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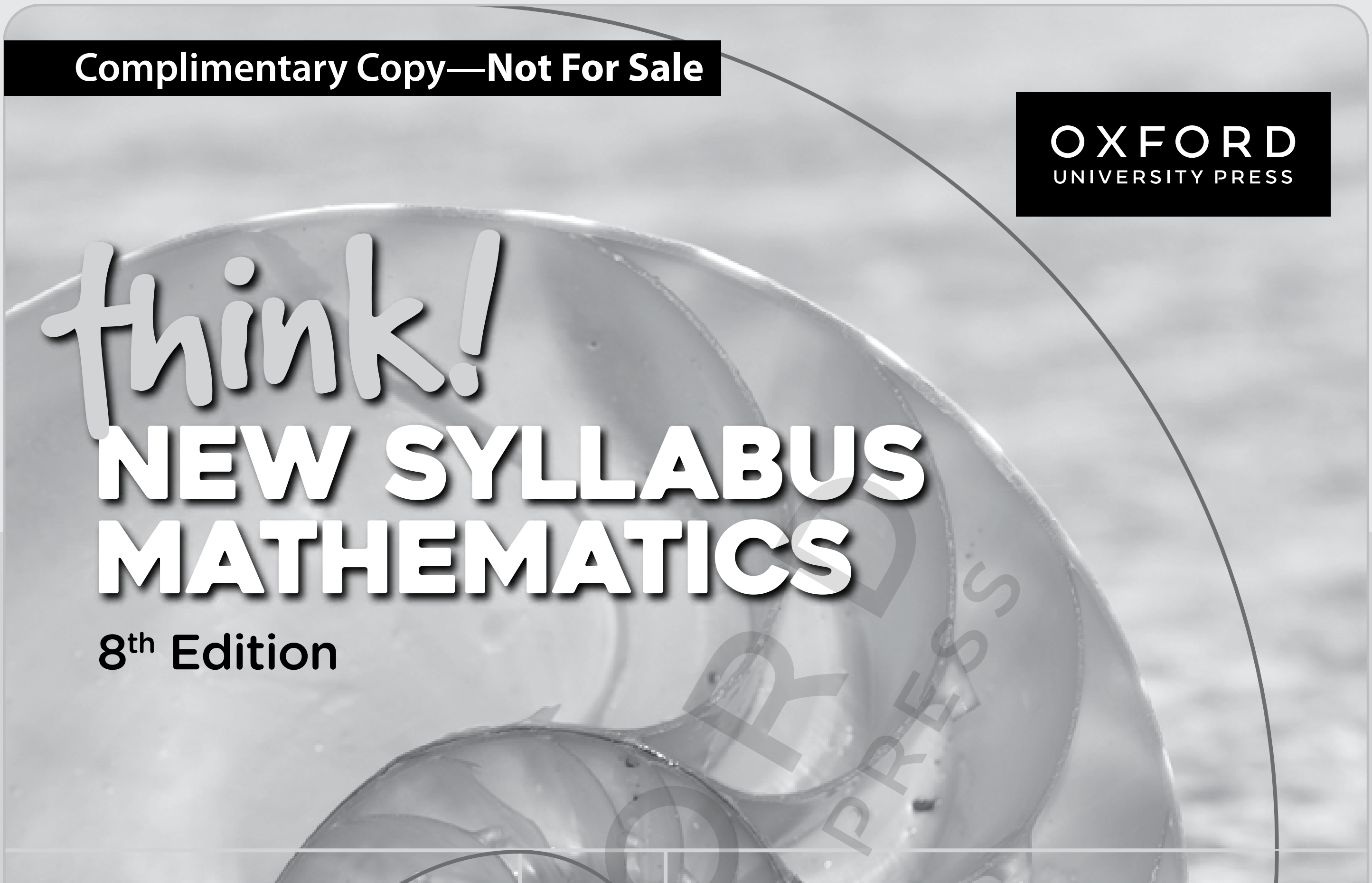


think! NEW SYLLABUS MATHEMATICS

8th Edition

Teacher's Resource Book 1

TV

CONTENTS

Syllabus Matching Grid

Cambridge O Level Mathematics (Syllabus D) 4024. Syllabus for examination in 2025, 2026 and 2027.

Theme or Topic	Subject Content	Reference
1. Number	Identify and use: • Natural numbers • Integers (positive, negative and zero) • Prime numbers • Square numbers • Cube numbers • Common factors and common multiples • Rational and irrational numbers (e.g. π, $\sqrt{2}$) • Reciprocals.	Book 1: Chapter 1 Chapter 2 Chapter 4
2. Sets	• Use set language, set notation and Venn diagrams to describe sets and represent relationships between sets	Book 2: Chapter 12 Book 4: Chapter 1
2. Powers and roots	Calculate with the following: • Squares • Square roots • Cubes and cube roots of numbers • Cube roots • other powers and roots of numbers	Book 1: Chapter 1 Chapter 2 Chapter 3 Chapter 4
4. Fractions, decimals and percentages	• Use the language and notation of the following in appropriate contexts: (a) proper fractions (b) improper fractions (c) mixed numbers (d) decimals (e) percentages. • Recognise equivalence and convert between these forms.	Book 1: Chapter 2 Chapter 3 Chapter 8
5. Ordering	• Order quantities by magnitude and demonstrate familiarity with the symbols $=, \neq, <, >, \leqslant, \geqslant$.	Book 1: Chapter 2 Chapter 3 Chapter 4 Book 2: Chapter 3
6. The four operations	• Use the four operations for calculations with integers, fractions and decimals, including correct ordering of operations and use of brackets.	Book 1: Chapter 2 Chapter 3 Chapter 4
7. Indices I	• Understand and use indices (positive, zero, negative and fractional). • Understand and use the rules of indices.	Book 3: Chapter 4
8. Standard form	• Use the standard form $A \times 10^n$, where n is a positive or negative integer, and $1 \leqslant A < 10$. • Convert numbers into and out of standard form. • Calculate with values in standard form.	Book 3: Chapter 4

9. Estimation	• Round values to a specified degree of accuracy. • Make estimates for calculations involving numbers, quantities and measurements. • Round answers to a reasonable degree of accuracy in the context of a given problem.	Book 1: Chapter 5
10. Limits of accuracy	• Give upper and lower bounds for data rounded to a specified accuracy. • Find upper and lower bounds of the results of calculations which have used data rounded to a specified accuracy.	Book 1: Chapter 5
11. Ratio and proportion	Understand and use ratio and proportion to: • give ratios in their simplest form • divide a quantity in a given ratio • use proportional reasoning and ratios in context.	Book 1: Chapter 9
12. Rates	• Use common measures of rate. • Apply other measures of rate. • Solve problems involving average speed.	Book 1: Chapter 9
13. Percentages	• Calculate a given percentage of a quantity. • Express one quantity as a percentage of another. • Calculate percentage increase or decrease. • Calculate with simple and compound interest. • Calculate using reverse percentages.	Book 1: Chapter 8
14. Using a calculator	• Use a calculator efficiently. • Enter values appropriately on a calculator. • Interpret the calculator display appropriately.	Book 1: Chapter 1 Chapter 4 Chapter 12
15. Time	• Calculate with time: seconds (s), minutes (min), hours (h), days, weeks, months, years, including the relationship between units. • Calculate times in terms of the 24-hour and 12-hour clock. • Read clocks and timetables.	Book 1: Chapter 9
16. Money	• Calculate with money. • Convert from one currency to another.	Book 1: Chapter 9 Book 2: Chapter 6
17. Exponential growth and decay	• Use exponential growth and decay.	Book 3: Chapter 4
18. Surds	• Understand and use surds, including simplifying expressions. • Rationalise the denominator.	Book 3: Chapter 4
19. Introduction to algebra	• Know that letters can be used to represent generalised numbers. • Substitute numbers into expressions and formulas.	Book 1: Chapter 6
20. Algebraic manipulation	• Simplify expressions by collecting like terms. • Expand products of algebraic expressions. • Factorise by extracting common factors. • Factorise expressions of the form: (a) $ax + bx + kay + kby$ (b) $a^2x^2 - b^2y^2$ (c) $a^2 + 2ab + b^2$ (d) $ax^2 + bx + c$ (e) $ax^2 + bx^2 + cx$ • Complete the square for expressions in the form $ax^2 + bx + c$.	Book 1: Chapter 6 Book 2: Chapter 4

21. Algebraic fractions	• Manipulate algebraic fractions. • Factorise and simplify rational expressions.	Book 1: Chapter 6 Book 3: Chapter 1
22. Indices II	• Understand and use indices (positive, zero, negative and fractional). • Understand and use the rules of indices.	Book 3: Chapter 4
23. Equations	• Construct expressions, equations and formulas. • Solve linear equations in one unknown. • Solve fractional equations with numerical and linear algebraic denominators. • Solve simultaneous linear equations in two unknowns. • Solve quadratic equations by factorisation, completing the square and by use of the quadratic formula. • Change the subject of formulas.	Book 1: Chapter 7 Book 2: Chapter 2 Book 3: Chapter 1 Chapter 2 Chapter 3
24. Inequalities	• Represent and interpret inequalities, including on a number line. • Construct, solve and interpret linear inequalities. • Represent and interpret linear inequalities in two variables graphically. • List inequalities that define a given region.	Book 2: Chapter 3
25. Sequences	• Continue a given number sequence or pattern. • Recognise patterns in sequences, including the term-to-term rule, and relationships between different sequences. • Find and use the nth term of sequences.	Book 2: Chapter 5
26. Proportion	• Express direct and inverse proportion in algebraic terms and use this form of expression to find unknown quantities.	Book 2: Chapter 7
27. Graphs in practical situations	• Use and interpret graphs in practical situations including travel graphs and conversion graphs. • Draw graphs from given data. • Apply the idea of rate of change to simple kinematics involving distance-time and speed-time graphs, acceleration and deceleration. • Calculate distance travelled as area under a speed-time graph.	Book 2: Chapter 1 Book 3: Chapter 6
28. Graphs of functions	• Construct tables of values, and draw, recognise and interpret graphs for functions of the following forms: (a) ax^n (includes sums of no more than three of these) (b) $ab^x + c$ where $n = -2, -1, -\frac{1}{2}, 0, \frac{1}{2}, 1, 2, 3$; a and c are rational numbers; and b is a positive integer. • Solve associated equations graphically, including finding and interpreting roots by graphical methods. • Draw and interpret graphs representing exponential growth and decay problems. • Estimate gradients of curves by drawing tangents.	Book 3: Chapter 6
29. Sketching curves	• Recognise, sketch and interpret graphs of the following functions: (a) linear (b) quadratic (c) cubic (d) reciprocal (e) exponential.	Book 2: Chapter 1 Book 3: Chapter 2 Chapter 3 Chapter 6

30.	Functions	• Understand functions, domain and range, and use function notation. • Understand and find inverse functions $f^{-1}(x)$. • Form composite functions as defined by $gf(x) = g(f(x))$.	Book 4: Chapter 5
31.	Coordinates	• Use and interpret Cartesian coordinates in two dimensions.	Book 2: Chapter 1
32.	Drawing linear graphs	• Draw straight-line graphs for linear equations.	Book 2: Chapter 1
33.	Gradient of linear graphs	• Find the gradient of a straight line. • Calculate the gradient of a straight line from the coordinates of two points on it.	Book 2: Chapter 1 Book 3: Chapter 5
34.	Length and midpoint	• Calculate the length of a line segment. • Find the coordinates of the midpoint of a line segment.	Book 3: Chapter 5
35.	Equations of linear graphs	• Interpret and obtain the equation of a straight-line graph.	Book 2: Chapter 1 Book 3: Chapter 5
36.	Parallel lines	• Find the gradient and equation of a straight line parallel to a given line.	Book 3: Chapter 5
37.	Perpendicular lines	• Find the gradient and equation of a straight line perpendicular to a given line.	Book 3: Chapter 5
38.	Geometrical terms	• Use and interpret the following geometrical terms: (a) point (b) vertex (c) line (d) plane (e) parallel (f) perpendicular (g) perpendicular bisector (h) bearing (i) right angle (j) acute, obtuse and reflex angles (k) interior and exterior angles (l) similar (m) congruent (n) scale factor. • Use and interpret the vocabulary of: (a) triangles (b) special quadrilaterals (c) polygons (d) nets (e) solids. • Use and interpret the vocabulary of a circle.	Book 1: Chapter 10 Chapter 11 Book 2: Chapter 8 Chapter 11
39.	Geometrical constructions	• Measure and draw lines and angles. • Construct a triangle, given the lengths of all sides, using a ruler and pair of compasses only. • Draw, use and interpret nets.	Book 1: Chapter 11

40. Scale drawings	• Draw and interpret scale drawings. • Use and interpret three-figure bearings.	Book 2: Chapter 8 Book 4: Chapter 7
41. Similarity	• Calculate lengths of similar shapes. • Use the relationships between lengths and areas of similar shapes and lengths, surface areas and volumes of similar solids. • Solve problems and give simple explanations involving similarity.	Book 2: Chapter 8 Book 4: Chapter 11
42. Symmetry	• Recognise line symmetry and order of rotational symmetry in two dimensions. • Recognise symmetry properties of prisms, cylinders, pyramids and cones.	Book 1: Chapter 11 Book 2: Chapter 11 Book 3: Chapter 7
43. Angles	• Calculate unknown angles and give simple explanations using the following geometrical properties: (a) sum of angles at a point = 360° (b) sum of angles at a point on a straight line = 180° (c) vertically opposite angles are equal (d) angle sum of a triangle = 180° and angle sum of a quadrilateral = 360°. • Calculate unknown angles and give geometric explanations for angles formed within parallel lines: (a) corresponding angles are equal (b) alternate angles are equal (c) co-interior (supplementary) angles sum to 180°. • Know and use angle properties of regular and irregular polygons.	Book 1: Chapter 10 Chapter 11
44. Circle theorems I	• Calculate unknown angles and give explanations using the following geometrical properties of circles: • angle in a semicircle = 90° • angle between tangent and radius = 90° • angle at the centre is twice the angle at the circumference • angles in the same segment are equal • opposite angles of a cyclic quadrilateral sum to 180° (supplementary) • alternate segment theorem.	Book 4: Chapter 9
45. Circle theorems II	• Use the following symmetry properties of circles: • equal chords are equidistant from the centre • the perpendicular bisector of a chord passes through the centre • tangents from an external point are equal in length.	Book 4: Chapter 9
46. Units of measure	• Use metric units of mass, length, area, volume and capacity in practical situations and convert quantities into larger or smaller units.	Book 1: Chapter 3 Chapter 12 Book 2: Chapter 11
47. Area and perimeter	• Carry out calculations involving the perimeter and area of a rectangle, triangle, parallelogram and trapezium.	Book 1: Chapter 12

48. Circles, arcs and sectors	• Carry out calculations involving the circumference and area of a circle. • Carry out calculations involving arc length and sector area as fractions of the circumference and area of a circle.	Book 1: Chapter 12 Book 4: Chapter 8
49. Surface area and volume	Carry out calculations and solve problems involving the surface area and volume of a: • cuboid • prism • cylinder • sphere • pyramid • cone.	Book 2: Chapter 11 Book 3: Chapter 7
50. Compound shapes and parts of shapes	• Carry out calculations and solve problems involving perimeters and areas of: (a) compound shapes (b) parts of shapes. • Carry out calculations and solve problems involving surface areas and volumes of: (a) compound solids (b) parts of solids.	Book 1: Chapter 12 Book 3: Chapter 7
51. Pythagoras' theorem	• Know and use Pythagoras' theorem.	Book 2: Chapter 9
52. Right-angled triangles	• Know and use the sine, cosine and tangent ratios for acute angles in calculations involving sides and angles of a right-angled triangle. • Solve problems in two dimensions using Pythagoras' theorem and trigonometry. • Know that the perpendicular distance from a point to a line is the shortest distance to the line. • Carry out calculations involving angles of elevation and depression.	Book 2: Chapter 9 Chapter 10 Book 4: Chapter 6 Chapter 7
53. Non-right-angled triangles	• Use the sine and cosine rules in calculations involving lengths and angles for any triangle. • Use the formula area of triangle = $\frac{1}{2}$ $ab \sin C$	Book 4: Chapter 6
54. Pythagoras' theorem and trigonometry in 3D	• Carry out calculations and solve problems in three dimensions using Pythagoras' theorem and trigonometry, including calculating the angle between a line and a plane.	Book 4: Chapter 7
55. Transformations	Recognise, describe and draw the following transformations: • Reflection of a shape in a straight line. • Rotation of a shape about a centre through multiples of 90°. • Enlargement of a shape from a centre by a scale factor. • Translation of a shape by a vector $\begin{pmatrix} x \\ y \end{pmatrix}$.	Book 4: Chapter 10
56. Vectors in two dimensions	• Describe a translation using a vector represented by $\begin{pmatrix} x \\ y \end{pmatrix}$, $\overrightarrow{AB}$ or $\mathbf{a}$ • Add and subtract vectors. • Multiply a vector by a scalar.	Book 4: Chapter 7
57. Magnitude of a vector	• Calculate the magnitude of a vectorvector $\begin{pmatrix} x \\ y \end{pmatrix}$ as $\sqrt{x^2 + y^2}$	Book 4: Chapter 4

58. Vector geometry	• Represent vectors by directed line segments. • Use position vectors. • Use the sum and difference of two or more vectors to express given vectors in terms of two coplanar vectors. • Use vectors to reason and to solve geometric problems.	Book 4: Chapter 4
59. Introduction to probability	• Understand and use the probability scale from 0 to 1. • Understand and use probability notation. • Calculate the probability of a single event. • Understand that the probability of an event not occurring = 1 – the probability of the event occurring.	Book 2: Chapter 12
60. Relative and expected frequencies	• Understand relative frequency as an estimate of probability. • Calculate expected frequencies.	Book 2: Chapter 12
61. Probability of combined events	Calculate the probability of combined events using, where appropriate: • sample space diagrams • Venn diagrams • tree diagrams.	Book 4: Chapter 2
62. Classifying statistical data	• Classify and tabulate statistical data.	Book 1: Chapter 13
63. Interpreting statistical data	• Read, interpret and draw inferences from tables and statistical diagrams. • Compare sets of data using tables, graphs and statistical measures. • Appreciate restrictions on drawing conclusions from given data.	Book 1: Chapter 13
64. Averages and measures of spread	• Calculate the mean, median, mode and range for individual data and distinguish between the purposes for which these are used. • Calculate an estimate of the mean for grouped discrete or grouped continuous data. • Identify the modal class from a grouped frequency distribution.	Book 3: Chapter 8
65. Statistical charts and diagrams	• Draw and interpret: (a) bar charts (b) pie charts (c) pictograms (d) simple frequency distributions.	Book 1: Chapter 13
66. Scatter diagrams	• Draw and interpret scatter diagrams. • Understand what is meant by positive, negative and zero correlation. • Draw by eye, interpret and use a straight line of best fit.	Book 4: Chapter 3
67. Cumulative frequency diagrams	• Draw and interpret cumulative frequency tables and diagrams. • Estimate and interpret the median, percentiles, quartiles and interquartile range from cumulative frequency diagrams.	Book 4: Chapter 3
68. Histograms	• Draw and interpret histograms. • Calculate with frequency density.	Book 2: Chapter 13

think! Mathematics Secondary 1 Mathematics Scheme of Work

Week	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Reasoning, Communication and Connection
1	**1 Primes, Highest Common Factor and Lowest Common Multiple**	**1.1 Prime Numbers** (pp. 2–8)	• Explain what a prime number is • Determine whether a whole number is prime • Express a composite number as a product of its prime factors • Express a composite number as a product of its prime factors	Identify and use prime numbers	Investigation – Classifying whole numbers Journal Writing (p. 3) Investigation – Sieve of Eratosthenes (p. 4) Class Discussion – Product of prime numbers (p. 4) Thinking Time (p. 5)		Journal Writing (p. 3) Investigation – Sieve of Eratosthenes (p. 4) Class Discussion – Product of prime numbers (p. 4) Worked Example 1 (p. 5) Practise Now 1 (p. 5) Thinking Time (p. 5) Worked Example 2 (p. 6) Practise Now 2 Q1 – 2 (p. 6) Ex 1A Q2, 3, 8 (p. 8)
		1.2 Square roots and cube roots (pp. 9–13)	• Find square roots and cube roots using prime factorisation, mental estimation and calculators	Identify and use • square numbers • cube numbers Calculate with • squares and square roots • cubes and cube roots Use a calculator efficiently			Attention (p. 10)
2		**1.3 Highest common factor and lowest common multiple** (pp. 14–21)	• Find the highest common factor (HCF) and lowest common multiple (LCM) of two or more numbers • Solve problems involving HCF and LCM in real-world contexts	Identify and use • common factors • common multiples			Practise Now 9 Q3 (p. 15) Practise Now 14 Q1 – 2 (p. 19) Ex 1C 9 Q8 (p. 20)

Week	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Reasoning, Communication and Connection
3	2 **Fractions**	2.1 **Fractions, improper fractions and mixed numbers** (pp. 24–29)	• Express fractions, improper fractions and mixed numbers in their equivalent forms	Use the language and notation of proper fractions, improper fractions and mixed numbers Recognise equivalence and convert between proper fractions, improper fractions and mixed numbers Order quantities by magnitude			Worked Example 5 (p. 28) Practise Now 5 (p. 28)
		2.2 **Adding and subtracting fractions and mixed numbers** (pp. 29–33)	• Add and subtract like and unlike fractions and mixed numbers	Use the four operations for calculations with fractions, including correct ordering of operations and use of brackets			
3, 4		2.3 **Multiplying fractions and mixed numbers** (pp. 34–38)	• Multiply fractions and mixed numbers		Investigation – Multiplying two fractions (p. 36)		Investigation – Multiplying two fractions (p. 36)
4		2.4 **Dividing fractions and mixed numbers** (pp. 34–38)	• Divide fractions and mixed numbers	Identify and use reciprocals Use the four operations for calculations with fractions, including correct ordering of operations and use of brackets			Introductory Problem Revisited (p. 41)

Week	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Reasoning, Communication and Connection
4, 5	**3** **Decimals**	**3.1 Decimals and fractions** (pp. 48–56)	• Convert between decimals and fractions	Recognise equivalence and convert between fractions and decimals	Investigation – Converting recurring decimal to fraction (pp. 51–52) Class Discussion – Identifying fractions with equivalent terminating fractions (p. 54)		Introductory Problem Revisited (p. 50) Investigation – Converting recurring decimal to fraction Q3, 4 (pp. 51–52) Class Discussion – Identifying fractions with equivalent terminating fractions (p. 54)
			• Compare and order decimals and fractions	Order quantities by magnitude			
5		**3.2 Operations involving decimals** (pp. 56–60)	• Add, subtract, multiply and divide decimals	Use the four operations for calculations with decimals	Class Discussion – Importance of place value (p. 60)	Class Discussion – Importance of place value (p. 60)	Class Discussion – Importance of place value (p. 60)
		3.3 Conversion of units of measurement for length, mass and volume (pp. 60–64)	• Convert units of measurement of length, mass and volume • Multiply and divide decimals by 10, 100, 1000	Use metric units of mass, length and capacity in practical situations and convert quantities into larger or smaller units	Class Discussion – Multiplying and dividing decimals by 10, 100 and 1000		
6	**4** **Integers, Rational Numbers and Real Numbers**	**4.1 Negative numbers** (pp. 69–74)	• Use negative numbers, rational numbers and real numbers in a real-world context	Identify and use integers (positive, zero and negative)	Class Discussion – Uses of negative numbers in real-world contexts (p. 71)		Class Discussion – Uses of negative numbers in real-world contexts (p. 71)
			• Represent real numbers on a number line and order the numbers	Order quantities by magnitude	Investigation – Ordering of numbers (pp. 71–72)		Investigation – Ordering of numbers (pp. 71–72) Ex 4A Q10 (p. 74)

Week	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Reasoning, Communication and Connection
		4.2 Addition and subtraction involving negative integers (pp. 74–84)	• Add and subtract integers	Use the four operations for calculations with integers, including correct ordering of operations and use of brackets	Investigation – Addition of two negative integers (p. 75) Investigation – Addition of a positive and a negative integer (p. 76) Investigation – Subtraction between two positive integers (p. 78) Investigation – Subtraction of a positive integer from a negative integer (p. 79) Investigation – Subtraction of a negative integers (p. 80)		Investigation – Addition of two negative integers (p. 75) Investigation – Addition of a positive and a negative integer (p. 76) Investigation – Subtraction between two positive integers (p. 78) Investigation – Subtraction of a positive integer from a negative integer (p. 79) Investigation – Subtraction of a negative integers (p. 80) Introductory Problem Revisited (p. 82)
6, 7		**4.3 Multiplication, division and combined operations involving negative integers** (pp. 74–84)	• Multiply and divide integers • Perform operations on integers, including using the calculator	Use the four operations for calculations with integers, including correct ordering of operations and use of brackets Use calculator efficiently	Investigation – Negative of an integer (p. 85) Investigation – Multiplication involving negative integers (p. 86) Thinking Time (p. 88)		Investigation – Negative of an integer (p. 85) Investigation – Multiplication involving negative integers (p. 86) Thinking Time (p. 88)

Week	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Reasoning, Communication and Connection
7		**4.4 Negative fractions and mixed numbers** (pp. 92–95)	• Perform operations on fractions and mixed numbers, including using the calculator	Use the four operations for calculations with integers, including correct ordering of operations and use of brackets Use calculator efficiently			
		4.5 Negative decimals (pp. 95–98)	• Perform operations on decimals including using the calculator	Use the four operations for calculations with decimals, including correct ordering of operations and use of brackets			
		4.6 Rational, irrational and real numbers (pp. 98–101)	• Perform operations on real numbers, including using the calculator	Identify and use rational and irrational numbers	Class Discussion – Rational and irrational numbers (p. 98) Investigation – Terminating, recurring and non-recurring decimals (pp. 99–100)		Class Discussion – Rational and irrational numbers (p. 98) Main text (p. 99) Investigation – Terminating, recurring and non-recurring decimals (pp. 99–100) Main text (p. 100)
8	**5 Approximation and Estimation**	**5.1 Rounding and significant figures** (pp. 107–113)	• Round off numbers to the nearest 10, 100, 1000, etc., to the nearest whole number and to a required number of decimal places	Round values to a specified degree of accuracy			Practise Now 2A Q2 (p. 108) Ex 5A Q18 and 19 (p. 117)
			• Round off numbers to a required number of significant figures	Round values to a specified degree of accuracy	Class Discussion – Which answer is more accurate? (pp. 108–109)		Class Discussion – Which answer is more accurate? (pp. 108–109) Ex 5A Q20 (p. 117)
		5.2 Limits of accuracy (pp. 114–116)	• Find the smallest and largest values that when rounded to the specified accuracy, is the given data.	Give appropriate upper and lower bounds for data given to a specified accuracy	Class Discussion – Upper and lower bounds of a rounded number (p. 114)		Class Discussion – Upper and lower bounds of a rounded number (p. 114)
			• Determine the smallest and largest values calculated from lower and upper bounds of data round to specified accuracy.	Find upper and lower bounds of the results of calculations which have used data rounded to a specified accuracy	Class Discussion – Determining the upper and lower bounds of calculated value (p. 115)		Class Discussion – Determining the upper and lower bounds of calculated value (p. 115)

Week	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Reasoning, Communication and Connection
8	**5** **Approximation and Estimation**	**5.3 Approximation and approximation errors in real-world contexts** (pp. 118–120)	• Compare follow-through errors arising from intermediate values that are rounded to different degrees of accuracy.	Round answers to a reasonable degree of accuracy in the context of a given problem	Investigation – Rounding in real life (p. 118) Investigation – The missing 0.1% votes (p. 118–119)		Investigation – Rounding in real life (p. 118) Investigation – The missing 0.1% votes (pp. 118–119) Ex 5B Q2 and 3 (p. 125)
			• Compare difference between value with specified degree of accuracy and approximate value		Thinking Time (p. 120)		Thinking Time (p. 120) Information (p. 120)
8		**5.4 Estimation and estimation errors in real-world contexts** (pp. 121–124)	• Make estimates and check the reasonableness of answers obtained from calculators.	Make estimates for calculations involving numbers, quantities and measurements			
			• Estimate quantities (numbers and measures) to an appropriate degree of accuracy in a variety of contexts, compare the estimates and share the estimation strategies.		Investigation – Using smaller quantity to estimate larger quantity (p. 124)		Investigation – Using smaller quantity to estimate larger quantity (p. 124)

Week	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Reasoning, Communication and Connection
9	**6** **Basic Algebra and Algebraic Manipulation**	**6.1 Basic algebra concepts and notations** (pp. 130–138)	• Use letters to represent numbers • Express basic arithmetical processes algebraically • Identify linear expressions	Know that letters can be used to represent generalised numbers	Class Discussion – Letters as generalised numbers (p. 132) Thinking Time (p. 132) Class Discussion – Interpreting meanings of algebraic notations (p. 133) Investigation – Comparing algebraic notations (pp. 133–134) Class Discussion – Expressing mathematical operations and simple real-world situations using algebraic expressions (p. 136) Class Discussion – Linear expressions (p. 138)	Investigation – Comparing algebraic notations (pp. 133–134)	Class Discussion – Letters as generalised numbers (p. 132) Thinking Time (p. 132) Class Discussion – Interpreting meanings of algebraic notations (p. 133) Investigation – Comparing algebraic notations (pp. 133–134) Class Discussion – Expressing mathematical operations and simple real-world situations using algebraic expressions (p. 136) Class Discussion – Linear expressions (p. 138)
			• Evaluate algebraic expressions	Substitute numbers into expressions	Journal Writing (p. 137)		Journal Writing (p. 137)
9		**6.2 Addition and subtraction of linear termss** (pp. 138–142)	• Identify like terms	Simplify expressions by collecting like terms	Class Discussion – Like and unlike terms (p. 138)		Main text (p. 138) Class Discussion – Like and unlike terms (p. 138)
			• Add and subtract linear expressions		Investigation – Addition and subtraction of like terms (p. 140)		Investigation – Addition and subtraction of like terms (p. 140)

Week	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Reasoning, Communication and Connection
10, 11		**6.3 Expansion and factorisation of linear expressions** (pp. 144–151)	• Expand and simplify linear expressions with integer coefficients	Expand products of algebraic expressions	Investigation – Expansion of linear expressions (p. 145)		Investigation – Expansion of linear expressions (p. 145) Main text (p. 146)
			• Factorise algebraic expressions by extracting common integer factors	Factorise by extracting common factors			Information (p. 147) Introductory Problem Revisited (pp. 149–150)
11		**6.4 Linear expressions with fractional coefficients** (pp. 151–156)	• Expand and simplify linear expressions with fractional coefficients	Manipulate algebraic fractions	Investigation – Comparing algebraic notations (pp. 152–153) Thinking Time (p. 155)	Investigation – Comparing algebraic notations (pp. 133–134)	Investigation – Comparing algebraic notations (pp. 152–153) Thinking Time (p. 155)
11	**7 Linear Equations**	**7.1 Linear equations** (pp. 160–166)	• Explore concept of equation • Solve linear equations in one variable	Solve linear equations in one unknown	Class Discussion – Expressions and equations (p. 161) Investigation – Solving linear equations (pp. 162– 163) Thinking Time (p. 165) Journal Writing (p. 165)		Class Discussion – Expressions and equations (p. 161) Investigation – Solving linear equations (pp. 162–163) Thinking Time (p. 165) Journal Writing (p. 165)
11		**7.2 Linear equations with fractional coefficients and fractional equations** (pp. 166–168)	• Solve fractional equations that can be reduced to linear equations	Solve fractional equations with numerical and linear algebraic denominators			
12		**7.3 Applications of linear equations in real-world contexts** (pp. 170–173)	• Construct linear equations to solve word problems	Construct equations	Introductory Problem Revisited (p. 172)		Introductory Problem Revisited (p. 172)

Week	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Reasoning, Communication and Connection
12		7.4 **Mathematical formulae** (pp. 174–176)	• Evaluate an unknown in a formula	Substitute numbers into formulas			
			• Construct a formula for a given problem	Construct formulas			Ex 7C Q14(ii) (p. 176)
13	**8 Percentage**	8.1 **Percentage** (pp. 179–194)	• Express a percentage as a fraction and vice versa • Express a percentage as a decimal and vice versa • Find the given percentage of a quantity	Calculate a given percentage of a quantity	Class Discussion – Identifying percentages used in daily life (p. 181) Thinking Time (p. 183) Class discussion – Interpreting percentages used in real life (pp. 183–184)		Class Discussion – Identifying percentages used in daily life (p. 181) Thinking Time (p. 183) Class discussion – Interpreting percentages used in real life (pp. 183–184)
			• Express one quantity as a percentage of another • Compare two quantities by percentage	Express one quantity as a percentage of another	Class Discussion – Expressing two quantities in equivalent forms (pp. 186–187) Introductory Problem Revisited (p. 189) Thinking Time (p. 191)		Class Discussion – Expressing two quantities in equivalent forms (pp. 186–187) Introductory Problem Revisited (p. 189) Thinking Time (p. 191) Practise Now 7 (p. 189) Practise Now 8 (p. 190) Practise Now 9 (p. 191) Ex 8A Q20, 21, 24, 25 (p. 193–194)
13	**8 Percentage**	8.2 **Percentage change, percentage point and reverse percentage** (pp. 194–203)	• Solve problems involving percentage change • Explain the difference between percent and percentage point	Calculate percentage increase or decrease	Thinking Time (p. 196) Investigation – Can we add percentages or take average of percentages? (p. 197) Investigation – Percentage point (pp. 198–199)		Thinking Time (p. 196) Investigation – Can we add percentages or take average of percentages? (p. 197) Investigation – Percentage point (pp. 198–199)

Week	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Reasoning, Communication and Connection
13	**8** **Percentage**		• Solve problems involving reverse percentage	Calculate using reverse percentages			Reflection (p. 199) Reflection (p. 200) Ex 8B Q21
13, 14	**9** **Ratio and Rate**	**9.1 Ratio** (pp. 194–216)	• Simplify a given ratio • Find ratios involving rational number • Find ratios involving three quantities • Solve problems involving ratio	Understand and use ratio to: • give ratios in their simplest form • divide a quantity in a given ratio • use proportional reasoning and ratios in contexts	Class Discussion – Relationship between ratios and fractions (pp. 207–208) Thinking Time (p. 208) Class Discussion – Making sense of ratios used in real-world contexts (p. 213)		Class Discussion – Relationship between ratios and fractions (pp. 207–208) Thinking Time (p. 208) Big Idea (p. 209) Class Discussion – Making sense of ratios used in real-world contexts (p. 213) Ex 9A Q22 (p. 216)
14, 15	**9** **Ratio and Rate**	**9.2 Rate** (pp. 216–277)	• Distinguish between constant and average rates • Solve problems involving rate	Apply common measures of rate Apply other measures of rate	Class Discussion – Different types of rates (pp. 217–218) Thinking Time (p. 221) Investigation – Average pulse rate (p. 222)		Class Discussion – Different types of rates (pp. 217–218) Reflection (p.219) Attention (p. 220) Thinking Time (p. 221) Investigation – Average pulse rate (p. 222)
			• Discuss real-life examples of rates such as currency exchange rate	Calculate with money Convert from one currency to another			
15		**9.3 Speed** (pp. 277–237)	• Read the time in 12-hour and 24-hour time formats	Read clocks and timetables	Main text (p. 228)		
			• Solve problems involving duration • Solve problems involving time zones and time differences	Calculate times in terms of the 24-hour and 12-hour clock	Class Discussion – Time zones (p. 229)		Class Discussion – Time zones (p. 229)

Week	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Reasoning, Communication and Connection
15	**9** **Ratio and Rate**		• Convert between different units of measurement for time	Calculate with time, including the relationship between units			Reflection (p. 231)
			• Discuss special types of rates such as speed • Solve problems involving speed and average speed	Solve problems involving average speed	Class Discussion – Making sense of speeds in real-world contexts (p. 233) Thinking Time (p. 234)		Class Discussion – Making sense of speeds in real-world contexts (p. 233) Thinking Time (p. 234)
16	**10** **Basic Geometry**	**10.1 Basic geometrical concepts and notations** (pp. 240–244)	• Use geometrical terms such as point, line segment, line, ray and plane • Identify various type of angles	Use and interpret the following geometrical terms: • point • vertex • line • plane • parallel • right angle • acute, obtuse and reflex angle	Thinking Time (p. 241)		Thinking Time (p. 241) Just for Fun (p. 243)
			• Measure a given angle using a protractor	Measure and draw lines and angles			
16, 17		**10.2 Properties of angles formed by intersecting lines** (pp. 224–247)	• Solve problems involving angles on a straight line, angles at a point and vertically opposite angles	Calculate unknown angles and give simple explanations using the following geometrical properties: • sum of angles at a point = 360° • sum of angles at a point on a straight line = 180° • vertically opposite angles are equal	Journal Writing – Vertically opposite angles (p. 247)		Journal Writing – Vertically opposite angles (p. 247) Reflection (p. 246)
17	**10** **Basic Geometry**	**10.3 Properties of angles formed by two parallel and transversal** (pp. 250–257)	• Solve problems involving angles formed by two parallel lines and a transversal, i.e. corresponding angles, alternate angles and interior angles	Calculate unknown angles and give geometric explanations for angles formed within parallel lines: • corresponding angles are equal • alternate angles are equal • supplementary angles sum to 180°	Investigation – Corresponding angles, alternate angles and interior angles (pp. 251–252)	Investigation – Corresponding angles, alternate angles and interior angles (pp. 251–252)	Investigation – Corresponding angles, alternate angles and interior angles (pp. 251 – 252) Ex 10B Q1(b)-(c) (p. 255)

Week	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Reasoning, Communication and Connection
18	**11** **Polygons and Geometrical Constructions**	**11.1 Triangles** (pp. 260–267)	• Identify different types of triangles and state their properties	Use and interpret the vocabulary of triangles	Investigation – Basic properties of triangle (pp. 261–262)	Investigation – Basic properties of triangle (pp. 261–262	Investigation – Basic properties of triangle (pp. 261–262
			• Solve problems involving the properties of triangles	Calculate unknown angles and give simple explanations using the following geometrical properties: • angle sum of a triangle = 180° Use and interpret the followng geometrical terms: interior and exterior angles	Thinking Time (p. 263) Thinking Time (p. 264)		Thinking Time (p. 263) Thinking Time (p. 264)
18, 19		**11.2 Quadrilaterals** (pp. 268–276)	• Solve problems involving the properties of special quadrilaterals	Calculate unknown angles and give simple explanations using the following geometrical properties: • angle sum of a quadrilateral = 360°	Thinking Time (p. 268)		Thinking Time (p. 268)
			• Identify different types of special quadrilaterals and state their properties	Use and interpret the vocabulary of special quadrilaterals Use and interpret the following geometrical terms: • interior and exterior angles	Investigation – Properties of special quadrilaterals (pp. 269–270)	Investigation – Properties of special quadrilaterals (pp. 269–270)	Investigation – Properties of special quadrilaterals (pp. 269–270)
19, 20		**11.3 Geometrical constructions of triangles and quadrilaterals** (pp. 276–284)	• Construct triangles and quadrilaterals using a ruler and a pair of compasses, and solve related problems	Use and interpret the following geometrical terms: • perpendicular • perpendicular bisector Construct a triangle, given the lengths of all sides, using a ruler and a pair of compasses only			Reflection (p. 281) Ex 11C Q 14, 15 (p. 287)

Week	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Reasoning, Communication and Connection
20	**11 Polygons and Geometrical Constructions**	**11.4 Polygons** (pp. 285–297)	• Identify different types of polygons and state their properties	Use and interpret the vocabulary of polygons	Class Discussion – What are polygons (p. 285) Investigation – Naming of polygons (p. 286)	Investigation – Naming of polygons (p. 286)	Class Discussion – What are polygons (p. 285) Investigation – Naming of polygons (p. 286)
			• Distinguish between regular and irregular polygons • Solve problems involving the properties of polygons	Know and use angle properties of regular and irregular polygons	Thinking Time (p. 286) Class Discussion – Definition of regular polygon (p. 286) Investigation – Sum of interior angles of polygon (p. 287) Investigation – Sum of exterior angles of a polygon (pp. 289–290) Thinking Time (p. 290) Class Discussion – Exterior angles of a polygon (p. 291)	Investigation – Sum of exterior angles of a polygon (pp. 289–290)	Thinking Time (p. 286) Class Discussion – Definition of regular polygon (p. 286) Investigation – Sum of interior angles of polygon (p. 287) Investigation – Sum of exterior angles of a polygon (pp. 289–290) Thinking Time (p. 290) Class Discussion – Exterior angles of a polygon (p. 291) Introductory Problem Revisited (p. 294)
20			• Identify lines of symmetry of 2D shapes, special quadrilaterals and regular polygons • State the order of rotational symmetry of 2D shapes, special quadrilaterals and regular polygons	Recognise line symmetry and order of rotational symmetry in two dimensions	Class Discussion – Line symmetry of special quadrilaterals and regular polygons (p. 295) Class Discussion – Rotational symmetry of special quadrilaterals and regular polygons (p. 296)		Class Discussion – Line symmetry of special quadrilaterals and regular polygons (p. 295) Class Discussion – Rotational symmetry of special quadrilaterals and regular polygons (p. 296)

Week	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Reasoning, Communication and Connection
21	**12 Perimeter and Area of Plane Figures**	**12.1 Conversion of units** (pp. 302–303)	• Convert between cm^2 and m^2	Use metric units of area in practical situations and convert quantities to larger or smaller units	Investigation – Converting between cm^2 and m^2 (p. 302)		Investigation – Converting between cm^2 and m^2 (p. 302)
		12.2 Perimeter and area of rectangles and triangles (pp. 304–310)	• Find the perimeter and area of rectangles and triangles	Carry out calculations involving the perimeter and area of rectangle and triangle	Investigation – Finding the area of a triangle (p. 306) Investigation ¬ Area of triangle using different sides as base (p. 308)		Investigation – Finding the area of a triangle (p. 306) Investigation – Area of triangle using different sides as base (p. 308)
22		**12.3 Perimeter and area of parallelograms** (pp. 304–310)	• Find the perimeter and area of parallelograms	Carry out calculations involving the perimeter and area of parallelogram	Investigation – Formula for area of parallelogram (p. 312) Journal Writing (p. 313) Thinking Time (p. 314)	Journal Writing (p. 313)	Investigation – Formula for area of parallelogram (p. 312) Journal Writing (p. 313) Thinking Time (p. 314) Worked Example 7 (pp. 315–316)
23		**12.4 Perimeter and area of trapeziums** (pp. 317–320)	• Find the perimeter and area of trapeziums	Carry out calculations involving the perimeter and area of trapezium	Investigation – Formula for area of trapezium (p. 318) Thinking Time (p. 319)		Investigation – Formula for area of trapezium (p. 318) Thinking Time (p. 319)
23		**12.4 Circumference and area of circles** (pp. 323–331)	• Find the circumference and area of circles	Carry out calculations involving the circumference and area of a circle	Investigation – Formula for circumference of circle (p. 324) Investigation – Formula for area of circle (p. 325)		Investigation – Formula for circumference of circle (p. 324) Investigation – Formula for area of circle (p. 325)
			• Solve problems involving the perimeter and area of composite figures	Carry out calculations and solve problems involving the perimeters and areas of compound shapes and parts of shapes	Class Discussion – Perimeter and area of composite figures in real-world contexts (p. 328)		Class Discussion – Perimeter and area of composite figures in real-world contexts

Week	Chapter	Section	Specific Instructional Objectives (SIOs)	Syllabus Subject Content	Activity	ICT	Reasoning, Communication and Connection
24	**13 Statistical Data Handling**	**13.1 Frequency table** (pp. 334–335)	• Collect, classify and tabulate data	Classify and tabulate statistical data			
		13.2 Pictogram (pp. 336–338)	• Construct and interpret data from pictograms	Draw and interpret pictograms	Class Discussion – Purposes and appropriateness of pictograms (pp. 336 – 337)		Class Discussion – Purposes and appropriateness of pictograms (pp. 336 – 337)
		13.3 Bar graph (pp. 338–341)	• Construct and interpret data from bar graphs • Evaluate the purposes and appropriateness of the use of different statistical diagrams	Draw and interpret bar graphs	Class Discussion – Purposes and appropriateness of bar graph (p. 339)		Class Discussion – Purposes and appropriateness of bar graph (p. 339) Practise Now 1B Q2 (pp. 340–341) Ex 13A Q6, 7 (pp. 343–344)
		13.4 Pie chart (pp. 345–348)	• Construct and interpret data from pie charts • Evaluate the purposes and appropriateness of the use of different statistical diagrams	Draw and interpret pie charts	Class Discussion – Construction and usefulness of pie chart (pp. 345–346) Journal Writing (p. 348)		Class Discussion – Construction and usefulness of pie chart (pp. 345–346) Journal Writing (p. 348) Ex 13B Q7 (p. 357)
25		**13.5 Evaluation of statistical representations** (pp. 349–351)	• Explain why some statistical information or diagrams can lead to a misinterpretation of data	Appreciate restrictions on drawing conclusions from given data	Class Discussion – Evaluation of statistical representations (pp. 349–351)		Class Discussion – Evaluation of statistical representations (pp. 349–351)
		13.6 Statistical investigation (pp. 351–355)	• Discuss and explain the choice of data collection and sampling methods to based on the statistical question to be investigated • Discuss and explain the choice of statistical diagram to present collated data for purpose of analysis and interpretation	Classify and tabulate statistical data Read, interpret and draw inferences from tables and statistical diagrams	Class Discussion – Stages of statistical investigation (pp. 352–355) Performance Task (p. 355)		Class Discussion – Stages of statistical investigation (pp. 352–355) Performance Task (p. 355)

Chapter 1 Primes, Highest Common Factor and Lowest Common Multiple

TEACHING NOTES

Suggested Approach

Students have learnt only whole numbers in primary school (they will only learn negative numbers and integers in Chapter 4). They have also learnt how to classify whole numbers into two groups, i.e. odd and even numbers. Teachers can introduce prime numbers as another way in which whole numbers can be classified (see Section 1.1). Traditionally, prime numbers apply to positive integers only, but the syllabus specifies whole numbers, which is not an issue since 0 is not a prime number. Teachers can also arouse students' interest in this topic by bringing in real-life applications (see chapter opener).

Section 1.1: Prime numbers

Teachers can build upon prerequisites, namely, factors, to introduce prime numbers by classifying whole numbers according to the number of factors they have (see Investigation: Classification of whole numbers). Since the concept of 0 may not be easily understood, it is dealt with separately in the last question of the investigation. Regardless of whether 0 is classified in the same group as 1 or in a new fourth group, 0 and 1 are neither prime nor composite. Teachers are to take note that 1 is not a prime number 'by choice', or else the uniqueness of prime factorisation will fail (see Information on page 6 of the textbook). Also, 0 is not a composite number because it cannot be expressed as a product of prime factors unlike e.g. $40 = 23 \times 5$.

To make the chapter more interesting, teachers can tell students about the largest known prime number (there is no largest prime number since there are infinitely many primes) and an important real-life application of prime numbers in the encryption of computer data (see chapter opener) in order to arouse their interest in this topic.

Section 1.2: Square roots and cube roots

Teachers can build upon what students have learnt about squares, square roots, cubes and cube roots in primary school. Perfect squares are also called square numbers and perfect cubes are also called cube numbers. Perfect numbers are not the same as perfect squares or perfect cubes. Perfect numbers are numbers which are equal to the sum of its proper factors, where proper factors are factors that are less than the number itself, e.g. $6 = 1 + 2 + 3$ and $28 = 1 + 2 + 4 + 7 + 14$ are the only two perfect numbers less than 100 (perfect numbers are not in the syllabus). After students have learnt negative numbers in Chapter 4, there is a need to revisit square roots and cube roots to discuss negative square roots and negative cube roots (see page 88 of the textbook). Teachers can impress upon students that the square root symbol $\sqrt{}$ refers to the positive square root only.

A common debate among some teachers is whether 0 is a perfect square. There is an argument that 0 is not a perfect square because 0 can multiply by any number (not necessarily itself) to give 0. However, this is not the definition of a perfect square. Since 0 is equal to 0 multiplied by itself, then 0 (the first 0, not the second 0, in this sentence) is a perfect square. Compare this with why 4 is a perfect square (4 is equal to the integer 2 multiplied by itself). Similarly, 0 is a perfect cube.

Section 1.3: Highest common factor and lowest common multiple

Teachers can build upon prerequisites, namely, common factors and common multiples, to develop the concepts of Highest Common Factor (HCF) and Lowest Common Multiple (LCM) respectively (HCF and LCM are no longer in the primary school syllabus although some primary school teachers teach their students HCF and LCM). Since the listing method (see pages 14 and 15 of the textbook) is not an efficient method to find the HCF and the LCM of two or more numbers, there is a need to learn the prime factorisation method and the ladder method (see Methods 1 and 2 in Worked Example 9 and in Worked Example 10). It is recommended that students familiarise themselves with both methods.

Introductory Problem

The solutions to this problem can be found in Introductory Problem Revisited (after Practise Now 14).

1.1 Prime numbers

Investigation (Classifying whole numbers)

1.

Number	Working	Factors
1	1 is divisible by 1 only.	1
2	2 = 1 × 2	1, 2
3	**3 = 1 × 3**	**1, 3**
4	4 = 1 × 4 = 2 × 2	1, 2, 4
5	**5 = 1 × 5**	**1, 5**
6	**6 = 1 × 6 = 2 × 3**	**1, 2, 3, 6**
7	**7 = 1 × 7**	**1, 7**
8	**8 = 1 × 8 = 2 × 4**	**1, 2, 4, 8**
9	**9 = 1 × 9 = 3 × 3**	**1, 3, 9**
10	**10 = 1 × 10 = 2 × 5**	**1, 2, 5, 10**
11	**11 = 1 × 11**	**1, 11**
12	**12 = 1 × 12 = 2 × 6 = 3 × 4**	**1, 2, 3, 4, 6, 12**
13	**13 = 1 × 13**	**1, 13**
14	**14 = 1 × 14 = 2 × 7**	**1, 2, 7, 14**
15	**15 = 1 × 15 = 3 × 5**	**1, 3, 5, 15**
16	**16 = 1 × 16 = 2 × 8 = 4 × 4**	**1, 2, 4, 8, 16**
17	**17 = 1 × 17**	**1, 17**
18	18 = 1 × 18 = 2 × 9 = 3 × 6	**1, 2, 3, 6, 9, 18**
19	**19 = 1 × 19**	**1, 19**
20	**20 = 1 × 20 = 2 × 10 = 4 × 5**	**1, 2, 4, 5, 10, 20**

Table 1.1

2. **Group A**: **1**
   Group B: **2, 3, 5, 7, 11, 13, 17, 19**
   Group C: **4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20**
3. 0 is divisible by 1, 2, 3, 4, …
   0 has an infinite number of factors.

Journal Writing (Page 3)

1. A prime number is a whole number that has exactly 2 different factors, 1 and itself.
   A composite number is a whole number that has more than 2 different factors. A composite number has a finite number of factors.
   Since 0 has an infinite number of factors, it is neither a prime nor a composite number.
   Since 1 has exactly 1 factor, it is also neither a prime nor a composite number.
2. **No**, I do not agree with Yasir. Consider the numbers 0 and 1. They are neither prime numbers nor composite numbers.

Investigation (Sieve of Erastosthenes)

1.

~~1~~	2	3	~~4~~	5	~~6~~	7	~~8~~	~~9~~	~~10~~
11	~~12~~	13	~~14~~	~~15~~	~~16~~	17	~~18~~	19	~~20~~
~~21~~	~~22~~	23	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	29	~~30~~
31	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	37	~~38~~	~~39~~	~~40~~
41	~~42~~	43	~~44~~	~~45~~	~~46~~	47	~~48~~	~~49~~	~~50~~
~~51~~	~~52~~	53	~~54~~	~~55~~	~~56~~	~~57~~	~~58~~	59	~~60~~
61	~~62~~	~~63~~	~~64~~	~~65~~	~~66~~	67	~~68~~	~~69~~	~~70~~
71	~~72~~	73	~~74~~	~~75~~	~~76~~	~~77~~	~~78~~	79	~~80~~
~~81~~	~~82~~	83	~~84~~	~~85~~	~~86~~	~~87~~	~~88~~	89	~~90~~
~~91~~	~~92~~	~~93~~	~~94~~	~~95~~	~~96~~	97	~~98~~	~~99~~	~~100~~

2. **(a)** The smallest prime number is **2**.
   (b) The largest prime number less than or equal to 100 is **97**.
   (c) There are **25** prime numbers which are less than or equal to 100.
   (d) **No**, not every odd number is a prime number, e.g. the number 9 is an odd number but it is a composite number.
   (e) **No**, not every even number is a composite number, e.g. the number 2 is an even number but it is a prime number.
   (f) For a number greater than 5, if its last digit is 0, 2, 4, 6 or 8, then the number is a multiple of 2, thus it is a composite number; if its last digit is 0 or 5, then the number is a multiple of 5, thus it is a composite number. Hence, for a prime number greater than 5, its last digit can only be **1, 3, 7 or 9**.

Class Discussion (Product of prime numbers)

(a) **Yes**, the product of two prime numbers can be an odd number, e.g. the product of the two prime numbers 3 and 5 is the odd number 15.

(b) **Yes**, the product of two prime numbers can be an even number, e.g. the product of the two prime numbers 2 and 3 is the even number 6.

(c) **No**, the product of two prime numbers P_1 and P_2 cannot be a prime number since P_1P_2 has 4 distinct factors, i.e. 1, P_1, P_2 and P_1P_2.

Practise Now 1

Since 534 is divisible by 2, then 534 is a **composite** number.

$\sqrt{1607} = 40.1$ (to 1 d.p.), so the largest prime number less than or equal to $\sqrt{1607}$ is 37.

Since 1607 is not divisible by any of the prime numbers 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31 and 37, then 1607 is a **prime** number.

Thinking Time (Page 5)

1.

	Number of dots	Number of different ways
(i)	4	**2**
(ii)	8	**2**
(iii)	12	**3**
(iv)	5	**1**
(v)	7	**1**
(vi)	1	**1**

2. A faster method would be to find the factors of the numbers.
3. The numbers are prime numbers or 1. Since a prime number has only two factors (1 and itself), it is only possible to arrange the dots in exactly 1 way.
4. The numbers are composite numbers. Since a composite number has more than 2 different factors, there is more than one way to arrange the dots.

Practise Now 2

1. Since 31 is a prime number, then 1 and 31 are its only two factors.
   $\therefore p + q = 1 + 31 = \mathbf{32}$
2. For $n \times (n + 28)$ to be a prime number, n and $n + 28$ are its only two factors.
   Since 1 has to be one of its two factors, $n = 1$.
   $\therefore$ the prime number is $1 \times (1 + 28) = \mathbf{29}$.

Practise Now 3

1. $126 = 2 \times 3 \times 3 \times 7$
   $= \mathbf{2 \times 3^2 \times 7}$
2. $792 = 2 \times 2 \times 2 \times 3 \times 3 \times 11$
   $= \mathbf{2^3 \times 3^2 \times 11}$
3. **(i)** $2021 = 43 \times 47$
   (ii) $2021 = 1 \times 2021$
   $= 43 \times 47$
   Since $a < b$, $\therefore$ the possible pairs are **(1, 2021)** and **(43, 47)**.

Practise Now 4

1. $195 = 3 \times 5 \times 13$
   Since each side of the cuboid is longer than 1 cm, the dimensions of the cuboid are **3 cm by 5 cm by 13 cm**.
2. $324 = 2^2 \times 3^4$
   Since each side of the cuboid is longer than 2 cm, the possible dimensions are as follows:

Side 1 (cm)	Side 2 (cm)	Side 3 (cm)
3	4	27
3	6	18
3	9	12
4	9	9
6	6	9

When the cuboid has a length and breadth of 6 cm and 3 cm respectively, the perimeter will be $2(6 + 3) = 18$ cm. $\therefore$ the height of the cuboid is **18 cm**.

Exercise 1A

1. **(a)** Since 87 is divisible by 3, then 87 is a **composite** number.
   (b) $\sqrt{67} = 8.2$ (to 1 d.p.), so the largest prime number less than or equal
   to $\sqrt{67}$ is 7.
   Since 67 is not divisible by any of the prime numbers 2, 3, 5 and 7, then 67 is a **prime** number.
   (c) $\sqrt{73} = 8.5$ (to 1 d.p.), so the largest prime number less than or equal
   to $\sqrt{73}$ is 7.
   Since 73 is not divisible by any of the prime numbers 2, 3, 5 and 7, then 73 is a **prime** number.
   (d) Since 91 is divisible by 7, then 91 is a **composite** number.
2. Since 37 is a prime number, then 1 and 37 are its only two factors.
   $\therefore p + q = 1 + 37 = \mathbf{38}$
3. For $n \times (n + 42)$ to be a prime number, n and $n + 42$ are its only two factors.
   Since 1 has to be one of its two factors, $n = 1$.
   $\therefore$ the prime number is $1 \times (1 + 42) = \mathbf{43}$.
4. **(a)** $72 = 2 \times 2 \times 2 \times 3 \times 3$
   $= \mathbf{2^3 \times 3^2}$
   (b) $756 = 2 \times 2 \times 3 \times 3 \times 3 \times 7$
   $= \mathbf{2^2 \times 3^3 \times 7}$
   (c) $187 = \mathbf{11 \times 17}$
   (d) $630 = 2 \times 3 \times 3 \times 5 \times 7$
   $= \mathbf{2 \times 3^2 \times 5 \times 7}$
5. **(i)** $2026 = \mathbf{2 \times 1013}$
   (ii) $2026 = 1 \times 2026$
   $= 2 \times 1013$
   Since $a < b$, $\therefore$ the possible pairs are **(1, 2026)** and **(2, 1013)**.
6. $273 = 3 \times 7 \times 13$
   Since each side of the cuboid is longer than 1 cm, the dimensions of the cuboid are **3 cm by 7 cm by 13 cm**.
7. **(a)** All the even numbers (2012, 2014, 2016, 2018 and 2020) are composite numbers since they are divisible by 2. 2013 and 2019 are divisible by 3. 2015 is divisible by 5.
   $\therefore$ the prime numbers are **2011** and **2017**.
   (b) All the even numbers (2022, 2024, 2026, 2028 and 2030) are composite numbers since they are divisible by 2. 2021 is divisible by 43, 2023 is divisible by 7 and 2025 is divisible by 5.
   $\therefore$ the prime numbers are **2027** and **2029**.

8. 2027 is a prime number, i.e. its only two factors are 1 and 2027.
$a \times b = 1 \times 2027 = 2027$
$\therefore a + b = 1 + 2027 = \mathbf{2028}$

9. **(a)** $8624 = 2 \times 2 \times 2 \times 2 \times 7 \times 7 \times 11$
$= \mathbf{2^4 \times 7^2 \times 11}$

(b) $6804 = 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 7$
$= \mathbf{2^2 \times 3^5 \times 7}$

(c) $26\,163 = 3 \times 3 \times 3 \times 3 \times 17 \times 19$
$= \mathbf{3^4 \times 17 \times 19}$

(d) $196\,000 = 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5 \times 7 \times 7$
$= \mathbf{2^5 \times 5^3 \times 7^2}$

10. Prime factorisation of $2022 = 2 \times 3 \times 337$
Since $x < y$, the possible pairs are **(1, 2022)**, **(2, 1011)**, **(3, 674)** and **(6, 337)**.

11. $210 = 2 \times 3 \times 5 \times 7$
$\therefore$ the possible dimensions are
2 cm by 3 cm by 35 cm, **2 cm by 5 cm by 21 cm**,
2 cm by 7 cm by 15 cm, **3 cm by 5 cm by 14 cm**,
3 cm by 7 cm by 10 cm and **5 cm by 6 cm by 7 cm**.

1.2 Square roots and cube roots

Practise Now 5

1. $784 = 2 \times 2 \times 2 \times 2 \times 7 \times 7$
$= (2 \times 2 \times 7) \times (2 \times 2 \times 7)$
$= (2 \times 2 \times 7)^2$
$\therefore \sqrt{784} = 2 \times 2 \times 7$
$= \mathbf{28}$
Alternatively,
$784 = 2 \times 2 \times 2 \times 2 \times 7 \times 7$
$= 2^4 \times 7^2$
$\therefore \sqrt{784} = \sqrt{2^4 \times 7^2}$
$= 2^2 \times 7$
$= \mathbf{28}$

2. $\sqrt{7056} = \sqrt{2^4 \times 3^2 \times 7^2}$
$= 2^2 \times 3 \times 7$
$= \mathbf{84}$

Practise Now 6

1. $2744 = 2 \times 2 \times 2 \times 7 \times 7 \times 7$
$= (2 \times 7) \times (2 \times 7) \times (2 \times 7)$
$= (2 \times 7)^3$
$\therefore \sqrt[3]{2744} = 2 \times 7$
$= \mathbf{14}$
Alternatively,
$2744 = 2 \times 2 \times 2 \times 7 \times 7 \times 7$
$= 2^3 \times 7^3$
$\therefore \sqrt[3]{2744} = \sqrt[3]{2^3 \times 7^4}$
$= 2 \times 7$
$= \mathbf{14}$

2. $\sqrt[3]{9261} = \sqrt[3]{3^3 \times 7^3}$
$= 3 \times 7$
$= \mathbf{21}$

3. **(i)** $15 \times 135 \times k = 3^4 \times 5^2 \times k$
For $3^4 \times 5^2 \times k$ to be a perfect cube, the index of each prime factor must be a multiple of 3.
For k to be the smallest, $k = 3^2 \times 5$ so that $3^4 \times 5^2 \times k = 3^4 \times 5^2 \times (3^2 \times 5)$
$= 3^6 \times 5^3$ is a perfect cube.
$\therefore$ smallest value of $k = 3^2 \times 5 = \mathbf{45}$

(ii) $15 \times 135 \times \frac{p}{q} = 3^4 \times 5^2 \times \frac{p}{q}$
For $3^4 \times 5^2 \times \frac{p}{q}$ to be a perfect cube, the index of each prime factor must be a multiple of 3. So, p and q must contain a factor of 3 or 5.
Since p and q are prime numbers, one of them must be 3 and the other 5.
$\therefore$ $\mathbf{p = 5}$ and $\mathbf{q = 3}$ so that $3^4 \times 5^2 \times \frac{p}{q} = 3^4 \times 5^2 \times \frac{5}{3} = 3^3 \times 5^3$ is a perfect cube.

Practise Now 7

(a) $\sqrt{123} \approx \sqrt{121} = \mathbf{11}$
(b) $\sqrt[3]{123} \approx \sqrt[3]{125} = \mathbf{5}$

Practise Now 8

1. **(a)** $23^2 + \sqrt{2025} - 7^3 = \mathbf{231}$
(b) $\dfrac{3^2 \times \sqrt{20}}{5^3 - \sqrt[3]{2013}} = \mathbf{0.3582}$ (to 4 d.p.)

2. Length of one side of poster $= \sqrt{987}$ cm
Perimeter of poster $= 4 \times \sqrt{987}$
$= \mathbf{125.7\ cm}$ (to 1 d.p.)

3. $\sqrt[3]{2020} = 12.6$ (to 1 d.p.)
Length of largest possible cube = 12 cm
Number of cubes used $= 12^3$
$= 1728$
Number of cubes left over $= 2020 - 1728$
$= \mathbf{292}$

Exercise 1B

1. **(a)** $1764 = 2 \times 2 \times 3 \times 3 \times 7 \times 7$
$= (2 \times 3 \times 7) \times (2 \times 3 \times 7)$
$= (2 \times 3 \times 7)^2$
$\therefore \sqrt{1764} = 2 \times 3 \times 7$
$= \mathbf{42}$
Alternatively,
$1764 = 2 \times 2 \times 3 \times 3 \times 7 \times 7$
$= 2^2 \times 3^2 \times 7^2$
$\therefore \sqrt{1764} = \sqrt{2^2 \times 3^2 \times 7^2}$
$= 2 \times 3 \times 7$
$= \mathbf{42}$

(b) $576 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3$
$= (2 \times 2 \times 2 \times 3) \times (2 \times 2 \times 2 \times 3)$
$= (2 \times 2 \times 2 \times 3)^2$
$\therefore \sqrt{576} = 2 \times 2 \times 2 \times 3$
$= \mathbf{24}$

Alternatively,

$576 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3$
$= 2^6 \times 3^3$
$\therefore \sqrt{576} = \sqrt{2^6 \times 3^2}$
$= 2^3 \times 3$
$= \mathbf{24}$

(c) $2916 = 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$
$= (2 \times 3 \times 3 \times 3) \times (2 \times 3 \times 3 \times 3)$
$= (2 \times 3 \times 3 \times 3)^2$
$\therefore \sqrt{2916} = 2 \times 3 \times 3 \times 3$
$= \mathbf{54}$

Alternatively,

$2916 = 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$
$= 2^2 \times 3^6$
$\therefore \sqrt{2916} = \sqrt{2^2 \times 3^6}$
$= 2 \times 3^3$
$= \mathbf{54}$

(d) $3136 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 7 \times 7$
$= (2 \times 2 \times 2 \times 7) \times (2 \times 2 \times 2 \times 7)$
$= (2 \times 2 \times 2 \times 7)^2$
$\therefore \sqrt{3136} = 2 \times 2 \times 2 \times 7$
$= \mathbf{56}$

Alternatively,

$3136 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 7 \times 7$
$= 2^6 \times 7^2$
$\therefore \sqrt{3136} = \sqrt{2^6 \times 7^2}$
$= 2^3 \times 7$
$= \mathbf{56}$

2. **(a)** $\sqrt{9801} = \sqrt{3^4 \times 11^2}$
$= 3^2 \times 11$
$= \mathbf{99}$

(b) $\sqrt{35\,721} = \sqrt{3^6 \times 7^2}$
$= 3^3 \times 7$
$= \mathbf{189}$

(c) $\sqrt{24\,336} = \sqrt{2^4 \times 3^2 \times 13^2}$
$= 2^2 \times 3 \times 13$
$= \mathbf{156}$

(d) $\sqrt{518\,400} = \sqrt{2^8 \times 3^4 \times 5^2}$
$= 2^4 \times 3^2 \times 5$
$= \mathbf{720}$

3. **(a)** $3375 = 3 \times 3 \times 3 \times 5 \times 5 \times 5$
$= (3 \times 5) \times (3 \times 5) \times (3 \times 5)$
$= (3 \times 5)^3$
$\therefore \sqrt[3]{3375} = 3 \times 5$
$= \mathbf{15}$

Alternatively,

$3375 = 3 \times 3 \times 3 \times 5 \times 5 \times 5$
$= 3^3 \times 5^3$
$\therefore \sqrt[3]{3375} = \sqrt[3]{3^3 \times 5^3}$
$= 3 \times 5$
$= \mathbf{15}$

(b) $1728 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$
$= (2 \times 2 \times 3) \times (2 \times 2 \times 3) \times (2 \times 2 \times 3)$
$= (2 \times 2 \times 3)^3$
$\therefore \sqrt[3]{1728} = 2 \times 2 \times 3$
$= \mathbf{12}$

Alternatively,

$1728 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$
$= 2^6 \times 3^3$
$\therefore \sqrt[3]{1728} = \sqrt[3]{2^6 \times 3^3}$
$= 2^2 \times 3$
$= \mathbf{12}$

(c) $5832 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$
$= (2 \times 3 \times 3) \times (2 \times 3 \times 3) \times (2 \times 3 \times 3)$
$= (2 \times 3 \times 3)^3$
$\therefore \sqrt[3]{5832} = 2 \times 3 \times 3$
$= \mathbf{18}$

Alternatively,

$5832 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$
$= 2^3 \times 3^6$
$\therefore \sqrt[3]{5832} = \sqrt[3]{2^3 \times 3^6}$
$= 2 \times 3^2$
$= \mathbf{18}$

(d) $8000 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5$
$= (2 \times 2 \times 5) \times (2 \times 2 \times 5) \times (2 \times 2 \times 5)$
$= (2 \times 2 \times 5)^3$
$\therefore \sqrt[3]{8000} = 2 \times 2 \times 5$
$= \mathbf{20}$

Alternatively,

$8000 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5$
$= 2^6 \times 5^3$
$\therefore \sqrt[3]{8000} = \sqrt[3]{2^6 \times 5^3}$
$= 2^2 \times 5$
$= \mathbf{20}$

4. **(a)** $\sqrt[3]{21\,952} = \sqrt[3]{2^6 \times 7^3}$
$= 2^2 \times 7$
$= \mathbf{28}$

(b) $\sqrt[3]{46\,656} = \sqrt[3]{2^6 \times 3^6}$
$= 2^2 \times 3^2$
$= \mathbf{36}$

(c) $\sqrt[3]{287\,496} = \sqrt[3]{2^3 \times 3^3 \times 11^3}$
$= 2 \times 3 \times 11$
$= \mathbf{66}$

(d) $\sqrt[3]{1\,728\,000} = \sqrt[3]{2^9 \times 3^3 \times 5^3}$
$= 2^3 \times 3 \times 5$
$= \mathbf{120}$

5. **(a)** $\sqrt{66} \approx \sqrt{64}$
$= \mathbf{8}$

(b) $\sqrt{80} \approx \sqrt{81}$
$= \mathbf{9}$

(c) $\sqrt[3]{218} \approx \sqrt[3]{216}$
$= \mathbf{6}$

(d) $\sqrt[3]{730} \approx \sqrt[3]{729}$
$= \mathbf{9}$

6. **(a)** $7^2 - \sqrt{361} + 21^3 = 9291$

(b) $\dfrac{\sqrt{555} + 5^2}{2^3 \times \sqrt[3]{222}} = \mathbf{1.0024}$ (to 4 d.p.)

(c) $\sqrt{4^3 + \sqrt[3]{4913}} = \mathbf{9}$

7. $112 = 2 \times 2 \times 2 \times 2 \times 7$
$= 2^4 \times 7$
For $112 \times h = 2^4 \times 7 \times h$ to be a square number, the index of each prime factor must be a multiple of 2.
For h to be the smallest, $h = 7$ so that $2^4 \times 7 \times 7 = 2^4 \times 7^2$ is a square number.
$\therefore$ the smallest whole number which can be multiplied is **7**.

8. $162 = 2 \times 3 \times 3 \times 3 \times 3$
$= 2 \times 34$
For $162 \times k$ to be a cube number, the index of each prime factor must be a multiple of 3.
For k to be the smallest, $k = 2^2 \times 3^2$ so that $2 \times 3^4 \times (2^2 \times 3^2)$ $= 2^3 \times 3^6$ is a cube number.

$\therefore$ the smallest whole number which can be multiplied is $2^2 \times 3^2 = \mathbf{36}$.

9. Let the pages that the textbook is opened to be n and $n + 1$.
$n(n + 1) = 420$
$\sqrt{420} \approx \sqrt{400} = 20$
If $n = 20$, $n + 1 = 21$.
Check: $20 \times 21 = 420$
$\therefore$ the textbook is opened to pages **20 and 21**.

10. Length of each side of photo frame $= \sqrt{250}$ cm
Perimeter of photo frame $= 4 \times \sqrt{250}$
$= \mathbf{63.2\ cm}$ (to 1 d.p.)

11. Length of each side of box $= \sqrt[3]{2197}$
$= 13$ cm
Area of one side of box $= 13^2$
$= \mathbf{169\ cm^2}$

12. $\sqrt{2020} = 44.9$ (to 1 d.p.)
Length of largest square = 44 cm
Number of tiles used $= 44^2$
$= 1936$
Number of tiles left over $= 2020 - 1936$
$= \mathbf{84}$

13. $6 \times 54 = (2 \times 3) \times (2 \times 3^3)$
$= 2^2 \times 3^4$
Since the index of each prime factor of 6×54 is even, then it is a perfect square.

14. **(i)** $6 \times 54 \times k = 2^2 \times 3^4 \times k$
For $2^2 \times 3^4 \times k$ to be a perfect cube, the index of each prime factor must be a multiple of 3.
For k to be the smallest, $k = 2 \times 3^2$ so that $2^2 \times 3^4 \times k = 2^2 \times 3^4 \times (2 \times 3^2) = 2^3 \times 3^6$ is a perfect cube.
$\therefore$ smallest value of $k = 2 \times 3^2 = \mathbf{18}$

(ii) $6 \times 54 \times \dfrac{p}{q} = 2^2 \times 3^4 \times \dfrac{p}{q}$

For $2^2 \times 3^4 \times \dfrac{p}{q}$ to be a perfect cube, the index of each prime factor must be a multiple of 3. So, p and q must contain a factor of 2 or 3.

Since p and q are prime numbers, one of them must be 2 and the other 3.
$\therefore$ $\boldsymbol{p = 2}$ and $\boldsymbol{q = 3}$ so that $2^2 \times 3^4 \times \dfrac{p}{q} = 2^2 \times 3^4 \times \dfrac{2}{3}$
$= 2^3 \times 3^3$ is a perfect cube.

1.3 Highest common factor and lowest common multiple

Practise Now 9

1. **Method 1:**
$56 = 2^3 \times 7$
$84 = 2^2 \times 3 \times 7$
HCF of 56 and 84 $= 2^2 \times 7$
$= \mathbf{28}$

Method 2:

2	56, 84
2	28, 42
7	14, 21
	2, 3

HCF of 56 and 84 $= 2 \times 2 \times 7$
$= \mathbf{28}$

2. $112 = 2^4 \times 7$
$140 = 2^2 \times 5 \times 7$
HCF of 112 and 140 $= 2^2 \times 7$
$= \mathbf{28}$

3. Greatest whole number that will divide both 504 and 588 exactly
= HCF of 504 and 588
$= 2^2 \times 3 \times 7$
$= 84$

Practise Now 10

1. **Method 1:**
$24 = 2^3 \times 3$
$90 = 2 \times 3^2 \times 5$
LCM of 24 and 90 $= 2^3 \times 3^2 \times 5$
$= \mathbf{360}$

Method 2:

2	24, 90
3	12, 45
	4, 15

HCF of 24 and 90 $= 2 \times 3 \times 4 \times 15$
$= \mathbf{360}$

2. Smallest whole number that is divisible by both 120 and 126
= LCM of 120 and 126
$= 2^3 \times 3^2 \times 5 \times 7$
$= \mathbf{2520}$

Practise Now 11

1. $27 = 3^3$

   $45 = 3^2 \times 5$

   LCM of 27 and 45 $= 3^3 \times 5$

   $= 135$

   $135 = 1 \times 135$

   $= 3 \times 45$

   $= 5 \times 27$

   $= 9 \times 15$

   $\therefore$ the number is 135 and its factors are **1, 3, 5, 9, 15, 27, 45 and 135**.

2. $4 = 2^2$

   $20 = 2^2 \times 5$

   LCM of 4 and 20 $= 2^2 \times 5$

   $= 20$

   Since 20 only has 6 factors, the number is more than 20.

   Try $2 \times 20 = 40$.

   $40 = 1 \times 40$

   $= 2 \times 20$

   $= 4 \times 10$

   $= 5 \times 8$

   $\therefore$ the number is 40 and its factors are **1, 2, 4, 5, 8, 10, 20 and 40**.

Practise Now 12

$$\begin{array}{rcccccccc} & n & = & \boxed{3} & \times & 3 & & & \\ & 15 & = & \boxed{3} & & & \times & 5 \\ \hline \text{LCM} = & 45 & = & 3 & \times & 3 & \times & 5 \end{array}$$

$\therefore n = 3 \times 3$

$= \mathbf{9}$

Practise Now 13

1. HCF $= 245 = 5 \times 7^2$

   LCM $= 4410 = 2 \times 3^2 \times 5 \times 7^2$

   Let the two numbers be a and b.

$$\begin{array}{rcccccccccc} & a & = & 2 & & & \times & 5 & \times & 7^2 \\ & b & = & & & 3^2 & \times & 5 & \times & 7^2 \\ \hline \text{HCF} = & 245 & = & & & & & 5 & \times & 7^2 \\ \text{LCM} = & 4410 & = & 2 & \times & 3^2 & \times & 5 & \times & 7^2 \end{array}$$

   $\therefore$ the two numbers are $2 \times 5 \times 7^2 =$ **490** and $3^2 \times 5 \times 7^2 =$ **2205**.

2. **(i)** For $240n = 252p$,

$$\begin{array}{rcccccccc} 240 & = & 2^4 & \times & \boxed{3} & \times & 5 & & \\ n & = & & & \boxed{3} & & & \times & 7 \\ \hline 252 & = & \boxed{2^2} & \times & 3^2 & & & \times & 7 \\ p & = & \boxed{2^2} & & & \times & 5 & & \end{array}$$

   $\therefore$ smallest non-zero whole number $n = 3 \times 7 =$ **21**

   (ii) For $\dfrac{240}{m} = \dfrac{2^4 \times 3 \times 5}{m}$ to be a factor of $252 = 2^2 \times 3^2 \times 7$,

   $m = 2^2 \times 5$ so that $\dfrac{2^4 \times 3 \times 5}{m} = \dfrac{2^4 \times 3 \times 5}{2^2 \times 5} = 2^2 \times 3$.

   $\therefore$ smallest non-zero whole number $m = 2^2 \times 5 =$ **20**

Practise Now 14

1. $15 = 3 \times 5$

   $36 = 2^2 \times 3^2$

   LCM of 15 and 36 $= 2^2 \times 3^2 \times 5$

   $= 180$

   180 minutes = 3 hours

   $\therefore$ the two bells will next toll together at 5.00 p.m.

2. **(i)** $140 = 2^2 \times 5 \times 7$

   $168 = 2^2 \times 3 \times 7$

   HCF of 140 and 168 $= 2^2 \times 7$

   $= 28$

   Greatest possible length of each of the smaller pieces of ropes = **28 cm**

   (ii) Number of smallest pieces of rope she can get altogether

   $= \dfrac{140}{28} + \dfrac{168}{28}$

   $= 5 + 6$

   $= \mathbf{11}$

Introductory Problem Revisited

$64 = 2^6$

$48 = 2^4 \times 3$

(i) Length of each square = HCF of 48 and 64

$= 2^4$

= **16 cm**

(ii) Number of squares Albert can cut $= (64 \div 16) \times (48 \div 16)$

$= 4 \times 3$

$= \mathbf{12}$

Exercise 1C

1. **(a)** $12 = 2^2 \times 3$

   $30 = 2 \times 3 \times 5$

   HCF of 12 and 30 $= 2 \times 3$

   $= \mathbf{6}$

   (b) $13 = 1 \times 13$

   $91 = 7 \times 13$

   HCF of 13 and 91 = **13**

   (c) $126 = 2 \times 3^2 \times 7$

   $240 = 2^4 \times 3 \times 5$

   HCF of 126 and 240 $= 2 \times 3$

   $= \mathbf{6}$

   (d) $180 = 2^2 \times 3^2 \times 5$

   $450 = 2 \times 3^2 \times 5^2$

   HCF of 180 and 450 $= 2 \times 3^2 \times 5$

   $= \mathbf{90}$

   (e) 11 and 31 are prime numbers.

   HCF of 11 and 31 = **1**

   (f) $64 = 2^6$

   $81 = 3^4$

   HCF of 64 and 81 = **1**

2. $156 = 2^2 \times 3 \times 13$

   $168 = 2^3 \times 3 \times 7$

   HCF of 156 and 168 $= 2^2 \times 3$

   $= \mathbf{12}$

3. **(a)** $45 = 3^2 \times 5$
$60 = 2^2 \times 3 \times 5$
LCM of 45 and 60 $= 2^2 \times 3^2 \times 5$
$= \mathbf{180}$

(b) $42 = 2 \times 3 \times 7$
$462 = 2 \times 3 \times 7 \times 11$
LCM of 42 and 462 $= 2 \times 3 \times 7 \times 11$
$= \mathbf{462}$

(c) $54 = 2 \times 3^3$
$240 = 2^4 \times 3 \times 5$
LCM of 54 and 240 $= 2^4 \times 3^3 \times 5$
$= \mathbf{2160}$

(d) 11 and 19 are prime numbers.
LCM 11 and 19 $= 11 \times 19$
$= \mathbf{209}$

(e) $27 = 3^3$
$32 = 2^5$
LCM 27 and 32 $= 3^3 \times 2^5$
$= \mathbf{864}$

(f) $78 = 2 \times 3 \times 13$
$352 = 2^5 \times 11$
LCM of 78 and 352 $= 2^5 \times 3 \times 11 \times 13$
$= \mathbf{13\,728}$

4. $80 = 2^4 \times 5$
$104 = 2^3 \times 13$
Smallest whole number divisible by 80 and 104
= LCM of 80 and 104
$= 2^4 \times 5 \times 13$
$= 1040$

5. $10 = 2 \times 5$
$125 = 5^3$
LCM of 10 and 125 $= 2 \times 5^3$
$= 250$
$250 = 1 \times 250$
$= 2 \times 125$
$= 5 \times 50$
$= 10 \times 25$
$\therefore$ the number is 250 and its factors are **1, 2, 5, 10, 25, 50, 125 and 250**.

6. $40 = 2^3 \times 5$
$100 = 2^2 \times 5^2$
LCM of 40 and 100 $= 2^3 \times 5^2$
$= 200$
$200 = 1 \times 200$
$= 2 \times 100$
$= 4 \times 50$
$= 5 \times 40$
$= 8 \times 25$
$= 10 \times 20$
$\therefore$ the number is 200 and its factors are **1, 2, 4, 5, 8, 10, 20, 25, 40, 50, 100 and 200**.

7.
$$\begin{array}{rcccccccc} k & = & \boxed{2} & \times & 2 & & & \times & 5 \\ 6 & = & \boxed{2} & & & \times & 3 & & \\ \hline \text{LCM} = 60 & = & 2 & \times & 2 & \times & 3 & \times & 5 \end{array}$$
$\therefore k = 2 \times 2 \times 5$
$= \mathbf{20}$

8. Greatest whole number that will divide 792 and 990 exactly
= HCF of 792 and 990
$= 2 \times 3^2 \times 11$
$= 198$

9. Smallest whole number divisible by 176 and 342
= LCM of 176 and 342
$= 2^4 \times 3^2 \times 11 \times 19$
$= \mathbf{30\,096}$

10. $4 = 2^2$
$26 = 2 \times 13$
LCM of 4 and 26 $= 2^2 \times 13$
$= 52$
Since 52 only has 6 factors, the number is more than 52.
Try $2 \times 52 = 104$.
$104 = 1 \times 104$
$= 2 \times 52$
$= 4 \times 26$
$= 8 \times 13$
$\therefore$ the number is 104 and its factors are **1, 2, 4, 8, 13, 26, 52 and 104**.

11. HCF $= 175 = 5^2 \times 7$
LCM $= 12\,600 = 2^3 \times 3^2 \times 5^2 \times 7$
Let the two numbers be a and b.
$$\begin{array}{rcccccccc} a & = & 2^3 & & & \times & 5^2 & \times & 7 \\ b & = & & & 3^2 & \times & 5^2 & \times & 7 \\ \hline \text{HCF} = 175 & = & & & & & 5^2 & \times & 7 \\ \text{LCM} = 12\,600 & = & 2^3 & \times & 3^2 & \times & 5^2 & \times & 7 \end{array}$$
$\therefore$ the two numbers are $2^3 \times 5^2 \times 7 = \mathbf{1400}$ and $3^2 \times 5^2 \times 7 = \mathbf{1575}$.

12. **(i)** $1050 = \mathbf{2 \times 3 \times 5^2 \times 7}$
(ii) Let the two numbers be a and b.
Since they are both greater than 40, a or $b \neq 21$.
$$\begin{array}{rcccccccc} a & = & 2 & \times & 3 & & & \times & 7 \\ b & = & & & 3 & \times & 5^2 & \times & 7 \\ \hline \text{HCF} = 21 & = & & & 3 & & & \times & 7 \\ \text{LCM} = 1050 & = & 2 & \times & 3 & \times & 5^2 & \times & 7 \end{array}$$
$\therefore$ the two numbers are $2 \times 3 \times 7 = \mathbf{42}$ and $3 \times 5^2 \times 7 = \mathbf{525}$.

13. **(i)** $171 = 32 \times 19$
$63 = 32 \times 7$
Largest number of gift bags that can be packed
= HCF of 171 and 63
$= 32$
$= \mathbf{9}$

(ii) Number of pens
$= \frac{171}{9}$
$= \mathbf{19}$
Number of pencils
$= \frac{63}{9}$
$= \mathbf{7}$

14. $120 = 2^3 \times 3 \times 5$

$18 = 2 \times 3^2$

Least number of sweets bought

$= \text{LCM of } 120 \text{ and } 18$

$= 2^3 \times 3^2 \times 5$

$= 360$

Least number of packs of sweets bought

$= \dfrac{360}{120}$

$= \mathbf{3}$

15. (i) $60 = 2^2 \times 3 \times 5$

$80 = 2^4 \times 5$

LCM of 60 and 80 $= 2^4 \times 3 \times 5$

$= 240$

$\therefore$ it will take **240 seconds** for both cars to be back at the starting point at the same time.

(ii) $5 \times 240 \text{ s} = 1200 \text{ s}$

$= 20 \text{ min}$

$\therefore$ it will take **20 minutes** for the faster car to be 5 laps ahead of the slower car.

16. (i) $65 = 5 \times 13$

$50 = 2 \times 5^2$

Length of each square

= HCF of 65 and 50

= **5 cm**

(ii) Number of squares he can cut altogether

$= \dfrac{65}{5} \times \dfrac{50}{5}$

$= 13 \times 10$

$= \mathbf{130}$

17. $8 = 2^3$

$12 = 2^2 \times 3$

LCM of 8 and 12 $= 2^3 \times 3$

$= 24$

Since 24 only has 8 factors, the numbers are more than 24.

We need to multiply 24 by a prime number to get the possible numbers.

Number	Factors	Number of factors
$5 \times 24 = 120$	1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20, 24, 30, 40, 60, 120	16
$7 \times 24 = 168$	1, 2, 3, 4, 6, 7, 8, 12, 14, 21, 24, 28, 42, 56, 84, 168	16

$\therefore$ two possible numbers are **120** and **168**.

Note: Numbers equivalent to any prime number > 5 multiplied by 24 will have exactly 16 factors.

18. (i) For $528h = 540m$,

$$\begin{array}{rcccccccc} 528 & = & 2^4 & \times & \boxed{3} & & & \times & 11 \\ h & = & & & \boxed{3^2} & \times & 5 & & \\ \hline 540 & = & \boxed{2^2} & \times & 3^3 & \times & 5 & & \\ m & = & \boxed{2^2} & & & & & \times & 11 \end{array}$$

$\therefore$ smallest whole number $h = 3^2 \times 5$

$= \mathbf{45}$

(ii) For $\dfrac{528}{k} = \dfrac{2^4 \times 3 \times 11}{k}$ to be a factor of $540 = 2^2 \times 3^3 \times 5$,

$k = 2^2 \times 11$ so that $\dfrac{2^4 \times 3 \times 11}{k} = \dfrac{2^4 \times 3 \times 11}{2^2 \times 11} = 2^2 \times 3$.

$\therefore$ smallest whole number $k = 2^2 \times 11 = \mathbf{44}$

19. (i) For $630n = 1248p$,

$$\begin{array}{rcccccccccc} 630 & = & \boxed{2} & \times & 3^2 & \times & 5 & \times & 7 & & \\ n & = & \boxed{2^4} & & & & & & & \times & 13 \\ \hline 1248 & = & 2^5 & \times & \boxed{3} & & & & & \times & 13 \\ p & = & & & \boxed{3} & \times & 5 & \times & 7 & & \end{array}$$

$\therefore$ smallest non-zero whole number $n = 2^4 \times 13$

$= \mathbf{208}$

(ii) For $\dfrac{1248}{m} = \dfrac{2^5 \times 3 \times 13}{m}$ to be a factor of 630

$= 2 \times 3^2 \times 5 \times 7$,

$m = 2^4 \times 13$ so that $\dfrac{2^5 \times 3 \times 13}{m} = \dfrac{2^5 \times 3 \times 13}{2^4 \times 13} = 2 \times 3$.

$\therefore$ smallest non-zero whole number $m = 2^4 \times 13 = \mathbf{208}$

20. (i) Let the number of boys in the class be n.

Then $15 \times n = 3 \times 5 \times n$ must be a multiple of $21 = 3 \times 7$.

Thus the possible values of n are multiples of 7.

Since the number of students in the class is between 30 and 40, $n = 14$ such that $14 + 20 = 34$.

$\therefore$ number of students in the class = **34**

(ii) Number of chocolate bars their form teacher receives

$= \dfrac{15 \times 14}{21}$

$= \mathbf{10}$

Chapter 2 Fractions

TEACHING NOTES

Suggested Approach

Students have been introduced to the concept of proper fractions and mixed numbers in primary school. They have also learnt to express a given fraction to an equivalent fraction and to perform simple arithmetic operations involving like fractions. Teachers can recap the concept of simple fractions using baking as an example (see Introductory Problem on page 24 of the textbook). Teachers can also get the students to intuit the number of $\frac{1}{4}$ –cups of white sugar the baker has and ask them to discuss how this number is obtained using arithmetic operations (i.e. division). Having derived that the baker can make 8 sets of cookies, the students can then discuss how they can obtain the total amount of other ingredients he needs. Teachers can get the students to discuss other real-life examples of the use of fractions. Diagrams are used at the beginning of each section to let students visualise the operations and understand how the results are obtained. However, teachers should guide students towards performing the operations without visual aids. Fractions and mixed numbers are revisited later in Chapter 4 after students have learnt negative numbers in Chapter 4 (section 4.4).

Section 2.1: Fractions, improper fractions and mixed numbers

Teachers can highlight that the highest common factor (HCF) of the numerator and denominator is used when reducing a given fraction to an equivalent fraction. When introducing improper fractions, teachers can build on students' knowledge of mixed numbers. Teachers are encouraged to revise the addition of like fractions that students have learnt in primary school when expressing a mixed number as an improper fraction and vice versa. When comparing fractions, it is more efficient when each fraction in the group is converted to an equivalent fraction sharing a common denominator. Teachers can highlight that the common denominator is the lowest common multiple (LCM) of the denominators of the fractions in the group.

Section 2.2: Adding and subtracting fractions and mixed numbers

Teachers can build on students' knowledge of addition and subtraction of like fractions – to add or subtract like fractions, the numerators are added or subtracted. Teachers can ask the students to suggest how they can add or subtract unlike fractions, starting with fractions with denominators that are multiples of each other (e.g. $\frac{3}{5}$ and $\frac{1}{10}$ on page 29 of the textbook). Two prerequisite concepts are used when adding or subtracting unlike fractions. 1) finding the lowest common multiple (LCM) of the denominators, and 2) expressing a fraction as an equivalent fraction.

Section 2.3: Multiplying fractions and mixed numbers

Students have learnt how to multiply a proper fraction by a whole number. The concept of fraction of a whole number and the concept of multiplication as 'groups of' are revisited in Worked Example 8(a) and (b). Teachers can draw the students' attention to the use of HCF to simplify the multiplication (see Method 3 of Worked Example 8(a)). Teachers should also impress upon students the commutative property of multiplication. In Worked Example 8(c), teachers can ease students into multiplication involving a mixed number using the concept of 'groups of' (see Method 1) and show that the same result can be obtained by expressing the mixed number as an improper fraction. When multiplying two fractions, the numerators and denominators are respectively multiplied (see Investigation: Multiplying two fractions). Teachers should highlight that a mixed number is converted to an improper fraction first before multiplying it with another fraction.

Section 2.4: Dividing fractions and mixed numbers

Teachers can begin this section by illustrating that dividing by a whole number is the same as multiplying by its reciprocal. This is also applied when dividing by fractions and mixed numbers. Here, students should reinforce this concept by revisiting part (a) of the Introductory Problem.

Introductory Problem

The solutions to this problem can be found in Introductory Problem Revisited (after Practise Now 12).

2.1 Fractions, improper fractions and mixed numbers

Practise Now 1

Simplest form: $\mathbf{\frac{1}{2}}, \mathbf{\frac{3}{4}}, \mathbf{\frac{5}{7}}, \mathbf{\frac{13}{23}}$

Not simplest form: $\frac{4}{6} = \mathbf{\frac{2}{3}}, \frac{6}{9} = \mathbf{\frac{2}{3}}, \frac{8}{10} = \mathbf{\frac{4}{5}}, \frac{20}{200} = \mathbf{\frac{1}{10}}$

Practise Now 2

(a) $\frac{2}{3} = \frac{6}{9}$

$\frac{6}{9} < \frac{7}{9}$ $\therefore \frac{2}{3} \mathbf{<} \frac{7}{9}$

(b) $\frac{3}{4} = \frac{15}{20}$

$\frac{7}{10} = \frac{14}{20}$

$\frac{15}{20} > \frac{14}{20}$ $\therefore \frac{3}{4} \mathbf{>} \frac{7}{10}$

(c) $\frac{9}{21} = \frac{3}{7}$

$\frac{27}{63} = \frac{3}{7}$

$\therefore \frac{9}{21} \mathbf{=} \frac{27}{63}$

(d) $\frac{2}{5} = \frac{16}{40}$

$\frac{3}{8} = \frac{15}{40}$

$\frac{16}{40} > \frac{15}{40}$ $\therefore \frac{2}{5} \mathbf{>} \frac{3}{8}$

Practise Now 3

(a) $2\frac{2}{3} = \frac{3}{3} + \frac{3}{3} + \frac{2}{3}$

$= \mathbf{\frac{8}{3}}$

(b) $5\frac{8}{10} = \frac{10}{10} + \frac{10}{10} + \frac{10}{10} + \frac{10}{10} + \frac{10}{10} + \frac{8}{10}$

$= \frac{58}{10}$

$= \mathbf{\frac{29}{5}}$

Practise Now 4

(a) $\frac{27}{4} = \frac{24}{4} + \frac{3}{4}$

$= \mathbf{6\frac{3}{4}}$

(b) $\frac{30}{9} = \frac{10}{3}$

$= \frac{9}{3} + \frac{1}{3}$

$= \mathbf{3\frac{1}{3}}$

Practise Now 5

(a) Since the whole numbers in $2\frac{2}{5}$ and $2\frac{12}{30}$ are the same, only the proper fractions are compared.

$\frac{2}{5} = \frac{12}{30}$

$\therefore 2\frac{2}{5} \mathbf{=} 2\frac{12}{30}$

(b) $\frac{37}{15}$ and $\frac{14}{5} = \frac{42}{15}$

Since $\frac{37}{15} < \frac{42}{15}$, $\therefore \frac{37}{15} \mathbf{<} \frac{42}{15}$

(c) $\frac{26}{7} = 3\frac{5}{7}$

Since the whole numbers in $3\frac{5}{7}$ and $3\frac{3}{5}$ are the same, only the proper fractions are compared.

$\frac{5}{7} = \frac{25}{35}$ and $\frac{3}{5} = \frac{21}{35}$

Since $\frac{25}{35} > \frac{21}{35}$, thus $\frac{5}{7} > \frac{3}{5}$.

$\therefore \frac{26}{7} \mathbf{>} 3\frac{3}{5}$

2.2 Adding and subtracting fractions and mixed numbers

Practise Now 6

1. **(a)** $\frac{2}{13} + \frac{6}{13} = \frac{(2 + 6)}{13}$

$= \mathbf{\frac{8}{13}}$

(b) $\frac{1}{3} + \frac{5}{9} = \frac{3}{9} + \frac{5}{9}$

$= \mathbf{\frac{8}{9}}$

(c) $\frac{3}{4} + \frac{5}{6} = \frac{9}{12} + \frac{10}{12}$

$= \frac{19}{12}$

$= \mathbf{1\frac{7}{12}}$

(d) $\frac{7}{2} + \frac{11}{6} = \frac{21}{6} + \frac{11}{6}$
$= \frac{32}{6}$
$= \frac{16}{3}$
$= \mathbf{5\frac{1}{3}}$

2. (a) $\frac{11}{15} - \frac{7}{15} = \frac{(11-7)}{15}$
$= \mathbf{\frac{4}{15}}$

(b) $2 - \frac{3}{4} = 1\frac{4}{4} - \frac{3}{4}$
$= \mathbf{1\frac{1}{4}}$

(c) $\frac{4}{5} - \frac{1}{4} = \frac{16}{20} - \frac{5}{20}$
$= \mathbf{\frac{11}{20}}$

(d) $\frac{15}{4} - \frac{5}{9} = 3\frac{3}{4} - \frac{5}{9}$
$= 3\frac{27}{36} - \frac{20}{36}$
$= \mathbf{3\frac{7}{36}}$

Practise Now 7

1. (a) $2\frac{2}{15} + 1\frac{8}{15} = 3\frac{(2+8)}{15}$
$= 3\frac{10}{15}$
$= \mathbf{3\frac{2}{3}}$

(b) $5\frac{3}{11} + 2\frac{9}{11} = 7\frac{(3+9)}{11}$
$= 7\frac{12}{11}$
$= \mathbf{8\frac{1}{11}}$

(c) $\frac{5}{16} + 5\frac{1}{6} = \frac{15}{48} + 5\frac{8}{48}$
$= 5\frac{(15+8)}{48}$
$= \mathbf{5\frac{23}{48}}$

(d) $1\frac{3}{7} + \frac{5}{3} = 1\frac{3}{7} + 1\frac{2}{3}$
$= 1\frac{9}{21} + 1\frac{14}{21}$
$= 2\frac{23}{21}$
$= \mathbf{3\frac{2}{21}}$

2. (a) $2\frac{5}{6} - 2\frac{1}{6} = \frac{(5-1)}{6}$
$= \frac{4}{6}$
$= \mathbf{\frac{2}{3}}$

(b) $5\frac{3}{7} - \frac{19}{7} = 2\frac{3}{7} - 2\frac{5}{7}$
$= 4\frac{10}{7} - 2\frac{5}{7}$
$= \mathbf{2\frac{5}{7}}$

(c) $5\frac{5}{9} - \frac{19}{6} = 5\frac{5}{9} - 3\frac{1}{6}$
$= 5\frac{10}{18} - 3\frac{3}{18}$
$= \mathbf{2\frac{7}{18}}$

(d) $2\frac{1}{6} - 1\frac{2}{5} = 2\frac{5}{30} - 1\frac{12}{30}$
$= 1\frac{35}{30} - 1\frac{12}{30}$
$= \mathbf{\frac{23}{30}}$

3. Total time taken $= \frac{1}{4} + \frac{1}{6} + 1\frac{2}{5}$
$= \frac{15}{60} + \frac{10}{60} + 1\frac{24}{60}$
$= 1\frac{(15+10+24)}{60}$
$= \mathbf{1\frac{49}{60}\ h}$

Exercise 2A

1. (a) $\frac{9}{81} = \mathbf{\frac{1}{9}}$

(b) $\frac{10}{12} = \mathbf{\frac{5}{6}}$

(c) $\frac{36}{90} = \mathbf{\frac{2}{5}}$

(d) $\frac{56}{84} = \mathbf{\frac{2}{3}}$

2. (a) $\frac{2}{5} = \frac{8}{20}$
$\frac{8}{20} < \frac{11}{20} \therefore \frac{2}{5} \mathbf{<} \frac{11}{20}$

(b) $\frac{3}{16} = \frac{15}{80}$
$\therefore \frac{3}{16} \mathbf{=} \frac{15}{80}$

(c) $\frac{2}{3} = \frac{16}{24}$ and $\frac{5}{8} = \frac{15}{24}$
$\frac{16}{24} > \frac{15}{24} \therefore \frac{2}{3} \mathbf{>} \frac{5}{8}$

(d) $\frac{13}{18} = \frac{26}{36}$ and $\frac{10}{12} = \frac{30}{36}$
$\frac{26}{36} < \frac{30}{36} \therefore 1\frac{13}{18} \mathbf{<} \frac{10}{12}$

3. (a) $12\frac{2}{3} = \frac{36}{3} + \frac{2}{3}$

$= \mathbf{\frac{38}{3}}$

(b) $4\frac{3}{8} = \frac{32}{8} + \frac{3}{8}$

$= \mathbf{\frac{35}{8}}$

(c) $5\frac{4}{6} = 5\frac{2}{3}$

$= \frac{15}{3} + \frac{2}{3}$

$= \mathbf{\frac{17}{3}}$

(d) $9\frac{72}{81} = 9\frac{8}{9}$

$= \frac{81}{9} + \frac{8}{9}$

$= \mathbf{\frac{89}{9}}$

4. (a) $\frac{35}{2} = \frac{34}{2} + \frac{1}{2}$

$= \mathbf{17\frac{1}{2}}$

(b) $\frac{21}{14} = \frac{3}{2}$

$= \frac{2}{2} + \frac{1}{2}$

$= \mathbf{1\frac{1}{2}}$

(c) $\frac{50}{6} = \frac{25}{3}$

$= \frac{24}{3} + \frac{1}{3}$

$= \mathbf{8\frac{1}{3}}$

(d) $\frac{30}{18} = \frac{5}{3}$

$= \frac{3}{3} + \frac{2}{3}$

$= \mathbf{1\frac{2}{3}}$

5. (a) Since the whole numbers in $1\frac{4}{12}$ and $1\frac{1}{3}$ are the same, only the proper fractions are compared.

$\frac{4}{12} = \frac{1}{3}$ and $\frac{1}{3}$

$\therefore 1\frac{4}{12} \mathbf{=} 1\frac{1}{3}$

(b) Since the whole numbers in $1\frac{2}{3}$ and $1\frac{9}{12}$ are the same, only the proper fractions are compared.

$\frac{2}{3} = \frac{8}{12}$ and $\frac{9}{12}$

Since $\frac{8}{12} < \frac{9}{12}$, thus $\frac{2}{3} < \frac{9}{12}$.

$\therefore 1\frac{2}{3} \mathbf{<} 1\frac{9}{12}$

(c) Since the whole numbers in $2\frac{12}{54}$ and $2\frac{6}{27}$ are the same, only the proper fractions are compared.

$\frac{12}{54} = \frac{2}{9}$ and $\frac{6}{27} = \frac{2}{9}$

$\therefore 2\frac{12}{54} \mathbf{=} 2\frac{6}{27}$

Alternatively,

$\frac{12}{54}$ and $\frac{6}{27} = \frac{12}{54}$

$\therefore 2\frac{12}{54} \mathbf{=} 2\frac{6}{27}$

(d) $\frac{70}{12}$ and $\frac{17}{3} = \frac{68}{12}$

$\frac{70}{12} > \frac{68}{12}$ $\therefore \frac{70}{12} \mathbf{>} \frac{17}{3}$

(e) $\frac{11}{7} = 1\frac{4}{7}$

$\frac{19}{12} = 1\frac{7}{12}$

$\frac{4}{7} = \frac{48}{84}$ and $\frac{7}{12} = \frac{49}{84}$

Since $\frac{48}{84} < \frac{49}{84}$, thus $\frac{4}{7} < \frac{7}{12}$ and $1\frac{4}{7} < 1\frac{7}{12}$.

$\therefore \frac{11}{7} \mathbf{<} \frac{19}{12}$

(f) $\frac{11}{9}$ and $\frac{77}{63} = \frac{11}{9}$

$\therefore \frac{11}{9} \mathbf{=} \frac{77}{63}$

(g) $\frac{12}{8} = \frac{3}{2} = 1\frac{1}{2}$

$\therefore 1\frac{1}{2} \mathbf{=} \frac{12}{8}$

(h) $\frac{105}{14} = \frac{98}{14} + \frac{7}{14}$

$= 7\frac{7}{14}$

$= 7\frac{1}{2}$

Since the whole numbers in $7\frac{4}{7}$ and $7\frac{1}{2}$ are the same, only the proper fractions are compared.

$\frac{4}{7} = \frac{8}{14}$ and $\frac{1}{2} = \frac{7}{14}$

Since $\frac{8}{14} > \frac{7}{14}$, thus $\frac{4}{7} > \frac{1}{2}$ and $7\frac{4}{7} > 7\frac{1}{2}$.

$\therefore 7\frac{4}{7} \mathbf{>} \frac{105}{14}$

6. (a) $\frac{8}{17} + \frac{5}{17} = \frac{(8+5)}{17}$

$= \mathbf{\frac{13}{17}}$

(b) $\frac{13}{20} + \frac{11}{20} = \frac{(13+11)}{20}$

$= \frac{24}{20}$

$= \frac{20}{20} + \frac{4}{20}$

$= 1\frac{4}{20}$

$= \mathbf{1\frac{1}{5}}$

(c) $\frac{2}{3} + \frac{5}{9} = \frac{6}{9} + \frac{5}{9}$

$= \frac{(6+5)}{9}$

$= \frac{11}{9}$

$= \frac{9}{9} + \frac{2}{9}$

$= \mathbf{1\frac{2}{9}}$

(d) $\frac{4}{6} + \frac{4}{9} = \frac{12}{18} + \frac{8}{18}$

$= \frac{(12+8)}{18}$

$= \frac{20}{18}$

$= \frac{18}{18} + \frac{2}{18}$

$= 1\frac{2}{18}$

$= \mathbf{1\frac{1}{9}}$

Alternatively,

$\frac{4}{6} + \frac{4}{9} = \frac{2}{3} + \frac{4}{9}$

$= \frac{6}{9} + \frac{4}{9}$

$= \frac{10}{9}$

$= \mathbf{1\frac{1}{9}}$

(e) $\frac{3}{4} + \frac{5}{6} = \frac{9}{12} + \frac{10}{12}$

$= \frac{(9+10)}{12}$

$= \frac{12}{12} + \frac{7}{12}$

$= \mathbf{1\frac{7}{12}}$

(f) $\frac{5}{13} - \frac{2}{13} = \frac{(5-2)}{13}$

$= \mathbf{\frac{3}{13}}$

(g) $\frac{7}{20} - \frac{8}{20} = \frac{(17-8)}{20}$

$= \mathbf{\frac{9}{20}}$

(h) $3 - \frac{2}{3} = 2\frac{3}{3} - \frac{2}{3}$

$= 2\frac{(3-2)}{3}$

$= \mathbf{2\frac{1}{3}}$

(i) $\frac{1}{2} - \frac{2}{5} = \frac{5}{10} - \frac{4}{10}$

$= \frac{(5-4)}{10}$

$= \mathbf{\frac{1}{10}}$

(j) $\frac{2}{3} - \frac{1}{7} = \frac{14}{21} - \frac{3}{21}$

$= \frac{(14-3)}{21}$

$= \mathbf{\frac{11}{21}}$

7. (a) $1\frac{1}{7} + 2\frac{5}{7} = 3\frac{(1+5)}{7}$

$= \mathbf{3\frac{6}{7}}$

(b) $1\frac{2}{9} + \frac{8}{9} = 1\frac{(2+8)}{9}$

$= 1\frac{10}{9}$

$= \mathbf{2\frac{1}{9}}$

(c) $2\frac{8}{15} + 5\frac{7}{15} = 7\frac{(8+7)}{15}$

$= 7\frac{15}{15}$

$= \mathbf{8}$

(d) $5\frac{2}{5} + 2\frac{3}{10} = 5\frac{4}{10} + 2\frac{3}{10}$

$= \mathbf{7\frac{7}{10}}$

(e) $1\frac{4}{15} + 3\frac{2}{5} = 1\frac{4}{15} + 3\frac{6}{15}$

$= 4\frac{10}{15}$

$= \mathbf{4\frac{2}{3}}$

(f) $1\frac{3}{10} + \frac{5}{6} = 1\frac{9}{30} + \frac{25}{30}$

$= 1\frac{34}{30}$

$= 2\frac{4}{30}$

$= \mathbf{2\frac{2}{15}}$

(g) $\frac{13}{4} + 1\frac{5}{6} = 3\frac{1}{4} + 1\frac{5}{6}$

$= 3\frac{3}{12} + 1\frac{10}{12}$

$= 4\frac{13}{12}$

$= \mathbf{5\frac{1}{12}}$

(h) $3\frac{5}{7} + \frac{9}{11} = 3\frac{55}{77} + \frac{63}{77}$

$= 3\frac{118}{77}$

$= \mathbf{4\frac{41}{77}}$

8. **(a)** $10\frac{3}{5} - 3\frac{1}{5} = 7\frac{(3-1)}{5}$

$= \mathbf{7\frac{2}{5}}$

(b) $2\frac{3}{7} - \frac{4}{7} = 1\frac{10}{7} - \frac{4}{7}$

$= \mathbf{1\frac{6}{7}}$

(c) $\frac{37}{18} - 1\frac{7}{18} = 2\frac{1}{18} - 1\frac{7}{18}$

$= 1\frac{19}{18} - 1\frac{7}{18}$

$= \frac{12}{18}$

$= \mathbf{\frac{2}{3}}$

(d) $2\frac{8}{11} - 2\frac{9}{22} = 2\frac{16}{22} - 2\frac{9}{22}$

$= \mathbf{\frac{7}{22}}$

(e) $10\frac{3}{8} - 6\frac{13}{24} = 10\frac{9}{24} - 6\frac{13}{24}$

$= 9\frac{33}{24} - 6\frac{13}{24}$

$= 3\frac{20}{24}$

$= \mathbf{3\frac{5}{6}}$

(f) $\frac{15}{4} - 2\frac{1}{6} \quad 3\frac{3}{4} - 2\frac{1}{6}$

$= 3\frac{9}{12} - 2\frac{2}{12}$

$= \mathbf{1\frac{7}{12}}$

(g) $3\frac{5}{8} - 1\frac{5}{6} = 3\frac{15}{24} - 1\frac{20}{24}$

$= 2\frac{39}{24} - 1\frac{20}{24}$

$= \mathbf{1\frac{19}{24}}$

(h) $3\frac{5}{7} - \frac{7}{5} = 3\frac{5}{7} - 1\frac{2}{5}$

$= 3\frac{25}{35} - 1\frac{14}{35}$

$= \mathbf{2\frac{11}{35}}$

9. The lowest common multiple of the denominators is 36.

$\frac{1}{6} = \frac{6}{36}$

$\frac{1}{9} = \frac{16}{36}$

$\frac{2}{3} = \frac{24}{36}$

$\frac{11}{18} = \frac{22}{36}$

$\frac{5}{12} = \frac{15}{36}$

In ascending order,

$\mathbf{\frac{1}{6}, \frac{5}{12}, \frac{4}{9}, \frac{11}{18}, \frac{2}{3}}$

10. **(i)** $3\frac{4}{5} = \frac{15}{5} + \frac{4}{5}$

$= \mathbf{\frac{19}{5}}$

$3\frac{2}{3} = \frac{9}{3} + \frac{2}{3}$

$= \mathbf{\frac{11}{3}}$

$3\frac{15}{25} = 3\frac{3}{5}$

$= \frac{15}{5} + \frac{3}{5}$

$= \mathbf{\frac{18}{5}}$

(ii) The lowest common multiple of the denominators is 30.

$3\frac{4}{5} = \frac{19}{5} = \frac{114}{30}$

$\frac{47}{15} = \frac{94}{30}$

$\frac{32}{10} = \frac{96}{30}$

$3\frac{2}{3} = \frac{11}{3} = \frac{110}{30}$

$3\frac{15}{25} = \frac{18}{5} = \frac{108}{30}$

In ascending order,

$\mathbf{\frac{47}{15}, \frac{32}{10}, 3\frac{15}{25}, 3\frac{2}{3}, 3\frac{4}{5}}$

11. **(a)** $\frac{3}{4} + \frac{3}{8} + \frac{5}{16}$

$= \frac{12}{16} + \frac{6}{16} + \frac{5}{16}$

$= \frac{(12 + 6 + 5)}{16}$

$= \frac{23}{16}$

$= \mathbf{1\frac{7}{16}}$

(b) $\frac{4}{6} - \frac{1}{9} - \frac{1}{3}$

$= \frac{12}{18} - \frac{2}{18} - \frac{6}{18}$

$= \frac{(12 - 2 - 6)}{18}$

$= \frac{4}{18}$

$= \mathbf{\frac{2}{9}}$

Alternatively,

$\frac{4}{6} - \frac{1}{9} - \frac{1}{3}$

$= \frac{2}{3} - \frac{1}{9} - \frac{1}{3}$

$= \frac{6}{9} - \frac{1}{9} - \frac{3}{9}$

$= \frac{(6 - 1 - 3)}{9}$

$= \mathbf{\frac{2}{9}}$

(c) $\frac{3}{4} + \frac{5}{6} - \frac{1}{2}$

$= \frac{9}{12} + \frac{10}{12} - \frac{6}{12}$

$= \frac{13}{12}$

$= \mathbf{1\frac{1}{12}}$

(d) $2\frac{1}{12} + 1\frac{1}{2} + 1\frac{5}{6}$

$= 2\frac{1}{12} + 1\frac{6}{12} + 1\frac{10}{12}$

$= 4\frac{17}{12}$

$= \mathbf{5\frac{5}{12}}$

(e) $5\frac{2}{3} + 4\frac{6}{7} + \frac{37}{21}$

$= 5\frac{2}{3} + 4\frac{6}{7} + 1\frac{16}{21}$

$= 5\frac{14}{21} + 4\frac{18}{21} + 1\frac{16}{21}$

$= 10\frac{48}{21}$

$= \mathbf{12\frac{2}{7}}$

(f) $4\frac{1}{2} + \frac{2}{7} + \frac{12}{35}$

$= 4\frac{35}{70} + \frac{20}{70} + \frac{24}{70}$

$= 4\frac{79}{70}$

$= \mathbf{5\frac{9}{70}}$

(g) $4\frac{5}{6} - 2\frac{1}{9} - 2\frac{1}{3}$

$= 4\frac{15}{18} - 2\frac{2}{28} - 2\frac{6}{18}$

$= \mathbf{\frac{7}{18}}$

(h) $3\frac{5}{21} - \frac{3}{4} - 1\frac{5}{7}$

$= 3\frac{20}{84} - \frac{63}{84} - 1\frac{60}{84}$

$= 1\frac{188}{84} - \frac{63}{84} - 1\frac{60}{84}$

$= \mathbf{\frac{65}{84}}$

(i) $3\frac{3}{4} + \frac{3}{8} - 1\frac{5}{16}$

$= 3\frac{12}{16} + \frac{6}{16} - 1\frac{5}{16}$

$= \mathbf{2\frac{13}{16}}$

(j) $10\frac{5}{8} - 3\frac{5}{6} + 1\frac{1}{3}$

$= 10\frac{15}{24} - 3\frac{20}{24} + 1\frac{8}{24}$

$= 8\frac{3}{24}$

$= \mathbf{8\frac{1}{8}}$

12. Mass of sugar used to bake muffins

$= 1 - \frac{1}{4} - \frac{3}{5}$

$= \frac{20}{20} - \frac{5}{20} - \frac{12}{20}$

$= \mathbf{\frac{3}{20}}$

13. Volume of water Nadia drank

$= \frac{7}{10} + \frac{1}{4}$

$= \frac{14}{20} + \frac{5}{20}$

$= \frac{19}{20}\ l$

Total volume of water

$= \frac{7}{10} + \frac{19}{20}$

$= \frac{14}{20} + \frac{19}{20}$

$= \frac{33}{20}$

$= \mathbf{1\frac{13}{20}\ l}$

14. Distance between library and Raju's house

$= 1\frac{2}{5} + 2\frac{1}{2}$

$= 1\frac{4}{10} + 2\frac{5}{10}$

$= 3\frac{9}{10}$ km

Total distance cycled

$= 3\frac{9}{10} + 1\frac{2}{5}$

$= 3\frac{9}{10} + 1\frac{4}{10}$

$= 4\frac{13}{10}$

$= \mathbf{5\frac{3}{10}}$ **km**

15. Volume of orange juice in each large bottle

$= 1\frac{1}{4} + 1\frac{3}{8}$

$= 1\frac{2}{8} + 1\frac{3}{8}$

$= 2\frac{5}{8}\ l$

Total volume of drinks

$= 1\frac{1}{4} + 1\frac{1}{4} + 1\frac{1}{4} + 2\frac{5}{8} + 2\frac{5}{8}$

$= 1\frac{2}{8} + 1\frac{2}{8} + 1\frac{2}{8} + 2\frac{5}{8} + 2\frac{5}{8}$

$= 7\frac{16}{8}$

$= \mathbf{9}\ \boldsymbol{l}$

(*An alternative method is presented below. Teachers may revisit this question after Section 2.3*)

Total volume of drinks

$= 3 \times 1\frac{1}{4} + 2 \times 2\frac{5}{8}$

$= 3\frac{3}{4} + 4\frac{5}{4}$

$= 7\frac{8}{4}$

$= \mathbf{9}\ \boldsymbol{l}$

16. Number of cakes in one box

$= 1\frac{1}{4} + 1\frac{1}{12}$

$= 1\frac{3}{12} + 1\frac{1}{12}$

$= 2\frac{4}{12}$

$= 2\frac{1}{3}$

Total number of cakes

$= 2\frac{1}{3} + 2\frac{1}{3} + 2\frac{1}{3} + 2\frac{1}{3} + 2\frac{1}{3} + 2\frac{1}{3}$

$= 12\frac{6}{3}$

$= \mathbf{14}$

2.3 Multiplying fractions and mixed numbers

Practise Now 8

(a) $\frac{3}{\cancel{7}} \times \cancel{28}^{\,4} = \frac{3}{1} \times 4$

$= \mathbf{12}$

(b) $\frac{1}{\cancel{6}_{\,2}} \times \cancel{81}^{\,27} = \frac{1}{2} \times 27$

$= \frac{27}{2}$

$= \mathbf{13\frac{1}{2}}$

(c) $36 \times 2\frac{7}{8} = 36 \times \frac{23}{8}$

$= 9 \times \frac{23}{2}$

$= \frac{207}{2}$

$= \mathbf{103\frac{1}{2}}$

Investigation (Multiplying two fractions)

1. **(b)** $\mathbf{\frac{1}{8}}$

(c) $\frac{1}{2}$ of $\frac{1}{4} = \frac{1}{2} \times \frac{1}{4}$

$= \mathbf{\frac{1}{8}}$

$= \frac{1}{(2 \times \mathbf{4})}$

2. **(a)** $\mathbf{\frac{1}{2}}$

(b) $\mathbf{\frac{1}{8}}$

(c) $\frac{1}{4}$ of $\frac{1}{2} = \mathbf{\frac{1}{4}} \times \frac{1}{2}$

$= \mathbf{\frac{1}{8}}$

$= \frac{1}{(\mathbf{4} \times 2)}$

3. **(a)** $\mathbf{\frac{3}{8}}$

(b) **Multiply the numerators** of the fractions to obtain the numerator of the product, i.e. $3 \times 1 = 3$.

Multiply the denominators of the fractions to obtain the denominator of the product, i.e. $4 \times 2 = 8$.

$\frac{3}{4} \times \frac{1}{2} = \frac{(3 \times 1)}{(4 \times 2)} = \frac{3}{8}$

Practise Now 9

(a) $\frac{1}{5} \times \frac{2}{3} = \frac{(1 \times 2)}{(5 \times 3)}$

$= \mathbf{\frac{2}{15}}$

(b) $\dfrac{1}{\cancel{3}} \times \dfrac{\cancel{3}}{13} = \dfrac{1}{1} \times \dfrac{1}{13}$

$= \mathbf{\dfrac{1}{13}}$

(c) $\dfrac{\cancel{4}}{\underset{3}{\cancel{15}}} \times \dfrac{\overset{7}{\cancel{35}}}{\underset{3}{\cancel{12}}} = \dfrac{1}{3} \times \dfrac{7}{3}$

$= \mathbf{\dfrac{7}{9}}$

(d) $1\dfrac{7}{15} \times 2\dfrac{10}{21} = \dfrac{22}{15} \times \dfrac{52}{21}$

$= \dfrac{(22 \times 52)}{(15 \times 21)}$

$= \dfrac{1144}{315}$

$= \mathbf{3\dfrac{199}{315}}$

Practise Now 10

(i) Fraction of muffins which are vanilla

$= 1 - \dfrac{3}{5}$

$= \dfrac{5}{5} - \dfrac{3}{5}$

$= \dfrac{2}{5}$

Fraction of muffins sold

$= \dfrac{1}{2} \times \dfrac{2}{5}$

$= \dfrac{1}{1} \times \dfrac{1}{5}$

$= \mathbf{\dfrac{1}{5}}$

(ii) $\dfrac{1}{5}$ of the muffins = 138

∴ total number of muffins

= 138 × 5

= **690**

2.4 Dividing fractions and mixed numbers

Practise Now 11

(a) $\dfrac{1}{4} \div 3 = \dfrac{1}{4} \times \dfrac{1}{3}$

$= \mathbf{\dfrac{1}{12}}$

(b) $\dfrac{11}{12} \div 11 = \dfrac{\cancel{11}}{12} \times \dfrac{1}{\cancel{11}}$

$= \mathbf{\dfrac{1}{12}}$

(c) $\dfrac{12}{7} \div 8 = \dfrac{\overset{3}{\cancel{12}}}{7} \times \dfrac{1}{\underset{2}{\cancel{8}}}$

$= \dfrac{3}{7} \times \dfrac{1}{2}$

$= \mathbf{\dfrac{3}{14}}$

(d) $2\dfrac{3}{4} \div 9 = \dfrac{11}{4} \times \dfrac{1}{9}$

$= \mathbf{\dfrac{11}{36}}$

Practise Now 12

1. **(a)** $9 \div \dfrac{1}{6} = 9 \times 6$

$= \mathbf{54}$

(b) $\dfrac{7}{8} \div \dfrac{21}{32} = \dfrac{\cancel{7}}{\cancel{8}} \times \dfrac{\overset{4}{\cancel{32}}}{\underset{3}{\cancel{21}}}$

$= \dfrac{1}{1} \times \dfrac{4}{3}$

$= \mathbf{\dfrac{4}{3}}$

(c) $\dfrac{12}{5} \div \dfrac{7}{3} = \dfrac{12}{5} \times \dfrac{3}{7}$

$= \dfrac{36}{35}$

$= \mathbf{1\dfrac{1}{35}}$

(d) $\dfrac{9}{4} \div \dfrac{15}{16} = \dfrac{\overset{3}{\cancel{9}}}{\cancel{4}} \times \dfrac{\overset{4}{\cancel{16}}}{\underset{5}{\cancel{15}}}$

$= \dfrac{3}{1} \times \dfrac{4}{5}$

$= \dfrac{12}{5}$

$= \mathbf{2\dfrac{2}{5}}$

(e) $2\dfrac{7}{10} \div \dfrac{6}{25} = \dfrac{27}{10} \div \dfrac{6}{25}$

$= \dfrac{\overset{9}{\cancel{27}}}{\underset{2}{\cancel{10}}} \times \dfrac{\overset{5}{\cancel{25}}}{\underset{2}{\cancel{6}}}$

$= \dfrac{9}{2} \times \dfrac{5}{2}$

$= \dfrac{45}{4}$

$= \mathbf{11\dfrac{1}{4}}$

(f) $4\dfrac{4}{5} \div 1\dfrac{1}{15} = \dfrac{24}{5} \div \dfrac{16}{15}$

$= \dfrac{\overset{3}{\cancel{24}}}{5} \times \dfrac{\overset{3}{\cancel{15}}}{\underset{2}{\cancel{16}}}$

$= \dfrac{3}{1} \times \dfrac{3}{2}$

$= \dfrac{9}{2}$

$= \mathbf{4\dfrac{1}{2}}$

2. (i) Distance ran by Li Ting's brother

$= 1\frac{3}{5} + 2\frac{11}{20}$

$= 1\frac{12}{20} + 2\frac{11}{20}$

$= 3\frac{23}{20}$

$= 4\frac{3}{20}$ km

Number of laps ran by Li Ting's brother $= 4\frac{3}{20} \div \frac{2}{5}$

$= \frac{83}{20} \times \frac{5}{2}$

$= \frac{83}{4} \times \frac{1}{2}$

$= \frac{83}{8}$

$= 10\frac{3}{8}$

Total distance ran by Li Ting and her brother $= 4\frac{3}{20} + 1\frac{3}{5}$

$= 4\frac{3}{20} + 1\frac{12}{20}$

$= 4\frac{15}{20}$

$= 5\frac{3}{4}$ km

Number of laps ran altogether

$= 5\frac{3}{4} \div \frac{2}{5}$

$= \frac{23}{4} \times \frac{5}{2}$

$= \frac{115}{8}$

$= \mathbf{14\frac{3}{8}}$

Introductory Problem Revisited

(a) Number of portions of white sugar $= 2 \div \frac{1}{4}$

$= 2 \times 4$

$= 8$

Number of cookies $= 8 \times 12 =$ **96**

(b) **2 cups of butter, 2 cups of brown sugar, 4 eggs, 4 teaspoons of vanilla extract, 2 teaspoon of baking soda, 6 cups of flour, 440 g of chocolate chips**

Exercise 2B

1. (a) $\frac{1}{6} \times 96$

$= \frac{1}{1} \times 16$

$= \mathbf{16}$

(b) $\frac{5}{3} \times 63 = 1\frac{2}{3} \times 63$

$1 \times 6 = 63$

$\frac{2}{3} \times 63 = \frac{2}{1} \times 21$

$= 42$

$\therefore \frac{5}{3} \times 63 = 63 + 42$

$= \mathbf{105}$

Alternatively,

$\frac{5}{3} \times 63 = \frac{5}{1} \times 21$

$= \mathbf{105}$

(c) $\frac{9}{4} \times 42 = 2\frac{1}{4} \times 42$

$2 \times 42 = 84$

$\frac{1}{4} \times 42 = \frac{1}{2} \times 21$

$= \frac{21}{2}$

$= 10\frac{1}{2}$

$\therefore \frac{9}{4} \times 42 = 84 + 10\frac{1}{2}$

$= \mathbf{94\frac{1}{2}}$

Alternatively,

$\frac{9}{4} \times 42 = \frac{9}{2} \times 21$

$= \frac{189}{2}$

$= \mathbf{94\frac{1}{2}}$

(d) $21 \times 2\frac{1}{3}$

$21 \times 2 = 42$

$21 \times \frac{1}{3} = 7 \times \frac{1}{1}$

$= 7$

$\therefore 21 \times 2\frac{1}{3} = 42 + 7$

$= \mathbf{49}$

(e) $\frac{1}{2} \times \frac{1}{5} = \frac{(1 \times 1)}{(2 \times 5)}$

$= \mathbf{\frac{1}{10}}$

(f) $\frac{3}{10} \times \frac{11}{12} = \frac{1}{10} \times \frac{11}{4}$

$= \mathbf{\frac{11}{40}}$

(g) $\frac{5}{9} \times \frac{15}{16} = \frac{5}{3} \times \frac{5}{16}$

$= \mathbf{\frac{25}{48}}$

(h) $\frac{15}{22} \times \frac{4}{9} = \frac{5}{11} \times \frac{2}{3}$

$= \mathbf{\frac{10}{33}}$

(i) $2\frac{2}{3} \times \frac{15}{26} = \frac{13}{5} \times \frac{15}{26}$

$= \frac{1}{1} \times \frac{3}{2}$

$= \mathbf{1\frac{1}{2}}$

(j) $1\frac{5}{8} \times \frac{4}{3} = \frac{5}{2} \times \frac{1}{1}$

$= \mathbf{2\frac{1}{2}}$

(k) $\frac{7}{11} \times 1\frac{4}{7} = \frac{7}{11} \times \frac{11}{7}$

$= \mathbf{1}$

(l) $1\frac{1}{9} \times 2\frac{5}{8} = \frac{10}{9} \times \frac{21}{8}$

$= \frac{5}{3} \times \frac{7}{4}$

$= \frac{35}{12}$

$= \mathbf{2\frac{11}{12}}$

2. **(a)** $\frac{1}{2} \div 3 = \frac{1}{2} \times \frac{1}{3}$

$= \mathbf{\frac{1}{6}}$

(b) $\frac{16}{21} \div 8 = \frac{16}{21} \times \frac{1}{8}$

$= \mathbf{\frac{2}{21}}$

(c) $\frac{24}{5} \div 2 = \frac{24}{5} \times \frac{1}{2}$

$= \frac{12}{5}$

$= \mathbf{2\frac{2}{5}}$

(d) $5\frac{25}{26} \div 10 = 5\frac{25}{26} \times \frac{1}{10}$

$= \frac{155}{26} \times \frac{1}{10}$

$= \frac{31}{26} \times \frac{1}{2}$

$= \mathbf{\frac{31}{52}}$

(e) $6 \div \frac{3}{5} = 6 \times \frac{5}{3}$

$= 2 \times 5$

$= \mathbf{10}$

(f) $45 \div \frac{18}{11} = 45 \times \frac{11}{18}$

$= 5 \times \frac{11}{2}$

$= \frac{55}{2}$

$= \mathbf{27\frac{1}{2}}$

(g) $\frac{3}{4} \div \frac{9}{10} = \frac{3}{4} \times \frac{10}{9}$

$= \frac{1}{2} \times \frac{5}{3}$

$= \mathbf{\frac{5}{6}}$

(h) $\frac{8}{15} \div \frac{32}{25} = \frac{8}{15} \times \frac{25}{32}$

$= \frac{1}{3} \times \frac{5}{4}$

$= \mathbf{\frac{5}{12}}$

(i) $\frac{15}{4} \div \frac{5}{2} = \frac{15}{4} \times \frac{2}{5}$

$= \frac{3}{2}$

$= \mathbf{1\frac{1}{2}}$

(j) $1\frac{7}{9} \div \frac{4}{3} = \frac{16}{9} \times \frac{3}{4}$

$= \frac{4}{3}$

$= \mathbf{1\frac{1}{3}}$

(k) $9\frac{2}{9} \div 9 = \frac{83}{9} \times \frac{1}{9}$

$= \frac{83}{81}$

$= \mathbf{1\frac{2}{81}}$

(l) $2\frac{2}{15} \div 1\frac{23}{25} = \frac{32}{15} \div \frac{48}{25}$

$= \frac{32}{15} \times \frac{25}{48}$

$= \frac{2}{3} \times \frac{5}{3}$

$= \frac{10}{9}$

$= \mathbf{1\frac{1}{9}}$

3. Amount of time spent visiting care home

$= \frac{4}{7} \times 8$

$= \frac{32}{7}$

$= \mathbf{4\frac{4}{7}\ h}$

4. Fraction of audience who are children

$= 1 - \frac{3}{4}$

$= \frac{1}{4}$

(i) Fraction of audience who are girls

$= \frac{2}{5} \times \frac{1}{4}$

$= \frac{2}{10}$

$= \mathbf{\frac{1}{10}}$

(ii) $\frac{1}{10}$ of audience = 90

$\therefore$ total number of people in audience

= 90 × 10

= 900

Fraction of audience who are boys

$= \frac{3}{5} \times \frac{1}{4}$

$= \frac{3}{20}$

Number of boys in the audience

$= \frac{3}{20} \times 900$

= 3 × 45

= **135**

5. Mass of each slice of butter

$= \frac{9}{10} \div 12$

$= \frac{9}{10} \times \frac{1}{12}$

$= \frac{3}{10} \times \frac{1}{4}$

$= \frac{3}{40}$ kg

Total mass of butter used for cupcakes

$= \frac{3}{40} \times 2\frac{1}{4}$

$= \frac{3}{40} \times \frac{9}{4}$

$= \mathbf{\frac{27}{160}}$ **kg**

6. Fraction of total cost Yasir and Nadia paid

$= 1 - \frac{1}{4} - \frac{1}{6}$

$= \frac{12}{12} - \frac{3}{12} - \frac{2}{12}$

$= \frac{7}{12}$

Yasir paid $\frac{3}{7}$ of the remaining cost of gift, thus fraction of remaining cost of Nadia paid

$= 1 - \frac{3}{7}$

$= \frac{4}{7}$

$\therefore$ fraction of total cost Nadia paid

$= \frac{4}{7} \times \frac{7}{12}$

$= \frac{1}{3}$

$\frac{1}{3}$ of total cost = \$16

$\therefore$ total cost

= \$16 × 3

= **\$48**

7. Fraction of spectators who are boys

$= \frac{1}{9} \times \frac{3}{4}$

$= \frac{1}{3} \times \frac{1}{4}$

$= \frac{1}{12}$

$= \frac{2}{24}$

Fraction of spectators who are girls

$= \frac{1}{6} \times \frac{1}{4}$

$= \frac{1}{24}$

$\therefore$ the difference between the number of boys and girls is $\frac{1}{24}$ of the spectators.

$\frac{1}{24}$ of the spectators = 56

Number of spectators

= 56 × 24

= **1344**

8. Area of overlapping region

$= \frac{1}{5}$ of the area of *PQRS*

$= \frac{1}{5} \times 13$ cm^2

$= \frac{13}{5}$ cm^2

$\frac{1}{8}$ of the area of *ABCD* = $\frac{13}{5}$ cm^2

$\therefore$ area of ABCD

$= \frac{13}{5} \times 8$

$= \frac{104}{5}$

$= 20\frac{4}{5}$ cm^2

Total area of the figure

= area of *ABCD* + area of *PQRS* – overlapping area

$= 20\frac{4}{5} + 13 - \frac{13}{5}$

$= 20\frac{4}{5} + 13 - 2\frac{3}{5}$

$= \mathbf{31\frac{1}{5}}$ **cm²**

9. Fraction of salary saved and donated

$= 1 - \frac{1}{4} - \frac{2}{5}$

$= \frac{20}{20} - \frac{5}{20} - \frac{8}{20}$

$= \frac{7}{20}$

Fraction of salary donated to 4 charities

$= \frac{2}{7} \times \frac{7}{20}$

$= \frac{1}{10}$

Fraction of salary donated to each charity

$= \frac{1}{10} \div 4$

$= \frac{1}{10} \times \frac{1}{4}$

$= \frac{1}{40}$

$\frac{1}{40}$ of salary = PKR 1400

∴ Albert's salary

= PKR 1400 × 40

= **PKR 56 000**

10. Fraction of lemonade poured into glasses

$= 1 - \frac{2}{5}$

$= \frac{3}{5}$

Volume of lemonade poured into glasses

$= \frac{3}{5} \times 10$

= 6 *l*

Total number of glasses needed

$= 6 \div \frac{3}{10}$

$= 6 \times \frac{10}{3}$

= 2 × 10

= 20

∴ she needs 20 – 12 = **8 more glasses**.

11. Fraction of pocket money left after spending on food

$= 1 - \frac{1}{3}$

$= \frac{2}{3}$

Fraction of remaining pocket money left after spending on the pen

$= 1 - \frac{1}{7}$

$= \frac{6}{7}$

∴ fraction of pocket money Imran has left

$= \frac{6}{7} \times \frac{2}{3}$

$= \frac{2}{7} \times 2$

$= \frac{4}{7}$

$\frac{4}{7}$ of total pocket money = PKR 240

$\frac{4}{7}$ × total pocket money = PKR 240

Total pocket money

= PKR 240 $\div \frac{4}{7}$

= PKR 240 $\times \frac{7}{4}$

= PKR 60 × 7

= **PKR 420**

Chapter 3 Decimals

TEACHING NOTES

Suggested Approach

Students have learnt decimals up to three decimal places and the four arithmetic operations involving decimals in primary school. This chapter opens with an introduction to terminating and recurring decimals, and the equivalence between fractions and decimals (see Introductory Problem on page 48 of the textbook). Teachers can ask for the students' observations about the decimals obtained in Question 1 (see Introductory Problem Revisited on page 50 of the textbook). Teachers can further pique students' curiosity by asking them how they could deduce if a decimal converted from a fraction is terminating or recurring by studying the fraction (this will be discussed further in the Class Discussion: Identifying fractions with equivalent terminating decimals on page 54). With reference to Question 2 of the Introductory Problem, teachers may also give a recurring decimal and ask the students if they are able to convert the recurring decimal to fraction by using the same method as Question 2 in the Introductory Problem.

Teachers should not use the term 'non-terminating decimals' when describing recurring decimals in this chapter. This is because non-terminating decimals also include irrational numbers. Terminating, recurring and non-recurring decimals will be further investigated in Chapter 4 when the students learn about rational and irrational numbers.

Section 3.1: Decimals and fractions

The terms terminating and recurring decimals are introduced in this section. In the first part of this section (Fractions and terminating decimals on page 48), teachers can build upon students' knowledge of decimals to show how a terminating decimal can be converted to a fraction. To convert a fraction into a terminating decimal, teachers can show how the fraction is first expressed as an equivalent fraction with a denominator of 10, 100, 1000, …. This will ease the students into identifying fractions with terminating decimals in the Class Discussion on page 54.

When converting a fraction to a recurring decimal, teachers can get the students to discuss why it is not possible to first convert the fraction to an equivalent fraction with a denominator of 10, 100, 1000, …. The technique to convert a recurring decimal to a fraction involves algebra. Since students may not have been introduced to algebra, teachers must explain the meaning of $10x$ and $10x - x$ used in the Investigation: Converting recurring decimal to fraction (page 51). Teachers can consolidate the students' findings from the Investigation by stressing the need to eliminate the recurring digits after the decimal point.

Section 3.2: Operations involving decimals

Now that the students have learnt how to convert decimals to fractions, teachers may use fractions when revisiting the arithmetic operations involving decimals in this section. This is to allow students to understand the steps they have learnt in primary school when performing arithmetic operations or provide an alternative method to perform the arithmetic operations. For instance, when adding or subtracting decimals with differing number of decimal places, teachers may use equivalent fractions to explain why the insertion of zero(s) after the last digit does not change the value of a decimal. For example, $0.70 = \frac{70}{100} = \frac{7}{10} = 0.7$.

Section 3.3: Conversion of units of measurement for length, mass and volume

Teachers may generalise the examples in Class Discussion: Multiplying the dividing decimals by 10, 100 and 1000 to any powers of ten. Teachers can pique students' interest by highlighting that the units of measurement in this section belong to the metric system, and to discuss other units of measurement within this system.

Introductory Problem

The solutions to this problem can be found in Introductory Problem Revisited (after Practise Now 1).

3.1 Decimals and fractions

Practise Now 1

1. **(a)** 0.05

$= \frac{5}{100}$

$= \mathbf{\frac{1}{20}}$

(b) 0.13

$= \mathbf{\frac{13}{100}}$

(c) 0.04

$= \frac{4}{100}$

$= \frac{1}{25}$

$\therefore 2.04 = \mathbf{2\frac{1}{25}}$

(d) 0.354

$= \frac{354}{1000}$

$= \frac{177}{500}$

$\therefore 7.354 = \mathbf{7\frac{177}{500}}$

2. **(a)** $\frac{2}{5}$

$= \frac{4}{10}$

$= \mathbf{0.4}$

(b) $\frac{9}{20}$

$= \frac{45}{100}$

$= \mathbf{0.45}$

(c) $\frac{78}{250}$

$= \frac{312}{1000}$

$= \mathbf{0.312}$

(d) $\frac{1}{20}$

$= \frac{5}{100}$

$= 0.05$

$\therefore 1\frac{1}{20} = \mathbf{1.05}$

(e) $\frac{3}{4}$

$= \frac{75}{100}$

$= 0.75$

$\therefore 2\frac{3}{4} = \mathbf{2.75}$

3. **(a)** $0.076 = 0.0760$

$\therefore 0.0760 \mathbf{>} 0.0706$

(b) $4.05 = 4.050$

$\therefore 4.050 \mathbf{=} 4.05$

(c) $\frac{33}{1000} = 0.033$

$2\frac{33}{1000} = 2.033$

$2.033 < 2.330$

$\therefore 2\frac{33}{1000} \mathbf{<} 2.330$

(d) $\frac{723}{8} = 90\frac{3}{8}$

$\frac{3}{8}$

$= \frac{375}{1000}$

$= 0.375$

$\frac{723}{8}$

$= 90\frac{3}{8}$

$= 90.375$

$90.37 < 90.375$

$\therefore 90.37 \mathbf{<} \frac{723}{8}$

Introductory Problem Revisited

1. **(a)** $\frac{1}{2}$

$= \frac{5}{10}$

$= \mathbf{0.5}$

(b) $\frac{3}{8}$

$= \frac{375}{1000}$

$= \mathbf{0.375}$

(c) $\frac{1}{3}$

$= 0.333\ 333\ 333\ 333 \ldots$

$= \mathbf{0.3333}$

(d) $\frac{1}{6}$

$= 0.166\ 666\ 666\ 666 \ldots$

$= \mathbf{0.1667}$

(e) $\frac{1}{7}$

$= 0.142\ 857\ 142\ 857\ \ldots$

$= \mathbf{0.142\ 857\ 142\ 9}$

2. (a) $0.3 = \mathbf{\frac{3}{10}}$

(b) 0.4

$= \frac{4}{10}$

$= \mathbf{\frac{2}{5}}$

(c) 0.25

$= \frac{25}{100}$

$= \mathbf{\frac{1}{4}}$

(d) $0.167 = \mathbf{\frac{167}{1000}}$

(e) 0.625

$= \frac{625}{1000}$

$= \mathbf{\frac{5}{8}}$

Method 1 in Worked Example 1(b) **cannot be used** to express the fractions in Question 1(c) to (e). This is because the denominator of each of these fractions cannot be converted to 10, 100, 1000, … .
The digits in the decimals in Question 1(c) to (e) repeat indefinitely, whereas those in Question 1(a) and (b) terminate.

Practise Now 2A

1. (a) $4.4444\ \ldots = \mathbf{4.\dot{4}}$

(b) $15.310\ 310\ 310\ \ldots = \mathbf{15.\dot{3}1\dot{0}}$

(c) $20.164\ 646\ 4\ldots = \mathbf{20.1\dot{6}\dot{4}}$

2. (a) $0.\dot{4}\dot{7} = \mathbf{0.474\ 747}$

(b) $0.0\dot{2}\dot{3} = \mathbf{0.023\ 232}$

(c) $0.\dot{2}0\dot{3} = \mathbf{1.203\ 203}$

Practise Now 2B

1. (a) $\frac{2}{9}$

$= 0.222\ 222\ \ldots$

$= \mathbf{0.\dot{2}}$

(b) $\frac{5}{6}$

$= 0.833\ 333\ \ldots$

$= \mathbf{0.8\dot{3}}$

(c) $\frac{1}{11}$

$= 0.090\ 909\ \ldots$

$= \mathbf{0.\dot{0}\dot{9}}$

(d) $\frac{3}{7}$

$= 0.428\ 571\ 428\ 571\ \ldots$

$= \mathbf{0.\dot{4}28\ 57\dot{1}}$

Investigation: Converting recurring decimal to fraction

1. (b) $10x - x = \mathbf{4.\dot{4}} - 0.\dot{4}$

(c) $9x = \mathbf{4}$

$x = \mathbf{\frac{4}{9}}$

2.

(a)	$0.\dot{8}$	$8.\dot{8}$	$\mathbf{8.\dot{8}} - 0.\dot{8} = 8$	$10x - x = \mathbf{8}$ $9x = \mathbf{8}$ $x = \mathbf{\frac{8}{9}}$
(b)	$1.\dot{5}$	$\mathbf{15.\dot{5}}$	$\mathbf{15.\dot{8} - 1.\dot{5} = 11}$	$\mathbf{10x - x = 11}$ $\mathbf{9x = 11}$ $\mathbf{x = \frac{11}{9}}$

3. (i) To create another recurring decimal that has the same recurring digits after the decimal point.

(ii) To eliminate the recurring digits after the decimal point.

4. (i) When $0.\dot{2}\dot{7}$ is multiplied by 10, the product is $2.\dot{7}\dot{2}$. The recurring decimal $10x$ has a different sequence of recurring digits as x. Thus, these digits cannot be eliminated when $0.\dot{2}\dot{7}$ is subtracted from $2.\dot{7}\dot{2}$, i.e.

$10x - x = 2.\dot{7}\dot{2} - 0.\dot{2}\dot{7}$

$9x = 2.\dot{4}\dot{5}$

(ii) Let x be $0.\dot{2}\dot{7}$
Multiply x by 100.
Subtract x from $100x$ to eliminate the recurring digits.
The left-hand side of the equation is now $99x$, which is equal to a value of 27. Divide both sides of the equation by 99.

(iii) $100x = 27.\dot{2}\dot{7}$

$100x - x = 27.\dot{2}\dot{7} - 0.\dot{2}\dot{7}$

$99x = 27$

$x = \frac{27}{99}$

Practise Now 3

(a) Let $x = 0.\dot{7}$

$10x - x = 7.\dot{7} - 0.\dot{7}$

$9x = 7$

$x = \mathbf{\frac{7}{9}}$

(b) Let $x = 0.\dot{3}\dot{6}$

$100x - x = 36.\dot{3}\dot{6} - 0.\dot{3}\dot{6}$

$99x = 36$

$x = \frac{36}{99}$

$= \mathbf{\frac{4}{11}}$

(c) Let $x = 0.\dot{0}\dot{5}$

$100x - x = 5.\dot{0}\dot{5} - 0.\dot{0}\dot{5}$

$99x = 5$

$x = \mathbf{\frac{5}{99}}$

(d) Let $x = 0.\dot{1}6\dot{7}$

$$1000x - x = 167.\dot{1}6\dot{7} - 0.\dot{1}6\dot{7}$$
$$999x = 167$$
$$x = \mathbf{\frac{167}{999}}$$

Practise Now 4

(a) Let $x = 0.8\dot{3}$

$$10x = 8.\dot{3}$$
$$100x = 83.\dot{3}$$
$$100x - 10x = 83.\dot{3} - 8.\dot{3}$$
$$90x = 75$$
$$x = \frac{75}{90}$$
$$= \mathbf{\frac{5}{6}}$$

(b) Let $x = 0.1\dot{3}\dot{6}$

$$10x = 1.\dot{3}\dot{6}$$
$$1000x = 136.\dot{3}\dot{6}$$
$$1000x - 10x = 136.\dot{3}\dot{6} - 1.\dot{3}\dot{6}$$
$$990x = 135$$
$$x = \frac{135}{990}$$
$$= \mathbf{\frac{3}{22}}$$

(c) Let $x = 0.41\dot{6}$

$$100x = 41.\dot{6}$$
$$1000x = 416.\dot{6}$$
$$1000x - 100x = 416.\dot{6} - 41.\dot{6}$$
$$900x = 375$$
$$x = \frac{375}{900}$$
$$= \mathbf{\frac{5}{12}}$$

(d) Let $x = 0.\dot{6}3\dot{0}$

$$1000x = 630.\dot{6}3\dot{0}$$
$$1000x - x = 630.\dot{6}3\dot{0} - 0.\dot{6}3\dot{0}$$
$$999x = 630$$
$$x = \frac{630}{999}$$
$$= \mathbf{\frac{70}{111}}$$

Practise Now 5

(a) Let $x = 1.\dot{5}\dot{3}$

$$100x = 153.\dot{5}\dot{3}$$
$$100x - x = 153.\dot{5}\dot{3} - 1.\dot{5}\dot{3}$$
$$99x = 152$$
$$x = \frac{152}{99}$$
$$= \mathbf{1\frac{62}{99}}$$

(b) Let $x = 2.6\dot{7}$

$$10x = 26.\dot{7}$$
$$100x = 267.\dot{7}$$
$$100x - 10x = 267.\dot{7} - 26.\dot{7}$$
$$90x = 241$$
$$x = \frac{241}{90}$$
$$= \mathbf{2\frac{61}{90}}$$

(c) Let $x = 2.4\dot{3}\dot{2}$

$$10x = 24.\dot{3}\dot{2}$$
$$1000x = 2432.\dot{3}\dot{2}$$
$$1000x - 10x = 2432.\dot{3}\dot{2} - 24.\dot{3}\dot{2}$$
$$990x = 2408$$
$$x = \frac{2408}{990}$$
$$= 2\frac{428}{990}$$
$$= \mathbf{2\frac{214}{495}}$$

(d) Let $x = 3.2\dot{4}8\dot{4}$

$$10x = 32.\dot{4}8\dot{4}$$
$$10000x = 32484.\dot{4}8\dot{4}$$
$$10000x - 10x = 32484.\dot{4}8\dot{4} - 32.\dot{4}8\dot{4}$$
$$9990x = 32452$$
$$x = \frac{32\,452}{9990}$$
$$= 3\frac{2482}{9990}$$
$$= \mathbf{3\frac{1241}{4995}}$$

Class Discussion: Identifying fractions with equivalent terminating decimals

- $\frac{1}{5} = \frac{2}{10} = \mathbf{0.2}$
- $\frac{1}{2} = \frac{5}{10} = \mathbf{0.5}$
- $\frac{2}{3}$

$$\begin{array}{r|l}
 & 0.666 \\
\hline
3 & 2.000 \\
 & -0 \\
\hline
 & 20 \\
 & -18 \\
\hline
 & 20 \\
 & -18 \\
\hline
 & 20 \\
 & -18 \\
\hline
 & 2 \\
\hline
\end{array}$$

$$\frac{2}{3} = \mathbf{0.\dot{6}}$$

- $\frac{5}{8} = \frac{625}{1000} = \mathbf{0.625}$
- $\frac{2}{7}$

$$
\begin{array}{r}
0.285714 \\
7\overline{)\,2.000000} \\
-\ 0 \\
\hline
20 \\
-\ 14 \\
\hline
60 \\
-\ 56 \\
\hline
40 \\
-\ 35 \\
\hline
50 \\
-\ 49 \\
\hline
10 \\
-\ 7 \\
\hline
30 \\
-\ 28 \\
\hline
2
\end{array}
$$

$\frac{2}{7} = \mathbf{0.\dot{2}8571\dot{4}}$

- $\frac{7}{12}$

$$
\begin{array}{r}
0.583 \\
12\overline{)\,7.000} \\
-\ 0 \\
\hline
70 \\
-\ 60 \\
\hline
100 \\
-\ 96 \\
\hline
40 \\
-\ 36 \\
\hline
4
\end{array}
$$

$\frac{7}{12} = \mathbf{0.58\dot{3}}$

- $\frac{11}{15}$

$$
\begin{array}{r}
0.73 \\
15\overline{)\,11.00} \\
-\ 0 \\
\hline
1\ 00 \\
-\ 1\ 05 \\
\hline
50 \\
-\ 45 \\
\hline
5
\end{array}
$$

$\frac{11}{15} = \mathbf{0.7\dot{3}}$

- $\frac{13}{50} = \frac{26}{100} = \mathbf{0.26}$
- $\mathbf{\frac{1}{5}}$, $\mathbf{\frac{1}{2}}$, $\mathbf{\frac{5}{8}}$ **and** $\mathbf{\frac{13}{50}}$ have equivalent terminating decimals.

2.

Fraction	Prime factorisation of denominator
$\frac{1}{5}$	**5**
$\frac{1}{2}$	**2**
$\frac{2}{3}$	**3**
$\frac{5}{8}$	$\mathbf{2^3}$
$\frac{2}{7}$	**7**
$\frac{7}{12}$	$\mathbf{2^2 \times 3}$
$\frac{11}{15}$	$\mathbf{3 \times 5}$
$\frac{13}{50}$	$\mathbf{2 \times 5^2}$

3. Prime factorisation of the denominators of fractions with equivalent terminating decimals consists only of prime factors 2 or 5. If the prime factorisation of the denominator also includes other prime factors (with or without 2 or 5), the decimal equivalent of the fraction will be recurring.

4. Express the denominator of the fraction as a product of its prime factors. If the list of prime factors only contains prime numbers 2 or 5, the fraction has an equivalent terminating decimal. Otherwise, it is recurring.
(Note: Prime factorisation of any power of ten consists only of prime factors 2 and 5. Thus, a fraction with a denominator whose prime factorisation consists only of prime factors 2 or 5 can be converted to an equivalent fraction with a denominator that is a power of ten, which has a terminating decimal equivalent.)

Exercise 3A

1. (a) 0.5
$= \frac{5}{10}$
$= \mathbf{\frac{1}{2}}$

(b) 0.47
$= \mathbf{\frac{47}{100}}$

(c) 0.36
$= \frac{36}{100}$
$= \mathbf{\frac{9}{25}}$

(d) 0.05
$= \frac{5}{100}$
$= \mathbf{\frac{1}{20}}$

(e) 0.04

$= \frac{4}{100}$

$= \frac{1}{25}$

$2.04 = \mathbf{2\frac{1}{25}}$

(f) 0.785

$= \frac{785}{1000}$

$= \mathbf{\frac{157}{200}}$

(g) 0.355

$= \frac{355}{1000}$

$= \frac{71}{200}$

$7.355 = \mathbf{7\frac{71}{200}}$

(h) 0.040

$= \frac{40}{1000}$

$= \frac{1}{25}$

$1.040 = \mathbf{1\frac{1}{25}}$

2. **(a)** $\frac{3}{5}$

$= \frac{6}{10}$

$= \mathbf{0.6}$

(b) $\frac{3}{20}$

$= \frac{15}{100}$

$= \mathbf{0.15}$

(c) $\frac{6}{20}$

$= \frac{30}{100}$

$= \mathbf{0.3}$

(d) $\frac{23}{25}$

$= \frac{91}{100}$

$= \mathbf{0.92}$

(e) $\frac{1}{2}$

$= \frac{5}{10}$

$= 0.5$

$3\frac{1}{2} = \mathbf{3.5}$

(f) $\frac{3}{4}$

$= \frac{75}{100}$

$= 0.75$

$5\frac{3}{4} = \mathbf{5.75}$

(g) $\frac{25}{4} = 6\frac{1}{4}$

$\frac{1}{4}$

$= \frac{25}{100}$

$= 0.25$

$\frac{25}{4} = 6\frac{1}{4} = \mathbf{6.25}$

(h) $\frac{5}{8}$

$= \frac{625}{1000}$

$= 0.625$

$5\frac{5}{8} = \mathbf{5.625}$

3. **(a)** $1.15 > 1.14$

(b) $1.15 < 1.2$

(c) $2.050 = 2.05$

$\therefore 2.05 = 2.050$

(d) $0.21 < 0.31$

(e) $3.1 > 3.099$

(f) $6\frac{3}{10} = 6.3$

$\therefore 6.3 = 6\frac{3}{10}$

(g) $\frac{11}{5}$

$= 2\frac{1}{5}$

$= 2\frac{2}{10}$

$= 2.2$

$\therefore \frac{11}{5} < 2.5$

(h) $\frac{95}{8}$

$= 11\frac{7}{8}$

$= 11\frac{875}{1000}$

$= 11.875$

$\therefore 11.8705 < \frac{95}{8}$

4. **(a)** $0.2222\ldots = \mathbf{0.\dot{2}}$

(b) $0.123\,123\,123\ldots = \mathbf{0.\dot{1}2\dot{3}}$

(c) $0.1777\ldots = \mathbf{0.1\dot{7}}$

(d) $0.234\,242\,424\ldots = \mathbf{0.23\dot{4}\dot{2}}$

5. **(a)** $0.\dot{1}\dot{2} = \mathbf{0.121\,212}$

(b) $0.7\dot{1} = \mathbf{0.711\,111}$

(c) $0.\dot{3}5\dot{3} = \mathbf{0.353\,353}$

(d) $0.33\dot{2}2\dot{4} = \mathbf{0.332\,242}$

6. **(a)** $\frac{7}{9}$

$$\begin{array}{r} 0.7 \\ 9\overline{)\,7.0} \\ -0 \\ \hline 7\;0 \\ -6\;3 \\ \hline 7 \end{array}$$

$\frac{7}{9}$
$= \mathbf{0.\dot{7}}$

(b) $\frac{1}{18}$

$$\begin{array}{r} 0.055 \\ 18\overline{)\,1.000} \\ -0 \\ \hline 100 \\ -90 \\ \hline 100 \\ -90 \\ \hline 10 \end{array}$$

$\frac{1}{18} = \mathbf{0.0\dot{5}}$

(c) $\frac{11}{15}$

$$\begin{array}{r} 0.73 \\ 15\overline{)\,11.00} \\ -0 \\ \hline 110 \\ -105 \\ \hline 50 \\ -45 \\ \hline 5 \end{array}$$

$\frac{11}{15} = \mathbf{0.7\dot{3}}$

(d) $\frac{5}{12}$

$$\begin{array}{r} 0.416 \\ 12\overline{)\,5.000} \\ -0 \\ \hline 50 \\ -48 \\ \hline 20 \\ -12 \\ \hline 80 \\ -72 \\ \hline 8 \end{array}$$

$\frac{5}{12} = \mathbf{0.41\dot{6}}$

7. **(a)** $x = 0.\dot{5}$

$10x = 5.\dot{5}$

$10x - x = 5.\dot{5} - 0.\dot{5}$

$9x = 5$

$x = \mathbf{\frac{5}{9}}$

(b) $x = 0.\dot{6}$

$10x = 6.\dot{6}$

$10x - x = 6.\dot{6} - 0.\dot{6}$

$9x = 6$

$x = \frac{6}{9}$

$= \mathbf{\frac{2}{3}}$

(c) $x = 0.\dot{8}\dot{7}$

$100x = 87.\dot{8}\dot{7}$

$100x - x = 87.\dot{8}\dot{7} - 0.\dot{8}\dot{7}$

$99x = 87$

$x = \frac{87}{99}$

$= \mathbf{\frac{29}{33}}$

(d) $x = 0.\dot{1}2\dot{1}$

$1000x = 121.\dot{1}2\dot{1}$

$1000x - x = 121.\dot{1}2\dot{1} - 0.\dot{1}2\dot{1}$

$999x = 121$

$x = \mathbf{\frac{121}{999}}$

(e) $x = 0.1\dot{2}$

$10x = 1.\dot{2}$

$100x = 12.\dot{2}$

$100x - 10x = 12.\dot{2} - 1.\dot{2}$

$90x = 11$

$x = \mathbf{\frac{11}{90}}$

(f) $x = 0.0\dot{7}$

$10x = 0.\dot{7}$

$100x = 7.\dot{7}$

$100x - 10x = 7.\dot{7} - 0.\dot{7}$

$90x = 7$

$x = \mathbf{\frac{7}{90}}$

(g) $x = 0.7\dot{7}\dot{4}$

$10x = 7.\dot{7}\dot{4}$

$1000x = 774.\dot{7}\dot{4}$

$1000x - 10x = 774.\dot{7}\dot{4} - 7.\dot{7}\dot{4}$

$990x = 767$

$x = \mathbf{\frac{767}{990}}$

(h) $x = 0.77\dot{4}$

$$100x = 77.\dot{4}$$
$$1000x = 774.\dot{4}$$
$$1000x - 100x = 774.\dot{4} - 77.\dot{4}$$
$$900x = 697$$
$$x = \mathbf{\frac{697}{900}}$$

8. **(a)** $x = 1.\dot{6}$

$$10x = 16.\dot{6}$$
$$10x - x = 16.\dot{6} - 1.\dot{6}$$
$$9x = 15$$
$$x = \frac{15}{9}$$
$$= 1\frac{6}{9}$$
$$= 1\mathbf{\frac{2}{3}}$$

(b) $x = 2.\dot{0}\dot{7}$

$$100x = 207.\dot{0}\dot{7}$$
$$100x - x = 207.\dot{0}\dot{7} - 2.\dot{0}\dot{7}$$
$$99x = 205$$
$$x = \frac{205}{99}$$
$$= \mathbf{2\frac{7}{99}}$$

(c) $x = 3.\dot{4}1\dot{5}$

$$1000x = 3415.\dot{4}1\dot{5}$$
$$1000x - x = 3415.\dot{4}1\dot{5} - 3.\dot{4}1\dot{5}$$
$$999x = 3412$$
$$x = \frac{3412}{999}$$
$$= \mathbf{3\frac{415}{999}}$$

(d) $x = 1.5\dot{7}$

$$10x = 15.\dot{7}$$
$$100x = 157.\dot{7}$$
$$100x - 10x = 157.\dot{7} - 15.\dot{7}$$
$$90x = 142$$
$$x = \frac{142}{90}$$
$$= 1\frac{52}{90}$$
$$= \mathbf{1\frac{26}{45}}$$

9. **(a)** $1.017\ 317\ 3173\ldots = \mathbf{1.0\dot{1}7\dot{3}}$

(b) $4.4444\ \ldots = \mathbf{4.\dot{4}}$

(c) $20.022\ 323\ 23\ \ldots = \mathbf{20.02\dot{2}\dot{3}}$

(d) $7.147\ 575\ 75\ \ldots = \mathbf{7.14\dot{7}\dot{5}}$

10. **(a)** $0.0\dot{1}0\dot{2} = 0.010\ 210\ 2102$

(b) $12.1\dot{1}1\dot{2} = 12.111\ 211\ 2112$

(c) $10.00\dot{3} = 10.003\ 333\ 3333$

(d) $3.\dot{0}33\dot{2} = 3.033\ 203\ 3203$

11. **(a)** $\frac{2}{11}$

$$\begin{array}{r l}
 & 0.181 \\
11 \,\big)\!\! & \overline{2.000} \\
 & -0 \\
 & 20 \\
 & -11 \\
 & 90 \\
 & -88 \\
 & 20 \\
 & -11 \\
 & 9
\end{array}$$

$$\frac{2}{11} = \mathbf{0.\dot{1}\dot{8}}$$

(b) $\frac{3}{22}$

$$\begin{array}{r l}
 & 0.136 \\
22 \,\big)\!\! & \overline{3.000} \\
 & -0 \\
 & 30 \\
 & -22 \\
 & 80 \\
 & -66 \\
 & 140 \\
 & -132 \\
 & 8
\end{array}$$

$$\frac{3}{22} = \mathbf{0.\dot{1}\dot{3}\dot{6}}$$

(c) $\frac{4}{7}$

$$\begin{array}{r l}
 & 0.571428 \\
22 \,\big)\!\! & \overline{3.000000} \\
 & -0 \\
 & 40 \\
 & -35 \\
 & 50 \\
 & -49 \\
 & 10 \\
 & -7 \\
 & 30 \\
 & -28 \\
 & 20 \\
 & -14 \\
 & 60 \\
 & -56 \\
 & 4
\end{array}$$

$$\frac{4}{7} = \mathbf{0.\dot{5}71\ 42\dot{8}}$$

(d) $\frac{1}{13}$

$$
\begin{array}{r|l}
 & 0.076923 \\
\hline
13 & 1.000000 \\
 & -0 \\
\hline
 & 100 \\
 & -\ \ 91 \\
\hline
 & 90 \\
 & -78 \\
\hline
 & 120 \\
 & -117 \\
\hline
 & 30 \\
 & -26 \\
\hline
 & 40 \\
 & -39 \\
\hline
 & 1 \\
\hline
\end{array}
$$

$\frac{1}{13} = \mathbf{0.\dot{0}76\,92\dot{3}}$

12. (a) $x = 0.\dot{1}\dot{2}$

$100x = 12.\dot{1}\dot{2}$

$100x - x = 12.\dot{1}\dot{2} - 0.\dot{1}\dot{2}$

$99x = 12$

$x = \frac{12}{99}$

$= \mathbf{\frac{4}{33}}$

(b) $x = 0.\dot{7}\dot{2}$

$100x = 72.7$

$100x - x = 72.\dot{7}\dot{2} - 0.\dot{7}\dot{2}$

$99x = 72$

$x = \frac{72}{99}$

$= \mathbf{\frac{8}{11}}$

(c) $x = 0.\dot{1}6\dot{2}$

$1000x = 162.\dot{1}6\dot{2}$

$1000x - x = 162.\dot{1}6\dot{2} - 0.\dot{1}6\dot{2}$

$999x = 162$

$x = \frac{162}{999}$

$= \mathbf{\frac{6}{37}}$

(d) $x = 0.\dot{0}79\dot{2}$

$10000x = 792.\dot{0}79\dot{2}$

$10000x - x = 792.\dot{0}79\dot{2} - 0.\dot{0}79\dot{2}$

$9999x = 792$

$x = \frac{792}{9999}$

$= \mathbf{\frac{8}{101}}$

(e) $x = 0.1\dot{5}$

$10x = 1.\dot{5}$

$100x = 15.\dot{5}$

$100x - 10x = 15.\dot{5} - 1.\dot{5}$

$90x = 14$

$x = \frac{14}{90}$

$= \mathbf{\frac{7}{45}}$

(f) $x = 0.00\dot{2}$

$100x = 0.\dot{2}$

$1000x = 2.\dot{2}$

$1000x - 100x = 2.\dot{2} - 0.\dot{2}$

$900x = 2$

$x = \frac{2}{900}$

$= \mathbf{\frac{1}{450}}$

(g) $x = 0.0\dot{0}\dot{2}$

$10x = 0.\dot{0}\dot{2}$

$1000x = 2.\dot{0}\dot{2}$

$1000x - 10x = 2.\dot{0}\dot{2} - 0.\dot{0}\dot{2}$

$990x = 2$

$x = \frac{2}{990}$

$= \mathbf{\frac{1}{495}}$

(h) $x = 0.16\dot{3}$

$100x = 16.\dot{3}$

$1000x = 163.\dot{3}$

$1000x - 100x = 163.\dot{3} - 16.\dot{3}$

$900x = 147$

$x = \frac{147}{900}$

$= \mathbf{\frac{49}{300}}$

13. (a) $x = 1.0\dot{7}$

$10x = 10.\dot{7}$

$100x = 107.\dot{7}$

$100x - 10x = 107.\dot{7} - 10.\dot{7}$

$90x = 97$

$x = \frac{97}{90}$

$= \mathbf{1\frac{7}{90}}$

(b) $x = 2.11\dot{5}$

$$\begin{aligned} 100x &= 211.\dot{5} \\ 1000x &= 2115.\dot{5} \\ 1000x - 100x &= 2115.\dot{5} - 211.\dot{5} \\ 900x &= 1904 \\ x &= \frac{1904}{900} \\ &= 2\frac{104}{900} \\ &= \mathbf{2\frac{26}{225}} \end{aligned}$$

(c) $x = 2.5\dot{0}\dot{7}$

$$\begin{aligned} 10x &= 25.\dot{0}\dot{7} \\ 1000x &= 2507.\dot{0}\dot{7} \\ 1000x - 10x &= 2507.\dot{0}\dot{7} - 25.\dot{0}\dot{7} \\ 990x &= 2482 \\ x &= \frac{2482}{990} \\ &= 2\frac{502}{990} \\ &= \mathbf{2\frac{251}{495}} \end{aligned}$$

14. Students should do long divisions that are similar to questions 6 and 11 until the digits repeat.

(a) $\frac{1}{17}$

$= 0.058\ 823\ 529\ 411\ 7647\ldots$

$= \mathbf{0.\dot{0}58\ 823\ 529\ 411\ 764\dot{7}}$

(b) $\frac{1}{19}$

$= 0.052\ 631\ 578\ 947\ 368\ldots$

$= \mathbf{0.\dot{0}52\ 631\ 578\ 947\ 36\dot{8}}$

15. (a) $x = 0.\dot{3}84\ 61\dot{5}$

$$\begin{aligned} 1000\ 000x &= 384\ 615.\dot{3}84\ 61\dot{5} \\ 1000\ 000x - x &= 384\ 615.\dot{3}84\ 61\dot{5} - 0.\dot{3}84\ 61\dot{5} \\ 999\ 999x &= 384\ 615 \\ x &= \frac{384\ 615}{999\ 999} \\ &= \frac{5}{13} \end{aligned}$$

(b) $x = 0.\dot{4}28\ 57\dot{1}$

$$\begin{aligned} 1000\ 000x &= 428\ 571.\dot{4}28\ 57\dot{1} \\ 1000\ 000x - x &= 428\ 571.\dot{4}28\ 57\dot{1} - 0.\dot{4}28\ 57\dot{1} \\ 999\ 999x &= 428\ 571 \\ x &= \frac{428\ 571}{999\ 999} \\ &= \frac{3}{7} \end{aligned}$$

(c) $x = 0.18\dot{9}\dot{7}$

$$\begin{aligned} 100x &= 18.\dot{9}\dot{7} \\ 10\ 000x &= 1897.\dot{9}\dot{7} \\ 10\ 000x - 100x &= 1897.\dot{9}\dot{7} - 18.\dot{9}\dot{7} \\ 9900x &= 1879 \\ x &= \mathbf{\frac{1879}{9900}} \end{aligned}$$

(d) $x = 0.1\dot{8}9\dot{7}$

$$\begin{aligned} 10x &= 1.\dot{8}9\dot{7} \\ 10\ 000x &= 1897.\dot{8}9\dot{7} \\ 10\ 000x - 10x &= 1897.\dot{8}9\dot{7} - 1.\dot{8}9\dot{7} \\ 9990x &= 1896 \\ x &= \frac{1896}{9900} \\ &= \mathbf{\frac{316}{1665}} \end{aligned}$$

16. (a) $x = 1.10\dot{8}\dot{7}$

$$\begin{aligned} 100x &= 110.\dot{8}\dot{7} \\ 10\ 000x &= 11\ 087.\dot{8}\dot{7} \\ 10\ 000x - 100x &= 11\ 087.\dot{8}\dot{7} - 110.\dot{8}\dot{7} \\ 9900x &= 10\ 977 \\ x &= 10\frac{977}{9900} \\ &= 1\frac{1077}{9900} \\ &= \mathbf{1\frac{359}{3300}} \end{aligned}$$

(b) $x = 2.2\dot{4}4\dot{6}$

$$\begin{aligned} 10x &= 22.\dot{4}4\dot{6} \\ 10\ 000x &= 22\ 446.\dot{4}4\dot{6} \\ 10\ 000x - 10x &= 22\ 446.\dot{4}4\dot{6} - 22.\dot{4}4\dot{6} \\ 9990x &= 22\ 424 \\ x &= 22\frac{424}{9990} \\ &= 2\frac{2444}{9990} \\ &= \mathbf{2\frac{1222}{4995}} \end{aligned}$$

17. $x = 0.\dot{9} = 0.999..$

Then $10x = 9.999..$

$$\begin{aligned} 10x - x &= 9 \\ 9x &= 9 \\ x &= 1 \\ 3 - 2.\dot{9} &= 3 - (2 + 0.\dot{9}) \\ &= 3 - (2 + 1) \\ &= 3 - 3 \\ &= \mathbf{0} \end{aligned}$$

3.2 Operations involving decimals

Practise Now 6A

1. (a)

$$\begin{array}{r} \overset{1}{2}.63 \\ +\ \ 8.50 \\ \hline 11.13 \\ \hline \end{array}$$

$\therefore\ 2.63 + 8.5 = \mathbf{11.13}$

(b)
$$\begin{array}{r} 5.70 \\ +\ 6.24 \\ \hline 11.94 \\ \hline \end{array}$$
$\therefore$ 5.7 + 6.24 = **11.94**

(c)
$$\begin{array}{r} \overset{1}{2}8.\overset{1}{0}90 \\ +\ \ 3.654 \\ \hline 31.744 \\ \hline \end{array}$$
$\therefore$ 28.09 + 3.654 = **31.744**

(d)
$$\begin{array}{r} 1\overset{1}{4}.\overset{1}{1}5 \\ +21.96 \\ \hline 36.11 \\ \hline \end{array}$$
$\therefore$ 14.15 + 21.96 = **36.11**

2. (a)
$$\begin{array}{r} 8.\overset{5}{\cancel{6}}\overset{1}{3} \\ -6.59 \\ \hline 2.04 \\ \hline \end{array}$$
$\therefore$ 8.63 – 6.59 = **2.04**

(b)
$$\begin{array}{r} \overset{6}{\cancel{7}}.\overset{1}{0} \\ -5.4 \\ \hline 1.6 \\ \hline \end{array}$$
$\therefore$ 7 – 5.4 = **1.6**

(c)
$$\begin{array}{r} 38.\overset{8}{\cancel{9}}\overset{1}{0} \\ -\ \ 3.65 \\ \hline 35.25 \\ \hline \end{array}$$
$\therefore$ 38.9 – 3.65 = **35.25**

(d)
$$\begin{array}{r} \overset{2}{\cancel{3}}\overset{9}{\cancel{0}}.\overset{\overset{9}{\cancel{1}}}{\cancel{0}}\overset{1}{0} \\ -\ \ 5.84 \\ \hline 24.16 \\ \hline \end{array}$$
$\therefore$ 30 – 5.84 = **24.16**

Practise Now 6B

(a)
$$\begin{array}{r} \overset{3}{1}.5 \\ \times\ \ \ 7 \\ \hline 10.5 \\ \hline \end{array}$$
$\therefore$ 1.5 × 7 = **10.5**

(b)
$$\begin{array}{r} 0.17 \\ \times\ \ 32 \\ \hline 34 \\ +\ 510 \\ \hline 5.44 \\ \hline \end{array}$$
$\therefore$ 35 × 0.17 = **5.44**

(c)
$$\begin{array}{r} \overset{2}{1}\overset{1}{5}.\overset{3}{4}8 \\ \times\ \ \ \ 40 \\ \hline 619.20 \\ \hline \end{array}$$
$\therefore$ 15.48 × 40 = **619.20**

Practise Now 7

(a)
$$\begin{array}{r} 13.56 \\ \times\ \ 2.4 \\ \hline 5424 \\ +2712\ \ \\ \hline 32.544 \\ \hline \end{array}$$
$\therefore$ 13.56 × 2.4 = **32.544**

(b)
$$\begin{array}{r} 137.8 \\ \times\ \ 0.351 \\ \hline 1378 \\ +\ \ 6890\ \ \\ +4134\ \ \ \ \\ \hline 48.3678 \\ \hline \end{array}$$
$\therefore$ 137.8 × 0.351 = **48.3678**

Practise Now 8

(a)
$$\begin{array}{r} 0.05 \\ 3\overline{)0.15} \\ -0\ \ \ \ \\ \hline 1\ \ \\ -0\ \ \\ \hline 15 \\ -15 \\ \hline 0 \\ \hline \end{array}$$
$\therefore$ 0.15 ÷ 3 = **0.05**

(b)
$$\begin{array}{r} 0.675 \\ 4\overline{)2.700} \\ -0\ \ \ \ \ \ \\ \hline 27\ \ \ \ \\ -24\ \ \ \ \\ \hline 30\ \ \\ -28\ \ \\ \hline 20 \\ -20 \\ \hline 0 \\ \hline \end{array}$$
$\therefore$ 2.7 ÷ 4 = **0.675**

(c)
$$\begin{array}{r} 0.26 \\ 12\overline{)3.12} \\ -0\ \ \ \ \\ \hline 31\ \ \\ -24\ \ \\ \hline 72 \\ -72 \\ \hline 0 \\ \hline \end{array}$$
$\therefore$ 3.12 ÷ 12 = **0.26**

Practise Now 9

(a) $0.92 \div 0.4 = \dfrac{0.92}{0.4}$

$= \dfrac{9.2}{4}$

$$\begin{array}{r}
2.3 \\
4\,\overline{)\;9.2} \\
-\;8 \\
\hline
1\;2 \\
-\;1\;2 \\
\hline
0
\end{array}$$

$\therefore\ 0.92 \div 0.4 = \mathbf{2.3}$

Alternatively,

$0.92 \div 0.4$

$= \dfrac{92}{100} \div \dfrac{4}{10}$

$= \dfrac{92}{100} \times \dfrac{10}{4}$

$= \dfrac{23}{10}$

$= 2.3$

(b) $1.845 \div 0.15 = \dfrac{1.845}{0.15}$

$= \dfrac{184.5}{15}$

$$\begin{array}{r}
1\;2\;.\;3 \\
15\,\overline{)\;1\;8\;4\;.\;5} \\
-\;1\;5 \\
\hline
3\;4 \\
-\;3\;0 \\
\hline
4\;5 \\
-\;4\;5 \\
\hline
0
\end{array}$$

$\therefore\ 1.845 \div 0.15 = \mathbf{12.3}$

Alternatively,

$1.845 \div 0.15$

$= \dfrac{1845}{1000} \div \dfrac{15}{100}$

$= \dfrac{1845}{1000} \times \dfrac{100}{15}$

$= \dfrac{123}{10}$

$= 12.3$

Class Discussion (Importance of place value)

In the division "proof", the number the "1" in the quotient is taken to be in the tens place. However, it has a value of 1.

In the multiplication "proof", the "1" in 14 is taken to have a value of 1 when it actually has a value of 10.

In the addition "proof", the "1"s in the five 14s are each taken to have a value of 1 when they actually have a value of 10.

3.3 Conversion of units of measurement for length, mass and volume

Class Discussion: Multiplying and dividing decimals by 10, 100 and 1000

	10	100	1000
Multiplication by	$0.1 \times 10 = \frac{1}{10} \times 10$ $= 1$ $0.01 \times 10 = \frac{\mathbf{1}}{\mathbf{100}} \times 10$ $= \mathbf{0.1}$ $0.001 \times 10 = \frac{\mathbf{1}}{\mathbf{1000}} \times 10$ $= \mathbf{0.01}$ • When a decimal is **multiplied by 10**, the decimal point shifts **1** **place to the right**.	$0.1 \times 100 = \frac{1}{10} \times 100$ $= \mathbf{10}$ $0.01 \times 100 = \frac{\mathbf{1}}{\mathbf{100}} \times 100$ $= \mathbf{0.01}$ $0.001 \times 100 = \frac{\mathbf{1}}{\mathbf{1000}} \times 100$ $= \mathbf{0.01}$ • When a decimal is **multiplied by 100**, the decimal point shifts **2** **places to the right**.	$0.1 \times 1000 = \frac{1}{10} \times 1000$ $= \mathbf{100}$ $0.01 \times 1000 = \frac{\mathbf{1}}{\mathbf{100}} \times 1000$ $= \mathbf{10}$ $0.001 \times 1000 = \frac{\mathbf{1}}{\mathbf{1000}} \times 1000$ $= \mathbf{1}$ • When a decimal is **multiplied by 1000**, the decimal point shifts **3** **places to the right**.
Division by	$0.1 \div 10 = \frac{1}{10} \times \frac{1}{10}$ $= \mathbf{0.01}$ $0.01 \div 10 = \frac{\mathbf{1}}{\mathbf{100}} \times \frac{1}{10}$ $= \mathbf{0.001}$ $0.001 \div 10 = \frac{\mathbf{1}}{\mathbf{1000}} \times \frac{1}{10}$ $= \mathbf{0.0001}$ • When a decimal is **divided by 10**, the decimal point shifts **1** **place to the left**.	$0.1 \div 100 = \frac{1}{10} \times \frac{1}{100}$ $= \mathbf{0.001}$ $0.01 \div 100 = \frac{\mathbf{1}}{\mathbf{100}} \times \frac{1}{100}$ $= \mathbf{0.0001}$ $0.001 \div 100 = \frac{\mathbf{1}}{\mathbf{1000}} \times \frac{1}{100}$ $= \mathbf{0.00001}$ • When a decimal is **divided by 100**, the decimal point shifts **2** **places to the left**.	$0.1 \div 1000 = \frac{1}{10} \times \frac{1}{1000}$ $= \mathbf{0.0001}$ $0.01 \div 1000 = \frac{\mathbf{1}}{\mathbf{100}} \times \frac{1}{1000}$ $= \mathbf{0.00001}$ $0.001 \div 1000 = \frac{\mathbf{1}}{\mathbf{1000}} \times \frac{1}{1000}$ $= \mathbf{0.000001}$ • When a decimal is **divided by 1000**, the decimal point shifts **2** **places to the left**.

Practise Now 10A

1. **(a)** 0.7 × 10 = **7**

(b) 0.063 × 100 = **6.3**

(c) 3.61 × 1000 = **3610**

(d) 32.9 × 1000 = **32 900**

2. **(a)** 0.9 ÷ 10 = **0.09**

(b) 137 ÷ 100 = **1.37**

(c) 143.1 ÷ 1000 = **0.1431**

(d) 2.7 ÷ 1000 = **0.0027**

Practise Now 10B

(a) 1 km = 1000 m
3.608 km
= 3.608 × 1000
= **3608 m**

(b) 1 km = 1000 m
7.055 km
= 7.055 × 1000
= 7055 m
1 m = 100 cm
7055 m
= 7055 × 100
= **705 500 cm**

(c) 1000 m = 1 km
1385 m
= 1385 ÷ 1000
= **1.385 km**

(d) 10 mm = 1 cm
485 mm
= 485 ÷ 10
= 48.5 cm
100 cm = 1 m
48.5 cm
= 48.5 ÷ 100
= **0.485 m**

Practise Now 11

(a) 1 tonne = 1000 kg
0.0575 tonnes
= 0.0575 × 1000
= **57.5 kg**

(b) 1 tonne = 1000 kg
3.04 tonnes
= 3.04 × 1000
= 3040 kg
1 kg = 1000 g
3040 kg
= 3040 × 1000
= **3 040 000 g**

(c) 1000 kg = 1 tonne
6975 kg
= 6975 ÷ 1000
= **6.975 kg**

(d) 1000 g = 1 kg
77 542 g
= 77 542 ÷ 1000
= 77.542 kg
1000 kg = 1 tonne
77.542 kg
= 77.542 ÷ 1000
= **0.077 542 tonnes**

Practise Now 12

(a) 1 *l* = 1000 ml
0.95 l
= 0.95 × 1000
= **950 ml**

(b) 1 *l* = 1000 ml
5423.05 l
= 5423.05 × 1000
= **5 423 050 ml**

(c) 1000 ml = 1 *l*
2765 ml
= 2765 ÷ 1000
= **2.765 *l***

(d) 1000 ml = 1 *l*
89.02 ml
= 89.02 ÷ 1000
= **0.089 02 *l***

Exercise 3B

1. **(a)**
$$\begin{array}{r} 2.5 \\ +\ 0.2 \\ \hline 2.7 \\ \hline \end{array}$$
∴ 2.5 + 0.2 = **2.7**

(b)
$$\begin{array}{r} \overset{1}{0}.72 \\ +\ 1.30 \\ \hline 2.02 \\ \hline \end{array}$$
∴ 0.72 + 1.3 = **2.02**

(c)
$$\begin{array}{r} 0.064 \\ +\ 1.530 \\ \hline 1.594 \\ \hline \end{array}$$
∴ 0.064 + 1.53 = **1.594**

(d)
$$\begin{array}{r} \overset{1}{2}.12 \\ +\ 0.97 \\ \hline 3.09 \\ \hline \end{array}$$
∴ 2.12 + 0.97= **3.09**

(e)
$$\begin{array}{r} \overset{1}{8}.28 \\ +\ 3.90 \\ \hline 12.18 \\ \hline \end{array}$$
∴ 8.28 + 3.9 = **12.18**

(f)
$$\begin{array}{r} 0.\overset{1}{0}64 \\ +\ 1.540 \\ \hline 1.604 \\ \hline \end{array}$$
∴ 0.064 + 1.54 = **1.604**

2. **(a)**
$$\begin{array}{r} 9.5 \\ -\ 8.2 \\ \hline 1.3 \\ \hline \end{array}$$
∴ 9.5 – 8.2 = **1.3**

(b)

$$\begin{array}{r} 7.\overset{7}{\cancel{8}}\overset{1}{0} \\ -\,0.42 \\ \hline 7.38 \\ \hline \end{array}$$

$\therefore$ 7.8 – 0.42 = **7.38**

(c)

$$\begin{array}{r} 9.813 \\ -\,1.600 \\ \hline 8.213 \\ \hline \end{array}$$

$\therefore$ 9.813 – 1.6 = **8.213**

(d)

$$\begin{array}{r} \overset{8}{9}.\overset{1}{\cancel{0}} \\ -\,8.2 \\ \hline 0.8 \\ \hline \end{array}$$

$\therefore$ 9.0 – 8.2 = **0.8**

(e)

$$\begin{array}{r} \overset{7}{\cancel{8}}.\overset{1}{0}3 \\ -\,4.90 \\ \hline 3.13 \\ \hline \end{array}$$

$\therefore$ 8.03 – 4.9 = **3.13**

(f)

$$\begin{array}{r} \overset{6}{\cancel{7}}.\overset{7}{8}\overset{1}{0} \\ -\,0.92 \\ \hline 6.88 \\ \hline \end{array}$$

$\therefore$ 7.8 – 0.92 = **6.88**

3. **(a)**

$$\begin{array}{r} \overset{3}{1}\overset{5}{4}.\overset{1}{7}2 \\ \times\quad 8 \\ \hline 117.76 \\ \hline \end{array}$$

$\therefore$ 14.72 × 8 = **117.76**

(b)

$$\begin{array}{r} 0.\overset{4}{0}\overset{8}{4}9 \\ \times\quad 9 \\ \hline 0.441 \\ \hline \end{array}$$

$\therefore$ 0.049 × 9 = **0.441**

(c)

$$\begin{array}{r} \overset{1}{3}.\overset{1}{6}52 \\ \times\quad 21 \\ \hline 3\ 652 \\ +\,73\ 040 \\ \hline 76.692 \\ \hline \end{array}$$

$\therefore$ 3.6520 × 21 = **76.692**

(d)

$$\begin{array}{r} 0.\overset{1}{0}\overset{2}{4}5 \\ \times\quad 4 \\ \hline 0.180 \\ \hline \end{array}$$

$\therefore$ 0.045 × 4 = **0.18**

4. **(a)**

$$\begin{array}{r} 14.72 \\ \times\quad 1.2 \\ \hline 2\ 944 \\ +\,14\ 72 \\ \hline 17.664 \\ \hline \end{array}$$

$\therefore$ 14.72 × 1.2 = **17.664**

(b)

$$\begin{array}{r} 130.4 \\ \times\quad 0.15 \\ \hline 6\ 520 \\ +\,13\ 04 \\ \hline 19.560 \\ \hline \end{array}$$

$\therefore$ 130.4 × 0.15 = **19.56**

(c)

$$\begin{array}{r} 0.27 \\ \times\quad 0.08 \\ \hline 0.0216 \\ \hline \end{array}$$

$\therefore$ 0.27 × 0.08 = **0.0216**

(d)

$$\begin{array}{r} 0.25 \\ \times\quad 1.963 \\ \hline 475 \\ 150 \\ 225 \\ 25 \\ \hline 0.49075 \\ \hline \end{array}$$

$\therefore$ 0.25 × 1.963 = **0.490 75**

5. **(a)**

$$\begin{array}{r} 0.27 \\ 3\,)\overline{0.81} \\ -\,0 \\ \hline 8 \\ -\,6 \\ \hline 21 \\ -\,21 \\ \hline 0 \\ \hline \end{array}$$

$\therefore$ 0.81 ÷ 3 = **0.27**

(b)

$$\begin{array}{r} 0.8565 \\ 3\,)\overline{3.4260} \\ -\,0 \\ \hline 34 \\ -\,32 \\ \hline 22 \\ -\,20 \\ \hline 26 \\ -\,24 \\ \hline 20 \\ -\,20 \\ \hline 0 \\ \hline \end{array}$$

$\therefore$ 3.426 ÷ 4 = **0.8565**

(c)

$$\begin{array}{r} 0.03325 \\ 3\,)\overline{0.26600} \\ -\quad 0 \\ \hline 26 \\ -\,24 \\ \hline 26 \\ -\,24 \\ \hline 20 \\ -\,16 \\ \hline 40 \\ -\,40 \\ \hline 0 \\ \hline \end{array}$$

$\therefore$ 0.266 ÷ 8 = **0.033 25**

(d)
$$
\begin{array}{r}
0.21825\\
32\overline{)\,6.98400}\\
-64\\
\hline
58\\
-32\\
\hline
264\\
-256\\
\hline
80\\
-64\\
\hline
160\\
-160\\
\hline
0
\end{array}
$$

$\therefore$ 6.984 ÷ 32 = **0.218 25**

8. **(a)** $0.81 \div 0.3 = \dfrac{0.81}{0.3}$

$= \dfrac{8.1}{3}$

$$
\begin{array}{r}
2.7\\
3\overline{)\,8.1}\\
-6\\
\hline
21\\
-21\\
\hline
0
\end{array}
$$

$\therefore$ 0.81 ÷ 0.3 = **2.7**

(b) $1.32 \div 0.12 = \dfrac{1.32}{0.12}$

$= \dfrac{132}{12}$

= **11**

(c) $3.426 \div 0.06 = \dfrac{3.426}{0.06}$

$= \dfrac{342.6}{6}$

$$
\begin{array}{r}
57.1\\
6\overline{)\,342.6}\\
-30\\
\hline
42\\
-42\\
\hline
6\\
-6\\
\hline
0
\end{array}
$$

$\therefore$ 3.436 ÷ 0.06 = **57.1**

(d) $4.35 \div 1.5 = \dfrac{4.35}{1.5}$

$= \dfrac{43.5}{15}$

$$
\begin{array}{r}
2.9\\
15\overline{)\,43.5}\\
-30\\
\hline
135\\
-135\\
\hline
0
\end{array}
$$

$\therefore$ 4.35 ÷ 1.5 = **2.9**

7. **(a)** 0.63 × 10 = **6.3**

(b) 40.125 × 100 = **4012.5**

(c) 0.0251 × 1000 = **25.1**

(d) 13.6 ÷ 10 = **1.36**

(e) 530.5 ÷ 100 = **5.305**

(f) 26.68 ÷ 1000 = **0.026 68**

8. **(a)** 1 km = 1000 m
0.123 km
= 0.123 × 1000
= **123 m**

(b) 1 m = 100 cm
83 m
= 83 × 100
= **8300 cm**

(c) 1 cm = 10 mm
0.1556 cm
= 0.1556 × 10
= **1.556 mm**

(d) 100 cm = 1 m
1037 cm
= 1037 ÷ 100
= **10.37 m**

(e) 1000 m = 1 km
503 m
= 503 ÷ 1000
= **0.503 km**

(f) 10 mm = 1 cm
2.5 mm
= 2.5 ÷ 10
= **0.25 cm**

9. **(a)** 1 tonne = 1000 kg
6.23 tonnes
= 6.23 × 1000
= **6230 kg**

(b) 1 kg = 1000 g
0.066 kg
= 0.066 × 1000
= **66 g**

(c) 1 tonne = 1000 kg
0.0256 tonnes
= 0.0256 × 1000
= **25.6 kg**

(d) 1000 g = 1 kg
365 g
= 365 ÷ 1000
= **0.365 kg**

(e) 1000 kg = 1 tonne
89 234 kg
= 89 234 ÷ 1000
= **89.234 tonnes**

(f) 1000 kg = 1 tonne
2.056 kg
= 2.056 ÷ 1000
= **0.002 056 tonnes**

10. **(a)** $1\ l = 1000$ ml
2.546 l
= 2.546 × 1000
= **2546 ml**

(b) $1\ l = 1000$ ml
45 l
= 45 × 1000
= **45 000 ml**

(c) 1000 ml = 1 l
8926 ml
= 8926 ÷ 1000
= **8.926 *l***

(d) 1000 ml = 1 l
3.02 ml
= 3.02 ÷ 1000
= **0.003 02 *l***

11. Amount of money
= 300 × \$0.05 + 80 × \$0.10 + 60 × \$0.20 + 100 × \$0.50
= \$15 + \$8 + \$12 + \$50
= **\$85**

12. **(a)** 1 m = 100 cm
0.15 m
= 0.15 × 100
= 15 cm
6.15 m
= 6 m + 0.15 m
= **6 m 15 cm**

(b) 1000 m = 1 km
55 m
= 55 ÷ 1000
= 0.055 km
6 km 55 m
= 6 km + 0.055 km
= **6.055 km**

(c) 1 m = 100 cm
54.44 m
= 54.44 × 100
= 5444 cm
1 cm = 10 mm
5444 cm
= 5444 × 10
= **54 440 mm**

(d) 100 cm = 1 m
462.23 cm
= 462.23 ÷ 100
= 4.6223 m
1000 m = 1 km
4.6223 m
= 4.6223 ÷ 1000
= **0.004 6223 km**

(e) 100 cm = 1 m
89 cm
= 89 ÷ 100
= 0.89 m
10 mm = 1 cm
6 mm
= 6 ÷ 10
= **0.6 cm**
100 cm = 1 m
0.6 cm
= 0.6 ÷ 100
= 0.006 m
89 cm 6 mm
= 0.89 m + 0.006 m
= **0.896 m**

(f) 1 kg = 1000 g
0.123 kg
= 0.123 × 1000
= 123 g
9.123 kg
= 9 kg + 0.123 kg
= **9 kg 123 g**

(g) 1000 g = 1 kg
365 g
= 365 ÷ 1000
= 0.365 kg
10 kg 365 g
= 10 kg + 0.365 kg
= **10.365 kg**

(h) 1 kg = 1000 g
42 kg
= 42 × 1000
= 42 000 g
42 kg 3 g
= 42 000 g + 3 g
= **42 003 g**

13. 100 cm = 1 m
513.1 cm
= 513.1 ÷ 100
= 5.131 m
Length of ribbon A = 5.131 m
Length of ribbon B
= 5.131 − 3.24
= 1.891 m

$$\begin{array}{r} \overset{4}{\cancel{5}}\,.\,\overset{\overset{1}{0}}{\cancel{1}}\,\overset{1}{3}\,1 \\ -\,3\,.\,2\,4\,0 \\ \hline 1\,.\,8\,9\,1 \\ \hline \end{array}$$

Total length of ribbons
= 5.131 + 1.891
= **7.022 m**

$$\begin{array}{r} \overset{1}{5}\,.\,\overset{1}{1}\,3\,1 \\ +\,1\,.\,8\,9\,1 \\ \hline 7\,.\,0\,2\,2 \\ \hline \end{array}$$

14. 1.1 kg
= 1.1 × 1000
= 1100 g
100 g of broccoli costs PKR 42.
Cost of 1100 g of broccoli
= 1100 ÷ 100 × 42
= 11 × 42
= PKR 462
100 g of carrots cost PKR 20.
Cost of 870 g of carrots
= 870 ÷ 100 × 20
= 8.7 × 20
= PKR 174
Total cost of 1.1 kg of broccoli and 870 g of carrots
= PKR 462 + PKR 174
= **PKR 636**

15. Volume of water in a glass
= volume of water in a cup + 0.15 *l*
Volume of water in a glass + volume of water in a cup = 0.842 *l*
(volume of water in a cup + 0.15 *l*) + volume of water in a cup = 0.842 *l*
2 × volume of water in a cup
= 0.842 − 0.15
= 0.692 l

$$\begin{array}{r} 0\,.\,\overset{7}{\cancel{8}}\overset{1}{4}2 \\ -\,0\,.\,1\,5\,0 \\ \hline 0\,.\,6\,9\,2 \\ \hline \end{array}$$

∴ Volume of water in a cup
= 0.692 ÷ 2
= 0.346 *l*
= 0.346 × 1000
= **346 ml**

$$\begin{array}{r} 0.\,3\,4\,6 \\ 2\,\big)\,\overline{0.\,6\,9\,2} \\ -\,0 \\ \hline 6 \\ -\,6 \\ \hline 9 \\ -\,8 \\ \hline 1\,2 \\ -\,1\,2 \\ \hline 0 \\ \hline \end{array}$$

16. 98 pens = (90 + 8) pens
Cost of 8 pens and 8 pencils
= 8 × 5.75
= $46
Cost of 98 pens and 8 pencils = $156.70
Cost of 90 pens + cost of 8 pens and 8 pencils = $156.70
Cost of 90 pens + $46 = $156.70
Cost of 90 pens = $156.70 − $46
= $110.70
Cost of each pen
= $110.70 ÷ 90
= $11.07 ÷ 9
= $1.23
Cost of each pencil
= $5.75 − $1.23
= $4.52
∴ cost of 20 pens and 40 pencils
= 20 × $1.23 + 40 × $4.52
= $24.60 + $180.80
= **$205.40**

17. Amount of money from 80 $0.20 and 40 $0.50 coins
= 80 × $0.20 + 40 × $0.50
= $16 + $20
= $36
Amount of money from $0.10 coins
= $143.80 × $36
= $107.80
∴ number of $0.10 coins
= 107.80 ÷ 0.10
= **1078**

Chapter 4 Integers, Rational Numbers and Real Numbers

TEACHING NOTES

Suggested Approach

Although the concept of negative numbers is new to most students as they have not learnt this in primary school, they do encounter negative numbers in their daily lives, e.g. in weather forecasts. Therefore, teachers can get students to discuss examples of the use of negative numbers in the real world to bring across the idea of negative numbers (see Class Discussion: Uses of negative numbers in real-world contexts). In this chapter, only number discs (or counters) showing the numbers 1 and –1 are needed. Since many Secondary 1 students are still in the concrete operational stage (according to Piaget), the use of number discs can help them to learn the concepts more easily. However, there is still a need to guide students to move from the 'concrete' to the 'abstract', partly because they cannot use number discs in examinations, and partly because they cannot use number discs to add or subtract large negative integers, fractions, and decimals (see Section 4.2).

Section 4.1: Negative numbers

Teachers should teach students to read the negative number –2 as negative 2, not minus 2 ('negative' is a state while 'minus' is an operation). For example, if you have \$5 and you owe your friend \$2, how much do you have left? Since nothing is mentioned about you returning money to your friend, you have \$5 left. Thus \$2 is a state of owing money. However, if you return \$2 to your friend, you have \$5 + (–\$2) = \$5 – \$2 = \$3 left, i.e. 5 minus 2 is an operation of returning money.

Students should also learn about the absolute value of a negative number (see page 71 of the textbook) because they will need it in Section 4.2.

In primary school, students have only learnt the terms 'less than' and 'more than', so there is a need to teach them how to use the symbols '<' and '>' when comparing numbers (see Investigation: Ordering of numbers). It is not necessary to teach them about 'less than or equal to' and 'more than or equal to' now.

Section 4.2: Addition and subtraction involving negative integers

Number discs cannot be used to add or subtract large negative integers, fractions and decimals, so there is a need to help students consolidate what they have learnt in the investigations on pages 75, 76, 78, 79 and 80 of the textbook by moving away from the 'concrete' to the following key 'abstract' concepts:

Key Concept 1. Adding two negative integers gives the same result as the negative of the sum of their absolute values. E.g. $(-2) + (-3) = -(2 + 3) = -5$.

Key Concept 2. Adding a negative integer gives the same result as subtracting the absolute value of the negative integer. E.g. $5 + (-2) = 5 - 2 = 3$ and $-2 + 5 = 5 - 2 = 3$.

Key Concept 3. Subtracting a greater positive integer from a smaller positive integer gives the same result as the negative of the difference of the two integers. E.g. $2 - 5 = -(5 - 2) = -3$.

Key Concept 4. Subtracting a positive integer from a negative integer gives the same result as the negative of the sum of the absolute values of the two integers. E.g. $-5 - 2 = -(5 + 2) = -7$.

Key Concept 5. Subtracting a negative integer gives the same result as adding the absolute value of the negative integer. E.g. $5 - (-2) = 5 + 2 = 7$.

To make the key concepts less abstract, numerical examples are used. Do not use algebra now because students are still unfamiliar with algebra even though they have learnt some basic algebra in primary school. Avoid teaching students '$- \times - = +$' now because the idea behind $5 - (-2)$ is subtraction, not multiplication. To make practice more interesting, a puzzle is designed on page 82 of the textbook.

Section 4.3: Multiplication and division involving negative integers

The idea of flipping over a disc to obtain the negative of a number, e.g. $-(-3) = 3$, is important in teaching multiplication involving negative numbers. Since number discs cannot be used to teach division involving negative numbers, another method is adopted (see page 87 of the textbook).

There is a need to revisit square roots and cube roots in this section to discuss negative square roots and negative cube roots (see page 88 of the textbook). Teachers can impress upon students that the square root symbol $\sqrt{}$ refers to the positive square root only.

Section 4.4 Negative fractions and mixed numbers

Students will apply the arithmetic operations involving negative integers in the previous sections to fractions. Since number discs cannot be used to add or subtract fractions, the abstract concepts in Sections 4.2 and 4.3 should be reiterated here. Other techniques when performing operations on fractions also apply here, namely, converting unlike fractions to like fractions, and mixed numbers to improper fractions.

Teachers are encouraged to introduce the use of calculators to perform calculations involving mixed numbers here.

Section 4.5 Negative decimals

Students will apply the arithmetic operations involving negative integers in the previous sections to decimals. Since number discs cannot be used to add or subtract decimals, the abstract concepts in Sections 4.2 and 4.3 are also applicable here.

Section 4.6: Rational, irrational and real numbers

Traditionally, real numbers are classified as either rational or irrational numbers. Another way to classify real numbers is according to whether their decimal forms are terminating, recurring, or non-recurring (see Investigation: Terminating, recurring and non-recurring decimals). If teachers show students the first million digits of π, many students may be surprised that π has so many digits! This suggests that students do not know that π has an infinite number of decimal places. Teachers may wish to celebrate Pi Day with students on March 14 by talking about p or singing the Pi song.

Introductory Problem

The solutions to this problem can be found in Introductory Problem Revisited (after Practise Now 6).

4.1 Negative numbers

Class Discussion (Uses of negative numbers in real-world contexts)

- Negative numbers are also used to tell time zones, which are based on Greenwich Mean Time (GMT). A country which is in the time zone of GMT –2 means that the time in that country is 2 hours behind the GMT. For example, Honolulu, Hawaii is in the time zone of GMT –10, while Liverpool, United Kingdom is in the time zone of GMT 0, therefore when it is 10 a.m. in Liverpool, it is 12 midnight in Honolulu.
- Latitude and longitude are a set of coordinates that allow for the specification of a geographical location on the Earth's surface and can be represented by positive and/or negative numbers. The latitude of a point is determined with reference to the equatorial plane; the North Pole has a latitude of +90°, which means that it is 90° north of the equator while the South Pole has a latitude of –90°, which means that it is 90° south of the equator. The longitude of a point gives its east-west position relative to the Prime Meridian (longitude 0°); a location with a longitude of +50° means that it is 50° east of the Prime Meridian while a location with a longitude of –50° means that it is 50° west of the Prime Meridian. The latitude and longitude of Rio Grande, Mexico are approximately –32° and –52° respectively, which means that it is 32° south of the equator and 52° west of the Prime Meridian.
- The use of negative numbers can also be seen in scoring systems, such as in golf. Each hole has a par score, which indicates the number of strokes required and a golfer's score for that hole is calculated based on the number of strokes played. A score of +3 on a hole shows that the golfer played three strokes above par, while a score of –3 on a hole shows that the golfer played three strokes under par.

Teachers may wish to note that the list is not exhaustive.

Practise Now 1A

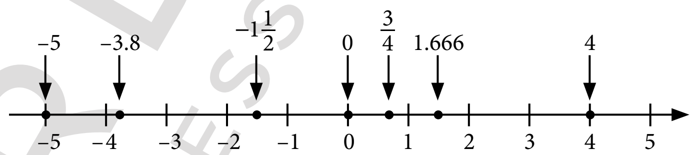
1. (i) **2020, 6**
   (ii) **–5, –17**
   (iii) **2020, 1.666, $\frac{3}{4}$, 6**
   (iv) **–5, $-\frac{1}{2}$, –3.8, –17, $-\frac{5}{3}$**
2. (a) **–43.6 °C**
   (b) **–10 994 m**
   (c) **–$10 000**
   (d) **–81°**

Investigation (Ordering of numbers)

1. (a) Point ***A***: **25 °C**, Point ***B***: **0 °C**, Point ***C***: **–10 °C**
   (b) Point ***A*** shows the highest temperature.
   (c) Point ***C*** shows the lowest temperature.
2. (b) **negative**
   (c) **more**; **greater**
   (d) **less**; **smaller**
3. (a) Since –3 is on the left of 2, we say '–3 is less than 2'. We write: **–3 < 2**.
   (b) Since –3 is on the right of –5, we say '–3 is more than –5'. We write: **–3 > –5**.

Practise Now 1B

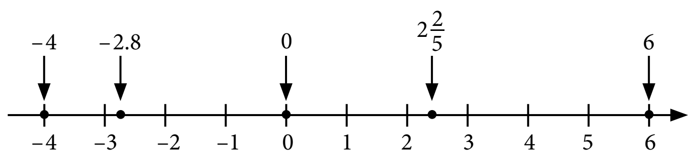
1. (b) **<** (c) **>** (d) **>**
2.



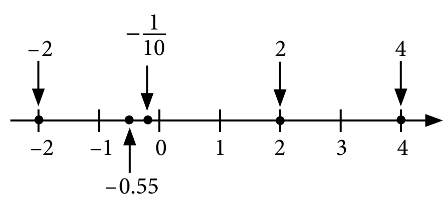
$\therefore$ **–5, –3.8, $-1\frac{1}{2}$, 0, $\frac{3}{4}$, 1.666, 4**

Exercise 4A

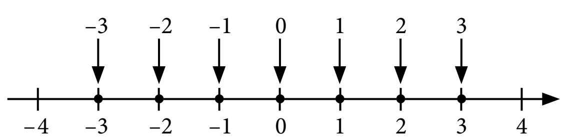
1. (i) **10 001, 4**
   (ii) **–12, –2026**
   (iii) **$\frac{1}{5}$, 4.33, 10 001, 4**
   (iv) **–0.3, $-\frac{9}{7}$, –12, $-1\frac{1}{2}$, –2026**
2. (a) **–273.15 °C** (b) **–86 m**
   (c) **–$85 000** (d) **–0.15 kg**
3. (a) **<** (b) **<**
   (c) **<** (d) **>**
   (e) **<** (f) **>**
4. (a)



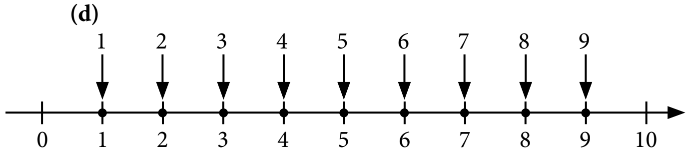
(b)



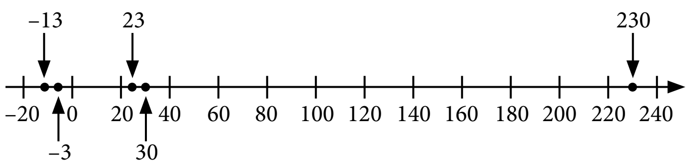
(c)



(d)

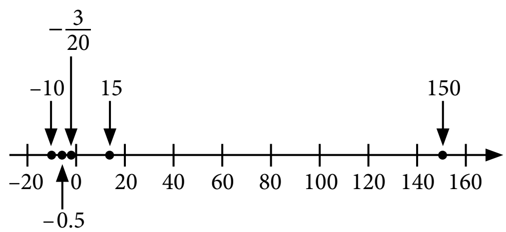


5. **(a)**



∴ **−13, −3, 23, 30, 230**

(b)

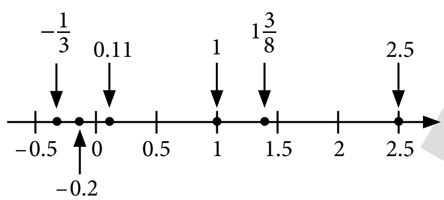


∴ **−10, −0.5, $-\frac{3}{20}$, 15, 150**

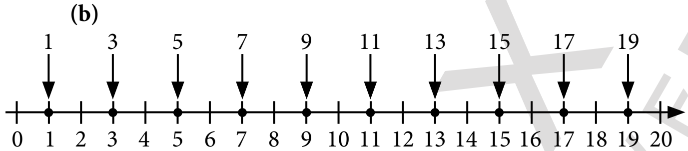
6. **(a)** 10 m below sea level
(b) A withdrawal of $25
(c) An anticlockwise rotation of 30°
(d) A deduction of 4 marks

7. **(a)** **>** **(b)** **>** **(c)** **<** **(d)** **>**

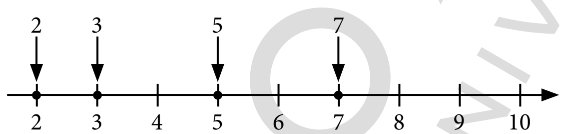
8. **(a)**



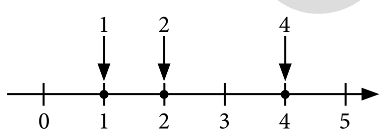
(b)



(c)

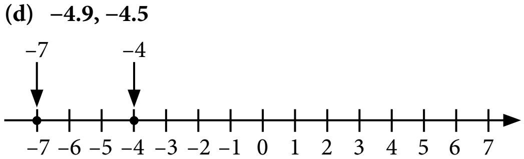


(d)



9. **(a)** **−101, −102**
OPEN **(b)** **0, 1, 2**
(c) **−7, 6, −1, 0**
(d) **−4.9, −4.5**

10.



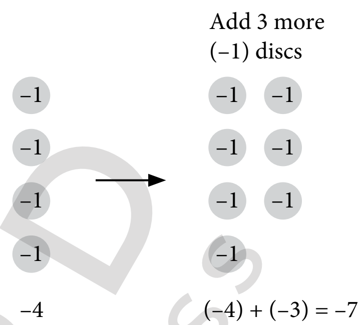
From the number line, −7 is on the left of −4.
∴ −7 is smaller than −4 and Bernard's statement is **false**.

11. The boiling points of liquid xenon and liquid oxygen are higher than −185 °C, while the boiling point of liquid nitrogen is less than −185 °C. Hence, only **liquid nitrogen** will evaporate.

4.2 Addition and subtraction involving negative integers

Investigation (Addition of two negative integers)

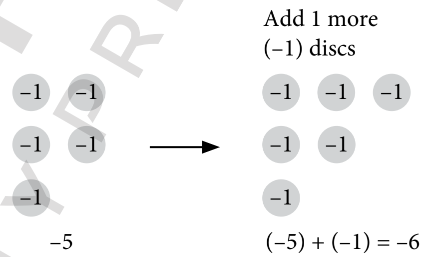
(a) **(i)**



∴ (−4) + (−3) = **−7**

(ii) 4 + 3 = **7**

(b) **(i)**



∴ (−5) + (−1) = **−6**

(ii) 5 + 1 = **6**

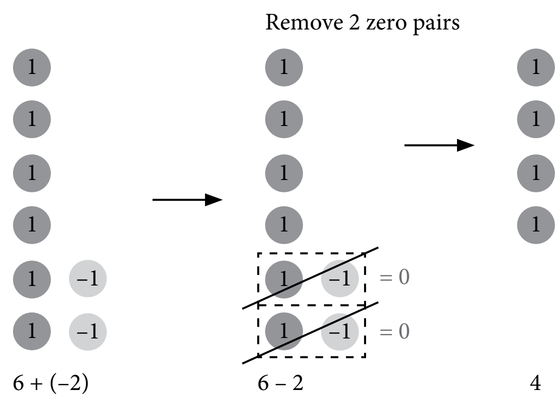
The absolute values of the results are the same.
The sign of the result in **(i)** is different from that in **(ii)**.

Practise Now 1C

(a) (−4) + (−5) = **−9**
(b) −9 + (−7) = **−16**
(c) (−21) + (−73) = **−94**
(d) (−67) + (−48) = **−115**

Investigation (Addition of a positive and a negative integer)

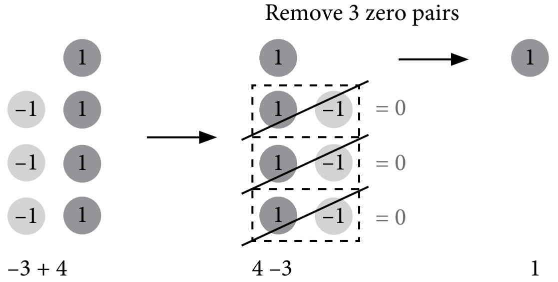
(a) **(i)**



∴ 6 + (−2) = 6 − 2
= **4**

(ii) 6 − 2 = **4**

(b) **(i)**



∴ −3 + 4 = 4 − 3
= **1**

(ii) 4 − 3 = **1**

The results are the same for **(i)** and **(ii)** in each part **(a)** and part **(b)**.

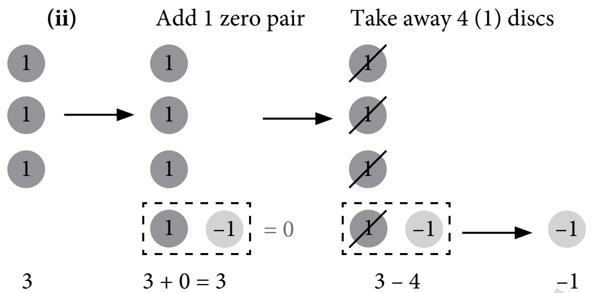
Practise Now 2

(a) 8 + (−3) = **5**
(b) 45 + (−17) = **28**
(c) −2 + 6 = **4**
(d) −12 + 35 = **23**

Investigation (Subtraction between two positive integers)

(a) **(i)** 4 − 3 = **1**

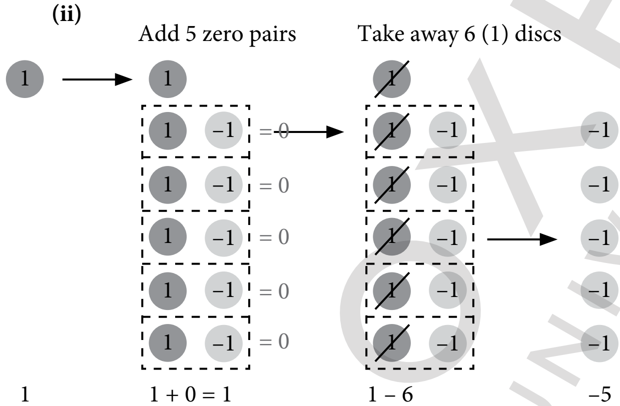
(ii)



∴ 3 − 4 = **−1**

(b) **(i)** 6 − 1 = **5**

(ii)



∴ 1 − 6 = **−5**

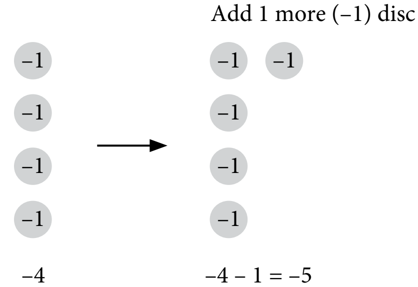
The absolute values of the results are the same.
The sign of the result in **(i)** is different from that in **(ii)**.

Practise Now 3

(a) 5 − 9 = **−4**
(b) 38 − 59 = **−21**
(c) 8 + (−11) = 8 − 11
= **−3**
(d) (−92) + 47 = 47 − 92
= **−45**

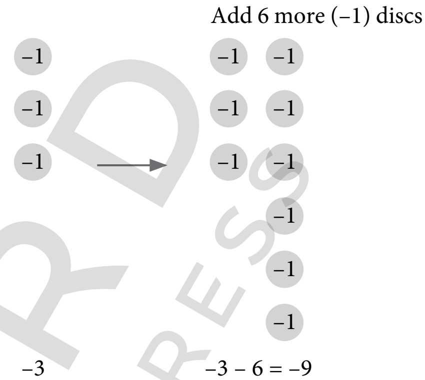
Investigation (Subtraction of a positive integer from a negative integer)

(a)



∴ −4 − 1 = **−5**

(b)



∴ −3 − 6 = **−9**

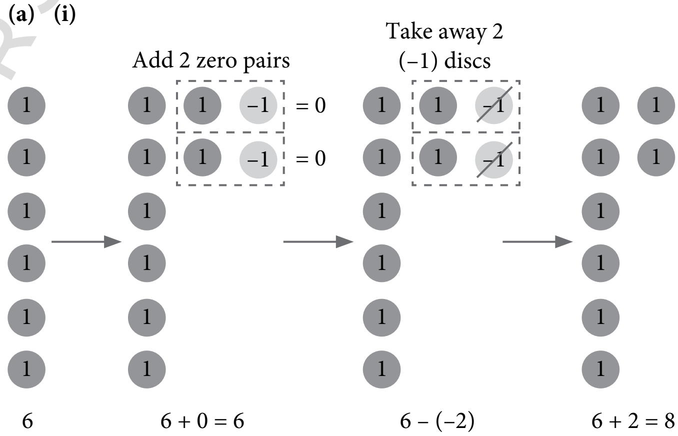
The result in each part has the same absolute value as the sum of the absolute values of the integers.

Practise Now 4

(a) −9 − 11 = **−20**
(b) −39 − 12 = **−51**
(c) −146 − 218 = **−364**

Investigation (Subtraction of a negative integer)

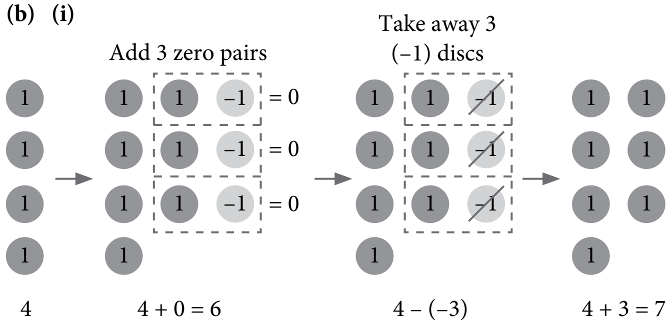
(a) **(i)**



∴ 6 − (−2) = 6 + 2
= **8**

(ii) 6 + 2 = **8**

(b) (i)



$\therefore$ 4 – (–3) = 4 + 3
= **7**

(ii) 4 + 3 = **7**

The results are the same.

Practise Now 5

(a) 4 – (–6) = 4 + 6
= **10**

(b) 30 – (–16) = 30 + 16
= **46**

(c) –8 – (–3) = –8 + 3
= **–5**

(d) –17 – (–35) = –17 + 35
= **18**

Practise Now 6

1. Point A shows –5 °C.
Point B shows 23 °C.
Difference in temperature = 23 °C – (–5 °C)
= 23 °C + 5 °C
= **28 °C**

2. Altitude at D = **–165 m**
Difference in altitude = 314 m – (–165 m)
= 314 m + 165 m
= **479 m**

Introductory Problem Revisited

(i) Minimum height of Mauna Kea = 5793 m + 4207 m
= **10 000 m**

(ii) **Mauna Kea**

Puzzle for consolidation

HE SPRAYS WATER ON YOUR HEAD

Exercise 4B

1. **(a)** (–8) + (–3) = **–11**
(b) –1 + (–6) = **–7**
(c) (–4) + (–7) = **–11**
(d) –5 + (–2) = **–7**

2. **(a)** 9 + (–4) = **5**
(b) 10 + (–3) = **7**
(c) –17 + 29 = **12**
(d) –20 + 20 = **0**

3. **(a)** 3 – 8 = **–5**
(b) 0 – 11 = **–11**
(c) 7 – 14 = **–7**
(d) 5 + (–8) = **–3**
(e) 0 + (–19) = **–19**
(f) –12 + 7 = **–5**

4. **(a)** –2 – 7 = **–9**
(b) –8 – 5 = **–13**
(c) –4 – 9 = **–13**
(d) –6 – 3 = **–9**

5. **(a)** 3 – (–9) = 3 + 9
= **12**
(b) 6 – (–5) = 6 + 5
= **11**
(c) –8 – (–2) = –8 + 2
= **–6**
(d) –4 – (–7) = –4 + 7
= **3**

6. Point A shows –7 °C.
Point B shows 16 °C.
Difference in temperature = 16 °C – (–7 °C)
= 16 °C + 7 °C
= **23 °C**

7. Temperature in the morning = –5 °C + 3 °C
= **–2 °C**

8. **A** 7 + (–3) = 4
B 19 – (–1) = 20
D –5 – (–5) = 0
E 8 – (–8) = 16
H –14 + 6 = –8
I –2 – (–9) = 7
L –1 + (–6) = –7
M 3 + (–12) = –9
N 8 – (–2) = 10
O –4 + 6 = 2
T –10 – (–5) = –5
Y –3 – 3 = –6

	2		7		4	
H	**O**	**L**	**I**	**D**	**A**	**Y**
–8		–7		0		–6

9. **(a)** (–14) + (–16) = **–30**
(b) –52 + (–27) = **–79**
(c) (–39) + (–65) = **–104**
(d) –138 + (–22) = **–160**

10. **(a)** 47 + (–13) = **34**
(b) 137 + (–24) = **113**
(c) –95 + 113 = **18**
(d) –78 + 139 = **61**

11. **(a)** 16 – 69 = **–53**
(b) 14 – 76 = **–62**
(c) 88 + (–123) = 88 – 123
= **–35**
(d) 76 + (–183) = 76 – 183
= **–107**
(e) –73 + 26 = **–47**
(f) –111 + 12 = **–99**

12. **(a)** $-84 - 23 = \mathbf{-107}$
(b) $-69 - 76 = \mathbf{-145}$
(c) $-714 - 716 = \mathbf{-1430}$
(d) $-767 - 697 = \mathbf{-1464}$

13. **(a)** $24 - (-11) = 24 + 11$
$= \mathbf{35}$
(b) $38 - (-57) = 38 + 57$
$= \mathbf{95}$
(c) $-69 - (-28) = -69 + 28$
$= \mathbf{-41}$
(d) $-34 - (-91) = -34 + 91$
$= \mathbf{57}$

14. Altitude of town = **–51 m**
Difference in altitude = 138 m – (–51 m)
= 138 m + 51 m
= **189 m**

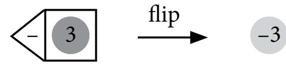
15. **(a)** Adding two negative integers will result in a **negative** value.
(b) Subtracting a greater positive integer from a smaller one will result in a **negative** value.
(c) Adding a negative integer to a positive integer is the same as subtracting the absolute value of the second integer from the first one. Then, subtracting a greater positive integer from a smaller one will result in a **negative** value.
(d) Subtracting a negative integer is the same as adding the absolute value of the integer. Then, the sum of two positive integers will result in a **positive** value.

16. **(i)** Difference between –2 and 3 = $3 - (-2)$
$= 3 + 2$
$= \mathbf{5}$
(ii) The timeline for BC and AD does not have a zero while the number line has a zero.
(iii) There are **4 years** between 2 BC and 3 AD.
Note: As there is no zero on the timeline, we cannot use $3 - (-2)$ to find the difference between 2 BC and 3 AD. In fact, the calculation should be $3 - (-2) - 1$, provided one year is in BC and the other year is in AD. If both are in BC, or both are in AD, the calculation is the same as that in **(i)**.
(iv) A real-life example is the floors in a building, i.e. we can consider B1 (Basement 1) as –1 but there is no floor with the number 0.

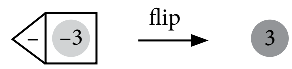
17. **(a)** Examples: $-3 + (-7) = -10$ (**$x = -3, y = -7$**);
$2 + (-12) = -10$ (**$x = 2, y = -12$**)
(b) Examples: $-3 - 7 = -10$ (**$x = -3, y = 7$**);
$2 - 12 = -10$ (**$x = 2, y = 12$**)

4.3 Multiplication, division and combined operations involving negative integers

Investigation (Negative of an integer)

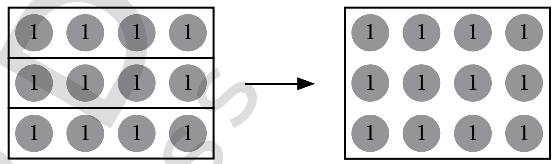
(a)


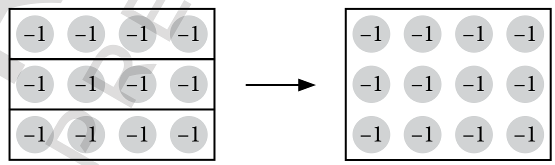
Negative of 3 $\qquad -(3) = \mathbf{-3}$

(b)


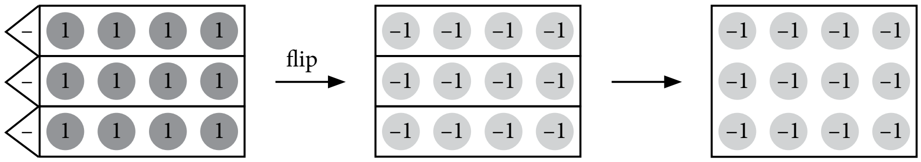
Negative of –3 $\qquad -(-3) = \mathbf{3}$

Investigation (Multiplication involving negative integers)

(a)

3×4 $\qquad 3 \times 4 = \mathbf{12}$

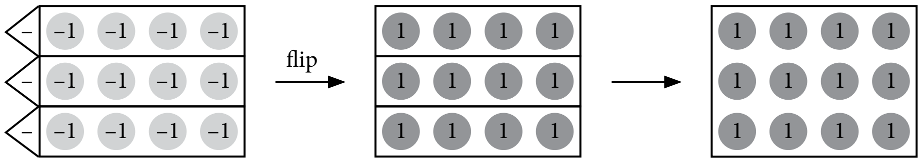
(b)

$3 \times (-4)$ $\qquad 3 \times (-4) = \mathbf{-12}$

(c)



$(-4) \times 3 = -(4 \times 3)$ $\qquad$ $(-4) \times 3 = \mathbf{-12}$

(d)



$(-4) \times (-3) = -[4 \times (-3)]$ $\qquad$ $(-4) \times (-3) = \mathbf{12}$

Practise Now 7A

(a) $2 \times (-9) = \mathbf{-18}$
(b) $(-8) \times 4 = \mathbf{-32}$
(c) $(-6)(-7) = \mathbf{42}$
(d) $-(-10) = \mathbf{10}$
(e) $11 \times (-3) = \mathbf{-33}$
(f) $-19 \times 10 = \mathbf{-190}$
(g) $-12 \times (-3) = \mathbf{36}$
(h) $-4(-7) = \mathbf{28}$
(i) $2020 \times (-1) = \mathbf{-2020}$

Practise Now 7B

(a) $(-8) \div 2 = \mathbf{-4}$
(b) $15 \div (-3) = \mathbf{-5}$
(c) $-21 \div (-7) = \mathbf{3}$
(d) $\dfrac{-16}{4} = \mathbf{-4}$
(e) $\dfrac{20}{-5} = \mathbf{-4}$
(f) $\dfrac{-24}{-3} = \mathbf{8}$

Practise Now 7C

1. **(a)** $8 = 1 \times 8$
$= 2 \times 4$
$\therefore$ positive and negative factors of 8 are **±1**, **±2**, **±4** and **±8**.

(b) $-9 = 1 \times (-9)$
$= -1 \times 9$
$= -3 \times 3$
$\therefore$ positive and negative factors of 9 are **±1**, **±3** and **±9**.

(c) $7 = 1 \times 7$
$\therefore$ positive and negative factors of 7 are **±1** and **±7**.

(d) $-1 = -1 \times 1$
$\therefore$ positive and negative factors of –1 are **1** and **–1** respectively.

2. **(a)** An example of a positive multiple of 2 is $2 \times 2 = \mathbf{4}$.
An example of a negative multiple of 2 is $-2 \times 2 = \mathbf{-4}$.

(b) An example of a positive multiple of –6 is $-2 \times (-6) = \mathbf{12}$.
An example of a negative multiple of –6 is $2 \times (-6) = \mathbf{-12}$.

(c) An example of a positive multiple of 1 is $5 \times 1 = \mathbf{5}$.
An example of a negative multiple of 1 is $-5 \times 1 = \mathbf{-5}$.

(d) An example of a positive multiple of –3 is $-2 \times (-3) = \mathbf{6}$.
An example of a negative multiple of –3 is $2 \times (-3) = \mathbf{-6}$.

Practise Now 8A

(a) Square roots of $64 = \pm\sqrt{64}$
$= \mathbf{\pm 8}$

(b) Negative square root of $9 = -\sqrt{9}$
$= \mathbf{-3}$

(c) $\sqrt{36} = \mathbf{6}$

Thinking Time (Page 88)

It is **not possible** to obtain the square roots of a negative number, e.g. $\pm\sqrt{-16}$, because the square of any number is more than or equal to 0.

Teachers may wish to take this opportunity to highlight to higher-ability students that even though $\pm\sqrt{-16}$ *is not defined in the set of real numbers, it is defined in the set of complex numbers.*

Practise Now 8B

(a) $(-3)^3 = \mathbf{-27}$
(b) $\sqrt[3]{64} = \mathbf{4}$
(c) $\sqrt[3]{-8} = \mathbf{-2}$
(d) $-\sqrt[3]{-27} = -(-3)$
$= \mathbf{3}$

Practise Now 9A

(a) $-2 \times \left(15 - \sqrt{49} + 2^3\right)$
$= -2 \times (15 - 7 + 8)$
$= -2 \times 16$
$= \mathbf{-32}$

(b) $4^3 - \left\{7 \times \left[16 - \left(\sqrt[3]{64} - 5\right)\right]\right\}$
$= 64 - \{7 \times [16 - (4 - 5)]\}$
$= 64 - \{7 \times [16 - (-1)]\}$
$= 64 - (7 \times 17)$
$= 64 - 119$
$= \mathbf{-55}$

Exercise 2C

1. **(a)** $5 \times (-7) = \mathbf{-35}$
   (b) $-8 \times 3 = \mathbf{-24}$
   (c) $(-9)(-4) = \mathbf{36}$
   (d) $-(-716) = \mathbf{716}$
   (e) $-1 \times (-697) = \mathbf{697}$
   (f) $-11(-8) = \mathbf{88}$
   (g) $-6 \times 0 = \mathbf{0}$
   (h) $42(-2) = \mathbf{-84}$
2. **(a)** $-12 \div 4 = \mathbf{-3}$
   (b) $16 \div (-2) = \mathbf{-8}$
   (c) $(-18) \div (-9) = \mathbf{2}$
   (d) $\frac{-14}{7} = \mathbf{-2}$
   (e) $\frac{45}{-5} = \mathbf{-9}$
   (f) $\frac{-18}{-3} = \mathbf{6}$
3. **(a)** $12 = 1 \times 12$
   $= 2 \times 6$
   $= 3 \times 4$
   $\therefore$ positive and negative factors of 12 are **±1**, **±2**, **±3**, **±4**, **±6** and **±12**.
   (b) $-23 = 1 \times (-23)$
   $= -1 \times 23$
   $\therefore$ positive and negative factors of –23 are **±1** and **±23**.
   (c) $16 = 1 \times 16$
   $= 2 \times 8$
   $= 4 \times 4$
   $\therefore$ positive and negative factors of 16 are **±1**, **±2**, **±4**, **±8** and **±16**.
   (d) $1 = 1 \times 1$
   $\therefore$ positive and negative factors of 1 are **1±**.
4. **(a)** An example of a positive multiple of 5 is $2 \times 5 = \mathbf{10}$.
   An example of a negative multiple of 5 is $-2 \times 5 = \mathbf{-10}$.
   (b) An example of a positive multiple of –8 is $-3 \times (-8) = \mathbf{24}$.
   An example of a negative multiple of –8 is $3 \times (-8) = \mathbf{-24}$.
   (c) An example of a positive multiple of 17 is $2 \times 17 = \mathbf{34}$.
   An example of a negative multiple of 17 is $-2 \times 17 = \mathbf{-34}$.
   (d) The product of any number and 0 is 0.
   $\therefore$ it is **not possible** to get a positive or a negative multiple of 0.
5. **(a)** Square roots of $81 = \pm\sqrt{81}$
   $= \mathbf{\pm 9}$
   (b) Square roots of $16 = \pm 16$
   $= \mathbf{\pm 4}$
   (c) Square roots of $25 = \pm\sqrt{25}$
   $= \mathbf{\pm 5}$
   (d) Square roots of $100 = \pm\sqrt{100}$
   $= \mathbf{\pm 10}$
6. **(a)** $\sqrt{81} = \mathbf{9}$
   (b) $\sqrt{4} = \mathbf{2}$
   (c) $-\sqrt{9} = \mathbf{-3}$
   (d) It is **not possible** to obtain a square root of a negative number since the square of any number is positive.
7. **(a)** $(-4)^3 = \mathbf{-64}$
   (b) $-4^3 = \mathbf{-64}$
   (c) $-(-4)^3 = -(-64)$
   $= \mathbf{64}$
   (d) $\sqrt[3]{8} = \mathbf{2}$
   (e) $-\sqrt[3]{125} = \mathbf{-5}$
   (f) $\sqrt[3]{-216} = \mathbf{-6}$
   (g) $-\sqrt[3]{-64} = -(-4)$
   $= \mathbf{4}$
   (h) $\sqrt[3]{-1000} = \mathbf{-10}$
8. **(a)** $-55 + (-10) - 12 = \mathbf{-77}$
   (b) $-12 - [(-8) - (-2)] + 3 = -12 -(-6) + 3$
   $= -6 + 3$
   $= \mathbf{-3}$
   (c) $-100 + (-45) + (-5) + 20 = -150 + 20$
   $= \mathbf{-130}$
   (d) $-2 + 3 \times 15 = -2 + 45$
   $= \mathbf{43}$
   (e) $(-5 - 2)(-3) = (-7)(-3)$
   $= \mathbf{21}$
   (f) $-25 \times (-4) \div (-12 + 32) = 100 \div 20$
   $= \mathbf{5}$
   (g) $3 \times (-3)^2 - (7 - 2)^2 = 3 \times 9 - 5^2$
   $= 27 - 25$
   $= \mathbf{2}$
   (h) $5[3 \times (-2) - 10] = 5[(-6) - 10]$
   $= 5(-16)$
   $= \mathbf{-80}$
   (i) $-12 \div [2^2 - (-2)] = -12 \div (4 + 2)$
   $= -12 \div 6$
   $= \mathbf{-2}$
   (j) $\sqrt{10 - 3 \times (-2)} = \sqrt{10 - (-6)}$
   $= \sqrt{16}$
   $= \mathbf{4}$
10. The value is **negative**. First, multiplying two negative numbers, i.e. $-987 \times (-654)$, will result in a positive number. Dividing this positive number by a negative number, i.e. (–321), will result in a negative number.
11. A positive factor of 0 is **1**. A negative factor of 0 is **–1**.
12. **No.** $\sqrt{n^2}$ refers to the positive square root of n^2. However, n can be negative.
    For example, $\sqrt{(-2)^2} = \sqrt{4} = 2 \neq -2$.
13. $198 = 2 \times 3^2 \times 11$
    Possible values of n^2 are 1 and 9.
    $\therefore$ the possible values of n are **±1** and **±3**.
14. **(a)** $24 \times (-2) \times 5 \div (-6) = -48 \times 5 \div (-6)$
    $= -240 \div (-6)$
    $= \mathbf{40}$
    (b) $4 \times 10 - 13 \times (-5) = 40 - (-65)$
    $= \mathbf{105}$
    (c) $160 \div \sqrt[3]{-8} - 20 \div (-5) = 160 \div (-2) - (-4)$
    $= -80 + 4$
    $= \mathbf{-76}$

(d) $\sqrt{5^2 - 3^2} - (57 - 77) \div 2 = \sqrt{25 - 9} - (-20) \div 2$
$= \sqrt{16} - (-10)$
$= 4 + 10$
$= \mathbf{14}$

(e) $[(12 - 18) \div 3 - (-1)] \times (-4)^3 = [(-6) \div 3 + 1] \times (-64)$
$= (-2 + 1) \times (-64)$
$= -1 \times (-64)$
$= \mathbf{64}$

(f) $(5 - 2)^3 \times 2 + [-4 + (-7)] \div (-2 + 4)^2 = 3^3 \times 2 + (-11) \div 2^2$
$= 27 \times 2 + \left(-\frac{11}{4}\right)$
$= 54 - 2\frac{3}{4}$
$= \mathbf{51\frac{1}{4}}$

(g) $\{-10 - [12 + (-3)^2] + 3^3\} \div (-3) = \{-10 - [12 + 9] + 27\} \div (-3)$
$= (-10 - 21 + 27) \div (-3)$
$= -4 \div (-3)$
$= \mathbf{1\frac{1}{3}}$

(h) $\sqrt[3]{-2 \times (-37) - [-2(-3) + 8 \times (-2) - 8 \times 2] + 5^2}$
$= \sqrt[3]{74 - [6 + (-16) - 16] + 25}$
$= \sqrt[3]{74 - (-26) + 25}$
$= \sqrt[3]{100 + 25}$
$= \sqrt[3]{125}$
$= \mathbf{5}$

15. 17 questions answered correctly = 17 × 3
= 51 marks
5 questions answered wrongly = 5 × (–1)
= –5 marks
Total scored = 51 – 5
= **46** marks

17. Prime numbers that are odd: 3 and 5
Prime number that is even: 2
Total points awarded = (7 + 1) × 5 – 4 × 9
= 40 – 36
= **4**

18. **(i)** Guess and Check:

Number of correct answers	Amount of marks awarded	Number of wrong answers	Amount of marks deducted	Total amount of marks
12	24	2	2	24 – 2 = 22
10	20	4	4	20 – 4 = 16
8	16	6	6	16 – 6 = 10

∴ she answered **8** questions correctly.

(ii) If he answered 3 questions correctly, he would be awarded 6 marks.
Number of marks for wrong answers = –8 – 6
= –14
Number of wrong answers = –14 ÷ (–1)
= 14
∴ he might have obtained **3 correct answers** and **14 wrong answers**.

19. A value of h represents the avatar moving h units forward.
A value of k represents the avatar moving $-k$ units backward.
∴ total distance moved = $(\boldsymbol{h - k})$ units

20. For $(n - 7) \times (n + 3)$ to be a prime number,
$n - 7 = 1$ or $n + 3 = -1$
$n = 8$ $\quad n = -4$
∴ the possible values of n are **8** and **–4**.

4.4 Negative fractions and mixed numbers

Practise Now 10

(a) $7\frac{1}{2} + \left(-3\frac{3}{5}\right) = 7\frac{1}{2} - 3\frac{3}{5}$
$= 7\frac{5}{10} - 3\frac{6}{10}$
$= \left(6 + \frac{10}{10}\right) + \frac{5}{10} - 3\frac{6}{10}$
$= \mathbf{3\frac{9}{10}}$

(b) $-3\frac{1}{3} + \left(-\frac{1}{4}\right) = -\frac{10}{3} - \frac{1}{4}$
$= -\frac{40}{12} - \frac{3}{12}$
$= -\frac{43}{12}$
$= \mathbf{-3\frac{7}{12}}$

(c) $-2\frac{3}{4} - \frac{5}{6} - \left(-\frac{8}{3}\right) = -\frac{11}{4} - \frac{5}{6} + \frac{8}{3}$
$= -\frac{33}{12} - \frac{10}{12} + \frac{32}{12}$
$= \frac{-33 - 10 + 32}{12}$
$= \mathbf{-\frac{11}{12}}$

(d) $\frac{5}{9} - 2\frac{1}{6} - \left(-\frac{4}{3}\right) = \frac{5}{9} - \frac{13}{6} + \frac{4}{3}$
$= \frac{10}{18} - \frac{39}{18} + \frac{24}{18}$
$= \frac{10 - 39 + 24}{18}$
$= \mathbf{-\frac{5}{18}}$

Practise Now 11

(a)
$-2\frac{2}{7} \times \frac{7}{32}$
$= -\frac{\cancel{16}^{1}}{\cancel{7}_{1}} \times \frac{\cancel{7}^{1}}{\cancel{32}_{2}}$
$= \mathbf{-\frac{1}{2}}$

(b) $\left(-\frac{11}{19}\right)\times\left(-1\frac{7}{22}\right)$

$=\left(-\frac{\cancel{11}^{1}}{19}\right)\times\left(-\frac{29}{\cancel{22}_{2}}\right)$

$=\mathbf{\frac{29}{38}}$

(c) $5\frac{1}{4}\div\left(-2\frac{4}{5}\right)$

$=\frac{21}{4}\times\left(-\frac{14}{5}\right)$

$=\frac{\cancel{21}^{3}}{4}\times\left(-\frac{5}{\cancel{14}_{2}}\right)$

$=\mathbf{-1\frac{7}{8}}$

(d) $\frac{20}{7}\div\left(-1\frac{4}{21}\right)$

$=\frac{20}{7}\div\left(-\frac{25}{21}\right)$

$=\frac{\cancel{20}^{4}}{7}\times\left(-\frac{\cancel{21}^{3}}{\cancel{25}_{5}}\right)$

$=-\frac{12}{5}$

$=\mathbf{-2\frac{2}{5}}$

Practise Now 12A

(a) $-2\frac{2}{11}\div\left[-\frac{18}{55}\times\left(-\frac{5}{6}\right)\right]$

$=-\frac{24}{11}\div\frac{3}{11}$

$=-\frac{\cancel{24}^{8}}{{}_{1}\cancel{11}}\div\frac{\cancel{11}^{1}}{\cancel{3}_{1}}$

$=\mathbf{-8}$

(b) $-\frac{4}{7}\times\left[\left(\frac{1}{2}\right)^2-\left(-1\frac{3}{4}\right)\right]=-\frac{4}{7}\times\left(\frac{1}{4}+\frac{7}{4}\right)$

$=-\frac{4}{7}\times\frac{8}{4}$

$=-\frac{4}{7}\times 2$

$=-\frac{8}{7}$

$=\mathbf{-1\frac{1}{7}}$

(c) $\left[-\frac{3}{4}+\left(-\frac{5}{2}\right)\right]\div\frac{2}{13}=\left(-\frac{3}{4}-\frac{10}{4}\right)\times\frac{13}{2}$

$=-\frac{13}{4}\times\frac{13}{2}$

$=-\frac{169}{8}$

$=\mathbf{-21\frac{1}{8}}$

(d) $-\frac{9}{14}\times\left[\left(-\frac{3}{2}\right)^2-2\frac{5}{6}\right]=-\frac{9}{14}\times\left[\frac{9}{4}-\frac{17}{6}\right]$

$=-\frac{9}{14}\times\left[\frac{27}{12}-\frac{34}{12}\right]$

$=-\frac{{}^{3}\cancel{9}}{{}_{2}\cancel{14}}\times\left(-\frac{\cancel{7}^{1}}{\cancel{12}_{4}}\right)$

$=\mathbf{\frac{3}{8}}$

4.5 Negative decimals

Practise Now 13

(a)
$$\begin{array}{rr} & 93.8 \\ + & 7.236 \\ \hline & 101.036 \\ \hline \end{array}$$

$\therefore\ -93.8-7.236=\mathbf{-101.036}$

(b)
$$\begin{array}{rr} & 15.0 \\ - & 5.4 \\ \hline & 9.6 \\ \hline \end{array}$$

$\therefore\ 5.4-15=\mathbf{-9.6}$

(c) $124.8+(-7.24)-(-22.44)=124.8-7.24+22.44$

$$\begin{array}{rr} & 124.80 \\ - & 7.24 \\ \hline & 117.56 \\ \hline \end{array}\qquad\begin{array}{rr} & 117.56 \\ + & 22.44 \\ \hline & 140.00 \\ \hline \end{array}$$

$\therefore\ 124.8+(-7.24)-(-22.44)=\mathbf{140}$

Practise Now 14

(a) $0.19\times(-4.5)$

$=\frac{19}{100}\times\left(-\frac{45}{10}\right)$

$=-\frac{855}{1000}$

$=\mathbf{-0.855}$

Alternatively,

$$\begin{array}{rr} & 0.19 \\ \times & 4.5 \\ \hline & 95 \\ + & 76 \\ \hline & 0.855 \\ \hline \end{array}$$

$\therefore 0.19\times(-4.5)=\mathbf{-0.855}$

(b) $-5.64\times(-0.678)$

$=-\frac{564}{100}\times\left(-\frac{678}{100}\right)$

$=\frac{382\ 392}{100\ 000}$

$=\mathbf{3.823\ 92}$

Alternatively,

$$\begin{array}{r} 5.64 \\ +\quad 0.678 \\ \hline 4512 \\ 3948 \\ +3384 \\ \hline 3.82392 \\ \hline \end{array}$$

$\therefore -5.64 \times (-0.678) =$ **3.823 92**

(c) $42 \div (-1.6)$

$= \frac{42}{(-1.6)}$

$= \frac{420}{(-16)}$

$= -\frac{105}{4}$

$$\begin{array}{r} 26.25 \\ 4\,\overline{)\,105.00} \\ -\ 8 \\ \hline 25 \\ -\ 24 \\ \hline 1\,0 \\ -\quad 8 \\ \hline 2\,0 \\ -\ 2\,0 \\ \hline 0 \end{array}$$

$\therefore 42 \div (-1.6) =$ **−26.25**

(d) $-120.254 \div 2.5$

$= -\frac{120.254}{2.5}$

$= -\frac{1\,202.54}{25}$

$$\begin{array}{r} 48.1016 \\ 25\,\overline{)\,1202.5400} \\ -100 \\ \hline 202 \\ -200 \\ \hline 2\,5 \\ -2\,5 \\ \hline 4 \\ -0 \\ \hline 40 \\ -25 \\ \hline 150 \\ -150 \\ \hline 0 \end{array}$$

$\therefore -120.254 \div 2.5 =$ **−48.1016**

Practise Now 15

(a) $\frac{0.12}{0.4} \times \left(\frac{-0.23}{0.6}\right) = \frac{1.2}{4} \times \left(\frac{-0.23}{0.6}\right)$

$= {}^{1}\cancel{0.3} \times \left(\frac{-0.23}{\cancel{0.6}_{\,2}}\right)$

$=$ **−0.115**

(b) $-7.2 \times [-1.3 + (-3.5)]$

$= -7.2 \times (-4.8)$

$=$ **34.56**

$$\begin{array}{r} 7\,.\,2 \\ \times\quad 4\,.\,8 \\ \hline 5\,76 \\ +28\,8 \\ \hline 34.56 \\ \hline \end{array}$$

(c) $-0.3^2 \times 4.5 \div (-2.7) - 0.65$

$= -0.09 \times \left(\frac{4.5}{-2.7}\right) - 0.65$

$= -9 \times \left(\frac{4.5}{-270}\right) - 0.65$

$= \frac{{}^{1.5}\cancel{4.5}}{{}_{10}\cancel{30}} - 0.65$

$= 0.15 - 0.65$

$=$ **−0.5**

(d) $3.2^3 \times (-0.625)^2 \div (-6.4)$

$= \frac{\overset{1}{\cancel{(3.2)}}\,\overset{1.6}{\cancel{(3.2)}}(3.2)(-0.625)^2}{\cancel{-6.4}\ \cancel{2}_{1}}$

$= -1.6 \times 3.2 \times 0.390\,625$

$= -5.12 \times 0.390\,625$

$=$ **−2**

4.6 Rational, irrational and real numbers

Class Discussion (Rational and irrational numbers)

1. **Yes**. They can be expressed as the ratio of two integers, e.g. $2 = \frac{2}{1}$ and $-3 = -\frac{3}{1}$.
2. **No**. Some square roots and cube roots can be expressed as the ratio of two integers, e.g. $\sqrt{\frac{1}{9}} = \frac{1}{3}$ and $\sqrt[3]{\frac{1}{8}} = \frac{1}{2}$.
3. **(a)** **3.142 857 143**
   (b) **3.141 592 654**
   Their values are different as $\frac{22}{7}$ is just an estimate of π.

Investigation (Terminating, recurring and non-recurring decimals)

Group 1	Group 2	Group 3
$\frac{7}{4}$ = **1.75**	$\frac{1}{3}$ = **0.333 333 333 3**	$\sqrt{2}$ = **1.414 213 562**
$-3\frac{1}{8}$ = **−3.125**	$-\frac{123}{99}$ = **−1.242 424 242**	$-\sqrt[3]{5}$ = **−1.709 975 947**
$\frac{63}{64}$ = **0.984 375**	$\frac{22}{7}$ = **3.142 857 143**	π = **3.141 592 654**

Table 4.1

1. **(i)** The numbers in Group 1 are **rational** numbers as they can all be expressed as the ratio of two integers.
   (ii) **Yes**, the digits after the decimal point terminate.
2. **(i)** The numbers in Group 2 are **rational** numbers as they can all be expressed as the ratio of two integers.
   (ii) Some digits after the decimal point **recur**.
3. **(i)** The numbers in Group 3 are **irrational** numbers as they cannot be expressed as the ratio of two integers.
   (ii) The digits after the decimal point **do not repeat** but they **continue indefinitely**.
4. **(i)** In the decimal representation of rational numbers, the digits after the decimal point either terminate or recur. In the decimal representation of irrational numbers, the digits after the decimal point do not repeat and continue indefinitely.
   (ii) The decimals representations of real numbers terminate, recur, or continue indefinitely without repetition.
5. We can express them in decimal form to compare their values.
   Since $\frac{7}{4} = 1.75$ and $\sqrt{2} = 1.41$ (to 2 d.p.), $\frac{7}{4}$ is larger.
   (i) $\mathbf{-3\frac{1}{8}, -\sqrt[3]{5}, -\frac{123}{99}, \frac{1}{3}, \frac{63}{64}, \sqrt{2}, \frac{7}{4}, \pi, \frac{22}{7}}$
   (ii)

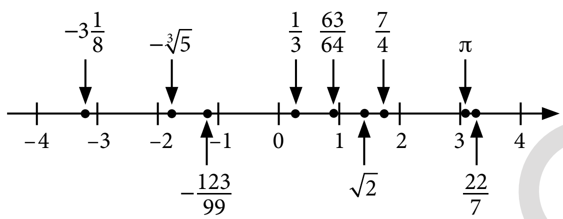


Fig. 4.17

Practise Now 16

$$\frac{\pi \times 0.7^2}{\sqrt[3]{2.4} + 1\frac{3}{10}} = \mathbf{0.583} \text{ (to 3 d.p.)}$$

Exercise 4D

1. **(a)** $-\frac{1}{2} + \left(-\frac{3}{4}\right) = -\frac{1}{2} - \frac{3}{4}$
   $= -\frac{2}{4} - \frac{3}{4}$
   $= \frac{-2-3}{4}$
   $= \frac{-5}{4}$
   $= \mathbf{-1\frac{1}{4}}$

   (b) $3\frac{1}{5} - 5\frac{1}{2} = \frac{16}{5} - \frac{11}{2}$
   $= \frac{32}{10} - \frac{55}{10}$
   $= \frac{-23}{10}$
   $= \mathbf{-2\frac{3}{10}}$

   (c) $3\frac{1}{8} + \left(-\frac{1}{4}\right) = 3\frac{1}{8} - \frac{1}{4}$
   $= 3\frac{1}{8} - \frac{2}{8}$
   $= \left(2 + \frac{8}{8}\right) + \frac{1}{8} - \frac{2}{8}$
   $= \mathbf{2\frac{7}{8}}$

   (d) $4\frac{1}{6} - \left(-4\frac{2}{3}\right) = 4\frac{1}{6} + 4\frac{4}{6}$
   $= \mathbf{8\frac{5}{6}}$

2. **(a)** $\frac{{}^{8}\cancel{64}}{{}_{5}\cancel{15}} \times \left(-\frac{\cancel{3}^{1}}{\cancel{8}_{1}}\right) = -\frac{8}{5}$
   $= \mathbf{-1\frac{3}{5}}$

   (b) $\frac{4}{15} \div \left(-\frac{10}{3}\right) = \frac{{}^{2}\cancel{4}}{{}_{5}\cancel{15}} \times \left(-\frac{\cancel{3}^{1}}{\cancel{10}_{5}}\right)$
   $= \mathbf{-\frac{2}{25}}$

   (c) $-6\frac{1}{8} \times \frac{3}{14} = -\frac{{}^{7}\cancel{49}}{8} \times \frac{3}{\cancel{14}_{2}}$
   $= -\frac{21}{16}$
   $= \mathbf{-1\frac{5}{16}}$

   (d) $-2\frac{1}{2} \times 4\frac{2}{5} = -\frac{{}^{1}\cancel{5}}{{}_{1}\cancel{2}} \times \frac{\cancel{22}^{11}}{\cancel{5}_{1}}$
   $= \mathbf{-11}$

   (e) $-1\frac{1}{4} \div \frac{3}{8} = -\frac{5}{4} \div \frac{3}{8}$
   $= -\frac{5}{{}_{1}\cancel{4}} \times \frac{\cancel{8}^{2}}{3}$
   $= -\frac{10}{3}$
   $= \mathbf{-3\frac{1}{3}}$

   (f) $-\frac{8}{9} \div \left(-1\frac{2}{3}\right) = -\frac{8}{9} \div \left(-\frac{5}{3}\right)$
   $= -\frac{8}{{}_{3}\cancel{9}} \times \left(-\frac{\cancel{3}^{1}}{5}\right)$
   $= \mathbf{\frac{8}{15}}$

4. **(a)** $-4.6 + (-3.2) = \mathbf{-7.8}$
   (b) $2.5 - 6.8 = -(6.8 - 2.5)$
   $= \mathbf{-4.3}$
   (c) $6.9 + (-4.7) = 6.9 - 4.7$
   $= \mathbf{2.2}$
   (d) $12.6 + (-35.7) = -(35.7 - 12.6)$
   $= \mathbf{-23.1}$
   (e) $-29.3 + 11.2 = -(29.3 - 11.2)$
   $= \mathbf{-18.1}$
   (f) $-15.3 - 27.9 = -(15.3 + 27.9)$
   $= \mathbf{-43.2}$

(g) $11.8 - (-2.3) = 11.8 + 2.3$
$= \mathbf{14.1}$

(h) $-0.7 - (-1.5) = -0.7 + 1.5$
$= \mathbf{0.8}$

5. **(a)** 14.72×1.3

$= \frac{1472}{100} \times \frac{13}{10}$

$= \frac{19\,136}{1000}$

$= \mathbf{19.136}$

Alternatively,

$$\begin{array}{r} 1\,4.\,7\,2 \\ \times \quad 1.\,3 \\ \hline 4\,4\,1\,6 \\ +1\,4\,7\,2 \\ \hline 1\,9\,1.\,3\,6 \\ \hline \end{array}$$

$\therefore 14.72 \times 1.3 = \mathbf{19.136}$

(b) $(-4.6) \times (-0.1833)$

$= -\frac{46}{10} \times \left(\frac{1833}{10\,000}\right)$

$= \frac{84\,318}{100\,000}$

$= \mathbf{0.843\ 18}$

Alternatively,

$$\begin{array}{r} 0.\,1\,8\,3\,3 \\ \times \quad 4.\,6 \\ \hline 1\,0\,9\,9\,8 \\ +\,7\,3\,3\,2 \\ \hline 0.\,8\,4\,3\,1\,8 \\ \hline \end{array}$$

$\therefore (-4.6) \times (-0.1833) = \mathbf{0.843\ 18}$

(c) $2.35 \times (-0.52)$

$= \frac{235}{100} \times \left(-\frac{52}{100}\right)$

$= \frac{12\,220}{10\,000}$

$= -1.2220$

$= \mathbf{-1.222}$

Alternatively,

$$\begin{array}{r} 2.\,3\,5 \\ \times 0.\,5\,2 \\ \hline 4\,7\,0 \\ +\,1\,1\,7\,5 \\ \hline 1.\,2\,2\,2\,0 \\ \hline \end{array}$$

$\therefore 2.35 \times (-0.52) = \mathbf{-1.222}$

(d) $3.426 \div (-0.06)$

$= \frac{3.426}{(-0.06)}$

$= -\frac{342.6}{6}$

$$\begin{array}{r} 5\,7.\,1 \\ 6\,\big)\overline{\;3\,4\,2.\,6} \\ -\,3\,0 \\ \hline 4\,2 \\ -\,4\,2 \\ \hline 6 \\ -\,6 \\ \hline 0 \end{array}$$

$\therefore 3.426 \div (-0.06) = \mathbf{-57.1}$

(e) $-1.32 \div 0.12$

$= \frac{(-1.32)}{0.12}$

$= -\frac{132}{12}$

$= \mathbf{-11}$

(f) $-0.16 \div 0.125$

$= \frac{(-0.16)}{0.125}$

$= -\frac{160}{125}$

$= -\frac{32}{25}$

$= \mathbf{-1.28}$

$$\begin{array}{r} 1.\,2\,8 \\ 25\,\big)\overline{\;3\,2\,0\,0} \\ -\,2\,5 \\ \hline 7\,0 \\ -\,5\,0 \\ \hline 2\,0\,0 \\ -\,2\,0\,0 \\ \hline 0 \end{array}$$

$\therefore -0.16 \div 0.125 = \mathbf{-1.28}$

6. **(a)** $4\frac{2}{7} - 6\frac{1}{3} - \left(-\frac{8}{21}\right)$

$= \frac{30}{7} - \frac{19}{3} + \frac{8}{21}$

$= \frac{90}{21} - \frac{133}{21} + \frac{8}{21}$

$= \frac{90 - 133 + 8}{21}$

$= \frac{-35}{21}$

$= -\frac{5}{3}$

$= \mathbf{-1\frac{2}{3}}$

(b) $2\frac{1}{5} - \left(-\frac{3}{4}\right) + \left(-7\frac{1}{10}\right)$

$= 2\frac{1}{5} + \frac{3}{4} - 7\frac{1}{10}$

$= \frac{11}{5} + \frac{3}{4} - \frac{71}{10}$

$= \frac{88}{40} + \frac{30}{40} - \frac{284}{40}$

$= \frac{88 + 30 - 284}{40}$

$= \frac{-166}{40}$

$= -\frac{83}{20}$

$= \mathbf{-4\frac{3}{20}}$

(c) $-9 + \left(-2\frac{1}{8}\right) + \left(-\frac{4}{3}\right) = -9 - \frac{17}{8} - \frac{4}{3}$

$= -9 - \frac{51}{24} - \frac{32}{24}$

$= -9 - \frac{83}{24}$

$= -9 - 3\frac{11}{24}$

$= \mathbf{-12\frac{11}{24}}$

(d) $-\frac{1}{5} + \frac{19}{4} + \left(-\frac{7}{2}\right) = -\frac{4}{20} + \frac{95}{20} - \frac{70}{20}$

$= \frac{-4 + 95 - 70}{20}$

$= \frac{21}{20}$

$= \mathbf{1\frac{1}{20}}$

(e) $-\frac{8}{3} - \frac{1}{6} - \left(-2\frac{1}{8}\right) = -\frac{8}{3} - \frac{1}{6} + \frac{17}{8}$

$= -\frac{64}{24} - \frac{4}{24} + \frac{51}{24}$

$= \frac{-64 - 4 + 51}{24}$

$= \mathbf{-\frac{17}{24}}$

(f) $5\frac{3}{4} - 2\frac{5}{6} + \left(-\frac{23}{15}\right) - \left(-4\frac{7}{10}\right) = 5\frac{3}{4} - 2\frac{5}{6} - \frac{23}{15} + 4\frac{7}{10}$

$= \frac{23}{4} - \frac{17}{6} - \frac{23}{15} + \frac{47}{10}$

$= \frac{345}{60} - \frac{170}{60} - \frac{92}{60} + \frac{282}{60}$

$= \frac{345 - 170 - 92 + 282}{60}$

$= \frac{365}{60}$

$= \frac{73}{12}$

$= \mathbf{6\frac{1}{12}}$

7. (a) $-\frac{5}{7} \times \left(-\frac{28}{15} + 1\frac{2}{3}\right) = -\frac{5}{7} \times \left(-\frac{28}{15} + \frac{5}{3}\right)$

$= -\frac{5}{7} \times \left(-\frac{28}{15} + \frac{25}{15}\right)$

$= -\frac{5}{7} \times \left(-\frac{3}{15}\right)$

$= -\frac{{}^{1}\cancel{5}}{7} \times \left(-\frac{1}{\cancel{5}_{1}}\right)$

$= \mathbf{\frac{1}{7}}$

(b) $\left[-\left(\frac{1}{2}\right)^2 - \left(-\frac{1}{3}\right)\right] \div \left(\frac{1}{4} - \frac{1}{3}\right)$

$= \left(-\frac{1}{4} + \frac{1}{3}\right) \div \left(\frac{1}{4} - \frac{1}{3}\right)$

$= \left(-\frac{3}{12} + \frac{4}{12}\right) \div \left(\frac{3}{12} - \frac{4}{12}\right)$

$= \frac{1}{12} \div \left(-\frac{1}{12}\right)$

$= \frac{1}{12} \times (-12)$

$= \mathbf{-1}$

(c) $10 - \frac{15}{8} \times \left(\frac{3}{2} \div 4\frac{1}{2}\right) + \left(-\frac{1}{4}\right)$

$= 10 - \frac{15}{8} \times \left(\frac{3}{2} \div \frac{9}{2}\right) + \left(-\frac{1}{4}\right)$

$= 10 - \frac{15}{8} \times \left(\frac{{}^{1}\cancel{3}}{{}_{1}\cancel{2}} \times \frac{\cancel{2}^{1}}{\cancel{9}_{3}}\right) + \left(-\frac{1}{4}\right)$

$= 10 - \frac{{}^{5}\cancel{15}}{8} \times \frac{1}{\cancel{3}_{1}} + \left(-\frac{1}{4}\right)$

$= 10 - \frac{5}{8} + \left(-\frac{1}{4}\right)$

$= 10 - \frac{5}{8} - \frac{1}{4}$

$= 10 - \frac{5}{8} - \frac{2}{8}$

$= \left(9 + \frac{8}{8}\right) - \frac{5}{8} - \frac{2}{8}$

$= \mathbf{9\frac{1}{8}}$

(d) $\left(-\frac{3}{2}\right)^2 \times \left(\frac{1}{15} - 2\frac{1}{3}\right) = \left(\frac{3}{2}\right)^2 \times \left(\frac{1}{15} - \frac{7}{3}\right)$

$= \frac{9}{4} \times \left(\frac{1}{15} - \frac{35}{15}\right)$

$= \frac{9}{4} \times \left(-\frac{34}{15}\right)$

$= \frac{{}^{3}\cancel{9}}{{}_{2}\cancel{4}} \times \left(-\frac{\cancel{34}^{17}}{\cancel{15}_{5}}\right)$

$= -\frac{51}{10}$

$= \mathbf{-5\frac{1}{10}}$

9. **(a)** $\left(\frac{1}{2}\right)^3 - \left(2\frac{3}{4}\right)^2 + \left(-\frac{3}{4}\right) = \mathbf{-8\frac{3}{16}}$

(b) $\frac{7}{3} + \frac{4}{9} \times \left(-\frac{1}{2}\right)^2 = \mathbf{2\frac{4}{9}}$

(c) $-\frac{11}{4} - \sqrt[3]{2 - 5\frac{3}{8}} = \mathbf{-1\frac{1}{4}}$

(d) $\sqrt{\frac{11}{12} \div \left(\frac{5}{2} - \frac{5}{6} \times \frac{19}{10}\right)} = \mathbf{1}$

10. **(a)** $-88.8 - 16.24 = -(88.8 + 16.24)$
$= \mathbf{-105.04}$

(b) $4.73 - 14 = -(14 - 4.73)$
$= \mathbf{-9.27}$

(c) $7.513 + (-18.9) = -(18.9 - 7.513)$
$= \mathbf{-11.387}$

(d) $-23.6 + 18.251 = -(23.6 - 18.251)$
$= \mathbf{-5.349}$

(e) $123.4 - (-5.67) = 123.4 + 5.67$
$= \mathbf{129.07}$

(f) $-53.9 - (-32.17) = -53.9 + 32.17$
$= \mathbf{-21.73}$

(g) $73.8 - (-5.79) + (-16.732) = 73.8 + 5.79 - 16.732$
$= 79.59 - 16.732$
$= \mathbf{62.858}$

(h) $-18.913 - (-2.78) - 7.8 = -18.913 + 2.78 - 7.8$
$= -16.133 - 7.8$
$= \mathbf{-23.933}$

11. **(a)** $\frac{0.15}{0.5} \times \left(\frac{-0.16}{1.2}\right) = \frac{1.5}{5} \times \left(\frac{-0.16}{1.2}\right)$
$= 0.3 \times \left(\frac{-0.16}{1.2}\right)$
$= {}^{1}\cancel{3} \times \left(\frac{-0.16}{\cancel{12}_{4}}\right)$
$= \mathbf{-0.04}$

(b) $\frac{0.027}{0.03} \times \left(\frac{1.4}{-0.18}\right) = \frac{2.7}{3} \times \left(\frac{1.4}{-0.18}\right)$
$= 0.9 \times \left(\frac{1.4}{-0.18}\right)$
$= {}^{1}\cancel{9} \times \left(\frac{14}{-\cancel{18}_{2}}\right)$
$= \mathbf{-7}$

(c) $-0.4^2 \times (-1.3) \div 0.8 - 0.62 = -0.4^2 \times \left(\frac{-1.3}{0.8}\right) - 0.62$
$= -0.4^2 \times \left(\frac{-13}{8}\right) - 0.62$
$= -\cancel{0.16}^{0.02} \times \left(\frac{-13}{\cancel{8}_1}\right) - 0.62$
$= 0.26 - 0.62$
$= \mathbf{-0.36}$

(d) $(-0.2)^3 \times \frac{27}{1.6} + 0.105 = (-0.2)^3 \times \frac{270}{16} + 0.105$
$= -\cancel{0.008}^{0.001} \times \frac{\cancel{270}^{135}}{\cancel{16}_{\cancel{8}_1}} + 0.105$
$= -0.135 + 0.105$
$= \mathbf{-0.03}$

(e) $-2.4 - (-1.6)^2 \div (-0.8)^3 = -2.4 - \frac{\cancel{(-1.6)}^{-2}\,\cancel{(-1.6)}^{-2}}{\cancel{(-0.8)}\,\cancel{(-0.8)}\,(-0.8)}$
$= -2.4 - \frac{4}{-0.8}$
$= -2.4 + \frac{40}{8}$
$= -2.4 + 5$
$= \mathbf{2.6}$

(f) $[-7.54 + (-5.79)] \times [7 \div (-4)]$
$= -13.33 \times (-1.75)$
$= \mathbf{23.3275}$

13. **(a)** $\left(\frac{\pi + 5\frac{1}{2}}{-2.1}\right)^2 = \mathbf{16.934}$ (to 3 d.p.)

(b) $-\frac{\pi^2 + \sqrt{2}}{7 - \sqrt[3]{4}} = \mathbf{-2.085}$ (to 3 d.p.)

(c) $\frac{\sqrt[3]{14^2 + 19^2}}{\pi - 4.55} = \mathbf{-5.842}$ (to 3 d.p.)

(d) $\sqrt{\frac{4.6^2 + 8.3^2 - \left(6\frac{1}{2}\right)^2}{2 \times 4.6 - 8.3}} = \mathbf{7.288}$ (to 3 d.p.)

14. Two other examples are a decimal with digits after the decimal point made up of consecutive positive odd integers, i.e. **0.135 791 113**…, and a decimal with digits after the decimal point made up of multiples of 3, i.e. **0.369 121 518**…

Chapter 5 Approximation and Estimation

TEACHING NOTES

Suggested Approach

Teachers can give students a real-life example when an approximated or estimated value is used before getting them to discuss occasions when they use approximation and estimation in their daily lives. In this chapter, they will first learn the five rules to identify the digits which are significant in a number before learning how to round off numbers to a specified number of significant figures. Students will also learn how to carry out estimation through worked examples that involve situations in real-world contexts. To make learning of mathematics relevant, students should know some reasons why they need to use approximations in their daily lives (Introductory Problem).

Section 5.1: Rounding and significant figures

After the recap on decimal places and rounding, teachers can build on students' knowledge of decimal places to study the accuracy of measurements (see Class Discussion: Which answer is more accurate?). From this class discussion, teachers should highlight that the accuracy of measurements does not depend on the number of digits nor the number of decimal places. Instead, accuracy is reflected in the number of significant figures of a measurement. Through the example on measuring cylinders on page 110 of the textbook, students will learn that a number is more accurate when it is given to a greater number of significant figures.

Section 5.2: Limits of Accuracy

After learning how to round off a given value, this section opens with a class discussion requiring students to deduce the smallest and largest possible actual values given a rounded number (see Class Discussion: Upper and lower bounds of a rounded number). Teachers should stress that the range of possible values of a rounded number does not included the upper bound.

Section 5.3: Approximation and approximation errors in real-world contexts

Teachers should tell students that the general instructions for O-level Mathematics examinations state, 'If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.' The investigation on page 118 of the textbook highlights the importance of giving intermediate values correct to four significant figures if we want the final answer to be accurate to three significant figures. Otherwise, a rounding error may occur. Students should also learn that there is a difference between 'approximately 500 000' and 'equal to 500 000 (to 1 s.f.)' (see the thinking time on page 120 of the textbook).

Teachers should highlight that in mathematics, the degree of accuracy is specified (e.g. = 11.4 cm (to 3 s.f.)) instead of using the symbol ≈ . The symbol is used when estimating an quantity to which we are unable to specify the degree of accuracy.

Section 5.4: Estimation and estimation errors in real-world contexts

Teachers can impress upon students that there are differences between approximation and estimation. Since students need to be aware when an answer is obviously wrong, estimation allows them to check the reasonableness of an answer obtained from a calculator (see Worked Example 6).

Students will also learn an important estimation strategy: use a smaller quantity to estimate a larger quantity (see Investigation: Using smaller quantity to estimate larger quantity).

Introductory Problem

1. The actual values that appear in the article are '7 267 582' and 'five'. '7 million' and '12 million' are approximate values while 'over 25' is an estimated value.
2. 'Approximation' rounds off a number to an approximate value while 'estimation' is a guess of an unknown quantity.
3. **(a)** It is not necessary to specify the actual number of airlines, as an approximation is sufficient to show that Jinnah International Airport serves several airlines.
   (b) A headline serves as a brief summary of the article to draw the attention of readers. Thus, it is more appropriate to use an approximate value instead of the actual value.

5.1 Rounding and significant figures

Practise Now 1

1. **(a)** 3 409 725 = **3 409 730** (to the nearest 10)
   (b) 3 409 725 = **3 409 700** (to the nearest 100)
   (c) 3 409 725 = 3 **410 000** (to the nearest 1000)
   (d) 3 409 725 = **3 410 000** (to the nearest 10 000)
2. Largest possible number of overseas visitors = **19 149 999**
   Smallest possible number of overseas visitors = **19 050 000**

Practise Now 2A

1. **(a)** 78.4695 = **78.5** (to 1 d.p.)
   (b) 78.4695 = **78** (to the nearest whole number)
   (c) 78.4695 = **78.47** (to the nearest hundredth)
   (d) 78.4695 = **78.470** (to the nearest 0.001)
2. No, I do not agree with Waseem. 8.4 is rounded off to 1 decimal place while 8.40 is rounded off to 2 decimal places.

Class Discussion (Which answer is more accurate?)

1. **(a)** Joyce should give the answer as **714.6 cm**.
   (b) **714.6 cm** is more accurate because an answer of 7.1 m can mean a length that ranges from 7.05 m to 7.15 m.
   (c) **No**, accuracy does not depend on the number of decimal places a number has. If accuracy depends on the number of decimal places, 714.6 cm and 7.1 m would be of the same accuracy since they both have one decimal place each.
2. **(a)** **7 267 582** is more accurate. This value gives the exact number of people, whereas the approximated value of 7 000 000 only implies that the number of passengers is between 6 500 000 to 7 500 000.
   (b) **No**, accuracy does not depend on the number of digits a number has. If accuracy depends on the number of digits, both 7 267 582 and 7 000 000 would be of the same accuracy since they both have 8 digits each.

Practise Now 2B

(a) The number 192 has **3** significant figures.
(b) The number 83.761 has **5** significant figures.
(c) The number 3 has **1** significant figure.
(d) The number 4.5 has **2** significant figures.

Practise Now 2C

(a) The number 5062 has **4** significant figures.
(b) The number 1.09 has **3** significant figures.
(c) The number 3.0024 has **5** significant figures.
(d) The number 70.8001 has **6** significant figures.

Practise Now 2D

(a) The number 0.021 has **2** significant figures.
(b) The number 0.603 has **3** significant figures.
(c) The number 0.001 74 has **3** significant figures.
(d) The number 0.109 08 has **5** significant figures.

Practise Now 2E

1. **(a)** The number 0.10 has **2** significant figures.
   (b) The number 0.0500 has **3** significant figures.
   (c) The number 41.0320 has **6** significant figures.
   (d) The number 6.090 has **4** significant figures.
2. **4.10 cm** is more accurate because 4.10 cm is measured to 3 significant figures, while 4.1 cm is measured to 2 significant figures.

Practise Now 2F

(a) 3800 m, which is corrected to the nearest 10 m, has **3** significant figures.
(b) 25 000 km, which is corrected to the nearest km, has **5** significant figures.
(c) 100 000 g, which is corrected to the nearest 10 000 g, has **2** significant figures.

Practise Now 3

1. **(a)** 3748 = **3750** (to 3 s.f.)
   (b) 0.004 709 89 = **0.004 710** (to 4 s.f.)
   (c) 4971 = **5000** (to 2 s.f.)
   (d) 0.099 99 = **0.10** (to 2 s.f.)
   0.099 99 = **0.100** (to 3 s.f.)
2. Largest possible number of people at the concert = **21 249**
   Smallest possible number of people at the concert = **21 150**

5.2 Limits of accuracy

Practise Now 4A

(a) $\mathbf{1.55} \leqslant a < \mathbf{1.65}$
(b) $\mathbf{0.55} \leqslant b < \mathbf{0.65}$
(c) $\mathbf{1.295} \leqslant c < \mathbf{1.305}$

Class Discussion: Determining the upper and lower bounds of calculated value

1.

Calculation	$p = 12.5$, $q = 0.35$	$p = 12.5$, $q = 0.45$	$p = 13.5$, $q = 0.35$	$p = 13.5$, $q = 0.45$
$p + q$	12.5 + 0.35 = 12.85	12.5 + 0.45 = **12.9**	**13.5 + 0.35 = 13.85**	**13.5 + 0.45 = 13.95**
$p \times q$	**12.5 × 0.35 = 4.38**	**12.5 × 0.45 = 5.63**	13.5 × 0.35 = 4.73	13.5 × 0.45 = **6.08**
$p - q$	12.5 – 0.35 = 12.15	**12.5 – 0.45 = 12.05**	**13.5 – 0.35 = 13.15**	**13.5 – 0.45 = 13.05**
$p \div q$	**12.5 ÷ 0.35 = 35.71**	12.5 ÷ 0.45 = **27.78**	**13.5 ÷ 0.35 = 38.57**	**13.5 ÷ 0.45 = 30**

2. *The smallest values are shown using ellipses. The largest values are shown using rectangles.*

3.

	Smallest value	Largest value
Addition $p + q$	$p_{lb} + q_{lb}$	$\boldsymbol{p_{ub} + q_{ub}}$
Multiplication $p \times q$	$\boldsymbol{p_{lb}} \times q_{lb}$	$\boldsymbol{p_{ub} \times q_{ub}}$
Subtraction $p - q$	$\boldsymbol{p_{lb} - q_{ub}}$	$p_{ub} - q_{lb}$
Division $p \div q$	$\boldsymbol{p_{lb} \div q_{ub}}$	$p_{ub} \div \boldsymbol{q_{lb}}$

Practise Now 4B

1. 0.565 h ⩽ time taken < 0.575 h
23.35 km/h ⩽ average speed < 23.45 km/h
Distance between Towns A and B
= average speed × time taken
Lower bound of distance
= 23.35 × 0.565
= 13.2 km
Upper bound of distance
= 23.45 × 0.575
= 13.5 km
∴ **13.2 km** ⩽ distance between A and B < **13.5 km**

2. 49.5 m ⩽ length of ribbon < 50.5 m
21.5 cm ⩽ length of cut ribbon < 22.5 cm
Smallest possible number of ribbons
= 49.5 m ÷ 22.5 cm
= 4950 ÷ 22.5 cm
= **220**

Exercise 5A

1. **(a)** 698 352 = **698 350** (to the nearest 10)
(b) 698 352 = **698 400** (to the nearest 100)
(c) 698 352 = **698 000** (to the nearest 1000)
(d) 698 352 = **700 000** (to the nearest 10 000)

2. **(a)** 44 974.8 = **44 970** (to the nearest 10)
(b) 44 974.8 = **45 000** (to the nearest 100)
(c) 44 974.8 = **45 000** (to the nearest 1000)
(d) 44 974.8 = **40 000** (to the nearest 10 000)

3. **(a)** 45.7395 = **45.7** (to 1 d.p.)
(b) 45.7395 = **46** (to the nearest whole number)
(c) 45.7395 = **45.74** (to the nearest 0.01)
(d) 45.7395 = **45.740** (to the nearest thousandth)

4. **(a)** 7.697 146 = **8** (to the nearest whole number)
(b) 7.697 146 = **7.70** (to 2 d.p.)
(c) 7.697 146 = **7.7** (to the nearest tenth)
(d) 7.697 146 = **7.697 15** (to the nearest 0.000 01)

5. **(a)** The number 39 018 has **5** significant figures.
(b) The number 0.028 030 has **5** significant figures.
(c) The number 700.406 00 has **8** significant figures.
(d) 2900, which is corrected to the nearest 10, has **3** significant figures.
(e) 2900, which is corrected to the nearest 100, has **2** significant figures.
(f) 2900, which is corrected to the nearest whole number, has **4** significant figures.

6. **(a)** 728 = **730** (to 2 s.f.)
(b) 503.98 = **504.0** (to 4 s.f.)
(c) 0.003 0195 = **0.003 020** (to 4 s.f.)
(d) 6396 = **6400** (to 2 s.f.)
6396 = **6400** (to 3 s.f.)
(e) 9.9999 = **10.0** (to 3 s.f.)
(f) 8.004 = **8.00** (to 3 s.f.)

7. **(a)** **45.5** ⩽ a < **46.5**
(b) **12.55** ⩽ b < **12.65**
(c) **0.395** ⩽ c < **0.405**
(d) **2945** ⩽ d < **2955**
(e) **295** ⩽ e < **305**
(f) **4495** ⩽ f < **4505**

8. 1.25 ⩽ a < 1.35
15 ⩽ b < 25
0.675 ⩽ c < 0.685
(a) Upper bound of $a + b$
= 1.35 + 25
= 26.35
= **26.4** (to 3 s.f.)

Lower bound of $a + b$
$= 1.25 + 15$
$= 16.25$
$= \mathbf{16.3}$ (to 3 s.f.)

(b) Upper bound of $b - a$
$= 25 - 1.25$
$= 23.75$
$= \mathbf{23.8}$ (to 3 s.f.)
Lower bound of $b - a$
$= 15 - 1.35$
$= 13.65$
$= \mathbf{13.7}$ (to 3 s.f.)

(c) Upper bound of $a \div c$
$= 1.35 \div 0.675$
$= 2$
Lower bound of $a \div c$
$= 1.25 \div 0.685$
$= \mathbf{1.82}$ (to 3 s.f.)

(d) Upper bound of $c \times b$
$= 0.685 \times 25$
$= 17.125$
$= \mathbf{17.1}$ (to 3 s.f.)
Lower bound of $c \times b$
$= 0.675 \times 15$
$= 10.125$
$= \mathbf{10.1}$ (to 3 s.f.)

9. Largest possible value of Pakistan's population = **235 824 999**
Smallest possible value of Pakistan's population = **235 815 000**

10. **(a)** 4.918 m = **4.9 m** (to the nearest 0.1 m)
(b) 9.71 cm = **10 cm** (to the nearest cm)
(c) \$10.982 = **\$11.00** (to the nearest ten cents)
(d) 6.489 kg = **6.49 kg** $\left(\text{to the nearest } \frac{1}{100} \text{ kg}\right)$

11. The actual timing, recorded to 0.01 s, must be between 50.35 s and 50.44 s,
inclusive.
$\therefore$ three possible values are **50.44 s**, **50.40 s** and **50.38 s**.

12. Possible values of x = **3**, **4**, **5** or **6**

13. Largest possible integer value of the area of Dolmen City = **320 400**
Smallest possible integer value of the area of Dolmen City = **319 500**

14. The actual diameter of the wire must be less than 0.1205 m and more than
or equal to 0.1195 m.
$\therefore$ three possible values are **0.1195**, **0.1201** and **0.120 08**.

15. Since 21 X09 = 22 000 (to 2 s.f.), then the possible values of X are 5, 6, 7, 8 or 9.
If 21 X09 is a perfect square, then by trial and error, X = **6**.

16. $155 \text{ mm} \leqslant l < 165 \text{ mm}$
$15.5 \text{ cm} \leqslant l < 16.5 \text{ cm}$
Lower bound of volume of cube
$= (15.5)^3$
$= \mathbf{3723.875\ cm^3}$
Upper bound of volume of cube
$= (16.5)^3$
$= \mathbf{4492.125\ cm^3}$

17. $18.95 \text{ m} \leqslant l < 19.05 \text{ m}$
$\Rightarrow 1895 \text{ cm} \leqslant l < 1905 \text{ cm}$
$1.75 \text{ m} \leqslant b < 1.85 \text{ m}$
$\Rightarrow 175 \text{ cm} \leqslant b < 185 \text{ cm}$

(i) Lower bound of area of vanguard sheet
$= 18.95 \text{ m} \times 1.75 \text{ m}$
$= \mathbf{33.1625\ m^2}$
Upper bound of area of vanguard sheet
$= 19.05 \text{ m} \times 1.85 \text{ m}$
$= \mathbf{35.2425\ m^2}$

(ii) $9.95 \text{ cm} \leqslant$ length of side of square < 10.05 cm
Upper bound of the number of squares that can be cut from l
$= 1905 \text{ cm} \div 9.95 \text{ cm}$
$= 191.5$ (1 decimal place)
$= 191$
Upper bound of the number of squares that can be cut from b
$= 185 \text{ cm} \div 9.95 \text{ cm}$
$= 18.6$ (1 decimal place)
$= 18$
$\therefore$ Largest possible number of squares that can be cut from vanguard sheet
$= 191 \times 18$
$= \mathbf{3438}$

18. **No**, I do not agree with Cheryl. She needs to put a '0' in the ones place as a place holder after dropping the digit '2', i.e. 5192.3 = 5190 (to the nearest 10).

19. **No**, I do not agree with Yasir. 27.0 is rounded off to 1 decimal place which is more accurate than 27 which is rounded off to the nearest whole number.

20. David thinks that 0.02 has 3 significant figures because he counted all the digits to be significant. He is wrong, because a measurement of 0.02 m is of the same accuracy as a measurement of 2 cm, i.e. there is only 1 significant figure.
Only the digit '2' is significant in 0.02. Two zeros need to be included after the digit '2' to indicate the 2nd and 3rd s.f.
$\therefore$ the correct answer is 0.019 95 = 0.0200 (to 3 s.f.).

21. $14.545 \leqslant a < 14.555$
$9.55 \leqslant b < 9.65$
$20.5 \leqslant c < 21.5$
Upper bound of $\dfrac{(180 \times b)}{(a \times b - c)}$
$= \dfrac{(\text{upper bound of } 180 \times b)}{(\text{lower bound of } a \times b - c)}$
Upper bound of $180 \times b$
$= 180 \times 9.65$
$= 1737$
Lower bound of $a \times b - c$
= lower bound of $a \times b$ – upper bound of c
$= 14.545 \times 9.55 - 21.5$
$= 117.404\ 75$
$\therefore$ upper bound of $\dfrac{(180 \times b)}{(a \times b - c)}$
$= \dfrac{1737}{(117.404\ 75)}$
$= \mathbf{14.8}$ (to 3 s.f.)

Lower bound of $\dfrac{(180 \times b)}{(a \times b - c)}$

$= \dfrac{(\text{lower bound of } 180 \times b)}{(\text{upper bound of } a \times b - c)}$

Lower bound of $180 \times b$

$= 180 \times 9.55$

$= 1719$

Upper bound of $a \times b - c$

$=$ upper bound of $a \times b$ – lower bound of c

$= 14.555 \times 9.65 - 20.5$

$= 119.955\ 75$

$\therefore$ lower bound of $\dfrac{(180 \times b)}{(a \times b - c)}$

$= \dfrac{1719}{(119.955\ 75)}$

$=$ **14.3** (to 3 s.f.)

22. Perimeter of rectangle = 2 × length + 2 × width

72.5 cm $\leqslant$ perimeter of rectangle $<$ 74.7 cm

11.5 cm $\leqslant$ length $<$ 12.5 cm

Lower bound of perimeter

= 2 × lower bound of length + 2 × lower bound of width

72.5 cm = 2 × 11.5 + 2 × lower bound of width

2 × lower bound of width = 72.5 – 2 × 11.5

= 49.5 cm

$\therefore$ lower bound of width = 49.5 ÷ 2

= **24.75 cm**

Upper bound of perimeter

= 2 × upper bound of length + 2 × upper bound of width

74.7 cm = 2 × 12.5 + 2 × upper bound of width

2 × upper bound of width = 74.7 – 2 × 12.5

= 49.7 cm

$\therefore$ upper bound of width = 49.7 ÷ 2

= 24.85 cm

24.75 cm $\leqslant$ width $<$ 24.85 cm

Width of rectangle to nearest 0.1 cm

= **24.8 cm**

5.3 Approximation and approximation errors in real-world contexts

Investigation (Rounding in real life)

1. Total number of passengers = 215 + 5

= 220

Number of buses required = 220 ÷ 30

$= 7\frac{1}{3}$

The nearest whole number to $7\frac{1}{3}$ is 7. However, 7 buses are not enough to carry 220 passengers, thus we round up to find the number of buses required to carry all the passengers.

$\therefore$ the number of buses required is **8**.

2. Maximum load of lift = 897 kg

= 900 kg (to the nearest 100 kg)

If the maximum load of the lift is given as 900 kg, it means that the lift is able to carry a load of $\leqslant$ 900 kg. However, the maximum load allowed is only 897 kg.

$\therefore$ the maximum load of the lift should be given as **800 kg**.

Investigation (The missing 0.1% votes)

1. The percentage of votes for each candidate is correct to 3 significant figures. Due to rounding errors in the intermediate steps, there is a follow-through error, resulting in the missing 0.1% of the votes. If the final answer is correct to 2 significant figures, we will obtain 100%. Hence, the final answer can only be accurate to 2 significant figures.

2. Percentage of votes for Albert $= \dfrac{188}{301} \times 100\%$

= 62.5% (to 3 s.f.)

Percentage of votes for Nadia $= \dfrac{52}{301} \times 100\%$

= 17.3% (to 3 s.f.)

Percentage of votes for Shaha $= \dfrac{61}{301} \times 100\%$

= 20.3% (to 3 s.f.)

Total percentage of votes = 62.5% + 17.3% + 20.3%

= 100.1%

The percentage of votes for each candidate is correct to 3 significant figures. Due to rounding errors in the intermediate steps, which results in a follow-through error, the total percentage of votes is 100.1%. If the final answer is correct to 2 significant figures, we will obtain 100%. Hence, the final answer can only be accurate to 2 significant figures.

Practise Now 5

(i) Length of square $= \sqrt{105}$

= **10.2 cm** (to 3 s.f.)

Note: Explain to students that they should not write $\sqrt{105}$ cm = 10.247 cm = 10.2 cm (to 3 s.f.). Firstly, it is confusing to write 10.247 = 10.2 (to 3 s.f.). Secondly, $\sqrt{105} \neq 10.247$, but $\sqrt{105}$ = 10.247 (to 5 s.f.). Thirdly, if we write $\sqrt{105}$ cm = 10.247 cm (to 5 s.f.) = 10.2 cm (to 3 s.f), it will defeat the purpose of leaving non-exact answers to 3 s.f. to prevent the working from being messy.

(ii) Perimeter of square $= \sqrt{105} \times 4$

= **41.0 cm** (to 3 s.f.)

Thinking Time (Page 120)

1. **(i)** When the population of City A is approximately 500 000, it is possible for the exact population size to be 450 000.

(ii) When the population of City A is approximately 500 000, it is possible for the exact population size to be 449 999.

2. **(i)** When the population of City B is equal to 500 000 (to 1 s.f.), it is possible for the exact population size to be 450 000 since 450 000 is equal to 500 000 when rounded off to 1 significant figure.

(ii) When the population of City B is equal to 500 000 (to 1 s.f.), it is not possible for the exact population size to be 449 999 since 449 999 is equal to 400 000 when rounded off to 1 significant figure.

5.4 Estimation and estimation errors in real-world contexts

Practise Now 6

1. $798 \times 195 \approx 800 \times 200$
$= 160\,000$
$\therefore$ Ali's answer, 15 561, is **not reasonable**.
Using a calculator, the actual answer is 155 610.
Hence, the estimated value, 160 000, is close to the actual value, 155 610.

2. **(a)** $5712 \div 297 \approx 5700 \div 300$
$= \mathbf{19}$
Using a calculator, the actual answer is 19.2 (to 3 s.f.).
Hence, the estimated value, 19, is close to the actual value, 19.2.
(b) $\sqrt{63} \times \sqrt[3]{129} \approx \sqrt{64} \times \sqrt[3]{125}$
$= 8 \times 5$
$= \mathbf{40}$
Using a calculator, the actual answer is 40.1 (to 3 s.f.).
Hence, the estimated value, 40, is close to the actual value, 40.1.

3. Time taken to drive from Islamabad to Kohat $= \dfrac{250}{80}$
$\approx \mathbf{\dfrac{240}{80}}$ **hours**

Practise Now 7

IDR 10 000 ≈ PKR 190
IDR 20 000 ≈ PKR 380
IDR 5000 ≈ PKR 95
IDR 25 000 = IDR 20 000 + IDR 5000
≈ PKR 380 + PKR 95
= PKR 475
$\therefore$ price of earrings ≈ **PKR 475**

Practise Now 8

Option A:
300 ml costs \$8.80 ≈ \$9.
100 ml costs about \$3.
50 ml costs about \$1.50.
300 ml + 50 ml costs about \$8.80 + \$1.50 = \$10.30.
Option B:
300 ml + 50 ml costs \$10.40.
$\therefore$ **Option A** gives the better value for money since \$10.30 < \$10.40.

Investigation (Using smaller quantity to estimate larger quantity)

For this investigation, the smaller box used is of length 9.2 cm, width 5.6 cm and height 2.7 cm.

Three trials are carried out to find the average number of PKR 1 coins that can fill the box. The result of each trial is shown in the table.

Trial	Number of PKR 1 coins
1	294
2	280
3	284

Average number of PKR 1 coins that can fill the smaller box
$= \dfrac{294 + 280 + 284}{3}$
$= \dfrac{858}{3}$
$= 286$

Volume of smaller box $= 9.2 \times 5.6 \times 2.7$
$= 139.104\ \text{cm}^3$

Volume of tank $= 50 \times 23 \times 13$
$= 14\,950\ \text{cm}^3$

Number of PKR 1 coins that can fill the tank
$= \dfrac{286}{139.104} \times 14\,950$
= 30 737 (to the nearest whole number)
$\therefore$ Amount of money in the tank = 30 737 × PKR 1
= **PKR 30 737**

Practise Now 9

Percentage of shaded region $= \dfrac{2}{3} \times 100\%$
$= \mathbf{66\dfrac{2}{3}\ \%}$

Exercise 5B

1. $\dfrac{200}{30} = 6.67$ (to 3 s.f.)
Greatest number of sweets that can be bought = **6**

2. The maximum height should be stated as **2 m**.
Although 2.51 m = 3 m (to the nearest metre), this value will allow vehicles of a height more than 2.51 m to enter. Stating 2 m as the maximum height ensures that vehicles entering the car park will definitely not exceed the height restriction.

3. **(i)** \$17.69 when rounded down to the nearest 5 cents will be \$17.65.
\$17.69 when rounded to the nearest 5 cents will be \$17.70.
(ii) Since 5 cents is the smallest denomination of currency in Singapore, a customer is unable to pay \$17.69 in cash. Rounding the amount to the nearest 5 cents will require the customer to pay an additional \$0.01. Thus, the amount is rounded down to \$17.65.

4. **(i)** Length of square $= \sqrt{264}$
$= \mathbf{16.2\ cm}$ (to 3 s.f.)
(ii) Perimeter of square $= \sqrt{264} \times 4$
$= \mathbf{65.0\ cm}$ (to 3 s.f.)

5. **(i)** Breadth of rectangle
$= \dfrac{25.6}{12}$
$= \mathbf{2.13\ m}$ (to 3 s.f.)
(ii) Perimeter of rectangle
$= 2 \times 12 + 2 \times 2.13\,33$
$= \mathbf{28.3\ m}$ (to 3 s.f.)

6. $218 \div 31 \approx 210 \div 30$
$= 7$
$\therefore$ Shaha's answer, 70.3, is **not reasonable**.
Using a calculator, the actual answer is 7.03 (to 3 s.f.).
Hence, the estimated value, 7, is close to the actual value, 7.03.

7. **(a)** $2013 \times 39 \approx 2000 \times 40$

$= \mathbf{80\ 000}$

Using a calculator, the actual answer is 78 507.

Yes, the estimated value, 80 000 is close to the actual value, 78 507.

(b) $\sqrt{145.6} \div \sqrt[3]{65.4} \approx \sqrt{144} \div \sqrt[3]{64}$

$= 12 \div 4$

$= \mathbf{3}$

Using a calculator, the actual answer is 2.99 (to 3 s.f.).

Yes, the estimated value, 3, is close to the actual value, 2.99.

8. **(i)** 3.612 = **3.6** (to 2 s.f.)

29.87 = **30** (to 2 s.f.)

(ii) $3.612 \div 29.87 \approx 3.6 \div 30$

$= \mathbf{0.12}$

(iii) Using a calculator, 3.612 ÷ 29.87 = **0.121** (to 3 s.f.)

(iv) **Yes**, the estimated value, 0.12, is close to the actual value 0.121.

9. Ratio of area of shaded region to that of unshaded region = **1 : 2**

10. **(i)** Area of field = 27.04 × 20.21

$= \mathbf{546.4784\ m^2}$

(ii) Cost of spraying insecticide = \$0.70 × 546.4784

= **\$383** (to the nearest dollar)

11. Amount of petrol used $= \dfrac{274}{9.1}$

$\approx \dfrac{270}{9}\ l$

12. Total amount the shopkeeper has to pay

= 32 × \$19 + 18 × \$9 + 49 × \$22 + 61 × \$18 + 52 × \$11

≈ 30 × \$20 + 20 × \$10 + 50 × \$20 + 60 × \$20 + 50 × \$10

= \$600 + \$200 + \$1000 + \$1200 + \$500

= **\$3500**

13. RM10 ≈ PKR 640

RM20 ≈ PKR 1280

RM5 ≈ PKR 320

RM25 = RM20 + RM5

≈ PKR 1280 + PKR 320

= PKR 1600

∴ price of bag ≈ **PKR 1600**

14. **Option A:**

300 g costs \$5.80 ≈ \$6.

100 g costs about \$2.

500 g costs about \$5.80 + \$2 + \$2 = \$9.80.

Option B:

500 g costs \$9.90.

∴ **Option A** gives the better value for money since \$9.80 < \$9.90.

15. This question tests students' sense of mass. The mass of an ordinary car is likely to be **1500 kg**.

Note: Teachers may wish to get students to give examples of objects with masses of 20 kg, 200 kg and 20 000 kg, e.g. 2 10-kg bags of rice have a total mass of 20 kg, 4 Secondary 1 students have a total mass of about 200 kg and a rocket has a mass of about 20 000 kg.

16. Measurement of height of man = 0.8 cm

Measurement of height of block of flats = 6.4 cm

∴ estimated height of block of flats $= \dfrac{6.4}{0.8} \times 1.7$ m

= **14 m** (to the nearest metre)

17. PKR 100 ≈ KRW 4500

∴ price of handbag

= KRW 26 700

≈ KRW 27 000

$\approx \text{PKR } \dfrac{27\ 000}{4500}$

= **PKR 6000**

18. Price of shoes in Shop A after a 20% discount $= \dfrac{80}{100} \times \$80.50$

$\approx \dfrac{80}{100} \times \$80$

$= \$64$

Price of shoes in Shop B after a 10% discount $= \dfrac{90}{100} \times \$69.50$

$\approx \dfrac{90}{100} \times \$70$

$= \$63$

∴ the shoes are cheaper in **Shop B** and should be bought from there.

Chapter 6 Basic Algebra and Algebraic Manipulation

TEACHING NOTES

Suggested Approach

Some students are still unfamiliar with algebra even though they have learnt some basic algebra in primary school. Thus for the lower ability students, teachers should teach this chapter as though they do not know algebra at all. In addition to the number discs showing the numbers 1 and –1 which students have encountered in Chapter 4, algebra discs showing x, $-x$, y and $-y$ are needed. Since many Secondary 1 students are still in the concrete operational stage (according to Piaget), the use of algebra discs can help them to learn the concepts more easily. However, there is still a need to guide students to move from the 'concrete' to the 'abstract', partly because they cannot use algebra discs in examinations, and partly because they cannot use algebra discs to manipulate algebraic expressions which consist of algebraic terms that have large or fractional coefficients.

Section 6.1: Basic algebraic concepts and notations

Teachers should teach students how to use letters to represent numbers and interpret basic algebraic notations such as $ab = a \times b$ (see Class Discussion: Letters as generalised numbers). Teachers should illustrate the definitions of mathematical terms such as 'algebraic term', 'coefficient', 'algebraic expression' and 'linear expression' using appropriate examples.

To make learning more interactive, students are given the opportunity to use a spreadsheet to explore the concept of variables (see Investigation: Comparing algebraic notations). Through this investigation, students should be able to observe that evaluating an algebraic expression means finding the value of the expression when the variables take on certain values. This investigation also provides students with an intuitive sense of the difference between pairs of expressions such as $2n$ and $2 + n$, n^2 and $2n$, and $2n^2$ and $(2n)^2$. In the class discussion on page 136 of the textbook, students are required to use algebraic expressions to express mathematical relationships. Students are expected to give a more rigorous mathematical explanation for the difference between such a pair of expressions in the journal writing on page 137 of the textbook.

It is recommended that students be taught distinguish between linear and non-linear algebraic expressions (see Class Discussion: Linear expressions) in the last part of this section.

Section 6.2: Addition and subtraction of linear terms

Algebra discs cannot be used to add or subtract algebraic terms with large coefficients, so there is a need to help students consolidate what they have learnt in the investigation on page 140 by drawing parallels parallels between the addition and subtraction of like terms and the addition and subtraction of real numbers that they have learnt in Chapter 4.

Section 6.3: Expansion and factorisation of linear expressions

The idea of flipping over a disc to obtain the negative of a number or variable, e.g. $-(-x) = x$, is needed to teach students how to obtain the negative of a linear expression. Algebra discs cannot be used to manipulate algebraic expressions which consist of algebraic terms that have large coefficients, so there is a need to help students consolidate what they have learnt in the investigation on page 145 of the textbook by moving away from the 'concrete' to the following 'abstract' concept:

Distributive Law: $a(b + c) = ab + ac$

Students should learn how to appreciate the factorisation process, i.e. it is the reverse of expansion. Teachers should tell students the difference between 'complete' and 'incomplete' factorisation. In Secondary 1, students only need to know how to factorise algebraic expressions by extracting the common factors. Teachers should make use of the opportunity when going through the introductory problem (see Introductory Problem Revisited) to highlight some common errors made by students when writing algebraic expressions.

Section 6.4: Linear expressions with fractional coefficients

Here, students can use the same spreadsheet in Section 6.1 to compare and conclude that the following pairs of expressions with fractional coefficients are equivalent: $\frac{n}{2}$ and $\frac{1}{2}n$, and $\frac{(n+1)}{3}$ and $\frac{1}{3}(n + 1)$. Teachers may remind the students that these are also linear terms, albeit with fractional coefficients. For review pusposes, teachers are encouraged to use the Thinking Time activity on page 155 to emphasise common errors made by students when manipulating algebraic expressions.

Introductory Problem

The solutions to this problem can be found in Introductory Problem Revisited (after Practise Now 7).

6.1 Basic algebraic concepts and notations

Class Discussion (Letters as generalised numbers)

1. **(i)** $2 \times 3 = 6$ $\quad 3 \times 2 = 6$

$6 = 6$, $\therefore$ $2 \times 3 = 3 \times 2$, i.e. the order of the two numbers does not matter.

(ii) $ab = ba$

2. **(i)** $5 - 2 = 3$ $\quad 2 - 5 = -3$

$3 \neq -3$, $\therefore$ $5 - 2 \neq 2 - 5$, i.e. the order of the two numbers matters.

(ii) $a - b \neq b - a$

3. For example: $10 \div 2 = 5$ $\quad 2 \div 10 = \frac{2}{10} = \frac{1}{5}$

$5 \neq \frac{1}{5}$, $\therefore$ $10 \div 2 \neq 2 \div 10$, i.e. the order of the two numbers matters.

$\therefore$ $a \div b \neq b \div a$

Thinking Time (Page 132)

(a) Area of a square = length × length

$\therefore$ $A = l \times l$

(b) Area of a rectangle = length × breadth

$\therefore$ $A = l \times b$

(c) Perimeter of a rectangle = 2 × (length + breadth)

$\therefore$ $P = 2 \times (l + b)$

Class Discussion (Interpreting meanings of algebraic notations)

	Notation	Meaning
(a)	$2n$	$2 \times n$
(b)	ab	$a \times b$
(c)	$\frac{a}{b}$	$a \div b$ or $a \times \frac{1}{b}$
(d)	n^2	$n \times n$
(e)	$4x^3$	$4 \times x \times x \times x$
(f)	x^2y	$x \times x \times y$

Investigation (Comparing algebraic notations)

2. The numbers were obtained by substituting $n = 2$ into the expressions in the cells C1 to F1.

3.

	A	B	C	D	E	F	G	H
1	n	$2n$	$2 + n$	$n + n$	$n \times n$	n^2		
2	1	2	3	2	1	1		
3	2	4	4	4	4	4		
4	3	6	5	6	9	9		
5	4	**8**	**6**	**8**	**16**	**16**		
6	5	**10**	**7**	**10**	**25**	**25**		

Table 6.5

4. **(i)** **No.** The values in Columns B and C are different.

(ii) **No.** The values in Columns C and D are different.

(iii) **Yes.** The values in Columns B and D are the same.

(iv) **No.** The values in Columns B and E are different.

(v) **Yes.** The values in Columns E and F are the same.

(vi) **No.** The values in Columns B and F are different.

5.

	A	B	C	D	E	F	G	H
1	n	$2n$	$2 + n$	$n + n$	$n \times n$	n^2	$\mathbf{2n^2}$	$\mathbf{(2n)^2}$
2	1	2	3	2	1	1	**2**	**4**
3	2	4	4	4	4	4	**8**	**16**
4	3	6	5	6	9	9	**18**	**36**
5	4	8	6	8	16	16	**32**	**64**
6	5	10	7	10	25	25	**50**	**100**

6. **(i)** **No.** The values in Columns G and H are different.

(ii) $(2n)^2$ means $(2 \times n) \times (2 \times n) = 4n^2$.

Class Discussion (Expressing mathematical operations and simple real-world situations using algebraic expressions)

1.

	In words	Algebraic expression
(a)	Sum of $2x$ and $3z$	$\mathbf{2x + 3z}$
(b)	Product of x and $7y$	$\mathbf{7xy}$
(c)	Divide $3ab$ by $2c$	$\mathbf{\frac{3ab}{2c}}$
(d)	Subtract $6q$ from $10z$	$\mathbf{10z - 6q}$
(e)	**Subtract the product of *x* and *y* from the sum of *p* and *q***	$(p + q) - xy$
(f)	**Divide the sum of 3 and *y* by 5**	$\frac{3 + y}{5}$
(g)	**Subtract the product of 2 and *c* from the positive square root of *b***	$\sqrt{b} - 2c$
(h)	There are three times as many girls as boys in a school. Find an expression, in terms of x, for the total number of students in the school, where x represents the number of boys in the school.	It is given that x represents the number of boys. $\therefore$ $\mathbf{3x}$ represents the number of girls. Total number of students $= x + 3x$ $= \mathbf{4x}$
(i)	The age of Nadia's father is thrice hers. The age of Nadia's brother is 5 years more than hers. Find an expression, in terms of y, for the sum of their ages, where y represents Nadia's age.	It is given that y represents Nadia's age. $\therefore$ Nadia's father is $\mathbf{3y}$ years old. Nadia's brother is $\mathbf{(y + 5)}$ years old. Sum of their ages $= y + 3y + y + 5$ $= \mathbf{(5y + 5)}$ years
(j)	The **length** is **three** times as long as the **breadth** of the rectangle. Find an expression, in terms of b, for the perimeter and the area of the rectangle, where b represents the breadth of the rectangle.	It is given that b represents the breadth of the rectangle in m. $3b$ represents the length of the rectangle in m. Perimeter of rectangle $= 2(3b + b)$ $= 2(4b)$ $= \mathbf{8b}$ m Area of rectangle $= 3b \times b$ $= \mathbf{3b^2}$ m^2

Table 6.6

Practise Now 1

1. **(a)** $5y - 4x = 5(4) - 4(-2)$
$= 20 + 8$
$= \mathbf{28}$

(b) $\frac{1}{x} - y + 3 = \frac{1}{-2} - 4 + 3$
$= -\frac{1}{2} - 4 + 3$
$= -4\frac{1}{2} + 3$
$= \mathbf{-1\frac{1}{2}}$

2. $p^2 + 3q^2 = \left(-\frac{1}{2}\right)^2 + 3(-2)^2$
$= \frac{1}{4} + 12$
$= \mathbf{12\frac{1}{4}}$

Journal Writing (Page 137)

By observation, the expressions $5 + n$ and $5n$ are equal only when $n = 1\frac{1}{4}$.
When $n < 1\frac{1}{4}$, $5n < 5 + n$. When $n > 1\frac{1}{4}$, $5n > 5 + n$.

Class Discussion (Linear expressions)

(a) $x - \frac{1}{2}$ is a **linear** expression with one term in x and one constant term.

(b) $-3y$ is a **linear** expression with one term in y.
(c) 10 is a **non-linear** expression since it is a constant.
(d) $5y - y + 9$, i.e. $4y + 9$, is a **linear** expression with one term in y and one constant term.
(e) $4y + y^2$ is a **non-linear** expression because it contains a term in y^2.
(f) $-2y + 3x + 7$ is a **linear** expression in two variables x and y.
(g) $6x - 2y + 3z + 7$ is a **linear** expression in three variables x, y and z.
(h) $x - 4y + 8xy$ is a **non-linear** expression because it contains a term in xy.

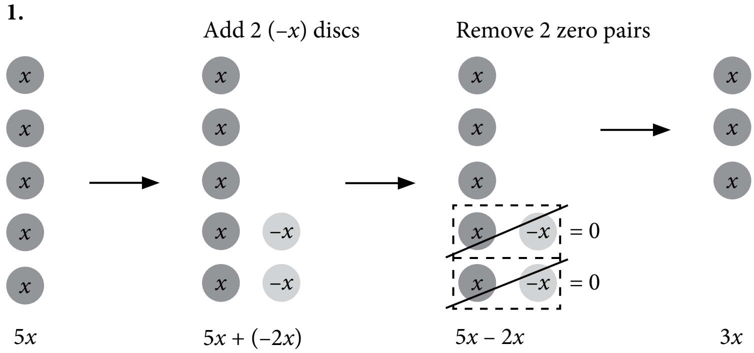
6.2 Addition and subtraction of linear terms

Class Discussion (Like and unlike terms)

(a) $4y$ and $-y$ are **like** terms because they contain the same variable y of the same power.
(b) -3 and 8 are **like** terms because they do not contain any variable.
(c) $5xy$ and $5x$ are **unlike** terms because they contain different variables.
(d) $7xy$ and $-2yx$ are **like** terms because $xy = yx$.
(e) $3y$ and y^2 are **unlike** terms because the variables y and y^2 are of different powers.
(f) $-6x^2$ and $4x^3$ are **unlike** terms because the variables x^2 and x^3 are of different powers.

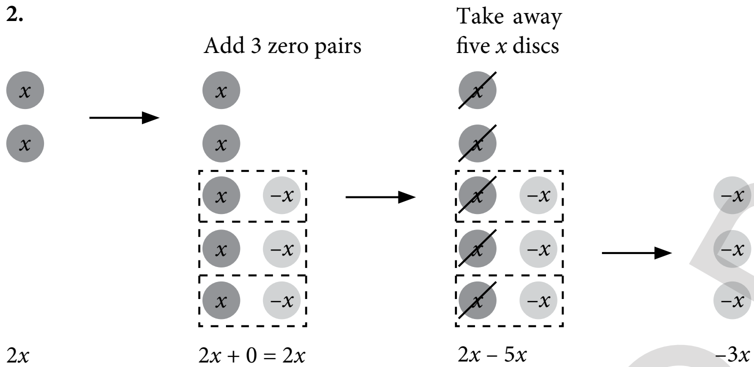
Investigation (Addition and subtraction of like terms)

1.



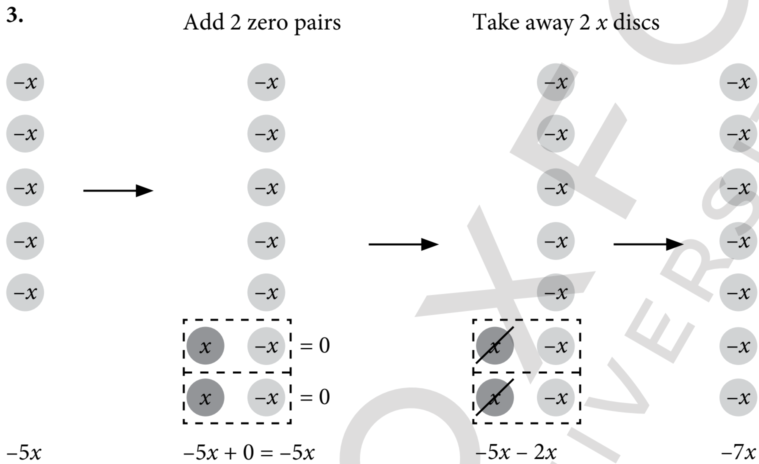
The coefficient of the answer is the same as $5 + (-2) = 3$.

2.



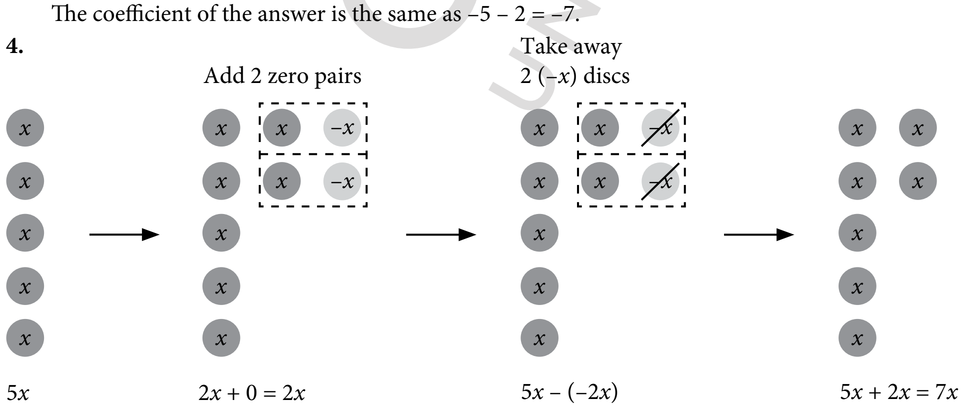
The coefficient of the answer is the same as $2 - 5 = -3$.

3.



The coefficient of the answer is the same as $-5 - 2 = -7$.

4.



The coefficient of the answer is the same as $5 - (-2) = 7$.

Practise Now 2

(a) $3x + (-7x) = 3x - 7x$
$= \mathbf{-4x}$

(b) $19x + (-33x) = 19x - 33x$
$= \mathbf{-14x}$

(c) $-6x + 2x = 2x + (-6x)$
$= 2x - 6x$
$= \mathbf{-4x}$

(d) $-35x + 12x = 12x + (-35x)$
$= 12x - 35x$
$= \mathbf{-23x}$

(e) $-9y - 11y = \mathbf{-20y}$

(f) $-39y - 12y = -39y + (-12y)$
$= \mathbf{-51y}$

(g) $-8z - (-3z) = -8z + 3z$
$= 3z - 8z$
$= \mathbf{-5z}$

(h) $-17z - (-35z) = -17z + 35z$
$= 35z - 17z$
$= \mathbf{18z}$

Practise Now 3

(a) $-4x + (-7y) - 2x - (-3y) = -4x + (-7y) + (-2x) + 3y$
$= -4x + (-2x) + (-7y) + 3y$
$= -6x + 3y + (-7y)$
$= -6x + (-4y)$
$= \mathbf{-6x - 4y}$

(b) $-8y + (-3x) - (-2y) + (-4x) = -8y + (-3x) + 2y + (-4x)$
$= -8y + 2y + (-3x) + (-4x)$
$= -6y + (-7x)$
$= \mathbf{-6y - 7x}$

(c) $9x + (-2z) + 7 - (-5z) + (-x) - 4 = 9x + (-2z) + 7 + 5z + (-x) - 4$
$= 9x + (-x) + (-2z) + 5z + 7 - 4$
$= 8x + 5z + (-2z) + 3$
$= 8x + 5z - 2z + 3$
$= \mathbf{8x + 3z + 3}$

(d) $6a - (-8) - 3b + a + (-7b) + 5 = 6a + 8 - 3b + a + (-7b) + 5$
$= 6a + a - 3b + (-7b) + 8 + 5$
$= 7a - 3b - 7b + 13$
$= \mathbf{7a - 10b + 13}$

Exercise 6A

1. **(a)** $\mathbf{ab + 5y}$
(b) $\mathbf{f^3 - 3}$
(c) $\mathbf{6kq}$
(d) $\mathbf{\dfrac{2w}{3xy}}$
(e) $\mathbf{3x - 4\sqrt{z}}$
(f) $\mathbf{\dfrac{2p}{5q}}$

2. **(a)** $4x - 7y = 4(6) - 7(-4)$
$= 24 + 28$
$= \mathbf{52}$

(b) $\dfrac{5x}{3y} + x = \dfrac{5(6)}{3(-4)} + 6$
$= \dfrac{30}{-12} + 6$
$= -2\dfrac{1}{2} + 6$
$= \mathbf{3\dfrac{1}{2}}$

(c) $2x^2 - y^3 = 2(6)^2 - (-4)^3$
$= 72 - (-64)$
$= 72 + 64$
$= \mathbf{136}$

(d) $3x + \dfrac{x}{y} - y^2 = 3(6) + \dfrac{6}{-4} - (-4)^2$
$= 18 - 1\dfrac{1}{2} - 16$
$= 16\dfrac{1}{2} - 16$
$= \mathbf{\dfrac{1}{2}}$

3. **(a)** $a(3c - b) = 3[3(6) - (-5)]$
$= 3(18 + 5)$
$= 3(23)$
$= \mathbf{69}$

(b) $ab^2 - ac = 3(-5)^2 - 3(6)$
$= 3(25) - 18$
$= 75 - 18$
$= \mathbf{57}$

(c) $\dfrac{b}{a} - \dfrac{c}{b} = \dfrac{-5}{3} - \dfrac{6}{-5}$
$= -1\dfrac{2}{3} + 1\dfrac{1}{5}$
$= \mathbf{-\dfrac{7}{15}}$

(d) $\dfrac{b+c}{a} + \dfrac{a+c}{b} = \dfrac{-5+6}{3} + \dfrac{3+6}{-5}$
$= \dfrac{1}{3} + \dfrac{9}{-5}$
$= \dfrac{1}{3} - 1\dfrac{4}{5}$
$= \mathbf{-\dfrac{7}{15}}$

4. **(a)** $5x + (-8x) = 5x - 8x$
$= \mathbf{-3x}$

(b) $-12x + 7x = 7x + (-12x)$
$= 7x - 12x$
$= \mathbf{-5x}$

(c) $13x + (-19x) = 13x - 19x$
$= \mathbf{-6x}$

(d) $-28x + 6x = 6x + (-28x)$
$= 6x - 28x$
$= \mathbf{-22x}$

(e) $-4y - 9y = \mathbf{-13y}$

(f) $-16y - 3y = \mathbf{-19y}$

(g) $-7z - (-2z) = -7z + 2z$
$= 2z - 7z$
$= \mathbf{-5z}$

(h) $-11z - (-24z) = -11z + 24z$
$= 24z - 11z$
$= \mathbf{13z}$

5. (a) $5x + 22 - 6x - 23 = 5x - 6x + 22 - 23$
$= \mathbf{-x - 1}$
(b) $x + 3y + 6x + 4y = x + 6x + 3y + 4y$
$= \mathbf{7x + 7y}$
(c) $6xy + 13x - 2yx - 5x = 6xy - 2yx + 13x - 5x$
$= \mathbf{4xy + 8x}$
(d) $6x - 20y + 7z - 8x + 25y - 11z$
$= 6x - 8x - 20y + 25y + 7z - 11z$
$= \mathbf{-2x + 5y - 4z}$

6. (a) Required answer $= 2x + 4y + (-5y)$
$= 2x + 4y - 5y$
$= \mathbf{2x - y}$
(b) Required answer $= -b - 4a + 7b - 6a$
$= -4a - 6a - b + 7b$
$= \mathbf{-10a + 6b}$
(c) Required answer $= 6d - 4c + (-7c + 6d)$
$= 6d - 4c - 7c + 6d$
$= -4c - 7c + 6d + 6d$
$= \mathbf{-11c + 12d}$
(d) Required answer $= 3pq - 6hk + (-3qp + 14kh)$
$= 3pq - 6hk - 3qp + 14kh$
$= 3pq - 3qp - 6hk + 14kh$
$= \mathbf{8hk}$

7. (a) $\mathbf{(p + q)^2 - \sqrt[3]{3hk}}$
(b) Total value = $\mathbf{(20x + 500y)}$ **cents**

8. (a) $\dfrac{3a - b}{2c} + \dfrac{3a - c}{c - b} = \dfrac{3(3) - (-4)}{2(-2)} + \dfrac{3(3) - (-2)}{-2 - (-4)}$
$= \dfrac{9 + 4}{-4} + \dfrac{9 + 2}{-2 + 4}$
$= \dfrac{13}{-4} + \dfrac{11}{2}$
$= -3\dfrac{1}{4} + 5\dfrac{1}{2}$
$= \mathbf{2\dfrac{1}{4}}$
(b) $\dfrac{2c - a}{3c + b} - \dfrac{5a + 4c}{c - a} = \dfrac{2(-2) - 3}{3(-2) + (-4)} - \dfrac{5(3) + 4(-2)}{-2 - 3}$
$= \dfrac{-4 - 3}{-6 - 4} - \dfrac{15 - 8}{-5}$
$= \dfrac{-7}{-10} - \dfrac{7}{-5}$
$= \dfrac{7}{10} + 1\dfrac{2}{5}$
$= \mathbf{2\dfrac{1}{10}}$
(c) $\dfrac{a + b + 2c}{3c - a - b} - \dfrac{5c}{4b} = \dfrac{3 + (-4) + 2(-2)}{3(-2) - 3 - (-4)} - \dfrac{5(-2)}{4(-4)}$
$= \dfrac{3 - 4 - 4}{-6 - 3 + 4} - \dfrac{-10}{-16}$
$= \dfrac{-5}{-5} - \dfrac{5}{8}$
$= 1 - \dfrac{5}{8}$
$= \mathbf{\dfrac{3}{8}}$
(d) $\dfrac{b - c}{3c + 4b} \div \left(\dfrac{bc}{a} + \dfrac{ac}{b}\right)$
$= \dfrac{-4 - (-2)}{3(-2) + 4(-4)} \div \left[\dfrac{(-4)(-2)}{3} + \dfrac{3(-2)}{-4}\right]$
$= \dfrac{-4 + 2}{-6 - 16} \div \left(\dfrac{8}{3} + \dfrac{-6}{-4}\right)$
$= \dfrac{-2}{-22} \div \left(2\dfrac{2}{3} + 1\dfrac{1}{2}\right)$
$= \dfrac{1}{11} \div 4\dfrac{1}{6}$
$= \mathbf{\dfrac{6}{275}}$

9. (a) $137x + (-24x) = 137x - 24x$
$= \mathbf{113x}$
(b) $76x + (-183x) = 76x - 183x$
$= \mathbf{-107x}$
(c) $-73x + 26x = 26x - 73x$
$= \mathbf{-47x}$
(d) $-95x + 113x = 113x - 95x$
$= \mathbf{18x}$
(e) $-84y - 23y = \mathbf{-107y}$
(f) $-714y - 716y = \mathbf{-1430y}$
(g) $-59z - (-48z) = -59z + 48z$
$= 48z - 59z$
$= \mathbf{-11z}$
(h) $-34z - (-191z) = -34z + 191z$
$= 191z - 34z$
$= \mathbf{157z}$

10. (a) $15a + (-7b) + (-18a) + 4b = 15a - 7b - 18a + 4b$
$= 15a - 18a - 7b + 4b$
$= \mathbf{-3a - 3b}$
(b) $-3h + (-5k) - (-10k) - 7h = -3h - 5k + 10k - 7h$
$= -3h - 7h - 5k + 10k$
$= \mathbf{-10h + 5k}$
(c) $9p - (-2q) - 8p - (-12q) = 9p + 2q - 8p + 12q$
$= 9p - 8p + 2q + 12q$
$= \mathbf{p + 14q}$
(d) $-7x - (-15y) - (-2x) + (-6y) = -7x + 15y + 2x - 6y$
$= -7x + 2x + 15y - 6y$
$= \mathbf{-5x + 9y}$
(e) $29x + (-13) - 7x - (-18y) + (-20) - 15y$
$= 29x - 13 - 7x + 18y - 20 - 15y$
$= 29x - 7x + 18y - 15y - 13 - 20$
$= \mathbf{22x + 3y - 33}$
(f) $-6y - 14 - (-8yz) - 6z + 23 + 9y - 8z + 2zy$
$= -6y - 14 + 8yz - 6z + 23 + 9y - 8z + 2zy$
$= -6y + 9y - 6z - 8z + 8yz + 2zy - 14 + 23$
$= \mathbf{3y - 14z + 10yz + 9}$

11. (i) $3p + (-q) - 7r - (-8p) - q + 2r = 3p - q - 7r + 8p - q + 2r$
$= 3p + 8p - q - q - 7r + 2r$
$= \mathbf{11p - 2q - 5r}$
(ii) When $p = 2$, $q = -1\dfrac{1}{2}$, $r = -5$,
$11p - 2q - 5r = 11(2) - 2\left(-1\dfrac{1}{2}\right) - 5(-5)$
$= 22 + 3 + 25$
$= 25 + 25$
$= \mathbf{50}$

12. **(i)** Raju's age 5 years later = $\mathbf{(12m + 5)}$ **years**

(ii) Present age of Raju's son $= 12m - 9m$

$= 3m$ years

Age of Raju's son 5 years later $= (3m + 5)$ years

Sum of their ages in 5 years' time $= 12m + 5 + 3m + 5$

$= 12m + 3m + 5 + 5$

$= \mathbf{(15m + 10)}$ **years**

13. Amount of money Li Ting had at first

$= 8 \times \$w + 7 \times \$m + \$(3w + 5m)$

$= \$8w + \$7m + \$(3w + 5m)$

$= \$(8w + 3w + 7m + 5m)$

$= \mathbf{\$(11w + 12m)}$

14. **(a)** Number of people who order chicken pizza $= \mathbf{\frac{5}{2}a}$

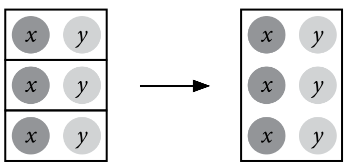
(b) Number of people who order cheese pizza $= \mathbf{\frac{2}{5}b}$

(c) Number of people who order cheese pizza $= \mathbf{\frac{2}{7}c}$

6.3 Expansion and factorisation of linear expressions

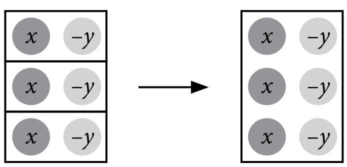
Investigation (Expansion of linear expressions)

(a)



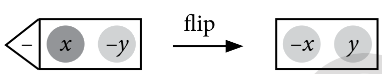
$3(x + y)$ $3x + 3y$

(b)



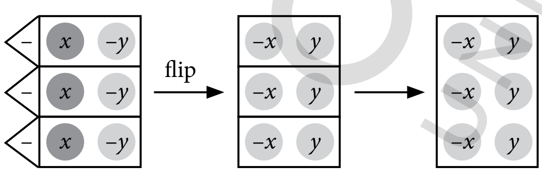
$3(x - y)$ $3x - 3y$

(c)



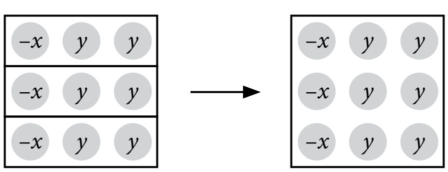
$-(x - y)$ $-x + y$

(d)



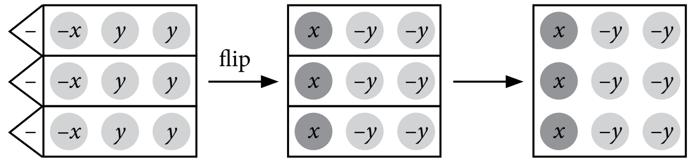
$-3(x - y) = -[3(x - y)]$ $-3x + 3y$

(e)



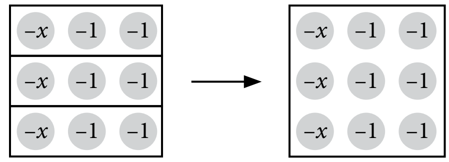
$3(-x + 2y)$ $-3x + 6y$

(f)



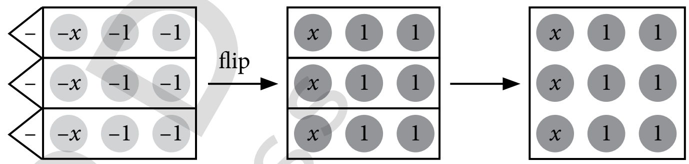
$-3(-x + 2y) = -[3(-x + 2y)]$ $3x - 6y$

(g)



$3(-x - 2)$ $-3x - 6$

(h)



$-3(-x - 2) = -[3(-x - 2)]$ $3x + 6$

Practise Now 4

(a) $2(x - 7) = \mathbf{2x - 14}$

(b) $-5(3x - 4y) = -15x + 20y$

$= \mathbf{20y - 15x}$

(c) $8 - a(-x + 2z) = 8 + [-a(-x + 2z)]$

$= \mathbf{8 + ax - 2az}$

(d) $6 - b(-3y - z) = 6 + [-b(-3y - z)]$

$= \mathbf{6 + 3by + bz}$

Practise Now 5

(a) $6(4x + y) + 2(x - y) = 24x + 6y + 2x - 2y$

$= 24x + 2x + 6y - 2y$

$= \mathbf{26x + 4y}$

(b) $x - [y - 3(2x - y)] = x - (y - 6x + 3y)$

$= x - (y + 3y - 6x)$

$= x - (4y - 6x)$

$= x - 4y + 6x$

$= \mathbf{7x - 4y}$

(c) $7x - 2[3(x - 2) - 2(x - 5)] = 7x - 2(3x - 6 - 2x + 10)$

$= 7x - 2(3x - 2x - 6 + 10)$

$= 7x - 2(x + 4)$

$= 7x - 2x - 8$

$= \mathbf{5x - 8}$

Practise Now 6

(a) $10x + 25 = \mathbf{5(2x + 5)}$

(b) $18 - 12x = \mathbf{6(3 - 2x)}$

(c) $21a - 14ay = \mathbf{7a(3 - 2y)}$

(d) $33ax + 27a + 3ay = \mathbf{3a(11x + 9 + y)}$

(e) $18x - 54xy + 36xz = \mathbf{18x(1 - 3y + 2z)}$

(f) $-10x + 30xy - 50xz = 30xy - 10x - 50xz$

$= \mathbf{10x(3y - 1 - 5z)}$

Practise Now 7

(a) $-5h - 25 = \mathbf{-5(h + 5)}$

(b) $-14ay - 21a = \mathbf{-7a(2y + 3)}$

(c) $-9z - 24bz - 15cz = \mathbf{-3z(3 + 8b + 5c)}$
(d) $-4 - 16x - 20xy = \mathbf{-4(1 + 4x + 5xy)}$

Introductory Problem Revisited

(a) x can be any number from 2 to 9;
y can be any number from 1 to 9
(b) Expression for two-digit number $= 10x + y$ — (1)
(c) Expression for sum of two digits $= x + y$ — (2)
(d) Subtracting (2) from (1), we get $9x$, which is a multiple of 9.
(e) The sum of the digits of all the multiples of 9 from 9 to 90 is 9.
Since $x < 10$, $\therefore$ the value of this final number is always 9.

Exercise 6B

1. **(a)** $-(x + 5) = \mathbf{-x - 5}$
(b) $-(4 - x) = -4 + x$
$= \mathbf{x - 4}$
(c) $2(3y + 7) = \mathbf{6y + 14}$
(d) $8(2y - 5) = \mathbf{16y - 40}$
(e) $-8(3a - 4b) = -24a + 32b$
$= \mathbf{32b - 24a}$
(f) $2a(x - y) = \mathbf{2ax - 2ay}$
(g) $5 - b(-6w + 2x) = 5 + [-b(-6w + 2x)]$
$= \mathbf{5 + 6bw - 2bx}$
(h) $13 + 3c(-y - 3z) = 13 + (-3cy - 9cz)$
$= \mathbf{13 - 3cy - 9cz}$

2. Present age of Ken's uncle $= 4(x + 5)$
$= \mathbf{(4x + 20)}$ **years**

3. **(a)** $5(a + 2b) - 3b = 5a + 10b - 3b$
$= \mathbf{5a + 7b}$
(b) $7(p + 10q) + 2(6p + 7q) = 7p + 70q + 12p + 14q$
$= 7p + 12p + 70q + 14q$
$= \mathbf{19p + 84q}$
(c) $a + 3b - (5a - 4b) = a + 3b - 5a + 4b$
$= a - 5a + 3b + 4b$
$= \mathbf{-4a + 7b}$
(d) $x + 3(2x - 3y + z) + 7z = x + 6x - 9y + 3z + 7z$
$= \mathbf{7x - 9y + 10z}$

4. Total cost $= 4x + 6(x - y)$
$= 4x + 6x - 6y$
$= \mathbf{(10x - 6y)}$ **cents**

5. **(a)** $12x + 9 = \mathbf{3(4x + 3)}$
(b) $27b - 36by = \mathbf{9b(3 - 4y)}$
(c) $-16pq + 6pr = 6pr - 16pq$
$= \mathbf{2p(3r - 8q)}$
(d) $8ax + 12a - 4az = \mathbf{4a(2x + 3 - z)}$
(e) $-6h + 30hk - 24hn = 30hk - 6h - 24hn$
$= \mathbf{6h(5k - 1 - 4n)}$
(f) $-4mx - 6my + 18mz = 18mz - 4mx - 6my$
$= \mathbf{2m(9z - 2x - 3y)}$

6. **(a)** $-25y - 35 = \mathbf{-5(5y + 7)}$
(b) $-64ax - 16ay = \mathbf{-16a(4x + y)}$
(c) $-81w - 9wx - 18wy = \mathbf{-9w(9 + x + 2y)}$
(d) $-24x - 36 - 12xz = \mathbf{-12(2x + 3 + xz)}$

7. **(a)** Required answer $= 2x - 5 - (-6x - 3)$
$= 2x - 5 + 6x + 3$
$= 2x + 6x - 5 + 3$
$= \mathbf{8x - 2}$
(b) Required answer $= 10x - 2y + z - (6x - y + 5z)$
$= 10x - 2y + z - 6x + y - 5z$
$= 10x - 6x - 2y + y + z - 5z$
$= \mathbf{4x - y - 4z}$
(c) Required answer $= -4p - 4q + 15sr - (8p + 9q - 5rs)$
$= -4p - 4q + 15sr - 8p - 9q + 5rs$
$= -4p - 8p - 4q - 9q + 15sr + 5rs$
$= \mathbf{-12p - 13q + 20rs}$
(d) Required answer $= 10a - b - 4c - 8d - (8a - 3b + 5c - 4d)$
$= 10a - b - 4c - 8d - 8a + 3b - 5c + 4d$
$= 10a - 8a - b + 3b - 4c - 5c - 8d + 4d$
$= \mathbf{2a + 2b - 9c - 4d}$

8. **(a)** $4u - 3(2u - 5v) = 4u - 6u + 15v$
$= \mathbf{-2u + 15v}$
(b) $-2a - 3(a - b) = -2a - 3a + 3b$
$= \mathbf{-5a + 3b}$
(c) $7m - 2n - 2(3n - 2m) = 7m - 2n - 6n + 4m$
$= 7m + 4m - 2n - 6n$
$= \mathbf{11m - 8n}$
(d) $5(2x + 4) - 3(-6 - x) = 10x + 20 + 18 + 3x$
$= 10x + 3x + 20 + 18$
$= \mathbf{13x + 38}$
(e) $-4(a - 3b) - 5(a - 3b) = -4a + 12b - 5a + 15b$
$= -4a - 5a + 12b + 15b$
$= \mathbf{-9a + 27b}$
(f) $5(3p - 2q) - 2(3p + 2q) = 15p - 10q - 6p - 4q$
$= 15p - 6p - 10q - 4q$
$= \mathbf{9p - 14q}$
(g) $x + y - 2(3x - 4y + 3) = x + y - 6x + 8y - 6$
$= x - 6x + y + 8y - 6$
$= \mathbf{-5x + 9y - 6}$
(h) $3(p - 2q) - 4(2p - 3q - 5) = 3p - 6q - 8p + 12q + 20$
$= 3p - 8p - 6q + 12q + 20$
$= \mathbf{-5p + 6q + 20}$
(i) $9(2a + 4b - 7c) - 4(b - c) - 7(-c - 4b)$
$= 18a + 36b - 63c - 4b + 4c + 7c + 28b$
$= 18a + 36b - 4b + 28b - 63c + 4c + 7c$
$= \mathbf{18a + 60b - 52c}$
(j) $-4[5(2x + 3y) - 4(x + 2y)] = -4(10x + 15y - 4x - 8y)$
$= -4(10x - 4x + 15y - 8y)$
$= -4(6x + 7y)$
$= \mathbf{-24x - 28y}$

9. **(a)** $-2\{3a - 4[a - (2 + a)]\} = -2[3a - 4(a - 2 - a)]$
$= -2[3a - 4(-2)]$
$= -2(3a + 8)$
$= \mathbf{-6a - 16}$
(b) $5\{3c - [d - 2(c + d)]\} = 5[3c - (d - 2c - 2d)]$
$= 5[3c - (-2c - d)]$
$= 5(3c + 2c + d)$
$= 5(5c + d)$
$= \mathbf{25c + 5d}$

10. **(a)** Required answer
$= 2x^2 - 3x + 11 + 7x^2 - 3x - 4 - (14x^2 - 12x - 6)$
$= 2x^2 - 3x + 11 + 7x^2 - 3x - 4 - 14x^2 + 12x + 6$
$= 2x^2 + 7x^2 - 14x^2 - 3x - 3x + 12x + 11 - 4 + 6$
$= \mathbf{-5x^2 + 6x + 13}$

(b) Required answer

$= 17x^3 - 12x + 11 + x^3 + 5 - (2x^3 + 7x^2 - 4x - 13)$

$= 17x^3 - 12x + 11 + x^3 + 5 - 2x^3 - 7x^2 + 4x + 13$

$= 17x^3 + x^3 - 2x^3 - 7x^2 - 12x + 4x + 11 + 5 + 13$

$= \mathbf{16x^3 - 7x^2 - 8x + 29}$

(c) Required answer

$= 4(2 - 5w) - (w^2 - 10w + 25 + 3w^2 - 7w)$

$= 8 - 20w - (w^2 + 3w^2 - 10w - 7w + 25)$

$= 8 - 20w - (4w^2 - 17w + 25)$

$= 8 - 20w - 4w^2 + 17w - 25$

$= -4w^2 - 20w + 17w + 8 - 25$

$= \mathbf{-4w^2 - 3w - 17}$

(d) Required answer

$= -2(-3 - 7w) - 3(6w^2 - 7w + 1)$

$= 6 + 14w - 18w^2 + 21w - 3$

$= -18w^2 + 14w + 21w + 6 - 3$

$= \mathbf{-18w^2 + 35w + 3}$

11. **(a)** $5x + 10x(b + c) = 5x[1 + 2(b + c)]$

$= \mathbf{5x(1 + 2b + 2c)}$

(b) $3xy - 6x(y - z) = 3x[y - 2(y - z)]$

$= 3x(y - 2y + 2z)$

$= \mathbf{3x(-y + 2z)}$

(c) $2x(7 + y) - 14x(y + 2) = 2x[7 + y - 7(y + 2)]$

$= 2x(7 + y - 7y - 14)$

$= 2x(y - 7y + 7 - 14)$

$= \mathbf{2x(-6y - 7)}$

(d) $-39b^2 - 13ab = \mathbf{-13b(3b + a)}$

(e) $-3a(2 + b) - 18a(b + 1) = -3a[(2 + b) + 6(b + 1)]$

$= -3a(2 + b + 6b + 6)$

$= \mathbf{-3a(7b + 8)}$

(f) $(-4xy^2 - 16xy)[a(3y + 2) - 2a(y - 1)]$

$= -4xy(y + 4)[a(3y + 2 - 2y + 2)]$

$= -4xy(y + 4)[a(y + 4)]$

$= \mathbf{-4axy(y + 4)^2}$

6.4 Linear expressions with fractional coefficients

Investigation (Comparing algebraic notations)

2. The numbers were obtained by substituting $n = 2$ into the expressions in the cells J1 to L1.

3.

	A	I	J	K	L
1	n	$\frac{n}{2}$	$\left(\frac{1}{2}\right)n$	$\frac{(n+1)}{3}$	$\left(\frac{1}{3}\right)$
2	1	0.5	0.5	0.666666667	0.666666667
3	2	1	1	1	1
4	3	1.5	1.5	1.333333333	1.333333333
5	4	**2**	**2**	**1.666666667**	**1.666666667**
6	5	**2.5**	**2.5**	**2**	**2**

Table 6.9

4. **(i)** **Yes**. The values in columns I and J are the same.

(ii) **Yes**. The values in columns K and L are the same.

Practise Now 8

(a) $\frac{1}{2}x + \frac{1}{4}y - \frac{2}{5}y - \frac{1}{3}x = \frac{1}{2}x - \frac{1}{3}x + \frac{1}{4}y - \frac{2}{5}y$

$= \frac{3}{6}x - \frac{2}{6}x + \frac{5}{20}y - \frac{8}{20}y$

$= \mathbf{\frac{1}{6}x - \frac{3}{20}y}$

(b) $\frac{1}{8}[-y - 3(16x - 3y)] = \frac{1}{8}(-y - 48x + 9y)$

$= \frac{1}{8}(-y + 9y - 48x)$

$= \frac{1}{8}(8y - 48x)$

$= \mathbf{y - 6x}$

Practise Now 9

1. **(a)** $\frac{1}{2}(x - 3) + \frac{2x - 5}{3} = \frac{3(x - 3)}{6} + \frac{2(2x - 5)}{6}$

$= \frac{3(x - 3) + 2(2x - 5)}{6}$

$= \frac{3x - 9 + 4x - 10}{6}$

$= \frac{3x + 4x - 9 - 10}{6}$

$= \mathbf{\frac{7x - 19}{6}}$

(b) $\frac{x - 2}{4} - \frac{2x - 7}{3} = \frac{3(x - 2)}{12} - \frac{4(2x - 7)}{12}$

$= \frac{3(x - 2) - 4(2x - 7)}{12}$

$= \frac{3x - 6 - 8x + 28}{12}$

$= \frac{3x - 8x - 6 + 28}{12}$

$= \mathbf{\frac{22 - 5x}{12}}$

2. **(a)** $\frac{x - 1}{3} + \frac{1}{2} - \frac{1}{4}(2x - 3) = \frac{4(x - 1)}{12} + \frac{6}{12} - \frac{3(2x - 3)}{12}$

$= \frac{4(x - 1) + 6 - 3(2x - 3)}{12}$

$= \frac{4x - 4 + 6 - 6x + 9}{12}$

$= \frac{4x - 6x - 4 + 6 + 9}{12}$

$= \mathbf{\frac{11 - 2x}{12}}$

(b) $2x + \frac{x - 4}{9} - \frac{2x - 5}{3} = \frac{9(2x)}{9} + \frac{x - 4}{9} - \frac{3(2x - 5)}{9}$

$= \frac{9(2x) + x - 4 - 3(2x - 5)}{9}$

$= \frac{18x + x - 4 - 6x + 15}{9}$

$= \frac{18x + x - 6x - 4 + 15}{9}$

$= \mathbf{\frac{13x + 11}{9}}$

Thinking Time (Page 155)

The five pairs of equivalent expressions are as follows:

1. **D** and **F**

$$\begin{aligned}3(x-2y)-2(3x-y) &= 3x-6y-6x+2y\\ &= 3x-6x-6y+2y\\ &= -3x-4y\end{aligned}$$

2. **A** and **E**

$$\begin{aligned}\frac{x-3}{2}-\frac{2x-5}{3} &= \frac{3(x-3)-2(2x-5)}{6}\\ &= \frac{3x-9-4x+10}{6}\\ &= \frac{3x-4x-9+10}{6}\\ &= \frac{-x+1}{6}\\ &= \frac{1-x}{6}\end{aligned}$$

3. **G** and **N**

$$\begin{aligned}\frac{3(x+3)}{4}-\frac{4(2x+3)}{4} &= \frac{9(x+3)-16(2x+3)}{12}\\ &= \frac{9x+27-32x-48}{12}\\ &= \frac{9x-32x+27-48}{12}\\ &= \frac{-23x-21}{12}\end{aligned}$$

4. **I** and **M**

$$\begin{aligned}2x-3[5x-y-2(7x-y)] &= 2x-3(5x-y-14x+2y)\\ &= 2x-3(5x-14x-y+2y)\\ &= 2x-3(-9x+y)\\ &= 2x+27x-3y\\ &= 29x-3y\end{aligned}$$

5. **C** and **J**, **C** and **K** or **J** and **K**

$7ay - 49y = 7(ay - 7y) = 7y(a - 7)$

Teachers may wish to get students to indicate the expression which is obtained when the expression 7ay – 49y is factorised completely.

Exercise 6C

1. (a)

$$\begin{aligned}\frac{1}{4}x+\frac{1}{5}y-\frac{1}{6}x-\frac{1}{10}y &= \frac{1}{4}x-\frac{1}{6}x+\frac{1}{5}y-\frac{1}{10}y\\ &= \frac{3}{12}x-\frac{2}{12}x+\frac{2}{10}y-\frac{1}{10}y\\ &= \boldsymbol{\frac{1}{12}x+\frac{1}{10}y}\end{aligned}$$

(b)

$$\begin{aligned}\frac{2}{3}a-\frac{1}{7}b+2a-\frac{3}{5}b &= \frac{2}{3}a+2a-\frac{1}{7}b-\frac{3}{5}b\\ &= \frac{2}{3}a+\frac{6}{3}a-\frac{5}{35}b-\frac{21}{35}b\\ &= \boldsymbol{\frac{8}{3}a-\frac{26}{35}b}\end{aligned}$$

(c)

$$\begin{aligned}\frac{5}{9}c+\frac{3}{4}d-\frac{7}{8}c-\frac{4}{3}d &= \frac{5}{9}c-\frac{7}{8}c+\frac{3}{4}d-\frac{4}{3}d\\ &= \frac{40}{72}c-\frac{63}{72}c+\frac{9}{12}d-\frac{16}{12}d\\ &= \boldsymbol{-\frac{23}{72}c-\frac{7}{12}d}\end{aligned}$$

(d)

$$\begin{aligned}&2f-\frac{5}{3}h+\frac{9}{4}k-\frac{1}{2}f-\frac{28}{5}k+\frac{5}{4}h\\ &= 2f-\frac{1}{2}f-\frac{5}{3}h+\frac{5}{4}h+\frac{9}{4}k-\frac{28}{5}k\\ &= \frac{4}{2}f-\frac{1}{2}f-\frac{20}{12}h+\frac{15}{12}h+\frac{45}{20}k-\frac{112}{20}k\\ &= \boldsymbol{\frac{3}{2}f-\frac{5}{12}h-\frac{67}{20}k}\end{aligned}$$

2. (a)

$$\begin{aligned}&5a+4b-3c-\left(2a-\frac{3}{2}b+\frac{3}{2}c\right)\\ &= 5a+4b-3c-2a+\frac{3}{2}b-\frac{3}{2}c\\ &= 5a-2a+4b+\frac{3}{2}b-3c-\frac{3}{2}c\\ &= 3a+\frac{8}{2}b+\frac{3}{2}b-\frac{6}{2}c-\frac{3}{2}c\\ &= \boldsymbol{3a+\frac{11}{2}b-\frac{9}{2}c}\end{aligned}$$

(b)

$$\begin{aligned}\frac{1}{2}[2x+2(x-3)] &= \frac{1}{2}(2x+2x-6)\\ &= \frac{1}{2}(4x-6)\\ &= \boldsymbol{2x-3}\end{aligned}$$

(c)

$$\begin{aligned}\frac{2}{5}[12p-(5+2p)] &= \frac{2}{5}(12p-5-2p)\\ &= \frac{2}{5}(12p-2p-5)\\ &= \frac{2}{5}(10p-5)\\ &= \boldsymbol{4p-2}\end{aligned}$$

(d)

$$\begin{aligned}\frac{1}{2}[8x+10-6(1-4x)] &= \frac{1}{2}(8x+10-6+24x)\\ &= \frac{1}{2}(8x+24x+10-6)\\ &= \frac{1}{2}(32x+4)\\ &= \boldsymbol{16x+2}\end{aligned}$$

3. (a)

$$\begin{aligned}\frac{1}{2}x+\frac{2x}{5} &= \frac{5x}{10}+\frac{4x}{10}\\ &= \boldsymbol{\frac{9}{10}x}\end{aligned}$$

(b)

$$\begin{aligned}\frac{a}{3}-\frac{1}{4}a &= \frac{4a}{12}-\frac{3a}{12}\\ &= \boldsymbol{\frac{1}{12}a}\end{aligned}$$

(c)

$$\begin{aligned}\frac{2h}{7}+\frac{1}{5}(h+1) &= \frac{10h}{35}+\frac{7(h+1)}{35}\\ &= \frac{10h+7(h+1)}{35}\\ &= \frac{10h+7h+7}{35}\\ &= \boldsymbol{\frac{17h+7}{35}}\end{aligned}$$

(d) $\frac{3x}{8} - \frac{1}{4}(x+2) = \frac{3x}{8} - \frac{2(x+2)}{8}$

$= \frac{3x - 2(x+2)}{8}$

$= \frac{3x - 2x - 4}{8}$

$= \mathbf{\frac{x-4}{8}}$

(e) $\frac{4x+1}{5} + \frac{3x-1}{2} = \frac{2(4x+1)}{10} + \frac{5(3x-1)}{10}$

$= \frac{2(4x+1)+5(3x-1)}{10}$

$= \frac{8x+2+15x-5}{10}$

$= \frac{8x+15x+2-5}{10}$

$= \mathbf{\frac{23x-3}{10}}$

(f) $\frac{3y-1}{4} - \frac{2y-3}{6} = \frac{3(3y-1)}{12} - \frac{2(2y-3)}{12}$

$= \frac{3(3y-1)-2(2y-3)}{12}$

$= \frac{9y-3-4y+6}{12}$

$= \frac{9y-4y-3+6}{12}$

$= \mathbf{\frac{5y+3}{12}}$

(g) $\frac{1}{4}(a-1) - \frac{a+7}{8} = \frac{2(a-2)}{8} - \frac{a+7}{8}$

$= \frac{2(a-2)-(a+7)}{8}$

$= \frac{2a-4-a-7}{8}$

$= \frac{2a-a-4-7}{8}$

$= \mathbf{\frac{a-11}{8}}$

(h) $\frac{1}{3}(3p-2p) - \frac{4p-5q}{4} = \frac{4(3p-2q)}{12} - \frac{3(4p-5q)}{12}$

$= \frac{4(3p-2q)-3(4p-5q)}{12}$

$= \frac{12p-8q-12p+15q}{12}$

$= \frac{12p-12p-8q+15q}{12}$

$= \mathbf{\frac{7}{12}q}$

4. (a) $y - \frac{2}{3}(9x-3y) = y - 2(3x-y)$

$= y - 6x + 2y$

$= -6x + y + 2y$

$= \mathbf{-6x + 3y}$

(b) $-\frac{1}{3}\{6(p+q) - 3[p - 2(p-3q)]\}$

$= -\frac{1}{3}[6(p+q) - 3(p - 2p + 6q)]$

$= -\frac{1}{3}[6(p+q) - 3(-p + 6q)]$

$= -\frac{1}{3}(6p + 6q + 3p - 18q)$

$= -\frac{1}{3}(6p + 3p + 6q - 18q)$

$= -\frac{1}{3}(9p - 12q)$

$= \mathbf{4q - 3p}$

5. (a) $\frac{7(x+3)}{2} + \frac{5(2x-5)}{3} = \frac{21(x+3)}{6} + \frac{10(2x-5)}{6}$

$= \frac{21(x+3)+10(2x-5)}{6}$

$= \frac{21x+63+20x-50}{6}$

$= \frac{21x+20x+63-50}{6}$

$= \mathbf{\frac{41x+13}{6}}$

(b) $\frac{3x-4}{5} - \frac{3(x-1)}{2} = \frac{2(3x-4)}{10} - \frac{15(x-1)}{10}$

$= \frac{2(3x-4)-15(x-1)}{10}$

$= \frac{6x-8-15x+15}{10}$

$= \frac{6x-15x-8+15}{10}$

$= \mathbf{\frac{7-9x}{10}}$

(c) $\frac{3}{4}(z-2) - \frac{4(2z-3)}{5} = \frac{15(z-2)}{20} - \frac{16(2z-3)}{20}$

$= \frac{15(z-2)-16(2z-3)}{20}$

$= \frac{15z-30-32z+48}{20}$

$= \frac{15z-32z-30+48}{20}$

$= \mathbf{\frac{18-17z}{20}}$

(d) $\frac{2(p-4q)}{3} - \frac{3(2p+q)}{2} = \frac{4(p-4q)}{6} - \frac{9(2p+q)}{6}$

$= \frac{4(p-4q)-9(2p+q)}{6}$

$= \frac{4p-16q-18p-9q}{6}$

$= \frac{4p-18p-16q-9q}{6}$

$= \mathbf{\frac{-14p-25q}{6}}$

(e) $-\frac{2b}{3} - \frac{3(a-2b)}{5} = -\frac{10b}{15} - \frac{9(a-2b)}{15}$

$= \frac{-10b-9(a-2b)}{15}$

$= \frac{-10b-9a+18b}{15}$

$= \frac{-9a-10b+18b}{15}$

$= \mathbf{\frac{8b-9a}{15}}$

(f) $\frac{2}{5}(x+3) - \frac{1}{2} + \frac{3x-4}{4} = \frac{8(x+3)}{20} - \frac{10}{20} + \frac{5(3x-4)}{20}$

$= \frac{8(x+3)-10+5(3x-4)}{20}$

$= \frac{8x+24-10+15x-20}{20}$

$= \frac{8x+15x+24-10-20}{20}$

$= \mathbf{\frac{23x-6}{20}}$

(g) $\frac{a+1}{2} - \frac{a+3}{3} - \frac{5a-2}{4}$

$= \frac{6(a+1)}{12} - \frac{4(a+3)}{12} - \frac{3(5a-2)}{12}$

$= \frac{6(a+1)-4(a+3)-3(5a-2)}{12}$

$= \frac{6a+6-4a-12-15a+6}{12}$

$= \frac{6a-4a-15a+6-12+6}{12}$

$= \mathbf{-\frac{13}{12}a}$

(h) $\frac{x+1}{2} + \frac{x+3}{3} - \frac{5x-1}{6} = \frac{3(x+1)}{6} + \frac{2(x+3)}{6} - \frac{5x-1}{6}$

$= \frac{3(x+1)+2(x+3)-(5x-1)}{6}$

$= \frac{3x+3+2x+6-5x+1}{6}$

$= \frac{3x+2x-5x+3+6+1}{6}$

$= \frac{10}{6}$

$= \frac{5}{3}$

$= \mathbf{1\frac{2}{3}}$

(i) $\frac{2(a-b)}{7} - \frac{2a+3b}{14} + \frac{a+b}{2}$

$= \frac{4(a-b)}{14} - \frac{2a+3b}{14} + \frac{7(a+b)}{14}$

$= \frac{4(a-b)-(2a+3b)+7(a+b)}{14}$

$= \frac{4a-4b-2a-3b+7a+7b}{14}$

$= \frac{4a-2a+7a-4b-3b+7b}{14}$

$= \mathbf{\frac{9}{14}a}$

(j) $\frac{x+3}{3} + \frac{5}{6}(3x+4) + 1 = \frac{2(x+3)}{6} + \frac{5(3x+4)}{6} + \frac{6}{6}$

$= \frac{2(x+3)+5(3x+4)+6}{6}$

$= \frac{2x+6+15x+20+6}{6}$

$= \frac{2x+15x+6+20+6}{6}$

$= \mathbf{\frac{17x+32}{6}}$

6. (a) $\frac{5(p-q)}{2} - \frac{2q-p}{14} - \frac{2(p+q)}{7}$

$= \frac{35(p-q)}{14} - \frac{2q-p}{14} - \frac{4(p+q)}{14}$

$= \frac{35(p-q)-(2q-p)-4(p+q)}{14}$

$= \frac{35p-35q-2q+p-4p-4q}{14}$

$= \frac{35p+p-4p-35q-2q-4q}{14}$

$= \mathbf{\frac{32p-41q}{14}}$

(b) $-\frac{2a+b}{3} - \left[\frac{3(a-3b)}{2} - \frac{4(a+2b)}{5}\right]$

$= -\frac{2a+b}{3} - \frac{3(a-3b)}{2} + \frac{4(a+2b)}{5}$

$= -\frac{10(2a+b)}{30} - \frac{45(a-3b)}{30} + \frac{24(a+2b)}{30}$

$= \frac{-10(2a+b)-45(a-3b)+24(a+2b)}{30}$

$= \frac{-20a-10b-45a+135b+24a+48b}{30}$

$= \frac{-20a-45a+24a-10b+135b+48b}{30}$

$= \mathbf{\frac{173b-41a}{30}}$

(c) $\frac{3(f-h)}{4} - \frac{7(h+k)}{6} + \frac{5(k-f)}{2}$

$= \frac{9(f-h)}{12} - \frac{14(h+k)}{12} + \frac{30(k-f)}{12}$

$= \frac{9(f-h)-14(h+k)+30(k-f)}{12}$

$= \frac{9f-9h-14h-14k+30k-30f}{12}$

$= \frac{9f-30f-9h-14h-14k+30k}{12}$

$= \mathbf{\frac{-21f-23h+16k}{12}}$

(d) $4 - \frac{x-y}{3} - \frac{3(y+4z)}{4} + \frac{5}{8}(x+3z)$

$= \frac{96}{24} - \frac{8(x-y)}{24} - \frac{18(y+z)}{24} + \frac{15(x+3z)}{24}$

$= \frac{96-8(x-y)-18(y+z)+15(x+3z)}{24}$

$= \frac{96-8x+8y-18y-18z+15x+45z}{24}$

$= \frac{96-8x+15x+8y-18y-18z+45z}{24}$

$= \mathbf{\frac{96+7x-10y+27z}{24}}$

Chapter 7 Linear Equations

TEACHING NOTES

Suggested Approach

In this chapter, algebra discs on a balance are used to teach students how to solve linear equations in one variable. Since many Secondary 1 students are still in the concrete operational stage (according to Piaget), this approach can help them to learn the concepts more easily. However, there is still a need to guide students to move from the 'concrete' to the 'abstract', partly because they cannot use this approach in examinations, and partly because they cannot use this approach to solve linear equations which consist of algebraic terms that have large or fractional coefficients (see Section 7.1). After students learn how to solve linear equations, they will learn how to evaluate an unknown in a formula and formulate linear equations to solve problems in real-world contexts.

Section 7.1: Linear equations

Students have learnt how to complete mathematical sentences such as $\square + 2 = 5$ in primary school. Teachers can introduce equations by telling students that when we replace $\square$ with x, we have $x + 2 = 5$, which is an equation. Teachers should illustrate the meaning of 'solving an equation' using appropriate examples. Students should know the difference between linear expressions and linear equations (see Class Discussion: Expressions and equations).

Teachers can use the 'Balance Method' to show how to solve linear equations which do not involve any brackets before illustrating how to solve those which involve brackets. As this approach cannot be used to solve linear equations which consist of algebraic terms that have large or fractional coefficients, so there is a need to help students consolidate what they have learnt from the Investigation activity on page 162 and Worked Example 1. Teachers can make use of the Thinking Time activity on page 165 to illustrate that some equations have no solutions or infinitely many solutions. Journal Writing on page 165 of the textbook reinforces students' understanding of the concept of equation. For example, since $x + 3 = 6$, $2x + 3 = 9$ and $10x - 4 = 5x + 11$ are equivalent equations that can be obtained from $x = 3$, then the value of x in each of the equations is 3.

Section 7.2 Linear equations with fractional coefficients and fractional equations

Teachers can help students consolidate their learning by highlighting that in Method 2 of Worked Example 2 and in Worked Example 3, the first step involves rewriting the given equation in its equivalent form with whole number coefficients. Meanwhile, Method 1 of Worked Example 2 uses the same steps as Worked Example 1.

Section 7.3: Applications of linear equations in real-world contexts

Teachers should illustrate how a word problem is solved using the model method before showing how the same problem can be solved using the algebraic method. Students should observe how the algebraic method is linked to the model method. Also, students should be aware why they need to learn the algebraic method. In this section, students are given ample opportunities to formulate linear equations to solve problems in real-world contexts.

Section 7.4: Mathematical formulae

Teachers can use simple formulae such as $A = lb$, where A, l and b are the area, the length and the breadth of the rectangle respectively, to let students understand that a formula uses variables to provide instructions for performing a calculation. Teachers may get students to provide examples of formulae which they have encountered in mathematics and the sciences.

Introductory Problem

The solutions can be found in Introductory Problem Revisited, after Practise Now 4.

7.1 Linear equations

Class Discussion (Expressions and equations)

1. **(a)** $x + 2$ is an **expression**.
   (b) $x + 2 = 5$ is an **equation**.
   (c) $6(y - 1)$ is an **expression**.
   (d) $6(y - 1) = 0$ is an **equation**.
   (e) $5x + 4 = 2x - 8$ is an **equation**.
   (f) $x^2 - 2 = 0$ is an **equation**.
   (g) $3x + y = -7$ is an **equation**.
   (h) $9\sqrt{x} - xy = 3$ is an **equation**.
   An expression has a different combination of numbers, variables and symbols, while an equation consists of expressions separated by an 'equal' sign. In an equation, the expressions on the left-hand side and right-hand side have the same value.
2. **(b)**, **(d)**, **(e)** and **(g)** are linear equations.
   Linear equations are equations that have only linear expressions on either side of the equation. A linear expression is an algebraic expression in which each variable term has a power of 1 and the variable terms are not multiplied by each other; a constant term may or may not be included.

Investigation (Solving linear equations)

Teachers may use the balance scale and algebra discs to help students visualise how the equations are solved.

Part 1

1. **(a)**
$$\begin{aligned} x + 3 &= 7 \\ x + 3 - 3 &= 7 - 3 \\ x &= \mathbf{4} \end{aligned}$$
(b)
$$\begin{aligned} x - 4 &= 6 \\ x - 4 + 4 &= 6 + 4 \\ x &= \mathbf{10} \end{aligned}$$
(c)
$$\begin{aligned} x + 2 &= -5 \\ x + 2 - 2 &= -5 - 2 \\ x &= \mathbf{-7} \end{aligned}$$
(d)
$$\begin{aligned} x - 8 &= -1 \\ x - 8 + 8 &= -1 + 8 \\ x &= \mathbf{7} \end{aligned}$$

Part 2

2. **(a)**
$$\begin{aligned} 2x - 5 &= 7 \\ 2x - 5 + 5 &= 7 + 5 \\ 2x &= 12 \\ \frac{2x}{2} &= \frac{12}{2} \\ x &= \mathbf{6} \end{aligned}$$
(b)
$$\begin{aligned} 3x + 8 &= 2 \\ 3x + 8 - 8 &= 2 - 8 \\ 3x &= -6 \\ \frac{3x}{3} &= -\frac{6}{3} \\ x &= \mathbf{-2} \end{aligned}$$
(c)
$$\begin{aligned} -4x + 3 &= -5 \\ -4x + 3 - 3 &= -5 - 3 \\ -4x &= -8 \\ 4x &= 8 \\ \frac{4x}{4} &= \frac{8}{2} \\ x &= \mathbf{2} \end{aligned}$$
(d)
$$\begin{aligned} -x - 8 &= -8 \\ -x - 8 + 8 &= -8 + 8 \\ -x &= 0 \\ x &= \mathbf{0} \end{aligned}$$

3. **Yes**, it is possible. However, it will be more complicated because fractions are involved since the constant terms '5' and '7' are not even numbers.
   Teachers may wish to show students that if we divide both sides of the equation by 2, every term will need to be divided:
   $2x - 5 = 7$
   $x - \frac{5}{2} = \frac{7}{2}$

Part 3

4. **(a)**
$$\begin{aligned} 5x + 3 &= 3x - 7 \\ 5x + 3 - 3x &= 3x - 7 - 3x \\ 2x + 3 &= -7 \\ 2x + 3 - 3 &= -7 - 3 \\ 2x &= -10 \\ \frac{2x}{2} &= -\frac{10}{2} \\ x &= \mathbf{-5} \end{aligned}$$
(b)
$$\begin{aligned} 4x - 2 &= x + 7 \\ 4x - 2 - x &= x + 7 - x \\ 3x - 2 &= 7 \\ 3x - 2 + 2 &= 7 + 2 \\ 3x &= 9 \\ \frac{3x}{3} &= \frac{9}{3} \\ x &= \mathbf{3} \end{aligned}$$
(c)
$$\begin{aligned} 3x - 2 &= -x + 14 \\ 3x - 2 + x &= -x + 14 + x \\ 4x - 2 &= 14 \\ 4x - 2 + 2 &= 14 + 2 \\ 4x &= 16 \\ \frac{4x}{4} &= \frac{16}{4} \\ x &= \mathbf{4} \end{aligned}$$

(d)
$$\begin{aligned} -2x - 5 &= 5x - 12 \\ -2x - 5 + 2x &= 5x - 12 + 2x \\ -5 &= 7x - 12 \\ 7x - 12 &= -5 \\ 7x - 12 + 12 &= -5 + 12 \\ 7x &= 7 \\ \frac{7x}{7} &= \frac{7}{7} \\ x &= \mathbf{1} \end{aligned}$$

Practise Now 1

(a)
$$\begin{aligned} 3x + 2 &= 8 \\ 3x + 2 - 2 &= 8 - 2 \\ 3x &= 6 \\ \frac{3x}{3} &= \frac{6}{3} \\ x &= \mathbf{2} \end{aligned}$$

(b)
$$\begin{aligned} 5x - 3 &= 3x - 12 \\ 5x - 3 + 3 &= 3x - 12 + 3 \\ 5x &= 3x - 9 \\ 5x - 3x &= 3x - 9 - 3x \\ 2x &= -9 \\ \frac{2x}{2} &= -\frac{9}{2} \\ x &= \mathbf{-4\frac{1}{2}} \end{aligned}$$

(c)
$$\begin{aligned} 2(2y + 1) &= 9y + 4 \\ 4y + 2 &= 9y + 4 \\ 9y + 4 &= 4y + 2 \\ 9y + 4 - 4y &= 4y + 2 - 4y \\ 5y + 4 &= 2 \\ 5y + 4 - 4 &= 2 - 4 \\ 5y &= -2 \\ \frac{5y}{5} &= -\frac{2}{5} \\ y &= \mathbf{-\frac{2}{5}} \end{aligned}$$

(d)
$$\begin{aligned} 2(z - 1) + 3(z - 1) &= 4 + 2z \\ 2z - 2 + 3z - 3 &= 4 + 2z \\ 5z - 5 &= 4 + 2z \\ 5z - 5 - 2z &= 4 + 2z - 2z \\ 3z - 5 &= 4 \\ 3z - 5 + 5 &= 4 + 5 \\ 3z &= 9 \\ \frac{3z}{3} &= \frac{9}{3} \\ z &= \mathbf{3} \end{aligned}$$

Thinking Time (Page 165)

(a) $3x + 4 = 2x + 1$

$x = -3$

(b) $3x + 4 = 3x + 1$ cannot be solved. The equation is not true for any value of x.

(c) LHS $= 3(x + 2)$

$= 3x + 6$

$=$ RHS

This equation is true for all values of x.

Journal Writing (Page 165)

Some equivalent equations that have the solution $x = -1$:

- $x + 1 = 0$
- $x - 1 = -2$
- $3x + 8 = 5$
- $2x - 1 = -3$
- $10x + 2 = 13x + 5$
- $2(2x - 3) = 5(x - 1)$

7.2 Linear equations with fractional coefficients and fractional equations

Practise Now 2

1. (a)
$$\begin{aligned} \frac{5}{7}y + 2 &= \frac{1}{2}y + 3\frac{1}{4} \\ \frac{5}{7}y - \frac{1}{2}y + 2 &= \frac{1}{2}y - \frac{1}{2}y + 3\frac{1}{4} \\ \frac{3}{14}y + 2 &= 3\frac{1}{4} \\ \frac{3}{14}y + 2 - 2 &= 3\frac{1}{4} - 2 \\ \frac{3}{14}y &= 1\frac{1}{4} \\ \frac{14}{3} \times \frac{3}{14}y &= \frac{14}{3} \times 1\frac{1}{4} \\ y &= \mathbf{5\frac{5}{6}} \end{aligned}$$

(b)
$$\begin{aligned} \frac{3z - 1}{2} &= \frac{z - 4}{3} \\ 6 \times \frac{3z - 1}{2} &= 6 \times \frac{z - 4}{3} \\ 3(3z - 1) &= 2(z - 4) \\ 9z - 3 &= 2z - 8 \\ 9z - 2z - 3 &= 2z - 2z - 8 \\ 7z - 3 &= -8 \\ 7z - 3 + 3 &= -8 + 3 \\ 7z &= -5 \\ \frac{7z}{7} &= -\frac{5}{7} \\ z &= \mathbf{-\frac{5}{7}} \end{aligned}$$

2. (a)
$$\begin{aligned} x + 0.7 &= 1.9 \\ x + 0.7 - 0.7 &= 1.9 - 0.7 \\ x &= \mathbf{1.2} \end{aligned}$$

(b)
$$\begin{aligned} 2y - 1.3 &= 2.8 \\ 2y - 1.3 + 1.3 &= 2.8 + 1.3 \\ 2y &= 4.1 \\ \frac{2y}{2} &= \frac{4.1}{2} \\ y &= \mathbf{2.05} \end{aligned}$$

Practise Now 3

(a)
$$\frac{8}{2x-3} = 4$$
$$(2x-3) \times \frac{8}{2x-3} = (2x-3) \times 4$$
$$8 = 4(2x-3)$$
$$8 = 8x-12$$
$$8x-12 = 8$$
$$8x-12+12 = 8+12$$
$$8x = 20$$
$$\frac{8x}{8} = \frac{20}{8}$$
$$x = \mathbf{2\frac{1}{2}}$$

(b)
$$\frac{y-3}{y+4} = \frac{3}{2}$$
$$2(y+4) \times \frac{y-3}{y+4} = 2(y+4) \times \frac{3}{2}$$
$$2(y-3) = 3(y+4)$$
$$2y-6 = 3y+12$$
$$2y-3y-6 = 3y-3y+12$$
$$-y-6 = 12$$
$$-y-6+6 = 12+6$$
$$-y = 18$$
$$y = \mathbf{-18}$$

Exercise 7A

1. **(a)**
$$x+8 = 15$$
$$x+8-8 = 15-8$$
$$x = \mathbf{7}$$

(b)
$$x+9 = -5$$
$$x+9-9 = -5-9$$
$$x = \mathbf{-14}$$

(c)
$$x-5 = 17$$
$$x-5+5 = 17+5$$
$$x = \mathbf{22}$$

(d)
$$y-7 = -3$$
$$y-7+7 = -3+7$$
$$y = \mathbf{4}$$

(e)
$$4x = -28$$
$$\frac{4x}{4} = \frac{-28}{4}$$
$$x = \mathbf{-7}$$

(f)
$$-24x = -144$$
$$24x = 144$$
$$\frac{24x}{24} = \frac{144}{24}$$
$$x = \mathbf{6}$$

(g)
$$3x-4 = 11$$
$$3x-4+4 = 11+4$$
$$3x = 15$$
$$\frac{3x}{3} = \frac{15}{3}$$
$$x = \mathbf{5}$$

(h)
$$9x+4 = 31$$
$$9x+4-4 = 31-4$$
$$9x = 27$$
$$\frac{9x}{9} = \frac{27}{9}$$
$$x = \mathbf{3}$$

(i)
$$12-7x = 6$$
$$12-12-7x = 6-12$$
$$-7x = -6$$
$$7x = 6$$
$$\frac{7x}{7} = \frac{6}{7}$$
$$x = \mathbf{\frac{6}{7}}$$

(j)
$$3-7y = -12$$
$$3-3-7y = -12-3$$
$$-7y = -15$$
$$7y = 15$$
$$\frac{7y}{7} = \frac{15}{7}$$
$$y = \mathbf{2\frac{1}{7}}$$

2. **(a)**
$$3x-7 = 4-8x$$
$$3x+8x-7 = 4-8x+8x$$
$$11x-7 = 4$$
$$11x-7+7 = 4+7$$
$$11x = 11$$
$$\frac{11x}{11} = \frac{11}{11}$$
$$x = \mathbf{1}$$

(b)
$$4x-10 = 5x+7$$
$$4x-5x-10 = 5x-5x+7$$
$$-x-10 = 7$$
$$-x-10+10 = 7+10$$
$$-x = 17$$
$$x = \mathbf{-17}$$

(c)
$$30+7y = -2y-6$$
$$30+7y+2y = -2y+2y-6$$
$$30+9y = -6$$
$$30-30+9y = -6-30$$
$$9y = -36$$
$$\frac{9y}{9} = \frac{-36}{9}$$
$$y = \mathbf{-4}$$

(d)
$$2y-7 = 7y-23$$
$$2y-7y-7 = 7y-7y-23$$
$$-5y-7 = -23$$
$$-5y-7+7 = -23+7$$
$$-5y = -16$$
$$5y = 16$$
$$\frac{5y}{5} = \frac{16}{5}$$
$$y = \mathbf{3\frac{1}{5}}$$

3. **(a)**
$$\begin{aligned}2(x+3)&=8\\2x+6&=8\\2x+6-6&=8-6\\2x&=2\\\frac{2x}{2}&=\frac{2}{2}\\x&=\mathbf{1}\end{aligned}$$

(b)
$$\begin{aligned}5(x-7)&=-15\\5x-35&=-15\\5x-35+35&=-15+35\\5x&=20\\\frac{5x}{5}&=\frac{20}{5}\\x&=\mathbf{4}\end{aligned}$$

(c)
$$\begin{aligned}7(-2x+4)&=-4x\\-14x+28&=-4x\\-14x+4x+28&=-4x+4x\\-10x+28&=0\\-10x+28-28&=0-28\\-10x&=-28\\10x&=28\\\frac{10x}{10}&=\frac{28}{10}\\x&=\mathbf{2\frac{4}{5}}\end{aligned}$$

(d)
$$\begin{aligned}3(2y+3)&=4y+3\\6y+9&=4y+3\\6y-4y+9&=4y-4y+3\\2y+9&=3\\2y+9-9&=3-9\\2y&=-6\\\frac{2y}{2}&=\frac{-6}{2}\\y&=\mathbf{-3}\end{aligned}$$

(e)
$$\begin{aligned}2(y+4)&=3(y+2)\\2y+8&=3y+6\\2y+8-3y&=3y+6-3y\\-y+8&=6\\-y+8-8&=6-8\\-y&=-2\\y&=\mathbf{2}\end{aligned}$$

(f)
$$\begin{aligned}5(5y-6)&=4(y-7)\\25y-30&=4y-28\\25y-4y-30&=4y-4y-28\\21y-30&=-28\\21y-30+30&=-28+30\\21y&=2\\\frac{21y}{21}&=\frac{2}{21}\\y&=\mathbf{\frac{2}{21}}\end{aligned}$$

(g)
$$\begin{aligned}5(b+6)&=2(3b-4)\\5b+30&=6b-8\\5b+30-6b&=6b-8-6b\\-b+30&=-8\\-b+30-30&=-8-30\\-b&=-38\\b&=\mathbf{38}\end{aligned}$$

(h)
$$\begin{aligned}3(2c+5)&=4(c-3)\\6c+15&=4c-12\\6c-4c+15&=4c-4c-12\\2c+15&=-12\\2c+15-15&=-12-15\\2c&=-27\\\frac{2c}{2}&=\frac{-27}{2}\\c&=\mathbf{-13\frac{1}{2}}\end{aligned}$$

(i)
$$\begin{aligned}9(2d+7)&=11(d+14)\\18d+63&=11d+154\\18d-11d+63&=11d-11d+154\\7d+63&=154\\7d+63-63&=154-63\\7d&=91\\\frac{7d}{7}&=\frac{91}{7}\\d&=\mathbf{13}\end{aligned}$$

(j)
$$\begin{aligned}28(f-1)&=5(7f-3)\\28f-28&=35f-15\\28f-28-35f&=35f-15-35f\\-7f-28&=-15\\-7f-28+28&=-15+28\\-7f&=13\\\frac{-7f}{-7}&=\frac{13}{-7}\\f&=\mathbf{-1\frac{6}{7}}\end{aligned}$$

4. **(a)**
$$\begin{aligned}7y-2\frac{3}{4}&=\frac{1}{2}\\7y-2\frac{3}{4}+2\frac{3}{4}&=\frac{1}{2}+2\frac{3}{4}\\7y&=3\frac{1}{4}\\\frac{7y}{7}&=3\frac{1}{4}\div 7\\y&=\mathbf{\frac{13}{28}}\end{aligned}$$

(b)
$$\begin{aligned}1\frac{1}{2}-2y&=\frac{1}{4}\\1\frac{1}{2}-1\frac{1}{2}-2y&=\frac{1}{4}-1\frac{1}{2}\\-2y&=-1\frac{1}{4}\\2y&=1\frac{1}{4}\\\frac{2y}{2}&=1\frac{1}{4}\div 2\\y&=\mathbf{\frac{5}{8}}\end{aligned}$$

(c)
$$\begin{aligned}\frac{1}{3}x&=7\\3\times\frac{1}{3}x&=3\times 7\\x&=\mathbf{21}\end{aligned}$$

(d) $\frac{3}{4}x = -6$

$\frac{4}{3} \times \frac{3}{4}x = \frac{4}{3} \times (-6)$

$x = \mathbf{-8}$

(e) $\frac{1}{3}x + 3 = 4$

$\frac{1}{3}x + 3 - 3 = 4 - 3$

$\frac{1}{3}x = 1$

$3 \times \frac{1}{3}x = 3 \times 1$

$x = \mathbf{3}$

(f) $\frac{y}{4} - 8 = -2$

$\frac{y}{4} - 8 + 8 = -2 + 8$

$\frac{y}{4} = 6$

$4 \times \frac{y}{4} = 4 \times 6$

$y = \mathbf{24}$

(g) $3 - \frac{1}{4}y = 2$

$3 - 3 - \frac{1}{4}y = 2 - 3$

$-\frac{1}{4}y = -1$

$\frac{1}{4}y = 1$

$4 \times \frac{1}{4}y = 4 \times 1$

$y = \mathbf{4}$

(h) $15 - \frac{2}{5}y = 11$

$15 - 15 - \frac{2}{5}y = 11 - 15$

$-\frac{2}{5}y = -4$

$\frac{2}{5}y = 4$

$\frac{5}{2} \times \frac{2}{5}y = \frac{5}{2} \times 4$

$y = \mathbf{10}$

5. **(a)** $y - 2.4 = 3.6$

$y - 2.4 + 2.4 = 3.6 + 2.4$

$y = \mathbf{6}$

(b) $y + 0.4 = 1.6$

$y + 0.4 - 0.4 = 1.6 - 0.4$

$y = \mathbf{1.2}$

(c) $-3y - 7.8 = -9.6$

$-3y - 7.8 + 7.8 = -9.6 + 7.8$

$-3y = -1.8$

$3y = 1.8$

$\frac{3y}{3} = \frac{1.8}{3}$

$y = \mathbf{0.6}$

(d) $4y - 1.9 = 6.3$

$4y - 1.9 + 1.9 = 6.3 + 1.9$

$4y = 8.2$

$\frac{4y}{4} = \frac{8.2}{4}$

$y = \mathbf{2.05}$

(e) $-2.7 + a = -6.4$

$-2.7 + 2.7 + a = -6.4 + 2.7$

$a = \mathbf{-3.7}$

(f) $2(2x - 2.2) = 4.6$

$4x - 4.4 = 4.6$

$4x - 4.4 + 4.4 = 4.6 + 4.4$

$4x = 9.0$

$\frac{4x}{4} = \frac{9.0}{4}$

$x = \mathbf{2.25}$

(g) $4(3y + 4.1) = 7.6$

$12y + 16.4 = 7.6$

$12y + 16.4 - 16.4 = 7.6 - 16.4$

$12y = -8.8$

$\frac{12y}{12} = \frac{-8.8}{12}$

$y = -\mathbf{\frac{11}{15}}$

(h) $3(2 - 0.4x) = 18$

$6 - 1.2x = 18$

$6 - 1.2x - 6 = 18 - 6$

$-1.2x = 12$

$\frac{-1.2x}{-1.2} = \frac{12}{-1.2}$

$x = \mathbf{-10}$

6. **(a)** $x = 12 - \frac{1}{3}x$

$x + \frac{1}{3}x = 12 - \frac{1}{3}x + \frac{1}{3}x$

$\frac{4}{3}x = 12$

$\frac{3}{4} \times \frac{4}{3}x = \frac{3}{4} \times 12$

$x = \mathbf{9}$

(b) $\frac{3}{5}x = \frac{1}{2}x + \frac{1}{2}$

$\frac{3}{5}x - \frac{1}{2}x = \frac{1}{2}x - \frac{1}{2}x + \frac{1}{2}$

$\frac{1}{10}x = \frac{1}{2}$

$10 \times \frac{1}{10}x = 10 \times \frac{1}{2}$

$x = \mathbf{5}$

(c) $\frac{y}{2} - \frac{1}{5} = 2 - \frac{y}{3}$

$\frac{y}{2} + \frac{y}{3} - \frac{1}{5} = 2 - \frac{y}{3} + \frac{y}{3}$

$\frac{5y}{6} - \frac{1}{5} = 2$

$$\frac{5y}{6} - \frac{1}{5} + \frac{1}{5} = 2 + \frac{1}{5}$$
$$\frac{5y}{6} = 2\frac{1}{5}$$
$$\frac{6}{5} \times \frac{5y}{6} = \frac{6}{5} \times 2\frac{1}{5}$$
$$y = \mathbf{2\frac{16}{25}}$$

(d)
$$\frac{2}{3}y - \frac{3}{4} = 2y + \frac{5}{8}$$
$$\frac{2}{3}y - 2y - \frac{3}{4} = 2y - 2y + \frac{5}{8}$$
$$-\frac{4}{3}y - \frac{3}{4} = \frac{5}{8}$$
$$-\frac{4}{3}y - \frac{3}{4} + \frac{3}{4} = \frac{5}{8} + \frac{3}{4}$$
$$-\frac{4}{3}y = 1\frac{3}{8}$$
$$\frac{4}{3}y = -1\frac{3}{8}$$
$$\frac{3}{4} \times \frac{4}{3}y = \frac{3}{4} \times \left(-1\frac{3}{8}\right)$$
$$y = \mathbf{-1}\,{}^{-}\tfrac{\mathbf{33}}{\mathbf{32}}$$

7. **(a)**
$$\frac{2}{x} = \frac{4}{5}$$
$$5x \times \frac{2}{x} = 5x \times \frac{4}{5}$$
$$10 = 4x$$
$$4x = 10$$
$$\frac{4x}{4} = \frac{10}{4}$$
$$x = \mathbf{2\frac{1}{2}}$$

(b)
$$\frac{12}{y-1} = \frac{2}{3}$$
$$3(y-1) \times \frac{12}{y-1} = 3(y-1) \times \frac{2}{3}$$
$$36 = 2(y-1)$$
$$36 = 2y - 2$$
$$2y - 2 = 36$$
$$2y - 2 + 2 = 36 + 2$$
$$2y = 38$$
$$\frac{2y}{2} = \frac{38}{2}$$
$$y = \mathbf{19}$$

8. **(a)**
$$-3(2 - x) = 6x$$
$$-6 + 3x = 6x$$
$$-6 + 3x - 6x = 6x - 6x$$
$$-6 - 3x = 0$$
$$-6 + 6 - 3x = 0 + 6$$
$$-3x = 6$$
$$3x = -6$$
$$\frac{3x}{3} = \frac{-6}{3}$$
$$x = \mathbf{-2}$$

(b)
$$5 - 3x = -6(x + 2)$$
$$5 - 3x = -6x - 12$$
$$5 - 3x + 6x = -6x + 6x - 12$$
$$5 + 3x = -12$$
$$5 - 5 + 3x = -12 - 5$$
$$3x = -17$$
$$\frac{3x}{3} = \frac{-17}{3}$$
$$x = \mathbf{-5\frac{2}{3}}$$

(c)
$$-3(9y + 2) = 2(-4y - 7)$$
$$-27y - 6 = -8y - 14$$
$$-27y + 8y - 6 = -8y + 8y - 14$$
$$-19y - 6 = -14$$
$$-19y - 6 + 6 = -14 + 6$$
$$-19y = -8$$
$$19y = 8$$
$$\frac{19y}{19} = \frac{8}{19}$$
$$y = \mathbf{\frac{8}{19}}$$

(d)
$$-3(4y - 5) = -7(-5 - 2y)$$
$$-12y + 15 = 35 + 14y$$
$$-12y - 14y + 15 = 35 + 14y - 14y$$
$$-26y + 15 = 35$$
$$-26y + 15 - 15 = 35 - 15$$
$$-26y = 20$$
$$26y = -20$$
$$\frac{26y}{26} = \frac{-20}{26}$$
$$y = -\mathbf{\frac{10}{13}}$$

(e)
$$3(5 - h) - 2(h - 2) = -1$$
$$15 - 3h - 2h + 4 = -1$$
$$15 + 4 - 3h - 2h = -1$$
$$19 - 5h = -1$$
$$19 - 19 - 5h = -1 - 19$$
$$-5h = -20$$
$$5h = 20$$
$$\frac{5h}{5} = \frac{20}{5}$$
$$h = \mathbf{4}$$

9. **(a)**
$$10x - \frac{5x + 4}{3} = 7$$
$$\frac{3(10x) - (5x + 4)}{3} = 7$$
$$\frac{30x - 5x - 4}{3} = 7$$
$$\frac{25x - 4}{3} = 7$$
$$3 \times \frac{25x - 4}{3} = 3 \times 7$$
$$25x - 4 = 21$$
$$25x - 4 + 4 = 21 + 4$$
$$25x = 25$$
$$\frac{25x}{25} = \frac{25}{25}$$
$$x = \mathbf{1}$$

(b)
$$\begin{aligned}
\frac{4x}{3} - \frac{x-1}{2} &= 1\frac{1}{4}\\
\frac{2(4x)-3(x-1)}{6} &= \frac{5}{4}\\
\frac{8x-3x+3}{6} &= \frac{5}{4}\\
\frac{5x+3}{6} &= \frac{5}{4}\\
12 \times \frac{5x+3}{6} &= 12 \times \frac{5}{4}\\
2(5x+3) &= 15\\
10x+6 &= 15\\
10x+6-6 &= 15-6\\
10x &= 9\\
\frac{10x}{10} &= \frac{9}{10}\\
x &= \mathbf{\frac{9}{10}}
\end{aligned}$$

(c)
$$\begin{aligned}
\frac{x-1}{3} - \frac{x+3}{4} &= -1\\
\frac{4(x-1)-3(x+3)}{12} &= -1\\
\frac{4x-4-3x-9}{12} &= -1\\
\frac{4x-3x-4-9}{12} &= -1\\
\frac{x-13}{12} &= -1\\
12 \times \frac{x-13}{12} &= 12 \times (-1)\\
x-13 &= -12\\
x-13+13 &= -12+13\\
x &= \mathbf{1}
\end{aligned}$$

(d)
$$\begin{aligned}
1 - \frac{y+5}{3} &= \frac{3(y-1)}{4}\\
\frac{3-(y+5)}{3} &= \frac{3(y-1)}{4}\\
\frac{3-y-5}{3} &= \frac{3(y-1)}{4}\\
\frac{-y+3-5}{3} &= \frac{3(y-1)}{4}\\
\frac{-y-2}{3} &= \frac{3(y-1)}{4}\\
12 \times \frac{-y-2}{3} &= 12 \times \frac{3(y-1)}{4}\\
4(-y-2) &= 9(y-1)\\
-4y-8 &= 9y-9\\
-4y-9y-8 &= 9y-9y-9\\
-13y-8 &= -9\\
-13y-8+8 &= -9+8\\
-13y &= -1\\
13y &= 1\\
\frac{13y}{13} &= \frac{1}{13}\\
y &= \mathbf{\frac{1}{13}}
\end{aligned}$$

(e)
$$\begin{aligned}
\frac{6(y-2)}{7} - 12 &= \frac{2(y-7)}{3}\\
\frac{6(y-2)-84}{7} &= \frac{2(y-7)}{3}\\
\frac{6y-12-84}{7} &= \frac{2(y-7)}{3}\\
\frac{6y-96}{7} &= \frac{2(y-7)}{3}\\
21 \times \frac{6y-96}{7} &= 21 \times \frac{2(y-7)}{3}\\
3(6y-96) &= 14(y-7)\\
18y-288 &= 14y-98\\
18y-14y-288 &= 14y-14y-98\\
4y-288 &= -98\\
4y-288+288 &= -98+288\\
4y &= 190\\
\frac{4y}{4} &= \frac{190}{4}\\
y &= \mathbf{47\frac{1}{2}}
\end{aligned}$$

(f)
$$\begin{aligned}
\frac{7-2y}{2} - \frac{2}{5}(2-y) &= 1\frac{1}{4}\\
\frac{5(7-2y)-4(2-y)}{10} &= \frac{5}{4}\\
\frac{35-10y-8+4y}{10} &= \frac{5}{4}\\
\frac{27-6y}{10} &= \frac{5}{4}\\
20 \times \frac{27-6y}{10} &= 20 \times \frac{5}{4}\\
2(27-6y) &= 25\\
54-12y &= 25\\
54-12y-54 &= 25-54\\
-12y &= -29\\
12y &= 29\\
\frac{12y}{12} &= \frac{29}{12}\\
y &= \mathbf{2\frac{5}{12}}
\end{aligned}$$

10. (a)
$$\begin{aligned}
\frac{5x+1}{3} &= 7\\
3 \times \frac{5x+1}{3} &= 3 \times 7\\
5x+1 &= 21\\
5x+1-1 &= 21-1\\
5x &= 20\\
\frac{5x}{5} &= \frac{20}{5}\\
x &= \mathbf{4}
\end{aligned}$$

(b)
$$\begin{aligned}
\frac{2x-3}{4} &= \frac{x-3}{3}\\
12\times\frac{2x-3}{4} &= 12\times\frac{x-3}{3}\\
3(2x-3) &= 4(x-3)\\
6x-9 &= 4x-12\\
6x-4x-9 &= 4x-4x-12\\
2x-9 &= -12\\
2x-9+9 &= -12+9\\
2x &= -3\\
\frac{2x}{2} &= \frac{-3}{2}\\
x &= \mathbf{-1\frac{1}{2}}
\end{aligned}$$

(c)
$$\begin{aligned}
\frac{3x-1}{5} &= \frac{x-1}{3}\\
15\times\frac{3x-1}{5} &= 15\times\frac{x-1}{3}\\
3(3x-1) &= 5(x-1)\\
9x-3 &= 5x-5\\
9x-5x-3 &= 5x-5x-5\\
4x-3 &= -5\\
4x-3+3 &= -5+3\\
4x &= -2\\
\frac{4x}{4} &= \frac{-2}{4}\\
x &= \mathbf{-\frac{1}{2}}
\end{aligned}$$

(d)
$$\begin{aligned}
\frac{1}{4}(5y+4) &= \frac{1}{3}(2y-1)\\
12\times\frac{1}{4}(5y+4) &= 12\times\frac{1}{3}(2y-1)\\
3(5y+4) &= 4(2y-1)\\
15y+12 &= 8y-4\\
15y-8y+12 &= 8y-8y-4\\
7y+12 &= -4\\
7y+12-12 &= -4-12\\
7y &= -16\\
\frac{7y}{7} &= \frac{-16}{7}\\
y &= \mathbf{-2\frac{2}{7}}
\end{aligned}$$

(e)
$$\begin{aligned}
\frac{2y-1}{5}-\frac{y+3}{7} &= 0\\
\frac{2y-1}{5}-\frac{y+3}{7}+\frac{y+3}{7} &= 0+\frac{y+3}{7}\\
\frac{2y-1}{5} &= \frac{y+3}{7}\\
35\times\frac{2y-1}{5} &= 35\times\frac{y+3}{7}\\
7(2y-1) &= 5(y+3)\\
14y-7 &= 5y+15\\
14y-5y-7 &= 5y-5y+15\\
9y-7 &= 15\\
9y-7+7 &= 15+7\\
9y &= 22\\
\frac{9y}{9} &= \frac{22}{9}\\
y &= \mathbf{2\frac{4}{9}}
\end{aligned}$$

(f)
$$\begin{aligned}
\frac{2y+3}{4}+\frac{y-5}{6} &= 0\\
\frac{2y+3}{4}+\frac{y-5}{6}-\frac{y-5}{6} &= 0-\frac{y-5}{6}\\
\frac{2y+3}{4} &= -\frac{y-5}{6}\\
\frac{2y+3}{4} &= \frac{5-y}{6}\\
12\times\frac{2y+3}{4} &= 12\times\frac{5-y}{6}\\
3(2y+3) &= 2(5-y)\\
6y+9 &= 10-2y\\
6y+2y+9 &= 10-2y+2y\\
8y+9 &= 10\\
8y+9-9 &= 10-9\\
8y &= 1\\
\frac{8y}{8} &= \frac{1}{8}\\
y &= \mathbf{\frac{1}{8}}
\end{aligned}$$

11. When $x = \frac{19}{20}$,

$$\begin{aligned}
\text{LHS} &= 2\left(\frac{19}{20}\right)-\frac{3}{4}\\
&= 1\frac{9}{10}-\frac{3}{4}\\
&= 1\frac{3}{20}\\
\text{RHS} &= \frac{1}{3}\left(\frac{19}{20}\right)+\frac{5}{6}\\
&= \frac{19}{60}+\frac{5}{6}\\
&= 1\frac{3}{20} = \text{LHS}
\end{aligned}$$

$\therefore\ x = \frac{19}{20}$ is the solution of the equation

$2x-\frac{3}{4} = \frac{1}{3}x+\frac{5}{6}$.

12. (a)
$$\begin{aligned}
\frac{12}{x+3} &= 2\\
(x+3)\times\frac{12}{x+3} &= (x+3)\times 2\\
12 &= 2(x+3)\\
12 &= 2x+6\\
2x+6 &= 12\\
2x+6-6 &= 12-6\\
2x &= 6\\
\frac{2x}{2} &= \frac{6}{2}\\
x &= \mathbf{3}
\end{aligned}$$

(b)
$$\begin{aligned}
\frac{11}{2x-1} &= 4\\
(2x-1)\times\frac{11}{2x-1} &= (2x-1)\times 4\\
11 &= 4(2x-1)\\
11 &= 8x-4\\
8x-4 &= 11\\
8x-4+4 &= 11+4\\
8x &= 15\\
\frac{8x}{8} &= \frac{15}{8}\\
x &= \mathbf{1\frac{7}{8}}
\end{aligned}$$

(c)
$$\begin{aligned}
\frac{32}{2x-5}-3 &= \frac{1}{4}\\
\frac{32}{2x-5}-3+3 &= \frac{1}{4}+3\\
\frac{32}{2x-5} &= \frac{13}{4}\\
4(2x-5)\times\frac{32}{2x-5} &= 4(2x-5)\times\frac{13}{4}\\
128 &= 13(2x-5)\\
128 &= 26x-65\\
26x-65 &= 128\\
26x-65+65 &= 128+65\\
26x &= 193\\
\frac{26x}{26} &= \frac{193}{26}\\
x &= \mathbf{7\frac{11}{26}}
\end{aligned}$$

(d)
$$\begin{aligned}
\frac{1}{2} &= \frac{1}{x+2}-1\\
\frac{1}{x+2}-1 &= \frac{1}{2}\\
\frac{1}{x+2}-1+1 &= \frac{1}{2}+1\\
\frac{1}{x+2} &= \frac{3}{2}\\
2(x+2)\times\frac{1}{x+2} &= 2(x+2)\times\frac{3}{2}\\
2 &= 3(x+2)\\
2 &= 3x+6\\
3x+6 &= 2\\
3x+6-6 &= 2-6\\
3x &= -4\\
\frac{3x}{3} &= \frac{-4}{3}\\
x &= \mathbf{-1\frac{1}{3}}
\end{aligned}$$

(e)
$$\begin{aligned}
\frac{y+5}{y-6} &= \frac{5}{4}\\
4(y-6)\times\frac{y+5}{y-6} &= 4(y-6)\times\frac{5}{4}\\
4(y+5) &= 5(y-6)\\
4y+20 &= 5y-30\\
4y-5y+20 &= 5y-5y-30\\
-y+20 &= -30\\
-y+20-20 &= -30-20\\
-y &= -50\\
y &= \mathbf{50}
\end{aligned}$$

(f)
$$\begin{aligned}
\frac{2y+1}{3y-5} &= \frac{4}{7}\\
7(3y-5)\times\frac{2y+1}{3y-5} &= 7(3y-5)\times\frac{4}{7}\\
7(2y+1) &= 4(3y-5)\\
14y+7 &= 12y-20\\
14y-12y+7 &= 12y-12y-20\\
2y+7 &= -20\\
2y+7-7 &= -20-7\\
2y &= -27\\
\frac{2y}{2} &= \frac{-27}{2}\\
y &= \mathbf{-13\frac{1}{2}}
\end{aligned}$$

(g)
$$\begin{aligned}
\frac{2}{y-2} &= \frac{3}{y+6}\\
(y-2)(y+6)\times\frac{2}{y-2} &= (y-2)(y+6)\times\frac{3}{y+6}\\
2(y+6) &= 3(y-2)\\
2y+12 &= 3y-6\\
2y-3y+12 &= 3y-3y-6\\
-y+12 &= -6\\
-y+12-12 &= -6-12\\
-y &= -18\\
y &= \mathbf{18}
\end{aligned}$$

(h)
$$\begin{aligned}
\frac{2}{7y-3} &= \frac{3}{9y-5}\\
(7y-3)(9y-5)\times\frac{2}{7y-3} &= (7y-3)(9y-5)\times\frac{3}{9y-5}\\
2(9y-5) &= 3(7y-3)\\
18y-10 &= 21y-9\\
18y-21y-10 &= 21y-21y-9\\
-3y-10 &= -9\\
-3y-10+10 &= -9+10\\
-3y &= 1\\
3y &= -1\\
\frac{3y}{3} &= \frac{-1}{3}\\
y &= \mathbf{-\frac{1}{3}}
\end{aligned}$$

13.
$$\begin{aligned}
4x+y &= 3x+5y\\
4x-3x+y &= 3x-3x+5y\\
x+y &= 5y\\
x+y-y &= 5y-y\\
x &= 4y\\
\frac{3}{16y}\times x &= \frac{3}{16y}\times 4y\\
\therefore\ \frac{3x}{16y} &= \mathbf{\frac{3}{4}}
\end{aligned}$$

14.
$$\begin{aligned}
\frac{3x-5y}{7x-4y} &= \frac{3}{4}\\
4(7x-4y)\times\frac{3x-5y}{7x-4y} &= 4(7x-4y)\times\frac{3}{4}\\
4(3x-5y) &= 3(7x-4y)\\
12x-20y &= 21x-12y\\
12x-21x-20y &= 21x-21x-12y\\
-9x-20y &= -12y
\end{aligned}$$

$$-9x - 20y + 20y = -12y + 20y$$
$$-9x = 8y$$
$$9x = -8y$$
$$\frac{9x}{9} = \frac{-8y}{9}$$
$$x = -\frac{8y}{9}$$
$$\frac{1}{y} \times x = \frac{1}{y} \times \left(-\frac{8y}{9}\right)$$
$$\therefore \frac{x}{y} = -\mathbf{\frac{8}{9}}$$

7.3 Applications of linear equations in real-world contexts

Practise Now 4

1. Let the smaller number be x.
   Then the larger number is $5x$.
   $$x + 5x = 24$$
   $$6x = 24$$
   $$x = \frac{24}{6}$$
   $$= 4$$
   $\therefore$ the two numbers are **4** and **20**.
2. Let the number of marks Cheryl obtains be x.
   Then the number of marks Shaha obtains is $x + 15$.
   $$x + 15 = 2x$$
   $$x - 2x = -15$$
   $$-x = -15$$
   $$x = 15$$
   $\therefore$ Cheryl obtains **15** marks.

Introductory Problem Revisited

Let the number of cousins Ken has be x.
$$6x + 5 = 7x - 4$$
$$5 + 4 = 7x - 6x$$
$$x = 9$$
$\therefore$ Ken has **9** cousins.

Practise Now 5

Let the number be x.
$$\frac{1}{5}x + 3\frac{7}{10} = 7$$
$$\frac{1}{5}x = 7 - 3\frac{7}{10}$$
$$\frac{1}{5}x = 3\frac{3}{10}$$
$$x = 5 \times 3\frac{3}{10}$$
$$= 16\frac{1}{2}$$
$\therefore$ the number is $\mathbf{16\frac{1}{2}}$.

Exercise 7B

1. Let the mass of the empty lorry be x kg.
   Then the mass of the bricks is $3x$ kg.
   $$x + 3x = 11\,600$$
   $$4x = 11\,600$$
   $$x = \frac{11\,600}{4}$$
   $$= 2900$$
   $\therefore$ the mass of the bricks is $3(2900)$ = **8700 kg**.
2. Let the smallest odd number be n.
   The next odd number will be $n + 2$.
   Then the next odd number will be $(n + 2) + 2 = n + 4$.
   The greatest odd number will be $(n + 4) + 2 = n + 6$.
   $$n + (n + 2) + (n + 4) + (n + 6) = 56$$
   $$n + n + n + n + 2 + 4 + 6 = 56$$
   $$4n + 12 = 56$$
   $$4n = 56 - 12$$
   $$4n = 44$$
   $$n = \frac{44}{4}$$
   $$= 11$$
   $\therefore$ the greatest of the 4 numbers is $11 + 6$ = **17**.
3. Let Sara's age be x years old.
   Then David's age is $(x + 4)$ years,
   Yasir's age is $(x - 2)$ years.
   $$x + (x + 4) + (x - 2) = 47$$
   $$x + x + x + 4 - 2 = 47$$
   $$3x + 2 = 47$$
   $$3x = 47 - 2$$
   $$3x = 45$$
   $$x = \frac{45}{3}$$
   $$= 15$$
   $\therefore$ Sara is **15 years old**, David is $15 + 4$ = **19 years old** and Yasir is $15 - 2$ = **13 years old**.
4. Let the number be x.
   $$3x = x + 28$$
   $$3x - x = 28$$
   $$2x = 28$$
   $$x = \frac{28}{2}$$
   $$= 14$$
   $\therefore$ the number is **14**.
5. Let the number of people going on the holiday be x.
   $$15x = 84 + 12x$$
   $$15x - 12x = 84$$
   $$3x = 84$$
   $$x = \frac{84}{3}$$
   $$= 28$$
   $\therefore$ there are **28** people going on the holiday.

6. Let the greater number be x.

Then the smaller number is $\frac{2}{3}x$.

$$x + \frac{2}{3}x = 45$$
$$\frac{5}{3}x = 45$$
$$x = \frac{3}{5} \times 45$$
$$= 27$$

$\therefore$ the smaller number is $\frac{2}{3}(27) = \mathbf{18}$.

7. Let the number be x.

$$68 - 4x = 3(x + 4)$$
$$68 - 4x = 3x + 12$$
$$-4x - 3x = 12 - 68$$
$$-7x = -56$$
$$x = \frac{-56}{-7}$$
$$= 8$$

$\therefore$ the number is **8**.

8. Let the number of boys who play badminton be x.

Then the number of boys who play soccer is $3x$.

$$3x - 12 = x + 12$$
$$3x - x = 12 + 12$$
$$2x = 24$$
$$x = \frac{24}{4}$$
$$= 12$$

$\therefore$ there are **12** boys who play badminton.

9. Let the son's age be x years.

Then the man's age is $6x$ years.

$$6x + 20 = 2(x + 20)$$
$$6x + 20 = 2x + 40$$
$$6x - 2x = 40 - 20$$
$$4x = 20$$
$$x = \frac{20}{4}$$
$$= 5$$

$\therefore$ the man was $6(5) - 5 =$ **25 years old** when his son was born.

10. Let the cost of a butter cake be $\$x$.

Then the cost of a chocolate cake is $\$(x + 2)$.

$$6(x + 2) + 5x = 130.8$$
$$6x + 12 + 5x = 130.8$$
$$6x + 5x = 130.8 - 12$$
$$11x = 118.8$$
$$x = \frac{118.8}{11}$$
$$= 10.8$$

$\therefore$ the cost of a chocolate cake is $\$(10.8 + 2) =$ **\$12.80**.

11. Let the number of 20-cent coins Albert has be x.

Then the number of 10-cent coins he has is $x + 12$.

$$10(x + 12) + 20x = 540$$
$$10x + 120 + 20x = 540$$
$$10x + 20x = 540 - 120$$
$$30x = 420$$
$$x = \frac{420}{30}$$
$$= 14$$

$\therefore$ Albert has $14 + (14 + 12) =$ **40** coins.

12. Let Li Ting's average speed for the first part of her journey be x km/h.

Then her average speed for the second part of her journey is $(x - 15)$ km/h.

Time taken for first part of journey $= \frac{350}{x}$ hours.

Time taken for second part of journey $= \frac{470 - 350}{x - 15}$

$= \frac{120}{x - 15}$ hours.

$$\frac{350}{x} = \frac{120}{x - 15}$$
$$350(x - 15) = 120x$$
$$350x - 5250 = 120x$$
$$350x - 120x = 5250$$
$$230x = 5250$$
$$x = \frac{5250}{230}$$
$$= 22\frac{19}{23}$$

$\therefore$ Li Ting's average speed for the second part of her journey is $22\frac{19}{23} - 15 =$ **7.83 km/h** (to 3 s.f.).

13. Let the number be x.

$$\frac{1}{2}x + 49 = \frac{9}{4}x$$
$$\frac{1}{2}x - \frac{9}{4}x = -49$$
$$-\frac{7}{4}x = -49$$
$$x = -\frac{4}{7} \times (-49)$$
$$= 28$$

$\therefore$ the number is **28**.

14. Let the denominator of the fraction be x.

Then the numerator of the fraction is $x - 5$.

$\therefore$ the fraction is $\frac{x-5}{x}$.

$$\frac{x - 5 + 1}{x + 1} = \frac{2}{3}$$
$$\frac{x - 4}{x + 1} = \frac{2}{3}$$
$$3(x - 4) = 2(x + 1)$$
$$3x - 12 = 2x + 2$$
$$3x - 2x = 2 + 12$$
$$x = 14$$

$\therefore$ the fraction is $\frac{14 - 5}{14} = \mathbf{\frac{9}{14}}$.

15. Let the number in the tens place be x.

Then the number in the ones place is $2.5x$.

$\therefore$ the number is $10x + 2.5x = 12.5x$.

$\therefore$ the number obtained when the digits are reversed is

$10(2.5x) + x = 25x + x = 26x$.

$$26x - 12.5x = 27$$
$$13.5x = 27$$
$$x = \frac{27}{13.5}$$
$$= 2$$

The number is $12.5(2) =$ **25**.

7.4 Mathematical formulae

Practise Now 6

1. $F = ma$

 (a) When $m = 1000$, $a = 0.05$,
 $F = 1000(0.05)$
 $= 50$ N
 Net force acting on body = **50 N**

 (b) When $F = 100$, $a = 0.1$,
 $100 = m(0.1)$
 $\therefore m = \dfrac{100}{0.1}$
 $= 1000$ kg
 Mass of body = **1000 kg**

2. $y + b = \dfrac{ay + c}{b}$

 When $y = 12$, $b = 3$, $a = 14$, $12 + 3 = \dfrac{14(12) + c}{3}$
 $15 = \dfrac{168 + c}{3}$
 $45 = 168 + c$
 $c = \mathbf{-123}$

Practise Now 7

(i) Let the smallest even number be n.
Then the other three consecutive even numbers will be $n + 2$, $n + 4$ and $n + 6$.
$\therefore$ sum of the three consecutive even numbers,
$S = n + (n + 2) + (n + 4) + (n + 6)$
$= \mathbf{4n + 12}$, where n is even

(ii) When $n = 14$,
$\therefore S = 4n + 12$
$= 4 \times (14) + 12$
$= \mathbf{68}$

Exercise 7C

1. $y = \dfrac{3}{5}x + 26$

 When $x = 12$,
 $y = \dfrac{3}{5}(12) + 26$
 $= \mathbf{33\frac{1}{5}}$

2. $S = 4\pi r^2$

 (i) When $r = 10\frac{1}{2}$,
 $S = 4\left(\dfrac{22}{7}\right)\left(10\dfrac{1}{2}\right)^2$
 $= \mathbf{1386}$

 (ii) When $S = 616$,
 $616 = 4\left(\dfrac{22}{7}\right)r^2$
 $616 = \dfrac{88}{7}r^2$
 $\dfrac{88}{7}r^2 = 616$
 $r^2 = \dfrac{7}{88} \times 616$
 $r^2 = 49$
 $\therefore r = \pm\sqrt{49}$
 $= \pm 7$
 $= \mathbf{7}$ or -7 (N.A. since $r > 0$)

3. $a = \dfrac{y^2 - xz}{5}$

 When $x = 2$, $y = -1$, $z = -3$,
 $a = \dfrac{(-1)^2 - 2(-3)}{5}$
 $= \dfrac{1 + 6}{5}$
 $= \dfrac{7}{5}$
 $= \mathbf{1\frac{2}{5}}$

4. $k = \dfrac{p + 2q}{3}$

 When $k = 7$, $q = 9$,
 $7 = \dfrac{p + 2(9)}{3}$
 $7 = \dfrac{p + 18}{3}$
 $3 \times 7 = p + 18$
 $21 = p + 18$
 $p + 18 = 21$
 $\therefore p = 21 - 18$
 $= \mathbf{3}$

5. **(a)** $\boldsymbol{P = xyz}$
 (b) $\boldsymbol{S = p^2 + q^3}$
 (c) $\boldsymbol{A = \dfrac{m + n + p + q}{4}}$
 (d) $\boldsymbol{T = 60a + b}$

6. $U = \pi(r + h)$

 When $U = 16\frac{1}{2}$, $h = 2$,
 $16\dfrac{1}{2} = \dfrac{22}{7}(r + 2)$
 $\dfrac{22}{7}(r + 2) = 16\dfrac{1}{2}$
 $r + 2 = \dfrac{7}{22} \times 16\dfrac{1}{2}$
 $r + 2 = 5\dfrac{1}{4}$
 $\therefore r = 5\dfrac{1}{4} - 2$
 $= \mathbf{3\frac{1}{4}}$

7. $v^2 = u^2 + 2gs$

When $v = 25$, $u = 12$, $g = 10$,

$$
\begin{aligned}
25^2 &= 12^2 + 2(10)s \\
625 &= 144 + 20s \\
144 + 20s &= 625 \\
20s &= 625 - 144 \\
20s &= 481 \\
\therefore s &= \frac{481}{20} \\
&= \mathbf{24\frac{1}{20}}
\end{aligned}
$$

8. $\frac{1}{a} - \frac{1}{b} = \frac{1}{c} + \frac{1}{d}$

When $a = \frac{1}{2}$, $b = \frac{1}{4}$, $d = -\frac{1}{5}$,

$$
\begin{aligned}
\frac{1}{\frac{1}{2}} - \frac{1}{\frac{1}{4}} &= \frac{1}{c} + \frac{1}{\left(\frac{-1}{5}\right)} \\
2 - 4 &= \frac{1}{c} - 5 \\
-2 &= \frac{1}{c} - 5 \\
-2 + 5 &= \frac{1}{c} \\
3 &= \frac{1}{c} \\
3c &= 1 \\
c &= \mathbf{\frac{1}{3}}
\end{aligned}
$$

9. $c = \frac{a}{b} - \frac{d - e}{f - d}$

When $a = 3$, $b = 4$, $c = -6$, $d = -5$ and $e = 2$,

$$
\begin{aligned}
-6 &= \frac{3}{4} - \frac{-5 - 2}{f - (-5)} \\
-6 &= \frac{3}{4} - \frac{-7}{f + 5} \\
-6 &= \frac{3}{4} + \frac{7}{f + 5} \\
-6 - \frac{3}{4} &= \frac{7}{f + 5} \\
-6\frac{3}{4} &= \frac{7}{f + 5} \\
-\frac{27}{4} &= \frac{7}{f + 5} \\
-27(f + 5) &= 4 \times 7 \\
-27(f + 5) &= 28 \\
-27f - 135 &= 28 \\
-27f &= 28 + 135 \\
-27f &= 163 \\
\therefore f &= \frac{163}{-27} \\
&= \mathbf{-6\frac{1}{27}}
\end{aligned}
$$

10. $\frac{m(nx^2 - y)}{z} = 5n$

When $m = 6$, $x = -2$, $y = -3$, $z = -5$,

$$
\begin{aligned}
\frac{6[n(-2)^2 - (-3)]}{-5} &= 5n \\
\frac{6(4n + 3)}{-5} &= 5n \\
6(4n + 3) &= -25n \\
24n + 18 &= -25n \\
24n + 25n &= -18 \\
49n &= -18 \\
n &= \mathbf{-\frac{18}{49}}
\end{aligned}
$$

11. **(i)** Let the smallest odd number be n.

Then the other two consecutive odd numbers will be $n + 2$, $n + 4$ and $n + 6$.

$\therefore$ sum of the four consecutive odd numbers,

$S = n + (n + 2) + (n + 4) + (n + 6)$

$\mathbf{S = 4n + 12}$, where n is odd

(ii) If the greatest odd number is -17, then $n + 6 = -17$

$$
\begin{aligned}
n &= -17 - 6 \\
&= -23 \\
\therefore S &= 4n + 12 \\
&= 4 \times (-23) + 12 \\
&= \mathbf{-80}
\end{aligned}
$$

12. **(i)** Let the youngest child be n years old.

Then the ages of the three other children will be $n + 2$, $n + 4$, $n + 6$ and $n + 8$.

$\therefore$ sum of total ages of Joyce and her siblings,

$S = n + n + 2 + n + 4 + n + 6 + n + 8$

$\mathbf{S = 5n + 20}$

(ii) If the youngest child is now 4 years old, then $n = 4$.

$$
\begin{aligned}
\therefore S &= 5n + 20 \\
S &= 5 \times 4 + 20 \\
&= 40
\end{aligned}
$$

$\therefore$ sum of their total ages at the end of the following year

$= 40 + 5$

$= \mathbf{45}$

13. **(i)** $T = c \times d + e \times \frac{f}{100}$

$\mathbf{T = cd + \frac{ef}{100}}$

(ii) $e = \frac{-145c}{4 - c}$

When $e = 150$,

$$
\begin{aligned}
150 &= \frac{-145c}{4 - c} \\
150(4 - c) &= -145c \\
600 - 150c &= -145c \\
-150c + 145c &= -600 \\
-5c &= -600 \\
c &= \frac{-600}{-5} \\
&= 120
\end{aligned}
$$

$d = \frac{f + 5}{50}$

When $d = 3$,

$$3 = \frac{f+5}{50}$$

$$50 \times 3 = f + 5$$

$$150 = f + 5$$

$$f + 5 = 150$$

$$f = 150 - 5$$

$$= 145$$

$$T = 120(3) + \frac{150(145)}{100}$$

$$= \mathbf{577.50}$$

14. $y = (x - 32) \times \frac{5}{9}$

(i) When $x = 134$,

$$y = (134 - 32) \times \frac{5}{9}$$

$$= 56.7 \text{ (to 3 s.f.)}$$

Required temperature = **56.7 °C**

(ii) When $x = 0$,

$$y = (0 - 32) \times \frac{5}{9}$$

$$= -17.8 \text{ (to 3 s.f.)}$$

Since 0 °F = –17.8 °C, it is **less common** for the temperature to fall below 0 °F because 0 °F is much lower than 0 °C.

(iii) When $y = -62.1$,

$$-62.1 = (x - 32) \times \frac{5}{9}$$

$$(x - 32) \times \frac{5}{9} = -62.1$$

$$x - 32 = -62.1 \times \frac{9}{5}$$

$$x - 32 = -111.78$$

$$\therefore x = -111.78 + 32$$

$$= -79.78$$

$$= -79.8 \text{ (to 3 s.f.)}$$

Required temperature = **–79.8 °F**

Chapter 8 Percentage

TEACHING NOTES

Suggested Approach

Although students have learnt percentage in primary school (i.e. how to express a part of a whole as a percentage, write fractions and decimals as percentages, and vice versa, find a percentage part of a whole and solve up to 2-step word problems involving percentage), many may still struggle with percentage. Teachers can introduce percentage as fractions by going right back to the fundamentals. Teachers can give students practical applications of percentages and show the changes in fractions and proportions through the examples to give them a better understanding of the concept.

Section 8.1: Percentage

Teachers can get students to work in pairs to interpret the various percentages found in the advertisement in the class discussion on page 181. Some guiding questions and other notes relating to this class discussion can be found in the worked solutions section on the next page. After the discussion, students should understand the meaning of percentage(s) better and interpret information more accurately. Students need to be able to comment critically on the usefulness of percentages before they can have a confident grasp of the topic.

Teachers can then build upon what students have learnt about percentage in primary school. Students may be able to learn how to accurately calculate a percentage but they might struggle to explain the meaning behind it. Teachers should emphasise on the basics of fractions and proportions before getting the students to calculate and interpret percentages. It is highly recommended that teachers use the Thinking Time activity on page 183 to emphasise the difference between 100 and 100%. This is a concept that some learners may still find challenging.

The Class Discussion activity on page 183 reinforces the concept of percentage as part of a whole. Students will then learn to express a quantity as a percentage of another quantity (see Class Discussion: Expressing two quantities in equivalent forms).

In Worked Example 8, students should learn that it is easy to see that more students from School A participated but it is not easy to see which school had a higher proportion of students participating in the walkathon. Teachers can highlight to the students that two quantities can be easily compared using percentages because the proportions are converted to the same base i.e. 100.

Section 8.2 Percentage change, percentage point and reverse percentage

Teachers should guide students on how to use algebra in percentage change and reverse percentage. Students may draw models, wherever applicable, to help them understand the problem. Since this section explores percentage change, teachers can highlight the use of the term 'percentage point' when referring to a change in percentages (see Investigation: Percentage point).

Through the worked examples in this section, students should be able to tackle percentage change and reverse percentage problems involving algebra. They should also learn how to identify whether the problem is a reverse percentage or a percentage change problem. Teachers can highlight to the students that percentage change is when they are given both the new value and the original value while a reverse percentage is when they need to find the original value given a quantity after a percentage increase or decrease. Teachers should stress to the students not to add or take the average of percentages when dealing with problems involving more than one percentage (see Investigation: Can we add percentages or take average of percentages?)

Introductory Problem

The solutions to this problem can be found in Introductory Problem Revisited (after Practise Now 7).

8.1 Percentage

Class Discussion (Identifying percentages used in daily life)

(i) Guiding Questions:

- What is the advertisement about?
- Are the percentages found in the advertisement expressed using the percentage symbol or in words?
- What do the percentages mean in the context of the advertisement?

Teachers may use this to assess students' prior knowledge of percentage, e.g. whether students are able to relate percentages to fractions and to perform relevant calculations using the given percentages to illustrate the meaning of the percentages in the context of the advertisement. Teachers may also use this as a trigger to show students the need to learn percentage, and link back to the different scenarios in the advertisement.

The percentages found in the advertisement are 75% and 160%, which are expressed using the percentage symbol. In the context of the advertisement, it means that the number of grapefruits produced in Florida amounts to up to three-quarters of the world's production, and the amount of vitamin C in one grapefruit is 1.6 times that of the recommended daily intake.

(ii) No. The term 'up to' in the phrase 'up to 75% of the world's supply of grapefruit' suggests that the greatest percentage of grapefruit produced in Florida is 75%. This means that the actual number produced may be less than 75% of that in the world.

Teachers may wish to ask students to list other instances where such phrases are used. (For example, a discount of up to 50% in a sale.) They may also want to take this opportunity to highlight to students the importance of being informed consumers. Students should not take information that appears on advertisements/articles at face value. Instead, they should learn how to interpret information accurately.

(iii) Yes. The phrase means that one grapefruit delivers 1.6 times more vitamin C than the recommended daily intake.

Practise Now 1A

1. (a) $76\% = \frac{76}{100}$

$= \mathbf{\frac{19}{25}}$

$76\% = \frac{76}{100}$

$= \mathbf{0.76}$

(b) $9\% = \mathbf{\frac{9}{100}}$

$9\% = \frac{9}{100}$

$= \mathbf{0.09}$

2. (a) $\frac{13}{20} = \frac{13}{20} \times 100\%$

$= \mathbf{65\%}$

(b) $0.25 = 0.25 \times 100\%$

$= \mathbf{25\%}$

Practise Now 1B

1. (a) $93\frac{5}{6}\% = 93\frac{5}{6} \div 100$

$= \frac{563}{6} \times \frac{1}{100}$

$= \mathbf{\frac{563}{600}}$

$93\frac{5}{6}\% = 93\frac{5}{6} \div 100$

$= 93.8 \div 100$ (to 3 s.f.)

$= \mathbf{0.938}$

(b) $14.7\% = \frac{14.7}{100}$

$= \mathbf{\frac{147}{1000}}$

$14.7\% = \frac{14.7}{100}$

$= \mathbf{0.147}$

2. (a) $\frac{4}{7} = \frac{4}{7} \times 100\%$

$= \frac{400}{7}\%$

$= \mathbf{57\frac{1}{7}\%}$ or $\mathbf{57.1\%}$ (to 3 s.f.)

(b) $0.5432 = 0.5432 \times 100\%$

$= \mathbf{54.32\%}$

Practise Now 2

1. (a) $318\% = \frac{318}{100}$

$= \mathbf{3\frac{9}{50}}$

$318\% = \frac{318}{100}$

$= \mathbf{3.18}$

(b) $407\frac{1}{4}\% = \dfrac{407\frac{1}{4}}{100}$

$= \mathbf{4\frac{29}{400}}$

$407\frac{1}{4}\% = \dfrac{407\frac{1}{4}}{100}$

$= \dfrac{1629}{400}$

$= \dfrac{407.25}{100}$

$= \mathbf{4.0725}$

(c) $0.2\% = \dfrac{0.2}{100}$

$= \mathbf{\dfrac{1}{500}}$

$0.2\% = \dfrac{0.2}{100}$

$= \mathbf{0.002}$

(d) $0.066\% = \dfrac{0.066}{100}$

$= \mathbf{\dfrac{33}{50\,000}}$

$0.066\% = \dfrac{0.066}{100}$

$= \mathbf{0.000\,66}$

2. **(a)** $49\frac{1}{3} = 49\frac{1}{3} \times 100\%$

$= \mathbf{4933\frac{1}{3}}\,\%$

(b) $5.468 = 5.468 \times 100\%$

$= \mathbf{546.8\%}$

(c) $0.0016 = 0.0016 \times 100\%$

$= \mathbf{0.16\%}$

(d) $\dfrac{17}{2000} = \dfrac{17}{2000} \times 100\%$

$= \mathbf{0.85\%}$

Thinking Time (Page 183)

$x\%$ is defined as $\dfrac{x}{100}$.

$\therefore\ 100\% = \dfrac{100}{100} = 1$

Multiplying $\dfrac{p}{q}$ by 1 will not change the value of $\dfrac{p}{q}$. However, multiplying $\dfrac{p}{q}$ by 100 will change the value of $\dfrac{p}{q}$.

Class Discussion (Interpreting percentages used in real life)

Part 1

(a) From Table 8.1, Neymar had a shooting accuracy of 81.9%. Hence, he scored $\mathbf{0.819x}$ goals.

(b) **No**, Giovani dos Santos may have scored less goals than Neymar. Shooting accuracy refers to the number of goals scored as a percentage of the number of goals attempted. A shooting accuracy of 100.0% means that Giovani dos Santos scored all the goals he attempted (e.g. if y shots were attempted, y goals were scored. However, Neymar might have attempted more goals than Giovani dos Santos, such that $0.819x > y$.

Part 2

(a) Total number of valid votes for both parties

$= 112\,677 + 49\,851$

$= 162\,528$

Percentage of total valid votes that Party P received

$= \dfrac{112\,677}{162\,528} \times 100\%$

$= 69.33\%$ (to 2 d.p.)

(b) Percentage of valid votes

$= \dfrac{162\,528}{179\,071} \times 100\%$

$= \mathbf{90.76\%}$ (to 2 d.p.)

Practise Now 3

1. **(a)** Percentage of cards that do not have a number printed

$= 100\% - 30\%$

$= 70\%$

Number of cards that do not have a number printed

$= 70\%$ of 50

$= \dfrac{70}{100} \times 50$

$= \mathbf{35}$

(b) Number of blue cards

$= 50 - 18$

$= 32$

Percentage of blue cards not laminated

$= 100\% - 75\%$

$= 25\%$

Number of blue cards not laminated

$= 25\%$ of blue cards

$= \dfrac{25}{100} \times 32$

$= \mathbf{8}$

2. **(a)** 2.5% of 30 cm $= \dfrac{2.5}{100} \times 30$

$= \mathbf{0.75\ cm}$

(b) $15\frac{3}{4}\%$ of 640 kg $= \dfrac{15\frac{3}{4}}{100} \times 640$

$= \mathbf{100.8\ kg}$

(c) 2500% of \$4.60 $= \dfrac{2500}{100} \times 4.60$

$= \mathbf{\$115}$

Practise Now 4

Since 100% = 1, then 100% of 1000 kg = 1000 kg.

$\therefore$ 3.6% of 1000 kg must be smaller than 1000 kg, so 3600 kg is incorrect.

Class Discussion (Expressing two quantities in equivalent forms)

1. **(a)** **(i)** Required percentage = $\frac{\mathbf{15}}{\mathbf{25}} \times 100\% = \mathbf{60\%}$

There are **60%** as many boys as girls in the class.
The number of boys is **60%** of the number of girls.

The number of boys is $\frac{\mathbf{3}}{\mathbf{5}}$ of the number of girls.

(iii) Required percentage = $\frac{\mathbf{25}}{\mathbf{15}} \times 100\% = \mathbf{166\frac{2}{3}}\ \%$

There are $\mathbf{166\frac{2}{3}}$ **%** as many girls as boys in the class.

The number of girls is $\mathbf{166\frac{2}{3}}$ **%** of the number of boys.

The number of girls is $\frac{\mathbf{5}}{\mathbf{3}}$ of the number of boys.

(b)

In words	A is **60%** of B.	B is $\mathbf{166\frac{2}{3}}$ % of A.
Percentage	$A = \mathbf{60\%} \times B$	$B = \mathbf{166\frac{2}{3}}\ \% \times A$
Fraction	$A = \frac{\mathbf{3}}{\mathbf{5}} \times B$	$B = \frac{\mathbf{5}}{\mathbf{3}} \times A$
Decimal	$A = \mathbf{0.6} \times B$	$B = \mathbf{1.67} \times A$
Ratio	$A : B = \mathbf{3 : 5}$	$B : A = \mathbf{5 : 3}$

Table 8.3

Practise Now 5

(i) Total number of students $= 450 + 750$
$= 1200$

Percentage of boys in the school $= \frac{450}{1200} \times 100\%$
$= \mathbf{37.5\%}$

(ii) Percentage of girls in the school $= \frac{750}{1200} \times 100\%$
$= \mathbf{62.5\%}$

Practise Now 6

1. **(i)** Number of pages in magazine A as a percentage of the number of pages
in magazine B

$= \frac{\text{number of pages in magazine A}}{\text{number of pages in magazine B}} \times \%100$

$= \frac{128}{200} \times \%100$

$= \mathbf{64\%}$

(ii) Difference in number of pages $= 200 - 128$
$= 72$

Percentage difference $= \frac{72}{128} \times \%100$
$= \mathbf{56.25\%}$

2. 1400 ml as a percentage of 2.1 l

$= \frac{1400 \text{ ml}}{2.1\, l} \times \%100$

$= \frac{1400}{2100} \times \%100$

$= \mathbf{66\frac{2}{3}}$ **%** or **66.7%** (to 3 s.f.)

Practise Now 7

If A is 20% more than B, then $A = 120\% \times B$
$= \frac{6}{5}B.$

So $B = \frac{5}{6}A$

$= \left(\frac{5}{6} \times 100\%\right) \times A$

$= 83\frac{1}{3}\%$ of A

B is $100\% - 83\frac{1}{3}\% = 16\frac{2}{3}\%$ less than A.

$\therefore$ I do not agree with Li Ting's statement.

Introductory Problem Revisited

1. No, we cannot find the quantity the percentage corresponds to as there are no numbers given.
2. No. The information provided is that of the students from Sindh, 23.5% are enrolled in pre-primary, 47.4% in primary and 14.7% in middle schools, while of the students from Punjab, 17.8% are enrolled in pre-primary, 47.1% in primary and 17.9% in middle schools. However, without taking into account the quantity of students from each province, we cannot compare the numbers using the percentages. For instance, 17.8% of students in Punjab might be more than 23.5% of students in Sindh.
3. Let the number of students in Punjab and Sindh be x and y respectively.
   Total percentage of students enrolled in high schools.
   $= \frac{0.082y + 0.094x}{x + y} \times 100\%$
   $\therefore \frac{0.082x + 0.094y}{x + y} = 0.091$

Practise Now 8

Percentage of people who attended the party in Village A

$= \frac{4000}{30\,000} \times 100\%$

$= 13\frac{1}{3}\%$

Percentage of people who attended the party in Village B

$= \frac{3500}{25\,000} \times 100\%$

$= 14\%$

$\therefore$ **Village B** had a higher percentage of people who attended its New Year Party.

Practise Now 9

Percentage of sugar in a blackberry $= \frac{0.439}{9} \times 100\%$
$= 4.88\%$ (to 3 s.f.)

Percentage of sugar in a blueberry $= \frac{0.149}{1.5} \times 100\%$
$= 9.93\%$ (to 3 s.f.)

Percentage of sugar in a raspberry $= \frac{0.084}{1.9} \times 100\%$
$= 4.42\%$ (to 3 s.f.)

Percentage of sugar in a strawberry $= \frac{0.326}{7} \times 100\%$
$= 4.66\%$ (to 3 s.f.)

Since the percentage of sugar in a raspberry is the lowest, then **raspberry** has the lowest sugar content.

Thinking Time (Page 191)

Guiding questions:

- Which is more? 100% or 10%?
- What is the context in this case?

Teachers should point out to students that 100% of Li Ting's money is not necessarily more than 10% of their money. It depends on the quantities in question. For example, if Li Ting has \$5 and you have \$60, 100% of Li Ting's money is \$5 while 10% of your money is \$6. In this case, Li Ting's proposal is not attractive because \$5 is less than \$6.

Exercise 8A

1. **(a)** $47\% = \mathbf{\frac{47}{100}}$

$47\% = \frac{47}{100} = \mathbf{0.47}$

(b) $64\% = \frac{64}{100} = \mathbf{\frac{16}{25}}$

$64\% = \frac{64}{100} = \mathbf{0.64}$

2. **(a)** $\frac{4}{5} = \frac{4}{5} \times 100\% = \mathbf{80\%}$

(b) $0.38 = 0.38 \times 100\% = \mathbf{38\%}$

3. **(a)** $5\frac{1}{8}\% = \frac{41}{8}\% = \mathbf{\frac{41}{800}}$

$5\frac{1}{8}\% = \frac{\frac{41}{8}}{100} = \frac{5.125}{100} = \mathbf{0.051\ 25}$

(b) $91.4\% = \frac{91.4}{100} = \mathbf{\frac{457}{500}}$

$91.4\% = \frac{91.4}{100} = \mathbf{0.914}$

4. **(a)** $\frac{2}{3} = \frac{2}{3} \times 100\% = \mathbf{66\frac{2}{3}}\%$ or $\mathbf{66.7\%}$ (to 3 s.f.)

(b) $0.776 = 0.776 \times 100\% = \mathbf{77.6\%}$

5. **(a)** $159\% = \frac{159}{100} = \mathbf{1\frac{59}{100}}$

$159\% = \frac{159}{100} = \mathbf{1.59}$

(b) $813\frac{2}{5}\% = \frac{813\frac{2}{5}}{100} = \mathbf{8\frac{67}{500}}$

$813\frac{2}{5}\% = \frac{813\frac{2}{5}}{100} = \mathbf{8.134}$

(c) $0.3\% = \frac{0.3}{100} = \mathbf{\frac{3}{1000}}$

$0.3\% = \mathbf{0.003}$

(d) $0.014\% = \frac{0.014}{100} = \mathbf{\frac{7}{50\ 000}}$

$0.014\% = \frac{0.014}{100} = \mathbf{0.000\ 14}$

6. **(a)** $81\frac{2}{3} = 81\frac{2}{3} \times 100\% = \mathbf{8166\frac{2}{3}}\%$

(b) $9.124 = 9.124 \times 100\% = \mathbf{912.4\%}$

(c) $0.0023 = 0.0023 \times 100\% = \mathbf{0.23\%}$

(d) $\frac{11}{4000} = \frac{11}{4000} \times 100\% = \mathbf{0.275\%}$

7. Percentage of cars which are not blue $= 100\% - 30\% = 70\%$

Number of cars which are not blue $= 70\% \times 120 = \frac{70}{100} \times 120 = \mathbf{84}$

8. Percentage of employees who were unaffected by the financial crisis
= 100% – 2.5% – 50.75%
= 46.75%
Number of employees who were unaffected by the finanacial crisis
= 46.75% × 12 000
$= \frac{46.75}{100} \times 12\,000$
= **5610**

9. **(a)** 0.8% of 4.5 m $= \frac{0.8}{100} \times 4.5$
= **0.036 m**

(b) $111\frac{4}{5}$ % of 24 kg $= \frac{111\frac{4}{5}}{100} \times 24$
= **26.832 kg**

(c) 312.5% of \$70 $= \frac{312.5}{100} \times 70$
= **\$218.75**

10. Since 100% = 1, then 100% of 10 000 m = 10 000 m.
∴ 7.5% of 10 000 m must be smaller than 10 000 m, so 75 000 m is incorrect.

11. Total number of atoms = 12 + 22 + 11
= 45
Percentage of hydrogen atoms in the compound
$= \frac{22}{45} \times 100\%$
= **48.9%** (to 3 s.f.)

12. **(a)** Required percentage $= \frac{45 \text{ minutes}}{1 \text{ hours}} \times 100\%$
$= \frac{45 \text{ minutes}}{60 \text{ minutes}} \times 100\%$
$= \frac{3}{4} \times 100\%$
= **75%**

(b) Required percentage $= \frac{25 \text{ seconds}}{3.5 \text{ minutes}} \times 100\%$
$= \frac{25 \text{ seconds}}{210 \text{ seconds}} \times 100\%$
$= \frac{5}{42} \times 100\%$
$= \mathbf{11\frac{19}{21}}$ **%** or **11.9%** (to 3 s.f.)

(c) Required percentage $= \frac{1 \text{ year}}{4 \text{ months}} \times 100\%$
$= \frac{12 \text{ months}}{4 \text{ months}} \times 100\%$
= 3 × 100%
= **300%**

(d) Required percentage $= \frac{15 \text{ mm}}{1 \text{ m}} \times 100\%$
$= \frac{15 \text{ mm}}{1000 \text{ mm}} \times 100\%$
$= \frac{3}{200} \times 100\%$
= **1.5%**

(e) Required percentage $= \frac{335 \text{ cm}}{5 \text{ m}} \times 100\%$
$= \frac{335 \text{ cm}}{500 \text{ cm}} \times 100\%$
$= \frac{67}{100} \times 100\%$
= **67%**

(f) Required percentage $= \frac{1 \text{ kg}}{800 \text{ g}} \times 100\%$
$= \frac{1000 \text{ g}}{800 \text{ g}} \times 100\%$
$= \frac{5}{4} \times 100\%$
= **125%**

(g) Required percentage $= \frac{60°}{360°} \times 100\%$
$= \frac{1}{6} \times 100\%$
$= \mathbf{16\frac{2}{3}}$ **%** or **16.7%** (to 3 s.f.)

(h) Required percentage $= \frac{63 \text{ cents}}{\$2.10} \times 100\%$
$= \frac{63 \text{ cents}}{210 \text{ cents}} \times 100\%$
$= \frac{3}{10} \times 100\%$
= **30%**

13. Percentage of annual income Waseem donated to charitable organisations
$= \frac{\$1200}{12 \times \$1600} \times 100\%$
$= \frac{\$1200}{\$19\,200} \times 100\%$
= 6.25%
Percentage of annual income Albert donated to charitable organisations
$= \frac{\$4500}{12 \times \$6800} \times 100\%$
$= \frac{\$4500}{\$81\,600} \times 100\%$
$= \frac{35}{68}$ %
∴ **Waseem** donated a higher percentage of his annual income to charitable organisations.

14. Number of boys $= \frac{25}{100} \times 40$
= 10
Number of boys who passed the test $= \frac{70}{100} \times 10$
= **7**

15. x% of 330 $= \frac{x}{100} \times 330$
$= 3.3x$
330% of $x = \frac{330}{100} \times x$
$= 3.3x$
∴ they have the same value.

16. Amount Ken spent on room rental $= 20.5\% \times \$1850$

$= \frac{20.5}{100} \times \$1850$

$= \$379.50$

Amount Ken overspent $= \$379.25 + \$690 + \$940 - \$1850$

$= \$159.25$

Required percentage $= \frac{\$159.25}{\$18.50} \times 100\%$

$=$ **8.61%** (to 2 d.p.)

17. Number of remaining pages after Friday $= 600 - 150$

$= 450$

Number of pages that remains to be read $= (100\% - 40\%) \times 450$

$= 60\% \times 450$

$= \frac{60}{100} \times 450$

$= 270$

Required percentage $= \frac{270}{600} \times 100\%$

$=$ **45%**

18. Percentage of marks Cheryl obtains $= \frac{40}{60} \times 100\%$

$= 66\frac{2}{3}\ \%$

$\therefore$ **Cheryl** gets a **bronze** award.

Percentage of marks David obtains $= \frac{46}{60} \times 100\%$

$= 76\frac{2}{3}\ \%$

$\therefore$ **David** gets a **silver** award.

Percentage of marks Vasi obtains $= \frac{49}{60} \times 100\%$

$= 81\frac{2}{3}\ \%$

$\therefore$ **Vasi** gets a **gold** award.

19. Let h be the height of the son.

$\therefore$ height of father will be 150% of $h = \frac{150}{100}h$

$= \frac{3}{2}h$

Difference in height $= \frac{3}{2}h - h$

$= \frac{h}{2}$

Percentage difference $= \frac{\frac{h}{2}}{\frac{3}{2}h} \times 100\%$

$=$ $\mathbf{33\frac{1}{3}}$ **%** or **33.3%** (to 3 s.f.)

20. **(i)** Total amount of money $= \$50 + \$60 + \$90$

$= \$200$

Percentage of sum of money that Li Ting received

$= \frac{\$60}{\$200} \times 100\%$

$=$ **30%**

(ii) Shaha's share as a percentage of Bernard's

$= \frac{90}{50} \times 100\%$

$= 180\%$

$\therefore$ I agree with Shaha's statement.

21. Percentage of typo errors for applicant A

$= \frac{17}{214} \times \%100$

$= 7.9439\%$ (to 5 s.f.)

Percentage of typo errors for applicant B

$= \frac{20}{252} \times 100\%$

$= 7.9365\%$ (to 5 s.f.)

Percentage of typo errors for applicant C

$= \frac{18}{229} \times 100\%$

$= 7.8603\%$ (to 5 s.f.)

Since the percentage of typo errors for applicant C is the lowest, then Siti should hire **applicant C** if she wants her secretary's typing to be as highly accurate as possible.

22. Area of space reserved for greenery in Town B

$= \frac{25}{100} \times 24\ \text{km}^2$

$= 6\ \text{km}^2$

Area of space reserved for greenery in Town A

$= \frac{1}{2} \times 6$

$= 3\ \text{km}^2$

Percentage of space in Town A reserved for greenery

$= \frac{3}{15} \times 100\%$

$=$ **20%**

23. Let x be the number of boys who must leave the theatre.

$$\frac{99 - x}{99 + 1 - x} \times 100\% = 98\%$$

$$\frac{99 - x}{100 - x} = \frac{49}{50}$$

$$4900 - 49x = 4950 - 50x$$

$$x = 50$$

$\therefore$ **50** boys must leave the theatre.

24. **(a)** She is not correct. The percentages 10% and 70% were obtained based on the number of students in Class A and Class B respectively.

(b) He is not correct. The percentages 10% and 70% were obtained based on the number of students in Class A and Class B respectively. We cannot assume that the number of students in each class is the same.

25. **(i)** Joyce's statement could be true if her annual income decreased from 2018 to 2019.

(ii) A possible amount for her annual income in 2018 is **\$38 400**.

Amount of donation $= \frac{10}{100} \times \$38\ 400$

$= \$3840$

Annual income in 2019 $= \frac{3840}{12} \times 100$

$=$ **\$32 000**

8.2 Percentage change, percentage point and reverse percentage

Practise Now 10

(i) New cost per CuM for households using more than 40 CuM of water

$= 122.9\% \times \$2.61$

$= \frac{122.9}{100} \times \$2.61$

$=$ **\$3.21** (to the nearest cent)

(ii) Percentage increase $= \frac{2.44 - 2.15}{2.15} \times 100\%$

$=$ **13.5%** (to 3 s.f.)

Thinking Time (Page 196)

A 110% decrease in its cost implies that the item costs less than \$0, which is not possible.

Practise Now 11

	Original cost	Percentage change	New cost
Retail space rental	PKR 700 000	−5%	$\frac{95}{100}$ × PKR 700 000 = PKR 665 000
Wages	PKR 525 000	−6%	$\frac{94}{100}$ × PKR 525 000 = PKR 493 500
Utilities	PKR 140 000	+7%	$\frac{107}{100}$ × PKR 140 000 = PKR 149 800
Business	PKR 1 365 000		PKR 1 308 300

Percentage **decrease** in monthly cost of running the business

$= \frac{\text{PKR } 1\,365\,000 - \text{PKR } 1\,308\,300}{\text{PKR } 1\,365\,000} \times 100\%$

$= \frac{\text{PKR } 56\,700}{\text{PKR } 1\,365\,000} \times 100\%$

$= \mathbf{4\frac{2}{13}\%}$ or **4.15%** (to 3 s.f.)

Investigation (Can we add percentages or take average of percentages?)

1. The bases are not the same. Hence, the numerical values must be computed and used to find the overall percentage change.
2. Yasir's monthly salary in 2018 $= 110\% \times \$x$

$= \frac{110}{100} \times \$x$

$= \$1.1x$

Yasir's monthly salary in 2019 $= 90\% \times \$1.1x$

$= \frac{90}{100} \times \$1.1x$

$= \$0.99x$

Hence, it is **not correct** to say that Yasir's monthly salary in 2019 was $\$x$.

3. **(i)** Average percentage pass for the two classes

$= \frac{80\% + 60\%}{2}$

$=$ **70%**

(ii) Number of pupils who passed $= \frac{80}{100} \times 35 + \frac{60}{100} \times 40$

$= 28 + 24$

$= 52$

Overall percentage pass for the two classes

$= \frac{52}{35 + 40} \times 100\%$

$= \mathbf{69\frac{1}{3}\%}$ or **69.3%** (to 3 s.f.)

(iii) The average percentage pass is higher than the overall percentage pass.

(iv) We cannot find overall percentage pass by taking $\frac{80\% + 60\%}{2} = 70\%$, because the total number of pupils in each of the two classes is different. We can only use this method if the 'whole' of both quantities is the same.

Practise Now 12

1. Let the breadth of the rectangle be x units.
Then the length of the rectangle is $2x$ units.

New length of rectangle $= \frac{120}{100} \times 2x = 2.4x$ units

New breadth of rectangle $= \frac{80}{100} \times x = 0.8x$ units

Original perimeter of rectangle $= 2(2x + x) = 6x$ units

New perimeter of rectangle $= 2(2.4x + 0.8x) = 6.4x$ units

Percentage increase in perimeter of rectangle

$= \frac{6.4x - 6x}{6x} \times 100\%$

$= \frac{0.4x}{6x} \times 100\%$

$= \frac{0.4}{6} \times 100\%$

$= \mathbf{6\frac{2}{3}\%}$ or **6.67%** (to 3 s.f.)

2. Let the breadth of the rectangle be x units.
Then the length of the rectangle is $2x$ units.

New length of rectangle $= \frac{110}{100} \times 2x = 2.2x$ units

New breadth of rectangle $= \frac{90}{100} \times x = 0.9x$ units

Original area of rectangle $= 2x \times x = 2x^2$ units^2

New area of rectangle $= 2.2x \times 0.9x = 1.98x^2$ units^2

Percentage decrease in area of rectangle $= \frac{2x^2 - 1.98x^2}{2x^2} \times 100\%$

$= \frac{0.02x^2}{2x^2} \times 100\%$

$= \frac{0.02}{2} \times 100\%$

$=$ **1%**

Investigation (Percentage point)

1. Increase from 17% to 18% = 18% – 17%
   = 1%

   Percentage increase from 17% to 18% $= \frac{1}{17} \times 100\%$
   $= \mathbf{6\%}$

2. **Yes**, Bernard is right in saying that the increase from 17% to 18% is an increase of 6%. Please see solution to Question 1.
3. While the percentage increase from 17% to 18% is 6%, the difference between 17% and 18% is 1%. The term 'percentage point' is used to describe the difference between two percentages.

Practise Now 13

70% of the books = 35

1% of the books $= \frac{35}{70}$

100% of the books $= \frac{35}{70} \times 100$
= 50

There are **50** books on the bookshelf.

Practise Now 14

1. **Method 1:**

   109% of original cost = PKR 141 700

   1% of original cost $= \frac{\text{PKR } 141\,700}{109}$

   100% of original cost $= \frac{\text{PKR } 141\,700}{109} \times 100$
   = PKR 130 000

   The original cost of the article is **PKR 130 000**.

   Method 2:

   Let the original cost of the article be $$x$.

   $109\% \times x = 141\,700$

   $1.09x = 141\,700$

   $x = 130\,000$

   The original cost of the article is **PKR 130 000**.

2. 120% of value in 2019 = $180 000

   1% of value in 2019 $= \frac{\$180\,000}{120}$

   100% of value in 2019 $= \frac{\$180\,000}{120} \times 100$
   = $150 000

   The value of the vase was $150 000 in 2019.

   120% of value in 2018 = $150 000

   1% of value in 2018 $= \frac{\$150\,000}{120}$

   100% of value in 2018 $= \frac{\$150\,000}{120} \times 100$
   = $125 000

   The value of the vase was **$125 000** in 2018.

Practise Now 15

1. **Method 1:**

   97% of original monthly salary = PKR 722 681

   1% of original monthly salary $= \frac{\text{PKR } 722\,681}{97}$

   100% of original monthly salary $= \frac{\text{PKR } 722\,681}{97} \times 100$
   = PKR 745 032

   Nadia's original monthly salary is **PKR 745 032**.

   Method 2:

   Let Nadia's original monthly salary be $$x$.

   $97\% \times x = 722\,681$

   $0.97x = 722\,681$

   $x = 745\,032$

   Nadia's original monthly salary is **PKR 745 032**.

2. 85% of value in 2019 = $86 700

   1% of value in 2019 $= \frac{\$86\,700}{85}$

   100% of value in 2019 $= \frac{\$86\,700}{85} \times 100$
   = $102 000

   The value of the car was $102 000 in 2019.

   85% of value in 2018 = $102 000

   1% of value in 2018 $= \frac{\$102\,000}{85}$

   100% of value in 2018 $= \frac{\$102\,000}{85} \times 100$
   = $120 000

   The value of the car was **$120 000** in 2018.

Exercise 8B

1. (a) Required value $= 135\% \times 60$
   $= \frac{135}{100} \times 60$
   $= \mathbf{81}$

   (b) Required value $= 225.7\% \times 28$
   $= \frac{225.7}{100} \times 28$
   $= \mathbf{63.196}$

   (c) Required value $= 55\% \times 120$
   $= \frac{55}{100} \times 120$
   $= \mathbf{66}$

   (d) Required value $= 62\frac{1}{2}\% \times 216$
   $= \frac{62\frac{1}{2}}{100} \times 216$
   $= \mathbf{135}$

2. Percentage increase in length of elastic band $= \frac{90 - 72}{72} \times 100\%$
   $= \frac{18}{72} \times 100\%$
   $= \mathbf{25\%}$

3. Percentage decrease in price of desktop computer

$= \frac{\text{PKR } 304\ 300 - \text{PKR } 228\ 225}{\text{PKR } 304\ 300} \times 100\%$

$= \frac{\text{PKR } 76\ 075}{\text{PKR } 304\ 300} \times 100\%$

= **25%**

4. 100% of original cost = \$333 000

$1\% \text{ of original cost} = \$\frac{333\ 000}{100}$

$136\% \text{ of original cost} = \$\frac{333\ 000}{100} \times 136$

$= \$452\ 880$

The cost of the house today is **\$452 880**.

5. Value of car at the end of 2021 = 80% × \$120 000

$= \frac{80}{100} \times \$120\ 000$

$= \$96\ 000$

Value of car at the end of 2022 = 90% × \$96 000

$= \frac{90}{100} \times \$96\ 000$

= **\$86 400**

6. 45% of the students = 135

$1\% \text{ of the students} = \frac{135}{45}$

$100\% \text{ of the students} = \frac{135}{45} \times 100$

$= 300$

There were **300** students who took part in the competition.

7. **(a)** 20% of number = 17

$1\% \text{ of number} = \frac{17}{20}$

$100\% \text{ of number} = \frac{17}{20} \times 100$

$= 85$

The number is **85**.

(b) 175% of number = 49

$1\% \text{ of number} = \frac{49}{175}$

$100\% \text{ of number} = \frac{49}{175} \times 100$

$= 28$

The number is **28**.

(c) 115% of number = 161

$1\% \text{ of number} = \frac{161}{115}$

$100\% \text{ of number} = \frac{161}{115} \times 100$

$= 140$

The number is **140**.

(d) 80% of number = 192

$1\% \text{ of number} = \frac{192}{80}$

$100\% \text{ of number} = \frac{192}{80} \times 100$

$= 240$

The number is **240**.

8. 106.2% of number of visitor arrivals in 2017 = 17.4 million

$1\% \text{ of number of visitor arrivals in 2017} = \frac{17.4}{106.2}$

$100\% \text{ of number of visitor arrivals in 2017} = \frac{17.4}{106.2} \times 100$

= **16.4 million** (to 1 d.p.)

9. 90% of original bill = PKR 13 050

$1\% \text{ of original bill} = \frac{\text{PKR } 13\ 050}{90}$

$100\% \text{ of original bill} = \frac{\text{PKR } 13\ 050}{90} \times 100$

= PKR 14 500

The original bill is **PKR 14 500**.

10. Value obtained after initial increase = 130% × 2400

$= \frac{130}{100} \times 2400$

$= 3120$

Final number = 80% × 3120

$= \frac{80}{100} \times 3120$

= **2496**

11. Let the number of train passengers in 2020 be x.

Number of train passengers in 2021 = 108% × x

$= \frac{108}{100} \times x$

$= 1.08x$

Number of train passengers in 2022 = 108% × $1.08x$

$= \frac{108}{100} \times 1.08x$

$= 1.1664x$

Percentage increase in number of train passengers from 2020 to 2022

$= \frac{1.1664x - x}{x} \times 100\%$

$= \frac{0.1664x}{x} \times 100\%$

$= 0.1664 \times 100\%$

= **16.64%**

12.

	Original cost	Percentage change	New cost
Raw materials	\$100	+11%	$\frac{111}{100} \times \$100 = \$111$
Overheads	\$80	+20%	$\frac{120}{100} \times \$80 = \$96$
Wages	\$120	−15%	$\frac{85}{100} \times \$120 = \$102$
Printer	\$300		\$309

Percentage increase in production cost of printer

$= \frac{\$309 - \$300}{\$300} \times 100\%$

$= \frac{\$9}{\$300} \times 100\%$

= **3%**

13. Let the initial cost of fuel be $\$x$ per litre.
After a 10% increase, cost of fuel per litre = $\$1.1x$
Albert now uses 0.9 litres for every 1 litre he intially used.
Difference in Albert's expenditure $= \$x - 0.9(\$1.1x)$
$= \$x - \$0.99x$
$= \$0.01x$
$\therefore$ Albert is **wrong**.

14. Let the breadth of the rectangle be x units.
Then the length of the rectangle is $2x$ units.
New length of rectangle $= \frac{90}{100} \times 2x = 1.8x$ units
New breadth of rectangle $= \frac{110}{100} \times x = 1.1x$ units
Original perimeter of rectangle $= 2(2x + x) = 6x$ units
New perimeter of rectangle $= 2(1.8x + 1.1x) = 5.8x$ units
Percentage decrease in perimeter of rectangle $= \frac{6x - 5.8x}{6x} \times 100\%$
$= \frac{0.2x}{6x} \times 100\%$
$= \frac{0.2}{6} \times 100\%$
$= \mathbf{3\frac{1}{3}}$**% or 3.33%** (to 3 s.f.)

15. Let the length of the rectangle be x units,
the breadth of the rectangle be y units.
New length of rectangle $= \frac{110}{100} \times x = 1.1x$ units
New breadth of rectangle $= \frac{90}{100} \times y = 0.9y$ units
Original area of rectangle $= xy$ units^2
New area of rectangle $= 1.1x \times 0.9y = 0.99xy$ units^2
Percentage decrease in area of rectangle $= \frac{xy - 0.99xy}{xy} \times 100\%$
$= \frac{0.01xy}{xy} \times 100\%$
$= 0.01 \times 100\%$
$=$ **1%**

16. 115% of value in 2021 = \$899 300
1% of value in 2021 $= \frac{\$899\ 300}{115}$
100% of value in 2021 $= \frac{\$899\ 300}{115} \times 100$
= \$782 000
The value of the condominium was \$782 000 in 2021.
115% of value in 2020 = \$782 000
1% of value in 2020 $= \frac{\$782\ 000}{115}$
100% of value in 2020 $= \frac{\$782\ 000}{115} \times 100$
= \$680 000
The value of the condominium was **\$680 000** in 2020.

17. 75% of value in 2021 = PKR 2 520 000
1% of value in 2021 $= \frac{\text{PKR } 2\ 520\ 000}{75}$
100% of value in 2021 $= \frac{\text{PKR } 2\ 520\ 000}{75} \times 100$
= PKR 3 360 000
The value of the surveying machine was PKR 3 360 000 in 2021.
75% of value in 2020 = PKR 3 360 000
1% of value in 2020 $= \frac{\text{PKR } 3\ 360\ 000}{75}$
100% of value in 2020 $= \frac{\text{PKR } 3\ 360\ 000}{75} \times 100$
= PKR 4 480 000
The value of the surveying machine was **PKR 4 480 000** in 2020.

18. 105% of value at the end of 2021 = \$61 824
1% of value at the end of 2021 $= \frac{\$61\ 824}{105}$
100% of value at the end of 2021 $= \frac{\$61\ 824}{105} \times 100$
= \$58 880
The value of the investment portfolio was **\$58 880** at the end of 2018.
92% of original value = \$58 880
1% of original value $= \frac{\$58\ 880}{92}$
100% of original value $= \frac{\$58\ 880}{92} \times 100$
= \$64 000
The original value of the investment portfolio was **\$64 000**.

19. Let Ali's height be x m.
108% of Yasir's height $= x$ m
1% of Yasir's height $= \frac{x}{108}$ m
100% of Yasir's height $= \frac{x}{108} \times 100$
$= \frac{25}{27}x$ m
Yasir's height is $\frac{25}{27}x$ m.
Vasi's height $= 90\% \times \frac{25}{27}x$
$= \frac{90}{100} \times \frac{25}{27}x$
$= \frac{5}{6}x$ m
Required percentage $= \frac{x}{\frac{5}{6}x} \times 100\%$
$= \frac{1}{\frac{5}{6}} \times 100\%$
$=$ **120%**

20. **(a)** Let x be the number of boys in the choir.

$\therefore$ there are $2x$ girls in the choir.

If the number of boys increased by 30%,

then the number of boys in the choir $= \frac{130}{100} \times x$

$= 1.3x$

Percentage decrease in number of girls $= \frac{2x - 1.3x}{2x} \times 100\%$

$= \frac{0.7}{2} \times 100\%$

$= \mathbf{35\%}$

(b) If the number of boys increased by 130%,

then the number of boys in the choir $= \frac{230}{100} \times x$

$= 2.3x$

Percentage increase in number of girls $= \frac{2.3x - 2x}{2x} \times 100\%$

$= \frac{0.3}{2} \times 100\%$

$= \mathbf{15\%}$

21. **(a)** **Scheme A:**

New annual salary $= \frac{112}{100} \times$ PKR 537 000 $\times$ 12

= PKR 7 217 280

Scheme B:

New annual salary $= \frac{108}{100} \times$ PKR 537 000 $\times$ 12 + PKR 223 800

= PKR 7 183 320

Since the new annual salary under Scheme A is higher, Shaha should accept **Scheme A**.

(b) Let x be Shaha's current monthly salary.

OPEN For both schemes to offer the same amount,

$$\frac{108}{100} \times x \times 12 + 223\,800 = \frac{112}{100} \times x \times 12$$

$$12.96x + 223\,800 = 13.44x$$

$$0.48x = 223\,800$$

$$x = 466\,250$$

$\therefore$ for Shaha to accept Scheme B because it is a better offer, a possible amount for her current monthly salary is **PKR 200 000**.

Chapter 9 Ratio and Rate

TEACHING NOTES

Suggested Approach

Students have learnt how to solve problems involving ratios and speed in primary school. A recap of the concept of ratio, and the relationship between ratios and fractions (see Class Discussion: Relationship between ratios and fractions) may be necessary for some students. Teachers can bring in real-life examples for ratio, rate and speed to arouse students' interest in this topic. Students will also learn how to solve problems involving ratio, rate and speed through worked examples that involve situations in real-world contexts.

Section 9.1: Ratio

Teachers can highlight the relationship between ratio and fractions (see Class Discussion: Relationship between ratios and fractions). Using this relationship, teachers can introduce equivalent ratios through a recap of equivalent fractions. Teachers should emphasise that ratio does not indicate the actual size of quantities involved. Practical examples can be given to the students to let them recognise what equivalent ratios are (e.g. using 2 different kinds of fruits).

Teachers should highlight some common errors in ratio (i.e. the ratio of a part of a whole with the ratio of two parts, incorrect order of numbers expressed when writing ratio and incorrect numerator expressed when writing ratio as a fraction).

To make learning interesting, students can explore more about the Golden Ratio (see chapter opener) or real-life applications of ratios (see Class Discussion: Making sense of ratios used in real-world contexts).

Section 9.2: Rate

Teachers should explain that rate is a relationship between two quantities with different units of measure (which is different from ratio). Teachers can give real life examples (e.g. rate of flow, consumption) for students to understand the concept of rate. Teachers can also get students to interpret using tables which show different kinds of rates (see Class Discussion: Different types of rates).

Students can get more practice by learning to calculate rates they are familiar with (see Investigation: Average Pulse Rate). Teachers should impress upon them to distinguish between constant and average rates.

Section 9.3: Speed

Teachers should inform students that speed is a special type of rate, i.e. speed is the distance covered per unit time. Teachers can get students to match appropriate speed to examples given (e.g. speed of a moving bicycle, lorry, car and aeroplane) to bring across the notion of speed.

Teachers can build upon what students have learnt about distance, time and speed. Students need to know that average speed is defined as the total distance travelled by the object per unit time and not the average of the speeds of the object. Teachers should also impress upon students that there are differences between average speed and constant speed.

Teachers should teach students the conversion of units of speed and time, and highlight to them to use appropriate units when solving problems. When calculating duration, students may need to take into account different time zones (see Class Discussion: Time zones).

Introductory Problem

Teachers may come back to the Introductory Problem later on in the chapter and get students to verify the charges in Introductory Problem Revisited (after Practise Now 11).

9.1 Ratio

Practise Now 1

(i) Ratio of the number of lemons to the number of pears
= **33 : 20**

(ii) Ratio of the number of pears to the number of fruits in the basket
= 20 : (33 + 20)
= **20 : 53**

Class Discussion (Relationships between ratios and fractions)

There are 5 green balls and 7 red balls in a bag.
Let A and B represent the number of green balls and red balls respectively.

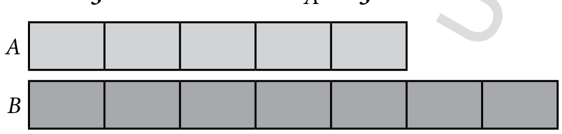
1. (a) If T is the total number of balls in the bag, find the ratio of A to T,
$A : T = \mathbf{5 : 12}$
The ratio of A to T is **5 : 12**.

(b) what fraction of T is A, i.e. what is $\frac{A}{T}$?
$\frac{A}{T} = \mathbf{\frac{5}{12}}$.

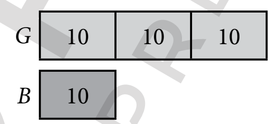
2. (a) Find the ratio of A to B.
$A : B = \mathbf{5 : 7}$
We can conclude that:
The ratio of A to B is **5 : 7**.
The following statement is equivalent to the above statement.
A is $\mathbf{\frac{5}{7}}$ (fraction) of B, i.e. $\frac{A}{B} = \mathbf{\frac{5}{7}}$ (fraction).

(b) Find the ratio of B to A.
$B : A = \mathbf{7 : 5}$
We can conclude that:
The ratio of B to A is **7 : 5**.
The following statement is equivalent to the above statement.
B is $\mathbf{\frac{7}{5}}$ (fraction) of A, i.e. $\frac{B}{A} = \mathbf{\frac{7}{5}}$ (fraction).

3.



4. *Teachers may use the specific example given here as a trigger for the next subsection on equivalent ratios.*
Example:
There are 30 girls and 10 boys in a class.
Let G and B represent the number of girls and boys respectively.
$G : B = 30 : 10$
$\quad = 3 : 1$
We can conclude that:
The ratio of G to B is 3 : 1.
The following statement is equivalent to the above statement.
G is $\frac{3}{1}$ (fraction) of B, i.e. $\frac{G}{B} = \frac{3}{1}$ (fraction).
OR
$B : G = 10 : 30$
$\quad = 1 : 3$
We can conclude that:
The ratio of B to G is 1 : 3.
The following statement is equivalent to the above statement.
B is $\frac{1}{3}$ (fraction) of G, i.e. $\frac{B}{G} = \frac{1}{3}$ (fraction).



Thinking Time (Page 208)

1. No. The ratio notation $a : b$ is equivalent to the fraction $\frac{a}{b}$.
A ratio 5 : 0 will give a fraction $\frac{5}{0}$, which is undefined.
A ratio 0 : 5 will give a fraction $\frac{0}{5}$, which is equal to 0, which will not enable the comparison of two quantities.

2. No. The ratio $a : b : c$ compares the three quantities a, b and c and cannot be expressed as a fraction directly. A fraction consists of only two numbers, whereby the denominator represents the total number of equal parts the whole is divided into.

Practise Now 2

(a) $2\frac{3}{5} : 1\frac{4}{9} = \frac{13}{5} \times 45 : \frac{13}{9} \times 45$
$= 117 : 65$
$= \mathbf{9 : 5}$

(b) $0.36 : 1.2 = 0.36 \times 100 : 1.2 \times 100$
$= 36 : 120$
$= \mathbf{3 : 10}$

Practise Now 3

700 g = 700 ÷ 1000
= 0.7 kg
= $\frac{7}{10}$ kg

$2\frac{1}{3} : \frac{7}{10} = \frac{7}{3} \times 30 : \frac{7}{10} \times 30$
$= 70 : 21$
$= \mathbf{10 : 3}$

Practise Now 4

1. **(a)** $3a : 7 = 8 : 5$

$$\frac{3a}{7} = \frac{8}{5}$$
$$15a = 56$$
$$a = \mathbf{3\frac{11}{15}}$$

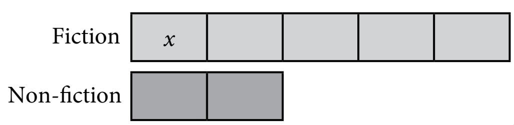
(b) $5 : b = 5^2 : 4^2$

$$b : 5 = 4^2 : 5^2$$
$$\frac{b}{5} = \frac{16}{25}$$
$$b = \frac{16}{25} \times 5$$
$$= \mathbf{3\frac{1}{5}}$$

2. $$\frac{3x}{8} = \frac{9y}{4}$$
$$12x = 72y$$
$$\frac{x}{y} = \frac{72}{12}$$
$$= 6$$
$\therefore x : y = \mathbf{6 : 1}$

Practise Now 5

1. Let the number of fiction books = $5x$.
Then the number of non-fiction books = $2x$.



From the model, we form the equation:
$$5x + 2x = 1421$$
$$7x = 1421$$
$$x = 203$$
There are $3 \times 203 =$ **609** more fiction than non-fiction books in the library.

2. Let the amount of money Cheryl had initially be $3x.
Then the amount of money Shaha had initially is $5x.

	Cheryl	Shaha
Before	$3x	$5x
After	$(3x + 150)	$(5x − 150)

$$\therefore \frac{3x+150}{5x-150} = \frac{7}{9}$$
$$9(3x + 150) = 7(5x - 150)$$
$$27x + 1350 = 35x - 1050$$
$$27x - 35x = -1050 - 1350$$
$$-8x = -2400$$
$$x = 300$$
$\therefore$ amount of money Cheryl had initially = $[3(300)]
= **$900**

Practise Now 6

1. $x : y = 5 : 6$ $\downarrow \times 2$ $= 10 : 12$ $\qquad y : z = 4 : 9$ $\downarrow \times 3$ $= 12 : 27$

(i) $x : y : z =$ **10 : 12 : 27**

(ii) $x : z =$ **10 : 27**

2. 0.9 km = 0.9 × 1000
= 900 m
30 000 cm = 30 000 100
= 300 m
600 : 900 : 300 = 6 : 9 : 3
= **2 : 3 : 1**

Practise Now 7

Let the amount of money Raju had initially be $6x.
Then the amount of money Albert and Vasi had initially is $4x and $5x respectively.

	Raju	Albert	Vasi
Before	$6x	$4x	$5x
After	$(6x − 45)	$(4x + 30)	$(5x + 15)

$$\therefore \frac{6x-45}{4x+30} = \frac{7}{6}$$
$$6(6x - 45) = 7(4x + 30)$$
$$36x - 270 = 28x + 210$$
$$36x - 28x = 210 + 270$$
$$8x = 480$$
$$x = 60$$
$\therefore$ amount of money Raju had initially = $[6(60)]
= **$360**

Class Discussion (Making sense of ratios used in real-world contexts)

1. When an image with a 4 : 3 aspect ratio is displayed on a screen with a 16 : 9 aspect ratio, the image is stretched horizontally and vertically by a different factor. Thus the image becomes distorted.

2. **(a)** **(i)** This means that for any amount of concentrate used, four times the amount of water will be needed.

(ii) Using the term "parts" enables easier calculation of the required amount of water depending on the total amount of mixture one needs to make.

(b) **(i)** **1 : 4**

(ii) $\frac{1}{5} \times 100\% =$ **20%**

(iii) $\mathbf{\frac{4}{5}}$

(c) **1 : 2**

(d) $$\frac{3}{3+7} = \frac{3}{10} = \frac{9}{30}$$
$$\frac{1}{1+2} = \frac{1}{3} = \frac{10}{30}$$
Since $\frac{10}{30} > \frac{9}{30}$, a drink with a **1 : 2 ratio** of concentrate to water will have a stronger flavour.

Practise Now 8

(i) Suggested amount of washing solution $= (1 + 4) \times 500$
$= 2500$ ml
$= 2.5\ l$

Number of bottles needed $= \dfrac{5}{2.5} \times 1$
$= \mathbf{2}$

(ii) Amount of detergent used $= 2 \times 500$
$= 1000$ ml
$= 1\ l$

Amount of water used in 10 l of washing solution $= 10 - 1$
$= 9\ l$

$\therefore$ mixing ratio is 1 part detergent to 9 parts water i.e. **1 : 9**.

Exercise 9A

1. **(i)** Ratio of the number of boys to the number of girls = **14 : 25**

(ii) Ratio of the number of girls to the total number of players in the team
= **25 : 39**

2. **(a)** $\dfrac{3}{8} : \dfrac{9}{4} = \dfrac{3}{8} \times 8 : \dfrac{9}{4} \times 8$
$= 3 : 18$
$= \mathbf{1 : 6}$

(b) $1 : \dfrac{3}{7} = 1 \times 7 : \dfrac{3}{7} \times 7$
$= \mathbf{7 : 3}$

(c) $0.45 : 0.85 = 0.45 \times 100 : 0.85 \times 100$
$= 45 : 85$
$= \mathbf{9 : 17}$

(d) $1.6 : 4 = \dfrac{8}{5} \times 5 : 4 \times 5$
$= 8 : 20$
$= \mathbf{2 : 5}$

3. **(a)** 1.5 m $= 1.5 \times 100$ cm
$= 150$ cm
$\therefore$ ratio = 150 : 300 = **1 : 2**

(b) $1.2\ l = 1.2 \times 1000$ ml
$= 1200$ ml
$\therefore$ ratio = 600 : 1200 = **1 : 2**

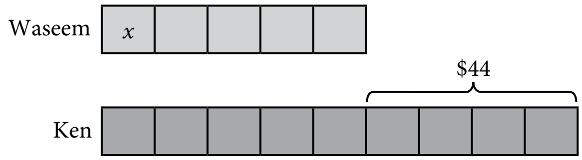
(c) \$1.25 = 1.25 × 100¢
= 125¢
$\therefore$ ratio = 50 : 125 = **2 : 5**

(d) 2.4 kg $= 2.4 \times 1000$ g
$= 2400$ g
$\therefore$ ratio = 2400 : 4000 = **3 : 5**

4. **(a)** $a : 400 = 6 : 25$
$\dfrac{a}{400} = \dfrac{6}{25}$
$25a = 2400$
$a = \mathbf{96}$

(b) $4^2 : 3^2 = 8 : 3b$
$3b : 8 = 3^2 : 4^2$
$\dfrac{3b}{8} = \dfrac{9}{16}$
$48b = 72$
$b = \mathbf{1\frac{1}{2}}$

5. Let the amount of money that Waseem gets be $5x$.
Then the amount of money that Ken gets is $9x$.



From the model, we form the equation:
$9x - 5x = 44$
$4x = 44$
$x = 11$
Total amount of money that is shared between the two boys
$= (5 + 9) \times 11$
= **\$154**

6. $a : b : c = 75 : 120 : 132$

(i) $a : b : c = \mathbf{25 : 40 : 44}$

(ii) $b : a = 40 : 25$
$= \mathbf{8 : 5}$

(iii) $b : c = 40 : 44$
$= \mathbf{10 : 11}$

7. **(a)** $\dfrac{2}{3} : \dfrac{3}{2} : \dfrac{5}{8} = \dfrac{2}{3} \times 24 : \dfrac{3}{2} \times 24 : \dfrac{5}{8} \times 24$
$= \mathbf{16 : 36 : 15}$

(b) $2 : \dfrac{7}{6} : \dfrac{7}{9} = 2 \times 18 : \dfrac{7}{6} \times 18 : \dfrac{7}{9} \times 18$
$= \mathbf{36 : 21 : 14}$

(c) $0.33 : 0.63 : 1.8 = 0.33 \times 100 : 0.63 \times 100 : 1.8 \times 100$
$= 33 : 63 : 180$
$= \mathbf{11 : 21 : 60}$

(d) $1.4 : 7 : 6.3 = 1.4 \times 10 : 7 \times 10 : 6.3 \times 10$
$= 14 : 70 : 63$
$= \mathbf{2 : 10 : 9}$

8. **(a)** $1.12\ l = 1.12 \times 1000$ ml
$= 1120$ ml
580 ml : 1.12 l : 104 ml = 580 : 1120 : 104
= **145 : 280 : 26**

(b) 2.8 kg $= 2.8 \times 1000$ g
$= 2800$ g
1.05 kg $= 1.05 \times 1000$ g
$= 1050$ g
2.8 kg : 700 g : 1.05 kg = 2800 : 700 : 1050
= **8 : 2 : 3**

(c) 2.4 km $= 2.4 \times 1000$ m
$= 2400$ m
32 m : 2.4 km : 64.8 m = 32 : 2400 : 64.8
$= 32 \times 10 : 2400 \times 10 : 64.8 \times 10$
= 320 : 24 000 : 648
= **40 : 3000 : 81**

(d) \$7.60 = 7.60 × 100¢
= 760¢
\$6 = 6 × 100¢
= 600¢
\$7.60 : 84¢ : 6\$ = 760 : 84 : 600
= **190 : 21 : 150**

9. (i) Number of toys Yasir makes $= \frac{1530}{12+16+17} \times 16$

$= \frac{1530}{45} \times 16$

$=$ **544**

(ii) Number of toys David makes $= \frac{1530}{12+16+17} \times 17$

$= \frac{1530}{45} \times 17$

$= 578$

Amount of money David earns = 578 × \$1.65

= **\$953.70**

10. (i) Number of patients : Number of doctors

= 510 000 : 12 000

= **85 : 2**

(ii) Number of doctors : Number of nurses : Number of volunteers

= 12 000 : 38 000 : 800

= 60 : 190 : 4

= **30 : 95 : 2**

11. (a) $0.75 : 3\frac{5}{16} = 0.75 \times 100 : 3\frac{5}{16} \times 100$

$= 75 : \frac{1325}{4}$

$= 75 \times 4 : \frac{1325}{4} \times 4$

$= 300 : 1325$

= **12 : 53**

(b) $7\frac{1}{7} : 4.5 = \frac{50}{7} : \frac{9}{2}$

$= \frac{50}{7} \times 14 : \frac{9}{2} \times 14$

= **100 : 63**

(c) $24\% : 1\frac{1}{5} = \frac{24}{100} : \frac{6}{5}$

$= \frac{24}{100} \times 100 : \frac{6}{5} \times 100$

$= 24 : 120$

= **1 : 5**

(d) $0.84 : 0.84\% = 0.84 : \frac{0.84}{100}$

$= 0.84 \times 100 : \frac{0.84}{100} \times 100$

$= 84 : 0.84$

$= 84 \times 100 : 0.84 \times 100$

$= 8400 : 84$

= **100 : 1**

12. (a) 2.475 m = 247.5 cm

∴ ratio = 247.5 : 33

= 247.5 × 10 : 33 × 10

= 2475 : 330

= **15 : 2**

(b) $4\frac{1}{5}$ kg $= \frac{21}{5} \times 1000$ g

= 4200 g

∴ ratio = 4200 : 61.6

= 4200 × 10 : 61.6 × 10

= 42 000 : 616

= **750 : 11**

(c) 250 ml $= \frac{1}{4}$ l

∴ ratio $= 3\frac{3}{4} : \frac{1}{4}$

$= \frac{15}{4} : \frac{1}{4}$

$= \frac{15}{4} \times 4 : \frac{1}{4} \times 4$

= **15 : 1**

(d) 75¢ = \$0.75

∴ ratio = 2.05 : 0.75

= 2.05 × 100 : 0.75 × 100

= 205 : 75

= **41 : 15**

13. (a) $\frac{2x}{5} = \frac{3y}{8}$

$16x = 15y$

$\frac{x}{y} = \frac{15}{16}$

∴ $x : y =$ **15 : 16**

(b) $\frac{7x}{9} = \frac{14y}{3}$

$21x = 126y$

$\frac{x}{y} = \frac{126}{21}$

$= 6$

∴ $x : y =$ **6 : 1**

14. (i) Let the initial number of teachers in the school be x.

Then the number of students in the school is $15x$.

$15x = 1200$

$x = 80$

The initial number of teachers in the school is **80**.

(ii) Let the number of teachers who join the school be y.

$\frac{80 + y}{1200} = \frac{3}{40}$

$40(80 + y) = 3(1200)$

$3200 + 40y = 3600$

$40y = 3600 - 3200$

$40y = 400$

$y = 10$

The number of teachers who join the school is **10**.

15. $p : q = \frac{3}{4} : 2 \quad \downarrow \times 4 \quad = 3 : 8 \quad \downarrow \quad = 6 : 16$

$p : r = \frac{1}{3} : \frac{1}{2} \quad \downarrow \times 6 \quad = 2 : 3 \quad \downarrow \quad = 6 : 9$

(i) $p : q : r =$ **6 : 16 : 9**

(ii) $q : r =$ **16 : 9**

16. $x : 3 : \frac{9}{2} = \frac{15}{4} : 4\frac{1}{2} : y$

$x \times 4 : 3 \times 4 : \frac{9}{2} \times 4 = \frac{15}{4} \times 4 : \frac{9}{2} \times 4 : y \times 4$

$4x : 12 : 18 = 15 : 18 : 4y$

$\frac{4x}{12} = \frac{15}{18}$ $\qquad$ $\frac{12}{18} = \frac{18}{4y}$

$\frac{x}{3} = \frac{5}{6}$ $\qquad$ $\frac{2}{3} = \frac{9}{2y}$

$6x = 15$ $\qquad$ $4y = 27$

$x = \frac{15}{6}$ $\qquad$ $y = \frac{27}{4}$

$x = \frac{5}{2}$ $\qquad$ $\boldsymbol{y = 6\frac{3}{4}}$

$\boldsymbol{x = 2\frac{1}{2}}$

17. Let the initial number of roses by $5x$.
Then the number of sunflowers and tulips is $6x$ and $9x$ respectively.

	Roses	**Sunflowers**	**Tulips**
Before	$5x$	$6x$	$9x$
After	$5x - 50$	$6x$	$9x$

$\therefore \frac{5x - 50}{6x} = \frac{3}{4}$

$4(5x - 50) = 3(6x)$

$20x - 200 = 18x$

$2x = 200$

$x = 100$

$\therefore$ number of roses left $= 5 \times 100 - 150$

$= \mathbf{450}$

18. **(i)** Amount of strawberry syrup needed $= \frac{6000}{7}$

$= 857\frac{1}{7}$ ml

Number of bottles of strawberry syrup needed $= 857\frac{1}{7} \div 240$

$= 3\frac{4}{7}$

$\therefore$ Vasi has to buy **4** bottles of strawberry syrup.

(ii) Amount of strawberry syrup $= 4 \times 240$

$= 960$ ml

Amount of lemonade needed $= 960 \times 3$

$= 2880$ ml

Initial amount of lemonade $= \frac{6000}{8} \times 3$

$= 2250$ ml

Additional amount of lemonade $= 2880 - 2250$

$=$ **630 ml**

Amount of carbonated water $= 960 \times 4$

$= 3840$ ml

Initial amount of carbonated water $= \frac{6000}{8} \times 4$

$= 3000$

Additional amount of carbonated water $= 3840 - 3000$

$=$ **840 ml**

19. Let the number that must be added be x.

$\frac{3 + x}{8 + x} = \frac{2}{3}$

$3(3 + x) = 2(8 + x)$

$9 + 3x = 16 + 2x$

$3x - 2x = 16 - 9$

$x = 7$

The number is **7**.

20. $\frac{x}{y} = \frac{3}{4}$ $\qquad$ $\frac{y}{z} = \frac{5}{8}$

$4x = 3y$ $\qquad$ $8y = 5z$

$x = \frac{3}{4}y$ $\qquad$ $z = \frac{8}{5}y$

$$\frac{2y}{3x - y + 2z} = \frac{2y}{3\left(\frac{3}{4}y\right) - y + 2\left(\frac{8}{5}y\right)}$$

$$= \frac{2y}{\frac{9}{4}y - y + \frac{16}{5}y}$$

$$= \frac{2y}{\frac{45}{20}y - \frac{20}{20}y + \frac{64}{20}y}$$

$$= \frac{2y}{\frac{89}{20}y}$$

$$= \boldsymbol{\frac{40}{89}}$$

21. **(a)** One possible set of values is $\boldsymbol{x = 10, y = 8, z = 6}$.

OPEN **(b)** One possible set of values is $\boldsymbol{x = 0.5, y = 0.4, z = 0.3}$.

22. **(a)** **No.** The number of voters in the respective classes is not known and thus, we are unable to determine the number of votes received.

OPEN **(b)** One possible scenario is that Class A had 42 voters and Class B had 30 voters. Then, Shaha would have obtained $\frac{42}{3} \times 2 + \frac{30}{5} \times 2 = 58$ votes and Bernard would have obtained $\frac{42}{3} \times 1 + \frac{30}{5} \times 3 = 32$ votes.

9.2 Rate

Class Discussion (Different types of rates]

Part 1

(a) Number of words typed and time.
(b) Different kinds. The units are different.
(c) Speed of a car.

Part 2

(d) Number of words spoken by the man and number of words spoken by the woman.
(e) Same kind. They are both the number of words.
(f) Number of steps walked by one person on average compared to another person.

Part 3

(g) In Practise Now 5 Question 1, the ratio 5 : 2 can be converted into a rate: there are 5 fiction books for every 2 non-fiction books in the library.

(h) **7 : 1**

Part 4

(i) **Method 1: Cost per gram of coffee powder**

Option A: $\dfrac{\$5.80}{200\text{ g}} = \$0.0290$ per g

Option B: $\dfrac{\$7.45}{250\text{ g}} = \$0.0298$ per g

Method 2: Cost per 100 g of coffee powder

Option A: $\dfrac{\$5.80}{2} = \$2.90$ per 100 g

Option B: $\dfrac{\$7.45}{2.5} = \$2.98$ per 100 g

(j) Cost per 100 g of cherries.

Part 5

(k) Cost of posting:

a large letter that weighs 280 g = **$3.38**
a medium parcel that weighs 10 kg = **$8.85**
a small parcel that weighs 1.8 kg = **$5.67**
a letter that weighs 120 g = **$2.60**

Practise Now 9

Number of words per minute that Li Ting can type

$= \dfrac{720}{16}$

$= 45$

Number of words per minute that Vasi can type

$= \dfrac{828}{18}$

$= 46$

Number of words per minute that Ali can type

$= \dfrac{798}{19}$

$= 42$

Thus, **Vasi** is the fastest typist.

Practise Now 10

A literacy rate of 86.3% means that there are 86.3 literate people per 100 people aged 15 years and over.

Practise Now 11

900-g tin: Cost per 100 g of chocolate drink powder

$= \dfrac{\text{PKR } 3870}{9}$

$= \text{PKR } 430$

1.45-kg tin: Cost per 100 g of chocolate drink powder

$= \dfrac{\text{PKR } 5800}{14.5}$

$= \text{PKR } 400$

Since the cost per 100 g for the 1.45-kg is less than that for the 900-g tin, the **1.45-kg tin** gives the better value for money.

Thinking Time (Page 221)

(i) **Yes.**

*Note: This is not shown as a method in Worked Example 11 because the comparison using ratios is not easy. However, the illustration below can help students see the relationship between ratio and rate to answer part **(ii)**.*

Let the volume and cost of body wash in Option A be x_A and y_A respectively, and in Option B be x_B and y_B respectively.

Then $x_B : x_A = 1200 : 950$
$= 24 : 19,$

and $y_B : y_A = 7.95 : 6.95.$

We can convert the ratios into fractions to compare them.

$\dfrac{x_B}{x_A} = \dfrac{24}{19} = 1.26$ (to 3 s.f.) and $\dfrac{y_B}{y_A} = \dfrac{7.95}{6.95} = 1.14$ (to 3 s.f.)

Since the ratio of the cost of Option B to the cost of Option A $\left(\dfrac{y_B}{y_A}\right)$ is lower than the ratio of the volume in Option B to the volume in Option A $\left(\dfrac{x_B}{x_A}\right)$, then Option B gives the better value for money.

(ii) The two methods used in Worked Example 11 compare two rates. In part **(i)**, we compared ratios. Either ratio or rate can be used to compare if two rates or two ratios are equal. The relationship between ratio and rate is that two equal rates can be converted into two equal ratios. Depending on the problem, it may be easier to use one of them, e.g. in Worked Example 11, using rates is preferred. We will learn more about this relationship in Secondary 2 when we study direct proportion.

Introductory Problem Revisited

Each value under 'Amount ($)' is calculated by multiplying the rate (under 'Rate ($)') and the number of units consumed (under 'Usage').

Total cost for:

Electricity services = 315 × 0.2215
= **$69.77** (to the nearest cent)

Gas services = 30 × 0.1853
= **$5.56** (to the nearest cent)

Water consumption (water services) = 15.0 × 1.19
= **$17.85**

Water Conservation Tax is calculated as a percentage (35%) of the cost of water consumption ($17.85).

∴ Water Conservation Tax (water services)
= 17.85 × 35%
= 17.85 × 0.35
= **$6.25** (to the nearest cent)

Cost of electricity, gas and water services
= $69.77 + $5.56 + $17.85 + $6.25
= **$99.43**

GST is calculated as a percentage (18%) of the cost of the three services.

∴ GST
= 99.43 × 18%
= 99.43 × 0.18
= **$17.90** (to the nearest cent)

Teachers may explain the purpose of GST (General Sales Tax) and build upon students' knowledge of percentage to calculate the amount of GST. GST will be covered in Chapter 6: Financial Transactions of Book 2.

Investigation (Average pulse rate)

6. Speed of walking from one point to another.

7. Rate at which water flows out from a tap.

Practise Now 12

1. **(a)** Cost per child

$= \dfrac{\$2.70 \times 32.5}{36}$

$=$ **\$2.44** (to the nearest cent)

(b) **(i)** Distance travelled on 1 litre of petrol

$= \dfrac{265}{25}$

$= \dfrac{265}{25}$

$= 10.6$ km

Distance travelled on 58 litres of petrol

$= 10.6 \times 58$

$=$ **614.8 km**

(ii) Amount of petrol required to travel a distance of 1007 km

$= \dfrac{1007}{10.6}$

$= 95$ litres

Amount that the car owner has to pay

$= 95 \times \$1.95$

$=$ **\$185.25**

2. In 1 minute, 5 people can finish $= 20 \div 3\dfrac{20}{60}$

$= 6$ buns

In 5 minutes, 5 people can finish $= 6 \times 5$

$= 30$ buns

In 5 minutes, 10 people can finish $= 30 \times 2$

$=$ **60** buns

Practise Now 13

1. **(a)** **(i)** NZD 1 = PKR 178.7524

NZD 2360 = PKR 178.7524 × 2360

= **PKR 421 856** (to the nearest PKR 1)

(ii) PHP 100 = PKR 535.6015

PHP 1 = $\dfrac{\text{PKR } 535.6015}{100}$

PHP 25 600 = $\dfrac{\text{PKR } 535.6015}{100} \times 25\,600$

= **PKR 137 114** (to the nearest PKR 1)

(b) **(i)** PKR 324.9140 = EUR 1

PKR 1 = EUR $\dfrac{1}{324.9140}$

PKR 5690 = EUR $\dfrac{1}{324.9140} \times 5690$

= **EUR 17.51** (to the nearest EUR 0.01)

(ii) PKR 768.5302 = THB 100

PKR 1 = THB $\dfrac{1}{768.5302}$

PKR 7460 = THB $\dfrac{1}{768.5302} \times 7460$

= **THB 971** (to the nearest THB 1)

2. HK\$100 = S\$16.988

HK\$1 = S\$ $\dfrac{16.988}{100}$

HK\$35 000 = S\$ $\dfrac{16.988}{100} \times 35\,000$

= S\$5945.80

Amount of remaining Singapore dollars

= S\$(5945.80 – 3500)

= S\$2445.80

S\$16.995 = HK\$100

S\$1 = HK\$ $\dfrac{100}{16.995}$

Amount of Hong Kong dollars they receive

= HK\$ $\left(\dfrac{100}{16.995} \times 2445.80\right)$

= **HK\$14 391** (to the nearest dollar)

Exercise 9B

1. **(a)** Number of words that she can type per minute

$= \dfrac{1800}{60}$

(1 hour = 60 minutes)

$=$ **30**

(b) Cost of one unit of electricity

$= \$\dfrac{120.99}{654}$

$=$ **\$0.19**

(c) His monthly rental rate

$= \$\dfrac{4800}{3}$

$=$ **\$1600**

(d) Its mass per metre

$= \dfrac{15}{3.25}$

$= \mathbf{4\dfrac{8}{13}}$ **kg** or **4.62 kg** (to 3 s.f.)

2. Time taken for Ali to fold 1 paper plane

$= \dfrac{20}{15}$

$= 1.\dot{3}$ minutes

Time taken for Nadia to fold 1 paper plane

$= \dfrac{25}{18}$

$= 1.3\dot{8}$ minutes

Time taken for Kumar to fold 1 paper plane

$= \dfrac{21}{16}$

$= 1.3125$ minutes

Thus, **Kumar** can fold paper planes at the fastest rate.

3. The rate of 0.004% means that there are 4 crimes recorded per 100 000 people

4. 3 hours = 180 minutes

Number of ornaments made in 3 hours

$= \dfrac{180}{15} \times 4$

$= 48$

Amount earned by the worker

$= 48 \times \$1.15$

$=$ **\$55.20**

5. (i) Distance travelled on 1 litre of petrol

$= \frac{259.6}{22}$

= 11.8 km

Distance travelled on 63 litres of petrol

= 11.8 × 63

= **743.4 km**

(ii) Amount of petrol required to travel a distance of 2013.2 km

$= \frac{2013.2}{11.8}$

$= 170\frac{36}{59}$ litres

Amount that the car owner has to pay

$= 170\frac{36}{59} \times \$1.99$

= **$339.51**

6. (i) Amount of fertiliser needed for a plot of land that has an area of 1 m^2

$= \frac{200}{8}$

= 25 g

Amount of fertiliser needed for a plot of land that has an area of 14 m^2

= 25 × 14

= **350 g**

(ii) Area of land that can be fertilised by 450 g of fertiliser

$= \frac{450}{25}$

= **18 m^2**

7. (a) USD 1 = PKR 295.1922

USD 765 = PKR 295.1922 × 765

= **PKR 225 822** (to the nearest PKR 1)

(b) IDR 100 = PKR 1.9201

IDR 1 = $\frac{\text{PKR } 1.9201}{100}$

IDR 2 560 000 = $\frac{\text{PKR } 1.9201}{100} \times 2\ 560\ 000$

= **PKR 49 155** (to the nearest PKR 1)

8. Brand A:

Cost of 10 packs = 2 × $5.75 − $1.05

= $10.45

Total number of pieces of facial tissues in 10 packs = 5 × 200 × 2

= 2000

Cost per piece = $\frac{\$10.45}{2000}$ = $0.005225

Brand B:

Total number of pieces of facial tissues in 5 packs = 5 × 150

= 750

Cost per piece = $\frac{\$3.95}{750}$ = $0.00527 (to 5 d.p.)

Brand C:

Total number of pieces of facial tissues in 5 packs = 5 × 200

= 1000

Cost per piece = $\frac{\$4.55}{1000}$ = $0.00445

Brand D:

Cost of 10 packs = 2 × $5.75 − $1.75

= $9.75

Total number of pieces of facial tissues in 10 packs = 5 × 200 × 2

= 2000

Cost per piece = $\frac{\$9.75}{2000}$ = $0.004875

Since the cost per piece for Brand C is the lowest, **Brand C** is the best buy.

9. (i) Temperature of the metal after 9 minutes

= 428 °C − [(23 °C × 3) + (15 °C × 6)]

= **269 °C**

(ii) Temperature of the metal after 18 minutes

= 428 °C − [(23 °C × 3) + (15 °C × 15)]

= 134 °C

Amount of temperature needed for the metal to fall so that it will reach a temperature of 25 °C

= 134 °C − 25 °C

= 109 °C

Time needed for the metal to reach a temperature of 25 °C

$= \frac{109}{8} + 18$

= **$31\frac{5}{8}$ minutes**

10. 4 weeks need 15 2-litre bottles of cooking oil.

1 week needs = $\frac{15 \times 2}{4}$ = 7.5 litres of cooking oil.

So 10-week period needs = 10 × 7.5 = 75 litres of cooking oil.

Number of 5-litre tins of cooking oil needed for a 10-week period

$= \frac{75}{5}$

= **15**

11. (i) Total amount to be paid to the men

= 224 × $7.50

= **$1680**

(ii) Number of normal working hours from 9 a.m. to 6 p.m. excluding lunch time

= 8 hours

Let the number of overtime hours needed to complete the project in 4 days by each worker be x.

$$4[4(8 + x)] = 224$$
$$16(8 + x) = 224$$
$$128 + 16x = 224$$
$$16x = 224 - 128$$
$$16x = 96$$
$$x = 6$$

Overtime hourly rate

= 1.5 × $7.5

= $11.25

Total amount to be paid to the 4 men if the project is to be completed in 4 days

= 4{4[(8 × $7.5) + (6 × $11.25)]}

= **$2040**

12. NZ$100 = S$94.85

$\text{NZ\$1} = \text{S\$}\,\dfrac{94.85}{100}$

$\text{NZ\$3200} = \text{S\$}\,\dfrac{94.85}{100} \times 3200 = \text{S\$3035.20}$

Amount of remaining Singapore dollars
= S$(3035.20 – 2560.20)
= S$475

S$97.65 = NZ$100

$\text{S\$1} = \text{NZ\$}\,\dfrac{100}{97.65}$

Amount of New Zealand dollars they receive

$= \text{NZ\$}\left(\dfrac{100}{97.65} \times 475\right)$

= **NZ$486.43** (to the nearest cent)

13. (a) **RS Forex**. With reference to the 'We Sell' column, Vasi will get NZD 1 for every PKR 204.4186 from PQ Money Changer and NZD 1 for every PKR 203.6634 from RS Forex. He will thus receive more New Zealand dollars if he changes his money at RS Forex, given the lower rate.

(b) **RS Forex**. With reference to the 'We Buy' column, Raju will get PKR 303.8489 for every USD 1 from PQ Money Changer and PKR 304.1395 for every USD 1 from RS Forex. He will thus receive more Pakistani rupees if he changes his money at RS Forex, given the higher rate.

14. (a) Website G:
Price of watch when converted to SGD

$= \text{PKR}\,\dfrac{430}{0.2650} \times 100$

= PKR 162 264 (to the nearest PKR 1)

Website U:
Price of watch when converted to SGD

$= \text{PKR}\,\dfrac{430}{0.3299} \times 100$

= PKR 181 873 (to the nearest PKR 1)

Since the price is lower on website G, **website G** offers a better buy.

(b) Maximum difference = PKR 181 873 – PKR 162 264
= **PKR 19 609**

9.3 Speed

Practise Now 14A

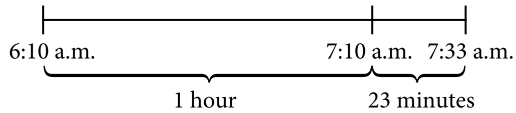
1. (a) 9.15 a.m. = **09 15**
(b) 8.59 p.m. = **20 59**
(c) 12.10 a.m. = **00 10**

2. (a) 00 08 = **12.08 a.m.**
(b) 02 10 = **2.10 a.m.**
(c) 12 56 = **12.56 p.m.**

3. 23 59 is 11.59 p.m.
One minute after 11.59 p.m. is 12 midnight. In 24-hour format, 12 midnight is **00 00**.

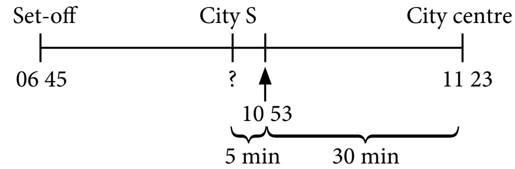
Practise Now 14B

1.



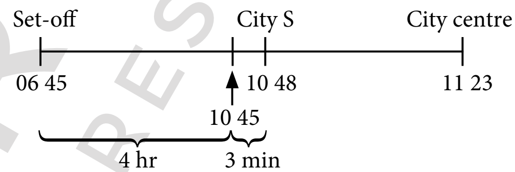
Duration between 6.10 a.m. and 7.10 a.m. = 1 hour
Duration between 7.10 a.m. and 7.33 a.m. = 23 minutes
∴ Duration of the run = **1 hour 23 minutes.**

2. (i)



30 minutes before 11 23 = 10 53
∴ Arrival time at City S
= 5 minutes before 11 53
= **10 48**

(ii)



∴ Duration of bus ride to City S = **4 hours 3 minutes**

Class Discussion: Time zones

1. Arrival time in Singapore = **09 55**

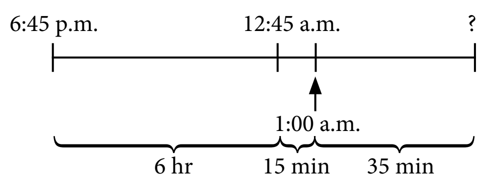
2. Singapore is in a **different time zone** as Pakistan. Thus, a person on the flight would arrive in Singapore at 09 55 according to local time in Pakistan.

3. Arrival time according to local time in Pakistan = 09 55
Arrival time according to local time in Singapore (shown in the flight itinerary) = 12 55
∴ time difference = **3 hours**
Local time in Singapore is **ahead** of that in Pakistan.
Teachers may show some examples of international news reports that involve the use of the term 'local time' and get the students to discuss its meaning.

4. Departure time in Singapore = 12 20
Departure time according to local time in Pakistan = 09 20.
Duration between 09 20 and 17 20 = 8 hours
Duration between 17 20 and 17 35 = 15 minutes
∴ flight duration = **8 hours 15 minutes**.

Practise Now 15A

Departure time in Tokyo = 8.45 p.m.
Departure time according to local time in Bangkok = 6.45 p.m.



Arrival time in Bangkok is 35 minutes after 1.00 a.m., i.e. **1.35 a.m. on 7 August 2023**.

Practise Now 15B

1. **(a)** 60 min = 1 h

$1 \text{ min} = \frac{1}{60} \text{ h}$

$18 \text{ min} = 18 \times \frac{1}{60}$

$= \frac{18}{60} \text{ h}$

$= \mathbf{\frac{3}{10}} \text{ h}$

(b) 60 min = 1 h

$1 \text{ min} = \frac{1}{60} \text{ h}$

$12 \text{ min} = 12 \times \frac{1}{60} \text{ h}$

$= \frac{12}{60} \text{ h}$

$= \frac{2}{10} \text{ h}$

= 0.2 h

∴ 4 h 23 min = **4.2 h**

(c) 60 s = 1 min

$1 \text{ s} = \frac{1}{60} \text{ min}$

$855 \text{ s} = 855 \times \frac{1}{60} \text{ min}$

$= \frac{57}{4} \text{ min}$

60 min = 1 h

$1 \text{ min} = \frac{1}{60} \text{ h}$

$\frac{57}{4} \text{ min} = \frac{57}{4} \times \frac{1}{60} \text{ h}$

$= \frac{19}{80} \text{ h}$

$= \frac{2375}{10\,000} \text{ h}$

= **0.2375 h**

2. **(a)** 1 h = 60 min

2 h = 2 × 60 min

= 120 min

∴ 2 h 9 min

= 120 min + 9 min

= **129 min**

(b) 1 min = 60 s

$\frac{2}{5} \text{ min} = \frac{2}{5} \times 60 \text{ s}$

= 24 s

∴ $4\frac{2}{5}$ min = **4 min 24 s**

(c) 1 h = 60 min

0.45 h = 0.45 × 60 min

= **27 min**

(d) 1 min = 60 s

1 h = 60 min

= 60 × 60 s

= 3600 s

$\frac{7}{20} \text{ h} = \frac{7}{20} \times 3600 \text{ s}$

= **1260 s**

Practise Now 16

1. **(i)** $25 \text{ minutes} = \frac{25}{60} \text{ hours}$

Speed of the train $= \dfrac{16.8}{\left(\frac{25}{60}\right)}$ = **40.32 km/h**

(ii) 16.8 km = 16 800 m

25 minutes = 25 × 60 = 1500 seconds

Speed of the train $= \frac{16\,800}{1500}$ = **11.2 m/s**

2. $55 \text{ km/h} = \frac{55 \text{ km}}{1 \text{ h}} = \frac{(55 \times 1000) \text{ m}}{3600 \text{ s}} = 15\frac{5}{18} \text{ m/s}$

12 minutes 30 seconds = (12 × 60) + 30 = 750 seconds

Distance travelled $= 15\frac{5}{18} \times 750 = \mathbf{11\,458\frac{1}{3}}$ **m**

3. **(i)** Highest speed limit for cars = 120 km/h

$= \frac{120 \text{ km}}{1 \text{ h}}$

$= \frac{120\,000 \text{ m}}{3600 \text{ s}}$

= **33.3 m/s** (to 3 s.f.)

(ii) Highest speed limit for cars = 120 km/h

$= \frac{120 \text{ km}}{1 \text{ h}}$

$= \frac{12\,000\,000 \text{ cm}}{60 \text{ min}}$

= **200 000 cm/min**

4. Speed of the swimmer

$= \frac{100 \text{ m}}{46.86 \text{ s}}$

$= \frac{(100 \div 1000) \text{ km}}{(46.86 \div 3600) \text{ h}}$

= 7.6825 km/h (to 5 s.f.)

No. of times the sailfish is faster than the swimmer

$= \frac{109}{7.6825}$

= **14.2** (to 3 s.f.)

Class Discussion (Making sense of speeds in real-world contexts)

1. Speed limit = 40 km/h

$= \frac{40\,000 \text{ m}}{60 \times 60 \text{ s}}$

$= 11\frac{1}{9} \text{ m/s}$

∴ Ali's speed of 30 m/s is too fast and he should slow down.

2. **(a)** **(i)** Average speed of Usain Bolt during the sprint

$= \frac{100 \text{ m}}{9.58 \text{ s}}$

= **10.4 m/s** (to 3 s.f.)

(ii) 10.4 m/s is the average speed for the whole distance of the sprint. 12.4 m/s is the fastest speed he attained at a specific instance in the sprint, within the 100 m.

(b) Maximum speed of zebra = 40 miles/h

$= \frac{40}{1.6} \text{ km/h}$

= 25 km/h

$= \frac{25\,000 \text{ m}}{60 \times 60 \text{ s}}$

= 6.94 m/s (to 3 s.f.)

This zebra is **not faster** than Usain Bolt.

Practise Now 17

Time taken for Raju to swim a distance of 1.5 km

$= \frac{1.5}{2.5}$ h

$= \frac{3}{5}$ h

Total time taken

$= \frac{3}{5} + 1\frac{1}{2} + 1\frac{1}{9}$

$= \frac{3}{5} + \frac{3}{2} + \frac{10}{9}$

$= \frac{54}{90} + \frac{135}{90} + \frac{100}{90}$

$= \frac{289}{90}$ hours

Distance that Raju runs

$= 9 \times 1\frac{1}{9}$

$= 9 \times \frac{10}{9}$

= 10 km

Total distance travelled

= 1.5 + 40 + 10

= 51.5 km

Average speed for the entire competition

$= \frac{\text{total distance travelled}}{\text{total time taken}}$

$= \frac{51.5}{\left(\frac{289}{90}\right)}$

= **16.0 km/h** (to 3 s.f.)

Thinking Time (Page 234)

1. The definition of average speed is the total distance travelled divided by the total time taken. Average speed is different from the general meaning of 'average' in statistics. The word 'average' here does not refer to the sum of all individual speeds divided by the total number of individual speeds.
2. The bases may be different. We need to compute the total distance and the total time to find the correct average speed.

Practise Now 18

Let the distance for the car to travel from Town A to Town B to meet the truck = x km.

Then the time taken for the car to travel from Town A to Town B to meet the truck at an average speed of 72 km/h = $\frac{x}{72}$ hour, and the time taken for the truck to travel from Town B to Town A to meet the car at an average speed of 38 km/h

$= \frac{550 - x}{38}$ hour

$\therefore \frac{x}{72} = \frac{550 - x}{38}$

$38x = 39\,600 - 72x$

$38x + 72x = 39\,600$

$110x = 39\,600$

$x = 360$ km

Hence, the time taken for the two vehicles to meet $= \frac{360}{72}$

= **5 hours**

Exercise 9C

1. **(a)** Duration between 4.15 p.m. and 9.15 p.m. = 5 h
   Duration between 9.15 p.m. and 9.29 p.m. = 14 min
   $\therefore$ Duration between 4.15 p.m. and 9.29 p.m. = **5 h 14 min**

   (b) Duration between 12 16 and 20 16 = 8 h
   Duration between 20 16 and 20 58 = 42 min
   $\therefore$ Duration between 12 16 and 20 58 = **8 h 42 min**

   (c) Duration between 04 35 and 09 35 = 5 h
   Duration between 09 32 and 09 35 = 3 min
   $\therefore$ Duration between 04 35 and 09 32 = **4 h 57 min**

   (d) 2.01 p.m. = 14 01
   Duration between 13 54 and 14 01
   = duration between 13 55 and 14 02
   = 5 min + 2 min
   = **7 min**

2. 3 hours after 14 20 = 17 20
   15 minutes after 17 20 = **17 35**
   Waseem left the fun fair at **17 35**.

3. Arrival time in Tokyo = 10.50 p.m.
   Arrival time according to local time in Hong Kong = 9.50 p.m.
   The duration will be calculated based on the local time in Hong Kong.
   Duration between 5.35 p.m. and 9.35 p.m. = 4 h
   Duration between 9.35 p.m. and 9.50 p.m. = 15 min
   $\therefore$ flight duration = **4 h 15 min**.

4. **(a)** 60 s = 1 min

   1 s $= \frac{1}{60}$ min

   49 s $= 496 \times \frac{1}{60}$ min

   $= \mathbf{\frac{49}{60}}$ **min**

   (b) 60 min = 1 h

   1 min $= \frac{1}{60}$ h

   36 min $= 36 \times \frac{1}{60}$ h

   $= \frac{36}{60}$ h

   $= \frac{6}{10}$ h

   = **0.6 h**

   (c) 60 s = 1 min

   1 s $= \frac{1}{60}$ min

   66 s $= 66 \times \frac{1}{60}$ min

   $= \frac{66}{60}$ min

   $= \frac{11}{10}$ min

   = **1.1 min**

(d) 60 s = 1 min

$1 \text{ s} = \frac{1}{60}$ min

$54 \text{ s} = 54 \times \frac{1}{60}$ min

$= \frac{54}{60}$ min

$= \frac{9}{10}$ min

= 0.9 min

$\therefore$ 2 min 54 s = **2.9 min**

(e) 1 min = 60 s

1 h = 60 min

= 60 × 60 s

= 3600 s

$1 \text{ s} = \frac{1}{3600}$ h

$945 \text{ s} = 945 \times \frac{1}{3600}$ h

$= \frac{21}{80}$ h

$= \frac{2625}{10\,000}$ h

= **0.2625 h**

(f) 60 min = 1 h

$1 \text{ min} = \frac{1}{60}$ h

$25 \text{ min} = 25 \times \frac{1}{60}$ h

$= \frac{25}{60}$ h

$= \frac{5}{12}$ h

1 min = 60 s

1 h = 60 min

= 60 × 60 s

= 3600 s

$1 \text{ s} = \frac{1}{3600}$ h

$3 \text{ s} = 3 \times \frac{1}{3600}$ h

$= \frac{3}{3600}$ h

$= \frac{1}{1200}$ h

$\therefore$ 25 min 3 s

$= \frac{5}{12} \text{ h} + \frac{1}{1200}$ h

$= \frac{500}{1200} \text{ h} + \frac{1}{1200}$ h

$= \frac{501}{1200}$ h

$= \mathbf{\frac{127}{400}}$ **h**

5. **(a)** 1 min = 60 s

0.13 min = 0.13 × 60 s

= **7.8 s**

(b) 1 h = 60 min

$\frac{1}{3} = \frac{1}{3} \times 60$ min

= **20 min**

(c) 1 min = 60 s

1 h = 60 min

= 60 × 60 s

= 3600 s

$1\frac{5}{6} \text{ h} = \frac{11}{6} \times 3600$ s

= **6600 s**

(d) 1 h = 3600 s

0.96 h = 0.96 × 3600 s

= **3456 s**

6. **(i)** $30 \text{ minutes} = \frac{30}{60} = \frac{1}{2}$ hour

Speed of the particle

$= \frac{24.6 \text{ km}}{\left(\frac{1}{2}\right) \text{hour}}$

= **49.2 km/h**

(ii) 24.6 km = 24.6 × 1000 = 24 600 m

30 minutes = 30 × 60 = 1800 s

Speed of the particle

$= \frac{24\,600 \text{ m}}{1800 \text{ s}}$

$= \mathbf{13\frac{2}{3}}$ **m/s** or **13.7 m/s** (to 3 s.f.)

7. $1224 \text{ hours} \xrightarrow{\text{1 hour 48 minutes}} 1412 \text{ hours}$

$1 \text{ hour } 48 \text{ minutes} = 1\frac{48}{60} = 1\frac{4}{5}$ hours

Distance between the two stations

$= 200 \times 1\frac{4}{5}$

= 360 km

= 360 × 1000 m

= **360 000 m**

8. **(a)** 8.4 km/min

$= \frac{8.4 \text{ km}}{1 \text{ min}}$

$= \frac{8.4 \text{ km}}{\left(\frac{1}{60}\right) \text{h}}$

= **504 km/h**

(b) 315 m/s

$= \frac{315 \text{ m}}{1 \text{ s}}$

$= \frac{\left(\frac{315}{1000}\right) \text{km}}{\left(\frac{1}{3600}\right) \text{h}}$

= **1134 km/h**

(c) 242 m/min

$= \frac{242 \text{ m}}{1 \text{ min}}$

$= \frac{\left(\frac{242}{1000}\right) \text{km}}{\left(\frac{1}{60}\right) \text{h}}$

= **14.52 km/h**

(d) 125 cm/s

$= \dfrac{125\text{ cm}}{1\text{ s}}$

$= \dfrac{\left(\dfrac{125}{100\,000}\right)\text{km}}{\left(\dfrac{1}{3600}\right)\text{h}}$

$=$ **4.5 km/h**

9. **(a)** 65 cm/s

$= \dfrac{65\text{ cm}}{1\text{ s}}$

$= \dfrac{\left(\dfrac{65}{100}\right)\text{m}}{1\text{ s}}$

$=$ **0.65 m/s**

(b) 367 km/h

$= \dfrac{367\text{ km}}{1\text{ h}}$

$= \dfrac{367 \times 1000\text{ m}}{3600\text{ s}}$

$= \dfrac{367\,000\text{ m}}{3600\text{ s}}$

$=$ **102 m/s** (to 3 s.f.)

(c) 1000 cm/min

$= \dfrac{1000\text{ cm}}{1\text{ min}}$

$= \dfrac{\left(\dfrac{1000}{100}\right)\text{m}}{60\text{ s}}$

$= \boldsymbol{\dfrac{1}{6}}$ **m/s** or **0.167 m/s** (to 3 s.f.)

(d) 86 km/min

$= \dfrac{86\text{ km}}{1\text{ min}}$

$= \dfrac{86 \times 1000\text{ m}}{60\text{ s}}$

$= \boldsymbol{1433\dfrac{1}{3}}$ **m/s**

10. Speed of Shajar Abbas

$= \dfrac{100\text{ m}}{10.37\text{ s}}$

$= \dfrac{\left(\dfrac{100}{1000}\right)\text{km}}{\left(\dfrac{10.37}{3600}\right)\text{h}}$

$= 34.716$ (to 5 s.f.)

Number of times the bullet train is as fast as the fastest Pakistani sprinter

$= \dfrac{365}{34.716}$

$=$ **10.5 times** (to 3 s.f.)

11. Time taken to travel the first part of the journey

$= \dfrac{19}{57}$

$= \dfrac{1}{3}$ h

Time taken to travel the remaining part of the journey

$= \dfrac{55}{110}$

$= \dfrac{1}{2}$ h

Average speed of the car for its entire journey

$= \dfrac{\text{total distance travelled}}{\text{total time taken}}$

$= \dfrac{19+55}{\dfrac{1}{3}+\dfrac{1}{2}}$

$= \dfrac{74}{\dfrac{5}{6}}$

$=$ **88.8 km/h**

12. Total duration of musical including intermission

= 2 h 12 min + 20 min

= 2 h 32 min

2 hours before 10.06 p.m.

= 8.06 p.m.

Time of commencement of musical

= 32 minutes before 8.06 p.m.

= 7.34 p.m.

Cheryl's arrival time

= 15 minutes before 7.34 p.m.

= **7.19 p.m.**

13. Departure time in Melbourne

= 20 minutes after 15 05

= 15 25

Departure time according to local time in Singapore = 13 25

7 hours after 13 25 = 20 25

35 minutes after 20 25 = 21 00

$\therefore$ arrival time in Singapore is **21 00**.

14. Time taken for car and bus to meet = 3 h

Distance travelled by car $= 90 \times 3$

$= 270$ km

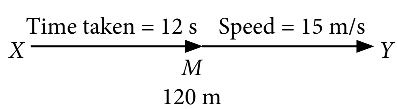
Distance travelled by bus $= 510 - 270$

$= 240$ km

Average speed of bus $= \dfrac{240}{3}$

$=$ **80 km/h**

15.



(i) Time taken to travel from M to Y

$= \dfrac{60}{15}$

$=$ **4 s**

(ii) Average speed of the object for its entire journey from X to Y

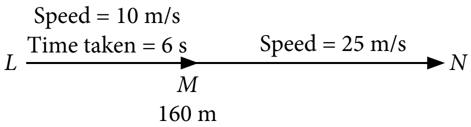
$= \dfrac{\text{total distance travelled}}{\text{total time taken}}$

$= \dfrac{120}{12+4}$

$= \dfrac{120}{16}$

$= \mathbf{7\frac{1}{2}\ m/s}$

16.



Distance travelled from L to M

$= 10 \times 6$

$= 60$ m

Thus, distance travelled from M to N

$= 160 - 60$

$= 100$ m

Time taken to travel from M to N

$= \dfrac{100}{25}$

$= 4$ s

Average speed of the object for its entire journey from L to N

$= \dfrac{\text{total distance travelled}}{\text{total time taken}}$

$= \dfrac{160}{6+4}$

$= \dfrac{160}{10}$

$=$ **16 m/s**

17. Time taken to travel the first 50 km of its journey

$= \dfrac{50 \times 1000 \text{ m}}{25 \text{ m/s}}$

$= 2000$ s

$= \dfrac{2000}{3600}$ h

$= \dfrac{5}{9}$ h

Time taken to travel the next 120 km of its journey

$= \dfrac{120}{80}$

$= 1\dfrac{1}{2}$ h

Distance travelled for the last part of its journey

$= 90 \times \dfrac{35}{60}$

$= 52.5$ km

Average speed of the object for its entire journey

$= \dfrac{\text{total distance travelled}}{\text{total time taken}}$

$= \dfrac{50+120+52.5}{\frac{5}{9}+1\frac{1}{2}+\frac{35}{60}}$

$=$ **84.3 km/h** (to 3 s.f.)

18. Average speed for the entire journey $= \dfrac{\text{total distance travelled}}{\text{total time taken}}$

$$\frac{4x+2y}{6} = 60$$

$$4x + 2y = 360$$

Possible values of x and y include $\mathbf{x = 50, y = 80}$.

19. Length of the goods train

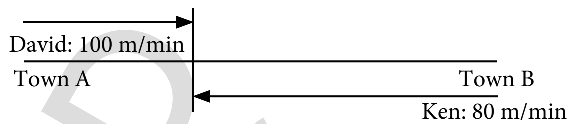
$= \left(72 \times \dfrac{8}{3600}\right) + \left(54 \times \dfrac{8}{3600}\right)$

$= \dfrac{7}{25}$ km

$= \dfrac{7}{25} \times 1000$

$=$ **280 m**

20.

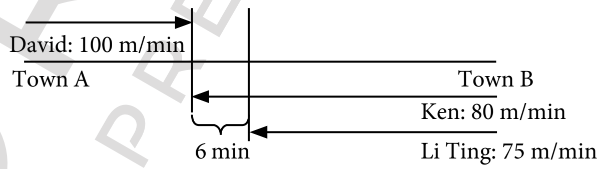


Let David meet Ken at x min.

So distance between Town A and Town B

$= 100 \times x + 80 \times x$

$= 180x$ m



Then David meets Li Ting at $(x + 6)$ min.

Distance between Town A and Town B

$= 100 \times (x + 6) + 75 \times (x + 6)$

$= 100x + 600 + 75x + 450$

$= (175x + 1050)$ m

Thus,

$$\begin{aligned} 180x &= 175x + 1050 \\ 180x - 175x &= 1050 \\ 5x &= 1050 \\ x &= 210 \end{aligned}$$

Distance between Town A and Town B

$= 180 \times 210$

$=$ **37 800 m**

21. Speed for return journey $= (s + 4)$ km/h

$$\begin{aligned} 2.5s &= \left(2.5 - \frac{1}{4}\right)(s+4) \\ &= 2.25s + 9 \\ 0.25s &= 9 \\ s &= 36 \end{aligned}$$

Average speed for the whole journey $= \dfrac{\text{total distance travelled}}{\text{total time taken}}$

$= \dfrac{2.5(36) \times 2}{2.5 + 2.5 - \frac{1}{4}}$

$= \dfrac{180}{4.75}$

$=$ **37.9 km/h** (to 3 s.f.)

Chapter 10 Basic Geometry

TEACHING NOTES

Suggested Approach

Students have learnt angle measurement in primary school. They have learnt the properties, namely, angles on a straight line, angles at a point and vertically opposite angles. However, students are unfamiliar with the types of angles and using algebraic terms in basic geometry. There is a need to guide students to apply basic algebra and linear equations in this topic. Students will learn how to do this through the worked examples in this topic. Teachers can introduce basic geometry by showing real-life applications (see Introductory Problem on page 240).

Section 10.1: Basic geometrical concepts and notations

Teachers should illustrate what a point, a line, intersecting lines and planes look like. Teachers can impress upon the students that there is a difference between a line and a ray. A ray has a direction while a line has no direction. Teachers can highlight to the students that for a ray, the arrowhead indicates the direction in which the ray extends while for a line, its arrowhead is to indicate that the line continues indefinitely.

The thinking time on page 241 of the textbook requires students to think and determine whether each of the statements is true or false. Teachers should make use of this opportunity to highlight and clear some common misconceptions about points, lines and planes.

Teachers can build upon prerequisites, namely angle measurement, to introduce the types of angles by classifying angle measurements according to their sizes.

To make practice more interesting, teachers can get the students to work in groups to measure and classify the various types of angles of different objects (i.e. scissors, set square, compass and the hands of a clock).

Section 10.2: Properties of angles formed by intersecting lines

Teachers should recap with students on what they have learnt in primary school, i.e. angles on a straight line, angles at a point and vertically opposite angles. Students are encouraged to show that vertically opposite angles of two intersecting lines are equal through Journal Writing on page 247. After going through Worked Examples 1 to 3, students should be able to identify the properties of angles and use algebraic terms to form and solve a linear equation to find the value of the unknowns. Students are expected to state reasons in their working.

Section 10.3: Properties of angles formed by two parallel lines and transversal

Teachers can get students to discuss examples where they encounter parallel lines in their daily lives and ask them what happens when a line or multiple lines cut the parallel lines.

To make learning more interactive, students are given the opportunity to explore the three angle properties observed when a pair of parallel lines is cut by a transversal (see Investigation: Corresponding angles, alternate angles and interior angles). Through this investigation, students should be able to observe the properties of angles associated with parallel lines. The investigation also helps students to learn how to solve problems involving angles formed by two parallel lines and a transversal. Students are expected to use appropriate algebraic variables to form and solve linear equations to find the value of the unknowns. Teachers should emphasise the importance of stating the properties when the students are solving questions on basic geometry.

Introductory Problem
Corresponding angles, e.g. $\angle x$ and $\angle y$, are measured and checked if they are equal. By converse of corresponding angles, if $\angle x$ and $\angle y$ are equal, then the lines are parallel.

10.1 Basic geometrical concepts and notations

Thinking Time (Page 241)
(a) **False**. There are an infinite number of points lying on a line segment.
(b) **True**.
(c) **False**. There is exactly one line that passes through any three distinct points which are collinear; it is not possible to draw a line that passes through three distinct points which are non-collinear.
(d) **False**. Two distinct lines intersect at one point; two coincident lines intersect at an infinite number of points; two parallel lines do not intersect at any point.
(e) **True**.

Practise Now 1A
(a) **Acute**
(b) **Reflex**
(c) **Obtuse**
(d) **Straight**
(e) **Right**
(f) **Reflex**
(g) **Obtuse**
(h) **Reflex**
(i) **Acute**

10.2 Properties of angles formed by intersecting lines

Practise Now 1B
1. **(a)** $b° + 122° = 180°$ (adj. $\angle$s on a str. line)
$b° = 180° - 122°$
$= 58°$
$\therefore b = \mathbf{58}$

(b) $95.3° + 64.8° + \angle COB = 180°$ (adj. $\angle$s on a str. line)
$\angle COB = 180° - 95.3° - 64.8°$
$= \mathbf{19.9°}$

2. $2c° + 100° + 3c° = 180°$ (adj. $\angle$s on a str. line)
$5c° + 100° = 180°$
$5c° = 180° - 100°$
$= 80°$
$c° = 16°$
$\therefore c = \mathbf{16}$

Practise Now 2
1. $58° + 148° + 7a° = 360°$ ($\angle$s at a point)
$7a° = 360° - 58° - 148°$
$= 154°$
$a° = 22°$
$\therefore a = \mathbf{22}$

2. $b° + 4b° + b° + 90° = 360°$ ($\angle$s at a point)
$6b° + 90° = 360°$
$6b° = 360° - 90°$
$= 270°$
$b° = 45°$
$\therefore b = \mathbf{45}$

Journal Writing (Vertically opposite angles)
$\angle p + \angle q = 180°$ (adj. $\angle$s on a str. line)
$\angle q + \angle r = \mathbf{180°}$ **(adj. $\angle$s on a str. line)**
$\therefore \angle p + \angle q = \angle q + \angle r$
$\therefore \angle p = \angle \boldsymbol{r}$
Similarly, $\angle q = \angle \boldsymbol{s}$.

Practise Now 3
(a) $B\hat{O}C = 90° + 53°$ (vert. opp. $\angle$s)
$= \mathbf{143°}$

(b) $3a° + 40° = a° + 60°$ (vert. opp. $\angle$s)
$3a° - a° = 60° - 40°$
$2a° = 20°$
$a° = 10°$
$\therefore \boldsymbol{a} = \mathbf{10}$
$a° + 60° + 4b° + 10° = 180°$
$10° + 60° + 4b° + 10° = 180°$
$4b° = 180° - 10° - 60° - 10°$
$= 100°$
$b° = 25°$
$\therefore \boldsymbol{b} = \mathbf{25}$

Exercise 10A
1. **(a)** $\boldsymbol{a} = \mathbf{79}$, $\boldsymbol{b} = \mathbf{106}$, $\boldsymbol{c} = \mathbf{98}$
(b) $\boldsymbol{d} = \mathbf{50}$, $\boldsymbol{e} = \mathbf{227}$
(c) $\boldsymbol{f} = \mathbf{117}$, $\boldsymbol{g} = \mathbf{45}$
(d) $\boldsymbol{h} = \mathbf{244}$, $\boldsymbol{i} = \mathbf{94}$, $\boldsymbol{j} = \mathbf{56}$

2. **(a)** **Obtuse**
(b) **Reflex**
(c) **Acute**
(d) **Right**
(e) **Straight**
(f) **Reflex**
(g) **Acute**
(h) **Obtuse**

3. **(a)** Complementary angle of $18° = 90° - 18°$
$= \mathbf{72°}$
(b) Complementary angle of $46° = 90° - 46°$
$= \mathbf{44°}$
(c) Complementary angle of $53° = 90° - 53°$
$= \mathbf{37°}$
(d) Complementary angle of $64° = 90° - 64°$
$= \mathbf{26°}$

4. (a) Supplementary angle of $36° = 180° - 36°$
$= \mathbf{144°}$

(b) Supplementary angle of $12° = 180° - 12°$
$= \mathbf{168°}$

(c) Supplementary angle of $102° = 180° - 102°$
$= \mathbf{78°}$

(d) Supplementary angle of $171° = 180° - 171°$
$= \mathbf{9°}$

5. (a) $a° + 33° = 180°$ (adj. $\angle$s on a str. line)
$a° = 180° - 33°$
$= 147°$
$\therefore \boldsymbol{a = 147}$

(b) $b° + 42° + 73° = 180°$ (adj. $\angle$s on a str. line)
$b° = 180° - 42° - 73°$
$= 65°$
$\therefore \boldsymbol{b = 65}$

(c) $4c° + 80° + c° = 180°$ (adj. $\angle$s on a str. line)
$4c° + c° = 180° - 80°$
$5c° = 100°$
$c° = 20°$
$\therefore \boldsymbol{c = 20}$

(d) $4d° + 16° + 2d° + 14° = 180°$ (adj. $\angle$s on a str. line)
$4d° + 2d° = 180° - 16° - 14°$
$6d° = 150°$
$d° = 25°$
$\therefore \boldsymbol{d = 25}$

6. (a) $x° + y° + z° = 180°$ (adj. $\angle$s on a str. line)
When $y° = 45°$, $z° = 86°$,
$x° + 45° + 86° = 180°$
$x° = 180° - 45° - 86°$
$= 49°$
$\therefore \boldsymbol{x = 49}$

(b) $x° + y° + z° = 180°$ (adj. $\angle$s on a str. line)
When $x° = 2y°$, $z° = 3y°$,
$2y° + y° + 3y° = 180°$
$6y° = 180°$
$y° = 30°$
$\therefore \boldsymbol{y = 30}$

7. (a) $a° + 67° + 52° + 135° = 360°$ ($\angle$s at a point)
$a° = 360° - 67° - 52° - 135°$
$= 106°$
$\therefore \boldsymbol{a = 106}$

(b) $5b° + 4b° + 3b° = 360°$ ($\angle$s at a point)
$12b° = 360°$
$b° = 30°$
$\therefore \boldsymbol{b = 30}$

(c) $16c° + 4c° + 90° + 4c° = 360°$ ($\angle$s at a point)
$16c° + 4c° + 4c° = 360° - 90°$
$24c° = 270°$
$c° = 11.25°$
$\therefore \boldsymbol{c = 11.25}$

(d) $(7d + 23)° + 6d° + 139° + 5d° = 360°$ ($\angle$s at a point)
$7d° + 23° + 6d° + 139° + 5d° = 360°$
$7d° + 6d° + 5d° = 360° - 23° - 139°$
$18d° = 198°$
$d° = 11°$
$\therefore \boldsymbol{d = 11}$

8. (i) $A\hat{O}C = \mathbf{48°}$ (vert. opp. $\angle$s)

(ii) $90° + D\hat{O}E + 48° = 180°$ (adj. $\angle$s on a str. line)
$D\hat{O}E = 180° - 90° - 48°$
$= \mathbf{42°}$

9. (a) $40° + 30° + a° = 117°$ (vert. opp. $\angle$s)
$a° = 117° - 40° - 30°$
$= 47°$
$\therefore \boldsymbol{a = 47}$

(b) $7b° + 3b° = 180°$ (adj. $\angle$s on a str. line)
$10b° = 180°$
$b° = 18°$
$\therefore \boldsymbol{b = 18}$
$c° = 7b°$ (vert. opp. $\angle$s)
$= 7(18°)$
$= 126°$
$\therefore \boldsymbol{c = 126}$

10. (a) $x° + y° + z° = 180°$ (adj. $\angle$s on a str. line)
$y° + x° + z° = 180°$
When $y° = x° + z°$,
$y° + y° = 180°$
$2y° = 180°$
$y° = 90°$
$\therefore \boldsymbol{y = 90}$

(b) $x° + y° + z° = 180°$ (adj. $\angle$s on a str. line)
When $x° = y° = z°$,
$z° + z° + z° = 180°$
$3z° = 180°$
$z° = 60°$
$\therefore \boldsymbol{z = 60}$

11. $A\hat{O}B + D\hat{O}A = 180°$ (adj. $\angle$s on a str. line)
$A\hat{O}B + 5A\hat{O}B = 180°$
$6A\hat{O}B = 180°$
$\therefore A\hat{O}B = \mathbf{30°}$
$B\hat{O}C = 2A\hat{O}B$
$= 2 \times 30°$
$= \mathbf{60°}$
$C\hat{O}D = 4A\hat{O}B$
$= 4 \times 30°$
$= \mathbf{120°}$
$D\hat{O}A = 5A\hat{O}B$
$= 5 \times 30°$
$= \mathbf{150°}$

12. (a) $7a° + 103° = 180°$ (adj. $\angle$s on a str. line)
$7a° = 180° - 103°$
$= 77°$
$a° = 11°$
$\therefore \boldsymbol{a = 11}$
$2b° + 13° = 103°$ (vert. opp. $\angle$s)
$2b° = 103° - 13°$
$= 90°$
$b° = 45°$
$\therefore \boldsymbol{b = 45}$

(b) $62° + 49° + 3c° = 180°$ (adj. ∠s on a str. line)
$3c° = 180° - 62° - 49°$
$= 69°$
$c° = 23°$
$\therefore$ **c = 23**
$d° = 3c°$ (vert. opp. ∠s)
$= 69°$
$\therefore$ **d = 69**
$e° = 62° + 49°$ (vert. opp. ∠s)
$= 111°$
$\therefore$ **e = 111**

(c) $24° + 90° + f° = 104° + 32°$ (vert. opp. ∠s)
$f° = 104° + 32° - 24° - 90°$
$= 22°$
$\therefore$ **f = 22**
$24° + 90° + f° + 2g° = 180°$ (adj. ∠s on a str. line)
$24° + 90° + 22° + 2g° = 180°$
$2g° = 180° - 24° - 90° - 22°$
$= 44°$
$g° = 22°$
$\therefore$ **g = 22**
$h° = 2g°$ (vert. opp. ∠s)
$= 44°$
$\therefore$ **h = 44**

(d) $7i° + 41° = 10i° + 23°$ (vert. opp. ∠s)
$10i - 7i° = 41° - 23°$
$3i° = 18°$
$i° = 6°$
$\therefore$ **i = 6**
$7i° + 41° + 5j° - 18° = 180°$ (adj. ∠s on a str. line)
$7(6°) + 41° + 5j° - 18° = 180°$
$42° + 41° + 5j° - 18° = 180°$
$5j° = 180° - 42° - 41° + 18°$
$= 115°$
$j° = 23°$
$\therefore$ **j = 23**

13. (i) $(186 - 4x)° + 34° = 6x°$ (vert. opp. ∠s)
$186° - 4x° + 34° = 6x°$
$6x° + 4x° = 186° + 34°$
$10x° = 220°$
$x° = 22°$
$\therefore$ **x = 22**
$6x° + 3y° = 180°$ (adj. ∠s on a str. line)
$6(22°) + 3y° = 180°$
$132° + 3y° = 180°$
$3y° = 180° - 132°$
$= 48°$
$y° = 16°$
$\therefore$ **y = 16**

(ii) Obtuse $A\hat{O}D = (186 - 4x)° + 34°$
$= [186 - 4(22)]° + 34°$
$= 98° + 34°$
= 132°
Reflex $C\hat{O}E = 180° + (186 - 4x)°$
$= 180° + 98°$
= 278°

10.3 Properties of angles formed by two parallel lines and transversal

Investigation (Corresponding angles, alternate angles and interior angles)

1. **∠*a* = ∠*b***
2. **∠*c* = ∠*d***
3. **∠*b* + ∠*d* = 180°**
4. **(a)** ∠*a* = ∠***b*** (corr. ∠s)
   (b) ∠*c* = ∠***d*** (alt. ∠s)
   (c) ∠*b* + ∠*d* = **180°** (int. ∠s)
5. **No.**
6. As long as the two parallel lines remain parallel, the size of ∠*a* will remain the same when the angle is shifted upwards to the other parallel line.
7. ∠*c* = ∠*RMY* (corr. ∠s)
   ∠*RMY* = ∠*d* (vert. opp. ∠s)
   $\therefore$ ∠*c* = ∠*d*
8. **Method 1:**
   ∠*b* = ∠*a* (corr. ∠s)
   ∠*a* + ∠*d* = 180° (adj. ∠s on a str. line)
   $\therefore$ ∠*b* + ∠*d* = 180°
   Method 2:
   ∠*c* = ∠*d* (alt. ∠s)
   ∠*c* + ∠*b* = 180° (adj. ∠s on a str. line)
   $\therefore$ ∠*b* + ∠*d* = 180°

Practise Now 4A

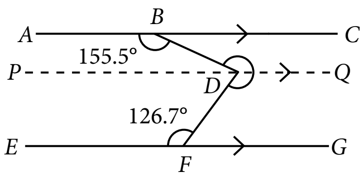
(a) (i) Any 1: ∠*a* and ∠*m*, ∠*b* and ∠*n*, ∠*c* and ∠*o*, ∠*d* and ∠*p*, ∠*e* and ∠*i*, ∠*f* and ∠*j*, ∠*g* and ∠*k*, ∠*h* and ∠*l*
(ii) Any 1: ∠*c* and ∠*m*, ∠*d* and ∠*n*, ∠*g* and ∠*i*, ∠*h* and ∠*j*
(iii) Any 1: ∠*c* and ∠*n*, ∠*d* and ∠*m*, ∠*g* and ∠*j*, ∠*h* and ∠*i*
(b) (i) **No**, ∠*e* ≠ ∠*a* as *PQ* is not parallel to *RS*.
(ii) **Yes**, ∠*g* = ∠*i* as *AB* is parallel to *CD* and ∠*g* and ∠*i* are alternate angles.
(iii) **No**, ∠*h* + ∠*c* ≠ 180° as *PQ* is not parallel to *RS*.

Practise Now 4B

1. $x° = 74°$ (corr. ∠s, *AB* // *CD*)
   $\therefore x =$ **74**
   Let the point of intersection of the lines *AB* and *RS* be *T*.
   Then ∠*BTS* = 80° (vert. opp. ∠s)
   $y° + 80° = 180°$ (int. ∠s, *AB* // *CD*)
   $y° = 180° - 80°$
   $= 100°$
   $\therefore y =$ **100**
   $z°$ = ∠*BTS* (corr. ∠s, *AB* // *CD*)
   $= 80°$
   $\therefore z =$ **80**
2. $2h° + 30° = 69°$ (corr. ∠s, *AB* // *CD*)
   $2h° = 39°$
   $h° = 19.5°$
   $\therefore h =$ **19.5**
   $2h° = k°$ (corr. ∠s, *AB* // *CD*)
   $k° = 39°$
   $\therefore k =$ **39**

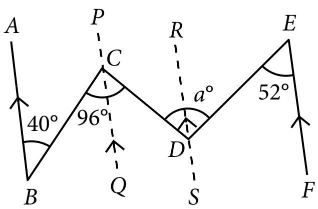
Practise Now 5

1.



$B\hat{D}Q = 155.5°$ (alt. $\angle$s, AC // PQ)
$F\hat{D}Q = 126.7°$ (alt. $\angle$s, PQ // EG)
reflex $B\hat{D}F = 155.5° + 126.7°$
$= \mathbf{282.2°}$

2.



$B\hat{C}Q = 40°$ (alt. $\angle$s, AB // PQ)
$Q\hat{C}D = 96° - 40°$
$= 56°$
$C\hat{D}R = 56°$ (alt. $\angle$s, PQ // RS)
$E\hat{D}R = 52°$ (alt. $\angle$s, RS // EF)
$a° = 56° + 52°$
$= 108°$
$\therefore a = \mathbf{108}$

Practise Now 6

Since $B\hat{W}Q = D\hat{Y}Q$ (= 122°), then AB // CD (converse of corr. $\angle$s).
$\therefore B\hat{X}S = C\hat{Z}R = \mathbf{65°}$ (alt. $\angle$s, AB // CD)

Exercise 10B

1. **(a)** **(i)** Any 2: $B\hat{X}R$ and $D\hat{Z}R$, $A\hat{X}R$ and $C\hat{Z}R$, $A\hat{X}S$ and $C\hat{Z}S$, $B\hat{X}S$ and $D\hat{Z}S$, $B\hat{W}P$ and $D\hat{Y}P$, $A\hat{W}P$ and $C\hat{Y}P$, $A\hat{W}Q$ and $C\hat{Y}Q$, $B\hat{W}Q$ and $D\hat{Y}Q$

(ii) Any 2: $A\hat{X}S$ and $D\hat{Z}R$, $B\hat{X}S$ and $C\hat{Z}R$, $A\hat{W}Q$ and $D\hat{Y}P$, $B\hat{W}Q$ and $D\hat{Y}P$

(iii) Any 2: $A\hat{X}S$ and $C\hat{Z}R$, $B\hat{X}S$ and $D\hat{Z}R$, $A\hat{W}Q$ and $C\hat{Y}P$, $B\hat{W}Q$ and $D\hat{Y}P$

(b) **No**, $B\hat{W}Q \neq A\hat{X}R$ as PQ is not parallel to RS.

(c) **No**, the sum of $D\hat{Y}P$ and $C\hat{Z}R$ is not equal to 180° as PQ is not parallel to RS.

2. **(a)** $a° = 117°$ (vert. opp. $\angle$s)
$\therefore \boldsymbol{a} = \mathbf{117}$
$b° = 117°$ (corr. $\angle$s, AB // CD)
$\therefore \boldsymbol{b} = \mathbf{117}$
$c° + a° = 180°$ (int. $\angle$s, AB // CD)
$c° + 117° = 180°$
$c° = 180° - 117°$
$= 63°$
$\therefore \boldsymbol{c} = \mathbf{63}$
$d° = 78°$ (corr. $\angle$s, AB // CD)
$\therefore \boldsymbol{d} = \mathbf{78}$

(b) $e° = 31°$ (alt. $\angle$s, AB // CD)
$\therefore \boldsymbol{e} = \mathbf{31}$
$f° = 35° + 31°$ (alt. $\angle$s, AB // CD)
$= 66°$
$\therefore \boldsymbol{f} = \mathbf{66}$

(c) $g° = 83°$ (alt. $\angle$s, AB // CD)
$\therefore \boldsymbol{g} = \mathbf{83}$
$h° = 69°$ (corr. $\angle$s, AB // CD)
$\therefore \boldsymbol{h} = \mathbf{69}$

(d) $i° + 75° + 60° = 180°$ (int. $\angle$s, AB // CD)
$i° = 180° - 75° - 60°$
$= 45°$
$\therefore \boldsymbol{i} = \mathbf{45}$
$j° = 60°$ (alt. $\angle$s, AB // CD)
$\therefore \boldsymbol{j} = \mathbf{60}$

3. **(a)** $a° = 38°$ (corr. $\angle$s, AB // CD)
$\therefore \boldsymbol{a} = \mathbf{38}$
$a° + 30° = 2b°$ (corr. $\angle$s, AB // CD)
$38° + 30° = 2b°$
$2b° = 68°$
$b° = 34°$
$\therefore \boldsymbol{b} = \mathbf{34}$

(b) $7c° = 140°$ (corr. $\angle$s, AB // CD)
$c° = 20°$
$\therefore \boldsymbol{c} = \mathbf{20}$
$2d° = 7c°$ (vert. opp. $\angle$s)
$= 140°$
$d° = 70°$
$\therefore \boldsymbol{d} = \mathbf{70}$

(c) $7e° + 3e° = 180°$ (int. $\angle$s, AB // CD)
$10e° = 180°$
$e° = 18°$
$\therefore \boldsymbol{e} = \mathbf{18}$

(d) $(2f + 6)° = (3f - 23)°$ (alt. $\angle$s, AB // CD)
$2f° + 6° = 3f° - 23°$
$3f° - 2f° = 6° + 23°$
$f° = 29°$
$\therefore \boldsymbol{f} = \mathbf{29}$

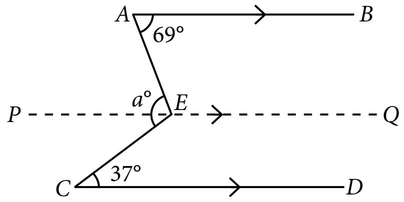
4. **(i)** $A\hat{E}B = \mathbf{68°}$ (alt. $\angle$s, BF // AD)

(ii) $E\hat{A}B = 58°$ (alt. $\angle$s, AB // CD)
$F\hat{B}A + E\hat{A}B = 180°$ (int. $\angle$s, BF // AD)
$F\hat{B}A + 58° = 180°$
$F\hat{B}A = 180° - 58°$
$= 122°$
$A\hat{B}E = F\hat{B}A - 68°$
$= 122° - 68°$
$= \mathbf{54°}$

5. **(i)** $C\hat{D}F = \mathbf{86°}$ (alt. $\angle$s, CE // FG)

(ii) $H\hat{D}E = 86°$ (vert. opp. $\angle$s)
$E\hat{D}A = H\hat{D}E - 47°$
$= 86° - 47°$
$= 39°$
$B\hat{A}D + E\hat{D}A = 180°$ (int. $\angle$s, AB // CE)
$B\hat{A}D + 39° = 180°$
$B\hat{A}D = 180° - 39°$
$= \mathbf{141°}$

6. **(a)**



$A\hat{E}P = 69°$ (alt. ∠s, AB // PQ)

$C\hat{E}P = 37°$ (alt. ∠s, PQ // CD)

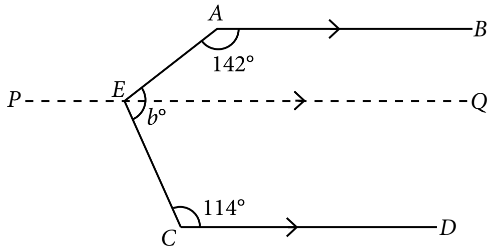
$a° = A\hat{E}P + C\hat{E}P$

$= 69° + 37°$

$= 106°$

$\therefore$ ***a* = 106**

(b)



$A\hat{E}Q + 142° = 180°$ (int. ∠s, AB // PQ)

$A\hat{E}Q = 180° - 142°$

$= 38°$

$CA\hat{E}Q + 114° = 180°$ (int. ∠s, PQ // CD)

$C\hat{E}Q = 180° - 114°$

$= 66°$

$b° = A\hat{E}Q + C\hat{E}Q$

$= 38° + 66°$

$= 104°$

$\therefore$ ***b* = 104**

7. $F\hat{D}C + 80° = 180°$ (adj. ∠s on a str. line)

$F\hat{D}C = 180° - 80°$

$= 100°$

$D\hat{C}A = F\hat{D}C$ (alt. ∠s, DF // AC)

$= 100°$

$D\hat{C}A + 4x° = 360°$ (∠s at a point)

$100° + 4x° = 360°$

$4x° = 360° - 100°$

$= 260°$

$x° = 65°$

$\therefore$ ***x* = 65**

$B\hat{A}C + D\hat{C}A = 180°$ (int. ∠s, AB // CE)

$B\hat{A}C + 100° = 180°$

$B\hat{A}C = 180° - 100°$

$= 80°$

$7y° + B\hat{A}C = 360°$

$7y° + 80° = 360°$

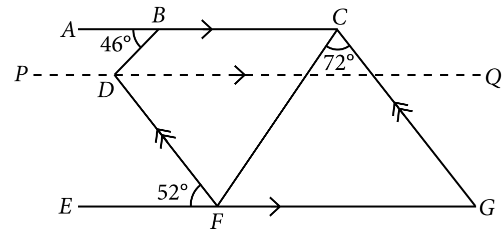
$7y° = 360° - 80°$

$= 280°$

$y° = 40°$

$\therefore$ ***y* = 40**

8.



(i) $C\hat{G}F =$ **52°** (corr. ∠s, FD // GC)

(ii) $B\hat{C}G + C\hat{G}F = 180°$ (int. ∠s, AC // EG)

$B\hat{C}G + 52° = 180°$

$B\hat{C}G = 180° - 52°$

$= 128°$

$B\hat{C}F = B\hat{C}G - 72°$

$= 128° - 72°$

$=$ **56°**

(iii) $B\hat{D}Q = 46°$ (alt. ∠s, AC // PQ)

$F\hat{D}Q = 52°$ (alt. ∠s, PQ // EG)

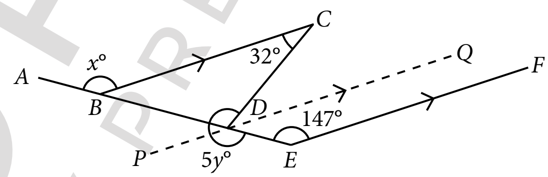
Reflex $B\hat{D}F + B\hat{D}Q + F\hat{D}Q = 360°$ (∠s at a point)

Reflex $B\hat{D}F + 46° + 52° = 360°$

Reflex $B\hat{D}F = 360° - 46° - 52°$

$=$ **262°**

9.



$x° = 147°$ (corr. ∠s, BC // EF)

$\therefore$ ***x* = 147**

$\angle CDQ = 32°$ (alt. ∠s, BC // PQ)

$\angle QDE + 147° = 180°$ (int. ∠s, PQ // EF)

$\angle QDE = 180° - 147°$

$= 33°$

$5y° + \angle CDQ + \angle QDE = 360°$ (∠s at a point)

$5y° + 32° + 33° = 360°$

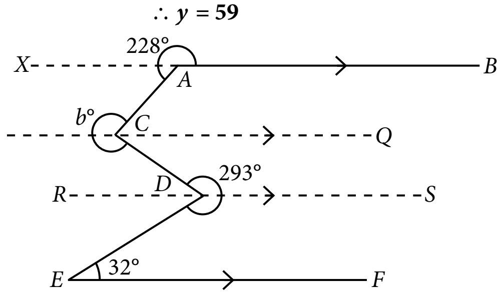
$5y° = 360° - 32° - 33°$

$= 295°$

$y° = 59°$

$\therefore$ ***y* = 59**

10.



$X\hat{A}C = 228° - 180°$

$= 48°$

$A\hat{C}Q = X\hat{A}C$ (alt. ∠s, XB // PQ)

$= 48°$

$E\hat{D}R = 32°$ (alt. ∠s, RS // EF)

$E\hat{D}R + C\hat{D}R + 293° = 360°$ (∠s at a point)

$32° + C\hat{D}R + 293° = 360°$

$C\hat{D}R = 360° - 32° - 293°$

$= 35°$

$D\hat{C}Q = C\hat{D}R = 35°$ (alt. $\angle$s, $PQ \parallel RS$)

$b° + A\hat{C}Q + D\hat{C}Q = 360°$ ($\angle$s at a point)

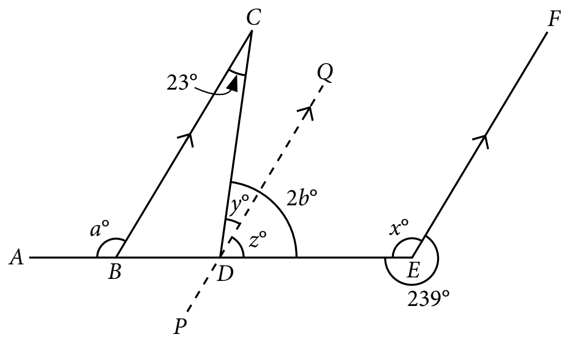
$b° + 48° + 35° = 360°$

$b° = 360° - 35° - 48°$

$= 277°$

$\therefore$ b = **277**

11.



$x° = 360° - 239°$ ($\angle$s at a point)

$= 121°$

$a° = 121°$ (corr. $\angle$s, $BC \parallel EF$)

$\therefore$ $\boldsymbol{a = 121}$

$y° = 23°$ (alt. $\angle$s, $BC \parallel PQ$)

$z° = 180° - 121°$ (int. $\angle$s, $PQ \parallel EF$)

$= 59°$

$2b° = y° + z°$

$= 23° + 59°$

$= 82°$

$b° = \dfrac{82°}{2}$

$= 41°$

$\therefore$ $\boldsymbol{b = 41}$

12. Since $\angle CRQ = \angle RQT = 42°$, then $AB \parallel CD$ (converse of alt. $\angle$s).

$\angle TUR = 180° - 157°$ (adj. $\angle$s on a str. line)

$= 23°$

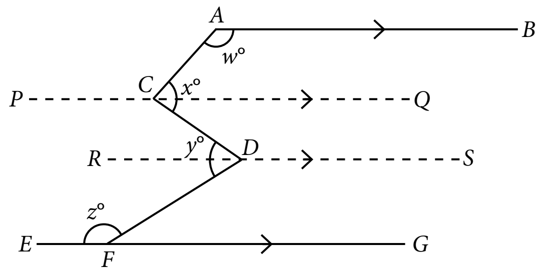
$\therefore$ $\angle PTQ$ = **23°** (corr. $\angle$s, $AB \parallel CD$)

13. Since $A\hat{X}S + C\hat{Z}R = 104° + 76° = 180°$,

then $AB \parallel CD$ (converse of int. $\angle$s).

$\therefore$ $B\hat{W}P = D\hat{Y}P$ = **46°** (corr. $\angle$s, $AB \parallel CD$)

14.



$Q\hat{C}A + w° = 180°$ (int. $\angle$s, $AB \parallel PQ$)

$Q\hat{C}A = 180° - w°$

$Q\hat{C}D = x° - Q\hat{C}A$

$= x° - (180° - w°)$

$= x° - 180° + w°$

$C\hat{D}R = Q\hat{C}D$ (alt. $\angle$s, $PQ \parallel RS$)

$= x° - 180° + w°$

$F\hat{D}R = y° - C\hat{D}R$

$= y° - (x° - 180° + w°)$

$= y° - x° + 180° - w°$

$F\hat{D}R + z° = 180°$ (int. $\angle$s, $RS \parallel EG$)

$y° - x° + 180° - w° + z° = 180°$

$w° + x° = y° + z°$

$\therefore$ $\boldsymbol{w + x = y + z}$

Chapter 11 Polygons and Geometrical Constructions

TEACHING NOTES

Suggested Approach

Students have learnt about triangles, and quadrilaterals such as parallelograms, rhombuses and trapeziums in primary school. In this chapter, students will learn the properties of these figures and how to find unknown angles, starting from triangles to quadrilaterals and eventually, polygons of *n* sides. The incremental approach is to ensure that students have a good understanding before they move on to a higher level. Teachers may want to dedicate more time and attention to the content on polygons in the final section of this chapter (see Section 11.4).

Section 11.3 is dedicated to the geometrical constructions of triangles and quadrilaterals. Students have learnt how to draw triangles and quadrilaterals using rulers, protractors and set squares in primary school. Teachers need to reintroduce these construction tools and demonstrate their use if students are still unfamiliar with them. When students are comfortable using these tools, teachers can then introduce the use of compasses in geometrical constructions and proceed to the subsections on construction of triangles and quadrilaterals.

Section 11.1: Triangles

Students have learnt about isosceles triangles, equilateral triangles and right-angled triangles in primary school. In this chapter, students should be aware that triangles can be classified by the number of equal sides or the types of angles. Teachers should highlight to the students that equilateral triangles are a special type of isosceles triangles while scalene triangles are triangles that are not isosceles and are not equilateral triangles.

Students should explore and discover that the longest side of a triangle is opposite the largest angle, and the sum of two sides is always larger than the third side (see Investigation: Basic properties of a triangle). A pair of activities is designed to help students apply their knowledge from Chapter 10 to prove that the angle sum of a triangle = 180° (Thinking Time, page 263) and that the exterior angle of a triangle is equal to the sum of the interior opposite angles (Thinking Time, page 264). Teachers should ensure students are clear about what exterior angles are before stating the relation between exterior angles and interior opposite angles of a triangle. Some may think that the exterior angle of a triangle is the same as the reflex angle at a vertex of a triangle.

Section 11.2: Quadrilaterals

Teachers may want to first recap students' knowledge of parallelograms, rhombuses and trapeziums based on what they have learnt in primary school. In this section, students will build on their knowledge of the angle sum of triangles to prove that the angle sum of a quadrilateral = 360° (see Thinking Time on page 268). Teachers can use what students have learnt about the properties of angles formed by parallel lines and transversal in Chapter 10 to reintroduce and build up their understanding of the different types of quadrilaterals and their properties (see Investigation: Properties of special quadrilaterals). The symmetric properties of special quadrilaterals will be discussed in Section 11.4 (Class Discussion: Line symmetry of special quadrilaterals and regular polygons and Class Discussion: Rotational symmetry of special quadrilaterals and regular polygons).

Before proceeding onto the next section, teachers may want to go through with the students the angle properties of triangles and quadrilaterals. This reinforces the students' knowledge as well as prepares them for the section on polygons.

Section 11.3: Geometrical constructions of triangles and quadrilaterals

Teachers may wish to recap with students how rulers, protractors and set squares are used for geometrical constructions. More emphasis should be placed on the use of protractors, such as the type of scale (inner or outer) to use, depending on the type of angle (acute or obtuse). Teachers need to impress upon students to avoid parallax errors when reading the length using a ruler, or an angle using a protractor.

Teachers should show and lead students on the use of compasses. Although the construction of perpendicular and angle bisectors is not required in the syllabus, the students should be guided here to construct the perpendicular bisector to familiarise the use of compasses in geometrical constructions. Teachers should also clearly define perpendicular bisectors. Stating what perpendicular and bisect mean individually will help students to remember their meanings. Students are to know and be familiar with the useful tips in using construction tools.

For the worked examples in this section, teachers are encouraged to go through the construction steps one by one with the students. Students should follow and construct the same figures as shown in the worked examples. Although the syllabus requires the construction of a triangle given the lengths of all sides, teachers are encouraged to ensure that the students can first construct all of the following types of triangles, before continuing with the construction of quadrilaterals:

- Given 3 sides
- Given 2 sides and an included angle
- Given 1 side and 2 angles

Proficiency in constructing of the listed types of triangles will help students successfully progress to more complex constructions of quadrilaterals.

As a rule of thumb, students should draw the longest line as a horizontal line. Teachers are to remind their students to mark all angles, vertices, lengths and other markings (same angles, same sides, right angles etc.) clearly. Students should not erase any arcs that they draw in the midst of construction and check their figure at the end.

Section 11.4: Polygons

Teachers should emphasise to the students that triangles and quadrilaterals are polygons so that they are aware that all the concepts which they have learnt in Sections 11.1 and 11.2 remain applicable in this section. Students should learn the different terms with regards to polygons. In this section, most polygons studied will be simple, convex polygons.

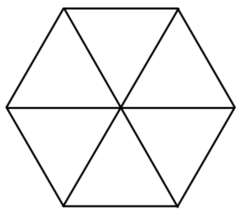
Students need to know the names of polygons with 10 sides or less and the general naming convention of polygons (see Class Discussion: Naming of polygons). Through the class discussion, students should be able to develop a good understanding of polygons and be able to name them. They should also know and appreciate the properties of regular polygons (see Class Discussion: Definition of a regular polygon, Class Discussion: Line symmetry of special quadrilaterals and regular polygons and Class Discussion: Rotational symmetry of special quadrilaterals and regular polygons).

Teachers can ask students to recall the properties of triangles and quadrilaterals during the investigation of the sum of interior angles and sum of exterior angles of a polygon. Students should be able to see a pattern in how the sum of interior angles differs as the number of sides increases and understand its formula (see Investigation: Sum of interior angles of a polygon). They should also discover that the sum of exterior angles is always equal to 360° regardless of the number of sides of the polygon (see Investigation: Sum of exterior angles of a pentagon). Teachers may highlight that the second angle property of a polygon applies regardless of the ways in which the exterior angles are labelled (see Section 11.4D on page 289 of the textbook).

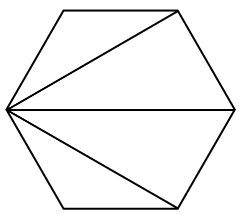
Introductory Problem

1. Encourage the class to think of more than one method, but there is no need for them to think of all the following methods at this stage.

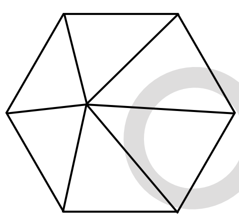
Method 1:
Students may see that a hexagon consists of 6 equilateral triangles. So, each interior angle is made up of two interior angles of two interior angles of an equilateral triangle, i.e. $2 \times 60° = 120°$.



Method 2:
Divide into triangles, find the sum of all the interior angles of the triangles, and then divide by 6.
Since there are 4 triangles, which gives a total of $180° \times 4 = 720°$, then each interior angle (being equal) will be $720° \div 6 = 120°$.

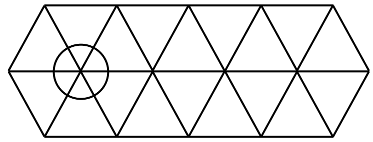


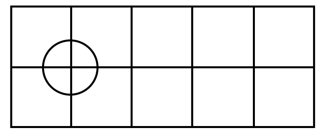
Method 3:
Connect a point inside the hexagon to all the vertices to form 6 triangles. Find the sum of all the interior angles of the 6 triangles and then take away 360° (= 6 angles around the chosen point). Finally, divide by 6, and the answer is 120°.

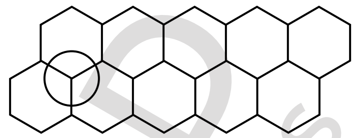


Method 4:
Similar to Method 2 but dissect the hexagon into two quadrilaterals.

2. There are only 2 other regular polygons that can be used for tessellating the plane without any gaps. They are equilateral triangles and squares. For tessellation to take place, we need to ensure the angle sum at the point of contact is 360°. Also, since each angle is equal, the number of pieces of the polygon joined at a point must be a whole number.







For an equilateral triangle, the interior angle = 60° and so, we can put six of these together to tessellate. For a square, the interior angle = 90°.
Therefore, we can put 4 squares together. To show that a regular pentagon cannot tessellate, we would need to know its interior angle.

3. *Note: Some of the 4 methods (e.g. Methods 2 and 3) mentioned in Prompt 1 above can be applied to other polygons. Encourage students to make an intelligent guess of the methods that can be used.*
We will revisit this when we come to the general formula for finding the sum of interior angles in Section 11.4.

11.1 Triangles

Investigation (Basic properties of triangle)

1. The side opposite $\angle B$ is $\boldsymbol{b}$ and the side opposite $\angle C$ is $\boldsymbol{c}$.
2. The largest angle is $\angle \boldsymbol{C}$ and the smallest angle is $\angle \boldsymbol{B}$.
The side opposite the largest angle, $\angle C$ is the longest side and the side opposite the smallest angle, $\angle B$ is the shortest side.
3. The bigger the angle, the longer the side opposite it. The angle opposite the shortest side is always the smallest angle and the angle opposite the longest side is always the largest angle.
4. The sum of the lengths of the two shorter sides of a triangle is always longer than the length of the longest side.
5. **Yes**, since the sum of the angles facing the two shorter sides are greater than the largest angle facing the longest side, hence, the sum of the lengths of the two shorter sides of a triangle is always longer than the length of the longest side.
6. No, it is **not possible** to form a triangle.
7. $\boldsymbol{a} + \boldsymbol{b} = \boldsymbol{c}$. It is still **not possible** to form a triangle.
8. The sum of the lengths of any two line segments has to be greater than the length of the third line segment.

Thinking Time (Page 263)

$\angle BAC = \angle ACP$ (alt. $\angle$s, PQ // AB)
$\angle ABC = \angle \boldsymbol{BCQ}$ (**alt.** $\angle$s, PQ // $\boldsymbol{AB}$)
Since $\angle ACP + \angle ACB + \angle BCQ = \mathbf{180°}$ (adj. $\angle$s on a str. line),
then $\angle BAC + \angle ACB + \angle \boldsymbol{ABC} = \mathbf{180°}$ ($\angle$ sum of $\triangle$)

Practise Now 1

1. $90° + 48° + a° = 180°$ ($\angle$ sum of $\triangle$)
$a° = 180° - 90° - 48°$
$= 42°$
$\therefore a = \mathbf{42}$

2. $\angle ABC = \angle BAC$ (base $\angle$s of isos. $\triangle$)
$2\angle ABC + \angle ACB = 180°$ ($\angle$ sum of $\triangle$)
$$\angle ABC = \frac{180° - \angle ACB}{2} = \frac{180° - 52°}{2} = \mathbf{64°}$$

Thinking Time (Page 264)

$\angle b + \angle d = \mathbf{180°}$ (adj. $\angle$s on a str. line)
$\angle a + \angle b + \angle c = \mathbf{180°}$ ($\angle$ sum of $\triangle$)
$\therefore \angle b + \angle d = \angle a + \angle b + \angle \boldsymbol{c}$
$\therefore$ exterior $\angle d = \angle \boldsymbol{a} + \angle \boldsymbol{c}$

Practise Now 2

1. (a) $y° = 34° + 75°$ (ext. $\angle$ of $\triangle$)
$= 109°$
$\therefore y = \mathbf{109}$

(b) $\angle CED + \angle CDE = y°$ (ext. $\angle$ of $\triangle$)
$\angle CED + 14° = 109°$
$\angle CED = 109° - 14°$
$= \mathbf{95°}$

2. (a) $\angle CBD = 93° + 41°$ (ext. $\angle$ of $\triangle$)
$= \mathbf{134°}$

(b) $\angle ADE = 180° - 93°$ (adj. $\angle$s on a str. line)
$= 87°$
$\angle DFE + \angle DEF = \angle ADE$ (ext. $\angle$ of $\triangle$)
$b° + 33° = 87°$
$\therefore b = 87 - 33$
$= \mathbf{54}$

Practise Now 3

1. $A\hat{D}E = 72°$ (vert. opp. $\angle$s)
$D\hat{A}E = 147° - A\hat{D}E$ (ext. $\angle$ of $\triangle$)
$= 147° - 72°$
$= \mathbf{75°}$

2. $Q\hat{R}S = 180° - 97° - 36°$ (int. $\angle$s, PQ // RS)
$= 47°$
$Q\hat{S}T = Q\hat{R}S + R\hat{Q}S$ (ext. $\angle$ of $\triangle$)
$= 47° + 63°$
$= \mathbf{110°}$

Exercise 11A

1. (a) $a° = 180° - 90° - 16°$ ($\angle$ sum of $\triangle$)
$= 74°$
$\therefore \boldsymbol{a = 74}$

(b) $b° = \dfrac{180° - 48°}{2}$ (base $\angle$ of isos. $\triangle$)
$= 66°$
$\therefore \boldsymbol{b = 66}$

(c) $2c° + 108° + 27° = 180°$ ($\angle$ sum of $\triangle$)
$2c° = 180 - 108° - 27°$
$= 45°$
$c° = 22.5°$
$\therefore \boldsymbol{c = 22.5}$

(d) $37° + 37° + 4d° = 180°$ (base $\angle$s of isos. $\triangle$, $\angle$ sum of $\triangle$)
$4d° = 180° - 37° - 37°$
$= 106°$
$d° = 26.5°$
$\therefore \boldsymbol{d = 26.5}$

(e) Since this is an equilateral triangle, all the angles are equal.
$3x° = 180°$ ($\angle$ sum of $\triangle$)
$x° = 60°$
$\therefore \boldsymbol{x = 60}$

(f) $2y° + 4y° + y° = 180°$ ($\angle$ sum of $\triangle$)
$7y° = 180°$
$\therefore \boldsymbol{y = 25.7}$ (to 1 d.p.)

2. $a° = \angle PRQ$ (vert. opp. $\angle$s)
$= 180° - 26° - 79°$ ($\angle$ sum of $\triangle PQR$)
$= 75°$
$\therefore \boldsymbol{a = 75}$
$b° = 180° - a° - 82°$ ($\angle$ sum of $\triangle RST$)
$= 180° - 75° - 82°$
$= 23°$
$\therefore \boldsymbol{b = 23}$

3. (i) $z° = 73° + 41°$ (ext. $\angle$ of $\triangle RST$)
$= 114°$
$\therefore z = \mathbf{114}$

(ii) $\angle PQS = z° - \angle QPR$ (ext. $\angle$ of $\triangle PQR$)
$= 114° - 35°$
$= \mathbf{79°}$

4. $a° + 90° = 115°$ (ext. $\angle$ of $\triangle BCE$)
$a° = 115° - 90°$
$= 25°$
$\therefore \boldsymbol{a = 25}$
$\angle FEG = 180° - 90° - 32°$ ($\angle$ sum of $\triangle EFG$)
$= 58°$
$a° + \angle FEG + b° = 180°$ (adj. $\angle$s on a str. line)
$25° + 58° + b° = 180°$
$b° = 180° - 25° - 58°$
$= 97°$
$\therefore \boldsymbol{b = 97}$

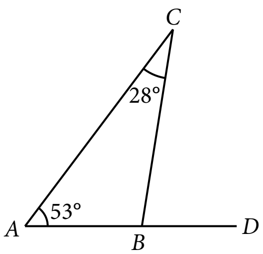
5. (i) Let $A\hat{D}B = B\hat{D}C = x°$
$90° + 20° + 2x° = 180°$ ($\angle$ sum of $\triangle ADC$)
$2x° = 180° - 90° - 20°$
$= 70°$
$x° = \dfrac{70°}{2}$
$= 35°$
$\therefore B\hat{D}C = \mathbf{35°}$

(ii) $C\hat{B}D + 20° + 35° = 180°$ ($\angle$ sum of $\triangle BCD$)
$C\hat{B}D = 180° - 20° - 35°$
$= \mathbf{125°}$

6. **Method 1:**
$M\hat{N}P = C\hat{N}Y = 90°$ (vert. opp. $\angle$s)
$N\hat{M}P = 180° - 90° - 31°$ ($\angle$ sum of $\triangle NMP$)
$= 59°$
$M\hat{O}N = 180° - 2 \times 59°$ ($\angle$ sum of isos. $\triangle MON$)
$= \mathbf{62°}$

Method 2:
Since $A\hat{M}N = C\hat{N}Y$ (= 90°), then AB // CD (converse of corr. $\angle$s).
$B\hat{M}O = 31°$ (alt. $\angle$s, AB // CD)
$O\hat{M}N = 180° - 90° - 31°$ (adj. $\angle$s on a str. line)
$= 59°$
$M\hat{O}N = 180° - 2 \times 59°$ ($\angle$ sum of isos. $\triangle MON$)
$= \mathbf{62°}$

7.



$\angle CBD = 53° + 28°$ (ext. $\angle$ of $\triangle$)
$= \mathbf{81°}$

8. $\angle ABE = \angle ABD = 89° + 27°$ (ext. $\angle$ of $\triangle BCD$)
$= 116°$
$p° = 116° + 22°$ (ext. $\angle$ of $\triangle ABE$)
$= 138°$
$\therefore p = \mathbf{138}$

9. $A\hat{D}E = 74°$ (vert. opp. $\angle$s)
$D\hat{A}E = 127° - 74°$ (ext. $\angle$ of $\triangle DAE$)
$= \mathbf{53°}$

10. $S\hat{T}U = Q\hat{R}U$ (alt. $\angle$s, QR // ST)
$= 44°$
$S\hat{U}T = 180° - 44° - 57°$ ($\angle$ sum of $\triangle SUT$)
$= \mathbf{79°}$
$Q\hat{T}R = 79° - 44°$ (ext. $\angle$ of $\triangle QRT$)
$= 35°$
$T\hat{R}S = Q\hat{T}R$ (alt. $\angle$s, QT // RS)
$= 35°$
$R\hat{S}U = 180° - 44° - 35° - 57°$ (int. $\angle$s, RQ // ST)
$= \mathbf{44°}$

11. $A\hat{H}F = 45°$ (vert. opp. $\angle$s)
$A\hat{H}I + C\hat{I}H = 180°$ (int. $\angle$s, AB // CD)
$(45° + 64°) + (32° + x°) = 180°$
$x° = 180° - 45° - 64° - 32°$
$= 39°$
$\therefore \boldsymbol{x = 39}$
$y° + 39° + 64° = 180°$ ($\angle$ sum of $\triangle$)
$y° = 180° - 39° - 64°$
$= 77°$
$\therefore \boldsymbol{y = 77}$

12. Let $C\hat{B}O$ be $x°$.
Then $C\hat{A}O = \frac{1}{2}x°$ and $B\hat{A}O = 1\frac{1}{2}x°$.
Since $OA = OC$, $\therefore A\hat{C}O = C\hat{A}O = \frac{1}{2}x°$.
Since $OB = OC$, $\therefore C\hat{B}O = B\hat{C}O = x°$.
Since $OA = OB$, $\therefore B\hat{A}O = A\hat{B}O = 1\frac{1}{2}x°$.
Hence,
$C\hat{A}B + A\hat{B}C + B\hat{C}A = 180°$($\angle$sum of $\triangle ABC$)
$$\left(\frac{1}{2}x° + 1\frac{1}{2}x°\right) + \left(1\frac{1}{2}x° + x°\right) + \left(\frac{1}{2}x° + x°\right) = 180°$$
$6x° = 180$
$x° = \frac{180°}{6}$
$= 30°$
$\therefore C\hat{A}O = \frac{1}{2}(30°) = \mathbf{15°}$.

13. Since $AB = AC$, then let $A\hat{B}C = A\hat{C}B = x°$.
$D\hat{B}E = 180° - x°$ (adj. $\angle$s on a str. line)
Since $BD = BE$, then
$B\hat{D}E = B\hat{E}D = \frac{180° - (180° - x°)}{2} = \frac{x°}{2}$.
Since $AF = DF$, $\therefore F\hat{A}D = F\hat{D}A$
$F\hat{A}D = F\hat{D}A = B\hat{D}E = \frac{x°}{2}$.
$\frac{x°}{2} + x° + x° = 180°$ ($\angle$ sum of $\triangle ABC$)
$2\frac{1}{2}x° = 180°$
$x° = \frac{180°}{2\frac{1}{2}}$
$= 72°$
$\therefore A\hat{B}C = \mathbf{72°}$

11.2 Quadrilaterals

Thinking Time (Page 268)

$\angle p + \angle q + \angle u = 180°$ ($\angle$ sum of $\triangle ABD$)
$\angle r + \angle s + \angle t = \mathbf{180°}$ ($\angle$ sum of $\boldsymbol{\triangle BCD}$)
Sum of interior angles of the quadrilateral $ABCD$
$= \angle p + \angle q + \angle r + \angle s + \angle t + \angle \boldsymbol{u}$
$= (\angle p + \angle q + \angle u) + (\angle r + \angle \boldsymbol{s} + \angle \boldsymbol{t})$
$= 180° + \mathbf{180°}$
$= \mathbf{360°}$

Investigation (Properties of special quadrilaterals)

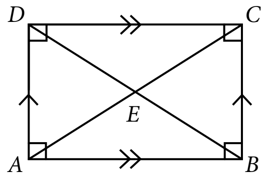
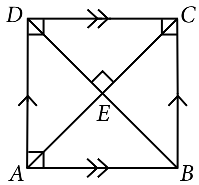
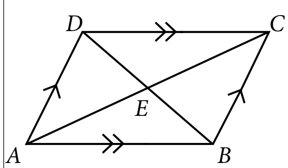
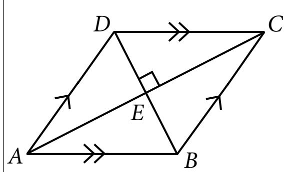
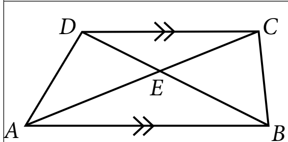
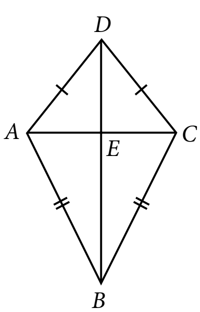
Quadrilateral	Parallel sides	Equal sides	Interior angles	Diagonals
 **Rectangle**	There are **two** pairs of parallel sides.	Opposite sides are equal in length.	All four angles are right angles.	• The two diagonals are **equal** in length. • Diagonals bisect each other, i.e. $AE = EC$ and $BE = ED$.
 **Square**	There are two pairs of parallel sides.	All **four** sides are equal in length.	All **four** angles are right angles.	• The two diagonals are equal in length. • Diagonals bisect each other at right angles, i.e. $AE = EC$, $BE = ED$ and $\angle AEB = \angle BEC = \angle CED = \angle AED = 90°$. • Diagonals **bisect** the interior angles, e.g. $\angle BAC = \angle CAD = 45°$ and $\angle ABD = \angle CBD = 45°$.
 **Parallelogram**	There are **two** pairs of parallel sides.	**Opposite** sides are equal in length.	**Opposite** angles are equal, i.e. $\angle BAD = \angle BCD$ and $\angle ABC = \angle ADC$.	Diagonals **bisect** each other, i.e. $AE = EC$ and $BE = ED$.
 **Rhombus**	There are **two** pairs of parallel sides.	All four sides are equal in length.	**Opposite** angles are equal, i.e. $\angle BAD = \angle BCD$ and $\angle ABC = \angle ADC$.	• Diagonals **bisect** each other at right angles, i.e. $AE = EC$, $BE = ED$ and $\angle AEB = \angle BEC = \angle AED = \angle CED = 90°$. • Diagonals **bisect** the interior angles, e.g. $\angle BAC = \angle CAD$ and $\angle ABD = \angle CBD$.
 **Trapezium**	There is at least* one pair of parallel sides, i.e. $AB // DC$.			
 **Kite**		There are at least** two pairs of equal adjacent sides, i.e. $AD = DC$ and $AB = BC$.		• Diagonals cut each other (not bisect) at **right** angles, i.e. $\angle AEB = \angle BEC = \angle AED = \angle CED = 90°$. • One diagonal **bisects** the interior angles, i.e. $\angle ADB = \angle CDB$ and $\angle ABD = \angle CBD$.

Table 11.3

Practise Now 4

1. (i) Since the diagonals of the rectangle bisect each other, then $CE = BE$.
$\therefore \angle BEC = 180° - 2 \times 63°$ ($\angle$ sum of isos. $\triangle$)
$= \mathbf{54°}$
(ii) $\angle DCE = 90° - 63°$
$= 27°$
$\angle CDE = \angle DCE$ (base $\angle$s of isos. $\triangle$)
$= \mathbf{27°}$
2. (i) $\angle PTS = 180° - 90° - 51°$ ($\angle$ sum of $\triangle PST$)
$= \mathbf{39°}$
(ii) $\angle RST = 90° - 51°$
$= 39°$
$\angle RTS = 180° - 39° - 68°$ ($\angle$ sum of $\triangle RST$)
$= \mathbf{73°}$

Practise Now 5

1. (i) $A\hat{D}C = 180° - 2 \times 32°$ ($\angle$ sum of isos. $\triangle ACD$)
$= 116°$
$A\hat{B}C = A\hat{D}C$ (opp. $\angle$s of rhombus)
$= \mathbf{116°}$
(ii) $C\hat{A}E = A\hat{C}D$ (alt. $\angle$s, AB // CD)
$= 32°$
$A\hat{C}E = 180° - 2 \times 32°$ ($\angle$ sum of isos. $\triangle ACE$)
$= 116°$
$A\hat{C}B = A\hat{C}D$ (diagonal AC of rhombus bisects BCD)
$= 32°$
$B\hat{C}E = 116° - 32°$
$= \mathbf{84°}$
2. (i) $S\hat{T}U = 108°$ (opp. $\angle$s of // gram)
$9x° = 108°$
$x = 12°$
$\therefore x = \mathbf{12}$
(ii) $(V\hat{U}W + 38°) + 108° = 180°$ (int. $\angle$s, SV // TU)
$= 180° - 38° - 108°$
$= \mathbf{34°}$

Practise Now 6

1. (i) $\angle DAE = \dfrac{50°}{2}$ (diagonal AC bisects $\angle DAB$)
$= \mathbf{25°}$
(ii) $\angle ADB = \dfrac{180° - 50°}{2}$ (base $\angle$s of isos. $\triangle ABD$)
$= 65°$
$\angle CDE = 137° - 65°$
$= \mathbf{72°}$
2. (i) Since the four smaller trapeziums are identical, the shorter parallel base of each smaller trapezium is equal, i.e. $EF = EH = FG$. Since $EH = FG$, the non-parallel sides of each smaller trapezium are equal, i.e. $AE = DH = BF = GC$.
$\therefore DH = GC$ (shown)
(ii) $DC = 8$ cm
$DH + HG + GC = 8$ cm
Since $DH = GC$ and $HG = AB = 4$ cm,
$2 \times DH + 4 = 8$ cm
$DH = \dfrac{8 - 4}{2}$
$= 2$ cm
$\therefore EF = \mathbf{2\ cm}$

Exercise 11B

1. (a) $a° + 54° = 90°$ (right angle of rectangle)
$a° = 90° - 54°$
$= 36°$
$\therefore \boldsymbol{a = 36}$
$b° = a°$ (alt. $\angle$s, AB // DC)
$\therefore \boldsymbol{b = 36}$
(b) $\angle EBC = 90°$ (right angle of rectangle)
$90° + 39° + c° = 180°$ ($\angle$ sum of $\triangle BCE$)
$c° = 180° - 90° - 39°$
$= 51°$
$\therefore \boldsymbol{c = 51}$
$\angle DCE = c° = 51°$ (alt. $\angle$s, EB // DC)
$d° + 78° + 51° = 180°$ ($\angle$ sum of $\triangle CDE$)
$d° = 180° - 78° - 51°$
$= 51°$
$\therefore \boldsymbol{d = 51}$
2. (i) $\angle AED = 52°$ (vert. opp. $\angle$s)
Since the diagonals of the rectangle bisect each other, then $AE = DE$.
$\therefore \angle ADB = \dfrac{180° - 52°}{2}$ (base $\angle$s of isos. $\triangle ADE$)
$= \mathbf{64°}$
(ii) $\angle CAD = \angle ADB = 64°$ (base $\angle$s of isos. $\triangle ADE$)
$64° + 90° + \angle ACD = 180°$ ($\angle$ sum of $\triangle ACD$)
$\angle ACD = 180° - 64° - 90°$
$= \mathbf{26°}$
3. (i) $\angle PTS + 90° + 28° = 180°$ ($\angle$ sum of $\triangle PTS$)
$\angle PTS = 180° - 90° - 28°$
$= \mathbf{62°}$
(ii) $\angle TSR + 28° = 90°$ (right angle of rectangle)
$\angle TSR = 90° - 28°$
$= 62°$
$\angle RTS + 62° + 43° = 180°$ ($\angle$ sum of $\triangle RST$)
$\angle RTS = 180° - 62° - 43°$
$= \mathbf{75°}$
4. (a) $p° = 38°$ (alt. $\angle$s, AD // BC)
$\therefore \boldsymbol{p = 38}$
$q° = 180° - 2 \times 38°$ ($\angle$ sum of isos. $\triangle ABD$)
$= 104°$
$\therefore \boldsymbol{q = 104}$
(b) $r° = \angle CAB = 42°$ (alt. $\angle$s, AB // DC)
$\therefore \boldsymbol{r = 42}$
$\angle DAB = 2 \times 42°$ (diagonal AC bisects $\angle DAB$)
$= 84°$
$s° = \dfrac{180° - 84°}{2}$ (base $\angle$s of isos. $\triangle ABD$)
$= 48°$
$\therefore \boldsymbol{s = 48}$
5. (a) $a° = 106°$ (opp. $\angle$s of // gram)
$\therefore \boldsymbol{a = 106}$
$b° = 48°$ (alt. $\angle$s, AD // BC)
$\therefore \boldsymbol{b = 48}$

(b) $4c° + 5c° = 180°$ (int. $\angle$s, AB // DC)

$9c° = 180°$

$c° = \dfrac{180°}{9}$

$= 20°$

$\therefore$ $\boldsymbol{c = 20}$

$2d° = 4(20°)$ (opp. $\angle$s of // gram)

$d° = \dfrac{80°}{2}$

$= 40°$

$\therefore$ $\boldsymbol{d = 40}$

6. **(a)** Since $ABCD$ is a kite, $\therefore$ $AD = CD$ and so $A\hat{C}D = C\hat{A}D = a°$

$a° + 100° + a° = 180°$ ($\angle$ sum of $\triangle ACD$)

$2a° = 180° - 100°$

$= 80°$

$a° = \dfrac{80°}{2}$

$= 40°$

$\therefore$ $\boldsymbol{a = 40}$

Since $ABCD$ is a kite, $\therefore$ $AB = CB$ and so $C\hat{A}B = A\hat{C}B = 61°$.

$61° + b° + 61° = 180°$ ($\angle$ sum of $\triangle ABC$)

$b° = 180° - 61° - 61°$

$= 58°$

$\therefore$ $\boldsymbol{b = 58}$

(b) $D\hat{A}C = B\hat{A}C = 40°$ (diagonal AC bisects $\angle BAD$)

$40° + 26° + c° = 180°$ ($\angle$ sum of $\triangle ACD$)

$c° = 180° - 40° - 26°$

$= 114°$

$\therefore$ $\boldsymbol{c = 114}$

7. **(i)** $\angle BAD = 2(25°)$ (diagonal AC bisects $\angle BAD$)

$= 50°$

$\angle ABD = \dfrac{180° - 50°}{2}$ (base $\angle$s of isos. $\triangle ABD$)

$= \mathbf{65°}$

(ii) $\angle BCD = 2(44°)$ (diagonal AC bisects $\angle BCD$)

$= 88°$

$\angle CBD = \dfrac{180° - 88°}{2}$ (base $\angle$s of isos. $\triangle BCD$)

$= \mathbf{46°}$

8. $5x° + x° = 180°$ (int. $\angle$s, AB // DC)

$6x° = 180°$

$x° = \dfrac{180°}{6}$

$= 30°$

$\therefore$ $\boldsymbol{x = 30}$

$2.2(30°) + y° = 180°$ (int. $\angle$s, AB // DC)

$y° = 180° - 66°$

$= 114°$

$\therefore$ $\boldsymbol{y = 114}$

9. **(a)** $\angle DAC = \angle BAC = 45°$ (diagonal AC bisects right angle DAB)

$\angle DAE = 45°$

$\angle ADE + \angle DAE = \angle DEC$ (ext. $\angle$ of $\triangle AED$)

$a° + 45° = 82°$

$a° = 82° - 45°$

$= 37°$

$\therefore$ $\boldsymbol{a = 37}$

$\angle AEF = 82°$ (vert. opp. $\angle$s)

$\angle EAF + \angle AEF = \angle EFB$ (ext. $\angle$ of $\triangle AEF$)

$45° + 82° = b°$

$b° = 127°$

$\therefore$ $\boldsymbol{b = 127}$

(b) $E\hat{C}F = 45°$ (diagonal AC bisects right angle DCB)

$c° + 45° + c° = 180°$ ($\angle$ sum of $\triangle CEF$)

$2c° = 180° - 45°$

$= 135°$

$c° = \dfrac{135°}{2}$

$= 67.5°$

$\therefore$ $\boldsymbol{c = 67.5}$

$C\hat{E}D = 90°$ (diagonals bisect each other at right angles)

Hence,

$d° + 67.5° = 90°$

$d° = 90° - 67.5°$

$= 22.5°$

$\therefore$ $\boldsymbol{d = 22.5}$

10. **(i)** $A\hat{B}D = 46°$ (alt. $\angle$s, AB // DC)

$46° + B\hat{A}D + 46° = 180°$ ($\angle$ sum of isos. $\triangle ABD$)

$B\hat{A}D = 180° - 46° - 46°$

$= \mathbf{88°}$

(ii) $D\hat{B}C = 46°$ (base $\angle$s of isos. $\triangle DBC$)

Since $BC = BE$, $\therefore$ $B\hat{C}E = B\hat{E}C = x°$.

$x° + x° = 46°$ (ext. $\angle$ of $\triangle BCE$)

$2x° = 46°$

$x° = \dfrac{46°}{2}$

$= 23°$

$\therefore$ $B\hat{C}E = \mathbf{23°}$

11. **(i)** $A\hat{D}E + 65° = 180°$ (int. $\angle$s, AB // DC)

$A\hat{D}E = 180° - 65°$

$= \mathbf{115°}$

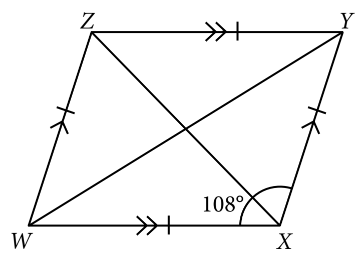
(ii) $B\hat{C}D = 65°$ (opp. $\angle$s of // gram)

$C\hat{B}E + 65° = 125°$ (ext. $\angle$ of $\triangle BCE$)

$C\hat{E}E = 125° - 65°$

$= \mathbf{60°}$

12.



(i) $W\hat{Z}Y = W\hat{X}Y = 108°$ (opp. $\angle$s of a // gram)

$X\hat{Z}Y = X\hat{Z}W = x°$ (diagonal ZX bisects $W\hat{Z}Y$)

$W\hat{Z}Y = 2x°$

$2x° = 108°$

$x° = \dfrac{108°}{2}$

$= 54°$

$\therefore$ $X\hat{Z}Y = \mathbf{54°}$

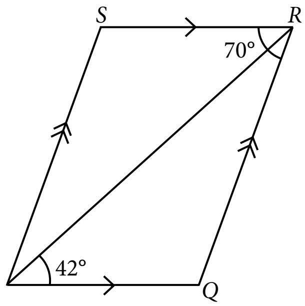
(ii) $X\hat{X}Z + 108° = 180°$ (int. $\angle$s, WX // ZY)

$X\hat{Y}Z = 180° - 108°$

$= \mathbf{72°}$

(iii) $X\hat{W}Z = X\hat{Y}Z = 72°$ (opp. $\angle$s of a // gram)
$X\hat{W}Y = Z\hat{W}Y = y°$ (diagonal YW bisects $X\hat{W}Z$)
$X\hat{W}Z = 2y°$
$2y° = 72°$
$y° = \frac{72°}{2}$
$= 36°$
$\therefore X\hat{W}Y = \mathbf{36°}$

13.

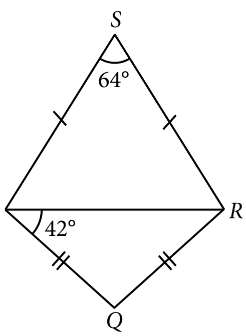


(i) $P\hat{Q}R + 70° = 180°$ (int. $\angle$s, PQ // SR)
$P\hat{Q}R = 180° - 70°$
$= \mathbf{110°}$

(ii) $42° + 110° + P\hat{R}Q = 180°$ ($\angle$ sum of $\triangle PQR$)
$P\hat{R}Q = 180° - 42° - 110°$
$= \mathbf{28°}$

14. $A\hat{D}B = (3x + 7)°$ (diagonal DB bisects $\angle ADC$)
$D\hat{A}C = (2x + 53)°$ (diagonal AC bisects $\angle BAD$)
$2(3x + 7)° + 2(2x + 53)° = 180°$ (int. $\angle$s, BA // CD)
$6x° + 14° + 4x° + 106° = 180°$
$10x° = 180° - 14° - 106°$
$10x° = 60°$
$x° = \frac{60°}{10}$
$= 6°$
$\therefore \boldsymbol{x = 6}$

15.

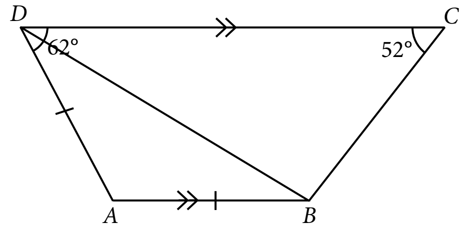


(i) Since $PS = RS$, $\therefore R\hat{P}S = P\hat{R}S = x°$.
$x° + 64° + x° = 180°$ ($\angle$ sum of $\triangle PRS$)
$2x° = 180° - 64°$
$= 116°$
$x° = \frac{116°}{2}$
$= 58°$
$\therefore P\hat{R}S = \mathbf{58°}$

(ii) Since $PQ = QR$, $\therefore Q\hat{P}R = Q\hat{R}P = 42°$.
$42° + P\hat{Q}R + 42° = 180°$ ($\angle$ sum of $\triangle PQR$)
$P\hat{Q}R = 180° - 42° - 42°$
$= \mathbf{96°}$

16. Since the smaller kites $ABEF$ and $ADGF$ are identical,
$F\hat{A}D = B\hat{A}F = 42°$ and $A\hat{D}C = A\hat{B}C = 118°$.
$B\hat{C}D + 118° + 42° + 42° + 118° = 360°$ ($\angle$ sum of quadrilateral)
$B\hat{C}D = 360° - 118° - 42° - 42° - 118°$
$= \mathbf{40°}$

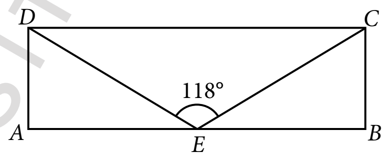
17.



(i) $B\hat{A}D + 62° = 180°$ (int. $\angle$s, AB // DC)
$B\hat{A}D = 180° - 62°$
$= 118°$
Since $AB = AD$, $\therefore A\hat{B}D = A\hat{D}B = x°$
$x° + 118° + x° = 180°$ ($\angle$ sum of $\triangle ABD$)
$2x° = 180° - 118°$
$= 62°$
$x° = \frac{62°}{2}$
$= 31°$
$\therefore A\hat{B}D = \mathbf{31°}$

(ii) $A\hat{B}C + 52° = 180°$ (int. $\angle$s, AB // DC)
$A\hat{B}C = 180° - 52°$
$= 128°$
From **(i)**, $A\hat{B}D = 31°$.
$31° + C\hat{B}D = 128°$
$C\hat{B}D = 128° - 31°$
$= \mathbf{97°}$

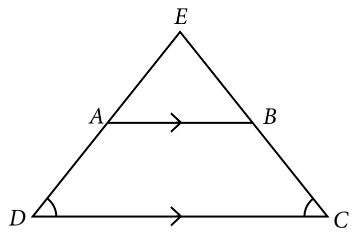
18.



(i) Since E is the midpoint of AB, $\therefore$ $CE = DE$ and hence
$C\hat{D}E = D\hat{C}E = x°$.
$x° + 118° + x° = 180°$ ($\angle$ sum of $\triangle CDE$)
$2x° = 180° - 118°$
$= 62°$
$x° = \frac{62°}{2}$
$= 31°$
$A\hat{D}E + 31° = 90°$ (right angle of rectangle)
$A\hat{D}E = \mathbf{59°}$

(ii) From **(i)**, $D\hat{C}E = x° = \mathbf{31°}$.

19. Extending DA and CB to intersect at E, we get:



Since $E\hat{D}C = E\hat{B}C$, $\triangle EDC$ is an isosceles triangle where $ED = EC$.
$E\hat{A}B = E\hat{D}C$ (corr. $\angle$s, AB // DC)
$E\hat{B}A = E\hat{C}D$ (corr. $\angle$s, AB // DC)
$\therefore$ $E\hat{A}B = E\hat{B}A$ and $\triangle EAB$ is an isosceles triangle where $EA = EB$.

$$\begin{aligned} AD &= ED - EA \\ &= EC - EB \\ &= BC \end{aligned}$$

$\therefore$ $AD = BC$ and trapezium $ABCD$ is an isosceles trapezium. (shown)

20. Sum of all angles in triangles ABC, DEF, GHI and JKL $= 4(180°) = 720°$

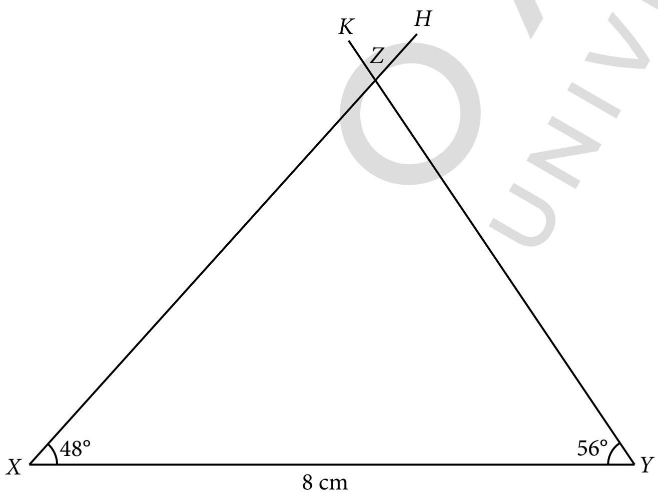
$\angle ACB = \angle ECJ$ (vert. opp. $\angle$s)
$\angle DEF = \angle CEG$ (vert. opp. $\angle$s)
$\angle IGH = \angle EGJ$ (vert. opp. $\angle$s)
$\angle KJL = \angle CJG$ (vert. opp. $\angle$s)

$$\begin{aligned} \angle ACB + \angle DEF + \angle IGH + \angle KJL &= \angle ECJ + \angle CEG + \angle EGJ + \angle CJG \\ &= 360° \text{ (}\angle\text{ sum of quadrilateral)} \end{aligned}$$

$$\begin{aligned} \therefore \angle a + \angle b + \angle c + \angle d + \angle e + \angle f + \angle g + \angle h &= 720° - 360° \\ &= \mathbf{360°} \end{aligned}$$

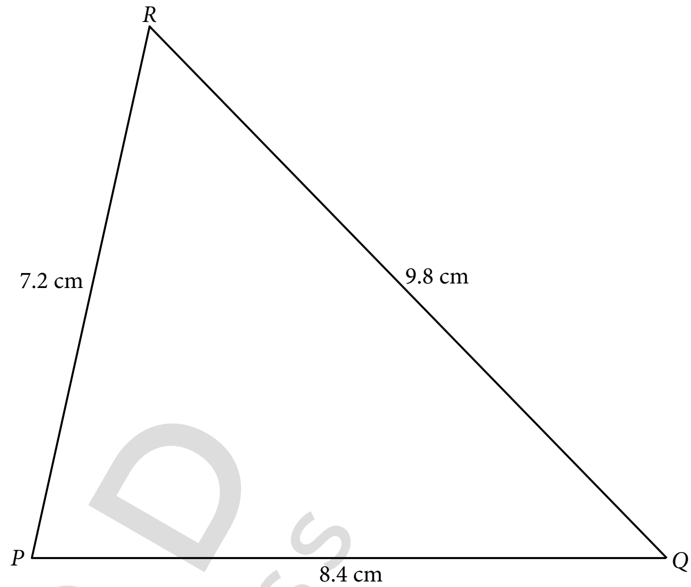
11.3 Geometrical constructions of triangles and quadrilaterals

Practise Now 7



$\therefore$ length of shortest side = $\boldsymbol{YZ = 6.1}$ **cm**

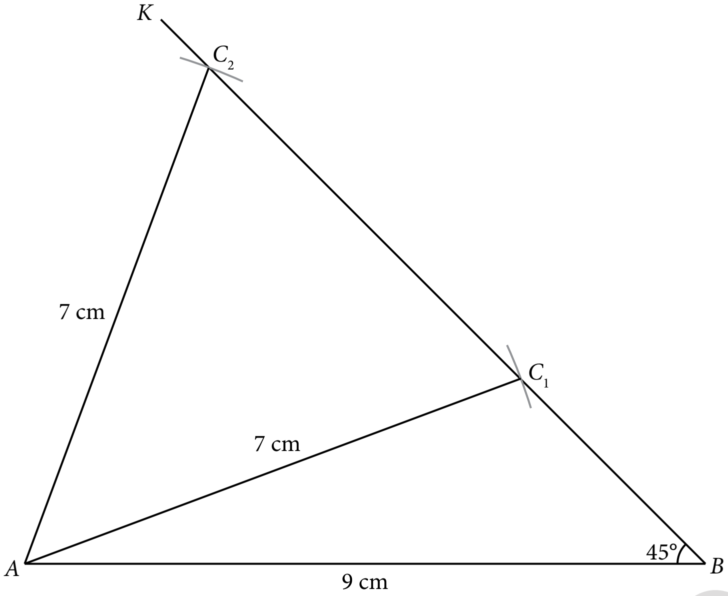
Practise Now 8



The angle facing the longest side QR is $\angle$***QPR*** **= °77**.

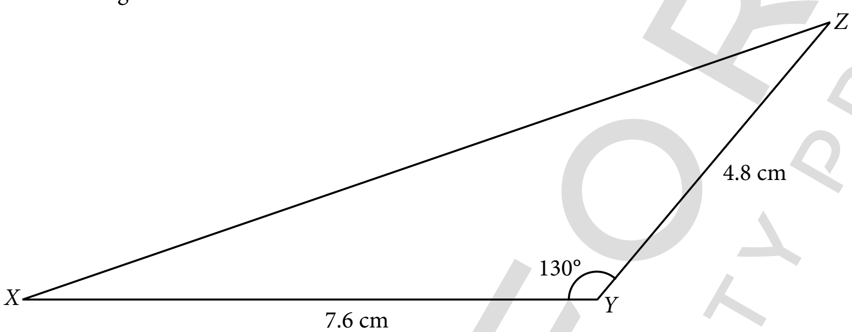
Practise Now 9

1.



Possible lengths of BC are **3.4 cm and 9.3 cm**.

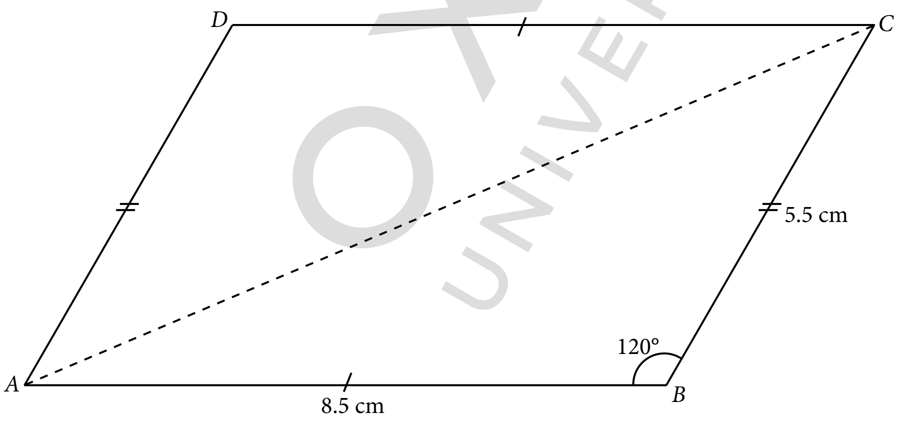
2.



Length of XZ = **11.3 cm**

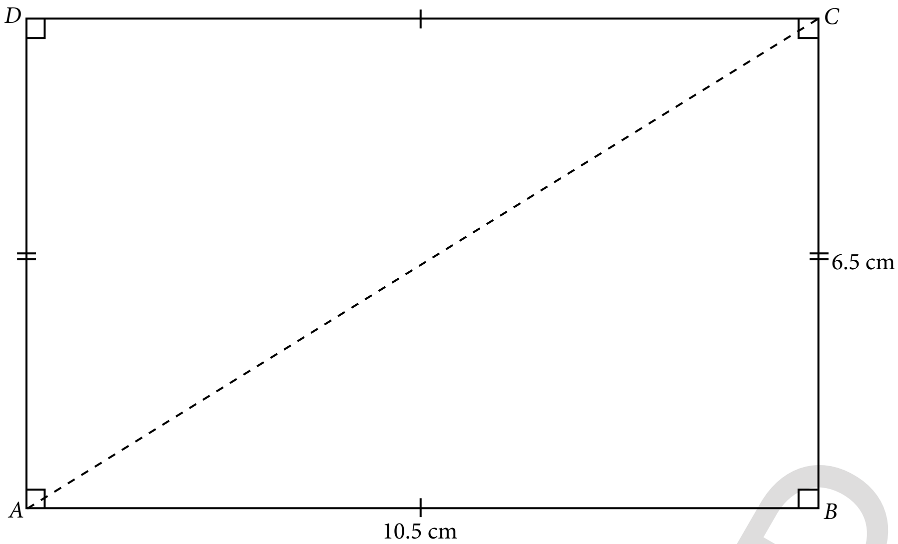
Practise Now 10

1.



Length of AC = **12.2 cm**

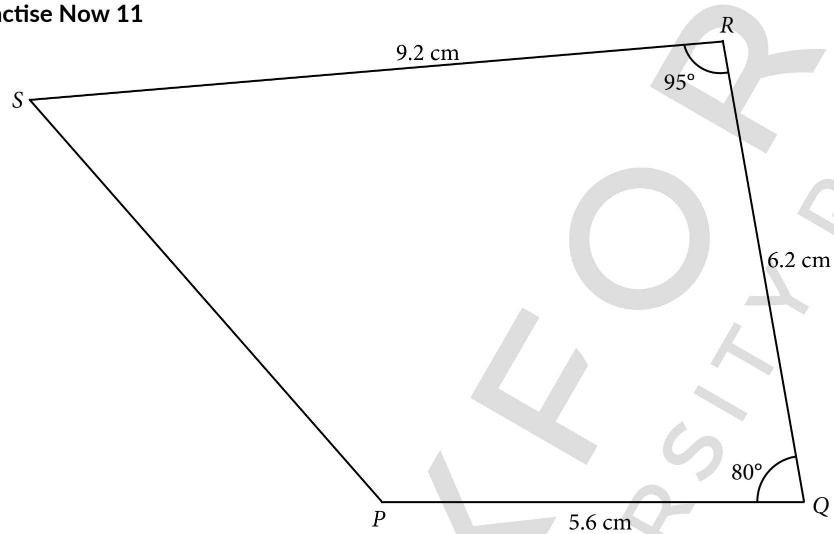
2.



Length of AC = **12.3 cm**

Practise Now 11

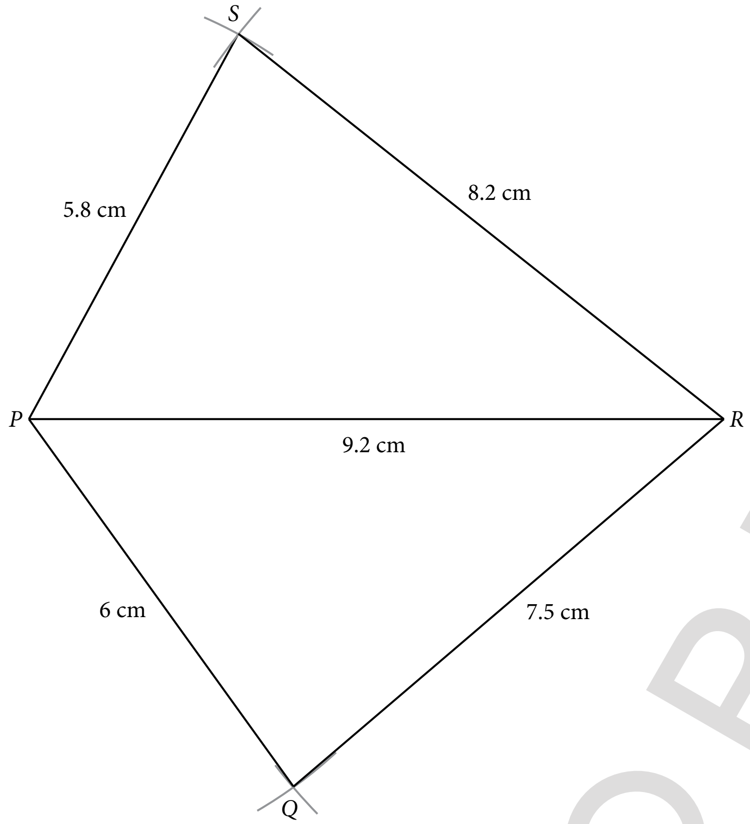
1.



(i) Length of PS = **7.0 cm**

(ii) $P\hat{S}R$ = **54°**

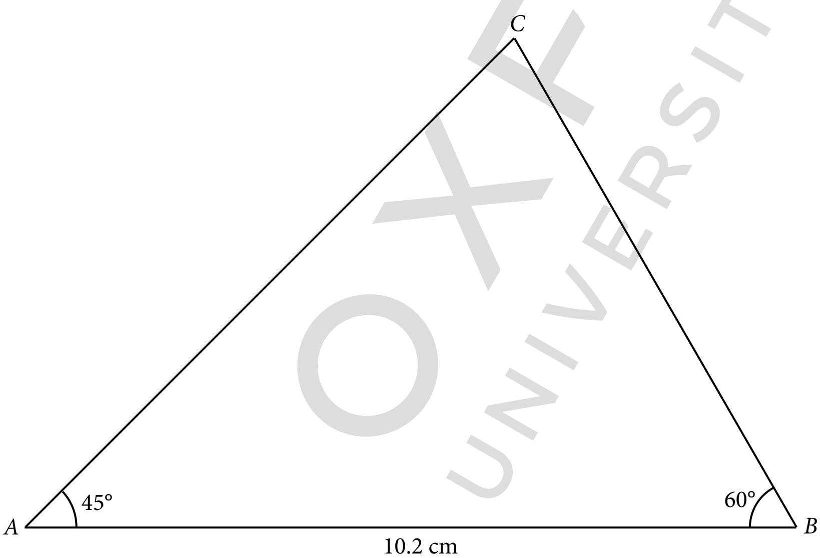
2.



$Q\hat{R}S$ = **79°**

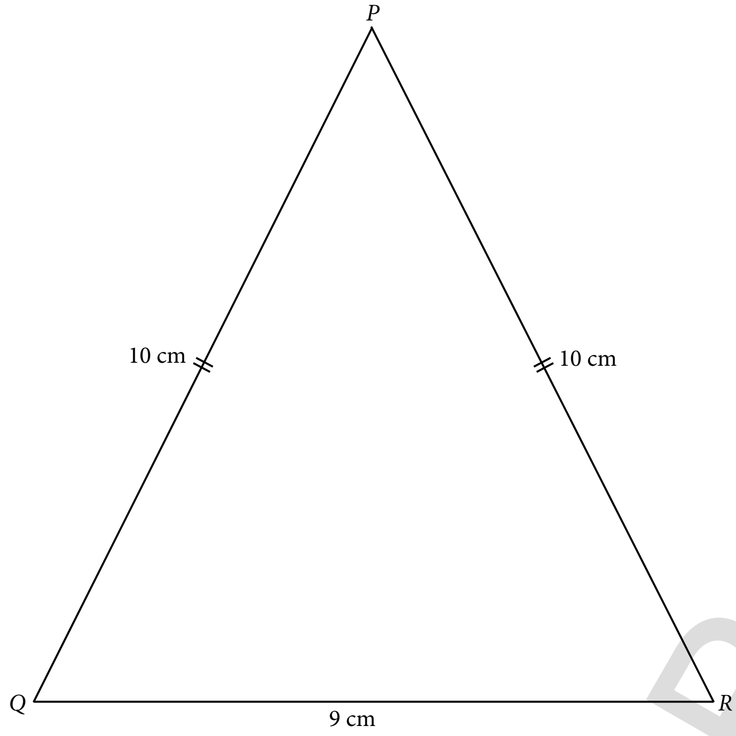
Exercise 11C

1.



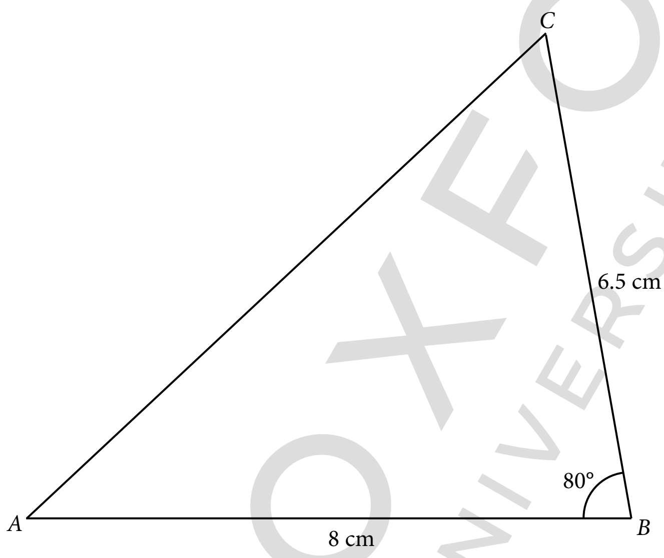
Length of AC = **9.1 cm**

2.



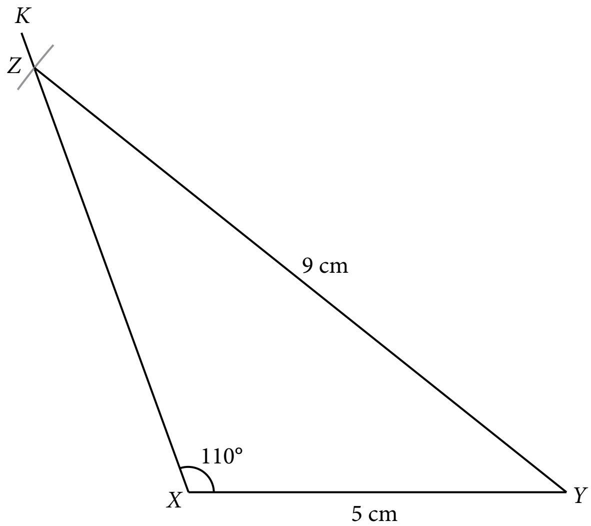
$Q\hat{P}R$ = **53°**

3. **(a)**



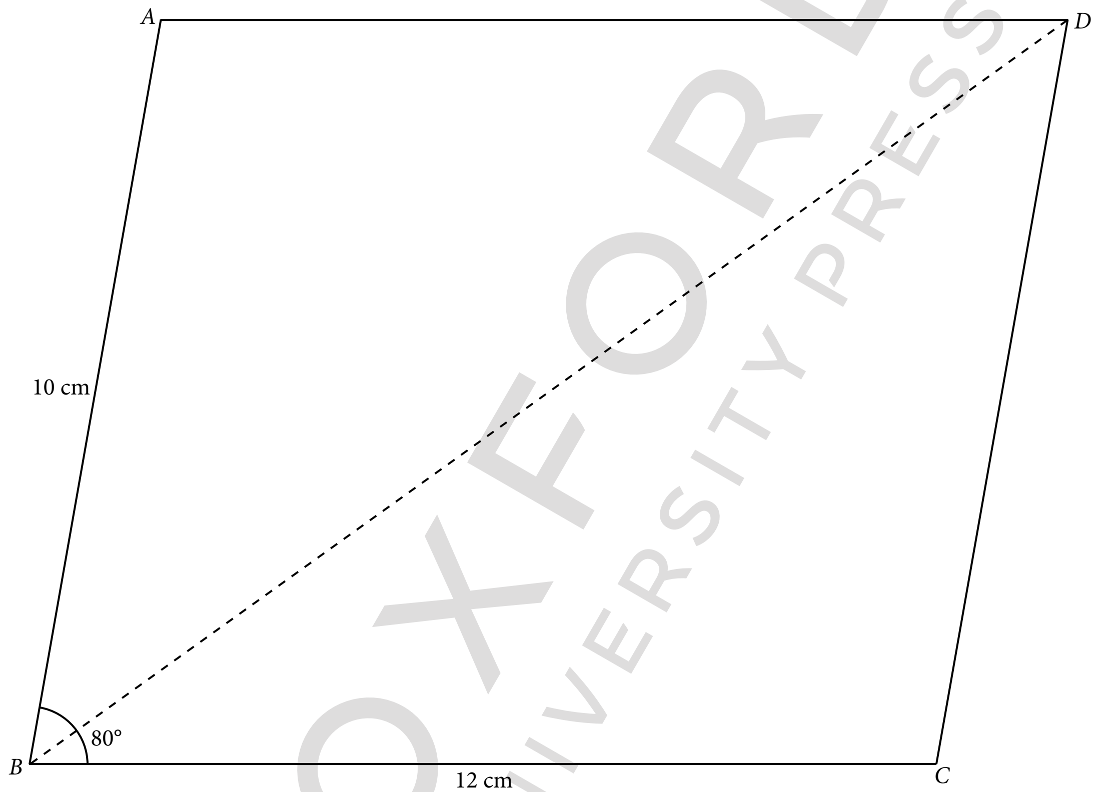
Length of AC = **9.4 cm**

(b)



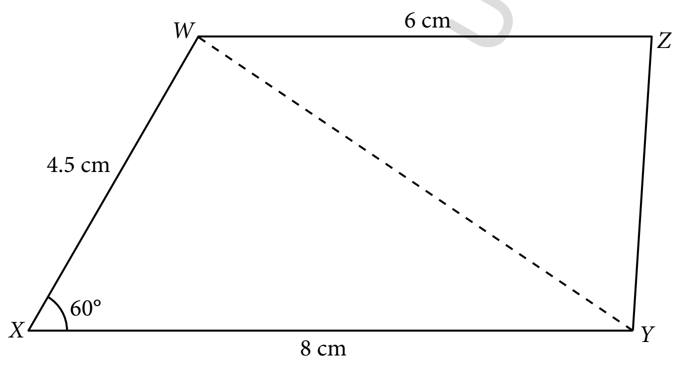
Length of XZ = **6.0 cm**

4.



Length of diagonal BD = **16.9 cm**

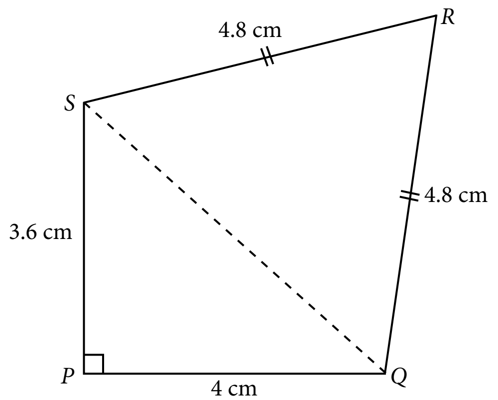
5.



Length of YZ = **3.9 cm**

Length of WY = **6.9 cm**

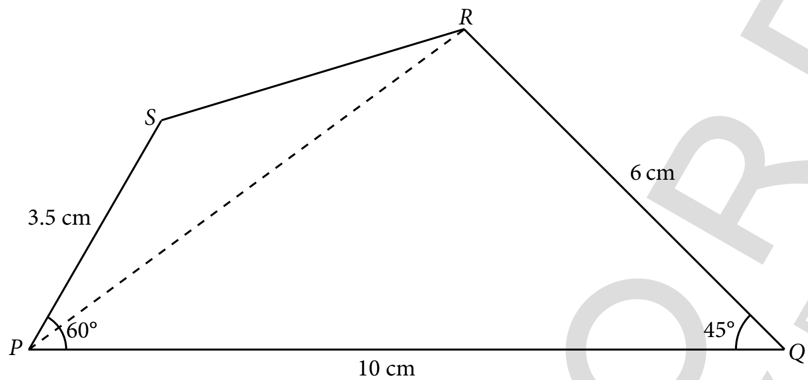
6.



(i) Length of QS = **5.4 cm**

(ii) $Q\hat{R}S$ = **68°**

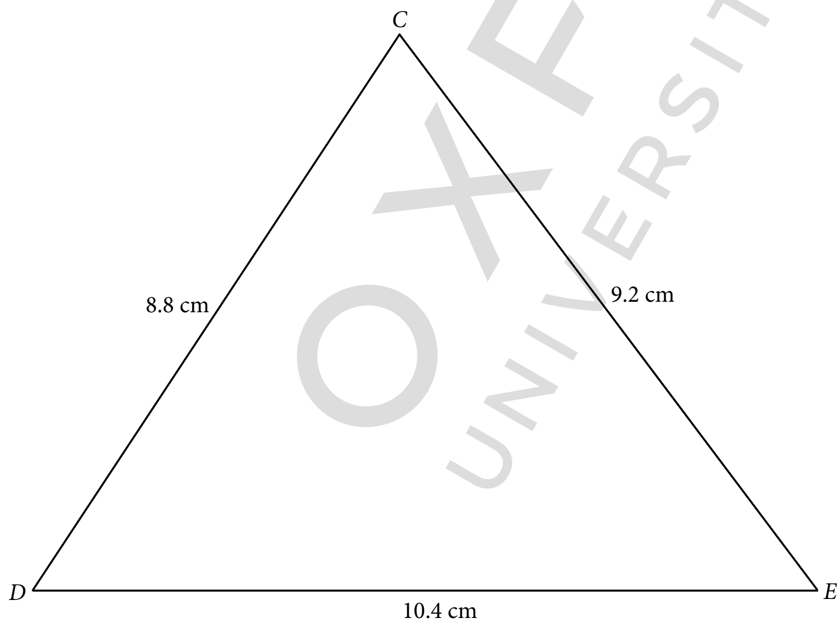
7.



(i) Length of SR = **4.2 cm**

(ii) $P\hat{R}S$ = **20°**

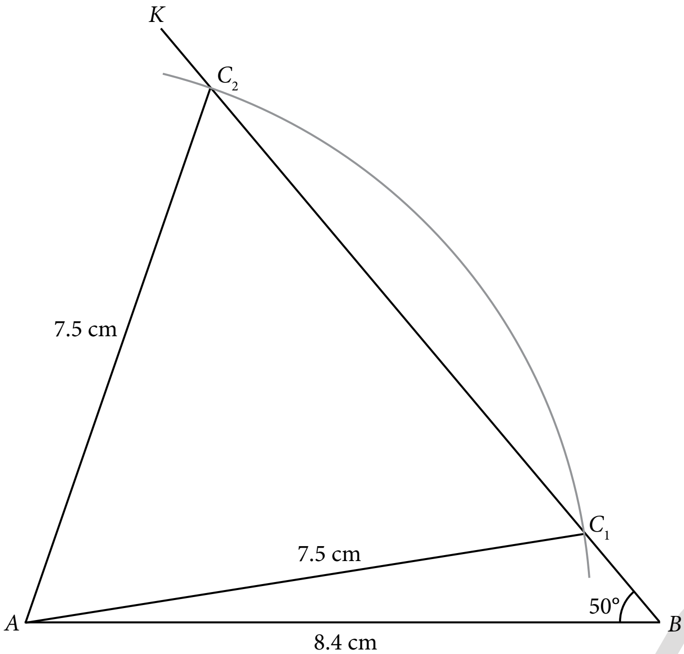
8.



The smallest angle is the angle opposite the shortest side CD.

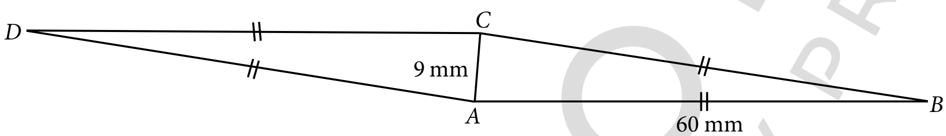
Required angle, $\angle$***CED* = 53°**

9.



Possible lengths of *BC* are **1.5 cm and 9.3 cm**.

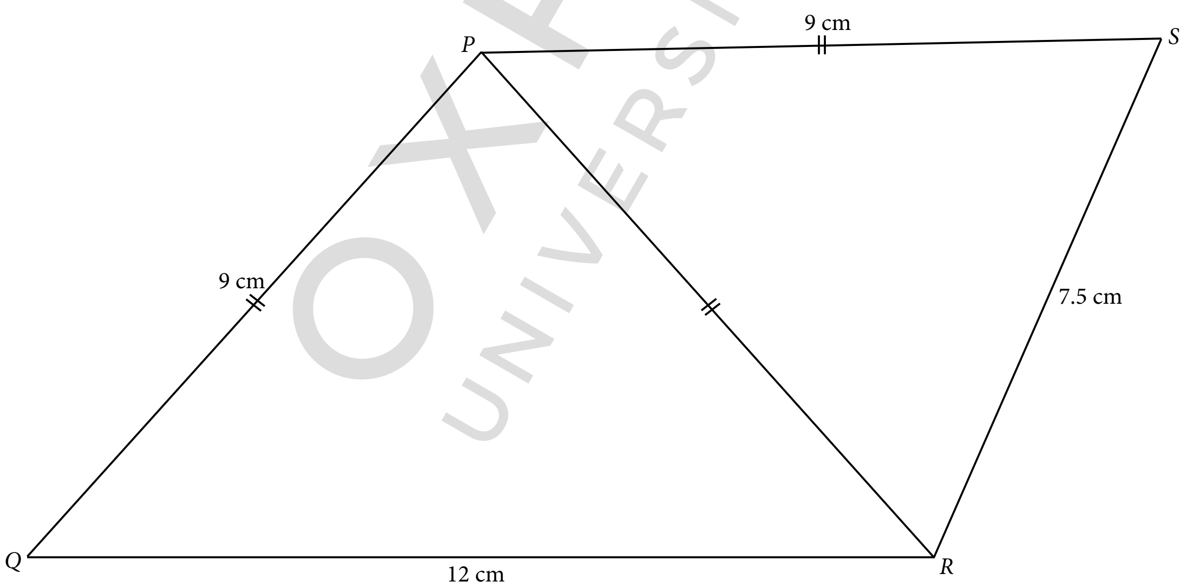
10.



(i) All four sides of a rhombus are equal in length.

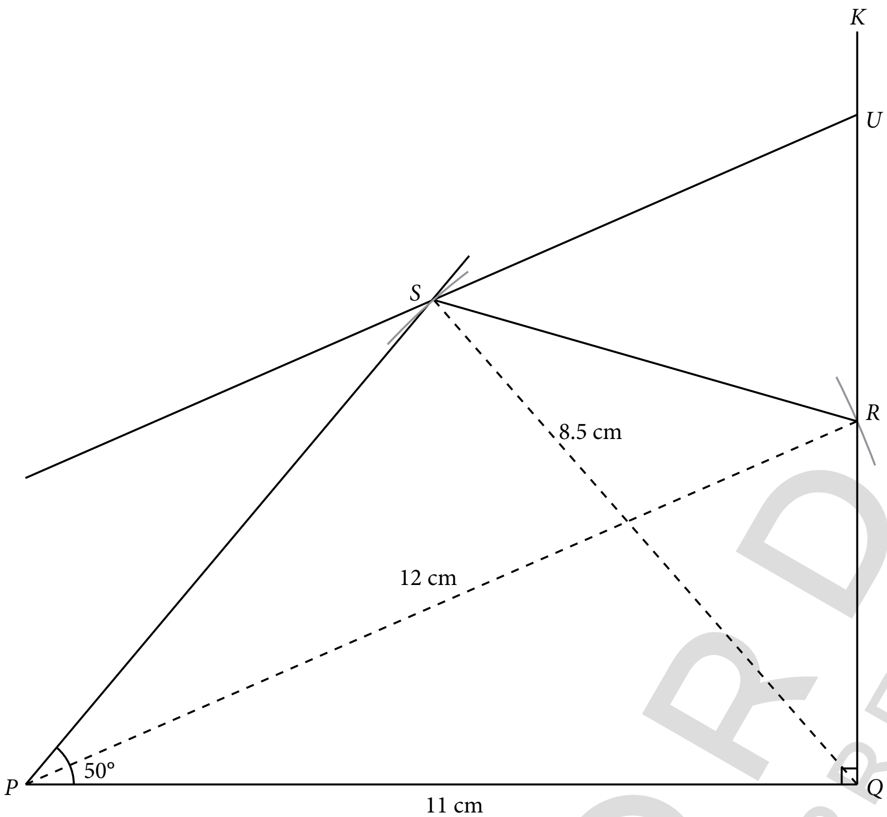
(ii) $B\hat{A}D$ = **171°**

11.



$Q\hat{P}S$ = **133°**

12.

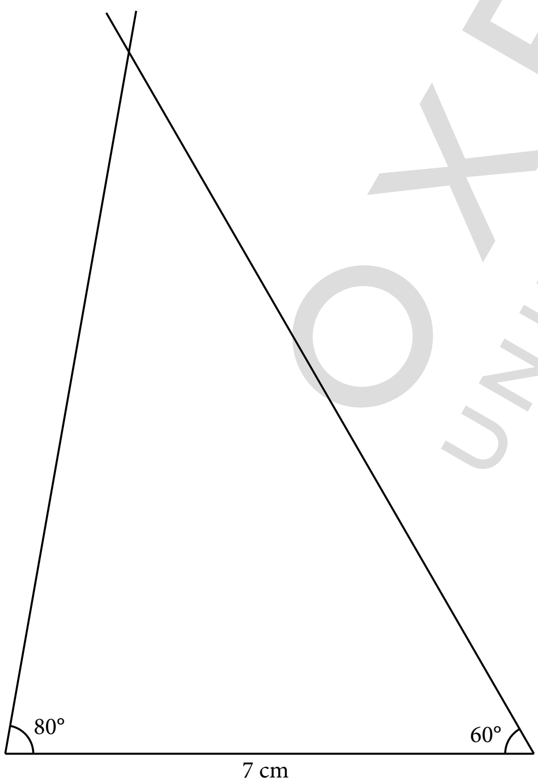


(i) $Q\hat{R}S$ = **106°**

(ii) Length of RU = **4.1 cm**

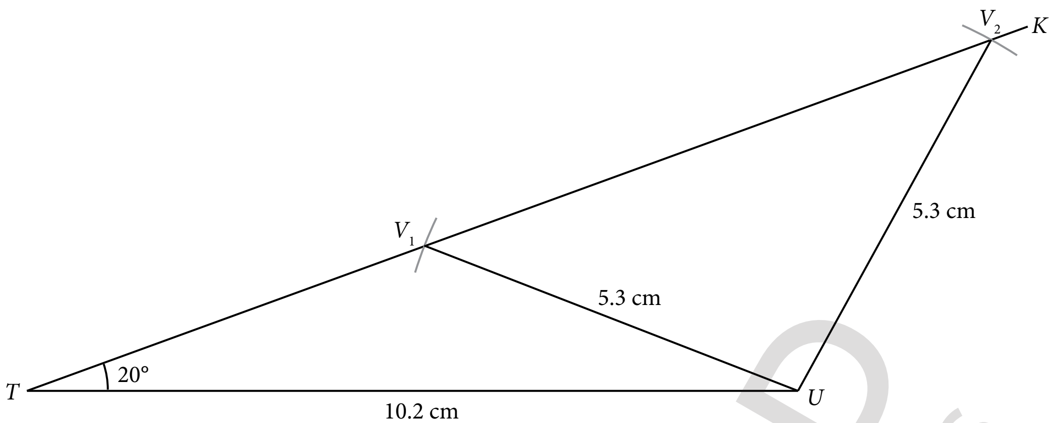
13. (i) **6** different triangles can be constructed.

(ii) The triangle with 7 cm as its shortest side is the one where 7 cm faces the smallest angle.



14. It is not possible to do so. The sum of the lengths of any two (shorter) sides of a triangle must be greater than the length of the third side. In this case, 1 cm + 3 cm = 4 cm is less than 5 cm. Thus, there is no such triangle.

15.



(i) 2 different triangles can be constructed.
A property of triangles is that the largest angle of a triangle is opposite the longest side and the smallest angle is opposite the shortest side.
Since $\angle UTV$ (= 20°) is opposite the side UV (with length 5.3 cm), then $\angle TVU$, which is opposite the longer side TU (with length 10.2 cm), must be larger than $\angle UTV$ (= 20°).
An angle larger than 20° can be an acute angle, a right angle or an obtuse angle.
If $\angle TVU$ is a right angle, there is only one way to draw the triangle TUV.
Otherwise, we can draw two different triangles such that $\angle TVU$ is an acute angle larger than 20°, or an obtuse angle.

Note: In Secondary 1, students will not know whether $\angle TVU$ is a right angle or otherwise unless they construct the triangle.

(ii) For V_1: length of longest side, TU = **10.2 cm**,
largest angle, $\angle TV_1U$ = **139°**
For V_2: length of longest side, TV_2 = **13.6 cm**,
largest angle, $\angle TUV_2$ = **119°**

11.4 Polygons

Class Discussion (What are polygons?)

1. The first shape has a curve, and the other two are not closed.
2. The third polygon is a simple polygon but the fourth one is not, as the boundaries cross each other.
3. The second and third polygons have interior angle(s) that are more than 180°.

Investigation (Naming of polygons)

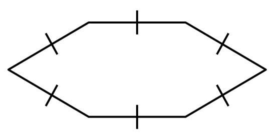
Pentagon (5-sided)
Hexagon (6-sided)
Octagon (8-sided)
Decagon (10-sided)

Thinking Time (Page 286)

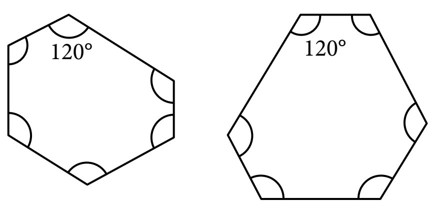
1. The **pentagon** and **hexagon** are regular polygons.
2. The name of a regular triangle is an **equilateral triangle** and the name of a regular quadrilateral is a **square**.

Class Discussion (Definition of a regular polygon)

1. **Yes**.
   (a) **Rhombus**
   (b)

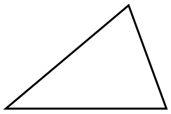
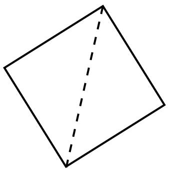
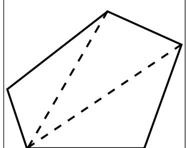
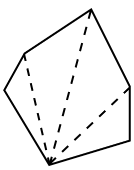
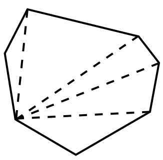
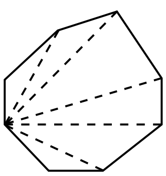


2. **Yes**.
   (a) **Rectangle**
   (b)



Investigation (Sum of interior angles of polygon)

1.

Polygon	Number of sides	Number of Triangle(s) formed	Sum of Interior Angles
 Triangle	3	1	$1 \times 180° = (3 - 2) \times 180°$
 Quadrilateral	4	2	$2 \times 180° = (4 - 2) \times 180°$
 Pentagon	**5**	**3**	$\mathbf{3 \times 180° = (5 - 2) \times 180°}$
 Hexagon	**6**	**4**	$\mathbf{4 \times 180° = (6 - 2) \times 180°}$
 Heptagon	**7**	**5**	$\mathbf{5 \times 180° = (7 - 2) \times 180°}$
 Octagon	**8**	**6**	$\mathbf{6 \times 180° = (8 - 2) \times 180°}$
n-gon	$\boldsymbol{n}$	$\boldsymbol{(n - 2)}$	$\boldsymbol{(n - 2) \times 180°}$

2. If a polygon has n sides, then it will form $(n - 2)$ triangles.

3. The general formula for the sum of interior angles of an n-sided polygon is $\boldsymbol{(n - 2) \times 180°}$.

4. Yes, a triangle will always be added when a point is added to change a pentagon into a hexagon.

Practise Now 12

1. (a) Sum of interior angles of a pentagon

$= (n - 2) \times 180°$

$= (5 - 2) \times 180°$

$= 540°$

$$a° + 121° + a° + a° + 107° = 540°$$
$$3a° = 540° - 121° - 107°$$
$$3a° = 312°$$
$$a° = \frac{312°}{2}$$
$$= 104°$$
$$\therefore a = \mathbf{104}$$

(b) Sum of interior angles of a hexagon

$= (n - 2) \times 180°$

$= (6 - 2) \times 180°$

$= 720°$

$$3b° + 4b° + 104° + 114° + 128° + 122° = 720°$$
$$7b° = 720° - 104° - 114° - 128° - 122°$$
$$7b° = 252°$$
$$b° = \frac{252°}{7}$$
$$= 36°$$
$$\therefore b = \mathbf{36}$$

2. Sum of interior angles of an n-sided polygon $= (n - 2) \times 180°$

$$74° + 136° + (n - 2) \times 110° = (n - 2) \times 180°$$
$$210° + n(110°) - 220° = n(180°) - 360°$$
$$n(70°) = 350°$$
$$n = \frac{350°}{70°}$$
$$= \mathbf{5}$$

Practise Now 13

(i) Sum of interior angles of a regular polygon with 24 sides

$= (n - 2) \times 180°$

$= (24 - 2) \times 180°$

$= \mathbf{3960°}$

(ii) Size of each interior angle of a regular polygon with 24 sides

$= \dfrac{3960°}{24}$

$= \mathbf{165°}$

Investigation (Sum of exterior angles of pentagon)

2. The exterior angles meet at a point. Since the sum of angles at a point is 360°, the sum of the exterior angles of a pentagon is 360°.

3. The exterior angles of an n-sided polygon also meet at a point. Since the sum of angles at a point is 360°, the sum of the exterior angles of an n-sided polygon is 360°.

Thinking Time (Page 290)

Since $\angle a + \angle p = 180°$, $\angle b + \angle q = 180°$, $\angle c + \angle r = \mathbf{180°}$, $\angle d + \angle s = \mathbf{180°}$ and $\angle e + \angle t = \mathbf{180°}$, then $\angle a + \angle p + \angle b + \angle q + \angle c + \angle r + \angle d + \angle s + \angle e + \angle t = \mathbf{5} \times 180°$

$\therefore (\angle a + \angle b + \angle c + \angle d + \angle e) + (\angle p + \angle q + \angle r + \angle s + \angle t) = 900°$

Since the sum of interior angles of a pentagon

$= \angle a + \angle b + \angle c + \angle d + \angle e$

$= (5 - 2) \times 180° = 540°$, then

$540° + (\angle p + \angle q + \angle r + \angle s + \angle t) = 900°$.

$\therefore \angle p + \angle q + \angle r + \angle s + \angle t = 900° - \mathbf{540°}$

$= \mathbf{360°}$

Class Discussion (Exterior angles of polygon)

1. **No.** Since 70° is not an exact divisor of 360°, hence a regular polygon to have an exterior angle of 70° is not possible.
2. **Since**

$360° = 3 \times 120°$,	$360° = 20 \times 18°$,
$360° = 4 \times 90°$,	$360° = 25 \times 15°$,
$360° = 6 \times 60°$,	$360° = 30 \times 12°$,
$360° = 8 \times 45°$,	$360° = 40 \times 9°$,
$360° = 9 \times 40°$,	$360° = 45 \times 8°$,
$360° = 10 \times 36°$,	$360° = 60 \times 6°$,
$360° = 12 \times 30°$,	$360° = 90 \times 4°$,
$360° = 15 \times 24°$,	$360° = 120 \times 3°$,
$360° = 18 \times 20°$,	$360° = 180 \times 2°$,

All the possible values of the angle are **2°, 3°, 4°, 6°, 8°, 9°, 12°, 15°, 18°, 20°, 24°, 30°, 36°, 40°, 45°, 60°, 90° and 120°**.

Practise Now 14

1. **(a)** The sum of exterior angles of the regular polygon is 360°.

$\therefore$ number of sides of the polygon

$= \dfrac{360°}{40°}$

$= \mathbf{9}$

(b) Size of each exterior angle of a regular polygon

$= 180° - 178°$

$= 2°$

The sum of exterior angles of the regular polygon is 360°.

$\therefore$ number of sides of the polygon

$= \dfrac{360°}{2°}$

$= \mathbf{180}$

2. The sum of exterior angles of the regular decagon is 360°.

$\therefore$ size of each exterior angle of the regular decagon

$= \dfrac{360°}{10°}$

$= 36°$

$\therefore$ size of each interior angle of the regular decagon

$= 180° - 36°$

$= \mathbf{144°}$

3. The sum of exterior angles of an n-sided polygon is 360°.

$25° + 26° + 3(180° - 161°) + (n - 5)(180° - 159°) = 360°$

$25° + 26° + 3(19°) + (n - 5)(21°) = 360°$

$25° + 26° + 57° + n(21°) - 105° = 360°$

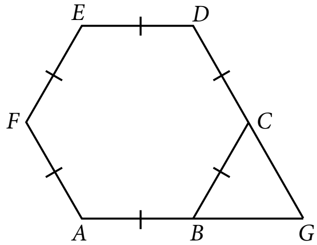
$n(21°) = 360° - 25° - 26° - 57° + 105°$

$= 357°$

$n = \dfrac{357°}{21°}$

$= \mathbf{17}$

Practise Now 15



Size of each exterior angle of the hexagon $= \dfrac{360°}{6}$

$= 60°$

$C\hat{B}G = B\hat{C}G = 60°$

$B\hat{G}C + 60° + 60° = 180°$ ($\angle$ sum of $\triangle BCG$)

$B\hat{G}C = 180° - 60° - 60°$

$= \mathbf{60°}$

Practise Now 16

(i) Sum of interior angles of a pentagon

$= (n - 2) \times 180°$

$= (5 - 2) \times 180°$

$= 540°$

Since $P\hat{B}C$ is an interior angle of a pentagon,

$\therefore P\hat{B}C = \dfrac{540°}{5} = \mathbf{108°}$.

(ii) Since $C\hat{R}Q$ is an interior angle of a pentagon,

$\therefore C\hat{R}Q = 108°$.

Let $Q\hat{C}R = C\hat{Q}R = x°$ (base $\angle$s of isos. $\triangle CQR$)

$x° + x° + 108° = 180°$ ($\angle$ sum of $\triangle CQR$)

$2x° = 180° - 108°$

$2x° = 72°$

$x° = \dfrac{72°}{2}$

$= 36°$

$\therefore Q\hat{C}R = \mathbf{36°}$

(iii) $B\hat{C}D + 108° + 90° = 360°$ ($\angle$s at a point)

$B\hat{C}D = 360° - 108° - 90°$

$= \mathbf{162°}$

(iv) Let $B\hat{D}C = B\hat{C}D = y°$ (base $\angle$s of isos. $\triangle BCD$)

$y° + y° + 162° = 180°$ ($\angle$ sum of $\triangle BCD$)

$2y° = 180° - 162°$

$2y° = 18°$

$y° = \dfrac{18°}{2}$

$= 9°$

$\therefore B\hat{D}C = \mathbf{9°}$

(v) Let the exterior angle of the n-sided polygon be $a°$.

$a° + 162° = 180°$ (adj. $\angle$s on a str. line)

$a° = 180° - 162°$

$= 18°$

Since the sum of the exterior angles of the n-sided polygon is 360°,

$\therefore n = \dfrac{360°}{18°}$

$= \mathbf{20}$

Introductory Problem Revisited

1. **Method 1:**

Size of each interior angle $= \dfrac{360°}{3}$ ($\angle$s at a point)

$= 120°$ (shown)

Method 2:

Sum of interior angles of a hexagon $= (6 - 2) \times 180°$

$= 720°$

Size of each interior angle $= \dfrac{720°}{6}$

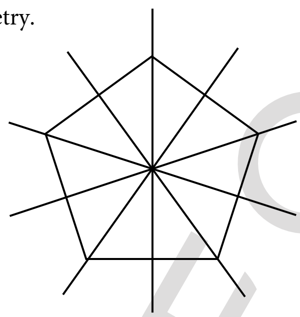
$= 120°$ (shown)

Class Discussion (Line symmetry of special quadrilaterals and regular polygons)

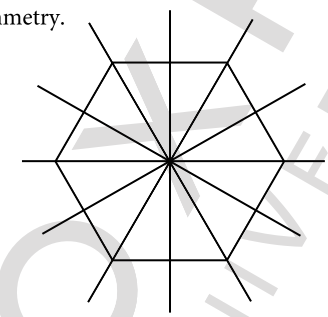
1. A **rectangle** has **2** lines of symmetry.
A **square** (that is a not a rectangle) has **4** lines of symmetry.
A **parallelogram** (that is not a rectangle or a rhombus) has no line of symmetry i.e. **0**.
A **rhombus** has **2** lines of symmetry.
A **kite** has **1** line of symmetry.

2. The angles opposite each other along the line of symmetry are equal.

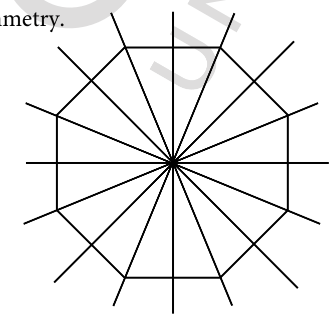
3. A **pentagon** has **5** lines of symmetry.



An **hexagon** has **6** lines of symmetry.



An **octagon** has **8** lines of symmetry.



Class Discussion (Rotational symmetry of special quadrilaterals and regular polygons)

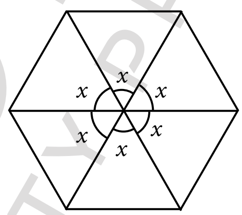
1. A **rectangle** has a rotational symmetry order of **2**.
A **square** (that is a not a rectangle) has a rotational symmetry order of **4**.
A **parallelogram** (that is not a rectangle or a rhombus) has a rotational symmetry order of **2**.
A **rhombus** has a rotational symmetry order of **2**.
A **kite** has a rotational symmetry order of **1**, i.e. no rotational symmetry.

2. The figure will only map onto itself when rotated about its centre.

3. For example:
- A square has 4 equal sides and angles given that it has a rotational symmetry of order 4.
- The opposite angles of a rhombus are equal.

4. **Yes**.
A **pentagon** has a rotational symmetry of order **5**.
An **hexagon** has a rotational symmetry of order **6**.
An **octagon** has a rotational symmetry of order **8**.

5. From the symmetries, it will indicate the number of identical triangles formed in each polygon.
A pentagon will have 5 identical triangles formed.
An octagon will have 8 identical triangles formed.
Other angles which are equal to $\angle x$ are indicated as below:



Exercise 11D

1. **(a)** Sum of interior angles of an 11-gon
$= (n - 2) \times 180°$
$= (11 - 2) \times 180°$
$= \mathbf{1620°}$

(b) Sum of interior angles of a 12-gon
$= (n - 2) \times 180°$
$= (12 - 2) \times 180°$
$= \mathbf{1800°}$

(c) Sum of interior angles of a 15-gon
$= (n - 2) \times 180°$
$= (15 - 2) \times 180°$
$= \mathbf{2340°}$

(d) Sum of interior angles of a 20-gon
$= (n - 2) \times 180°$
$= (20 - 2) \times 180°$
$= \mathbf{3240°}$

2. **(a)** Sum of interior angles of a quadrilateral
$= (n - 2) \times 180°$
$= (4 - 2) \times 180°$
$= 360°$

$$78° + 62° + a° + 110° = 360°$$
$$a° = 360° - 78° - 62° - 110°$$
$$= 110°$$
$$\therefore a = \mathbf{110}$$

(b) Sum of interior angles of a quadrilateral

$= (n - 2) \times 180°$

$= (4 - 2) \times 180°$

$= 360°$

$b° + 78° + 2b° + 84° = 360°$

$3b° = 360° - 78° - 84°$

$= 198°$

$b° = \dfrac{198°}{3}$

$= 66°$

$\therefore$ ***b* = 66**

(c) Sum of interior angles of a pentagon

$= (n - 2) \times 180°$

$= (5 - 2) \times 180°$

$= 540°$

$c° + 152° + 38° + 2c° + 101° = 540°$

$3c° = 540° - 152° - 38° - 101°$

$3c° = 249°$

$c° = \dfrac{249°}{3}$

$= 83°$

$\therefore$ ***c* = 83**

(d) Sum of interior angles of a hexagon

$= (n - 2) \times 180°$

$= (6 - 2) \times 180°$

$= 720°$

$102° + 5d° + 4d° + 4d° + 108° + 4d° = 720°$

$17d° = 720° - 102° - 108°$

$= 510°$

$d° = \dfrac{510°}{17}$

$= 30°$

$\therefore$ ***d* = 30**

3. (a) (i) Sum of interior angles of a regular octagon

$= (n - 2) \times 180°$

$= (8 - 2) \times 180°$

= **1080°**

(ii) Hence, size of each interior angle of a regular octagon

$= \dfrac{1080°}{8}$

= **135°**

(b) (i) Sum of interior angles of a regular polygon with 18 sides

$= (n - 2) \times 180°$

$= (18 - 2) \times 180°$

= **2880°**

(ii) Hence, size of each interior angle of a regular polygon with 18 sides

$= \dfrac{2880°}{18}$

= **160°**

4. (a) The sum of exterior angles of the regular polygon is 360°.

$\therefore$ number of sides of the polygon $= \dfrac{360°}{45°}$

= **8**

(b) The sum of exterior angles of the regular polygon is 360°.

$\therefore$ number of sides of the polygon $= \dfrac{360°}{90°}$

= **4**

(c) The sum of exterior angles of the regular polygon is 360°.

$\therefore$ number of sides of the polygon $= \dfrac{360°}{4°}$

= **90**

(d) The sum of exterior angles of the regular polygon is 360°.

$\therefore$ number of sides of the polygon $= \dfrac{360°}{120°}$

= **3**

5. (a) Size of each exterior angle of the regular polygon

$= 180° - 140°$

$= 40°$

The sum of exterior angles of a regular polygon is 360°.

$\therefore$ number of sides of the polygon

$= \dfrac{360°}{40°}$

= **9**

(b) Size of each exterior angle of the regular polygon

$= 180° - 162°$

$= 18°$

The sum of exterior angles of a regular polygon is 360°.

$\therefore$ number of sides of the polygon

$= \dfrac{360°}{18°}$

= **20**

(c) Size of each exterior angle of the regular polygon

$= 180° - 172°$

$= 8°$

The sum of exterior angles of a regular polygon is 360°.

$\therefore$ number of sides of the polygon

$= \dfrac{360°}{8°}$

= **45**

(d) Size of each exterior angle of the regular polygon

$= 180° - 175°$

$= 5°$

The sum of exterior angles of a regular polygon is 360°.

$\therefore$ number of sides of the polygon

$= \dfrac{360°}{5°}$

= **72**

6. (a) The sum of exterior angles of the regular polygon is 360°.

$\therefore$ Size of each exterior angle of the regular polygon

$= \dfrac{360°}{24}$

$= 15°$

$\therefore$ Size of each interior angle of a regular polygon with 24 sides

$= 180° - 15°$

= **165°**

(b) The sum of exterior angles of the regular polygon is 360°.

$\therefore$ Size of each exterior angle of the regular polygon

$= \dfrac{360°}{36}$

$= 10°$

$\therefore$ Size of each interior angle of a regular polygon with 36 sides

$= 180° - 10°$

= **170°**

7. Sum of interior angles of n-sided polygon = $(n-2) \times 180°$

$$76° + 169° + 105° + (n-3) \times 146° = (n-2) \times 180°$$
$$350° + n(146°) - 438° = n(180°) - 360°$$
$$n(34°) = 272°$$
$$n = \frac{272°}{34°}$$
$$= \mathbf{8}$$

8. The sum of exterior angles of an sided polygon is 360°.

$$3(50°) + (180° - 127°) + (180° - 135°) + (n-5)(180° - 173°) = 360°$$
$$150° + 53° + 45° + (n-5)(7°) = 360°$$
$$150° + 53° + 45° + n(7°) - 35° = 360°$$
$$n(7°) = 360° - 150° - 53° - 45° + 35°$$
$$= 147°$$
$$n = \frac{147°}{7°}$$
$$= \mathbf{21}$$

9. Let each interior angle and exterior angle be $13x°$ and $2x°$ respectively.

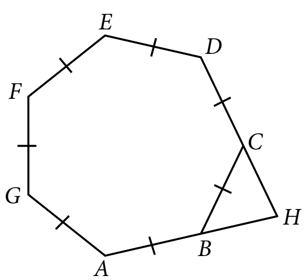
$13x° + 2x° = 180°$ (adj. $\angle$s on a str. line)

$$15x° = 180°$$
$$x° = \frac{180°}{15}$$
$$= 12°$$

Size of each exterior angle, $2x° = 2(12°)$
$$= 24°$$

$\therefore$ number of sides of polygon, $n = \dfrac{360°}{24°}$
$$= \mathbf{15}$$

10.



Size of each exterior angle of the heptagon $= \dfrac{360°}{7}$
$$= 51.429° \text{ (to 3 d.p.)}$$

$B\hat{H}C = 180° - 51.429° - 51.429°$ ($\angle$ sum of $\triangle BCH$)
$= \mathbf{77.1°}$ (to 1 d.p.)

11. (i) Sum of interior angles of a hexagon
$= (n-2) \times 180°$
$= (6-2) \times 180°$
$= 720°$
$\therefore$ size of each interior angle of a hexagon
$= \dfrac{720°}{6}$
$= 120°$
Since $A\hat{B}P$ is an interior angle of a hexagon,
$\therefore A\hat{B}P = \mathbf{120°}$.

(ii) Since $P\hat{Q}R$ is an interior angle of a hexagon,
$\therefore P\hat{Q}R = 120°$.
$P\hat{Q}X = \dfrac{120°}{2}$ (QA is a line of symmetry)
$= \mathbf{60°}$

(iii) $A\hat{X}B = \dfrac{360°}{6}$ ($\angle$s at a point)
$= \mathbf{60°}$

(iv) Sum of interior angles of a pentagon
$= (n-2) \times 180°$
$= (5-2) \times 180°$
$= 540°$
$\therefore$ size of each interior angle of a pentagon
$= \dfrac{540°}{5}$
$= 108°$
Since $A\hat{B}C$ is an interior angle of a pentagon,
$\therefore A\hat{B}C = \mathbf{108°}$.

(v) Since size of each interior angle of a pentagon = 108°,
$\therefore B\hat{C}D = 108°$
Let $B\hat{A}C = B\hat{C}A = x°$ (base $\angle$s of isos. $\triangle ABC$)
$x° + x° + 108° = 180°$ ($\angle$ sum of $\triangle ABC$)
$$2x° = 180° - 108°$$
$$2x° = 72°$$
$$x° = \frac{72°}{2}$$
$$= 36°$$
$$\therefore x = 36$$
Hence,
$A\hat{C}D = B\hat{C}D - B\hat{C}A$
$= 108° - 36°$
$= \mathbf{72°}$

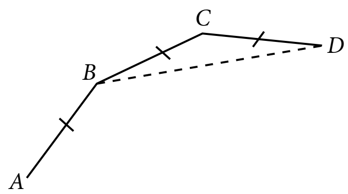
(vi) Since size of each interior angle of a hexagon = 120°,
$\therefore B\hat{A}S = 120°$
Since size of each interior angle of a pentagon = 108°,
$\therefore B\hat{A}E = 108°$
$120° + 108° + S\hat{A}E = 360°$ ($\angle$s at a point)
$$S\hat{A}E = 360° - 120° - 108°$$
$$= 132°$$
Let $A\hat{S}E = A\hat{E}S = x°$ (base $\angle$ of isos. $\triangle AES$)
$x° + x° + 132° = 180°$ ($\angle$ sum of $\triangle AES$)
$$2x° = 180° - 132°$$
$$2x° = 48°$$
$$x° = \frac{48°}{2}$$
$$= 24°$$
$$\therefore A\hat{S}E = \mathbf{24°}$$

12. (i) Since sum of exterior angles of an n-sided polygon is 360°,
$\therefore n = \dfrac{360°}{36°} = \mathbf{10}$.

(ii) Size of an interior angle of the polygon
$= 180° - 36°$ (adj. $\angle$s on a str. line)
$= 144°$
$\angle CDB = \dfrac{180° - 144°}{2}$ (base $\angle$s of isos. $\triangle BCD$)
$= 18°$
$\angle BDE = 144° - 18°$
$= \mathbf{126°}$

(iii) $\angle DCE = \dfrac{180° - 144°}{2}$ (base $\angle$s of isos. $\triangle CDE$)
$= 18°$
$\angle CXD = 180° - 18° - 18°$ ($\angle$ sum of $\triangle CXD$)
$= 144°$
$\angle BXE = \angle CXD = \mathbf{144°}$ (vert. opp. $\angle$s)

13.



(i) Sum of interior angles of a regular polygon with 20 sides
$= (n - 2) \times 180°$
$= (20 - 2) \times 180°$
$= 3240°$
Hence, size of each interior angles of a regular polygon with 20 sides
$= \dfrac{3240°}{20}$
$= 162°$
$\therefore\ A\hat{B}C = \mathbf{162°}$

(ii) Since size of each interior angle of a regular polygon with 20 sides
$= 162°$,
$\therefore\ B\hat{C}D = 162°$
Let $C\hat{B}D = C\hat{D}B = x°$ (base $\angle$s of isos. $\triangle BCD$)
$x° + x° + 162° = 180°$ ($\angle$ sum of $\triangle BCD$)
$2x° = 180° - 162°$
$2x° = 18°$
$x° = \dfrac{18°}{2}$
$= 9°$
$\therefore\ x = 9$
Hence,
$A\hat{B}D = A\hat{B}C - C\hat{B}D$
$= 162° - 9°$
$= \mathbf{153°}$

14. $a_1 + x_1 = 180°$ (adj. $\angle$s on a str. line)
$a_2 + x_2 = 180°$ (adj. $\angle$s on a str. line)
$a_3 + x_3 = 180°$ (adj. $\angle$s on a str. line)
$a_4 + x_4 = 180°$ (adj. $\angle$s on a str. line)
$a_n + x_n = 180°$ (adj. $\angle$s on a str. line)
Hence,
$a_1 + x_1 + a_2 + x_2 + a_3 + x_3 + a_4 + a_4 + \cdots + a_n + x_n = n \times 180°$
$\underbrace{a_1 + a_2 + a_3 + a_4 + \cdots + a_n} + \underbrace{x_1 + x_2 + x_3 + x_4 + \cdots + x_n} = n \times 180°$
$(n - 2) \times 180° + x_1 + x_2 + x_3 + x_4 + \cdots + x_n = n \times 180°$
$x_1 + x_2 + x_3 + x_4 + \cdots + x_n = n \times 180° - (n - 2) + 360°$
$x_1 + x_2 + x_3 + x_4 + \cdots + x_n = 180°n - 180°n + 360°$
$\therefore\ x_1 + x_2 + x_3 + x_4 + \cdots + x_n = 360°$ (shown)

15. Sum of interior angles of a pentagon $= 540°$
Let the exterior angle of the pentagon be $x°$.
$5(180° - x°) = 540°$
$900° - 5x° = 540°$
$-5x° = 540° - 900°$
$-5x° = -360°$
$x° = \dfrac{360°}{5}$
$= 72°$
$\angle a + 72° + 72° = 180°$
$\angle a = 180° - 72° - 72°$
$= 36°$
Hence, $\angle a + \angle b + \angle c + \angle d + \angle e = 5 \times 36° = \mathbf{180°}$

16. (i) Let the interior angle be $5x°$ and the exterior angle be $x°$.
$5x° + x° = 180°$ (adj. $\angle$s on a str. line)
$6x° = 180°$
$x° = \dfrac{180°}{6}$
$= 30°$
Since sum of exterior angles of a n-sided polygon is $360°$,
$\therefore\ n = \dfrac{360°}{30°} = \mathbf{12}$

(ii) $A\hat{B}C = 5(30°) = 150°$ (int. $\angle$ of a 12-sided polygon)
Let $B\hat{A}C = B\hat{C}A = x°$ (base $\angle$s of isos. $\triangle ABC$)
$x° + x° + 150° = 180°$ ($\angle$ sum of $\triangle ABC$)
$2x° = 180° - 150°$
$2x° = 30°$
$x° = \dfrac{30°}{2}$
$= 15°$
Hence, $A\hat{C}D = B\hat{C}D - B\hat{C}A$
$= 150° - 15°$
$= \mathbf{135°}$

(iii) $A\hat{B}C = B\hat{C}D = 150°$ (int. $\angle$ of a 12-sided polygon)
$B\hat{A}D = A\hat{D}C = y°$ (base $\angle$s of isos. quadrilateral, $BA = CD$)
$y° + y° + 150° + 150° = 360°$ ($\angle$ sum of quadrilateral)
$2y° = 360° - 150° - 150°$
$2y° = 60°$
$y° = \dfrac{60°}{2}$
$= 30°$
$\therefore\ A\hat{D}C = 30°$
$C\hat{D}E = 150°$ (int. $\angle$ of a 12-sided polygon)
Hence, $A\hat{D}E = C\hat{D}E - A\hat{D}C$
$= 150° - 30°$
$= \mathbf{120°}$

17. Let the number of sides of the polygon the robot travels in be n.
Then, the distance travelled by the robot in one round is $3n$ m.
The size of each exterior angle of the polygon is $x°$.
$\therefore\ \dfrac{360°}{n} = x°$
Since x is an integer, n must be a factor of 360.
For $3n$ m to be between 100 m and 150 m, possible values of n are 36 (where $3n = 108$), 40 (where $3n = 120$) and 45 (where $3n = 135$).
When $n = 36$,
$x° = \dfrac{360°}{36°}$
$= 10°$
$\therefore\ \boldsymbol{x = 10}$
Perimeter, $3n$ m $=$ **108 m**
When $n = 40$,
$x° = \dfrac{360°}{40}$
$= 9°$
$\therefore\ \boldsymbol{x = 9}$
Perimeter, $3n$ m $=$ **120 m**
When $n = 45$,
$x° = \dfrac{360°}{45}$
$= 8°$
$\therefore\ \boldsymbol{x = 8}$
Perimeter, $3n$ m $=$ **135 m**

Chapter 12 Perimeter and Area of Plane Figures

TEACHING NOTES

Suggested Approach

In the previous chapter, students have learnt the construction of plane figures such as triangles and quadrilaterals with the aid of compasses. Here, they will learn how to convert units of area, and to find the perimeter and area of triangles, quadrilaterals and circles. Students will review how to find the area of rectangles and then progress to calculating the area of triangles, parallelograms and trapeziums. Once the students are familiar with the concept of perimeter as a measure of boundary and area as a measure of the amount of space within the boundary of a two-dimensional figure, they will learn how to determine the circumference and area of circles in Section 12.5, where the constant π will be formally introduced. Thus, teachers should ensure that students are able to solve problems relating to base, height and area of triangles before progressing to parallelograms, trapeziums and finally, circles.

Section 12.1: Conversion of units

Teachers may wish to recap with the students the conversion of unit lengths from one unit of measurement to another (i.e. mm, cm, m and km) before moving onto the conversion of units for areas.

Teachers may ask students to remember simple calculations such as $1 \text{ cm}^2 = 1 \text{ cm} \times 1 \text{ cm}$ to help them in their calculations when they solve problems involving the conversion of units (see Investigation: Converting between cm^2 and m^2).

Section 12.2: Perimeter and area of rectangles and triangles

This section opens with a revision of the perimeter and area of rectangles and squares. Students are reminded to be clear of the difference in the units used for perimeter and area (e.g. cm and cm^2).

In Section 12.C, teachers should highlight that any of the three sides of a triangle can be designated as the base, and the height of a triangle is the perpendicular distance between the base and the apex (opposite vertex) of the triangle. The height may lie within, or outside of the triangle. In the latter case, a dotted line is extended from the base in order to find the height of the triangle (see Fig. 12.2(c) on page 305). Teachers should stress that the base, however, does not include the length of the extension.

Two activities are designed to help students can conclude that the area of a triangle is $\frac{1}{2} \times \text{base} \times \text{height}$ and emphasise that the area of a triangle is the same regardless of the side chosen as its base (see Investigation: Finding the area of a triangle and Investigation: Area of triangle using different sides as base). Teachers should ensure that students are confident in solving problems relating to finding the base, height and area of a triangle before proceeding to the next section.

Section 12.3: Perimeter and area of parallelograms

Teachers should ensure that the students are able to identify the base and height of parallelograms. It is important to emphasise to the students that the height of a parallelogram is with reference to the base and it must be perpendicular to the chosen base. Also, the height may lie within, or outside of the parallelogram. Teachers can highlight to the students that identifying the height of a parallelogram is similar to identifying the height of a triangle.

Teachers can guide students in finding the formula for the area of a parallelogram (see Investigation: Formula for area of parallelogram). Both possible methods may be shown to students (The second method involves drawing the diagonal of the parallelogram and finding the area of the two triangles). It is important that students realise that this formula applies even when the height of a parallelogram cannot be drawn within the base (see Journal Writing on page 313). Students can also discover for themselves that the area of a parallelogram is invariant with the same height and base regardless of how slanted they are (see Thinking Time on page 314).

Section 12.4: Perimeter and area of trapeziums

Teachers should recap with students the properties of a trapezium. Unlike the parallelogram, the base of the trapezium is not required, and the height must be with reference to the two parallel sides of the trapezium. Thus, the height lies either inside the trapezium, or it is one of its sides (this occurs in a right trapezium, where two adjacent angles are right angles).

Teachers should guide students in finding the formula for the area of a trapezium (see Investigation: Formula for area of a trapezium). Both possible methods should be shown to students (The second method again involves drawing the diagonal of the trapezium and finding the area of the two triangles).

Teachers can enhance the students' understanding and appreciation of the areas of parallelograms and trapeziums by showing them the link between the area of a trapezium, a parallelogram and a triangle (see Thinking Time on page 319).

Section 12.5 Circumference and area of circles

Teachers should familiarise students with the terms used when describing a circle (centre, radius and diameter), and highlight that the diameter of a circle is twice its radius.

Teachers should guide the students in finding the formulae for the circumference and the area of a circle (see Investigation: Formula for circumference of circle and Investigation: Formula for area of circle). From the first investigation, the constant π is introduced as the ratio of the circumference to the diameter of a circle.

Teachers may consolidate what the students have learnt in this chapter and pique their interest by showing the relevance of area and perimeter of plane figures in real-world contexts (see Class Discussion: Perimeter and area of composite figures in real-world contexts).

Introductory Problem

(a) Counting the number of square units, the area of $ABCD$ is 20 square units.

(b) The trapezium can be divded into two triangles and one rectangle.

$$\text{Area of } ABCD = \frac{1}{2} \times 2 \times 4 + 3 \times 4 + \frac{1}{2} \times 2 \times 4$$
$$= 4 + 12 + 4$$
$$= 20 \text{ units}^2$$

(c) We can find the area of a trapezium by decomposing it into other figures.

(d) Any trapezium can be decomposed into a rectangle and one or two triangles.

12.1 Conversion of units

Investigation (Converting between cm² and m²)

1. **Yes**, 1 m = 100 cm. Thus the length of each side of the two squares is equal and the squares are identical.
2. **Yes**.
3. Area of square P = **1 m²**
4. Area of square Q = **10 000 cm²**
5. Since the two squares have equal areas, then 10 000 cm² = 1 m².
6. Area of square P = 10 000 cm²
   Area of each square tile = 1 cm²
   $$\text{Number of square tiles needed} = \frac{10\,000}{1} = \mathbf{10\,000}$$

Practise Now 1

(a)
$$1 \text{ m} = 100 \text{ cm}$$
$$(1 \text{ m})^2 = (100 \text{ cm})^2$$
$$= 100 \text{ cm} \times 100 \text{ cm}$$
$$1 \text{ m}^2 = 10\,000 \text{ cm}^2$$
$$10 \text{ m}^2 = 10 \times 10\,000 \text{ cm}^2$$
$$= \mathbf{100\,000 \text{ cm}^2}$$

(b)
$$1 \text{ m} = 100 \text{ cm}$$
$$(1 \text{ m})^2 = (100 \text{ cm})^2$$
$$= 100 \text{ cm} \times 100 \text{ cm}$$
$$1 \text{ m}^2 = 10\,000 \text{ cm}^2$$
$$22.5 \text{ m}^2 = 22.5 \times 10\,000 \text{ cm}^2$$
$$= \mathbf{225\,000 \text{ cm}^2}$$

(c)
$$1 \text{ m} = 100 \text{ cm}$$
$$(1 \text{ m})^2 = (100 \text{ cm})^2$$
$$= 100 \text{ cm} \times 100 \text{ cm}$$
$$1 \text{ m}^2 = 10\,000 \text{ cm}^2$$
$$0.16 \text{ m}^2 = 0.16 \times 10\,000 \text{ cm}^2$$
$$= \mathbf{1600 \text{ cm}^2}$$

(d)
$$100 \text{ cm} = 1 \text{ m}$$
$$1 \text{ cm} = \frac{1}{100} \text{ m}$$
$$(1 \text{ cm})^2 = \left(\frac{1}{100} \text{ m}\right)^2$$
$$= \frac{1}{100} \text{ m} \times \frac{1}{100} \text{ m}$$
$$1 \text{ cm}^2 = \frac{1}{10\,000} \text{ m}$$
$$300 \text{ cm}^2 = 300 \times \frac{1}{10\,000} \text{ m}^2$$
$$= \mathbf{0.03 \text{ m}^2}$$

(e)
$$100 \text{ cm} = 1 \text{ m}$$
$$1 \text{ cm} = \frac{1}{100} \text{ m}$$
$$(1 \text{ cm})^2 = \left(\frac{1}{100} \text{ m}\right)^2$$
$$= \frac{1}{100} \text{ m} \times \frac{1}{100} \text{ m}$$
$$1 \text{ cm}^2 = \frac{1}{10\,000} \text{ m}$$
$$7146 \text{ cm}^2 = 7146 \times \frac{1}{10\,000} \text{ m}^2$$
$$= \mathbf{0.7146 \text{ m}^2}$$

(f)
$$100 \text{ cm} = 1 \text{ m}$$
$$1 \text{ cm} = \frac{1}{100} \text{ m}$$
$$(1 \text{ cm})^2 = \left(\frac{1}{100} \text{ m}\right)^2$$
$$= \frac{1}{100} \text{ m} \times \frac{1}{100} \text{ m}$$
$$1 \text{ cm}^2 = \frac{1}{10\,000} \text{ m}$$
$$0.1 \text{ cm}^2 = 0.1 \times \frac{1}{10\,000} \text{ m}^2$$
$$= \mathbf{0.000\,01 \text{ m}^2}$$

12.2 Perimeter and area of rectangles and triangles

Practise Now 2

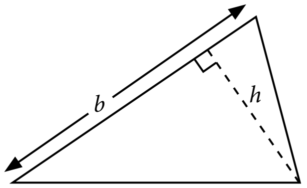
Let the length of the square field = x m.

$$2[(x + 5.5 + 5.5) + (x + 3.5 + 3.5)] = 64$$
$$2x + 18 = 32$$
$$2x = 14$$
$$x = 7$$

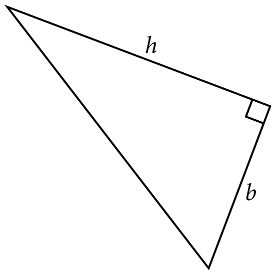
Area of path
$= (7 + 5.5 + 5.5)(7 + 3.5 + 3.5) - 7^2$
$= 18(14) - 49$
$= \mathbf{203 \text{ m}^2}$

Practise Now 3A

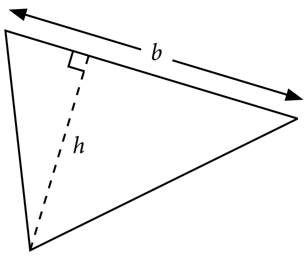
(a)



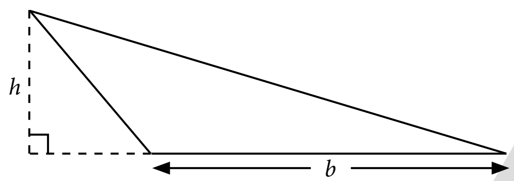
(b)



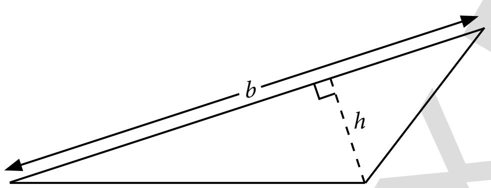
(c)



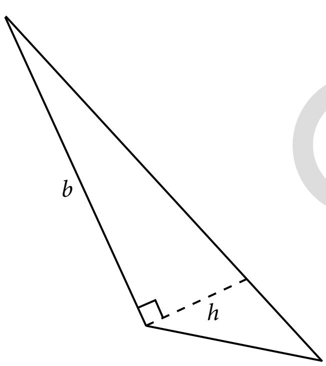
(d)



(e)



(f)



Investigation (Finding area of a triangle)

Part 1:

1. Area of $ABCD$
$= AB \times \boldsymbol{BC}$
$= \mathbf{6} \times \mathbf{10}$
$= \mathbf{60}\text{ cm}^2$

Area of $\triangle ABC = \frac{1}{2} \times$ area of $ABCD$
$= \frac{1}{2} \times 6 \times \mathbf{10}$
$= \mathbf{30}\text{ cm}^2$

2. Area of $\triangle JKG = \frac{1}{2} \times$ area of $JKGH$
$= \frac{1}{2} \times \mathbf{6} \times \mathbf{7}$
$= \mathbf{21}\text{ cm}^2$

Area of $\triangle JFG$ = Area of $\triangle JFK$ + Area of $\triangle JKG$
$= \mathbf{30}\text{ cm}^2$

3. **(a)** The area of $\triangle ABC$ is **half** of the area of rectangle $ABCD$.
(b) The area of $\triangle JFG$ is **half** of the area of rectangle $EFGH$.

4. Area of $\triangle ABC = \frac{1}{2} \times \boldsymbol{AB} \times BC$

Area of $\triangle JFG = \frac{1}{2} \times \boldsymbol{JK} \times FG$

5. Area of $\triangle LMN = \frac{1}{2} \times \mathbf{6} \times \mathbf{8}$
$= \mathbf{24}\text{ cm}^2$

At the conclusion that area of triangle = $\frac{1}{2}$ × base × height, teachers may point out that △ABC and △JFG have the same area. Although the shapes of △ABC and △JFG are different, they have the same base and height. Teachers may ask students why △LMN has a smaller area than △ABC and △JFG.

Practise Now 3B

(a) Base = 4 cm and height = 1.5 cm

Area of triangle $= \frac{1}{2} \times 4 \times 1.5$
$= \mathbf{3\ cm^2}$

(b) Base = 6.75 cm and height = 2 cm

Area of triangle $= \frac{1}{2} \times 6.75 \times 2$
$= \frac{1}{2} \times 2 \times 6.75$
$= \mathbf{6.75\ cm^2}$

(c) Base = 3 m and height = 320 cm = 3.2 m

Area of triangle $= \frac{1}{2} \times 3 \times 3.2$
$= \mathbf{4.8\ m^2}$

Investigation (Area of triangle using different sides as base)

2. $AC = \mathbf{2.35}$ cm
$h_1 = \mathbf{1.45}$ cm

3. Area of $\triangle ABC = \mathbf{1.7}\text{ cm}^2$ (to 1 d.p.)

4. **(i)** $BC = \mathbf{2.15}$ cm
$h_2 = \mathbf{1.55}$ cm
Area of $\triangle ABC = \mathbf{1.7}\text{ cm}^2$ (to 1 d.p.)

(ii) $AB = \mathbf{1.6}$ cm
$h_3 = \mathbf{2.1}$ cm
Area of $\triangle ABC = \mathbf{1.7}\text{ cm}^2$ (to 1 d.p.)

Yes, the area of $\triangle ABC$ obtained using different pairs of heights and bases are the same.

Practise Now 4

1. Area of triangle $= \frac{1}{2} \times \text{base} \times \text{height}$

$$62.3 = \frac{1}{2} \times QR \times 7.4$$
$$= 3.7 \times QR$$
$$QR = \frac{62.3}{3.7}$$
$$= \mathbf{16.8\ m} \text{ (to 3 s.f.)}$$

2. **(i)** Area of $\triangle ACD = \frac{1}{2} \times 18 \times 28$
$$= \mathbf{252\ cm^2}$$

(ii) Area of $\triangle ACD = \frac{1}{2} \times AC \times BD$
$$252 = \frac{1}{2} \times 30 \times BD$$
$$= 15 \times BD$$
$$BD = \frac{252}{15}$$
$$= \mathbf{16.8\ cm}$$

3. Area of shaded region
= area of rectangle $ABCD$ – area of $\triangle ARQ$ – area of $\triangle BRS$ – area of $\triangle CPS$ – area of $\triangle DPQ$
$$= 25 \times 17 - \frac{1}{2} \times (25 - 14) \times 5 - \frac{1}{2} \times 14 \times 3$$
$$- \frac{1}{2} \times (25 - 8) \times (17 - 3) - \frac{1}{2} \times (17 - 5) \times 8$$
$$= 425 - \frac{1}{2} \times 11 \times 5 - 21 - \frac{1}{2} \times 17 \times 14 - \frac{1}{2} \times 12 \times 8$$
$$= 425 - 27\frac{1}{2} - 21 - 119 - 48$$
$$= \mathbf{209\frac{1}{2}\ m^2}$$

Exercise 12A

1. **(a)**
$$1 \text{ m} = 100 \text{ cm}$$
$$(1 \text{ m})^2 = (100 \text{ cm})^2$$
$$= 100 \text{ cm} \times 100 \text{ cm}$$
$$1 \text{ m}^2 = 10\ 000 \text{ cm}^2$$
$$40 \text{ m}^2 = 40 \times 10\ 000 \text{ cm}^2$$
$$= \mathbf{400\ 000\ cm^2}$$

(b)
$$1 \text{ m} = 100 \text{ cm}$$
$$(1 \text{ m})^2 = (100 \text{ cm})^2$$
$$= 100 \text{ cm} \times 100 \text{ cm}$$
$$1 \text{ m}^2 = 10\ 000 \text{ cm}^2$$
$$89.2 \text{ m}^2 = 89.2 \times 10\ 000 \text{ cm}^2$$
$$= \mathbf{892\ 000\ cm^2}$$

(c)
$$1 \text{ m} = 100 \text{ cm}$$
$$(1 \text{ m})^2 = (100 \text{ cm})^2$$
$$= 100 \text{ cm} \times 100 \text{ cm}$$
$$1 \text{ m}^2 = 10\ 000 \text{ cm}^2$$
$$0.03 \text{ m}^2 = 0.03 \times 10\ 000 \text{ cm}^2$$
$$= \mathbf{300\ cm^2}$$

(d)
$$1 \text{ m} = 100 \text{ cm}$$
$$(1 \text{ m})^2 = (100 \text{ cm})^2$$
$$= 100 \text{ cm} \times 100 \text{ cm}$$
$$1 \text{ m}^2 = 10\ 000 \text{ cm}^2$$
$$5.176 \text{ m}^2 = 5.176 \times 10\ 000 \text{ cm}^2$$
$$= \mathbf{51\ 760\ cm^2}$$

2. **(i)** Area of rectangle = length × breadth
$$259 = 18.5 \times \text{breadth}$$
Breadth of rectangle $= \frac{259}{18.5}$
$$= \mathbf{14\ cm}$$

(ii) Perimeter of rectangle $= 2(18.5 + 14)$
$$= \mathbf{65\ cm}$$

3. **(a)** Base = 4.2 cm and height = 1.5 cm

Area of triangle $= \frac{1}{2} \times \text{base} \times \text{height}$
$$= \frac{1}{2} \times 4.2 \times 1.5$$
$$= \mathbf{3.15\ cm^2}$$

(b) Base = 6 cm and height = 4.5 cm

Area of triangle $= \frac{1}{2} \times \text{base} \times \text{height}$
$$= \frac{1}{2} \times 6 \times 4.5$$
$$= \mathbf{13.5\ cm^2}$$

4. Area of triangle $= \frac{1}{2} \times \text{base} \times \text{height}$
$$2.31 = \frac{1}{2} \times 2.48 \times WX$$
$$= 1.24 \times WX$$
$$WX = \frac{2.31}{1.24}$$
$$= \mathbf{1.86\ cm} \text{ (to 3 s.f.)}$$

5. Let the breadth of the rectangular field be x m.
Then the length of the field is $(x + 15)$ m.
$$2[(x + 15) + x] = 70$$
$$2(2x + 15) = 70$$
$$2x + 15 = 35$$
$$2x = 20$$
$$x = 10$$
$\therefore$ breadth of field = 10 m

Length of field = 10 + 15
= 25 m

Area of field = 25 × 10
= 250 m²

Area of path = (25 + 2.5 + 2.5) × (10 + 5 + 5) – 250
= 30 × 20 – 250
= 600 – 250
= **350 m²**

6. Area of shaded region = area of $\triangle ABC$ – area of $\triangle ADE$
$$= \frac{1}{2} \times 20 \times 21 - \frac{1}{2} \times 10 \times 10.5$$
$$= 210 - 52.5$$
$$= \mathbf{157.5\ m^2}$$

7. Area of $\triangle ACD = \frac{1}{2} \times AC \times BD = \frac{1}{2} \times CD \times AE$
$$\frac{1}{2} \times 20 \times BD = \frac{1}{2} \times 22 \times 16$$
$$10 \times BD = 176$$
$$BD = 17.6$$
Length of BD = **17.6 cm**

8. Area of shaded region = area of $\triangle PQR$ + area of $\triangle PSR$

$$\begin{aligned}
&= \frac{1}{2} \times AR \times RP + \frac{1}{2} \times RB \times RP \\
&= \frac{1}{2} \times RP \times (AR + RB) \\
&= \frac{1}{2} \times AD \times AB \\
&= \frac{1}{2} \times 23 \times (7 + 13.5) \\
&= \frac{1}{2} \times 20.5 \times 23 \\
&= \mathbf{235.75\ m^2}
\end{aligned}$$

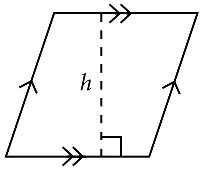
9. **(a)** Example: The possible dimensions of rectangles with an area of 30 cm² are
6 cm by 5 cm (perimeter = 22 cm),
10 cm by 3 cm (perimeter = 26 cm), and
15 cm by 2 cm (perimeter = 34 cm).

(b) Example: The possible dimensions of rectangles with a perimeter of 40 cm are
12 cm by 8 cm (area = 96 cm²),
15 cm by 5 cm (area = 75 cm²), and
17 cm by 3 cm (area = 51 cm²).

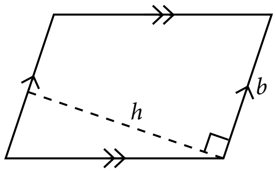
12.3 Perimeter and area of parallelograms

Practise Now 5A

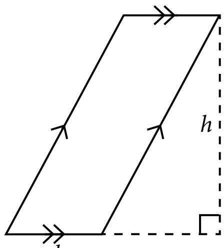
(a)



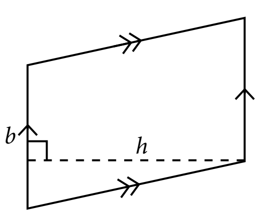
(b)



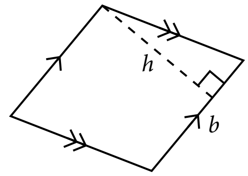
(c)



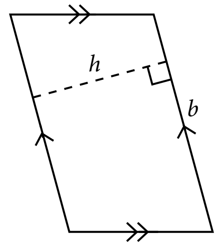
(d)



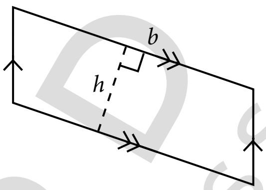
(e)



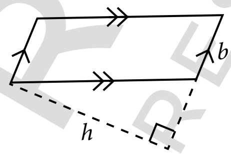
(f)



(g)

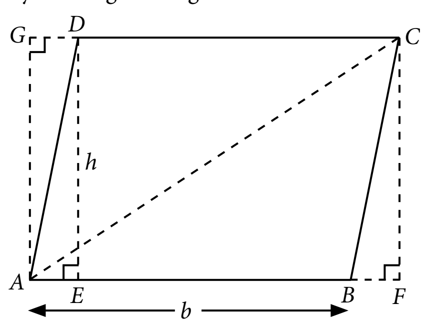


(h)



Investigation (Formula for area of parallelogram)

1. The new quadrilateral $CDEF$ is a **rectangle**.
2. Length of CF = length of DE = $\boldsymbol{h}$
Length of EF = length of EB + length of BF
= length of EB + length of AE
= $\boldsymbol{b}$
3. **Area of parallelogram *ABCD*** = area of rectangle $CDEF$
= $EF \times CF$
= $\boldsymbol{bh}$
4. Divide the parallelogram $ABCD$ into two triangles ABC and ADC by drawing the diagonal AC as shown below:



Length of CF = length of AG = length of DE = h
Area of parallelogram $ABCD$ = area of $\triangle ABC$ + area of $\triangle ADC$

$$\begin{aligned}
&= \frac{1}{2} \times AB \times CF + \frac{1}{2} \times DC \times AG \\
&= \frac{1}{2} bh + \frac{1}{2} bh \\
&= bh
\end{aligned}$$

Journal Writing (Page 313)

From the geometry software template 'Area of Parallelogram', we can conclude that the formula for the area of parallelogram is also applicable to oblique parallelograms. The height of the parallelogram is taken as the perpendicular distance between the two parallel sides.

Practise Now 5B

1. **(i)** Perimeter of parallelogram $= 2(30 + 7)$
$= \mathbf{74\ m}$

(ii) Area of parallelogram $= 24 \times 7$
$= \mathbf{168\ m^2}$

2. **(i)** Perimeter of rhombus $= 4 \times 8$
$= \mathbf{32\ cm}$

(ii) Area of rhombus $= 8 \times 7.4$
$= \mathbf{59.2\ cm^2}$

Thinking Time (Page 314)

(i) Area of parallelogram X $= 6 \times 8$
$= \mathbf{48\ cm^2}$
Area of parallelogram Y $= 6 \times 8$
$= \mathbf{48\ cm^2}$
Area of parallelogram Z $= 6 \times 8$
$= \mathbf{48\ cm^2}$

(ii) They have the same areas because the lengths of their bases and heights are the same.

Practise Now 6

Area of $\triangle PRS = \frac{1}{2} \times RS \times$ height

$48 = \frac{1}{2} \times 10 \times$ height
$= 5 \times$ height

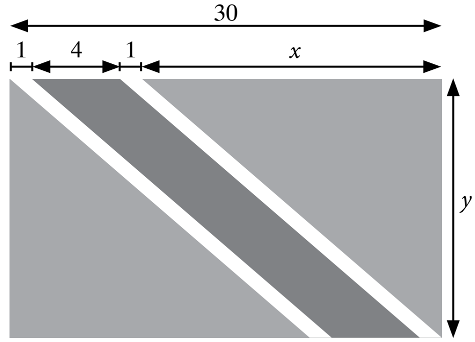
Height of $\triangle PRS = \frac{48}{5}$
$= 9.6$ m

Height of $\triangle PRS$ = height of rhombus $= RT$
$\therefore$ length of $RT = \mathbf{9.6\ m}$

Practise Now 7

Area of parallelogram $ABCD = AD \times$ height
Area of parallelogram $PQRS = PS \times$ height
Since parallelograms $ABCD$ and $PQRS$ have a common height,
area of $ABCD$: area of $PQRS = AD : PS$
$= \mathbf{5 : 1}$

Practise Now 8



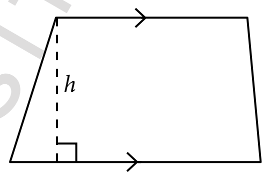
$x = 30 - 1 - 4 - 1$
$= 24$ units

Area of red portion $= 2 \times \frac{1}{2} \times 24 \times y$
$= 24y$ units^2
Area of black portion $= 4 \times y$
$= 4y$ units^2
Area of white portion $= 2 \times 1 \times y$
$= 2y$ units^2
$\therefore$ required ratio $= 24y : 4y : 2y$
$= \mathbf{12 : 2 : 1}$

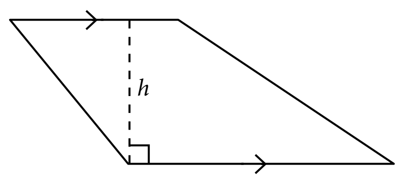
12.4 Perimeter and area of trapeziums

Practise Now 9A

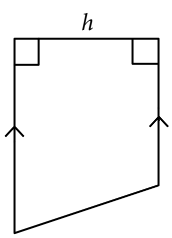
(a)



(b)

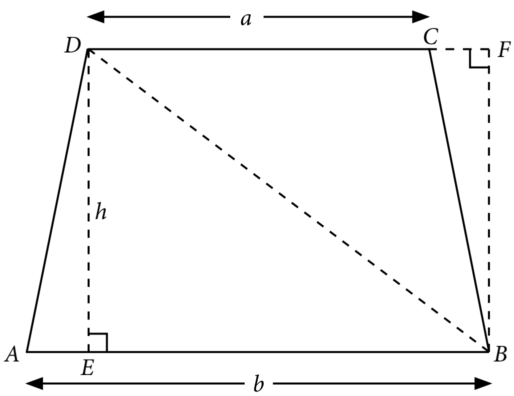


(c)



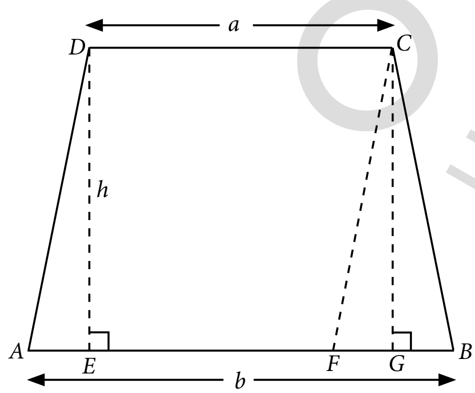
Investigation (Formula for area of trapezium)

1. The new quadrilateral $AFGD$ is a **parallelogram**.
2. Length of AF = length of AB + length of EF
$= b + a$
$= \boldsymbol{a + b}$
3. **Area of trapezium $ABCD$** $= \frac{1}{2} \times$ area of parallelogram $AFGD$
$= \frac{1}{2} \times AF \times h$
$= \boldsymbol{\frac{1}{2}(a + b)h}$
4. **Method 1:**
Divide the trapezium $ABCD$ into two triangles ABD and DCB by drawing the diagonal BD as shown below:



Length of FB = length of $DE = h$
Area of trapezium $ABCD$ = area of $\triangle ABD$ + area of $\triangle DCB$
$= \frac{1}{2} \times AB \times DE + \frac{1}{2} \times DC \times FB$
$= \frac{1}{2} \times b \times h + \frac{1}{2} \times a \times h$
$= \frac{1}{2}(b + a)h$
$= \frac{1}{2}(a + b)h$

Method 2:
Divide the trapezium $ABCD$ into a parallelogram $AFCD$ and a triangle FBC by drawing a line FC // AD as shown below:



Length of CG = length of $DE = h$
Length of AF = length of $DC = a$
$\therefore$ Length of FB = length of AB – length of AF
$= b - a$

Area of trapezium $ABCD$
= area of parallelogram $AFCD$ + area of $\triangle FBC$
$= AF \times DE + \frac{1}{2} \times FB \times CG$
$= a \times h + \frac{1}{2} \times (b - a) \times h$
$= \frac{1}{2}(2a + b - a)h$
$= \frac{1}{2}(a + b)h$

Teachers may wish to get higher-ability students to come up with more methods to find a formula for the area of a trapezium.

Thinking Time (Page 319)

1. (i) The new figure is a **parallelogram**.
(ii) Area of trapezium $= \frac{1}{2}(a + b)h$
When $a = b$,
$\frac{1}{2}(a + b)h = \frac{1}{2}(b + b)h$
$= \frac{1}{2}(2b)h$
$= \boldsymbol{bh}$
= **area of parallelogram**
2. (i) The new figure is a **triangle**.
(ii) Area of trapezium $= \frac{1}{2}(a + b)h$
When $a = 0$,
$\frac{1}{2}(a + b)h = \frac{1}{2}(0 + b)h$
$= \boldsymbol{\frac{1}{2}bh}$
= **area of triangle**

Practise Now 9B

(i) Perimeter of trapezium $= 5 + 6 + 13.2 + 5.5$
$= \mathbf{29.7\ m}$

(ii) Area of trapezium $= \frac{1}{2} \times (13.2 + 5) \times 4$
$= \mathbf{36.4\ m^2}$

Practise Now 10

(i) Area of trapezium $= \frac{1}{2} \times (SR + PQ) \times PS$
$72 = \frac{1}{2} \times (10 + 14) \times PS$
$= 12 \times PS$
$PS = \frac{72}{12}$
$= 6$
$\therefore$ length of PS = **6 m**

(ii) Perimeter of trapezium $= PQ + QR + SR + PS$
$37.2 = 14 + QR + 10 + 6$
$QR = 37.2 - 14 - 10 - 6$
$= 7.2$
$\therefore$ length of QR = **7.2 m**

Exercise 12B

1. (i) Perimeter of parallelogram $= 2(10 + 6)$ $= \mathbf{32\ cm}$

   (ii) Area of parallelogram $= 6 \times 9$ $= \mathbf{54\ cm^2}$

2. (i) Perimeter of rhombus $= 4 \times 5.6$ $= \mathbf{22.4\ cm}$

   (ii) Area of rhombus $= 5.6 \times 5$ $= \mathbf{28\ cm^2}$

3. (a) Area of parallelogram $= 12 \times 7$ $= \mathbf{84\ cm^2}$

   (b) Base of parallelogram $= \dfrac{42}{6}$ $= \mathbf{7\ m}$

   (c) Height of parallelogram $= \dfrac{42.9}{7.8}$ $= \mathbf{5.5\ mm}$

4. Area of parallelogram $= QR \times SU$
   $= 10 \times 11.2$
   $= 112\ m^2$

   $PQ \times ST$ = area of parallelogram

   $PQ \times 8 = 112$

   $PQ = \dfrac{112}{8}$
   $= 14$

   $\therefore$ length of PQ = **14 m**

5. Area of rhombus $= BD \times AE$

   $44.8 = BD \times 6.4$

   $BD = \dfrac{44.8}{6.4}$
   $= 7$ cm

   Perimeter of rhombus $= 4 \times 7$ $= \mathbf{28\ cm}$

6. (i) Perimeter of trapezium $= 35.5 + 18 + 20 + 16$ $= \mathbf{89.5\ cm}$

   (ii) Area of trapezium $= \dfrac{1}{2} \times (35.5 + 20) \times 15$ $= \mathbf{416.25\ cm^2}$

7. (a) Area of trapezium $= \dfrac{1}{2} \times (7 + 11) \times 6$
   $= \dfrac{1}{2} \times 18 \times 6$
   $= \mathbf{54\ cm^2}$

   (b) Height of trapezium $= \dfrac{126}{\frac{1}{2} \times (8 + 10)}$
   $= \dfrac{126}{\frac{1}{2} \times 18}$
   $= \dfrac{126}{9}$
   $= \mathbf{14\ m}$

   (c) Length of parallel side 2 of trapezium $= \dfrac{72}{\frac{1}{2} \times 8} - 5$
   $= \dfrac{72}{4} - 5$
   $= 18 - 5$
   $= \mathbf{13\ mm}$

8. (i) Area of trapezium $= \dfrac{1}{2} \times (PQ + RS) \times PT$

   $185 = \dfrac{1}{2} \times (12 + RS) \times 10$
   $= 5 \times (12 + RS)$

   $12 + RS = \dfrac{185}{5}$
   $= 37$

   $RS = 37 - 12$
   $= 25$

   $\therefore$ length of RS = **25 m**

   (ii) Perimeter of trapezium $= PQ + QR + RS + PS$

   $61 = 12 + QR + 25 + 13$

   $QR = 61 - 12 - 25 - 13$
   $= 11$

   $\therefore$ length of QR = **11 m**

9. Let the perpendicular height from F to AD be h cm.

   Area of parallelogram $ABFG = \dfrac{702}{2}$
   $= 351\ cm^2$

   $AB \times h = 351\ cm^2$

   Area of shaded region $= \dfrac{1}{2} \times BC \times h$

   Since $AB = \dfrac{1}{2} \times BC$, $\dfrac{1}{2} \times BC \times h = AB \times h$.

   $\therefore$ area of shaded region = $\mathbf{351\ cm^2}$

10. Total area of duct tape needed
    = total area of 3 parallelograms of base 5 cm and height 18 cm
    $= 3 \times (5 \times 18)$
    $= \mathbf{270\ cm^2}$

11. (i) Let the height of the parallelogram $ABCD$ with reference to the base BC be h cm.

    Area of parallelogram $ABCD = BC \times h = 80\ cm^2$

    Area of $\triangle ABE = \dfrac{1}{2} \times BE \times h$
    $= \dfrac{1}{2} \times 2BC \times h$
    $= BC \times h$
    $= \mathbf{80\ cm^2}$

    (ii) Let the height of the parallelogram $ABCD$ with reference to the base DC be h' cm.

    Area of parallelogram $ABCD = DC \times h' = 80\ cm^2$

    Area of $\triangle ADF = \dfrac{1}{2} \times DF \times h'$
    $= \dfrac{1}{2} \times \dfrac{1}{2} DC \times h'$
    $= \dfrac{1}{4} \times DC \times h'$
    $= \dfrac{1}{4} \times 80$
    $= \mathbf{20\ cm^2}$

12. Area of $\triangle AED = \frac{1}{2} \times AE \times ED = 25 \text{ cm}^2$

$AE \times ED = 50$

Area of trapezium $BCDE = \frac{1}{2} \times (EB + DC) \times ED$

$= \frac{1}{2} \times (3AE + 4AE) \times ED$

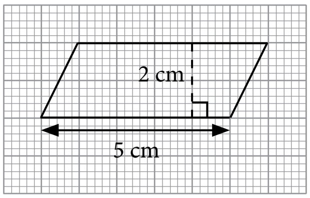
$= \frac{1}{2} \times 7AE \times ED$

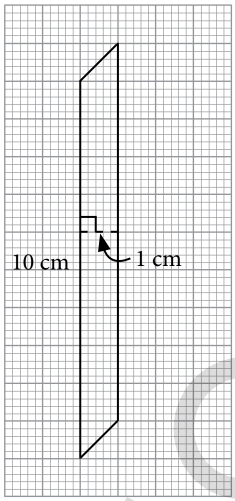
$= \frac{7}{2} \times AE \times ED$

$= \frac{7}{2} \times 50$

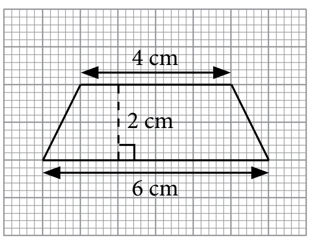
$= \mathbf{175\ cm^2}$

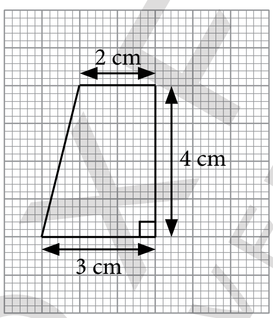
13. **(a)**





(b)





12.5 Circumference and area of circles

Practise Now 11A

(i) ***AD***, ***CF***

(ii) ***OA***, ***OC***, ***OD*** and ***OF***

(iii) **(a)** $OA = OF$

Both OA and OF are radii of the circle.

(b) $AD = CF$

Both AD and CF are the diameters of the circle.

(c) $BE < CF$

The diameter is the longest line segment connecting two points on the boundary of a circle. BE is not the diameter of the circle.

Investigation: Formula for circumference of circle

1.

diameter	circumference	$\frac{\text{circumference}}{\text{diameter}}$ (to 1 d.p.)
3 cm	9.4 cm	3.1
4 cm	12.5 cm	**3.1**
5 cm	**15.7 cm**	**3.1**
6 cm	**18.8 cm**	**3.1**

2. $\frac{\text{circumference}}{\text{diameter}} = 3.1$ (to 1 d.p.)

circumference = 3.1 × diameter

∴ the circumference of a circle is approximately 3.1 times its diameter.

Teachers may highlight that since π is a ratio of two lengths (circumference and diameter), it does not have units.

Investigation: Formula for area of circle

1. $ABCD$ resembles a **rectangle**.

2. Circumference $= \pi d$

$AB = \frac{1}{2} \times$ circumference

$= \frac{1}{2} \times \pi d$

$= \frac{\pi d}{2}$

$= \pi \boldsymbol{r}$

3. Area of rectangle $ABCD = (\frac{1}{2} \times \text{circumference}) \times r$

$= \pi r \times \text{r}$

$= \pi \boldsymbol{r^2}$

Practise Now 11B

1. Circumference of circle $= \pi d$

$= \pi \times 12$

$= \mathbf{37.7\ cm}$ (to 3.s.f.)

Radius of circle = 6 cm

Area of circle $= \pi r^2$

$= \pi \times (6)^2$

$= \pi \times 36$

$= \mathbf{113\ cm^2}$ (to 3 s.f.)

2. Area of semicircle $= \frac{1}{2} \times$ area of circle

$= \frac{1}{2} \times \pi r^2$

$= \frac{1}{2} \times \pi \times (8)^2$

$=$ **101 cm²** (to 3 s.f.)

Length of arc AB $= \frac{1}{2} \times$ circumference

$= \frac{1}{2} \times \pi d$

$= \frac{\pi d}{2}$

$= \pi r$

$= \pi \times 8$

$= 25.133$ cm (to 5 s.f.)

$\therefore$ Perimeter of semicircle

= length of arc $AB + AB$

$= 25.133 + 16$

$=$ **41.1 cm** (to 3 s.f.)

Practise Now 12

1. Area of the circle $= 804$ cm^2

$\pi r^2 = 804$

$r^2 = \frac{804}{\pi}$

$r = \sqrt{\frac{804}{\pi}}$

$= 15.996$ (to 5 s.f.)

$\therefore$ circumference of the circle $= 2\pi r$

$= 2 \times \pi \times 15.996$

$=$ **101 cm** (to 3 s.f.)

2. (i) Circumference of the larger circle $= 75$ m

$2\pi r = 75$

$r = \frac{75}{2\pi}$

$r = 11.937$ m (to 5 s.f.)

$\therefore AB = r$

$=$ **11.9 m** (to 3 s.f.)

(ii) Circumference of the smaller circle $= \pi d$

$= \pi \times 11.937$

$=$ **37.5 m** (to 3 s.f.)

(iii) Radius of smaller circle $= AB \div 2$

$= 11.937 \div 2$

$= 5.9685$ m

Area of the circle $= \pi r^2$

$= \pi \times (5.9685)^2$

$=$ **112 m²** (to 3 s.f.)

Practise Now 13

(i) Perimeter of unshaded region $= \frac{3}{4} \times 2\pi(14) + 2(14)$

$= 21\pi + 28$

$=$ **94.0 cm** (to 3 s.f.)

(ii) Area of unshaded region $= \frac{3}{4} \times \pi(14)^2$

$= 147\pi$

$=$ **462 cm²** (to 3 s.f.)

(iii) Area of shaded region = area of square – area of unshaded region

$= (2 \times 14)^2 - 147\pi$

$= 28^2 - 147\pi$

$= 784 - 147\pi$

$=$ **322 cm²** (to 3 s.f.)

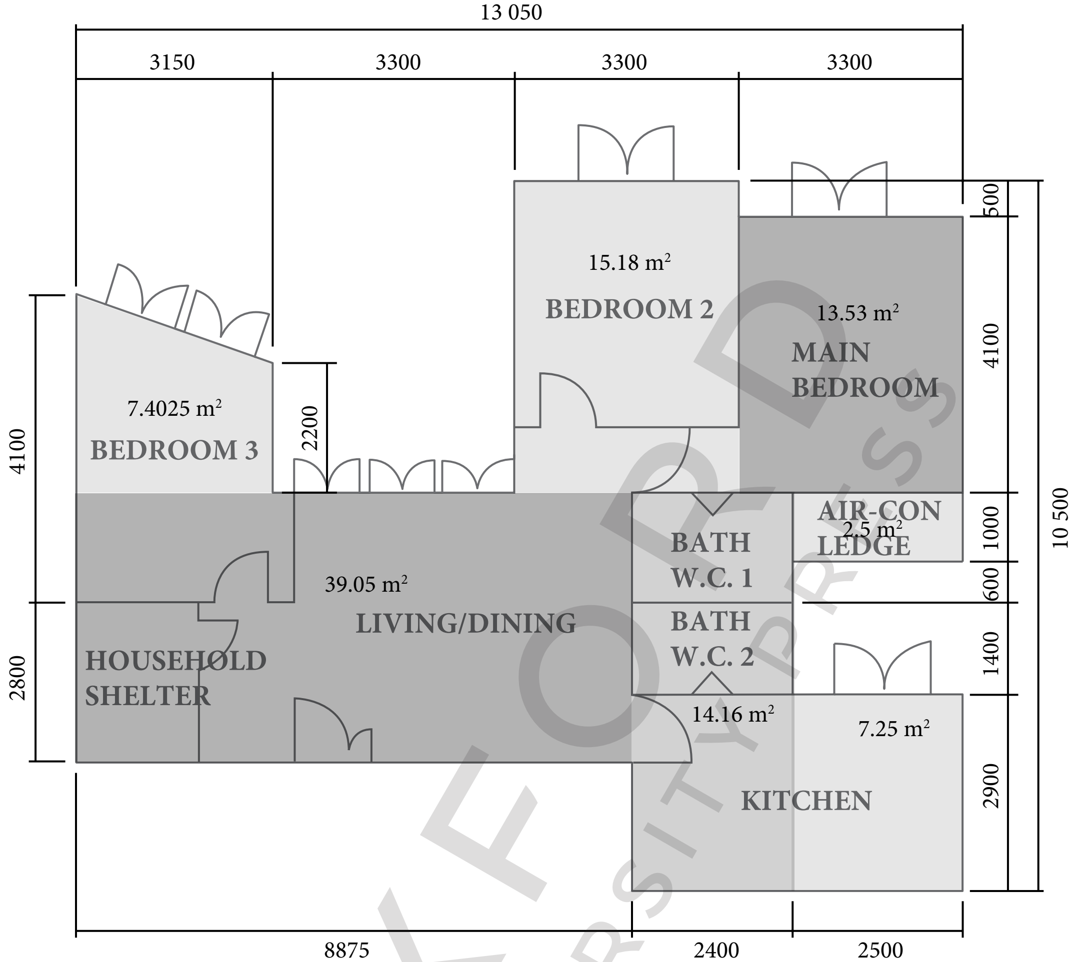
Class Discussion: Perimeter and area of composite figures in real-world contexts

1. Number of square tiles needed $= \frac{660}{60} \times \frac{360}{60}$

$= 11 \times 6$

$= \mathbf{66}$

2. **(a)**



Total floor area = $\mathbf{99.0725\ m^2}$

(b) Using mm^2 will result in very large numbers. When measuring area, we should use the most appropriate unit that will make computation more manageable and convenient.

3. The **rectangle** is the most useful as the formula to calculate its area is the most straightforward and it can be easily marked out from most composite plane figures.

Exercise 12C

1.

	Radius	Circumference	Area
(a)	8 cm	**50.3 cm**	**201 cm²**
(b)	**5.51 cm**	34.6 cm	**95.3 cm²**
(c)	**9.80 mm**	**61.6 mm**	302 mm²

(a) Circumference $= \pi d$

$= \pi \times (2r)$

$= 2\pi r$

$= 2\pi(8)$

$=$ **50.3 cm** (to 3 s.f.)

Area $= \pi r^2$

$= \pi\,(8^2)$

$=$ **201 cm²** (to 3 s.f.)

(b) Circumference $= \pi d$

$= 2\pi r$

$\therefore r = \dfrac{\text{circumference}}{2\pi}$

$= \dfrac{34.6}{2\pi}$

$= 5.5068$ cm (to 5 s.f.)

$=$ **5.51 cm** (to 3 s.f.)

Area $= \pi r^2$

$= \pi(5.5068)^2$

$=$ **95.3 cm²** (to 3 s.f.)

(c) Area $= \pi r^2$

$= \pi r^2$

$= \dfrac{\text{Area}}{\pi}$

$r = \sqrt{\dfrac{\text{Area}}{\pi}}$

$= \sqrt{\dfrac{302}{\pi}}$

$= 9.8046$ mm (to 5 s.f.)

$=$ **9.80 mm** (to 3 s.f.)

Circumference $= \pi d$

$= 2\pi r$

$= 2\pi(9.8046)$

$=$ **61.6 mm** (to 3 s.f.)

3. **(i)** Let x be the diameter of the smaller circle.

Circumference of smaller circle $= \pi x$

$x = \dfrac{\text{circumference}}{\pi}$

$= \dfrac{31.4}{\pi}$

$= 9.9949$ cm (to 5 s.f.)

Radius of semicircle $= x$

$=$ **9.99 cm** (to 3 s.f.)

(ii) Area of semicircle $= \dfrac{1}{2} \times \pi x^2$

$= \dfrac{1}{2} \times \pi \times (9.9949)^2$

$= 156.92$ cm² (to 5 s.f.)

Radius of smaller circle $= 9.9949 \div 2$

$= 4.9975$ cm (to 5 s.f.)

Area of smaller circle $= \pi(4.9975)^2$

$= 78.461$ cm² (to 5 s.f.)

$\therefore$ area of shaded region

= area of semicircle – area of smaller circle

$= 156.92 - 78.461$

$=$ **78.5 cm²** (to 3 s.f.)

4. **(a)** Perimeter of figure $= 5 + 3 + \dfrac{1}{2} \times 2\pi(2)$

$= 8 + 2\pi$

$=$ **14.3 m** (to 3 s.f.)

Area of figure = area of triangle + area of semicircle

$= \dfrac{1}{2} \times 2(2) \times 3 + \dfrac{1}{2} \times \pi \times 2^2$

$= 6 + 2\pi$

$=$ **12.3 m²** (to 3.s.f)

(b) Perimeter of figure $= 2\left[\dfrac{1}{2} \times \pi \times \left(\dfrac{18}{2}\right)\right] + \dfrac{1}{2} \times \pi \times 18$

$= 9\pi + 9\pi$

$=$ **56.5 cm** (to 3 s.f.)

Area of figure $= 2\left[\dfrac{1}{2} \times \pi \times \left(\dfrac{18}{4}\right)^2\right] + \dfrac{1}{2} \times \pi \times \left(\dfrac{18}{2}\right)^2$

$= \dfrac{81}{4}\pi \times \dfrac{81}{2}\pi$

$=$ **191 cm²** (to 3 s.f.)

5. **(i)** Perimeter of figure $= 2\pi(2) + 2(9 - 2 \times 2) + 2(3)$

$= 4\pi + 2(5) + 6$

$= 4\pi + 10 + 6$

$= 4\pi + 16$

$=$ **28.6 m** (to 3 s.f.)

(ii) Area of figure = area of rectangle – area of four quadrants

$= 9 \times [2(2) + 3] - \pi(2)^2$

$= 9 \times 7 - 4\pi$

$= 63 - 4\pi$

$=$ **50.4 m²** (to 3 s.f.)

6. Total area of shaded regions

= area of rectangle – area of parallelogram – area of circle – area of triangle

$= (12 + 14) \times (15 + 10) - (12 + 14 - 5 - 2) \times 10 - \pi(4)^2 - \dfrac{1}{2} \times 12 \times 15$

$= 26 \times 25 - 19 \times 10 - 16\pi - 90$

$= 650 - 190 - 16\pi - 90$

$= 370 - 16\pi$

$=$ **320 cm²** (to 3 s.f.)

7. Area of figure = area of trapezium $ABCE$

– area of parallelogram $GHDE$ – area of semicircle

$= \dfrac{1}{2} \times (12 + 13 + 15) \times 24 - 13 \times 16 - \dfrac{1}{2} \times \pi\left(\dfrac{15}{2}\right)^2$

$= \dfrac{1}{2} \times 40 \times 24 - 208 - \dfrac{1}{2} \times \pi(7.5)^2$

$= 480 - 208 - 28.125\pi$

$= 272 - 28.125\pi$

$=$ **184 cm²** (to 3 s.f.)

8. **(i)** Area of cross section of stool

= area of bigger trapezium – area of smaller trapezium

$= \frac{1}{2} \times (35 + 45) \times 30 - \frac{1}{2} \times (15 + 30 \times 15)$

$= 1200 - 337.5$

$= \mathbf{862.5\ cm^2}$

(ii) Area of cross section of new stool

= area of trapezium – area of semicircle

$= 1200 - \frac{1}{2} \times \pi \times \left(\frac{30}{2}\right)^2$

$= 1200 - \frac{1}{2} \times \pi \times 15^2$

$= 1200 - 112.5\pi$

$= 846.57$ cm^2 (to 5 s.f.)

Percentage **decrease** in cross-sectional area

$= \frac{862.5 - 846.57}{862.5} \times 100\%$

$= \mathbf{1.85\%}$ (to 3 s.f.)

9. Area of figure

$= \frac{1}{2} \times 15 \times 3 + \left[\frac{1}{2} \times (3 + 6.6) \times 6 - \frac{1}{2} \times (1.5 + 3) \times 3.6\right]$

$+ \left[\frac{1}{2} \times (6.6 + 13.5) \times 6 - \frac{1}{2}\pi\left(\frac{9.6}{2}\right)^2\right]$

$= 22.5 + (28.8 - 8.1) + (60.2 - 11.52\pi)$

$= 103.4 - 11.52\pi$

$= 67.209$ cm^2 (to 5 s.f.)

Area of paper = 210 mm × 297 mm

= 21 cm × 29.7 cm

= 623.7 cm^2

Percentage of paper used $= \frac{67.209}{623.7} \times 100\%$

$= \mathbf{10.8\%}$ (to 3 s.f.)

10. Radius of each circle $= \sqrt{\frac{0.785}{\pi}}$ cm

Area of shaded region $= \frac{1}{2} \times 2\sqrt{\frac{0.785}{\pi}} \times \sqrt{\frac{0.785}{\pi}}$

$= \frac{0.785}{\pi}$

$= \mathbf{0.250\ cm^2}$ (to 3 s.f.)

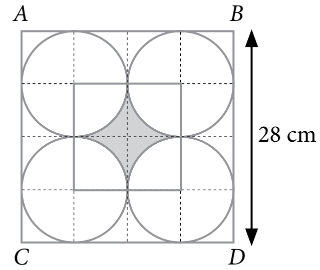
11. Area of grass within the goat's reach $= \pi(1.5)^2$

$= 2.25\pi$ m^2

Time the goat needs to eat the grass $= 2.25\pi \times 14$

$= \mathbf{99.0\ minutes}$ (to 3 s.f.)

12. The square *ABCD* is divided into 16 equal smaller squares as shown.



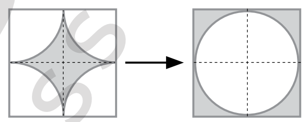
Perimeter of the shaded region

= circumference of one circle

$= \pi(14)$

$= \mathbf{44.0\ cm}$ (to 3 s.f.)

The squares containing the shaded region can be rearranged as shown.



$\therefore$ Area of shaded region

$= 14^2 - \pi\left(\frac{14}{2}\right)^2$

$= \mathbf{42.1\ cm^2}$ (to 3 s.f.)

Chapter 13 Statistical Data Handling

TEACHING NOTES

Suggested Approach

In primary school, students have learnt statistical diagrams such as pictograms, bar graphs, pie charts and line graphs. In this chapter, students revisit pictograms, bar graphs and pie charts. They are expected to know and appreciate the advantages and disadvantages of each diagram. With such knowledge, students can choose the most appropriate diagram to display the results of a given statistical investigation. Teachers may want to provide more examples when introducing the various stages of a statistical study and involve the students in evaluating and discussing the issues that arise in each stage. Knowledge from past chapters may be required, such as percentages.

Section 13.1: Frequency table

Teachers should define statistics as the collection, organisation, display and interpretation of data. Teachers may highlight that a frequency table is primarily used to organise the data collected. Terms such as 'statistical data', 'frequency' and 'total frequency' should be clearly defined. An example of how the data in Table 13.1 is organised vertically can also be shown. Teachers can get the students to give some examples of categorical and numerical data.

Section 13.2: Pictogram

Using the class discussion, students can be guided to interpret pictograms and identify its advantages and disadvantages (see Class Discussion: Purpose and appropriateness of pictogram). Teachers should remind students that when drawing a pictogram, the key must always be included. The same picture can be used to represent all the categories (see, for example, Exercise 13A Question 1). Teachers may ask the students to construct a frequency table to represent the data given in the pictogram.

To support struggling learners, teachers can recap multiplication of mixed numbers, e.g. $2\frac{1}{2}$ pictures of pears represent $2\frac{1}{2} \times 20 = 50$ pears.

Section 13.3: Bar graph

Teachers can build upon what students have learnt about pictograms to introduce bar graphs (see Fig. 13.4 and 13.5). Teachers may involve students in discussing the advantages of the bar graph over a pictogram, and vice versa (see Class Discussion: Purposes and appropriateness of bar graph). Teachers should stress that the frequency axis of a bar graph should start from zero, and the bars are spaced at regular intervals.

Section 13.4: Pie chart

Some students may still be unfamiliar with calculating the size of the angle of each sector in a pie chart when attempting to construct a pie chart (see Class Discussion: Construction and usefulness of pie chart). As such, teachers may wish to support struggling students by reminding them that the sum of angles at a point is 360°, and 360° represents the total frequency of a data set.

To guide the students in identifying the advantages and disadvantages of a pie chart, teachers may ask the students to compare the information displayed in Fig. 13.5 or Fig. 13.6 with those in Fig. 13.8(b).

Section 13.5 Evaluation of statistical representations

Teachers can use the examples in the class discussion (see Class Discussion: Evaluation of statistical representations) to guide students to evaluate statistical data and to discuss the instances of statistical misuse. The importance of not engaging in any unethical behaviors, ensuring objectivity and providing the complete picture without omitting any forms of misrepresentation need to be inculcated in the students at the end of the class discussion.

Section 13.6 Statistical investigation

Teachers can show the four stages of investigation randomly and ask the students to arrange them in the order that each one is carried out. Using the class discussion as a guide (see Class Discussion: Stages of statistical investigation), teachers can get the students to discuss considerations and potential issues that can arise at each stage of a statistical study.

Teachers may want to assign small-scale projects for students where they conduct their own statistical studies (see Performance Task on page 355). Such projects allow students to apply what they have learnt about statistical data handling in real-world contexts.

Introductory Problem

100 units were sold in January while 120 units were sold in February. Since $120 \neq 2 \times 100$, Bernard is wrong.

13.2 Pictogram

Class Discussion (Purposes and appropriateness of pictogram)

Part 1:

1. The favourite fruit is the **watermelon**.
2. **50** students chose pear.
3. **75** students chose honeydew.
4. Not every picture can be divided equally into 4.

Part 2:

5. **Bernard** is correct. According to the key, each picture represents 5 vehicles. Hence, $3 \times 5 = 15$ students travel by bus and $4 \times 5 = 20$ students travel by car.
$\therefore$ more students travel by car and the length of the picture does not affect the number of vehicles.
6. To avoid a misinterpretation of the data, we can replace each bus and each car in the pictogram with a standard icon. Alternatively, the bus and the car should be of the same length.
7. It might look odd as a bus is much bigger than a car in real life. However, it is just a representation.
8. An advantage is that the pictures are colourful and appealing. Disadvantages include the difficulties involved in drawing pictures and a fraction of a picture. Inaccurate drawings would also likely result in misinterpretation.

Practise Now 1A

(i) **(a)** Profit earned by the company in 2020 $= 5.5 \times \$1\,000\,000$
$= \mathbf{\$5\,500\,000}$

(b) Profit earned by the company in 2022 $= 7 \times \$1\,000\,000$
$= \mathbf{\$7\,000\,000}$

(ii) The company earned the least profit in **2019**.
Decrease in profit from 2018 to 2019 $= 1.5 \times \$1\,000\,000$
$= \mathbf{\$1\,500\,000}$

13.3 Bar graph

Class Discussion (Purposes and appropriateness of bar graph)

Part 1:

1. A pictogram is more visually appealing.
2. There is no need to draw pictures. The values can also be read off the axis, so there is no need to count the number of pictures unlike in the interpretation of a pictogram.

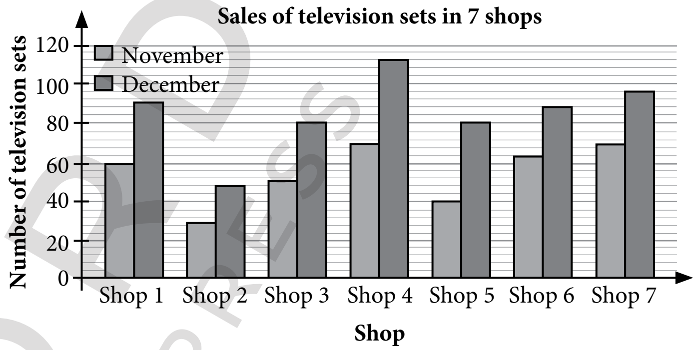
Part 2:

3. When the frequency axis does not start from zero, the graph can be easily misinterpreted.
4. The range of values of the data set will affect whether or not the frequency axis should start from zero.

Practise Now 1B

1. **(i)** **Black** is the least popular colour.
(ii) **No**, the bar graph does not have a mistake. The y-axis of the graph shows that the corresponding values are in thousands, i.e. there are 3500 grey cars and 1500 black cars.

2. **(i)**



(ii) **(a)** Total number of television sets sold in the seven shops in November
$= 60 + 30 + 50 + 70 + 40 + 64 + 70$
$= \mathbf{384}$

(b) Total number of television sets sold in the seven shops in December
$= 90 + 48 + 80 + 112 + 80 + 88 + 96$
$= \mathbf{594}$

(iii) Required percentage $= \dfrac{384}{384 + 594} \times 100\%$
$= \dfrac{384}{978} \times 100\%$
$= \mathbf{39.3\%}$ (to 3 s.f.)

(iv) **(a)** Required percentage $= \dfrac{70 + 96}{978} \times 100\%$
$= \dfrac{166}{978} \times 100\%$
$= \mathbf{17.0\%}$ (to 3 s.f.)

(b) **No**, I do not agree with the manager. Since Shop 2 sold the least number of televsion sets in November and December, it should be closed down.

(v) The company performed better in terms of sales in **December**. The graph shows that more television sets were sold in December than November in all seven shops.

Exercise 13A

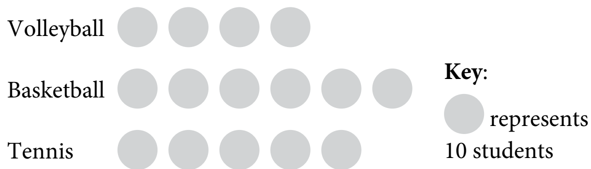
1. **(i)** The greatest number of buses were registered in **June and July**.
Number of buses registered $= 5 \times 50$
$= \mathbf{250}$

(ii) Total number of buses registered
= (2.5 + 5 + 5 + 3.5 + 2) × 50
= 18 × 50
= **900**

(iii) Total amount collected = 900 × $220
= **$198 000**

(iv) Percentage decrease in number of buses = $\frac{1.5 \times 50}{250} \times 100\%$
= **30%**

2. (i) Students who play volleyball, basketball or tennis

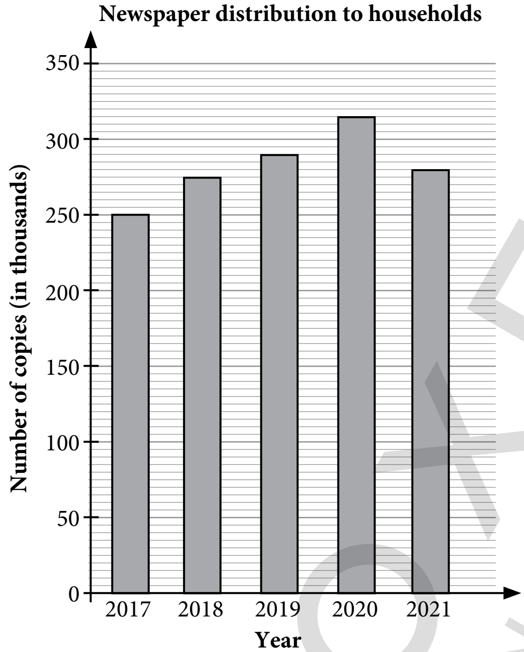


(ii) Required ratio = **4 : 5**

(iii) Required percentage = $\frac{5}{6} \times 100\%$
= $\mathbf{83\frac{1}{3}}$ **%** or **83.3%** (to 3 s.f.)

3.

Newspaper distribution to households



4. (i) Total number of gold medals attained
= 17 + 7 + 10 + 22 + 39
= **95**

(ii) Percentage of gold medals attained by France
= $\frac{10}{95} \times 100\%$
= **10.5%** (to 3 s.f.)

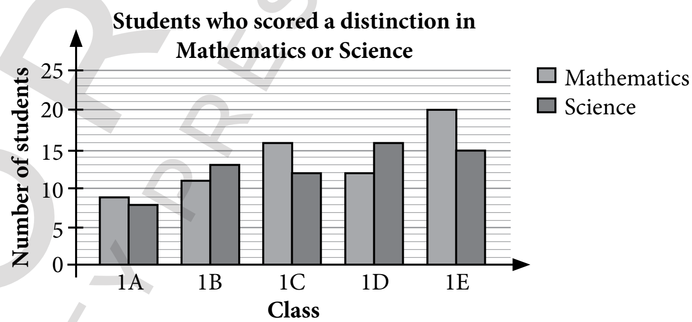
5. (i) Total number of workers employed in the town
= 4 × 1 + 6 × 2 + 5 × 3 + 3 × 4 + 2 × 5
= 4 + 12 + 15 + 12 + 10
= **53**

(ii) Total number of shops in the town
= 4 + 6 + 5 + 3 + 2
= 20
Number of shops hiring 3 or more workers = 5 + 3 + 2
= 10

∴ Required percentage = $\frac{10}{20} \times 100\%$
= **50%**

6. (i)

Class	Class 1A	Class 1B	Class 1C	Class 1D	Class 1E
Number of students who scored a distinction in Mathematics	9	11	16	12	20
Number of students who scored a distinction in Science	8	**13**	12	**16**	15



(ii) (a) Total number of students in the 5 classes who score a distinction in Mathematics
= 9 + 11 + 16 + 12 + 20
= **68**

(b) Total number of students in the 5 classes who score a distinction in Science
= 8 + 13 + 12 + 16 + 15
= **64**

(iii) Required percentage = $\frac{12}{68} \times 100\%$
= **17.6%** (to 3 s.f.)

(iv) Percentage of students in Class 1D who score a distinction in Science
= $\frac{16}{40} \times 100\%$
= **40%**

(v) **No**, Waseem is not correct to say that there are 35 students in Class 1E. There may be students in the class who did not score distinctions in both Mathematics and Science. There may also be students in the class who scored distinctions in both Mathematics and Science.

7. (i) Number of candidates who took the test in March
= **950**

(ii) Number of candidates who failed the test in June
= **500**

(iii) Total number of candidates who failed the test in the six months
$= 400 + 350 + 350 + 400 + 450 + 500$
$= 2450$

$\therefore$ Required percentage $= \frac{500}{2450} \times 100\%$
$= \mathbf{20.4\%}$ (to 3 s.f.)

(iv) The percentage of successful candidates increases over the six months as candidates who failed in the first few months could have learnt from their mistakes and passed the test on their next attempt in the later months.

13.4 Pie chart

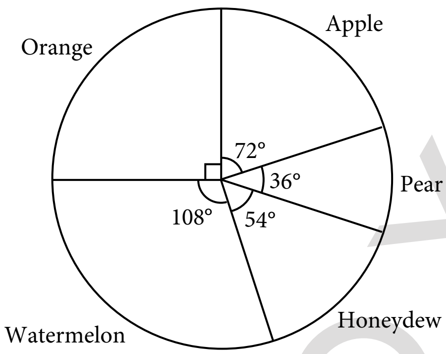
Class Discussion (Construction and usefulness of pie chart)

Part 1: Construction

1.

Fruit	Apple	Pear	Honeydew	Watermelon	Orange
Number of students	100	50	75	150	125
Angle of sector	$\frac{100}{500} \times 360°$ $= 72°$	$\frac{50}{500} \times 360°$ $= \mathbf{36°}$	$\frac{75}{500} \times 360°$ $= \mathbf{54°}$	$\frac{150}{500} \times 360°$ $= \mathbf{108°}$	$\frac{125}{500} \times 360°$ $= \mathbf{90°}$

2.



3. From the pie chart, watermelon is the most popular fruit among the 500 students. It has the largest sector size (or largest angle).

4. **(i)** Fraction of students that like orange the most $= \mathbf{\frac{1}{4}}$

(ii) **No**, it is difficult to tell from the corresponding bar graphs that orange is the favourite fruit of $\frac{1}{4}$ of the 500 students.

Part 2: Advantages and disadvantages

5. It is easier to compare the relative size of each category (sector) with the whole using a pie chart.

6. It is difficult to compare sectors if they are similar in size.

7. Based on the 3-dimensional pie chart, Raju spends the most on **entertainment**.

8. Based on the 2-dimensional pie chart, Raju spends the most on **rent**.

9. In a 3-dimensional pie chart, the sizes of the sectors will look distorted. The sectors towards the back of the pie chart will appear smaller than those towards the front.

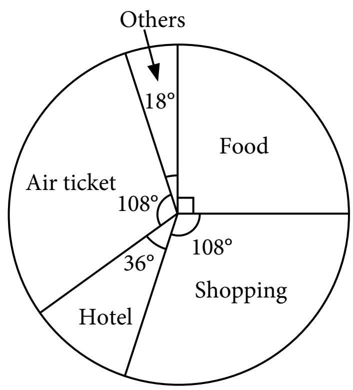
10. We should be aware that a sector at the back of a 3-dimensional pie chart may appear smaller than a sector in the front, even if it corresponds to a larger quantity. Therefore, we should compare the actual frequency of each sector when a 3-dimensional pie chart is used.

11. It is not easy to tell by just looking at the pie chart. However, a protractor can be used to measure the angles of the sectors.

Practise Now 1C

Imran's total expenditure on the holiday
$= \$1000 + \$1200 + \$400 + \$1200 + \$200$
$= \$4000$

Item	Angle of sector
Food	$\frac{\$1000}{\$4000} \times 360° = 90°$
Shopping	$\frac{\$1200}{\$4000} \times 360° = 108°$
Hotel	$\frac{\$400}{\$4000} \times 360° = 36°$
Air ticket	$\frac{\$1200}{\$4000} \times 360° = 108°$
Others	$\frac{\$200}{\$4000} \times 360° = 18°$



Practise Now 1D

(i) $4x° + 2x° + 237.6° = 360°$ ($\angle$s at a point)
$4x° + 2x° = 360° - 237.6°$
$6x° = 122.4°$
$x° = 20.4°$
$\therefore x = \mathbf{20.4}$

(ii) Required percentage $= \frac{4(20.4°)}{360°} \times 100\%$
$= \frac{81.6°}{360°} \times 100\%$
$= \mathbf{22\frac{2}{3}\%}$ or $\mathbf{22.7\%}$ (to 3 s.f.)

(iii) Angle of sector representing cranberry juice = 237.6°

237.6° represent 759 ml

$$1° \text{ represents } \frac{759}{237.6} \text{ ml}$$

$$360° \text{ represent } \frac{759}{237.6} \times 360 \text{ ml} = 1150 \text{ ml}$$

∴ amount of fruit punch in the jar = **1150 ml**

Journal Writing (Page 348)

(a) A **bar graph or pictogram** can be used, where the months are treated as 'labels' for the last two. Twelve sections will be too cluttered for a pie chart. (For small frequency, a pictogram without scales can be used, so that fractions of pictures do not need to be drawn.)

(b) A **bar graph or pie chart** can be used. The same issue as in part **(b)** applies for the use of a pictograms with scales. Line graphs are not suitable for categorical data.

(c) A **pie chart** is ideal to display fractions and categorical data.

13.5 Evaluation of statistical representations

Class Discussion (Evaluation of statistical representations)

Part 1: Collection of data

2. It is stated in the article that the poll was conducted on the UEFA website. As such, the voters who took part in the poll were most likely to belong to the younger generation who are more computer-savvy and hence, the voters were unlikely to be representative of all football fans.

3. As shown in the article, the number of votes for the three footballers were close, with 123 582 votes for Zidane, 122 569 votes for Beckenbauer and 119 332 votes for Cruyff. This is despite the fact that most of the younger generation, who were most likely to have voted in the poll, may not know who Beckenbauer and Cruyff are as they were at the peak of their careers in the 1970s. Hence, if older football fans were to participate in the poll, Zidane would probably not have come in first place.

4. The choice of a sample is important as if the sample chosen for collection of data is not representative of the whole population, the figures that are obtained may be misleading. Hence, a representative sample should be chosen whenever possible.

Part 2: Organisation of data

5. Banks and insurance firms, timeshare companies and motor vehicle companies received the most number of complaints.

6. The article states that banks and insurance firms, which were grouped together, received the most number of complaints. If banks and insurance firms were not grouped together, it is possible that timeshare companies received the most number of companies. For example, if the 1416 complaints were split equally between banks and insurance firms, they would have received 708 complaints each, then the number of complaints received by timeshare companies, i.e. 1238 complaints, would have been the greatest.

7. This shows that when organising data, it is important to consider whether to group separate entities as doing so might mislead consumers and result in inaccurate conclusions.

Part 3: Display of data

8. Although the height of the bar for Company E appears to be twice that of the bar for Company C, Company E's claim is not valid as the bars do not start from 0. By reading off the bar graph, Company E sold 160 light bulbs in a week, which is not twice as many as the 130 light bulbs sold by Company C in a week.

9. For bar graphs, if the vertical axis does not start from 0, the height of each bar will not be proportional to its corresponding frequency, i.e. number of light bulbs sold by each company in a week. Such display of statistical data may mislead consumers.

Part 4: Interpretation of data

10. The conclusion was obtained based on a simple majority, i.e. since more than 50% of the employees were satisfied with working in the company, the survey concluded that the employees were satisfied with the company and that the company was a good place to work in.

11. $40\% \times 300 = \frac{40}{100} \times 300$

$= 120$ employees

It is stated in the article that 40% of the employees, i.e. 120 employees were not satisfied with working in the company. As such, even though a simple majority of the employees was satisfied with working in the company, it cannot be concluded that most of the employees were satisfied. This shows that we should not use simple majorities to arrive at conclusions or make decisions.

12. The amendment of the constitution of a country is a very serious matter where the agreement of a simple majority is insufficient, therefore there is a need for a greater percentage of elected Members of Parliament (MPs) to agree before the constitution can be amended. As a result, the Singapore government requires the agreement of at least a two-third majority before the constitution can be amended.

13. It is important to have a basis or contention in order to decide on an issue, and that in some occasions, it is insufficient to make decisions based on a simple majority.

Part 5: Ethical issues

It is unethical to use statistics to mislead others as it is essentially a form of misrepresentation and people may arrive at the wrong conclusions or make the wrong decisions.

13.6 Statistical investigation

Class Discussion (Stages of statistical investigation)

1. They have to decide how many and which sports to play. This can be based on the resources that they have, e.g. they may have the resources to organise only three sports or the equipment for certain types of sports.

2. A possible way is to look at the number of students in the various sports co-curricular activities (CCA) – which is not a statistical investigation. However, this is not ideal because a CCA is restricted by the number of students it can accommodate, and does not demonstrate the overall popularity of the sport. Another way is to survey the students.
3. It is usually not practical, unless the school population is small. We can survey a number of students, e.g. 100.
4. Some students may go and play some sports such as soccer or basketball during recess, and these are the students who may be more likely to take part in the Games Day.
5. In this case, we want the sample to be representative of students who may be more likely to take part in the Games Day.
6. If we want it to be representative of students who may be more likely to take part in the Games Day, we should survey students who are more active in sports or those who play sports during recess; if we want it to be representative of entire student population, we have to try to survey students from a variety of locations in the school during recess.
7. It is harder for a sample to be representative if its size is too small.
8. If the sample size is too big, the student council may not find enough students to survey, and/or they may not have the resources to organise and analyse such a large amount of data.
9. Include all sports offered by the school, or all sports the school council has the resources to carry out, or just the more popular sports. Exclude sports where the competition is too long to be completed in one Games Day.
10. The list can include a category called 'Others (please specify)'.
11. Although the question of the statistical investigation posed in Stage 1 might ask for 3 sports, it is sufficient for the survey question to ask for 1. The 3 sports should be chosen based on the 3 most popular sports among the student population, and not to individuals. This will ensure that there will be at least 1 sport for majority of the students to play.
12. This facilitates easy counting in multiples of five.
13.

Sport	Tally	Number of students
Soccer	卌 I	176
Volleyball	卌 IIII	144
Basketball	卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌	**100**
Hockey	卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 III	**88**
Netball	卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 II	**92**
Total		**600**

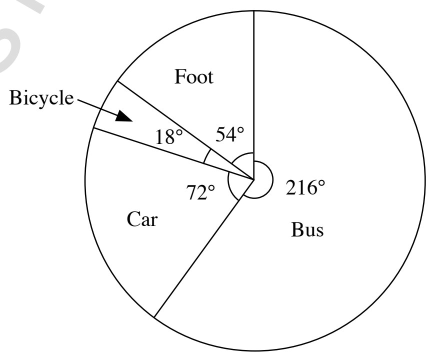
Table 13.3

14. Check if the total frequency tallies with the number of survey participants (or sample size).
15. Yes, a bar graph is an appropriate choice. A pie chart is not suitable as the aim is not to find the fraction of each category out of the whole. The likelihood of having to draw fractions of pictures makes a pictogram with scales unsuitable. Line graphs are for numerical data and we are not looking at trends over time.
16. Opinions may be divided, but in general, it may be easier to draw a conclusion from the bar graph unless the heights of two bars are almost the same.
17. Soccer, followed by volleyball and then basketball.
18. Yes, because the sample may not be representative of the population.
19. The student council can conduct another survey to choose between basketball and netball, but this is time consuming. They can just go ahead with basketball since it does not entail serious repurcussions.

Exercise 13B

1. Total number of students surveyed = 768 + 256 + 64 + 192
= 1280

Mode of transport	Angle of sector
Bus	$\frac{768}{1280} \times 360° = 216°$
Car	$\frac{256}{1280} \times 360° = 72°$
Bicycle	$\frac{64}{1280} \times 360° = 18°$
Foot	$\frac{192}{1280} \times 360° = 54°$



2. **(i)** Angle of sector that represents number of students who prefer yam
= **90°**

(ii) Angle of sector that represents number of students who prefer vanilla
= 360° – 120° – 90° – 50° (∠s at a point)
= **100°**

(iii) Required percentage = $\frac{100°}{360°} \times 100\%$
= $\mathbf{27\frac{7}{9}\%}$ or **27.8%** (to 3 s.f.)

(iv) Total number of students in the class = $\frac{360°}{50°} \times 5$
= **36**

3. (i) Required percentage $= \frac{180°}{360°} \times 100\%$

$= \mathbf{50\%}$

(ii) Required percentage $= \frac{72°}{360°} \times 100\%$

$= \mathbf{20\%}$

(iii) $x° = \frac{17\frac{1}{2}}{100} \times 360°$

$= 63°$

$\therefore x = \mathbf{63}$

4. (i) Total number of cars in the survey $= 20 + 25 + 20 + 30 + 25$

$= \mathbf{120}$

(ii) Total number of people in all the cars

$= 20 \times 1 + 25 \times 2 + 20 \times 3 + 30 \times 4 + 25 \times 5$

$= 20 + 50 + 60 + 120 + 125$

$= \mathbf{375}$

(iii) Number of cars with 4 or more people $= 30 + 25$

$= 55$

$\therefore$ required percentage $= \frac{55}{120} \times 100\%$

$= \mathbf{45\frac{5}{6}\%}$ or $\mathbf{45.8\%}$ (to 3 s.f.)

(iv) Angle of sector that represents number of cars with 1 person

$= \frac{20}{120} \times 360°$

$= \mathbf{60°}$

Angle of sector that represents number of cars with 2 people

$= \frac{25}{120} \times 360°$

$= \mathbf{75°}$

Angle of sector that represents number of cars with 3 people

$= \frac{20}{120} \times 360°$

$= \mathbf{60°}$

Angle of sector that represents number of cars with 4 people

$= \frac{30}{120} \times 360°$

$= \mathbf{90°}$

Angle of sector that represents number of cars with 5 people

$= \frac{25}{120} \times 360°$

$= \mathbf{75°}$

5. (i) Total angle of sectors that represent number of female students and teachers in the school

$= 360° - 240°$ ($\angle$s at a point)

$= 120°$

Angle of sector that represents number of teachers in the school

$= \frac{1}{6} \times 120°$

$= \mathbf{20°}$

(ii) (a) Number of female students in the school $= 5 \times 45$

$= \mathbf{225}$

(b) Number of male students in the school $= \frac{240°}{20°} \times 45$

$= \mathbf{540}$

(iii) Total school population $= 45 + 225 + 540$

$= 810$

Number of female teachers in the school $= \frac{2}{3} \times 45$

$= 30$

Number of females in the school $= 225 + 30$

$= 255$

$\therefore$ required percentage $= \frac{255}{810} \times 100\%$

$= \mathbf{31.5\%}$ (to 3 s.f.)

6. $\frac{5}{1 + x + 5} \times 360° = 120°$

$\frac{5}{6 + x} = \frac{120°}{360°}$

$\frac{5}{6 + x} = \frac{1}{3}$

$15 = 6 + x$

$\therefore x = \mathbf{9}$

7. **No.** The pie chart may lead one to conclude that Brand A has a very significant market share. Based on the percentage of market share, Brand A + Brand B = 35% + 15% = 50% and Brand C + Brand D = 28% + 22% = 50%. Thus, an accurate chart should show a division of exactly half.