

NEW

# COUNTDOWN

THIRD EDITION

A Comprehensive Mathematics  
Series for Grade 7

7

Teaching Guide

OXFORD  
UNIVERSITY PRESS



# CONTENTS

<b>Introduction</b>	<b>iv</b>
<b>Curriculum</b>	<b>1</b>
✔ Strands and Benchmarks	
✔ Syllabus Matching Grid	
<b>Teaching and Learning</b>	<b>8</b>
✔ Guiding Principles	
✔ Mathematical Practices	
✔ Lesson Planning	
✔ Features of the Teaching Guide	
✔ Sample Lesson Plans	
<b>Assessment</b>	<b>70</b>
✔ Types of Assessment	
✔ Bloom's Cognitive Domains	
✔ Evaluation	

# Introduction

Welcome, users of the Countdown series. Countdown has been the choice of Mathematics teachers for many years. This Teaching Guide has been specially designed to help them teach mathematics in the best possible manner. It will serve as a reference book to streamline the teaching and learning experience in the classroom.

Teachers are entrusted with the task of providing support and motivation to their students, especially those who are at the lower end of the spectrum of abilities. In fact, their success is determined by the level of understanding demonstrated by the least able students.

Teachers regulate their efforts and develop a teaching plan that corresponds to the previous knowledge of the students and difficulty of the subject matter. The more well-thought out and comprehensive a teaching plan is, the more effective it is. This teaching guide will help teachers streamline the development of a lesson plan for each topic and guide the teacher on the level of complexity and amount of practice required for each topic. It also helps the teacher introduce effective learning tools to the students to complete their learning process.

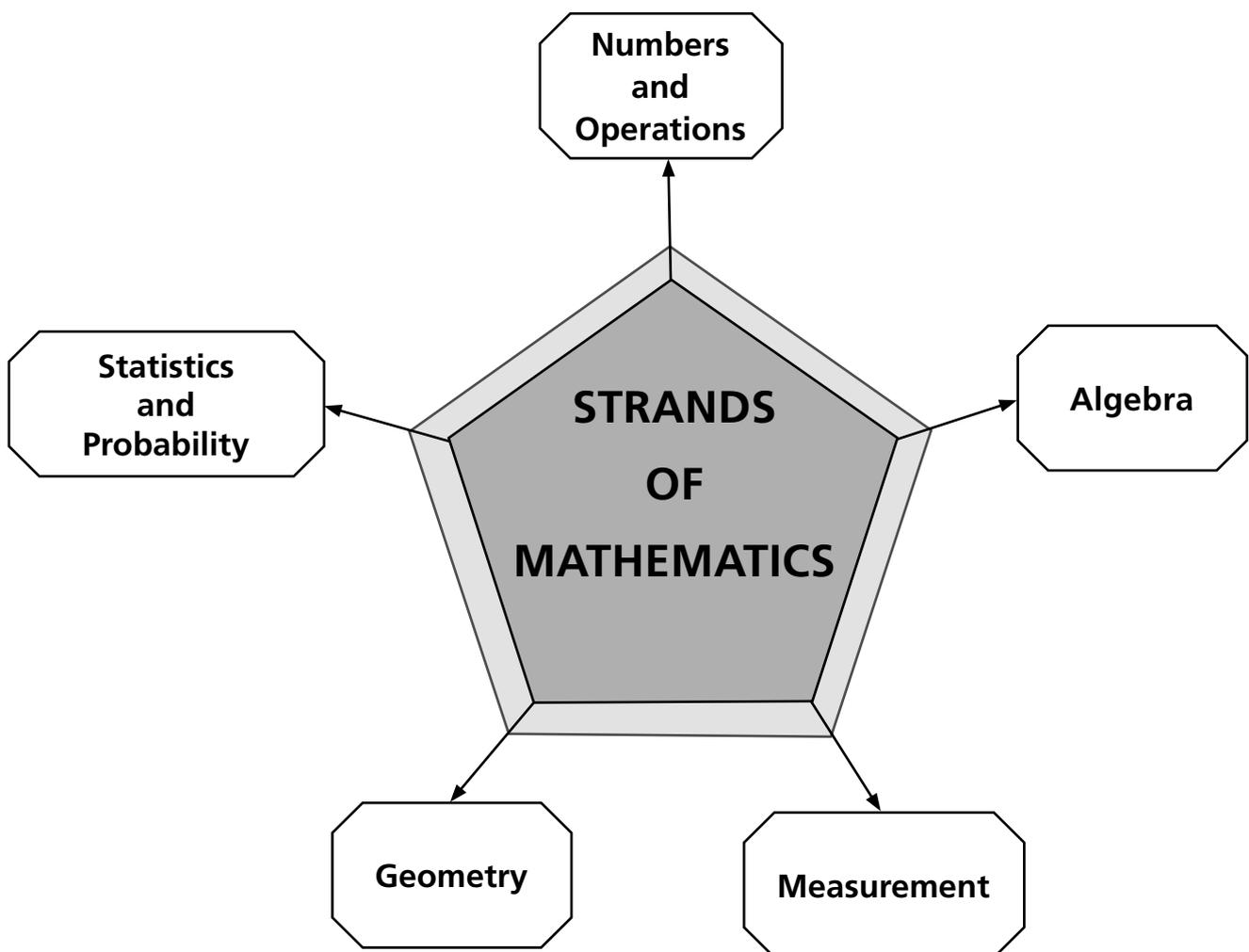
Shazia Asad

# Curriculum

## ► Strands and Benchmarks

Pakistan National Curriculum for Mathematics 2022

The Pakistan National Curriculum for Mathematics 2022 is based on these five strands:



## Towards greater focus and coherence of a mathematical programme

A comprehensive and coherent mathematical programme needs to allocate proportional time to all strands. A composite strand covers number, measurement and geometry, algebra, and information handling.

Each strand requires a focussed approach to avoid the pitfall of a broad general approach. If, say, an algebraic strand is approached, coherence and intertwining of concepts within the strand at all grade levels is imperative. The aims and objectives of the grades below and above should be kept in mind.

“What and how students are taught should reflect not only the topics that fall within a certain academic discipline, but also the key ideas that determine how knowledge is organised and generated within that discipline.”

William Schmidt and Richard Houang (2002)

### Strands and Bench marks of the Pakistan National Curriculum 2022

	Grade 6	Grade 7	Grade 8
<b>Strand</b>	<b>Benchmarks:</b>		
<b>Domain A: Numbers and Operations</b>	Students will be able use language, notation and Venn diagrams to describe sets and their elements, operate with real numbers, their properties and identify absolute value of real numbers, apply commutative ,associative and distributive laws on real numbers , compare, arrange and round off real numbers to required degree of accuracy, calculate factors, multiples, HCF and LCM, square roots and cube roots, ratio, rate, proportion, percentages, profit, loss, discount, Zakat, Ushr, commission, Taxes, insurance, partnership and Inheritance and apply all of these concepts in real life contexts.		
<b>Domain B:</b>	<b>Benchmarks:</b>		
<b>Algebra</b>	Students will be able to recognise and manipulate number patterns, use letters to represent numbers, expand, simplify, factorise, evaluate and manipulate algebraic expressions, use algebraic identities, interpret and plot graphs of linear equations, solve linear and simultaneous linear equations and linear inequalities and apply all these concepts in real life context.		
<b>Domain C:</b>	<b>Benchmarks:</b>		
<b>Measurement</b>	Students will be able to convert between different units of measure, solve problems involving speed, distance, time, area and perimeter of 2D shapes, surface area and volume of 3D shapes and apply the Pythagorean Theorem.		



<b>Domain D:</b>	<b>Benchmarks:</b>
<b>Geometry</b>	Students will be able to construct lines, angles of different measure, bisectors of angles, line segments, triangles and quadrilaterals, use the properties of triangles, quadrilaterals, polygons and circles to calculate unknown angles and lengths, apply facts of congruence and similarity and analyse and apply concepts of symmetry and transformations from two and three-dimensional perspectives.
<b>Domain E:</b>	<b>Benchmarks:</b>
<b>Statistics and Probability</b>	Students will be able to collect, classify and tabulate statistical data, interpret, construct and use statistical graphs, calculate and interpret measures of central tendency and solve problems using various concepts pertaining to Experimental and Theoretical Probability.

## Syllabus Matching Grid

SLOs	Domain A: Numbers and Operations	Covered in NCD 7
M-07-A-01	With increasing degree of challenge, use the concept of place value for whole numbers, integers, rational numbers and decimal numbers	Unit 3
M-07-A-02	Round off whole numbers, integers, rational numbers, and decimal numbers to a required degree of accuracy, significance or decimal places (up to 3 decimal places)	Unit 2
M-07-A-03	Use knowledge of rounding off to give an estimate to a calculation; to check the reasonableness of the solution	Unit 2
M-07-A-04	Recall HCF and LCM	Covered in Grade 6
M-07-A-05	Recall - Recognise, identify, and represent integers (positive, negative, and neutral integers) and their absolute or numerical value	Unit 2
M-07-A-06	Identify and represent (on a number line) rational numbers	Unit 2
M-07-A-07	Represent whole numbers, integers, and decimal numbers on a number line	Unit 3
M-07-A-08	Identify and convert between various types of fractions	Unit 3
M-07-A-09	Compare (using symbols $<$ , $>$ , $=$ , $\leq$ and $\geq$ ) and arrange (in ascending or descending order) whole numbers, integers, rational numbers and decimal numbers	Unit 2 and Unit 3
M-07-A-10	Verify associative and commutative properties of rational numbers	Unit 2
M-07-A-11	Verify associative, commutative, and distributive properties of rational numbers	Unit 2
M-07-A-12	Solve real-world word problems involving operations on rational numbers	Unit 2
M-07-A-13	Recognise the order of operations and use it to solve mathematical expressions involving whole numbers, decimals, fractions, and integers	Revision: Numbers
M-07-A-14	Calculate rate and average rate of quantities	Unit 5
M-07-A-15	Calculate increase and decrease in a ratio based on change in quantities	Unit 5
M-07-A-16	Explain and calculate direct and inverse proportion and solve real-world word problems related to direct and inverse proportion	Unit 5
M-07-A-17	Identify and differentiate between selling price, cost price, loss, discount, profit percentage, and loss percentage	Unit 6
M-07-A-18	Explain income tax, property tax, general sales tax, value-added tax, zakat, and ushr	Unit 6
M-07-A-19	Solve real-world word problems involving profit, loss, discount, commission, tax, zakat, and ushr	Unit 6 and Revision: Arithmetic
M-07-A-20	Recognise and calculate squares of numbers up to 3-digits.	Unit 4
M-07-A-21	Find the square root of perfect squares of (up to 3 digits) natural numbers, fractions, and decimals	Unit 4
M-07-A-22	Solve real-world word problems involving squares and square roots	Unit 4



M-07-A-23	Use language, notation, and Venn diagrams to represent different sets and their elements. (natural numbers, whole numbers, integers, even numbers, odd numbers, prime numbers)	Unit 1
M-07-A-24	Identify and differentiate between: <ul style="list-style-type: none"> <li>- subset and superset</li> <li>- proper and improper</li> <li>- equal and equivalent</li> <li>- disjoint and overlapping</li> </ul>	Unit 1
M-07-A-25	Describe and perform operations on sets (union, intersection, difference and complement)	Unit 1
M-07-A-26	Verify the following: $A \cap A' = \emptyset$ $A \cup A' = U$ $(A \cup B)' = A' \cap B'$ $(A \cap B)' = A' \cup B'$	Unit 1
<b>SLOs</b>	<b>Domain B: Algebra</b>	
M-07-B-01	Recall recognising simple patterns from various number sequences	Unit 7
M-07-B-02	Recall how to continue a given number sequence and find: <ul style="list-style-type: none"> <li>- term to term rule</li> <li>- position to term rule</li> </ul>	Unit 7
M-07-B-03	Find terms of a sequence when the general term ( $n^{\text{th}}$ term) is given	Unit 7
M-07-B-04	Solve real-life problems involving number sequences and patterns	Unit 7 and Revision: Algebra
M-07-B-05	Students will know Muhammad bin Musa Al- Khwarizmi as the founding father of Algebra	Unit 7
M-07-B-06	Recall variables as a quantity which can take various numerical values	Unit 7
M-07-B-07	Recognise open and close sentences, like and unlike terms, variable, constant, expression, equation, and inequality	Unit 7
M-07-B-08	Recognise polynomials as algebraic expressions in which the powers of variables are whole numbers	Unit 7
M-07-B-09	Identify a monomial, a binomial, and a trinomial as a polynomial	Unit 7
M-07-B-10	Add and subtract two or more polynomials	Unit 7 and Revision: Algebra
M-07-B-11	Find the product of: <ul style="list-style-type: none"> <li>- monomial with monomial</li> <li>- monomial with binomial/trinomial</li> <li>- binomials with binomial/trinomial</li> </ul>	Unit 7
M-07-B-12	Simplify algebraic expressions (by expanding products of algebraic expressions by a number, a variable or an algebraic expression) involving addition, subtraction, multiplication, and division	Unit 7



M-07-B-13	Explore the following algebraic identities and use them to expand expressions: $(a + b)^2 = a^2 + b^2 + 2ab$ $(a - b)^2 = a^2 + b^2 - 2ab$ $a^2 - b^2 = (a + b)(a - b)$	Unit 8 and Revision: Algebra
M-07-B-14	Factorise algebraic expressions (by taking out common terms and by regrouping)	Unit 9
M-07-B-15	Factorise quadratic expressions (by middle term breaking method)	Unit 9
M-07-B-16	Construct linear equations in two variables such as; $ax + by = c$ , where $a$ and $b$ are not zero	Unit 10
M-07-B-17	Recall solving linear equations in one variable	Unit 10
M-07-B-18	Introduction to Cartesian coordinate system	Unit 10
M-07-B-19	Plot the graph of the linear equation $ax + b = 0$ where $a \neq 0$ and of linear equations in two variables	Unit 10
M-07-B-20	Recognise and state the equation of a horizontal line and a vertical line	Unit 10
M-07-B-21	Find values of 'x' and 'y' from the graph	Unit 10
<b>SLOs</b>	<b>Domain C: Measurement</b>	
M-07-C-01	Convert different units of distance	Unit 14
M-07-C-02	Convert 12-hour clock to 24-hour clock and vice versa	Unit 14
M-07-C-03	Convert between different units of time and speed	Unit 14
M-07-C-04	Calculate arrival time, departure time, and journey time in a given situation (on the previous day and the next day)	Unit 14
M-07-C-05	Solve real-world word problems involving distance, time, and average speed	Unit 14
M-07-C-06	Differentiate between uniform and average speeds	Unit 14
M-07-C-07	Calculate the area and perimeter of the shaded/ unshaded region in composite shapes	Unit 14 and Revision Mensuration
M-07-C-08	Calculate the circumference and area of a circle	Unit 14
M-07-C-09	Calculate the surface area and volume of any simple 3-D shape including right prisms and cylinders	Unit 15
M-07-C-10	Convert between standard units of area ( $m^2$ , $cm^2$ , $mm^2$ , and vice versa) and volume ( $m^3$ , $cm^3$ , and $mm^3$ and vice versa)	Unit 14 and Unit 15
M-07-C-11	Solve real-life word problems involving the surface area and volume of right prisms and cylinders	Unit 15
<b>SLOs</b>	<b>Domain D: Geometry</b>	
M-07-D-01	Recognise quadrilaterals and their characteristics (parallel sides, equal sides, equal angles, right angles, lines of symmetry etc) Square, rectangle, parallelogram, rhombus, trapezium, and kite	Unit 12

M-07-D-02	Differentiate between convex and concave polygons	Unit 11
M-07-D-03	Translate an object and give precise description of transformation	Unit 12
M-07-D-04	Know that the perpendicular distance from a point to a line is the shortest distance to the line	Unit 12
M-07-D-05	Describe the properties of a circle; centre, radius, diameter, chord, arcs, major and minor arc, semi-circle, and segment of a circle	Unit 13
M-07-D-06	Calculate unknown angles in quadrilaterals using the properties of quadrilaterals (square, rectangle, parallelogram, rhombus, trapezium, and kite)	Unit 12
M-07-D-07	Understand the relationship between interior and exterior angles of polygons and between opposite interior and exterior angles in a triangle	Unit 11
M-07-D-08	Calculate the interior and exterior angles of a polygon and the sum of interior angles of a polygon	Unit 11
M-07-D-09	Recognise identity and draw lines of symmetry in 2D shapes and rotate objects using rotational symmetry; and find the order of rotational symmetry	Unit 12
M-07-D-10	Calculate unknown angles in a triangle	Unit 11
M-07-D-11	Construct different types of triangles (equilateral, isosceles, scalene, acute-angled, right-angled, and obtuse-angled)	Unit 12
<b>SLOs</b>	<b>Domain E: Statistics and Probability</b>	
M-07-E-01	<ul style="list-style-type: none"> <li>- Recognise drawing and interpreting of bar graphs, line graphs, and pie charts</li> <li>- Differentiate between a histogram and a bar graph</li> <li>- Construct and compare histograms for both discrete and continuous data with equal interval range</li> <li>- Select and justify the most appropriate graph(s) for a given data set and draw simple conclusions based on the shape of the graph</li> </ul>	Unit 16
M-07-E-02	Recognise the difference between discrete, continuous, grouped and ungrouped data	Unit 16
M-07-E-03	Calculate the mean, median, and mode for ungrouped data and the mean for grouped data and solve related real-world problems; Compare, choose, and justify the appropriate measures of central tendency for a given set of data	Unit 16
M-07-E-04	Construct frequency distribution tables for given data (i.e., frequency, lower class limit, upper class limit, class interval and mid-point) and solve related real-world problems	Unit 16
M-07-E-05	Explain and compute the probability of: certain events, impossible events, and complement of an event (including real-world word problems)	Unit 16

# Teaching and Learning

## Guiding Principles

1. Students explore mathematical ideas in ways that maintain their enjoyment of and curiosity about mathematics, help them develop depth of understanding, and reflect real-world applications.
2. All students have access to high quality mathematics programmes.
3. Mathematics learning is a lifelong process that begins and continues in the home and extends to school, community settings, and professional life.
4. Mathematics instruction both connects with other disciplines and moves toward integration of mathematical domains.
5. Working together in teams and groups enhances mathematical learning, helps students communicate effectively, and develops social and mathematical skills.
6. Mathematics assessment is a multifaceted tool that monitors student performance, improves instruction, enhances learning, and encourages student self-reflection.

## Principle 1

Students explore mathematical ideas in ways that maintain their enjoyment of and curiosity about mathematics, help them develop depth of understanding, and reflect real-world applications.

- The understanding of mathematical concepts depends not only on what is taught, but also hinges on the way the topic is taught.
- In order to plan developmentally appropriate work, it is essential for teachers to familiarise themselves with each individual student's mathematical capacity.
- Students can be encouraged to muse over their learning and express their reasoning through questions such as;
  - *How did you work through this problem?*
  - *Why did you choose this particular strategy to solve the problem?*
  - *Are there other ways? Can you think of them?*
  - *How can you be sure you have the correct solution?*
  - *Could there be more than one correct solution?*
  - *How can you convince me that your solution makes sense?*
- For effective development of mathematical understanding students should undertake tasks of inquiry, reasoning, and problem solving which are similar to real-world experiences.
- Learning is most effective when students are able to establish a connection between the activities within the classroom and real-world experiences.
- Activities, investigations, and projects which facilitate a deeper understanding of mathematics should be strongly encouraged as they promote inquiry, discovery, and mastery.
- Questions for teachers to consider when planning an investigation:
  - *Have I identified and defined the mathematical content of the investigation, activity, or project?*
  - *Have I carefully compared the network of ideas included in the curriculum with the students' knowledge?*
  - *Have I noted discrepancies, misunderstandings, and gaps in students' knowledge as well as evidence of learning?*

## Principle 2

All students have access to high quality mathematics programmes.

- Every student should be fairly represented in a classroom and be ensured access to resources.
- Students develop a sense of control of their future if a teacher is attentive to each student's ideas.

## Principle 3

Mathematics learning is a lifelong process that begins and continues in the home and extends to school, community settings, and professional life.

- The formation of mathematical ideas is a part of a natural process that accompanies pre-kindergarden students' experience of exploring the world and environment around them. Shape, size, position, and symmetry are ideas that can be understood by playing with toys that can be found in a child's playroom, for example, building blocks.
- Gathering and itemising objects such as stones, shells, toy cars, and erasers, leads to discovery of patterns and classification. At secondary level research data collection, for example, market reviews of the stock market and world economy, is an integral continued learning process. Within the environs of the classroom, projects and assignments can be set which help students relate new concepts to real-life situations.

## Principle 4

Mathematics instruction connects with other disciplines and moves toward integration of mathematical domains.

An evaluation of maths textbooks considered two critical points. The first was, did the textbook include a variety of examples and applications at different levels so that students could proceed from simple to more complex problem-solving situations?

And the second was whether algebra and geometry were truly integrated rather than presented alternately.

- It is important to understand that students are always making connections between their mathematical understanding and other disciplines in addition to the connections with their world.
- An integrated approach to mathematics may include activities which combine sorting, measurement, estimation, and geometry. Such activities should be introduced at primary level.
- At secondary level, connections between algebra and geometry, ideas from discrete mathematics, statistics, and probability, establish connections between mathematics and life at home, at work, and in the community.
- What makes integration efforts successful is open communication between teachers. By observing each other and discussing individual students teachers improve the mathematics programme for students and support their own professional growth.

## Principle 5

Working together in teams and groups enhances mathematical learning, helps students communicate effectively, and develops social and mathematical skills.

- The Common Core of Learning suggests that teachers 'develop, test, and evaluate possible solutions'.
- Team work can be beneficial to students in many ways as it encourages them to interact with others and thus enhances self-assessment, exposes them to multiple strategies, and teaches them to be members of a collective workforce.

- Teachers should keep in mind the following considerations when dealing with a group of students:
  - *High expectations and standards should be established for all students, including those with gaps in their knowledge bases.*
  - *Students should be encouraged to achieve their highest potential in mathematics.*
  - *Students learn mathematics at different rates, and the interest of different students' in mathematics varies.*
- Support should be made available to students based on individual needs.
- Levels of mathematics and expectations should be kept high for all students.

## Principle 6

Mathematics assessment in the classroom is a multifaceted tool that monitors student performance, improves instruction, enhances learning, and encourages student self-reflection.

- An open-ended assessment facilitates multiple approaches to problems and creative expression of mathematical ideas.
- Portfolio assessments imply that teachers have worked with students to establish individual criteria for selecting work for placement in a portfolio and judging its merit.
- Using observation for assessment purposes serves as a reflection of a students' understanding of mathematics, and the strategies he/she commonly employs to solve problems and his/her learning style.

## Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Use appropriate tools strategically.
5. Attend to precision and format.
6. Express regularity in repetitive reasoning.
7. Analyse mathematical relationships and use them to solve problems.
8. Apply and extend previous understanding of operations.
9. Use properties of operations to generate equivalent expressions.
10. Investigate, process, develop, and evaluate data.

### ▶ Lesson Planning

Before starting lesson planning, it is imperative to consider teaching and the art of teaching.

#### FURL

First Understand by Relating to day-to-day routine, and then Learn. It is vital for teachers to relate fine teaching to real-life situations and routine.

'R' is re-teaching and revising, which of course falls under the supplementary/continuity category. Effective teaching stems from engaging every student in the classroom. This is only possible if you have a comprehensive lesson plan.

There are three integral facets to lesson planning: curriculum, instruction, and evaluation.

## 1. Curriculum

A syllabus should pertain to the needs of the students and objectives of the school. It should be neither over-ambitious, nor lacking. (One of the major pitfalls in school curricula arises in planning of mathematics.)

## 2. Instructions

Any method of instruction, for example verbal explanation, material aided explanation, or teach-by-asking can be used. The method adopted by the teacher reflects his/her skills. Experience alone does not work, as the most experienced teachers sometime adopt a short-sighted approach; the same could be said for beginner teachers. The best teacher is the one who works out a plan that is customised to the needs of the students, and only such a plan can succeed in achieving the desired objectives.

## 3. Evaluation

The evaluation process should be treated as an integral teaching tool that tells the teachers how effective they have been in their attempt to teach the topic. No evaluation is just a test of student learning; it also assesses how well a teacher has taught.

Evaluation has to be an ongoing process; during the course of study formal teaching should be interspersed with thought-provoking questions, quizzes, assignments, and classwork.

## Long-term Lesson Plan

A long-term lesson plan extends over the entire term. Generally schools have coordinators to plan the big picture in the form of Core Syllabus and Unit Studies.

Core syllabi are the topics to be covered during a term. Two things which are very important during planning are the 'Time Frame' and the 'Prerequisites' of the students.

An experienced coordinator will know the depth of the topic and the ability of the students to grasp it in the assigned time frame.

## Suggested Unit Study Format

Weeks	Dates	Months	Days	Remarks

## Short-term Lesson Planning

A short-term plan is a day-to-day lesson plan, based on the sub-topics chosen from the long-term plan.

### Features of the Teaching Guide

The Teaching Guide contains the following features. The headings through which the teachers will be led are explained as follows.



#### Specific Learning Objectives

Each topic is explained clearly by the author in the textbook with detailed explanation, supported by worked examples. The guide will define and highlight the objectives of the topic. It will also outline the learning outcomes and objectives.



### **Suggested Time Frame**

Timing is important in each of the lesson plans. The guide will provide a suggested time frame. However, every lesson is important in shaping the behavioural and learning patterns of the students. The teacher has the discretion to either extend or shorten the time frame as required.



### **Prior Knowledge and Revision**

It is important to highlight any background knowledge of the topic in question. The guide will identify concepts taught earlier or, in effect, revise the prior knowledge. Revision is essential, otherwise the students may not understand the topic fully.

The initial question when planning for a topic should be how much do the students already know about the topic? If it is an introductory lesson, then a preceding topic could be touched upon, which could lead on to the new topic. In the lesson plan, the teacher can note what prior knowledge the students have of the current topic.



### **Real-life Application and Activities**

Today's students are very proactive. The study of any topic, if not related to practical real-life, will not excite them. Their interest can easily be stimulated if we relate the topic at hand to real-life experiences. Activities and assignments will be suggested which will do just that.

Flash cards based on the concept being taught will have more impact.



### **Summary of Key Facts**

Facts and rules mentioned in the text are listed for quick reference.



### **Frequently Made Mistakes**

It is important to be aware of students' common misunderstandings of certain concepts. If the teacher is aware of these they can be easily rectified during the lessons. Such topical misconceptions are mentioned.



### **Sample Lesson Plan**

Planning your work and then implementing your plan are the building blocks of teaching. Teachers adopt different teaching methods/approaches to a topic.

A sample lesson plan is provided in every unit as a preliminary structure that can be followed. A topic is selected and a lesson plan written under the following headings:

#### **Topic**

This is the main topic/sub-topic.

#### **Specific Learning Objectives**

This identifies the specific learning objective/s of the sub-topic being taught in that particular lesson.

## Suggested Duration

Suggested duration is the number of periods required to cover the topic. Generally, class dynamics vary from year to year, so flexibility is important.

The teacher should draw his/her own parameters, but can adjust the teaching time depending on the receptivity of the class to that topic. Note that introduction to a new topic takes longer, but familiar topics tend to take less time.

## Key vocabulary

List of mathematical words and terms related to the topic that may need to be pre-taught.

## Method and Strategy

This suggests how you could demonstrate, discuss, and explain a topic.

The introduction to the topic can be done through starter activities and recap of previous knowledge which can be linked to the current topic.

## Resources (Optional)

This section includes everyday objects and models, exercises given in the unit, worksheets, assignments, and projects.

## Written Assignments

Finally, written assignments can be given for practice. It should be noted that classwork should comprise sums of all levels of difficulty, and once the teacher is sure that students are capable of independent work, homework should be handed out. For continuity, alternate sums from the exercises may be done as classwork and homework.

**Supplementary Work (Optional):** A project or assignment could be given. It could involve group work or individual research to complement and build on what students have already learnt in class. The students will do the work at home and may present their findings in class.

## Evaluation

At the end of each sub-topic, practice exercises should be done. For further practice, the students can be given a practice worksheet or a comprehensive marked assessment.



# Sets



## Specific Learning Objectives

In this unit students will learn:

- about sets and use language of sets
- to use set notation: Descriptive, Tabular, and Set-builder notation
- how to represent a set by a Venn diagram
- about different types of sets and their elements (natural numbers, whole numbers, integers, even numbers, odd numbers, and prime numbers),
- to identify and differentiate between subsets and supersets, proper and improper subsets, equal and equivalent sets, disjoint and overlapping sets
- to describe and perform the operations of union of sets, intersection of sets, difference of two sets, complement of a set
- to verify the following:
  - $A \cap A' = \emptyset$
  - $A \cup A' = U$
  - $(A \cup B)' = A' \cap B'$
  - $(A \cap B)' = A' \cup B'$



## Suggested Time Frame

4 to 5 periods



## Prior Knowledge and Revision

Students have been introduced to the concepts of sets and their descriptive and tabular notations. A quick review quiz can be conducted where the teacher writes the various types of sets from the list below and encourages the students to come up with the correct terminology.

- finite and infinite sets
- empty set
- singleton sets
- universal set



## Real-life Application and Activities

The symbols table on page 7 needs to be highlighted in the lesson. The teacher can display the signs on chart paper on the soft board. At the beginning of every lesson, the symbols could be revised orally by looking at the chart presentation for a minute.

This will ensure that the students are well-versed in the symbols. They need to be able to read the symbols as a language of mathematics. They can feel confident when a teacher writes a set notation on the board and the students are able to explain it in words.

### Example

#### Symbols used in sets

	Symbol	Meaning	Example
1.	=	is equal to	$\{1, 2, 5\} = \{5, 1, 2\}$ (Sets having exactly the same members are equal sets.)
2.	$\neq$	is not equal to	$\{1, 2, 5\} \neq \{1, 2, 4\}$
3.	$\in$	is a member of	$3 \in \{1, 2, 3\}$
4.	$\notin$	is not a member of	$3 \notin \{0, 1, 2\}$
5.	$\emptyset$	is the empty set or the null set	$\{ \}$

### Starter activity

The teacher can make a table of a fictional set of students who got A<sup>+</sup> in mathematics and English. While doing so, the students will observe that some got A<sup>+</sup> in both subjects.

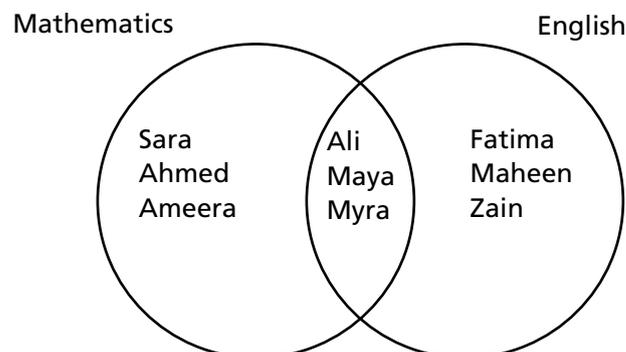
Set of students who got an A<sup>+</sup> in mathematics:

{Ali, Maya, Myra, Sara, Ahmed, Ameera}

Set of students who got an A<sup>+</sup> in English:

{Fatima, Maheen, Zain, Ali, Maya, Myra}

The teacher will highlight the fact that three students got an A<sup>+</sup> in both subjects, through Venn diagram.



## Activity

An interesting ten minute activity can be conducted by bringing two hoola hoops to class. Place them on the floor and label English and Mathematics with flash cards placed on the floor just outside the hoops. Make sure the hoops overlap.

Call out 'Begin' and the students should scramble to take their respective positions. Change the labelling of the hoops for another set of subjects and start again.

This activity will not only explain the concept of sets, but will also be helpful in the understanding of Venn diagrams and their intersections and unions.



## Summary of Key Facts

- There are different types of numbers, which can be expressed in the form of a set.
- Subsets and supersets, proper and improper subsets, equal and equivalent sets, and disjoint and overlapping sets can be differentiated from each other according to their definitions.
- The operations applied on the sets are union of two sets, intersection of two sets, difference of two sets, and complement of a set.
- The intersection of a set with its complement is a null set.
- The union of a set with its complement is the universal set.
- The complement of the union of sets A and B is equal to the intersection of the complements of the sets A and B.
- The complement of the intersection of set A and set B is equal to the union of the complements of the sets A and B.



## Frequently Made Mistakes

Students generally get confused with the symbols. If the suggestions stated earlier are implemented they surely will not find it difficult to decode the language of sets.



## Sample Lesson Plan

### Topic

Operations on sets

### Specific Learning Objectives

Students will be able to describe and perform operations on sets (union, intersection, difference, and complement).

### Suggested Duration

1 period

### Key Vocabulary

union, intersection, complement

## Method and strategy

### Activity

Consider an example of natural numbers sets.

Universal set =  $\{1, 2, 3, 4, \dots, 10\}$

Set A =  $\{2, 4, 6, 8\}$

Set B =  $\{2, 3, 5, 7\}$

Set C =  $\{1, 2, 3, 4, 5\}$

Emphasise that operations are also used on sets just like on different type of numbers. Ask them to find union and intersection of different combinations. The teacher should do similar examples on the board and reinforce the concept of union and intersection of sets. This can also be represented by a Venn diagram by shading the entire union.

### Written Assignment

Questions 4, 5, and 6 of Exercise 1 can be done as classwork. Five similar sums can be given for homework.

### Evaluation

A marked assignment can be done in class for the entire Exercise 1 as the students progress during the course of the week.

This unit is more presentation-based and the symbols are of utmost importance. Marks should be awarded for the correct use of symbols.

# 2

## Rational Numbers



### Specific Learning Objectives

In this unit students will learn:

- to identify a rational number as a number that can be expressed in the form of  $\frac{p}{q}$  where  $p$  and  $q$  are integers and  $q \neq 0$
- to identify and represent rational numbers on a number line
- to compare (using symbols  $<$ ,  $>$ ,  $=$ ) and arrange rational numbers in ascending and descending order
- to round off the rational numbers to a required degree of accuracy
- to round off to give an estimate to a calculation; to check the reasonableness of the solution
- to use number line for comparing and ordering
- to verify commutative, associative, and distributive properties of rational numbers



### Suggested Time Frame

5 to 6 periods



### Prior Knowledge and Revision

Students are already aware of natural and whole numbers as taught in earlier classes. They have been introduced to the number line and understand the laws of addition, subtraction, multiplication, and division of integers.

It would be advisable to revise the rules using a number line drawn on the board.

- |                  |   |
|------------------|---|
| $(+) + (+)$      | [Add and write a positive (+) sign in the answer.]                |
| $(-) + (-)$      | [Add and write a negative (-) sign in the answer.]                |
| $(+) + (-)$      | [Subtract and write the sign of the larger number in the answer.] |
| $(+) \times (+)$ | [Multiply and write a positive (+) sign in the answer.]           |
| $(-) \times (-)$ | [Multiply and write a positive (+) sign in the answer.]           |
| $(+) \times (-)$ | [Multiply and write a negative (-) sign in the answer.]           |
| $(+) \div (+)$   | [Divide and write a positive (+) sign in the answer.]             |
| $(-) \div (-)$   | [Divide and write a positive (+) sign in the answer.]             |
| $(+) \div (-)$   | [Divide and write a negative (-) sign in the answer.]             |



## Real-life Application and Activities

The following activity can be done on the board as a fun game.

Divide the students into groups of three.

Write a sum.

*Example:*

$$\frac{1}{4} \div \left( \frac{1}{2} - \frac{3}{4} + \frac{1}{4} \right)$$

Ask one group to attempt the sum left to right. Ask the next group to follow the order of operation of BODMAS. See who gets the higher value and point out that order of operation matters as they end up with two different answers.

This activity will not only make the students practise together, but will also make them appreciate the significance of BODMAS. Since they will be working in groups they can help each other by pointing out any mistakes and giving the right clue if anyone is unable to grasp the concept.



## Summary of Key Facts

Rational numbers are expressed in the form of  $p/q$ .

- They can be presented on a number line and can be arranged in ascending and descending order.
- The sum of a rational number and its additive inverse is always zero (0).
- The product of a rational number with its reciprocal is always 1.
- The commutative and associative properties with respect to addition and multiplication are satisfied by the rational numbers.
- The distributive property with respect to multiplication over addition is satisfied by the rational numbers.



## Frequently Made Mistakes

Students generally get confused with the terminology (or vocabulary) of rational and irrational numbers and their reciprocals. It is important that the earlier terminology (or vocabulary) and concepts of natural, whole numbers and integers are thoroughly revised before the concept of rational numbers is introduced. This is important as this unit forms the basis of algebra. The students should recognise the significance of the order of operations and the rules of the signs.



## Sample Lesson Plan

### Topic

Comparing rational numbers

### Specific Learning Objective

Students will be able to compare rational numbers

### Suggested Duration

1 period

### Key Vocabulary

rational number, unlike denominators

### Method and Strategy

Students should understand that in order to compare rational numbers, the rational numbers should have common denominators. Ask them to rewrite the given rational numbers with positive denominators to get common denominators. To obtain common denominators the rational numbers are multiplied by the common factor. Once this is done, the numerator which is smaller is placed first and then the inequality sign is written.

### Example

Write  $<$ ,  $>$  or  $=$  in the box

$$\frac{3}{4} \square \frac{5}{7}$$

$$\frac{21}{28} \square \frac{20}{28}$$

$$\text{Therefore, } \frac{3}{4} \square \frac{5}{7}$$

### Written Assignment

Questions 3 to 8 of Exercise 2B can be done in class. The sums that are not completed can be given for homework.

### Evaluation

An assessment will be planned along the lines of Exercise 2B. Since this unit is technical, students will also be given a 'Fill in the blanks test'. The blanks will be based on the definitions, rules, and properties taught in this unit.

# 3 Decimal Numbers



## Specific Learning Objectives

In this unit students will learn:

- to identify decimal numbers as a fraction which have denominators as power of 10
- to convert a fraction into an equivalent decimal fraction and vice versa
- to convert percentages into decimal numbers and vice versa
- to compare (using symbols  $<$ ,  $>$ ,  $=$ ) and arrange decimal numbers in ascending and descending order
- to use number line for comparing and ordering decimal numbers
- to round off the decimal numbers (up to 3 decimal places)
- to estimate a calculation and check the validity of the solution



## Suggested Time Frame

5 to 6 periods



## Prior Knowledge and Revision

Students are aware of decimals and the identification of the place value of decimals. Teachers can revise the place value of decimals by mentioning tenths, hundredths, and thousandths as the first, second, and third decimal places. Operations in decimals should also be revised as students can do story sums containing all the four operations of finding the sum, difference, product, and division of decimals.

Revision of place value of decimals can be called 'pinning the decimal point', along the lines of pinning the donkey's tail game. It is a short five-minute activity where the students can scramble into groups and the teacher divides the board into as many columns as the number of groups. The teacher writes 5 sums in each column and each group sends a volunteer. It can become a rowdy game as the students are allowed to help their volunteers. The group that finishes all five sums first and correctly gains a point. The sums on the board can be as follows:

Place the decimal point as required.

1) 7643            (3 hundredths)

Decimal point will be placed after 6. Therefore, 76.43.

2) 807945        (5 thousandths)

Decimal point will be placed after 7. Therefore, 807.945.



## Real-life Application and Activities

Decimals are associated with money. The teacher can ask for newspaper clippings where the growth rate or national reserves of the country are mentioned in decimals. Similarly, money conversions have decimal points.

### *Example*

Dollars can be converted to rupees according to the conversion rate.

A list of currencies and their conversions could be shared in class by the teacher.

When explaining terminating and non-terminating or recurring decimals, the literal meanings of the words could be explained. Terminating means to end; therefore terminating decimals have decimal places that are fixed and complete. Recurring decimals have decimal places that keep on repeating indefinitely. These decimal places can go on to infinity and the recurrence can sometimes be in groups or sequences of numbers.



## Summary of Key Facts

- Decimals are fractions which have denominators with powers of 10. Therefore, decimals can be expressed as rational numbers.
- Terminating decimals have a finite number of decimal places.
- Decimal numbers with an infinite number of digits after the decimal point are known as nonterminating decimal numbers.
- The denominator of a fraction, which results in a terminating decimal will have only 2 or 5 or both, as factors.
- If a set of digits is repeated again and again in a nonterminating decimal, it is called a recurring decimal.



## Frequently Made Mistakes

When doing long division, students sometimes get confused about when to introduce the decimal point and hence the zero to the dividend. A lot of practice questions to be solved by long division to create decimals. Sums to be done in class, on the board initially and then in students' notebooks.



## Sample Lesson Plan

### Topic

Rounding off and approximation

### Specific Learning Objectives

Students will be able to round off of decimals.

### Suggested Duration

1 period

## Key Vocabulary

rounding off, approximation, decimal places, decimal point

## Method and Strategy

A digit next to the number that needs to be rounded off has to be circled. If the number to its right is 5 or more, then the circled digit is increased by 1 and the remaining digits are replaced by zero.

### *Example*

Round off 28.89 to the nearest whole number.

2(8).89 there 28 is the whole number. Therefore, look at the digit to its right since,  $8 > 5$  the whole number will be increased by 1.

28.89 rounded off to the nearest whole number will be 29.

This rounding off can also be explained by using a number line. Whenever a decimal has to be rounded off to the nearest whole number, a number line comes in handy.

A number line can be made on the floor in the corner of the class room and kept during the duration of this unit. Coloured electrical tape can be taped to the ground to form the number line and the numbers can be placed as flash cards that can be replaced according to the demands of the sums or number sets. The gradings or the dashes on the number line can also be made semi-permanent by making the markings with a different coloured electrical tape.

## Written Assignments

Questions 4 and 5 of Exercise 3 can be done in class. Similar sums can be given for homework.

## Evaluation

A comprehensive test can be conducted where learning of all concepts taught can be assessed. Story sums involving rounding off and conversions can be asked. This will develop critical thinking skills.



# Squares and Square Roots



## Specific Learning Objectives

In this unit students will learn:

- to recognise and calculate squares of numbers up to three digits
- to calculate square roots of perfect squares (up to 3 digits) of natural numbers, fractions, and decimals
- that the square of a proper fraction is less than itself
- that the square of a decimal number less than 1, is less than itself
- to solve real-world word problems involving squares and square roots



## Suggested Time Frame

4 to 5 periods



## Prior Knowledge and Revision

Students are aware of square numbers and the area of a square. The correlation of the fact that a square number and area of a square are the same is important. The teacher can hold a quick one minute quiz where he/ she calls out a number and the students multiply it by itself and state its square.

A game of snap can also be played.

Make a set of 20 flash cards with the squares of 1 to 10 written on each card twice. Cards are shuffled and distributed between two students; when one of them calls out snap as he/ she gets the same square number, he/ she needs to call out the number it is a square of. This can be played by all students in turn and can be a five minute fun time with the rest cheering.

The multiplication rule of two negative numbers can be recalled and it can be pointed out that when two negative numbers are multiplied, we get a positive square.

### *Example*

$$13 \times 13 = 169$$

$$(-13) \times (-13) = 169$$



## Real-life Application and Activities

Although this is an entirely computation-based unit, you can create a game so that students can follow the steps of the prime factorisation faster.

You will require a white board, different coloured board markers, a stop watch, and flash cards. The students select a square number from the flash cards. They also pick out the option of prime factorisation or division method. The student goes to the board, solves the sum, and his/ her finishing time in the case of a correct answer is written in a column at the side of the board. All students take turns till the whole class had a turn. This activity will involve the whole class while each sum is being solved. This will result in a lot of practice as students will all follow each sum done on the board and will be encouraged to point out any mistakes. This will also quicken their mathematical computation.



## Summary of Key Facts

- When a number is multiplied by itself, the product is known as the square of that number.
- For integers of like signs, the product (or the quotient) is positive.
- For integers of unlike signs, the product or the quotient is negative.
- All perfect squares have two sets of square roots.
- The square of a proper fraction is less than itself.
- The square of a decimal number less than 1, is less than itself.
- The number that forms a square, when multiplied by itself is called the square root of the product.
- The positive square root is expressed with the symbol  $\sqrt{\quad}$ .
- Numbers which represent the area of a square are called square numbers. The length of the side of the square is the square root of the number.



## Frequently Made Mistakes

This is an entirely mathematical concept with steps to be learnt. Students sometimes make mistakes if they don't remember the steps. Therefore they should focus on learning the steps. The steps of prime factorisation method, along with a worked example, can be displayed on chart paper for the students' perusal during the course of the week.



## Sample Lesson Plan

### Topic

Squares and square roots

### Specific Learning Objectives

Students will be able to find the smallest whole number to be multiplied to make a number a square number.

### Suggested Duration

1 period.

### Key Vocabulary

square numbers, prime factorisation, exponents, power

### Method and Strategy

In order to find the number which will make the number a complete square, the students first have to revise the prime factorisation method. The exponential form representation is important.

### Example

$$120 = 2^3 \times 3 \times 5$$

2, 3, and 5 need to be introduced to make complete pairs.

Hence 30 is the smallest number to be multiplied to 120 to make it a perfect square.

This is a difficult and conceptual topic. A lot of sums should be done on the board and then, once the teacher feels that the students can work independently, they can do sums in their exercise notebooks.

### Written Assignment

Exercise 4 Q 6, 7 will be done in class and Q 12 will be given for homework.

Find the smallest whole number by which the following numbers will be multiplied to make them a perfect square.

1)	120	3)	66	5)	260	7)	95	9)	45
2)	325	4)	35	6)	180	8)	21	10)	500

### Evaluation

As mentioned earlier, this is an extremely conceptual unit with steps of mathematical computation. A written quiz will also be given for the steps of factorisation method. A comprehensive test including word problems will be given at the end of the unit. Assessment of learning during the course of the topic is important and will be implemented in the form of five minute quizzes.



# Rate, Ratio, and Proportion



## Specific Learning Objectives

In this unit students will learn:

- to calculate rate and average rate of quantities.
- to calculate increase and decrease in a ratio based on change in quantities
- to explain and calculate direct and inverse proportion
- to solve real-world word problems related to direct and inverse proportion



## Suggested Time Frame

4 to 5 periods



## Prior Knowledge and Revision

This unit is a continuation of the topic of ratios. The teacher should conduct a recall session in which rules of ratio are revised. The facts to be revised are:

- Ratios are always expressed in their simplest form.
- Quantities are in the same units.
- Ratios are placed in the following order new : old

### *Examples*

a.  $4 : 8$

$1 : 2$

b.  $400 \text{ g} : 1000 \text{ kg}$

$400 \text{ g} : 1\,000\,000 \text{ g}$

$4 : 10\,000$

$1 \text{ g} : 2500 \text{ g}$



## Real-life Application and Activities

Real-life examples on pages 55 and 56 can be discussed in class and a brainstorming session can be conducted.

**Example**

- Cost and number of apples
- Speed of the car and time
- Amount of food and the days it will last
- Number of pipes filling up a tank and the time taken

With the help of these examples, students should be encouraged to explore parity. If one quantity increases, the other also increases. Sometimes if one quantity increases, the other quantity or value decreases.

The difference between direct and inverse proportion should be explained through real-life examples and applications. Only when the students are able to distinguish between the two, should the teacher proceed.

**Summary of Key Facts**

- Rate is a special ratio in which the two terms are in different units.
- The Average Rate is defined as the rate at which one quantity is changing with respect to something else changing.
- Proportion is an equation in which two ratios are equal to each other.
- Proportion explains the comparison between the size, number, or amount of one thing to another thing.
- To increase a quantity in the ratio of  $a : b$ , and  $a > b$ , we multiply the quantity by fraction  $\frac{a}{b}$ .
- To decrease an amount in a given ratio, i.e.  $b : a$ , and  $b < a$ , we multiply the amount by a fraction  $\frac{b}{a}$ .
- There are two types of variations: direct variation and inverse variation.
- When an increase (or decrease) in one quantity leads to an increase (or decrease) in other quantity, these quantities are in direct proportion or variation to each other.
- When an increase (or decrease) in one quantity leads to a corresponding decrease (or increase) in the other quantity, the two quantities are said to vary inversely.

**Frequently Made Mistakes**

This is an easy unit and the students enjoy it as long as they can differentiate between direct and inverse proportion.

**Sample Lesson Plan****Topic**

Direct and inverse variation

**Specific Learning Objective**

Students will be able to calculate direct and inverse proportion.

## Suggested Duration

1 period

## Key Vocabulary

proportion, direct proportion, inverse proportion

## Method and Strategy

Real-life examples of inverse proportions will be given in class. The teacher will highlight the fact that if one quantity increases, the other decreases.

A very logical deduction is that if the speed is greater, the car will take less time to finish a journey.

## Activity

A simple activity will be done in class, in which two identical toy cars are brought into the lesson and are pushed with different forces to travel a given distance. The students will record the times on the stop watch and see that more force results in less time, and vice versa.

Once the students have decided the proportion, whether direct or inverse, the teacher will then explain the method. When the operation is inverse, horizontal multiplication is done.

## Written Assignment

Questions 3 to 16 of Exercise 5 will be given together so that students can distinguish between direct and inverse variation and then carry out the mathematical computation.

## Evaluation

A quiz will be given after each concept to assess whether to move on to the next concept or reinforce earlier learning. Quizzes are assessment of learning which are very beneficial.

A comprehensive assessment will be given along the lines of Exercise 5 to evaluate learning.



# Financial Arithmetic



## Specific Learning Objectives

In this unit students will learn:

- to identify and differentiate between selling price, cost price, loss, discount, profit percentage and loss percentage
- to calculate income tax, property tax, general sales tax, value-added tax, zakat and ushr
- to solve real-world word problems involving profit loss, discount, commission, tax, zakat, and ushr



## Suggested Time Frame

At least 10 periods



## Prior Knowledge and Revision

Students studied percentages in Grade 6. Financial transactions are the application of percentage in real-life scenarios. The teacher should first have a revision worksheet prepared for the lesson in which the concepts of percentages are revised. The teacher should not embark on this new unit until the students have thoroughly revised and revisited the concepts.



## Real-life Application and Activities

Profit and loss is a real-life application. Students can be shown newspaper clippings of sale advertisements and taught how to calculate the discount from the marked price.

Students can be encouraged to create their own business plan and present it in class.

### Activity

A class outing can be organised to a manufacturing unit e.g. a shoe factory.

The manager could be asked to present a simple break-down of the production costs to the students.

$$\begin{aligned} \text{Overhead costs} &= \text{Rs } x \\ \text{Material costs} &= \text{Rs } y \\ \text{Labour} &= \text{Rs } w \\ \text{Total costs} &= \text{Rs } z \\ \text{Sale price} &= \text{Rs } v \\ \text{Profit} &= v - z \\ \text{Profit \%} &= \frac{v - z}{z} \times 100 \end{aligned}$$

It should be highlighted that the above values are for one shoe or per unit. It should also be pointed out that the sale price has to be higher than the cost price to make a profit.

### Activity

The teacher can ask parents to take their child to a discount store. They can write an essay on their experience and findings.

### Example

Item 1

$$\begin{aligned} \text{Marked price:} & \quad \text{RS 400} \\ \text{Discount :} & \quad 30\% \\ & \quad = \frac{30}{100} \times 400 \\ \text{Discount} & \quad = \text{Rs 120} \\ \text{Sale price after discount} & \quad = 400 - 120 = \text{Rs 280} \end{aligned}$$

They can make a list of various items in this format, but it is important that they visit a sale shop and have a hands-on experience.

### Activity

Students should be asked to prepare a mock report of assets and savings, e.g. gold jewellery, savings, where they calculate their value and then work out the *zakat* on the assets.

All the activities could be recorded on chart paper and displayed in class for all to view. In this way mathematics can be made interesting and relevant.



## Summary of Key Facts

- The price at which an object is purchased is known as its cost price (CP) and the price at which the article is sold is called its selling price (SP).
- A profit is earned when the selling price is greater than the cost price (SP > CP).
- If the cost price is greater than the selling price (CP > SP), there will be a loss.
- The reduction made on the marked price of an article is called the discount.
- Net Selling price = Marked price – Discount
- Marked price = the price printed on the tag of an article

- Discount = reduction on the marked price
- Discount = marked price – selling price
- People also pay taxes on the money they earn during a year. This is called income tax.
- The government allows people who earn certain types of income to exclude those amounts from their annual income. Such income is called exempt income.
- The amount or income on which income tax has to be paid (which is not exempted) is called taxable income.
- The tax paid on property is called property tax. Different governments require the property tax to be calculated at different rates.
- The tax paid by the buyer to the seller on the selling price is called general sales tax.
- Value-added tax is a general tax, which is applied to all commercial activities. It includes production and distribution of goods and provision of services.
- Value-added tax is often called by its initial letters VAT.
- Commission is a fee given to sales person or agent for his services. The commission may be a flat fee or as a percentage of the total cost.
- Every Muslim who owns wealth equivalent to the value of 7.5 tola of gold or 52.5 tola of silver is obligated to pay zakat.
- The amount of zakat is calculated at the rate of  $2\frac{1}{2}\%$  or  $\frac{1}{40}$  of the total yearly savings.
- Ushr is a tax that is applied on a Muslim's agricultural assets.



### Frequently Made Mistakes

Students usually think that if a discount is given, there will be a loss. This is not true as the discount is given on the marked price to which some percentage of profit has already been added.



### Sample Lesson Plan

#### Topic

Profit and Loss

#### Specific Learning Objective

Students will be able to calculate profit or loss.

#### Suggested Duration

1 period

#### Key Vocabulary

cost price, selling price, discount, profit, loss, profit percentage, loss percentage

## Method and Strategy

### Activity

Students will be divided in groups and asked to set up a market corner in the class with the items they already have with them. For example, stationery items, books, lunch box, bags etc.

Each item will have a price tag showing the original price and the reduced price after 10%, 20%, 25%, etc.

One team member will become a cashier and others sales representatives. Students will move around to buy items with (fake money) discounts.

At the end of the activity all groups will calculate whether they made a profit or a loss has incurred.

### Written Assignments

Question 1 of Exercise 6A will be done in class, and Q 2 - 7 to be done at home.

### Evaluation

To assess learning a test based on the topic will be chosen from Revision 2.

# 7

## Algebraic Polynomials



### Specific Learning Objectives

In this unit students will learn:

- to recognise simple number patterns from various number sequences
- how to continue a given number sequence and find term to term rule and position to term rule
- to find terms of sequence when  $n$ th term is given
- to solve real-life problems involving number sequences and patterns
- about Muhammad bin Musa Al-Khwarizmi as the founding father of algebra
- to recognise variables as a quantity which can take several numerical values
- to recognise open and close sentences, like and unlike terms, variable, constant, expression, equation, and inequality
- to recognise polynomials as algebraic expressions in which the power of variables are whole numbers
- to identify monomial, binomial, and trinomial as a polynomial
- to add and subtract two or more polynomials
- to find the product of monomial with monomial, monomial with binomial/ trinomial, and binomial with binomial/ trinomial
- to simplify algebraic expressions involving addition, subtraction, multiplication, and division



### Suggested Time Frame

6 to 8 periods



### Prior Knowledge and Revision

The students have a basic knowledge of algebra. They should to recall that it is a branch of mathematics where quantities are expressed in letters and variables, and unknown values are figured out by forming algebraic expressions and equations.

The teacher can have a recap session in class by writing sums on the board and eliciting responses from the students who have to say the algebraic expressions out loud.

**Example**

- 1) Saima's age after 5 years =  $s + 5$
- 2) The cost of  $x$  apples if one costs 50 cents =  $50x$
- 3) The weight of a full truck with bricks is  $w$  and the weight of empty truck is  $z$ , then the weight of the bricks =  $w - z$ .

Also explain to the students that the power notation studied earlier is linked with algebra.

**Example**

2 to the power of 2 is the value of 4.

$$2^2 = 4$$

$$y \times y \times y = y^3$$

$w \times w \times w \times w \times w = w^5$  ( $w$  to the power or exponent of 5, where  $w$  is the base variable)

**Real-life Application and Activities**

Algebra should be considered as a language of mathematics where the unknown value is denoted by a letter. The rules of operations that were taught in class 6 can be revised again by playing a game with flash cards.

Each child is given a set of flash cards with signs of plus and minus written separately on each card. The teacher calls out the operation of addition, subtraction, multiplication, and division.

The child picks up two flash cards, look at the signs and figures out the sign of the result of the product, sum, or difference.

The student can be timed or the teacher can give 30 seconds and they have to write as many operations and answers as possible in their notebooks.

This encourages them to be quick with their laws and operations.

**Summary of Key Facts**

- A sequence is a list of numbers in a certain order. Each number in a sequence is called a term. Each term in a sequence has a position (first, second, third, and so on).
- A rule that defines the value of each term in a sequence about its position, is known as position to term rule.
- The  $n^{\text{th}}$  term of a sequence is the position to term rule using  $n$  to represent the position number.
- Variables are varying numbers which can take various numerical values.
- Constant is a symbol with fixed numerical value.
- Coefficient is a number which is placed before the variable.
- A closed sentence is always true or always false.
- Like terms have same variables and powers. They can be added or subtracted together.
- Unlike terms cannot be added or subtracted together, they have different variables.
- Equation is a statement of equality between two expressions consisting of variables and numbers. It contains a symbol of equality (=) between the two expressions.

- Inequality compares any two values and shows that one value is less than, greater than, or equal to the value on the other side of the equation.
- The exponent of a number indicates the number of times a particular number is multiplied by itself.
- A polynomial is an algebraic expression consisting of one or more terms, in each of which the exponent of the variable is zero or a positive integer.
- A monomial is a polynomial consisting of a single term.
- A binomial is a polynomial consisting of two terms.
- A trinomial is a polynomial consisting of three terms.
- Terms containing the same variables and the same corresponding exponents are known as like terms, and those having different variables or the same variables but different corresponding exponents are called unlike terms.
- To add algebraic polynomials, arrange the like terms horizontally and then add.
- To subtract, change the sign of each of the terms to be subtracted.
- When we multiply two powers having the same base, we add the exponents.
- The product of two terms with like signs is positive and that of two terms with unlike signs is negative.
- To find the power of a power, we multiply the exponents.
- To divide one monomial by another, the numerical coefficient of the dividend is divided by that of the divisor and the literal dividend is divided by the literal divisor.
- The sign rules of addition and subtraction and the rules of multiplication and division should be revised before simplifying polynomials.



## Frequently Made Mistakes

Students' most common mistake is placing the incorrect sign while applying the four operations. Teachers need to be extremely careful when explaining the sign concept.



## Sample Lesson Plan

### Topic

Division of polynomials

### Specific Learning Objectives

Students will be able to divide polynomials

### Suggested Duration

1 period

### Key Vocabulary

polynomial, coefficient, power, exponent

## Method and Strategy

Students will be introduced to this operation by relating it to regular division. The only difference is that all terms are divided or cancelled by the divisor term but only the powers of the same base variable are subtracted.

Lots of practice worksheets on this operation should be given and the rules should be revised.

(+) and (+) = +

(+) and (-) = -

(-) and (-) = +

## Written Assignments

Questions 10 (i) to (v) of Exercise 7B will be done in class in the students' notebooks. While, Q 10 (vi), (vii), and (viii) will be given for homework.

## Evaluation

This unit is relatively easy and a comprehensive assessment by using sums from Exercise 7A and 7B can be given.

Board quizzes will also be done for all the operations.

# 8

## Algebraic Identities



### Specific Learning Objectives

In this unit students will learn:

- to explore the following algebraic identities and use them to expand expressions:
  - $(a + b)^2 = a^2 + b^2 + 2ab$
  - $(a - b)^2 = a^2 + b^2 - 2ab$
  - $(a + b)(a - b) = a^2 - b^2$



### Suggested Time Frame

6 to 7 periods



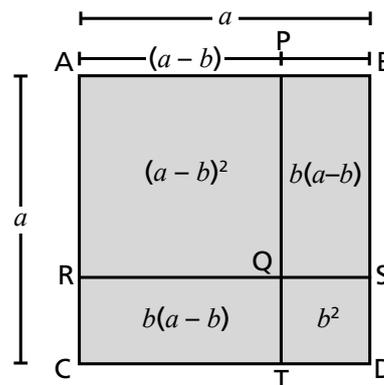
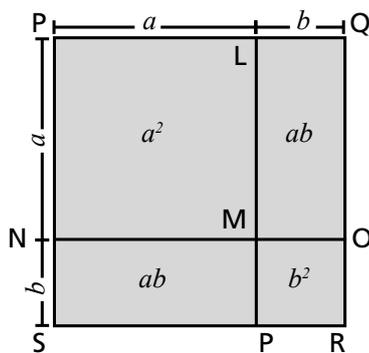
### Prior Knowledge and Revision

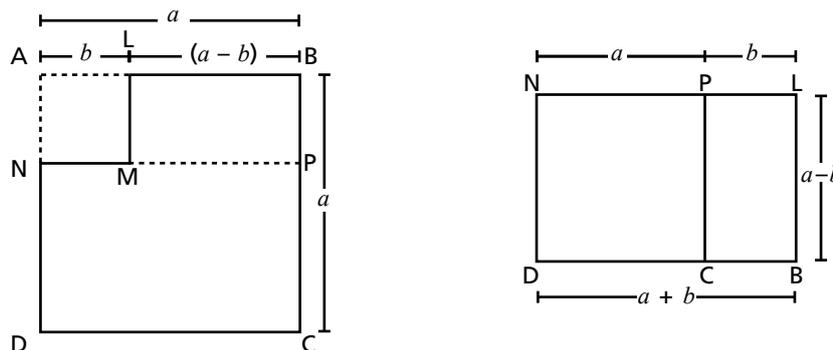
Students have not studied algebraic identities before. This topic is an extension of their understanding of the rules and laws governing algebraic polynomials.



### Real-life Application and Activities

The identities can be explained geometrically through the areas of rectangles and squares.





The teacher should make cut-outs of these diagrams on chart paper and then explain them by proving the identities geometrically. The sides of the squares and rectangles should be denoted by variables 'a' and 'b'.



## Summary of Key Facts

- Square of the sum of two terms is given as  
 $(a + b)^2 = a^2 + 2ab + b^2$ .
- Square of the difference of two terms is given as  
 $(a - b)^2 = a^2 - 2ab + b^2$ .
- Product of sum and difference of two terms is given as  
 $(a + b)(a - b) = a^2 - b^2$   
 $(a + b)(a - b) = a^2 - b^2$



## Frequently Made Mistakes

When solving the first and the second identity, the students get confused with the  $2ab$  expression. It is to be highlighted that this product has a minus sign in the difference of the whole square identity. The fact that  $b$  squared can never have a minus sign should also be pointed out.

$$(-b)^2 = (-b) \times (-b) = + b^2$$



## Sample Lesson Plan

### Topic

Algebraic identities

### Specific learning Objectives

Students will be able to solve arithmetic expressions using the first algebraic identity.

### Suggested Duration

1 period

## Key Vocabulary

numerical value, algebraic identity

## Method and Strategy

When explaining the fact that arithmetic expressions do not have to be evaluated manually with arithmetic operations, it should be pointed out that algebraic identity is a quicker way.

### Example

$$105 \times 105$$

The students will be told that the product can be found arithmetically but this will take longer. Also highlight the fact that the question mentions the use of algebraic identity to evaluate.

$$105 \times 105 = 105^2$$

Using the first algebraic identity:

$$a^2 + 2ab + b^2$$

$$= (100 + 5)^2$$

$$= (100)^2 + 2(100)(5) + (5)^2$$

$$= 10000 + 1000 + 25$$

$$= 11025$$

## Written Assignments

Questions 2 and 3 of Exercise 8 will be given as a class assignment. Worksheet as shown in the example below will be prepared and given as home assignment.

### Examples

1)  $206 \times 206$

2)  $505 \times 505$

3)  $101 \times 101$

4)  $702 \times 702$

5)  $4001 \times 4001$

### Answers

1) 42 436

2) 255 025

3) 10 201

4) 492 804

5) 16 008 001

## Evaluation

Assessment of learning will play a key role during this unit. Short, five minute quizzes will be given after each identity and concept taught. This will inform the teacher whether to proceed or not. It is important for the teacher to understand that algebra is a relatively easy and enjoyable branch of Mathematics but the identities are a critical step in this area. The concept of identities and their application is very important. The students will be given enough mini-tests to ensure that they are well versed in each identity before a comprehensive assessment including all concepts is given.

Revision 3 exercise will be used as comprehensive test for this unit.



# Factorisation of Algebraic Expressions



## Specific Learning Objectives

In this unit students will learn:

- to factorise algebraic expressions by taking out common terms and by regrouping
- to factorise quadratic expressions by breaking middle term



## Suggested Time Frame

4 to 5 periods



## Prior Knowledge and Revision

Students have just studied algebraic identities and this unit is a continuation of its application. It is also an extension and build-up of the concepts of identities. The teacher should not have any issues with this unit as it is a progression of the earlier topic.



## Real-life Application and Activities

The teacher should be aware of the complexities of this unit and should display the geometric cut-outs of the identities on the soft board during the week. The three identities should also be written on chart paper and displayed on the soft board. This is latent learning where the students are encouraged to use it for reference while doing sums and in this way the identities become embedded in their minds.



## Summary of Key Facts

- An algebraic expression can also be expressed in the form of its factors.
- Algebraic expressions can be factorised:
  - using common factors
  - using middle term breaking
  - by regrouping terms
- If an expression containing two or more terms possesses a common factor, then that factor is a factor of each term.

- An expression which can be written as the difference of two squared terms can be factorised into their sum and their difference.
- If, in an algebraic expression, common factors are not apparent, we can rearrange the terms to form groups of common factors.



## Frequently Made Mistakes

The fact that sometimes the sums need to be factorised multiple times is a bit challenging for students. It is important that the teacher highlights that checking whether a common factor can be found should be done before proceeding with the factorisation using identities.



## Sample Lesson Plan

### Topic

Factorisation of algebraic expressions

### Specific Learning Objective

Students will be able to factorise algebraic expressions.

### Suggested Duration

1 period

### Key Vocabulary

factorisation, difference of two squares, common factor

### Method and Strategy

The first and foremost rule to teach is to check for common numbers as factors and then common variables. The common variables and numbers have to be the smallest for them to be a factor for all terms.

#### *Example*

$$4xy + 6x^2y + 10xyz$$

The common factors that are variables are  $x$  and  $y$ . 2 is the common number factor for all. Hence  $2xy$  is the common term.

$$2xy(2 + 3x + 5z)$$

Furthermore, for multiple factorisation, the difference of two squares will be applied after the common factorisation.

#### *Example*

$$27x^3 - 3x$$

$$3x(9x^2 - 1)$$

$$3x(3x + 1)(3x - 1)$$

$3x$  is the common factor.

## Written Assignment

Selected sums from Exercise 9B will be done in class once the example provided above has been done on the board.

A worksheet will be provided as a homework assignment.

### *Example*

	Answers
1) $90x^2y^2 - 10$	1) $10(3xy + 1)(3xy - 1)$
2) $2x^2 + 10x + 50$	2) $2(x + 5)^2$
3) $49 - 4a^2$	3) $(7 + 2a)(7 - 2a)$
4) $81a^4 - 1$	4) $(9a^2 + 1)(3a + 1)(3a - 1)$
5) $(x - 4)^2 - 25$	5) $(x + 1)(x - 9)$

## Evaluation

At the beginning of each lesson there will be a two-minute recap test of the concepts taught in the previous lesson. Peer checking will be done and the sums solved on the board. Conducting this activity throughout this unit will not take more than five minutes in every lesson and will ensure that there are no gaps in the understanding and application of the concepts.

# 10

## Linear Equations



### Specific Learning Objectives

In this unit students will learn:

- to construct linear equations in two variables such as;  $ax + by = c$ , where  $a$  and  $b$  are not zero
- to solve simultaneous linear inequalities
- about cartesian system and coordinate plane
- to plot the graph of linear equation  $ax + b = 0$ , where  $a \neq 0$
- to plot the graph of linear equations in two variables
- to recognise and state the equation of horizontal and vertical line
- to find the value of  $x$  and  $y$  from the graph



### Suggested Time Frame

6 to 7 periods



### Prior Knowledge and Revision

Students have been forming algebraic expressions out of sentences and statements. The algebraic equation has been introduced in previous lessons. The concept of transposing should be revised. The key properties should also be done on the board.

The fact that 'LHS = RHS' should be revised.

An algebraic equation is a combination of terms that are connected by an 'equal to' sign stating that the variables and numbers on each sides are equal. This a perfect way of finding the value of an unknown variable. The formation of such mathematical statements is fundamental to problem solving in arithmetic and geometry. It has to be highlighted that algebra is not an isolated branch of mathematics, but the structural base of mathematics.



### Real-life Application and Activities

There are different types of equations: linear equations, equations with brackets, and equations with denominators. Each type of equation should be taught separately in a different lesson. A mixed exercise such as Exercise 10A can then be given.

Word problems can be converted to real-life situations by substituting the names of the students in the questions. Similarly, real-life word problems can be written on the board and can be role played.

**Example**

If Ali has two brothers in real-life, the teacher will make a word problem knowing that they all like to play cricket.

**Question**

If Ali scored 53 runs and his brother Amir scored 23 runs and the collective score of all three brothers was 212, then how many runs did Umar, the third brother, score?

Solution

$$x + 53 + 23 = 212$$

$$x + 76 = 212$$

$$x = 212 - 76$$

$$x = 136$$

Umar scored 136 runs in the cricket match.

**Summary of Key Facts**

- Two algebraic expressions connected by a sign of equality constitute an algebraic equation.
- A first-degree equation involving only one unknown quantity is known as a simple equation.
- When equal quantities are added to equal quantities, the results are also equal.
- When equal quantities are subtracted from equal quantities, the remainders are also equal.
- When equal quantities are multiplied by equal quantities, the products are equal.
- When equal quantities are divided by equal quantities, the quotients are also equal.
- A term of an equation can be transferred from one side to the other with a change of sign. This process is known as transposition.
- A Coordinate plane is defined by two perpendicular number lines, the x-axis, which is horizontal, and the y-axis, which is vertical.
- A linear equation can be represented as a line graph.
- A gradient is a measure of how steep a slope is. The slopes of parallel lines are equal, and the slopes of perpendicular lines are opposite reciprocals.
- The equation of a straight line is given in the form  $y = mx + c$ , where, the constant  $m$  is the gradient of the line and the constant  $c$  is the y-intercept.
- Horizontal lines go left and right and are in the form of  $y = b$ , where  $b$  represents the y-intercept, while vertical lines go up and down and are in the form of  $x = a$ , where  $a$  represents the x-coordinate of all points.
- Let  $m_1$  and  $m_2$  be the slopes of two parallel lines then  $m_1 = m_2$ .
- Let  $m_1$  and  $m_2$  be the slopes of two perpendicular lines then  $m_1 = \frac{1}{m_2}$ .



## Frequently Made Mistakes

Students who do not understand algebraic rules will have difficulty in transposing and solving the equation. If that is the case, the teacher can revisit the number line concept and explain how the rules of algebraic signs are derived.



## Sample Lesson Plan

### Topic

Algebraic equations

### Specific Learning Objectives

Students will be able to solve equations with fractional terms.

### Suggested Duration

1 period

### Key Vocabulary

denominators, lcm, transpose

### Method and Strategy

#### Example

$$\text{Solve } \frac{3x+1}{16} + \frac{2x-3}{7} = \frac{x+3}{8} + \frac{3x-1}{14}.$$

Solution:

$$\frac{3x+1}{16} + \frac{2x-3}{7} = \frac{x+3}{8} + \frac{3x-1}{14}$$

$$\frac{3x+1}{16} - \frac{x+3}{8} = \frac{3x-1}{14} - \frac{2x-3}{7}$$

$$\frac{3x+1-2(x+3)}{16} = \frac{3x-1-2(2x-3)}{14}$$

$$\frac{3x+1-2x-6}{16} = \frac{3x-1-4x+6}{14}$$

$$\frac{x-5}{16} = \frac{-x+5}{14}$$

$$14(x-5) = 16(5-x)$$

$$14x-70 = 80-16x$$

$$30x = 150$$

$$x = 5$$

This is the most complex form of solving an equation at this level. The concept of transposing comes right at the end. Initially the LCM is found and the numerators are multiplied by the number found by the division of the LCM and the denominator. When doing so, the students have to be

very careful of the minus sign as all signs will change when multiplied by a negative number. The denominators are then cross-multiplied once the LHS and RHS both have single terms, keeping in mind the rules of transposing, and the equation is then solved.

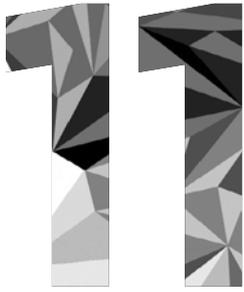
These types of sums should be done on the board and students can take turns to solve them. A lot of practice worksheets should be handed out for homework and classwork assignments.

### **Written Assignment**

All even-numbered sums of Exercise 10A will be done in class and the odd-numbered sums will be given for homework. This will ensure that sums of all levels of difficulty are done in class and then parallel sums are given for homework.

### **Evaluation**

This is quite a comprehensive chapter and two assessments will be given. One can involve all types of equations and the other can be completely based on word problems where the students will be expected to form the equation and then solve it.



# Lines and Angles



## Specific Learning Objectives

In this unit students will learn:

- that the sum of interior and exterior angles of a triangle is  $180^\circ$
- that the exterior angle of a triangle is equal to the sum of the opposite two interior angles of the triangle
- to calculate unknown angles in a triangle
- to differentiate between convex and concave polygons
- about the relationship between interior and exterior angles of polygons
- to calculate the sum of interior angles of a polygon
- to calculate each interior angle of a polygon
- that the sum of exterior angles of polygons is  $360^\circ$
- to calculate each exterior angle of a polygon



## Suggested Time Frame

2 to 3 periods



## Prior Knowledge and Revision

Students are well aware of the geometry strand of mathematics. It is important to recall the correct use of the geometric instruments.

- A protractor is used to construct angles.
- A pair of compasses is used to construct line segments.

Recognition and definitions of lines, rays, and line segments should also be quickly revisited in class. While revising the three types, the difference between the three with regard to the end point should be emphasised.



## Real-life Application and Activities

### Activity

A hands-on activity will be done in class.

Each student requires sheets of coloured paper, marker, ruler, glue stick and a pair of scissors.

A set of flash cards can be made with the name of a different shape written on each flash card. For example, pentagon, hexagon, octagon etc.

Ask students to paste the cut-outs in their notebooks and measure the interior and exterior angles. Give examples where polygons are used, such as tessellation, in architecture etc.



## Summary of Key Facts

- The sum of interior angles of a triangle equals to  $180^\circ$ .
- The interior and exterior angles of a triangle are supplementary, their sum is  $180^\circ$ .
- An exterior angle of a triangle is equal to the sum of the two opposite interior angles.
- A polygon is a closed, flat, two-dimensional (2D) shape with straight sides.
- Angles made inside the polygons are known as interior angles
- In a convex polygon, all the interior angles are less than  $180^\circ$ .
- In concave polygon, there is at least one angle more than  $180^\circ$ .
- If the side of a polygon is extended, the angle formed outside the polygon is the exterior angle
- Sum of interior angles of a polygon is  $(n - 2) \times 180^\circ$  (where n is the number of sides)
- Each interior angle of a polygon =  $\frac{(n - 2) \times 180^\circ}{n}$
- Sum of exterior angles of a polygon =  $360^\circ$
- Each exterior angle of a polygon =  $\frac{360^\circ}{n}$



## Frequently Made Mistakes

Students sometimes confuse interior angles with exterior angles of polygons. Sum of interior angles of polygons depends on number of sides and it varies accordingly. While, the sum of exterior angles of polygons is always  $360^\circ$ .



## Sample Lesson Plan

### Topic

Polygons

### Specific learning objectives

Students will be able to calculate sum of interior angles of a polygon.

### Suggested duration

1 period

### Key vocabulary

polygon, interior angle, exterior angle.

## Method and Strategy

Polygons are 2-D flat shapes. Students are already familiar with these shapes. They have studied these shapes in previous classes. Therefore, calculating angles of polygons will be of interest.

To calculate sum of interior angles or each interior angle the table given on page 151 of NCD 7 will be very helpful. Emphasis will be laid upon the fact that, while calculating how many triangles are formed in each polygon, the students must join vertices from one vertex only as shown in the table. They will be reminded that the number of triangles formed will always be two less than the number of sides of the Polygon.

## Activity

Distribute cut-outs of different polygons to students and tell them to choose different vertices to check whether same number of triangles will be formed, that is  $(n - 2)$  where 'n' is the number of sides.

Sum of interior angles =  $(n - 2) \times 180^\circ$

Each interior angle =  $\frac{(n - 2) \times 180^\circ}{n}$

Sum of exterior angles =  $360^\circ$

Each exterior angle =  $\frac{360^\circ}{n}$

## Written Assignment

Exercise 11 Q6 will be done in class. A worksheet based on interior angles will be given for homework.

## Evaluation

This unit is based on manipulation, therefore, marked assignment will be given to evaluate learning.



# Practical Geometry



## Specific Learning Objectives

In this unit students will learn:

- to construct a scalene triangle, an equilateral triangle, and an isosceles triangle
- to construct an acute-angled triangle, a right-angled triangle, and an obtuse-angled triangle
- to recognise quadrilaterals and their characteristics (square, rectangle, parallelogram, rhombus, trapezium, and kite)
- calculate unknown angles in quadrilaterals using the properties of quadrilaterals (square, rectangle, parallelogram, rhombus, trapezium, and kite)
- to construct the quadrilaterals:
  - square
  - rectangle
  - parallelogram
  - rhombus
  - kite
  - trapezium
- to recognise, identify, and draw lines of symmetry in 2D shapes
- to rotate objects using rotational symmetry and find the order of rotational symmetry
- to translate an object and give the precise description of translation



## Suggested Time Frame

6 to 8 periods



## Prior Knowledge and Revision

This unit involves the use of geometric instruments. Prior to beginning this chapter, the students should revise the key words, bisection, equilateral, and isosceles triangles. The properties of types of angles and triangles are also important.

Board geometric instruments should be used to teach the students the correct handling of the instruments.

The concept of shapes made by 3 or more line segments has already been introduced to the students. A brainstorming session on identification of various shapes can be done on the board. After the identification of various quadrilaterals, the teacher should prompt the students to identify the properties of each shape.

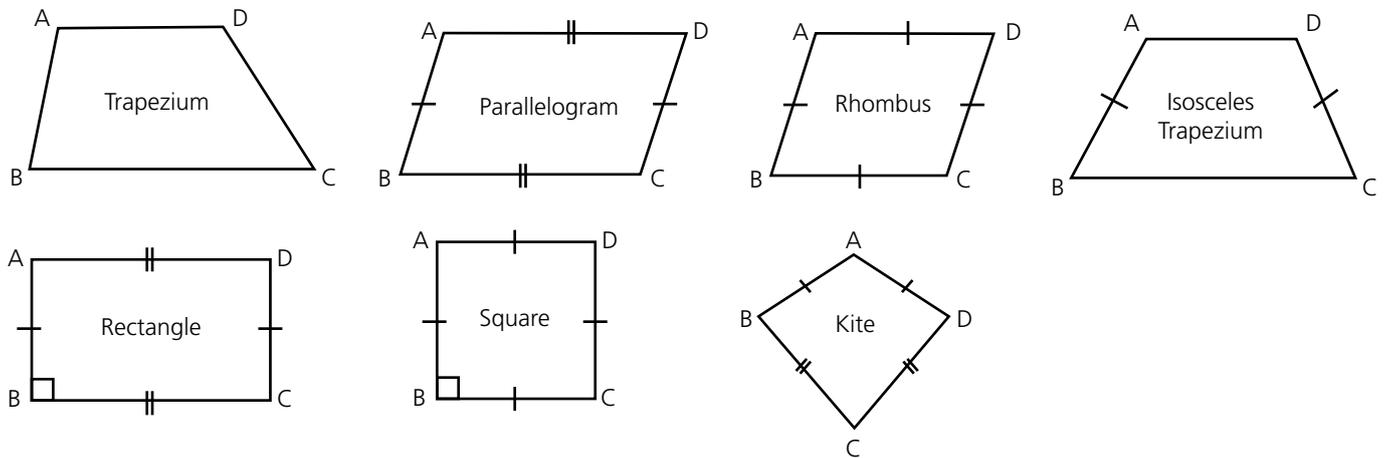
### Example

A rhombus has all sides equal but it is not a square. Why? (The angles are not right angles.)

A parallelogram has length and breadth but it is not a rectangle. Why? (There are no right angles at the vertices.)

A kite is an unusual quadrilateral with equal sides adjacent to each other (two small adjacent sides equal and two longer adjacent sides equal).

A trapezium is different from a parallelogram. Give two properties supporting the statement (only one set of parallel lines and the parallel lines are not equal in length).



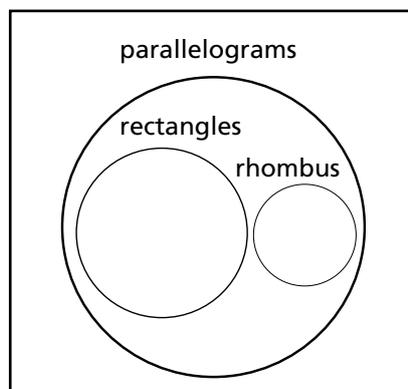
## Real-life Application and Activities

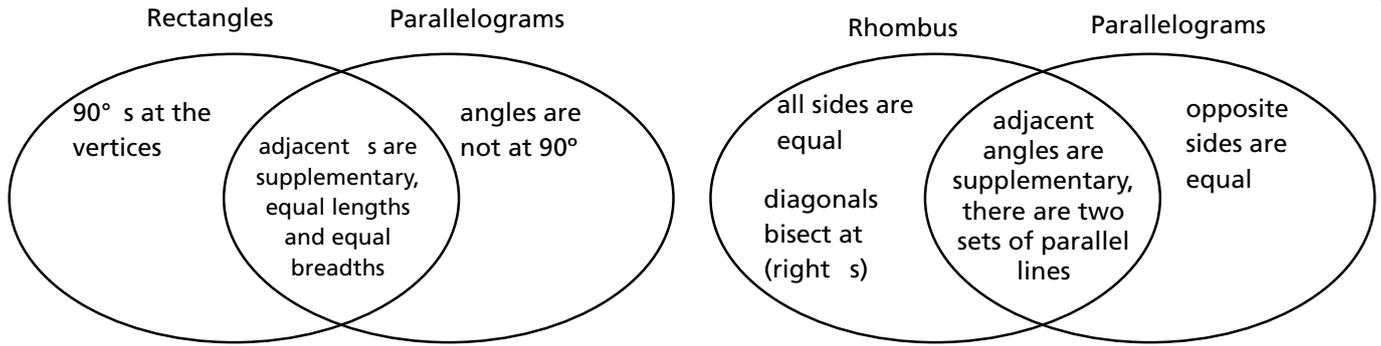
The teacher can stimulate the interest of the students by informing them that we can role-play by acting as architects and can plan designs using the geometric instruments.

Henceforth, all work is hands-on and as the teacher explains the steps of construction, the students should write them in their notebooks and construct figures accordingly.

To reinforce knowledge of the properties, the students can be divided into groups and each group can be assigned a quadrilateral. The group then makes a cut-out of the shape assigned from chart paper and give a minute-long presentation on the properties of the assigned quadrilateral.

### Quadrilaterals





## Summary of Key Facts

- A quadrilateral is a closed, four-sided, plane figure.
- A diagonal of a quadrilateral is a line segment joining its two opposite pairs of vertices.
- A parallelogram is a quadrilateral in which the opposite sides are parallel and equal.
- A rhombus is a parallelogram in which all the four sides are equal.
- A rectangle is a parallelogram in which all the angles are right angles.
- A square is a rectangle with four equal sides.
- A trapezium is a quadrilateral with two and only two, parallel sides.
- An isosceles trapezium is a trapezium in which the non-parallel sides are equal.
- A kite is a quadrilateral in which the two pairs of adjacent sides are equal. (In general, the opposite sides are not parallel or equal).
- The order of Rotational Symmetry tells us how many times a shape looks the same when it rotates 360 degrees, and it is equal to the number of sides.
- Translation means the shifting of a figure from one place to another to a new position on a grid.



## Frequently Made Mistakes

Students should be made aware that the correct use of geometric instruments for example how to hold and place the instruments, is important to produce accurate drawings.

Students find the relationship between the shapes of quadrilaterals a bit challenging. If the shapes are taught in a way where the overlapping properties are first pointed out and then the additional properties which differentiate one shape from the other are explained it will help the students immensely. Venn diagrams of similarities and differences will also help.



## Sample Lesson Plan

### Topic

Geometric constructions

### Specific Learning Objectives

Students will be able to construct triangles with the ratio of the sides and perimeter given.

**Suggested Duration**

1 period

**Key vocabulary**

ratios, perimeter, pair of compasses

**Method and Strategy**

To construct a triangle with a given set of ratios, first revise the concept of proportional ratios.

*Example:*

Triangle ABC has sides in the ratio of 1 : 4 : 5.

If perimeter is 30 cm, then the sides will be calculated as:

$$\overline{AB} = \frac{1}{10} \times 30 = 3 \text{ cm}$$

$$\overline{BC} = \frac{4}{10} \times 30 = 12 \text{ cm}$$

$$\overline{AC} = \frac{5}{10} \times 30 = 15 \text{ cm}$$

Draw  $\overline{AC}$ , the longest side, as the base line. With A and C as centres draw two arcs with radius 3 cm and 12 cm respectively, cutting each other at B.

Join B to A and C.

It should be pointed out that sometimes mathematical computations are done before proceeding with construction of the triangle. Similarly, to construct a triangle with given altitude or vertical angle, mathematical working will be required.

**Written Assignment**

Practice sums will be given in class and for homework from Exercise 12A. The teacher will approach each child individually to help them use the geometric instruments along with helping with the mathematical concepts.

**Evaluation**

Marked assignments will be given in class and homework periodically before taking a comprehensive assessment of this unit.

The test will have a choice of options of different cases of constructions and at least 5 sums of constructions should be given for the duration of a one-period test.

# 13 Circles



## Specific Learning Objectives

In this unit students will learn:

- to define elements of a circle:
  - centre
  - radius
  - diameter
  - chord
  - arcs
  - major and minor arc
  - semi-circle
  - major and minor segments of a circle
- to describe the properties of circle



## Suggested Time Frame

2 to 3 periods.



## Prior Knowledge and Revision

Parts of a circle have been taught earlier; however, the difference between a chord, diameter, and radius should be explained with the help of diagrams.

The radius is the distance from the centre to the circumference of the circle, whereas the diameter is the measure of the circle across, passing through the centre.

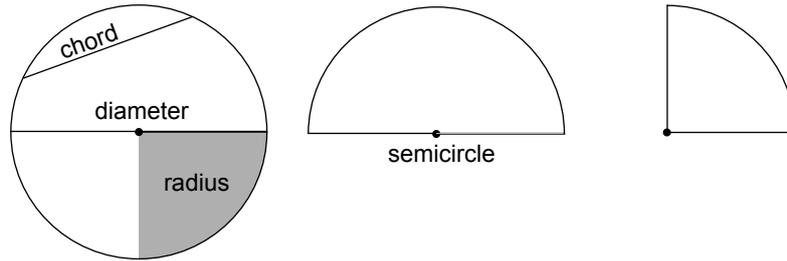
The radius touches the circumference of a circle at one point, while the diameter touches it at two points.

The radius and diameter are constant values.

A chord touches the circle at two points but does not pass through the centre.

A semicircle is half a circle, subtended (meeting at two points) by a diameter. A quadrant is a quarter of a circle subtended by two radii.

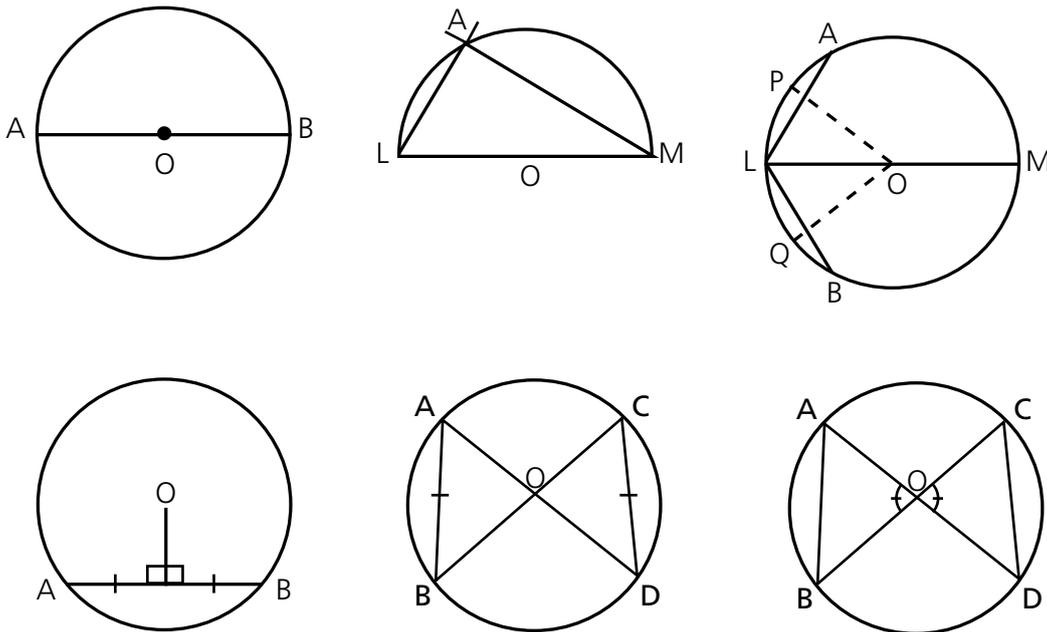
The circumference of a circle is its perimeter and the circular measure of its boundary.



## Real-life Application and Activities

Construction of circles and semicircles is relatively easy as only the use of compasses is required and the students need to get the value of the radius on the compasses and the circle or semicircle can be drawn.

The properties of circles are extremely critical and these can only be explained if done practically.



### Activity

You will need chart paper, drawing pins, and thread.

Cut out a big circle with the width of the chart paper as the diameter.

Put the drawing pins at the end point of the diameter. Put a thread around the drawing pins to make a loop. Pull the thread and pin it opposite the diameter on the semicircle.

Measure the angle formed on the circumference with a protractor or a set square: it will be  $90^\circ$ .

Similarly, on the same chart paper loop a thread around the two drawing pins and pin it on the circle at two points on the circumference, this time to create a chord and not a diameter. Measure the distance of the chord from the centre and use the same measurement to tie another chord on the other side of the centre of the circle. Measure the length of the threads forming the chord: they will be equal.

By this method all properties can be proved. The students write the properties on the chart paper. Help students to prove all the properties practically.



## Summary of Key Facts

- A circle is a plane figure consisting of points that lie at the same distance from its centre.
- The centre of a circle is the fixed point on the plane from which the distance of the moving point is always constant.
- The circumference of a circle is the boundary or perimeter of the circle.
- The distance between the centre and a point on the circumference is called the radius.
- The diameter is the distance between two points on the circumference along a straight line that passes through the centre.
- Any diameter divides a circle into two equal parts; each part is known as a semicircle.
- A line segment that joins the endpoints of an arc is called a chord.
- Any part of the circumference or perimeter of a circle is known as an arc of the circle.
- When a circle is divided into two unequal parts by a chord, the arc that forms the smaller part is called the minor arc, the arc that forms the larger part is called the major arc.
- A segment of a circle is the area enclosed between an arc and the corresponding chord.
- When a circle is divided into two parts by a chord, the smaller segment formed is called the minor segment, the larger segment formed is called the major segment.
- A circle divides the plane in three distinct parts, namely the interior region, the exterior region, and the circle itself.
- Equal chords of a circle are equidistant from the centre.
- The perpendicular, drawn from the centre of a circle to a chord, bisects the chord.
- Equal chords of a circle subtend equal angles at the centre.
- Every semicircle has an angle that measures  $90^\circ$ .
- All the angles subtended by a chord in the same segment of a circle are equal.



## Frequently Made Mistakes

Students even at a higher grade mix up the concepts of chords and diameters. This causes further confusion later on while working on the theorems.



## Sample Lesson Plan

### Topic

Circles

### Specific Learning Objective

Students will be able to use the given properties of the circle:

- equal chords are equidistant from the centre,
- the perpendicular line from the centre bisects the chord, and
- equal chords subtend equal angles at the circumference.

### Suggested Duration

1 period

### Key Vocabulary

chords, equidistant, subtend, perpendicular, bisect, bisector

### Method and Strategy

The activity stated earlier will be shown to the students to revise the theorems. However, for the theorems to be more effective in application, a lot of practice sums will be done.

### Written Assignments

Exercise 13B will be done in class on the board and then given for homework.

### Evaluation

A comprehensive assessment on this unit should be given. It should be pointed out to students that inaccuracy in construction will result in the loss of marks.



# Perimeter and Area



## Specific Learning Objectives

In this unit students will learn:

- to convert between standard units of area:  $m^2$ ,  $cm^2$ ,  $mm^2$  and vice versa
- to convert 12-hour clock to 24-hour clock and vice versa
- to convert different units of time and speed
- about the relation between speed, distance, and time
- to differentiate between uniform speed and average speed
- to calculate arrival time, departure time, and journey time in each situation
- solve the real-world word problems involving distance, time, and average speed
- to calculate the area and perimeter of shaded and unshaded regions in composite shapes
- find the circumference of a circle
- find the area of a circle
- solve the real-world word problems involving perimeter and area



## Suggested Time Frame

6 to 8 periods



## Prior Knowledge and Revision

This unit is a continuation of concepts taught in the earlier grades. The students are aware of the concepts of area and perimeter, so no formal introduction is needed.

Revision of shapes and calculating the area and perimeter of composite shapes made up of squares and rectangles can be done.

### Activity

The teacher can make the revision fun by bringing cut-outs on chart papers and dividing the class into groups and asking them to calculate the area and perimeter of the cut-outs. These cut-outs can be put on the floor and the groups can work on the floor. This activity should not take more than five minutes. The calculations can be done in their exercise notebooks.

Shapes are everywhere; architecture involves spatial geometry, and the construction of a house involves calculation of materials required, areas, etc. Even something as relatively simple as making a wooden table or cupboard requires knowledge of the concepts taught in this unit.

If there is an in-house handy man or carpenter in school, he can be invited to the lesson to explain the dimensions and material requirements of making a desk.

Students should not be given the formulae as mathematical computations alone. They need to understand the derivation to appreciate the real-life application.



## Real-life Application and Activities

The value of pi ( $\pi$ ) can easily be explained with an interesting hands-on activity.

### Activity

You will need: a 1 metre length of yarn, (any thick thread will also do), a marker, different everyday objects that are circular e.g. a CD, circular plate, circular sharpener, a 30 cm ruler, and play dough or any adhesive.

Fasten the yarn **around** the circular object with the play dough so it stays in position.

Measure the yarn and record the length in centimetre.

Now place the yarn **across**, making sure it passes through the centre.

Measure and record the length.

Ask the students to calculate the value: around/across.

They should come to a value close to 3.142.

They should repeat this process with two more circular objects of different sizes.

The teacher should then point out the constant value of  $\pi$  that it is 3.142 for all circles.

Once the formula for the circumference is introduced he/she will relate **around** to the circumference and **across** to the diameter.



## Summary of Key Facts

- The International System of Units (SI), commonly known as the metric system, is the international standard for measurement.
- For measuring time, we can either use the 12-hour clock or 24-hour clock. We denote a.m. from midnight to just before noon and p.m. from afternoon till just before midnight.
- The speed of an object tells how fast it is moving. Speed is a special type of rate.
- $\text{Speed} = \frac{\text{Distance travelled}}{\text{Time taken}}$
- $\text{Distance} = \text{Speed} \times \text{Time}$
- $\text{Time} = \frac{\text{Distance}}{\text{Speed}}$
- When an object covers equal distance in equal time intervals, it is said to be in uniform speed.
- The ratio of total distance travelled by an object to the total time taken by the object with uniform speed is defined as the average speed of the object.
- The distance around a plane figure is called its perimeter.

- The area of the shaded region is the difference between the area of the entire polygon and the area of the unshaded part inside the polygon.
- The distance around the boundary of a circle is called its circumference.
- Circumference of a circle =  $2\pi r$
- Area of a circle =  $\pi r^2$
- To find the areas that are borders the concepts of external area and internal area should be made clear. Once these have been found, the areas are subtracted to get the area of the borders.



## Frequently Made Mistakes

Students generally get confused with the identification of the altitude and perpendicular lines. The earlier unit on this can be revised to obtain correct values which are substituted in the formulae.



## Sample Lesson Plan

### Topic

Circumference and area of a circle

### Specific Learning Objectives

Students will be able to find the circumference and area of a circle.

### Suggested Duration

1 period

### Key Vocabulary

radius, diameter, perimeter, circumference, area, pi

### Method and Strategy

This is a new and interesting topic, therefore, the lesson will begin with an activity. Students will do the Activity 1, 2, and 3 given on pages 206 and 207 to create an interest and understanding of the concept of a area of a circle.

### Written Assignment

Exercise 14A Q 1 - 4 will be done as class work and Q 5 - 9 will be given for homework.

### Evaluation

A comprehensive test along the lines of Exercises 14B will be given on the completion of the unit.

# 15

## Volume and Surface Area



### Specific Learning Objectives

In this unit students will learn:

- to convert between standard units of volume:  $m^3$ ,  $cm^3$ ,  $mm^3$ , and vice versa
- to calculate the surface area and volume of any simple 3D shape including right prisms and cylinders
- solve real-life word problems involving volume and surface area of right prisms and cylinders



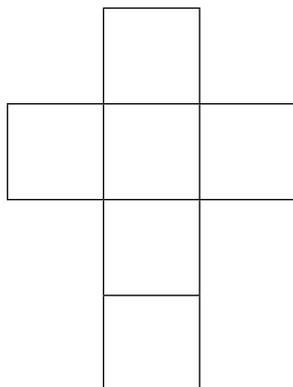
### Prior Knowledge and Revision

Students calculated the surface area and volume of cubes and cuboids in the previous grade.

A brief revision at the beginning of the lesson can be done where the faces and edges of a cube and a cuboid are identified and the surface area and volume formulae are revised.

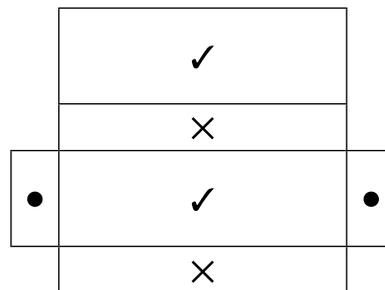
### Activity

Net diagrams of a cube and cuboid can be photocopied and handed to the students. They can cut them out and tape the edges to create a 3D shape out of a 2D cut-out. This will highlight the fact that a 2D shape can be converted into a 3D shape that will have a volume.



**Net diagram of a cube**

Since all the sides are equal, the faces are all equal in area and dimensions.



**Net diagram of a cuboid**

Since the dimensions are different, 2 faces each have the same dimensions and same area.



## Real-life Application and Activities

### Activity

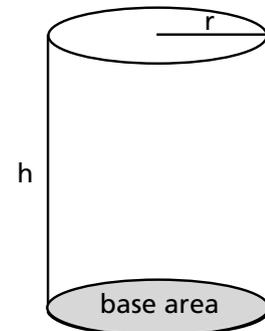
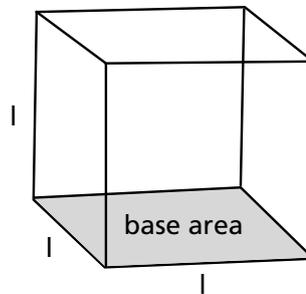
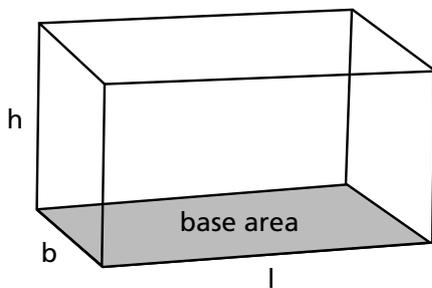
The relationship between volume and base area of cubes, cuboids, and cylinders can be explained. The basic formula for volume is:

Volume = base area  $\times$  height

The shaded region in each diagram below is the base.

Therefore the volume of each shape can now be easily calculated.

- i) Volume of a cuboid = base area  $\times$  height  
 $V = (l \times b) \times h$  (base is a rectangle, therefore the base area =  $l \times b$ )
- ii) Volume of a cube = base area  $\times$  height  
 $V = (l \times l) \times l = l^3$  (base is a square, therefore the base area =  $l \times l$ )
- iii) Volume of a cylinder = base area  $\times$  height  
 $V = (\pi r^2)h$  (base is a circle, therefore the base area =  $\pi r^2$ )



### Summary of Key Facts

- The tiniest of dots on the paper has only one dimension. It indicates a position.
- A line has only one dimension—length.
- Squares, circles, triangles, and ovals have two dimensions—length and breadth.
- Cubes, cuboids, and spheres are three-dimensional objects. They have depth as well as length and breadth.
- The area of the surfaces of a 3D solid is known as its surface area.
- The length, breadth, and height of a rectangular solid are called its dimensions.
- Lengths are measured in millimetres, centimetres, and metres.
- Area is measured in square centimetres, square metres, or square kilometres.
- Volume of 3D objects is measured in cubic centimetres, cubic metres, or cubic kilometres.
- The most convenient unit of volume is the cubic centimetre ( $\text{cm}^3$ ), which is a solid region formed by a cube of side one centimetre.

- A geometric solid that has a polygon as its base and vertical sides perpendicular to the base is known as a right prism because the angles between the base and sides are right angles.
- A cylinder which stands on a circular base at right angles is a right circular cylinder.
- A cylinder has a flat base and a flat top and one curved surface.
- Area of the curved surface of a right circular cylinder =  $2\pi rh$
- Total surface area of a right circular cylinder =  $2\pi r(h + r)$
- Volume of a right circular cylinder =  $\pi r^2 h$



## Frequently Made Mistakes

The identification of dimensions when applying the formula is very important. The students tend to put in the value of the diameter instead of the radius. Similar mistakes also occur in the case of cubes and cuboids.

The hands-on activity of the net diagram will ensure that the concepts of the dimensions and their shapes are clear.



## Sample Lesson Plan

### Topic

Volume and surface area of a cylinder

### Specific Learning Objectives

Students will be able to calculate the surface area of a cylinder.

### Suggested Duration

1 period

### Key Vocabulary

radius, circumference, height, curved surface area, total surface area

### Method and Strategy

#### Activity

The most effective way of teaching the formula of the surface area of a cylinder is to take a piece of paper and show the class the length and breadth of the paper.

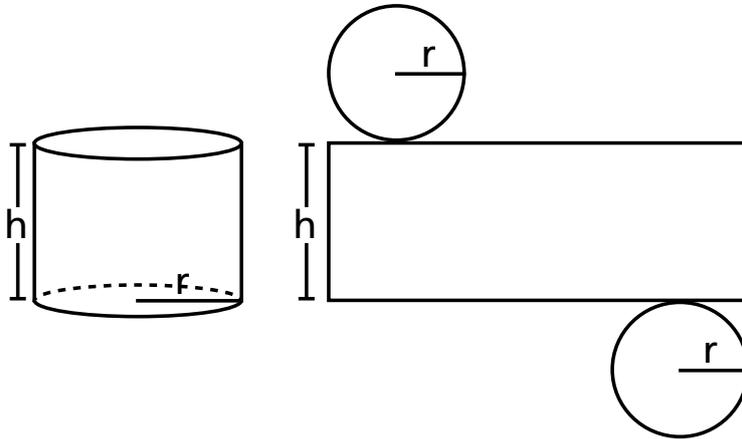
Highlight the fact that the rectangular paper is actually the curved surface area of the cylinder as it folds to form a cylinder.

Put two circle cut-outs on the top and bottom of the paper cylinder. Make sure that the circumference of the two circles is equal to the length of the rectangle; only then they will be placed perfectly.

This fact can be pointed out to the students.

Curved surface area of a cylinder =  $2\pi r \times h$

Total surface area of a closed cylinder =  $2\pi rh + 2\pi r^2$



### Written Assignment

Questions 8 and 9 from Exercise 15 will be done in class. They will note the formulae down with markers in their notebooks before they proceed to do the sums. Question 10 and 11 will be given for homework.

### Evaluation

This is an extremely conceptual topic. Quizzes to find the area or volume of any one shape will be given at the beginning of each lesson. This way the concepts will be further enhanced as the unit progresses. A comprehensive assessment covering all concepts will be given once the students are confident. The Revision 5 on pages 218 to 220 can be used to assess mensuration.

# 16

## Data Handling



### Specific Learning Objectives

In this unit students will learn:

- to recognise drawing and interpretation of bar graphs, line graphs, and pie chart
- differentiate between a histogram and a bar graph
- draw and interpret bar graph, line graph, and pie chart
- to construct and compare histograms for both discrete and continuous data with equal interval range
- to select and justify the most appropriate graph for a data set and draw simple conclusions based on the shape of the graph
- to recognise the difference between discrete, continuous, grouped, and ungrouped data
- to calculate mean, median, and mode for ungrouped data
- to calculate mean for grouped data
- solve real-life word problems; compare, choose, and justify the appropriate measures of central tendency for a given set of data
- to construct frequency distribution tables for given data: frequency, lower class limit, upper class limit, class interval and mid-point)
- to solve related real-world problems
- to explain and compute the probability of: certain events, impossible events and complement of an event
- to solve real-world word problems related to probability



### Suggested Time Frame

4 to 5 periods



### Prior Knowledge and Revision

Students are aware of line graphs and bar graphs. The teacher can give a Power Point presentation and show colourful slides of various bar graphs and line graphs. A list of questions can be read out and the students can answer by looking at the slides. This activity will not only help them revise the concepts, but will also add variety to mathematics lessons..



## Real-life Application and Activities

Students are aware of simple distribution of data where frequency is not mentioned. The facts that the data is now grouped and the quantity is within a range have to be explained clearly. The steps of converting raw data into grouped data and that of constructing a bar graph, have to be explained clearly and highlighted as a soft board presentation.

Students should be encouraged to draw on chart paper representations of a bar graph. They can work in groups. This will enhance their understanding as they will benefit from peer cooperation.



## Summary of Key Facts

- Discrete data is countable and contains distinct values. It is represented by a bar graph.
- Continuous data is measurable and includes any value within the range. It is graphically represented by a histogram.
- Ungrouped data is data in raw form.
- A table that shows class intervals and their corresponding frequencies is called a frequency distribution table.
- The difference between the greatest and smallest data values is called the range of the data.
- Each class interval consists of a smallest value and a greatest value. These are called the lower class limit and upper class limit, respectively.
- Numerical data also known as quantitative data is represented as a line graph by plotting dots on a graph paper and then joining them by line segments.
- A pie chart is a circular chart on which data is represented in the form of sectors of a circle.
- Measure of central tendency can be defined as a 'middle' value of the data.
- It is a way to find or describe the centre of a set of data.
- The mean value of a set of data is the average value of the given data.
- The weighted mean:  $xw = \frac{x_1w_1 + x_2w_2 + x_3w_3 + \dots + x_nw_n}{w_1 + w_2 + w_3 + \dots + w_n}$
- The median value of a data set is the value that lies in the central position after the data has been arranged in ascending or descending order.
- The mode is the value that is repeated the most number of times in the data set.
- The probability of an event is a number between 0 and 1, where 0 indicates impossible events and 1 indicates the events which are certain.
- Probability of an event to happen  $P(E) = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$



## Frequently Made Mistakes

Students often make mistakes adding frequency. In the case of a pie chart, if the angles of the sector are to be calculated, then they should check that their sum is  $360^\circ$ , as angles at a point add up to  $360^\circ$ . Similarly, care should be taken while calculating percentages as their total should be 100%.



## Sample Lesson Plan

### Topic

Pie charts

### Specific Learning Objectives

Students will be able to calculate the value of the angles of a pie chart.

### Suggested Duration

1 period

### Key Vocabulary

sectors, circles, frequency distribution, pie charts

### Method and Strategy

A pie chart represents information in a circle. Each distribution is represented by a sector. All the sectors together form one complete circle. The angle of each sector is calculated arithmetically and it should be pointed out that since angles at a point add up to  $360^\circ$ , the angles of all sectors should also add up to  $360^\circ$ .

Any distribution can be presented in the form of a bar graph or a pie chart.

### Example

The number of students in different classes of a school who like to play hockey are given below. Draw a pie chart to represent the same.

$$\text{Grade VI: } \frac{4}{36} \times 360^\circ = 40^\circ$$

$$\text{Grade VII: } \frac{2}{36} \times 360^\circ = 20^\circ$$

$$\text{Grade VIII: } \frac{10}{36} \times 360^\circ = 100^\circ$$

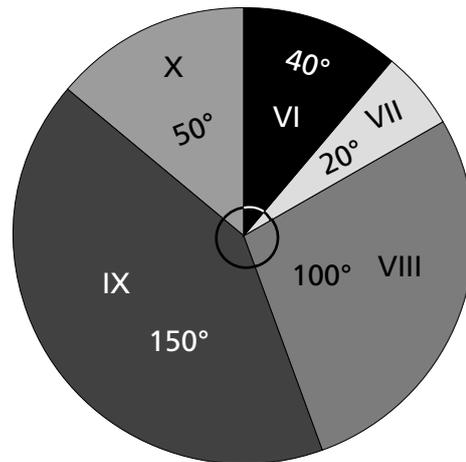
$$\text{Grade IX: } \frac{15}{36} \times 360^\circ = 150^\circ$$

$$\text{Grade X: } \frac{5}{36} \times 360^\circ = 50^\circ$$

$$\text{To check : } 40^\circ + 20^\circ + 100^\circ + 150^\circ + 50^\circ = 360^\circ$$

It should be explained that in order to calculate the values of the angles of a pie chart, we take the frequency of the subject over the total to create a fraction and then multiply by  $360^\circ$  as it is going to be a fraction of a full circle.

$$\text{Angle of a sector} = \frac{\text{frequency}}{\text{total frequency}} \times 360^\circ$$



## Written Assignment

Question 9 of Exercise 16A will be done in class and Question 10 will be given for homework. Only when the students are clear about the calculations of 'how' and 'why' will the teacher ask the students to bring protractors to the next lesson and proceed to teach the construction of pie charts.

## Evaluation

This is a presentation-based unit. Marks can be awarded on assignments and classwork involving bar graphs and pie charts.

# Assessment

A teacher's journey involves three stages Exposition, Practice, and Consolidation.

**Exposition** is the setting forth of content, and the quality and extent of the information relayed.

**Practice** involves problem solving, reasoning and proof, communication, representations, and correction.

Assessment is the final stage of **consolidation** of the process of learning.

Assessment of teaching means taking a measure of its effectiveness.

## Assessments

Students can be evaluated on various criteria and by multiple methods (Oral or written, projects, tests/ examinations, etc.) during or at the end of a session year.

Assessment is a mandatory part of the teaching and learning process. It cannot be treated isolated from the teaching and learning process. It helps both teachers and learners to judge and evaluate their efforts and pace of learning.

In mathematics it becomes more essential, as mathematical concepts are linked with each other. Concepts grasped during one teaching session serve as a basis for the learning of upcoming concepts. Teachers use assessments for several purposes such as pre-assessing the learners' need, providing relevant instruction, assessing the intended learning outcomes, placement of the learners in different groups, diagnosis of weaknesses and strengths of the learners, adjustment of teaching strategies/ techniques and promotion of the learners to the next grade. Major functions of the assessment are instructional planning, feedback, making decision, and selection of appropriate resources and strategies to move forward. In short the prime purpose of any assessment is to improve students' learning.

## Types of Assessments

Assessment is classified according to its purpose, such as:

- Assessment for Learning (AFL)
- Assessment of Learning (AOL)

## Formative Assessments:

These are commonly used as '**assessments for learning.**' Formative assessments are conducted throughout teaching practice. They show evidence of student's learning and are helpful feedback for the teachers to adjust their instructional methods to reduce the learning gaps for students.

In **assessment for learning** the teacher provides students with a feedback and support for improvement. The purpose for teachers is to:

- gather evidence of student achievement consistently, fairly, and over short periods of learning time, basically through informal methods
- monitor students' progress towards the defined learning goals
- define teaching adjustments and next steps for teaching to help students reach their potential
- adjust teaching to help students according to their potential



The most common forms of **assessment for learning (formative assessment)** are:

In-class activities where students present their findings informally and provide feedback on peer assessments, observations of students non-verbal feedback, homework exercises, questioning (open and closed), quiz, projects, selected responses (may include MCQs, true: false, matching short answers, fill-in-the-blanks, etc), open-ended tasks, performance assessments, process-focused assessments, discussions between student and teacher, answering specific questions, students reflections, students feedback collected through self-assessments etc.

### **Summative Assessments**

These are also known as 'assessments of learning.' Summative assessments check for learners' achievement at the end of the lesson, unit/ unit, or course. Usually, although not necessarily, these involve formal tests or exams. They are commonly used for grading and ranking students.

### **Assessment of Learning (Summative)**

This assessment leads to the evaluation of student learning. It accurately summarises and communicates to parents, individual students, teachers, other teachers, school leaders and policymakers what students know and can do concerning the overall curriculum expectations.

The teacher assesses a student's summative work at the end of a learning period, to determine to what degree (at what level) the student has achieved the learning goal.

The purpose for teachers is to:

- provide evidence of students' achievement during a specific class and often at the end of a learning unit
- provide assessment data for evaluation
- make judgments about the quality of students learning on the set curriculum expectations
- provide a value (pass/ fail) to that quality of learning achieved by the students
- record and report student's achievements to all stakeholders including parents, teachers, school and senior management as well as students themselves
- use this data as assessment data for the evaluation of student learning

The most common forms of **assessment of learning (summative assessment)** are:

class tests, end of unit tests, monthly tests, mid-year/ annual examinations, standardized tests, multiple choice questions (MCQ), structured papers, presentations (peer or tutor – assessed in controlled environments etc.

### **Bloom's Cognitive Domains**

The cognitive domains given below are used for assessment purpose:

- Knowing: Knowledge
- Applying: Understanding and Application
- Reasoning: Analysis, Synthesis, and Evaluation

### **Knowing:**

Knowing refers to students need to be efficient with the basic knowledge or concept on the recall of mathematical language, basic facts or mathematical concepts, symbolic representation, spatial relations, simple procedures and application of the definitions.



Action verbs for **knowing** are:

- Recall
- Identify
- Interpret
- Describe
- Recognize
- Measure
- Represent
- Explain
- State
- Arrange/ Order

**Applying:**

Applying refers to students need to be efficient with the application of mathematics in range of contexts. Students need to apply mathematical knowledge of facts, skills and procedures or understanding of mathematical concepts to create representations.

Problem solving is central to applying domain, with an emphasis on more familiar and routine tasks. Problem solving is referred to the real-life problems or concerned with the purely mathematical questions involving numeric or algebraic expressions, functions, equations, geometrical shapes or figures and statistical data sets.

Action verbs for **applying** are:

- Examine
- Compute
- Collect
- Differentiate
- Add
- Subtract
- Multiply
- Divide
- Rotate
- Reflect
- Translate
- Enlarge
- Interpret
- Manipulate
- Plot
- Factorise

## Reasoning:

Reasoning involves logical and systematic thinking. It includes intuitive and deductive reasoning based on patterns and regularities that can be used to arrive at solutions to problems set in unfamiliar situations. Such problems may be referred to purely mathematical or may have real life settings. For example, the reasoning involves ability to observe and make conjectures. It also involves logical deductions based on specific assumptions and rules.

Action verbs for **reasoning** are:

- Analyse
- Predict
- Construct
- Evaluate
- Compare
- Express
- Demonstrate
- Verify
- Solve
- Differentiate

## Content Domain

Content domain is the body of knowledge, skills or abilities that are being measured or examined by a test, experiment or research study. It may cover all aspects of the subject area as well as be well-defined objectives.

In secondary level mathematics (Grade VI – VIII), strands and bench marks of the National Curriculum (2022) are based on the following content domains:

### Numbers and Operations

- Algebra
- Measurement
- Geometry
- Statistics and Probability

## Evaluation:

An ideal and fair evaluation involves a plan that is comprehensive. It covers a broad spectrum of all aspects of mathematics. The assessment papers should test every aspect of the topics thought. These can be demarcated into categories: basic, intermediate, and advanced content. The advanced content should be minimal as it tests the most able students only.

Multiple choice questions, also known as fixed choice or selected response items, required students to identify the correct answer from a given set of possible options.

Structured questions assess various aspects of students' understanding: knowledge of content and vocabulary, reasoning skills, and mathematical proofs.

All-in-all the teaching's assessment of students' ability must be based on classroom activity, informal assessment, and final evaluation at the end of a topic and/or the year.

Cognitive domains play vital role in the development of assessment. To assess the student's in secondary classes the following ratio of cognitive domains are used.

Cognitive Domains/ Skills	Percentage weightage	Comprises of	Covers
<b>Knowing:</b>	<b>20%</b>	Recall	Recall definition, terminology, unit of measurement, geometric shapes and notations
		Describe	Description of numbers, expressions, quantities and shapes by their attributes and properties
		Convert	Conversion of numbers and quantities from one form to another
		Recognise/ Identify	Recognition of numbers, expressions, quantities, shapes and properties
		Arrange/order	Arrange numbers, expressions, quantities and shapes by common properties
		Measures	Measure geometrical shapes, lines, angles and graphs
<b>Applying:</b>	<b>40%</b>	Determine	Determine appropriate operations, strategies and tools for solving problems for which there are commonly used methods of solution
		Apply	Application of some rules, algorithm/ formula
		Manipulate	Manipulation of terms, and rules in to simpler form
		Compute	Carry out algorithmic procedure for $+$ , $-$ , $\times$ , $\div$ or combination of these with numbers, fractions, decimal and carry out straight forward algebraic expressions
<b>Reasoning:</b>	<b>40%</b>	Construct	Construction of tables, geometrical figures and graphs
		Demonstrate	Demonstration of properties of numbers and geometrical figures
		Evaluate	Evaluation of numerical values from expressions, equations, formulas and graphs
		Explain	Explanation of terminologies, formulas, algorithms and properties with reasoning
		Calculate	Calculation of quantities, expressions by using appropriate mathematical operations, formulas and techniques
		Solve	Solution of real-life situations using various mathematical strategies
		Verify	Verification of rules, identities and properties

To develop an assessment tool, a Table of Specification is used to align objectives, instructions and assessment. For example, following table explain weightage of specific topics with respect to different strands in accordance with the curriculum.

### Unit Wise Weightage to be used for Table of Specification for Grade VI

Sr. #	Strand	Title	Weightage	Total	Cognitive Domains/ Skills
1	<b>Numbers and Operations</b>	Sets	7%	37%	<b>K: 20%</b> <b>A: 40%</b> <b>R: 40%</b>
2		Real Numbers	6%		
3		Squares and Square Roots, Cubes and Cube Roots	9%		
4		Variations	4%		
5		Financial Mathematics	11%		
6	<b>Algebra</b>	Algebra	11%	18%	<b>K: 20%</b> <b>A: 40%</b> <b>R: 40%</b>
7		Linear Equations	7%		
8	<b>Geometry</b>	Geometry	9%	18%	<b>K: 20%</b> <b>A: 40%</b> <b>R: 40%</b>
9		Practical Geometry	5%		
10		Transformations	4%		
11	<b>Measurement</b>	Mensuration	9%	9%	<b>K: 20%</b> <b>A: 40%</b> <b>R: 40%</b>
12	<b>Statistics and Probability</b>	Data Handling	7%	18%	<b>K: 20%</b> <b>A: 40%</b> <b>R: 40%</b>
13		Probability	11%		
		Total Weightage	100%	100%	

**Key:**                      **Knowing (K)**                      **Applying (A)**                      **Reasoning (R)**

[Acknowledgement: Text related to assessment is with reference to Pakistan National Curriculum 2022.]

