

8

**Step by Step
Solution Guide**

NEW
COUNTDOWN
ENHANCED BLENDED EDITION

OXFORD
UNIVERSITY PRESS

OXFORD
UNIVERSITY PRESS

Oxford University Press is a department of the University of Oxford.
It furthers the University's objective of excellence in research, scholarship,
and education by publishing worldwide. Oxford is a registered trade mark of
Oxford University Press in the UK and in certain other countries

Published in Pakistan by
Oxford University Press
No. 38, Sector 15, Korangi Industrial Area,
PO Box 8214, Karachi-74900, Pakistan

© Oxford University Press 2027

The moral rights of the author have been asserted

First Edition published in 2021

Third Edition (Enhance Blended) published in 2027

All rights reserved. No part of this publication may be reproduced, stored in
a retrieval system, or transmitted, used for text and data mining, or used for
training artificial intelligence, in any form or by any means, without the
prior permission in writing of Oxford University Press, or as expressly permitted
by law, by licence, or under terms agreed with the appropriate reprographics
rights organisation. Enquiries concerning reproduction outside the scope of the
above should be sent to the Rights Department, Oxford University Press, at the
address above

You must not circulate this work in any other form
and you must impose this same condition on any acquirer

ISBN 9786275228172

Preface

New Countdown Third Edition (Enhanced Blended Edition) is a carefully structured and graded mathematics course, comprising eleven books for Classes Pre-Primary to Class 8. The pattern followed in the entire series ensures development in all areas of a child's growth through basic multi-focal knowledge, emphasising number skills and mathematical concepts.

The **Step by Step Solution Guide** is a comprehensive resource that complements the New Countdown series to provide a holistic framework within which students are able to understand, grasp, approach, and apply the learned mathematical concepts, and to successfully implement the objectives of the mathematics curriculum.

This guide highlights the patterns, approaches, functions, and relationships between the curriculum strands, so that the students can apply their mathematical knowledge and develop a holistic understanding of the subject that can then be translated into real-life application. The main objective of this guide is not to simply cross-reference the answers, but to guide the students through the thinking process upon approaching a mathematical problem, to reaching the correct answer. This guide therefore provides the extensive breakdown of not only solving the equation, but also the mental strategies, appropriate reasoning and formatting, and the ability to decipher what mathematical concepts can be applied to the particular question, in order to work towards the answer.

This in-depth breakdown of solving questions encompasses all the questions in each exercise, as well as the questions in the revision exercises. There are also helpful hints available in this guide that supplements a student's thinking process when approaching a certain problem. The helpful hints will help to avoid pre-emptive misconceptions that will be beneficial to students and teachers. They help guide the student towards the correct formula by effectively contextualising the mathematical concept and linking it to real-life application. The mathematical proofing, format and reasoning is in line with the assessment expectations.

Finally, apart from the step-by-step worked solutions themselves, the end of this guide also includes a direct answer key that can be used for cross-referencing purposes by the teacher. These answers correlate to the model paper in the Assessment Resource Pack.

The Step by Step Guide provides thorough insight and furthers one's understanding of what is expected of a student in an examination beyond simply arriving at the right answer. This guide helps ensure that the process comes from a place of deep understanding and reasoning of mathematical concepts by guiding the students' approach and thinking process during problem solving, and therefore reaching the desired answer.

CONTENTS

1	Operations on Sets	1	8	Algebra: Factorisation and Expansion	48
2	Real Numbers	6		Revision 2: Algebra	58
3	Squares and Square Roots, Cubes and Cube Roots	9	9	Linear Equations	63
4	Proportions	21	10	Mensuration	78
	Revision 1: Numbers	25	11	Polygons	71
5	Financial Arithmetic	30	12	Practical Geometry and Transformation	86
6	Algebra: Laws of Indices/Exponents	35	13	Data Handling	98
7	Algebra: Polynomials	38			



OPERATIONS ON SETS

Exercise 1

1. (i) $A \cup B$
(ii) $(A \cup B) \cup C$
(iii) $(A \cup B) \cap (A \cup C)$
(iv) common elements
(v) 2^n
2. (i) False
Reason: A set containing 3 elements has $2^3 = 8$ elements.
(ii) True
(iii) False
Reason: Power set is the set of all the subsets of a given set.
(iv) False
Reason: A super set contains all the elements of its subset and it contains at least one elements not present in subset.
(v) True
3. (i) The number of elements is 1
 $\therefore$ the number of subsets is $2^n = 2^1 = 2$
 $\{\}, \{a\}$
- Helpful Hint**
Null set is a subset of every set.
- (ii) The number of elements is 2
 $\therefore$ the number of subsets is $2^n = 2^2 = 4$
 $\{\}, \{x\}, \{y\}, \{x, y\}$
(iii) The number of elements is 3
 $\therefore$ the number of subsets is $2^3 = 8$
 $\{\}, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}$
- (iv) There is no element.
 $\therefore$ number of subsets is $2^0 = 1$
- (v) The number of elements is 4
 $\therefore$ the number of subsets is $2^4 = 16$
 $\{\}, \{a\}, \{b\}, \{c\}, \{d\},$
 $\{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\},$
 $\{a, b, c\}, \{a, b, d\}, \{b, c, d\}, \{c, d, a\},$
 $\{a, b, c, d\}$
- (vi) There is only one element.
 $\therefore$ number of subsets is $2^n = 2^1 = 2$
 $\{\}, \{0\}$
4. (i) $\{a, b, c\} \cap \{b\} = \{b\}$
only b is common in both the sets
(ii) $\{2, 4, 6, 8\} \cap \{1, 2, 3, 4, 5\} = \{2, 4\}$
(iii) $\{\text{the vowels in the English alphabet}\} \cap \{\text{the first four letters of the English alphabet}\}$
 $= \{a, e, i, o, u\} \cap \{a, b, c, d\}$
 $= \{a\}$
because only a is common in both sets
(iv) $\{c, a, t\} \cap \{d, o, g\} = \{\}$
There is are no common elements in the two sets.
(v) $\{3, 6, 9, 12\} \cap \{2, 3, 4, 5, 6\} = \{3, 6\}$
(vi) $\{O\} \cap \{E\} = \{\}$
There are no common elements in the two sets.
5. (i) $\{a, c, d\} \cup \{a, b, c, d\} = \{a, b, c, d\}$
Write all the elements of both the sets.
- Helpful Hint**
Write common elements only once.

(ii) $\{3, 8, 4\} \cup \phi = \{3, 8, 4\}$

Helpful Hint

ϕ represents empty or null set.

(iii) $\{3, 6, 9, 12\} \cup \{2, 4, 6, 8\}$

$= \{2, 3, 4, 6, 8, 9, 12\}$

6. (i) $\{0, 2, 4, 6, 8\} \cup \{1, 2, 3, 8\}$

$= \{0, 1, 2, 3, 4, 6, 8\}$

(ii) $\{0, 4, 6, 8\} \cup \{4, 5, 6, 7\}$

$= \{0, 2, 4, 5, 6, 7, 8\}$

(iii) $\{1, 2, 3, 8\} \cup \{4, 5, 6, 7\}$

$= \{1, 2, 3, 4, 5, 6, 7, 8\}$

(iv) $\{1, 2, 3, 8\} \cup \{0, 1, 2, \dots, 9\}$

$= \{0, 1, 2, \dots, 9\}$

(v) $\{0, 2, 4, 6, 8\} \cup \{\}$

$= \{0, 2, 4, 6, 8\}$

(vi) $\{0, 2, 4, 6, 8\} \cap \{1, 3, 5, 7, 9\}$

$= \{\}$ or ϕ

(vii) $\{1, 3, 5, 7, 9\} \cap \{1, 2, 3, 8\}$

$= \{1, 3\}$

(viii) $\{1, 2, 3, 8\} \cap \{4, 5, 6, 7\}$

$= \{\}$ or ϕ

(ix) $\{1, 3, 5, 7, 9\} \cap \{\}$

$= \{\}$ or ϕ

(x) $\{1, 3, 5, 7, 9\} \cap \{0, 1, 2, \dots, 9\}$

$= \{1, 3, 5, 7, 9\}$

7. (i) $A' = \{1, 2, 3, 4, 5, 6, 7\} - \{1, 2, 5, 7\}$

$A' = \{3, 4, 6\}$

(ii) $B' = \{1, 2, 3, 4, 5, 6, 7\} - \{1, 3, 6, 7\}$

$B' = \{2, 4, 5\}$

(iii) $A \cap B = \{1, 2, 5, 7\} \cap \{1, 3, 6, 7\}$

$= \{1, 7\}$

$(A \cap B)' = \{1, 2, 3, 4, 5, 6, 7\} - \{1, 7\}$

$= \{2, 3, 4, 5, 6\}$

(iv) $A \cup B = \{1, 2, 5, 7\} \cup \{1, 3, 6, 7\}$

$= \{1, 2, 3, 5, 6, 7\}$

$(A \cup B)' = \{1, 2, 3, 4, 5, 6, 7\} - \{1, 2, 3,$

$5, 6, 7\}$

$= \{4\}$

(v) $A' = \{1, 2, 3, 4, 5, 6, 7\} - \{1, 2, 5, 7\}$

$= \{3, 4, 6\}$

$B' = \{1, 2, 3, 4, 5, 6, 7\} - \{1, 3, 6, 7\}$

$= \{2, 4, 5\}$

$A' \cup B' = \{3, 4, 6\} \cup \{2, 4, 5\}$

$= \{2, 3, 4, 5, 6\}$

(vi) $A' = \{3, 4, 6\}$

$B' = \{2, 4, 5\}$

$A' \cap B' = \{3, 4, 6\} \cap \{2, 4, 5\}$

$= \{4\}$

8. (a) (i) $\{1, 3\} \cup \{2, 4\} \cup \{5\} = \{1, 2, 3, 4, 5\}$

(ii) $A \cup B = \{1, 3\} \cup \{2, 4\} = \{1, 2, 3, 4\}$

$(A \cup B) \cap C = \{1, 2, 3, 4\} \cap \{5\} = \{\}$

(iii) $A \cap B = \{1, 3\} \cap \{2, 4\} = \{\}$

$(A \cap B) \cup C = \{\} \cup \{5\} = \{5\}$

(iv) $A \cap B = \{1, 3\} \cap \{2, 4\} = \{\}$

$(A \cap B) \cap C = \{\} \cap \{5\} = \{\}$

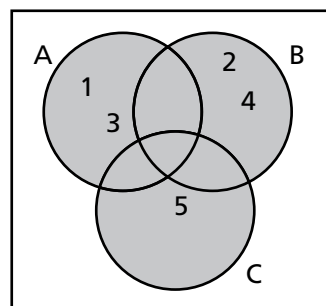
(v) $B \cap C = \{2, 4\} \cap \{5\} = \{\}$

$A \cup (B \cap C) = \{1, 3\} \cup \{\} = \{1, 3\}$

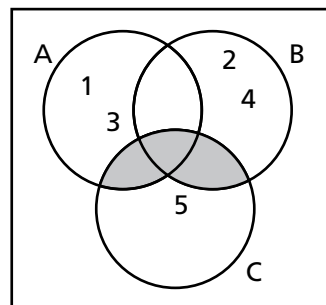
(vi) $B \cup C = \{2, 4\} \cup \{5\} = \{2, 4, 5\}$

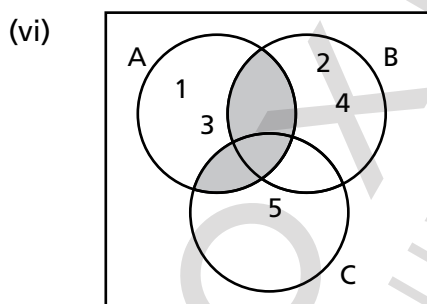
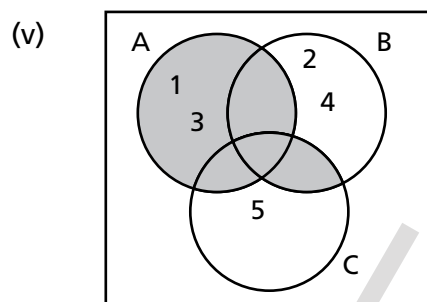
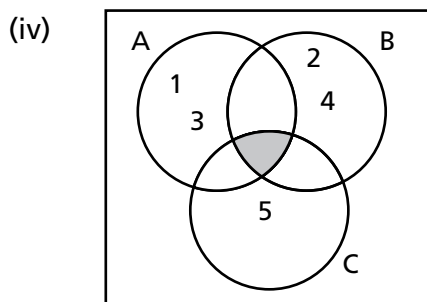
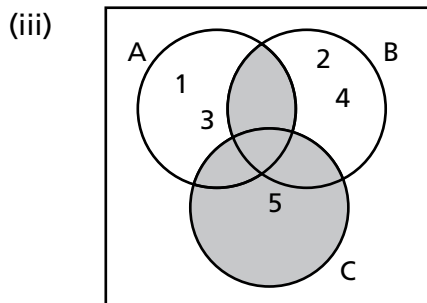
$A \cap (B \cup C) = \{1, 3\} \cap \{2, 4, 5\} = \{\}$

(b) (i)



(ii)





- (c) (i) LHS
 $A' \cup B'$
 $A' = \{1, 2, 3, 4, 5\} - \{1, 3\} = \{2, 4, 5\}$
 $B' = \{1, 2, 3, 4, 5\} - \{2, 4\} = \{1, 3, 5\}$
 $A' \cup B' = \{1, 2, 3, 4, 5\}$
 RHS
 $(A \cap B)'$
 $A \cap B = \{1, 3\} \cap \{2, 4\} = \{\}$

$$(A \cap B)' = \{1, 2, 3, 4, 5\} - \{\} = \{1, 2, 3, 4, 5\}$$

$$\therefore \text{LHS} = \text{RHS}$$

Thus $A' \cup B' = (A \cap B)'$ is proved

- (ii) LHS
 $A' \cap B' = \{2, 4, 5\} \cap \{1, 3, 5\} = \{5\}$
 RHS
 $(A \cup B)'$
 $A \cup B = \{1, 3\} \cup \{2, 4\} = \{1, 2, 3, 4\}$
 $(A \cup B)' = \{1, 2, 3, 4, 5\} - \{1, 2, 3, 4\} = \{5\}$
 $\therefore \text{LHS} = \text{RHS}$
 Thus $A' \cap B' = (A \cup B)'$ is proved

9. (i) $A \cap B = \{6, 8\}$ shows the common elements. That means 6 and 8 are elements of set B.
 $A \cup B = \{2, 4, 5, 6, 7, 8\}$ shows all the elements of A and B both.
 The elements, 5 and 7 do not belong to set A. This shows that 5 and 7 are also the elements of set B.
 $\therefore B = \{5, 6, 7, 8\}$
 (ii) $A \cap B = \{3\}$
 $\Rightarrow$ Set B only has 3 as an element.
 $A \cup B = \{1, 2, 3, 4\}$
 Since 1 and 2 do not belong to set A
 $\therefore B = \{1, 2, 3\}$

10. (i) =
 (Commutative property)
 (ii) =
 Since ϕ is a null set, it does not have any element hence its union with any other set gives the set itself as a result.
 (iii) $\neq$
 The result will be the set A.
 (iv) =
 Since the elements are written only once if they are common in both the sets. Thus the resultant will be the set A itself.

(v) =

Since union and intersection both gives the same elements as set B.
Thus both the sets are equal.

11. Commutative property of union of three sets

$$A \cup (B \cap C) = (A \cup B) \cap C$$

Consider LHS

$$A \cup (B \cap C)$$

$$(B \cap C) = \{3, 5, 6\} \cap \{5, 7, 9\} = \{3, 5, 6, 7, 9\}$$

$$\begin{aligned} A \cup (B \cap C) &= \{2, 3, 4\} \cup \{3, 5, 6, 7, 9\} \\ &= \{2, 3, 4, 5, 7, 9\} \end{aligned}$$

RHS

$$(A \cup B) \cap C$$

$$A \cup B = \{2, 3, 4\} \cup \{3, 5, 6\} = \{2, 3, 4, 5, 6\}$$

$$\begin{aligned} (A \cup B) \cap C &= \{2, 3, 4, 5, 6\} \cap \{5, 7, 9\} \\ &= \{2, 3, 4, 5, 6, 7, 9\} \end{aligned}$$

$\therefore$ LHS = RHS

Thus associative property of union is proved.

12. $A \cap (B \cap C) = (A \cap B) \cap C$

LHS

$$\begin{aligned} B \cap C &= \{p, q, r, s\} \cap \{n, o, p, q\} \\ &= \{p, q\} \end{aligned}$$

$$\begin{aligned} A \cap (B \cap C) &= \{m, n, o\} \cap \{p, q\} \\ &= \{ \} \end{aligned}$$

RHS

$$A \cap B = \{m, n, o\} \cap \{p, q, r, s\} = \{ \}$$

$$\begin{aligned} (A \cap B) \cap C &= \{ \} \cap \{n, o, p, q\} \\ &= \{ \} \end{aligned}$$

$\therefore$ LHS = RHS

Thus the associative property of intersection is proved.

13. First Law

$$(A \cup B)' = A' \cap B'$$

LHS

$$\begin{aligned} A \cup B &= \{1, 2, 3, 4\} \cup \{3, 4, 5, 6\} \\ &= \{1, 2, 3, 4, 5, 6\} \end{aligned}$$

$$\begin{aligned} (A \cup B)' &= \{1, 2, 3, 4, 5, 6\} - \{1, 2, 3, 4, 5, 6\} \\ &= \{ \} \end{aligned}$$

RHS

$$A' = \{1, 2, 3, 4, 5, 6\} - \{1, 2, 3, 4\} = \{5, 6\}$$

$$B' = \{1, 2, 3, 4, 5, 6\} - \{3, 4, 5, 6\} = \{1, 2\}$$

$$A' \cap B' = \{5, 6\} \cap \{1, 2\} = \{ \}$$

$\therefore$ LHS = RHS

Thus De Morgan's first law is proved.

2nd Law

$$(A \cap B)' = A' \cup B'$$

LHS

$$\begin{aligned} A \cap B &= \{1, 2, 3, 4\} \cap \{3, 4, 5, 6\} \\ &= \{3, 4\} \end{aligned}$$

$$\begin{aligned} (A \cap B)' &= \{1, 2, 3, 4, 5, 6\} - \{3, 4\} \\ &= \{1, 2, 5, 6\} \end{aligned}$$

RHS

$$A' = \{5, 6\} \quad B' = \{1, 2\}$$

$$\begin{aligned} A' \cup B' &= \{5, 6\} \cup \{1, 2\} \\ &= \{1, 2, 5, 6\} \end{aligned}$$

$\therefore$ LHS = RHS

Thus De Morgan's 2nd law is proved.

14. (a) Swimming = {B, E, F, H}

$$\text{Coding} = \{A, C, F, G\}$$

$$\text{Painting} = \{B, C, D, F\}$$

(b) B, F

(f) F

Multiple Choice Questions 1

1. A

All other sets have elements other than the elements of universal set.

2. D

Integers include negative numbers, positive numbers and zero, whereas whole numbers include only positive numbers and zero. Even numbers consist of multiples of 2 only, and odd numbers are the numbers other than multiples of 2. Hence, the set of integers has all the members of whole numbers, even numbers and odd numbers sets.

3. B

$$2^n = 2^4 = 16$$

4. C

Options A and D are not the subset of A as they have other element than those in set A. Option B is improper subset of A.

5. C

None of the other options satisfy De Morgan's law.

OXFORD
UNIVERSITY PRESS



REAL NUMBERS

Exercise 2A

1. 45 → has two significant figures because all non zero digits are significant.
- 0.046 → has two significant figures because all leading zeros are non significant figures.
- 7.4220 → has five significant figures because the zeros after non zero digits after decimal are significant.
- 5002 → has four significant figures because all zeros between two non-zero digits are significant.
- 3800 → has two significant figures because the zeros at the end are not significant.

Helpful Hint

In whole number, the zeros at the end may or may not be significant.

- 0003006 → has four significant figures. The zeros in the beginning are non-significant where as the zeros between two non-zero digits are significant.
- 143.000 → has six significant figures because the zeros at the end of a decimal number are significant.

Exercise 2B

1. (i) $2 - 7.2 = 5.2$
 $|-5.2| = 5.2$

Helpful Hint

Absolute value of any real number is positive.

- (ii) terminating
- (iii) Zero (0)
- (iv) five
- (v) $-\frac{2}{5}$
 $-\frac{5}{2} * -\frac{2}{5} = 1$
2. (i) True
Since 2.315796 is a non-terminating decimal number, and hence it is an irrational number.
- (ii) True
- (iii) False
 $1.03\bar{2}$ is a recurring decimal number, hence it is a rational number.
- (iv) True
The value of π is a non-terminating decimal number, hence it is an irrational number.
3. (i) 49
Since 6 is greater than 5, the digit 8 will round up to 9.
- (ii) 0.005
Since 3 is less than 5, hence the required digit will remain the same.
- (iii) 390200
Since the digit next to 4th significant figure is 5, the digit will round up.
- (iv) 535.01

Since the last digit is 8, the digit will round up.

6. (i) $\frac{1}{7}$ (ii) $-\frac{1}{2.5}$ (iii) $\frac{1}{\sqrt{3}}$
 (iv) $\frac{11}{6}$ (v) $-\sqrt{2}$ (vi) $\frac{1}{347.99}$

7. (i) -29 (ii) 7875 (iii) $-\frac{47}{50}$
 (iv) $\sqrt{7}$ (v) 6712.04 (vi) $-\frac{3}{10}$

8. (i) $15, 30, 2.5$

Associative property of addition states that:

$$a + (b + c) = (a + b) + c$$

$$15 + (30 + 2.5) = (15 + 30) + 2.5$$

Helpful Hint

To prove the law the left hand side should be equal to the right hand side

LHS

$$15 + (30 + 2.5)$$

$$= 15 + 32.5$$

$$= 47.5$$

RHS

$$(15 + 30) + 2.5$$

$$= 45 + 2.5$$

$$= 47.5$$

Since LHS = RHS

Hence, the associative property of addition is proved.

(ii) $\frac{3}{5}, 24, 11$

LHS

$$\frac{3}{5} + (24 + 11)$$

$$= \frac{3}{5} + 35 = \frac{3 + 175}{5}$$

$$= \frac{178}{5} = 35\frac{3}{5}$$

RHS

$$\left(\frac{3}{5} + 24\right) + 11$$

$$= \frac{3 + 120}{5} + 11 = \frac{123}{5} + 11$$

$$= \frac{123 + 55}{5} = \frac{178}{5} = 35\frac{3}{5}$$

Since LHS = RHS

Hence, the associative property of addition is proved.

9. (i) $1.5, 3, 8$

Distributive property of multiplication over addition states that:

$$a * (b + c) = a * b + a * c$$

LHS

$$1.5 * (3 + 8) = 1.5 * 11 = 16.5$$

RHS

$$1.5 * 3 + 1.5 * 8 = 4.5 + 12.0 = 16.5$$

Since, LHS = RHS

Hence, distributive property of multiplication over addition is proved.

(ii) $2, \frac{4}{15}, \frac{7}{15}$

$$2 * \left(\frac{4}{15} + \frac{7}{15}\right) = 2 * \frac{4}{15} + 2 * \frac{7}{15}$$

LHS

$$2 * \left(\frac{4}{15} + \frac{7}{15}\right) = 2 * \frac{11}{15} = 1\frac{7}{15}$$

RHS

$$2 * \frac{4}{15} + 2 * \frac{7}{15} = \frac{8}{15} + \frac{14}{15} = \frac{22}{15} = 1\frac{7}{15}$$

Since, LHS = RHS

Hence, distributive law of multiplication over addition is proved.

10. (i) $100, 25, 3000$

Associative property of multiplication states that:

$$a * (b * c) = (a * b) * c$$

$$100 * (25 * 3000) = (100 * 25) * 3000$$

LHS

$$100 * (25 * 3000) = 100 * 75000 = 7500000$$

RHS

$$(100 * 25) * 3000 = 2500 * 3000 = 7500000$$

Since, LHS = RHS

Hence, associative property of multiplication is proved.

$$(ii) \quad -5 * (20 * \frac{4}{5}) = (-5 * 20) * \frac{4}{5}$$

LHS

$$-5 * (20 * \frac{4}{5}) = -5 * 16 = -80$$

RHS

$$(-5 * 20) * \frac{4}{5} = -100 * \frac{4}{5} = -80$$

Since, LHS = RHS

Hence, associative property of multiplication is proved.

$$11. (i) \quad 7 * (-4.5 + 12) = 7 * (-4.5) + 7 * 12$$

LHS

$$7 * (-4.5 + 12) = 7 * 7.5 = 52.5$$

RHS

$$7 * (-4.5) + 7 * 12 = -31.5 + 84 = 52.5$$

Since LHS = RHS

Hence, distributive property of multiplication over addition is proved.

$$(ii) \quad \frac{1}{3} * (4 + \frac{2}{3}) = \frac{1}{3} * 4 + \frac{1}{3} * \frac{2}{3}$$

LHS

$$\frac{1}{3} * (4 + \frac{2}{3}) = \frac{1}{3} * \frac{14}{3} = \frac{14}{9} = 1\frac{5}{9}$$

RHS

$$\frac{1}{3} * 4 + \frac{1}{3} * \frac{2}{3} = \frac{4}{3} + \frac{2}{9} = \frac{12 + 2}{9}$$

$$= \frac{14}{9} = 1\frac{5}{9}$$

Since, LHS = RHS

Hence, distributive property of multiplication over addition is proved.

$$12. (i) \quad \left| -\frac{2}{5} \right| = \frac{2}{5}z$$

$$(ii) \quad \left| -2(4.62) \right| = \left| -9.24 \right| = 9.24$$

$$(iii) \quad \left| \sqrt{2} \right| = \sqrt{2}$$

13. Saima had 593.66 cm long cloth. She cut off 28.5 cm from it. The length of remaining cloth is
- $$593.66 \text{ cm} - 28.5 \text{ cm} = 565.16 \text{ cm}$$
- Approximation upto 4 significant figures
- 565.2 cm

14. Maheen jogged 2.23 km
Faheem jogged $3.25 * 2.23 \text{ km}$
 $= 7.2475 \text{ km}$
Faheem jogged approximately 7.24 km

$$15. \quad \frac{1}{4} + \frac{1}{4} + \frac{3}{8}$$

$$= \frac{2 + 2 + 3}{8} = \frac{7}{8}$$

$$16. \quad \frac{3}{5} + \frac{1}{4}$$

$$= \frac{12 + 5}{20} = \frac{17}{20}$$

Haris moved $\frac{17}{20}$ of his house lawn left to

$$\text{now is } 1 - \frac{17}{20} = \frac{20 - 17}{20}$$

$$= \frac{3}{20}$$

17. We have radius = 35 cm
Take radius = 34.5 cm, 35 cm and 35.5 cm
Area of circle = πr^2
 $r = 34.5 \text{ cm}$
Area of circle = $\frac{22}{7} * 345$
 $= 108.43 \text{ cm}^2$
 $r = 35 \text{ cm}$
Area of circle = $\frac{22}{7} * 35$
 $= 110 \text{ cm}^2$
 $r = 35.5 \text{ cm}$
Area of circle = $\frac{22}{7} * 35.5$
 $= 111.57 \text{ cm}^2$
If radius = 34.5 cm², then the error is
 $110 - 108.43 = 1.57 \text{ cm}^2$
If radius = 35.5 cm², then the error is
 $111.57 - 110 = 1.57 \text{ cm}^2$

Multiple choice question 2

1. B 2. B 3. C 4. C 5. D



SQUARES AND SQUARE ROOTS CUBES AND CUBE ROOTS

Exercise 3A

- radical
 - two
 - less
 - division
 - 4.4
- False
100 is a perfect square but 200 is not a perfect square.
 - True
 - True

Helpful Hint

If the number is non-terminating and non-recurring, it is an irrational number.

- False
345 is not a perfect square.
- True

3. (i)

$$\begin{array}{r}
 4.7 \\
 4 \overline{) 22.09} \\
 + 4 \quad - 16 \rightarrow 4 \times 4 \\
 \hline
 87 \quad 609 \\
 - 609 \rightarrow 87 \times 7 \\
 \hline
 000
 \end{array}$$

$\therefore \sqrt{22.09} = 4.7$

(ii)

$$\begin{array}{r}
 2.71 \\
 2 \overline{) 7.3441} \\
 + 2 \quad - 4 \rightarrow 2 \times 2 \\
 \hline
 47 \quad 334 \\
 + 7 \quad - 329 \rightarrow 47 \times 7 \\
 \hline
 541 \quad 541 \\
 - 541 \rightarrow 541 \times 1 \\
 \hline
 0
 \end{array}$$

$\therefore \sqrt{6.3441} = 2.71$

(iii)

$$\begin{array}{r}
 7.12 \\
 7 \overline{) 50.6944} \\
 + 7 \quad - 49 \rightarrow 7 \times 7 \\
 \hline
 141 \quad 169 \\
 + 1 \quad - 141 \rightarrow 141 \times 1 \\
 \hline
 1422 \quad 2844 \\
 - 2844 \rightarrow 1422 \times 2 \\
 \hline
 0
 \end{array}$$

$\therefore \sqrt{50.6944} = 7.12$

(iv)

$$\begin{array}{r}
 18.47 \\
 1 \overline{) 341.1409} \\
 + 1 \quad - 1 \rightarrow 1 \times 1 \\
 \hline
 28 \quad 241 \\
 + 8 \quad - 224 \rightarrow 28 \times 8 \\
 \hline
 364 \quad 1741 \\
 + 4 \quad - 1456 \rightarrow 364 \times 4 \\
 \hline
 3687 \quad 25809 \\
 - 25809 \rightarrow 3687 \times 7 \\
 \hline
 0
 \end{array}$$

$\therefore \sqrt{341.1409} = 18.47$

(v)

$$\begin{array}{r}
 0.0907 \\
 0 \overline{) 0.00822649} \\
 + 0 \quad - 0 \rightarrow 0 \times 0 \\
 \hline
 09 \quad 82 \\
 + 9 \quad - 81 \rightarrow 9 \times 9 \\
 \hline
 180 \quad 126 \\
 + 0 \quad 0 \rightarrow 180 \times 0 \\
 \hline
 1807 \quad 12649 \\
 - 12649 \rightarrow 1807 \times 1 \\
 \hline
 0
 \end{array}$$

$\therefore \sqrt{0.00822649} = 0.0907$

4. (i) $\sqrt{2\frac{9}{49}} = \sqrt{\frac{107}{49}} = \sqrt{\frac{107}{7 \times 7}} = \sqrt{\frac{107}{7}}$

$$\begin{array}{r} 10.34 \\ 1 \overline{) 107} \\ + 1 \quad - 1 \\ \hline 20 \quad 007 \\ + 0 \quad - 0 \\ \hline 203 \quad 700 \\ + 3 \quad - 609 \\ \hline 2064 \quad 9100 \\ - \quad 8256 \\ \hline \quad \quad 844 \end{array}$$

$$\therefore \sqrt{\frac{107}{7}} = \frac{10.34}{7}$$

$$\begin{array}{r} 1.477 \\ 7 \overline{) 10.34} \\ - 7 \\ \hline 33 \\ - 28 \\ \hline 54 \\ - 49 \\ \hline 50 \\ - 49 \\ \hline 1 \end{array}$$

$$\therefore \sqrt{2\frac{9}{49}} = 1.48$$

(ii) $\sqrt{7\frac{9}{16}}$
 $= \sqrt{\frac{121}{16}}$
 $= \frac{11}{4}$
 $= 2.75 \text{ or } 2\frac{3}{4}$

(iii) $\sqrt{6\frac{145}{256}} = \sqrt{\frac{1681}{256}}$

$$\begin{array}{r} 41 \\ 4 \overline{) 1681} \\ + 4 \quad - 16 \\ \hline 81 \quad 81 \\ - \quad 81 \\ \hline 0 \end{array}$$

$$\therefore \sqrt{1681} = 41$$

$$\begin{array}{r} 16 \\ 1 \overline{) 256} \\ + 1 \quad - 1 \\ \hline 26 \quad 156 \\ - \quad 156 \\ \hline 0 \end{array}$$

$$\therefore \sqrt{256} = 16$$

Thus $\sqrt{\frac{1681}{256}} = \frac{41}{16}$
 $= 2.5625 \text{ or } 2\frac{9}{16}$

(iv) $\sqrt{9\frac{67}{121}} = \sqrt{\frac{1156}{121}}$

$$\begin{array}{r} 34 \\ 3 \overline{) 1156} \\ + 3 \quad - 9 \\ \hline 64 \quad 256 \\ - \quad 256 \\ \hline 0 \end{array}$$

$$\therefore \sqrt{1156} = 34$$

$$\sqrt{121} = 11 \text{ or } 3\frac{1}{11}$$

(v) $\sqrt{\frac{0.324}{72.9}}$

$$\begin{array}{r} 0.566 \\ 5 \overline{) 0.324} \\ +5 \quad -25 \\ \hline 106 \quad 740 \\ +6 \quad -636 \\ \hline 1129 \quad 10400 \\ -10161 \\ \hline 239 \end{array}$$

$$\sqrt{0.324} = 0.566$$

$$\begin{array}{r} 8.538 \\ 8 \overline{) 72.9} \\ +8 \quad -64 \\ \hline 165 \quad 890 \\ +5 \quad -825 \\ \hline 1703 \quad 6500 \\ +3 \quad -5109 \\ \hline 17068 \quad 139100 \\ -136544 \\ \hline 2556 \end{array}$$

$$\sqrt{72.9} = 8.538$$

$$\therefore \sqrt{\frac{0.324}{72.9}} = \frac{0.566}{8.538} = 0.066$$

(vi) $\sqrt{8\frac{1}{6}} = \sqrt{\frac{6}{49\frac{6}{6}}}$

$$= \sqrt{\frac{6 \times 6}{49}} = \sqrt{\frac{36}{49}}$$

$$= \sqrt{\frac{6 \times 6}{7 \times 7}} = \frac{6}{7}$$

5. (i) $\sqrt{2}$

$$\begin{array}{r} 1.414 \\ 1 \overline{) 2.0000} \\ +1 \quad -1 \\ \hline 24 \quad 100 \\ +4 \quad -96 \\ \hline 281 \quad 400 \\ +1 \quad -281 \\ \hline 2824 \quad 11900 \\ -11296 \\ \hline 604 \end{array}$$

$$\therefore \sqrt{2} = 1.414$$

(ii) $\sqrt{\frac{1}{3}} = \sqrt{0.3333\ldots}$

$$\begin{array}{r} 0.577 \\ 5 \overline{) 0.3333} \\ +5 \quad -25 \\ \hline 107 \quad 833 \\ +7 \quad -749 \\ \hline 1147 \quad 8400 \\ -8029 \\ \hline 371 \end{array}$$

$$\therefore \sqrt{\frac{1}{3}} = 0.577$$

$$\begin{array}{r} 0.3333 \\ 3 \overline{) 10} \\ -9 \\ \hline 10 \\ -9 \\ \hline 10 \\ -9 \\ \hline 10 \\ -9 \\ \hline 1 \end{array}$$

(iii) $\sqrt{0.1}$

$$\begin{array}{r} 0.316 \\ 3 \overline{) 0.10} \\ +3 \quad -9 \\ \hline 61 \quad 100 \\ +1 \quad -61 \\ \hline 626 \quad 3900 \\ -3756 \\ \hline 144 \end{array}$$

$$\therefore \sqrt{0.1} = 0.316$$

(iv)

$$\sqrt{1 + (0.021)^2}$$

$$\sqrt{1 + 0.000441} = \sqrt{1.000441}$$

$$\sqrt{\frac{1000441}{1000000}} = \frac{\sqrt{1000441}}{1000}$$

		1000.42
1		100 04 41
+ 1		- 1
20		000
+ 0		- 0
200		004
+ 0		- 0
2000		441
+ 0		- 0
20004		44100
+ 4		- 40004
200082		409600
		- 400164
		9436

$$\therefore \sqrt{\frac{1000441}{1000}} = \frac{1000.42}{1000}$$

$$= 100042$$

$$= 1.000$$

(v)

$$\sqrt{\sqrt{32} - \sqrt{128} + \sqrt{50}}$$

$$\sqrt{32} = ?$$

		5.657
5		32
+ 5		- 25
106		700
+ 6		- 636
1125		6400
+ 5		- 5625
11306		77500
		- 67836
		8664

$$\therefore \sqrt{32} = 5.656$$

$$\sqrt{128} = ?$$

		11.313
1		1 28
+ 1		- 1
21		028
+ 1		- 21
223		700
+ 3		- 669
2261		3100
+ 1		- 2261
22673		83900
		- 68019
		15881

$$\therefore \sqrt{128} = 11.313$$

$$\sqrt{50} = ?$$

		7.071
7		50
+ 7		- 49
140		100
+ 0		- 0
1407		10000
+ 7		- 9849
14141		15100
		- 14141
		951

$$\therefore \sqrt{50} = 7.071$$

$$\sqrt{\sqrt{32} - \sqrt{128} + \sqrt{50}}$$

$$= \sqrt{5.656 - 11.313 + 7.071} = \sqrt{1.414}$$

		1.189
1		1.41 4
+ 1		- 1
21		0 41
+ 1		- 21
228		2040
+ 8		- 1824
236 9		21600
		- 21321
		279

$$\sqrt{1.414} = 1.189$$

$$\therefore \sqrt{\sqrt{32} - \sqrt{128} + \sqrt{50}} = 1.189$$

$$(vi) \sqrt{\frac{17}{25}} = \frac{\sqrt{17}}{\sqrt{5 \times 5}} = \frac{\sqrt{17}}{5}$$

$$\begin{array}{r} 4.123 \\ 4 \overline{) 17} \\ +4 \quad -16 \\ \hline 81 \quad 100 \\ +1 \quad -81 \\ \hline 822 \quad 1900 \\ +2 \quad -1644 \\ \hline 8243 \quad 25600 \\ \quad -24729 \\ \hline \quad \quad 871 \end{array}$$

$$\therefore \sqrt{\frac{17}{25}} = \frac{4.123}{5} = 0.824$$

$$(vii) \sqrt{1 + 0.002116} = \sqrt{1.002116}$$

$$\begin{array}{r} 1.001 \\ 1 \overline{) 1.00 \ 21 \ 16} \\ +1 \quad -1 \\ \hline 20 \quad 00 \\ +0 \quad -0 \\ \hline 200 \quad 21 \\ +0 \quad -0 \\ \hline 2001 \quad 2116 \\ \quad -2001 \\ \hline \quad \quad 115 \end{array}$$

$$\therefore \sqrt{1 + 0.002116} = 1.001$$

$$6. (i) \sqrt{\frac{9}{2}} + \sqrt{\frac{49}{9}} = \sqrt{\frac{9}{2 \times 2}} + \sqrt{\frac{49}{4 \times 9}}$$

Helpful Hint

$$\frac{9}{2} \div 2 = \frac{9}{2} \times \frac{1}{2}$$

$$= \sqrt{\frac{9}{4}} + \sqrt{\frac{49}{36}} = \frac{3}{2} + \frac{7}{6}$$

$$= \frac{9 \times 7}{6} + \frac{16}{6} = \frac{8}{3} = 2\frac{2}{3}$$

$$\begin{aligned} (ii) \quad & \sqrt{\frac{16}{5}} + \sqrt{\frac{25}{8}} + \sqrt{\frac{49}{10}} \\ &= \sqrt{\frac{16}{5 \times 5}} + \sqrt{\frac{25}{8 \times 8}} - \sqrt{\frac{49}{10 \times 10}} \\ &= \frac{4}{5} - \frac{5}{8} - \frac{7}{10} \\ &= \frac{32 + 25 - 28}{40} = \frac{29}{40} \end{aligned}$$

7. Let x and $3x$ be the required numbers.

$$(x)(3x) = 9\frac{18}{25}$$

$$3x^2 = \frac{243}{25}$$

$$x^2 = \frac{243}{25 \times 3} = \frac{81}{25}$$

$$x^2 = \frac{81}{25}$$

Take square root on both the sides.

$$\sqrt{x^2} = \sqrt{\frac{81}{25}}$$

$$x^2 = \sqrt{\frac{9 \times 9}{5 \times 5}}$$

$$x = \frac{9}{5} = 1\frac{4}{5}$$

$$3x = \frac{3 \times 9}{5} = \frac{27}{5} = 5\frac{2}{5}$$

$\therefore$ The two numbers are

$$1\frac{4}{5} \text{ and } 5\frac{2}{5}$$

8.

Helpful Hint

Product of HCF and LCM of two numbers is equal to the product of those two numbers.

$$\text{HCF} \times \text{LCM} = 5\frac{5}{64}$$

$$3\frac{1}{4} \times \text{LCM} = 5\frac{5}{64}$$

$$\frac{13}{4} \times \text{LCM} = \frac{325}{64}$$

$$\frac{13}{4} \times \text{LCM} = \frac{325}{64}$$

$$\text{LCM} = \frac{325}{\cancel{64}^{25}_{16}} \times \frac{1}{\cancel{4}_1^{13}}$$

$$\text{LCM} = \frac{25}{16}$$

$$\sqrt{\text{LCM}} = \sqrt{\frac{25}{16}}$$

$$= \sqrt{\frac{5 \times 5}{4 \times 4}}$$

$$= \pm \frac{5}{4} = \pm 1\frac{1}{4}$$

9.

$$\sqrt{944.578756}$$

$$\sqrt{\frac{944578756}{1000000}} = \pm \sqrt{\frac{944578756}{1000}}$$

	30734
3	9 44 57 87 56
+ 3	- 9
60	44
+ 0	- 00
607	4457
+ 7	- 4249
6143	20887
+ 3	- 18429
61464	245856
	- 245856
	0

$$\therefore \pm \sqrt{\frac{944578756}{1000}} = \pm \frac{30734}{1000}$$

$$= \pm 30.734$$

10. Area of a square = l^2

$$l^2 = 331.24 \text{ sq. metres}$$

Take square root to find the length of each side

$$\sqrt{l^2} = \sqrt{331.24}$$

$$l = \sqrt{\frac{33124}{100}} = \sqrt{\frac{33124}{10}}$$

	182
1	3 31 24
+ 1	- 1
28	231
+ 8	- 224
362	724
	- 724
	0

$$\therefore l = \frac{182}{10} = 18.2$$

$$\text{Perimeter} = 4l = 4 \times 18.2 = 72.8 \text{ metres}$$

11. Area of a square = 105 square metres

$$l^2 = 105$$

Take square root on both the sides.

$$\sqrt{l^2} = \sqrt{105}$$

	10.246
1	1 05
+ 1	- 1
20	05
+ 0	- 0
202	500
+ 2	- 404
2044	9600
+ 4	- 8176
20486	142400
	122916
	18484

$$\therefore l = 10.246$$

$$= 10.25 \text{ m}$$

12. Area of a square = $\frac{289}{64}$ square metres

$$l^2 = \frac{289}{64}$$

$$\sqrt{l^2} = \sqrt{\frac{289}{64}}$$

$$l = \frac{\sqrt{17 \times 17}}{\sqrt{8 \times 8}} = \frac{17}{18}$$

$$= 2\frac{1}{8} \text{ m or } 2.125 \text{ m}$$

13. Area of square = 4m^2

$$4\text{m}^2 = 40000 \text{ cm}^2$$

Helpful Hint

$$1\text{m} = 100 \text{ cm}$$

$$1 \text{ m}^2 = 10000 \text{ cm}^2$$

Number of paper

$$\text{squares needed} = \frac{40000}{40} = 1000$$

Exercise 3B

1. (i) 3 (ii) perfect cube
(iii) 9 (iv) $3^3 = 27$ (v) Index

2. (i) True
Even numbers are multiples of 2. So the cube of any even number is also a multiple of 2.

- (ii) False

Lets consider a natural number n

$$(-n)^3 = (-n) \times (-n) \times (-n)$$

$$= +n^2 \times (-n) = -n^3$$

$\therefore$ cube of a negative number is always negative.

- (iii) True

$$6 \times 6 \times 6 = 216$$

- (iv) False

$$= \sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}}$$

$$\therefore \sqrt[3]{\frac{a}{b}} \neq \sqrt[3]{a} \times \sqrt[3]{a}$$

- (v) True

Because $6^3 = 216$, $7^3 = 343$ and 250 lies between 216 and 343.

3. (i)

$$\begin{array}{r} 3 \overline{) 729} \\ 3 \overline{) 243} \\ 3 \overline{) 81} \\ 3 \overline{) 27} \\ 3 \overline{) 9} \\ 3 \overline{) 3} \\ 1 \end{array}$$

The prime factors of 729 are

$$3 \times 3 \times 3 \times 3 \times 3 \times 3$$

$$= 3^3 \times 3^3 = (3 \times 3)^3$$

$$= 9^3$$

The prime factors of 729 can be grouped into triplets of equal factors
 $\therefore$ 729 is a perfect cube.

- (ii)

$$\begin{array}{r} 2 \overline{) 2700} \\ 2 \overline{) 1350} \\ 3 \overline{) 675} \\ 3 \overline{) 225} \\ 3 \overline{) 75} \\ 5 \overline{) 25} \\ 5 \overline{) 5} \\ 1 \end{array}$$

The prime factors of 2700 are

$$2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 5 = 2^2 \times 3^3 \times 5^2$$

Its prime factors can not be grouped into triplets.

$\therefore$ 2700 is not a perfect cube.

(iii)

2	27000
2	13500
2	6750
3	3375
3	1125
3	375
5	125
5	25
5	5
	1

The prime factors are

$$2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 5 \times 5 \\ = 2^3 \times 3^3 \times 5^3 = (2 \times 3 \times 5)^3 = 30^3$$

The prime factors of 27000 can be grouped into triplets of equal factors.

$\therefore$ 27000 is a perfect cube.

(iv)

2	34128
2	17064
2	8532
2	4266
3	2133
3	711
3	237
79	79
	1

Prime factors of 34128 are

$$2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 79$$

These prime factors can not be grouped into triplet of equal factors.

$\therefore$ 34128 is not a perfect cube.

4.

2	864
2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

$$864 = 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \\ = 2^5 \times 3^3$$

One more 2 is required to make another triplet of 2.

Multiply 864 by 2.

$$2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$$

This is a perfect cube.

Thus the required smallest integer is 2.

5.

2	13500
2	6750
3	3375
3	1125
3	375
5	125
5	25
5	5
	1

$$13500 = 2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 5 \times 5$$

3 and 5 occur thrice but 2 occurs only twice.

$$2 \times 2 = 4$$

Divide 13500 by 4

$$\therefore 3 \times 3 \times 3 \times 5 \times 5 \times 5 = 3375$$

This is a perfect cube.

The required smallest integer is 4.

6. (i)

2	5832
2	2916
2	1458
3	729
3	243
3	81
3	27
3	9
3	3
	1

$$\begin{aligned} \sqrt[3]{5832} &= \sqrt[3]{2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3} \\ &= \sqrt[3]{2^3 \times 3^3 \times 3^3} \\ &= \sqrt[3]{18^3} \\ &= 18 \end{aligned}$$

$$\begin{array}{r}
 3 \overline{) 9261} \\
 3 \overline{) 3087} \\
 3 \overline{) 1029} \\
 7 \overline{) 343} \\
 7 \overline{) 49} \\
 7 \overline{) 7} \\
 \hline
 1
 \end{array}$$

$$\begin{aligned}
 \sqrt[3]{9261} &= \sqrt[3]{3 \times 3 \times 3 \times 7 \times 7 \times 7} \\
 &= \sqrt[3]{3^3 \times 7^3} \\
 &= \sqrt[3]{21^3} \\
 &= 21
 \end{aligned}$$

$$\begin{array}{r}
 2 \overline{) 21952} \\
 2 \overline{) 10976} \\
 2 \overline{) 5488} \\
 2 \overline{) 2744} \\
 2 \overline{) 1372} \\
 2 \overline{) 686} \\
 7 \overline{) 343} \\
 7 \overline{) 49} \\
 7 \overline{) 7} \\
 \hline
 1
 \end{array}$$

$$\begin{aligned}
 \sqrt[3]{21952} &= \sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 7 \times 7 \times 7} \\
 &= \sqrt[3]{2^3 \times 2^3 \times 7^3} \\
 &= \sqrt[3]{28^3} \\
 &= 28
 \end{aligned}$$

$$\begin{array}{r}
 5 \overline{) 42875} \\
 5 \overline{) 8575} \\
 5 \overline{) 1715} \\
 7 \overline{) 343} \\
 7 \overline{) 49} \\
 7 \overline{) 7} \\
 \hline
 1
 \end{array}$$

$$\begin{aligned}
 \sqrt[3]{42875} &= \sqrt[3]{5 \times 5 \times 5 \times 7 \times 7 \times 7} \\
 &= \sqrt[3]{5^3 \times 7^3} \\
 &= \sqrt[3]{35^3} \\
 &= 35
 \end{aligned}$$

$$\begin{array}{r}
 2 \overline{) 74088} \\
 2 \overline{) 37044} \\
 2 \overline{) 18522} \\
 3 \overline{) 9261} \\
 3 \overline{) 3087} \\
 3 \overline{) 1029} \\
 7 \overline{) 343} \\
 7 \overline{) 49} \\
 7 \overline{) 7} \\
 \hline
 1
 \end{array}$$

$$\begin{aligned}
 \sqrt[3]{74088} &= \sqrt[3]{2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 7 \times 7 \times 7} \\
 &= \sqrt[3]{2^3 \times 3^3 \times 7^3} \\
 &= \sqrt[3]{42^3} \\
 &= 42
 \end{aligned}$$

$$\begin{array}{r}
 5 \overline{) 274625} \\
 5 \overline{) 54925} \\
 5 \overline{) 10985} \\
 13 \overline{) 2197} \\
 13 \overline{) 169} \\
 13 \overline{) 13} \\
 \hline
 1
 \end{array}$$

$$\begin{aligned}
 \sqrt[3]{274625} &= \sqrt[3]{5 \times 5 \times 5 \times 13 \times 13 \times 13} \\
 &= \sqrt[3]{5^3 \times 13^3} \\
 &= \sqrt[3]{65^3} \\
 &= 65
 \end{aligned}$$

7. Volume of a cube = l^3

$$l^3 = 46656 \text{ cubic metres}$$

$$l = \sqrt[3]{46656}$$

2	46656
2	23328
2	11664
2	5832
2	2916
2	1458
3	729
3	243
3	81
3	27
3	9
3	3
	1

$$\sqrt[3]{46656} =$$

$$\sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3}$$

$$= \sqrt[3]{2^3 \times 2^3 \times 3^3 \times 3^3}$$

$$= \sqrt[3]{36^3}$$

$$= 36 \text{ metres}$$

8. (i)

2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

$$\sqrt[3]{512} = \sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}$$

$$= \sqrt[3]{2^3 \times 2^3 \times 2^3}$$

$$= \sqrt[3]{8^3} = 8$$

Helpful Hint

Cube root of a negative number is always negative. $-8 \times -8 \times -8 = -512$

$$\therefore \sqrt[3]{-512} = -8$$

Thus, -512 is a cube of negative integer.

(ii)

2	1296
2	648
2	324
2	162
3	81
3	27
3	9
3	3
	1

$$2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3$$

$$= 2^3 \times 2 \times 3^3 \times 3$$

Prime factors can not be grouped into triplets of equal numbers.

Thus, -1296 is not the cube of negative integer.

(iii)

2	1372
2	686
7	343
7	49
7	7
	1

$$2 \times 2 \times 7 \times 7 \times 7$$

$$= 2^2 \times 7^3$$

-1372 is not a cube of negative integer.

(iv)

13	2197
13	169
13	13
	1

$$13 \times 13 \times 13 = 13^3$$

$$\therefore \sqrt[3]{-2197} = -13$$

Thus, -2197 is a cube of negative integer.

(v)

$$\begin{array}{r}
 2 \overline{) 17576} \\
 2 \overline{) 8788} \\
 2 \overline{) 4394} \\
 13 \overline{) 2197} \\
 13 \overline{) 179} \\
 13 \overline{) 13} \\
 \hline
 1
 \end{array}$$

$$2 \times 2 \times 2 \times 13 \times 13 \times 13$$

$$= 2^3 \times 13^3 = 26^3$$

$$\therefore \sqrt[3]{-17576} = -26$$

Thus, -17576 is a cube of negative integer.

(ii)

$$\sqrt[3]{343} = \sqrt[3]{7 \times 7 \times 7} = 7$$

$$\sqrt[3]{8} = \sqrt[3]{2 \times 2 \times 2} = 2$$

$$\begin{array}{r}
 2 \overline{) 2744} \\
 2 \overline{) 1372} \\
 2 \overline{) 686} \\
 7 \overline{) 343} \\
 7 \overline{) 49} \\
 7 \overline{) 7} \\
 \hline
 1
 \end{array}$$

$$\sqrt[3]{-2744} = \sqrt[3]{2 \times 2 \times 2 \times 7 \times 7 \times 7}$$

$$= -14$$

$$\sqrt[3]{343} \times \sqrt[3]{8} \div \sqrt[3]{-2744}$$

$$= 7 \times 2 \div (-14)$$

$$= 14 \div -14$$

$$= -1$$

(iii)

$$\sqrt[3]{-125} \times \sqrt[3]{64} \times \sqrt[3]{27}$$

$$= -5 \div 4 \times 3$$

$$= \frac{-5}{4} \times 3 = \frac{15}{4} = -3 \frac{3}{4}$$

$$(iv) \sqrt[3]{1331} \times \sqrt[3]{8} \times \sqrt[3]{1000}$$

$$= 11 \times 2 \times 10$$

$$= 220$$

(v)

$$\left(\sqrt[3]{1728} \div \sqrt[3]{125} \right) \times \left(\sqrt[3]{1000} \div \sqrt[3]{64} \right)$$

$$= (12 \div 5) \times (10 \div 4)$$

$$= \frac{12^3}{5^3} \times \frac{10^2}{4^3}$$

$$= 6$$

9.

Helpful HintVolume of a cube = l^3

$$\text{Volume of a cube} = 0.064 \text{ m}^3$$

$$l^3 = 0.064 \text{ m}^3$$

$$l^3 = \frac{64}{1000}$$

$$l = \sqrt[3]{\frac{64}{1000}} = \frac{\sqrt[3]{64}}{\sqrt[3]{1000}}$$

$$= \frac{\sqrt[3]{4 \times 4 \times 4}}{\sqrt[3]{10 \times 10 \times 10}}$$

$$= \frac{4}{10} = 0.4 \text{ m}$$

10. (i)

$$\sqrt[3]{27} = \sqrt[3]{3 \times 3 \times 3} = 3$$

$$\sqrt[3]{216} = \sqrt[3]{6 \times 6 \times 6} = 6$$

$$\sqrt[3]{729} = \sqrt[3]{9 \times 9 \times 9} = 9$$

$$\sqrt[3]{27} \times \sqrt[3]{216} \div \sqrt[3]{729}$$

$$= 3 \times 6 \div 9 = 18 \div 9 = 2$$

11. Volume of a cube = l^3

$$= 11^3 = 11 \times 11 \times 11$$

$$= 1331 \text{ cm}^3$$

12. Minimum volume of tank = 729 m^3

$$l^3 = 729$$

$$\sqrt[3]{l^3} = \sqrt[3]{729}$$

$$l = 9 \text{ cm}$$

13. Volume = 1728 cm^3

$$l^3 = 1728$$

$$\sqrt[3]{l^3} = \sqrt[3]{1728}$$

$$l = 12 \text{ cm}$$

Since the length height, and width each is greater than 12 cm, hence, Baneen can keep the box in her drawer.

1300 is not a perfect cube

$$\sqrt[3]{1331} = 11$$

$$\sqrt[3]{1728} = 12$$

$$\sqrt[3]{125} = 5$$

5. A

$$16 \times 16 \times 16 = 4096$$

Multiple Choice Questions 3

1. B

$$512 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$$

$$\sqrt[3]{512} = \sqrt[3]{2^3 \times 2^3 \times 2^3} = 8$$

2. D

$$3375 = 3 \times 3 \times 3 \times 5 \times 5 \times 5$$

$$\sqrt[3]{3375} = \sqrt[3]{3^3 \times 5^3} = 15$$

3. C

In $\sqrt[3]{2197}$, 3 is the index,

$\sqrt{\quad}$, is a radical and

13 is the cube root of 2197.

4. D



PROPORTIONS

Exercise 4

1. (i) Direct

If we increase the number of books, total cost will also increase.

- (ii) directly
(iii) inversely
(iv) constant
(v) curved

2. (i) True

- (ii) False

More petrol is consumed to cover more distance.

- (iii) True

- (iv) False

The graph of direct proportion always passes through the origin.

- (v) False

If x and y are directly proportional then $y = kx$

- 3.

x	4	7	11	20
y	20	35	55	100

We can notice that y is obtained by multiplying the value of x by 5

$$4 \times 5 = 20$$

$$7 \times 5 = 35$$

Hence

$$11 \times 5 = 55$$

and

$$\frac{100}{5} = 20$$

- 4.

x	9	27	81
y	27	9	3

5. Distance

15 km

time

3 hrs

x km

2 hrs

$$\frac{x}{2} = \frac{15}{3}$$

$$x = 5 \times 2$$

$$x = 10 \text{ km}$$

6. Number of men

11

length

$$6 \frac{3}{4} \text{ m} = \frac{27}{4} \text{ m}$$

x

27 m

Since both the quantities are directly proportional

$$\frac{x}{27} = \frac{11}{\frac{27}{4}}$$

$$\frac{x}{27} = \frac{4 \times 11}{27}$$

$$x = \frac{44 \times 27}{27}$$

$$x = 44 \text{ men}$$

7. number of taps

6

time

30 min

x

20 min

Since both the quantities are inversely proportional.

$$20 \times x = 6 \times 30$$

$$x = \frac{6 \times 30}{20}$$

$$x = 9 \text{ taps}$$

8. number of girls number of days

50

40

$$50 + 30 = 80$$

x

Both the quantities are inversely proportional

$$80 \times x = 40 \times 50$$

$$x = \frac{40 \times 50}{80}$$

$$x = 25 \text{ days}$$

9. length of rod weight of rod

12 m

42 kg

6 m

x kg

Both the quantities are directly proportional

$$\frac{x}{6} = \frac{42}{12}$$

$$x = \frac{42 \times 6}{12}$$

$$x = 21 \text{ Kg}$$

10. Width Length Price

12

25

2500

50

30

x

$$\frac{x}{2500} = \frac{50}{12} \times \frac{30}{25}$$

$$x = \frac{50 \times 30 \times 2500}{12 \times 25} = 12500$$

∴ Rs 12500 is the price of a carpet measuring 50 feet by 30 feet

11. Men Time Days

100

8

35

x

10

25

$$\frac{x}{100} = \frac{8}{10} \times \frac{35}{25}$$

$$x = \frac{8 \times 35 \times 100}{10 \times 25} = 112$$

∴ 112 men should be employed to finish the job in 25 days if they work 10 hours a day.

12. (a) Since y is directly proportional to x , the constant is given by

$$k = \frac{y}{x}$$

$$k = \frac{9}{6} = \frac{3}{2}$$

- (b) for direct proportion, we have

$$y = kx$$

Substitute the value of k in above equation

$$y = \frac{3}{2}x$$

- (c)

x	2	6	10	14	18
y	3	9	15	21	27

$$y = \frac{3}{2}x$$

$$15 = \frac{3}{2}x$$

$$x = 15 \times \frac{2}{3}$$

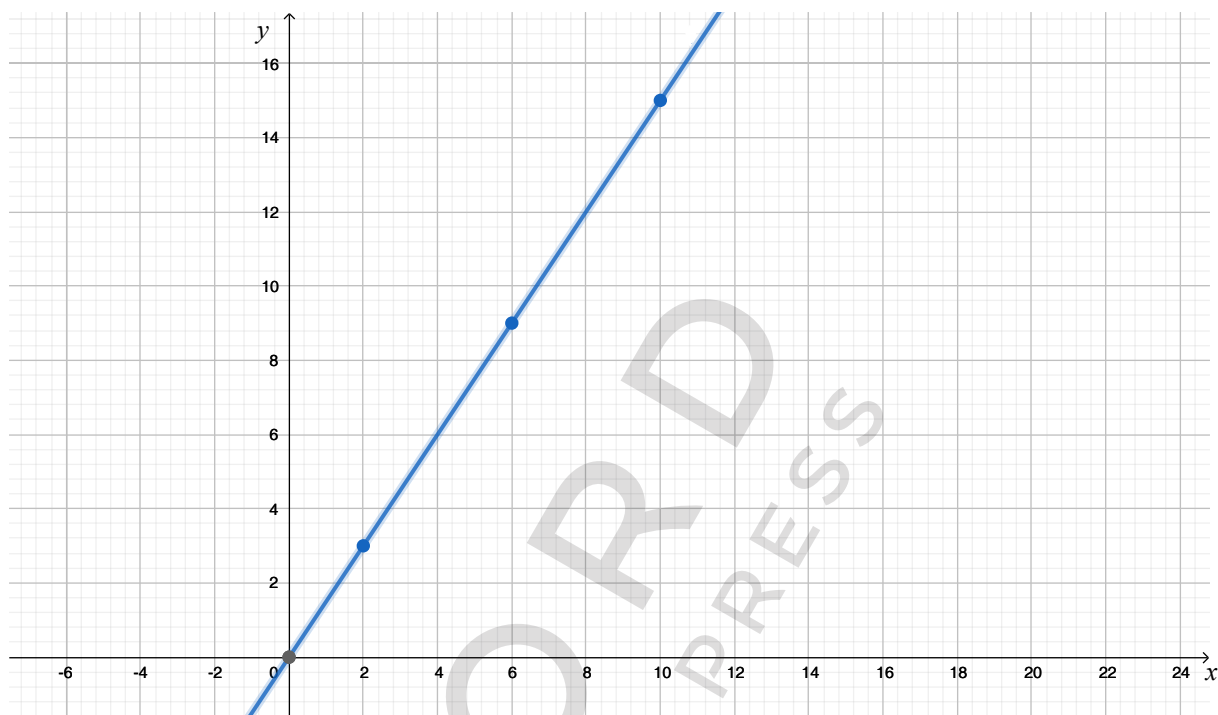
$$x = 10$$

$$y = \frac{3}{2}x$$

$$y = \frac{3}{2} \times 14$$

$$y = 21$$

(d) Use the table of values given in part c to draw the graph.





REVISION: NUMBERS

Q1. Distributive Law of union over intersection states that

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

Let's RHS; $A \cup (B \cap C)$

$$B \cap C = \{1, 3, 5, 7, 9, 11, 13, 15\}$$

$$\cap \{0, 1, 5, 6, 10, 13\}$$

$$= \{1, 5, 13\}$$

$$A \cup (B \cap C) = \{0, 5, 10, 15\} \cup \{1, 5, 13\}$$

$$= \{0, 1, 5, 10, 13, 15\}$$

Now take LHS; $(A \cup B) \cap (A \cup C)$

$$A \cup B = \{0, 5, 10, 15\} \cup \{0, 1, 5, 6, 10, 13\}$$

$$= \{0, 1, 5, 6, 10, 13, 15\}$$

$$B \cup C = \{1, 3, 5, 7, 9, 11, 13, 15\}$$

$$\cup \{0, 1, 5, 6, 10, 13\}$$

$$= \{0, 1, 3, 5, 6, 7, 9, 10, 11, 13, 15\}$$

$$(A \cup B) \cap (A \cup C) = \{0, 1, 5, 6, 10, 13, 15\}$$

$$\cap \{0, 1, 3, 5, 6, 7, 9, 10, 11, 13, 15\}$$

$$= \{0, 1, 5, 6, 10, 13, 15\}$$

Since LHS = RHS,

Thus, the distributive law of union over intersection is proved.

2. additive inverse Multiplicative inverse

(i) -354 $\frac{1}{354}$

(ii) 0.005 $-\frac{1}{0.005}$

(iii) $-\frac{1}{3\sqrt{5}}$ $\frac{3}{\sqrt{5}}$

(iv) $2\frac{5}{9}$ $\frac{9}{25}$

(v) $-\frac{17}{23}$ $\frac{22}{17}$ or $2\frac{6}{17}$

3. (i)

$$\begin{array}{r} 2.5 \\ 2 \overline{) 6.25} \\ + 2 \quad - 4 \quad \\ \hline 45 \quad 225 \\ - 225 \quad \\ \hline 0 \end{array}$$

$$\therefore \sqrt{6.25} = 2.5$$

(ii)

$$\begin{array}{r} 27 \\ 2 \overline{) 729} \\ + 2 \quad - 4 \quad \\ \hline 47 \quad 329 \\ - 329 \quad \\ \hline 0 \end{array}$$

$$\therefore \sqrt{729} = 27$$

(iii)

$$\begin{array}{r} 19 \\ 1 \overline{) 361} \\ + 1 \quad - 1 \quad \\ \hline 29 \quad 261 \\ - 261 \quad \\ \hline 0 \end{array}$$

$$\therefore \sqrt{361} = 19$$

(iv)

$$\begin{array}{r} 111 \\ 1 \overline{) 12321} \\ + 1 \quad - 1 \quad \\ \hline 21 \quad 23 \\ + 1 \quad - 21 \quad \\ \hline 221 \quad 221 \\ - 221 \quad \\ \hline 0 \end{array}$$

$$\therefore \sqrt{12321} = 111$$

(v)

$$\begin{array}{r}
 1.452 \\
 1 \overline{) 2.10 \, 83 \, 04} \\
 + 1 \quad - 1 \\
 \hline
 21 110 \\
 + 1 \quad - 96 \\
 \hline
 285 1483 \\
 + 5 \quad - 1425 \\
 \hline
 2902 5804 \\
 - 5804 \\
 \hline
 0
 \end{array}$$

$$\therefore \sqrt{2.108304} = 1.452$$

(vi)

$$\begin{array}{r}
 22.2 \\
 2 \overline{) 492.84} \\
 + 2 \quad - 4 \\
 \hline
 42 092 \\
 + 2 \quad - 84 \\
 \hline
 442 884 \\
 - 884 \\
 \hline
 0
 \end{array}$$

$$\therefore \sqrt{492.84} = 22.2$$

(vii)

$$\begin{array}{r}
 8.8 \\
 8 \overline{) 77.44} \\
 + 8 \quad - 64 \\
 \hline
 168 1344 \\
 - 1344 \\
 \hline
 0
 \end{array}$$

$$\therefore \sqrt{77.44} = 8.8$$

(viii)

$$\begin{array}{r}
 0.71 \\
 0 \overline{) 0.50 \, 41} \\
 + 0 \quad - 0 \\
 \hline
 7 50 \\
 + 7 \quad - 49 \\
 \hline
 141 141 \\
 - 141 \\
 \hline
 0
 \end{array}$$

$$\therefore \sqrt{0.5041} = 0.71$$

$$4. \quad (i) \quad \sqrt{1\frac{29}{49}} = \sqrt{\frac{78}{49}} = \frac{\sqrt{78}}{\sqrt{49}} = \frac{\sqrt{78}}{7}$$

$$\begin{array}{r}
 8.83 \\
 8 \overline{) 78} \\
 + 8 \quad - 64 \\
 \hline
 168 1400 \\
 + 8 \quad - 1344 \\
 \hline
 1763 5600 \\
 - 5289 \\
 \hline
 311
 \end{array}$$

$$\therefore \sqrt{\frac{78}{49}} = \frac{8.83}{7} = 1.26$$

$$(ii) \quad \sqrt{2\frac{42}{75}} = \sqrt{\frac{192}{49}} = \sqrt{2.56}$$

$$\begin{array}{r}
 1.6 \\
 1 \overline{) 2.56} \\
 + 1 \quad - 1 \\
 \hline
 26 156 \\
 - 156 \\
 \hline
 0
 \end{array}$$

$$\therefore \sqrt{2\frac{42}{75}} = 1.6$$

$$(iii) \sqrt{3\frac{334}{3024}} = \sqrt{\frac{9406}{3024}} = \sqrt{3.11}$$

$$\begin{array}{r} 1 \overline{) 3.11} \\ + 1 \quad - 1 \quad \\ \hline 27 \quad 211 \\ 7 \quad - 189 \\ \hline 346 \quad 2200 \\ - 2076 \\ \hline 124 \end{array}$$

$$\therefore \sqrt{3\frac{334}{3024}} = 1.77$$

5. (i) $\sqrt{12}$

$$\begin{array}{r} 3.46 \overline{) 12} \\ + 3 \quad - 9 \quad \\ \hline 64 \quad 300 \\ + 4 \quad - 256 \\ \hline 686 \quad 4400 \\ - 4116 \\ \hline 284 \end{array}$$

$$\therefore \sqrt{12} = 3.46$$

(ii)

$$\begin{array}{r} 7.28 \overline{) 53} \\ + 7 \quad - 49 \quad \\ \hline 142 \quad 400 \\ + 2 \quad - 284 \\ \hline 1448 \quad 11600 \\ - 11584 \\ \hline 16 \end{array}$$

$$\therefore \sqrt{53} = 7.28$$

(iii)

$$\begin{array}{r} 10.44 \overline{) 109} \\ 1 \overline{) 109} \\ + 1 \quad - 1 \quad \\ \hline 20 \quad 09 \\ + 0 \quad 0 \quad \\ \hline 204 \quad 900 \\ - 4 \quad - 816 \\ \hline 2084 \quad 8400 \\ - 8336 \\ \hline 64 \end{array}$$

$$\therefore \sqrt{109} = 10.44$$

(iv) $\sqrt{26}$

$$\begin{array}{r} 5.09 \overline{) 26} \\ 5 \overline{) 26} \\ + 5 \quad - 25 \quad \\ \hline 100 \quad 100 \\ + 0 \quad - 0 \quad \\ \hline 1009 \quad 10000 \\ - 9081 \\ \hline 1919 \end{array}$$

$$\therefore \sqrt{26} = 5.09$$

6. (i)

$$\begin{array}{r|l} 3 & 3375 \\ 3 & 1125 \\ 3 & 375 \\ 5 & 125 \\ 5 & 25 \\ 5 & 5 \\ & 1 \end{array}$$

$$\sqrt[3]{3375} = \sqrt[3]{3^3 \times 5^3} = 3 \times 5$$

$$\sqrt[3]{3375} = 15$$

(ii)

2	17576
2	8788
2	4394
13	2197
13	169
13	13
	1

$$\sqrt[3]{17576} = \sqrt[3]{2 \times 2 \times 2 \times 13 \times 13 \times 13}$$

$$\sqrt[3]{17576} = 2 \times 13 = 26$$

(iii)

61	226981
61	3721
61	61
	1

$$\sqrt[3]{226981} = \sqrt[3]{61 \times 61 \times 61}$$

$$\therefore \sqrt[3]{226981} = 61$$

(iv)

2	2744
2	1372
2	686
7	343
7	49
7	7
	1

$$\sqrt[3]{2744} = \sqrt[3]{2 \times 2 \times 2 \times 7 \times 7 \times 7}$$

$$\therefore \sqrt[3]{2744} = 2 \times 7 = 14$$

(v)

3	35937
3	11979
3	3993
11	1331
11	121
11	11
	1

$$\therefore \sqrt[3]{35937} = \sqrt[3]{3^3 \times 11^3}$$

$$\therefore \sqrt[3]{35937} = 3 \times 11 = 33$$

7.

(i) Calculate $\sqrt{15.21}$

Since $30^2 = 900$ and $40^2 = 1600$, the root is near $4.3.9^2 = 15.21$.

Result: 3.9

(ii) Calculate $\sqrt{73.96}$

Since $80^2 = 6400$ and $90^2 = 8100$, the root is between 8 and 9. $8.6^2 = 73.96$.

Result: 8.6

(iii) Calculate $\sqrt{8213.05}$

Using long division or estimation $90^2 = 8100$, we find the approximate root.

≈ 90.626

8. Rationalize and calculate

$$\frac{3}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3} \text{ Knowing } \sqrt{3} \approx 1.73205:$$

$$\frac{2 \times 1.73205}{3} \approx \frac{3.4641}{3} \approx 1.1547$$

9. 1.15

$$(i) \frac{\sqrt{13.69} + \sqrt{18.49}}{\sqrt{151.29} - \sqrt{5.29}}$$

$$3.7 + 4.3 = 8.0 \text{ (Numerator)}$$

$$12.3 - 2.3 = 10.0 \text{ (Denominator)}$$

$$\frac{8}{10} = 0.8$$

$$0.8$$

$$(ii) \frac{\sqrt{77.44} + \sqrt{43.56}}{\sqrt{77.44} - \sqrt{44.89}}$$

$$8.8 + 6.6 = 15.4$$

$$8.8 - 6.7 = 2.1$$

$$\frac{15.4}{2.1} = \frac{154}{21} \times \frac{22}{3} = 7.33$$

$$7.33$$

10.

$\sqrt{999999} \approx 999.99$. The largest integer root is 999.

$$999^2 = (1000 - 1)^2 = 1000000 - 2000 + 1 = 998001.$$

$$998001$$

11.

The volume V is given as 32.768 m^3 .
Since $V = s^3$ we have:

$$s^3 = 32.768$$

$$s = \sqrt[3]{32.768}$$

$$32.768 = \frac{32768}{1000}$$

$$\sqrt{\frac{32768}{1000}} = \frac{\sqrt[3]{32768}}{\sqrt[3]{1000}}$$

$$s = \frac{32}{10} = 3.2$$

The length of the side of the box is 3.2 m.

12.

$$26244 = 2^2 \times 3^8.$$

To be a cube, powers must be multiples of 3. We have 2^2 (needs to be removed) and $3^8 = 3^6 \times 3^2$

$$\text{Divisor} = 2^2 \times 3^2 = 4 \times 9 = 36.$$

$$\text{Quotient} = \frac{26244}{36} = 729. \sqrt[3]{729} = 9.$$

Divisor is 36; Cube root is 9.

13.

$$(i) \sqrt{9.61} = 3.1$$

$$(ii) \sqrt{92.16} = 9.6$$

$$(iii) \sqrt{1867.1041} = 43.21$$

16.

$$(i) \sqrt{5.4756} = 2.34$$

$$(ii) \sqrt{547.56} = 2.34 \times 10 = 23.4$$

$$(iii) \sqrt{0.054756} = 2.34/10 = 0.234$$

18.

(i)

7	456533
7	65219
7	9317
11	1331
11	121
11	11
	1

$$456533 = 7 \times 7 \times 7 \times 11 \times 11 \times 11$$

$$= 7 \times 11$$

$$\sqrt{456533} = 77$$

(ii)

3	59319
3	19773
3	6591
13	2197
13	169
13	13
	1

$$59319 = 3 \times 3 \times 3 \times 13 \times 13 \times 13$$

$$= 3 \times 13$$

$$\sqrt[3]{59319} = 39.$$

(iii)

1520.875	3	1520875
		1000

5	1520875
5	304175
5	60835
23	12167
23	529
23	23
	1

10	1000
10	100
10	10
	1

$$= \sqrt[3]{\frac{1520875}{1000}}$$

$$= \frac{5 \times 5 \times 5 \times 23 \times 23 \times 23}{10 \times 10 \times 10} = \frac{5 \times 23}{10}$$

$$= \frac{115}{10}$$

$$= \sqrt[3]{1520.875} = 11.5$$

19.

$$\sqrt[3]{512}$$

$$= \overline{2 \times 2 \times 2} \times \overline{2 \times 2 \times 2} \times \overline{2 \times 2 \times 2}$$

$$= 2 \times 2 \times 2$$

$$= 8\text{m}$$

20.

(i) $\sqrt[3]{792}$

Prime factorize 792:

$$792 = 2 \times 396 = 2^2 \times 198$$

$$= 2^3 \times 99 = 2^3 \times 3^2 \times 11.$$

Extract perfect cube factors:

$$\sqrt[3]{2^3 \times 3^2 \times 11} = 2 \times \sqrt[3]{99}.$$

Approximating $\sqrt[3]{99} \approx 4.626$.

Result: $792 \approx 9.252$.

(ii) $\sqrt[3]{29600}$

Prime factorize 29600:

$$29600 = 100 \times 296$$

$$= (2^2 \times 5^2) \times (2^3 \times 37) =$$

$$29600 = 2^3 \times (2^2 \times 5^2 \times 37)$$

$$= 2^3 \times 3700.$$

$$\sqrt[3]{29600} = 2 \times \sqrt[3]{3700}.$$

Approximating $\sqrt[3]{3700} \approx 15.465$.

Result: $\approx 2 \times 15.465 \approx 30.93$.

(iii) $\sqrt[3]{\frac{31}{216}} \frac{\sqrt[3]{a}}{\sqrt[3]{b}}$

$$\frac{\sqrt[3]{31}}{\sqrt[3]{216}}.$$

$$\sqrt[3]{29600} = 2 \times \sqrt[3]{3700}.$$

Since $216 = 6^3$, $\sqrt[3]{216} = 6$.

Approximating $\sqrt[3]{31} \approx 3.141$.

$$\frac{3.141}{6} \approx 0.5235.19.$$



FINANCIAL ARITHMETIC

Exercise 5

1. (i) Health (ii) 20% (iii) $\frac{1}{8}$
(iv) Markup = Profit – Principal
(v) Marked Price – Selling Price
2. (i) False
Life insurance premiums are paid on monthly or yearly basis.
(ii) False
A son inherits twice the share of a daughter.
(iii) True
(iv) False
The profit is divided equally if the investment is equal.
(v) True
- 3.

Helpful Hint

Profit is earned if selling price is greater than the cost price.

$$\text{CP} = \text{Rs } 3000$$

$$\text{SP} = \text{Rs } 3500$$

$$\text{Profit} = \text{SP} - \text{CP}$$

$$= 3500 - 3000 = \text{Rs } 500$$

$$\text{Profit Percentage} = \frac{\text{Profit}}{\text{CP}} \times 100\%$$

$$= \frac{\text{Rs } 500}{\text{Rs } 3000} \times 100$$

$$= 16.66\% \text{ or } 16.7\%$$

4.

Helpful Hint

Loss is occurred if selling price is less than the cost price.

$$\text{CP} = \text{Rs } 10000$$

$$\text{SP} = \text{Rs } 9800$$

$$\text{Loss} = \text{CP} - \text{SP}$$

$$= \text{Rs } 10000 - \text{Rs } 9800$$

$$= \text{Rs } 200$$

$$\text{Loss percentage} = \frac{\text{Loss}}{\text{CP}} \times 100\%$$

$$= \frac{200}{10000} \times 100\% = 2\%$$

$$\therefore \text{Loss percentage} = 2\%$$

5. CP of desk = Rs 15000

CP of chair = Rs 5000

Total CP = 15000 + 5000

$$= \text{Rs } 20000$$

SP of desk = Rs 15500

SP of chair = Rs 1000

Total SP = 15500 + 1000

$$= \text{Rs } 16500$$

Since CP is greater than SP hence Jawaid bears an over loss.

$$\text{Loss} = \text{CP} - \text{SP}$$

$$= 20000 - 16500$$

$$= \text{Rs } 3500$$

$$\text{Loss percentage} = \frac{\text{Loss}}{\text{CP}} \times 100\%$$

$$= \frac{3500}{20000} \times 100\% = 17.5\%$$

$$\therefore \text{Loss percentage} = 17.5\%$$

6. CP = Rs 750000
 SP = Rs 200000
 Loss = CP – SP
 = 750000 – 200000
 = Rs 550000
 Loss percentage = $\frac{\text{Loss}}{\text{CP}} \times 100\%$
 = $\frac{550000}{750000} \times 100\%$
 = 73.3%
7. The most expensive item in the shop is priced at Rs 100.
 Since,
 10% of 100 is Rs 10.
 Hence, the shopkeeper should offer 10% discount on all the items, so that he does not lose more than Rs 10 on any item.
8. Total purchase = Rs 12,000
 1% of Rs 5000 = $\frac{1}{100} \times 50000 = \text{Rs } 50$
 2% of Rs 5000 = $\frac{2}{100} \times 500000 = \text{Rs } 200$
 Remaining amount = 12000 – 10000
 = Rs 2000
 3% of Rs 2000 = $\frac{3}{100} \times 2000$
 = Rs 60
 Total Discount = Rs 50 + Rs 100 + Rs 60
 = Rs 210
9. Rate of Premium = 2%
 Insurance amount = Rs 12000000
 Amount of Premium = ?
 Amount of premium = 2% of Rs 12000000
 = $\frac{2}{100} \times 12000000$
 = Rs 240000
10. Market value of car = Rs 1000000
 Rate of premium = 5%
 Amount of insurance premium paid in the 1st year = 5% of Rs 1000000

$$= \frac{5}{100} \times 1000000$$

$$= \text{Rs } 50000$$

Amount of insurance premium paid in

$$\text{2nd year} = \frac{5}{100} \times (1000000 - 50000)$$

$$= \frac{5}{100} \times 950000$$

$$= \text{Rs } 47500$$

Amount of insurance premium paid in

$$\text{3rd year} = \frac{5}{100} \times (950000 - 47500)$$

$$= \frac{5}{100} \times 902500$$

$$= \text{Rs } 45125$$

11. Total amount = Rs 250000
 widow's share = $\frac{1}{8}$ of Rs 250000
 = $\frac{1}{8} \times 250000$
 = Rs 31250
 Remaining amount = 250000 – 31250
 = Rs 218850

Helpful Hint

Son inherits twice the share of a daughter.

Ratio of shares of sons to share of daughter

$$2 : 1$$

$$2 : 1 \times 2 \quad [2 \text{ daughters}]$$

$$2 : 2$$

$$1 : 1$$

$$\text{Sum of ratios} = 1 + 1 = 2$$

$$\text{1 son's share} = \frac{1}{2} \times 218750$$

$$= \text{Rs } 109375$$

$$\text{2 daughter's share} = \frac{1}{2} \times 218750$$

$$= \text{Rs } 109375$$

$$\text{1 daughter's share} = \text{Rs } \frac{109375}{2}$$

$$= \text{Rs } 54687.5$$

12. Total amount = Rs 30000

Ratio of shares of sons to the shares of daughters

$$2 : 1$$

$$2 \times 2 : 1 \quad [2 \text{ sons}]$$

$$4 : 1$$

$$\text{Sum of ratios} = 4 + 1 = 5$$

$$2 \text{ sons' share} = \frac{4}{5} \times 30000 = \text{Rs } 24000$$

$$1 \text{ son's share} = \text{Rs } \frac{24000}{2} = \text{Rs } 12000$$

$$\begin{aligned} \text{the daughter's share} &= \frac{1}{5} \times 30000 \\ &= \text{Rs } 6000 \end{aligned}$$

13. Ratio of share in profit = ratio of their investments

Maria's investment: Samia's investment

$$22500 : 35000$$

$$9 : 14$$

$$\text{Sum of ratios} = 9 + 14 = 23$$

$$\begin{aligned} \text{Maria's share in profit} &= \frac{9}{23} \times 13800 \\ &= \text{Rs } 5400 \end{aligned}$$

$$\begin{aligned} \text{Samia's share in profit} &= \frac{14}{23} \times 13800 \\ &= \text{Rs } 8400 \end{aligned}$$

Or we can simply subtract Maria's share from total profit Rs 13800 – 5400 = Rs 8400

14. Total amount = Rs 240000

Ratio of inheritance = 5 : 4 : 3

$$\text{Sum of the ratios} = 5 + 4 + 3 = 12$$

$$\begin{aligned} \text{Aslam receives inheritance} &= \frac{5}{12} \times \text{Rs } 240000 \\ &= \text{Rs } 100000 \end{aligned}$$

$$\begin{aligned} \text{Pervaiz receives} &= \frac{4}{12} \times \text{Rs } 240000 \\ &= \text{Rs } 80,000 \end{aligned}$$

$$\begin{aligned} \text{Sana receives} &= \frac{3}{12} \times \text{Rs } 240000 \\ &= \text{Rs } 60000 \end{aligned}$$

15. Amount in US\$ = \$59

Exchange rate for \$1 = Rs 104

$$\begin{aligned} \text{Amount in rupees} &= 59 \times 104 \\ &= \text{Rs } 6136 \end{aligned}$$

16. Total amount in rupees = Rs 200000

$$\begin{aligned} \text{Half of amount} &= \text{Rs } \frac{200000}{2} \\ &= \text{Rs } 100000 \end{aligned}$$

Exchange rate for \$1 = Rs 107

$$\begin{aligned} \text{Amount in US\$} &= \frac{\text{Rs } 100000}{\text{Rs } 107} \\ &= \$ 934.579 \end{aligned}$$

Exchange rate for \$1 = Rs 106 for rest of the amount

$$\begin{aligned} \text{Amount in US\$} &= \frac{\text{Rs } 100000}{\text{Rs } 106} \\ &= \$ 934.396 \end{aligned}$$

$$\begin{aligned} \text{Total amount in US\$} &= \$ 934.579 + \$ 934.396 \\ &= \$ 1877.98 \approx \$ 1878 \end{aligned}$$

17. (i) P = Rs 400

$$R = 5\%$$

$$T = 4 \text{ years}$$

$$\text{Markup} = ?$$

$$\begin{aligned} \text{Markup} &= \frac{P \times R \times T}{100} \\ &= \frac{400 \times 5 \times 4}{100} \\ &= \text{Rs } 80 \end{aligned}$$

Hence markup is Rs 80.

(ii) P = Rs 50

$$R = 3\frac{1}{2}\%$$

$$T = 7 \text{ years}$$

$$\text{Markup} = ?$$

$$\begin{aligned} \text{Markup} &= \frac{50 \times 7 \times 7}{2 \times 100} \\ &= \frac{49}{4} \\ &= \text{Rs } 12.25 \end{aligned}$$

Hence markup is Rs 12.25

$$\begin{aligned}
 \text{(iii)} \quad P &= \text{Rs } 450 \\
 R &= 2\% \\
 T &= 2 \text{ years } 9 \text{ month} = 2 \frac{3}{4} \text{ years} \\
 \text{Markup} &= ? \\
 \text{Markup} &= \frac{P \times R \times T}{100} \\
 &= \frac{450 \times 2 \times 2 \frac{3}{4}}{100} \\
 &= \frac{99}{2} \\
 &= \text{Rs } 49.50
 \end{aligned}$$

Hence, markup is Rs 49.50

$$\begin{aligned}
 18. \text{ (i)} \quad P &= \text{Rs } 700 \\
 \text{Markup} &= \text{Rs } 210 \\
 T &= 3 \text{ years} \\
 R\% &= ? \\
 \text{Markup} &= \frac{P \times R \times T}{100} \\
 R &= \frac{\text{Markup} \times 100}{P \times T} \\
 R &= \frac{210 \times 100}{700 \times 3} \\
 R\% &= 10\%
 \end{aligned}$$

Hence, the rate% per annum is Rs 10%

$$\begin{aligned}
 \text{(ii)} \quad P &= \text{Rs } 1200 \\
 \text{Markup} &= \text{Rs } 144 \\
 T &= 2 \text{ years} \\
 R\% &= ? \\
 R &= \frac{\text{Markup} \times 100}{P \times T} \\
 &= \frac{144 \times 100}{1200 \times 2} \\
 &= 6 \\
 R\% &= 6\%
 \end{aligned}$$

Hence the rate% per annum is 6%

$$\begin{aligned}
 19. \text{ (i)} \quad P &= \text{Rs } 600 \\
 \text{Markup} &= \text{Rs } 90 \\
 R\% &= 5\% \text{ per anum} \\
 T &= ? \\
 T &= \frac{\text{Markup} \times 100}{P \times R}
 \end{aligned}$$

$$= \frac{90 \times 100}{600 \times 5} = 3$$

$$T = 3 \text{ years}$$

Hence the required time is 3 years.

$$\begin{aligned}
 \text{(ii)} \quad P &= \text{Rs } 1500 \\
 \text{Markup} &= \text{Rs } 450 \\
 R\% &= 6\% \text{ per annum} \\
 T &= ? \\
 T &= \frac{M \times 100}{P \times R} \\
 &= \frac{450 \times 100}{1500 \times 6} \\
 T &= 5 \text{ years}
 \end{aligned}$$

Hence the required time is 5 years.

$$\begin{aligned}
 20. \quad P &= \text{Rs } 660 \\
 R\% &= 4 \frac{1}{2}\% = \frac{9}{2}\% \\
 T &= 3 \text{ year and } 4 \text{ months} = 3 \frac{1}{3} \text{ year} \\
 \text{Markup} &= ? \\
 \text{Markup} &= \frac{P \times R \times T}{100} \\
 &= \frac{660 \times 9 \times 3 \frac{1}{3}}{100} \\
 &= 99
 \end{aligned}$$

$$\text{Markup} = \text{Rs } 99.$$

Hence the required markup is Rs 99.

$$\begin{aligned}
 21. \quad P &= \text{Rs } 600 \\
 R\% &= 6\% \\
 T &= 3 \text{ year and } 6 \text{ months and } 20 \text{ days} \\
 \text{Markup} &= ? \\
 \text{Markup} &= \frac{P \times R \times T}{100} \\
 T &= 3 \text{ years } 6 \text{ months and } 20 \text{ days.}
 \end{aligned}$$

Converting into years

$$20 \text{ days} = \frac{20}{30} = \frac{2}{3} \text{ months}$$

$$\text{Total months} = 6 \frac{2}{3} = \frac{20}{3} \text{ months}$$

$$\frac{20}{3} \text{ months} = \frac{20}{3 \times 12} \text{ years}$$

$$= \frac{5}{9} \text{ years}$$

$$\text{Total years} = 3 \frac{5}{9} \text{ years}$$

$$= \frac{32}{9} \text{ years}$$

$$\text{Markup} = \frac{600 \times 6^2 \times 32}{100 \times 9}$$

$$\text{Markup} = \text{Rs } 128$$

Hence the markup is Rs 128.

22. $P = \text{Rs } 850$

$$R\% = 5 \frac{1}{2}\% = \frac{11}{2}\% \text{ per annum}$$

$$T = 6 \text{ months} = \frac{1}{2} \text{ years}$$

$$\text{Markup} = ?$$

$$\text{Markup} = \frac{P \times R \times T}{100}$$

$$= \frac{850 \times 11 \times 1}{2 \times 100 \times 2}$$

$$\text{Markup} = \frac{187}{8}$$

$$= \text{Rs } 23.375$$

Hence the markup is Rs 23.375

23. $P = \text{Rs } 560$

$$R\% = 6\% \text{ per annum}$$

$$T = 6 \text{ months} = \frac{1}{2} \text{ year}$$

$$A = ?$$

$$\text{Markup} = \frac{P \times R \times T}{100}$$

$$= \frac{560 \times 6 \times 1}{2 \times 100}$$

$$= \frac{84}{10}$$

$$= \text{Rs } 8.40$$

$$\text{Profit} = \text{Principal} + \text{Markup}$$

$$= 560 + 8.40$$

$$= \text{Rs } 568.40$$

Hence the profit is Rs 568.40.

24. $P = ?$

$$\text{Markup} = \text{Rs } 130$$

$$R\% = 3 \frac{1}{4}\% = \frac{13}{4}\%$$

$$T = 5 \text{ years}$$

$$P = \frac{I \times 100}{R \times T}$$

$$= \frac{130 \times 100}{13 \times 5}$$

$$= \text{Rs } 800$$

Hence, principal amount is Rs 800.

Multiple Choice Questions 5

- D
- D
Loss = CP – SP = Rs 2000 – Rs 18000 = Rs 200
Loss percentage = $\frac{\text{Loss}}{\text{CP}} \times 100\%$
 $= \frac{200}{2000} \times 100\% = 10\%$
- C
Profit = SP – CP = Rs 3000 – Rs 2000 = Rs 1000
Profit percentage = $\frac{\text{Profit}}{\text{CP}} \times 100\%$
 $= \frac{1000}{2000} \times 100\% = 50\%$
- C
Dis = Dis rate $\times$ Marked price
 $= 3\% \times \text{Rs } 500 = \frac{3}{100} \times \text{Rs } 500 = \text{Rs } 15$
- B
Amount of premium
 $= \text{rate of premium} \times \text{insurance amount}$
Rs 4000 = rate of premium $\times$ Rs 200,000
rate of premium = $\frac{\text{Rs } 4000}{\text{Rs } 200000} \times 100\%$
 $= 2\%$



ALGEBRA: LAWS OF INDICES/EXPONENTS

Exercise 6

1. (i) 125
(ii) exponent
(iii) 0 (zero)
(iv) 3.456×10^2
(v) 0.0001357

2. (i) False
 $x^0 = 1$
(ii) True
(iii) False

Helpful Hint

$$a^m \times a^n = a^{m+n}$$

- (iv) False
 $x^3 \div x^5 = \frac{x^3}{x^5} = x^{3-5} = x^{-2}$
- (v) False
 $(5^4)^2 = 5^{4 \times 2}$
3. (i) 3^4
(ii) $8 = 2 \times 2 \times 2 = 2^3$
(iii) $\left(\frac{2}{3}\right)^1$
(iv) $\frac{8}{27} = \frac{2 \times 2 \times 2}{3 \times 3 \times 3} = \frac{2^3}{3^3} = \left(\frac{2}{3}\right)^3$
(v) $2^{-3} \times 2^{-3} = 2^{-3-3} = 2^{-6}$

4.

Helpful Hint

Give answers without simplification of exponents.

- (i) base = , 3 exponent = 4

- (ii) base = 4, exponent = 1
- (iii) base = 2^{-3} , exponent = 5
- (iv) base = x , exponent = -4
- (v) base = 5, exponent = 0
- (vi) base = 1, exponent = 1
- (vii) base = 3, exponent = $\frac{1}{2}$
- (viii) base = 5, exponent = $\frac{1}{2}$
- (ix) base = $3a^2$, exponent = 0
- (x) base = a^2 , exponent = 1

5. (i) $4(a^3)^0$
 $= 4 \times 1 \quad [a^0 = 1]$

- (ii) $(3)^{-4}$
 $= \frac{1}{3^4} = \frac{1}{81}$

- (iii) $\frac{4}{2^0} = 4$

- (iv) $(2)^6$
 $= 64$

- (v) $(2^{\frac{1}{2}})^8$
 $= 2^{\frac{1}{2} \times 8}$
 $= 2^4 = 16$

6. (i) $(4x^3)(2x^3)$
 $= 8x^{3+3} \quad [a^m \times a^n = a^{m+n}]$
 $= 8x^6$

- (ii) $8^0 = 1 \quad [a^0 = 1]$

- (iii) $(9x)^0$
 $= -1$

$$\begin{aligned}
 \text{(iv)} \quad & (y^4)^3 \\
 &= y^{4 \times 3} \\
 &= y^{12} [(a^0)^n = a^{mn}] \\
 \text{(v)} \quad & (x^2y)^4 \\
 &= x^{2 \times 4} y^4 = x^8 y^4 \\
 \text{(vi)} \quad & (2cd^4)(cd)^5 \\
 &= (2cd^4)(c^5d^5) = 2c^{1+5}d^{4+5} \\
 &= 2c^6d^9 \\
 \text{(vii)} \quad & (2fg^4)^4 (fg)^6 \\
 &= (16f^4g^{16})(f^6g^6) \\
 &= 16f^{4+6}g^{16+6} \\
 &= 16f^{10}g^{22} \\
 \text{(viii)} \quad & \frac{x^5y^6}{xy^2} \\
 &= x^{5-1}y^{6-2} = x^4y^4 \\
 \text{(ix)} \quad & \frac{x^2y^5}{xy^4} \\
 &= x^{2-1}y^{5-4} = xy \\
 \text{(x)} \quad & \frac{x^{-2}}{x^{-8}} \\
 &= x^{-2-(-8)} = x^6 \\
 \text{(xi)} \quad & \frac{24x^6}{12x^{-8}} \\
 &= 2x^{6-(-8)} = 2x^{14} \\
 \text{(xii)} \quad & (2x^3y^{-3})^{-2} \\
 &= \frac{y^6}{4x^6}
 \end{aligned}$$

$$\begin{aligned}
 7. \quad \text{(i)} \quad & \left(\frac{-2}{7}\right)^{-4} \times \left(\frac{-2}{7}\right)^2 \\
 &= \left(\frac{-7}{2}\right)^4 \times \left(\frac{-5}{7}\right)^2 \\
 &= \frac{7^{4-2} \times 25}{(-2)^4} \\
 &= \frac{7^{4-2} \times 25}{(-2)^4} \\
 &= \frac{7^2 \times 25}{16} = \frac{49 \times 25}{16} = \frac{1225}{16}
 \end{aligned}$$

$$= 76 \frac{9}{16}$$

$$\begin{aligned}
 \text{(ii)} \quad & \left(\frac{-1}{4}\right)^{-3} \times \left(\frac{-1}{4}\right)^{-2} \\
 &= (-4)^3 \times (-4)^2 \\
 &= (-4)^{3+2} = (-4)^5 \\
 &= -64
 \end{aligned}$$

Helpful Hint

If the exponent on a negative number is an odd number, the result is always a negative number.

$$\begin{aligned}
 \text{(iii)} \quad & \left[\left(\frac{-3}{2}\right)^2\right]^{-3} \\
 &= \left[\frac{-3}{2}\right]^{-6} \\
 &= \left[\frac{-2}{3}\right]^6 = \frac{64}{729}
 \end{aligned}$$

$$\begin{aligned}
 \text{(iv)} \quad & (2)^{-2} \times (4)^2 \\
 &= \frac{1}{2^2} \times 16 \\
 &= \frac{1}{4} \times 16 = 4
 \end{aligned}$$

$$\begin{aligned}
 8. \quad \text{(i)} \quad & \left(\frac{1}{2}\right)^{-2} \times \left(\frac{1}{3}\right)^{-2} \times \left(\frac{1}{4}\right)^{-2} \\
 &= 2^2 + 3^2 + 4^2 \\
 &= 4 + 9 + 16 = 29
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad & \left(\frac{2}{5}\right)^{-2} \div \left(\frac{9}{5}\right)^0 \\
 &= \frac{2^2}{5^2} \div 1 \\
 &= \frac{4}{25}
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii)} \quad &= (2^{-1} \times 5^{-1})^{-1} \div 4^{-1} \\
 &= \left(\frac{1}{2} \times \frac{1}{5}\right)^{-1} \div \frac{1}{4} \\
 &= \left(\frac{1}{10}\right)^{-1} \times 4 = 10 \times 4 = 40
 \end{aligned}$$

$$\begin{aligned}
 \text{(iv)} &= (4^{-1} + 8^{-1}) \div \left(\frac{2}{3}\right)^{-1} \\
 &= \left(\frac{1}{4} + \frac{1}{8}\right) \div \left(\frac{3}{2}\right) \\
 &= \frac{2+1}{8} \times \frac{2}{3} \\
 &= \frac{\overset{1}{\cancel{3}}}{8_4} \times \frac{\overset{1}{\cancel{2}}}{3_1} = \frac{1}{4}
 \end{aligned}$$

9. (i) $\overleftarrow{700}$ (2 places towards left)
 7.00×10^2
- (ii) $\overleftarrow{5100000000}$ (9 places towards left)
 5.10×10^9
- (iii) $\overleftarrow{30812}$ (4 places towards left)
 3.0812×10^4
- (iv) $\overrightarrow{0.003187}$ (3 places towards right)
 3.187×10^{-3}

Helpful Hint

Use negative sign for exponent of 10 when the decimal point moves towards right

10. (i) $\overrightarrow{3.18 \times 10^6}$ (move decimal point 6 places towards right)
 $= 3180000$

Helpful Hint

Move decimal point toward right if the exponent of 10 is positive and vice versa

- (ii) $\overrightarrow{0.65 \times 10^{-3}}$ (move decimal point 3 places towards left)
 $= 0.00065$
- (iii) $\overrightarrow{1.80 \times 10^2}$ (move decimal point 2 places towards right)
 $= 180$

$$\begin{aligned}
 \text{(iv)} & \overrightarrow{6.21 \times 10^4} \text{ (move decimal point 4 places towards right)} \\
 &= 62100
 \end{aligned}$$

Multiple Choice Question 6

1. A
2. C
3. A
4. D
5. B



ALGEBRA

POLYNOMIALS

Exercise 7A

1. (i) 1, 4, 7, 10, 13, ...

$$T_1 = 1, d = 4 - 1 = 3$$

$$n^{\text{th}} \text{ term} = T_1 + (n - 1)d$$

$$T_n = 1 + (n - 1)3$$

$$= 1 + 3n - 3$$

$$T_n = 3n - 2$$

Helpful Hint

T_1 is the first term of the number sequence and d is the common difference

- (ii) 2, 6, 10, 14, 18, ...

$$T_1 = 2$$

$$d = 6 - 2 = 4$$

$$T_n = T_1 + (n - 1)d$$

$$= 2 + (n - 1)4$$

$$= 2 + 4n - 4$$

$$T_n = 4n - 2$$

- (iii) -4, 0, 4, 8, 12, ...

$$T_1 = -4$$

$$d = 0 - (-4) = 0 + 4 = 4$$

$$T_n = T_1 + (n - 1)d$$

$$= -4 + (n - 1)4$$

$$= -4 + 4n - 4$$

- (iv) 26, 20, 14, 8, 2, ...

$$T_1 = 26, d = 20 - 26 = -6$$

$$T_n = 26 + (n - 1)(-6)$$

$$= 26 - 6n + 6$$

$$= 32 - 6n$$

2. (i) 13, 16, 19, ...

$$T_n = T_1 + (n - 1)d$$

$$T_n = 13 + (n - 1)3$$

$$= 13 + 3n - 3$$

$$T_n = 3n + 4$$

(ii) $T_{10} = 3(10) + 4$

$$= 30 + 4$$

$$T_{10} = 34$$

Helpful Hint

To find 10th term substitute $n = 10$ in the n^{th} term formula.

3. 1, 5, 9, 13, 17, ...

$$T_1 = 1, d = 5 - 1 = 4$$

(i) $T_n = T_1 + (n - 1)d$

$$= 1 + (n - 1)4$$

$$= 1 + 4n - 4$$

$$T_n = 4n - 3$$

(ii) $T_{10} = 4(10) - 3$

$$= 40 - 3 = 37$$

- (iii) Keep 95 equal to the n^{th} term

$$4n - 3 = 95$$

Now find the value of n .

$$4n = 95 + 3$$

$$n = \frac{98}{4} = 24.5$$

Since, the value of n is not a natural number hence, 95 will not be a term of this sequence.

Helpful Hint

Here ' n ' represents natural numbers.

4. (i) $T_n = 5n + 2$

$$\text{1st term } T_1 = 5(1) + 2 = 5 + 2 = 7$$

$$\text{2nd term } T_2 = 5(2) + 2 = 10 + 2 = 12$$

$$\text{3rd term } T_3 = 5(3) + 2 = 15 + 2 = 17$$

- (ii) 5, 11, 17, 23, 29, ...

$$T_1 = 5, d = 11 - 5 = 6$$

$$T_n = T_1 + (n - 1)d$$

$$= 5 + (n - 1)6$$

$$= 6n - 1$$

5. (i) 2, 9, 16, 23, 30,
 $T_1 = 2, d = 9 - 2 = 7$
 $T_n = 2 + (n - 1) 7$
 $= 2 + 7n - 7$
 $= 7n - 5$

(ii) $T_{100} = 7(100) - 5$
 $= 700 - 5$
 $T_{100} = 695$

6. (i) 10, 7, 4, 1, -2
 $T_1 = 10, d = 7 - 10 = -3$
 $T_n = 10 + (n - 1)(-3)$
 $= 10 - 3n + 3$

$T_n = 13 - 3n$
(ii) $T_{50} = 13 - 3(50)$
 $= 13 - 150$
 $T_{50} = -137$

7. 12, 22, 32, 42, 52,
 $T_1 = 12, d = 22 - 12 = 10$
 $T_n = 12 + (n - 1) 10$
 $= 12 + 10n - 10$
 $T_n = 2 + 10n$

8. (i) $T_n = 3n - 2$
1st term $T_1 = 3(1) - 2$
 $= 3 - 2 = 1$
2nd term $T_2 = 3(2) - 2$
 $= 6 - 2 = 4$
(ii) $3n - 2 = 70$
 $3n = 70 + 2$
 $n = \frac{72}{3}$
 $n = 24$
70 is the 24th term of this sequence.

(iii) $3n - 2 = 101$
 $3n = 101 + 2$
 $n = \frac{103}{3}$
 $n = 34.33$

Since, n is not a natural number hence, 111 is not a term of the given sequence.

9. $T_n = 5 - 3n$
1st term $= T_1 = 5 - 3(1)$
 $= 5 - 3 = 2$

2nd term $= T_2 = 5 - 3(2)$
 $= 5 - 6 = -1$

3rd term $= T_3 = 5 - 3(3)$
 $= 5 - 9 = -4$

10. $T_n = 4n - 7$

(i) $T_1 = 4(1) - 7 = 4 - 7 = -3$
 $T_2 = 4(2) - 7 = 8 - 7 = 1$
 $T_3 = 4(3) - 7 = 12 - 7 = 5$

(ii) $T_{50} = 4(50) - 7 = 200 - 7 = 193$
 $T_{51} = 4(51) - 7 = 204 - 7 = 197$
Difference between 50th and 51st terms is,

$T_{51} - T_{50} = 197 - 193 = 4$

(iii) Last term = 393

$T_n = 393$
 $4n - 7 = 393$
 $4n = 393 + 7$

$n = \frac{400}{4}$
 $n = 100$

There are 100 terms in this sequence.

11. 30, 25, 20, 15, 10,
 $T_1 = 30, d = 25 - 30 = -5$
 $T_n = T_1 + (n - 1)d$
 $= 30 + (n - 1)(-5)$
 $= 30 - 5n + 5$
 $= 35 - 5n$

12. 2, 2.5, 3, 3.5, 4,
(i) $T_1 = 2, d = 2.5 - 2 = 0.5$
 $T_n = T_1 + (n - 1)d$
 $= 2 + (n - 1)(0.5)$
 $= 2 + 0.5n - 0.5$
 $= 0.5n + 1.5$

(ii) $T_{20} = 0.5(20) + 1.5$
 $= 10 + 1.5$
 $= 11.5$

13.

(i) $\begin{array}{|c|} \hline 22 \\ \hline \end{array}$ $\begin{array}{|c|} \hline 26 \\ \hline \end{array}$
 (5) (6)

(ii)

Fig #	# of dots
1	6
2	10
3	14
4	18

(iii) $T_n = 4n + 2$
 $T_{100} = 4(100) + 2$
 $= 400 + 2$
 $T_{100} = 402$

difference = 4 : first term = 6

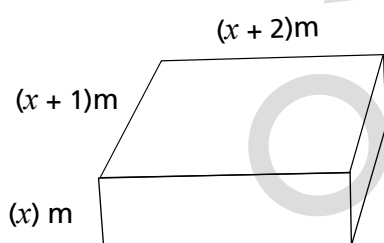
So, $T_n = T_1 + (n - 1)9$
 $= 6 + (n - 1)4$
 $= 6 + 4n - 4$
 $= 4n + 2$

Exercise 7B

- $(m^2 + mn + n^2)(m^2 + n)$
 $= m^2(m^2 + n) + mn(m^2 + n) + n^2(m^2 + n)$
 $= m^4 + m^2n + m^3n + mn^2 + m^2n^2 + n^3$
 or $m^4 + m^3n + m^2n + m^2n^2 + mn^2 + n^3$
- $(ab + 1)(5a^2 - 2ab + 3b^2)$
 $= ab(5a^2 - 2ab + 3b^2) + 1(5a^2 - 2ab + 3b^2)$
 $= 5a^3b - 2a^2b^2 + 3ab^3 + 5a^2 - 2ab + 3b^2$
- $(a^4 - a^2 + 1)(a^4 + a^2 - 1)$
 $= a^4(a^4 + a^3 - 1) - a^2(a^4 + a^3 - 1) + 1(a^4 + a^3 - 1)$
 $= a^8 + a^7 - a^4 - a^6 - a^5 + a + a^4 + a^3 - 1$
 $= a^8 + a^7 - a^6 - a^5 + a^3 + a - 1$
- $(px^2 + qx + r)(ax^2 + bx + c)$
 $= px^2(ax^2 + bx + c) + qx(ax^2 + bx + c) + r(ax^2 + bx + c)$
 $= apx^4 + pbx^3 + px^2c + aqx^3 + bqx^2 + qxc + arx^2 + brx + rc$
 $= apx^4 + (pb + aq)x^3 + (bc + pq + qr)x^2 + (qc + br)x + rc$
- $(a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$
 $= a(a^2 + b^2 + c^2 - ab - bc - ca) + b(a^2 + b^2 + c^2 - ab - bc - ca) + c(a^2 + b^2 + c^2 - ab - bc - ca)$
 $= a^3 + ab^2 + ac^2 - a^2b - abc - ca^2 + a^2b + b^3 + bc^2 - ab^2 - b^2c - abc + a^2c + b^2c + c^3 - abc - bc^2 - c^2a$
 $= a^3 + b^3 + c^3 - 3abc$
- $(x^3 - x^2 + x - 1)(1 + x + x^2 + x^3)$
 $= x^3(1 + x + x^2 + x^3) - x^2(1 + x + x^2 + x^3) + x(1 + x + x^2 + x^3) - 1(1 + x + x^2 + x^3)$
 $= x^3 + x^4 + x^5 + x^6 - x^2 - x^3 - x^4 - x^5 + x + x^2 + x^3 + x^4 - 1 - x - x^2 - x^3$
 $= x^6 + x^4 - x^2 - 1$

7. $(m+1)(m-2)(m+3)$
 $= \{m(m-2) + 1(m-2)\}(m+3)$
 $= (m^2 - 2m + m - 2)(m+3)$
 $= (m^2 - m - 2)(m+3)$
 $= m(m^2 - m - 2) + 3(m^2 - m - 2)$
 $= m^3 - m^2 - 2m + 3m^2 - 3m - 6$
 $= m^3 + 2m^2 - 5m - 6$
8. $(x+y)(x^2-xy+y^2)(x^3-y^3)$
9. $(m^2+mn+n^2)(m^2-mn+n^2)(m^4-m^2n^2+n^4)$
 $= \{m^2(m^2-mn+n^2) + mn(m^2-mn+n^2) + n^2(m^2-mn+n^2)\}(m^4-m^2n^2+n^4)$
 $= (m^4 - m^3n + m^2n^2 + m^3n - m^2n^2 + mn^3 + m^2n^2 - mn^3 + n^4)(m^4-m^2n^2+n^4)$
 $= (m^4 + m^2n^2 + n^4)(m^4-m^2n^2+n^4)$
 $= \{m^4(m^4-m^2n^2+n^4) + m^2n^2(m^4-m^2n^2+n^4) + n^4(m^4-m^2n^2+n^4)\}$
 $= m^8 - m^6n^2 + m^4n^4 + m^6n^2 - m^4n^4 + m^2n^6 + m^4n^4 - m^2n^6 + n^8$
 $= m^8 + m^4n^4 + n^8$
10. $(1+x+x^2)(1-x+x^2)(1-x^2+x^4)$
 $= \{1(1-x+x^2) + x(1-x+x^2) + x^2(1-x+x^2)\}(1-x^2+x^4)$
 $= (1 - x + x^2 + x - x^2 + x^3 + x^2 - x^3 + x^4)(1-x^2+x^4)$
 $= (1 + x^2 + x^4)(1-x^2+x^4)$
 $= 1(1-x^2+x^4) + x^2(1-x^2+x^4) + x^4(1-x^2+x^4)$
 $= 1 - x^2 + x^4 + x^2 - x^4 + x^6 + x^4 - x^6 + x^8$
 $= 1 + x^4 + x^8$

11.



Helpful Hint

Volume of the block is equal to the volume of cuboid.

Length of the block = $l = (x+2)m$

Breadth of the block = $b = (x+1)m$

Height of the block = $h = (x)m$

Volume = ?

$$\begin{aligned}\text{Volume} &= l \times b \times h = x(x+1)(x+2) \\ &= [x(x) + x(1)](x+2) = (x^2+x)(x+2) \\ &= (x^2+3x^2+2x)m^3 \\ &= (x^2+3x^2+2x)m^3\end{aligned}$$

Since, $x = 20$

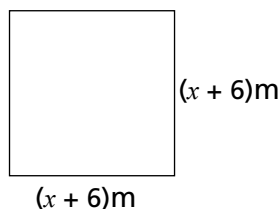
hence,

$$\begin{aligned}\text{Volume} &= (20)^3 + 3(20)^2 + 2(20) \\ &= 8000 + 3 \times 400 + 40 \\ &= 8000 + 1200 + 40 = 9240 \text{ m}^3\end{aligned}$$

12.

Helpful Hint

The area of square pool = the area of a square.



Length of a side of a square = $l = (x + 6)\text{m}$

Area of a square

$$= l^2 = l \times l$$

$$= (x + 6)(x + 6)$$

$$= x(x + 6) + 6(x + 6)$$

$$= x^2 + 6x + 6x + 36 = x^2 + 12x + 36$$

$$\therefore \text{Area of the pool} = (x^2 + 12x + 36) \text{ m}^2$$

13. Price of one doll = Rs $(a + 5)$

Number of dolls = $(a + 6)$

Total price to buy $(a + 6)$ dolls

$$= \text{number of dolls} \times \text{price of one doll}$$

$$= (a + 6)(a + 5)$$

$$= a(a + 5) + 6(a + 5)$$

$$= a^2 + a + 6a + 30$$

$$= a^2 + 11a + 30$$

$\therefore$ Rahila will have to spend Rs $(a^2 + 11a + 30)$ to buy $(a + 6)$ dolls.

14. Number of flowers = $25x^2 + 5x + 5$

Price of one flower = Rs $5x$

Total money at the end of 5 days = ?

Money at the end of each day

$$= \text{price of one flower} \div \text{number of flowers}$$

$$= 5x(25x^2 + 5x + 5)$$

$$= \text{Rs } (125x^3 + 25x^2 + 25x)$$

Total money at the end of 5 days

$$= 5 \times (125x^3 + 25x^2 + 25x)$$

$$= \text{Rs } (625x^3 + 125x^2 + 125x)$$

or = Rs $125x(5x^2 + x + 1)$

15. Length = $l = (2x - 1) \text{ m}$

Width or breadth = $b = (x + 2) \text{ m}$

Area = ?

$$\text{Area} = l \times b = (2x - 1)(x + 2)$$

$$= 2x(x + 2) - 1(x + 2)$$

$$= 2x^2 + 4x - x - 2$$

$$= (2x^2 + 3x - 2) \text{ m}^2$$

16. $l = (2x - 1)\text{m}$

$$b = (x + 2)\text{m}$$

Parking area = ?

Cost of clearing parking area = ?

$$\text{Cost per m}^2 = \text{Rs } 50$$

$$\text{Area} = l \times b$$

$$= (2x - 1)(x + 2)$$

$$= (2x^2 + 3x - 2) \text{ m}^2$$

$$\text{Total Cost} = 50(2x^2 + 3x - 2)$$

$$= \text{Rs } (100x^2 + 150x - 100)$$

17. $l = (2x + 1) \text{ m}$

$$b = (5x) \text{ m}$$

Area = ?

$$\text{Area} = 5x(2x + 1) = (10x^2 + 5x) \text{ m}^2$$

If $x = 3$,

$$\text{Area} = 10(3)^2 + 5(3) = 90 + 15 = 105 \text{ m}^2$$

Since $105 \text{ m}^2 > 100 \text{ m}^2$,

Hence, Hamid can have his garden.

18. Length of first board = $l_1 = (2.5x + 3.4y) \text{ m}$

length of second board = $l_2 = (3.5x + 4.2y) \text{ m}$

$$x = 2$$

$$y = 1$$

Assembling, $x = 2$ and $y = 1$,

$$l_1 = 2.5(2) + 3.4(1) = 5 + 3.4 = 8.4 \text{ m}$$

$$l_2 = 3.5(2) + 4.2 = 7 + 4.2 = 11.2 \text{ m}$$

Since, $11.2 \text{ m} > 8.4 \text{ m}$
Hence, 2nd board is larger.

Exercise 7C

1. (i) distributive (ii) a and b
(iii) 2 (two) (iv) $8x^4 - 3x$
(v) $x^2 - 14x + 45$.

2. (i) False

$$\begin{aligned} & a(a + 2) + 1(a + 2) \\ &= a^2 + 2a + 2 \\ &= a^2 + 3a + 2 \end{aligned}$$

- (ii) True

- (iii) False

$$\begin{aligned} & (3x^2 + 5x)(2x + y) \\ &= 3x^2(2x + y) + 5x(2x + y) \\ &= 6x^3 + 3x^2y + 10x^2 + 5xy \end{aligned}$$

- (iv) True

$$\begin{array}{r} 3m - 2 \xrightarrow{\text{quotient}} \\ m + 6 \overline{) 3m^2 + 16m - 12} \\ \underline{3m^2 + 18m} \\ 0 - 2m - 12 \\ \underline{- 2m - 12} \\ 0 \end{array}$$

- (v) True

$$\begin{array}{r} 5x^2 - 5x + 3 \\ x^2 \overline{) 5x^4 - 5x^3 + 3x^2} \\ \underline{5x^4} \\ 0 - 5x^3 \\ \underline{- 5x^3} \\ 0 + 3x^2 \\ \underline{+ 3x^2} \\ 0 \end{array} \rightarrow \text{no remainder}$$

3.

$$\begin{array}{r} a + b \\ a + b \overline{) a^2 + 2ab + b^2} \\ \underline{a^2 + ab} \\ ab + b^2 \\ \underline{ab + b^2} \\ 0 \end{array}$$

4.

$$\begin{array}{r} a - b \\ a - b \overline{) a^2 - 2ab + b^2} \\ \underline{a^2 - ab} \\ -ab + b^2 \\ \underline{-ab + b^2} \\ 0 \end{array}$$

5.

Helpful Hint

$$\begin{aligned} a^2(a - b) &= a^3 - a^2b, & ab(a - b) &= a^2b - ab^2 \\ b^2(a - b) &= ab^2 - b^3 \end{aligned}$$

$$\begin{array}{r} a^2 + ab + b^2 \\ a - b \overline{) a^3 - b^3} \\ \underline{a^3} \\ -b^3 + a^2b \\ \underline{+ a^2b - ab^2} \\ \underline{- ab^2} \\ \underline{+ ab^2} \\ 0 \end{array}$$

6.

$$\begin{array}{r} a^2 - ab + b^2 \\ a + b \overline{) a^3 + b^3} \\ \underline{a^3} \\ b^3 - a^2b \\ \underline{- a^2b - ab^2} \\ \underline{+ ab^2} \\ \underline{+ ab^2} \\ 0 \end{array}$$

7.

$$\begin{array}{r}
 x^2 - x - 1 \\
 x + 2 \overline{) x^3 + x^2 - 3x - 2} \\
 \underline{x^3 + 2x^2} \\
 -x^2 - 3x \\
 \underline{-x^2 - 2x} \\
 +x \\
 \underline{+x} \\
 -x - 2 \\
 \underline{-x - 2} \\
 0
 \end{array}$$

8.

$$\begin{array}{r}
 8x^2 - 2x - 3 \\
 x - 1 \overline{) 8x^3 - 10x^2 - x + 3} \\
 \underline{8x^3 - 8x^2} \\
 -2x^2 - x + 3
 \end{array}$$

9.

$$\begin{array}{r}
 m^2 - 16 \\
 m^2 - 16 \overline{) m^3 - m^2 - 16m + 16} \\
 \underline{m^3 - m^2} \\
 0 \\
 \underline{0} \\
 0
 \end{array}$$

10.

$$\begin{array}{r}
 a^2 - ab - ac + b^2 + c^2 - bc = a^2 + b^2 + c^2 - ab - bc - ca \\
 a + b + c \overline{) a^3 + b^3 + c^3 - 3abc} \\
 \underline{a^3 + a^2b + a^2c} \\
 -a^2b - a^2c \\
 \underline{-a^2b - a^2c} \\
 0 \\
 \underline{+ b^3 + c^3 - 2abc} \\
 -abc \\
 \underline{-abc} \\
 0 \\
 \underline{+ b^3 + c^3 - abc} \\
 +b^3 \\
 \underline{+b^3} \\
 0 \\
 \underline{+ c^3 - abc} \\
 +c^3 \\
 \underline{+c^3} \\
 0 \\
 \underline{-abc} \\
 -abc \\
 \underline{-abc} \\
 0
 \end{array}$$

0

15.

$$\begin{array}{r}
 a^2 - 2a + 4 \\
 a-2 \overline{) a^3 + 0a^2 + 0a - 8} \\
 \underline{a^3 - 2a^2} \\
 2a^2 - 0a \\
 \underline{2a^2 - 4a} \\
 4a - 8 \\
 \underline{4a - 8} \\
 0
 \end{array}$$

16.

$$\begin{array}{r}
 a^2 + 2ab + b^2 \\
 a+2 \overline{) a^3 + 3a^2b + 3ab^2 + b^3} \\
 \underline{a^3 + a^2b} \\
 2a^2b + 3ab^2 \\
 \underline{2a^2b + 2ab^2} \\
 ab^2 + b^3 \\
 \underline{ab^2 + b^3} \\
 0
 \end{array}$$

17.

$$\begin{array}{r}
 3x^2 - 5xy + y^2 \\
 x-2y \overline{) 3x^3 - 11x^2y + 11xy^2 - 2y^3} \\
 \underline{3x^3 - 6x^2y} \\
 -5x^2y + 11xy^2 \\
 \underline{-5x^2y + 10xy^2} \\
 xy^2 - 2y^3 \\
 \underline{xy^2 - 2y^3} \\
 0
 \end{array}$$

18.

$$\begin{array}{r}
 2x - 3 \\
 3x-4 \overline{) 6x^2 - 17x + 12} \\
 \underline{6x^2 - 8x} \\
 -9x + 12 \\
 \underline{-9x + 12} \\
 0
 \end{array}$$

∴ there are $2x - 3$ pencils in a box.

19.

$$\begin{array}{r}
 a + 6 \\
 a+5 \overline{) a^2 + 11a + 30} \\
 \underline{a^2 + 5a} \\
 6a + 30 \\
 \underline{6a + 30} \\
 0
 \end{array}$$

∴ Sana can buy $a + 6$ dolls in Rs $(a^2 + 11a + 30)$

Multiple Choice Questions 7

1. B

$$\begin{aligned}
 (a+1)(a+2) &= a(a+2) + 1(a+2) \\
 &= a^2 + 2a + a + 2 \\
 &= a^2 + 3a + 2
 \end{aligned}$$

2. A

Helpful Hint

$$(x+5)(x-3)$$

$$\begin{aligned}
 (x+5)(x-3) &= x^2 - 3x + 5x - 15 \\
 &= x^2 + 2x - 15
 \end{aligned}$$

3. D

4. C

Helpful Hint

Divide first term of the dividend with the first term of the divisor.

$$\frac{2x^2}{x} = 2x$$

5. A

$$(x + 1)(x^2 + 2x)$$

$$= x^3 + 2x^2 + x^2 + 2x$$

$$= x^3 + 3x^2 + 2x$$



FACTORISATION OF ALGEBRAIC EXPRESSIONS

Exercise 8A

1. (i) $2a + 1$
 $(2a + 1)^2 = (2a)^2 + 2(2a)(1) + (1)^2$ [using the identity $(a+b)^2 = a^2 + 2ab + b^2$]
 $(2a + 1)^2 = 4a^2 + 4a + 1$
- (ii) $3b + 2c$
 $(3b + 2c)^2 = (3b)^2 + 2(3b)(2c) + (2c)^2$
 $= 9b^2 + 12bc + 4c^2$
- (iii) $2p^2 + 3q^2$
 $(2p^2 + 3q^2)^2 = (2p^2)^2 + 2(2p^2)(3q^2) + (3q^2)^2$
 $= 4p^4 + 12p^2q^2 + 9q^4$

2

Helpful Hint:

All the sums have been solved by applying the identity $(a+b)^2 = a^2 + 2ab + b^2$

- (i) $9p^2 + 12pq + 4q^2$
 $= (3p)^2 + 2(3p)(2q) + (2q)^2$
 $= (3p + 2q)^2$
 $= (3p + 2q)(3p + 2q)$
- (ii) $4x^2 + 4xy + y^2$
 $= (2x)^2 + 2(x)(y) + (y)^2$
 $= (2x + y)^2$
 $= (2x + y)(2x + y)$
- (iii) $25p^2 + 10pq + q^2$
 $= (5p)^2 + 2(5p)(q) + (q)^2$
 $= (5p + q)^2$
 $= (5p + q)(5p + q)$
- (iv) $36x^2 + 24xy + 4y^2$
 $= (6x)^2 + 2(6x)(2y) + (2y)^2$
 $= (6x + 2y)^2$
 $= (6x + 2y)(6x + 2y)$

(v) $x^2 + 2xy + y^2$
 $= (x)^2 + 2(x)(y) + (y)^2$
 $= (x + y)^2$
 $= (x + y)(x + y)$

3.

(i) $16a^2 + 24ab + 9b^2$, When $a = 4$, $b = 3$
 $= (4a)^2 + 2(4a)(3b) + (3b)^2$
 $= (4a + 3b)^2$ [using formula $a^2 + 2ab + b^2 = (a + b)^2$]
 $= (4 \times 4 + 3 \times 3)^2$ [substituting $a = 4$ and $b = 3$]
 $= (16 + 9)^2$
 $= (25)^2$
 $= 625$

(ii) $4m^4 + 12m^2n^2 + 9n^4$, When $m = \frac{1}{2}$ and $n = \frac{1}{3}$
 $= 4m^4 + 12m^2n^2 + 9n^4$
 $= (2m^2)^2 + 2(2m^2)(3n^2) + (3n^2)^2$
[using formula]
 $= (2m^2 + 3n^2)^2$
[Substitute $m = \frac{1}{2}$, $n = \frac{1}{3}$]
 $= \left(2 \times \left(\frac{1}{2}\right)^2 + 3 \times \left(\frac{1}{3}\right)^2\right)^2$
 $= \left(2 \times \frac{1}{4} + 3 \times \frac{1}{9}\right)^2$
 $= \left(\frac{1}{2} + \frac{1}{3}\right)^2$
 $= \left(\frac{3+2}{6}\right)^2$
 $= \left(\frac{5}{6}\right)^2$
 $= \frac{25}{36}$

$$4. \quad p + \frac{1}{p} = 5$$

Squaring both the sides,

$$\left(p + \frac{1}{p}\right)^2 = (5)^2$$

$$(p^2) + 2 \times p \times \frac{1}{p} + \left(\frac{1}{p}\right)^2 = 25$$

$$p^2 + 2 + \frac{1}{p^2} = 25$$

$$p^2 + \frac{1}{p^2} = 25 - 2$$

$$p^2 + \frac{1}{p^2} = 23$$

Hence, the required result is proved.

5. Given that,

$$a + b = 5$$

Squaring both the sides

$$(a + b)^2 = (5)^2$$

$$a^2 + 2ab + b^2 = 25 \quad [\text{using formula}]$$

$$\therefore \text{ we have } ab = 4$$

$$a^2 + 2 \times 4 + b^2 = 25$$

$$a^2 + b^2 = 25 - 8$$

$$a^2 + b^2 = 17$$

$$(a^2 + b^2)^2 = (17)^2$$

Squaring both the sides

$$a^4 + 2a^2b^2 + b^4 = 289 \quad [\text{using formula}]$$

$$a^4 + 2 \times (4)^2 + b^4 = 289$$

$$a^4 + 32 \times b^4 = 289$$

$$a^4 + b^4 = 289 - 32 = 257$$

$$\text{Hence, } a^2 + b^2 = 17 \text{ and } a^4 + b^4 = 257$$

6. Given that,

$$2ab + 5cd = 5 \text{ and } abcd = 1$$

Squaring both the sides

$$(2ab + 5cd)^2 = (5)^2$$

$$(2ab)^2 + 2(2ab)(5cd) + (5cd)^2 = 25,$$

$$4a^2b^2 + 20abcd + 25c^2d^2 = 25$$

$$\therefore \text{ we have } abcd = 1,$$

$$4a^2b^2 + 25c^2d^2 + 20 = 25$$

$$4a^2b^2 + 25c^2d^2 = 25 - 20$$

$$4a^2b^2 + 25c^2d^2 = 5$$

7. (i) 503×503

$$= (503)^2$$

$$= (500 + 3)^2$$

$$= (500)^2 + 2(500)(3) + (3)^2$$

[using identity $(a + b)^2$]

$$= 250000 + 3000 + 9$$

$$= 253009$$

(ii) 1005×1005

$$= (1005)^2$$

$$= (1000 + 5)^2$$

$$= (1000)^2 + 2(1000)(5) + (5)^2$$

$$= 1000000 + 10000 + 25$$

$$= 1010025$$

(iii) 904×904

$$= (904)^2$$

$$= (900 + 4)^2$$

$$= (900)^2 + 2(900)(4) + (4)^2$$

$$= 810000 + 7200 + 16$$

$$= 817216$$

8.

Helpful Hint:

Use identity $a^2 - 2ab + b^2 = (a - b)^2$

(i) $9x^2 - 12xy + 4y^2$

$$= (3x)^2 - 2 \times 3x \times 2y + (2y)^2$$

$$= (3x - 2y)^2$$

(ii) $36a^2 - 24a + 4$

$$= (6a)^2 - 2 \times 6a \times 2 + (2)^2$$

$$= (6a - 2)^2$$

(iii) $16x^2 - 8xy + y^2$

$$= (4x)^2 - 2 \times 4x \times y + (y)^2$$

$$= (4x - y)^2$$

(iv) $4a^2 - 20ab + 25b^2$

$$= (2a)^2 - 2 \times 2a \times 5b + (5b)^2$$

$$= (2a - 5b)^2$$

(v) $x^2 - 6xy + 9y^2$

$$= (x)^2 - 2 \times x \times 3y + (3y)^2$$

$$= (x - 3y)^2$$

9. (i) $25a^2 - 10a + 1$, When $a = \frac{1}{5}$

$$25a^2 - 10a + 1 = (5a)^2 - 2(5a)(1) + (1)^2$$

$$= (5a - 1)^2 \text{ [using formula]}$$

$$= \left(5 \times \frac{1}{5} - 1\right)^2 \text{ [putting } a = \frac{1}{5}]$$

$$= (1 - 1)^2 = 0$$

$$\begin{aligned}
 \text{(ii)} \quad & 4(a+b)^2 - 20(a+b) + 25 \text{ when } a=2, b=1 \\
 &= \{2(a+b)\}^2 - 2 \times 2(a+b) \times 5 + (5)^2 \\
 &= \{2(a+b) - 5\}^2
 \end{aligned}$$

Substitute $a=2, b=1$

$$= (6-5)^2$$

$$\text{(iii)} \quad 36(l+m)^2 - 48n(l+m) + 16n^2,$$

When $l = \frac{1}{2}, m = \frac{1}{3}$ and $n = \frac{1}{4}$

$$\begin{aligned}
 & 36(l+m)^2 - 48n(l+m) + 16n^2 \\
 &= \{6(l+m)\}^2 - 2 \times 6(l+m) \times 4n + (4n)^2 \\
 &= \{6(l+m) - 4n\}^2 \text{ [using formula]} \\
 &\left\{ \text{Substitute } l = \frac{1}{2}, m = \frac{1}{3}, n = \frac{1}{4} \right\} \\
 &= \left\{ 6 \times \left(\frac{1}{2} + \frac{1}{3} \right) - 4 \times \frac{1}{4} \right\}^2 \\
 &= \left\{ 6 \times \frac{5}{6} - 4 \times \frac{1}{4} \right\}^2 \\
 &= (5-1)^2 \\
 &= (4)^2 \\
 &= 16
 \end{aligned}$$

$$\begin{aligned}
 10. \text{ (i)} \quad & 57 \times 57 \\
 &= (60-3)(60-3) \\
 &= (60-3)^2 \\
 &\quad \text{(applying formula)} \\
 &= (60)^2 - 2 \times 60 \times 3 + (3)^2 \\
 &= 3600 - 360 + 9 \\
 &= 3249
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad & 994 \times 994 \\
 &= (1000-6)(1000-6) \\
 &= (1000-6)^2 \\
 &= (1000)^2 - 2 \times 1000 \times 6 + (6)^2 \\
 &= 1000000 - 12000 + 36 \\
 &= 988036
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii)} \quad & 9997 \times 9997 \\
 &= (10000-3)(10000-3) \\
 &= (10000-3)^2 \\
 &= (10000)^2 - 2(10000)(3) + (3)^2 \\
 &= 100000000 - 60000 + 9 \\
 &= 99940009
 \end{aligned}$$

11. Given $a - \frac{1}{a} = 2$ then show that

$$\text{(i)} \quad \text{Show that } a^2 + \frac{1}{a^2} = 6$$

$$a - \frac{1}{a} = 2$$

$$\left(a - \frac{1}{a}\right)^2 = (2)^2 \text{ [squaring both the sides]}$$

$$a^2 - 2 \times a \times \frac{1}{a} + \frac{1}{a^2} = 4$$

$$a^2 + \frac{1}{a^2} = 4 + 2$$

$$a^2 + \frac{1}{a^2} = 6$$

Hence proved.

$$\text{(ii)} \quad a^4 + \frac{1}{a^4} = 34$$

$$a - \frac{1}{a} = 2$$

(squaring both the sides)

$$\left(a - \frac{1}{a}\right)^2 = 4$$

$$a^2 + \frac{1}{a^2} = 6$$

(squaring both the sides)

$$\left(a^2 + \frac{1}{a^2}\right)^2 = (6)^2$$

$$a^4 + 2 + \frac{1}{a^4} = 36$$

$$a^4 + \frac{1}{a^4} = 36 - 2 = 34$$

Hence, proved.

$$\text{(iii)} \quad \left(a + \frac{1}{a}\right)^2 = 8$$

we have

$$\left(a - \frac{1}{a}\right) = 2$$

(squaring both the sides)

$$a^2 - 2 + \frac{1}{a^2} = 4$$

$$a^2 - 2 + \frac{1}{a^2} + 4 = 4 + 4$$

(adding 4 on both the sides)

$$a^2 + 2 + \frac{1}{a^2} = 8$$

$$\left(a + \frac{1}{a}\right)^2 = 8$$

Hence, proved.

Helpful Hint:Use the identity $(a + b)(a - b) = a^2 - b^2$

$$\begin{aligned}
 12. \quad (i) \quad & (a + 1)(a - 1)(a^2 + 1) \\
 &= \{(a)^2 - (1)^2\}(a^2 + 1) \text{ [using identity]} \\
 &= (a^2 - 1)(a^2 + 1) \\
 &= (a^2)^2 - (1)^2 \\
 &= a^4 - 1
 \end{aligned}$$

$$\begin{aligned}
 (ii) \quad &= (a + b)(a - b)(a^2 + b^2)(a^4 + b^4) \\
 &= (a^2 - b^2)(a^2 + b^2)(a^4 + b^4) \\
 &= (a^4 - b^4)(a^4 + b^4) \text{ [using identity]} \\
 &= a^8 - b^8
 \end{aligned}$$

$$\begin{aligned}
 (iii) \quad &= (2p + 3q)(2p - 3q)(4p^2 + 9q^2) \\
 &\quad (16p^2 + 81q^2) \\
 &= (4p^2 - 9q^2)(4p^2 + 9q^2)(16p^4 + 81q^4) \\
 &= (16p^4 - 81q^4)(16p^4 + 81q^4) \\
 &= 256p^8 - 6561q^8
 \end{aligned}$$

$$\begin{aligned}
 13. \quad &81p^2 - 49q^2 \\
 &81p^2 - 49q^2 = (9p - 7q)(9p + 7q) \\
 &\text{using identity } a^2 - b^2 = (a - b)(a + b)
 \end{aligned}$$

$$\begin{aligned}
 14. \quad (i) \quad &(3a + b)(3a - b) \\
 &= (3a)^2 - (b)^2 \\
 &= 9a^2 - b^2
 \end{aligned}$$

$$\begin{aligned}
 (ii) \quad &(5a + 3b)(5a - 3b) \\
 &= (5a)^2 - (3b)^2 \\
 &= 25a^2 - 9b^2
 \end{aligned}$$

$$\begin{aligned}
 (iii) \quad &(2x + 3y)(2x - 3y) \\
 &= (2x)^2 - (3y)^2 \\
 &= 4x^2 - 9y^2
 \end{aligned}$$

Exercise 8B

$$\begin{aligned}
 1. \quad &p^2q + pq^2 \\
 &= pq(p + q) \\
 &\text{Taking out common factors.}
 \end{aligned}$$

$$\begin{aligned}
 2. \quad &3x^3 - 15x^2y \\
 &= 3(x^3 - 5x^2y) \\
 &= 3x^2(x - 5y)
 \end{aligned}$$

$$\begin{aligned}
 3. \quad &45y^4 - 9xy^3 \\
 &= 9(5y^4 - xy^3) \\
 &= 9y^3(5y - x)
 \end{aligned}$$

$$\begin{aligned}
 4. \quad &6p^2q + 12pq^2 \\
 &= 6(p^2q + 2pq^2) \\
 &= 6pq(p + 2q)
 \end{aligned}$$

$$\begin{aligned}
 5. \quad &5a^4b^2 + 15a^2b^4 \\
 &= 5(a^4b^2 + 3a^2b^4) \\
 &= 5a^2b^2(a^2 + 3b^2)
 \end{aligned}$$

$$\begin{aligned}
 6. \quad &xy + x + y + 1 \\
 &= (xy + x) + (y + 1)
 \end{aligned}$$

Helpful Hint:

Grouping the terms

$$\begin{aligned}
 &= x(y + 1) + (y + 1) \\
 &= (x + 1)(y + 1)
 \end{aligned}$$

$$\begin{aligned}
 7. \quad &am^3 - am^2 - m + 1 \\
 &= am^2(m - 1) - (m - 1) \\
 &= (m - 1)(am^2 - 1)
 \end{aligned}$$

$$8. \quad 4a^2 + 12ab + 9b^2$$

Helpful Hint:Using formula $a^2 + 2ab + b^2 = (a + b)^2$

$$\begin{aligned}
 &= (2a)^2 + 2(2a)(3b) + (3b)^2 \\
 &= (2a + 3b)^2 \\
 &= (2a + 3b)(2a + 3b)
 \end{aligned}$$

$$\begin{aligned}
 9. \quad &x^2 - (p + q)x + pq \\
 &= x^2 - px - qx + pq \\
 &= x(x - p) - q(x - p) \\
 &= (x - p)(x - q)
 \end{aligned}$$

$$10. \quad abc - ab - c + 1$$

Helpful Hint:Taking ab common, from the first two terms.

$$= ab(c - 1) - (c - 1)$$

Taking $(c - 1)$ common,

$$= (c - 1)(ab - 1)$$

$$11. \quad (a + b)(p + q + r) + (b + c)(p + q + r) + (c + a)(p + q + r)$$

Taking $(p + q + r)$ common, we get

$$= (p + q + r)(a + b + b + c + c + a)$$

$$= (p + q + r)(2a + 2b + 2c)$$

$$= 2(a + b + c)(p + q + r)$$

Exercise 8C

Helpful Hint:

$$a^2 - b^2 = (a + b)(a - b)$$

- $9x^2 - 25y^2$
 $= (3x + 5y)(3x - 5y)$
- $1 - 9c^2$
 $= (1 - 3c)(1 + 3c)$
- $m^2n^2 - p^2$
 $= (mn)^2 - (p)^2$
 $= (mn - p)(mn + p)$
- $1 - (p + q)^2$
 $= (1)^2 - (p + q)^2$
 $= (1 - p - q)(1 + p + q)$
- $(x + y)^2 - (x - y)^2$
 $= (x + y + x - y)(x + y - x + y)$
 $= (2y)(2x)$
 $= 4xy$
- $9x^2 - (2x - 3y)^2$
 $= (3x)^2 - (2x - 3y)^2$
 $= (3x - 2x + 3y)(3x + 2x - 3y)$
 $= (x + 3y)(5x - 3y)$
- $(a + b - c)^2 - (a - b + c)^2$
 $= (a + b - c + a - b + c)(a + b - c - a + b - c)$
 $= (2a)(2b - 2c)$
 $= 2(2a)(b - c)$
 $= 4a(b - c)$
- $3a(3a - 2b) + b^2 - c^2$
 $= 9a^2 - 6ab + b^2 - c^2$
 $= (3a - b)^2 - (c)^2$
 $= (3a - b + c)(3a - b - c)$
- $9(p - q)^2 - 25(q - r)^2$
 $= \{3(p - q)\}^2 - \{5(q - r)\}^2$
 $= \{3(p - q) + 5(q - r)\} \{3(p - q) - 5(q - r)\}$
 $= (3p - 3q + 5q - 5r)(3p - 3q - 5q + 5r)$
 $= (3p + 2q - 5r)(3p - 8q + 5r)$
- $a^8 - b^8$

Helpful Hint:

$$a^2 - b^2 = (a - b)(a + b)$$

$$= (a^4)^2 - (b^4)^2$$

$$= (a^4 - b^4)(a^4 + b^4)$$

$$= (a^2 - b^2)(a^2 + b^2)(a^4 + b^4)$$

$$= (a - b)(a + b)(a^2 + b^2)(a^4 + b^4)$$

$$11. a^2 - 1 + 2b - b^2$$

$$= a^2 - (b^2 - 2b + 1)$$

$$= a^2 - (b - 1)^2$$

Helpful Hint:

$$(using a^2 - 2ab + b^2 = (a - b)^2)$$

$$= (a - b + 1)(a + b - 1)$$

$$12. 4x^4 + 81 = (2x^2)^2 + (9)^2$$

[completing the square].

$$= (2x^2)^2 + 2 \times 2x^2 \times 9 + (9)^2 - 2 \times 2x^2 \times 9$$

$$= (2x^2 + 9)^2 - 36x^2$$

$$= (2x^2 + 9)^2 - (6x)^2$$

(using $a^2 - b^2 = (a + b)(a - b)$)

$$= (2x^2 + 9 + 6x)(2x^2 + 9 - 6x)$$

$$= (2x^2 + 6x + 9)(2x^2 - 6x + 9)$$

$$13. 9x^4 - 3x^2 + 1$$

Helpful Hint:

Split the middle term to complete the square.

$$9x^4 - 3x^2 + 1$$

$$\begin{array}{c} \swarrow \quad \searrow \\ -9x^2 \quad +6x^2 \end{array}$$

Helpful Hint:

$$-9x^2 + 6x^2 = -3x^2$$

$$= 9x^4 - 9x^2 + 6x^2 + 1$$

[by rearranging the terms]

$$= 9x^4 + 6x^2 + 1 - 9x^2$$

$$= (9x^4 + 6x^2 + 1) - 9x^2$$

[By grouping]

$$= (3x^2 + 1)^2 - 9x^2$$

$$= (3x + 1)^2 - (3x)^2$$

$$= (3x^2 + 1 + 3x)(3x^2 + 1 - 3x)$$

$$= (3x^2 + 3x + 1)(3x^2 - 3x + 1)$$

14. $x^4 + x^2 + 1$

Splitting the middle term:

Helpful Hint:

$$2x^2 - x^2 = x^2$$

$$\begin{aligned} & x^4 + x^2 + 1 \\ = & \begin{array}{c} x^4 + x^2 + 1 \\ \swarrow \quad \searrow \\ 2x^2 \quad -x^2 \end{array} \\ = & x^4 + 2x^2 - x^2 + 1 \\ = & x^4 + 2x^2 + 1 - x^2 \text{ (rearranging the terms)} \\ = & (x^4 + 2x^2 + 1) - x^2 \text{ (grouping the terms)} \\ = & (x^2 + 1)^2 - (x)^2 \\ \therefore & a^2 + 2ab + b^2 = (a + b)^2 \\ = & (x^2 + x + 1)(x^2 - x + 1) \end{aligned}$$

15. $x^4 + 3x^2y^2 + 4y^4$

$$= x^4 + 3x^2y^2 + 4y^4$$

[splitting the middle term]

$$\begin{array}{c} 4x^2y^2 \quad -x^2y^2 \end{array}$$

Helpful Hint:

(using $3x^2y^2 = 4x^2y^2 - x^2y^2$)

$$\begin{aligned} & = x^4 + 4x^2y^2 - x^2y^2 + 4y^4 \\ & = x^4 + 4x^2y^2 + 4y^4 - x^2y^2 \\ & \quad \text{(rearranging the terms)} \\ & = (x^4 + 4x^2y^2 + 4y^2) - x^2y^2 \\ & \quad \text{(grouping the terms)} \\ & = (x^2 + 2y^2)^2 - (xy)^2 \\ & = (x^2 + 2y^2 + xy)(x^2 + 2y^2 - xy) \\ & = (x^2 + xy + 2y^2)(x^2 - xy + 2y^2) \end{aligned}$$

16. $(a + b)^4 + 4$

$$\begin{aligned} & = \text{Let } a + b = x, \text{ then} \\ & = (a + b)^4 + 4 = x^4 + 4 \\ & = (x^2)^2 + (2)^2 \\ & = (x^2)^2 + 4x^2 + (2)^2 - 4x^2 \end{aligned}$$

Helpful Hint:

Add $4x^2$ to make a complete square and subtract the same.

$$\begin{aligned} & = (x^2 + 2)^2 - (2x)^2 \\ & = (x^2 + 2x + 2)(x^2 - 2x + 2) \end{aligned}$$

[Substituting $x = a + b$]

$$= \{(a + b)^2 + 2(a + b) + 2\}$$

17. $x^4 + 8x^2 + 144$

$$x^4 + 8x^2 + 144$$

$$\begin{array}{c} 24x^2 \quad -16x^2 \end{array}$$

$$= x^4 + 24x^2 + 144 - 16x^2$$

$$= (x^2)^2 + 24x^2 + (12)^2 - 16x^2$$

[completing the square]

$$= (x^2 + 12)^2 - 16x^2$$

$$= (x^2 + 12)^2 - (4x)^2$$

$$= (x^2 + 12 + 4x)(x^2 + 12 - 4x)$$

$$= (x^2 + 4x + 12)(x^2 - 4x + 12)$$

18. $3a^4 - 18a^2b^2 + 3b^4$

$\therefore 3$ is a common factor.

$$= 3(a^4 - 6a^2b^2 + b^4)$$

$$= 3(a^4 - 2a^2b^2 - 4a^2b^2 - b^4)$$

[breaking the middle term to obtain a perfect square]

$$= 3(a^4 - 2a^2b^2 + b^4 - 4a^2b^2)$$

[rearranging the terms]

$$= 3\{(a^4 - 2a^2b^2 + b^4) - 4a^2b^2\}$$

$$\therefore a^2 - 2ab + b^2 = (a - b)^2$$

$$= 3\{(a^2 - b^2)^2 - (2ab)^2\}$$

$$= 3\{a^2 - b^2 + 2ab\}(a^2 - b^2 - 2ab)$$

$$= 3(a^2 + 2ab - b^2)(a^2 - 2ab - b^2)$$

19. $9a^2 - 4b^2 + 16c^2 - 1 - 4b - 24ac$

$$= (9a^2 - 24ac + 16c^2) - (4b^2 + 4b + 1)$$

[rearranging and grouping the terms]

$$= \{(3a)^2 - 2 \times 3a \times 4c + (4c)^2\} - \{(2b)^2 + 2 \times 2b \times 1 + (1)^2\}$$

$$= (3a - 4c)^2 - (2b + 1)^2$$

$$= (3a - 4c + 2b + 1)(3a - 4c - 2b - 1)$$

$$= (3a + 2b - 4c + 1)(3a - 2b - 4c - 1)$$

20. Option A is correct.

21. Option C is correct.

22. Option D is correct.

23. Option D is correct.

24. Option A is correct.

Exercise 8D

1. (i) $(x + 3)^3 = ?$

Helpful Hint

$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$\begin{aligned}(x + 3)^3 &= x^3 + 3x^2(3) + 3x(3)^2 + 3^3 \\ &= x^3 + 9x^2 + 27x + 27\end{aligned}$$

(ii) $(a + 4b)^3$
 $= a^3 + 3a^2(4b) + 3a(4b)^2 + (4b)^3$
 $= a^3 + 12a^2b + 48ab^2 + 64b^3$

(iii) $(3x + 2y)^3$
 $= (3x)^3 + 3(3x)^2(2y) + 3(3x)(2y)^2 + (2y)^3$
 $= 27x^3 + 54x^2y + 36xy^2 + 8y^3$

(iv) $(x^2 + 2y)^3$
 $= (x^2)^3 + 3(x^2)^2(2y) + 3x^2(2y)^2 + (2y)^3$
 $= x^6 + 6x^4y + 12x^2y^2 + 8y^3$

(v) $(ax + by)^3$
 $= (ax)^3 + 3(ax)^2(by) + 3(ax)(by)^2 + (by)^3$
 $= a^3x^3 + 3a^2x^2by + 3axb^2y^2 + b^3y^3$

(vi) $(a^2 + bc)^3$
 $= (a^2)^3 + 3(a^2)^2(bc) + 3(a^2)(bc)^2 + (bc)^3$
 $= a^6 + 3a^4bc + 3a^2b^2c^2 + b^3c^3$

(vii) $(a^2 + b^2)^3$
 $= (a^2)^3 + 3(a^2)^2(b^2) + 3(a^2)(b^2)^2 + (b^2)^3$
 $= a^6 + 3a^4b^2 + 3a^2b^4 + b^6$

(viii) $42^3 = (40 + 2)^3$
 $= (40)^3 + 3(40)^2(2) + 3(40)(2)^2 + (2)^3$
 $= 64000 + 9600 + 480 + 8$
 $= 74088$

(ix) $(105)^3 = (100 + 5)^3$
 $= (100)^3 + 3(100)^2(5) + 3(100)(5)^2 + (5)^3$
 $= 1000000 + 150000 + 7500 + 125$
 $= 1157625$

2. (i) $a^3 + 9a^2 + 27a + 30$
 Substitute $a = 3$
 $(3)^3 + 9(3)^2 + 27(3) + 30$
 $= 27 + 81 + 81 + 30$
 $= 219$

(ii) $(31)^3 + 3 \times (31)^2 \times 19 + 3 \times 31 \times (19)^2 + (19)^3$
 $= 29791 + 3 \times 961 \times 19 + 93 \times 361 + 6859$
 $= 29791 + 54777 + 33573 + 6859$
 $= 125000$

3. $x^3 + y^3 + 24xy = ?$
 Given that $x + y = 8$
 [Cubing both the sides]
 $(x + y)^3 = 8^3$
 by using formula,
 $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$
 $(x + y)^3 = 8^3$
 $x^3 + y^3 + 3xy(x + y) = 512$
 $x^3 + y^3 + 3xy(8) = 512$
 $\therefore x^3 + y^3 + 24xy = 512$

4. $m^3 + n^3 - 9mn = ?$
 Given that $m + n + 3 = 0$
 $m + n = -3$
 [by cubing both the sides]
 $(m + n)^3 = (-3)^3$
 $m^3 + n^3 + 3mn(m + n) = -27$
 $m^3 + n^3 + 3mn(-3) = -27$
 $m^3 + n^3 - 9mn = -27$

5. (i) $(2x + 5)^3$
 $= (2x)^3 + 3(2x)^2(5) + 3(2x)(5)^2 + (5)^3$
 $= 8x^3 + 60x^2 + 150x + 125$
 (ii) $(ax - by)^3$
 $= (ax)^3 - 3(ax)^2(by) + 3(ax)(by)^2 - (by)^3$
 $= a^3x^3 - 3a^2x^2by + 3axb^2y^2 - b^3y^3$
 (iii) $(a^2 - b^2)^3$
 $= (a^2)^3 - 3(a^2)^2b^2 + 3a^2(b^2)^2 - (b^2)^3$
 $= a^6 - 3a^4b^2 + 3a^2b^4 - b^6$
 (iv) $(-7a + 2b^2)^3 = (2b^2 - 7a)^3$
 $= (2b^2)^3 - 3(2b^2)^2(7a) + 3(2b^2)(7a)^2 - (7a)^3$
 $= 8b^6 - 84b^4a + 294b^2a^2 - 343a^3$

$$\begin{aligned}
 \text{(v)} \quad & (2a - 3bc)^3 \\
 &= (2a)^3 - 3(2a)^2 (3bc) + 3(2a) (3bc)^2 - (3bc)^3 \\
 &= 8a^3 - 36a^2bc + 54ab^2c - 27b^3c^3
 \end{aligned}$$

$$\text{(vi)} \quad (1 + a - 2b)^3 = ?$$

Helpful Hint:

In this case make $(1 + a)$ one term and $2b$ the second term.

Let $x = 1 + a$ and $y = 2b$, then

$$x^2 = (1 + a)^2 = 1 + 2a + a^2$$

$$x^3 = (1 + a)^3 = 1 + 3a + 3a^2 + a^3$$

$$y^2 = (2b)^2 = 4b^2$$

$$y^3 = 8b^3$$

$$(1 + a - 2b)^3 = (x - y)^3$$

$$= x^3 - 3x^2y + 3xy^2 - y^3$$

Substitute

$$x = 1 + a, \quad x^2 = 1 + 2a + a^2, \quad x^3 = 1 + 3a + 3a^2 + a^3,$$

$$y = 2b, \quad y^2 = 4b^2 \quad \text{and} \quad y^3 = 8b^3$$

$$\begin{aligned}
 \therefore (1 + a - 2b)^3 &= 1 + 3a + 3a^2 + a^3 - 3(1 + 2a + a^2) 2b + 3(1 + a) (4b^2) - 8b^3 \\
 &= 1 + 3a + 3a^2 + a^3 - 6b - 12ab - 6a^2b + 12b^2 + 12ab^2 - 8b^3
 \end{aligned}$$

$$\text{(vii)} \quad (x^2 - y - z)^3 = (x^2 - (y + z))^3$$

Let $a = x^2$ and $b = y + z$,

then

$$a^2 = x^4, \quad a^3 = x^6$$

$$b^2 = (y + z)^2 = y^2 + 2yz + z^2$$

$$b^3 = (y + z)^3 = y^3 + 3y^2z + 3yz^2 + z^3$$

$$[(a - b)^3 = a^3 - 3a^2b + 3ab^2 + b^3]$$

$$(x^2 - y - z)^3 = x^6 - 3x^4(y + z) + 3x^2(y^2 + 2yz + z^2) - (y^3 + 3y^2z + 3yz^2 + z^3)$$

$$= x^6 - 3x^4y - 3x^4z + 3x^2y^2 + 6x^2yz + 3x^2z^2 - y^3 - y^2z - yz^2 - z^3$$

$$\text{(viii)} \quad (a - 2b - 3c)^3 = ((a - 2b) - 3c)^3$$

Let $x = a - 2b$ and $y = 3c$, then

$$x^2 = a^2 - 4ab + b^2$$

$$x^3 = a^3 - 3a^2(2b) + 3a(2b)^2 - 8b^3$$

$$= a^3 - 6a^2b + 6ab^2 - 8b^3$$

$$y^2 = 9c^2$$

$$y^3 = 27c^3$$

The equation becomes,

$$(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$$

$$\begin{aligned}
 (a - 2b - 3c)^3 &= a^3 - 6a^2b + 6ab^2 - 8b^3 - 3(a^2 - 4ab + b^2) 3c + 3(a - 2b) (9c^2) - 27c^3 \\
 &= a^3 - 6a^2b + 6ab^2 - 8b^3 - 9ca^2 + 36abc - 9b^2c + 27ac^2 - 54bc^2 - 27c^3
 \end{aligned}$$

$$(ix) \quad (p^2 - q^2 - r^2)^3 = ((p^2 - q^2) - r^2)^3$$

Let $x = p^2 - q^2$ and $y = r^2$

$$x^2 = (p^2)^2 - 2(p^2)(q^2) + (q^2)^2 = p^4 - 2p^2q^2 + q^4$$

$$x^3 = (p^2)^3 - 3(p^2)^2(q^2) + 3p^2(q^2)^2 - (q^2)^3 = p^6 - 3p^4q^2 + 3p^2q^4 - q^6$$

$$y^2 = r^4, \quad y^3 = r^6$$

The equation is

$$(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$$

$$(p^2 - q^2 - r^2)^3 = p^6 - 3p^4q^2 + 3p^2q^4 - q^4 - q^6 - 3(p^2 - q^2)r^4 - r^6$$

$$= p^6 - 3p^4q^2 + 3p^2q^4 - q^6 - 3p^2r^2 + 6p^2q^2r^2 - 3q^4r^2 + 3p^2r^4 - 3q^2r^4 - r^6$$

$$(x) \quad (198)^3 = (200 - 2)^3$$

Helpful Hint

$$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

$$= (200)^3 - 3(200)^2(2) + 3(200)(2)^2 - (2)^3$$

$$= 8000000 - 6(40000) + 600(4) - 8$$

$$= 8000000 - 240000 + 2400 - 8$$

$$= 7762392$$

$$(xi) \quad (399)^3 = (400 - 1)^3$$

$$= (400)^3 - 3(400)^2(1) + 3(400)(1)^2 - (1)^3$$

$$= 64000000 - 3(160000) + 1200 - 1$$

$$= 64000000 - 480000 + 1200 - 1$$

$$= 63521199$$

$$(xii) \quad 999 = (1000 - 1)^3$$

$$= (1000)^3 - 3(1000)^2(1) + 3(1)^2(1000) - (1)^3$$

$$= 1000000000 - 3000000 + 3000 - 1$$

$$= 997002999$$

$$6. (i) \quad \text{Let } 51 = a \text{ and } 46 = b$$

So the expression becomes

$$a^3 - 3a^2b + 3ab^2 - b^3$$

and we know that

$$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

$\therefore$ we have

$$51^3 - 3 \times 51^2 \times 46 + 3 \times 46^2 \times 51 - 46^3$$

$$= (51 - 46)^3 = 5^3 = 125$$

$$(ii) \quad \text{Let } 31.6 = a \text{ and } 28.6 = b$$

$$31.6^3 - 3 \times 31.6^2 \times 28.6 + 3 \times 28.6^2 \times 31.6 - 28.6^3$$

(we know that $31.6 \times 31.6 \times 31.6 = 31.6^3$)

$$= (31.6 - 28.6)^3$$

$$= 3^3 = 27$$

(iii) Let $5.83 = a$ and $3.83 = b$

$$\begin{aligned} & 5.83^3 - 3 \times 5.83^2 \times 3.83 + 3 \times 5.83 \times 3.83^2 - 3.83^3 \\ &= (5.83 - 3.83)^3 \\ &= 2^3 = 8 \end{aligned}$$

7. $p = 2q + 4$
 $p - 2q = 4$
 [Cubing both the sides]
 $(p - 2q)^3 = 4^3$
 $p^3 - (2q)^3 - 3(p)(2q)(p - 2q) = 64$
 $p^3 - 8q^3 - 6pq(4) = 64$ [$p - 2q = 4$]
 $\therefore p^3 - 8q^3 - 24pq = 64$

8. $\frac{a^2 - 1}{a} = 1$
 $\frac{a^2}{a} - \frac{1}{a} = 1$
 $a - \frac{1}{a} = 1$
 Cubing both the sides
 $(a - \frac{1}{a})^3 = 1^3$
 $[(a - b)^3 = a^3 - b^3 - 3ab(a - b)]$
 Using above formula we have
 $a^3 - \frac{1}{a^3} - 3(a)(\frac{1}{a})(a - \frac{1}{a}) = 1$
 $a^3 - \frac{1}{a^3} - 3(1) = 1$
 $a^3 - \frac{1}{a^3} - 3 = 1$
 $\frac{a^6 - 1}{a^3} - 4 = 0$

9. $x - y = 4$, $xy = 21$
 $x^3 - y^3 = ?$
 $x - y = 4$ [Cubing both the sides]
 $(x - y)^3 = 4^3$
 $x^3 - y^3 - 3xy(x - y) = 64$
 [substitute $xy = 21$ and $x - y = 4$]
 $x^3 - y^3 - 3(21)(4) = 64$
 $x^3 - y^3 - 252 = 64$
 $x^3 - y^3 = 64 + 252$
 $\therefore x^3 - y^3 = 316$

10. $a^3 - b^3 = ?$
 $a - b = 2$, $a^2 + b^2 = 4$
 $a - b = 2$ square both the sides
 $(a - b)^2 = 2^2$
 $a^2 - 2ab + b^2 = 4$
 $a^2 + b^2 = 4 + 2ab$
 $4 = 4 + 2ab$ [$a^2 + b^2 = 4$]
 $2ab = 4 - 4 = 0$
 $ab = 0$
 Now cube both the sides of
 $a - b = 2$
 $(a - b)^3 = 2^3$
 $a^3 - b^3 - 3ab(a - b) = 8$
 $a^3 - b^3 - 3(0)(2) = 8$ [$ab = 0$, $a - b = 2$]
 $\therefore a^3 - b^3 = 8$

Multiple Choice Questions 8

- Option A is correct.
- Option C is correct.
- Option D is correct.
- Option D is correct.
- Option A is correct.



REVISION: ALGEBRA

1. $(x + y)(x^3 - y^3)$

$$= (x + y)(x - y)(x^2 + xy + y^2)$$

$$= (x^2 - y^2)(x^2 + xy + y^2)$$

2. (i) $(x + 2y)(x - 5y)(x - 9y)$

$$= (x^2 - 5xy + 2xy - 10y^2)(x - 9y)$$

$$= (x^2 - 3xy - 10y^2)(x - 9y)$$

$$= x^3 - 9x^2y - 3x^2y + 27xy^2 - 10xy^2 + 90y^3$$

$$= x^3 - 12x^2y + 17xy^2 + 90y^3$$

(ii) $(a + b + c)(b + c - a)(c + a - b)(a + b - c)$

$$= (ab + ac - a^2 + b^2 + bc - ab + bc + c^2 - ac)(ac + bc - c^2 + a^2 + ab - ac - ab - b^2 + bc)$$

$$= (-a^2 + b^2 + c^2 + 2bc)(a^2 - b^2 - c^2 + 2bc)$$

$$= -a^4 + a^2b^2 + a^2c^2 - 2a^2bc + a^2b - b^4 - b^2c^2 + 2b^3c + a^2c^2 - b^2c^2 - c^4 + 2bc^3 + 2a^2bc - 2b^3c - 2bc^3 + 4b^2c^2$$

$$= 2a^2b^2 + 2b^2c^2 + 2a^2c^2 - a^4 - b^4 - c^4$$

(iii) $(m^4 - 2m^2n^2 + n^4)(m^3 + 2m^2n - n^3)$

$$= m^7 + 2m^6n - m^4n^3 - 2m^5n^2 - 4m^4n^3 + 2m^2n^5 + m^3n^4 + 2m^2n^5 - n^7$$

$$= m^7 + 2m^6n - 5m^4n^3 - 2m^5n^2 + 4m^2n^5 + m^3n^4 - n^7$$

3.

$$\begin{array}{r}
 a^3 + 2a^2 + 7a + 20 \\
 a^2 + 2a - 3 \overline{) \begin{array}{l} a^5 + 0a^4 + 0a^3 + 0a^2 - 61a - 60 \\ a^5 - 2a^4 - 3a^3 \\ \hline 2a^4 + 3a^3 \\ 2a^4 - 4a^3 - 6a^2 \\ \hline 7a^3 + 6a^2 - 61a \\ 7a^3 - 14a^2 - 21a \\ \hline 20a^2 - 40a - 60 \\ 20a^2 - 40a - 60 \\ \hline 0 \end{array} }
 \end{array}$$

$$\begin{aligned}
 4. \quad & (2p + 3q)^3 - 18q(4p^2 - 9q^2) - (2p - 3q)^3 \\
 &= (2p)^3 + 3(2p)^2(3q) + 3(2p)(3q)^2 + (3q)^3 - 72p^2q + 162q^3 - \left\{ (2p)^3 - 3(2p)^2(3q) + 3(2p)(3q)^2 - (3q)^3 \right\} \\
 &= 8p^3 + 36p^2q + 54pq^2 + 27q^3 - 72p^2q + 162q^3 - 8p^3 + 36p^2q - 54pq^2 + 27q^3 \\
 &= 216q^3
 \end{aligned}$$

$$\begin{aligned}
 5. \quad & x - y = 6 \\
 & x^3 - y^3 - 18xy = ? \\
 &= (x - y)(x^2 + xy + y^2) - 18xy \quad [a^3 - b^3 = (a - b)(a^2 + ab + b^2)] \\
 &= 6(x^2 + xy + y^2) - 18xy \\
 &= 6x^2 + 6xy + 6y^2 - 18xy \\
 &= 6x^2 - 12xy + 6y^2 \\
 &= 6(x^2 - 2xy + y^2) \\
 &= 6(x - y)^2 = 6 \times 6^2 \quad (x - y = 6) \\
 &= 6 \times 36 = 216
 \end{aligned}$$

$$\begin{aligned}
 6. \quad & \text{Since } 8x^3 - 12x^2 + 6x + 5x = \frac{1}{2} \\
 &= 8\left(\frac{1}{2}\right)^3 - 12\left(\frac{1}{2}\right)^2 + 6\left(\frac{1}{2}\right) + 5 \\
 &= 8 \times \frac{1}{8} - 12 \times \frac{1}{4} + 6 \times \frac{1}{2} + 5 \\
 &= 1 - 3 + 3 + 5 \\
 &= 6
 \end{aligned}$$

$$\begin{aligned}
 7. \quad & (x^2 + 1)(x^4 - x^2 + 1) \\
 &= x^6 - x^4 + x^2 + x^4 - x^2 + 1 \\
 &= x^6 + 1
 \end{aligned}$$

$$\begin{aligned}
 8. \quad & (x + 1)(x^2 - x + 1) + (2x - 1)(4x^2 + 2x + 1) - (x - 1)(x^2 + x + 1) \\
 &= x^3 + 13 + (2x)^3 - (1)^3 - (x^3 - 3) \quad [(a + b)(a^2 - ab + b) = a^3 + b^3] \\
 &= x^3 + 1 + 8x^3 - 1 - x^3 + 1 \\
 &= 8x^3 + 1
 \end{aligned}$$

$$\begin{aligned}
 9. \quad & \text{(i) } a^3 + 27 \\
 &= a^3 + (3)^3 \\
 &= (a + 3)(a^2 - 3a + 9) \quad [(a^3 + b^3) = (a + b)(a^2 - ab + b^2)] \\
 & \text{(ii) } 8a^3 + 12a^2 + 6a + 2 \\
 &= 8a^3 + 12a^2 + 6a + 1 + 1 \\
 &= (2a + 1)^3 + 1 \quad [a^3 + b^3 = (a + b)(a^2 - ab + b^2)]
 \end{aligned}$$

$$\begin{aligned}
 &= (2a + 1 + 1) \{ (2a + 1)^2 - (2a + 1)(1) + 2 \} \\
 &= (2a + 2)(4a^2 + 4a + 1 - 2a - 1 + 1) \\
 &= 2(a + 1)(4a^2 + 2a + 1)
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii)} \quad &a^3 - 3a^2b + 3ab^2 - b^3 + c^3 \\
 &= (a - b)^3 + c^3 \\
 &= (a - b + c) \{ (a - b)^2 - (a - b)(c) + c^2 \} \\
 &= (a - b + c)(a^2 - 2ab + b^2 - ac + bc + c^2) \\
 &= (a - b + c)(a^2 + b^2 + c^2 - 2ab + bc - ac)
 \end{aligned}$$

$$\begin{aligned}
 \text{(iv)} \quad &x^2 - 7x + 10 \\
 &= x^2 - (5 + 2)x + 10 \\
 &= x^2 - 5x - 2x + 10 \\
 &= x(x - 5) - 2(x - 5) \\
 &= (x - 5)(x - 2)
 \end{aligned}$$

$$\begin{aligned}
 \text{(v)} \quad &a^2 + ab - 6b^2 \\
 &= a^2 - (3 - 2)ab - 6b^2 \\
 &= a^2 - 3ab + 2ab - 6b^2 \\
 &= a(a - 3b) + 2b(a - 3b) \\
 &= (a - 3b)(a - 2b)
 \end{aligned}$$

$$\begin{aligned}
 10. \text{ (i)} \quad &\frac{x-6}{x-8} + \frac{x-2}{x-4} = \frac{x-5}{x-7} + \frac{x-3}{x-5} \\
 &\frac{(x-6)(x-4) + (x-2)(x-8)}{(x-8)(x-4)} = \frac{(x-5)(x-5) + (x-3)(x-7)}{(x-7)(x-5)} \\
 &\frac{x^2 - 10x + 24 + x^2 - 10x + 16}{x^2 - 12x + 32} = \frac{x^2 - 10x + 25 + x^2 - 10x + 21}{x^2 - 12x + 35} \\
 &(2x^2 - 20x + 40)(x^2 - 12x + 35) = (2x^2 - 20x + 46)(x^2 - 12x + 32) \\
 &\cancel{2x^4} - \cancel{24x^3} + 70x^2 - \cancel{20x^3} + \cancel{240x^2} - 700x + 40x^2 - 480x + 1400 \\
 &= \cancel{2x^4} - \cancel{24x^3} + 64x^2 - \cancel{20x^3} + \cancel{240x^2} - 640x + 46x^2 - 552x + 1472 \\
 &\cancel{110x^2} - 1180x + 1400 = \cancel{110x^2} - 1192x + 1472 \\
 &1192x - 1180x = 1472 - 1400 \\
 &12x = 72 \\
 &x = \frac{72}{12} \\
 &x = 6
 \end{aligned}$$

$$(ii) \frac{a}{x+a} + \frac{b}{x+a} = 2$$

$$a(x+b) + b(x+a) = 2(x+a)(x+6)$$

$$ax + ab + bx + ab = 2(x^2 + bx + ax + ab)$$

$$ax + 2ab + bx = 2x^2 + 2bx + 2ax + 2ab$$

$$2x^2 + 2bx - bx + 2ax - ax + 2ab - 2ab = 0$$

$$2x^2 + bx + ax = 0$$

$$x(2x + b + a) = 0$$

$$2x + b + a = 0$$

$$2x = -(a+b)$$

$$x = -\frac{(a+b)}{2}$$

$$(iii) \frac{1}{(x-3)(x-4)} = \frac{1}{(x-5)(x-6)}$$

$$(x-5)(x-6) = (x-3)(x-4)$$

$$x^2 - 11x + 30 = x^2 - 7x + 12$$

$$11x - 7x = 30 - 12$$

$$4x = 18$$

$$x = \frac{18}{4}$$

$$x = \frac{9}{2}$$

$$\begin{aligned} 11. (i) \quad & 2x^4 - 5x^2 - 3 \\ &= 2x^4 - (6-1)x^2 - 3 \\ &= 2x^4 + 1x^2 - 6x^2 - 3 \\ &= x^2(2x^2 + 1) - 3(2x^2 + 1) \\ &= (x^2 - 3)(2x^2 + 1) \end{aligned}$$

$$\begin{aligned} (ii) \quad & a^2 - 8a - 20 \\ &= a^2 - (10-2)a - 20 \\ &= a^2 - 10a + 2a - 20 \\ &= a(a-10) + 2(a-10) \\ &= (a-10)(a+2) \end{aligned}$$

$$\begin{aligned} (iii) \quad & 21a^2 - 58a + 21 \\ &= 21a^2 - (49+9)a + 21 \\ &= 21a^2 - 49a - 9a + 21 \\ &= 7a(3a-7) - 3(3a-7) \\ &= (7a-3)(3a-7) \end{aligned}$$

$$\begin{aligned} 12. (i) \quad & \frac{x^3 - y^3}{x^2 - xy + y^2} \\ &= \frac{(x-y)(x^2 + xy + y^2)}{x^2 - xy + y^2} = x - y \\ & [a^3 - b^3 = (a-b)(a^2 - ab + b^2)] \end{aligned}$$

$$\begin{aligned} (ii) \quad & \frac{x^2 + 2x - 8}{x^3 + x^2 - 12x} \\ &= \frac{x^2 + 4x - 2x - 8}{x(x^2 + x - 12)} = \frac{x(x+4) - 2(x+4)}{x\{x^2 + 4x - 3x - 12\}} \\ &= \frac{(x-2)(x+4)}{x\{x(x+4) - 3(x+4)\}} = \frac{(x-2)(x+4)}{x(x+4)(x-3)} \\ &= \frac{x-2}{x(x-3)} \end{aligned}$$

$$\begin{aligned} (iii) \quad & \frac{x^2 - 2}{6xy} \times \frac{18y^3}{5x^4 - 10x^2} \\ &= \frac{x^2 - 2}{xy} \times \frac{3y \times y \times y}{5x^2(x^2 - 2)} \\ &= \frac{3y^2}{5x^3} \end{aligned}$$

$$\begin{aligned} (iv) \quad & \frac{8x^3 - 4y^2}{2a^2 - 3a + 1} \\ &= \frac{4(2x^3 - y^2)}{2a^2 - 2a - a + 1} \\ &= \frac{4(2x^3 - y^2)}{2a(a-1) - 1(a-1)} \\ &= \frac{4(2x^3 - y^2)}{(2a-1)(a-1)} \\ & \text{(can not be reduced further)} \end{aligned}$$

$$\begin{aligned} (v) \quad & \frac{a^2 + 4a}{a^2 - 9a} \div \frac{a^2 + 2a - 8}{x^2 - xy} \\ &= \frac{a(a+4)}{a(a-9)} \times \frac{x(x-y)}{(a+4)(a-4)} \\ &= \frac{x(x-y)}{(a-9)(a-2)} \end{aligned}$$

13. Let x be the required number, then

$$2x - 3.5 = x + 7.5$$

$$2x - x = 7.5 + 3.5$$

$$x = 11.0$$

14. Let x be the age of Arshad,
then
Sajid's age = $x + 10$
Sohail's age = $x - 3$
Sajid's age + Sohail's age = $4 \times$ Arshad's age + 2

$$x + 10 + x - 3 = 4x + 2$$

$$4x - 2x = 7 - 2$$

$$x = 5$$

 Arshad's age = 5 years
 Sajid's age = $5 + 10 = 15$ years
 Sohail's age = $5 - 3 = 2$ years

15. Let x° be the smallest angle,
Largest angle = $2x^\circ$
3rd angle = $x^\circ + 8^\circ$
Since the sum of the angles of a triangle is 180° ,

$$\therefore x + 2x + x + 8 = 180^\circ$$

$$4x = 180 - 8$$

$$x = \frac{172}{4}$$

$$x = 43^\circ$$

$$\text{Largest angle} = 2x = 2(43) = 86^\circ$$

$$\text{3rd angle} = x + 8 = 43 + 8 = 51^\circ$$

16. Let x be the denominator

$$\frac{x-7}{x} = \frac{x-7+2}{x+9}$$

$$\frac{x-7}{x} = \frac{x-5}{x+9}$$

$$(x-7)(x+9) = (x-5)$$

$$x^2 + 2x - 63 = x^2 - 5x$$

$$2x + 5x = 63$$

$$7x = 63$$

$$x = \frac{63}{7}$$

$$x = 9$$

$$\therefore \text{The required fraction is} = \frac{x-7}{x}$$

$$= \frac{9-7}{9}$$

$$= \frac{2}{9}$$

17. (i) $(4a^4 - 5a^2 + 7)(a^2 - 3)$

$$= 4a^6 - 12a^4 - 5a^4 + 15a^2 + 7a^2 - 21$$

$$= 4a^6 - 17a^4 + 22a^2 - 21$$

(ii) $(5x + 4)(x^2 - 3x + 7)$

$$= 5x^3 - 15x^2 + 35x + 4x^2 - 12x + 28$$

$$= 5x^3 - 11x^2 + 23x + 28$$

(iii) $(9x^5 - 3x^4 + 2x^3 - 1)(3x - 7)$

$$= 27x^6 - 63x^5 - 9x^5 + 21x^4 + 6x^4 - 14x^3 - 3x + 7$$

$$= 27x^6 - 72x^5 + 27x^4 - 14x^3 - 3x + 7$$

18. From $a + b - 3 = 0$, we get $a + b = 3$.

$$a^3 + b^3 = (a + b)^3 - 3ab(a + b)$$

$$= (3)^3 - 3ab(3) = 27 - 9ab$$

$$(27 - 9ab) + 9ab - 26 = 27 - 26 = 1$$

19. Use the identity $x^3 + y^3 = (x + y)^3 - 3xy(x + y)$.

Substitute $x + y = 4$:

$$x^3 + y^3 = (4)^3 - 3xy(4)$$

$$x^3 + y^3 = 64 - 12xy$$

$$(64 - 12xy) + 12xy - 64 = 0$$

$$= 0$$

20. Find $a^3 + b^3$ using $(a + b)^3 - 3ab(a + b)$:

$$a^3 + b^3 = (7)^3 - 3(12)(7) = 343 - 252 = 91$$

$$= 91 + 4(12)(7) = 91 + 336 = 427$$

$$= 427$$

21. The following identity is used:

$$(a - b)(a^2 + 2a + 4) = a^3 -$$

(i) $(a - 2)(a^2 + 2a + 4) = a^3 - 2^3 = a^3 - 8$

(ii) $(2x - 3y)(4x^2 + 6xy + 9y^2) = (2x)^3 - (3y)^3$

$$= 8x^3 - 27y^3$$

(iii) $(ab - cd)(a^2b^2 + abcd + c^2d^2) = (ab)^3 - (cd)^3$

$$= a^3b^3 - c^3d^3$$

22.

(i) $(x^2)^3 + (1)^3$

$$(x^2 + 1)((x^2)^2 - (x^2)(1) + (1)^2)$$

$$(x^2 + 1)(x^4 - x^2 + 1)$$

(ii)

$$3(27x^3 - 64y^3)$$

$$27x^3 = (3x)^3$$

$$64y^3 = (4y)^3$$

$$3((3x)^3 - (4y)^3)$$

Use the Difference of Cubes formula with

$$A = 3x \text{ and } B = 4y:$$

$$3(3x - 4y)((3x)^2 + (3x)(4y) + (4y)^2)$$

$$= 3(3x - 4y)(9x^2 + 12xy + 16y^2)$$

(iii) We can rewrite $a^3 b^3$ as $(ab)^3$. So, the expression is:

$$(ab)^3 + c^3$$

Using the Sum of Cubes formula with,
 $a = ab$ and $b = c$:

$$(ab + c)((ab)^2 - (ab)(c) + c^2)$$

$$(ab + c)(a^2 b^2 - abc + c^2)$$

(iv) Factorise $a^3 b^3 - 64c^3$

First, rewrite the terms as cubes:

$$a^3 b^3 = (ab)^3$$

$$64c^3 = (4c)^3$$

$$= (ab)^3 - (4c)^3$$

Using the Difference of Cubes formula with
 $A = ab$ and $B = 4c$:

$$(ab - 4c)((ab)^2 + (ab)(4c) + (4c)^2)$$

23. So, the common factor is $6p$. let's divide both terms by $6p$:

$$6p(p + 3(q - r))$$

Now, multiply 3 by the terms inside the small parentheses:

$$6p(p + 3q - 3r)$$

$$= 6p(p + 3q - 3r)$$

24.

(i) $x^4 - 5x^2 + 4$

$$x^4 - 4x^2 - 1x^2 + 4$$

$$(x^4 - 4x^2) - (x^2 - 4)$$

$$x^2(x^2 - 4) - 1(x^2 - 4)$$

$$(x^2 - 1)(x^2 - 4)$$

We can factorize these even further using the difference of squares: $(x^2 - 1) = (x - 1)(x + 1)$ and $(x^2 - 4) = (x - 2)(x + 2)$.

$$= (x - 1)(x + 1)(x - 2)(x + 2)$$

iii.

(iv) $28 + 3a - a^2$

$$-a^2 + 3a + 28.$$

$$-(a^2 - 3a - 28)$$

$$-(a^2 - 7a + 4a - 28)$$

$$-[a(a - 7) + 4(a - 7)]$$

$$-(a + 4)(a - 7)$$

$$= -(-(a + 4)(a - 7))$$

$$= (a + 4)(a - 7)$$

iv.

$$-(x^2 + 4x - 5)$$

$$-(x^2 + 5x - 1x - 5)$$

$$-[x(x + 5) - 1(x + 5)]$$

$$-(x - 1)(x + 5)$$

To remove the negative sign at the front, multiply it into $(x - 1)$ to get $(1 - x)$.

$$= (1 - x)(x + 5)$$

Let the present age of the son be x and the mother be y .

Twenty years ago:

$$\text{Son's age: } (x - 20)$$

$$\text{Mother's age: } (y - 20)$$

$$\text{According to the problem: } y - 20 = 4(x - 20)$$

$$y - 20 = 4x - 80$$

$$y = 4x - 60 \quad \text{--- (Equation 1)}$$

Four years hence (in the future):

$$\text{Son's age: } (x + 4)$$

$$\text{Mother's age: } (y + 4)$$

$$\text{According to the problem: } y + 4 = 2(x + 4)$$

$$y + 4 = 2x + 8$$

$$y = 2x + 4 \quad \text{--- (Equation 2)}$$

Equating the two expressions for y :

$$4x - 60 = 2x + 4$$

$$2x = 64$$

$$x = 32$$

For y :

$$y = 2(32) + 4 = 64 + 4 = 68$$

The mother's present age is 68 years and the son's present age is 32 years.

25. Area of a rectangle = length $\times$ breadth

$$\text{Area} = (2x + 5)(x - 3)$$

$$\text{Area} = 2x(x - 3) + 5(x - 3)$$

$$\text{Area} = 2x^2 - 6x + 5x - 15$$

$$\text{Area} = \{2x^2 - x - 15\} \text{ square metres}$$

We know the identity:

$$x^3 + y^3 = (x + y)^3 - 3xy(x + y).$$

Substitute $x + y = 4$:

$$x^3 + y^3 = (4)^3 - 3xy(4)$$

$$x^3 + y^3 = 64 - 12xy$$

$$(64 - 12xy) + 12xy - 64 = 0$$

$$= 0$$

26.

Let Son = s , Father = $4s$.

In 5 years: Son = $s + 5$. Father = $4s + 5$.

$$(s + 5) + (4s + 5) = 55$$

$$5s + 10 = 55 \Rightarrow 5s = 45 \Rightarrow s = 9$$

$$\text{Father} = 4 \times 9 = 36.$$

Son is 9, Father is 36.

27.

Let the number be x .

$$3x - 4.2 = 2x + 2.8$$

$$3x - 2x = 2.8 + 4.2$$

$$x = 7$$

The number is 7.

28. (i)

$$(y^2 + 2y + 3)(3y^2 + 5y - 7)$$

$$y^2(3y^2 + 5y - 7) + 2y(3y^2 + 5y - 7) + 3(3y^2 + 5y - 7)$$

$$3y^4 + 5y^3 - 7y^2 + 6y^3 + 10y^2 - 14y + 9y^2 + 15y - 21$$

Grouping the like terms (terms with the same power of y):

$$3y^4 + (5y^3 + 6y^3) + (-7y^2 + 10y^2 + 9y^2) + (-14y + 15y) - 21$$

$$= 3y^4 + 11y^3 + 12y^2 + y - 21$$

(ii) $(6x + 5)(3x - 7)$

$$(6x \times 3x) + (6x \times -7) + (5 \times 3x) + (5 \times -7)$$

$$18x^2 - 42x + 15x - 35$$

$$18x^2 - 27x - 35$$

(iii) $(5x^2 - 3x + 7)^2$

$$(5x^2 - 3x + 7)(5x^2 - 3x + 7).$$

$$5x^2(5x^2 - 3x + 7) - 3x(5x^2 - 3x + 7) + 7(5x^2 - 3x + 7)$$

$$25x^4 - 15x^3 + 35x^2 - 15x^3 + 9x^2 - 21x + 35x^2 - 21x + 49$$

$$25x^4 + (-15x^3 - 15x^3) + (35x^2 + 9x^2 + 35x^2) + (-21x - 21x) + 49$$

$$25x^4 - 30x^3 + 79x^2 - 42x + 49$$

29. (i)

$$(i) (x + 2)(x^2 - 2x + 4) - (x - 2)(x^2 + 2x + 4)$$

The first part follows the "sum of cubes" identity, $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$. where $a = x$ and $b = 2$.

$$(x + 2)(x^2 - 2x + 4) = x^3 + 2^3 = x^3 + 8$$

The second part follows the “difference of cubes” identity, $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$, where $a = x$ and $b = 2$.

$$(x - 2)(x^2 + 2x + 4) = x^3 - 2^3 = x^3 - 8$$

Subtracting the two results:

$$(x^3 + 8) - (x^3 - 8) = x^3 + 8 - x^3 + 8$$

$$(ii) (5x - 6)(2x + 3) + (3x + 5)^2$$

First, multiply the binomials:

$$(5x)(2x) + (5x)(3) + (-6)(2x) + (-6)(3) \\ = 10x^2 + 15x - 12x - 18 = 10x^2 + 3x - 18$$

Next, expand the squared binomial:

$$(3x + 5)^2 = (3x)^2 + 2(3x)(5) + 5^2 \\ = 9x^2 + 30x + 25$$

Finally, add the two results together:

$$(10x^2 + 3x - 18) + (9x^2 + 30x + 25)$$

$$= (10x^2 + 9x^2) + (3x + 30x) + (-18 + 25)$$

Final Answer:

$$19x^2 + 33x + 7$$

$$(iii) (a^2 - 2a + 4)(a^2 + 2a + 4)$$

We can group the terms to look like a “difference of squares” pattern, $(a - b)(a + b) = a^2 - b^2$, by grouping

$(a^2 + 4)$ together:

$$(a^2 + 4) - 2a)((a^2 + 4) + 2a)$$

This simplifies to:

$$(a^2 + 4)^2 - (2a)^2$$

Expanding $(a^2 + 4)^2$ gives $a^4 + 8a^2 + 16$.

Subtracting the squared term $(2a)^2 = 4a^2$

$$a^4 + 8a^2 + 16 - 4a^2$$

$$= a^4 + 4a^2 + 16$$



LINEAR EQUATIONS

Exercise 9A

1. (i) $\text{gradient} = \frac{\text{rise}}{\text{run}}$

As we move from left to right the rise is from -12 to -4 , i.e. 8 units.

Similarly the run is from -1 to 1 , i.e. 2 units

$$\therefore \text{gradient} = \frac{8}{2} = 4$$

Helpful Hint

Since the line is going upwards if we move from left to right, the gradient is positive.

The line intersects y -axis at -8 , therefore, y -intercept $= -8$

(ii) As we move from left to right the rise is from 10 to 6 i.e. -4 units and run is -18 to -9 , i.e. 9 units

Therefore,

$$\text{gradient} = \frac{\text{rise}}{\text{run}} = -\frac{4}{9}$$

Helpful Hint

If we move from left to right, the line goes downward, and the gradient will be negative.

$$y\text{-intercept} = 2$$

(iii) $\text{gradient} = \frac{15}{24} = \frac{5}{8}$

$$y\text{-intercept} = 5$$

(iv) $\text{gradient} = \frac{3}{3} = 1$

$$y\text{-intercept} = 0$$

(v) $\text{gradient} = \frac{12}{5}$

$$y\text{-intercept} = 16$$

2. (i) To write equation of a line, we need to find its gradient and y -intercept.

Helpful Hint

The general form of equation of a line is $y = mx + c$, where m is the gradient and c is the y -intercept of the line.

$$\text{In this graph gradient, } m = \frac{4}{2} = 2$$

$$\text{and } y\text{-intercept, } c = -3$$

So, the equation is

$$y = 2x - 3$$

(ii) $m = -\frac{5}{4}$

Helpful Hint

The line is going downwards as we move from left to right so the gradient is negative.

$$y\text{-intercept, } c = -2$$

The equation is

$$y = -\frac{5}{4}x - 2$$

(iii) $m = \frac{7}{5}, c = 0$

The equation is

$$y = \frac{7}{5}x$$

(iv) $m = -\frac{7}{4}, c = 3$

The equation is

$$y = -\frac{7}{4}x + 3$$

(v) $m = \frac{3}{5}, c = 0$

The equation is

$$y = \frac{3}{5}x$$

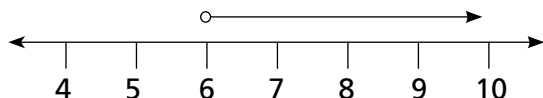
Exercise 9B

1. (i) $x - 2 > 4$

[add 2 to both the sides]

$$x > 4 + 2$$

$$x > 6$$



Helpful Hint

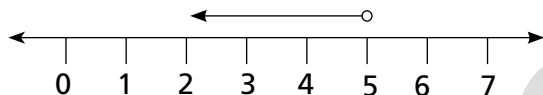
Use open circle for $>$ and $<$

(ii) $6x < 30$

[divide both the sides by 6]

$$x < \frac{30}{6}$$

$$x < 5$$

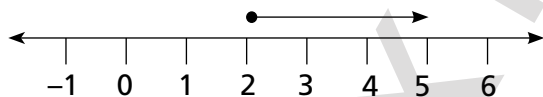


(iii) $6x - 9 \geq 11$

[subtract 9 from both the sides]

$$6x \geq 11 - 9$$

$$6x \geq 2$$



Helpful Hint

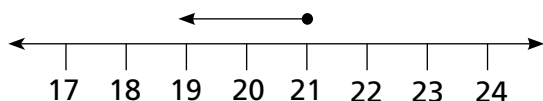
Use close circle for $\leq$ and $\geq$

(iv) $\frac{x}{3} \leq 7$

multiply both sides by 3

$$x \leq 7 \times 3$$

$$x \leq 21$$

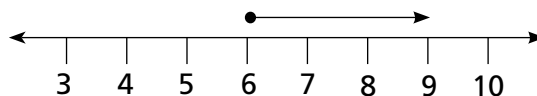


(v) $\frac{x}{2} \geq 3$

[multiply both sides by 2]

$$x \geq 3 \times 2$$

$$x \geq 6$$



(vi) $3 < -5n + 2n$

$$3 < -3n$$

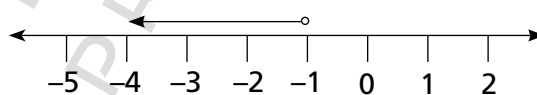
$$-3n > 3$$

[divide both sides by -3]

$$n < -1$$

Helpful Hint

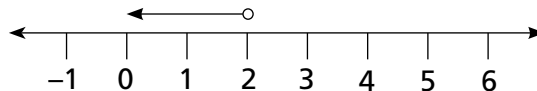
Change/flip the sign when multiplying with any negative number



(vii) $-p - 4p > -10$

$$-5p > -10$$

$$p < 2$$



(viii) $9 \geq -2m + 2 - 3$

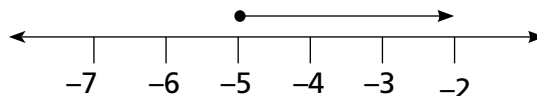
$$9 \geq -2m - 1$$

$$9 + 1 \geq -2m$$

$$10 \geq m$$

$$-5 \leq m$$

$$m \geq -5$$



(ix) $-3 - 6(4x + 6) > -111$

$$-6(4x + 6) > -111 + 3$$

$$-6(4x + 6) > -108$$

$$4x + 6 < -\frac{108}{-6}$$

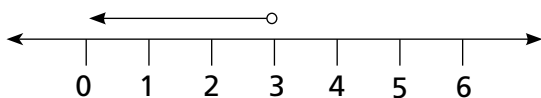
$$4x + 6 < 18$$

$$4x < 18 - 6$$

$$4x < 12$$

$$x < \frac{12}{4}$$

$$x < 3$$



(x) $5(6 + 3r) + 7 \geq 127$

$$5(6 + 3r) \geq 127 - 7$$

$$5(6 + 3r) \geq 120$$

$$6 + 3r \geq \frac{120}{5}$$

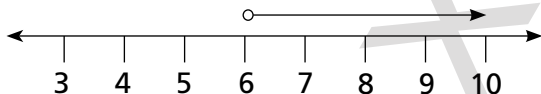
$$6 + 3r \geq 24$$

$$3r \geq 24 - 6$$

$$3r \geq 18$$

$$r \geq \frac{18}{3}$$

$$r \geq 6$$



Exercise 9C

- coefficient, constant
 - $5x + 2y$
 - 2
 - elimination, substitution
 - $x = 9, y = 4$
- True
 - True
 - False

Reason:

Both equation will have a solution for particular values of x and y

For example, if $x = 1$ and $y = 2$

$$\begin{aligned} x + y &= 1 + 2 \\ &= 3 \neq 0 \end{aligned}$$

(iv) True

(v) False

Reason: LHS RHS

$$x - 3y = 9$$

by substituting the values, $x = 4$ and $y = 3$, we have

$$\begin{aligned} \text{LHS } 4 - 3\left(\frac{5}{3}\right) &= 4 - 5 = -1 \\ \text{LHS is not equal to RHS} \end{aligned}$$

3. (i) $7x + 3y = 25$ _____ (i)

$-2x + y = 4$ _____ (ii)

Eliminate y from the equations.

$(-2x + y = 4) \times 3$ [multiply equation (ii) by 3 to make the coefficients of y same]

We have

$$\begin{array}{rcl} 7x + 3y = 25 & \text{_____} & \text{(i)} \\ -6x + 3y = 12 & \text{_____} & \text{(i)[subtract eq} \\ + & - & - \\ \hline 13x = 13 & \text{_____} & \text{(ii) (ii) from (i)]} \end{array}$$

$$x = \frac{13}{13} = 1$$

Now, substitute this value of x in any one of the given equations.

Equation (ii)

$$-2x + y = 4$$

$$-2(1) + y = 4$$

$$-2 + y = 4$$

$$y = 4 + 2$$

$$\boxed{y = 6}$$

$$\therefore x = 1, y = 6$$

(ii) $5x - y = 7$ _____ (i)

$2x + y = 7$ _____ (ii)

The coefficients of y are already same.

Helpful Hint

If the signs of the coefficients of eliminating variables are different, add the equation.

Add eq (i) and (ii)

$$\begin{array}{r} 5x - y = 7 \\ + \quad 2x + y = 7 \\ \hline 7x = 14 \end{array}$$

$$x = \frac{14}{7}$$

$$x = 2$$

Substitute $y = 2$ eq (i)

$$5x - y = 7$$

$$5(2) - y = 7$$

$$10 - y = 7$$

$$-y = 7 - 10$$

$$-y = -3$$

$$y = +3$$

$$\therefore x = 2, y = 3$$

(iii) $2x - y = 2$ _____ (i)

$$\begin{array}{r} 5x + y = -9 \\ \hline 7x = -7 \end{array} \quad \text{_____ (ii)}$$

Add equations (i) and (ii)

$$x = \frac{-7}{7} = -1$$

Substitute $x = -1$ in eq (i)

$$2x - y = 2$$

$$2(-1) - y = 2$$

$$-2 - y = 2$$

$$-y = 2 + 2$$

$$-y = 4$$

$$y = -4$$

$$\therefore x = -1, y = -4$$

(iv) $4x + 4y = -4$ _____ (i)

$x + 7y = -19$ _____ (ii)

$$(x + 7y = -19) \times 4$$

$$4x + 28y = -76$$
 _____ (ii)

Now subtract eq (ii) from eq (i)

$$\begin{array}{r} 4x + 4y = -4 \\ 4x + 28y = -76 \\ - \quad - \quad + \\ \hline -24y = 72 \end{array}$$

$$y = -\frac{72}{24}$$

$$y = -3$$

Substitute in eq (i)

$$4x + 4(-3) = -4$$

$$4x - 12 = -4$$

$$4x = 8$$

$$x = 2$$

$$\therefore x = 2, y = -3$$

(v) Add both the equations to eliminate y

$3x - y = 7$ _____ (i)

$$\begin{array}{r} \frac{1}{2}x + y = 7 \\ + \quad \left(3 + \frac{1}{2}\right)x = 14 \\ \hline \end{array} \quad \text{_____ (ii)}$$

$$\frac{6+1}{2}x = 14$$

$$\frac{7}{2}x = 14$$

$$7x = 14 \times 2 = 28$$

$$x = \frac{28}{7} = 4$$

Substitute $x = 4$ in eq (i)

$$3(4) - y = 7$$

$$12 - y = 7$$

$$-y = 7 - 12$$

$$-y = -5$$

$$y = 5$$

$$\therefore x = 4, y = 5$$

(vi) $4a - 3b = 10$ — (i)

$2a + b = 10$ — (ii)

Multiply eq (ii) by 3 to make coefficients of b same.

$(2a + b = 10) \times 3$

$6a + 3b = 30$ — (ii)

Add both the equations (i) and (ii)

$$\begin{array}{r} 4a - 3b = 10 \\ + 6a + 3b = 30 \\ \hline 10a = 40 \end{array}$$

$a = \frac{40}{10}$

$a = 4$

Substitute $a = 4$ in eq (i)

$4(4) - 3b = 10$

$16 - 3b = 10$

$-3b = 10 - 16 = -6$

$b = \frac{-6}{-3}$

$b = 2$

$\therefore a = 4, b = 2$

(vii) $a - 5b = 10$ — (i)

$5a - 3b = 17$ — (ii)

Multiply eq (i) by 5

$(a - 5b = 10) \times 5$

Subtract eq (ii) from eq (i)

$5a - 25b = 50$ — (i)

$5a - 3b = 17$ — (ii)

$$\begin{array}{r} - + - \\ \hline -22b = 33 \end{array}$$

$b = -\frac{33}{22} = -\frac{3}{2}$

Substitute $b = -\frac{3}{2}$ in eq (i)

$a - 5\left(-\frac{3}{2}\right) = 10$

$a + \frac{15}{2} = 10$

$a = 10 - \frac{15}{2} = \frac{20 - 15}{2}$

$2a = 5$

$a = \frac{5}{2}$

$\therefore a = \frac{5}{2}, b = -\frac{3}{2}$

(viii) $a + 1\frac{1}{2}b = 8\frac{1}{2}$

$a + \frac{3}{2}b = \frac{17}{2}$ — (i)

$2a - b = 1$ — (ii)

Multiply eq (i) by 2

$\left(a + \frac{3}{2}b = \frac{17}{2}\right) \times 2$

$2a + 3b = 17$

$2a - b = 1$

$$\begin{array}{r} - + - \\ \hline 4b = 16 \end{array}$$

$b = \frac{16}{4}$

$b = 4$

Substitute $b = 4$ in eq (ii)

$2a - b = 1$

$2a - 4 = 1$

$2a = 1 + 4 = 5$

$a = \frac{5}{2}$

$\therefore a = \frac{5}{2}, b = 4$

(ix) $3x - 5 = y$

$3x - y = 5$ — (i)

$x = y - 1\frac{1}{3}$

$x - y = -\frac{4}{3}$ — (ii) Subtract eq (ii) from (i)

$3x - y = 5$

$x - y = -\frac{4}{3}$

$$\begin{array}{r} - + + \\ \hline 2x = 5 + \frac{4}{3} \end{array}$$

$2x = \frac{15 + 4}{3}$

$2x = \frac{19}{3}$

$x = \frac{19}{6} \times \frac{1}{2}$

$x = \frac{19}{6}$

Substitute $\frac{19}{6}$ in eq (i)

$3x - y = 5$

$3\left(\frac{19}{6}\right) - y = 5$

$$\frac{19}{2} - y = 5$$

$$y = \frac{19}{2} - 5$$

$$y = \frac{19 - 10}{2}$$

$$y = \frac{9}{2}$$

$$\therefore x = \frac{19}{6}, y = \frac{9}{2}$$

(x) $x + y = 4$ — (i)

$3x - 2y = 7.5$ — (ii)

Multiply eq (i) by (ii)

$$(x + y = 4) \times 2$$

Add both the equation

$$2x + 2y = 8$$
 — (i)

$$3x - 2y = 7.5$$
 — (ii)

$$5x = 15.5$$

$$x = 3.1$$

Substitute $x = 3.1$ in eq (i)

$$3.1 + y = 4$$

$$y = 4 - 3.1$$

$$y = 0.9$$

$$x = 3.1, y = 0.9$$

4. (i) $5x - 2y = 8$ — (i)

$3x - 2y = 4$ — (ii)

Helpful Hint

Make 'y' the subject of the equation.

From eq (i)

$$-2y = 8 - 5x$$

$$y = -\frac{(8 - 5x)}{2}$$

$$y = -\frac{5x - 8}{2}$$

Substitute in eq (ii)

$$3x - 2\left(-\frac{5x - 8}{2}\right) = 4$$

$$3x - 5x + 8 = 4$$

$$-2x = 4 - 8$$

$$-2x = -4$$

$$x = 2$$

Substitute $x = 2$ in eq (i)

$$5(2) - 2y = 8$$

$$10 - 2y = 8$$

$$-2y = 8 - 10$$

$$-2y = -2$$

$$y = 1$$

$$\therefore x = 2, y = 1$$

(ii) $5b + 14a = 31$ — (i)

$2a - 3b = -29$ — (ii)

From eq (ii)

$$2a = -29 + 3b$$

$$a = \frac{-29 + 3b}{2}$$

Substitute in eq (i)

$$5b + 14\left(\frac{-29 + 3b}{2}\right) = 31$$

$$5b - 203 + 21b = 31$$

$$26b = 31 + 203$$

$$26b = 234$$

$$b = \frac{234}{26}$$

$$b = 9$$

Substitute $b = 9$ in eq (ii)

$$2a - 3(9) = -29$$

$$2a - 27 = -29$$

$$2a = -29 + 27$$

$$2a = -2$$

$$a = -1$$

$$\therefore a = -1, b = 9$$

(iii) $2a - 3b = 1.5$ — (i)

$2a - b = 8.5$ — (ii)

From eq (ii)

$$-b = 8.5 - 2a$$

$$b = 2a - 8.5$$

Substitute $a = 6$ in eq (i)

$$2a - 3(2a - 8.5) = 1.5$$

$$2a - 6a + 25.5 = 1.5$$

$$-4a = 1.5 - 25.5$$

$$-4a = -24.0$$

$$a = \frac{24}{4}$$

$$a = 6$$

Substitute in eq (ii)

$$2(6) - b = 85$$

$$12 - b = 8.5$$

$$b = 12 - 8.5$$

$$b = 3.5$$

$$\therefore a = 6, b = 3.5$$

$$(iv) \quad x - y = 3 \quad \text{--- (i)}$$

$$5x + y = 33 \quad \text{--- (ii)}$$

From eq (i)

$$x = 3 + y$$

Substitute $y = +3$ in eq (ii)

$$5(3 + y) + y = 33$$

$$15 + 5y + y = 33$$

$$6y = 33 - 15 = 18$$

$$y = \frac{18}{6} = 3$$

Substitute in eq (i)

$$x - 3 = 3$$

$$x = 3 + 3$$

$$x = 6$$

$$\therefore x = 6, y = 3$$

$$(v) \quad 2z + x = 15 \quad \text{--- (i)}$$

$$2z + 3x = 9 \quad \text{--- (ii)}$$

From eq (i)

$$x = 15 - 2z$$

Substitute in eq (ii)

$$2z + 3(15 - 2z) = 9$$

$$2z + 45 - 6z = 9$$

$$-4z = 9 - 45 = -36$$

$$z = \frac{36}{4} = 9$$

Substitute in eq (i)

$$2(9) + x = 15$$

$$18 + x = 15$$

$$x = 15 - 18$$

$$x = -3$$

$$\therefore x = -3, z = 9$$

$$(vi) \quad 5m - 7n = 5 \quad \text{--- (i)}$$

$$m - 2n = -2 \quad \text{--- (ii)}$$

From eq (ii)

$$m = -2 + 2n$$

Substitute in eq (i)

$$5(-2 + 2n) - 7n = 5$$

$$-10 + 10n - 7n = 5$$

$$3n = 15$$

$$n = 5$$

Substitute $n = 5$ in eq (ii)

$$m - 2(5) = -2$$

$$m - 10 = -2$$

$$m = -2 + 10$$

$$m = 8$$

$$\therefore m = 8, n = 5$$

$$(vii) \quad m + 1.5n = 23 \quad \text{--- (i)}$$

$$1.5m - 2n = -8 \quad \text{--- (ii)}$$

From eq (i)

$$m = 23 - 1.5n$$

Substitute in eq (ii)

$$1.5(23 - 1.5n) - 2n = -8$$

$$34.5 - 2.25n - 2n = -8$$

$$4.25n = 42.5$$

$$n = \frac{42.5}{4.25}$$

$$4.25$$

$$n = 10$$

Substitute $n = 10$ in eq (i)

$$m + 1.5(10) = 23$$

$$m + 15 = 23$$

$$m = 23 - 15 = 8$$

$$\therefore m = 8, n = 10$$

$$(viii) \quad \frac{5a}{2} - \frac{13}{3}b = -29 \quad \text{--- (i)}$$

$$\frac{4}{3}b - \frac{3}{2}a = 6 \quad \text{--- (ii)}$$

From eq (i)

$$\frac{5}{2}a = -29 + \frac{13}{3}b$$

$$a = \frac{2}{5} \left(-29 + \frac{13}{3}b \right)$$

$$a = -\frac{58}{5} + \frac{26}{15}b$$

Substitute in eq (ii)

$$\frac{4}{3}b - \frac{3}{2} \left(-\frac{58}{5} + \frac{26}{15}b \right) = +6$$

$$\frac{4}{3}b + \frac{174}{10} - \frac{26}{10}b = 6$$

$$\left(\frac{4}{3} - \frac{26}{10} \right)b = 6 - \frac{174}{10}$$

$$\frac{40 - 78}{30}b = \frac{60 - 174}{10}$$

$$-\frac{38}{30}b = -\frac{114}{10}$$

$$b = \frac{114}{10} \times \frac{30}{38}$$

$$b = 9$$

Substitute $b = 9$ in eq (i)

$$\frac{5}{2}a - \frac{13}{3} \times 9 = -29$$

$$\frac{5}{2}a = -29 + 39 = 10$$

$$a = 10 \times \frac{2}{5}$$

$$a = 4$$

$$\therefore a = 4, \quad b = 9$$

$$(ix) \quad c - \frac{1}{5}d = 9 \quad \text{--- (i)}$$

$$c - 2d = -9 \quad \text{--- (ii)}$$

From eq (i)

$$c = 9 + \frac{1}{5}d$$

Substitute in eq (ii)

$$9 + \frac{1}{5}d - 2d = -9$$

$$\frac{d - 10d}{5} = -9 - 9$$

$$-\frac{9d}{5} = -18$$

$$9d = 5 \times 18$$

$$d = \frac{5 \times 18}{9}$$

$$d = 10$$

Substitute $d = 10$ in eq (i)

$$c - \frac{1}{5} \times 10 = 9$$

$$c = 9 + 2$$

$$c = 11$$

$$\therefore c = 11, \quad d = 10.$$

$$(x) \quad x - y = 7 \quad \text{--- (i)}$$

$$2y + 3x = 11 \quad \text{--- (i)}$$

From eq (i)

$$x = 7 + y$$

Substitute in eq (ii)

$$2y + 3(7 + y) = 11$$

$$2y + 21 + 3y = 11$$

$$5y = 11 - 21$$

$$5y = -10$$

$$y = -2$$

Substitute in eq (i)

$$x - (-2) = 7$$

$$x + 2 = 7$$

$$x = 7 - 2$$

$$x = 5$$

$$\therefore x = 5, \quad y = -2$$

$$5. (i) \quad x + y = 3 \quad \text{--- eq 1}$$

$$4x - y = 2 \quad \text{--- eq 2}$$

Make table of values for equation 1 by substituting various values of x to find corresponding values of y .

$$x + y = 3$$

$$y = 3 - x$$

Helpful Hint

Make y the subject of the equation. Keep y at left hand side of the equation and all other terms at right hand side to find the value of y .

x	-2	-1	0	1	2
y	5	4	3	2	1

Now plot these points on a coordinate plane (graph sheet).

Join all points to form a straight line and name it as equation 1 on the graph.

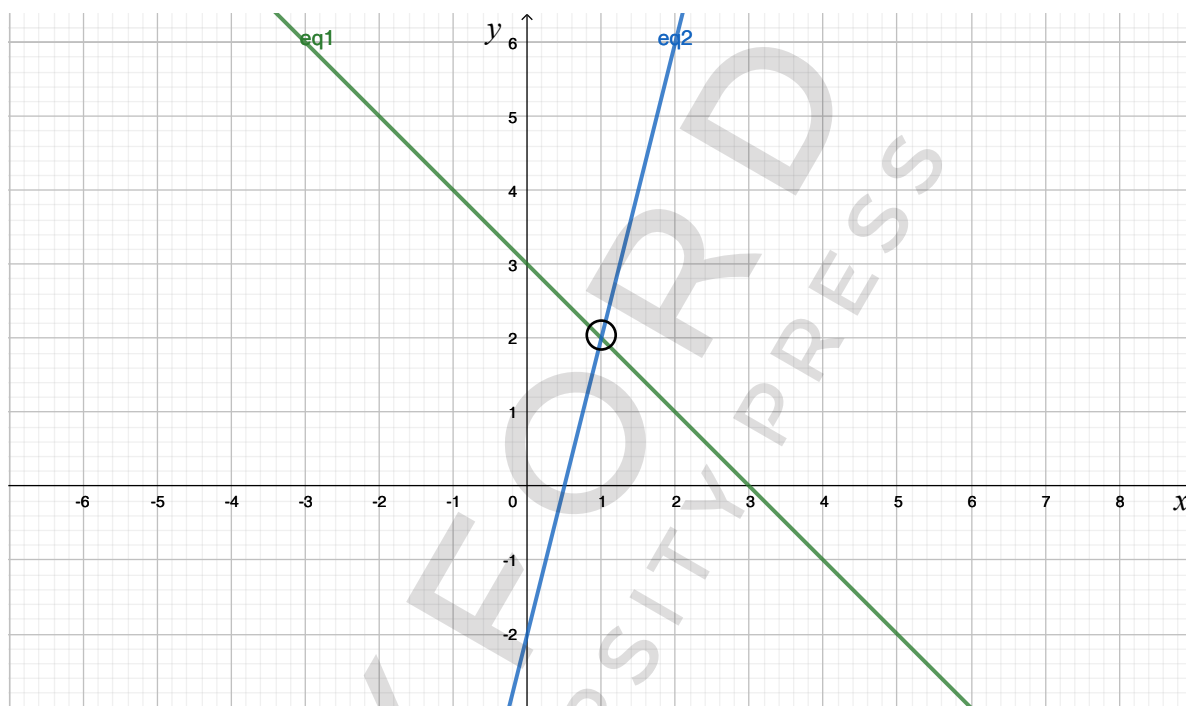
Now, follow same steps for equation 2.

$$4x - y = 2$$

$$y = 4x - 2$$

x	-2	-1	0	1	2
y	-10	-6	-2	2	6

Plot the points and draw line for equation. Label it equation 2



Encircle the point of intersection of the two lines and note down the values of x and y .

This point of intersection is the solution of the two equations i.e.

$$x = 1 \text{ and } y = 2$$

or $(1, 2)$ is the solution.

(ii) $x + y = 3$ _____ eq 1

$$2x - 2y = 10$$
 _____ eq 2

Make table of values for equation 1.

$$x + y = 3$$

$$y = 3 - x$$

x	-2	-1	0	1	2
y	5	4	3	2	1

Make table of values for equation 2

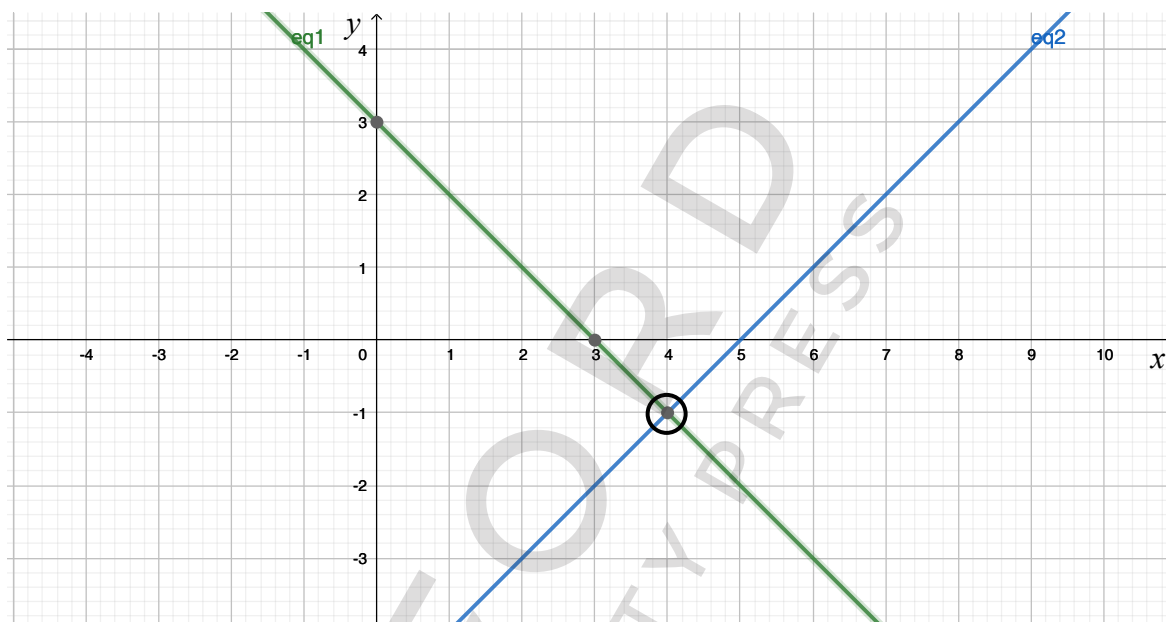
$$2x - 2y = 10$$

$$x - y = 5$$

$$y = x - 5$$

x	2	3	4	5	6	7
y	-3	-2	-1	0	1	2

Draw graphs for both the equations on same plane



Solution: $x = 4, y = -1$

(iii) $5x - y = 6$ _____ eq 1

$2x + y = 8$ _____ eq 2

Take equation 1

$$5x - y = 6$$

$$y = 5x - 6$$

x	0	1	2	3	4
y	-6	-1	4	9	14

Take equation 2

$$2x + y = 8$$

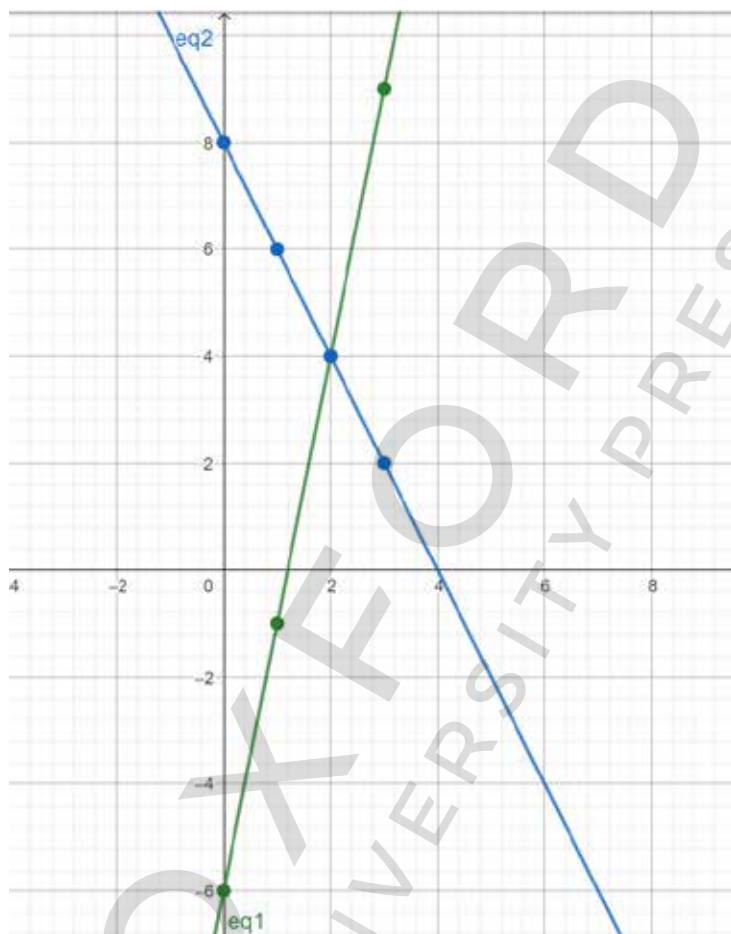
$$y = 8 - 2x$$

x	0	1	2	3	4
y	8	6	4	2	0

Draw the graphs for equations 1 and 2 on the same plane.

Helpful Hint

Choose scale as 1 cm = 5 units on both the axes.



Solution $x = 2, y = 4$

(iv) $3x - y = 2$ _____ eq 1

$9x - 3y = 6$ _____ eq 2

For equation 1

$3x - y = 2$

$y = 3x - 2$

x	-1	0	1	2	3
y	-5	-2	1	4	7

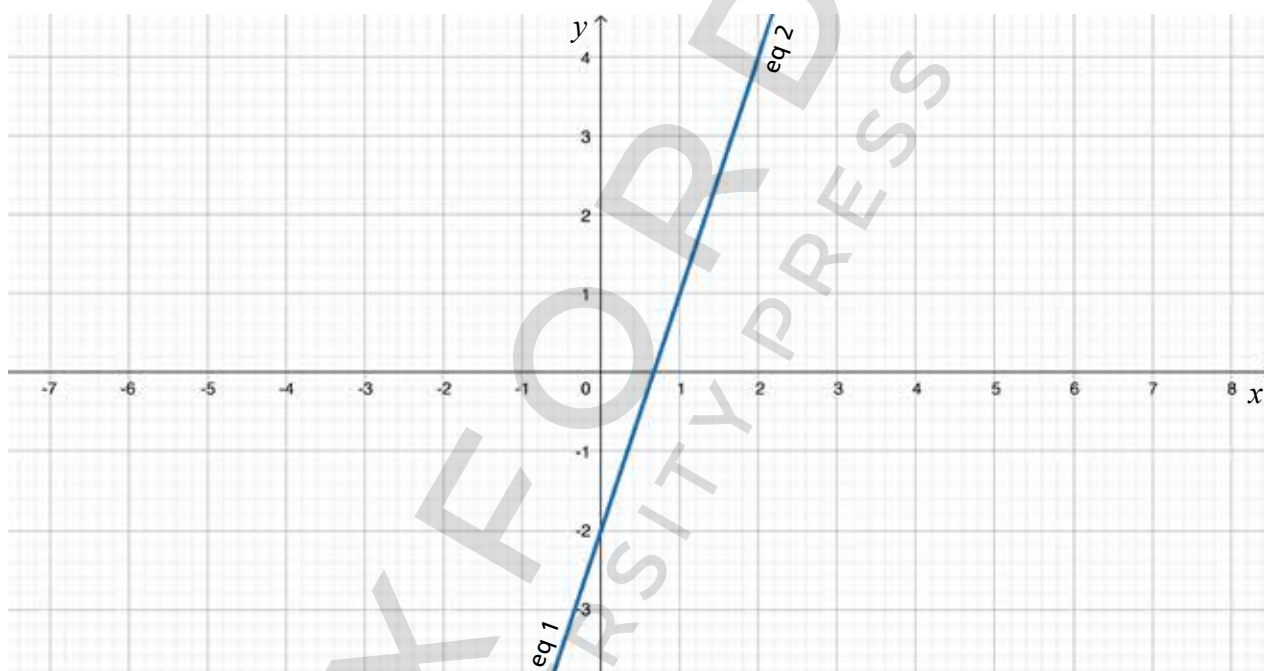
For equation 2

$$9x - 3y = 6$$

$$3y = 9x - 6$$

$$y = \frac{9x - 6}{3} = 3x - 2$$

x	-1	0	1	2	3
y	-5	-2	1	4	7



You can notice that the graph is same for both the equations. The lines coincide each other. Hence, the two equations have infinite number of solutions.

(v) $y - 2x = 3$ _____ eq 1

$2x - y = 5$ _____ eq 2

Make table of values for equation 1.

$$y - 2x = 3$$

$$y = 2x + 3$$

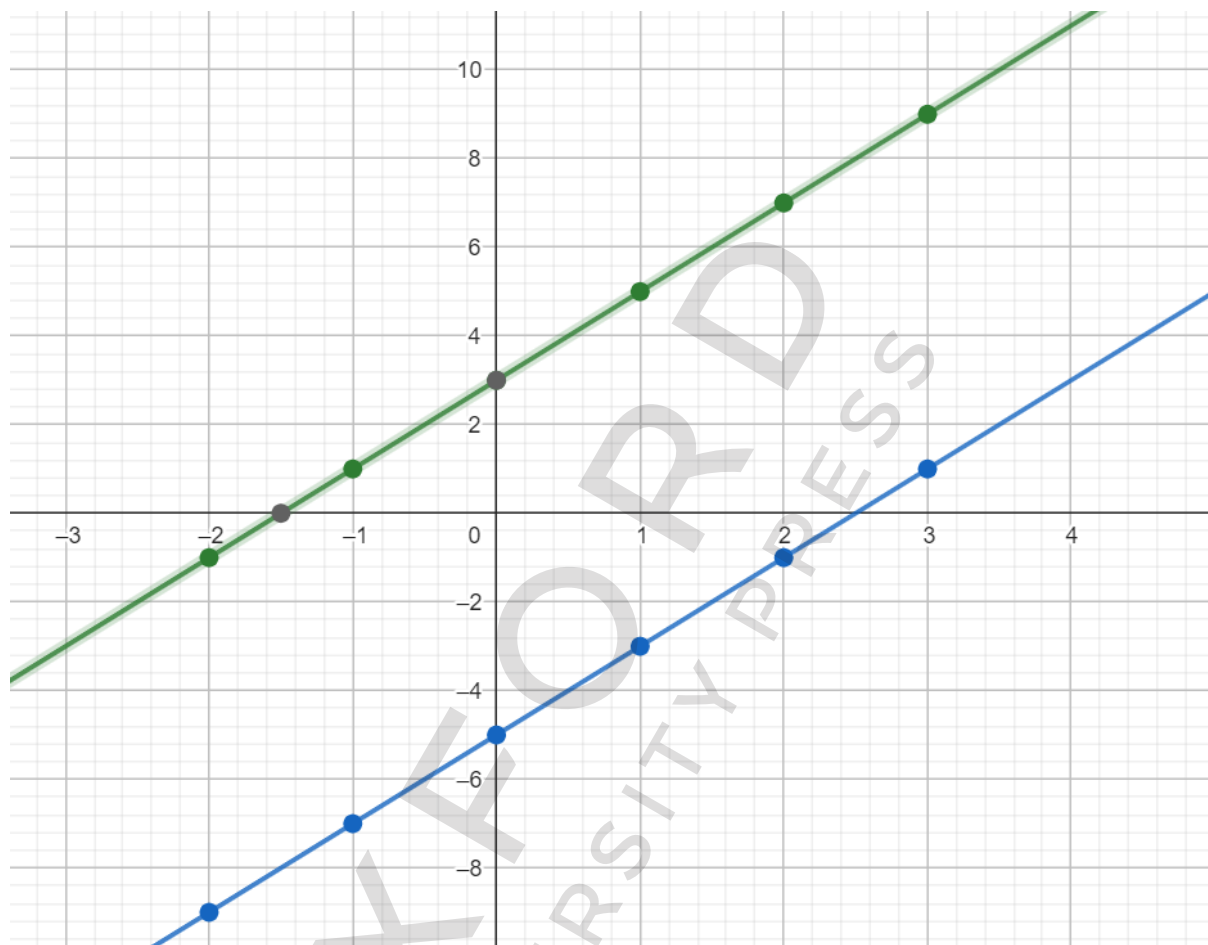
$x - 2$	-1	0	1	2	3
$y - 1$	1	3	5	7	9

For equation 2

$$2x - y = 5$$

$$y = 2x - 5$$

$x - 2$	-1	0	1	2	3
$y - 9$	-7	-5	-3	-1	1



Since, the two lines are parallel, they do not intersect each other at any point, hence, the two equations have no solution.

(vi) $x + y = 1$ _____ eq 1

$3x + y = 15$ _____ eq 2

Equation 1:

$$x + y = 1$$

$$y = 1 - x$$

x	-2	-1	0	1	2
y	3	2	1	0	-1

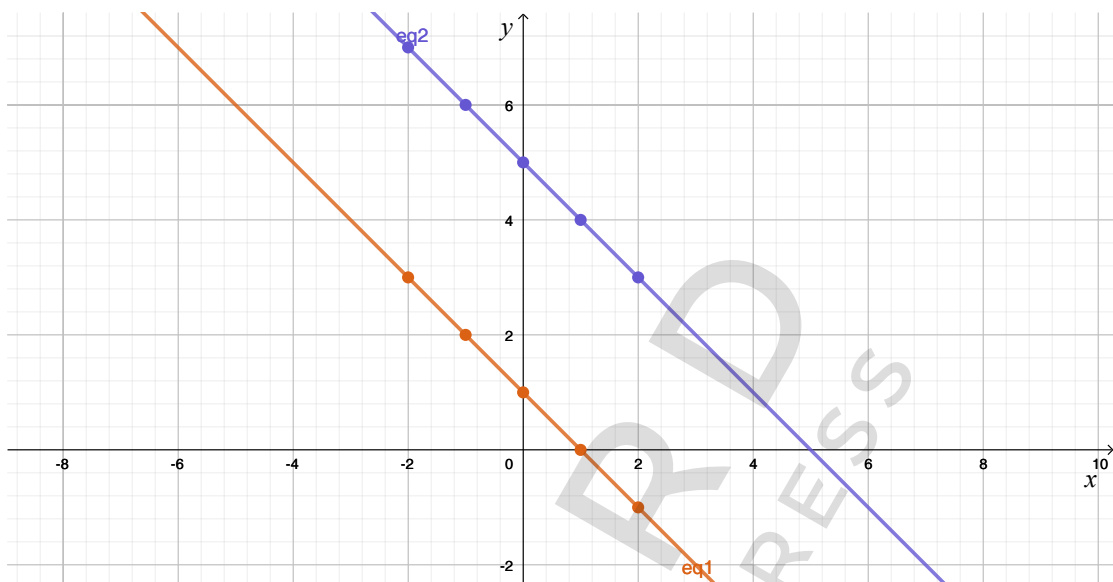
For Equation 2:

$$3x + 3y = 15$$

$$x + y = 5$$

$$y = 5 - x$$

x	-2	-1	0	1	2
y	7	6	5	4	3



Since, the two lines are parallel, they have no solution.

6. Let x and y be the numbers of days Ahmed and Manu each worked for, respectively.

$$200x + 250y = 3900 \text{ [divide both sides by 10]}$$

$$20x + 25y = 390 \quad \text{--- (i)}$$

$$250y = 200x + 1100$$

$$250y - 200y = 1100 \text{ [divide both side by 10]}$$

$$-20x + 25y = 110 \quad \text{--- (i)}$$

Add eq (i) and eq (ii)

$$20x + 25y = 390$$

$$-20x + 25y = 110$$

$$50y = 500$$

$$y = \frac{500}{50} = 10$$

Substitute $y = 10$ in eq (i)

$$20x + 25(10) = 390$$

$$20x + 250 = 390$$

$$20x = 390 - 250 = 140$$

$$x = \frac{140}{20} = 7$$

$\therefore$ Ahmed worked for 7 days and Manu

worked for 10 days.

Helpful Hint

Total payment = number of days $\times$ payment per day

$$\text{Ahmed's payment} = 200 \times 7 = \text{Rs } 1400$$

$$\text{Manu's payment} = 250 \times 10 = \text{Rs } 2500$$

$$-20x + 25y = 110 \quad \text{--- (i)}$$

Add eq (i) and eq (ii)

$$20x + 25y = 390$$

$$-20x + 25y = 110$$

$$50y = 500$$

$$y = \frac{500}{50} = 1$$

7. Let x and y be the amounts in Anum's purse and money box, respectively.

$$2x + y = 1700 \quad \text{--- (i)}$$

$$3x + y = 2200 \quad \text{--- (ii)}$$

Subtract eq (i) from eq (ii)

$$3x + y = 2200$$

$$2x + y = 1700$$

$$\begin{array}{r} - \quad - \quad - \\ 3x + y = 2200 \\ 2x + y = 1700 \\ \hline x = 500 \end{array}$$

Substitute in eq (i)

$$2(500) + y = 1700$$

$$y = 1700 - 1000$$

$$y = 700$$

$\therefore$ Anum has Rs 700 in her money box

8. Let x and y be the ages of Rehana and Sonia respectively.

$$x + y = 20 \quad \text{--- (i)}$$

$$x - y = 8 \quad \text{--- (ii)}$$

add both the equations

$$2x = 28$$

$$x = 14$$

Substitute in eq (i)

$$14 + y = 20$$

$$y = 20 - 14 = 6$$

$\therefore$ Rehana is 14 years old and Sonia is 6 years old.

9. Let x and y be the cost of one cup of cappuccino and one cup of cold coffee, respectively.

$$4x + 4y = 1400 \quad \text{--- (i)}$$

$$4x = 3y$$

$$4x - 3y = 0 \quad \text{--- (ii)}$$

$$7y = 1400 \quad \text{Subtract eq (ii) from eq (i)}$$

$$y = 200$$

Substitute in eq (ii)

$$4x - 3(200) = 0$$

$$4x = 600$$

$$x = \frac{600}{4}$$

$$x = 150$$

$\therefore$ 1 cup of cappuccino costs Rs 150 and 1 cup of cold coffee costs Rs 200

5 cups of cappuccino cost $5 \times 150 = \text{Rs } 750$

5 glasses of cold coffee cost

$$= 5 \times 200 = \text{Rs } 1000$$

Hence, Abbas will pay

$$\text{Rs } 1000 + \text{Rs } 750 = \text{Rs } 1750$$

Multiple Choice Questions 9

1. C
2. C
3. C
4. C
5. D

Exercise 10A

- diameter
 - circumference
 - segment
 - interior
 - chord
- True
 - False

A diameter of a circle is a line segment passing through the centre of the circle and its end points lie on the circle. There can be many lines passing through the centre and its end points touching the boundary of the circle.

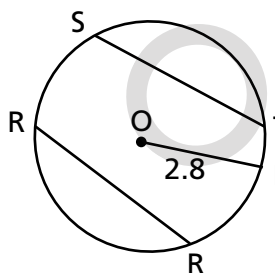
(iii) False

A radius is a straight line from the centre to the circumference of a circle, while the chord is straight line whose end points lie on the circle.

(iv) True

(v) True

3.



Steps of construction:

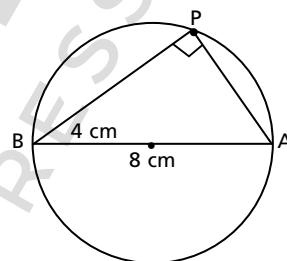
- Step 1: Using a compass, draw a circle of radius 2.8 cm.
- Step 2: O is the centre of the circle.
- Step 3: Take points S and T on the boundary of the circle. Join S and T. $\overline{ST}$ is a

chord.

Step 4: Take points Q and R on the boundary of the circle. Join Q and R. $\overline{QR}$ is another chord.

$\therefore \overline{ST}$ and $\overline{QR}$ are two chords not passing through the centre.

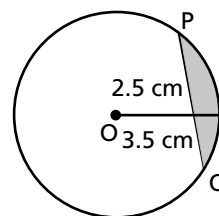
4.



Steps of construction:

- Step 1: Draw a circle of diameter 8 cm or radius 4 cm.
- Step 2: Draw the diameter $\overline{AB}$ of the circle.
- Step 3: Take another point P on the circle.
- Step 4: Join A to P and B to P.
- Step 5: Measure $\angle APB$ m
 $\angle APB = 90^\circ$

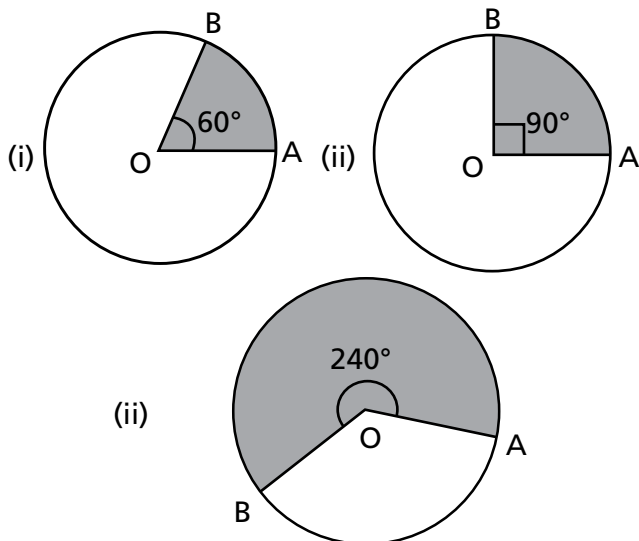
5.



Steps of construction:

- Step 1: Draw a circle with radius 3.5 cm.
- Step 2: Draw the chord $\overline{PQ}$ of length 2.5 cm touching the circle at P and Q.
- Step 3: Shade the minor segment PQ. $\overline{PQ}$ is the required segment.

6.

**Steps of construction:**

Step 1: Draw a circle of suitable radius with centre O.

Step 2: Take a point A on the circle and join A and O.

Step 3: Construct an angle of 60° on $\overline{OA}$, $\angle AOB = 60^\circ$; area between $\overline{AO}$ and $\overline{OB}$ is the sector of the circle.

Repeat the same steps for (ii) and (iii)

7. Angle formed by minor arc, $x = 120^\circ$ radius of the circle, $r = 6$ cm

$$\begin{aligned}\text{arc length} &= \frac{x}{360} \times 2\pi r \\ &= \frac{120}{360} \times 2 \times 3.14 \times 6 \\ &= 4 \times 3.14 \\ &= 12.56 \text{ cm}\end{aligned}$$

8. Diameters = 36 cm

$$\begin{aligned}\text{radius} \quad r &= \frac{d}{2} = \frac{36}{2} \text{ cm} \\ r &= 18 \text{ cm}\end{aligned}$$

$$x = 60^\circ$$

$$\text{arc length} = \frac{x}{360} \times 2\pi r$$

$$\begin{aligned}&= \frac{60}{360} \times 2 \times 3.14 \times 18 \\ &= 6 \times 3.14 \\ &= 18.84 \text{ cm}\end{aligned}$$

9. Angle formed by minor arc:

$$x = 360 - 110^\circ$$

$$x = 250^\circ$$

$$\begin{aligned}\text{arc length} &= \frac{250}{360} \times 2\pi r \\ &= \frac{250}{360} \times 2 \times 3.14 \times 18 \\ &= 25 \times 3.14 \\ &= 78.5 \text{ cm}\end{aligned}$$

Helpful Hint

Sum of angles of all minor and major sectors is equal to 360° . The complete rotation in a circle is 360° degrees.

10. (a) $x = 90^\circ$

$$r = 16 \text{ cm}$$

$$\begin{aligned}\text{Sector area} &= \frac{x}{360} \times \pi r^2 \\ &= \frac{90}{360} \times 3.14 \times 16^2 \\ &= 64 \times 3.14 \\ &= 200.96 \text{ cm}^2\end{aligned}$$

b. $x = 135^\circ$

$$r = 8 \text{ m}$$

$$\begin{aligned}\text{Sector area} &= \frac{x}{360} \times \pi r^2 \\ &= \frac{135}{360} \times 3.14 \times 8^2 \\ &= 24 \times 3.14 \\ &= 75.36 \text{ m}^2\end{aligned}$$

c. $x = 40^\circ$

$$r = 18 \text{ m}$$

$$\begin{aligned}\text{Sector area} &= \frac{x}{360} \times \pi r^2 \\ &= \frac{40}{360} \times 3.14 \times 18^2 \\ &= 36 \times 3.14 \\ &= 113.04 \text{ m}^2\end{aligned}$$

Exercise 10B

1. (i) In the given $\triangle ABC$,
 $= 3$ cm
 $= 5$ cm
 $= ?$

Helpful Hint

Hypotenuse is always the side opposite to the right angle of a triangle.

According to Pythagoras' theorem:

$$c^2 = a^2 + b^2$$

$$c^2 = 3^2 + 5^2$$

$$c^2 = 9 + 25$$

$$c^2 = 34$$

$$\sqrt{c^2} = \sqrt{34}$$

$$c = 5.8 \text{ cm}$$

- (ii) In the given $\triangle ABC$,
 $= ?$

$$= 6 \text{ cm}$$

$$= 10 \text{ cm}$$

$$10^2 = 6^2 + b^2$$

$$100 = 36 + b^2$$

$$100 - 36 = b^2$$

$$b^2 = 64$$

$$\sqrt{b^2} = \sqrt{64}$$

$$b = 8 \text{ cm}$$

- (iii) In the given $\triangle ABC$,

$$= 2 \text{ cm}$$

$$= 6 \text{ cm}$$

$$= ?$$

$$c^2 = a^2 + b^2$$

$$c^2 = 2^2 + 6^2$$

$$c^2 = 4 + 36$$

$$c^2 = 40$$

$$\sqrt{c^2} = \sqrt{40}$$

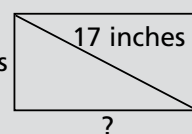
$$c = 6.32 \text{ cm}$$

2.

Helpful Hint

Draw the figure to understand its dimensions.

8 inches



$$= 8 \text{ inches}$$

$$= ?$$

$$= 17 \text{ inches}$$

$$c^2 = a^2 + b^2$$

$$17^2 = 8^2 + b^2$$

$$289 - 64 = b^2$$

$$b^2 = 225$$

$$b = 15$$

$\therefore$ The length is 15 inches

3. $= 8 \text{ cm}$

$$= 6 \text{ cm}$$

$$= ?$$

$$c^2 = a^2 + b^2$$

$$c^2 = 8^2 + 6^2$$

$$c^2 = 64 + 36$$

$$c^2 = 100$$

$$\sqrt{c^2} = \sqrt{100}$$

$$c = 10 \text{ cm}$$

4.

Helpful Hint

Hypotenuse is always the longest side of a right-angled triangle.

$$= 13 \text{ cm}$$

$$= 5 \text{ cm}$$

$$= 12 \text{ cm}$$

LHS RHS

$$c^2 = a^2 + b^2$$

To verify Pythagoras' theorem, right hand side of the theorem should be equal to its left hand side.

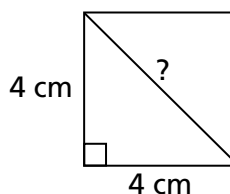
$$\text{LHS } c^2 = 13^2 = 169$$

$$\text{RHS } a^2 + b^2 = 5^2 + 12^2 = 25 + 144 = 169$$

Since LHS = RHS

Pythagoras' theorem is verified.

5.



$$= 4 \text{ cm}$$

$$= 4 \text{ cm}$$

$$= ?$$

$$^2 = 2^2 + 2^2$$

$$^2 = 4^2 + 4^2$$

$$^2 = 16 + 16$$

$$^2 = 32$$

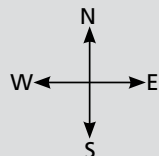
$$= \sqrt{32}$$

$$= 5.66 \text{ cm}$$

6.

Helpful Hint

Cardinal directions



Lets A be the starting point of the truck then C is the find point where the truck stops. The distance between starting point and ending point is represented by hypotenuse.

According to Pythagoras' theorem:

$$^2 = ^2 + ^2$$

$$^2 = 100^2 + 75^2$$

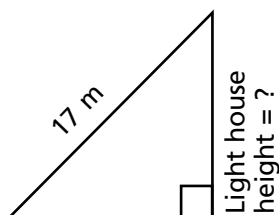
$$^2 = 10000 + 5625$$

$$^2 = 15625$$

$$\sqrt{c^2} = \sqrt{15625}$$

$$= 125 \text{ km}$$

7.



Shadow 15 m long

$$= ?$$

$$= 15 \text{ m}$$

$$= 17 \text{ m}$$

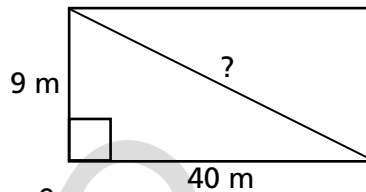
$$^2 = 2^2 + 2^2$$

$$17^2 = 2^2 + 15^2$$

$$^2 = 289 - 225 = 64$$

$$= 8 \text{ m}$$

8.



$$= 9 \text{ m}$$

$$= 40 \text{ m}$$

$$= ?$$

$$^2 = 2^2 + 2^2$$

$$^2 = 9^2 + 40^2$$

$$^2 = 81 + 1600$$

$$^2 = 1681$$

$$= 41 \text{ m}$$

9.

$$= 8 \text{ m}$$

$$= 15 \text{ m}$$

$$= ?$$

$$^2 = 2^2 + 2^2$$

$$^2 = 8^2 + 15^2$$

$$^2 = 64 + 225$$

$$^2 = 289$$

$$= 17 \text{ m}$$

Exercise 10C

1. (i) isosceles

(ii) four

(iii) regular polygon

(iv) $\frac{4}{3} \pi^3$

(v) 2 (two)

2. (i) False

volume of a pyramid = $\frac{1}{3} \times \text{base area} \times \text{height}$

- (ii) True
(iii) True
(iv) False

Area of the base of cone is πr^2

- (v) True

3. Volume of a pyramid = $\frac{1}{3} \times \text{base area} \times \text{height}$

$$v = \frac{1}{3} \times 18 \times 18 \times 26$$

Helpful Hint

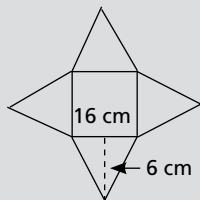
Since the base of the pyramid is square so the area of square is calculated.

$$v = 2808 \text{ m}^3$$

- (ii) There are 5 surfaces of the given pyramid. There are four triangles and one square.

Helpful Hint

You can draw net of the pyramid to calculate its surface area:



$$\begin{aligned} \text{Total surface area} &= 4 \times \text{area of triangle} + \text{base area} \\ &= 4 \times \frac{1}{2} \times 16 \times 6 + 16 \times 16 \\ &= 192 + 256 \\ &= 960 \text{ cm}^2 \end{aligned}$$

4. Volume of pyramid = 12000 cm^3
 $\frac{1}{3} \times \text{base area} \times \text{height} = 12000$
 $\frac{1}{3} \times 20 \times 20 \times \text{height} = 12000$

$$= \frac{12000 \times 3}{400}$$

$$= 90 \text{ cm}$$

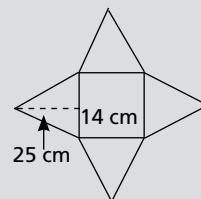
The height of the pyramid should be 90 cm to be filled with 12000 cm^3 of sand.

5. Volume = $\frac{1}{3} \times \text{base area} \times \text{height}$

$$\begin{aligned} &= \frac{1}{3} \times 14 \times 14 \times 24 \\ &= 1568 \text{ cm}^3 \end{aligned}$$

Total surface area = 4 x area of triangle + base area

Helpful Hint



$$\begin{aligned} &= 4 \times \frac{1}{2} \times 14 \times 25 + 14 \times 14 \\ &= 700 + 196 \\ &= 896 \text{ cm}^2 \end{aligned}$$

6. Volume = $\frac{1}{3} \times \text{base area} \times \text{height}$

$$= \frac{1}{3} \times 24 \times 24 \times 12$$

$$= 2304 \text{ cm}^3$$

7. Base area = 36000 cm^2

$$\text{Volume} = 7200 \text{ cm}^3$$

$$\begin{aligned} \text{Volume} &= \frac{1}{3} \times \text{base area} \times \text{height} \\ \frac{1}{3} \times \text{base area} \times \text{height} &= 7200 \text{ cm}^3 \\ \frac{1}{3} \times \frac{1200}{3600} \times \text{height} &= 7200 \\ &= \frac{7200}{1200} \\ &= 6 \text{ cm} \end{aligned}$$

8. Area of base of a cone = $\pi r^2 = 38.50 \text{ sq.cm}$

$$\pi r^2 = 38.50$$

$$\frac{22}{7} r^2 = 38.50$$

$$r^2 = 38.50 \times \frac{7}{22} = 12.25$$

$$\begin{aligned} \sqrt{r^2} &= \sqrt{12.25} \\ &= 3.5 \text{ cm} \end{aligned}$$

Given that, height = 3 times of radius

$$\therefore \text{height} = 3 \times 3.5 = 10.5 \text{ cm}$$

$$\text{volume of cone} = \frac{1}{3} \times \text{area of base} \times \text{height}$$

$$= \frac{1}{3} \times 30.50 \times 10.5$$

$$= 134.75 \text{ cm}^3$$

9.

$$= 3 \text{ cm}$$

$$= 5 \text{ cm}$$

total surface area = ?

$$\text{total surface area} = \pi + \pi^2$$

$$= \pi (+)$$

$$= \pi \times 3 (5 + 3) = \frac{22}{7} \times 3 \times 8$$

$$= \pi 24 = 24\pi \text{ cm}^2$$

$$\text{or } 24 \times \frac{22}{7} = 75.43 \text{ cm}^2$$

$$10. = 12 \text{ cm}$$

$$= 5 \text{ cm}$$

volume of a cone = ?

$$\text{volume} = \frac{1}{3} \pi^2$$

$$= \frac{1}{3} \times \frac{22}{7} \times (5)^2 \times 12$$

$$= \frac{6600}{21} = 314.3 \text{ cm}^3$$

$$11. = 5 \text{ m}$$

$$= 12 \text{ m}$$

$$= 13 \text{ m}$$

(i) area of canvas = curved surface area

$$= \pi$$

$$= 3.14 \times 12 \times 13$$

$$= 489.84 \text{ cm}^2$$

13. Surface area of sphere = 616 sq.cm

$$4 \pi^2 = 616$$

$$4 \times \frac{22}{7} \times^2 = 616$$

$$^2 = \frac{616 \times 7}{4 \times 22} = 49$$

$$= 7 \text{ cm}$$

$$(ii) \text{ volume} = \frac{1}{3} \pi^2$$

$$= \frac{1}{3} \times 3.14 \times (12)^2 \times 5$$

$$= 753.6 \text{ cm}^3$$

12. Surface area of sphere = 324π sq.cm

Volume = ?

Helpful Hint

To find out volume of sphere we need to find out the radius first.

$$\text{Surface area of sphere} = 4 \pi^2$$

$$4 \pi^2 = 324\pi$$

$$^2 = \frac{324}{4} = 81$$

$$\sqrt{r^2} = \sqrt{81}$$

$$= 9 \text{ cm}$$

$$\text{Volume of sphere} = \frac{4}{3} \pi^3$$

$$= \frac{4}{3} (9 \times 9 \times 9) \pi = \frac{4}{3} \times 729\pi$$

$$= 972\pi \text{ cm}^3$$

Volume of sphere with diameter 1 cm ,
= 0.5 cm

$$\text{volume} = \frac{4}{3} \pi^3$$

$$= \frac{4}{3} \pi (0.5)^3$$

$$= 0.167\pi \text{ cm}^3$$

Helpful Hint

$$\text{no. of small spheres} = \frac{\text{volume of big sphere}}{\text{volume of small sphere}}$$

$$\text{no. of small spheres} = \frac{324\pi}{0.1667\pi}$$

$$= 5832$$

$$\begin{aligned}
 \text{volume of the sphere} &= \frac{4}{3} \pi r^3 \\
 &= \frac{4}{3} \pi \times 7 \times 7 \times 7 \times 3 \\
 &= \frac{4}{3} \times \frac{22}{7} \times 7 \times 7 \times 7 \times 3 \\
 &= 4312 \text{ cm}^3
 \end{aligned}$$

Multiple Choice Questions

1. D

$$\begin{aligned}
 \text{Volume} &= \frac{4}{3} \pi r^3 \\
 &= \frac{4}{3} \times \frac{22}{7} \times (10)^3 \\
 &= 4190.48 \text{ cm}^3
 \end{aligned}$$

2. B

$$\begin{aligned}
 \text{Surface area} &= 4 \pi r^2 \\
 &= 4 \times \frac{22}{7} \times 52 \\
 &= 314
 \end{aligned}$$

3. C

$$\begin{aligned}
 \text{Volume} &= \frac{1}{3} \pi r^2 h \\
 &= \frac{1}{3} \times \frac{22}{7} \times (2) \times 4 \\
 &= 16.76 \text{ cm}^3
 \end{aligned}$$

4. D

$$\begin{aligned}
 \text{Surface area} &= \pi r^2 + \pi r h \\
 &= \pi (r^2 + r h) = \frac{22}{7} \times 5 (5 + 7) \\
 &= 188.57 \text{ cm}^2
 \end{aligned}$$

5. A



GEOMETRY: CONGRUENCE AND SIMILARITY

Exercise 11

1. (i) congruent
(ii) four
(iii) similar
(iv) congruent
(v) similar

2. (i) False

Reason: Any two triangles are congruent, if their angles and corresponding sides are equal.

- (ii) True

Reason: Triangle can be enlarged by increasing the length of its sides, that will change the area of the triangle.

- (iii) False

Reason: Any two triangles are congruent if two sides and included angle are equal.

- (iv) True

Reason: Two triangles are similar if all the corresponding angles are equal and corresponding sides are in the same ratio.

- (v) False

Reason: Congruent triangles are of the same shape and same size, while similar triangles have same shape but different size.

3. $m\angle A = m\angle Q$
 $m\angle B = m\angle R$
 $m\angle C = m\angle P$
 $\overline{mAB} = \overline{mQR}$
 $\overline{mBC} = \overline{mRP}$
 $\overline{mCA} = \overline{mPQ}$

4. (i) Congruent (SSS)
(ii) Congruent (SAS)
(iii) Not congruent
(iv) Congruent (RHS)
(v) Congruent (ASA)

5. (i) $m\overline{AC} = m\overline{XZ}$
 $x = 6$

- (ii) $m\angle RPQ = m\angle WUV$
 $x = 25^\circ$

- (iii) $m\angle XZY = m\angle RTS$
 $a = 45^\circ$

- (iv) $m\angle XZY = m\angle BAC$
 $x = 60^\circ$

$$m\angle XYZ = m\angle BCA$$

$$y = 80^\circ$$

6. (i) Similar, because the sides are in ratio 1:2

- (ii) Similar, because ratio of the sides is 1:3

- (iii) Not Similar

The ratio of the length of sides is not equal.

- (iv) Similar, the ratios of the length of the sides are proportional as 1:2

Multiple Choice Questions 15

1. D

2. C

The case of congruency is ASA.

3. D

$m\angle S$ should be equal to $m\angle E$.

4. D

5. B



PRACTICAL GEOMETRY AND TRANSFORMATION

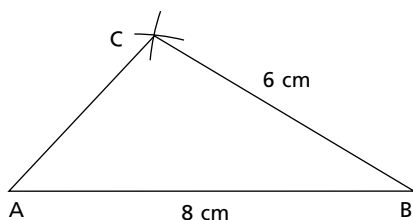
Exercise 12A

1.

Helpful Hint:

(i), (ii) and (iii) follow the same steps of construction.

- (i) $m \overline{AB} = 8 \text{ cm}$, $m \overline{BC} = 6 \text{ cm}$, $\overline{CA} = 4 \text{ cm}$



Steps of construction:

Step 1: Draw $\overline{AB} = 8 \text{ cm}$.

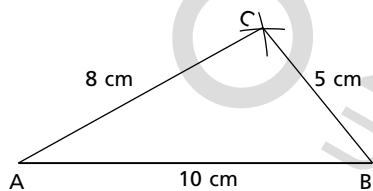
Step 2: With B as centre draw an arc with radius 6 cm.

Step 3: With A as centre, draw an arc with radius 4 cm.

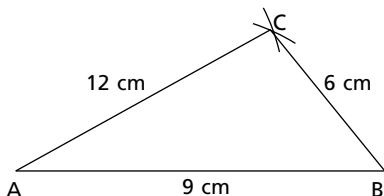
Step 4: Join C to A and B

$\triangle ABC$ is the required triangle.

- (ii) $m \overline{AB} = 10 \text{ cm}$, $m \overline{BC} = 5 \text{ cm}$, $\overline{CA} = 8 \text{ cm}$.



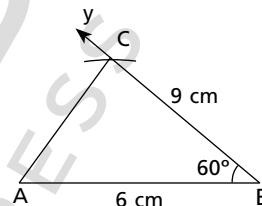
- (iii) $\overline{AB} = 9 \text{ cm}$, $m \overline{BC} = 6 \text{ cm}$, $\overline{CA} = 12 \text{ cm}$



Helpful Hint:

(iv), (v) and (vi) follow the same steps of construction.

- (iv) $m \overline{AB} = 6 \text{ cm}$, $m \overline{BC} = 9 \text{ cm}$, $m \angle B = 60^\circ$



Steps of construction:

Steps 1: Draw $\overline{AB} = 6 \text{ cm}$

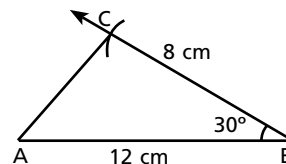
Steps 2: At point B, draw $m \angle ABC = 60^\circ$

Step 3: With B as center, draw an arc of radius 9 cm to cut BY at C.

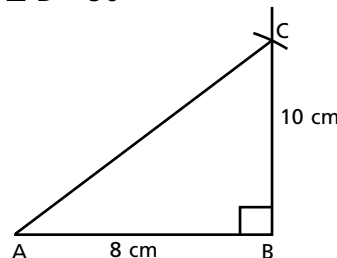
Step 4: Join A to C.

$\triangle ABC$ is the required triangle.

- (v) $m \overline{AB} = 12 \text{ cm}$, $m \overline{BC} = 8 \text{ cm}$
 $m \angle B = 30^\circ$



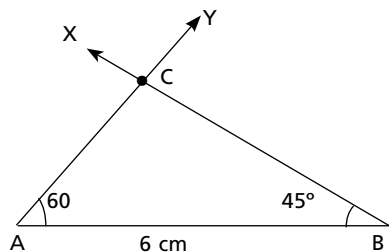
- (vi) $m \overline{AB} = 8 \text{ cm}$, $m \overline{BC} = 10 \text{ cm}$,
 $m \angle B = 90^\circ$



Helpful Hint:

(vii) and (viii) follow the same steps of construction.

(vii) $m \overline{AB} = 6 \text{ cm}$, $m \angle A = 60^\circ$, $m \angle B = 45^\circ$

**Steps of construction:**

Step 1: Draw $m \overline{AB} = 6 \text{ cm}$

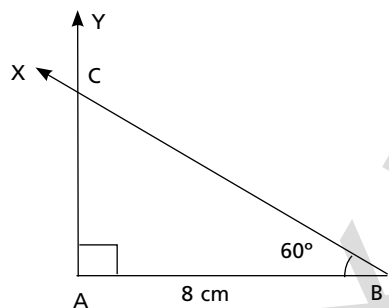
Step 2: At point A, draw $m \angle BAX = 60^\circ$

Step 3: At point B, draw $m \angle ABY = 45^\circ$

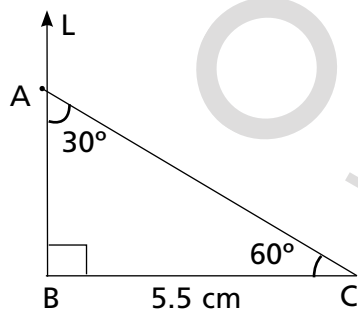
Step 4: Mark point C at the intersection of AY and BX.

$\triangle ABC$ is the required triangle.

(viii) $m \overline{AB} = 8 \text{ cm}$, $m \angle A = 90^\circ$, $m \angle B = 60^\circ$



(ix) $m \overline{BC} = 5.5 \text{ cm}$, $m \angle A = 30^\circ$, $m \angle B = 90^\circ$

**Steps of construction:**

Step 1: Draw $\overline{BC} = 5.5 \text{ cm}$

Step 2: Construct an angle of 90° at B.

Step 3: Since sum of the angles in a triangle is

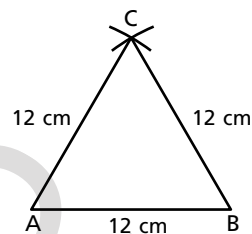
180° , i.e. $\angle B + \angle A + \angle C = 180^\circ$, Make angle of 60° at C meeting BL at A.

Step 4: Measure $\angle A$. It will be 30° .

$\triangle ABC$ is the required triangle.

2. Construct the following equilateral triangles:

(i) $m \overline{AB} = 12 \text{ cm}$

**Steps of construction:**

Step 1: Draw $m \overline{AB} = 12 \text{ cm}$

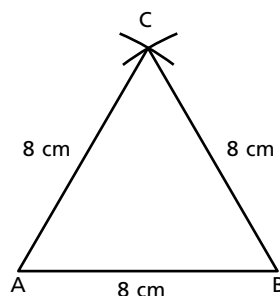
Step 2: With A as centre, draw an arc with radius 12 cm.

Step 3: With B as centre, draw another arc with radius 12 cm. So that it intersects the previous arc at C.

Step 4: Join C to A and B.

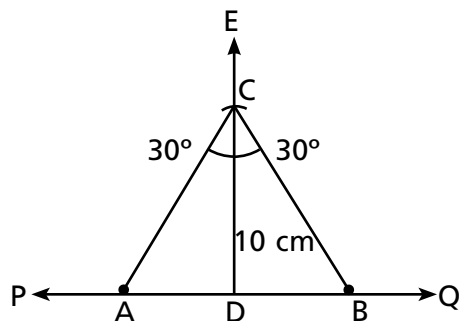
$\triangle ABC$ is the required triangle.

(ii) $m \overline{AB} = 8 \text{ cm}$



Follow the same steps as given in Q4 (i)

(iii) altitude $\overline{CD} = 10 \text{ cm}$



Steps of construction:

Step 1: Draw $m \overline{PQ} = 10$ cm

Step 2: Mark a point D at the centre $\overline{AB}$.

Step 3: Draw a perpendicular $\overline{DE}$ on $\overline{AB}$.

Step 4: With D as centre, draw an arc with radius 10 cm intersecting the perpendicular at C.

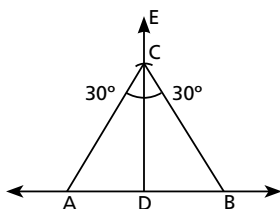
Step 5: At C draw two angles of 30° on either side of CD. $m \angle BCD$ and

$$m \angle ACD = 30^\circ$$

Step 6: Mark point A and B, where the arms of the angles intersect $\overline{PQ}$.

$\triangle ABC$ is the required equilateral triangle.

(iv) altitude $\overline{CD} = 15$ cm



Steps of construction:

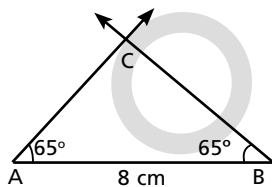
Follow the same steps as given in Q4 (iii).

3.

Helpful Hint:

In an isosceles triangle base angles are equal, therefore, $\angle B = 65^\circ$

(i) $m \overline{AB} = 8$ cm, $m \angle A = 65^\circ$



Steps of construction:

Step 1: Draw $m \overline{AB} = 8$ cm

Step 2: Construct $m \angle BAC = 65^\circ$

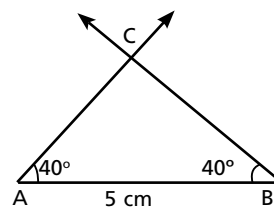
Step 3: Construct $m \angle ABC = 65^\circ$

Step 4: Extend the arms of the angles, so that they meet each other at point C.

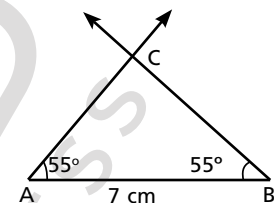
$\triangle ABC$ is the required triangle.

Follow the steps of construction given in Q5 (i), for (ii), (iii), (iv).

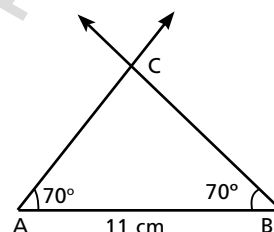
(ii) $m \overline{AB} = 5$ cm, $m \angle A = 40^\circ$



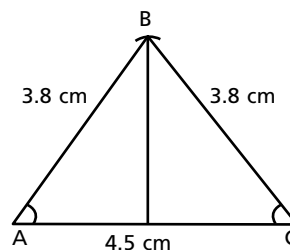
(iii) $m \overline{AB} = 7$ cm, $m \angle A = 70^\circ$



(iv) $m \overline{AB} = 11$ cm, $m \angle A = 70^\circ$



4.



Steps of construction:

Step 1: Draw $\overline{AC} = 4.5$ cm

Step 2: With centre C and radius 3.8 cm draw an arc above $\overline{AC}$.

Step 3: with centre A, draw another arc with the same radius, intersecting the previous arc with at point B.

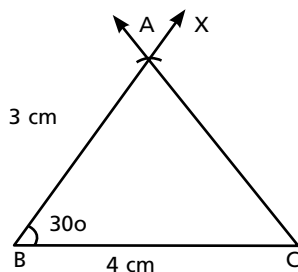
Step 4: Join A and B. Join B and C.

$\triangle ABC$ is the required triangle.

Since, $\triangle ABC$ is an isosceles triangle, its base angles are equal.

$$\therefore \angle BAC = \angle ACB$$

5.



Steps of construction:

Step 1: Draw $\overline{BC} = 4$ cm

Step 2: At point B, draw $m\angle ABC = 30^\circ$

Step 3: With B as centre, draw an arc with radius 3 cm to cut BX at A.

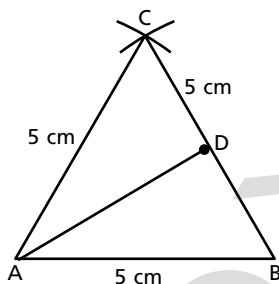
Step 4: Join A to C.

$\triangle ABC$ is the required triangle.

6. Follow the steps of construction as in Q2(i) taking each side = 5 cm

$\triangle ABC$ is an equilateral triangle.

Now, Take D as the centre of $\overline{BC}$.

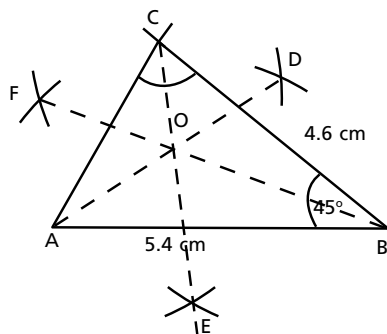


Join A and D.

Measure $\angle ADB$.

$$m\angle ADB = 90^\circ$$

7.



Steps of construction:

Step 1: Draw $\overline{AB} = 5.4$ cm

Step 2: Make an angle of 45° at point B.

Helpful Hint:

Using compass and ruler construct an angle of 90° and bisect it to get 45° .

Step 3: Using a compass cut $\overline{BC} = 4.6$ cm.

Step 4: Join A and C.

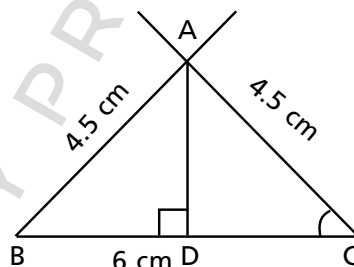
$\triangle ABC$ is the required triangle.

Step 5: Using a pair of compasses draw the angle bisectors at $\angle A$, $\angle B$, and $\angle C$.

Step 6: Join A and D, B and F, C and E.

Angle bisectors $\overline{AD}$, $\overline{BF}$, and $\overline{CE}$ pass through the same point O.

8.



Steps of construction:

Step 1: Draw $\overline{BC} = 6$ cm.

Step 2: With B as centre and radius 4.5 cm draw an arc above $\overline{BC}$.

Step 3: With C as centre and radius 4.5 cm draw another arc, intersecting the previous arc at point A.

Step 4: Join A to B and A to C.

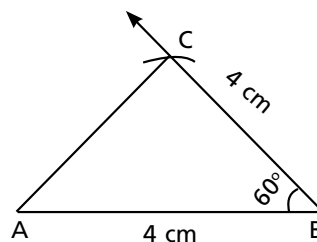
$\triangle ABC$ is the required triangle.

Step 5: Mark a point D in the centre of $\overline{BC}$.

Step 6: Join A and D. Measure $\angle ADB$.

$$\angle ADB = 90^\circ$$

9.



Step of construction:

Step 1: Draw $\overline{AB} = 4 \text{ cm}$.

Step 2: Draw $\angle ABL = 60^\circ$ at point B.

Step 3: With centre B and radius 4 cm draw an arc on $\overline{BL}$ at C.

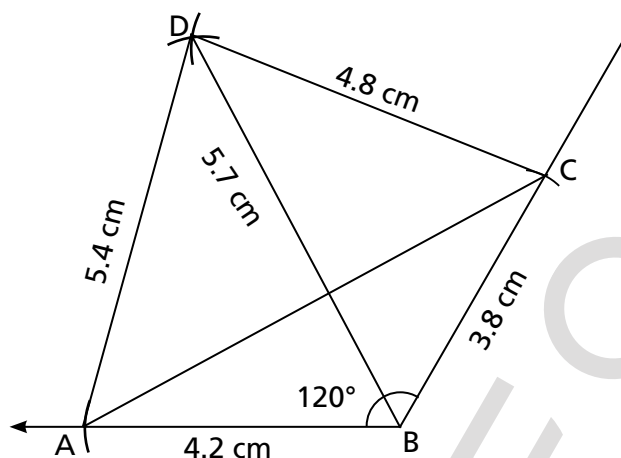
$$\angle ABC = 60^\circ$$

Step 4: Join A to C.

$\triangle ABC$ is the required triangle.

Exercise 12B

1.



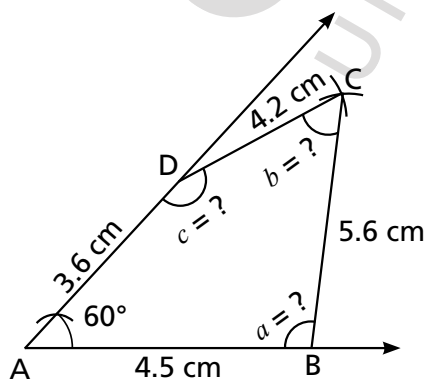
Measurement of the diagonals:

$m \overline{AC} = 6.9 \text{ cm}$

$m \overline{BD} = 5.7 \text{ cm}$

Follow the steps given in the book (construction 1) according to the measurements given in the question.

2.



Follow the steps given in the book (construction 1) according to the measurements given in the question.

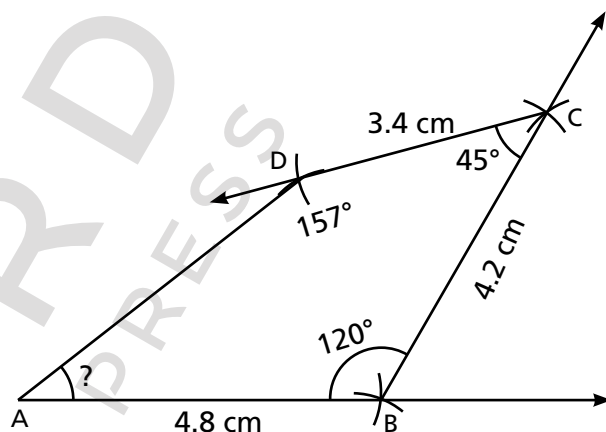
The other three angles are

$$a = 97^\circ, \quad b = 47^\circ, \quad \text{and } c = 156^\circ$$

$\therefore$ the sum of the angles is

$$97^\circ + 47^\circ + 156^\circ = 300^\circ$$

3. Let $\overline{AB} = 4.8 \text{ cm}$, $\overline{BC} = 4.2 \text{ cm}$, $\overline{CD} = 3.4 \text{ cm}$ then $\angle B = 120^\circ$, $\angle C = 45^\circ$

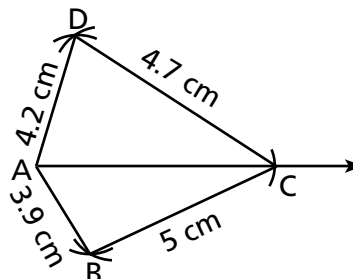


Follow the steps given in the book (construction 2) according to the measurements given in the question.

$$a = 38^\circ, \quad d = 157^\circ$$

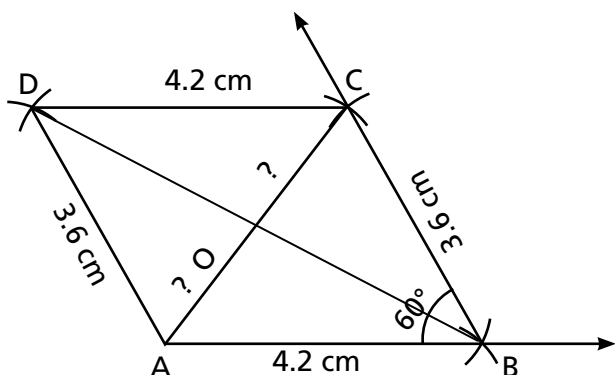
the sum is $39^\circ + 157^\circ = 195^\circ$

4. Let following be the sides of the quadrilateral $\overline{AB} = 3.9 \text{ cm}$, $\overline{BC} = 5 \text{ cm}$, $\overline{CD} = 4.7 \text{ cm}$, and $\overline{DA} = 4.2 \text{ cm}$



Follow the steps given in the book (construction 3) according to the measurements given in the question.

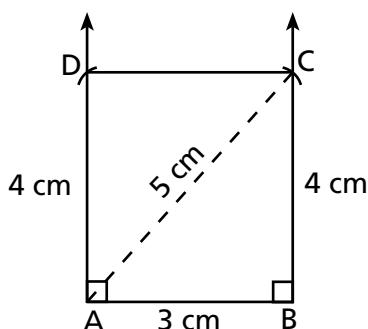
5.



Follow the steps given in the book (construction 11) according to the measurements given in the question.

$$\left. \begin{array}{l} m \overline{AO} = 2 \text{ cm} \\ m \overline{CO} = 2 \text{ cm} \end{array} \right\} \text{yes, both are equal}$$

6. Let $m \overline{AB} = 3 \text{ cm}$ and $m \overline{BC} = 4 \text{ cm}$



Follow the steps given in the book (construction 9) according to the measurements given in the question.

Proof :

$$AB^2 + BC^2 = AC^2,$$

LHS

$$AB^2 + BC^2 = 3^2 + 4^2 = 9 + 16 = 25$$

RHS

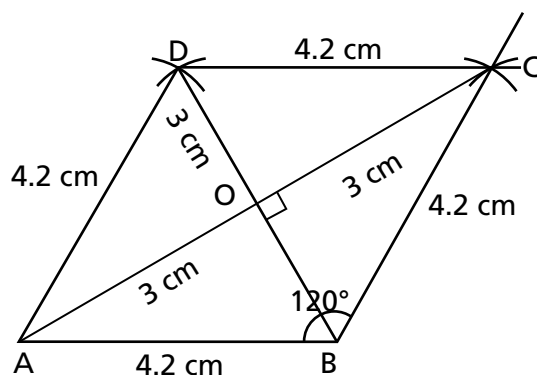
$$AC^2 = 5^2 = 25$$

Since LHS = RHS

Hence, it is proved that

$$AB^2 + BC^2 = AC^2$$

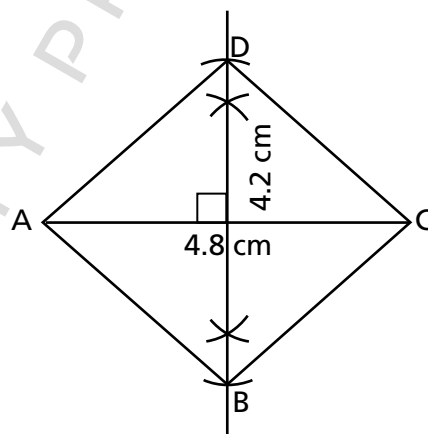
7.



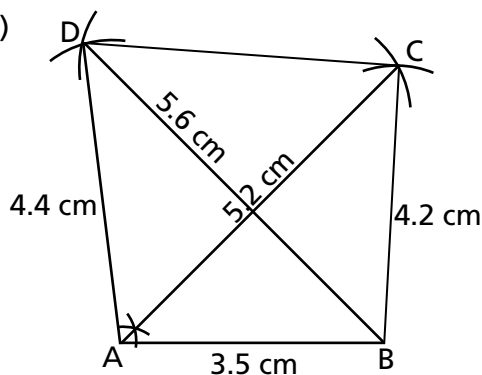
Follow the steps given in the book (construction 13) according to the measurements given in the question.

$$\begin{array}{l} m \overline{AO} = m \overline{CO} = m \overline{BO} = m \overline{DO} = 3 \text{ cm} \\ m \angle BOC = 90^\circ \end{array}$$

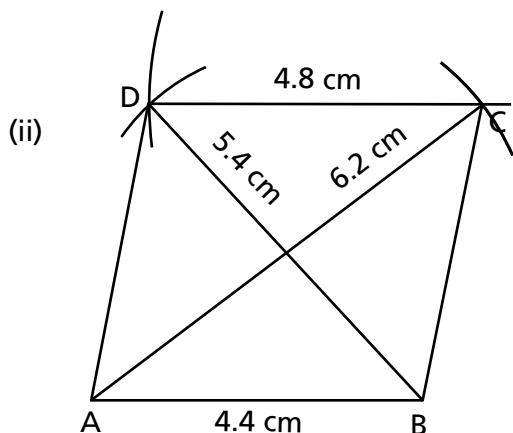
8. Let the rhombus be ABCD, and its diagonals are $\overline{AC}$ and $\overline{BD}$



9. (i)

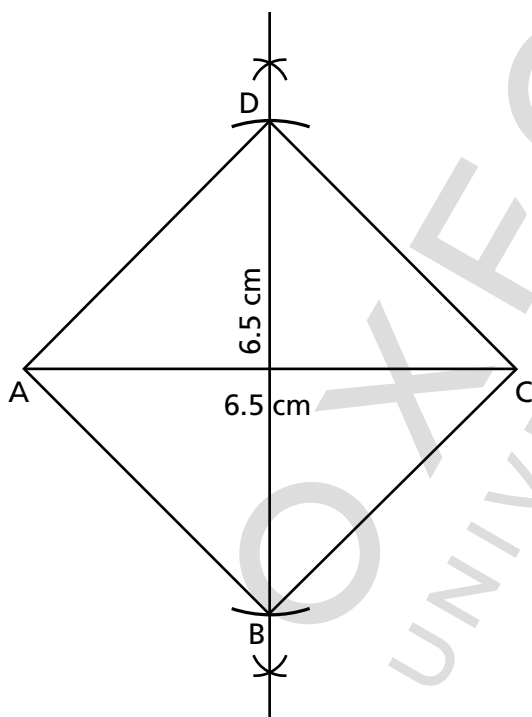


Follow the steps given in the book (construction 4) according to the measurements given in the question.



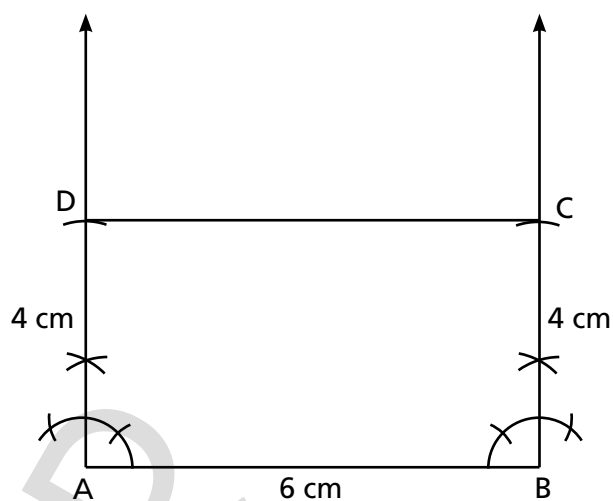
Follow the steps given in the book (construction 3) according to the measurements given in the question.

10.



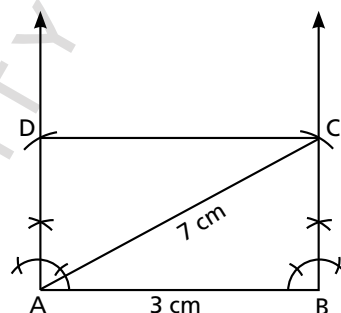
Follow the steps given in the book (construction 11) according to the measurements given in the question.

11.



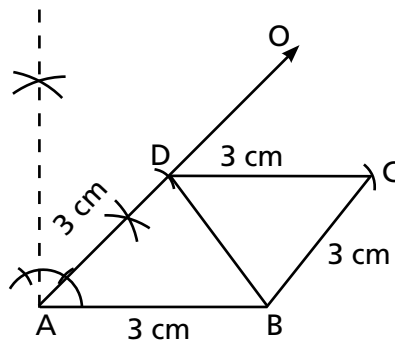
Follow the steps given in the book (construction 9) according to the measurements given in the question.

12. Let the rectangle be ABCD, and $AB = 3$ cm, $AC = 7$ cm



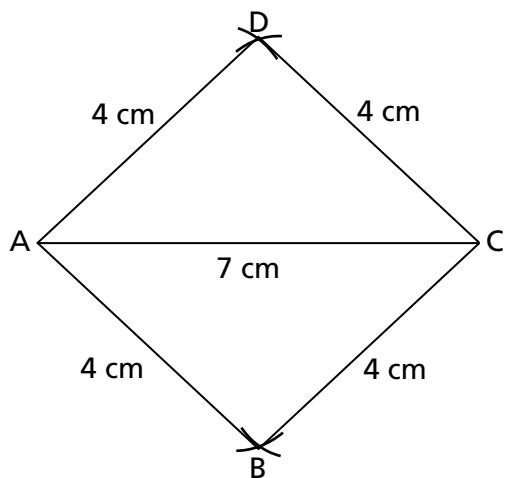
Follow the steps given in the book (construction 10) according to the measurements given in the question.

13.



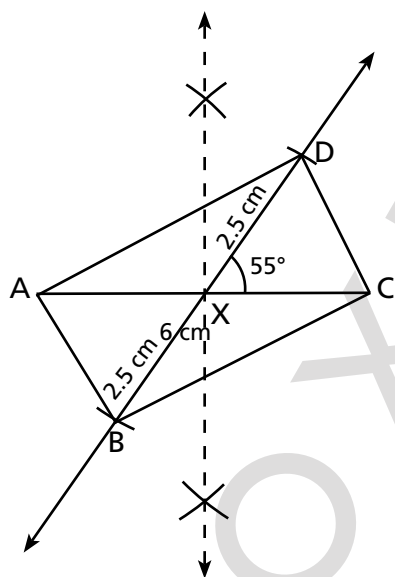
Follow the steps given in the book (construction 13) according to the measurements given in the question.

14.



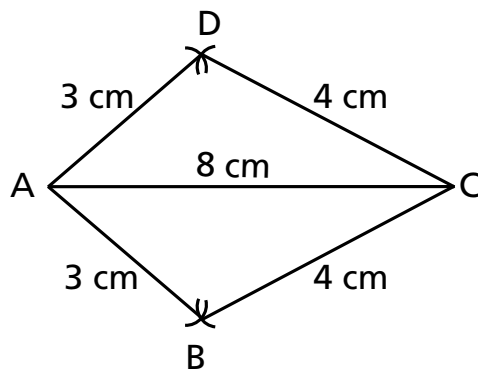
Follow the steps given in the book (construction 14) according to the measurements given in the question.

15.



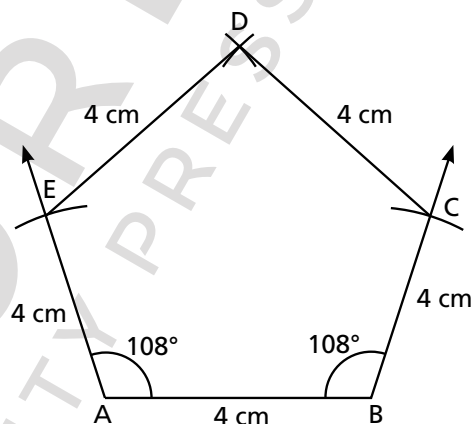
Follow the steps given in the book (construction 12) according to the measurements given in the question.

16.



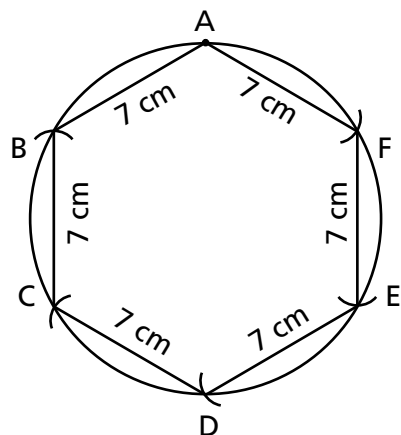
Follow the steps given in the book (construction 15) according to the measurements given in the question.

17.



Follow the steps given in the book (construction 17) according to the measurements given in the question.

18.



Follow the steps given in the book

(construction 17) according to the measurements given in the question.

Exercise 12C

1. (i) Since A is common vertex in both the images, the centre of rotation is A.
To find the angle of rotation, you need to join corresponding vertices of both the images with centre of rotation. In this case they are already joined.

Now find out the angle formed by BAB' and CAC' .

Since the measure of angle is 90° , the angle of rotation is 90° anticlockwise.

- (ii) U is the centre of rotation. The angle of rotation is 90° clockwise.
(iii) B is the centre of rotation. The angle of rotation is 180° clockwise.

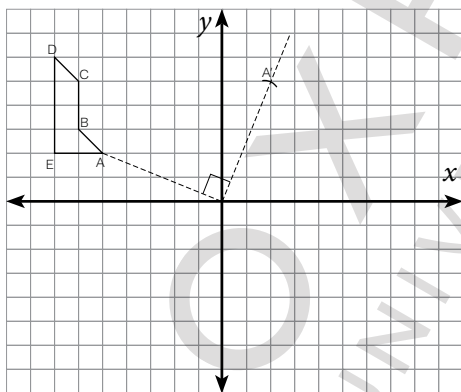
2. (i) Name the vertices of given shape as A, B, C, D, and E.

Join A with origin O.

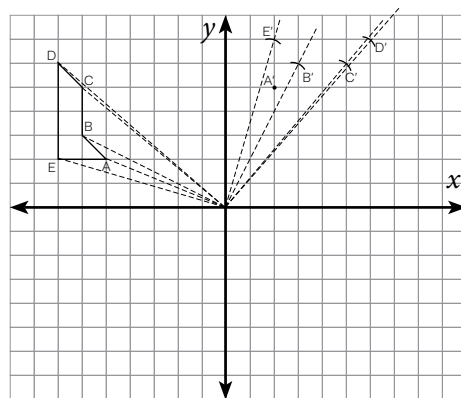
Make an angle of 90° at point O.

Mark an arc at the same distance as from O to A, on the line.

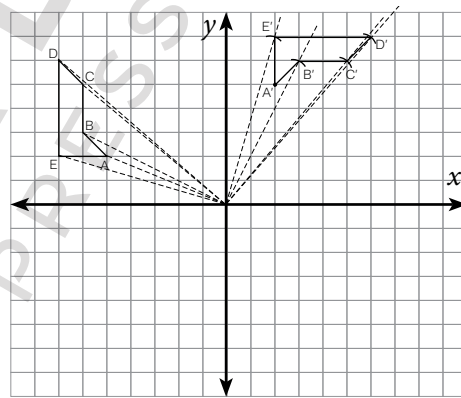
This new point is A'



Now, repeat above steps for B' , C' , D' , and E' .



Join A' to B' , B' to C' , C' to D' , D' to E' , and E' to A' to draw the new rotated image.



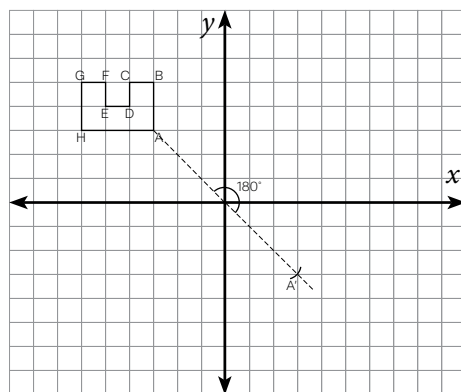
- (ii) Name the vertices of image as A, B, C, D, E, F, G, and H.

Join A to origin O.

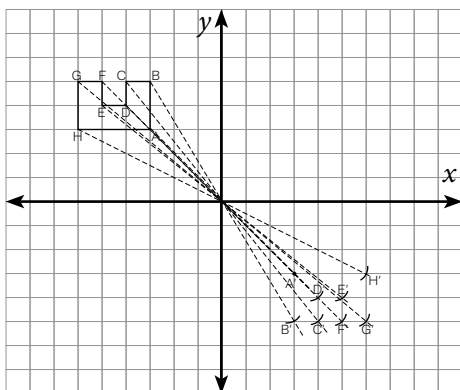
Make angle of 180° at O.

Make an arc at the same distance as from O to A on the line

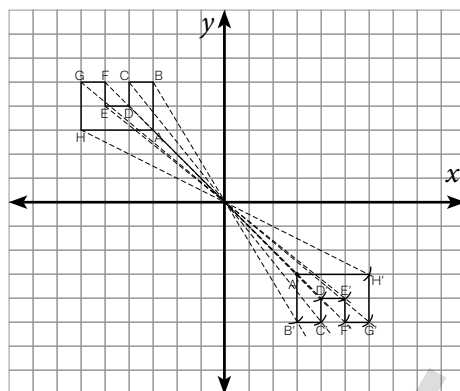
This new point is A'



Now repeat above steps for B' , C' , D' , E' , F' , G' , and H' .

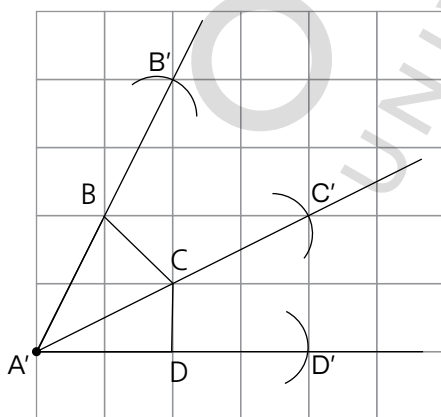


Join A' to B' , B' to C' , C' to D' , D' to E' , E' to F' , F' to G' , G' to H' , and H' to A' , to draw the new rotated image



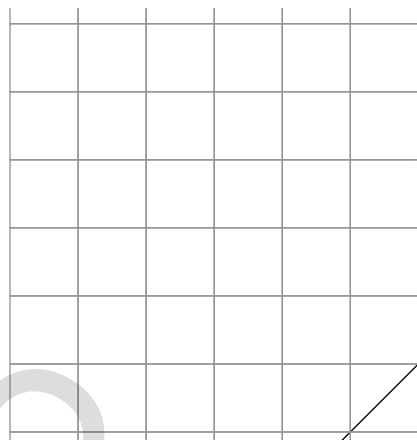
3. (i) The centre of enlargement is A. measure the length from A to B. Join A to B and produce the line further.

Since the scale factor is 2, multiply the measurement of length by 2 and mark a new point B' with the measurement.

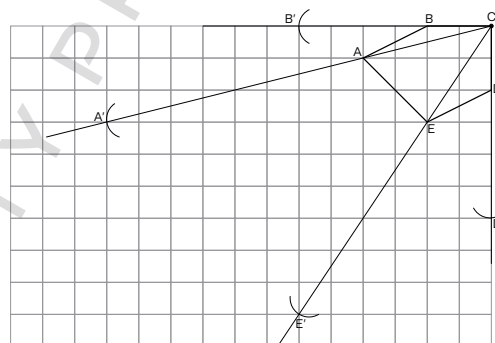


Repeat above, steps to mark the point C' D' .

Join A to B' , B' to C' , C' to D' , and D' to A .

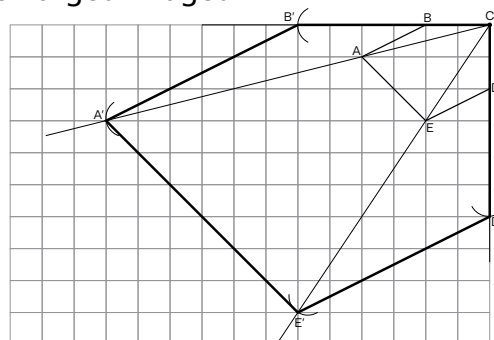


- (ii) The centre of enlargement is C. Join C to A and produce it further. Measure length from C to A and multiply it by 3. Make new point A' with new length from C.



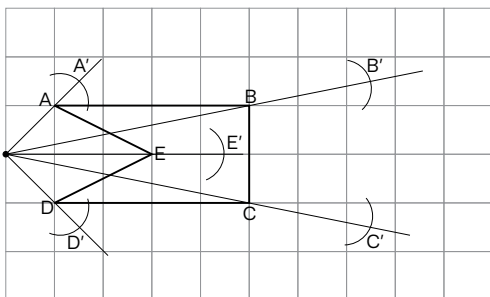
Repeat above, steps to mark the point B' , D' , and E' .

Join A' to B' , B' to C' , C' to D' , D' to E' , and E' to A' to get the new enlarged imaged.

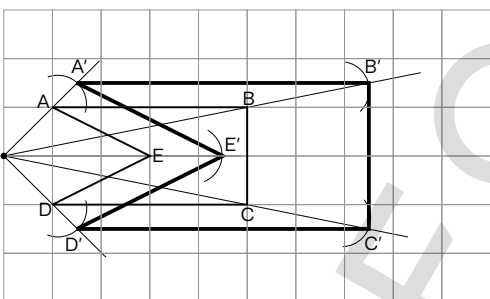


- (iii) Given name ' P ' to the centre of enlargement.

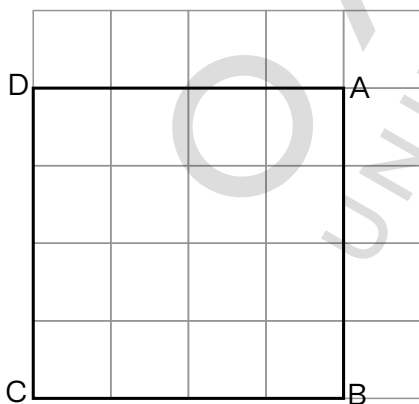
Join P to A and produce it further.
Measure the length from P to A and multiply it by 1.5.
Mark a point A' on the line with the calculated measurement.



Repeat above steps to mark the points B', C', D', and E'.
Join A' to B', B' to C', C' to D', D' to E', and E' to A' to make the enlarged image.



- (iv) Draw guidelines from centre of enlargement to each vertex. Insert the same shape with bigger square only.

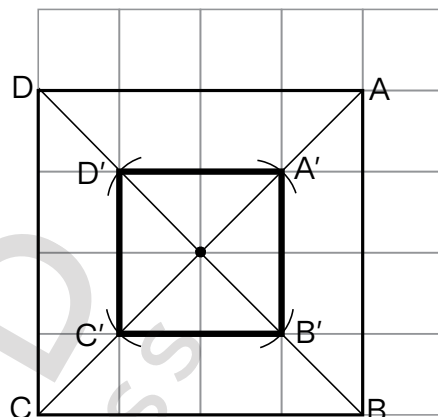


Measure the length from centre of enlargement to A and multiply it by 0.5.

Mark A' with calculated measurement.

Repeat for B', C', and D'.

Join A to B', B' to C', C' to D', and D' to A'.



4. (i) $ABC \rightarrow AB'C'$

The scale factor is 2 in this case
Since $AB = BB'$
hence $AB' = 2 AB$
The scale factors is 2.

- (ii) $ABC \rightarrow AB''C''$

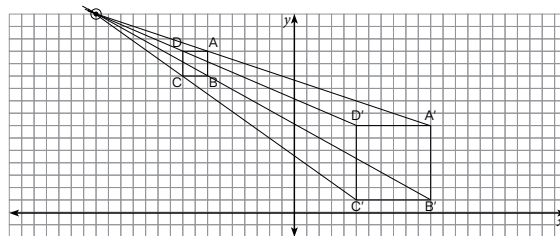
The scale factor is 3

- (iii) $AB'C' \rightarrow ABC$

The scale factor is $\frac{1}{2}$

- (iv) The scale factor is $\frac{2}{3}$

5. Join A to A', B to B', C to C', and D to D'.



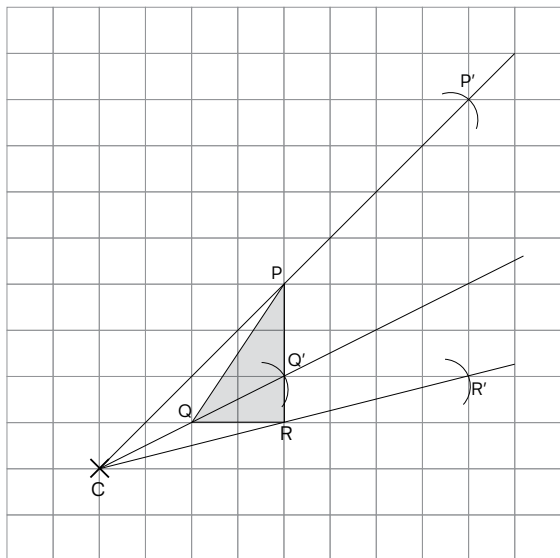
The point where all these lines intersect each other is the centre of enlargement.
Name the given triangle as $\triangle PQR$

6. Draw guidelines from C to each vertex of the image.
Produce the lines further.

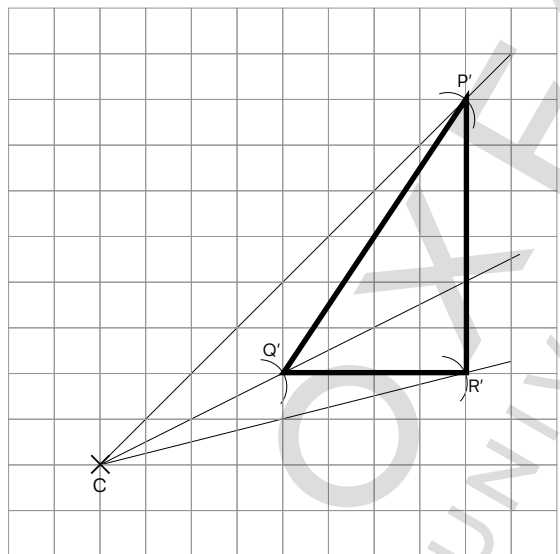
Measure length from C to P and multiply it by 2.

Make point P' with new length.

Repeat the steps for Q' and R'.



Join P' to Q', Q' to R', and R' to P' to get the enlarged image.



Multiple Choice Questions 12

1. D

2. B

Since all side lengths are equal hence, triangle is equilateral.

3. A

Since the measurement of one of three angles is 90° hence, the triangle is a right-angled triangle.

4. B

None of the sides are equal in length.

5. A

13

DATA HANDLING

Exercise 13A

- (i) Primary data (ii) class intervals
(iii) range (iv) histogram
(v) mode
- (i) True
(ii) False

Helpful Hint

The data needs to be arranged in ascending order to determine the median.

(iii) True

(iv) True

(v) False

Reason: Sum of fx is equal to the sum of the product of data value and its frequency

3. $42 - 36 = 6$

$48 - 42 = 6$

$54 - 48 = 6$

$60 - 54 = 6$

$\therefore$ class size is 6

4. Arrange data in ascending order.

2, 5, ⑥, ⑦, 11, 17

$$\text{Median} = \frac{6 + 7}{2}$$

$$= \frac{13}{2}$$

$$= 6.5$$

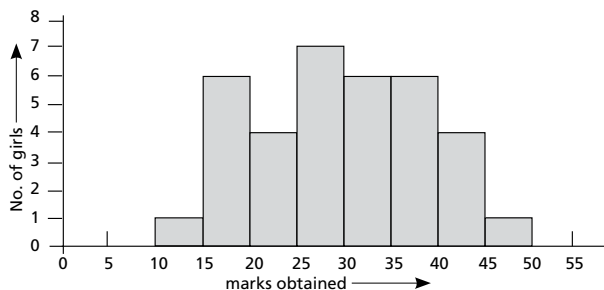
5. Since the lowest value is 68, start the class interval with 60.

Class width = 20 (given in the question)

Class interval	Frequency
$60 \leq x < 80$	5
$80 \leq x < 100$	10
$100 \leq x < 120$	9
$120 \leq x < 140$	4
$140 \leq x < 160$	2

6.

Class Interval	Frequency
$10 \leq x < 15$	1
$15 \leq x < 20$	6
$20 \leq x < 25$	4
$25 \leq x < 30$	7
$30 \leq x < 35$	6
$35 \leq x < 40$	6
$40 \leq x < 45$	4
$45 \leq x < 50$	1

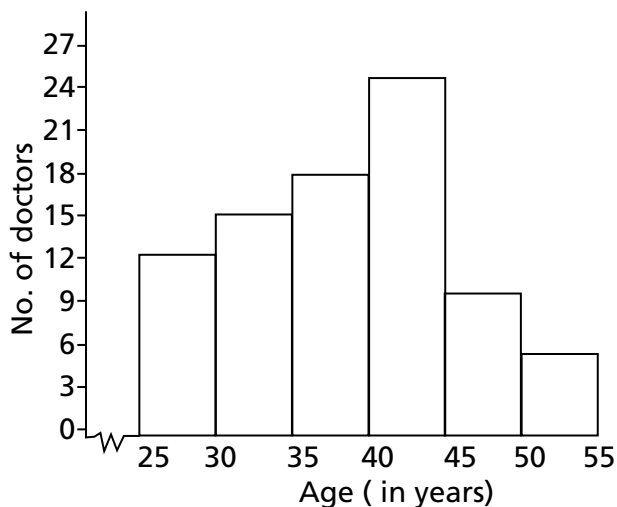


7. Range = highest score – lowest score
 $= 45 - 18$
 $= 27$

$$\text{Mean} = \frac{\text{sum of values}}{\text{number of values}}$$

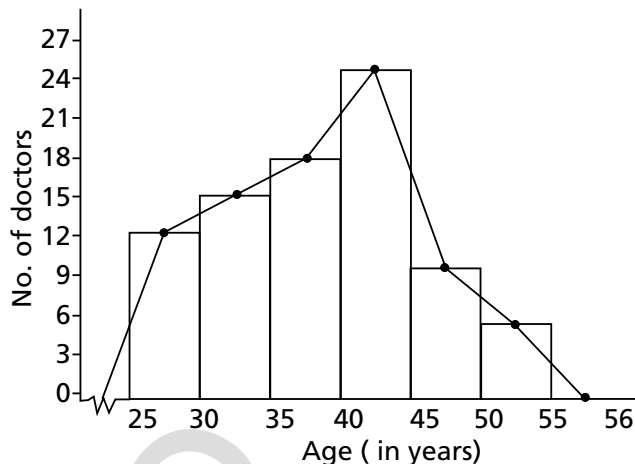
$$\begin{aligned}
 &= \frac{40 + 35 + 24 + 18 + 32 + 22 + 45 + 38 + 30 + 20}{10} \\
 &= \frac{304}{10} \\
 &= 30.4
 \end{aligned}$$

8. (a) Ages of doctors working in a city



(b)

Age (in years)	No. of doctor	Mid value
$25 \leq x < 30$	12	$\frac{25 + 30}{2} = 27.5$
$30 \leq x < 35$	15	$\frac{30 + 35}{2} = 32.5$
$35 \leq x < 40$	18	$\frac{35 + 40}{2} = 37.5$
$40 \leq x < 45$	25	$\frac{40 + 45}{2} = 42.5$
$45 \leq x < 50$	10	$\frac{45 + 50}{2} = 47.5$
$50 \leq x < 55$	5	$\frac{50 + 55}{2} = 52.5$



9. First ten natural numbers are

1 2 3 4 5 6 7 8 9 10

$$\begin{aligned}
 \text{Mean} &= \frac{1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10}{10} \\
 &= \frac{55}{10} \\
 &= 5.5
 \end{aligned}$$

10. First eight odd natural numbers are

1 3 5 7 9 11 13 15

$$\begin{aligned}
 \text{Mean} &= \frac{1 + 3 + 5 + 7 + 9 + 11 + 13 + 15}{8} \\
 &= \frac{64}{8} \\
 &= 8.
 \end{aligned}$$

11. Mean of 15 salaries = 12 500

No. of salaries = 15

managers salary = 36 500

Mean of 16 salaries = ?

Mean of 15 salaries = 12 500

$$\frac{\text{sum of 15 salaries}}{15} = 12\,500$$

$$\text{Sum of 15 salaries} = 12\,500 \times 15 = 187\,500$$

$$\begin{aligned}
 \text{Mean of 16 salaries} &= \frac{\text{sum of 16 salaries}}{16} \\
 &= \frac{\text{sum of 15 salaries} + \text{manager's salary}}{16} \\
 &= \frac{187\,500 + 36\,500}{16} = \frac{224\,000}{16} \\
 &= \text{Rs } 14\,000
 \end{aligned}$$

$$\begin{aligned}
 12. \text{ Mean} &= \frac{\sum fx}{f} \\
 &= \frac{(4 \times 11) + (3 \times 13) + (7 \times 15) + (16 \times 18) + (4 \times 19) + (3 \times 20) + (2 \times 21) + (1 \times 22)}{4 + 3 + 7 + 16 + 4 + 3 + 2 + 1} \\
 &= \frac{44 + 39 + 105 + 288 + 76 + 60 + 42 + 22}{40} \\
 &= \frac{676}{40} = 16.9
 \end{aligned}$$

13. Since class intervals are given instead of actual values, find and the mid value for each that will be served as x .

Kg	No. of students f	Mid value x
$28 < x \leq 30$	4	$\frac{28 + 30}{2} = 29$
$30 < x \leq 32$	8	$\frac{30 + 32}{2} = 31$
$32 < x \leq 34$	10	$\frac{32 + 34}{2} = 33$
$34 < x \leq 36$	5	$\frac{34 + 36}{2} = 35$
$36 < x \leq 38$	4	$\frac{36 + 38}{2} = 37$
$38 < x \leq 40$	1	$\frac{38 + 40}{2} = 39$

$$\begin{aligned}
 \text{Mean} &= \frac{\sum fx}{f} = \frac{(4 \times 29) + (8 \times 31) + (10 \times 33) + (5 \times 35) + (4 \times 37) + (1 \times 39)}{4 + 8 + 10 + 5 + 4 + 1} \\
 &= \frac{116 + 248 + 330 + 175 + 148 + 39}{32} \\
 &= \frac{1056}{32} = 33 \text{ kg}
 \end{aligned}$$

$$14. \text{ Mean} = \frac{22 + 54 + 100 + 4 + 5 + 29 + 51 + 33 + 8 + 5 + 13 + 85 + 40 + 65 + 5 + 73 + 84}{17}$$

$$= \frac{676}{17} = 39.76$$

To find median, arrange the data in ascending order

4, 5, 5, 5, 8, 13, 22, 29, 33, 40, 51, 54, 65, 73, 84, 85, 100

The value in the middle is 33.

Median = 33

Mode = 5 (5 is the most occurring value)

15. (i)

Marks	Frequency
$50 < x \leq 60$	3
$60 < x \leq 70$	5
$70 < x \leq 80$	8
$80 < x \leq 90$	8
$90 < x \leq 100$	6

(ii) $70 < x \leq 80$ and $81 < x \leq 90$

(iii) $70 < x \leq 80$

$$16. \text{ Mean} = \frac{16 + 16 + 12 + 14 + 13 + 15 + 13 + 13 + 16 + 12 + 15 + 10}{12}$$

$$= \frac{165}{12}$$

$$= 13.75 \text{ mm}$$

Helpful Hint

Arrange data in ascending order to determine the median.

10 12 12 13 13 13 14 15 15 16 16 16

There are two middle values, 13 and 14.

$$\text{Median} = \frac{13 + 14}{2}$$

$$= \frac{27}{2}$$

$$= 13.5 \text{ mm}$$

mode = 13 and 16

17. (a) 10, 16, 12, 15, 8, 16, 10, 17, 12, 14

$$\text{mean} = \frac{\Sigma x}{n}$$

$$\text{mean} = \frac{10 + 16 + 12 + 15 + 8 + 16 + 10 + 17 + 12 + 14}{10} = \frac{130}{10}$$

$$\text{mean} = 13$$

variance

$$= \frac{(10 - 13)^2 + (16 - 13)^2 + (12 - 13)^2 + (15 - 13)^2 + (8 - 13)^2 + (16 - 13)^2 + (10 - 13)^2 + (17 - 13)^2 + (12 - 13)^2 + (14 - 13)^2}{10}$$

$$= \frac{(-3)^2 + (3)^2 + (-1)^2 + (2)^2 + (-5)^2 + (3)^2 + (-3)^2 + (4)^2 + (-1)^2 + (1)^2}{10}$$

$$= \frac{9 + 9 + 1 + 4 + 25 + 9 + 9 + 16 + 1 + 1}{10}$$

$$= \frac{84}{10} = 8.4$$

$$\text{Standard deviation} = \sqrt{8.4} = 2.9$$

- (b) 74, 72, 83, 96, 64, 79, 88, 68

$$\text{Mean} = \frac{\Sigma x}{n}$$

$$\text{Mean} = \frac{74 + 72 + 83 + 96 + 64 + 79 + 88 + 68}{8}$$

$$= \frac{624}{8} = 78$$

$$= \frac{(74 - 78)^2 + (72 - 78)^2 + (83 - 78)^2 + (96 - 78)^2 + (64 - 78)^2 + (79 - 78)^2 + (88 - 78)^2 + (68 - 78)^2}{8}$$

$$= \frac{(-4)^2 + (-6)^2 + (5)^2 + (18)^2 + (-14)^2 + (10)^2 + (-10)^2}{8}$$

$$= \frac{16 + 36 + 25 + 324 + 196 + 1 + 100 + 100}{8}$$

$$= \frac{798}{8} = 99.75$$

$$\text{Standard deviation} = \sqrt{99.75} \\ = 9.99$$

$$(c) \text{ mean} = \frac{\Sigma x}{n}$$

$$= \frac{326 + 437 + 374 + 366 + 419 + 424}{6} = \frac{3246}{6}$$

$$= 391$$

$$\text{variance} = \frac{(326 - 391)^2 + (437 - 391)^2 + (374 - 391)^2 + (366 - 391)^2 + (419 - 391)^2 + (424 - 391)^2}{6}$$

$$= \frac{(-65)^2 + (46)^2 + (-17)^2 + (-25)^2 + (28)^2 + (33)^2}{6}$$

$$= \frac{4225 + 2116 + 289 + 625 + 784 + 1089}{6}$$

$$= \frac{9128}{6} = 1521.33$$

$$\text{Standard deviation} = \sqrt{1521.33}$$

$$= 93.00$$

Exercise 13B

1. i and iv are examples of independent events.

2. (i) $P(\text{yellow}) = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$

$$= \frac{2}{8}$$

$$= \frac{1}{4}$$

(ii) $P(\text{odd}) = \frac{4}{8} = \frac{1}{2}$

(iii) Numbers greater than 2 = {3, 4, 5, 6, 7, 8}

$$P(\text{greater than 2}) = \frac{6}{8} = \frac{3}{4}$$

(iv) Numbers less than 9 = {1, 2, 3, 4, 5, 6, 7, 8}

$$P(\text{less than 9}) = \frac{8}{8} = 1$$

$$\begin{aligned}
 3. \text{ Experimental probability} &= \frac{\text{Number of times event occurs}}{\text{Total number of trials}} \\
 &= \frac{64}{126} \\
 &= \frac{32}{63}
 \end{aligned}$$

$$\begin{aligned}
 4. \text{ Experimental probability} &= \frac{\text{Number of times event occurs}}{\text{Total number of trials}} \\
 &= \frac{128}{200} \\
 &= \frac{16}{25}
 \end{aligned}$$

$$\begin{aligned}
 5. \text{ Experimental probability } P(6) &= \frac{24}{60} \\
 &= \frac{2}{5}
 \end{aligned}$$

$$\begin{aligned}
 6. \text{ Experimental probability } P(\text{dislike orange juice}) &= \frac{118}{200} \\
 &= \frac{59}{100}
 \end{aligned}$$

7. Sample space = {GH, GT, WH, WT, PH, PT}

Helpful Hint

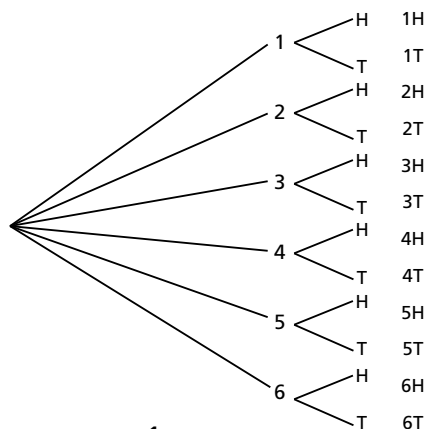
Tree diagram or possibility diagram can be used to list down the sample space.

8.

		Snacks			
		Muffin (M)	Brownie (B)	Crisps (Cr)	Pastry (P)
Drink	Tea (T)	TM	TB	TCr	TP
	Coffee (C)	CM	CB	CCr	CP
	Juice	JM	JB	JCr	JP

9. Peas Carrots
 Peas Turnip
 Peas Spinach
 Carrots Turnip
 Carrots Spinach
 Turnip Spinach

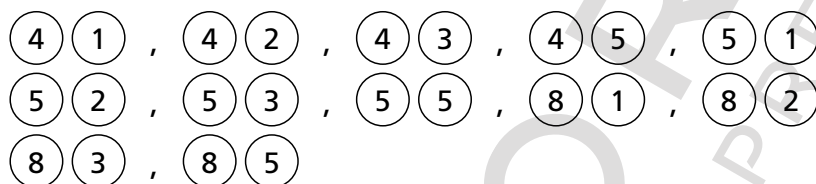
10. (i)



(ii) $P(3H) = \frac{1}{12}$

(iii) $P(\text{tail and a prime number}) = \frac{3}{12} = \frac{1}{4}$

11. (i)



(ii)

Helpful Hint

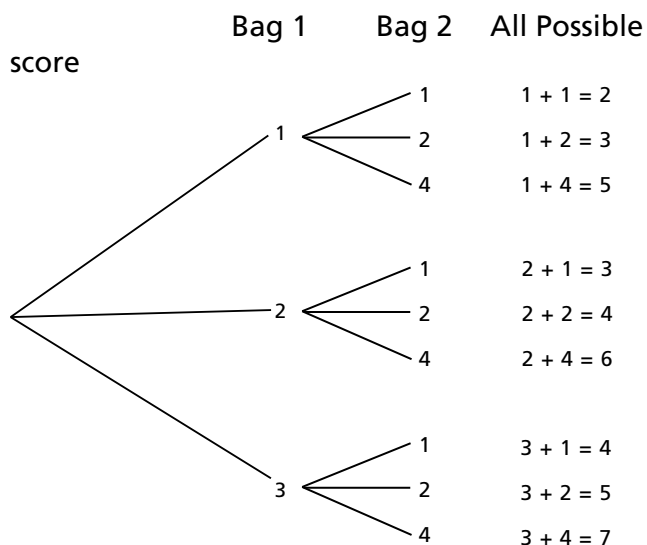
Possibility Diagram is a useful tool in such cases.

		Box 2				
		+	1	2	3	5
Box 1	4	5	6	7	9	
	5	6	7	8	10	
	8	9	10	11	13	

Encircled results are the required outcomes

$$P(10) = \frac{2}{12} = \frac{1}{6}$$

12. (i)



(ii) $P(4) = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$

$$= \frac{2}{9}$$

(iii) $P(5) = \frac{5}{9}$

13. (i)

		Dice 1						
		+	1	2	3	4	5	6
Dice 2	1	1	2	3	4	5	6	
	2	2	4	6	8	10	12	
	3	3	6	9	12	15	18	
	4	4	8	12	16	20	24	
	5	5	10	15	20	25	30	
	6	6	12	18	24	30	36	

(ii) $P(12) = \frac{4}{36} = \frac{1}{9}$

(iii) $P(10 \text{ or more}) = \frac{19}{36}$

(iv) $P(\text{even number}) = \frac{27}{36} = \frac{3}{4}$

14. (i)

		Spinner 1			
Score		1	2	3	4
Coin	Heads	2	3	4	5
	Tails	2	4	6	8

(ii) (a) $P(4) = \frac{2}{8} = \frac{1}{4}$

(b) $P(5 \text{ or more}) = \frac{3}{8}$

(c) $P(\text{Prime number}) = \frac{4}{8} = \frac{1}{2}$

15. (i)

		Spinner 1				
		+	1	3	4	5
Spinner 2	5	8	8	9	10	
	6	7	9	10	11	
	7	8	10	11	12	

(ii) (a) $P(8) = \frac{2}{12} = \frac{1}{6}$

(b) $P(\text{odd number}) = \frac{5}{12}$

16. (i)

		Bag 1		
Score		P	G	Y
Bag 2	2	$2 \times 2 = 4$	$2 + 1 = 3$	2
	3	$3 \times 2 = 6$	$3 + 1 = 4$	3
	4	$4 \times 2 = 8$	$4 + 1 = 5$	4
	5	$5 \times 2 = 10$	$5 + 1 = 6$	5

(ii) $P(\text{multiples of 3}) = \frac{4}{15}$

17. Probability of getting red marble from bag 1 is given by;

$$P(\text{red}) = \frac{1}{4}$$

Probability of getting yellow marble from bag 2 is given by;

$$P(\text{yellow}) = \frac{1}{4}$$

$$\therefore P(\text{red and yellow}) = P(\text{red}) \cdot P(\text{yellow})$$

$$= \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{16}$$

18. $P(3) = \frac{1}{6}$

$$P(\text{even number}) = \frac{3}{6} = \frac{1}{2}$$

$$\therefore P(3 \text{ and even number})$$

$$= P(3) \cdot P(\text{even number})$$

$$= \frac{1}{6} \cdot \frac{1}{2} = \frac{1}{12}$$

19. $P(\text{red}) = \frac{5}{20} = \frac{1}{4}$

$$P(\text{red}) = \frac{8}{20} = \frac{2}{5}$$

$$P(\text{red or blue}) = P(\text{red}) + P(\text{blue})$$

$$= \frac{1}{4} + \frac{2}{5}$$

$$= \frac{5+8}{20} = \frac{13}{20}$$

20. (i) $P(W) = \frac{12}{30}$

$$P(B) = \frac{8}{30}$$

$$P(W \text{ or } B) = P(W) + P(B)$$

$$= \frac{12}{30} + \frac{8}{30} = \frac{20}{30}$$

$$= \frac{5+8}{10} = \frac{13}{10} = \frac{13}{10}$$

- (ii) If 7 cows have no horns then

$$30 - 7 = 23 \text{ cows have horns.}$$

$$P(\text{Cows with horns}) = \frac{23}{30}$$

- (iii) Number of cows of different colours than black or white = $30 - 20 = 10$

$$P(\text{Cow with different colour})$$

$$= \frac{10}{30} = \frac{1}{3}$$

OR $P(\text{Cow with different colour})$

$$= 1 - P(W \text{ or } B)$$

$$= 1 - \frac{2}{3} = \frac{1}{3}$$

Multiple Choice Questions 13

(1) A

(2) B

(3) B

(4) D

(5) D