 m

$\therefore$ the actual length L of the apartment is **12 m**.

Practise Now 11

1. (i)

Map		Actual
1 cm	represents	5 km
6.5 cm	represents	(6.5×5) km = 32.5 km

$\therefore$ the actual length of the road is **32.5 km**.

(ii)

Actual		Map
5 km	is represented by	1 cm
25 km	is represented by	(5×1) cm = 5 cm

$\therefore$ the corresponding distance on the map is **5 cm**.

(iii) 5 km = 500 000 cm

i.e. the scale of the map is $\mathbf{\frac{1}{500\ 000}}$.

2. (i)

Map		Actual
1 cm	represents	50 000 cm = 0.5 km
2 cm	represents	(2×0.5) km = 1 km

$\therefore$ the actual length is **1 km**.

(ii)

Actual		Map
1 km	is represented by	2 cm
14.5 km	is represented by	(14.5×2) cm = 29 cm

$\therefore$ the length on the map is **29 cm**.

Practise Now 12

1. (i)

Map		Actual
1 cm	represents	2 km
1 cm^2	represents	$(2\text{ km})^2 = 4\text{ km}^2$
3 cm^2	represents	$(3 \times 4)\text{ km}^2 = 12\text{ km}^2$

$\therefore$ the actual area of the plot of land is **12 km^2**.

(ii)

Actual		Map
2 km = 2000 m	is represented by	1 cm
1000 m	is represented by	$(1 \div 2)$ cm = 0.5 cm
$1000^2\text{ m}^2 = 1\ 000\ 000\text{ m}^2$	is represented by	$(0.5\text{ cm})^2 = 0.25\text{ cm}^2$
18 000 000 m^2	is represented by	$(18 \times 0.25)\text{ cm}^2 = 4.5\text{ cm}^2$

$\therefore$ the area on the map is **4.5 cm^2**.

2. (i)

Map		Actual
5 cm	represents	1 km
1 cm	represents	$(1 \div 5)$ km = 0.2 km
		= 20 000 cm

$\therefore$ the scale of the map is **1 : 20 000**.

(ii)

Map		Actual
1 cm	represents	0.2 km
1 cm^2	represents	$(0.2\text{ km})^2 = 0.04\text{ km}^2$
14 cm^2	represents	$(14 \times 0.04)\text{ km}^2 = 0.56\text{ km}^2$

$\therefore$ the actual area of the plot of land is **0.56 km^2**.

Exercise 8C

1. $\triangle XYZ$ is similar to $\triangle X'Y'Z'$ under enlargement.

$\frac{X'Y'}{XY} = \frac{Y'Z'}{YZ} = 2.5$

$\frac{X'Y'}{4} = 2.5$ and $\frac{8.75}{YZ} = 2.5$

$\therefore X'Y' = \mathbf{10\text{ cm}}$ and $YZ = \mathbf{3.5\text{ cm}}$.

2. (i) $PQRS$ is similar to $P'Q'R'S'$ under enlargement.

$k = \frac{P'Q'}{PQ}$

$= \frac{16}{8}$

$= \mathbf{2}$

(ii) $\frac{Q'R'}{QR} = \frac{S'R'}{SR} = 2$

$\frac{Q'R'}{4} = 2$ and $\frac{14}{SR} = 2$

$\therefore Q'R' = \mathbf{8\text{ cm}}$ and $SR = \mathbf{7\text{ cm}}$.

3. (i) Actual dimensions of Bedroom 1

$= (3 \times 1.5)$ m by (2.5×1.5) m

= 4.5 m by 3.75 m

(ii) Actual area of kitchen

$= (2 \times 1.5)\text{ m} \times (1.5 \times 1.5)\text{ m}$

= 6.75 m^2

(iii) Actual total area of apartment

$= [(3 + 2) \times 1.5]\text{ m} \times [(3 + 1.5 + 2.5) \times 1.5]\text{ m}$

$= 7.5\text{ m} \times 10.5\text{ m}$

= 78.75 m^2

4. **(i)** By measuring the vertical width that represents 27 km, we obtain 3 cm.

Map		**Actual**
3 cm	represents	27 km
1 cm	represents	$(27 \div 3)$ km = 9 km

$\therefore$ the scale is **1 cm to 9 km**.

(ii)

Map		**Actual**
1 cm	represents	9 km
5.6 cm	represents	(5.6×9) km = 50.4 km

$\therefore$ the actual distance is **50.4 km**.

5. **(i)**

Map		**Actual**
1 cm	represents	50 m
26 cm	represents	(26×50) m = 1300 km

$\therefore$ the actual length of the Karakoram Highway is **1300 km**.

(ii) 50 km = 50 000 m = 5 000 000 cm

i.e. the scale of the map is $\dfrac{1}{5\,000\,000}$.

6. **(i)**

Map		**Actual**
1 cm	represents	20 000 cm = 0.2 km
$5\frac{1}{2}$ cm	represents	$\left(5\frac{1}{2} \times 0.2\right)$ km = 1.1 km

$\therefore$ the actual length is **1.1 km**.

(ii)

Actual		**Map**
0.2 km = 200 m	is represented by	1 cm
100 m	is represented by	$(1 \div 2) = 0.5$ cm

$\therefore$ the length on the map is **0.5 cm**.

7. **(i)**

Map		**Actual**
1 cm	represents	8 km
1 cm^2	represents	$(8\text{ km})^2 = 64\text{ km}^2$
5 cm^2	represents	$(5 \times 64)\text{ km}^2 = 320\text{ km}^2$

$\therefore$ the actual area of the forest is **320 km^2**.

(ii)

Actual		**Map**
8 km	is represented by	1 cm
64 km^2	is represented by	1 cm^2
128 km^2	is represented by	$(2 \times 1)\text{ cm}^2 = 2\text{ cm}^2$

$\therefore$ the area of the park on the map is **2 cm^2**.

8. $\triangle ABC$ is similar to $\triangle AB'C'$ under enlargement.

$$\frac{AB'}{AB} = \frac{B'C'}{BC} = 3$$

$$\frac{AB+6}{AB} = 3 \quad \text{and} \quad \frac{12}{BC} = 3$$

$$AB + 6 = 3AB \qquad BC = 4$$

$$2AB = 6$$

$$AB = 3$$

$$AB' = 3 + 6$$

$$= 9$$

$\therefore$ BC = **4 cm** and AB' = **9 cm**.

9. Let the height of the tin of milk on the screen be h cm.

$$\frac{h}{24} = \frac{25}{75}$$

$$h = \frac{25}{75} \times 24$$

$$= 8$$

$\therefore$ the height of the tin of milk on the screen is **8 cm**.

10. **(i)**

Plan		**Actual**
12 cm	represents	3 m
1 cm	represents	$(3 \div 12)$ m = 0.25 m

$\therefore$ the scale is **1 cm to 0.25 m**.

(ii)

Actual		**Plan**
0.25 m = 25 cm	is represented by	1 cm
1 cm	is represented by	$(1 \div 25)$ cm = 0.04 cm
425 cm	is represented by	(425×0.04) cm = 17 cm

$\therefore$ the width of the living room on the floor plan is **17 cm**.

11. **(i)**

Model		**Actual**
1 cm	represents	15 m
18.2 cm	represents	(18.2×15) m = 273 m

$\therefore$ the actual height of the tower is **273 m**.

(ii)

Actual		**Model**
12 m	is represented by	1 cm
1 m	is represented by	$(1 \div 12)$ cm = $\frac{1}{12}$ cm
273 m	is represented by	$\left(273 \times \frac{1}{12}\right)$ cm = 22.8 cm (to the nearest 0.1 cm)

$\therefore$ the height of the model tower will be **22.8 cm**.

12. **(i)**

Map		**Actual**
4 cm	represents	5 km
1 cm	represents	$(5 \div 4)$ km = 1.25 km
21.04 cm	represents	(21.04×1.25) km = 26.3 km

$\therefore$ the actual distance between the two shopping centres is **26.3 km**.

(ii)

Actual		**Map**
175 000 cm = 1.75 km	is represented by	1 cm
1 km	is represented by	$(1 \div 1.75)$ cm = $\frac{4}{7}$ cm
26.3 km	is represented by	$\left(26.3 \times \frac{4}{7}\right)$ cm = 15.1 cm (to the nearest 0.1 cm)

$\therefore$ the distance between the two shopping centres on this map will be **0.8 cm**.

13. **(i)** By measuring the bar on the map that represents 300 m, we obtain 2.4 cm.

Map		**Actual**
2.4 cm	represents	300 m
1 cm	represents	$(300 \div 2.4)$ m = 125 m = 12 500 cm

$\therefore$ the scale is **1 : 12 500**.

(ii) By measuring XY on the map, we get 2.5 cm.

Map		**Actual**
1 cm	represents	125 m
2.5 cm	represents	(2.5×125) m = 312.5 m

$\therefore$ the actual distance XY of the biking trail is **312.5 m**.

(iii) The actual trail XY is not completely straight.

14. (i)

Map		Actual
1 cm	represents	240 000 cm = 2.4 km
1 cm^2	represents	$(2.4\text{ km})^2 = 5.76\text{ km}^2$
3.8 cm^2	represents	$(3.8 \times 5.76)\text{ km}^2 = 21.9\text{ km}^2$ (to 3.s.f)

$\therefore$ the actual area of the lake is **21.9 km²**.

(ii) 2 908 800 m^2 = 2.9088 km^2

Actual		Map
5.76 km^2	is represented by	1 cm^2
1 km^2	is represented by	$(1 \div 5.76)\text{ cm}^2 = \frac{25}{144}\text{ cm}^2$
2.9088 km^2	is represented by	$\left(2.9088 \times \frac{25}{144}\right)\text{ cm}^2 = 0.505\text{ cm}^2$

$\therefore$ the area of the plot of land on the map is **0.505 cm²**.

15. (i) 1 cm represents 500 m = 50 000 cm

$\therefore$ the scale is **1 : 50 000**.

(ii)

Actual		Map
500 m = 0.5 km	is represented by	1 cm
1 km	is represented by	$(1 \div 0.5)$ cm = 2 cm
28 km	is represented by	(28×2) cm = 56 cm

$\therefore$ the corresponding distance on the map is **56 cm**.

(iii)

Map		Actual
1 cm	represents	0.5 km
1 cm^2	represents	$(0.5\text{ km})^2 = 0.25\text{ km}^2$
12 cm^2	represents	$(12 \times 0.25)\text{ km}^2 = 3\text{ km}^2$

$\therefore$ the actual area of the jungle is **3 km²**.

16. (i) By measuring the bar on the map that represents 60 km, we obtain 1 cm.

1 cm represents 60 km = 6 000 000 cm

i.e. the scale of the map is $\mathbf{\frac{1}{6\ 000\ 000}}$.

(ii) By measuring the straight-line distance between Singapore and Kuantan on the map, we obtain 5.2 cm.

Map		Actual
1 cm	represents	60 km
5.2 cm	represents	(5.2×60) km = 312 km

$\therefore$ the actual straight-line distance is **312 km**.

(iii) By measuring the straight-line distance between Melaka and Kuala Lumpur on the map, we obtain 2.3 cm.

Map		Actual
1 cm	represents	60 km
2.3 cm	represents	(2.3×60) km = 138 km

Taxi fare = $0.60 × 138
= $82.80

$\therefore$ it would cost **$82.80**.

(iv) By measuring the straight-line distance between Batu Pahat and Port Dickson on the map, we obtain 2.7 cm.

Map		Actual
1 cm	represents	60 km
2.7 cm	represents	(2.7×60) km = 162 km

$$\text{Time taken} = \frac{162}{60} = 2\frac{7}{10}\text{ h} = 2\text{ h }42\text{ min}$$

$\therefore$ the time taken is **2 hours 42 minutes**.

(v) By measuring the straight-line distance between Johor Bahru and Segamat on the map, we obtain 2.8 cm.

Map		Actual
1 cm	represents	60 km
2.8 cm	represents	(2.8×60) km = 168 km

$$\text{Average speed} = \frac{168}{4} = 42\text{ km/h}$$

$\therefore$ the average speed of the train is **42 km/h**.

17. 28 hectares = 280 000 m^2

Map		Actual
112 cm^2	represents	280 000 m^2
1 cm^2	represents	$(280\ 000 \div 112)\text{ m}^2 = 2500\text{ m}^2$
1 cm	represents	$\sqrt{2500}$ m = 50 m

$\therefore$ x = **50**

Chapter 9 Pythagoras' Theorem

TEACHING NOTES

Suggested Approach

There are many ways of proving the Pythagoras' Theorem. An unofficial tally shows more than 300 ways of doing this. Teachers may use this opportunity to ask students to do a project of finding the best or the easiest method of doing this and get the students to present them to their class.

Students should be able to easily recall the previous lesson on similar triangles and apply their understanding in this chapter.

Section 9.1 Pythagoras' Theorem

Students are expected to know that the longest side of a right-angled triangle is known as the hypotenuse. The condition that the triangle must be a right-angled triangle has to be highlighted.

Teachers may wish to prove the Pythagoras' Theorem by showing the activity on the pages 273 and 274 (see Investigation: Pythagoras' Theorem). Again, it is important to state the theorem applies only to right-angled triangles. The theorem does not hold for other types of triangles.

Section 9.2 Applications of Pythagoras' Theorem in real-world contexts

There are many real-life application of Pythagoras' Theorem that the teachers can show the students. The worked examples and exercises should be more than enough for students to appreciate how the theorem is frequently present in real life. Teachers should always remind students to check before applying the theorem that the triangle is a right-angled triangle and that the longest side refers to the hypotenuse.

Section 9.3 Converse of Pythagoras' Theorem

Worked Example 8 provides an example of the converse of Pythagoras' Theorem. Some students should find the converse of the theorem easily manageable while teachers should take note of struggling learners who may face challenges in understanding the use of Pythagoras' Theorem to show that a triangle is right-angled. Students should be guided of the importance of giving reasons to justify their answers.

Introductory Problem
The solutions to this problem can be found in Introductory Problem Revisited (after Practise Now 7).

9.1 Pythagoras' Theorem

Practise Now 1A
(a) AB is the hypotenuse of $\triangle ABC$.
(b) DE is the hypotenuse of $\triangle DEF$.
(c) PQ is the hypotenuse of $\triangle PQR$.

Investigation (Pythagoras' Theorem)
Part 1
AB is the hypotenuse of each triangle.
1.

	BC	AC	AB	BC^2	AC^2	AB^2	$BC^2 + AC^2$
(a)	3 cm	**4 cm**	**5 cm**	9 cm^2	**16 cm^2**	**25 cm^2**	**25 cm^2**
(b)	6 cm	**8 cm**	**10 cm**	36 cm^2	**64 cm^2**	**100 cm^2**	**100 cm^2**
(c)	5 cm	**12 cm**	**13 cm**	25 cm^2	**144 cm^2**	**169 cm^2**	**169 cm^2**

2. $AB^2 = BC^2 + AC^2$

Part 2
1. AB is the hypotenuse of the right-angled triangle ABC.
2.

	BC	AC	AB	BC^2	AC^2	AB^2	$BC^2 + AC^2$
(a)	**5.17 cm**	**7.55 cm**	**9.15 cm**	**26.72 cm^2**	**56.99 cm^2**	**83.71 cm^2**	**83.71 cm^2**
(b)	**1.97 cm**	**3.45 cm**	**3.97 cm**	**3.87 cm^2**	**11.89 cm^2**	**15.76 cm^2**	**15.76 cm^2**
(c)	**3.86 cm**	**4.05 cm**	**5.59 cm**	**14.87 cm^2**	**16.43 cm^2**	**31.3 cm^2**	**31.3 cm^2**
(d)	**4.79 cm**	**5.03 cm**	**6.94 cm**	**22.9 cm^2**	**25.3 cm^2**	**48.2 cm^2**	**48.2 cm^2**
(e)	**7.84 cm**	**8.24 cm**	**11.38 cm**	**61.5 cm^2**	**67.95 cm^2**	**129.45 cm^2**	**129.45 cm^2**
(f)	**12 cm**	**5 cm**	**13 cm**	**144.01 cm^2**	**25.04 cm^2**	**169.06 cm^2**	**169.06 cm^2**

3. $AB^2 = BC^2 + AC^2$

Thinking Time (Page 275)
The hypotenuse of a right-angled triangle is the longest side of the triangle.
Since square R has sides of length equivalent to that of the hypotenuse of the right-angled triangle shown in Fig. 9.4, its length is longest, i.e. area of plot R is the largest among the 3 plots of land P, Q and R.
Using Pythagoras' Theorem, area of plot R is equal to the sum of the areas of plots P and Q.
Therefore, options **(c)** and **(d)** are equal and both give the largest possible plot of land among the 4 options given.

Practise Now 1B

1. In $\triangle ABC$, $\angle ACB = 90°$.
Using Pythagoras' Theorem,
$$\begin{aligned} AB^2 &= BC^2 + AC^2 \\ &= 8^2 + 6^2 \\ &= 64 + 36 \\ &= 100 \end{aligned}$$
$$\begin{aligned} \therefore AB &= \sqrt{100} \quad (\text{since } AB > 0) \\ &= \mathbf{10\ m} \end{aligned}$$

2. In $\triangle ABC$, $\angle ACB = 90°$.
Using Pythagoras' Theorem,
$$\begin{aligned} AB^2 &= BC^2 + AC^2 \\ &= 7^2 + 24^2 \\ &= 49 + 576 \\ &= 625 \end{aligned}$$
$$\begin{aligned} \therefore AB &= \sqrt{625} \quad (\text{since } AB > 0) \\ &= \mathbf{25\ cm} \end{aligned}$$

Practise Now 2

1. In $\triangle PQR$, $\angle PRQ = 90°$.
Using Pythagoras' Theorem,
$$\begin{aligned} PQ^2 &= PR^2 + RQ^2 \\ 15^2 &= PR^2 + 12^2 \\ PR^2 &= 15^2 - 12^2 \\ &= 225 - 144 \\ &= 81 \end{aligned}$$
$$\begin{aligned} \therefore PR &= \sqrt{81} \quad (\text{since } PR > 0) \\ &= \mathbf{9\ cm} \end{aligned}$$

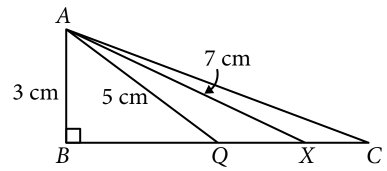
2. In $\triangle PQR$, $\angle PRQ = 90°$.
Using Pythagoras' Theorem,
$$\begin{aligned} PQ^2 &= PR^2 + QR^2 \\ 35^2 &= 28^2 + QR^2 \\ QR^2 &= 35^2 - 28^2 \\ &= 1225 - 784 \\ &= 441 \end{aligned}$$
$$\begin{aligned} \therefore QR &= \sqrt{441} \quad (\text{since } QR > 0) \\ &= \mathbf{21\ m} \end{aligned}$$

Practise Now 3

1. (a) (i) In $\triangle ABQ$, $\angle ABQ = 90°$.
Using Pythagoras' Theorem,
$$\begin{aligned} AQ^2 &= AB^2 + BQ^2 \\ 5^2 &= 3^2 + BQ^2 \\ BQ^2 &= 5^2 - 3^2 \\ &= 25 - 9 \\ &= 16 \end{aligned}$$
$$\begin{aligned} \therefore BQ &= \sqrt{16} \quad (\text{since } BQ > 0) \\ &= \mathbf{4\ cm} \end{aligned}$$

(ii) In $\triangle ABC$, $\angle ABC = 90°$.
Using Pythagoras' Theorem,
$$\begin{aligned} AC^2 &= AB^2 + BC^2 \\ &= 3^2 + (4 + 4)^2 \\ &= 3^2 + 8^2 \\ &= 9 + 64 \\ &= 73 \end{aligned}$$
$$\begin{aligned} \therefore AC &= \sqrt{73} \quad (\text{since } AC > 0) \\ &= \mathbf{8.54\ cm} \text{ (to 3 s.f.)} \end{aligned}$$

(b)



In $\triangle ABX$, $\angle ABX = 90°$.
Using Pythagoras' Theorem,
$$\begin{aligned} AX^2 &= AB^2 + BX^2 \\ 7^2 &= 3^2 + BX^2 \\ BX^2 &= 7^2 - 3^2 \\ &= 49 - 9 \\ &= 40 \end{aligned}$$
$$\therefore BX = \sqrt{40} \quad (\text{since } BX > 0)$$
$$\begin{aligned} CX &= BC - BX \\ &= (4 + 4) - \sqrt{40} \\ &= 1.6754 \text{ cm (to 5 s.f.)} \end{aligned}$$
$$\begin{aligned} QX &= BX - BQ \\ &= \sqrt{40} - 4 \\ &= 2.3246 \text{ cm (to 5 s.f.)} \end{aligned}$$
Since length of $CX <$ length of QX, X lies closer to $\boldsymbol{C}$.

2. (i) In $\triangle GHI$, $\angle GIH = 90°$.
Using Pythagoras' Theorem,
$$\begin{aligned} GH^2 &= GI^2 + HI^2 \\ 61^2 &= 11^2 + HI^2 \\ HI^2 &= 61^2 - 11^2 \\ &= 3721 - 121 \\ &= 3600 \end{aligned}$$
$$\begin{aligned} \therefore HI &= \sqrt{3600} \quad (\text{since } HI > 0) \\ &= \mathbf{60\ cm} \end{aligned}$$

(ii) In $\triangle GRI$, $\angle GIR = 90°$.
Using Pythagoras' Theorem,
$$\begin{aligned} GR^2 &= GI^2 + RI^2 \\ &= 11^2 + (60 - 21)^2 \\ &= 11^2 + 39^2 \\ &= 121 + 1521 \\ &= 1642 \end{aligned}$$
$$\begin{aligned} \therefore GR &= \sqrt{1642} \quad (\text{since } GR > 0) \\ &= \mathbf{40.5\ cm} \text{ (to 3 s.f.)} \end{aligned}$$

3. (i) In $\triangle HKR$, $\angle HRK = 90°$.
Using Pythagoras' Theorem,
$$\begin{aligned} HK^2 &= KR^2 + HR^2 \\ 19^2 &= 13^2 + HR^2 \\ HR^2 &= 19^2 - 13^2 \\ &= 361 - 169 \\ &= 192 \end{aligned}$$
$$\begin{aligned} \therefore HR &= \sqrt{192} \quad (\text{since } HR > 0) \\ &= \mathbf{13.9\ cm} \text{ (to 3 s.f.)} \end{aligned}$$

(ii) In $\triangle PQR$, $\angle PRQ = 90°$.
Using Pythagoras' Theorem,
$$PQ^2 = QR^2 + PR^2$$
$$33^2 = (QK + 13)^2 + \left(6 + \sqrt{192}\right)^2$$
$$(QK + 13)^2 = 33^2 - \left(6 + \sqrt{192}\right)^2$$
$$QK + 13 = \sqrt{33^2 - \left(6 + \sqrt{192}\right)^2} \quad (\text{since } QR > 0)$$
$$\begin{aligned} \therefore QK &= \sqrt{33^2 - \left(6 + \sqrt{192}\right)^2} - 13 \\ &= \mathbf{13.4\ cm} \end{aligned}$$

Exercise 9A

1. **(a)** Using Pythagoras' Theorem,

$a^2 = 20^2 + 21^2$
$= 400 + 441$
$= 841$
$\therefore a = \sqrt{841}$ (since $a > 0$)
$= \mathbf{29}$

(b) Using Pythagoras' Theorem,

$b^2 = 35^2 + 12^2$
$= 1225 + 144$
$= 1369$
$\therefore b = \sqrt{1369}$ (since $b > 0$)
$= \mathbf{37}$

(c) Using Pythagoras' Theorem,

$c^2 = 10^2 + 12^2$
$= 100 + 144$
$= 244$
$\therefore c = \sqrt{244}$ (since $c > 0$)
$= \mathbf{15.6}$ (to 3 s.f.)

(d) Using Pythagoras' Theorem,

$d^2 = 23^2 + 29^2$
$= 529 + 841$
$= 1370$
$\therefore d = \sqrt{1370}$ (since $d > 0$)
$= \mathbf{37.0}$ (to 3 s.f.)

2. In $\triangle ABC$, $\angle ABC = 90°$.
Using Pythagoras' Theorem,

$AC^2 = AB^2 + BC^2$
$= 8^2 + 15^2$
$= 64 + 225$
$= 289$
$\therefore AC = \sqrt{289}$ (since $AC > 0$)
$= \mathbf{17\ cm}$

3. In $\triangle DEF$, $\angle DEF = 90°$.
Using Pythagoras' Theorem,

$DF^2 = DE^2 + EF^2$
$= 6.7^2 + 5.5^2$
$= 44.89 + 30.25$
$= 75.14$
$\therefore DF = \sqrt{75.14}$ (since $DF > 0$)
$= \mathbf{8.67\ m}$ (to 3 s.f.)

4. **(a)** Using Pythagoras' Theorem,

$39^2 = 15^2 + a^2$
$a^2 = 39^2 - 15^2$
$= 1521 - 225$
$= 1296$
$\therefore a = \sqrt{1296}$ (since $a > 0$)
$= \mathbf{36}$

(b) Using Pythagoras' Theorem,

$19^2 = 14^2 + b^2$
$b^2 = 19^2 - 14^2$
$= 361 - 196$
$= 165$
$\therefore b = \sqrt{165}$ (since $b > 0$)
$= \mathbf{12.8}$ (to 3 s.f.)

(c) Using Pythagoras' Theorem,

$9.8^2 = 6.5^2 + c^2$
$c^2 = 9.8^2 - 6.5^2$
$= 96.04 - 42.25$
$= 53.79$
$\therefore c = \sqrt{53.79}$ (since $c > 0$)
$= \mathbf{7.33}$ (to 3 s.f.)

(d) Using Pythagoras' Theorem,

$24.7^2 = 14.5^2 + d^2$
$d^2 = 24.7^2 - 14.5^2$
$= 610.09 - 210.25$
$= 399.84$
$\therefore d = \sqrt{399.84}$ (since $d > 0$)
$= \mathbf{20.0}$ (to 3 s.f.)

5. In $\triangle GHI$, $\angle GHI = 90°$.
Using Pythagoras' Theorem,

$GI^2 = GH^2 + HI^2$
$65^2 = 33^2 + HI^2$
$HI^2 = 65^2 - 33^2$
$= 4225 - 1089$
$= 3136$
$\therefore HI = \sqrt{3136}$ (since $HI > 0$)
$= \mathbf{56\ cm}$

6. In $\triangle MNO$, $\angle MNO = 90°$.
Using Pythagoras' Theorem,

$MO^2 = MN^2 + NO^2$
$14.2^2 = MN^2 + 11^2$
$MN^2 = 14.2^2 - 11^2$
$= 201.64 - 121$
$= 80.64$
$\therefore MN = \sqrt{80.64}$ (since $MN > 0$)
$= \mathbf{8.98\ m}$ (to 3 s.f.)

7. **(i)** In $\triangle PQS$, $\angle PQS = 90°$.
Using Pythagoras' Theorem,

$PS^2 = PQ^2 + QS^2$
$53^2 = 45^2 + QS^2$
$QS^2 = 53^2 - 45^2$
$= 2809 - 2025$
$= 784$
$\therefore QS = \sqrt{784}$ (since $QS > 0$)
$= \mathbf{28\ cm}$

(ii) In $\triangle QSR$, $\angle QSR = 90°$.
Using Pythagoras' Theorem,

$QR^2 = QS^2 + SR^2$
$30^2 = 28^2 + SR^2$
$SR^2 = 30^2 - 28^2$
$= 900 - 784$
$= 116$
$\therefore SR = \sqrt{116}$ (since $SR > 0$)
$= \mathbf{10.8\ cm}$ (to 3 s.f.)

8. Let the lengths of the other two sides of the triangle be a cm and b cm respectively.

Using Pythagoras' Theorem,

$\left(\sqrt{34}\right)^2 = a^2 + b^2$

$34 = a^2 + b^2 \quad — (1)$

A possible set of lengths of a and b such that equation (1) will be valid:

$a =$ **5 cm** and $b =$ **3 cm**

Substitute $a = 5$ and $b = 3$ into (1):

$$\begin{aligned} \text{RHS} &= 5^2 + 3^2 \\ &= 25 + 9 \\ &= 34 \\ &= \text{LHS} \end{aligned}$$

9. **(a)** Since $\triangle TUV$ is an isosceles triangle, the line segment TH bisects UV, i.e. $HU = HV$.

In $\triangle THU$, $\angle THU = 90°$.

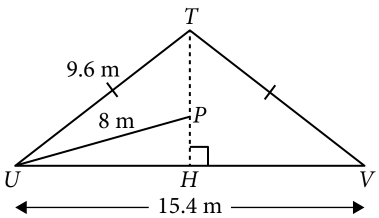
Using Pythagoras' Theorem,

$$\begin{aligned} TU^2 &= TH^2 + HU^2 \\ 9.6^2 &= TH^2 + \left(\frac{15.4}{2}\right)^2 \\ 9.6^2 &= TH^2 + 7.7^2 \\ TH^2 &= 9.6^2 - 7.7^2 \\ &= 92.16 - 59.29 \\ &= 32.87 \end{aligned}$$

$\therefore TH = \sqrt{32.87}$ (since $TH > 0$)

$= \mathbf{5.73\ m}$ (to 3 s.f.)

(b)



In $\triangle PHU$, $\angle PHU = 90°$.

Using Pythagoras' Theorem,

$$\begin{aligned} UP^2 &= HU^2 + PH^2 \\ 8^2 &= \left(\frac{15.4}{2}\right)^2 + PH^2 \\ PH^2 &= 8^2 - \left(\frac{15.4}{2}\right)^2 \\ &= 8^2 - 7.7^2 \\ &= 64 - 59.29 \\ &= 4.71 \end{aligned}$$

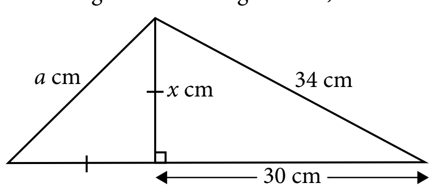
$\therefore PH = \sqrt{4.71}$ (since $PH > 0$)

$= 2.1703$ m (to 5 s.f.)

$$\begin{aligned} PT &= TH - PH \\ &= \sqrt{32.87} - \sqrt{4.71} \\ &= 3.5630 \text{ m (to 5 s.f.)} \end{aligned}$$

Since length of $PH <$ length of PT, P lies closer to $\boldsymbol{H}$.

10. **(a)**



Using Pythagoras' Theorem on the right-angled triangle on the right,

$$\begin{aligned} 34^2 &= x^2 + 30^2 \\ x^2 &= 34^2 - 30^2 \\ &= 1156 - 900 \\ &= 256 \end{aligned}$$

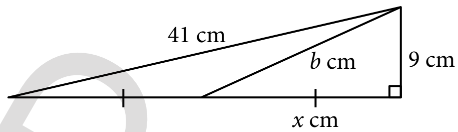
Using Pythagoras' Theorem on the right-angled triangle on the left,

$$\begin{aligned} a^2 &= x^2 + x^2 \\ &= 256 + 256 \\ &= 512 \end{aligned}$$

$\therefore a = \sqrt{512}$ (since $a > 0$)

$= \mathbf{22.6}$ (to 3 s.f.)

(b)



Using Pythagoras' Theorem on the larger right-angled triangle,

$$\begin{aligned} 41^2 &= (x + x)^2 + 9^2 \\ &= 4x^2 + 9^2 \\ 4x^2 &= 41^2 - 9^2 \\ &= 1681 - 81 \\ &= 1600 \\ x^2 &= 400 \end{aligned}$$

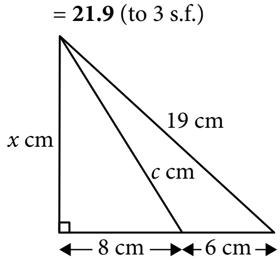
Using Pythagoras' Theorem on the smaller right-angled triangle,

$$\begin{aligned} b^2 &= x^2 + 9^2 \\ &= 400 + 81 \\ &= 481 \end{aligned}$$

$\therefore b = \sqrt{481}$ (since $b > 0$)

$= \mathbf{21.9}$ (to 3 s.f.)

(c)



Using Pythagoras' Theorem on the larger right-angled triangle,

$$\begin{aligned} 19^2 &= (8 + 6)^2 + x^2 \\ &= 14^2 + x^2 \\ x^2 &= 19^2 - 14^2 \\ &= 361 - 196 \\ &= 165 \end{aligned}$$

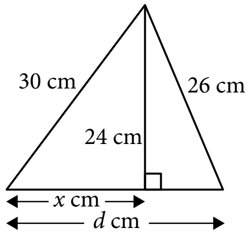
Using Pythagoras' Theorem on the smaller right-angled triangle,

$$\begin{aligned} c^2 &= x^2 + 8^2 \\ &= 165 + 64 \\ &= 229 \end{aligned}$$

$\therefore c = \sqrt{229}$ (since $c > 0$)

$= \mathbf{15.1}$ (to 3 s.f.)

(d)



Using Pythagoras' Theorem on the right-angled triangle on the left,

$30^2 = x^2 + 24^2$

$x^2 = 30^2 - 24^2$

$= 900 - 576$

$= 324$

$x = \sqrt{324}$ (since $x > 0$)

$= 18$

Using Pythagoras' Theorem on the right-angled triangle on the right,

$26^2 = (d - x)^2 + 24^2$

$= (d - 18)^2 + 24^2$

$(d - 18)^2 = 26^2 - 24^2$

$= 676 - 576$

$= 100$

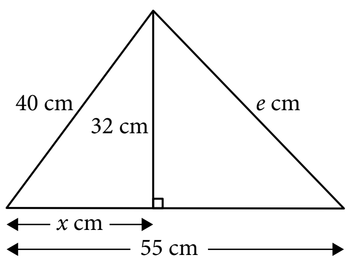
$d - 18 = \sqrt{100}$ (since $d - 18 > 0$)

$= 10$

$\therefore d = 10 + 18$

$= \mathbf{28}$

(e)



Using Pythagoras' Theorem on the right-angled triangle on the left,

$40^2 = x^2 + 32^2$

$x^2 = 40^2 - 32^2$

$= 1600 - 1024$

$= 576$

$x = \sqrt{576}$ (since $x > 0$)

$= 24$

Using Pythagoras' Theorem on the right-angled triangle on the right,

$e^2 = (55 - x)^2 + 32^2$

$= (55 - 24)^2 + 32^2$

$= 31^2 + 32^2$

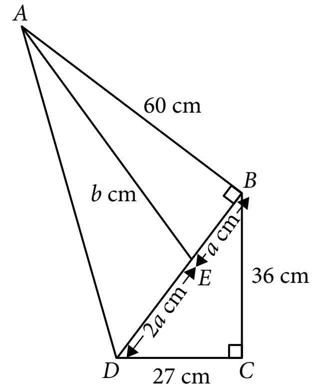
$= 961 + 1024$

$= 1985$

$\therefore e = \sqrt{1985}$ (since $e > 0$)

$= \mathbf{44.6}$ (to 3 s.f.)

11. (a)



In $\triangle BCD$, $\angle BCD = 90°$.

Using Pythagoras' Theorem,

$BD^2 = BC^2 + CD^2$

$(2a + a)^2 = 36^2 + 27^2$

$(3a)^2 = 1296 + 729$

$9a^2 = 2025$

$a^2 = 225$

$a = \sqrt{225}$ (since $a > 0$)

$= \mathbf{15}$

In $\triangle ABE$, $\angle ABE = 90°$.

Using Pythagoras' Theorem,

$AE^2 = AB^2 + BE^2$

$b^2 = 60^2 + a^2$

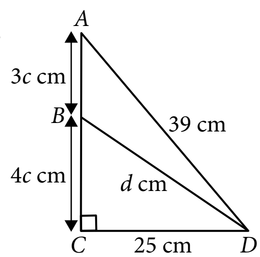
$= 3600 + 225$

$= 3825$

$b = \sqrt{3825}$ (since $b > 0$)

$= \mathbf{61.8}$ (to 3 s.f.)

(b)



In $\triangle ACD$, $\angle ACD = 90°$.

Using Pythagoras' Theorem,

$AD^2 = AC^2 + CD^2$

$39^2 = (3c + 4c)^2 + 25^2$

$= (7c)^2 + 25^2$

$= 49c^2 + 25^2$

$49c^2 = 39^2 - 25^2$

$= 1521 - 625$

$= 896$

$c^2 = \dfrac{128}{7}$

$c = \sqrt{\dfrac{128}{7}}$ (since $c > 0$)

$= \mathbf{4.28}$ (to 3 s.f.)

In $\triangle BCD$, $\angle BCD = 90°$.
Using Pythagoras' Theorem,

$BD^2 = BC^2 + CD^2$

$d^2 = (4c)^2 + 25^2$

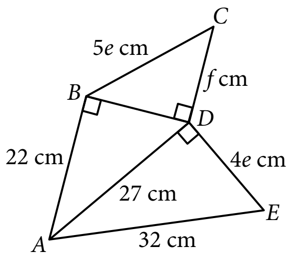
$= 16\left(\frac{128}{7}\right) + 625$

$= \frac{6423}{7}$

$d = \sqrt{\frac{6423}{7}}$ (since $d > 0$)

$=$ **30.3** (to 3 s.f.)

(c)



In $\triangle ADE$, $\angle ADE = 90°$.
Using Pythagoras' Theorem,

$AE^2 = AD^2 + DE^2$

$32^2 = 27^2 + (4e)^2$

$= 27^2 + 16e^2$

$16e^2 = 32^2 - 27^2$

$= 1024 - 729$

$= 295$

$e^2 = 18.4375$

$e = \sqrt{18.4375}$ (since $e > 0$)

$=$ **4.29** (to 3 s.f.)

In $\triangle ABD$, $\angle ABD = 90°$.
Using Pythagoras' Theorem,

$AD^2 = AB^2 + BD^2$

$27^2 = 22^2 + BD^2$

$BD^2 = 27^2 - 22^2$

In $\triangle BDC$, $\angle BDC = 90°$.
Using Pythagoras' Theorem,

$BC^2 = BD^2 + DC^2$

$(5e)^2 = (27^2 - 22^2) + f^2$

$25(18.4375) = 729 - 484 + f^2$

$460.9375 = 245 + f^2$

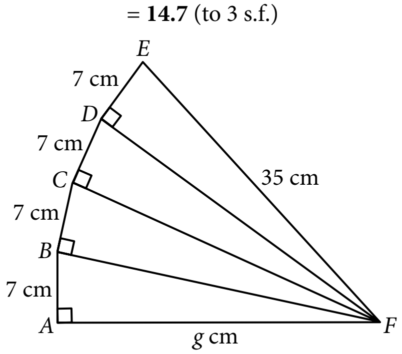
$f^2 = 460.9375 - 245$

$= 215.9375$

$f = \sqrt{215.9375}$ (since $f > 0$)

$=$ **14.7** (to 3 s.f.)

(d)



In $\triangle FDE$, $\angle FDE = 90°$.
Using Pythagoras' Theorem,

$FE^2 = FD^2 + DE^2$

$35^2 = FD^2 + 7^2$

$FD^2 = 35^2 - 7^2$

In $\triangle FCD$, $\angle FCD = 90°$.
Using Pythagoras' Theorem,

$FD^2 = FC^2 + CD^2$

$35^2 - 7^2 = FC^2 + 7^2$

$FC^2 = 35^2 - 7^2 - 7^2$

In $\triangle FBC$, $\angle FBC = 90°$.
Using Pythagoras' Theorem,

$FC^2 = FB^2 + BC^2$

$35^2 - 7^2 - 7^2 = FB^2 + 7^2$

$FB^2 = 35^2 - 7^2 - 7^2 - 7^2$

In $\triangle FAB$, $\angle FAB = 90°$.
Using Pythagoras' Theorem,

$FB^2 = FA^2 + AB^2$

$35^2 - 7^2 - 7^2 - 7^2 = g^2 + 7^2$

$g^2 = 35^2 - 7^2 - 7^2 - 7^2 - 7^2$

$= 1225 - 49 - 49 - 49 - 49$

$= 1029$

$g = \sqrt{1029}$ (since $g > 0$)

$=$ **32.1** (to 3 s.f.)

12. **(i)** In $\triangle WYX$, $\angle WYX = 90°$.
Using Pythagoras' Theorem,

$WX^2 = WY^2 + YX^2$

$(18 + 14)^2 = 24^2 + (YQ + 9.8)^2$

$32^2 = 24^2 + (YQ + 9.8)^2$

$(YQ + 9.8)^2 = 32^2 - 24^2$

$= 1024 - 576$

$= 448$

$YQ + 9.8 = \sqrt{448}$ (since $YQ + 9.8 > 0$)

$\therefore\ YQ = \sqrt{448} - 9.8$

$=$ **11.4 m** (to 3 s.f.)

(ii) In $\triangle XPY$, $\angle XPY = 90°$ (adj. $\angle$s on a str. line).
Using Pythagoras' Theorem,

$YX^2 = YP^2 + PX^2$

$448 = YP^2 + 14^2$

$YP^2 = 448 - 14^2$

$= 448 - 196$

$= 252$

$YP = \sqrt{252}$ (since $YP > 0$)

$\therefore$ area of $\triangle XPY = \frac{1}{2} \times YP \times PX$

$= \frac{1}{2} \times \sqrt{252} \times 14$

$=$ **111 m²** (to 3 s.f.)

13. In $\triangle HBK$, $\angle HBK = 90°$.
Using Pythagoras' Theorem,

$HK^2 = HB^2 + BK^2$
$22^2 = 15^2 + BK^2$
$BK^2 = 22^2 - 15^2$
$= 484 - 225$
$= 259$
$BK = \sqrt{259}$ (since $BK > 0$)

In $\triangle ABC$, $\angle ABC = 90°$.
Using Pythagoras' Theorem,

$AC^2 = AB^2 + BC^2$
$43^2 = (AH + 15)^2 + \left(\sqrt{259} + 19\right)^2$
$(AH + 15)^2 = 43^2 - \left(\sqrt{259} + 19\right)^2$
$AH + 15 = \sqrt{43^2 - \left(\sqrt{259} + 19\right)^2}$ (since $AH + 15 > 0$)
$\therefore AH = \sqrt{43^2 - \left(\sqrt{259} + 19\right)^2} - 15$
$=$ **9.85 cm** (to 3 s.f.)

14. In $\triangle EPF$, $\angle EPF = 90°$.
Using Pythagoras' Theorem,

$EF^2 = EP^2 + PF^2$
$23^2 = EP^2 + 13^2$
$EP^2 = 23^2 - 13^2$
$= 529 - 169$
$= 360$
$EP = \sqrt{360}$ (since $EP > 0$)

In $\triangle EPD$, $\angle EPD = 90°$.
Using Pythagoras' Theorem,

$ED^2 = EP^2 + PD^2$
$31^2 = 360 + PD^2$
$PD^2 = 31^2 - 360$
$= 961 - 360$
$= 601$
$PD = \sqrt{601}$ (since $PD > 0$)

In $\triangle DGF$, $\angle DGF = 90°$.
Using Pythagoras' Theorem,

$DF^2 = DG^2 + GF^2$
$(PD + 13)^2 = 32^2 + GF^2$
$\left(\sqrt{601} + 13\right)^2 = 32^2 + GF^2$
$GF^2 = \left(\sqrt{601} + 13\right)^2 - 32^2$
$GF = \sqrt{\left(\sqrt{601} + 13\right)^2 - 32^2}$ (since $GF > 0$)

Area of the figure
= area of $\triangle DGF$ + area of $\triangle DEF$
$= \frac{1}{2} \times DG \times GF + \frac{1}{2} \times DF \times EP$
$= \frac{1}{2} \times 32 \times \sqrt{\left(\sqrt{601} + 13\right)^2 - 32^2} + \frac{1}{2} \times \left(\sqrt{601} + 13\right) \times \sqrt{360}$
$=$ **669 m²** (to 3 s.f.)

9.2 Applications of Pythagoras' Theorem in real-world contexts

Practise Now 4

1. Let the length of the cable be x m.
Using Pythagoras' Theorem,

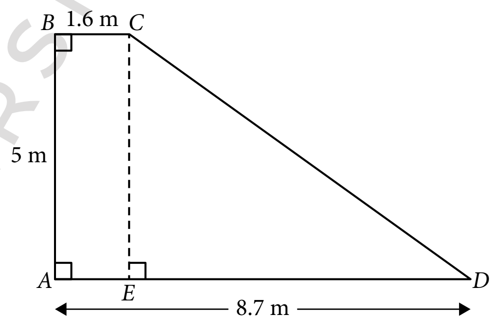
$x^2 = 24^2 + 14^2$
$= 576 + 196$
$= 772$
$x = \sqrt{772}$ (since $x > 0$)
$= 27.8$ (to 3 s.f.)
$\therefore$ the cable is **27.8 m** long.

2. Let the vertical distance from the top of the ladder to the base of the wall be x m.
Using Pythagoras' Theorem,

$2.5^2 = 1.5^2 + x^2$
$x^2 = 2.5^2 - 1.5^2$
$= 6.25 - 2.25$
$= 4$
$x = \sqrt{4}$ (since $x > 0$)
$= 2$

Shufen's height above the ground when standing at the top of the ladder
$= x + 1.6$
$= 2 + 1.6$
$= 3.6$ m
Since 3.6 m > 3.5 m, Shufen **will be able to** hang the frame on the wall.

Practise Now 5

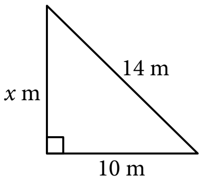


In $\triangle CED$, $\angle CED = 90°$.
Using Pythagoras' Theorem,

$CD^2 = CE^2 + ED^2$
$= 5^2 + (8.7 - 1.6)^2$
$= 5^2 + 7.1^2$
$= 25 + 50.41$
$= 75.41$
$\therefore CD = \sqrt{75.41}$ (since $CD > 0$)
$=$ **8.68 m** (to 3 s.f.)

Class Discussion (Modelling real-world phenomena)

(a) This problem can be modelled using a right-angled triangle, which then allows us to utilise Pythagoras' Theorem to solve for the height of the tree.



Let the vertical distance from the top of the tree to Kumar's eyes be x m.

Using Pythagoras' Theorem,

$$14^2 = x^2 + 10^2$$
$$x^2 = 14^2 - 10^2$$
$$= 196 - 100$$
$$= 96$$
$$x = \sqrt{96} \text{ (since } x > 0)$$

$\therefore$ the height of the tree $= \sqrt{96} + 1.8$

$= \mathbf{11.6\ m}$ (to 3 s.f.)

(b) Some assumptions of the model include:

- The tree has a constant width.
- Both Kumar and the tree are on level ground.
- For each reference region, a constant point is used for measurement.

Note that the list given is not exhaustive.

Practise Now 6

In $\triangle ABD$, $\angle BAD = 90°$.

Using Pythagoras' Theorem,

$$BD^2 = DA^2 + AB^2$$
$$(2x + 18)^2 = x^2 + (2x + 12)^2$$
$$4x^2 + 72x + 324 = x^2 + 4x^2 + 48x + 144$$
$$x^2 - 24x - 180 = 0$$
$$(x - 30)(x + 6) = 0$$
$$x = 30 \text{ or } x = -6 \text{ (N.A. since } x > 0)$$

Surface area of the table $= AD \times AB$

$= x(2x + 12)$

$= 30(2 \times 12 + 30)$

$= 30(60 + 12)$

$= 30 \times 72$

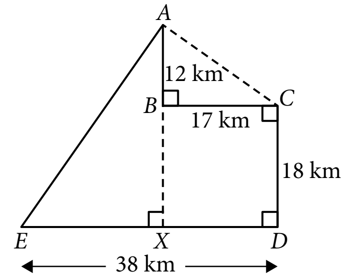
$= 2160\ \text{cm}^2 > 2000\ \text{cm}^2$

$\therefore$ the tablecloth **cannot** cover the entire surface of the table.

Practise Now 7

(i) $AB = 10 \times 1.2 = 12$ km

$BC = 10 \times 1.7 = 17$ km



In $\triangle ABC$, $\angle ABC = 90°$.

Using Pythagoras' Theorem,

$$AC^2 = AB^2 + BC^2$$
$$= 12^2 + 17^2$$
$$= 144 + 289$$
$$= 433$$
$$AC = \sqrt{433} \text{ (since } AC > 0)$$
$$= 20.8 \text{ km (to 3 s.f.)}$$

$\therefore$ the shortest distance between Port A and Jetty C is **20.8 km**.

(ii) Draw a perpendicular line from B to DE cutting DE at X.

In $\triangle AXE$, $\angle AXE = 90°$.

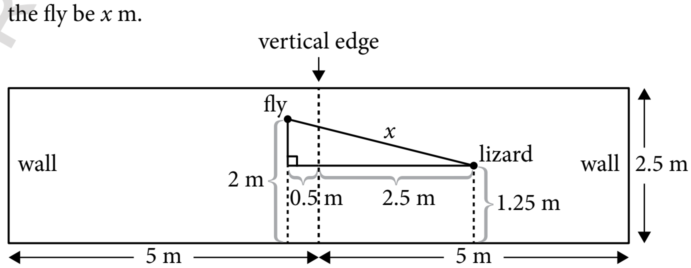
Using Pythagoras' Theorem,

$$AE^2 = AX^2 + EX^2$$
$$= (12 + 18)^2 + (38 - 17)^2$$
$$= 30^2 + 21^2$$
$$= 900 + 441$$
$$= 1341$$
$$AE = \sqrt{1341} \text{ (since } AE > 0)$$
$$= 36.6 \text{ km (to 3 s.f.)}$$

$\therefore$ the shortest distance between Port A and Island E is **36.6 km**.

Introductory Problem Revisited

Let the shortest distance that the lizard would have to crawl to catch the fly be x m.



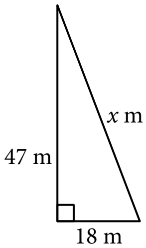
Using Pythagoras' Theorem,

$$x^2 = (2 - 1.25)^2 + (0.5 + 2.5)^2$$
$$= 0.75^2 + 3^2$$
$$= 0.5625 + 9$$
$$= 9.5625$$
$$x = \sqrt{9.5625} \text{ (since } x > 0)$$
$$= 3.09 \text{ (to 3 s.f.)}$$

$\therefore$ the shortest distance that the lizard would have to crawl in order to catch the fly = **3.09 m**.

Exercise 9B

1. Let the length of each cable be x m.

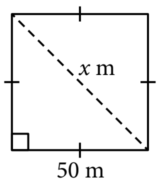


Using Pythagoras' Theorem,

$x^2 = 47^2 + 18^2$
$\quad = 2209 + 324$
$\quad = 2533$
$x = \sqrt{2533}$ (since $x > 0$)
$\quad = 50.3$ (to 3 s.f.)

$\therefore$ each cable is **50.3 m** long.

2. Let the length of the barricade be x m.

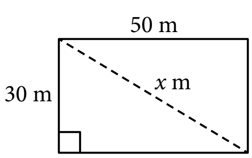


Using Pythagoras' Theorem,

$x^2 = 50^2 + 50^2$
$\quad = 2500 + 2500$
$\quad = 5000$
$x = \sqrt{5000}$ (since $x > 0$)
$\quad = 70.7$ (to 3 s.f.)

$\therefore$ the length of the barricade is **70.7 m**.

3. Let the distance Bernard has to swim be x m.

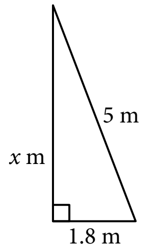


Using Pythagoras' Theorem,

$x^2 = 50^2 + 30^2$
$\quad = 2500 + 900$
$\quad = 3400$
$x = \sqrt{3400}$ (since $x > 0$)
$\quad = 58.3$ (to 3 s.f.)

$\therefore$ the distance Bernard has to swim is **58.3 m**.

4. Let the top of the ladder be x m above the ground.

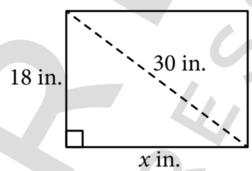


Using Pythagoras' Theorem,

$5^2 = 1.8^2 + x^2$
$x^2 = 5^2 - 1.8^2$
$\quad = 25 - 3.24$
$\quad = 21.76$
$x = \sqrt{21.76}$ (since $x > 0$)
$\quad = 4.66$ (to 3 s.f.)

$\therefore$ the top of the ladder is **4.66 m** above the ground.

5. Let the breadth of the television screen be x in.

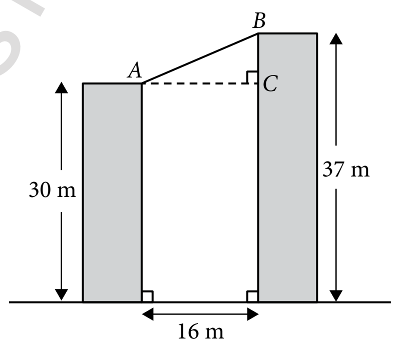


Using Pythagoras' Theorem,

$30^2 = x^2 + 18^2$
$x^2 = 30^2 - 18^2$
$\quad = 900 - 324$
$\quad = 576$
$x = \sqrt{576}$ (since $x > 0$)
$\quad = 24$

$\therefore$ the breadth of the television screen is **24 inches**.

6.



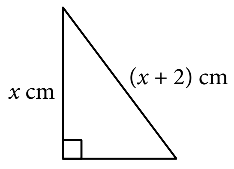
In $\triangle ABC$, $\angle C = 90°$.

Using Pythagoras' Theorem,

$AB^2 = AC^2 + BC^2$
$\quad = 16^2 + (37 - 30)^2$
$\quad = 16^2 + 7^2$
$\quad = 256 + 49$
$\quad = 305$
$AB = \sqrt{305}$ (since $AB > 0$)
$\quad = 17.5$ m (to 3 s.f.)

$\therefore$ the length of the cable is **17.5 m**.

7. Since $x > 0$, then the length of the hypotenuse must be $(x + 2)$ cm.



Using Pythagoras' Theorem,

$$(x + 2)^2 = x^2 + (x + 1)^2$$
$$x^2 + 4x + 4 = x^2 + x^2 + 2x + 1$$
$$x^2 - 2x - 3 = 0$$
$$(x - 3)(x + 1) = 0$$
$$\therefore x = \mathbf{3} \quad \text{or} \quad x = -1 \text{ (N.A. since } x > 0)$$

8. In $\triangle AED$, $\angle E = 90°$.

Using Pythagoras' Theorem,

$$AD^2 = AE^2 + DE^2$$
$$= 8^2 + 8^2$$
$$= 64 + 64$$
$$= 128$$
$$AD = \sqrt{128} \quad (\text{since } AD > 0)$$

In $\triangle BCD$, $\angle C = 90°$.

Using Pythagoras' Theorem,

$$BD^2 = BC^2 + CD^2$$
$$= 14^2 + 14^2$$
$$= 196 + 196$$
$$= 392$$
$$BD = \sqrt{392} \quad (\text{since } BD > 0)$$

$\therefore$ the total length of the sides along which the glue has to be applied

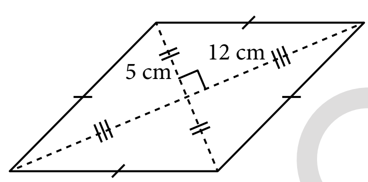
$= AD + BD$

$= \sqrt{128} + \sqrt{392}$

$=$ **31.1 cm** (to 3 s.f.)

9. The diagonals of a rhombus are perpendicular bisectors of each other.

Let the length of each side of the coaster be x cm.



Using Pythagoras' Theorem,

$$x^2 = 5^2 + 12^2$$
$$= 25 + 144$$
$$= 169$$
$$x = \sqrt{169} \quad (\text{since } x > 0)$$
$$= 13$$

$\therefore$ the length of each side of the coaster is **13 cm**.

10. **(i)** In $\triangle PQR$, $\angle PQR = 90°$.

Using Pythagoras' Theorem,

$$PR^2 = PQ^2 + QR^2$$
$$= 4.2^2 + 1.1^2$$
$$= 17.64 + 1.21$$
$$= 18.85$$
$$PR = \sqrt{18.85} \quad (\text{since } PR > 0)$$
$$= 4.34 \text{ m (to 3 s.f.)}$$

$\therefore$ the length of the pole is **4.34 m**.

(ii) In $\triangle XQY$, $\angle XQY = 90°$.

Using Pythagoras' Theorem,

$$XY^2 = XQ^2 + QY^2$$
$$QY^2 = XY^2 - XQ^2$$
$$= PR^2 - XQ^2$$
$$= 18.85 - (4.2 - 0.9)^2$$
$$= 18.85 - 3.3^2$$
$$= 18.85 - 10.89$$
$$= 7.96$$
$$QY = \sqrt{7.96} \quad (\text{since } QY > 0)$$
$$RQ + RY = \sqrt{7.96}$$
$$\therefore RY = \sqrt{7.96} - RQ$$
$$= \sqrt{7.96} - 1.1$$
$$= \mathbf{1.72\ m} \text{ (to 3 s.f.)}$$

11. In $\triangle FGH$, $\angle FGH = 90°$.

Using Pythagoras' Theorem,

$$FH^2 = FG^2 + GH^2$$
$$(4x + 1)^2 = (3x + 6)^2 + (x + 1)^2$$
$$16x^2 + 8x + 1 = 9x^2 + 36x + 36 + x^2 + 2x + 1$$
$$6x^2 - 30x - 36 = 0$$
$$x^2 - 5x - 6 = 0$$
$$(x - 6)(x + 1) = 0$$
$$x = 6 \quad \text{or} \quad x = -1 \text{ (N.A. since } x + 1 > 0)$$

Length of $FG = 3(6) + 6$

$= 18 + 6$

$= 24$ m

$= 2400$ cm

$\therefore$ number of stools required $= \dfrac{2400}{40}$

$=$ **60**

12. **(i)** Using Pythagoras' Theorem,

$$\left(5\sqrt{x}\right)^2 = x^2 + 12^2$$
$$25x = x^2 + 144$$
$$x^2 - 25x + 144 = 0$$
$$(x - 16)(x - 9) = 0$$
$$x = 16 \quad \text{or} \quad x = 9$$

Let $x = 9$.

$\therefore$ possible area of the garden $= 12 \times 9$

$=$ **108 m²**

(ii) Area of half of the garden $= \dfrac{108}{2}$

$= 54 \text{ m}^2$

Cost of landscaping half of the garden $= 54 \times 110$

$=$ **\$5940**

13. (i) In $\triangle HLO$, $\angle HLO = 90°$.
Using Pythagoras' Theorem,

$$OH^2 = OL^2 + LH^2$$
$$= 6^2 + (9 - 2)^2$$
$$= 6^2 + 7^2$$
$$= 36 + 49$$
$$= 85$$
$$OH = \sqrt{85} \text{ (since } OH > 0)$$
$$= 9.22 \text{ cm (to 3 s.f.)}$$

$\therefore$ the length of the zip is **9.22 cm**.

(ii) Let the length of NK and OK be x cm and y cm respectively.
In $\triangle HMN$, $\angle HMN = 90°$.
Using Pythagoras' Theorem,

$$HN^2 = HM^2 + MN^2$$
$$= 2^2 + 6^2$$
$$= 4 + 36$$
$$= 40$$

In $\triangle HKN$, $\angle HKN = 90°$.
Using Pythagoras' Theorem,

$$HN^2 = HK^2 + NK^2$$
$$40 = (OH - OK)^2 + x^2$$
$$= \left(\sqrt{85} - y\right)^2 + x^2$$
$$= 85 - 2\sqrt{85}y + y^2 + x^2$$
$$y^2 = -x^2 + 2\sqrt{85}y - 45 \quad \text{— (1)}$$

In $\triangle OKN$, $\angle OKN = 90°$.
Using Pythagoras' Theorem,

$$ON^2 = OK^2 + NK^2$$
$$9^2 = y^2 + x^2$$
$$y^2 = 9^2 - x^2 \quad \text{— (2)}$$

Substitute (1) into (2):

$$-x^2 + 2\sqrt{85}y - 45 = 9^2 - x^2$$
$$= 81 - x^2$$
$$2\sqrt{85}y = 126$$
$$y = \frac{126}{2\sqrt{85}}$$
$$= \frac{63}{\sqrt{85}} \quad \text{— (3)}$$

Substitute (3) into (2):

$$\left(\frac{63}{\sqrt{85}}\right)^2 = 9^2 - x^2$$
$$\frac{3969}{85} = 81 - x^2$$
$$x^2 = 81 - \frac{3969}{85}$$
$$x = \sqrt{81 - \frac{3969}{85}} \text{ (since } x > 0)$$
$$= 5.86 \text{ (to 3 s.f.)}$$

$\therefore$ the length of the second zip is **5.86 cm**.

14. (i) Area of $\triangle DAP = \frac{1}{2} \times AD \times AP$

$$= \frac{1}{2} \times 15 \times (28 - 6)$$
$$= \frac{1}{2} \times 15 \times 22$$
$$= 165 \text{ m}^2$$

Area of $\triangle RCD = \frac{1}{2} \times RC \times CD$

$$= \frac{1}{2} \times (15 - 6) \times 28$$
$$= \frac{1}{2} \times 9 \times 28$$
$$= 126 \text{ m}^2$$

Area of square $PBRQ = 6^2$
$= 36$ m^2

Area of rectangle $ABCD = 28 \times 15$
$= 420$ m^2

$\therefore$ area of shaded region $DPQR$
= area of rectangle $ABCD$ – area of square $PBRQ$ – area of $\triangle RCD$ – area of $\triangle DAP$
= 420 – 36 – 126 – 165
= **93 m^2**

(ii) In $\triangle DAP$, $\angle DAP = 90°$.
Using Pythagoras' Theorem,

$$DP^2 = DA^2 + AP^2$$
$$= 15^2 + 22^2$$
$$= 225 + 484$$
$$= 709$$
$$DP = \sqrt{709} \text{ (since } DP > 0)$$
$$= 26.6 \text{ m}$$

$\therefore$ the length of DP is **26.6 m**.

(iii) **Method 1:**
From part **(i)**, area of $\triangle DAP = 165$ m^2

$$\frac{1}{2} \times DP \times AX = 165$$
$$\frac{1}{2} \times \sqrt{709} \times AX = 165$$
$$AX = 12.4 \text{ m (to 3 s.f.)}$$

$\therefore$ the length of AX is **12.4 m**.

Method 2:
Let the length of XD be x m.
In $\triangle AXD$, $\angle AXD = 90°$.
Using Pythagoras' Theorem,

$$AD^2 = AX^2 + XD^2$$
$$15^2 = AX^2 + x^2$$
$$AX^2 = 15^2 - x^2 \quad \text{— (1)}$$

In $\triangle AXP$, $\angle AXP = 90°$.
Using Pythagoras' Theorem,

$$AP^2 = AX^2 + XP^2$$
$$22^2 = AX^2 + (DP - XD)^2$$
$$22^2 = AX^2 + \left(\sqrt{709} - x\right)^2$$
$$AX^2 = 22^2 - \left(\sqrt{709} - x\right)^2 \quad \text{— (2)}$$

Substitute (1) into (2):

$$15^2 - x^2 = 22^2 - \left(\sqrt{709} - x\right)^2$$
$$= 484 - \left(709 - 2\sqrt{709}x + x^2\right)$$
$$= 484 - 709 + 2\sqrt{709}x - x^2$$
$$2\sqrt{709}x = 15^2 - 484 + 709$$
$$= 450$$
$$x = \frac{450}{2\sqrt{709}} \text{ — (3)}$$

Substitute (3) into (1):

$$AX^2 = 15^2 - \left(\frac{450}{2\sqrt{709}}\right)^2$$
$$AX = \sqrt{15^2 - \left(\frac{450}{2\sqrt{709}}\right)^2} \quad (\text{since } AX > 0)$$
$$= 12.4 \text{ m (to 3 s.f.)}$$

$\therefore$ the length of AX is **12.4 m**.

15. (a) (i) Length of each side of the square tabletop

$$= \frac{132}{4}$$
$$= \mathbf{33 \text{ cm}}$$

(ii) Perimeter of round tabletop $= 2\pi r$

$$132 = 2\pi r$$
$$r = \frac{132}{2\pi}$$
$$= \frac{132}{2\left(\frac{22}{7}\right)}$$
$$= \mathbf{21 \text{ cm}}$$

(b) Area of square tabletop $= 33^2$

$$= \mathbf{1089 \text{ cm}^2}$$

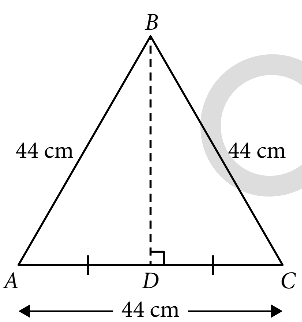
Area of round tabletop $= \pi r^2$

$$= \frac{22}{7} \times 21^2$$
$$= \mathbf{1386 \text{ cm}^2}$$

(c) (i) Length of each side of equilateral triangle

$$= \frac{132}{3}$$
$$= \mathbf{44 \text{ cm}}$$

(ii)



The tabletop in the shape of an equilateral triangle can be modelled as $\triangle ABC$ as shown. Since $\triangle ABC$ is an equilateral triangle, the line segment BD is the perpendicular bisector of AC, i.e. $AD = CD$.

In $\triangle ADB$, $\angle ADB = 90°$.

Using Pythagoras' Theorem,

$$AB^2 = AD^2 + BD^2$$
$$44^2 = \left(\frac{44}{2}\right)^2 + BD^2$$
$$44^2 = 22^2 + BD^2$$
$$BD^2 = 44^2 - 22^2$$
$$= 1936 - 484$$
$$= 1452$$
$$BD = \sqrt{1452} \quad (\text{since } BD > 0)$$

$\therefore$ area of tabletop $= 2 \times$ area of $\triangle ABD$

$$= 2 \times \frac{1}{2} \times 22 \times \sqrt{1452}$$
$$= \mathbf{838 \text{ cm}^2} \text{ (to 3 s.f.)}$$

(d) Since the round tabletop has the largest area, the designer should make the shape of the tabletop as a **circle** if he wants to have the most tabletop space.

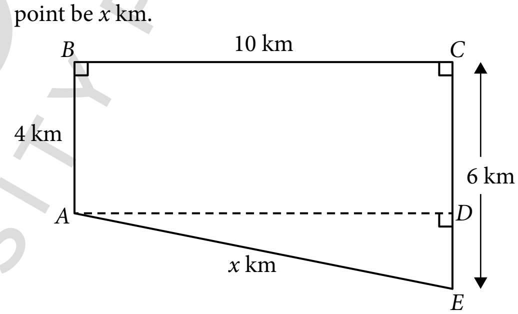
16. When the courier travels due North at an average speed of 40 km/h for 6 minutes, distance travelled

$$= 40 \times \frac{6}{60}$$
$$= 4 \text{ km}$$

When the courier travels due South at an average speed of 30 km/h for 12 minutes, distance travelled

$$= 30 \times \frac{12}{60}$$
$$= 6 \text{ km}$$

Let the shortest distance between the courier and his starting point be x km.



In $\triangle ADE$, $\angle ADE = 90°$.

Using Pythagoras' Theorem,

$$AE^2 = AD^2 + DE^2$$
$$x^2 = 10^2 + (6 - 4)^2$$
$$= 10^2 + 2^2$$
$$= 100 + 4$$
$$= 104$$
$$x = \sqrt{104} \quad (\text{since } x > 0)$$
$$= 10.2 \text{ km (to 3 s.f.)}$$

$\therefore$ the shortest distance between the courier and his starting point is **10.2 km**.

9.3 Converse of Pythagoras' Theorem

Practise Now 8

1. **(a)** AB is the longest side of $\triangle ABC$.

$AB^2 = 12^2$
$= 144$
$BC^2 + AC^2 = 10^2 + 8^2$
$= 100 + 64$
$= 164$

Since $AB^2 \neq BC^2 + AC^2$, $\triangle ABC$ is **not a right-angled triangle**.

(b) PQ is the longest side of $\triangle PQR$.

$PQ^2 = 34^2$
$= 1156$
$QR^2 + PR^2 = 16^2 + 30^2$
$= 256 + 900$
$= 1156$

Since $PQ^2 = QR^2 + PR^2$, then by the converse of Pythagoras' Theorem, $\triangle PQR$ is a **right-angled triangle** where $\angle \boldsymbol{R} = 90°$.

2. **(i)** XZ is the longest side of $\triangle XYZ$.

$XZ^2 = 51^2$
$= 2601$
$XY^2 + YZ^2 = 45^2 + 24^2$
$= 2025 + 576$
$= 2601$

Since $XZ^2 = XY^2 + YZ^2$, then by the converse of Pythagoras' Theorem, $\triangle XYZ$ is a right-angled triangle where $\angle XYZ = 90°$. (shown)

(ii) In $\triangle XYT$, $\angle XYT = 90°$.
Using Pythagoras' Theorem,

$TX^2 = YT^2 + XY^2$
$= (24 - 14)^2 + 45^2$
$= 10^2 + 45^2$
$= 100 + 2025$
$= 2125$
$TX = \sqrt{2125}$ (since $TX > 0$)
$= 46.1$ m (to 3 s.f.)

$\therefore$ the shortest distance of the tree from X is **46.1 m**.

Exercise 9C

1. **(a)** AC is the longest side of $\triangle ABC$.

$AC^2 = 65^2$
$= 4225$
$AB^2 + BC^2 = 16^2 + 63^2$
$= 256 + 3969$
$= 4225$

Since $AC^2 = AB^2 + BC^2$, then by the converse of Pythagoras' Theorem, $\triangle ABC$ is a **right-angled triangle** where $\angle \boldsymbol{B} = 90°$.

(b) EF is the longest side of $\triangle DEF$.

$EF^2 = 27^2$
$= 729$
$DE^2 + DF^2 = 24^2 + 21^2$
$= 576 + 441$
$= 1017$

Since $EF^2 \neq DE^2 + DF^2$, $\triangle DEF$ is **not a right-angled triangle**.

(c) GH is the longest side of $\triangle GHI$.

$GH^2 = 7.8^2$
$= 60.84$
$HI^2 + GI^2 = 7.1^2 + 2.4^2$
$= 50.41 + 5.76$
$= 56.17$

Since $GH^2 \neq HI^2 + GI^2$, $\triangle GHI$ is **not a right-angled triangle**.

(d) MN is the longest side of $\triangle MNO$.

$MN^2 = \left(\frac{5}{13}\right)^2$
$= \frac{25}{169}$
$NO^2 + MO^2 = \left(\frac{3}{13}\right)^2 + \left(\frac{4}{13}\right)^2$
$= \frac{9}{169} + \frac{16}{169}$
$= \frac{25}{169}$

Since $MN^2 = NO^2 + MO^2$, then by the converse of Pythagoras' Theorem, $\triangle MNO$ is a **right-angled triangle** where $\angle \boldsymbol{O} = 90°$.

2. PR is the longest side of $\triangle PQR$.

$PR^2 = 30^2$
$= 900$
$PQ^2 + QR^2 = 19^2 + 24^2$
$= 361 + 576$
$= 937$

Since $PR^2 \neq PQ^2 + QR^2$, $\triangle PQR$ is not a right-angled triangle. (shown)

3. $TU = \frac{5}{6} = \frac{10}{12}$

$SU = \frac{1}{3} = \frac{4}{12}$

TU is the longest side of $\triangle STU$.

$TU^2 = \left(\frac{5}{6}\right)^2$
$= \frac{25}{36}$
$ST^2 + SU^2 = \left(\frac{7}{12}\right)^2 + \left(\frac{1}{3}\right)^2$
$= \frac{49}{144} + \frac{1}{9}$
$= \frac{49}{144} + \frac{16}{144}$
$= \frac{65}{144}$

Since $TU^2 \neq ST^2 + SU^2$, $\triangle STU$ is **not a right-angled triangle**.

4. In $\triangle PQS$, $\angle P = 90°$.

Using Pythagoras' Theorem,

$$\begin{aligned} SQ^2 &= SP^2 + PQ^2 \\ &= 30^2 + 40^2 \\ &= 900 + 1600 \\ &= 2500 \\ SQ &= \sqrt{2500} \quad (\text{since } SQ > 0) \\ &= 50 \text{ m} \end{aligned}$$

$$\begin{aligned} SX &= \frac{16}{16+9} \times 50 \\ &= \frac{16}{25} \times 50 \\ &= 32 \text{ m} \end{aligned}$$

SR is the longest side of $\triangle SXR$.

$$\begin{aligned} SR^2 &= 40^2 \\ &= 1600 \end{aligned}$$

$$\begin{aligned} SX^2 + XR^2 &= 32^2 + 24^2 \\ &= 1024 + 576 \\ &= 1600 \end{aligned}$$

Since $SR^2 = SX^2 + XR^2$, then by the converse of Pythagoras' Theorem, $\triangle SXR$ is a right-angled triangle where $\angle X = 90°$.

Let the point at which Imran stops at be I.

Since IR represents the shortest distance from line segment SQ to the point R, then IR must be perpendicular to SQ.

As there is only 1 point along SQ that forms a perpendicular with point R with respect to line segment SQ, $I = X$.

$\therefore$ Imran stops at X. (shown)

5. Since m and n are positive integers,

$m^2 + n^2 > m^2 - n^2$

Also,

$$\begin{aligned} (m-n)^2 &> 0 \\ m^2 - 2mn + n^2 &> 0 \\ m^2 + n^2 &> 2mn \end{aligned}$$

$\therefore$ c is the longest side of the triangle.

$$\begin{aligned} c^2 &= (m^2 + n^2)^2 \\ &= m^4 + 2m^2n^2 + n^4 \end{aligned}$$

$$\begin{aligned} a^2 + b^2 &= (m^2 - n^2)^2 + (2mn)^2 \\ &= m^4 - 2m^2n^2 + n^4 + 4m^2n^2 \\ &= m^4 + 2m^2n^2 + n^4 \end{aligned}$$

Since $c^2 = a^2 + b^2$, then by the converse of Pythagoras' Theorem, the triangle is a right-angled triangle. (shown)

Chapter 10 Trigonometric Ratios

TEACHING NOTES

Suggested Approach

Teachers may want to introduce this topic by stating some of the uses of trigonometry such as surveying, engineering, physics and other physical sciences etc. Teachers can also introduce this chapter from a historical perspective. For instance, teachers can show old trigonometric tables to students, and explaining how the people in the past studied trigonometry before calculators became common. Depending on the profiles on the students, teachers may want to introduce the basic trigonometric ratios one at a time, or present them together.

Section 10.1 Trigonometric ratios

Teachers should guide the students through the activity on page 233 (see Investigation: Trigonometric ratios). Just like Pythagoras' Theorem, it is important to emphasise that trigonometric ratios are applicable only to right-angled triangles. Students should not attempt to use trigonometric ratios in other types of triangles.

To help students to memorise the trigonometric ratios easily, teachers may wish to use the mnemonic 'TOA-CAH-SOH' (means 'Big–foot lady' in a Chinese dialect). Students may need practice to identify the opposite, adjacent and hypotenuse sides with reference to a given angle as they may find the ratios confusing at the initial stage.

In using a calculator, it is important to remind pupils to check and see that the MODE is set as DEG.

The examination requirements state students are to give answers correct to 3 significant figures and angles in degree to correct to 1 decimal place. Therefore, students should develop the habit of working with 4 or 5 significant figures and angles in degree to 2 decimal places and give the final answer correct to the required accuracy.

Section 10.2 Applications of trigonometric ratios to find unknown sides of right-angled triangles

Students are required to solve simple right-angled triangles in this section. They are expected to understand, write and express their working in explicit form before using a calculator to evaluate the expression. For example, when $\sin 72° = \frac{x}{12}$, they must first write

$$\begin{aligned} x &= 12 \sin 72° \\ &= 11.4 \text{ (to 3 s.f.)} \end{aligned}$$

and not state the answer 11.4, corrected to 3 significant figures, outright (see Worked Example 2).

Section 10.3 Applications of trigonometric ratios to find unknown angles in right-angled triangles

Previously, students are taught to find the sides given the angles of a right-angled triangle. Here, they will do the reverse, i.e. finding the angles of a right-angled triangle, given the sides.

Teachers should remind students to choose the correct trigonometric ratio that can be used to find the angle.

In finding an unknown side or angle, the angle properties and Pythagoras' Theorem that the students have learnt in the previous chapters can be used as well. Students should practise using different approaches to a particular problem so as to appreciate the concepts they have learnt thus far.

In Journal Writing on page 309, some students will discover that when $\angle x = 45°$, $\tan x° = 1$ and thus $a = b$. Teachers can highlight that a right-angled triangle where $\angle x = 45°$ is also an isosceles triangle.

Section 10.4 Applications of trigonometric ratios in real-world contexts

In this section, students will learn how trigonometry is used in real-life situations. Teachers are encouraged to work through as many worked examples as possible. Students should also work through some questions of similar type.

Teachers can consolidate important concepts inthis chapter by getting the students to revisit the Introductory Problem (see Introductory Problem Revisited on page 316).

Introductory Problem

The solutions to this problem can be found in Introductory Problem Revisited (after Practise Now 9).

10.1 Trigonometric ratios

Investigation (Trigonometric ratios)

2. The triangles obtained are **similar right-angled triangles**.

3. **(a), (b)**

Triangle	Angle A	Length of AB	Length of BC	Length of AC	$\frac{BC}{AB}$	$\frac{AC}{AB}$	$\frac{BC}{AC}$
1	30°	4.33	2.16	3.75	0.50	0.87	0.58
2	30°	**9.76**	**4.88**	**8.46**	**0.50**	**0.87**	**0.58**
3	30°	**3.13**	**1.56**	**2.71**	**0.50**	**0.87**	**0.58**

4. **(a)** For different sets of lengths of AB, BC and AC, the ratios of $\frac{BC}{AB}$, $\frac{AC}{AB}$ and $\frac{BC}{AC}$ remain constant.

(b) Since the triangles obtained are similar right-angled triangles, then all the ratios of the corresponding sides are equal.

For two similar right-angled triangles ABC and $AB'C'$:

$$\frac{BC}{B'C'} = \frac{AB}{AB'}$$

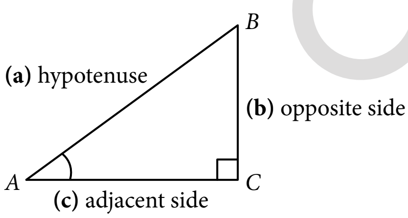
$$\frac{BC}{AB} = \frac{B'C'}{AB'}$$

Hence, the ratio $\frac{BC}{AB}$ will always be equal.

5. **(a), (b)**

Angle A	Length of AB	Length of BC	Length of AC	$\frac{BC}{AB}$	$\frac{AC}{AB}$	$\frac{BC}{AC}$
50°	**4.67**	**3.58**	**3.00**	**0.77**	**0.64**	**1.2**
50°	**8.52**	**6.52**	**5.47**	**0.77**	**0.64**	**1.2**
50°	**10.51**	**8.05**	**6.75**	**0.77**	**0.64**	**1.2**

6.



7. $\frac{AC}{AB} = \frac{\textbf{adj}}{\textbf{hyp}}$

$\frac{BC}{AC} = \frac{\textbf{opp}}{\textbf{adj}}$

8. With reference to angle B in Fig. 10.2, ***AC* is the opposite side** and ***BC* is the adjacent side**.

Thinking Time (Page 298)

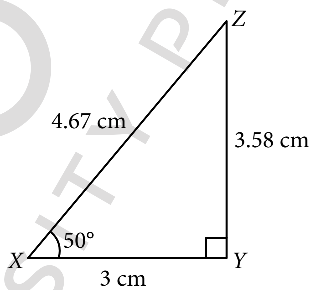
For $\triangle ABC$ with $\angle A$ fixed at 30° shown in the Investigation (Trigonometric ratios) on page 60,

$$\begin{aligned}\sin \angle A &= \sin 30^\circ\\ &= \frac{\text{opp}}{\text{hyp}}\\ &= \frac{BC}{AB}\\ &= 0.50\end{aligned}$$

$$\begin{aligned}\cos \angle A &= \cos 30^\circ\\ &= \frac{\text{adj}}{\text{hyp}}\\ &= \frac{AC}{AB}\\ &= 0.87\end{aligned}$$

$$\begin{aligned}\tan \angle A &= \tan 30^\circ\\ &= \frac{\text{opp}}{\text{adj}}\\ &= \frac{BC}{AC}\\ &= 0.58\end{aligned}$$

One possible $\triangle XYZ$, where $\angle X = 50°$ and $\angle Y = 90°$, is shown below.



$$\begin{aligned}\sin \angle X &= \sin 50^\circ\\ &= \frac{\text{opp}}{\text{hyp}}\\ &= \frac{YZ}{XZ}\\ &= \frac{3.58}{4.67}\\ &= 0.77 \text{ (to 2 s.f.)}\end{aligned}$$

$$\begin{aligned}\cos \angle X &= \cos 50^\circ\\ &= \frac{\text{adj}}{\text{hyp}}\\ &= \frac{XY}{XZ}\\ &= \frac{3}{4.67}\\ &= 0.64 \text{ (to 2 s.f.)}\end{aligned}$$

$$\begin{aligned}\tan \angle X &= \tan 50^\circ\\ &= \frac{\text{opp}}{\text{adj}}\\ &= \frac{YZ}{XY}\\ &= \frac{3.58}{3}\\ &= 1.2 \text{ (to 2 s.f.)}\end{aligned}$$

From the calculations shown, sin 30° ≠ sin 50°, cos 30° ≠ cos 50° and tan 30° ≠ tan 50°. Hence the trigonometric ratios of $\angle X$ are **not the same** as those of $\angle A$.

In general, the lengths of the sides of the triangle are affected by the value of the angle, which implies that the **trigonometric ratios are dependent on the value of the angle**.

Practise Now 1A

1. **(i)** The hypotenuse of $\triangle ABC$ is the side ***AC***.

(ii) The side opposite $\angle A$ is ***BC***.

(iii) The side adjacent to $\angle A$ is ***AB***.

2. **(i)** $\sin P = \dfrac{\text{opp}}{\text{hyp}} = \mathbf{\dfrac{3}{5}}$

(ii) $\cos P = \dfrac{\text{adj}}{\text{hyp}} = \mathbf{\dfrac{4}{5}}$

(iii) $\tan P = \dfrac{\text{opp}}{\text{adj}} = \mathbf{\dfrac{3}{4}}$

(iv) $\sin Q = \dfrac{\text{opp}}{\text{hyp}} = \mathbf{\dfrac{4}{5}}$

(v) $\cos Q = \dfrac{\text{adj}}{\text{hyp}} = \mathbf{\dfrac{3}{5}}$

(vi) $\tan Q = \dfrac{\text{opp}}{\text{adj}} = \mathbf{\dfrac{4}{3}}$

3. **(a)** **(i)** $\sin X = \dfrac{\text{opp}}{\text{hyp}} = \boldsymbol{\dfrac{a}{c}}$

(ii) $\cos X = \dfrac{\text{adj}}{\text{hyp}} = \boldsymbol{\dfrac{b}{c}}$

(iii) $\tan X = \dfrac{\text{opp}}{\text{adj}} = \boldsymbol{\dfrac{a}{b}}$

(iv) $\sin Y = \dfrac{\text{opp}}{\text{hyp}} = \boldsymbol{\dfrac{b}{c}}$

(v) $\cos Y = \dfrac{\text{adj}}{\text{hyp}} = \boldsymbol{\dfrac{a}{c}}$

(vi) $\tan Y = \dfrac{\text{opp}}{\text{adj}} = \boldsymbol{\dfrac{b}{a}}$

(b) $\tan X = \tan Y$

$$\frac{a}{b} = \frac{b}{a}$$

$$a^2 = b^2$$

$$a = b \ (a > 0 \text{ and } b > 0)$$

Let $a = 5$ and $b = 5$.

In $\triangle XYZ$, $\angle XZY = 90°$.

Using Pythagoras' Theorem,

$$\begin{aligned} XY^2 &= XZ^2 + YZ^2 \\ &= b^2 + a^2 \\ &= 5^2 + 5^2 \\ &= 25 + 25 \\ &= 50 \\ XY &= \sqrt{50} \ (\text{since } XY > 0) \\ &= 7.07 \text{ m} \end{aligned}$$

$\therefore$ $\boldsymbol{a = 5}$, $\boldsymbol{b = 5}$ and $\boldsymbol{c = 7.07}$.

Practise Now 1B

(a) Sequence of calculator keys:

cos (2 4) =

$\therefore$ cos 24° = **0.914** (to 3 s.f.)

(b) Sequence of calculator keys:

tan (7 4 . 6) =

$\therefore$ tan 74.6° = **3.63** (to 3 s.f.)

(c) Sequence of calculator keys:

sin (7 2 . 1 5) =

$\therefore$ sin 72.15° = **0.952** (to 3 s.f.)

(d) Sequence of calculator keys:

3 sin (4 8) +
2 cos (3 9) =

$\therefore$ 3 sin 48° + 2 cos 39° = **3.78** (to 3 s.f.)

(e) Sequence of calculator keys:

5 ÷ tan (1 8 . 3) =

$\therefore$ $\dfrac{5}{\tan 18.3°}$ = **15.1** (to 3 s.f.)

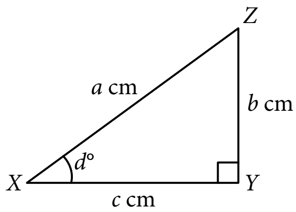
(f) Sequence of calculator keys:

(tan (4 8 . 3) – sin (2 8
. 7)) ÷ (cos (1 5) +
cos (3 5)) =

$\therefore$ $\dfrac{\tan 48.3° - \sin 28.7°}{\cos 15° + \cos 35°}$ = **0.360** (to 3 s.f.)

Thinking Time (Page 301)

We can determine the trigonometric ratios of acute angles using right-angled triangles. Consider a right-angled triangle XYZ with acute angle ZXY.



(a) $\cos \angle ZXY = \dfrac{\text{adj}}{\text{hyp}}$

$= \dfrac{c}{a}$

Since the length of the hypotenuse of $\triangle XYZ$ is always greater than the length of either of the other two sides of the triangle, $\dfrac{c}{a} < 1$. Hence,

I agree that the cosine of an acute angle is always less than 1.

(b) $\sin \angle ZXY = \dfrac{\text{opp}}{\text{hyp}}$

$= \dfrac{b}{a}$

For any acute angle ZXY, the length of ZY in $\triangle XYZ$ will always be greater than 0, i.e. $b > 0$. Hence, **I agree** that the sine of an acute angle can never be 0.

(c) $\tan \angle ZXY = \dfrac{\text{opp}}{\text{adj}}$

$= \dfrac{b}{c}$

When $\triangle XYZ$ is an isosceles triangle such that $b = c$, then $\dfrac{b}{c} = 1$. Hence,

I agree that the tangent of an acute angle is sometimes equal to 1.

Exercise 10A

1. **(a)** **(i)** The hypotenuse of $\triangle PQR$ is the side ***PQ***.

(ii) The side opposite $\angle a$ is ***PR***.

(iii) The side adjacent to $\angle a$ is ***QR***.

(b) **(i)** The hypotenuse of $\triangle XYZ$ is the side ***XY***.

(ii) The side opposite $\angle a$ is ***XZ***.

(iii) The side adjacent to $\angle a$ is ***YZ***.

2. **(a)** **(i)** $\sin A = \dfrac{\text{opp}}{\text{hyp}}$

$= \mathbf{\dfrac{5}{13}}$

(ii) $\cos A = \dfrac{\text{adj}}{\text{hyp}}$

$= \mathbf{\dfrac{12}{13}}$

(iii) $\tan A = \dfrac{\text{opp}}{\text{adj}}$

$= \mathbf{\dfrac{5}{12}}$

(iv) $\sin B = \dfrac{\text{opp}}{\text{hyp}}$

$= \mathbf{\dfrac{12}{13}}$

(v) $\cos B = \dfrac{\text{adj}}{\text{hyp}}$

$= \mathbf{\dfrac{5}{13}}$

(vi) $\tan B = \dfrac{\text{opp}}{\text{adj}}$

$= \mathbf{\dfrac{12}{5}}$

(b) **(i)** $\sin A = \dfrac{\text{opp}}{\text{hyp}}$

$= \mathbf{\dfrac{24}{25}}$

(ii) $\cos A = \dfrac{\text{adj}}{\text{hyp}}$

$= \mathbf{\dfrac{7}{25}}$

(iii) $\tan A = \dfrac{\text{opp}}{\text{adj}}$

$= \mathbf{\dfrac{24}{7}}$

(iv) $\sin B = \dfrac{\text{opp}}{\text{hyp}}$

$= \mathbf{\dfrac{7}{25}}$

(v) $\cos B = \dfrac{\text{adj}}{\text{hyp}}$

$= \mathbf{\dfrac{24}{25}}$

(vi) $\tan B = \dfrac{\text{opp}}{\text{adj}}$

$= \mathbf{\dfrac{7}{24}}$

3. **(i)** $\sin P = \dfrac{\text{opp}}{\text{hyp}}$

$= \boldsymbol{\dfrac{y}{z}}$

(ii) $\cos P = \dfrac{\text{adj}}{\text{hyp}}$

$= \boldsymbol{\dfrac{x}{z}}$

(iii) $\tan P = \dfrac{\text{opp}}{\text{adj}}$

$= \boldsymbol{\dfrac{y}{x}}$

(iv) $\sin Q = \dfrac{\text{opp}}{\text{hyp}}$

$= \boldsymbol{\dfrac{x}{z}}$

(v) $\cos Q = \dfrac{\text{adj}}{\text{hyp}}$

$= \boldsymbol{\dfrac{y}{z}}$

(vi) $\tan Q = \dfrac{\text{opp}}{\text{adj}}$

$= \boldsymbol{\dfrac{x}{y}}$

4. **(a)** Sequence of calculator keys:

tan (4 7) =

$\therefore$ tan 47° = **1.07** (to 3 s.f.)

(b) Sequence of calculator keys:

sin (7 5 . 3) =

$\therefore$ sin 75.3° = **0.967** (to 3 s.f.)

(c) Sequence of calculator keys:

cos (3 0 . 1 9) =

$\therefore$ cos 30.19° = **0.864** (to 3 s.f.)

(d) Sequence of calculator keys:

sin (3 5) + cos (4 9) =

$\therefore$ sin 35° + cos 49° = **1.23** (to 3 s.f.)

(e) Sequence of calculator keys:

2 cos (4 2 . 3) +
3 sin (1 6 . 8) =

$\therefore$ 2 cos 42.3° + 3 sin 16.8° = **2.35** (to 3 s.f.)

(f) Sequence of calculator keys:

sin (7 1 . 6) ×
tan (1 6 . 7) =

$\therefore$ sin 71.6° × tan 16.7° = **0.285** (to 3 s.f.)

5. **(a)** **(i)** $\sin M = \frac{\text{opp}}{\text{hyp}}$

$= \boldsymbol{\frac{c}{a}}$

(ii) $\cos M = \frac{\text{adj}}{\text{hyp}}$

$= \boldsymbol{\frac{b}{a}}$

(iii) $\tan M = \frac{\text{opp}}{\text{adj}}$

$= \boldsymbol{\frac{c}{b}}$

(iv) $\sin N = \frac{\text{opp}}{\text{hyp}}$

$= \boldsymbol{\frac{b}{a}}$

(v) $\cos N = \frac{\text{adj}}{\text{hyp}}$

$= \boldsymbol{\frac{c}{a}}$

(vi) $\tan N = \frac{\text{opp}}{\text{adj}}$

$= \boldsymbol{\frac{b}{c}}$

(b) $\tan M = \tan N$

$\frac{c}{b} = \frac{b}{c}$

$c^2 = b^2$

$c = b$ ($b > 0$ and $c > 0$)

Let $b = 3$ and $c = 3$.

In $\triangle MON$, $\angle MON = 90°$.

Using Pythagoras' Theorem,

$$\begin{aligned} MN^2 &= MO^2 + ON^2 \\ &= b^2 + c^2 \\ &= 3^2 + 3^2 \\ &= 9 + 9 \\ &= 18 \\ MN &= \sqrt{18} \quad (\text{since } MN > 0) \\ &= 4.24 \text{ m} \end{aligned}$$

$\therefore$ $\boldsymbol{a = 4.24}$, $\boldsymbol{b = 3}$ and $\boldsymbol{c = 3}$.

6. **(a)** Sequence of calculator keys:

(5 tan (6 1 . 4)) ÷
(2 cos (1 0 . 3)) =

$\therefore$ $\frac{5\tan 61.4°}{2\cos 10.3°}$ = **4.66** (to 3 s.f.)

(b) Sequence of calculator keys:

(4 (sin (2 2 . 5)) x^2)
÷ cos (6 7 . 5) =

$\therefore$ $\frac{4(\sin 22.5°)^2}{\cos 67.5°}$ = **1.53** (to 3 s.f.)

(c) Sequence of calculator keys:

(tan (1 5) +
cos (3 3)) ÷
sin (7 8 . 4) =

$\therefore$ $\frac{\tan 15° + \cos 33°}{\sin 78.4°}$ = **1.13** (to 3 s.f.)

(d) Sequence of calculator keys:

tan (4 7 . 9) ÷
(cos (8 4) –
sin (6 3)) =

$\therefore$ $\frac{\tan 47.9°}{\cos 84° - \sin 63°}$ = **–1.41** (to 3 s.f.)

(e) Sequence of calculator keys:

(cos (6 7) +
sin (8 9)) ÷
(tan (6 3 . 4) ×
cos (1 5 . 5)) =

$\therefore$ $\frac{\cos 67° + \sin 89°}{\tan 63.4° \times \cos 15.5°}$ = **0.723** (to 3 s.f.)

(f) Sequence of calculator keys:

(sin (2 4 . 6) ÷
cos (6 2 . 1)) ÷
(tan (2 1) +
cos (1 4)) =

$\therefore$ $\frac{\sin 24.6° \div \cos 62.1°}{\tan 21° + \cos 14°}$ = **0.657** (to 3 s.f.)

(g) Sequence of calculator keys:

(sin (5 7) −
cos (7 3)) ÷
(tan (1 5 . 3) ×
sin (8 3 . 4)) =

$\therefore \dfrac{\sin 57° - \cos 73°}{\tan 15.3° \times \sin 83.4°} = \mathbf{2.01}$ (to 3 s.f.)

(h) Sequence of calculator keys:

(cos (2 4 . 7) ×
sin (3 5 . 1)) ÷
(tan (5 7) −
cos (1 5)) =

$\therefore \dfrac{\cos 24.7° \times \sin 35.1°}{\tan 57° - \cos 15°} = \mathbf{0.910}$ (to 3 s.f.)

10.2 Applications of trigonometric ratios to find unknown sides of right-angled triangles

Practise Now 2

1. **(a)** $\sin \angle BAC = \dfrac{\text{opp}}{\text{hyp}} = \dfrac{BC}{AB}$

$\sin 30° = \dfrac{a}{11}$

$\therefore a = 11 \sin 30°$

$= \mathbf{5.5}$

(b) $\sin \angle QPR = \dfrac{\text{opp}}{\text{hyp}} = \dfrac{QR}{PQ}$

$\sin 67° = \dfrac{8.7}{r}$

$\therefore r = \dfrac{8.7}{\sin 67°}$

$= \mathbf{9.45}$ (to 3 s.f.)

2. **(a)** $\cos \angle YXZ = \dfrac{\text{adj}}{\text{hyp}} = \dfrac{XZ}{XY}$

$\cos 58° = \dfrac{4.9}{z}$

$\therefore z = \dfrac{4.9}{\cos 58°}$

$= \mathbf{9.25}$ (to 3 s.f.)

(b) $\cos \angle ABC = \dfrac{\text{adj}}{\text{hyp}} = \dfrac{BC}{AB}$

$\cos 47° = \dfrac{a}{18}$

$\therefore a = 18 \cos 47°$

$= \mathbf{12.3}$ (to 3 s.f.)

3. **(a)** $\tan \angle YXZ = \dfrac{\text{opp}}{\text{adj}} = \dfrac{YZ}{XZ}$

$\tan 35° = \dfrac{12}{y}$

$\therefore y = \dfrac{12}{\tan 35°}$

$= \mathbf{17.1}$ (to 3 s.f.)

(b) $\tan \angle QPR = \dfrac{\text{opp}}{\text{adj}} = \dfrac{QR}{PR}$

$\tan 53° = \dfrac{p}{20}$

$\therefore p = 20 \tan 53°$

$= \mathbf{26.5}$ (to 3 s.f.)

Practise Now 3

1. **(i)** $\cos \angle ABC = \dfrac{\text{adj}}{\text{hyp}} = \dfrac{BC}{AB}$

$\cos 27° = \dfrac{BC}{23}$

$\therefore BC = 23 \cos 27°$

$= \mathbf{20.5\ m}$ (to 3 s.f.)

(ii) $\sin \angle ABC = \dfrac{\text{opp}}{\text{hyp}} = \dfrac{AC}{AB}$

$\sin 27° = \dfrac{AC}{23}$

$\therefore AC = 23 \sin 27°$

$= \mathbf{10.4\ m}$ (to 3 s.f.)

2. **(i)** In $\triangle PQR$,

$\cos \angle QPR = \dfrac{\text{adj}}{\text{hyp}} = \dfrac{PQ}{PR}$

$\cos 46° = \dfrac{PQ}{9.2}$

$\therefore PQ = 9.2 \cos 46°$

$= \mathbf{6.39\ m}$ (to 3 s.f.)

(ii) In $\triangle PQR$,

$\sin \angle QPR = \dfrac{\text{opp}}{\text{hyp}} = \dfrac{QR}{PR}$

$\sin 46° = \dfrac{QR}{9.2}$

$\therefore QR = 9.2 \sin 46°$

$= \mathbf{6.62\ m}$ (to 3 s.f.)

(iii) In $\triangle PRS$,

$\sin \angle PSR = \dfrac{\text{opp}}{\text{hyp}} = \dfrac{PR}{PS}$

$\sin 48° = \dfrac{9.2}{PS}$

$\therefore PS = \dfrac{9.2}{\sin 48°}$

$= \mathbf{12.4\ m}$ (to 3 s.f.)

(iv) In $\triangle PRS$,

$\tan \angle PSR = \dfrac{\text{opp}}{\text{adj}} = \dfrac{PR}{RS}$

$\tan 48° = \dfrac{9.2}{RS}$

$\therefore RS = \dfrac{9.2}{\tan 48°}$

$= \mathbf{8.28\ m}$ (to 3 s.f.)

(v) Perimeter of quadrilateral $PQRS$

$= PQ + QR + RS + PS$

$= 9.2 \cos 46° + 9.2 \sin 46° + \dfrac{9.2}{\tan 48°} + \dfrac{9.2}{\sin 48°}$

$= \mathbf{33.7\ m}$ (to 3 s.f.)

(vi) Area of quadrilateral $PQRS$

= area of $\triangle PQR$ + area of $\triangle PRS$

$= \dfrac{1}{2} \times PQ \times QR + \dfrac{1}{2} \times PR \times RS$

$= \dfrac{1}{2} \times 9.2 \cos 46° \times 9.2 \sin 46° + \dfrac{1}{2} \times 9.2 \times \dfrac{9.2}{\tan 48°}$

$= \mathbf{59.3\ m^2}$

Exercise 10B

1. **(a)** $\sin 67° = \frac{\text{opp}}{\text{hyp}}$

$= \frac{a}{15}$

$\therefore a = 15 \sin 67°$

$= \mathbf{13.8}$ (to 3 s.f.)

(b) $\sin 15° = \frac{\text{opp}}{\text{hyp}}$

$= \frac{9.7}{b}$

$\therefore b = \frac{9.7}{\sin 15°}$

$= \mathbf{37.5}$ (to 3 s.f.)

2. **(a)** $\cos 36° = \frac{\text{adj}}{\text{hyp}}$

$= \frac{a}{13.5}$

$\therefore a = 13.5 \cos 36°$

$= \mathbf{10.9}$ (to 3 s.f.)

(b) $\cos 61° = \frac{\text{adj}}{\text{hyp}}$

$= \frac{17}{b}$

$\therefore b = \frac{17}{\cos 61°}$

$= \mathbf{35.1}$ (to 3 s.f.)

3. **(a)** $\tan 28° = \frac{\text{opp}}{\text{adj}}$

$= \frac{a}{14}$

$\therefore a = 14 \tan 28°$

$= \mathbf{7.44}$ (to 3 s.f.)

(b) $\tan 62.5° = \frac{\text{opp}}{\text{adj}}$

$= \frac{13}{b}$

$\therefore b = \frac{13}{\tan 62.5°}$

$= \mathbf{6.77}$ (to 3 s.f.)

4. **(a)** $\sin 34° = \frac{\text{opp}}{\text{hyp}}$

$= \frac{a}{12}$

$\therefore a = 12 \sin 34°$

$= \mathbf{6.71}$ (to 3 s.f.)

$\cos 34° = \frac{\text{adj}}{\text{hyp}}$

$= \frac{b}{12}$

$\therefore b = 12 \cos 34°$

$= \mathbf{9.95}$ (to 3 s.f.)

(b) $\cos 43° = \frac{\text{adj}}{\text{hyp}}$

$= \frac{c}{16}$

$\therefore c = 16 \cos 43°$

$= \mathbf{11.7}$ (to 3 s.f.)

$\sin 43° = \frac{\text{opp}}{\text{hyp}}$

$= \frac{d}{16}$

$\therefore d = 16 \sin 43°$

$= \mathbf{10.9}$ (to 3 s.f.)

(c) $\tan 44.2° = \frac{\text{opp}}{\text{adj}}$

$= \frac{e}{7}$

$\therefore e = 7 \tan 44.2°$

$= \mathbf{6.81}$ (to 3 s.f.)

$\cos 44.2° = \frac{\text{adj}}{\text{hyp}}$

$= \frac{7}{f}$

$\therefore f = \frac{7}{\cos 44.2°}$

$= \mathbf{9.76}$ (to 3 s.f.)

(d) $\tan 21.5° = \frac{\text{opp}}{\text{adj}}$

$= \frac{8.9}{g}$

$\therefore g = \frac{8.9}{\tan 21.5°}$

$= \mathbf{22.6}$ (to 3 s.f.)

$\sin 21.5° = \frac{\text{opp}}{\text{hyp}}$

$= \frac{8.9}{h}$

$\therefore h = \frac{8.9}{\sin 21.5°}$

$= \mathbf{24.3}$ (to 3 s.f.)

5. **(i)** In $\triangle ABH$,

$\sin \angle ABH = \frac{\text{opp}}{\text{hyp}} = \frac{AH}{AB}$

$\sin 56° = \frac{AH}{8.9}$

$\therefore AH = 8.9 \sin 56°$

$= \mathbf{7.38\ m}$ (to 3 s.f.)

(ii) $\angle ACH = 180° - 90° - 56°$ ($\angle$ sum of $\triangle ABC$)

$= 34°$

In $\triangle ACH$,

$\tan \angle ACH = \frac{\text{opp}}{\text{adj}} = \frac{AH}{HC}$

$\tan 34° = \frac{8.9 \sin 56°}{HC}$

$\therefore HC = \frac{8.9 \sin 56°}{\tan 34°}$

$= \mathbf{10.9\ m}$ (to 3 s.f.)

6. **(i)** $\angle QTS = 180° - 90°$ (adj. $\angle$s on a str. line)

$= 90°$

In $\triangle QST$,

$\sin \angle QST = \frac{\text{opp}}{\text{hyp}} = \frac{TQ}{QS}$

$\sin 60° = \frac{TQ}{25}$

$\therefore TQ = 25 \sin 60°$

$= \mathbf{21.7\ cm}$ (to 3 s.f.)

(ii) $\angle SQT = 180° - 90° - 60°$ ($\angle$ sum of $\triangle QST$)

$= 30°$

$\angle PQT = 180° - 90° - 30°$ (adj. $\angle$s on a str. line)

$= 60°$

In $\triangle PQT$,

$\tan \angle PQT = \frac{\text{opp}}{\text{adj}} = \frac{PT}{TQ}$

$\tan 60° = \frac{PT}{25 \sin 60°}$

$\therefore PT = \tan 60° \times 25 \sin 60°$

$= \mathbf{37.5\ cm}$

(iii) In $\triangle PQT$,

$$\cos \angle PQT = \frac{\text{adj}}{\text{hyp}} = \frac{TQ}{PQ}$$

$$\cos 60° = \frac{25\sin 60°}{PQ}$$

$$PQ = \frac{25\sin 60°}{\cos 60°}$$

$$= 43.301 \text{ cm (to 5 s.f.)}$$

In $\triangle QRS$,

$$\tan \angle RSQ = \frac{\text{opp}}{\text{adj}} = \frac{QR}{QS}$$

$$\tan 45° = \frac{QR}{25}$$

$$QR = 25 \tan 45°$$

$$= 25 \text{ cm}$$

$\therefore PR = PQ + QR$

$= 43.301 + 25$

$=$ **68.3 cm** (to 3 s.f.)

7. **(i)** In $\triangle VWX$,

$$\cos \angle VWX = \frac{\text{adj}}{\text{hyp}} = \frac{WX}{VW}$$

$$\cos 63° = \frac{WX}{154}$$

$$WX = 154 \cos 63°$$

$$= 69.915 \text{ m (to 5 s.f.)}$$

$$\sin \angle VWX = \frac{\text{opp}}{\text{hyp}} = \frac{VX}{VW}$$

$$\sin 63° = \frac{VX}{154}$$

$$VX = 154 \sin 63°$$

$$= 137.22 \text{ m (to 5 s.f.)}$$

$\angle VYX = 180° - 90°$ (adj. $\angle$s on a str. line)

$= 90°$

In $\triangle VYX$, $\angle VYX = 90°$.

Using Pythagoras' Theorem,

$$VX^2 = VY^2 + XY^2$$

$$137.22^2 = 88^2 + XY^2$$

$$XY^2 = 137.22^2 - 88^2$$

$$XY = \sqrt{137.22^2 - 88^2} \quad (\text{since } XY > 0)$$

$$= 105.29 \text{ m (to 5 s.f.)}$$

In $\triangle VYZ$,

$$\tan \angle VZY = \frac{\text{opp}}{\text{adj}} = \frac{VY}{YZ}$$

$$\tan 46° = \frac{88}{YZ}$$

$$YZ = \frac{88}{\tan 46°}$$

$$= 84.981 \text{ m (to 5 s.f.)}$$

$$\sin \angle VZY = \frac{\text{opp}}{\text{hyp}} = \frac{VY}{VZ}$$

$$\sin 46° = \frac{88}{VZ}$$

$$VZ = \frac{88}{\sin 46°}$$

$$= 122.33 \text{ m}$$

Perimeter of the figure

$= VW + WX + XY + YZ + VZ$

$= 154 + 69.915 + 105.29 + 84.981 + 122.33$

$=$ **537 m** (to 3 s.f.)

(ii) Area of the figure

$=$ area of $\triangle VWX$ + area of $\triangle VXY$ + area of $\triangle VYZ$

$$= \frac{1}{2} \times WX \times VX + \frac{1}{2} \times XY \times VY + \frac{1}{2} \times YZ \times VY$$

$$= \frac{1}{2} \times 69.915 \times 137.22 + \frac{1}{2} \times 105.29 \times 88 + \frac{1}{2} \times 84.981 \times 88$$

$=$ **13 200 m²** (to 3 s.f.)

8. Since y is inversely proportional to $(\tan x)^2$,

$$y_1(\tan x_1)^2 = y_2(\tan x_2)^2$$

$$y\,[\tan (2 \times 30)°]^2 = 2\,(\tan 30°)^2$$

$$y = \frac{2(\tan 30°)^2}{[\tan(2 \times 30°)]^2}$$

$$= \mathbf{\frac{2}{9}}$$

10.3 Applications of trigonometric ratios to find unknown angles in right-angled triangles

Practise Now 4A

(a) Sequence of calculator keys:

sin⁻¹

SHIFT sin (0 . 7 8) =

$\therefore \angle A = \sin^{-1}(0.78)$

$=$ **51.3°** (to 1 d.p.)

(b) Sequence of calculator keys:

cos⁻¹

SHIFT cos (0 . 3 5) =

$\therefore \angle B = \cos^{-1}(0.35)$

$=$ **69.5°** (to 1 d.p.)

(c) Sequence of calculator keys:

tan⁻¹

SHIFT tan (1 . 2 3) =

$\therefore \angle C = \tan^{-1}(1.23)$

$=$ **50.9°** (to 1 d.p.)

Practise Now 4B

(a) $\sin x° = \frac{11}{19}$

$$x° = \sin^{-1}\left(\frac{11}{19}\right)$$

$$= 35.4° \text{ (to 1 d.p.)}$$

$\therefore$ $\boldsymbol{x = 35.4}$

(b) $\cos y° = \frac{9.1}{21}$

$$y° = \cos^{-1}\left(\frac{9.1}{21}\right)$$

$$= 64.3° \text{ (to 1 d.p.)}$$

$\therefore$ $\boldsymbol{y = 64.3}$

(c) $\tan z° = \frac{29.2}{18.8}$

$$z° = \tan^{-1}\left(\frac{29.2}{18.8}\right)$$

$$= 57.2° \text{ (to 1 d.p.)}$$

$\therefore$ $\boldsymbol{z = 57.2}$

Practise Now 5

1. **(i)** In $\triangle BCK$,

$$\tan \angle BKC = \frac{BC}{CK}$$
$$\tan 42^\circ = \frac{BC}{7.6}$$
$$BC = 7.6 \tan 42^\circ$$
$$= 6.8431 \text{ m (to 5 s.f.)}$$

In $\triangle ABC$,

$$\sin \angle BAC = \frac{6.8431}{17.3}$$
$$\therefore \angle BAC = \sin^{-1}\left(\frac{6.8431}{17.3}\right)$$
$$= \mathbf{23.3^\circ} \text{ (to 1 d.p.)}$$

(ii) In $\triangle ABC$, $\angle ACB = 90^\circ$.
Using Pythagoras' Theorem,

$$AB^2 = BC^2 + AC^2$$
$$= BC^2 + (CK + KA)^2$$
$$17.3^2 = 6.8431^2 + (7.6 + KA)^2$$
$$(7.6 + KA)^2 = 17.3^2 - 6.8431^2$$
$$7.6 + KA = \sqrt{17.3^2 - 6.8431^2} \quad (\text{since } 7.6 + KA > 0)$$
$$\therefore KA = \sqrt{17.3^2 - 6.8431^2} - 7.6$$
$$= \mathbf{8.29 \text{ m}} \text{ (to 3 s.f.)}$$

2. **(i)** In $\triangle PQS$,

$$\tan \angle PQS = \frac{3}{4}$$
$$\therefore \angle PQS = \tan^{-1}\left(\frac{3}{4}\right)$$
$$= \mathbf{36.9^\circ} \text{ (to 1 d.p.)}$$

(ii) In $\triangle PQS$, $\angle QPS = 90^\circ$.
Using Pythagoras' Theorem,

$$QS^2 = PQ^2 + PS^2$$
$$= 4^2 + 3^2$$
$$= 16 + 9$$
$$= 25$$
$$QS = \sqrt{25} \quad (\text{since } QS > 0)$$
$$= 5 \text{ cm}$$

$\angle QSR = \angle PQS$ (alt. $\angle$s, PQ // SR)

From part **(i)**, $\tan \angle PQS = \frac{3}{4}$

In $\triangle QRS$,

$$\tan \angle QSR = \frac{QR}{5}$$
$$\frac{3}{4} = \frac{QR}{5}$$
$$\therefore QR = \frac{3}{4} \times 5$$
$$= \mathbf{3.75 \text{ cm}}$$

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$\tan 30^\circ = \frac{1}{\sqrt{3}}$

$\tan 45^\circ = 1$

$\tan 60^\circ = \sqrt{3}$

With reference to the values of tan 30°, tan 45° and tan 60°, as the value of the acute angle y increases, the value of $\tan y$ increases.

Since $a < b$, $\frac{a}{b} < 1$. Hence, for the acute angle x, $\tan x < \tan 45^\circ$ and hence x is **less than 45°**.

Exercise 10C

1. **(a)** Sequence of calculator keys:

$\sin^{-1}$
SHIFT | sin | (| 0 | . | 5 | 2 | 7 |) | =

$$\therefore \angle A = \sin^{-1}(0.527)$$
$$= \mathbf{31.8^\circ} \text{ (to 1 d.p.)}$$

(b) Sequence of calculator keys:

$\cos^{-1}$
SHIFT | cos | (| 0 | . | 7 | 2 | 5 |) | =

$$\therefore \angle B = \cos^{-1}(0.725)$$
$$= \mathbf{43.5^\circ} \text{ (to 1 d.p.)}$$

(c) Sequence of calculator keys:

$\tan^{-1}$
SHIFT | tan | (| 2 | . | 5 | 6 |) | =

$$\therefore \angle C = \tan^{-1}(2.56)$$
$$= \mathbf{68.7^\circ} \text{ (to 1 d.p.)}$$

2. **(a)**
$$\sin a^\circ = \frac{12}{26}$$
$$a^\circ = \sin^{-1}\left(\frac{12}{26}\right)$$
$$= 27.5^\circ \text{ (to 1 d.p.)}$$
$$\therefore \boldsymbol{a} = \mathbf{27.5}$$

(b)
$$\cos b^\circ = \frac{10}{17}$$
$$b^\circ = \cos^{-1}\left(\frac{10}{17}\right)$$
$$= 54.0^\circ \text{ (to 1 d.p.)}$$
$$\therefore \boldsymbol{b} = \mathbf{54.0}$$

(c)
$$\tan c^\circ = \frac{27}{11}$$
$$c^\circ = \tan^{-1}\left(\frac{27}{11}\right)$$
$$= 67.8^\circ \text{ (to 1 d.p.)}$$
$$\therefore \boldsymbol{c} = \mathbf{67.8}$$

(d)
$$\cos d^\circ = \frac{17.6}{20}$$
$$d^\circ = \cos^{-1}\left(\frac{17.6}{20}\right)$$
$$= 28.4^\circ \text{ (to 1 d.p.)}$$
$$\therefore \boldsymbol{d} = \mathbf{28.4}$$

(e)
$$\sin e^\circ = \frac{15}{22.7}$$
$$e^\circ = \sin^{-1}\left(\frac{15}{22.7}\right)$$
$$= 41.4^\circ \text{ (to 1 d.p.)}$$
$$\therefore \boldsymbol{e} = \mathbf{41.4}$$

(f)
$$\tan f^\circ = \frac{12.5}{14}$$
$$f^\circ = \tan^{-1}\left(\frac{12.5}{14}\right)$$
$$= 41.8^\circ \text{ (to 1 d.p.)}$$
$$\therefore \boldsymbol{f} = \mathbf{41.8}$$

(g)
$$\tan g^\circ = \frac{14.7}{12.9}$$
$$g^\circ = \tan^{-1}\left(\frac{14.7}{12.9}\right)$$
$$= 48.7^\circ \text{ (to 1 d.p.)}$$
$$\therefore \boldsymbol{g} = \mathbf{48.7}$$

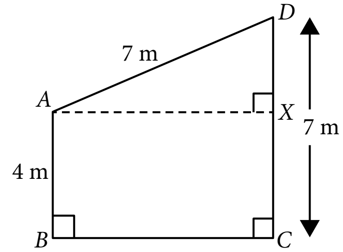
(h) $\cos h° = \frac{15.8}{21.2}$

$$h° = \cos^{-1}\left(\frac{15.8}{21.2}\right)$$
$$= 41.8° \text{ (to 1 d.p.)}$$
$$\therefore \boldsymbol{h = 41.8}$$

(i) $\sin i° = \frac{32.75}{41.62}$

$$i° = \sin^{-1}\left(\frac{32.75}{41.62}\right)$$
$$= 51.9° \text{ (to 1 d.p.)}$$
$$\therefore \boldsymbol{i = 51.9}$$

3. (i)



In $\triangle ADX$,

$$\cos \angle ADX = \frac{DX}{AD}$$
$$= \frac{7-4}{7}$$
$$= \frac{3}{7}$$
$$\angle ADX = \cos^{-1}\left(\frac{3}{7}\right)$$
$$= 64.6° \text{ (to 1 d.p.)}$$
$$\angle ADC = \angle ADX$$
$$= \mathbf{64.6°}$$

(ii) In $\triangle ADX$, $\angle AXD = 90°$.

Using Pythagoras' Theorem,

$$AD^2 = AX^2 + DX^2$$
$$7^2 = AX^2 + 3^2$$
$$AX^2 = 7^2 - 3^2$$
$$= 49 - 9$$
$$= 40$$
$$AX = \sqrt{40} \text{ (since } AX > 0)$$
$$= 6.32 \text{ m (to 3 s.f.)}$$

Since $ABCX$ is a rectangle,

$$AB = AX$$
$$= \mathbf{6.32\ m}$$

4. (i) In $\triangle HMN$,

$$\cos \angle HMN = \frac{MN}{HM}$$
$$\cos 38° = \frac{MN}{9.2}$$
$$MN = 9.2 \cos 38°$$
$$= 7.2497 \text{ cm (to 5 s.f.)}$$

In $\triangle LMN$,

$$\sin \angle MLN = \frac{7.2497}{15.5}$$
$$\angle MLN = \sin^{-1}\left(\frac{7.2497}{15.5}\right)$$
$$= \mathbf{27.9}° \text{ (to 1 d.p.)}$$

(ii) In $\triangle HMN$,

$$\sin \angle HMN = \frac{HN}{HM}$$
$$\sin 38° = \frac{HN}{9.2}$$
$$HN = 9.2 \sin 38°$$
$$= 5.6641 \text{ cm (to 5 s.f.)}$$

In $\triangle LMN$, $\angle LNM = 90°$.

Using Pythagoras' Theorem,

$$ML^2 = MN^2 + LN^2$$
$$= MN^2 + (HN + HL)^2$$
$$15.5^2 = 7.2497^2 + (5.6641 + HL)^2$$
$$(5.6641 + HL)^2 = 15.5^2 - 7.2497^2$$
$$5.6641 + HL = \sqrt{15.5^2 - 7.2497^2} \text{ (since } 5.6641 + HL > 0)$$
$$HL = \sqrt{15.5^2 - 7.2497^2} - 5.6641$$
$$= \mathbf{8.04\ cm} \text{ (to 3 s.f.)}$$

5. (i) In $\triangle PQR$,

$$\cos \angle QPR = \frac{7.6}{17.4}$$
$$\angle QPR = \cos^{-1}\left(\frac{7.6}{17.4}\right)$$
$$= 64.101° \text{ (to 3 d.p.)}$$
$$\angle PKR = 180° - 137° \text{ (adj. } \angle\text{s on a str. line)}$$
$$= 43°$$
$$\angle KPR = 180° - 90° - 43° \text{ } (\angle \text{ sum of } \triangle KPR)$$
$$= 47°$$
$$\angle QPK = \angle QPR - \angle KPR$$
$$= 64.101° - 47°$$
$$= \mathbf{17.1}° \text{ (to 1 d.p.)}$$

(ii) In $\triangle PKR$,

$$\tan \angle PKR = \frac{PR}{KR}$$
$$\tan 43° = \frac{7.6}{KR}$$
$$KR = \frac{7.6}{\tan 43°}$$
$$= 8.1500 \text{ m (to 5 s.f.)}$$

In $\triangle PQR$, $\angle PRQ = 90°$.

Using Pythagoras' Theorem,

$$PQ^2 = PR^2 + QR^2$$
$$= PR^2 + (QK + KR)^2$$
$$17.4^2 = 7.6^2 + (QK + 8.1500)^2$$
$$(QK + 8.1500)^2 = 17.4^2 - 7.6^2$$
$$QK + 8.1500 = \sqrt{17.4^2 - 7.6^2} \text{ (since } QK + 8.1500 > 0)$$
$$\therefore QK = \sqrt{17.4^2 - 7.6^2} - 8.1500$$
$$= \mathbf{7.50\ m} \text{ (to 3 s.f.)}$$

6. $TU = TH + HU$

$$11 = 1.2HU + HU$$
$$= 2.2HU$$
$$HU = 5 \text{ cm}$$

Area of $\triangle STH = 21 \text{ cm}^2$

$$\frac{1}{2} \times TH \times HS = 21$$
$$HS = \frac{42}{TH}$$
$$= \frac{42}{1.2HU}$$
$$= \frac{42}{1.2(5)}$$
$$= 7 \text{ cm}$$

In $\triangle HSU$,

$$\tan \angle HSU = \frac{HU}{HS} = \frac{5}{7}$$

$$\angle HSU = \tan^{-1}\left(\frac{5}{7}\right) = 35.538° \text{ (to 3 d.p.)}$$

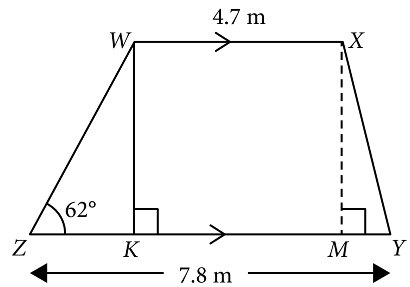
In $\triangle HST$,

$$\tan \angle HST = \frac{TH}{HS} = \frac{6}{7}$$

$$\angle HST = \tan^{-1}\left(\frac{6}{7}\right) = 40.601° \text{ (to 3 d.p.)}$$

$$\angle TSU = \angle HSU + \angle HST = 35.538° + 40.601° = \mathbf{76.1}° \text{ (to 1 d.p.)}$$

7. **(i)**



$$\frac{WK}{ZY} = \frac{6}{13}$$

$$\frac{WK}{7.8} = \frac{6}{13}$$

$$WK = \frac{6}{13} \times 7.8 = 3.6 \text{ m}$$

In $\triangle KWZ$,

$$\tan \angle WZK = \frac{WK}{ZK}$$

$$\tan 62° = \frac{3.6}{ZK}$$

$$ZK = \frac{3.6}{\tan 62°} = 1.9142 \text{ m (to 5 s.f.)}$$

Since $KMXW$ is a rectangle, $MX = WK = 3.6$ m and $WX = KM = 4.7$ m.

$$MY = ZY - ZK - KM = 7.8 - 1.9142 - 4.7 = 1.1858 \text{ m}$$

In $\triangle MXY$,

$$\tan \angle XYM = \frac{3.6}{1.1858}$$

$$\angle XYM = \tan^{-1}\left(\frac{3.6}{1.1858}\right) = 71.8° \text{ (to 1 d.p.)}$$

$$\angle XYZ = \angle XYM = \mathbf{71.8}°$$

(ii) In $\triangle KWZ$,

$$\sin \angle WZK = \frac{WK}{WZ}$$

$$\sin 62° = \frac{3.6}{WZ}$$

$$WZ = \frac{3.6}{\sin 62°} = 4.0773 \text{ m (to 5 s.f.)}$$

In $\triangle MXY$, $\angle XMY = 90°$.

Using Pythagoras' Theorem,

$$XY^2 = MX^2 + MY^2 = 3.6^2 + 1.1858^2$$

$$XY = \sqrt{3.6^2 + 1.1858^2} \text{ (since } XY > 0)$$

$$= 3.7903 \text{ m (to 5 s.f.)}$$

$\therefore$ perimeter of trapezium $WXYZ = WX + XY + YZ + WZ$

$$= 4.7 + 3.7903 + 7.8 + 4.0773$$

$$= \mathbf{20.4 \text{ m}} \text{ (to 3 s.f.)}$$

8. In $\triangle ACH$,

$$\tan \angle ACH = \frac{AH}{HC}$$

$$AH = HC \tan \angle ACH \quad \text{— (1)}$$

In $\triangle ABH$,

$$\tan \angle ABH = \frac{AH}{BH}$$

$$\tan 35° = \frac{AH}{BH}$$

$$AH = BH \tan 35° \quad \text{— (2)}$$

Substituting (1) into (2):

$$HC \tan \angle ACH = BH \tan 35°$$

$$2BH \tan \angle ACH = BH \tan 35°$$

$$2BH \tan \angle ACH - BH \tan 35° = 0$$

$$BH(2 \tan \angle ACH - \tan 35°) = 0$$

$2 \tan \angle ACH - \tan 35° = 0$ or $BH = 0$ (reject since $BH > 0$)

$$2 \tan \angle ACH = \tan 35°$$

$$\tan \angle ACH = \frac{\tan 35°}{2}$$

$$\angle ACH = \tan^{-1}\left(\frac{\tan 35°}{2}\right) = 19.3° \text{ (to 1 d.p.)}$$

Since $\angle ACH = \angle ACB$, $\angle ACB = \mathbf{19.3}°$.

10.4 Applications of trigonometric ratios in real-world contexts

Practise Now 6

1. $\tan 54° = \frac{PQ}{2.8}$

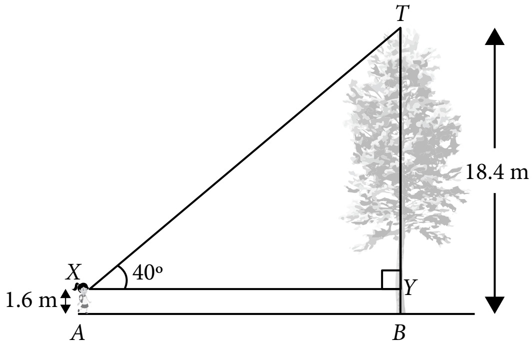
$$PQ = 2.8 \tan 54° = 3.85 \text{ m (to 3 s.f.)}$$

$\therefore$ the height of the flagpole is **3.85 m**.

2. $\tan 37° = \frac{40}{FA}$

$$\therefore FA = \frac{40}{\tan 37°} = \mathbf{53.1 \text{ m}} \text{ (to 3 s.f.)}$$

3.



In $\triangle TXY$,

$$\tan \angle TXY = \frac{TY}{XY}$$

$$\tan 40° = \frac{TB - XA}{AB}$$

$$= \frac{18.4 - 1.6}{AB}$$

$$= \frac{16.8}{AB}$$

$$AB = \frac{16.8}{\tan 40°}$$

$$= 20.0 \text{ m (to 3 s.f.)}$$

$\therefore$ the distance AB between Li Ting and the foot of the tree is **20.0 m**.

Practise Now 7

$$\tan \angle TAB = \frac{13}{24}$$

$$\therefore \angle TAB = \tan^{-1}\left(\frac{13}{24}\right)$$

$$= \mathbf{28.4°} \text{ (to 1 d.p.)}$$

Practise Now 8

$BC = WH = 3.2$ m

$\angle BWC = \angle WCH$ (alt. $\angle$s, BW // CH)

$= 32.4°$

In $\triangle BCW$,

$$\tan \angle BWC = \frac{BC}{BW}$$

$$\tan 32.4° = \frac{3.2}{BW}$$

$$BW = \frac{3.2}{\tan 32.4°}$$

$$= 5.0424 \text{ m (to 5 s.f.)}$$

In $\triangle ABW$,

$$\tan \angle AWB = \frac{AB}{BW}$$

$$\tan 49.6° = \frac{AB}{5.0424}$$

$$AB = 5.0424 \tan 49.6°$$

$$= 5.9248 \text{ m (to 5 s.f.)}$$

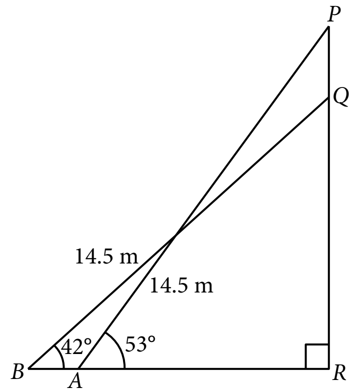
$AC = AB + BC$

$= 5.9248 + 3.2$

$= 9.12$ m (to 3 s.f.)

$\therefore$ the height of the mast is **9.12 m**.

Practise Now 9



In the figure, P and Q represent the top and lower edge of the signboard respectively. AP and BQ represent the pole when it is at two different positions.

In $\triangle APR$,

$$\sin 53° = \frac{PR}{14.5}$$

$$PR = 14.5 \sin 53°$$

$$= 11.580 \text{ m (to 5 s.f.)}$$

In $\triangle BQR$,

$$\sin 42° = \frac{QR}{14.5}$$

$$QR = 14.5 \sin 42°$$

$$= 9.7024 \text{ m (to 5 s.f.)}$$

$PQ = PR - QR$

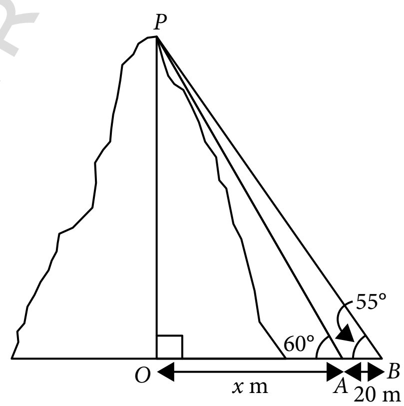
$= 11.580 - 9.7024$

$= 1.88$ m (to 3 s.f.)

$\therefore$ the height of the signboard is **1.88 m**.

Introductory Problem Revisited

A possible way to determine the height of Bukit Timah Hill without climbing the hill is to use trigonometric ratios. An example of the method is illustrated below:



Let the height of the hill, denoted by OP, be h metres.

A surveyor starts from point A, where OA is perpendicular to OP, and measures $\angle OAP$ using a theodolite. Suppose $\angle OAP = 60°$. The surveyor then moves towards point B, taken to be 20 metres away from A in this example, and measures $\angle OBP$ using a theodolite. Suppose $\angle OBP = 55°$.

In $\triangle AOP$,

$\tan 60° = \dfrac{OP}{x}$

$OP = x \tan 60°$ — (1)

In $\triangle BOP$,

$\tan 55° = \dfrac{OP}{x+20}$

$OP = (x + 20) \tan 55°$ — (2)

Substitute (1) into (2):

$$x \tan 60° = (x + 20) \tan 55°$$
$$x \tan 60° - x \tan 55° = 20 \tan 55°$$
$$x(\tan 60° - \tan 55°) = 20 \tan 55°$$
$$x = \frac{20 \tan 55°}{\tan 60° - \tan 55°}$$
$$= 93.987 \text{ (to 5 s.f.)}$$

Substitute $x = 93.987$ into (1):

$OP = 93.987 \tan 60°$

$= 163$ m (to 3 s.f.)

$\therefore$ ***h* = 163**

Some assumptions made when calculating the height of the hill are:

- The peak of the hill is a visible point from ground level.
- Points O, A and B are level, i.e. OAB is a straight line.

Exercise 10D

1. $\tan 32° = \dfrac{TB}{34}$

$TB = 34 \tan 32°$

$= 21.2$ m (to 3 s.f.)

$\therefore$ the height of the Christmas tree is **21.2 m**.

2. $\tan 27° = \dfrac{7.7}{AQ}$

$\therefore AQ = \dfrac{7.7}{\tan 27°}$

$=$ **15.1 m** (to 3 s.f.)

3. $\cos 53° = \dfrac{AB}{120}$

$\therefore AB = 120 \cos 53°$

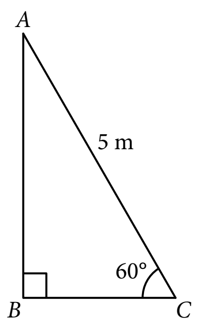
$=$ **72.2 m** (to 3 s.f.)

4. $\tan \angle PRQ = \dfrac{82}{62}$

$\therefore \angle PRQ = \tan^{-1}\left(\dfrac{82}{62}\right)$

$=$ **52.9°** (to 1 d.p.)

5. **(i)**



Let the length of the ladder be AC and the height of the nail above the ground be AB.

$\sin 60° = \dfrac{AB}{5}$

$AB = 5 \sin 60°$

$= 4.33$ m (to 3 s.f.)

$\therefore$ the nail is **4.33 m** above the ground.

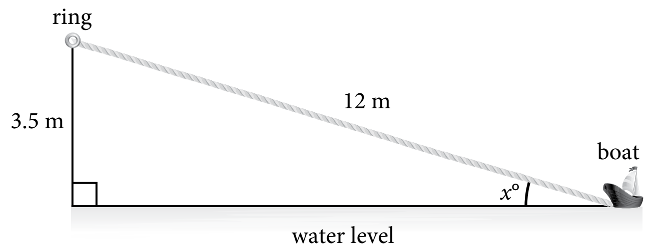
(ii) $\cos 60° = \dfrac{BC}{5}$

$BC = 5 \cos 60°$

$= 2.5$ m

$\therefore$ the foot of the ladder is **2.5 m** from the base of the wall.

6.



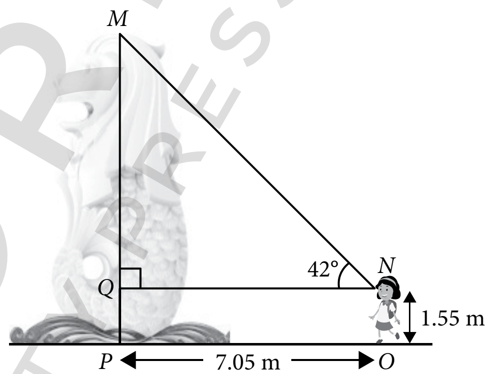
Let the angle the rope makes with the water be $x°$.

$\sin x° = \dfrac{3.5}{12}$

$\therefore x° = \sin^{-1}\left(\dfrac{3.5}{12}\right)$

$=$ **17.0°** (to 1 d.p.)

7.



In $\triangle MNQ$,

$\tan \angle MNQ = \dfrac{MQ}{NQ}$

$\tan 42° = \dfrac{MQ}{OP}$

$= \dfrac{MQ}{7.05}$

$MQ = 7.05 \tan 42°$

$= 6.3478$ m (to 5 s.f.)

$\therefore$ height of the statue, $MP = MQ + QP$

$= MQ + NO$

$= 6.3478 + 1.55$

$=$ **7.90 m** (to 3 s.f.)

8. $QR = WH = 8$ m

$\angle QWR = \angle WRH$ (alt. $\angle$s, QW // RH)

$= 24.3°$

In $\triangle QRW$,

$\tan \angle QWR = \dfrac{QR}{QW}$

$\tan 24.3° = \dfrac{8}{QW}$

$QW = \dfrac{8}{\tan 24.3°}$

$= 17.718$ m (to 5 s.f.)

In $\triangle PQW$,

$\tan \angle PWQ = \dfrac{PQ}{QW}$

$\tan 35.4° = \dfrac{PQ}{17.718}$

$PQ = 17.718 \tan 35.4°$

$= 12.592$ m (to 5 s.f.)

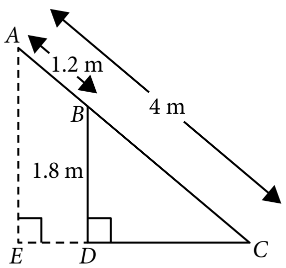
$PR = PQ + QR$

$= 12.592 + 8$

$= 20.6$ m (to 3 s.f.)

$\therefore$ the height of the flagpole is **20.6 m**.

9.



Let the length of the plank be represented by the length of AC and the height of the wall be represented by the length of BD.

$BC = AC - AB$

$= 4 - 1.2$

$= 2.8$ m

In $\triangle BCD$,

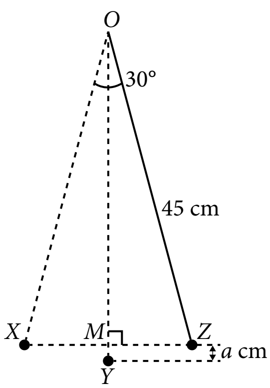
$\cos \angle CBD = \dfrac{1.8}{2.8}$

$\angle CBD = \cos^{-1}\left(\dfrac{1.8}{2.8}\right)$

$= 50.0°$ (to 1 d.p.)

$\therefore$ the angle the plank makes with the wall is **50.0°**.

10.



As the length of the pendulum is constant, $OX = OY = OZ = 45$ cm. X and Z are at the extreme ends of the oscillation and Y is the equilibrium position. Hence XZ is a straight line and OMY is the perpendicular bisector of XZ. OMY is also the angle bisector of $\angle XOZ$, i.e. $\angle MOZ = 15°$.

In $\triangle MOZ$,

$\cos \angle MOZ = \dfrac{OM}{OZ}$

$\cos 15° = \dfrac{OM}{45}$

$OM = 45 \cos 15°$

$= 43.467$ cm (to 5 s.f.)

$OY = OM + MY$

$= OM + a$

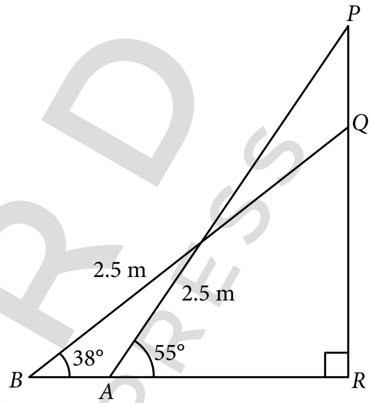
$45 = 43.467 + a$

$a = 45 - 43.467$

$= 1.53$ (to 3 s.f.)

$\therefore$ the height in which the pendulum bob rises above Y is **1.53 cm**.

11.



In the figure, P and Q represent the top and lower edge of the window respectively. AP and BQ represent the ladder when it is at two different positions.

In $\triangle APR$,

$\sin 55° = \dfrac{PR}{2.5}$

$PR = 2.5 \sin 55°$

$= 2.0479$ m (to 5 s.f.)

In $\triangle BQR$,

$\sin 38° = \dfrac{QR}{2.5}$

$QR = 2.5 \sin 38°$

$= 1.5392$ m (to 5 s.f.)

$PQ = PR - QR$

$= 2.0479 - 1.5392$

$= 0.509$ m (to 3 s.f.)

$= 50.9$ cm

$\therefore$ the height of the window is **50.9 cm**.

12. (i) $HD = AB = 18$ m

In $\triangle ACH$,

$\sin \angle CAH = \dfrac{CH}{AC}$

$\sin 35° = \dfrac{CH}{36}$

$CH = 36 \sin 35°$

$= 20.649$ m

$CD = CH + HD$

$= 20.649 + 18$

$=$ **38.6 m** (to 3 s.f.)

(ii) In $\triangle ACH$,

$\cos \angle CAH = \dfrac{AH}{AC}$

$\cos 35° = \dfrac{AH}{36}$

$AH = 36 \cos 35°$

$= 29.489$ m

$AF = AH - FH$

$= AH - GD$

$= 29.489 - 20$

$= 9.489$ m

$\angle AFE = \angle BGF$ (corr. $\angle$s, AF // BG)

$= 90°$

In $\triangle AEF$, $\angle AFE = 90°$.

Using Pythagoras' Theorem,

$AE^2 = AF^2 + EF^2$

$36^2 = 9.489^2 + EF^2$

$EF^2 = 36^2 - 9.489^2$

$EF = \sqrt{36^2 - 9.489^2}$ (since $EF > 0$)

$=$ **34.7 m** (to 3 s.f.)

(iii) In $\triangle AEF$,

$\cos \angle EAF = \dfrac{9.489}{36}$

$\angle EAF = \cos^{-1}\left(\dfrac{9.489}{36}\right)$

$= 74.717°$ (to 3 d.p.)

$\angle EAC = \angle EAF - \angle CAH$

$= 74.717° - 35°$

$= 39.7°$ (to 1 d.p.)

$\therefore$ the jib has rotated **39.7°** in the anticlockwise direction about A.

13. Let the length of QB be x m.

In $\triangle BQT$,

$\tan \angle BQT = \dfrac{TB}{x}$

$\tan 32° = \dfrac{TB}{x}$

$TB = x \tan 32°$ — (1)

In $\triangle BPT$,

$\tan \angle BPT = \dfrac{TB}{x+10}$

$\tan 23° = \dfrac{TB}{x+10}$

$TB = (x + 10) \tan 23°$ — (2)

Substituting (1) into (2):

$x \tan 32° = (x + 10) \tan 23°$

$x \tan 32° - x \tan 23° = 10 \tan 23°$

$x (\tan 32° - \tan 23°) = 10 \tan 23°$

$x = \dfrac{10 \tan 23°}{\tan 32° - \tan 23°}$

$= 21.182$ (to 5 s.f.)

Substitute $x = 21.182$ into (1):

$TB = 21.182 \tan 32°$

$= 13.2$ m (to 3 s.f.)

$\therefore$ the height of the tree is **13.2 m**.

Chapter 11 Volume, Surface Area and Symmetry of Prisms and Cylinders

TEACHING NOTES

Suggested Approach

Students have learnt the conversion of unit area and perimeter and area of plane figures in Book 1. This chapter will be dealing with the conversion of unit volumes and the volume and surface area of solids. Students are reminded that for a two-dimensional shape, the perimeter is the measure of its boundary and the area is the measure of the space enclosed within the boundary. A transition is then made to surface area and volume for a three-dimensional object. To assist in the students' understanding, teachers should continually remind students to be aware of the linkages between both topics, as well as introducing real-life applications that can reinforce learning.

Section 11.1 Conversion of units

Teachers should recap the unit conversion of lengths and areas, proceed to introduce of volume by stating actual applications (see Class Discussion: Measurements in daily lives), and then stating the different units associated with volume (e.g. m, cm^3 and m^3).

Students should recognise how the number of dimensions and the unit representation for lengths, areas and volumes are related (e.g. cm, cm^2 and cm^3). Students should recall calculations such as 1 cm3 = 1 cm × 1 cm × 1 cm and solve problems involving conversion of unit volumes.

Section 11.2 Three-dimensional solids

The investigation on page 324 is aimed at helping the students visualise a three-dimensional solid from a drawing. It is crucial that students realise that angles and lengths of a solid appear distorted when drawn in perspective in a two-dimensional diagram. Some solids such as cuboid boxes can be brought to class to aid in visualisation of these solids.

In the subsection on nets of solids, teachers should first define and explain that nets are basically flattened figures that can be folded to its threedimensional solids. Teachers should show the nets of the various solids. Students are encouraged to make their own nets and form the different three-dimensional solids. They should also be able to visualise the solids from different viewpoints.

Section 11.3 Volume and surface area of cubes and cuboids

Teachers can state that the volume of an object refers to the space it occupies, so the greater the volume, the more space the object occupies.

Students should be informed and know that the volume of cubes and cuboids is the product of its three sides (base × height = (length × breadth) × height).

The formulas for the total surface area of cubes and cuboids can be explored and discovered by students (see Class Discussion: Surface area of cubes and cuboids). It is important for the students observe that the total surface area is the total area of all its faces.

Section 11.4 Volume and surface area of prisms

Teachers can introduce prisms to the students by stacking a few cubes to form a prism and show them how a prism looks like. Students should know terms like lateral faces and cross-sections, and learn that prisms are solids with uniform polygonal cross-sections. Teachers can ask the students to name some real-life examples of prisms and use this opportunity to get them to explain why certain objects are not prisms so that they can get a better understanding about prisms.

Observant students should realise that cuboids are prisms. Teachers can highlight to the students that prisms do not necessarily have square bases and challenge students to think of bases of other possible shapes (see Fig. 11.5 on page 333).

Teachers should illustrate and derive the formulas for the volume and total surface area. Students need to understand the definitions of volume and total surface area rather than memorise the formulas.

Section 11.5 Volume and surface area of cylinders

Similar to the last section, teachers can introduce cylinders by stacking coins or showing students real-life examples of cylindrical objects. Only right circular cylinders are covered in this syllabus.

Some students may think that cylinders are also prisms since both have uniform cross-sections. Teachers need to impress upon students that this is not the case even though cylinders and prisms share similarities (see Investigation: Comparison between a cylinder and a prism).

Teachers should also cover the formulas for the volume and total surface area of cylinders. Again, students need to understand the definitions of volume and total surface area rather than memorise formulas.

Section 11.6 Volume and surface area of composite solids

Teachers should go through Worked Example 10 closely with students. Other than assessing their understanding, teachers can inform students to be aware of any sides that should be omitted in finding total surface areas.

Section 11.7 Symmetry in right prisms and cylinders

Teachers can recap and build upon students' knowledge of line and rotational symmetry of two-dimensional shapes to explain plane and rotational symmetry of three-dimensional solids. In Worked Example 11, teachers can draw the students' attention to the relationship between line symmetry of the base of a prism to its plane symmetry. Teachers can use the mirror to help struggling students to visualise the symmetry of a solid about the plane of symmetry.

Students should then be able to reason that a cylinder, and later a cone, has infinite number of plane and rotational symmetries due to the infinite number of line and rotational symmetries of its circular base (see Investigation: Plane and rotational symmetries of a cylinder). The Thinking Time on page 351 can be used to consolidate the meaning of symmetry about a plane and an axis.

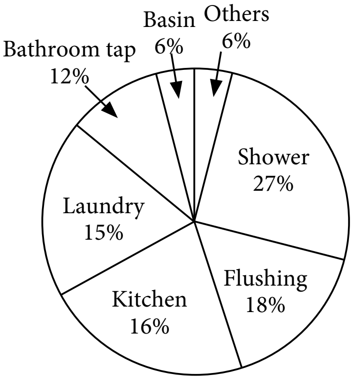
Introductory Problem

The solutions to this problem can be found in Introductory Problem Revisited (after Practise Now 3).

11.1 Conversion of units

Class Discussion (Measurements in daily lives)

1. **(i)** Household water consumption in 2016/2017:



Source: https://www.pub.gov.sg/savewater/athome

The activity which requires the greatest amount of water is shower.

(ii) –

Some measures:

- Take shorter showers.
- Turn off the shower tap while soaping.
- Use a tumbler when brushing your teeth.
- Do not thaw food under running water. Let it defrost overnight inside the refrigerator instead.
- Wash vegetables and dishes in a sink or container filled with water.
- Install thimbles or water saving devices at taps with high flow rate.
- Turn off taps tightly to ensure they do not drip.
- Do not leave the tap running when not in use.

2. **(i)** The volume of one teaspoon of liquid is **5 ml**.

(ii) This corresponds to **2 litres** of water.

Practise Now 1

(a) **(i)**

$$\begin{aligned} 1 \text{ m} &= 100 \text{ cm} \\ (1 \text{ m})^3 &= (100 \text{ cm})^3 \\ &= 100 \text{ cm} \times 100 \text{ cm} \times 100 \text{ cm} \\ 1 \text{ m}^3 &= 1\,000\,000 \text{ cm}^3 \\ 10 \text{ m}^3 &= 10 \times 1\,000\,000 \\ &= \mathbf{10\,000\,000 \text{ cm}^3} \end{aligned}$$

(ii) $10\,000\,000 \text{ cm}^3 = \mathbf{10\,000\,000 \text{ ml}}$

(b) **(i)** $1\,000\,000 \text{ cm}^3 = 1 \text{ m}^3$

$$\begin{aligned} 1 \text{ cm}^3 &= \frac{1}{1\,000\,000} \text{ m}^3 \\ 165\,000 \text{ cm}^3 &= \frac{165\,000}{1\,000\,000} \text{ m}^3 \\ &= \mathbf{0.165 \text{ m}^3} \end{aligned}$$

(ii)

$$\begin{aligned} 1 \text{ cm} &= 1 \text{ ml} \\ &= \frac{1}{1000}\, l \\ 165\,000 \text{ cm}^3 &= \frac{165\,000}{1000}\, l \\ &= \mathbf{165}\ \boldsymbol{l} \end{aligned}$$

11.2 Three-dimensional solids

Investigation (Visualising 3D solids)

Part 2:

1. **(i)** and **(ii)**

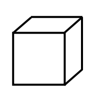
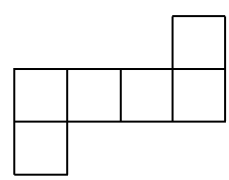
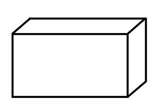
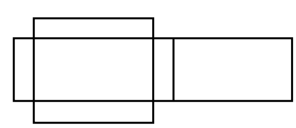
	AB	*AD*	*CD*	*BC*	*DX*	*CY*	*XY*	*BZ*	*YZ*
AB		⊥	//	⊥	—	—	//	⊥	—
AD	⊥		⊥	//	⊥	—	—	—	//
CD	//	⊥		⊥	⊥	⊥	//	—	—
BC	⊥	//	⊥		—	⊥	—	⊥	//
DX	—	⊥	⊥	—		//	⊥	//	—
CY	—	—	⊥	⊥	//		⊥	//	⊥
XY	//	—	//	—	⊥	⊥		—	⊥
BZ	⊥	—	—	⊥	//	//	—		⊥
YZ	—	//	—	//	—	⊥	⊥	⊥	

(iii) ***ABCD*; *DCXY*; *BCYZ***

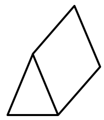
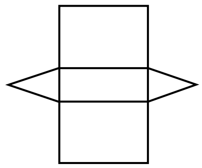
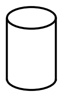
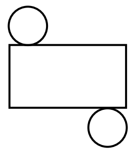
2. **No**. The plane *ADYZ* is a rectangle, because $AD \neq DY$.

Investigation (Cubes, cuboids, prisms and cylinders)

Part 2:

Name	Figure	Net
Cube		
Cuboid		

Part 3:

Name	Figure	Net
Triangular prism		
Cylinder		

11.3 Volume and surface area of cubes and cuboids

Practise Now 2

1. **(i)** Volume of the cuboid = $l \times 18 \times 38 = 35\ 568$

$l = \dfrac{35\ 568}{18 \times 38}$

$= 52$

$\therefore$ the length of the cuboid is **52 cm**.

(ii) Volume of each small cube = $2 \times 2 \times 2 = 8$ cm^3

Number of cubes to be obtained

$= \dfrac{35\ 568}{8}$

= 4446

(iii) Maximum number of cubes obtained

$= (52 \div 2) \times (18 \div 2) \times (38 \div 2)$

$= 26 \times 9 \times 19$

$= 4446$

$\therefore$ **Yes**, Ali is correct.

2. Volume of the open rectangular tank

$= 55 \times 35 \times 36$

$= 69\ 300$ cm^3

Volume of water in the open rectangular tank initally

$= \dfrac{1}{2} \times 69\ 300$

$= 34\ 650$ cm^3

Total volume of water in the open rectangular tank after 7700 cm^3 of water are added to it

$= 34\ 650 + 7700$

$= 42\ 350$ cm^3

Let the depth of water in the tank be d cm.

$55 \times 35 \times d = 42\ 350$

$1925d = 42\ 350$

$d = 22$

Depth of water = **22 cm**

Practise Now 3

External volume = $(180 + 30 + 30) \times (120 + 30 + 30) \times (80 + 30)$

$= 4\ 752\ 000$ cm^3

Internal volume = $180 \times 120 \times 80$

$= 1\ 728\ 000$ cm^3

$\therefore$ Volume of concrete used = $4\ 752\ 000 - 1\ 728\ 000$

$= 3\ 024\ 000$ cm^3

= 3.024 m^3

Class Discussion (Surface area of cubes and cuboids):

1. A cube has **6** surfaces. Each surface is in the shape of a **square**.
The area of each face is **equal**.
$\therefore$ the total surface area of a cube is **$6l^2$**.
A cuboid has **6** surfaces. Each surface is in the shape of a **rectangle**.
$\therefore$ the total surface area of a cube is **$2(b \times l + b \times h + l \times h)$**.
2. The total surface of the object is equal to the total area of all the faces of the net.

Practise Now 4

1. **(i)** Volume of cuboid = $8 \times 5 \times 10$

= 400 cm^3

(ii) Surface area of the cuboid = $2(8 \times 5 + 8 \times 10 + 5 \times 10)$

= 340 cm^2

2. **(i)** Volume of water in the tank

$= 16 \times 9 \times 8$

$= 1152$ cm^3

$= 1152$ ml

$= \dfrac{1152}{1000}\ l$

= 1.152 l

(ii) Surface area of the tank that is in contact with the water

$= (16 \times 9) + 2(16 \times 8 + 9 \times 8)$

= 544 cm^2

3. Let the length of the cube be l cm.

$l \times l \times l = 27$ cm^3

$l^3 = 27$

$l = \sqrt[3]{27}$

$l = 3$

Total area of the faces that will be coated with paint

$= 6(3 \times 3)$

= 54 cm^2

Exercise 11A

1. **(a)** **(i)** 1 m^3 = 1 000 000 cm^3

4 m^3 = $4 \times 1\ 000\ 000$ cm^3

= 4 000 000 cm^3

(ii) 1 m^3 = 1 000 000 cm^3

0.5 m^3 = $0.5 \times 1\ 000\ 000$ cm^3

= 500 000 cm^3

(b) **(i)** 1 000 000 cm^3 = 1 m^3

250 000 cm^3 = $\dfrac{250\ 000}{1\ 000\ 000}$ m^3

= 0.25 m^3

(ii) 1 000 000 cm^3 = 1 m^3

67 800 cm^3 = $\dfrac{67\ 800}{1\ 000\ 000}$ m^3

= 0.0678 m^3

2. (a) (i) $1\text{ m}^3 = 1\,000\,000\text{ cm}^3$

$0.84\text{ m}^3 = 0.84 \times 1\,000\,000\text{ cm}^3$

$= \mathbf{840\,000\text{ cm}^3}$

(ii) $1\text{ cm}^3 = 1\text{ ml}$

$840\,000\text{ cm}^3 = \mathbf{840\,000\text{ ml}}$

(b) (i) $1\,000\,000\text{ cm}^3 = 1\text{ m}^3$

$2560\text{ cm}^3 = \frac{2560}{1\,000\,000}\text{ m}^3$

$= \mathbf{0.002\,56\text{ m}^3}$

(ii) $1\text{ cm}^3 = 1\text{ ml}$

$2560\text{ cm}^3 = 2560\text{ ml}$

$= \frac{2560}{1000}\ l$

$= \mathbf{2.56\ l}$

3. (i) Volume of the cuboid $= 28 \times b \times 15 = 6720\text{ cm}^3$

$\therefore b = \frac{6720}{28 \times 15}$

$= 16$

$\therefore$ breadth = **16 cm**

(ii) Volume of each small cube $= 4 \times 4 \times 4 = 64\text{ cm}^3$

Number of cubes to be obtained

$= \frac{6720}{64}$

$= \mathbf{105}$

4. (a) (i) Volume of the cuboid $= 6 \times 8 \times 10$

$= \mathbf{480\text{ cm}^3}$

(ii) Surface area of the cuboid $= 2(6 \times 8 + 8 \times 10 + 6 \times 10)$

$= \mathbf{376\text{ cm}^2}$

(b) (i) Volume of the cuboid $= 7 \times 12 \times 5$

$= \mathbf{420\text{ cm}^3}$

(ii) Surface area of the cuboid $= 2(7 \times 12 + 5 \times 7 + 5 \times 12)$

$= \mathbf{358\text{ cm}^2}$

(c) (i) Volume of the cuboid $= 120 \times 10 \times 96$

$= \mathbf{115\,200\text{ mm}^3}$

(ii) Surface area of the cuboid

$= 2(120 \times 10 + 96 \times 10 + 120 \times 96)$

$= \mathbf{27\,360\text{ mm}^2}$

(d) (i) Volume of the cuboid $= 1\frac{1}{2} \times \frac{1}{2} \times 10$

$= \mathbf{7\frac{1}{2}\text{ cm}^3}$

(ii) Surface area of the cuboid

$= 2\left(1\frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times 10 + 1\frac{1}{2} \times 10\right)$

$= \mathbf{41\frac{1}{2}\text{ cm}^2}$

(e) (i) Volume of the cuboid $= 1\frac{2}{5} \times \frac{3}{8} \times \frac{5}{8}$

$= \mathbf{\frac{21}{64}\text{ cm}^3}$

(ii) Surface area of the cuboid

$= 2\left(1\frac{2}{5} \times \frac{3}{8} + \frac{3}{8} \times \frac{5}{8} + 1\frac{2}{5} \times \frac{5}{8}\right)$

$= \mathbf{3\frac{43}{160}\text{ cm}^2}$

(f) (i) Volume of the cuboid $= 3.9 \times 0.7 \times 1.5$

$= \mathbf{4.095\text{ cm}^3}$

(ii) Surface area of the cuboid

$= 2(3.9 \times 0.7 + 0.7 \times 1.5 + 3.9 \times 1.5)$

$= \mathbf{19.26\text{ cm}^2}$

5.

	Length	Breadth	Height	Volume	Total surface area
(a)	24 mm	18 mm	5 mm	**2160 mm^3**	**1284 mm^2**
(b)	5 cm	3 cm	**8 cm**	120 cm^3	**158 cm^2**
(c)	**2.5 cm**	6 cm	3.5 cm	52.5 cm^3	**89.5 cm^2**
(d)	12 m	**8 m**	6 m	576 m^3	**432 m^2**

(a) Volume $= 24 \times 18 \times 5$

$= \mathbf{2160\text{ mm}^3}$

Surface area $= 2(24 \times 18 + 24 \times 5 + 18 \times 5)$

$= \mathbf{1284\text{ mm}^2}$

(b) Let the height of the cuboid be h cm.

Volume $= 5 \times 3 \times h = 120\text{ cm}^3$

$\therefore h = \frac{120}{5 \times 3} = \mathbf{8\text{ cm}}$

Surface area $= 2(5 \times 3 + 5 \times 8 + 3 \times 8)$

$= \mathbf{158\text{ cm}^2}$

(c) Let the length of the cuboid be l cm.

Volume $= l \times 6 \times 3.5 = 52.5\text{ cm}^3$

$\therefore l = \frac{52.5}{6 \times 3.5} = \mathbf{2.5\text{ cm}}$

Surface area $= 2(2.5 \times 6 + 6 \times 3.5 + 2.5 \times 3.5)$

$= \mathbf{89.5\text{ cm}^2}$

(d) Let the breadth of the cuboid be b m.

Volume $= 12 \times b \times 6 = 576\text{ m}^3$

$\therefore b = \frac{576}{12 \times 6} = \mathbf{8\text{ m}}$

Surface area $= 2(12 \times 8 + 6 \times 8 + 12 \times 6)$

$= \mathbf{432\text{ m}^2}$

6. (a) Volume of block of cheese $= 0.24\text{ m} \times 0.19\text{ m} \times 0.15\text{ m}$

$= 0.006\,84\text{ m}^3$

Length of each side of cube $= \sqrt[3]{0.006\,84}$

$= \mathbf{0.190\text{ m}}$ (to 3 s.f.)

(b) $24 \div 2 = 12$

$19 \div 2 = 9\text{ R }1$

$15 \div 2 = 7\text{ R }1$

Number of cubes that can be cut $= 12 \times 9 \times 7$

$= \mathbf{756}$

7. Volume of the open rectangular tank

$= 4 \times 2 \times 4.8$

$= 38.4\text{ m}^3$

Volume of water in the open rectangular tank initally

$= \frac{3}{4} \times 38.4$

$= 28.8\text{ m}^3$

$4000\ l = 4000 \times 1000\text{ ml}$

$= 4\,000\,000\text{ ml}$

$= 4\,000\,000\text{ cm}^3$

$= \frac{4\,000\,000}{1\,000\,000}\text{ m}^3$

$= 4\text{ m}^3$

Total volume of water in the open rectangular tank after 4000 litres of water are added to it
$= 28.8 + 4$
$= 32.8 \text{ m}^3$
Let the depth of water in the tank be d m.
$4 \times 2 \times d = 32.8$
$8d = 32.8$
$d = 4.1$
$\therefore$ depth $=$ **4.1 m**

8. External volume $= (3.2 + 0.2 + 0.2) \times (2.2 + 0.2 + 0.2) \times (1.5 + 0.2)$
$= 3.6 \times 2.6 \times 1.7$
$= 15.912 \text{ m}^3$
Internal volume $= 3.2 \times 2.2 \times 1.5$
$= 10.56 \text{ m}^3$
Volume of wood used $= 15.912 - 10.56$
$= \mathbf{5.352 \text{ m}^3}$

9. External volume $= 15 \times 10 \times 45$
$= 6750 \text{ cm}^3$
Internal volume $= 3 \times 2 \times 45$
$= 270 \text{ cm}^3$
Volume of the hollow glass structure $= 6750 - 270$
$= \mathbf{6480 \text{ cm}^3}$

10. **(i)** Number of trips required to fill the entire quarry
$= \dfrac{2.85 \times 1\,000\,000}{6.25}$
$= \mathbf{456\,000}$
(ii) Cost to fill the quarry
$= 456\,000 \times \$55$
$= \mathbf{\$25\,080\,000}$
(iii) 3 hectares $= 30\,000 \text{ m}^2$
Cost to fill 1 m^2 of the land
$= \$\dfrac{25\,080\,000}{30\,000}$
$= \mathbf{\$836}$

11. **(i)** Volume of water in the tank
$= 0.2 \times 0.15 \times 0.16$
$= 0.0048 \text{ m}^3$
$= 0.0048 \times 1\,000\,000 \text{ cm}^3$
$= 4800 \text{ cm}^3$
$= 4800 \text{ ml}$
$= \dfrac{4800}{1000}\ l$
$= \mathbf{4.8\ l}$
(ii) Surface area of the tank that is in contact with the water
$= (0.2 \times 0.15) + 2(0.2 \times 0.16 + 0.15 \times 0.16)$
$= \mathbf{0.142 \text{ m}^2}$

12. **(i)** Volume of water in the tank
$= 80 \times 40 \times 35$
$= 112\,000 \text{ cm}^3$
$= 112\,000 \text{ ml}$
$= \dfrac{112\,000}{1000}\ l$
$= \mathbf{112\ l}$
(ii) Surface area of the tank that is in contact with the water
$= (80 \times 40) + 2(80 \times 35 + 40 \times 35)$
$= 11\,600 \text{ cm}^2$
$= \dfrac{11\,600}{10\,000} \text{ m}^2$
$= \mathbf{1.16 \text{ m}^2}$

13. Let the length of the cube be l cm.
$l^3 = 1331$
$l = \sqrt[3]{1331}$
$= 11$
Total area of faces coated with paint $= 6(11 \times 11)$
$= \mathbf{726 \text{ cm}^2}$

14. Let the length of the cube be l cm.
$6l^2 = 433.5$
$l^2 = 72.25$
$l = \sqrt{72.25}$
$= 8.5$
Volume of cube $= (8.5)^3$
$= \mathbf{614.125 \text{ cm}^3}$

15. Volume of wood used to make this trough
$= (185 \times 45 \times 28) - [(185 - 2.5 - 2.5) \times (45 - 2.5 - 2.5) \times (28 - 2.5)]$
$= (185 \times 45 \times 28) - (180 \times 40 \times 25.5)$
$= 233\,100 - 183\,600$
$= 49\,500 \text{ cm}^3$
$= \dfrac{49\,500}{1\,000\,000} \text{ m}^3$
$= \mathbf{0.0495 \text{ m}^3}$

16. In one minute, the water will flow through $22 \times 60 = 1320$ cm along the drain.
Amount of water that will flow through in one minute
$= 30 \times 3.5 \times 1320$
$= 138\,600 \text{ cm}^3$
$= 138\,600 \text{ ml}$
$= \dfrac{138\,600}{1000}\ l$
$= \mathbf{138.6\ l}$

17. **(i)** Let the height of the cuboid be h cm.
Surface area of the cuboid $= 2(12 \times 9 + 12 \times h + 9 \times h)$
$= 426 \text{ cm}^2$
$2(108 + 12h + 9h) = 426$
$2(108 + 21h) = 426$
$108 + 21h = 213$
$21h = 213 - 108$
$21h = 105$
$h = 5$
$\therefore$ height of cuboid $=$ **5 cm**
(ii) Volume of the cuboid
$= 12 \times 9 \times 5$
$= \mathbf{540 \text{ cm}^3}$
(iii) Volume of each smaller cuboid
$= 5 \times 3 \times 2$
$= 30 \text{ cm}^3$
Number of smaller cuboids that can be obtained
$= \dfrac{540}{30}$
$= \mathbf{18}$
(iv) **No**, Bernard is not correct.
$12 \div 3 = 4$
$9 \div 3 = 3$
$5 \div 3 = 1 \text{ R } 2$
Maximum number of cubes that can be obtained
$= 4 \times 3 \times 1$
$= 12$
$\dfrac{\text{Volume of cuboid}}{27} = \dfrac{540}{27} = 20 \neq 12$

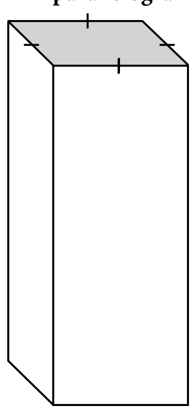
18. **(i)** Floor area of Room A $= 26 \times 1$
$= \mathbf{26\ m^2}$
Volume of Room A $= 26 \times 1 \times 3$
$= \mathbf{78\ m^3}$
Floor area of Room B $= 5 \times 5$
$= \mathbf{25\ m^2}$
Volume of Room B $= 5 \times 5 \times 3$
$= \mathbf{75\ m^3}$
Floor area of Room C $= 6 \times 6$
$= \mathbf{36\ m^2}$
Volume of Room C $= 6 \times 6 \times 1.8$
$= \mathbf{64.8\ m^3}$

(ii) **No**. If both rooms, A and B, have the same height, then we will use the floor area as the gauge. If the rooms do not have the same height, then we will use the volume to decide.

11.4 Volume and surface area of prisms

Thinking Time (Page 334)

1. (i) The shape of all the lateral faces of a right prism is a **rectangle**.
   (ii) The shape of all the lateral faces of an oblique prism is a **parallelogram**.
2.



Investigation (Volume of prism)

1.

Prism	Area of cross-sectional base (base area)	Distance between cross-sectional bases (height)	Volume of prism (by counting unit cubes)
A	**7 units²**	**2 units**	**14 units³**
B	**5 units²**	**3 units**	**15 units³**
C	**12 units²**	**2 units**	**24 units³**

2. Volume of prism = base area × height
3. Volume of cuboid with dimensions 3 m by 4 m by 2 m
$= 3 \times 4 \times 2$
$= 24\ m^3$
Since two identical right-angled triangular prisms can form a cuboid,
volume of one such prism $= \dfrac{24}{2}$
$= 12\ m^3$.

Using the formula for volume of prism given,
volume of prism $= \left(\dfrac{1}{2} \times 3 \times 4\right) \times 2$
$= 12\ m^3$.
The volumes found are equal for both methods. Hence, the formula is true.

Practise Now 5

1. Base area = area of square
$= 4 \times 4$
$= 16\ m^2$
Volume of the prism = base area × height
$= 16 \times 10$
$= \mathbf{160\ m^3}$
2. Base area = area of triangle
$= \dfrac{1}{2} \times 5.6 \times x$
$= 2.8x\ cm^2$
Volume of the prism = base area × height $= 2.8x \times 12 = 151.2$
$33.6x = 151.2$
$x = \mathbf{4.5}$

Practise Now 6

(i) Volume of the prism = base area × height
$= \left[\left(\dfrac{1}{2} \times 3 \times 4\right) + (6 \times 5)\right] \times 4.5$
$= 36 \times 4.5$
$= \mathbf{162\ cm^3}$

(ii) Total surface area of the prism
= perimeter of the base × height + 2 × base area
$= (3 + 4 + 6 + 5 + 6) \times 4.5 + 2 \times 36$
$= \mathbf{180\ cm^2}$

Exercise 11B

1. (a) Volume of the prism = base area × height
$= \left[\dfrac{1}{2} \times (75 + 59) \times 46\right] \times 120$
$= 3028 \times 120$
$= \mathbf{369\ 840\ cm^3}$

(b) Volume of the prism
= base area × height
$= \left[\dfrac{1}{2} \times (16 + 28) \times (18 - 7) + 7 \times 28\right] \times 38$
$= 438 \times 38$
$= \mathbf{16\ 644\ cm^3}$

(c) Volume of the prism = base area × height
$= [9 \times 5 + 9 \times 3 + (16 - 8) \times (9 - 6)] \times 10$
$= 96 \times 10$
$= \mathbf{960\ cm^3}$

(d) Volume of the prism = base area × height
$= \left[\dfrac{1}{2} \times (14 + 18) \times 6\right] \times 12$
$= 96 \times 12$
$= \mathbf{1152\ cm^3}$

(e) Volume of the prism = base area × height
$= \left[\dfrac{1}{2} \times 6 \times 8 + 13 \times 10\right] \times 5$
$= 154 \times 5$
$= \mathbf{770\ cm^3}$

(f) Volume of the prism = base area × height

$= \left[\frac{1}{2}\times 18\times(12-3)+3\times 18\right]\times 35$

$= 135 \times 35$

$= \mathbf{4725\ cm^3}$

2.

	AB	BC	BC	Area of $\triangle ABC$	Volume of prism
(a)	3 cm	4 cm	7 cm	**6 cm²**	**42 cm³**
(b)	9 cm	**14 cm**	11 cm	63 cm²	**693 cm³**
(c)	**32 cm**	15 cm	300 cm	**240 cm²**	72 000 cm³
(d)	24.6 cm	7.8 cm	**400 cm**	**95.94 cm²**	38 376 cm³

(a) Area of $\triangle ABC$

$= \frac{1}{2} \times 4 \times 3$

$= \mathbf{6\ cm^2}$

Volume of prism

$= 6 \times 7$

$= \mathbf{42\ cm^3}$

(b) Area of $\triangle ABC$

$= \frac{1}{2} \times BC \times 9 = 63$

$4.5BC = 63$

$BC = \mathbf{14\ cm}$

Volume of prism

$= 63 \times 11$

$= \mathbf{693\ cm^3}$

(c) Volume of prism

$=$ Area of $\triangle ABC \times 300 = 72\ 000$

Area of $\triangle ABC = \mathbf{240\ cm^2}$

Area of $\triangle ABC$

$= \frac{1}{2} \times 15 \times AB = 240$

$7.5AB = 240$

$AB = \mathbf{32\ cm}$

(d) Area of $\triangle ABC$

$= \frac{1}{2} \times 7.8 \times 24.6$

$= \mathbf{95.94\ cm^2}$

Volume of prism

$= 95.94 \times CD = 38\ 376$

$CD = \mathbf{400\ cm}$

3. Air space in the hall = Volume of the prism

= base area × height

$= \left[\frac{1}{2}\times 42\times(38-23)+42\times 23\right]\times 80$

$= 1281 \times 80$

$= \mathbf{102\ 480\ m^3}$

4. **(a)** **(i)** Volume of the prism = base area × height

$= \left(\frac{1}{2}\times 6\times 4\right)\times 15$

$= 12 \times 15$

$= \mathbf{180\ cm^3}$

(ii) Total surface area of the prism

= perimeter of the base × height + 2 × base area

$= (5 + 5 + 6) \times 15 + 2 \times 12$

$= \mathbf{264\ cm^2}$

(b) **(i)** Volume of the prism = base area × height

$= [2 \times 7 + (5 - 2) \times (7 - 6)] \times 9$

$= 17 \times 9$

$= \mathbf{153\ cm^3}$

(ii) Total surface area of the prism

= perimeter of the base × height + 2 × base area

$= (7 + 2 + 6 + 3 + 1 + 5) \times 9 + 2 \times 17$

$= \mathbf{250\ cm^2}$

5. **(i)** Volume of water in the pool when it is full

= Volume of the prism

= base area × height

$= \left[\frac{1}{2}\times(1.2+2)\times 50\right]\times 25$

$= 80 \times 25$

$= \mathbf{2000\ m^3}$

(ii) Area of the pool which is in contact with the water

$= [(1.2 + 50 + 2 + 50.01) \times 25 + 2 \times 80] - (25 \times 50)$

$= \mathbf{1490.25\ m^2}$

11.5 Volume and surface area of cylinders

Thinking Time (Page 340)

No, a cylinder is not a prism. The base of a prism must be a polygon, but the base of a cylinder is a circle.

Investigation (Comparison between cylinder and prism)

1. The polygon will start to look like a **circle**.
2. The prism will start to look like a **cylinder**.

Practise Now 7

1. Base radius = 18 ÷ 2 = 9 cm

Height of the cylinder = 2.5 × 9 = 22.5 cm

Volume of the cylinder $= \pi r^2 h$

$= \pi(9)^2(22.5)$

$= \mathbf{5730\ cm^3}$ (to 3 s.f.)

2. Base radius = 12 ÷ 2 = 6 cm

Volume of the cylinder $= \pi(6)^2 h = 1000$

$h = \frac{1000}{\pi(6)^2}$

$h = \mathbf{8.84\ cm}$ (to 3 s.f.)

Practise Now 8

1. Since petrol is discharged through the pipe at a rate of 2.45 m/s, i.e. 245 cm/s, in 1 second, the volume of petrol discharged is the volume of petrol that fills the pipe to a length of 245 cm.

In 1 second, volume of petrol discharged

= volume of pipe of length 245 cm

$= \pi r^2 h$

$= \pi(0.6)^2(245)$

$= 88.2\pi\ cm^3$

In 3 minutes, volume of petrol discharged

$= 88.2\pi \times 3 \times 60$

$= 49\ 900\ cm^3$

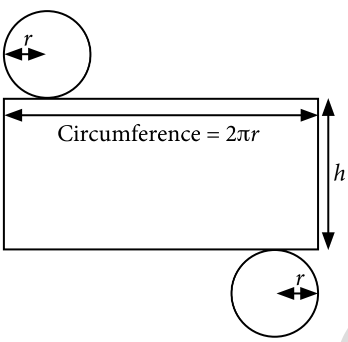
$= \mathbf{49.9}$ ***l*** (to 3 s.f.)

2. Base radius = 0.036 ÷ 2 = 0.018 m
Since water is discharged through the pipe at a rate of 1.6 m/s, i.e. in 1 second, the volume of water discharged is the volume of water that fills the pipe to a length of 1.6 m.
In 1 second, volume of water discharged
= volume of pipe of length 1.6 m
$= \pi r^2 h$
$= \pi(0.018)^2(1.6)$
$= 0.000\ 518\ 4\pi$ cm^3
Volume of the cylindrical tank
$= \pi r^2 h$
$= \pi(3.4)^2(1.4)$
$= 16.184\pi$ cm^3
Time required to fill the tank
$= \dfrac{16.184\pi}{0.000\ 518\ 4\pi}$
$= 3\ 1219\dfrac{11}{81}$ s
= **520 min** (to the nearest minute)

Journal Writing (Page 342)

For example, Pearl Bank Apartments is an iconic horse-shoe shaped building in Singapore, which was built in 1976. The shape of the building resembles part of a cylinder and its unique design was meant to represent community. The presence of a common exposed corridor served to facilitate interaction among neighbours. The design of the new building, which will replace it by 2023, draws inspiration from the old C-shaped design.

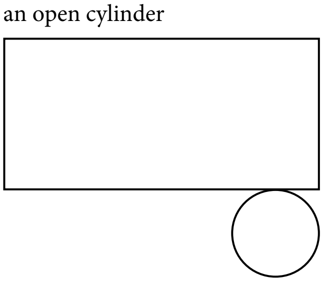
Thinking Time (Page 343)



Total outer surface of a closed cylinder
$= \pi r^2 + 2\pi rh + \pi r^2$
$= 2\pi r^2 + 2\pi rh$

Class Discussion (Total surface area of other types of cylinders)

(a) an open cylinder



Total outer surface of an open cylinder $= 2\pi rh + \pi r^2$

(b) a pipe of negligible thinkness

Total outer surface of a pipe of negligible thickness $= 2\pi rh$

Practise Now 9

1. **(i)** Total surface area of the can
$= 2\pi r^2 + 2\pi rh$
$= 2\pi(3.5)^2 + 2\pi(3.5)(10)$
$= 24.5\pi + 70\pi$
$= 94.5\pi$
= **297 cm²** (to 3 s.f.)

(ii) Area of the can that is painted
$= \pi r^2 + 2\pi rh$
$= \pi(3.5)^2 + 2\pi(3.5)(10)$
$= 12.25\pi + 70\pi$
$= 82.25\pi$
Ratio of the area of the can that is painted, to the total surface area found in **(i)**.
$= 82.25\pi : 94.5\pi$
$= 82.25 : 94.5$
$= \mathbf{47 : 54}$

2. **(i)** Area of the cross section of the pipe
$= \pi(2.5)^2 - \pi(2.1)^2$
$= 6.25\pi - 4.41\pi$
$= 1.84\pi$ cm^2 (shown)

(ii) Internal curved surface area of the pipe
$= 2\pi(2.1)(12)$
$= 50.4\pi$
= **158 cm²** (to 3 s.f.)

(iii) Total surface area of the pipe
$= 2(1.84\pi) + 50.4\pi + 0\pi(2.5)(12)$
$= 3.68\pi + 50.4\pi + 60\pi$
$= 114.08\pi$
= **358 cm²** (to 3 s.f.)

Exercise 11C

1. Base radius = 0.4 ÷ 2 = 0.2 m
Height of the cylinder $= \dfrac{3}{4} \times 0.2 = 0.15$ m
Volume of the cylinder $= \pi r^2 h$
$= \pi(0.2)^2(0.15)$
$= 0.006\pi$ m^3
$= 6000\pi$ cm^3
$= \dfrac{6000\pi}{1000}\ l$
= **18.8 *l***

2. Let the depth of water in the drum be d cm.
Base radius = 48 ÷ 2 = 24 cm
150 l = 150 000 ml = 150 000 cm^3

Volume of water in the drum $= \pi r^2 d = 150\ 000\ \text{cm}^3$

$$\pi(24)^2 d = 150\ 000$$
$$d = \frac{150\ 000}{\pi(24)^2}$$
$$= 82.9$$

$\therefore$ depth of water = **82.9 cm**

3. Base radius = 15 ÷ 2 = 7.5 cm

Capacity of the drinking trough

$= \frac{1}{2} \times \pi \times (7.5)^2 \times 84$

$= 7420\ \text{cm}^3$ (to 3 s.f.)

$=$ **7.42 *l***

4. **(a)** **(i)** Volume of the closed cylinder

$= \pi r^2 h$

$= \pi(7)^2(12)$

= **1850 cm³** (to 3 s.f.)

(ii) Total surface area of the closed cylinder

$= 2\pi r^2 + 2\pi rh$

$= 2\pi(7)^2 + 2\pi(7)(12)$

$= 98\pi + 168\pi$

$= 266\pi$

= **836 cm²** (to 3 s.f.)

(b) Base radius = 1.2 ÷ 2 = **0.6 m**

(i) Volume of the closed cylinder

$= \pi r^2 h$

$= \pi(0.6)^2(4)$

= **4.52 m³** (to 3 s.f.)

(ii) Total surface area of the closed cylinder

$= 2\pi r^2 + 2\pi rh$

$= 2\pi(0.6)^2 + 2\pi(0.6)(4)$

$= 0.72\pi + 4.8\pi$

$= 5.52\pi$

= **17.3 m²** (to 3 s.f.)

(c) **(i)** Volume of the closed cylinder

$= \pi r^2 h$

$= \pi(15)^2(63)$

= **44 500 mm³** (to 3 s.f.)

(ii) Total surface area of the closed cylinder

$= 2\pi r^2 + 2\pi rh$

$= 2\pi(15)^2 + 2\pi(15)(63)$

$= 450\pi + 1890\pi$

$= 2340\pi$

= **7350 mm²** (to 3 s.f.)

5.

	Diameter	Radius	Height	Volume	Total surface area
(a)	**8.00 cm**	**4.00 cm**	14 cm	704 cm³	**453 cm²**
(b)	**28.0 cm**	**14.0 cm**	20 cm	12 320 cm³	**2990 cm²**
(c)	4 cm	**2 cm**	**42.0 cm**	528 cm³	**553 cm²**
(d)	**8 m**	4 m	**21.0 m**	1056 m³	**629 m²**

(a) Volume $= 704\ \text{cm}^3$

$$\pi r^2(14) = 704$$
$$r^2 = \frac{704}{14\pi}$$
$$r = \sqrt{\frac{704}{14\pi}}$$

= **4.00 cm** (to 3 s.f.)

$\therefore d = 2 \times 4.0008 =$ **8.00 cm** (to 3 s.f.)

Total surface area

$= 2\pi r^2 + 2\pi rh$

$= 2\pi(4.0008)^2 + 2\pi(4.0008)(14)$

= **453 cm²**

(b) Volume $= 12\ 320\ \text{cm}^3$

$$\pi r^2(20) = 12\ 320$$
$$r^2 = \frac{12\ 320}{20\pi}$$
$$r = \sqrt{\frac{12\ 320}{20\pi}}$$

= **14.0 cm** (to 3 s.f.)

$\therefore d = 2 \times 14.003 =$ **28.0 cm** (to 3 s.f.)

Total surface area

$= 2\pi r^2 + 2\pi rh$

$= 2\pi(14.003)^2 + 2\pi(14.003)(20)$

= **2990 cm²** (to 3 s.f.)

(c) $r = 4 \div 2 =$ **2 cm**

Volume $= 528\ \text{cm}^3$

$$\pi(2)^2 h = 528$$
$$h = \frac{528}{4\pi}$$

= **42.0 cm** (to 3 s.f.)

Total surface area

$= 2\pi r^2 + 2\pi rh$

$= 2\pi(2)^2 + 2\pi(2)(42.017)$

= **553 cm²** (to 3 s.f.)

(d) $d = 4 \times 2 =$ **8 m**

Volume $= 1056\ \text{m}^3$

$$\pi(4)^2 h = 1056$$
$$h = \frac{1056}{16\pi}$$

= **21.0 m** (to 3 s.f.)

Total surface area

$= 2\pi r^2 + 2\pi rh$

$= 2\pi(4)^2 + 2\pi(4)(21.008)$

= **629 m²** (to 3 s.f.)

6. 35 mm = 35 ÷ 10 = 3.5 cm

Base radius = 3.5 ÷ 2 = 1.75 cm

Total surface area that needs to be painted for 1 wooden closed cylinder

$= 2\pi r^2 + 2\pi rh$

$= 2\pi(1.75)^2 + 2\pi(1.75)(7)$

$= 6.125\pi + 24.5\pi$

$= 30.625\pi$

Total surface area that needs to be painted for 200 wooden closed cylinders

$= 200 \times 30.625\pi$

= **19 244.75 cm²**

7. Base radius = 2.4 ÷ 2 = 1.2 m

Volume of the tank $= \pi r^2 h$
$= \pi(1.2)^2(6.4)$
$= 9.216\pi \text{ m}^3$
$= 9\,216\,000\pi \text{ cm}^3$

Volume of the cylinder container $= \pi r^2 h$
$= \pi(8.2)^2(28)$
$= 1882.72\pi \text{ cm}^3$

Number of complete cylindrical containers which can be filled by the oil in the tank

$= \dfrac{9\,216\,000\pi}{1882.72\pi}$

= **4895** (to the nearest whole number)

8. Base radius of the copper cylindrical rod = 14 ÷ 2 = 7 cm

Volume of the copper cylindrical rod
$= \pi r^2 h$
$= \pi(7)^2(47)$
$= 2303\pi$

Let the length of the wire be l.

Base radius of the wire = 8 ÷ 2 = 4 mm = 0.4 cm

Volume of the wire $= \pi(0.4)^2 l = 2303\pi$

$(0.4)^2 l = 2303$

$l = \dfrac{2303}{0.16}$

= 14 400 cm (to 3 s.f.)

= **144 m**

9. External base radius = 28 ÷ 2 = 14 mm = 1.4 cm

Internal base radius = 20 ÷ 2 = 10 mm = 1 cm

Volume of the metal used in making the pipe
$= \pi(1.4)^2(35) - \pi(1)^2(35)$
$= 68.6\pi - 35\pi$
$= 33.6\pi$
= **106 cm³** (to 3 s.f.)

10. Base radius = 2.4 ÷ 2 = 1.2 cm

Since water is discharged through the pipe at a rate of 2.8 m/s, i.e. 280 cm/s, in 1 second, the volume of water discharged is the volume of water that fills the pipe to a length of 280 cm.

In 1 second, volume of water discharged
= volume of pipe of length 280 cm
$= \pi r^2 h$
$= \pi(1.2)^2(280)$
$= 403.2\pi \text{ cm}^3$

Half an hour = 30 minutes

In 30 minutes, volume of water discharged
$= 403.2\pi \times 30 \times 60$
$= 725\,760\pi \text{ cm}^3$
$= 2\,280\,000 \text{ cm}^3$ (to 3 s.f.)
= **2280 *l***

11. Base radius of the pipe = 64 ÷ 2 = 32 mm

Since water is discharged through the pipe at a rate of 2.05 mm/s, i.e. in 1 second, the volume of water discharged is the volume of water that fills the pipe to a length of 2.05 mm.

In 1 second, volume of water discharged
= volume of pipe of length 2.05 mm
$= \pi r^2 h$
$= \pi(32)^2(2.05)$
$= 2099.2\pi \text{ mm}^3$

Base radius of the cylindrical tank = 7.6 ÷ 2 = 3.8 cm = 38 mm

2.3 m = 230 cm = 2300 mm

Volume of the cylindrical tank
$= \pi r^2 h$
$= \pi(38)^2(2300)$
$= 3\,321\,200\pi \text{ cm}^3$

Time required to fill the tank

$= \dfrac{3\,321\,200\pi}{2099.2\pi}$

$= 1582\dfrac{83}{656}$ s

= **26 min** (to the nearest minute)

12. **(i)** Volume of water in the tank = 18 × 16 × 13
= **3744 cm³**

(ii) Let the height of water in the cylindrical container be h.

Base radius of the cylindrical container = 17 ÷ 2 = 8.5 cm

Volume of water in the cylindrical container $= \pi(8.5)^2 h$
$= 3744$

$h = \dfrac{3744}{\pi(8.5)^2}$
= 16.5 (to 3 s.f.)

∴ height of water = **16.5 cm**

(iii) Surface area of the cylindrical container that is in contact with the water
$= \pi r^2 + 2\pi rh$
$= \pi(8.5)^2 + 2\pi(8.5)(16.495)$
$= 72.25\pi + 280.415\pi$
$= 352.665\pi$
= **1110 cm²** (to 3 s.f.)

13. **(i)** Base radius = 186 ÷ 2 = 93 mm = 9.3 cm

Height $= \dfrac{1}{3} \times 93 = 31$ mm = 3.1 cm

Total surface area of the container
$= 2\pi r^2 + 2\pi rh$
$= 2\pi(9.3)^2 + 2\pi(9.3)(3.1)$
$= 172.98\pi + 57.66\pi$
$= 230.64\pi$
= **725 cm²** (to 3 s.f.)

(ii) Area of the container that is painted
$= \pi r^2 + 2\pi rh$
$= \pi(9.3)^2 + 2\pi(9.3)(3.1)$
$= 86.49\pi + 57.66\pi$
$= 144.15\pi$

Required fraction $= \dfrac{144.15\pi}{230.64\pi}$

$= \boldsymbol{\dfrac{5}{8}}$

14. 72.0 mm = 7.20 cm = 0.0720 m

32 km² = 32 000 000 m²

Volume of rain = 32 000 000 × 0.0720
= 2 304 000 m³

Volume of rainwater flow per channel in 1 second = 18 × 26.4
= 475.2 m³

Time required for the channels to drain off the rain

$= \dfrac{2\,304\,000}{2 \times 475.2}$

= 2424.2 s (to 5 s.f.)

= **40 min** (to the nearest minute)

15. (i) Base radius = 23 ÷ 2 = 11.5 mm = 1.15 cm

Height = 4 mm = 0.4 cm

Volume of water and metal discs in the tank

$= (32 \times 28 \times 19) + 2580[\pi \times (1.15)^2 \times 0.4]$

$= (17\,024 + 1364.82\pi)$ cm^3

Let the new height in the tank be h.

$$\begin{aligned} \text{Volume in the tank} &= 32 \times 28 \times h \\ &= 17\,024 + 1364.82\pi \\ 896h &= 17\,024 + 1364.82\pi \\ h &= \frac{17\,024 + 1364.82\pi}{896} \\ &= 23.8 \text{ (to 3 s.f.)} \end{aligned}$$

∴ new height of water in the tank = **23.8 cm**

(ii) Surface area of the tank that is in contact with the water after the discs have been added

$= 2(32 \times 23.79 + 28 \times 23.79) + 32 \times 28$

= **3750 cm²** (to 3 s.f.)

16. Total surface area of the pipe

$= 2[\pi(3.8 + 0.8)^2 - \pi(3.8)^2] + 2\pi(3.8 + 0.8)(15) + 2\pi(3.8)(15)$

$= 13.44\pi + 138\pi + 114\pi$

$= 265.44\pi$

= **834 cm²** (to 3 s.f.)

11.6 Volume and surface area of composite solids

Practise Now 10

1. (i) Volume of the container

$= 20 \times 9 \times 14 + \frac{1}{4} \times \pi(14)^2(20)$

$= 2520 + 980\pi$

= **5600 cm³** (to 3 s.f.)

(ii) Total surface area of the container

$= 2\left[\frac{1}{4} \times \pi(14)^2\right] + 2(9 \times 14) + 2(20 \times 9) + 2(14 \times 20) + \frac{1}{4} \times 2\pi(14)(20)$

$= 98\pi + 252 + 360 + 560 + 140\pi$

$= 238\pi + 1172$

= **1920 cm²** (to 3 s.f.)

2. (i) Volume of the solid

$= 6 \times 12 \times 8 - \frac{1}{2} \times \pi(3)^2(12)$

$= 576 - 54\pi$

= **406 cm³** (to 3 s.f.)

(ii) Total surface area of the solid

$= 2\left[8 \times 6 - \frac{1}{2} \times \pi(3)^2\right] + 2(8 \times 12) + \frac{1}{2} \times 2\pi(3)(12) + 6 \times 12$

$= 96 - 9\pi + 192 + 36\pi + 72$

$= 360 + 27\pi$

= **445 cm²** (to 3 s.f.)

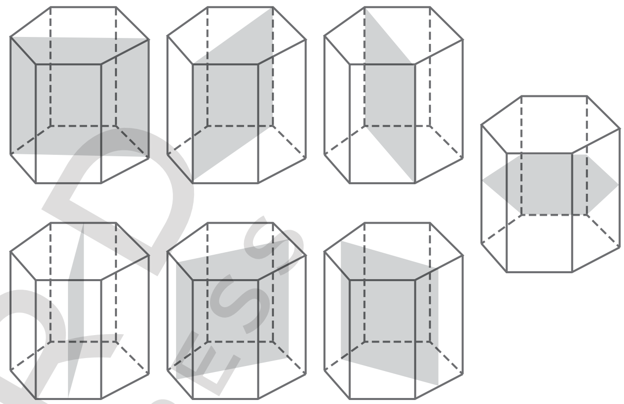
11.7 Symmetry in right prisms and cylinders

Practise Now 11

(a) There are 7 planes of symmetry.

The 7 planes of symmetry are:

[*can take figure from OUP Bk 2 C11 pg 350. Remove the red crosses and dotted line]



(b) Order of rotational symmetry about XY = **6**.

Investigation: Plane and rotational symmetry of a cylinder

1. (a) A circle has an infinite number of lines of symmetry. The lines of symmetry are **diameters** of the circle.

*Any **diameter** of a circle cuts the circle into two halves that are mirror images of each other.*

(b) A circle has an **infinite** order of rotational symmetry about its centre.

Any infinitesimal angle of rotation about the centre of a circle maps the circle onto itself.

2. (a) A cylinder has an infinite number of planes of symmetry.

(b) A cylinder has an infinite order of rotational symmetry about its axis.

From Question 1, we know that a circle has an infinite number of lines of symmetry and infinite order of rotational symmetry about its centre. Thus, a cylinder has an infinite number of planes of symmetry that are perpendicular to its base and an infinite order of rotational symmetry about the axis,

Thinking Time (Page 351)

(a) The two halves of the cuboid are not symmetrical about the given plane.

(b) No. The cuboid maps onto itself only once during a full rotation about the given axis, i.e. the order of rotational symmetry of the cuboid about the given axis = 1.

Exercise 11D

1. **(i)** Volume of the solid
$= 7 \times 3 \times 2 + 12 \times 8 \times 5$
$= 42 + 480$
$= \mathbf{522\ cm^3}$

(ii) Total surface area of the solid
$= 2(5 \times 12) + 2(5 \times 8) + 2(12 \times 8) + 2(2 \times 7) + 2(3 \times 2)$
$= 120 + 80 + 192 + 28 + 12$
$= \mathbf{432\ cm^2}$

2. **(i)** Volume of the solid
$= \pi(2.5)^2(8) + 7 \times 11 \times 3$
$= 50\pi + 231$
$= \mathbf{388\ cm^3}$ (to 3 s.f.)

(ii) Total surface area of the solid
$= 7 \times 11 + 2\pi(2.5)(8) + 2(3 \times 7) + 2(3 \times 11) + (7 \times 11)$
$= 77 + 40\pi + 42 + 66 + 77$
$= 262 + 40\pi$
$= \mathbf{388\ cm^2}$ (to 3 s.f.)

3. **(i)** Volume of the solid
$= \pi(5)^2(3) + \pi(12.5)^2(2)$
$= 75\pi + 312.5\pi$
$= 387.5\pi$
$= \mathbf{1220\ cm^3}$ (to 3 s.f.)

(ii) Total surface area of the solid
$= \pi(12.5)^2 + 2\pi(5)(3) + 2\pi(12.5)(2) + \pi(12.5)^2$
$= 156.25\pi + 30\pi + 50\pi + 156.25\pi$
$= 392.5\pi$
$= \mathbf{1230\ cm^2}$ (to 3 s.f.)

4. **(i)** Volume of the glass block
$= \frac{1}{4} \times \pi 24)^2(56) + 24 \times 56 \times 40$
$= 8064\pi + 53\,760$
$= \mathbf{79\,100\ cm^3}$ (to 3 s.f.)

(ii) Total surface area of the glass block
$= 2\left[\frac{1}{4} \times \pi(24)^2\right] + 2(40 \times 24) + 2(40 \times 56) + 2(24 \times 56)$
$+ \frac{1}{4} \times 2\pi(24)(56)$
$= 288\pi + 1920 + 4480 + 2688 + 672\pi$
$= 960\pi + 9088$
$= \mathbf{12\,100\ cm^2}$ (to 3 s.f.)

5. **(i)** Volume of the solid
$= 10 \times 12 \times 7 - \frac{1}{2} \times \pi(2)^2(12)$
$= 840 - 24\pi$
$= \mathbf{765\ cm^3}$ (to 3 s.f.)

(ii) Total surface area of the solid
$= 2\left[7 \times 10 - \frac{1}{2} \times \pi(2)^2\right] + 2(7 \times 12) + 2(3 \times 12)$
$+ \frac{1}{2} \times 2\pi(2)(12) + 10 \times 12$
$= 140 - 4\pi + 168 + 72 + 24\pi + 120$
$= 500 + 20\pi$
$= \mathbf{563\ cm^2}$ (to 3 s.f.)

6. **(i)** Volume of the remaining solid
$= \pi(12)^2(32) - \pi(5)^2(14)$
$= 4608\pi - 350\pi$
$= 4258\pi$
$= \mathbf{13\,400\ cm^3}$ (to 3 s.f.)

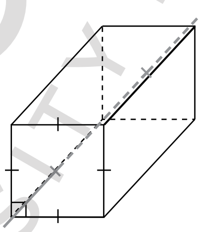
(ii) Area that will covered in paint
$= 2\pi(12)(32) + 2\pi(5)(14) + 2[\pi(12)^2]$
$= 768\pi + 140\pi + 288\pi$
$= 1196\pi$
$= \mathbf{3760\ cm^2}$ (to 3 s.f.)

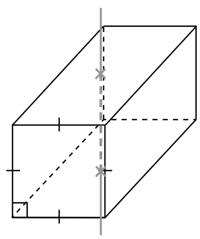
7. **(i)** Volume of the solid
$= \left[\frac{1}{2} \times (40 + 88) \times 70\right] \times 25 - \pi(15)^2(25)$
$= 112\,000 - 5625\pi$
$= \mathbf{94\,300\ cm^3}$ (to 3 s.f.)

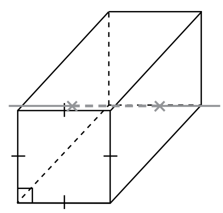
(ii) Total surface area of the solid
$= (74 + 40 + 74 + 88) \times 25 + 2\left[\frac{1}{2} \times (40 + 88) \times 70 - \pi(15)^2\right]$
$+ 2\pi(15)(25)$
$= 6900 + 8960 - 450\pi + 750\pi$
$= 15\,860 + 300\pi$
$= \mathbf{16\,800\ cm^2}$ (to 3 s.f.)

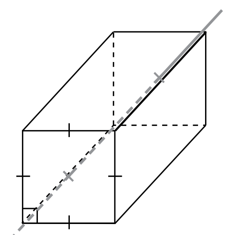
8. **(a)** **(i)** Number of planes of symmetry = 5

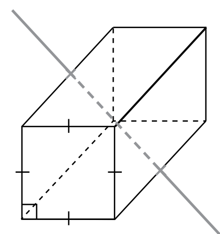
(ii) The axes of symmetry and the order of rotational symmetry about each axis are:
[*this question might be a bit problematic. Might have updates]


Order of rotational symmetry = **4**


Order of rotational symmetry = **2**

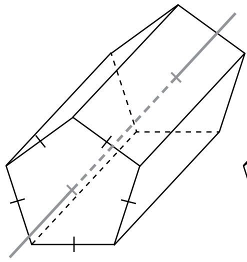

Order of rotational symmetry = **2**


Order of rotational symmetry = **2**

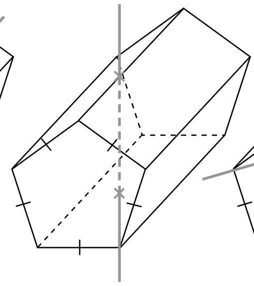

Order of rotational symmetry = **2**

(b) (i) Number of planes of symmetry = 6

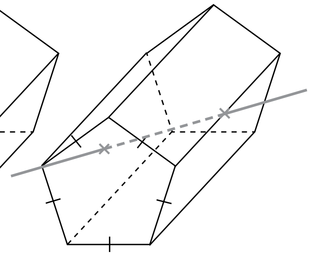
(ii) The axes of symmetry and the order of rotational symmetry about each axis are:



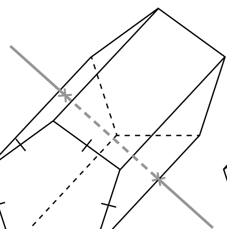
Order of rotational symmetry = **5**



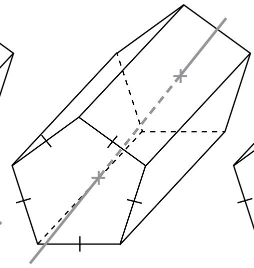
Order of rotational symmetry = **2**



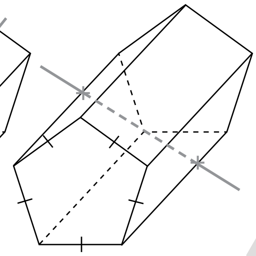
Order of rotational symmetry = **2**



Order of rotational symmetry = **2**



Order of rotational symmetry = **2**



Order of rotational symmetry = **2**

9. (i) Volume of the solid

$= \frac{1}{2} \times [\pi(6 + 1.5)^2 - \pi(6)^2] \times 8$

$= 4(56.25\pi - 36\pi)$

$= 4(20.25\pi)$

$= 81\pi$

$= \mathbf{254\ cm^3}$ (to 3 s.f.)

(ii) Total surface area of the solid

$= 2 \times \frac{1}{2} \times [\pi(7.5)^2 - \pi(6)^2] + \frac{1}{2} \times 2\pi(7.5) \times 8$

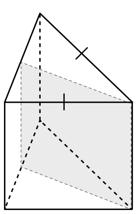
$+ \frac{1}{2} \times 2\pi(6) \times 8 + 2(1.5 \times 8)$

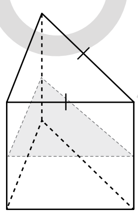
$= 20.25\pi + 60\pi + 48\pi + 24$

$= 24 + 128.25\,\pi$

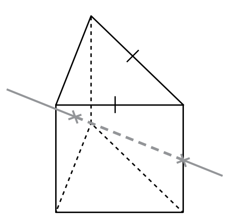
$= \mathbf{427\ cm^2}$ (to 3 s.f.)

10. (a) (i) The planes of symmetry are:



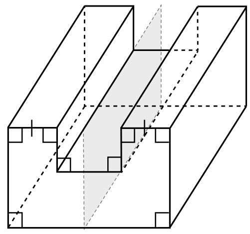


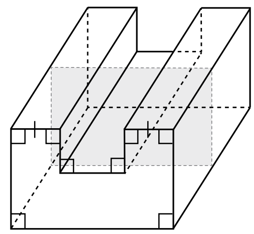
The axis of symmetry is:



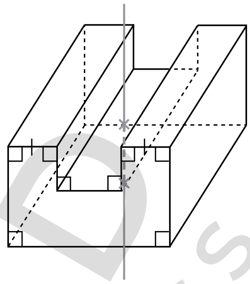
Order of rotational symmetry about axis = **2**

(b) The planes of symmetry are:



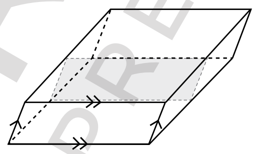


The axis of symmetry is:



Order of rotational symmetry about axis = **2**

(c) The plane of symmetry is:



There is no axis of symmetry.

Chapter 12 Introduction to Set Notation and Probability

TEACHING NOTES

Suggested Approach

Students would not have learnt sets and probability in primary school so the concept will be entirely new for them. Teachers may begin the lesson by first arousing the students' interest in the topic of probability. This new topic can be introduced by using the chapter opener on Page 355 and discussing statements that are often used in our daily life (see Introductory Problem on page 356).

The first two sections of this chapter focus on set notations. Although students are likely unfamiliar with the term 'sets', they may have encountered real-life uses of sets and subsets. Teachers may use examples such as classification of animals or triangles to allow students to relate better to the topic and to familiarise them with set notations.

Section 12.1 Sets and set notations

The notations used in set theory, such as $\in$, $\notin$ and $n(S)$, and the ways to describe a set (see page 358) are new to students. Thus, teachers should take the time to familiarise the students with these notations before moving on to the next section. Students may struggle with interpreting the notations. It is recommended that they go through the Worked Examples and Practise Now exercises to build confidence in using these terms. Additionally, concepts such as 'well-defined' and 'distinct' are to be explored (see Class Discussion: Well-defined and distinct objects in a set). The Thinking Time activity on page 359 helps to hone students' logical thinking skills and test their understanding of the meaning of $n(S)$ and of equal sets.

Section 12.2 Venn diagrams, universal sets, complements of sets and subsets

Here, the use of a Venn diagram to represent a set is introduced. Other notations are also introduced in this section (see page 364). Teachers should take time to familiarise the students with these notations.

The definitions of other terms associated with set theory, such as universal set, complement of a set and subset, should be made clear to the students. Teachers can make use of the class discussion (see Class Discussion: Understanding subset) to explain the concept of subsets.

Section 12.3 Probability experiment and sample space

From the statements discussed in our daily life (see Introductory Problem on page 356), teachers can build upon them and guide the students to determine the chance of each event happening. This measure of chance is the definition of probability.

This can then lead to relating the chance of any event happening to a number line taking on values between 0 and 1 inclusive. Teachers should further discuss the reason for a 'certain' event to take the value of 1 as well as the reason for an 'impossible' event to take the value of 0.

At the end of this section, a simple class activity can be carried out by encouraging students to form statements to describe an event. Words such as 'unlikely', 'likely', 'impossible' or 'certain' are encouraged to be used in the statements. Students can then mark the chance of the event occurring on a number line. At the end of this section, students should be able to define probability as a measure of chance.

Teacher can do a simple experiment such as tossing a coin to introduce the words 'outcomes' and 'sample space'. Going through the different experiments listed in the table on page 369 can help to reinforce the meanings of 'outcomes' and 'sample space'.

For the last experiment, teachers are to let the students know that there is a difference between drawing the first and second black ball. Thus, there is a need to differentiate the two black balls. Similarly, there is a need to differentiate the three white balls. Hence, the outcomes are two individual black balls and three individual white balls.

Teachers can assess students' understanding of the terms using the Practise Now examples available.

Section 12.4 Probability of single events

In the previous sections, students would have learnt events with a probability of 0 or 1. Here, they are to grasp that for any event E, $0 \leqslant P(E) \leqslant 1$ and later $P(\text{not } E) = 1 - P(E)$.

As a recommended technique for solving problems involving probability, teachers should encourage the students to always list all the possible outcomes in a simple chance situation to calculate the probability. Doing so will allow better visualization of the outcomes for a particular event to happen.

Since the topic on probability is new to students, it is recommended that teachers should not use set notations at this stage, but to ensure that students are confident in the various terms used in probability (e.g. event, favourable outcome, sample space). Later, teachers may use a Venn diagram to show that the sample space containing all possible outcomes of a probability experiment is the universal set, while the event of a favourable outcome is the subset of the sample space.

Section 12.5 Further examples on probability of single events

In this section, there are more calculations of probability using real-life examples. Teachers should use this section to reinforce the concept of probability.

Section 12.6 Experimental approach to finding probability

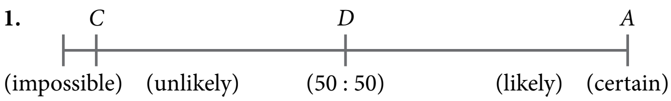
To start off this section, students should attempt an activity (see Investigation: Tossing a coin). The terms 'experimental probability' and 'theoretical probability' and their relationship should be highlighted and explained to students. Students are expected to be able to conclude that as the number of trials increase, the relative frequency of an event occurring tends towards the theoretical probability of the event.

At the same time, teachers can illustrate the meanings of 'fair' and 'unbiased'. If any of the two terms are used, then the chance of any outcome happening in an event is exactly equal.

Teachers should also wrap up the investigation by highlighting how the expected frequency of an event can be calculated from the theoretical probability and the total number of trials.

Introductory Problem

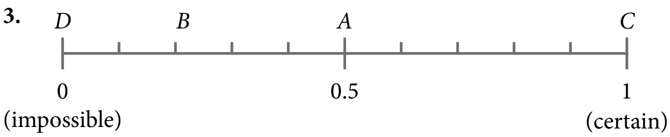
1.



Event B generally depends on the time of the year. For example, if it is the rainy season, then students may conclude that it is more likely to rain tomorrow, i.e. B lies between 50 : 50 and certain.

2. **0.5**

3.



Event *A*: Obtaining a 'head' when tossing an ordinary coin once.
Event *B*: Drawing a blue ball at random from a bag containing 1 blue ball and 4 identical red balls.
Event *C*: Obtaining a number less than 8 when rolling an ordinary six-sided die.
Event *D*: Drawing 5 Aces from a standard pack of 52 playing cards.

12.1 Sets and set notations

Practise Now 1A

1. (i) A = **{2, 4, 6, 8}**
   (ii) (a) **True**
   (b) **True**
   (c) **False**
   (d) **True**
   (iii) (a) $2 \in A$
   (b) $5 \notin A$
   (c) $11 \notin A$
   (d) $6 \in A$
   (iv) n(A) = **4**
2. n(B) = **10**

Class Discussion (Well-defined and distinct objects in a set)

1. H is not a set because it is not well-defined. The definition of "tall" is subjective.
2. T contains identical pens which are distinct. We can list the elements of T as $T = \{P_1, P_2\}$.
3. There are repeated letters in the word 'CLEVER'. Since the letter 'E' is not distinct, we can list the elements of S as S = {C, L, E, V, R}.

Practise Now 1B

(i) **C = {11, 12, 13, 14, 15, 16, 17}**
D = {10, 11, 12, 13, 14, 15, 16, 17}
(ii) **No**.
n(C) = 7 and n(D) = 8
The number of elements belonging to the sets C and D are different.
$\therefore$ n(C) $\neq$ n(D)
(iii) **No**, $C \neq D$ because they do not contain exactly the same elements. $10 \in D$ but $10 \notin C$.

Thinking Time (Page 359)

(a) **Yes**. If $A = B$, it means that both sets contain exactly the same elements.
Thus, the number of elements belonging to the sets A and B will be the same.
$\therefore$ n(A) = n(B)
(b) **No**. If n(A) = n(B), it does not imply that $A = B$.
For example, let A = {1, 3, 5, 7, 9} and B = {2, 4, 6, 8, 10}.
n(A) = n(B) = 5, but the elements in A are all different from the elements in B.

Practise Now 2

(i) ***P* and *Q*** are empty sets. **P = Ø** and **Q = Ø**.
(ii) **Yes**, P and Q are equal sets because both of them are empty sets.
(iii) **No**, Q and R are not equal sets because Q is an empty set but R is not an empty set. R is the set containing one element Ø.

Exercise 12A

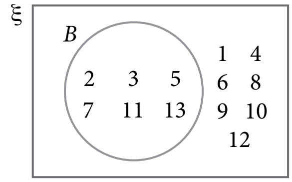
1. (i) **A = {1, 3, 5, 7, 9}**
   (ii) (a) **True**
   (b) **True**
   (c) **False**
   (d) **True**
   (iii) n(A) = **5**
2. (a) **B = {2, 3, 4, 5, 6, 7, 8, 9}**
   n(B) = 8
   (b) **C = {–10, –9, –8, –7, –6, –5, –4, –3, –2, –1}**
   n(C) = 10
   (c) **D = {2, 4, 6, 8, 10, 12}**
   n(D) = 6
   (d) **E = {A}**
   n(E) = 1
3. (a) **F = {red, orange, yellow, green, blue, indigo, violet}**
   n(F) = 7
   (b) **G = {Kashmir Solidarity Day, Pakistan Day, Eid al-Fitr, Labour Day, Eid al-Adha, Ashura, Independence Day of Pakistan, Mawlid, Iqbal Day, Christmas Day, Quaid-e Azam's Birthday}**
   n(G) = 11
   (c) **H = {S, Y, M, T, R}**
   n(H) =5
4. (a) **K = Ø**
   (b) **L = {2}**
   (c) **M = Ø**
   (d) **N = Ø**

5. (i) P = **{Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday}**
(ii) (a) **Tuesday** $\in P$
(b) **Sunday** $\in P$
(c) **March** $\notin P$
(d) **Holiday** $\notin P$
(iii) $n(P) = 7$
6. (i) **No**
(ii) $Q = \{4, 9, 16, 25, 36, 49\}$
(iii) $n(Q) = 6$
7. (i) R is the set of non-negative even integers.
S is the set of non-negative even integers less than 10.
(ii) **No**. $n(S) = 5$ while $n(R)$ is infinite.
(iii) **No**. $R \neq S$ because they do not contain exactly the same elements. $10 \in R$ but $10 \notin S$.
8. (i) T is an empty set. $T = \varnothing$.
(ii) **No**, T and U are not equal sets because T is an empty set but U is not an empty set.
(iii) **No**, U and V are not equal sets because they do not contain exactly the same elements. $U = \{1\}$ and $V = \{0\}$.
9. (a) **False**, because 'c' is an element of the set containing the letters 'c', 'a' and 'r'.
Therefore, c $\in$ {c, a, r}.
(b) **False**, because the word 'car' is not an element of the set containing the letters 'c', 'a' and 'r'.
(c) **False**, because the set containing the letter 'c' is not an element of the set containing the letters 'c', 'a' and 'r'.
{c} and {c, a, r} are both sets.
(d) **False**, because the set containing the letters 'c', 'a' and 'r' is a set and cannot be equal to a number. But the number of elements of the set containing the letters 'c', 'a' and 'r' is 3.
10. (a) $X = \{x : x$ is a prime number$\}$
(b) $Y = \{x : x$ is a non-negative multiple of four$\}$
(c) $Z = \{x : x$ is a positive integer and a factor of 12$\}$
11. (a) **False**, because {0} is a set containing one element, which is 0. Therefore, it is not an empty set and $\{0\} \neq \varnothing$.
(b) **True**, because the set { } does not contain any elements, hence it is an empty set.
(c) **False**, because $\{\varnothing\}$ is a set containing one element, which is the symbol $\varnothing$.
Therefore, it is not an empty set.
(d) **True**, because $\varnothing$ is the empty set which contains no elements. Therefore, the number of elements in the empty set is 0.

1.2 Venn diagrams, universal sets, complements of sets and subsets

Practise Now 3

(i) $\xi = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13\}$
$B = \{2, 3, 5, 7, 11, 13\}$
(ii)



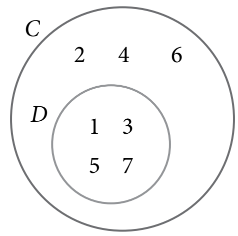
(iii) $B' = \{1, 4, 6, 8, 9, 10, 12\}$
(iv) B' is the set of integers between 1 and 13 inclusive which are not prime numbers.
(v) $n(\xi) = 13$, $n(B) = 6$ and $n(B') = 7$
(vi) **Yes**. Since B and B' contain all the elements of ξ, and B and B' do not contain any common elements, $n(B) + n(B') = n(\xi)$.

Thinking Time (Page 363)

(i) **No**. If the universal set ξ is not defined, we do not know what elements lie outside of the set A, if any.
(ii) **Yes**. The set S and its complement S' will not contain any common elements. Hence given any set S in a universal set ξ, the number of elements in the set S and the number of elements outside the set S will add up to the number of elements in the universal set. Therefore, $n(S) + n(S') = n(\xi)$.

Practise Now 4A

1. (i)



(ii) **Yes**, $D \subset C$ because every element of D is an element of C and $D \neq C$.
(iii) **Yes**, $C \not\subseteq D$ because there are elements in C that are not elements of D.
2. (a) **True**
(b) **True**
(c) **True**
(d) **False**
3. (i) $P = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13\}$
$Q = \{2, 3, 5, 7, 11\}$
(ii) $Q \subset P$ because every element of Q is an element of P and $Q \neq P$.
$P \not\subset Q$ because there are elements in P that are not elements of Q.
(iii) $R = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13\}$
(iv) $P \subseteq R$ **and** $R \subseteq P$ because every element of P is an element of R and vice versa. P and R are equal sets.

Class Discussion (Understanding subset)

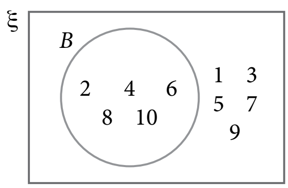
1. **Yes**
2. **Yes**. For A to be a subset of B, then A is either a proper subset of B or A and B are equal sets.
3. **No**. If C is a subset of D, it means that every element of C is an element of D. It is possible that there no other elements in D that are outside C, which will mean that $C = D$. If this is the case, then $C \not\subset D$. Hence, if $C \subseteq D$, it does not mean that $C \subset D$.
4. **No**
5. **(a)** Since there are elements in P that are not elements of Q and there are also elements in Q that are not elements of P, then $P \not\subset Q$ and $Q \not\subset P$.

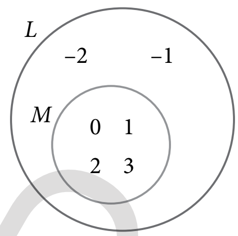
   (b) Since there are elements in P that are not elements of Q, then $P \not\subset Q$.
   Since every element of Q is an element of P and $Q \neq P$, then $Q \subset P$.

Practise Now 4B

(a) **Ø, {7}, {8} and {7, 8}**
(b) **Ø, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, {a, b, c}**

Exercise 12B

1. **(i)** **ξ = {cat, hamster, lion, mouse, tiger}**
   A **= {cat, hamster, mouse}**
   (ii) ***A′*** **= {lion, tiger}**
2. **(i)** **ξ = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}**
   B **= {2, 4, 6, 8, 10}**
   (ii)
   

   (iii) ***B′*** **= {1, 3, 5, 7, 9}**
   (iv) B' is the set of integers between 1 and 10 inclusive which are odd numbers.
3. **(i)** ***C*** **= {s, t, u}**
   D **= {s, t, u, v, w, x, y, z}**
   (ii) **Yes**, $C \subset D$ because every element of C is an element of D and $C \neq D$.
4. **(a)** **True**
   (b) **True**
   (c) **True**
   (d) **False**
5. **(a)** **Ø, {a}, {b}, {a, b}**
   (b) **Ø, {Singapore}, {Malaysia}, {Singapore, Malaysia}**
   (c) **Ø, {14}, {16}, {14, 16}**
   (d) **Ø, {7}**
6. **(i)** **ξ = {1, 2, 3, 4, 5, 6, 7, 8, 9}**
   J **= {1, 4, 6, 8, 9}**
   J′ **= {2, 3, 5, 7}**
   (ii) J' is the set of integers between 0 and 10 which are prime numbers.
   (iii) n(ξ) = **9**, n(J) = **5** and n(J') = **4**
   (iv) **Yes**. Since J and J' contain all the elements of ξ, and J and J' do not contain any common elements, n(J) + n(J') = n(ξ).
7. **(i)** **ξ = {a, b, c, d, e, f, g, h, i, j}**
   K **= {b, c, d, f, g, h, j}**
   K′ **= {a, e, i}**
   (ii) K' is the set of the first 10 letters of the English alphabet which are vowels.
   (iii) n(ξ) = **10**, n(K) = **7** and n(K') = **3**
   (iv) **Yes**. Since K and K' contain all the elements of ξ, and K and K' do not contain any common elements, n(K) + n(K') = n(ξ).
8. **(i)**
   

   (ii) **Yes**, $M \subset L$ because every element of M is an element of L and $M \neq L$.
   (iii) **Yes**, $L \not\subseteq M$ because there are elements in L that are not elements of M.
9. **(i)** ***N*** **= {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20}**
   P **= {4, 8, 12, 16}**
   (ii) $\boldsymbol{P \subset N}$ because every element of P is an element of N and $P \neq N$.
   $N \not\subset P$ because there are elements in N that are not elements of P.
   (iii) ***Q*** **= {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20}**
   (iv) $\boldsymbol{N \subseteq Q}$ **and** $\boldsymbol{Q \subseteq N}$ because every element of N is an element of Q and vice versa. N and Q are equal sets.
10. **(a)** **Ø, {7}, {8}, {9}, {7, 8}, {7, 9}, {8, 9}, {7, 8, 9}**
    (b) **Ø, {2}, {3}, {5}, {2, 3}, {2, 5}, {3, 5}, {2, 3, 5}**
    (c) **Ø, {a}, {b}, {c}, {d}, {a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}, {a, b, c, d}**
    (d) **Ø, {I}, {N}, {O}, {U}, {I, N}, {I, O}, {I, U}, {N, O}, {N, U}, {O, U}, {I, N, O}, {I, N, U}, {I, O, U}, {N, O, U}, {I, N, O, U}**
11. **(i)** ***V′*** **= {1, 2, 4, 5, 7, 8, 10, 11, 13, 14, 16, 17, 19, 20}**
    (ii) $V' = \{x : x$ is an integer that is not a multiple of 3 such that $0 < x < 21\}$
12. **(i)** **Yes**, $X \subset W$ because every element of X is an element of W and $X \neq W$.
    (ii) **Yes**, $W \not\subset X$ because there are elements in W that are not elements of X.
13. Number of subsets of $Y = \mathbf{2^a}$

12.3 Probability experiment and sample space

Practise Now 5

Sample space = **{Orange, Purple, Green, Blue, Red}**
Total number of possible outcomes = **5**

Practise Now 6

(a) Let B_1, B_2, B_3, B_4 and B_5 represent the five blue marbles; and R_1, R_2, R_3 and R_4 represent the four red marbles.
Sample space = $\{\boldsymbol{B_1, B_2, B_3, B_4, B_5, R_1, R_2, R_3, R_4}\}$
Total number of possible outcomes = **9**

(b) Sample space = {$\mathbf{N_1, A_1, T, I, O, N_2, A_2, L}$}
Total number of possible outcomes = **8**

(c) Sample space = {**357, 358, 359, 360, …, 389**}
Total number of possible outcomes
= number of integers from 1 to 389 – number of integers from 1 to 356
= 389 – 356
= **33**

12.4 Probability of single events

Practise Now 7

(i) Total number of possible outcomes
= number of integers from 1 to 24 – number of integers from 1 to 9
= 24 – 9
= 15

P(drawing a '21') = $\mathbf{\frac{1}{15}}$

(ii) There are 7 odd numbers from 10 to 24, i.e. 11, 13, 15, 17, 19, 21 and 23.

P(drawing an odd number) = $\mathbf{\frac{7}{15}}$

(iii) There are 5 prime numbers from 10 to 24, i.e. 11, 13, 17, 19 and 23.

P(drawing a prime number) = $\frac{5}{15}$

= $\mathbf{\frac{1}{3}}$

(iv) There are no perfect cubes from 10 to 24.

P(drawing a perfect cube) = $\frac{0}{15}$

= **0**

Practise Now 8

(i) Total number of possible outcomes = 52
There are 26 red cards in the pack.

P(drawing a red card) = $\frac{26}{52}$

= $\mathbf{\frac{1}{2}}$

(ii) There are 4 Aces in the pack.

P(drawing an Ace) = $\frac{4}{52}$

= $\mathbf{\frac{1}{13}}$

(iii) There is 1 three of clubs in the pack.

P(drawing the three of clubs) = $\mathbf{\frac{1}{52}}$

(iv) Since a card is either the three of clubs or not the three of clubs, then P(drawing the three of clubs) + P(drawing a card which is not the three of clubs) = 1
∴ P(drawing a card which is not the three of clubs)
= 1 – P(drawing the three of clubs)

= $1 - \frac{1}{52}$

= $\mathbf{\frac{51}{52}}$

Practise Now 9

1. (i) Total number of letters = 8
There is one 'D'.

P(a 'D' is chosen) = $\mathbf{\frac{1}{8}}$

(ii) There are 6 consonants, i.e. 1 'C', 1 'H', 1 'L', 1 'D', 1 'R' and 1 'N'.

P(letter chosen is a consonant) = $\frac{6}{8}$

= $\mathbf{\frac{3}{4}}$

(iii) P(letter chosen is not a consonant)
= 1 – P(letter chosen is a consonant)

= $1 - \frac{3}{4}$

= $\mathbf{\frac{1}{4}}$

2. (i) Total number of possible outcomes = 9 + 6 + 4 + 5
= 24

P(drawing a purple marble) = $\frac{4}{24}$

= $\mathbf{\frac{1}{6}}$

(ii) Total number of red and blue marbles = 9 + 5
= 14

P(drawing a red or a blue marble) = $\frac{14}{24}$

= $\mathbf{\frac{7}{12}}$

(iii) There are no white marbles.

P(drawing a white marble) = $\frac{0}{24}$

= **0**

(iv) P(drawing a marble that is not white)
= 1 – P(drawing a white marble)
= 1 – 0
= **1**

3. P(draw a blue ball) = 1 – P(draw a ball that is not blue)
= 1 – P(draw a red ball) – P(draw a green ball)

= $1 - \frac{1}{3} - \frac{1}{6}$

= $\frac{6}{6} - \frac{2}{6} - \frac{1}{6}$

= $\frac{3}{6}$

= $\frac{1}{2}$

Number of blue balls = $\frac{1}{2} \times 24$

= **12**

Exercise 12C

1. Sample space = **{1, 2, 3, 4, 5, 6}**
Total number of possible outcomes = **6**

2. **(a)** Sample space = **{2, 3, 4, 5}**
Total number of possible outcomes = **4**

(b) Sample space = **{A, B, C, D, E, F, G, H, I, J}**
Total number of possible outcomes = **10**

(c) Let R_1, R_2, R_3, R_4 and R_5 represent the five red discs; B_1, B_2 and B_3 represent the three blue discs; and G_1 and G_2 represent the two green discs.
Sample space = $\{\boldsymbol{R_1, R_2, R_3, R_4, R_5, B_1, B_2, B_3, G_1, G_2}\}$
Total number of possible outcomes = **10**

(d) Sample space = $\{\mathbf{T, E_1, A, C, H, E_2, R}\}$
Total number of possible outcomes = **7**

(e) Sample space = **{100, 101, 102, 103, …, 999}**
Total number of possible outcomes
= number of integers from 1 to 999 – number of integers from 1 to 99
= 999 – 99
= **900**

3. **(i)** Total number of possible outcomes = 8
P(rolling a '7') = $\mathbf{\frac{3}{8}}$

(ii) There are two '3's and one '4'.
P(rolling a '3' or a '4') = $\frac{2+1}{8}$
= $\mathbf{\frac{3}{8}}$

(iii) All the 8 numbers on the 8-sided fair die are less than 10.
P(rolling a number less than 10) = $\frac{8}{8}$
= **1**

(iv) There is one '2'.
Since the number rolled is either '2' or not '2', then
P(rolling a '2') + P(rolling a number which is not '2') = 1
∴ P(rolling a number which is not '2') = 1 – $\frac{1}{8}$
= $\mathbf{\frac{7}{8}}$

4. **(i)** Total number of possible outcomes
= number of integers from 1 to 22 – number of integers from 1 to 9
= 22 – 9
= 13
There are 7 even numbers from 10 to 22, i.e. 10, 12, 14, 16, 18, 20 and 22.
P(drawing an even number) = $\mathbf{\frac{7}{13}}$

(ii) There are 7 numbers between 13 and 19 inclusive, i.e. 13, 14, 15, 16, 17, 18 and 19.
P(drawing a number between 13 and 19 inclusive) = $\mathbf{\frac{7}{13}}$

(iii) There are 5 composite numbers from 10 to 17, i.e. 10, 12, 14, 15 and 16.
P(drawing a composite number that is less than 18) = $\mathbf{\frac{5}{13}}$

(iv) There is no number in the box that is greater than 22.
P(drawing a number greater than 22) = $\frac{0}{13}$
= **0**

(v) There are 3 numbers from 10 to 22 that are divisible by 4, i.e. 12, 16 and 20.
P(drawing a number that is divisible by 4) = $\mathbf{\frac{3}{13}}$

5. **(i)** Total number of possible outcomes = 52
There is 1 Ace of spades in the pack.
P(drawing the Ace of spades) = $\mathbf{\frac{1}{52}}$

(ii) There are 13 hearts and 13 clubs in the pack.
P(drawing a heart or a club) = $\frac{13+13}{52}$
= $\frac{26}{52}$
= $\mathbf{\frac{1}{2}}$

(iii) There are 12 picture cards in the pack.
P(drawing a picture card) = $\frac{12}{52}$
= $\mathbf{\frac{3}{13}}$

(iv) Since a card is either a picture card or a non-picture card, then P(drawing a picture card) + P(drawing a non-picture card) = 1
∴ P(drawing a non-picture card)
= 1 – P(drawing a picture card)
= 1 – $\frac{3}{13}$
= $\mathbf{\frac{10}{13}}$

6. **(i)** Total number of letters = 11
There is one 'A'.
P(the letter 'A' is chosen) = $\mathbf{\frac{1}{11}}$

(ii) There are 2 'B's.
P(the letter 'B' is chosen) = $\mathbf{\frac{2}{11}}$

(iii) There are 4 vowels, i.e. 1 'O', 1 'A' and 2 'I's.
P(letter chosen is a vowel) = $\mathbf{\frac{4}{11}}$

(iv) There are 7 consonants, i.e. 1 'P', 1 'R', 2 'B's, 1 'L', 1 'T' and 1 'Y'.
P(letter chosen is a consonant) = $\mathbf{\frac{7}{11}}$

7. **(i)** Total number of possible outcomes = 5
There is 1 '♥' on the spinner.
P(pointer pointing to ♥) = $\mathbf{\frac{1}{5}}$

(ii) There are 3 letters of the English alphabet on the spinner, i.e. A, F and V.
P(pointer pointing to a letter of the English alphabet) = $\frac{3}{5}$

(iii) There is 1 vowel on the spinner, i.e. A.
P(pointer pointing to a vowel) = $\mathbf{\frac{1}{5}}$

(iv) There are 2 consonants on the spinner, i.e. F and V.
P(pointer pointing to a consonant) = $\mathbf{\frac{2}{5}}$

8. (i) Total number of possible outcomes = 30

Number of girls in the class = 8 + 3 + 1

= 12

$$\text{P(a girl is chosen)} = \frac{12}{30} = \mathbf{\frac{2}{5}}$$

(ii) Number of students with brown hair in the class = 11 + 8

= 19

P(student chosen does not have brown hair)

= 1 – P(student chosen does not have brown hair)

$= 1 - \frac{19}{30}$

$= \mathbf{\frac{11}{30}}$

(iii) P(student chosen is not a boy with red hair)

= 1 – P(student chosen is a boy with red hair)

$= 1 - \frac{3}{30}$

$= \frac{30}{30} - \frac{3}{30}$

$= \frac{27}{30}$

$= \mathbf{\frac{9}{10}}$

(iv) There are no students with black hair in the class.

$$\text{P(student chosen is a student with black hair)} = \frac{0}{30} = \mathbf{0}$$

9. (i) Total number of possible outcomes = 5 + 5

= 10

$$\text{P(choosing a book in Japanese)} = \mathbf{\frac{3}{10}}$$

(ii) Number of novels in English = 4

$$\text{P(choosing a novel which is in English)} = \frac{4}{10} = \mathbf{\frac{2}{5}}$$

10. (i) Total number of possible outcomes

= number of integers from 1 to 99 – number of integers from 1 to 9

= 99 – 9

= 90

Number of integers from 10 to 19

= number of integers from 1 to 19 – number of integers from 1 to 9

= 19 – 9

= 10

$$\text{P(two-digit number chosen is less than 20)} = \frac{10}{90} = \mathbf{\frac{1}{9}}$$

(ii) There are 6 perfect squares from 10 to 99, i.e. 16, 25, 36, 49, 64 and 81.

$$\text{P(two-digit number chosen is a perfect square)} = \frac{6}{90} = \mathbf{\frac{1}{15}}$$

11. (i) Total number of possible outcomes = 52 + 2

= 54

There are 26 red cards in the pack.

$$\text{P(drawing a red card)} = \frac{26}{54} = \mathbf{\frac{13}{27}}$$

(ii) There are 4 twos in the pack.

$$\text{P(drawing a two)} = \frac{4}{54} = \mathbf{\frac{2}{27}}$$

(iii) There are 2 Jokers in the pack.

$$\text{P(drawing a Joker)} = \frac{2}{54} = \mathbf{\frac{1}{27}}$$

(iv) There are 4 Queens and 4 Kings in the pack.

$$\text{P(drawing a Queen or a King)} = \frac{4+4}{54} = \frac{8}{54} = \mathbf{\frac{4}{27}}$$

12. (i) Total number of possible outcomes = 52 – 13

= 39

There are 13 black cards remaining in the pack, i.e. 13 cards which are spades.

$$\text{P(drawing a black card)} = \frac{13}{39} = \mathbf{\frac{1}{3}}$$

(ii) There are 13 diamonds in the pack.

$$\text{P(drawing a diamond)} = \frac{13}{39} = \mathbf{\frac{1}{3}}$$

(iii) There are 9 picture cards remaining in the pack.

$$\text{P(drawing a picture card)} = \frac{9}{39} = \mathbf{\frac{3}{13}}$$

(iv) There are 3 Aces remaining in the pack.

P(drawing a card which is not an Ace)

= 1 – P(drawing an Ace)

$= 1 - \frac{3}{39}$

$= \frac{39}{39} - \frac{3}{39}$

$= \frac{36}{39}$

$= \mathbf{\frac{12}{13}}$

13. (i) There are 6 possible outcomes, i.e. integers from 0 to 5.

$$\text{P(the number in column } A \text{ is a 4)} = \mathbf{\frac{1}{6}}$$

(ii) There are 10 possible outcomes, i.e. integers from 0 to 9.

$$\text{P(the number in column } B \text{ is an 8)} = \mathbf{\frac{1}{10}}$$

(iii) There are 6 possible outcomes, i.e. integers from 0 to 5.

Since all the integers from 0 to 5 are less than 6,

$$\text{P(the number in column } A \text{ is less than 6)} = \frac{6}{6} = \mathbf{1}$$

(iv) There are 10 possible outcomes, i.e. integers from 0 to 9. From 0 to 9, there are 4 integers greater than 5, i.e. 6, 7, 8 and 9.

P(the number in column B is greater than 5) $= \frac{4}{10}$

$= \mathbf{\frac{2}{5}}$

14. (i) Total number of possible outcomes = 38

P(student chosen is a girl who did not check in her luggage)

$= \frac{8}{38}$

$= \mathbf{\frac{4}{19}}$

(ii) Number of girls = 38 – 18

= 20

Number of girls who checked in their luggage

= 20 – 8

= 12

Number of students who checked in their luggage

= 6 + 12

= 18

P(student chosen has checked in his/her luggage)

$= \frac{18}{38}$

$= \mathbf{\frac{9}{19}}$

15. (a) (i) Total number of possible outcomes = 16 + 24

= 40

P(student chosen is a boy) $= \frac{16}{40}$

$= \mathbf{\frac{2}{5}}$

(ii) Number of students who are left-handed = 3 + 2

= 5

P(student chosen is left-handed) $= \frac{5}{40}$

$= \mathbf{\frac{1}{8}}$

(b) (i) Total number of possible outcomes = 40 – 1

= 39

P(student chosen is a boy who is left-handed) $= \frac{3}{39}$

$= \mathbf{\frac{1}{13}}$

(ii) Number of girls remaining who are right-handed

= 24 – 2 – 1

= 21

P(student chosen is a girl who is right-handed) $= \frac{21}{39}$

$= \mathbf{\frac{7}{13}}$

16. (i) Number of pairs of socks in the bin which are yellow

$= \frac{2}{9} \times 117$

= **26**

(ii) P(pair of socks selected is neither yellow nor grey)

$= 1 - \frac{2}{9} - \frac{3}{13}$

$= \frac{117}{117} - \frac{26}{117} - \frac{27}{117}$

$= \frac{64}{117}$

Number of pairs of socks in the bin that are neither yellow nor grey

$= \frac{64}{117} \times 117$

= **64**

17. (i) Total number of possible outcomes = 80

There are 9 questions from 1 to 80 with a single digit question number, i.e. integers from 1 to 9.

P(question number of question selected contains only a single digit)

$= \mathbf{\frac{9}{80}}$

(ii) There are 13 numbers from 1 to 80 that are greater than 67, i.e. integers from 68 to 80.

P(question number of question selected is greater than 67)

$= \mathbf{\frac{13}{80}}$

(iii) There are 16 numbers from 1 to 80 that contain exactly one '7', i.e. 7, 17, 27, 37, 47, 57, 67, 70, 71, 72, 73, 74, 75, 76, 78, 79.

P(question number of question selected contains exactly one '7')

$= \frac{16}{80}$

$= \mathbf{\frac{1}{5}}$

(iv) A number that is divisible by both 2 and 5 is a multiple of 10.

There are 8 numbers from 1 to 80 that are multiples of 10, i.e. 10, 20, 30, 40, 50, 60, 70 and 80.

P(question number of question selected is divisible by both 2 and 5)

$= \frac{8}{80}$

$= \mathbf{\frac{1}{10}}$

18. (i) Sample space = {23, 25, 27, 32, 35, 37, 52, 53, 57, 72, 73, 75}

Total number of possible outcomes = 12

There are 3 numbers in the sample space that are divisible by 4, i.e. 32, 52 and 72.

P(two-digit number is divisible by 4) $= \frac{3}{12}$

$= \mathbf{\frac{1}{4}}$

(ii) There are 4 prime numbers in the sample space, i.e. 23, 37, 53 and 73.

P(two-digit number is a prime number) $= \frac{4}{12}$

$= \mathbf{\frac{1}{3}}$

19. P(getting a '3') = 2 × P(getting a '1')

P(getting a '2') = 3 × P(getting a '3')

= 6 × P(getting a '1')

P(getting a '4') = P(getting a '2')

= 6 × P(getting a '1')

P(getting a '1') + P(getting a '2') + P(getting a '3') + P(getting a '4') = 1

15 × P(getting a '1') = 1

P(getting a '1') $= \frac{1}{15}$

Of the 4 possible outcomes, '2' and '3' are prime numbers.
P(getting a prime number)
= P(getting a '2' or '3')
= 6 × P(getting a '1') + 2 × P(getting a '1')
= 8 × P(getting a '1')
$= 8 \times \frac{1}{15}$
$= \mathbf{\frac{8}{15}}$

20. There are 11 two-digit numbers that are divisible by 8, i.e. 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96.

Given that P(number selected is not divisible by 8) = $\frac{33}{38}$, then

P(number selected is divisible by 8) $= 1 - \frac{33}{38}$
$= \frac{5}{38}$

Since the number of multiples of 8 in the sample space must be an integer, then the size of the sample space can either be 38 or 76.

When the size of the sample space is 38,

Number of multiples of 8 in the sample space $= \frac{5}{38} \times 38$
$= 5$

Number of two-digit numbers in the sample space that are not divisible by 8
= 38 − 5
= 33

∴ a possible sample space = **{16, 17, 18, 19, …, 53}**

12.5 Further examples of probability of single events

Practise Now 10

(i) P(point selected lies in the red sector)
$= \frac{\text{area of red sector}}{\text{area of circle}}$
$= \frac{\text{angle of red sector}}{\text{angle of circle}}$
$= \frac{135°}{360°}$
$= \mathbf{\frac{3}{8}}$

(ii) Angle of the pink sector = 360° − 90° − 135° − 45° − 30°
= 60°

P(point selected lies in the pink sector)
$= \frac{\text{area of pink sector}}{\text{area of circle}}$
$= \frac{\text{angle of pink sector}}{\text{angle of circle}}$
$= \frac{60°}{360°}$
$= \mathbf{\frac{1}{6}}$

(iii) P(point selected lies in the yellow sector) = **0**

(iv) Sum of angles of blue and orange sector = 30° + 45°
= 75°

P(point selected lies in the blue or orange sector)
$= \frac{\text{area of blue sector} + \text{area of orange sector}}{\text{area of circle}}$
$= \frac{\text{angle of blue sector} + \text{angle of orange sector}}{\text{angle of circle}}$
$= \frac{75°}{360°}$
$= \mathbf{\frac{5}{24}}$

Practise Now 11

1. (i) Total number of balls in the box = 12 + (x + 2)
= $\mathbf{14 + x}$

(ii) P(drawing a yellow ball) = $\mathbf{\frac{x+2}{14+x}}$

(iii) Given that $\frac{x+2}{14+x} = \frac{2}{5}$,
$5(x + 2) = 2(14 + x)$
$5x + 10 = 28 + 2x$
$3x = 18$
$x = \mathbf{6}$

2. Total number of students remaining in the school hall
$= 28 + 25 - y$
$= 53 - y$
Number of girls remaining in the school hall = $25 - y$
P(selecting a girl) = $\frac{25-y}{53-y}$

Given that $\frac{25-y}{53-y} = \frac{3}{7}$,
$7(25 - y) = 3(53 - y)$
$175 - 7y = 159 - 3y$
$-4y = -16$
$y = \mathbf{4}$

12.6 Experimental approach to finding probability

Investigation (Tossing a coin)

1. (i) *Teachers may guide the students to fill in the necessary information in the table.*
(ii) The results of each student are likely to be different. The results obtained are those of a random experiment.

2. (i) *Teachers may guide the students to fill in the necessary information in the table.*
(ii) *Teachers may guide the students to fill in the necessary information in the table.*

3. When the number of tosses increases, the relative frequency of obtaining a 'head' or 'tail' will generally approach the theoretical value of $\frac{1}{2}$.

4. **No**. The relative frequency of obtaining a 'head' or a 'tail' in a probability experiment is not always equal to the theoretical value, in this case $\frac{1}{2}$.

5. **(i)** **No**. When the number of tosses is small, it is still possible for the relative frequency of obtaining a 'head' to be equal to the theoretical value of $\frac{1}{2}$, e.g. see the point (100, 0.5) in Fig. 12.2.

(ii) **No**. For more tosses, the difference between the fraction of obtaining a 'head' and $\frac{1}{2}$ can become smaller or larger. But the fraction of obtaining a 'head' will approach $\frac{1}{2}$ for more tosses.

Exercise 12D

1. **(i)** $$\begin{aligned} \text{P(student selected prefers apple)} &= \frac{\text{area of red sector}}{\text{area of circle}} \\ &= \frac{\text{angle of red sector}}{\text{angle of circle}} \\ &= \frac{150°}{360°} \\ &= \boldsymbol{\frac{5}{12}} \end{aligned}$$

(ii) $$\begin{aligned} \text{Angle of the orange sector} &= 360° - 150° - 90° - 45° \\ &= 75° \end{aligned}$$

P(student selected prefers mango)

$$\begin{aligned} &= \frac{\text{area of orange sector}}{\text{area of circle}} \\ &= \frac{\text{angle of orange sector}}{\text{angle of circle}} \\ &= \frac{75°}{360°} \\ &= \boldsymbol{\frac{5}{24}} \end{aligned}$$

(iii) $$\begin{aligned} \text{Sum of angles of blue and green sector} &= 90° + 45° \\ &= 135° \end{aligned}$$

P(student selected prefers papaya or guava)

$$\begin{aligned} &= \frac{\text{area of blue sector} + \text{area of green sector}}{\text{area of circle}} \\ &= \frac{\text{angle of blue sector} + \text{angle of green sector}}{\text{angle of circle}} \\ &= \frac{135°}{360°} \\ &= \boldsymbol{\frac{3}{8}} \end{aligned}$$

2. **(i)** $$\begin{aligned} \text{P(point lies in region } R) &= \frac{\text{area of region } R}{\text{area of octagon}} \\ &= \boldsymbol{\frac{1}{8}} \end{aligned}$$

(ii) $$\begin{aligned} \text{P(point lies in region } S) &= \frac{\text{area of region } S}{\text{area of octagon}} \\ &= \boldsymbol{\frac{3}{8}} \end{aligned}$$

(iii) P(point lies in region P or Q)

$$\begin{aligned} &= \frac{\text{area of region } P + \text{area of region } Q}{\text{area of octagon}} \\ &= \frac{4}{8} \\ &= \boldsymbol{\frac{1}{2}} \end{aligned}$$

3. **(i)** Total number of students at the school parade square $= \boldsymbol{15 + x}$

(ii) P(student selected is a girl) $= \boldsymbol{\frac{15}{15+x}}$

(iii) Given that $\frac{15}{15+x} = \frac{1}{5}$,

$$\begin{aligned} 5(15) &= 15 + x \\ 75 &= 15 + x \\ x &= \mathbf{60} \end{aligned}$$

4. $$\begin{aligned} \text{Total number of presents} &= (3h + 11) + (h + 5) \\ &= 4h + 16 \end{aligned}$$

P(Albert obtains a red present) $= \frac{3h+11}{4h+16}$

Given that $\frac{3h+11}{4h+16} = \frac{19}{26}$,

$$\begin{aligned} 26(3h + 11) &= 19(4h + 16) \\ 78h + 286 &= 76h + 304 \\ 2h &= 18 \\ h &= \mathbf{9} \end{aligned}$$

5. Let A be the event that there is no change in bone mass density.
Let B be the event that there is a slight reduction in bone mass density.
Let C be the event that there is a significant reduction in bone mass density.

$$\begin{aligned} P(A) + P(B) + P(C) &= 1 \\ \frac{7}{13} + \frac{1}{k} + \frac{1}{2k} &= 1 \\ \frac{14k+26+13}{26k} &= 1 \\ 14k + 26 + 13 &= 26k \\ -12k &= -39 \\ k &= \boldsymbol{\frac{13}{4}} \end{aligned}$$

6. $$\begin{aligned} \text{Total number of toothbrushes} &= 15 + 5 \\ &= 20 \end{aligned}$$

P(draw a toothbrush with soft bristles) $= \frac{p+5}{20}$

Given that $\frac{p+5}{20} = \frac{3}{4}$,

$$\begin{aligned} 4(p + 5) &= 3(20) \\ 4p + 20 &= 60 \\ 4p &= 40 \\ p &= \mathbf{10} \end{aligned}$$

7. Total number of students remaining on the track and field team
$= 23 + 35 - q - (q + 4)$
$= 54 - 2q$
Number of boys remaining on the track and field team
$= 23 - q$

P(selecting a boy to represent the school) $= \frac{23-q}{54-2q}$

Given that $\frac{23-q}{54-2q} = \frac{2}{5}$,

$$\begin{aligned} 5(23 - q) &= 2(54 - 2q) \\ 115 - 5q &= 108 - 4q \\ -q &= -7 \\ q &= \mathbf{7} \end{aligned}$$

8. Total number of marbles in bottle
$= 2 + 8 + 10 + x$
$= 20 + x$

P(draw a red marble) $= \dfrac{2+x}{20+x}$

Given that $\dfrac{2+x}{20+x} \geqslant \dfrac{1}{2}$ and that x is a positive integer,

let $\dfrac{2+x}{20+x} = \dfrac{1}{2}$.

$2(2 + x) = 20 + x$
$4 + 2x = 20 + x$
$x = 16$

$\therefore$ $x \geqslant 16$ and a possible value of x is **18**.

9. P(point lies in the triangle MAN) $= \dfrac{\text{area of triangle } MAN}{\text{area of square } ABCD}$

$= \dfrac{\frac{1}{2}\times\frac{1}{2}AB\times\frac{1}{2}AD}{AB\times AD}$

$= \mathbf{\dfrac{1}{8}}$

10. (i) P(draw a black ball)
$= 1 -$ P(draw a ball that is not black)
$= 1 -$ P(draw a red ball) $-$ P(draw a yellow ball)
$= 1 - \dfrac{1}{4} - \dfrac{2}{5}$
$= \dfrac{20}{20} - \dfrac{5}{20} - \dfrac{8}{20}$
$= \mathbf{\dfrac{7}{20}}$

(ii) Total number of balls in bag now
$= 40 + (2x + 1) + (x + 2) - (x - 3)$
$= \mathbf{2x + 46}$

(iii) Initial number of yellow balls $= \dfrac{2}{5} \times 40$
$= 16$

P(draw yellow ball) $= \dfrac{16+(x+2)}{2x+46}$

$= \dfrac{x+18}{2x+46}$

Given that $\dfrac{x+18}{2x+46} = \dfrac{3}{7}$,

$7(x + 18) = 3(2x + 46)$
$7x + 126 = 6x + 138$
$x = 12$

$\therefore$ number of yellow balls in the bag now $= 16 + (x + 2)$
$= 16 + 12 + 2$
$= \mathbf{30}$

11. Initial number of students in the auditorium,
$2x + y = 50$ — (1)
Total number of students in the auditorium now
$= 50 - (y - 6) + (2x - 5)$
$= 51 + 2x - y$

P(selecting a girl) $= \dfrac{y+(2x-5)}{51+2x-y}$

$= \dfrac{y+2x-5}{51+2x-y}$

Given that $\dfrac{y+2x-5}{51+2x-y} = \dfrac{9}{13}$,

$13(y + 2x - 5) = 9(51 + 2x - y)$
$13y + 26x - 65 = 459 + 18x - 9y$
$8x + 22y = 524$ — (2)

From (1), $8x + 4y = 200$ — (3)

(2) – (3): $8x + 22y - (8x + 4y) = 524 - 200$
$18y = 324$
$y = 18$

Substitute $y = 18$ into (1):
$2x + 18 = 50$
$2x = 32$
$x = 16$

$\therefore$ $\boldsymbol{x = 16}$ and $\boldsymbol{y = 18}$

Chapter 13 Histograms

TEACHING NOTES

Suggested Approach

In primary school and secondary one, students have explored the use of statistical diagrams such as pictograms, bar graphs, pie charts and line graphs.

In this chapter, students will learn about histograms. Teachers are encouraged to begin the chapter by reviewing the statistical diagrams that students have learnt (refer to Introductory Problem on page 388). Students should also be reminded about the difference between categorical and numerical data. Teachers can help students understand that bar graphs, pictograms, and pie charts are suitable for representing categorical data, while line graphs are used for certain types of numerical data. Additionally, a histogram, which students will learn in this chapter, is the most appropriate statistical diagram for representing the data set shown in Table 13.1.

Section 13.1 Histograms for ungrouped data

Teachers can introduce the construction of histograms using Worked Example 1. The students can be grouped to discuss the purpose of histograms (see Class Discussion: Purposes and appropriateness of histograms for ungrouped data). Students should be made aware of the potential misinterpretation of data. It is also important for students to understand that histograms and bar graphs, while similar in appearance, represent data in different ways.

Section 13.2 Histograms for grouped data

It is important to highlight the differences between ungrouped data and grouped data from the start. Grouped data differs from ungrouped data in that it uses class intervals to group similar data together. In this section, students must be given the opportunity to create frequency tables. They should also investigate how the choice of class intervals for grouping the data affects the shape of the histogram, and consequently affects the interpretation and conclusion (see Thinking Time on page 394).

The last activity in this section exposes students to the use of histograms for grouped data with unequal class intervals (see Class Discussion: Histograms for Grouped Data with Unequal Class Intervals). A key takeaway from this activity is that the frequency is represented by the area of each column, and students should take care not to read off the vertical axis when analysing the data. Teachers may inform students that most questions of histograms are those with equal class intervals.

Introductory Problem

1. Name of diagram: **Bar graph**
   Type of data: **Categorical data**
   Name of diagram: **Pie chart**
   Type of data: **Categorical data**
   Name of diagram: **Line graph**
   Type of data: **Numerical data**
2.

	Advantage	Disadvantage
Pictogram	It is colourful and appealing.	The actual frequency in each category may be distorted or misinterpreted if a fraction of a picture is not drawn accurately.
Bar graph	It is easy to draw and will not be distorted based on different sizes of pictures.	The data may be distorted or misinterpreted if the frequency axis does not start from zero.
Pie chart	It facilitates the comparison of the relative size of each category against the whole.	It is difficult to compare sectors that are about the same size and will look cluttered if there are too many categories.
Line graph	It allows the observation of the rising or falling trend of a set of numerical data over a period of time.	The data may be distorted and misinterpreted as the vertical axis may not start from zero. On the contrary, having the vertical axis start at zero will likely prevent the observation of minor fluctuations in the data.

3. **(i)** It displays **numerical data**.
   (ii) The data set does not show a variation in data over a period of time, unlike that in the line graph in Fig. 13.1. Instead, it shows individual data values that are not related to one another.
   (ii) **No**, the four statistical diagrams are not suitable to represent the data set. Since the data are not categorical data, a pictogram, bar graph and pie chart cannot be used. A line graph is also not suitable, as explained in part **(ii)**.

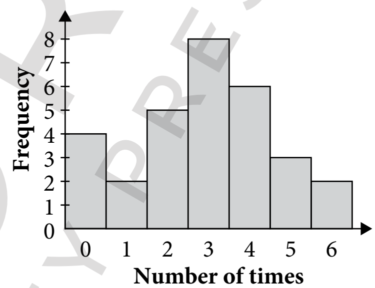
13.1 Histograms for ungrouped data

Practise Now 1

(i)

Number of times	Tally	Frequency
0	||||	4
1	||	2
2	~~||||~~	5
3	~~||||~~ |||	8
4	~~||||~~ |	6
5	|||	3
6	||	2
Total frequency		30

(ii)



(iii) The most common number of times is **3**.
(iv) Number of students who drank bubble tea $\leqslant$ 3 times
$= 4 + 2 + 5 + 8$
$= 19$
Required fraction $= \mathbf{\frac{19}{30}}$

Class Discussion (Purposes and appropriateness of histogram for ungrouped data)

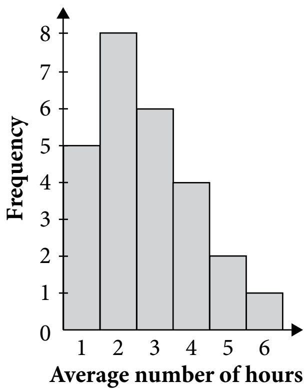
1. Both a histogram and a bar chart can be used to show the frequency of a certain variable, which is represented by a rectangle. However, the values of the horizontal axis in a histogram have to be arranged in a certain order, while the categories of a bar chart do not have to be arranged in any ascending or descending order. There are spaces between the rectangles of a bar chart while there are no spaces between the rectangles in a histogram.
2. **No**, he is not correct. From the graph, there are twice, and not thrice, as many households which own two smartphones (200) as that which own one smartphone (100). Albert misinterpreted the data as he took the height of the bar to be proportional to the frequency. However, it must be noted that the vertical axis starts from 50 and not 0.

Exercise 13A

1. (i)

Average number of hours	Tally	Frequency
1	𝍸	5
2	𝍸 |||	8
3	𝍸		6
4						4
5				2
6			1
Total frequency		26

(ii)



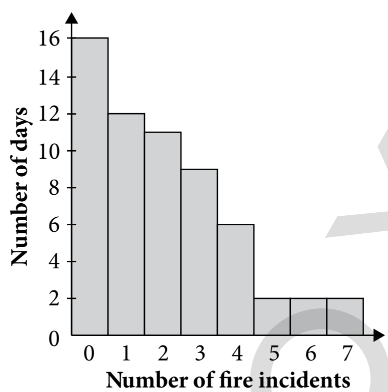
(iii) The most common average number of hours is **2 hours**.

(iv) Number of teachers who participated in the survey = **26**

(v) Fraction of teachers who spend an average of 3 hours $= \frac{6}{26}$

$= \mathbf{\frac{3}{13}}$

2. (i)



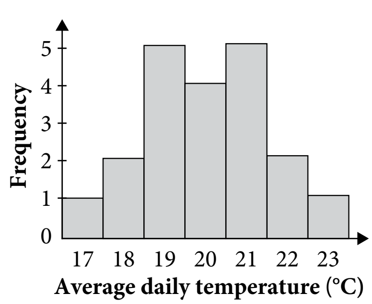
(ii) Required ratio = 16 : (2 + 2)

= 16 : 4

= **4 : 1**

(iii) **No**. The most common number of fire incidents is 0, of which 16 days out of 60 days had 0 fire incidents.

3. (i)



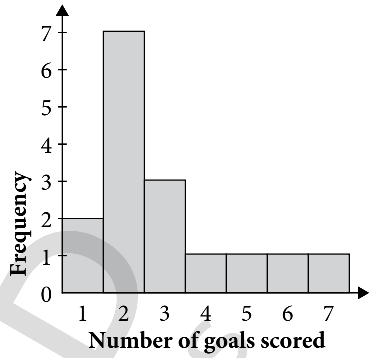
(ii) Percentage of temperatures recorded that are at most 20 °C

$= \frac{12}{20} \times 100\%$

= 60%

(iii) The average daily temperature of a city in autumn for 20 days ranges from 17 °C to 23 °C. The temperatures are symmetrical about 20 °C.

4. (i)



(ii) The most common number of goals scored was **2**.

(iii) Percentage of matches with $\geqslant 3$ goals

$= \frac{7}{16} \times 100\%$

$= \mathbf{43\frac{3}{4}\%}$ or **43.75%**

(iv) Number of matches with $\leqslant 1$ goal in 2018 = 2

Number of matches with $\leqslant 1$ goal in 2010

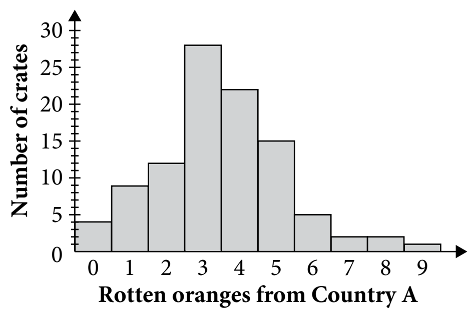
$= \frac{2}{(100-60)\%} \times 100\%$

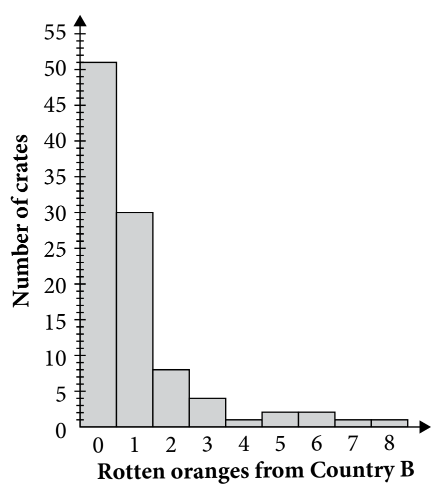
= 5

Number of matches with $>$ 1 goal in 2010 = 16 – 5

= **11**

5. (i)





(ii) Largest number of rotten oranges found in a crate from Country A
= **9**
Largest number of rotten oranges found in a crate from Country B
= **8**

(iii) Total number of rotten oranges from Country A
$= (4 \times 0) + (9 \times 1) + (12 \times 2) + (28 \times 3) + (22 \times 4) + (15 \times 5) + (5 \times 6) + (2 \times 7) + (2 \times 8) + (1 \times 9)$
$= 0 + 9 + 24 + 84 + 88 + 75 + 30 + 14 + 16 + 9$
$= \mathbf{349}$
Total number of rotten oranges from Country B
$= (51 \times 0) + (30 \times 1) + (8 \times 2) + (4 \times 3) + (1 \times 4) + (2 \times 5) + (2 \times 6) + (1 \times 7) + (1 \times 8)$
$= 0 + 30 + 16 + 12 + 4 + 10 + 12 + 7 + 8$
$= \mathbf{99}$

(iv) P(crate contains no fewer than p rotten oranges) $= \frac{3}{4}$
Number of crates with no fewer than p rotten oranges
$= \frac{3}{4} \times 100$
$= 75$
Since $1 + 2 + 2 + 5 + 15 + 22 + 28 = 75$,
$\therefore p = \mathbf{3}$

13.2 Histograms for grouped data

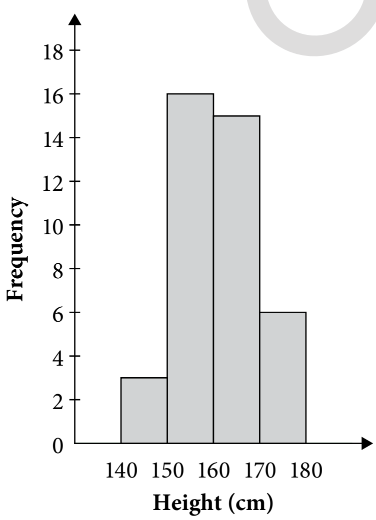
Thinking Time (Page 394)

1. **(i)** The first and last value that falls in each interval is different for the two tables. For example, a data value of 150 cm will be in the second interval of $150 \leqslant x < 160$ in Table 13.6 but in the first interval of $140 < x \leqslant 150$ in Table 13.7.

(ii)

Height (x cm)	Tally	Frequency
$140 < x \leqslant 150$	|||	**3**
$150 < x \leqslant 160$	𝍸 𝍸 𝍸 |	**16**
$160 < x \leqslant 170$	𝍸 𝍸 𝍸	**15**
$170 < x \leqslant 180$	𝍸 |	**6**

(iii)



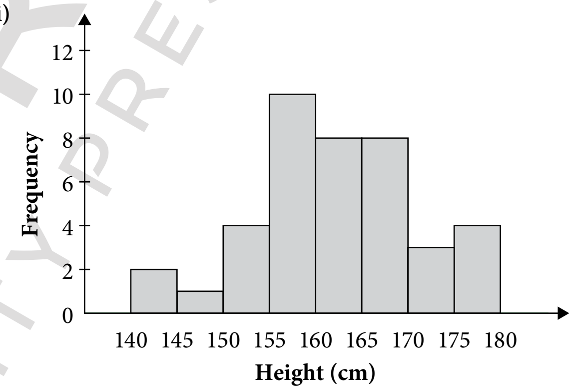
2. **(i)** The values of 160 cm and 170 cm are in different intervals for the two histograms.
(ii) The class interval **160–169** contains the most data values.
(iii) The class interval **151–160** contains the most data values.
(iv) There is **no correct answer** as the way statistics is represented and the grouping of data may affect the interpretation and conclusion.

3. **(i)** The class width of the class intervals in Table 13.8 is 5 cm while those in Table 13.6 is 10 cm.

(ii)

Height (x cm)	Tally	Frequency
$140 \leqslant x < 145$	||	**2**
$145 \leqslant x < 150$	|	**1**
$150 \leqslant x < 155$	||||	**4**
$155 \leqslant x < 160$	𝍸 𝍸	**10**
$\mathbf{160 \leqslant x < 165}$	𝍸 |||	**8**
$\mathbf{165 \leqslant x < 170}$	𝍸 |||	**8**
$\mathbf{170 \leqslant x < 175}$	|||	**3**
$175 \leqslant x < 180$	||||	**4**

(iii)

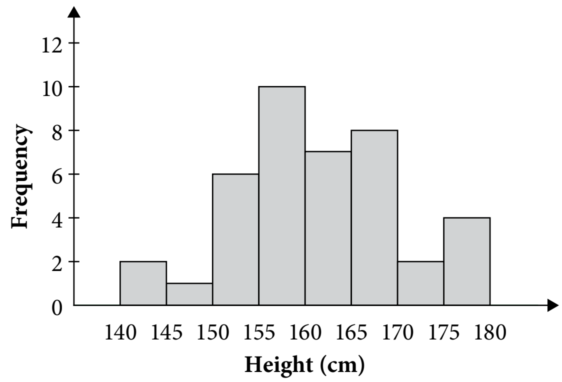


4. **(i)** The frequency in each class interval is the same for the histogram and the stem-and-leaf diagram. However, the stem-and-leaf diagram shows the individual heights, while the histogram does not.
(ii) The class interval **155–159** contains the most data values.
(iii) The answers are **different** as the data values are grouped differently, thus the interpretation of the data and the conclusions drawn are different.

5. **(i)**

Height (x cm)	Tally	Frequency
$140 < x \leqslant 145$	||	**2**
$\mathbf{145 < x \leqslant 150}$	|	**1**
$\mathbf{150 < x \leqslant 155}$	𝍸 |	**6**
$\mathbf{155 < x \leqslant 160}$	𝍸 𝍸	**10**
$\mathbf{160 < x \leqslant 165}$	𝍸 ||	**7**
$\mathbf{165 < x \leqslant 170}$	𝍸 |||	**8**
$\mathbf{170 < x \leqslant 175}$	||	**2**
$175 < x \leqslant 180$	||||	**4**

(ii)



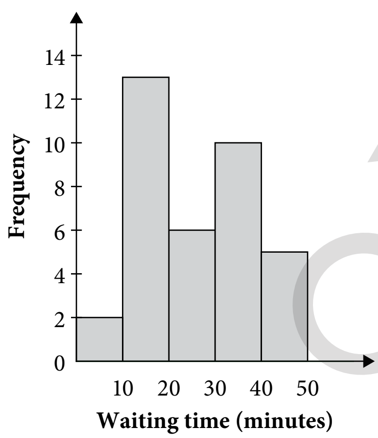
6. (i) The values of 155 cm, 160 cm, 165 cm and 170 cm are in different intervals for the two histograms.
   (ii) The class interval **156–160** contains the most data values.
   (iii) The answers are **different** as the data values are grouped differently, thus the interpretation of the data and the conclusions drawn are different.
7. All the histograms are correct as there is more than one way to construct a histogram for a data set, depending on how the values are grouped.

Practise Now 2A

(i)

Waiting time (x minutes)	Tally	Frequency
$0 \leqslant x < 10$	||	2
$10 \leqslant x < 20$	𝍸 𝍸 |||	13
$20 \leqslant x < 30$	𝍸 |	6
$30 \leqslant x < 40$	𝍸 𝍸	10
$40 \leqslant x < 50$	𝍸	5

(ii)



(iii) The class interval **$10 \leqslant x < 20$** contains the most data values.

Class Discussion (Histogram for grouped data with unequal class intervals)

Part 1:

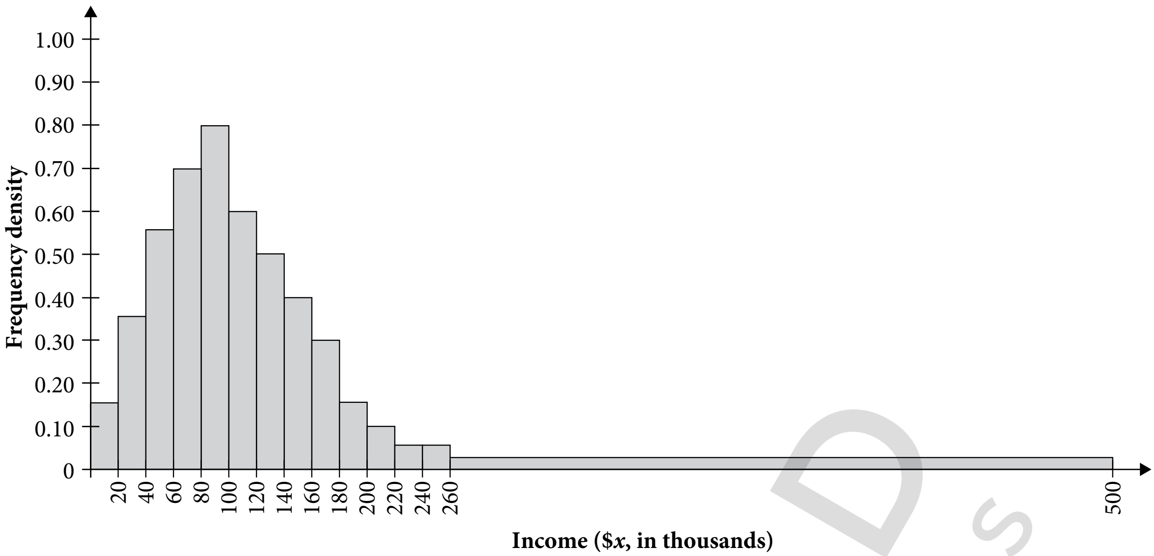
1. **No**, it is not a suitable representation. Constructing the column as such makes it seem like there are many households whose incomes fall in the class interval $260 \leqslant x < 500$ as the rectangle looks much bigger than the other class intervals.
2. The number of households is divided by the number of equal class intervals that are combined to form the last class interval of $260 \leqslant x < 500$, which is 12.
3. In statistics, it is possible to have a fraction of a household.
4. **Yes**, the histogram in Fig. 13.9 is a more suitable representation of the data than that in Fig. 13.8. The area of each column is proportional to the frequency of the class interval.

Part 2:

5.

Income (\$$x$, in thousands)	Frequency (No. of households)	Frequency density $= \dfrac{\text{Frequency}}{\text{Size of class interval}}$
$0 \leqslant x < 20$	3	$\frac{3}{20} = 0.15$
$20 \leqslant x < 40$	7	$\frac{7}{20} = 0.35$
$40 \leqslant x < 60$	11	$\frac{11}{20} = \mathbf{0.55}$
$60 \leqslant x < 80$	14	$\frac{14}{20} = \mathbf{0.70}$
$80 \leqslant x < 100$	16	$\frac{16}{20} = \mathbf{0.80}$
$100 \leqslant x < 120$	12	$\frac{12}{20} = \mathbf{0.60}$
$120 \leqslant x < 140$	10	$\frac{10}{20} = \mathbf{0.50}$
$140 \leqslant x < 160$	8	$\frac{8}{20} = \mathbf{0.40}$
$160 \leqslant x < 180$	6	$\frac{6}{20} = \mathbf{0.30}$
$180 \leqslant x < 200$	3	$\frac{3}{20} = \mathbf{0.15}$
$200 \leqslant x < 220$	2	$\frac{2}{20} = \mathbf{0.10}$
$220 \leqslant x < 240$	1	$\frac{1}{20} = \mathbf{0.05}$
$240 \leqslant x < 260$	1	$\frac{1}{20} = \mathbf{0.05}$
$260 \leqslant x < 500$	6	$\frac{6}{240} = \mathbf{0.025}$

6.



7.

Income ($x, in thousands)	**Frequency (No. of households)**	**Frequency density** $= \dfrac{\textbf{Frequency}}{\textbf{Size of class interval}}$	**Area of rectangle**
$0 \leqslant x < 20$	3	$\frac{3}{20} = \mathbf{0.15}$	$20 \times 0.15 = \mathbf{3}$
$20 \leqslant x < 40$	7	$\frac{7}{20} = \mathbf{0.35}$	$20 \times 0.35 = \mathbf{7}$
$40 \leqslant x < 60$	11	$\frac{11}{20} = \mathbf{0.55}$	$20 \times 0.55 = \mathbf{11}$
$60 \leqslant x < 80$	14	$\frac{14}{20} = \mathbf{0.70}$	$20 \times 0.70 = \mathbf{14}$
$80 \leqslant x < 100$	16	$\frac{16}{20} = \mathbf{0.80}$	$20 \times 0.80 = \mathbf{16}$
$100 \leqslant x < 120$	12	$\frac{12}{20} = \mathbf{0.60}$	$20 \times 0.60 = \mathbf{12}$
$120 \leqslant x < 140$	10	$\frac{10}{20} = \mathbf{0.50}$	$20 \times 0.50 = \mathbf{10}$
$140 \leqslant x < 160$	8	$\frac{8}{20} = \mathbf{0.40}$	$20 \times 0.40 = \mathbf{8}$
$160 \leqslant x < 180$	6	$\frac{6}{20} = \mathbf{0.30}$	$20 \times 0.30 = \mathbf{6}$
$180 \leqslant x < 200$	3	$\frac{3}{20} = \mathbf{0.15}$	$20 \times 0.15 = \mathbf{3}$
$200 \leqslant x < 220$	2	$\frac{2}{20} = \mathbf{0.10}$	$20 \times 0.15 = \mathbf{2}$
$220 \leqslant x < 240$	1	$\frac{1}{20} = \mathbf{0.05}$	$20 \times 0.05 = \mathbf{1}$
$240 \leqslant x < 260$	1	$\frac{1}{20} = \mathbf{0.05}$	$20 \times 0.05 = \mathbf{1}$
$260 \leqslant x < 500$	6	$\frac{6}{240} = \mathbf{0.025}$	$240 \times 0.025 = \mathbf{6}$

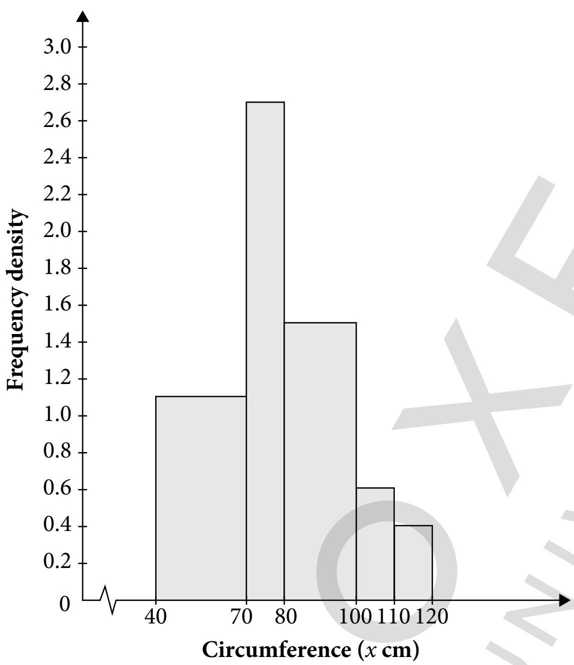
The area of each rectangle is the same value as the frequency in each class interval.

8. The histogram with unequal class intervals is preferred over the histogram with equal class interval because the use of unequal class intervals ensures that no classes have zero frequency. Thus, gaps between classes are eliminated, providing a more accurate representation of the general shape of data distribution.
9. Some examples where unequal class intervals are used in histograms, beside income, are mass, height, temperature, age etc, where there are extreme values and gaps in between the data.

Practise Now 2B

(a)

Circumference (x cm)	Frequency	Class width	Frequency density
$40 \leqslant x < 70$	3	**30**	$\frac{33}{30} = \mathbf{1.1}$
$70 \leqslant x < 80$	7	**10**	$\frac{27}{10} = \mathbf{2.7}$
$80 \leqslant x < 100$	11	**20**	$\frac{30}{20} = \mathbf{1.5}$
$100 \leqslant x < 110$	14	**10**	$\frac{6}{10} = \mathbf{0.6}$
$110 \leqslant x < 120$	16	**10**	$\frac{4}{10} = \mathbf{0.4}$



(b) Since frequency = frequency density × class width, the number of trees can be estimated by finding the areas of the relevant sections of the histograms.

Estimated frequency between:

- 75 cm and 80 cm circumference $= 2.7 \times 5$
  $= 13.5$
- 80 cm and 97 cm circumference $= 1.5 \times 17$
  $= 25.5$

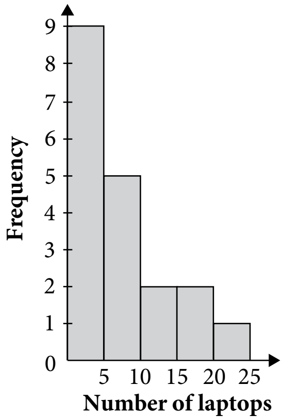
$\therefore$ estimated number of beech trees that have a circumference between 75 cm and 97 cm

$= 13.5 + 25.5$

$= \mathbf{39}$

Exercise 13B

1. **(i)**



(ii) The class interval **$20 \leqslant x < 25$** contains the fewest data values.

(iii) Number of days number of laptops sold $\geqslant 15$
= 2 + 1
= **3** days

2. **(i)**

Time (x hours)	Frequency
$0 \leqslant x < 2$	0
$2 \leqslant x < 4$	30
$4 \leqslant x < 6$	40
$6 \leqslant x < 8$	60
$8 \leqslant x < 10$	20

(ii) The class interval **$6 \leqslant x < 8$** contains the most students.

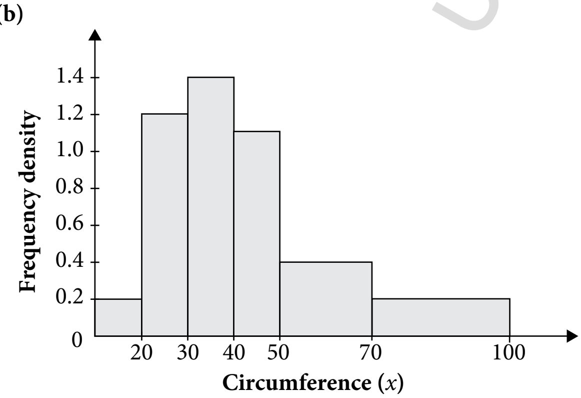
(iii) Number of students who spent an average of more than or equal to 4, but less than 6 hours = **40**

(iv) Total number of students = 30 + 40 + 60 + 20
= **150**

3. **(a)**

Class interval	Frequency	Class width	Frequency density = $\dfrac{\text{Frequency}}{\text{Class width}}$
$0 \leqslant x < 20$	4	20	$4 \div 20 = 0.2$
$20 \leqslant x < 30$	12	**10**	**$12 \div 10 = 1.2$**
$30 \leqslant x < 40$	14	10	$14 \div 10 = 1.4$
$40 \leqslant x < 50$	11	**10**	**$11 \div 10 = 1.1$**
$50 \leqslant x < 70$	8	**20**	**$8 \div 20 = 0.4$**
$70 \leqslant x < 100$	6	**30**	**$6 \div 30 = 0.2$**

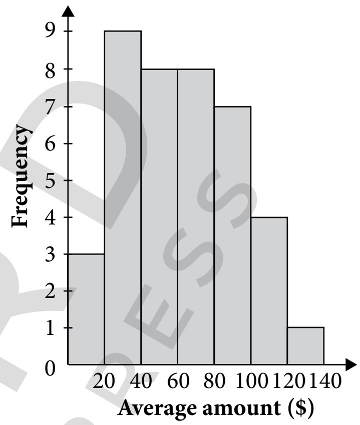
(b)



4. **(i)**

Average amount ($\$x$)	Tally	Frequency
$0 \leqslant x < 20$	|||	3
$20 \leqslant x < 40$	~~||||~~ ||||	9
$40 \leqslant x < 60$	~~||||~~ |||	8
$60 \leqslant x < 80$	~~||||~~ |||	8
$80 \leqslant x < 100$	~~||||~~ ||	7
$100 \leqslant x < 120$	||||	4
$120 \leqslant x < 140$	|	1
Total frequency		40

(ii)



(iii) The class interval **$20 \leqslant x < 40$** contains the most people.

(iv) Required percentage $= \dfrac{20}{40} \times 100\%$
$=$ **50%**

5. **(i)**

Length of service (x years)	Frequency	Class width	Frequency density
$50 \leqslant x < 15$	32	**5**	$\dfrac{32}{5} = \mathbf{6.4}$
$15 \leqslant x < 20$	40	**5**	$\dfrac{40}{5} = \mathbf{8.0}$
$20 \leqslant x < 25$	25	**5**	$\dfrac{25}{5} = \mathbf{5.0}$
$25 \leqslant x < 30$	12	**5**	$\dfrac{12}{5} = \mathbf{2.4}$
$30 \leqslant x < 40$	7	**10**	$\dfrac{7}{10} = \mathbf{0.7}$
$40 \leqslant x < 50$	4	**10**	$\dfrac{4}{10} = \mathbf{0.4}$

(ii) Estimated number of teachers who have been teaching between:

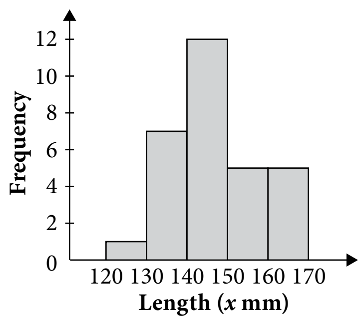
- 26 and 30 years
  $= 2.4 \times 4$
  $= 9.6$
- 30 and 40 years
  $= 7$
- 40 and 46 years
  $= 0.4 \times 6$
  $= 2.4$

$\therefore$ number of teachers who have been teaching between 26 and 46 years
$= 9.6 + 7 + 2.4$
$=$ **19**

6. (i)

Length (x mm)	Tally	Frequency
$120 \leqslant x < 130$	|	1
$130 \leqslant x < 140$	𝍸 ||	7
$140 \leqslant x < 150$	𝍸 𝍸 ||	12
$150 \leqslant x < 160$	𝍸	5
$160 \leqslant x < 170$	𝍸	5
Total frequency		30

(ii)



(iii) Required percentage $= \frac{8}{10} \times 100\%$
$=$ **80%**

OPEN **(iv)** For $130 \leqslant x < 140$ to be the class interval with the most leaves, **at least 6 more leaves in this interval** must have been measured. $\therefore$ a possible set of leaves measured is 130 cm, 131 cm, 132 cm, 133 cm, 134 cm and 135 cm.

Notes