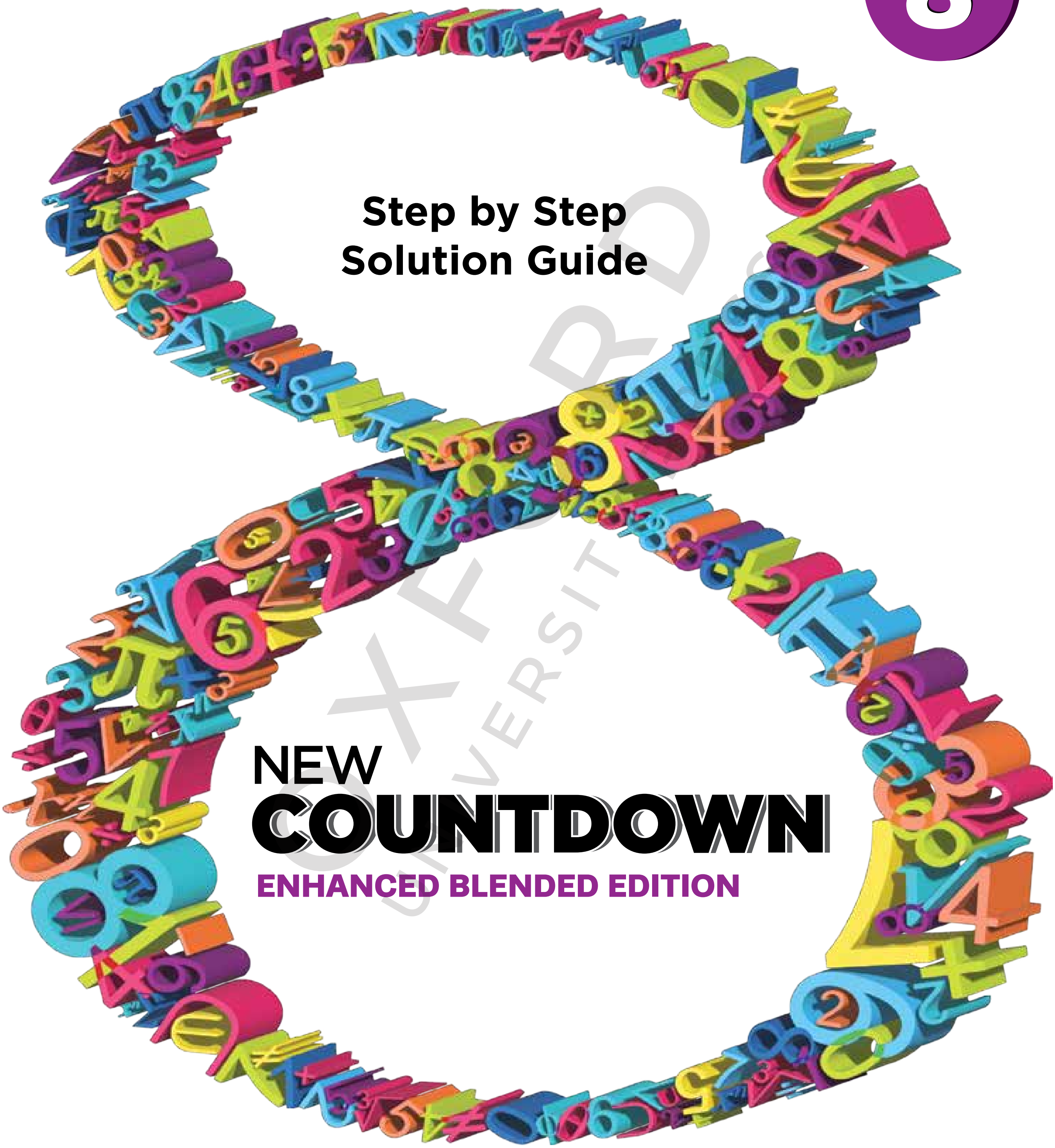

8

Step by Step Solution Guide

NEW COUNTDOWN

ENHANCED BLENDED EDITION

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Preface

New Countdown Third Edition (***Enhanced Blended Edition***) is a carefully structured and graded mathematics course, comprising eleven books for Classes Pre-Primary to Class 8. The pattern followed in the entire series ensures development in all areas of a child's growth through basic multi-focal knowledge, emphasising number skills and mathematical concepts.

The ***Step by Step Solution Guide*** is a comprehensive resource that complements the New Countdown series to provide a holistic framework within which students are able to understand, grasp, approach, and apply the learned mathematical concepts, and to successfully implement the objectives of the mathematics curriculum.

This guide highlights the patterns, approaches, functions, and relationships between the curriculum strands, so that the students can apply their mathematical knowledge and develop a holistic understanding of the subject that can then be translated into real-life application. The main objective of this guide is not to simply cross-reference the answers, but to guide the students through the thinking process upon approaching a mathematical problem, to reaching the correct answer. This guide therefore provides the extensive breakdown of not only solving the equation, but also the mental strategies, appropriate reasoning and formatting, and the ability to decipher what mathematical concepts can be applied to the particular question, in order to work towards the answer.

This in-depth breakdown of solving questions encompasses all the questions in each exercise, as well as the questions in the revision exercises. There are also helpful hints available in this guide that supplements a student's thinking process when approaching a certain problem. The helpful hints will help to avoid pre-emptive misconceptions that will be beneficial to students and teachers. They help guide the student towards the correct formula by effectively contextualising the mathematical concept and linking it to real-life application. The mathematical proofing, format and reasoning is in line with the assessment expectations.

Finally, apart from the step-by-step worked solutions themselves, the end of this guide also includes a direct answer key that can be used for cross-referencing purposes by the teacher. These answers correlate to the model paper in the Assessment Resource Pack.

The Step by Step Guide provides thorough insight and furthers one's understanding of what is expected of a student in an examination beyond simply arriving at the right answer. This guide helps ensure that the process comes from a place of deep understanding and reasoning of mathematical concepts by guiding the students' approach and thinking process during problem solving, and therefore reaching the desired answer.

CONTENTS

1 OPERATIONS ON SETS

Exercise 1

1. (i) $A \cup B$
   (ii) $(A \cup B) \cup C$
   (iii) $(A \cup B) \cap (A \cup C)$
   (iv) common elements
   (v) 2^n

2. (i) False
   Reason: A set containing 3 elements has $2^3 = 8$ elements.
   (ii) True
   (iii) False
   Reason: Power set is the set of all the subsets of a given set.
   (iv) False
   Reason: A super set contains all the elements of its subset and it contains at least one elements not present in subset.
   (v) True

3. (i) The number of elements is 1
   $\therefore$ the number of subsets is $2^n = 2^1 = 2$
   { }, {a}

Helpful Hint
Null set is a subset of every set.

   (ii) The number of elements is 2
   $\therefore$ the number of subsets is $2^n = 2^2 = 4$
   { }, {×}, {y}, {x, y}
   (iii) The number of elements is 3
   $\therefore$ the number of subsets is $2^3 = 8$
   { } {1}, {2}, {3}, {1, 2} {1,3}, {2, 3}, {1, 2, 3}
   (iv) There is no element.
   $\therefore$ number of subsets is $2^0 = 1$
   (v) The number of elements is 4
   $\therefore$ the number of subsets is $2^4 = 16$
   { }, {a}, {b}, {c}, {d},
   {a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {c,d},
   {a, b, c}, {a, b, d}, {b, c, d}, {c, d, a},
   {a, b, c, d}
   (vi) There is only one element.
   $\therefore$ number of subsets is $2^n = 2^1 = 2$
   { }, {0}

4. (i) $\{a, b, c\} \cap \{b\} = \{b\}$
   only b is common in both the sets
   (ii) $\{2, 4, 6, 8\} \cap \{1, 2, 3, 4, 5\} = \{2, 4\}$
   (iii) {the vowels in the English alphabet} $\cap$ {the first four letters of the English alphabet}
   $= \{a, e, i, o, u\} \cap \{a, b, c, d\}$
   $= \{a\}$
   because only a is common in both sets
   (iv) $\{c, a, t\} \cap \{d, o, g\} = \{\ \}$
   There is are no common elements in the two sets.
   (v) $\{3, 6, 9, 12\} \cap \{2, 3, 4, 5, 6\} = \{3, 6\}$
   (vi) $\{O\} \cap \{E\} = \{\ \}$
   There are no common elements in the two sets.

5. (i) $\{a, c, d\} \cup \{a, b, c, d\} = \{a, b, c, d\}$
   Write all the elements of both the sets.

Helpful Hint
Write common elements only once.

(ii) $\{3, 8, 4\} \cup \phi = \{3, 8, 4\}$

Helpful Hint

ϕ represents empty or null set.

(iii) $\{3, 6, 9, 12\} \cup \{2, 4, 6, 8\}$
$= \{2, 3, 4, 6, 8, 9, 12\}$

6. (i) $\{0, 2, 4, 6, 8\} \cup \{1, 2, 3, 8\}$
$= \{0, 1, 2, 3, 4, 6, 8\}$

(ii) $\{0, 4, 6, 8\} \cup \{4, 5, 6\ 7\}$
$= \{0, 2, 4, 5, 6, 7, 8\}$

(iii) $\{1, 2, 3, 8\} \cup \{4, 5, 6, 7\}$
$= \{1, 2, 3, 4, 5, 6, 7, 8\}$

(iv) $\{1, 2, 3, 8\} \cup \{0, 1, 2, ..., 9\}$
$= \{0, 1, 2, ..., 9\}$

(v) $\{0, 2, 4, 6, 8\} \cup \{\ \}$
$= \{0, 2, 4, 6, 8\}$

(vi) $\{0, 2, 4, 6, 8\} \cap \{1, 3, 5, 7, 9\}$
$= \{\ \}$ or ϕ

(vii) $\{1, 3, 5, 7, 9\} \cap \{1, 2, 3, 8\}$
$= \{1, 3\}$

(viii) $\{1, 2, 3, 8\} \cap \{4, 5, 6, 7\}$
$= \{\ \}$ or ϕ

(ix) $\{1, 3, 5, 7, 9\} \cap \{\ \}$
$= \{\ \}$ or ϕ

(x) $\{1, 3, 5, 7, 9\} \cap \{0, 1, 2, ..., 9)$
$= \{1, 3, 5, 7, 9\}$

7. (i) $A' = \{1, 2, 3, 4, 5, 6, 7\} - \{1, 2, 5, 7\}$
$A' = \{3, 4, 6\}$

(ii) $B' = \{1, 2, 3, 4, 5, 6, 7\} - \{1, 3, 6, 7\}$
$B' = \{2, 4, 5\}$

(iii) $A \cap B = \{1, 2, 5, 7\} \cap \{1, 3, 6, 7\}$
$= \{1, 7\}$
$(A \cap B)' = \{1, 2, 3, 4, 5, 6, 7\}\ \{1, 7\}$
$= \{2, 3, 4, 5, 6\}$

(iv) $A \cup B = \{1, 2, 5, 7\} \cup \{1, 3, 6, 7\}$
$= \{1, 2, 3, 5, 6, 7\}$
$(A \cup B)' = \{1, 2, 3, 4, 5, 6, 7\} - \{1, 2, 3, 5, 6, 7\}$
$= \{4\}$

(v) $A' = \{1, 2, 3, 4, 5, 6, 7\} - \{1, 2, 5, 7\}$
$= \{3, 4, 6\}$
$B' = \{1, 2, 3, 4, 5, 6, 7\} - \{1, 3, 6, 7\}$
$= \{2, 4, 5\}$
$A' \cup B' = \{3, 4, 6\} \cup \{2, 4, 5\}$
$= \{2, 3, 4, 5, 6\}$

(vi) $A' = \{3, 4, 6\}$
$B' = \{2, 4, 5\}$
$A' \cap B' = \{3, 4, 6\} \cap \{2, 4, 5\}$
$= \{4\}$

8. (a) (i) $\{1, 3\} \cup \{2, 4\} \cup \{5\} = \{1, 2, 3, 4, 5\}$

(ii) $A \cup B = \{1, 3\} \cup \{2, 4\} = \{1, 2, 3, 4\}$
$(A \cup B) \cap C = \{1, 2, 3, 4\} \cap \{5\} = \{\ \}$

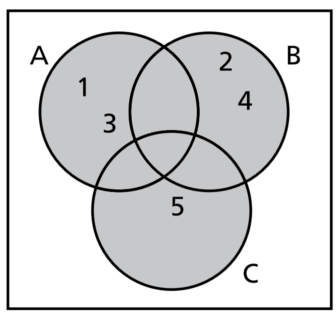
(iii) $A \cap B = \{1, 3\} \cap \{2, 4\} = \{\ \}$
$(A \cap B) \cup C = \{\ \} \cup \{5\} = \{5\}$

(iv) $A \cap B = \{1, 3\} \cap \{2, 4\} = \{\ \}$
$(A \cap B) \cap C = \{\ \} \cap \{5\} = \{\}$

(v) $B \cap C = \{2, 4\} \cap \{5\} = \{\ \}$
$A \cup (B \cap C) = \{1, 3\} \cup \{\ \} = \{1, 3\}$

(vi) $B \cup C = \{2, 4\} \cup \{5\} = \{2, 4, 5\}$
$A \cap (B \cup C) = \{1, 3\} \cap \{2, 4, 5\} = \{\ \}$

(b) (i)



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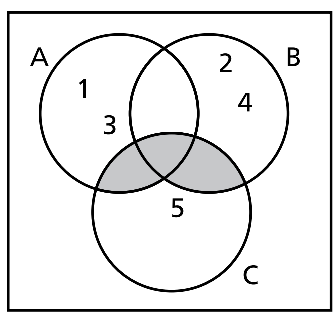

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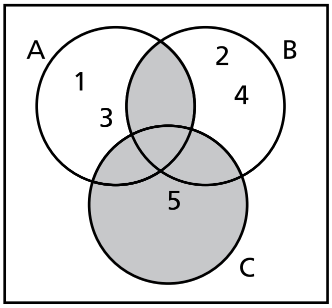

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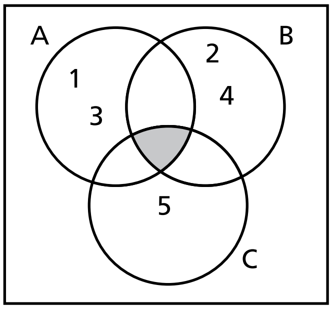

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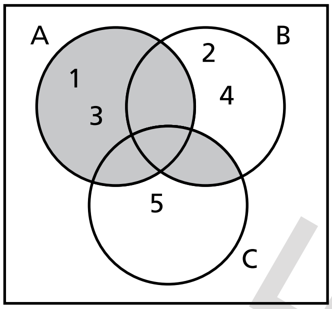

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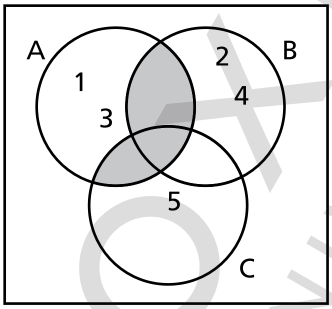


(c) (i) LHS

A'∪B'

A' = {1, 2, 3, 4, 5} – {1, 3} = {2, 4,5}

B' = {1, 2, 3, 4, 5} – {2, 4} = {1, 3, 5}

A'∪B' = {1, 2, 3, 4, 5}

RHS

(A∩B)'

A∩B = {1, 3} ∩ {2, 4} = { }

(A∩B)' = {1, 2, 3, 4,5} – { } = {1, 2, 3, 4, 5}

∴ LHS = RHS

Thus A'∪B' = (A∩B)' is proved

(ii) LHS

A'∩B' = {2, 4, 5} ∩ {1, 3, 5} = {5}

RHS

(A∪B)'

A∪B = {1, 3} ∪ {2 4} = {1, 2, 3, 4}

(A∪B)' = {1, 2, 3, 4, 5} – {1, 2, 3, 4} = {5}

∴ LHS = RHS

Thus A'∩B' = (A∪B)' is proved

9. (i) A∩B = {6, 8} shows the common elements. That means 6 and 8 are elements of set B.

A∪B = {2, 4, 5, 6, 7, 8} shows all the elements of A and B both.

The elements, 5 and 7 do not belong to set A. This shows that 5 and 7 are also the elements of set B.

∴ B = {5, 6, 7,8}

(ii) A∩B = {3}

⇒ Set B only has 3 as an element.

A∪B = {1, 2, 3, 4}

Since 1 and 2 do not belong to set A

∴ B = {1, 2, 3}

10. (i) =

(Commutative property)

(ii) =

Since ϕ is a null set, it does not have any element hence its union with any other set gives the set itself as a result.

(iii) ≠

The result will be the set A.

(iv) =

Since the elements are written only once if they are common is both the sets. Thus the resultant will be the set A itself.

(v) =

Since union and intersection both gives the same elements as set B. Thus both the sets are equal.

11. Commutative property of union of three sets

A∪ (B∪C) = (A∪B) ∪C

Consider LHS

A∪ (B∪C)

(B∪C) = {3, 5, 6} ∪ {5, 7, 9} = {3, 5, 6, 7, 9}

A∪ (B∪C) = {2, 3, 4} ∪ {3, 5, 6, 7, 9}

= {2, 3, 4, 5, 7 9}

RHS

(A∪B) ∪C

A∪B = {2, 3, 4} ∪ {3, 5, 6} = {2, 3, 4, 5, 6}

(A∪B) ∪C = {2, 3, 4, 5, 6} ∪ {5, 7, 9}

= {2, 3, 4, 5, 6, 7, 9}

∴ LHS = RHS

Thus associative property of union is proved.

12. A∩ (B∩C) = (A∩B) ∩C

LHS

B∩C = {p, q, r, s} ∩ {n, o, p, q}

= {p, q}

A∩ (B∩C) = {m, n, o} ∩ {p, q]

= { }

RHS

A∩B = {m,n, o} ∩ {p, q, r, s} = { }

(A∩B) ∩C = { } ∩ {n, o, p, q}

= { }

∴ LHS = RHS

Thus the associative property of intersection is proved.

13. First Law

(A∪B)' = A'∩B'

LHS

A∪B = {1, 2, 3, 4,} ∪ {3, 4, 5, 6}

= {1, 2, 3, 4, 5, 6}

(A∪B)' = {1, 2, 3, 4, 5, 6} – {1, 2, 3, 4, 5, 6}

= { }

RHS

A' = {1, 2, 3, 4, 5, 6} – {1, 2, 3, 4} = {5, 6}

B' = {1, 2, 3, 4, 5, 6} – {3, 4, 5, 6} = {1, 2}

A'∩B' = {5, 6} ∩ {1, 2} = { }

∴ LHS = RHS

Thus De Morgan's first law is proved.

2nd Law

(A∩B)' = A'∪B'

LHS

A∩B = {1, 2, 3, 4} ∩ {3, 4, 5, 6}

= {3, 4}

(A∩B)' = {1, 2, 3, 4, 5, 6} – {3, 4}

= {1, 2, 5, 6}

RHS

A' = {5, 6}   B' = {1,2}

A'∪B' = {5, 6} ∪ {1, 2}

= {1, 2, 5, 6}

∴ LHS = RHS

Thus De Morgan's 2nd law is proved.

14. (a) Swimming = {B, E, F, H}

Coding = {A, C, F, G}

Painting = {B, C, D, F}

(b) B, F

(f) F

Multiple Choice Questions 1

1. A

All other sets have elements other than the elements of universal set.

2. D

Integers include negative numbers, positive numbers and zero, whereas whole numbers include only positive numbers and zero. Even numbers consist of multiples of 2 only, and odd numbers are the numbers other than multiples of 2. Hence, the set of integers has all the members of whole numbers, even numbers and odd numbers sets.

3. B

$2^n = 2^4 = 16$

4. C

Options A and D are not the subset of A as they have other element than those in set A. Option B is improper subset of A.

5. C

None of the other options satisfy De Morgan's law.

2 REAL NUMBERS

Exercise 2A

1. 45 → has two significant figures because all non zero digits are significant.

   0.046 → has two significant figures because all leading zeros are non significant figures.

   7.4220 → has five significant figures because the zeros after non zero digits after decimal are significant.

   5002 → has four significant figures because all zeros between two non-zero digits are significant.

   3800 → has two significant figures because the zeros at the end are not significant.

> **Helpful Hint**
> In whole number, the zeros at the end may or may not be significant.

   0003006 → has four significant figures. The zeros in the beginning are non-significant where as the zeros between two non-zero digits are significant.

   143.000 → has six significant figures because the zeros at the end of a decimal number are significant.

Exercise 2B

1. (i) $2 - 7.2 = 5.2$

   $|-5.2| = 5.2$

> **Helpful Hint**
> Absolute value of any real number is positive.

   (ii) terminating

   (iii) Zero (0)

   (iv) five

   (v) $-\frac{2}{5}$

   $-\frac{5}{2} * -\frac{2}{5} = 1$

2. (i) True

   Since 2.315796 .... is a non-terminating decimal number, and hence it is an irrational number.

   (ii) True

   (iii) False

   $1.03\overline{2}$ is a recurring decimal number, hence it is a rational number.

   (iv) True

   The value of π is a non-terminating decimal number, hence it is an irrational number.

3. (i) 49

   Since 6 is greater than 5, the digit 8 will round up to 9.

   (ii) 0.005

   Since 3 is less than 5, hence the required digit will remain the same.

   (iii) 390200

   Since the digit next to 4^{th} significant figure is 5, the digit will round up.

   (iv) 535.01

Since the last digit is 8, the digit will round up.

6. (i) $\frac{1}{7}$ (ii) $-\frac{1}{2.5}$ (iii) $\frac{1}{\sqrt{3}}$

(iv) $\frac{11}{6}$ (v) $-\sqrt{2}$ (vi) $\frac{1}{347.99}$

7. (i) –29 (ii) 7875 (iii) $-\frac{47}{50}$

(iv) $\sqrt{7}$ (v) 6712.04 (vi) $-\frac{3}{10}$

8. (i) 15, 30, 2.5

Associative property of addition states that:

$a + (b + c) = (a + b) + c$

15 + (30 + 2.5) = (15 + 30) + 2.5

Helpful Hint

To prove the law the left hand side should be equal to the right hand side

LHS

15 + (30 + 2.5)

= 15 + 32.5

= 47.5

RHS

(15 + 30) + 2.5

= 45 + 2.5

= 47.5

Since LHS = RHS

Hence, the associative property of addition is proved.

(ii) $\frac{3}{5}$, 24, 11

LHS

$\frac{3}{5} + (24 + 11)$

$= \frac{3}{5} + 35 = \frac{3 + 175}{5}$

$= \frac{178}{5} = 35\frac{3}{5}$

RHS

$\left(\frac{3}{5} + 24\right) + 11$

$= \frac{3 + 120}{5} + 11 = \frac{123}{5} + 11$

$= \frac{123 + 55}{5} = \frac{178}{5} = 35\frac{3}{5}$

Since LHS = RHS

Hence, the associative property of addition is proved.

9. (i) 1.5, 3, 8

Distributive property of multiplication over addition states that:

$a * (b + c) = a * b + a * c$

LHS

1.5 * (3 + 8) = 1.5 * 11 = 16.5

RHS

1.5 * 3 + 15 * 8 = 4.5 + 12.0 = 16.5

Since, LHS = RHS

Hence, distributive property of multiplication over addition is proved.

(ii) 2, $\frac{4}{15}$, $\frac{7}{15}$

$2 * \left(\frac{4}{15} + \frac{7}{15}\right) = 2 * \frac{4}{15} + 2 * \frac{7}{15}$

LHS

$2 * \left(\frac{4}{15} + \frac{7}{15}\right) = 2 * \frac{11}{15} = 1\frac{7}{15}$

RHS

$2 * \frac{4}{15} + 2 * \frac{7}{15} = \frac{8}{15} + \frac{14}{15} = \frac{22}{15} = 1\frac{7}{15}$

Since, LHS = RHS

Hence, distributive law of multiplication over addition is proved.

10. (i) 100, 25, 3000

Associative property of multiplication states that:

$a * (b * c) = (a * b) * c$

100 * (25 * 3000) = (100 * 25) * 3000

LHS

100 * (25 * 3000) = 100 * 75000 = 7500000

RHS

(100 * 25) * 3000 = 2500 * 3000 = 7500000

Since, LHS = RHS

Hence, associative property of multiplication is proved.

(ii) $-5 * (20 * \frac{4}{5}) = (-5 * 20) * \frac{4}{5}$

LHS

$-5 * (20 * \frac{4}{5}) = -5 * 16 = -80$

RHS

$(-5 * 20) * \frac{4}{5} = -100 * \frac{4}{5} = -80$

Since, LHS = RHS

Hence, associative property of multiplication is proved.

11. (i) $7 * (-4.5 + 12) = 7 * (-4.5) + 7 * 12$

LHS

$7 * (-4.5 + 12) = 7 * 7.5 = 52.5$

RHS

$7 * (-4.5) + 7 * 12 = -31.5 + 84 = 52.5$

Since LHS = RHS

Hence, distributive property of multiplication over addition is proved.

(ii) $\frac{1}{3} * (4 + \frac{2}{3}) = \frac{1}{3} * 4 + \frac{1}{3} * \frac{2}{3}$

LHS

$\frac{1}{3} * (4 + \frac{2}{3}) = \frac{1}{3} * \frac{14}{3} = \frac{14}{9} = 1\frac{5}{9}$

RHS

$\frac{1}{3} * 4 + \frac{1}{3} * \frac{2}{3} = \frac{4}{3} + \frac{2}{9} = \frac{12 + 2}{9}$

$= \frac{14}{9} = 1\frac{5}{9}$

Since, LHS = RHS

Hence, distributive property of multiplication over addition is proved.

12. (i) $\left|-\frac{2}{5}\right| = \frac{2}{5}z$

(ii) $|-2(4.62)| = |-9.24| = 9.24$

(iii) $|\sqrt{2}| = \sqrt{2}$

13. Saima had 593.66 cm long cloth. She cut off 28.5 cm from it. The length of remaining cloth is
593.66 cm – 28.5 cm = 565.16 cm

Approximation upto 4 significant figures
565.2 cm

14. Maheen jogged 2.23 km
Faheem jogged 3.25 * 2.23 km
= 7.2475 km
Faheem jogged approximately 7.24 km

15. $\frac{1}{4} + \frac{1}{4} + \frac{3}{8}$

$= \frac{2 + 2 + 3}{8} = \frac{7}{8}$

16. $\frac{3}{5} + \frac{1}{4}$

$= \frac{12 + 5}{20} = \frac{17}{20}$

Haris moved $\frac{17}{20}$ of his house lawn left to now is $1 - \frac{17}{20} = \frac{20 - 17}{20}$

$= \frac{3}{20}$

17. We have radius = 35 cm
Take radius = 34.5 cm, 35 cm and 35.5 cm
Area of circle = πr^2
r = 34.5 cm
Area of circle = $\frac{22}{7} * 345$
= 108.43 cm^2
r = 35 cm
Area of circle = $\frac{22}{7} * 35$
= 110 cm^2
r = 35.5 cm
Area of circle = $\frac{22}{7} * 35.5$
= 111.57 cm^2
If radius = 34.5 cm^2, then the error is
110 – 108.43 = 1.57 cm^2
If radius = 35.5 cm^2, then the error is
111.57 – 110 = 1.57 cm^2

Multiple choice question 2

1. B 2. B 3. C 4. C 5 D

3 SQUARES AND SQUARE ROOTS CUBES AND CUBE ROOTS

Exercise 3A

1. (i) radical (ii) two (iii) less
   (iv) division (v) 4.4

2. (i) False
   100 is a perfect square but 200 is not a perfect square.
   (ii) True
   (iii) True

> **Helpful Hint**
> If the number is non-terminating and non-recurring, it is an irrational number.

   (iv) False
   345 is not a perfect square.
   (v) True

3. (i)

	4.7	
4	$\overline{22}\ \overline{09}$	
+ 4	− 16 → 4 × 4	
87	609	
	− 609 → 87 × 7	
	000	

$\therefore \sqrt{22.09} = 4.7$

(ii)

	2.71	
2	$\overline{7}.\overline{34}\ \overline{41}$	
+ 2	− 4	→ 2 × 2
47	334	
+ 7	− 329	→ 47 × 7
541	541	
	− 541	→ 541 × 1
	0	

$\therefore \sqrt{6.3441} = 2.71$

(iii)

	7.12	
7	$\overline{50}.\overline{69}\ \overline{44}$	
+ 7	− 49	→ 7 × 7
141	169	
+ 1	− 141	→ 141 × 1
1422	2844	
	− 2844	→ 1422 × 2
	0	

$\therefore \sqrt{50.6944} = 7.12$

(iv)

	18.47	
1	$\overline{3}\overline{41}.\overline{14}\ \overline{09}$	
+ 1	− 1	→ 1 × 1
28	241	
+ 8	− 224	→ 28 × 8
364	1741	
+ 4	− 1456	→ 364 × 4
3687	25809	
	− 25809	→ 3687 × 7
	0	

$\therefore \sqrt{341.1409} = 18.47$

(v)

	0.0907	
0	$0.\overline{00}.\overline{82}\ \overline{26}\ \overline{49}$	
+ 0	− 0	→ 0 × 0
09	82	
+ 9	− 81	→ 9 × 9
180	126	
+ 0	0	→ 180 × 0
1807	12649	
	− 12649	→ 1807 × 1
	0	

$\therefore \sqrt{0.00822649} = 0.0907$

4. (i) $\sqrt{2\frac{9}{49}} = \sqrt{\frac{107}{49}} = \sqrt{\frac{107}{7 \times 7}} = \sqrt{\frac{107}{7}}$

	10.34
1	$\overline{1}\,\overline{07}$
+ 1	− 1
20	007
+ 0	− 0
203	700
+ 3	− 609
2064	9100
	− 8256
	844

$\therefore \sqrt{\frac{107}{7}} = \frac{10.34}{7}$

	1.477
7	10.34
	− 7
	33
	− 28
	54
	− 49
	50
	− 49
	1

$\therefore \sqrt{2\frac{9}{49}} = 1.48$

(ii) $\sqrt{7\frac{9}{16}}$

$= \sqrt{\frac{121}{16}}$

$= \frac{11}{4}$

$= 2.75$ or $2\frac{3}{4}$

(iii) $\sqrt{6\frac{145}{256}} = \sqrt{\frac{1681}{256}}$

	41
4	$\overline{16}\,\overline{81}$
+ 4	− 16
81	81
	− 81
	0

$\therefore \sqrt{1681} = 41$

	16
1	$\overline{2}\,\overline{56}$
+ 1	− 1
26	156
6	− 156
	0

$\therefore \sqrt{256} = 16$

Thus $\sqrt{\frac{1681}{256}} = \frac{41}{16}$

$= 2.5625$ or $2\frac{9}{16}$

(iv) $\sqrt{9\frac{67}{121}} = \sqrt{\frac{1156}{121}}$

	34
3	$\overline{11}\,\overline{56}$
+ 3	− 9
64	256
	− 256
	0

$\therefore \sqrt{1156} = 34$

$\sqrt{121} = 11$ or $3\frac{1}{11}$

(v) $\sqrt{\dfrac{0.324}{72.9}}$

	0.566
5	$0.\overline{32}\ \overline{4}$
+ 5	− 25
106	740
+ 6	− 636
1129	10400
	− 10161
	239

$\sqrt{0.324} = 0.566$

	8.538
8	$\overline{72}.\overline{9}$
+ 8	− 64
16 5	890
+ 5	− 825
170 3	6500
+ 3	− 5109
17068	139100
	− 136544
	2556

$\sqrt{72.9} = 8.538$

$\therefore \sqrt{\dfrac{0.324}{72.9}} = \dfrac{0.566}{8.538} = 0.066$

(vi) $\sqrt{\dfrac{6}{8\frac{1}{6}}} = \sqrt{\dfrac{6}{\frac{49}{6}}}$

$= \sqrt{\dfrac{6 \times 6}{49}} = \sqrt{\dfrac{36}{49}}$

$= \sqrt{\dfrac{6 \times 6}{7 \times 7}} = \dfrac{6}{7}$

5. (i) $\sqrt{2}$

	1.414
1	$2.\overline{00}\ \overline{00}$
+ 1	− 1
2 4	100
+ 4	− 96
281	400
+1	− 281
2824	11900
	− 11296
	604

$\therefore \sqrt{2} = 1.414$

(ii) $\sqrt{\dfrac{1}{3}} = \sqrt{0.3333\ 3}$

	0.577
5	$0.\overline{33}\ \overline{33}$
+ 5	− 25
107	833
+ 7	− 749
1147	8400
	− 8029
	371

	0.3333
3	10
	− 9
	10
	− 9
	10
	− 9
	10
	− 9
	1

$\therefore \sqrt{\dfrac{1}{3}} = 0.577$

(iii) $\sqrt{0.1}$

	0.316
3	$0.\overline{10}$
+ 3	− 9
61	100
+ 1	− 61
626	3900
	− 3756
	144

$\therefore \sqrt{0.1} = 0.316$

(iv)

$\sqrt{1 + (0.021)^2}$

$\sqrt{1 + 0.000441} = \sqrt{1.000441}$

$\sqrt{\frac{1000441}{1000000}} = \frac{\sqrt{1000441}}{1000}$

	1000.42
1	1 00 04 41
+ 1	− 1
20	000
+ 0	− 0
200	004
+ 0	− 0
2000	441
+ 0	− 0
20004	44100
+ 4	− 40004
200082	409600
	− 400164
	9436

$\therefore \sqrt{\frac{1000441}{1000}} = \frac{1000.42}{1000}$

= 100042
= 1.000

(v)

$\sqrt{\sqrt{32} - \sqrt{128} + \sqrt{50}}$

$\sqrt{32} = ?$

	5.657
5	32
+ 5	− 25
106	700
+ 6	− 636
1125	6400
+ 5	− 5625
11306	77500
	− 67836
	8664

$\therefore \sqrt{32} = 5.656$

$\sqrt{128} = ?$

	11.313
1	1 28
+ 1	− 1
2 1	028
+ 1	− 21
223	700
+ 3	− 669
2261	3100
+ 1	− 2261
22673	83900
	− 68019
	15881

$\therefore \sqrt{128} = 11.313$

$\sqrt{50} = ?$

	7.071
7	50
+ 7	− 49
140	100
+ 0	− 0
1407	10000
+ 7	− 9849
14141	15100
	− 14141
	951

$\therefore \sqrt{50} = 7.071$

$\sqrt{\sqrt{32} - \sqrt{128} + \sqrt{50}}$

$= \sqrt{5.656 - 11.313 + 7.071} = \sqrt{1.414}$

	1.189
1	1.41 4
+ 1	− 1
21	0 41
+ 1	− 21
228	2040
+ 8	− 1824
236 9	21600
	− 21321
	279

$\sqrt{1.414} = 1.189$

$\therefore \sqrt{\sqrt{32} + \sqrt{128} + \sqrt{50}} = 1.189$

(vi) $\sqrt{\frac{17}{25}} = \frac{\sqrt{17}}{\sqrt{5 \times 5}} = \frac{\sqrt{17}}{5}$

	4.123
4	17
+ 4	− 16
8 1	100
+ 1	− 81
82 2	1900
+ 2	− 1644
8243	25600
	− 24729
	871

$\therefore \sqrt{\frac{17}{25}} = \frac{4.123}{5} = 0.824$

(vii) $\sqrt{1 + 0.002116} = \sqrt{1.002116}$

	1.001
1	$1.\overline{00}\ \overline{21}\ \overline{16}$
+ 1	− 1
20	0 0
+ 0	− 0
200	21
+ 0	− 0
2001	2116
	− 2001
	115

$\therefore \sqrt{1 + 0.002116} = 1.001$

6. (i) $\sqrt{\frac{\frac{9}{2}}{2}} + \sqrt{\frac{\frac{49}{4}}{9}} = \sqrt{\frac{9}{2 \times 2}} + \sqrt{\frac{49}{4 \times 9}}$

Helpful Hint

$\frac{9}{2} \div 2 = \frac{9}{2} \times \frac{1}{2}$

$= \sqrt{\frac{9}{4}} + \sqrt{\frac{49}{36}} = \frac{3}{2} + \frac{7}{6}$

$= \frac{9 \times 7}{6} + \frac{16}{6} = \frac{8}{3} = 2\frac{2}{3}$

(ii) $\sqrt{\frac{\frac{16}{5}}{5}} + \sqrt{\frac{\frac{25}{8}}{8}} + \sqrt{\frac{\frac{49}{10}}{10}}$

$= \sqrt{\frac{16}{5 \times 5}} + \sqrt{\frac{25}{8 \times 8}} - \sqrt{\frac{49}{10 \times 10}}$

$= \frac{4}{5} - \frac{5}{8} - \frac{7}{10}$

$= \frac{32 + 25 - 28}{40} = \frac{29}{40}$

7. Let x and $3x$ be the required numbers.

$(x)(3x) = 9\frac{18}{25}$

$3x^2 = \frac{243}{25}$

$x^2 = \frac{\overset{81}{\cancel{243}}}{25 \times \cancel{3}} = \frac{81}{25}$

$x^2 = \frac{81}{25}$

Take square root on both the sides.

$\sqrt{x^2} = \sqrt{\frac{81}{25}}$

$x^2 = \sqrt{\frac{9 \times 9}{5 \times 5}}$

$x = \frac{9}{5} = 1\frac{4}{5}$

$3x = \frac{3 \times 9}{5} = \frac{27}{5} = 5\frac{2}{5}$

$\therefore$ The two numbers are

$1\frac{4}{5}$ and $5\frac{2}{5}$

8.

Helpful Hint

Product of HCF and LCM of two numbers is equal to the product of those two numbers.

HCF × LCM $= 5\frac{5}{64}$

$$3\frac{1}{4} \times \text{LCM} = 5\frac{5}{64}$$

$$\frac{13}{4} \times \text{LCM} = \frac{325}{64}$$

$$\frac{13}{4} \times \text{LCM} = \frac{325}{64}$$

$$\text{LCM} = \frac{\overset{25}{\cancel{325}}}{\underset{16}{\cancel{64}}} \times \frac{\overset{1}{\cancel{4}}}{\cancel{13}_1}$$

$$\text{LCM} = \frac{25}{16}$$

$$\sqrt{\text{LCM}} = \sqrt{\frac{25}{16}}$$

$$= \sqrt{\frac{5 \times 5}{4 \times 4}}$$

$$= \pm \frac{5}{4} = \pm 1\frac{1}{4}$$

9.

$$\sqrt{944.578756}$$

$$\sqrt{\frac{944578756}{1000000}} = \pm \frac{\sqrt{944578756}}{1000}$$

	30734
3	$9\ \overline{44}\ \overline{57}\ \overline{87}\ \overline{56}$
+ 3	− 9
60	44
+ 0	− 00
607	4457
+ 7	− 4249
6143	20887
+ 3	− 18429
61464	245856
	− 245856
	0

$$\therefore \pm \frac{\sqrt{944578756}}{1000} = \pm \frac{30734}{1000}$$

$$= \pm 30.734$$

10. Area of a square = l^2

l^2 = 331.24 sq. metres

Take square root to find the length of each side

$$\sqrt{l^2} = \sqrt{331.24}$$

$$l = \sqrt{\frac{33124}{100}} = \frac{\sqrt{33124}}{10}$$

	182
1	$3\ \overline{31}\ \overline{24}$
+ 1	− 1
28	231
+ 8	− 224
362	724
	− 724
	0

$$\therefore l = \frac{182}{10} = 18.2$$

Perimeter = $4l$ = 4 × 18.2 = 72.8 metres

11. Area of a square = 105 square metres

l^2 = 105

Take square root on both the sides.

$$\sqrt{l^2} = \sqrt{105}$$

	10.246
1	$1\ \overline{05}$
+ 1	− 1
20	05
+ 0	− 0
202	500
+ 2	− 404
2044	9600
+ 4	− 8176
20486	142400
	122916
	18484

$$\therefore l = 10.246$$

$$= 10.25 \text{ m}$$

12. Area of a square = $\frac{289}{64}$ square metres

$$l^2 = \frac{289}{64}$$

$$\sqrt{l^2} = \sqrt{\frac{289}{64}}$$

$$l = \frac{\sqrt{17 \times 17}}{\sqrt{8 \times 8}} = \frac{17}{18}$$

$$= 2\frac{1}{8} \text{ m or } 2.125 \text{ m}$$

13. Area of square = $4m^2$

$4m^2 = 40000\ cm^2$

Helpful Hint
1m = 100 cm
1 m^2 = 10000 cm^2

Number of paper squares needed = $\frac{40000}{40}$ = 1000

Exercise 3B

1. (i) 3 (ii) perfect cube
(iii) 9 (iv) $3^3 = 27$ (v) Index

2. (i) True
Even numbers are multiples of 2. So the cube of any even number is also a multiple of 2.

(ii) False
Lets consider a natural number n
$(-n)^3 = (-n) \times (-n) \times (-n)$
$= +n^2 \times (-n) = -n^3$

∴ cube of a negative number is always negative.

(iii) True
6 × 6 × 6 = 216

(iv) False

$$= \sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}}$$

$$\therefore \sqrt[3]{\frac{a}{b}} \neq \sqrt[3]{a} \times \sqrt[3]{a}$$

(v) True
Because $6^3 = 216$, $7^3 = 343$ and 250 lies between 216 and 343.

3. (i)

3	729
3	243
3	81
3	27
3	9
3	3
	1

The prime factors of 729 are
3 × 3 × 3 × 3 × 3 × 3
$= 3^3 \times 3^3 = (3 \times 3)^3$
$= 9^3$
The prime factors of 729 can be grouped into triplets of equal factors
∴ 729 is a perfect cube.

(ii)

2	2700
2	1350
3	675
3	225
3	75
5	25
5	5
	1

The prime factors of 2700 are
$2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 5 = 2^2 \times 3^3 \times 5^2$
Its prime factors can not be grouped into triplets.
∴ 2700 is not a perfect cube.

(iii)

2	27000
2	13500
2	6750
3	3375
3	1125
3	375
5	125
5	25
5	5
	1

The prime factors are

$2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 5 \times 5$

$= 2^3 \times 3^3 \times 5^3 = (2 \times 3 \times 5)^3 = 30^3$

The prime factors of 27000 can be grouped into triplets of equal factors.

$\therefore$ 27000 is a perfect cube.

(iv)

2	34128
2	17064
2	8532
2	4266
3	2133
3	711
3	237
79	79
	1

Prime factors of 34128 are

$2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 79$

These prime factors can not be grouped into triplet of equal factors.

$\therefore$ 34128 is not a perfect cube.

4.

2	864
2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

$864 = 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$

$= 2^3 \times 2^2 \times 3^3$

One more 2 is required to make another triplet of 2.

Multiply 864 by 2.

$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$

This is a perfect cube.

Thus the required smallest integer is 2.

5.

2	13500
2	6750
3	3375
3	1125
3	375
5	125
5	25
5	5
	1

$13500 = 2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 5 \times 5$

3 and 5 occur thrice but 2 occurs only twice.

$2 \times 2 = 4$

Divide 13500 by 4

$\therefore 3 \times 3 \times 3 \times 5 \times 5 \times 5 = 3375$

This is a perfect cube.

The required smallest integer is 4.

6. (i)

2	5832
2	2916
2	1458
3	729
3	243
3	81
3	27
3	9
3	3
	1

$$\sqrt[3]{5832} = \sqrt[3]{2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3}$$

$$= \sqrt[3]{2^3 \times 3^3 \times 3^3}$$

$$= \sqrt[3]{18^3}$$

$$= 18$$

(ii)

3	9261
3	3087
3	1029
7	343
7	49
7	7
	1

$$\sqrt[3]{9261} = \sqrt[3]{3 \times 3 \times 3 \times 7 \times 7 \times 7}$$
$$= \sqrt[3]{3^3 \times 7^3}$$
$$= \sqrt[3]{21^3}$$
$$= 21$$

(iii)

2	21952
2	10976
2	5488
2	2744
2	1372
2	686
7	343
7	49
7	7
	1

$$\sqrt[3]{21952} = \sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 7 \times 7 \times 7}$$
$$= \sqrt[3]{2^3 \times 2^3 \times 7^3}$$
$$= \sqrt[3]{28^3}$$
$$= 28$$

(iv)

5	42875
5	8575
5	1715
7	343
7	49
7	7
	1

$$\sqrt[3]{42875} = \sqrt[3]{5 \times 5 \times 5 \times 7 \times 7 \times 7}$$
$$= \sqrt[3]{5^3 \times 7^3}$$
$$= \sqrt[3]{35^3}$$
$$= 35$$

(v)

2	74088
2	37044
2	18522
3	9261
3	3087
3	1029
7	343
7	49
7	7
	1

$$\sqrt[3]{74088} = \sqrt[3]{2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 7 \times 7 \times 7}$$
$$= \sqrt[3]{2^3 \times 2^3 \times 7^3}$$
$$= \sqrt[3]{42^3}$$
$$= 42$$

(vi)

5	274625
5	54925
5	10985
13	2197
13	169
13	13
	1

$$\sqrt[3]{274625} = \sqrt[3]{5 \times 5 \times 5 \times 13 \times 13 \times 13}$$
$$= \sqrt[3]{5^3 \times 13^3}$$
$$= \sqrt[3]{65^3}$$
$$= 65$$

7. Volume of a cube = l^3

l^3 = 46656 cubic metres

$l = \sqrt[3]{46656}$

2	46656
2	23328
2	11664
2	5832
2	2916
2	1458
3	729
3	243
3	81
3	27
3	9
3	3
	1

$\sqrt[3]{46656} =$

$\sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3}$

$= \sqrt[3]{2^3 \times 2^3 \times 3^3 \times 3^3}$

$= \sqrt[3]{36^3}$

= 36 metres

8. (i)

2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

$\sqrt[3]{512} = \sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}$

$= \sqrt[3]{2^3 \times 2^3 \times 2^3}$

$= \sqrt[3]{8^3} = 8$

Helpful Hint

Cube root of a negative number is always negative. $-8 \times -8 \times -8 = -512$

$\therefore \sqrt[3]{-512} = -8$

Thus, – 512 is a cube of negative integer.

(ii)

2	1296
2	648
2	324
2	162
3	81
3	27
3	9
3	3
	1

$2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3$

$= 2^3 \times 2 \times 3^3 \times 3$

Prime factors can not be grouped into triplets of equal numbers.

Thus, – 1296 is not the cube of negative integer.

(iii)

2	1372
2	686
7	343
7	49
7	7
	1

$2 \times 2 \times 7 \times 7 \times 7$

$= 2^2 \times 7^3$

– 1372 is not a cube of negative integer.

(iv)

13	2197
13	169
13	13
	1

$13 \times 13 \times 13 = 13^3$

$\therefore \sqrt[3]{-2197} = -13$

Thus, – 2197 is a cube of negative integer.

(v)

2	17576
2	8788
2	4394
13	2197
13	179
13	13
	1

$2 \times 2 \times 2 \times 13 \times 13 \times 13$

$= 2^3 \times 13^3 = 26^3$

$\therefore \sqrt[3]{-17576} = -26$

Thus, – 17576 is a cube of negative integer.

9.

Helpful Hint

Volume of a cube = l^3

Volume of a cube = 0.064 m^3

$l^3 = 0.064\text{m}^3$

$l^3 = \frac{64}{1000}$

$l = \sqrt[3]{\frac{64}{1000}} = \frac{\sqrt[3]{64}}{\sqrt[3]{1000}}$

$= \frac{\sqrt[3]{4 \times 4 \times 4}}{\sqrt[3]{10 \times 10 \times 10}}$

$= \frac{4}{10} = 0.4 \text{ m}$

10. (i)

$\sqrt[3]{27} = \sqrt[3]{3 \times 3 \times 3} = 3$

$\sqrt[3]{216} = \sqrt[3]{6 \times 6 \times 6} = 6$

$\sqrt[3]{729} = \sqrt[3]{9 \times 9 \times 9} = 9$

$\sqrt[3]{27} \times \sqrt[3]{216} \div \sqrt[3]{729}$

$= 3 \times 6 \div 9 = 18 \div 9 = 2$

(ii)

$\sqrt[3]{343} = \sqrt[3]{7 \times 7 \times 7} = 7$

$\sqrt[3]{8} = \sqrt[3]{2 \times 2 \times 2} = 2$

2	2744
2	1372
2	686
7	343
7	49
7	7
	1

$\sqrt[3]{-2744} = \sqrt[3]{2 \times 2 \times 2 \times 7 \times 7 \times 7}$

$= -14$

$\sqrt[3]{343} \times \sqrt[3]{8} \div \sqrt[3]{-2744}$

$= 7 \times 2 \div (-14)$

$= 14 \div -14$

$= -1$

(iii)

$\sqrt[3]{-125} \times \sqrt[3]{64} \times \sqrt[3]{27}$

$= -5 \div 4 \times 3$

$= \frac{-5}{4} \times 3 = \frac{15}{4} = -3\frac{3}{4}$

(iv) $\sqrt[3]{1331} \times \sqrt[3]{8} \times \sqrt[3]{1000}$

$= 11 \times 2 \times 10$

$= 220$

(v)

$\left(\sqrt[3]{1728} \div \sqrt[3]{125}\right) \times \left(\sqrt[3]{1000} \div \sqrt[3]{64}\right)$

$= (12 \div 5) \times (10 \div 4)$

$= \frac{\cancel{12}^{3}}{\cancel{5}_{1}} \times \frac{\cancel{10}^{2}}{\cancel{4}_{1}}$

$= 6$

11. Volume of a cube = l^3

$= 11^3 = 11 \times 11 \times 11$

$= 1331\ cm^3$

12. Minimum volume of tank = $729\ m^3$

$l^3 = 729$

$\sqrt[3]{l^3} = \sqrt[3]{729}$

$l = 9$ cm

13. Volume = $1728\ cm^3$

$l^3 = 1728$

$\sqrt[3]{l^3} = \sqrt[3]{1728}$

$l = 12$ cm

Since the length height, and width each is greater than 12 cm, hence, Baneen can keep the box in her drawer.

Multiple Choice Questions 3

1. B

$512 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$\sqrt[3]{512} = \sqrt[3]{2^3 \times 2^3 \times 2^3} = 8$

2. D

$3375 = 3 \times 3 \times 3 \times 5 \times 5 \times 5$

$\sqrt[3]{3375} = \sqrt[3]{3^3 \times 5^3} = 15$

3. C

In $\sqrt[3]{2197}$, 3 is the index,

$\sqrt{}$, is a radical and

13 is the cube root of 2197.

4. D

1300 is not a perfect cube

$\sqrt[3]{1331} = 11$

$\sqrt[3]{1728} = 12$

$\sqrt[3]{125} = 5$

5. A

$16 \times 16 \times 16 = 4096$

4 PROPORTIONS

Exercise 4

1. (i) Direct
   If we increase the number of books, total cost will also increase.
   (ii) directly
   (iii) inversely
   (iv) constant
   (v) curved

2. (i) True
   (ii) False
   More petrol is consumed to cover more distance.
   (iii) True
   (iv) False
   The graph of direct proportion always passes through the origin.
   (v) False
   If x and y are directly proportional then $y = kx$

3.

x	4	7	11	(20)
y	20	35	(55)	100

We can notice that y is obtained by multiplying the value of x by 5

$4 \times 5 = 20$

$7 \times 5 = 30$

Hence

$11 \times 5 = 55$

and $\frac{100}{5} = 20$

4.

x	9	27	81
y	27	9	(3)

5. Distance — time
   15 km — 3 hrs
   x km — 2 hrs

$\frac{x}{2} = \frac{15}{3}$

$x = 5 \times 2$

$x = 10$ km

6. Number of men — length
   11 — $6\frac{3}{4}$ m $= \frac{27}{4}$ m
   x — 27 m

Since both the quantities are directly proportional

$\frac{x}{27} = \frac{11}{\frac{27}{4}}$

$\frac{x}{27} = \frac{4 \times 11}{27}$

$x = \frac{44 \times \cancel{27}}{\cancel{27}}$

$x = 44$ men

7. number of taps — time
   6 — 30 min
   x — 20 min

Since both the quantities are inversely proportional.

$20 \times x = 6 \times 30$

$x = \frac{\overset{3}{\cancel{6}} \times \cancel{30}}{\cancel{20}}$

$x = 9$ taps

8. number of girls — number of days

number of girls	number of days
50	40
50 + 30 = 80	x

Both the quantities are inversely proportional

$80 \times x = 40 \times 50$

$$x = \frac{\overset{1}{\cancel{40}} \times \overset{25}{\cancel{50}}}{\cancel{80}}$$

$x = 25$ days

9.

length of rod	weight of rod
12 m	42 kg
6 m	x kg

Both the quantities are directly proportional

$$\frac{x}{6} = \frac{42}{12}$$

$$x = \frac{\overset{21}{\cancel{42}} \times \overset{1}{\cancel{6}}}{\underset{\cancel{2}\,1}{\cancel{12}}}$$

$x = 21$ Kg

10.

Width	Length	Price
12 ↓	25 ↓	2500 ↓
50	30	x

$$\frac{x}{2500} = \frac{50}{12} \times \frac{30}{25}$$

$$x = \frac{50 \times 30 \times 2500}{12 \times 25} = 12500$$

∴ Rs 12500 is the price of a carpet measuring 50 feet by 30 feet

11.

Men	Time	Days
100 ↑	8 ↓	35 ↓
x	10	25

$$\frac{x}{100} = \frac{8}{10} \times \frac{35}{25}$$

$$x = \frac{8 \times 35 \times 100}{10 \times 25} = 112$$

∴ 112 men should be employed to finish the job in 25 days if they work 10 hours a day.

12. (a) Since y is directly proportional to x, the constant is given by

$$k = \frac{y}{x}$$

$$k = \frac{9}{6} = \frac{3}{2}$$

(b) for direct proportion, we have

$y = kx$

Substitute the value of k in above equation

$$y = \frac{3}{2}x$$

(c)

x	2	6	(10)	14	18
y	3	9	15	(21)	27

$$y = \frac{3}{2}x$$

$$15 = \frac{3}{2}x$$

$$x = 15 \times \frac{2}{3}$$

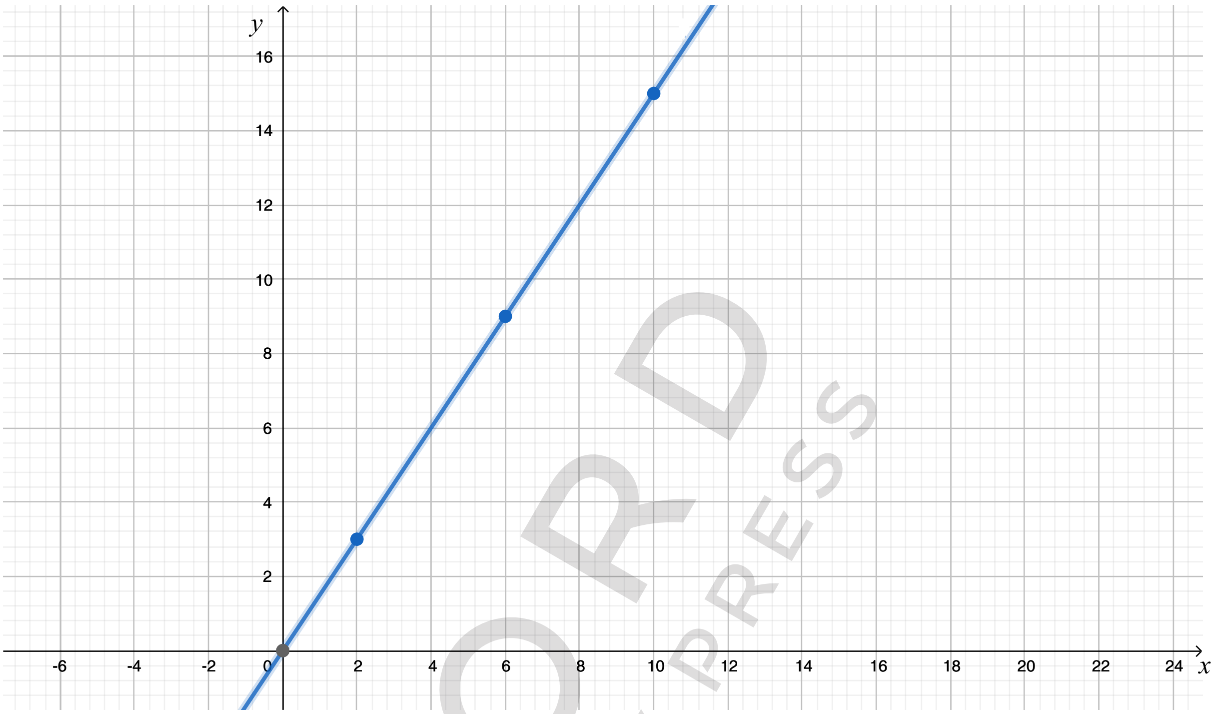
$x = 10$

$$y = \frac{3}{2}x$$

$$y = \frac{3}{2} \times 14$$

$y = 21$

(d) Use the table of values given in part c to draw the graph.



1 REVISION: NUMBERS

Q1. Distributive Law of union order intersection states that

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Let's RHS; $A \cup (B \cap C)$

$B \cap C = \{1, 3, 5, 7, 9, 11, 13, 15\} \cap \{0, 1, 5, 6, 10, 13\}$

$= \{1, 5, 13\}$

$A \cup (B \cap C) = \{0, 5, 10, 15\} \cup \{1, 5, 13\}$

$= \{0, 1, 5, 10, 13, 15\}$

Now take LHS; $(A \cup B) \cap (A \cup C)$

$A \cup B = 0, 5, 10, 15\} \cup \{0, 1, 5, 6, 10, 13\}$

$= \{0, 1, 5, 6, 10, 13, 15\}$

$B \cup C = \{1, 3, 5, 7, 9, 11, 13, 15\} \cup \{0, 1, 5, 6, 10, 13\}$

$= \{0, 1, 3, 5, 6, 7, 9, 10, 11, 13, 15\}$

$(A \cup B) \cap (A \cup C) = (0, 1, 5, 6, 10, 13, 15\} \cap \{0, 1, 3, 5, 6, 7, 9, 10, 11, 13, 15\}$

$= \{0, 1, 5, 6, 10, 13, 15\}$

Since LHS = RHS,

Thus, the distributive law of union over intersection is proved.

2.

	additive inverse	Multiplicative inverse
(i)	-354	$\frac{1}{354}$
(ii)	0.005	$-\frac{1}{0.005}$
(iii)	$-\frac{1}{\sqrt[3]{5}}$	$\sqrt[3]{5}$
(iv)	$2\frac{5}{9}$	$\frac{9}{25}$
(v)	$-\frac{17}{23}$	$\frac{22}{17}$ or $2\frac{6}{17}$

3. (i)

$$\begin{array}{r|l} & 2.5 \\ \hline 2 & 6.25 \\ +2 & -4 \\ \hline 45 & 225 \\ & -225 \\ \hline & 0 \end{array}$$

$\therefore \sqrt{6.25} = 2.5$

(ii)

$$\begin{array}{r|l} & 27 \\ \hline 2 & \overline{7}\,\overline{29} \\ +2 & -4 \\ \hline 47 & 329 \\ & -329 \\ \hline & 0 \end{array}$$

$\therefore \sqrt{729} = 27$

(iii)

$$\begin{array}{r|l} & 19 \\ \hline 1 & \overline{3}\,\overline{61} \\ +1 & -1 \\ \hline 29 & 261 \\ & -261 \\ \hline & 0 \end{array}$$

$\therefore \sqrt{361} = 19$

(iv)

$$\begin{array}{r|l} & 111 \\ \hline 1 & \overline{1}\,\overline{23}\,\overline{21} \\ +1 & -1 \\ \hline 21 & 23 \\ +1 & -21 \\ \hline 221 & 221 \\ & -221 \\ \hline & 0 \end{array}$$

$\therefore \sqrt{12321} = 111$

(v)

	1.452
1	2.10 83 04
+ 1	− 1
21	110
+ 1	− 96
285	1483
+ 5	− 1425
2902	5804
	− 5804
	0

$\therefore \sqrt{2.108304} = 1.452$

(vi)

	22.2
2	492.84
+ 2	− 4
42	092
+ 2	− 84
442	884
	− 884
	0

$\therefore \sqrt{492.84} = 22.2$

(vii)

	8.8
8	77.44
+ 8	− 64
168	1344
	− 1344
	0

$\therefore \sqrt{77.44} = 8.8$

(viii)

	0.71
0	0.50 41
+ 0	− 0
7	50
+ 7	− 49
141	141
	− 141
	0

$\therefore \sqrt{0.5041} = 0.71$

4. (i) $\sqrt{1\frac{29}{49}} = \sqrt{\frac{78}{49}} = \frac{\sqrt{78}}{\sqrt{49}} = \frac{\sqrt{78}}{7}$

	8.83
8	78
+ 8	− 64
168	1400
+ 8	− 1344
1763	5600
	− 5289
	311

$\therefore \sqrt{\frac{78}{49}} = \frac{8.83}{7} = 1.26$

(ii) $\sqrt{2\frac{42}{75}} = \sqrt{\frac{192}{49}} = \sqrt{2.56}$

	1.6
1	2.56
+ 1	− 1
26	156
	− 156
	0

$\therefore \sqrt{2\frac{42}{75}} = 1.6$

(iii) $\sqrt{3\frac{334}{3024}} = \sqrt{\frac{9406}{3024}} = \sqrt{3.11}$

	1.77
1	$3.\overline{11}$
+ 1	– 1
27	211
7	– 189
346	2200
	– 2076
	124

$\therefore \sqrt{3\frac{334}{3024}} = 1.77$

5. (i) $\sqrt{12}$

	3.46
3	12
+ 3	– 9
64	300
+ 4	– 256
686	4400
	– 4116
	284

$\therefore \sqrt{12} = 3.46$

(ii)

	7.28
7	53
+ 7	– 49
142	400
+ 2	– 284
1448	11600
	– 11584
	16

$\therefore \sqrt{53} = 7.28$

(iii)

	10.44
1	$1\overline{09}$
+ 1	– 1
20	09
+ 0	0
204	900
– 4	– 816
2084	8400
	– 8336
	64

$\therefore \sqrt{109} = 10.44$

(iv) $\sqrt{26}$

	5.09
5	26
+ 5	– 25
100	100
+ 0	– 0
1009	10000
	– 9081
	1919

$\therefore \sqrt{26} = 5.09$

6. (i)

3	3375
3	1125
3	375
5	125
5	25
5	5
	1

$\sqrt[3]{3375} = \sqrt[3]{3^3 \times 5^3} = 3 \times 5$

$\sqrt[3]{3375} = 15$

(ii)

2	17576
2	8788
2	4394
13	2197
13	169
13	13
	1

$\sqrt[3]{17576} = \sqrt[3]{2 \times 2 \times 2 \times 13 \times 13 \times 13}$

$\sqrt[3]{17576} = 2 \times 13 = 26$

(iii)

61	226981
61	3721
61	61
	1

$\sqrt[3]{226981} = \sqrt[3]{61 \times 61 \times 61}$

$\therefore \sqrt{226981} = 61$

(iv)

2	2744
2	1372
2	686
7	343
7	49
7	7
	1

$\sqrt[3]{2744} = \sqrt[3]{2 \times 2 \times 2 \times 7 \times 7 \times 7}$

$\therefore \sqrt[3]{2744} = 2 \times 7 = 14$

(v)

3	35937
3	11979
3	3993
11	1331
11	121
11	11
	1

$\therefore \sqrt[3]{35937} = \sqrt[3]{3^3 \times 11^3}$

$\therefore \sqrt[3]{35937} = 2 \times 11 = 33$

7.

(i) Calculate $\sqrt{15.21}$

Since $30^2 = 900$ and 40^2 1600, the root is near $4.3.9^2 = 15.21$.

Result: 3.9

(ii) Calculate $\sqrt{73.96}$

Since $80^2 = 6400$ and 90^2 8100, the root is between 8 and $9.8.6^2 = 73.96$.

Result: 8.6

(iii) Calculate $\sqrt{8213.05}$

Using long division or estimation $90^2 =$ 8100), we find the approximate root.

≈ 90.626

8. Rationalize and calculate

$\frac{3}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$ Knwing $\sqrt{3} \approx 1.73205$:

$\frac{2 \times 1.72305}{3} \approx \frac{3.4641}{3} \approx 1.1547$

9. 1.15

(i) $\frac{\sqrt{13.69} + \sqrt{18.49}}{\sqrt{151.29} - \sqrt{5.29}}$

$3.7 + 4.3 = 8.0$ (Numerator)

$12.3 - 2.3 = 10.0$ (Denominator)

$\frac{8}{10} = 0.8$

0.8

(ii) $\frac{\sqrt{77.44} + \sqrt{43.56}}{\sqrt{77.44} - \sqrt{44.89}}$

$8.8 + 6.6 = 15.4$

$8.8 - 6.7 = 2.1$

$\frac{15.4}{2.1} = \frac{154}{21} \frac{22}{3} = 7.33$

7.33

10.

$\sqrt{999999} \approx 999.99$. The largest integer root is 999.

$999^2 = (1000 - 1)^2 = 1000000 - 2000 + 1$
$= 998001.$

998001

11.

The volume V is given as 32.768 m³. Since $V = s^3$ we have:

$s^3 = 32.768$

$s \sqrt[3]{32.768}$

$32.768 = \frac{32768}{1000}$

$\sqrt{\frac{32768}{1000}} = \frac{\sqrt[3]{32768}}{\sqrt[3]{1000}}$

$s = \frac{32}{10}$ 3.2

The length of the side of the box is 3.2 m.

12.

$26244 = 2^2 \times 3^8$.

To be a cube, powers must be multiples of 3. We have 2^2 (needs to be removed) and $3^8 = 3^6 \times 3^2$

Divisor $= 2^2 \times 3^2 = 4 \times 9 = 36$.

Quotient $= \frac{26244}{36} = 729$. $\sqrt[3]{729} = 9$.

Divisor is 36; Cube root is 9.

13.

(i) $\sqrt{9.61} = 3.1$

(ii) $\sqrt{92.16} = 9.6$

(iii) $\sqrt{1867.1041} = 43.21$

16.

(i) $\sqrt{5.4756} = 2.34$

(ii) $\sqrt{547.56} = 2.34 \times 10 = 23.4$

(iii) $\sqrt{0.054756} = 2.34/10 = 0.234$

18.

(i)

7	456533
7	65219
7	9317
11	1331
11	121
11	11
	1

$456533 = 7 \times 7 \times 7 \times 11 \times 11 \times 11$

$= 7 \times 11$

$\sqrt{456533} = 77$

(ii)

3	59319
3	19773
3	6591
13	2197
13	169
13	13
	1

$59319 = 3 \times 3 \times 3 \times 13 \times 13 \times 13$

$= 3 \times 13$

$\sqrt[3]{59319} = 39.$

(iii)

1520.875

3	1520875
	1000

5	1520875
5	304175
5	60835
23	12167
23	529
23	23
	1

10	1000
10	100
10	10
	1

$= \sqrt[3]{\frac{1520875}{1000}}$

$= \frac{5 \times 5 \times 5 \times 23 \times 23 \times 23}{10 \times 10 \times 10} = \frac{5 \times 23}{10}$

$= \frac{115}{10}$

$= \sqrt[3]{1520.875} = 11.5$

19.

$\sqrt[3]{512}$

$= \overline{2 \times 2 \times 2} \times \overline{2 \times 2 \times 2} \times \overline{2 \times 2 \times 2}$

$= 2 \times 2 \times 2$

$= 8m$

20.

(i) $\sqrt[3]{792}$

Prime factorize 792:

$792 = 2 \times 396 = 2^2 \times 198$

$= 2^3 \times 99 = 2^3 \times 3^2 \times 11.$

Extract perfect cube factors:

$\sqrt[3]{2^3 \times 3^2 \times 11} = 2 \times \sqrt[3]{99}.$

Approximating $\sqrt[3]{99} \approx 4.626.$

Result: $792 \approx 9.252.$

(ii) $\sqrt[3]{29600}$

Prime factorize 29600:

$29600 = 100 \times 296$

$= (2^2 \times 5^2) \times (2^3 \times 37) =.$

$29600 = 2^3 \times (2^2 \times 5^2 \times 37)$

$= 2^3 \times 3700.$

$\sqrt[3]{29600} = 2 \times \sqrt[3]{3700}.$

Approximating $\sqrt[3]{3700} \approx 15.465.$

Result: $\approx 2 \times 15.465 \approx 30.93.$

(iii) $\sqrt[3]{\frac{31}{216}}$ $\frac{\sqrt[3]{a}}{\sqrt[3]{b}}$

$\frac{\sqrt[3]{31}}{\sqrt[3]{216}}.$

$\sqrt[3]{29600} = 2 \times \sqrt[3]{3700}.$

Since $216 = 6^3$, $\sqrt[3]{216} = 6.$

Approximating $\sqrt[3]{31} \approx 3.141.$

$\frac{3.141}{6} \approx 0.5235.19.$

5 FINANCIAL ARITHMETIC

Exercise 5

1. (i) Health (ii) 20% (iii) $\frac{1}{8}$

   (iv) Markup = Profit – Principal

   (v) Marked Price – Selling Price

2. (i) False

   Life insurance premiums are paid on monthly or yearly basis.

   (ii) False

   A son inherits twice the share of a daughter.

   (iii) True

   (iv) False

   The profit is divided equally if the investment is equal.

   (v) True

3.

Helpful Hint
Profit is earned if selling price is greater than the cost price.

CP = Rs 3000

SP = Rs 3500

Profit = SP – CP

= 3500 – 3000 = Rs 500

Profit Percentage = $\frac{\text{Profit}}{\text{CP}} \times 100\%$

$= \frac{\text{Rs } 500}{\text{Rs } 3000} \times 100$

= 16.66% or 16.7%

4.

Helpful Hint
Loss is occurred if selling price is less than the cost price.

CP = Rs 10000

SP = Rs 9800

Loss = CP – SP

= Rs 10000 – Rs 9800

= Rs 200

Loss percentage = $\frac{\text{Loss}}{\text{CP}} \times 100\%$

$= \frac{200}{10000} \times 100\% = 2\%$

∴ Loss percentage = 2%

5. CP of desk =Rs 15000

   CP of chair = Rs 5000

   Total CP = 15000 + 5000
   = Rs 20000

   SP of desk = Rs 15500

   SP of chair = Rs 1000

   Total SP = 15500 + 100
   = Rs 16500

   Since CP is greater than SP hence Jawaid bears an over loss.

   Loss = CP – SP

   = 20000 – 16500

   = Rs 3500

   Loss percentage = $\frac{\text{Loss}}{\text{CP}} \times 100\%$

   $= \frac{3500}{20000} \times 100\% = 17.5\%$

   ∴ Loss percentage = 17.5%

6. CP = Rs 750000

SP = Rs 200000

Loss = CP – SP

= 750000 – 200000

= Rs 550000

Loss percentage = $\frac{\text{Loss}}{\text{CP}} \times 100\%$

$= \frac{550000}{750000} \times 100\%$

= 73.3%

7. The most expensive item in the shop is priced at Rs 100.

Since,

10% of 100 is Rs 10.

Hence, the shopkeeper should offer 10% discount on all the items, so that he does not lose more than Rs 10 on any item.

8. Total purchase = Rs 12,000

1% of Rs 5000 = $\frac{1}{100} \times 50000$ = Rs 50

2% of Rs 5000 = $\frac{2}{100} \times 500000$ = Rs 200

Remaining amount = 12000 – 10000

= Rs 2000

3% of Rs 2000 = $\frac{3}{100} \times 2000$

= Rs 60

Total Discount = Rs 50 + Rs 100 + Rs 60

= Rs 210

9. Rate of Premium = 2%

Insurance amount = Rs 12000000

Amount of Premium = ?

Amount of premium = 2% of Rs 12000000

$= \frac{2}{100} \times 12000000$

= Rs 240000

10. Market value of car= Rs 1000000

Rate of premium = 5%

Amount of insurance premium paid in the 1st year = 5% of Rs 1000000

$= \frac{5}{100} \times 1000000$

= Rs 50000

Amount of insurance premium paid in 2nd year = $\frac{5}{100} \times (1000000 - 50000)$

$= \frac{5}{100} \times 950000$

= Rs 47500

Amount of insurance premium paid in 3rd year = $\frac{5}{100} \times (950000 - 47500)$

$= \frac{5}{100} \times 902500$

= Rs 45125

11. Total amount = Rs 250000

widow's share = $\frac{1}{8}$ of Rs 250000

$= \frac{1}{8} \times 250000$

= Rs 31250

Remaining amount = 250000 – 31250

= Rs 218850

Helpful Hint

Son inherits twice the share of a daughter.

Ratio of shares of sons to share of daughter

2 : 1

2 : 1 × 2 [2 daughters]

2 : 2

1 : 1

Sum of ratios = 1 + 1 = 2

1 son's share = $\frac{1}{2} \times 218750$

= Rs 109375

2 daughter's share = $\frac{1}{2} \times 218750$

= Rs 109375

1 daughter's share = Rs $\frac{109375}{2}$

= Rs 54687.5

12. Total amount = Rs 30000

Ratio of shares of sons to the shares of daughters

2 : 1

2 × 2 : 1 [2 sons]

4 : 1

Sum of ratios = 4 + 1 = 5

2 sons' share = $\frac{4}{5} \times 30000$ = Rs 24000

1 son's share = Rs $\frac{240000}{2}$ = Rs 12000

the daughter's share = $\frac{1}{5} \times 30000$

= Rs 6000

13. Ratio of share in profit = ratio of their investments

Maria's investment: Samia's investment

22500 : 35000

9 : 14

Sum of ratios = 9 + 14 = 23

Maria's share in profit = $\frac{9}{23} \times 13800$

= Rs 5400

Samia's share in profit = $\frac{14}{23} \times 13800$

= Rs 8400

Or we can simply subtract Maria's share from total profit Rs 13800 – 5400 = Rs 8400

14. Total amount = Rs 240000

Ratio of inheritance = 5 : 4 : 3

Sum of the ratios = 5 + 4 + 3 = 12

Aslam receives inheritance = $\frac{5}{12}$ × Rs 240000

= Rs 100000

Pervaiz receives = $\frac{4}{12}$ × Rs 240000

= Rs 80,000

Sana receives = $\frac{3}{12}$ × Rs 240000

= Rs 60000

15. Amount in US$\$$ = $\$59$

Exchange rate for $\$1$ = Rs 104

Amount in rupees = 59 x 104

= Rs 6136

16. Total amount in rupees = Rs 200000

Half of amount = Rs $\frac{200000}{2}$

= Rs 100000

Exchange rate for $\$1$ = Rs 107

Amount in US$\$$ = $\frac{\text{Rs } 100000}{\text{Rs } 107}$

= $\$\ 934.579$

Exchange rate for $\$1$ = Rs 106 for rest of the amount

Amount in US$\$$ = $\frac{\text{Rs } 100000}{\text{Rs } 106}$

= $\$\ 934.396$

Total amount in US$\$$ = $\$\ 934.579 + \$\ 943.396$

= $\$\ 1877.98 \cong \$\ 1878$

17. (i)

P = Rs 400

R = 5%

T = 4 years

Markup = ?

Markup = $\frac{P \times R \times T}{100}$

= $\frac{\cancel{400} \times 5 \times 4}{\cancel{100}}$

= Rs 80

Hence markup is Rs 80.

(ii)

P = Rs 50

R = $3\frac{1}{2}\%$

T = 7 years

Markup = ?

Markup = $\frac{\cancel{50} \times 7 \times 7}{2 \times \cancel{100}_2}$

= $\frac{49}{4}$

= Rs 12.25

Hence markup is Rs 12.25

(iii) P = Rs 450

R = 2%

T = 2 years 9 month = $2\frac{3}{4}$ years

Markup = ?

$$\text{Markup} = \frac{P \times R \times T}{100}$$

$$= \frac{\cancel{450}^{9} \times \cancel{4} \times 11}{{}_{2}\cancel{100} \times \cancel{4}}$$

$$= \frac{99}{2}$$

= Rs 49.50

Hence, markup is Rs 49.50

18. (i) P = Rs 700

Markup = Rs 210

T = 3 years

R% = ?

$$\text{Markup} = \frac{P \times R \times T}{100}$$

$$R = \frac{\text{Markup} \times 100}{P \times T}$$

$$R = \frac{\cancel{210}^{\cancel{70}\,10} \times \cancel{100}}{\cancel{700}_{\cancel{7}} \times \cancel{3}_{1}}$$

R% = 10%

Hence, the rate% per annum is Rs 10%

(ii) P = Rs 1200

Markup = Rs 144

T = 2 years

R% = ?

$$R = \frac{\text{Markup} \times 100}{P \times T}$$

$$= \frac{\cancel{144}^{\cancel{12}\,6} \times \cancel{100}}{\cancel{1200}_{\cancel{12}\,1} \times \cancel{2}}$$

= 6

R% = 6%

Hence the rate% per annum is 6%

19. (i) P = Rs 600

Markup = Rs 90

R% = 5% per anum

T = ?

$$T = \frac{\text{Markup} \times 100}{P \times T}$$

$$= \frac{\cancel{90}^{\cancel{15}\,3} \times \cancel{100}^{1}}{\cancel{600}_{6} \times \cancel{5}_{1}}$$

T = 3 years

Hence the required time is 3 years.

(ii) P = Rs 1500

Markup = Rs 450

R% = 6% per annum

T = ?

$$T = \frac{M \times 100}{P \times R}$$

$$= \frac{\cancel{450}^{\cancel{75}\,5} \times \cancel{100}^{1}}{\cancel{1500}_{\cancel{15}\,1} \times \cancel{6}_{1}}$$

T = 5 years

Hence the required time is 5 years.

20. P = Rs 660

R% = $4\frac{1}{2}\% = \frac{9}{2}\%$

T = 3 year and 4 months = $3\frac{1}{3}$ year

Markup = ?

$$\text{Markup} = \frac{P \times R \times T}{100}$$

$$= \frac{\cancel{660}^{\cancel{66}\,\cancel{22}\,11} \times 9 \times \cancel{10}^{1}}{\cancel{2} \times \cancel{100}_{\cancel{10}\,1} \times \cancel{3}}$$

Markup = Rs 99.

Hence the required markup is Rs 99.

21. P = Rs 600

R% = 6%

T = 3 year and 6 months and 20 days

Markup = ?

$$\text{Markup} = \frac{P \times R \times T}{100}$$

T = 3 years 6 months and 20 days.

Converting into years

20 days = $\frac{20}{30} = \frac{2}{3}$ months

Total months = $6\frac{2}{3} = \frac{20}{3}$ months

$\frac{20}{3}$ months = $\frac{\cancel{20}^{5}}{3 \times \cancel{12}_{3}}$ years

$$= \frac{5}{9} \text{ years}$$

$$\text{Total years} = 3\frac{5}{9} \text{ years}$$

$$= \frac{32}{9} \text{ years}$$

$$\text{Markup} = \frac{\cancel{600}^{\cancel{6}\,2} \times \cancel{6}^{2} \times 32}{\cancel{100}_{1} \times \cancel{9}_{\cancel{3}\,1}}$$

$$\text{Markup} = \text{Rs } 128$$

Hence the markup is Rs 128.

22.
$$P = \text{Rs } 850$$

$$R\% = 5\frac{1}{2}\% = \frac{11}{2}\% \text{ per annum}$$

$$T = 6 \text{ months} = \frac{1}{2} \text{ years}$$

$$\text{Markup} = ?$$

$$\text{Markup} = \frac{P \times R \times T}{100}$$

$$= \frac{\cancel{850}^{\cancel{85}\,17} \times 11 \times 1}{2 \times \cancel{100}_{\cancel{10}\,2} \times 2}$$

$$\text{Markup} = \frac{187}{8}$$

$$= \text{Rs } 23.375$$

Hence the markup is Rs 23.375

23.
$$P = \text{Rs } 560$$

$$R\% = 6\% \text{ per annum}$$

$$T = 6 \text{ months} = 1/2 \text{ year}$$

$$A = ?$$

$$\text{Markup} = \frac{P \times R \times T}{100}$$

$$= \frac{\cancel{560}^{\cancel{56}\,28} \times 1 \times 3}{\cancel{2}_{1} \times \cancel{100}_{10}}$$

$$= \frac{84}{10}$$

$$= \text{Rs } 8.40$$

$$\text{Profit} = \text{Principal} + \text{Markup}$$

$$= 560 + 8.40$$

$$= \text{Rs } 568.40$$

Hence the profit is Rs 568.40.

24.
$$P = ?$$

$$\text{Markup} = \text{Rs } 130$$

$$R\% = 3\frac{1}{4}\% = \frac{13}{4}\%$$

$$T = 5 \text{ years}$$

$$P = \frac{I \times 100}{R \times T}$$

$$= \frac{\cancel{130}^{\cancel{10}\,2} \times 100 \times 4}{\cancel{13} \times \cancel{5}_{1}}$$

$$= \text{Rs } 800$$

Hence, principal amount is Rs 800.

Multiple Choice Questions 5

1. D
2. D

Loss = CP – SP = Rs 2000 – Rs 18000 = Rs 200

$$\text{Loss percentage} = \frac{\text{Loss}}{\text{CP}} \times 100\%$$

$$= \frac{200}{2000} \times 100\% = 10\%$$

3. C

Profit = SP – CP = Rs 3000 – Rs 2000 = Rs 1000

$$\text{Profit percentage} = \frac{\text{Profit}}{\text{CP}} \times 100\%$$

$$= \frac{1000}{2000} \times 100\% = 50\%$$

4. C

Dis = Dis rate × Marked price

$$= 3\% \times \text{Rs } 500 = \frac{3}{100} \times \text{Rs } 500 = \text{Rs } 15$$

5. B

Amount of premium

= rate of premium × insurance amount

Rs 4000 = rate of premium × Rs 200,000

$$\text{rate of premium} = \frac{\text{Rs } 4000}{\text{Rs } 200000} \times 100\%$$

= 2%

6 ALGEBRA: LAWS OF INDICES/EXPONENTS

Exercise 6

1. (i) 125
   (ii) exponent
   (iii) 0 (zero)
   (iv) 3.456×10^2
   (v) 0.0001357

2. (i) False
   $x^0 = 1$
   (ii) True
   (iii) False

> **Helpful Hint**
> $a^m \times a^n = a^{m+n}$

   (iv) False
   $x^3 \div x^5 = \frac{x^3}{x^5} = x^{3-5} = x^{-2}$
   (v) False
   $(5^4)^2 = 5^{4\times2}$

3. (i) 3^4
   (ii) $8 = 2 \times 2 \times 2 = 2^3$
   (iii) $\left(\frac{2}{3}\right)^1$
   (iv) $\frac{8}{27} = \frac{2 \times 2 \times 2}{3 \times 3 \times 3} = \frac{2^3}{3^3} = \left(\frac{2}{3}\right)^3$
   (v) $2^{-3} \times 2^{-3} = 2^{-3-3} = 2^{-6}$

4.

> **Helpful Hint**
> Give answers without simplification of exponents.

   (i) base = , 3 exponent = 4
   (ii) base = 4, exponent = 1
   (iii) base = 2^{-3}, exponent = 5
   (iv) base = x, exponent = –4
   (v) base = 5, exponent = 0
   (vi) base = 1, exponent = 1
   (vii) base = 3, exponent = $\frac{1}{2}$
   (viii) base = 5, exponent = $\frac{1}{2}$
   (ix) base = $3a^2$, exponent = 0
   (x) base= a^2, exponent = 1

5. (i) $4\,(a^3)^0$
   $= 4 \times 1$ $[a^o = 1]$
   (ii) $(3)^{-4}$
   $= \frac{1}{3^4} = \frac{1}{81}$
   (iii) $\frac{4}{2^0} = 4$
   (iv) $(2)^6$
   $= 64$
   (v) $(2^{\frac{1}{2}})^8$
   $= 2^{\frac{1}{\cancel{4}} \times \cancel{8}^{4}}$
   $=2^4 = 16$

6. (i) $(4x^3)\,(2x^3)$
   $= 8x^{3+3}$ $[a^m \times a^n = a^{mn}]$
   $= 8x^6$
   (ii) $8^0 = 1$ $[a^0 = 1]$
   (iii) $(9x)^0$
   $= -1$

(iv) $(y^4)^3$

$= y^{4\times3}$

$= y^{12}[(a^0)^n = a^{mn}]$

(v) $(x^2y)^4$

$= x^{2\times4}\,y^{4} = x^8\,y^4$

(vi) $(2cd^4)\,(cd)^5$

$= (2cd^4)\,(c^5d^5) = 2\,c^{1+5}\,d^{4+5}$

$= 2c^6d^9$

(vii) $(2fg^4)^4\,(fg)^6$

$= (15f^4g^{16})\,(f^6\,g^6)$

$=16f^{4+6}\,g^{16+6}$

$=16f^{10}\,g^{22}$

(viii) $\dfrac{x^5\,y^6}{xy^2}$

$= x^{5-1}\,y^{6-2} = x^4\,y^4$

(ix) $\dfrac{x^2\,y^5}{xy^4}$

$= x^{2-1}\,y^{5-4} = x\,y$

(x) $\dfrac{x^{-2}}{x^{-8}}$

$= x^{8-1} = x^7$

(xi) $\dfrac{24x^6}{12x^{-8}}$

$=2x^{8+6} = 2x^{14}$

(xii) $(2x^32y^{-3})^{-2}$

$= \dfrac{y^6}{4x^6}$

7. (i) $\left(\dfrac{-2}{7}\right)^{-4} \times \left(\dfrac{-2}{7}\right)^{2}$

$= \left(\dfrac{-7}{23}\right)^{4} \times \left(\dfrac{-5}{7}\right)^{2}$

$= \dfrac{7^{4-2}\times 25}{(-2)^4}$

$= \dfrac{7^{4-2}\times 25}{(-2)^4}$

$= \dfrac{7^2\times 25}{16} = \dfrac{49\times 25}{16} = \dfrac{1225}{16}$

$= 76\,\dfrac{9}{16}$

(ii) $\left(\dfrac{-1}{4}\right)^{-3} \times \left(\dfrac{-1}{4}\right)^{-2}$

$= (-4)^3 \times (-4)^2$

$= (-4)^{3+2} = (-4)^5$

$= -\,64$

Helpful Hint

If the exponent on a negative number is an odd number, the result is always a negative number.

(iii) $\left[\left(\dfrac{-3}{2}\right)^{2}\right]^{-3}$

$= \left[\dfrac{-3}{2}\right]^{-6}$

$= \left[\dfrac{-2}{3}\right]^{6} = \dfrac{64}{729}$

(iv) $(2)^{-2} \times (4)^2$

$= \dfrac{1}{2^2} \times 16$

$= \dfrac{1}{4} \times 16 = 4$

8. (i) $\left(\dfrac{1}{2}\right)^{-2} \times \left(\dfrac{1}{3}\right)^{-2} \times \left(\dfrac{1}{4}\right)^{-2}$

$= 2^2 + 3^2 + 4^2$

$= 4 + 9 + 16 = 29$

(ii) $\left(\dfrac{2}{5}\right)^{-2} \div \left(\dfrac{9}{5}\right)^{0}$

$= \dfrac{2^2}{5^2} \div 1$

$= \dfrac{4}{25}$

(iii) $= (2^{-1} \times 5^{-1})^{-1} \div 4^{-1}$

$= \left(\dfrac{1}{2} \times \dfrac{1}{5}\right)^{-1} \div \dfrac{1}{4}$

$= \left(\dfrac{1}{10}\right)^{-1} \times 4 = 10 \times 4 = 40$

(iv) $= (4^{-1} + 8^{-1}) \div \left(\frac{2}{3}\right)^{-1}$

$= \left(\frac{1}{4} + \frac{1}{8}\right) \div \left(\frac{3}{2}\right)$

$= \frac{2+1}{8} \times \frac{2}{3}$

$= \frac{\cancel{3}^{1}}{\cancel{8}_{4}} \times \frac{\cancel{2}^{1}}{\cancel{3}_{1}} = \frac{1}{4}$

9. (i) 700 (2 places towards left)
7.00 × 10^2

(ii) 5100000000 (9 places towards left)
5.10 × 10^9

(iii) 30812 (4 places towards left)
3.0812 × 10^4

(iv) 0.003187 (3 places towards right)
3.187 × 10^{-3}

Helpful Hint
Use negative sign for exponent of 10 when the decimal point moves towards right

10. (i) 3.18 ×10^6 (move decimal point 6 places towards right)
= 3180000

Helpful Hint
Move decimal point toward right if the exponent of 10 is positive and vice versa

(ii) 0.65 ×10^{-3} (move decimal point 3 places towards left)
= 0.00065

(iii) 1.80 ×10^2 (move decimal point 2 places towards right)
= 180

(iv) 6.21 ×10^4 (move decimal point 4 places towards right)
= 62100

Multiple Choice Question 6

1. A
2. C
3. A
4. D
5. B

7 ALGEBRA POLYNOMIALS

Exercise 7A

1. (i) 1, 4, 7, 10, 13,...

$T_1 = 1$, $d = 4 - 1 = 3$

n^{th} term = $T_1 + (n - 1)d$

$T_n = 1 + (n -)3$

$= 1 + 3n - 3$

$T_n = 3n - 2$

> **Helpful Hint**
> T_1 is the first term of the number sequence and d is the common difference

(ii) 2, 6, 10, 14, 18,...

$T_1 = 2$

$d = 6 - 2 = 4$

$T_n = T_1 + (n - 1)d$

$= 4 + (n - 1)4$

$= 4 + 4n - 4$

$T_n = 4n$

(iii) – 4, 0, –4 –8, 12, ...

$T_1 = -4$

$d = 0 - (+4) = 0 - 4 = -4$

$T_n = 4\ 3 + (n - 1)\ 4\ (-4)$

$= 4 - 4n + 4$

$= 8 - 4n$

(iv) 26, 20, 14, 8, 2,...

$T_1 = 26$, $d = 20 - 26 = 6$

$T_n = 26 + (n - 1)(6)$

$= 26 - 6n + 6$

$= 32 - 6n$

2. (i) 13, 16, 19,...

$T_n = T_1 + (n - 1)d$

$T_n = 7 + (n - 1)\ 3$

$= 7 + 3n - 3$

$T_n = 3n + 4$

(ii) $T_{10} = 3\ (10) + 4$

$= 30 + 4$

$T_{10} = 34$

> **Helpful Hint**
> To find 10th term substitute $n = 10$ in the n^{th} term formula.

3. 1, 5, 9, 13, 17,

$T_1 = 1$, $d = 5 - 1 = 4$

(i) $T_n = T_1 + (n - 1)\ d$

$= 1 + (n - 1)\ 4$

$= 1 + 4n - 4$

$T_n = 4n - 3$

(ii) $T_{10} = 4\ (`10) - 3$

$= 40 - 3 = 37$

(iii) Keep 95 equal to the nth term

$4n - 3 = 95$

Now find the value of n.

$4n = 95 + 3$

$n = \frac{98}{4} = 24.5$

Since, the value of n is not a natural number hence, 95 will not be a term of this sequence.

> **Helpful Hint**
> Here 'n' represents natural numbers.

4. (i) $Tn = 5n + 2$

1st term $T_1 = 5\ (1) + 2 = 5 + 2 = 7$

2nd term $T_2 = 5\ (2) + 2 = 10 + 2 = 12$

3rd term $T_3 = 5\ (3) + 2 = 15 + 2 = 17$

(ii) 5, 11, 17, 23, 29,

$T_1 = 5$, $d = 11 - 5 = 6$

$T_n = T_1 + (n - 1)d$

$= 5 + (n - 1)\ 6$

$= 6n - 1$

5. (i) 2, 9, 16, 23, 30,

$T_1 = 2,\ d = 9 - 2 = 7$

$T_n = 2 + (n - 1)\ 7$

$= 2 + 7n - 7$

$= 7n - 5$

(ii) $T_{100} = 7\ (100) - 5$

$= 700 - 5$

$T_{100} = 695$

6. (i) 10, 7, 4, 1, – 2

$T_1 = 10,\ d = 7 - 10 = -3$

$T_n = 10 + (n - 1)\ (-3)$

$= 10 - 3n + 3$

$T_n = 13 - 3n$

(ii) $T_{50} = 13 - 3\ (50)$

$= 13 - 150$

$T_{50} = -137$

7. 12, 22, 32, 42, 52,

$T_1 =,\ d = 22 - 12 = 10$

$T_n = 12 + (n - 1)\ 10$

$= 12 + 10n - 10$

$T_n = 2 + 10n$

8. (i) $T_n = 3n - 2$

1st term T_1 3 (1) – 2

$= 3 - 2 = 1$

2nd term $T_2 = 3\ (2) - 2$

$= 6 - 2 = 4$

(ii) $3n - 2 = 70$

$3n = 70 + 2$

$n = \frac{72}{3}$

$n = 24$

70 is the 24th term of this sequence.

(iii) $3n - 2 = 101$

$3n = 101 + 2$

$n = \frac{103}{3}$

$n = 34.33$

Since, n is not a natural number hence, 111 is not a term of the given sequence.

9. $T_n = 5 - 3n$

1st term $= T_1 = 5 - 3\ (1)$

$= 5 - 3 = 2$

2nd term $= T_2 = 5 - 3\ (2)$

$= 5 - 6 = -1$

3rd term $= T_3 = 5 - 3\ (3)$

$= 5 - 9 = -4$

10. $T_n = 4n - 7$

(i) $T_1 = 4(1) - 7 = 4 - 7 = -3$

$T_2 = 4\ (2) - 7 = 8 - 7 = 1$

$T_3 = 4\ (3) - 7 = 12 - 7 = 5$

(ii) $T_{50} = 4\ (50) - 7 = 200 - 7 = 193$

$T_{51} = 4\ (51) - 7 = 204 - 7 = 197$

Difference between 50th and 51st terms is,

$T_{51} - T_{50} = 197 - 193 = 4$

(iii) Last term = 393

$T_n = 393$

$4n - 7 = 393$

$4n = 393 + 7$

$n = \frac{400}{4}$

$n = 100$

There are 100 terms in this sequence.

11. 30, 25, 20, 15, 10,

$T_1 = 30,\ d = 25 - 30 = -5$

$T_n = T_1 + (n - 1)\ d$

$= 30 + (n - 1)(-5)$

$= 30 - 5n + 5$

$= 35 - 5n$

12. 2, 25, 3, 3.5, 4,

(i) $T_1 = 2\ ,\ d = 2.5 - 2 = 0.5$

$T_n = T_1 + (n - 1)d$

$= 2 + (n - 1)\ (0.5)$

$= 2 + 0.5n - 0.5$

$= 0.5n + 1.5$

(ii) $T_{20} = 0.5(20) + 1.5$

$= 10 + 1.5$

$= 11.5$

13.

(i) 22 (5) 26 (6)

(ii)

Fig #	# of dots
1	6
2	10
3	14
4	18

(iii) $T_n = 4n + 2$

$T_{100} = 4(100) + 2$

$= 400 + 2$

$T_{100} = 402$

difference = 4 : first term = 6

So, $T_n = T_1 + (n - 1)9$

$= 6 + (n - 1)4$

$= 6 + 4n - 4$

$= 4n + 2$

Exercise 7B

1. $(m^2 + mn + n^2)(m^2 + n)$

   $= m^2(m^2 + n) + mn(m^2 + n) + n^2(m^2 + n)$

   $= m^4 + m^2n + m^3n + mn^2 + m^2n^2 + n^3$

   or $m^4 + m^3n + m^2n + m^2n^2 + mn^2 + n^3$

2. $(ab + 1)(5a^2 - 2ab + 3b^2)$

   $= ab(5a^2 - 2ab + 3b^2) + 1(5a^2 - 2ab + 3b^2)$

   $= 5a^3b - 2a^2b^2 + 3ab^3 + 5a^2 - 2ab + 3b^2$

3. $(a^4 - a^2 + 1)(a^4 + a^2 - 1)$

   $= a^4(a^4 + a^3 - 1) - a^2(a^4 + a^3 - 1) + 1(a^4 + a^3 - 1)$

   $= a^8 + a^7 - a^4 - a^6 - a^5 + a + a^4 + a^3 - 1$

   $= a^8 + a^7 - a^6 - a^5 + a^3 + a - 1$

4. $(px^2 + qx + r)(ax^2 + bx + c)$

   $= px^2(ax^2 + bx + c) + qx(ax^2 + bx + c) + r(ax^2 + bx + c)$

   $= apx^4 + pbx^3 + px^2c + aqx^3 + bqx^2 + qxc + arx^2 + brx + rc$

   $= apx^4 + (pb + aq)x^3 + (bc + pq + qr)x^2 + (qc + br)x + rc$

5. $(a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

   $= a(a^2 + b^2 + c^2 - ab - bc - ca) + b(a^2 + b^2 + c^2 - ab - bc - ca) + c(a^2 + b^2 + c^2 - ab - bc - ca)$

   $= a^3 + ab^2 + ac^2 - a^2b - abc - ca^2 + a^2b + b^3 + bc^2 - ab^2 - b^2c - abc + a^2c + b^2c + c^3 - abc - bc^2 - c^2a$

   $= a^3 + b^3 + c^3 - 3abc$

6. $(x^3 - x^2 + x - 1)(1 + x + x^2 + x^3)$

   $= x^3(1 + x + x^2 + x^3) - x^2(1 + x + x^2 + x^3) + x(1 + x + x^2 + x^2) - 1(1 + x + x^2 + x^3)$

   $= x^3 + x^4 + x^5 + x^6 - x^2 - x^3 - x^4 - x^5 + x + x^2 + x^3 + x^4 - 1 - x - x^2 - x^3$

   $= x^6 + x^4 - x^2 - 1$

7. $(m + 1)(m - 2)(m + 3)$

$= \{m(m - 2) + 1\,(m - 2)\}\,(m + 3)$

$= (m^2 - 2m + m - 2)(m + 3)$

$= (m^2 - m - 2)(m + 3)$

$= m(m^2 - m - 2) + 3\,(m^2 - m - 2)$

$= m^3 - m^2 - 2m + 3m^2 - 3m - 6$

$= m^3 + 2m^2 - 5m - 6$

8. $(x + y)(x^2 - xy + y^2)(x^3 - y^3)$

$= \{x(x^2 - xy + y^2) + y(x^2 - xy + y^2)\}\,(x^3 - y^3)$

$= (x^3 - x^2y + xy^2 + x^2y - xy^2 + y^3)(x^3 - y^3)$

$= (x^3 + y^3)(x^3 - y^3)$

$= x^3(x^3 - y^3) + y^3(x^3 - y^3)$

$= x^6 - x^3y^3 + x^3y^3 - y^6$

$= x^6 - y^6$

9. $(m^2 + mn + n^2)(m^2 - mn + n^2)(m^4 - m^2n^2 + n^4)$

$= \{m^2(m^2 - mn + n^2) + mn(m^2 - mn + n^2) + n^2(m^2 - mn + n^2)\}\,(m^4 - m^2\,n^2 + n^4)$

$= (m^4 - m^3n + m^2n^2 + m^3n - m^2n^2 + mn^3 + m^2n^2 - mn^3 + n^4)(m^4 - m^2n^2 + n^4)$

$= (m^4 + m^2n^2 + n^4)(m^4 - m^2n^2 + n^4)$

$= \{m^4(m^4 - m^2n^2 + n^4) + m^2n^2(m^4 - m^2n^2 + n^4) + n^4(m^4 - m^2n^2 + n^4)\}$

$= m^8 - m^6n^2 + m^4n^4 + m^6n^2 - m^4n^4 + m^2n^6 + m^4n^4 - m^2n^6 + n^8$

$= m^8 + m^4n^4 + n^8$

10. $(1 + x + x^2)(1 - x + x^2)(1 - x^2 + x^4)$

$= \{1(1 - x + x^2) + x\,(1 - x + x^2) + x^2\,(1 - x + x^2)\}\,(1 - x^2 + x^4)$

$= (1 - x + x^2 + x - x^2 + x^3 + x^2 - x^3 + x^4)(1 - x^2 + x^4)$

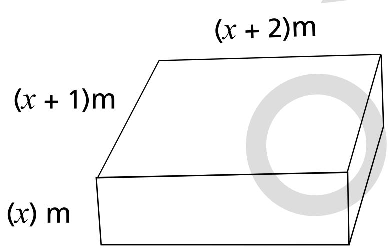
$= (1 + x^2 + x^4)(1 - x^2 + x^4)$

$= 1(1 - x^2 + x^4) + x^2\,(1 - x^2 + x^4) + x^4\,(1 - x^2 + x^4)$

$= 1 - x^2 + x^4 + x^2 - x^4 + x^6 + x^4 - x^6 + x^8$

$= 1 + x^4 + x^8$

11.



Helpful Hint
Volume of the block is equal to the volume of cuboid.

Length of the block = $l = (x + 2)$m

Breadth of the block = $b = (x + 1)$m

Height of the block = $h = (x)$m

Volume = ?

Volume = $l \times b \times h = x(x + 1)(x + 2)$

$= [\,x(x) + x\,(1)\,]\,(x + 2) = (x^2 + x)(x + 2)$

$= (x^2 + 3x^2 + 2x)\text{m}^3$

$= (x^2 + 3x^2 + 2x)\text{m}^3$

Since, $x = 20$

hence,

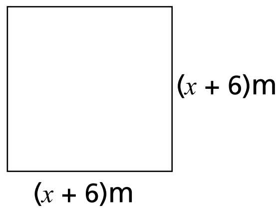
Volume = $(20)^3 + 3(20)^2 + 2(20)$

$= 8000 + 3 \times 400 + 40$

$= 8000 + 1200 + 40 = 9240\ \text{m}^3$

12.

Helpful Hint
The area of square pool = the area of a square.



Length of a side of a square = $l = (x + 6)$m

Area of a square

$= l^2 = l \times l$

$= (x + 6)(x + 6)$

$= x(x + 6) + 6(x + 6)$

$= x^2 + 6x + 6x + 36 = x^2 + 12x + 36$

$\therefore$ Area of the pool = $(x^2 + 12x + 36)$ m^2

13. Price of one doll = Rs $(a + 5)$

Number of dolls = $(a + 6)$

Total price to buy $(a + 6)$ dolls

= number of dolls × price of one doll

$= (a + 6)(a + 5)$

$= a(a + 5) + 6(a + 5)$

$= a^2 + a + 6a + 30$

$= a^2 + 11a + 30$

$\therefore$ Rahila will have to spend Rs $(a^2 + 11a + 30)$ to buy $(a + 6)$ dolls.

14. Number of flowers = $25x^2 + 5x + 5$

Price of one flower = Rs $5x$

Total money at the end of 5 days = ?

Money at the end of each day

= price of one flower ÷ number of flowers

$= 5x(25x^2 + 5x + 5)$

= Rs $(125x^3 + 25x^2 + 25x)$

Total money at the end of 5 days

$= 5 \times (125x^3 + 25x^2 + 25x)$

= Rs $(625x^3 + 125x^2 + 125x)$

or = Rs $125x(5x^2 + x + 1)$

15. Length = $l = (2x – 1)$ m

Width or breadth = $b = (x + 2)$ m

Area = ?

Area = $l \times b = (2x – 1)(x + 2)$

$= 2x(x + 2) – 1(x + 2)$

$= 2x^2 + 4x – x – 2$

$= (2x^2 + 3x – 2)$ m^2

16. $l = (2x – 1)$m

$b = (x + 2)$m

Parking area = ?

Cost of clearing parking area = ?

Cost per m^2 = Rs 50

Area = $l \times b$

$= (2x – 1)(x + 2)$

$= (2x^2 + 3x – 2)$ m^2

Total Cost = $50(2x^2 + 3x – 2)$

= Rs $(100x^2 + 150x – 100)$

17. $l = (2x + 1)$ m

$b = (5x)$ m

Area =?

Area = $5x(2x + 1) = (10x^2 + 5x)$ m^2

If $x = 3$,

Area = $10(3)^2 + 5(3) = 90 + 15 = 105$ m^2

Since 105 m^2 > 100 m^2,

Hence, Hamid can have his garden.

18. Length of first board = $l_1 = (2.5x + 3.4y)$ m

length of second board = $l_2 = (3.5x + 4.2y)$ m

$x = 2$

$y = 1$

Asseming , $x = 2$ and $y = 1$,

$l_1 = 2.5(2) + 3.4(1) = 5 + 3.4 = 8.4$ m

$l_2 = 3.5(2) + 4.2 = 7 + 4.3 = 11.2$ m

Since, 11.2 m > 8.4 m

Hence, 2nd board is larger.

Exercise 7C

1. (i) distributive (ii) a and b

   (iii) 2 (two) (iv) $8x^4 - 3x$

   (v) $x^2 - 14x + 45$.

2. (i) False

   $a(a + 2) + 1(a + 2)$

   $= a^2 + 2a + 2$

   $= a^2 + 3a + 2$

   (ii) True

   (iii) False

   $(3x^2 + 5x)(2x + y)$

   $= 3x^2(2x + y) + 5x(2x + y)$

   $= 6x^3 + 3x^2y + 10x^2 + 5xy$

   (iv) True

   $3m - 2 \longrightarrow$ quotient

   $m + 6 \,\overline{)\,3m^2 + 16m - 12}$

   $3m^2 + 18m$

   $- \quad -$

   $0 \quad -2m - 12$

   $-2m - 12$

   $+ \quad +$

   0

   (v) True

   $5x^2 - 5x + 3$

   $x^2 \,\overline{)\,5x^4 - 5x^3 + 3x^2}$

   $5x^4$

   $-$

   $0 \quad -5x^3$

   $-5x$

   $+$

   $0 + 3x^2$

   $+3x^2$

   $-$

   $00 \longrightarrow$ no remainder

3. $a + b$

   $a + b \,\overline{)\,a^2 + 2ab + b^2}$

   $a^2 + ab$

   $- \quad -$

   $ab + b^2$

   $ab + b^2$

   $- \quad -$

   0

4. $a - b$

   $a - b \,\overline{)\,a^2 - 2ab + b^2}$

   $a^2 - ab$

   $- \quad +$

   $-ab + b^2$

   $-ab + b^2$

   $+ \quad -$

   0

5. **Helpful Hint**

   $a^2(a - b) = a^3 - a^2b, \quad ab(a - b) = a^2b - ab^2$

   $b^2(a - b) = ab^2 - b^3$

   $a^2 + ab + b^2$

   $a - b \,\overline{)\,a^3 - b^3}$

   $a^3 \quad - a^2b$

   $- \quad +$

   $-b^3 + a^2b$

   $+a^2b - ab^2$

   $- \quad +$

   $-b^3 \quad + ab^2$

   $-b^3 \quad + ab^2$

   0

6. $a^2 - ab + b^2$

   $a + b \,\overline{)\,a^3 + b^3}$

   $a^3 \quad a^2b$

   $- \quad -$

   $b^3 - a^2b$

   $-a^2b - ab^2$

   $+ \quad +$

   $b^3 \quad + ab^2$

   $b^3 \quad + ab^2$

   $- \quad -$

   0

7.

$$x + 2 \overline{)\, x^3 + x^2 - 3x - 2 \,} \quad \text{Quotient: } x^2 - x - 1$$

$$x^3 + 2x^2$$
$$- \quad -$$
$$-x^2 - 3x$$
$$-x^2 - 2x$$
$$+ \quad +$$
$$-x - 2$$
$$-x - 2$$
$$+ \quad +$$
$$0$$

8.

$$x - 1 \overline{)\, 8x^3 - 10x^2 - x + 3 \,} \quad \text{Quotient: } 8x^2 - 2x - 3$$

$$8x^3 - 8x^2$$
$$- \quad +$$
$$-2x^2 - x$$
$$-2x^2 + 2x$$
$$+ \quad -$$
$$-3x + 3$$
$$-3x + 3$$
$$+ \quad -$$
$$0$$

9.

$$m^2 - 16 \overline{)\, m^3 - m^2 - 16m + 16 \,} \quad \text{Quotient: } m - 1$$

$$m^3 - 16m$$
$$- \quad +$$
$$-m^2 + 16$$
$$-m^2 + 16$$
$$+ \quad -$$
$$0$$

10.

$$a + b + c \overline{)\, a^3 + b^3 + c^3 - 3abc \,}$$

$$\text{Quotient: } a^2 - ab - ac + b^2 + c^2 - bc = a^2 + b^2 + c^2 - ab - bc - ca$$

$$a^3 + a^2b + a^2c$$
$$- \quad - \quad -$$
$$b^3 + c^3 - 3abc - a^2b - a^2c$$
$$-abc - a^2b - ab^2$$
$$+ \quad + \quad +$$
$$+b^3 + c^3 - 2abc - a^2c + ab^2$$
$$-abc - a^2c - ac^2$$
$$+ \quad +$$
$$+b^3 + c^3 - abc + ab^2 + ac^2$$
$$+b^3 + ab^2 + cb^2$$
$$- \quad - \quad -$$
$$+c^3 - abc + ac^2 - cb^2$$
$$+c^3 + ac^2 + bc^2$$
$$- \quad - \quad -$$
$$-abc - cb^2 - bc^2$$
$$-abc - cb^2 - bc^2$$
$$+ \quad + \quad +$$
$$0$$

11.

Helpful Hint
Leave the space for decreasing powers of x

$$x^2 - xy + y^2 + x + y + 1$$

$x + y - 1 \overline{)\; x^3 \qquad\qquad + y^3 + 3xy - 1}$

$+ x^3 + x^2y \qquad\qquad - x^2$

$- \quad - \qquad\qquad +$

$- x^2y \qquad + y^3 + 3xy - 1 + x^2$

$- x^2y \quad - xy^2 \qquad + xy$

$+ \quad + \qquad -$

$+ xy^2 \qquad + y^3 + 2xy - 1 \quad + x^2$

$+ xy^2 \qquad + y^3 \qquad - y^2$

$- \quad - \qquad +$

$+ 2xy - 1 \quad + x^2 \quad + y^2$

$+ xy \quad + x^2 \qquad - x$

$- \quad - \quad - \quad +$

$+ xy - 1 \qquad + y^2 + x$

$+ xy \qquad + y^2 \qquad - y$

$- \quad - \quad +$

$- 1 \qquad + x + y$

$- 1 \qquad + x + y$

$+ \qquad - \quad -$

0

12.

$$- 2x + 1$$

$8x^3 + 4x^2 + 2x + 1 \overline{)\; - 16x^4 \qquad\qquad + 1}$

$+ 16x^4 - 8x^3 - 4x^2 - 2x$

$- \quad - \quad +$

$+ 8x^3 + 4x^2 + 2x + 1$

$+ 8x^3 + 4x^2 + 2x + 1$

$- \quad - \quad - \quad -$

0

13.

$$x + 12y$$

$2x + 5y \overline{)\; 2x^2 + 29xy + 60y^2}$

$+ 2x^2 + 5xy$

$- \quad - \quad +$

$+ 24xy + 60y^2$

$+ 24xy + 60y^2$

$- \quad -$

0

14.

$$x - 3$$

$2x + 3x + 9 \overline{)\; x^3 \qquad\qquad - 27}$

$+ x^3 + 3x^2 + 9x$

$- \quad - \quad +$

$- 3x^2 - 9x - 27$

$- 3x^2 - 9x - 27$

$+ \quad + \quad +$

0

15.

$$a - 2 \,\overline{)\, a^3 + 0a^2 + 0a - 8} \quad \text{quotient: } a^2 - 2a + 4$$
$a^3 - 2a^2$
$- \quad +$
$2a^2 - 0a$
$2a^2 - 4a$
$+ \quad +$
$4a - 8$
$4a - 8$
$- \quad +$
0

16.

$$a + 2 \,\overline{)\, a^3 + 3a^3b + 3ab^2 + b^3} \quad \text{quotient: } a^2 + 2ab + b^2$$
$a^3 + a^2b$
$- \quad -$
$+ 2a^2b + 3ab^2$
$+ 2a^2b + 2ab^2$
$- \quad -$
$+ ab^2 + b^3$
$+ ab^2 + b^3$
$- \quad -$
0

17.

$$x - 2y \,\overline{)\, 3x^3 - 11x^2y + 11xy^2 - 2y^3} \quad \text{quotient: } 3x^2 - 5xy + y^2$$
$3x^3 - 6x^2y$
$- \quad +$
$- 5x^2y + 11xy^2$
$- 5x^2y + 10xy^2$
$+ \quad -$
$+ xy^2 - 2y^3$
$+ xy^2 - 2y^3$
$- \quad +$
0

18.

$$3x - 4 \,\overline{)\, 6x^2 - 17x + 12} \quad \text{quotient: } 2x - 3$$
$+ 6x^2 - 8x$
$- \quad + \quad +$
$- 9x + 12$
$- 9x + 12$
$+ \quad -$
0

$\therefore$ there are $2x - 3$ pencils in a box.

19.

$$a + 5 \,\overline{)\, a^2 + 11a + 30} \quad \text{quotient: } a + 6$$
$+ a^2 + 5a$
$- \quad -$
$+ 6a + 30$
$+ 6a + 30$
$- \quad -$
0

$\therefore$ Sana can buy $a + 6$ dolls in Rs $(a^2 + 11a + 30)$

Multiple Choice Questions 7

1. B

$(a + 1)(a + 2) = a(a + 2) + 1\,(a + 2)$

$= a^2 + 2a + a + 2$

$= a^2 + 3a + 2$

2. A

Helpful Hint

$(x + 5)\,(x - 3)$

$(x + 5)\,(x - 3) = x^2 - 3x + 5x - 15$

$= x^2 + 2x - 15$

3. D

4. C

Helpful Hint
Divide first term of the dividend with the first term of the divisor.

$$\frac{2x^2}{x} = 2x$$

5. A

$$(x + 1)(x^2 + 2x) = x^3 + 2x^2 + x^2 + 2x$$
$$= x^3 + 3x^2 + 2x$$

8 FACTORISATION OF ALGEBRAIC EXPRESSIONS

Exercise 8A

1. (i) $2a + 1$

$(2a + 1)^2 = (2a)^2 + 2(2a) + (1)^2$ [using the identity $(a+b)^2 = a^2 + 2ab + b^2$]

$(2a + 1)^2 = 4a^2 + 4a + 1$

(ii) $3b + 2c$

$(3b + 2c)^2 = (3b)^2 + 2(3b)(2c) + (2c)^2$

$= 9b^2 + 12bc + 4c^2$

(iii) $2p^2 + 3q^2$

$(2p^2 + 3q^2)^2 = (2p^2)^2 + 2(2p^2)(3q^2) + (3q^2)^2$

$= 4p^4 + 12p^2q^2 + 9q^4$

2

Helpful Hint:
All the sums have been solved by applying the identity $(a+b)^2 = a^2 + 2ab + b^2$

(i) $9p^2 + 12pq + 4q^2$

$= (3p)^2 + 2(3p)(2q) + (2q)^2$

$= (3p + 2q)^2$

$= (3p + 2q)(3p + q)$

(ii) $4x^2 + 4xy + y^2$

$= (2x)^2 + 2(x)(y) + (y)^2$

$= (2x + y)^2$

$= (2x + y)(2x + y)$

(iii) $25p^2 + 10pq + q^2$

$= (5p)^2 + 2(5p)(q) + (q)^2$

$= (5p + q)^2$

$= (5p + q)(5p + q)$

(iv) $36x^2 + 24xy + 4y^2$

$= (6x)^2 + 2(6x)(2y) + (2y)^2$

$= (6x + 2y)^2$

$= (6x + 2y)(6x + 2y)$

(v) $x^2 + 2xy + y^2$

$= (x)^2 + 2(x)(y) + (y)^2$

$= (x + y)^2$

$= (x + y)(x + y)$

3.

(i) $16a^2 + 24ab + 9b^2$, When $a = 4$, $b = 3$

$= (4a)^2 + 2(4a)(3b) + (3b)^2$

$= (4a + 3b)^2$ [using formula $a^2 + 2ab + b^2 = (a + b)^2$]

$= (4 \times 4 + 3 \times 3)^2$ [substituting $a = 4$ and $b = 3$]

$= (16 + 9)^2$

$= (25)^2$

$= 625$

(ii) $4m^4 + 12m^2n^2 + 9n^4$, When $m = \frac{1}{2}$ and $n = \frac{1}{3}$

$= 4m^4 + 12m^2n^2 + 9n^4$

$= (2m^2)^2 + 2(2m^2)(3n^2) + 3n^2)^2$ [using formula]

$= (2m^2 + 3n^2)^2$

[Substitute $m = \frac{1}{2}$, $n = \frac{1}{3}$]

$= \left(2 \times (\frac{1}{2})^2 + 3 \times (\frac{1}{3})^2\right)^2$

$= (2 \times \frac{1}{4} + 3 \times \frac{1}{9})^2$

$= (\frac{1}{2} + \frac{1}{3})^2$

$= \left(\frac{3 + 2}{6}\right)^2$

$= \left(\frac{5}{6}\right)^2$

$= \frac{25}{36}$

4. $p + \frac{1}{p} = 5$

Squaring both the sides,

$\left(p + \frac{1}{p}\right)^2 = (5)^2$

$(p^2) + 2 \times p \times \frac{1}{p} + \left(\frac{1}{p}\right)^2 = 25$

$p^2 + 2 + \frac{1}{p^2} = 25$

$p^2 + \frac{1}{p^2} = 25 - 2$

$p^2 + \frac{1}{p^2} = 23$

Hence, the required result is proved.

5. Given that,

$a + b = 5$

Squaring both the sides

$(a + b)^2 = (5)^2$

$a^2 + 2ab + b^2 = 25$ [using formula]

$\therefore$ we have $ab = 4$

$a^2 + 2 \times 4 + b^2 = 25$

$a^2 + b^2 = 25 - 8$

$a^2 + b^2 = 17$

$(a^2 + b^2)^2 = (17)^2$

Squaring both the sides

$a^4 + 2a^2b^2 + b^4 = 289$ [using formula]

$a^4 + 2 \times (4)^2 + b^4 = 289$

$a^4 + 32 \times b^4 = 289$

$a^4 + b^4 = 289 - 32 = 257$

Hence, $a^2 + b^2 = 17$ and $a^4 + b^4 = 257$

6. Given that,

$2ab + 5cd = 5$ and $abcd = 1$

Squaring both the sides

$(2ab + 5cd)^2 = (5)^2$

$(2ab)^2 + 2(2ab)(5cd) + (5cd)^2 = 25,$

$4a^2b^2 + 20abcd + 25c^2d^2 = 25$

$\therefore$ we have $abcd = 1$,

$4a^2b^2 + 25c^2d^2 + 20 = 25$

$4a^2b^2 + 25c^2d^2 = 25 - 20$

$4a^2b^2 + 25c^2d^2 = 5$

7. (i) 503×503

$= (503)^2$

$= (500 + 3)^2$

$= (500)^2 + 2(500)(3) + (3)^2$ [using identity $(a + b)^2$]

$= 250000 + 3000 + 9$

$= 253009$

(ii) 1005×1005

$= (1005)^2$

$= (1000 + 5)^2$

$= (1000)^2 + 2(1000)(5) + (5)^2$

$= 1000000 + 10000 + 25$

$= 1010025$

(iii) 904×904

$= (904)^2$

$= (900 + 4)^2$

$= (900)^2 + 2(900)(4) + (4)^2$

$= 810000 + 7200 + 16$

$= 817216$

8.

Helpful Hint:

Use identity $a^2 - 2ab + b^2 = (a - b)^2$

(i) $9x^2 - 12xy + 4y^2$

$= (3x)^2 - 2 \times 3x \times 2y + (2y)^2$

$= (3x - 2y)^2$

(ii) $36a^2 - 24a + 4$

$= (6a)^2 - 2 \times 6a \times 2 + (2)^2$

$= (6a - 2)^2$

(iii) $16x^2 - 8xy + y^2$

$= (4x)^2 - 2 \times 4x \times y + (y)^2$

$= (4x - y)^2$

(iv) $4a^2 - 20ab + 25b^2$

$= (2a)^2 - 2 \times 2a \times 5b + (5b)^2$

$= (2a - 5b)^2$

(v) $x^2 - 6xy + 9y^2$

$= (x)^2 - 2 \times x \times 3y + (3y)^2$

$= (x - 3y)^2$

9. (i) $25a^2 - 10a + 1$, When $a = \frac{1}{5}$

$25a^2 - 10a + 1 = (5a)^2 - 2(5a)(1) + (1)^2$

$= (5a - 1)^2$ [using formula]

$= \left(5 \times \frac{1}{5} - 1\right)^2$ [putting $a = \frac{1}{5}$]

$= (1 - 1)^2 = 0$

(ii) $4(a + b)^2 - 20\,(a + b) + 25$ when $a = 2, b = 1$

$= \{2(a + b)\}^2 - 2 \times 2(a + b) \times 5 + (5)^2$

$= \{2\,(a + b) - 5\}^2$

Substitute a = 2, $b = 1$

$= (6 - 5)^2$

(iii) $36\,(l + m)^2 - 48n\,(l + m) + 16n^2$,

When $l = \frac{1}{2}$, $m = \frac{1}{3}$ and $n = \frac{1}{4}$

$36\,(l + m)^2 - 48n\,(l + m) + 16n^2$

$= \{6\,(l + m)\}^2 - 2 \times 6\,(l + m) \times 4n + (4n)^2$

$= \{6\,(l + m) - 4n\}^2$ [using formula]

$\{$Substitute $l = \frac{1}{2}$, $m^2 \frac{1}{3}$, $n^2 \frac{1}{4}\}$

$= \{6 \times (\frac{1}{2} + \frac{1}{3}) - 4 \times \frac{1}{4}\}^2$

$= \{6 \times \frac{5}{6} - 4 \times \frac{1}{4}\}^2$

$= (5 - 1)^2$

$= (4)^2$

$= 16$

10. (i) 57×57

$= (60 - 3)\,(60 - 3)$

$= (60 - 3)^2$

(applying formula)

$= (60)^2 - 2 \times 60 \times 3 + (3)^2$

$= 3600 - 360 + 9$

$= 3249$

(ii) 994×994

$= (1000 - 6)\,(1000 - 6)$

$= (1000 - 6)^2$

$= (1000)^2 - 2 \times 1000 \times 6 + (6)^2$

$= 1000000 - 12000 + 36$

$= 988036$

(iii) 9997×9997

$= (10000 - 3)\,(10000 - 3)$

$= (10000 - 3)^2$

$= (10000)^2 - 2\,(10000)\,(3) + (3)^2$

$= 100000000 - 60000 + 9$

$= 99940009$

11. Given $a - \frac{1}{a} = 2$ then show that

(i) Show that $a^2 + \frac{1}{a^2} = 6$

$a - \frac{1}{a} = 2$

$\left(a - \frac{1}{a}\right)^2 = (2)^2$ [squaring both the sides]

$a^2 - 2 \times a \times \frac{1}{a} + \frac{1}{a^2} = 4$

$a^2 + \frac{1}{a^2} = 4 + 2$

$a^2 + \frac{1}{a^2} = 6$

Hence proved.

(ii) $a^4 + \frac{1}{a^4} = 34$

$a - \frac{1}{a} = 2$

(squaring both the sides)

$\left(a - \frac{1}{a}\right)^2 = 4$

$a^2 + \frac{1}{a^2} = 6$

(squaring both the sides)

$\left(a^2 + \frac{1}{a^2}\right)^2 = (6)^2$

$a^4 + 2 + \frac{1}{a^4} = 36$

$a^4 + \frac{1}{a^4} = 36 - 2 = 34$

Hence, proved.

(iii) $\left(a + \frac{1}{a}\right)^2 = 8$

we have

$\left(a - \frac{1}{a}\right) = 2$

(squaring both the sides)

$a^2 - 2 + \frac{1}{a^2} = 4$

$a^2 - 2 + \frac{1}{a^2} + 4 = 4 + 4$

(adding 4 on both the sides)

$a^2 + 2 + \frac{1}{a^2} = 8$

$\left(a + \frac{1}{a}\right)^2 = 8$

Hence, proved.

Helpful Hint:
Use the identity $(a + b)(a - b) = a^2 - b^2$

12. (i) $(a + 1)(a - 1)(a^2 + 1)$

$= \{(a)^2 - (1)^2\}(a^2 + 1)$ [using identity]

$= (a^2 - 1)(a^2 + 1)$

$= (a^2)^2 - (1)^2$

$= a^4 - 1$

(ii) $= (a + b)(a - b)(a^2 + b^2)(a^4 + b^4)$

$= (a^2 - b^2)(a^2 + b^2)(a^4 + b^4)$

$= (a^4 - b^4)(a^4 + b^4)$ [using identity]

$= a^8 - b^8$

(iii) $= (2p + 3q)(2p - 3q)(4p^2 + 9q^2)(16p^2 + 81q^2)$

$= (4p^2 - 9q^2)(4p^2 + 9q^2)(16p^4 + 81q^4)$

$= (16p^4 - 81q^4)(16p^4 + 81q^4)$

$= 256p^8 - 6561q^8$

13. $81p^2 - 49q^2$

$81p^2 - 49q^2 = (9p - 7q)(9p + 7q)$
using identity $a^2 - b^2 = (a - b)(a + b)$

14. (i) $(3a + b)(3a - b)$

$= (3a)^2 - (b)^2$

$= 9a^2 - b^2$

(ii) $(5a + 3b)(5a - 3b)$

$= (5a)^2 - (3b)^2$

$= 25a^2 - 9b^2$

(iii) $(2x + 3y)(2x - 3y)$

$= (2x)^2 - (3y)^2$

$= 4x^2 - 9y^2$

Exercise 8B

1. $p^2q + pq^2$
$= pq(p + q)$
Taking out common factors.

2. $3x^3 - 15x^2y$
$= 3(x^3 - 5x^2y)$
$= 3x^2(x - 5y)$

3. $45y^4 - 9xy^3$
$= 9(5y^4 - xy^3)$
$= 9y^3(5y - x)$

4. $6p^2q + 12pq^2$
$= 6(p^2q + 2pq^2)$
$= 6pq(p + 2q)$

5. $5a^4b^2 + 15a^2b^4$
$= 5(a^4b^2 + 3a^2b^4)$
$= 5a^2b^2(a^2 + 3b^2)$

6. $xy + x + y + 1$
$= (xy + x) + (y + 1)$

Helpful Hint:
Grouping the terms

$= x(y + 1) + (y + 1)$
$= (x + 1)(y + 1)$

7. $am^3 - am^2 - m + 1$
$= am^2(m - 1) - (m - 1)$
$= (m - 1)(am^2 - 1)$

8. $4a^2 + 12ab + 9b^2$

Helpful Hint:
Using formula $a^2 + 2ab + b^2 = (a + b)^2$

$= (2a)^2 + 2(2a)(3b) + (3b)^2$
$= (2a + 3b)^2$
$= (2a + 3b)(2a + 3b)$

9. $x^2 - (p + q)x + pq$
$= x^2 - px - qx + pq$
$= x(x - p) - q(x - p)$
$= (x - p)(x - q)$

10. $abc - ab - c + 1$

Helpful Hint:
Taking ab common, from the first two terms.

$= ab(c - 1) - (c - 1)$
Taking $(c - 1)$ common,
$= (c - 1)(ab - 1)$

11. $(a + b)(p + q + r) + (b + c)(p + q + r) + (c + a)(p + q + r)$
Taking $(p + q + r)$ common, we get
$= (p + q + r)(a + b + b + c + c + a)$
$= (p + q + r)(2a + 2b + 2c)$
$= 2(a + b + c)(p + q + r)$

Exercise 8C

Helpful Hint:
$a^2 - b^2 = (a + b)(a - b)$

1. $9x^2 - 25y^2$
 $= (3x ^1 5y)\ 3x + 5y)$
2. $1 - 9c^2$
 $= (1 - 3c)(1 + 3c)$
3. $m^2n^2 - p^2$
 $= (mn)^2 - (p)^2$
 $= (mn - p)(mn + p)$
4. $1 - (p + q)^2$
 $= (1)^2 - (p + q)^2$
 $= (1 - p - q)(1 + p + q)$
5. $(x + y)^2 - (x - y)^2$
 $= (x + y - \times + y)(\times + y + x - y)$
 $= (2y)(2x)$
 $= 4xy$
6. $9x^2 - (2x - 3y)^2$
 $= (3x)^2 - (2x - 3y)^2$
 $= (3x - 2x + 3y)(3x + 2x - 3y)$
 $= (x + 3y)(5x - 3y)$
7. $(a + b - c)^2 - (a - b + c)^2$
 $= (a + b - c + a - b + c)(a + b - c - a + b - c)$
 $= (2a)(2b - 2c)$
 $= 2(2a)(b - c)$
 $= 4a(b - c)$
8. $3a(3a - 2b) + b^2 - c^2$
 $= 9a^2 - 6ab + b^2 - c^2$
 $= (3a - b)^2 - (c)^2$
 $= (3a - b + c)(3a - b - c)$
9. $9(p-q)^2 - 25(q - r)^2$
 $= \{3(p - q)\}^2 - \{5(q - r)\}^2$
 $= \{3(p - q) + 5(q - r)\}\{3(p - q) - 5(q - r)\}$
 $= (3p - 3q + 5q - 5r)(3p - 3q - 5q + 5r)$
 $= (3p + 2q - 5r)(3p - 8q + 5r)$
10. $a^8 - b^8$

Helpful Hint:
$a^2 - b^2 = (a - b)(a + b)$

 $= (a^4)^2 - (b^4)^2$
 $= (a^4 - b^4)(a^4 + b^4)$
 $= (a^2 - b^2)(a^2 + b^2)(a^4 + b^4)$
 $= (a - b)(a + b)(a^2 + b^2)(a^4 + b^4)$

11. $a^2 - 1 + 2b - b^2$
 $= a^2 - (b^2 - 2b + 1)$
 $= a^2 - (b - 1)^2$

Helpful Hint:
(using $a^2 - 2ab + b^2 = (a - b)^2$)

 $= (a - b + 1)(a + b - 1)$

12. $4x^4 + 81 = (2x^2) + (9)^2$
 [completing the square].
 $= (2x^2)^2 + 2 \times 2x^2 \times 9 + (9)^2 - 2 \times 2x^2 \times 9$
 $= (2x^2 + 9)^2 - 36x^2$
 $= (2x^2 + 9)^2 - (6x)^2$
 (using $a^2 - b^2 = (a + b)(a - b)$)
 $= (2x^2 + 9 + 6x)(2x^2 + 9 - 6x)$
 $= (2x^2 + 6x + 9)(2x^2 - 6x + 9)$

13. $9x^4 - 3x^2 + 1$

Helpful Hint:
Split the middle term to complete the square.

$9x^4 - 3x^2 + 1$

$-9x^2$ $+6x^2$

Helpful Hint:
$-9x^2 + 6x^2 = -3x^2$

 $= 9x^4 - 9x^2 + 6x^2 + 1$
 [by rearranging the terms]
 $= 9x^4 + 6x^2 + 1 - 9x^2$
 $= (9x^4 + 6x^2 + 1) - 9x^2$ [By grouping]
 $= (3x^2 + 1)^2 - 9x^2$
 $= (3x + 1)^2 - (3x)^2$
 $= (3x^2 + 1 + 3x)(3x^2 + 1 - 3x)$
 $= (3x^2 + 3x + 1)(3x^2 - 3x + 1)$

14. $x^4 + x^2 + 1$

Splitting the middle term:

Helpful Hint:
$2x^2 - x^2 = x^2$

$= x^4 + x^2 + 1$ (splitting x^2 into $2x^2$ and $-x^2$)

$= x^4 + 2x^2 - x^2 + 1$

$= x^4 + 2x^2 + 1 - x^2$ (rearranging the terms)

$= (x^4 + 2x^2 + 1) - x^2$ (grouping the terms)

$= (x^2 + 1)^2 - (x)^2$

$\because a^2 + 2ab + b^2 = (a + b)^2$

$= (x^2 + x + 1)(x^2 - x + 1)$

15. $x^4 + 3x^2y^2 + 4y^4$

$= x^4 + 3x^2y^2 + 4y^4$ (splitting $3x^2y^2$ into $4x^2y^2$ and $-x^2y^2$) [splitting the middle term]

Helpful Hint:
(using $3x^2y^2 = 4x^2y^2 - x^2y^2$)

$= x^4 + 4x^2y^2 - x^2y^2 + 4y^4$

$= x^4 + 4x^2y^2 + 4y^4 - x^2y^2$

(rearranging the terms)

$= (x^4 + 4x^2y^2 + 4y^2) - x^2y^2$

(grouping the terms)

$= (x^2 + 2y^2)^2 - (xy)^2$

$= (x^2 + 2y^2 + xy)(x^2 + 2y^2 - xy)$

$= (x^2 + xy + 2y^2)(x^2 - xy + 2y^2)$

16. $(a + b)^4 + 4$

= Let $a + b = x$, then

$= (a + b)^4 + 4 = x^4 + 4$

$= (x^2)^2 + (2)^2$

$= (x^2)^2 + 4x^2 + (2)^2 - 4x^2$

Helpful Hint:
Add $4x^2$ to make a complete square and subtract the same.

$= (x^2 + 2)^2 - (2x)^2$

$= (x^2 + 2x + 2)(x^2 - 2x + 2)$

[Substituting $x = a + b$]

$= \{(a + b)^2 + 2(a + b) + 2\}$

17. $x^4 + 8x^2 + 144$

$x^4 + 8x^2 + 144$ (splitting $8x^2$ into $24x^2$ and $-16x^2$)

$= x^4 + 24x^2 + 144 - 16x^2$

$= (x^2)^2 + 24x^2 + (12)^2 - 16x^2$

[completing the square]

$= (x^2 + 12)^2 - 16x^2$

$= (x^2 + 12)^2 - (4x)^2$

$= (x^2 + 12 + 4x)(x^2 + 12 - 4x)$

$= (x^2 + 4x + 12)(x^2 - 4x + 12)$

18. $3a^4 - 18a^2b^2 + 3b^4$

$\because$ 3 is a common factor.

$= 3(a^4 - 6a^2b^2 + b^4)$

$= 3(a^4 - 2a^2b^2 - 4a^2b^2 - b^4)$

[breaking the middle term to obtain a perfect square]

$= 3(a^4 - 2a^2b^2 + b^4 - 4a^2b^2)$

[rearranging the terms]

$= 3\{(a^4 - 2a^2b^2 + b^4) - 4a^2b^2\}$

$\because a^2 - 2ab + b^2 = (a - b)^2$

$= 3\{(a^2 - b^2)^2 - (2ab)^2\}$

$= 3\{a^2 - b^2 + 2ab)(a^2 - b^2 - 2ab)]$

$= 3(a^2 + 2ab - b^2)(a^2 - 2ab - b^2)$

19 $9a^2 - 4b^2 + 16c^2 - 1 - 4b - 24ac$

$= (9a^2 - 24ac + 16c^2) - (4b^2 + 4b + 1)$

[rearranging and grouping the terms]

$= \{(3a)^2 - 2 \times 3a \times 4c + (4c)^2\} - \{(2b)^2 + 2 \times 2b \times 1 + (1)^2\}$

$= (3a - 4c)^2 - (2b + 1)^2$

$= (3a - 4c + 2b + 1)(3a - 4c - 2b - 1)$

$= (3a + 2b - 4c + 1)(3a - 2b - 4c - 1)$

20. Option A is correct.

21. Option C is correct.

22. Option D is correct.

23. Option D is correct.

24. Option A is correct.

Exercise 8D

1. (i) $(x + 3)^3 = ?$

Helpful Hint
$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(x + 3)^3 = x^3 + 3x^2(3) + 3x(3)^2 + 3^3$
$= x^3 + 9x^2 + 27x + 27$

(ii) $(a + 4b)^3$
$= a^3 + 3a^2(4b) + 3a(4b)^2 + (4b)^3$
$= a^3 + 12a^2b + 48ab^2 + 64b^3$

(iii) $(3x + 2y)^3$
$= (3x)^3 + 3(3x)^2(2y) + 3(3x)(2y)^2 + (2y)^3$
$= 27x^3 + 54x^2y + 36xy^2 + 8y^3$

(iv) $(x^2 + 2y)^3$
$= (x^2)^3 + 3(x^2)^2(2y) + 3x^2(2y)^2 + (2y)^3$
$= x^6 + 6x^4y + 12x^2y^2 + 8y^{3\backslash}$

(v) $(ax + by)^3$
$= (ax)^3 + 3(ax)^2(by) + 3(ax)(by)^2 + (by)^3$
$= a^3x^3 + 3a^2x^2\,by + 3ax\,b^2y^2 + b^3y^3$

(vi) $(a^2 + bc)^3$
$= (a^2)^3 + 3(a^2)^2(bc) + 3(a^2)(bc)^2 + (bc)^3$
$= a^6 + 3a^4bc + 3a^2b^2c^2 + b^3c^3$

(vii) $(a^2 + b^2)^3$
$= (a^2)^3 + 3(a^2)^2(b^2) + 3(a^2)(b^2)^2 + (b^2)^3$
$= a^6 + 3a^4b^2 + 3a^2b^4 + b^6$

(viii) $42^3 = (40 + 2)^3$
$= (40)^3 + 3(40)^2(2) + 3(40)(2)^2 + (2)^3$
$= 64000 + 9600 + 480 + 8$
$= 74088$

(ix) $(105)^3 = (100 + 5)^3$
$= (100)^3 + 3(100)^2(5) + 3(100(5)^2 + (5)^3$
$= 1000000 + 150000 + 7500 + 125$
$= 1157625$

2. (i) $a^3 + 9a^2 + 27a + 30$
Substitute $a = 3$
$(3)^3 + 9(3)^2 + 27(3) + 30$
$= 27 + 81 + 81 + 30$
$= 219$

(ii) $(31)^3 + 3 \times (31)^2 \times 19 + 3 \times 31 \times (19)^2 + (19)^3$
$= 29791 + 3 \times 961 \times 19 + 93 \times 361 + 6859$
$= 29791 + 54777 + 33573 + 6859$
$= 125000$

3. $x^3 + y^3 + 24xy = ?$
Given that $x + y = 8$
[Cubing both the sides]
$(x + y)^3 = 8^3$
by using formula,
$(a + b)^3 = a^3 + b^3 + 3ab(a + b)$
$(x + y)^3 = 8^3$
$x^3 + y^3 + 3xy(x + y) = 512$
$x^3 + y^3 + 3xy(8) = 512$
$\therefore x^3 + y^3 + 24xy = 512$

4. $m^3 + n^3 - 9mn = ?$
Given that $m + n + 3 = 0$
$m + n = -3$
[by cubing both the sides]
$(m + n)^3 = (-3)^3$
$m^3 + n^3 + 3mn(m + n) = -27$
$m^3 + n^3 + 3mn(-3) = -27$
$m^3 + n^3 - 9mn = -27$

5. (i) $(2x + 5)^3$
$= (2x)^3 + 3(2x)^2(5) + 3(2x)(5)^2 + (5)^3$
$= 8x^3 + 60x^2 + 150x + 125$

(ii) $(ax - by)^3$
$= (ax)^3 - 3(ax)^2(by) + 3(ax)(by)^2 - (by)^3$
$= a^3x^3 - 3a^2x^2\,by + 3axb^2y^2 - b^3y^3$

(iii) $(a^2 - b^2)^3$
$= (a^2)^3 - 3(a^2)^2b^2 + 3a^2(b^2)^2 - (b^2)^3$
$= a^6 - 3a^4b^2 + 3a^2b^4 - b^6$

(iv) $(-7a + 2b^2)^3 = (2b^2 - 7a)^3$
$= (2b^2)^3 - 3(2b^2)^2(7a) + 3(^2b^2)(7a)^2 - (7a)^3$
$= 8b^6 - 84b^4a + 294\,b^2a^2 - 343a^3$

(v) $(2a - 3bc)^3$

$= (2a)^3 - 3(2a)^2 (3bc) + 3(2a) (3bc)^2 - (3bc)^3$

$= 8a^3 - 36a^2bc + 54ab^2c - 27b^3c^3$

(vi) $(1 + a - 2b)^3 = ?$

Helpful Hint:
In this case make $(1 + a)$ one term and $2b$ the second term.

Let $x = 1 + a$ and $y = 2b$, then

$x^2 = (1 + a)^2 = 12 + 2a + a^2$

$x^3 = (1 + a)^3 = 13 + 3a + 3a^2 + a^3$

$y^2 = (2b)^2 = 4b^2$

$y^3 = 8b^3$

$(1 + a + 2b)^3 = (x - y)^3$

$= x^3 - 3x^2y + 3xy^2 - y^3$

Substitute

$x = 1 + 9$, $x^2 = 1 + 2a + a^2$, $x^3 = 1 + 3a + 3a^2 + a^3$,

$y = 2b$, $y^2 = 4b^2$ and $y^3 = 8b^3$

$\therefore (1 + a - 2b)^3 = 1 + 3a + 3a^2 + a^3 - 3(1 + 2a + a^2)\, 2b + 3(1 + a)\, (4b^2) - 8b^3$

$= 1 + 3a + 3a^2 + a^3 - 6b - 12ab - 6a^2b + 12b^2 + 12ab^2 - 8b^3$

(vii) $(x^2 - y - z)^3 = (x^2 - (y + z))^3$

Let $a = x^2$ and $b = y + z$,

then

$a^2 = x^4, a^3 = x^6$

$b^2 = (y + z)^2 = y^2 + 2yz + z^2$

$b^3 = (y + z)^3 = y^3 + 3y^2z + 3yz^2 + z^3$

$[(a - b)^3 = a^3 - 3a^2b + 3ab^2 + b^3]$

$(x^2 - y - z)^3 = x^6 - 3x^4 (y + z) + 3x^2 (y^2 + 2yz + z^2) - (y^3 + 3y^2z + 3yz^2 + z^3)$

$= x^6 - 3x^4y - 3x^4z + 3x^2y^2 + 6x^2yz + 3x^2z^2 - y^2 - y^3 - 3y^2z - 3yz^2 - z^3$

(viii) $(a - 2b - 3c)^3 = ((a - 2b) - 3c)^3$

Let $x = a - 2b$ and $y = 3c$, then

$x^2 = a^2 - 4ab + b^2$

$x^3 = a^3 - 3a^2 (2b) + 3a (2b)^2 - 8b^3$

$= a^3 - 6a^2 b + 6ab^2 - 8b^3$

$y^2 = 9c^2$

$y^3 = 27c^3$

The equation becomes,

$(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$

$(a - 2b - 3c)^3 = a^3 - 6a^2b + 6ab^2 - 8b^3 - 3(a^2 - 4ab + b)\, 3c + 3(a - 2b)\, (9c^2) - 27c^3$

$= a^3 - 6a^2b + 6ab^2 - 8b^3 - 9ca^2 + 36abc - 9b^2c + 27ac^2 - 54bc^2 \;\; 27c^3$

(ix) $(p^2 - q^2 - r^2)^3 = ((p^2 - q^2) - r^2)^3$

Let $x = p^2 - q^2$ and $y = r^2$

$x^2 = (p^2)^2 - 2(p^2)(q^2)^2 + (q^2)^2 = p^4 - 2p^2q^2 + q^4$

$x^3 = (p^2)^3 - 3(p^2)^2 (q^2) + 3p^2 (q^2)^2 - (q^2)^3 = p^6 - 3p^4q^2 + 3p^2q^4 - q^6$

$y^2 = r^4$, $y^3 = r^6$

The equation is

$(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$

$(p^2 - q^2 - r^2) = p^6 - 3p^4q^2 + 3p^2q^4 - q^4 - q^6 - 3(p^2 - q^2)r^4 - r^6$

$= p^6 - 3p^4q^2 + 3p^2q^4 - q^6 - 3p^2r^2 + 6p^2q^2r^2 - 3q^4r^2 + 3p^2r^4 - 3q^2r^4 - r^6$

(x) $(198)^3 = (200 - 2)^3$

Helpful Hint

$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$= (200)^3 - 3(200)^2 (2) + 3(200)(2)^2 - (2)^3$

$= 8000000 - 6(40000) + 600 (4) - 8$

$= 8000000 - 240000 + 2400 - 8$

$= 7762392$

(xi) $(399)^3 = (400 - 1)^3$

$= (400)^3 - 3(400)^2 (1) + 3(400) (1)^2 - (1)^3$

$= 64000000 - 3(160000) + 1200 - 1$

$= 64000000 - 480000 + 1200 - 1$

$= 63521199$

(xii) $999 = (1000 - 1)^3$

$= (1000)^3 - 3(1000)^2 (1) + 3(1)^2 (1000) - (1)^3$

$= 1000000000 - 3000000 + 3000 - 1$

$= 997002999$

6. (i) Let $51 = a$ and $46 = b$

So the expression becomes

$a^3 - 3a^2b + 3ab^2 - b^3$

and we know that

$(a - b)^3 = a^3 - 3a^2 b + 3ab^2 - b^3$

$\therefore$ we have

$51^3 - 3 \times 51^2 \times 46 + 3 \times 46^2 \times 51 \times 46^3$

$= (51 - 46) = 5^3 = 125$

(ii) Let $31.6 = a$ and $28.6 = b$

$31.6^3 - 3 \times 31.6^2 \times 28.6 + 3 \times 28.6^2 \times 31.6 - 28.6^3$

(we know that $31.6 \times 31.6 \text{ x } 31.6 = 31.6^3$)

$= (31.6 - 28.6)^3$

$= 3^3 = 27$

(iii) Let 5.83 = a and 3.83 = b

$5.83^3 - 3 \times 5.83^2 \times 3.83 + 3 \times 5.83 \times 3.83^2 - 3.83^3$

$= (5.83 - 3.83)^3$

$= 2^3 = 8$

---

7. $p = 2q + 4$

$p - 2q = 4$

[Cubing both the sides]

$(p - 2q)^3 = 4^3$

$p^3 - (2q)^3 - 3(p)(2q)(p - 2q) = 64$

$p^3 - 8q^3 - 6pq\,(4) = 64$ $\quad [p - 2q = 4]$

$\therefore\ p^3 - 8q^3 - 24pq = 64$

8. $\dfrac{\frac{a^2 - 1}{a}}{} = 1$

$\dfrac{a}{\not a} - \dfrac{1}{a} = 1$

$a - \dfrac{1}{a} = 1$

Cubing both the sides

$(a - \dfrac{1}{a})^3 = 1^3$

$[(a - b)^3 = a^3 - b^3 - 3ab\,(a - b)]$

Using above formula we have

$a^3 \dfrac{1}{a^3} - 3\,(\not a)\left(\dfrac{1}{\not a}\right)(a - \dfrac{1}{a}) = 1$

$a^3 - \dfrac{1}{a^3} - 3(1) = 1$

$a^3 - \dfrac{1}{a^3} - 3 = 1$

$\dfrac{a^6 - 1}{a^3} - 4 = 0$

9. $x - y = 4$, $x\,y = 21$

$x^3 - y^3 = ?$

$x - y = 4$ $\quad$ [Cubing both the sides]

$(x - y)^3 = 4^3$

$x^3 - y^3 - 3xy\,(x - y) = 64$

[substitute $xy = 21$ and $x - y = 4$]

$x^3 - y^3 - 3(21)\,(4) = 64$

$x^3 - y^3 - 252 = 64$

$x^3 - y^3 = 64 + 252$

$\therefore\ x^3 - y^3 = 316$

10. $a^3 - b^3 = ?$

$a - b = 2,$ $\quad a^2 + b^2 = 4$

$a - b = 2$ $\quad$ square both the sides

$(a - b)^2 = 2^2$

$a^2 - 2ab + b^2 = 4$

$a^2 + b^2 = 4 + 2ab$

$4 = 4 + 2ab$ $\quad [a^2 + b^2 = 4]$

$2ab = 4 - 4 = 0$

$ab = 0$

Now cube both the sides of

$a - b = 2$

$(a - b)^3 = 2^3$

$a^3 - b^3 - 3\,ab\,(a - b) = 8$

$a^3 - b^3 - 3(0)(2) = 8$ $\quad [ab = 0,\ a - b = 2]$

$\therefore\ a^3 - b^3 = 8$

Multiple Choice Questions 8

1. Option A is correct.
2. Option C is correct.
3. Option D is correct.
4. Option D is correct.
5. Option A is correct.

7 REVISION: ALGEBRA

1. $(x + y)(x^3 - y^3)$

   $= (x + y)(x - y)(x^2 + ny + y^2)$

   $= (x^2 - y^2)(x^2 + xy + y^2)$

2. (i) $(x + 2y)(x - 5y)(x - 9y)$

   $= (x^2 - 5xy + 2xy - 10y^2)(x - 9y)$

   $= (x^2 - 3xy - 10y^2)(x - 9y)$

   $= x^3 - 9x^2y - 3x^2y + 27xy^2 - 10xy^2 + 90y^3$

   $= x^3 - 12x^2y + 17xy^2 + 90y^3$

   (ii) $(a + b + c)(b + c - a)(c + a - b)(a + b - c)$

   $= (\cancel{ab} + \cancel{ac} - a^2 + b^2 + bc - \cancel{ab} + bc + c^2 - \cancel{ac})(\cancel{ac} + bc - c^2 + a^2 + \cancel{ab} - \cancel{ac} - \cancel{ab} - b^2 + bc)$

   $= (-a^2 + b^2 + c^2 + 2bc)(a^2 - b^2 - c^2 + 2bc)$

   $= -a^4 + a^2b^2 + a^2c^2 - \cancel{2a^2bc} + a^2b - b^4 - b^2c^2 + \cancel{2b^3c} + a^2c^2 - b^2c^2 - c^4 + \cancel{2bc^3} + \cancel{2a^2bc} - \cancel{2b^3c} - \cancel{2bc^3} + 4b^2c^2$

   $= 2a^2b^2 + 2b^2c^2 + 2a^2c^2 - a^4 - b^4 - c^4$

   (iii) $(m^4 - 2m^2n^2 + n^4)(m^3 + 2m^2n - n^3)$

   $= m^7 + 2m^6n - m^4n^3 - 2m^5n^2 - 4m^4n^3 + 2m^2n^5 + m^3n^4 + 2m^2n^5 - n^7$

   $= m^7 + 2m^6n - 5m^4n^3 - 2m^5n^2 + 4m^2n^5 + m^3n^4 - n^7$

3.

$$
\begin{array}{r|l}
 & a^3 + 2a^2 + 7a + 20 \\
\hline
a^2 + 2a - 3 & a^5 + 0a^4 + 0a^3 + 0a^2 - 61a - 60 \\
 & a^5 - 2a^4 - 3a^3 \\
 & -\quad +\quad + \\
\hline
 & 2a^4 + 3a^3 \\
 & 2a^4 - 4a^3 - 6a^2 \\
 & -\quad +\quad + \\
\hline
 & 7a^3 + 6a^2 - 61a \\
 & 7a^3 - 14a^2 - 21a \\
 & -\quad +\quad + \\
\hline
 & 20a^2 - 40a - 60 \\
 & 20a^2 - 40a - 60 \\
 & -\quad +\quad + \\
\hline
 & 0
\end{array}
$$

4. $(2p + 3q)^3 - 18q\,(4p^2 - 9q^2) - (2p - 3q)^3$

$= (2p)^3 + 3\,(2p)^2\,(3q) + 3(2p)(3q)^2 + (3q)^3 - 72\,p^2q + 162q^3 - \left\{(2p)^3 - 3\,(2p)^2(3q) + 3(2p)(3q)^2 - (3q)^3\right\}$

$= 8\cancel{p^3} + 36\cancel{p^2}q + 54\cancel{pq^2} + 27q^3 - \cancel{72}p^2q + 16\;2q^3 \;\; - 8\cancel{p^3} + 36\cancel{p^2}q - \cancel{54}pq^2 + 27q^3$

$= 216q^3$

5. $x - y = 6$

$x^3 - y^3 - 18xy = ?$

$= (x - y)\,(x^2 + xy + y^2) - 18xy \qquad [a^3 - b^3 = (a - b)(a^2 + ab + b^2)]$

$= 6(x^2 + xy + y^2) - 18xy$

$= 6x^2 + 6xy + 6y^2 - 18xy$

$= 6x^2 - 12xy + 6y^2$

$= 6(x^2 - 2xy + y^2)$

$= 6(x - y)^2 = 6 \times 6^2 \qquad (x - y = 6)$

$= 6 \times 36 = 216$

6. Since $8x^3 - 12x^2 + 6x + 5\;x = \frac{1}{2}$

$= 8\left(\frac{1}{2}\right)^3 - 12\left(\frac{1}{2}\right)^2 + 6\left(\frac{1}{2}\right) + 5$

$= \cancel{8} \times \frac{1}{\cancel{8}} - \overset{3}{\cancel{12}} \times \frac{1}{\cancel{4}_1} + \overset{3}{\cancel{6}} \times \frac{1}{\cancel{2}} + 5$

$= 1 - 3 + 3 + 5$

$= 6$

7. $(x^2 + 1)(x^4 - x^2 + 1)$

$= x^6 - \cancel{x^4} + \cancel{x^2} + \cancel{x^4} - \cancel{x^2} + 1$

$= x^6 + 1$

8. $(x + 1)\,(x^2 - x + 1) + (2x - 1)(4x^2 + 2x + 1) - (x - 1)(x^2 + x + 1)$

$= x^3 + 13 + (2x)^3 - (1)^3 - (x^3 - 3) \qquad \left[(a + b)(a^2 - ab + b) = a^3 + b^3\right]$

$= x^3 + 1 + 8x^3 - 1 - x^3 + 1$

$= 8x^3 + 1$

9. (i) $a^3 + 27$

$= a^3 + (3)^3$

$= (a + 3)(a^2 - 3\,a + 9) \qquad \left[(a^3 + b^3 = (a + b)(a^2 - ab + b^2)\right]$

(ii) $8a^3 + 12a^2 + 6a + 2$

$= 8a^3 + 12a^2 + 6a + 1 + 1$

$= (2a + 1)^3 + 1 \qquad \left[a^3 + b^3 = (a + b)(a^2 - ab + b^2)\right]$

$= (2a + 1 + 1)\{(2a + 1)^2 - (2a + 1)(1) + 2\}$

$= (2a + 2)(4a^2 + 4a + 1 - 2a - 1 + 1)$

$= 2(a + 1)(4a^2 + 2a + 1)$

(iii) $a^3 - 3a^2b + 3ab^2 - b^3 + c^3$

$= (a - b)^3 + c^3$

$= (a - b + c)\{(a - b)^2 - (a - b)(c) + c^2\}$

$= (a - b + c)(a^2 - 2ab + b^2 - ac + bc + c^2)$

$= (a - b + c)(a^2 + b^2 + c^2 - 2ab + bc - ac)$

(iv) $x^2 - 7x + 10$

$= x^2 - (5 + 2)x + 10$

$= x^2 - 5x - 2x + 10$

$= x(x - 5) - 2(x - 5)$

$= (x - 5)(x - 2)$

(v) $a^2 + ab - 6b^2$

$= a^2 - (3 - 2)ab - 6b^2$

$= a^2 - 3ab + 2ab - 6b^2$

$= a(a - 3b) + 2b(a - 3b)$

$= (a - 3b)(a - 2b)$

10. (i) $\frac{x-6}{x-8} + \frac{x-2}{x-4} = \frac{x-5}{x-7} + \frac{x-3}{x-5}$

$$\frac{(x-6)(x-4) + (x-2)(x-8)}{(x-8)(x-4)} = \frac{(x-5)(x-5) + (x-3)(x-7)}{(x-7)(x-5)}$$

$$\frac{x^2 - 10x + 24 + x^2 - 10x + 16}{x^2 - 12x + 32} = \frac{x^2 - 10x + 25 + x^2 - 10x + 21}{x^2 - 12x + 35}$$

$(2x^2 - 20x + 40)(x^2 - 12x + 35) = (2x^2 - 20x + 46)(x^2 - 12x + 32)$

$\cancel{2x^4} - \cancel{24x^3} + 70x^2 - \cancel{20x^3} + \cancel{240x^2} - 700x + 40x^2 - 480x + 1400$

$= \cancel{2x^4} - \cancel{24x^3} + 64x^2 - \cancel{20x^3} + \cancel{240x^2} - 640x + 46x^2 - 552x + 1472$

$\cancel{110x^2} - 1180x + 1400 = \cancel{110x^2} - 1192x + 1472$

$1192x - 1180x = 1472 - 1400$

$12x = 72$

$x = \frac{72}{12}$

$x = 6$

(ii) $\frac{a}{x+a} + \frac{b}{x+a} = 2$

$a(x+b) + b(x+a) = 2(x+a)(x+6)$

$ax + ab + bx + ab = 2(x^2 + bx + ax + ab)$

$ax + 2ab + bx = 2x^2 + 2bx + 2ax + 2ab$

$2x^2 + 2bx - bx + 2ax - ax + 2ab - 2ab = 0$

$2x^2 + bx + ax = 0$

$x(2x + b + a) = 0$

$2x + b + a = 0$

$2x = -(a+b)$

$x = -\frac{(a+b)}{2}$

(iii) $\frac{1}{(x-3)(x-4)} = \frac{1}{(x-5)(x-6)}$

$(x-5)(x-6) = (x-3)(x-4)$

$\cancel{x^2} - 11x + 30 = \cancel{x^2} - 7x + 12$

$11x - 7x = 30 - 12$

$4x = 18$

$x = \frac{18}{4}$

$x = \frac{9}{2}$

11. (i) $2x^4 - 5x^2 - 3$

$= 2x^4 - (6-1)x^2 - 3$

$= 2x^4 + 1x^2 - 6x^2 - 3$

$= x^2(2x^2 + 1) - 3(2x^2 + 1)$

$= (x^2 - 3)(2x^2 + 1)$

(ii) $a^2 - 8a - 20$

$= a^2 - (10-2)a - 20$

$= a^2 - 10a + 2a - 20$

$= a(a-10) + 2(a-10)$

$= (a-10)(a+2)$

(iii) $21a^2 - 58a + 21$

$= 21a^2 - (49+9)a + 21$

$= 21a^2 - 49a - 9a + 21$

$= 7a(3a-7) - 3(3a-7)$

$= (7a-3)(3a-7)$

12. (i) $\frac{x^3 - y^3}{x^2 - xy + y^2}$

$= \frac{(x-y)\cancel{(x^2+xy+y^2)}}{\cancel{x^2+xy+y^2}} = x - y$

$\left[a^3 - b^3 = (a-b)(a^2 - ab + b^2)\right]$

(ii) $\frac{x^2 + 2x - 8}{x^3 + x^2 - 12x}$

$= \frac{x^2 + 4x - 2x - 8}{x(x^2 + x - 12)} = \frac{x(x+4) - 2(x+4)}{x\{x^2 + 4x - 3x - 12\}}$

$= \frac{(x-2)(x+4)}{x\{x(x+4) - 3(x+4)\}} = \frac{(x-2)\cancel{(x+4)}}{x\cancel{(x+4)}(x-3)}$

$= \frac{x-2}{x(x-3)}$

(iii) $\frac{x^2-2}{6xy} \times \frac{18y^3}{5x^4 - 10x^2}$

$= \frac{\cancel{x^2-2}}{x\cancel{y}} \times \frac{3\cancel{y} \times y \times y}{5x^2\cancel{(x^2-2)}}$

$= \frac{3y^2}{5x^3}$

(iv) $\frac{8x^3 - 4y^2}{2a^2 - 3a + 1}$

$= \frac{4(2x^3 - y^2)}{2a^2 - 2a - a + 1}$

$= \frac{4(2x^3 - y^2)}{2a(a-1) - 1(a-1)}$

$= \frac{4(2x^3 - y^2)}{(2a-1)(a-1)}$

(can not be reduced further)

(v) $\frac{a^2 + 4a}{a^2 - 9a} \div \frac{a^2 + 2a - 8}{x^2 - xy}$

$= \frac{\cancel{a}\cancel{(a+4)}}{\cancel{a}(a-9)} \times \frac{x(x-y)}{\cancel{(a+4)}(a-4)}$

$= \frac{x(x-y)}{(a-9)(a-2)}$

13. Let x be the required number, then

$2x - 3.5 = x + 7.5$

$2x - x = 7.5 + 3.5$

$x = 11.0$

14. Let x be the age of Arshad, then

Sajid's age = $x + 10$

Sohail's age = $x - 3$

Sajid's age + Sohail's age = 4 × Arshad's age + 2

$$x + 10 + x - 3 = 4x + 2$$
$$4x - 2x = 7 - 2$$
$$x = 5$$

Arshad's age = 5 years

Sajid's age = 5 + 10 = 15 years

Sohail's age = 5 – 3 = 2 years

15. Let $x°$ be the smallest angle,

Largest angle = $2x°$

3rd angle = $x° + 8°$

Since the sum of the angles of a triangle is 180°,

$$\therefore x + 2x + x + 8 = 180°$$
$$4x = 180 - 8$$
$$x = \frac{172}{4}$$
$$x = 43°$$

Largest angle = $2x = 2(43) = 86°$

3rd angle = $x + 8 = 43 + 8 = 51°$

16. Let x be the denominator

$$\frac{x-7}{x} = \frac{x-7+2}{x+9}$$
$$\frac{x-7}{x} = \frac{x-5}{x+9}$$
$$(x-7)(x+9) = (x-5)$$
$$\cancel{x^2} + 2x - 63 = \cancel{x^2} - 5x$$
$$2x + 5x = 63$$
$$7x = 63$$
$$x = \frac{63}{7}$$
$$x = 9$$

$\therefore$ The required fraction is $= \frac{x-7}{x}$

$$= \frac{9-7}{9}$$
$$= \frac{2}{9}$$

17. (i) $(4a^4 - 5a^2 + 7)(a^2 - 3)$

$= 4a^6 - 12a^4 - 5a^4 + 15a^2 + 7a^2 - 21$

$= 4a^6 - 17a^4 + 22a^2 - 21$

(ii) $(5x + 4)(x^2 - 3x + 7)$

$= 5x^3 - 15x^2 + 35x + 4x^2 - 12x + 28$

$= 5x^3 - 11x^2 + 23x + 28$

(iii) $(9x^5 - 3x^4 + 2x^3 - 1)(3x - 7)$

$= 27x^6 - 63x^5 - 9x^5 + 21x^4 + 6x^4 - 14x^3 - 3x + 7$

$= 27x^6 - 72x^5 + 27x^4 - 14x^3 - 3x + 7$

18. From $a + b - 3 = 0$, we get $a + b = 3$.

$a^3 + b^3 = (a+ b)^3 - 3ab\,(a + b)$

$= (3)^3 - 3ab(3) = 27 - 9ab$

$(27 - 9ab) + 9ab$- $26 = 27 - 26 = 1$

19. Use the identity $x^3 + y^3 = (x + y)^3 - 3xy(x + y)$.

Substitute $x + y = 4$:

$x^3 + y^3 = (4)^3 - 3xy(4)$

$x^3 + y^3 = 64 - 12xy$

$(64 - 12xy) + 12xy - 64 = 0$

$= 0$

20. Find $a^3 + b^3$ using $(a + b)^3 - 3ab(a + b)$:

$a^3 + b^3 = (7)^3 - 3(12)(7) = 343 - 252 = 91$

$= 91 + 4(12)(7) = 91 + 336 = 427$

$= 427$

21. The following identity is used:

$(a - b)\,(a^2 + 2a + 4:) = a^3$ -

(i) $(a - 2)(a^2 + 2a + 4) = a^3 - 2^3 = a^3 - 8$

(ii) $(2x - 3y)(4x^2 + 6xy + 9y^2\,) = (2x)^3 - (3y)^3$
$= 8x^3 - 27y^3$

(iii) $(ab - cd)(a^2b^2 + abcd + c^2d^2) = (ab)^3 - (cd)^3$
$= a^3b^3 - c^3d^3$

22.

(i) $(x^2)^3 + (1)^3$

$(x^2 + 1)((x^2)^2 - (x^2)(1) + (1)^2)$

$(x^2 + 1)(x^4 - x^2 + 1)$

(ii)

$3(27x^3 - 64y^3)$

$27x^3 = (3x)^3$

$64y^3 = (4y)^3$

$3((3x)^3 - (4y)^3)$

Use the Difference of Cubes formula with

A = $3x$ and B = $4y$:

$3(3x - 4y)((3x)^2 + (3x)(4y) + (4y)^2)$

$= 3(3x - 4y)(9x^2 + 12xy + 16y^2)$

(iii) We can rewrite $a^3\, b^3$ as $(ab)^3$. So, the expression is:

$(ab)^3 + c^3$

Using the Sum of Cubes formula with, $a = ab$ and $b = c$:

$(ab+ c)((ab)^2 - (ab)(c) + c^2)$

$(ab+ c)(a^2 b^2 - abc + c^2)$

(iv) Factorise $a^3\, b^3 - 64c^3$

First, rewrite the terms as cubes:

$a^3\, b^3 = (ab)^3$

$64c^3 = (4c)^3$

$= (ab)^3 - (4c)^3$

Using the Difference of Cubes formula with A = ab and B = $4c$:

$(ab - 4c)((ab)^2 + (ab)(4c) + (4c)^2)$

23. So, the common factor is $6p$. let's divide both terms by $6p$:

$6p\,(p + 3(q - r))$

Now, multiply 3 by the terms inside the small parentheses:

$6p(p + 3q - 3r)$

$= 6p(p+ 3q - 3r)$

24.

(i) $x^4 - 5x^2 + 4$

$x^4 - 4x^2 - 1x^2 + 4$

$(x^4 - 4x^2) - (x^2 - 4)$

$x^2\,(x^2 - 4) - 1(x^2 - 4)$

$(x^2 - 1)\,(x^2 - 4)$

We can factorize these even further using the difference of squares: $(x^2 - 1) = (x - 1)\,(x + 1)$ and $(x^2 - 4) = (x - 2)(x + 2)$.

$= (x - 1)\,(x + 1)(x - 2)\,(x + 2)$

iii.

(iv) $28 + 3a - a^2$

$- a^2 + 3a + 28.$

$- (a^2 - 3a - 28)$

$- (a^2 - 7a + 4a - 28)$

$- [a(a - 7) + 4(a - 7)]$

$- (a + 4)(a - 7)$

$= - (-(a + 4)(a - 7))$

$= (a + 4)(a - 7)$

iv.

$- (x^2 + 4x - 5)$

$- (x^2 + 5x - 1x - 5)$

$- [x(x + 5) - 1(x + 5)]$

$- (x - l)(x + 5)$

To remove the negative sign at the front, multiply it into $(x - 1)$ to get $\{1 - x)$.

$= (1 - x)(x + 5)$

Let the present age of the son be x and the mother be y.

Twenty years ago:

Son's age: $(x - 20\}$

Mother's age: $(y - 20)$

According to the problem: $y - 20 = 4(x - 20)$

$y - 20 = 4x - 80$

$y = 4x - 60$ --- (Equation 1)

Four years hence (in the future):

Son's age: $(x + 4)$

Mother's age: $(y + 4)$

According to the problem: $y + 4 = 2(x + 4)$

$y + 4 = 2x + 8$

$y = 2x + 4$ --- (Equation 2)

Equating the two expressions for y:

$4x - 60 = 2x + 4$

$2x = 64$

$x = 32$

For y:

$y = 2(32) + 4 = 64 + 4 = 68$

The mother's present age is 68 years and the son's present age is 32 years.

25. Area of a rectangle = length × breadth

Area = $(2x + 5)$ $(x - 3\}$

Area = $2x(x - 3) + 5(x - 3)$

Area = $2x^2 - 6x + 5x - 15$

Area = $\{2x^2 - x - 15)$ square metres

We know the identity:
$x^3 + y3 = (x + y)^3 - 3xy(x + y)$.

Substitute $x + y = 4$:

$x^3 + y^3 = (4)^3 - 3xy(4)$

$x^3 + y^3 = 64 - 12xy$

$(64 - 12xy) + 12xy - 64 = 0$

$= 0$

26.

Let Son = s, Father = $4s$.

In 5 years: Son = $s + 5$. Father = 4s + 5.

$(s + 5) + (4s + 5) = 55$

$5s + 10 = 55 \Rightarrow 5s = 45 \Rightarrow s = 9$

Father = 4 × 9 = 36.

Son is 9, Father is 36.

27.

Let the number be x.

$3x - 4.2 = 2x + 2.8$

$3z - 2x = 2.8 + 4.2$

$x = 7$

The number is 7.

28. (i)

$(y^2 + 2y + 3)(3y^2 + 5y - 7)$

$y^2 (3y^2 + 5y - 7) + 2y (3y^2 + 5y - 7) + 3(3y^2 + 5y - 7)$

$3y^4 + 5y^3 - 7y^2 + 6y^3 + 10y^2 - 14y + 9y^2 + 15y - 21$

Grouping the like terms (terms with the same power of y):

$3y^4 + (5y^3 + 6y^3) + (-7y^2 + 10y^2 + 9y^2) + (-14y + 15y) - 21$

$= 3y^4 + 11y^3 + 12y^2 + y - 21$

(ii) $(6x + 5)(3x - 7)$

$(6x \times 3x) + (6x \times -7) + (5 \times 3x) + (5x - 7)$

$18x^2 - 42x + 15x - 35$

$18x^2 - 27x - 35$

(iii) $(5x^2 - 3x + 7)^2$

$(5x^2 - 3x + 7)(5x^2 - 3x + 7)$.

$5x^2 (5x^2 - 3x + 7) - 3x(5x^2 - 3x + 7) + 7(5x^2 - 3x + 7)$

$25x^4 - 15x^3 + 35x^2 - 15x^3 + 9x^2 - 21x + 35x^2 - 21x + 49$

$25x^4 + (-15x^3 - 15x^3) + (35x^2 + 9x^2 + 35x^2) + (-21x - 21x) + 49$

$25x^4 - 30x^3 + 79x^2 - 42x + 49$

29. (i)

(i) $(x + 2)(x^2 - 2x + 4) - (x - 2)(x^2 + 2x + 4)$

The first part follows the "sum of cubes" identity, $a^3 + b^3 = (a + b)(a^2 - a^b + b^2)$. where $a = x$ and $b = 2$.

$(x + 2)(x^2 - 2x + 4) = x^3 + 2^3 = x^3 + 8$

The second part follows the ''difference of cubes" identity, $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$, where $a = x$ and $b = 2$.

$(x - 2)(x^2 + 2x + 4) = x^3 - 2^3 = x^3 - 8)$

Subtracting the two results:

$(x^3 + 8) - (x^3 - 8) = x^3 + 8 - x^3 + 8)$

(ii) $(5x - 6)(2x + 3) + (3x + 5)^2$

First, multiply the binomials:

$(5x)(2x) + (5x)(3) + (-6)(2x) + (-6)(3)$
$= 10x^2 + 15x - 12x - 18 = 10x^2 + 3x - 18$

Next, expand the squared binomial:

$(3x + 5)^2 = (3x)^2 + 2(3x)(5) + 5^2$
$= 9x^2 + 30x + 25$

Finally, add the two results together:

$(10x^2 + 3x - 18) + (9x^2 + 30x + 25)$

$= (10x^2 + 9x^2) + (3x + 30x) + (-18 + 25)$

Final Answer:

$19x^2 + 33x + 7$

(iii) $(a^2 - 2a + 4)(a^2 + 2a + 4)$

We can group the terms to look like a "difference of squares· pattern, $(a - b)(a + b) = a^2 - b^2$, by grouping

$(a^2 + 4)$ together:

$(a^2 + 4) - 2a)((a^2 + 4) + 2a)$

This simplifies to:

$(a^2 + 4)^2 - (2a)^2$

Expanding $(a^2 + 4)^2$ gives $a^4 + 8a^2 + 16$.

Subtracting the squared term $(2a)^2 = 4a^2$

$a^2 + 8a^2 + 16 - 4a^2$

$= a^4 + 4a^2 + 16$

9 LINEAR EQUATIONS

Exercise 9A

1. (i) gradient = $\frac{\text{rise}}{\text{run}}$

As we move from left to right the rise is from –12 to –4, i.e 8 units.

Similarly the run is from –1 to 1, i.e 2 units

$\therefore$ gradient = $\frac{8}{2}$ = 4

> **Helpful Hint**
> Since the line is going upwards if we move from left to right, the gradient is positive.

The line intersects y – axis at –8, there fore, y– intercept = – 8

(ii) As we move from left to right the rise is from 10 to 6 i.e. –4 units and run is –18 to –9, i.e. 9 units

Therefore,

$$\text{gradient} = \frac{\text{rise}}{\text{run}} = -\frac{4}{9}$$

> **Helpful Hint**
> If we move from left to right, the line goes downward, and the gradient will be negative.

y – intercept = 2

(iii) gradient = $\frac{15}{24} = \frac{5}{8}$

y – intercept = 5

(iv) gradient = $\frac{3}{3}$ = 1

y – intercept = 0

(v) gradient = $\frac{12}{5}$

y – intercept = 16

2. (i) To write equation of a line, we need to find its gradient and y–intercept.

> **Helpful Hint**
> The general form of equation of a line is $y = mx + c$, where m is the gradient and c is the y–intercept of the line.

In this graph gradient, m = $\frac{4}{2}$ = 2

and y–intercept, c = –3

So, the equation is

$$y = 2x - 3$$

(ii) $m = -\frac{5}{4}$

> **Helpful Hint**
> Theline is going downwards as we move from left to right so the gradient is negative.

y–intercept, c = –2

The equation is

$$y = -\frac{5}{4}x - 2$$

(iii) $m = \frac{7}{5}$, c = 0

The equation is

$y = \frac{7}{5}x$

(iv) $m = -\frac{7}{4}$, c = 3

The equation is

$y = -\frac{7}{4}x + 3$

(v) $m = \frac{3}{5}$, c = 0

The equation is

$y = \frac{3}{5}x$

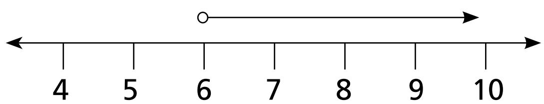
Exercise 9B

1. (i) $x - 2 > 4$

[add 2 to both the sides]

$x > 4 + 2$

$x > 6$



Helpful Hint

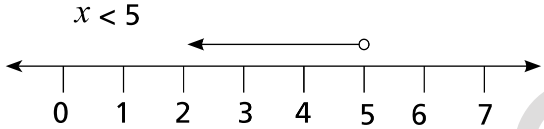
Use open circle for > and <

(ii) $6x < 30$

[divide both the sides by 6]

$x < \frac{30}{6}$

$x < 5$

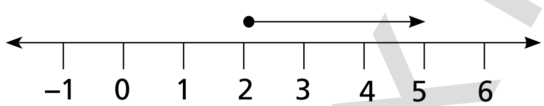


(iii) $6\ x\ 9 \geq 11$

[subtract 9 from both the sides]

$x \geq 11 - 9$

$x \geq 2$



Helpful Hint

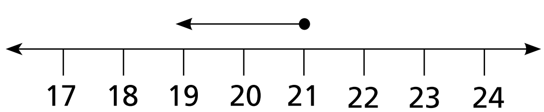
Use close circle for ≤ and ≥

(iv) $\frac{x}{3} \leq 7$

multiply both sides by 3

$x \leq 7 * 3$

$x \leq 21$

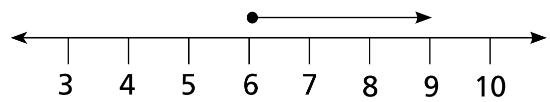


(v) $\frac{x}{2} \geq 3$

[multiply both sides by 2]

$x \geq 3 * 2$

$x \geq 6$



(vi) $3 < -5n + 2n$

$3 < -3n$

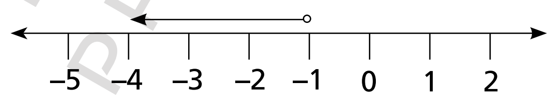
$-3n > 3$

[divide both sides by – 3]

$n < -1$

Helpful Hint

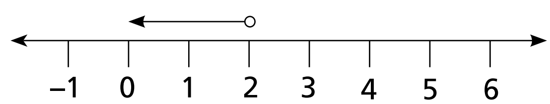
Change/flip the sign when multiplying with any negative number



(vii) $-p - 4p > -10$

$-5p > -10$

$p < 2$



(viii) $9 \geq -2m + 2 - 3$

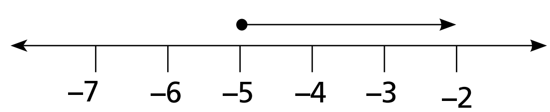
$9 \geq -2m - 1$

$9 + 1 \geq -2m$

$10 \geq m$

$-5 \leq m$

$m \geq -5$



(ix) $-3 - 6(4x + 6) > -111$

$-6(4x + 6) > -111 + 3$

$-6(4x + 6) > -108$

$4x + 6 < -\frac{108}{-6}$

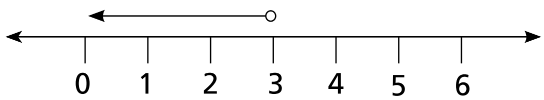
$4x + 6 < 18$

$4x < 18 - 6$

$4x < 12$

$x < \frac{12}{4}$

$x < 3$



(x) $5(6 + 3r) + 7 \geq 127$

$5(6 + 3r) \geq 127 - 7$

$5(6 + 3r) \geq 120$

$6 + 3r \geq \frac{120}{5}$

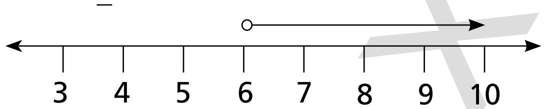
$6 + 3r \geq 24$

$3r \geq 24 - 6$

$3r \geq 18$

$r \geq \frac{18}{3}$

$r \geq 6$



Exercise 9C

1. (i) coefficient, constant
   (ii) $5x + 2y$
   (iii) 2
   (iv) elimination, substitution
   (v) $x = 9, y = 4$

2. (i) True
   (ii) True
   (iii) False

Reason:

Both equation will have a solution for particular values of x and y

For example, if $x = 1$ and $y = 2$

$x + y$

$= 1 + 2$

$= 3 \neq 0$

(iv) True

(v) False

Reason: LHS RHS

$x - 3y = 9$

by substituting the values, $x = 4$ and $y = 3$, we have

LHS $4 - \cancel{3}\left(\frac{5}{\cancel{3}}\right)$

$= 4 - 5 = -1$

LHS is not equal to RHS

3. (i) $7x + 3y = 25$ ______ (i)

$-2x + y = 4$ ______ (ii)

Eliminate y from the equations.

$(-2x + y = 4) \times 3$ [multiply equation (ii) by 3 to make the coefficients of y same]

We have

$7x + 3y = 25$ ______ (i) (i)[subtract eq

$-6x + 3y = 12$

$+ \quad - \quad -$ ______ (ii) (ii) from (i)]

$13x = 13$

$x = \frac{13}{13} = 1$

Now, substitute this value of x in any one of the given equations.

Equation (ii)

$-2x + y = 4$

$-2(1) + y = 4$

$-2 + y = 4$

$y = 4 + 2$

$\boxed{y = 6}$

$\therefore x = 1, y = 6$

(ii) $5x - y = 7$ ____________ (i)

$2x + y = 7$ ____________ (ii)

The coefficients of y are already same.

Helpful Hint

If the signs of the coefficients of eliminating variables are different, add the equation.

Add eq (i) and (ii)

$$\begin{array}{rl} & 5x - y = 7 \\ + & 2x + y = 7 \\ \hline & 7x \quad = 14 \end{array}$$

$$x = \frac{14}{7}$$

$$x = 2$$

Substitute $y = 2$ eq (i)

$5x - y = 7$

$5(2) - y = 7$

$10 - y = 7$

$-y = 7 - 10$

$-y = -3$

$y = +3$

$\therefore x = 2, y = 3$

(iii) $2x - y = 2$ ____________ (i)

$5x + y = -9$ ____________ (ii)

$7x \quad = -7$

Add equations (i) and (ii)

$$x = \frac{-7}{7} = -1$$

Substitute $x = 1$ in eq (i)

$2x - y = 2$

$2(-1) - y = 2$

$-2 - y = 2$

$-y = 2 + 2$

$-y = 4$

$y = -4$

$\therefore x = -1, \ y = -4$

(iv) $4x + 4y = -4$ __________ (i)

$x + 7y = -19$ __________ (ii)

$(x + 7y = -19) \times 4$

$4x + 28y = -76$ __________ (ii)

Now subtract eq (ii) from eq (i)

$$\begin{array}{l} 4x + 4y = -4 \\ 4x + 28y = -76 \\ - \quad - \qquad + \\ \hline \quad -24y = 72 \end{array}$$

$$y = -\frac{72}{24}$$

$$y = -3$$

Substitute in eq (i)

$4x + 4(-3) = -3$

$4x - 12 = -4$

$4x = 8$

$x = 2$

$\therefore x = 2, y = -3$

(v) Add both the equations to eliminate y

$3x - y = 7$ __________ (i)

$\frac{1}{2}x + y = 7$ __________ (ii)

$$+\ \left(3 + \frac{1}{2}\right)x = 14$$

$$\frac{6+1}{2}x = 14$$

$$\frac{7}{2}x = 14$$

$$7x = 14 \times 2 = 28$$

$$x = \frac{28}{7} = 4$$

Substitute $x = 4$ in eq (i)

$3(4) - y = 7$

$12 - y = 7$

$-y = 7 - 12$

$-y = -5$

$y = 5$

$\therefore x = 4, y = 5$

(vi) $4a - 3b = 10$ ——— (i)

$2a + b = 10$ ——— (ii)

Multiply eq (ii) by 3 to make coefficients of b same.

$(2a + b = 10) \times 3$

$6a + 3b = 30$ —— (ii)

Add both the equations (i) and (ii)

$$\begin{array}{r} 4a - 3b = 10 \\ + \; 6a + 3b = 30 \\ \hline 10a \quad\quad = 40 \end{array}$$

$a = \frac{40}{10}$

$a = 4$

Substitute $a = 4$ in eq (i)

$4(4) - 3b = 10$

$16 - 3b = 10$

$-3b = 10 - 16 = -6$

$b = \frac{-6}{-3}$

$b = 2$

$\therefore a = 4, b = 2$

(vii) $a - 5b = 10$ —— (i)

$5a - 3b = 17$ —— (i)

Multiply eq (i) by 5

$(a - 5b = 10) \times 5$

Subtract eq (ii) from eq (i)

$5a - 25b = 50$ —— (i)

$5a - 3b = 17$ —— (ii)

$$\begin{array}{r} 5a - 25b = 50 \\ 5a - 3b \;\; = 17 \\ - \quad + \quad\quad - \\ \hline -22b = 33 \end{array}$$

$b = -\frac{33}{22} = -\frac{3}{2}$

Substitute $b = \frac{3}{2}$ in eq (i)

$a - 5\left(-\frac{3}{2}\right) = 10$

$a + \frac{15}{2} = 10$

$a = 10 - \frac{15}{2} = \frac{20 - 15}{2}$

$2a = 5$

$a = \frac{5}{2}$

$\therefore a = \frac{5}{2}, \; b = \frac{-3}{2}$

(viii) $a + 1\frac{1}{2}\; b = 8\frac{1}{2}$

$a + \frac{3}{2}\; b = \frac{17}{2}$ —— (i)

$2a - b = 1$ —— (ii)

Multiply eq (i) by 2

$\left(a + \frac{3}{2}\; b = \frac{17}{2}\right) \times 2$

$$\begin{array}{r} 2a + 3b = 17 \\ 2a - \;\; b = 1 \\ - \quad + \quad\;\; - \\ \hline 4b = 16 \end{array}$$

$b = \frac{16}{4}$

$b = 4$

Substitute $b = 4$ in eq (ii)

$2a - b = 1$

$2a - 4 = 1$

$2a = 1 + 4 = 5$

$a = \frac{5}{2}$

$\therefore a = \frac{5}{2}, b = 4$

(ix) $3x - 5 = y$

$3x - y = 5$ —— (i)

$x = y - 1\frac{1}{3}$

$x - y = -\frac{4}{3}$ —— (ii) Subtract eq (ii) from (i)

$$\begin{array}{r} 3x - y = 5 \\ x - y = -\frac{4}{3} \\ - \quad + = + \\ \hline 2x = 5 + \frac{4}{3} \end{array}$$

$2x = \frac{15 + 4}{3}$

$2x = \frac{19}{3}$

$x = \frac{19}{6} \times \frac{1}{2}$

$x = \frac{19}{6}$

Substitute $\frac{19}{16}$ in eq (i)

$3x - y = 5$

$3\left(\frac{19}{6}\right) - y = 5$

$$\frac{19}{2} - y = 5$$

$$y = \frac{19}{2} - 5$$

$$y = \frac{19-10}{2}$$

$$y = \frac{9}{2}$$

$$\therefore x = \frac{19}{6}, \; y = \frac{9}{2}$$

(x) $x + y = 4$ —— (i)

$3x - 2y = 7.5$ —— (ii)

Multiply eq (i) by (ii)

$(x + y = 4) \times 2$

Add both the equation

$2x + 2y = 8$ —— (i)

$\underline{3x - 2y = 7.5}$ —— (ii)

$5x = 15.5$

$x = 3.1$

Substitute $x = 3.1$ in eq (i)

$3.1 + y = 4$

$y = 4 - 3.1$

$y = 0.9$

$x - 3.1 \; , \; y = 0.09$

4. (i) $5x - 2y = 8$ —— (i)

$3x - 2y = 4$ —— (ii)

Helpful Hint
Make 'y' the subject of the equation.

From eq (i)

$-2y = 8 - 5x$

$$y = -\frac{(8-5x)}{2}$$

$$y = -\frac{5x-8}{2}$$

Substitute in eq (ii)

$$3x - \cancel{2}\left(\frac{5x-8}{\cancel{2}}\right) = 4$$

$3x - 5x + 8 = 4$

$-2x = 4 - 8$

$-2x = -4$

$x = 2$

Substitute $x = 2$ in eq (i)

$5(2) - 2y = 8$

$10 - 2y = 8$

$-2y = 8 - 10$

$-2y = -2$

$y = 1$

$\therefore x = 2 \; , \; y = 1$

(ii) $5b + 14a = 31$ —— (i)

$2a - 3b = -29$ —— (ii)

From eq (ii)

$2a = -29 + 3b$

$$a = \frac{-29 + 3b}{2}$$

Substitute in eq (i)

$$5b + \overset{7}{\cancel{14}}\left(\frac{-29 + 3b}{\cancel{2}}\right) = 31$$

$5b - 203 + 21b = 31$

$26b = 31 + 203$

$26b = 234$

$$b = \frac{234}{26}$$

$b = 9$

Substitute $b = 9$ in eq (ii)

$2a - 3(9) = -29$

$2a - 27 = -29$

$2a = -29 + 27$

$2a = -2$

$a = -1$

$\therefore a = -1 \; , \; b = 9$

(iii) $2a - 3b = 1.5$ —— (i)

$2a - b = 8.5$ —— (ii)

From eq (ii)

$-b = 8.5 - 2a$

$b = 2a - 8.5$

Substitute $a = 6$ in eq (i)

$2a - 3\,(2a - 8.5) = 1.5$

$2a - 6a + 25.5 = 1.5$

$-4a = 1.5 - 25.5$

$-4a = -24.0$

$a = \frac{24}{4}$

$a = 6$

Substitute in eq (ii)

$2(6) - b = 85$

$12 - b = 8.5$

$b = 12 - 8.5$

$b = 3.5$

$\therefore a = 6\ ,\ b = 3.5$

(iv) $x - y = 3$ —— (i)

$5x + y = 33$ —— (ii)

From eq (i)

$x = 3 + y$

Substitute $y = + 3$ in eq (ii)

$5(3 + y) + y = 33$

$15 + 5y + y = 33$

$6y = 33 - 15 = 18$

$y = \frac{18}{6} = 3$

Substitute in eq (i)

$x - 3 = 3$

$x = 3 + 3$

$x = 6$

$\therefore x = 6\ ,\ y = 3$

(v) $2z + x = 15$ —— (i)

$2z + 3x = 9$ —— (ii)

From eq (i)

$x = 15 - 2z$

Substitute in eq (ii)

$2z + 3(15 - 2z) = 9$

$2z + 45 - 6z = 9$

$-4z = 9 - 45 = -36$

$z = \frac{36}{4} = 9$

Substitute in eq (i)

$2(9) + x = 15$

$18 + x = 15$

$x = 15 - 18$

$x = -3$

$\therefore x = -3\ ,\ z = 9$

(vi) $5m - 7n = 5$ —— (i)

$m - 2n = -2$ —— (ii)

From eq (ii)

$m = -2 + 2n$

Substitute in eq (i)

$5(-2 + 2n) - 7n = 5$

$-10 + 10n - 7n = 5$

$3n = 15$

$n = 5$

Substitute $n = 5$ in eq (ii)

$m - 2(5) = -2$

$m - 10 = -2$

$m = -2 + 10$

$m = 8$

$\therefore m = 8,\ n = 5$

(vii) $m + 1.5n = 23$ —— (i)

$1.5m - 2n = -8$ —— (ii)

From eq (i)

$m = 23 - 1.5n$

Substitute in eq (ii)

$1.5(23 - 1.5n) - 2n = -8$

$34.5 - 2.25n - 2n = -8$

$4.25n = 42.5$

$n = \frac{42.5}{4.25}$

$n = 10$

Substitute $n = 10$ in eq (i)

$m + 1.5(10) = 23$

$m + 15 = 23$

$m = 23 - 15 = 8$

$\therefore m = 8\ ,\ n = 10$

(viii) $\frac{5a}{2} - \frac{13}{3}b = -29$ —— (i)

$\frac{4}{3}b - \frac{3}{2}a = 6$ —— (ii)

From eq (i)

$\frac{5}{2}a = -29 + \frac{13}{3}b$

$$a = \frac{2}{5}\left(-29 + \frac{13}{3}b\right)$$

$$a = -\frac{58}{5} + \frac{26}{15}b$$

Substitute in eq (ii)

$$\frac{4}{3}b - \frac{3}{2}\left(-\frac{58}{5} + \frac{26}{15}b\right) = +6$$

$$\frac{4}{3}b + \frac{174}{10} - \frac{26}{10}b = 6$$

$$\left(\frac{4}{3} - \frac{26}{10}\right)b = 6 - \frac{174}{10}$$

$$\frac{40-78}{30}b = \frac{60-174}{10}$$

$$-\frac{38}{30}b = -\frac{114}{10}$$

$$b = \frac{114}{10} \times \frac{30}{38}$$

$$b = 9$$

Substitute $b = 9$ in eq (i)

$$\frac{5}{2}a - \frac{13}{3} \times 9 = -29$$

$$\frac{5}{2}a = -29 + 39 = 10$$

$$a = 10 \times \frac{2}{5}$$

$$a = 4$$

$$\therefore a = 4 \quad , \quad b = 9$$

(ix) $c - \frac{1}{5}d = 9$ —— (i)

$c - 2d = -9$ —— (ii)

From eq (i)

$$c = 9 + \frac{1}{5}d$$

Substitute in eq (ii)

$$9 + \frac{1}{5}d - 2d = -9$$

$$\frac{d - 10d}{5} = -9 - 9$$

$$-\frac{9d}{5} = -18$$

$$9d = 5 \times 18$$

$$d = \frac{5 \times 18}{9}$$

$$d = 10$$

Substitute $d = 10$ in eq (i)

$$c - \frac{1}{5} \times 10 = 9$$

$$c = 9 + 2$$

$$c = 11$$

$$\therefore c = 11, d = 10.$$

(x) $x - y = 7$ —— (i)

$2y + 3x = 11$ —— (i)

From eq (i)

$$x = 7 + y$$

Substitute in eq (ii)

$$2y + 3(7 + y) = 11$$

$$2y + 21 + 3y = 11$$

$$5y = 11 - 21$$

$$5y = -10$$

$$y = -2$$

Substitute in eq (i)

$$x - (-2) = 7$$

$$x + 2 = 7$$

$$x = 7 - 2$$

$$x = 5$$

$$\therefore x = 5 \quad , \quad y = -2$$

5. (i) $x + y = 3$ ______ eq 1

$4x - y = 2$ ______ eq 2

Make table of values for equation 1 by substituting various values of x to find corresponding values of y.

$x + y = 3$

$y = 3 - x$

Helpful Hint

Make y the subject of the equation. Keep y at left hand side of the equation and all other terms at right hand side to find the value of y.

x	–2	–1	0	1	2
y	5	4	3	2	1

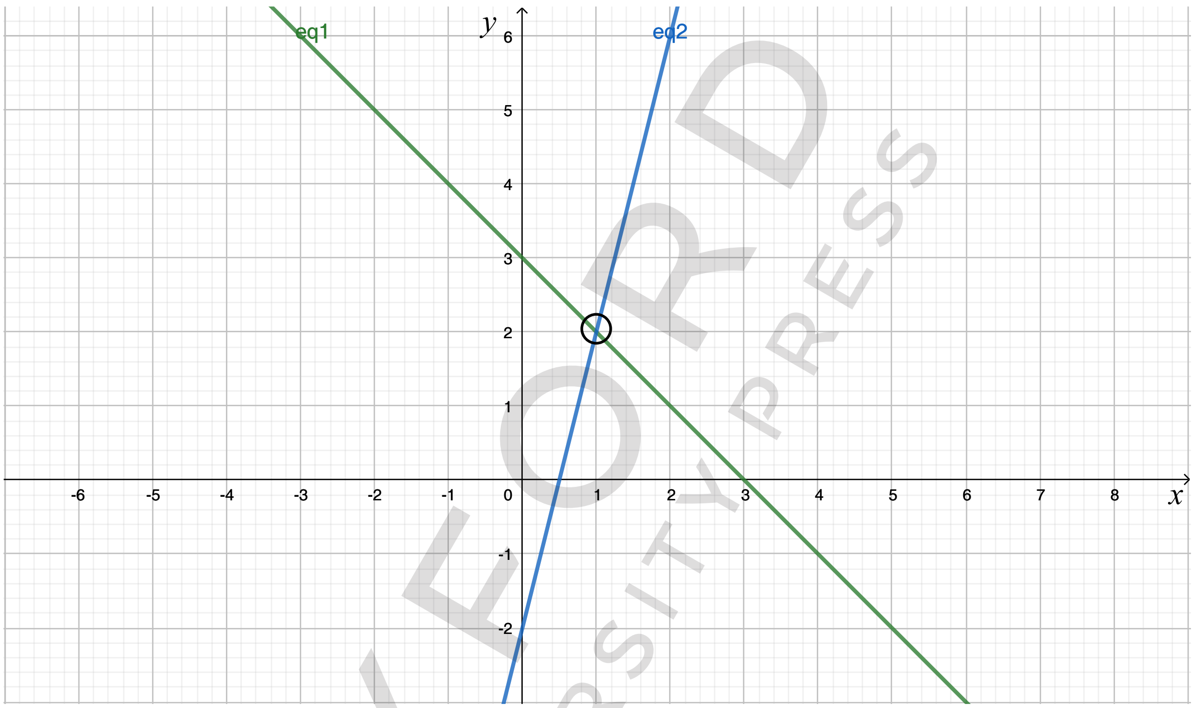
Now plot these pints on a coordinate plane (graph sheet).
Join all points to form a straight line and name it as equation 1 on the graph.
Now, follow same steps for equation 2.

$4x - y = 2$

$y = 4x - 2$

x	− 2	− 1	0	1	2
y	−10	−6	−2	2	6

Plot the points and draw line for equation. Label it equation 2



Encircle the point of intersection of the two lines and note down the values of x and y.
This point of intersection is the solution of the two equations i.e.
$x = 1$ and $y = 2$
or (1, 2) is the solution.

(ii) $x + y = 3$ ______ eq 1

$2x - 2y = 10$ ______ eq 2

Make table of values for equation 1.

$x + y = 3$

$y = 3 - x$

x	− 2	− 1	0	1	2
y	5	4	3	2	1

Make table of values for equation 2

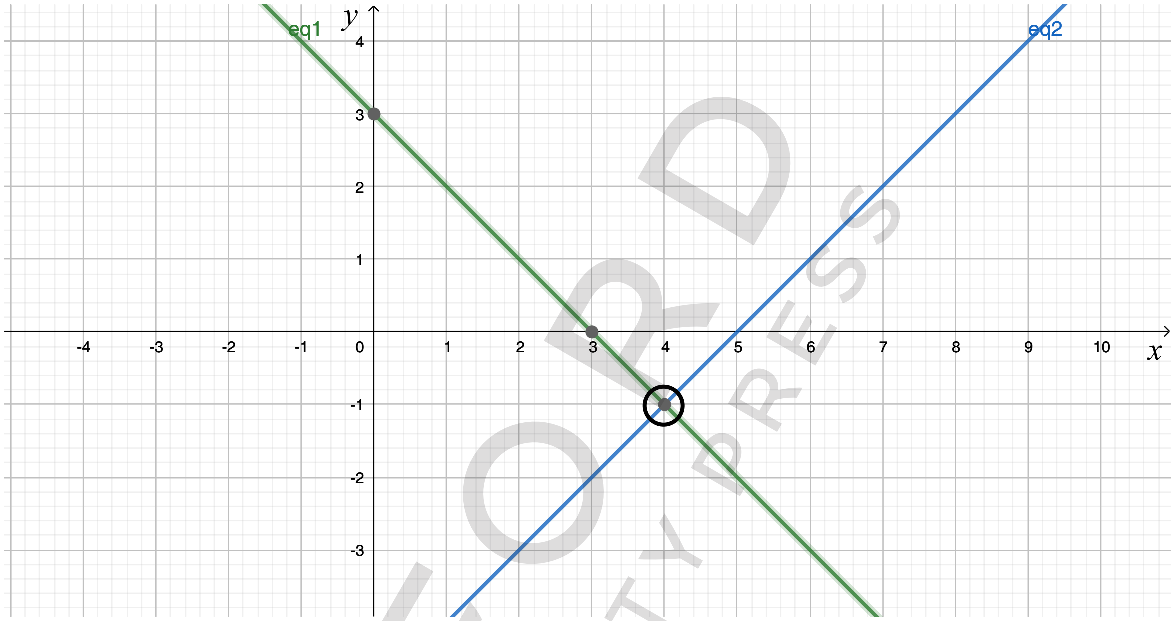
$2x - 2y = 10$

$x - y = 5$

$y = x - 5$

x	2	3	4	5	6	7
y	–3	–2	–1	0	1	2

Draw graphs for both the equations on same plane



Solution: $x = 4$, $y = -1$

(iii) $5x - y = 6$ ______ eq 1

$2x + y = 8$ ______ eq 2

Take equation 1

$5x - y = 6$

$y = 5x - 6$

x	0	1	2	3	4
y	–6	–1	4	9	74

Take equation 2

$2x + y = 8$

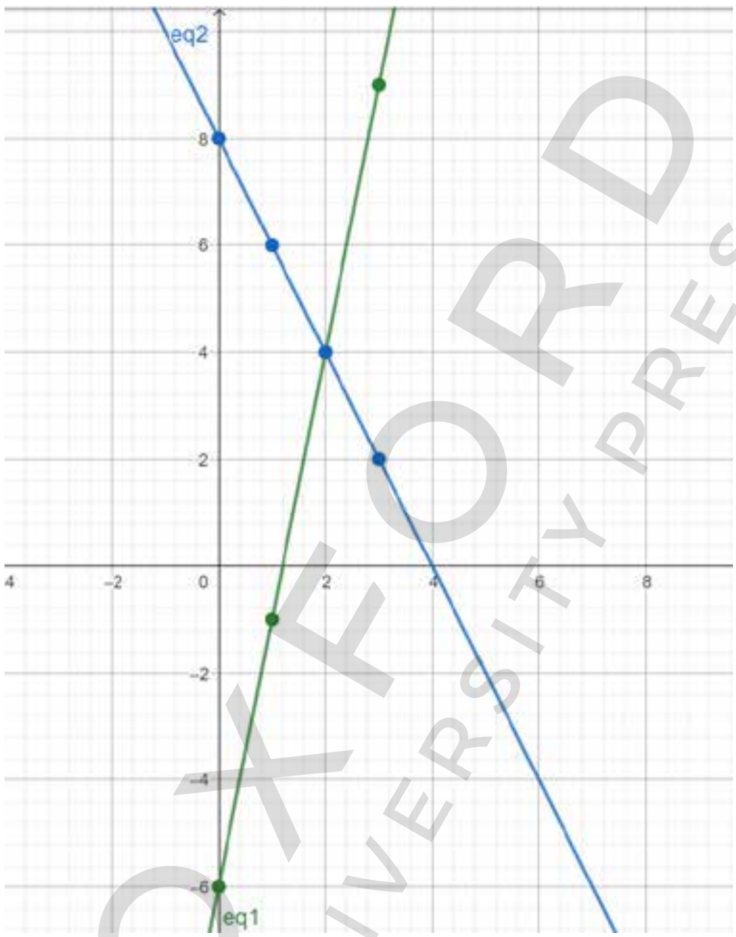
$y = 8 - 2x$

x	0	1	2	3	4
y	8	6	4	2	0

Draw the graphs for equations 1 and 2 on the same plane.

Helpful Hint

Choose scale as 1 cm = 5 units on both the axes.



Solution $x = 2, y = 4$

(iv) $3x - y = 2$ ______ eq 1

$9x - 3y = 6$ ______ eq 2

For equation 1

$3x - y = 2$

$y = 3x - 2$

x	–1	0	1	2	3
y	–5	–2	1	4	7

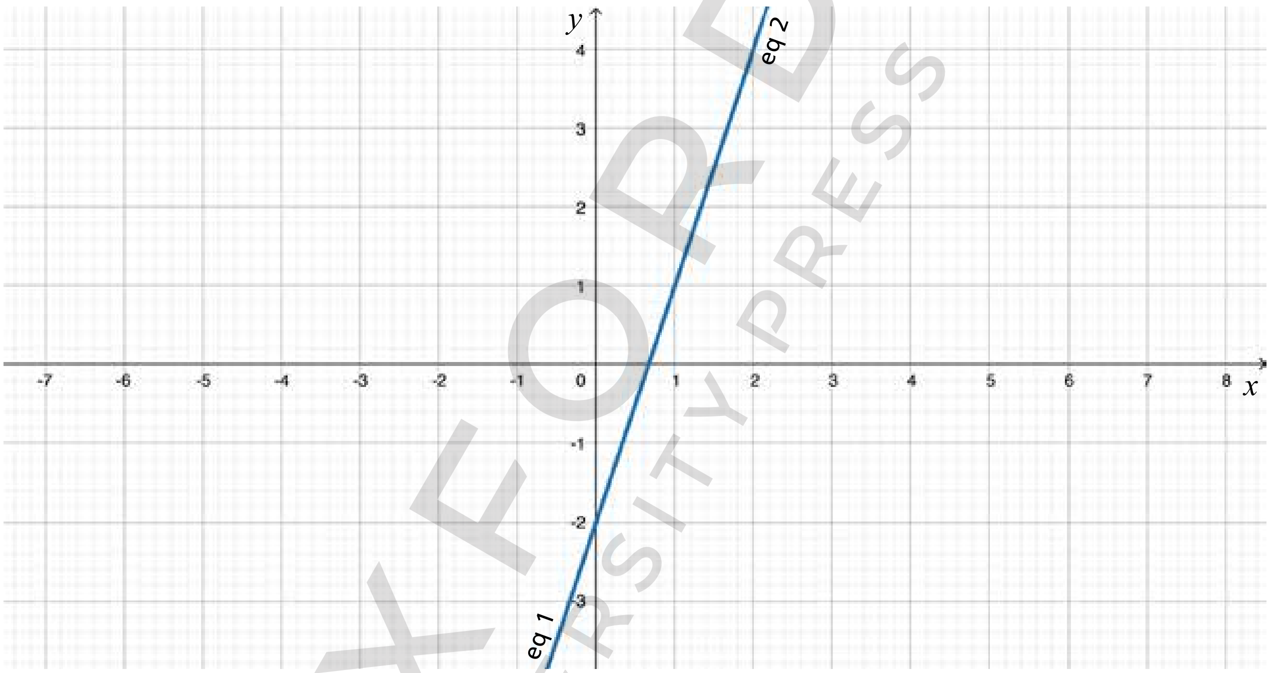
For equation 2

$9x - 3y = 6$

$3y = 9x - 6$

$y = \frac{9x - 6}{3} = 3x - 2$

x	–1	0	1	2	3
y	–5	–2	1	4	7



You can notice that the graph is same for both the equations. The lines coincide each other. Hence, the two equations have infinite number of solutions.

(v) $y - 2x = 3$ ______ eq 1

$2x - y = 5$ ______ eq 2

Make table of values for equation 1.

$y - 2x = 3$

$y = 2x + 3$

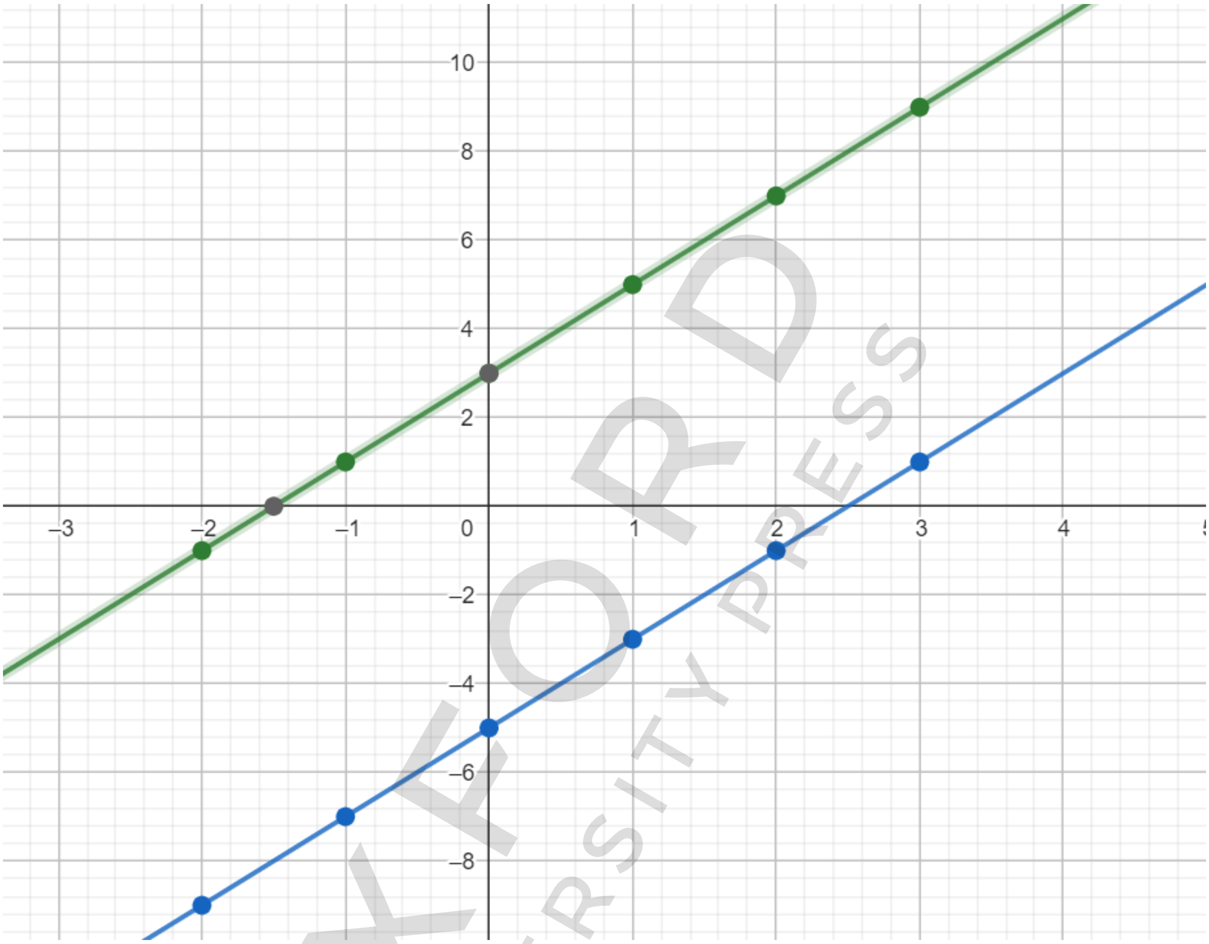
x – 2	–1	0	1	2	3
y – 1	1	3	5	7	9

For equation 2

$2x - y = 5$

$y = 2x - 5$

x – 2	–1	0	1	2	3
y – 9	–7	–5	–3	–1	1



Since, the two lines are parallel, they do not intersect each other at any point, hence, the two equations have no solution.

(vi) $x + y = 1$ ______ eq 1

$3x + y = 15$ ______ eq 2

Equation 1:

$x + y = 1$

$y = 1 - x$

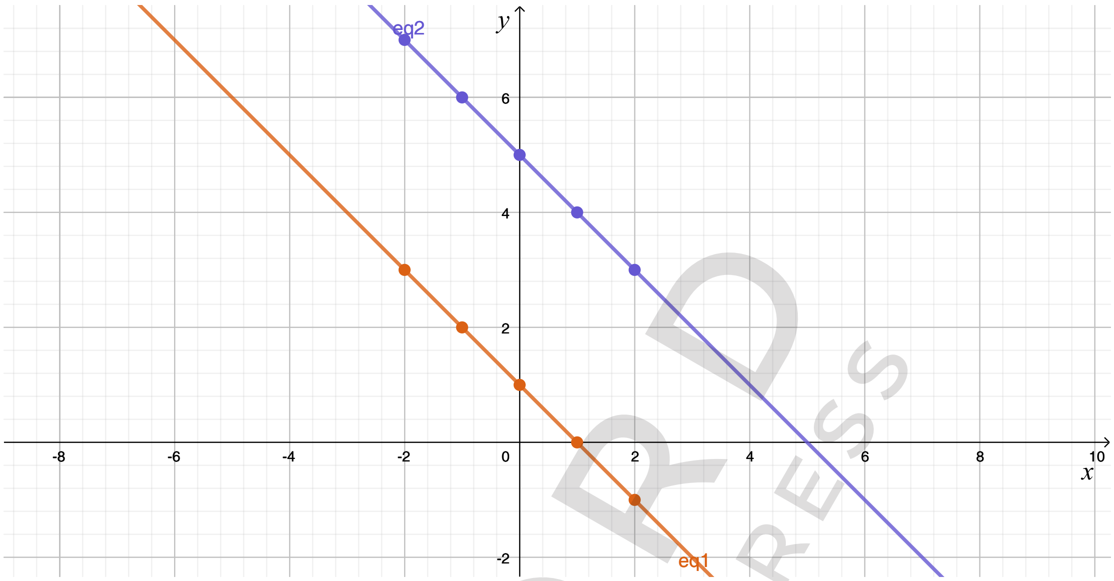
x	–2	–1	0	1	2
y	3	2	1	0	–1

For Equation 2: $3x + 3y = 15$

$x + y = 5$

$y = 5 - x$

x	–2	–1	0	1	2
y	7	6	5	4	3



Since, the two lines are parallel, they have no solution.

6. Let x and y be the numbers of days Ahmed and Manu each worked for, respectively.

$200x + 250y = 3900$ [divide both sides by 10]

$20x + 25y = 390$ —— (i)

$250y = 200x + 1100$

$250y - 200y = 1100$ [divide both side by 10]

$-20x + 25y = 110$ —— (i)

Add eq (i) and eq (ii)

$20x + 25y = 390$

$-20x + 25y = 110$

$50y = 500$

$y = \frac{500}{50} = 10$

Substitute $y = 10$ in eq (i)

$20x + 25(10) = 390$

$20x + 250 = 390$

$20x = 390 - 250 = 140$

$x = \frac{140}{20} = 7$

∴ Ahmed worked for 7 days and Many worked for 10 days.

Helpful Hint

Total payment = number of days × payment per day

Ahmed's payment = 200 × 7 = Rs 1400

Manu's payment = 250 × 10 = Rs 2500

$-20x + 25y = 110$ —— (i)

Add eq (i) and eq (ii)

$20x + 25y = 390$

$-20x + 25y = 110$

$50y = 500$

$y = \frac{500}{50} = 1$

7. Let x and y be the amounts in Anum's purse and money box, respectively.

$2x + y = 1700$ —— (i)

$3x + y = 2200$ —— (ii)

Subtract eq (i) from eq (ii)

$3x + y = 2200$

$2x + y = 1700$

$- \quad - \quad\quad -$

$x = 500$

Substitute in eq (i)

$2(500) + y = 1700$

$y = 1700 - 1000$

$y = 700$

$\therefore$ Anum has Rs 700 in her money box

8. Let x and y be the ages of Rehana and Sonia respectively.

   $x + y = 20$ —— (i)

   $x - y = 8$ —— (ii)

   add both the equations

   $2x = 28$

   $x = 14$

   Substitute in eq (i)

   $14 + y = 20$

   $y = 20 - 14 = 6$

   $\therefore$ Rehana is 14 years old and Sonia is 6 years old.

9. Let x and y be the cost of one cup of cappuccino and one cup of cold coffee, respectively.

   $4x + 4y = 1400$ —— (i)

   $4x = 3y$

   $4x - 3y = 0$ —— (ii)

   $7y = 1400$ Subtract eq (ii) from eq (i)

   $y = 200$

   Substitute in eq (ii)

   $4x - 3(200) = 0$

   $4x = 600$

   $x = \frac{600}{4}$

   $x = 150$

   $\therefore$ 1 cup of cappuccino costs Rs 150 and 1 cup of cold coffee costs Rs 200

   5 cups of cappuccino cost $5 \times 150 =$ Rs 750

   5 glasses of cold coffee cost

   $= 5 \times 200 =$ Rs 1000

   Hence, Abbas will pay

   Rs 1000 + Rs 750 = Rs 1750

Multiple Choice Questions 9

1. C
2. C
3. C
4. C
5. D

10 MENSURATION

Exercise 10A

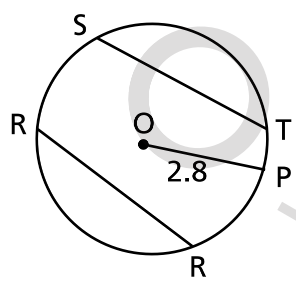
1. (i) diameter
   (ii) circumference
   (iii) segment
   (iv) interior
   (v) chord
2. (i) True
   (ii) False

   A diameter of a circle is a line segment passing through the centre of the circle and its end points lie on the circle. There can be many lines passing through the centre and its end points touching the boundary of the circle.

   (iii) False

   A radius is a straight line from the centre to the circumference of a circle, while the chord is straight line whose end points lie on the circle.

   (iv) True
   (v) True
3.



Steps of construction:

Step 1: Using a compass, draw a circle of radius 2.8 cm.

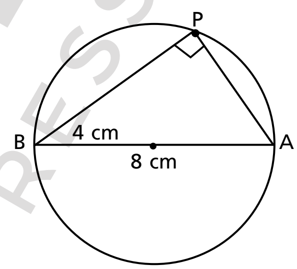
Step 2: O is the centre of the circle.

Step 3: Take points S and T on the boundary of the circle. Join S and T. $\overline{ST}$ is a chord.

Step 4: Take points Q and R on the boundary of the circle. Join Q and R $\overline{QR}$ is another chord.

$\therefore$ $\overline{ST}$ and $\overline{QR}$ are two chords not passing through the centre.

4.



Steps of construction:

Step 1: Draw a circle of diameter 8 cm or radius 4 cm.

Step 2: Draw the diameter $\overline{AB}$ of the circle.

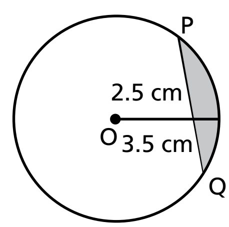
Step 3: Take another point P on the circle

Step 4: Join A to P and B to P.

Step 5: Measure $\angle$ APB m

$\angle$ APB = 90°

5.



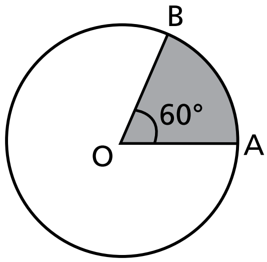
Steps of construction:

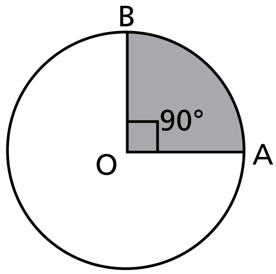
Step 1: Draw a circle with radius 3.5 cm.

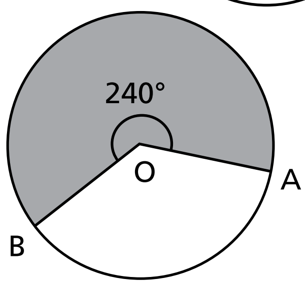
Step 2: Draw the chord $\overline{PQ}$ of length 2.5 cm touching the circle at P and Q.

Step 3: Shade the minor segment PQ. PQ is the required segment.

6.

(i) 

(ii) 

(ii) 

Steps of construction:

Step 1: Draw a circle of suitable radius with centre O.

Step 2: Take a point A on the circle and join A and O.

Step 3: Construct an angle of 60° on $\overline{OA}$, $\angle$AOB = 60°; area between $\overline{AO}$ and $\overline{OB}$ is the sector of the circle.

Repeat the same steps for (ii) and (iii)

7. Angle formed by minor arc, x = 120° radius of the circle, r = 6 cm

arc length $= \frac{x}{360} \times 2\,\pi\, x$

$= \frac{120}{360} \times 2 \times 3.14 \times 6$

$= 4 \times 3.14$

$= 12.56$ cm

8. Diameters = 36 cm

radius $= r = \frac{d}{2} = \frac{36}{2}$ cm

r = 18 cm

x = 60°

arc length $= \frac{x}{360} \times 2\,\pi\, r$

$= \frac{60}{360} \times 2 \times 3.14 \times 18$

$= 6 \times 3.14$

$= 18.84$ cm

9. Angle formed by minor arc:

$x = 360 - 110°$

$x = 250°$

arc length $= \frac{250}{360} \times 2xr$

$= \frac{250}{360} \times 2 \times 3.14 \times 18$

$= 25 \times 3.14$

$= 78.5$ cm

Helpful Hint

Sum of angles of all minor and major sectors is equal to 360°. The complete rotation in a circle is 360° degrees.

10. (a) $x = 90°$

r = 16 cm

Sector area $= \frac{x}{360} \times \pi\, r^2$

$= \frac{90}{360} \times 3.14 \times 16^2$

$= 64 \times 3.14$

$= 200.96\ cm^2$

b. $x = 135°$

r = 8 m

Sector area $= \frac{x}{360} \times \pi\, r^2$

$= \frac{135}{360} \times 3.14 \times 8^2$

$= 24 \times 3.14$

$= 75.36\ m^2$

c. $x = 40°$

r = 18 m

Sector area $= \frac{x}{360} \times \pi\, r^2$

$= \frac{40}{360} \times 3.14 \times 18^2$

$= 36 \times 3.14$

$= 113.04\ m^2$

Exercise 10B

1. (i) In the given $\triangle ABC$,

$= 3$ cm

$= 5$ cm

$= ?$

Helpful Hint

Hypotenuse is always the side opposite to the right angle of a triangle.

According to Pythagoras' theorem:

$^2 = {}^2 + {}^2$

$^2 = 3^2 + 5^2$

$^2 = 9 + 25$

$^2 = 34$

$\sqrt{c^2} = \sqrt{34}$

$= 5.8$ cm

(ii) In the given $\triangle ABC$,

$= ?$

$= 6$ cm

$= 10$ cm

$10^2 = {}^2 + 6^2$

$100 = {}^2 + 36$

$100 - 36 = {}^2$

$^2 = 64$

$\sqrt{a^2} = \sqrt{64}$

$= 8$ cm

(iii) In the given $\triangle ABC$,

$= 2$ cm

$= 6$ cm

$= ?$

$^2 = {}^2 + {}^2$

$^2 = 2^2 + {}^2$

$^2 = 4 + 36$

$^2 = 40$

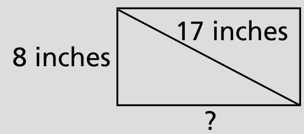
$\sqrt{c^2} = \sqrt{40}$

$= 6.32$ cm

2.

Helpful Hint

Draw the figure to understand its dimensions.



$= 8$ inches

$= ?$

$= 17$ inches

$^2 = {}^2 + {}^2$

$17^2 = 8^2 + {}^2$

$289 - 64 = {}^2$

$^2 = 225$

$= 15$

$\therefore$ The length is 15 inches

3. $= 8$ cm

$= 6$ cm

$= ?$

$^2 = {}^2 + {}^2$

$^2 = 8^2 + 6^2$

$^2 = 64 + 36$

$^2 = 100$

$\sqrt{c} = \sqrt{100}$

$= 10$ cm

4.

Helpful Hint

Hypotenuse is always the longest side of a right-angled triangle.

$= 13$ cm

$= 5$ cm

$= 12$ cm

LHS RHS

$^2 = {}^2 + {}^2$

To verity Pythagoras' theorem, right hand side of the theorem should be equal to its left hand side.

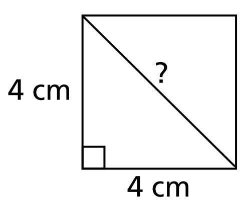
LHS $^2 = 13^2 = 169$

RHS $^2 + {}^2 = 5^2 + 12^2 = 25 + 144 = 169$

Since LHS = RHS

Pythagoras' theorem is verified.

5.



= 4 cm

= 4 cm

= ?

$^2 = {}^2 + {}^2$

$^2 = 4^2 + 4^2$

$^2 = 16 + 16$

$^2 = 32$

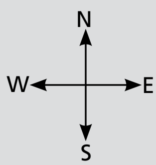
$= \sqrt{32}$

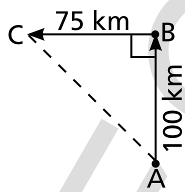
$= 5.66$ cm

6.

Helpful Hint

Cardinal directions





Lets A be the starting point of the truck then C is the find point where the truck stops. The distance between starting point and ending point is represented by hypotenuse.

According to Pythagoras' theorem:

$^2 = {}^2 + {}^2$

$^2 = 100^2 + 75^2$

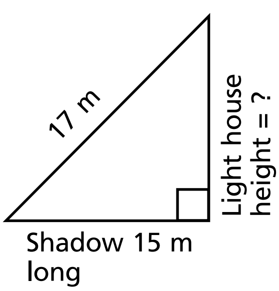
$^2 = 10000 + 5625$

$^2 = 15625$

$\sqrt{c^2} = \sqrt{15625}$

$= 125$ km

7.



= ?

= 15 m

= 17 m

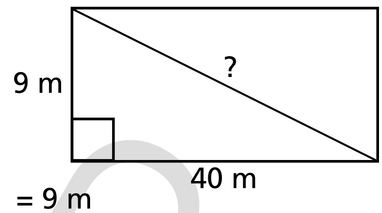
$^2 = {}^2 + {}^2$

$17^2 = {}^2 + 15^2$

$^2 = 289 - 225 = 64$

= 8 m

8.



= 9 m

= 40 m

= ?

$^2 = {}^2 + {}^2$

$^2 = 9^2 + 40^2$

$^2 = 81 + 1600$

$^2 = 1681$

= 41 m

9. = 8 m

= 15 m

= ?

$^2 = {}^2 + {}^2$

$^2 = 8^2 + 15^2$

$^2 = 64 + 225$

$^2 = 289$

= 17 m

Exercise 10C

1. (i) isosceles
   (ii) four
   (iii) regular polygon
   (iv) $\frac{4}{3}\pi^3$
   (v) 2 (two)
2. (i) False
   volume of a pyramid = $\frac{1}{3}$ * base area * height

(ii) True

(iii) True

(iv) False

Area of the base of cone is π^2

(v) True

3. Volume of a pyramid = $\frac{1}{3}$ * base area * height

$v = \frac{1}{3} * 18 * 18 * 26$

Helpful Hint

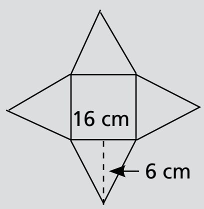
Since the base of the pyramid is square so the area of square is calculated.

$v = 2808\ m^3$

(ii) There are 5 surfaces of the given pyramid. There are four 4 triangles and one square.

Helpful Hint

You can draw net of the pyramid to calculate its surface area:



Total surface area = 4 * area of triangle + base area

$= 4 * \frac{1}{2} * 16 * 6 + 16 * 16$

$= 192 + 256$

$= 960\ cm^2$

4. Volume of pyramid = $12000\ cm^3$

$\frac{1}{3}$ * base area * height = 12000

$\frac{1}{3} * 20 * 20 * \ = 12000$

$= \frac{\cancel{12000}^{3} * 3}{\cancel{400}_{1}}$

= 90 cm

The height of the pyramid should be 90 cm to be filled with $12000\ cm^3$ of sand.

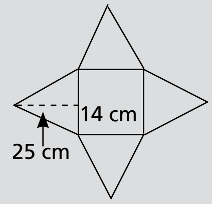
5. Volume = $\frac{1}{3}$ * base area * height

$= \frac{1}{3} * 14 * 14 * 24$

$= 1568\ cm^3$

Total surface area = 4 x area of triangle + base area

Helpful Hint



$= 4 * \frac{1}{2} * 14 * 25 + 14 * 145$

$= 700 + 196$

$= 896\ cm^2$

6. Volume = $\frac{1}{3}$ * base area * height

$= \frac{1}{3} * 24 * 24 * 12$

$= 2304\ cm^3$

7. Base area = $36000\ cm^2$

Volume = $7200\ cm^2$

Volume = $\frac{1}{3}$ * base area * height

$\frac{1}{3}$ * base area * height = $7200\ cm^3$

$\frac{1}{\cancel{3}} * \frac{1200}{\cancel{3600}} * \ = 7200$

$= \frac{7200}{1200}$

= 6 cm

8. Area of base of a cone = π^2 = 38.50 sq.cm

$\pi^2 = 38.50$

$\frac{22}{7}\ ^2 = 38.50$

$^2 = 38.50 \times \frac{7}{22} = 12.25$

$\sqrt{r^2} = \sqrt{12.25}$

= 3.5 cm

Given that, height = 3 times of radius

$\therefore$ = 3 = 3 × 3.5 = 10.5 cm

volume of cone = $\frac{1}{3}$ area of base × height

$= \frac{1}{3} \times 30.50 \times 10.5$

$= 134.75\ cm^3$

9.

= 3 cm

= 5 cm

total surface area = ?

total surface area = $\pi + \pi^2$

$= \pi\ (\ +\ )$

$= \pi \times 3\ (5 + 3) = \frac{22}{7} \times 3 \times 8$

$= \pi 24 = 24\pi\ cm^2$

or $24 \times \frac{22}{7} = 75.43\ cm^2$

10. = 12 cm

= 5 cm

volume of a cone = ?

volume $= \frac{1}{3}\ \pi^2$

$= \frac{1}{3} \times \frac{22}{7} \times (5)^2 \times 12$

$= \frac{6600}{21} = 314.3\ cm^3$

11. = 5 m

= 12 m

= 13 m

(i) area of canvas = curved surface area

$= \pi$

$= 3.14 \times 12 \times 13$

$= 489.84\ cm^2$

13. Surface area of sphere = 616 sq.cm

$4\ \pi^2 = 616$

$4 \times \frac{22}{7} \times {}^2 = 616$

${}^2 = \frac{616 \times 7}{4 \times 22} = 49$

= 7 cm

(ii) volume $= \frac{1}{3}\ \pi^2$

$= \frac{1}{3} \times 3.14 \times (12)^2 \times 5$

$= 753.6\ cm^3$

12. Surface area of sphere = 324π sq.cm

Volume = ?

Helpful Hint

To find out volume of sphere we need to find out the radius first.

Surface area of sphere = $4\ \pi^2$

$4\ \not{\pi}^2 = 324\not{\pi}$

${}^2 = \frac{324}{4} = 81$

$\sqrt{r^2} = \sqrt{81}$

= 9 cm

Volume of sphere $= \frac{4}{3}\ \pi^3$

$= \frac{4}{3}\ (9 \times 9 \times 9)\pi = \frac{4}{3} \times 729\pi$

$= 972\pi\ cm^3$

Volume of sphere with diameter 1 cm ,

= 0.5 cm

volume $= \frac{4}{3}\ \pi^3$

$= \frac{4}{3}\ \pi\ (0.5)^3$

$= 0.167\pi\ cm^3$

Helpful Hint

no. of small spheres = $\frac{\text{volume of big sphere}}{\text{volume of small sphere}}$

no. of small spheres = $\frac{324\not{\pi}}{0.1667\not{\pi}}$

= 5832

volume of the sphere = $\frac{4}{3} \pi\, ^{3}3$

$= \frac{4}{3} \pi \times 7 \times 7 \times 7 \times 3$

$= \frac{4}{\cancel{3}} \times \frac{22}{\cancel{7}} \times 7 \times 7 \times \cancel{7} \times \cancel{3}$

$= 4312 \text{ cm}^3$

Multiple Choice Questions

1. D

   Volume = $\frac{4}{3} \pi^3$

   $= \frac{4}{3} \times \frac{22}{7} \times (10)^3$

   $= 4190.48 \text{ cm}^3$

2. B

   Surface area = $4 \pi^2$

   $= 4 \times \frac{22}{7} \times 52$

   $= 314$

3. C

   Volume = $\frac{1}{3} \pi^2$

   $= \frac{1}{3} \times \frac{22}{7} \times (2) \times 4$

   $= 16.76 \text{ cm}^3$

4. D

   Surface area = $\pi + \pi^2$

   $= \pi\ (+) = \frac{22}{7} \times 5\ (5 + 7)$

   $= 188.57 \text{ cm}^2$

5. A

11 GEOMETRY: CONGRUENCE AND SIMILARITY

Exercise 11

1. (i) congruent
   (ii) four
   (iii) similar
   (iv) congruent
   (v) similar

2. (i) False

   Reason: Any two triangles are congruent, if their angles and corresponding sides are equal.

   (ii) True

   Reason: Triangle can be enlarged by increasing the length of tis sides, that will change the area of the triangle.

   (iii) False

   Reason: Any two triangles are congruent if two sides and included angle are equal.

   (iv) True

   Reason: Two triangles are similar if all the corresponding angles are equal and corresponding sides are in the same ratio.

   (v) False

   Reason: Congruent triangles are of the same shape and same size, while similar triangles have same shape but deferent size.

3. $m\angle A = m\angle Q$

   $m\angle B = m\angle R$

   $m\angle C = m\angle P$

   $m\overline{AB} = m\overline{QR}$

   $m\overline{BC} = m\overline{RP}$

   $m\overline{CA} = m\overline{PQ}$

4. (i) Congruent (SSS)
   (ii) Congruent (SAS)
   (iii) Not congruent
   (iv) Congruent (RHS)
   (v) Congruent (ASA)

5. (i) $m\overline{AC} = m\overline{XZ}$

   $x = 6$

   (ii) $m\angle RPQ = m\angle WUV$

   $x = 25^\circ$

   (iii) $m\angle XZY = m\angle RTS$

   $a = 45^\circ$

   (iv) $m\angle XZY = m\angle BAC$

   $x = 60^\circ$

   $m\angle XYZ = m\angle BCA$

   $y = 80^\circ$

6. (i) Similar, because the sides are in ratio 1:2

   (ii) Similar, because ratio of the sides is 1:3

   (iii) Not Similar

   The ratio of the length of sides is not equal.

   (iv) Similar, the ratios of the length of the sides are proportional as 1:2

Multiple Choice Questions 15

1. D
2. C

   The case of congruency is ASA.

3. D

   $m\angle S$ should be equal to $m\angle E$.

4. D
5. B

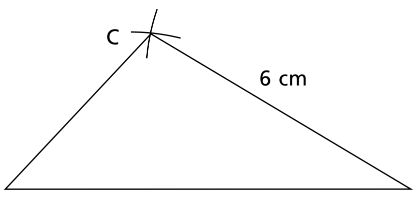
12 PRACTICAL GEOMETRY AND TRANSFORMATION

Exercise 12A

1.

Helpful Hint:
(i), (ii) and (iii) follow the same steps of construction.

(i) $m\overline{AB} = 8$ cm, $m\overline{BC} = 6$ cm, $\overline{CA} = 4$ cm



Steps of construction:

Step 1: Draw $\overline{AB} = 8$ cm.

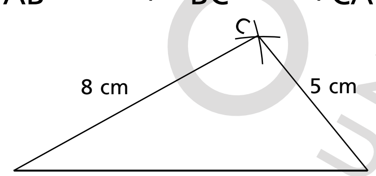
Step 2: With B as centre draw an arc with radius 6 cm.

Step 3: With A as centre, draw an arc with radius 4 cm.

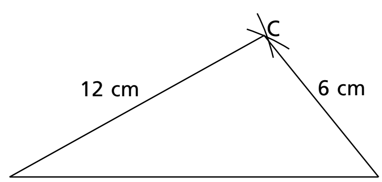
Step 4: Join C to A and B

Δ ABC is the required triangle.

(ii) $m\overline{AB} = 10$ cm, $m\overline{BC} = 5$ cm, $\overline{CA} = 8$ cm.

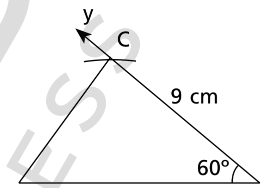


(iii) $\overline{AB} = 9$ cm, $m\overline{BC} = 6$ cm, $\overline{CA} = 12$ cm



Helpful Hint:
(iv), (v) and (vi) follow the same steps of construction.

(iv) $m\overline{AB} = 6$ cm, $m\overline{BC} = 9$ cm, $m\angle B = 60°$



Steps of construction:

Steps 1: Draw $m\overline{AB} = 6$ cm

Steps 2: At point B, draw $m\angle ABC = 60°$

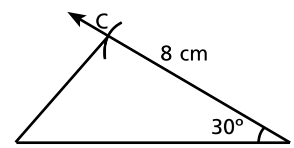
Step 3: With B as center, draw an arc of radius 9 cm to cut $\overline{BY}$ at C.

Step 4: Join A to C.

Δ ABC is the required triangle.

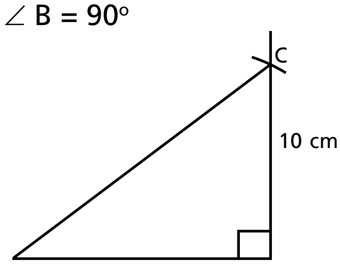
(v) $m\overline{AB} = 12$ cm, $m\overline{BC} = 8$ cm

$m\angle B = 30°$



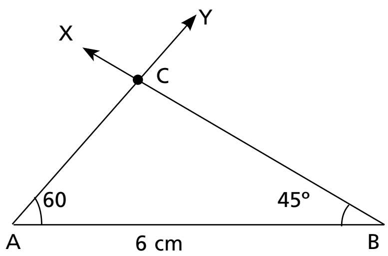
(vi) $m\overline{AB} = 8$ cm, $m\overline{BC} = 10$ cm,

$m\angle B = 90°$



Helpful Hint:
(vii) and (viii) follow the same steps of construction.

(vii) $m\overline{AB} = 6$ cm, $m\angle A = 60^\circ$, $m\angle B = 45^\circ$



Steps of construction:

Step 1: Draw $m\overline{AB} = 6$ cm BAY

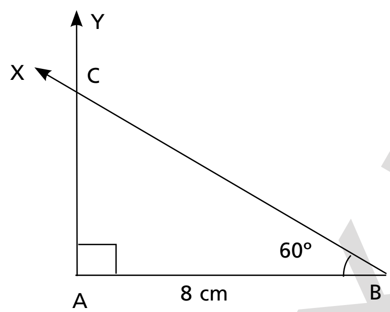
Step 2: At point A, draw $m\angle BAY = 60^\circ$

Step 3: At point B, draw $m\angle ABX = 45^\circ$

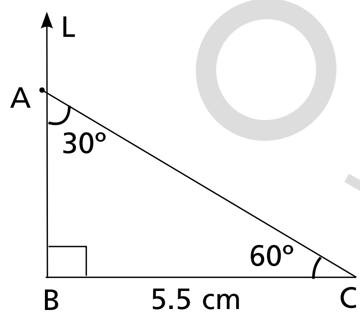
Step 4: Mark point C at the intersection of $\overline{AY}$ and $\overline{BX}$.

Δ ABC is the required triangle.

(viii) $m\overline{AB} = 8$ cm, $m\angle A$ 90°, $m\angle B = 60^\circ$



(ix) $m\overline{BC} = 5.5$ cm, $m\angle A = 30^\circ$, $m\angle B = 90^\circ$



Steps of construction:

Step 1: Draw $\overline{BC} = 5.5$ cm

Step 2: Construct an angle of 90° at B.

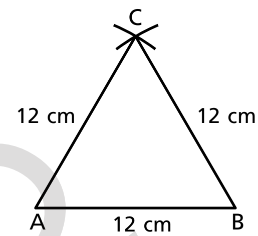
Step 3: Since sum of the angles in a triangle is 180°, i.e $\angle B + \angle A + \angle C = 180^\circ$, Make angle of 60° at C meeting $\overline{BL}$ at A.

Step 4: Measure $\angle$ A. It will be 30°.

Δ ABC is the required triangle.

2. Construct the following equilateral triangles:

(i) $m\overline{AB} = 12$ cm



Steps of construction:

Step 1: Draw $m\overline{AB} = 12$ cm

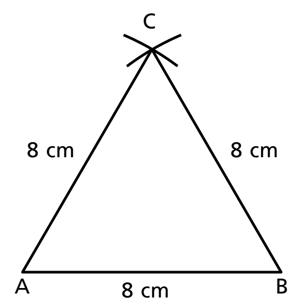
Step 2: With A as centre, draw an arc with radius 12 cm.

Step 3: With B as centre, draw another arc with radius 12 cm. So that it intersects the previous arc at C.

Step 4: Join C to A and B.

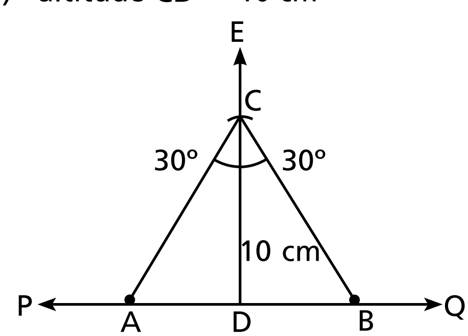
Δ ABC is the required triangle.

(ii) $m\overline{AB} = 8$ cm



Follow the same steps as given in Q4 (i)

(iii) altitude $\overline{CD} = 10$ cm



Steps of construction:

Step 1: Draw $m\,\overline{PQ}$ = 10 cm

Step 2: Mark a point D at the centre $\overline{AB}$.

Step 3: Draw a perpendicular $\overline{DE}$ on $\overline{AB}$.

Step 4: With D as centre, draw an arc with radius 10 cm intersecting the perpendicular at C.

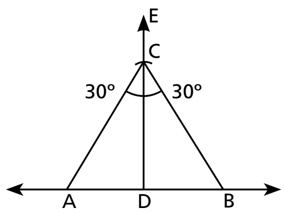
Step 5 At C draw two angles of 30° on either side of CD. $m\angle$ BCD and

$m\angle$ ACD = 30°

Step 6: Mark point A and B, where the arms of the angles interest $\overline{PQ}$.

Δ ABC is the required equilateral triangle.

(iv) altitude $\overline{CD}$ = 15 cm



Steps of construction:

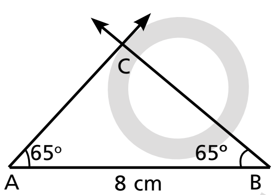
Follow the same steps as given in Q4 (iii).

3.

Helpful Hint:

In an isosceles triangle base angles are equal, therefore, $\angle$ B = 65°

(i) $m\,\overline{AB}$ = 8 cm, $m\angle$ A = 65°



Steps of construction:

Step 1: Draw m AB = 8 cm

Step 2: Construct $m\angle$ BAC = 65°

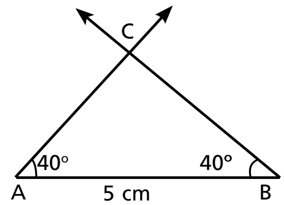
Step 3: Construct $m\angle$ ABC = 65°

Step 4: Extend the arms of the angles, so that they meet each other at point C.

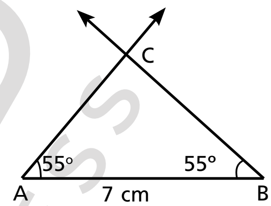
Δ ABC is the required triangle.

Follow the steps of construction given in Q5 (i), for (ii), (iii), (iv).

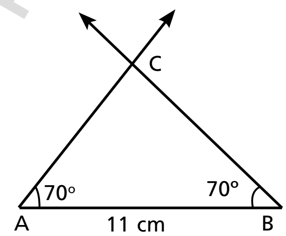
(ii) $m\,\overline{AB}$ = 5 cm, $m\angle$ A = 40°



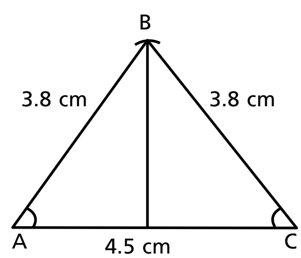
(iii) $m\,\overline{AB}$ = 7 cm, $m\angle$A = 70°



(iv) $m\,\overline{AB}$ = 11 cm, $m\angle$ A = 70°



4.



Steps of construction:

Step 1: Draw $\overline{AC}$ = 4.5 cm

Step 2: With centre C and radius 3.8 cm draw an arc above $\overline{AC}$.

Step 3: with centre A, draw another arc with the same radius, intersecting the previous arc with at point B.

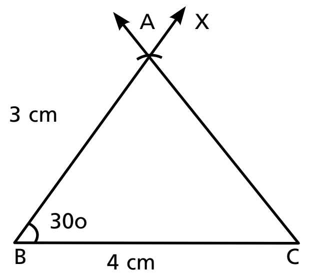
Step 4: Join A and B. Join B and C.

Δ ABC is the required triangle.

Since, ∆ ABC is an isosceles triangle, its base angle are equal.

∴ ∠ BAC = ∠ ACB

5.



Steps of construction:

Step 1: Draw $\overline{BC}$ = 4 cm

Step 2: At point B, draw *m* ∠ ABC = 30°

Step 3: With B as centre, draw an arc with radius 3 cm to cut $\overline{BX}$ at A.

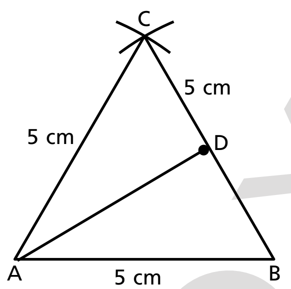
Step 4: Join A to C.

∆ ABC is the required triangle.

6. Follow the steps of construction as in Q2(i) taking each side = 5 cm

∆ ABC is an equilateral triangle.

Now, Take D as the centre of $\overline{BC}$.

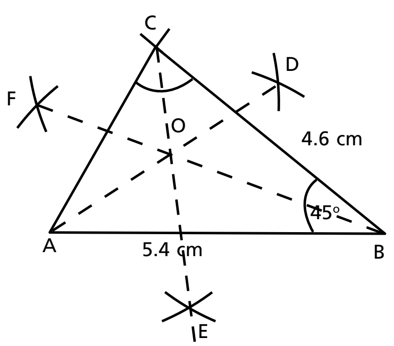


Join A and D.

Measure ∠ ADB.

m ∠ ADB = 90°

7.



Steps of construction:

Step 1: Draw $\overline{AB}$ = 5.4 cm

Step 2: Make an angle of 45° at point B.

Helpful Hint:
Using compass and ruler construct an angle of 90° and bisect it to get 45°.

Step 3: Using a compass cut $\overline{BC}$ = 4.6 cm.

Step 4: Join A and C.

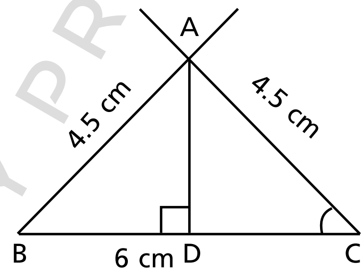
∆ ABC is the required triangle.

Step 5: Using a pair of compasses draw the angle bisectors at ∠ A, ∠ B, and ∠ C.

Step 6: Join A and D, B and F, C and E.

Angle bisectors $\overline{AD}$, $\overline{BF}$, and $\overline{CE}$ pass through the same point O.

8.



Steps of construction:

Step 1: Draw $\overline{BC}$ = 6 cm.

Step 2: With B as centre and radius 4.5 cm draw an arc above $\overline{BC}$.

Step 3: With C as centre and radius 4.5 cm draw another arc, intersecting the previous arc at point A.

Step 4: Join A to B and A to C.

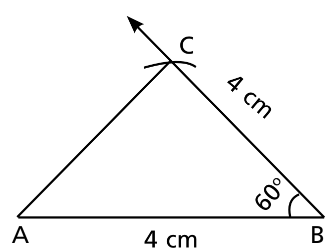
∆ ABC is the required triangle.

Step 5: Mark a point D in the centre of $\overline{BC}$.

Step 6: Join A and D. Measure ∠ ADB.

∠ADB = 90°

9.



Step of construction:

Step 1: Draw $\overline{AB}$ = 4 cm.

Step 2: Draw ∠ ABL = 60° at point B.

Step 3: With centre B and radius 4 cm draw an aare on $\overline{BL}$ at C.

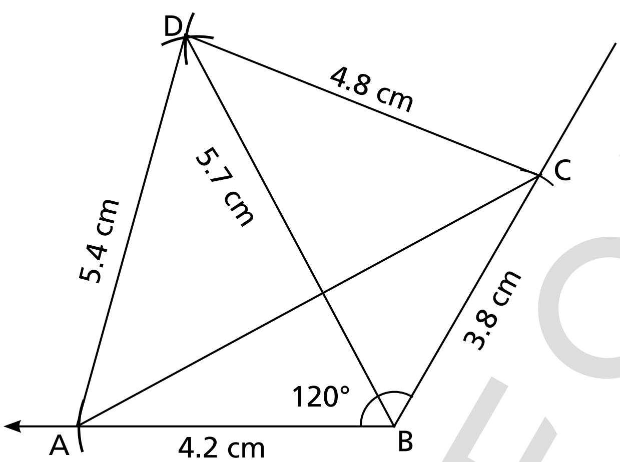
∠ ABC = 60°

Step 4: Join A to C.

∆ ABC is the required triangle.

Exercise 12B

1.



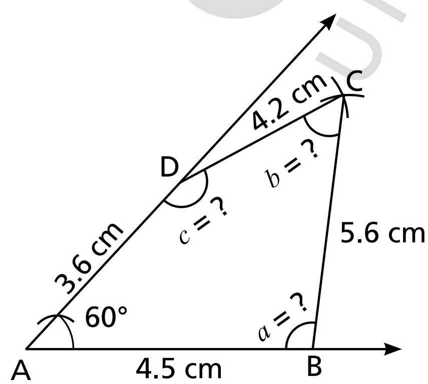
Measurement of the diagonals:

m $\overline{AC}$ 6.9 cm

m $\overline{BD}$ = 5.7 cm

Follow the steps given in the book (construction 1) according to the measurements given in the question.

2.



Follow the steps given in the book (construction 1) according to the measurements given in the question.

The other three angles are

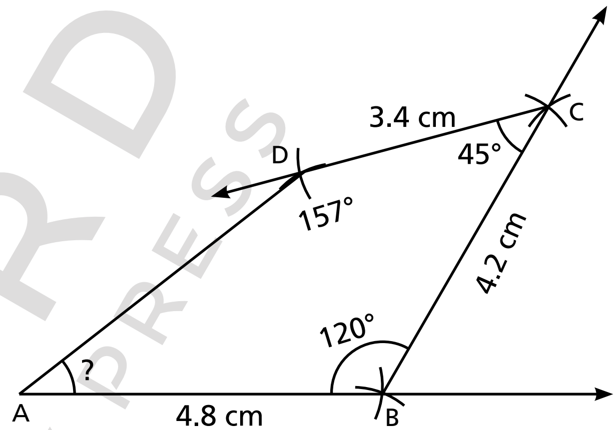
a = 97° , b = 47°, and c = 156°

∴ the sum of the angles is

97° + 47° + 156° = 300°

3. Let $\overline{AB}$ = 4.8 cm, $\overline{BC}$ = 4.2 cm , CD = 3.4 cm

then ∠ B = 120° , ∠ C = 45°

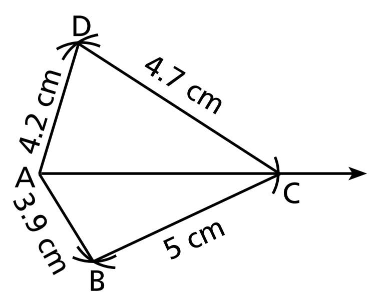


Follow the steps given in the book (construction 2) according to the measurements given in the question.

a = 38° , d = 157°

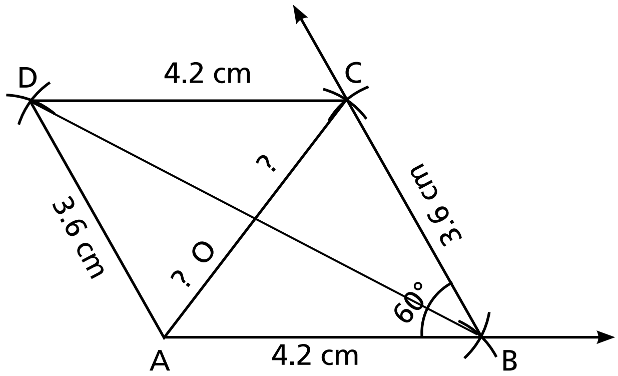
the sum is 39° + 157° = 195°

4. Let following be the sides of the quadrilateral $\overline{AB}$ = 3.9 cm, $\overline{BC}$ = 5 cm, $\overline{CD}$ = 4.7 cm, and $\overline{DA}$ = 4.2 cm



Follow the steps given in the book (construction 3) according to the measurements given in the question.

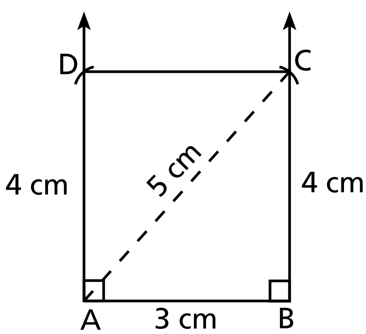
5.



Follow the steps given in the book (construction 11) according to the measurements given in the question.

m $\overline{AO}$ = 2 cm
m $\overline{CO}$ = 2 cm } yes, both are equal

6. Let m $\overline{AB}$ = 3 cm and m $\overline{BC}$ = 4 cm



Follow the steps given in the book (construction 9) according to the measurements given in the question.

Proof :

$AB^2 + BC^2 = AC^2$,

LHS

$AB^2 + BC^2 = 3^2 + 4^2 = 9 + 16 = 25$

RHS

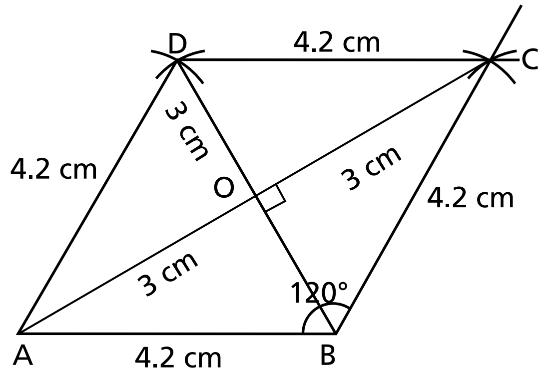
$AC^2 = 5^2 = 25$

Since LHS = RHS

Hence, it is proved that

$$AB^2 + BC^2 = AC^2$$

7.

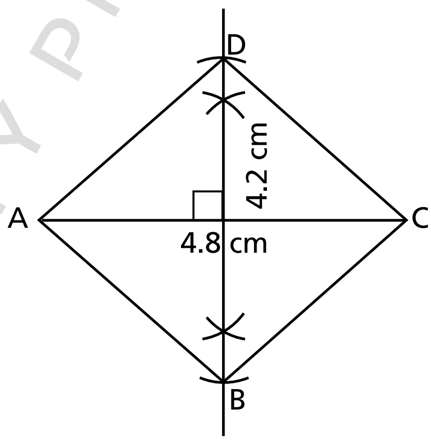


Follow the steps given in the book (construction 13) according to the measurements given in the question.

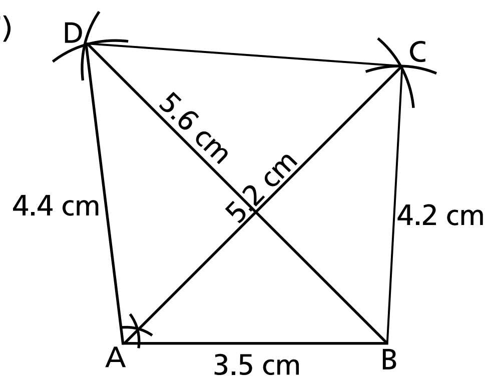
m $\overline{AO}$ = m $\overline{CO}$ = m $\overline{BO}$ = m $\overline{DO}$ = 3 cm

m ∠ BOC = 90°

8. Let the rhombus be ABCD, and its diagonals are $\overline{AC}$ and $\overline{BD}$

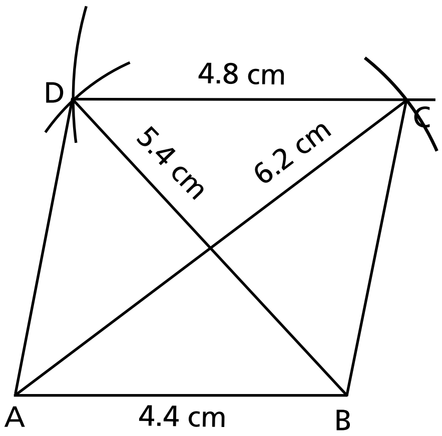


9. (i)



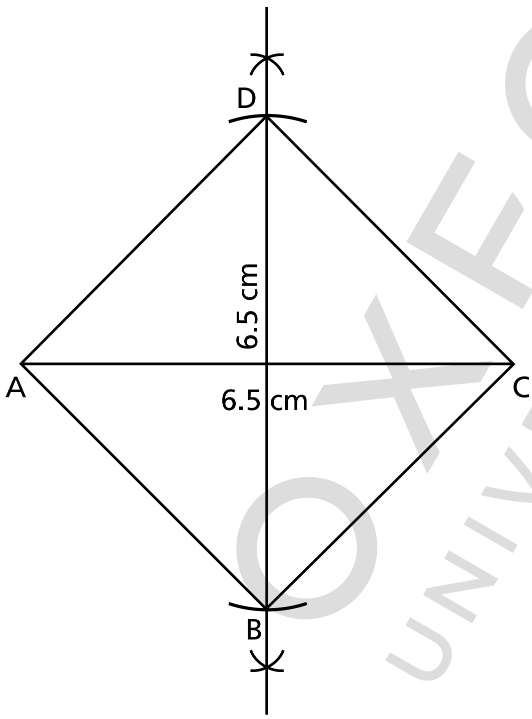
Follow the steps given in the book (construction 4) according to the measurements given in the question.

(ii)



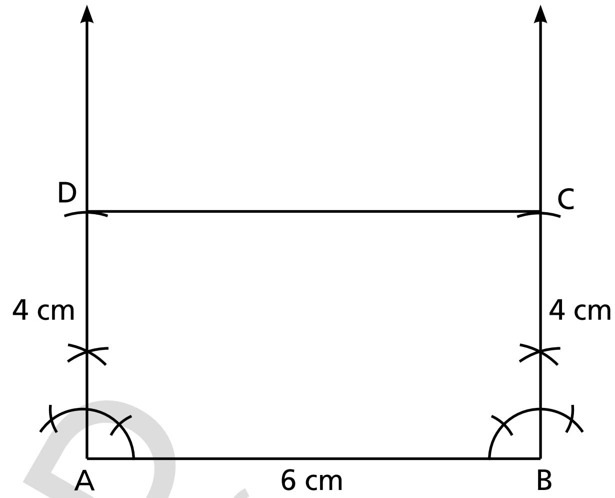
Follow the steps given in the book (construction 3) according to the measurements given in the question.

10.



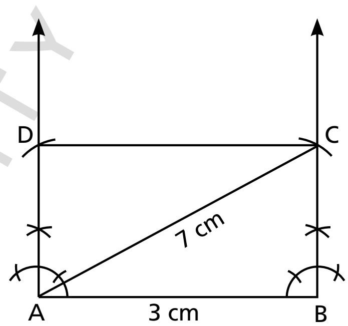
Follow the steps given in the book (construction 11) according to the measurements given in the question.

11.



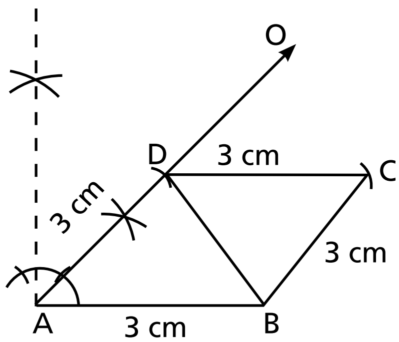
Follow the steps given in the book (construction 9) according to the measurements given in the question.

12. Let the rectangle be ABCD, and AB = 3 cm, AC = 7 cm



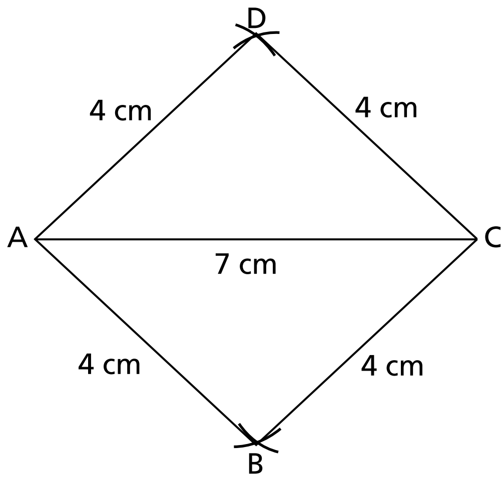
Follow the steps given in the book (construction 10) according to the measurements given in the question.

13.



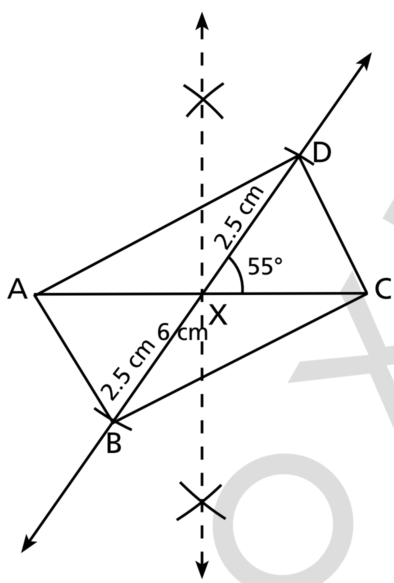
Follow the steps given in the book (construction 13) according to the measurements given in the question.

14.



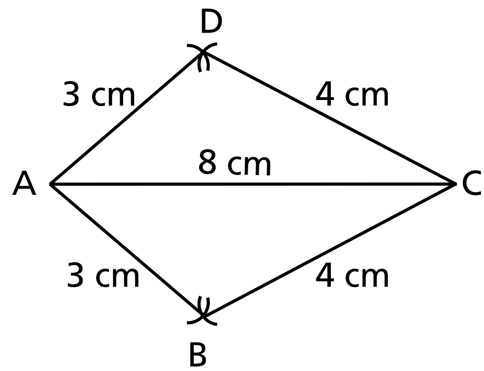
Follow the steps given in the book (construction 14) according to the measurements given in the question.

15.



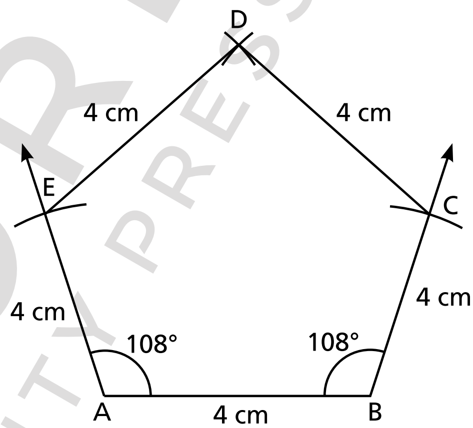
Follow the steps given in the book (construction 12) according to the measurements given in the question.

16.



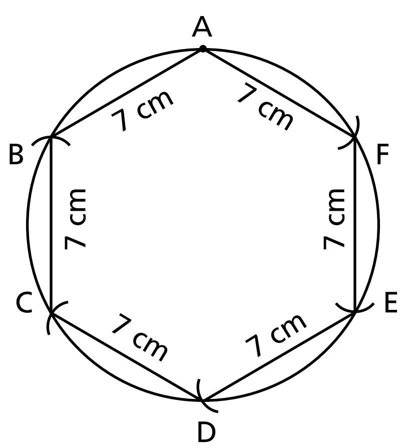
Follow the steps given in the book (construction 15) according to the measurements given in the question.

17.



Follow the steps given in the book (construction 17) according to the measurements given in the question.

18.



Follow the steps given in the book

(construction 17) according to the measurements given in the question.

Exercise 12C

1. (i) Since A is common vertex in both the images, the centre of rotation is A.

   To find the angle of rotation, you need to join corresponding vertices of both the images with centre of rotation. In this case they are alreading joined.

   Now find out the angle formed by BAB' and CAC'.

   Since the measure of angle is 90°, the angle of rotation is 90 anticlockwise.

   (ii) U is the centre of rotation. The angle of rotation is 90° clockwise.

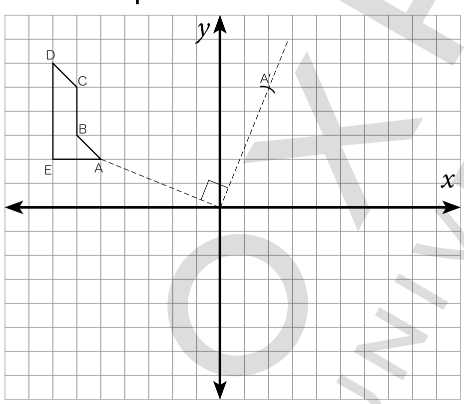
   (iii) B is the centre of rotation. The angle of rotation is 180° clockwise.

2. (i) Name the vertices of given shape as A,B, C, D, and E.

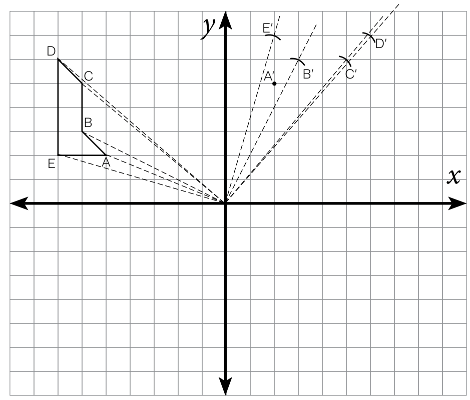
   Join A with origin O.

   Make an angle of 90° at point O. Mark an arc at the same distance as from O to A, on the line.

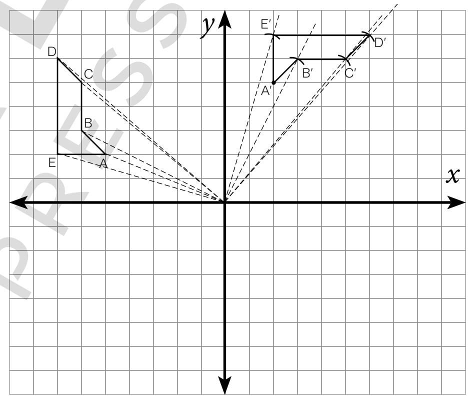
   This new point is A'



Now, repeat above steps for B' , C', D', and E'.



Join A' to B', B' to C', C' to D', D' to E', and E' to A' to draw the new rotated image.



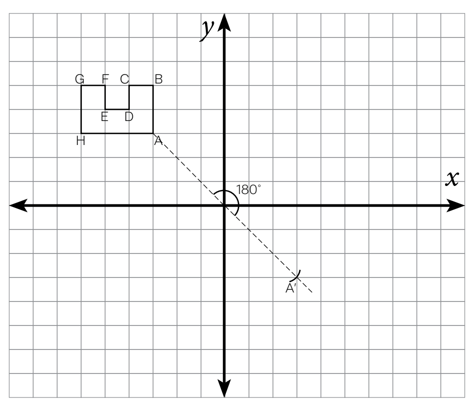
(ii) Name the vertices of image as A, B, C, D, E, F, G, and H.

Join A to origin O.

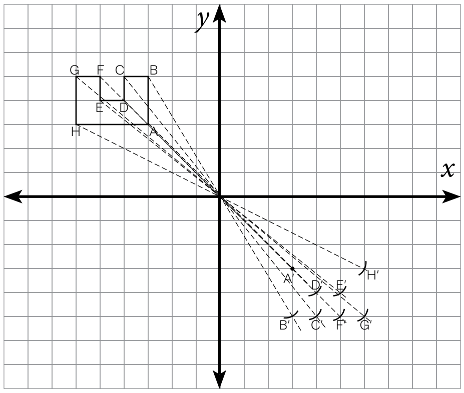
Make angle of 180° at O.

Make an arc at the same distance as form O to A on the line

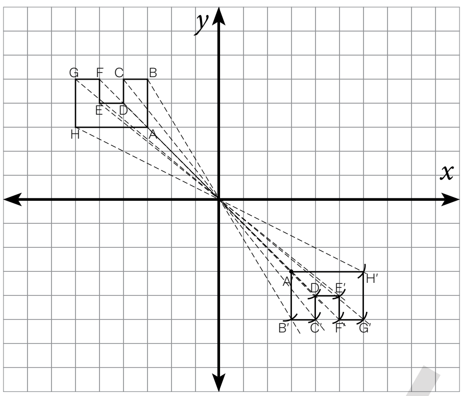
This new point is A'



Now repeat above steps for B', C', D', E', F', G', and H'.

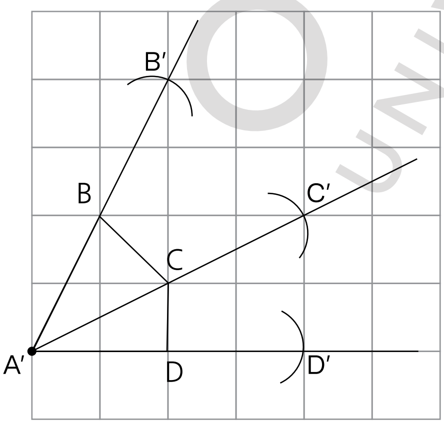


Join A' to B', B' to C', C' to D', D' to E', E' to F', F' to G', G' to H', and H' to A', to draw the new rotated image



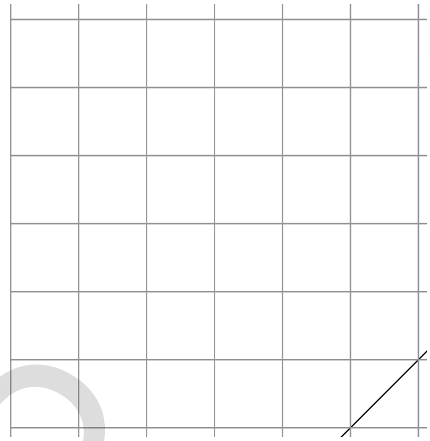
3. (i) The centre of enlargement is A. measure the length from A to B. Join A to B and produce the line further.

Since the scale factor is 2, multiply the measurement of length by 2 and mark a new point B' with the measurement.



Repeat above, steps to mark the point C' D'.

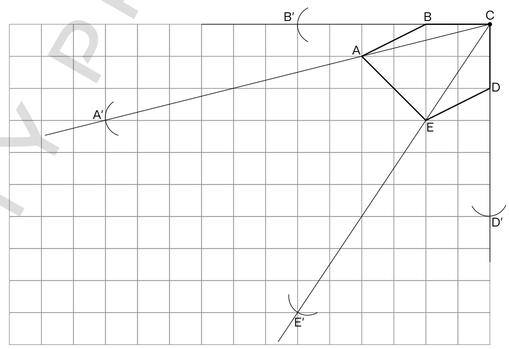
Joint A to B', B' to C', C' to D', and D' to A.



(ii) The centre of enlargement is C.

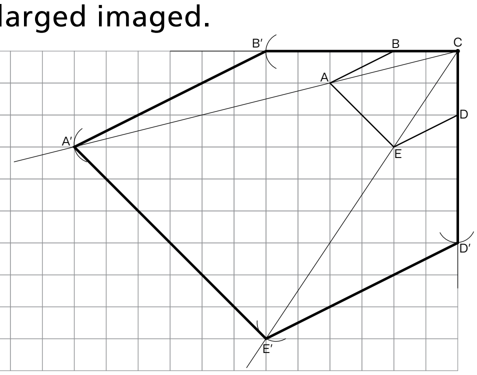
Join C to A and produce it further Measure length from C to A and multiply it by 3.

Make new point A' with new length from C.



Repeat above, steps to mark the point B', D', and E'.

Join A' to B', B' to C, C to D', D' to E', and E' to A' to get the new enlarged imaged.

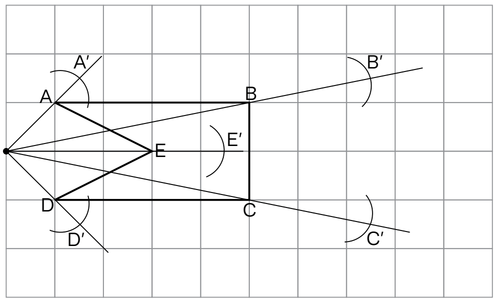


(iii) Given name 'P' to the centre of enlargement.

Join P to A and produce it further.

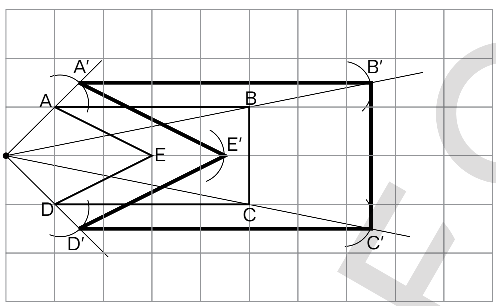
Measure the length from P to A and multiply it by 1.5.

Mark a point A' on the line with the calculated measurement.

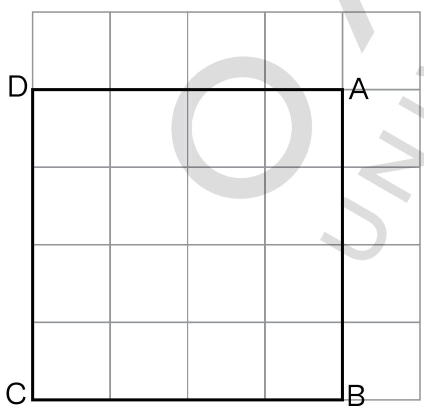


Repeat above steps to mark the points B', C', D', and E'.

Join A' to B', B' to C' , C', to D', D' to E', and E' to A' to make the enlarged image.



(iv) Draw guidelines from centre of enlargement to each vertex. Insert the same shape with bigger square only.

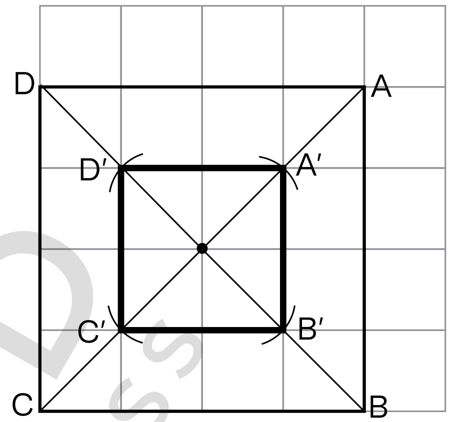


Measure the length from centre of enlargement to A and multiply it by 0.5.

Mark A′ with calculated measurement.

Repeat for B′, C′, and D′.

Join A to B′, B′ to C′, C′ to D′, and D′ to A′.



4. (i) ABC → AB′C′

The scale factor is 2 in this case

Since AB = BB′

hence AB′ = 2 AB

The scale factors is 2.

(ii) ABC → AB″ C″

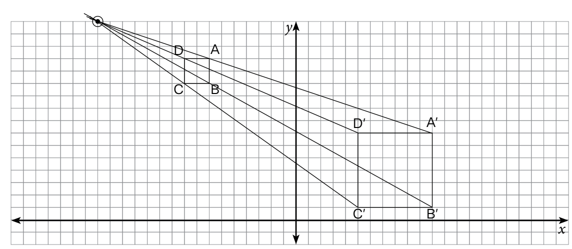
The scale factor is 3

(iii) AB′C′ → ABC

The scale factor is $\frac{1}{2}$

(iv) The scale factor is $\frac{2}{3}$

5. Join A to A′ , B to B′ , C to C′ , and D to D′.



The point where all these lines intersect each other is the centre of enlargement.

Name the given triangle as ∆PQR

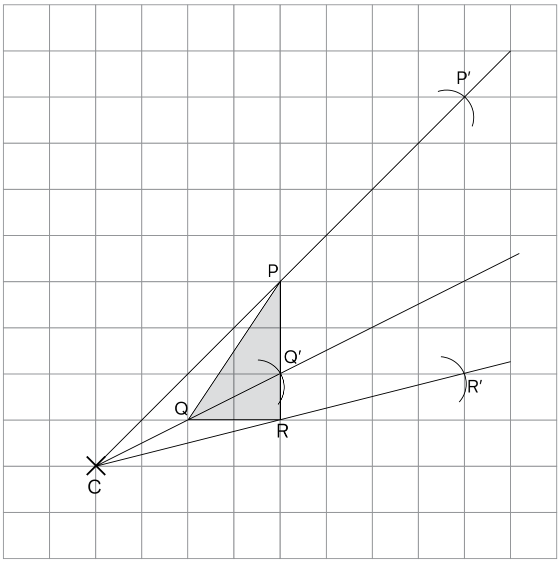
6. Draw guidelines from C to each vertex of the image.

Produce the lines further.

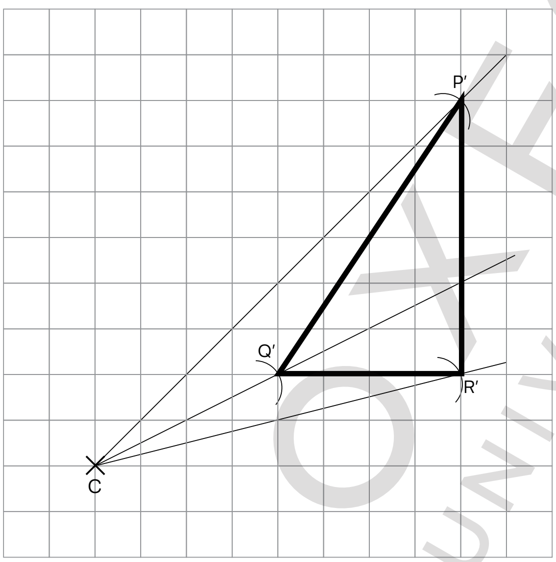
Measure length from C to P and multiply it by 2.

Make point P' with new length.

Repeat the steps for Q' and R' .



Join P' to Q', Q' to R', and R' to P' to get the enlarged image.



Multiple Choice Questions 12

1. D
2. B
   Since all side lengths are equal hence, triangle is equilateral.
3. A
   Since the measurement of one of three angles is 90° hence, the triangle is a right-angled triangle.
4. B
   None of the sides are equal in length.
5. A

13 DATA HANDLING

Exercise 13A

1. (i) Primary data (ii) class intervals
   (iii) range (iv) histogram
   (v) mode

2. (i) True
   (ii) False

Helpful Hint
The data needs to be arranged in ascending order to determine the median.

   (iii) True
   (iv) True
   (v) False

   Reason: Sum of fx is equal to the sum of the product of data value and its conquering

3. $42 - 36 = 6$
   $48 - 42 = 6$
   $54 - 48 = 6$
   $60 - 54 = 6$
   $\therefore$ class size is 6

4. Arrange data in ascending order.

   2, 5, ⑥, ⑦, 11, 17

   $$\text{Median} = \frac{6+7}{2} = \frac{13}{2} = 6.5$$

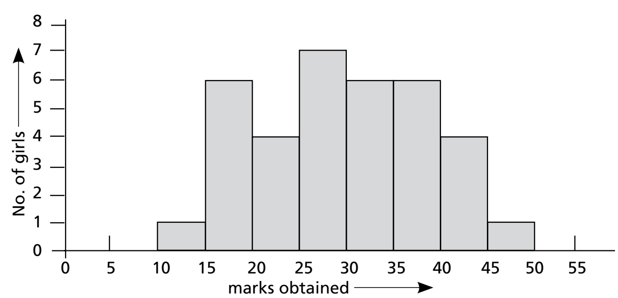
5. Since the lowest value is 68, start the class interval with 60.

   Class width = 20 (given in the question)

Class interval	Frequency
$60 \le x < 80$	5
$80 \le x < 100$	10
$100 \le x < 120$	9
$120 \le x < 140$	4
$140 \le x < 160$	2

6.

Class Interval	**Frequency**
$10 \le x < 15$	1
$15 \le x < 20$	6
$20 \le x < 25$	4
$25 \le x < 30$	7
$30 \le x < 35$	6
$35 \le x < 40$	6
$40 \le x < 45$	4
$45 \le x < 50$	1



7. Range = highest score – lowest score
   $= 45 - 18$
   $= 27$

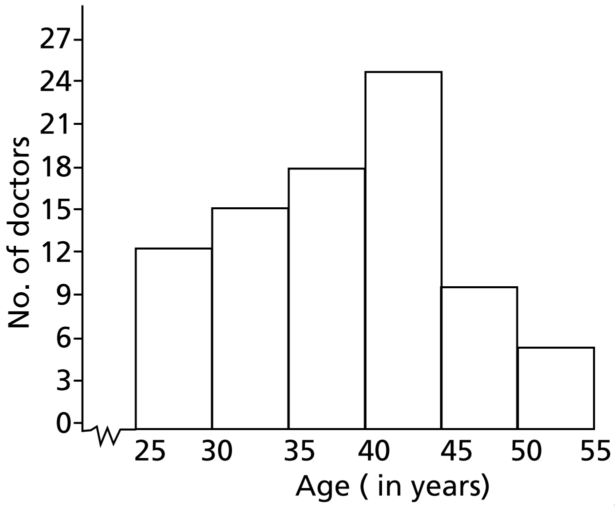
   $$\text{Mean} = \frac{\text{sum of values}}{\text{number of values}}$$

$$= \frac{40 + 35 + 24 + 18 + 32 + 22 + 45 + 38 + 30 + 20}{10}$$

$$= \frac{304}{10}$$

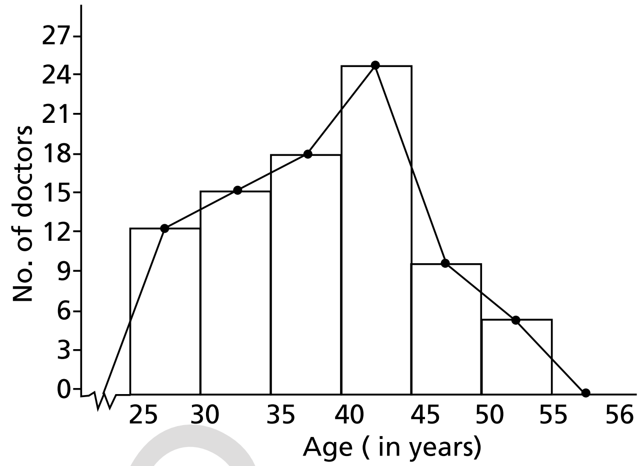
$$= 30.4$$

8. (a) Ages of doctors working in a city



(b)

Age (in years)	No. of doctor	Mid value
$25 \le x < 30$	12	$\frac{25 + 30}{2} = 27.5$
$30 \le x < 35$	15	$\frac{30 + 35}{2} = 32.5$
$35 \le x < 40$	18	$\frac{35 + 40}{2} = 37.5$
$40 \le x < 45$	25	$\frac{40 + 45}{2} = 42.5$
$45 \le x < 50$	10	$\frac{45 + 50}{2} = 47.5$
$50 \le x < 55$	5	$\frac{50 + 55}{2} = 52.5$



9. First ten natural numbers are

1 2 3 4 5 6 7 8 9 10

$$\text{Mean} = \frac{1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10}{10}$$

$$= \frac{55}{10}$$

$$= 5.5$$

10. First eight odd natural numbers are

1 3 5 7 9 11 13 15

$$\text{Mean} = \frac{1 + 3 + 5 + 7 + 9 + 11 + 13 + 15}{8}$$

$$= \frac{64}{8}$$

$$= 8.$$

11. Mean of 15 salaries = 12 500

No. of salaries = 15

managers salary = 36 500

Mean of 16 salaries = ?

Mean of 15 salaries = 12 500

$$\frac{\text{sum of 15 salaries}}{15} = 12\,500$$

Sum of 15 salaries = 12 500 × 15 = 187 500

$$\text{Mean of 16 salaries} = \frac{\text{sum of 16 salaries}}{16}$$

$$= \frac{\text{sum of 15 salaries + manager's salary}}{16}$$

$$= \frac{187\,500 + 36\,500}{16} = \frac{224\,000}{16}$$

$= \text{Rs } 14\,000$

12. Mean $= \frac{\Sigma fx}{f}$

$$= \frac{(4 \times 11) + (3 \times 13) + (7 \times 15) + (16 \times 18) + (4 \times 19) + (3 \times 20) + (2 \times 21) + (1 \times 22)}{4 + 3 + 7 + 16 + 4 + 3 + 2 + 1}$$

$$= \frac{44 + 39 + 105 + 288 + 76 + 60 + 42 + 22}{40}$$

$$= \frac{676}{40} = 16.9$$

13. Since class intervals are given instead of actual values, find and the mid value for each that will be served as x.

Kg	No. of students f	Mid value x
$28 < x \leq 30$	4	$\frac{28+30}{2} = 29$
$30 < x \leq 32$	8	$\frac{30+32}{2} = 31$
$32 < x \leq 34$	10	$\frac{32+34}{2} = 33$
$34 < x \leq 36$	5	$\frac{34+36}{2} = 35$
$36 < x \leq 38$	4	$\frac{36+38}{2} = 37$
$38 < x \leq 40$	1	$\frac{38+40}{2} = 39$

$$\text{Mean} = \frac{\Sigma fx}{f} = \frac{(4 \times 29) + (8 \times 31) + (10 \times 33) + (5 \times 35) + (4 \times 37) + (39 \times 1) + (1 \times 22)}{4 + 8 + 10 + 5 + 4 + 1}$$

$$= \frac{116 + 248 + 330 + 175 + 148 + 39}{32}$$

$$= \frac{1056}{32} = 33 \text{ kg}$$

14. Mean = $\frac{22 + 54 + 100 + 4 + 5 + 29 + 51 + 33 + 8 + 5 + 13 + 85 + 40 + 65 + 5 + 73 + 84}{17}$

$= \frac{676}{17} = 39.76$

To find median, arrange the data in ascending order

4, 5, 5, 5, 8, 13, 22, 29, 33, 40, 51, 54, 65, 73, 84, 85, 100

The value in the middle is 33.

Median = 33

Mode = 5 (5 is the most occurring value)

15. (i)

Marks	Frequency
$50 < x \leq 60$	3
$60 < x \leq 70$	5
$70 < x \leq 80$	8
$80 < x \leq 90$	8
$90 < x \leq 100$	6

(ii) $70 < x \leq 80$ and $81 < x \leq 90$

(iii) $70 < x \leq 80$

16. Mean = $\frac{16 + 16 + 12 + 14 + 13 + 15 + 13 + 13 + 16 + 12 + 15 + 10}{12}$

$= \frac{165}{12}$

= 13.75 mm

Helpful Hint

Arrange data in ascending order to determine the median.

10 12 12 13 13 13 14 15 15 16 16 16

There are two middle values, 13 and 14.

Median = $\frac{13 + 14}{2}$

$= \frac{27}{2}$

= 13.5 mm

mode = 13 and 16

17. (a) 10, 16, 12, 15, 8, 16, 10, 17, 12, 14

mean = $\frac{\Sigma x}{n}$

mean = $\frac{10 + 16 + 12 + 15 + 8 + 16 + 10 + 17 + 12 + 14}{10} = \frac{130}{10}$

mean = 13

variance

$$= \frac{(10-13)^2 + (16-13)^2 + (12-13)^2 + (15-13)^2 + (8-13)^2 + (16-13)^2 + (10-13)^2 + (17-13)^2 + (12-13)^2 + (14-13)^2}{10}$$

$$= \frac{(-3)^2 + (3)^2 + b\,(-1)^2 + (2)^2 + (-5)^2 + (3)^2 + (-3)^2 + (4)^2 + (-1)^2 + (1)^2}{10}$$

$$= \frac{9 + 9 + 1 + 4 + 25 + 9 + 9 + 16 + 1 + 1}{10}$$

$$= \frac{84}{10} = 8.4$$

Standard deviation $= \sqrt{8.4} = 2.9$

(b) 74, 72, 83, 96, 64, 79, 88, 68

Mean = $\frac{\Sigma x}{n}$

Mean $= \frac{74 + 72 + 83 + 96 + 64 + 79 + 88 + 68}{8}$

$$= \frac{624}{8} = 78$$

$$= \frac{(74-78)^2 + (72-78)^2 + (83-78)^2 + (96-78)^2 + (64-78)^2 + (79-78)^2 + (88-78)^2 + (68-78)^2}{8}$$

$$= \frac{(-4)^2 + (-6)^2 + (5)^2 + (18)^2 + (-14)^2 + (10)^2 + (-10)^2}{8}$$

$$= \frac{16 + 36 + 25 + 324 + 196 + 1 + 100 + 100}{8}$$

$$= \frac{798}{8} = 99.75$$

Standard deviation $= \sqrt{99.75}$

$= 9.99$

(c) mean = $\frac{\Sigma x}{n}$

$$= \frac{326 + 437 + 374 + 366 + 419 + 424}{6} = \frac{3246}{6}$$

$= 391$

variance $= \frac{(326-391)^2 + (437-391)^2 + (374-391)^2 + (366-391)^2 + (419-391)^2 + (424-391)^2}{6}$

$$= \frac{(-65)^2 + (46)^2 + (-17)^2 + (-25)^2 + (28)^2 + (33)^2}{6}$$

$$= \frac{4225 + 2116 + 289 + 625 + 784 + 1089}{6}$$

$$= \frac{9128}{6} = 1521.33$$

Standard deviation $= \sqrt{1521.33}$

$= 93.00$

Exercise 13B

1. i and iv are examples of independent events.

2. (i) P(yellow) $= \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$

$$= \frac{2}{8}$$

$$= \frac{1}{4}$$

(ii) P(odd) $= \frac{4}{8} = \frac{1}{2}$

(iii) Numbers greater than 2 = {3, 4, 5, 6, 7, 8}

P (greater than 2) $= \frac{6}{8} = \frac{3}{4}$

(iv) Numbers less than 9 = {1, 2, 3, 4, 5, 6, 7, 8}

P (less than 9) $= \frac{8}{8} = 1$

3. Experimental probability = $\frac{\text{Number of times event occurs}}{\text{Total number of trials}}$

$$= \frac{64}{126}$$

$$= \frac{32}{63}$$

4. Experimental probability = $\frac{\text{Number of times event occurs}}{\text{Total number of trials}}$

$$= \frac{128}{200}$$

$$= \frac{16}{25}$$

5. Experimental probability P(6) = $\frac{24}{60}$

$$= \frac{2}{5}$$

6. Experimental probability P(dislike orange juice) = $\frac{118}{200}$

$$= \frac{59}{100}$$

7. Sample space = {GH, GT, WH, WT, PH, PT}

Helpful Hint

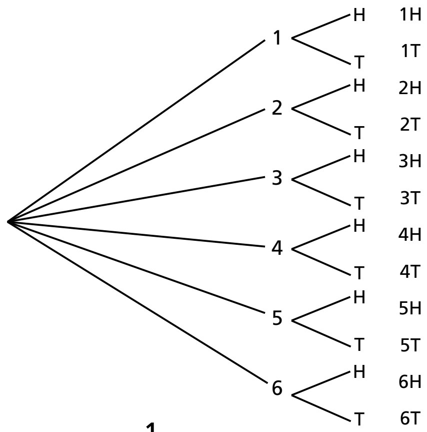
Tree diagram or possibility diagram can be used to list down the sample space.

8.

		Snacks			
		Muffin (M)	Brownie (B)	Crisps (Cr)	Pastry (P)
Drink	Tea (T)	TM	TB	TCr	TP
	Coffee (C)	CM	CB	CCr	CP
	Juice	JM	JB	JCr	JP

9. Peas Carrots
Peas Turnip
Peas Spinach
Carrots Turnip
Carrots Spinach
Turnip Spinach

10. (i)



(ii) P (3H) = $\frac{1}{12}$

(iii) P (tail and a prime number) = $\frac{3}{12}$ = $\frac{1}{4}$

11. (i) (4)(1), (4)(2), (4)(3), (4)(5), (5)(1)
(5)(2), (5)(3), (5)(5), (8)(1), (8)(2)
(8)(3), (8)(5)

(ii)

Helpful Hint

Possibility Diagram is a useful tool in such cases.

Box 2

+	1	2	3	5
4	5	6	7	9
5	6	7	8	(10)
8	9	(10)	11	13

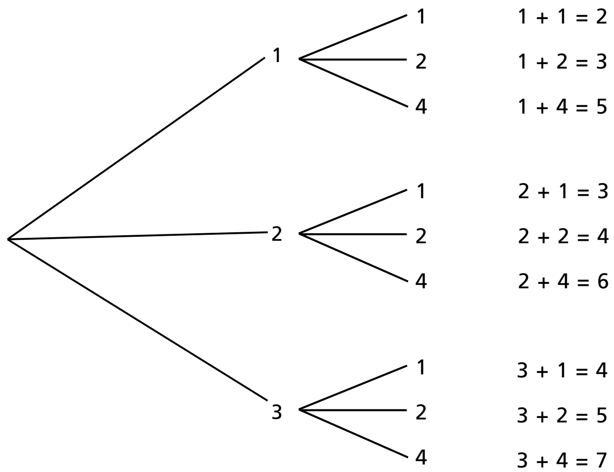
Box 1

Encircled results are the required outcomes

P (10) = $\frac{2}{12}$ = $\frac{1}{6}$

12. (i)

Bag 1 Bag 2 All Possible score



(ii) P(4) = $\frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$

$= \frac{2}{9}$

(iii) P (5) = $\frac{5}{9}$

13. (i)

Dice 1 (columns), Dice 2 (rows)

+	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18`
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

(ii) P (12) = $\frac{4}{36} = \frac{1}{9}$

(iil) P (10 or more) = $\frac{19}{36}$

(iv) P (even number) = $\frac{27}{36} = \frac{3}{4}$

14. (i)

Spinner 1 (columns), Coin (rows)

Score	1	2	3	4
Heads	2	3	4	5
Tails	2	4	6	8

(ii) (a) P (4) = $\frac{2}{8} = \frac{1}{4}$

(b) P(5 or more) = $\frac{3}{8}$

(c) P(Prime number) = $\frac{4}{8} = \frac{1}{2}$

15. (i)

Spinner 1 (columns), Spinner 2 (rows)

+	1	3	4	5
5	8	8	9	10
6	7	9	10	11
7	8	10	11	12

(ii) (a) P (8) = $\frac{2}{12} = \frac{1}{6}$

(b) P(odd number) = $\frac{5}{12}$

16. (i)

Bag 1 (columns), Bag 2 (rows)

Score	P	G	Y
2	$2 \times 2 = 4$	2 + 1 = 3	2
3	$3 \times 2 = 6$	3 + 1 = 4	3
4	$4 \times 2 = 8$	4 + 1 = 5	4
5	$5 \times 2 = 10$	5 + 1 = 6	5

(ii) P (multiples of 3) = $\frac{4}{15}$

17. Probability of getting red marble form bag 1 is is given by;

$P\ (red) = \frac{1}{4}$

Probability of getting yellow marble fro bag 2 is given by;

$P\ (yellow) = \frac{1}{4}$

$\therefore$P (red and yellow) = P)red) P(yellow)

$= \frac{1}{4}\ \ \frac{1}{4} = \frac{1}{16}$

18. $P(3) = \frac{1}{6}$

$P(even\ number) = \frac{3}{6} = \frac{1}{2}$

$\therefore$P(3 and even number)

= P(3) P(even number)

$= \frac{1}{6}\ \ \frac{1}{2} = \frac{1}{12}$

19. $P(red) = \frac{5}{20} = \frac{1}{4}$

$P(red) = \frac{8}{20} = \frac{2}{5}$

P (red or blue) = P)red) P(blue)

$= \frac{1}{4}\ \ \frac{2}{5}$

$= = \frac{5+8}{20} = \frac{13}{20}$

20. (i) $P(W) = \frac{12}{30}$

$P(B) = \frac{8}{30}$

P (W or B) = P(W) P(B)

$= \frac{12}{30}\ \ \frac{8}{30} = \frac{13}{20}$

$= \frac{5+8}{20} = \frac{20}{30} = \frac{2}{3}$

(ii) If 7 cows has no horns then

30 – 7 = 23 cows have horns.

$P(Cows\ with\ horns) = \frac{23}{30}$

(iii) Number of cows of different colours than black or white = 30 – 20 = 10

P(Cow with different colour)

$= \frac{10}{30} = \frac{1}{3}$

OR P(Cow with different colour)

= 1 – P(W or B)

$= 1 - \frac{2}{3} = \frac{1}{3}$

Multiple Choice Questions 13

(1) A

(2) B

(3) B

(4) D

(5) D