7th
EDITION

NEW SYLLABUS
MATHEMATICS
WORKBOOK FULL SOLUTIONS

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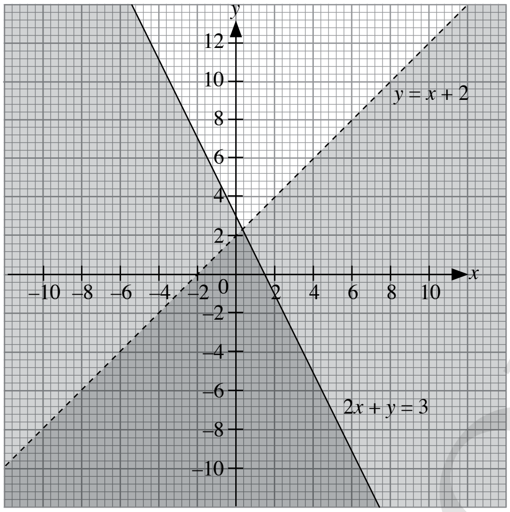
ANSWERS

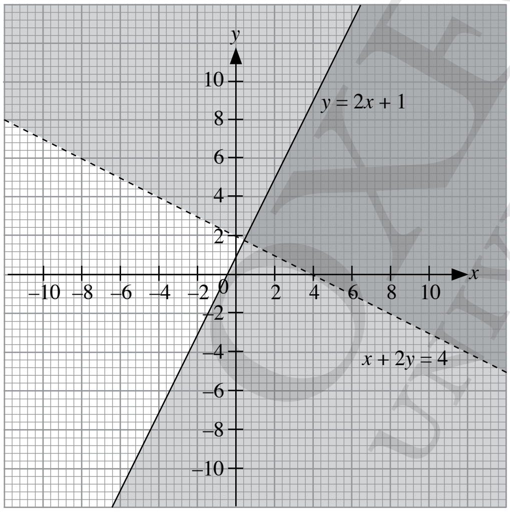
Chapter 1 Linear Inequalities In Two Variables

Basic

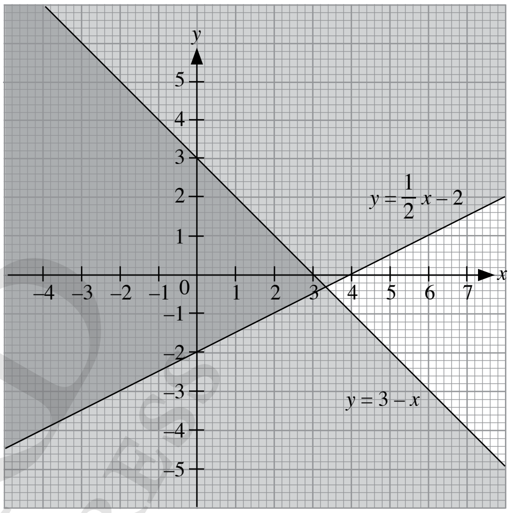
1. **(a)** $2x + y \geqslant 3, y > x + 2$



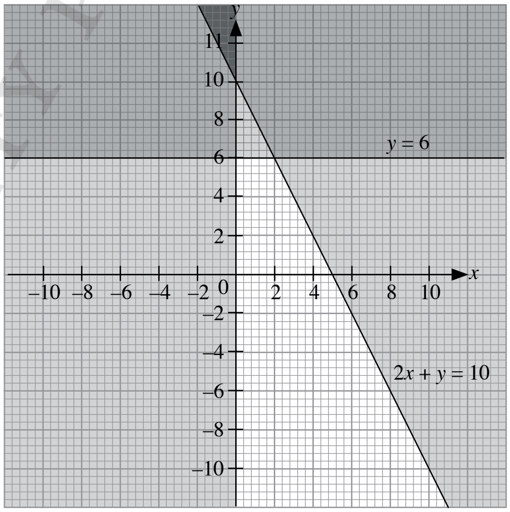
(b) $y \geqslant 2x + 1, x + 2y < 4$



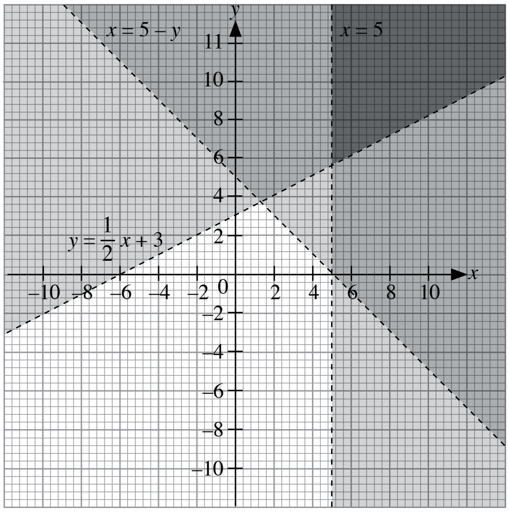
(c) $y \geqslant 3 - x, y \leqslant \frac{1}{2}x - 2$



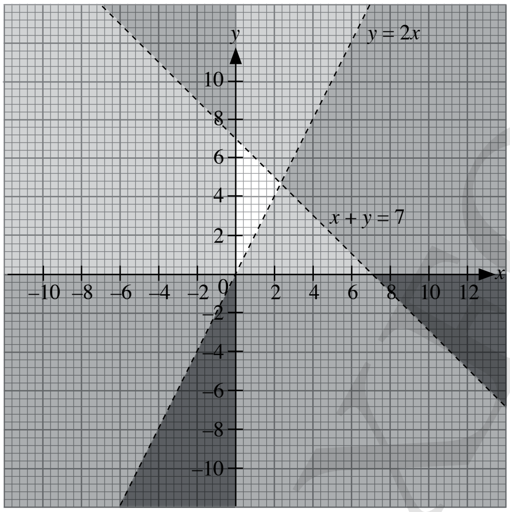
(d) $x > 0, 2x + y \leqslant 10, y \leqslant 6$



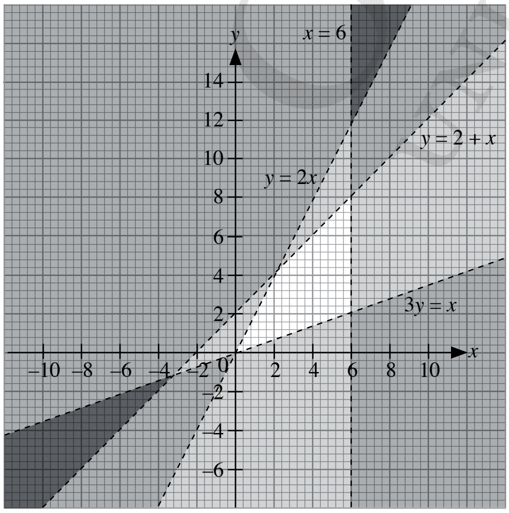
(e) $x - 5 < 0, x < 5 - y, y < \frac{1}{2}x + 3$



(f) $x \geqslant 0, y \geqslant 0, x + y < 7, y > 2x$



(g) $3y > x, y < 2x, x < 6, y < 2 + x$



Intermediate

2. **(a)** $y \geqslant 1, 2y \geqslant x + 2, 4x + 3y \leqslant 12, y \leqslant 2x + 4$

(b) $x \geqslant 1, x \leqslant 4, 2y + x \geqslant 4, 2y < x + 4$

3. Substitute the coordinates of the vertices of the unshaded region into $x + 3y$.

$x = 0, y = 1: 0 + 3(1) = 3$

$x = 1, y = 0: 1 + 3(0) = 1$

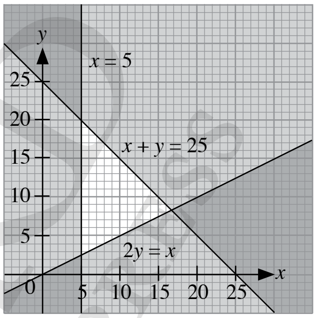
$x = 5, y = 5: 5 + 3(5) = 20$

$x = 0, y = 5: 0 + 3(5) = 15$

$\therefore$ The greatest value of $x + 3y$ is 20 and the least value is 1.

4. **(a)** $x + y \leqslant 25, x \geqslant 5, y \geqslant \frac{1}{2}x$

(b)



(c) Let the amount of syrup that Kate has be C litres.

$C = x + 0.5y$

Substitute the coordinates of the vertices of the unshaded region into $C = x + 0.5y$.

$x = 5, y = 2.5$: $C = 5 + 0.5(2.5)$
$= 6.25$

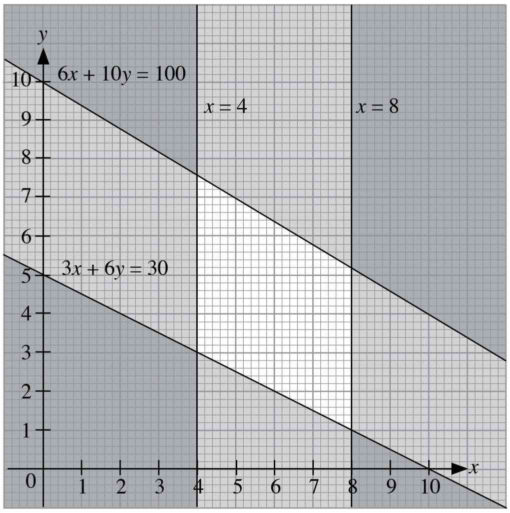
$x = 5, y = 20$: $C = 5 + 0.5(20)$
$= 15$

$x = 16, y = 9$: $C = 16 + 0.5(9)$
$= 20.5$

$\therefore$ The maximum possible amount of syrup Kate has is 20.5 litres.

5. **(a)** $6x + 10y \leqslant 100, 3x + 6y \geqslant 30, 4 \leqslant x \leqslant 8$

(b)



(c) Let the restaurant owner's expenses on flour be \$$E$.

$E = 6x + 10y$

Substitute the coordinates of the vertices of the unshaded region into $E = 6x + 10y$.

$x = 4, y = 3$: $E = 6(4) + 10(3)$

$= 54$

$x = 4, y = 7.6$: $E = 6(4) + 10(7.6)$

$= 100$

$x = 8, y = 5.2$: $E = 6(8) + 10(5.2)$

$= 100$

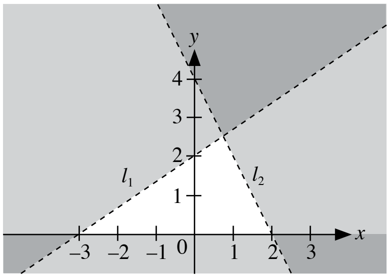
$x = 8, y = 1$: $E = 6(8) + 10(1)$

$= 58$

$\therefore$ The restaurant owner should buy 4 "Economy" packets and 3 "Giant" packets to minimise his expenses on flour.

Advanced

6. (a)



Equation of l_1: $y = \frac{2}{3}x + 2$

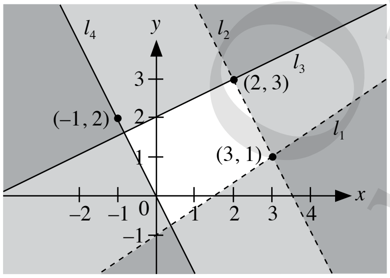
$3y = 2x + 6$

Equation of l_2: $y = -2x + 4$

$2x + y = 4$

$\therefore$ The inequalities are $y \geqslant 0$, $3y < 2x + 6$ and $2x + y < 4$.

(b)



Equation of l_1: $\frac{y-1}{x-3} = \frac{1-(-1)}{3-0}$

$\frac{y-1}{x-3} = \frac{2}{3}$

$3y - 3 = 2x - 6$

$3y = 2x - 3$

Equation of l_2: $\frac{y-1}{x-3} = \frac{3-1}{2-3}$

$\frac{y-1}{x-3} = -2$

$y - 1 = 6 - 2x$

$2x + y = 7$

Equation of l_3: $\frac{y-3}{x-2} = \frac{3-2}{2-0}$

$\frac{y-3}{x-2} = \frac{1}{2}$

$2y - 6 = x - 2$

$2y = x + 4$

Equation of l_4: $y = -2x$

$2x + y = 0$

$\therefore$ The inequalities are $3y > 2x - 3$, $2x + y < 7$, $2y \leqslant x + 4$ and $2x + y \geqslant 0$.

7. Draw the lines $y = 2x$, $2y = x$ and $x + y = 6$.

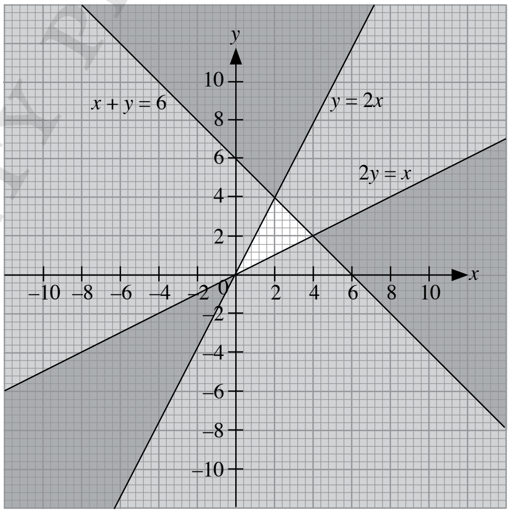
Shade the regions not required by the inequalities:

$y \leqslant 2x$, $2y \geqslant x$ and $x + y \leqslant 6$

(i) Above $y = 2x$

(ii) Below $2y = x$

(iii) Above $x + y = 6$



Substitute the coordinates of the vertices of the unshaded region into $2x + y$:

$x = 0, y = 0$: $2(0) + 0 = 0$

$x = 2, y = 4$: $2(2) + 4 = 8$

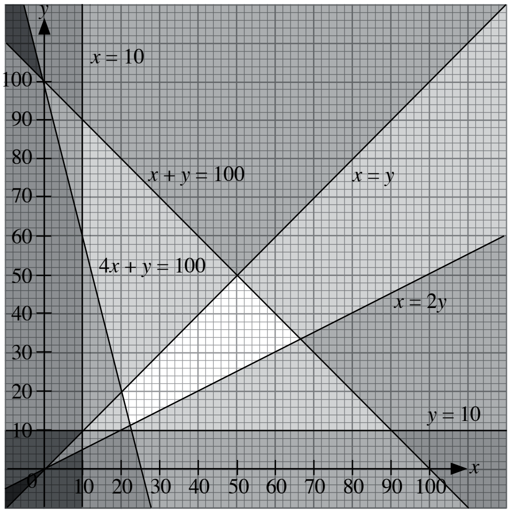
$x = 4, y = 2$: $2(4) + 2 = 10$

$\therefore$ The maximum value of $2x + y$ is 10

8. Draw the lines $x = 10$, $y = 10$, $x = y$, $x = 2y$, $x + y = 100$ and $4x + y = 100$.

Shade the regions not required by the inequalities:

$x \geqslant 10$, $y \geqslant 10$, $x \geqslant y$, $x \leqslant 2y$, $x + y \leqslant 100$ and $4x + y \geqslant 100$.



We obtain the greatest value of x from the intersection of $x = 2y$ and $x + y = 100$.

$$x + \frac{1}{2}x = 100$$

$$x = 66\frac{2}{3}$$

Least value of $x = 20$

Greatest value of $y = 50$

We obtain the least value of y from the intersection of $4x + y = 100$ and $x = 2y$.

$$4(2y) + y = 100$$

$$9y = 100$$

$$y = 11\frac{1}{9}$$

9. Draw the lines $2x = y$, $2x = 3y$, $y = 8 - 2x$ and $y = 16 - 2x$.

Shade the regions not required by the inequalities:

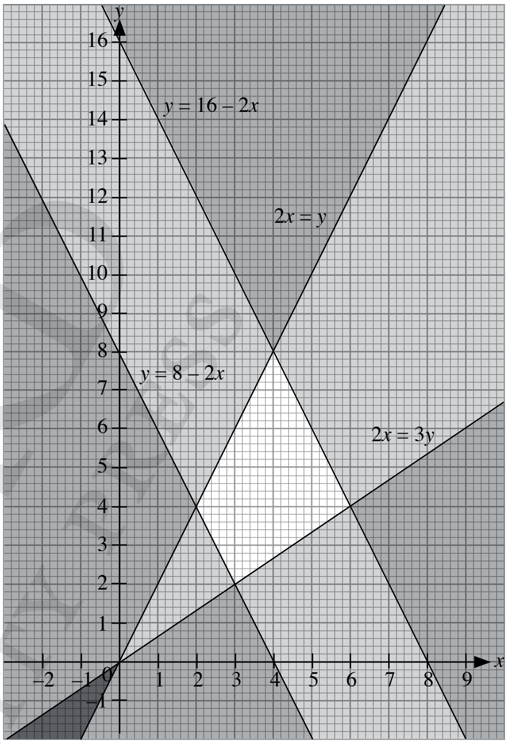
$2x \geqslant y$, $2x \leqslant 3y$, $y \geqslant 8 - 2x$ and $y \leqslant 16 - 2x$.

(i) Above $2x = y$

(ii) Below $2x = 3y$

(iii) Below $y = 8 - 2x$

(iv) Above $y = 16 - 2x$



Substitute the coordinates of the vertices of the unshaded region into $5x + 2y$.

$x = 3$, $y = 2$: $5(3) + 2(2) = 19$

$x = 2$, $y = 4$: $5(2) + 2(4) = 18$

$x = 4$, $y = 8$: $5(4) + 2(8) = 36$

$x = 6$, $y = 4$: $5(6) + 2(4) = 38$

$\therefore$ The greatest value of $5x + 2y$ is 38 and the least value is 18.

10. Draw the lines $y = 2x + 4$ and $y = 10 - 2x$.

Shade the regions not required by the inequalities:

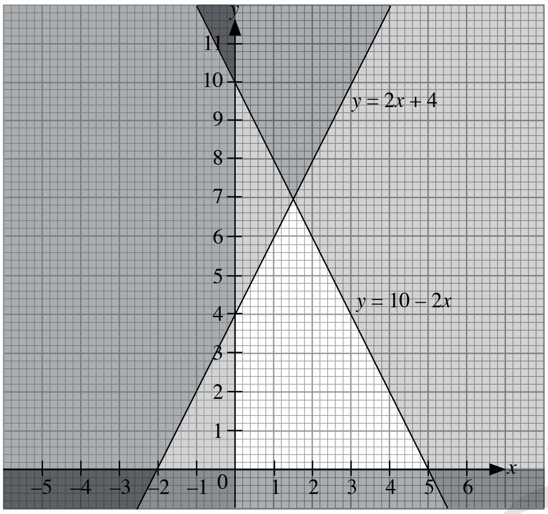
$x \geqslant 0$, $y \geqslant 0$, $y \leqslant 2x + 4$ and $y \leqslant 10 - 2x$

(i) Left of the y-axis

(ii) Below the x-axis

(iii) Above $y = 2x + 4$

(iv) Above $y = 10 - 2x$



Substitute the coordinates of the vertices of the unshaded region into $x + 2y$.

$x = 0, y = 0$: $0 + 2(0) = 0$

$x = 0, y = 4$: $0 + 2(4) = 8$

$x = 1.5, y = 7$: $1.5 + 2(7) = 15.5$

$x = 5, y = 0$: $5 + 2(0) = 5$

$\therefore$ The greatest value of $x + 2y$ is 15.5 and the least value is 0.

11. Draw the lines $2y + 3x = 40$ and $y + 2x = 24$.

Shade the regions not required by the inequalities:

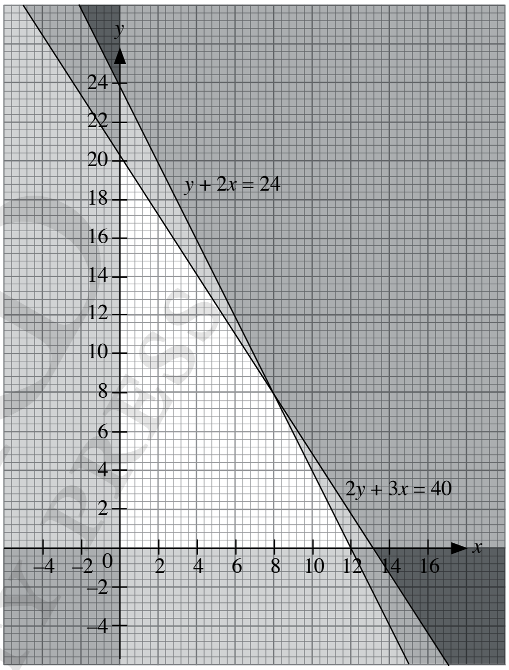
$x \geqslant 0$, $y \geqslant 0$, $2y + 3x \leqslant 40$ and $y + 2x \leqslant 24$

(i) Left of the y-axis

(ii) Below the x-axis

(iii) Above $2y + 3x = 40$

(iv) Above $y + 2x = 24$



Substitute the coordinates of the vertices of the unshaded region into $20x + 12y$.

$x = 0, y = 0$: $20(0) + 12(0) = 0$

$x = 0, y = 20$: $20(0) + 12(20) = 240$

$x = 8, y = 8$: $20(8) + 12(8) = 256$

$x = 12, y = 0$: $20(12) + 12(0) = 240$

$\therefore$ The greatest value of $20x + 12y$ is 256.

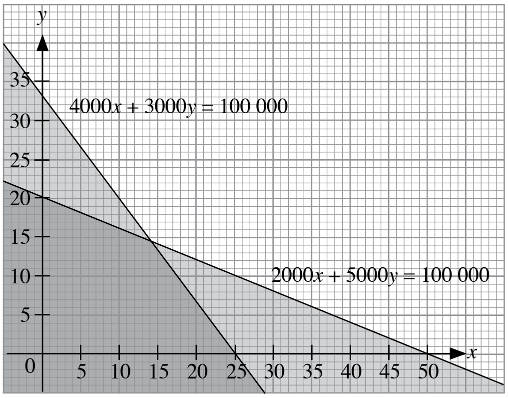
12. **(a)** $2000x + 5000y \geqslant 100\ 000$

$$y \geqslant 20 - \frac{2}{5}x$$

$4000x + 3000y \geqslant 100\ 000$

$$y \geqslant 33\frac{1}{3} - \frac{4}{3}x$$

(b)

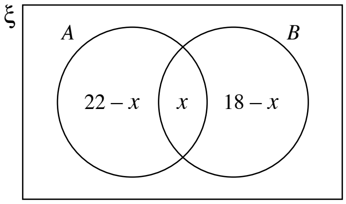


(c) From the unshaded region, the least number of machines that can package the required quantity of canned drinks and cup noodles is 15 + 14 = 29.

Chapter 2 Further Sets

Basic

1. Let A = {pupils who forgot to bring their compasses} and
B = {pupils who forgot to bring their protractors}.
Let x be the number who forgot to bring both their compasses and protractors.



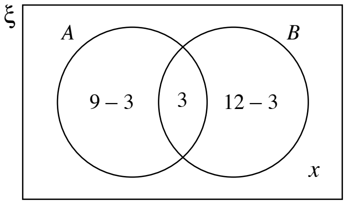
$(22 - x) + x + (18 - x) = 27$

$40 - x = 27$

$x = 13$

$\therefore$ 13 pupils had forgotten to bring both instruments.

2. Let A = {students without school badges} and
B = {students without proper school shoes}.
Let x be the number of students who did not commit any of the offences.



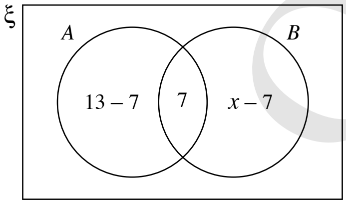
$(9 - 3) + 3 + (12 - 3) + x = 39$

$18 + x = 39$

$x = 21$

$\therefore$ 21 students did not commit any of the offences.

3. Let x be $n(B)$.



Since $n(A \cup B) = 50$

$(13 - 7) + 7 + (x - 7) = 50$

$6 + x = 50$

$x = 44$

$\therefore n(B) = 44$

4. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 22 + 5 - 13$

$= 14$

5. **(i)** $n(A \cap B)$ will have the greatest value when $B \subseteq A$.
Greatest value of $n(A \cap B) = n(B)$

$= 10$

A and B cannot be disjoint sets since
$n(A) + n(B) > n(\xi)$
Least value of $n(A \cap B) = (24 + 10) - 32$

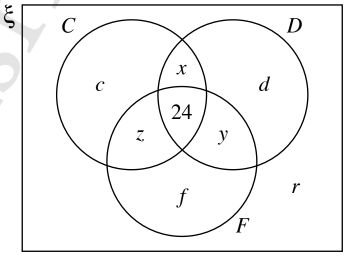
$= 2$

(ii) Since A and B cannot be disjoint sets, greatest value of $n(A \cup B) = 32$
$n(A \cup B)$ will have the least value when $B \subseteq A$.
Least value of $n(A \cup B) = n(A)$

$= 24$

Intermediate

6. Let C = {customers who ordered chicken rice}
D = {customers who ordered duck rice}
F = {customers who ordered fried rice}
$\therefore n(C) = 45, n(D) = 52, n(F) = 51$
Let x represent the customers who ordered chicken rice and duck rice only, y to represent the customers who ordered duck rice and fried rice only, z to represent the customers who ordered chicken rice and fried rice only, c to represent the customers who ordered chicken rice only, d to represent the customers who ordered duck rice only, f to represent the customers who ordered fried rice only and r for the customers who did not order anything.



$n(C \cap D) = 31$

$x + 24 = 31$

$x = 7$

$n(D \cap F) = 33$

$y + 24 = 33$

$y = 9$

$n(C \cap F) = 37$

$z + 24 = 37$

$z = 13$

$n(C) = 45$

$c + 7 + 13 + 24 = 45$

$c = 1$

$n(D) = 52$

$d + 7 + 24 + 9 = 52$

$d = 12$

$n(F) = 51$

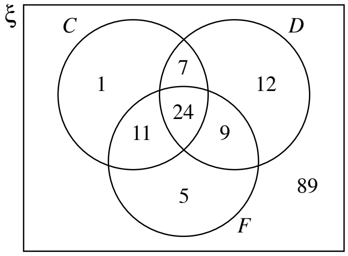
$f + 13 + 24 + 9 = 51$

$f = 5$

$n(\xi) = 160$

$r = 160 - 24 - 7 - 9 - 13 - 1 - 12 - 5$

$= 89$



(i) Number of customers who ordered more than one type of rice = 7 + 13 + 9 + 24

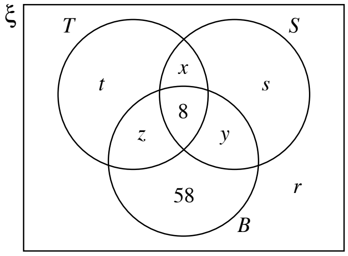
= 53

(ii) Number of customers who visited but decided not to patronise the stall = 89

7. Let T = {members who play tennis},

S = {members who play squash},

B = {members who play badminton}.



Let x represent the number of members who play tennis and squash only, y to represent the number of members who play squash and badminton only, z to represent the number of members who play tennis and badminton only, t to represent the number of members who play tennis only and s to represent the number of members who play squash only.

(i) $n(T \cap S) = 22$

$x + 8 = 22$

$x = 14$

$n(T \cap B) = 15$

$z + 8 = 15$

$z = 7$

$n(T) = 68$

$t + 14 + 7 + 8 = 68$

$t = 39$

∴ There are 39 members who play tennis only.

(ii) $n(S \cap B) = 20$

$y + 8 = 20$

$y = 12$

$n(S) = 62$

$s + 14 + 12 + 8 = 62$

$s = 28$

∴ There are 28 members who play squash only.

(iii) $n(B) = 7 + 12 + 8 + 58$

$= 85$

∴ There are 85 members who play badminton.

(iv) $n(\xi) = 200$

$r = 200 - 68 - 28 - 12 - 58$

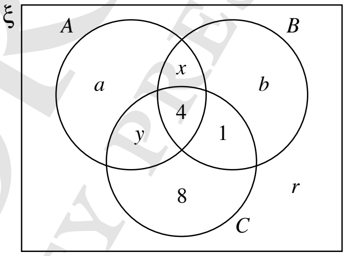
$= 34$

∴ There are 34 members who do not play any of the games.

8. Let A = {schools without proper fire exit signs},

B = {schools with insufficient fire extinguishers},

C = {schools with faulty fire alarm systems}.



Let x represent the number of schools without proper fire exit signs and insufficient fire extinguishers only, y to represent the number of schools without proper fire exit signs and faulty fire alarm systems only, a to represent the number of schools without proper fire exit signs only, b to represent the number of schools who had insufficient fire extinguishers only and r to represent the number of schools without any faults.

(i) Since there are 5 schools with exactly two faults,

$x + y + 1 = 5$

$x + y = 4$

$n(A) = 18$

$a = 18 - 4 - 4$

$= 10$

∴ There are 10 schools without proper fire exits only.

(ii) n(C) = 15

$y = 15 - 4 - 1 - 8$

$= 2$

$x + 2 = 4$

$x = 2$

n(B) = 24

$b = 24 - 2 - 4 - 1$

$= 17$

∴ There are 17 schools who had only insufficient fire extinguishers.

(iii) n(ξ) = 120

$r = 120 - 18 - 17 - 1 - 8$

$= 76$

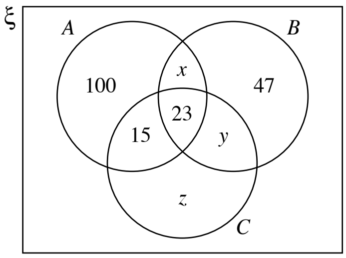
∴ There are 76 schools without any of the three faults.

9. **(i)** According to this survey, A is the most popular product.

(ii) Let A = {respondents who use product A},

B = {respondents who use product B},

C = {respondents who use product C}.



Let x represent the number of respondents who use A and B but not C, y to represent number of respondents who use B and C but not A, and z to represent the number of respondents who use only C.

n($B \cap C$) = 40

$y + 23 = 40$

$y = 17$

n(ξ) = 280

$158 + 47 + 17 + z = 280$

$z = 58$

∴ 58 respondents use product C only.

(iii) n(A) = 158

$x = 158 - 100 - 15 - 23$

$= 20$

Number of respondents who use at least two products

$= 20 + 15 + 17 + 23$

$= 75$

$$\text{Required fraction} = \frac{75}{280}$$

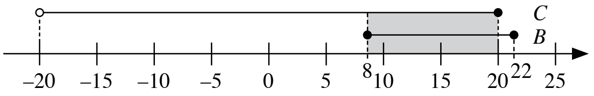
$$= \frac{15}{56}$$

(iv) Number of respondents who use only one of the products = 100 + 47 + 58

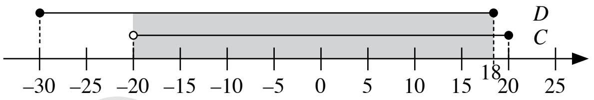
$= 205$

10. **(i)** $A' = \varnothing$

(ii) $B \cap C = \{x\text{: } 8 \leqslant x \leqslant 20\}$

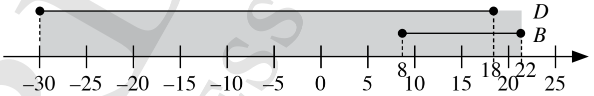


(iii) $C \cap D = \{-20 < x \leqslant 18\}$

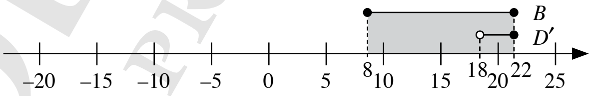


(iv) $B \cup D = \{x : -30 \leqslant x \leqslant 22\}$

$= \xi$



(v) $B \cup D' = \{x\text{: } 8 < x \leqslant 22\}$



11. A is the set of points on the straight line $y = \frac{1}{2}x + 9$ with gradient $\frac{1}{2}$ and y-intercept 9.

B is the set of points on the line $y = px + q$.

Since n($A \cap B$) = 0, the two lines do not intersect.

Therefore, they must be parallel, i.e. $p = \frac{2}{3}$.

q can take any value except 9, one possible value of q is 5.

12. **(i)** Since A and B are disjoint sets,

n($A \cap B$) = 0

(ii) Since A and B are disjoint sets,

n($A \cup B$) = 55 + 12

$= 67$

13. **(i)** n($A \cup B$) = n(A) + n(B) – n($A \cap B$)

$= 75 + 28 - 15$

$= 88$

(ii) n($P \cup Q$) = 46 + 24

$= 70$

(iii) n($R \cap S$) = n(R)

$= 43$

Advanced

14. **(i)** If $d = 0$,

$n(\xi) = n(X) + n(Y) - n(X \cap Y)$

$120 = (a + b) + (b + c) - b$

$b = 80 + 48 - 120$

$= 8$

(ii) $n(X) = a + b$

$a + b = 80$

$n(Y) = b + c$

$2b = 48$

$b = 24$

$\therefore c = 24$

Substitute $b = 24$ into $a + b = 80$:

$a + 24 = 80$

$a = 56$

$d = 120 - 56 - 24 - 24$

$= 16$

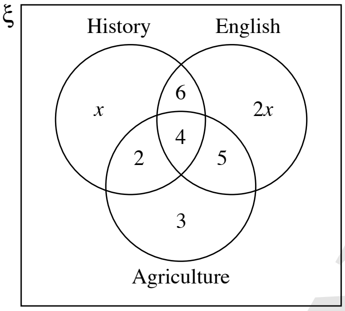
(iii) d will have the greatest value when $Y \subseteq X$.

Greatest value of $d = 120 - n(X)$

$= 120 - 80$

$= 40$

15.



(i) Since there are 42 students who do not study Agriculture,

$x + 6 + 2x = 42$

$3x = 36$

$x = 12$

(ii) Total number of pupils studying English

$= 6 + 4 + 5 + 2(12)$

$= 39$

16. Let K = {pupils proficient in Korean} and
G = {pupils proficient in German}.

(i) The number of pupils proficient in both languages is at its maximum when $K \subseteq G$.

Maximum possible number of pupils proficient in both Korean and German

$= n(K)$

$= 14$

(ii) The number of pupils proficient in both languages is at its least when $K \cap G = \varnothing$.

Least possible number of pupils proficient in both Korean and German = 0

(iii) There is a maximum number of pupils proficient in only one foreign language when $K \cap G = \varnothing$.

Maximum number of pupils proficient in only one foreign language $= K \cup G$

$= 14 + 17$

$= 31$

Chapter 3 Probability of Combined Events

Basic

1. The fifteen cards are labelled 16, 17, 18, …, 30.

(a) $\text{P(contains 7)} = \frac{2}{15}$

(b) $\text{P(contains at least a 2)} = \frac{10}{15} = \frac{2}{3}$

(c) $\text{P(multiple of 3)} = \frac{5}{15} = \frac{1}{3}$

(d) $\text{P(prime)} = \frac{4}{15}$

(e) $\text{P(divisible by 5)} = \frac{3}{15} = \frac{1}{5}$

2. There are 5 red balls, 6 white balls and 9 green balls.

(a) $\text{P(green)} = \frac{9}{20}$

(b) $\text{P(red and white)} = \frac{11}{20}$

(c) There are no yellow balls.
P(yellow) = 0

(d) P(red, green or white) = 1

3. The ten cards are numbered: 1, 2, 2, 3, 3, 3, 5, 7, 8, 9

(a) $\text{P(prime number)} = \frac{7}{10}$

(b) $\text{P(divisible by 3)} = \frac{4}{10} = \frac{2}{5}$

4. There are x white marbles (W), y blue marbles (B) and 8 red marbles (R).

$$P(B) = \frac{y}{x+y+8} = \frac{8}{15}$$
$$8x + 8y + 64 = 15y$$
$$8x + 64 = 7y \text{ — (1)}$$
$$P(W) = \frac{x}{x+y+8} = \frac{1}{5}$$
$$5x = x + y + 8$$
$$4x - 8 = y \text{ — (2)}$$

Substitute (2) into (1):

$$8x + 64 = 7(4x - 8)$$
$$8x + 64 = 28x - 56$$
$$20x = 120$$
$$x = 6$$
$$\therefore x = 6$$
$$y = 4(6) - 8 = 16$$

$\therefore$ Total number of marbles = 6 + 16 + 8 = 30

5. **(a)** $\text{P(a ‘5’)} = \frac{1}{12}$

(b) $\text{P(a heart)} = \frac{2}{12} = \frac{1}{6}$

(c) $\text{P(a spade)} = \frac{6}{12} = \frac{1}{2}$

(d) $\text{P(a picture card)} = \frac{6}{12} = \frac{1}{2}$

(e) P(the ace of diamond) = 0

6. **(a)**

		y					
		1	2	3	4	5	6
x	1	0	–1	–2	–3	–4	–5
	2	1	0	–1	–2	–3	–4
	3	2	1	0	–1	–2	–3
	4	3	2	1	0	–1	–2
	5	4	3	2	1	0	–1
	6	5	4	3	2	1	0

(b) **(i)** $\text{P(negative)} = \frac{15}{36} = \frac{5}{12}$

(ii) $\text{P(positive and even)} = \frac{6}{36} = \frac{1}{6}$

(iii) $\text{P(non-zero)} = \frac{30}{36} = \frac{5}{6}$

(iv) $P(\geqslant 2) = \frac{10}{36} = \frac{5}{18}$

(v) $\text{P(not a multiple of 3)} = \frac{24}{36} = \frac{2}{3}$

7. There are 9 men (M), 6 women (W), 12 boys (B) and 3 girls (G).

(a) $\text{P(male)} = \frac{9+12}{30} = \frac{7}{10}$

(b) $P(W \text{ or } B \text{ or } G) = \frac{6+12+3}{30} = \frac{7}{10}$

8. There are x red balls and $(35 - x)$ blue balls.

(a) $\text{P(red)} = \frac{x}{35}$

(b) After 5 red balls are removed, there are $(x - 5)$ red balls and $(30 - x)$ blue balls.

$$\text{P(red)} = \frac{x-5}{30} = \frac{x}{35} - \frac{1}{14}$$
$$\frac{x-5}{30} = \frac{2x-5}{70}$$
$$70x - 350 = 60x - 150$$
$$10x = 200$$
$$\therefore x = 20$$

9. There are x red balls (R), $(x+3)$ blue balls (B) and $(3x-1)$ white balls (W).

(a) $P(R) = \frac{x}{11} = \frac{x}{x+(x+3)+(3x-1)}$

$11x = 10x + 4$

$\therefore x = 4$

(b) There are $4R$, $7B$, $11W$.

(i) $P(W) = \frac{11}{22} = \frac{1}{2}$

(ii) $P(BB) = \frac{7}{22} \times \frac{6}{21} = \frac{1}{11}$

Intermediate

10. (a) (i) $P(< 4) = P(1, 2 \text{ or } 3) = \frac{3}{8}$

(ii) $P(\text{a prime number}) = P(2, 3, 5, 7) = \frac{4}{8} = \frac{1}{2}$

(iii) $P(6 \text{ or } 8) = \frac{2}{8} = \frac{1}{4}$

(b)

×	1	2	3	4	5	6	7	8
1	1	2	3	4	5	6	7	8
2	2	4	6	8	10	12	14	16
3	3	6	9	12	15	18	21	24
4	4	8	12	16	20	24	28	32
5	5	10	15	20	25	30	35	40
6	6	12	18	24	30	36	42	48
7	7	14	21	28	35	42	49	56
8	8	16	24	32	40	48	56	64

(i) $P(\text{odd}) = \frac{16}{64} = \frac{1}{4}$

(ii) $P(\text{even}) = 1 - P(\text{odd}) = 1 - \frac{1}{4} = \frac{3}{4}$

(iii) $P(\text{a perfect square}) = \frac{12}{64} = \frac{3}{16}$

(iv) $P(\text{not a perfect cube}) = 1 - P(\text{a perfect cube})$

$= 1 - \frac{6}{64}$

$= \frac{29}{32}$

(v) $P(\text{a prime number}) = \frac{8}{64} = \frac{1}{8}$

(vi) $P(\text{a multiple of } 6) = \frac{21}{64}$

(vii) $P(\leqslant 20) = \frac{38}{64} = \frac{19}{32}$

(viii) $P(\text{divisible by 3 or 5}) = \frac{39}{64}$

(ix) $P(\text{divisible by 3 and 4}) = \frac{11}{64}$

11. (a)

+	1	2	3	4	5	6	8
3	4	5	6	7	8	9	11
5	6	7	8	9	10	11	13
7	8	9	10	11	12	13	15
9	10	11	12	13	14	15	17

×	1	2	3	4	5	6	8
3	3	6	9	12	15	18	24
5	5	10	15	20	25	30	40
7	7	14	21	28	35	42	56
9	9	18	27	36	45	54	72

(b) (i) $P(\text{sum} > 5) = \frac{26}{28} = \frac{13}{14}$

(ii) $P(\text{sum} \leqslant 9) = \frac{12}{28} = \frac{3}{7}$

(iii) $P(\text{sum is prime}) = \frac{11}{28}$

(iv) $P(\text{sum is a multiple of 5}) = \frac{6}{28} = \frac{3}{14}$

(v) $P(\text{product is odd}) = \frac{12}{28} = \frac{3}{7}$

(vi) $P(\text{product is even}) = \frac{16}{28} = \frac{4}{7}$

(vii) $P(\text{product consists of two digits}) = \frac{22}{28} = \frac{11}{14}$

(viii) $P(\text{product is divisible by 4}) = \frac{8}{28} = \frac{2}{7}$

(ix) $P(\text{product} \geqslant 20) = \frac{15}{28}$

(x) $P(\text{product is a perfect square}) = \frac{3}{28}$

12. $A = \{2, 3\}$, $B = \{1, 3, 9\}$, $C = \{2, 4, 6, 8, 10\}$

(a) $A \cap B = \{3\}$

(b) $P(\text{number is in } C) = \frac{5}{8}$

(c) $P(\text{number is in } B) = \frac{1}{2}$

13. $\xi = \{41, 42, 43, \ldots, 59, 60\}$

(a) P(an even number) $= \frac{10}{20} = \frac{1}{2}$

(b) P(a perfect square) $= \frac{1}{20}$

(c) P(a multiple of 7) $= \frac{3}{20}$

(d) P(product of its two digits is odd)

$= \text{P}(51, 53, 55, 57, 59)$

$= \frac{5}{20}$

$= \frac{1}{4}$

(e) **(i)** $\text{P}(\text{sum} > 10) = \text{P}(47, 48, 49, 56, 57, 58, 59)$

$= \frac{7}{20}$

(ii) $\text{P}(\text{sum} > 4) = 1$

(iii) $\text{P}(\text{sum} > 15) = 0$

14. **(a)** P(Michael does not proceed to JC or Poly)

$= 1 - \frac{3}{8} - \frac{1}{3}$

$= \frac{7}{24}$

(b) P(Michael proceeds to JC while Shirley proceeds to neither JC nor Poly)

$= \frac{3}{8} \times \left[1-\left(\frac{5}{8}+\frac{1}{4}\right)\right]$

$= \frac{3}{64}$

(c) P(only one proceeds to JC)

= P(Michael proceeds to JC and Shirley does not) or P(Shirley proceeds to JC and Michael does not)

$= \frac{3}{8} \times \left(1-\frac{5}{8}\right) + \left(1-\frac{3}{8}\right) \times \frac{5}{8}$

$= \frac{17}{32}$

15. There are 8 white discs (W), 12 green discs (G) and x yellow discs (Y).

(a) $\text{P}(Y) = \frac{x}{8+12+x} = \frac{2}{7}$

$7x = 40 + 2x$

$5x = 40$

$x = 8$

(b) **(i)** $\text{P}(WW) = \frac{8}{28} \times \frac{7}{27} = \frac{2}{27}$

(ii) $\text{P}(GG) = \frac{12}{28} \times \frac{11}{27} = \frac{11}{63}$

(iii) $\text{P}(WY) = \text{P}(WY \text{ or } YW)$

$= \frac{8}{28} \times \frac{8}{27} + \frac{8}{28} \times \frac{8}{27}$

$= \frac{32}{189}$

(iv) $\text{P}(G \text{ and black}) = 0$

16. **(a)** $\{(5C, 0W), (4C, 1W), (3C, 2W), (2C, 3W), (1C, 4W), (0C, 5W)\}$

$= \{20, 15, 13, 5, 0, -5\}$

(b) **(i)** P(20 marks) $= \frac{1}{6}$

(ii) P(0 marks) $= \frac{1}{6}$

(iii) P(> 6 marks) $= \frac{3}{6} = \frac{1}{2}$

(iv) P(< –3 marks) $= \frac{1}{6}$

17. **(a)**

+	1	2	3	4	5	6	7	8
1	2	3	4	5	6	7	8	9
2	3	4	5	6	7	8	9	10
3	4	5	6	7	8	9	10	11
4	5	6	7	8	9	10	11	12
5	6	7	8	9	10	11	12	13
6	7	8	9	10	11	12	13	14
7	8	9	10	11	12	13	14	15
8	9	10	11	12	13	14	15	16

(b) **(i)** P(even) $= \frac{32}{64} = \frac{1}{2}$

(ii) P(odd) $= \frac{32}{64} = \frac{1}{2}$

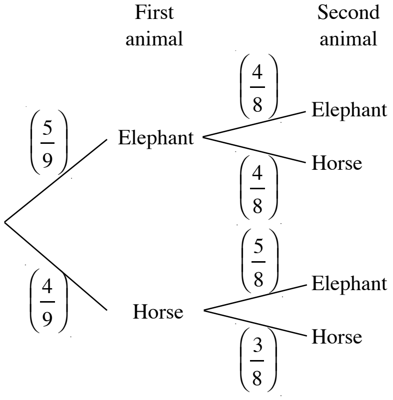
(iii) P(prime) $= \frac{23}{64}$

(iv) $\text{P}(\leqslant 10) = \frac{43}{64}$

(v) $\text{P}(> 5) = \frac{54}{64} = \frac{27}{32}$

(vi) P(multiple of 3) $= \frac{22}{64} = \frac{11}{32}$

18. **(a)**



(b) (i) P(first animal is horse and second is elephant)

$= \frac{4}{9} \times \frac{5}{8}$

$= \frac{5}{18}$

(ii) P(at least one of the animals is an elephant)

= 1 – P(both horses)

$= 1 - \frac{4}{9} \times \frac{3}{8}$

$= \frac{5}{6}$

Alternatively,

P(at least one of the animals is an elephant)

= P(Elephant, Elephant) or P(Elephant, Horse) or P(Horse, Elephant)

$= \frac{5}{9} \times \frac{4}{8} + \frac{5}{9} \times \frac{4}{8} + \frac{4}{9} \times \frac{5}{8}$

$= \frac{5}{6}$

(iii) P(second animal chosen is a horse)

= P(Elephant, Horse) or P(Horse, Horse)

$= \frac{5}{9} \times \frac{4}{8} + \frac{4}{9} \times \frac{3}{8}$

$= \frac{4}{9}$

19. There are x red marbles (R), y yellow marbles (Y) and 55 blue marbles (B).

(a) $P(R) = \frac{1}{8} = \frac{x}{x+y+55}$

$8x = x + y + 55$

$y = 7x - 55$ — (1)

$P(Y) = \frac{5}{12} = \frac{x}{x+y+55}$

$12y = 5x + 5y + 275$

$7y = 5x + 275$ — (2)

(b) Substitute (1) into (2) :

$7(7x - 55) = 5x + 275$

$49x - 385 = 5x + 275$

$44x = 660$

$x = 15$

Substitute $x = 15$ into (1) :

$y = 7(15) - 55$

$= 50$

(c) Now, there are 15 R, 50 Y and 55 B.

(i) $P(RR) = \frac{15}{120} \times \frac{14}{119} = \frac{1}{68}$

(ii) P(one R and one B)

= P(RB or BR)

$= \frac{15}{120} \times \frac{55}{119} + \frac{55}{120} \times \frac{15}{119}$

$= \frac{55}{476}$

(iii) P(2 marbles of different colours)

= P(RB , RY, YB, BR, YR, BY)

$= \left(\frac{15}{120} \times \frac{55}{119} + \frac{15}{120} \times \frac{50}{110} + \frac{50}{120} \times \frac{55}{119}\right) \times 2$

$= \frac{865}{1428}$

20. There are x red balls (R) and $(15 - x)$ white balls (W).

(a) $P(R) = \frac{x}{15}$

(b) $P(RR) = \frac{x}{15} \times \frac{x-1}{14} = \frac{x(x-1)}{210}$

(c) $\frac{x}{15} \times \frac{x-1}{14} = \frac{12}{35}$

$35x(x - 1) = 12 \times 210$

$x(x - 1) = 72$

$x^2 - x = 72$

(d) $x^2 - x - 72 = 0$

$(x + 8)(x - 9) = 0$

$\therefore x = -8$ (NA) or $x = 9$

∴ There are 6 white balls in the bag.

21. (a) $P(Y) = \frac{60°}{360°} = \frac{1}{6}$

(b) (i) $P(RB) = \frac{120}{360} \times \frac{120}{360} = \frac{1}{9}$

(ii) P(G at second spin)

= P(GG, RG, BG, YG)

$= \frac{60}{360} \times \frac{60}{360} + \frac{120}{360} \times \frac{60}{360} + \frac{120}{360} \times \frac{60}{360} + \frac{60}{360} \times \frac{60}{360}$

$= \frac{1}{6}$

(iii) P(Y or R)

= P(YY, YR, RY, RR)

$= \frac{1}{6} \times \frac{1}{6} + \frac{1}{6} \times \frac{120}{360} + \frac{120}{360} \times \frac{1}{6} + \frac{120}{360} \times \frac{120}{360}$

$= \frac{1}{4}$

(iv) P(different colours at both spins)

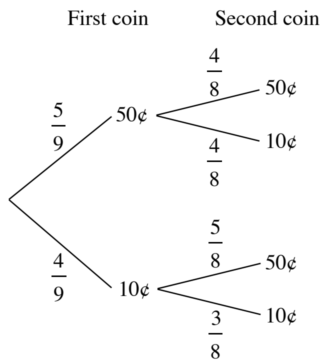
= 1 – P(same colour at both spins)

= 1 – P(RR or YY or BB or GG)

$= 1 - \left(\frac{1}{3} \times \frac{1}{3} + \frac{1}{6} \times \frac{1}{6} + \frac{1}{3} \times \frac{1}{3} + \frac{1}{6} \times \frac{1}{6}\right)$

$= \frac{13}{18}$

22. (a)



(b) (i) P(20 cents in total) = P(10 cents, 10 cents)

$= \frac{4}{9} \times \frac{3}{8}$

$= \frac{1}{6}$

(ii) P(60 cents in total)

= P(10 cents, 50 cents) or P(50 cents, 10 cents)

$= \frac{4}{9} \times \frac{5}{8} + \frac{5}{9} \times \frac{4}{8}$

$= \frac{5}{9}$

(c) (i) P(70 cents in total)

= P(50 cents, 10 cents, 10 cents) or
P(10 cents, 50 cents, 10 cents) or
P(10 cents, 10 cents, 50 cents)

$= \frac{5}{9} \times \frac{4}{8} \times \frac{3}{7} + \frac{4}{9} \times \frac{5}{8} \times \frac{3}{7} + \frac{4}{9} \times \frac{3}{8} \times \frac{5}{7}$

$= \frac{5}{14}$

(ii) P(at least \$1.10)

= P(50 cents, 50 cents, 50 cents) or
P(50 cents, 50 cents, 10 cents) or
P(10 cents, 50 cents, 50 cents) or
P(50 cents, 10 cents, 50 cents)

$= \frac{5}{9} \times \frac{4}{8} \times \frac{3}{7} + \frac{5}{9} \times \frac{4}{8} \times \frac{4}{7} + \frac{4}{9} \times \frac{5}{8} \times \frac{4}{7}$

$+ \frac{5}{9} \times \frac{4}{8} \times \frac{4}{7}$

$= \frac{25}{42}$

23. (a) P(box *B* is chosen) $= \frac{1}{2}$

(b) P(even number on ball) = P(*A* even or *B* even)

$= \frac{1}{2} \times \frac{4}{9} + \frac{1}{2} \times \frac{3}{6}$

$= \frac{17}{36}$

(c) P(box *A* is chosen and even number on ball)

$= \frac{1}{2} \times \frac{4}{9}$

$= \frac{2}{9}$

(d) P(box *B* is chosen and prime number on ball)

$= \frac{1}{2} \times \frac{3}{6}$

$= \frac{1}{4}$

24. (a) P(both alive) $= 0.45 \times 0.5 = \frac{9}{40}$

(b) P(only wife alive) = P(man dies and wife survives)

$= (1 - 0.45) \times 0.5$

$= \frac{11}{40}$

(c) P(at least one of them survives)

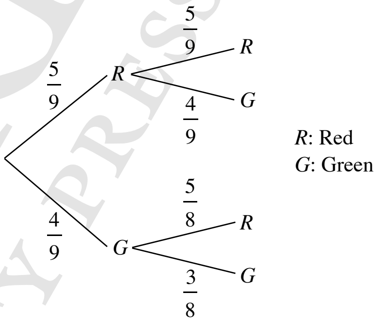
= 1 – P(both do not survive)

$= 1 - (1 - 0.45) \times (1 - 0.5)$

$= 1 - \frac{11}{40}$

$= \frac{29}{40}$

25. (a)



(b) (i) $P(RR) = \frac{5}{9} \times \frac{5}{9} = \frac{25}{81}$

(ii) P(different colours) = P(*RG* or *GR*)

$= \frac{5}{9} \times \frac{4}{9} + \frac{4}{9} \times \frac{5}{8}$

$= \frac{85}{162}$

(iii) P(at least three green balls are left)

$= 1 - P(RR) - P(RG) - P(GR)$

$= 1 - \frac{5}{9} \times \frac{5}{9} - \frac{5}{9} \times \frac{4}{9} - \frac{4}{9} \times \frac{5}{8}$

$= \frac{1}{6}$

26. (a) P(only *LL* solves)

= P(*LL* solves and *LS* does not solve)

$= \frac{1}{2} \times \left(1 - \frac{2}{5}\right)$

$= \frac{3}{10}$

(b) P(at least one of them solves)

= 1 – P(both do not solve)

$= 1 - \frac{3}{5} \times \frac{1}{2}$

$= \frac{7}{10}$

27. **(a)** P(two diamonds) = $\frac{13}{52} \times \frac{12}{51} = \frac{1}{17}$

(b) P(two Queens) = $\frac{4}{52} \times \frac{3}{51} = \frac{1}{221}$

(c) P(one heart and one spade)

= P(heart, spade or spade, heart)

$= \frac{13}{52} \times \frac{13}{51} + \frac{13}{52} \times \frac{13}{51}$

$= \frac{13}{102}$

28. There are 7 toffees in green paper (*TG*), 4 barley sugar in red paper (*BR*), 3 toffees in red paper (*TR*) and 6 barley sugar in green paper (*BG*).

(a) P(*T* and *BR*) = $\frac{10}{20} \times \frac{4}{19} = \frac{2}{19}$

(b) P(*TT*) = $\frac{10}{20} \times \frac{9}{19} = \frac{9}{38}$

(c) P(*BG*, *BG*) = $\frac{6}{20} \times \frac{5}{19} = \frac{3}{38}$

(d) P(same flavour) = P(*TT* or *BB*)

$= \frac{10}{20} \times \frac{9}{19} + \frac{10}{20} \times \frac{9}{19}$

$= \frac{9}{19}$

(e) P(different colour) = P(*GR* or *RG*)

$= \frac{13}{20} \times \frac{7}{19} + \frac{7}{20} \times \frac{13}{19}$

$= \frac{91}{190}$

29. There are 6 yellow marbles (*Y*) and 3 green marbles (*G*).

(a) P(*YY* with replacement) = $\frac{6}{9} \times \frac{6}{9} = \frac{4}{9}$

(b) P(*YY* without replacement) = $\frac{6}{9} \times \frac{5}{8} = \frac{5}{12}$

Advanced

30. **(a)** P(to *Q*) = $\frac{1}{3}$

(b) P(to *T*) = P(straight and right)

$= \frac{1}{2} \times \frac{1}{6}$

$= \frac{1}{12}$

(c) P(to *U*) = P(straight and straight)

$= \frac{1}{2} \times \frac{1}{2}$

$= \frac{1}{4}$

31. There are 3 red socks (*R*) and 5 green socks (*G*) in the first bag and 6 red socks (*R*) and 4 green socks (*G*) in the second bag.

(a) P(both *R*) = P(*RR*)

$= \frac{3}{8} \times \frac{6}{5}$

$= \frac{9}{40}$

(b) P(at least one is *G*) = 1 – P(*RR*)

$= 1 - \frac{9}{40}$

$= \frac{31}{40}$

(c) P(different colours) = P(*RG* or *GR*)

$= \frac{3}{8} \times \frac{4}{10} + \frac{5}{8} \times \frac{6}{10}$

$= \frac{21}{40}$

32. P(getting distinction in English) = P(*E*) = $\frac{5}{7}$

P(getting distinction in Maths) = P(*M*) = $\frac{3}{4}$

P(getting distinction in Science) = P(*S*) = $\frac{5}{6}$

(a) P(no distinction) = $\frac{2}{7} \times \frac{1}{4} \times \frac{1}{6} = \frac{1}{84}$

(b) P(exactly one distinction)

= P(*EM'S'* or *E'MS'* or *E'M'S*)

$= \frac{5}{7} \times \frac{1}{4} \times \frac{1}{6} + \frac{2}{7} \times \frac{3}{4} \times \frac{1}{6} + \frac{2}{7} \times \frac{1}{4} \times \frac{5}{6}$

$= \frac{1}{8}$

(c) P(qualify for entry)

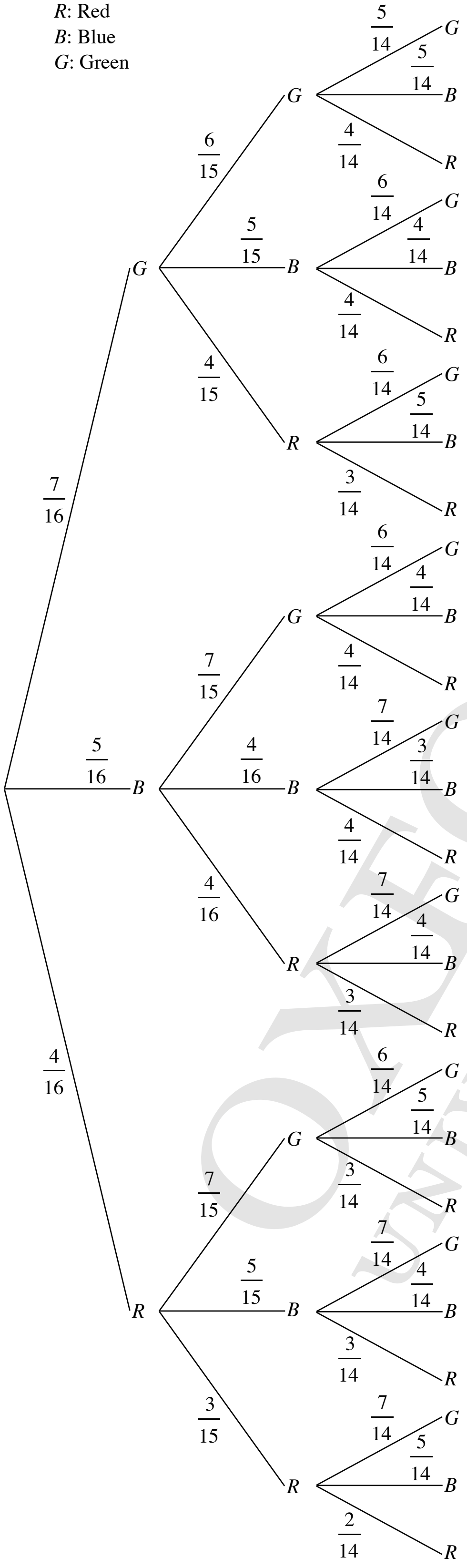
= 1 – P(no distinction or exactly one distinction)

$= 1 - \frac{1}{84} - \frac{1}{8}$

$= \frac{145}{168}$

33. (a)

R: Red
B: Blue
G: Green



(b) (i) P(one ball of each colour)

= P(*GBR* or *GRB* or *BGR* or *BRG* or *RGB* or *RBG*)

$= \left(\frac{7}{16}\times\frac{5}{15}\times\frac{4}{14}\right)\times 6$

$= \frac{1}{4}$

(ii) P(exactly one is blue)

= P(*BB*'*B*' or *B*'*BB*' or *B*'*B*'*B*)

$= \frac{5}{16}\times\frac{11}{15}\times\frac{10}{14}+\frac{11}{16}\times\frac{5}{15}\times\frac{10}{14}$

$+\frac{11}{16}\times\frac{10}{15}\times\frac{5}{14}$

$= \frac{55}{112}$

(iii) P(no red balls) = P(*R*'*R*'*R*')

$= \frac{12}{16}\times\frac{11}{15}\times\frac{10}{14}$

$= \frac{11}{28}$

(iv) P(second ball is *G*)

= P(*GG* any, *BG* any, *RG* any)

$= \frac{7}{16}\times\frac{6}{15}\times 1+\frac{5}{16}\times\frac{7}{15}\times 1+\frac{4}{16}\times\frac{7}{15}\times 1$

$= \frac{7}{16}$

34. There are 4 white counters (*W*) and 3 black counters (*B*).

P(two counters of each colour are left)

= P(*WWB* or *WBW* or *BWW*)

$= \frac{4}{7}\times\frac{3}{6}\times\frac{3}{5}+\frac{4}{7}\times\frac{3}{6}\times\frac{3}{5}+\frac{3}{7}\times\frac{4}{6}\times\frac{3}{5}$

$= \frac{18}{35}$

New Trend

35. (a) P(both balls are black) $= \left(\frac{15-n}{15}\right)\left(\frac{14-n}{14}\right)$

$= \frac{210-29n+n^2}{210}$

(b) $\frac{210-29n+n^2}{210} = \frac{2}{35}$

$210 - 29n + n^2 = 12$

$n^2 - 29n + 198 = 0$ (shown)

(c) $n^2 - 29n + 198 = 0$

$(n - 11)(n - 18) = 0$

$n = 11$ or $n = 18$ (NA)

∴ There are 15 – 11 = 4 black balls.

36. (a) (i) P(student from School *A* who obtains > 30 marks)

$= \frac{23+19}{160}$

$= \frac{21}{80}$

(ii) P(student gets a score $\leqslant$ 20 marks)

$= \frac{17+9}{160}$

$= \frac{13}{80}$

(b) P(both students from School *B* who obtain > 40 marks)

$= \frac{22}{160} \times \frac{21}{159}$

$= 0.0182$ (to 3 s.f.)

37. (a) P(prime) $= \frac{5}{10}$

$= \frac{1}{2}$

(b) P(both even) $= \frac{1}{2} \times \frac{1}{2}$

$= \frac{1}{4}$

(c) P(sum is 3) = P(1, 2) + P(2, 1)

$= \frac{2}{10} \times \frac{2}{10} + \frac{2}{10} \times \frac{2}{10}$

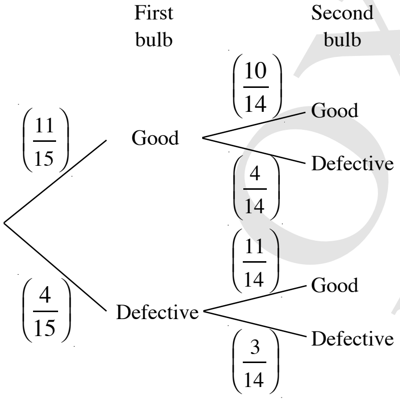
$= \frac{2}{25}$

P(sum is not 3) = 1 – P(sum is 3)

$= 1 - \frac{2}{25}$

$= \frac{23}{25}$

38. (a)



(b) (i) P(first bulb is good and second bulb is defective)

$= \frac{11}{15} \times \frac{4}{14}$

$= \frac{22}{105}$

(ii) P(both bulbs are good) $= \frac{11}{15} \times \frac{10}{14}$

$= \frac{11}{21}$

(iii) P(neither bulb is good) $= \frac{4}{15} \times \frac{3}{14}$

$= \frac{2}{35}$

(iv) P(one bulb is defective)

= P(first is good and second is defective)

+ P(first is defective and second is good)

$= \frac{11}{15} \times \frac{4}{14} + \frac{4}{15} \times \frac{11}{14}$

$= \frac{44}{105}$

39. (a)

		First Outcome					
		1	2	3	4	5	6
Second Outcome	6	(1, 6)	(2, 6)	(3, 6)	(4, 6)	(5, 6)	☒
	5	(1, 5)	(2, 5)	(3, 5)	(4, 5)	☒	(6, 5)
	4	(1, 4)	(2, 4)	(3, 4)	☒	(5, 4)	(6, 4)
	3	(1, 3)	(2, 3)	☒	(4, 3)	(5, 3)	(6, 3)
	2	(1, 2)	☒	(3, 2)	(4, 2)	(5, 2)	(6, 2)
	1	☒	(2, 1)	(3, 1)	(4, 1)	(5, 1)	(6, 1)

(b) Total number of outcomes = 30

(i) P(both numbers more than 4) $= \frac{2}{30}$

$= \frac{1}{15}$

(ii) P(sum of numbers is 12) = 0

(iii) P(product is less than 6) $= \frac{8}{30}$

$= \frac{4}{15}$

(iv) P(neither counter has an odd number)

= P(both counters have even numbers)

$= \frac{6}{30}$

$= \frac{1}{5}$

40. **(i)** **(a)** P(girl who comes to school by public transport)

$$= \frac{8}{40}$$

$$= \frac{1}{5}$$

(b) P(boy who comes to school by private transport)

$$= \frac{7}{40}$$

(c) P(pupil who comes to school by public transport)

$$= \frac{20}{40}$$

$$= \frac{1}{2}$$

(d) P(pupil is a boy) $= \frac{19}{40}$

(ii) **(a)** P(both female) $= \frac{21}{40} \times \frac{20}{39}$

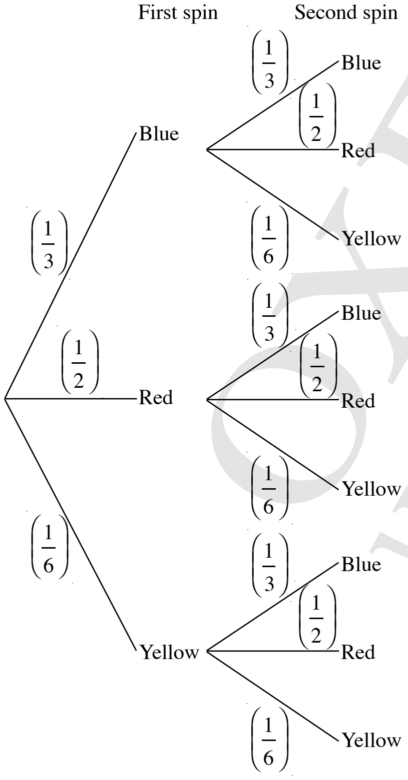
$$= \frac{7}{26}$$

(b) P(neither are boys taking public transport)

$$= \frac{28}{40} \times \frac{27}{39}$$

$$= \frac{63}{130}$$

41. **(a)**



(b) **(i)** P(blue, red) $= \frac{1}{3} \times \frac{1}{2}$

$$= \frac{1}{6}$$

(ii) P(same colour at both spins)

= P(blue, blue) or P(red, red) or P(yellow, yellow)

$$= \frac{1}{3} \times \frac{1}{3} + \frac{1}{2} \times \frac{1}{2} + \frac{1}{6} \times \frac{1}{6}$$

$$= \frac{7}{18}$$

(iii) P(different colours at both spins)

= 1 – P(same colour at both spins)

$$= 1 - \frac{7}{18}$$

$$= \frac{11}{18}$$

Chapter 4 Statistical Data Analysis

Basic

1. **(a)**

Period of time (t min)	Cumulative Frequency
$x \leqslant 30$	12
$x \leqslant 45$	30
$x \leqslant 60$	57
$x \leqslant 75$	96
$x \leqslant 90$	112
$x \leqslant 105$	120

(b) **(i)** Number of cars which stayed $\leqslant 60$ min = 57

(ii) Number of cars which stayed > 75 min

= 120 – 96 = 24

(iii) Number of cars which stayed $45 < t < 90$

= 112 – 30 = 82

2. **(a)**

Price of motorcycles (\$$p$)	Number of motorcycles
$p < 10\,000$	25
$p < 20\,000$	99
$p < 30\,000$	228
$p < 40\,000$	272
$p < 50\,000$	289
$p < 60\,000$	297
$p < 70\,000$	300

(b) **(i)** Number of 'Best selling' motorcycles

= 300 – 272

= 28

(ii) Number of 'Average selling' motorcycles

= 300 – 99

= 201

(iii) Number of 'Worst selling' motorcycles

= 300 – 25

= 275

3. **(a)** Arrange the given data in ascending order.

$$\overbrace{2, \underset{\overset{\uparrow}{Q_1}}{3}, 4}^{\text{lower half}}, \underset{\overset{\uparrow}{Q_2}}{6}, \overbrace{7, \underset{\overset{\uparrow}{Q_3}}{9}, 10}^{\text{upper half}}$$

For the given data, $n = 7$.

$\therefore Q_2 = 6$

$Q_1 = 3$

$Q_3 = 9$

Range = 10 – 2 = 8

Lower quartile = 3

Median = 6

Upper quartile = 9

Interquartile range = 9 – 3 = 6

(b) Arrange the given data in ascending order.

$$\overbrace{48, 55 \underset{\overset{\uparrow}{Q_1}}{,} 60, 66}^{\text{lower half}} \underset{\overset{\uparrow}{Q_2}}{,} \overbrace{76, 82 \underset{\overset{\uparrow}{Q_3}}{,} 87, 95}^{\text{upper half}}$$

For the given data, $n = 8$.

$\therefore Q_2 = \dfrac{66 + 76}{2} = 71$

$Q_1 = \dfrac{55 + 60}{2} = 57.5$

$Q_3 = \dfrac{82 + 87}{2} = 84.5$

Range = 95 – 48 = 47

Lower quartile = 57.5

Median = 71

Upper quartile = 84.5

Interquartile range = 84.5 – 57.5 = 27

(c) Arrange the given data in ascending order.

$$\overbrace{96, 140 \underset{\overset{\uparrow}{Q_1}}{,} 152, 167}^{\text{lower half}}, \underset{\overset{\uparrow}{Q_2}}{170}, \overbrace{181, 208 \underset{\overset{\uparrow}{Q_3}}{,} 219, 223}^{\text{upper half}}$$

For the given data, $n = 9$.

$\therefore Q_2 = 170$

$Q_1 = \dfrac{140 + 152}{2} = 146$

$Q_3 = \dfrac{208 + 219}{2} = 213.5$

Range = 223 – 96 = 127

Lower quartile = 146

Median = 170

Upper quartile = 213.5

Interquartile range = 213.5 – 146 = 67.5

4. **(a)** **(i)** From the graph, median time = 51 minutes.

(ii) From the graph, lower quartile = 47 minutes.

(iii) From the graph, upper quartile = 53.5 minutes.

$\therefore$ Interquartile range = 53.5 – 47 = 6.5 minutes

(iv) Number of participants who took longer than 56 minutes = 120 – 104 = 16

(b) 40% of the participants = $\frac{40}{100} \times 120 = 48$,

i.e. 48 participants were given a merit certificate. From the graph, 48 participants completed the race in less than 50 minutes.

$\therefore x = 50$.

(c) Generally, the performance of the participants is not too good because only 16 out of 200 were given a merit certificate.

5. **(a)** From the graph, median diameter = 29 cm.

(b) From the graph, lower quartile = 26 cm,

upper quartile = 33 cm.

$\therefore$ Interquartile range = 33 – 26 = 7 cm

(c) Fraction with diameters < 18 cm or > 37 cm = $\frac{21}{100}$

6. **(a)** Fraction which sold the watch for < \$56.50

$= \frac{94}{100}$

$= \frac{47}{50}$

(b) Median price = \$55.30

(c) From the graph, lower quartile = \$54.40,

upper quartile = \$55.70.

$\therefore$ Interquartile range = 55.70 – 54.50 = \$1.30

(d) The 10th percentile = \$53.60

(e) The 90th percentile = \$56.10

7. **(i)** Median = $\frac{186 + 188}{2} = 187$

(ii) Range = 236 – 155 = 81

(iii) Lower quartile = $\frac{169 + 170}{2} = 169.5$

Upper quartile = $\frac{196 + 197}{2} = 196.5$

$\therefore$ Interquartile range = 196.5 – 169.5 = 27

8. **(a)** Arrange the given data in ascending order.

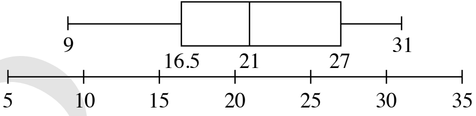
$$\overbrace{9, \underset{\substack{\uparrow \\ Q_1}}{16, 17}, 19}^{\text{lower half}}, \underset{\substack{\uparrow \\ Q_2}}{21}, \overbrace{23, \underset{\substack{\uparrow \\ Q_3}}{26, 28}, 31}^{\text{upper half}}$$

For the given data, $n = 9$.

$\therefore Q_2 = 21$

$Q_1 = \frac{16 + 17}{2} = 16.5$

$Q_3 = \frac{26 + 28}{2} = 27$



(b) Arrange the given data in ascending order.

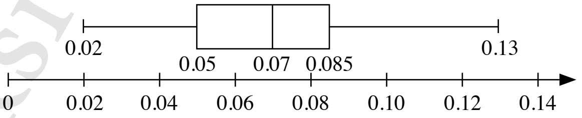
$$\overbrace{0.02, \underset{\substack{\uparrow \\ Q_1}}{0.04, 0.06}, 0.06}^{\text{lower half}}, \underset{\substack{\uparrow \\ Q_2}}{\,}\overbrace{0.08, \underset{\substack{\uparrow \\ Q_3}}{0.08, 0.09}, 0.13}^{\text{upper half}}$$

For the given data, $n = 8$.

$\therefore Q_2 = 0.07$

$Q_1 = \frac{0.04 + 0.06}{2} = 0.05$

$Q_3 = \frac{0.08 + 0.09}{2} = 0.085$



(c) Arrange the given data in ascending order.

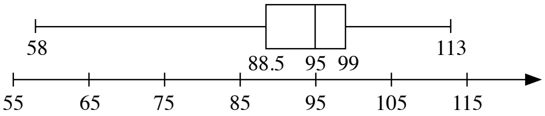
$$\overbrace{58, 80, \underset{\substack{\uparrow \\ Q_1}}{85, 92}, 92, 95}^{\text{lower half}}, \underset{\substack{\uparrow \\ Q_2}}{\,}\overbrace{95, 96, \underset{\substack{\uparrow \\ Q_3}}{98, 100}, 105, 113}^{\text{upper half}}$$

For the given data, $n = 12$.

$\therefore Q_2 = 95$

$Q_1 = \frac{85 + 92}{2} = 88.5$

$Q_3 = \frac{98 + 100}{2} = 99$



9. **(a)** Median = 3.4 hours

(b) Range = 6 – 0.5 = 5.5 hours

(c) Lower quartile = 5 hours

Upper quartile = 3 hours

∴ Interquartile range = 5 – 3 = 2 hours

10. **(a)**

x	x^2
0	0
2	4
23	529
19	361
5	25
16	256
24	576
8	64
$\Sigma x = 97$	$\Sigma x^2 = 1815$

Mean, $\bar{x} = \frac{\Sigma x}{n} = \frac{97}{8} = 12.125$

Standard deviation $= \sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2}$

$= \sqrt{\frac{1815}{8} - 12.125^2}$

= 8.94 (to 3 s.f.)

(b)

x	x^2
45.5	2070.25
75.6	5715.36
40.7	1656.49
66.3	4395.69
18.9	357.21
27.1	734.41
52.8	2787.84
$\Sigma x = 326.9$	$\Sigma x^2 = 17\,717.25$

Mean, $\bar{x} = \frac{\Sigma x}{n} = \frac{326.9}{7} = 46.7$

Standard deviation $= \sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2}$

$= \sqrt{\frac{17\,717.25}{7} - 46.7^2}$

= 18.7 (to 3 s.f.)

11.

x	f	fx	fx^2
0	8	0	0
1	11	11	11
2	5	10	20
3	7	21	63
4	4	16	64
5	0	0	0
6	2	12	72
Sum	$\Sigma f = 37$	$\Sigma fx = 70$	$\Sigma fx^2 = 230$

Mean, $\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{70}{37} = 1.89$

Standard deviation $= \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \bar{x}^2}$

$= \sqrt{\frac{230}{37} - 1.89^2}$

= 1.63 (to 3 s.f.)

Intermediate

12. **(a)** **(i)** From the graph, median = 13.7 km.

(ii) From the graph, the 10th percentile = 5.5 km.

(iii) From the graph, lower quartile = 11 km,

upper quartile = 19 km.

∴ Interquartile range = 19 – 11

= 8 km

(b) From the graph, number of workers who travelled less than 30 km = 82

∴ Number of workers who stay far away = 90 – 82

= 8

Percentage = $\frac{8}{90} \times 100\% = 8.89\%$ (to 3 s.f.)

(c)

Distance in km	**Number of workers**
$\leqslant 5$	7
$\leqslant 10$	22
$\leqslant 15$	52
$\leqslant 20$	72
$\leqslant 25$	78
$\leqslant 30$	82
$\leqslant 35$	86
$\leqslant 40$	88
$\leqslant 45$	90

13. (a) (i) From the graph, median = 71 kg.

(ii) From the graph, lower quartile = 64.5 kg,

upper quartile = 77 kg.

$\therefore$ Interquartile range $= 77 - 64.5$

$= 12.5$ kg

(iii) From the graph, number of students with weights greater than 78 kg $= 500 - 390$

$= 110$

(iv) From the graph, number of students with weights 62 kg or less = 75.

Percentage $= \frac{75}{500} \times 100\% = 15\%$

(b) For College *A*,

number of students with weights 75 kg and above

$= 500 - 460$

$= 40$

Percentage overweight $= \frac{40}{500} \times 100\% = 8\%$

For College *B*,

number of students with weights 75 kg and above

$= 500 - 330$

$= 170$

Percentage overweight $= \frac{170}{500} \times 100\% = 34\%$

(c) Generally, students from College *B* weigh heavier as compared to students from College *A* as the median weight is higher in College *B* as compared to that of College *A*. College *B* also has a higher percentage of students who are overweight as computed in **(b)**.

14. (a) From the graph, median mark = 64 marks.

(b) From the graph, lower quartile = 54 marks,

upper quartile = 73 marks.

$\therefore$ Interquartile range $= 73 - 54$

$= 19$ marks

(c) From the graph, 20th percentile = 50 marks.

(d) From the graph, 90th percentile = 79 marks.

(e) 75% of students passed the test means 25% of the students failed.

25% of 60 students $= \frac{25}{100} \times 60 = 15$ students failed the test.

From the graph, 15 students scored less than 54 marks.

$\therefore$ The pass mark is 54 marks.

15. (a) (i) From the graph for Geography,

median mark = 44 marks.

From the graph for Geography,

lower quartile = 39 marks,

upper quartile = 52 marks.

$\therefore$ Interquartile range $= 52 - 39 = 13$ marks

(ii) From the graph for Geography, number of students who scored >75 marks

$= 200 - 190$

$= 10$

$\therefore$ Percentage $= \frac{10}{200} \times 100\% = 5\%$

(b) (i) From the graph for History,

median mark = 51 marks.

From the graph for History,

lower quartile = 37 marks,

upper quartile = 62 marks.

$\therefore$ Interquartile range $= 62 - 37 = 25$ marks

(ii) From the graph for History, number of students who scored > 75 marks

$= 200 - 180$

$= 20$

$\therefore$ Percentage $= \frac{20}{200} \times 100\% = 10\%$

(c) The Geography test is more difficult as compared to the History test since the median score for the former is lower, i.e. students generally scored lower for the Geography test as compared to the History test.

16. (a) From the graph, lower quartile = 10.5 marks,

median = 15 marks,

upper quartile = 21 marks.

(b) From the graph, number of students who took the test in School *Y* = 35.

(c) Interquartile range $= 35 - 21.5 = 13.5$ marks

(d) Number of students who scored < 40 marks = 31

Percentage who received a distinction

$= \frac{35 - 31}{35} \times 100\%$

$= 11.4\%$ (to 3 s.f.)

(e) Yes, I agree because School *Y* has a higher median score as compared to School *X*.

17. (a) The minimum length is 10 cm and the maximum length is 110 cm. The lengths of the objects have a lower quartile of 30 cm, a median of 42 cm and an upper quartile of 50 cm.

(b) Interquartile range $= 50 - 30 = 20$ cm

(c) Range $= 110 - 10 = 100$ cm

18. (a) For City *A*,

lower quartile = 116

median = 136

upper quartile = 152

For City *B*,

lower quartile = 40

median = 64

upper quartile = 88

(b) For City *A*, interquartile range = 152 – 116 = 36

For City *B*, interquartile range = 88 – 40 = 48

(c) City *B* shows a greater spread.

(d) City *A* has worse air pollution than City *B* since the median PSI for City *A* is much higher than that of City *B*.

19. (a) Arrange the given data in ascending order.

$$\overbrace{17, 20, \underset{\underset{Q_1}{\uparrow}}{27}, 33, 46}^{\text{lower half}}, \underset{\underset{Q_2}{\uparrow}}{} \overbrace{50, 56, \underset{\underset{Q_3}{\uparrow}}{67}, 80, 91}^{\text{upper half}}$$

For the given data, $n = 10$.

$$\therefore Q_2 = \frac{46 + 50}{2} = 48$$

$Q_1 = 27$

$Q_3 = 67$

$\therefore x_1 = 17, x_2 = 27, x_3 = 48, x_4 = 67, x_5 = 91$

(b) Range = 91 – 17 = 74

(c) Interquartile range = 67 – 27 = 40

(d) Percentage of cities whose API is considered unhealthy

$$= \frac{4}{10} \times 100\%$$

$= 40\%$

20. (a) For City *A*:

Arrange the given data in ascending order.

$$\overbrace{68, 76, \underset{\underset{Q_1}{\uparrow}}{89}, 91, 110}^{\text{lower half}}, \underset{\underset{Q_2}{\uparrow}}{} \overbrace{112, 122, \underset{\underset{Q_3}{\uparrow}}{124}, 153, 167}^{\text{upper half}}$$

For the given data, $n = 10$.

$$\therefore Q_2 = \frac{110 + 112}{2} = 111$$

$Q_1 = 89$

$Q_3 = 124$

(i) Range = 167 – 68 = 99

(ii) Median = 111

(iii) Interquartile range = 124 – 89 = 35

For City *B* :

Arrange the given data in ascending order.

$$\overbrace{15, 19, \underset{\underset{Q_1}{\uparrow}}{24}, 29, 31}^{\text{lower half}}, \underset{\underset{Q_2}{\uparrow}}{} \overbrace{44, 51, \underset{\underset{Q_3}{\uparrow}}{55}, 77, 80}^{\text{upper half}}$$

For the given data, $n = 10$.

$$\therefore Q_2 = \frac{31 + 44}{2} = 37.5$$

$Q_1 = 24$

$Q_3 = 55$

(i) Range = 80 – 15 = 65

(ii) Median = 37.5

(iii) Interquartile range = 55 – 24 = 31

(b) City *A* shows a greater spread.

(c) The air pollution of City *A* is worse than City *B* since the median PSI for City *A* is much higher than that of City *B*.

21. (a) For Class *B*,

Marks	**Mid-value (x)**	f	fx	fx^2
$10 < x \leqslant 30$	20	4	80	1600
$30 < x \leqslant 50$	40	9	360	14 400
$50 < x \leqslant 70$	60	12	720	43 200
$70 < x \leqslant 90$	80	5	400	32 000
Sum		$\Sigma f = 30$	Σfx $= 1560$	Σfx^2 $= 91\ 200$

(i) Mean, $\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{1560}{30} = 52$ marks

(ii) Standard deviation $= \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \bar{x}^2}$

$$= \sqrt{\frac{91\ 200}{30} - 52^2}$$

$= 18.3$ marks (to 3 s.f.)

(b) Class *A* performed better since its mean mark is higher than that of Class *B*.

22.

Time (min)	Mid-value (x)	f	fx	fx^2
$30 < x \leqslant 35$	32.5	4	130	4225
$35 < x \leqslant 40$	37.5	2	75	2812.5
$40 < x \leqslant 45$	42.5	4	170	7225
$45 < x \leqslant 50$	47.5	5	237.5	11 281.25
$50 < x \leqslant 55$	52.5	3	157.5	8268.75
$55 < x \leqslant 60$	57.5	3	172.5	9918.75
$60 < x \leqslant 65$	62.5	4	250	15 625
$65 < x \leqslant 70$	67.5	5	337.5	22 781.25
Sum		$\Sigma f = 30$	Σfx $= 1530$	Σfx^2 $= 82\ 137.5$

(i) Mean, $\bar{x} = \dfrac{\Sigma fx}{\Sigma f} = \dfrac{1530}{30} = 51$ min

(ii) Standard deviation $= \sqrt{\dfrac{\Sigma fx^2}{\Sigma f} - \bar{x}^2}$

$$= \sqrt{\frac{82\ 137.5}{30} - 51^2}$$

$$= 11.7 \text{ min (to 3 s.f.)}$$

23. $15 + 6 + 18 + 9 + 2 + x = 9 \times 6$

$$50 + x = 54$$

$$x = 4$$

Standard deviation

$$= \sqrt{\frac{(15-9)^2 + (6-9)^2 + (18-9)^2 + (9-9)^2 + (2-9)^2 + (4-9)^2}{6}}$$

$= 5.77$ (to 3 s.f.)

24. $145 + 126 + 137 + 150 + x + 2x = 130 \times 6$

$$558 + 3x = 780$$

$$x = 74$$

Standard deviation

$$= \sqrt{\frac{(145-130)^2 + (126-130)^2 + (137-130)^2 + (150-130)^2 + (74-130)^2 + (148-130)^2}{6}}$$

$= 26.3$ (to 3 s.f.)

25. (a) For Mr Lim,

(i) mean distance $= \dfrac{52 + 21 + 37 + 6 + 24 + 40}{6}$

$= 30$

(ii) standard deviation

$$= \sqrt{\frac{(52-30)^2 + (21-30)^2 + (37-30)^2 + (6-30)^2 + (24-30)^2 + (40-30)^2}{6}}$$

$= 14.9$

For Mr Tan,

(i) mean distance $= \dfrac{25 + 14 + 21 + 48 + 18 + 9}{6}$

$= 22.5$

(ii) standard deviation

$$= \sqrt{\frac{(25-22.5)^2 + (14-22.5)^2 + (21-22.5)^2 + (48-22.5)^2 + (18-22.5)^2 + (9-22.5)^2}{6}}$$

$= 12.5$

(b) Mr Tan's performance was more consistent since his standard deviation is smaller which means a smaller spread in data.

(c) Mr Tan was a better shooter since the mean distance from the centre of target each shot hit is smaller.

26. (a) (i) Median $= \dfrac{33 + 34}{2} = 33.5$ months

(ii) Range $= 44 - 12 = 32$ months

(iii) Lower quartile $= 24$ months

Upper quartile $= 37$ months

$\therefore$ Interquartile range $= 37 - 24 = 13$ months

(iv) Mean

$$= \frac{12 + 15 + 16 + 22 + 24 \times 2 + 25 \times 2 + 33 \times 2 + 34 + 36 + 37 \times 3 + 38 + 40 \times 2 + 41 + 44}{20}$$

$= 30.65$

$= 30.7$ (to 3 s.f.)

(v) Standard deviation

$$= \sqrt{\frac{(12-30.65)^2 + (15-30.65)^2 + (16-30.65)^2 + (22-30.65)^2 + (24-30.65)^2 \times 2 + (25-30.65)^2 \times 2 + (33-30.65)^2 \times 2 + (34-30.65)^2 + (36-30.65)^2 + (37-30.65)^2 \times 3 + (38-30.65)^2 + (40-30.65)^2 \times 2 + (41-30.65)^2 + (44-30.65)^2}{20}}$$

$= 9.27$ (to 3 s.f.)

(b) Brand Y because the mean lifespan of the lightbulbs is higher.

27. (a) (i) Mean

$= \dfrac{11 + 13 + 13 + 20 + 27 + 35 + 36 + 37 + 37 + 41}{10}$

= 27 hours

(ii)

x	$x - \overline{x}$	$(x - \overline{x})^2$
11	–16	256
13	–14	196
13	–14	196
20	–7	49
27	0	0
35	8	64
36	9	81
37	10	100
37	10	100
41	14	196
Sum		$\Sigma f(x - \overline{x})^2 = 1238$

Standard deviation $= \sqrt{\dfrac{\Sigma(x - \overline{x})^2}{n}}$

$= \sqrt{\dfrac{1238}{10}}$

= 11.1 (to 3 s.f.)

(b) Brand B since it has a higher mean and a smaller standard deviation indicating a longer lifespan and a more consistent performance as compared to Brand A.

(c) (i) Median $= \dfrac{27 + 35}{2} = 31$

Lower quartile = 13

Upper quartile = 37

$\therefore x_1 = 13, x_2 = 31, x_3 = 37$

(ii) $x_3 - x_1 = 37 - 13 = 24$

It represents the interquartile range.

28. (a) Number of pupils = 4 + 3 + 2 + 3 + 3 + 6 + 4 + 1 + 1

= 27

(b) The most common number of correct answers is 7.

(c) Percentage who answered less than 6 correctly

$= \dfrac{4 + 3 + 2 + 3}{27} \times 100\%$

= 44.4% (to 3 s.f.)

(d) (i) Mean

$= \dfrac{1 \times 4 + 2 \times 3 + 4 \times 2 + 5 \times 3 + 6 \times 3 + 7 \times 6 + 8 \times 4 + 9 + 10}{27}$

= 5.33 (to 3 s.f.)

(ii)

x	f	fx	fx^2
1	4	4	4
2	3	6	12
3	0	0	0
4	2	8	32
5	3	15	75
6	3	18	108
7	6	42	294
8	4	32	256
9	1	9	81
10	1	10	100
Sum	$\Sigma f = 27$	$\Sigma fx = 144$	$\Sigma fx^2 = 962$

Standard deviation $= \sqrt{\dfrac{\Sigma fx^2}{\Sigma f} - \overline{x}^2}$

$= \sqrt{\dfrac{962}{27} - 5.33^2}$

= 2.69 (to 3 s.f.)

29. (a) (i) For Class X,

x	f	fx	fx^2
2	2	4	8
3	3	9	27
4	6	24	96
5	11	55	275
6	10	60	360
7	7	49	343
8	1	8	64
Sum	$\Sigma f = 40$	$\Sigma fx = 209$	$\Sigma fx^2 = 1173$

mean, $\overline{x} = \dfrac{\Sigma fx}{\Sigma f}$

$= \dfrac{209}{40}$

= 5.225

= 5.23 hours (to 3 s.f.)

standard deviation $= \sqrt{\dfrac{\Sigma fx^2}{\Sigma f} - \overline{x}^2}$

$= \sqrt{\dfrac{1173}{40} - 5.225^2}$

= 1.42 hours (to 3 s.f.)

(ii) For Class Y,

x	f	fx	fx^2
2	4	8	16
3	4	12	36
4	9	36	144
5	8	40	200
6	7	42	252
7	5	35	245
8	3	24	192
Sum	$\Sigma f = 40$	$\Sigma fx = 197$	$\Sigma fx^2 = 1085$

$$\begin{aligned} \text{mean}, \bar{x} &= \frac{\Sigma fx}{\Sigma f} \\ &= \frac{197}{40} \\ &= 4.925 \\ &= 4.93 \text{ hours (to 3 s.f.)} \end{aligned}$$

$$\begin{aligned} \text{standard deviation} &= \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \bar{x}^2} \\ &= \sqrt{\frac{1085}{40} - 4.925^2} \\ &= 1.69 \text{ hours (to 3 s.f.)} \end{aligned}$$

(b) Class Y spends less time on surfing the Internet since the mean time spent by pupils on the Internet is less as compared to Class X.

30. **(a)** Mean of Dodo = 20

$$\frac{21 + 43 + x + 8 + 34 + 24 + 12 + 2}{8} = 20$$

$144 + x = 160$

$x = 16$

Mean of Nana = y

$$\frac{6 + 9 + 15 + 26 + 10 + 14 + 21 + 3}{8} = y$$

$y = 13$

$\therefore x = 16, y = 13$

(b) Dodo was more careless because the mean number of mistakes she made is higher than Nana's.

(c) Nana was more consistent because her standard deviation is smaller than Dodo's, i.e. the number of mistakes is not as widely spread as Dodo's.

31. **(a)** For Factory F,

Lifespan	**Mid-value (x)**	f	fx	fx^2
$600 < x \leqslant 699$	649.5	2	1299	843 700.5
$700 < x \leqslant 799$	749.5	9	6745.5	5 055 752.25
$800 < x \leqslant 899$	849.5	16	13 592	11 546 404
$900 < x \leqslant 999$	949.5	21	19 939.5	18 932 555.25
$1000 < x \leqslant 1099$	1049.5	29	30 435.5	31 942 057.25
$1100 < x \leqslant 1199$	1149.5	18	20 691	23 784 304.5
$1200 < x \leqslant 1299$	1249.5	5	6247.5	7 806 251.25
Sum		$\Sigma f =$ 100	$\Sigma fx =$ 98 950	$\Sigma fx^2 =$ 99 911 025

$$p = \text{Mean} = \frac{\Sigma fx}{\Sigma f} = \frac{98\,950}{100} = 989.5$$

$$\begin{aligned} q = \text{Standard deviation} &= \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \bar{x}^2} \\ &= \sqrt{\frac{99\,911\,025}{100} - 989.5^2} \\ &= 141.5 \end{aligned}$$

$$\begin{aligned} 8 + 10 + 12 + 16 + r + 18 + 12 &= 100 \\ 76 + r &= 100 \\ r &= 24 \end{aligned}$$

(b) The two factories produced light bulbs of the same life span because their means are the same.

(c) Factory F's light bulbs have more consistent life spans because its standard deviation is smaller indicating a smaller spread in data.

32. **(a)**

Score	**Frequency for Group *A***	**Frequency for Group *B***
$10 < x \leqslant 20$	1	6
$20 < x \leqslant 30$	2	0
$30 < x \leqslant 40$	3	0
$40 < x \leqslant 50$	1	0
$50 < x \leqslant 60$	3	2
$60 < x \leqslant 70$	3	8
$70 < x \leqslant 80$	3	5
$80 < x \leqslant 90$	4	4
$90 < x \leqslant 100$	2	0

(b) (i) For Group A,

mean, $\bar{x}$

$$= \frac{\Sigma fx}{\Sigma f}$$

$$= \frac{(1 \times 15) + (2 \times 25) + (3 \times 35) + (1 \times 45) + (3 \times 55) + (3 \times 65) + (3 \times 75) + (4 \times 85) + (2 \times 95)}{1 + 2 + 3 + 1 + 3 + 3 + 3 + 4 + 2}$$

$= 60.5$ (to 3 s.f.)

$$\text{standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \bar{x}^2}$$

$$= \sqrt{\frac{92\ 750}{22} - 60.5^2}$$

$= 23.6$ (to 3 s.f.)

(ii) For Group B,

mean, $\bar{x}$

$$= \frac{\Sigma fx}{\Sigma f}$$

$$= \frac{(6 \times 15) + (2 \times 55) + (8 \times 65) + (5 \times 75) + (4 \times 85)}{6 + 2 + 8 + 5 + 4}$$

$= 57.4$ (to 3 s.f.)

$$\text{standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \bar{x}^2}$$

$$= \sqrt{\frac{98\ 225}{25} - 57.4^2}$$

$= 25.2$ (to 3 s.f.)

(c) Group A performed better because it has a higher mean as compared to Group B.

Advanced

33. (a) From the diagram, there were 20 children.

(b) Mass of lightest child = 30.2 kg

(c) (i) Min = 30.2 kg

$$\text{Lower quartile} = \frac{40.2 + 40.6}{2} = 40.4 \text{ kg}$$

Upper quartile = 50.6 kg

$$\text{Median} = \frac{40.9 + 50.3}{2} = 45.6$$

$\therefore x_1 = 30.2, x_2 = 40.4, x_3 = 45.6, x_4 = 50.6$

(ii) $x_4 - x_2 = 50.6 - 40.4 = 10.2$

It represents the interquartile range.

(d) (i) Median after 3 months = 42 kg

Decrease in median = 45.6 – 42 = 3.6 kg

(ii) Upper quartile after 3 months = 46 kg

Decrease in upper quartile = 50.6 – 46 = 4.6 kg

(iii) The masses of the children decreased after being encouraged to exercise as indicated by the decrease in median and upper quartile.

34. (a) (i) From the graph, the number of pellets that take 25.5 seconds or less to dissolve in water = 710

(ii) Upper quartile = 24.3 seconds

(iii) From the graph, the number of pellets that take 21.5 seconds or less to dissolve in water = 90

Percentage of pellets that take more than 21.5 seconds

$$= \frac{760 - 90}{90} \times 100\%$$

$= 88.2\%$ (to 3 s.f.)

(b) (i) From the graph, median = 23.7,

lower quartile = 22.5,

upper quartile = 24.3.

$\therefore x_1 = 22.5, x_2 = 23.7, x_3 = 24.3$

(ii) $x_3 - x_1 = 24.3 - 22.5 = 1.8$

It represents the interquartile range.

35. (a) (i) From the graph,

median length of service = 19 years.

(ii) From the graph, lower quartile = 15 years,

upper quartile = 23 years.

$\therefore$ Interquartile range = 23 – 15

= 8 years

(iii) From the graph, number of men whose length of service $\leqslant$ 18 years = 85

$$\therefore \text{Proportion} = \frac{85}{200} = \frac{17}{40}$$

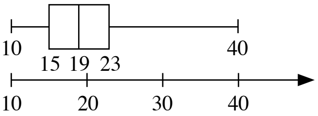
(iv) Percentage of men whose length of service is > 33 years

$$= \frac{12}{200} \times 100\%$$

$= 6\%$

(b) $a = 40, b = 20, c = 20$

(c)



36. $16 + 21 + 22 + 18 + x + y = 19 \times 7$

$97 + x + y = 133$

$x + y = 36$ — (1)

$$\frac{(16 - 19)^2 + (21 - 19)^2 + (22 - 19)^2 + (18 - 19)^2 + (20 - 19)^2 + (x - 19)^2 + (y - 19)^2}{7} = 3.742$$

$24 + (x - 19)^2 + (y - 19)^2 = 98$

$(x - 19)^2 + (y - 19)^2 = 74$ — (2)

Solving (1) and (2),

$x = 12, y = 24$.

37. (a) No, we cannot because they are two different individual sets of data.

(b) No, we cannot because they are two different individual sets of data.

(c)

Number of hours (x)	**Number of students (f)**	fx	fx^2
10	3 + 16 = 19	190	1900
15	12 + 22 = 34	510	7650
20	19 + 34 = 53	1060	21 200
25	36 + 18 = 54	1350	33 750
30	22 + 10 = 32	960	28 800
35	5	280	9800
Sum	$\Sigma f = 200$	$\Sigma fx = 200$	$\Sigma fx^2 = 103\ 100$

$$\text{Combined mean} = \frac{4350}{200} = 21.75 = 21.8 \text{ (to 3 s.f.)}$$

$$\text{Standard deviation} = \sqrt{\frac{103\,100}{200} - 21.75^2}$$

$$= 6.51 \text{ (to 3 s.f.)}$$

38. (a) (i) Candy, lowest mean.

(ii) Dodi, highest mean.

(b) (i) Eifer, interquartile range = 30 seconds

(ii) Candy, interquartile range = 140 seconds

(c) From fastest to slowest,
Candy, Beggy, Eifer, Afi, Dodi.

(d) No. A histogram or bar graph does not show the lower and upper quartiles thus not allowing us to obtain the interquartile range.

39. (a) (i) No.

(ii) Exact cost price

$= 1 \times 11 + 2 \times 23 + 3 \times 68 + 4 \times 54 + 5 \times 32 + 6 \times 7 + 7 \times 76 + 8 \times 5 + 9 \times 15 + 10 \times 9$

= \$14.76 million

Exact loss = 14.76 – 13.5 = \$1.26 million

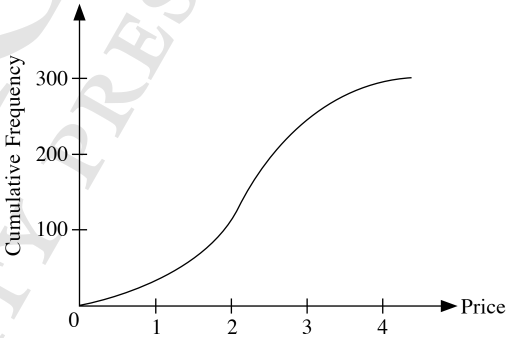
The exact cost price is lower than \$15 million and the exact cost is also less than \$1.5 million as claimed by NSG. Hence, the calculation of NSG is misleading.

(b) (i) Profit $= 5.50 \times 3 - 14.76$ = \$1.74 million (Proven)

(ii) No, the price rise should not be approved because NSG is actually making a profit instead of loss of \$1.5 million as claimed.

(c)

Wholesale price of items (\$ x)	**Number of items sold (f)**	**Cumulative Frequency**
1	11	11
2	23	34
3	68	102
4	54	156
5	32	188
6	7	195
7	76	271
8	5	276
9	15	291
10	9	300



(i) Median = \$3.9 million

(ii) Lower quartile = \$2.5 million

Upper quartile = \$6.3 million

∴ Interquartile range = 6.3 – 2.5 = \$3.8 million

Yes, NSG should represent the data with a cumulative frequency curve.

40. (a) No, it was misleading.

(b) For Math quiz,
standard deviation

$$= \sqrt{\frac{(2-6)^2 \times 5 + (10-6)^2 \times 5}{10}}$$

= 4

For Science quiz,
standard deviation

$$= \sqrt{\frac{(2-6)^2 + (3-6)^2 + (4-6)^2 + (6-6)^2 \times 2 + (7-6)^2 \times 3 + (8-6)^2 + (10-6)^2}{10}}$$

= 2.28 (to 3 s.f.)

(c) The scores for the Science quiz had a better spread but the scores for Math had greater variability.

New Trend

41. **(a)**

Temperature (x °C)	Mid-value (x)	City G			City P		
		f	fx	fx^2	f	fx	fx^2
$15.0 < x \leqslant 15.5$	15.25	3	45.75	697.69	22	335.5	5116.385
$15.5 < x \leqslant 16.0$	15.75	14	220.5	3472.88	27	425.25	6697.69
$16.0 < x \leqslant 16.5$	16.25	26	422.5	6865.63	19	308.75	5017.19
$16.5 < x \leqslant 17.0$	16.75	33	552.75	9258.56	20	335	5611.25
$17.0 < x \leqslant 17.5$	17.25	21	362.25	6248.81	16	276	4761
$17.5 < x \leqslant 18.0$	17.75	10	177.5	3150.63	5	88.75	1575.31
$18.0 < x \leqslant 18.5$	18.25	3	54.75	999.19	1	18.25	333.06
Sum		$\Sigma f = 110$	$\Sigma fx = 1836$	$\Sigma fx^2 = 30\,693.39$	$\Sigma f = 110$	$\Sigma fx = 1787.5$	$\Sigma fx^2 = 29\,111.88$

For City G,

(i) mean, $\bar{x} = \frac{\Sigma fx}{\Sigma f}$

$= \frac{1836}{110}$

$= 16.7$°C (to 3 s.f.)

(ii) standard deviation $= \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \bar{x}^2}$

$= \sqrt{\frac{30\,693.39}{110} - 16.69^2}$

$= 0.689$°C (to 3 s.f.)

For City P,

(i) mean, $\bar{x} = \frac{\Sigma fx}{\Sigma f}$

$= \frac{1787.5}{110}$

$= 16.25$°C

(ii) standard deviation $= \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \bar{x}^2}$

$= \sqrt{\frac{29\,111.88}{110} - 16.25^2}$

$= 0.769$°C (to 3 s.f.)

(b) City G is warmer because its mean temperature is higher.

(c) City G's temperature is more consistent because its standard deviation is smaller.

(d) For City G, mean = 19.7°C

standard deviation = 0.689°C

For City P, mean = 19.25°C

standard deviation = 0.769°C

42. **(a)** **(i)** From the graph, median mark = 68 marks.

(ii) From the graph, the 75th percentile = 73 marks.

(iii) From the graph, lower quartile = 62 marks,

upper quartile = 73 marks.

$\therefore$ Interquartile range = 73 – 62

= 11 marks

(iv) From the graph, the number of students who scored 60 marks or less is 18.

(b) Number of students who were not given the grade 'Distinction' = 80 – 10 = 70

From the graph, the lowest mark required to qualify for 'Distinction' = 77 marks.

(c) The cumulative frequency curve for School *B* will be on the right of School *A*'s.

43. **(a)** Lower quartile = 69 marks

Median = 76 marks

Upper quartile = 94 marks

(b) Minimum = 62 marks

Maximum = 100 marks

(c) Interquartile range = 94 – 69

= 25 marks

44. **(a)** For Bank *A*,

Time (min)	**Mid-value (t)**	f	ft	ft^2
$10 < t \leqslant 12$	11	5	55	605
$12 < t \leqslant 14$	13	29	377	4901
$14 < t \leqslant 16$	15	10	150	2250
$16 < t \leqslant 18$	17	12	204	3468
$18 < t \leqslant 20$	19	4	76	1444
Sum		$\Sigma f = 60$	Σft = 862	Σft^2 = 12 668

(i) mean, $\bar{t} = \dfrac{\Sigma ft}{\Sigma f} = \dfrac{862}{60} = 14.4$ min (to 3 s.f.)

(ii) standard deviation $= \sqrt{\dfrac{\Sigma ft^2}{\Sigma f} - \bar{t}^2}$

$= \sqrt{\dfrac{12\,668}{60} - 14.4^2}$

= 1.94 min (to 3 s.f.)

(b) Bank *B* is more efficient since its mean waiting time is shorter than that of Bank *A*.

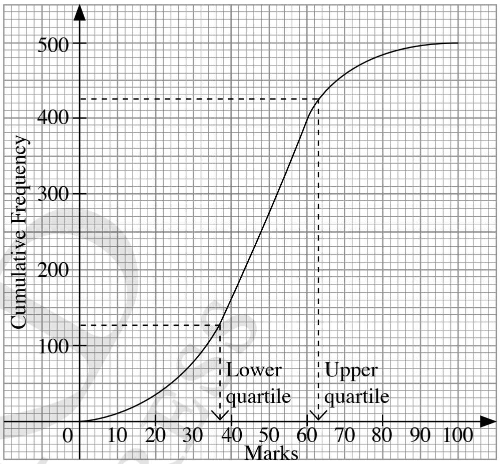
45. **(a)** **(i)** From the graph, median = 51 marks.

(ii) 100% – 65% = 35% of the students failed.

35% of 500 students $= \dfrac{35}{100} \times 500 = 175$ students scored less than 43 marks.

$\therefore$ The passing mark is 43.

(b) Interquartile range = 63 – 37 = 26



(c) **(i)** Median = 42 marks

Lower quartile = 34 marks

Upper quartile = 48 marks

$\therefore$ Interquartile range = 48 – 34 = 14 marks

(ii) Yes, because the median score is higher for the Geography test as compared to the Science test which indicates that students generally did better for Geography as compared to Science.

46. (a) (i) Median mark = $\frac{46 + 46}{2}$ = 46 marks

(ii) Range = 87 – 7 = 80 marks

(iii) Lower quartile = $\frac{34 + 35}{2}$ = 34.5 marks

Upper quartile = $\frac{62 + 62}{2}$ = 62 marks

∴ Interquartile range = 62 – 34.5 = 27.5 marks

(iv) Mean = $\frac{7+12+24+28+30+34+35+39+43+45+46+46+46+47+49+50+51+62+62+66+78+83+85+87}{24}$

= 48.125

Standard deviation = $\sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2}$

$= \sqrt{\frac{66\,099}{24} - 48.125^2}$

= 20.9 (to 3 s.f.)

(b) The correct median is now 46 + 3 = 49 whereas the standard deviation is not affected by the error.

47.

Blood pressure (mm Hg)	Mid-value (x)	f	fx	fx^2
$55 < x \leqslant 60$	57.5	1	57.5	3306.25
$60 < x \leqslant 65$	62.5	4	250	15 625
$65 < x \leqslant 70$	67.5	10	675	45 562.5
$70 < x \leqslant 75$	72.5	21	1522.5	110 381.25
$75 < x \leqslant 80$	77.5	35	2712.5	210 218.75
$80 < x \leqslant 85$	82.5	29	2392.5	197 381.25
$85 < x \leqslant 90$	87.5	13	1137.5	99 531.25
$90 < x \leqslant 95$	92.5	7	647.5	59 893.75
Sum		$\Sigma f = 120$	Σfx = 9395	Σfx^2 = 741 900

Mean, $\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{9395}{120}$ = 78.3 mm Hg (to 3 s.f.)

Standard deviation = $\sqrt{\frac{\Sigma fx^2}{\Sigma f} - \bar{x}^2}$

$= \sqrt{\frac{741\,900}{120} - 78.29^2}$

= 7.29 mm Hg (to 3 s.f.)

48. (a) (i) From the graph,
number of cars whose speed ⩽ 35 km/h = 320

(ii) From the graph, median = 33 km/h.

(iii) From the graph, lower quartile = 29,
upper quartile = 38.

∴ Interquartile range = 38 – 29
= 9 km/h

(iv) Number of cars whose speed > 43 km/h
= 500 – 430
= 70

Fraction = $\frac{70}{500} = \frac{7}{50}$

(b) (i) Median = 41 km/h

Interquartile range = 50 – 37
= 13

(ii) Yes, because the median speed of the second group of cars is higher than that of the first group.

49. (a) (i) From the graph, median = \$237.50.

(ii) From the graph, 90th percentile = \$345.

(iii) From the graph, lower quartile = \$182.50,

upper quartile = \$295.

$\therefore$ Interquartile range = 295 – 182.50

= \$112.50

(iv) From the graph, number of workers whose weekly wage $\geqslant$ \$220 = 500 – 205

= 295

(b) From the graph, lower quartile = \$145,

upper quartile = \$230.

$\therefore$ Interquartile range = 230 – 145 = \$85

(c) From the graph, he would have been given \$190 if he had been employed by Factory *B*.

(d) Agree. Factory *B* paid the workers better than Factory *A* because the median wage for Factory *B* is higher than that of Factory *A*.

50. (a)

Distance (x km)	**Mid-value (x)**	f	fx	fx^2
$0 < x \leqslant 5$	2.5	12	30	75
$5 < x \leqslant 10$	7.5	21	157.5	1181.25
$10 < x \leqslant 15$	12.5	46	575	7187.5
$15 < x \leqslant 20$	17.5	27	472.5	8268.75
$20 < x \leqslant 25$	22.5	10	225	5062.5
$25 < x \leqslant 30$	27.5	7	192.5	5293.75
$30 < x \leqslant 35$	32.5	5	162.5	5281.25
$35 < x \leqslant 40$	37.5	2	75	2812.5
Sum		$\Sigma f = 130$	Σfx = 1890	Σfx^2 = 35 162.5

Mean, $\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{1890}{130}$ = 14.5 km (to 3 s.f.)

Standard deviation $= \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \bar{x}^2}$

$= \sqrt{\frac{35\,162.5}{130} - 14.5^2}$

= 7.76 km (to 3 s.f.)

(b) Percentage of commuters

$= \frac{5+2}{130} \times 100\%$

$= 5\frac{5}{13}\%$

(c) $P = 33 + 46 = 79$

$Q = 116 + 7 = 123$

Chapter 5 Matrices

Basic

1. **(a)** $\begin{pmatrix} 2 & 3 \\ 1 & -4 \end{pmatrix} + \begin{pmatrix} 7 & 1 \\ -4 & 3 \end{pmatrix} = \begin{pmatrix} 9 & 4 \\ -3 & -1 \end{pmatrix}$

(b) NA

(c) NA

(d) $\begin{pmatrix} 5 & 3 & 2 \\ 1 & -1 & 3 \\ -4 & -3 & 5 \end{pmatrix} + \begin{pmatrix} -4 & 0 & 3 \\ -3 & 2 & 5 \\ 11 & 3 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 3 & 5 \\ -2 & 1 & 8 \\ 7 & 0 & 4 \end{pmatrix}$

(e) $\begin{pmatrix} 2 & 1 \\ -1 & 3 \end{pmatrix} + \begin{pmatrix} 1 & -1 \\ 0 & -2 \end{pmatrix} - \begin{pmatrix} -1 & -2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 4 & 2 \\ -4 & -3 \end{pmatrix}$

(f) $\begin{pmatrix} 2 \\ 3 \end{pmatrix} - 2\begin{pmatrix} 4 \\ 6 \end{pmatrix} + \begin{pmatrix} 7 \\ 5 \end{pmatrix} = \begin{pmatrix} 1 \\ -4 \end{pmatrix}$

(g) $(3 \quad 2 \quad 4) + (1 \quad 5 \quad 6) = (4 \quad 7 \quad 10)$

(h) $(1 \quad 3) + (9 \quad 5) - 2(7 \quad 8) = (-4 \quad -8)$

2. **(a)** $\mathbf{A} + \begin{pmatrix} 3 & 2 \\ 1 & 7 \end{pmatrix} = \begin{pmatrix} 6 & 1 \\ 5 & 4 \end{pmatrix}$

$\mathbf{A} = \begin{pmatrix} 6 & 1 \\ 5 & 4 \end{pmatrix} - \begin{pmatrix} 3 & 2 \\ 1 & 7 \end{pmatrix}$

$= \begin{pmatrix} 3 & -1 \\ 4 & -3 \end{pmatrix}$

(b) $2\mathbf{B} - \begin{pmatrix} 9 & 1 \\ -3 & 4 \end{pmatrix} = \begin{pmatrix} 10 & 1 \\ 3 & 6 \end{pmatrix}$

$2\mathbf{B} = \begin{pmatrix} 10 & 1 \\ 3 & 6 \end{pmatrix} + \begin{pmatrix} 9 & 1 \\ -3 & 4 \end{pmatrix}$

$= \begin{pmatrix} 19 & 2 \\ 0 & 10 \end{pmatrix}$

$\mathbf{B} = \frac{1}{2}\begin{pmatrix} 19 & 2 \\ 0 & 10 \end{pmatrix}$

(c) $\begin{pmatrix} 4 & 1 \\ 3 & 5 \end{pmatrix} - 3\mathbf{C} = \begin{pmatrix} 1 & -8 \\ 0 & -4 \end{pmatrix}$

$3\mathbf{C} = \begin{pmatrix} 4 & 1 \\ 3 & 5 \end{pmatrix} - \begin{pmatrix} 1 & -8 \\ 0 & -4 \end{pmatrix}$

$= \begin{pmatrix} 3 & 9 \\ 3 & 9 \end{pmatrix}$

$\mathbf{C} = \begin{pmatrix} 1 & 3 \\ 1 & 3 \end{pmatrix}$

(d) $\begin{pmatrix} -5 & -9 \\ 7 & 5 \end{pmatrix} - 4\mathbf{D} = \begin{pmatrix} 3 & 11 \\ -5 & 5 \end{pmatrix}$

$4\mathbf{D} = \begin{pmatrix} -5 & -9 \\ 7 & 5 \end{pmatrix} - \begin{pmatrix} 3 & 11 \\ -5 & 5 \end{pmatrix}$

$= \begin{pmatrix} -8 & -20 \\ 12 & 0 \end{pmatrix}$

$\mathbf{D} = \begin{pmatrix} -2 & -5 \\ 3 & 0 \end{pmatrix}$

(e) $\begin{pmatrix} 4 & 5 \\ 3 & -2 \end{pmatrix} - 2\mathbf{E} = \begin{pmatrix} -4 & 1 \\ 0 & -5 \end{pmatrix} - 3\mathbf{E}$

$\mathbf{E} = \begin{pmatrix} -4 & 1 \\ 0 & -5 \end{pmatrix} - \begin{pmatrix} 4 & 5 \\ 3 & -2 \end{pmatrix}$

$= \begin{pmatrix} -8 & -4 \\ -3 & -3 \end{pmatrix}$

(f) $2\mathbf{F} + 3\begin{pmatrix} 4 & 2 \\ 3 & -4 \end{pmatrix} = \begin{pmatrix} 6 & -4 \\ 1 & 8 \end{pmatrix}$

$2\mathbf{F} = \begin{pmatrix} 6 & -4 \\ 1 & 8 \end{pmatrix} - \begin{pmatrix} 12 & 6 \\ 9 & -12 \end{pmatrix}$

$= \begin{pmatrix} -6 & -10 \\ -8 & 20 \end{pmatrix}$

$\mathbf{F} = \begin{pmatrix} -3 & -5 \\ -4 & 10 \end{pmatrix}$

3. **(a)** $\begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}\begin{pmatrix} 2 \\ 6 \end{pmatrix} = \begin{pmatrix} 18 \\ 26 \end{pmatrix}$

(b) $\begin{pmatrix} 1 \\ 3 \end{pmatrix}\begin{pmatrix} 2 & -1 \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ 6 & -3 \end{pmatrix}$

(c) $\begin{pmatrix} 4 & -1 \end{pmatrix}\begin{pmatrix} 7 \\ 6 \end{pmatrix} = (22)$

(d) NA

(e) $\begin{pmatrix} 3 & 1 & 4 \\ -1 & -2 & 1 \end{pmatrix}\begin{pmatrix} 2 & 3 \\ 1 & -4 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 3 & 5 \\ -5 & 5 \end{pmatrix}$

(f) $\begin{pmatrix} 1 & -1 \\ 3 & 2 \\ -2 & 4 \end{pmatrix}\begin{pmatrix} 4 & -3 & 1 \\ -2 & -1 & 2 \end{pmatrix} = \begin{pmatrix} 6 & -2 & -1 \\ 8 & -11 & 7 \\ -16 & 2 & 6 \end{pmatrix}$

(g) $\begin{pmatrix} 0 & 1 \\ 2 & 3 \\ -1 & 4 \end{pmatrix}\begin{pmatrix} 3 \\ -2 \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \\ -11 \end{pmatrix}$

(h) $\begin{pmatrix} 1 & 2 & 3 \end{pmatrix}\begin{pmatrix} 0 & 3 \\ 1 & -2 \\ -1 & 4 \end{pmatrix} = \begin{pmatrix} -1 & 11 \end{pmatrix}$

4. **(a)** $$\begin{pmatrix}3&-1\\-4&6\end{pmatrix}+2\begin{pmatrix}3&a\\b&c\end{pmatrix}=\begin{pmatrix}9&7\\8&-24\end{pmatrix}$$

$$2\begin{pmatrix}3&a\\b&c\end{pmatrix}=\begin{pmatrix}9&7\\8&-24\end{pmatrix}-\begin{pmatrix}3&-1\\-4&6\end{pmatrix}$$

$$\begin{pmatrix}3&a\\b&c\end{pmatrix}=\frac{1}{2}\begin{pmatrix}6&8\\12&-30\end{pmatrix}$$

$$\begin{pmatrix}3&a\\b&c\end{pmatrix}=\begin{pmatrix}3&4\\6&-15\end{pmatrix}$$

Equating the corresponding elements, we have

$a = 4, b = 6, c = -15$

(b) $$\begin{pmatrix}-6&h\\5&2\end{pmatrix}-\begin{pmatrix}1&2h\\k&-3\end{pmatrix}=\begin{pmatrix}-7&-9\\3k&5\end{pmatrix}$$

$$\begin{pmatrix}-7&-h\\5-k&5\end{pmatrix}=\begin{pmatrix}-7&-9\\3k&5\end{pmatrix}$$

Equating the corresponding elements, we have

$-h = -9$

$h = 9$

$5 - k = 3k$

$k = 1\frac{1}{4}$

$\therefore h = 9, k = 1\frac{1}{4}$

(c) $$\begin{pmatrix}3&2\\1&4\end{pmatrix}+3\begin{pmatrix}7&5\\-1&a\end{pmatrix}=\begin{pmatrix}b&c\\d&2a\end{pmatrix}$$

$$\begin{pmatrix}21&15\\-3&3a\end{pmatrix}-\begin{pmatrix}b&c\\d&2a\end{pmatrix}=-\begin{pmatrix}3&2\\1&4\end{pmatrix}$$

$$\begin{pmatrix}21-b&15-c\\-3-d&a\end{pmatrix}=\begin{pmatrix}-3&-2\\-1&-4\end{pmatrix}$$

Equating the corresponding elements, we have

$21 - b = -3$

$b = 24$

$15 - c = -2$

$c = 17$

$-3 - d = -1$

$d = -2$

$a = -4$

$\therefore a = -4, b = 24, c = 17, d = -2$

(d) $$\begin{pmatrix}1&-1&3\\-2&-3&4\end{pmatrix}-\begin{pmatrix}a&b&c\\4&3&9\end{pmatrix}=\begin{pmatrix}6&-7&3\\h&k&t\end{pmatrix}$$

$$\begin{pmatrix}6&-7&3\\h&k&t\end{pmatrix}+\begin{pmatrix}a&b&c\\4&3&9\end{pmatrix}=\begin{pmatrix}1&-1&3\\-2&-3&4\end{pmatrix}$$

$$\begin{pmatrix}6+a&-7+b&3+c\\h+4&k+3&t+9\end{pmatrix}=\begin{pmatrix}1&-1&3\\-2&-3&4\end{pmatrix}$$

Equating the corresponding elements, we have

$6 + a = 1$

$a = -5$

$-7 + b = -1$

$b = 6$

$3 + c = 3$

$c = 0$

$h + 4 = -2$

$h = -6$

$k + 3 = -3$

$k = -6$

$t + 9 = 4$

$t = -5$

$\therefore a = -5, b = 6, c = 0, h = -6, k = -6, t = -5$

(e) $$\begin{pmatrix}2&1\\-4&-6\\-3&8\end{pmatrix}+3\begin{pmatrix}x&y\\-1&4\\9&2\end{pmatrix}=\begin{pmatrix}x&6\\h&2k\\3t&14\end{pmatrix}$$

$$\begin{pmatrix}3x&3y\\-3&12\\27&6\end{pmatrix}-\begin{pmatrix}x&6\\h&2k\\3t&14\end{pmatrix}=-\begin{pmatrix}2&1\\-4&-6\\-3&8\end{pmatrix}$$

$$\begin{pmatrix}2x&3y-6\\-3-h&12-2k\\27-3t&-8\end{pmatrix}=\begin{pmatrix}-2&-1\\4&6\\3&-8\end{pmatrix}$$

Equating the corresponding elements, we have

$2x = -2$

$x = -1$

$3y - 6 = -1$

$y = 1\frac{2}{3}$

$-3 - h = 4$

$h = -7$

$12 - 2k = 6$

$k = 3$

$27 - 3t = 3$

$t = 8$

$\therefore x = -1, y = 1\frac{2}{3}, h = -7, k = 3, t = 8$

(f) $$\begin{pmatrix}-7&a&b\\-1&-2&3\end{pmatrix}=\begin{pmatrix}-7\\-1\end{pmatrix}\begin{pmatrix}1&2&-3\end{pmatrix}$$

$$\begin{pmatrix}-7&a&b\\-1&-2&3\end{pmatrix}=\begin{pmatrix}-7&-14&21\\-1&-2&3\end{pmatrix}$$

Equating the corresponding elements, we have

$a = -14, b = 21$

(g) $\begin{pmatrix}7 & -2\\ 10 & 3\end{pmatrix}\begin{pmatrix}p & q\\ r & s\end{pmatrix}=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$

$\begin{pmatrix}7p-2r & 7q-2s\\ -10p+3r & -10q+3s\end{pmatrix}=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$

Equating the corresponding elements, we have

$7p - 2r = 1$ — (1)

$7q - 2s = 0$ — (2)

$-10p + 3r = 0$ — (3)

$-10q + 3s = 1$ — (4)

From (2) : $q = \frac{2}{7}s$ — (5)

Substitute (5) into (4) : $-10\left(\frac{2}{7}s\right) + 3s = 1$

$\frac{1}{7}s = 1$

$s = 7$

Substitute $s = 7$ into (5) : $q = \frac{2}{7} \times 7 = 2$

From (3): $r = \frac{10}{3}p$ — (6)

Substitute (6) into (1) : $7p - 2\left(\frac{10}{3}p\right) = 1$

$\frac{1}{3}p = 1$

$p = 3$

Substitute $p = 3$ into (6) : $r = \frac{10}{3}(3) = 10$

$\therefore p = 3, q = 2, r = 10, s = 7$

(h) $\begin{pmatrix}a & -2\\ -5 & 1\end{pmatrix}\begin{pmatrix}-1\\ b\end{pmatrix}=\begin{pmatrix}-11\\ 10\end{pmatrix}$

$\begin{pmatrix}-a-2b\\ 5+b\end{pmatrix}=\begin{pmatrix}-11\\ 10\end{pmatrix}$

Equating the corresponding elements, we have

$-a - 2b = -11$ — (1)

$5 + b = 10$ — (2)

From (2) : $b = 10 - 5 = 5$

Substitute $b = 5$ into (1) : $-a - 2(5) = -11$

$a = 1$

$\therefore a = 1, b = 5$

(i) $\begin{pmatrix}3a & -1\\ a & b\end{pmatrix}\begin{pmatrix}1 & c\\ 4 & -3\end{pmatrix}=\begin{pmatrix}-7 & 6\\ 21 & d\end{pmatrix}$

$\begin{pmatrix}3a-4 & 3ac+3\\ a+4b & ac-3b\end{pmatrix}=\begin{pmatrix}-7 & 6\\ 21 & d\end{pmatrix}$

Equating the corresponding elements, we have

$3a - 4 = -7$

$a = -1$

$3ac + 3 = 6$

$c = -1$

$a + 4b = 21$

$b = \frac{22}{4}$

$= 5\frac{1}{2}$

$ac - 3b = d$

$d = -15\frac{1}{2}$

$\therefore a = -1, b = 5\frac{1}{2}, c = -1, d = -15\frac{1}{2}$

(j) $\begin{pmatrix}2 & 1\\ -3 & h\end{pmatrix}\begin{pmatrix}k & t & x\\ -1 & 2 & -3\end{pmatrix}=\begin{pmatrix}-7 & 6 & -5\\ 5 & t & 3k\end{pmatrix}$

$\begin{pmatrix}2k-1 & 2t+2 & 2x-3\\ -3k-h & -3t+2h & -3x-3h\end{pmatrix}=\begin{pmatrix}-7 & 6 & -5\\ 5 & t & 3k\end{pmatrix}$

Equating the corresponding elements, we have

$2k - 1 = -7$

$k = -3$

$2t + 2 = 6$

$t = 2$

$2x - 3 = -5$

$x = -1$

$-3k - h = 5$

$h = -3(-3) - 5$

$= 4$

$\therefore h = 4, k = -3, t = 2, x = -1$

5. **(a)** $\mathbf{X} + \begin{pmatrix}2 & 1\\ 4 & -3\end{pmatrix}=\begin{pmatrix}3 & 5\\ 7 & -9\end{pmatrix}$

$\mathbf{X} = \begin{pmatrix}3 & 5\\ 7 & -9\end{pmatrix} - \begin{pmatrix}2 & 1\\ 4 & -3\end{pmatrix}$

$= \begin{pmatrix}1 & 4\\ 3 & -6\end{pmatrix}$

(b) $\mathbf{Y} - \begin{pmatrix}2 & -4\\ -5 & -6\end{pmatrix}=\begin{pmatrix}1 & 7\\ 9 & 3\end{pmatrix}$

$\mathbf{Y} = \begin{pmatrix}1 & 7\\ 9 & 3\end{pmatrix} + \begin{pmatrix}2 & -4\\ -5 & -6\end{pmatrix}$

$= \begin{pmatrix}3 & 3\\ 4 & -3\end{pmatrix}$

(c) $$\mathbf{Z} + 2\begin{pmatrix} 3 & 1 \\ -4 & 5 \end{pmatrix} = \begin{pmatrix} 6 & 10 \\ 12 & 3 \end{pmatrix}$$

$$\mathbf{Z} = \begin{pmatrix} 6 & 10 \\ 12 & 3 \end{pmatrix} - \begin{pmatrix} 6 & 2 \\ -8 & 10 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 8 \\ 20 & -7 \end{pmatrix}$$

(d) $$\mathbf{P} - 3\begin{pmatrix} 1 & -3 \\ 4 & -2 \end{pmatrix} = 2\begin{pmatrix} 5 & 7 \\ 3 & -4 \end{pmatrix}$$

$$\mathbf{P} = \begin{pmatrix} 10 & 14 \\ 6 & -8 \end{pmatrix} + \begin{pmatrix} 3 & -9 \\ 12 & -6 \end{pmatrix}$$

$$= \begin{pmatrix} 13 & 5 \\ 18 & -14 \end{pmatrix}$$

(e) $$\mathbf{Q} + 3\begin{pmatrix} 1 & 2 & 5 \\ 4 & -6 & 7 \end{pmatrix} = \begin{pmatrix} 4 & 7 & -9 \\ -13 & 12 & 21 \end{pmatrix}$$

$$\mathbf{Q} = \begin{pmatrix} 4 & 7 & -9 \\ -13 & 12 & 21 \end{pmatrix}$$

$$- \begin{pmatrix} 3 & 6 & 15 \\ 12 & -18 & 21 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 1 & -24 \\ -25 & 30 & 0 \end{pmatrix}$$

(f) $$\mathbf{R} - 2\begin{pmatrix} -1 & -3 & -9 \\ 5 & 8 & -10 \end{pmatrix} = \begin{pmatrix} 7 & 3 & -5 \\ 8 & -9 & 4 \end{pmatrix}$$

$$\mathbf{R} = \begin{pmatrix} 7 & 3 & -5 \\ 8 & -9 & 4 \end{pmatrix}$$

$$+ 2\begin{pmatrix} -1 & -3 & -9 \\ 5 & 8 & -10 \end{pmatrix}$$

$$= \begin{pmatrix} 5 & -3 & -23 \\ 18 & 7 & -16 \end{pmatrix}$$

6. $$\mathbf{A} = \begin{pmatrix} 10 & 8 \\ -5 & 4 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 4 & -1 \\ 7 & -3 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -1 & -3 \\ 0 & 4 \end{pmatrix}$$

(a) $$\mathbf{A} + \mathbf{B} - 2\mathbf{C} = \begin{pmatrix} 10 & 8 \\ -5 & 4 \end{pmatrix} + \begin{pmatrix} 4 & -1 \\ 7 & -3 \end{pmatrix} - 2\begin{pmatrix} -1 & -3 \\ 0 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 16 & 13 \\ 2 & -7 \end{pmatrix}$$

(b) $2\mathbf{A} + 5\mathbf{B} - 3\mathbf{C}$

$$= 2\begin{pmatrix} 10 & 8 \\ -5 & 4 \end{pmatrix} + 5\begin{pmatrix} 4 & -1 \\ 7 & -3 \end{pmatrix} - 3\begin{pmatrix} -1 & -3 \\ 0 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 20 & 16 \\ -10 & 8 \end{pmatrix} + \begin{pmatrix} 20 & -5 \\ 35 & -15 \end{pmatrix} - \begin{pmatrix} -3 & -9 \\ 0 & 12 \end{pmatrix}$$

$$= \begin{pmatrix} 43 & 20 \\ 25 & -19 \end{pmatrix}$$

(c) $$\mathbf{AB} - \mathbf{C} = \begin{pmatrix} 10 & 8 \\ -5 & 4 \end{pmatrix}\begin{pmatrix} 4 & -1 \\ 7 & -3 \end{pmatrix} - \begin{pmatrix} -1 & -3 \\ 0 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 96 & -34 \\ 8 & -7 \end{pmatrix} - \begin{pmatrix} -1 & -3 \\ 0 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 97 & -31 \\ 8 & -11 \end{pmatrix}$$

(d) $$\mathbf{AC} + \mathbf{B} = \begin{pmatrix} 10 & 8 \\ -5 & 4 \end{pmatrix}\begin{pmatrix} -1 & -3 \\ 0 & 4 \end{pmatrix} + \begin{pmatrix} 4 & -1 \\ 7 & -3 \end{pmatrix}$$

$$= \begin{pmatrix} -10 & 2 \\ 5 & 31 \end{pmatrix} + \begin{pmatrix} 4 & -1 \\ 7 & -3 \end{pmatrix}$$

$$= \begin{pmatrix} -6 & 1 \\ 12 & 28 \end{pmatrix}$$

(e) $$\mathbf{BC} - 2\mathbf{A} = \begin{pmatrix} 4 & -1 \\ 7 & -3 \end{pmatrix}\begin{pmatrix} -1 & -3 \\ 0 & 4 \end{pmatrix} - 2\begin{pmatrix} 10 & 8 \\ -5 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} -4 & -16 \\ -7 & -33 \end{pmatrix} - \begin{pmatrix} 20 & 16 \\ -10 & 8 \end{pmatrix}$$

$$= \begin{pmatrix} -24 & -32 \\ 3 & -41 \end{pmatrix}$$

(f) $$\mathbf{ABC} = \begin{pmatrix} 10 & 8 \\ -5 & 4 \end{pmatrix}\begin{pmatrix} 4 & -1 \\ 7 & -3 \end{pmatrix}\begin{pmatrix} -1 & -3 \\ 0 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 96 & -34 \\ 8 & -7 \end{pmatrix}\begin{pmatrix} -1 & -3 \\ 0 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} -96 & -424 \\ -8 & -52 \end{pmatrix}$$

7. **(i)** **(a)** Determinant = (3 × 2) – (2 × 3)

= 0

(b) Since determinant = 0, the matrix has no inverse.

(ii) **(a)** Determinant = (2 × 3) – (0 × 1)

= 6

(b) $$\text{Inverse} = \frac{1}{6}\begin{pmatrix} 3 & 0 \\ -1 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{2} & 0 \\ -\frac{1}{6} & \frac{1}{3} \end{pmatrix}$$

(iii) **(a)** Determinant = (1 × 6) – (4 × 5)

= –14

(b) $$\text{Inverse} = \frac{1}{-14}\begin{pmatrix} 6 & -4 \\ -5 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -\frac{3}{7} & \frac{2}{7} \\ \frac{5}{7} & -\frac{1}{14} \end{pmatrix}$$

(iv) (a) Determinant $= (2 \times 8) - (5 \times 3)$
$= 1$

(b) Inverse $= \begin{pmatrix} 8 & -5 \\ -3 & 2 \end{pmatrix}$

(v) (a) Determinant $= (3 \times 9) - (8 \times 4)$
$= -5$

(b) Inverse $= \dfrac{1}{-5}\begin{pmatrix} 9 & -8 \\ -4 & 3 \end{pmatrix}$

$$= \begin{pmatrix} -\frac{9}{5} & \frac{8}{5} \\ \frac{4}{5} & -\frac{3}{5} \end{pmatrix}$$

(vi) (a) Determinant $= \left(\frac{1}{2} \times -\frac{1}{4}\right) - \left(\frac{1}{4} \times \frac{1}{8}\right)$
$= -\dfrac{5}{32}$

(b) Inverse $= -\dfrac{32}{5}\begin{pmatrix} -\frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{8} & \frac{1}{2} \end{pmatrix}$

$$= \begin{pmatrix} \frac{8}{5} & \frac{8}{5} \\ \frac{4}{5} & -\frac{16}{5} \end{pmatrix}$$

(vii) (a) Determinant $= \left(\frac{1}{2} \times \frac{1}{4}\right) - \left(\frac{3}{8} \times \frac{1}{8}\right)$
$= \dfrac{5}{64}$

(b) Inverse $= \dfrac{64}{5}\begin{pmatrix} \frac{1}{4} & -\frac{3}{8} \\ -\frac{1}{8} & \frac{1}{2} \end{pmatrix}$

$$= \begin{pmatrix} \frac{16}{5} & -\frac{24}{5} \\ -\frac{8}{5} & \frac{32}{5} \end{pmatrix}$$

(viii) (a) Determinant $= (1 \times 1) - (-1 \times -1)$
$= 0$

(b) Since determinant = 0, the matrix has no inverse.

8. (i) Determinant $= \left(0 \times \frac{1}{p}\right) - (p \times 0)$
$= 0$

$\therefore$ The inverse does not exist.

(ii) Determinant $= (a \times a) - (-a \times a)$
$= 2a^2$

$\therefore$ The inverse exists.

Inverse $= \dfrac{1}{2a^2}\begin{pmatrix} a & a \\ -a & a \end{pmatrix}$

$$= \begin{pmatrix} \frac{1}{2a} & \frac{1}{2a} \\ -\frac{1}{2a} & \frac{1}{2a} \end{pmatrix}$$

(iii) Determinant $= (3 \times 3) - (0 \times 0)$
$= 9$

$\therefore$ The inverse exists.

Inverse $= \dfrac{1}{9}\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$

$$= \begin{pmatrix} \frac{1}{3} & 0 \\ 0 & \frac{1}{3} \end{pmatrix}$$

(iv) Determinant $= (0 \times 0) - (-1 \times -1)$
$= -1$

$\therefore$ The inverse exists.

Inverse $= -\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$$= \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$$

9. $|\mathbf{A}| = (2 \times 3) - (4 \times 1)$
$= 2$

$\therefore$ $\mathbf{A}^{-1}$ exists.

$$\mathbf{A}^{-1} = \frac{1}{2}\begin{pmatrix} 3 & -4 \\ -1 & 2 \end{pmatrix}$$

$$\mathbf{AX} = \mathbf{B}$$

$$\mathbf{A}^{-1}\mathbf{AX} = \mathbf{A}^{-1}\mathbf{B}$$

$$(\mathbf{A}^{-1}\mathbf{A})\mathbf{X} = \mathbf{A}^{-1}\mathbf{B}$$

$$\mathbf{IX} = \mathbf{A}^{-1}\mathbf{B}$$

$$\mathbf{X} = \frac{1}{2}\begin{pmatrix} 3 & -4 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 16 \\ 13 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} -4 \\ 10 \end{pmatrix}$$

$$= \begin{pmatrix} -2 \\ 5 \end{pmatrix}$$

10. (a) The simultaneous equations may be written in matrix form as

$$\begin{pmatrix} 3 & 2 \\ 4 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 9 \\ 24 \end{pmatrix}$$

Determinant of $\begin{pmatrix} 3 & 2 \\ 4 & 3 \end{pmatrix} = (3 \times 3) - (2 \times 4)$

$= 1$

$$\begin{aligned} \begin{pmatrix} x \\ y \end{pmatrix} &= \begin{pmatrix} 3 & 2 \\ 4 & 3 \end{pmatrix}^{-1}\begin{pmatrix} 9 \\ 24 \end{pmatrix} \\ &= \begin{pmatrix} 3 & -2 \\ -4 & 3 \end{pmatrix}\begin{pmatrix} 9 \\ 24 \end{pmatrix} \\ &= \begin{pmatrix} -21 \\ 36 \end{pmatrix} \end{aligned}$$

$\therefore x = -21, y = 36$

(b) The simultaneous equations may be written in matrix form as

$$\begin{pmatrix} 3 & 2 \\ -3 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 10 \\ -4 \end{pmatrix}$$

Determinant of $\begin{pmatrix} 3 & 2 \\ -3 & 2 \end{pmatrix} = (3 \times 2) - (2 \times -3)$

$= 12$

$$\begin{aligned} \begin{pmatrix} x \\ y \end{pmatrix} &= \begin{pmatrix} 3 & 2 \\ -3 & 2 \end{pmatrix}^{-1}\begin{pmatrix} 10 \\ -4 \end{pmatrix} \\ &= \frac{1}{12}\begin{pmatrix} 2 & -2 \\ 3 & 3 \end{pmatrix}\begin{pmatrix} 10 \\ -4 \end{pmatrix} \\ &= \frac{1}{12}\begin{pmatrix} 28 \\ 18 \end{pmatrix} \\ &= \begin{pmatrix} 2\frac{1}{3} \\ 1\frac{1}{2} \end{pmatrix} \end{aligned}$$

$\therefore x = 2\frac{1}{3}, y = 1\frac{1}{2}$

(c) The simultaneous equations may be written in matrix form as

$$\begin{pmatrix} 4 & 5 \\ 2 & 5 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Determinant of $\begin{pmatrix} 4 & 5 \\ 2 & 5 \end{pmatrix} = (4 \times 5) - (5 \times 2)$

$= 10$

$$\begin{aligned} \begin{pmatrix} x \\ y \end{pmatrix} &= \begin{pmatrix} 4 & 5 \\ 2 & 5 \end{pmatrix}^{-1}\begin{pmatrix} 0 \\ 1 \end{pmatrix} \\ &= \frac{1}{10}\begin{pmatrix} 5 & -5 \\ -2 & 4 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} \\ &= \frac{1}{10}\begin{pmatrix} -5 \\ 4 \end{pmatrix} \\ &= \begin{pmatrix} -\frac{1}{2} \\ \frac{2}{5} \end{pmatrix} \end{aligned}$$

$\therefore x = -\frac{1}{2}, y = \frac{2}{5}$

(d) The simultaneous equations may be written in matrix form as

$$\begin{pmatrix} 3 & 2 \\ -4 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 6 \\ 14 \end{pmatrix}$$

Determinant of $\begin{pmatrix} 3 & 2 \\ -4 & 1 \end{pmatrix} = (3 \times 1) - (2 \times -4)$

$= 11$

$$\begin{aligned} \begin{pmatrix} x \\ y \end{pmatrix} &= \begin{pmatrix} 3 & 2 \\ -4 & 1 \end{pmatrix}^{-1}\begin{pmatrix} 6 \\ 14 \end{pmatrix} \\ &= \frac{1}{11}\begin{pmatrix} 1 & -2 \\ 4 & 3 \end{pmatrix}\begin{pmatrix} 6 \\ 14 \end{pmatrix} \\ &= \frac{1}{11}\begin{pmatrix} -22 \\ 66 \end{pmatrix} \\ &= \begin{pmatrix} -2 \\ 6 \end{pmatrix} \end{aligned}$$

$\therefore x = -2, y = 6$

(e) The simultaneous equations may be written in matrix form as

$$\begin{pmatrix} 5 & 1 \\ 10 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$$

Determinant of $\begin{pmatrix} 5 & 1 \\ 10 & 2 \end{pmatrix} = (5 \times 2) - (1 \times 10)$

$= 0$

Hence $\begin{pmatrix} 5 & 1 \\ 10 & 2 \end{pmatrix}$ is a singular matrix and its inverse matrix does not exist.

$5x + y = 2$ and $10x + 2y = 4$ represent the same line. There is an infinite number solutions.

(f) The simultaneous equations may be written in matrix form as

$$\begin{pmatrix}6 & -1\\ 6 & -1\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix} = \begin{pmatrix}3\\ 4\end{pmatrix}$$

$$\text{Determinant of }\begin{pmatrix}6 & -1\\ 6 & -1\end{pmatrix} = (6 \times -1) - (-1 \times 6)$$

$$= 0$$

Hence $\begin{pmatrix}6 & -1\\ 6 & -1\end{pmatrix}$ is a singular matrix and its inverse matrix does not exist.

$5x + y = 2$ and $10x + 2y = 4$ represent two parallel lines. There is no solution.

Intermediate

11. (a) $\begin{pmatrix}p & q\end{pmatrix}\begin{pmatrix}3 & 0\\ -4 & 2\end{pmatrix} = \begin{pmatrix}-3 & 6\end{pmatrix}$

$$\begin{pmatrix}3p - 4q & 2q\end{pmatrix} = \begin{pmatrix}-3 & 6\end{pmatrix}$$

Equating the corresponding elements, we have

$3p - 4q = -3$ — (1)

$2q = 6$ — (2)

From (2) : $q = 3$

Substitute $q = 3$ into (1) : $3p - 4(3) = -3$

$p = 3$

$\therefore p = 3, q = 3$

(b) $\begin{pmatrix}a & b\\ 3 & 2a\end{pmatrix}\begin{pmatrix}1\\ 4\end{pmatrix} = \begin{pmatrix}15\\ 11\end{pmatrix}$

$$\begin{pmatrix}a + 4b\\ 3 + 8a\end{pmatrix} = \begin{pmatrix}15\\ 11\end{pmatrix}$$

Equating the corresponding elements, we have

$a + 4b = 15$ — (1)

$3 + 8a = 11$ — (2)

From (2) : $a = 1$

Substitute $a = 1$ into (1) : $1 + 4b = 15$

$b = 3\frac{1}{2}$

$\therefore a = 1, b = 3\frac{1}{2}$

(c) $\begin{pmatrix}x & 2\\ 2y & 0\end{pmatrix}\begin{pmatrix}4\\ -1\end{pmatrix} = \begin{pmatrix}-2\\ 8\end{pmatrix}$

$$\begin{pmatrix}4x - 2\\ 8y\end{pmatrix} = \begin{pmatrix}-2\\ 8\end{pmatrix}$$

Equating the corresponding elements, we have

$4x - 2 = -2$

$x = 0$

$8y = 8$

$y = 1$

$\therefore x = 0, y = 1$

(d) $\begin{pmatrix}3 & x\\ 5 & 0\end{pmatrix}\begin{pmatrix}x\\ x\end{pmatrix} = \begin{pmatrix}y\\ -10\end{pmatrix}$

$$\begin{pmatrix}3x + x^2\\ 5x\end{pmatrix} = \begin{pmatrix}y\\ -10\end{pmatrix}$$

Equating the corresponding elements, we have

$3x + x^2 = y$ — (1)

$5x = -10$ — (2)

From (2) : $x = -2$

Substitute $x = -2$ into (1) : $y = 3(-2) + (-2)^2 = -2$

$\therefore x = -2, y = -2$

(e) $\begin{pmatrix}x & 3\\ -1 & 4\end{pmatrix}\begin{pmatrix}4 & y\\ 1 & x\end{pmatrix} = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$

$$\begin{pmatrix}4x + 3 & xy + 3x\\ 0 & 4x - y\end{pmatrix} = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$

Equating the corresponding elements, we have

$4x + 3 = 1$

$x = -\frac{1}{2}$

$xy + 3x = 0$

$-\frac{1}{2}y = -3\left(-\frac{1}{2}\right)$

$y = -3$

Substitute $x = -\frac{1}{2}$ and $y = -3$ into $4x - y$:

$4\left(-\frac{1}{2}\right) - (-3) = 1$

$\therefore x = -\frac{1}{2}$ and $y = -3$

12. (a) $\mathbf{AB} = \begin{pmatrix}2\\ 3\end{pmatrix}(-1 \quad 4)$

$$= \begin{pmatrix}-2 & 8\\ -3 & 12\end{pmatrix}$$

(b) $2\mathbf{BA} = 2(-1 \quad 4)\begin{pmatrix}2\\ 3\end{pmatrix}$

$= 2(10)$

$= (20)$

13. $\mathbf{AB} = \begin{pmatrix}1\\ 3\end{pmatrix}(2 \quad 4)$

$$= \begin{pmatrix}2 & 4\\ 6 & 12\end{pmatrix}$$

$\mathbf{BA} = (2 \quad 4)\begin{pmatrix}1\\ 3\end{pmatrix}$

$= (14)$

14. **(a)** $3\mathbf{A} - 2\mathbf{X} = \mathbf{B}$

$$3\begin{pmatrix}-6 & -16\\ 13 & 9\end{pmatrix} - 2\mathbf{X} = \begin{pmatrix}-1 & -5\\ 4 & 6\end{pmatrix}$$

$$2\mathbf{X} = \begin{pmatrix}-18 & -48\\ 39 & 27\end{pmatrix} - \begin{pmatrix}-1 & -5\\ 4 & 6\end{pmatrix}$$

$$\mathbf{X} = \frac{1}{2}\begin{pmatrix}-17 & -43\\ 35 & 21\end{pmatrix}$$

(b) Let $\mathbf{Y} = \begin{pmatrix}a & b\\ c & d\end{pmatrix}$

$$\mathbf{YB} = \mathbf{A}$$

$$\begin{pmatrix}a & b\\ c & d\end{pmatrix}\begin{pmatrix}-1 & -5\\ 4 & 6\end{pmatrix} = \begin{pmatrix}-6 & -16\\ 13 & 9\end{pmatrix}$$

$$\begin{pmatrix}-a+4b & -5a+6b\\ -c+4d & -5c+6d\end{pmatrix} = \begin{pmatrix}-6 & -16\\ 13 & 9\end{pmatrix}$$

Equating the corresponding elements, we have

$-a + 4b = -6$ — (1)

$-5a + 6b = -16$ — (2)

$-c + 4d = 13$ — (3)

$-5c + 6d = 9$ — (4)

From (1) : $a = 4b + 6$ — (5)

Substitute (5) into (2) : $-5(4b + 6) + 6b = -16$

$$-14b = 14$$

$$b = -1$$

Substitute $b = -1$ into (5) : $a = 4(-1) + 6 = 2$

From (3) : $c = 4d - 13$ — (6)

Substitute (6) into (4) : $-5(4d - 13) + 6d = 9$

$$-14d = -56$$

$$d = 4$$

Substitute $d = 4$ into (6) : $c = 4(4) - 13 = 3$

$\therefore \mathbf{Y} = \begin{pmatrix}2 & -1\\ 3 & 4\end{pmatrix}$

15. $(1 \quad 3)\begin{pmatrix}x\\ 3\end{pmatrix} = \begin{pmatrix}1 & 3 & -1\end{pmatrix}\begin{pmatrix}x\\ 2x\\ 3\end{pmatrix}$

$(x + 9) = (x + 6x - 3)$

Equating the corresponding elements, we have

$x + 9 = x + 6x - 3$

$6x = 12$

$x = 2$

16. $\begin{pmatrix}x\\ 1\end{pmatrix}(3 \quad y) = \begin{pmatrix}-4 & 2\\ 1 & -3\end{pmatrix}\begin{pmatrix}z & t\\ 3 & 5\end{pmatrix}$

$$\begin{pmatrix}3x & xy\\ 3 & y\end{pmatrix} = \begin{pmatrix}-4z+6 & -4t+10\\ z-9 & t-15\end{pmatrix}$$

Equating the corresponding elements, we have

$3x = -4z + 6$ — (1)

$xy = -4t + 10$ — (2)

$3 = z - 9$ — (3)

$y = t - 15$ — (4)

From (3) : $z = 12$

Substitute $z = 12$ into (1) : $3x = -4(12) + 6 = -42$

$$x = -14$$

Substitute (4) and $x = -14$ into (2) : $-14(t - 15) = -4t + 10$

$$10t = 200$$

$$t = 20$$

Substitute $t = 20$ into (4) : $y = 20 - 15 = 5$

$\therefore t = 20, x = -14, y = 5, z = 12$

17. $\begin{pmatrix}3 & 0\\ -1 & 4\end{pmatrix}\begin{pmatrix}a & b\\ c & 4\end{pmatrix} = \begin{pmatrix}x\\ 4\end{pmatrix}(2 \quad 1)$

$$\begin{pmatrix}3a & 3b\\ -a+4c & -b+16\end{pmatrix} = \begin{pmatrix}2x & x\\ 8 & 4\end{pmatrix}$$

Equating the corresponding elements, we have

$3a = 2x$ — (1)

$3b = x$ — (2)

$-a + 4c = 8$ — (3)

$-b + 16 = 4$ — (4)

From (4): $b = 12$

Substitute $b = 12$ into (2): $x = 36$

Substitute $x = 36$ into (1): $a = \frac{1}{3} \times 2(36) = 24$

Substitute $a = 24$ into (3): $c = \frac{1}{4} \times (8 + 24) = 8$

$\therefore a = 24, b = 12, c = 8, x = 36$

18. $\begin{pmatrix} 2x & 0 \\ 3y & -z \end{pmatrix}\begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 6 \\ 11 \end{pmatrix}$

$$\begin{pmatrix} 6x \\ 9y-4z \end{pmatrix} = \begin{pmatrix} 6 \\ 11 \end{pmatrix}$$

Equating the corresponding elements, we have

$6x = 6$

$x = 1$

$9y - 4z = 11$ — (1)

$$\begin{pmatrix} 2x & 0 \\ 3y & -z \end{pmatrix}\begin{pmatrix} 5 \\ -15 \end{pmatrix} = \begin{pmatrix} 10 \\ -15 \end{pmatrix}$$

$$\begin{pmatrix} 10x \\ 15y+15z \end{pmatrix} = \begin{pmatrix} 10 \\ -15 \end{pmatrix}$$

Equating the corresponding elements, we have

$10x = 10$

$x = 1$

$15y + 15z = -15$

$y + z = -1$ — (2)

From (2): $y = -1 - z$ — (3)

Substitute (3) into (1): $9(-1 - z) - 4z = 11$

$-13z = 20$

$z = -\dfrac{20}{13}$

Substitute $z = -\dfrac{20}{13}$ into (3): $y = \dfrac{7}{13}$

$\therefore x = 1, y = \dfrac{7}{13}, z = -\dfrac{20}{13}$

19. $\mathbf{L} = \begin{pmatrix} 8 & -6 \\ -5 & 2 \end{pmatrix}$, $\mathbf{M} = \begin{pmatrix} 4 & -3 \\ 9 & -5 \end{pmatrix}$ and $\mathbf{N} = \begin{pmatrix} 3 & -3 \\ -2 & 5 \end{pmatrix}$

(a) $2\mathbf{L} + 3\mathbf{M} - \mathbf{N}$

$$= 2\begin{pmatrix} 8 & -6 \\ -5 & 2 \end{pmatrix} + 3\begin{pmatrix} 4 & -3 \\ 9 & -5 \end{pmatrix} - \begin{pmatrix} 3 & -3 \\ -2 & 5 \end{pmatrix}$$

$$= \begin{pmatrix} 16 & -12 \\ -10 & 4 \end{pmatrix} + \begin{pmatrix} 12 & -9 \\ 27 & -15 \end{pmatrix} - \begin{pmatrix} 3 & -3 \\ -2 & 5 \end{pmatrix}$$

$$= \begin{pmatrix} 25 & -18 \\ 19 & -16 \end{pmatrix}$$

(b) $\mathbf{LMN} = \begin{pmatrix} 8 & -6 \\ -5 & 2 \end{pmatrix}\begin{pmatrix} 4 & -3 \\ 9 & -5 \end{pmatrix}\begin{pmatrix} 3 & -3 \\ -2 & 5 \end{pmatrix}$

$$= \begin{pmatrix} -22 & 6 \\ -2 & 5 \end{pmatrix}\begin{pmatrix} 3 & -3 \\ -2 & 5 \end{pmatrix}$$

$$= \begin{pmatrix} -78 & 96 \\ -16 & 31 \end{pmatrix}$$

20. $\begin{pmatrix} 3 & 0 \\ 5 & 2 \end{pmatrix}\begin{pmatrix} k & 0 \\ 3 & 3h \end{pmatrix} = \begin{pmatrix} 6 & 0 \\ m & 2h-7 \end{pmatrix}$

$$\begin{pmatrix} 3k & 0 \\ 5k+6 & 6h \end{pmatrix} = \begin{pmatrix} 6 & 0 \\ m & 2h-7 \end{pmatrix}$$

Equating the corresponding elements, we have

$3k = 6$

$k = 2$

$5k + 6 = m$

$m = 16$

$6h = 2h - 7$

$h = -1\dfrac{3}{4}$

$\therefore h = -1\dfrac{3}{4}, k = 2, m = 16$

21. The simultaneous equations may be written in matrix form as

$$\begin{pmatrix} 9 & q \\ 18 & -6 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 12 \\ 4 \end{pmatrix}$$

Determinant of $\begin{pmatrix} 9 & q \\ 18 & -6 \end{pmatrix} = (9 \times -6) - (q \times 18)$

$= -54 - 18q$

Since the equations have no solution, $-54 - 18q = 0$

$q = -3$

Advanced

22. Let $\mathbf{C} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$.

$$\begin{pmatrix} 2 & 1 \\ -1 & 3 \end{pmatrix}\begin{pmatrix} 3 & -1 \\ 4 & -2 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 10 & -4 \\ 9 & -5 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 10a-4c & 10b-4d \\ 9a-5c & 9b-5d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Equating the corresponding elements, we have

$10a - 4c = 1$ — (1)

$10b - 4d = 0$ — (2)

$9a - 5c = 0$ — (3)

$9b - 5d = 1$ — (4)

From (2): $b = \dfrac{2}{5}d$ — (5)

Substitute (5) into (4): $9\left(\dfrac{2}{5}d\right) - 5d = 1$

$d = -\dfrac{5}{7}$

Substitute $d = -\frac{5}{7}$ into (5): $b = -\frac{2}{7}$

From (3): $a = \frac{5}{9}c$ — (6)

Substitute (6) into (1): $10\left(\frac{5}{9}c\right) - 4c = 1$

$$c = \frac{9}{14}$$

Substitute $c = \frac{9}{14}$ into (6): $a = \frac{5}{14}$

$\therefore \mathbf{C} = \frac{1}{14}\begin{pmatrix} 5 & -4 \\ 9 & -10 \end{pmatrix}$

23. $\mathbf{AB} = \mathbf{BA}$

$$\begin{pmatrix} -3 & h \\ -1 & 1 \end{pmatrix}\begin{pmatrix} k & 0 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} k & 0 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} -3 & h \\ -1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} -3k+h & 2h \\ -k+1 & 2 \end{pmatrix} = \begin{pmatrix} -3k & kh \\ -5 & h+2 \end{pmatrix}$$

Equating the corresponding elements, we have

$-3k + h = -3k$ — (1)

$h = 0$

$2h = kh$ — (2)

$h = 0$ or $k = 2$

$-k + 1 = -5$ — (3)

$k = 6$

$\therefore$ $k = 2$ is rejected since it does not satisfy (3).

$2 = h + 2$ — (4)

$h = 0$

$\therefore h = 0, k = 6$

24. $\mathbf{A}^2 = \begin{pmatrix} -1 & -1 \\ 3 & 3 \end{pmatrix}\begin{pmatrix} -1 & -1 \\ 3 & 3 \end{pmatrix}$

$= \begin{pmatrix} -2 & -2 \\ 6 & 6 \end{pmatrix}$

$\mathbf{A}^3 = \begin{pmatrix} -2 & -2 \\ 6 & 6 \end{pmatrix}\begin{pmatrix} -1 & -1 \\ 3 & 3 \end{pmatrix}$

$= \begin{pmatrix} -4 & -4 \\ 12 & 12 \end{pmatrix}$

$\therefore \mathbf{A}^5 = \begin{pmatrix} -16 & -16 \\ 48 & 48 \end{pmatrix}$ and $\mathbf{A}^7 \begin{pmatrix} -64 & -64 \\ 192 & 192 \end{pmatrix}$

25. $(x \quad y)\begin{pmatrix} x \\ y \end{pmatrix} = (13)$

$(x^2 + y^2) = (13)$

$x^2 + y^2 = 13$

$\therefore x^2 = 4,\ y^2 = 9$ or $x^2 = 9,\ y^2 = 4$

$\therefore x = 2, y = 3$ or $x = 3, y = 2$

26. $\mathbf{CBA} = \begin{pmatrix} 1 & 0 & 2 \\ 3 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}(1 \quad 0 \quad 2 \quad 3)$

$= \begin{pmatrix} 5 \\ 3 \end{pmatrix}(1 \quad 0 \quad 2 \quad 3)$

$= \begin{pmatrix} 5 & 0 & 10 & 15 \\ 3 & 0 & 6 & 9 \end{pmatrix}$

27. (a) $\begin{pmatrix} 0.25 & 0.2 & 0.3 & 5 \\ 0.2 & 0.15 & 0.25 & 4 \\ 0.3 & 0.25 & 0.25 & 5 \\ 0.25 & 0.3 & 0.28 & 3 \end{pmatrix}\begin{pmatrix} 1.20 \\ 2.50 \\ 1.10 \\ 0.12 \end{pmatrix} = \begin{pmatrix} 1.73 \\ 1.37 \\ 1.86 \\ 1.27 \end{pmatrix}$

(b) $(350 \quad 380 \quad 420 \quad 290)\begin{pmatrix} 1.73 \\ 1.37 \\ 1.86 \\ 1.27 \end{pmatrix} = (2406.10)$

$\therefore$ The total cost is \$2406.10.

28. (a) $\begin{pmatrix} 2500 & 1400 & 1200 \\ 4600 & 2800 & 2600 \end{pmatrix}\begin{pmatrix} 8 \\ 12 \\ 14 \end{pmatrix} = \begin{pmatrix} 53\ 600 \\ 106\ 800 \end{pmatrix}$

$(9.8 \quad 12)\begin{pmatrix} 53\ 600 \\ 106\ 800 \end{pmatrix} = (1\ 806\ 880)$

$\therefore$ The total revenue is 1 806 880 cents.

(b) $(x \quad x)\begin{pmatrix} 53\ 600 \\ 106\ 800 \end{pmatrix} = (1\ 806\ 880)$

$(160400x) = (1\ 806\ 880)$

$x = 11.26$ cents (to nearest 0.01 cents)

29. (a) $\begin{pmatrix} 60 & 65 & 95 & 50 \\ 20 & 25 & 30 & 15 \\ 25 & 20 & 30 & 15 \end{pmatrix}\begin{pmatrix} 250 \\ 200 \\ 150 \\ 430 \end{pmatrix} = \begin{pmatrix} 63\ 750 \\ 20\ 950 \\ 21\ 200 \end{pmatrix}$

$(1 \quad 1 \quad 1)\begin{pmatrix} 63\ 750 \\ 20\ 950 \\ 21\ 200 \end{pmatrix} = (105\ 900)$

$\therefore$ The total cost of preparing the food is 105 900 cents.

(b) $(2 \quad 1.5 \quad 2.5 \quad 1.2)\begin{pmatrix} 250 \\ 200 \\ 150 \\ 430 \end{pmatrix} = (1691)$

$\therefore$ $\mathbf{H} = (2 \quad 1.5 \quad 2.5 \quad 1.2)$, $\mathbf{K} = \begin{pmatrix} 250 \\ 200 \\ 150 \\ 430 \end{pmatrix}$ and the total time in preparing the food is 1691 minutes.

30. $$(24\quad 36\quad 28)\begin{pmatrix}3 & 1.5 & 6 & 2 & 0\\ 2 & 2 & 8 & 0 & 2\\ 1 & 2 & 0 & 2 & 3\end{pmatrix}$$

$$= (172\quad 164\quad 432\quad 104\quad 156)$$

$$(172\quad 164\quad 432\quad 104\quad 156)\begin{pmatrix}3.2\\ 12\\ 1.85\\ 50\\ 72\end{pmatrix} = (19\,749.6)$$

∴ The total cost of the hampers is \$19 749.60.

31. $$\mathbf{A}^2 = \begin{pmatrix}1 & -1\\ 2 & -1\end{pmatrix}\begin{pmatrix}1 & -1\\ 2 & -1\end{pmatrix}$$

$$= \begin{pmatrix}-1 & 0\\ 0 & -1\end{pmatrix}$$

$$\mathbf{B}^2 = \begin{pmatrix}1 & 1\\ 4 & -1\end{pmatrix}\begin{pmatrix}1 & 1\\ 4 & -1\end{pmatrix}$$

$$= \begin{pmatrix}5 & 0\\ 0 & 5\end{pmatrix}$$

$$\mathbf{AB} = \begin{pmatrix}1 & -1\\ 2 & -1\end{pmatrix}\begin{pmatrix}1 & 1\\ 4 & -1\end{pmatrix}$$

$$= \begin{pmatrix}-3 & 2\\ -2 & 3\end{pmatrix}$$

$$\mathbf{BA} = \begin{pmatrix}1 & 1\\ 4 & -1\end{pmatrix}\begin{pmatrix}1 & -1\\ 2 & -1\end{pmatrix}$$

$$= \begin{pmatrix}3 & -2\\ 2 & -3\end{pmatrix}$$

$$(\mathbf{AB} + \mathbf{BA}) = \begin{pmatrix}-3 & 2\\ -2 & 3\end{pmatrix} + \begin{pmatrix}3 & -2\\ 2 & -3\end{pmatrix}$$

$$= \begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix}$$

$$\mathbf{A} + \mathbf{B} = \begin{pmatrix}1 & -1\\ 2 & -1\end{pmatrix} + \begin{pmatrix}1 & 1\\ 4 & -1\end{pmatrix}$$

$$= \begin{pmatrix}2 & 0\\ 6 & -2\end{pmatrix}$$

$$(\mathbf{A} + \mathbf{B})^2 = \begin{pmatrix}2 & 0\\ 6 & -2\end{pmatrix}\begin{pmatrix}2 & 0\\ 6 & -2\end{pmatrix}$$

$$= \begin{pmatrix}4 & 0\\ 0 & 4\end{pmatrix}$$

$$\text{Determinant of } (\mathbf{A} + \mathbf{B}) = (2 \times -2) - (0 \times 6)$$

$$= -4$$

$$\text{Inverse} = \frac{1}{-4}\begin{pmatrix}-2 & 0\\ -6 & 2\end{pmatrix}$$

$$= \begin{pmatrix}\frac{1}{2} & 0\\ \frac{3}{2} & -\frac{1}{2}\end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix}1 & 0\\ 3 & -1\end{pmatrix}$$

New Trend

32. **(a)** $$\mathbf{D} = \begin{pmatrix}5 & 3 & 2\\ 0 & 4 & 1\end{pmatrix}$$

(b) $\mathbf{E} = \mathbf{DC}$

$$= \begin{pmatrix}5 & 3 & 2\\ 0 & 4 & 1\end{pmatrix}\begin{pmatrix}1.6 & 0.2\\ 1.1 & -0.1\\ 3.2 & -0.3\end{pmatrix}$$

$$= \begin{pmatrix}17.7 & 0.1\\ 7.6 & -0.7\end{pmatrix}$$

(c) Amount of savings = \$0.70

(d) $$\text{Amount Yan pays} = \frac{92}{100} \times \$17.70$$

$$= \$16.28 \text{ (to 2 d.p.)}$$

33. **(a)** The duration of time frame is not specified.
The intervals of number of hours are not consistent.
0 or 1 hour option not available.
Unable to capture the range of 3-4 or 5-6 hours.
The question does not specify the subject of the tuition.
Any two of the above.

(b) $$\mathbf{Q} = \begin{pmatrix}24 & 15 & 17\\ 18 & 22 & 16\end{pmatrix}$$

(c) **(i)** $$\mathbf{QS} = \begin{pmatrix}24 & 15 & 17\\ 18 & 22 & 16\end{pmatrix}\begin{pmatrix}15\\ 18\\ 20\end{pmatrix}$$

$$= \begin{pmatrix}970\\ 986\end{pmatrix}$$

(ii) $$\mathbf{QC} = \begin{pmatrix}24 & 15 & 17\\ 18 & 22 & 16\end{pmatrix}\begin{pmatrix}10\\ 15\\ 18\end{pmatrix}$$

$$= \begin{pmatrix}771\\ 798\end{pmatrix}$$

$$\mathbf{P} = \mathbf{QS} - \mathbf{QC}$$

$$= \begin{pmatrix}970\\ 986\end{pmatrix} - \begin{pmatrix}771\\ 798\end{pmatrix}$$

$$= \begin{pmatrix}199\\ 188\end{pmatrix}$$

(iii) **P** represents the profits earned by each shop respectively. \$199 represents the profit earned by shop *A* and \$188 represents the profit earned by shop *B*.

(iv) Percentage profit of $A = \frac{199}{771} \times 100\%$
$= 25.8\%$ (to 3 s.f.)

Percentage profit of $B = \frac{188}{798} \times 100\%$
$= 23.6\%$ (to 3 s.f.)

∴ Shop *A* performed better that day since it has a higher percentage profit.

(v) $\mathbf{T} = \begin{pmatrix} 24+18 & 15+22 & 17+16 \end{pmatrix}$

$= \begin{pmatrix} 42 & 37 & 33 \end{pmatrix}$

34. **(a)** $\begin{pmatrix} 27 & 9 & 6 \\ 26 & 12 & 4 \end{pmatrix}\begin{pmatrix} 5 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 168 \\ 170 \end{pmatrix}$

(b) The elements represent the total points gained by each soccer team from the matches they played in a recent season.

35. **(a)** $\mathbf{T} = 4\mathbf{A} + 3\mathbf{B}$

$$= 4\begin{pmatrix} 84 & 37 & 38 \\ 48 & 24 & 27 \end{pmatrix} + 3\begin{pmatrix} 26 & 95 & 70 \\ 15 & 82 & 43 \end{pmatrix}$$

$$= \begin{pmatrix} 336 & 148 & 152 \\ 192 & 96 & 108 \end{pmatrix} + \begin{pmatrix} 78 & 285 & 210 \\ 45 & 246 & 129 \end{pmatrix}$$

$$= \begin{pmatrix} 414 & 433 & 362 \\ 237 & 342 & 237 \end{pmatrix}$$

(b) $\mathbf{P} = \begin{pmatrix} 1 \\ 1.5 \\ 2 \end{pmatrix}$

(c) $\mathbf{E} = \mathbf{TP}$

$$= \begin{pmatrix} 414 & 433 & 362 \\ 237 & 342 & 237 \end{pmatrix}\begin{pmatrix} 1 \\ 1.5 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} 1787.5 \\ 1224 \end{pmatrix}$$

(d) They represent the total amount of money earned by each shop respectively for a week.

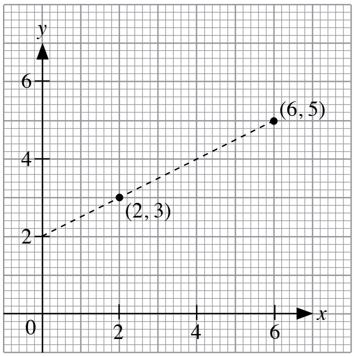
36. **(a)** $\begin{pmatrix} 48 & 22 & 10 \\ 95 & 101 & 4 \end{pmatrix}\begin{pmatrix} 10 \\ 7 \\ 5 \end{pmatrix} = \begin{pmatrix} 684 \\ 1677 \end{pmatrix}$

(b) It represents the total amount of money collected for each day.

Chapter 6 Further Geometrical Constructions

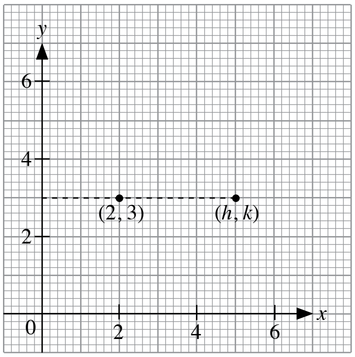
Basic

1.



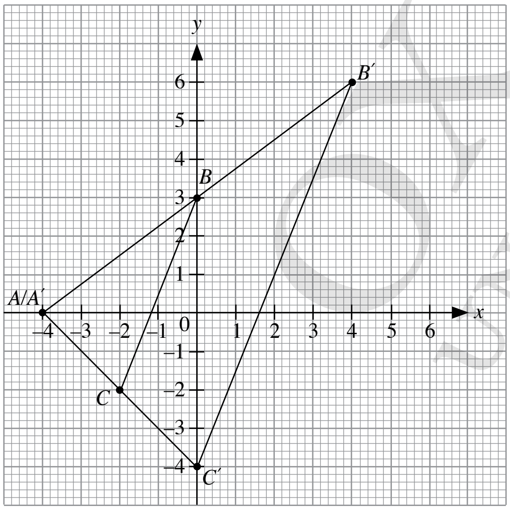
$\therefore\ k = 5$

2.



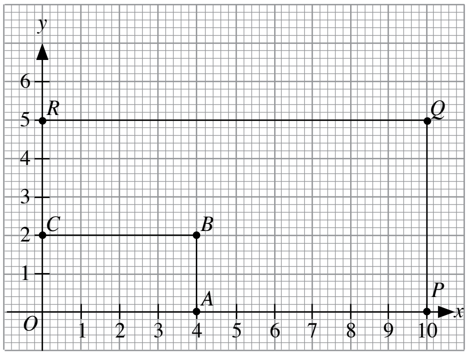
$\therefore\ h = 5, k = 3$

3.



$\therefore\ A'(-4, 0), B'(4, 6)$ and $C'(0, -4)$

4.



$\therefore\ P(10, 0), Q(10, 5)$ and $R(0, 5)$

Area of $OABC = 2 \times 4$

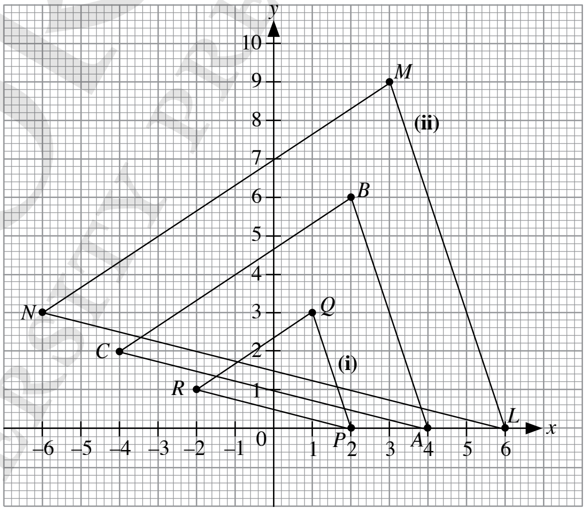
$= 8$ units^2

Area of $OPQR = 5 \times 10$

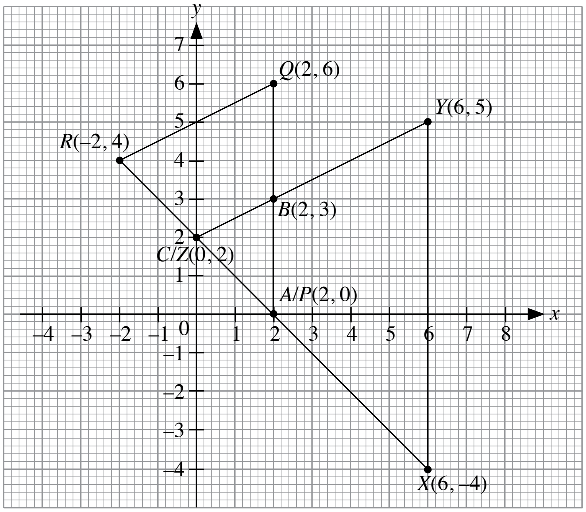
$= 50$ units^2

$$\frac{\text{Area of } OABC}{\text{Area of } OPQR} = \frac{8}{50} = \frac{4}{25}$$

5.

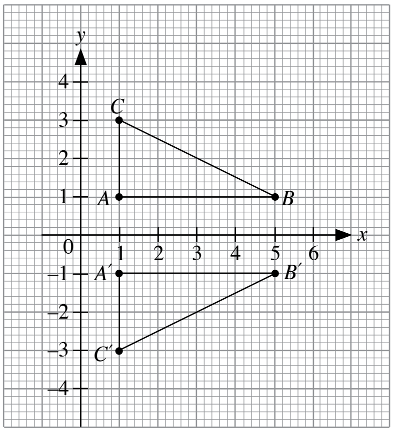


6.



7.

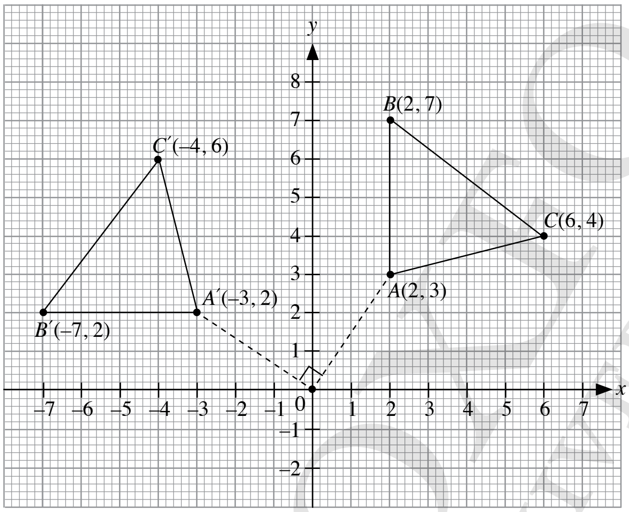
$$\begin{pmatrix}1&0\\0&-1\end{pmatrix}\begin{array}{c}\begin{array}{ccc}A&B&C\end{array}\\\begin{pmatrix}1&5&1\\1&1&3\end{pmatrix}\end{array}=\begin{array}{c}\begin{array}{ccc}A'&B'&C'\end{array}\\\begin{pmatrix}1&5&1\\-1&-1&-3\end{pmatrix}\end{array}$$



The matrix represents a reflection in the *x*-axis.

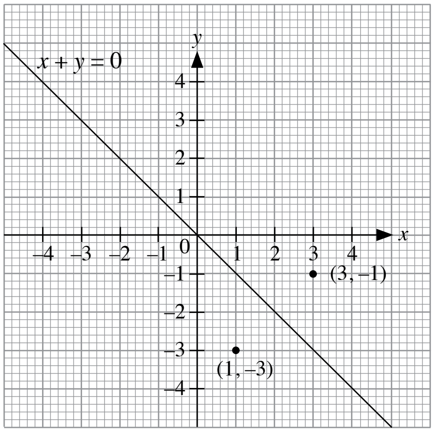
8.

$$\begin{pmatrix}0&-1\\1&0\end{pmatrix}\begin{array}{c}\begin{array}{ccc}A&B&C\end{array}\\\begin{pmatrix}2&2&6\\3&7&4\end{pmatrix}\end{array}=\begin{array}{c}\begin{array}{ccc}A'&B'&C'\end{array}\\\begin{pmatrix}-3&-7&-4\\2&2&6\end{pmatrix}\end{array}$$



The transformation is a 90° anticlockwise rotation about the origin, (0, 0).

9. **(a)**



The coordinates of the image of (3, –1) is (1, –3).

(b) The matrix is $\begin{pmatrix}0&-1\\-1&0\end{pmatrix}$.

(c) The coordinates of the point are (–5, 1).

10. **(i)** $E^2\begin{pmatrix}1\\-1\end{pmatrix}=\begin{pmatrix}2&0\\0&2\end{pmatrix}\begin{pmatrix}2&0\\0&2\end{pmatrix}\begin{pmatrix}1\\-1\end{pmatrix}$

$$=\begin{pmatrix}4&0\\0&4\end{pmatrix}\begin{pmatrix}1\\-1\end{pmatrix}$$

$$=\begin{pmatrix}4\\-4\end{pmatrix}\text{ i.e. }(4,-4)$$

(ii) $R\begin{pmatrix}1\\-1\end{pmatrix}=\begin{pmatrix}-1&0\\0&-1\end{pmatrix}\begin{pmatrix}1\\-1\end{pmatrix}$

$$=\begin{pmatrix}-1\\1\end{pmatrix}\text{ i.e. }(-1,1)$$

(iii) $ER\begin{pmatrix}1\\-1\end{pmatrix}=\begin{pmatrix}2&0\\0&2\end{pmatrix}\begin{pmatrix}-1\\1\end{pmatrix}=\begin{pmatrix}-2\\2\end{pmatrix}$ i.e. (–2, 2)

(iv) $RE\begin{pmatrix}1\\-1\end{pmatrix}=\begin{pmatrix}-1&0\\0&-1\end{pmatrix}\begin{pmatrix}2\\-2\end{pmatrix}=\begin{pmatrix}-2\\2\end{pmatrix}$ i.e. (–2, 2)

11. **(i)** $E^2\begin{pmatrix}-2\\2\end{pmatrix}=\begin{pmatrix}1\frac{1}{2}&0\\0&1\frac{1}{2}\end{pmatrix}\begin{pmatrix}1\frac{1}{2}&0\\0&1\frac{1}{2}\end{pmatrix}\begin{pmatrix}-2\\2\end{pmatrix}$

$$=\begin{pmatrix}1\frac{1}{2}&0\\0&1\frac{1}{2}\end{pmatrix}\begin{pmatrix}-3\\3\end{pmatrix}$$

$$=\begin{pmatrix}-4\frac{1}{2}\\4\frac{1}{2}\end{pmatrix}\text{ i.e. }\left(-4\frac{1}{2},4\frac{1}{2}\right)$$

(ii) $T^2\begin{pmatrix}-2\\2\end{pmatrix}=\begin{pmatrix}-2\\3\end{pmatrix}+\begin{pmatrix}-2\\3\end{pmatrix}+\begin{pmatrix}-2\\2\end{pmatrix}$

$$=\begin{pmatrix}-6\\8\end{pmatrix}\text{ i.e. }(-6,8)$$

(iii) $\text{TE}\begin{pmatrix}-2\\2\end{pmatrix}=\text{T}\begin{pmatrix}-3\\3\end{pmatrix}$

$$=\begin{pmatrix}-2\\3\end{pmatrix}+\begin{pmatrix}-3\\3\end{pmatrix}$$

$$=\begin{pmatrix}-5\\6\end{pmatrix}\text{ i.e. }(-5, 6)$$

(iv) $\text{ET}\begin{pmatrix}-2\\3\end{pmatrix}=\text{E}\begin{pmatrix}-4\\5\end{pmatrix}$

$$=\begin{pmatrix}1\frac{1}{2}&0\\0&1\frac{1}{2}\end{pmatrix}\begin{pmatrix}-4\\5\end{pmatrix}$$

$$=\begin{pmatrix}-6\\7\frac{1}{2}\end{pmatrix}\text{ i.e. }\left(-6,\ 7\frac{1}{2}\right)$$

12. **(i)** $\text{M}\begin{pmatrix}2\\2\end{pmatrix}=\begin{pmatrix}2\\0\end{pmatrix}$ i.e. $(2, 0)$

(ii) $\text{E}^2\begin{pmatrix}2\\2\end{pmatrix}=\text{E}\begin{pmatrix}3\\4\end{pmatrix}=\begin{pmatrix}5\\8\end{pmatrix}$ i.e. $(5, 8)$

(iii) $\text{EM}\begin{pmatrix}2\\2\end{pmatrix}=\text{E}\begin{pmatrix}2\\0\end{pmatrix}=\begin{pmatrix}3\\0\end{pmatrix}$ i.e. $(3, 0)$

(iv) $\text{ME}\begin{pmatrix}2\\2\end{pmatrix}=\text{M}\begin{pmatrix}3\\4\end{pmatrix}=\begin{pmatrix}3\\-2\end{pmatrix}$ i.e. $(3, -2)$

13. **(i)** $\text{R}^2\begin{pmatrix}3\\2\end{pmatrix}=\text{R}\begin{pmatrix}2\\-3\end{pmatrix}=\begin{pmatrix}-3\\-2\end{pmatrix}$ i.e. $(-3, -2)$

(ii) $\text{E}^2\begin{pmatrix}3\\2\end{pmatrix}=\text{E}\begin{pmatrix}5\\6\end{pmatrix}=\begin{pmatrix}8\\12\end{pmatrix}$ i.e. $(8, 12)$

(iii) $\text{ER}\begin{pmatrix}3\\2\end{pmatrix}=\text{E}\begin{pmatrix}2\\-3\end{pmatrix}=\begin{pmatrix}2\\-9\end{pmatrix}$ i.e. $(2, -9)$

(iv) $\text{RE}\begin{pmatrix}3\\2\end{pmatrix}=\text{R}\begin{pmatrix}5\\6\end{pmatrix}=\begin{pmatrix}6\\-5\end{pmatrix}$ i.e. $(6, -5)$

14. **(i)** $\text{E}^2\begin{pmatrix}3\\1\end{pmatrix}=\text{E}\begin{pmatrix}5\\3\end{pmatrix}=\begin{pmatrix}8\\6\end{pmatrix}$ i.e. $(8, 6)$

(ii) $\text{R}^2\begin{pmatrix}3\\1\end{pmatrix}=\text{R}\begin{pmatrix}2\\2\end{pmatrix}=\begin{pmatrix}3\\3\end{pmatrix}$ i.e. $(3, 3)$

(iii) $\text{ER}\begin{pmatrix}3\\1\end{pmatrix}=\text{E}\begin{pmatrix}2\\2\end{pmatrix}=\begin{pmatrix}2\\6\end{pmatrix}$ i.e. $(2, 6)$

(iv) $\text{RE}\begin{pmatrix}3\\1\end{pmatrix}=\text{R}\begin{pmatrix}5\\3\end{pmatrix}=\begin{pmatrix}4\\0\end{pmatrix}$ i.e. $(4, 0)$

15. **(i)** A reflection in the line *OB* followed by a 60° clockwise rotation about *O*.

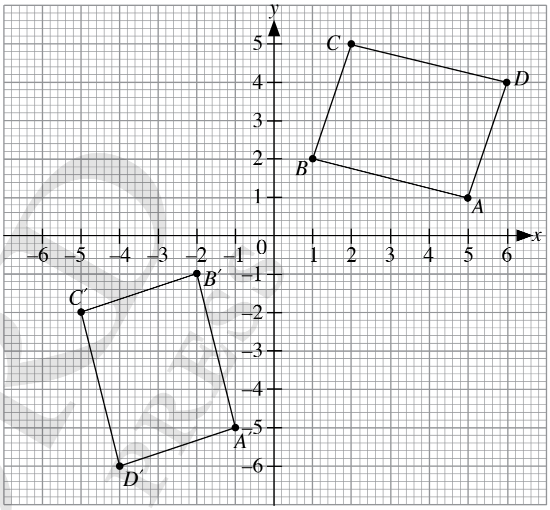
(ii) A reflection in the line *FC* followed by a 120° clockwise rotation about *O*.

16. A 180° rotation about the mid-point of *AQ* followed by an enlargement with centre at *A* and scale factor 2.

17. A reflection in the line $x = 2\frac{1}{2}$ followed by an enlargement with centre at *Q* and scale factor $1\frac{1}{2}$.

18.

$$\begin{pmatrix}0&-1\\-1&0\end{pmatrix}\begin{array}{c}\begin{matrix}A&B&C&D\end{matrix}\\\begin{pmatrix}5&1&2&6\\1&2&5&4\end{pmatrix}\end{array}=\begin{array}{c}\begin{matrix}A'&B'&C'&D'\end{matrix}\\\begin{pmatrix}-1&-2&-5&-4\\-5&-1&-2&-6\end{pmatrix}\end{array}$$



The matrix represents a reflection in the line $y + x = 0$.

19. **(a)** $\begin{pmatrix}0&-1\\1&0\end{pmatrix}$ **(b)** $\begin{pmatrix}-1&0\\0&-1\end{pmatrix}$

(c) $\begin{pmatrix}0&1\\-1&0\end{pmatrix}$ **(d)** $\begin{pmatrix}1&0\\0&1\end{pmatrix}$

(e) $\begin{pmatrix}0&-1\\1&0\end{pmatrix}$ **(f)** $\begin{pmatrix}-1&0\\0&-1\end{pmatrix}$

20. **(a)** A 90° anticlockwise rotation about *O*.

(b) A translation represented by $\begin{pmatrix}-4\\-3\end{pmatrix}$.

(c) A reflection in the *x*-axis.

(d) An enlargement with centre at $(2, 0)$ and scale factor of -2.

21. **(a)** A translation of 4 cm along *AC*.

(b) A 180° rotation about the midpoint of *BE*.

(c) A reflection in the line *BE*.

22. **(i)** A 120° clockwise rotation about *O*.

(ii) A 60° clockwise rotation about *O* followed by a reflection in the perpendicular bisector of *CD*.

23. **(a)** $\text{EM}\begin{pmatrix}2\\3\end{pmatrix} = \text{E}\begin{pmatrix}2\\-3\end{pmatrix} = \begin{pmatrix}6\\-9\end{pmatrix}$

$\text{EM}\begin{pmatrix}-2\\-1\end{pmatrix} = \text{E}\begin{pmatrix}-2\\1\end{pmatrix} = \begin{pmatrix}-6\\3\end{pmatrix}$

(b) $\text{E} = \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}, \text{M} = \begin{pmatrix}3 & 0\\0 & 3\end{pmatrix}$

24. **(a)** $3 = 2x - 1$

$2x = 4$

$x = 2$

(b) $\mathbf{A}^2 = \begin{pmatrix}3 & 0\\0 & 2x-1\end{pmatrix}\begin{pmatrix}3 & 0\\0 & 2x-1\end{pmatrix} = \begin{pmatrix}9 & 0\\0 & (2x-1)^2\end{pmatrix}$

(c) $3(2x - 1) = 9(2x - 1)^2$

$9(2x - 1)^2 - 3(2x - 1) = 0$

$3(2x - 1)[3(2x - 1) - 1] = 0$

$3(2x - 1)(6x - 4) = 0$

$x = \frac{1}{2}$ or $x = \frac{2}{3}$

25. **(i)** $\triangle OED$

(ii) $\triangle ORS$

(iii) $\triangle OQR$

(iv) $\triangle OAB$

(v) $\triangle OQR$

(vi) $\triangle OBC$

(vii) $\triangle OST$

(viii) $\triangle OAF$

26. **(i)** A 180° rotation about O.

(ii) An enlargement centre at B and scale factor 2.

(iii) An enlargement centre at B and scale factor 2 followed by a reflection in the line AC.

(iv) Area of $\triangle OAB = 4x$ cm^2

$\therefore$ Area of rhombus $ABCD = 4(4x) = 16x$ cm^2

27. **(i)** A reflection in the line AD.

(ii) A 180° clockwise rotation about O.

(iii) An enlargement centre at O and scale factor 2.

(iv) A reflection in the line AD followed by an enlargement centre at O and scale factor 2.

(v) A 90° clockwise rotation about O followed by an enlargement centre at O and scale factor $\frac{1}{2}$.

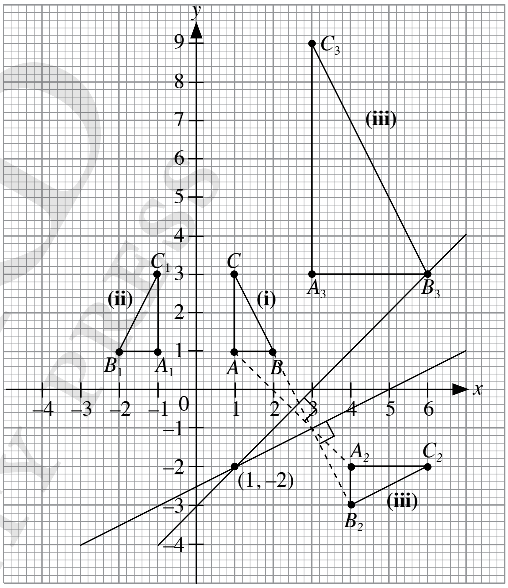
28. $\text{T} + \begin{pmatrix}4\\5\end{pmatrix} = \begin{pmatrix}1\\2\end{pmatrix}$

$\therefore \text{T} = \begin{pmatrix}1\\2\end{pmatrix} - \begin{pmatrix}4\\5\end{pmatrix} = \begin{pmatrix}-3\\-3\end{pmatrix}$

$A' = \begin{pmatrix}1\\1\end{pmatrix} + \begin{pmatrix}-3\\-3\end{pmatrix} = \begin{pmatrix}-2\\-2\end{pmatrix}$

$B' = \begin{pmatrix}2\\1\end{pmatrix} + \begin{pmatrix}-3\\-3\end{pmatrix} = \begin{pmatrix}-1\\-2\end{pmatrix}$

$C' = \begin{pmatrix}1\\3\end{pmatrix} + \begin{pmatrix}-3\\-3\end{pmatrix} = \begin{pmatrix}-2\\0\end{pmatrix}$



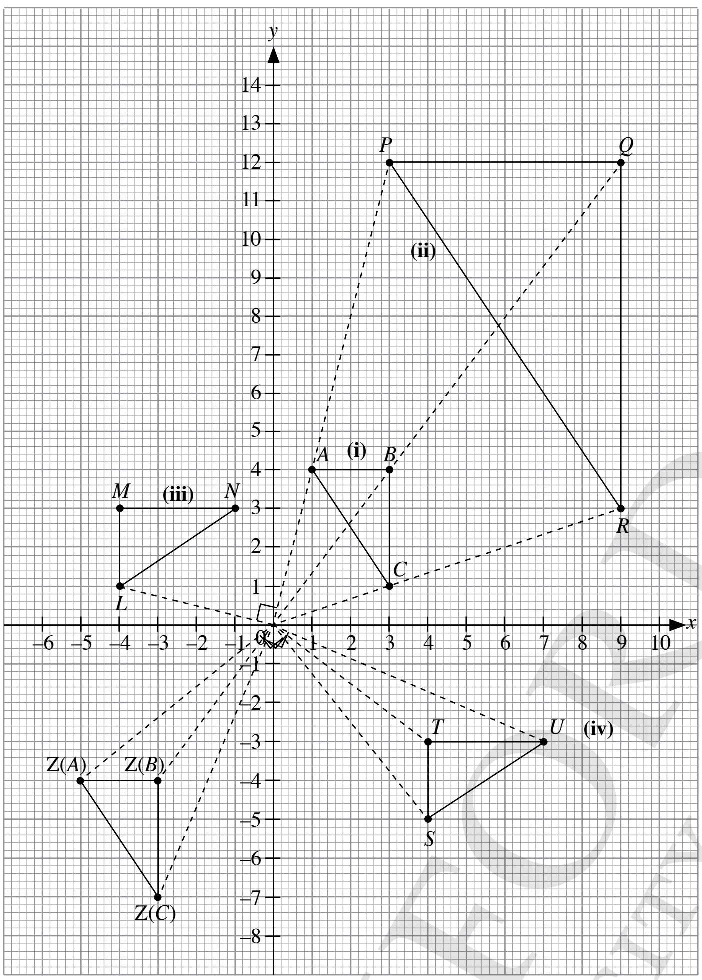
(ii) X is a reflection in the y-axis.

(iii) Y is a 90° clockwise rotation about (1, –2).

(iv) $\begin{pmatrix}3 & 0\\0 & 3\end{pmatrix}\begin{pmatrix}1 & 2 & 1\\1 & 1 & 3\end{pmatrix} = \begin{pmatrix}3 & 6 & 3\\3 & 3 & 9\end{pmatrix}$

$\therefore A_3(3, 3), B_3(6, 3)$ and $C_3(3, 9)$.

29.



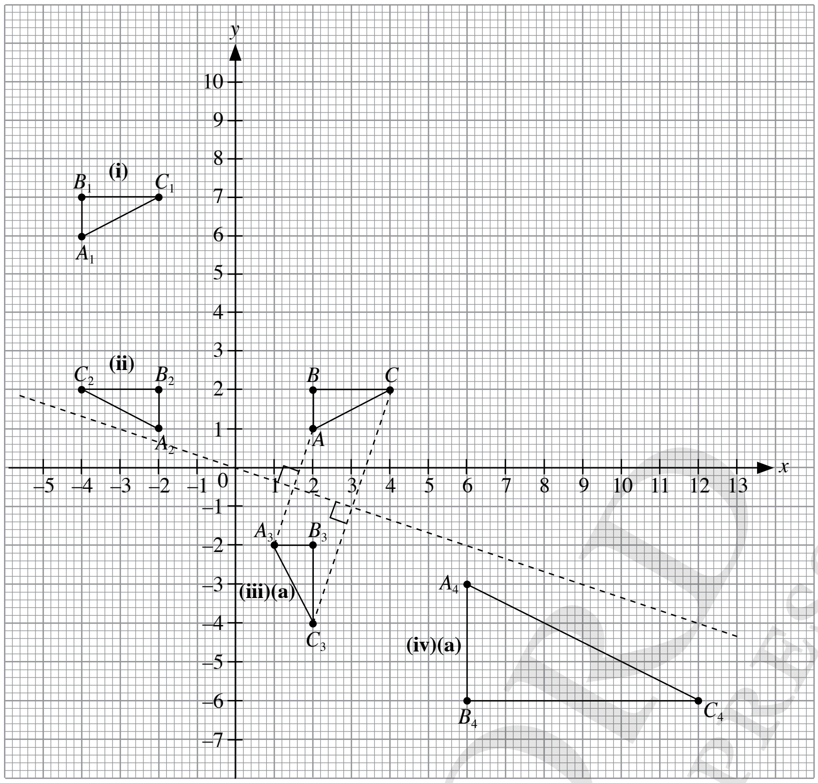
(ii) X is an enlargement with centre at O and scale factor 3.

(iv) A to S: $\text{YZ}\begin{pmatrix} 1 \\ 4 \end{pmatrix} = \text{Y}\begin{pmatrix} -5 \\ -4 \end{pmatrix} = \begin{pmatrix} 4 \\ -5 \end{pmatrix}$

B to T: $\text{YZ}\begin{pmatrix} 3 \\ 4 \end{pmatrix} = \text{Y}\begin{pmatrix} -3 \\ -4 \end{pmatrix} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$

C to U: $\text{YZ}\begin{pmatrix} 3 \\ 1 \end{pmatrix} = \text{Y}\begin{pmatrix} -3 \\ -7 \end{pmatrix} = \begin{pmatrix} 7 \\ -3 \end{pmatrix}$

30.



(ii) $\mathrm{X} + \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} -4 \\ 6 \end{pmatrix}$

$$\mathrm{X} = \begin{pmatrix} -4 \\ 6 \end{pmatrix} - \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} -6 \\ 5 \end{pmatrix}$$

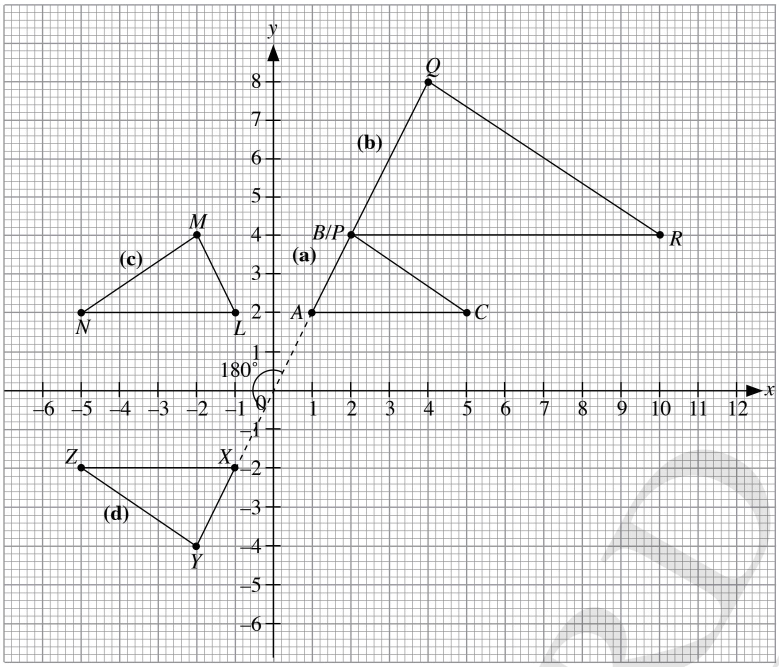
B to B_1: $\mathrm{X}\begin{pmatrix} 2 \\ 2 \end{pmatrix} = \begin{pmatrix} -4 \\ 7 \end{pmatrix}$

C to C_1: $\mathrm{X}\begin{pmatrix} 4 \\ 2 \end{pmatrix} = \begin{pmatrix} -2 \\ 7 \end{pmatrix}$

(iii) (b) Z is a 90° clockwise rotation about O.

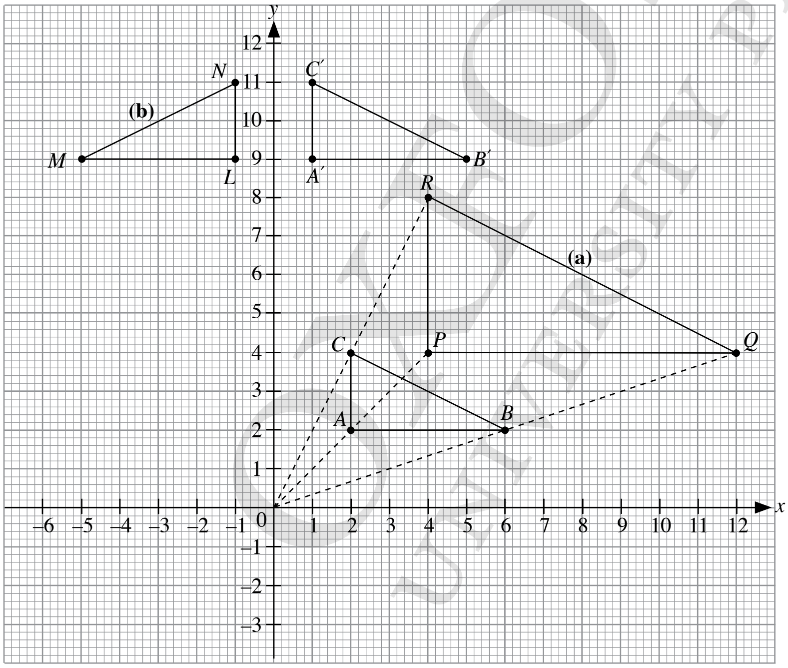
(iv) (b) S is an enlargement with centre at O and scale factor 3. T is a reflection in the x-axis.

31.



(d) A reflection in the *x*-axis.

32.

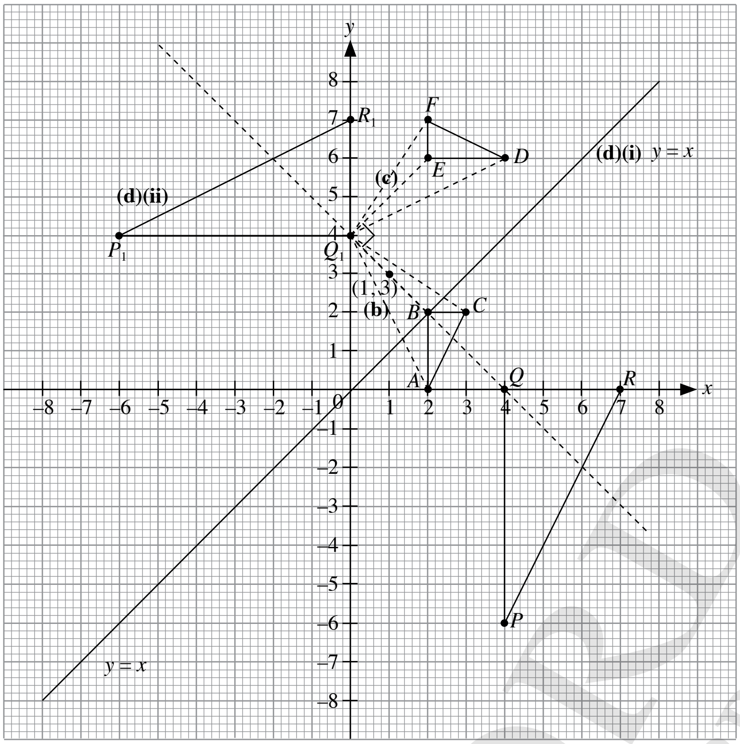


(b) *A* to *A*′: $\begin{pmatrix}-1\\7\end{pmatrix}+\begin{pmatrix}2\\2\end{pmatrix}=\begin{pmatrix}1\\9\end{pmatrix}$

B to *B*′: $\begin{pmatrix}-1\\7\end{pmatrix}+\begin{pmatrix}6\\2\end{pmatrix}=\begin{pmatrix}5\\9\end{pmatrix}$

C to *C*′: $\begin{pmatrix}-1\\7\end{pmatrix}+\begin{pmatrix}2\\4\end{pmatrix}=\begin{pmatrix}1\\11\end{pmatrix}$

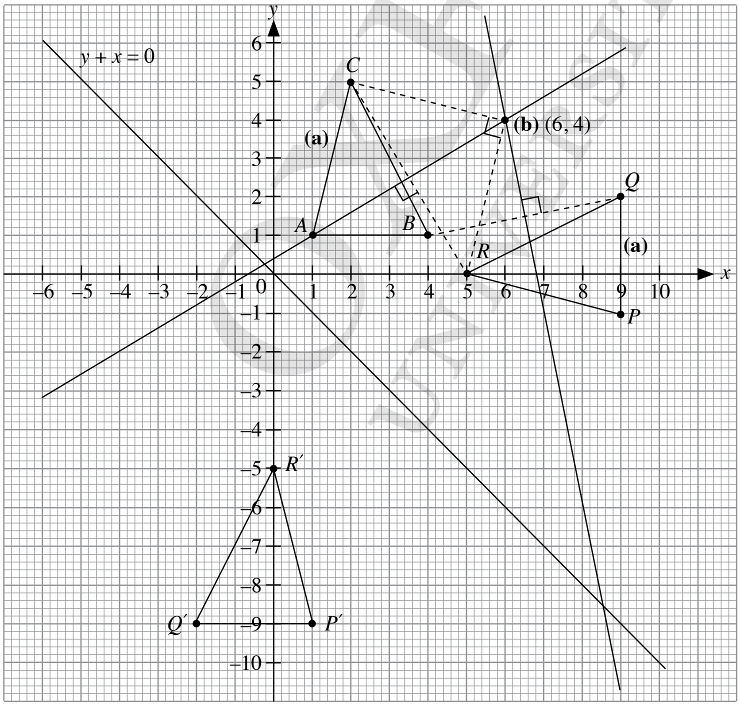
33.



(b) X is an enlargement with centre at (1, 3) and scale factor 3.

(c) Centre of rotation is (0, 4), angle of rotation is 90° anti-clockwise.

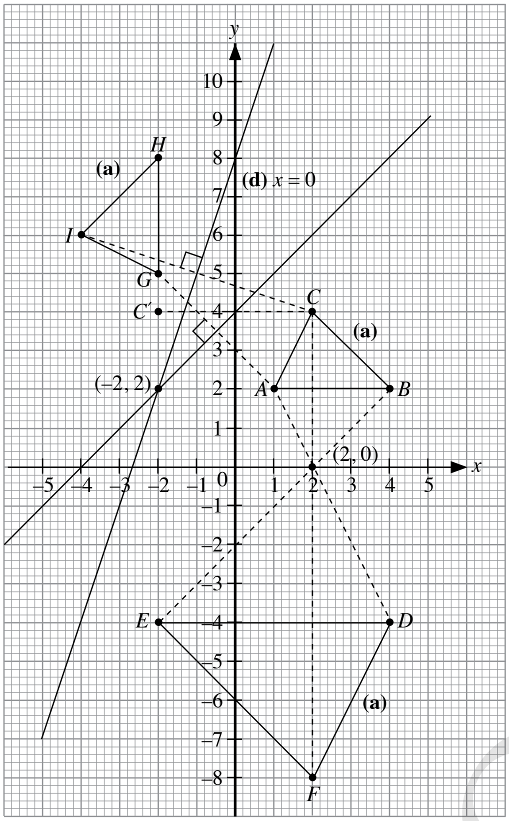
34.



(b) X is a 90° anticlockwise rotation about (6, 4).

(c) The matrix representing Y is $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$.

35.

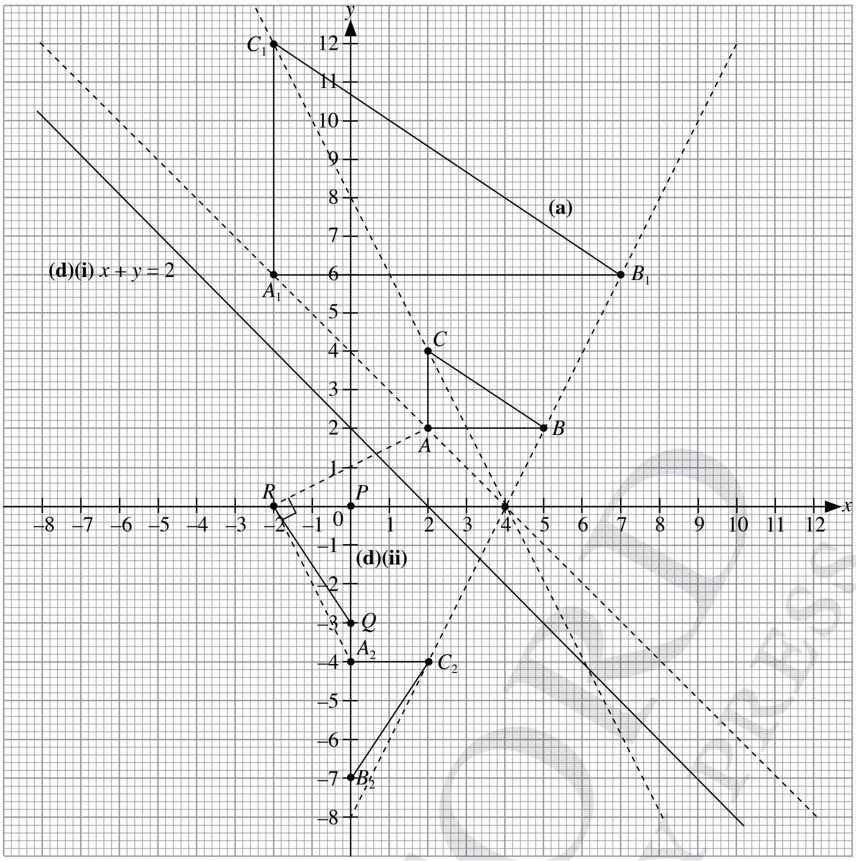


(b) X is an enlargement at $(2, 0)$ and scale factor -2.

(c) Y is a 90° anticlockwise rotation about $(-2, 2)$.

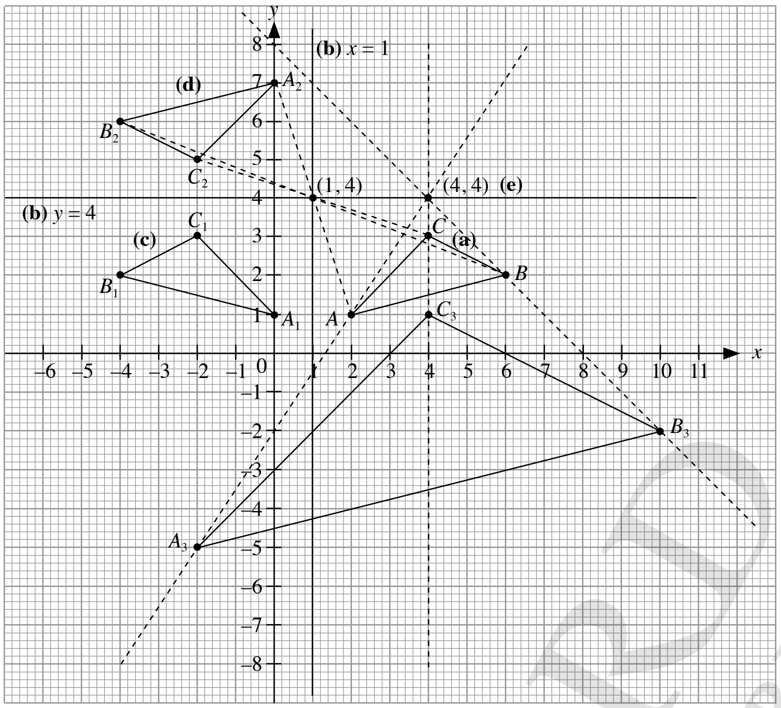
(d) The equation of line l is $x = 0$ and the matrix is $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$.

36.



(b) X is an enlargement centre at (4, 0) and scale factor 3.

37.

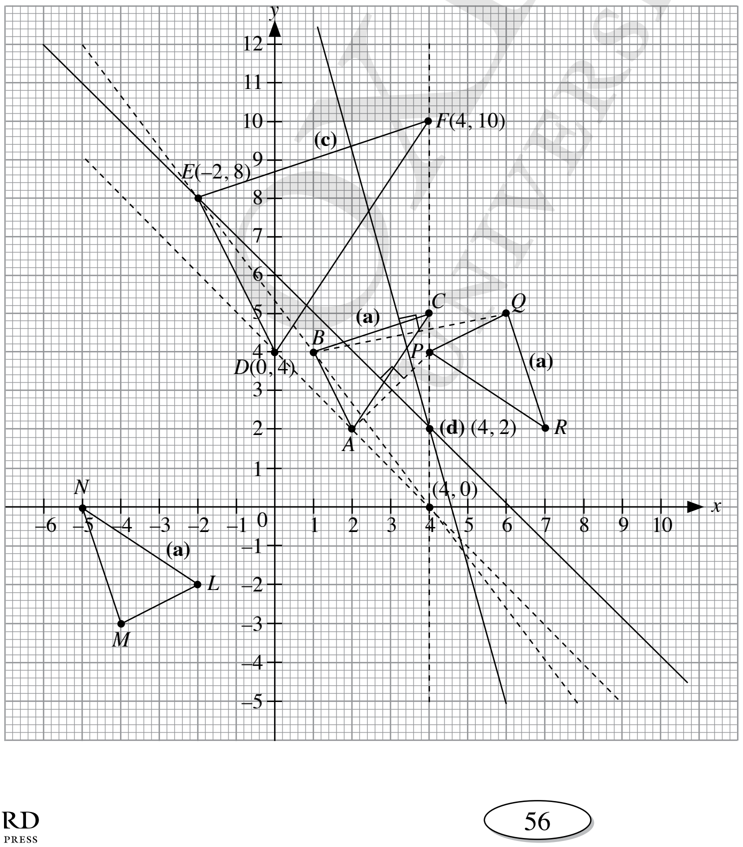


(d) R is a 180° rotation about (1, 4) or an enlargement centre at (1, 4) and scale factor –1.

(e) The centre of enlargement is (4, 4) with scale factor 3. The coordinates of C_3 are (4, 1).

Advanced

38.



(b) **(i)** The matrix which represents the rotation is $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

(ii) The matrix which represents the translation is $\begin{pmatrix} 0 \\ -4 \end{pmatrix}$

(c) **(i)** The coordinates of D are $(0, 4)$.

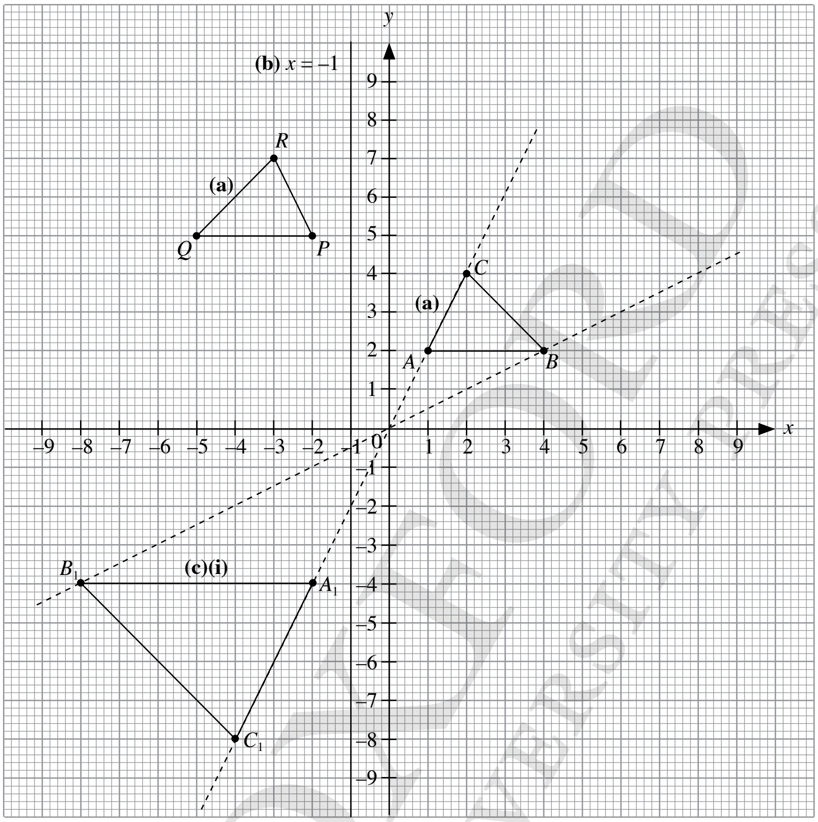
(iii) Scale factor = 2

(iv) $\dfrac{\text{area of } \triangle DEF}{\text{area of } \triangle ABC} = 4$

(d) **(i)** Centre of rotation = (4, 2)

(ii) 90° clockwise rotation

39.

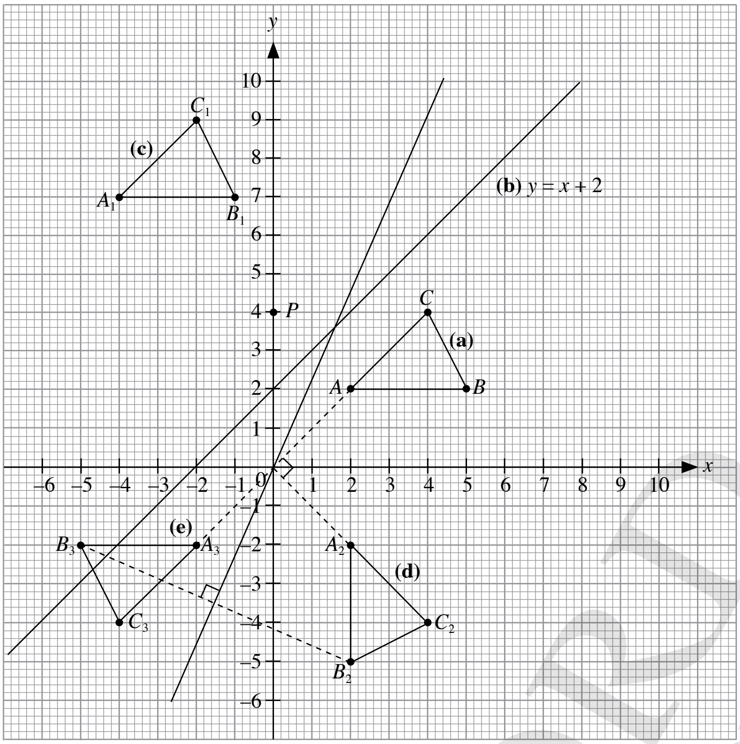


(b) Equation of l: $x = -1$

(c) **(ii)** $\begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$

(iii) $\dfrac{\text{area of } \triangle A_1B_1C_1}{\text{area of } \triangle ABC} = 4$

40.



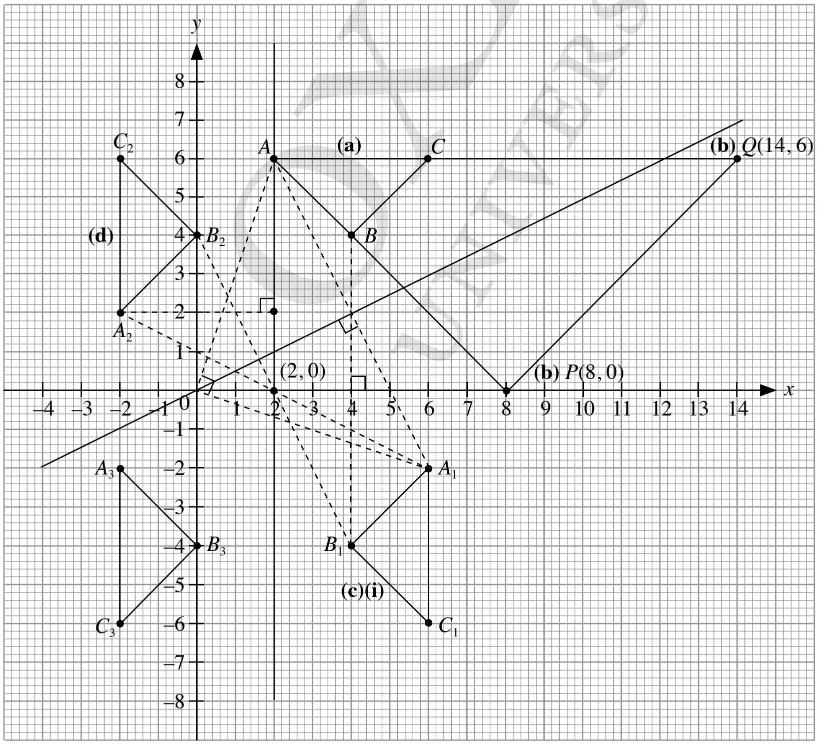
(b) The equation of the line is $y = x + 2$.

(e) The matrix is $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$.

(f) H is a 90° clockwise rotation about the origin.

The matrix is $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$.

41.



(b) From the graph, $P(8, 0)$ and $Q(14, 6)$.

$\therefore m = 8, h = 14$ and $k = 6$

(c) **(ii)** X is a 90° clockwise rotation about the origin (0, 0).

(iii) The matrix is $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$.

(e) R is a 180° rotation about (2, 0).

(f) **(ii)** A reflection in the x-axis.

42. **(a)** A 90° anticlockwise rotation about the origin.

(b) $\begin{pmatrix} 6 \\ 2 \end{pmatrix}$

(c) The coordinates of P are (3, 11).

(d) **(i)** The centre of enlargement is $A(6, 12)$.

(ii) **(a)** $k = 9$

(b) Scale factor = $1\frac{1}{2}$

(c) The coordinates of E are (0, –3).

(d) The ratio is 4 : 9.

43. **(a)** **(i)** Scale factor = 2

(ii) The centre of enlargement is (–10, –4).

(b) A reflection in the y-axis.

(c) A reflection in the line $y = x$.

(d) V is a 90° clockwise rotation about (2, –4).

44. **(a)** Gradient of $DC = \frac{4 - 0}{1 - 9}$

$= -\frac{1}{2}$

(b) Area of $ABCD = 2 \times 8$

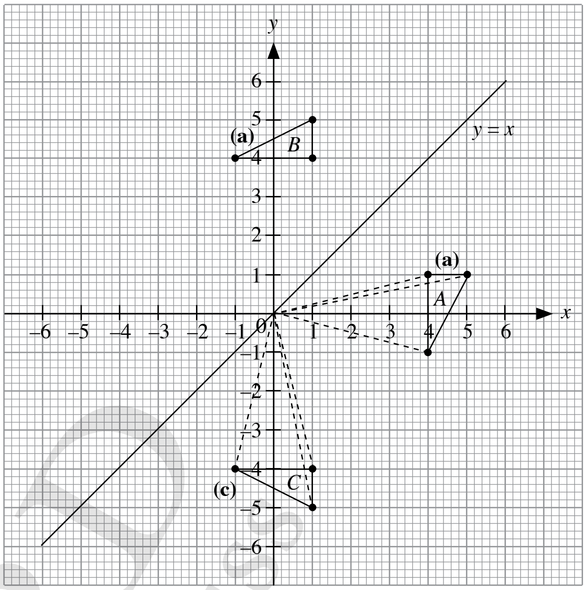
$= 16$ units^2

(c) The coordinates of the image of D is (1, 0).

(d) The coordinates of the point are $\left(\frac{1+9}{2}, \frac{6+0}{2}\right)$

i.e. (5, 3)

45.



(b) A reflection in the line $y = x$.

(d) A reflection in the x-axis.

Chapter 7 Vectors

1. $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$= \begin{pmatrix} 2 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ 6 \end{pmatrix}$

$= \begin{pmatrix} 1 \\ -3 \end{pmatrix}$

$\overrightarrow{CD} = \overrightarrow{OD} - \overrightarrow{OC}$

$= \begin{pmatrix} 6 \\ 6 \end{pmatrix} - \begin{pmatrix} 3 \\ 4 \end{pmatrix}$

$= \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

$\overrightarrow{EF} = \overrightarrow{OF} - \overrightarrow{OE}$

$= \begin{pmatrix} 6 \\ 4 \end{pmatrix} - \begin{pmatrix} 9 \\ 4 \end{pmatrix}$

$= \begin{pmatrix} -3 \\ 0 \end{pmatrix}$

$\overrightarrow{GH} = \overrightarrow{OH} - \overrightarrow{OG}$

$= \begin{pmatrix} 3 \\ 3 \end{pmatrix} - \begin{pmatrix} 9 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} -6 \\ 2 \end{pmatrix}$

$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$

$= \begin{pmatrix} 11 \\ 6 \end{pmatrix} - \begin{pmatrix} 11 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} 0 \\ 4 \end{pmatrix}$

$\overrightarrow{LM} = \overrightarrow{OM} - \overrightarrow{OL}$

$= \begin{pmatrix} 12 \\ 1 \end{pmatrix} - \begin{pmatrix} 13 \\ 7 \end{pmatrix}$

$= \begin{pmatrix} -1 \\ -6 \end{pmatrix}$

$\overrightarrow{UV} = \overrightarrow{OV} - \overrightarrow{OU}$

$= \begin{pmatrix} 14 \\ 5 \end{pmatrix} - \begin{pmatrix} 15 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} -1 \\ 2 \end{pmatrix}$

$\overrightarrow{XY} = \overrightarrow{OY} - \overrightarrow{OX}$

$= \begin{pmatrix} 7 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} 6 \\ 0 \end{pmatrix}$

2. **a** $= \begin{pmatrix} 3 \\ 5 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} 2 \\ 4 \end{pmatrix}$

b $= \begin{pmatrix} 5 \\ 3 \end{pmatrix} - \begin{pmatrix} 3 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} 2 \\ 0 \end{pmatrix}$

c $= \begin{pmatrix} 4 \\ 6 \end{pmatrix} - \begin{pmatrix} 7 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} -3 \\ 3 \end{pmatrix}$

d $= \begin{pmatrix} 3 \\ 1 \end{pmatrix} - \begin{pmatrix} 8 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} -5 \\ -2 \end{pmatrix}$

e $= \begin{pmatrix} 9 \\ 1 \end{pmatrix} - \begin{pmatrix} 9 \\ 6 \end{pmatrix}$

$= \begin{pmatrix} 0 \\ -5 \end{pmatrix}$

f $= \begin{pmatrix} 11 \\ 1 \end{pmatrix} - \begin{pmatrix} 10 \\ 6 \end{pmatrix}$

$= \begin{pmatrix} 1 \\ -5 \end{pmatrix}$

g $= \begin{pmatrix} 11 \\ 6 \end{pmatrix} - \begin{pmatrix} 17 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} -6 \\ 3 \end{pmatrix}$

h $= \begin{pmatrix} 15 \\ 3 \end{pmatrix} - \begin{pmatrix} 12 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} 3 \\ 1 \end{pmatrix}$

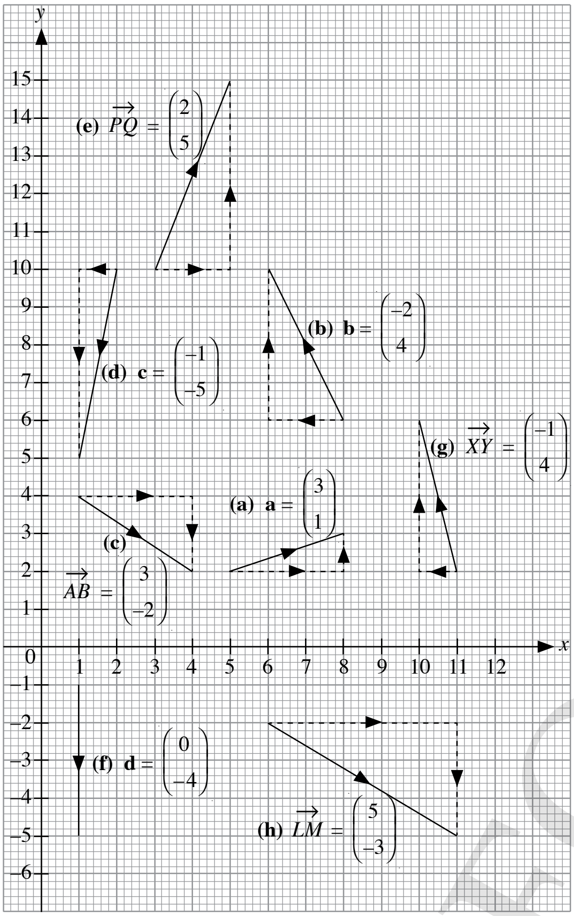
k $= \begin{pmatrix} 14 \\ 2 \end{pmatrix} - \begin{pmatrix} 16 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} -2 \\ 0 \end{pmatrix}$

l $= \begin{pmatrix} 16 \\ 6 \end{pmatrix} - \begin{pmatrix} 16 \\ 4 \end{pmatrix}$

$= \begin{pmatrix} 0 \\ 2 \end{pmatrix}$

3.



4. **(a)** $\begin{pmatrix} 3 \\ -7 \end{pmatrix}$

(b) $\begin{pmatrix} -1 \\ -5 \end{pmatrix}$

(c) $\begin{pmatrix} -2 \\ 6 \end{pmatrix}$

(d) $\begin{pmatrix} 8 \\ 5 \end{pmatrix}$

(e) $\begin{pmatrix} -1 \\ -9 \end{pmatrix}$

(f) $\begin{pmatrix} 4 \\ -5 \end{pmatrix}$

(g) $\begin{pmatrix} -7 \\ 9 \end{pmatrix}$

(h) $\begin{pmatrix} 4 \\ 10 \end{pmatrix}$

5. $\mathbf{d} = \mathbf{e} + \mathbf{a} + \mathbf{b} + \mathbf{c}$

$$= \begin{pmatrix} -1 \\ 5 \end{pmatrix} + \begin{pmatrix} 4 \\ 1 \end{pmatrix} + \begin{pmatrix} 2 \\ -3 \end{pmatrix} + \begin{pmatrix} -3 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ 4 \end{pmatrix}$$

6. $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$

$$= \begin{pmatrix} -2 \\ 4 \end{pmatrix} + \begin{pmatrix} 1 \\ 5 \end{pmatrix}$$

$$= \begin{pmatrix} -1 \\ 9 \end{pmatrix}$$

$|\overrightarrow{AC}| = \sqrt{(-1)^2 + 9^2}$ = 9.1 units (to 1 d.p.)

7. $\overrightarrow{LM} = \overrightarrow{LK} + \overrightarrow{KM}$

$$= \begin{pmatrix} 8 \\ 11 \end{pmatrix} + \begin{pmatrix} -4 \\ -8 \end{pmatrix}$$

$$= \begin{pmatrix} 4 \\ 3 \end{pmatrix}$$

$|\overrightarrow{LM}| = \sqrt{4^2 + 3^2}$ = 5 units

8. $\overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD}$

$$= \begin{pmatrix} -1 \\ 2 \end{pmatrix} + \begin{pmatrix} 4 \\ -3 \end{pmatrix} + \begin{pmatrix} 3 \\ 9 \end{pmatrix}$$

$$= \begin{pmatrix} 6 \\ 8 \end{pmatrix}$$

$|\overrightarrow{AD}| = \sqrt{6^2 + 8^2}$ = 10 units

Intermediate

9. **(a)** $\overrightarrow{XY} = \begin{pmatrix} 0 \\ -13 \end{pmatrix}$

$|\overrightarrow{XY}| = \sqrt{0^2 + (-13)^2}$ = 13 units

(b) $\overrightarrow{PQ} = \begin{pmatrix} a \\ 12 \end{pmatrix}$

$|\overrightarrow{XY}| = |\overrightarrow{PQ}|$

$13 = \sqrt{a^2 + 12^2}$

$169 = a^2 + 144$

$a = \pm\sqrt{25}$

$= \pm 5$

10. **(a)** $|\overrightarrow{AB}| = \sqrt{12^2 + (-5)^2} = 13$ units

(b) $\overrightarrow{CD} = \begin{pmatrix} t \\ 24 \end{pmatrix}$

$$|\overrightarrow{CD}| = 2|\overrightarrow{AB}|$$

$$\sqrt{t^2 + 24^2} = 26$$

$$t^2 + 576 = 676$$

$$t = \pm\sqrt{100}$$

$$= \pm 10$$

11. **(a)** **(i)** $\overrightarrow{TQ}$

(ii) $\overrightarrow{KF}$

(iii) $\overrightarrow{LH}$

(iv) $\overrightarrow{MP}$

(v) $\overrightarrow{NQ}$

(vi) $\overrightarrow{PN}$

(vii) $\overrightarrow{TM}$

(viii) $\overrightarrow{MQ}$

(ix) $\overrightarrow{QL}$

(b) **(i)** $3\overrightarrow{AB}$

(ii) $3\overrightarrow{AB}$

(iii) $4\overrightarrow{AB}$

(iv) $2\overrightarrow{AB}$

(v) $4\overrightarrow{AB}$

(vi) $7\overrightarrow{AB}$

(vii) $4\overrightarrow{AB}$

(c) **(i)** $\overrightarrow{KF} = 3\overrightarrow{AB} = 3\begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 6 \\ 3 \end{pmatrix}$

(ii) $\overrightarrow{LH} = 4\overrightarrow{AB} = 4\begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 8 \\ 4 \end{pmatrix}$

(iii) $\overrightarrow{HS} = -7\overrightarrow{AB} = -7\begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} -14 \\ -7 \end{pmatrix}$

(iv) $\overrightarrow{PQ} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$

(v) $\overrightarrow{CA} = \begin{pmatrix} -4 \\ -1 \end{pmatrix}$

(vi) $\overrightarrow{KH} = \begin{pmatrix} 10 \\ 3 \end{pmatrix}$

(vii) $\overrightarrow{LD} = \begin{pmatrix} 0 \\ 4 \end{pmatrix}$

(viii) $\overrightarrow{NM} = \begin{pmatrix} -6 \\ 0 \end{pmatrix}$

(ix) $\overrightarrow{DQ} = \begin{pmatrix} 4 \\ -5 \end{pmatrix}$

(x) $\overrightarrow{QG} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$

(xi) $\overrightarrow{PN} = \begin{pmatrix} 8 \\ 1 \end{pmatrix}$

(xii) $\overrightarrow{UM} = \begin{pmatrix} -6 \\ 3 \end{pmatrix}$

(xiii) $\overrightarrow{AT} = \begin{pmatrix} 6 \\ -6 \end{pmatrix}$

(xiv) $\overrightarrow{TG} = \begin{pmatrix} 6 \\ 7 \end{pmatrix}$

(xv) $\overrightarrow{PB} = \begin{pmatrix} -4 \\ 5 \end{pmatrix}$

(d) **(i)** $\overrightarrow{AD}$

(ii) $\overrightarrow{BK}$

(iii) $\overrightarrow{FL}$ or $\overrightarrow{GM}$ or $\overrightarrow{KS}$

(iv) $\overrightarrow{LE}$ or $\overrightarrow{MF}$

(v) $\overrightarrow{SA}$

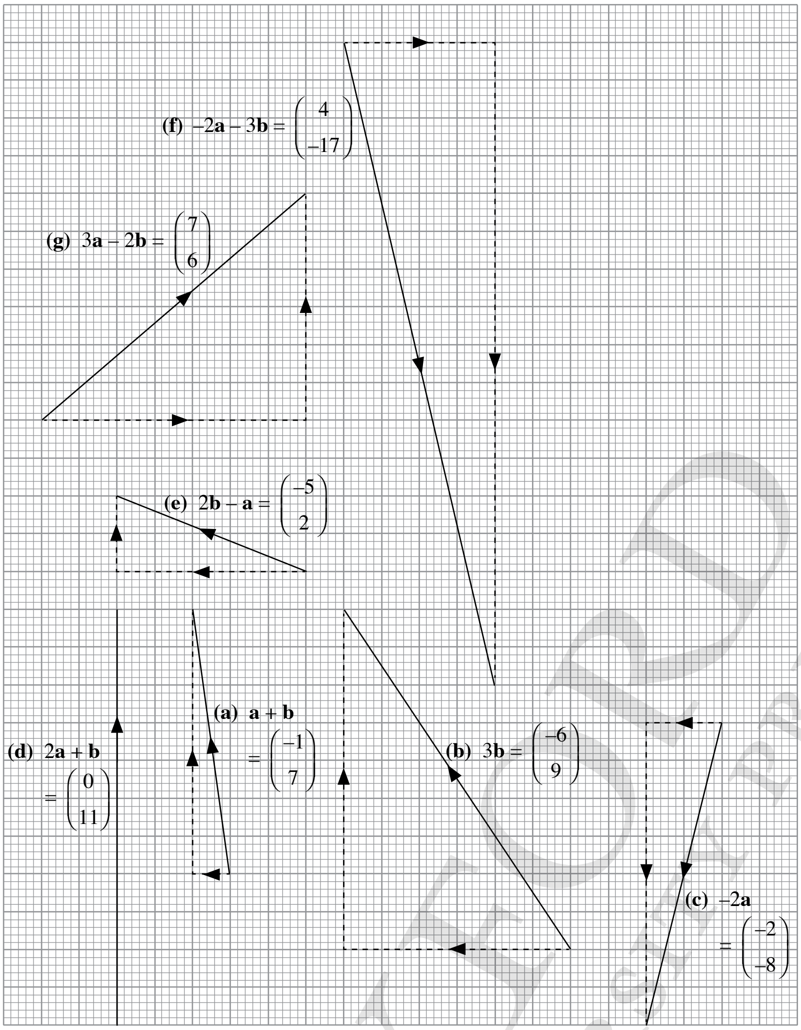
(vi) $\overrightarrow{SU}$

(vii) $\overrightarrow{KR}$

(viii) $\overrightarrow{TA}$

(ix) $\overrightarrow{SD}$

12.



13. **(a)** $-3\mathbf{q} = \begin{pmatrix} 6 \\ 15 \end{pmatrix}$

(b) $2\mathbf{p} - \mathbf{q} = 2\begin{pmatrix} 3 \\ 4 \end{pmatrix} - \begin{pmatrix} -2 \\ -5 \end{pmatrix} = \begin{pmatrix} 8 \\ 13 \end{pmatrix}$

∴ The coordinates are (8, 13).

(c) $-2(2\mathbf{p} + \mathbf{q}) = -4\begin{pmatrix} 3 \\ 4 \end{pmatrix} - 2\begin{pmatrix} -2 \\ -5 \end{pmatrix} = \begin{pmatrix} -8 \\ -6 \end{pmatrix}$

(d) $\frac{1}{2}(\mathbf{p} - \mathbf{q}) = \left[\begin{pmatrix} 3 \\ 4 \end{pmatrix} - \begin{pmatrix} -2 \\ -5 \end{pmatrix}\right] = \frac{1}{2}\begin{pmatrix} 5 \\ 9 \end{pmatrix} = \begin{pmatrix} 2\frac{1}{2} \\ 4\frac{1}{2} \end{pmatrix}$

(e) $\frac{1}{7}(5\mathbf{p} - 3\mathbf{q}) = \frac{1}{7} \times \left[5\begin{pmatrix} 3 \\ 4 \end{pmatrix} - 3\begin{pmatrix} -2 \\ -5 \end{pmatrix}\right]$

$$= \frac{1}{7}\begin{pmatrix} 21 \\ 35 \end{pmatrix}$$

$$= \begin{pmatrix} 3 \\ 5 \end{pmatrix}$$

14. **(a)** $\overrightarrow{OR} = \overrightarrow{OP} - \overrightarrow{OQ} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} - \begin{pmatrix} -2 \\ 4 \end{pmatrix} = \begin{pmatrix} 5 \\ -3 \end{pmatrix}$

∴ The coordinates of R are (5, –3).

(b) $\overrightarrow{OS} = \overrightarrow{OP} + 2\overrightarrow{OQ} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} - 2\begin{pmatrix} -2 \\ 4 \end{pmatrix} = \begin{pmatrix} -1 \\ 9 \end{pmatrix}$

∴ The coordinates $S(x, y)$ are (–1, 9), where $x = -1$ and $y = 9$.

15. **(a)** $\overrightarrow{OA} = \overrightarrow{OL} - 2\overrightarrow{OM} = \begin{pmatrix} 5 \\ -2 \end{pmatrix} - 2\begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ -8 \end{pmatrix}$

∴ The coordinates of A are (1, –8).

(b) $|\overrightarrow{OA}| = \sqrt{1^2 + (-8)^2} = 8.1$ units (to 2 s.f.)

16. **(a)** $\overrightarrow{PR} = \overrightarrow{PQ} + \overrightarrow{QR} = \begin{pmatrix}1\\4\end{pmatrix}+\begin{pmatrix}8\\6\end{pmatrix}=\begin{pmatrix}9\\10\end{pmatrix}$

(b) $|\overrightarrow{QR}| = \sqrt{8^2 + 6^2} = 10$ units

(c) **(i)** $\overrightarrow{ST} = \overrightarrow{OT} - \overrightarrow{OS} = \begin{pmatrix}5\\k\end{pmatrix}-\begin{pmatrix}1\\-3\end{pmatrix}=\begin{pmatrix}4\\k+3\end{pmatrix}$

(ii) If $\overrightarrow{ST} \mathbin{/\!/} \overrightarrow{PQ}$,

$$\overrightarrow{ST} = \begin{pmatrix}4\\k+3\end{pmatrix} = 4\begin{pmatrix}1\\\frac{k+3}{4}\end{pmatrix} = 4\overrightarrow{PQ}$$

$$\begin{pmatrix}1\\\frac{k+3}{4}\end{pmatrix} = \begin{pmatrix}1\\4\end{pmatrix}$$

$\therefore k = 13$

(iii) If $|\overrightarrow{ST}| = |\overrightarrow{PQ}|$,

$$\sqrt{4^2 + (k+3)^2} = \sqrt{1^2 + 4^2}$$
$$16 + (k+3)^2 = 1 + 16$$
$$k + 3 = \pm 1$$
$$k = 1 - 3 \text{ or } k = -1 - 3$$
$$= -2 \qquad = -4$$

$\therefore k = -2$ or -4.

17. $\mathbf{a} = \begin{pmatrix}-3\\2\end{pmatrix}$, $\mathbf{b} = \begin{pmatrix}2\\-10\end{pmatrix}$, $\mathbf{p} = \begin{pmatrix}h\\5\end{pmatrix}$ and $\mathbf{q} = \begin{pmatrix}7\\k\end{pmatrix}$

(a) $3\mathbf{a} + 2\mathbf{b} = 3\begin{pmatrix}-3\\2\end{pmatrix}+2\begin{pmatrix}2\\-10\end{pmatrix}=\begin{pmatrix}-5\\-14\end{pmatrix}$

(b) $\mathbf{a} - \mathbf{b} = \begin{pmatrix}-3\\2\end{pmatrix}-\begin{pmatrix}2\\-10\end{pmatrix}=\begin{pmatrix}-5\\12\end{pmatrix}$

(c) $|\mathbf{a} - \mathbf{b}| = \sqrt{(-5)^2 + 12^2} = 13$ units

18. **(a)** $\mathbf{m} = \overrightarrow{OM} = \overrightarrow{OL} + \overrightarrow{LM} = 2\begin{pmatrix}4\\1\end{pmatrix}+\begin{pmatrix}-3\\1\end{pmatrix}=\begin{pmatrix}-2\\3\end{pmatrix}$

(b) $|\overrightarrow{LM}| = \sqrt{(-6)^2 + 2^2} = 6.3$ units (to 2 s.f.)

(c) $\overrightarrow{ON} = \overrightarrow{ML} = 2\begin{pmatrix}-3\\1\end{pmatrix}=\begin{pmatrix}6\\-2\end{pmatrix}$

$\therefore$ The coordinates of the point N are $(6, -2)$.

19. **(a)** $\mathbf{b} = \overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB} = \begin{pmatrix}7\\4\end{pmatrix}+\begin{pmatrix}-9\\12\end{pmatrix}=\begin{pmatrix}-2\\16\end{pmatrix}$

(b) $|\overrightarrow{AB}| = \sqrt{(-9)^2 + 12^2} = 15$ units

(c) $\overrightarrow{OC} = \overrightarrow{BA} = \begin{pmatrix}-9\\12\end{pmatrix}$

$\therefore$ The coordinates of the point C are $(9, -12)$.

20. **(a)** $\overrightarrow{DA} = -3\mathbf{q}$

(b) $\overrightarrow{AC} = \overrightarrow{AD} + \overrightarrow{DC} = 3\mathbf{q} + \mathbf{p}$

(c) $\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AD} + \overrightarrow{DC}$

$= -4\mathbf{p} + 3\mathbf{q} + \mathbf{p}$

$= 3(\mathbf{q} - \mathbf{p})$

(d) $\overrightarrow{BM} = \frac{1}{2}\overrightarrow{BC} = \frac{3}{2}(\mathbf{q} - \mathbf{p})$

(e) $\overrightarrow{AM} = \overrightarrow{AB} + \overrightarrow{BM}$

$= 4\mathbf{p} + \frac{3}{2}(\mathbf{q} - \mathbf{p})$

$= \frac{1}{2}(5\mathbf{p} + 3\mathbf{q})$

21. **(a)** $\overrightarrow{PA} = -\overrightarrow{AP} = -\frac{1}{2}\overrightarrow{AB} = -2\mathbf{s}$

(b) Since $\overrightarrow{BQ} = \frac{1}{3}\overrightarrow{BC}$,

$\overrightarrow{QC} = \frac{2}{3}\overrightarrow{BC} = \frac{2}{3}(-6\mathbf{t}) = -4\mathbf{t}$

(c) $\overrightarrow{PD} = \overrightarrow{PA} + \overrightarrow{AD} = -2\mathbf{s} - 6\mathbf{t} = -2(3\mathbf{t} + \mathbf{s})$

(d) $\overrightarrow{PQ} = \overrightarrow{PD} + \overrightarrow{DC} + \overrightarrow{CQ}$

$= -2\mathbf{s} - 6\mathbf{t} + 4\mathbf{s} + 4\mathbf{t}$

$= 2(\mathbf{s} - \mathbf{t})$

(e) $\overrightarrow{QD} = \overrightarrow{QC} + \overrightarrow{CD} = -4\mathbf{t} - 4\mathbf{s} = -4(\mathbf{t} + \mathbf{s})$

22. **(a)** $\overrightarrow{RU} = 2\mathbf{b}$

(b) $\overrightarrow{QU} = \overrightarrow{QP} + \overrightarrow{PU} = 2\mathbf{b} + \mathbf{a}$

(c) $\overrightarrow{PT} = \overrightarrow{PU} + \overrightarrow{UT} = \mathbf{a} + \mathbf{d}$

(d) $\overrightarrow{SQ} = \overrightarrow{SR} + \overrightarrow{RQ} = -\mathbf{c} - \mathbf{a} = -(\mathbf{a} + \mathbf{c})$

(e) $\overrightarrow{RT} = \overrightarrow{RU} + \overrightarrow{UT} = 2\mathbf{b} + \mathbf{d}$

(f) $\overrightarrow{QT} = \overrightarrow{QU} + \overrightarrow{UT} = \mathbf{a} + 2\mathbf{b} + \mathbf{d}$

(g) $\overrightarrow{SP} = \overrightarrow{SR} + \overrightarrow{RQ} + \overrightarrow{QP} = 2\mathbf{b} - \mathbf{a} - \mathbf{c}$

23. **(a)** $\overrightarrow{EF} = \overrightarrow{ED} + \overrightarrow{DC} + \overrightarrow{CF} = 2\mathbf{p} + 2\mathbf{q} - 3\mathbf{p} = 2\mathbf{q} - \mathbf{p}$

(b) $\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AF} + \overrightarrow{FC} = -\mathbf{p} - 2\mathbf{q} + 3\mathbf{p} = 2(\mathbf{p} - \mathbf{q})$

(c) $\overrightarrow{EC} = \overrightarrow{EF} + \overrightarrow{FC} = 2\mathbf{q} - \mathbf{p} + 3\mathbf{p} = 2(\mathbf{p} + \mathbf{q})$

(d) $\overrightarrow{AD} = \overrightarrow{AF} + \overrightarrow{FE} + \overrightarrow{ED}$

$= -2\mathbf{q} + \mathbf{p} - 2\mathbf{q} + 2\mathbf{p}$

$= 3\mathbf{p} - 4\mathbf{q}$

(e) $\overrightarrow{EB} = \overrightarrow{EF} + \overrightarrow{FA} + \overrightarrow{AB}$

$= 2\mathbf{q} - \mathbf{p} + 2\mathbf{q} + \mathbf{p}$

$= 4\mathbf{q}$

24. **(a)** $\overrightarrow{AB} = \overrightarrow{AE} + \overrightarrow{EB} = \mathbf{a} - \mathbf{b}$

(b) $\overrightarrow{EC} = \overrightarrow{EB} + \overrightarrow{BC} = \mathbf{a} + \mathbf{a} - \mathbf{b} = 2\mathbf{a} - \mathbf{b}$

(c) $\overrightarrow{DE} = \overrightarrow{DC} + \overrightarrow{CE} = 4\mathbf{a} + \mathbf{b} - 2\mathbf{a} = 2\mathbf{a} + \mathbf{b}$

(d) $\overrightarrow{AD} = \overrightarrow{AE} + \overrightarrow{ED} = -\mathbf{b} - 2\mathbf{a} - \mathbf{b} = -2(\mathbf{a} + \mathbf{b})$

25. **(a)** $\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC} = \mathbf{a} + 3\mathbf{c}$

(b) $\overrightarrow{AE} = \overrightarrow{AF} + \overrightarrow{FE} = \mathbf{b} + \mathbf{c}$

(c) $\overrightarrow{ED} = \overrightarrow{EC} + \overrightarrow{CD} = \frac{5}{2}\mathbf{b} - 2\mathbf{a}$

(d) $\overrightarrow{FD} = \overrightarrow{FE} + \overrightarrow{ED} = \mathbf{c} + \frac{5}{2}\mathbf{b} - 2\mathbf{a}$

26. **(a)** **(i)** From the diagram,
$\overrightarrow{ST} + \overrightarrow{TQ} = \overrightarrow{SQ}$

(ii) From the diagram,
$\overrightarrow{PR} + \overrightarrow{RT} = \overrightarrow{PT}$

(iii) From the diagram,
$\overrightarrow{PS} + \overrightarrow{SQ} + \overrightarrow{QR} = \overrightarrow{PR}$

(iv) From the diagram,
$\overrightarrow{RQ} + \overrightarrow{QT} + \overrightarrow{TP} + \overrightarrow{PS} = \overrightarrow{RS}$

(b) **(i)** $\overrightarrow{PT} + \mathbf{u} = \overrightarrow{PQ}$

From the diagram, $\mathbf{u} = \overrightarrow{TQ}$.

(ii) $\overrightarrow{ST} + \mathbf{u} = \overrightarrow{SQ}$

From the diagram, $\mathbf{u} = \overrightarrow{TQ}$.

(iii) $\overrightarrow{SQ} + \mathbf{u} = \mathbf{0}$

From the diagram, $\mathbf{u} = \overrightarrow{QS}$.

(iv) $\overrightarrow{SQ} + \mathbf{u} = \overrightarrow{RQ}$

From the diagram, $\mathbf{u} = \overrightarrow{RS}$.

(v) $\mathbf{u} + \overrightarrow{SP} = \overrightarrow{RP}$

From the diagram, $\mathbf{u} = \overrightarrow{RS}$.

(vi) $\overrightarrow{TQ} + \mathbf{u} = \overrightarrow{TS}$

From the diagram, $\mathbf{u} = \overrightarrow{QS}$.

(vii) $\overrightarrow{PS} + \mathbf{u} + \overrightarrow{TR} = \overrightarrow{PR}$

From the diagram, $\mathbf{u} = \overrightarrow{ST}$.

(viii) $\overrightarrow{SR} + \mathbf{u} + \overrightarrow{TS} = \mathbf{0}$

From the diagram, $\mathbf{u} = \overrightarrow{RT}$.

(ix) $\overrightarrow{PT} + \overrightarrow{TS} + \mathbf{u} = \overrightarrow{PS}$

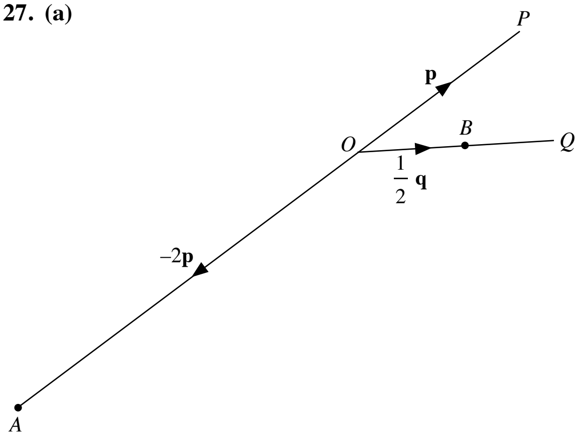
From the diagram, $\mathbf{u} = \mathbf{0}$.

(c) **(i)** From the diagram,
$\overrightarrow{SQ} + (-\overrightarrow{QT}) = \overrightarrow{SQ}$

(ii) From the diagram,
$\overrightarrow{PR} + (-\overrightarrow{QR}) = \overrightarrow{PQ}$

(iii) From the diagram,
$\overrightarrow{RS} + \overrightarrow{QP} + (-\overrightarrow{QS}) = \overrightarrow{RP}$

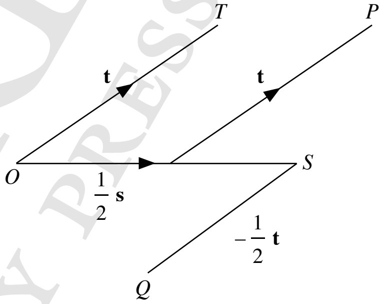
27. **(a)**



(b) $\overrightarrow{QA} = \overrightarrow{OA} - \overrightarrow{OQ} = -2\mathbf{p} - \mathbf{q}$

$\overrightarrow{PB} = \overrightarrow{PO} + \overrightarrow{OB} = \frac{1}{2}\mathbf{q} - \mathbf{p}$

28. **(a)**



(b) $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP} = \mathbf{s} - \frac{1}{2}\mathbf{t} - \frac{1}{2}\mathbf{s} - \mathbf{t} = \frac{1}{2}(\mathbf{s} - 3\mathbf{t})$

29. **(a)** $\overrightarrow{OQ} = \overrightarrow{OP} + \overrightarrow{PQ} = 4\mathbf{a} + 4\mathbf{b}$

(b) $\overrightarrow{OX} = \overrightarrow{OR} + \overrightarrow{RX} = 2\mathbf{a} + 4\mathbf{b}$

(c) $\overrightarrow{QS} = \overrightarrow{QR} + \overrightarrow{RS} = 8\mathbf{b} - 4\mathbf{a}$

Advanced

30. **(a)** **(i)** $\overrightarrow{EF} = -\overrightarrow{BA} = -\mathbf{p}$

(ii) $\overrightarrow{BE} = \overrightarrow{BA} + \overrightarrow{AE} = \mathbf{q} - \mathbf{p}$

(b) $|\overrightarrow{AE}| = 2$ units

$|\overrightarrow{AO}| = |\overrightarrow{OE}| = 1$ unit

Since the figure is a regular octagon, $|\overrightarrow{OB}| = |\overrightarrow{OD}|$
$= 1$ unit

Using Pythagoras' Theorem,

$|\overrightarrow{BD}| = \sqrt{1+1} = \sqrt{2}$ units

(c) $\overrightarrow{BD} = \sqrt{2}\,\mathbf{q}$

31. (a) $\overrightarrow{RP} = \overrightarrow{RO} + \overrightarrow{OQ} = \mathbf{p} - \mathbf{q}$

$\overrightarrow{OQ} = \overrightarrow{OR} + \overrightarrow{RQ} = \mathbf{p} + \mathbf{q}$

(b) (i) Rhombus

(ii) 90°

(c) (i) $\overrightarrow{AQ} = \overrightarrow{OQ} - \overrightarrow{OA} = \mathbf{p} + \mathbf{q} - 2\mathbf{p} = \mathbf{q} - \mathbf{p}$

$\overrightarrow{BQ} = \overrightarrow{OQ} - \overrightarrow{OB} = \mathbf{p} + \mathbf{q} - 2\mathbf{q} = \mathbf{p} - \mathbf{q}$

(ii) $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = 2\mathbf{q} - 2\mathbf{p} = 2(\mathbf{q} - \mathbf{p})$

$$\frac{AB}{QB} = \frac{|2(\mathbf{q} - \mathbf{p})|}{|\mathbf{p} - \mathbf{q}|} = 2$$

32. (a) (i) $\overrightarrow{XC} = \frac{1}{3}\overrightarrow{DC} = \mathbf{p}$

$\overrightarrow{QR} = \overrightarrow{AB} = 3\mathbf{p}$

$XC : QR = 1 : 3$

(ii) $\triangle AXC$ is similar to $\triangle AQR$ and $XC : QR = 1 : 3$, $AC : AR = 1 : 3$ since the ratio of all corr. sides is equal for similar figures.

(iii) $\triangle AXC$ is similar to $\triangle AQR$.

$\therefore AX : AQ = 1 : 3$ since the ratio of all corr. sides is equal for similar figures.

$\triangle ADX$ is similar to $\triangle APQ$.

$\therefore DX : PQ = AX : AQ = 1 : 3$

(b) (i) $\overrightarrow{DX} = \frac{2}{3}\overrightarrow{DC} = 2\mathbf{p}$

(ii) $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = 3\mathbf{p} + 2\mathbf{q}$

(iii) Since $AC : AR = 1 : 3$, $\overrightarrow{AR} = 3(3\mathbf{p} + 2\mathbf{q})$

(iv) $\overrightarrow{BR} = \overrightarrow{AR} - \overrightarrow{BA}$

$= 3(3\mathbf{p} + 2\mathbf{q}) - 3\mathbf{p}$

$= 6(\mathbf{p} + \mathbf{q})$

(v) $\overrightarrow{RQ} = \overrightarrow{BA} = -3\mathbf{p}$

(vi) Since $DX : PX = 1 : 3$, $\overrightarrow{PQ} = 3(2\mathbf{p}) = 6\mathbf{p}$

(vii) Since $AD : AP = 1 : 3$, $\overrightarrow{AP} = 3(2\mathbf{q}) = 6\mathbf{q}$

(viii) $\overrightarrow{PB} = \overrightarrow{PA} + \overrightarrow{AB} = 3(\mathbf{p} - 2\mathbf{q})$

(ix) $\overrightarrow{BQ} = \overrightarrow{BR} + \overrightarrow{RQ}$

$= 6(\mathbf{p} + \mathbf{q}) - 3\mathbf{p}$

$= 3(2\mathbf{q} + \mathbf{p})$

(x) $\overrightarrow{DR} = \overrightarrow{DP} + \overrightarrow{PR} = 4\mathbf{q} + 6\mathbf{p} + 3\mathbf{p} = 4\mathbf{q} + 9\mathbf{p}$

33. (a) (i) $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP} = 2(\mathbf{b} - \mathbf{a})$

(ii) $\overrightarrow{PM} = \frac{1}{2}\overrightarrow{PQ} = \mathbf{b} - \mathbf{a}$

(iii) $\overrightarrow{OM} = \overrightarrow{OQ} + \overrightarrow{QM} = 2\mathbf{b} + \mathbf{a} - \mathbf{b} = \mathbf{a} + \mathbf{b}$

(iv) $\overrightarrow{RS} = \overrightarrow{OS} - \overrightarrow{OR} = 4\mathbf{b} - \frac{3}{2}(2\mathbf{a}) = 4\mathbf{b} - 3\mathbf{a}$

(v) Since $RN = \frac{3}{7}RS$, $NS = \frac{4}{7}RS$

$\therefore \overrightarrow{NS} = \frac{4}{7}\overrightarrow{RS} = \frac{4}{7}(4\mathbf{b} - 3\mathbf{a})$

(vi) $\overrightarrow{ON} = \overrightarrow{OS} + \overrightarrow{SN}$

$= 4\mathbf{b} - \frac{4}{7}(4\mathbf{b} - 3\mathbf{a})$

$= \frac{12}{7}(\mathbf{a} + \mathbf{b})$

(b) $$\frac{OM}{MN} = \frac{|\mathbf{a} + \mathbf{b}|}{\left|\frac{12}{7}(\mathbf{a} + \mathbf{b}) - (\mathbf{a} + \mathbf{b})\right|} = \frac{|\mathbf{a} + \mathbf{b}|}{\left|\frac{5}{7}(\mathbf{a} + \mathbf{b})\right|} = \frac{7}{5}$$

$\therefore OM : MN = 7 : 5$

(c) $\overrightarrow{MN} = \frac{5}{7}\overrightarrow{OM} = \frac{5}{7}(\mathbf{a} + \mathbf{b})$

New Trend

34. (a) $\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$

$$= \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} 7 \\ -3 \end{pmatrix}$$

$$= \begin{pmatrix} 8 \\ -1 \end{pmatrix}$$

$\therefore$ The coordinates of B are $(8, -1)$.

(b) (i) $\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC}$

$$= -\begin{pmatrix} 7 \\ -3 \end{pmatrix} + \begin{pmatrix} 1 \\ 5 \end{pmatrix}$$

$$= \begin{pmatrix} -6 \\ 8 \end{pmatrix}$$

Since M is the midpoint of $\overrightarrow{BC}$, $\overrightarrow{BM} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$

$\overrightarrow{AM} = \overrightarrow{AB} + \overrightarrow{BM} = \begin{pmatrix} 7 \\ -3 \end{pmatrix} + \begin{pmatrix} -3 \\ 4 \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$

(ii) $|\overrightarrow{AM}| = \sqrt{4^2 + 1^2} = 4.12$ units (to 3 s.f.)

(c) For *ABPC* to be a parallelogram,

$$\overrightarrow{CP} = \overrightarrow{AB} = \begin{pmatrix} 7 \\ -3 \end{pmatrix}.$$

$$\overrightarrow{OP} - \overrightarrow{OC} = \begin{pmatrix} 7 \\ -3 \end{pmatrix}$$

$$\overrightarrow{OP} - (\overrightarrow{OA} + \overrightarrow{AC}) = \begin{pmatrix} 7 \\ -3 \end{pmatrix}$$

$$\overrightarrow{OP} = \begin{pmatrix} 7 \\ -3 \end{pmatrix} + \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} 1 \\ 5 \end{pmatrix}$$

$$= \begin{pmatrix} 9 \\ 4 \end{pmatrix}$$

$\therefore$ The coordinates of P are (9, 4).

35. (a) (i) $\overrightarrow{AB} = \mathbf{b} - 2\mathbf{a}$

(ii) $\overrightarrow{AC} = \overrightarrow{MC} - \overrightarrow{MA}$

$= 6\mathbf{a} - 2\mathbf{b} - (2\mathbf{a} - 2\mathbf{b})$

$= 4\mathbf{a}$

(iii) $\overrightarrow{CD} = \overrightarrow{OD} - \overrightarrow{OC}$

$= 3\mathbf{b} - 6\mathbf{a}$

$= 3(\mathbf{b} - 2\mathbf{a})$

(b) $\overrightarrow{CD} = 3\overrightarrow{AB}$ so $\overrightarrow{CD} \,//\, \overrightarrow{AB}$.

$\angle AOB = \angle COD$ (common $\angle$)

$\angle OAB = \angle OCD$ (corr. $\angle$s, CD // AB)

$\therefore$ $\triangle OAB$ is similar to $\triangle OCD$. (2 pairs of corr. $\angle$s equal)

(c) $\dfrac{\text{Area of } \triangle OAB}{\text{Area of } \triangle OCD} = \left(\dfrac{1}{3}\right)^2 = \dfrac{1}{9}$

$\therefore$ The ratio of area of $\triangle OAB$ to area of *ACDB* is 1 : 8.

36. (a) $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$

$= 2\mathbf{q} - \mathbf{p}$

$\overrightarrow{RQ} = \dfrac{1}{3}(2\mathbf{q} - \mathbf{p})$

(b) $\overrightarrow{SQ} = \dfrac{1}{4}\overrightarrow{OQ}$

$= \dfrac{1}{4}(2\mathbf{q})$

$= \dfrac{1}{2}\mathbf{q}$

$\overrightarrow{SR} = \overrightarrow{SQ} + \overrightarrow{QR}$

$= \dfrac{1}{2}\mathbf{q} + \dfrac{1}{3}(-2\mathbf{q} + \mathbf{p})$

$= -\dfrac{1}{6}\mathbf{q} + \dfrac{1}{3}\mathbf{p}$

Since $\overrightarrow{SR} \neq k\mathbf{p}$, *OP* and *SR* are not parallel.

37. (a) $\overrightarrow{XY} = \overrightarrow{XO} + \overrightarrow{OY}$

$$= -\begin{pmatrix} 3 \\ 7 \end{pmatrix} + \begin{pmatrix} -12 \\ -9 \end{pmatrix}$$

$$= \begin{pmatrix} -15 \\ -16 \end{pmatrix}$$

(b) $|\overrightarrow{XY}| = \sqrt{(-15)^2 + (-16)^2}$ = 21.9 units (to 3 s.f.)

(c) $\overrightarrow{YX} = 2\overrightarrow{XZ}$

$$\begin{pmatrix} 15 \\ 16 \end{pmatrix} = 2(\overrightarrow{XO} + \overrightarrow{OZ})$$

$$\begin{pmatrix} 7.5 \\ 8 \end{pmatrix} = -\begin{pmatrix} 3 \\ 7 \end{pmatrix} + \overrightarrow{OZ}$$

$$\overrightarrow{OZ} = \begin{pmatrix} 7.5 \\ 8 \end{pmatrix} + \begin{pmatrix} 3 \\ 7 \end{pmatrix}$$

$$= \begin{pmatrix} 10.5 \\ 15 \end{pmatrix}$$

$\therefore$ The coordinates of the point Z are (10.5, 15).

38. (a) Gradient of $AB = -\dfrac{2}{3}$

Equation of AB is $y = -\dfrac{2}{3}x + c$

When $x = 10$, $y = -4$

$-4 = -\dfrac{2}{3}(10) + c$

$c = \dfrac{8}{3}$

$\therefore$ Equation of AB is $y = -\dfrac{2}{3}x + \dfrac{8}{3}$.

(b) $y = -\dfrac{2}{3}x + \dfrac{8}{3}$ —(1)

$4y = x + 18$ —(2)

Substitute (1) into (2):

$4\left(-\dfrac{2}{3}x + \dfrac{8}{3}\right) = x + 18$

$-\dfrac{8}{3}x + \dfrac{32}{3} = x + 18$

$\dfrac{11}{3}x = -\dfrac{22}{3}$

$x = -2$

$y = -\dfrac{2}{3}(-2) + \dfrac{8}{3}$

$= 4$

$\therefore$ The coordinates of the point of intersection are (−2, 4).

39. (a) (i) $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$= \mathbf{b} - \mathbf{a}$

(ii) $\overrightarrow{OP} = \overrightarrow{OB} + \overrightarrow{BP}$

$= \mathbf{b} + \frac{1}{4}(\mathbf{a} - \mathbf{b})$

$= \frac{1}{4}\mathbf{a} + \frac{3}{4}\mathbf{b}$

(iii) $\overrightarrow{PC} = \overrightarrow{OC} - \overrightarrow{OP}$

$= (\mathbf{a} - \mathbf{b}) - \left(\frac{1}{4}\mathbf{a} + \frac{3}{4}\mathbf{b}\right)$

$= \frac{3}{4}\mathbf{a} - \frac{7}{4}\mathbf{b}$

(b) $\triangle APX$ is similar to $\triangle OCX$.

$\therefore$ $PX : CX = 3 : 4$ since the ratio of all corr. sides is equal for similar figures.

$\overrightarrow{PX} = \frac{3}{7}\overrightarrow{PC}$

$= \frac{3}{7}\left(\frac{3}{4}\mathbf{a} - \frac{7}{4}\mathbf{b}\right)$

$= \frac{9}{28}\mathbf{a} - \frac{3}{4}\mathbf{b}$

$\overrightarrow{OX} = \overrightarrow{OP} + \overrightarrow{PX}$

$= \frac{1}{4}\mathbf{a} + \frac{3}{4}\mathbf{b} + \frac{9}{28}\mathbf{a} - \frac{3}{4}\mathbf{b}$

$= \frac{4}{7}\mathbf{a}$ (shown)

(c) $\dfrac{\text{Area of } \triangle APQ}{\text{Area of } \triangle APC} = \dfrac{\frac{1}{2} \times AQ \times AP \times \sin\angle PAQ}{\frac{1}{2} \times AC \times AP \times \sin\angle PAC}$

$= \frac{2}{3}$

$\therefore$ The ratio of area of $\triangle APQ$ to area of $\triangle APC$ is 2 : 3.

40. (a) (i) $\overrightarrow{PM} = \overrightarrow{PS} + \overrightarrow{SM}$

$= \mathbf{a} + 2\mathbf{b}$

(ii) $\overrightarrow{TR} = \overrightarrow{TS} + \overrightarrow{SR} = 2\mathbf{a} - 2\mathbf{b}$

(iii) $\overrightarrow{MN} = \overrightarrow{MR} + \overrightarrow{RN}$

$= \overrightarrow{MR} - \overrightarrow{NR}$

$= \overrightarrow{MR} - \frac{1}{3}\overrightarrow{TR}$

$= \mathbf{a} - \frac{1}{3}(2\mathbf{a} - 2\mathbf{b})$

$= \frac{1}{3}(\mathbf{a} + 2\mathbf{b})$

(b) $\overrightarrow{MN} = \frac{1}{3}\overrightarrow{PM}$

P, M and N are collinear, with M being the common point.

(c) (i) $\dfrac{PM}{PN} = \dfrac{|\mathbf{a} + 2\mathbf{b}|}{\left|\frac{4}{3}(\mathbf{a} + 2\mathbf{b})\right|} = \dfrac{3}{4}$

$\therefore PM : PN = 3 : 4$

(ii) $\dfrac{\text{Area of } \triangle MTP}{\text{Area of } \triangle MTN} = \dfrac{\frac{1}{2} \times PM \times h}{\frac{1}{2} \times MN \times h}$

$= \frac{3}{1}$

$\therefore$ The ratio of area of $\triangle MTP$ to area of $\triangle MTN$ is 3 : 1.

41. (a) $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$= \begin{pmatrix} -3 \\ 4 \end{pmatrix} - \begin{pmatrix} 5 \\ 8 \end{pmatrix}$

$= \begin{pmatrix} -8 \\ -4 \end{pmatrix}$

(b) $|\overrightarrow{AB}| = \sqrt{(-8)^2 + (-4)^2} = 8.94$ units (to 3 s.f.)

(c) $4\overrightarrow{AX} = \overrightarrow{AB}$

$\overrightarrow{AX} = \frac{1}{4}\overrightarrow{AB}$

$= \frac{1}{4}\begin{pmatrix} -8 \\ -4 \end{pmatrix}$

$= \begin{pmatrix} -2 \\ -1 \end{pmatrix}$

42. (a) (i) $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$= 4\mathbf{b} - 4\mathbf{a}$

(ii) $\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC}$

$= 4\mathbf{a} + \frac{3}{2}(4\mathbf{b} - 4\mathbf{a})$

$= 6\mathbf{b} - 2\mathbf{a}$

(iii) $\overrightarrow{NC} = \overrightarrow{OC} - \overrightarrow{ON}$

$= 6\mathbf{b} - 2\mathbf{a} - \frac{3}{4}(4\mathbf{b})$

$= 3\mathbf{b} - 2\mathbf{a}$

(iv) $\overrightarrow{MC} = \overrightarrow{MA} + \overrightarrow{AC}$

$= 2\mathbf{a} + 6\mathbf{b} - 6\mathbf{a}$

$= 6\mathbf{b} - 4\mathbf{a}$

(b) $\overrightarrow{MC} = 6\mathbf{b} - 4\mathbf{a}$

$= 2(3\mathbf{b} - 2\mathbf{a})$

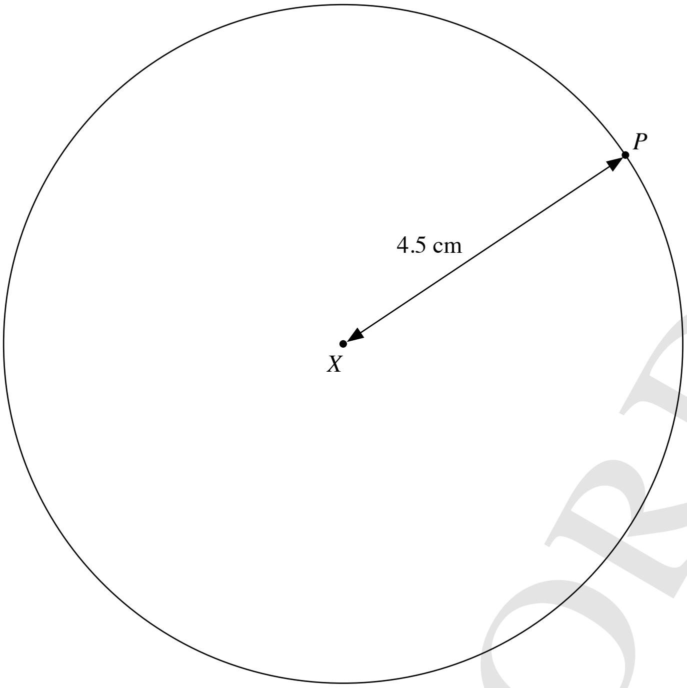
$= 2\overrightarrow{NC}$

$MC = 2NC$ and M, N and C are collinear.

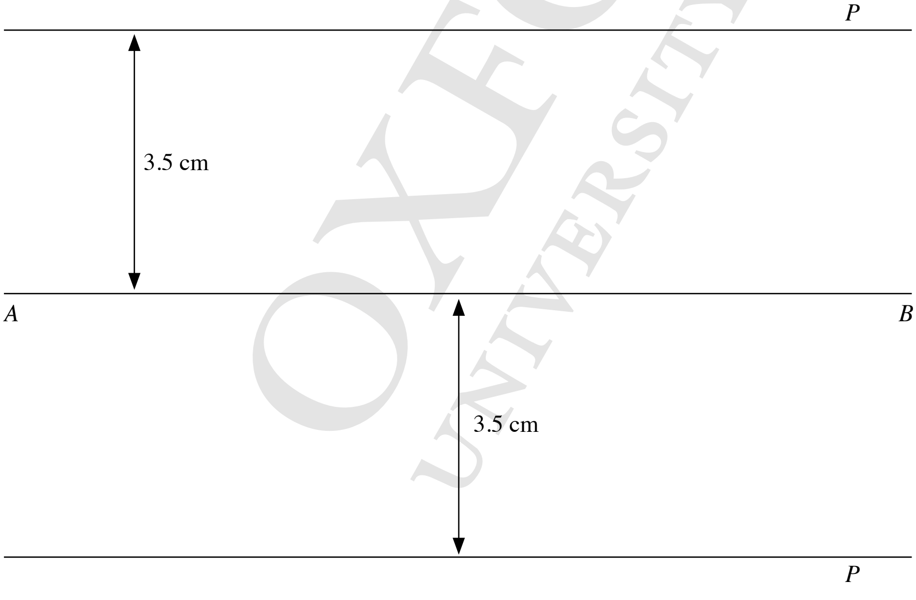
Basic

1. Locus of *P* is a circle with centre at *O* and radius 8 cm.

2.

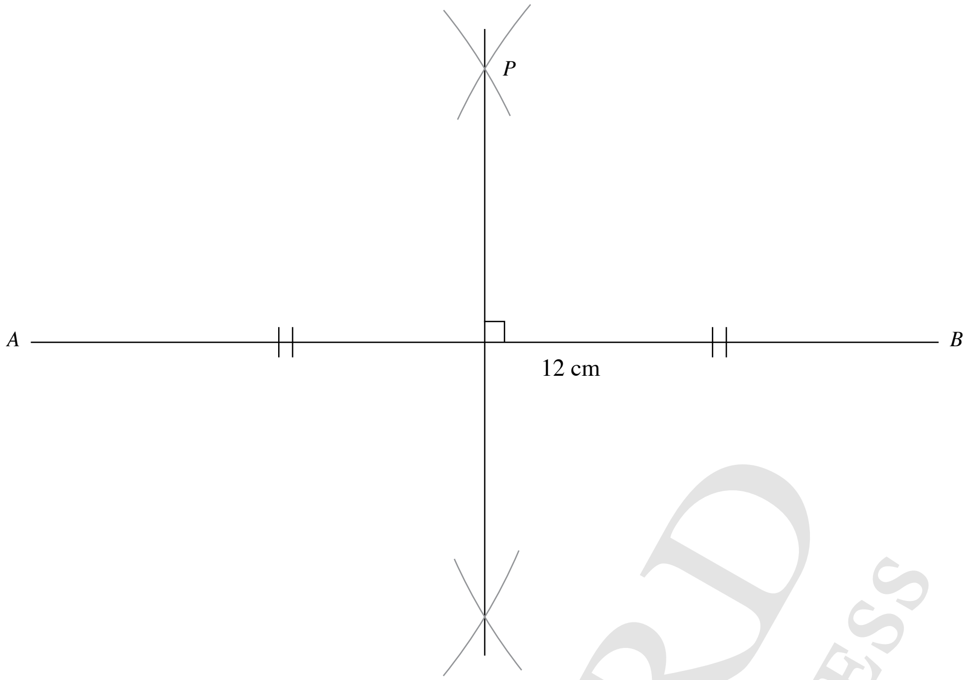


3.

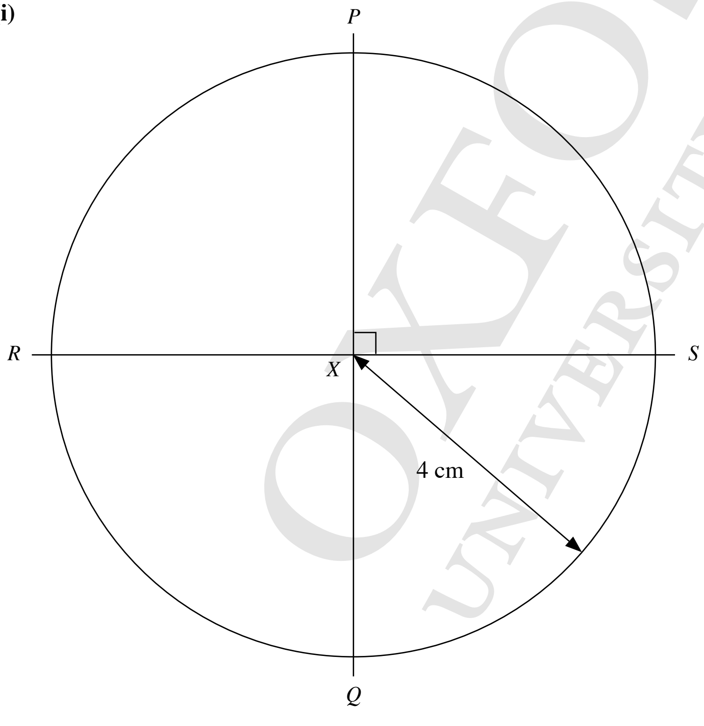


4. *Q* is a point on two parallel lines, one above and the other below and parallel to *l*.

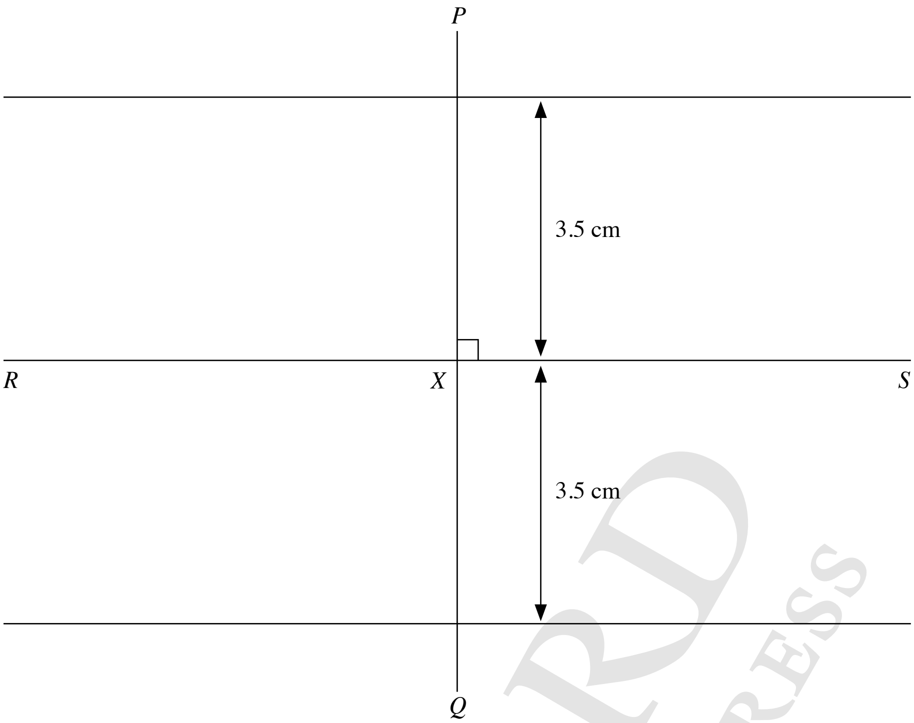
5.



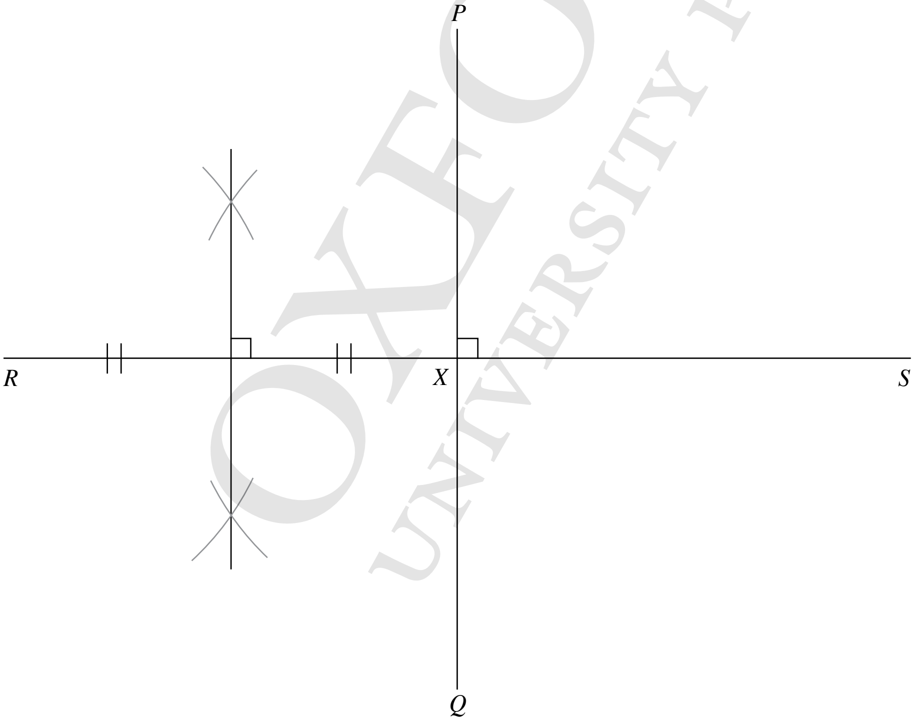
6. **(i)**



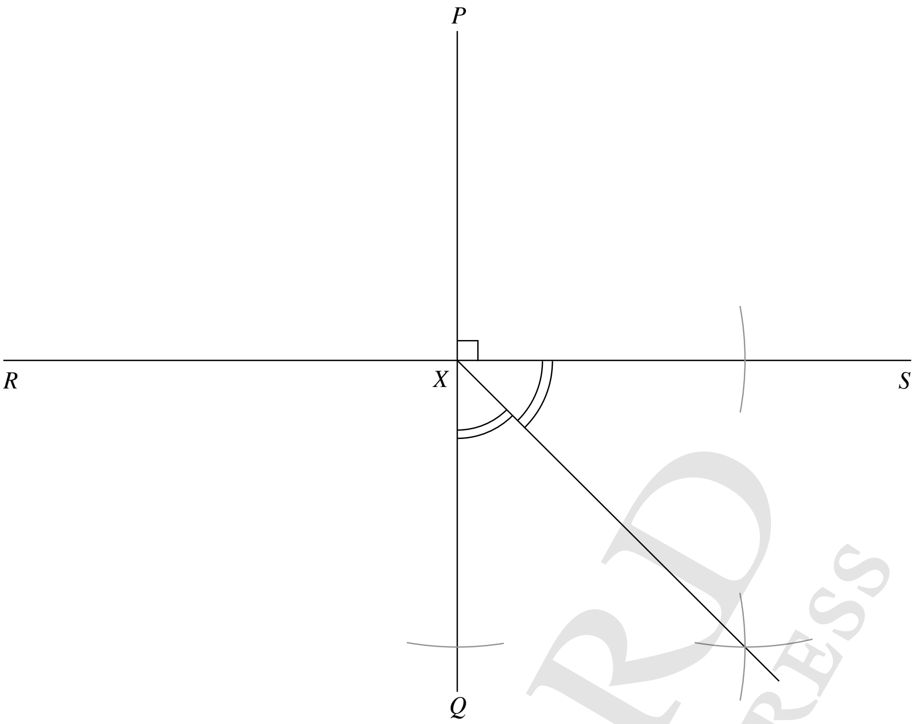
(ii)



(iii)

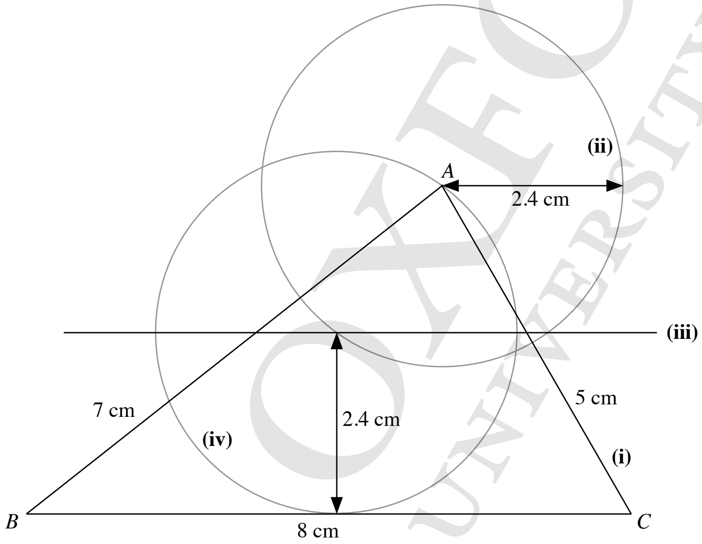


(iv)

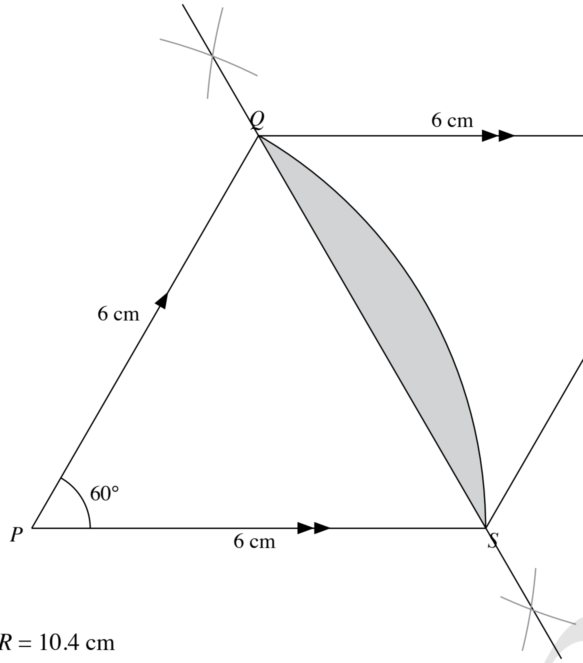


Intermediate

7.

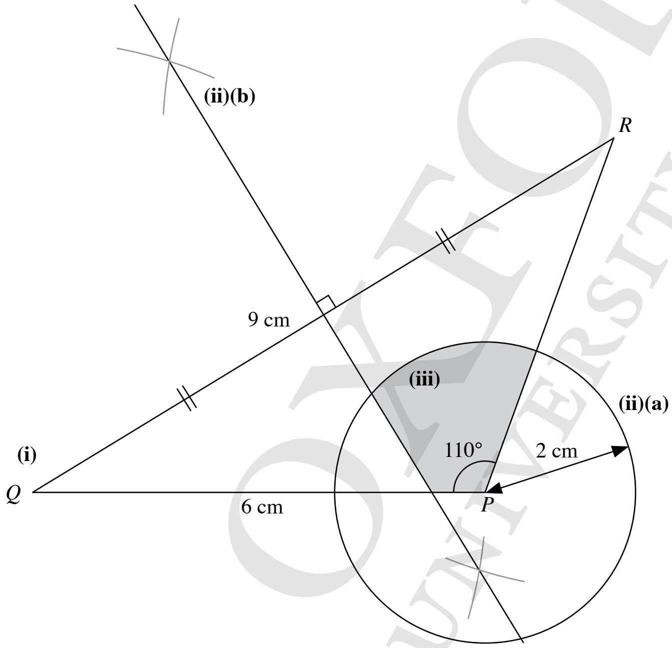


8.

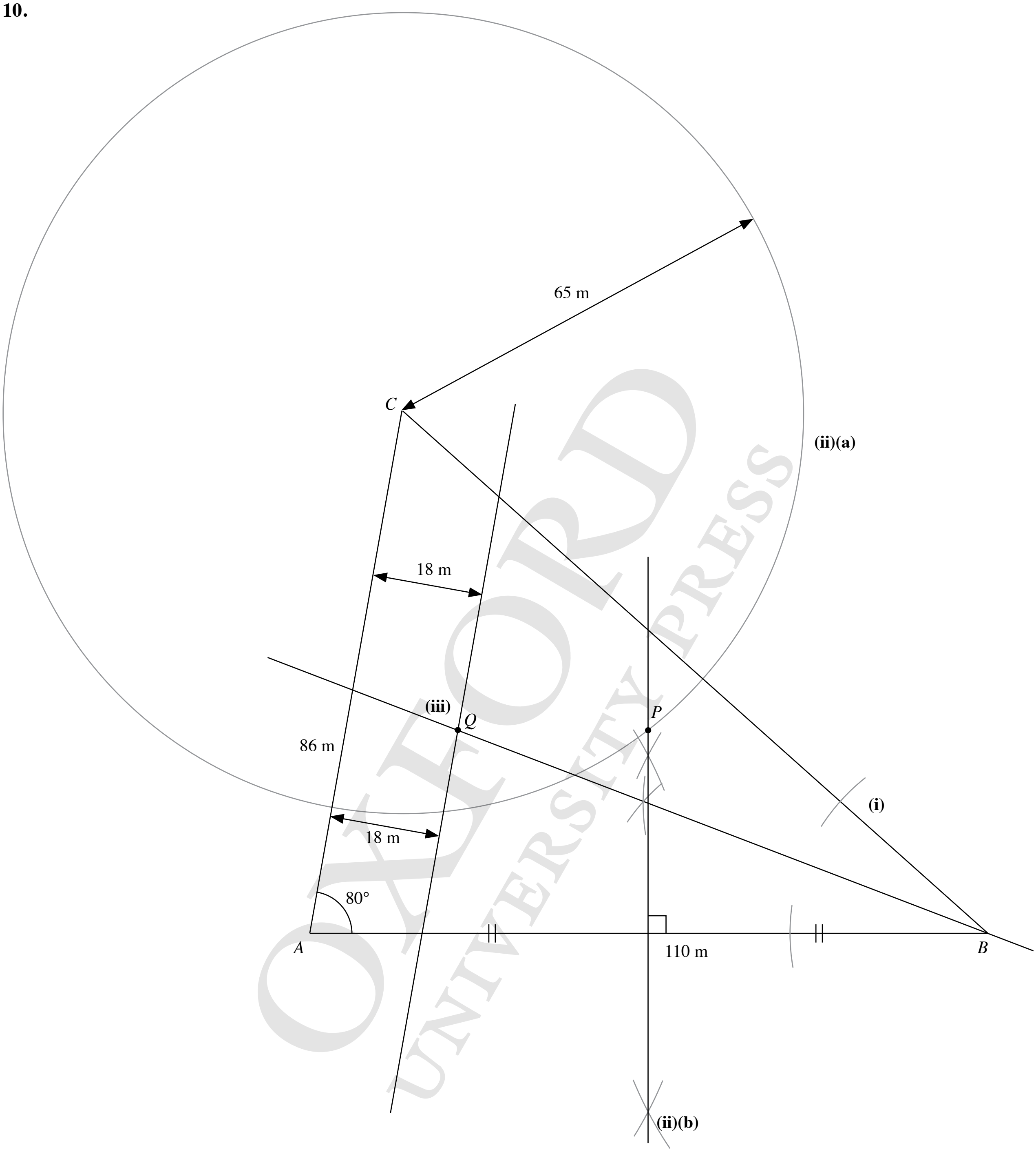


PR = 10.4 cm

9.

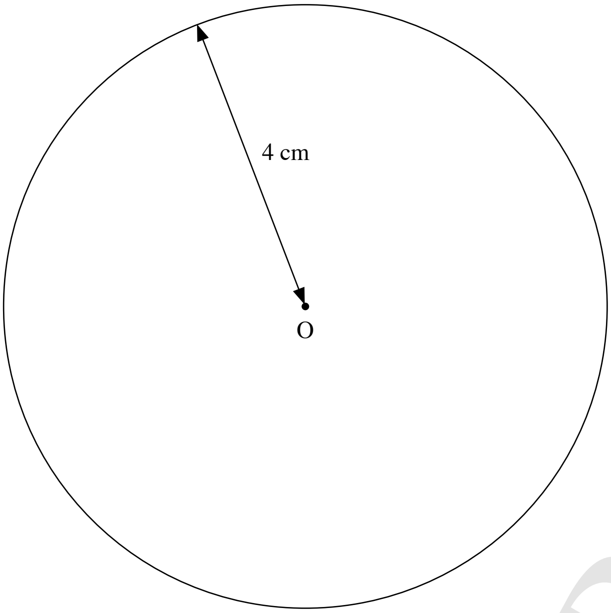


10.

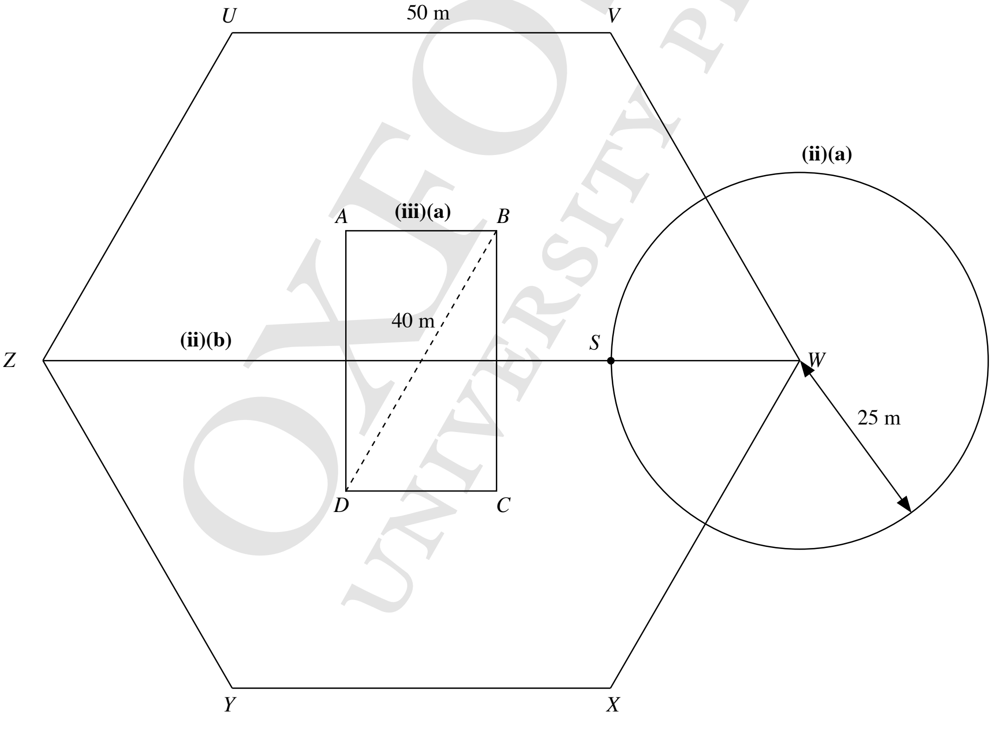


Advanced

11.



12.

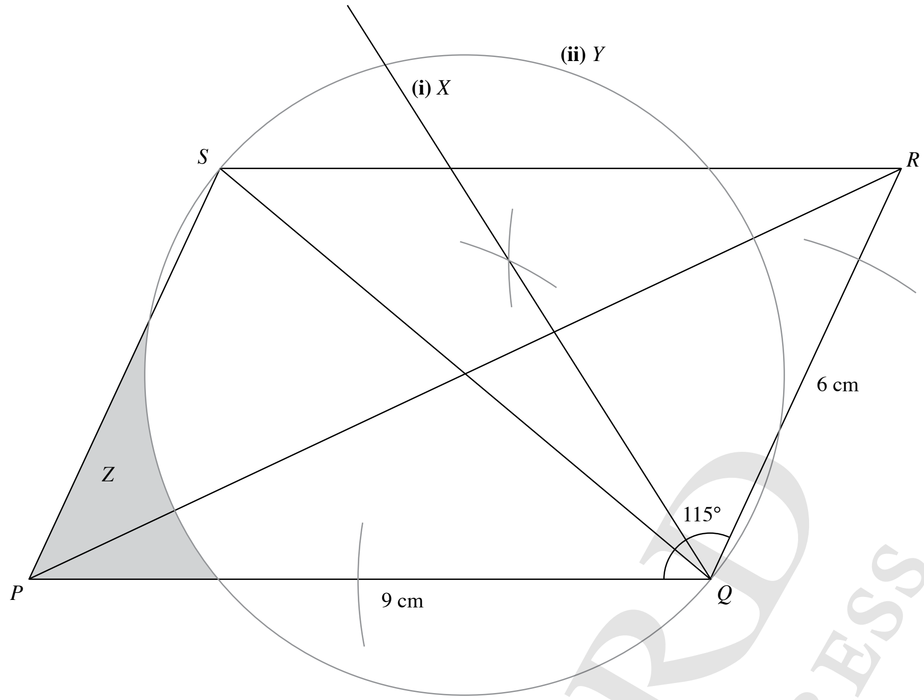


(iii) (b) Area $= 20 \times 2\sqrt{20^2 - 10^2}$

$= 693 \text{ m}^2$

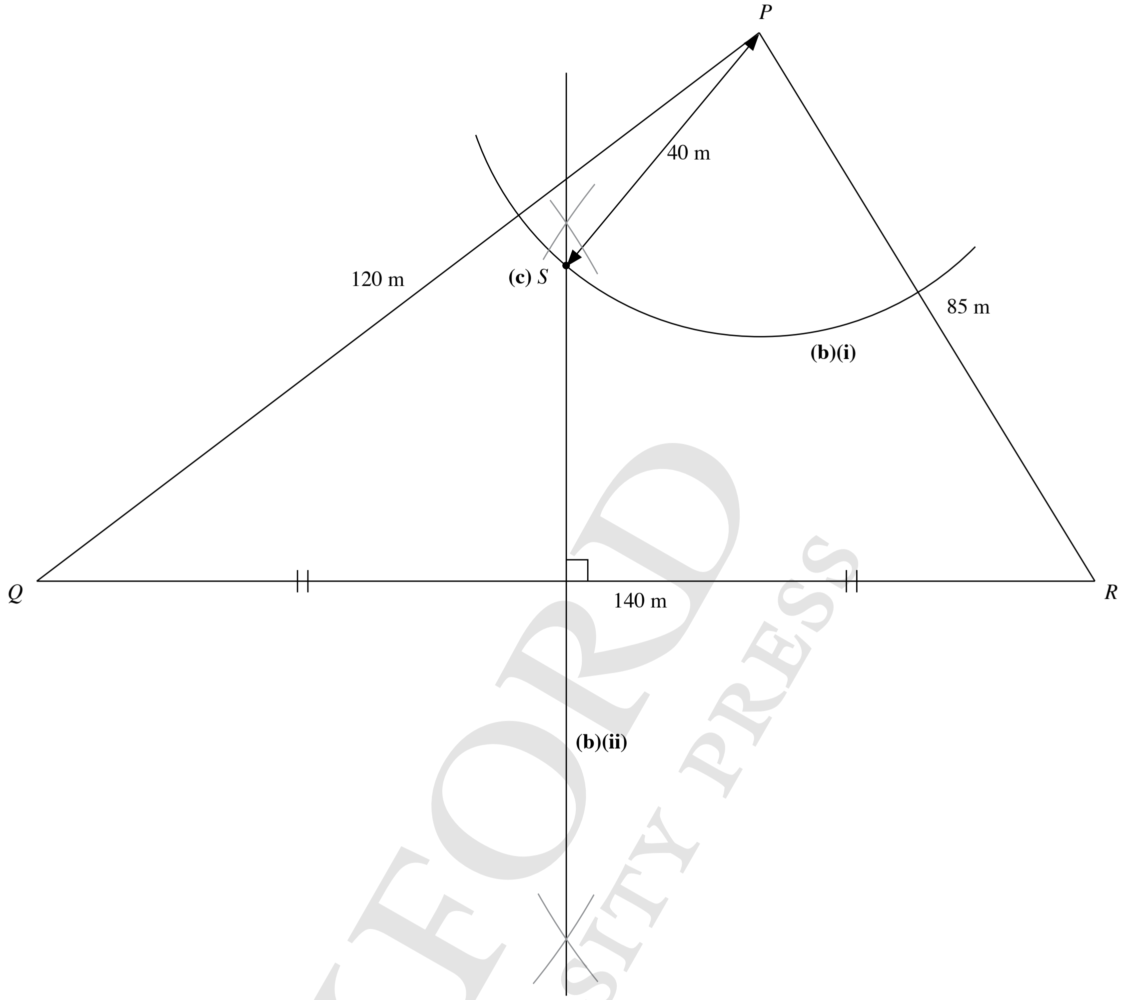
The area of the seating space is 693 m^2.

13.



PR = 12.8 cm

14.

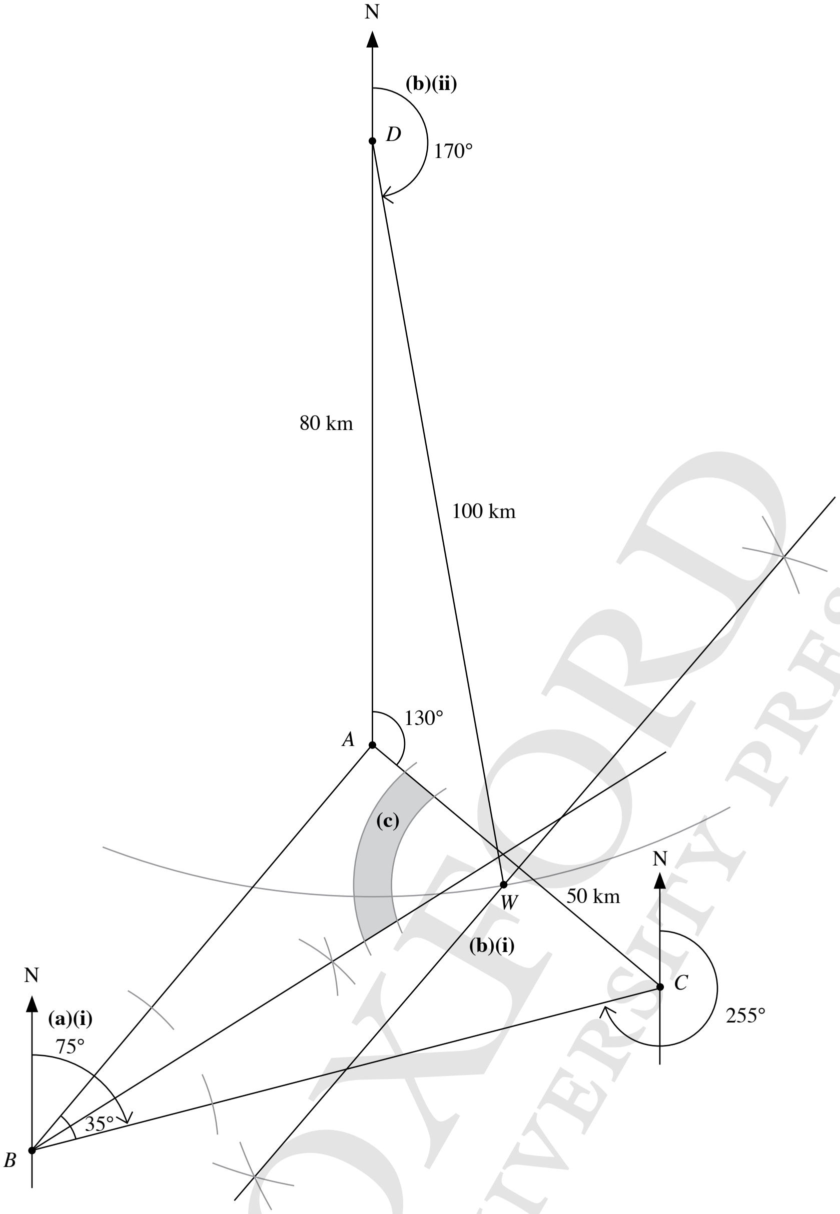


(a) Smallest angle = 37.5°

(d) RS = 8.2 cm

∴ The distance of the security post from R is 82 m.

15.



(a) **(i)** $A\hat{B}C = 35°$,

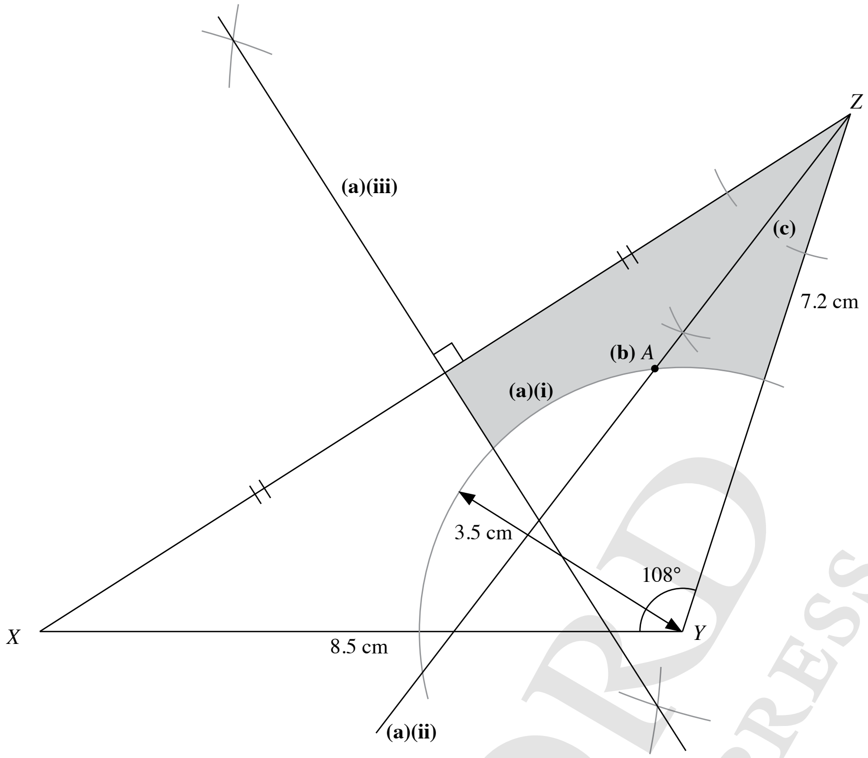
The bearing of *C* from *B* is 075°.

(ii) $AB = 7$ cm,

The distance of Bangpra from Arunya is 70 km.

(b) **(ii)** The bearing of Watpoo from Daogam is 170°.

16.



Chapter 9 Numbers and Algebra

Exercise 9A

1. **(a)** 46×5 nanometres $= 46 \times 5 \times 10^{-9}$

$= 2.30 \times 10^{-7}$ m

(b) Volume $= \dfrac{4}{3}\pi(5 \times 10^{-9})^3$

$= 5.24 \times 10^{-24}$ m^3

2. **(a)** $4.5 \times 10^9 = 4.5 \times 10^6 \times 10^3$

$= (4.5 \times 10^6)$ thousands

$\therefore k = 4.5 \times 10^6$

(b) Time taken $= \dfrac{4.5 \times 10^9 \times 10^3}{3 \times 10^8}$ s

$= 1.5 \times 10^4$ s

$= \dfrac{1.5 \times 10^4}{60}$ min

$= 250$ min

3. **(a)** Number of thumb drives $= \dfrac{24.8 \times 10^{12}}{2.58 \times 10^9}$

$= 9600$ (to nearest 100)

(b) Number of pages $= \dfrac{24.8 \times 10^{12}}{4.85 \times 10^3}$

$= 5113 \times 10^6$ (to nearest million)

4. **(a)** 310 nanometres $= 310 \times 10^{-9}$ m

$= 3.1 \times 10^{-7}$ m

(b) Number of protons $= \dfrac{62}{3.1 \times 10^{-7}}$

$= 2 \times 10^8$

5. **(a)** 2.2 femtoseconds $= 2.2 \times 10^{-15}$ s

(b) Number of times $= \dfrac{5}{2.2 \times 10^{-15}}$

$= 2.3 \times 10^{15}$ (to 2 s.f.)

6. **(a)** 120 gigabytes $= 120 \times 10^9$

$= 1.2 \times 10^{11}$ bytes

(b) Number of files $= \dfrac{1.2 \times 10^{11}}{540 \times 10^3}$

$= 2.22 \times 10^5$ (to 3 s.f.)

7. **(a)** $\dfrac{3.507 \times 0.089}{0.0029 + \sqrt{0.036}} = 1.62$ (to 3 s.f.)

(b) $3p + \dfrac{2}{q} = 3(3.5 \times 10^7) + \dfrac{2}{7.98 \times 10^{-8}}$

$= 1.30 \times 10^8$ (to 3 s.f.)

(c) $25^{-0.5} + \left(\dfrac{1}{27}\right)^{\frac{1}{3}} = \dfrac{1}{\sqrt{25}} + \sqrt[3]{\dfrac{1}{27}}$

$= \dfrac{1}{5} + \dfrac{1}{3}$

$= \dfrac{8}{15}$

8. **(a)** 700 billion $= 700 \times 10^9$

$= 7 \times 10^{11}$

$= 7 \times 10^n$

$\therefore n = 11$

(b) 700 billion $- 9.5 \times 10^9 = 7 \times 10^{11} - 9.5 \times 10^9$

$= 6.905 \times 10^{11}$

9. **(a)** 00 40 – 21 47 = 2 hours 53 minutes

= 173 minutes

(b) 22 20 + 7 h + 12 h 35 min = 17 55 on Wednesday

(c) **(i)** 03 45 – 6 h = 21 45 on Sunday

(ii) 10 45 – 6 h + 11 h 50 min = 16 35 on Monday

10. **(a)** $\left(\dfrac{5}{7} - \dfrac{1}{3}\right)$of tank $= 56\ l$

Full tank $= 147\ l$

$\therefore$ Additional volume $= \dfrac{2}{7} \times 147$

$= 42\ l$

(b) Time left = 04 18 – 8 h 43 min = 19 35 on Sunday

(c) $\dfrac{5.4^3}{7.85 \times \sqrt{23.4}} = 4.147$ (to 4 s.f.)

11. **(a)** 3.3 trillion $\times$ 1.46 = S\$$3.3 \times 10^{12} \times 1.46$

= S\$$4.82 \times 10^{12}$ (to 3 s.f.)

(b) Number of apartments $= \dfrac{4.818 \times 10^{12}}{1.25 \times 10^6}$

= 3 854 400 (to nearest 1000)

12. **(a)** Number of litres $= \dfrac{385}{13.5}$

$= 29\ l$ (to nearest litre)

(b) Number of km $= \dfrac{50}{2.17} \times 13.5$

= 311 km (to nearest km)

(c) Difference $= \left(\dfrac{50}{1.58} - \dfrac{50}{2.17}\right) \times 13.5$

= 116 km (to nearest km)

(d) Number of km $= \dfrac{3}{4}x \times 13.5$

$= 10\dfrac{1}{8}x$ km

13. **(a)** $A = 3.8 \times 10^{12} \times 2.4 \times 10^{56} \times 5.6 \times 10^{74}$

$= 5.107 \times 10^{143}$ (to 4 s.f.)

(b) $\dfrac{100 \text{ m}}{9.58 \text{ s}} = \dfrac{100}{1000}$ km $\div \dfrac{9.58}{3600}$ h

$= 37.58$ km/h (to 2 d.p.)

(c) $4116 = 2^2 \times 3 \times 7^3$

$$\begin{array}{r|r} 2 & 4116 \\ \hline 2 & 2058 \\ \hline 7 & 1029 \\ \hline 7 & 147 \\ \hline 7 & 21 \\ \hline & 3 \end{array}$$

$\therefore$ Smallest whole number needed $= 3 \times 7$

$= 21$

14. **(a)** 7.77 (to 3 s.f.)

(b) 0.770 (to 3 s.f.)

(c) –1.74 (to 3 s.f.)

15. **(a)** Total time $= 35 \times 58$ min

$= 2030$ min

(b) Number of DVD needed $= \dfrac{2030}{120}$

≈ 17

Total cost $= 17 \times 0.90$

$= \$15.30$

16. **(a)** $8500\left(1+\dfrac{x}{100}\right)^3 = 9355.98$

$1 + \dfrac{x}{100} = \sqrt[3]{\dfrac{9355.98}{8500}}$

$= 1.0325$ (to 5 s.f.)

$\therefore x = 3.25$ (to 3 s.f.)

(b) Total amount of interest

$= \dfrac{8200 \times 3.88 \times 5}{100} + 9400\left(1+\dfrac{3.85}{100}\right)^5 - 9400$

$= \$3545.10$ (to 2 d.p.)

17. Number of people $= \dfrac{12 \times 1000}{0.75}$

$= 16\,000$

18. **(a)** Total amount left $= \dfrac{15\,000}{1-\dfrac{1}{8}-\dfrac{1}{4}}$

$= \$24\,000$

(b) Amount left behind $= \dfrac{12\,000 \times 7}{1-\dfrac{5}{12}-\dfrac{2}{3}\left(1-\dfrac{5}{12}\right)}$

$= \$450\,000$

(c) $\left(\dfrac{58}{100}-\dfrac{42}{100}\right)$ of total number of votes = 400 votes

$\therefore$ Total number of votes = 2500.

19. **(a)** 1 h turns 30°

$16\;40 - 10\;10 = 6\dfrac{1}{2}$ h

$\therefore 6\dfrac{1}{2}$ h turn $30° \times 6.5 = 195°$

(b) $25 + 8 + 7 = 40$ min

07 20 – 40 min = 06 40

20. **(a)** Greatest perimeter $= (6.35 + 7.85) \times 2$

$= 28.4$ cm

(b) Least area $= 6.25 \times 7.75$

$= 48.4\text{ cm}^2$ (to 3 s.f.)

21. **(a)** Highest fuel consumption $= \dfrac{575}{46.5}$

$= 12.4$ km/litre (to 1 d.p.)

(b) Lowest fuel consumption $= \dfrac{585}{45.5}$

$= 12.9$ km/litre (to 1 d.p.)

22. **(a)** When $F = 158$, $C = \dfrac{5}{9} \times (158 - 32)$

$= 70$ °C

(b) When $C = 16$, $16 = \dfrac{5}{9} \times (F - 32)$

$F - 32 = 28.8$

$F = 60.8$ °F

(c) When $C = F$, $F = \dfrac{5}{9}(F - 32)$

$\dfrac{4}{9}F = \dfrac{5}{9} \times (-32)$

$F = -40$ °F

(d) $F = (15 + C) \times 2 = 2(15 + C)$

(e) When $C = 16$, $F = 2(15 + 16)$

$= 62$ °F

(f) Percentage error $= \dfrac{62 - 60.8}{60.8} \times 100\%$

$= 1.97\%$ (to 2 d.p.)

23. **(a)** 85% of total = \$3570

$\therefore$ 15% of downpayment $= \dfrac{3570}{85} \times 15$

$= \$630$

(b) 16% of income = \$216

$\therefore$ Amount spent $= \dfrac{216}{16} \times 84 \times 12 = \$13\,608$

(c) Let x be the number of apples bought originally.

$(x - 200)\left(\dfrac{15\,000}{x} + 10\right) - 15\,000 = 5000$

$15\,000 + 10x - \dfrac{3\,000\,000}{x} - 2000 - 15\,000 = 5000$

$10x^2 - 3\,000\,000 - 7000x = 0$

$x^2 - 700x - 300\,000 = 0$

$(x + 300)(x - 1000) = 0$

$x = -300$ (NA) or $x = 1000$

$\therefore$ 1000 apples were bought originally.

(d) Let the cost be \$$x$.

$x \times \dfrac{85}{100} = 209.10$

$x = 246$

$\therefore$ He must sell it for $\$246 \times \dfrac{105}{100} = \$258.30$.

24. **(a)** Simple interest $= \dfrac{5600 \times 2.85 \times 4}{100}$

$= \$638.40$ (to 2 d.p.)

(b) Interest earned $= 5600\left(1+\dfrac{2.79}{100}\right)^4 - 5600$

$= \$651.60$ (to 2 d.p.)

25. **(a)** Property tax payable $= \$13\,500 \times \dfrac{4}{100}$

$= \$540$

(b) Income liable for tax

$= 34\,500 - 3000 - 3(2000) - 6380$

$= \$19\,120$

$\therefore$ Tax payable

$= (8000 \times 0.03) + (19\,120 - 8000) \times 0.05$

$= \$796$

26. **(a)** Let $x be the selling price of each book.

$48 \times x - 48 \times 18.50 = 264$

$x = 24$

$\therefore$ Each book was sold at $24.

(b) $\text{Cost price} \times \left(1 + \frac{45}{100}\right) = 174$

$\therefore$ Cost price $= \frac{174}{1.45} = \$120.$

27. **(a)** Let $x be the amount Jun Wei gets.

$\therefore$ Devi gets $\$(x - 4)$ and Khairul gets $\$(x - 4 - 1)$

$= \$(x - 5)$.

$x + x - 4 + x - 5 = 42$

$3x - 9 = 42$

$3x = 51$

$x = 17$

$\therefore$ Jun Wei gets $17.

(b) Simple interest $= \dfrac{680 \times 1.25 \times \frac{10}{12}}{100}$

$= \$7.08$ (to 3 d.p.)

(c) 30 months = 2.5 years

Interest rate compounded half-yearly = 1.3%

Compound interest $= 2800\left(1 + \frac{1.3}{100}\right)^5 - 2800$

$= \$186.79$

28. **(a)** $225 = 200\left(1 + \frac{x}{100}\right)$

$1.125 = 1 + \frac{x}{100}$

$x = 100(0.125)$

$= 12.5$

(b) New salary $= 2400 \times 1.08$

$= \$2592$

New expenditure $= (2400 - 350) \times 1.09$

$= \$2234.50$

He was able to save $\$(2592 - 2234.50) = \$357.50$

29. **(a)** $t = \dfrac{100 \times I}{PR}$

$= \dfrac{100 \times 337.50}{2500 \times 3}$

$= 4\frac{1}{2}$ years

(b) S$1 = RM$3.09

RM$2880 $= \text{S\$}\,\frac{2880}{3.09}$

$= \text{S\$}932.04$ (to nearest cent)

(c) After 1st month $= 50\,000\left(1 + \frac{1}{100}\right) - 4200$

$= 46\,300$

After 2nd month $= 46\,300\left(1 + \frac{1}{100}\right) - 4200$

$= 42\,563$

After 3rd month $= 42\,563\left(1 + \frac{1}{100}\right) - 4200$

$= 38\,788.63$

$= \$38\,788.60$ (to nearest 10 cents)

30. **(a)** $3412.50 = \dfrac{42\,000 \times R \times \frac{30}{12}}{100}$

$R = \dfrac{341\,250}{42\,000 \times 2.5} = 3.25$

$\therefore$ Rate is 3.25% per annum.

(b) $T = \dfrac{100 \times (14\,490 - 12\,000)}{12\,000 \times 3\frac{3}{4}}$

= 5 years 6 months and 12 days

31. Total amount $= 7600\left(1 + \frac{4.8}{100}\right)^8$

$= \$8494.10$ (to nearest 10 cents)

32. **(a)** $\text{Marked price} \times \frac{110}{100} = 1200 \times \frac{110}{100}$

$\therefore$ Marked price $= \dfrac{1200 \times 1.1}{0.8}$

$= \$1650$

(b) Number of pencils left $= 1440 - 40 \times 12 - 30 \times 12$

$= 600$

$1440 \times 0.25 \times \dfrac{116\frac{2}{3}}{100}$

$= 40 \times 3.60 + 30 \times 4.20 + 600 \times x$ cents

$x = \dfrac{420 - 144 - 126}{600}$

$= 0.25$

$\therefore$ He must sell the remaining pencils at 25 cents each.

33. **(a)** Amount $= 50\,000\left(1 + 7.6 \times \frac{100}{4}\right)^8$

$= \text{A\$}58\,125.07$

(b) Amount of S$ he got $= 58\,125.07 \times 0.9876$

$= \text{S\$}57\,404.32$

Amount of S$ he paid for $= 50\,000 \times 1.2789$

$= \text{S\$}63\,945$

$\therefore$ He lost a total of $63\,945 - 57\,404.32$

$= \text{S\$}6540.50$ (to nearest 50 cents)

34. **(a)** Number of days $= \dfrac{92}{4} = 23$

(b) On Wednesday, 3rd December 2008

(c) After 2^{nd} week, $92 - 40 = 52$ man-days are needed.

(d) Number of days needed now $= \dfrac{52}{6}$

$$= 8\frac{2}{3}$$

$\therefore$ Number of days less needed $= 13 - 8\dfrac{2}{3}$

$$= 4\frac{1}{3}$$

35. **(a)** Amount of salt $= 6 \times 0.4$

$$= 2.4 \text{ kg}$$

Let x kg be the amount of salt to add.

$$\frac{2.4 + x}{6 + x} = \frac{6}{10}$$

$$36 + 6x = 24 + 10x$$

$$12 = 4x$$

$$x = 3$$

$\therefore$ 3 kg of salt is needed.

(b) Percentage $= \dfrac{920 \times 0.55 + 880 \times 0.45}{1800}$

$= 50.11\%$ (to 2 d.p.)

36. Number of months = 10

Amount needed to pay $= 4500\left(1 + \dfrac{2}{100}\right)^{10}$

$= \$5485.47$ (to 2 d.p.)

37. **(a)** Cost $\times \dfrac{125}{100} = \$24$

$\therefore$ Cost $= 24 \div 1.25$

$= \$19.20$

(b) Let x be the number of apples thrown away.

$$(240 - x)(0.35 - 0.25) = 19.80$$

$$(240 - x)(0.1) = 19.80$$

$$240 - x = 198$$

$$= 42$$

$\therefore$ Percentage of apples thrown away $= \dfrac{42}{240} \times 100\%$

$= 17.5\%$

38. Extra cost $= 100 + 24 \times 28 - 620$

$= \$152$

Percentage of cash price $= \dfrac{152}{620} \times 100\%$

$= 24.5\%$ (to 3 s.f.)

39. **(a)** Amount of £ bought $= \dfrac{200\ 000}{2.105}$

$= £95\ 011.88$ (to 2 d.p.)

(b) Total amount at the end of 3 months

$= 95\ 011.88 + \dfrac{95011.88 \times 4.5 \times 3}{100 \times 12}$

$= £96\ 080.76$ (to 2 d.p.)

(c) Amount in S\$ $= 96\ 080.76 \times 2.096$

$= \text{S}\$201\ 385.28$ (to 2 d.p.)

40. Cost price $\times 1.25 = 240$

Cost price $= \dfrac{240}{1.25}$

$= \$192$

Percentage profit sold at \$212 $= \dfrac{212 - 192}{192} \times 100\%$

$= 10.4\%$ (to 3 s.f.)

41. **(a)** Expenditure in June

$= 520 \times \dfrac{105}{100} + 120 \times \dfrac{105}{100} + 480 \times \dfrac{110}{100} + 360 \times \dfrac{75}{100}$

$= \$1470$

(b) Difference $= 520 + 120 + 480 + 300 - 1470$

$= 10$

$\therefore$ Percentage decrease $= \dfrac{10}{1480} \times 100\%$

$= 0.676\%$ (to 3 s.f.)

42. Profit per dozen $= 12 \times 15 - 150$

$= 30$ cents

$\therefore$ Percentage profit $= \dfrac{30}{150} \times 100\%$

$= 20\%$

43. **(a)** Amount in A\$ initially $= \dfrac{72\ 000}{1.263}$

$$R = \frac{5.8}{2} = 2.9$$

$$T = \frac{90}{6} = 15$$

After 90 months, amount $= \dfrac{72\ 000}{1.263}\left(1 + \dfrac{2.9}{100}\right)^{15}$

$= \text{A}\$87\ 530.57$ (to 2 d.p.)

(b) Amount in S\$ $= \dfrac{87\ 530.57}{0.848}$

$= \text{S}\$103\ 220.01$ (to 2 d.p.)

44. **(a)** Cost $\times \dfrac{80}{100} = 240$

$\therefore$ Cost $= 240 \div 0.8$

$= \$300$

$\therefore$ He must sell at $\$300 \times \dfrac{115}{100} = \$345$.

(b) 107% of price = 285

$\therefore$ 7% GST $= \dfrac{285}{107} \times 7$

$= \$18.64$

(c) Selling price $= (650 - 62) \times \dfrac{2}{14}$

$= \$84$

$\therefore$ Percentage gain $= \dfrac{84 - 70}{70} \times 100\%$

$= 20\%$

45. **(a)** Extra cost $= (160 + 24 \times 40) - 940$

$= \$180$

(b) Percentage difference $= \dfrac{180}{940} \times 100\%$

$= 19.1\%$ (to 3 s.f.)

46. **(a)** Cost $= 24 + 50 \times 0.25$

$= \$36.50$

(b) $44 = 24 + 0.25 \times x$ units

$x = 80$ units

(c) $A = 24 + 0.25n$

(d) When $n = 50$,

$A = 24 + 0.25(50)$

$= \$36.50$

When $A = 44$,

$44 = 24 + 0.25n$

$n = \dfrac{44 - 24}{0.25}$

$= 80$ units

47. **(a)** $A = 320 \times \dfrac{105}{100}$

$= \$336$

$B = 8 + 320 \times \dfrac{103}{100}$

$= \$337.60$

$\therefore$ He should choose Scheme B.

(b) $x \times \dfrac{105}{100} = 8 + x \times \dfrac{103}{100}$

$x\left(\dfrac{2}{100}\right) = 8$

$x = 400$

$\therefore$ He is earning \$400 now.

(c) $8x + 512\ 000 \times \dfrac{103}{100} = 529\ 440$

$8x = 2080$

$x = 260$

$\therefore$ There are 260 employees.

48. **(a)** Number of boxes $= \dfrac{1840 - 52}{6}$

$= 298$

(b) Profit $= 298 \times 1.85 - 350 - 298 \times 0.04$

$= \$189.38$

49. Let the original mixture be 1 kg.

$\therefore$ New weight of paint = 450 g and of turpentine is 550 g + 40×10 = 950 g

$\therefore$ Percentage of paint $= \dfrac{450}{1400} \times 100\%$

$= 32.1\%$ (to 3 s.f.)

50. Let the percentage gain/loss be x.

$$\left(540\ 000 \times \frac{100 - x}{100}\right) \times \left(\frac{100 + x}{100}\right) = 535\ 000$$

$$\frac{10\ 000 - x^2}{10\ 000} = \frac{535}{540}$$

$$= \frac{107}{108}$$

$$1\ 080\ 000 - 108x^2 = 1\ 070\ 000$$

$$x = \sqrt{\frac{10\ 000}{108}}$$

$= 9.6225$ (to 5 s.f.)

$\therefore$ Mr Ong paid $540\ 000 \times \dfrac{100 - 9.6225}{100} = \$488\ 038.50$

51. **(a)** Bill for food only $= 12 + 8 + 28 + 16 + 5(0.5 + 0.3)$

$= \$68$

(i) Bill with service charge $= 68 \times 1.1$

$= \$74.80$

(ii) Bill with service charge and GST $= 74.80 \times 1.07$

$= \$80.04$

(b) There is no difference.

Bill with GST then service charge $= (68 \times 1.07) \times 1.1$

$= \$80.04$

52. **(a)** \$11 050 = 5 shares

$\therefore$ Total profit $= \dfrac{11\ 050}{5} \times 15$

$= \$33\ 150$

(b) Simple interest $= \dfrac{288\ 000 \times 7 \times \frac{35}{12}}{100}$

$= \$58\ 800$

(c) **(i)** $33\frac{1}{3}\%$ of cost = \$5.40

$\therefore$ Cost = \$16.20

$\therefore$ Selling price $= 16.20 + 5.40$

$= \$21.60$

(ii) $\dfrac{80}{5.45} = 14.7$ (to 3 s.f.)

$\therefore$ 14 cakes can be bought.

(d) Let x be the profit in 2014.

$\therefore$ Profit in 2015 is $x \times \dfrac{118}{100} = 1.18x$

Profit in 2016 $= 1.18x\left(\dfrac{75}{100}\right)$

$= 0.885x$

$\therefore$ Percentage of profit $= \dfrac{0.885x}{x} \times 100\%$

$= 88.5\%$.

Exercise 9B

1. **(a)** $y = k(2x + 1)$, where k is constant.

When $x = 2$, $y = k(5)$

When $x = 7$, $y = k(15)$

$15k - 5k = 4$

$k = \dfrac{2}{5}$

$\therefore\ y = \dfrac{2}{5}(2x + 1)$

(b) When $y = 5$,

$5 = \dfrac{2}{5}(2x+1)$

$25 = 4x + 2$

$x = \dfrac{23}{4}$

$= 5\dfrac{3}{4}$

2. **(a)** $y = \dfrac{k}{3x+2}$, where k is constant.

When $x = 0$, $y = \dfrac{k}{2}$

When $x = 2$, $y = \dfrac{k}{8}$

$\dfrac{k}{2} - \dfrac{k}{8} = 9$

$k = 24$

$\therefore\ y = \dfrac{24}{3x+2}$

(b) When $x = 6$,

$y = \dfrac{24}{3(6)+2}$

$= 1.2$

3. **(a)** $y = \dfrac{k}{x^2 + 3}$, where k is a constant.

When $x = 1$, $y = 15$,

$15 = \dfrac{k}{1+3}$

$k = 60$

$\therefore\ y = \dfrac{60}{x^2 + 3}$

(b) When $x = 5$,

$y = \dfrac{60}{5^2 + 3}$

$= 2\dfrac{1}{7}$

4. **(a)** $v = \dfrac{k}{\sqrt{p}}$, where k is a constant.

When $p = 25$, $v = 40$,

$40 = \dfrac{k}{\sqrt{25}}$

$k = 200$

$\therefore v = \dfrac{200}{\sqrt{p}}$

(b) When $p = 36$,

$v = \dfrac{200}{\sqrt{36}}$

$= 33\dfrac{1}{3}$

5. 1 : 600 000 i.e. 1 cm represents 6 km

(a) 288 km is represented by $\dfrac{28}{6}$ cm i.e. 48 cm on the map

(b) 1 cm^2 represents $(6\text{ km})^2 = 36\text{ km}^2$

$\therefore$ Actual area of the lake $= 24 \times 36$

$= 864\text{ km}^2$

6. 4 cm^2 represents 1 km^2

$\therefore$ 2 cm represents 1 km

1 cm represents 500 m, i.e. 50 000 cm

(a) 45 km^2 is represented by $45 \times 4 = 180\text{ cm}^2$

(b) 9.2 cm represents $9.2 \times \dfrac{1}{2} = 4.6$ km

(c) 1 : 50 000

7. **(a)** $6x + 2y = 13x - 3y$

$5y = 7x$

$\dfrac{x}{y} = \dfrac{5}{7}$

$\therefore\ x : y = 5 : 7$

(b) $\dfrac{3}{4}(x - 2y) = 3y - x$

$3x - 6y = 12y - 4x$

$7x = 18y$

$\dfrac{x}{y} = \dfrac{18}{7}$

$\therefore\ x : y = 18 : 7$

8. 24 men working 9 hr/day take 45 days

(a) 18 men working 9 hr/day will take $45 \div \dfrac{18}{24} = 60$ days

18 men working 8 hr/day will take $60 \div \dfrac{8}{9} = 67\dfrac{1}{2}$ days

18 men will take $67\dfrac{1}{2}$ days to build the same house if they work 8 hr/day.

(b) 20 men working 9 h/day will take $45 \div \frac{20}{24} = 54$ days

20 men working $9 \div \frac{48}{54} = 10\frac{1}{8}$ h/day will take 48 days

$\therefore$ 20 men must work $10\frac{1}{8}$ h/day for the house to be completed in 48 days.

9. **(a)** Total time $= \frac{56}{16} + \frac{30}{18}$

$= 5\frac{1}{6}$ h

Total distance $= 56 + 30$

$= 86$ km

$\therefore$ Average speed $= \frac{86}{5\frac{1}{6}}$

$= 16\frac{20}{31}$ km/h

(b) 8 men take 12 days to repair 4 km

1 man takes $12 \times 8 = 96$ days to repair 4 km

6 men take $96 \div 6 = 16$ days to repair 4 km

6 men take $16 \times \frac{3}{4} = 12$ days to repair 3 km

$\therefore$ It will take 6 men 12 days to repair a road of length 3 km.

10. JW : K : L = 7 : 8 : 6

1 unit = 18 sweets

(a) Total number of sweets $= 18 \times (7 + 8 + 6)$

$= 378$

(b) Khairul receives $8 \times 18 = 144$ sweets

11. A : B : C = 5 : 9 : 7

2 units = 12 sweets

(a) Original number $= \frac{12}{2}\ (5 + 9 + 7)$

$= 126$ sweets

(b) Basu receives $6 \times 9 = 54$ sweets

12. J : B : H = 2 : 3: 11

(a) 1 unit = \$2.50

$\therefore$ Original sum $= 2.50(2 + 3 + 11)$

= \$40

(b) John has 2×2.50 = \$5, Bala has 3×2.50 = \$7.50

John now has \$5 + \$5 = \$10

J : B = 10 : 7.50

= 4 : 3

13. **(a)** Average speed $= \frac{3 \times 6 + 2 \times 8}{3 + 2}$

$= 6.8$ km/h

(b) 6.8 km/h $= \frac{6.8 \times 1000}{60 \times 60}$ m/s

$= 1\frac{8}{9}$ m/s

14. **(a)** Distance moved $= 286 \times \frac{8}{3600}$ km

$= \frac{143}{225}$ km

$= 635\frac{5}{9}$ m

$635\frac{5}{9} = \frac{\theta}{360} \times 2\pi \times 364$

$\therefore \theta = 100.0°$ (to 1 d.p.)

(b) Average speed $= \frac{55 \times 2 + 70 \times 3}{2 + 3}$

$= 64$ km/h

15. **(a)** $A : B : C = 5 : 7 : 13$

2 units = \$5200

$\therefore$ C receives $13 \times \frac{5200}{2}$ = \$33 800

(b) Average speed $= \frac{(200 + 160)\text{ km}}{\left(\frac{200}{60} + \frac{160}{48}\right)\text{h}}$

$= 54$ km/h

16. **(a)** $\frac{8 \times 60}{20}$ articles = 24 articles

(b) 72 boys need 120 loaves of bread for 5 days

1 boy needs $120 \div 72 = \frac{5}{3}$ loaves for 5 days

$72 + 12 = 84$ boys need $\frac{5}{3} \times 84$

$= 140$ loaves for 5 days

84 boys need $\frac{140}{5} \times 4 = 112$ loaves for 4 days

$\therefore$ 112 loaves of bread are needed for the next 4 days.

17. **(a)** $y = kx$, where k is a constant.

When $x = 3, y = 21$,

$21 = k(3)$

$k = 7$

$\therefore y = 7x$

(b) **(i)** When $x = 2\frac{2}{7}$,

$y = 7 \times 2\frac{2}{7}$

$= 16$

(ii) When $y = 42.7$,

$42.7 = 7x$

$x = 6.1$

18. **(a)** $p = k(nq + 1)$, where k is a constant.

When $q = 1, p = 16$,

$16 = k[n(1) + 1]$

$16 = k(n + 1)$ — (1)

When $q = 2, p = 28$,

$28 = k[n(2) + 1]$

$28 = k(2n + 1)$ — (2)

$\frac{(2)}{(1)} : \frac{28}{16} = \frac{2n+1}{n+1}$

$28n + 28 = 32n = 16$

$12 = 4n$

$\therefore n = 3$

(b) When $q = 1, p = 16$,

$16 = k[3(1) + 1]$

$k = 4$

$\therefore p = 4(3q + 1)$

When $q = 3$,

$p = 4[3(3) + 1]$

$= 40$

19. **(a)** $y = \frac{k}{x^3 - 1}$, where k is a constant.

When $x = 2, y = 28$,

$28 = \frac{k}{2^3 - 1}$

$k = 196$

$\therefore y = \frac{196}{x^3 - 1}$

When $x = 1.5$,

$y = \frac{196}{1.5^3 - 1}$

$= 82\frac{10}{19}$

(b) $R = \frac{k}{r^2}$, where k is a constant.

When $R = 0.45, r = 0.4$,

$0.45 = \frac{k}{0.4^2}$

$k = 0.072$

$\therefore R = \frac{0.072}{r^2}$

When $r = 0.6$,

$R = \frac{0.072}{0.6^2}$

$= 0.2$

$\therefore$ The resistance when the radius is 0.4 cm is 0.2 ohms.

(c) $I = \frac{k}{d^2}$, where k is a constant.

When $d = 4, I = 4\frac{1}{2}$,

$4\frac{1}{2} = \frac{k}{4^2}$

$k = 72$

$\therefore I = \frac{72}{d^2}$

When $d = 6$,

$I = \frac{72}{6^2}$

$= 2$

$\therefore$ The illumination at a distance of 6 m is 2 candle-power.

20. 1 : 25 000, i.e. 1 cm represents 250 m.

(a) 8 cm represents 8×250 m = 2 km

(b) Length of side = $\frac{2 \text{ km}}{4} = \frac{1}{2}$ km

Area of lake = $\left(\frac{1}{2}\right)^2$ km^2 = $\frac{1}{4}$ km^2

21. **(a)** 1 : 20 000, i.e. 1 cm represents 200 m or $\frac{1}{5}$ km.

1 cm^2 represents $\left(\frac{1}{5}\right)^2$ km^2 = $\frac{1}{25}$ km^2

40 cm^2 represents $\frac{40}{25}$ km^2

1 : 40 000, i.e. 1 cm represents $\frac{2}{5}$ km

$\therefore$ 1 km^2 is represented by $\frac{25}{4}$ cm^2

$\frac{40}{25}$ km^2 is represented by $\frac{40}{25} \times \frac{25}{4} = 10$ cm^2

Or

$\therefore$ Area in new map $= 40 \times \frac{25}{4} \times \frac{1}{25}$

$= 10$ cm^2

(b) 1 : 75 000, i.e. 1 cm represents $\frac{3}{4}$ km.

1 cm^2 represents $\left(\frac{3}{4}\right)^2$ km^2 = $\frac{9}{16}$ km^2

$\therefore$ 54 km^2 is represented by $54 \times \frac{16}{9}$

= 96 cm^2 on the map.

22. 1 cm represents 250 m or 1 cm represents $\frac{1}{4}$ km.

(a) 13.5 km is represented by $13.5 \div \frac{1}{4} = 54$ cm

(b) 1 cm^2 represents $\left(\frac{1}{4}\right)^2$ km^2

$\therefore$ 240 cm^2 represents $240 \times \frac{1}{16} = 15$ km^2

23. $y = \dfrac{k}{2x+3}$, where k is a constant.

(a) When $x = 1$,

$$y = \frac{k}{2(1)+3} = \frac{k}{5}$$

When $x = 3$,

$$y = \frac{k}{2(3)+3} = \frac{k}{9}$$

$$\frac{k}{5} - \frac{k}{9} = 4$$

$$4k = 180$$

$$k = 45$$

$$\therefore y = \frac{45}{2x+3}$$

(b) When $x = 7$,

$$y = \frac{45}{2(7)+3} = 2\frac{11}{17}$$

24. **(a)** Let C represent copper, T represent tin and Z represent zinc.

$C : T = 3 : 5$, $T : Z = 3 : 7$

$$C : T : Z = \frac{3}{8} : \left(\frac{5}{8} + \frac{3}{10}\right) : \frac{7}{10} = 15 : 37 : 28$$

(b) Weight of $Z = \dfrac{28}{15} \times 90$

$= 168$ kg

25. $T = k\sqrt{l}$, where k is a constant.

When $l = 8$, $T = 3.1$,

$$3.1 = k\sqrt{8}$$

$$k = \frac{3.1}{\sqrt{8}}$$

$$\therefore T = \frac{3.1}{\sqrt{8}}\sqrt{l}$$

When $l = 10$,

$$T = \frac{3.1}{\sqrt{8}}\sqrt{10}$$

$= 3.47$ s (to 3 s.f.)

26. 4 cm : x km

$\therefore$ 16 cm^2 : x^2 km^2

52 cm^2 represent $\dfrac{x^2}{16} \times 52 = 3\dfrac{1}{4}\,x^2$ km^2

27. 1 : x, i.e. 1 cm^2 represents x^2 cm^2

40 cm^2 represents 144 000 m^2

1 cm^2 represents 3600 m^2

1 cm represents 60 m = 6000 cm

$\therefore x = 6000$

28. **(a)** 19 46

(b) 20 30

(c) 1 hour

(d) Time taken = 19 58 – 18 48

= 1 h 10 min

(i) Speed $= \dfrac{42 \text{ km}}{1\frac{1}{6} \text{ h}}$

= 36 km/h

(ii) Speed in m/s $= \dfrac{36 \times 1000 \text{ m}}{60 \times 60 \text{ s}}$

= 10 m/s

29. **(a)** Total fees $= 5 \times 10.50 + 15 \times 6.50 + 2 \times 7.50$

= \$240

(b) Total fees $= 5 \times 8.50 + 8 \times 5.50 + 16 \times 6.50$

= \$190.50

30. **(a)** In a day, John paints $\dfrac{1}{15}$, Peter paints $\dfrac{1}{x}$ of a house.

$$\frac{1}{15} + \frac{1}{x} = \frac{1}{10}$$

$\therefore x = 30$

(b) Let the distance be d m.

$$\text{Total time} = \frac{d}{u} + \frac{d}{v} = \frac{d(u+v)}{uv}$$

$$\text{Average speed} = \frac{2d}{\frac{d(u+v)}{uv}} = \frac{2uv}{u+v} \text{ m/s}$$

31. **(a)** In one day, Ahmad can do $\dfrac{1}{6}$, Ali can do $\dfrac{1}{3}$ of work.

Together they can do $\dfrac{1}{6} + \dfrac{1}{3} = \dfrac{1}{2}$ of work.

$\therefore$ They will take 2 days if they work on it together.

(b) A worked 32 man-days,

B worked 36 man-days,

C worked 40 man-days.

$\therefore$ A is paid \$675 $\times \dfrac{32}{32 + 36 + 40} = \$200$

32. $V = \dfrac{X}{T}$ and $v = \dfrac{x}{t}$

$$\frac{X}{x} = \frac{VT}{vt} = \frac{V}{v}\left(\frac{T}{t}\right)$$

$$\frac{5}{2} = \frac{7}{4}\left(\frac{T}{t}\right)$$

$$\frac{T}{t} = \frac{5 \times 4}{2 \times 7} = \frac{10}{7}$$

$\therefore T : t = 10 : 7$

33. $A = kd^2$, i.e. $\dfrac{A_1}{A_2} = \left(\dfrac{d_1}{d_2}\right)^2$

(a) $\dfrac{A_1}{7.06 \times 10^9} = \left(\dfrac{2.95}{1}\right)^2$

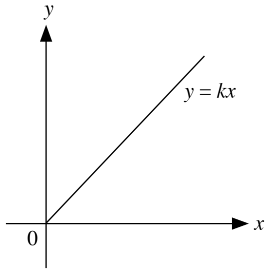
$\therefore$ Surface area of Jupiter $= 7.06 \times 10^9 \times 2.95^2$

$= 6.1 \times 10^{10}$ km^2

(b) $\dfrac{5.2 \times 10^8}{3.8 \times 10^7} = \left(\dfrac{n}{1}\right)^2$

$\therefore n = \sqrt{\dfrac{52}{3.8}} = 3.7$ (to 1 d.p.)

34. $y = kx^n$



(a) **(i)** $n = 3$

(ii) $n = -\dfrac{1}{2}$

(iii) $n = \dfrac{1}{4}$

(b) $y = kx^2$

When $x = \dfrac{1}{4}$, $y = \dfrac{3}{4}$,

$\dfrac{3}{4} = k\left(\dfrac{1}{4}\right)^2$

$= 12$

$\therefore y = 12x^2$

When $x = 2.5$,

$y = 12(2.5)^2 = 75$

35. 1 : 20 000

(a) $PQ = 1.35 \times 20\,000$ cm

$= 270$ m

(b) Angle of elevation $= \tan^{-1} \dfrac{315}{270}$

$= 49.4°$ (to 1 d.p.)

36. **(a)** Amount $= \$320 + (435 - 300) \times \dfrac{50}{100}$

$= \$387.50$

(b) Amount $= \$60 + 450 \times 0.7$

$= \$375$

(c) $320 + (x - 300) \times \dfrac{1}{2} = 60 + x \times \dfrac{7}{10}$

$$170 + \frac{1}{2}x = 60 + \frac{7}{10}x$$

$$x = \frac{110}{\frac{7}{10} - \frac{1}{2}}$$

$= 550$

$\therefore$ The cost will be the same for 550 km travelled.

37. $25^x = 125^y$

$5^{2x} = 5^{3y}$

$\therefore 2x = 3y$

$4^x = 16^z = 4^{2z}$

$\therefore x = 2z$

$x : y = 3 : 2$

$= 6 : 4$

$x : z = 2 : 1$

$= 6 : 3$

$\therefore x : y : z = 6 : 4 : 3$

Exercise 9C

1. **(a)** $3x - 2(x - 3) + 3(5 - 4x) = 3x - 2x + 6 + 15 - 12x$

$= 21 - 11x$

(b) $\dfrac{1}{2x} + \dfrac{3}{4x} - \dfrac{5}{6x} = \dfrac{6 + 9 - 10}{12x}$

$= \dfrac{5}{12x}$

(c) $\dfrac{x}{3} + \dfrac{x}{4} + \dfrac{x}{5} = x\left(\dfrac{1}{3} + \dfrac{1}{4} + \dfrac{1}{5}\right)$

$= \dfrac{47x}{60}$

(d) $\dfrac{x+1}{3} - \dfrac{x}{6} - \dfrac{x-2}{4} = \dfrac{4x + 4 - 2x - 3x + 6}{12}$

$= \dfrac{10 - x}{12}$

(e) $\dfrac{2x-1}{3} - \dfrac{x+5}{4} = \dfrac{8x - 4 - 3x - 15}{12}$

$= \dfrac{5x - 19}{12}$

(f) $\dfrac{3(a-b)}{5} - \dfrac{3a + 4b}{10} + \dfrac{a - b}{2}$

$= \dfrac{6a - 6b - 3a - 4b + 5a - 5b}{10}$

$= \dfrac{8a - 15b}{10}$

(g) $\frac{x+y}{2} - \frac{x-y}{3} + \frac{2(3x-2y)}{3}$

$= \frac{3x+3y-2x+2y+12x-8y}{6}$

$= \frac{13x-3y}{6}$

(h) $\frac{5}{x-y} - \frac{4}{y-x} = \frac{5}{x-y} + \frac{4}{x-y}$

$= \frac{9}{x-y}$

2. **(a)** $\frac{8a^2b^3}{3c^3} \div \frac{4a^3b^2}{9bc} = \frac{8a^2b^3}{3c^3} \times \frac{9bc}{4a^3b^2}$

$= \frac{6b^2}{ac^2}$

(b) $\frac{4xy^3}{12x^2yz} \div \frac{9x^3}{6y^2z^3} = \frac{4xy^3}{12x^2yz} \times \frac{6y^2z^3}{9x^3}$

$= \frac{2y^4z^2}{9x^4}$

(c) $\frac{m^2+2m-3}{m^2+8m+15} = \frac{(m-1)(m+3)}{(m+3)(m+5)}$

$= \frac{m-1}{m+5}$

(d) $\frac{x}{x-2} + \frac{1}{x+2} = \frac{x(x+2)+(x-2)}{(x-2)(x+2)}$

$= \frac{x^2+3x-2}{(x-2)(x+2)}$

(e) $\frac{1}{x+1} - \frac{1}{2x-1} = \frac{2x-1-(x+1)}{(x+1)(2x-1)}$

$= \frac{x-2}{(x+1)(2x-1)}$

(f) $\frac{1}{x+2} + \frac{1}{x^2+3x+2} = \frac{1}{x+2} + \frac{1}{(x+1)(x+2)}$

$= \frac{(x+1)+1}{(x+1)(x+2)}$

$= \frac{1}{x+1}$

(g) $\frac{2x}{x+7} - \frac{5x}{x-3} = \frac{2x(x-3)-5x(x+7)}{(x+7)(x-3)}$

$= \frac{-3x^2-41x}{(x+7)(x-3)}$

(h) $\frac{3}{x-2} + \frac{1}{x+2} - \frac{2x-5}{x^2-4}$

$= \frac{3(x+2)+(x-2)-2x+5}{(x+2)(x-2)}$

$= \frac{2x+9}{(x+2)(x-2)}$

3. **(a)** $T = \frac{k}{y}(x-y)$

When $k = 3$, $y = 5$ and $x = 7$,

$T = \frac{3}{5}(7-5)$

$= 1\frac{1}{5}$

(b) Given that $a = 2b$,

$\frac{2a+7b}{25b-2a} = \frac{2(2b)+7b}{25b-2(2b)}$

$= \frac{11}{21}$

(c) $x^2 - y^2 = 48$

$(x+y)(x-y) = 48$

$4(x-y) = 48$

$(x-y) = 12$

$\therefore 5(x-y) = 5(12) = 60$

(d) $\frac{2}{a} + \frac{2}{b} = \frac{2(b+a)}{ab}$

$= \frac{2(5)}{4}$

$= 2\frac{1}{2}$

(e) $\frac{2x+3y}{2x-3y} = \frac{5}{3}$

$6x + 9y = 10x - 15y$

$24y = 4x$

$\frac{x}{y} = 6$

$\therefore \frac{3x}{5y} = \frac{3}{5} \times \frac{x}{y}$

$= \frac{3}{5}(6)$

$= 3\frac{3}{5}$

4. **(a)** $(-2)^{-2} + \left(\frac{1}{8}\right)^{\frac{1}{3}} + 2^{-3} = \frac{1}{(-2)^2} + \sqrt[3]{\frac{1}{8}} + \frac{1}{2^3}$

$= \frac{1}{4} + \frac{1}{2} + \frac{1}{8}$

$= \frac{7}{8}$

(b) $6 = \sqrt{2^2 + c^2 + (-17)}$

$36 = 4 + c^2 - 17$

$c^2 = 36 + 17 - 4$

$= 49$

$\therefore c = -7$ (since $c < 0$)

(c) $12(3)(3) + 42(3)(3) - 8(3)(-2) + 13(-2)(-2)$

$= 586$

5. **(a)** $4x^7 \times 7x^{-4} = 28x^3$

(b) $7p^5 \times 2p^{-\frac{1}{2}} = 14p^{4\frac{1}{2}}$

(c) $(2x^3)^4 \div (4\sqrt{x})^3 = \dfrac{2^4 x^{12}}{4^4 x^{\frac{3}{2}}}$

$= \dfrac{x^{10\frac{1}{2}}}{4}$

(d) $(5a^3b^2)^2 \div \dfrac{b^2}{a^2} = 25a^6b^4 \times \dfrac{b^2}{a^2}$

$= 25a^4b^6$

(e) $(2x)^3 \div (3x)^2 = \dfrac{8x^3}{9x^2}$

$= \dfrac{8x}{9}$

(f) $\left(\dfrac{1}{2}x\right)^4 \times (4x)^3 = \dfrac{1}{16}x^4 \times 64x^3$

$= 4x^7$

6. **(a)** $27^{-\frac{2}{3}} \times 8^{\frac{1}{3}} = \dfrac{1}{(\sqrt[3]{27})^2} \times \sqrt[3]{8}$

$= \dfrac{1}{9} \times 2$

$= \dfrac{2}{9}$

(b) $5.3^0 + 16^{-\frac{3}{4}} = 1 + \dfrac{1}{(\sqrt[4]{16})^3}$

$= 1 + \dfrac{1}{8}$

$= 1\dfrac{1}{8}$

(c) $8^{-\frac{2}{3}} + 32^{\frac{1}{2}} = \dfrac{1}{(\sqrt[3]{8})^2} + \sqrt[5]{32}$

$= \dfrac{1}{2} + 2$

$= 2\dfrac{1}{2}$

(d) $\left(2\dfrac{3}{4}\right)^{-1} \div 16^{-1.25} = \dfrac{1}{\frac{11}{4}} \div \dfrac{1}{(\sqrt[4]{16})^5}$

$= \dfrac{4}{11} \div \dfrac{1}{32}$

$= 11\dfrac{7}{11}$

(e) $27^{\frac{2}{3}} + 81^{\frac{3}{4}} \times 16^{\frac{1}{2}} = (\sqrt[3]{27})^2 + (\sqrt{16})^3 \times \sqrt{16}$

$= 9 + 27 \times 4$

$= 117$

(f) $4^{-11} \times 4^9 \div \left(\dfrac{8}{27}\right)^{-1\frac{1}{3}} = 4^{-2} \times \left(\sqrt[3]{\dfrac{27}{8}}\right)^4$

$= \dfrac{1}{4^2} \times \left(\dfrac{3}{2}\right)^4$

$= \dfrac{1}{16} \times \dfrac{81}{16}$

$= \dfrac{81}{256}$

(g) $3^{\frac{1}{3}} \times 12^{\frac{2}{3}} \times 4^{\frac{1}{3}} = (3 \times 4)^{\frac{1}{3}} \times 12^{\frac{2}{3}}$

$= 12^1$

$= 12$

7. **(a)** $y = \dfrac{5x+3}{6-7x}$

$5x + 3 = 6y - 7xy$

$x(5 + 7y) = 6y - 3$

$\therefore x = \dfrac{6y-3}{7y+5}$

(b) $\dfrac{w}{k} = \dfrac{2x}{a-x}$

$2xk = wa - wx$

$x(2k + w) = wa$

$\therefore x = \dfrac{wa}{2k+w}$

(c) $A = \dfrac{p(1+3x)}{2x-k}$

$2Ax - Ak = p + 3px$

$x(2A - 3p) = p + Ak$

$x = \dfrac{p+Ak}{2A-3p}$

(d) $p = \dfrac{q-x}{1+qx}$

$p + pqx = q - x$

$x(pq + 1) = q - p$

$\therefore x = \dfrac{q-p}{pq+1}$

(e) $y = \sqrt{\dfrac{x}{x+2}}$

$y^2 = \dfrac{x}{x+2}$

$x = xy^2 + 2y^2$

$x(1 - y^2) = 2y^2$

$\therefore x = \dfrac{2y^2}{1-y^2}$

(f) $y = \dfrac{x}{2a-3x}$

$x = 2ay - 3xy$

$x(1 + 3y) = 2ay$

$\therefore x = \dfrac{2ay}{1+3y}$

(g) $T = \dfrac{y(x-a)}{x-b}$

$Tx - Tb = yx - ya$

$x(T - y) = Tb - ya$

$\therefore x = \dfrac{Tb - ya}{T - y}$

(h) $y = \sqrt{\dfrac{2x + y}{3x - 5}}$

$y^2 = \dfrac{2x + y}{3x - 5}$

$3xy^2 – 5y^2 = 2x + y$

$x(3y^2 - 2) = y + 5y^2$

$\therefore x = \dfrac{y + 5y^2}{3y^2 - 2}$

(i) $t = 2\pi\sqrt{\dfrac{x^2 + g^2}{x^2}}$

$t^2 = \dfrac{4\pi^2(x^2 + g^2)}{x^2}$

$x^2t^2 = 4\pi^2x^2 + 4\pi^2g^2$

$x^2(t^2 - 4\pi^2) = 4\pi^2g^2$

$x^2 = \dfrac{4\pi^2g^2}{t^2 - 4\pi^2}$

$x = \dfrac{2\pi g}{\sqrt{t^2 - 4\pi^2}}$

(j) $\dfrac{y}{2} = \sqrt[3]{\dfrac{x + 5}{2x + 1}}$

$\dfrac{y^3}{8} = \dfrac{x + 5}{2x + 1}$

$8x + 40 = 2xy^3 + y^3$

$x(8 - 2y^3) = y^3 - 40$

$x = \dfrac{y^3 - 40}{8 - 2y^3}$

8. **(a)** **(i)** $x = \dfrac{y + b}{y + 5c} = \dfrac{3 + 4(-2)}{3 + 5(1)} = -\dfrac{5}{8}$

(ii) $xy + 5cx = y + 4b$

$y(x - 1) = 4b - 5cx$

$y = \dfrac{4b - 5cx}{x - 1}$

(b) **(i)** $a = \sqrt{\dfrac{b(c + d)}{c}} = \sqrt{\dfrac{-8(-1 + 9)}{-1}} = \sqrt{64} = 8$

(ii) $a^2 = \dfrac{bc + bd}{c}$

$a^2c = bc + bd$

$c(a^2 - b) = bd$

$\therefore c = \dfrac{bd}{a^2 - b}$

9. **(a)**
$$\begin{array}{r|rrrr} 2 & 120 & 168 & 288 & 360 \\ \hline 2 & 60 & 84 & 144 & 180 \\ \hline 2 & 30 & 42 & 72 & 90 \\ \hline 3 & 15 & 21 & 36 & 45 \\ \hline & 5 & 7 & 12 & 15 \end{array}$$

Greatest possible length = $2^3 \times 3 = 24$ cm

(b) Number of pieces = $24 \times 5 \times 7 \times 12 \times 15$

= 151 200

10. **(a)** 4, 5, 7, 10, 14, 19, $\underline{25}$, $\underline{32}$

(b) 2, 5, 10, 17, 26, 37, $\underline{50}$, $\underline{65}$

(c) 3, 4, 6, 10, 18, 34, $\underline{66}$, $\underline{130}$

11. **(a)** 2, 6, 12, 20, 30, 42, …

$\therefore T_n = n(n + 1)$

(b) 3, 6, 11, 20, 37, 70, …

$\therefore T_n = n + 2^n$

(c) 2, 3, 5, 9, 17, 33, …

$\therefore T_n = 1 + 2^{n-1}$

12. **(a)** 3, 5, 9, 15, 23, $\underline{33}$, $\underline{45}$

(b) 15, 18, 16, 19, 17, $\underline{20}$, $\underline{18}$

(c) $\dfrac{4}{5}, \dfrac{3}{4}, \dfrac{8}{11}, \dfrac{5}{7}, \dfrac{12}{17}$

$= \dfrac{4}{5}, \dfrac{6}{8}, \dfrac{8}{11}, \dfrac{10}{14}, \dfrac{12}{17}, \dfrac{14}{20} = \underline{\dfrac{7}{10}}, \underline{\dfrac{16}{23}}$

13. **(a)** $7^2 - 6^2 = 7 + 6$

$8^2 - 7^2 = 8 + 7$

(b) $156^2 - 155^2 = 156 + 155 = 311$

(c) $10^2 - 11^2 + 12^2 - 13^2 + 14^2 - 15^2 + 16^2$

$= 16^2 - 15^2 + 14^2 - 13^2 + 12^2 - 11^2 + 10^2$

$= 16 + 15 + 14 + 13 + 12 + 11 + 100$

$= 181$

14. **(a)** **(i)**
$$\begin{array}{r|ll} xy & 3x^2y & 2xy^3 \\ \hline & 3x & 2y^2 \end{array}$$

$\therefore$ LCM $= xy(3x)(2y^2)$

$= 6x^2y^3$

(ii)
$$\begin{array}{r|rrr} 4b & 4a^2bc & 8a^3b^2 & 12bc^3 \\ \hline a^2c & a^2c & 2a^3b & 3c^3 \\ \hline & 1 & 2ab & 3c^2 \end{array}$$

$\therefore$ LCM $= 4a^2bc(2ab)3c^2$

$= 24a^3b^2c^3$

(b) **(i)** $n^4 - 4n^2m^2 = n^2(n^2 - 4m^2)$

$= n^2(n + 2m)(n - 2m)$

$m^2n + 2m^3 = m^2(n + 2m)$

(ii) HCF $= (n + 2m)$

15. $\dfrac{2x - 3y}{2y - 5x} = \dfrac{2(2t - 4) - 3(3t + 5)}{2(3t + 5) - 5(2t - 4)}$

$= \dfrac{-5t - 23}{-4t + 30}$

16. (a) $7A - 3B = 7(2x^2 - 3x + 5) - 3(7x^2 - 4x + 9)$

$= 8 - 9x - 7x^2$

(b) Let Peter be x years old and Jane be $(36 - x)$ years old 6 years ago,

Peter's age = $(x - 6)$ years old

Jane's age = $(36 - x - 6)$

$= (30 - x)$ years old

$x - 6 = 2(30 - x)$

$3x = 66$

$x = 22$

i.e. Jane is 14 years old now.

$\therefore$ In 5 years' time, Peter will be 27 years old and Jane will be 19 years old.

17. (a)
$$u = \frac{v}{6uw^2 - 1}$$
$$6u^2w^2 - u = v$$
$$w^2 = \frac{v + u}{6u^2}$$
$$\therefore w = \pm\sqrt{\frac{v + u}{6u^2}}$$

(b)
$$k = \sqrt{\frac{m(y + z)}{4}}$$
$$k^2 = \frac{my + mz}{4}$$
$$my + mz = 4k^2$$
$$mz = 4k^2 - my$$
$$\therefore z = \frac{4k^2 - my}{m}$$

(c)
$$p = \frac{kb - a}{k - b}$$
$$pk - bp = kb - a$$
$$k(p - b) = bp - a$$
$$k = \frac{bp - a}{p - b}$$

(d)
$$x^2 = \sqrt{\frac{a + 2y}{b - 3y}}$$
$$x^4 = \frac{a + 2y}{b - 3y}$$
$$a + 2y = bx^4 - 3x^4y$$
$$y(2 + 3x^4) = bx^4 - a$$
$$y = \frac{bx^4 - a}{3x^4 + 2}$$

18. (a) $4 - 16x^2 = 4(1 - 4x^2) = 4(1 + 2x)(1 - 2x)$

(b) $2a - 2b + 3c(b - a) = 2(a - b) - 3c(a - b)$

$= (a - b)(2 - 3c)$

(c) $x^3 + x^2 - 6x = x(x^2 + x - 6)$

$= x(x + 3)(x - 2)$

(d) $4b^2 - 6b + 6bk - 9k = 2b(2b - 3) + 3k(2b - 3)$

$= (2b - 3)(2b + 3k)$

(e) $9t^2 + 18t - 16 = (3t + 8)(3t - 2)$

(f) $6a^2 - 3a - 30 = 3(2a^2 - a - 15)$

$= 3(2a + 5)(a - 3)$

(g) $6t^2 - 18t = 6t(t - 3)$

(h) $6ax - 3by - 6ay + 3bx = 6a(x - y) + 3b(x - y)$

$= 3(2a + b)(x - y)$

(i) $x^2y^2 - 15xy + 56 = (xy - 7)(xy - 8)$

(j) $(x + 2y)^2 - 2(x + 2y) - 15 = [(x+2y)+3][(x+2y)-5]$

$= (x + 2y + 3)(x + 2y - 5)$

19. (a) (i) $a^2 - 2ab + b^2 = 19$ — (1)

$a^2 + 2ab + b^2 = 37$ — (2)

$(1) - (2) : 4ab = 18$

$\therefore 8ab = 36$

(ii) $(1) + (2) : 2a^2 + 2b^2 = 56$

$$\therefore 3a^2 + 3b^2 = \frac{56}{2} \times 3$$

$= 84$

(b) $(x + y)(x - y) = 42$

Since $x - y = 7$,

$7(x + y) = 42$

$\therefore x + y = 6$

$4x + 4y = 24$

(c) $3(x - y)^2 = 3(x^2 + y^2 - 2xy)$

$= 3[84 - 2(14)]$

$= 168$

20. (a) $2(x + 5) + x^2 - 25 = 2(x + 5) + (x + 5)(x - 5)$

$= (x + 5)(2 + x - 5)$

$= (x + 5)(x - 3)$

(b) $4x^3 + 4x^2 - 3x = x(4x^2 + 4x - 3)$

$= x(2x + 3)(2x - 1)$

(c) $x^3 + x^2 - 1 - x = x^2(x + 1) - (1 + x)$

$= (x + 1)(x^2 - 1)$

$= (x + 1)(x + 1)(x - 1)$

$= (x + 1)^2(x - 1)$

(d) $x^2 - 2x + 2xy - 4y = x(x - 2) + 2y(x - 2)$

$= (x - 2)(x + 2y)$

(e) $5(3x - 4)^2 - 45 = 5[(3x - 4)^2 - 9]$

$= 5[(3x - 4) + 3][(3x - 4) - 3]$

$= 5(3x - 1)(3x - 7)$

(f) $2(x^2 + 3) + 7(x^2 - 1) = 9x^2 - 1$

$= (3x + 1)(3x - 1)$

(h) $21 - x - 2x^2 = (7 + 2x)(3 - x)$

(i) $1 - a^2 - a^2b^2 + b^2 = (1 - a^2) + b^2(1 - a^2)$

$= (1 - a^2)(1 + b^2)$

$= (1 - a)(1 + a)(1 + b^2)$

(j) $3pq - 6pr + 2r - q = 3p(q - 2r) - (q - 2r)$

$= (q - 2r)(3p - 1)$

21. Given that $5^x = 3$,

(a) $125^x = (5^3)^x$
$= (5^x)^3$
$= 27$

(b) $25^{2x+1} = (5^2)^{2x+1}$
$= 5^{4x} \times 5^2$
$= (5^x)^4 \times 25$
$= 3^4 \times 25$
$= 2025$

22. $5a - 7b = 2a + 9b$
$3a = 16b$
$\frac{a}{b} = \frac{16}{3}$
$\therefore \frac{5a}{13b} = \frac{5}{3}\left(\frac{16}{3}\right) = 8\frac{8}{9}$

23. X = 18, Y = 15

24. **(a)** 2, 3, 5, 8, 13, 21, 34, <u>55</u>, <u>89</u>

(b) **(i)** 36, 44

(ii) $180 = 12 + 8n$
$n = 21$
$\therefore$ It would have 21 cubes.

(iii) Number of rods $= 12 + 49(8)$
$= 404$

25. **(a)** $x^3 + y - x^2y - x = x^2(x - y) - (x - y)$
$= (x - y)(x^2 - 1)$
$= (x - y)(x + 1)(x - 1)$

(b) $16x^4 - 81y^8 = (4x^2 - 9y^4)(4x^2 + 9y^4)$
$= (2x + 3y^2)(2x - 3y^2)(4x^2 + 9y^4)$

26. **(a)** $a = 15, b = 21$

(b) $h = n - 1, k = \frac{1}{2}n(n + 1)$

(c) $105 = \frac{1}{2}n(n + 1)$
$n^2 + n - 210 = 0$
$(n + 15)(n - 14) = 0$
$\therefore$ It is figure 14.

(d) $435 = \frac{1}{2}n(n + 1)$
$n^2 + n - 870 = 0$
$(n - 29)(n + 30) = 0$
$n = 29$
$h = 28$
$\therefore$ 28 lines have been added to A.

27. **(a)** $p = 30, q = 61, r = 42, s = 85$

(b) **(i)** $m = ab$

(ii) $n = 2m + 1$

(c) When $a = 101$,
$m = 101 \times 102$
$n = 2(101 \times 102) + 1$
$= 20\ 605$

(d) All the numbers in the n column are odd.

28. **(a)**

		Fig. 1	Fig. 2	Fig. 3	Fig. 4	Fig. 5
	n	1	2	3	4	5
Number of vertices	V	6	9	12	15	18
Number of edges	E	7	12	17	22	27
Number of 1 × 1 unit squares	X	2	4	6	8	10
Number of 2 × 2 unit squares	Y	0	1	2	3	4

(b) **(i)** $X = 2n$

(ii) $Y = n - 1$

(iii) $V = 3(n + 1)$

(c) **(i)** Fig. 23

(ii) 46

(d) **(i)** $V + X = E + 1$

(ii) $E = 7 + 5(n - 1)$
$= 2 + 5n$

(iii) $5V - 3E = 9$
$E = \frac{5}{3}V - 3$

(e) $5(111) - 3E = 9$
$E = 182$
$\therefore$ The figure which has 111 vertices has 182 edges.

Exercise 9D

1. **(a)** $5(3x - 2) - 2(4x - 1) = 20$
$15x - 10 - 8x + 2 = 20$
$7x = 28$
$x = 4$

(b) $\frac{x}{4} - \frac{x}{5} = \frac{1}{3}$
$\frac{1}{20}x = \frac{1}{3}$
$x = 6\frac{2}{3}$

(c) $\frac{x - 2}{7} - \frac{x + 3}{9} = \frac{1}{11}$
$99(x - 2) - 77(x + 3) = 63$
$22x = 63 + 198 + 231$
$x = 22\frac{4}{11}$

(d) $\frac{2}{x} = \frac{7}{x + 4}$
$7x = 2x + 8$
$5x = 8$
$x = 1\frac{3}{5}$

(e) $\sqrt{x} = 4$

$x = 16$

(f) $\frac{a}{3} = \frac{27}{a}$

$a^2 = 81$

$a = \pm 9$

(g) $4\left(\frac{1}{3x} - 1\right) = 3\left(\frac{1}{9x} - 2\right)$

$\frac{4}{3x} - 4 = \frac{1}{3x} - 6$

$\frac{1}{x} = -2$

$x = -\frac{1}{2}$

(h) $\frac{1}{4}(3t - 3) + 2 = t - \frac{1}{3}(2t - 10)$

$\frac{3}{4}t - \frac{3}{4} + 2 = t - \frac{2}{3}t + \frac{10}{3}$

$t = 5$

(i) $(x + 2)^2 = 25$

$x + 2 = 5$ or $x + 2 = -5$

$\therefore x = 3$ or -7

(j) $(3x - 4)^2 = 5(3x - 4)$

$(3x - 4)[(3x - 4) - 5] = 0$

$3x - 4 = 0$ or $3x - 9 = 0$

$\therefore x = 1\frac{1}{3}$ or 3

2. **(a)** $(2x - 7)(x - 2) = 9$

$2x^2 - 11x + 14 - 9 = 0$

$2x^2 - 11x + 5 = 0$

$(2x - 1)(x - 5) = 0$

$\therefore x = \frac{1}{2}$ or 5

(b) $\frac{12}{(x+1)^2} - \frac{1}{x+1} = 1$

$12 - (x + 1) = (x + 1)^2$

$12 - x - 1 = x^2 + 2x + 1$

$x^2 + 3x - 10 = 0$

$(x - 2)(x + 5) = 0$

$\therefore x = 2$ or $x = -5$

(c) $x = 3 + \frac{5}{x}$

$x^2 = 3x + 5$

$x^2 - 3x - 5 = 0$

$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-5)}}{2(1)}$

$= 4.19$ or -1.19

(d) $\left(x + \frac{6}{x}\right)^2 + \left(x + \frac{6}{x}\right) - 30 = 0$

$\left[\left(x + \frac{6}{x}\right) + 6\right]\left[\left(x + \frac{6}{x}\right) - 5\right] = 0$

$\therefore x + \frac{6}{x} + 6 = 0$ or $x + \frac{6}{x} - 5 = 0$

$x^2 + 6x + 6 = 0$ $\qquad$ $x^2 - 5x + 6 = 0$

$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(6)}}{2(1)}$ $\qquad$ $(x - 2)(x - 3) = 0$

$x = -1.27$ or -4.73 $\qquad$ $x = 2$ or 3

$\therefore x = -1.27, -4.73, 2$ or 3

3. **(a)** $2x^2 - 5x - 3 = 0$

$(2x + 1)(x - 3) = 0$

$\therefore x = -\frac{1}{2}$ or 3

(b) $(x - 6)^2 = 25$

$x - 6 = 5$ or $x - 6 = -5$

$\therefore x = 11$ or 1

(c) $3x^2 - 7x - 5 = 0$

$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(3)(-5)}}{2(3)}$

$= 2.91$ or -0.57

(d) $(3x + 1)^2 = (x + 4)^2$

$3x + 1 = x + 4$ or $3x + 1 = -x - 4$

$\therefore x = 1\frac{1}{2}$ or $-1\frac{1}{4}$

(e) $x + 3 = \frac{10}{x}$

$x^2 + 3x - 10 = 0$

$(x - 2)(x + 5) = 0$

$\therefore x = 2$ or -5

(f) $\frac{8}{x} - 3 = \frac{5}{2x+1}$

$8(2x + 1) - 3x(2x + 1) = 5x$

$6x^2 - 8x - 8 = 0$

$3x^2 - 4x - 4 = 0$

$(3x + 2)(x - 2) = 0$

$\therefore x = -\frac{2}{3}$ or 2

(g) $\frac{1}{x} - \frac{3}{2x+1} = 2$

$2x + 1 - 3x = 2x(2x + 1)$

$4x^2 + 3x - 1 = 0$

$(4x - 1)(x + 1) = 0$

$\therefore x = \frac{1}{4}$ or -1

(h) $5x^2 - 7x - 13 = 0$

$x = \dfrac{-(-7) \pm \sqrt{(-7)^2 - 4(5)(-13)}}{2(5)}$

$= 2.46$ or -1.06

(i) $2x^2 + 9x - 7 = 0$

$x = \dfrac{-(9) \pm \sqrt{9^2 - 4(2)(-7)}}{2(2)}$

$= 0.68$ or -5.18

(j) $3x^2 - 14x - 9 = 0$

$x = \dfrac{-(-14) \pm \sqrt{(-14)^2 - 4(3)(-9)}}{2(3)}$

$= 5.24$ or -0.57

4. **(a)** $3x^2 + 4x - 7 = 0$

$(3x + 7)(x - 1) = 0$

$x = -2\dfrac{1}{3}$ or 1

(b) $11x^2 + 14x - 1 = 0$

$x = \dfrac{-14 \pm \sqrt{14^2 - 4(11)(-1)}}{2(11)}$

$= 0.07$ or -1.34

(c) $11 + 3x - 3x^2 = 0$

$x = \dfrac{-3 \pm \sqrt{3^2 - 4(-3)(11)}}{2(-3)}$

$= -1.48$ or 2.48

(d) $2 - 5x - 7x^2 = 0$

$(2 - 7x)(1 + x) = 0$

$x = \dfrac{2}{7}$ or -1

(e) $\dfrac{1}{x-1} + \dfrac{1}{x+2} = \dfrac{1}{2}$

$2(x + 2) + 2(x - 1) = (x + 2)(x - 1)$

$4x + 2 = x^2 + x - 2$

$x^2 - 3x - 4 = 0$

$(x + 1)(x - 4) = 0$

$x = -1$ or 4

(f) $\dfrac{3}{x+1} = 3 - \dfrac{5}{x-3}$

$3(x - 3) = 3(x + 1)(x - 3) - 5(x + 1)$

$3x - 9 = 3x^2 - 6x - 9 - 5x - 5$

$3x^2 - 14x - 5 = 0$

$(3x + 1)(x - 5) = 0$

$x = -\dfrac{1}{3}$ or 5

(g) $\dfrac{x+1}{3x-2} = \dfrac{3x}{x+6}$

$3x(3x - 2) = (x + 1)(x + 6)$

$9x^2 - 6x = x^2 + 7x + 6$

$8x^2 - 13x - 6 = 0$

$(8x + 3)(x - 2) = 0$

$x = -\dfrac{3}{8}$ or 2

(h) $\dfrac{5}{x-3} = x - 3$

$(x - 3)^2 = 5$

$x - 3 = \sqrt{5}$ or $x - 3 = -\sqrt{5}$

$x = 5.24$ or 0.76

(i) $\dfrac{5}{x} + 4 = 3x$

$5 + 4x = 3x^2$

$3x^2 - 4x - 5 = 0$

$x = \dfrac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(-5)}}{2(3)}$

$= 2.12$ or -0.79

(j) $\dfrac{x}{x-3} = 2 + \dfrac{8}{x}$

$x^2 = 2x(x - 3) + 8(x - 3)$

$x^2 = 2x^2 - 6x + 8x - 24$

$x^2 + 2x - 24 = 0$

$(x + 6)(x - 4) = 0$

$x = -6$ or 4

5. **(a)** $5x + 4y = 49$ — (1)

$4x - 5y = -10$ — (2)

$(1) \times 5 : 25x + 20y = 245$ — (3)

$(2) \times 4 : 16x - 20y = -4$ — (4)

$(3) + (4) : 41x = 205$

$x = 5$

Substitute $x = 5$ into (1) : $5(5) + 4y = 49$

$y = 6$

$\therefore x = 5, y = 6$

(b) $x - y = 4$ — (1)

$2x + 3y = 13$ — (2)

$(1) \times 3 : 3x - 3y = 12$ — (3)

$(2) + (3) : 5x = 25$

$x = 5$

Substitute $x = 5$ into (1) : $5 - y = 4$

$y = 1$

$\therefore x = 5, y = 1$

(c) $\dfrac{2x + y + 3}{2} = \dfrac{3x - y + 1}{12}$

$24x + 12y + 36 = 6x - 2y + 2$

$18x + 14y = -34$ — (1)

$3x - 2y = 16$ — (2)

$(2) \times 6 :\ 18x - 12y = 96$ — (3)

$(1) - (3) :\ 26y = -130$

$y = -5$

Substitute $y = 5$ into (2) : $3x - 2(-5) = 16$

$x = 2$

$\therefore x = 2, y = -5$

(d) $3x + 5y = 60$ — (1)

$2x - y = 14$ — (2)

$(2) \times 5 :\ 10x - 5y = 70$ — (3)

$(1) + (3) :\ 13x = 130$

$x = 10$

Substitute $x = 10$ into (2) : $2(10) - y = 14$

$y = 6$

$\therefore x = 10, y = 6$

(e) $5x + 3y - 12 + 3 = 0$ — (1)

$18x - 12 + 10y - 25 + 3 = 0$ — (2)

$(1) \times 10 :\ 50x + 30y = 90$ — (3)

$(2) \times 3 :\ 54x + 30y = 102$ — (4)

$(4) - (3) :\ 4x = 12$

$x = 3$

Substitute $x = 3$ into (1) : $5(3) + 3y = 9$

$y = -2$

$\therefore x = 3, y = -2$

(f) $5x - 2y = 29$ — (1)

$x + 4y = -3$ — (2)

$(1) \times 2 :\ 10x - 4y = 58$ — (3)

$(2) + (3) :\ 11x = 55$

$x = 5$

Substitute $x = 5$ into (2) : $5 + 4y = -3$

$y = -2$

$\therefore x = 5, y = -2$

(g) $4x - 3y = 17$ — (1)

$5x + 6y = -8$ — (2)

$(1) \times 2 :\ 8x - 6y = 34$ — (3)

$(2) + (3) :\ 13x = 26$

$x = 2$

Substitute $x = 2$ into (1) : $4(2) - 3y = 17$

$y = -3$

$\therefore x = 2, y = -3$

(h) $2x + 3y = 2$ — (1)

$6x - y = 2\dfrac{2}{3}$ — (2)

$(2) \times 3 :\ 18x - 3y = 8$ — (3)

$(1) + (3) :\ 20x = 10$

$x = \dfrac{1}{2}$

Substitute $x = \dfrac{1}{2}$ into (2) : $6\left(\dfrac{1}{2}\right) - y = 2\dfrac{2}{3}$

$y = \dfrac{1}{3}$

$\therefore x = \dfrac{1}{2}, y = \dfrac{1}{3}$

6. **(a)** $5x - 4 \leqslant 26$ and $26 < 7x - 1$

$5x \leqslant 30 \qquad 27 < 7x$

$x \leqslant 6 \qquad 3\dfrac{6}{7} < x$

$\therefore 3\dfrac{6}{7} < x \leqslant 6$

(b) $2x + 6 \leqslant 5x$ and $5x < 25$

$6 \leqslant 3x \qquad x < 5$

$2 \leqslant x$

$\therefore 2 \leqslant x < 5$

(c) $2x - 3 < 21$ and $21 \leqslant 4x - 3$

$2x < 2 \qquad 24 \leqslant 4x$

$x < 12 \qquad 6 \leqslant x$

$\therefore 6 \leqslant x < 12$

(d) $9 < 2x - 1$ and $2x - 1 < 15$

$10 < 2x \qquad 2x < 16$

$5 < x \qquad x < 8$

$\therefore 5 < x < 8$

7. **(a)** $\dfrac{x}{2} + 3x < 23$

$x < 6\dfrac{4}{7}$

$\therefore x = 6$

(b) $x - 1 \leqslant 25 \qquad 25 \leqslant 3x - 1$

$x \leqslant 26 \qquad 26 \leqslant 3x$

$x \geqslant 8\dfrac{2}{3}$

$\therefore x = 11, 13, 17, 19, 23$

(c) $3x - 4 \geqslant 5\dfrac{1}{2}$

$3x \geqslant 9\dfrac{1}{2}$

$x \geqslant 3\dfrac{1}{6}$

$\therefore x = 4$

(d) $2k + 8 > 3k + 3$

$5 > k$

$\therefore k = 4$

8. $\frac{2}{3}x - \frac{x-4}{4} < 5$

$\frac{2}{3}x - \frac{1}{4}x + 1 < 5$

$\frac{5}{12}x < 4$

$x < 9\frac{3}{5}$

$\therefore x = 9$

9. **(a)** $6p < 39$

$p < 6\frac{1}{2}$

$\therefore p = 6$

(b) $3q - 1 > 35$

$3q > 36$

$q > 12$

$\therefore q = 13$

10. **(a)** $5 - 7x = 27 - 12x - 11x$

$16x = 22$

$x = 1\frac{3}{8}$

(b) $2x^2 + 10x - 3x - 15 = 2x^2 + 22x - 9x - 99$

$6x = 84$

$x = 14$

(c) $9x^2 - 12x + 4 = 0$

$(3x - 2)(3x - 2) = 0$

$x = \frac{2}{3}$

(d) $4^{x(x+1)} = 8^{x+1}$

$2^{2x(x+1)} = 2^{3(x+1)}$

$2x^2 + 2x = 3x + 3$

$2x^2 - x - 3 = 0$

$(2x - 3)(x + 1) = 0$

$x = 1\frac{1}{2}$ or -1

11. **(a)** $4^{3x-2} = 32$

$2^{2(3x-2)} = 2^5$

$6x - 4 = 5$

$x = \frac{9}{6}$

$= 1\frac{1}{2}$

(b) $4^{x+2} = 8^{2x-7}$

$2^{2(x+2)} = 2^{3(2x-7)}$

$2x + 4 = 6x - 21$

$4x = 25$

$\therefore x = 6\frac{1}{4}$

(c) $6x^3 = 384$

$x^3 = 64$

$x = \sqrt[3]{64}$

$= 4$

(d) $9^{3x+4} = 27^{3x-5}$

$3^{2(3x+4)} = 3^{3(3x-5)}$

$6x + 8 = 9x - 15$

$3x = 23$

$x = 7\frac{2}{3}$

(e) $25^{x-4} = 125^{3x+1}$

$5^{2(x-4)} = 5^{3(3x+1)}$

$2x - 8 = 9x + 3$

$7x = -11$

$x = -1\frac{4}{7}$

(f) $49^{3x-5} = 343^{7-2x}$

$7^{2(3x-5)} = 7^{3(7-2x)}$

$6x - 10 = 21 - 6x$

$12x = 31$

$x = 2\frac{7}{12}$

12. $4 < x - 2$ and $x - 2 < 8$

$6 < x$ $\quad$ $x < 10$

$6 < x < 10$

$x = 7, 8, 9$

$9 < 2x + 1$ and $2x + 1 < 17$

$8 < 2x$ $\quad$ $2x < 16$

$4 < x$ $\quad$ $x < 8$

$4 < x < 8$

$x = 5, 6, 7$

$\therefore$ The integer value of x is 7.

13. $x + 1 < 7$ and $7 < 2x + 5$

$x < 6$ $\quad$ $2 < 2x$

$1 < x$

$\therefore 1 < x < 6$

$\therefore$ The integer values of x include 2, 3, 4 and 5.

14. $6xy + 9x - 4y = 6$

$6xy - 4y + 9x - 6 = 0$

$2y(3x - 2) + 3(3x - 2) = 0$

$(3x - 2)(2y + 3) = 0$

$\therefore x = \frac{2}{3}$ or $y = -1\frac{1}{2}$

15. $-5 \leqslant x \leqslant 3, \quad -8 \leqslant y \leqslant 6$

(a) Largest $2x + 3y = 2(3) + 3(6) = 24$

(b) Least $3xy = 3(-5)(6) = -90$

(c) Greatest $x^2 + y^2 = (-5)^2 + (-8)^2 = 89$

(d) Least $x^2 - y^2 = 0^2 - (-8)^2 = -64$

16. $-5 \leqslant 3x + 1$ and $3x + 1 \leqslant 2x + 13$

$-2 \leqslant x$ $\qquad$ $x \leqslant 12$

$\therefore -2 \leqslant x \leqslant 12$

$-3 \leqslant y \leqslant 4$

(a) Greatest $x = 12$

(b) Smallest $y = -3$

(c) Smallest $(x + y)(x - y) = x^2 - y^2$

$= 0^2 - 4^2$

$= -16$

17. $2 \leqslant x \leqslant 6, \quad -2 \leqslant y \leqslant 3$

(a) Smallest $xy = 6(-2) = -12$

(b) Greatest $\frac{y}{x} = \frac{3}{2} = 1\frac{1}{2}$

(c) Greatest $x^2 - y^2 = 6^2 - 0^2 = 36$

18. $-7 \leqslant x \leqslant 4, \quad -5 \leqslant y \leqslant 2$

(a) Greatest $x^2 + y = (-7)^2 + 2 = 51$

(b) Least $xy = 4(-5) = -20$

(c) Least $x^2 - y^2 = 0^2 - (-5)^2 = -25$

19. $-2 \leqslant x \leqslant 4, \quad -6 \leqslant y \leqslant 5$

(a) Greatest $2x - y = 2(4) - (-6) = 14$

(b) Greatest $2x^2 - y^2 = 2(4)^2 - 0^2 = 32$

(c) Smallest $(x - y)^3 = (-2 - 5)^3 = -343$

20. $2 \leqslant x \leqslant 8, \quad -4 \leqslant y \leqslant -2$

(a) Greatest $x - y = 8 - (-4) = 12$

(b) Least $\frac{x}{y} = \frac{8}{-2} = -4$

(c) Least $\frac{x^2 + y^2}{y - x} = \frac{8^2 + (-4)^2}{-2 - 2} = -20$

21. $-4 \leqslant x \leqslant 1, \quad 2 \leqslant y \leqslant 9$

(a) Least $x^2 - y = 0^2 - 9 = -9$

(b) Greatest $\frac{x^2}{y} = \frac{(-4)^2}{2} = 8$

(c) Least $y - x^2 = 2 - (-4)^2 = -14$

(d) Greatest $x^2 - 2xy + y^2 = (x - y)^2 = (-4 - 9)^2 = 169$

22. Upper bound of perimeter $= 3 \times 6.45$

$= 19.35$ cm

Lower bound of perimeter $= 3 \times 6.35$

$= 19.05$ cm

23. Upper bound of $\frac{7.6 + 4.5}{2.4} = \frac{7.65 + 4.55}{2.35}$

$= 5.19$ (to 3 s.f.)

Lower bound of $\frac{7.6 + 4.5}{2.4} = \frac{7.55 + 4.45}{2.45}$

$= 4.90$ (to 3 s.f.)

24. Upper bound of combined mass $= 4.55 + 5.65 + 6.25$

$= 16.45$ kg

Lower bound of combined mass $= 4.45 + 5.55 + 6.15$

$= 16.15$ kg

25. Upper bound of $20 \times 25 = 20.5 \times 25.5$

$= 522.75$

Lower bound of $20 \times 25 = 19.5 \times 24.5$

$= 477.75$

26. Let x and y be number of people who bought \$5 tickets and \$3 tickets respectively.

$5x + 3y = 760 \quad$ — (1)

$\frac{3}{5}x + \frac{2}{3}y = 128 \quad$ — (2)

$(2) \times 4\frac{1}{2}$: $2\frac{7}{10}x + 3y = 576$ — (3)

$(1) - (3)$: $5x - 2\frac{7}{10}x = 760 - 576$

$2\frac{3}{10}x = 184$

$x = 80$

$\therefore$ 80 bought \$5 tickets.

27. Let p represent pencils and r represent rulers.

$4p + r = 1.00$ — (1)

$6p + 3r = 2.10$ — (2)

$(1) \times 3$: $12p + 3r = 3.00$ — (3)

$(3) - (2)$: $6p = 0.9$

$p = 0.15$

Substitute $p = 0.15$ into (1): $4(0.15) + r = 1.00$

$r = 0.40$

$3p + 13r = 3(0.15) + 13(0.4) = \$5.65$

28. $ax + by + 1 = 0$ — (1)

$(b - 1)x + 5y + 3x = 0$ — (2)

Substitute $x = 2$ and $y = -5$ into (1) and (2) :

$a(2) + b(-5) + 1 = 0$

$2a - 5b + 1 = 0$ — (3)

$2(b - 1) + 5(-5) + 3(2) = 0$

$2b = 25 - 6 + 2$

$b = 10\frac{1}{2}$

Substitute $b = 10\frac{1}{2}$ into (3) : $2a - 5\left(10\frac{1}{2}\right) + 1 = 0$

$a = 25\frac{3}{4}$

$\therefore a = 25\frac{3}{4}, b = 10\frac{1}{2}$

29. $x = 11y + 6$ — (1)

$x + 8 = 13y$ — (2)

$11y + 6 + 8 = 13y$

$y = 7$

Substitute $y = 7$ into (1) : $x = 11(7) + 6$

$= 83$

$\therefore x = 83, y = 7$

30. Let the number be $10x + y$.

$$\begin{aligned} x + y &= 7 \quad \text{— (1)} \\ (10y + x) - (10x + y) &= 9 \quad \text{— (2)} \\ 9y - 9x &= 9 \\ y - x &= 1 \quad \text{— (3)} \end{aligned}$$

$(1) + (3) : 2y = 8$

$y = 4$

Substitute $y = 4$ into $(1) : x + 4 = 7$

$x = 3$

$\therefore$ The number is 34 or 43.

31. **(a)** $2x + 3y = 2$ — (1)

$6x - 5y = 48$ — (2)

$(1) \times 3 : 6x + 9y = 6$ — (3)

$(3) - (2) : 14y = -42$

$y = -3$

Substitute $y = -3$ into $(1) : 2x + 3(-3) = 2$

$x = 5\frac{1}{2}$

$\therefore x = 5\frac{1}{2}, y = -3$

(b) $2x + 3y = 2$ — (1)

$4x + 6y = 4$ — (2)

There will be an infinite number of solutions since the two lines are parallel and do not intersect each other.

32. $\frac{5}{6}x - \frac{5}{16}x = 750$

$$\begin{aligned} x &= 750 \div \frac{25}{48} \\ &= 144 \end{aligned}$$

33. $(x + 1) + (x + 3) = 70$

$x = 33$

$\therefore 2x + 5 = 2(33) + 5 = 71$

34. Let x days be the number of days a mechanic takes to do his job alone.

In 1 day, the mechanic can do $\frac{1}{x}$ of his job.

In 1 day, the helper can do $\frac{1}{x + 6}$ of his job.

$$\begin{aligned} \frac{1}{x} + \frac{1}{x + 6} &= \frac{1}{4} \\ x + 6 + x &= \frac{1}{4}x(x + 6) \\ x^2 + 6x &= 8x + 24 \\ x^2 - 2x - 24 &= 0 \\ (x + 4)(x - 6) &= 0 \end{aligned}$$

$\therefore x = 6$ (since number of days > 0)

$\therefore$ The mechanic takes 6 days while his helper takes 12 days.

35. **(a)** $\frac{80}{x - 0.6} - \frac{80}{x} = 14.5$

$80x - 80(x - 0.6) = 14.5x(x - 0.6)$

$14.5x^2 - 8.7x - 48 = 0$

$$x = \frac{-(-8.7) \pm \sqrt{(-8.7)^2 - 4(14.5)(-48)}}{2(14.5)}$$

$= 2.144$ or -1.544 (to 3 d.p.)

(b) Cost in Dec 2008 $= 2.144 - 0.60$

$= \$1.54$ (to nearest cent)

36. **(a)** Let x be the present age of the son.

$$\begin{aligned} (x - 8)(3x - 8) &= 112 \\ 3x^2 - 32x + 64 - 112 &= 0 \\ 3x^2 - 32x - 48 &= 0 \\ (3x + 4)(x - 12) &= 0 \end{aligned}$$

$\therefore x = -\frac{4}{3}$ (NA) or $x = 12$

$\therefore$The son is 12 years old and his father is 36 years old.

(b) $3x^2 - 7x - 17 = 0$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(3)(-17)}}{2(3)}$$

$= 3.82$ or -1.48 (to 3 s.f.)

37. **(a)** $2a + b = 1\frac{1}{2}$ — (1)

$3a - 2b = -4\frac{3}{4}$ — (2)

$(1) \times 2 : 4a + 2b = 3$ — (3)

$(2) + (3) : 7a = -1\frac{3}{4}$

$a = -\frac{1}{4}$

Substitute $a = -\frac{1}{4}$ into $(1) : 2\left(-\frac{1}{4}\right) + b = 1\frac{1}{2}$

$b = 2$

$\therefore a = -\frac{1}{4}, b = 2$

(b) $\frac{1}{x} = -\frac{1}{4}$

$\therefore x = -4$

$\frac{1}{y} = 2$

$\therefore y = \frac{1}{2}$

38. **(a)** $14x + 9y = 90$ — (1)

(b) $4x + 6y = 36$ — (2)

(c) $(1) \times 2 : 28x + 18y = 180$ — (3)

$(2) \times 3 : 12x + 18y = 108$ — (4)

$(3) - (4) : 16x = 72$

$x = 4.5$

Substitute $x = 4.5$ into (2) : $4(4.5) + 6y = 36$

$y = 3$

$\therefore PQ = y + y = 6$ cm, $QR = x + x = 9$ cm

39. Let x min be the time taken by the larger pipe.

$$\frac{1}{x} + \frac{1}{x+8} = \frac{1}{3}$$

$3(x + 8 + x) = x(x + 8)$

$x^2 + 2x - 24 = 0$

$(x - 4)(x + 6) = 0$

$x = 4$ or -6 (NA)

$\therefore$ The larger pipe takes 4 minutes to fill the water cistern alone.

40. $x(x + 2) = 483$

$x^2 + 2x - 483 = 0$

$(x - 21)(x + 23) = 0$

$x = 21$ or -23

$\therefore$ The two numbers are 21 and 23.

41. **(a)** $9x^2 - 1 = 15x + 5$

$9x^2 - 15x - 6 = 0$

$3x^2 - 5x - 2 = 0$

$(3x + 1)(x - 2) = 0$

$\therefore x = -\frac{1}{3}$ or 2

(b) Let the two numbers be x and y.

$x + y = 10$ — (1)

$xy = 24$ — (2)

From (1) : $x = 10 - y$ — (3)

Substitute (3) into (2) : $(10 - y)y = 24$

$10y - y^2 = 24$

$y^2 - 10y + 24 = 0$

$(y - 6)(y - 4) = 0$

$y = 6$ or 4

$x = 10 - 6$ or $10 - 4$

$= 4$ or 6

$\therefore$ The two numbers are 4 and 6.

42. $x(x + 3) = 108$

$x^2 + 3x - 108 = 0$

$(x + 12)(x - 9) = 0$

$x = -12$ or 9

$\therefore$ The two numbers are 9 and 12 or -12 and -9.

43. $x(x + 2) = 7(x + 2)$

$x^2 + 2x = 7x + 14$

$x^2 - 5x - 14 = 0$

$(x - 7)(x + 2) = 0$

$x = 7$ or -2 (NA)

$\therefore$ The two numbers are 7 and 9.

44. **(a)** Time taken = $\frac{80}{x}$ h

(b) Time taken = $\frac{80}{x-3}$ h

(c) $\frac{80}{x-3} - \frac{80}{x} = \frac{4}{3}$

$4x(x - 3) = 3[80x - 80(x - 3)]$

$4x^2 - 12x - 720 = 0$

$x^2 - 3x - 180 = 0$

$(x - 15)(x + 12) = 0$

$x = 15$ or -12 (NA)

$\therefore x = 15$

45. Let width be x cm.

$x(x + 4) = 96$

$x^2 + 4x - 96 = 0$

$(x + 12)(x - 8) = 0$

$x = -12$ (NA) or 8

$\therefore$ The length of the rectangle is 12 cm.

46. $(x + 2)^2 - x^2 = 48$

$x^2 + 4x + 4 - x^2 = 48$

$4x = 44$

$x = 11$

$\therefore$ The two numbers are 11 and 13.

47. **(a)** $\frac{130}{x} + \frac{36}{x-25} = 3\frac{1}{4} = \frac{13}{4}$

$13x(x - 25) = 4[130(x - 25) + 36x]$

$13x^2 - 325x = 664x - 13000$

$13x^2 - 989x + 13000 = 0$

(b) $x = \frac{-(-989) \pm \sqrt{(-989)^2 - 4(13)(13\,000)}}{2(13)}$

$= 59.18$ or 16.90 (to 4 s.f.) (NA)

Time saved = $3\frac{1}{4}$ h $- \frac{166}{59.18}$ h $= 26.7$ min (to 3 s.f.)

48. **(a)** $(3x + 5)(x + 3) = \frac{1}{2}(11x - 9)(2x + 1)$

$2(3x^2 + 14x + 15) = 22x^2 - 7x - 9$

$16x^2 - 35x - 39 = 0$

$(16x + 13)(x - 3) = 0$

$x = -\frac{13}{16}$ (NA) or $x = 3$

(b) $PQ = 24, QR = 7, AD = 14, CD = 6$

Using Pythagoras′ Theorem,

$$PR^2 = PQ^2 + QR^2$$
$$PR = \sqrt{24^2 + 6^2}$$
$$= 25 \text{ cm}$$

Using Pythagoras′ Theorem,

$$AC^2 = AD^2 + CD^2$$
$$AC = \sqrt{14^2 + 7^2}$$
$$= 15.2 \text{ cm (to 3 s.f.)}$$

49. (a) $\frac{600}{x}$, $\frac{600}{x} + 220$

(b)
$$x\left(\frac{600}{x}\right) = \left(x - 5\frac{1}{2}\right)\left(\frac{600}{x} + 220\right)$$
$$600 = 600 - \frac{3300}{x} + 220x - 1210$$
$$220x^2 - 1210x - 3300 = 0$$
$$2x^2 - 11x - 30 = 0$$

(c) $(2x - 15)(x + 2) = 0$

$x = 7\frac{1}{2}$ or -2 (NA)

$\therefore$ Speed of car = $\frac{600}{7.5}$ = 80 km/h

50. (a) $\frac{20}{v}$ h

(b) $\frac{20}{v - 1}$ h

(c) (i) $\frac{20}{v-1} - \frac{20}{v} = \frac{3}{4}$
$$3v(v - 1) = 4[20v - 20(v - 1)]$$
$$3v^2 - 3v - 80 = 0$$

(ii)
$$v = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(3)(-80)}}{2(3)}$$
$$= 5.7 \text{ or } -4.7 \text{ (to 1 d.p.) (NA)}$$

(d) Total time = $\frac{20}{5.7} + \frac{20}{4.7}$ = 7 h 46 min

51. (a) $\frac{3000}{x}$

(b) $\frac{3000}{x - 6}$

(c)
$$\frac{3000}{x-6} - \frac{3000}{x} = 5\frac{5}{9}$$
$$= \frac{50}{9}$$
$$50x(x - 6) = 9[3000x - 3000(x - 6)]$$
$$50x^2 - 300x - 162\,000 = 0$$
$$x^2 - 6x - 3240 = 0$$

(d) $(x - 60)(x + 54) = 0$

$x = 60$ or -54 (NA)

$$\text{Number of kg bought} = \frac{5000}{60 - 6}$$
$$= 55\frac{5}{9}$$

52.
$$\text{Max. number} = \frac{1.2 \times 10^6}{2000}$$
$$= 600$$
$$\text{Min. number} = \frac{1.2 \times 10^6}{3000}$$
$$= 400$$

53. (a) (i) $\frac{320}{x}$

(ii) $\frac{320}{x + 0.2}$

(b) $\frac{320}{x} - \frac{320}{x + 0.2} = 20$
$$20x\left(x + \frac{1}{5}\right) = 320\left(x + \frac{1}{5}\right) - 320x$$
$$20x^2 + 4x - 64 = 0$$
$$5x^2 + x - 16 = 0$$

(c)
$$x = \frac{-1 \pm \sqrt{1^2 - 4(5)(-16)}}{2(5)}$$

$x = 1.692$ or -1.892 (to 3 d.p.) (NA)

(d)
$$\text{Number of litres} = \frac{320}{1.892}$$
$$= 169.1 \text{ (to nearest 0.1 litre)}$$

54. (a)
$$QP = 2x + 1\frac{1}{2}(x - 3)$$
$$= 3\frac{1}{2}x - 4\frac{1}{2}$$

(b) $3(x + 1)$

(c)
$$3\frac{1}{2}x - 4\frac{1}{2} = 3x + 3$$
$$x = 15$$

(d)
$$\text{Total distance} = 3(16) \times 2$$
$$= 96 \text{ km}$$
$$\text{Average speed} = \frac{96}{3\frac{1}{2} + 3\frac{1}{2}} = 13\frac{5}{7} \text{ km/h}$$

55. (i) $y \leqslant 4x, x + y \leqslant 8, 4y \geqslant x$

(ii) $y \geqslant 2, y + 2x \leqslant 8, 2y \leqslant 3x + 12$

(iii) $y \leqslant 8, y \geqslant 2, y + 2x \geqslant 0, y \geqslant 2x$

(iv) $y \geqslant 2x - 4, x + y + 4 \geqslant 0, 2y \leqslant x + 4$

56. Draw the lines $x = 10$, $x + 2y = 30$ and $3x + 2y = 60$.

Shade the regions not required by the inequalities:

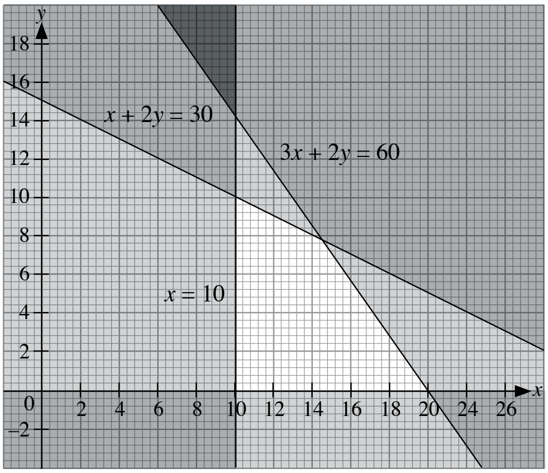
$x \geqslant 10, y \geqslant 0, x + 2y \leqslant 30$ and $3x + 2y \leqslant 60$

(i) Left of $x = 10$

(ii) Below the x-axis

(iii) Above $x + 2y = 30$

(iii) Above $3x + 2y = 60$



$2x + y$ must be satisfied by the unshaded region.

If $x = 20$, $y = 0$, we obtain the maximum value of $2x + y$.

Maximum value of $2x + y = 2(20) + 0$

$= 40$

57. Draw the lines $y = 2x$, $y = x$, $x + y = 13$ and $3x + y = 24$.

Shade the regions not required by the inequalities:

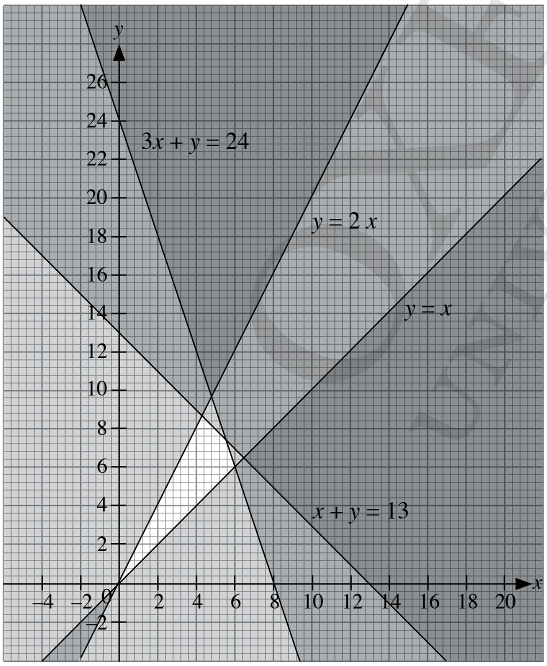
$y \leqslant 2x$, $y \geqslant x$, $x + y \leqslant 13$ and $3x + y \leqslant 24$

(i) Above $y = 2x$

(ii) Below $y = x$

(iii) Above $x + y = 13$

(iv) Above $3x + y = 24$



$x + 2y$ must be satisfied by the unshaded region.

If $x = 4.3$, $y = 8.7$, we obtain the maximum value of $x + 2y$.

Maximum value of $x + 2y = 4.3 + 2(8.7)$

$= 21.7$

58. Draw the lines $x + 2y = 12$, $2x + y = 10$, $4y = x$ and $2y = x - 8$.

Shade the regions not required by the inequalities:

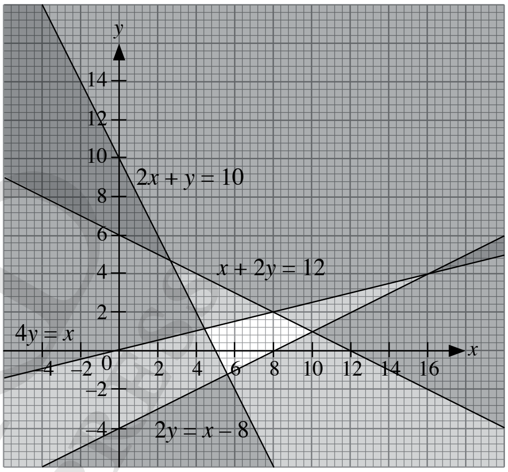
$x + 2y \leqslant 12$, $2x + y \geqslant 10$, $4y \leqslant x$ and $2y \geqslant x - 8$

(i) Above $x + 2y = 12$

(ii) Below $2x + y = 10$

(iii) Above $4y = x$

(iv) Below $2y = x - 8$



$6x + 2y$ must be satisfied by the unshaded region.

If $x = 10$, $y = 1$, we obtain the greatest value of $6x + 2y$.

Greatest value of $6x + 2y = 6(10) + 2(1)$

$= 62$

59. (i) Equation of MN: $y = 4$

Equation of PR:

$$\frac{y-6}{x-0} = \frac{6-0}{0-6}$$

$$y - 6 = -x$$

$$x + y = 6$$

When $y = 4$,

$x = 2$

$\therefore$ The coordinates of Q are $(2, 4)$.

(ii) Equation of PM:

$$\frac{y-4}{x-0} = \frac{4-0}{0-6}$$

$$y - 4 = -\frac{2}{3}x$$

$3y + 2x = 12$

The unshaded region lies above PM.

Hence $3y + 2x \leqslant 12$ defines a part of the unshaded region.

The unshaded region lies below PR and below the line $y = 4$. Hence $x + y \leqslant 6$ and $y \geqslant 4$ defines a part of the unshaded region.

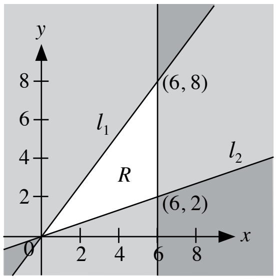
$\therefore$ The unshaded region is defined by the three inequalities

$3y + 2x \geqslant 12, y \leqslant 4, x + y \leqslant 6.$

(iii) Area of $\triangle MPQ = \frac{1}{2} \times 2 \times 4$

$= 4$ units^2

60.



(i) Equation of l_1:

$$\frac{y-0}{x-0} = \frac{8-0}{6-0}$$

$$6y = 8x$$

$$3y = 4x$$

The unshaded region lies below l_1.

Hence $3y \leqslant 4x$ defines a part of the unshaded region.

Equation of l_2:

$$\frac{y-0}{x-0} = \frac{2-0}{6-0}$$

$$6y = 2x$$

$$3y = x$$

The unshaded region lies above l_2.

Hence $3y \geqslant x$ defines a part of the unshaded region.

The unshaded region lies to the left of $x = 6$.

Hence $x \leqslant 6$ defines a part of the unshaded region.

$\therefore$ The unshaded region is defined by the three inequalities $x \leqslant 6, 3y \leqslant 4x, 3y \geqslant x$.

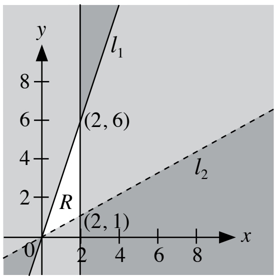
(ii) If $x = 6, y = 8$, we obtain the largest value of $3x + 2y$.

Maximum value of $3x + 2y = 3(6) + 2(8)$

$= 34$

$\therefore c = 34$

61.



(i) Equation of l_1:

$$\frac{y-0}{x-0} = \frac{6-0}{2-0}$$

$$2y = 6x$$

$$y = 3x$$

The unshaded region lies below l_1.

Hence $y \leqslant 3x$ defines a part of the unshaded region.

Equation of l_2:

$$\frac{y-0}{x-0} = \frac{1-0}{2-0}$$

$$2y = x$$

The unshaded region lies above l_2.

Hence $2y > x$ defines a part of the unshaded region.

$\therefore$ The two other inequalities are $y \leqslant 3x$ and $2y > x$.

(ii) If $x = 2$, $y = 6$, we obtain the maximum value of $2x + y$.

Maximum value of $2x + y = 2(2) + 6$

$= 10$

62. Equation of AB:

$$\frac{y-5}{x-0} = \frac{5-0}{0-4}$$

$$y - 5 = -\frac{5}{4}x$$

$$4y + 5x = 20$$

The unshaded region lies above AB. Hence $4y + 5x \geqslant 20$ defines a part of the unshaded region.

Equation of BC:

$$\frac{y-5}{x-0} = \frac{5-9}{0-7}$$

$$y - 5 = \frac{4}{7}x$$

$$7y = 4x + 35$$

The unshaded region lies below BC.

Hence $7y \geqslant 4x + 35$ defines a part of the unshaded region.

Equation of AC:

$$\frac{y-0}{x-4} = \frac{9-0}{7-4}$$

$$y = 3x - 12$$

The unshaded region lies above AC.

Hence $y \geqslant 3x - 12$ defines a part of the unshaded region.

$\therefore$ The unshaded region is defined by the three inequalities:

$4y + 5x \geqslant 20$, $y \geqslant 3x - 12$, $7y \leqslant 4x + 35$.

Exercise 9E

1. $f(x) = 4x^2 - 2x + 1$

$f(-3) = 4(-3)^2 - 2(-3) + 1$

$= 43$

$f\left(\frac{1}{2}\right) = 4\left(\frac{1}{2}\right)^2 - 2\left(\frac{1}{2}\right) + 1$

$= 1$

$f(2) = 4(2)^2 - 2(2) + 1$

$= 13$

2. $f(x) = 6x - 1$

Let $y = 6x - 1$.

$6x = y + 1$

$x = \frac{y+1}{6}$

$\therefore f^{-1}(x) = \frac{x+1}{6}$

3. $f(x) = mx + c$

$f(-1) = -m + c = 4$ — (1)

$f(3) = 3m + c = 8$ — (2)

(1) – (2):

$-4m = -4$

$m = 1$

Substitute $m = 1$ into (1):

$-1 + c = 4$

$c = 5$

$\therefore f(x) = x + 5$

$f(2) = 2 + 5$

$= 7$

$f(-10) = -10 + 5$

$= -5$

4. $f(x) = \frac{3x}{2x-1}$

Let $y = \frac{3x}{2x-1}$.

$y(2x - 1) = 3x$

$2xy - 3x = y$

$x(2y - 3) = y$

$x = \frac{y}{2y-3}$

$\therefore f^{-1}(x) = \frac{x}{2x-3}$, undefined when $x = \frac{3}{2}$.

$f^{-1}(4) = \frac{4}{2(4)-3}$

$= \frac{4}{5}$

5. $g(x) = 7x^2 - 3x - 5$

(i) $g(-2x) = 7(-2x)^2 - 3(-2x) - 5$

$= 28x^2 + 6x - 5$

(ii) $g(x + 2) = 7(x + 2)^2 - 3(x + 2) - 5$

$= 7(x^2 + 4x + 4) - 3x - 6 - 5$

$= 7x^2 + 28x + 28 - 3x - 6 - 5$

$= 7x^2 + 25x + 17$

(iii) $g(x + 1) - g(x - 1)$

$= [7(x + 1)^2 - 3(x + 1) - 5] - [7(x - 1)^2 - 3(x - 1) - 5]$

$= [7(x^2 + 2x + 1) - 3x - 3 - 5] - [7(x^2 - 2x + 1) - 3x + 3 - 5]$

$= 7x^2 + 11x - 1 - (7x^2 - 17x + 5)$

$= 28x - 6$

6. $h(x) = \frac{1-9x}{3-x}$

Let $y = \frac{1-9x}{3-x}$.

$y(3 - x) = 1 - 9x$

$3y - xy = 1 - 9x$

$9x - xy = 1 - 3y$

$x(9 - y) = 1 - 3y$

$x = \frac{1-3y}{9-y}$

$\therefore h^{-1}(x) = \frac{1-3x}{9-x}$, undefined when $x = 9$.

7. $y = (2x + 3)(5 - 2x)$

(a) When $y = 0$,

$(2x + 3)(5 - 2x) = 0$

$2x + 3 = 0$ or $5 - 2x = 0$

$x = -1\frac{1}{2}$ or $x = 2\frac{1}{2}$

$\therefore A\left(-1\frac{1}{2}, 0\right)$

When $x = 0$,

$y = (3)(5) = 15$

$\therefore C(0, 15)$

(b) Length of $AB = 2\frac{1}{2} - \left(-1\frac{1}{2}\right)$

$= 4$ units

Midpoint of $AB = 2$ units from $A\left(-1\frac{1}{2}, 0\right)$

x-coordinate of line of symmetry $= -1\frac{1}{2} + 2$

$= \frac{1}{2}$

$\therefore$ Line of symmetry has the equation $x = \frac{1}{2}$

8. $y = 20 + 3x - 2x^2$

(a) When $y = 0$,

$20 + 3x - 2x^2 = 0$

$2x^2 - 3x - 20 = 0$

$(x - 4)(2x + 5) = 0$

$x = 4$ or $-\frac{5}{2}$

$\therefore P\left(-\frac{5}{2}, 0\right), R(4, 0)$

When $x = 0$,

$y = 20$

$\therefore Q(0, 20)$

(b) Length of $PR = 4 - \left(-\frac{5}{2}\right)$

$= \frac{13}{2}$ units

Midpoint of $PR = \frac{13}{4}$ units from $P\left(-\frac{5}{2}, 0\right)$

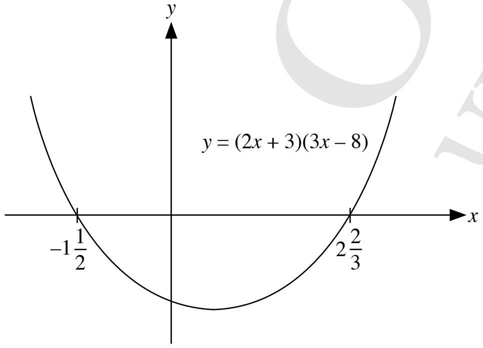
x-coordinate of line of symmetry $= -\frac{5}{2} + \frac{13}{4}$

$= \frac{3}{4}$

$\therefore$ Line of symmetry has the equation $x = \frac{3}{4}$

(c) $y = 8 + 5x$

9.



Line of symmetry of this curve is $x = \frac{7}{12}$

10. (a) $y = (x - 4)(2x + h)$

When $x = 0, y = 4$,

$4 = (-4)(h)$

$\therefore h = -1$

(b) When $y = 0$,

$(x - 4)(2x - 1) = 0$

$x = 4$ or $\frac{1}{2}$

$\therefore P\left(\frac{1}{2}, 0\right), Q(4, 0)$

11. (a)

x	–4	–3	–2	–1	0	1	2
y	14	3	–4	–7	–6	–1	8

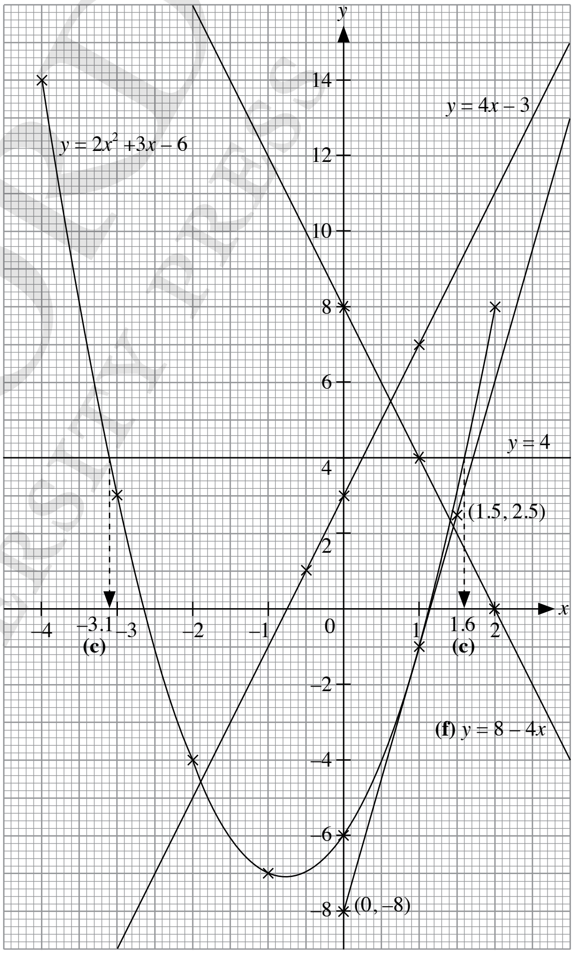
$y = 2x^2 + 3x - 6$

When $x = -3, y = 2(-3)^2 + 3(-3) - 6 = 3$

When $x = -2, y = 2(-2)^2 + 3(-2) - 6 = -4$

When $x = 1, y = 2(1)^2 + 3(1) - 6 = -1$

(b)



(c) $2x^2 + 3x - 6 = 10 - 6$

Draw $y = 4$

$\therefore x = -3.1, 1.6$

(d) Gradient $= \frac{2.5 - (-8)}{1.5 - 0}$

$= 7$

(e) $2x^2 - x + 4x - 3 - 3 < 4x - 3$

Draw $y = 4x - 3$

$\therefore -1 < x < 1.5$

(f) $8 - 4x = 2x^2 + 3x - 6$

$2x^2 + 7x - 14 = 0$

12. (a)

x	–3	–2	–1	0	1	2	3	4	5
y	–6.1	–2.4	0.3	2	2.7	2.4	1.1	–1.2	–4.5

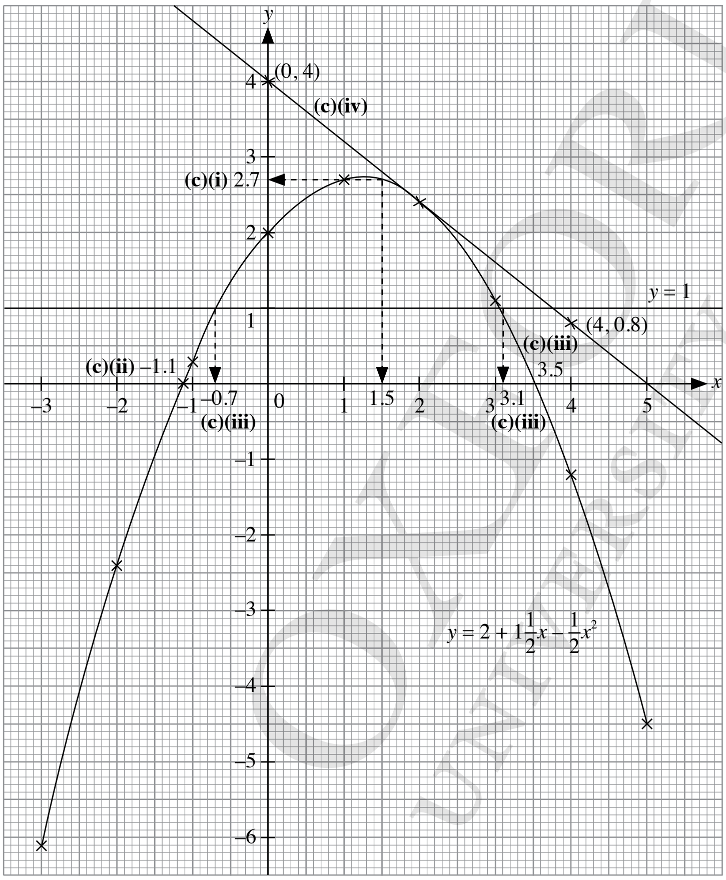
$y = 2 + 1\frac{1}{5}x - \frac{1}{2}x^2$

When $x = -1$, $y = 2 + 1\frac{1}{5}(-1) - \frac{1}{2}(-1)^2 = 0.3$

When $x = 3$, $y = 2 + 1\frac{1}{5}(3) - \frac{1}{2}(3)^2 = 1.1$

When $x = 5$, $y = 2 + 1\frac{1}{5}(5) - \frac{1}{2}(5)^2 = -4.5$

(b)



(c) **(i)** When $x = 1.5$, $y = 2.7$

(ii) When $y = 0$, $x = -1.1$ or 3.5

(iii) $2 + 1\frac{1}{5}x - \frac{1}{2}x^2 = 1$

Draw $y = 1$

$\therefore x = -0.7$ or 3.1

(iv) Gradient $= \frac{4 - 0.8}{0 - 4}$

$= -0.8$

13. **(a)**

x	–4	–3	–2	–1	0	1	2	2.5	3
y	4.2	3	2	1.3	1	1.4	3	4.6	7

$y = \dfrac{2x^2 - x + 6}{6 - x}$

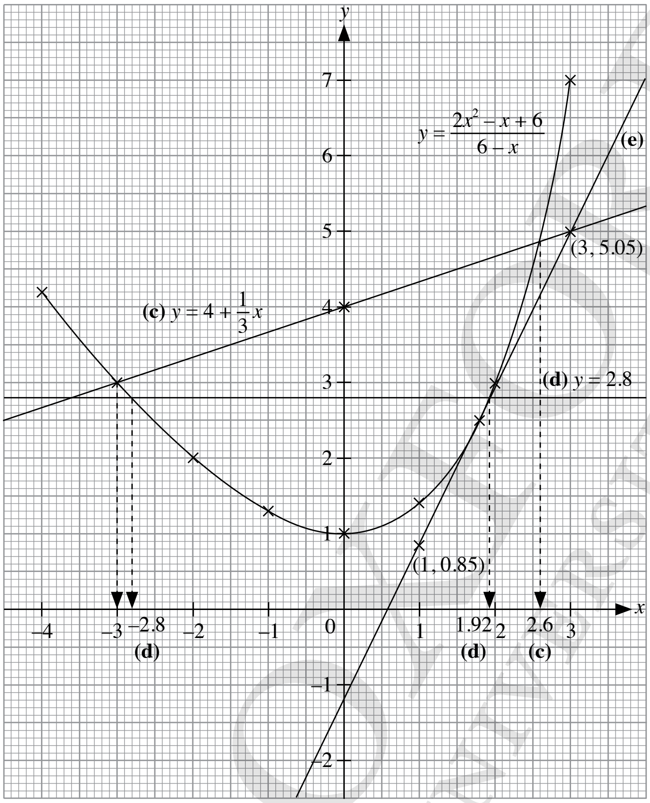
When $x = -2$, $y = \dfrac{2(-2)^2 - (-2) + 6}{6 - (-2)} = 2$

When $x = 1$, $y = \dfrac{2(1)^2 - 1 + 6}{6 - 1} = 1.4$

When $x = 2$, $y = \dfrac{2(2)^2 - 2 + 6}{6 - 2} = 3$

When $x = 3$, $y = \dfrac{2(3)^2 - 3 + 6}{6 - 3} = 7$

(b)



(c) $\dfrac{2x^2 - x + 6}{6 - x} = 4 + \dfrac{1}{3}x$

Draw $y = 4 + \dfrac{1}{3}x$

$\therefore x = -3$ or 2.6

(d) $\dfrac{2x^2 - x + 6}{6 - x} < 2.8$

Draw $y = 2.8$

$-2.8 < x < 1.9$

(e) Gradient $= \dfrac{5.05 - 0.85}{3 - 1}$

$= 2.1$

14. **(a)**

x	0.5	1	1.5	2	2.5	3	3.5	4
y	–3.75	–1.5	–0.58	0	0.45	0.83	1.18	1.5

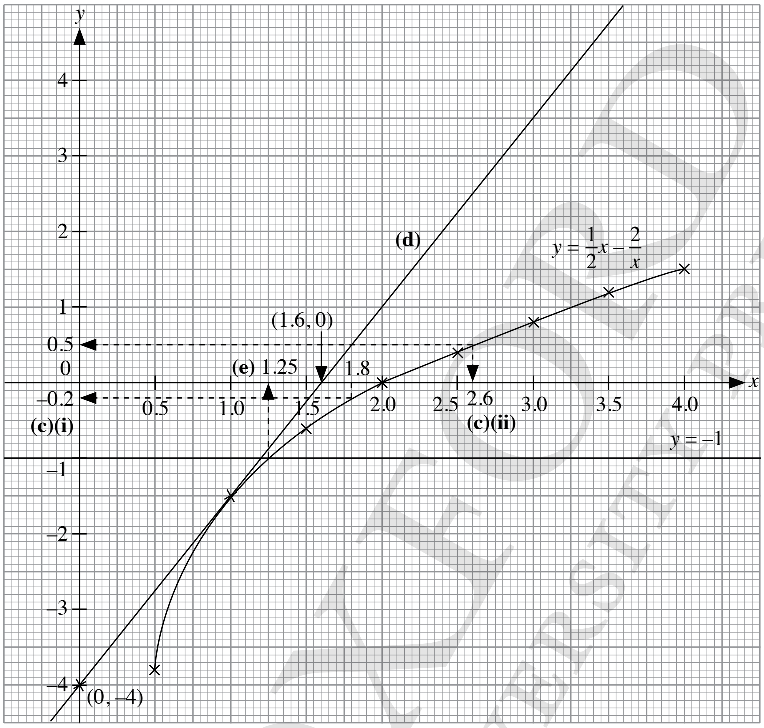
$y = \frac{1}{2}x - \frac{2}{x}$

When $x = 2$, $y = \frac{1}{2}(2) - \frac{2}{2} = 0$

When $x = 2.5$, $y = \frac{1}{2}(2.5) - \frac{2}{2.5} = 0.45$

When $x = 3.5$, $y = \frac{1}{2}(3.5) - \frac{2}{3.5} = 1.18$ (to 2 d.p.)

(b)



(c) **(i)** When $x = 1.8$, $y = –0.2$

(ii) When $y = 0.5$, $x = 2.6$

(d) Gradient $= \frac{0 - (-4)}{1.6 - 0}$

$= 2.5$

(e) $x^2 + 2x = 4$

$\frac{x}{2} + 1 = \frac{2}{x}$

$\frac{x}{2} - \frac{2}{x} = -1$

Draw $y = -1$

$\therefore x = 1.25$

15. **(a)** $y = 2x + \frac{8}{x} - 7$

When $x = 3.5$,

$y = 2(3.5) + \frac{8}{3.5} - 7$

$= 2.3$ (to 1 d.p.)

$\therefore h = 2.3$

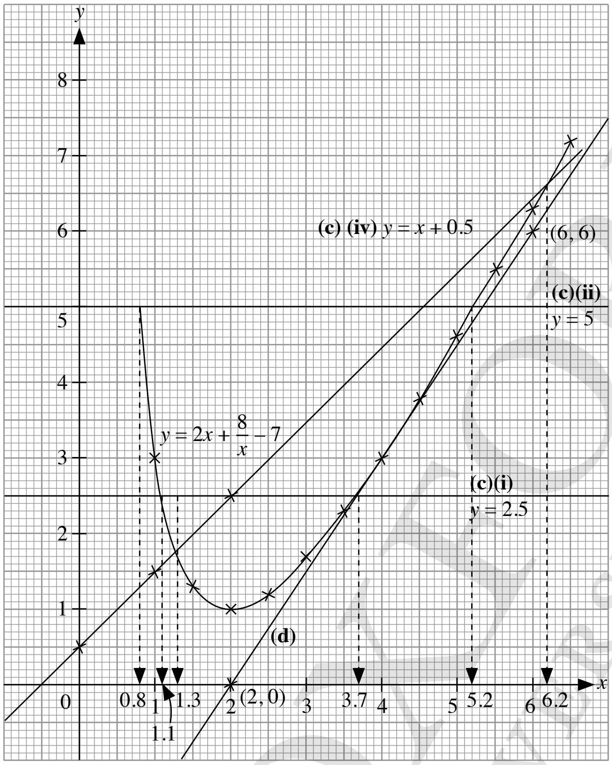
When $x = 6.5$,

$y = 2(6.5) + \frac{8}{6.5} - 7$

$= 7.2$ (to 1 d.p.)

$\therefore k = 7.2$

(b)



(c) **(i)** $2x + \frac{8}{x} - 7 = 2.5$

Draw $y = 2.5$

$\therefore x = 1.1$ or 3.7

(ii) $2x + \frac{8}{x} = 12$

$2x + \frac{8}{x} - 7 = 12 - 7 = 5$

Draw $y = 5$

$\therefore x = 0.8$ or 5.2

(iii) $x + \frac{4}{x} = 3.5$

$2\left(x + \frac{4}{x}\right) - 7 = 2(3.5) - 7 = 0$

Draw $y = 0$

$\therefore$ There are no solutions.

(iv) $x + \frac{8}{x} = 7.5$

$x + x + \frac{8}{x} - 7 = x + 7.5 - 7$

$= x + 0.5$

Draw $y = x + 0.5$

$\therefore x = 1.3$ or 6.2

(d) Gradient $= \frac{6 - 0}{6 - 2}$

$= 1.5$

16. **(a)** $y = \frac{1}{10}(x^3 + 3x + 20)$

When $x = -2$,

$$y = \frac{1}{10}[(-2)^3 + 3(-2) + 20]$$
$$= 0.6$$

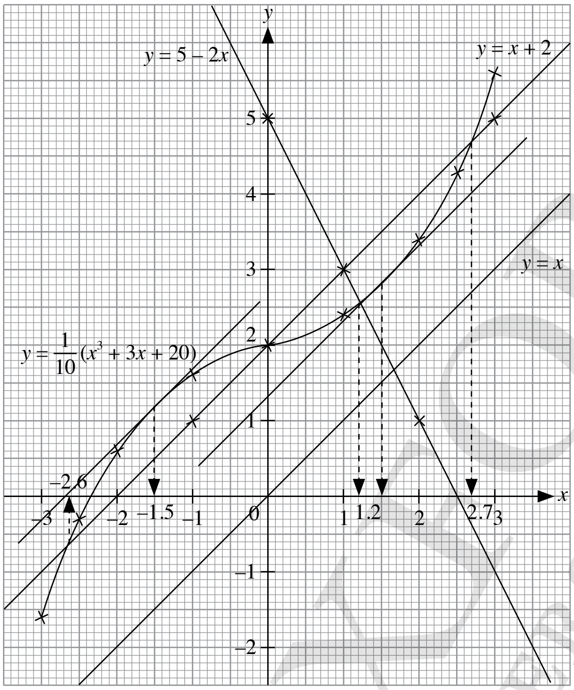
$\therefore h = 0.6$

When $x = 2.5$,

$$y = \frac{1}{10}[(2.5)^3 + 3(2.5) + 20]$$
$$= 4.3$$

$\therefore k = 4.3$

(b)



(c)
$$\begin{aligned} x^3 + 23x &= 30 \\ x^3 + 3x + 20 &= 30 - 20x + 20 \\ \frac{1}{10}(x^3 + 3x + 20) &= 5 - 2x \end{aligned}$$

Draw $y = 5 - 2x$

$\therefore x = 1.2$

(d) From the graph, $x = -1.5$ or 1

(e)
$$\begin{aligned} x^3 - 7x &\geqslant 0 \\ x^3 - 7x + 10x + 20 &\geqslant 10x + 20 \\ \frac{1}{10}(x^3 - 7x + 20) &\geqslant x + 2 \end{aligned}$$

$\therefore x \geqslant 2.6$ or $-2.6 \leqslant x \leqslant 0$

17. (a) $y = 1.8 + x - \frac{4}{x^2}$

When $x = 1$,

$y = 1.8 + 1 - \frac{4}{1^2}$

$= -1.2$

$\therefore h = -1.2$

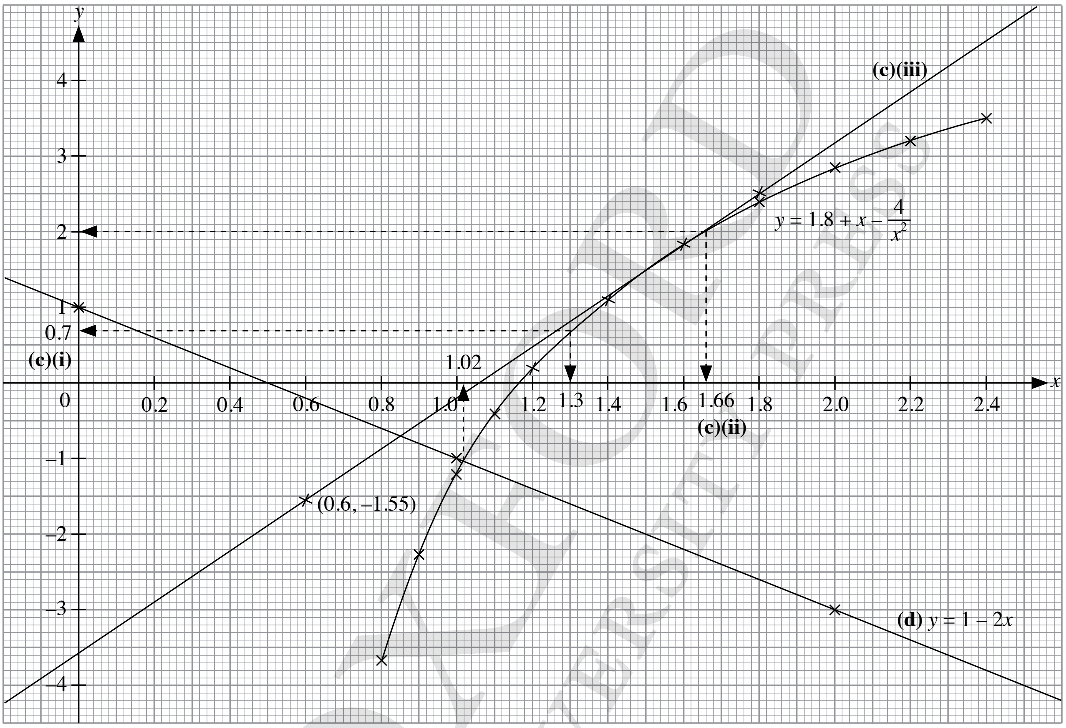
When $x = 1.1$,

$y = 1.8 + 1.1 - \frac{4}{1.1^2}$

$= -0.41$ (to 2 d.p.)

$\therefore k = -0.41$

(b)



(c) (i) When $x = 1.3$, $y = 0.70$

(ii) When $y = 2$, $x = 1.66$

(iii) Gradient $= \frac{2.55 - (-1.55)}{1.8 - 0.6}$

$= 3.42$ (to 3 s.f.)

(d) $\frac{4}{x^2} = 3x + 0.8$

$-\frac{4}{x^2} = -3x - 0.8$

$1.8 + x - \frac{4}{x^2} = 1.8 + x - 3x - 0.8$

Draw $y = 1 - 2x$

$\therefore x = 1.02$

18. **(a)** $y = 6x^2 - x^3$

When $x = 2$,

$y = 6(2)^2 - (2)^3$

$= 16$

$\therefore p = 16$

When $x = 3.5$,

$y = 6(3.5)^2 - (3.5)^3$

$= 30.6$ (to 3 s.f.)

$\therefore q = 30.6$

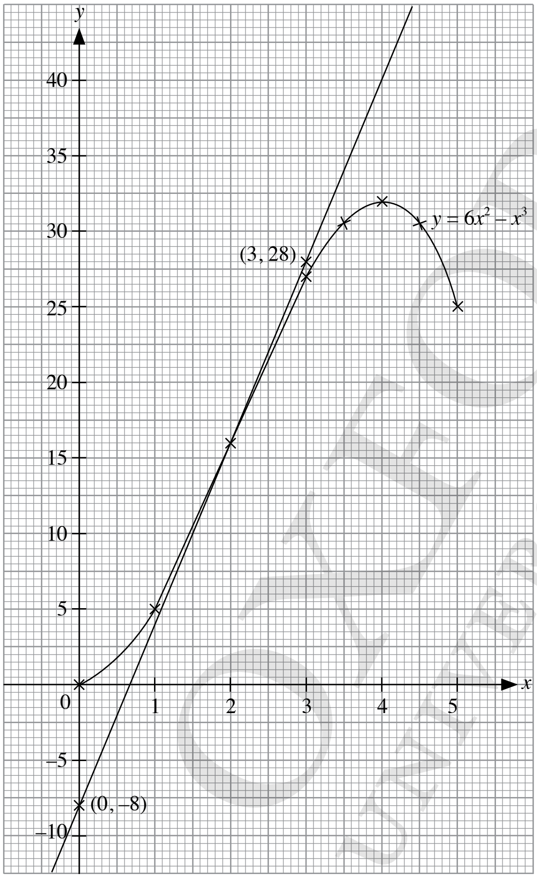
When $x = 2$,

$y = 6(4.5)^2 - (4.5)^3$

$= 30.4$ (to 3 s.f.)

$\therefore r = 30.4$

(b)



(c) **(i)** From the graph, $x = 4$ for which y is the greatest.

(ii) $2.3 \leqslant x \leqslant 5.7$

(iii) Gradient $= \dfrac{28 - (-8)}{3 - 0}$

$= 12$

(iv) $6x^2 - x^3 - 2x = 5$

$6x^2 - x^3 = 2x + 5$

Draw $y = 2x + 5$

$\therefore x = 1.25$

19. **(a)** $y = 8 - 4^x$

When $x = 0.6$,

$y = 8 - 4^{0.6}$

$= 5.7$ (to 1 d.p.)

$\therefore h = 5.7$

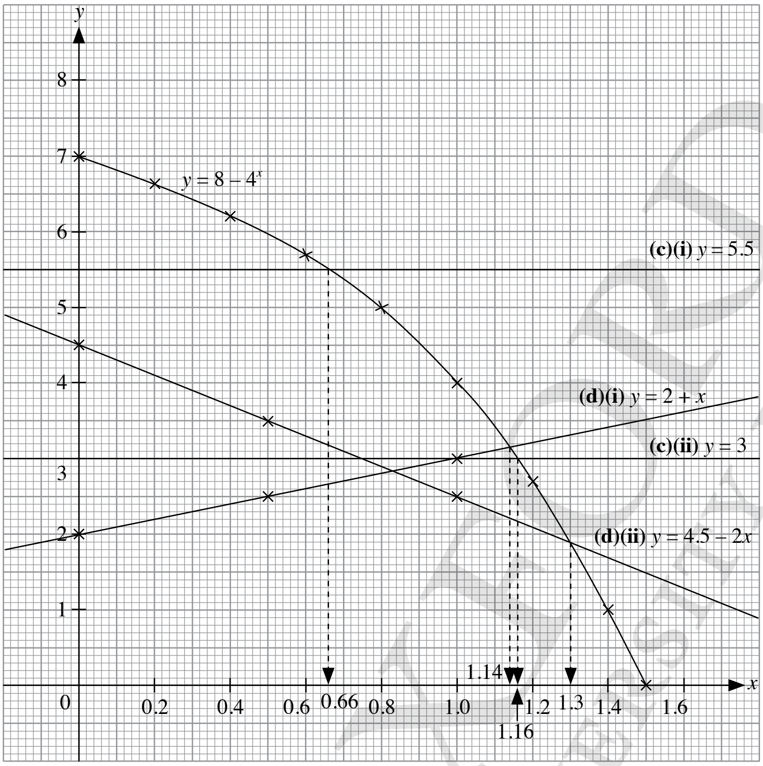
When $x = 1.2$,

$y = 8 - 4^{1.2}$

$= 2.7$ (to 1 d.p.)

$\therefore k = 2.7$

(b)



(c) **(i)** $8 - 4^x = 5.5$

Draw $y = 5.5$

$\therefore x = 0.66$

(ii) $8 - 4^x = 3$

Draw $y = 3$

$\therefore x = 1.16$

(d) **(i)** $4^x = 6 - x$

$-4^x = -6 + x$

$8 - 4^x = 2 + x$

Draw $y = 2 + x$

$\therefore x = 1.14$

(ii) $4^x = 2x + 3.5$

$-4^x = -2x - 3.5$

$8 - 4^x = 4.5 - 2x$

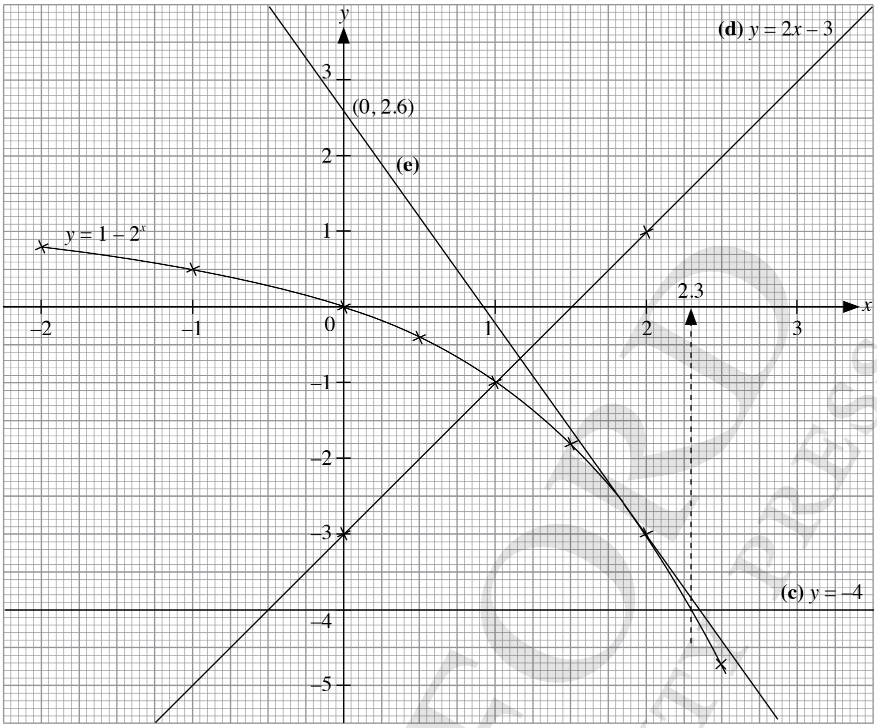
Draw $y = 4.5 - 2x$

$\therefore x = 1.3$

20. **(a)**

x	–2	–1	0	0.5	1	1.5	2	2.5
y	0.8	0.5	0	–0.4	–1	–1.8	–3	–4.7

(b)



(c) $2^x = 5$

$-2^x = -5$

$1 - 2^x = 1 - 5$

$= -4$

Draw $y = -4$

$\therefore\ x = 2.3$

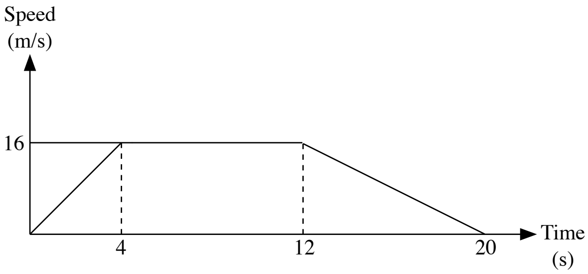
(d) The point where $y = 1 - 2^x$ and $y = 2x - 3$ meet is (1, –1).

(e) $$\text{Gradient} = \frac{2.6 - (-4)}{0 - 2.3}$$

$= -2.9$ (to 2 s.f.)

Exercise 9F

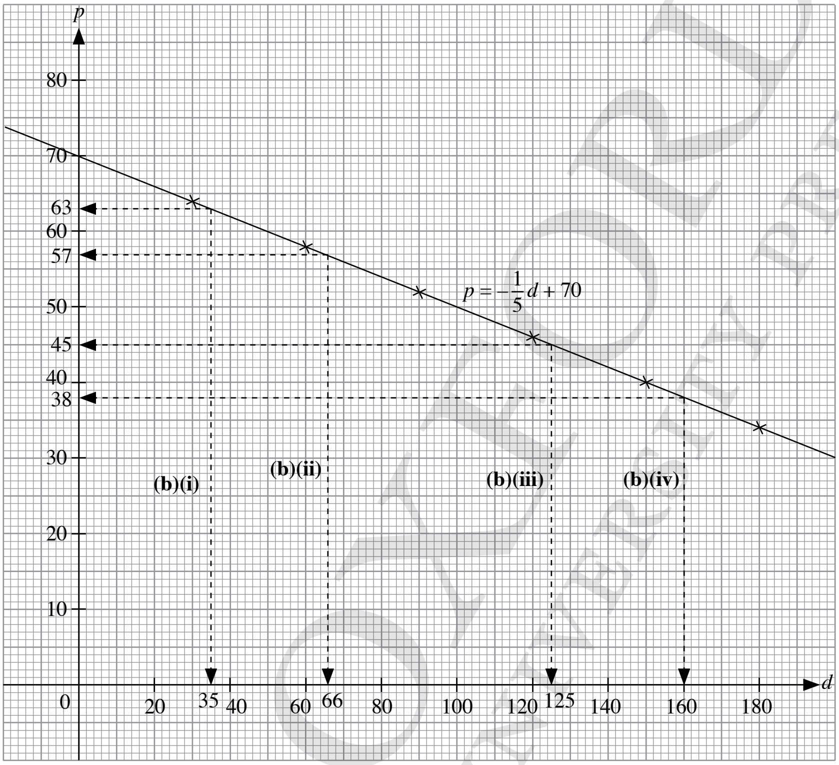
1. **(a)**



(b) Acceleration $= \dfrac{16 - 0}{4 - 0}$

$= 4 \text{ m/s}^2$

2. **(a)**



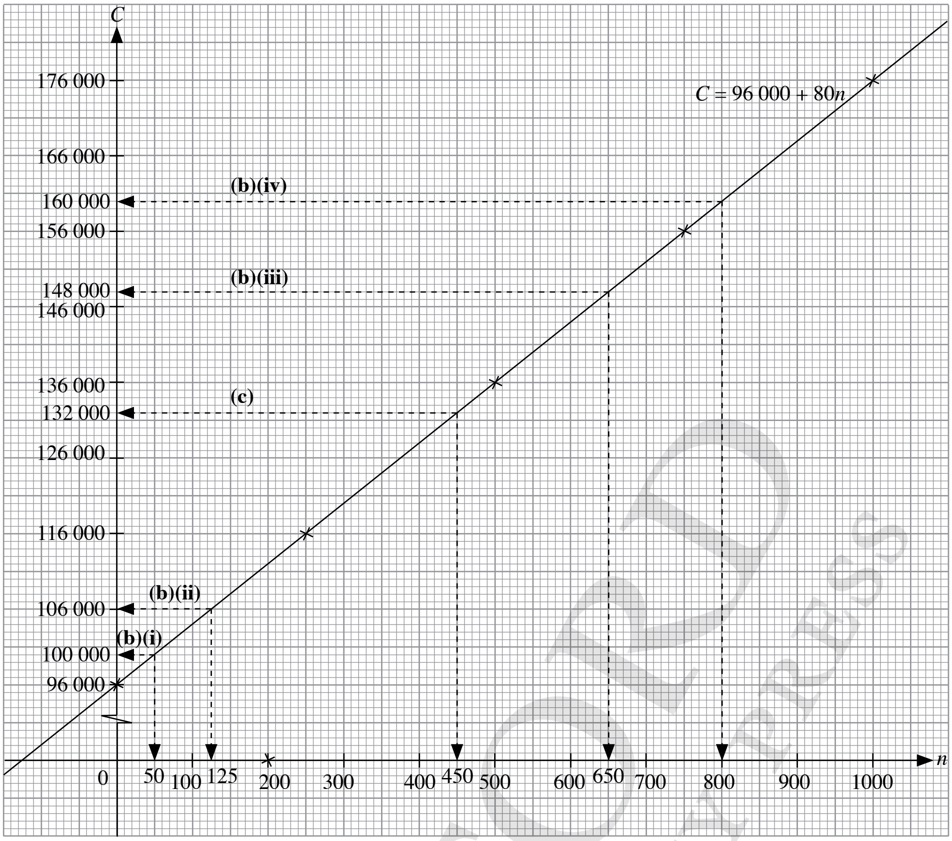
(b) **(i)** When $p = 35$, $d = 63$

(ii) When $p = 66$, $d = 57$

(iii) When $p = 125$, $d = 45$

(iv) When $p = 160$, $d = 38$

3. **(a)**



(b) **(i)** When $n = 50$, $C = \$100\ 000$

(ii) When $n = 125$, $C = \$106\ 000$

(iii) When $n = 650$, $C = \$148\ 000$

(iv) When $n = 800$, $C = \$160\ 000$

(c) From the graph,
$m = 450$

4. **(a)** Distance travelled by Train 1 = 100 km
Distance travelled by Train 2 = 500 – 360
= 140 km

(b) Speed of Train 1 = $\frac{100}{1.3}$
= 76.9 km/h (to 3 s.f.)

Speed of Train 2 = $\frac{140}{1.5}$
= 93.3 km/h (to 3 s.f.)

(c) 10 24. 280 km from Town *P*.

(d) Average speed of Train 1 = $\frac{500}{6}$

$= 83\frac{1}{3}$ km/h

Average speed of Train 2 = $\frac{500}{7}$

$= 71\frac{3}{7}$ km/h

5. **(a)** Time taken = 1 h 30 min

$$\text{Speed} = \frac{60}{1.5}$$
$$= 40 \text{ km/h}$$

(b) Time when Steven overtakes Harry = 08 42

Distance from Town A = 27 km

(c) Distance Harry travelled after 1 h = 10 km

(d) Distance Harry travelled when Steven reaches his destination = 35 km

(e) Time taken by Steven to travel 45 km = 1 h 6 min

(f) Harry′s average speed $= \frac{60}{6}$

$$= 10 \text{ km/h}$$

6. **(a)** Acceleration $= \frac{30-0}{15-0}$

$$= 2 \text{ m/s}^2$$

(b) Average speed $= \frac{10 \times 30 + \frac{1}{2} \times 20 \times 30}{30}$

$$= 20 \text{ m/s}$$

7. **(a)** Acceleration $= \frac{10-5}{4-0}$

$$= 1.25 \text{ m/s}^2$$

(b) Total distance = 180 m

$$\left[\frac{1}{2} \times (5+10) \times 4\right] + (4 \times 10)$$
$$+ \left[\frac{1}{2} \times (10+v) \times 4\right] = 180$$
$$70 + 20 + 2v = 180$$
$$2v = 90$$
$$v = 45$$

(c) Since retardation is 5 m/s^2,

$$\frac{45-0}{12-t} = -5$$
$$-9 = 12 - t$$
$$t = 21$$

8. **(a)** Acceleration $= \frac{15-8}{15-0}$

$$= \frac{7}{15} \text{ m/s}^2$$

(b) Total distance travelled

$$= (8 \times 15) + \left(\frac{1}{2} \times 15 \times 7\right) + \left(\frac{1}{2} \times 25 \times 15\right)$$
$$= 360 \text{ m}$$

(c) Average speed $= \frac{360}{40}$

$$= 9 \text{ m/s}$$

9. **(a)** Gradient at $t = 4 = \frac{40-10}{0-5}$

$$= -6$$

Let s represent the speed at $t = 4$,

$$\frac{10-s}{5-4} = -6$$
$$s = 16 \text{ m/s}$$

(b) Let x represent the speed at $t = 28$,

$$\frac{10-x}{15-28} = 2$$
$$x = 36 \text{ m/s}$$

10. **(a)** Since acceleration = 2 m/s^2,

$$\frac{v-0}{5-0} = 2$$
$$\therefore v = 10$$

(b) Since retardation = 2 × acceleration,

$$-\frac{v}{15-t} = 2 \times \frac{v}{5}$$
$$-5v = 30v - 2vt$$
$$-35v + 2vt = 0$$
$$v(2t - 35) = 0$$
$$v = 0 \text{ (NA) or } t = 17.5$$

Since total distance travelled = 275 m,

$$\frac{1}{2} \times 5 \times v + 10v + \frac{1}{2} \times 2.5 \times v = 275$$
$$2.5v + 10v + 1.25v = 275$$
$$13.75v = 275$$
$$v = 20$$

11. **(a)** **(i)** Distance travelled = 80 × 2

$$= 160 \text{ m}$$

(ii) Distance travelled

$$= \frac{1}{2} \times 20 \times 4 + \frac{1}{2} \times 40 \times 4 + 20 \times 4$$
$$= 200 \text{ m}$$

(iii) Acceleration $= \frac{4-0}{60-20}$

$$= 0.1 \text{ m/s}^2$$

(b) **(i)** In first 20 s, cyclist travelled 40 m and jogger travelled 40 m.

After $t = 20$, speed of jogger > speed of cyclist

$\therefore t_1 = 20$

(ii) From $t = 20$ to $t = 40$,

Distance travelled by cyclist $= \frac{1}{2} \times 20 \times 2$

$$= 20 \text{ m}$$

Distance travelled by jogger $= 20 \times 2$

$$= 40$$

$\therefore t_2 = 20$

(iii) The cyclist overtook the jogger at $t = 60$

$\therefore t_3 = 60$

12. **(a)** Gradient for (1) $= \dfrac{45 - 10}{0 - 300}$

$= -\dfrac{7}{60}$

Gradient for (2) $= \dfrac{40 - 30}{440 - 600}$

$= -\dfrac{1}{16}$

The gradients represent the rate of petrol usage.

(b) The difference in values of the gradient is because the rate of petrol usage during descent is lower as compared to that of ascent.

(c) Amount of petrol used $= 35 + 19$

$= 54l$

Amount he paid in S\$ $= \dfrac{54 \times 2.75}{2.99}$

$=$ S\$49.67 (to 2 d.p.)

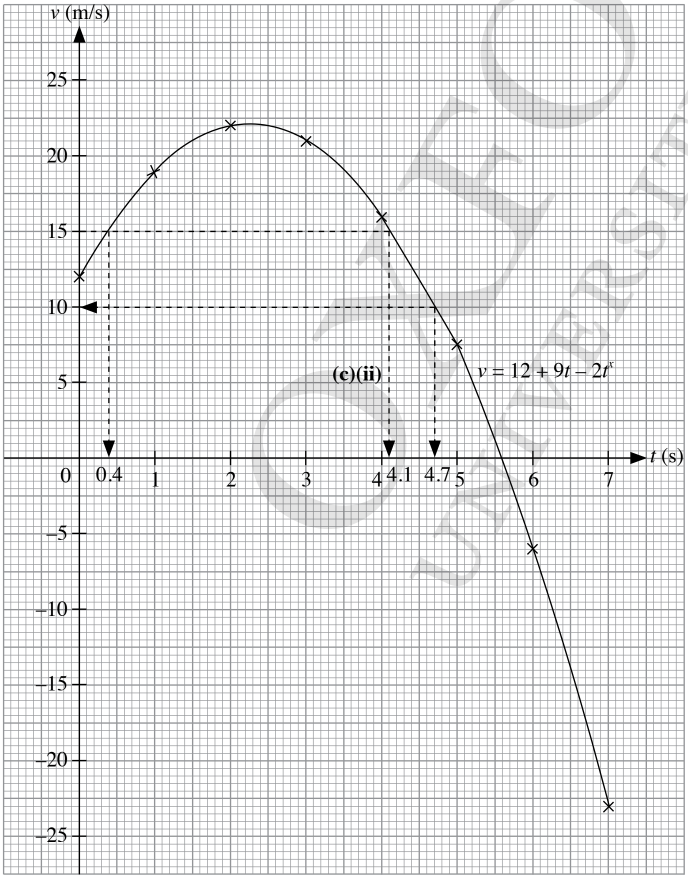
13. **(a)** $v = 12 + 9t - 2t^2$

When $t = 6$,

$v = 12 + 9(6) - 2(6)^2$

$= -6$

(b)



(c) **(i)** When $v = 0$, $t = 5.6$

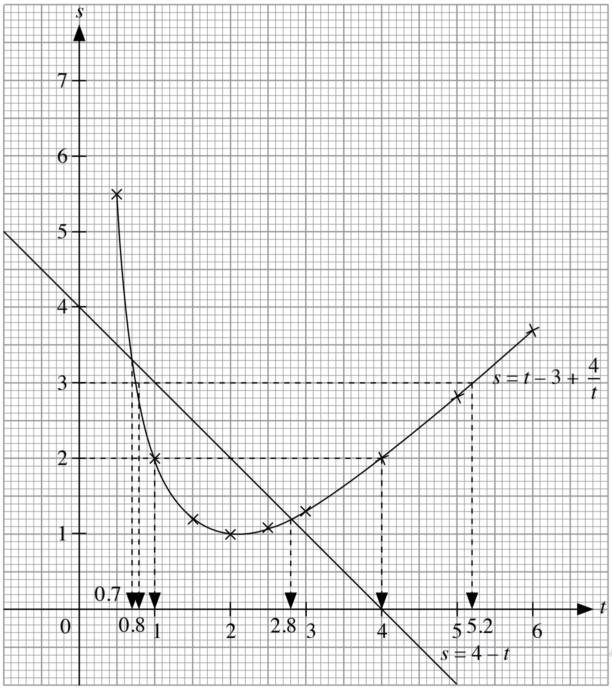
(ii) When $v = 10$, $t = 4.7$

(iii) Direction QP

(iv) Acceleration is zero at $t = 2.3$

(v) From the graph, $v > 15$ when $0.4 < t < 4$

14. (a)



(b) $t - 3 + \frac{4}{t} = 4 - t$

Draw $s = 4 - t$

$\therefore t = 0.7$ or 4

(c) (i) When $s = 3, t = 0.8$ or 5.2

(ii) $t - 3 + \frac{4}{t} < 2$

$\therefore 1 < t < 4$

$\therefore$ The particle is less than 2 m from t for 3 seconds.

15. (a) $8x - 4y = 20$ — (1)

$y = 2x - 3$ — (2)

From (1) : $2x - y = 5$ — (3)

From (2) : $2x - y = 3$ — (4)

There are no solutions because (3) and (4) contradict each other.

$\therefore$ We can also say that the straight lines representing these two equations do not intersect each other.

(b) $y = -\frac{2}{3}x + \frac{4}{3}$ — (1)

$12x + 18y = 24$ — (2)

From (1) : $3y + 2x = 4$ — (3)

From (2) : $3y + 2x = 4$ — (4)

Since (3) = (4), there is only one equation to solve for two unknowns hence, there will be an infinite number of solutions. The straight lines representing these two equations is the same line hence there are no intersections which gives rise to an infinite number of solutions.

Exercise 9G

1. (a) $A' = \{2, 4, 7, 8\}$

(b) $B' = \{1, 3, 5, 7\}$

(c) $A \cup B = \{1, 2, 3, 4, 5, 6, 8\}$

(d) $A \cap B = 6$

(e) $A \cap B' = \{1, 3, 5\}$

2. $A = \{1, 3, 5, 7, \ldots 17, 19\}$

$B = \{2, 3, 5, 7, 11, 13, 17, 19\}$

(a) $A \cap B = \{3, 5, 7, 11, 13, 17 19\}$

(b) $A \cup B = \{1, 2, 3, 5, 7, 9, 11, 13, 15, 17, 19\}$

3. (a) $A \cap B = \{x, y\}$

(b) $A \cup B = \{a, b, c, d, m, n, x, y, z, s\}$

(c) $A \cap C = \{a\}$

(d) $A \cup C = \{a, b, c, x, y, z, s\}$

(e) $A \cap D = \varnothing$

(f) $B \cap C = \{s\}$

(g) $B \cup C = \{a, m, n, x, y, s\}$

(h) $C \cap D = \{s\}$

(i) $A \cap (B \cap C) = \varnothing$

4. $\xi = \{1, 2, 3, \ldots 19, 20\}$

$A = \{2, 4, 6, 8, 10, 12\}$

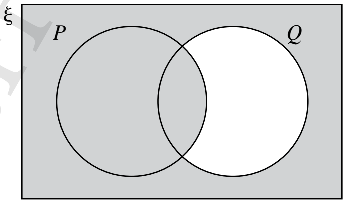
$B = \{1, 4, 9, 16\}$

(a) $A \cap B' = \{2, 6, 8, 10, 12\}$

(b) $A' \cap B = \{1, 9, 16\}$

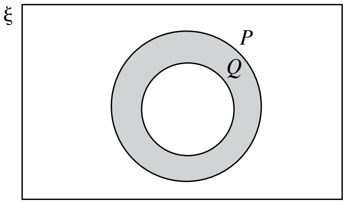
(c) $A' \cup B' = (A \cap B)' = \{1, 2, 3, 5, 6, 7, \ldots 19, 20\}$

5. (a)



$P \cup Q'$

(b)



$P \cap Q'$

6. $A = \{M, O, D, E, R, N\}, B = \{M, A, T, H, E, I, C, L\}$, $C = \{M, E, T, H, O, D\}$

(a) $B \cap C = \{M, E, T, H\}$

(b) $A \cap C' = \{R, N\}$

7. $P = \{x : x$ is a prime number less than $18\}$

$= \{2, 3, 5, 7, 11, 13, 17\}$

$E = \{x : x$ is an even number between 1 and $15\}$

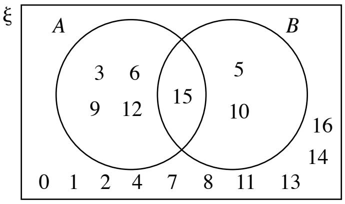
$= \{2, 4, 6, 8, 10, 12, 14\}$

$P \cup E = \{2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 17\}$

8. **(a)** $(P \cap Q)' = \{b, c\}' = \{a, d\}$

(b) $P \cup Q = \{a, b, c, d\}$

9. **(a)**



(b) $A \cap B' = \{3, 6, 9, 12\}$

(c) The set of numbers divisible by 3 and 5 are multiples of 15.

10. **(a)** $B = \{2, 4, 6, 8\}$

(b) $A \cap B = \{2, 4, 6, 8\}$

(c) $A \cup B' = \{2, 4, 6, 8, 10\}$

11. **(a), (c), (e)** are true.

12. $\xi = \{3, 4, 5, \dots 17, 18\}$

$A = \{5, 6, \dots 15, 16\}$

$P = \{3, 4, 17, 18\}$

$Q = \{3, 5, 7, 11, 13, 17\}$

(a) $A = \{5, 6, 7, 8, 9, 10, 11, 12, 13, 14\ 15, 16\}$

(b) $P \cap Q = \{3, 17\}$

13. $A = \{11, 13, 17, 19, 23, 29, 31, 37, 41\}$

$B = \{23, 29, 31, 37, 41, 43, 47\}$

(a) $A \cap B = \{2, 4, 6, 8\}$

(b) $A \cup B = \{11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47\}$

14. $\xi = \{1, 3, 5, 7, \dots 21, 23, 25\}$

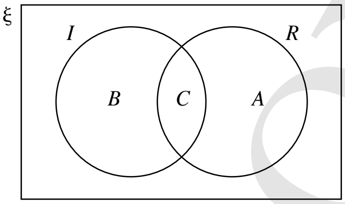
$A = \{3, 9, 15, 21\}$

$B = \{5, 15, 25\}$

(a) $A \cap B = \{15\}$

(b) $A \cap B' = \{3, 9, 21\}$

15.



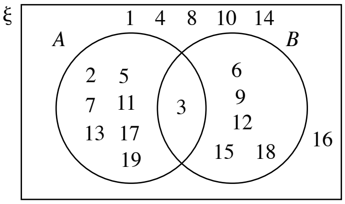
16. $\xi = \{4, 5, 6, \dots 14, 15\}$

$A = \{5, 7, 11, 13\}$

$B = \{7, 1\}$

$(A \cup B)' = \{4, 6, 8, 9, 10, 12, 15\}$

17. **(a)**



(b) **(i)** $A \cap B = \{3\}$

(ii) $A' \cup B = \{1, 3, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18\}$

18. $\xi = \{1, 2, 3, \dots 18, 19\}$

$A = \{5, 6, 7, \dots 13, 14\}$

$2x - 1 < 36, \qquad 2x - 1 > 16$

$x < 18\frac{1}{2} \qquad x > 8\frac{1}{2}$

$\therefore B = \{9, 10, 11, 12, 13, 14, 15, 16, 17, 18\}$

(a) $A' = \{1, 2, 3, 4, 15, 16, 17, 18, 19\}$

(b) $A \cup B' = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 19\}$

(c) $A' \cup B = \{15, 16, 17, 18\}$

19. **(a)** $A \cap B = A$

(b) $A \cup B = B$

20. $\xi = \{1, 2, 3, 4, 5, \dots 20, 21, 22\}$

$A = \{2, 3, 5, 7, 11, 13, 17, 19\}$

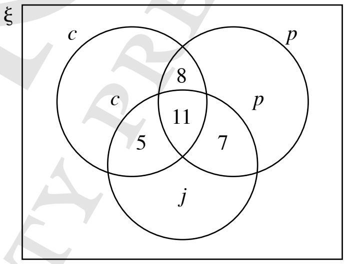
$B = \{h < x < k\}$

$\therefore$ When $h = 8$, $k = 10$ or $h = 14$, $k = 16$, $A \cap B = \varnothing$

21. Let C = {people who listen to Classical}

P = {people who listen to Pop}

J = {people who listen to Jazz}



Let c, p and j represent the number of people who listen to Classical only, Pop only and Jazz only respectively.

$c = 37 - 8 - 11 - 5$

$= 13$

$p = 46 - 8 - 11 - 7$

$= 20$

$j = 28 - 11 - 5 - 7$

$= 5$

Total number of people surveyed

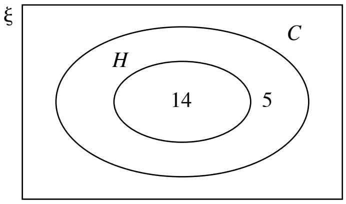
$= 13 + 8 + 11 + 5 + 20 + 7 + 5$

$= 69$

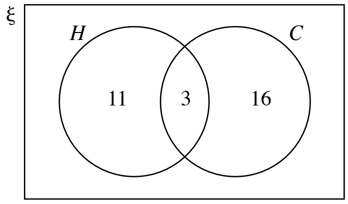
22. Let H = {students who enjoy watching horror movies}

C = {students who enjoy watching comedies}

(i) $n(H \cap C)$ will have the greatest value when $H \subseteq C$. Greatest possible number of students who enjoy both horror movies and comedies = 14



(ii) $n(H \cap C)$ will have the least value when $H \cap C \neq \varnothing$.
Least possible number of students who enjoy both horror movies and comedies = 3



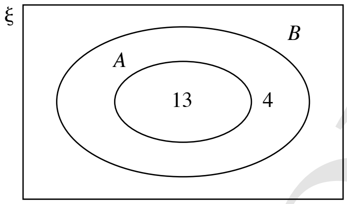
(iii) Greatest possible number of students who enjoy only one genre of movie
$= 11 + 16$
$= 27$

23. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$22 = 19 + n(B) - 9$
$n(B) = 12$

24. **(i)** $n(A \cap B)$ will have the greatest value when $A \subseteq B$.
Greatest value of $n(A \cap B) = n(A)$
$= 11$
$n(A \cap B)$ will have the least value when $A \cap B \neq \varnothing$.
Least value of $n(A \cap B) = 4$

(ii) $n(A \cup B)$ will have the greatest value when $A \cap B \neq \varnothing$.
Greatest value of $n(A \cup B) = n(\xi)$
$= 25$
$n(A \cup B)$ will have the least value when $A \subseteq B$.
Least value of $n(A \cup B) = n(B)$
$= 18$

25.



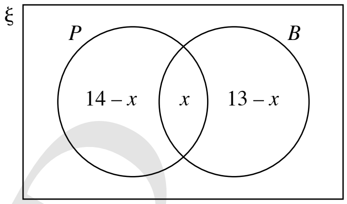
(i) $n(A \cap B) = n(A)$
$= 13$

(ii) $n(A \cup B) = n(B)$
$=17$

26. $n(A \cup B)'$ will have the greatest value when $n(A \cup B)$ has the least value.
$n(A \cup B)$ will have the least value when $B \subseteq A$.
Least value of $n(A \cup B) = n(A)$
$= 32$
Greatest value of $n(A \cup B)' = 45 - 32$
$= 13$

27. $n(A \cap B)'$ will have the least value when $n(A \cap B)$ has the greatest value.
$n(A \cap B)$ will have the greatest value when $B \subseteq A$.
Greatest value of $n(A \cap B) = n(B)$
$= 25$
Least value of $n(A \cap B)' = 33 - 25$
$= 8$

28. Let P = {people who do not consume pork}
B = {people who do not consume beef}



Let x represent the number of people who do not consume both beef and pork.
$(14 - x) + x + (13 - x) = 25$
$27 - x = 25$
$x = 2$
∴ There are 2 people who do not consume both beef and pork.

Exercise 9H

1. **(a)** $3 + 2 = y$ $\quad x + 2 - 5 = 7$
$h + 3 = -4$ $\quad k + 3k + 1 = 9$
$\therefore x = 10, y = 5, h = -7, k = 2$

(b) $m - 5 = -4m + 9$
$n + 7 = 3n - n - 4$
$\therefore m = 2.8, n = 11$

(c) $x - 3y = 6$ — (1)
$y - x = -8$ — (2)
$3z = x$ — (3)
$9 = 2x - 9y$ — (4)
$xy = 9$ — (5)
From (2) : $y = -8 + x$ — (6)
Substitute (6) into (1) : $x - 3(-8 + x) = 6$
$x = 9$
Substitute $x = 9$ into (6) : $y = 1$
Substitute $x = 9$ into (3) : $z = 3$
$\therefore y = 1, x = 9, z = 3$

(d) $p + q - 2p - 14 = q + 2 - 5p \quad — (1)$

$3h - p - 5 = 7 + h - p - 2 \quad — (2)$

$4 - k - 3h = 2h - 3k \quad — (3)$

From (1) : $4p = 16$

$p = 4$

From (2) : $2h = 10$

$h = 5$

From (3) : $2k = 5(5) - 5$

$k = 10$

$\therefore$ $p = 4$, $h = 5$, $k = 10$ and q can be of any value.

2. **(a)** $\begin{pmatrix}1&2\\3&5\end{pmatrix}\begin{pmatrix}0&-1\\3&-2\end{pmatrix}=\begin{pmatrix}6&-5\\15&-13\end{pmatrix}$

(b) $\begin{pmatrix}4&2\\-1&-3\end{pmatrix}\begin{pmatrix}2\\-1\end{pmatrix}=\begin{pmatrix}6\\1\end{pmatrix}$

(c) $\begin{pmatrix}-2&-1&0\\3&4&-3\end{pmatrix}\begin{pmatrix}1&2\\0&4\\-1&3\end{pmatrix}=\begin{pmatrix}-2&-8\\6&13\end{pmatrix}$

(d) $\begin{pmatrix}1&2\\-1&0\end{pmatrix}\begin{pmatrix}2&1&5\\3&4&0\end{pmatrix}=\begin{pmatrix}8&9&5\\-2&-1&-5\end{pmatrix}$

(e) $\begin{pmatrix}2&-3\\1&-4\\0&-2\end{pmatrix}\begin{pmatrix}3&2&0\\1&-1&4\end{pmatrix}=\begin{pmatrix}3&7&-12\\-1&6&-16\\-2&2&-8\end{pmatrix}$

(f) $(4\quad 3)\begin{pmatrix}2&-3&6\\1&4&-1\end{pmatrix}=(11\quad 0\quad 21)$

3. **(a)** $\begin{pmatrix}2&3\\1&4\end{pmatrix}\begin{pmatrix}-1\\5\end{pmatrix}=\begin{pmatrix}13\\19\end{pmatrix}$

(b) $\begin{pmatrix}2&1\\5&3\end{pmatrix}\begin{pmatrix}3&-1\\2&-5\end{pmatrix}=\begin{pmatrix}8&-7\\21&-20\end{pmatrix}$

(c) NA

(d) $\begin{pmatrix}2&-3\\4&7\end{pmatrix}\begin{pmatrix}3&5\\7&9\end{pmatrix}=\begin{pmatrix}-15&-17\\61&83\end{pmatrix}$

(e) NA

(f) $(1\quad 5)\begin{pmatrix}2\\1\end{pmatrix}=(7)$

(g) $\begin{pmatrix}3\\1\end{pmatrix}(2\quad 4)=\begin{pmatrix}6&12\\2&4\end{pmatrix}$

(h) NA

(i) $(3\quad 1\quad -2)\begin{pmatrix}1\\3\\-1\end{pmatrix}=(8)$

(j) $\begin{pmatrix}2\\5\\-1\end{pmatrix}(1\quad 5\quad 7)=\begin{pmatrix}2&10&14\\5&25&35\\-1&-5&-7\end{pmatrix}$

(k) NA

(l) $\begin{pmatrix}2&1&3\\4&-1&2\end{pmatrix}\begin{pmatrix}1\\4\\-1\end{pmatrix}=\begin{pmatrix}3\\-2\end{pmatrix}$

(m) $\begin{pmatrix}2&1\\3&5\\4&2\end{pmatrix}\begin{pmatrix}2&1\\-1&3\end{pmatrix}=\begin{pmatrix}3&5\\1&18\\6&10\end{pmatrix}$

(n) $\begin{pmatrix}1&4\\-1&-3\\4&2\end{pmatrix}\begin{pmatrix}1&2&4\\-3&-1&-2\end{pmatrix}=\begin{pmatrix}-11&-2&-4\\8&1&2\\-2&6&12\end{pmatrix}$

4. **(a)** $\begin{pmatrix}3&-1\\1&4\end{pmatrix}\begin{pmatrix}1&x\\-2&y\end{pmatrix}=\begin{pmatrix}1&x\\-2&y\end{pmatrix}\begin{pmatrix}3&-1\\1&4\end{pmatrix}$

$\begin{pmatrix}5&3x-y\\-7&x+4y\end{pmatrix}=\begin{pmatrix}3+x&-1+4x\\-6+y&2+4y\end{pmatrix}$

$5 = 3 + x$

$\therefore\ x = 2$

$-7 = -6 + y$

$\therefore\ y = -1$

(b) $\begin{pmatrix}3&0&1\\-1&2&4\end{pmatrix}\begin{pmatrix}1&2\\0&-1\\-1&3\end{pmatrix}+\begin{pmatrix}3\\1\end{pmatrix}(2\quad 3)=\begin{pmatrix}a&b\\2x&3y\end{pmatrix}$

$\begin{pmatrix}2&9\\-5&8\end{pmatrix}+\begin{pmatrix}6&9\\2&3\end{pmatrix}=\begin{pmatrix}a&b\\2x&3y\end{pmatrix}$

$\begin{pmatrix}8&18\\-3&11\end{pmatrix}=\begin{pmatrix}a&b\\2x&3y\end{pmatrix}$

$\therefore\ a = 8, b = 18, x = -1.5, y = 3\frac{2}{3}$

5. $\begin{pmatrix}3&x\\4&y\end{pmatrix}=\begin{pmatrix}3&x\\4&y\end{pmatrix}\begin{pmatrix}3&x\\4&y\end{pmatrix}$

$=\begin{pmatrix}9+4x&3x+xy\\12+4y&4x+y^2\end{pmatrix}$

$3 = 9 + 4x$

$\therefore\ x = -1.5$

$4 = 12 + 4y$

$\therefore\ y = -2$

6. **(a)** $\begin{pmatrix}3&1\\0&4\end{pmatrix}\begin{pmatrix}1\\-3\end{pmatrix}+\begin{pmatrix}4\\-1\end{pmatrix}=\begin{pmatrix}2x\\y\end{pmatrix}$

$\begin{pmatrix}0\\-12\end{pmatrix}+\begin{pmatrix}4\\-1\end{pmatrix}=\begin{pmatrix}2x\\y\end{pmatrix}$

$\therefore\ x = 2, y = -13$

(b) $\begin{pmatrix}3&h&2h+1\\7&2k&-1\end{pmatrix}\begin{pmatrix}k\\-1\\0\end{pmatrix}=\begin{pmatrix}7\\15\end{pmatrix}$

$\begin{pmatrix}3k-h\\7k-2k\end{pmatrix}=\begin{pmatrix}7\\15\end{pmatrix}$

$\therefore\ k = 3, h = 2$

7. **(a)** $(p \quad q)\begin{pmatrix} 3 & 0 \\ -4 & 2 \end{pmatrix} = (-3 \quad 6)$

$(3p - 4q \quad 2q) = (-3 \quad 6)$

$\therefore q = 3, p = 3$

(b) $\begin{pmatrix} a & b \\ 3 & 2a \end{pmatrix}\begin{pmatrix} 1 \\ 4 \end{pmatrix} = \begin{pmatrix} 15 \\ 11 \end{pmatrix}$

$\begin{pmatrix} a + 4b \\ 3 + 8a \end{pmatrix} = \begin{pmatrix} 15 \\ 11 \end{pmatrix}$

$\therefore a = 1, b = 3\frac{1}{2}$

(c) $\begin{pmatrix} x & 2 \\ 2z & 0 \end{pmatrix}\begin{pmatrix} 4 \\ -1 \end{pmatrix} = \begin{pmatrix} -2 \\ 8 \end{pmatrix}$

$\begin{pmatrix} 4x - 2 \\ 8z \end{pmatrix} = \begin{pmatrix} -2 \\ 8 \end{pmatrix}$

$\therefore z = 1, x = 0$

(d) $\begin{pmatrix} 3 & x \\ 5 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} y \\ -10 \end{pmatrix}$

$\begin{pmatrix} 3x + x^2 \\ 5x \end{pmatrix} = \begin{pmatrix} y \\ -10 \end{pmatrix}$

$\therefore x = -2, y = -6 + 4 = -2$

(e) $\begin{pmatrix} x & 3 \\ -1 & 4 \end{pmatrix}\begin{pmatrix} 4 & y \\ 1 & x \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$\begin{pmatrix} 4x + 3 & xy + 3x \\ 0 & -y + 4x \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$4x + 3 = 1$

$\therefore x = -0.5$

$-y - 2 = 1$

$\therefore y = -3$

8. **(i)** If $p = \frac{1}{2}$, $\mathbf{A} = \begin{pmatrix} 2 & 6 \\ -1 & \frac{1}{2} \end{pmatrix}$

$$\mathbf{A}^2 = \begin{pmatrix} 2 & 6 \\ -1 & \frac{1}{2} \end{pmatrix}\begin{pmatrix} 2 & 6 \\ -1 & \frac{1}{2} \end{pmatrix}$$

$$= \begin{pmatrix} -2 & 15 \\ -2\frac{1}{2} & -5\frac{3}{4} \end{pmatrix}$$

(ii) If $p = 4$, $\mathbf{A} = \begin{pmatrix} 2 & 6 \\ -1 & 4 \end{pmatrix}$

$|\mathbf{A}| = (2 \times 4) - (6 \times -1)$

$= 14$

$$\mathbf{A}^{-1} = \frac{1}{14}\begin{pmatrix} 4 & -6 \\ 1 & 2 \end{pmatrix}$$

(iii) For **A** to not have an inverse, $|\mathbf{A}| = 0$.

$|\mathbf{A}| = 2p + 6 = 0$

$p = -3$

9. **(a)** **(i)** $2\begin{pmatrix} 1 & -2 \\ 3 & 0 \end{pmatrix} - \begin{pmatrix} -5 & 2 \\ 0 & 3 \end{pmatrix}$

$$= \begin{pmatrix} 2 & -4 \\ 6 & 0 \end{pmatrix} - \begin{pmatrix} -5 & 2 \\ 0 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 7 & -6 \\ 6 & -3 \end{pmatrix}$$

(ii) $\begin{pmatrix} 1 & 3 & 4 \end{pmatrix}\begin{pmatrix} 0 & 4 \\ 3 & 1 \\ 5 & 0 \end{pmatrix}$

$$= \begin{pmatrix} 29 & 7 \end{pmatrix}$$

(b) **(i)** Since determinant = 15,

$w(w + 1) - (-1)(2w + 5) = 15$

$w^2 + w + 2w + 5 = 15$

$w^2 + 3w - 10 = 0$ (shown)

(ii) $w^2 + 3w - 10 = 0$

$(w - 2)(w + 5) = 0$

$w - 2 = 0$ or $w + 5 = 0$

$w = 2 \qquad w = -5$

10. $|\mathbf{A}| = (3 \times 4) - (11 \times 1)$

$= 1$

$$\mathbf{A}^{-1} = \begin{pmatrix} 4 & -11 \\ -1 & 3 \end{pmatrix}$$

$\mathbf{AP} = \mathbf{B}$

$\mathbf{A}^{-1}\mathbf{AP} = \mathbf{A}^{-1}\mathbf{B}$

$$\mathbf{P} = \begin{pmatrix} 4 & -11 \\ -1 & 3 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} -3 & -18 \\ 1 & 5 \end{pmatrix}$$

11. **(a)** The simultaneous equations may be written in matrix form as

$$\begin{pmatrix} 3 & 9 \\ 6 & 18 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

Determinant of $\begin{pmatrix} 3 & 9 \\ 6 & 18 \end{pmatrix} = (3 \times 18) - (9 \times 6)$

$= 0$

Hence $\begin{pmatrix} 3 & 9 \\ 6 & 18 \end{pmatrix}$ is a singular matrix and its inverse matrix does not exist.

The graphs of $3x + 9y = 1$ and $6x + 18y = 2$ represent the same line. There is an infinite number of solutions since the two lines coincide. Some solutions include $\left(0, \frac{1}{9}\right)$ and $\left(1, -\frac{2}{9}\right)$.

(b) The simultaneous equations may be written in matrix form as

$$\begin{pmatrix} 8 & 2 \\ 12 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 7 \end{pmatrix}$$

Determinant of $\begin{pmatrix} 8 & 2 \\ 12 & 3 \end{pmatrix} = (8 \times 3) - (2 \times 12)$

$= 0$

Hence $\begin{pmatrix} 8 & 2 \\ 12 & 3 \end{pmatrix}$ is a singular matrix and its inverse matrix does not exist. Then graphs of $8x + 2y = 5$ and $12x + 3y = 7$ represent two parallel lines. There is no solution since there is no intersection between the two lines.

(c) The simultaneous equations may be written in matrix form as

$$\begin{pmatrix} 6 & -1 \\ 4 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 9 \\ -10 \end{pmatrix}$$

Determinant of $\begin{pmatrix} 6 & -1 \\ 4 & -2 \end{pmatrix} = (6 \times -2) - (-1 \times 4)$

$= -8$

$$\begin{aligned} \begin{pmatrix} x \\ y \end{pmatrix} &= \begin{pmatrix} 6 & -1 \\ 4 & -2 \end{pmatrix}^{-1}\begin{pmatrix} 9 \\ -10 \end{pmatrix} \\ &= \frac{1}{-8}\begin{pmatrix} -2 & 1 \\ -4 & 6 \end{pmatrix}\begin{pmatrix} 9 \\ -10 \end{pmatrix} \\ &= \frac{1}{-8}\begin{pmatrix} -28 \\ -96 \end{pmatrix} \\ &= \begin{pmatrix} 3.5 \\ 12 \end{pmatrix} \end{aligned}$$

$\therefore x = 3.5, y = 12$

The graphs of $6x - y - 9 = -$ and $4x - 2y + 10 = 0$ intersect at the point (3.5, 12).

12. (a) $\mathbf{B} = \begin{pmatrix} 35 & 40 & 28 \\ 16 & 65 & 38 \end{pmatrix}$

(b) $$\begin{aligned} \frac{1}{2}(\mathbf{A} + \mathbf{B}) &= \frac{1}{2}\begin{pmatrix} 25+35 & 30+40 & 40+28 \\ 32+16 & 15+65 & 20+38 \end{pmatrix} \\ &= \begin{pmatrix} 30 & 35 & 34 \\ 24 & 40 & 29 \end{pmatrix} \end{aligned}$$

It represents the average number of assessment books sold over two weeks.

(c) $\mathbf{S} = \begin{pmatrix} 25 & 30 & 40 \\ 32 & 15 & 20 \end{pmatrix}\begin{pmatrix} 8.50 \\ 6.50 \\ 9.80 \end{pmatrix} = \begin{pmatrix} 799.50 \\ 565.50 \end{pmatrix}$

It represents the total sales by A star and Excellence books.

(d) $\mathbf{C} = (1 \quad 1)\begin{pmatrix} 35 & 40 & 28 \\ 16 & 65 & 38 \end{pmatrix} = (51 \quad 105 \quad 66)$

It represents the total number of Math, A Math and Physics books sold in week 2.

13. (a) $\mathbf{Q} = \begin{pmatrix} 430 & 370 & 520 \\ 250 & 360 & 280 \end{pmatrix}$

(b) $\mathbf{P} + \mathbf{Q} = \begin{pmatrix} 750 & 790 & 900 \\ 660 & 900 & 670 \end{pmatrix}$

(c) It represents the total collection from each level from the two schools.

(d) $\mathbf{S} = (1 \quad 1)\begin{pmatrix} 320 & 420 & 380 \\ 410 & 540 & 390 \end{pmatrix} = (730 \quad 960 \quad 770)$

(e) It represents the total collection from each level from school *ABC* for the two homes.

(f) $\mathbf{T} = \begin{pmatrix} 320 & 420 & 380 \\ 410 & 540 & 390 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1120 \\ 1340 \end{pmatrix}$

It represents the total collection from the three levels for each of the homes.

14. (a) $\mathbf{T} = \mathbf{P} + \mathbf{Q} = \begin{pmatrix} 70 & 50 & 41 \\ 55 & 72 & 89 \end{pmatrix}$

The elements represent the total sales of the different types of coffee in the morning and afternoon.

(b) $\mathbf{PC} = \begin{pmatrix} 28 & 20 & 15 \\ 20 & 30 & 35 \end{pmatrix}\begin{pmatrix} 1.50 \\ 1.60 \\ 1.70 \end{pmatrix} = \begin{pmatrix} 99.50 \\ 137.50 \end{pmatrix}$

The elements represent the cost of coffee in the two outlets in the morning.

(c) $\mathbf{T}(\mathbf{S} - \mathbf{C}) = \begin{pmatrix} 70 & 50 & 41 \\ 55 & 72 & 89 \end{pmatrix}\begin{pmatrix} 2.00 \\ 2.20 \\ 2.40 \end{pmatrix} = \begin{pmatrix} 388.40 \\ 482.00 \end{pmatrix}$

The elements represent the total profit made by the two outlets in a day.

15. $$\begin{pmatrix} 20 & 45 & 55 \\ 35 & 40 & 46 \\ 42 & 28 & 64 \\ 54 & 48 & 38 \\ 60 & 74 & 50 \end{pmatrix}\begin{pmatrix} 14 \\ 20 \\ 18 \end{pmatrix} = \begin{pmatrix} 2170 \\ 2118 \\ 2300 \\ 2400 \\ 3220 \end{pmatrix}$$

$$(24 \quad 32 \quad 36 \quad 18 \quad 12)\begin{pmatrix} 2170 \\ 2118 \\ 2300 \\ 2400 \\ 3220 \end{pmatrix} = (284\,496)$$

The total cost of the floor tiles used for all units is \$284 496.

16. $\begin{pmatrix} 9 & 1 & 8 & 12 \\ 10 & 2 & 5 & 9 \\ 7 & 4 & 9 & 7 \\ 11 & 2 & 6 & 11 \\ 8 & 3 & 3 & 8 \end{pmatrix}\begin{pmatrix} 7 \\ 3 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 86 \\ 90 \\ 77 \\ 100 \\ 76 \end{pmatrix}$

17. (a) $\mathbf{AD} = \begin{pmatrix} 8 & 3 & 4 \\ 7 & 9 & 7 \\ 4 & 5 & 9 \\ 6 & 8 & 5 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 15 \\ 23 \\ 18 \\ 19 \end{pmatrix}$

It represents the total number of medals won by each house.

(b) $\mathbf{EB} = (8 \quad 3 \quad 4)\begin{pmatrix} 10 \\ 8 \\ 5 \end{pmatrix} = (124)$

It represents the total amount of book vouchers won by Red House.

(c) $\mathbf{CAB} = (1 \quad 1 \quad 1 \quad 1)\begin{pmatrix} 8 & 3 & 4 \\ 7 & 9 & 7 \\ 4 & 5 & 9 \\ 6 & 8 & 5 \end{pmatrix}\begin{pmatrix} 10 \\ 8 \\ 5 \end{pmatrix} = (575)$

It represents the total amount of book vouchers given out to all the four houses.

18. (a) $\mathbf{PQ} = \begin{pmatrix} 32 & 26 & 20 \\ 42 & 46 & 38 \end{pmatrix}\begin{pmatrix} 3.20 \\ 4.50 \\ 4.80 \end{pmatrix} = \begin{pmatrix} 315.40 \\ 523.80 \end{pmatrix}$

It represents the total sales for each of the 2 days.

(b) $\mathbf{R} = \begin{pmatrix} 0.75 & 0 & 0 \\ 0 & 0.7 & 0 \\ 0 & 0 & 0.6 \end{pmatrix}$

$\mathbf{RQ} = \begin{pmatrix} 0.75 & 0 & 0 \\ 0 & 0.7 & 0 \\ 0 & 0 & 0.6 \end{pmatrix}\begin{pmatrix} 3.20 \\ 4.50 \\ 4.80 \end{pmatrix} = \begin{pmatrix} 2.40 \\ 3.15 \\ 2.88 \end{pmatrix}$

(c) $\mathbf{PRQ} = \begin{pmatrix} 32 & 26 & 20 \\ 42 & 46 & 38 \end{pmatrix}\begin{pmatrix} 2.40 \\ 3.15 \\ 2.88 \end{pmatrix} = \begin{pmatrix} 216.30 \\ 355.14 \end{pmatrix}$

It represents the sales after discounts were given for each of the two days.

(d) $\mathbf{MPRQ} = (1 \quad 1)\begin{pmatrix} 216.30 \\ 355.14 \end{pmatrix} = (571.44)$

It represents the total sales for the two days.

19. (a) $\mathbf{P} = \mathbf{AB} = \begin{pmatrix} 20 & 18 & 16 \\ 24 & 12 & 20 \\ 30 & 25 & 15 \\ 18 & 30 & 20 \end{pmatrix}\begin{pmatrix} 50 \\ 40 \\ 70 \end{pmatrix} = \begin{pmatrix} 2840 \\ 3080 \\ 3550 \\ 3500 \end{pmatrix}$

It represents the total cost (in cents) of the fruits in each of the four weeks.

(b) (i) $\mathbf{Q} = \mathbf{CA} = (1 \quad 1 \quad 1 \quad 1)\begin{pmatrix} 20 & 18 & 16 \\ 24 & 12 & 20 \\ 30 & 25 & 15 \\ 18 & 30 & 20 \end{pmatrix}$

$= (92 \quad 85 \quad 71)$

It represents the total number of apples, oranges and pears bought in the four weeks.

(ii) $\mathbf{CP} = (1 \quad 1 \quad 1 \quad 1)\begin{pmatrix} 2840 \\ 3080 \\ 3550 \\ 3500 \end{pmatrix} = (12\,970)$

It represents the total amount spent (in cents) in the four weeks on fruits by Mr Ong.

20. (a) $\mathbf{AB} = \begin{pmatrix} 85 & 150 & 90 & 75 \\ 110 & 180 & 100 & 105 \\ 140 & 300 & 105 & 125 \end{pmatrix}\begin{pmatrix} 70 \\ 80 \\ 120 \\ 90 \end{pmatrix}$

$= \begin{pmatrix} 35\,500 \\ 43\,550 \\ 49\,650 \end{pmatrix}$

It represents the sales (in cents) in each of the three days in July.

(b) $\mathbf{DN} = \begin{pmatrix} 80 & 160 & 100 & 70 \\ 110 & 170 & 110 & 130 \\ 120 & 210 & 100 & 120 \end{pmatrix}\begin{pmatrix} 80 \\ 90 \\ 120 \\ 100 \end{pmatrix}$

$= \begin{pmatrix} 39\,800 \\ 50\,300 \\ 52\,500 \end{pmatrix}$

It represents the sales (in cents) in each of the three days in November.

(c) $\mathbf{K} = (1 \quad 1 \quad 1)$

$\mathbf{KAB} = (1 \quad 1 \quad 1)\begin{pmatrix} 35\,500 \\ 43\,550 \\ 49\,650 \end{pmatrix} = (128\,700)$

21. (a) $\mathbf{AB} = \begin{pmatrix} 350 & 820 & 320 \\ 280 & 920 & 250 \end{pmatrix}\begin{pmatrix} 2800 & 1200 \\ 3800 & 1800 \\ 2600 & 2200 \end{pmatrix}$

$= \begin{pmatrix} 4\,928\,000 & 2\,600\,000 \\ 4\,930\,000 & 2\,542\,000 \end{pmatrix}$

It represents the total profits earned by the two plantations in the years 2007 and 2008.

(b) $\mathbf{AC} = \begin{pmatrix} 350 & 820 & 320 \\ 280 & 920 & 250 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} 1490 \\ 1450 \end{pmatrix}$

It represents the total number of hectares planted with the three crops in the two plantations.

(c) $(280 \quad 920 \quad 250)\begin{pmatrix} 1200 \\ 1800 \\ 2200 \end{pmatrix} = (2\,542\,000)$

Chapter 10 Geometry and Measurement

Exercise 10A

1. $\angle OAC = 2.5x$ ($\angle$ at centre $= 2\ \angle$ at circumference)

$2.5x° + 2x° + 90° = 180°$ ($\angle$ sum of a $\triangle$)

$x = 20$

Since $OA = OB$ (radii of circle),

$\angle OAC = \angle OBC$

$2y° = 2.5x°$

$y = 1.25(20)$

$= 25$

$\therefore x = 20, y = 25$

2. **(a)** $\angle ABC = 180° - 28° - 28° = 124°$ ($\angle$ sum of a $\triangle$)

(b) $\angle ACD = 93° - 28° = 65°$

$\angle ADK = \angle CDK$

$= 180° - 90° - 65°$ ($\angle$ sum of a $\triangle$)

$= 25°$

3. $\angle BEC = 57°$ (alt. $\angle$s)

$x° = 180° - 57° - 73°$

$= 50°$ ($\angle$ sum of a $\triangle$)

$y° = 180° - 22° - 57°$

$= 101°$ (alt. $\angle$s, adj. $\angle$s on a str. line)

$y° = z° + 73°$ (ext. $\angle$ = sum of int. opp. $\angle$s)

$z° = 101° - 73°$

$= 28°$

$\therefore x = 50, y = 101, z = 28$

4. **(a)** $m = (180° - 80°) + 25°$

$= 125°$ (int. $\angle$s, alt. $\angle$s)

(b) $s = 140° - (120° - 65°)$

$= 85°$ (alt. $\angle$s)

5. **(a)** $85° = 43° + e$ (corr. $\angle$s, ext. $\angle$ = sum of int. opp. $\angle$s)

$e = 42°$

(b) $k = 180° - 52° - 78°$

$= 50°$ (vert. opp. $\angle$s, adj. $\angle$s on a str. line)

$l = 52°$ (alt. $\angle$s)

6. $x° + 25° = 4x° - 20°$ (opp. $\angle$s of //gram)

$3x° = 45°$

$x = 15$

$\angle ABC = 180° - 15° - 25°$

$= 140°$ (int. $\angle$s, $AD \parallel BC$)

$2y° = 140°$ (base $\angle$s of isos. $\triangle$, ext. $\angle$ = sum of

$y = 70$ int. opp. $\angle$s)

7. $y = 70°$ (alt. $\angle$s)

$x + 70° = 135°$ (ext. $\angle$ = sum of int. opp. $\angle$s)

$z = x$

$= 65°$ (corr. $\angle$s)

8. **(a)** $\angle ABC = \dfrac{(6-2)\times 180°}{6} = 120°$

(b) $\angle ACB = \dfrac{180° - 120°}{2}$

$= 30°$ ($AB = AC$, base $\angle$s of isos. $\triangle$)

(c) $\angle ADE = \dfrac{120°}{2} = 60°$

9. Sum of ext. $\angle$s $= 360°$

$37° + 47° + 73° + (n-3)\times 29° = 360°$

$29n = 290°$

$n = 10$

10. **(a)** $\angle EBC = 40°$ (alt. $\angle$s)

$\angle AED = 180° - 68° - 40°$ ($\angle$ sum of a $\triangle$,

$= 72°$ vert. opp. $\angle$s)

(b) $\angle EAB = 180° - 68° - 38° - 40°$

$= 34°$ ($\angle$ sum of a $\triangle$)

(c) $\angle ABC = 38° + 40°$

$= 78°$

11. **(a)** Number of sides of polygon $= \dfrac{360°}{18°} = 20$

(b) Sum of int. $\angle$s $= (9-2)\times 180°$

$3x° + 6(145°) = 1260°$

$x = 130$

(c) $\angle ABC = \dfrac{(5-2)\times 180°}{5}$

$= 108°$

$\angle ACB = \dfrac{180° - 108°}{2}$

$= 36°$

$= \angle ECD$

$\angle ACE = 108° - 36° - 36°$

$= 36°$

$\angle CAE = 108° - 36°$

$= 72°$

12. Sum of int. $\angle$s $= (6-2)\times 180°$

$3x + 3(x + 20) = 720$

$6x = 660$

$\therefore x = 110$

13. **(a)** $\angle ABC = \dfrac{(12-2)\times 180°}{12}$

$= 150°$

(b) $\angle BAC = \dfrac{180° - 150°}{12}$

$= 15°$ ($AB = BC$, base $\angle$s of isos. $\triangle$)

14. **(a)** $n = \dfrac{360°}{20°} = 18$

(b) $\dfrac{(18-2)\times 180°}{18} + 2° = \dfrac{(2n-2)\times 180°}{2n}$

$162 \times 2n = 360n - 360$

$n = 10$

15. $\angle BCD = 180° - 75°$

$= 105°$ (int. $\angle$s, $AB \,//\, DC$)

$(5 - 2) \times 180° = 105° + 75° + 105° + 2x°$

$x = 127.5$

16. **(a)** $x = \angle FRS$

$= 180° - 125°$

$= 55°$ (corr. $\angle$s, $AB \,//\, EF$, adj. $\angle$s on a str. line)

(b) $y = 180° - 65° - 55°$

$= 60°$ (corr. $\angle$s, adj. $\angle$s on a str. line)

(c) $\angle TQU = 125° - y$

$= 65°$ (vert. opp. $\angle$s)

$z = 180° - 2 \times 65°$

$= 50°$

17. $\angle ABC = 68° - 19°$

$= 49°$

$= \angle ACB$ (corr. $\angle$s, base $\angle$s of isos. $\triangle$)

$\angle CDA + \angle CAD = 49°$ (ext. $\angle$ = sum of int. opp. $\angle$s)

$\angle CDA = \dfrac{49°}{2}$

$= 24.5°$

18. **(a)** **(i)** Let x be size of each ext. $\angle$

$9x = 180°$

$x = 20°$

$\therefore$ Each exterior angle is 20°.

(ii) Sum of ext. $\angle$s = 360°

$n = \dfrac{360°}{20°}$

$= 18$

(b) $\dfrac{\text{exterior angle of a hexagon}}{\text{exterior angle of an octagon}} = \dfrac{360° \div 6}{360° \div 8}$

$= \dfrac{4}{3}$

19. **(a)** $\angle ABC = 180° - 2 \times 36°$

$= 108°$

(b) $\angle KDA = 180° - 108° - 36°$ (opp. $\angle$s of a rhombus,

$= 36°$ alt. $\angle$s, adj. $\angle$s on a

str. line)

$\angle KAD + 36° = 64°$ (ext. $\angle$ = sum of int. opp. $\angle$s)

$\therefore \angle KAD = 28°$

(c) $\angle BQK = 36°$ (alt. $\angle$s)

$\angle KQR = 180° - 64° - 88°$

$= 28°$ ($\angle$ sum of a $\triangle$)

$\therefore \angle BQR = 36° + 28°$

$= 64°$

20. **(a)** Since $AC = BC$,

$\angle ABC = \angle BAC$

$= 56°$ (base $\angle$s of isos. $\triangle$)

$\angle ACB = 180° - 56° - 56°$

$= 68°$ ($\angle$ sum of a $\triangle$)

(b) $\angle QCR = 180° - 56° - 38°$

$= 86°$ (corr. $\angle$s, adj. $\angle$s on a str. line)

(c) $\angle CDR = \angle BCQ$

$= 180° - 38° - 86°$

$= 56°$ (corr. $\angle$s, adj. $\angle$s on a str. line)

(d) $\angle SRT = 180° - 38° - 56°$

$= 86°$ ($\angle$ sum of a $\triangle$, vert. opp. $\angle$s)

21. **(a)** $\angle ARS = 180° - 135° = 45°$ (opp. $\angle$s of // gram)

$\angle QRS = 70°$ (alt. $\angle$s)

$\therefore \angle QRA = 70° - 45°$

$= 25°$

(b) $\angle ARS = 45°$

22. $(7x + 13)° - 25° + (5x - 24)° = 180°$

$12x = 216$

$x = 18$

23. **(a)** Sum of int. $\angle$s = $(5 - 2) \times 180°$

$87° + 104° + (2x - 3)° + (3x - 16)° + (200 - 2x)°$

$= 720°$

$3x = 168$

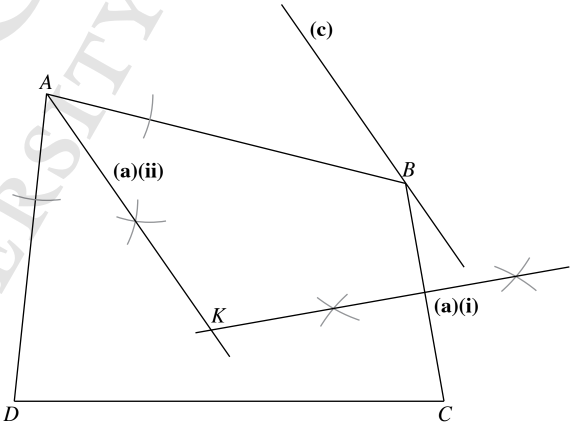
$x = 56$

(b) The interior angles are 87°, 104°, 19°, 152° and 88°.

$\therefore$ The largest exterior angle = $180° - 87°$

$= 93°$

24. **(b)** $AK = 3.2$ cm



25. $\dfrac{(2n - 2) \times 180°}{2n} = \dfrac{(n - 2) \times 180°}{n} + 30°$

$180n - 180 = 180n - 360 + 30n$

$30n = 180$

$\therefore n = 6$

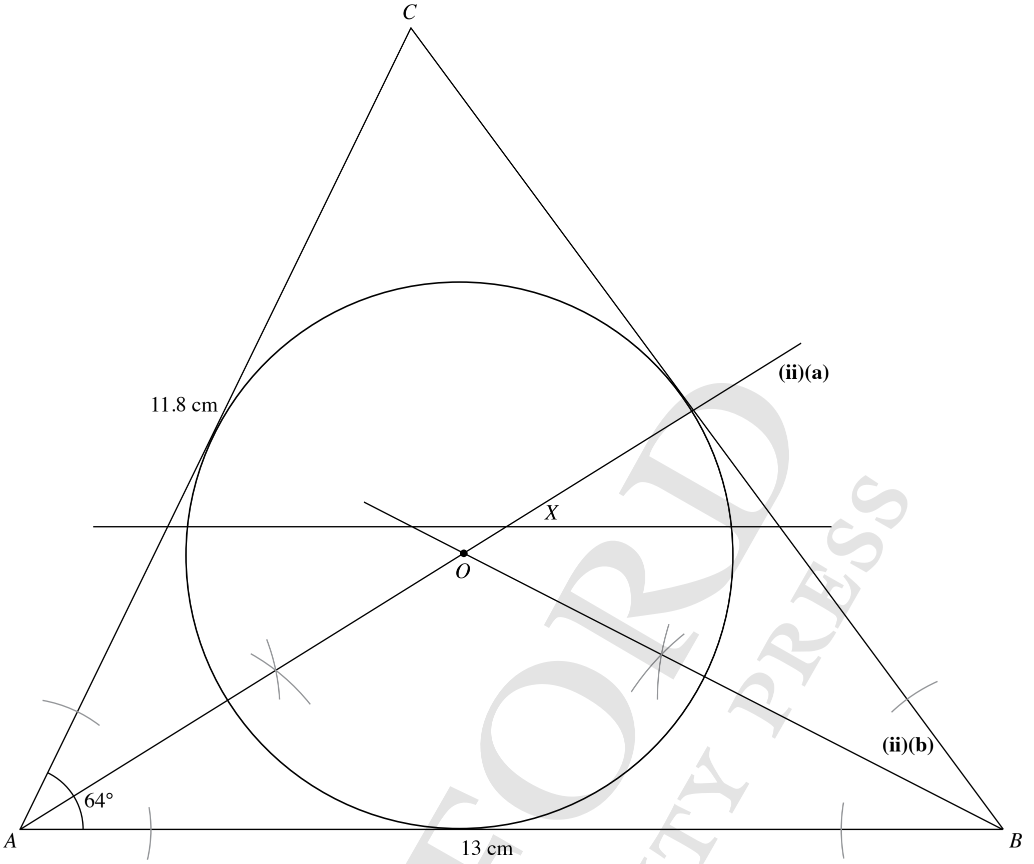
26. $\angle RQS = 180° - 84° \times 2$

$= 12°$

$\therefore n = \dfrac{360°}{12°} = 30$

27. A circle of radius 7.5 cm with centre B.

28.



(i) $BC = 13.2$ cm

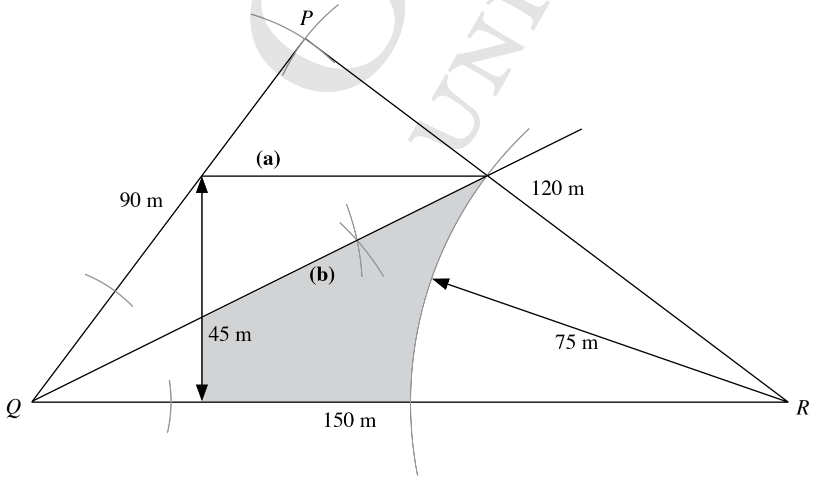
(ii) Radius = 3.6 cm

(iii) Let h be the distance of X from AB.

$$\frac{1}{2} \times 13 \times h = 26$$

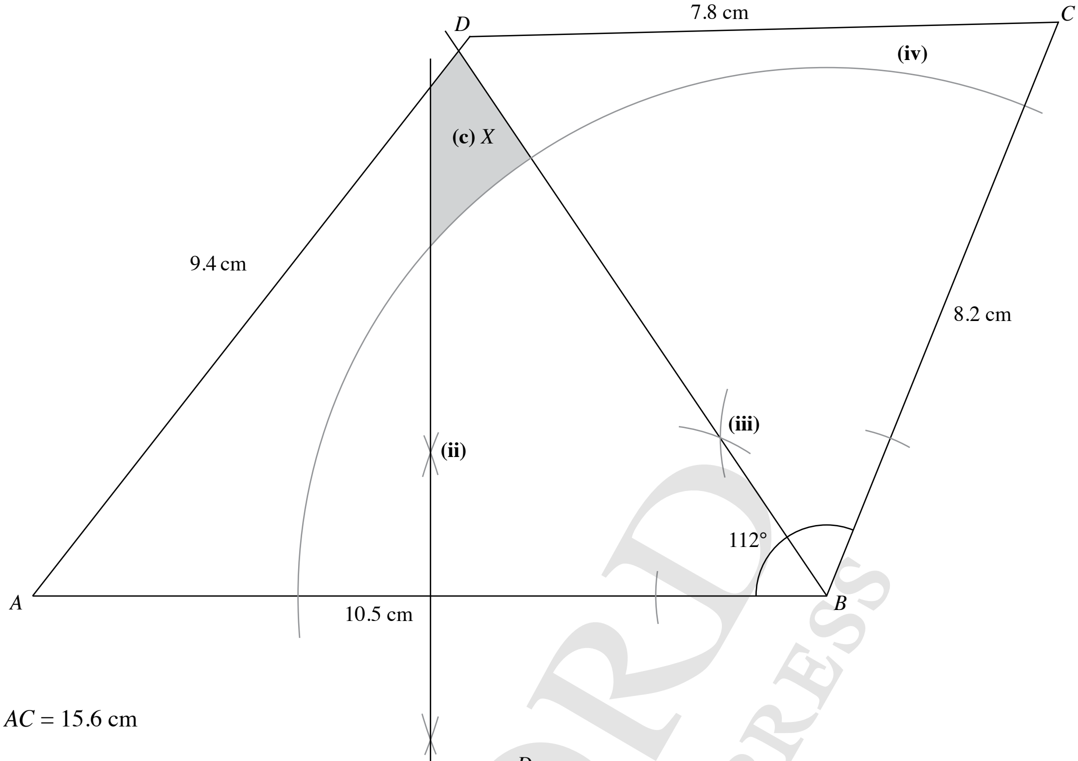
$$h = 4$$

29.



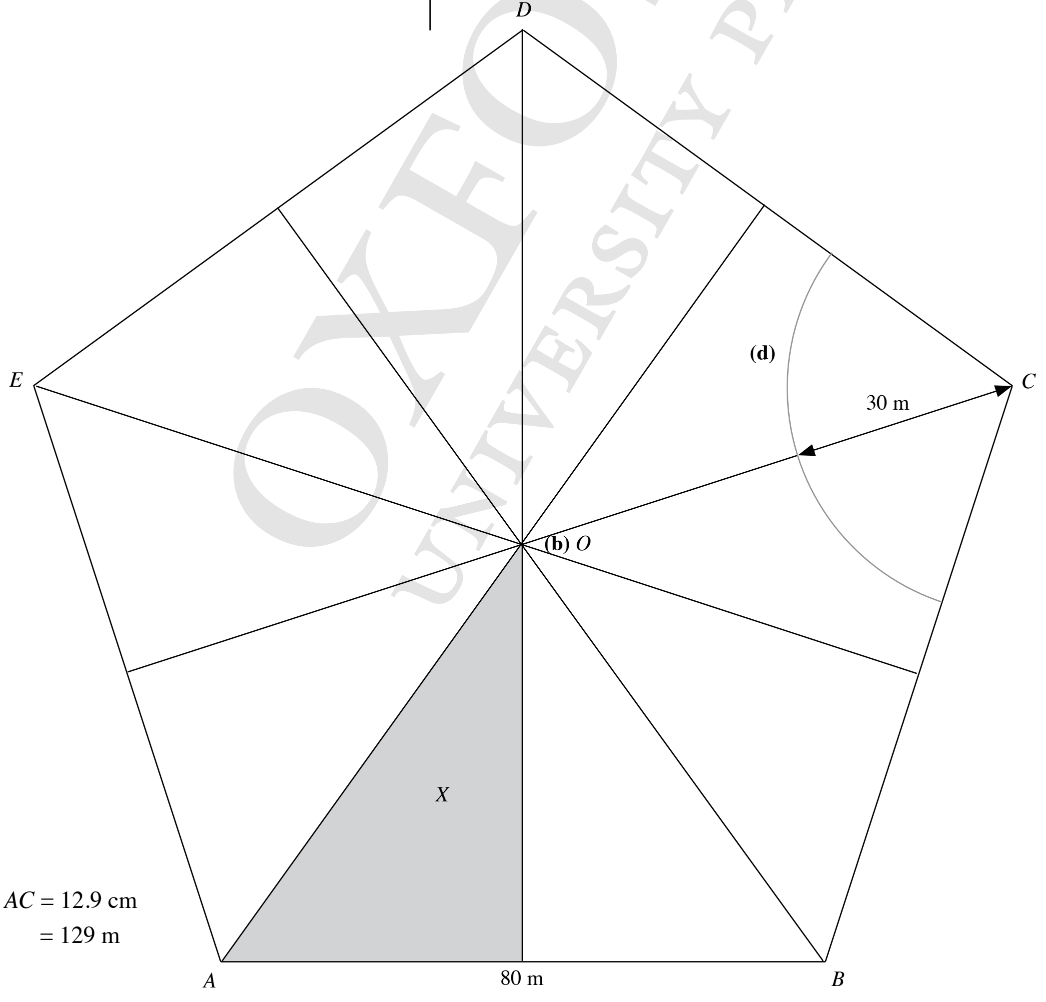
$\angle PRQ = 37°$

30. **(a)**



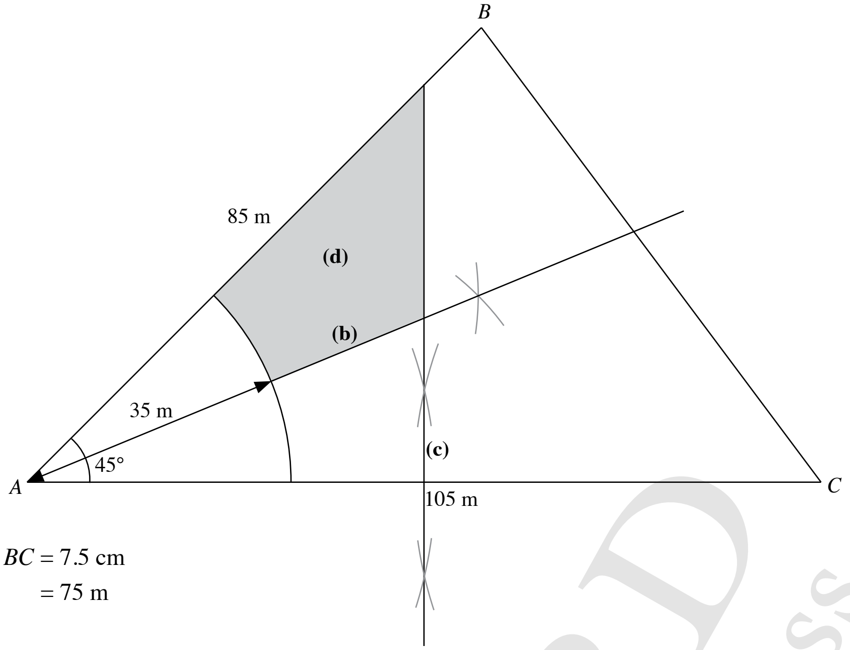
(b) $AC = 15.6$ cm

31.



(a) $AC = 12.9$ cm
$= 129$ m

32.



(a) $BC = 7.5$ cm

$= 75$ m

Exercise 10B

1. **(a)** F
   (b) T
   (c) F
   (d) F

2. $ABCD \equiv PQRS$ (given).

x cm $= PQ$

$= AB$

$= 8$ cm (corr. sides of congruent figures)

$y° = \angle RQP$

$= \angle CBA$

$= 82°$ (corr. $\angle$s of congruent figures)

$z° = \angle BCD$

$= \angle QRS$

$= 38°$ (corr. $\angle$s of congruent figures)

$\therefore x = 8, y = 82, z = 38$

3. **(a)** $\triangle ABC$ and $\triangle ADF$ are similar.
   (b) $\triangle ABC$ and $\triangle BDE$ are congruent, $\triangle BCE$ and $\triangle CEF$ are congruent.

4. $\triangle ABP$ and $\triangle EDP$ are congruent, $\triangle PBC$ and $\triangle PDC$ are congruent, $\triangle PCA$ and $\triangle PCE$ are congruent.

5. **(a)** Yes, all corresponding angles are equal.
   (b) No, ratio of corresponding sides are not equal.

6. $\triangle ABC$ is similar to $\triangle APQ$ (given).

$$\frac{BC}{PQ} = \frac{AB}{AP} = \frac{AC}{AQ}$$

$$\frac{5}{12} = \frac{6}{y+6} = \frac{x}{x+9}$$

$6 \times 12 = 5y + 30$

$\therefore y = 8\frac{2}{5}$

$12x = 5x + 45$

$\therefore x = 6\frac{3}{7}$

7. $$\frac{A_1}{A_2} = \frac{25}{81} = \left(\frac{S_1}{S_2}\right)^2$$

$$\frac{S_1}{S_2} = \sqrt{\frac{25}{81}} = \frac{5}{9}$$

$$\frac{14}{\text{Height}} = \frac{5}{9}$$

$\therefore$ Height of the larger trophy $= 25\frac{1}{5}$ cm

$$\frac{420}{\text{Volume}} = \left(\frac{5}{9}\right)^3$$

$\therefore$ Volume of the larger trophy $= 420 \times \left(\frac{9}{5}\right)^3$

$= 2450$ cm^3 (to 3 s.f.)

8. $\frac{1}{3}\pi r^2 h = 250$ cm^3

(a) Volume $= \frac{1}{3}(2r)^2 h$

$= 4(250)$

$= 1000$ cm^3

(b) Volume $= \frac{1}{3}(3r)^2(3h)$

$= 27 \times 250$

$= 6750$ cm^3

9. **(a)** $$\frac{1}{400} = \frac{\text{Height}}{120 \times 100}$$

Height of model $= 30$ cm

(b) 1 cm represents 4 m

100 cm^3 represents $100 \times 4^3 = 6400$ m^3

10. $\frac{S_1}{S_2} = \frac{4}{160} = \frac{1}{40}$

$\frac{A_1}{A_2} = \left(\frac{1}{40}\right)^2 = \frac{1}{1600}$

$\therefore$ The ratio of their total surface areas is 1 : 1600.

11. (a) $\frac{V_1}{V_2} = \left(\frac{S_1}{S_2}\right)^3 = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$

$\frac{200}{V_2} = \frac{1}{8}$

$\therefore$ Volume of similar cone $= 200 \times 8$

$= 1600 \text{ cm}^3$

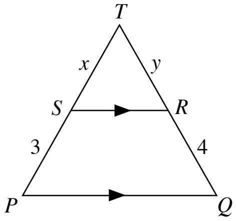
(b) Volume of cone $= \frac{1}{3}\pi(2r)^2\frac{1}{3}h$

$= \frac{4}{3}\left(\frac{1}{3}\pi r^2 h\right)$

$= \frac{4}{3} \times 200$

$= 266\frac{2}{3} \text{ cm}^3$

12. (i)



$\triangle TSR$ is similar to $\triangle TPQ$.

$\frac{TS}{TP} = \frac{TR}{TQ}$

$\therefore \frac{x}{x+3} = \frac{y}{y+4}$

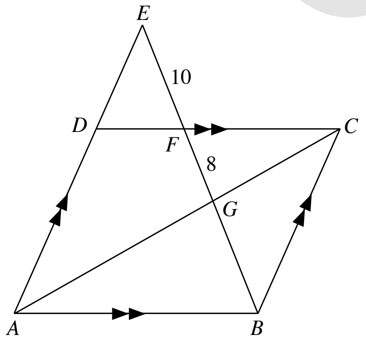
(ii) $\frac{SR}{PQ} = \frac{1}{2}$

$\therefore \frac{x}{x+3} = \frac{1}{2}$ and $\frac{y}{y+4} = \frac{1}{2}$

$2x = x + 3$ and $2y = y + 4$

$x = 3$ $\qquad$ $y = 4$

13.



Let $GB = z$, $AB = y$ and $CF = x$

$\therefore DF = y - x$

$\triangle EDF$ and $\triangle EAB$ are similar.

$\triangle FGC$ and $\triangle BGA$ are similar.

$\therefore \frac{y-x}{y} = \frac{10}{18+z}$ and $\frac{x}{y} = \frac{8}{z}$

$xz = 8y$ — (1)

$10y = (y - x)(18 + z)$

$10y = 18y + yz - 18x - xz$ — (2)

Substitute (1) into (2) : $10y = 18y + yz - 18x - 8y$

$18x = yz$

$x = \frac{yz}{18}$ — (3)

Substitute (3) into (1) : $\left(\frac{yz}{18}\right)z = 8y$

$z^2 = 18 \times 8$

$z = \sqrt{144}$

$= 12$

$\therefore GB$ is 12 cm.

14. $\frac{1}{3}\pi r^2 h = 200 \text{ cm}^3$

(a) Volume of new cone $= \frac{1}{3}\pi(r)^2(6h)$

$= \frac{3}{2}\left(\frac{1}{3}\pi r^2 h\right)$

$= 300 \text{ cm}^3$

(b) Volume of new cone $= 3^3\left(\frac{1}{3}\pi r^2 h\right)$

$= 27 \times 200$

$= 5400 \text{ cm}^3$

(c) Let A_1 and A_2 represent the curved surface areas of new cone and cone C respectively.

$\frac{A_1}{A_2} = \left(\frac{r_1}{r_2}\right)^2 = 4$

$\frac{r_1}{r_2} = 2$

$\therefore$ Radius of new cone $= 2r$

$\frac{V_1}{V_2} = \left(\frac{r_1}{r_2}\right)^3 = 8$

Since $V_2 = 200 \text{ cm}^3$,

Volume of new cone $= V_1$

$= 8 \times 200$

$= 1600 \text{ cm}^3$

15. (a) $\frac{V_1}{V_2} = \left(\frac{h_1}{h_2}\right)^3 = \frac{24}{375}$

$\frac{h_1}{h_2} = \sqrt[3]{\frac{24}{375}} = \frac{2}{5}$

$\therefore h_1 : h_2 = 2 : 5$

(b) $\frac{A_1}{A_2} = \left(\frac{h_1}{h_2}\right)^2$

$= \left(\frac{2}{5}\right)^2$

$= \frac{4}{25}$

$\therefore A_1 : A_2 = 4 : 25$

16. $\frac{6}{4} = \frac{8}{x}$ or $\frac{6}{8} = \frac{x}{8}$

$\therefore x = \frac{32}{6}$ or $x = \frac{48}{4}$

$= 5\frac{1}{3}$ $= 12$

17. (a) $\triangle ABC$ and $\triangle QRC$

(b) $PQ : BC = 3 : 7$

$\therefore BR : BC = 3 : 7$

(c) $\frac{\text{Area of } \triangle QRC}{\text{Area of } \triangle ABC} = \left(\frac{4}{7}\right)^2 = \frac{16}{49}$

Area of $\triangle QRC$ is represented by 16 units^2.

9 units^2 represent 20 cm^2

16 units^2 represent $\frac{20}{9} \times 16 = 35\frac{5}{9}$ cm^2

$\therefore$ Area of $\triangle QRC = 35\frac{5}{9}$ cm^2

$\frac{\text{Area of } \triangle APQ}{\text{Area of } \triangle ABC} = \left(\frac{3}{7}\right)^2 = \frac{9}{49}$

Area of $PQRB$ is represented by (49 – 9 –16) units^2

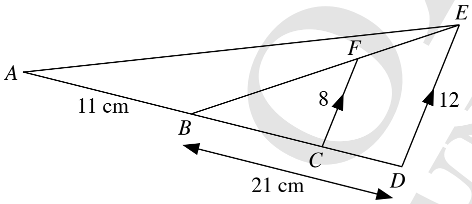
= 24 units^2

9 units^2 represent 20 cm^2

24 units^2 represent $\frac{20}{9} \times 24 = 53\frac{1}{3}$ cm^2

$\therefore$ Area of parallelogram $PQRB = 53\frac{1}{3}$ cm^2

18.



(a) Let $BC = x$ and $CD = y$.

$\frac{8}{12} = \frac{x}{x+y} = \frac{2}{3}$

$3x = 2x + 2y$

$\therefore x = 2y$

$2y + y = 21$

$y = 7$

$\therefore BC = x = 14$ cm.

(b) (i) $\frac{\text{Area of } \triangle BCF}{\text{Area of } \triangle BDE} = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$

(ii) Let h be the common height between the two triangles.

$\frac{\text{Area of } \triangle ABE}{\text{Area of } \triangle ADE} = \frac{\frac{1}{2} \times 11 \times h}{\frac{1}{2} \times (11+21) \times h}$

$= \frac{11}{32}$

(iii) Let h be the common height between the two triangles.

Area of $CDEF$ as a fraction of $\triangle BDE = 1 - \frac{4}{9}$

$= \frac{5}{9}$

$\frac{\text{Area of } \triangle ABE}{\text{Area of } CDEF} = \frac{\frac{1}{2} \times 11 \times h}{\frac{1}{2} \times 21 \times h \times \frac{5}{9}}$

$= \frac{33}{35}$

19. $T_1 : T_2 = 2 : 5$

$= 4 : 10$

$T_2 : T_3 = 10 : 9$

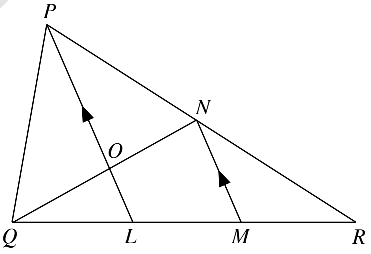
$T_1 : T_2 : T_3 = 4 : 10 : 9$

$T_1 : T_3 = 4 : 9$

$\frac{\text{Area of } T_1}{\text{Area of } T_3} = \left(\frac{4}{9}\right)^2 = \frac{16}{81}$

$\therefore$ The ratio of area of T_1 to that of triangle T_3 is 16 : 81.

20.



Let $QL = x$ cm. $\therefore$ $LR = 2x$ and $LM = MR = x$ cm.

$\frac{\text{Area of } \triangle QOL}{\text{Area of } \triangle QNM} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

$\frac{\text{Area of } \triangle NMR}{\text{Area of } \triangle NQM} = \frac{\frac{1}{2}(x)(\text{height})}{\frac{1}{2}(2x)(\text{height})} = \frac{1}{2}$

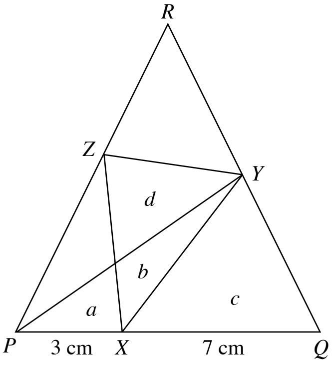
Area of $\triangle NMR = \frac{1}{2} \triangle QNM$

Area of $\triangle OQL$: Area of $OLMN$: Area of $\triangle NMR$

= 1 : 3 : 2

$\therefore$ The ratio of $\triangle OQL$ to the area of quadrilateral $OLRN$ is 1 : 5.

21.



(a) **(i)** Let the area be a, b, c and d as indicated above. Since $a + b + c = b + c + d, a = d$ and $\triangle PXY$ and $\triangle XYZ$ have equal areas.

(ii) Let XY be the base of $\triangle PXY$ and $\triangle XYZ$.

Since $\triangle PXY = \triangle XYZ = \frac{1}{2} \times XY \times$ height, this means that both triangles have the same perpendicular height and $XY \,//\, PR$.

(b)
$$\frac{\text{Area of } \triangle PXY}{\text{Area of } \triangle QXY} = \frac{\frac{1}{2} \times PY \times \text{height}}{\frac{1}{2} \times XQ \times \text{height}} = \frac{3}{7} = \frac{21}{49}$$

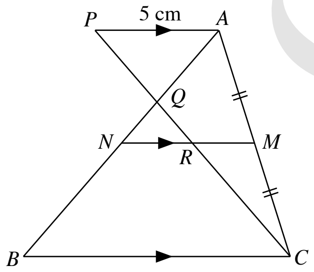
and $\frac{\text{Area of } \triangle QXY}{\text{Area of } \triangle QPY} = \left(\frac{7}{10}\right)^2 = \frac{49}{100}$

100 units^2 represent 20 cm^2

$XQYZ = (21 + 49)$ units^2
$= 70$ units^2

Area of quadrilateral $XQYZ = \frac{20}{100} \times 70$
$= 14$ cm^2

22.



M is the midpoint of AC.

$\therefore \frac{MR}{AP} = \frac{CM}{CA} = \frac{1}{2}$

$MR = \frac{1}{2} \times AP = 2.5$ cm

$\frac{CR}{CP} = \frac{1}{2}$ implies that R is the midpoint of PC.

Let $PQ = x$ cm. Since it is given that $PC = 4PQ$, $PC = 4x$

$CR = \frac{1}{2} \times PC = 2x$ and $QR = x$

$NR = PA = 5$ cm

$\therefore MN = 5 + 2.5 = 7.5$ cm

23. **(a)** Let h be the height of the larger tin.

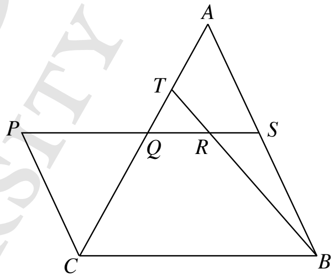
$$\frac{h}{8} = \frac{8}{6}$$
$$h = \frac{64}{6} = 10\frac{2}{3} \text{ cm}$$

(b)
$$\frac{\text{Surface area of smaller tin}}{\text{Surface area of larger tin}} = \left(\frac{6}{8}\right)^2 = \frac{9}{16}$$

(c)
$$\frac{\$1.35}{\text{cost of larger tin}} = \left(\frac{6}{8}\right)^3$$

$\therefore$ Cost of larger tin $= \$1.35 \times \frac{64}{27}$
$= \$3.20$

24.



(a) $\angle PQC = \angle SQA$ (vert. opp. $\angle$s)
$\angle ASQ = \angle CPQ$ (alt. $\angle$s)
$\therefore \triangle PQC$ and $\triangle SAQ$ are similar (2 pairs of corr. $\angle$s equal)

(b) $\triangle TCB$

(c) $\triangle BAC$

(d) **(i)** $\frac{\text{Area of } \triangle TQR}{\text{Area of } \triangle TCB} = \left(\frac{1}{4}\right)^2 = \frac{12}{\triangle TCB}$

$\therefore$ Area of $\triangle TCB = 16 \times 12$
$= 192$ cm^2

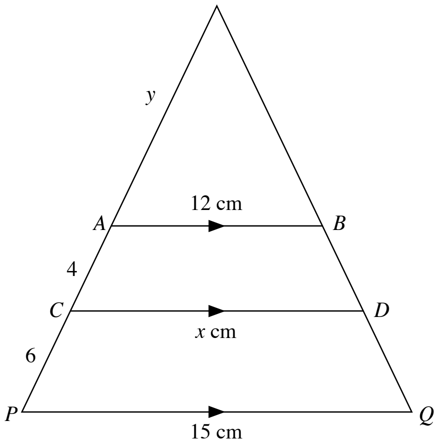
(ii) Area of $\triangle RSB = 3$(Area of $\triangle TQR$)
$= 3(12)$
$= 36$ cm^2

(iii) Area of trapezium $QRBC = 192 - 12$
$= 180$ cm^2

(iv) $\frac{\text{Area of } \triangle PQC}{\text{Area of } \triangle TQR} = \left(\frac{2}{1}\right)^2 = 4$

$\therefore$ Area of $\triangle PQC = 4 \times 12 = 48$ cm^2

25.



$$\frac{y}{y+10} = \frac{12}{15}$$

$$15y = 12y + 120$$

$\therefore y = 40$ cm

$$\frac{40}{44} = \frac{12}{x}$$

$$40x = 44 \times 12$$

$\therefore x = 13.2$

26. **(a)** Volume of hemisphere $= \frac{2}{3}\pi(3)^3$

$= 56.556$

$= 56.6$ cm^3 (to 3 s.f.)

(b) Let h be height of the cone.

Volume of cone $= \frac{2}{3} \times$ volume of hemisphere

$$\frac{1}{3}\pi(3)^2h = \frac{2}{3}\left[\frac{2}{3}\pi(3)^3\right]$$

$\therefore h = 4$ cm

(c) $\dfrac{2.5}{\frac{1}{2}AB} = \dfrac{4}{3}$

$\therefore AB = 3.75$ cm

(d) **(i)** Volume of empty space $= \frac{1}{3}\pi\left(\frac{3.75}{2}\right)^2(2.5)$

$= 9.21$ cm^3

(ii) Volume of liquid in container

$= \frac{1}{3}\pi(3)^2(4) - 9.21 + 56.556$

$= 85.1$ cm^3

(e) **(i)** $\dfrac{\text{Volume of smaller container}}{\text{Volume of larger container}} = \left(\dfrac{6}{24}\right)^3$

$= \dfrac{1}{64}$

$\therefore$ The ratio is 1 : 64.

(ii) Volume of the hemisphere of the larger container

$= 56.6 \times 64$

$= 3620$ cm^3

27. $\pi r^2h = 400$

New radius $= 1.5r$

New height $= \frac{1}{2}h$

New volume $= \pi(1.5r)^2\left(\frac{1}{2}h\right)$

$= \frac{9}{8}\pi r^2h$

$= \frac{9}{8} \times 400$

$= 450$ cm^3

Exercise 10C

1. **(a)** Using Pythagoras' Theorem,

$AB = \sqrt{11^2 - 8^2}$

$= 7.55$ cm (to 3 s.f.)

(b) $\tan 1.2 = \dfrac{8}{CD}$

$CD = \dfrac{8}{\tan 1.2}$

$= 3.11$ cm (to 3 s.f.)

(c) $\angle BAD = \sin^{-1} \dfrac{8}{11}$

$= 0.814$ rad (to 3 s.f.)

(d) $\angle CAB = \tan^{-1} \dfrac{8}{7.55 + 3.11}$

$= 0.644$ rad (to 3 s.f.)

2. **(a)** Using Pythagoras' Theorem,

$AC = \sqrt{17^2 - 8^2}$

$= 15$

(b) **(i)** $\sin \angle BAC = \dfrac{8}{17}$

(ii) $\cos \angle CBQ = -\dfrac{8}{17}$

(iii) $\tan \angle PAC = -\dfrac{8}{15}$

3. **(a)** $\cos \angle DAC = -\cos 62°$

$= -0.469$ (to 3 s.f.)

(b) $\cos 62° = \dfrac{AB}{25}$

$AB = 25 \cos 62°$

$= 11.7$ m (to 3 s.f.)

(c) Area of $\triangle ABC = \frac{1}{2} \times 11.74 \times 25 \sin 62°$

$= 130$ m^2 (to 3 s.f.)

4. **(a)** $\tan 0.8 = \dfrac{BC}{9.8}$

$\therefore BC = 9.8 \tan 0.8$

$= 10.1$ cm (to 3 s.f.)

(b) $\sin 0.8 = \dfrac{BD}{9.8}$

$\therefore BD = 9.8 \sin 0.8$

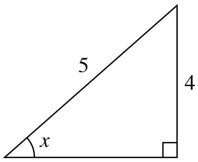
$= 7.03$ cm (to 3 s.f.)

(c) $\cos 0.8 = \dfrac{AD}{9.8}$

$\therefore AD = 9.8 \cos 0.8$

$= 6.83$ cm (to 3 s.f.)

5. **(a)**



Using Pythagoras' Theorem,

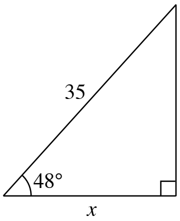
base of triangle $= \sqrt{5^2 - 4^2}$

$= 3$

$\sin x + 2 \cos x + 3 \tan x = \dfrac{4}{5} + 2\left(\dfrac{3}{5}\right) + 3\left(\dfrac{4}{3}\right)$

$= 6$

(b)



Unknown angle $= 180° - 90° - 48°$

$= 42°$ ($\angle$ sum of a $\triangle$)

$\sin 42° = \dfrac{x}{35}$

$\therefore x = 35 \sin 42° = 23.4$ cm (to 3 s.f.)

6. **(a)** Using Pythagoras' Theorem,

$BC = \sqrt{1 - k^2}$

$\sin \angle CDB = \dfrac{\sqrt{1-k^2}}{1}$

$= \sqrt{1-k^2}$

(b) $\cos \angle CBD = \dfrac{\sqrt{1-k^2}}{1}$

$= \sqrt{1-k^2}$

(c) $\tan \angle ABC = -\dfrac{k}{\sqrt{1-k^2}}$

7. **(a)** $\tan 58° = \dfrac{AC}{CT}$

$= \dfrac{15 + BC}{CT}$

$= \dfrac{15}{CT} + \dfrac{BC}{CT}$

$= \dfrac{15}{CT} + \tan 44°$

$CT = \dfrac{15}{\tan 58° - \tan 44°}$

$= 23.64$ (to 4 s.f.)

$\therefore BC = 23.64 \times \tan 44°$

$= 22.8$ cm (to 3 s.f.)

(b) $CT = 23.6$ cm (to 3 s.f.)

(c) Using Pythagoras' Theorem,

$AT = \sqrt{23.64^2 + (15 + 22.82)^2}$

$= 44.6$ cm (to 3 s.f.)

8. **(a)** $\angle BAC = 30° = \angle LAC$

$\therefore \angle ACB = 180° - 90° - 60°$

$= 30°$ ($\angle$ sum of a $\triangle$)

(b) Since $\angle LAC = \angle ACB = 30°$,

$\triangle LAC$ is an isosceles triangle.

$\therefore CL = AL = 4$ cm

(c) $\sin 60° = \dfrac{AB}{4}$

$\therefore AB = 4 \sin 60°$

$= 3.46$ cm (to 3 s.f.)

9. **(a)** $KB = 4.2$ cm, $AB = 6.4$ cm

$\angle ABK = \cos^{-1} \dfrac{4.2}{6.4}$

$= 49.0°$ (to 1 d.p.)

(b) Area of parallelogram $ABCD = AD \times AK$

$= 8.4 \times \sin 48.99°$

$= 40.6 \text{ cm}^2$ (to 3 s.f.)

10. **(a)** Bearing of $R = 360° - 20°$

$= 340°$

(b) Bearing of O from $R = 180° - 20° - 20°$

$= 140°$

11. **(a)** **(i)** Bearing of A from $B = 360° - 45°$

$= 315°$

(ii) Bearing of B from $C = \dfrac{180° - 45°}{2}$

$= 067.5°$

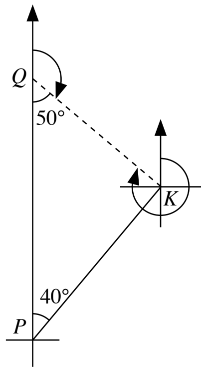
(ii) Bearing of C from $B = 270° - (67.5° - 45°)$

$= 247.5°$

(b) Area of $\triangle ABC = \dfrac{1}{2} \times 10 \times 10 \times \sin 45°$

$= 35.4 \text{ cm}^2$ (to 3 s.f.)

12.



(a) Bearing of K from $Q = 180° - 50°$
$= 130°$

(b) Bearing of Q from $K = 360° - 50°$
$= 310°$

13. (a) Bearing of O from $A = 180° + 66°$
$= 246°$

(b) Bearing of B from $O = 66° + 60°$
$= 126°$

(c) Bearing of B from $A = 180° + 66° - 60°$
$= 186°$

14. (a) Bearing of C from $A = 116° + 180° - 2 \times 42°$
$= 212°$

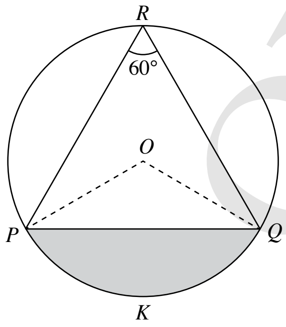
(b) Bearing of A from $B = 360° - 64°$
$= 296°$

(c) Bearing of B from $C = 32° + 42°$
$= 074°$

15. (a) Using cosine rule,

$$PQ = \sqrt{8^2 + 8^2 - 2 \times 8 \times 8 \times \cos 120°}$$
$$= 13.9 \text{ cm}$$

(b)



$$\angle POQ = 2 \times 60°$$
$$= 120° \text{ } (\angle \text{ at centre} = 2 \angle \text{ at circumference})$$

Area of minor segment PKQ

$$= \frac{120°}{360°} \times \pi \times 8^2 - \frac{1}{2} \times 8^2 \times \sin 120°$$
$$= 39.3 \text{ cm}^2 \text{ (to 3 s.f.)}$$

16. (a) $\cos 48° = \dfrac{AB}{8}$

$\therefore AB = 8 \cos 48° = 5.35$ cm (to 3 s.f.)

(b) Using Pythagoras' Theorem,

$$AD = \sqrt{8^2 + 15^2}$$
$$= 17$$

$$\sin \angle ADC + \cos \angle ADC = \frac{8}{17} + \frac{15}{17}$$
$$= 1\frac{6}{17}$$

17. (a) Using cosine rule,

$$\angle ADC = \cos^{-1} \frac{6^2 + 15^2 - 17^2}{2 \times 6 \times 15}$$
$$= 98.9° \text{ (to 1 d.p.)}$$

(b) Using sine rule,

$$\frac{BC}{\sin 38°} = \frac{17}{\sin 58°}$$

$$\therefore BC = \frac{17 \sin 38°}{\sin 58°}$$
$$= 12.34 \text{ cm (to 4 s.f.)}$$
$$= 12.3 \text{ cm (to 3 s.f.)}$$

(c) Area of $ABCD$

$$= \frac{1}{2} \times 6 \times 15 \times \sin 98.9°$$
$$+ \left[\frac{1}{2} \times 17 \times 12.34 \times \sin (180° - 58° - 38°)\right]$$
$$= 149 \text{ cm}^2 \text{ (to 3 s.f.)}$$

18. (a) Using cosine rule,

$$AD = \sqrt{8^2 + 11.6^2 - 2 \times 8 \times 11.6 \times \cos 2.18}$$
$$= 17.46 \text{ cm (to 4 s.f.)}$$

Using sine rule,

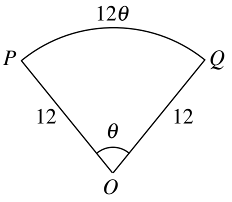
$$\frac{\sin \triangle ADO}{8} = \frac{\sin 2.18}{17.46}$$

$$\angle ADO = \sin^{-1} \frac{8 \sin 2.18}{17.46}$$
$$= 0.385 \text{ rad (to 3 s.f.)}$$

(b) Perimeter of $ABCDA = 8(2\pi - 2.18) + 3.6 + 17.46$
$= 53.9$ cm (to 3 s.f.)

(c) Area of $OABCO = \dfrac{1}{2} \times 8^2 \times (2\pi - 2.18)$
$= 131 \text{ cm}^2$ (to 3 s.f.)

19. (a)



Given that the area of the sector = 36 cm^2,

$$\frac{1}{2} \times 12^2 \times \theta = 36$$
$$\theta = 0.5 \text{ rad}$$

$\therefore$ Length of wire $= 12 + 12 + 12(0.5) = 30$ cm

(b) Given that the wire is bent to form a circle, let r be radius of the circle.

$2\pi r = 30$

$r = 4.75$ cm (to 3 s.f.)

20. (a) $\triangle AOQ$ or $\triangle OBQ$

(b) $\angle OAP = 90°$ (tangent $\perp$ radius)

Using Pythagoras' Theorem,

$$OP = \sqrt{5^2 + 12^2}$$
$$= 13 \text{ cm}$$

$$\angle AOQ = \tan^{-1} \frac{12}{5}$$
$$= 67.38° \text{ (to 2 d.p.)}$$

$$\therefore \angle ABQ = \frac{1}{2} \times 67.38°$$
$$= 33.7° \text{ (to 1 d.p.)}$$

(c) $PQ = 13 - 5 = 8$ cm

21. (a) Using cosine rule,

$$QR = \sqrt{8^2 + 8^2 - 2 \times 8 \times 8 \times \cos 30°}$$
$$= 4.14 \text{ cm (to 3 s.f.)}$$

(b) Area of $\triangle PQR = \frac{1}{2} \times 8 \times 8 \times \sin 30°$

$$= 16 \text{ cm}^2$$

(c) $\angle PQX = \tan^{-1} \frac{6}{8}$

$$= 36.9° \text{ (to 1 d.p.)}$$

(d) Volume of $PQRX = \frac{1}{3} \times 16 \times 6$

$$= 32 \text{ cm}^3$$

22. (a) $\angle ABC = (180° - 124°) + 17° = 73°$

Using cosine rule,

$$AC = \sqrt{72^2 + 93^2 - 2 \times 72 \times 93 \times \cos 73°}$$
$$= 99.6 \text{ m (to 3 s.f.)}$$

(b) Using sine rule,

$$\frac{\sin \triangle BAK}{93} = \frac{\sin 73°}{99.59}$$

$$\angle BAK = \sin^{-1} \frac{93 \sin 73°}{99.59}$$
$$= 63.3° \text{ (to 1 d.p.)}$$

$\therefore$ Bearing of C from $A = 124° - 63.3°$

$$= 060.7° \text{ (to 1 d.p.)}$$

(c) Let h be the shortest distance from AC to B.

$$\sin 63.3° = \frac{h}{72}$$
$$h = 72 \sin 63.3°$$
$$= 64.32 \text{ m (to 4 s.f.)}$$

$\therefore$ Greatest $\angle$ of elevation $= \tan^{-1} \frac{38}{64.32}$

$$= 30.6° \text{ (to 1 d.p.)}$$

23. (a) $\angle ABC = 42° + 35°$

$$= 77°$$

(b) Using sine rule,

$$\frac{BC}{\sin 48°} = \frac{95}{\sin 55°}$$
$$BC = \frac{95 \sin 48°}{\sin 55°}$$
$$= 86.2 \text{ m (to 3 s.f.)}$$

(c) $\angle$ of elevation $= \tan^{-1} \frac{24}{86.19}$

$$= 15.6° \text{ (to 1 d.p.)}$$

(d) Using cosine rule,

$$BK = \sqrt{95^2 + 35^2 - 2 \times 95 \times 35 \times \cos 48°}$$
$$= 74.28 \text{ m (to 4 s.f.)}$$

$\angle$ of elevation $= \tan^{-1} \frac{24}{74.28}$

$$= 17.9° \text{ (to 1 d.p.)}$$

24. (a) $\cos 22° = \frac{AF}{240}$

$$\therefore AF = 240 \cos 22°$$
$$= 222.52 \text{ m (to 5 s.f.)}$$

$$\cos 46° = \frac{FE}{350}$$
$$\therefore FE = 350 \cos 46°$$
$$= 243.13 \text{ m (to 5 s.f.)}$$
$$\therefore AE = 222.52 + 243.13$$
$$= 466 \text{ m (to 3 s.f.)}$$

(b) $BF = 240 \sin 22°$

$$= 89.91 \text{ m (to 4 s.f.)}$$

$\therefore$ Area of $BFEC$

$$= 89.91 \times 243.13 + \frac{1}{2} \times 243.13 \times 350 \times \sin 46°$$
$$= 52\,500 \text{ m}^2 \text{ (to 3 s.f.)}$$

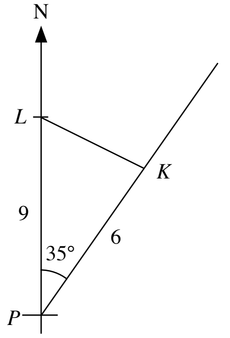
(c) $CD = 350 \sin 46°$

$$= 251.77 \text{ m (to 5 s.f.)}$$

$$\angle CAE = \tan^{-1} \frac{251.77 + 89.91}{465.65}$$
$$= 36.3° \text{ (to 1 d.p.)}$$
$$\therefore \angle CAB = 36.3° - 22°$$
$$= 14.3° \text{ (to 1 d.p.)}$$

25. Distance of $PK = 8 \times \frac{3}{4}$

$= 6$ km

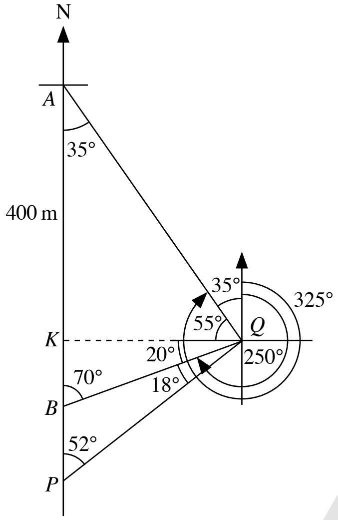


Using cosine rule,

Distance of $LK = \sqrt{9^2 + 6^2 - 2 \times 9 \times 6 \times \cos 35°}$

$= 5.34$ km (to 3 s.f.)

26.



(a) Using sine rule,

$$\frac{BQ}{\sin 35°} = \frac{400}{\sin 75°}$$

$$BQ = \frac{400 \sin 35°}{\sin 75°}$$

$= 237.52$ m (to 5 s.f.)

$$\frac{PB}{\sin 18°} = \frac{237.52}{\sin 52°}$$

$$PB = \frac{237.52 \sin 18°}{\sin 52°}$$

$= 93.1$ m (to 3 s.f.)

(b) Using cosine rule,

$$PQ = \sqrt{93.14^2 + 237.52^2 - 2 \times 93.14 \times 237.52 \times \cos 110°}$$

$= 283$ m (to 3 s.f.)

(c) $\sin 20° = \frac{BK}{237.52}$

$BK = 237.52 \sin 20°$

$= 81.24$ m (to 4 s.f.)

$\therefore AK = 400 - 81.24$

$= 319$ m (to 3 s.f.)

(d) $\cos 20° = \frac{KQ}{237.52}$

$KQ = 237.52 \cos 20°$

$= 223$ m (to 3 s.f.)

27. **(a)** Using Pythagoras' Theorem,

$AC = \sqrt{24^2 + 8^2 + 6^2}$

$= 26$

$\sin \angle CAE = \frac{6}{26}$

$= \frac{3}{13}$

(b) Using Pythagoras' Theorem,

$BC = \sqrt{8^2 + 6^2}$

$= 10$

$\sin \angle CBE = \frac{6}{10}$

$= \frac{3}{5}$

(c) $\cos \angle DAC = \frac{10}{26}$

$= \frac{5}{13}$

28. **(a)** Area of sector $OABC = \frac{70°}{360°} \times \frac{22}{7} \times 6^2$

$= 22$ cm^2

(b) Area of $\triangle OAC = \frac{1}{2} \times 6^2 \times \sin 70°$

$= 16.9$ cm^2

(c) Area of minor segment $ABC = 22 - 16.91$

$= 5.09$ cm^2

(d) $\angle CDA = \frac{1}{2} \times 70°$

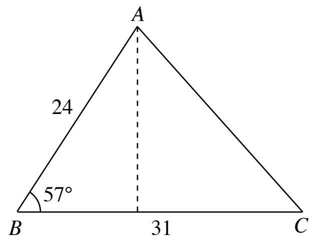
$= 35°$ ($\angle$ at centre = 2 $\angle$ at circumference)

(e) $\tan 35° = \frac{CT}{12}$

$CT = 12 \tan 35°$

$= 8.40$ cm (to 3 s.f.)

29. **(a)**



Area of $\triangle ABC = \frac{1}{2} \times 24 \times 31 \times \sin 57°$

$= 311.99$ (to 5 s.f.)

$= 312 \text{ cm}^2$ (to 3 s.f.)

(b) Let h be the perpendicular distance from A to BC.

$311.99 = \frac{1}{2} \times 31 \times h$

$\therefore h = 20.1$ cm (to 3 s.f.)

30. **(a)** $12\theta = 11$

$\therefore \theta = \frac{11}{12}$ radians

(b) Using cosine rule,

$AC = \sqrt{12^2 + 7.5^2 - 2 \times 12 \times 7.5 \times \cos \frac{11}{12}}$

$= 9.52$ cm (to 3 s.f.)

(c) Area of shaded region

$= \frac{1}{2} \times 12^2 \times \frac{11}{12} - \frac{1}{2} \times 12 \times 7.5 \times \sin \frac{11}{12}$

$= 30.3 \text{ cm}^2$ (to 3 s.f.)

31. **(a)** $20 = (2d)\theta$

$\therefore \theta = \frac{10}{d}$

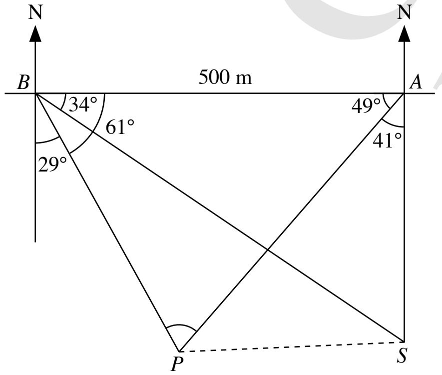
(b) $\dfrac{\frac{1}{2} \times 2d^2 \times \theta}{\frac{1}{2} \times OD^2\theta - \frac{1}{2} \times 2d^2 \times \theta} = \frac{1}{3}$

$\therefore 3(4d^2) = OD^2 - 4d^2$

$OD^2 = 16d^2$

$OD = 4d$

32.



(a) Consider $\triangle ABS$.

$\tan 34° = \frac{AS}{500}$

$AS = 500 \tan 34°$

$= 337.3$ m (to 4 s.f.)

$= 337$ m (to 3 s.f.)

(b) $\angle APB = 180° - 61° - 49°$ ($\angle$ sum of a $\triangle$)

$= 70°$

Using sine rule,

$\frac{AP}{\sin 61°} = \frac{500}{\sin 70°}$

$AP = \frac{500 \sin 61°}{\sin 70°}$

$= 465.4$ m (to 4 s.f.)

$= 465$ m (to 3 s.f.)

(c) Using cosine rule,

$SP = \sqrt{33.3^2 + 465.4^2 - 2 \times 337.3 \times 465.4 \times \cos 41°}$

$= 306$ m (to 3 s.f.)

33. **(a)** $\sin 44° = \frac{BC}{45}$

$BC = 45 \sin 44°$

$= 31.26$ (to 4 s.f.)

$= 31.3$ m (to 3 s.f.)

(b) $\cos 44° = \frac{CD}{45}$

$CD = 45 \cos 44°$

$= 32.37$ (to 4 s.f.)

$= 32.4$ m (to 3 s.f.)

(c) $\sin 42° = \frac{BC}{AC}$

$AC = \frac{31.26}{\sin 42°}$

$= 46.72$ (to 4 s.f.)

$= 46.7$ m (to 3 s.f.)

(d) Using cosine rule,

$AD = \sqrt{32.37^2 + 46.72^2 - 2 \times 32.37 \times 46.72 \times \cos 138°}$

$= 74.01$ m (to 4 s.f.)

Using sine rule,

$\frac{\sin \angle CAD}{32.37} = \frac{\sin 138°}{74.01}$

$\angle CAD = \sin^{-1} \frac{32.37 \sin 138°}{74.01}$

$= 17.0°$ (to 1 d.p.)

$\therefore$ Bearing of D from A is $48° + 17° = 065°$.

34. **(a)** $\angle OCP = 90°$ as PQ is a tangent to the circle.

(b) **(i)** $PC = 4$ cm

$\therefore PQ = 12$ cm

$PA = 4$ cm

$\therefore PR = 16$ cm

Consider $\triangle PQR$.

Using Pythagoras' Theorem,

$QR = \sqrt{12^2 + 16^2}$

$= 20$ cm

(ii) Consider $\triangle OCQ$.

Using Pythagoras' Theorem,

$OQ = \sqrt{4^2 + 8^2}$

$= \sqrt{80}$

$= 8.94$ cm (to 3 s.f.)

(c) $\angle BOC = \angle QOC$

$= 2 \tan^{-1} \frac{8}{4}$

$= 126.9°$ (to 1 d.p.)

(d) Area of sector $CAB = \frac{360° - 126.9°}{360°} \times \pi \times 4^2$

$= 32.5\text{ cm}^2$ (to 3 s.f.)

35. **(a)** $\angle OAB = \frac{360°}{6}$

$= 60°$

(b) $\sin 30° = \frac{5}{AO}$

$\therefore AO = \frac{5}{\sin 30°}$

$= 10$

Area of hexagon $= 6\left(\frac{1}{2} \times 10^2 \times \sin 60°\right)$

$= 260\text{ cm}^2$ (to 3 s.f.)

(c) **(i)** Bearing of B from $D = 180° - 60°$

$= 120°$

(ii) Bearing of B from A is 060°

36. **(a)** Using Pythagoras' Theorem,

$TC = \sqrt{10^2 + 24^2}$

$= 26$ m

(b) **(i)** $\frac{3}{4} = \frac{10}{AB}$

$\therefore AB = \frac{40}{3} = 13\frac{1}{3}$ m

(ii) Using Pythagoras' Theorem,

$AT = \sqrt{10^2 + 13\frac{1}{3}^2}$

$= 16\frac{2}{3}$ m

(c) Volume of $TABC = \frac{1}{3} \times \left(\frac{1}{2} \times 24 \times 13\frac{1}{3}\right) \times 10$

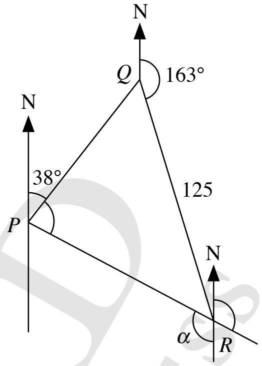
$= 533\frac{1}{3}\text{ m}^3$

(d) $\angle BAC = \tan^{-1} \frac{24}{13\frac{1}{3}}$

$= 60.9°$ (to 1 d.p.)

$\therefore$ Bearing of A from C is $180° + 60.9° = 240.9°$

37.



(a) $\alpha = 118°$

$\therefore$ The bearing of P from R is $118° + 180° = 298°$

(b) $\angle QPR = 118° - 38°$

$= 80°$ (alt. $\angle$s)

$\angle PQR = 360° - (180° - 38°) - 163°$

$= 55°$ (PN // QN, corr. $\angle$s, $\angle$s at a point)

Using sine rule,

$\frac{PR}{\sin 55°} = \frac{125}{\sin 80°}$

$PR = \frac{125 \sin 55°}{\sin 80°}$

$= 103.97$

$= 104$ m (to 3 s.f.)

(c) $\angle PRQ = 180° - 80° - 55°$

$= 45°$ ($\angle$ sum of a $\triangle$)

Area of $\triangle PQR = \frac{1}{2} \times 103.97 \times 125 \times \sin 45°$

$= 4594.9$ (to 4 s.f.)

$= 4590\text{ m}^2$ (to 3 s.f.)

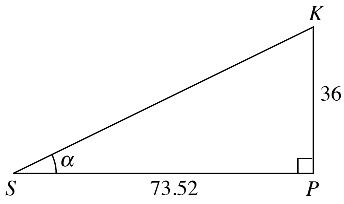
(d) Let the shortest distance from P to QR be h.

$459.4 = \frac{1}{2} \times 125 \times h$

$\therefore h = 73.52$ (to 4 s.f.)

$= 73.5$ m (to 3 s.f.)

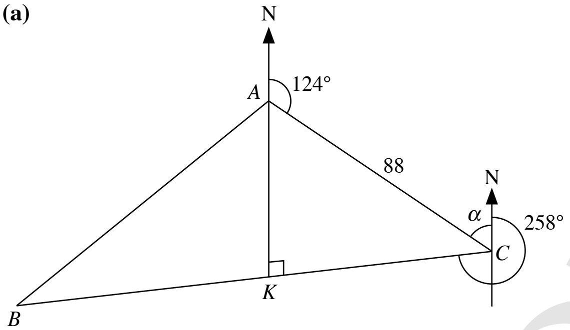
(e)



To make the angle of elevation greatest, the length of PS must be the shortest, that is the shortest distance from P to QR.

$$\text{So } \alpha = \tan^{-1} \frac{36}{73.52}$$
$$= 26.1° \text{ (to 1 d.p.)}$$

38. (a)

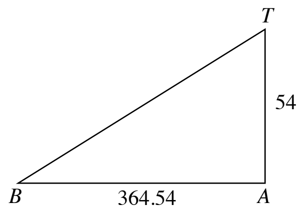


$$\angle BAC = 248° - 124°$$
$$= 124°$$
$$\alpha = 180° - 124°$$
$$= 56° \text{ (adj. } \angle\text{s on a str. line, alt. } \angle\text{s)}$$
$$\therefore \angle ACB = 360° - \alpha - 258°$$
$$= 46° \text{ (}\angle\text{s at a point)}$$
$$\angle ABC = 180° - 124° - 46°$$
$$= 10° \text{ (}\angle\text{ sum of a } \triangle\text{)}$$

(b) Using sine rule,

$$\frac{AB}{\sin 46°} = \frac{88}{\sin 10°}$$
$$AB = \frac{88 \sin 46°}{\sin 10°}$$
$$= 364.54 \text{ (to 4 s.f.)}$$
$$= 365 \text{ m (to 3 s.f.)}$$

(c)



$$\tan \angle ABT = \frac{54}{364.54}$$
$$\angle ABT = \tan^{-1} \frac{54}{364.54}$$
$$= 8.4° \text{ (to 1 d.p.)}$$

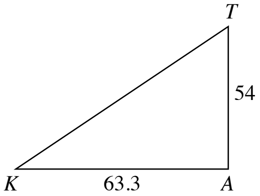
$\therefore$ The angle of elevation of T from B is 8.43°.

(d) Area of $\triangle ABC = \frac{1}{2} \times 88 \times 364.54 \times \sin 124°$

$$= 13\,300 \text{ m}^2 \text{ (to 3 s.f.)}$$

(e) $\sin 46° = \frac{AK}{88}$

$\therefore AK = 63.3$ m (to 3 s.f.)



$\therefore$ Greatest angle of elevation of T is

$$\tan^{-1} \frac{54}{63.3} = 40.5° \text{ (to 1 d.p.)}$$

39. (a) Using Pythagoras' Theorem,

$$PB^2 = 12^2 + 6^2$$
$$= 180$$
$$= AQ^2,$$
$$PC^2 = PB^2 + BC^2$$
$$= 180 + 5^2$$
$$= 205$$
$$\therefore PC = \sqrt{205}$$
$$= 14.3 \text{ m (to 3 s.f.)}$$

(b) Using Pythagoras' Theorem,

$$QC = \sqrt{6^2 + 5^2}$$
$$= \sqrt{61}$$

(i) $\sin \angle CQB = \frac{BC}{CQ}$

$$= \frac{5}{\sqrt{61}}$$
$$= 0.640 \text{ (to 3 s.f.)}$$

(ii) $\tan \angle CPB = \frac{BC}{PB}$

$$= \frac{5}{\sqrt{180}}$$
$$= 0.373 \text{ (to 3 s.f.)}$$

(iii) $\cos \angle AQD = \frac{AQ}{QD}$

$$= \frac{PB}{PC}$$
$$= \frac{\sqrt{180}}{\sqrt{205}}$$
$$= 0.937 \text{ (to 3 s.f.)}$$

Exercise 10D

1. Area of shaded region $= \frac{135°}{360°} \times \pi[56^2 - (56 - 35)^2]$

$= 3180 \text{ cm}^2$ (to 3 s.f.)

2. **(a)** Area of shaded region

$= \frac{1}{2} \times 7 \times 6 - \frac{1}{2} \times 7^2 \times \tan^{-1} \frac{6}{7}$

$= 3.64 \text{ cm}^2$ (to 3 s.f.)

(b) $OB = \sqrt{6^2 + 7^2}$

$= 9.2195$ cm (to 5 s.f.)

$AC = 7\left(\tan^{-1}\frac{6}{7}\right)$

$= 4.9604$ cm (to 5 s.f.)

$\therefore$ Perimeter $= 6 + 4.9604 + (9.2195 - 7)$

$= 13.2$ cm (to 3 s.f.)

3. **(a)** $AD = 18\theta, BC = 30\theta, AB = 12$ cm

$\therefore 18\theta + 30\theta + 12 + 12 = 62.4$

$\theta = \frac{38.4}{48}$

$= 0.8$ rad

(b) Area $= \frac{1}{2}(30^2 - 18^2) \times 0.8$

$= 230.4 \text{ cm}^2$

4. **(a)** Perimeter $= 2\pi(2.8)$

$= 17.584$ cm (to 5 s.f.)

$= 17.6$ cm (to 3 s.f.)

(b) Side of square $= \frac{17.584}{4}$

$= 4.396$ cm

Area of square $= 4.396^2$

$= 19.3 \text{ cm}^2$ (to 3 s.f.)

5. Space available $= 14 \times 14 \times 5$

$= 980 \text{ cm}^2$

Number of spheres needed $= \frac{980}{\frac{4}{3}\pi(1)^3}$

$= 233.96$ (to 5 s.f.)

$\therefore$ The maximum number of balls that can be put in is 233.

6. Perimeter $= \frac{240°}{360°} \times 2\pi(6) + 6 + 6$

$= (12 + 8\pi)$ cm

7. **(a)** $\cos 45° = \frac{OC}{14}$

$\therefore OC = 14 \cos 45°$

$= 9.8995$ cm (to 5 s.f.)

Area of $\triangle OAC = \frac{1}{2} \times 14 \times 9.8995 \times \sin 45°$

$= 49 \text{ cm}^2$

(b) Area of shaded region $= \frac{45°}{360°} \times \pi(14)^2 - 49$

$= 28.0 \text{ cm}^2$ (to 3 s.f.)

8. **(a)** $6(x)^2 = 150$

$x^2 = 25$

$\therefore x = 5$ (since length > 0)

(b) Volume $= 5^3$

$= 125 \text{ cm}^3$

$\therefore$ Mass = volume × density

$= 125 \times 4$

$= 500$ g

9. Let radius of smallest circle be $3x$ cm.

$\pi(3x)^2 = 9\pi x^2 = 18\text{cm}^2$

$\pi x^2 = 2 \text{ cm}^2$

Area of largest circle $= \pi(12x)^2$

$= 144\pi x^2$

Area of second largest circle $= \pi(5x)^2$

$= 25\pi x^2$

Area of shaded region

$= \frac{1}{2}(144\pi x^2) - \frac{1}{2}(25\pi x^2) - \frac{1}{2}(9\pi x^2)$

$= 55\pi x^2$

$= 55(2)$

$= 110 \text{ cm}^2$

10. Total volume $= \pi(60)^2 \times 24$

$= 86\,400\pi \text{ cm}^3$

Volume drained in 1 min $= \pi(1.2)^2 \times 15000$

$= 21\,600\pi \text{ cm}^3$

$\therefore$ Time taken $= \frac{86\,400\pi}{21\,600\pi}$

$= 4$ min

11. **(a)** Volume $= 150 \times 58 \times 65$

$= 565\,500 \text{ cm}^3$

$= 565.5\ l$

(b) Depth $= \frac{565\,500}{200 \times 100}$

$= 28.\ 3$ cm (to 3.s.f.)

12. $\frac{4}{3}\pi \times 6^3 = \frac{1}{3}\pi \times 4^2 \times \text{height}$

$\therefore$ height $= \frac{4 \times 6^3}{4^2} = 54$ cm

13. **(a)** Let r be the radius of the circle.

$\frac{144°}{360°} \times 2\pi r = 4.4$

$\therefore r = 1\frac{3}{4}$ cm

(b) Area of minor sector $OAB = \frac{144°}{360°} \times \pi\left(\frac{7}{4}\right)^2$

$= 3.85 \text{ cm}^2$

14. Number of cones to be made

$$= \frac{\text{volume of cylindrical rod}}{\text{volume of cone}}$$

$$= \frac{\pi \times 5^2 \times 30}{\frac{1}{3} \times \pi \times 1^2 \times 2}$$

$$= 1125$$

15. Let h be rise in water level.

$$\frac{1}{3}\pi \times 4^2 \times 8 = \pi \times 6^2 \times h$$

$$h = \frac{\frac{1}{3} \times 4^2 \times 8}{6^2}$$

$$= 1.185 \text{ cm}$$

16. **(a)** Surface area $= 4\pi(21)^2$

$$= 5542.5 \text{ (to 5 s.f.)}$$

$$= 5540 \text{ cm}^2 \text{ (to 3 s.f.)}$$

(b) Total area to be painted $= \frac{2500 \times 5542.49}{100 \times 100}$ m^2

$$= 1385.6 \text{ m}^2$$

$\therefore$ Number of tins of paint needed $= \frac{1385.6}{100}$

$$= 14 \text{ tins}$$

17. **(a)** Length of arc $AB = 16 \times 1.25$

$$= 20 \text{ cm}$$

(b) $AC = \sqrt{16^2 + 10^2 - 2(16)(10)\cos 1.25}$

$$= 16.0 \text{ cm (to 3 s.f.)}$$

(c) Area of region

$$= \frac{1}{2} \times 16^2 \times 1.25 - \frac{1}{2} \times 16 \times 10 \times \sin 1.25$$

$$= 84.1 \text{ cm}^2 \text{ (to 3 s.f.)}$$

18. **(a)** $10 \times \angle AOB = 7.5$

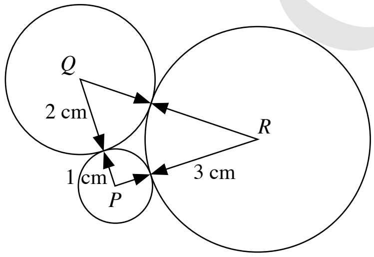
$$\angle AOB = 0.75 \text{ rad}$$

(b) Shaded area

$$= \frac{1}{2} \times 10 \times 14 \times \sin 0.75 - \frac{1}{2} \times 10^2 \times 0.75$$

$$= 10.2 \text{ cm}^2 \text{ (to 3 s.f.)}$$

19.



$PQ = 3$ cm, $PR = 4$ cm and $QR = 5$ cm

Since $3^2 + 4^2 = 5^2$,

$\angle QPR = 90°$ (Converse of Pythagoras' Theorem)

20. Shaded area $= 2\left[\frac{1}{2}r^2\left(\frac{\pi}{2}\right) - \frac{1}{2}r^2\right]$

$$= r^2\left(\frac{1}{2}\pi - 1\right) \text{ cm}^2$$

21. **(a)** Length of arc $PAQ = \frac{60°}{360°} \times 2\pi \times 21$

$$= 22 \text{ cm}$$

(b) $\angle SOR = \frac{16.5}{21}$ rad

$$= \frac{16.5}{21} \times \frac{180}{\pi}$$

$$= 45°$$

(c) Area of shaded sector OQR

$$= \frac{1}{2} \times \pi \times 21^2 - \frac{(60 + 45)°}{360°} \times \pi \times 21^2$$

$$= 288\frac{3}{4} \text{ cm}^2$$

22. Area of shaded region $= \frac{90°}{360°} \times \pi \times 20^2 - \frac{1}{2} \times 20^2$

$$= 114.2 \text{ cm}^2$$

23. Area of shaded region $= \frac{1}{2}\left[\pi a^2 - \pi\left(\frac{a}{2}\right)^2\right]$

$$= \frac{3}{8}\pi a^2$$

Area of unshaded region $= \frac{1}{2}\left[\pi\left(\frac{3a}{2}\right)^2\right] - \frac{3}{8}\pi a^2$

$$= \frac{3}{4}\pi a^2$$

$$\frac{\text{Area of shaded region}}{\text{Area of unshaded region}} = \frac{\frac{3}{8}}{\frac{3}{4}} = \frac{1}{2}$$

$\therefore$ The ratio of the shaded region to that of the unshaded region is 1 : 2.

24. **(a)** $AB = BC$ (given)

$AD = CD$ (given)

$\angle BAD = \angle BCD$ (property of kite)

$\therefore \triangle ABD \equiv \triangle CBD$ (SAS)

(b) **(i)** $\angle QAD = 60°$

$\angle PCB = 60°$

Arc length $QD = \frac{60°}{360°} \times 2\pi \times 21$

$$= 22 \text{ cm}$$

Arc length $PB = \frac{60°}{360°} \times 2\pi \times 14$

$$= 14\frac{3}{4} \text{ cm}$$

$\therefore$ Perimeter of $QABPCD$

$$= 21 + 14 + 14\frac{3}{4} + 14 + 21 + 22$$

$$= 106\frac{2}{3} \text{ cm}$$

(ii) Total area of shaded regions

$$= \frac{60°}{360°} \times [\pi(14)^2 + \pi(21)^2]$$

$$= 333\frac{2}{3} \text{ cm}^2$$

25. Let h be the height of water in tank.

$$\pi \times 14^2 \times 20 = 40 \times 30 \times h$$

$$h = \frac{\pi \times 14^2 \times 20}{40 \times 30}$$

$$= 10.3 \text{ cm (to 3 s.f.)}$$

26. **(a)** Rise in water level $= \dfrac{18 \times \frac{4}{3} \times \pi \times 1^3}{\pi \times 8^2}$

$$= \frac{3}{8} \text{ cm}$$

(b) Surface area in contact with water

$$= \pi \times 8^2 + 2\pi \times 8 \times 9\frac{3}{8}$$

$$= 672 \text{ cm}^2 \text{ (to 3 s.f.)}$$

27. Volume of wood remaining $= \dfrac{4}{3}\pi(9)^3 - \dfrac{1}{3}\pi(3)^2(8)$

$$= 948\pi \text{ cm}^3$$

28. **(a)** Volume of sphere $= 381\dfrac{6}{7}$

$$= \frac{3}{8}\pi r^3$$

$$\therefore r = \sqrt[3]{\frac{3 \times 381\frac{6}{7}}{4\pi}}$$

$$= 4.5 \text{ cm}$$

(b) Surface area $= 4\pi(4.5)^2$

$$= 254\frac{4}{7} \text{ cm}^2$$

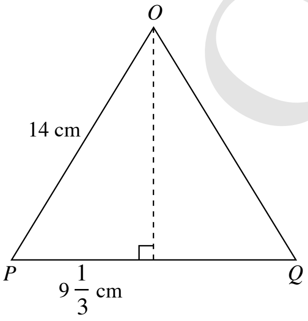
29. **(a)** Length of arc $PQR = \dfrac{240°}{360°} \times 2\pi \times 14$

$$= 2\pi r$$

$$r = \frac{240°}{360°} \times 14$$

$$= 9\frac{1}{3} \text{ cm}$$

(b)



$$\cos \angle OPQ = \frac{9\frac{1}{3}}{14} = \frac{2}{3}$$

(c) Curved surface area $= \pi \times 9\dfrac{1}{3} \times 14$

$$= 411 \text{ cm}^2 \text{ (to 3 s.f.)}$$

30. Volume of concrete used

= volume of pyramid + volume of cuboid

$$= \frac{1}{3} \times 30 \times 30 \times 9 + 30 \times 30 \times 150$$

$$= 1.4 \times 10^5 \text{ cm}^3$$

31. Volume of hemispherical bowl $= \dfrac{2}{3}\pi\left(\dfrac{45}{2}\right)^3 \text{ cm}^3$

$$= 7593\frac{3}{4}\pi \text{ cm}^3$$

Amount of water flow in 1 second

$$= \frac{\pi \times 0.6^2 \times 16 \times 1000 \times 100}{60 \times 60}$$

$$= 160\pi \text{ cm}^3$$

Time taken $= \dfrac{7593\frac{3}{4}\pi}{160\pi} = 47.46$

= 47 seconds (to nearest second)

∴ It will take 47 seconds to fill the bowl.

32. **(a)** Height of cone $= \dfrac{38 - 14}{2}$

$$= 12 \text{ cm}$$

Volume of solid $= \pi \times 5^2 \times 14 + 2\left(\dfrac{1}{3} \times \pi \times 5^2 \times 12\right)$

$$= 1730 \text{ cm}^3 \text{ (to 3 s.f.)}$$

(b) Total surface area of solid

$$= 2(\pi \times 5 \times 13) + 2\pi \times 5 \times 14$$

$$= 848 \text{ cm}^2 \text{ (to 3 s.f.)}$$

33. **(a)** $l = 5$ cm

(b) $\dfrac{216°}{360°} \times 2\pi \times 5 = 2\pi r$

$$\therefore r = 3 \text{ cm}$$

(c) Curved surface area $= \pi rl$

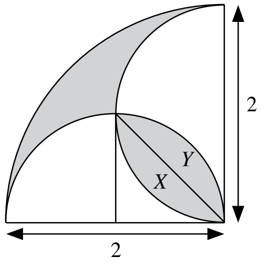
$$= \pi \times 3 \times 5$$

$$= 15\pi$$

(d) Capacity of cone $= \dfrac{1}{3}\pi \times 3^2 \times \sqrt{5^2 - 3^2}$

$$= 37.7 \text{ cm}^3 \text{ (to 3 s.f.)}$$

34. **(a)**



Area of Y = area of small quadrant – area of triangle

$$= \frac{1}{4} \times \pi \times 1^2 - \frac{1}{2} \times 1 \times 1$$

$$= \frac{1}{4}\pi - \frac{1}{2} \text{ cm}^2$$

Area of unshaded region

$= 2(\text{area of semicircle} - XY)$

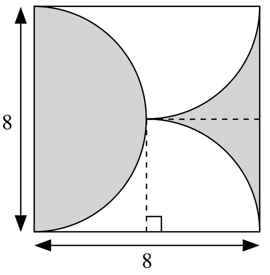
$= 2\left[\frac{1}{2}\times\pi\times 1^2 - 2\left(\frac{1}{4}\pi - \frac{1}{2}\right)\right]$

$= 2\text{ cm}^2$

$\therefore$ Area of shaded region $= \frac{1}{4}\times\pi\times 2^2 - 2$

$= 1.14\text{ cm}^2$ (to 3 s.f.)

(b)

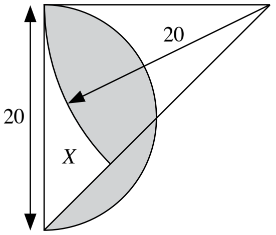


Area of shaded region

$= \frac{1}{2}\times\pi\times 4^2 + 2\left[4^2 - \frac{1}{4}\times\pi\times 4^2\right]$

$= 32\text{ cm}^2$

(c)



Unshaded area X = area of triangle – area of sector

$= \frac{1}{2}\times 20^2 - \frac{45°}{360°}\times\pi\times 20^2$

$= 42.92\text{ cm}^2$ (to 4 s.f.)

$\therefore$ Shaded area

= area of semicircle – unshaded area X

$= \frac{1}{2}\times\pi\times 10^2 - 42.92$

$= 114\text{ cm}^2$ (to 3 s.f.)

35. (a) (i) Height of cone $= \sqrt{125^2 - 35^2}$

$= 120$ cm

(ii) Height of buoy $= 120 + 35$

$= 155$ cm

(iii) Volume of buoy

= volume of hemisphere + volume of cone

$= \frac{2}{3}\pi\times 35^3 + \frac{1}{3}\pi\times 35^2\times 120$

$= 244\ 000\text{ cm}^3$ (to 3 s.f.)

(iv) Total surface area

= curved surface area of hemisphere + cone

$= 2\pi\times 35^2 + \pi\times 35\times 125$

$= 21\ 400\text{ cm}^2$ (to 3 s.f.)

(b) Total area for painting

$= \pi\times 3.5\times\sqrt{1.2^2 + 3.5^2} + 2\pi\times 3.5$

$= 128.65\text{ m}^2$ (to 5 s.f.)

$\therefore$ Cost of painting $= 128.65\times 1.6$

$= \$206$

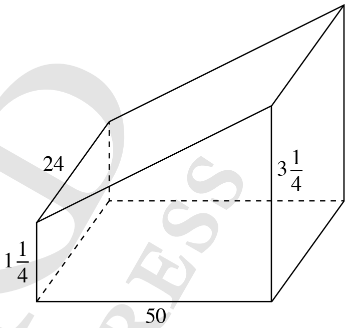
36. (a) Let h be the increase in height of the remaining part of the field.

$800\times 400\times 200 = h(6000\times 4200 - 800\times 400)$

$h = \frac{800\times 400\times 200}{24\ 880\ 000}$

$= 2.57$ cm (to 3 s.f.)

(b)



Volume $= \frac{1}{2}\left(1\frac{1}{4} + 3\frac{1}{4}\right)\times 50\times 24$

$= 2700\text{ m}^3$

$= 2700\times 1000$ litres

$= 2.7\times 10^6$ litres

37. (a) $\frac{r}{60} = \frac{60}{140}$

$\therefore r = 25$ cm

(b) $l = \sqrt{\left(25\frac{5}{7}\right)^2 + 60^2} = 65.278$ (to 5 s.f.)

Surface area $= \pi\left(25\frac{5}{7}\right)(65.278)$

$= 5270\text{ cm}^2$ (to 3 s.f.)

(c) Volume of empty container $= \frac{1}{3}\pi\times 60^2\times 140$

$= 168\ 000\pi\text{ cm}^3$

$\frac{168\ 000\pi - 41\ 546}{168\ 000\pi} = \left(\frac{140 - h}{140}\right)^3$

$\therefore \frac{140 - h}{140} = \sqrt[3]{0.921\ 28}$

$= 0.973\ 04$ (to 5 s.f.)

$136.2257 = 140 - h$

$\therefore h = 3.77$(to 3 s.f.)

38. (a) Volume of container $= 29\times 29\times 37$

$= 31\ 117\text{ cm}^3$

$= 31.1$ litres (to 3 s.f.)

(b) Total area $= (2\times 29\times 29 + 4\times 29\times 37)\times 1.06$

$= 6330\text{ cm}^2$

(c) $29 \times 29 = \pi r^2$

$r = \sqrt{\dfrac{29^2}{\pi}}$ = 164 mm (to nearest mm)

(d) Area $= (2 \times \pi \times 16.36^2 + 2 \times \pi \times 1636 \times 37) \times 1.11$

$= 6090$ cm^2 (to 3 s.f.)

(e) Difference $= (6330 - 6090) \times 5000$ cm^2

$= 120$ m^2

$\therefore$ Amount saved $= 120 \times \$8.50$

$= \$1020$

Exercise 10E

1. **(i)** A reflection in the line *FOC* followed by another reflection in the line *OD*.

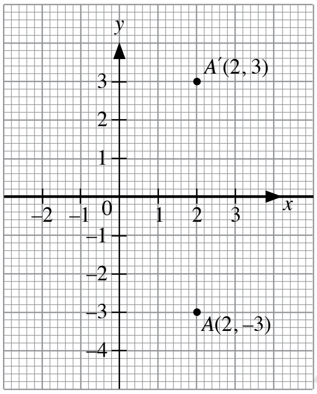
(ii) A reflection in the line *FOC* followed by a 120° clockwise rotation about *O*.

2. **(a)** A translation of 4 cm along *AC*.

(b) A 180° rotation about the midpoint.

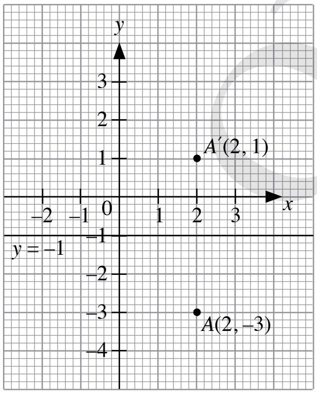
(c) A reflection in the line *BE*.

3. **(a)**



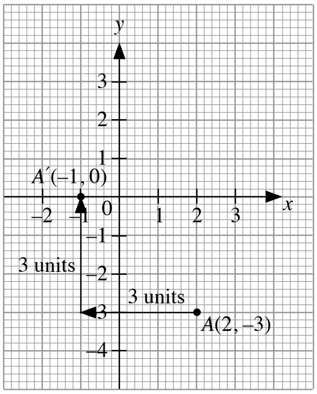
$\therefore$ The coordinates of the image of *A* is (2, 3)

(b)



$\therefore$ The coordinates of the image of *A* is (2, 1).

(c)



$$\begin{pmatrix} 2 \\ -3 \end{pmatrix} + \begin{pmatrix} -3 \\ 3 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$\therefore$ The coordinates of the image of *A* is (–1, 0).

4. **(i)** Let the translation vector of T be $\begin{pmatrix} a \\ b \end{pmatrix}$.

$$\begin{pmatrix} 5 \\ 6 \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 10 \\ 13 \end{pmatrix}$$

$$\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 10 \\ 13 \end{pmatrix} - \begin{pmatrix} 5 \\ 6 \end{pmatrix}$$

$$= \begin{pmatrix} 5 \\ 7 \end{pmatrix}$$

Let $\begin{pmatrix} h \\ k \end{pmatrix}$ be the image of (5, 6) under translation T.

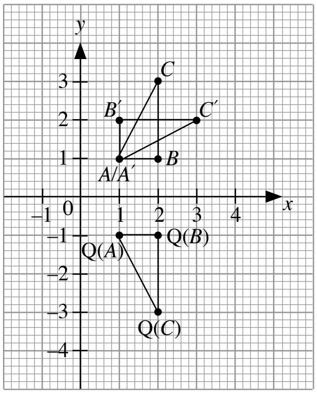
$$\begin{pmatrix} h \\ k \end{pmatrix} + \begin{pmatrix} 5 \\ 7 \end{pmatrix} = \begin{pmatrix} 9 \\ 5 \end{pmatrix}$$

$$\begin{pmatrix} h \\ k \end{pmatrix} = \begin{pmatrix} 9 \\ 5 \end{pmatrix} - \begin{pmatrix} 5 \\ 7 \end{pmatrix}$$

$$= \begin{pmatrix} 4 \\ -2 \end{pmatrix}$$

$\therefore$ The coordinates of the point are (4, –2).

(ii)



5. **(i)** Let the translation vector of T be $\begin{pmatrix} a \\ b \end{pmatrix}$.

$$\begin{pmatrix}3\\5\end{pmatrix}+\begin{pmatrix}a\\b\end{pmatrix}=\begin{pmatrix}5\\9\end{pmatrix}$$

$$\begin{pmatrix}a\\b\end{pmatrix}=\begin{pmatrix}5\\9\end{pmatrix}-\begin{pmatrix}3\\5\end{pmatrix}$$

$$=\begin{pmatrix}2\\4\end{pmatrix}$$

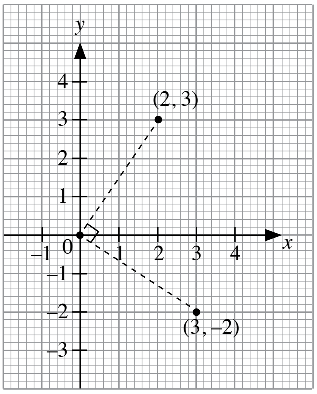
Let $\begin{pmatrix}h\\k\end{pmatrix}$ be the image of (2, 3) under translation T.

$$\begin{pmatrix}2\\3\end{pmatrix}+\begin{pmatrix}2\\4\end{pmatrix}=\begin{pmatrix}h\\k\end{pmatrix}$$

$$\begin{pmatrix}h\\k\end{pmatrix}=\begin{pmatrix}4\\7\end{pmatrix}$$

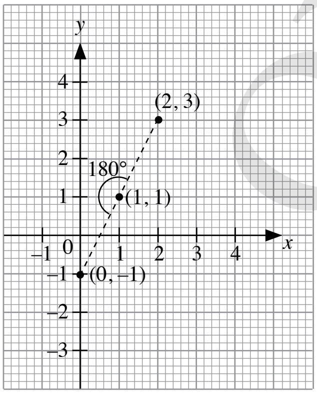
∴ The coordinates of the image of (2, 3) under T is (4, 7).

(ii)



∴ The coordinates of the image of (2, 3) are (3, –2).

(iii)



∴ The coordinates of the image of (2, 3) are (0, –1).

6. **(i)** $T^2\begin{pmatrix}6\\-3\end{pmatrix}=\begin{pmatrix}3\\5\end{pmatrix}+\begin{pmatrix}3\\5\end{pmatrix}+\begin{pmatrix}6\\-3\end{pmatrix}=\begin{pmatrix}12\\7\end{pmatrix}$

(ii) $M\begin{pmatrix}6\\-3\end{pmatrix}=\begin{pmatrix}-1&0\\0&1\end{pmatrix}\begin{pmatrix}6\\-3\end{pmatrix}$

$=\begin{pmatrix}-6\\-3\end{pmatrix}$ i.e. (–6, –3)

(iii) $TM\begin{pmatrix}6\\-3\end{pmatrix}=T\begin{pmatrix}-6\\-3\end{pmatrix}$

$=\begin{pmatrix}3\\5\end{pmatrix}+\begin{pmatrix}-6\\-3\end{pmatrix}$

$=\begin{pmatrix}-3\\2\end{pmatrix}$ i.e. (–3, 2)

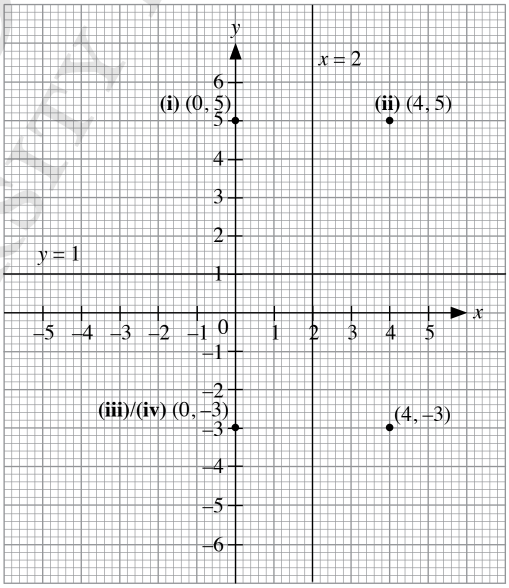
(iv) $MT\begin{pmatrix}6\\-3\end{pmatrix}=M\left[\begin{pmatrix}3\\5\end{pmatrix}+\begin{pmatrix}6\\-3\end{pmatrix}\right]$

$=M\begin{pmatrix}9\\2\end{pmatrix}$

$=\begin{pmatrix}-1&0\\0&1\end{pmatrix}\begin{pmatrix}9\\2\end{pmatrix}$

$=\begin{pmatrix}-9\\2\end{pmatrix}$ i.e. (–9, 2)

7.



(i) $P\begin{pmatrix}4\\5\end{pmatrix}=\begin{pmatrix}0\\5\end{pmatrix}$ i.e. (0, 5)

(ii) $R^2\begin{pmatrix}4\\5\end{pmatrix}=\begin{pmatrix}4\\5\end{pmatrix}$ i.e. (4, 5)

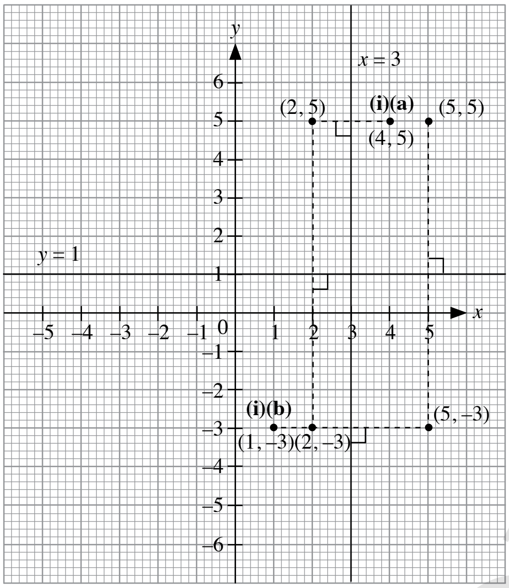
(iii) $RP\begin{pmatrix}4\\5\end{pmatrix}=R\begin{pmatrix}0\\5\end{pmatrix}$

$=\begin{pmatrix}0\\-3\end{pmatrix}$ i.e. (0, –3)

(iv) $\text{PR}\begin{pmatrix} 4 \\ 5 \end{pmatrix} = \text{P}\begin{pmatrix} 4 \\ -3 \end{pmatrix}$

$= \begin{pmatrix} 0 \\ -3 \end{pmatrix}$ i.e. (0, –3)

8. **(i)**



(a) Image of (4, 5) = (2, –3)

(b) Image of (1, –3) = (5, 5)

(ii) The coordinates of the point which remains invariant are (3, 1).

9. **(a)** $y = a + bx - x^2$ passes through (0, 3) and (3, 0).

$3 = a + b(0) - 0^2$

$a = 3$

$0 = 3 + b(3) - 3^2$

$b = 2$

$\therefore a = 3, b = 2$

(b) $y = 3 + 2x - 2x^2 = (3 - x)(1 + x)$

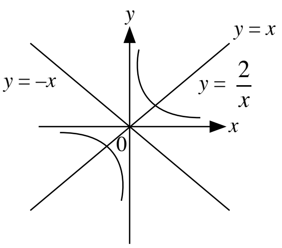
When $y = 0$,

$x = 3$ and $x = -1$

Line of symmetry is $x = \dfrac{3 + (-1)}{2} = 1$ i.e. $x = 1$

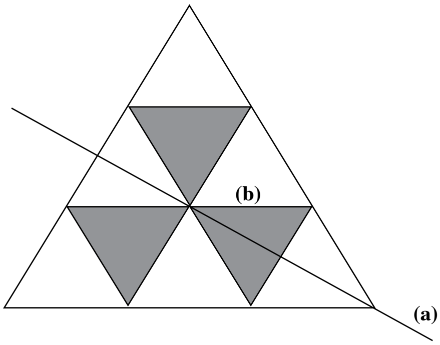
10. **(a)** The curve has rotational symmetry of order 2.

(b)



The line of symmetry is $y = x$ or $y = -x$.

11.



12. **(a)** $y = ax^2 + bx + c$ passes through (–2, 0), (4, 0) and (0, –8).

$-8 = a(0)^2 + b(0) + c$

$c = -8$

$0 = a(-2)^2 + b(-2) - 8$

$4a - 2b = 8$ — (1)

$0 = a(4)^2 + b(4) - 8$

$16a + 4b = 8$ — (2)

(1) × 2: $8a - 4b = 16$ — (3)

(2) + (3): $24a = 24$

$a = 1$

Substitute $a = 1$ into (1): $4(1) - 2b = 8$

$b = -2$

$\therefore a = 1, b = -2, c = -8$

(b) Equation of line of symmetry is $x = \dfrac{4 + (-2)}{2} = 1$

i.e. $x = 1$

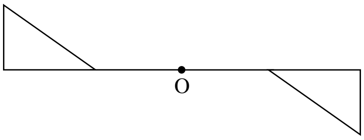
(c) $y = x^2 - 2x - 8$

When $x = 5$, $y = k$.

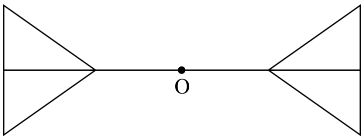
$k = 5^2 - 2(5) - 8$

$= 7$

13. **(a)**



(b)



14. **(a)** $y = 2(x + 1)(x - 3)$

When $y = 0$, $x = -1$ or $x = 3$.

$\therefore A(-1, 0)$ and $B(3, 0)$

When $x = 0$, $y = 2(0 + 1)(0 - 3)$

$= -6$

$\therefore C(0, -6)$

(b) $24 = 2(k + 1)(k - 3)$

$k^2 - 2k - 3 = 12$

$k^2 - 2k - 15 = 0$

$(k - 5)(k + 3) = 0$

$k = 5$ or $k = -3$

(c) Equation of line of symmetry is $x = \dfrac{-1 + 3}{2} = 1$

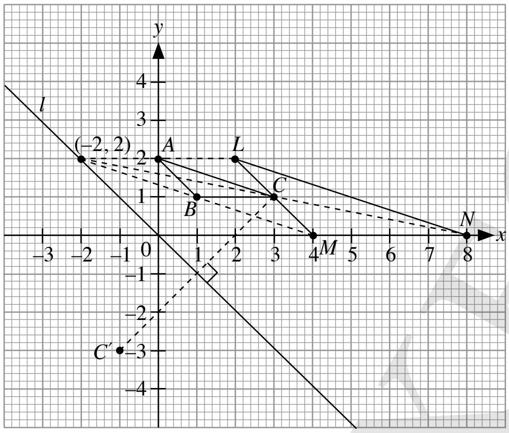
i.e. $x = 1$

15. **(a)** $\begin{pmatrix} 9 \\ -3 \end{pmatrix}$

(b) $(3, 4)$

(c) centre $(10, 11)$, $k = -\dfrac{1}{2}$

16.



(a) **(i)** Gradient $= -1$

(ii) Equation of line AB is $y = -x + 2$ or $y + x = 2$.

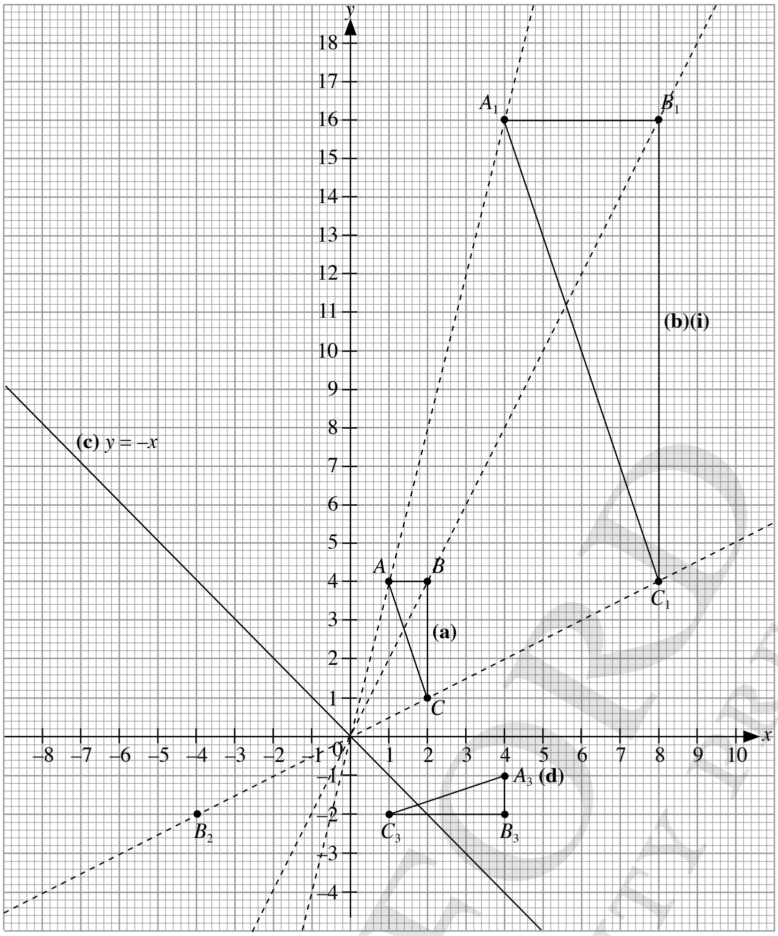
(iii) The coordinates of D are $(4, 2)$.

(b) The coordinates of the image of the point C are $(-1, -3)$.

(c) **(i)** The coordinates of the centre of the enlargement is $(-2, 2)$.

(ii) The coordinates of N are $(8, 0)$.

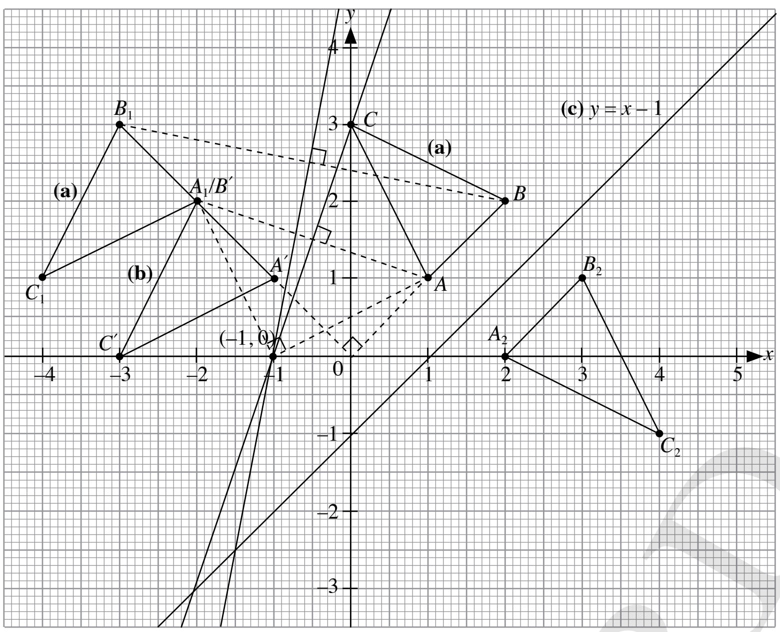
17.



(b) (ii) The scale factor of enlargement, $k = 4$.

(c) Line l is $y = -x$.

18.

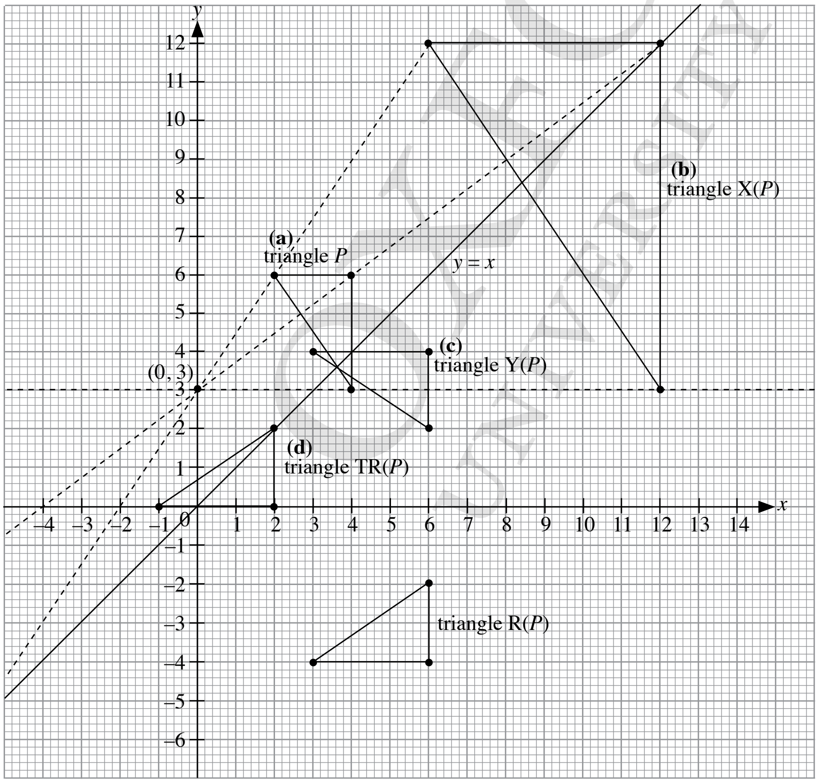


(a) The transformation is a 90° anticlockwise rotation about (–1, 0).

(b) The translation is $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$.

(c) The equation of l is $y = x - 1$.

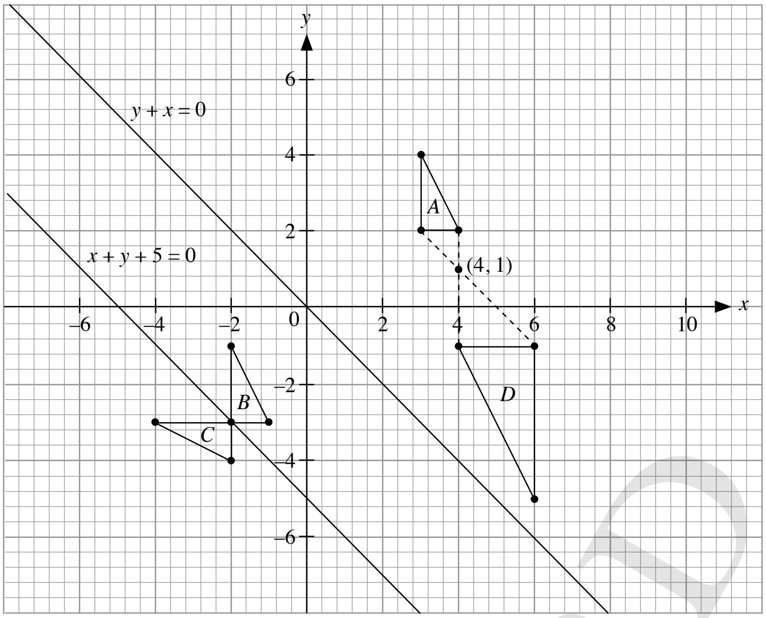
19.



(b) The transformation is an enlargement with scale factor 3 at centre (0, 3).

(c) The transformation is a reflection in the line $y = x$.

20.



(a) The translation is represented by the column vector $\begin{pmatrix} -5 \\ -5 \end{pmatrix}$.

(b) **(i)** The line l is $y = -x$ or $y + x = 0$.

(ii) The line m is $y = -x - 5$ or $x + y + 5 = 0$.

(c) **(i)** The coordinates of the centre of enlargement are (4, 1).

(ii) The scale factor of the enlargement is –2.

(iii) $\dfrac{\text{area of triangle } A}{\text{area of triangle } D} = \dfrac{1}{(-2)^2} = \dfrac{1}{4}$

Exercise 10F

1. $3(4h + 5) - 4(2h - 1) = 23$

$12h + 15 - 8h + 4 = 23$

$4h = 4$

$\therefore h = 1$

2. $3y = 4x + 47$

$y = \dfrac{4}{3}x + \dfrac{47}{3}$

$a = \dfrac{4}{3} = 1\dfrac{1}{3}$

$3 = 1\dfrac{1}{3}(1) + b$

$\therefore b = 1\dfrac{2}{3}$

3. $5x + 6y = 30$

On x-axis, $y = 0$.

$x = 6$

$\therefore P$ is (6, 0).

On y-axis, $x = 0$.

$y = 5$

$\therefore Q$ is (0, 5).

Area of $\triangle PQR = \dfrac{1}{2} \times 7 \times 6$

$= 21$ units^2

4. $y = kx + 6$ — (1)

$y = 2x + h$ —(2)

Since the point of intersection is (–5, 1),

Substitute $x = -5$, $y = 1$ into (1): $1 = k(-5) + 6$

$k = 1$

Substitute $x = -5$, $y = 1$ into (2): $1 = 2(-5) + h$

$h = 11$

$\therefore k = 1, h = 11$

5. **(a)** Gradient = $\frac{8-2}{3-1} = 3$

Equation of the line is $y = 3x + c$

$8 = 3(3) + c$

$c = -1$

$\therefore$ Equation of the line is $y = 3x - 1$.

(b) At x-axis, $y = 0$.

$3x - 1 = 0$

$x = \frac{1}{3}$

$\therefore A$ is $\left(\frac{1}{3}, 0\right)$.

6. **(a)** Gradient = $\frac{11-3}{2-0} = 4$

(b) Equation of the line is $y = 4x + 3$

7. **(a)** Gradient = $\frac{4-1}{2-1} = 3$

Equation is $y = 3x + c$

Since l passes through $(1, 1)$,

$1 = 3(1) + c$

$c = -2$

$\therefore$ Equation of l is $y = 3x - 2$.

(b) Since l passes through $(3, k)$,

$k = 3(3) - 2 = 7$

8. **(a)** Area of $\triangle ABC = \frac{1}{2} \times 7 \times (3 + 4)$

$= 24\frac{1}{2}$ units^2

(b) Let H be (h, k).

For $ABHC$ to be a parallelogram,

$$\left(\frac{-3+4}{2}, \frac{4+6}{2}\right) = \left(\frac{-3+h}{2}, \frac{3+k}{2}\right)$$

$$-3 + 4 = -3 + h$$

$$h = 4$$

$$-4 + 6 = 3 + k$$

$$k = -1$$

$\therefore H$ is $(4, -1)$.

(c) Gradient = $\frac{6-(-4)}{4-(-3)} = \frac{10}{7}$

Equation of the line BC is $y = \frac{10}{7}x + c$.

Since $(4, 6)$ lies on BC,

$6 = \frac{10}{7}(4) + c$

$c = \frac{2}{7}$

$\therefore$ Equation of the line BC is $y = \frac{10}{7}x + \frac{2}{7}$.

(d) Area of trapezium $ABCD = 35$ units^2

$\frac{1}{2} \times [7 + (k - 6)] \times 7 = 35$

$\therefore k = 9$

9. **(a)** **(i)** Area of $\triangle ABC = \frac{1}{2} \times (4 + 2) \times (5 - 1)$

$= 12$ units^2

(ii) Let K be (h, k).

For $CABK$ to be a parallelogram,

$$\left(\frac{4+(-4)}{2}, \frac{1+5}{2}\right) = \left(\frac{-2+h}{2}, \frac{1+k}{2}\right)$$

$$4 + (-4) = -2 + h$$

$$h = 2$$

$$1 + 5 = 1 + k$$

$$k = 5$$

$\therefore K$ is $(2, 5)$.

(iii) $\tan \angle BAC = \frac{5-1}{-4-(-2)}$

$= -2$

(b) $AB = 6$ units

Since the area of trapezium $ABCH$

$= 2 \times$ Area of $\triangle ABC$,

$CH = 6$ units

$\therefore h = -4 - 6 = -10$ or $h = -4 + 6 = 2$ (NA)

10. **(a)** $3x + 4y - 24 = 0$

$y = \frac{-3x + 24}{4}$

$= -\frac{3}{4}x + 6$

$\therefore$ Gradient $= -\frac{3}{4}$

(b) **(i)** At A, $y = 0$.

$x = 8$

$\therefore A$ is $(8, 0)$.

At B, $x = 0$

$y = 6$

$\therefore B$ is $(0, 6)$.

(ii) $AB = \sqrt{8^2 + 6^2}$

$= 10$ units

(iii) Area of $\triangle OAB = \frac{1}{2} \times 6 \times 8$

$= 24$ units^2

11. **(a)** Gradient of $PR = \frac{8-2}{0-12}$

$= -\frac{1}{2}$

(b) $y = -\frac{1}{2}x + 8$

$2y + x = 16$

Equation of PR is $2y + x = 16$.

(c) At S, $y = 0$.

$x = 16$

$\therefore S$ is $(16, 0)$.

(d) Area of $\triangle POR = \frac{1}{2} \times 16 \times 8 - \frac{1}{2} \times 16 \times 2$

$= 48$ units^2

12. **(a)** $AB = \sqrt{(-3-0)^2 + (14-5)^2}$

$= 9.49$ units (to 3 s.f.)

(b) Gradient of $AB = \frac{14-5}{-3}$

$= -3$

$\therefore$ Equation of AB is $y = -3x + 5$.

(c) At C, $y = 0$.

$-3x + 5 = 0$

$x = \frac{5}{3}$

$\therefore C$ is $\left(1\frac{2}{3}, 0\right)$.

(d) Area of $\triangle BCD = 15$ units^2

$\frac{1}{2} \times CD \times 5 = 15$

$CD = 6$

$h = 1\frac{2}{3} + 6$ or $1\frac{2}{3} - 6$

$\therefore h = 7\frac{2}{3}$ or $-4\frac{1}{3}$

13. **(a)** Equation of line l is $3y = mx + 15$.

$y = \frac{m}{3}x + 5$

$\therefore K$ is $(0, 5)$.

(b) $2y + 8x = 13$

$y = -4x + \frac{13}{2}$

Since line l has the same gradient as the second line.

$\frac{m}{3} = -4$

$m = -12$

(c) Since $(2, 6)$ lies on line l,

$3(6) = m(2) + 15$

$\therefore m = 1\frac{1}{2}$

(d) $HK = \sqrt{(2-0)^2 + (6-5)^2}$

$= 2.24$ units (to 3 s.f.)

14. **(a)** Gradient of $AB = \frac{9-3}{8-0}$

$= \frac{3}{4}$

(b) Equation of AB is $y = \frac{3}{4}x + 3$.

(c) $AB = \sqrt{(8-0)^2 + (9-3)^2}$

$= 10$ units

(d) At C, $x = 5$.

$y = \frac{3}{4}(5) + 3$

$= 6\frac{3}{4}$

$\therefore C$ is $\left(5, 6\frac{3}{4}\right)$.

15. **(a)** Equation of BC is $5y - 4x = 20$

At C, $y = 0$.

$-4x = 20$

$x = -5$

$\therefore C$ is $(-5, 0)$.

(b) Gradient of $AB = \frac{4-0}{0-3}$

$= -\frac{4}{3}$

(c) Equation of AB is $y = -\frac{4}{3}x + 4$.

(d) Area of $\triangle ABC = \frac{1}{2} \times (3+5) \times 4$

$= 16$ units^2

16. **(a)** Since $y = ax^2 + bx + c$ cuts the y-axis at $(0, -8)$,

$c = -8$

Since $y = ax^2 + bx + c$ cuts the x-axis at $(-2, 0)$ and $(4, 0)$,

$0 = a(-2)^2 + b(-2) - 8$

$4a - 2b = 8$ — (1)

$0 = a(4)^2 + b(4) - 8$

$16a + 4b = 8$ — (2)

$(1) \times 2 : 8a - 4b = 16$ — (3)

$(2) + (3) : 24a = 24$

$a = 1$

Substitute $a = 1$ into (1): $4 - 2b = 8$

$b = -2$

$\therefore a = 1, b = -2, c = -8$.

(b) $x = \frac{4-2}{2}$

$= 1$

$\therefore$ Equation of the line of symmetry is $x = 1$.

(c) Since $(5, k)$ lies on the curve, $y = x^2 - 2x - 8$

$k = (5)^2 - 2(5) - 8$

$= 7$

17. **(a)** $y = 2(x + 1)(x - 3)$

At A and B, $y = 0$.

$2(x + 1)(x - 3) = 0$

$x = -1$ or $x = 3$

$\therefore A$ is $(-1, 0)$, B is $(3, 0)$

At C, $x = 0$.

$y = 2(1)(-3)$

$= -6$

$\therefore C$ is $(0, -6)$

(b) Since $(k, 24)$ lies on the curve,

$24 = 2(k + 1)(k - 3)$

$k^2 - 2k - 3 = 12$

$k^2 - 2k - 15 = 0$

$(k + 3)(k - 5) = 0$

$\therefore k = -3$ or $k = 5$

(c) $x = \dfrac{3-1}{2}$

$= 1$

$\therefore$ Equation of line of symmetry is $x = 1$.

18. (a) $\dfrac{k-3}{6-8} = \dfrac{-8-k}{-3-6}$

$9(k-3) = -2(8+k)$

$9k - 27 = -16 - 2k$

$11k = 11$

$\therefore k = 1$

(b) $\dfrac{PQ}{QR} = \dfrac{\sqrt{(8-6)^2 + (3-1)^2}}{\sqrt{(6+3)^2 + (1+8)}}$

$= \dfrac{\sqrt{8}}{\sqrt{162}}$

$= \dfrac{\sqrt{4}}{\sqrt{81}}$

$= \dfrac{2}{9}$

(c) $PQ = 2RS$

Let midpoint of QR be M and S be (h, k), then R is the midpoint of MS.

$$M = \left(\frac{6-3}{2}, \frac{1-8}{2}\right)$$

$$= \left(\frac{3}{2}, -\frac{7}{2}\right)$$

$$(-3, -8) = \left(\frac{h+\frac{3}{2}}{2}, \frac{k-\frac{7}{2}}{2}\right)$$

$h + \dfrac{3}{2} = -6$ and $k - \dfrac{7}{2} = -16$

$h = -7\dfrac{1}{2}$ $\qquad k = -12\dfrac{1}{2}$

$\therefore S$ is $\left(-7\dfrac{1}{2}, -12\dfrac{1}{2}\right)$.

19. (a) Gradient of $AB = \dfrac{7-(-2)}{-5-6}$

$= -\dfrac{9}{11}$

(b) Gradient of line parallel to AB = Gradient of AB

$= -\dfrac{9}{11}$

Equation of line parallel to AB: $y = -\dfrac{9}{11}x + c$

When $x = 2$, $y = -8$

$-8 = -\dfrac{9}{11}(2) + c$

$c = -\dfrac{70}{11}$

$\therefore$ Equation of line parallel to AB: $y = -\dfrac{9}{11}x - \dfrac{70}{11}$

$11y + 9x + 70 = 0$

(c) When $y = 0$,

$9x = -70$

$x = -7\dfrac{7}{9}$

When $x = 0$,

$11y = -70$

$y = -6\dfrac{4}{11}$

$\therefore P\left(-7\dfrac{7}{9}, 0\right)$ and $Q\left(0, -6\dfrac{4}{11}\right)$

20. Since PQ is parallel to the line $x = 13$,

x-coordinate of Q = x-coordinate of P

$= -4$

$3y - x = 31$

$y = \dfrac{1}{3}x + \dfrac{31}{3}$

Since QR is parallel to the line $3y - x = 31$,

Gradient of $QR = \dfrac{1}{3}$

Let y-coordinate of Q be q i.e. $Q(-4, q)$.

$\dfrac{q-(-4)}{-4-5} = \dfrac{1}{3}$

$3q + 12 = -9$

$3q = -21$

$q = -7$

$\therefore Q(-4, -7)$

x-coordinate of S = x-coordinate of Ri

$= 5$

Let y-coordinate of S be s i.e. $S(5, s)$.

Gradient of PS = Gradient of QR

$\dfrac{y-7}{5-(-4)} = \dfrac{1}{3}$

$3y - 21 = 9$

$y = 10$

$\therefore S(5, 10)$

21. Gradient of $PQ = \dfrac{7-(-5)}{-9-4}$

$= -\dfrac{12}{13}$

Line l: $3y = hx + k$

$y = \dfrac{h}{3}x + \dfrac{k}{3}$

Since the line is perpendicular to PQ,

$\dfrac{h}{3} \times -\dfrac{12}{13} = -1$

$\dfrac{h}{3} = \dfrac{13}{12}$

$h = 3\dfrac{1}{4}$

Substitute $h = 3\frac{1}{4}$, $x = 2$ and $y = -1$ into line l:

$-1 = \frac{3\frac{1}{4}}{3}(2) + \frac{k}{3}$

$-3 = \frac{13}{2} + k$

$k = -9\frac{1}{2}$

$\therefore h = 3\frac{1}{4}$ and $k = -9\frac{1}{2}$

22. **(a)** Gradient of $AC = \frac{9 - 5}{2 - 5}$

$= -\frac{4}{3}$

Equation of AC: $y = -\frac{4}{3}x + c$

When $x = 2$, $y = 9$

$9 = -\frac{4}{3}(2) + c$

$c = \frac{35}{3}$

$\therefore$ Equation of AC: $y = -\frac{4}{3}x + \frac{35}{3}$

$3y + 4x = 35$

Since the diagonals of a rhombus cut one another at 90°, gradient of $AC \times$ gradient of $BD = -1$

gradient of $BD = -1 \div -\frac{4}{3}$

$= \frac{3}{4}$

Equation of BD: $y = \frac{3}{4}x + c$

When $x = 5$, $y = 5$

$5 = \frac{3}{4}(5) + c$

$c = \frac{5}{4}$

$\therefore$ Equation of BD: $y = \frac{3}{4}x + \frac{5}{4}$

$4y = 3x + 5$

(b) $x + 7y = 17$

$y = -\frac{1}{7}x + \frac{17}{7}$

Gradient of BC = Gradient of AD

$= -\frac{1}{7}$

Equation of BC: $y = -\frac{1}{7}x + c$

When $x = 8$, $y = 1$

$1 = -\frac{1}{7}(8) + c$

$c = \frac{15}{7}$

$\therefore$ Equation of BC: $y = -\frac{1}{7}x + \frac{15}{7}$

$7y + x = 15$

23. **(a)** When $x = 2$, $y = h$

$2h = 3(2) - 14$

$2h = -8$

$h = -4$

When $x = k$, $y = 8$

$2(8) = 3k - 14$

$3k = 30$

$k = 10$

$\therefore h = -4$ and $k = 10$

(b) Gradient of CP = Gradient of OB

$= \frac{0 - (-4)}{0 - 2}$

$= -2$

Equation of CP: $y = -2x + c$

When $y = 0$,

$3x - 14 = 0$

$x = \frac{14}{3}$

$C(4\frac{2}{3}, 0)$

When $x = 4\frac{2}{3}$, $y = 0$

$-2(4\frac{2}{3}) + c = 0$

$c = 9\frac{1}{3}$

$\therefore$ Equation of CP: $y = -2x + 9\frac{1}{3}$

$y + 2x = 9\frac{1}{3}$

(c) Area of $\triangle OCP = \frac{1}{2} \times 9\frac{1}{3} \times 4\frac{2}{3}$

$= 21\frac{7}{9}$ units^2

24. **(a)** Gradient of $AP = \frac{6 - (-4)}{4 - 10}$

$= -\frac{5}{3}$

Equation of AP: $y = -\frac{5}{3}x + c$

When $x = 4$, $y = 6$

$6 = -\frac{5}{3}(4) + c$

$c = \frac{38}{3}$

$\therefore$ Equation of AP: $y = -\frac{5}{3}x + \frac{38}{3}$

$3y + 5x = 38$

When $y = 0$,

$5x = 38$

$x = 7.6$

$\therefore B(7.6, 0)$

(b) Since gradient of BC = gradient of OA

$$= \frac{3}{2}$$

Equation of BC: $y = \frac{3}{2}x + c$

When $x = 7.6$, $y = 0$

$$c = -\frac{6}{4}(7.6)$$

$$= -11\frac{2}{5}$$

$\therefore$ Equation of BC: $y = 1\frac{1}{2}x - 11\frac{2}{5}$.

(c) Since gradient of OA = gradient of BC,

$$\frac{0 - t}{7.6 - 15} = \frac{3}{2}$$

$$-2t = -22\frac{1}{5}$$

$$t = 11\frac{1}{10}$$

(d) Area of $\triangle OAB = \frac{1}{2} \times 7.6 \times 6$

$$= 22.8 \text{ units}^2$$

25. (a) Using Pythagoras' Theorem,

$OB^2 = AB^2 - OA^2$

$$OB = \sqrt{17^2 - 15^2}$$

$$= 8$$

$\therefore B(0, 8)$

Since $OB = BQ$, $Q(0, 16)$.

Since $OA = OP$, $P(-15, 0)$.

(b) Gradient of $PQ = \frac{16 - 0}{0 - (-15)}$

$$= \frac{16}{15}$$

Equation of PQ: $y = \frac{16}{15}x + 16$

$$15y = 16x + 240$$

(c) Using Pythagoras' Theorem,

$PQ^2 = OP^2 + OQ^2$

$$PQ = \sqrt{15^2 + 16^2}$$

$$\approx 21.931$$

$$= 21.9 \text{ units (to 3 s.f.)}$$

(d) Area of $\triangle OPQ = \frac{1}{2} \times 16 \times 15$

$$= 120 \text{ units}^2$$

$$\frac{1}{2} \times PQ \times OR = 120$$

$$OR = \frac{120 \times 2}{21.931}$$

$$= 10.9 \text{ units (to 3 s.f.)}$$

Exercise 10G

1. (a) $|\overrightarrow{AB}| = \sqrt{5^2 + (-3)^2}$

$$= \sqrt{34}$$

(b) $k\begin{pmatrix} 5 \\ -3 \end{pmatrix} = \begin{pmatrix} 20 \\ 6x \end{pmatrix}$

$5k = 20$

$\therefore k = 4$

$-3k = 6x$

$\therefore x = -2$

2. (a) $|\overrightarrow{OA}| = \sqrt{3^2 + 4^2}$

$$= 5$$

(b) $2\overrightarrow{OA} + 3\overrightarrow{OB} = 2\begin{pmatrix} 3 \\ 4 \end{pmatrix} + 3\begin{pmatrix} 1 \\ 7 \end{pmatrix}$

$$= \begin{pmatrix} 9 \\ 29 \end{pmatrix}$$

3. (a) $\overrightarrow{OQ} = \overrightarrow{OP} + \overrightarrow{PQ}$

$$= \begin{pmatrix} 3 \\ 4 \end{pmatrix} + \begin{pmatrix} -4 \\ 5 \end{pmatrix}$$

$$= \begin{pmatrix} -1 \\ 9 \end{pmatrix}$$

$\therefore Q$ is $(-1, 9)$.

(b) $\overrightarrow{AB} = \begin{pmatrix} -2 \\ 12 \end{pmatrix} - \begin{pmatrix} 6 \\ 8 \end{pmatrix}$

$$= \begin{pmatrix} -8 \\ 4 \end{pmatrix}$$

$$\overrightarrow{AC} = \begin{pmatrix} h \\ 0 \end{pmatrix} - \begin{pmatrix} 6 \\ 8 \end{pmatrix}$$

$$= \begin{pmatrix} h - 6 \\ -8 \end{pmatrix}$$

$$\sqrt{(-8)^2 + 4^2} = \sqrt{(h - 6)^2 + (-8)^2}$$

$$h - 6 = \pm 4$$

$\therefore h = 10$ or 2

4. $\begin{pmatrix} m \\ 3 \end{pmatrix} + 2\begin{pmatrix} 11 \\ n \end{pmatrix} = \begin{pmatrix} 23 \\ 9 \end{pmatrix}$

$m + 22 = 23$

$\therefore m = 1$

$3 + 2n = 9$

$\therefore n = 3$

5. (a) $\overrightarrow{EA} = \overrightarrow{OA} - \overrightarrow{OE}$

$$= \mathbf{a} - \mathbf{b}$$

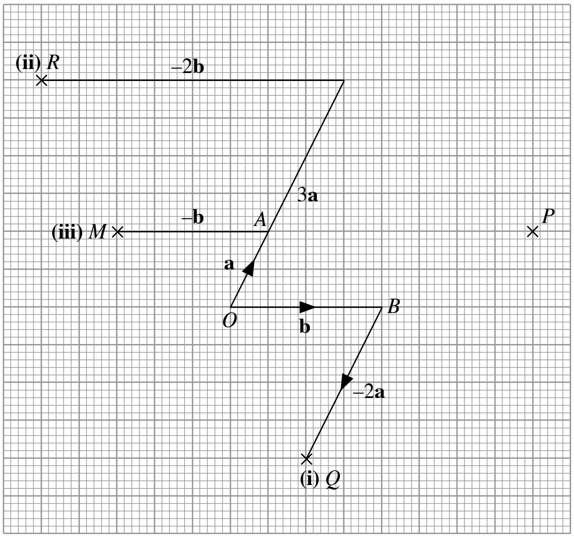
(b) $\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AD}$

$$= \mathbf{a} + 2\mathbf{b}$$

(c) $\overrightarrow{AD} = 2\mathbf{b}$. Quadrilateral $OADE$ is known as a trapezium.

6. **(a)** $\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP} = \mathbf{a} + 2\mathbf{b}$

(b)



7. **(a)** $|\overrightarrow{OP}| = \sqrt{6^2 + 8^2}$

$= 10$

(b) $10 = \sqrt{m^2 + 0^2}$

$m = \pm 10$

8. **(a)** $\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC}$

$= \begin{pmatrix} -2 \\ 3 \end{pmatrix} + \begin{pmatrix} -5 \\ -4 \end{pmatrix}$

$= \begin{pmatrix} -7 \\ -1 \end{pmatrix}$

(b) $\overrightarrow{AM} = \overrightarrow{AB} + \frac{1}{2}\overrightarrow{BC}$

$= \begin{pmatrix} 2 \\ -3 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} -7 \\ -1 \end{pmatrix}$

$= \begin{pmatrix} -1.5 \\ -3.5 \end{pmatrix}$

(c) $\overrightarrow{AB} = \overrightarrow{CP}$

$\begin{pmatrix} 2 \\ -3 \end{pmatrix} = \overrightarrow{OP} - \overrightarrow{OC}$

$= \overrightarrow{OP} - (\overrightarrow{OA} + \overrightarrow{AC})$

$\overrightarrow{OP} = \begin{pmatrix} 2 \\ -3 \end{pmatrix} + \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} -5 \\ -4 \end{pmatrix}$

$= \begin{pmatrix} -2 \\ -5 \end{pmatrix}$

$\therefore$ P is (–2, –5).

9. **(a)** $2\overrightarrow{PQ} + 3\overrightarrow{QR} = 2\begin{pmatrix} -2 \\ -5 \end{pmatrix} + 3\begin{pmatrix} 2 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} 2 \\ 19 \end{pmatrix}$

(b) Let $\begin{pmatrix} k \\ 12.5 \end{pmatrix} = h\begin{pmatrix} -2 \\ 5 \end{pmatrix}$

$12.5 = 5h$

$\therefore h = \frac{12.5}{5}$

$= 2.5$

$\therefore k = -2h$

$= -5$

(c) $|\overrightarrow{PR}| = \left|\begin{pmatrix} -2 \\ 5 \end{pmatrix} + \begin{pmatrix} 2 \\ 3 \end{pmatrix}\right|$

$= \left|\begin{pmatrix} 0 \\ 8 \end{pmatrix}\right|$

= 8 units

10. **(a)** $|\overrightarrow{PQ}| = \sqrt{(-9)^2 + 12^2}$

= 15 units

(b) $\overrightarrow{RS} = \frac{4}{3}\begin{pmatrix} -9 \\ 12 \end{pmatrix}$

$= \begin{pmatrix} -12 \\ 16 \end{pmatrix}$

(c) $|\overrightarrow{RS}| = \sqrt{(-12)^2 + 16^2}$

= 20 units

(d) $\overrightarrow{OQ} = \begin{pmatrix} 5 \\ -8 \end{pmatrix} + \begin{pmatrix} -9 \\ 12 \end{pmatrix}$

$= \begin{pmatrix} -4 \\ 4 \end{pmatrix}$

$\therefore Q$ is (–4, 4).

11. **(a)** $|\overrightarrow{PQ}| = \sqrt{(-10)^2 + 24^2}$

= 26 units

(b) $\overrightarrow{OQ} = \begin{pmatrix} 8 \\ 19 \end{pmatrix} + \begin{pmatrix} -10 \\ 24 \end{pmatrix}$

$= \begin{pmatrix} -2 \\ 43 \end{pmatrix}$

$\therefore Q$ is (–2, 43).

(c) $\overrightarrow{RS} = \frac{1}{4}\begin{pmatrix} -10 \\ 24 \end{pmatrix}$

$= \begin{pmatrix} -2.5 \\ 6 \end{pmatrix}$

12. **(a)** $2\mathbf{s} - \frac{1}{5}\mathbf{t} = 2\begin{pmatrix} 2 \\ 5 \end{pmatrix} - \frac{1}{5}\begin{pmatrix} 5 \\ -10 \end{pmatrix}$

$= \begin{pmatrix} 3 \\ 12 \end{pmatrix}$

(b) **(i)** $|\mathbf{t}| = \sqrt{5^2 + (-10)^2}$

= 11.2 units (to 3 s.f.)

(ii) $\left|2\mathbf{s} - \frac{1}{5}\mathbf{t}\right| = \sqrt{3^2 + 12^2}$

= 12.4 units (to 3 s.f.)

13. (a) $\overrightarrow{EB} = \overrightarrow{EA} + \overrightarrow{AB}$

$= -\frac{1}{2}\mathbf{c} + \mathbf{a}$

(b) $\overrightarrow{AK} = \overrightarrow{AE} + \frac{1}{3}\overrightarrow{EB}$

$= \frac{1}{2}\mathbf{c} + \frac{1}{3}\left(\mathbf{a} - \frac{1}{2}\mathbf{c}\right)$

$= \frac{1}{3}(\mathbf{a} + \mathbf{c})$

14. (a) $\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}$

$= \begin{pmatrix} 4 \\ 6 \end{pmatrix} - \begin{pmatrix} 2 \\ 4 \end{pmatrix}$

$= \begin{pmatrix} 2 \\ 2 \end{pmatrix}$

$\overrightarrow{AD} = \overrightarrow{OD} - \overrightarrow{OA}$

$= \begin{pmatrix} 6 \\ 7 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} 5 \\ 5 \end{pmatrix}$

(b) (i) $\overrightarrow{BC}$ and $\overrightarrow{AD}$ are parallel.

(ii) $\frac{BC}{AD} = \frac{\sqrt{2^2 + 2^2}}{\sqrt{5^2 + 5^2}}$

$= \frac{2}{5}$

(c) $\overrightarrow{AB} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$|\overrightarrow{AB}| = \sqrt{1^2 + 2^2}$

$= \sqrt{5}$

$\overrightarrow{CD} = \begin{pmatrix} 6 \\ 7 \end{pmatrix} - \begin{pmatrix} 4 \\ 6 \end{pmatrix}$

$= \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$|\overrightarrow{CD}| = \sqrt{2^2 + 1^2}$

$= \sqrt{5}$

$\therefore |\overrightarrow{AB}| = |\overrightarrow{CD}|$ (shown)

15. (a) $\overrightarrow{AP} = 2\mathbf{b}$

(b) $\overrightarrow{PD} = -2\mathbf{a}$

(c) $\overrightarrow{AD} = \overrightarrow{AP} + \overrightarrow{PB}$

$= 2\mathbf{b} - 2\mathbf{a}$

$= 2(\mathbf{b} - \mathbf{a})$

(d) $\overrightarrow{CB} = \overrightarrow{CP} + \overrightarrow{PB}$

$= \mathbf{b} - \mathbf{a}$

AD and *CB* are parallel and $AD = 2CB$.

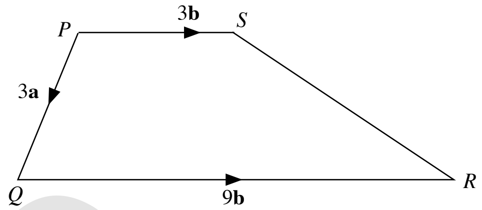
16. (a) Trapezium

(b) $\overrightarrow{SP} = \overrightarrow{SR} + \overrightarrow{RQ} + \overrightarrow{QP}$

$= 2\mathbf{a} - \mathbf{b} - 3\mathbf{a}$

$= -\mathbf{a} - \mathbf{b}$

17. (a)



(b) $PS \,//\, QR$, $QR = 3PS$

(c) $\overrightarrow{SR} = \overrightarrow{SP} + \overrightarrow{PQ} + \overrightarrow{QR}$

$= -3\mathbf{b} + 3\mathbf{a} + 9\mathbf{b}$

$= 3\mathbf{a} + 6\mathbf{b}$

(d) $\frac{\overrightarrow{AP}}{\overrightarrow{AP} + 3\mathbf{a}} = \frac{3\mathbf{b}}{9\mathbf{b}}$

$= \frac{1}{3}$

$3\overrightarrow{AP} = \overrightarrow{AP} + 3\mathbf{a}$

$\therefore \overrightarrow{AP} = 1\frac{1}{2}\mathbf{a}$

18. (a) (i) $\overrightarrow{DB} = \overrightarrow{DC} + \overrightarrow{CB}$

$= \mathbf{a} - \mathbf{b}$

(ii) $\overrightarrow{DE} = \frac{3}{5}\mathbf{a}$

(iii) $\overrightarrow{AE} = \mathbf{b} + \frac{3}{5}\mathbf{a}$

(b) (i) $\overrightarrow{BF} = h\overrightarrow{BD}$

$= h(\mathbf{b} - \mathbf{a})$

(ii) $\overrightarrow{BF} = \overrightarrow{BA} + \overrightarrow{AF}$

$= -\mathbf{a} + k\left(\mathbf{b} + \frac{3}{5}\mathbf{a}\right)$

$= k\mathbf{b} + \mathbf{a}\left(\frac{3}{5}k - 1\right)$

19. (a) (i) $\overrightarrow{UR} = 2\mathbf{a}$

(ii) $\overrightarrow{QV} = \frac{1}{2}\overrightarrow{QT}$

$= \frac{1}{2}(\overrightarrow{QR} + \overrightarrow{RT})$

$= \frac{1}{2}(3\mathbf{a} + \mathbf{b})$

(iii) $\overrightarrow{QP} = \overrightarrow{QR} + 2\overrightarrow{RT}$

$= 3\mathbf{a} + 2\mathbf{b}$

(b) $\overrightarrow{PV} = \overrightarrow{PQ} + \overrightarrow{QV}$

$= -3\mathbf{a} - 2\mathbf{b} + \frac{3}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$

$= -\frac{3}{2}(\mathbf{a} + \mathbf{b})$

(c) $\overrightarrow{PU} = -3\mathbf{a} - 2\mathbf{b} + \mathbf{a}$

$= -2(\mathbf{a} + \mathbf{b})$

(d) (i) $\frac{PV}{PU} = \frac{-\frac{3}{2}(\mathbf{a}+\mathbf{b})}{-2(\mathbf{a}+\mathbf{b})}$

$= \frac{3}{4}$

(ii) $\frac{\text{Area of } \triangle PQV}{\text{Area of } \triangle PQU} = \frac{\frac{1}{2}PV \times \text{height}}{\frac{1}{2}PU \times \text{height}}$

$= \frac{3}{4}$

(iii) $\frac{\text{Area of } \triangle PQU}{\text{Area of } PQRS} = \frac{\frac{1}{2}QU \times \text{height}}{3QU \times \text{height}}$

$= \frac{1}{6}$

$= \frac{4}{24}$

$\frac{\text{Area of } \triangle PQV}{\text{Area of } PQRS} = \frac{3}{24} = \frac{1}{8}$

20. (a) $\overrightarrow{AQ} = \overrightarrow{OQ} - \overrightarrow{OA}$

$= \frac{3}{7}\mathbf{b} - \mathbf{a}$

(b) $\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP}$

$= \mathbf{a} + \frac{1}{4}\overrightarrow{AB}$

$= \mathbf{a} + \frac{1}{4}(\mathbf{b} - \mathbf{a})$

$= \frac{1}{4}(3\mathbf{a} + \mathbf{b})$

(c) $\overrightarrow{OR} = \overrightarrow{OA} + \overrightarrow{AR}$

$= \mathbf{a} + k\left(\frac{3}{7}\mathbf{b} - \mathbf{a}\right)$

$= (1 - k)\mathbf{a} + \frac{3}{7}k\mathbf{b}$

21. (a) (i) $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$= 3\mathbf{b} - 4\mathbf{a}$

(ii) $\overrightarrow{AM} = \frac{1}{2}(3\mathbf{b} - 4\mathbf{a})$

(iii) $\overrightarrow{OM} = \overrightarrow{OA} + \overrightarrow{AM}$

$= 4\mathbf{a} + \frac{3}{2}\mathbf{b} - 2\mathbf{a}$

$= 2\mathbf{a} + \frac{3}{2}\mathbf{b}$

(iv) $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$

$= 6\mathbf{b} - 7\mathbf{a}$

(v) $\overrightarrow{MQ} = \overrightarrow{MO} + \overrightarrow{OQ}$

$= -2\mathbf{a} - \frac{3}{2}\mathbf{b} + 6\mathbf{b}$

$= 4\frac{1}{2}\mathbf{b} - 2\mathbf{a}$

(vi) $\overrightarrow{MP} = \overrightarrow{MA} + \overrightarrow{AP}$

$= -\frac{3}{2}\mathbf{b} + 2\mathbf{a} + 3\mathbf{a}$

$= 5\mathbf{a} - \frac{3}{2}\mathbf{b}$

(b) (i) $\frac{\text{Area of } \triangle OAM}{\text{Area of } \triangle AMP} = \frac{\frac{1}{2}(4\mathbf{a}) \times \text{height}}{\frac{1}{2}(3\mathbf{a}) \times \text{height}}$

$= \frac{4}{3}$

(ii) $\frac{\text{Area of } \triangle OMB}{\text{Area of } \triangle OMQ} = \frac{\frac{1}{2}(3\mathbf{b}) \times \text{height}}{\frac{1}{2}(6\mathbf{b}) \times \text{height}}$

$= \frac{1}{2}$

(c) $\overrightarrow{OR} = \overrightarrow{OP} + \overrightarrow{PR}$

$= 7\mathbf{a} + \frac{7}{15}\overrightarrow{PQ}$

$= 7\mathbf{a} + \frac{7}{15}(6\mathbf{b} - 7\mathbf{a})$

$= \frac{28}{15}\left(2\mathbf{a} + \frac{3}{2}\mathbf{b}\right)$

$= \frac{28}{15}\overrightarrow{OM}$

$\therefore h = \frac{15}{28}$

Exercise 10H

1. (a) $\angle OCA = \angle OBA = 90°$ (tangent ⊥ radius)

Obtuse $\angle BOC = 360° - 90° - 90° - 70°$

$= 110°$

(b) $\angle BOC = 360° - 110°$

$= 250°$ (∠s at a point)

Obtuse $\angle BPC = \frac{1}{2} \times 250°$

$= 125°$ (∠ at centre = 2 ∠ at circumference)

2. (a) $\angle BOA = 2 \times 62°$

$=124°$ (∠ at centre = 2 ∠ at circumference)

$\angle BAO = \frac{180° - 124°}{2}$

$= 28°$ (base ∠s of isos. △)

(b) $\angle AOD = 2 \times 28°$

$= 56°$ (∠ at centre = 2 ∠ at circumference)

(c) $\angle CBX = 32°$ (∠s in the same segment)

$\therefore \angle BXC = 180° - 62° - 32°$

$= 86°$ (∠ sum of a △)

(d) $\angle OAD = \dfrac{180° - 56°}{2}$

$= 62°$ (base ∠s of isos. △)

$\therefore \angle TAD = 90° - 62°$

$= 28°$

3. **(a)** $\angle AOB = 2x$ (∠ at centre = 2 ∠ at circumference)

(b) $\angle BOC = \angle OBA$

$= \dfrac{180° - 2x}{2}$

$= 90° - x$ (alt. ∠s, base ∠s of isos. △)

4. **(a)** $\angle OTA = 90°$ (tangent ⊥ radius)

$\angle OBA = 360° - 72° - 90° - 38°$

$= 160°$

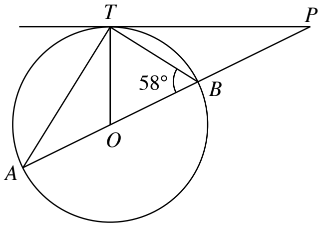
(b) $\angle BCT = \dfrac{1}{2} \times 72°$

$= 36°$ (∠ at centre = 2 ∠ at circumference)

(c) $\angle OTC = 180° - 38° - 36° - 90°$

$= 16°$ (∠ sum of a △)

5. **(a)**



$\angle OTB = \angle ABT = 58°$ (base ∠s of isos. △)

$\angle OTP = 90°$ (rt. ∠ in a semicircle)

$\angle BTP = 90° - 58° = 32°$

(b) $\angle BPT = 58° - 32° = 26°$

6. **(a)** $\angle ABC = \dfrac{360° - 162°}{2}$

$= 99°$ (∠ at centre = 2 ∠ at circumference)

(b) $\angle BAO = 360° - 99° - 162° - 63°$

$= 36°$

7. **(a)** Let r represent radius of the circle.

Using Pythagoras' Theorem,

$r^2 + 16^2 = (r + 8)^2$

$r^2 + 256 = 8^2 + 16r + 64$

$\therefore r = 12$ cm

(b) $\angle POA = \tan^{-1} \dfrac{16}{12}$

$= 0.927$ radians (to 3 s.f.)

8. **(a)** $x° = \dfrac{108°}{2}$

$= 54°$ (∠ at centre = 2 ∠ at circumference)

$y° = \dfrac{180° - 108°}{2}$

$= 36°$ (base ∠s of isos. △)

$\therefore x = 54, y = 36.$

(b) $x° = 90° - 34° = 56°$

$y° = 90°$ (rt. ∠ in a semicircle)

$\therefore x = 56, y = 90.$

9. **(a)** $\angle QOR = 90° - 34°$

$= 56°$ (tangent ⊥radius, ∠ sum of a △)

$\angle OCQ = \dfrac{180° - 56°}{2}$

$= 62°$ (base ∠s of isos. △)

(b) $\angle OBA = \angle OAB$

$= 31°$ (base ∠s of isos. △)

(c) $\angle OCA = \dfrac{1}{2} \times 56°$

$= 28°$ (ext. ∠ = sum of opp. int. ∠s, base ∠s of isos. △)

10. **(a)** $\angle ADC = \left(\dfrac{\pi}{2} - 0.7\right)$ rad (rt. ∠ in a semicircle, ∠ sum of a △)

$\therefore \angle ABC = \pi - \left(\dfrac{\pi}{2} - 0.7\right)$

$= \left(\dfrac{\pi}{2} + 0.7\right)$ rad

(b) $\angle OCD = \angle ADC$

$= \left(\dfrac{\pi}{2} - 0.7\right)$ rad (base ∠s of isos. △)

11. **(a)** $\angle OAB = \angle OBA$

$= 48°$ (base ∠s of isos. △)

(b) $\angle BCD = 180° - 48°$

$= 132°$ (∠s in opp. segments)

$\angle CBD = \dfrac{180° - 132°}{2}$

$= 24°$ (base ∠s of isos. △)

$\angle ABD = 90°$ (rt. ∠ in a semicircle)

$\therefore \angle ABC = 90° + 24°$

$= 114°$

(c) $\angle BCD = 132°$

(d) $\angle ADC = 180° - 114°$

$= 66°$ (∠s in opp. segments)

12. $\angle OBP = 90°$ (tangent ⊥radius)

Using Pythagoras' Theorem,

$4^2 + 8^2 = (4 + x)^2$

$4 + x = \sqrt{80}$

$x = \sqrt{80} - 4$

$= 4.94$ (to 3 s.f.)

13. $\angle ADB = \frac{42^\circ}{2}$

$= 21^\circ$ ($\angle$ at centre = 2 $\angle$ at circumference)

$\angle OBC = \frac{88^\circ}{2}$

$= 44^\circ$ ($\angle$ at centre = 2 $\angle$ at circumference)

$\angle APB + 21^\circ = 44^\circ$ (ext. $\angle$ = sum of opp. int. $\angle$s)

$\therefore \angle APB = 23^\circ$

14. **(a)** $\angle COD = 2 \times 52^\circ$

$= 104^\circ$ ($\angle$ at centre = 2 $\angle$ at circumference)

(b) $\angle OCD = \frac{180^\circ - 104^\circ}{2}$

$= 38^\circ$ (base $\angle$s of isos. $\triangle$)

$\angle BCD = 180^\circ - 64^\circ$

$= 116^\circ$ ($\angle$s in opp. segments)

$\therefore \angle BCO = 116^\circ - 38^\circ$

$= 78^\circ$

15. **(a)** $x^\circ = 90^\circ - 65^\circ$

$= 25^\circ$ (rt. $\angle$ in semicircle, $\angle$ sum of a $\triangle$)

y cm = 16 cm

$\therefore x = 25, y = 16$

(b) $x = 180^\circ - 52^\circ$

$= 128^\circ$ (tangent $\perp$ radius, $\angle$ sum of a quadrilateral)

$y = \frac{128^\circ}{2}$

$= 64^\circ$ ($\angle$ at centre = 2 $\angle$ at circumference)

16. **(a)** $\angle ADB = 90^\circ$ (rt. $\angle$ in semicircle)

$\angle ADC = 90^\circ + 15^\circ$

$= 105^\circ$

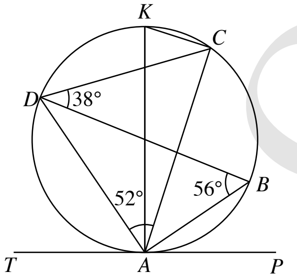
(b) $\angle BAD = 90^\circ - 35^\circ$

$= 55^\circ$ (rt. $\angle$ in semicircle, $\angle$ sum of a $\triangle$)

$\angle BCD = 180^\circ - 55^\circ$

$= 125^\circ$ ($\angle$s in opp. segments)

17.



Draw a diameter AK. Join KC.

$\angle CAB = 38^\circ$ ($\angle$s in same segment)

$\angle ADB = 180^\circ - 56^\circ - 38^\circ - 52^\circ$

$= 34^\circ$ ($\angle$ sum of a $\triangle$)

$\angle KCA = 90^\circ$ (rt. $\angle$ in semicircle)

$\angle CKA = 38^\circ + 34^\circ$

$= 72^\circ$ ($\angle$s in same segment)

$\angle KAC = 90^\circ - 72^\circ$

$= 18^\circ$ ($\angle$ sum of a $\triangle$)

$\angle KAP = 90^\circ$ (tangent $\perp$ radius)

$\therefore \angle PAB = 90^\circ - 18^\circ - 38^\circ$

$= 34^\circ$

18. **(a)** $\angle ADB = 41^\circ$ ($\angle$s in same segment)

$\angle ABC = 180^\circ - 41^\circ - 29^\circ$

$= 110^\circ$ ($\angle$s in opp. segments)

(b) $\angle OAD = \angle ADB$

$= 41^\circ$ (base $\angle$s of isos. $\triangle$)

$\angle OAT = 90^\circ$ (tangent $\perp$radius)

$\angle DAT = 90^\circ - 41^\circ$

$= 49^\circ$ (ext. $\angle$ = sum of opp. int. $\angle$s,
base $\angle$s of isos. $\triangle$)

(c) $\angle ACD = 90^\circ - 41^\circ$

$= 49^\circ$

19. **(a)** $\angle ADB = 74^\circ$ ($\angle$s in same segment)

$\angle DAT = \frac{74^\circ}{2}$

$= 37^\circ$ (base $\angle$s of isos. $\triangle$)

$\angle CBD = \angle OAD$

$= 90^\circ - 37^\circ$

$= 53^\circ$ (alt. $\angle$s)

$\angle ABC = 90^\circ$ (rt. $\angle$ in semicircle)

$\angle ABD = 90^\circ - 53^\circ$

$= 37^\circ$

(b) $\angle BAC = 180^\circ - 90^\circ - 74^\circ$

$= 16^\circ$ ($\angle$ sum of a $\triangle$)

20. **(a)** Let O be the centre of the circle.

$\angle ADB = 68^\circ$ ($\angle$s in same segment)

$\angle AOB = 2 \times 68^\circ$

$= 136^\circ$ ($\angle$ at centre = 2 $\angle$ at circumference)

$\angle OAB = \angle OBA$

$= \frac{180^\circ - 136^\circ}{2}$

$= 22^\circ$ (base $\angle$s of isos. $\triangle$)

$\angle TBA = \angle TAB$

$= 90^\circ - 22^\circ$

$= 68^\circ$ (tangent $\perp$radius)

(b) $\angle BAC = 22^\circ + 8^\circ = 30^\circ$

$\angle ABC = 180^\circ - 68^\circ - 30^\circ$

$= 82^\circ$ ($\angle$ sum of a $\triangle$)

(c) $\angle CBD = \angle CAD$

$= 96^\circ - 68^\circ$

$= 28^\circ$ ($\angle$s in same segment, ext. $\angle$ = sum of
opp. int. $\angle$s)

$\angle PAD = 180^\circ - 98^\circ - 28^\circ$

$= 54^\circ$ (adj. $\angle$s on a str. line)

21. **(a)** Radius perpendicular to tangent.

(b) $\triangle AOB$ or $\triangle TAB$

(c) $\angle AOB = 2 \times 58^\circ$

$= 116^\circ$ ($\angle$ at centre = 2 $\angle$ at circumference)

(d) $\angle ABO = \frac{180^\circ - 116^\circ}{2}$

$= 32^\circ$ (base $\angle$s of isos. $\triangle$)

(e) $\angle ATB = 180^\circ - 116^\circ$

$= 64^\circ$

22. **(a)** 90° (rt. $\angle$ in semicircle)

(b) **(i)** $\angle SOP = 2 \times 48^\circ$

$= 96^\circ$ ($\angle$ at centre = 2 $\angle$ at circumference)

$\angle QOS = 180^\circ - 96^\circ$

$= 84^\circ$ (adj. $\angle$s on a str. line)

(ii) $\angle QPS = 180^\circ - 90^\circ - 48^\circ$

$= 42^\circ$ ($\angle$s in opp. segments)

23. **(a)** $\angle AOD = 180^\circ - 2x^\circ$ (base $\angle$s of isos. $\triangle$,
$\angle$ sum of a $\triangle$)

(b) $\angle ACD = (180^\circ - 2x^\circ)$

$= 90^\circ - x^\circ$ ($\angle$ at centre = 2 $\angle$ at circumference)

(c) $\angle BDC = y^\circ$ ($\angle$s in same segment)

$\angle ACD = 90^\circ - y^\circ$ ($\angle$ sum of a $\triangle$)

$90^\circ - x^\circ = 90^\circ - y^\circ$

$\therefore x = y$

24. **(a)** $\angle PSR = 52^\circ$

$\therefore \angle PTR = 52^\circ$ ($\angle$s in same segment)

(b) $\angle TRS = 52^\circ$ (alt. $\angle$s)

$\therefore \angle SOT = 2 \times 52^\circ$

$= 104^\circ$ ($\angle$ at centre = 2 $\angle$ at circumference)

25. **(a)** $\angle ACB = \dfrac{180^\circ - 130^\circ}{2}$

$= 25^\circ$ (base $\angle$s of isos. $\triangle$)

$\therefore \angle AEB = 25^\circ$ ($\angle$s in same segment)

(b) $\angle BCD = 130^\circ$

$\angle ACD = 130^\circ - 25^\circ = 105^\circ$

$\therefore \angle AED = 180^\circ - 105^\circ$

$= 75^\circ$ ($\angle$s in opp. segments)

(c) $\angle CAD = 25^\circ$

$\therefore \angle COD = 2 \times 25^\circ$

$= 50^\circ$ ($\angle$ at centre = 2 $\angle$ at circumference)

26. **(a)** $\angle AOB = 180^\circ - 34^\circ - 34^\circ$

$= 112^\circ$ (base $\angle$s of isos. $\triangle$, $\angle$ sum of a $\triangle$)

$\angle APB = 180^\circ - 112^\circ = 68^\circ$ ($\angle$s in opp. segments)

(b) $\angle ACB = \dfrac{112^\circ}{2}$

$= 56^\circ$ ($\angle$ at centre = 2 $\angle$ at circumference)

27. **(a)** Let $\angle PAD = 4x^\circ$ and $\angle DAC = 5x^\circ$.

$\angle ACD = \angle PAD$

$= 4x^\circ$ ($\angle$s in alt. segment)

$4x^\circ + 4x^\circ + 5x^\circ + 37^\circ = 180^\circ$

$13x^\circ + 37^\circ = 180^\circ$

$13x^\circ = 143^\circ$

$x^\circ = 11^\circ$

$\angle ACD = 4 \times 11^\circ$

$= 44^\circ$

(b) $\angle ABC = \angle PAC$

$= 9 \times 11^\circ$

$= 99^\circ$ ($\angle$s in alt. segment)

$\angle ACB = \dfrac{180^\circ - 99^\circ}{2}$

$= 40.5^\circ$ (base $\angle$s of isos. $\triangle$)

(c) $\angle ADC = 180^\circ - 99^\circ$

$= 81^\circ$ ($\angle$s in opp. segments)

28. **(i)** $\angle QPX = \angle BXQ$

$= 4x^\circ$ ($\angle$s in alt. segment)

$4x^\circ + 5x^\circ + 63^\circ = 180^\circ$ ($\angle$ sum of a $\triangle$)

$9x^\circ = 117^\circ$

$x^\circ = 13^\circ$

$\therefore x = 13$

(ii) $\angle AXP = 180^\circ - 63^\circ - (4 \times 13^\circ)$

$= 65^\circ$ (adj. $\angle$s on a str. line)

(iii) $\angle PRX = \angle PXB$

$= 63^\circ + (4 \times 13^\circ)$

$= 115^\circ$ ($\angle$s in alt. segment)

29. **(i)** $\angle ADB = \angle ACB$

$= 41^\circ$

$\angle ABC = 180^\circ - \angle ADB$

$= 180^\circ - 41^\circ - 29^\circ$

$= 110^\circ$ ($\angle$s in opp. segments)

(ii) $\angle OAD = \angle ODA$

$= 41^\circ$ ($OA = OD$, base $\angle$s of isos. $\triangle$)

$\angle OAT = 90^\circ$ (tangent $\perp$ radius)

$\angle DAT = 90^\circ - 41^\circ$

$= 49^\circ$

(iii) $\angle ACD = \angle DAT$

$= 49^\circ$ ($\angle$s in alt. segment)

30. **(i)** $\angle APB = 180^\circ - 58^\circ - 61^\circ$

$= 61^\circ$ (adj. $\angle$s on a str. line)

$\angle BAP = \angle TPB$

$= 58^\circ$ ($\angle$s in alt. segment)

$\angle ABP = 180^\circ - 58^\circ - 61^\circ$

$= 61^\circ$ ($\angle$ sum of a $\triangle$)

$\angle ABR = 180^\circ - 61^\circ$

$= 119^\circ$ (adj. $\angle$s on a str. line)

$\angle PQR = 180^\circ - 119^\circ$

$= 61^\circ$ ($\angle$s in opp. segments)

(ii) $\angle BAQ = 180^\circ - 58^\circ$

$= 122^\circ$ (adj. $\angle$s on a str. line)

$\angle PRQ = 180^\circ - 122^\circ$

$= 58^\circ$ ($\angle$s in opp. segments)

Chapter 11 Probability and Statistics

Exercise 11A

1. **(a)** $P(\text{A is } 2) = \frac{1}{6}$

(b) $P(A < 6) = 1$

(c) $P(\text{B is } 9) = \frac{1}{10}$

(d) $P(B > 7) = \frac{2}{10}$

$= \frac{1}{5}$

2. **(a)** $P(\text{C is } 0) = \frac{1}{3}$

(b) $P(C > 1) = \frac{1}{3}$

(c) $P(\text{D is } 7) = \frac{1}{10}$

(d) $P(D < 3) = \frac{3}{10}$

3. **(a)**

	A	**A**	**K**	**B**	**B**
A	AA	AA	AK	AB	AB
A	AA	AA	AK	AB	AB
A	AA	AA	AK	AB	AB
P	PA	PA	PK	PB	PB
P	PA	PA	PK	PB	PB

(b) **(i)** $P(2A) = \frac{6}{25}$

(ii) $P(AB) = \frac{6}{25}$

(iii) $P(PK) = \frac{2}{25}$

4. **(a)**

+	**5**	**10**	**20**	**50**	**100**
5	10	15	25	55	105
10	15	20	30	60	110
20	25	30	40	70	120
50	55	60	70	100	150
100	105	110	120	150	200

(b) **(i)** $P(\leqslant 60¢) = \frac{13}{25}$

(ii) $P(> 95¢) = \frac{10}{25}$

(c) **(i)** $P(\text{exactly } 10¢) = \frac{2}{25}$

(ii) $P(< 110¢) = \frac{18}{25}$

5. **(a)**

×	**3**	**4**	**5**	**6**	**7**	**8**
3	9	12	15	18	21	24
4	12	16	20	24	28	32
5	15	20	25	30	35	40
6	18	24	30	36	42	48
7	21	28	35	42	49	56
8	24	32	40	48	56	64

(b) **(i)** $P(\text{odd}) = \frac{9}{36}$

$= \frac{1}{4}$

(ii) $P(\leqslant 23) = \frac{12}{36}$

$= \frac{1}{3}$

(iii) $P(\text{prime}) = 0$

(iv) $P(\text{divisible by } 14) = \frac{6}{36}$

$= \frac{1}{6}$

(v) $P(> 8) = 1$

6. There are x red balls (R), 8 yellow balls (Y) and 10 blue balls (B).

(a) $P(R) = \frac{x}{x+18}$

(b) $\frac{10}{x+18+15} = \frac{2}{7}$

$$2x + 66 = 70$$

$$2x = 4$$

$\therefore x = 2$

7. There are 18 boys (B) and 32 girls (G).

$$\frac{18 + x + 3}{18 + 32 + 2x + 12} = \frac{3}{8}$$

$$\frac{x + 21}{2x + 62} = \frac{3}{8}$$

$$8x + 168 = 6x + 186$$

$$2x = 18$$

$\therefore x = 9$

8. There are x blue balls (B), $(2x + 5)$ red balls (R) and $(3x + 25)$ yellow balls (Y).

$$\frac{2x + 5}{2x + 5 + x + 3x + 25} = \frac{11}{36}$$

$$36(2x + 5) = 11(6x + 30)$$

$$72x + 180 = 66x + 330$$

$$6x = 150$$

$\therefore x = 25$

9. There are 6 red pens (R), 2 yellow pens (Y) and 4 green pens (G).

(a) $\text{P}(G) = \frac{4}{12}$

$= \frac{1}{3}$

(b) P(second pen drawn is R)

$= \text{P}(RR, YR, GR)$

$= \frac{6}{12} \times \frac{5}{11} + \frac{2}{12} \times \frac{6}{11} + \frac{4}{12} \times \frac{6}{11}$

$= \frac{1}{2}$

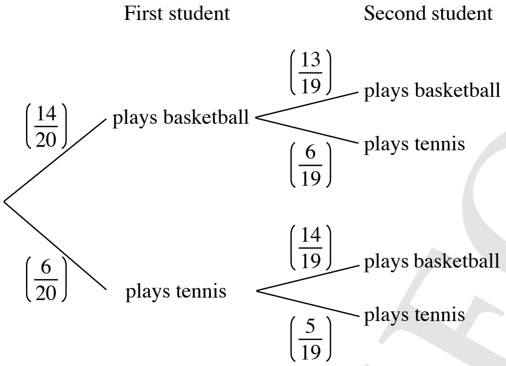
(c) P(all of the same colour)

$= \text{P}(RRR \text{ or } GGG)$

$= \frac{6}{12} \times \frac{5}{11} \times \frac{4}{10} + \frac{4}{12} \times \frac{3}{11} \times \frac{2}{10}$

$= \frac{6}{55}$

10. (a)



(b) (i) P(first plays tennis and second plays basketball)

$= \frac{6}{20} \times \frac{14}{19}$

$= \frac{21}{95}$

(ii) P(at most one play tennis)

= 1 – P(both play tennis)

$= 1 - \frac{6}{20} \times \frac{5}{19}$

$= \frac{35}{38}$

(iii) P(second student plays basketball)

$= \frac{14}{20} \times \frac{13}{19} + \frac{6}{20} \times \frac{14}{19}$

$= \frac{7}{10}$

11. (a) $p = \frac{1}{4}, q = \frac{1}{3}, r = \frac{7}{15}, s = \frac{4}{15}, t = \frac{4}{15},$

$u = \frac{7}{15}, v = \frac{1}{3}, w = \frac{1}{5}$

(b) (i) $\text{P}(\$1) = \frac{4}{16} \times \frac{1}{5}$

$= \frac{1}{20}$

(ii) P(70¢) = P(20, 50 or 35, 35 or 50, 20)

$= \frac{7}{16} \times \frac{4}{15} + \frac{5}{16} \times \frac{4}{15} + \frac{4}{16} \times \frac{7}{15}$

$= \frac{19}{60}$

(iii) P(> 60¢) = 1 – P(< 60¢)

= 1 – P(20, 20 or 20, 35 or 35, 20)

$= 1 - \left(\frac{7}{16} \times \frac{6}{15} + \frac{7}{15} \times \frac{5}{15} + \frac{5}{16} \times \frac{7}{15}\right)$

$= \frac{8}{15}$

(c) P(90¢) = P(20, 20, 50 or 20, 50, 20 or 20, 35, 35 or 35, 35, 20 or 35, 20, 35 or 50, 20, 20)

$= \frac{7}{16} \times \frac{6}{15} \times \frac{4}{14} + \frac{7}{16} \times \frac{4}{15} \times \frac{3}{14}$

$+ \frac{7}{16} \times \frac{5}{15} \times \frac{4}{14} + \frac{5}{16} \times \frac{4}{15} \times \frac{7}{14}$

$+ \frac{5}{16} \times \frac{7}{15} \times \frac{4}{14} + \frac{4}{16} \times \frac{7}{15} \times \frac{6}{14}$

$= \frac{1}{4}$

12. There are 5 beef dishes (B), 2 fish dishes (F) and 4 chicken dishes (C).

(a) P(at least one chicken dish)

$= \frac{4}{11} \times \frac{3}{10} + \frac{4}{11} \times \frac{7}{10} + \frac{7}{11} \times \frac{4}{10}$

$= \frac{35}{55}$

(b) P(same dish)

$= \text{P}(BBB \text{ or } CCC)$

$= \frac{5}{11} \times \frac{4}{10} \times \frac{5}{11} + \frac{4}{11} \times \frac{3}{10} \times \frac{2}{9}$

$= \frac{14}{165}$

(c) P(different dishes)

$= \text{P}(BFC, BCF, CBF, CFB, FCB, FBC)$

$= \left(\frac{5}{11} \times \frac{2}{10} \times \frac{4}{9}\right) \times 6$

$= \frac{8}{33}$

13. There are x sweets. $\frac{2x}{5}$ are yellow (Y), $\frac{x}{10}$ are green (G).

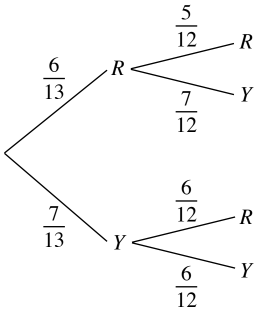
(a) Number of red sweets $= x - \frac{2x}{5} - \frac{x}{10}$

$$= \frac{1}{2}x$$

(b) $P(YG) = \frac{2}{5} \times \frac{1}{10}$

$$= \frac{1}{25}$$

14. **(a)** There are 6 red balls (R) and 7 yellow balls (Y).



$P(RY) = \frac{6}{13} \times \frac{7}{12}$

$$= \frac{7}{26}$$

(b) P(two balls are of different colours)

$= P(RY \text{ or } YR)$

$= \frac{7}{26} + \frac{7}{13} \times \frac{6}{12}$

$= \frac{14}{26}$

$= \frac{7}{13}$

15. **(a)** A has the numbers 1, 2, 3, …, 24, 25

(i) P(multiples of 3 and 7) $= \frac{1}{25}$

(ii) P(multiples of 3 or 7)

= P(3, 6, 9,... 24 or 7, 14,... 21)

$= \frac{10}{25}$

$= \frac{2}{5}$

(b) B has the numbers 3, 6, 9, 12, 15, 18, 21, 24, 4, 8, 16, 20

(i) P(multiple of 7) $= \frac{1}{12}$

(ii) P(multiple of 3, 4 or 7) = 1

16. **(a)** **(i)** $x = \frac{2}{3}, y = \frac{5}{12}, z = \frac{7}{12}$

(ii) **(a)** P(15¢) = P(5¢, 10¢ or 10¢, 5¢)

$$= \frac{5}{13} \times \frac{2}{3} + \frac{8}{13} \times \frac{5}{12}$$

$$= \frac{20}{39}$$

(b) P(>20¢) = 0

(b) P(5 balls chosen > 25¢)

= 1 – P(5¢ for all draws)

$= 1 - \frac{5}{13} \times \frac{4}{12} \times \frac{3}{11} \times \frac{2}{10} \times \frac{1}{9}$

$= \frac{1286}{1287}$

17. There are 24 white marbles (W), x red marbles (R) and y blue marbles (B).

(a) $P(R) = \frac{1}{5}$

$$= \frac{x}{x + y + 24}$$

$$5x = x + y + 24$$

$$y = 4x - 24 \quad \text{— (1)}$$

$P(B) = \frac{2}{5}$

$$= \frac{y}{x + y + 24}$$

$$5y = 2x + 2y + 48$$

$$3y = 2x + 48 \quad \text{— (2)}$$

Substitute (1) into (2) : $3(4x - 24) = 2x + 48$

$$10x = 120$$

$$x = 12$$

Substitute $x = 12$ into (1) : $y = 4(12) - 24 = 24$

$\therefore x = 12$ and $y = 24$

(b) **(i)** P(two marbles are of the same colour)

$= P(RR \text{ or } BB \text{ or } WW)$

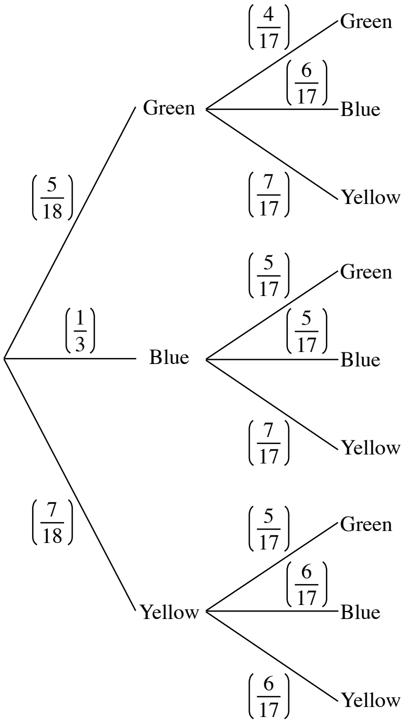
$= \frac{12}{60} \times \frac{11}{59} + \frac{24}{60} \times \frac{23}{59} + \frac{24}{60} \times \frac{23}{59}$

$= \frac{103}{295}$

(ii) $P(WR) = \frac{24}{60} \times \frac{12}{59}$

$$= \frac{24}{295}$$

18. **(a)** There are 5 green (G), 6 blue (B) and 7 yellow marbles (Y).



(b) **(i)** $P(BY \text{ or } YB) = \frac{1}{3} \times \frac{7}{17} + \frac{7}{18} \times \frac{6}{17}$

$= \frac{14}{51}$

(ii) P(two marbles are of the same colour)

$= P(GG \text{ or } YY \text{ or } BB)$

$= \frac{5}{18} \times \frac{4}{17} + \frac{7}{18} \times \frac{6}{17} + \frac{1}{3} \times \frac{5}{17}$

$= \frac{46}{153}$

(iii) $P(\text{not } G)$

$= P(BB, BY, YB, YY)$

$= \frac{1}{3} \times \frac{5}{17} + \frac{1}{3} \times \frac{7}{17} + \frac{7}{18} \times \frac{6}{17} + \frac{7}{18} \times \frac{6}{17}$

$= \frac{26}{51}$

19. **(a)** **(i)** Probability $= \frac{5}{10} \times \frac{5}{9} + \frac{5}{10} \times \frac{4}{9}$

$= \frac{5}{18} + \frac{4}{18}$

$= \frac{9}{18}$

(ii) Probability $= \frac{9}{18} \times \frac{9}{17} + \frac{9}{18} \times \frac{8}{17}$

$= \frac{9}{34} + \frac{8}{34}$

$= \frac{1}{2}$

(b) $a = n, p = 2n, q = 2n - 1, b = n - 1, r = 4n - 2$

(c) $P(WB) = \frac{10}{20} \times \frac{10}{19}$

$= \frac{5}{19}$

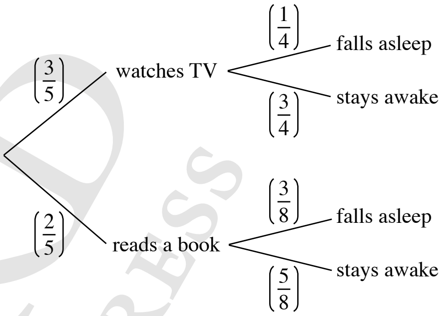
(d) $P(\text{not } BB) = 1 - P(BB)$

$= 1 - \frac{12}{24} \times \frac{11}{23}$

$= \frac{35}{46}$

(e) $c = \frac{1998 + 2}{4} = 500, d = 499$

20. **(a)**



(b) P(stays awake) = P(watches TV, stays awake or reads a book, stay awake)

$= \frac{3}{5} \times \frac{3}{4} + \frac{2}{5} \times \frac{5}{8}$

$= \frac{7}{10}$

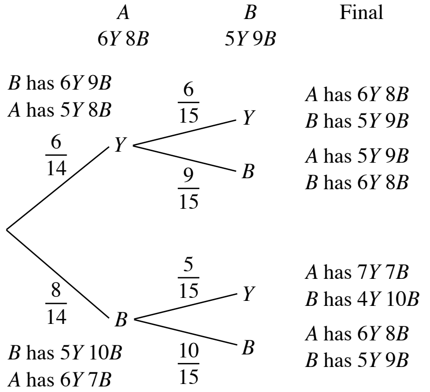
(c) P(awake for at least one evening)

= 1 – P(asleep for both evenings)

$= 1 - \frac{3}{10} \times \frac{3}{10}$

$= \frac{91}{100}$

21. Bag A contains 6 yellow balls (Y) and 8 blue balls (B).
Bag B contains 5 yellow balls (Y) and 9 blue balls (B).



(a) $\text{P}(A \text{ has } 7Y\,7B) = \frac{8}{14} \times \frac{5}{15}$

$= \frac{4}{21}$

(b) $\text{P}(B \text{ has } 6Y\,8B) = \frac{6}{14} \times \frac{9}{15}$

$= \frac{9}{35}$

(c) $\text{P}(A \text{ has } 6Y\,8B) = \frac{6}{14} \times \frac{6}{15} + \frac{8}{14} \times \frac{10}{15}$

$= \frac{58}{105}$

(d) $\text{P}(B \text{ has } 5Y\,10B) = 0$

Exercise 11B

1. **(a)** $\frac{72}{360} \times 100\% = 20\%$

(b) $360° - 120° - 72° - 72° = 96°$

96° represents 288 pupils

$\therefore$ Whole school population $= \frac{288}{96} \times 360$

$= 1080$

2. **(a)** STV 5

(b) $50 - 35 = 15$

(c) $20 + 40 + 35 + 10 + 50 + 5 = 160$

(d) $\frac{20}{160} \times 100\% = 12.5\%$

3. **(a)** $\frac{1(x) + 2 \times 9 + 3 \times 5 + 4 \times 3 + 5 \times 7 + 6 \times 6}{x + 9 + 5 + 3 + 7 + 6} = 3\frac{3}{20}$

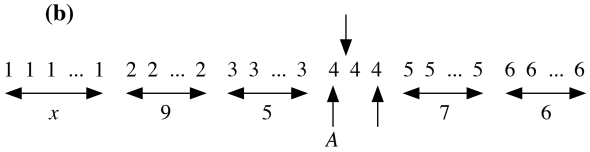
$\frac{x + 116}{x + 30} = 3\frac{3}{20}$

$x + 116 = 3\frac{3}{20}x + 94\frac{1}{2}$

$2\frac{3}{20}x = 21\frac{1}{2}$

$x = 10$

(b)



$x + 14 = 2 + 13$

$x = 1$

$\therefore$ Possible values of x are 0 and 1.

4. **(a)** Modal height = 137 cm

(b) Median height $= \frac{134 + 136}{2}$

$= 135$ cm

(c) Lower quartile = 124 cm

Upper quartile = 138 cm

$\therefore$ Interquartile range $= 138 - 124$

$= 14$ cm

5. **(a)** $\frac{y}{12x} = \frac{5}{3}$

$3y = 60x$

$y = 20x$

$12x + 8x + 2y = 360$

$20x + 2(20x) = 360$

$x = 6$

(b) $8 \times 6° = 48°$ represents 240 g

$\therefore$ Mass of eggs used $= \frac{240}{48} \times 120$

$= 600$ g

6. **(a)** Modal number of goals scored = 19

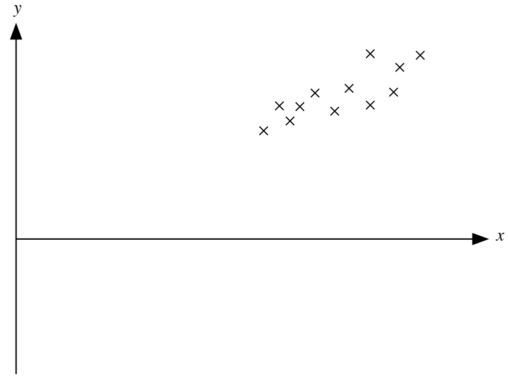
(b) Lower quartile = 15

Upper quartile = 23

$\therefore$ Interquartile range $= 23 - 15$

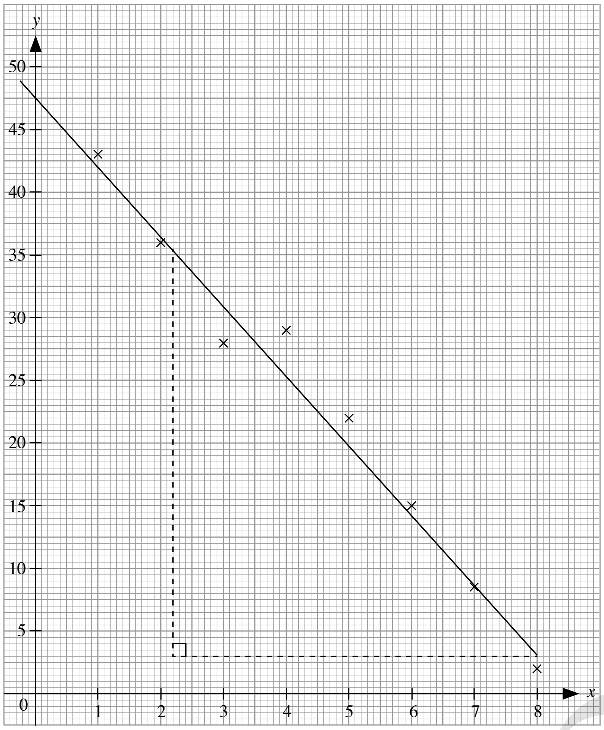
$= 8$

7. **(i)**



(ii) There is a strong, positive correlation.

8. **(i)** and **(ii)**



(iii) The data displays strong negative correlation.

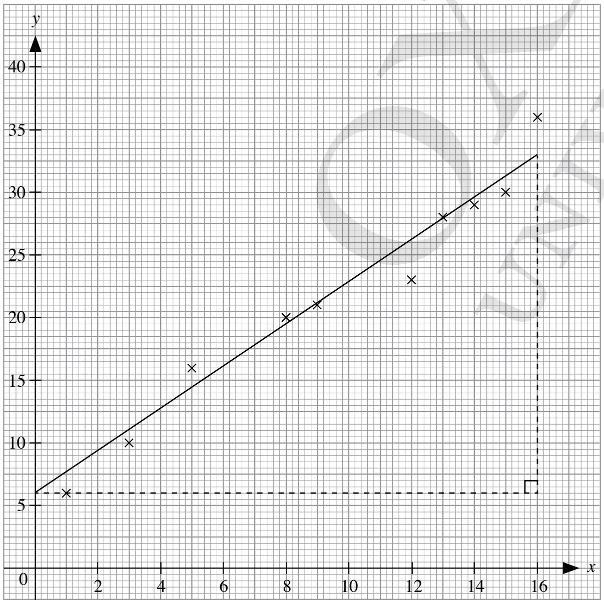
(iv) Gradient $= \dfrac{35.5 - 3}{2.2 - 8}$

$= -5.60$ (to 3 s.f.)

(v) y-intercept $= 47.5$

Equation of line of best fit: $y = -5.60x + 47.5$

9. **(i)**



(iii) The data displays a strong, positive correlation.

(iv) Gradient $= \dfrac{33 - 5}{16 - 0}$

$= 1.75$

(v) y-intercept $= 5$

Equation of line of best fit: $y = 1.75x + 5$

10. **(a)** Total number of games $= 1 + 4 + 3 + 2 + 3 + 2$

$= 15$

(b) Mode = 1

(c) Mean number of goals $= \dfrac{4 + 6 + 6 + 12 + 10}{15}$

$= 2\dfrac{8}{15}$

11. **(a)** $x = 3, 2, 1$ or 0

(b)

$$\underbrace{0\ 0\ 1\ 1\ 1\ 2\ 2\ 2\ 2}_{9}\ \ \underset{A}{\overset{3}{\uparrow}}\ \uparrow\ \underset{B}{\overset{3}{\uparrow}}\ \ \underbrace{4\ 4}_{2}\ \ \underbrace{5\ 5\ \ldots\ 5}_{x}$$

For A, $9 = 1 + 2 + x$

$x = 6$

For B, $9 + 1 = 2 + x$

$x = 8$

$\therefore$ Possible values of x include 6, 7 or 8.

(c) $\dfrac{3 + 8 + 6 + 8 + 5x}{2 + 3 + 4 + 2 + 2 + x} = 3$

$5x + 25 = 3x + 39$

$2x = 14$

$x = 7$

12. **(a)** Mode = 106 min

(b) Median = 106 min

(c) Lower quartile $= \dfrac{93 + 94}{2}$

$= 93.5$ min

Upper quartile $= \dfrac{108 + 110}{2}$

$= 109$ min

Interquartile range $= 109 - 93.5$

$= 15.5$ min

13. **(a)** Mean temperature

$$= \frac{\begin{array}{c}28 \times 3 + 29 \times 3 + 30 \times 6 + 31 \times 7 + 32 \times 5 \\ + 33 \times 3 + 34 \times 4\end{array}}{31}$$

$= 31.1$°C (to 3 s.f.)

(b) Standard deviation = 1.78 °C (to 3 s.f.)

14. **(a)** 20 30 – 04 20 = 16 h 10 min

$\therefore$ Angle $= \dfrac{16\frac{1}{6}}{24} \times 360°$

$= 242.5°$

(b) 14 10 – 09 35 = 4 h 35 min

$$\therefore \text{ Angle for daylight } = \frac{4\frac{35}{60}}{24} \times 360°$$

$$= 68\frac{3}{4}°$$

$\therefore$ Angle for darkness $= 360° - 68.75°$

$= 291.25°$

15. (a) $x = 3$

(b) Mode = 2

(c) $55 + x + y = 100$

$y = 45 - x$

$7 + 72 + 3x + 4y + 25 + 12 = 2.7 \times 100$

$3x + 4y = 154$

$3x + 4(45 - x) = 154$

$\therefore x = 180 - 154 = 26$

$y = 45 - 26 = 19$

16. (a) $\frac{110}{360} \times 100\% = 30\frac{5}{9}\ \%$

(b) $\frac{a}{360} \times 100\% = 17\frac{1}{2}\ \%$

$a = 63°$

$b = 360° - 110° - 78° - 63°$

$= 109°$

17. (a) Amount raised

$= [30 + 66 + 96 + 91 + 70 + 135 + (4 + 5 + 6) \times 15]$

$\times 10 + (4 + 10 + 18) \times 25$

$= \$7930$

(b) $$\text{Mean} = \frac{30 + 66 + 96 + 91 + 70 + 135 + 64 + 85 + 54}{3 + 6 + 8 + 7 + 5 + 9 + 4 + 5 + 3}$$

$= 13.82$

(c) $3 + 6 + 8 + 7 + 5 = 9 + 4 + 5 + x$

$\therefore x = 11$

18. (a) Mode = 2

(b) Median = 2

(c) $$\text{Mean} = \frac{28 + 70 + 60 + 52 + 25 + 30}{28 + 35 + 20 + 13 + 5 + 5}$$

$= 2.5$

19. (a) (i) From the diagram, there are 40 pupils in the class.

(ii) 35 marks

(iii) 75% of 70 marks = 52.5 marks

Number of pupils who scored above 52.5 marks = 7

$\therefore$ 7 students scored distinction.

(b) (i) $x = 45 - 4 - 12 - 16 - 5 - 3$

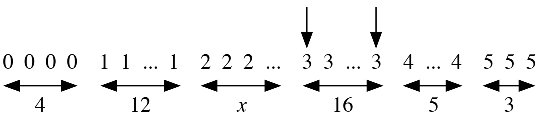
$= 5$

$$\text{Mean} = \frac{12 + 10 + 48 + 20 + 15}{45}$$

$= 2.33$ (to 3 s.f.)

(ii) $x < 16$

(iii)



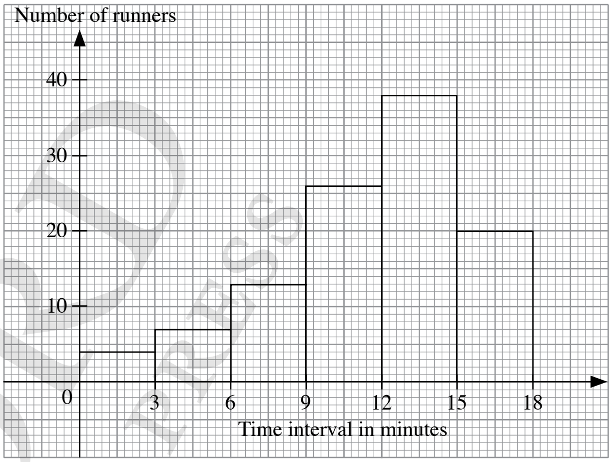
If A is the median,

$4 + 12 + x = 15 + 5 + 3$

$x = 7$

$\therefore$ Possible values of x are 0, 1, 2, 3, 4, 5, 6, 7.

20. (a)



(b) Modal interval = 12 – 15

(c) Mean time of first 50 runners

$$= 1\text{ h }15\text{ min} + \frac{1.5 \times 3 + 4.5 \times 7 + 7.5 \times 13 + 10.5 \times 26}{49}$$

$= 1$ h 15 min + 8.13 min

$= 1$ h 23 min (to nearest min)

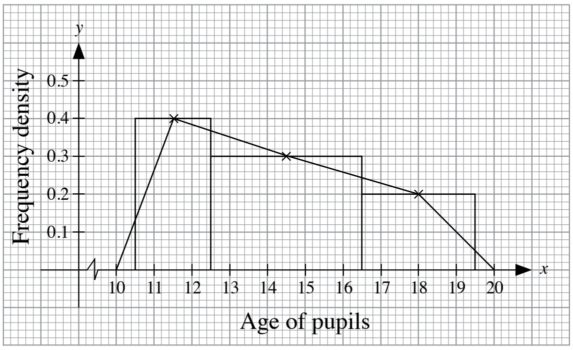
(d) Let x be number of runners who finished in the 7th interval.

$$\frac{38 + 20 + x}{3} = 31$$

$x = 93 - 38 - 20$

$= 35$

21. (a) and **(b)**



22. **(a)** $7 \quad 10 \quad \underbrace{A \quad \ldots}_{13} \quad \underset{\substack{\uparrow \\ \text{median}}}{13} \quad 8 \quad x$

$7 + 10 + 12 = 8 + x$

$x = 21$

$\therefore$ Possible values of x are 0, 1, 2… 21

(b) Total number $= 7 + 10 + 13 + 8 + 2$

$= 40$

New mean $= \dfrac{30.5 \times 40 - 12}{40}$

$= 30.2$ years

23. For boys,

mean = 16.8, standard deviation = 4.5

For girls,

mean = 15.1, standard deviation = 8.3

The boys performed better as the mean is higher. The spread of the performance of the girls is larger indicating greater variability in their marks.

24. **(a)** **(i)** 295.5 days

(ii) 337.5 days

(iii) Lower quartile = 270 days

Interquartile range $= 337.5 - 270$

$= 67.5$ days

(iv) 80th percentile = 342.5 days

(b) 70% of 300 = 210

$\therefore x = 325$ days

(c) It will be more steep with a narrower range.

25. **(a)** Arrange the given data in ascending order.

$a = 109$, $b = 125$, $c = 131$, $d = 141.5$, $e = 153$

(b) Range $= 153 - 109$

$= 44$ cm

Interquartile range $= 141.5 - 125$

$= 16.5$ cm

26. **(a)** **(i)** Median = 3.5 min,

Lower quartile = 3.2 min,

Upper quartile = 4 min.

(ii) Percentage of customers whose service time

> 3.8 min $= \dfrac{325 - 224}{325} \times 100\%$

$= 31.1\%$ (to 3 s.f.)

(iii) Fraction of customers taking > 4.5 min

$= \dfrac{15}{325}$

$= \dfrac{3}{65}$

(b) **(i)** Median = 3.4 min

Interquartile range $= 4 - 2.5$

$= 1.5$ min

(ii) Service time at the two post offices are about the same. The spread at post office *A* is larger.

27. **(a)** Mean = 42.75 marks

Standard deviation = 12.8 marks (to 3 s.f.)

(b) The pupils in 4B performed better as their mean is higher. The spread of performance in 4B is narrower as the standard deviation is smaller.

28. Arrange the given data in ascending order.

15, 32, 35, 47, 56, 59, 62, 64, 65, 73, 76, 80, 82, 93

(a) Median $= \dfrac{62 + 64}{2}$

$= 63$ marks

Lower quartile = 47 marks

Upper quartile = 76 marks

(b) Interquartile range $= 76 - 47$

$= 29$ marks

(c) Secondary 4B performed better than Secondary 4A as the median of 4B is higher than that of 4A. The spread of performance in Secondary 4A is also greater as the interquartile range and range are higher than that of Secondary 4B.

29. **(a)** Median = 69 marks

Interquartile range $= 87 - 52.5$

$= 34.5$ marks

(b) **(i)** Median = 75 marks

(ii) Upper quartile = 80 marks

(iii) Interquartile range $= 80 - 67$

$= 13$ marks

(iv) 80th percentile = 81.5 marks

(c) Secondary 4Q performed better as the median is higher. The spread of performance of Secondary 4P is greater as the interquartile range is higher.

(d) Yes. The median of Secondary 4Q is higher and the lowest mark scored in Secondary 4Q is higher than that scored by pupils in Secondary 4P.

30. **(a)** For Brand *X*,

median = 320 minutes

interquartile range = 90 minutes

For Brand *Y*,

median = 390 minutes

interquartile range = 320 minutes

(b) Brand *Y* lasts longer than Brand *X* as the median is higher. The spread of time of Brand *Y* is larger than Brand *X* as the interquartile range is higher.

31. **(a)** **(i)** Median = 140.5 cm

(ii) Upper quartile = 146 cm

(iii) Interquartile range = 146 – 134.5
= 11.5 cm

(b)

Height (x cm)	$120 \leqslant x < 130$	$130 \leqslant x < 140$	$140 \leqslant x < 150$	$150 \leqslant x < 160$	$160 \leqslant x < 170$
Frequency	14	44	48	12	2

(c) **(i)** Mean = 140.3 cm

(ii) Standard deviation = 8.84 cm (to 3 s.f.)

(d) Pupils in *ABC* school are generally taller as the mean is larger. The spread of heights of pupils in *XYZ* school is greater as the standard deviation is bigger.

(e) Pupils in *PCK* school are generally taller as the median is larger. The spread of heights of pupils in *PCK* school is greater as the interquartile range is greater than that of *ABC* school.

32. **(a)** **(i)** Median travelling time = 26 minutes

(ii) Interquartile range = 32 – 21
= 11 minutes

(iii) Number of teachers taking > 40 min = 7

(iv) Percentage of teachers taking < 15 min

$$= \frac{3}{65} \times 100\%$$

= 4.62% (to 3 s.f.)

(b) Number of teachers taking $\geqslant$ 30 min = 20

$$\therefore \text{P(both teachers take} \geqslant 30 \text{ min)} = \frac{20}{65} \times \frac{19}{64}$$

$$= \frac{19}{208}$$

(c) **(i)** Median travelling time = 34 minutes

(ii) Interquartile range = 41 – 23 = 18 minutes

ABC school is more accessible by public transport as the median travelling time is smaller than that of *XYZ* school.

Specimen Paper

Paper 1

1. **(a)** 18 36 + 14 h = 08 36

08 36 + 24 min = 09 00

09 00 + 18 min = 09 18

$\therefore$ Total travelling time = 14 h + (24 + 18) min

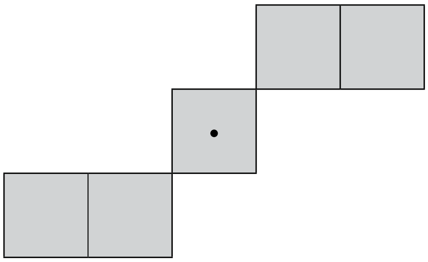
= 14 h 42 min

(b) Arranged in ascending order,

$-\frac{1}{3}, -0.33, 3.4\%, 3.28 \times 10^{-1}$

2. **(a)**
$$\begin{aligned}\left(\frac{64}{125}\right)^{-\frac{1}{3}} \div \left(2-\frac{3}{4}\right)^{0} &= \left(\frac{125}{64}\right)^{\frac{1}{3}} \div 1\\ &= \frac{5}{4}\\ &= 1\frac{1}{4}\end{aligned}$$

(b)



3.
$$\begin{aligned}3x + 4x + 6x + 7x &= 360°\\ x &= 18°\end{aligned}$$

Interior angles are 54°, 72°, 108° and 126°.

$\therefore$ Smallest exterior angle = 180° – 126°

= 54°

4. **(a)**
$$\begin{aligned}8x^3y^3 - 18xy^3 &= 2xy^3(4x^2 - 9)\\ &= 2xy^3(2x + 3)(2x - 3)\end{aligned}$$

(b)
$$\begin{aligned}a^2 - b^2 + 4b - 4 &= a^2 - (b^2 - 4b + 4)\\ &= a^2 - (b - 2)^2\\ &= (a + b - 2)(a - b + 2)\end{aligned}$$

5.
$$\begin{aligned}p\sqrt{\frac{p^2 - q}{q}} &= \frac{1 - p}{2}\\ p^2\left(\frac{p^2 - q}{q}\right) &= \frac{1 - 2p + p^2}{4}\\ q(1 - 2p + p^2) &= 4(p^4 - p^2q)\\ q[(1 - 2p + p^2) + 4p^2] &= 4p^4\\ \therefore q &= \frac{4p^4}{1 - 2p + 5p^2}\end{aligned}$$

6. **(a)** $y = \frac{k}{\sqrt{x + 3}}$ where k is a constant.

When $x = 6$, $y = 6$.

$$\begin{aligned}6 &= \frac{k}{\sqrt{6 + 3}}\\ k &= 18\\ \therefore y &= \frac{18}{\sqrt{x + 3}}\end{aligned}$$

(b) When $y = 9$,

$$\begin{aligned}9 &= \frac{18}{\sqrt{x + 3}}\\ \sqrt{x + 3} &= 2\\ x + 3 &= 4\\ x &= 1\end{aligned}$$

7. $x \geqslant 12$, $\frac{3x + 7}{3} < 39$

$$\begin{aligned}x + \frac{7}{3} &< 39\\ x &< 36\frac{2}{3}\end{aligned}$$

$\therefore x = 13, 17, 19, 23, 29, 31$

8.
$$\begin{aligned}&\frac{3}{1 - 2x} - \frac{7}{4x - 2}\\ &= \frac{3}{1 - 2x} - \frac{7}{2(2x - 1)}\\ &= \frac{3}{1 - 2x} + \frac{7}{2(1 - 2x)}\\ &= \frac{13}{2(1 - 2x)}\end{aligned}$$

9. **(a)** **(i)** $\angle ABD = \angle CDB$ (alt. $\angle$s)

$\angle ADB = \angle ECD$ (given)

$\therefore \triangle ABD$ is similar to $\triangle ECD$ (2 pairs of corr. $\angle$s equal).

(ii) $\angle BDC = 73° - 44°$ (ext. $\angle$ = 2 int. opp. $\angle$s)

= 29°

$\angle ABD = 29°$ (alt. $\angle$s)

$\angle DAB = 180° - 44° - 29°$ ($\angle$ sum of a $\triangle$)

= 107°

Reflex $\angle DAB = 360° - 107°$ ($\angle$s at a point)

= 253°

(b) $\angle EBC = 180° - 73° - 18°$ ($\angle$ sum of a $\triangle$)

= 89°

$\neq$ 90°

$\therefore$ A semicircle, with DC as diameter, does not pass through B.

10. **(a)**
$$\begin{aligned}\cos \angle BAD &= \frac{2}{5}\\ &= \frac{AD}{25}\\ AD &= 10 \text{ cm}\\ \therefore AC &= 2 \times 10\\ &= 20 \text{ cm}\end{aligned}$$

(b) Using Pythagoras' Theorem,

$$BD = \sqrt{25^2 - 10^2}$$
$$= \sqrt{525}$$
$$= 5\sqrt{21} \text{ cm}$$

$$\therefore \tan \angle BAS = -\frac{5\sqrt{21}}{10}$$
$$= -\frac{\sqrt{21}}{2}$$

(c) Area of $\triangle ABC = \frac{1}{2} \times 20 \times 5\sqrt{21}$

$$= 50\sqrt{21} \text{ cm}^2$$

11. New price to sell

$$= \frac{108}{96} \times 264$$
$$= \$297$$

12. (a) Gradient of $PQ = \frac{0-(-6)}{5-0}$

$$= \frac{6}{5}$$

$$y = \frac{6}{5}x - 6$$
$$5y = 6x - 30$$

$\therefore$ Equation of PQ is $5y = 6x - 30$.

(b) Area of $\triangle PQR = \frac{1}{2} \times (4 + 6) \times 5$

$$= 25 \text{ units}^2$$

13. $\xi = \{4, 5, 6, \ldots, 15, 16\}$

$P = \{5, 8\}$

$Q = \{4, 5\}$

(a) $R = \{6, 9, 12, 15\}$

(b) $P \cap Q = \{5\}$

(c) $Q' \cap R = \{6, 9, 12, 15\}$

14. (a) Volume $= \frac{1}{3}\pi r^2 h$

$$= 80 \text{ cm}^3$$

New volume $= \frac{1}{3}\pi(2r)^2\left(\frac{1}{3}h\right)$

$$= \frac{4}{3}\left(\frac{1}{3}\pi r^2 h\right)$$
$$= \frac{4}{3}(80)$$
$$= 106\frac{2}{3} \text{ cm}^3$$

(b) New volume $= 3^3 \times 80$

$$= 2160 \text{ cm}^3$$

15. (a) Equation of curve is $y = (x + 1.5)(x - 6)$

$$= x^2 - 4.5x - 9$$

$\therefore b = -4.5, c = -9$

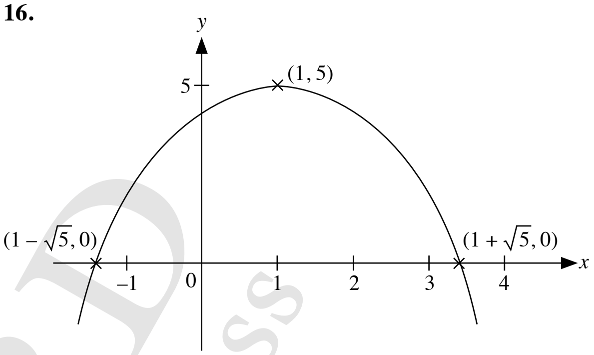
(b) At C, $x = 0$,

$y = -9$

$\therefore$ Coordinates of C are is $(0, -9)$.

(c) Equation of line of symmetry is $x = \frac{6-1.5}{2} = 2\frac{1}{4}$.

16.



$\therefore$ Maximum point is $(1, 5)$.

17. (i) $\triangle AOB$ is mapped onto $\triangle EOD$ by a reflection in FC.

(ii) $\triangle AOB$ is mapped onto $\triangle BOC$ by a 60° anticlockwise rotation about O.

(iii) $\triangle AOB$ is mapped onto $\triangle EOF$ by a 120° clockwise rotation about O.

(iv) $\triangle EOF$ is mapped onto $\triangle DCO$ by a translation of 3 cm along FC.

18. (a) P(20 points) $= \frac{120}{360}$

$$= \frac{1}{3}$$

(b) (i) P(20 points) = P(10 points, 10 points)

$$= \frac{60}{360} \times \frac{60}{360}$$
$$= \frac{1}{36}$$

(ii) P(50 points)

= P(10 points, 40 points) or P(40 points, 10 points)

$$= \frac{1}{6} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{6}$$
$$= \frac{1}{6}$$

(iii) P(more than 30 points)

= 1 – P(20 points or 30 points)

= 1 – P(10 points, 10 points) or P(10 points, 20 points) or P(20 points, 10 points)

$$= 1 - \left(\frac{1}{36} + \frac{1}{6} \times \frac{1}{3} + \frac{1}{3} \times \frac{1}{6}\right)$$
$$= \frac{31}{36}$$

(c) P(60 points)

$= \text{P}((10, 10, 40) \text{ or } (20, 20, 20) \text{ or } (10, 40, 10) \text{ or } (40, 10, 10))$

$$= \frac{1}{6} \times \frac{1}{6} \times \frac{1}{2} + \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} + \frac{1}{6} \times \frac{1}{2} \times \frac{1}{6} + \frac{1}{2} \times \frac{1}{6} \times \frac{1}{6}$$

$$= \frac{17}{216}$$

19. **(a)** $\frac{2}{7 \times 8 \times 9} = \frac{1}{7} - \frac{2}{8} + \frac{1}{9}$

(b) $k = 20 \times 21 \times 22$

$= 9240$

(c) $\frac{2}{4080} = \frac{1}{p} - \frac{2}{q} + \frac{1}{r}$

Try $\sqrt[3]{4080} = 15.9$

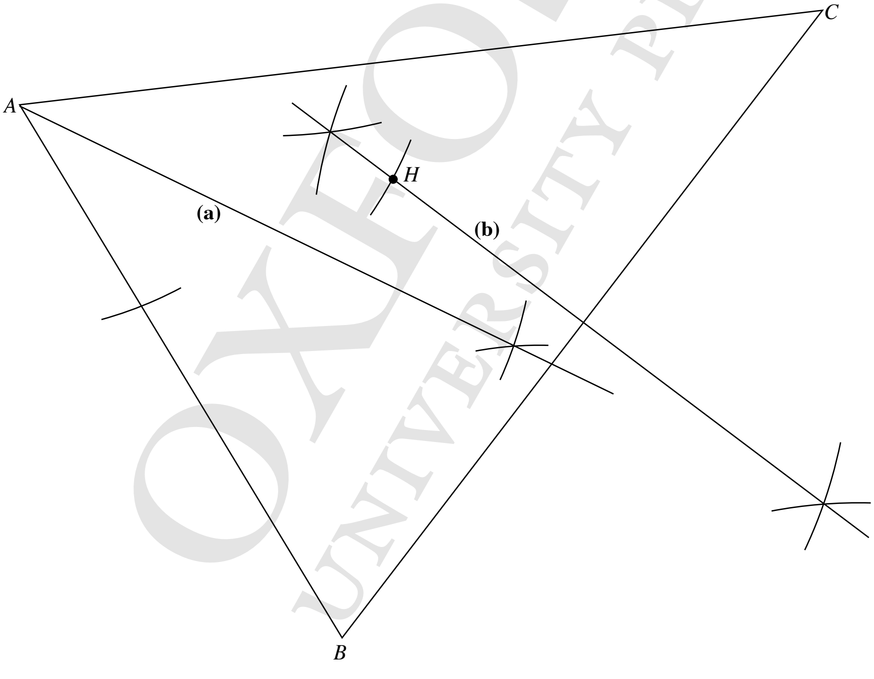
$\therefore p = 15, q = 16, r = 17$

(d) $\frac{2}{a} = \frac{1}{b} - \frac{2}{n} + \frac{1}{c}$

$c = n + 1, b = n - 1$

$a = n(n + 1)(n - 1) = n(n^2 - 1)$

20.



21. Let $\$t$ represent the cost of a cup of tea and $\$c$ represent the cost of a piece of cake.

$8t + 6c = 19.80$ — (1)

$12t + 11c = 33.10$ — (2)

$(1) \times 3 : 24t + 18c = 59.40$ — (3)

$(2) \times 2 : 24t + 22c = 66.20$ — (4)

$(4) - (3) : 4c = 6.8$

$c = 1.70$

Substitute $c = 1.70$ into (2) :

$12t + 11(1.70) = 33.10$

$$t = \frac{33.10 - 11(1.70)}{12}$$

$$= 1.20$$

∴ The cost of a cup of tea is $1.20 and a piece of cake is $1.70.

22.
$$\frac{5000 \times 2.25 \times 4}{100} = 5000\left(1 + \frac{x}{100}\right)^4 - 5000$$

$$5450 = 5000\left(1 + \frac{x}{100}\right)^4$$

$$\left(1 + \frac{x}{100}\right)^4 = \frac{5450}{5000}$$

$$1 + \frac{x}{100} = \sqrt[4]{\frac{5450}{5000}}$$

$$\frac{x}{100} = 0.021\ 778 \text{ (to 5 s.f.)}$$

∴ $x = 2.178$ (to 4 s.f.)

Paper 2

1. **(a)** $(380 - 32) \times \frac{5}{9} = 193.3$ °C (to 1 d.p.)

(b) $3.90 + $3.00 + $3.20 + $2.20 + $1 + 2 × $19 + $2.90
= $54.20

(c) 360 ÷ 24 = 15 sets of ingredients

15 × 250 g = 3750 g butter	8 × $6.20 = $49.60 7 × $6.20 + $3.90 = $47.30
15 × 250 g = 3750 g sugar	3 × $3.60 + $3.00 = $13.80
15 × 250 g = 3750 g flour	4 × $3.20 = $12.80
15 × 4 eggs = 60 eggs	4 × $2.70 = $10.80
15 × 4 × 15 ml = 900 ml milk	1 × $3.20 = $3.20
15 × 24 = 360 paper cups	4 × $2.90 = $11.60

Cost for ingredients = $47.30 + $13.80 + $12.80 + $10.80
+ $3.20 + $11.60
= $99.50

$99.50 ÷ 360 = $0.276 (to 3 d.p.)

Betty will also incur the cost of water and electricity for baking and washing the utensils. The rounded off cost per cupcake may be $0.30. Betty should charge $1.50 since it is a charity fundraising fair, she can raise 360 × $1.20 = $4.32 for the charity.

Accept any valid answers.

2. **(a)** $\left(\frac{1}{x} + \frac{6}{x+1}\right)$ h

(b)
$$\frac{7}{x} - \left(\frac{1}{x} + \frac{6}{x+1}\right) = \frac{1}{2}$$

$$\frac{6}{x} - \frac{6}{x+1} = \frac{1}{2}$$

$$12(x+1) - 12x = x^2 + x$$

$$x^2 + x - 12 = 0 \text{ (shown)}$$

(c) $(x - 3)(x + 4) = 0$

∴ $x = 3$ or $x = -4$ (NA since speed cannot be negative)

(d) $\frac{7}{3}$ h = 2 h 20 min

(e)
$$\text{Average speed} = \frac{7}{2\frac{1}{3} - \frac{1}{2}}$$

$$= 3\frac{9}{11} \text{ km/h}$$

3. **(a)** $\dfrac{68 \text{ kg}}{\text{min}} = \dfrac{68 \times 1000 \text{ g}}{60 \text{ s}}$

$= 1133\dfrac{1}{3}$ g/s

(b) Time taken $= \dfrac{8000}{68}$ min

$= 1$ h 58 min (to nearest min)

(c) Volume $= \dfrac{8000 \times 1000}{0.85}$ cm^3

$= 9412$ litres (to 4 s.f.)

(d) Number of cars $= \dfrac{9412}{35}$

≈ 268

Amount left over $= 9412 - 268 \times 35$

$= 32$ litres

4. **(a)** **(i)** Let $\angle PAD = 4x°$ and $\angle CAD = 5x°$

$81° = 37° + 4x°$

$x = 11$

$\angle ACD = 180° - 81° - 5(11)°$ ($\angle$ sum of a $\triangle$)

$= 44°$

(ii) $\angle ABC = 180° - 81°$ ($\angle$s in opp. segments)

$= 99°$

$\angle ACB = \dfrac{180° - 99°}{2}$ (base $\angle$s of isos. $\triangle$)

$= 40.5°$

(iii) $\angle BAQ = 180° - 4(11°) - 5(11°) - 40.5°$

$= 40.5°$ (adj. $\angle$s on a str. line)

(b) Using Pythagoras' Theorem,

$AC = \sqrt{7.4^2 - 4.6^2}$

$= 5.7966$ cm (to 3 s.f.)

$\therefore$ Area of $ABCD$

$= \dfrac{1}{2} \times 5.7966 \times 4.6 + \dfrac{1}{2} \times 5.7966 \times 9.6$

$= 41.2$ cm^2 (to 3 s.f.)

5. **(a)** $\mathbf{P} = \begin{pmatrix} 1 & 4 & 0 \\ 2 & 1 & 3 \end{pmatrix}\begin{pmatrix} 45 \\ 20 \\ 40 \end{pmatrix} = \begin{pmatrix} 125 \\ 230 \end{pmatrix}$

The elements in **P** represent the cost of fruits required to make 1 cup of Juice *X* and 1 cup of Juice *Y* respectively.

(b) $\mathbf{Q} = (12 \quad 15)\begin{pmatrix} 1 & 4 & 0 \\ 2 & 1 & 3 \end{pmatrix}$

$= (42 \quad 63 \quad 45)$

The elements in **Q** represent the total number of different types of fruit needed to make 12 cups of Juice *X* and 15 cups of Juice *Y*.

(c) $\mathbf{RP} = (12 \quad 15)\begin{pmatrix} 125 \\ 230 \end{pmatrix}$

$= (4950)$

The element in **RP** represents the total cost of fruits required to make 12 cups of Juice *X* and 15 cups of Juice *Y*.

6. **(a)** **(i)** $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$= \begin{pmatrix} -3 \\ 1 \end{pmatrix} + \begin{pmatrix} 4 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} 1 \\ 4 \end{pmatrix}$

$\overrightarrow{CD} = -\overrightarrow{AB}$

$= \begin{pmatrix} -1 \\ -4 \end{pmatrix}$

$\overrightarrow{DA} = \overrightarrow{DO} + \overrightarrow{OA}$

$= \begin{pmatrix} 9 \\ -2 \end{pmatrix} + \begin{pmatrix} 3 \\ -1 \end{pmatrix}$

$= \begin{pmatrix} 12 \\ -3 \end{pmatrix}$

$\overrightarrow{BC} = -\overrightarrow{DA}$

$= \begin{pmatrix} -12 \\ 3 \end{pmatrix}$

(ii) $\overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{BC}$

$= \begin{pmatrix} 4 \\ 3 \end{pmatrix} + \begin{pmatrix} -12 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} -8 \\ 6 \end{pmatrix}$

$\therefore$ *C* is the point $(-8, 6)$.

(b) **(i)** $\overrightarrow{PS} = \overrightarrow{PQ} + \overrightarrow{QS}$

$= \mathbf{a} + \dfrac{3}{4}\mathbf{b}$

(ii) $\overrightarrow{PT} = \overrightarrow{PQ} + \overrightarrow{QT}$

$= \mathbf{a} - \dfrac{1}{2}\mathbf{b}$

(iii) $\overrightarrow{TR} = \overrightarrow{TQ} + \overrightarrow{QR}$

$= \dfrac{1}{2}\mathbf{b} + \mathbf{b}$

$= 1\dfrac{1}{2}\mathbf{b}$

7. **(a)** Angle of elevation $= \tan^{-1} \dfrac{7}{28}$

$= 14.0°$ (to 1 d.p.)

(b) Using Pythagoras' Theorem,

$OB = \sqrt{28^2 + 9^2}$

$= \sqrt{865}$

$\tan 22° = \dfrac{TB}{\sqrt{865}}$

$\therefore$ $TB = 11.9$ m (to 3 s.f.)

(c) Angle of depression $= \tan^{-1} \left(\dfrac{11.88 - 7}{9}\right)$

$= 28.5°$ (to 1 d.p.)

(d) Let d m be the shortest distance from A to OB.

Area of $\triangle OAB = \dfrac{1}{2} \times 28 \times 9$

$= \dfrac{1}{2} \times \sqrt{865} \times d$

$d = \dfrac{28 \times 9}{\sqrt{865}}$

$= 8.57$ (to 3 s.f.)

$\therefore$ The shortest distance from A to OB is 8.57 m.

(e) Greatest angle of elevation $= \tan^{-1} \dfrac{7}{8.568}$

$= 39.2°$ (to 1 d.p.)

(f) Area of $\triangle OAB$ on the map $= \dfrac{1}{2} \times \dfrac{28}{2} \times \dfrac{9}{2}$

$= 31.5 \text{ cm}^2$

8. **(a)** $y = x^2 + \dfrac{1}{x} + 1$

When $x = 0.8$,

$y = 0.8^2 + \dfrac{1}{0.8} + 1$

$= 2.9$

$\therefore$ $a = 2.9$

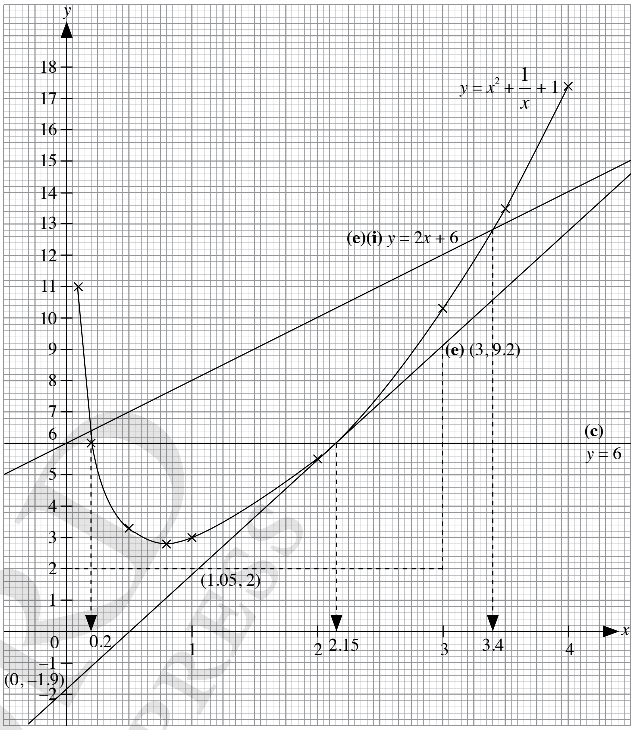
When $x = 3.5$,

$y = 3.5^2 + \dfrac{1}{3.5} + 1$

$= 13.5$

$\therefore$ $b = 13.5$

(b)



(c) $x^2 + \dfrac{1}{x} \leqslant 5$

$x^2 + \dfrac{1}{x} + 1 \leqslant 6$

Draw $y = 6$

$0.2 \leqslant x \leqslant 2.15$

(d) At $x = 2$,

gradient $= \dfrac{9.2 - 2}{3 - 1.05}$

$= 3.69$

(e) **(ii)** $x = 0.18$ or 3.4

(iii) $x^2 + \dfrac{1}{x} + 1 = 2x + 6$

$x^3 + 1 + x = 2x^2 + 6x$

$x^3 - 2x^2 - 5x + 1 = 0$

$\therefore$ $A = -2, B = -5$

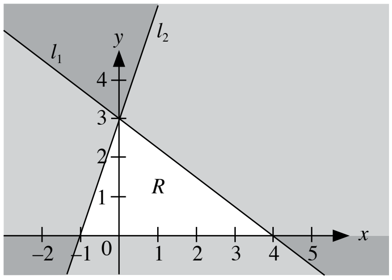
9. **(a)** S\$ $\dfrac{3800 \times 33.15}{100}$ = S\$1259.70

(b) RM 580 × 2.965 = RM1720 (to the nearest RM)

(c) At money changer B, RM100 can get him

$\dfrac{100}{3.017}$ = S\$33.15 (to 2 d.p.)

$\therefore$ He can go to either one of the money changers since the rates are the same.

10. 

Equation of l_1:

$$\frac{y-3}{x-0} = \frac{3-0}{0-4}$$

$$\frac{y-3}{x} = -\frac{3}{4}$$

$$4y - 12 = -3x$$

$$4y + 3x = 12$$

The unshaded region lies below l_1. Hence $4y + 3x \leqslant 12$ defines a part of the unshaded region.

Equation of l_2:

$$\frac{y-3}{x-0} = \frac{3-0}{0-(-1)}$$

$$y - 3 = 3x$$

$$y = 3x + 3$$

The unshaded region lies below l_2. Hence $y \leqslant 3x + 3$.

The unshaded region lies above the x-axis. Hence $y \geqslant 0$ defines a part of the unshaded region.

∴ The unshaded region is defined by the three inequalities:

$y \geqslant 0$, $4y + 3x \leqslant 12$ and $y \leqslant 3x + 3$

11. **(i)** From the graph, the number of workers earning ⩽ \$320 a week = 320.

(ii) From the graph, the median weekly wage = \$335.

(iii) From the graph, the lower quartile = \$250.

(iv) From the graph, the upper quartile = \$395.

∴ Interquartile range = 395 – 250
= \$145

(v) From the graph, the number of workers earning ⩽ \$450 a week = 650

Percentage who earn > \$450 a week

$$= \frac{720 - 650}{720} \times 100\%$$

= 9.72% (to 3 s.f.)

NOTES

NOTES