7th EDITION

NEW SYLLABUS MATHEMATICS
WORKBOOK FULL SOLUTIONS

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ANSWERS

Chapter 1 Quadratic Equations and Functions

Basic

1. (a) $(x+3)^2 = 4$

$x + 3 = \pm\sqrt{4}$

$= \pm 2$

$x + 3 = 2$ or $x + 3 = -2$

$x = 2 - 3$ $\quad$ $x = -2 - 3$

$x = -1$ $\quad$ $x = -5$

(b) $(2x+1)^2 = \frac{4}{25}$

$2x + 1 = \pm\sqrt{\frac{4}{25}}$

$= \pm\frac{2}{5}$

$2x + 1 = \frac{2}{5}$ or $2x + 1 = -\frac{2}{5}$

$2x = \frac{2}{5} - 1$ $\quad$ $2x = -\frac{2}{5} - 1$

$2x = -\frac{3}{5}$ $\quad$ $2x = -\frac{7}{5}$

$x = -\frac{3}{10}$ $\quad$ $x = -\frac{7}{10}$

(c) $(3x-4)^2 = 5$

$3x - 4 = \pm\sqrt{5}$

$3x - 4 = \sqrt{5}$ or $3x - 4 = -\sqrt{5}$

$3x = \sqrt{5} + 4$ $\quad$ $3x = -\sqrt{5} + 4$

$x = \frac{\sqrt{5}+4}{3}$ $\quad$ $x = \frac{-\sqrt{5}+4}{3}$

$x = 2.08$ (to 2 d.p.) $\quad$ $x = 0.59$ (to 2 d.p.)

(d) $\left(\frac{1}{2}x - 1\right)^2 = 17$

$\frac{1}{2}x - 1 = \pm\sqrt{17}$

$\frac{1}{2}x - 1 = \sqrt{17}$ or $\frac{1}{2}x - 1 = -\sqrt{17}$

$\frac{1}{2}x = \sqrt{17} + 1$ $\quad$ $\frac{1}{2}x = 1 - \sqrt{17}$

$x = 2(\sqrt{17} + 1)$ $\quad$ $x = 2(1 - \sqrt{17})$

$x = 10.25$ (to 2 d.p.) $\quad$ $x = -6.25$ (to 2 d.p.)

2. (a) $x^2 + 5x - 1 = 0$

$x^2 + 5x = 1$

$x^2 + 5x + \left(\frac{5}{2}\right)^2 = 1 + \left(\frac{5}{2}\right)^2$

$\left(x + \frac{5}{2}\right)^2 = 1 + \frac{25}{4}$

$\left(x + \frac{5}{2}\right)^2 = \frac{29}{4}$

$x + \frac{5}{2} = \pm\sqrt{\frac{29}{4}}$

$x + \frac{5}{2} = \sqrt{\frac{29}{4}}$ or $x + \frac{5}{2} = -\sqrt{\frac{29}{4}}$

$x = \sqrt{\frac{29}{4}} - \frac{5}{2}$ $\quad$ $x = -\sqrt{\frac{29}{4}} - \frac{5}{2}$

$x = 0.193$ (to 3 s.f.) $\quad$ $x = -5.19$ (to 3 s.f.)

(b) $x^2 - 7x + 3 = 0$

$x^2 - 7x = -3$

$x^2 - 7x + \left(-\frac{7}{2}\right)^2 = -3 + \left(-\frac{7}{2}\right)^2$

$\left(x - \frac{7}{2}\right)^2 = -3 + \frac{49}{4}$

$\left(x - \frac{7}{2}\right)^2 = \frac{37}{4}$

$x - \frac{7}{2} = \pm\sqrt{\frac{37}{4}}$

$x - \frac{7}{2} = \sqrt{\frac{37}{4}}$ or $x - \frac{7}{2} = -\sqrt{\frac{37}{4}}$

$x = \sqrt{\frac{37}{4}} + \frac{7}{2}$ $\quad$ $x = -\sqrt{\frac{37}{4}} + \frac{7}{2}$

$x = 6.54$ (to 3 s.f.) $\quad$ $x = 0.459$ (to 3 s.f.)

(c) $x^2 + 8x + 2.5 = 0$

$x^2 + 8x = -2.5$

$x^2 + 8x + \left(\frac{8}{2}\right)^2 = -2.5 + \left(\frac{8}{2}\right)^2$

$x^2 + 8x + 4^2 = -2.5 + 4^2$

$(x+4)^2 = 13.5$

$x + 4 = \pm\sqrt{13.5}$

$x + 4 = \sqrt{13.5}$ or $x + 4 = -\sqrt{13.5}$

$x = \sqrt{13.5} - 4$ $\quad$ $x = -\sqrt{13.5} - 4$

$x = -0.326$ (to 3 s.f.) $\quad$ $x = -7.67$ (to 3 s.f.)

(d) $x^2 - 11x = 7$

$$x^2 - 11x + \left(-\frac{11}{2}\right)^2 = 7 + \left(-\frac{11}{2}\right)^2$$

$$\left(x - \frac{11}{2}\right)^2 = \frac{149}{4}$$

$$x - \frac{11}{2} = \pm\sqrt{\frac{149}{4}}$$

$$x - \frac{11}{2} = \sqrt{\frac{149}{4}} \text{ or } x - \frac{11}{2} = -\sqrt{\frac{149}{4}}$$

$$x = \sqrt{\frac{149}{4}} + \frac{11}{2} \qquad x = -\sqrt{\frac{149}{4}} + \frac{11}{2}$$

$x = 11.6$ (to 3 s.f.) $x = -0.603$ (to 3 s.f.)

3. (a) $2x^2 + 3x + 1 = 0$

$a = 2, b = 3, c = 1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-3 \pm \sqrt{3^2 - 4(2)(1)}}{2(2)}$$

$$= \frac{-3 \pm \sqrt{1}}{4}$$

$$= \frac{-3 \pm 1}{4}$$

$$= \frac{-3+1}{4} \text{ or } \frac{-3-1}{4}$$

$$= -\frac{1}{2} \text{ or } -1$$

(b) $3x^2 = 5x + 1$

$3x^2 - 5x - 1 = 0$

$a = 3, b = -5, c = -1$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(3)(-1)}}{2(3)}$$

$$= \frac{5 \pm \sqrt{25 + 12}}{6}$$

$$= \frac{5 \pm \sqrt{37}}{6}$$

$$= \frac{5 + \sqrt{37}}{6} \text{ or } \frac{5 - \sqrt{37}}{6}$$

$= 1.85$ (to 3 s.f.) or -0.180 (to 3 s.f.)

(c) $7x = 2x^2 + 4$

$-2x^2 + 7x - 4 = 0$

$a = -2, b = 7, c = -4$

$$x = \frac{-7 \pm \sqrt{7^2 - 4(-2)(-4)}}{2(-2)}$$

$$= \frac{-7 \pm \sqrt{49 - 32}}{-4}$$

$$= \frac{-7 \pm \sqrt{17}}{-4}$$

$$= \frac{-7 + \sqrt{17}}{-4} \text{ or } \frac{-7 - \sqrt{17}}{-4}$$

$= 0.719$ (to 3 s.f.) or 2.78 (to 3 s.f.)

(d) $3 - 4x^2 = 7x$

$-4x^2 - 7x + 3 = 0$

$a = -4, b = -7, c = 3$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(-4)(3)}}{2(-4)}$$

$$= \frac{7 \pm \sqrt{49 + 48}}{-8}$$

$$= \frac{7 \pm \sqrt{97}}{-8}$$

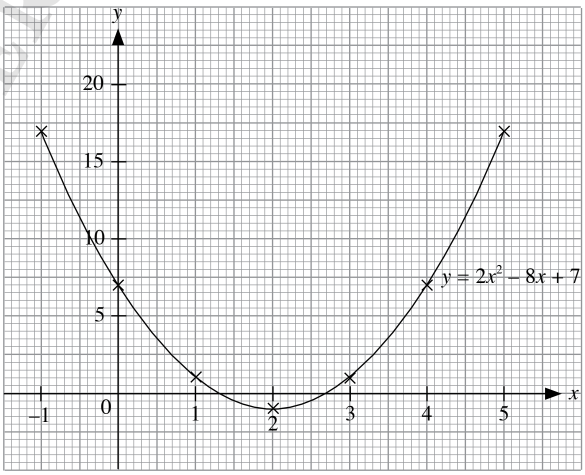
$$= \frac{7 + \sqrt{97}}{-8} \text{ or } \frac{7 - \sqrt{97}}{-8}$$

$= -2.11$ (to 3 s.f.) or 0.356 (to 3 s.f.)

4. (i)

x	-1	0	1	2	3	4	5
y	17	7	1	-1	1	7	17

(ii)

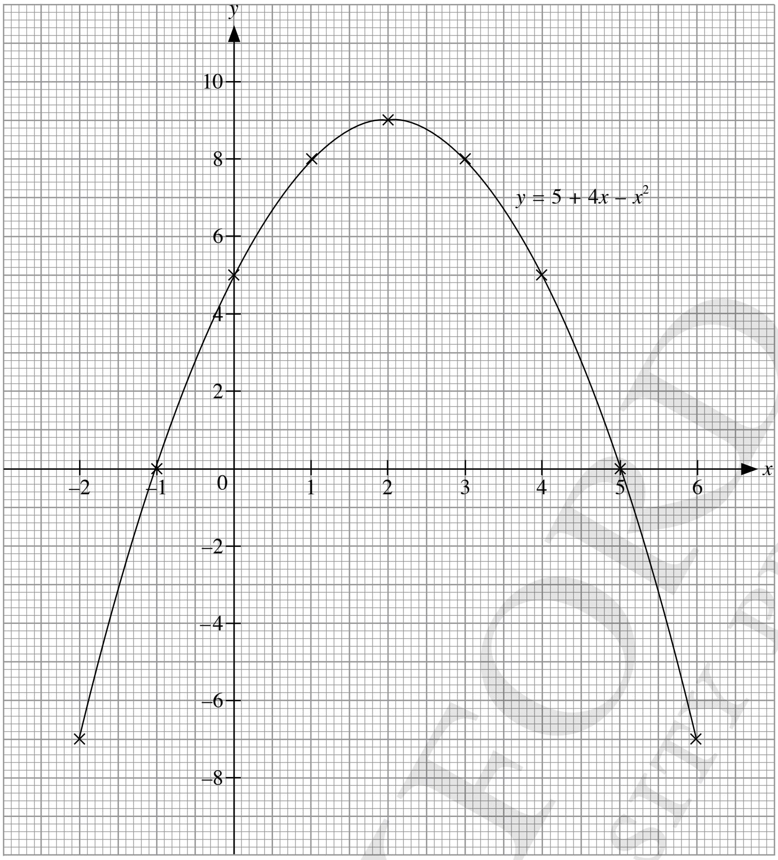


(iii) $x = 1.3$ or 2.7

5. (i)

x	–2	–1	0	1	2	3	4	5	6
y	–7	0	5	8	9	8	5	0	–7

(ii)



(iii) $x = -1$ or 5

6. (i) When $y = 0$,

$(x-2)^2 - 7 = 0$

$x - 2 = \pm\sqrt{7}$

$x - 2 = \sqrt{7}$ or $x - 2 = -\sqrt{7}$

$x = \sqrt{7} + 2$ $\quad$ $x = -\sqrt{7} + 2$

$x = 4.65$ (to 3 s.f.) $\quad$ $x = -0.646$ (to 3 s.f.)

$\therefore$ The graph cuts the x-axis at $(4.65, 0)$ and $(-0.646, 0)$.

When $x = 0$,

$y = (0-2)^2 - 7$

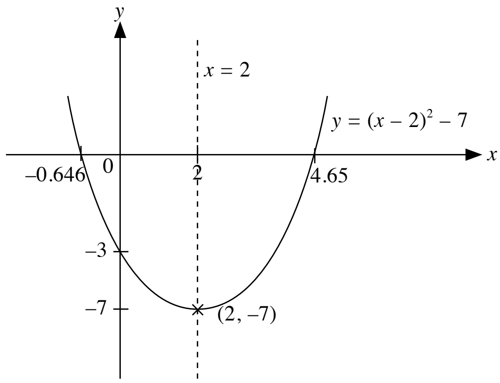
$y = 4 - 7$

$y = -3$

$\therefore$ The graph cuts the y-axis at $(0, -3)$.

(ii) Coordinates of the minimum point are $(2, -7)$.

(iii)



(iv) The equation of the line of symmetry $x = 2$.

7. **(i)** When $y = 0$,

$$-(2x+1)^2 + 5 = 0$$
$$-(2x+1)^2 = -5$$
$$(2x+1)^2 = 5$$
$$2x + 1 = \pm\sqrt{5}$$
$$2x + 1 = \sqrt{5} \quad \text{or} \quad 2x + 1 = -\sqrt{5}$$
$$2x = \sqrt{5} - 1 \qquad 2x = -\sqrt{5} - 1$$
$$x = \frac{\sqrt{5}-1}{2} \qquad x = \frac{-\sqrt{5}-1}{2}$$
$$= 0.618 \text{ (to 3 s.f.)} \qquad = -1.62 \text{ (to 3 s.f.)}$$

$\therefore$ The graph cuts the x-axis at $(0.618, 0)$ and $(-1.62, 0)$.

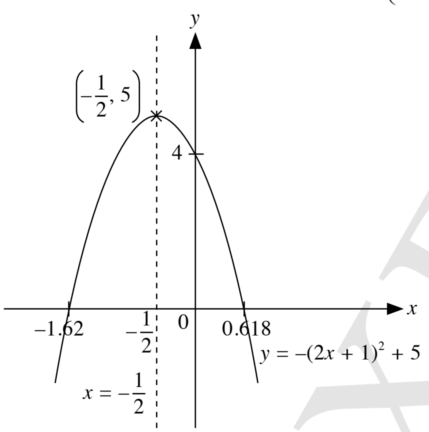
When $x = 0$,

$$y = -[2(0) + 1]^2 + 5$$
$$= 4$$

$\therefore$ The graph cuts the y-axis at $(0, 4)$.

(ii) Coordinates of the maximum point are $\left(-\frac{1}{2}, 5\right)$.

(iii)



(iv) The equation of the line of symmetry is $x = -\frac{1}{2}$.

8. **(i)** When $y = 0$,

$$(x+1)(x-3) = 0$$
$$x = -1 \text{ or } x = 3$$

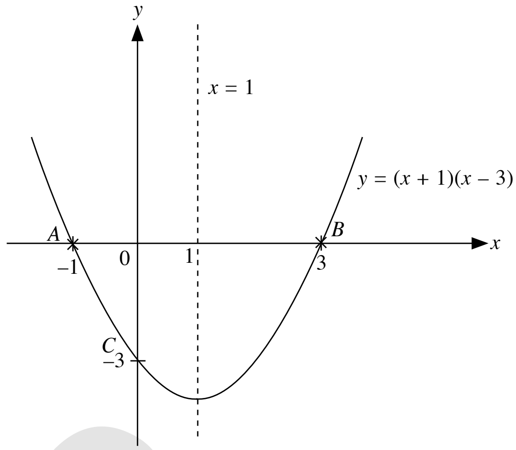
$\therefore$ Coordinates of A are $(-1, 0)$ and coordinates of B are $(3, 0)$.

When $x = 0$,

$$y = (0+1)(0-3)$$
$$= -3$$

$\therefore$ Coordinates of C are $(0, -3)$.

(ii)



(iii) The equation of the line of symmetry is $x = 1$.

Intermediate

9. **(a)**
$$5x^2 - 2x = 0$$
$$5x(x-2) = 0$$
$$5x = 0 \quad \text{or} \quad x - 2 = 0$$
$$x = 0 \qquad x = 2$$

(b)
$$(x-1)(x+1) = 15$$
$$x^2 - 1 = 15$$
$$x^2 - 16 = 0$$
$$(x+4)(x-4) = 0$$
$$x + 4 = 0 \quad \text{or} \quad x - 4 = 0$$
$$x = -4 \qquad x = 4$$

(c)
$$x^2 + 4x = 17$$
$$x^2 + 4x - 17 = 0$$
$$a = 1, b = 4, c = -17$$
$$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(-17)}}{2(1)}$$
$$= \frac{-4 \pm \sqrt{16 + 68}}{2}$$
$$= \frac{-4 \pm \sqrt{84}}{2}$$
$$= \frac{-4 + \sqrt{84}}{2} \quad \text{or} \quad \frac{-4 - \sqrt{84}}{2}$$
$$= 2.58 \text{ (to 3 s.f.)} \quad \text{or} \quad -6.58 \text{ (to 3 s.f.)}$$

(d)
$$x(2x+7) - 3(x+2) = 0$$
$$2x^2 + 7x - 3x - 6 = 0$$
$$2x^2 + 4x - 6 = 0$$
$$x^2 + 2x - 3 = 0$$
$$(x+3)(x-1) = 0$$
$$x + 3 = 0 \quad \text{or} \quad x - 1 = 0$$
$$x = -3 \qquad x = 1$$

(e)

$$\begin{aligned}3x(x+4) &= 2x(x-4)\\ 3x^2+12x &= 2x^2-8x\\ 3x^2-2x^2+12x+8x &= 0\\ x^2+20x &= 0\\ x(x+20) &= 0\end{aligned}$$

$x = 0$ or $x + 20 = 0$

$x = -20$

(f)

$$\begin{aligned}(3x-1)^2 &= 12x+18\\ 9x^2-6x+1-12x-18 &= 0\\ 9x^2-18x-17 &= 0\end{aligned}$$

$a = 9, b = -18, c = -17$

$$x = \frac{-(-18) \pm \sqrt{(-18)^2 - 4(9)(-17)}}{2(9)}$$

$$= \frac{18 \pm \sqrt{324+612}}{18}$$

$$= \frac{18 \pm \sqrt{936}}{18}$$

$= 2.70$ (to 3 s.f.) or -0.700 (to 3 s.f.)

10. (a)

$$\begin{aligned}(x-4)\left(x+\frac{2}{3}\right) &= 0\\ x^2+\frac{2}{3}x-4x-\frac{8}{3} &= 0\\ 3x^2+2x-12x-8 &= 0\\ 3x^2-10x-8 &= 0\end{aligned}$$

(b)

$$\begin{aligned}(x+0.5)(x-2) &= 0\\ x^2-2x+0.5x-1 &= 0\\ x^2-1.5x-1 &= 0\\ 2x^2-3x-2 &= 0\end{aligned}$$

11. (a)

$$\begin{aligned}1+\frac{4x+2}{2x} &= \frac{5}{x}\\ \frac{2x+4x+2}{2x} &= \frac{5}{x}\\ \frac{6x+2}{2x} &= \frac{5}{x}\\ 6x^2+2x &= 10x\\ 6x^2-8x &= 0\\ 2x(3x-4) &= 0\end{aligned}$$

$2x = 0$ $\quad 3x - 4 = 0$

$x = 0$ (rejected) $\quad x = \frac{4}{3}$

$= 1\frac{1}{3}$

(b)

$$\begin{aligned}2x-\frac{2+3x}{4} &= \frac{2}{x}\\ \frac{8x-(2+3x)}{4} &= \frac{2}{x}\\ \frac{8x-2-3x}{4} &= \frac{2}{x}\\ \frac{5x-2}{4} &= \frac{2}{x}\end{aligned}$$

$$\begin{aligned}5x^2-2x &= 8\\ 5x^2-2x-8 &= 0\end{aligned}$$

$a = 5, b = -2, c = -8$

$$x = \frac{-(-2) \pm \sqrt{2^2 - 4(5)(-8)}}{2(5)}$$

$$= \frac{2 \pm \sqrt{4+160}}{10}$$

$$= \frac{2 \pm \sqrt{164}}{10}$$

$$= \frac{2+\sqrt{164}}{10} \text{ or } \frac{2-\sqrt{164}}{10}$$

$= 1.48$ (to 3 s.f.) or -1.08 (to 3 s.f.)

(c)

$$\begin{aligned}\frac{6}{x}+\frac{1}{x-6} &= 1\\ \frac{6(x-6)+x}{x(x-6)} &= 1\\ 6x-36+x &= x^2-6x\\ 7x-36 &= x^2-6x\\ x^2-13x+36 &= 0\\ (x-9)(x-4) &= 0\end{aligned}$$

$x - 9 = 0$ or $x - 4 = 0$

$x = 9$ $\quad x = 4$

(d)

$$\begin{aligned}2x-1 &= \frac{8x-7}{x+1}\\ (2x-1)(x+1) &= 8x-7\\ 2x^2+2x-x-1 &= 8x-7\\ 2x^2+x-1 &= 8x-7\\ 2x^2-7x+6 &= 0\\ (2x-3)(x-2) &= 0\end{aligned}$$

$2x - 3 = 0$ or $x - 2 = 0$

$2x = 3$ $\quad x = 2$

$x = \frac{3}{2}$

$= 1\frac{1}{2}$

(e)

$$\begin{aligned}\frac{x}{x-1}-\frac{2}{2-3x} &= \frac{5}{2}\\ \frac{x(2-3x)-2(x-1)}{(x-1)(2-3x)} &= \frac{5}{2}\\ \frac{2x-3x^2-2x+2}{(x-1)(2-3x)} &= \frac{5}{2}\\ \frac{2-3x^2}{(x-1)(2-3x)} &= \frac{5}{2}\\ 2(2-3x^2) &= 5(x-1)(2-3x)\\ 4-6x^2 &= 5(2x-3x^2-2+3x)\\ &= 5(5x-3x^2-2)\\ &= 25x-15x^2-10\end{aligned}$$

$$9x^2 - 25x + 14 = 0$$
$$(x - 2)(9x - 7) = 0$$
$$x - 2 = 0 \quad \text{or} \quad 9x - 7 = 0$$
$$x = 2 \qquad 9x = 7$$
$$x = \frac{7}{9}$$

(f)
$$\frac{1}{x^2 - 9} - \frac{2}{3 - x} = 0$$
$$\frac{1}{(x + 3)(x - 3)} - \frac{2}{3 - x} = 0$$
$$\frac{1}{(x + 3)(x - 3)} + \frac{2}{x - 3} = 0$$
$$\frac{1 + 2(x + 3)}{(x + 3)(x - 3)} = 0$$
$$1 + 2x + 6 = 0$$
$$2x + 7 = 0$$
$$2x = -7$$
$$x = -\frac{7}{2}$$
$$= -3\frac{1}{2}$$

12. (a)
$$6x^2 - x - 15 = 0$$
$$(3x - 5)(2x + 3) = 0$$
$$3x - 5 = 0 \quad \text{or} \quad 2x + 3 = 0$$
$$3x = 5 \qquad 2x = -3$$
$$x = \frac{5}{3} \qquad x = -\frac{3}{2}$$
$$= 1\frac{2}{3} \qquad = -1\frac{1}{2}$$

(b) $6(y - 3)^2 - (y - 3) - 15 = 0$

Let $y - 3 = x$.

Using **(a)**,

$$\therefore y - 3 = 1\frac{2}{3} \qquad y - 3 = -1\frac{1}{2}$$
$$y = 4\frac{2}{3} \qquad y = 1\frac{1}{2}$$

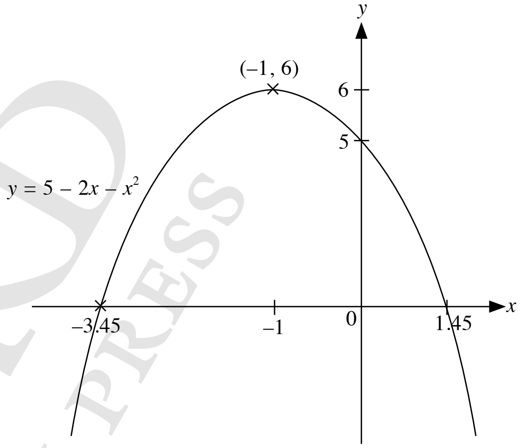
13. $5 - 2x - x^2 = 5 - (x^2 + 2x)$
$$= 5 - \left[x^2 + 2x + \left(\frac{2}{2}\right)^2 - \left(\frac{2}{2}\right)^2\right]$$
$$= 5 - [(x + 1)^2 - 1]$$
$$= 5 - (x + 1)^2 + 1$$
$$= 6 - (x + 1)^2$$

(i) Maximum value is 6.

(ii) $5 - 2x - x^2 = 0$
$$6 - (x + 1)^2 = 0$$
$$(x + 1)^2 = 6$$
$$x + 1 = \pm\sqrt{6}$$
$$x = \sqrt{6} - 1 \quad \text{or} \quad -\sqrt{6} - 1$$
$$= 1.45 \text{ (to 3 s.f.)} \qquad -3.45 \text{ (to 3 s.f.)}$$

(iii) When $x = 0$,
$$y = 5 - 2(0) - 0^2$$
$$= 5$$

The graph cuts the y-axis at (0, 5).

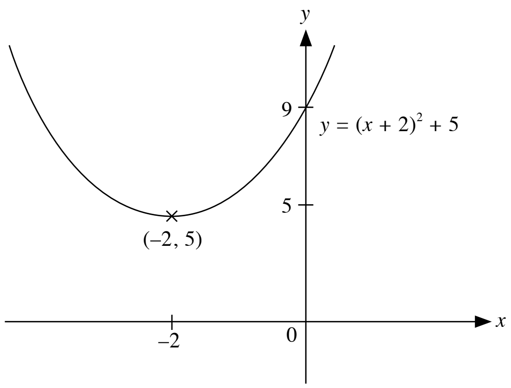


14. (i) $h = -2, k = 5$

(ii) $y = (x + 2)^2 + 5$

When $x = 0$,
$$y = (0 + 2)^2 + 5$$
$$= 9$$

The graph cuts the y-axis at (0, 9).



15. $y = (3 - 2x)(2x + 7)$

When $x = 0$,

$y = (3)(7)$

$= 21$

The graph cuts the y-axis at $(0, 21)$.

When $y = 0$,

$(3 - 2x)(2x + 7) = 0$

$3 - 2x = 0$ or $2x + 7 = 0$

$2x = 3$ $\quad$ $2x = -7$

$x = 1\frac{1}{2}$ $\quad$ $x = -3\frac{1}{2}$

The graph cuts the x-axis at $\left(1\frac{1}{2}, 0\right)$ and $\left(-3\frac{1}{2}, 0\right)$.

$$x = \frac{1\frac{1}{2} + \left(-3\frac{1}{2}\right)}{2}$$

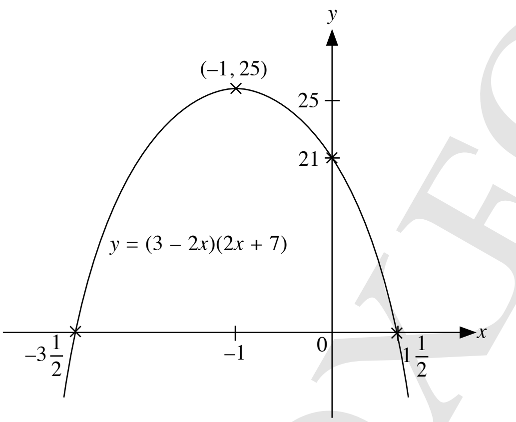
$= -1$

$\therefore y = [3 - 2(-1)][2(-1) + 7]$

$= 5(5)$

$= 25$

The coordinates of the maximum point are $(-1, 25)$.



16. **(i)** $y = (x - 1)(x - 5)$

$= x^2 - 6x + 5$

$\therefore a = -6, b = 5$

(ii) When $x = 0$,

$y = 0^2 - 6(0) + 5$

$= 5$

$\therefore$ Coordinates of M are $(0, 5)$.

17. (i) When $x = -1$,

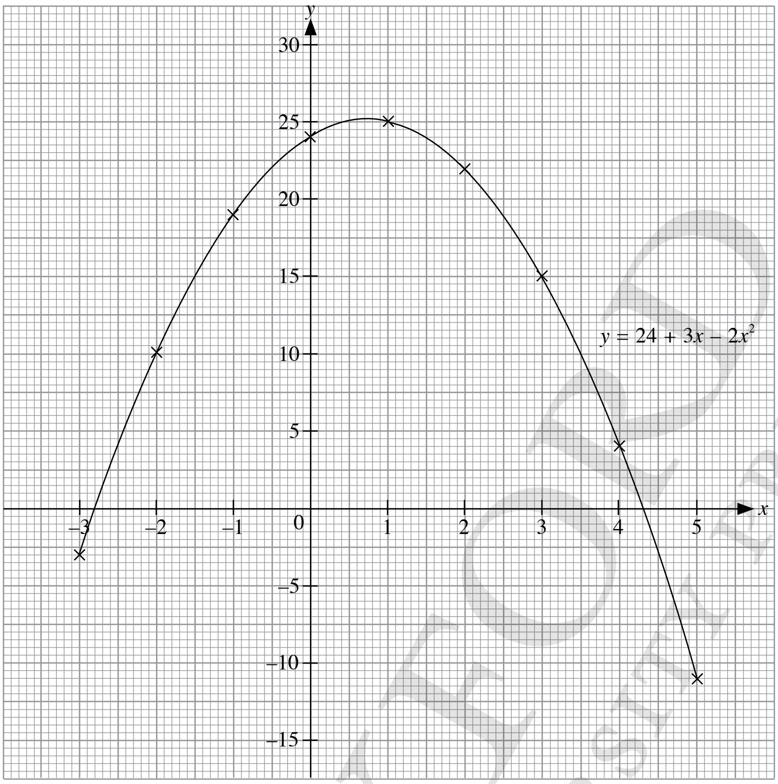
$p = 24 + 3(-1) - 2(-1)^2$

$= 24 - 3 - 2$

$= 19$

(ii)

x	–3	–2	–1	0	1	2	3	4	5
y	–3	10	19	24	25	22	15	4	–11



(iii) $x = -2.85$ or 4.25

18. (i) $AC = (8 - x)$ m

(ii) Using Pythagoras' Theorem,

$AC^2 + BC^2 = AB^2$

$(8 - x)^2 + x^2 = 7^2$

$64 - 16x + x^2 + x^2 = 49$

$2x^2 - 16x + 15 = 0$ (shown)

(iii) $2x^2 - 16x + 15 = 0$

$a = 2,\ b = -16,\ c = 15$

$$x = \frac{-(-16) \pm \sqrt{(-16)^2 - 4(2)(15)}}{2(2)}$$

$$= \frac{16 \pm \sqrt{256 - 120}}{4}$$

$$= \frac{16 \pm \sqrt{136}}{4}$$

$$= \frac{16 + \sqrt{136}}{4} \quad \text{or} \quad \frac{16 - \sqrt{136}}{4}$$

$= 6.92$ (to 2 d.p.) or 1.08 (to 2 d.p.)

(iv) 1.08

19. (i) Number of boxes of cookies he bought

$$= \frac{600}{x}$$

(ii) Total sum received from the sale of the cookies

$$= \$\left(\frac{600}{x} - 2\right)(x + 2)$$

(iii)

$$\left(\frac{600}{x} - 2\right)(x + 2) - 600 = 72$$

$$\left(\frac{600}{x} - 2\right)(x + 2) = 672$$

$(600 - 2x)(x + 2) = 672x$

$600x + 1200 - 2x^2 - 4x - 672x = 0$

$1200 - 76x - 2x^2 = 0$

$2x^2 + 76x - 1200 = 0$

$x^2 + 38x - 600 = 0$ (shown)

(iv) $x^2 + 38x - 600 = 0$

$(x - 12)(x + 50) = 0$

$x = 12$ or $x = -50$ (rejected)

(v) Selling price of each box of cookies $= 12 + 2$

$= \$14$

20. (i) Number of concert tickets = $\frac{2820}{x}$

(ii) Number of concert tickets = $\frac{2820}{x-20}$

(iii)
$$\frac{2820}{x-20} - \frac{2820}{x} = 3$$
$$2820x - 2820(x-20) = 3x(x-20)$$
$$2820x - 2820x + 56\,400 = 3x^2 - 60x$$
$$3x^2 - 60x - 56\,400 = 0$$
$$x^2 - 20x - 18\,800 = 0 \text{ (shown)}$$

(iv) $x^2 - 20x - 18\,800 = 0$

$a = 1, b = -20, c = -18\,800$

$$x = \frac{-(-20) \pm \sqrt{(-20)^2 - 4(1)(-18\,800)}}{2(1)}$$
$$= \frac{20 \pm \sqrt{75\,600}}{2}$$
$= 147.5$ (to 1 d.p.) or -127.5 (to 1 d.p.)

(v) Maximum number of tickets

$= \frac{3000}{147.5}$

$= 20$ (round down to the nearest integer)

21. (i) Numbers of litres of petrol used = $\frac{100}{x}$

(ii) Number of litres of petrol used = $\frac{100}{x+4}$

(iii)
$$\frac{100}{x} - \frac{100}{x+4} = 5$$
$$100x + 400 - 100x = 5x(x+4)$$
$$400 = 5x^2 + 20x$$
$$5x^2 + 20x - 400 = 0$$
$$x^2 + 4x - 80 = 0 \text{ (shown)}$$

(iv) $x^2 + 4x - 80 = 0$

$a = 1, b = 4, c = -80$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(-80)}}{2(1)} = \frac{-4 \pm \sqrt{336}}{2}$$
$= 7.17$ (to 3 s.f.) or -11.2 (to 3 s.f.)

(v) Number of litres of petrol used = $\frac{100}{7.17 + 4}$

$= 8.95$ (to 3 s.f.)

22. (i) Amount of Japanese yen that he received

$= \frac{4000}{x}$

(ii) Amount of Japanese yen that he now received

$= \frac{2500}{x - 0.05}$

(iii)
$$\frac{4000}{x} + \frac{2500}{x-0.05} = 92\,000$$
$$4000x - 200 + 2500x = 92\,000x(x - 0.05)$$
$$6500x - 200 = 92\,000x^2 - 4600x$$
$$92\,000x^2 - 11\,100x + 200 = 0$$
$$920x^2 - 111x + 2 = 0 \text{ (shown)}$$

(iv) $920x^2 - 111x + 2 = 0$

$a = 920, b = -111, c = 2$

$$x = \frac{-(-111) \pm \sqrt{(-111)^2 - 4(920)(2)}}{2(920)}$$
$$= \frac{111 \pm \sqrt{4961}}{1840}$$
$= 0.0986$ (to 3 s.f.) or 0.0220 (to 3 s.f.)

(v) S\$0.0986 – S\$0.05 = S\$0.0486

Exchange rate on the day of his journey is

¥1 = S\$0.0486

($x \neq 0.0220$ because the exchange rate cannot be negative)

23. (a) (i) $\frac{1}{x}$

(ii) $\frac{1}{x-4}$

(b)
$$\frac{1}{x} + \frac{1}{x-4} = \frac{1}{6}$$
$$\frac{x - 4 + x}{x(x-4)} = \frac{1}{6}$$
$$6(2x - 4) = x(x - 4)$$
$$12x - 24 = x^2 - 4x$$
$$x^2 - 16x + 24 = 0 \text{ (shown)}$$

(c) (i) $x^2 - 16x + 24 = 0$

$a = 1, b = -16, c = 24$

$$x = \frac{-(-16) \pm \sqrt{(-16)^2 - 4(1)(24)}}{2(1)}$$
$$= \frac{16 \pm \sqrt{160}}{2}$$
$= 14.3$ (to s.f.) or 1.68 (to 3 s.f.)

(ii) $x \neq 1.68$ because the time taken by Tap B to fill the pool cannot be negative.

(d) Time taken by Tap B = 14.3 – 4

= 10.3 h

= 10 h 18 min

24. (a) $\dfrac{300}{x}$ h

(b) (i) $\dfrac{300}{x+4}$ h

(ii)
$$\frac{300}{x} - \frac{300}{x+4} = \frac{50}{60}$$
$$\frac{300(x+4) - 300x}{x(x+4)} = \frac{5}{6}$$
$$\frac{300x + 1200 - 300x}{x(x+4)} = \frac{5}{6}$$
$$\frac{1200}{x(x+4)} = \frac{5}{6}$$
$$7200 = 5x^2 + 20x$$
$$5x^2 + 20x - 7200 = 0$$
$$x^2 + 4x - 1440 = 0 \text{ (shown)}$$

(iii) $x^2 + 4x - 1440 = 0$

$(x - 36)(x + 40) = 0$

$x = 36$ or $x = -40$

$x = -40$ is rejected since speed cannot be negative.

(iv) Time taken $= \dfrac{300}{36+4}$

$= 7.5$ h

$\therefore$ Time at which the ship reached Port Q = 16 30

25. (a) (i) Breadth of rectangle = $2x$ m

Area of rectangle = $(4x)(2x)$

$= 8x^2$ m^2

(ii) Length of square $= \dfrac{80 - 12x}{4}$

$= (20 - 3x)$ m

(iii) Area of square $= (20 - 3x)^2$

$= (400 - 120x + 9x^2)$ m^2

(b) $y = 400 - 120x + 9x^2 + 8x^2$

$= 17x^2 - 120x + 400$ (shown)

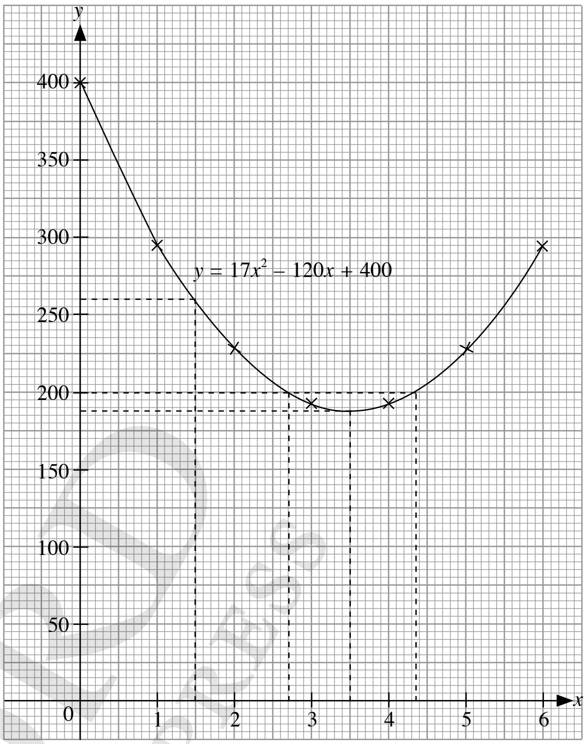
(c) When $x = 6$,

$y = 17(6)^2 - 120(6) + 400$

$= 292$

$\therefore a = 292$

(d)

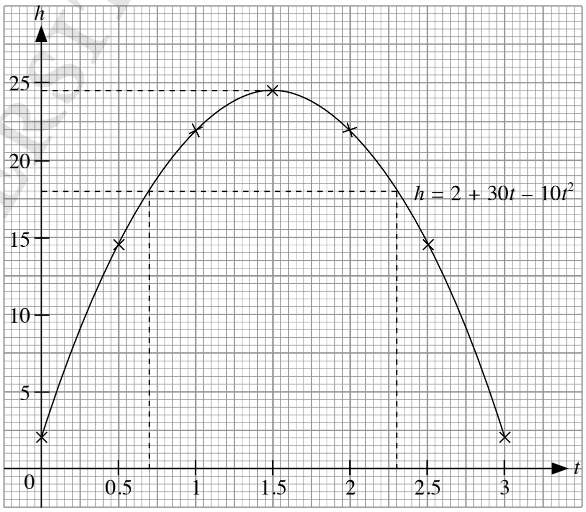


(e) (i) When $x = 1.5$, $y = 260$.

(ii) When $y = 200$, $x = 2.70$ or $x = 4.35$.

(iii) $x = 3.50$

26. (a)



(b) (i) Maximum height reached by the ball = 24.5 m

(ii) When $h = 18$ m, $t = 0.7$ s and $t = 2.3$ s

(c) Extend the curve from $t = 3$ such that it meets the horizontal t-axis.

The value of t at the intersection gives the approximate time when the ball hits the ground.

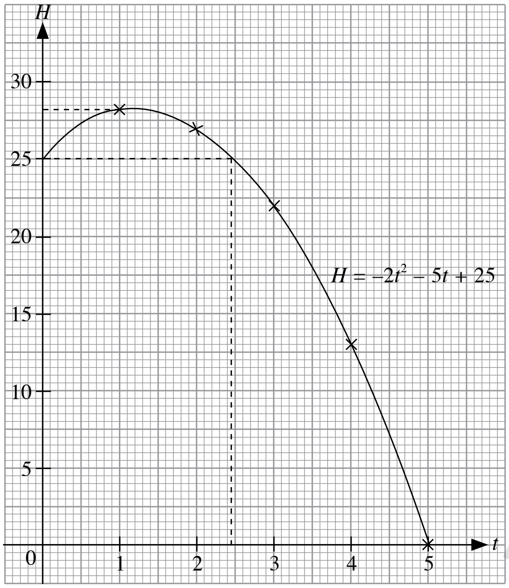
27. (a) When $t = 3$,

$H = -2(3)^2 + 5(3) + 25$

$= 22$

$\therefore p = 22$

(b)



(c) **(i)** When $t = 0$, $H = 25$.

$\therefore$ The height of the cliff is 25 m.

(ii) Maximum height of the stone above ground

$= 28.25$ m

(iii) When $H = 25$,

$t = 0$ or $t = 2.45$

(rejected)

$\therefore$ The stone is again at the same vertical level as the top of the cliff at $t = 2.45$s.

Advanced

28. (i) Consider the hot water tap.

x m^3 in 1 minute

1 m^3 in $\frac{1}{x}$ minutes

2 m^3 in $\frac{2}{x}$ minutes

Consider the cold water tap.

y m^3 in 1 minute

1 m^3 in $\frac{1}{y}$ minutes

2 m^3 in $\frac{2}{y}$ minutes

Consider both taps.

$(x + y)$ m^3 in 1 minute

1 m^3 in $\frac{1}{x+y}$ minutes

2 m^3 in $\frac{2}{x+y}$ minutes

$$\frac{2}{x} - \frac{2}{y} = 4$$

$$2y - 2x = 4xy$$

$$y - x = 2xy$$

$$y - 2xy = x$$

$$y(1 - 2x) = x$$

$$y = \frac{x}{1-2x} \quad \text{— (1)}$$

$$\frac{2}{x+y} = 3\frac{1}{3}$$

$$= \frac{10}{3}$$

$$6 = 10x + 10y$$

$$5x + 5y = 3 \quad \text{— (2)}$$

Substitute (1) into (2):

$$5x + \frac{5x}{1-2x} = 3$$

$$5x(1 - 2x) + 5x = 3(1 - 2x)$$

$$5x - 10x^2 + 5x = 3 - 6x$$

$$10x^2 - 16x + 3 = 0 \text{ (shown)}$$

(ii) $10x^2 - 16x + 3 = 0$

$a = 10$, $b = -16$, $c = 3$

$$x = \frac{-(-16) \pm \sqrt{(-16)^2 - 4(10)(3)}}{2(10)}$$

$$= \frac{16 \pm \sqrt{136}}{20}$$

$= 1.38$ (to 2 d.p.) or 0.22 (to 2 d.p.)

(iii) When $x = 1.38$,

$$y = \frac{1.38}{1 - 2(1.38)} < 0$$

Since the time taken cannot be a negative value, $x \neq 1.38$.

When $x = 0.22$,

time taken $= \frac{2}{0.22}$

$= 9$ minutes (to the nearest minute)

New Trend

29. (i) $x^2 - 5x + 2 = x^2 - 5x + \left(-\frac{5}{2}\right)^2 + 2 - \left(-\frac{5}{2}\right)^2$

$= \left(x - \frac{5}{2}\right)^2 + 2 - \frac{25}{4}$

$= \left(x - \frac{5}{2}\right)^2 - \frac{17}{4}$

(ii) $x^2 - 5x + 2 = 0$

$\left(x - \frac{5}{2}\right)^2 - \frac{17}{4} = 0$

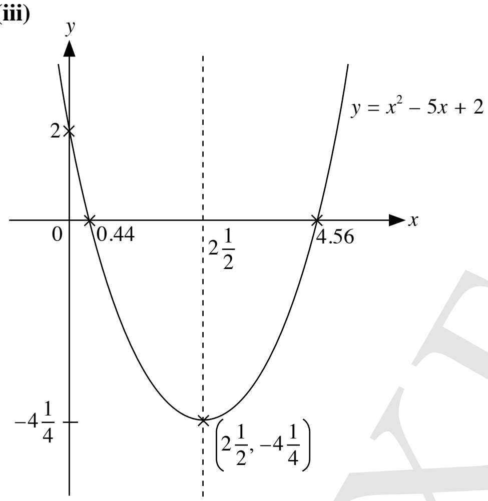
$\left(x - \frac{5}{2}\right)^2 = \frac{17}{4}$

$x - \frac{5}{2} = \pm\sqrt{\frac{17}{4}}$

$x = \sqrt{\frac{17}{4}} + \frac{5}{2}$ or $x = -\sqrt{\frac{17}{4}} + \frac{5}{2}$

$= 4.56$ $\quad$ $= 0.44$ (to 2 d.p.)

(iii)



30. (a) $50 \text{ km} \div \left(\frac{x}{60}\right) \text{h}$

$= \frac{3000}{x}$ km/h

(b) $70 \text{ km} \div \left(\frac{x+25}{60}\right) \text{h}$

$= \frac{4200}{x+25}$ km/h

(c) $\frac{3000}{x} - \frac{4200}{x+25} = 10$

$\frac{3000x + 75\ 000 - 4200x}{x(x+25)} = 10$

$75\ 000 - 1200x = 10(x^2 + 25x)$

$x^2 + 25x = 7500 - 120x$

$x^2 + 145x - 7500 = 0$ (proven)

(d) $x^2 + 145x - 7500 = 0$

$x = \frac{-145 \pm \sqrt{145^2 - 4(1)(-7500)}}{2}$

$= \frac{-145 \pm \sqrt{51\ 025}}{2}$

$= 40.444$ (to 3 d.p.) or -185.444 (to 3 d.p.)

(e) Since the time taken cannot be a negative value, $x \neq -185.444$.

When $x = 40.444$,

total time taken $= \left(\frac{40.444 + 40.444 + 25}{60}\right)$ h

$= 1.7648$ h

Average speed $= \frac{120 \text{ km}}{1.7648 \text{ h}}$

$= 68.0$ km/h (to 3 s.f.)

31. (a) $x^2 - 10x + 16 = x^2 - 10x + \left(-\frac{10}{2}\right)^2 + 16 - \left(-\frac{10}{2}\right)^2$

$= (x - 5)^2 + 16 - 25$

$= (x - 5)^2 - 9$

(b) Minimum value is -9.

(c) Line of symmetry is $x = 5$.

32. (a) Using Pythagoras' Theorem,

$AC^2 = AB^2 + BC^2$

$= x^2 + \left(\frac{2x}{3}\right)^2$

$= x^2 + \frac{4x^2}{9}$

$= \frac{13x^2}{9}$ (shown)

(b) Using Pythagoras' Theorem,

$AC^2 + CG^2 = AG^2$

$\frac{13x^2}{9} + (x + 4)^2 = 20^2$

$\frac{13x^2}{9} + x^2 + 8x + 16 = 400$

$13x^2 + 9x^2 + 72x + 144 = 3600$

$22x^2 + 72x - 3456 = 0$

$11x^2 + 36x - 1728 = 0$ (shown)

(c) $x = \frac{-36 \pm \sqrt{36^2 - 4(11)(-1728)}}{2(11)}$

$= \frac{-36 \pm \sqrt{77\ 328}}{22}$

$= 11.0036$ (to 4 d.p.) or -14.2763 (to 4 d.p.)

(d) Since the length of the cuboid cannot be a negative value, $x \neq -14.2763$.

Volume $= 11.0036\left(\frac{2}{3} \times 11.0036\right)(15.0036)$

$= 1210 \text{ cm}^3$ (to 3 s.f.)

33. (a) $\dfrac{900}{x}$ min

(b) $\dfrac{900}{x+12}$ min

(c) $\dfrac{900}{x} - \dfrac{900}{x+12} = 18$

$900x + 10\,800 - 900x = 18(x^2 + 12x)$

$18x^2 + 216x - 10\,800 = 0$

$x^2 + 12x - 600 = 0$ (shown)

(d) $x^2 + 12x - 600 = 0$

$x = \dfrac{-12 \pm \sqrt{12^2 - 4(1)(-600)}}{2(1)}$

$= \dfrac{-12 \pm \sqrt{2544}}{2}$

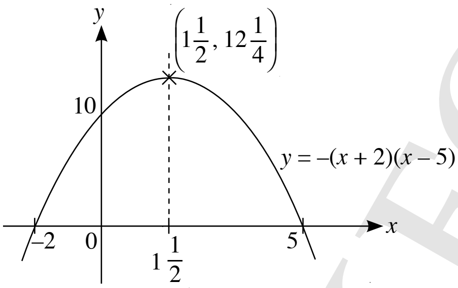
$= 19.2$ (to 1 d.p.) or -31.2 (to 1 d.p.)

(e) Since the rate of filling the mattress with water cannot be a negative value, $x \neq -31.2$

Time taken using the large tap $= \dfrac{900}{19.2+12}$ min

$= 28$ min 51 s (to the nearest second)

34. (a)



(b) Equation of the line of symmetry: $x = \dfrac{-2+5}{2}$

$= 1\dfrac{1}{2}$

(c) When $x = 1\dfrac{1}{2}$,

$y = -\left(1\dfrac{1}{2} + 2\right)\left(1\dfrac{1}{2} - 5\right)$

$= 12\dfrac{1}{4}$

$\therefore$ Coordinates of the turning point are $\left(1\dfrac{1}{2}, 12\dfrac{1}{4}\right)$.

35. (a) Total surface area

$= 2(x)\left(\dfrac{x}{2}\right) + 2(x)\left(\dfrac{x}{2} - 0.1\right) + 2\left(\dfrac{x}{2}\right)\left(\dfrac{x}{2} - 0.1\right)$

$= x^2 + x^2 - 0.2x + 0.5x^2 - 0.1x$

$= 2.5x^2 - 0.3x$

(b) $2.5x^2 - 0.3x = 6$

$25x^2 - 3x - 60 = 0$

(c) $x = \dfrac{-(-3) \pm \sqrt{(-3)^2 - 4(25)(-60)}}{2(25)}$

$= \dfrac{3 \pm \sqrt{6009}}{50}$

$= \dfrac{3 + \sqrt{6009}}{50}$ or $\dfrac{3 - \sqrt{6009}}{50}$

$= 1.61$ (to 2 d.p.) or -1.49 (to 2 d.p.)

(d) Since the height of the pedestal cannot be a negative value, $x \neq -1.49$.

Width of the pedestal $= \dfrac{1.61}{2} - 0.1$

$= 0.705$ m

Chapter 2 Further Functions

Basic

1. $g(x) = 3x^2 + 5$

(i) $g(a) = 3a^2 + 5$

(ii) $g(a + 2) = 3(a + 2)^2 + 5$
$= 3(a^2 + 4a + 4) + 5$
$= 3a^2 + 12a + 12 + 5$
$= 3a^2 + 12a + 17$

(iii) $g(a + 2) - g(a - 2)$
$= 3a^2 + 12a + 17 - [3(a - 2)^2 + 5]$
$= 3a^2 + 12a + 17 - 3(a^2 - 4a + 4) - 5$
$= 3a^2 + 12a + 17 - 3a^2 + 12a - 12 - 5$
$= 24a$

2. $h(x) = 12x^2 - 11x + 2$

(i) $h(2c) - h(c)$
$= [12(2c)^2 - 11(2c) + 2] - [12c^2 - 11c + 2]$
$= (48c^2 - 22c + 2) - 12c^2 + 11c - 2$
$= 36c^2 - 11c$

(ii) $12c^2 - 11c + 2 = 0$
$(3c - 2)(4c - 1) = 0$
$3c - 2 = 0$ or $4c - 1 = 0$
$c = \frac{2}{3}$ $\quad c = \frac{1}{4}$

(iii) $h(c^2) + h(c) = 12c^4 - 11c^2 + 2 + 12c^2 - 11c + 2$
$= 12c^4 + c^2 - 11c + 4$

3. $f(x) = mx + c$
$f(2) = 2m + c = 7$ — (1)
$f(-3) = -3m + c = -8$ — (2)
(1) – (2): $5m = 15$
$m = 3$
Substitute $m = 3$ into (1):
$2(3) + c = 7$
$c = 1$
$\therefore m = 3, c = 1$
$f(x) = 3x + 1$
$f(5) = 3(5) + 1$
$= 16$
$f(-11) = 3(-11) + 1$
$= -33 + 1$
$= -32$

4. $f(x) = -2x + 3$
Let $y = -2x + 3$.
$-2x = y - 3$
$x = -\frac{1}{2}(y - 3)$
$f^{-1}(x) = -\frac{1}{2}(x - 3)$

5. $f(x) = \frac{5}{7}x - 2$
Let $y = \frac{5}{7}x - 2$.
$\frac{5}{7}x = y + 2$
$x = \frac{7}{5}(y + 2)$
$f^{-1}(x) = \frac{7}{5}(x + 2)$

6. $f(x) = 9x - 3$
Let $y = 9x - 3$.
$9x = y + 3$
$x = \frac{1}{9}(y + 3)$
$f^{-1}(x) = \frac{1}{9}(x + 3)$
$f^{-1}(5) = \frac{1}{9}(5 + 3)$
$= \frac{8}{9}$

7. $g(x) = 8x - 12$
When $g(x) = 52$,
$8x - 12 = 52$
$8x = 64$
$x = 8$
When $g(x) = -14$,
$8x - 12 = -14$
$8x = -2$
$x = -\frac{1}{4}$

8. $h(x) = ax^2 + bx + 5$
$h(4) = a(4)^2 + b(4) + 5$
$= 16a + 4b + 5$
$16a + 4b + 5 = 33$
$16a + 4b = 28$
$4a + b = 7$ — (1)
$h(-2) = a(-2)^2 + b(-2) + 5$
$= 4a - 2b + 5$
$4a - 2b + 5 = 25$
$4a - 2b = 20$ — (2)
From (1): $b = 7 - 4a$ — (3)
Substitute (3) into (2):
$4a - 2(7 - 4a) = 20$
$4a - 14 + 8a = 20$
$12a = 34$
$a = 2\frac{5}{6}$

Substitute $a = 2\frac{5}{6}$ into (3):

$b = 7 - 4\left(2\frac{5}{6}\right)$

$= -4\frac{1}{3}$

$h(3) = \left(2\frac{5}{6}\right)(3)^2 + \left(-4\frac{1}{3}\right)(3) + 5$

$= 17\frac{1}{2}$

$h(-3) = \left(2\frac{5}{6}\right)(-3)^2 + \left(-4\frac{1}{3}\right)(-3) + 5$

$= 43\frac{1}{2}$

9. $f(x) = ax^2 + bx$

$f(-3) = a(-3)^2 + b(-3)$

$= 9a - 3b$

$9a - 3b = 18$

$3a - b = 6$ — (1)

$f(6) = a(6)^2 + b(6)$

$= 36a + 6b$

$36a + 6b = 24$

$6a + b = 4$ — (2)

(1) + (2):

$9a = 10$

$a = 1\frac{1}{9}$

Substitute $a = 1\frac{1}{9}$ into (1):

$3\left(1\frac{1}{9}\right) - b = 6$

$b = -2\frac{2}{3}$

10. $f(x) = \frac{2}{x} + 8$

Let $y = \frac{2}{x} + 8$.

$xy = 2 + 8x$

$xy - 8x = 2$

$x(y - 8) = 2$

$x = \frac{2}{y - 8}$

$\therefore f^{-1}(x) = \frac{2}{x - 8}$

$f^{-1}(-1) = \frac{2}{-1 - 8}$

$= -\frac{2}{9}$

$f^{-1}(3) = \frac{2}{3 - 8}$

$= -\frac{2}{5}$

11. $f(x) = \frac{5x - 2}{x - 6}$

Let $y = \frac{5x - 2}{x - 6}$.

$y(x - 6) = 5x - 2$

$yx - 6y = 5x - 2$

$5x - yx = 2 - 6y$

$x(5 - y) = 2 - 6y$

$x = \frac{2 - 6y}{5 - y}$

$f^{-1}(x) = \frac{2 - 6x}{5 - x}$

$f^{-1}(2) = \frac{2 - 6(2)}{5 - 2}$

$= -3\frac{1}{3}$

$f^{-1}(8) = \frac{2 - 6(8)}{5 - 8}$

$= 15\frac{1}{3}$

12. $f(x) = ax + b$

$f(3) = 3a + b = 7$ — (1)

Let $y = ax + b$.

$ax = y - b$

$x = \frac{y - b}{a}$

$f^{-1}(x) = \frac{x - b}{a}$

$f^{-1}(2) = \frac{2 - b}{a} = 4$

$4a + b = 2$ — (2)

(2) – (1):

$a = -5$

Substitute $a = -5$ into (1):

$3(-5) + b = 7$

$b = 22$

$\therefore a = -5, b = 22$

$f^{-1}(x) = -\frac{x - 22}{5}$

$f^{-1}(9) = -\frac{9 - 22}{5}$

$= 2\frac{3}{5}$

$f^{-1}\left(-8\frac{1}{4}\right) = -\frac{-8\frac{1}{4} - 22}{5}$

$= 6\frac{1}{20}$

13. $f(x) = px + q$

$f(3) = 3p + q = -8 \qquad — (1)$

$f(-1) = -p + q = -11 \qquad — (2)$

(1) – (2):

$4p = 3$

$p = \frac{3}{4}$

Substitute $p = \frac{3}{4}$ into (1):

$3\left(\frac{3}{4}\right) + q = -8$

$\frac{9}{4} + q = -8$

$q = -10\frac{1}{4}$

$\therefore f(x) = \frac{3}{4}p - 10\frac{1}{4}$

Let $y = \frac{3}{4}x - 10\frac{1}{4}$.

$\frac{3}{4}x = y + 10\frac{1}{4}$

$3x = 4y + 41$

$x = \frac{1}{3}(4y + 41)$

$\therefore f^{-1}(x) = \frac{1}{3}(4x + 41)$

Chapter 3 Linear Inequalities

Basic

1. **(a)** $15 < 30$

(b) $-2 > -5$

(c) $(-3)^2 > -9$

(d) $-2^4 = -16$

(e) $\left(-\frac{1}{3}\right)^{11} < \left(-\frac{1}{3}\right)^4$

(f) $\sqrt{16} > \sqrt{10}$

(g) $h - 3 > h - 4$

(h) $k + 10 > k + 7$

(i) $12 - p < 14 - p$

(j) $16 - 4q < 2(8 - q)$

2. **(a)** $a < b$

(b) $d > -3$

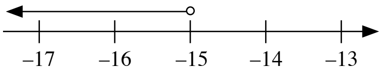
(c) $-\frac{h}{2} < -\frac{k}{2}$

(d) $3m \geqslant 3n$

(e) $-6p \geqslant -6q$

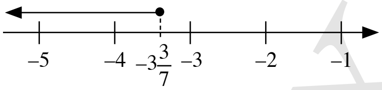
3. **(a)** $-5x > 75$

$x < -15$



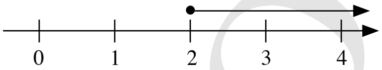
(b) $-7x \geqslant 24$

$x \leqslant -3\frac{3}{7}$



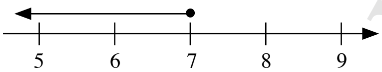
(c) $a + 1 \geqslant 3$

$a \geqslant 2$



(d) $b - 2 \leqslant 5$

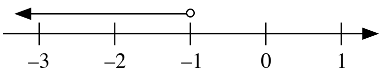
$b \leqslant 7$



(e) $-c + 1 > 2$

$-c > 1$

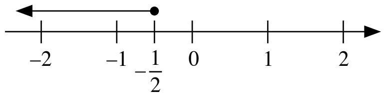
$c < -1$



(f) $-6d - 3 \geqslant 0$

$-6d \geqslant 3$

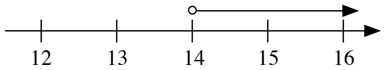
$d \leqslant -\frac{1}{2}$



(g) $12 - e < -2$

$-e < -14$

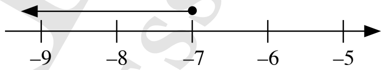
$e > 14$



(h) $20 + 4f \leqslant f - 1$

$3f \leqslant -21$

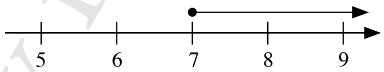
$f \leqslant -7$



(i) $3 - 2g \leqslant -4 - g$

$-g \leqslant -7$

$g \geqslant 7$

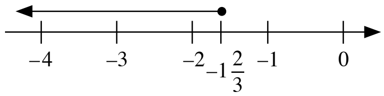


(j) $2(1 - 5h) \geqslant 4(3 - h)$

$2 - 10h \geqslant 12 - 4h$

$-6h \geqslant 10$

$h \leqslant -1\frac{2}{3}$

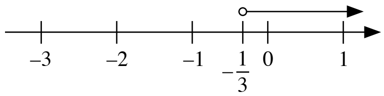


(k) $2(i + 3) > 4(1 - i)$

$2i + 6 > 4 - 4i$

$6i > -2$

$i > -\frac{1}{3}$

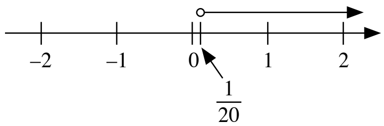


(l) $8j - 1 > 4(-3j)$

$8j - 1 > -12j$

$20j > 1$

$j > \frac{1}{20}$

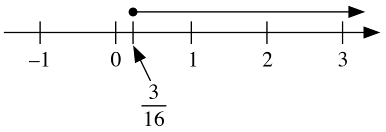


(m) $9(1 - 2k) \leqslant 2(3 - k)$

$9 - 18k \leqslant 6 - 2k$

$-16k \leqslant -3$

$k \geqslant \dfrac{3}{16}$

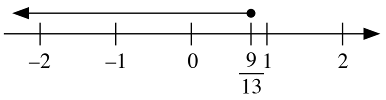


(n) $2(5 - 4l) \geqslant 5l + 1$

$10 - 8l \geqslant 5l + 1$

$-13l \geqslant -9$

$l \leqslant \dfrac{9}{13}$



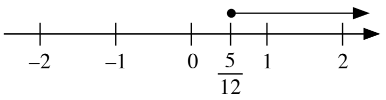
(o) $4(2m + 3) \geqslant 2(m + 7) - 3(2m - 1)$

$8m + 12 \geqslant 2m + 14 - 6m + 3$

$8m + 12 \geqslant 17 - 4m$

$12m \geqslant 5$

$m \geqslant \dfrac{5}{12}$



4. **(a)** $-2x < 13$

$x > -6\dfrac{1}{2}$

$\therefore$ Smallest integer value of x is –6.

(b) $2x + 1 > 16$

$2x > 15$

$x > 7\dfrac{1}{2}$

$\therefore$ Smallest integer value of x is 8.

(c) $9x + 12 > 30$

$9x > 18$

$x > 2$

$\therefore$ Smallest integer value of x is 3.

(d) $10x + 2 \geqslant 20$

$10x \geqslant 18$

$x \geqslant 1.8$

$\therefore$ Smallest integer value of x is 2.

5. **(a)** $-4y \geqslant 27$

$y \leqslant -6\dfrac{3}{4}$

$\therefore$ Largest integer value of y is –7.

(b) $3y - 2 < 13$

$3y < 15$

$y < 5$

$\therefore$ Largest integer value of y is 4.

(c) $16y + 1 \leqslant 31$

$16y \leqslant 30$

$y \leqslant 1\dfrac{7}{8}$

$\therefore$ Largest integer value of y is 1.

(d) $4(2y + 3) < 24$

$2y + 3 < 6$

$2y < 3$

$y < 1\dfrac{1}{2}$

$\therefore$ Largest integer value of y is 1.

6. $\dfrac{1}{2}h + \dfrac{1}{3}(h - 6) \geqslant 13$

$\dfrac{1}{2}h + \dfrac{1}{3}h - 2 \geqslant 13$

$\dfrac{5}{6}h \geqslant 15$

$h \geqslant 18$

(a) Least integer value of h is 18.

(b) Least prime number h is 19.

7. $3(x + 2) \geqslant 5(x - 1)$

$3x + 6 \geqslant 5x - 5$

$-2x \geqslant -11$

$x \leqslant 5\dfrac{1}{2}$

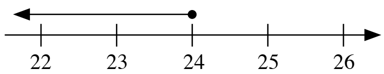
(a) $5\dfrac{1}{2}$

(b) 5

(c) 5

8. $6 + x \leqslant 30$

$x \leqslant 24$



(a) 2, 3, 5, 7, 11, 13, 17, 19, 23

(b) 16

9. Let x be the number of \$2 notes.

$2x + 10(21 - x) < 110$

$2x + 210 - 10x < 110$

$-8x < -100$

$x > 12.5$

$\therefore$ Minimum number of \$2 notes is 13.

10. Let x be the mark Shirley scores for her third History test.

$\dfrac{72 + 58 + x}{3} \geqslant 70$

$130 + x \geqslant 210$

$x \geqslant 80$

$\therefore$ Minimum mark is 80.

11. Let \$$x$ be the amount that Nathan pays.

$$x + 50 + x \leqslant 220$$
$$2x + 50 \leqslant 220$$
$$2x \leqslant 170$$
$$x \leqslant 85$$

$\therefore$ Greatest amount that Vishal pays is \$135.

12. Let x be the number of kiwi fruits he sells.

$$0.55x - 66.50 \geqslant 20$$
$$0.55x \geqslant 86.5$$
$$x \geqslant 157\frac{3}{11}$$

$\therefore$ Least number of kiwi fruits is 158.

13. **(i)** Maximum amount $= \$1.50 \times 12 = \$18$

Minimum amount $= \$1.20 \times 12 = \$14.40$

(ii) Let x be the number of cups of ice-cream.

$$(1.50)x + (1.20)(2) + (1.20)(10 - x) \leqslant 16$$
$$1.5x + 2.4 + 12 - 1.2x \leqslant 16$$
$$0.3x \leqslant 1.6$$
$$x \leqslant 5\frac{1}{3}$$

$\therefore$ Maximum number of cups of ice-cream is 5.

14. Let the length of the square be x cm.

$$4x \leqslant 50$$
$$x \leqslant 12.5$$

Largest possible area $= 12.5^2$

$= 156.3\ \text{cm}^2$ (to 4 s.f.)

15. **(a)** $x + 1 \leqslant 5$ and $2x > -8$

$x \leqslant 4$ $\qquad$ $x > -4$

$\therefore -4 < x \leqslant 4$

(b) $4x + 2 < 10$ and $3x - 1 \geqslant 11$

$4x < 8$ $\qquad$ $3x \geqslant 12$

$x < 2$ $\qquad$ $x \geqslant 4$

$\therefore$ No solution

(c) $x + 1 < 14$ and $2x + 3 > 12$

$x < 13$ $\qquad$ $2x > 9$

$x > 4\frac{1}{2}$

$\therefore 4\frac{1}{2} < x < 13$

(d) $6 + 2x > 0$ and $20 - 4x > 1 - 2x$

$2x > -6$ $\qquad$ $-2x > -19$

$x > -3$ $\qquad$ $x < 9\frac{1}{2}$

$\therefore -3 < x < 9\frac{1}{2}$

(e) $x + 3 < 22$ and $14 \leqslant 5x - 2$

$x < 19$ $\qquad$ $-5x \leqslant -16$

$x \geqslant 3\frac{1}{5}$

$\therefore 3\frac{1}{5} \leqslant x < 19$

(f) $x - 1 < 10$ and $4x + 1 > 7$

$x < 11$ $\qquad$ $4x > 6$

$x > 1\frac{1}{2}$

$\therefore 1\frac{1}{2} < x < 11$

(g) $2x - 3 \leqslant 5$ and $7 - 6x \leqslant -3$

$2x \leqslant 8$ $\qquad$ $-6x \leqslant -10$

$x \leqslant 4$ $\qquad$ $x \geqslant 1\frac{2}{3}$

$\therefore 1\frac{2}{3} \leqslant x \leqslant 4$

(h) $10x - 7 < 11$ and $5x - 2 > -4$

$10x < 18$ $\qquad$ $5x > -2$

$x < 1\frac{4}{5}$ $\qquad$ $x > -\frac{2}{5}$

$\therefore -\frac{2}{5} < x < 1\frac{4}{5}$

(i) $2x - 9 < 14$ and $3x - 8 > 11$

$2x < 23$ $\qquad$ $3x > 19$

$x < 11\frac{1}{2}$ $\qquad$ $x > 6\frac{1}{3}$

$\therefore 6\frac{1}{3} < x < 11\frac{1}{2}$

(j) $14 - x > 3$ and $1 - 2x < 10$

$-x > -11$ $\qquad$ $-2x < 9$

$x < 11$ $\qquad$ $x > -4\frac{1}{2}$

$\therefore -4\frac{1}{2} < x < 11$

16. Maximum length $= 6\ \text{cm} + \frac{1\ \text{cm}}{2}$

$= 6.5$ cm

Minimum length $= 6\ \text{cm} - \frac{1\ \text{cm}}{2}$

$= 5.5$ cm

17. Upper bound of length = 18.5 m

Upper bound of breadth = 7.5 m

Upper bound of area $= 18.5 \times 7.5$

$= 138.75\ \text{m}^2$

Lower bound of length = 17.5 m

Lower bound of breadth = 6.5 m

Lower bound of area $= 17.5\ \text{m} \times 6.5\ \text{m}$

$= 113.75\ \text{m}^2$

18. 1 mm = 0.1 cm

Maximum length $= 9.0 \text{ cm} + \frac{0.1 \text{ cm}}{2}$

$= 9.05$ cm

Maximum breadth $= 7.5 \text{ cm} + \frac{0.1 \text{ cm}}{2}$

$= 7.55$ cm

Maximum area $= 9.05 \times 7.55$

$= 68.3275 \text{ cm}^2$

Minimum length $= 9.0 \text{ cm} - \frac{0.1 \text{ cm}}{2}$

$= 8.95$ cm

Minimum breadth $= 7.5 \text{ cm} - \frac{0.1 \text{ cm}}{2}$

$= 7.45$ cm

Minimum area $= 8.95 \times 7.45$

$= 66.6775 \text{ cm}^2$

19. Least possible total height

$= 154.5 + 156.5 + 159.5 + 159.5 + 164.5$

$= 794.5$ cm

20. Largest possible volume $= 7.5 \times 7.5 \times 7.5$

$= 421.875 \text{ cm}^3$

Intermediate

21. (a) $\frac{3x}{6} \leqslant -8$

$x \leqslant -16$

(b) $\frac{x+1}{4} \geqslant \frac{x}{3}$

$3x + 3 \geqslant 4x$

$-x \geqslant -3$

$x \leqslant 3$

(c) $\frac{1}{4} + \frac{1}{3}x > 3x - \frac{1}{2}$

$\frac{3+4x}{12} > \frac{6x-1}{2}$

$6 + 8x > 72x - 12$

$-64x > -18$

$x < \frac{9}{32}$

(d) $\frac{x-1}{2} - \frac{x+1}{3} < 1\frac{1}{6}$

$\frac{3(x-1)-2(x+1)}{6} < \frac{7}{6}$

$3x - 3 - 2x - 2 < 7$

$x - 5 < 7$

$x < 12$

(e) $\frac{2x+1}{3} < \frac{3x-4}{5} + \frac{2}{3}$

$\frac{2x+1}{3} < \frac{3(3x-4)+2(5)}{15}$

$\frac{2x+1}{3} < \frac{9x-12+10}{15}$

$\frac{2x+1}{3} < \frac{9x-2}{15}$

$30x + 15 < 27x - 6$

$3x < -21$

$x < -7$

(f) $\frac{2x-1}{4} - \frac{2x-7}{3} < \frac{5}{7}$

$\frac{3(2x-1)-4(2x-7)}{12} < \frac{5}{7}$

$\frac{6x-3-8x+28}{12} < \frac{5}{7}$

$\frac{25-2x}{12} < \frac{5}{7}$

$175 - 14x < 60$

$-14x < -115$

$x > 8\frac{3}{14}$

(g) $\frac{5x}{6} - \frac{7}{9} \leqslant 2x - 4\frac{1}{2}$

$\frac{15x-14}{18} \leqslant \frac{4x-9}{2}$

$30x - 28 \leqslant 72x - 162$

$-42x \leqslant -134$

$x \geqslant 3\frac{4}{21}$

(h) $\frac{2-4x}{5} \geqslant 2\frac{1}{2} - 6x$

$\frac{2-4x}{5} \geqslant \frac{5-12x}{2}$

$4 - 8x \geqslant 25 - 60x$

$52x \geqslant 21$

$x \geqslant \frac{21}{52}$

(i) $\frac{2x-7}{8} + \frac{x-3}{4} \leqslant \frac{2x+3}{6} + 1$

$\frac{2x-7+2(x-3)}{8} \leqslant \frac{2x+3+6}{6}$

$\frac{2x-7+2x-6}{8} \leqslant \frac{2x+9}{6}$

$\frac{4x-13}{8} \leqslant \frac{2x+9}{6}$

$24x - 78 \leqslant 16x + 72$

$8x \leqslant 150$

$x \leqslant 18\frac{3}{4}$

(j) $\frac{x}{5} - 4 < 3 - \frac{5}{4}x$

$\frac{x}{5} + \frac{5}{4}x < 7$

$\frac{4x + 25x}{20} < 7$

$\frac{29x}{20} < 7$

$x < 4\frac{24}{29}$

(k) $\frac{1}{3}(4x - 3) > \frac{1}{2}(x + 5)$

$8x - 6 > 3x + 15$

$5x > 21$

$x > 4\frac{1}{5}$

22. (a) $2 - x < 2x + 3 \leqslant x + 6$

$2 - x < 2x + 3$ and $2x + 3 \leqslant x + 6$

$-3x < 1$; $x \leqslant 3$

$x > -\frac{1}{3}$

$\therefore -\frac{1}{3} < x \leqslant 3$

(b) $x + 2 < 14 < 3x + 1$

$x + 2 < 14$ and $14 < 3x + 1$

$x > 12$; $-3x < -13$

$x > 4\frac{1}{3}$

$\therefore 4\frac{1}{3} < x < 12$

(c) $8x + 1 \leqslant 2x + 1 \leqslant 3x + 2$

$8x + 1 \leqslant 2x + 2$ and $2x + 1 \leqslant 3x + 2$

$6x \leqslant 0$; $-x \leqslant 1$

$x \leqslant 0$; $x \geqslant -1$

$\therefore -1 \leqslant x \leqslant 0$

(d) $3x - 3 \leqslant 5x + 9 \leqslant x + 35$

$3x - 3 \leqslant 5x + 9$ and $5x + 9 \leqslant x + 35$

$-2x \leqslant 12$; $4x \leqslant 26$

$x \geqslant -6$; $x \leqslant 6\frac{1}{2}$

$\therefore -6 \leqslant x \leqslant 6\frac{1}{2}$

(e) $6x + 4 < 4x - 2 \leqslant 2x + 1$

$6x + 4 < 4x - 2$ and $4x - 2 \leqslant 2x + 1$

$2x < -6$; $2x \leqslant 3$

$x < -3$; $x \leqslant 1\frac{1}{2}$

$\therefore x < -3$

(f) $x + 2 \geqslant 1 - 3x > x - 11$

$x + 2 \geqslant 1 - 3x$ and $1 - 3x > x - 11$

$4x \geqslant -1$; $-4x > -12$

$x \geqslant -\frac{1}{4}$; $x < 3$

$\therefore -\frac{1}{4} \leqslant x < 3$

(g) $3x - 5 < 26 \leqslant 4x - 6$

$3x - 5 < 26$ and $26 \leqslant 4x - 6$

$3x < 31$; $-4x \leqslant -32$

$x < 10\frac{1}{3}$; $x \geqslant 8$

$\therefore 8 \leqslant x < 10\frac{1}{3}$

23. (a) $x - \frac{3}{2} < \frac{5 - 6x}{4} < x + \frac{1}{2}$

$x - \frac{3}{2} < \frac{5 - 6x}{4}$ and $\frac{5 - 6x}{4} < x + \frac{1}{2}$

$\frac{2x - 3}{2} < \frac{5 - 6x}{4}$; $\frac{5 - 6x}{4} < \frac{2x + 1}{2}$

$8x - 12 < 10 - 12x$; $10 - 12x < 8x + 4$

$20x < 22$; $-20x < -6$

$x < 1\frac{1}{10}$; $x > \frac{3}{10}$

$\therefore \frac{3}{10} < x < 1\frac{1}{10}$

(b) $2 + \frac{3x}{2} \leqslant \frac{5x + 1}{3} \leqslant \frac{3x + 11}{2}$

$2 + \frac{3x}{2} \leqslant \frac{5x + 1}{3}$ and $\frac{5x + 1}{3} \leqslant \frac{3x + 11}{2}$

$\frac{4 + 3x}{2} \leqslant \frac{5x + 1}{3}$; $10x + 2 \leqslant 9x + 33$

$12 + 9x \leqslant 10x + 2$; $x \leqslant 31$

$-x \leqslant -10$

$x \geqslant 10$

$\therefore 10 \leqslant x \leqslant 31$

(c) $2x + 3 > \frac{7x + 6}{4} \geqslant 3x + 2$

$2x + 3 > \frac{7x + 6}{4}$ and $\frac{7x + 6}{4} \geqslant 3x + 2$

$8x + 12 > 7x + 6$; $7x + 6 \geqslant 12x + 8$

$x > -6$; $-5x \geqslant 2$

$x \leqslant -\frac{2}{5}$

$\therefore -6 < x \leqslant -\frac{2}{5}$

(d) $2x - 15\frac{1}{2} > x + \frac{1}{2} \geqslant 2x - 25\frac{1}{2}$

$2x - 15\frac{1}{2} > x + \frac{1}{2}$ and $x + \frac{1}{2} \geqslant 2x - 25\frac{1}{2}$

$x > 16$ $\quad$ $-x \geqslant -26$

$x \leqslant 26$

$\therefore 16 < x \leqslant 26$

(e) $\frac{x}{2} + \frac{1}{5} \geqslant \frac{2x}{5} > x - 5$

$\frac{x}{2} + \frac{1}{5} \geqslant \frac{2x}{5}$ and $\frac{2x}{5} > x - 5$

$\frac{5x+2}{10} \geqslant \frac{2x}{5}$ $\quad$ $2x > 5x - 25$

$25x + 10 \geqslant 20x$ $\quad$ $-3x > -25$

$5x \geqslant -10$ $\quad$ $x < 8\frac{1}{3}$

$x \geqslant -2$

$\therefore -2 \leqslant x < 8\frac{1}{3}$

(f) $\frac{1}{2}x + 6 < \frac{1}{4}x + 10 < x + 5$

$\frac{1}{2}x + 6 < \frac{1}{4}x + 10$ and $\frac{1}{4}x + 10 < x + 5$

$\frac{1}{4}x < 4$ $\quad$ $-\frac{3}{4}x < -5$

$x < 16$ $\quad$ $x > 6\frac{2}{3}$

$\therefore 6\frac{2}{3} < x < 16$

(g) $-2x + 4 \leqslant \frac{3x-5}{3} \leqslant 5x - 6$

$-2x + 4 \leqslant \frac{3x-5}{3}$ and $\frac{3x-5}{3} \leqslant 5x - 6$

$-6x + 12 \leqslant 3x - 5$ $\quad$ $3x - 5 \leqslant 15x - 18$

$-9x \leqslant -17$ $\quad$ $-12x \leqslant -13$

$x \geqslant 1\frac{8}{9}$ $\quad$ $x \geqslant 1\frac{1}{12}$

$\therefore x \geqslant 1\frac{8}{9}$

(h) $\frac{2}{5}x < 2x - 1 \leqslant \frac{10+2x}{15}$

$\frac{2}{5}x < 2x - 1$ and $2x - 1 \leqslant \frac{10+2x}{15}$

$2x < 10x - 5$ $\quad$ $30x - 15 \leqslant 10 + 2x$

$-8x < -5$ $\quad$ $28x \leqslant 25$

$x > \frac{5}{8}$ $\quad$ $x \leqslant \frac{25}{28}$

$\therefore \frac{5}{8} < x \leqslant \frac{25}{28}$

24. $\frac{1}{2}(y - 4) > \frac{2y}{3}$

$3y - 12 > 4y$

$-y > 12$

$y < -12$

$\therefore$ Largest integer value of y is -13.

25. $3 - 3x \leqslant 2 + 2x < 5x + 1$

$3 - 3x \leqslant 2 + 2x$ and $2 + 2x < 5x + 1$

$-5x \leqslant -1$ $\quad$ $-3x < -1$

$x \geqslant \frac{1}{5}$ $\quad$ $x > \frac{1}{3}$

$\therefore x > \frac{1}{3}$

(a) 1

(b) 2

26. $3x + 5 < 4x - 2 \leqslant 3x + 7$

$3x + 5 < 4x - 2$ and $4x - 2 \leqslant 3x + 7$

$-x < -7$ $\quad$ $x \leqslant 9$

$x > 7$

$\therefore 7 < x \leqslant 9$

Integer values of x are 8 and 9.

27. $\frac{q+8}{3} \leqslant \frac{4q}{3} - 4$

$\frac{q+8}{3} \leqslant \frac{4q-12}{3}$

$q + 8 \leqslant 4q - 12$

$-3q \leqslant -20$

$q \geqslant 6\frac{2}{3}$

(a) 7

(b) 7

28. $\frac{1}{4}x - \frac{3}{5}\left(x + \frac{1}{3}\right) \leqslant \frac{1}{2}(x - 9)$

$\frac{1}{4}x - \frac{3}{5}x - \frac{1}{5} \leqslant \frac{1}{2}x - \frac{9}{2}$

$-\frac{7}{20}x - \frac{1}{5} \leqslant \frac{1}{2}x - \frac{9}{2}$

$-\frac{17}{20}x \leqslant -\frac{43}{10}$

$x \geqslant 5\frac{1}{17}$

(a) $5\frac{1}{17}$

(b) 6

29. $\frac{y+8}{3} \leqslant \frac{4y}{5} - 1$

$\frac{y+8}{3} \leqslant \frac{4y-5}{5}$

$5y + 40 \leqslant 12y - 15$

$-7y \leqslant -55$

$y \geqslant 7\frac{6}{7}$

(a) 8

(b) 11

30. $40 < 60 - 50t < 50$

$40 < 60 - 50t$ and $60 - 50t < 50$

$50t < 20$ $\qquad -50t < -10$

$t < \frac{2}{5}$ $\qquad t > \frac{1}{5}$

$\therefore \frac{1}{5} < t < \frac{2}{5}$

31. $5 < x - 1 < 9$ and $9\frac{1}{2} < 2x + 1\frac{1}{2} < 18$

$6 < x < 10$ $\qquad 8 < 2x < 16\frac{1}{2}$

$4 < x < 8\frac{1}{4}$

$\therefore 6 < x < 8\frac{1}{4}$

Integers are 7 and 8.

32. $x < 3 + 8$

$\therefore x < 11$

33. Let the integers be x, $x + 1$ and $x + 2$.

$x + x + 1 + x + 2 \leqslant 370$

$3x + 3 \leqslant 370$

$3x \leqslant 367$

$x \leqslant 122\frac{1}{3}$

(a) 123

(b) $\sqrt{124} = 11.1$ (to 3 s.f.)

34. Let x m be the breadth of the plot.

$2(4x + x) \leqslant 220$

$10x \leqslant 220$

$x \leqslant 22$

Largest possible area = (88)(22)

= 1936 m^2

35. Let Farhan's age be x years.

$x + 2x \geqslant 53$

$3x \geqslant 53$

$x \geqslant 17\frac{2}{3}$

$\therefore$ Minimum age of Farhan is 18 years.

36. Let the number of questions he answered correctly be x.

$2x - (18 - x) > 30$

$2x - 18 + x > 30$

$3x > 48$

$x > 16$

$\therefore$ Minimum number of questions he answered correctly is 17.

37. Let x be the number of strawberries.

$x + \frac{2}{3}x \leqslant 65$

$\frac{5}{3}x \leqslant 65$

$x \leqslant 39$

$\therefore$ Maximum number of strawberries is 39.

38. Let the number of 50-cent coins be x.

$3(50) + 20(2) + x(0.5) \leqslant 200$

$150 + 40 + 0.5x \leqslant 200$

$0.5x \leqslant 10$

$x \leqslant 20$

$\therefore$ Maximum number of 50-cent coins is 20.

39. (a) Greatest possible value of $a + b = 3 + (-2)$

$= 1$

(b) Least possible value of $a - b = -5 - (-2)$

$= -3$

(c) Largest possible value of $ab = (-5)(-8)$

$= 40$

(d) Smallest possible value of $\frac{a}{b} = \frac{3}{-2}$

$= -1\frac{1}{2}$

(e) Greatest possible value of $a^2 = (-5)^2$

$= 25$

Least possible value of $a^2 = 0^2$

$= 0$

40. Upper bound of total mass = 60.5 kg

Lower bound of total number of potatoes = 45

Upper bound of average mass of one potato = $\frac{60.5}{45}$

= 1.34 kg (to 3 s.f.)

Lower bound of total mass = 59.5 kg

Upper bound of total number of potatoes = 55

Lower bound of average mass of one potato = $\frac{59.5}{55.5}$

= 1.08 kg (to 3 s.f.)

41. Maximum distance travelled = 900.5 m

Minimum time taken = 4.5 s

Maximum speed = $\dfrac{900.5}{4.5}$

= 200 m/s (to 3 s.f.)

Minimum distance travelled = 899.5 m

Maximum time = 5.5 s

Minimum speed = $\dfrac{899.5}{5.5}$

= 164 m/s (to 3 s.f.)

42. (a) Greatest possible value of perimeter = 2(22.5 + 57.5)

= 160 m

Least possible value of perimeter = 2(21.5 + 56.5)

= 156 m

(b) Greatest possible value of area = 22.5 × 57.5

= 1293.75 m^2

Least possible value of area = 21.5 × 56.5

= 1214.75 m^2

Advanced

43. (a) Greatest possible value of $(x - y)^2 = [8 - (-5)]^2$

$= 169$

(b) Least possible value of $(x + y)^2 = [5 + (-5)]^2$

$= 0$

(c) Largest possible value of $\dfrac{2y}{x} = \dfrac{2(2)}{2}$

$= 2$

(d) Largest possible value of $\dfrac{y^2}{x} = \dfrac{(-5)^2}{2}$

$= 12\frac{1}{2}$

(e) Greatest possible value of $x^3 - y^3 = 8^3 - (-5)^3$

$= 637$

Least possible value of $x^3 - y^3 = 2^3 - 2^3$

$= 0$

44. (a) Least possible value of $p^2 - q^2 = \left(-\frac{1}{2}\right)^2 - 6^2$

$= -35\frac{3}{4}$

(b) Least possible value of $p^2 + q^2 = \left(-\frac{1}{2}\right)^2 + 0^2$

$= \frac{1}{4}$

(c) Largest possible value of $pq = (-2)(-1)$

$= 2$

(d) Smallest possible value of $\dfrac{q}{p} = \dfrac{6}{-\frac{1}{2}}$

$= -12$

(e) Greatest possible value of $p^3 + q^3 = \left(-\frac{1}{2}\right)^2 + 6^3$

$= 215\frac{7}{8}$

Least possible value of $p^3 + q^3 = (-2)^3 + (-1)^3$

$= -9$

New Trend

45. (i) $-10 < 7 - 2x \leqslant -1$

$-10 < 7 - 2x$ and $7 - 2x \leqslant -1$

$2x < 17$ $\quad$ $-2x \leqslant -8$

$x < 8\frac{1}{2}$ $\quad$ $x \geqslant 4$

$\therefore 4 \leqslant x < 8\frac{1}{2}$

(ii) Integers are 4, 5, 6, 7 and 8.

46. $2(x + 1) > \frac{3}{5}(x - 4)$

$10(x + 1) > 3(x - 4)$

$10x + 10 > 3x - 12$

$7x > -22$

$x > -3\frac{1}{7}$

47. (a) $-5 < x \leqslant 3$

Integers are –4, –3, –2, –1, 0, 1, 2 and 3.

(b) $x - 3 < 2x - 1 < 5 + x$

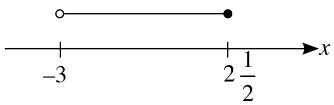
$x - 3 < 2x - 1$ and $2x - 1 < 5 + x$

$-x < 2$ $\quad$ $x < 6$

$x > -2$

$\therefore -2 < x < 6$

48. (a)



(b) $4x + 5 \leqslant 5x - 2 \leqslant 4x + 7$

$4x + 5 \leqslant 5x - 2$ and $5x - 2 \leqslant 4x + 7$

$-x \leqslant -7$ $\quad$ $x \leqslant 9$

$x \geqslant 7$

$\therefore 7 \leqslant x \leqslant 9$

Chapter 4 Indices and Standard Form

Basic

1. **(a)** $a^4 \div a^{-2} \times a^7$

$= a^{4-(-2)+7}$

$= a^{13}$

(b) $2b^7 \times 4b^{-3}$

$= 8b^{7+(-3)}$

$= 8b^4$

(c) $c^{-2} \times (c^{\frac{1}{2}})^6 \times c^{-1}$

$= c^{-2} \times c^3 \times c^{-1}$

$= c^{-2+3+(-1)}$

$= c^0$

$= 1$

(d) $\sqrt[3]{d^2} \times \sqrt{d^3} \div d^2$

$= d^{\frac{2}{3}} \times d^{\frac{3}{2}} \div d^2$

$= d^{\frac{2}{3}+\frac{3}{2}-2}$

$= d^{\frac{1}{6}}$

$\dfrac{e^{-5} \times e^9}{e}$

$= e^{-5+9-1}$

$= e^3$

(e) $\dfrac{e^{-5} \times e^9}{e}$

$= e^{-5+9-11}$

$= e^3$

(f) $\dfrac{f^{-\frac{1}{2}} \times f^4}{f^0 \times \sqrt{f} \div f^{-2}}$

$= \dfrac{f^{-\frac{1}{2}+4}}{f^{\frac{1}{2}-(-2)}}$

$= \dfrac{f^{3\frac{1}{2}}}{f^{2\frac{1}{2}}}$

$= f$

2. **(a)** $\left(\dfrac{3w}{5}\right)^{-2}$

$= \left(\dfrac{5}{3w}\right)^2$

$= \dfrac{25}{9w^2}$

(b) $\left(\dfrac{3}{7x}\right)^{-2}$

$= \left(\dfrac{7x}{3}\right)^2$

$= \dfrac{49x^2}{9}$

(c) $3 \div 9y^{-2}$

$= 3 \div \dfrac{9}{y^2}$

$= 3 \times \dfrac{y^2}{9}$

$= \dfrac{y^2}{3}$

(d) $(5z)^0 \div 8z^{-4}$

$= 1 \div \dfrac{8}{z^4}$

$= 1 \times \dfrac{z^4}{8}$

$= \dfrac{z^4}{8}$

3. **(a)** $(-27)^{\frac{2}{3}}$

$= (\sqrt[3]{-27})^2$

$= (-3)^2$

$= 9$

(b) $8^{-\frac{2}{3}}$

$= \dfrac{1}{8^{\frac{2}{3}}}$

$= \dfrac{1}{(\sqrt[3]{8})^2}$

$= \dfrac{1}{2^2}$

$= \dfrac{1}{4}$

(c) $\sqrt[3]{0.027}$

$= \sqrt[3]{\dfrac{27}{1000}}$

$= \sqrt[3]{\left(\dfrac{3}{10}\right)^3}$

$= \dfrac{3}{10}$

(d) $3^4 - 3^3$

$= 81 - 27$

$= 54$

4. **(a)** $2^{2a-1} = 128$

$= 2^7$

$2a - 1 = 7$

$2a = 8$

$a = 4$

(b) $6^{3b} = 216$

$= 6^3$

$3b = 3$

$b = 1$

(c) $3^{c+1} = 27^{-1}$
$= (3^3)^{-1}$
$= 3^{-3}$
$c + 1 = -3$
$c = -4$

(d) $8^{3d-1} = 1$
$3d - 1 = 0$
$3d = 1$
$d = \frac{1}{3}$

5. **(a)** $0.0231 = 2.31 \times 10^{-2}$
(b) $62\ 500 = 6.25 \times 10^4$
(c) $5\ 390\ 000 = 5.39 \times 10^6$
(d) $0.000\ 005\ 345 = 5.345 \times 10^{-6}$

6. **(a)** $9.43 \times 10^{-4} = 0.000\ 943$
(b) $6.1 \times 10^4 = 61\ 000$
(c) $2.795 \times 10^6 = 2795\ 000$
(d) $7 \times 10^{-7} = 0.000\ 0007$

7. **(a)** $(8.59 \times 10^{-7}) \times (0.392 \times 10^5)$
$= 3.37 \times 10^{-2}$ (to 3 s.f.)
(b) $(8.05 \times 10^6) \div (7 \times 10^{-2})$
$= 1.15 \times 10^8$
(c) $3.2 \times 10^6 + 1.8 \times 10^4$
$= 3.22 \times 10^6$ (to 3 s.f.)
(d) $1.97 \times 10^7 - 2.02 \times 10^5$
$= 1.95 \times 10^7$ (to 3 s.f.)

8. 750 gigabytes $= 750 \times 10^9$ bytes
$= 7.5 \times 10^{11}$ bytes

9. 0.5 MHz $= 0.5 \times 10^6$ hertz
$= 5 \times 10^5$ hertz

10. 76 µg $= 76 \times 10^{-6}$ g
$= 7.6 \times 10^{-5}$ g

11. **(i)** 273 picograms $= 273 \times 10^{-12}$ g
$= 2.73 \times 10^{-10}$ g
(ii) Total mass $= (0.3 \times 10^9) \times (2.73 \times 10^{-10})$
$= 8.19 \times 10^{-2}$

12. **(a)** $q : p = 1.2 \times 10^6 : 9.6 \times 10^5$
$= 12 : 9.6$
$= 5 : 4$
(b) Distance between Beijing and Tokyo
$= 9.6 \times 10^5 + 1.2 \times 10^6$
$= 2.16 \times 10^6$ m

13. **(a)** Difference in mass $= 2.66 \times 10^{-23} - 1.99 \times 10^{-23}$
$= 0.67 \times 10^{-23}$
$= 6.7 \times 10^{-24}$ g
(b) Mass of one molecule $= 1.99 \times 10^{-23} + 2(2.66 \times 10^{-23})$
$= 1.99 \times 10^{-23} + 5.32 \times 10^{-23}$
$= 7.31 \times 10^{-23}$ g

Intermediate

14. **(a)** $(5a^2b^3)^3$
$= 125a^{2\times3}b^{3\times3}$
$= 125a^6b^9$

(b) $5a^4b^8 \times 9a^2b^3$
$= 45a^{4+2}b^{8+3}$
$= 45a^6b^{11}$

(c) $\frac{a^4 \times (ab^2)^2}{(a^8b)^2}$
$= \frac{a^4 \times a^{1\times2}b^{2\times2}}{a^{8\times2}b^{1\times2}}$
$= \frac{a^{4+2}b^4}{a^{16}b^2}$
$= \frac{a^6b^4}{a^{16}b^2}$
$= \frac{b^{4-2}}{a^{16-6}}$
$= \frac{b^2}{a^{10}}$

(d) $-(4a^3b)^2 \times \frac{3a^5}{8b^4}$
$= -16a^{3\times2}b^{1\times2} \times \frac{3a^5}{8b^4}$
$= -16a^6b^2 \times \frac{3a^5}{8b^4}$
$= -\frac{6a^{6+5}}{b^{4-2}}$
$= -\frac{6a^{11}}{b^2}$

15. **(a)** $3^0 + \left(\frac{1}{3}\right)^{-4}$
$= 1 + 3^4$
$= 1 + 81$
$= 82$

(b) $8^{-2} + 8^0 - 8^1$
$= \frac{1}{8^2} + 1 - 8$
$= \frac{1}{64} - 7$
$= -6\frac{63}{64}$

(c) $9^{-1} + 9^0 + 9^{\frac{1}{2}}$
$= \frac{1}{9} + 1 + 3$
$= 4\frac{1}{9}$

(d) $16^{-\frac{3}{4}} \times 8^2 \div 2^{-1}$

$= (2^4)^{-\frac{3}{4}} \times (2^3)^2 \div 2^{-1}$

$= 2^{-3} \times 2^6 \div 2^{-1}$

$= 2^{-3+6-(-1)}$

$= 2^4$

$= 16$

(e) $\left(\frac{3}{4}\right)^{-2} + 3^{-1} - 3$

$= \left(\frac{4}{3}\right)^2 + \frac{1}{3} - 3$

$= \frac{16}{9} - 2\frac{2}{3}$

$= -\frac{8}{9}$

(f) $(81^{\frac{1}{2}} - 4^0) \times 3^{-2}$

$= (9 - 1) \times \frac{1}{3^2}$

$= 8 \times \frac{1}{9}$

$= \frac{8}{9}$

(g) $\left(\frac{1}{27}\right)^0 \times \left(\frac{27}{8}\right)^{-\frac{2}{3}} \div \frac{1}{3^2}$

$= 1 \times \left(\frac{27}{8}\right)^{-\frac{2}{3}} \times 3^2$

$= \left[\left(\frac{2}{3}\right)^3\right]^{\frac{2}{3}} \times 9$

$= \left(\frac{2}{3}\right)^2 \times 9$

$= \frac{4}{9} \times 9$

$= 4$

(h) $\left(\frac{2}{5}\right)^{-2} \div 125^{\frac{1}{3}}$

$= \left(\frac{2}{5}\right)^{-2} \div (5^3)^{\frac{1}{3}}$

$= \frac{25}{4} \div 5$

$= \frac{25}{4} \times \frac{1}{5}$

$= \frac{5}{4}$

$= 1\frac{1}{4}$

16. (a) $\frac{(-2x^2y)^3}{4x^{-1}(y^2)^3}$

$= \frac{-8x^{2\times3}y^{1\times3}}{4x^{-1}y^{2\times3}}$

$= -\frac{2x^6y^3}{x^{-1}y^6}$

$= -\frac{2x^{6-(-1)}}{y^{6-3}}$

$= -\frac{2x^7}{y^3}$

(b) $\frac{(2x^2y)^3 \times \sqrt{x^8}}{x^{-2}y^5}$

$= \frac{8x^{2\times3}y^{1\times3} \times x^4}{x^{-2}y^5}$

$= \frac{8x^{6+4}y^3}{x^{-2}y^5}$

$= \frac{8x^{10-(-2)}}{y^{5-3}}$

$= \frac{8x^{12}}{y^2}$

(c) $\frac{(2xy)^2}{35xy^7} \div \left(\frac{x^{-1}y^{-2}}{4}\right)^{-2}$

$= \frac{4x^{1\times2}y^{1\times2}}{35xy^7} \div \left(\frac{4}{x^{-1}y^{-2}}\right)^2$

$= \frac{4x^2y^2}{35xy^7} \div (4xy^2)^2$

$= \frac{4x^{2-1}}{35y^{7-2}} \div 16x^{1\times2}y^{2\times2}$

$= \frac{4x}{35y^5} \times \frac{1}{16x^2y^4}$

$= \frac{1}{140xy^9}$

(d) $\left(\frac{2x}{y^{-1}}\right)^2 \div \left(\frac{2}{x^{-2}y}\right)^{-2}$

$= \frac{4x^{1\times2}}{y^{-1\times2}} \div \left(\frac{x^{-2}y}{2}\right)^2$

$= \frac{4x^2}{y^{-2}} \div \left(\frac{y}{2x^2}\right)^2$

$= 4x^2y^2 \div \frac{y^{1\times2}}{4x^{2\times2}}$

$= 4x^2y^2 \div \frac{y^2}{4x^4}$

$= 4x^2y^2 \times \frac{4x^4}{y^2}$

$= 16x^6$

17. $\dfrac{5^p}{\sqrt{5}} = 5^{-p}$

$\dfrac{5^p}{5^{\frac{1}{2}}} = 5^{-p}$

$5^{p-\frac{1}{2}} = 5^{-p}$

$p - \dfrac{1}{2} = -p$

$2p = \dfrac{1}{2}$

$p = \dfrac{1}{4}$

18. $\dfrac{a^3 \times \sqrt[3]{a}}{\sqrt{a^5}} = a^w$

$\dfrac{a^3 \times a^{\frac{1}{3}}}{a^{\frac{5}{2}}} = a^w$

$a^{3+\frac{1}{3}-\frac{5}{2}} = a^w$

$w = \dfrac{5}{6}$

19. $10^{3p+2q-r}$

$= \dfrac{(10^{3p})(10^{2q})}{10^r}$

$= \dfrac{(10^p)^3(10^q)^2}{10^r}$

$= \dfrac{(2)^3(3)^2}{1250}$

$= 5.76 \times 10^{-2}$

20. **(a)** $10^{-4} - 3.12 \times 10^{-5}$

$= 6.88 \times 10^{-5}$

(b) $\dfrac{0.26 \times 10^{-4}}{2.31 \times 23 \times 10^{-2}}$

$= 4.89 \times 10^{-5}$ (to 3 s.f.)

(c) $1.2 \times 10^8 + 2(3.5 \times 10^7)$

$= 1.9 \times 10^8$

(d) $\sqrt[4]{1600 \times 10^{-4}}$

$= 6.32 \times 10^{-1}$ (to 3 s.f.)

(e) $\dfrac{7.5 \times 10^6}{1.5 \times 10^3} + 4.1 \times 10^4$

$= 4.6 \times 10^4$

(f) $\dfrac{(4 \times 10^2)^5 - (5 \times 10^6)}{\sqrt{16 \times 10^{-4}}}$

$= 2.56 \times 10^{14}$ (to 3 s.f.)

21. **(a)** $\dfrac{2b}{a} = \dfrac{2(2 \times 10^2)}{5 \times 10^{-3}}$

$= 8 \times 10^4$

(b) $\dfrac{3}{a} - b = \dfrac{3}{5 \times 10^{-3}} - 2 \times 10^2$

$= 4 \times 10^2$

22. **(a)** $p \times 2q = 4 \times 10^9 \times 2 \times 3 \times 10^5$

$= 2.4 \times 10^{15}$

(b) $\dfrac{q^2}{p} = \dfrac{(3 \times 10^3)^2}{4 \times 10^9}$

$= 2.25 \times 10^1$

23. 3.3 nanoseconds $= 3.3 \times 10^{-9}$ seconds

4.2 billion km $= 4.2 \times 10^9$ km

$= 4.2 \times 10^{12}$ m

Time taken $= \dfrac{4.2 \times 10^{12}}{1 \div (3.3 \times 10^{-9})}$

$= 1.386 \times 10^4$ seconds

24. **(a)** Difference in population $= 50 \times 10^6 - 5.18 \times 10^6$

$= 4.482 \times 10^7$

(b) $5.18 \times 10^6 : 6.97 \times 10^9$

$1 : 1350$ (to 3 s.f.)

25. **(i)** 0.000 001 654 cm $= 1.654 \times 10^{-6}$ cm

(ii) Volume $= \dfrac{4}{3}\pi\left(\dfrac{1.654 \times 10^{-6}}{2}\right)^3 \times 10^6$

$= 2.37 \times 10^{-12}$ cm^3 (to 3 s.f.)

26. $x = 1, y = -2$

Advanced

27. **(i)** Number of daughter cells at the end of 1 hour $= 2^3$

(ii) Number of daughter cells at the end of 1 day $= 2^{72}$

(iii) Number of daughter cells at the end of 1 week $= 2^{504}$

28. $\dfrac{8(9^{3x}) - 27^{2x}}{3^{2x+1} \times 81^{x-1}} = \dfrac{8(3^2)^{3x} - (3^3)^{2x}}{3(3^{2x}) \times (3^4)^{x-1}}$

$= \dfrac{8(3^{6x}) - 3^{6x}}{3(3^{2x}) \times 3^{4x} \times 3^{-4}}$

$= \dfrac{7(3^{6x})}{3^{-3}(3^{6x})}$

$= 189$

29. **(a)** $\dfrac{2^{15}}{8^5} = \dfrac{(2^3)^5}{8^5}$

$= \dfrac{8^5}{8^5}$

$= 1$

(b) $2^8 \times 5^4 = (2^2)^4 \times 5^4$

$= 4^4 \times 5^4$

$= 20^4$

$= 160\ 000$

30. $9^n + 9^n + 9^n = 243$

$3(9^n) = 243$

$9^n = 81$

$= 9^2$

$n = 2$

New Trend

31. $16 \times 64^n = 1$

$4^2 \times (4^3)^n = 4^0$

$4^{2+3n} = 4^0$

$2 + 3n = 0$

$n = -\frac{2}{3}$

32. (a) $2^n \times 2^{-2} = \frac{1}{32}$

$2^{n-2} = \frac{1}{2^5}$

$= 2^{-5}$

$n - 2 = -5$

$n = -3$

(b) $\frac{1}{36} = 6^k$

$\frac{1}{6^2} = 6^k$

$6^k = 6^{-2}$

$k = -2$

33. $\left(\frac{2x}{y^{-1}}\right)^2 \div \frac{1}{3x^{-3}y^{-3}}$

$= \frac{4x^2}{y^{-1\times 2}} \div \frac{x^3y^3}{3}$

$= \frac{4x^2}{y^{-2}} \times \frac{3}{x^3y^3}$

$= \frac{12}{xy}$

34. (a) $(x^9y^{-3})^{\frac{1}{3}} \times (x^8y^{-2})^{\frac{3}{2}}$

$= x^{9\times\frac{1}{3}}y^{-3\times\frac{1}{3}} \times x^{8\times\frac{3}{2}}y^{-2\times\frac{3}{2}}$

$= x^3y^{-1} \times x^{12}y^{-3}$

$= x^{3+12}y^{-1+(-3)}$

$= x^{15}y^{-4}$

$= \frac{x^{15}}{y^4}$

(b) $\left(\frac{125}{x^{27}}\right)^{-\frac{1}{3}} = \left(\frac{x^{27}}{125}\right)^{\frac{1}{3}}$

$= \frac{x^9}{5}$

35. (a) (i) $11^{20} \div 11^5 = 11^{20-5}$

$= 11^{15}$

(ii) $\frac{1}{121} = \frac{1}{11^2}$

$= 11^{-2}$

(iii) $\sqrt[6]{11} = 11^{\frac{1}{6}}$

(b) $5^{-3} \times 5^k = 1$

$5^{-3+k} = 5^0$

$-3 + k = 0$

$k = 3$

36. (i) $46\ \mu\text{m} = 46 \times 10^{-6}$ m

$= 4.6 \times 10^{-5}$ m

(ii) Area $= \pi(4.6 \times 10^{-5})^2$

$= 6.65 \times 10^{-9}$ m^2 (to 3 s.f.)

37. (a) $12\,000 = 1.2 \times 10^4$

(b) Percentage increase in speed

$= \frac{1.14\times10^7 - 9.7\times10^6}{9.7\times10^6} \times 100\%$

$= \frac{10^6(1.14\times10 - 9.7)}{9.7\times10^6} \times 100\%$

$= \frac{1.7}{9.7} \times 100\%$

$= 17.5\%$ (to 3 s.f.)

(c) $29\ \text{m/s} = \frac{29\ \text{m}}{1\ \text{s}}$

$= \frac{(29 \div 1000)\ \text{km}}{(1 \div 3600)\ \text{h}}$

$= 104.4$ km/h

$= 1.044 \times 10^2$ km/h

38. (a) Difference in population $= 6.64 \times 10^7 - 5.077 \times 10^6$

$= 6.64 \times 10^7 - 0.5077 \times 10^7$

$= 6.1323 \times 10^7$

(b) 100% represent the population of Thailand in 1950.

338% represent the population of Thailand in 2010 $= 6.64 \times 10^7$

Population of Thailand in 1950 $= \frac{6.64\times10^7}{338} \times 100$

$= 1.96 \times 10^7$ (to 3 s.f.)

39. (a) $50\,197.4 \times 10^9$ Wh $= 50\,197.4 \times 10^6$ kWh

$= 5.019\,74 \times 10^{10}$ kWh

(b) Mean domestic electricity consumed per person

$= \frac{4716.1 \times 10^9}{3.111\times10^6}$

$= 1516$ kWh (to the nearest kWh)

(c) 100% represent electricity consumption in 2000.

Electricity consumption in 2015 is represented by $100 - 41.6 = 58.4\%$

Electricity consumption in 2000

$= \frac{5471.2}{58.4} \times 100$

$= 9368$ GWh (to the nearest GWh)

40. (i) When $t = 0$,

$V = 20\,000 \times 1.1^0$

$= 20\,000$

∴ The value of the flat when it was first built was \$20 000.

(ii) When $t = 2$,

$\$V = 20\,000 \times 1.1^2$
$= 24\,200$

Percentage increase $= \dfrac{24\,200 - 20\,000}{20\,000} \times 100\%$
$= 21\%$

$\therefore$ The value of the flat increased by 21% after two years.

41. **(a)** $P = 35\,480 \times 1.0125^5$
$= \$37\,753.63$ (to the nearest cent)

(b) Percentage increase in the balance
$= \dfrac{37\,753.63 - 35\,480}{35\,480} \times 100\%$
$= 6.41\%$ (to 3 s.f.)

42. 200 ha = 200 000 m^2

Number of trees on 200 000 m^2 $= \dfrac{200\,000}{10} \times 4$
$= 80\,000$

Total number of fruits on trees $= 60 \times 80\,000$
$= 4\,800\,000$
$= 4.8 \times 10^6$

Average number of seeds produced by these fruits $= \dfrac{1.44 \times 10^7}{4.8 \times 10^6}$
$= 3$

43. **(a)** 8.48 light years $= 8.48 \times 9.46 \times 10^{15}$ m
$= 80.2208 \times 10^{15}$ m
$= 8.02208 \times 10^{13}$ km

(b) 4.35 light years $= 4.35 \times 9.46 \times 10^{15}$ m
$= 41.151 \times 10^{15}$ m
$= 4.1151 \times 10^{16}$ m
$= 4.1151 \times 10^{13}$ km

Time taken $= \dfrac{4.1151 \times 10^{13}}{50\,000}$
$= 0.823\,02 \times 10^9$ h
$= \dfrac{0.823\,02 \times 10^9 \text{ h}}{(365 \times 24) \text{ h}}$
$= 0.000\,093\,952\,05 \times 10^9$ years
$= 94\,000$ years (to 2 s.f.)

Revision Test A1

1. **(a)** $(x + 2)(3x - 7) = 0$

$x + 2 = 0$ or $3x - 7 = 0$

$x = -2$ or $3x = 7$

$x = \frac{7}{3}$

$= 2\frac{1}{3}$

(b) $(x + 2)(3x - 7) = 4$

$3x^2 - x - 14 - 4 = 0$

$3x^2 - x - 18 = 0$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(3)(-18)}}{2(3)}$$

$$= \frac{1 \pm \sqrt{217}}{6}$$

$= -2.62$ (to 2 d.p.) or -2.29 (to 2 d.p.)

2. $g(x) = 2x + 1$

Let $y = 2x + 1$

$x = \frac{y - 1}{2}$

$\therefore g^{-1}(x) = \frac{x - 1}{2}$

(i) $g^{-1}(5) = \frac{5 - 1}{2}$

$= 2$

(ii) $g^{-1}\left(-\frac{1}{3}\right) = \frac{-\frac{1}{3} - 1}{2}$

$= -\frac{2}{3}$

3. $f(x) = ax + b$

$f(3) = 3a + b = 12$ — (1)

$f(-4) = -4a + b = 6$ — (2)

(1) – (2): $7a = 6$

$a = \frac{6}{7}$

Substitute $a = \frac{6}{7}$ into (1):

$3\left(\frac{6}{7}\right) + b = 12$

$b = 9\frac{3}{7}$

$\therefore f(x) = \frac{6}{7}x + 9\frac{3}{7}$

$f(8) = \frac{6}{7}(8) + \frac{66}{7} = 16\frac{2}{7}$

$f(-2) = \frac{6}{7}(-2) + \frac{66}{7} = 7\frac{5}{7}$

4. $7.8 \times 10^{-6} : 2.5 \times 10^{-3}$

$1 : 321$ (to 3 s.f.)

5. $2^{3x} \times 8^{2x-1} \times 16^x = 1$

$2^{3x} \times (2^3)^{2x-1} \times (2^4)^x = 2^0$

$2^{3x + 6x - 3 + 4x} = 2^0$

$\therefore 3x + 6x - 3 + 4x = 0$

$13x - 3 = 0$

$x = \frac{3}{13}$

6. **(a)** $(0.064)^{-\frac{1}{3}} = \frac{1}{\sqrt[3]{0.064}}$

$= \frac{1}{0.4}$

$= 2.5$

(b) $\left(\frac{1}{8}\right)^{-\frac{1}{3}} - \left(\frac{1}{8}\right)^{-\frac{1}{4}} \times \sqrt[4]{2}$

$= (2^{-3})^{-\frac{1}{3}} - (2^{-3})^{-\frac{1}{4}} \times 2^{\frac{1}{4}}$

$= 2^1 - 2^{\frac{3}{4} + \frac{1}{4}}$

$= 2^1 - 2^1$

$= 0$

7. Number of nuclei needed $= \frac{0.5(9.5 \times 10^{-9})}{88 \times 10^{-12}}$

$= 5.4 \times 10$

8. **(a)** $5^{-2} + 5^{-1} + 5^0 + 5^1$

$= \frac{1}{5^2} + \frac{1}{5^1} + 1 + 5$

$= 6\frac{6}{25}$

(b) $2\sqrt{2} \times 3\sqrt{6} \times 4\sqrt{12}$

$= 2\sqrt{2} \times 3\sqrt{6} \times 4\sqrt{2}\sqrt{6}$

$= 2 \times 3 \times 4 \times (\sqrt{2})^2 \times (\sqrt{6})^2$

$= 288$

9. **(a)** $(a + a^{\frac{1}{2}}) \div (a^{\frac{1}{3}} + a^0)$

$= \frac{64 + \sqrt{64}}{\sqrt[3]{64} + 64^0}$

$= \frac{64 + 8}{4 + 1}$

$= 14.4$

(b) $4x + 1 < 28 < 9x - 10$

$4x + 1 < 28$ and $28 < 9x - 10$

$4x < 27$ | $-9x < -38$

$x < \frac{27}{4}$ | $x > \frac{38}{9}$

$x < 6\frac{3}{4}$ | $x > 4\frac{2}{9}$

$\therefore 4\frac{2}{9} < x < 6\frac{3}{4}$

$\therefore$ Possible integer values of x are 5 and 6.

10. (a) $2 < 5x - 7 \leqslant 34$

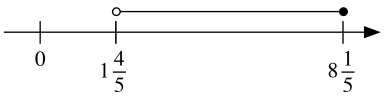
$2 < 5x - 7$ and $5x - 7 \leqslant 34$

$-5x < -9$ $\qquad$ $5x \leqslant 41$

$x > \frac{9}{5}$ $\qquad$ $x \leqslant \frac{41}{5}$

$x > 1\frac{4}{5}$ $\qquad$ $x \leqslant 8\frac{1}{5}$

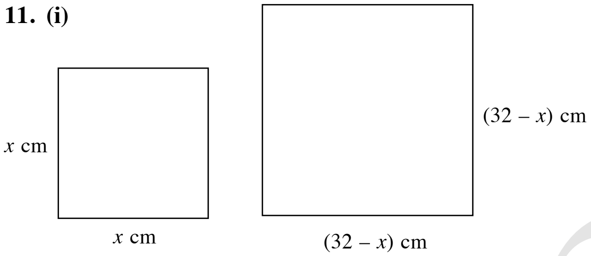
$\therefore 1\frac{4}{5} < x \leqslant 8\frac{1}{5}$



(b) (i) $8\frac{1}{5}$

(ii) 2

11. (i)



Perimeter of the larger square = $128 - 4x$

$\therefore$ Length of larger square $= \frac{128 - 4x}{4}$

$= 32 - x$

Since area of larger square is six times the area of smaller square,

$(32 - x)^2 = 6(x^2)$

$1024 - 64x + x^2 = 6x^2$

$\therefore 5x^2 + 64x - 1024 = 0$ (shown)

(ii) $5x^2 + 64x - 1024 = 0$

$a = 5, b = 64, c = -1024$

$x = \frac{-64 \pm \sqrt{64^2 - 4(5)(-1024)}}{2(5)}$

$= \frac{-64 \pm \sqrt{24\ 576}}{10}$

$= -22.076$ or $x = 9.276$

$= -22.08$ (to 2 d.p.) or $x = 9.28$ (to 2 d.p.)

(iii) As the length cannot be a negative value, $x = -22.08$ is rejected.

(iv) $32 - 9.276 = 22.724$ cm

Difference in the lengths of diagonals

$= \sqrt{2(22.724)^2} - \sqrt{2(9.276)^2}$

$= 19.0$ cm (to 3 s.f.)

Revision Test A2

1. (a) Diameter of a microorganism $= 2 \times (32.6 \times 10^{-6})$ m.

$\therefore$ Number of microorganisms needed

$= \frac{0.75}{65.2 \times 10^{-6}}$

$= 11\ 500$

$= 1.15 \times 10^4$

(b) $(3x - 5)^2 - 57 = 0$

$(3x - 5)^2 = 57$

$3x - 5 = \pm\sqrt{57}$

$x = \frac{5 \pm \sqrt{57}}{3}$

$x = \frac{5 - \sqrt{57}}{3}$ or $\frac{5 + \sqrt{57}}{3}$

$x = -0.85$ (to 2 d.p.) or 4.18 (to 2 d.p.)

2. Number of times $= \frac{148 \times 10^9}{380 \times 10^6} = 389$ (to 3 s.f.)

3. $[3^{-1} - (-1)^0] \times \left(\frac{8}{27}\right)^{-\frac{1}{3}} \div \sqrt{\left(\frac{4}{9}\right)^{-1}}$

$= \left(\frac{1}{3} - 1\right) \times \sqrt[3]{\frac{27}{8}} \div \sqrt{\frac{9}{4}}$

$= -\frac{2}{3} \times \frac{3}{2} \div \frac{3}{2}$

$= -\frac{2}{3}$

4. $\frac{2}{5}x + 1 \leqslant \frac{1}{6}x + 5\frac{1}{2}$

$\frac{2}{5}x - \frac{1}{6}x \leqslant -1 + 5\frac{1}{2}$

$\frac{7}{30}x \leqslant 4\frac{1}{2}$

$x \leqslant 19\frac{2}{7}$

(i) $x = 19$

(ii) $x = 19$

(iii) $x = 19\frac{2}{7}$

5. **(a)** $\sqrt[4]{\left(\dfrac{y^3x^{-6}}{x^2y^5}\right)^{-2}} = \sqrt[4]{(x^{-8}y^{-2})^{-2}}$

$= (x^{16}y^4)^{\frac{1}{4}}$

$= x^4y$

(b) $\dfrac{3}{\sqrt[3]{3}} = 3^x$

$\dfrac{3^1}{3^{\frac{1}{3}}} = 3^x$

$3^{1-\frac{1}{3}} = 3^x$

$\therefore 1 - \dfrac{1}{3} = x$

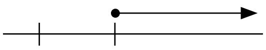
$x = \dfrac{2}{3}$

6. **(a)** $\dfrac{3x-2}{5} \geqslant \dfrac{4x+1}{7}$

$7(3x-2) \geqslant 5(4x+1)$

$21x - 14 \geqslant 20x + 5$

$x \geqslant 19$



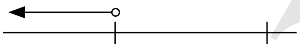
(b) $\dfrac{1}{2}(x+3) - \dfrac{1}{4} < \dfrac{1}{5}(2x-5)$

$\dfrac{1}{2}x + \dfrac{3}{2} - \dfrac{1}{4} < \dfrac{2}{5}x - 1$

$\dfrac{1}{2}x - \dfrac{2}{5}x < -1 + \dfrac{1}{4} - \dfrac{3}{2}$

$\dfrac{1}{10}x < -2\dfrac{1}{4}$

$\therefore x < -22\dfrac{1}{2}$



7. $f(x) = \dfrac{3-x}{x}$

Let $y = \dfrac{3-x}{x}$

$xy = 3 - x$

$xy + x = 3$

$x(y+1) = 3$

$x = \dfrac{3}{y+1}$

$\therefore f^{-1}(x) = \dfrac{3}{x+1}, x \neq -1.$

$f^{-1}(2) = \dfrac{3}{2+1}$

$= 1$

$f^{-1}(-7) = \dfrac{3}{-7+1}$

$= -\dfrac{1}{2}$

8. $h(x) = ax^2 + bx - 5$

$h(3) = a(3)^2 + b(3) - 5$

$9a + 3b - 5 = 30$

$9a + 3b = 35 \quad — (1)$

$h(-2) = a(-2)^2 + b(-2) - 5$

$4a - 2b - 5 = 10$

$4a - 2b = 15 \quad — (2)$

From (1):

$a = \dfrac{35 - 3b}{9} \quad — (3)$

Substitute (3) into (2):

$4\left(\dfrac{35-3b}{9}\right) - 2b = 15$

$4(35 - 3b) - 18b = 135$

$140 - 12b - 18b = 135$

$-30b = -5$

$b = \dfrac{1}{6}$

Substitute $b = \dfrac{1}{6}$ into (3):

$a = \dfrac{35 - 3\left(\frac{1}{6}\right)}{9}$

$= 3\dfrac{5}{6}$

$\therefore a = 3\dfrac{5}{6}, b = \dfrac{1}{6}$

$h(-5) = \dfrac{23}{6}(-5)^2 + \dfrac{1}{6}(-5) - 5$

$= 90$

$h\left(\dfrac{1}{4}\right) = \dfrac{23}{6}\left(\dfrac{1}{4}\right) + \dfrac{1}{6}\left(\dfrac{1}{4}\right) - 5$

$= -4\dfrac{23}{32}$

9. Let x h be the time taken for the larger pipe to fill the pool on its own.

$\therefore$ The smaller pipe will take $(x + 5)$ h to fill the pool on its own.

In 1 h, the larger pipe can fill $\frac{1}{x}$ of the pool.

In 1 h, the smaller pipe can fill $\frac{1}{x+5}$ of the pool.

In 1 h, both pipes can fill $\frac{1}{6}$ of the pool.

$$\frac{1}{x} + \frac{1}{x+5} = \frac{1}{6}$$

$$\frac{(x+5)+x}{x(x+5)} = \frac{1}{6}$$

$$6(2x + 5) = x(x + 5)$$

$$12x + 30 = x^2 + 5x$$

$$x^2 - 7x - 30 = 0$$

$$(x - 10)(x + 3) = 0$$

$x - 10 = 0$ or $x + 3 = 0$

$x = 10$ or $x = -3$ (rejected)

$\therefore$ The larger pipe will take 10 h to fill the pool on its own and the smaller pipe will take 15 h to fill the pool on its own.

10. (i) → Speed = $(x + 4)$ km/h → downstream

← Speed = $(x - 4)$ km/h

Time taken to go downstream = $\frac{18}{x+4}$ h

Time taken to go upstream = $\frac{18}{x-4}$ h

$$\frac{18}{x+4} - \frac{18}{x-4} = 2\frac{1}{6}$$

$$\frac{18(x-4)+18(x+4)}{(x-4)(x+4)} = \frac{13}{6}$$

$$6(18x - 72 + 18x + 72) = 13(x^2 - 16)$$

$$216x = 13x^2 - 208$$

$$13x^2 - 216x - 208 = 0 \text{ (shown)}$$

(ii) $13x^2 - 216x - 208 = 0$

$a = 13$, $b = -216$, $c = -208$

$$x = \frac{-(-216) \pm \sqrt{(-216)^2 - 4(13)(-208)}}{2(13)}$$

$$= \frac{216 \pm \sqrt{57\,472}}{26}$$

$x = \frac{216 - \sqrt{57\,472}}{26}$ or $\frac{216 + \sqrt{57\,472}}{26}$

$= -0.91$ (to 2 d.p.) or 17.53 (to 2 d.p.)

(iii) $x = -0.91$ is rejected because the speed of the boat cannot be negative.

(iv) Difference in time taken $= \frac{18}{17.53 - 4} - \frac{18}{17.53 + 4}$

$= 0.4942$ h

$= 29.7$ min (to 1 d.p.)

Chapter 5 Application of Mathematics in Practical Situations

Basic

1. **(i)** $\frac{\text{Profit}}{\text{Original price}} \times 100\% = 15\%$

$\frac{150}{\text{Original price}} \times 100\% = 15\%$

Original price $= \frac{150 \times 100}{15} = \$1000$

(ii) Profit = selling price – cost price

150 = selling price – 1000

Selling price of watch = 150 + 1000 = $1150

2. **(i)** $\frac{\text{Discount}}{\text{Marked price}} \times 100\% = 8\%$

$\frac{112}{\text{Marked price}} \times 100\% = 8\%$

Marked price $= \frac{112 \times 100}{8} = \$1400$

(ii) Discount = marked price – sale price

$112 = $1400 – sale price

Sale price = $1400 – $112 = $1288

3. Selling price of the iPads = 900 × 15

= $13 500

Increase = $13 500 – $10 000

= $3500

(i) Percentage increase $= \frac{\text{Increase}}{\text{Cost price}} \times 100\%$

$= \frac{3500}{10\,000} \times 100\%$

$= 35\%$

(ii) Percentage increase $= \frac{\text{Increase}}{\text{Selling price}} \times 100\%$

$= \frac{3500}{13\,500} \times 100\%$

$= 25.9\%$ (to 3 s.f.)

4. 107% of the marked price = $27.20

$\frac{107}{100} \times \text{marked price} = 27.20$

Marked price $= 27.20 \div \frac{107}{100}$

$= 27.20 \times \frac{100}{107}$

= $25.42

5. **(a)** Amount of commission he receives

= 15% of $50 000

$= \frac{15}{100} \times 50\,000$

= $7500

(b) Let $x be the signing bonus.

15% of $x = $4800

$\frac{15}{100} \times x = 4800$

$x = 4800 \div \frac{15}{100}$

$= 4800 \times \frac{100}{15}$

$= 32\,000$

The amount of signing bonus is $32 000.

6. **(a)** **(i)** Simple interest = 6% of $700

$= \frac{6}{100} \times 700$

= $42

Simple interest for 5 years = 5 × $42 = $210

(ii) Total amount of money loaned after 5 years

= $700 + $210

= $910

(b) **(i)** Simple interest = 8% of $360

$= \frac{8}{100} \times 360$

= $28.80

Simple interest for 3.5 years = 3.5 × $28.80

= $100.80

(ii) Total amount of money loaned after 3.5 years

= $360 + $100.80

= $460.80

(c) **(i)** Simple interest $= 4\frac{1}{4}\%$ of $480

$= \frac{4\frac{1}{4}}{100} \times 480$

= $20.40

Convert 4 years and 8 months to years.

4 years and 8 months $= 4 + \frac{8}{12}$

$= 4\frac{2}{3}$ years

Simple interest for $4\frac{2}{3}$ years

$= 4\frac{2}{3} \times \$20.40$

= $95.20

(ii) Total amount of money loaned after 4 years 8 months

= $480 + $95.20

= $575.20

(d) (i) Simple interest = $9\frac{3}{8}$% of \$1600

$$= \frac{9\frac{3}{8}}{100} \times 1600$$

$$= \$150$$

Convert 18 months to years.

$$18 \text{ months} = \frac{18}{12}$$

$$= 1\frac{1}{2} \text{ years}$$

Simple interest for $1\frac{1}{2}$ years

$$= 1\frac{1}{2} \times \$150$$

$$= \$225$$

(ii) Total amount of money loaned after 18 months
= \$1600 + \$225
= \$1825

7. Amount of interest given to Ethan
= \$5355 – \$4500
= \$855

Let T years denote the time taken for the investment to grow to \$5355.

$$855 = \frac{4500 \times 4\frac{3}{4} \times T}{100}$$

$$855 = 213.75 \times T$$

$$T = 4$$

The time taken for Ethan's investment to grow to \$5355 is 4 years.

8. **(a)** $A = 2500\left(1 + \frac{3}{100}\right)^2$

$= \$2652.25$

$I = \$2652.25 - \$2500$

$= \$152.25$

(b) $A = 2500\left(1 + \frac{\frac{3}{12}}{100}\right)^{24}$

$= \$2654.39$ (to 2 d.p.)

$I = \$2654.39 - \$2500$

$= \$154.39$ (to 2 d.p.)

9. €1 = £0.62

£1 = $\frac{1}{0.62}$ = €1.61 (to the nearest cent)

£126 = €1.61 × 126 = €203.23 (to the nearest cent)

Intermediate

10. (i) Percentage point is the difference between two percentages.

Percentage point of candidate C and A = 42 – 7
= 35%

Percentage point of candidate B and A = 39 – 7
= 32%

(ii) *To find the total number of voters*,

Method 1

Number of people who did not vote
= 20% of 15 000

$$= \frac{20}{100} \times 15\,000$$

= 3000

Number of people who voted
= 15 000 – 3000
= 12 000

Method 2

Percentage of people who voted
= 100% – 20% = 80%

Number of people who voted
= 80% of 15 000

$$= \frac{80}{100} \times 15\,000$$

= 12 000

To find the number of votes for each candidate

Number of people who voted for candidate A
= 7% of 12 000

$$= \frac{7}{100} \times 12\,000$$

= 840

Number of people who voted for candidate B
= 39% of 12 000

$$= \frac{39}{100} \times 12\,000$$

= 4680

Number of people who voted for candidate C
= 42% of 12 000

$$= \frac{42}{100} \times 12\,000$$

= 5040

11. Subscription + service charge

$= 110\%$ of \$59.90

$= \frac{110}{100} \times 59.90$

= \$65.89

Amount payable before GST

$= 113\%$ of \$65.89

= \$74.4557

Total cost of the bill

$= 107\%$ of \$74.4557

$= \frac{107}{100} \times \$74.4557$

= \$79.67 (to the nearest cent)

12. Original price of the coffee $= 3 \times 9 + 1 \times 13$

$= \$40$

Selling price of the mixture of coffee $= \frac{1.25}{0.1} \times 4$

$= \$50$

Profit $= 50 - 40 = \$10$

Percentage profit $= \frac{10}{50} \times 100\%$

$= 20\%$

13. Original cost of tea $= 30 \times 32 + 20 \times 35$

$= \$1660$

Selling price of tea $= 40 \times (30 + 20)$

$= \$2000$

Profit $= 2000 - 1660 = \$340$

Percentage profit $= \frac{340}{1660} \times 100\%$

$= 20.5\%$ (to 3 s.f.)

14. Price of an item after 8% discount

$= 92\%$ of \$45

$= \frac{92}{100} \times 45$

= \$41.40

Price of an item after a further discount of 9%

$= 91\%$ of \$41.40

$= \frac{91}{100} \times 41.40$

= \$37.67

She paid \$37.67 for the item.

15. 90% of the price which Teck Meng paid for the camera

= \$414

Price Teck Meng paid for the camera $= 414 \div \frac{90}{100}$

$= 414 \times \frac{100}{90}$

$= \$460$

115% of the original price of the camera = \$460

Original price of the camera $= 460 \div \frac{115}{100}$

$= 460 \times \frac{100}{115}$

$= \$400$

The original price of the camera is \$400.

16. Let the number of peaches be x.

Cost price of 1 peach $= \$\frac{294}{x}$

Selling price of 1 peach

$= 140\%$ of $\$\frac{294}{x}$

$= \frac{140}{100} \times \frac{294}{x}$

$= \$\frac{411.6}{x}$

Amount collected from selling all the good peaches

$= 294 + 84 = \$378$

$(x - 16) \times \frac{411.6}{x} = 378$

$411.6(x - 16) = 378x$

$411.6x - 378x = 6585.6$

$33.6x = 6585.6$

$x = 196$

$\therefore$ Mr Ong bought 196 peaches.

17. **(a)** $116\frac{2}{3}\%$ of the marked price = \$420

1% of the marked price $= \frac{420}{116\frac{2}{3}}$

100% of the marked price $= \frac{420}{116\frac{2}{3}} \times 100$

$= \$360$

The price paid by Mr Tan is \$360.

(b) (i) Selling price of the display set

$= \left(100 - 10\frac{1}{2}\right)\%$ of \$420

$= 89\frac{1}{2}\%$ of \$420

$= \frac{179}{2}\%$ of \$420

$= \left(\frac{179}{2} \div 100\right) \times 420$

$= \frac{179}{2} \times \frac{1}{100} \times 420$

= \$375.90

The selling price of the display set is \$375.90.

(ii) Percentage profit $= \frac{375.90 - 360}{360} \times 100\%$

= 4.42% (to 3 s.f.)

18. Amount of commission the salesman got

= 25% of \$5264

$= \frac{25}{100} \times 5264$

= \$1316

Total income

= basic salary + commission

= 520 + 1316

= \$1836

19. Total cost of the materials for building the fence without discount and goods tax

$= 5 \times 25 + 6 \times 12 + 1 \times 10 + 12 \times \frac{15}{6} + 300 \times \frac{10}{1000}$

= 125 + 72 + 10 + 30 + 3

= \$240

Cost of the materials after discount = 90% of 240

$= \frac{90}{100} \times 240$

= \$216

Cost of the materials with goods tax

= 115% of \$216

$= \frac{115}{100} \times 216$

= \$248.40

The total amount that he has to pay, after discount and goods tax, is \$248.40.

20. (i) Total reliefs

= \$1000 + \$2000 + \$5000 + \$4500 + \$6000

= \$18 500

Taxable income = \$56 000 – \$18 500

= \$37 500

(ii)

		Tax
\$37 500 →	First \$30 000	: \$200
→	Next \$7500	: 3.5% of \$7500 = \$262.50

∴ Income tax payable = \$200 + \$262.50

= \$462.50

(iii) Percentage of tax $= \frac{\$462.50}{\$37\ 500} \times 100\%$

= 1.23% (to 3 s.f.)

21. (i) Commission earned for selling the HDB four-room flat

$= \left(\$15\ 000 \times \frac{5}{100}\right) + \left(\$45\ 000 \times \frac{3}{100}\right) + \left(\$40\ 000 \times \frac{2.5}{100}\right) + (\$58\ 500 - \$15\ 000 - \$45\ 000 - \$40\ 000) \times \frac{2}{100}$

= \$12 800

Commission earned for selling a private house

$= \left(\$15\ 000 \times \frac{5}{100}\right) + \left(\$45\ 000 \times \frac{3}{100}\right) + \left(\$40\ 000 \times \frac{2.5}{100}\right) + (\$1\ 085\ 000 - \$15\ 000 - \$45\ 000 - \$40\ 000) \times \frac{2}{100}$

= \$22 800

(ii) Total commission received

= \$22 800 + \$12 800

= \$35 600

22. Let the initial invested amount be \$$P$.

$I = \frac{PRT}{100}$

$25.20 = \frac{P \times 4 \times \frac{9}{12}}{100}$

$25.20 = 0.03P$

$P = 25.2 \div 0.03$

$= 840$

For the new interest rate,

$44.80 + 25.20 = \frac{840 \times x \times \frac{20}{12}}{100}$

$70 = 14x$

$x = 5$

23. $A = 20\ 000\left(1 + \frac{\frac{3.2}{12}}{100}\right)^{48}$

= \$22 727.19 (to 2 d.p.)

24. $A = 6050\left(1 + \frac{\frac{4}{4}}{100}\right)^{8}$

= \$6551 (to the nearest dollar)

25. $28\ 121.60 = 25\ 000\left(1+\frac{r}{100}\right)^3$

$\left(1+\frac{r}{100}\right)^3 = 1.124\ 864$

$1 + \frac{r}{100} = \sqrt[3]{1.124\ 864}$

$\frac{r}{100} = \sqrt[3]{1.124\ 864} - 1$

$r = 4$

26. $P + 11\ 798.38 = P\left(1+\frac{\frac{6}{2}}{100}\right)^6$

$11\ 798.38 = P(1.03)^6 - P$

$= P(1.03^6 - 1)$

$P = \frac{11\ 798.38}{1.03^6 - 1}$

= \$60 800 (to the nearest dollar)

27. (i) Deposit = 25% of \$1300

$= \frac{25}{100} \times \$1300$

= \$325

Remaining amount = \$1300 – \$325

= \$975

Amount of interest the man owes at the end of 1 year

$= \$975 \times \frac{18}{100}$

= \$175.50

Amount of interest the man has to pay at the end of 2 years

= \$175.50 × 2

= \$351

Total amount to be paid in monthly instalments

= \$975 + \$351

= \$1326

Monthly instalment

$= \frac{\$1326}{24}$

= \$55.25

(ii) Total amount the man has to pay for the TV set

= \$325 + \$1326

= \$1651

(iii) Difference in the amount paid with hire purchase

= \$1651 – \$1300

= \$351

28. (i) US\$1.59 = S\$2.02

US\$5400 = $\frac{\text{S\$2.02}}{\text{US\$1.59}} \times$ US\$5400

= S\$6860.377

= S\$6860.38 (to the nearest cent)

(ii) Amount, in Singapore dollars, left after his stay

= S\$6860.377 – S\$4500

= S\$2360.377

= S\$2360.38 (to the nearest cent)

(iii) Amount of pounds he received from the money changer at the end of his stay

= $\frac{£1}{\text{S\$1.94}} \times$ S\$2360.377

= £1216.6892

= £1216.69 (to the nearest cent)

If he exchanged the remaining amount of Singapore dollars using the rate £1 to S\$2.02, then he had

$\frac{£1}{\text{S\$2.02}} \times$ S\$2360.377 = £1168.503.

Difference in the amount exchanged

= £1216.6892 – £1168.503

= £48.1862

= £48.19 (to the nearest cent)

∴ He had £48.19 more based on the new rate.

Advanced

29. (a) Number of packets = $\frac{24\ 000}{4}$

= 6000

Total selling price = 6000 × \$1.20

= \$7200

(b) Costs of labour and materials = \$0.17 × 24 000

= \$4080

Total cost of production

= cost of administration

+ cost of labour and materials

= 1545 + 4080

= \$5625

Profit = 7200 – 5625

= \$1575

Percentage profit made = $\frac{1575}{5625} \times 100\%$

= 28%

(c) Number of packets $= \frac{212\,000}{4}$

$= 53\,000$

Total selling price $= 53\,000 \times \$1.20$

$= \$63\,600$

$123\frac{1}{2}$ % of the cost of production = \$63 600

Cost of production $= 63\,600 \div 123\frac{1}{2}\,\%$

$= 63\,600 \div \frac{247}{2}\,\%$

$= 63\,600 \times \frac{247}{2} \times 100$

$= \$51\,500$ (to 3 s.f.)

The cost of producing 212 000 rubber pieces is \$51 000.

30. (i) Selling price of the condominium

= 90% of \$950 000

$= \frac{90}{100} \times 950\,000$

= \$855 000

(ii) Amount Mei Shan received after paying the agent

= 98% of \$855 000

$= \frac{98}{100} \times 855\,000$

= \$837 900

(iii) Amount agent received from seller

= \$855 000 – \$837 900

= \$17 100

Amount agent received from buyer

= 5% of \$855 000

$= \frac{5}{100} \times 855\,000$

= \$42 750

Total amount received by the agent

= 42 750 + 17 100

= \$59 850

31. (i) Number of litres used $= \frac{\$3600}{\$2.00} = 1800$ litres

(ii) Total distance travelled $= 1800 \times 16$

= 28 800 km

(iii) Total cost in 2011

= \$3600 + \$2000 + \$850 + \$880

= \$7330

(iv) Total cost in 2012

$= \$880 + \left(\$3600 \times \frac{100+5}{100}\right) + \left(\$850 \times \frac{100+15}{100}\right) + \left(\$2000 \times \frac{100-10}{100}\right)$

= \$880 + \$3780 + \$977.50 + \$1800

= \$7437.50

Increase = \$7437.50 – \$7330 = \$107.50

Percentage increase $= \frac{\$107.50}{\$7330} \times 100\%$

= 1.5% (to 2 s.f.)

32. (i) Total cash price

= \$580 + \$380 + \$140 + \$480 + \$240

= \$1820

(ii) (a) Deposit = 20% of \$1820

$= \frac{12}{100} \times \$1820$

= \$364

Remaining amount = \$1820 – \$364

= \$1456

Credit charge = 12% of \$1456

$= \frac{12}{100} \times \$1456$

= \$174.72

Total amount to be paid in instalments

= \$1456 + \$174.72

= \$1630.72

Monthly instalment

$= \frac{\$1630.72}{12}$

= \$135.893

= \$135.89 (to the nearest cent)

(b) Total hire purchase = \$364 + \$1630.72

= \$1994.72

(iii) Total cash price after reduction

$= \left(\$580 \times \frac{100-10}{100}\right) + \left(\$380 \times \frac{100-5}{100}\right) + \left(\$480 \times \frac{100-3}{100}\right) + \$140 + \$240$

= \$522 + \$361 + \$465.60 + \$140 + \$240

= \$1728.60

33. (i) Number of litres of petrol required to drive around France

$= \frac{1920}{12}$

= 160 litres

(ii) Total cost of the petrol used in Euros

= €1.47 × 160

= €235.20

(iii) Total cost of the petrol in Singapore dollars
= €235.20 × S$1.5599
= S$366.888
= S$367 (to the nearest dollar)

(iv) Cost of each adult ferry ticket in Singapore dollars
= £100 × S$1.9399
= S$193.99

New Trend

34. Let the original price of the toy be $\$x$.

$$180\% \text{ of } x = 900$$
$$\frac{180}{100} \times x = 900$$
$$x = 900 \div \frac{180}{100}$$
$$= 900 \times \frac{100}{180}$$
$$= 500$$

The original price of the toy is \$500.

35. $$9004.07 = P\left(1+\frac{3}{100}\right)^4$$
$$P = \frac{9004.07}{(1.03)^4}$$
= \$8000 (to the nearest dollar)

36. **(a)** Amount he is paid for working 45 hours
= 45 × \$11.80
= \$531

Bonus = 4% of \$12 680
$= \frac{4}{100} \times 12\,680$
= \$514.40

Total earnings for the week = \$531 + \$514.40
= \$1045.40

(b) **(i)** Downpayment = $\frac{2}{5} \times \$4999$
= \$1999.60

Total amount paid in monthly instalments
= 24 × \$125.75
= \$3018.00

Total amount Lisa pays for the laptop
= \$1999.60 + \$3018.00
= \$5017.60

(ii) Original price = $\frac{119}{85} \times 100$
= \$140

37. Deposit = 20% of \$1299
$= \frac{20}{100} \times \$1299$
= \$259.80

Let $\$x$ be one monthly payment.
$$1348.80 = 259.80 + 18x$$
$$18x = 1089$$
$$x = 60.5$$

One monthly payment is \$60.50.

38. Extra charge for making monthly payments
= 8% of \$1280
= \$102.40

Monthly payment
$= \frac{\$(1280 + 102.40)}{12}$
= \$115.20

39. If he changes in Singapore,
S\$1 = £0.51
S\$2400 = 2400 × 0.51
= £1224

If he changes in London,
S\$2.02 = £1
S\$2400 = $\frac{2400}{2.02}$
= £1188.1188 (to 4 d.p.)

Difference in the amount exchanged
= 1224 – 1188.1188
= £35.88 (to the nearest cent)

40. **(a)** US\$1 = S\$1.38
US\$500 = S\$(500 × 1.38)
= S\$690

(b) S\$1.38 = US\$1
S\$800 = $\frac{\text{S\$800}}{\text{S\$1.38}}$ × US\$1
= US\$579 $\frac{49}{69}$

US\$1.12 = €1

US\$579 $\frac{49}{69}$ = $\dfrac{\text{US\$579}\frac{49}{69}}{\text{US\$1.12}}$ × €1
= €518 (to the nearest euro)

41. **(a)** $$A = 1668\left(1+\frac{2.6}{100}\right)^3$$
= \$1801.52 (to 2 d.p.)

I = 1801.52 – 1668
= \$133.52

(b) Amount to be paid in euros = $799 + \left(\frac{0.8}{100} \times 799\right)$
= €805.392

€0.65 = S\$1
€805.632 = S\$ $\frac{805.632}{0.65}$
= S\$1239.43 (to the nearest cent)

Chapter 6 Coordinate Geometry

Basic

1. **(a)** Gradient $= \frac{9-3}{7-5}$

$= 3$

(b) Gradient = 0

(c) Gradient $= \frac{-1-5}{1-(-2)}$

$= -2$

(d) Gradient $= \frac{-13-(-4)}{-1-15}$

$= \frac{9}{16}$

(e) Gradient $= \frac{-14-6}{-5-3}$

$= \frac{5}{2}$

(f) Gradient $= \frac{-10-(-3)}{-4-8}$

$= \frac{7}{12}$

2. Gradient of $AB = \frac{4-0}{0-2}$

$= -2$

Gradient of AC = undefined

Gradient of $AD = \frac{3-0}{7-2}$

$= \frac{3}{5}$

Gradient of $BE = \frac{1-4}{6-0}$

$= -\frac{1}{2}$

Gradient of $CD = \frac{3-5}{7-2}$

$= -\frac{2}{5}$

3. $\frac{5-a}{7-4} = 2$

$\frac{5-a}{3} = 2$

$5 - a = 6$

$a = -1$

4. $\frac{5-(-2)}{b-6} = 14$

$\frac{7}{b-6} = 14$

$b - 6 = \frac{1}{2}$

$b = 6\frac{1}{2}$

5. $\frac{4-7}{-k-2k} = -2$

$\frac{-3}{-3k} = -2$

$\frac{1}{k} = -2$

$k = -\frac{1}{2}$

6. **(a)** Length $= \sqrt{(7-2)^2 + (2-4)^2}$

$= \sqrt{29}$

= 5.39 units (to 3 s.f.)

(b) Length $= \sqrt{(3-1)^2 + [5-(-2)]^2}$

$= \sqrt{53}$

= 7.28 units (to 3 s.f.)

(c) Length $= \sqrt{[6-(-2)]^2 + (-1-1)^2}$

$= \sqrt{68}$

= 8.25 units (to 3 s.f.)

(d) Length $= \sqrt{[1-(-2)]^2 + [4-(-3)]^2}$

$= \sqrt{58}$

= 7.62 units (to 3 s.f.)

(e) Length $= 4 - (-7)$

= 11 units

(f) Length $= \sqrt{[7-(-2)]^2 + (-1-3)^2}$

$= \sqrt{97}$

= 9.85 units (to 3 s.f.)

7. $\sqrt{(0-3)^2 + (k-5)^2} = 5$

$9 + (k-5)^2 = 25$

$(k-5)^2 = 16$

$k - 5 = 4$ or $k - 5 = -4$

$k = 9$ $\quad$ $k = 1$

8. $y = -2x + c$

When $x = 2$, $y = 0$,

$0 = -2(2) + c$

$= -4 + c$

$c = 4$

9. $y = 3x + k$

When $x = 2$, $y = -5$,

$-5 = 3(2) + k$

$= 6 + k$

$k = -11$

10. **(a)** Gradient $= \frac{5-3}{1-0}$

$= 2$

Equation of line: $y = 2x + 3$

(b) Gradient $= \dfrac{3-(-3)}{5-0}$

$= \dfrac{6}{5}$

Equation of line: $y = \dfrac{6}{5}x - 3$

(c) Equation of line: $y = 1$

(d) Equation of line: $x = 5$

(e) Gradient $= \dfrac{3-(-2)}{-5-(-3)}$

$= -\dfrac{5}{2}$

Substitute $x = -5$, $y = 3$ and $m = -\dfrac{5}{2}$ into $y = mx + c$:

$3 = -\dfrac{5}{2}(-5) + c$

$= \dfrac{25}{2} + c$

$c = -\dfrac{19}{2}$

Equation of line: $y = -\dfrac{5}{2}x - \dfrac{19}{2}$

(f) Gradient $= \dfrac{7-0}{0-6}$

$= -\dfrac{7}{6}$

Equation of line: $y = -\dfrac{7}{6}x + 7$

11. (a) Substitute $x = 5$, $y = 4$ and $m = 2$ into $y = mx + c$:

$4 = 2(5) + c$

$= 10 + c$

$c = -6$

Equation of line: $y = 2x - 6$

(b) Substitute $x = -1$, $y = 3$ and $m = \dfrac{1}{2}$ into $y = mx + c$:

$3 = \dfrac{1}{2}(-1) + c$

$= -\dfrac{1}{2} + c$

$c = \dfrac{7}{2}$

Equation of line: $y = \dfrac{1}{2}x + \dfrac{7}{2}$

(c) Equation of line: $y = -5x$

(d) Substitute $x = 7$, $y = 6$ and $m = -\dfrac{1}{3}$ into $y = mx + c$:

$6 = -\dfrac{1}{3}(7) + c$

$= -\dfrac{7}{3} + c$

$c = \dfrac{25}{3}$

Equation of line: $y = -\dfrac{1}{3}x + \dfrac{25}{3}$

(e) Equation of line: $y = 9$

(f) Equation of line: $y = 4x + 3$

12. (a) Gradient $= \dfrac{1-0}{0-(-1)}$

$= 1$

Equation of line: $y = x + 1$

(b) Gradient $= \dfrac{0-2}{2-0}$

$= -1$

Equation of line: $y = -x + 2$

(c) Equation of line: $y = 2$

(d) Equation of line: $x = 1$

(e) Gradient $= \dfrac{0-(-1)}{2-0}$

$= \dfrac{1}{2}$

Equation of line: $y = \dfrac{1}{2}x - 1$

(f) Gradient $= \dfrac{0-1\frac{1}{2}}{2-0}$

$= -\dfrac{3}{4}$

Equation of line: $y = -\dfrac{3}{4}x + \dfrac{3}{2}$

13. (a) Gradient of line $= 9$

(b) Gradient of line $= -\dfrac{4}{5}$

14. (a) Gradient of line $= -1 \div 3$

$= -\dfrac{1}{3}$

(b) Gradient of line $= -1 \div -\dfrac{2}{9}$

$= 4\dfrac{1}{2}$

Intermediate

15. Gradient of AB = Gradient of AC

$$\frac{4-1}{5-2} = \frac{h-1}{7-2}$$
$$1 = \frac{h-1}{5}$$
$$h - 1 = 5$$
$$h = 6$$

16. $$\frac{3-(-5)}{0-2} = \frac{k-3}{-3-0}$$
$$-4 = \frac{k-3}{-3}$$
$$k - 3 = 12$$
$$k = 15$$

17. Consider $mx = 5y + 4$.

$5y = mx - 4$

$y = \frac{m}{5}x - \frac{4}{5}$

Consider $7x + 6y + 5 = 0$.

$6y = -7x - 5$

$y = -\frac{7}{6}x - \frac{5}{6}$

Since the gradients are the same,

$\frac{m}{5} = -\frac{7}{6}$

$m = -\frac{35}{6}$

$= -5\frac{5}{6}$

18. **(a)** $\frac{x}{3} + \frac{y}{5} = 1$

$\frac{y}{5} = -\frac{x}{3} + 1$

$y = -\frac{5}{3}x + 5$

$\therefore$ Gradient $= -\frac{5}{3}$

(b) $\frac{x}{4} - \frac{y}{3} = 1$

$\frac{y}{3} = \frac{x}{4} - 1$

$y = \frac{3}{4}x - 3$

$\therefore$ Gradient $= \frac{3}{4}$

(c) $\frac{2x}{3} - \frac{4y}{5} = 1$

$\frac{4y}{5} = \frac{2x}{3} - 1$

$y = \frac{5}{6}x - \frac{5}{4}$

$\therefore$ Gradient $= \frac{5}{6}$

(d) $\frac{3x}{5} + \frac{y}{2} = 1$

$\frac{y}{2} = -\frac{3x}{5} + 1$

$y = -\frac{6}{5}x + 2$

$\therefore$ Gradient $= -\frac{6}{5}$

(e) $\frac{x}{7} - \frac{y}{11} = 1$

$\frac{y}{11} = \frac{x}{7} - 1$

$y = \frac{11}{7}x - 11$

$\therefore$ Gradient $= \frac{11}{7}$

(f) $\frac{y}{2} - \frac{x}{5} = 1$

$\frac{y}{2} = \frac{x}{5} + 1$

$y = \frac{2}{5}x + 2$

$\therefore$ Gradient $= \frac{2}{5}$

19. $2y = kx + h$

When $x = -3$, $y = 6$,

$2(6) = k(-3) + h$

$12 = -3k + h$ — (1)

When $x = 1$, $y = 11$,

$2(11) = k(1) + h$

$22 = k + h$ — (2)

(2) – (1): $4k = 10$

$k = 2.5$

$h = 19.5$

$\therefore k = 2.5, h = 19.5$

20. **(i)** $5x + 7y = 35$

When $y = 0$,

$5x = 35$

$x = 7$

$\therefore$ Coordinates of H are $(7, 0)$.

When $x = 0$,

$7y = 35$

$y = 5$

$\therefore$ Coordinates of K are $(0, 5)$.

(ii) $HK = \sqrt{(0-7)^2 + (5-0)^2}$

$= \sqrt{74}$

$= 8.60$ units (to 3 s.f.)

21. $\frac{x}{a} + \frac{y}{b} = 1$

When $x = 0$, $y = 3$,

$\frac{3}{b} = 1$

$b = 3$

When $x = 5$, $y = 1$,

$\frac{5}{a} + \frac{1}{3} = 1$

$\frac{5}{a} = \frac{2}{3}$

$a = 7\frac{1}{2}$

$\therefore a = 7\frac{1}{2}, b = 3$

$\frac{x}{a} + \frac{y}{b} = 1$

$\frac{y}{b} = -\frac{x}{a} + 1$

$y = -\frac{b}{a}x + b$

Gradient $= -\frac{b}{a}$

$= -\frac{3}{7\frac{1}{2}}$

$= -\frac{2}{5}$

22. Consider $2y = kx + 6c$.

$y = \frac{k}{2}x + 3c$

Consider $5x + 4y = 7$.

$4y = -5x + 7$

$y = -\frac{5}{4}x + \frac{7}{4}$

Since the gradients are the same,

$\frac{k}{2} = -\frac{5}{4}$

$k = -\frac{5}{2}$

$= -2\frac{1}{2}$

When $x = 1$, $y = 8$,

$2(8) = -2\frac{1}{2}(1) + 6c$

$16 = -2\frac{1}{2} + 6c$

$6c = 18\frac{1}{2}$

$c = 3\frac{1}{12}$

$\therefore k = -2\frac{1}{2}, c = 3\frac{1}{12}$

23. **(i)** Area of $\triangle ABC = \frac{1}{2}(7)(7)$

$= 24.5 \text{ units}^2$

(ii) $BC = \sqrt{(7-9)^2 + (11-4)^2}$

$= \sqrt{53}$

$= 7.28$ units (to 3 s.f.)

(iii) Let the length of the perpendicular from A to BC be h.

Area of $\triangle ABC = \frac{1}{2} \times BC \times h$

$24.5 = \frac{1}{2} \times \sqrt{53} \times h$

$h = \frac{2 \times 24.5}{\sqrt{53}}$

$= 6.73$ units (to 3 s.f.)

24. **(i)** $AB = \sqrt{[3-(-2)]^2 + (1-6)^2}$

$= \sqrt{50}$

$= 7.071$ units (to 4 s.f.)

$BC = \sqrt{(8-3)^2 + (6-1)^2}$

$= \sqrt{50}$

$= 7.071$ units (to 4 s.f.)

$AC = 10$ units

Perimeter of $\triangle ABC = AB + BC + AC$

$= \sqrt{50} + \sqrt{50} + 10$

$= 24.1$ units (to 3 s.f.)

Area of $\triangle ABC = \frac{1}{2}(10)(5)$

$= 25 \text{ units}^2$

(ii) Since $AB = BC$, $\triangle ABC$ is an isosceles triangle.

(iii) By symmetry, $D(3, 11)$.

25. **(i)** $AB = \sqrt{(4-2)^2 + (2-6)^2}$

$= \sqrt{20}$

$= 4.47$ units (to 3 s.f.)

$BC = \sqrt{(12-4)^2 + (6-2)^2}$

$= \sqrt{80}$

$= 8.94$ units (to 3 s.f.)

$AC = 10$ units

(ii) $AC^2 = 10^2$

$= 100$

$AB^2 + BC^2 = (\sqrt{20})^2 + (\sqrt{80})^2$

$= 100$

Since $AC^2 = AB^2 + BC^2$, $\triangle ABC$ is a right-angled triangle.

26. Gradient of AB = Gradient of AC

$$\frac{-2-8}{-5-(-3)} = \frac{13-8}{k-(-3)}$$
$$5 = \frac{5}{k+3}$$
$$k+3 = 1$$
$$k = -2$$

Substitute $x = -2$, $y = 13$ and $m = 5$ into $y = mx + c$:

$13 = 5(-2) + c$
$= -10 + c$
$c = 23$

Equation of line: $y = 5x + 23$

27. Gradient $= \frac{-6-3}{4-(-4)}$
$= -\frac{9}{8}$

Substitute $x = -4$, $y = 3$ and $m = -\frac{9}{8}$ into $y = mx + c$:

$3 = -\frac{9}{8}(-4) + c$
$= \frac{9}{2} + c$
$c = -\frac{3}{2}$

Equation of line: $y = -\frac{9}{8}x - \frac{3}{2}$

28. **(i)** Gradient of $AB = \frac{8-0}{7-5}$
$= 4$

(ii) Substitute $x = 5$, $y = 0$ and $m = 4$ into $y = mx + c$:
$0 = 4(5) + c$
$= 20 + c$
$c = -20$
Equation of AB: $y = 4x - 20$

(iii) Area of $\triangle ABC = \frac{1}{2}(5)(8)$
$= 20 \text{ units}^2$

29. Consider $(k+2)x + 5 = 3y$.

$y = \frac{k+2}{3}x + \frac{5}{3}$

Consider $(k+3)y = 2x - 6$.

$y = \frac{2}{k+3}x - \frac{6}{k+3}$

Since the gradients are the same,

$$\frac{k+2}{3} = \frac{2}{k+3}$$
$$(k+2)(k+3) = 6$$
$$k^2 + 5k + 6 = 6$$
$$k^2 + 5k = 0$$
$$k(k+5) = 0$$

$k = 0$ or $k = -5$

30. $\sqrt{[2-(2-k)]^2 + (k-1)^2} = \sqrt{5-4k}$
$$k^2 + (k-1)^2 = 5 - 4k$$
$$k^2 + k^2 - 2k + 1 = 5 - 4k$$
$$2k^2 + 2k - 4 = 0$$
$$k^2 + k - 2 = 0$$
$$(k+2)(k-1) = 0$$

$k = -2$ or $k = 1$

31. **(i)** Gradient of $AB = \frac{3-1}{4-(-1)}$
$= \frac{2}{5}$

Substitute $x = 4$, $y = 3$ and $m = \frac{2}{5}$ into $y = mx + c$:

$3 = \frac{2}{5}(4) + c$
$= \frac{8}{5} + c$
$c = \frac{7}{5}$

Equation of AB: $y = \frac{2}{5}x + \frac{7}{5}$

Equation of BC: $x = 4$

Gradient of $AC = \frac{-2-1}{4-(-1)}$
$= -\frac{3}{5}$

Substitute $x = -1$, $y = 1$ and $m = -\frac{3}{5}$ into $y = mx + c$:

$1 = -\frac{3}{5}(-1) + c$
$= \frac{3}{5} + c$
$c = \frac{2}{5}$

Equation of AC: $y = -\frac{3}{5}x + \frac{2}{5}$

(ii) Area of $\triangle ABC = \frac{1}{2}(5)(5)$
$= 12\frac{1}{2} \text{ units}^2$

(iii) $\frac{1}{2} \times AE \times 5 = 10$
$AE = 4$
$\therefore k = 5$ or $k = -3$

32. (i) Gradient $= \dfrac{9-0}{0-7}$

$= -\dfrac{9}{7}$

Equation of AB: $y = -\dfrac{9}{7}x + 9$

(ii) Gradient $= \dfrac{4\frac{1}{2}-0}{3\frac{1}{2}-0}$

$= \dfrac{9}{7}$

Equation of OC: $y = \dfrac{9}{7}x$

(iii) Equation of line: $y = 4\dfrac{1}{2}$

(iv) Equation of line: $x = 3\dfrac{1}{2}$

33. (a) Equation of required line: $y = 7x + c$

Since the line passes through (1, 9),

$9 = 7(1) + c$

$c = 2$

$\therefore$ Equation of line is $y = 7x + 2$

(b) Gradient of required line $= -1 \div 6$

$= -\dfrac{1}{6}$

Equation of required line: $y = -\dfrac{1}{6}x + c$

Since the line passes through (2, 6),

$6 = -\dfrac{1}{6}(2) + c$

$c = \dfrac{19}{3}$

$\therefore$ Equation of line is $y = -\dfrac{1}{6}x + \dfrac{19}{3}$

(c) Equation of required line: $y = -4x + c$

Since the line passes through (3, 1),

$1 = -4(3) + c$

$c = 13$

$\therefore$ Equation of line is $y = -4x + 13$

(d) Gradient of required line $= -1 \div -5$

$= \dfrac{1}{5}$

Equation of required line: $y = \dfrac{1}{5}x + c$

Since the line passes through (–2, –4),

$-4 = \dfrac{1}{5}(-2) + c$

$c = -\dfrac{18}{5}$

$\therefore$ Equation of line is $y = \dfrac{1}{5}x - \dfrac{18}{5}$

34. $3y - x = 19$

$y = \dfrac{1}{3}x + \dfrac{19}{3}$

Gradient of $PQ = (-1) \div \left(\dfrac{1}{3}\right)$

$= -3$

Equation of PQ: $y = -3x + c$

Since (2, –3) lies on PQ,

$-3 = -3(2) + c$

$c = 3$

$\therefore$ Equation of PQ: $y = -3x + 3$

35. (i) Gradient of $AB = \dfrac{5-(-3)}{2-1}$

$= 8$

(ii) Equation of line: $y = 8x + c$

Since $C(9, 9)$ lies on the line,

$9 = 8(9) + c$

$c = -63$

$\therefore$ Equation: $y = 8x - 63$

36. (i) Gradient of $BC = \dfrac{8-(-4)}{3-(-3)}$

$= 2$

Equation of BC: $y = 2x + c$

Since $B(-3, -4)$ lies on BC,

$-4 = 2(-3) + c$

$c = 2$

$\therefore$ Equation of BC: $y = 2x + 2$

(ii) Gradient of $AD = (-1) \div 2$

$= -\dfrac{1}{2}$

Equation of AD: $y = -\dfrac{1}{2}x + c$

Since $A(7, -2)$ lies on AD,

$-2 = -\dfrac{1}{2}(7) + c$

$c = \dfrac{3}{2}$

$\therefore$ Equation of AD: $y = -\dfrac{1}{2}x + \dfrac{3}{2}$

(iii) Length of $AB = \sqrt{(-3-7)^2 + [-4-(-2)]^2}$

$= \sqrt{104}$

$= 2\sqrt{26}$ units

Length of $BC = \sqrt{[3-(-3)]^2 + [8-(-4)]^2}$

$= \sqrt{180}$

$= 6\sqrt{5}$ units

37. **(a)** $3y + x = 25$

$$y = -\frac{1}{3}x + \frac{25}{3}$$

Gradient of AD = Gradient of BC

$$= -\frac{1}{3}$$

Equation of AD: $y = -\frac{1}{3}x + c$

Since $A(-1, 2)$ lies on AD,

$$2 = -\frac{1}{3}(-1) + c$$

$$c = \frac{5}{3}$$

$\therefore$ Equation of AD: $y = -\frac{1}{3}x + \frac{5}{3}$

$$3y = 5 - x$$

(b) Gradient of $AB = (-1) \div \left(-\frac{1}{3}\right)$

$$= 3$$

Equation of AB: $y = 3x + c$

Since $A(-1, 2)$ lies on AB

$2 = 3(-1) + c$

$c = 5$

$\therefore$ Equation of AB: $y = 3x + 5$

Advanced

38. **(i)** Gradient of $AB = \frac{12 - 3}{7 - (-2)}$

$$= 1$$

Gradient of $BC = \frac{-5 - 12}{11 - 7}$

$$= -\frac{17}{4}$$

Gradient of $CD = \frac{-14 - (-5)}{2 - 11}$

$$= 1$$

Gradient of $DA = \frac{3 - (-14)}{-2 - 2}$

$$= -\frac{17}{4}$$

They are equal.

(ii) Substitute $x = -2$, $y = 3$ and $m = -\frac{17}{4}$ into $y = mx + c$:

$$3 = -\frac{17}{4}(-2) + c$$

$$= \frac{17}{2} + c$$

$$c = -\frac{11}{2}$$

Equation of AD: $y = -\frac{17}{4}x - \frac{11}{2}$

Substitute $x = 11$, $y = -5$ and $m = 1$ into $y = mx + c$:

$-5 = 1(11) + c$

$= 11 + c$

$c = -16$

Equation of CD: $y = x - 16$

(iii) $BD = \sqrt{(2 - 7)^2 + (-14 - 12)^2}$

$= \sqrt{701}$

$= 26.5$ units (to 3 s.f.)

39. $x + y = 0$ — (1)

$x = 0$ — (2)

$y = x - 1$ — (3)

Substitute (2) into (1):

$y = 0$

$\therefore$ Coordinates of A are $(0, 0)$.

Substitute (2) into (3):

$y = -1$

$\therefore$ Coordinates of B are $(0, -1)$.

Substitute (3) into (1):

$x + x - 1 = 0$

$2x = 1$

$$x = \frac{1}{2}$$

$$y = -\frac{1}{2}$$

$\therefore$ Coordinates of C are $\left(\frac{1}{2}, -\frac{1}{2}\right)$.

Area of $\triangle ABC = \frac{1}{2}(1)\left(\frac{1}{2}\right)$

$$= \frac{1}{4} \text{ units}^2$$

40. **(i)** Gradient $= \frac{3 - 0}{0 - (-1)}$

$$= 3$$

Equation of AB: $y = 3x + 3$

(ii) $\sqrt{[0 - (-1)]^2 + (3 - 0)^2} = \sqrt{h}$

$1^2 + 3^2 = h$

$h = 10$

(iii) By symmetry, $C(2, 1)$.

(iv) Let X be the point where BC intersects $y = x + 1$, i.e. $X(1, 2)$

$AX = \sqrt{[1 - (-1)]^2 + (2 - 0)^2}$

$= \sqrt{8}$ units

$BC = \sqrt{(2 - 0)^2 + (1 - 3)^2}$

$= \sqrt{8}$ units

Area of $\triangle ABC = \frac{1}{2} \times \sqrt{8} \times \sqrt{8}$

$= 4 \text{ units}^2$

41. (i) Equation of QR: $y = 3x + c$

Since $R(4, 6)$ lies on QR,

$6 = 3(4) + c$

$c = -6$

$\therefore$ Equation of QR: $y = 3x - 6$

(ii) $y = 3x - 6$

At x-axis, $y = 0$,

$3x - 6 = 0$

$x = 2$

$\therefore q = 2$

(iii) Gradient of $PQ \times$ Gradient of $PR = -1$

$$\frac{0-p}{2-0} \times \frac{6-p}{4-0} = -1$$

$$-\frac{p}{2} \times \frac{6-p}{4} = -1$$

$$p(6-p) = 8$$

$$p^2 - 6p + 8 = 0$$

$$(p-2)(p-4) = 0$$

$\therefore p = 2$ or $p = 4$

42. (i) If $\triangle ABC$ has a right angle at B,

Gradient of $AB \times$ Gradient of $BC = -1$

$$\frac{7-(-2)}{2-(-4)} \times \frac{k-7}{20-2} = -1$$

$$\frac{3}{2}\left(\frac{k-7}{18}\right) = -1$$

$$k - 7 = -12$$

$\therefore k = -5$

(ii) Length of $BC = \sqrt{(20-2)^2 + (-5-7)^2}$

$= \sqrt{468}$

Length of $AB = \sqrt{[2-(-4)]^2 + [7-(-2)]^2}$

$= \sqrt{117}$

$$\frac{BC}{AB} = \frac{\sqrt{468}}{\sqrt{117}}$$

$$= \sqrt{\frac{468}{117}}$$

$$= \sqrt{4}$$

$$= 2$$

$\therefore BC = 2AB$ (shown)

(iii) Gradient of $AB = \frac{3}{2}$

Equation of AB: $y = \frac{3}{2}x + c$

Since $A(-4, -2)$ lies on AB,

$-2 = \frac{3}{2}(-4) + c$

$c = 4$

$\therefore$ Equation of AB: $y = \frac{3}{2}x + 4$

$2y = 3x + 8$

Gradient of $BC = -\frac{2}{3}$

Equation of BC: $y = -\frac{2}{3}x + c$

Since $B(2, 7)$ lies on BC,

$7 = -\frac{2}{3}(2) + c$

$c = \frac{25}{3}$

$\therefore$ Equation of BC: $y = -\frac{2}{3}x + \frac{25}{3}$

$3y = 25 - 2x$

(iv) Line AB: $2y = 3x + 8$

At y-axis, $x = 0$,

$2y = 3(0) + 8$

$y = 4$

Line BC: $3y = 25 - 2x$

At x-axis, $y = 0$,

$3(0) = 25 - 2x$

$x = 12.5$

$\therefore P(0, 4)$, $Q(12.5, 0)$

(v) Gradient of $AP = \frac{4-(-2)}{0-(-4)}$

$= \frac{3}{2}$

Gradient of $PQ = \frac{0-4}{12.5-0}$

$= -\frac{8}{25}$

Gradient of $AP \times$ Gradient of PQ

$= \frac{3}{2} \times \left(-\frac{8}{25}\right)$

$= -\frac{12}{25} \neq -1$

$\therefore \angle APQ$ is not a right angle.

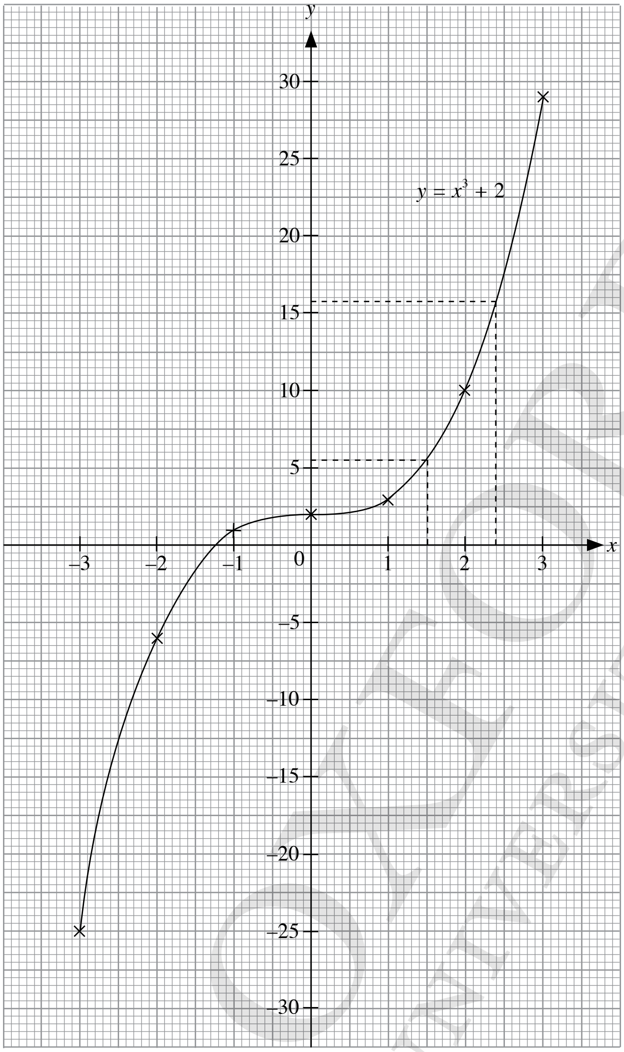
Chapter 7 Graphs of Functions and Graphical Solution

Basic

1. **(a)**

x	–3	–2	–1	0	1	2	3
y	–25	–6	1	2	3	10	29

(b)



(c) **(i)** When $x = 1.5$, $y = 5.5$.

(ii) When $y = 16$, $x = 2.4$.

2. **(a)** When $x = -3$, $y = -2.7$.

$\therefore a = -2.7$

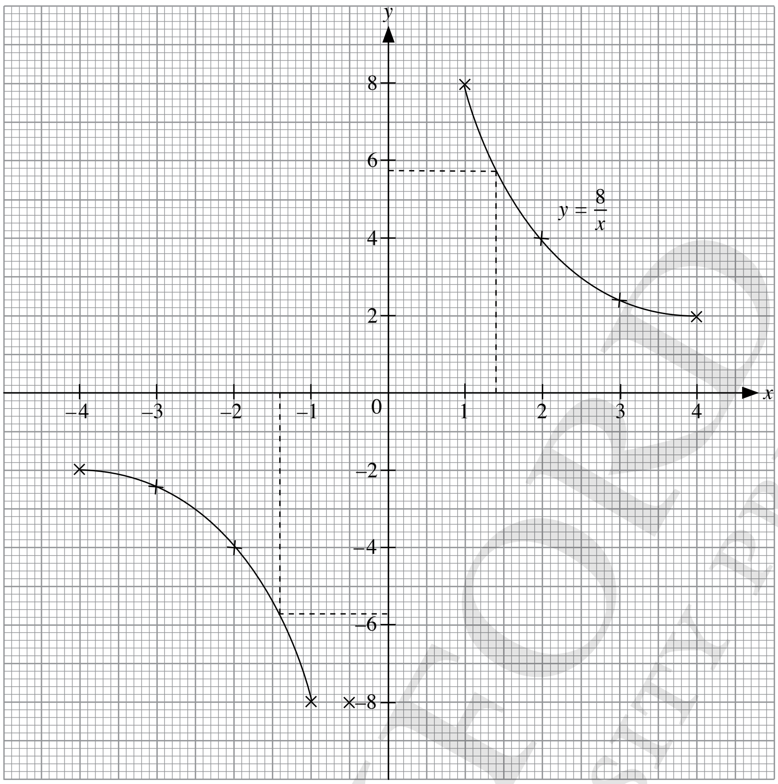
When $x = 2$, $y = 4$

$\therefore b = 4$

When $x = 4$, $y = 2$.

$\therefore c = 2$

(b)



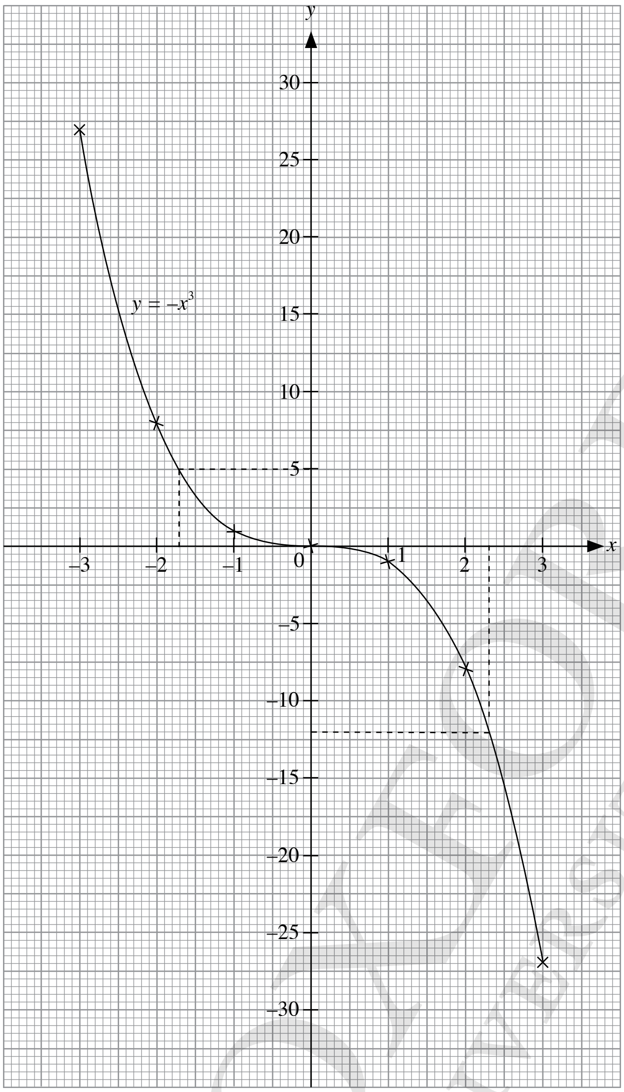
(c) **(i)** When $x = 1.4$, $y = 5.7$.

(ii) When $y = -5.7$, $x = -1.4$.

3. **(a)**

x	–3	–2	–1	0	1	2	3
y	27	8	1	0	–1	–8	–27

(b)



(c) **(i)** When $x = -1.7$, $y = 5$.

(ii) When $y = -12$, $x = 2.3$.

4. **(a)** When $x = -2$,

$$y = -\frac{5}{(-2)}$$
$$= 2.5$$

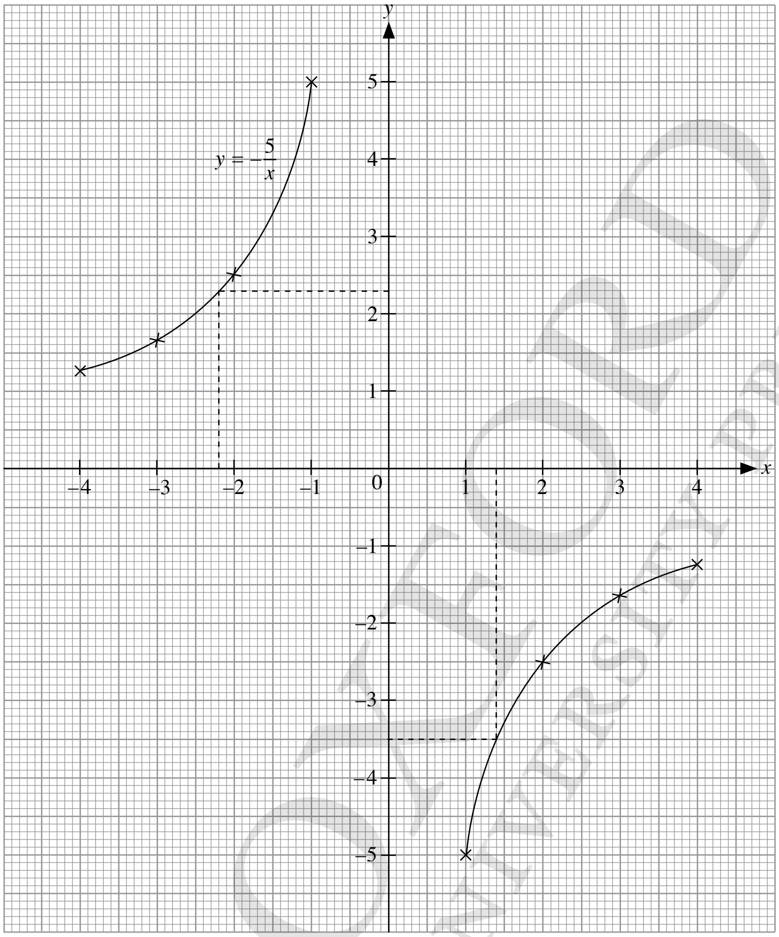
$\therefore a = 2.5$

When $x = 3$,

$$y = -\frac{5}{3}$$
$$= -1.67 \text{ (to 3 s.f.)}$$

$\therefore b = -1.67$

(b)



(c) **(i)** When $x = -2.2$, $y = 2.3$.

(ii) When $y = -3.5$, $x = 1.4$.

5. (a) When $x = -3$,

$$y = \frac{4}{(-3)^2}$$
$$= 0.44 \text{ (to 2 d.p.)}$$

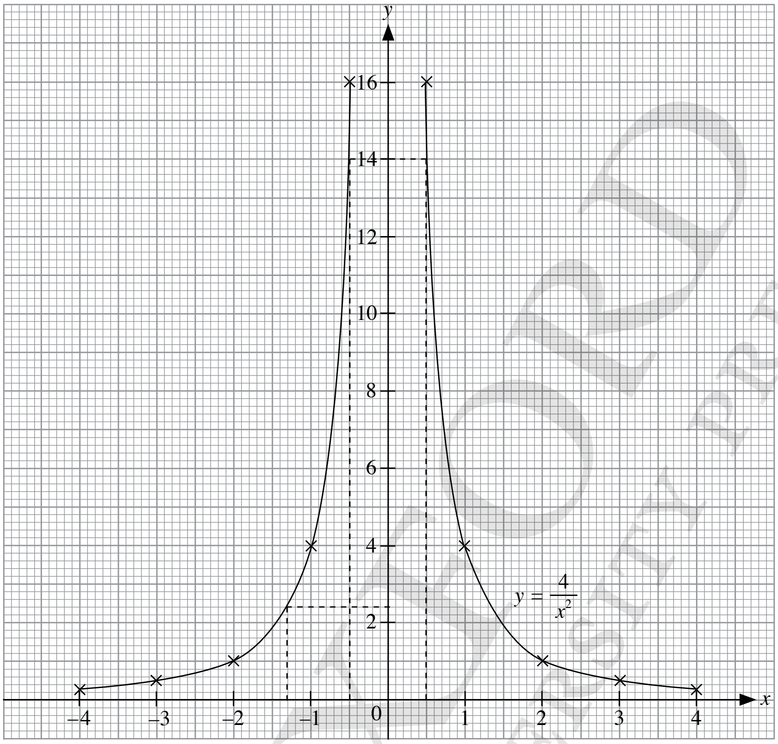
$\therefore a = 0.44$

When $x = 3$,

$$y = \frac{4}{3^2}$$
$$= 0.44 \text{ (to 2 d.p.)}$$

$\therefore b = 0.44$

(b)



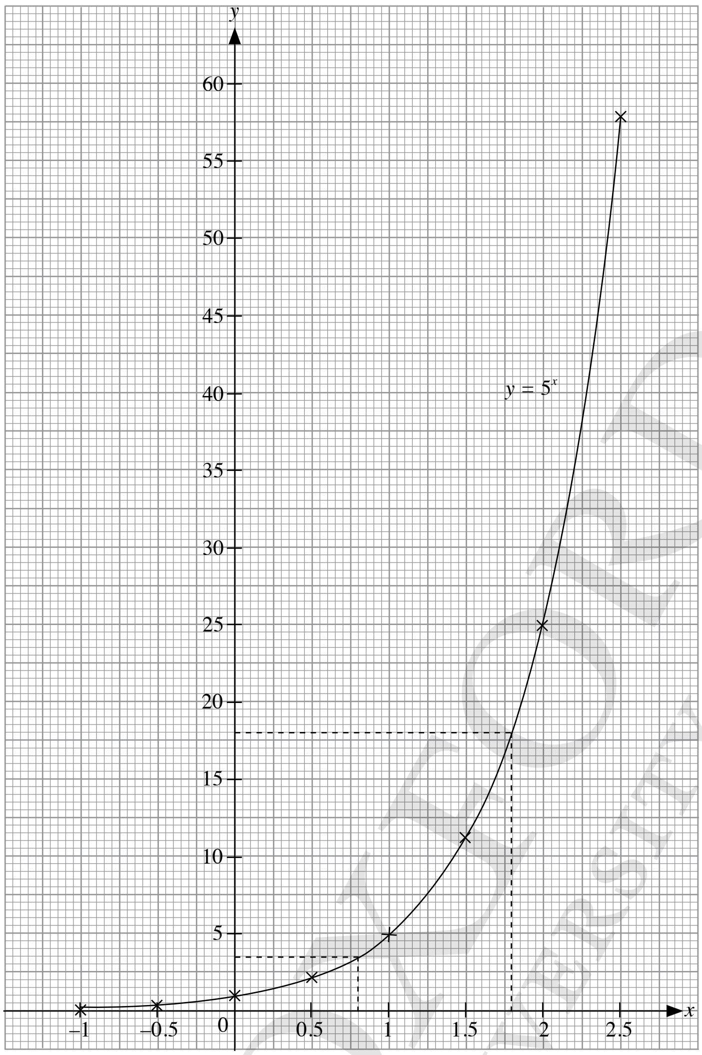
(c) (i) When $x = -1.3$, $y = 2.4$.

(ii) When $y = 14$, $x = 0.5$ or $x = -0.5$.

6. (a)

x	–1	–0.5	0	0.5	1	1.5	2	2.5
y	0.2	0.45	1	2.24	5	11.18	25	55.90

(b)



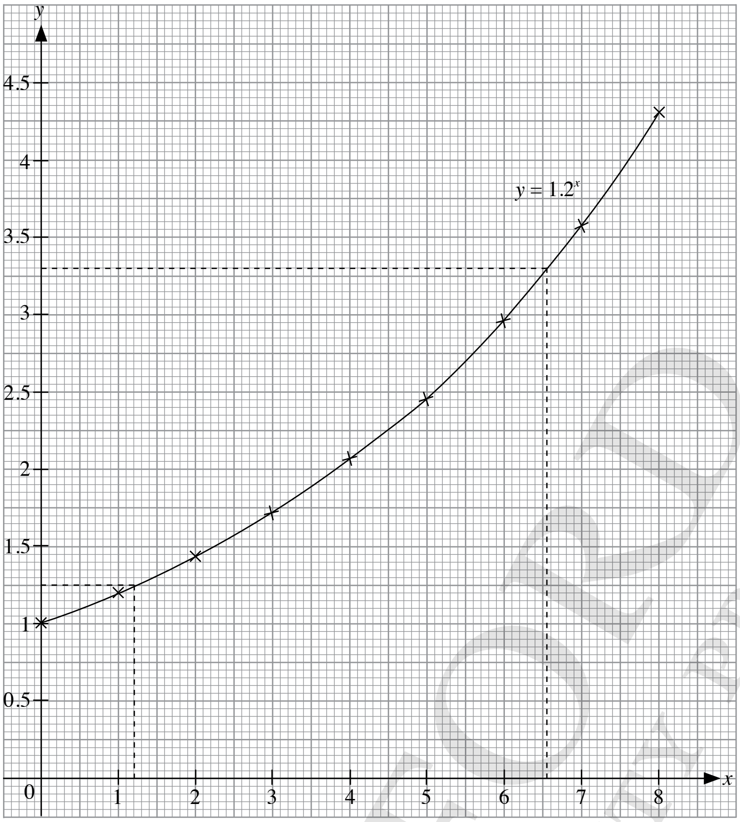
(c) (i) When $x = 0.8$, $y = 3.5$.

(ii) When $y = 18$, $x = 1.8$.

7. (a)

x	0	1	2	3	4	5	6	7	8
y	1.00	1.20	1.44	1.73	2.07	2.49	2.99	3.58	4.30

(b)



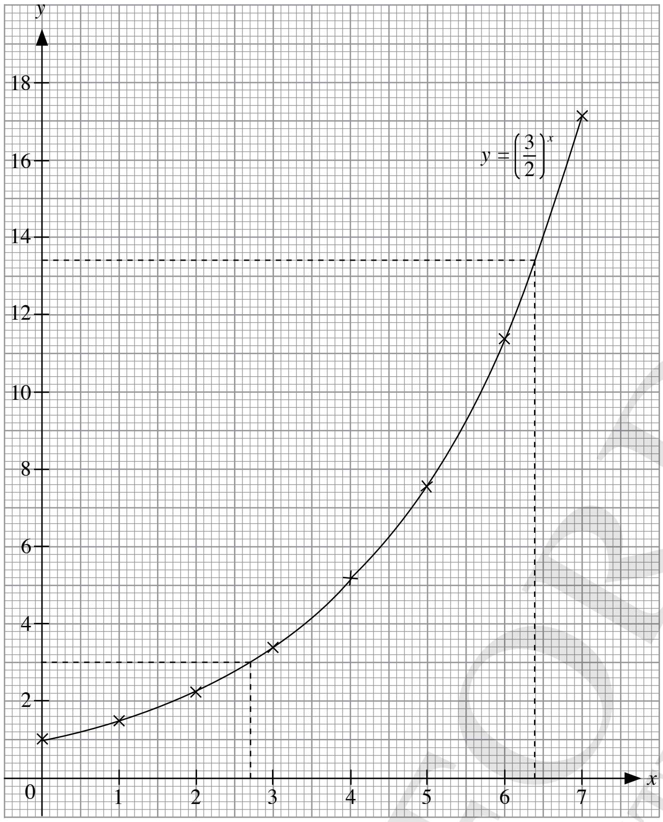
(c) (i) When $x = 1.2$, $y = 1.25$.

(ii) When $y = 3.3$, $x = 6.55$.

8. (a)

x	0	1	2	3	4	5	6	7
y	1.00	1.50	2.25	3.37	5.06	7.59	11.39	17.09

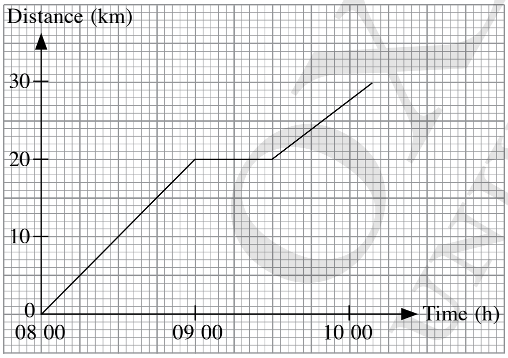
(b)



(c) (i) When $x = 6.4$, $y = 13.4$.

(ii) When $y = 3$, $x = 2.7$.

9. (i)



(ii) 10 10

10. (i) Duration = 5 – 3.4

= 1.6 h

(ii) Average speed in the first 2 hours = $\frac{10}{2}$

= 5 km/h

(iii) Average speed for the whole journey = $\frac{60}{6}$

= 10 km/h

11. (i) Acceleration in the last 2 seconds $= \dfrac{0-12}{2-0}$

$= -6 \text{ m/s}^2$

(ii) Average speed for the whole journey $= \dfrac{54}{6}$

$= 9 \text{ m/s}$

12. (i) It represents the acceleration of the car.

(ii) When $t = 10$,

Speed $= \dfrac{30-12}{20-0}$

$= 0.9 \text{ m/s}$

(iii) Average speed for the whole journey $= \dfrac{680}{50}$

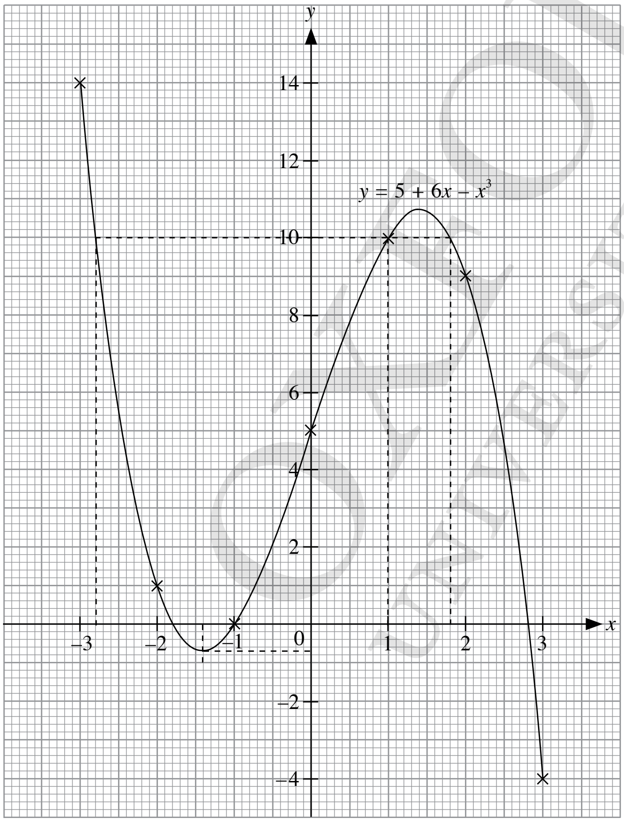
$= 13.6 \text{ m/s}$

Intermediate

13. (a)

x	–3	–2	–1	0	1	2	3
y	14	1	0	5	10	9	–4

(b)

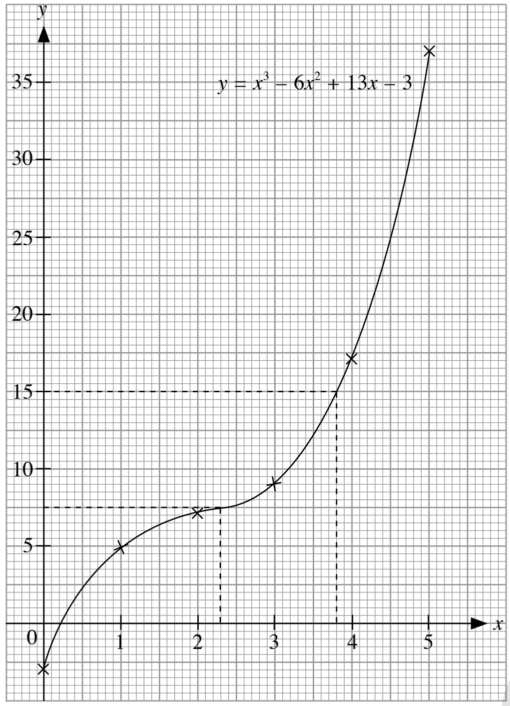


(c) (i) When $x = -1.4$, $y = -0.7$.

(ii) When $y = 10$, $x = -2.8$ or $x = 1$ or $x = 1.8$.

14.

x	0	1	2	3	4	5
y	–3	5	7	9	17	37



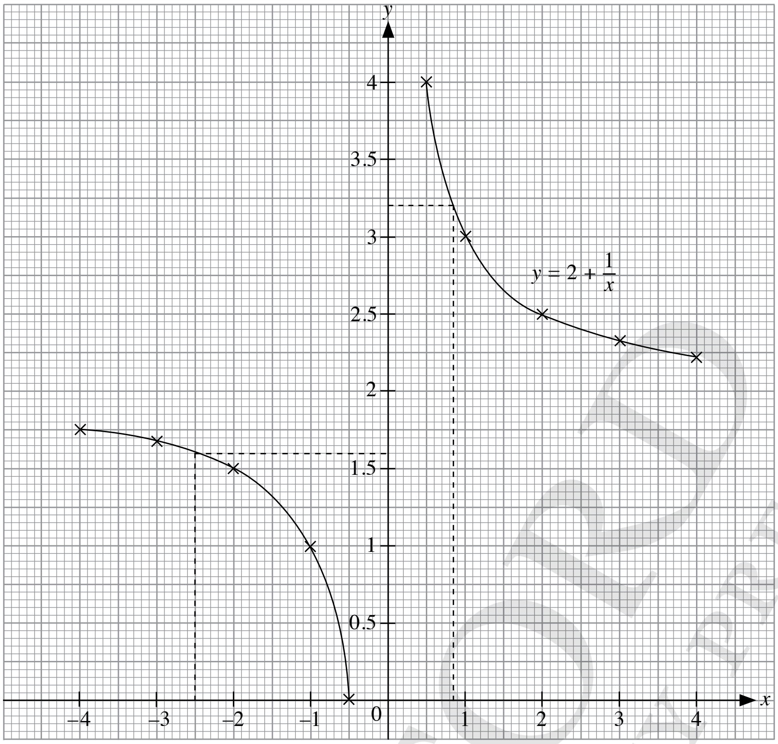
(i) When $x = 2.3$, $y = 7.25$.

(ii) When $y = 15$, $x = 3.8$.

15.

x	–4	–3	–2	–1	$-\frac{1}{2}$	$\frac{1}{2}$	1	2	3	4
y	1.75	1.67	1.5	1	0	4	3	2.5	2.33	2.25

(a)

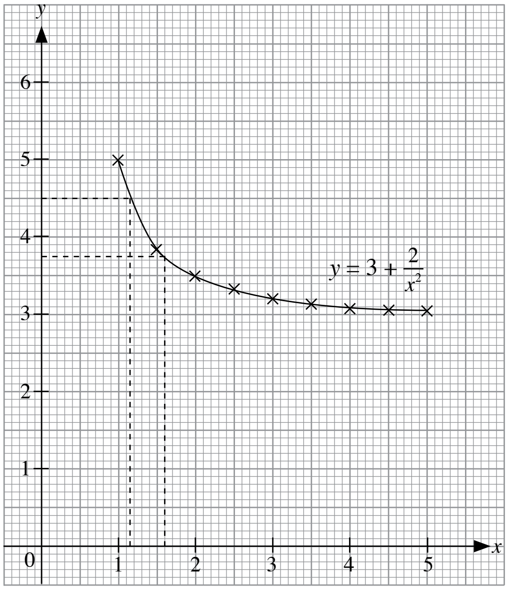


(b) (i) When $x = -2.5$, $y = 1.6$.

(ii) When $y = 3.2$, $x = 0.85$.

16.

x	1	1.5	2	2.5	3	3.5	4	4.5	5
y	5	3.89	3.5	3.32	3.22	3.16	3.13	3.10	3.08

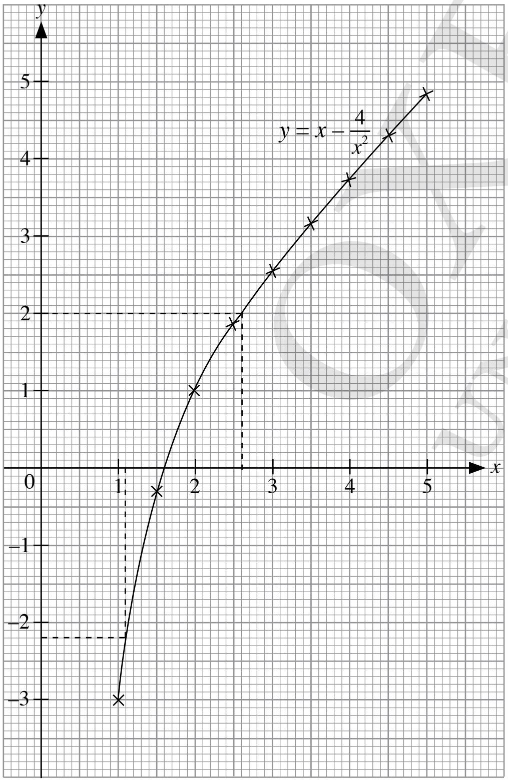


(i) When $x = 1.6$, $y = 3.775$.

(ii) When $y = 4.5$, $x = 1.15$.

17.

x	1	1.5	2	2.5	3	3.5	4	4.5	5
y	–3	–0.28	1	1.86	2.56	3.17	3.75	4.30	4.84

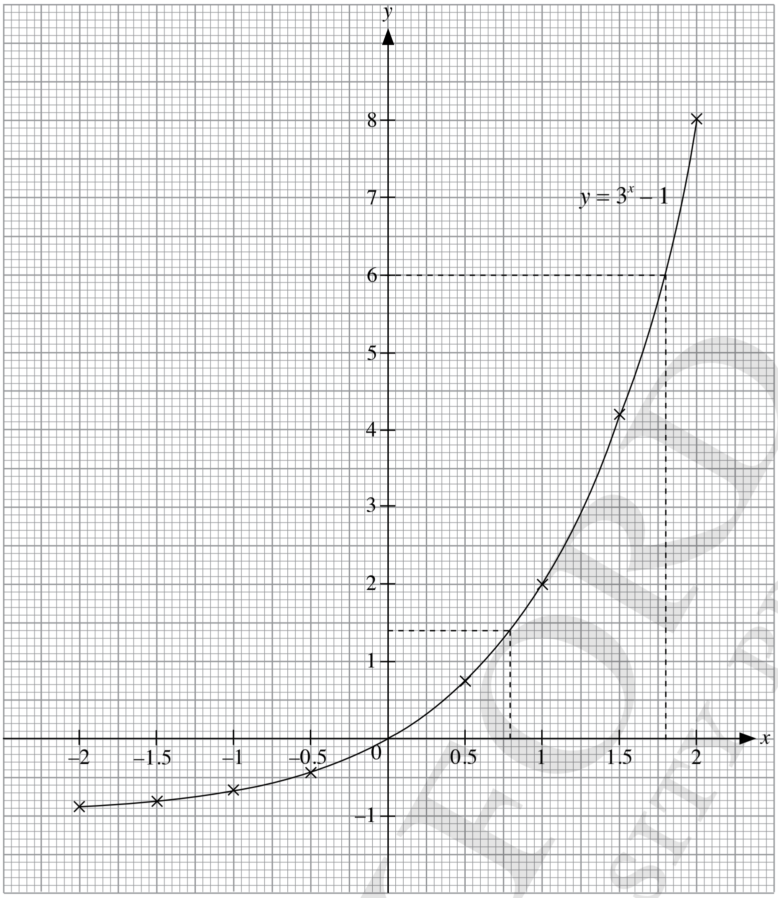


(i) When $x = 2.6$, $y = 2$.

(ii) When $y = -2.2$, $x = 1.1$.

18.

x	–2	–1.5	–1	–0.5	0	0.5	1	1.5	2
y	–0.89	–0.81	–0.67	–0.42	0	0.73	2	4.20	8

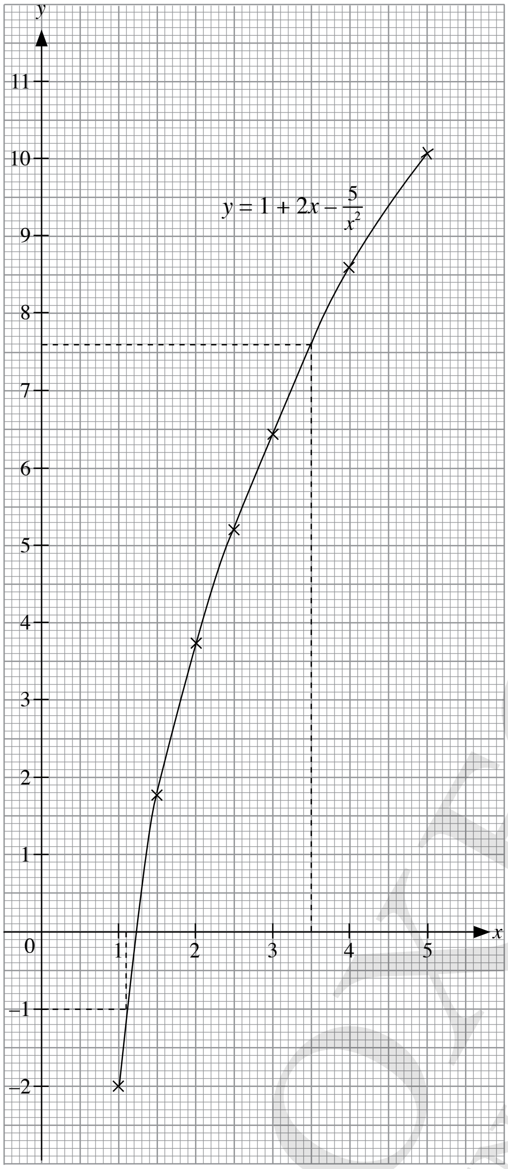


(i) When $x = 0.8$, $y = 1.4$.

(ii) When $y = 6$, $x = 1.8$.

19.

x	1	1.5	2	2.5	3	3.5	4	4.5	5
y	–2	1.78	3.75	5.2	6.44	7.59	8.69	9.75	10.8

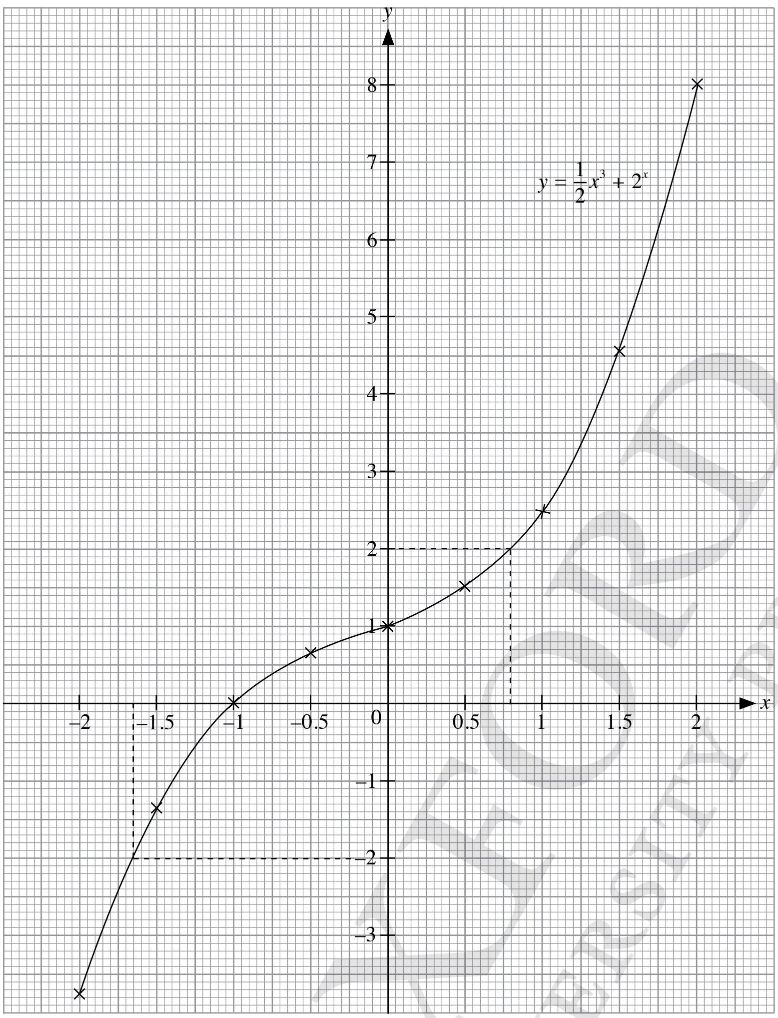


(i) When $x = 3.5$, $y = 7.6$.

(ii) When $y = -1$, $x = 1.1$.

20.

x	–2	–1.5	–1	–0.5	0	0.5	1	1.5	2
y	–3.75	–1.33	0	0.64	1	1.48	2.5	4.52	8

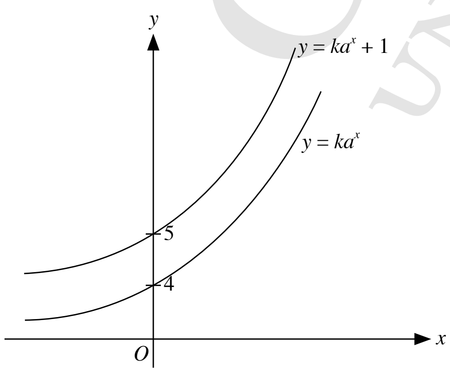


(i) When $x = 0.8$, $y = 2$.

(ii) When $y = -2$, $x = -1.65$.

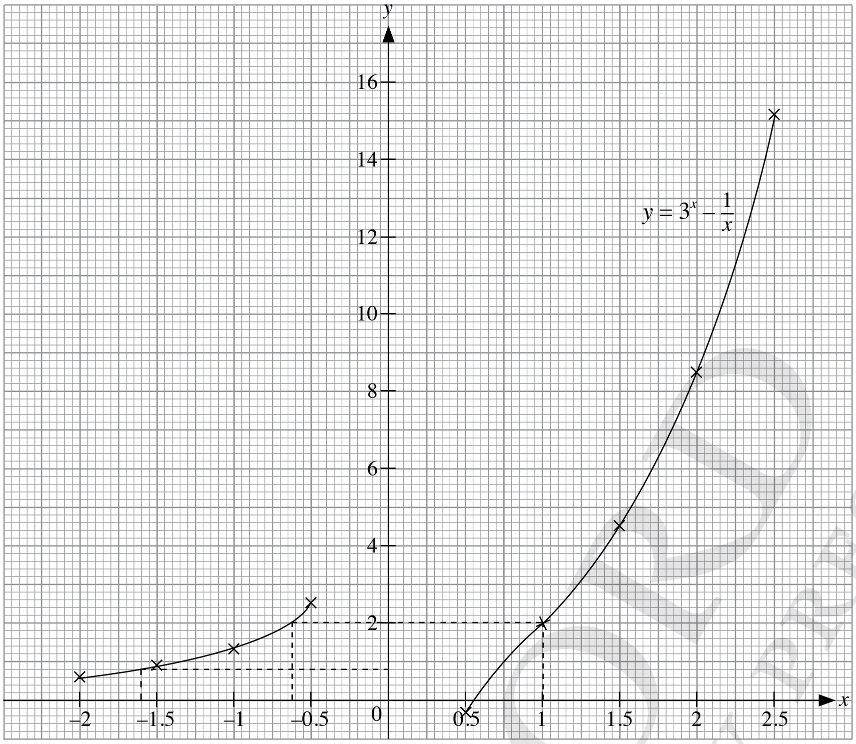
21. (i) $k = 4$

(ii)



22.

x	–2	–1.5	–1	–0.5	0.5	1	1.5	2	2.5
y	0.61	0.86	1.33	2.58	–0.27	2	4.53	8.5	15.19



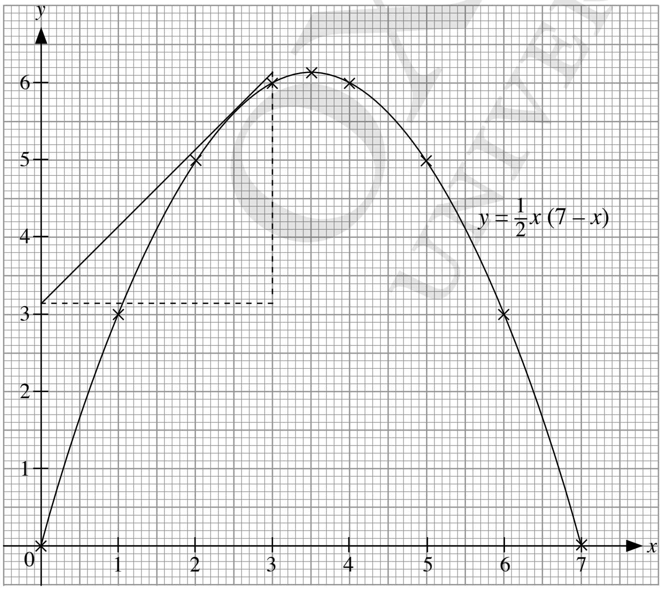
(i) When $x = -1.6$, $y = 0.8$.

(ii) When $y = 2$, $x = -0.65$ and $x = 1$.

23.

x	0	1	2	3	3.5	4	5	6	7
y	0	3	5	6	6.125	6	5	3	0

(a)



(b) Using points (0, 3.15) and (3, 6.15),

$$\text{gradient of tangent} = \frac{6.15 - 3.15}{3 - 0}$$

$$= 1$$

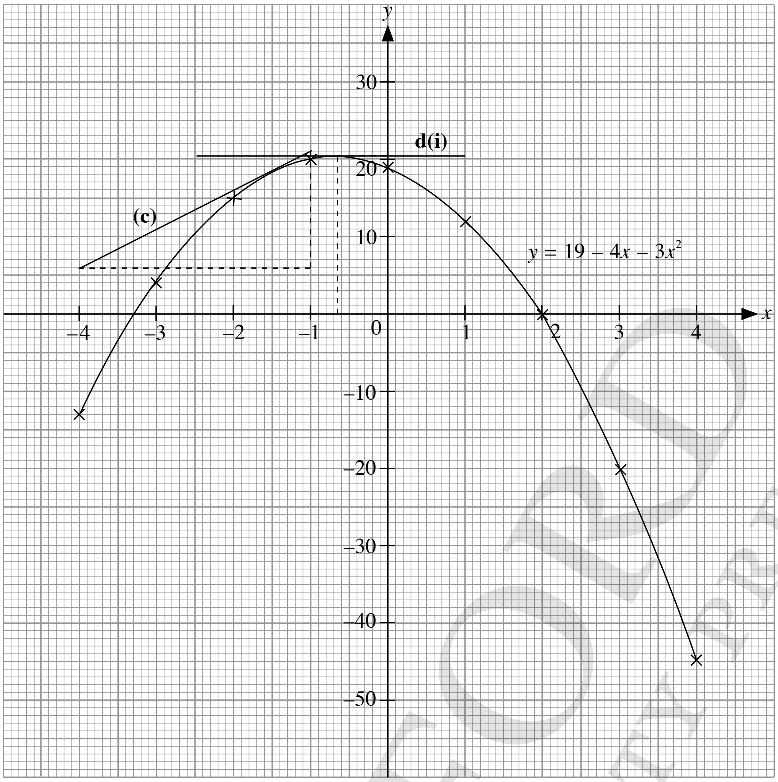
24. **(a)** When $x = -1$, $y = 20$.

$\therefore p = 20$

When $x = 3$, $y = -20$.

$\therefore q = -20$

(b)



(c) Using points (–4, 6) and (–1, 21),

$$\text{gradient of tangent} = \frac{21 - 6}{-1 - (-4)} = 5$$

(d) **(ii)** From the graph, $h = -0.65$ and $k = 20.5$.

25. (a) When $x = -2$, $y = 3.5$.

$\therefore p = 3.5$

When $x = -1.5$, $y = 6.2$.

$\therefore q = 6.2$

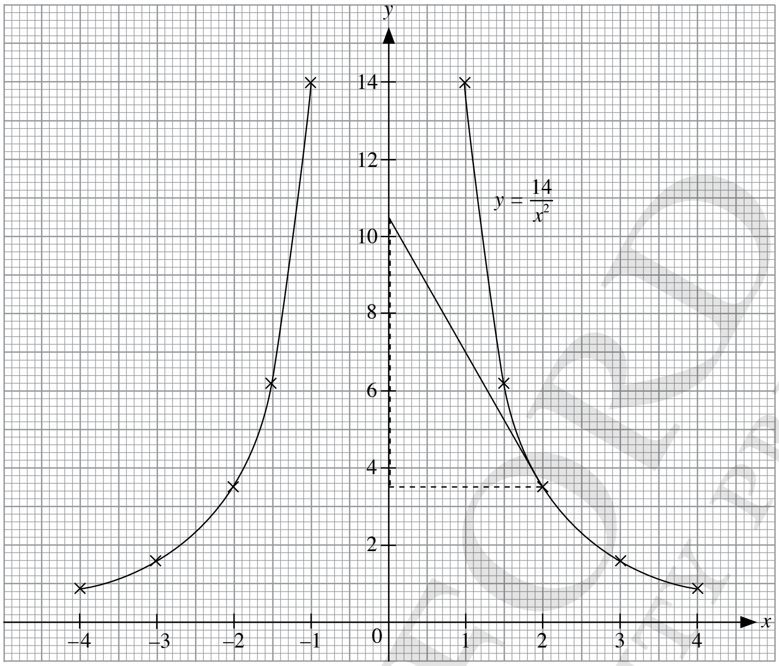
When $x = 1.5$, $y = 6.2$.

$\therefore r = 6.2$

When $x = 2$, $y = 3.5$.

$\therefore s = 3.5$

(b)



(c) $x = 0$

(d) Using points (0, 10.5) and (2, 3.5),

gradient of tangent $= \dfrac{3.5 - 10.5}{2 - 0}$

$= -3.5$

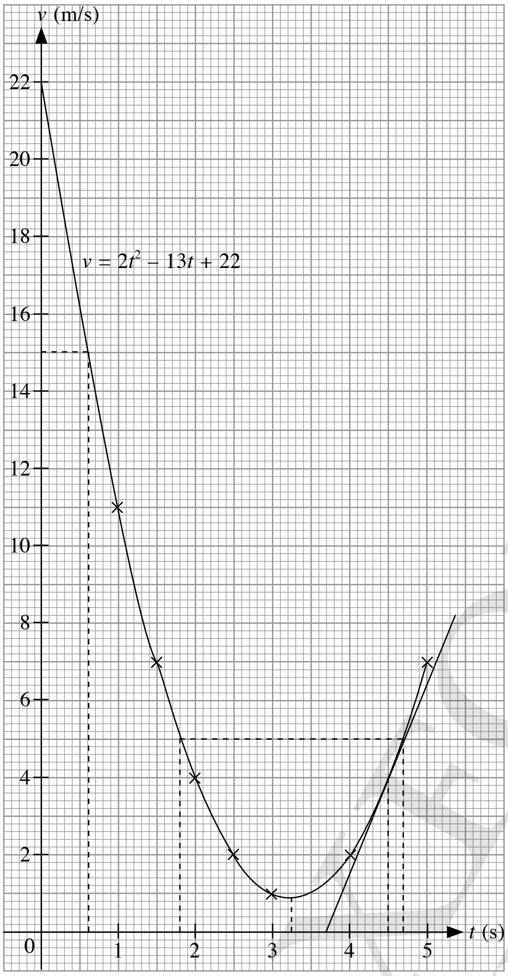
26. **(a)** When $t = 2.5$, $v = 2$.

$\therefore a = 2$

When $t = 5$, $v = 7$.

$\therefore b = 7$

(b)



(c) **(i)** When $v = 15$, $t = 0.6$.

(ii) When the acceleration is zero, the time is 3.25 s.

(iii) Using points (3.7, 0) and (4.5, 4),

$$\text{Gradient of tangent} = \frac{4 - 0}{4.5 - 3.7} = 5$$

When $t = 4.5$, the acceleration of the particle is 5 m/s^2.

(iv) When $v < 5$, the time interval is $1.8 < t < 4.7$.

27. **(a)** When $x = 4$, $y = 56$.

$\therefore p = 56$

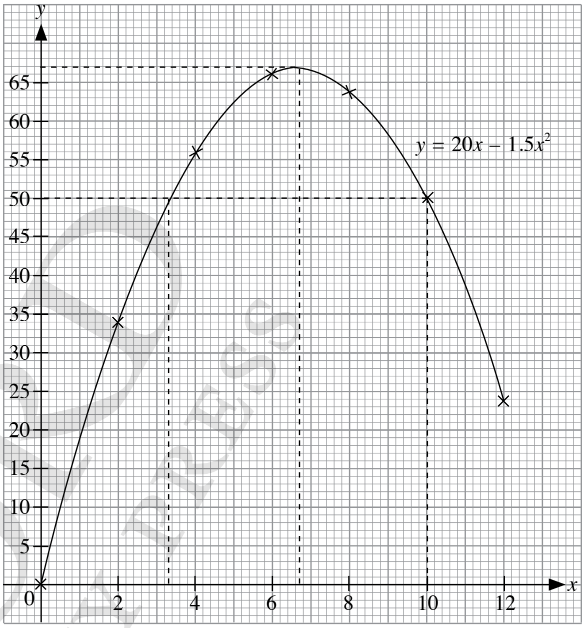
When $x = 8$, $y = 64$.

$\therefore q = 64$

When $x = 10$, $y = 50$.

$\therefore r = 50$

(b)



(c) **(i)** Largest possible area is 67 cm^2 when $x = 6.7$.

(ii) When $y > 50$, the range of values of x is $3.3 < x < 10$.

28. **(a)** When $x = 1.5$, $y = 2.5$.

$\therefore p = 2.25$

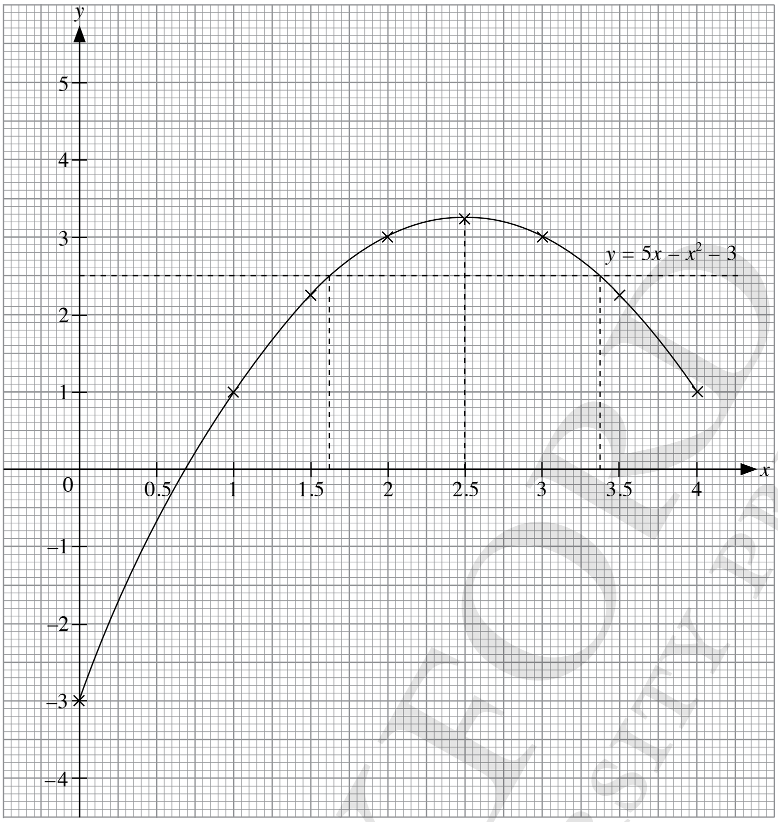
When $x = 2.5$, $y = 3.25$.

$\therefore q = 3.25$

When $x = 4$, $y = 1$.

$\therefore r = 1$

(b)

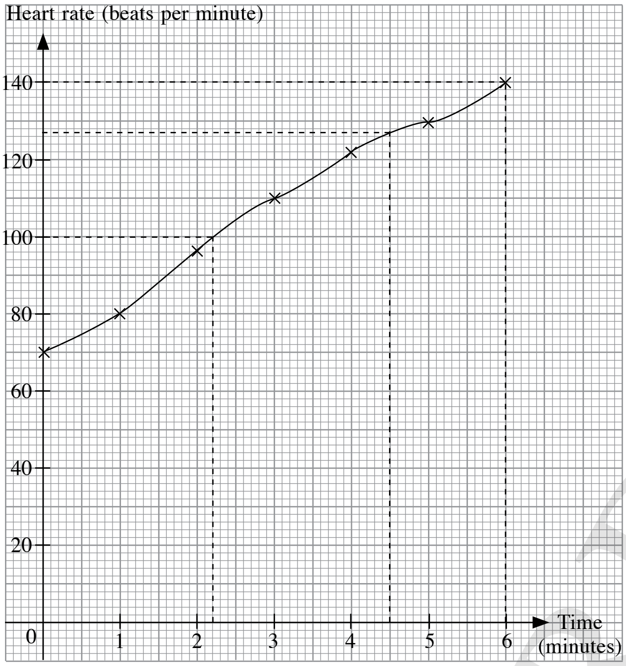


(c) **(i)** Number of mugs = 2500

(ii) Number of mugs = 700

(iii) When $y \geqslant 2.5$, the range of values of x is $1.625 \leqslant x \leqslant 3.375$.

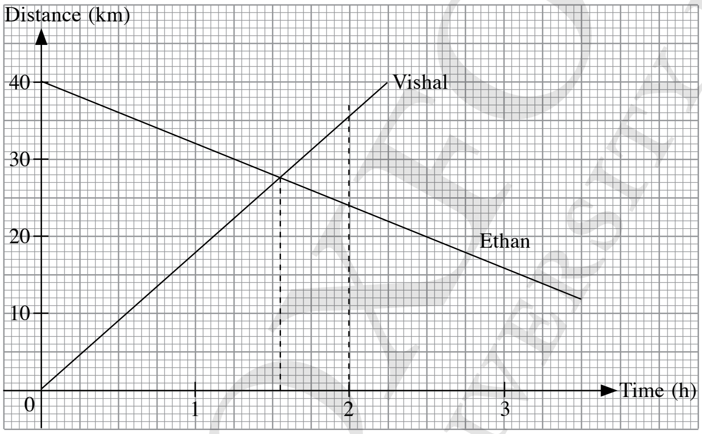
29. **(a)**



(b) **(i)** Approximate time taken is 2.2 minutes.

(ii) Range of heart rate in the last 1.5 minutes is $127 \leqslant t \leqslant 140$.

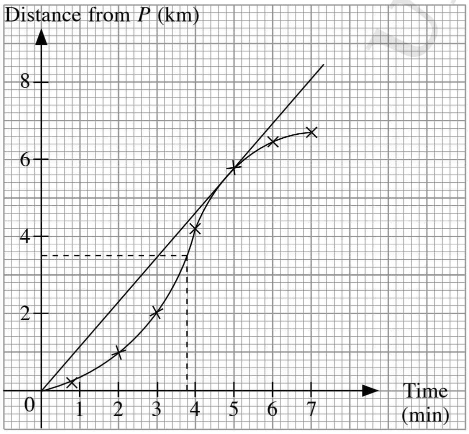
30. **(a)**



(b) **(i)** Time taken for them to pass each other is 1.55 h.

(ii) They will be 12 km apart at 1.1 h and 2 h.

31. **(a)**

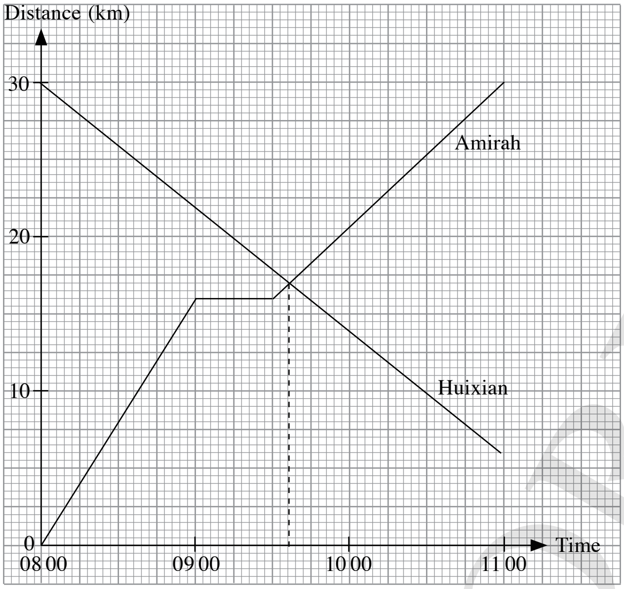


(b) Time taken to travel the first 3.5 km of the journey is 3.8 min.

(c) Using points (1, 0.8) and (6.8, 7.4),

$$\text{Gradient of tangent} = \frac{7.4 - 0.8}{6.8 - 1}$$
$$= 1.137 \text{ km/min}$$
$$= 68.3 \text{ km/h}$$

32. (a)



(b) (i) Amirah and Huixian met at 09 36.

(ii) They were 13 km away from Huixian's home.

33. (i) $\text{Acceleration} = \frac{30 - 0}{t - 0}$

$$1.5 = \frac{30}{t}$$
$$t = 20$$

(ii) $\text{Total distance travelled} = \frac{1}{2} \times (20 + 80) \times 30$

$$= 1500 \text{ m}$$

$$\text{Average speed} = \frac{1500}{80}$$
$$= 18.75 \text{ m/s}$$

34. (i) $\text{Acceleration during the first 21 seconds} = \frac{21 - 70}{21 - 0}$

$$= -2\frac{1}{3} \text{ m/s}^2$$

$\therefore \text{Deceleration} = 2\frac{1}{3} \text{ m/s}^2$

(ii) Acceleration during the second part of the journey

$= -0.5 \text{ m/s}^2$

$$\frac{0 - 21}{t - 21} = -0.5$$
$$42 = t - 21$$
$$t = 63$$

35. **(i)** When $t = 0$, $v = 5$.

$\therefore$ Initial speed is 5 m/s.

(ii) When $t = 8$, $v = 21$.

$\therefore$ Speed is 21 m/s.

(iii) Acceleration of the particle = 2 m/s^2

(iv) Average speed $= \frac{104}{8}$

$= 13$ m/s

36. **(i)** Acceleration of police car $= \frac{40 - 0}{15 - 0}$

$= 2\frac{2}{3}$ m/s^2

(ii) Time taken $= \frac{30}{\frac{8}{3}}$

$= 11\frac{1}{4}$ s

(iii) Distance travelled $= 30 \times 30$

$= 900$ m

37. **(i)** Average speed $= \frac{350}{14}$

$= 25$ m/s

(ii) Acceleration of the particle $= \frac{40 - 22}{10 - 2}$

$= 2.25$ m/s^2

(iii) Using similar triangles,

$\frac{v}{40} = \frac{3}{4}$

$v = 30$

$\therefore$ Speed of the particle is 30 m/s.

(iv) Acceleration of the particle in the last 4 seconds

$= \frac{0 - 40}{14 - 10}$

$= -10$ m/s^2

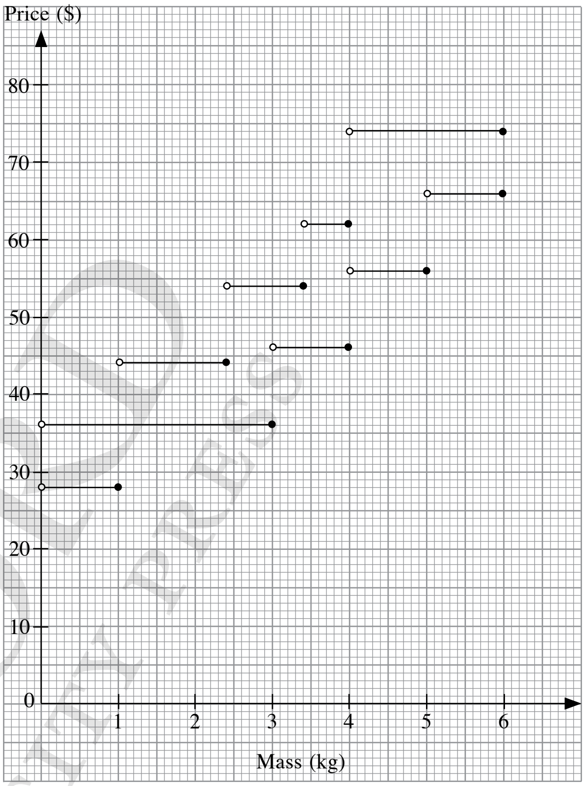
$\therefore$ Deceleration of the particle in the last 4 seconds

$= 10$ m/s^2

38. **(a)**

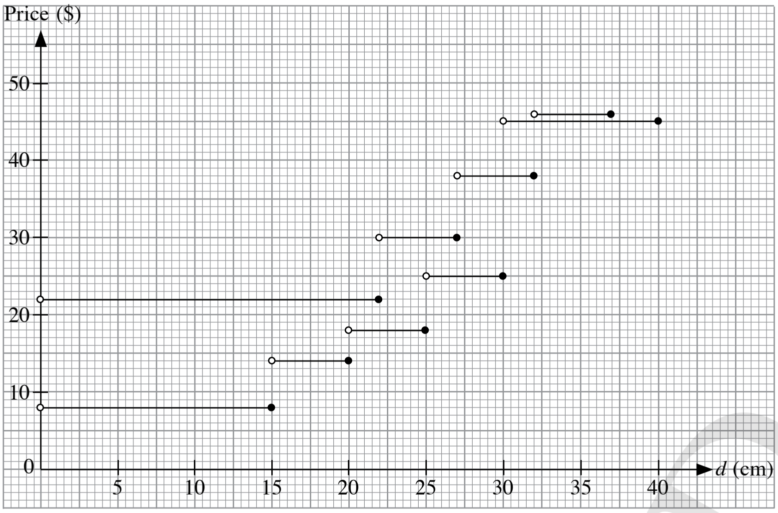
Mass (kg)	1	2.8	4	5.5
Price ($)	28	54	62	76

(b)



(c) Tea Break Café offers a lower price. The price difference is $20.

39. **(a)** and **(c)**



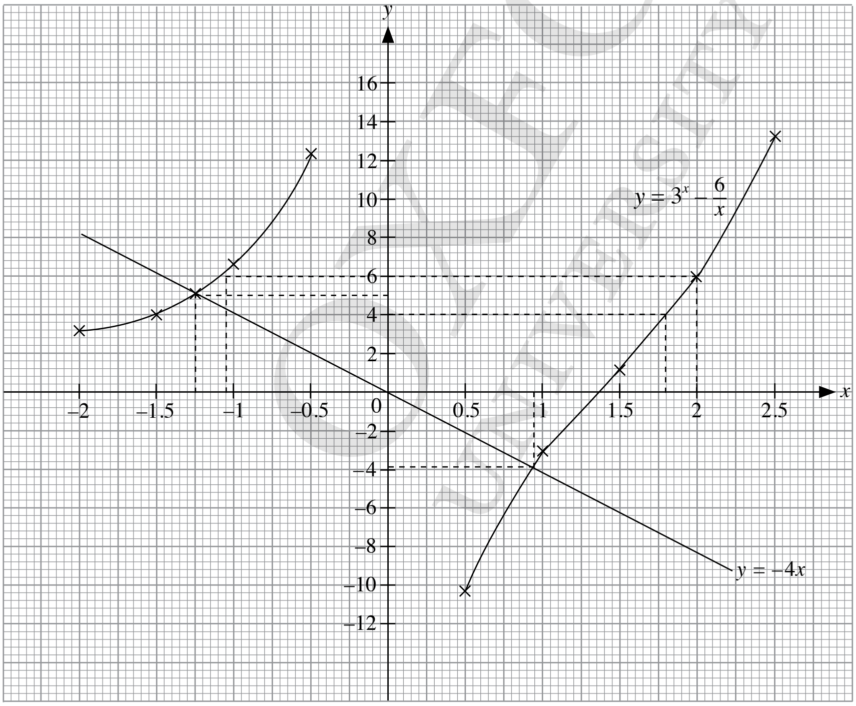
(b) Price of a pizza of diameter 25 cm = $18

(d) Pizza Place offers a lower price for a pizza of diameter 32 cm. The difference in the prices is $7.

Advanced

40.

x	–2	–1.5	–1	–0.5	0.5	1	1.5	2	2.5
y	3.11	4.19	6.33	12.6	–10.3	–3	1.20	6	13.2



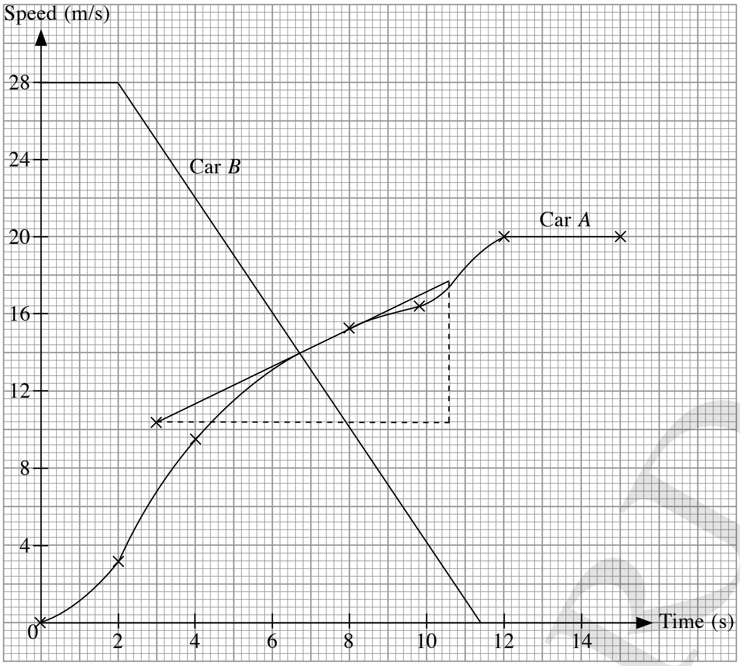
(a) **(i)** When $x = 1.8$, $y = 4$.

(ii) When $y = 6$, $x = -1.05$ or $x = 2$.

(b) **(i)** From the graph, the points of intersection are $(-1.25, 5)$ and $(0.95, -3.8)$.

(ii) $x = -1.25$ and $x = 0.95$

41. **(a)** and **(b)**



(c) **(i)** 6.7 s

(ii) Distance moved by car B in the first 2 seconds

$= 28 \times 2$

$= 56$ m

(iii) Using the points (3, 10.8) and (10.6, 17.6),

$$\text{gradient} = \frac{17.6 - 10.8}{10.6 - 3}$$

$$= 0.895 \text{ m/s}^2 \text{ (to 3 s.f.)}$$

$\therefore$ Acceleration of car A is 0.895 m/s^2.

42. **(i)** Acceleration of the train $= \frac{0 - 20}{90 - 0}$

$$= -\frac{2}{9} \text{ m/s}^2$$

$\therefore$ Deceleration of train $= \frac{2}{9}$ m/s^2.

(ii) Distance travelled by train from P and $Q = \frac{1}{2}(t - 180 + t - 150)(40)$

$$760 = \frac{1}{2}(2t - 330)(40)$$

$$38 = 2t - 330$$

$$2t = 368$$

$$t = 184$$

43. (i) Substitute $v = 18$ into $v = \frac{18t}{35}$:

$$18 = \frac{18t}{35}$$
$$t = 35$$

$\therefore$ Coordinates of A are (35, 18).

Substitute $v = 18$ into $\frac{t}{240} + \frac{v}{54} = 1$:

$$\frac{t}{240} + \frac{18}{54} = 1$$
$$\frac{t}{240} = \frac{2}{3}$$
$$t = 160$$

$\therefore$ Coordinates of B are (160, 18).

Substitute $v = 0$ into $\frac{t}{240} + \frac{v}{54} = 1$:

$$\frac{t}{240} = 1$$
$$t = 240$$

$\therefore$ Coordinates of C are (240, 0).

(ii) Length of time $= 160 - 35$
$= 125$ s

(iii) Distance travelled $= 18 \times 125$
$= 2250$ m

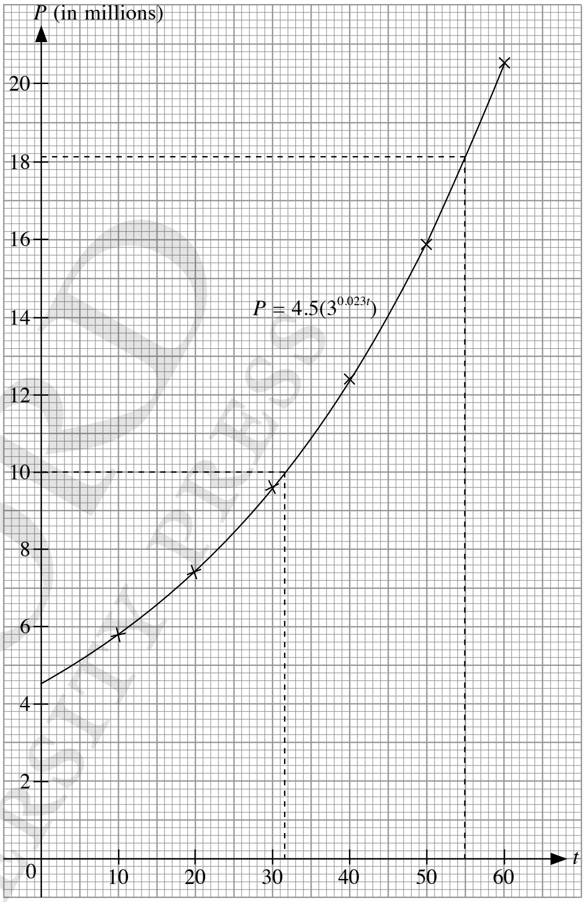
(iv) Gradient of $BC = \frac{0 - 18}{240 - 160}$
$= -0.225$

$\therefore$ Deceleration of bus is 0.225 m/s^2.

44. (a)

t	0	10	20	30	40	50	60
Year	1990	2000	2010	2020	2030	2040	2050
***P*(in millions)**	4.5	5.8	7.4	9.6	12.4	15.9	20.5

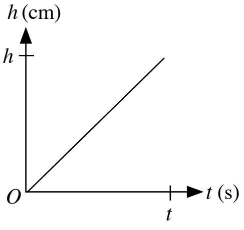
(b)



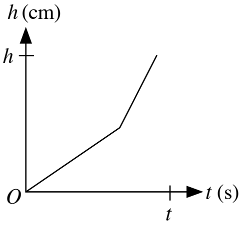
(c) (i) In 2045, $t = 55$.
From the graph, $P = 18.1$.
$\therefore$ Population in the year 2045 is 18.1 million.

(ii) From the graph, when $P = 10$, $t = 31.5$.

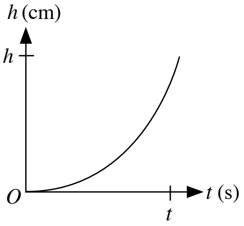
45.



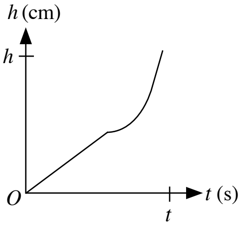
A



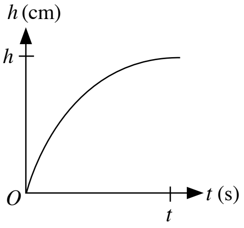
B



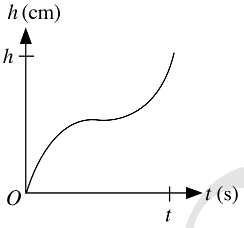
C



D



E



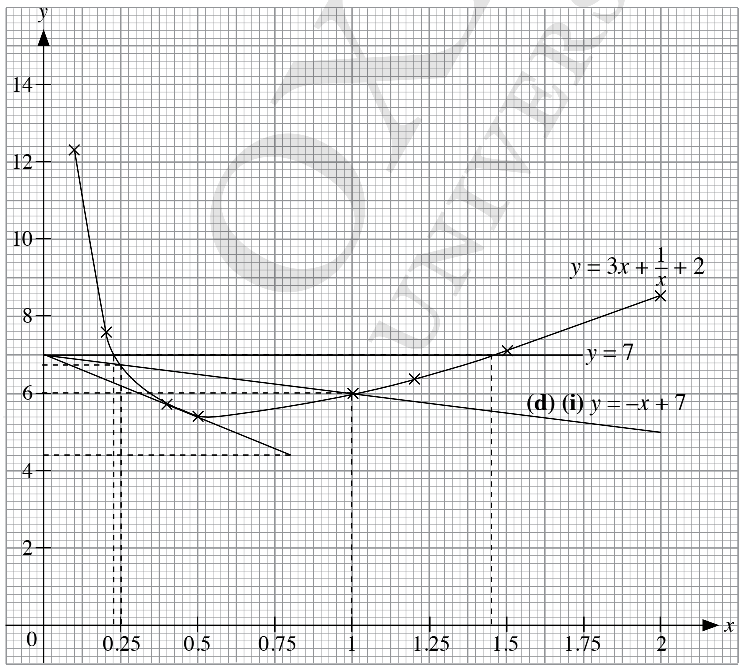
F

New Trend

46.

x	0.1	0.2	0.4	0.5	1	1.2	1.5	2
y	12.3	7.6	5.7	5.5	6	6.4	7.2	8.5

(a)



(b) $3x + \frac{1}{x} = 5$

$3x + \frac{1}{x} + 2 = 7$

When $y = 7$,

$x = 0.23$ or $x = 1.45$

(c) Using points (0, 7) and (0.8, 4.4)

Gradient of tangent $= \frac{7-4.4}{0-0.8}$

$= -3.25$

(d) **(ii)** Since gradient = −1,

$y = -x + c$

When $x = 2$, $y = 5$

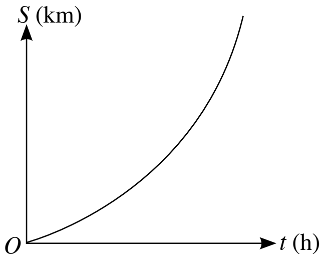
$5 = -2 + c$

$c = 7$

$\therefore$ The equation of the line is $y = -x + 7$.

(iii) From the graph, the points of intersection are (0.25, 6.75) and (1, 6).

47. **(a)**



(b) $S = kt^3$

When $t = 4$, $S = 128$

$128 = k(4)^3$

$64k = 128$

$k = 2$

$\therefore S = 2t^3$

(c) When $S = 182\frac{1}{4}$,

$182\frac{1}{4} = 2t^3$

$t^3 = 91\frac{1}{8}$

$t = 4.5$

$\therefore$ The time taken to travel $182\frac{1}{4}$ km is 4.5 hours.

48. **(a)** When $x = 2$,

$y = 6(2)^2 - 2^3$

$= 16$

$\therefore p = 16$

When $x = 4$,

$y = 6(4)^2 - 4^3$

$= 32$

$\therefore q = 32$

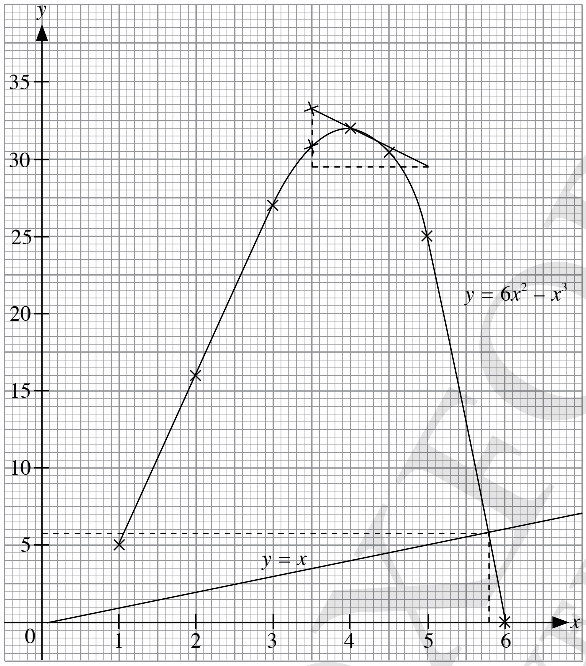
When $x = 5$,

$y = 6(5)^2 - 5^3$

$= 25$

$\therefore r = 25$

(b)



(c) Using the points (3.5, 33.5) and (5, 29.5),

gradient $= \dfrac{29.5 - 33.5}{5 - 3.5}$

$= -2.67$ (to 3 s.f.)

(d) **(i)** From the graph, the point of intersection is (5.8, 5.75)

(ii) 5.8

49. **(a)** $y = x^2 - 5x - 6$

(b) $y = 10^x$

(c) $y = 3 - x^3$

50. **(a)** **(i)** Acceleration during the first 15 seconds

$= \dfrac{20 - 0}{15 - 0}$

$= 1\dfrac{1}{3}$ m/s^2

(ii) Acceleration during the last 10 seconds

$= \dfrac{0 - 20}{35 - 25}$

$= -2$ m/s^2

$\therefore$ Deceleration of the car during the last 10 seconds is 2m/s^2.

(iii) Average speed $= \dfrac{450}{35}$

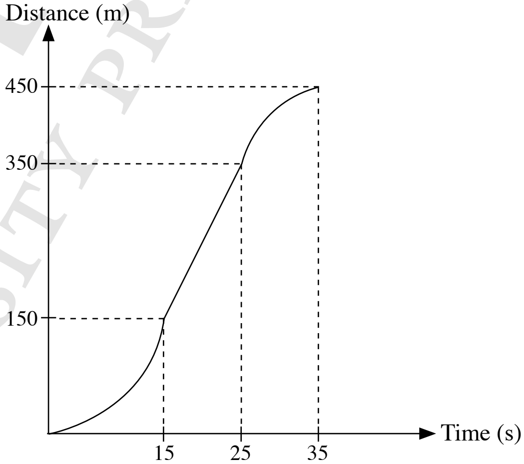
$= 12.9$ m/s (to 3 s.f.)

(b) Distance travelled from 0 s to 15 s $= \dfrac{1}{2} \times 15 \times 20$

$= 150$ m

Distance travelled from 15 s to 25 s $= 10 \times 20$

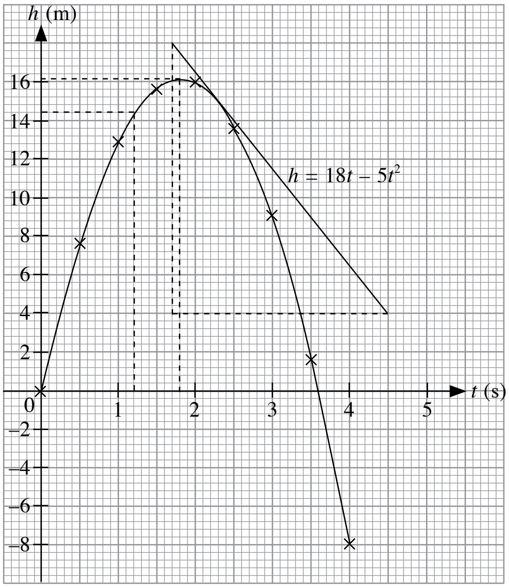
$= 200$ m



51. (a)

x	0	0.5	1	1.5	2	2.5	3	3.5	4
y	0	7.75	13	15.75	16	13.75	9	1.75	–8

(b)



(c) **(i)** Maximum height = 16.2 m

(ii) Time taken = 1.8 s

(iii) Vertical height of the ball at 1.2 s = 14.4 m

(iv) Time taken to hit the ground = 3.6 s

(d) Using points (1.7, 18) and (4.5, 4),

$$\text{gradient} = \frac{18-4}{1.7-4.5}$$

$$= -5 \text{ m/s}^2$$

52. $y = ka^x$

When $x = 0$, $y = \frac{1}{2}$.

$$\frac{1}{2} = ka^0$$

$$k = \frac{1}{2}$$

When $x = 4$, $y = 648$.

$$648 = \frac{1}{2}a^4$$

$$a^4 = 1296$$

$$a = \pm\sqrt[4]{1296}$$

$a = 6$ or -6 (rejected since $a > 0$)

$$\therefore a = 6, k = \frac{1}{2}$$

53. (i) It represents the acceleration of the motorcylist.

(ii) Duration = 2 h

(iii) Acceleration in the first hour $= \frac{100 - 40}{1}$

$= 60 \text{ km/h}^2$

(iv) Acceleration in the last hour $= \frac{0 - 100}{1}$

$= -100 \text{ km/h}^2$

$\therefore$ Deceleration in the last hour is 100 km/h^2.

(v) Total distance travelled

$$= \left[\frac{1}{2} \times (40 + 100) \times 1\right] + (100 \times 2) + \left(\frac{1}{2} \times 100 \times 1\right)$$

= 320 km

(vi) $100 \text{ km/h} = \frac{100 \text{ km}}{1 \text{ h}}$

$$= \frac{100\ 000 \text{ m}}{3600 \text{ s}}$$

$$= 27\frac{7}{9} \text{ m/s}$$

54. (a) When $x = 1.5$,

$$y = 1.5 + \frac{12}{1.5} - 5$$

$= 4.5$

$\therefore p = 4.5$

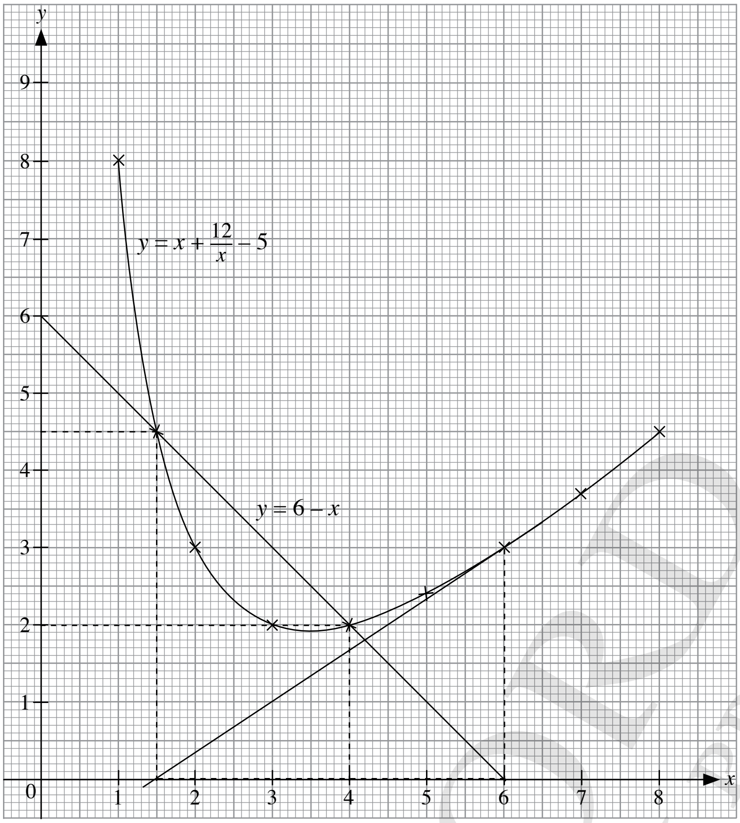
When $x = 5$,

$$y = 5 + \frac{12}{5} - 5$$

$= 2.4$

$\therefore q = 2.4$

(b)



(c) Using the points (6, 3) and (1.5, 0),

$$\text{gradient} = \frac{0-3}{1.5-6}$$

$$= \frac{2}{3}$$

(d) **(i)** From the graph, the points of intersection are (1.5, 4.5) and (4, 2).

(ii) $2x + \frac{12}{x} - 11 = 0$

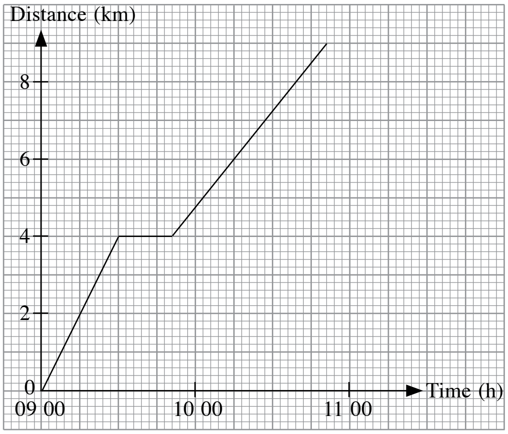
$x + \frac{12}{x} - 5 = 6 - x$

Draw $y = 6 - x$.

$x = 1.5$ or $x = 4$

(e) Coordinates of minimum point are (3.5, 1.9).

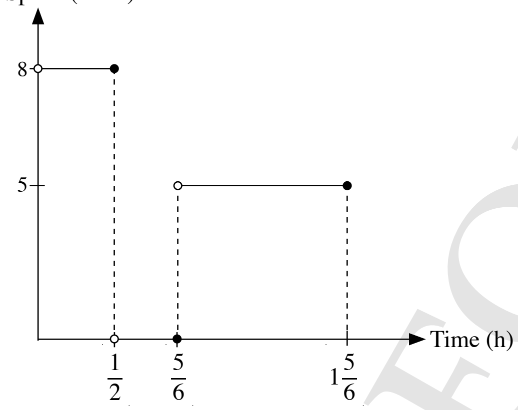
55. (i)



(ii) 10 50

(iii) 9 km

(iv)



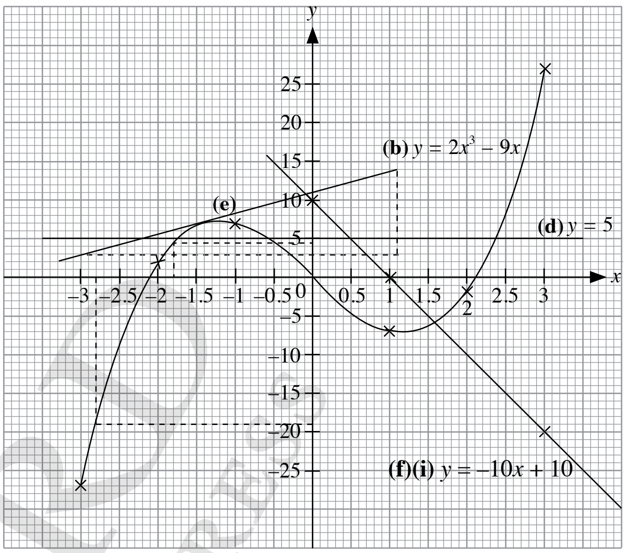
56. (a) When $x = -2$, $y = 2$.

$\therefore p = 2$

When $x = 2$, $y = -2$.

$\therefore q = -2$

(b)



(c) (i) From the graph,

when $x = -1.8$, $y = 4.5$.

(ii) From the graph,

when $y = -19$, $x = 2.8$.

(d) $2x^3 - 9x = 5$

Draw $y = 5$.

$x = -1.75$ or $x = -0.6$ or $x = 2.35$

(e) Draw tangent at $(-1.4, 7.112)$ on graph.

Using points $(-2.9, 3)$ and $(1.1, 14)$,

gradient $= \dfrac{3-14}{-2.9-1.1}$

$= 2.75$

(f) (ii) $x = 1.6$

(iii) $2x^3 - 9x = -10x + 10$

$2x^3 + x - 10 = 0$

$x^3 + \frac{1}{2}x - 5 = 0$

Comparing $x^3 + \frac{1}{2}x - 5 = 0$

with $x^3 + Ax^2 + \frac{1}{2}x + B = 0$,

$A = 0$, $B = -5$

Revision Test B1

1. Cost price of durians

$= 480 \times \$2.50$

$= \$1200$

Amount received from selling the durians

$= \left(\frac{20}{100} \times 480\right) \times \$6 + \left(\frac{75}{100} \times 384\right) \times \$4$

$= 576 + 1152$

$= \$1728$

Gain = selling price – cost price

$= 1728 - 1200$

$= \$528$

Percentage gain

$= \frac{528}{1200} \times 100\%$

$= 44\%$

2. $P = \$8800$, $r = 2.75$

Amount at the end of 3 years $= 8800\left(1 + \frac{\frac{2.75}{4}}{100}\right)^{12}$

$= \$9554.09$

$\therefore$ Interest earned $= 9554.09 - 8800$

$= \$754$ (to 3 s.f.)

3. Let the total distance of the journey be x km.

Time taken for 30% of journey $= \frac{3x}{10} \div 80 \text{ h} = \frac{3x}{800} \text{ h}$

Time taken for 20% of journey $= \frac{2x}{10} \div 40 \text{ h} = \frac{x}{200} \text{ h}$

Time taken for remaining 50% of journey $= \frac{x}{2} \div 60 \text{ h}$

$= \frac{x}{120} \text{ h}$

$\therefore$ Total time taken $= \frac{3x}{800} \text{ h} + \frac{x}{200} \text{ h} + \frac{x}{120} \text{ h}$

$= \frac{9x + 12x + 20}{2400} \text{ h}$

$= \frac{41x}{2400} \text{ h}$

$\therefore$ Average speed $= x \div \frac{41x}{2400}$

$= 58.54$ km/h (to 2 d.p.)

4. (a) Gradient $= \frac{3}{4} = \frac{7 - a}{2a - 8}$

$3(2a - 8) = 4(7 - a)$

$6a - 24 = 28 - 4a$

$10a = 52$

$a = 5.2$

(b) $5x + 7y = 13$

$7y = -5x + 13$

$y = -\frac{5}{7}x + \frac{13}{7}$

Gradient of $5x + 7y = 13$ is $-\frac{5}{7}$.

$\therefore m = -\frac{5}{7}$

$(7, 4)$ lies on $y = -\frac{5}{7}x + c$

When $x = 7$ and $y = 4$,

$4 = -\frac{5}{7}(7) + c$

$\therefore c = 9$

5. (a) Since gradient $= -\frac{4}{3}$, the equation is $y = -\frac{4}{3}x + c$.

$y = -\frac{4}{3}x + c$ passes through $(3, -2)$.

When $x = 3$, $y = -2$

$-2 = -\frac{4}{3}(3) + c$

$-2 = -4 + c$

$c = 2$

$\therefore$ Equation of straight line is $y = -\frac{4}{3}x + 2$.

$y = -\frac{4}{3}x + 2$ passes through $\left(-\frac{3}{2}, a\right)$.

When $x = -\frac{3}{2}$, $y = a$

$a = -\frac{4}{3}\left(-\frac{3}{2}\right) + 2$

$a = 2 + 2$

$\therefore a = 4$

(b) (i) Length of $PQ = \sqrt{(0 - 5)^2 + (12 - 0)^2}$

$= 13$ units

(ii) Gradient of $PQ = \frac{12 - 0}{0 - 5} = -\frac{12}{5}$

Equation of line is $y = -\frac{12}{5}x + c$

$y = -\frac{12}{5}x + c$ passes through $R(2, 1)$.

When $x = 2$, $y = 1$

$1 = -\frac{12}{5}(2) + c$

$= -\frac{24}{5} + c$

$c = \frac{29}{5}$

$\therefore$ Equation of the line is $y = -\frac{12}{5}x + \frac{29}{5}$.

6. (i) From the graph, the rate of decrease in battery level

$= \frac{84 - 60}{12 - 0}$

$= 2\%$/min (to 3 s.f.)

(ii) From the graph, the rate of increase in battery level

$= \frac{100 - 48}{100 - 60}$

$= 1.3\%$/min

7. **(a)** **(i)** Acceleration at $t = 2$ is $\frac{20}{5} = 4$ m/s^2

(ii) Acceleration at $t = 23$ is $-\frac{40}{10} = -4$ m/s^2

(b) Distance travelled from $t = 0$ to $t = 5$

$= \frac{1}{2}(5)(2)$

$= 50$ m

Distance travelled from $t = 5$ to $t = 20$

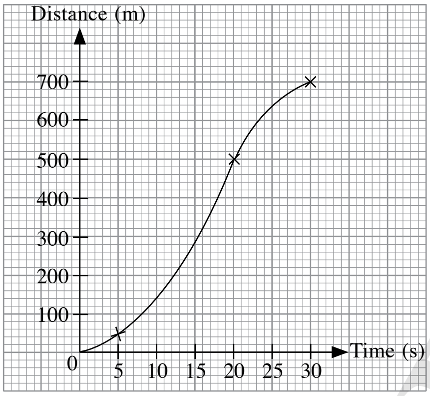
$= \frac{1}{2}(20 + 40) \times 15$

$= 450$ m

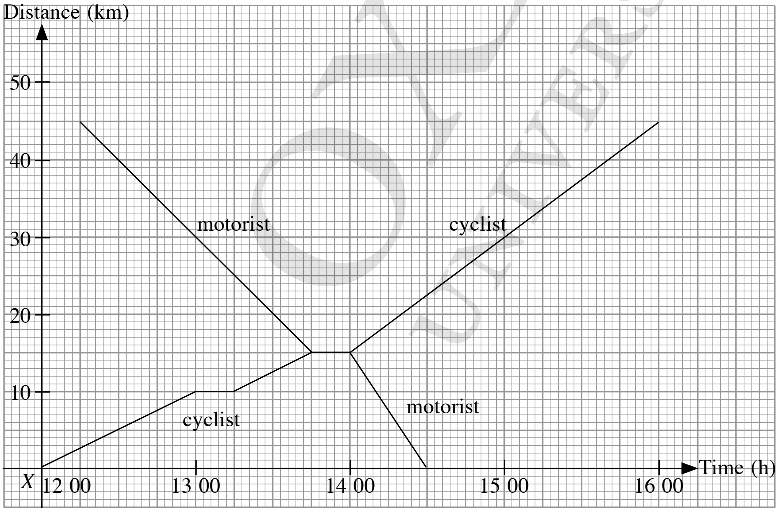
Distance travelled from $t = 20$ to $t = 30$

$= \frac{1}{2}(10)(40)$

$= 200$ m



8.



(i) From the graph, the travellers met at 13 45.

(ii) From the graph, the motorist had travelled 30 km.

(iii) The cyclist reached X at 16 00.

The motorist reached Y at 14 30.

9. **(a)** $y = x^3 - 4x^2$

When $x = -1$,

$h = (-1)^3 - 4(-1)^2 = -5$

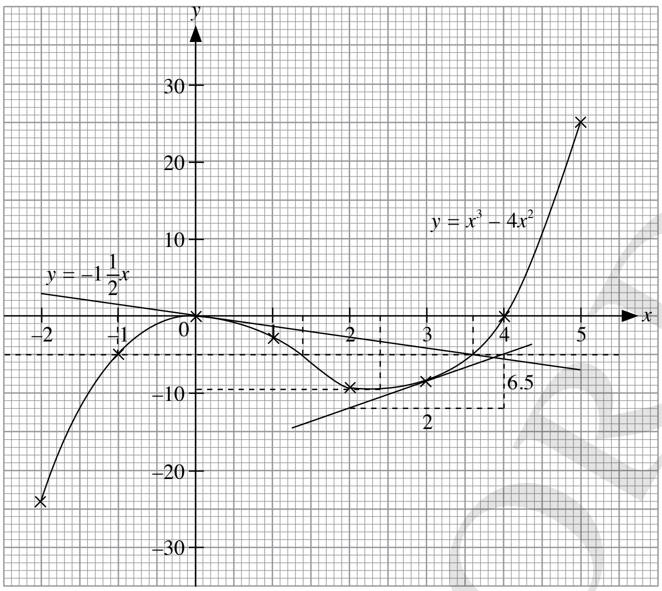
$\therefore h = -5$

When $x = 4$,

$k = (4)^3 - 4(4)^2 = 0$

$\therefore k = 0$

(b)



(c) **(i)** From the graph, when $x = 2.4$, $y = -9.5$.

(ii) From the graph, when $y = -5$,

$x = -1$ or 1.4 or 3.6

(d) From the graph, gradient $= \dfrac{\text{vertical change}}{\text{horizontal change}}$

$= \dfrac{6.5}{2}$

$= 3.25$

(e) $2x^3 - 8x^2 + 3x = 0$

$2x^3 - 8x^2 = -3x$

$x^3 - 4x^2 = -1\dfrac{1}{2}x$

$\therefore$ Draw $y = -1\dfrac{1}{2}x$ on the same diagram.

From the graph, $x = 0$ or 0.4 or 3.6.

Revision Test B2

1. Marked price of couch $= \frac{255}{107} \times 100$

$= \$238.32$ (to nearest cent)

2. **(a)** Interest earned $= 2200 - 1480$

$= \$740$

$$740 = \frac{1480 \times 15 \times t}{100}$$

$$\therefore t = \frac{740 \times 100}{1480 \times 15}$$

$$= 3\frac{1}{3} \text{ years}$$

$= 3$ years 4 months

(b) Amount at the end of 2 years $= 8000\left(1 + \frac{\frac{4.5}{2}}{100}\right)^4$

$= \$8740$ (to 3 s.f.)

3. **(i)** Gradient $= \frac{2 - (-3)}{6 - 0}$

$$= \frac{5}{6}$$

$$\therefore m = \frac{5}{6}$$

y-intercept $= -3$

$\therefore c = -3$

(ii) Area of $\triangle ABC = \frac{1}{2}[5 - (-3)] \times 6$

$= 24$ units^2

(iii) $AB = \sqrt{(6-0)^2 + (2-(-3))^2}$

$= 7.81$ units (to 3 s.f.)

(iv) Let the perpendicular distance from C to AB be x.

Area of $\triangle ABC = 24$ units^2

$$\frac{1}{2}(AB)(x) = 24$$

$$\frac{1}{2}(7.810)(x) = 24$$

$$x = \frac{24 \times 2}{7.810}$$

$= 6.15$ units (to 3 s.f.)

(v) Gradient of perpendicular from C to $AB = -1 \div \frac{5}{6}$

$$= -\frac{6}{5}$$

4. **(i)** Gradient of $AC = \frac{6 - (-2)}{-6 - 1} = -\frac{8}{7}$

Gradient of $AB = \frac{6 - (-2)}{-6 - 5} = -\frac{8}{13}$

(ii) $BC = \sqrt{(-6-5)^2 + [6-(-2)]^2}$

$= 13.6$ units (to 3 s.f.)

(iii) Area of $\triangle ABC = \frac{1}{2}(5 - 1) \times [6 - (-2)]$

$= 16$ units^2

(iv) $AB = CD$

$\therefore D$ has coordinates $(-10, 6)$.

(v) $AB = KC$

$\therefore K$ has coordinates $(-2, 6)$.

(vi) Equation of line AC is $y = -\frac{8}{7}x + c$.

The point $A(1, -2)$ lies on line AC.

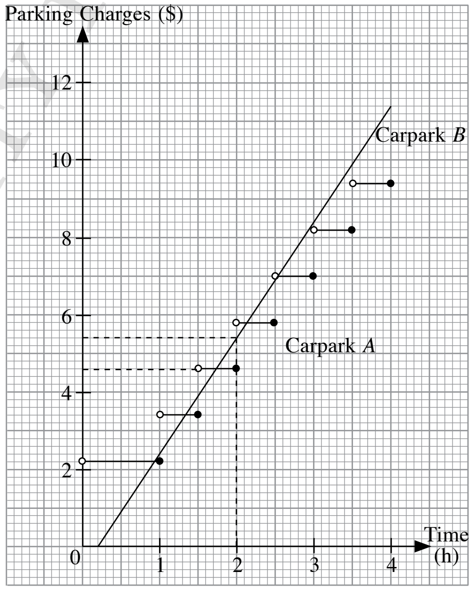
When $x = 1$, $y = -2$

$$-2 = -\frac{8}{7}(1) + c$$

$$c = -\frac{6}{7}$$

$\therefore$ Equation of straight line is $y = -\frac{8}{7}x - \frac{6}{7}$.

5. **(i)**



From the graph, parking charges at Carpark $A = \$4.60$.

From the graph, parking charges at Carpark $B = \$5.40$.

$\therefore$ Shirley should park at Carpark A, which offers lower parking charges for 2 hours.

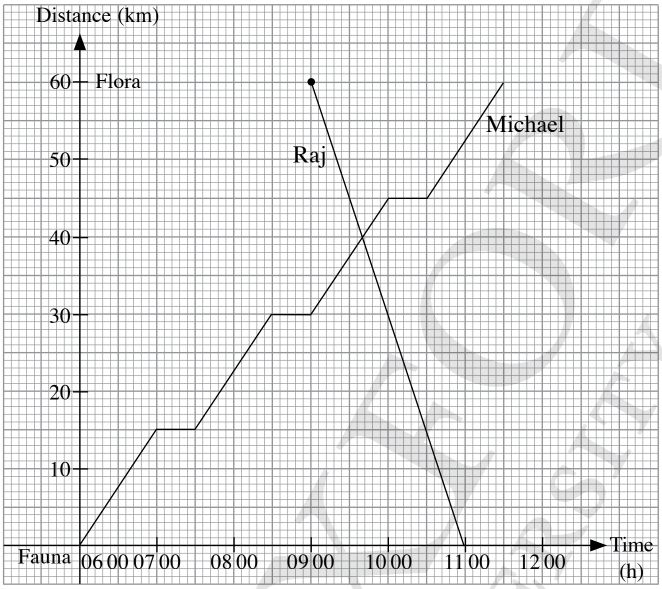
(ii) For Carpark A,

$$\text{average parking rate} = \frac{\text{parking charges}}{\text{time}}$$
$$= \frac{4.60}{120}$$
$$= 0.0383$$
$$= 3.8 \text{ cents/min}$$
(to the nearest 0.1 cent)

For Carpark B,

$$\text{average parking rate} = \frac{\text{parking charges}}{\text{time}}$$
$$= \frac{5.40}{120}$$
$$= 0.045$$
$$= 4.5 \text{ cents/min}$$
(to the nearest 0.1 cent)

6. (a) (i)



(ii) Michael's arrival time at Flora is 11 30.

(b) (ii) The two men pass each other at 09 40 and they are 20 km away from Flora.

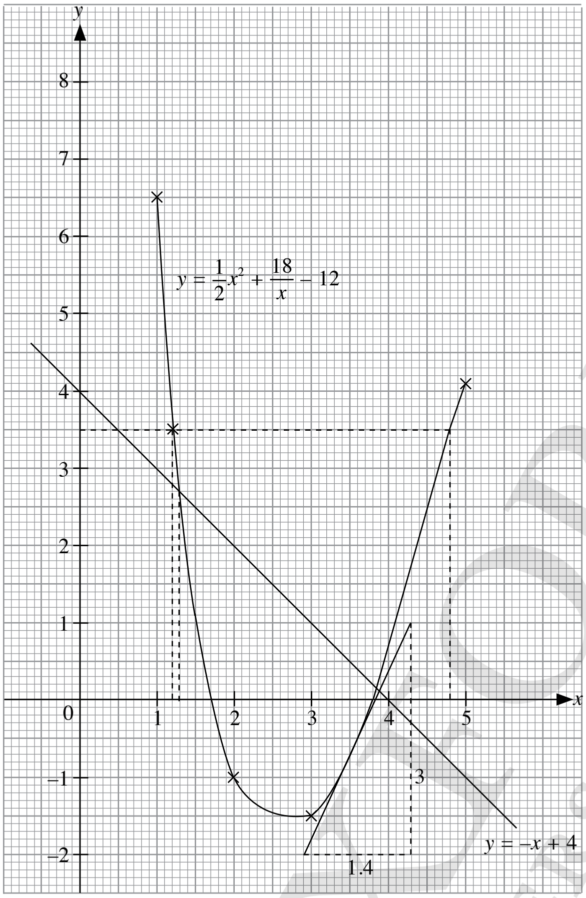
7. (i) When $x = 4.5$,

$h = \frac{1}{2}(4.5)^2 + \frac{18}{4.5} - 12$

$= 2.1$

$\therefore h = 2.1$

(ii)



(iii) From the graph, when $y = 3.5$,

$x = 1.2$ and $x = 4.8$.

(iv) Gradient at $x = 3.5 = \frac{\text{vertical change}}{\text{horizontal change}}$

$= \frac{3}{1.4}$

$= 2.1$

(v) For $1 < x \leqslant 5$,

$x^3 + 2x^2 - 32x + 36 = 0$

Dividing throughout by x,

$x^2 + 2x - 32 + \frac{36}{x} = 0$

$\frac{1}{2}x^2 + x - 16 + \frac{18}{x} = 0$

$\frac{1}{2}x^2 + \frac{18}{x} + x - 12 - 4 = 0$

$\frac{1}{2}x^2 + \frac{18}{x} - 12 = -x + 4$

$\therefore$ Draw $y = -x + 4$.

From the graph, the intersections are at $x = 1.3$ and $x = 3.9$.

$\therefore$ For $1 < x \leqslant 5$, the solutions are $x = 1.3$ and $x = 3.9$.

Mid-Year Examination Specimen Paper A

Part I

1. Total number of tweets

$$= \frac{3 \times 365 \times \frac{24}{2} \times 3.56 \times 10^7}{10^6} \text{ million}$$

$= 468\ 000$ million

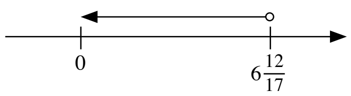
2. **(a)**
$$27 - 3x^2 = 0$$
$$3(9 - x^2) = 0$$
$$3(3 + x)(3 - x) = 0$$
$$\therefore x = -3 \text{ or } x = 3$$

(b)
$$5x^2 - 10x - (x - 2) = 0$$
$$5x(x - 2) - (x - 2) = 0$$
$$(x - 2)(5x - 1) = 0$$
$$\therefore x = 2 \text{ or } x = \frac{1}{5}$$

(c)
$$\frac{1}{x} - \frac{3}{2x+1} = 2$$
$$2x + 1 - 3x = 2x(x + 1)$$
$$1 - x = 4x^2 + 2x$$
$$4x^2 + 3x - 1 = 0$$
$$(4x - 1)(x + 1) = 0$$
$$\therefore x = \frac{1}{4} \text{ or } x = -1$$

3. The equation is
$$\left(x - \frac{4}{5}\right)\left(x - \frac{3}{4}\right) = 0$$
$$(5x - 4)(4x - 3) = 0$$
$$20x^2 - 15x - 16x + 12 = 0$$
$$\therefore 20x^2 - 31x + 12 = 0$$

4.
$$1\frac{1}{2}x - \frac{2}{3}(3 - 2x) < 17$$
$$1\frac{1}{2}x - 2 + \frac{4}{3}x < 17$$
$$2\frac{5}{6}x < 19$$
$$x < 6\frac{12}{17}$$



$\therefore$ Largest prime number is 5.

5. $2x - 8 < 13 \leqslant 3x - 10$

$2x - 8 < 13$ and $13 \leqslant 3x - 10$

$2x < 21$ and $23 \leqslant 3x$

$x < 10\frac{1}{2}$ and $7\frac{2}{3} \leqslant x$

$\therefore 7\frac{2}{3} \leqslant x < 10\frac{1}{2}$

$\therefore$ The integer values satisfying the inequality are 8, 9 and 10.

6. $y = 2x^2 + 7x - 13$

When $y = 6$,

$2x^2 + 7x - 13 = 6$

$2x^2 + 7x - 19 = 0$

$a = 2$, $b = 7$ and $c = -19$

$$x = \frac{-7 \pm \sqrt{7^2 - 4(2)(-19)}}{2(2)}$$

$$x = \frac{-7 - \sqrt{201}}{4} \quad \text{or} \quad x = \frac{-7 + \sqrt{201}}{4}$$
$$= -5.294 \qquad\qquad = 1.794$$

$\therefore x = 1.79$ or -5.29 (to 2 d.p.)

7. **(a)** $7xy^2 = 7(3.4 \times 10^{-5})(6.35 \times 10^{-10})^2$

$= 9.597 \times 10^{-23}$ (to 4 s.f.)

(b) $$\frac{5xy}{3z^3} = \frac{5(3.4 \times 10^{-5})(6.35 \times 10^{-10})}{3(8.46 \times 10^6)^3}$$

$= 5.943 \times 10^{-35}$ (to 4 s.f.)

(c) $$\frac{9\sqrt{xz}}{\sqrt[3]{y}} = \frac{9(3.4 \times 10^{-5})^{\frac{1}{2}}(8.46 \times 10^6)}{(6.35 \times 10^{-10})^{\frac{1}{3}}}$$

$= 5.165 \times 10^8$ (to 4 s.f.)

8. **(i)** Gradient $= \frac{3 - (-11)}{-5 - 3}$

$= -\frac{7}{4}$

(ii) Equation of line is $y = -\frac{7}{4}x + c$

$-11 = -\frac{7}{4}(3) + c$

$c = -5\frac{3}{4}$

$\therefore$ Equation is $y = -\frac{7}{4}x - 5\frac{3}{4}$.

$4y = -7x - 23$

$4y + 7x = -23$

9. Let the amount of savings be \$$P$.

For Bank A, $n = 8$, $R = 1.4$.

Interest earned from Bank $A = P\left(1+\frac{R}{100}\right)^n - P$

$= P\left(1+\frac{1.4}{100}\right)^8 - P$

$= \$0.1176P$

Interest earned from Bank $B = \frac{P \times 2.92 \times 4}{100}$

$= \$0.1168P$

Hence, it is better to deposit your savings in Bank A as the interest earned is higher (\$$0.1176P >$ \$$0.1169P$).

10. $7x^2 - 5 = 8x$

$7x^2 - 8x - 5 = 0$

$a = 7, b = -8, c = -5$

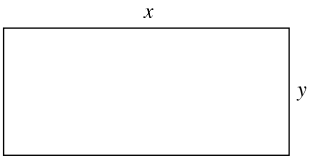
$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(7)(-5)}}{2(7)}$

$x = \frac{8 - \sqrt{204}}{14}$ or $x = \frac{8 + \sqrt{204}}{14}$

$= 0.448$ $\quad$ $= 1.591$

$\therefore x = 0.45$ or 1.59 (to 2 d.p.)

11.



Let x m be the length and y m be the width such that $x > y$.

Perimeter $= 500$

$2x + 2y = 500$

$y = 250 - x$

Area $= 14\,400$ m^2

$xy = 14\,400$

$x(250 - x) = 14\,400$

$x^2 - 250x + 14\,400 = 0$

$(x - 90)(x - 160) = 0$

$\therefore x = 90$ or $x = 160$

When $x = 160$, $y = 250 - 160 = 90$

$\therefore$ Width of rectangle = 90 m

Length of diagonal $= \sqrt{90^2 + 160^2}$

$= 184$ m (to 3 s.f.)

12. $x + 7 > 5$ and $2x - 7 < 2$

$x > -2$ and $2x < 9$

$x < 4\frac{1}{2}$

$\therefore -2 < x < 4\frac{1}{2}$

Integer values of x satisfying the inequalities are $-1, 0, 1, 2, 3$ and 4.

13. $3y + 8x = 2$

$y = -\frac{8}{3}x - \frac{2}{3}$

Gradient of required line $= -1 \div -\frac{8}{3}$

$= \frac{3}{8}$

Equation of required line: $y = \frac{3}{8}x + c$

Since the line passes through $(-2, 3)$,

$3 = \frac{3}{8}(-2) + c$

$c = \frac{15}{4}$

$\therefore$ Equation of line is $y = \frac{3}{8}x + \frac{15}{4}$

14. Least possible total height $= 152.5 + 154.5 + 159.5 + 174.5$

$= 641$ cm

15. $f(x) = 4x^2 - 2x + 1$

$f(3x - 1) = 4(3x - 1)^2 - 2(3x - 1) + 1$

$= 4(9x^2 - 6x + 1) - 6x + 2 + 1$

$= 36x^2 - 24x + 4 - 6x + 3$

$= 36x^2 - 30x + 7$

Part II

Section A

1. (a) $6x^2 - x = 2$

$6x^2 - x - 2 = 0$

$(3x - 2)(2x + 1) = 0$

$x = \frac{2}{3}$ or $x = -\frac{1}{2}$

(b) $\frac{x^2}{2x + 3} - \frac{x}{2} = 1$

$2x^2 - x(2x + 3) = 2(2x + 3)$

$2x^2 - 2x^2 - 3x = 4x + 6$

$-3x = 4x + 6$

$-7x = 6$

$x = -\frac{6}{7}$

2. (a) Greatest value of $2x - y = 2(5) - 6$

$= 4$

(b) Least value of $y - 2x = 6 - 2(5)$

$= -4$

(c) Least value of $xy - x = (2)(6) - 5$

$= 7$

(d) Greatest value of $(x + y)(x - y) = x^2 - y^2$

$= 5^2 - 10^2$

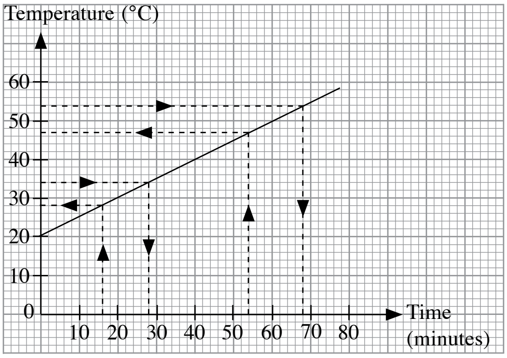
$= 25 - 100$

$= -75$

(e) Least value of $\frac{x}{y - x} = \frac{2}{10 - 2}$

$= \frac{1}{4}$

3.



From the graph,

(a) **(i)** 28 °C

(ii) 47 °C

(b) **(i)** 28 minutes

(ii) 68 minutes

(c) When t (time) = 0, temperature = 20 °C.

4. **(i)** Let the speed of the cargo train be x km/h.

$\therefore$ The speed of the passenger train is $(x + 10)$ km/h.

Time taken by cargo train = $\dfrac{250}{x}$ h

Time taken by passenger train = $\dfrac{250}{x+10}$ h

$$\frac{250}{x} - \frac{250}{x+10} = \frac{75}{60}$$

$$250(x + 10) - 250x = \frac{5}{4}(x)(x + 10)$$

$$2500 = \frac{5}{4}(x^2 + 10x)$$

$$x^2 + 10x - 2000 = 0$$

$$(x - 40)(x + 50) = 0$$

$x = 40$ or $x = -50$ (rejected)

$\therefore$ Speed of cargo train is 40 km/h.

(ii) Time taken by passenger train = $\dfrac{250}{40+10}$

$= 5$ h

Section B

5. Number of times $= \dfrac{4230 \times 10^{12}}{0.38 \times 10^{9}}$

$= 1.113 \times 10^{7}$ (to 3 d.p.)

6. **(a)** Gradient of line $AC = \dfrac{4-0}{0-(-2)}$

$= 2$

Gradient of line $AB = \dfrac{4-2}{0-4}$

$= -\dfrac{1}{2}$

(b) **(i)** Equation of line AC is $y = 2x + 4$.

(ii) Equation of line AB is $y = -\dfrac{1}{2}x + c$

Since AB passes through the point (4, 2),

$2 = -\dfrac{1}{2}(4) + c$

$2 = -2 + c$

$c = 4$

$\therefore$ Equation of line AB is $y = -\dfrac{1}{2}x + 4$

(or $2y = 8 - x$).

(iii) Equation of line is $y = 2x + d$.

Since the line passes through the point (4, 2),

$2 = 2(4) + d$

$d = -6$

$\therefore$ Equation of line is $y = 2x - 6$.

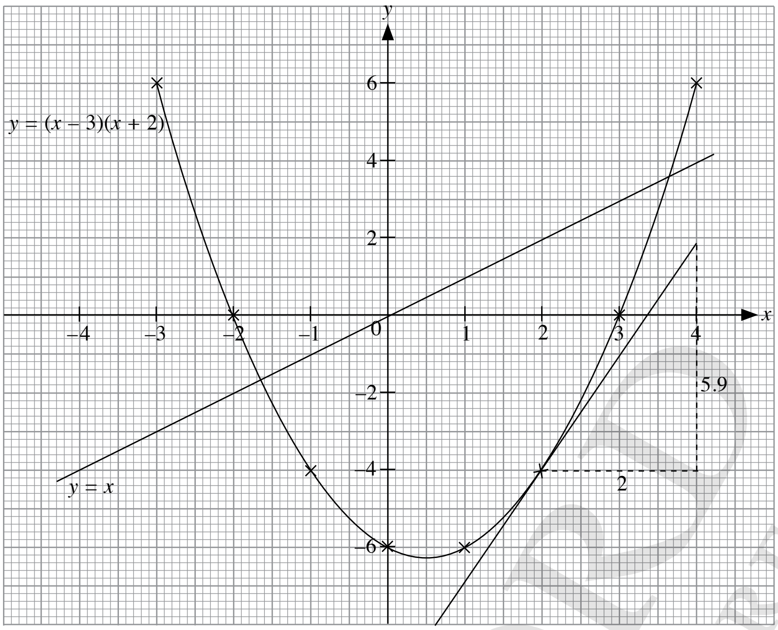
(c) D is the point (–6, 2).

(d) K is the point (6, 6).

7.

x	–3	–2	–1	0	1	2	3	4
y	6	0	–4	–6	–6	–4	0	6

(i)



(ii) From the graph, when $y \leqslant 0$, $-2 \leqslant x \leqslant 3$.

(iii) Equation of the axis of symmetry is $x = \dfrac{1}{2}$.

(iv)
$$x^2 - 2x - 6 = 0$$
$$x^2 - 2x - 6 + x = x$$
$$x^2 - x - 6 = x$$

$\therefore$ Draw the line $y = x$.

From the graph, $x = -1.6$ or $x = 3.6$.

(v) Gradient (at $x = 2$) $= \dfrac{\text{Vertical change}}{\text{Horizontal change}}$

$= \dfrac{1.9 - (-4)}{4 - 2}$

$= 2.95$

Mid-Year Examination Specimen Paper B

Part I

1. $\dfrac{x^{\frac{1}{2}} \times x^{\frac{3}{8}}}{(8x^2)^{\frac{2}{3}}} = x^{\frac{1}{2}+\frac{3}{8}} \div (8^{\frac{2}{3}}x^{\frac{4}{3}})$

$= \dfrac{x^{\frac{1}{2}+\frac{3}{8}-\frac{4}{3}}}{(\sqrt[3]{8})^2}$

$= \dfrac{1}{4}x^{-\frac{11}{24}}$

$= \dfrac{1}{4x^{\frac{11}{24}}}$

2. **(i)** Interest earned $= 8000\left(1+\dfrac{\frac{3.68}{2}}{100}\right)^{10} - 8000$

$= \$1600.06$

(ii) Interest earned $= 8000\left(1+\dfrac{3.68}{100}\right)^{5} - 8000$

$= \$1584.40$

3. Sale price $= \dfrac{75}{100} \times \$459$

$= \$344.25$

4. $f(x) = \dfrac{6x-4}{x-7}$

Let $y = \dfrac{6x-4}{x-7}$.

$y(x-7) = 6x - 4$

$xy - 7y = 6x - 4$

$xy - 6x = 7y - 4$

$x(y-6) = 7y - 4$

$x = \dfrac{7y-4}{y-6}$

$\therefore f^{-1}(x) = \dfrac{7x-4}{x-6}, x \neq 6$

5. **(a)** $2a^3b^{-5} \times (3a^{-1}b^2)^2 = 2a^3b^{-5} \times 9a^{-2}b^4$

$= 18a^{3-2}b^{-5+4}$

$= \dfrac{18a}{b}$

(b) $\dfrac{72x^5y^2}{z^7} \div \left(\dfrac{4x^4z^{-3}}{y^2}\right)^3$

$= \dfrac{72x^5y^2}{z^7} \times \left(\dfrac{y^2}{4x^4z^{-3}}\right)^3$

$= \dfrac{72x^5y^2}{z^7} \times \dfrac{y^6}{64x^{12}z^{-9}}$

$= \dfrac{9y^8}{8x^7z^{-2}}$

$= \dfrac{9y^8z^2}{8x^7}$

6. **(a)** $2x + y = 2(4.8 \times 10^{-15}) + 2.4 \times 10^{-17}$

$= 9.6 \times 10^{-15} + 2.4 \times 10^{-17}$

$= 10^{-17}(9.6 \times 10^2 + 2.4)$

$= 9.62 \times 10^{-15}$ (to 3 s.f.)

(b) $5xy = 5(4.8 \times 10^{-15})(2.4 \times 10^{-17})$

$= 57.6 \times 10^{-32}$

$= 5.76 \times 10^{-31}$

(c) $\dfrac{3x}{y} = \dfrac{3(4.8 \times 10^{-15})}{2.4 \times 10^{-17}}$

$= 6 \times 10^{-15-(-17)}$

$= 6 \times 10^2$

7. Least possible value of

(a) $x - y = -5 - 8$

$= -13$

(b) $\dfrac{4x}{y} = \dfrac{4(-5)}{3}$

$= -6\dfrac{2}{3}$

(c) $(3x + y)^2 = [3(-2) + 6]^2$

$= 0$

(d) $x^2 - y^2 = (-2)^2 - 8^2$

$= 4 - 64$

$= -60$

8. **(a)** $\dfrac{x+1}{x+2} = \dfrac{2}{x+5} + \dfrac{x}{x+3}$

$\dfrac{x+1}{x+2} = \dfrac{2(x+3)+x(x+5)}{(x+5)(x+3)}$

$(x+1)(x+5)(x+3) = (x+2)(x^2+5x+2x+6)$

$(x+1)(x^2+8x+15) = (x+2)(x^2+7x+6)$

$x^3 + 8x^2 + 15x + x^2 + 8x + 15$

$= x^3 + 7x^2 + 6x + 2x^2 + 14x + 12$

$23x + 15 = 20x + 12$

$3x = -3$

$x = -1$

(b) $\dfrac{3}{x-5} - \dfrac{2x}{3-x} = 5$

$3(3-x) - 2x(x-5) = 5(x-5)(3-x)$

$9 - 3x - 2x^2 + 10x = 5(3x - x^2 - 15 + 5x)$

$-2x^2 + 7x + 9 = -5x^2 + 40x - 75$

$3x^2 - 33x + 84 = 0$

$x^2 - 11x + 28 = 0$

$(x-4)(x-7) = 0$

$\therefore x = 4$ or $x = 7$

(c) $(4x-5)^2 = 17$

$4x - 5 = \sqrt{17}$ or $4x - 5 = -\sqrt{17}$

$x = \dfrac{5+\sqrt{17}}{4}$ $\qquad$ $x = \dfrac{5-\sqrt{17}}{4}$

$x = 2.28$ (to 2 d.p.) or $x = 0.44$ (to 2 d.p.)

9. Let the numerator be x.

$\therefore$ The denominator is $(8 - x)$ and the fraction is $\frac{x}{8-x}$.

$$\frac{x+1}{(8-x)+1} - \frac{x}{8-x} = \frac{1}{15}$$

$$\frac{x+1}{9-x} - \frac{x}{8-x} = \frac{1}{15}$$

$$\frac{(x+1)(8-x) - x(9-x)}{(9-x)(8-x)} = \frac{1}{15}$$

$$15(7x - x^2 + 8 - 9x + x^2) = 72 - 17x + x^2$$

$$x^2 - 17x + 30x + 72 - 120 = 0$$

$$x^2 + 13x - 48 = 0$$

$$(x-3)(x+16) = 0$$

$x = 3$ or $x = -16$ (rejected)

$\therefore$ The fraction is $\frac{3}{5}$.

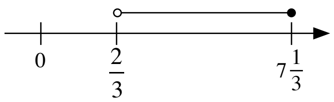
10. (a) $-5 < 3x - 7 \leqslant 15$

$-5 < 3x - 7$ and $3x - 7 \leqslant 15$

$2 < 3x$ $\qquad$ $3x \leqslant 22$

$x > \frac{2}{3}$ $\qquad$ $x \leqslant 7\frac{1}{3}$

$\therefore \frac{2}{3} < x \leqslant 7\frac{1}{3}$



(b) (i) Greatest rational number is $7\frac{1}{3}$.

(ii) Least prime number is 2.

11. (i) Gradient $= \frac{2-(-9)}{-4-2}$

$= -\frac{11}{6}$

(ii) Equation of line is $y = -\frac{11}{6}x + c$.

Since the line passes through the point $(-4, 2)$,

$2 = -\frac{11}{6}(-4) + c$

$c = -\frac{16}{3}$

$\therefore$ Equation of the line is $y = -\frac{11}{6}x - \frac{16}{3}$

(or $6y + 11x + 32 = 0$).

(iii) Gradient of required line $= -1 \div -\frac{11}{6}$

$= \frac{6}{11}$

Equation of required line: $y = \frac{6}{11}x + c$

Since the line passes through $(2, 5)$,

$5 = \frac{6}{11}(2) + c$

$c = \frac{43}{11}$

$\therefore$ Equation of line is $y = \frac{6}{11}x + \frac{43}{11}$

i.e. $11y = 6x + 43$

12. (a) $2x + 5y = 51$

Since the point $(p, 3p)$ lies on the line,

$2(p) + 5(3p) = 51$

$17p = 51$

$p = 3$

(b) (i) $2x + 3y = k$

Since the line passes through the point $(3, 2)$,

$2(3) + 3(2) = k$

$k = 12$

(ii) Gradient of $2x + 3y = 12$ is $-\frac{2}{3}$.

Gradient of $7x - hy = 97$ is $\frac{7}{h}$.

$\therefore \frac{7}{h} = -\frac{2}{3}$

$-2h = 21$

$h = -10\frac{1}{2}$

13. Interest earned from Bank $A = \frac{7500 \times 4.2 \times 10}{100}$

$= \$3150$

Interest earned from Bank $B = 7500\left(1 + \frac{\frac{4.08}{4}}{100}\right)^{40}$

$= 7500\left(1 + \frac{1.02}{100}\right)^{40}$

$= \$3755.27$

$\therefore$ Nora should invest her money in Bank B as the interest earned is higher.

Part II

Section A

1. **(a)** $8^{-\frac{1}{3}} + 625^{\frac{1}{4}} = \dfrac{1}{\sqrt[3]{8}} + \sqrt[4]{625}$

$= \dfrac{1}{2} + 5$

$= 5\dfrac{1}{2}$

(b) $36^{1.5} - 32^{-0.2} = (6^2)^{1.5} - \dfrac{1}{\sqrt[5]{32}}$

$= 6^3 - \dfrac{1}{2}$

$= 216 - \dfrac{1}{2}$

$= 215\dfrac{1}{2}$

2. **(a)** $\sqrt[3]{\dfrac{27p^6q^9}{8r^{12}}} \times \left(\dfrac{3r}{2pq^3}\right)^2$

$= \dfrac{3p^2q^3}{2r^4} \times \dfrac{9r^2}{4p^2q^6}$

$= \dfrac{27}{8r^2q^3}$

(b) $\sqrt{2\dfrac{1}{4}x^8y^2} \div \sqrt[4]{\dfrac{81}{16x^{12}y^{20}}}$

$= \dfrac{3}{2}x^4y \times \dfrac{2x^3y^5}{3}$

$= x^7y^6$

3. **(i)** $h = 9$ and $k = 2$

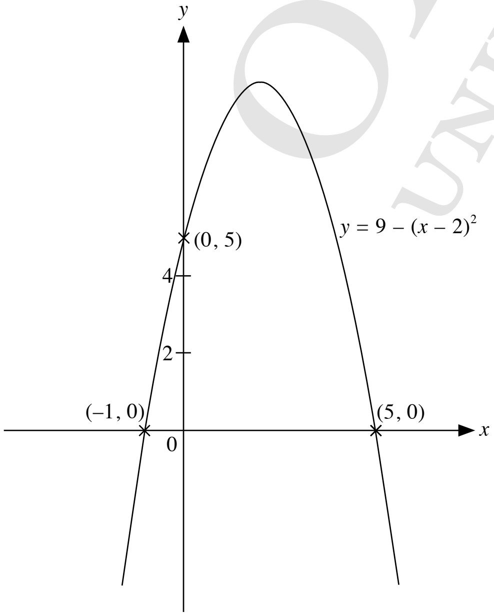
(ii) $9 - (x - 2)^2 = 0$

$(x - 2)^2 = 9$

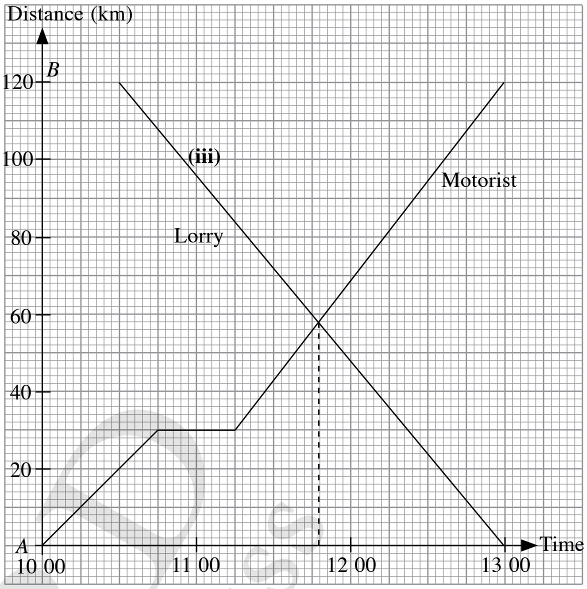
$x - 2 = 3$ or $x - 2 = -3$

$x = 5$ or $x = -1$

(iii)



4. **(i)**



(ii) Speed, x km/h $= \dfrac{90 \text{ km}}{1.75 \text{ h}}$

$= 51.42$ km/h

$\therefore x = 51.4$ km/h (to 3 s.f.)

(iv) From the graph, they met at 11 48 at a point 57 km from Town A.

Section B

5. **(i)** $\dfrac{v}{x} = \dfrac{20}{15} \Rightarrow v = \dfrac{4}{3}x$

(ii) $\dfrac{20}{t} = \dfrac{4}{5} \Rightarrow t = 25$

$\therefore$ Duration for which car is travelling at constant speed $= 80 - 15 - 25$

$= 40$ s

(iii) Total distance moved $= \dfrac{1}{2}(40 + 80) \times 20$

$= 1200$ m

$\therefore$ Average speed $= \dfrac{1200}{80}$

$= 15$ m/s

6. **(i)** Time taken $= \dfrac{380}{v}$ h

(ii) Time taken for return journey $= \dfrac{380}{v - 15}$ h

(iii) $\dfrac{380}{v-15} - \dfrac{380}{v} = \dfrac{40}{60}$

$380v - 380(v - 15) = \dfrac{2}{3}(v)(v - 15)$

$3(5700) = 2v^2 - 30v$

$v^2 - 15v - 8550 = 0$ (shown)

(iv) $v^2 - 15v - 8550 = 0$

$a = 1, b = -15, c = -8550$

$$v = \frac{-(-15) \pm \sqrt{(-15)^2 - 4(1)(-8550)}}{2(1)}$$

$$v = \frac{15 - \sqrt{34\ 425}}{2} \quad \text{or} \quad v = \frac{15 + \sqrt{34\ 425}}{2}$$

$= -85.27$ (rejected) or $= 100.269$ (to 3 d.p.)

$\therefore v = 100.27$ km/h (to 2.d.p.)

(v) Time taken from Singapore to Selangor

$= \frac{380}{100.269}$ h

$= 3.789$ h

$= 3$ h 47 min (to the nearest minute)

Time taken for return journey

$= \frac{380}{100.269 - 15}$

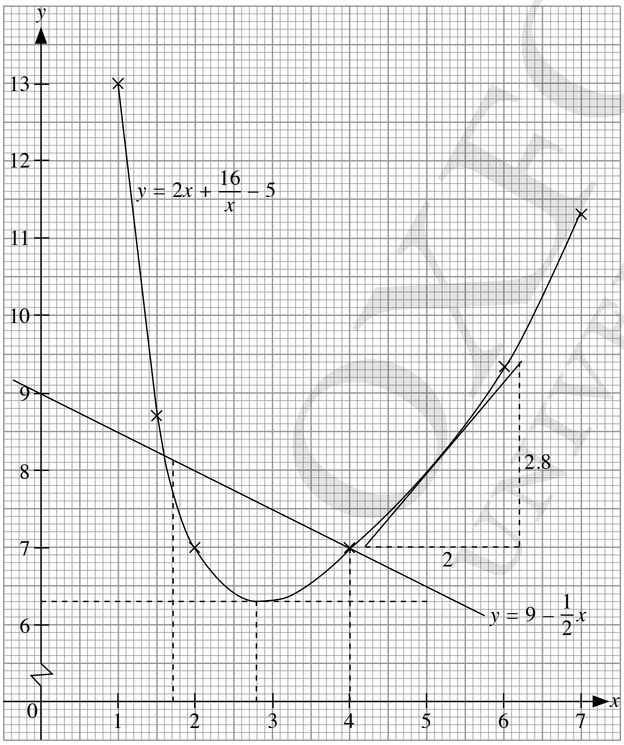
$= 4.456$ h

$= 4$ h 27 min (to the nearest minute)

7. (i) $h = 2(3) + \frac{16}{3} - 5$

$= 6.3$

(ii)



(iii) From the graph, the gradient is equal to 0 at $x = 2.8$.

(iv) At $x = 5$, gradient $= \frac{\text{vertical change}}{\text{horizontal change}}$

$= \frac{2.8}{2}$

$= 1.4$

(v) $\frac{16}{x} + 2\frac{1}{2}x = 14$

$\frac{16}{x} + 2x - 5 = 14 - 5 - \frac{1}{2}x$

$= 9 - \frac{1}{2}x$

$\therefore$ Draw $y = 9 - \frac{1}{2}x$.

From the graph, $x = 1.7$ or $x = 4$.

Chapter 8 Further Trigonometry

Basic

1. **(a)** $\sin 140° = \sin(180° - 40°)$
$= \sin 40°$
$= 0.643$

(b) $\cos 66° = -\cos(180° - 66°)$
$= -\cos 114°$
$= -(-0.407)$
$= 0.407$

(c) $2\cos 114° + 3\sin 140° = 2(-0.407) + 3(0.643)$
$= 1.115$

2. **(a)** $3\sin 55° + \cos 105° = 3(0.819) + (-0.259)$
$= 2.198$

(b) $5\cos 105° - 2\sin 55° = 5(-0.259) - 2(0.819)$
$= -2.933$

(c) $5\sin 125° - 4\sin 55° = 5(0.819) - 4(0.819)$
$= 0.819$

(d) $7\cos 75° - 3\cos 105° = 7(0.259) - 3(-0.259)$
$= 2.59$

3. **(a)** $\sin x = 0.453$
$x = 26.93°$ (to 2 d.p.) or $x = 180° - 26.93°$
$= 26.9°$ (to 1 d.p.) $= 153.1°$ (to 1 d.p.)

(b) $\sin x = 0.729$
$x = 46.80°$ (to 2 d.p.) or $x = 180° - 46.80°$
$= 46.8°$ (to 1 d.p.) $= 133.2°$ (to 1 d.p.)

(c) $\tan x = 0.568$
$x = 29.6°$ (to 1 d.p.)

(d) $\tan x = 1.387$
$x = 54.2°$ (to 1 d.p.)

(e) $\cos x = -0.763$
$x = 139.7°$ (to 1 d.p.)

(f) $\cos x = -0.624$
$x = 128.6°$ (to 1 d.p.)

4. Using Pythagoras' Theorem,
$AB^2 + BC^2 = AC^2$
$AB^2 + 5^2 = 13^2$
$AB^2 = 144$
$AB = 12$ units

(a) $3\sin\angle PAC + 2\cos\angle PAC = 3\left(\frac{5}{13}\right) + 2\left(-\frac{12}{13}\right)$
$= -\frac{9}{13}$

(b) $3\tan\angle BAC + \cos\angle ACB = 3\left(\frac{5}{12}\right) + \frac{5}{13}$
$= 1\frac{33}{52}$

(c) $\cos\angle PAC - \tan\angle ACB = -\frac{12}{13} - \frac{12}{5}$
$= -3\frac{21}{65}$

5. Let H be the point $(12, -2)$,
i.e. $QH = 8$ units and $RH = 6$ units.
Using Pythagoras' Theorem,
$QR^2 = QH^2 + RH^2$
$= 8^2 + 6^2$
$= 100$
$QR = 10$ units

(a) $\sin\angle PQR = \sin\angle RQH$
$= \frac{6}{10}$
$= \frac{3}{5}$

(b) $\cos\angle PQR = -\cos\angle RQH$
$= -\frac{8}{10}$
$= -\frac{4}{5}$

(c) $\tan\angle QPR = \frac{6}{18}$
$= \frac{1}{3}$

6. **(a)** Using Pythagoras' Theorem,
$PQ^2 = 3^2 + 4^2$
$= 25$
$PQ = 5$ units

(b) Let H be the point $(-3, 4)$.
$\sin\angle PQR = \sin\angle PQH$
$= \frac{4}{5}$
$\cos\angle PQR = -\cos\angle PQH$
$= -\frac{3}{5}$

(c) **(i)** Area of $\triangle PQR = \frac{1}{2} \times PQ \times QR \times \sin\angle PQR$
$= \frac{1}{2} \times 5 \times 6 \times \frac{4}{5}$
$= 12$ units^2

(ii) Using Cosine Rule,
$PR^2 = PQ^2 + QR^2 - 2(PQ)(QR)\cos\angle PQR$
$= 5^2 + 6^2 - 2(5)(6)\left(-\frac{3}{5}\right)$
$= 97$
$PR = 9.85$ units (to 3 s.f.)

7. Using Pythagoras' Theorem,
$AB^2 = AK^2 + BK^2$
$= 8^2 + 15^2$
$= 289$
$AB = 17$ units

(a) $\sin \angle ABC = \sin \angle ABK$

$= \frac{8}{17}$

(b) $\cos \angle ABC = -\cos \angle ABK$

$= -\frac{15}{17}$

(c) $\sin \angle ABK + \tan \angle ACB = \frac{8}{17} + \frac{8}{32}$

$= \frac{49}{68}$

8. **(a)** Area of $\triangle ABC = \frac{1}{2} \times 9.2 \times 7.6 \times \sin 56°$

$= 29.0 \text{ cm}^2$ (to 3 s.f.)

(b) Area of $\triangle PQR = \frac{1}{2} \times 13 \times 12 \times \sin 108°$

$= 74.2 \text{ cm}^2$ (to 3 s.f.)

(c) $\angle LMN = 180° - 76° - 41°$ ($\angle$ sum of a $\triangle$)

$= 63°$

Area of $\triangle LMN = \frac{1}{2} \times 6.8 \times 11.3 \times \sin 63°$

$= 34.2 \text{ cm}^2$ (to 3 s.f.)

(d) $\angle XYZ = 180° - 38° - 29°$ ($\angle$ sum of a $\triangle$)

$= 113°$

Area of $\triangle XYZ = \frac{1}{2} \times 14 \times 19 \times \sin 113°$

$= 122 \text{ cm}^2$ (to 3 s.f.)

(e) Area of $ABCD = 2 \times \frac{1}{2} \times 23 \times 15 \times \sin 114°$

$= 315 \text{ cm}^2$ (to 3 s.f.)

(f) Area of $PQRS = 2 \times \frac{1}{2} \times 9.8 \times 16.8 \times \sin 94°$

$= 164 \text{ cm}^2$ (to 3 s.f.)

9. **(a)** Using Sine Rule,

$\frac{\sin \angle B}{11} = \frac{\sin 59°}{13}$

$\sin \angle B = \frac{11 \sin 59°}{13}$

$\angle B = 46.5°$ (to 1 d.p.)

$\angle C = 180° - 59° - 46.49°$ ($\angle$ sum of a $\triangle$)

$= 74.5°$ (to 1 d.p.)

Using Cosine Rule,

$AB^2 = 11^2 + 13^2 - 2(11)(13) \cos 74.50°$

$AB = 14.6$ cm (to 3 s.f.)

(b) Using Sine Rule,

$\frac{\sin \angle P}{9.5} = \frac{\sin 84°}{15}$

$\sin \angle P = \frac{9.5 \sin 84°}{15}$

$\angle P = 39.0°$ (to 1 d.p.)

$\angle R = 180° - 84° - 39.04°$ ($\angle$ sum of a $\triangle$)

$= 57.0°$ (to 1 d.p.)

Using Cosine Rule,

$PQ^2 = 9.5^2 + 15^2 - 2(9.5)(15) \cos 56.95°$

$PQ = 12.6$ cm (to 3 s.f.)

(c) $\angle N = 180° - 46° - 73°$ ($\angle$ sum of a $\triangle$)

$= 61°$

Using Sine Rule,

$\frac{MN}{\sin 46°} = \frac{17.6}{\sin 61°}$

$MN = \frac{17.6 \sin 46°}{\sin 61°}$

$= 14.5$ cm (to 3 s.f.)

Using Sine Rule,

$\frac{LN}{\sin 73°} = \frac{17.6}{\sin 61°}$

$LN = \frac{17.6 \sin 73°}{\sin 61°}$

$= 19.2$ cm (to 3 s.f.)

(d) Using Sine Rule,

$\frac{\sin \angle X}{14} = \frac{\sin 128°}{26}$

$\sin \angle X = \frac{14 \sin 128°}{26}$

$\angle X = 25.1°$ (to 1 d.p.)

$\angle Z = 180° - 128° - 25.10°$ ($\angle$ sum of a $\triangle$)

$= 26.9°$ (to 1 d.p.)

Using Cosine Rule,

$XY^2 = 26^2 + 14^2 - 2(26)(14) \cos 26.89°$

$XY = 14.9$ cm (to 3 s.f.)

10. **(a)** Using Cosine Rule,

$BC^2 = 8.9^2 + 7.7^2 - 2(8.9)(7.7) \cos 68°$

$BC = 9.34$ cm (to 3 s.f.)

Using Sine Rule,

$\frac{\sin \angle B}{8.9} = \frac{\sin 68°}{9.335}$

$\angle B = 62.1°$ (to 1 d.p.)

$\angle C = 180° - 68° - 62.11°$ ($\angle$ sum of a $\triangle$)

$= 49.9°$ (to 1 d.p.)

(b) Using Cosine Rule,

$PR^2 = 12^2 + 19^2 - 2(12)(19) \cos 132°$

$PR = 28.5$ cm (to 3 s.f.)

Using Sine Rule,

$\frac{\sin \angle P}{19} = \frac{\sin 132°}{28.46}$

$\angle P = 29.7°$ (to 1 d.p.)

$\angle R = 180° - 132° - 29.74°$ ($\angle$ sum of a $\triangle$)

$= 18.3°$ (to 1 d.p.)

(c) Using Cosine Rule,

$LN^2 = 16^2 + 13.5^2 - 2(16)(13.5)\cos 106°$

$LN = 23.6$ cm (to 3 s.f.)

Using Sine Rule,

$\frac{\sin \angle L}{13.5} = \frac{\sin 106°}{23.60}$

$\angle L = 33.3°$ (to 1 d.p.)

$\angle N = 180° - 106° - 33.34°$ (∠ sum of a △)

$= 40.7°$ (to 1 d.p.)

(d) Using Cosine Rule,

$YZ^2 = 16.8^2 + 24.7^2 - 2(16.8)(24.7)\cos 23°$

$YZ = 11.3$ cm (to 3 s.f.)

Using Sine Rule,

$\frac{\sin \angle Z}{16.8} = \frac{\sin 23°}{11.33}$

$\angle Z = 35.4°$ (to 1 d.p.)

$\angle Y = 180° - 23° - 35.40°$ (∠ sum of a △)

$= 121.6°$ (to 1 d.p.)

Intermediate

11. $\sin x = -\cos 108°$

$= \cos 72°$

$= \sin 18°$ or $\sin 162°$

$\therefore x = 18°$ or $x = 162°$

12. **(a)** Using Pythagoras' Theorem,

$x^2 + 6^2 = 10^2$

$x^2 = 10^2 - 6^2$

$= 64$

$x = 8$

(b) $2\sin \angle ABD + \cos \angle BDC = 2\left(\frac{6}{10}\right) + \frac{6}{10}$

$= 1\frac{4}{5}$

(c) $3\cos \angle ABD + 5\cos \angle CBD = 3\left(-\frac{8}{10}\right) + 5\left(\frac{8}{10}\right)$

$= 1\frac{3}{5}$

13. **(a)** $\cos x = -\cos 40°$

$= \cos 140°$

$x = 140°$

(b) $\sin x = \sin 72°$

$x = 72°$ or $x = 180° - 72°$

$= 108°$

(c) $\cos x = -\cos 107°$

$= \cos 73°$

$x = 73°$

(d) $\cos x = \cos 126°$

$x = 126°$

(e) $\sin x = \sin 134°$

$= \sin 46°$

$x = 46°$ or $x = 134°$

(f) $\sin(180° - x) = \sin 20°$

$\sin x = \sin 20°$

$x = 20°$ or $x = 160°$

14. Using Pythagoras' Theorem,

$PR^2 = 24^2 + 7^2$

$= 625$

$PR = 25$ units

(a) $\sin P\hat{R}Q = \frac{24}{25}$

(b) $\cos S\hat{P}R = -\frac{24}{25}$

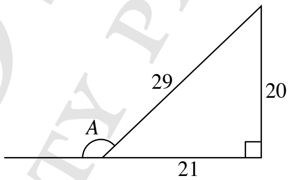
(c) $\sin S\hat{P}R + \tan P\hat{R}Q = \frac{7}{25} + \frac{24}{7}$

$= 3\frac{124}{175}$

(d) $4\cos Q\hat{P}R + 3\cos S\hat{P}R = 4\left(\frac{24}{25}\right) + 3\left(-\frac{24}{25}\right)$

$= \frac{24}{25}$

15.



(a) $\cos A = -\frac{21}{29}$

(b) $2\cos A + \tan(180° - A) = 2\left(-\frac{21}{29}\right) + \frac{20}{21}$

$= -\frac{302}{609}$

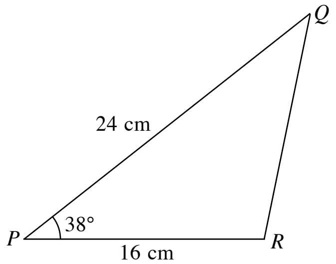
(c) $5\cos A + 4\cos(180° - A) = 5\left(-\frac{21}{29}\right) + 4\left(\frac{21}{29}\right)$

$= -\frac{21}{29}$

(d) $7\sin A - 6\sin(180° - A) = 7\left(\frac{20}{29}\right) - 6\left(\frac{20}{29}\right)$

$= \frac{20}{29}$

16.



(i) Using Cosine Rule,

$QR^2 = 24^2 + 16^2 - 2(24)(16) \cos 38°$

$QR = 15.1$ cm (to 3 s.f.)

(ii) Using Sine Rule,

$$\frac{\sin PQR}{16} = \frac{\sin 38°}{15.06}$$

$$\sin \angle PQR = \frac{16 \sin 38°}{15.06}$$

$\angle PQR = 40.9°$ (to 1 d.p.)

17. Using Cosine Rule,

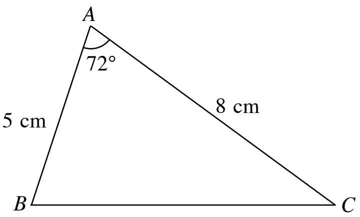
$QR^2 = PQ^2 + PR^2 - 2(PQ)(PR) \cos Q\hat{P}R$

$12^2 = 11.8^2 + 9.8^2 - 2(11.8)(9.8) \cos Q\hat{P}S$

$$\cos \angle QPR = \frac{11.8^2 + 9.8^2 - 12^2}{2(11.8)(9.8)}$$

$\angle QPR = 66.8°$ (to 1 d.p.)

18.



(i) Using Cosine Rule,

$BC^2 = 5^2 + 8^2 - 2(5)(8) \cos 72°$

$BC = 8.02$ cm (to 3 s.f.)

(ii) Using Sine Rule,

$$\frac{\sin ABC}{8} = \frac{\sin 72°}{8.017}$$

$$\sin \angle ABC = \frac{8 \sin 72°}{8.017}$$

$\angle ABC = 71.6°$ (to 1 d.p.)

19. Using Cosine Rule,

$y^2 = x^2 + z^2 - 2xz \cos Y$

$6.9^2 = 7.8^2 + 11.2^2 - 2(7.8)(11.2) \cos Y$

$$\cos Y = \frac{7.2^2 + 11.2^2 - 6.9^2}{2(7.8)(11.2)}$$

$Y = 37.5°$ (to 1 d.p.)

$$\text{Area of } \triangle XYZ = \frac{1}{2}(7.8)(11.2) \sin 37.47°$$

$= 26.6 \text{ cm}^2$ (to 3 s.f.)

20. (i) Using Cosine Rule,

$21^2 = 14^2 + 15^2 - 2(14)(15) \cos \angle BAC$

$$\cos B\hat{A}C = \frac{14^2 + 15^2 - 21^2}{2(14)(15)}$$

$$= -\frac{1}{21}$$

(ii) Since $\cos B\hat{A}C = -\frac{1}{21}$,

$B\hat{A}C = 92.72°$ (to 2 d.p.)

$$\text{Area of } \triangle ABC = \frac{1}{2}(14)(15) \sin 92.72°$$

$= 105 \text{ cm}^2$ (to 3 s.f.)

21. (i) Using Cosine Rule,

$AD^2 = 9^2 + 20^2 - 2(9)(20) \cos 128°$

$AD = 26.5$ cm (to 3 s.f.)

(ii) $\angle DBC = 180° - 128°$ (adj. $\angle$s on a str. line)

$= 52°$

Using Sine Rule,

$$\frac{\sin \angle BCD}{20} = \frac{\sin 52°}{17.5}$$

$$\sin \angle BCD = \frac{20 \sin 52°}{17.5}$$

$\angle BCD = 64.23°$ (to 2 d.p.) or $B\hat{C}D = 180° - 64.23° = 115.76°$ (to 2 d.p.)

$\angle BDC = 180° - 52° - 115.76°$

$= 12.2°$ (to 1 d.p.)

(iii) Using Cosine Rule,

$BC^2 = 20^2 + 17.5^2 - 2(20)(17.5) \cos 12.23°$

$BC = 4.71$ cm (to 3 s.f.)

(iv) $$\frac{\text{Area of } \triangle ABD}{\text{Area of } \triangle BCD} = \frac{AB}{BC}$$

$$= \frac{9}{4.71}$$

$= 1.91$ (to 3s.f.)

22. (i) $\angle ADB = 180° - 27° - 136°$ ($\angle$ sum of a $\triangle$)

$= 17°$

Using Sine Rule,

$$\frac{AB}{\sin 17°} = \frac{9}{\sin 27°}$$

$$AB = \frac{9 \sin 17°}{\sin 27°}$$

$= 5.80$ cm (to 3 s.f.)

(ii) Using Sine Rule,

$$\frac{AD}{\sin 136°} = \frac{9}{\sin 27°}$$

$$AD = \frac{9 \sin 136°}{\sin 27°}$$

$= 13.8$ cm (to 3 s.f.)

(iii) $\angle DBC = 180° - 136°$ (adj. $\angle$s on a str. line)

$= 44°$

Using Cosine Rule,

$CD^2 = 9^2 + 5^2 - 2(9)(5) \cos 44°$

$CD = 6.42$ cm (to 3 s.f.)

(iv) $$\text{Area of } \triangle ACD = \frac{1}{2}(5.796)(9) \sin 136° + \frac{1}{2}(9)(5) \sin 44°$$

$= 33.7 \text{ cm}^2$ (to 3 s.f.)

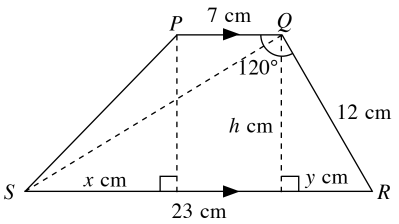
23. Let the perpendicular distance from A to BD be h cm.

$$\text{Area of } \triangle ACD = \frac{1}{2}(24)h$$
$$178 = 12h$$
$$h = 14\frac{5}{6} \text{ cm}$$
$$\text{Area of } \triangle ABC = \frac{1}{2}(36)\left(14\frac{5}{6}\right)$$
$$= 267 \text{ cm}^2$$

24. $$\text{Area of } PQBC = \frac{1}{2}(21.7)(15.9)\sin 74° - \frac{1}{2}(15)(5.5)\sin 74°$$
$$= 126 \text{ cm}^2 \text{ (to 3 s.f.)}$$

Using Cosine Rule,
$$BC^2 = 21.7^2 + 15.9^2 - 2(21.7)(15.9)\cos 74°$$
$$BC = 23.1 \text{ cm (to 3 s.f.)}$$

25. **(i)**



Let the distance between PQ and SR be h cm.

$\angle QRS = 180° - 120°$ (int. ∠s, PQ // SR)
$= 60°$

$$\sin 60° = \frac{h}{12}$$
$$h = 12 \sin 60°$$
$$= 10.4 \text{ cm (to 3 s.f.)}$$

(ii) Using Cosine Rule,
$$SQ^2 = 23^2 + 12^2 - 2(23)(12)\cos 60°$$
$$SQ = 19.9 \text{ cm (to 3 s.f.)}$$

(iii) $$\cos 60° = \frac{y}{12}$$
$$y = 12\cos 60°$$
$$= 6$$
$$x = 23 - 7 - 6$$
$$= 10$$
$$\tan \angle PSR = \frac{10.39}{10}$$
$$\angle PSR = 46.1° \text{ (to 1 d.p.)}$$

(iv) Using Pythagoras' Theorem,
$$PS^2 = 10.49^2 + 10^2$$
$$PS = 14.4 \text{ cm (to 3 s.f.)}$$

26. Using Cosine Rule,
$$BC^2 = 8^2 + 7^2 - 2(8)(7)\left(\frac{11}{16}\right)$$
$$BC = 6 \text{ cm}$$

27. **(i)** $\angle ACB = 180° - 31° - 36° - 44°$ (∠ sum of a △)
$= 69°$

Using Sine Rule,
$$\frac{BC}{\sin 31°} = \frac{80}{\sin 69°}$$
$$BC = \frac{80 \sin 31°}{\sin 69°}$$
$$= 44.1 \text{ cm (to 3 s.f.)}$$

(ii) $\angle ADB = 180° - 44° - 31° - 37°$ (∠ sum of a △)
$= 68°$

Since △ABD is an isosceles triangle, $BD = 80$ cm.

(iii) Using Cosine Rule,
$$CD^2 = 44.13^2 + 80^2 - 2(44.13)(80)\cos 36°$$
$$CD = 51.3 \text{ cm (to 3 s.f.)}$$

28. **(i)** $$\sin \angle PSR = \frac{5}{8}$$
$$\angle PSR = 38.7° \text{ (to 1 d.p.)}$$

(ii) $$\text{Area of } \triangle PQR = \frac{1}{2}(3)(5)\sin 55°$$
$$= 6.1 \text{ cm}^2 \text{ (to 1 d.p.)}$$

29. **(i)** Using Cosine Rule,
$$AC^2 = 58^2 + 35^2 - 2(58)(35)\cos 82°$$
$$AC = 63.4 \text{ m (to 3 s.f.)}$$

(ii) Using Sine Rule,
$$\frac{\sin ADC}{63.43} = \frac{\sin 48°}{60}$$
$$\sin \angle ADC = \frac{63.43 \sin 48°}{60}$$
$$\angle ADC = 51.78° \text{ (to 2 d.p.)}$$
$$\angle DAC = 180° - 48° - 51.78°$$
$$= 80.2° \text{ (to 1 d.p.)}$$

(iii) $$\text{Area of } \triangle ACD = \frac{1}{2}(60)(63.43)\sin 80.21°$$
$$= 1880 \text{ m}^2 \text{ (to 3 s.f.)}$$

30. $\angle RQP = 180° - 50°$ (adj. ∠s on a str. line)
$= 130°$

Using Cosine Rule,
$$PR^2 = 4^2 + 6^2 - 2(4)(6)\cos 130°$$
$$PR = 9.10 \text{ cm (to 3 s.f.)}$$

Using Sine Rule,
$$\frac{\sin PRQ}{4} = \frac{\sin 130°}{9.102}$$
$$\sin \angle PRQ = \frac{4 \sin 130°}{9.102}$$
$$\angle PRQ = 19.67° \text{ (to 2 d.p.)}$$

Using Sine Rule,

$$\frac{\sin QSR}{6} = \frac{\sin 50°}{8}$$

$$\sin \angle QSR = \frac{6 \sin 50°}{8}$$

$\angle QSR = 35.06°$ (to 2 d.p.)

$\angle QRS = 180° - 50° - 35.06°$ ($\angle$ sum of a $\triangle$)

$= 94.39°$ (to 2 d.p.)

$\angle PRS = 19.67° + 94.93°$

$= 114.6°$ (to 1 d.p.)

31. Let the radius of the circle be r cm.

Using Pythagoras' Theorem,

$(r - 8)^2 + 12^2 = r^2$

$r^2 - 16r + 64 + 144 = r^2$

$16r = 208$

$r = 13$

$\therefore$ Radius of circle = 13 cm

$$\sin \angle AOB = \frac{12}{13}$$

$\angle AOB = 67.38°$ (to 2 d.p.)

$$\text{Area of } \triangle AOB = \frac{1}{2}(13)(13) \sin 67.38°$$

$= 78 \text{ cm}^2$

32. (i) $\tan \angle PAB = \dfrac{7.2}{22.4}$

$\angle PAB = 17.81°$ (to 2 d.p.)

$\angle CAB = 2(17.81°)$

$= 35.63°$ (to 2 d.p.)

$$\tan 35.63° = \frac{CP + 7.2}{22.4}$$

$CP = 22.4 \tan 35.63° - 7.2$

$= 8.86$ cm (to 3 s.f.)

(ii) $\angle ACB = 180° - 90° - 35.63°$ ($\angle$ sum of a $\triangle$)

$= 54.36°$ (to 2 d.p.)

Using Cosine Rule,

$PQ^2 = 6.8^2 + 8.859^2 - 2(6.8)(8.859) \cos 54.36°$

$PQ = 7.38$ cm (to 3 s.f.)

(iii) Using Sine Rule,

$$\frac{\sin CQP}{8.859} = \frac{\sin 54.36°}{7.383}$$

$$\sin \angle CQP = \frac{8.859 \sin 54.36°}{7.383}$$

$\angle CQP = 77.18°$ (to 2 d.p.)

$\angle AQP = 180° - 77.18°$ (adj. $\angle$s on a str. line)

$= 102.8°$ (to 1 d.p.)

(iv) Using Pythagoras' Theorem,

$AC^2 = 22.4^2 + (7.2 + 8.859)^2$

$AC = 27.56$ cm (to 4 s.f.)

$AQ = 27.56 - 6.8$

$= 20.76$ cm (to 4 s.f.)

$$\text{Area of } \triangle APQ = \frac{1}{2}(20.76)(7.383) \sin 102.81°$$

$= 74.7 \text{ cm}^2$ (to 3 s.f.)

Advanced

33. (i) Using Cosine Rule,

$(\sqrt{127})^2 = (2x - 1)^2 + (x + 2)^2$
$- 2(2x - 1)(x + 2) \cos 120°$

$127 = 4x^2 - 4x + 1 + x^2 + 4x + 4 + 2x^2$
$+ 4x - x - 2$

$7x^2 + 3x - 124 = 0$ (shown)

(ii) $7x^2 + 3x - 124 = 0$

$(7x + 31)(x - 4) = 0$

$x = -4\frac{3}{7}$ or $x = 4$

$x = -4\frac{3}{7}$ is rejected since length cannot take a negative value.

(iii) $AB = 7$ cm

$BC = 6$ cm

$$\text{Area of } \triangle ABC = \frac{1}{2}(7)(6) \sin 120°$$

$= 18.2 \text{ cm}^2$ (to 3 s.f.)

New Trend

34. Let the angle be x.

$\sin x = 0.672$

$x = 42.2°$ (to 1 d.p.)

Obtuse $\angle x = 180° - 42.2°$

$= 137.8°$

$\therefore$ Two possible angles are 42.2° and 137.8°.

35. (i) Using Cosine Rule,

$AC^2 = 70^2 + 80^2 - 2(70)(80) \cos 120°$

$AC = 130$ m

(ii) $\text{Area of } \triangle ABC = \dfrac{1}{2}(70)(80) \sin 120°$

$= 2420 \text{ m}^2$ (to 3 s.f.)

(iii) Using Cosine Rule,

$142^2 = 130^2 + 130^2 - 2(130)(130) \cos A\hat{C}D$

$$\cos \angle ACD = \frac{130^2 + 130^2 - 142^2}{2(130)(130)}$$

$\angle ACD = 66.2°$ (to 1 d.p.)

36. **(a)** **(i)** $\angle ADB = 180° - 103° - 42°$ (alt. $\angle$s, AB // DC,
$= 35°$ $\angle$ sum of a $\triangle$)

(ii) Using Sine Rule,

$$\frac{BD}{\sin 103°} = \frac{14}{\sin 35°}$$

$$BD = \frac{14 \sin 103°}{\sin 35°}$$

$= 23.8$ mm (to 3 s.f.)

(iii) Area of $ABCD$

$$= \left(\frac{1}{2} \times 14 \times 23.783 \times \sin 42°\right) + \left(\frac{1}{2} \times 19 \times 23.783 \times \sin 42°\right)$$

$= 262.57\,9898$

$= 263$ mm^2 (to 3 s.f.)

$= 2.63$ cm^2

(b) Let A_2 cm^2 be the area of the enlarged school badge.

19 mm = 1.9 cm

0.95 m = 95 cm

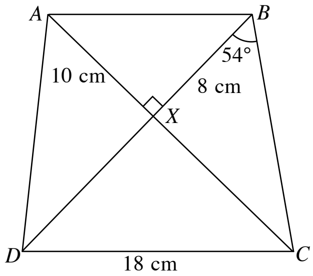
$$\left(\frac{1.9}{95}\right)^2 = \frac{2.6258}{A_2}$$

$$\frac{2.6258}{A_2} = \frac{1}{2500}$$

$A_2 = 6560$ (to 3 s.f.)

$\therefore$ The area of the enlarged school badge is 6560 cm^2.

37.



(i) $\tan 54° = \frac{XC}{8}$

$XC = 8 \tan 54°$

$= 11.0$ cm (to 3 s.f.)

$\cos 54° = \frac{8}{BC}$

$BC = \frac{8}{\cos 54°}$

$= 13.6$ cm (to 3 s.f.)

(ii) $\tan \angle BAX = \frac{8}{10}$

$\angle BAX = 38.7°$ (to 1 d.p.)

(iii) Using Sine Rule,

$$\frac{\sin BDC}{13.61} = \frac{\sin 54°}{18}$$

$$\sin \angle BDC = \frac{13.61 \sin 54°}{18}$$

$\angle BDC = 37.7°$ (to 1 d.p.)

38. **(a)** $\cos y° = -\frac{6}{x}$

(b) Using similar triangles, $\left(\frac{AC}{6}\right)^2 = \frac{1}{3}$

$$\frac{AC^2}{36} = \frac{1}{3}$$

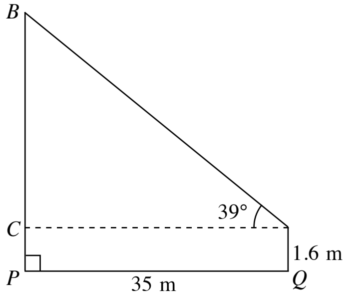
$AC^2 = 12$

$AC = 3.46$ cm (to 3 s.f.)

Chapter 9 Applications of Trigonometry

Basic

1.



$\tan 39° = \dfrac{BC}{35}$

$BC = 35 \tan 39°$

$= 28.34$ m (to 4 s.f.)

$\therefore$ Height of the tree $= 28.34 + 1.6$

$= 29.9$ m (to 3 s.f.)

2. $\tan 18° = \dfrac{72}{x}$

$x = \dfrac{72}{\tan 18°}$

$= 222$ (to 3 s.f.)

3. **(a)** **(i)** $101° + 49° = 150°$ (ext. $\angle$ = sum of int. opp. $\angle$s)

Bearing of Q from $R = 150°$

(ii) $180° + 49° + 101° = 330°$

Bearing of R from $Q = 330°$

(b) Using Sine Rule,

$\dfrac{PR}{\sin 101°} = \dfrac{1.45}{\sin (180° - 49° - 101°)}$

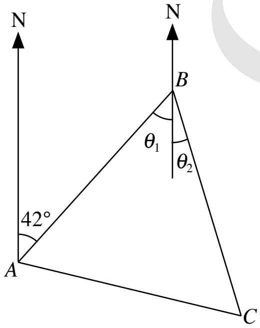
$PR = \dfrac{1.45 \sin 101°}{\sin 30°}$

$= 2.85$ km (to 3 s.f.)

4. **(i)** $42° + 60° = 102°$

Bearing of C from A is 102°

(ii)



$\theta_1 = 42°$ (alt. $\angle$s)

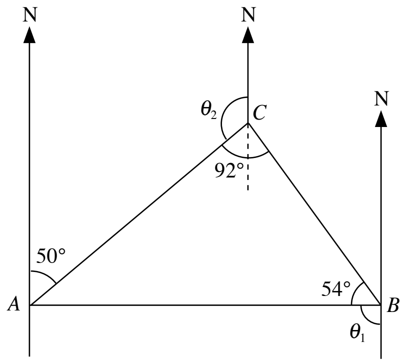
$\theta_2 = 60° - 42°$

$= 18°$

$180° - 18° = 162°$ ($\angle$s on a str. line)

Bearing of C from B is 162°.

5.



(a) $\angle CAB = 180° - 92° - 54°$ ($\angle$ sum of a $\triangle$)

$= 34°$

$50° + 34° = 84°$

Bearing of B from A is 084°.

(b) Bearing of C from A is 050°.

(c) $\theta_1 = 84°$

$180° + 84° = 264°$

Bearing of A from B is 264°.

(d) $264° + 54° = 318°$

Bearing of C from B is 318°.

(e) $180° + 50° = 230°$

Bearing of A from C is 230°.

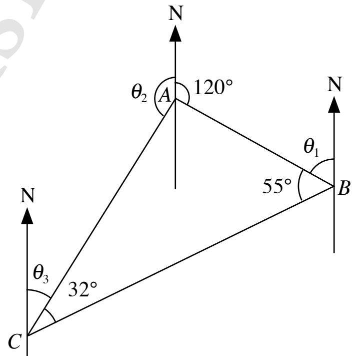
(f) $\theta_2 = 180° - 50°$ (int. $\angle$s)

$= 130°$

$360° - 130° - 92° = 138°$ ($\angle$s at a pt.)

Bearing of B from C is 138°.

6.



(a) Bearing of B from A is 120°.

(b) $\angle CAB = 180° - 55° - 32°$ ($\angle$ sum of a $\triangle$)

$= 93°$

$120° + 93° = 213°$

Bearing of C from A is 213°.

(c) $\theta_1 = 180° - 120°$ (int. $\angle$s)

$= 60°$

$360° - 60° = 300°$ ($\angle$s at a pt.)

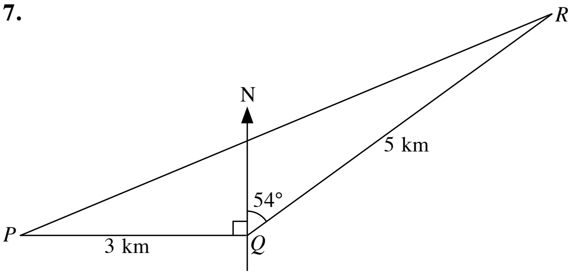
Bearing of A from B is 300°.

(d) $360° - 55° - 60° = 245°$ (∠s at a pt.)
Bearing of C from B is 245°.

(e) $\theta_2 = 360° - 120° - 93°$ (∠s at a pt.)
$= 147°$
$\theta_3 = 180° - 147°$ (int. ∠s)
$= 33°$
Bearing of A from C is 033°.

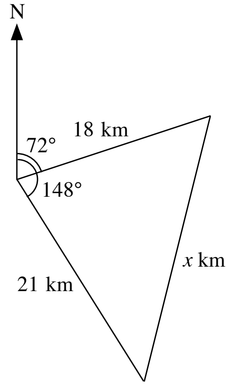
(f) $33° + 32° = 65°$
Bearing of B from C is 065°.

7.



Using Cosine Rule,
$PR^2 = 3^2 + 5^2 - 2(3)(5)\cos 144°$
$PR = 7.63$ km (to 3 s.f.)
Using Sine Rule,
$$\frac{\sin \angle RPQ}{5} = \frac{\sin 144°}{7.633}$$
$$\sin \angle RPQ = \frac{5 \sin 144°}{7.633}$$
$\angle RPQ = 22.64°$ (to 2 d.p.)
$90° - 22.64° = 67.4°$ (to 1 d.p.)
Bearing of R from P is 067.4°.

8.



Using Cosine Rule,
$x^2 = 18^2 + 21^2 - 2(18)(21)\cos 76°$
$x = 24.1$ (to 3 s.f.)
They will be 24.1 km apart.

Intermediate

9. $\tan 24° = \frac{65}{x}$
$$x = \frac{65}{\tan 24°}$$
$= 146$ (to 3 s.f.)
Let the height of the lighthouse be h m.
$$\tan 32° = \frac{h + 65}{145.9}$$
$h = 145.9 \tan 32° - 65$
$= 26.2$ (to 3 s.f.)
∴ Height of lighthouse is 26.2 m.

10. Let $PH = x$ m and $TH = h$ m.
$$\tan 31° = \frac{h}{x}$$
$$x = \frac{h}{\tan 31°} \quad \text{— (1)}$$
$$\tan 18° = \frac{h}{x + 28}$$
$$x \tan 18° + 28 \tan 18° = h \quad \text{— (2)}$$
Substitute (1) into (2):
$$\left(\frac{\tan 18°}{\tan 31°}\right)h + 28 \tan 18° = h$$
$$h - \left(\frac{\tan 18°}{\tan 31°}\right)h = 28 \tan 18°$$
$$\left(1 - \frac{\tan 18°}{\tan 31°}\right)h = 28 \tan 18°$$
$$h = \frac{28 \tan 18°}{1 - \frac{\tan 18°}{\tan 31°}}$$
$= 19.8$ (to 3 s.f.)
∴ Height of the building is 19.8 m.

11. (i) Let $QR = x$ m and $TR = h$ m.
$$\tan 38° = \frac{h}{x}$$
$$x = \frac{h}{\tan 38°} \quad \text{— (1)}$$
$$\tan 27° = \frac{h}{x + 580}$$
$$x \tan 27° + 580 \tan 27° = h \quad \text{— (2)}$$

Substitute (1) into (2):

$$\left(\frac{\tan 27°}{\tan 38°}\right)h + 580 \tan 27° = h$$

$$h - \left(\frac{\tan 27°}{\tan 38°}\right)h = 580 \tan 27°$$

$$\left(1 - \frac{\tan 27°}{\tan 38°}\right)h = 580 \tan 27°$$

$$h = \frac{580 \tan 27°}{1 - \frac{\tan 27°}{\tan 38°}}$$

$$= 850 \text{ (to 3 s.f.)}$$

$\therefore TR = 850$ m

(ii) Substitute $h = 849.6$ into (1):

$$x = \frac{849.6}{\tan 38°}$$

$$= 1090 \text{ (to 3 s.f.)}$$

$\therefore QR = 1090$ m

12. (i) Let $BC = x$ m and $PB = h$ m.

$$\tan 22° = \frac{h}{x}$$

$$h = x \tan 22° \quad \text{— (1)}$$

$$\tan 28° = \frac{h + 20}{x}$$

$$x \tan 28° = h + 20 \quad \text{— (2)}$$

Substitute (1) into (2):

$$x \tan 28° = x \tan 22° + 20$$

$$x \tan 28° - x \tan 22° = 20$$

$$x(\tan 28° - \tan 22°) = 20$$

$$x = \frac{20}{\tan 28° - \tan 22°}$$

$$= 157 \text{ (to 3 s.f.)}$$

$\therefore BC = 157$ m

(ii) Substitute $x = 156.6$ into (1):

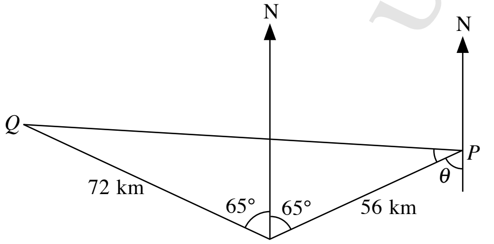
$$h = 156.6 \tan 22°$$

$$= 63.28 \text{ (to 4 s.f.)}$$

$$\therefore AB = 63.28 + 20$$

$$= 83.3 \text{ m (to 3 s.f.)}$$

13. (i)



Using Cosine Rule,

$$PQ^2 = 72° + 56^2 - 2(72)(56) \cos 127°$$

$$PQ = 115 \text{ km (to 3 s.f.)}$$

The ships are 115 km apart.

(ii) Using Sine Rule,

$$\frac{\sin \angle P}{72} = \frac{\sin 127°}{114.7}$$

$$\sin \angle P = \frac{72 \sin 127°}{114.7}$$

$$\angle P = 30.06° \text{ (to 2 d.p.)}$$

$$\theta = 65° \text{ (alt. } \angle\text{s)}$$

$$180° + 65° + 30.06° = 275.1° \text{ (to 1 d.p.)}$$

Bearing of Q from P is 275.1°.

14. In $\triangle CPB$,

$$\angle CPB = 17°$$

$$\angle PCB = 30° \text{ (alt. } \angle\text{s)}$$

Using Sine Rule,

$$\frac{BC}{\sin 17°} = \frac{1200}{\sin 30°}$$

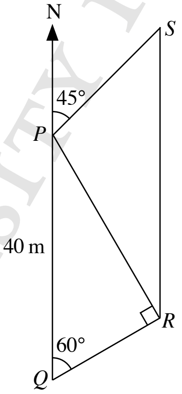
$$BC = \frac{1200 \sin 17°}{\sin 30°}$$

$$= 701.6 \text{ m (to 4 s.f.)}$$

$$x = \frac{\frac{701.6}{1000}}{\frac{15}{60}}$$

$$= 2.81 \text{ (to 3 s.f.)}$$

15.



(i)

$$\cos 60° = \frac{QR}{40}$$

$$QR = 40 \cos 60°$$

$$= 20 \text{ m}$$

$$\sin 60° = \frac{PR}{40}$$

$$PR = 40 \sin 60°$$

$$= 34.6 \text{ m}$$

$$\angle PRS = 180° - 60° - 90° \text{ (int. } \angle\text{s, } QP \parallel RS)$$

$$= 30°$$

$$\angle PSR = 45° \text{ (alt. } \angle\text{s, } QP \parallel RS)$$

Using Sine Rule,

$$\frac{PS}{\sin 30°} = \frac{34.64}{\sin 45°}$$

$$PS = \frac{34.64 \sin 30°}{\sin 45°}$$

$= 24.5$ m (to 3 s.f.)

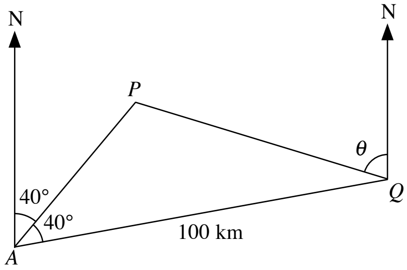
(ii) $\angle QPS = 180° - 45°$ ($\angle$s on a str. line)

$= 135°$

Area of $\triangle PQS = \frac{1}{2}(40)(24.49) \sin 135°$

$= 346 \text{ m}^2$ (to 3 s.f.)

16.



(i) Using Cosine Rule,

$PQ^2 = 50^2 + 100^2 - 2(50)(100) \cos 40°$

$PQ = 69.6$ km (to 3 s.f.)

(ii) Using Sine Rule,

$$\frac{\sin \angle PQA}{50} = \frac{\sin 40°}{69.56}$$

$$\sin \angle PQA = \frac{50 \sin 40°}{69.56}$$

$\angle PQA = 27.5°$ (to 1 d.p.)

(iii) $\theta = 180° - 80° - 27.51°$ (int. $\angle$s)

$= 72.48°$ (to 2 d.p.)

$360° - 72.48° = 287.5°$ ($\angle$s at a pt.) (to 1 d.p.)

Bearing of P from Q is $287.5°$.

17. (i) Using Cosine Rule,

$AC^2 = 70^2 + 80^2 - 2(70)(80) \cos 115°$

$AC = 127$ m (to 3 s.f.)

(ii) Using Cosine Rule,

$126.6^2 = 190^2 + 110^2 - 2(190)(110) \cos \angle ADC$

$$\cos \angle ADC = \frac{190^2 + 110^2 - 126.6^2}{2(190)(110)}$$

$\angle ADC = 39.7°$ (to 1 d.p.)

(iii) Using Sine Rule,

$$\frac{\sin \angle ACB}{70} = \frac{\sin 115°}{126.6}$$

$$\sin \angle ACB = \frac{70 \sin 115°}{126.6}$$

$\angle ACB = 30.1°$ (to 1 d.p.)

(iv) $\angle BAC = 180° - 115° - 30.06°$ ($\angle$ sum of a $\triangle$)

$= 34.93°$ (to 2 d.p.)

Using Sine Rule,

$$\frac{\sin \angle CAD}{110} = \frac{\sin 39.68°}{126.6}$$

$$\sin \angle CAD = \frac{110 \sin 39.68°}{126.6}$$

$\angle CAD = 33.69°$ (to 2 d.p.)

$90° + 33.69° + 34.93° = 158.6°$ (to 1 d.p.)

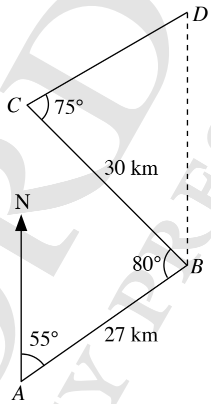
Bearing of B from A is $158.6°$.

(v) Area of $ABCD$

$= \frac{1}{2}(70)(80) \sin 115° + \frac{1}{2}(190)(110) \sin 39.68°$

$= 9210 \text{ m}^2$ (to 3 s.f.)

18.



(i) $\angle DBC = 180° - 80° - 55°$ (int. $\angle$s)

$= 45°$

$360° - 45° = 315°$ ($\angle$s at a pt.)

Bearing of C from B is $315°$.

(ii) Using Cosine Rule,

$AC^2 = 30^2 + 27^2 - 2(30)(27) \cos 80°$

$AC = 36.7$ km (to 3 s.f.)

(iii) Using Sine Rule,

$$\frac{CD}{\sin 45°} = \frac{30}{\sin(180° - 45° - 75°)}$$

$$CD = \frac{30 \sin 45°}{\sin 60°}$$

$= 24.49$ km (to 4 s.f.)

Time taken to sail from B to D

$= \frac{30}{60} + \frac{30}{12} + \frac{45}{60} + \frac{24.49}{14}$

$= 5.499$ h (to 4 s.f.)

$= 5$ h 30 min (to the nearest minute)

The ship reached port D at 16 45.

(iv) Using Cosine Rule,

$BD^2 = 24.49^2 + 30^2 - 2(24.49)(30) \cos 75°$

$BD = 33.5$ km (to 3 s.f.)

(v) Using Cosine Rule,

$AD^2 = 27^2 + 33.46^2 - 2(27)(33.46)\cos 125°$

$AD = 53.71$ km (to 4 s.f.)

Using Sine Rule,

$\frac{\sin \angle DAB}{33.46} = \frac{\sin 125°}{53.71}$

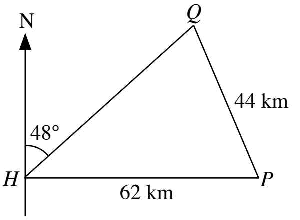
$\sin \angle DAB = \frac{33.46 \sin 125°}{53.71}$

$\angle DAB = 30.68°$ (to 2 d.p.)

$55° - 30.68° = 24.3°$ (to 1 d.p.)

Bearing of D from A is 024.3°.

19.



(a) (i) $\angle QHP = 90° - 48°$

$= 42°$

Using Sine Rule,

$\frac{\sin \angle HQP}{62} = \frac{\sin 42°}{44}$

$\sin \angle HQP = \frac{62 \sin 42°}{44}$

$\angle HQP = 70.53°$ (to 2 d.p.)

$\angle QPH = 180° - 42° - 70.53°$ ($\angle$ sum of a $\triangle$)

$= 67.46°$ (to 2 d.p.)

$270° + 67.46° = 337.5°$ (to 1 d.p.)

Bearing of Q from P is 337.5°.

(ii) Using Cosine Rule,

$HQ^2 = 62^2 + 44^2 - 2(62)(44)\cos 67.46°$

$HQ = 60.7$ km (to 3 s.f.)

(b) Time taken $= 2\left(\frac{45}{15}\right) + \frac{40}{60}$

$= 6\frac{2}{3}$ h

$= 6$ h 40 minutes

It returns to H At 17 55.

(c) Using Cosine Rule,

$45^2 = 61^2 + 60.73^2 - 2(61)(60.73)\cos \angle HQR$

$\cos \angle HQR = \frac{62^2 + 60.73^2 - 45^2}{2(61)(60.73)}$

$\angle HQR = 43.4°$ (to 1 d.p.)

(d) Let the shortest distance from R to HQ to x km.

$\sin 43.38° = \frac{x}{61}$

$x = 61 \sin 43.38°$

$= 41.9$ (to 3 s.f.)

The shortest distance from R to HQ is 41.9 km.

(e) Area of $HPQR = \frac{1}{2}(60.73)(41.90)$

$+ \frac{1}{2}(62)(44)\sin 67.46°$

$= 2530$ km^2 (to 3 s.f.)

20. (i) Using Cosine Rule,

$DF^2 = 1^2 + 1^2 - 2(1)(1)\cos 50°$

$DF = 0.845$ m (to 3 s.f.)

(ii) Using Pythagoras' Theorem,

$BD^2 = 1^2 + 2^2$

$BD = 2.24$ m (to 3 s.f.)

(iii) Using Cosine Rule,

$0.8452^2 = 2.236^2 + 2.236^2$

$- 2(2.236)(2.236)\cos \angle DBF$

$\cos \angle DBF = \frac{2.236^2 + 2.236^2 - 0.8452^2}{2(2.236)(2.236)}$

$\angle DBF = 21.8°$ (to 1 d.p.)

21. (i) $AB = AC = 12$ m

Using Pythagoras' Theorem,

$AT^2 + 12^2 = 16^2$

$AT^2 = 16^2 - 12^2$

$AT = 10.6$ m (to 3 s.f.)

(ii) $\tan \angle TCA = \frac{10.58}{12}$

$\angle TCA = 41.4°$ (to 1 d.p.)

$\therefore$ Angle of elevation is 41.4°.

(iii) Area of $\triangle ABC = \frac{1}{2}(12)(12)\sin 120°$

$= 62.4$ m^2 (to 3 s.f.)

22. (i) $BP = AQ = 32$ cm

Using Pythagoras' Theorem,

$CP^2 + 22^2 = 32^2$

$CP^2 = 32^2 - 22^2$

$CP = 23.2$ cm (to 3 s.f.)

(ii) Using Pythagoras' Theorem,

$AC^2 = 35^2 + 22^2$

$AC = 41.34$ cm (to 4 s.f.)

$\tan \angle PAC = \frac{23.23}{41.34}$

$\angle PAC = 29.3°$ (to 1 d.p.)

(iii) $AD = BC = 22$ cm

$\cos \angle QAD = \frac{22}{32}$

$\angle QAD = 46.6°$ (to 1 d.p.)

23. (i) $\tan 24° = \frac{BT}{80}$

$BT = 80 \tan 24°$
$= 35.6$ m (to 3 s.f.)

(ii) Using Pythagoras' Theorem,
$AC^2 + 80^2 = 110^2$
$AC^2 = 110^2 - 80^2$
$AC = 75.5$ m (to 3 s.f.)

(iii) $\tan \angle TCB = \frac{35.61}{110}$
$\angle TCB = 17.9°$ (to 1 d.p.)

24. (i) $\cos 55° = \frac{BQ}{48}$
$BQ = 48 \cos 55°$
$= 27.5$ cm (to 3 s.f.)

(ii) $\sin 28° = \frac{CQ}{27.53}$
$CQ = 27.53 \sin 28°$
$= 12.9$ cm (to 3 s.f.)

(iii) $PD = CQ = 12.92$ cm
$\sin \angle PBD = \frac{12.92}{48}$
$\angle PBD = 15.6°$ (to 1 d.p.)

25. (i) $\tan 32° = \frac{CT}{17}$
$CT = 17 \tan 32°$
$= 10.6$ m (to 3 s.f.)

(ii) $DC = AB = 22$ m
$\tan \angle TDC = \frac{10.62}{22}$
$\angle TDC = 25.8°$ (to 1 d.p.)

(iii) Using Pythagoras' Theorem,
$AC^2 = 22^2 + 17^2$
$AC = 27.80$ m (to 4 s.f.)
$\tan \angle TAC = \frac{10.62}{27.80}$
$\angle TAC = 20.9°$ (to 1 d.p.)

26. (i) $\tan 38° = \frac{AT}{80}$
$AT = 80 \tan 38°$
$= 62.5$ m (to 3 s.f.)

(ii) $\tan 48° = \frac{AB}{80}$
$AB = 80 \tan 48°$
$= 88.8$ m (to 3 s.f.)

(iii) $\tan \angle TBA = \frac{62.50}{88.84}$
$\angle TBA = 35.1°$ (to 1 d.p.)
$\therefore$ Angle of elevation is 35.1°.

Advanced

27. (i) $\tan 22.3° = \frac{AT}{24}$
$AT = 24 \tan 22.3°$
$= 9.84$ m (to 3 s.f.)

(ii) $\tan \angle TCA = \frac{9.843}{22}$
$\angle TCA = 24.1°$ (to 1 d.p.)
$\therefore$ Angle of elevation is 24.1°.

(iii) $\angle BAC = 360° - 90° - 146°$ ($\angle$s at a pt.)
$= 124°$
Using Cosine Rule,
$BC^2 = 24^2 + 22^2 - 2(24)(22) \cos 124°$
$BC = 40.63$ m (to 4 s.f.)
Using Sine Rule,
$\frac{\sin \angle ACB}{24} = \frac{\sin 124°}{40.63}$
$\sin \angle ACB = \frac{24 \sin 124°}{40.63}$
$\angle ACB = 29.32°$ (to 2 d.p.)
$\sin 29.32° = \frac{AP}{22}$
$AP = 22 \sin 29.32°$
$= 10.77$ m (to 4 s.f.)
$\tan \angle TPA = \frac{9.843}{10.77}$
$\angle TPA = 42.4°$ (to 1 d.p.)
$\therefore$ Angle of elevation is 42.4°.

28. (a) (i) $\sin 38.5° = \frac{AT}{14.6}$
$AT = 14.6 \sin 38.5°$
$= 9.09$ m (to 3 s.f.)

(ii) $\cos 38.5° = \frac{AC}{14.6}$
$AC = 14.6 \cos 38.5°$
$= 11.42$ m (to 4 s.f.)
$\tan 32.6° = \frac{11.42}{AB}$
$AB = \frac{11.42}{\tan 32.6°}$
$= 17.9$ m (to 3 s.f.)

(iii) $\tan \angle TBA = \frac{9.088}{17.86}$
$\angle TBA = 27.0°$ (to 1 d.p.)
$\therefore$ Angle of elevation is 27.0°.

(b) (i) $\sin 32.6° = \frac{AP}{17.86}$
$AP = 17.86 \sin 32.6°$
$= 9.63$ m (to 3 s.f.)

(ii) $\tan \angle TPA = \frac{9.088}{9.625}$
$\angle TPA = 43.4°$ (to 1 d.p.)

29. (i) $PX = QR = 8$ m

$\tan 18° = \dfrac{SX}{8}$

$SX = 8 \tan 18°$

$= 2.599$ m (to 3 s.f.)

$XR = PQ = 5$ m

$SR = 5 + 2.599$

$= 7.60$ m (to 3 s.f.)

(ii) Using Pythagoras' Theorem,

$MR^2 = 20^2 + 8^2$

$MR = 21.54$ m (to 4 s.f.)

$\tan \angle SMR = \dfrac{7.599}{21.54}$

$\angle SMR = 19.4°$ (to 1 d.p.)

(iii) $\tan \angle QMR = \dfrac{8}{20}$

$\angle QMR = 21.8°$ (to 1 d.p.)

Bearing of R from M is 021.8°.

(iv) $\cos 18° = \dfrac{8}{PS}$

$PS = \dfrac{8}{\cos 18°}$

$= 8.41$ m (to 3 s.f.)

New Trend

30. (i) Using Cosine Rule,

$60^2 = 80^2 + 30^2 - 2(80)(30) \cos \angle CBP$

$\cos \angle CBP = \dfrac{80^2 + 30^2 - 60^2}{2(80)(30)}$

$\angle CBP = 39.6°$ (to 1 d.p.)

(ii) Using Cosine Rule,

$AC^2 = 80^2 + 80^2 - 2(80)(80) \cos 39.57°$

$AC = 54.16$ m (to 4 s.f.)

Using Sine Rule,

$\dfrac{\sin \angle ACB}{80} = \dfrac{\sin 39.57°}{54.16}$

$\sin \angle ACB = \dfrac{80 \sin 39.57°}{54.16}$

$\angle ACB = 70.2°$ (to 1 d.p.)

$90° - 70.2° = 19.8°$

Bearing of A from C is 019.8°.

31. (a) (i) Using Cosine Rule,

$QS^2 = 84^2 + 130^2 - 2(84)(130) \cos 68°$

$QS = 126$ m (to 3 s.f.)

(ii) Let the shortest distance from R to QS be x m.

$\sin 50° = \dfrac{x}{90}$

$x = 90 \sin 50°$

$= 68.9$ (to 3 s.f.)

The shortest distance from R to QS is 68.9 m.

(iii) Using Sine Rule,

$\dfrac{130}{\sin \angle PQS} = \dfrac{125.60}{\sin 68°}$

$\sin \angle PQS = \dfrac{130 \sin 68°}{125.60}$

$\angle PQS = 73.671°$ (to 3 d.p.)

$\angle PQR = 73.671° + 50°$

$= 123.7°$ (to 1 d.p.)

(b) Area of land

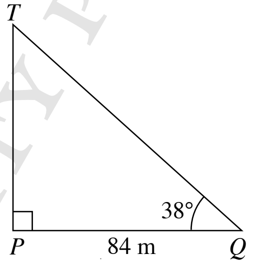
$= \dfrac{1}{2}(84)(130) \sin 68° + \dfrac{1}{2}(90)(125.60) \sin 50°$

$= 9392.1$ m^2 (to 5 s.f.)

Value $= 50\ 000 \times \dfrac{9392.1}{10\ 000}$

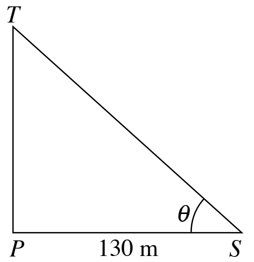
$= \$46\ 960.50$

(c)



$\tan 38° = \dfrac{TP}{84}$

$TP = 84 \tan 38°$



Let the angle of elevation of T from S be θ.

$\tan \theta = \dfrac{84 \tan 38°}{130}$

$\theta = 26.8°$ (to 1 d.p.)

32. (a) $\tan\theta = \frac{1}{9}$

$\theta = 6.3402°$ (to 5 s.f.)

(b) Length of horizontal distance $= 0.18 \times 9$

$= 1.62$ m

Volume of cement needed $= \frac{1}{2}(0.18)(1.62)(2)$

$= 0.2916 \text{ m}^3$

(c) Let the total length of the handrail be l m.

Using Pythagoras' Theorem,

$(l - 0.4)^2 = 0.18^2 + 1.62^2$

$l - 0.4 = 1.629\ 969\ 325$

$l = 2.03$ (to 3 s.f.)

Total length of metallic material $= 2.03 + 2(1.5)$

$= 5.03$ m

33. (a) $\tan\angle BAC = \frac{30}{40}$

$\angle BAC = 36.9°$ (to 1 d.p.)

(b) (i) Using Sine Rule,

$\tan 50° = \frac{AD}{40}$

$AD = 47.7$ m (to 3 s.f.)

(ii) Using Pythagoras' Theorem,

$AC^2 = 40^2 + 30^2$

$AC = 50$ m

$\tan\angle ACD = \frac{47.67}{50}$

$\angle ACD = 43.6°$

(iii) $\cos 43.63° = \frac{50}{DC}$

$DC = \frac{50}{\cos 43.63°}$

$= 69.08$ m (to 4 s.f.)

Area of $\triangle BCD = \frac{1}{2} \times 69.08 \times 30 \times \sin 64.26°$

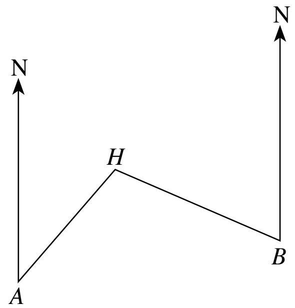
$= 933 \text{ m}^2$ (to 3 s.f.)

(c) $\frac{1}{2} \times DX \times 30 = 933.382$

$DX = 62.2$ (to 3 s.f.)

$\therefore$ The shortest possible length of cable DX is 62.2 m.

34. (a)



(b) 1 cm represents 20 km

2.4 cm represent $2.4 \times 20 = 48$ km

$\therefore$ The actual distance of the helicopter from B is 48 km.

(c) Bearing of the helicopter from $B = 360° - 66°$

$= 294°$

35. (i) $\sin\angle CAD = \frac{24}{43}$

$\angle CAD = 33.93°$ (to 2 d.p.)

$90° - 33.93° = 56.1°$ (to 1 d.p.)

Bearing of C from A is 056.1°.

(ii) $\angle BXA = 180° - 108° - 40°$ ($\angle$ sum of a $\triangle$)

$= 32°$

Using Sine Rule,

$\frac{AX}{\sin 40°} = \frac{25}{\sin 32°}$

$AX = \frac{25\sin 40°}{\sin 32°}$

$= 30.32$ m (to 4 s.f.)

$CX = 43 - 30.32$

$= 12.7$ m (to 3 s.f.)

(iii) Using Cosine Rule,

$BC^2 = 25^2 + 43^2 - 2(25)(43)\cos 108°$

$BC = 56.0$ m (to 3 s.f.)

(iv) Area of $ABCD = \frac{1}{2}(25)(43)\sin 108°$

$+ \frac{1}{2}(43)(24)\sin 56.07°$

$= 939 \text{ m}^2$ (to 3 s.f.)

Chapter 10 Arc Length, Area of Sector, and Radian Measure

Basic

1. **(a)** **(i)** Perimeter $= \frac{70°}{360°}(2\pi)(6) + 2(6)$

$= 19\frac{1}{3}$ cm

(ii) Area $= \frac{70°}{360°}(\pi)(6)^2$

$= 22$ cm^2

(b) **(i)** Perimeter $= \frac{280°}{360°}(2\pi)(9) + 2(9)$

$= 62$ cm

(ii) Area $= \frac{280°}{360°}(\pi)(9)^2$

$= 198$ cm^2

(c) **(i)** Perimeter $= \frac{360° - 36°}{360°}(2\pi)(35) + 2(35)$

$= 268$ cm

(ii) Area $= \frac{360° - 36°}{360°}(\pi)(35)^2$

$= 3465$ cm^2

2. **(a)** Perimeter $= \frac{140°}{360°}(2\pi)(27) + 2(27)$

$= (21\pi + 54)$ cm

Area $= \frac{140°}{360°}(\pi)(27)^2$

$= 283.5\pi$ cm^2

(b) Perimeter $= \frac{72°}{360°}(2\pi)(15) + 2(15)$

$= (6\pi + 30)$ cm

Area $= \frac{72°}{360°}(\pi)(15)^2$

$= 45\pi$ cm^2

(c) Perimeter $= \frac{240°}{360°}(2\pi)(6) + 2(6)$

$= (8\pi + 12)$ cm

Area $= \frac{240°}{360°}(\pi)(6)^2$

$= 24\pi$ cm^2

3. **(a)** Arc length = 8 cm

$\frac{\theta}{360}(2\pi)(24) = 8$

$\theta = 19.1$ (to 1 d.p.)

(b) Arc length = 10.6 cm

$\frac{\theta}{360}(2\pi)(24) = 10.6$

$\theta = 25.3$ (to 1 d.p.)

(c) Arc length = 6.5 cm

$\frac{\theta}{360}(2\pi)(24) = 6.5$

$\theta = 15.5$ (to 1 d.p.)

4. **(a)** Area of sector = 25.5 m^2

$\frac{\theta}{360}(\pi)(8)^2 = 25.5$

$\theta = 45.7$ (to 1 d.p.)

(b) Area of sector = 6.6 m^2

$\frac{\theta}{360}(\pi)(8)^2 = 6.6$

$\theta = 11.8$ (to 1 d.p.)

(c) Area of sector = 8 m^2

$\frac{\theta}{360}(\pi)(8)^2 = 8$

$\theta = 14.3$ (to 1 d.p.)

5. **(a)** $76° = \frac{76°}{180°} \times \pi$

$= 1.33$ rad (to 3 s.f.)

(b) $124.8° = \frac{124.8°}{180°} \times \pi$

$= 2.18$ rad (to 3 s.f.)

(c) $257.3° = \frac{257.3°}{180°} \times \pi$

$= 4.49$ rad (to 3 s.f.)

(d) $345.4° = \frac{345.4°}{180°} \times \pi$

$= 6.03$ rad (to 3 s.f.)

6. **(a)** $\frac{2\pi}{9}$ rad $= \frac{2\pi}{9} \times \frac{180°}{\pi}$

$= 40°$

(b) $\frac{5\pi}{17}$ rad $= \frac{5\pi}{17} \times \frac{180°}{\pi}$

$= 52.9°$ (to 1 d.p.)

(c) 2.16 rad $= 2.16 \times \frac{180°}{\pi}$

$= 123.8°$ (to 1 d.p.)

(d) 3.07 rad $= 3.07 \times \frac{180°}{\pi}$

$= 175.9°$ (to 1 d.p.)

7. **(a)** $\sin 0.47 = 0.453$ (to 3 s.f.)

(b) $\cos 0.128 = 0.992$ (to 3 s.f.)

(c) $\tan 1.175 = 2.39$ (to 3 s.f.)

(d) $\sin \frac{2\pi}{7} = 0.782$ (to 3 s.f.)

(e) $\cos 0.85\pi = -0.891$ (to 3 s.f.)

(f) $\tan \frac{15\pi}{37} = 3.27$ (to 3 s.f.)

8. **(a)** $\sin x = 0.69$

$x = 0.761$ (to 3 s.f.)

(b) $\cos x = 0.476$

$x = 1.07$ (to 3 s.f.)

(c) $\tan x = 0.369$

$x = 0.354$ (to 3 s.f.)

(d) $\sin x = 0.137$

$x = 0.137$ (to 3 s.f.)

9. **(i)** $\tan 1.07 = \frac{9.4}{PQ}$

$PQ = \frac{9.4}{\tan 1.07}$

$= 5.14$ cm (to 3 s.f.)

(ii) $\sin 1.07 = \frac{9.4}{PR}$

$PR = \frac{9.4}{\sin 1.07}$

$= 10.7$ cm (to 3 s.f.)

10. **(a)** **(i)** Arc length $= 16(1.75)$

$= 28$ cm

(ii) Perimeter $= 28 + 2(16)$

$= 60$ cm

(iii) Area $= \frac{1}{2}(16)^2(\pi - 1.75)$

$= 178\ \text{cm}^2$ (to 3 s.f.)

(b) **(i)** Arc length $= 5(0.7)$

$= 3.5$ cm

(ii) Perimeter $= 3.5 + 2(5)$

$= 13.5$ cm

(iii) Area $= \frac{1}{2}(5)^2(\pi - 0.7)$

$= 30.5\ \text{cm}^2$ (to 3 s.f.)

(c) **(i)** Arc length $= 10(2.09)$

$= 20.9$ m

(ii) Perimeter $= 20.9 + 2(10)$

$= 40.9$ m

(iii) Area $= \frac{1}{2}(10)^2(\pi - 2.09)$

$= 52.6\ \text{m}^2$ (to 3 s.f.)

(d) **(i)** Arc length $= 20(2.62)$

$= 52.4$ cm

(ii) Perimeter $= 52.4 + 2(20)$

$= 92.4$ cm

(iii) Area $= \frac{1}{2}(20)^2(\pi - 2.62)$

$= 104\ \text{cm}^2$ (to 3 s.f.)

11. **(a)** $r = \frac{45}{0.63}$

$= 71$ (to the nearest integer)

$E = \frac{1}{2}(71.4)^2(0.63)$

$= 1606$ (to the nearest integer)

(b) $r = \frac{72.5}{0.87}$

$= 83$ (to the nearest integer)

$E = \frac{1}{2}(83.3)^2(0.87)$

$= 3018$ (to the nearest integer)

(c) $r = \frac{48.6}{1.29}$

$= 38$ (to the nearest integer)

$E = \frac{1}{2}(37.674)^2(1.29)$

$= 915$ (to the nearest integer)

(d) $r = \frac{95}{2.16}$

$= 44$ (to the nearest integer)

$E = \frac{1}{2}(43.981)^2(2.16)$

$= 2089$ (to the nearest integer)

12. **(i)** $s = r\theta$

$\theta = \frac{s}{r}$ — (1)

$A = \frac{1}{2}r^2\theta$ — (2)

Substitute (1) into (2):

$A = \frac{1}{2}r^2\left(\frac{s}{r}\right)$

$= \frac{rs}{2}$ (shown)

(ii) **(a)** $A = \frac{7 \times 12}{2}$

$= 42\ \text{cm}^2$

(b) $A = \frac{8 \times 5}{2}$

$= 20\ \text{m}^2$

(c) $A = \frac{10 \times 11.8}{2}$

$= 59\ \text{cm}^2$

(d) $A = \frac{9 \times 30}{2}$

$= 135\ \text{cm}^2$

13. **(a)** Perimeter $= \frac{30°}{360°}(2\pi)(20) + \frac{30°}{360°}(2\pi)(30) + 2(10)$

$= 46.2$ cm (to 3 s.f.)

Area $= \frac{30°}{360°}(\pi)(30)^2 - \frac{30°}{360°}(\pi)(20)^2$

$= 131\ \text{cm}^2$ (to 3 s.f.)

(b) Perimeter $= \frac{120°}{360°}(2\pi)(21)^2 + \frac{120°}{360°}(2\pi)(11) + 2(10)$

$= 87.0$ cm (to 3 s.f.)

Area $= \frac{120°}{360°}(\pi)(21)^2 - \frac{120°}{360°}(\pi)(11)^2$

$= 335$ cm^2 (to 3 s.f.)

Intermediate

14. (i) $\sin \angle ACB = \frac{8.9}{23.4}$

$\angle ACB = 0.390$ rad (to 3 s.f.)

(ii) $\angle BAC = \pi - \frac{\pi}{2} - 0.3901$ ($\angle$ sum of a $\triangle$)

$= 1.180$ rad (to 4 s.f.)

$\angle BAD = 1.180 + 0.24$

$= 1.420$ rad (to 4 s.f.)

$\cos 1.420 = \frac{8.9}{AD}$

$AD = \frac{8.9}{\cos 1.420}$

$= 59.5$ m (to 3 s.f.)

(iii) Using Cosine Rule,

$CD^2 = 23.4^2 + 59.49^2 - 2(23.4)(59.49)\cos 0.24$

$= 37.2$ m (to 3 s.f.)

15. (a) Let the angle subtended at the centre of the circle be θ rad.

$\theta = \frac{17.6}{21}$

$= 0.838$ (to 3 s.f.)

$\therefore$ Angle subtended is 0.838 rad.

(b) Let the angle subtended at the centre of the circle be θ rad.

$\frac{1}{2}(12)^2\theta = 128$

$\theta = 1.78$ (to 3 s.f.)

$\therefore$ Angle subtended is 1.78 rad.

16. (i) Perimeter $= \frac{50°}{360°}(2\pi)(20) + \frac{50°}{360°}(2\pi)(36) + 2(16)$

$= 80.9$ m (to 3 s.f.)

(ii) Using Cosine Rule,

$AC^2 = 20^2 + 36^2 - 2(20)(36)\cos 50°$

$AC = 27.8$ m (to 3 s.f.)

17. (i) Circumference of circle $= 35.2 + 52.8$

$= 88$ cm

Let the radius of the circle be r cm.

$2\pi r = 88$

$r = 14.0$ (to 3 s.f.)

$\therefore$ Radius of circle is 14.0 cm.

(ii) Let the angle subtended at the centre of the circle be θ rad.

$\theta = \frac{35.2}{14.00}$

$= 2.51$ (to 3 s.f.)

$\therefore$ Angle subtended is 2.51 rad.

18. (a) Time taken $= \frac{156°}{360°} \times 60$

$= 26$ minutes

(b) (i) Distance moved $= \frac{12}{60}(\pi)(42)$

$= 26.4$ cm

(ii) Distance moved $= \frac{45}{60}(\pi)(42)$

$= 99$ cm

19. Arc length $= \left(\frac{42}{2}\right)(25 \times 4)$

$= 2100$ cm

$= 21$ m

20. (i) Let the angle subtended at the centre of the circle be θ rad.

$\frac{1}{2}(6)^2\theta = 72$

$\theta = 4$

Length of wire $= 6(4)$

$= 24$ cm

(ii) Let the radius of the circle be r cm.

$2\pi r = 24$

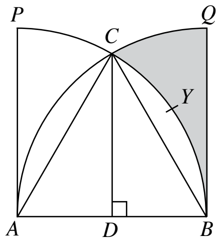
$r = \frac{12}{\pi}$

$\therefore$ Radius of circle is $\frac{12}{\pi}$ cm.

21. (i) Perimeter $= \frac{1}{4}(2\pi)(8) + 8$

$= 20.6$ cm (to 3 s.f.)

(ii)



$\cos \angle CAD = \frac{4}{8}$

$\angle CAD = 60°$

Area of segment $CYB = \frac{60°}{360°}(\pi)(8)^2 - \frac{1}{2}(8)^2 \sin 60°$

$= 5.797$ cm^2 (to 4 s.f.)

$\angle CBQ = 90° - 60°$

$= 30°$

Area of sector $CBQ = \frac{30°}{360°}(\pi)(8)^2$

$= 16.76$ cm^2 (to 4 s.f.)

Area of shaded region $= 16.76 - 5.797$

$= 11.0$ cm^2 (to 3 s.f.)

22. (a) (i) Perimeter $= \frac{180° - 60°}{360°}(2\pi)(21) + 2(21)$

$= 86.0$ cm (to 3 s.f.)

(ii) Area $= \frac{180° - 60°}{360°}(\pi)(21)^2$

$= 462$ cm^2 (to 3 s.f.)

(b) (i) Perimeter $= \frac{75° + 75°}{360°}(2\pi)(63) + 2(63)$

$= 291$ cm^2 (to 3 s.f.)

(ii) Area $= \frac{75° + 75°}{360°}(\pi)(63)^2$

$= 5200$ cm^2 (to 3 s.f.)

(c) (i) Perimeter $= 35(1.26) + 2(35)$

$= 114.1$ cm

(ii) Area $= \frac{1}{2}(35)^2(1.26)$

$= 771.75$ cm^2

23. Total area $= \left[\frac{120°}{360°}(\pi)(10)^2 - \frac{120°}{360°}(\pi)(6)^2\right] + \frac{360° - 120°}{360°}(\pi)(6)^2$

$= 142$ cm^2 (to the nearest cm^2)

24. (i) Volume $= \frac{1}{3}\pi(10)^2(24)$

$= 2510$ cm^3 (to 3 s.f.)

(ii) Curved surface area $= \pi(10)(26)$

$= 817$ cm^2 (to 3 s.f.)

(iii) Perimeter of net $= 2\pi(10) + 2(26)$

$= 115$ cm (to 3 s.f.)

25. $\frac{1}{2}(5)^2\theta = 20$

$\theta = 1.6$ rad

Using Cosine Rule,

$PQ^2 = 5^2 + 5^2 - 2(5)(5)\cos 1.6$

$PQ = 7.17$ cm (to 3 s.f.)

26. (i) $OP = OQ = 7$ cm

$\tan 60° = \frac{OR}{7}$

$OR = 7 \tan 60°$

$= 12.1$ cm (to 3 s.f.)

(ii) $\cos 60° = \frac{7}{PR}$

$PR = \frac{7}{\cos 60°}$

$= 14$ cm

(iii) Area $= \frac{120°}{360°}(\pi)(7)^2$

$= 51.3$ cm^2 (to 3 s.f.)

27. (i) $10(\angle AOC) = 5$

$\angle AOC = 0.5$ rad

(ii) Area $= \frac{1}{2}(10)(15)\sin 0.5 - \frac{1}{2}(10)^2(0.5)$

$= 11.0$ cm^2 (to 3 s.f.)

28. (i) $\cos\frac{\pi}{3} = \frac{OA}{8}$

$OA = 8\cos\frac{\pi}{3}$

$= 4$ cm

$\sin\frac{\pi}{3} = \frac{AC}{8}$

$AC = 8\sin\frac{\pi}{3}$

$= 6.928$ cm (to 4 s.f.)

Perimeter $= 6.928 + (8 - 4) + 4\left(\frac{\pi}{3}\right)$

$= 15.1$ cm (to 3 s.f.)

(ii) Area $= \frac{1}{2}(4)(6.928) - \frac{1}{2}(4)^2\left(\frac{\pi}{3}\right)$

$= 5.48$ cm^2 (to 3 s.f.)

29. (i) Area $= \frac{1}{2}rs$

(ii) $r\left(\frac{1}{3}\right) = s$

$s = \frac{r}{3}$ — (1)

$\frac{1}{2}r^2\left(\frac{1}{3}\right) = 8$

$r^2 = 48$

$r = 6.93$ (to 3 s.f.)

Substitute $r = 6.928$ into (1):

$s = \frac{6.928}{3}$

$= 2.31$ (to 3 s.f.)

30. Using Cosine Rule,

$10^2 = 12^2 + 12^2 - 2(12)(12) \cos \angle PRQ$

$\cos \angle PRQ = \dfrac{12^2 + 12^2 - 10^2}{2(12)(12)}$

$\angle PRQ = 0.8595$ rad (to 4 s.f.)

$\angle POQ = 2\pi - \dfrac{\pi}{2} - \dfrac{\pi}{2} - 0.8595$ (∠ sum of a quadrilateral)

$= 2.282$ rad (to 4 s.f.)

$\sin \dfrac{2.282}{2} = \dfrac{5}{OP}$

$OP = \dfrac{5}{\sin 1.141}$

$= 5.500$ cm (to 4 s.f.)

Area $= (12)(5.500) - \dfrac{1}{2}(5.500)^2(2.282)$

$= 31.5 \text{ cm}^2$ (to 3 s.f.)

31. (i) $\tan \angle PST = \dfrac{5}{12}$

$\angle PST = 0.395$ rad (to 3 s.f.)

$\angle POT = 0.3947 + 0.3947$ (ext. ∠ = sum of int. opp. ∠s)

$= 0.790$ rad (to 3 s.f.)

(ii) $\angle POS = \pi - 0.7895$ (∠ on a str. line)

$= 2.352$ (to 4 s.f.)

Using Pythagoras' Theorem,

$ST^2 = 12^2 + 5^2$

$ST = 13$ cm

$OS = 6.5$ cm

Area $= \dfrac{1}{2}(6.5)^2(2.352) - \dfrac{1}{2}(6.5)^2 \sin 2.352$

$= 34.7 \text{ cm}^2$ (to 3 s.f.)

32. (i) $8(\angle AOB) = 6$

$\angle AOB = 0.75$ rad

(ii) Area $= \dfrac{1}{2}(8)^2(0.75) - \dfrac{1}{2}(8)(6) \sin 0.75$

$= 7.64 \text{ cm}^2$ (to 3 s.f.)

33. (i) $\dfrac{1}{2}(5x)^2(1.25) - \dfrac{1}{2}(3x)^2(1.25) = 250$

$15.625x^2 - 5.625x^2 = 250$

$10x^2 = 250$

$x^2 = 25$

$x = 5$

(ii) Perimeter $= 15(1.25) + 25(1.25) + 2(10)$

$= 70$ cm

Advanced

34. (i) $\cos \angle ABQ = \dfrac{5}{7 + 4}$

$= \dfrac{5}{11}$

$\angle ABQ = 1.10$ rad (to 3 s.f.)

$\angle BAR = \pi - 1.098$ (int. ∠s, $BQ // AR$)

$= 2.04$ rad (to 3 s.f.)

(ii) Using Pythagoras' Theorem,

$(QS + HR)^2 + 5^2 = 11^2$

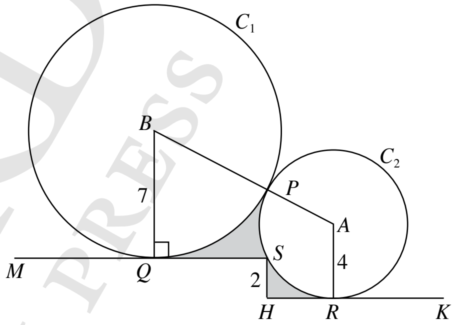
$(QS + HR)^2 = 11^2 - 5^2$

$QS + HR = 9.797$ cm (to 4 s.f.)

Perimeter $= 7(1.098) + 4(2.042) + 9.797 + 2$

$= 27.7$ cm (to 3 s.f.)

(iii)



Area of trapezium $ABXR = \dfrac{1}{2}(9 + 4)(9.797)$

$= 63.68 \text{ cm}^2$ (to 4 s.f.)

Using Pythagoras' Theorem,

$HR^2 + 2^2 = 4^2$

$HR^2 = 4^2 - 2^2$

$HR = 3.464$ cm (to 4 s.f.)

$XH = 9.797 - 3.464$

$= 6.333$ cm (to 4 s.f.)

Area of shaded region $PQSHRSP$

$= 63.68 - \dfrac{1}{2}(7)^2(1.098) - \dfrac{1}{2}(4)^2(2.042) - (6.333)(2)$

$= 7.75 \text{ cm}^2$ (to 3 s.f.)

35. (i) $\cos \angle PAB = \dfrac{3.5}{5}$

$\angle PAB = 0.7953$ rad (to 4 s.f.)

$\angle PAQ = \dfrac{\pi}{2} - 0.7953$

$= 0.775$ rad (to 3 s.f.)

(ii) Area $= 9(3.5) - \dfrac{1}{2}(5)(3.5) \sin 0.7953$

$- \dfrac{1}{2}(5)^2 \sin 0.7753$

$= 16.5 \text{ cm}^2$ (to 3 s.f.)

36. (i) $r\theta + 2r = 4$

$r\theta = 4 - 2r$

$\theta = \left(\frac{4}{r} - 2\right)$

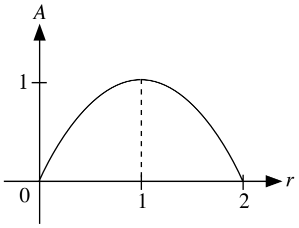
(ii) Area $= \frac{1}{2}r^2\theta$

$= \frac{1}{2}r^2\left(\frac{4}{r} - 2\right)$

$= 2r - r^2$

Let $A = 2r - r^2$

$= r(2 - r)$



When the area is a maximum,
$r = 1$.

(iii) When $r = 1$,

Area $= 1(2 - 1)$

$= 1 \text{ m}^2$

$\theta = \frac{4}{1} - 2$

$= 2$ rad

37. (i) Area of $\triangle PQR = \frac{1}{2}(2r)(r)$

$= r^2$

Using Pythagoras' Theorem,

$PR^2 = r^2 + r^2$

$= 2r^2$

$PR = \sqrt{2}r$ cm

Area of unshaded segment PTQ

$= \frac{1}{4}\pi(\sqrt{2}r)^2 - r^2$

$= \left(\frac{1}{2}\pi r^2 - r^2\right)$ cm^2

Area of shaded region $= \frac{1}{2}\pi r^2 - \left(\frac{1}{2}\pi r^2 - r^2\right)$

$= r^2$ cm^2

The two areas are equal.

(ii) Area of segment $QSR = \frac{1}{4}\pi r^2 - \frac{1}{2}(r)(r)$

$= \left(\frac{1}{4}\pi r^2 - \frac{1}{2}r^2\right)$ cm^2

$$\frac{\text{Area of segment } PTQ}{\text{Area of segment } QSR} = \frac{\frac{1}{2}\pi r^2 - r^2}{\frac{1}{4}\pi r^2 - \frac{1}{2}r^2}$$

$$= \frac{\frac{1}{2}r^2(\pi - 2)}{\frac{1}{4}r^2(\pi - 2)}$$

$= 2$

38. Area of sector $OAB = \frac{1}{2}(24)^2\left(\frac{\pi}{3}\right)$

$= 96\pi$ cm^2

$\sin\frac{\pi}{6} = \frac{QX}{24 - QX}$

$\frac{1}{2} = \frac{QX}{24 - QX}$

$24 - QX = 2QX$

$3QX = 24$

$QX = 8$ cm

$\angle QXR = \frac{\pi}{2} + \frac{\pi}{6}$ (ext. $\angle$ = sum of int. opp. $\angle$s)

$= \frac{2\pi}{3}$ rad

Area of unshaded sector $QRPX = \frac{1}{2}(8)^2\left(\frac{2\pi}{3}\right) \times 2$

$= \frac{128\pi}{3}$ cm^2

$\angle QXO = \pi - \frac{2\pi}{3}$ ($\angle$s on a str. line)

$= \frac{\pi}{3}$ rad

Area of quadrilateral $OQXP = \frac{1}{2}(8)(16)\sin\frac{\pi}{3} \times 2$

$= 110.8$ cm^2 (to 4 s.f.)

Area of shaded region $= 96\pi - 110.8 - \frac{128\pi}{3}$

$= 56.7$ cm^2 (to 3 s.f.)

39. (i)
$$\sin 0.6 = \frac{CQ}{12 - CQ}$$
$$12 \sin 0.6 - CQ \sin 0.6 = CQ$$
$$CQ + CQ \sin 0.6 = 12 \sin 0.6$$
$$CQ(1 + \sin 0.6) = 12 \sin 0.6$$
$$CQ = \frac{12 \sin 0.6}{1 + \sin 0.6}$$
$$= 4.33 \text{ cm (to 3 s.f.)}$$
$\therefore$ Radius of enclosed circle is 4.33 cm.

(ii) $\angle PCQ = 2\pi - \frac{\pi}{2} - \frac{\pi}{2} - 1.2$ ($\angle$ sum of a quadrilateral)
$= 1.941$ rad (to 4 s.f.)
$$\tan 0.6 = \frac{4.330}{OQ}$$
$$OQ = \frac{4.330}{\tan 0.6}$$
$= 6.329$ cm (to 4 s.f.)
Perimeter $= 4.330(1.941) + 2(6.329)$
$= 21.1$ cm (to 3 s.f.)

(iii) Area of shaded region
$= \frac{1}{2}(6.329)(4.330) \times 2 - \frac{1}{2}(4.330)^2(1.941)$
$= 9.21 \text{ cm}^2$ (to 3 s.f.)

New Trend

40. $\cos \angle BOC = \frac{OB}{OC}$
$= \frac{3.5}{5}$
$\angle BOC = 0.7954$ rad (to 4 s.f.)
$\angle AOC = 2\angle BOC$
$= 1.5908$ rad
Area of minor segment
$= \frac{1}{2}(5)^2(1.5908) - \frac{1}{2}(5)^2 \sin 1.5908$
$= 7.388 \text{ cm}^2$ (to 4 s.f.)
Area of coin pouch $= 2[\pi(5)^2 - 7.388]$
$= 142 \text{ cm}^2$ (to 3 s.f.)

41. (i) $(2d)\theta = 20$
$$\theta = \frac{10}{d}$$

(ii) Area of $R_1 = \frac{1}{2}(2d)^2\theta$
$= 2d^2\theta \text{ cm}^2$
Area of $R_2 = 6d^2\theta \text{ cm}^2$
$$\frac{1}{2}(OD)^2\theta = 6d^2\theta + 2d^2\theta$$
$$OD^2 = 16d^2$$
$$OD = 4d \text{ cm}$$

42. (a) $AO = 50$ cm
Using Pythagoras' Theorem,
$OB^2 + 40^2 = 50^2$
$OB^2 = 50^2 - 40^2$
$OB = 30$ cm
$\sin \angle BOA = \frac{40}{50}$
$\angle BOA = 0.927\ 30$ (to 5 s.f.)
$\angle BOC = \pi - 2(0.92730)$
$= 1.2874$
Total area $= 40(30) + \frac{1}{2}(30)^2(1.2874)$
$= 1779 \text{ cm}^2$ (to the nearest cm^2)

(b) Total area of 100 kites $= 100 \times 1779$
$= 177\ 900 \text{ cm}^2$
$= 17.79 \text{ m}^2$
Cost $= \$23 \times 17.79$
$= \$409.17$

43. (a) Perimeter $= 4(2.5) + 2(4)$
$= 18$ cm
Time taken $= \frac{18}{0.2}$
$= 90$ s

(b) Let the radius of the base of the cone be r cm.
$2\pi r = 4(2.5)$
$$r = \frac{4(2.5)}{2\pi}$$
$= 1.59$ (to 3 s.f.)
$\therefore$ The radius of the base of the cone is 1.59 cm.

44. Let the radius of the circle be r cm.
$$6 = \frac{35°}{360°}(2\pi r)$$
$$r = \frac{216}{7\pi} \text{ cm}$$
$$\text{Volume} = \frac{35°}{360°} \times \pi\left(\frac{216}{7\pi}\right)^2 \times 9$$
$= 265 \text{ cm}^3$ (to 3 s.f.)

45. (i) Length of major arc $PQ = 120 - 16$
$= 104$ cm
Reflex $\angle POQ = \frac{104}{120} \times 2\pi$
$= 5.45$ rad (to 3 s.f.)

(ii) Number of complete revolutions
$= \frac{10}{1.2}$
$= 8$ (round down to the nearest integer)

(iii) Let the radius of the wheel be r cm.
$2\pi r = 120$
$r = 19.1$ (to 3 s.f.)
$\therefore$ Radius of wheel is 19.1 cm.

(iv) Area $= \frac{1}{2}(19.09)^2(2\pi - 5.445)$
$= 153 \text{ cm}^2$ (to 3 s.f.)

Revision Test C1

1. **(a)** $CD = \sqrt{41^2 - 40^2}$
$= 9$ cm

(b) **(i)** $\tan \angle BDC = \dfrac{40}{9}$
$= 4\dfrac{4}{9}$

(ii) $\sin \angle ABD = \dfrac{9}{41}$

(iii) $\cos \angle BDE = -\dfrac{9}{41}$

2. **(i)** $QR = \sqrt{7.8^2 + 10.4^2 - 2(7.8)(10.4)\cos 82°}$
$= 12.1$ cm (to 3 s.f.)

(ii) $\dfrac{\sin PQR}{10.4} = \dfrac{\sin 82°}{12.12}$

$\angle PQR = \sin^{-1}\left(\dfrac{10.4 \sin 82°}{12.12}\right)$
$= 58.3°$ (to 1 d.p.)

3. **(i)** $\angle ACB = 180° - 47° - 51°$
$= 82°$

$\dfrac{AC}{\sin 51°} = \dfrac{16.8}{\sin 82°}$

$AC = \dfrac{16.8 \times \sin 51°}{\sin 82°}$
$= 13.18$
$= 13.2$ cm (to 3 s.f.)

(ii) Area of $\triangle ABC = \dfrac{1}{2}(AB)(AC)\sin 47°$
$= \dfrac{1}{2}(16.8)(13.18)\sin 47°$
$= 81.0 \text{ cm}^2$ (to 3 s.f.)

4. **(i)** Length of O to $AB = \sqrt{13^2 - 12^2}$
$= 5$ cm

$\angle AOB = 2 \times \sin^{-1}\left(\dfrac{12}{13}\right)$
$= 2.35$ rad (to 3 s.f.)

(ii) Length of arc $APB = 2.35 \times 13$
$= 30.6$ cm (to 3 s.f.)

(iii) Area of segment APB
$= \dfrac{1}{2}(13)^2(2.35) - \dfrac{1}{2}(24)(5)$
$= 139 \text{ cm}^2$ (to 3 s.f.)

(iv) Area of segment BQC
$= \dfrac{1}{2}(13)^2(\pi - 2.35) - \dfrac{1}{2}(10)(12)$
$= 6.89 \text{ cm}^2$ (to 3 s.f.)

5. **(i)** In $\triangle PQR$,
$\cos 50° = \dfrac{6}{PQ}$
$PQ = \dfrac{6}{\cos 50°}$
$= 9.33$ cm (to 3 s.f.)

(ii) In $\triangle VQR$,
$\tan \angle VQR = \dfrac{9}{6}$
$\angle VQR = \tan^{-1} 1.5$
$= 56.3°$ (to 1 d.p.)

(iii) $\sin 50° = \dfrac{RT}{6}$
$RT = 6 \sin 50°$
$\angle VTR = \tan^{-1}\left(\dfrac{9}{6 \sin 50°}\right)$
$= 62.9°$ (to 1 d.p.)

6. $\cos \angle BAC = \dfrac{(5x + 12y)^2 + (12x + 5y)^2 - (13x + 13y)^2}{2(5x + 12y)(12x + 5y)}$

Evaluating the numerator,
$(25x^2 + 144y^2 + 120xy) + (144x^2 + 25y^2 + 120xy)$
$- [169x^2 + 169y^2 + 338xy]$
$= 240xy - 338xy$
$= -98xy$

Since x and y are positive, $\therefore$ the numerator is negative and the denominator is positive.
$\therefore$ $\cos \angle BAC$ is negative and $\angle BAC$ is an obtuse angle.

7. **(a)** **(i)** By Sine Rule,
$\dfrac{\sin \angle ADB}{80} = \dfrac{\sin(180° - 47°)}{170}$
$\angle ADB = \sin^{-1}\left[\dfrac{80 \sin 133°}{170}\right]$
$= 20.13°$ (to 2 d.p.)
$\therefore \angle ABD = 47° - 20.13°$
$= 26.9°$ (to 1 d.p.)
$\therefore$ The bearing of D from B is 026.9°.

(ii) $BC = \sqrt{80^2 + 70^2 - 2(80)(70)\cos 133°}$
$= 138$ m (to 3 s.f.)

(b) Let the height of the building be h m.
$\tan \theta = \dfrac{h}{BD}$
$= \dfrac{h}{170}$
$h = 170 \times \tan 33°$
$= 110$ (to 3 s.f.)
$\therefore$ The height of the building is 110 m.

Revision Test C2

1. **(i)** $RS = \sqrt{5^2 + 7^2}$
$= 8.60$ cm (to 3 s.f.)

(ii) $\sin 50° = \dfrac{5}{PQ}$
$\therefore PQ = \dfrac{5}{\sin 50°}$
$= 6.53$ cm (to 3 s.f.)

(iii) $\tan 50° = \dfrac{5}{PL}$
$\therefore PL = \dfrac{5}{\tan 50°}$
$= 4.20$ cm
$\therefore PS = 4.20 + 4 + 7$
$= 15.2$ cm (to 3 s.f.)

(iv) $\angle MSR = \tan^{-1}\left(\dfrac{5}{7}\right)$
$= 35.5°$ (to 1 d.p.)

(v) Area of $PQRS = \dfrac{1}{2}(4 + 15.2) \times 5$
$= 48 \text{ cm}^2$

2. $\tan 62° = \dfrac{AB}{46}$
$\therefore AB = 46 \tan 62°$
$= 86.5$
$= 87$ m (to the nearest metre)

$\tan 64° = \dfrac{AC}{46}$
$\therefore AC = 46 \tan 64°$
$= 94.3$ m
$\therefore BC = 94.3 - 86.5$
$= 7.8$
$= 8$ m (to the nearest metre)

3. **(a)** Area of sector $AOC = \dfrac{1}{2}(8)^2\left(\pi - \dfrac{3}{4}\right)$
$= 76.5 \text{ cm}^2$ (to 3 s.f.)

(b) Arc $AC = 8(\pi - \theta)$
Arc $BC = 8\theta$
$8\pi - 8\theta = 8\theta + 16$
$16\theta = 8\pi - 16$
$\theta = \dfrac{8\pi - 16}{16}$
$= 0.571$ rad (to 3 s.f.)

4. **(i)** $NR = \sqrt{26^2 - 10^2}$
$= 24$ cm

(ii) $\angle QRP = \sin^{-1}\left(\dfrac{10}{26}\right)$
$= 22.6°$ (to 1 d.p.)

(iii) $\cos 34° = \dfrac{10}{PQ}$
$\therefore PQ = \dfrac{10}{\cos 34°}$
$= 12.1$ cm (to 3 s.f.)

(iv) $\tan 34° = \dfrac{NQ}{10}$
$NQ = 10 \tan 34°$
$= 6.75$ cm (to 3 s.f.)

(v) Area of $\triangle PQR = \dfrac{1}{2}(25 + 6.75) \times 10$
$= 154 \text{ cm}^2$

5. **(i)** $OB = \sqrt{12^2 + 16^2}$
$= 20$ cm

(ii) $\sin 45° = \dfrac{AM}{16}$
$\therefore AM = 16 \sin 45°$
$= 11.3$ cm (to 3 s.f.)

(iii) $OM = \sqrt{12^2 + 11.31^2}$
$= 16.5$ cm (to 3 s.f.)

(iv) $\angle OMA = \tan^{-1}\left(\dfrac{12}{11.31}\right)$
$= 46.7°$ (to 1 d.p.)

6. **(i)** Arc $ED = \dfrac{40°}{360°} \times 2\pi(7)$
$= 4.89$ cm (to 3 s.f.)

(ii) $\sin 20° = \dfrac{5}{AB}$
$\therefore AB = \dfrac{5}{\sin 20°}$
$= 14.62$ cm (to 4 s.f.)
$\therefore$ Area of $BCDE = \dfrac{1}{2}(14.62)^2 \sin 40° - \dfrac{40°}{360°}\pi(7)^2$
$= 51.6 \text{ cm}^2$ (to 3 s.f.)

(iii) Volume $= 51.58 \times 0.4$
$= 20.6 \text{ cm}^3$ (to 3 s.f.)

7. **(i)** $\angle ACB = \cos^{-1}\left(\dfrac{(23x)^2 + (17x)^2 - (26x)^2}{2(23x)(17x)}\right)$
$= 79.5°$ (to 1 d.p.)

(ii) $\dfrac{1}{2}(23x)(17x) \sin 79.54° = 325$
$\therefore x = \sqrt{\dfrac{325 \times 2}{(23)(17) \sin 79.54°}}$
$= 1.30$ (to 2 d.p.)

Chapter 11 Congruence and Similarity Tests

Basic

1. **(a)** $AB = ZY$
$BC = YX$
$AC = ZX$
$\therefore \triangle ABC \equiv \triangle ZYX$ (SSS)

(b) $PQ = LM$
$\angle QPR = \angle MLN$
$\angle PRQ = \angle LNM$
$\therefore \triangle PQR \equiv \triangle LMN$ (AAS)

(c) $AB = XY$
$AC = XZ$
$\angle BAC = \angle YXZ$
$\therefore \triangle ABC \equiv \triangle XYZ$ (SAS)

(d) $TP = SR$
$TQ = SQ$
$PQ = RQ$
$\therefore \triangle TPQ \equiv \triangle SRQ$ (SSS)

(e) $\angle CAB = \angle CBA$ (base $\angle$s of isos. $\triangle$)
$\angle FED = \angle FDE$ (base $\angle$s of isos. $\triangle$)
$\angle CAB = \angle FED$
$\angle CBA = \angle FDE$
$CA = FE$
$\therefore \triangle CAB \equiv \triangle FED$ (AAS)

(f) $ML = PQ$
$MO = PO$
$\angle LMO = \angle QPO$
$\therefore \triangle MLO \equiv \triangle PQO$ (SAS)

(g) $AB = ED$
$AC = EC$
$\angle ACB = \angle ECD$, which is not the included angle.
$\therefore$ The triangles may not be congruent.

(h) $PQ = PS$
$QR = SR$
PR is a common side.
$\therefore \triangle PQR \equiv \triangle PSR$ (SSS)

(i) $OL = OP$
$\angle OLM = \angle OPQ$
$\angle LOM = \angle POQ$
$\therefore \triangle OLM \equiv \triangle OPQ$ (AAS)

(j) $AB = CB$
BD is a common side.
$\angle BAD = \angle BCD = 90°$
$\therefore \triangle ABD \equiv \triangle CBD$ (RHS)

(k) $PQ = AB$
$\angle OPQ = \angle OAB$ (alt. $\angle$s, QP // AB)
$\angle POQ = \angle AOB$ (vert. opp. $\angle$s)
$\therefore \triangle OPQ \equiv \triangle OAB$ (AAS)

(l) $BC = EF$
$\angle BAC = \angle EDF$
$\angle BCA = \angle EFD$
$\therefore \triangle ABC \equiv \triangle DEF$ (AAS)

2. **(a)** $\angle BAC = \angle ZXY$
$\angle ACB = \angle XYZ$
$\therefore \triangle ABC$ is similar to $\triangle XZY$
(2 pairs of corr. $\angle$s equal).

(b) $\angle ABC = 180° - 90° - 30°$ ($\angle$ sum of a $\triangle$)
$= 60°$
$\angle ABC = \angle YZX$
$\angle CAB = \angle XYZ$
$\therefore \triangle ABC$ is similar to $\triangle YZX$
(2 pairs of corr. $\angle$s equal).

(c) $\dfrac{AC}{ZX} = \dfrac{13}{13} = 1$
$\dfrac{AB}{ZY} = \dfrac{13}{12}$
$\dfrac{BC}{YX} = \dfrac{5}{10} = \dfrac{1}{2}$
Since the ratios of the corresponding sides are not equal, the triangles are not similar.

(d) $\dfrac{AB}{XY} = \dfrac{14}{7} = 2$
$\dfrac{BC}{YZ} = \dfrac{6}{2} = 3$
Since the ratios of the corresponding sides are not equal, the triangles are not similar.

3. Let the height of the lamp post be h m.
Using similar triangles,
$$\frac{h}{1.7} = \frac{2.3 + 1.7}{1.7} = \frac{4.0}{1.7}$$
$h = 4.0$
$\therefore$ Height of lamp post is 4.0 m.

Intermediate

4. **(a)** $\triangle APD \equiv \triangle DSC \equiv \triangle BQA \equiv \triangle CRB$

(b) $\triangle AQP \equiv \triangle BSR$
$\triangle AQR \equiv \triangle BSP$
$\triangle ABP \equiv \triangle ABR$

(c) $\triangle RSX \equiv \triangle RQX$
$\triangle PSX \equiv \triangle PQX$
$\triangle PSR \equiv \triangle PQR$

(d) $\triangle PQT \equiv \triangle SRT$

5. $\angle EAB = \angle EDC$ (base $\angle$s of isos. $\triangle$)
$\angle EBA = \angle ECD$ (adj. $\angle$s. on a str. line)
$EA = ED$
$\therefore \triangle EAB \equiv \triangle EDC$ (AAS)

6. $\angle ABE + \angle EBD = \angle EBD + \angle DBC$

i.e. $\angle ABD = \angle CBE$

$\angle ADB = \angle CEB$

$AB = CB$

$\therefore \triangle ABD \equiv \triangle CBE$ (AAS)

7. **(a)** $\triangle AQR$

(b) $\triangle ASP$

8. $\triangle QZS$ and $\triangle YZX$

9. **(a)** $\triangle CAX$

(b) $\triangle EYZ$

10. **(a)** **(i)** $\triangle DXC$

(ii) $\triangle CDB$

(b) $\triangle DXA$

11. **(a)** $\triangle TSR$

Using similar triangles,

$$\frac{x}{18} = \frac{5}{9}$$
$$x = \frac{5}{9} \times 18$$
$$= 10$$
$$\frac{y}{6} = \frac{9}{5}$$
$$y = \frac{9}{5} \times 6$$
$$= 10.8$$

(b) $\triangle ABR$

Using similar triangles,

$$\frac{x+5}{5} = \frac{6}{2}$$
$$x+5 = \frac{6}{2} \times 5$$
$$= 15$$
$$x = 10$$
$$\frac{y+4}{4} = \frac{6}{2}$$
$$y+4 = \frac{6}{2} \times 4$$
$$= 12$$
$$y = 8$$

(c) $\triangle QAR$

Using similar triangles,

$$\frac{x+12}{15} = \frac{15}{12}$$
$$x + 12 = \frac{15}{12} \times 15$$
$$= 18\frac{3}{4}$$
$$x = 6\frac{3}{4}$$
$$\frac{y}{9} = \frac{15}{12}$$
$$y = \frac{15}{12} \times 9$$
$$= 11\frac{1}{4}$$

(d) $\triangle PXQ$

Using similar triangles,

$$\frac{x}{12} = \frac{12}{18}$$
$$x = \frac{12}{18} \times 12$$
$$= 8$$
$$\frac{y}{10} = \frac{18}{12}$$
$$y = \frac{18}{12} \times 10$$
$$= 15$$

(e) $\triangle ARB$

Using similar triangles,

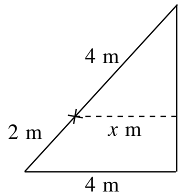
$$\frac{x}{6} = \frac{15}{12}$$
$$x = \frac{15}{12} \times 6$$
$$= 7\frac{1}{2}$$
$$\frac{y}{10} = \frac{12}{15}$$
$$y = \frac{12}{15} \times 10$$
$$= 8$$

(f) $\triangle MLR$

Using similar triangles,

$$\frac{x}{12-x} = \frac{6}{9}$$
$$9x = 72 - 6x$$
$$15x = 72$$
$$x = 4\frac{4}{5}$$

12.



Let the horizontal distance between the lizard and the wall be x m.

Using similar triangles,

$$\frac{x}{4} = \frac{4}{6}$$

$$x = \frac{4}{6} \times 4$$

$$= 2\frac{2}{3}$$

$\therefore$ Horizontal distance is $2\frac{2}{3}$ m.

13. **(i)** $\angle PQR = \angle PXZ$ (corr. $\angle$s, QR // XZ)

$\angle PQR = \angle XQY$ (common $\angle$)

$\angle QPR = \angle QXY$ (corr. $\angle$s. PR // XY)

$\angle QPR = \angle XPZ$ (common $\angle$)

$\therefore$ $\triangle PQR$, $\triangle PXZ$ and $\triangle XQY$ are similar (2 pairs of corr. $\angle$s equal).

(ii) Using similar triangles,

$$\frac{XY}{PR} = \frac{QY}{QR}$$

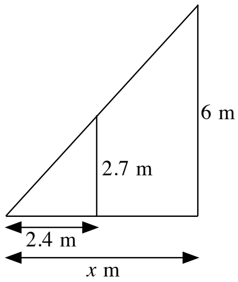
$$\frac{XY}{8.5} = \frac{QR - XZ}{6.75}$$

$$\frac{XY}{8.5} = \frac{6.75 - 3}{6.75}$$

$XY = 4.72$ (to 3 s.f.)

$\therefore$ The length of XY is 4.72 cm.

14.



Using similar triangles,

$$\frac{x}{2.4} = \frac{6}{2.7}$$

$$x = \frac{6}{2.7} \times 2.4$$

$$= 5\frac{1}{3}$$

15. $7OP = 5PQ$

$$\frac{OP}{PQ} = \frac{5}{7}$$

$$\frac{OP}{OQ} = \frac{5}{12}$$

$$\frac{QB}{PA} = \frac{12}{5}$$

$\therefore$ $QB : PA = 12 : 5$

16. $\triangle ABC$ is similar to $\triangle CDE$.

$$\frac{BC}{12 - BC} = \frac{5}{7}$$

$7BC = 60 - 5BC$

$12BC = 60$

$BC = 5$ cm

17. $\triangle LMN$ is similar to $\triangle LCB$.

$$\frac{BC}{6} = \frac{10}{4}$$

$$BC = \frac{10}{4} \times 6$$

$= 15$ cm

$\triangle AMN$ is similar to $\triangle ABC$.

$$\frac{AM}{AM + 10} = \frac{6}{15}$$

$15AM = 6AM + 60$

$9AM = 60$

$AM = 6\frac{2}{3}$ cm

18. **(i)** $\angle SQT = \angle RPT$ (given)

$\angle STQ = \angle RTP$ (common $\angle$)

$\therefore$ $\triangle SQT$ is similar to $\triangle RPT$ (2 pairs of corr. $\angle$s equal).

(ii) Using similar triangles,

$$\frac{QS}{9} = \frac{7}{6}$$

$$QS = \frac{7}{6} \times 9$$

$= 10.5$ cm

19. **(a)** $\angle BAC = \angle CBD$ (given)

$\angle ABC = \angle BCD$ (alt. $\angle$s, AB // CD)

$\therefore$ $\triangle ABC$ is similar to $\triangle BCD$ (2 pairs of corr. $\angle$s equal).

(b) **(i)** Using similar triangles,

$$\frac{BC}{9} = \frac{16}{BC}$$

$BC^2 = 144$

$BC = 12$ cm

(ii) $$\frac{AC}{BD} = \frac{9}{12}$$

$$= \frac{3}{4}$$

20. (i) $\angle PSQ = \angle PQR$ (given)

$\angle QPS = \angle RPQ$ (common $\angle$)

$\therefore \triangle PSQ$ is similar to $\triangle PQR$ (2 pairs of corr. $\angle$s equal).

(ii) Using similar triangles,

$$\frac{PQ}{9+16} = \frac{9}{PQ}$$

$$PQ^2 = 225$$

$$PQ = 15 \text{ cm}$$

21. $\triangle AXZ$ is similar to $\triangle BYZ$.

$$\frac{YZ}{YZ+10} = \frac{4}{6}$$

$$6YZ = 4YZ + 40$$

$$2YZ = 40$$

$$YZ = 20 \text{ cm}$$

22. (a) $\triangle ZAB$ is similar to $\triangle ZYX$.

Using similar triangles,

$$\frac{BZ}{3\frac{1}{2}} = \frac{6}{3}$$

$$BZ = \frac{6}{3} \times 3\frac{1}{2}$$

$$= 7 \text{ cm}$$

(b) $\triangle ZXY$ is similar to $\triangle ZQR$.

Using similar triangles,

$$\frac{YZ}{YZ+16} = \frac{3}{11}$$

$$11YZ = 3YZ + 48$$

$$8YZ = 48$$

$$YZ = 6 \text{ cm}$$

23. $\triangle ZXY$ is similar to $\triangle ZCB$.

Using similar triangles,

$$\frac{BC}{2.8} = \frac{2}{1.4}$$

$$BC = \frac{2}{1.4} \times 2.8$$

$$= 4 \text{ m}$$

$\triangle AXY$ is similar to $\triangle ABC$.

$$\frac{CY + 3.2}{3.2} = \frac{4}{2.8}$$

$$CY + 3.2 = \frac{4}{2.8} \times 3.2$$

$$CY = 1.37 \text{ m (to 3 s.f.)}$$

$$\frac{CZ}{1.2} = \frac{2}{1.4}$$

$$CZ = \frac{2}{1.4} \times 1.2$$

$$= 1.71 \text{ m (to 3 s.f.)}$$

Advanced

24. $\triangle MQP$ is similar to $\triangle MRS$.

$$\frac{QP}{RS} = \frac{6}{10}$$

$$= \frac{3}{5}$$

$\triangle PML$ is similar to $\triangle PSR$.

$PM : MS$

$3 : 5$

$\therefore PM : PS$

$3 : 8$

$$\frac{PM}{PS} = \frac{LM}{RS}$$

$$\frac{3}{8} = \frac{LM}{10}$$

$$LM = \frac{3 \times 10}{8}$$

$$= 3.75 \text{ cm}$$

25. $\triangle PQR$ is similar to $\triangle YXR$.

$$\frac{2x-y}{7} = \frac{2x+3y}{9}$$

$$18x - 9y = 14x + 21y$$

$$4x = 30y$$

$$\frac{x}{y} = \frac{15}{2}$$

$\therefore x : y = 15 : 2$

New Trend

26. (a) $\angle DPQ = \angle APB$ (common $\angle$)

$\angle PDQ = \angle PAB$ (corr. $\angle$s, DC // AB)

$\therefore \triangle PDQ$ is similar to $\triangle PAB$ (2 pairs of corr. $\angle$s equal).

(b) $\triangle BCQ$

(c) Using similar triangles,

$$\frac{DQ}{AB} = \frac{PD}{PA}$$

$$= \frac{1}{3}$$

$\therefore DQ : AB = 1 : 3$

(d) $$\frac{10}{10+8+RB} = \frac{1}{3}$$

$$30 = 18 + RB$$

$$RB = 12 \text{ cm}$$

27. (a) $\angle CAB = \angle NCB$ (given)

$\angle ABC = \angle CBN$ (common $\angle$)

$\therefore \triangle ABC$ is similar to $\triangle CBN$ (2 pairs of corr. $\angle$s equal).

(b) Using similar triangles,

$$\frac{BC}{25} = \frac{13}{BC}$$

$$BC^2 = 325$$

$$BC = 18 \text{ cm}$$

Chapter 12 Area and Volume of Similar Figures and Solids

Basic

1. (a) $\frac{A_2}{50} = \left(\frac{2}{10}\right)^2$

$A_2 = \left(\frac{2}{10}\right)^2 \times 50$

$= 2 \text{ cm}^2$

(b) $\frac{A_1}{0.7} = \left(\frac{0.8}{0.4}\right)^2$

$A_1 = \left(\frac{0.8}{0.4}\right)^2 \times 0.7$

$= 2.8 \text{ m}^2$

(c) $\frac{A_2}{96} = \left(\frac{3}{12}\right)^2$

$A_2 = \left(\frac{3}{12}\right)^2 \times 96$

$= 6 \text{ cm}^2$

(d) $\frac{A_1}{172} = \left(\frac{22.2}{14.8}\right)^2$

$A_1 = \left(\frac{22.2}{14.8}\right)^2 \times 172$

$= 387 \text{ m}^2$

(e) $\frac{A_2}{56} = \left(\frac{4p}{6p}\right)^2$

$A_2 = \left(\frac{4p}{6p}\right)^2 \times 56$

$= 24\frac{8}{9} \text{ cm}^2$

(f) $\frac{A_1}{125p} = \left(\frac{6}{10}\right)^2$

$A_1 = \left(\frac{6}{10}\right)^2 \times 125p$

$= 45p \text{ m}^2$

2. (a) $\left(\frac{a}{4}\right)^2 = \frac{16}{64}$

$\frac{a}{4} = \sqrt{\frac{16}{64}}$

$a = \sqrt{\frac{16}{64}} \times 4$

$= 2$

(b) $\left(\frac{b}{5}\right)^2 = \frac{480}{125}$

$\frac{b}{5} = \sqrt{\frac{480}{125}}$

$b = \sqrt{\frac{480}{125}} \times 5$

$= 9.80$ (to 3 s.f.)

(c) $\left(\frac{c}{7}\right)^2 = \frac{4.9}{38.5}$

$\frac{c}{7} = \sqrt{\frac{4.9}{38.5}}$

$c = \sqrt{\frac{4.9}{38.5}} \times 7$

$= 2.50$ (to 3 s.f.)

(d) $\left(\frac{d}{0.6}\right)^2 = \frac{6p}{24p}$

$\frac{d}{0.6} = \sqrt{\frac{6p}{24p}}$

$d = \sqrt{\frac{6p}{24p}} \times 0.6$

$= 0.3$

(e) $\left(\frac{e}{\frac{35p}{2}}\right)^2 = \frac{128}{50}$

$\frac{e}{\frac{35p}{2}} = \sqrt{\frac{128}{50}}$

$e = \sqrt{\frac{128}{50}} \times \frac{35p}{2}$

$= 28p$

(f) $\left(\frac{f}{8}\right)^2 = \frac{87\frac{1}{2}}{55}$

$\frac{f}{8} = \sqrt{\frac{87\frac{1}{2}}{55}}$

$f = \sqrt{\frac{87\frac{1}{2}}{55}} \times 8$

$= 10.1$ (to 3 s.f.)

3. (a) $\frac{V_2}{32} = \left(\frac{4}{8}\right)^3$

$V_2 = \left(\frac{4}{8}\right)^3 \times 32$

$= 4 \text{ mm}^3$

(b) $\frac{V_1}{81} = \left(\frac{4}{6}\right)^3$

$V_1 = \left(\frac{4}{6}\right)^3 \times 81$

$= 24 \text{ m}^3$

(c) $\frac{V_2}{480} = \left(\frac{5}{10}\right)^3$

$V_2 = \left(\frac{5}{10}\right)^3 \times 480$

$= 60 \text{ cm}^3$

(d) $\frac{V_1}{375p} = \left(\frac{9.6}{12}\right)^3$

$V_1 = \left(\frac{9.6}{12}\right)^3 \times 375p$

$= 192p \text{ cm}^3$

(e) $\frac{V_1}{95} = \left(\frac{12}{6}\right)^3$

$V_1 = \left(\frac{12}{6}\right)^3 \times 95$

$= 760 \text{ cm}^3$

(f) $\frac{V_2}{40} = \left(\frac{13.5}{4.5}\right)^3$

$V_2 = \left(\frac{13.5}{4.5}\right)^3 \times 40$

$= 1080 \text{ cm}^3$

4. (a) $\left(\frac{a}{0.5}\right)^3 = \frac{270}{10}$

$\frac{a}{0.5} = \sqrt[3]{\frac{270}{10}}$

$a = \sqrt[3]{\frac{270}{10}} \times 0.5$

$= 1.5$

(b) $\left(\frac{b}{8}\right)^3 = \frac{13}{104}$

$\frac{b}{8} = \sqrt[3]{\frac{13}{104}}$

$b = \sqrt[3]{\frac{13}{104}} \times 8$

$= 4$

(c) $\left(\frac{c}{22}\right)^3 = \frac{7a}{56a}$

$\frac{c}{22} = \sqrt[3]{\frac{7a}{56a}}$

$c = \sqrt[3]{\frac{7a}{56a}} \times 22$

$= 11$

(d) $\left(\frac{d}{14}\right)^3 = \frac{54p}{16p}$

$\frac{d}{14} = \sqrt[3]{\frac{54p}{16p}}$

$d = \sqrt[3]{\frac{54p}{16p}} \times 14$

$= 21$

(e) $\left(\frac{e}{20.4}\right)^3 = \frac{81p}{192p}$

$\frac{e}{20.4} = \sqrt[3]{\frac{81p}{192p}}$

$e = \sqrt[3]{\frac{81p}{192p}} \times 20.4$

$= 15.3$

(f) $\left(\frac{f}{8.4}\right)^3 = \frac{7}{448}$

$\frac{f}{8.4} = \sqrt[3]{\frac{7}{448}}$

$f = \sqrt[3]{\frac{7}{448}} \times 8.4$

$= 2.1$

5. Let the areas of the similar circle and larger circle be A_1 cm^2 and A_2 cm^2 respectively.

$\frac{A_1}{A_2} = \left(\frac{3}{8}\right)^2$

$= \frac{9}{64}$

$\therefore A_1 : A_2 = 9 : 64$

6. (a) $\triangle ABX$ is similar to $\triangle DCX$.

$\frac{\text{Area of } \triangle ABX}{\text{Area of } \triangle CDX} = \left(\frac{5}{9}\right)^2$

$= \frac{25}{81}$

(b) $\triangle ABX$ is similar to $\triangle CDX$.

$\frac{\text{Area of } \triangle ABX}{\text{Area of } \triangle CDX} = \left(\frac{14}{10}\right)^2$

$= \frac{49}{25}$

7. (i) $\frac{\text{Area of } \triangle XAB}{490} = \left(\frac{12}{21}\right)^2$

$\text{Area of } \triangle XAB = \left(\frac{12}{21}\right)^2 \times 490$

$= 160 \text{ m}^2$

(ii) Area of $ABZY = 490 - 160$

$= 330 \text{ m}^2$

8. $\left(\frac{25 + CE}{25}\right)^2 = \frac{288}{50}$

$\frac{25 + CE}{25} = \sqrt{\frac{288}{50}}$

$= \frac{12}{5}$

$25 + CE = 60$

$CE = 35 \text{ m}$

9. **(i)** Let the surface areas of the smaller ball and larger ball be A_1 cm^2 and A_2 cm^2 respectively.

$$\frac{A_1}{A_2} = \left(\frac{2}{5}\right)^2$$
$$= \frac{4}{25}$$

$\therefore A_1 : A_2 = 4 : 25$

(ii) Let the volumes of the balls be V_1 cm^3 and V_2 cm^3 respectively.

$$\frac{V_1}{V_2} = \left(\frac{2}{5}\right)^3$$
$$= \frac{8}{125}$$

$\therefore V_1 : V_2 = 8 : 125$

10. Let the mass of the larger sphere be m kg.

$$\frac{m}{27} = \left(\frac{2.8}{1.2}\right)^3$$
$$m = \left(\frac{2.8}{1.2}\right)^3 \times 27$$
$$= 343$$

$\therefore$ Mass of larger sphere is 343 kg.

Intermediate

11. **(a)** **(i)** $1^2 : 3^2 = 1 : 9$

(ii) $1^3 : 3^3 = 1 : 27$

(b) **(i)** $1.44 : 7.84 = 9 : 49$

$\sqrt{9} : \sqrt{49} = 3 : 7$

(ii) $3^3 : 7^3 = 27 : 343$

(c) $\sqrt[3]{1^2} : \sqrt[3]{64^2} = 1 : 16$

(d) $\sqrt[3]{1} : \sqrt[3]{2.197} = 1 : 1.3$

$= 10 : 13$

12. **(i)** Let the total surface areas of the small marker and large marker be A_1 cm^2 and A_2 cm^2 respectively.

$$\frac{A_1}{A_2} = \left(\frac{14}{28}\right)^2$$
$$= \frac{1}{4}$$

$\therefore A_1 : A_2 = 1 : 4$

(ii) $$\frac{A_1}{208} = \frac{1}{4}$$
$$A_1 = \frac{1}{4} \times 208$$
$$= 52$$

$\therefore$ Total surface area of the small marker is 52 cm^2.

13. **(i)** Height of original statue $= \frac{5}{2} \times 24$

$= 60$ cm

(ii) Let the mass of the original statue be m kg.

$$\frac{m}{1.6} = \left(\frac{5}{2}\right)^3$$
$$m = \left(\frac{5}{2}\right)^3 \times 1.6$$
$$= 25$$

$\therefore$ Mass of original statue is 25 kg.

14. **(i)** Let the circumferences of the top of the smaller glass and larger glass be C_1 cm and C_2 cm respectively.

$$\left(\frac{C_1}{C_2}\right)^2 = \frac{9}{49}$$
$$\frac{C_1}{C_2} = \frac{3}{7}$$

$\therefore C_1 : C_2 = 3 : 7$

(ii) Let the capacity of the smaller glass be V cm^3.

$$\frac{V}{857.5} = \left(\frac{3}{7}\right)^3$$
$$V = \left(\frac{3}{7}\right)^3 \times 857.5$$
$$= 67.5$$

$\therefore$ Capacity of smaller glass is 67.5 cm^3.

15. **(i)** Height of replica $= \frac{7}{300} \times 860$

$= 20.1$ cm (to 3 s.f.)

(ii) Let the mass of the statue be m kg.

$$\frac{m}{0.4} = \left(\frac{300}{7}\right)^3$$
$$m = \left(\frac{300}{7}\right)^3 \times 0.4$$
$$= 31\ 500 \text{ (to 3 s.f.)}$$

$\therefore$ Mass of statue is 31 500 kg.

16. $\triangle PQR$ is similar to $\triangle PTS$.

$$\frac{\text{Area of } \triangle PTS}{27} = \left(\frac{5}{3}\right)^2$$
$$\text{Area of } \triangle PTS = \left(\frac{5}{3}\right)^2 \times 27$$
$$= 75 \text{ cm}^2$$
$$\text{Area of } QRST = 75 - 27$$
$$= 48 \text{ cm}^2$$

17. $$\frac{\text{Area of } AFGHI}{245} = \left(\frac{4}{7}\right)^2$$
$$\text{Area of } AFGHI = \left(\frac{4}{7}\right)^2 \times 245$$
$$= 80 \text{ cm}^2$$
$$\text{Area of shaded region} = 245 - 80$$
$$= 165 \text{ cm}^2$$

18. Let the actual area occupied be A m^2.

$$\frac{A}{396 \times 10^{-4}} = \left(\frac{26}{22 \times 10^{-2}}\right)^2$$

$$A = \left(\frac{26}{22 \times 10^{-2}}\right)^2 \times 396 \times 10^{-4}$$

$$= 553 \text{ (to 3 s.f.)}$$

$\therefore$ Actual area is 553 m^2.

19. Let the actual area occupied be A m^2.

$$\frac{A}{24 \times 10^{-4}} = \left(\frac{120}{1.5 \times 10^{-2}}\right)^2$$

$$A = \left(\frac{120}{1.5 \times 10^{-2}}\right)^2 \times 24 \times 10^{-4}$$

$$= 153\ 600$$

$\therefore$ Actual area is 153 600 m^2.

20. (i) $\frac{\text{Area of } \triangle PMN}{\text{Area of } \triangle PQR} = \left(\frac{4}{5}\right)^2$

$$= \frac{16}{25}$$

(ii) Area of $MNRQ = \frac{1}{2}(5 + 4)(2)$

$$= 9 \text{ m}^2$$

Let the area of $\triangle PMN$ be A m^2.

$$\frac{A}{A + 9} = \frac{16}{25}$$

$$25A = 16A + 144$$

$$9A = 144$$

$$A = 16$$

$\therefore$ Area of $\triangle PMN = 16$ m^2

21. (i) $ST : QR = a : a + b$

(ii) Area of $\triangle PST$: Area of $\triangle PQR = a^2 : (a + b)^2$

22. (i) $\triangle PBA$ is similar to $\triangle PSQ$.

$$\frac{AB}{10} = \frac{1}{2}$$

$$AB = \frac{1}{2} \times 10$$

$$= 5 \text{ cm}$$

(ii) $\frac{\text{Area of } \triangle PQS}{45} = \left(\frac{2}{1}\right)^2$

$$\text{Area of } \triangle PQS = \left(\frac{2}{1}\right)^2 \times 45$$

$$= 180 \text{ cm}^2$$

(iii) $RC : RQ = p : p + q$

(iv) Area of $\triangle RCD$: Area of $\triangle RQS = p^2 : (p + q)^2$

23. (i) $\frac{\text{Area of } \triangle PRS}{20} = \left(\frac{6}{3}\right)^2$

$$\text{Area of } \triangle PRS = \left(\frac{6}{3}\right)^2 \times 20$$

$$= 80 \text{ cm}^2$$

(ii) Area of $\triangle PQS = \frac{10}{6} \times 80$

$$= 133\frac{1}{3} \text{ cm}^2$$

24. (i) Let the dimeter of the larger container be d cm.

$$\frac{d}{7} = \frac{12}{8}$$

$$d = \frac{12}{8} \times 7$$

$$= 10\frac{1}{2}$$

$\therefore$ Diameter of larger container is $10\frac{1}{2}$ cm.

(ii) Volume of smaller container

: Volume of larger container

$= 8^3 : 12^3$

$= 8 : 27$

25. (i) Mass $= 8640(6.5)$

$= 56\ 160$ g

$= 56.16$ kg

(ii) Let the height of the model be h cm.

$$\left(\frac{h}{86}\right)^3 = \frac{135}{8640}$$

$$\frac{h}{86} = \sqrt[3]{\frac{135}{8640}}$$

$$h = \sqrt[3]{\frac{135}{8640}} \times 86$$

$$= 21.5$$

$\therefore$ Height of model is 21.5 cm.

26. Let the volumes of the original ingot and the smaller ingot be V_1 cm^3 and V_2 cm^3 respectively.

$$\frac{V_1}{V_2} = \frac{216}{1}$$

If the lengths of the ingots are l_1 cm and l_2 cm respectively, then,

$$\frac{l_1}{l_2} = \sqrt[3]{\frac{216}{1}}$$

$$= \frac{6}{1}$$

$$\frac{l_1}{4.24} = \frac{6}{1}$$

$$l_1 = \frac{6}{1} \times 4.24$$

$$= 25.44$$

$\therefore$ Length of original ingot is 25.44 cm.

27. **(i)** Let the circumference of the small tin be C cm.

$$\frac{C}{48} = \frac{7.5}{11.25}$$
$$C = \frac{7.5}{11.25} \times 48$$
$$= 32$$

$\therefore$ Circumference of small tin is 32 cm.

(ii) Volume of large tin : Volume of small tin

$= 11.25^3 : 7.5^3$

$= 27 : 8$

(iii) $\text{Cost} = \frac{8}{27} \times \$10.80$

$= \$3.20$

28. **(i)** Let the length of the actual boat l m.

$$\frac{l}{1.6} = \frac{2.1}{0.14}$$
$$l = \frac{2.1}{0.14} \times 1.6$$
$$= 24$$

$\therefore$ Length of actual boat is 24 m.

(ii) $$\frac{\text{Surface area of model boat}}{\text{Surface area of actual boat}} = \left(\frac{2.1}{0.14}\right)^2$$
$$= \frac{225}{1}$$

(iii) Let the cost of painting the actual boat be $\$C$.

$$\frac{C}{3.2} = \frac{225}{1}$$
$$C = \frac{225}{1} \times 3.2$$
$$= 720$$

$\therefore$ Cost of painting the actual boat is \$720.

29. **(i)** Let the volume of the whole cone be V_1 cm^3.

$$\frac{V_1}{V} = \left(\frac{2}{1}\right)^3$$
$$V_1 = \left(\frac{2}{1}\right)^3 \times V$$
$$= 8V$$

$\therefore$ Volume of the whole cone is $8V$ cm^3.

(ii) $$\frac{V}{V + 3500} = \left(\frac{1}{2}\right)^3$$
$$= \frac{1}{8}$$
$$8V = V + 3500$$
$$7V = 3500$$
$$V = 500$$

$\therefore$ Volume of solid A is 500 cm^3.

30. **(i)** Let the depth of the water be h cm.

$$\left(\frac{h}{12}\right)^3 = \frac{1}{8}$$
$$\frac{h}{12} = \sqrt[3]{\frac{1}{8}}$$
$$h = \sqrt[3]{\frac{1}{8}} \times 12$$
$$= 6$$

$\therefore$ Depth of water is 6 cm.

(ii) Area of top surface of the water

: Area of top surface of the container

$= 6^2 : 12^2$

$= 1 : 4$

31. **(i)** Let the height of the container be h cm.

$$\left(\frac{h}{5}\right)^3 = \frac{27}{1}$$
$$\frac{h}{5} = \sqrt[3]{\frac{27}{1}}$$
$$h = \sqrt[3]{\frac{27}{1}} \times 5$$
$$= 15$$

$\therefore$ Height of container is 15 cm.

(ii) Area of top surface of the water

: Area of top surface of the container

$= 5^2 : 15^2$

$= 1 : 9$

32. Let the lengths of the lighter box and heavier box be l_1 m and l_2 m respectively.

$$\left(\frac{l_1}{l_2}\right)^3 = \frac{8.58}{68.64}$$
$$\frac{l_1}{l_2} = \sqrt[3]{\frac{8.58}{68.64}}$$
$$= \frac{1}{2}$$

Let the base area of the ligher box be A m^2.

$$\frac{A}{23.72} = \left(\frac{1}{2}\right)^2$$
$$A = \left(\frac{1}{2}\right)^2 \times 23.72$$
$$= 5.93$$

$\therefore$ Base area of the lighter box is 5.93 m^2.

Advanced

33. **(i)** $$\frac{\text{Area of } SQRT + 20}{20} = \left(\frac{5}{2}\right)^2$$
$$\text{Area of } SQRT + 20 = \left(\frac{5}{2}\right)^2 \times 20$$
$$= 125$$
$$\text{Area of } SQRT = 105 \text{ cm}^2$$

(ii) $\dfrac{\text{Area of } \triangle TUR}{20} = \left(\dfrac{3}{2}\right)^2$

$\text{Area of } \triangle TUR = \left(\dfrac{3}{2}\right)^2 \times 20$

$= 45 \text{ cm}^2$

$\text{Area of } STUQ = 105 - 45$

$= 60 \text{ cm}^2$

34. (a) $\triangle ARQ$ is similar to $\triangle ABP$.

$\dfrac{9 + RB}{9} = \dfrac{8}{6}$

$9 + RB = 12$

$RB = 3$ cm (shown)

(b) (i) $\dfrac{\text{Area of } \triangle ARQ}{\text{Area of } \triangle ABP} = \left(\dfrac{6}{8}\right)^2$

$= \dfrac{9}{16}$

$\therefore \dfrac{\text{Area of } \triangle ARQ}{\text{Area of trapezium } RBPQ} = \dfrac{9}{7}$

(ii) $\dfrac{\text{Area of } \triangle BQR}{\text{Area of } \triangle ARQ} = \dfrac{3}{9}$

$= \dfrac{1}{3}$

(iii) $\dfrac{\text{Area of } \triangle ABC}{\text{Area of } \triangle BQR} = \left(\dfrac{12}{3}\right)^2$

$= \dfrac{16}{1}$

(c) (i) From **(b)(i)**,

$\text{Area of } \triangle ARQ = \dfrac{9}{7} \times 21$

$= 27 \text{ cm}^2$

(ii) $\text{Area of } \triangle BQR = \dfrac{3}{7} \times 21$

$= 9 \text{ cm}^2$

(iii) $\text{Area of } \triangle ABC = 16 \times 9$

$= 144 \text{ cm}^2$

35. (i) $\angle HMN = \angle APN$ (alt. $\angle$s, MH // AP)

$\angle MNH = \angle PNA$ (vert. opp. $\angle$s)

$\therefore \triangle MNH$ is similar to $\triangle PNA$.

(2 pairs of corr. $\angle$s equal).

(ii) $MH : AP = 24 : 8$

$= 3 : 1$

(iii) Area of $\triangle PNA$: Area of $\triangle MNH$

$= 1 : 9$

Area of $\triangle MAN$: Area of $\triangle MNH$

$= 1 : 3$

$= 3 : 9$

$\therefore$ Area of $\triangle MAN$: Area of $HNPT$

$= 3 : 11$

36. $\left(\dfrac{40}{30}\right)^3 = \dfrac{x^2}{x + 0.4}$

$\dfrac{64}{27} = \dfrac{x^2}{x + 0.4}$

$64x + 25.6 = 27x^2$

$27x^2 - 64x - 25.6 = 0$

$x = \dfrac{-(-64) \pm \sqrt{(-64)^2 - 4(27)(-25.6)}}{2(27)}$

$= 2.719$ or -0.349

$\therefore x = 2.72$ (to 3 s.f.)

37. (i) Let the total slant surface areas of A, B and C be S_A cm^2, S_B cm^2 and S_C cm^2 respectively.

$\dfrac{S_A}{S_A + S_B} = \left(\dfrac{2}{5}\right)^2$

$= \dfrac{4}{25}$

$\dfrac{S_A}{S_A + S_B + S_C} = \left(\dfrac{2}{9}\right)^2$

$= \dfrac{4}{81}$

$\therefore S_A : S_B : S_C = 4 : 21 : 56$

(ii) Let the volumes of A, B and C be V_A cm^3, V_B cm^3 and and V_C cm^3 respectively.

$\dfrac{V_A}{V_A + V_B} = \left(\dfrac{2}{5}\right)^3$

$= \dfrac{8}{125}$

$\dfrac{V_A}{V_A + V_B + V_C} = \left(\dfrac{2}{9}\right)^3$

$= \dfrac{8}{729}$

$\therefore V_A : V_B : V_C = 8 : 117 : 604$

38. (i) $\dfrac{\text{Volume of whole iceberg}}{V} = \left(\dfrac{72}{6}\right)^3$

$\text{Volume of whole iceberg} = \left(\dfrac{72}{6}\right)^3 \times V$

$= 1728V \text{ m}^3$

(ii) Volume of tip of iceberg : Volume of submerged part

$= 1 : 1727$

39. Let the radius of cylinders A and B be r_A and r_B respectively.
Let the volume of metal balls P and Q be V.
Since the water level in B increases by 3 cm,

$$V = \pi r_B^2(3)$$
$$= 3\pi r_B^2$$

Let the increase in the water level in A be x cm,

$$V = \pi r_A^2(x)$$
$$= x\pi r_A^2$$

$$\therefore\ 3\pi r_B^2 = x\pi r_A^2$$
$$x = \left(\frac{r_B}{r_A}\right)^2 \times 3$$
$$= \left(\frac{6}{1}\right)^2 \times 3$$
$$= 36 \times 3$$
$$= 108 \text{ cm}$$

$\therefore$ The increase in the water level in A is 108 cm.

New Trend

40. (a) Let the height of the 40-litre backpack be h cm.

$$\left(\frac{h}{80}\right)^3 = \frac{40}{65}$$
$$\frac{h}{80} = \sqrt[3]{\frac{40}{65}}$$
$$h = \sqrt[3]{\frac{40}{65}} \times 80$$
$$= 68.0 \text{ cm (to 3 s.f.)}$$

(b) Let the surface areas of the 65-litre and 40-litre backpacks be A_1 cm^2 and A_2 cm^2 respectively.

$$\frac{A_1}{A_2} = \left(\sqrt[3]{\frac{65}{40}}\right)^2$$
$$= 1.38 \text{ (to 2 d.p.)}$$

The ratio is 1.38 : 1.

$\therefore\ k = 1.38$

41. Let the heights of the smaller oxygen tank and larger oxygen tank be h_1 cm and h_2 cm respectively.

$$\left(\frac{h_1}{h_2}\right)^3 = \frac{42}{164}$$
$$\frac{h_1}{h_2} = \sqrt[3]{\frac{42}{164}}$$
$$= 0.635 \text{ (to 3 s.f.)}$$

$$\text{Percentage} = 0.635 \times 100\%$$
$$= 63.5\%$$

$\therefore$ The height of the smaller tank is 63.5% of that of the larger tank.

42. (a) $\angle AOM = 360° \div 10$

$$= 36°$$
$$\tan 36° = \frac{3}{OM}$$
$$OM = \frac{3}{\tan 36°}$$
$$= 4.1291 \text{ cm (to 5 s.f.)}$$
$$\text{Area of pentagon} = 5 \times \frac{1}{2} \times 6 \times \frac{3}{\tan 36°}$$
$$= 61.94 \text{ cm}^2 \text{ (to 4 s.f.) (shown)}$$

(b) Using Pythagoras' Theorem,

$$OX^2 + 4.1291^2 = 10^2$$
$$OX^2 = 10^2 - 4.1291^2$$
$$OX = 9.1077 \text{ cm (to 5 s.f.)}$$
$$\text{Volume} = \frac{1}{3} \times 61.94 \times 9.1077$$
$$= 188 \text{ cm}^3 \text{ (to 3 s.f.)}$$

(c) Let the volume of the second pyramid be V_2 cm^3.

$$\frac{V_2}{188.04} = \left(\frac{10}{6}\right)^3$$
$$V_2 = \left(\frac{10}{6}\right)^3 \times 188.04$$
$$= 871 \text{ cm}^3 \text{ (to 3 s.f.)}$$

$\therefore$ Volume of the second pyramid is 871 cm^3.

Chapter 13 Geometrical Properties of Circles

Basic

1. $\angle BOA = 2\angle ACB$ ($\angle$ at centre = 2 $\angle$ at $\bigcirc^{ce}$)
$= 2(47°)$
$= 94°$
$\angle CAO = 180° - 68° - 94°$ ($\angle$ sum of a $\triangle$)
$= 18°$

2. **(i)** $\angle ACD = 90°$ (rt. $\angle$ in a semicircle)
$\angle ADC = 180° - 90° - 48°$ ($\angle$ sum of a $\triangle$)
$= 42°$

(ii) $\angle ABC = 180° - 42°$ ($\angle$s in opp. segments)
$= 138°$
$\angle ACB = \dfrac{180° - 138°}{2}$ (base $\angle$s of isos. $\triangle$)
$= 21°$

3. $\angle BAD = 180° - 80°$ ($\angle$s in opp. segments)
$= 100°$
$\angle ABD = \dfrac{180° - 100°}{2}$ (base $\angle$s of isos. $\triangle$)
$= 40°$
$\angle DBC = 180° - 64° - 80°$ ($\angle$ sum of a $\triangle$)
$= 36°$
$\angle ABC = \angle ABD + \angle DBC$
$= 40° + 36°$
$= 76°$

4. **(i)** $\angle ADC = \dfrac{1}{2}\angle AOC$ ($\angle$ at centre = 2 $\angle$ at $\bigcirc^{ce}$)
$= \dfrac{1}{2}(42°)$
$= 21°$

(ii) $\angle OAD = \angle ADC$ (alt. $\angle$s, $OA \,//\, DC$)
$= 21°$

5. **(i)** $\angle BAD = 180° - 72°$ (adj. $\angle$s on a str. line)
$= 108°$
$\angle BCD = 180° - 108°$ ($\angle$s in opp. segments)
$= 72°$

(ii) $\angle BAC = \angle BDC$ ($\angle$s in same segment)
$= 38°$
$\angle DAC = 108° - 38°$
$= 70°$

(iii) $\angle DBA + \angle MAB = \angle AMD$ (ext. $\angle$ = sum of int. opp. $\angle$s)
$\angle DBA + 38° = 64°$
$\angle DBA = 26°$

6. **(i)** $\angle AOB = 2\angle ACB$ ($\angle$ at centre = 2 $\angle$ at $\bigcirc^{ce}$)
$= 2(45°)$
$= 90°$

(ii) $\angle OBA = \dfrac{180° - 90°}{2}$ (base $\angle$s of isos. $\triangle$)
$= 45°$
$\angle OBC = 61°$ (base $\angle$s of isos. $\triangle$)
$\angle ABC = \angle OBA + \angle OBC$
$= 45° + 61°$
$= 106°$

(iii) $\angle BOC = 180° - 61° - 61°$ ($\angle$ sum of a $\triangle$)
$= 58°$

7. **(i)** $\angle PSQ = 90°$ (rt. $\angle$ in a semicircle)
$\angle SPQ = 180° - 90° - 38°$ ($\angle$ sum of a $\triangle$)
$= 52°$

(ii) $\angle QST + \angle STQ = \angle PQS$ (ext. $\angle$ = sum of int. opp. $\angle$s)
$\angle QST + 22° = 38°$
$\angle QST = 16°$

(iii) $\angle PQS + \angle SQR = \angle PSQ + \angle QSR$ ($\angle$s in opp. segments)
$38° + \angle SQR = 90° + 16°$
$\angle SQR = 68°$

8. $\angle BAD = \dfrac{1}{2}\angle BOD$ ($\angle$ at centre = 2 $\angle$ at $\bigcirc^{ce}$)
$= \dfrac{1}{2}(132°)$
$= 66°$
$\angle BCD + \angle BAD = 180°$ ($\angle$s in opp. segments)
$\angle BCD + 66° = 180°$
$\angle BCD = 114°$
$\angle DCE = 180° - 114°$ (adj. $\angle$s on a str. line)
$= 66°$

9. $\angle BAD = \dfrac{1}{2}\angle BOD$ ($\angle$ at centre = 2 $\angle$ at $\bigcirc^{ce}$)
$= \dfrac{1}{2}(130°)$
$= 65°$
$\angle BAX = 180° - 65°$ (adj. $\angle$s on a str. line)
$= 115°$

10. $\angle AQB = 90°$ (rt. $\angle$ in a semicircle)
$\angle PQB = 90° - 38°$
$= 62°$
$\angle PBQ = 180° - 32° - 62°$ (adj. $\angle$ sum of a $\triangle$)
$= 86°$

11. **(i)** $\angle ABC = 90°$ (rt. $\angle$ in a semicircle)
$\angle DBC = 90° - 24°$
$= 66°$

(ii) $\angle BAP + \angle ABP = \angle BPC$ (ext. $\angle$ = sum of int. opp. $\angle$s)
$\angle BAP + 24° = 58°$
$\angle BAP = 34°$
$\angle BDC = \angle BAP$
$= 34°$ ($\angle$s in same segment)

12. **(i)** reflex $\angle AOC = 360° - 132°$ ($\angle$s at a pt.)
$= 228°$

$\angle ABC = \frac{1}{2} \times$ reflex $\angle AOC$ ($\angle$ at centre = 2 $\angle$ at $\bigcirc^{ce}$)

$= \frac{1}{2}(228°)$

$= 114°$

(ii) $\angle OAB = 360° - 114° - 64° - 132°$ ($\angle$ sum of a quadrilateral)
$= 50°$

(iii) $\angle OAC = \frac{180° - 132°}{2}$ (base $\angle$s of isos. $\triangle$)
$= 24°$

13. $\angle PQR = \frac{1}{2} \times$ reflex $\angle POR$ ($\angle$ at centre = 2$\angle$ at $\bigcirc^{ce}$)

$= \frac{1}{2}(230°)$

$\angle OPQ = 180° - 115°$ (int. $\angle$s, $PO \parallel QR$)
$= 65°$

14. **(i)** $\angle BCD = 90°$ (rt. $\angle$ in a semicircle)

$\angle BCA = \angle ADB$ ($\angle$s in same segment)
$= 25°$

$\angle ACD = 90° - 25°$
$= 65°$

(ii) $\angle CAD + \angle ADB = \angle CKD$ (ext. $\angle$ = sum of int. opp. $\angle$s)
$\angle CAD + 25° = 64°$
$\angle CAD = 39°$

15. **(i)** $\angle ADC = 180° - 124°$ (adj. $\angle$s on a str. line)
$= 56°$

$\angle ACD = 180° - 56° - 56°$ ($\angle$ sum of a $\triangle$)
$= 68°$

(ii) $\angle ABC + \angle ADC = 180°$ ($\angle$s in opp. segments)
$\angle ABC + 56° = 180°$
$\angle ABC = 124°$

$\angle BAC = \frac{180° - 124°}{2}$ (base $\angle$s of isos. $\triangle$)
$= 28°$

16. **(i)** $\angle BCD + \angle BAD = 180°$ ($\angle$s in opp. segments)
$\angle BCD + 70° = 180°$
$\angle BCD = 110°$

(ii) $\angle BOD = 2\angle BAD$ ($\angle$ at centre = 2 $\angle$ at $\bigcirc^{ce}$)
$= 2(70°)$
$= 140°$

$\angle OBD = \frac{180° - 140°}{2}$ (base $\angle$s of isos. $\triangle$)
$= 20°$

17. **(i)** $\angle ACD = 180° - 56° - 78°$ (adj. $\angle$s on a str. line)
$= 46°$

$\angle ABD = \angle ACD$ ($\angle$s in same segment)
$= 46°$

(ii) $\angle CAD = 180° - 28° - 78° - 46°$ ($\angle$ sum of a $\triangle$)
$= 28°$

$\angle CBD = \angle CAD$ ($\angle$s in same segment)
$= 28°$

18. **(i)** $\angle ADC + \angle ABC = 180°$ ($\angle$s in opp. segments)
$\angle ADC + 107° = 180°$
$\angle ADC = 73°$

(ii) $\angle BCD + \angle BAD = 180°$ ($\angle$s in opp. segments)
$\angle BCD + 80° = 180°$
$\angle BCD = 100°$
$\angle BCP = 180° - 100°$ (adj. $\angle$s on a str. line)
$= 80°$

(iii) $\angle BPC = 180° - 80° - 73°$ ($\angle$ sum of a $\triangle$)
$= 27°$

19. **(i)** $\angle ADE = \angle ACE$ ($\angle$s in same segments)
$= 52°$

$\angle ADC = 124° - 52°$
$= 72°$

(ii) $\angle EAC + \angle CDE = 180°$ ($\angle$s in opp. segments)
$\angle EAC + 124° = 180°$
$\angle EAC = 56°$

$\angle ACB + \angle ACE + \angle EAC + \angle CAB = 180°$ ($\angle$s in opp. segments)
$\angle ACB + 52° + 56° + 30° = 180°$
$\angle ACB = 42°$

20. **(i)** $\angle CAB = \angle BDC$ ($\angle$s in same segments)
$= 22°$

$\angle BDA = 180° - 46° - 64° - 22°$ ($\angle$ sum of a $\triangle$)
$= 48°$

$\angle ATD = 180° - 22° - 48° - 64° - 22°$ ($\angle$ sum of a $\triangle$)
$= 24°$

(ii) $\angle ABC + \angle ADC = 180°$ ($\angle$s in opp. segments)
$\angle ABC + 48° + 22° = 180°$
$\angle ABC = 110°$
$\angle TBC = 180° - 110°$ ($\angle$s on a str. line)
$= 70°$

21. **(i)** $\angle BOC = 2\angle BAC$ ($\angle$ at centre = 2 $\angle$ at $\bigcirc^{ce}$)
$= 2(24°)$
$= 48°$

(ii) $\angle OCA = \angle BAC$ (alt. $\angle$s, $CO \parallel BA$)
$= 24°$

$\angle OAC = \angle OCA$ (base $\angle$s of isos. $\triangle$)

$\angle OBA = \angle OAB$ (base $\angle$s of isos. $\triangle$)
$= 24° + 24°$
$= 48°$

Intermediate

22. $\angle ABC = \frac{1}{2}\angle AOC$ ($\angle$ at centre = 2 $\angle$ at $\bigcirc^{ce}$)

$= \frac{1}{2}(62°)$

$= 31°$

$\angle OAP = 90°$ (rt. $\angle$ in a semicircle)

$\angle OAB = 180° - 31° - 34° - 90°$ ($\angle$ sum of a $\triangle$)

$= 25°$

23. (i) $\angle ABD = 90°$ (rt. $\angle$ in a semicircle)

$\angle CBD = \angle CAD$ ($\angle$s in same segment)

$= 30°$

$\angle ABC = 90° + 30°$

$= 120°$

(ii) $\angle ACB = \angle BAQ$ ($\angle$s in alt. segment)

$= 28°$

(iii) $\angle DAQ = 90°$ (rt. $\angle$ in a semicircle)

$\angle DAC + \angle CAB + \angle BAQ = 90°$

$30° + \angle CAB + 28° = 90°$

$\angle CAB = 32°$

$\angle BDC = \angle BAC$ ($\angle$s in same segment)

$= 32°$

24. (i) $\angle OCT = 90°$ (tangent $\perp$ radius)

$\angle ACT + \angle OCA = 90°$

$36° + \angle OCA = 90°$

$\angle OCA = 54°$

$\angle OAC = \angle OCA$ (base $\angle$s of isos. $\triangle$)

$= 54°$

(ii) $\angle CAB = 36° + 50°$ (ext. $\angle$ = sum of int. opp. $\angle$s)

$= 86°$

$\angle OAB = \angle CAB - \angle OAC$

$= 86° - 54°$

$= 32°$

$\angle AOB = 180° - 2(32)°$ ($\angle$ sum of a $\triangle$)

$= 116°$

25. (i) $\angle OAT = 90°$ (tangent $\perp$ radius)

Using Pythagoras' Theorem,

$OT^2 = OA^2 + AT^2$

$= 4.6^2 + 7.2^2$

$= 73$

$OT = \sqrt{73}$ cm

$TC = \sqrt{73} - 4.6$

$= 3.94$ cm (to 3 s.f.)

(ii) $\tan \angle ATO = \frac{4.6}{7.2}$

$\angle ATO = \tan^{-1} \frac{4.6}{7.2}$

$= 32.57°$ (to 2 d.p.)

$\angle ATB = 2(32.57°)$

$= 65.1°$ (to 1 d.p.)

26. $\angle COD = 180° - 57° - 57°$ ($\angle$ sum of a $\triangle$)

$= 66°$

$\angle OAC = \frac{1}{2}\angle COD$ ($\angle$ at centre = 2 $\angle$ at $\bigcirc^{ce}$)

$= \frac{1}{2}(66°)$

$= 33°$

reflex $\angle AOC = 180° + 66°$

$= 246°$

$\angle ABC = \frac{1}{2}\angle AOC$ ($\angle$ at centre = 2 $\angle$ at $\bigcirc^{ce}$)

$= \frac{1}{2}(246°)$

$= 123°$

$\angle BAC = \frac{180° - 123°}{2}$ (base $\angle$s of isos. $\triangle$)

$= 28.5°$

$\angle OAB = \angle OAC + \angle BAC$

$= 33° + 28.5°$

$= 61.5°$

27. $\angle AOC = \frac{1}{2}\angle ABC$ ($\angle$ at centre = 2 $\angle$ at $\bigcirc^{ce}$)

$= \frac{1}{2}(98°)$

$= 49°$

$\angle OAC + \angle OCA + 98° = 180°$ ($\angle$ sum of a $\triangle$)

$\angle OAC + \angle OCA = 82°$

$\angle OCB = 180° - 49° - 25° - 82°$ ($\angle$ sum of a $\triangle$)

$= 24°$

28. $\angle ACB = 90°$ (rt. $\angle$ in a semicircle)

$\angle DCA = \angle ABD$ ($\angle$s in same segment)

$= 13°$

$\angle BAC = 25° + 13°$ (ext. $\angle$ = sum of int. opp. $\angle$s)

$= 38°$

29. (i) $\angle ADB = 90°$ (rt. $\angle$ in a semicircle)

$\angle ABD + \angle ACD = 180°$ ($\angle$s in opp. segments)

$\angle ABD + 108° = 180°$

$\angle ABD = 72°$

$\angle BAD = 180° - 90° - 72°$ ($\angle$ sum of a $\triangle$)

$= 18°$

(ii) $\angle BCD = \angle BAD$ ($\angle$s in same segment)

$= 18°$

(iii) $\angle BDX + \angle BXD = 72°$ (ext. $\angle$ = sum of int. opp. $\angle$)

$\angle BDX + 24° = 72°$

$\angle BDX = 48°$

30. (i) $\angle COD = 2\angle CAD$ ($\angle$ at centre = 2 $\angle$ at $\bigcirc^{ce}$)

$= 2(28°)$

$= 56°$

$\angle OCD = \frac{180° - 56°}{2}$ (base $\angle$s of isos. $\triangle$)

$= 62°$

(ii) $\angle BAC = \frac{1}{2}\angle BAC$ ($\angle$ at centre = 2 $\angle$ at $\bigcirc^{ce}$)
$= \frac{1}{2}(114°)$
$= 57°$
$\angle BAD = 28° + 57°$
$= 85°$

31. (i) reflex $\angle AOC = 360° - 122°$ ($\angle$s at a pt.)
$= 238°$
$\angle ABC = \frac{1}{2} \times$ reflex $\angle AOC$ ($\angle$ at centre = 2 $\angle$ at $\bigcirc^{ce}$)
$= \frac{1}{2}(238°)$
$= 119°$

(ii) $\angle DXC = 96°$ (vert. opp. $\angle$s)
$\angle ADC = \frac{1}{2}\angle AOC$ ($\angle$ at centre = 2 $\angle$ at $\bigcirc^{ce}$)
$= \frac{1}{2}(122°)$
$= 61°$
$\angle XCD = 180° - 96° - 61°$ ($\angle$ sum of a $\triangle$)
$= 23°$

32. (i) $\angle AQB = 90°$ (rt. $\angle$ in a semicircle)
$\angle ABQ = 180° - 90° - 26°$ ($\angle$ sum of a $\triangle$)
$= 64°$
$\angle APQ + \angle ABQ = 180°$ ($\angle$s in opp. segments)
$\angle APQ + 64° = 180°$
$\angle APQ = 116°$

(ii) $\angle APQ + \angle PAQ + \angle QAB = 180°$ (int. $\angle$s, $PQ \parallel AB$)
$116° + \angle PAQ + 26° = 180°$
$\angle PAQ = 38°$

33. (i) $\angle ABC = 180° - 38° - 76°$ ($\angle$ sum of a $\triangle$)
$= 66°$
$\angle AQB + \angle ACB = 180°$ ($\angle$s in opp. segments)
$\angle AQB + 76° = 180°$
$\angle AQB = 104°$
$\angle ABQ = \frac{180° - 104°}{2}$ (base $\angle$s of isos. $\triangle$)
$= 38°$
$\angle QBC = 66° + 38°$
$= 104°$

(ii) $\angle PBQ = 90°$ (rt. $\angle$ in a semcircle)
$\angle PBC = 104° - 90°$
$= 14°$

34. (i) reflex $\angle AOC = 2\angle ABC$ ($\angle$ at centre = 2 $\angle$ at $\bigcirc^{ce}$)
$= 2(118°)$
$= 236°$
obtuse $\angle AOC = 360° - 236°$ ($\angle$s at a pt.)
$= 124°$

(ii) Let the radius of the circle be r cm.
Using Cosine Rule,
$24^2 = r^2 + r^2 - 2(r)(r)\cos 124°$
$576 = 2r^2 - 2r^2 \cos 124°$
$= 2r^2(1 - \cos 124°)$
$288 = r^2(1 - \cos 124°)$
$r^2 = \frac{288}{1 - \cos 124°}$
$r = \sqrt{\frac{288}{1 - \cos 124°}}$
$= 13.6$ (to 3 s.f.)
$\therefore$ Radius of circle = 13.6 cm

35. (i) $\angle PQD + \angle PCD = 180°$ ($\angle$s in opp. segments)
$\angle PQD + 86° = 180°$
$\angle PQD = 94°$
$\angle PQA = 180° - 94°$ ($\angle$s on a str. line)
$= 86°$
$\angle ABC + \angle PQA = 180°$ ($\angle$s in opp. segments)
$\angle ABC + 86° = 180°$
$\angle ABC = 94°$

(ii) $\angle BPQ + \angle BAQ = 180°$ ($\angle$s in opp. segments)
$\angle BPQ + 95° = 180°$
$\angle BPQ = 85°$
$\angle QPC = 180° - 85°$ ($\angle$s on a str. line)
$= 95°$
$\angle ADC + \angle QPC = 180°$ ($\angle$s in opp. segments)
$\angle ADC + 95° = 180°$
$\angle ADC = 85°$

36. $\angle ACD = \frac{1}{2}\angle AOD$ ($\angle$ at centre = 2 $\angle$ at $\bigcirc^{ce}$)
$= \frac{1}{2}(104°)$
$= 52°$
$\angle ACP = 180° - 52°$ (adj. $\angle$s on a str. line)
$= 128°$
$\angle ABD = \angle ACD$ ($\angle$s in same segment)
$= 52°$
$\angle DBP = 180° - 52°$ (adj. $\angle$s on a str. line)
$= 128°$
$\angle BKC = 360° - 32° - 128° - 128°$ ($\angle$ sum of a quadrilateral)
$= 72°$

37. $\angle AED + \angle ACD = 180°$ ($\angle$s in opp. segments)
$\angle ACB = 90°$ (rt. $\angle$ in a semicircle)
$\angle BCD + \angle AED = \angle ACB + \angle ACD + \angle AED$
$= 90° + 180°$
$= 270°$

38. (i) $\angle ABD = 21° + 29°$ (ext. $\angle$ = sum of int. opp. $\angle$s)
$= 50°$

(ii) $\angle BAC = \angle BDC$ ($\angle$s in same segment)
$= 21°$

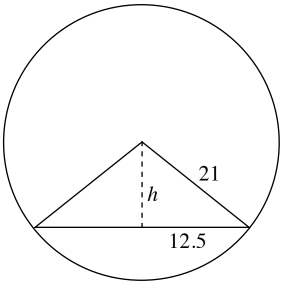
$\angle AKD = 21° + 50°$ (ext. $\angle$ = sum of int. opp. $\angle$s)
$= 71°$

(iii) $\angle ACD = \angle ABD$ ($\angle$s in same segment)
$= 50°$

$\angle PBC + 29° = 50° + 55°$ (ext. $\angle$ = sum of int. opp. $\angle$s)
$\angle PBC = 76°$

39. $\angle ACD = 90°$ (rt. $\angle$ in a semicircle)
$\angle BAD + \angle BCD = 180°$ ($\angle$s in opp. segments)
$\angle BAD + 38° + 90° = 180°$
$\angle BAD = 52°$
$\angle BED = 52° + 38°$ (ext. $\angle$ = sum of int. opp. $\angle$s)
$= 90°$

40.



Using Pythagoras' Theorem,
$h^2 + 12.5^2 = 21^2$
$h^2 = 284.75$
$h = 16.9$ (to 3 s.f.)
$\therefore$ Perpendicular distance from the centre of circle to the chord is 16.9 cm.

41. (i) $\angle AOD = 2\angle ABD$ ($\angle$ at centre = 2 $\angle$ at $\bigcirc^{ce}$)
$= 2(42°)$
$= 84°$

$\angle OAD = \dfrac{180° - 84°}{2}$ (base $\angle$s of isos. $\triangle$)
$= 48°$

(ii) $\angle ACD = \angle ABD$ ($\angle$s in same segment)
$= 42°$

$\angle BDC + 42° = 64°$ (ext. $\angle$ = sum. of int. opp. $\angle$s)
$\angle BDC = 22°$
$\angle ADC = 90°$ (rt. $\angle$ in a semicircle)
$\angle ADB = 90° - 22°$
$= 68°$

42. $\angle ACD = \dfrac{1}{2}\angle AOD$ ($\angle$ at centre = 2 $\angle$ at $\bigcirc^{ce}$)
$= \dfrac{1}{2}(114°)$
$= 57°$

$\angle AKD + 18° = 57°$ (ext. $\angle$ = sum of int. opp. $\angle$s)
$\angle AKD = 39°$

43. (i) $\angle AOB = 180° - 2(40°)$ ($\angle$ sum of a $\triangle$)
$= 100°$

$\angle ACB = \dfrac{1}{2}\angle AOB$ ($\angle$ at centre = 2 $\angle$ at $\bigcirc^{ce}$)
$= \dfrac{1}{2}(100°)$
$= 50°$

(ii) $\angle OBC = 180° - 50° - 69° - 40°$ ($\angle$ sum of a $\triangle$)
$= 21°$

$\angle OBD = 180° - 21°$ ($\angle$s on a str. line)
$= 159°$

44. (i) $\angle BAC = \angle BDC$ ($\angle$s in same segment)
$= 19°$

$\angle ACP = 180° - 19° - 35°$ ($\angle$ sum of a $\triangle$)
$= 126°$

(ii) $\angle DBP = 180° - 19° - 35°$ ($\angle$ sum of a $\triangle$)
$= 126°$

$\angle BXC = 360° - 126° - 126° - 35°$ ($\angle$ sum of a quadrilateral)
$= 73°$

45. $\angle ACB = 90°$ (rt. $\angle$ in a semicircle)
$\angle CBE = 90° + 28°$ (ext. $\angle$ = sum of int. opp. $\angle$s)
$= 118°$
$\angle ADE + \angle CBE = 180°$ ($\angle$s in opp. segments)
$\angle ADE + 118° = 180°$
$\angle ADE = 62°$

46. $\angle ADC + \angle ABC = 180°$ ($\angle$s in opp. segments)
$\angle ADC + 108° = 180°$
$\angle ADC = 72°$
$\angle ACD = 180° - 72° - 71°$ ($\angle$ sum of a $\triangle$)
$= 37°$

47. $\angle ACB = \dfrac{1}{2}\angle AOB$ ($\angle$ at centre = 2 $\angle$ at $\bigcirc^{ce}$)
$= \dfrac{1}{2}(96°)$
$= 48°$

$\angle OAB + \angle OBA = 180° - 96°$ ($\angle$ sum of a $\triangle$)
$= 84°$
$\angle OBC = 180° - 48° - 32° - 84°$ ($\angle$ sum of a $\triangle$)
$= 16°$

48. (i) $\angle ACB = \dfrac{1}{2}\angle AOB$ ($\angle$ at centre = 2 $\angle$ at $\bigcirc^{ce}$)
$= \dfrac{1}{2}(48°)$
$= 24°$

$\angle BXC = 180° - 24° - 46°$ ($\angle$ sum of a $\triangle$)
$= 110°$

(ii) $\angle OXA = \angle BXC$ (vert. opp. $\angle$s)
$= 110°$

$\angle OAC = 180° - 48° - 110°$ ($\angle$ sum of a $\triangle$)
$= 22°$

49. $\angle OAT = 90°$ (tangent $\perp$ radius)
$\angle OAB = 90 - 26°$
$= 64°$
$\angle COA = 180° - 64°$ (int. $\angle$s, CO // BA)
$= 116°$
reflex $\angle COA = 360° - 116°$($\angle$s at a pt.)
$= 244°$
$\angle ABC = \frac{1}{2} \times$ reflex $\angle COA$ ($\angle$ at centre = 2 $\angle$ at ○ce)
$= \frac{1}{2}(244°)$
$= 122°$
$\angle OCB = 180° - 122°$ (int. $\angle$s, CO // BA)
$= 58°$

50. $\angle ORP = \angle OPR$ (base $\angle$s of isos. △)
$= x°$
$\angle POQ = x° + x°$ (ext. $\angle$ = sum of int. opp. $\angle$s)
$= 2x°$
$\angle OPT = \angle OQT$ (tangent $\perp$ radius)
$= 90°$
$\angle PTQ = 360° - 90° - 2x° - 90°$
$= 180° - 2x°$
$= (180 - 2x)°$

51. **(i)** $\angle OAT = 90°$ (tangent $\perp$ radius)
$\angle OAB = 180° - 32° - 40° - 90°$ ($\angle$ sum of a △)
$= 18°$
(ii) $\angle AOC = 2\angle ABC$ ($\angle$ at centre = 2 $\angle$ at ○ce)
$= 2(32°)$
$= 64°$
$\angle OAC + \angle OCA = 180° - 64°$ ($\angle$ sum of a △)
$= 116°$
$\angle OCB = 180° - 116° - 18° - 32°$ ($\angle$ sum of a △)
$= 14°$

52. **(i)** $\angle ADC + \angle ABC = 180°$ ($\angle$s in opp. segments)
$\angle ADC + 124° = 180°$
$\angle ADC = 56°$
$\angle DAC = 180° - 63° - 56°$ ($\angle$ sum of a △)
$= 61°$
(ii) $\angle DOA = 2\angle DCA$ ($\angle$ at centre = 2 $\angle$ at ○ce)
$= 2(63°)$
$= 126°$
$\angle ODA = \frac{180° - 126°}{2}$ (base $\angle$s of isos. △)
$= 27°$
$\angle ODC = 56° - 27°$
$= 29°$

53. **(i)** $\angle OXC = \angle AXD$ (vert. opp. $\angle$s)
$= 128°$
$\angle OCA = \angle OAC$ (base $\angle$s of isos. △)
$= 17°$
$\angle COD = 180° - 128° - 17°$ ($\angle$ sum of a △)
$= 35°$
(ii) $\angle DAX = \frac{1}{2}\angle COD$ ($\angle$ at centre = 2 $\angle$ at ○ce)
$= \frac{1}{2}(35°)$
$= 17.5°$

54. **(a)** Since BD is a diameter of the circle,
$\angle BCD = 90°$ (rt. $\angle$ in a semicircle).
(b) **(i)** $\angle ACD = \frac{1}{2}\angle AOD$ ($\angle$ at centre = 2 $\angle$ at ○ce)
$= \frac{1}{2}(124°)$
$= 62°$
(ii) $\angle OAT = \angle ODT$ (tangent $\perp$ radius)
$= 90°$
$\angle ATD = 360° - 90° - 124° - 90°$ ($\angle$ sum of a quadrilateral)
$= 56°$
(iii) $TA = TD$ (symmetric properties of tangents to circle)
$\angle TAD = \frac{180° - 56°}{2}$ (base $\angle$s of isos. △)
$= 62°$
(iv) $\angle BXC = 62° + 32°$ (ext. $\angle$ = sum of int. opp. $\angle$s)
$= 94°$

55. **(i)** $\angle ADP = 180° - 36° - x°$ ($\angle$ sum of a △)
$= 144° - x°$
$= (144 - x)°$
(ii) $\angle ABQ + 34° = x°$ (ext. $\angle$ = sum of int. opp. $\angle$s)
$\angle ABQ = x° - 34°$
$= (x - 34)°$
(iii) $\angle ABC = 180° - (x - 34)°$ ($\angle$s on a str. line)
$= 214° - x°$
$\angle ABC + \angle ADP = 180°$ ($\angle$s in opp. segments)
$214° - x° + 144° - x° = 180°$
$2x° = 178°$
$x° = 89°$
$\therefore x = 89$

56. **(i)** $\angle PRS = \angle PQS$ ($\angle$s in same segment)
$= 40°$
(ii) $\angle PQR = 90°$ (rt. $\angle$ in a semicircle)
$\angle QPR = 180° - 38° - 90°$ ($\angle$ sum of a △)
$= 52°$
(iii) $\angle PXQ = 180° - 40° - 52°$ ($\angle$ sum of a △)
$= 88°$
$\angle SXR = \angle PXQ$ (vert. opp. $\angle$s)
$= 88°$

57. **(i)** $\angle OCA = \angle OAC$ (base $\angle$s of isos. △)
$= 32°$
$\angle BAC = 90°$ (rt. $\angle$ in a semicircle)
$\angle OBA = 180° - 90° - 32°$ ($\angle$ sum of a △)
$= 58°$

(ii) $\angle OAB = \angle OBA$ (base $\angle$s of isos. $\triangle$)

$= 58°$

$\angle OAT = 90°$ (tangent $\perp$ radius)

$\angle BAT = 90° - 58°$

$= 32°$

(iii) $\angle ATC = 180° - 90° - 32° - 32°$ ($\angle$ sum of a $\triangle$)

$= 26°$

58. (i) $\angle ACB = 90°$ (rt. $\angle$ in a semicirle)

$\angle OAC = \angle OCA$ (base $\angle$s of isos. $\triangle$)

$= 90° - 58°$

$= 32°$

(ii) $\angle OBC = \angle OCB$ (base $\angle$s of isos. $\triangle$)

$= 58°$

$\angle ADC + \angle ABC = 180°$ ($\angle$s in opp. segments)

$\angle ADC + 58° = 180°$

$\angle ADC = 122°$

$\angle CAD = \dfrac{180° - 122°}{2}$ (base $\angle$s of isos. $\triangle$)

$= 29°$

59. (i) $\angle ADB = \angle TAB$

$= 60°$ ($\angle$s in alt. segment)

(ii) $\angle OAT = 90°$ (tangent $\perp$ radius)

$\angle OAB = 90° - 60°$

$= 30°$

$\angle BCD = 180° - 45° - 30°$

$= 105°$ ($\angle$s in opp. segments are supp.)

60. (i) $\angle TAB = \dfrac{180° - 48°}{2}$

$= 66°$ (base $\angle$ of isos. $\triangle$)

(ii) $\angle ACB = \angle TAB$

$= 66°$ ($\angle$s in alt. segment)

(iii) $\angle CAP = 180° - 66° - 66°$

$= 48°$ (adj. $\angle$s on a str. line)

Advanced

61. $\angle QPR = \dfrac{1}{2}\angle QOR$ ($\angle$ at centre = 2 $\angle$ $\bigcirc^{ce}$)

$= \dfrac{1}{2}(54°)$

$= 27°$

$\angle PRS = \dfrac{1}{2}\angle POS$ ($\angle$ at centre = 2 $\angle$ at $\bigcirc^{ce}$)

$= \dfrac{1}{2}(116°)$

$= 58°$

$\angle PRT = 180° - 58°$ ($\angle$s on a str. line)

$= 122°$

$\angle PTS = 180° - 122° - 27°$ ($\angle$ sum of a $\triangle$)

$= 31°$

62. (i) $\angle BAC = \angle BCA$ (base $\angle$s of isos. $\triangle$)

$= 33°$

$\angle BDC = \angle BAC$ ($\angle$s in same segment)

$= 33°$

(ii) $\angle BDA = \angle BCA$ ($\angle$s in same segment)

$= 33°$

$\angle BEA = \angle BCA$ ($\angle$s in same segment)

$= 33°$

$\angle ABE = \angle AEB$ (base $\angle$s of isos. $\triangle$)

$= 33°$

$\angle ADE = \angle ABE$ ($\angle$s in same segment)

$= 33°$

$\angle CDE = 33° + 33° + 33°$

$= 99°$

(iii) $\angle AOE = 2\angle ADE$ ($\angle$ at centre = 2 $\angle$ at $\bigcirc^{ce}$)

$= 2(33°)$

$= 66°$

$\angle OEA = \dfrac{180° - 66°}{2}$ (base $\angle$s of isos. $\triangle$)

$= 57°$

63. $\angle DBC = 90°$ (rt. $\angle$ in a semicircle)

$\angle YBC = \angle YCB$ (base $\angle$ of isos. $\triangle$)

$= 48°$

$\angle DBY = 90° - 48°$

$= 42°$

$\angle XBD = \dfrac{180° - 80°}{2}$ (base $\angle$s of isos. $\triangle$)

$= 50°$

$\angle XBY = 50° + 42°$

$= 92°$

64. Let $\angle DFE = x°$.

$\angle AFB = \angle DFE$ (vert. opp. $\angle$s)

$= x°$

$\angle FDE = x° + 68°$ (ext. $\angle$ = sum of int. opp. $\angle$s)

$\angle FBC = x° + 26°$ (ext. $\angle$ = sum of int. opp. $\angle$s)

$\angle FDE + \angle FBC = 180°$ ($\angle$s in opp. segments)

$x° + 68° + x° + 26° = 180°$

$2x° = 86°$

$x° = 43°$

$\therefore \angle DFE = 43°$

65. (i) $\angle ACB = 90°$ (rt. $\angle$ in a semicircle)

$\angle CAB = 180° - \angle ABC - 90°$ ($\angle$ sum of a $\triangle$)

$= 90° - \angle ABC$

$\angle CDA = 180° - \angle ABC$ ($\angle$s in opp. segments)

$\angle CDA + \angle DAT + \angle ATD = 180°$

($\angle$ sum of a $\triangle$)

$180° - \angle ABC + 46° + 90° - \angle ABC + 22° = 180°$

$2\angle ABC = 158°$

$\angle ABC = 79°$

(ii) $\angle CDA = 180° - 79°$
$= 101°$
$\angle ACD = 180° - 101° - 46°$ ($\angle$ sum of a $\triangle$)
$= 33°$

66. (i) $\angle CAT = \angle ABC$ ($\angle$s in alt. segment)
$= 40°$

(ii) Let O be the centre of the circle.
$\angle OAB = \angle OBA$ (base $\angle$s of isos. $\triangle$)
$= 40°$
$\angle OAT = 90°$ (tangent $\perp$ radius)
$\angle ATC = 180° - 90° - 40° - 40°$
$= 10°$

67. $\angle AOC = 180° - 18° - 18°$ ($\angle$ sum of a $\triangle$)
$= 144°$
$\angle OAQ = 90°$ (tangent $\perp$ radius)
$\angle AQB = 360° - 90° - 144° - 18° - 41°$ ($\angle$ sum of a quadrilateral)
$= 67°$

68. (i) $\angle CPT = \angle PTQ$ (alt. $\angle$s, CP // AQ)
$= 58°$
$PB = PC$ (symmetric properties of tangents to circle)
$\angle PCB = \dfrac{180° - 58°}{2}$ (base $\angle$s of isos. $\triangle$)
$= 61°$

(ii) $\angle TAB = \dfrac{58°}{2}$ (ext. $\angle$ = sum of int. opp. $\angle$s)
$= 29°$

(iii) $\angle PBC = \angle PCB$ (base $\angle$s of isos. $\triangle$)
$= 61°$
$\angle TBA = \angle TAB$ (base $\angle$s of isos. $\triangle$)
$= 29°$
$\angle ABC = 180° - 61° - 29°$ ($\angle$s on a str. line)
$= 90°$
i.e. AC is a diameter of the circle.
$\angle ACP = 90°$ (tangent $\perp$ radius)
$\angle ACB = 90° - 61°$
$= 29°$

New Trend

69. (a) (i) $\angle BDC = 34°$ ($\angle$s in same segment)

(ii) $\angle ACD = \dfrac{1}{2}\angle AOD$ ($\angle$ at centre = 2 $\angle$ at $\bigcirc^{ce}$)
$= \dfrac{1}{2}(118°)$
$= 59°$

(iii) $\angle BOA = 180° - 118°$ (adj. $\angle$s on a str. line)
$\angle OBA = \dfrac{180° - 62°}{2}$ (base $\angle$s of isos. $\triangle$)
$= 59°$
$\angle BXA = 180° - 59° - 34°$ ($\angle$ sum of a $\triangle$)
$= 87°$
$\angle CXD = 87°$ (vert. opp. $\angle$s)

(iv) $\angle BAT = 90° - 59°$ (tangent $\perp$ radius)
$= 31°$

(b) Let the radius of the other circle be r cm.
$r + r + r(1.8) = 22.8$
$3.8r = 22.8$
$r = 6$
Area of sector $= \dfrac{1}{2}(6)^2(1.8)$
$= 32.4\text{ cm}^2$

70. (i) $\angle AOB = 180° - 36° - 36°$ ($\angle$ sum of a $\triangle$)
$= 108°$
$\angle ADB = \dfrac{1}{2}\angle AOB$ ($\angle$ at centre = 2 $\angle$ at $\bigcirc^{ce}$)
$= \dfrac{1}{2}(108°)$
$= 54°$

(ii) $\angle OBD = 180° - 54° - 28° - 36° - 36°$ ($\angle$ sum of a $\triangle$)
$= 26°$

(iii) $\angle ADC = 90°$ (rt. $\angle$ in a semicircle)
$\angle ACD = 180° - 90° - 28°$ ($\angle$ sum of a $\triangle$)
$= 62°$
$\angle AED + \angle ACD = 180°$ ($\angle$s in opp. segments)
$\angle AED + 62° = 180°$
$\angle AED = 118°$

71. (a) $\angle ABC = 180° - 43° - 44°$ ($\angle$ sum of a $\triangle$)
$= 93°$
Since $\angle ABC \neq 90°$, AB is not perpendicular to BC.
$\therefore$ BC is not a tangent to the circle.

(b) (i) $\angle ACT = 48° - 28°$ (ext. $\angle$ = sum of int. opp. $\angle$s)
$= 20°$
$\angle ABD = \angle ACT$ ($\angle$s in same segment)
$= 20°$

(ii) $\angle ADC = 28° + 75°$ (ext. $\angle$ = sum of int. opp. $\angle$s)
$= 103°$
$\angle ABD + \angle DBC + \angle ADC = 180°$ ($\angle$s in opp. segments)
$20° + \angle DBC + 103° = 180°$
$\angle DBC = 57°$

72. (a) $\triangle OAT \cong \triangle OBT$ (RHS)

(b) (i) $TA = TB$ (symmetric properties of tangents to circle)
$\angle TAB = \dfrac{180° - 50°}{2}$ (base $\angle$s of isos. $\triangle$)
$= 65°$

(ii) $\angle OAT = \angle OBT$ (tangent $\perp$ radius)
$= 90°$
$\angle AOB = 360° - 90° - 50° - 90°$
($\angle$ sum of a quadrilateral)
$= 130°$
$\angle ACB = \dfrac{1}{2}\angle AOB$ ($\angle$ at centre = 2 $\angle$ at $\bigcirc^{ce}$)
$= \dfrac{1}{2}(130°)$
$= 65°$

(iii) $\angle OAB + \angle OBA = 180° - 130°$ ($\angle$ sum of a $\triangle$)
$= 50°$
$\angle OBC = 180° - 50° - 20° - 65°$ ($\angle$ sum of a $\triangle$)
$= 45°$

73. (i) $TA = TB$ (symmetric properties of tangents to circle)
$\angle BAT = \dfrac{180° - 46°}{2}$ (base $\angle$s of isos. $\triangle$)
$= 67°$
$\angle OAT = 90°$ (tangent $\perp$ radius)
$\angle OAB = 90° - 67°$
$= 23°$

(ii) $\angle AOB = 180° - 2(23°)$ (base $\angle$s of isos. $\triangle$)
$= 134°$
$\angle ACB = \dfrac{1}{2}\angle AOB$ ($\angle$ at centre = 2 $\angle$ at $\bigcirc^{ce}$)
$= \dfrac{1}{2}(134°)$
$= 67°$

(iii) $\angle OBA = \angle OAB$ (base $\angle$s of isos. $\triangle$)
$= 23°$
$\angle OBK = 90°$ (tangent $\perp$ radius)
$\angle OCB = \angle OBC$ (base $\angle$s of isos. $\triangle$)
$= 90° - 58°$
$= 32°$
$\angle CAO + \angle OCA$
$= 180° - 23° - 23° - 32° - 32°$ ($\angle$ sum of a $\triangle$)
$= 70°$
$\angle OCA = \angle CAO$ (base $\angle$s of isos. $\triangle$)
$2\angle CAO = 70°$
$\angle CAO = 35°$

(iv) In $\triangle OAT$,
$\tan 23° = \dfrac{OA}{TA}$
$= \dfrac{8}{TA}$
$TA = \dfrac{8}{\tan 23°}$
$= 18.8$ cm (to 3 s.f.)

74. (a) $\angle CBD = 180° - 90° - 42°$
(rt. $\angle$ in semicircle, $\angle$ sum of a $\triangle$)
$= 48°$
$\angle BAC = 42°$ ($\angle$s in same segment)
$\angle ACD = 180° - 72° - 42°$ (corr. $\angle$s, BD // AE,
$= 66°$ $\angle$ sum of a $\triangle$)

(b) (i) $\angle DBA = \angle ACD$ ($\angle$s in same segment)
$= 66°$

(ii) $\angle CDE + \angle CAE = 180°$ ($\angle$s in opp. segments)
$42° + \angle BDE + 72° = 180°$
$\angle BDE = 66°$

75. $\angle ABP = 180° - 54° - 42°$ ($\angle$ sum of a $\triangle$)
$= 84°$
$\angle ADC + \angle ABC = 180°$ ($\angle$s in opp. segments)
$\angle ADC + 84° = 180°$
$\angle ADC = 96°$
$\angle AQD = 180° - 96° - 54°$ ($\angle$ sum of a $\triangle$)
$= 30°$

76. (a) (i) $\angle AOD = 2\angle ABO$ ($\angle$ at centre = 2 $\angle$ at $\bigcirc^{ce}$)
$= 2(36°)$
$= 72°$
$\angle OAD = \dfrac{180° - 72°}{2}$ (base $\angle$s of isos. $\triangle$)
$= 54°$

(ii) $\angle OCT = \angle OAT$ (tangent $\perp$ radius)
$= 90°$
$\angle COT = \angle AOT$
$= 72°$
$\angle ATC = 360° - 90° - 72 - 72° - 90°$ ($\angle$ sum of a quadrilateral)
$= 36°$

(b) $\triangle OCT \equiv \triangle OAT$ (RHS)
$\tan 72° = \dfrac{CT}{8}$
$CT = 8 \tan 72°$
$= 24.621$ cm (to 5 s.f.)
Area of $OATC = 2 \times \dfrac{1}{2}(24.621)(8)$
$= 197\text{ cm}^2$ (to 3 s.f.)

Revision Test D1

1. $\triangle ABC$ and $\triangle APQ$ are similar.

$$\frac{AB}{AP} = \frac{AC}{AQ} = \frac{BC}{PQ}$$

$$\therefore \frac{x}{x+5} = \frac{8}{14} = \frac{4}{y}$$

$8x + 40 = 14x$ and $8y = 4 \times 14$

$40 = 6x$ $\quad y = 7$

$$x = \frac{40}{6}$$

$$= 6\frac{2}{3}$$

2. **(a)** Scale 45 : 3000

i.e. 3 : 200

(b) Area scale $3^2 : 200^2$

i.e. 9 : 40 000

$$\text{Actual floor area} = \frac{40\,000}{9} \times 810 \text{ cm}^2$$

$$= \frac{3\,600\,000}{10\,000} \text{ m}^2$$

$$= 360 \text{ m}^2$$

(c) 3 cm represent 2 m

$(3 \text{ cm})^3$ represent $(2 \text{ m})^3$

$\therefore$ 162 cm^3 represent $\frac{8 \text{ m}^3}{27} \times 162 = 48 \text{ m}^3$

3. Let $AQ = x$ cm and $AP = y$ cm.

(a)
$$\frac{\text{Area of } \triangle APQ}{\text{Area of } \triangle ABC} = \frac{\frac{1}{2}(x)(y) \sin A}{\frac{1}{2}(4x)(2y) \sin A}$$

$$= \frac{1}{8}$$

(b)
$$\frac{\text{Area of } \triangle ABR}{\text{Area of } \triangle ABS} = \frac{\frac{1}{2}(2x)(2y) \sin A}{\frac{1}{2}(3x)(2y) \sin A}$$

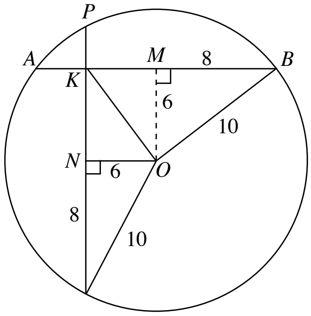
$$= \frac{2}{3}$$

(c) Draw a line parallel to AC passing through B.

$$\frac{\text{Area of } \triangle BRS}{\text{Area of } \triangle ABC} = \frac{\frac{1}{2}(x)(\text{height})}{\frac{1}{2}(4x)(\text{height})}$$

$$= \frac{1}{4}$$

4.



$O\hat{M}B = O\hat{N}Q = 90°$

Using Pythagoras' Theorem,

$$OM = \sqrt{10^2 - 8^2}$$

$$= 6 \text{ cm}$$

$$ON = \sqrt{10^2 - 8^2}$$

$$= 6 \text{ cm}$$

$\therefore OK = \sqrt{6^2 + 6^2} = 8.49$ cm (to 2 d.p.)

5. $B\hat{D}C = 180° - 141° - 25°$ ($\angle$ sum of a $\triangle$)

$= 14°$

$\therefore B\hat{A}C = 14°$ ($\angle$s in same segment)

$A\hat{B}Q = 180° - 141°$

$= 39°$ (adj. $\angle$s on a str. line)

$\therefore A\hat{Q}D = 39° + 14°$ (ext. $\angle$ = sum of int. opp. $\angle$s)

$= 53°$

6. **(a)**
$$\frac{\text{Volume of larger cup}}{\text{Volume of smaller cup}} = \left(\frac{5}{3\frac{1}{3}}\right)^3$$

$$= \frac{\text{Volume of larger cup}}{48 \text{ cm}^3}$$

$$\therefore \text{Volume of larger cup} = 48 \times \left(\frac{5}{3\frac{1}{3}}\right)^3$$

$$= 162 \text{ cm}^3$$

(b)
$$\left(\frac{\text{Height of larger container}}{\text{Height of smaller container}}\right)^3 = \frac{76.8}{32.4}$$

$$\therefore \frac{\text{Height of larger container}}{\text{Height of smaller container}} = \sqrt[3]{\frac{76.8}{32.4}}$$

$$= \frac{4}{3}$$

$$\frac{\text{Surface area of larger container}}{\text{Surface area of smaller container}} = \left(\frac{4}{3}\right)^2$$

$$= \frac{168}{\text{Surface area of smaller container}} = \frac{16}{9}$$

$$\therefore \text{Area of smaller container} = 168 \div \frac{16}{9}$$

$$= 94\frac{1}{2} \text{ cm}^2$$

7. (i) $\dfrac{\text{Surface area of smaller statue}}{\text{Surface area of larger statue}} = \left(\dfrac{\text{Height of smaller statue}}{\text{Height of larger statue}}\right)^2$

$= \left(\dfrac{20}{20 \times 100}\right)^2$

$= \dfrac{1}{100}$

(ii) $\dfrac{\text{Weight of larger statue}}{\text{Weight of smaller statue}}$

$= \left(\dfrac{\text{Height of larger statue}}{\text{Height of smaller statue}}\right)^3$

$\dfrac{\text{Weight of larger statue}}{355 \text{ g}} = \left(\dfrac{10}{1}\right)^3$

$\therefore$ Weight of larger statue $= \dfrac{355 \times 10^3}{1000}$

$= 355$ kg

8. (i) $Q\hat{S}R = R\hat{P}Q = 26°$ ($\angle$s in the same segment)

(ii) $T\hat{Q}P = T\hat{P}Q$

$= \dfrac{180° - 48°}{2}$ (base $\angle$s of isos. $\triangle$)

$= 66°$

$\therefore X\hat{Q}S = 180° - 66° - 42°$ (adj. $\angle$s on a str. line)

$= 72°$

(iii) $T\hat{P}R = 66° - 26°$

$= 40°$

(iv) $P\hat{X}S = 72° + 26°$ (ext. $\angle$ = sum of int. opp. $\angle$s)

$= 98°$

9. Let $P\hat{T}Q = \theta°$, $TP = \sqrt{3}x$ and $OQ = x$.

$\dfrac{\text{Area of } \triangle PTQ}{\text{Area of } \triangle POQ} = \dfrac{\frac{1}{2}(\sqrt{3}x)(\sqrt{3}x)\sin\theta°}{\frac{1}{2}(x)(x)\sin(180 - \theta°)}$

$= 3$

Revision Test D2

1. **(a)** Given $\frac{1}{3}x^2h = 120 \text{ cm}^3$,

$$\frac{\text{Volume of larger cone}}{120} = \left(\frac{3x}{x}\right)^3$$

$\therefore$ Volume of larger cone $= 120 \times 27$

$= 3240 \text{ cm}^3$

(b) Volume of new cone $= \frac{1}{3}(2x)^2(3h)$

$= 12 \times \left(\frac{1}{3}x^2h\right)$

$= 12 \times 120$

$= 1440 \text{ cm}^3$

2. $AP = 6 \text{ cm} = BP$

Using Pythagoras' Theorem,

$OP = \sqrt{25^2 - 18^2}$

$= \sqrt{301}$

$= 17.35$ cm (to 4 s.f.)

$OA = \sqrt{6^2 + 301}$

$= 18.38$ cm (to 2 d.p.)

3. $T\hat{O}Q = 2(40°) = 80°$ (∠ at centre = 2 ∠ at ○^{ce})

$\therefore O\hat{T}Q = \frac{180° - 80°}{2} = 50°$ (base ∠s of isos. △)

$\therefore P\hat{T}Q = 90° - 50°$ (tangent ⊥ radius)

$= 40°$

$\therefore T\hat{P}Q = 75° - 40°$

$= 35°$ (ext. ∠ = sum of int. opp. ∠s)

4. $A\hat{B}D = 29° + 38°$

$= 67°$ (ext. ∠ = sum of int. opp. ∠s)

$B\hat{A}C = 29°$ (∠s in same segment)

$\therefore C\hat{Q}D = A\hat{Q}B$

$= 180° - 29° - 67°$ (∠ sum of a △)

$= 84°$

5. $A\hat{C}D = 90°$ (rt. ∠ in semicircle)

$\therefore A\hat{D}C = 180° - 90° - 14°$ (∠ sum of a △)

$= 76°$

$A\hat{B}C = 180° - 76°$(∠s in opp. segments.)

$= 104°$

$\therefore B\hat{C}A = \frac{180° - 104°}{2}$ (base ∠s of isos. △)

$= 38°$

$38° = A\hat{P}B + 14°$ (ext. ∠ = sum of int. opp. ∠s)

$\therefore A\hat{P}B = 38° - 14°$

$= 24°$

6. **(a)** $PA = QB = 4$ cm (Given)

$\angle APC = \angle BQA = 60°$

$PC = QA$

$= 16 - 4$

$= 12$ cm

$\therefore \triangle APC$ is congruent to $\triangle BQA$ (SAS).

(b) $\triangle CRB$

From part **(a)**, $AC = BA$ and from part **(b)**, $AC = CB$.

$\therefore \triangle ABC$ is an equilateral triangle.

7. Reflex $\angle AOB = 360° - 128°$

$= 232°$

$\therefore A\hat{Q}B = \frac{1}{2}(232°)$ (∠ at centre = 2 ∠ at ○^{ce})

$= 116°$

$116° + 3a° + 5a° = 180°$ (∠ sum of a △)

$a = \frac{180 - 116}{8}$

$= 8$

$B\hat{O}Q = 2(3a°)$

$= 48°$ (∠ at centre = 2 ∠ at ○^{ce})

$B\hat{Q}O = \frac{180° - 48°}{2}$

$= 66°$ (base ∠s of isos. △)

$B\hat{Q}P = 90° - 66°$

$= 34°$ (tangent ⊥ radius)

$\therefore 5(8°) = 34° + b° \Rightarrow b = 16$ (ext. ∠ = sum of int. opp. ∠s)

8. **(a)** $S\hat{Q}A = R\hat{A}Q$ (alt. ∠s, AR // SQ)

$S\hat{A}Q = R\hat{Q}A$ (alt. ∠s, AR // SQ)

AQ is a common side.

$\therefore \triangle ASQ \equiv \triangle QRA$ (ASA) (shown)

(b) $\triangle ASQ$ is similar to $\triangle ABC$.

$\therefore \frac{AS}{AB} = \frac{SQ}{BC}$

$\frac{2}{6} = \frac{SQ}{15}$

$\therefore SQ = \frac{2 \times 15}{6}$

$= 5$ cm

(c) $\triangle BCA$ and $\triangle RAQ$

(i) $\frac{\text{Area of } \triangle PCQ}{\text{Area of } \triangle BCA} = \left(\frac{4}{6}\right)^2$

$= \frac{4}{9}$

$\frac{\text{Area of } \triangle PCQ}{36} = \frac{4}{9}$

$\therefore$ Area of $\triangle PCQ = \frac{36 \times 4}{9}$

$= 16 \text{ cm}^2$

(ii) $\triangle BPQ = \frac{1}{2} \times BP \times \text{height}$

$= \frac{1}{2} \times \left(\frac{1}{2} \times PC \times \text{height}\right)$

$= \frac{1}{2}(16)$

$= 8 \text{ cm}^2$

(iii) $\frac{\text{Area of } \triangle ASQ}{\text{Area of } \triangle ABC} = \frac{\text{Area of } \triangle ASQ}{36}$

$= \left(\frac{2}{6}\right)^2$

$= \frac{1}{9}$

$\therefore$ Area of $\triangle ASQ = 4 \text{ cm}^2$

$\therefore$ Area of quadrilateral $ASQR = 2 \times 4$

$= 8 \text{ cm}^2$

End-of-Year Examination Specimen Paper A

Part I

1. Since $\cos A < 0$ and $0° < A < 180°$, A is an obtuse angle.

(i) $\sin A = -\frac{\sqrt{13^2 - 5^2}}{13}$

$= -\frac{12}{13}$

(ii) $\tan A = -\frac{12}{5}$

2. $x + 4 < 10 < 5x - 1$

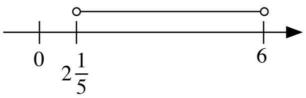
$x + 4 < 10$ and $10 < 5x - 1$

$x < 6$ and $11 < 5x$

$x > \frac{11}{5}$

$x > 2\frac{1}{5}$

$\therefore 2\frac{1}{5} < x < 6$



3. **(a)** $5x^2 - 7x = 1$

$5x^2 - 7x - 1 = 0$

$a = 5, b = -7, c = -1$

$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(5)(-1)}}{2(5)}$

$x = \frac{7 - \sqrt{69}}{10}$ or $x = \frac{7 + \sqrt{69}}{10}$

$= -0.1306$ or $= 1.530$

$x = -0.131$ or $x = 1.53$ (to 3 s.f.)

(b) $2x^{\frac{3}{4}} = 54$

$x^{\frac{3}{4}} = 27$

$x = 27^{\frac{4}{3}}$

$= (\sqrt[3]{27})^4$

$\therefore x = 81$

(c) $\frac{x-3}{2} - \frac{x+5}{3} = 1$

$3(x - 3) - 2(x + 5) = (2)(3)$

$3x - 9 - 2x - 10 = 6$

$\therefore x = 25$

4. 3 years 9 months = 3.75 years

Let the interest rate be r.

$6000 + [3.75 \times (6000 \times r)] = 7237.50$

$22\ 500r = 1237.50$

$r = \frac{1237.50}{22\ 500}$

$= 0.055$

$= 5.5\%$

5. Let s and b denote the small and big cups respectively.

(i) $\frac{A_s}{A_b} = \left(\frac{c_s}{c_b}\right)^2$

$\left(\frac{c_s}{c_b}\right)^2 = \frac{16}{25}$

$\frac{c_s}{c_b} = \sqrt{\frac{16}{25}}$

$= \frac{4}{5}$

The ratio is 4 : 5.

(ii) $\frac{V_s}{V_b} = \left(\frac{c_s}{c_b}\right)^3$

$\frac{120.5}{V_b} = \left(\frac{4}{5}\right)^3$

$V_b = \frac{125}{64} \times 120.5$

$= 235\ \text{cm}^3$ (to 3 s.f.)

$\therefore$ The volume of the larger (big) cup is $235\ \text{cm}^3$.

6. **(i)** $\tan \angle PRQ = \frac{70}{90}$

$\angle PRQ = 37.9°$ (to 1 d.p.)

Bearing of Q from R is 037.9°.

(ii) Using Cosine Rule,

$PS^2 = 90^2 + 25^2 - 2(90)(25) \cos 37.87°$

$PS = 71.9$ m (to 3 s.f.)

(iii) Using Sine Rule,

$\frac{\sin \angle RPS}{25} = \frac{\sin 37.87°}{71.92}$

$\sin \angle RPS = \frac{25 \sin 37.87°}{71.92}$

$\angle RPS = 12.32°$ (to 2 d.p.)

Area of $\triangle PQS = \frac{1}{2}(70)(90)$

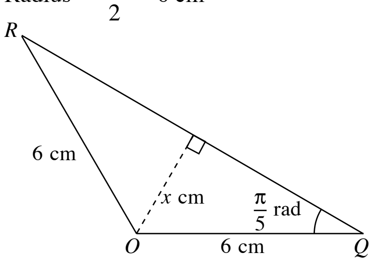
$- \frac{1}{2}(71.92)(90) \sin 12.32°$

$= 2460\ \text{m}^2$ (to 3 s.f.)

(iv) $\tan \angle TSP = \frac{15}{71.92}$

$\angle TSP = 11.8°$ (to 1 d.p.)

7. (i) Radius $= \frac{12}{2} = 6$ cm



$\cos\left(\frac{\pi}{5}\right) = \frac{x}{6}$

$x = 6 \times \cos\left(\frac{\pi}{5}\right)$

$= 4.854$

$\therefore QR = 2 \times 4.854$

$= 9.708$

$= 9.71$ cm (to 3 s.f.)

(ii) $Q\hat{O}R = \pi - 2\left(\frac{\pi}{5}\right)$ ($\angle$ sum of a $\triangle$)

$= \frac{3\pi}{5}$ radians

(iii) Length of arc $RQ = 6 \times \frac{3\pi}{5}$

$= \frac{18\pi}{5}$ cm

Length of arc $RTQ = \dfrac{2\pi \times \left(\frac{9.708}{2}\right)}{2}$

$= 4.854\pi$ cm

$\therefore$ Perimeter of shaded region $= \left(\frac{18\pi}{5} + 4.854\pi\right)$ cm

$= 26.55$ cm

$= 26.6$ cm (to 3 s.f.)

(iv) Area of sector $ORQ = \frac{1}{2}(6)^2\left(\frac{3\pi}{5}\right)$

$= \left(\frac{54\pi}{5}\right)$ cm^2

Area of $\triangle ORQ = \frac{1}{2}(6)(6)\sin\left(\frac{3\pi}{5}\right)$

$= 17.119$ cm^2

Area of semicircle $(RTQ) = \dfrac{\pi\left(\frac{9.708}{2}\right)^2}{2}$

$= 11.780\pi$ cm^2

$\therefore$ Area of shaded region

$= 11.780\pi - \left(\frac{54\pi}{5} - 17.119\right)$

$= 20.2$ cm^2 (to 3 s.f.)

8. (a) $4x^2 + x = 4(3.6 \times 10^{-2})^2 + (3.6 \times 10^{-2})$

$= 0.041\ 184$ (to 3 s.f.)

$= 4.1 \times 10^{-2}$

(b) $\sqrt{x+1} = \sqrt{(3.6 \times 10^{-2}) + 1}$

$= 1.02 \times 10^0$ (to 3 s.f.)

9. (a) (i) $A\hat{C}B = 180° - 85° - 40°$

$= 55°$

Using Sine Rule,

$\frac{AB}{\sin 55°} = \frac{CB}{\sin 40°}$

$AB = \frac{8}{\sin 40°} \times \sin 55°$

$= 10.2$ m (to 3 s.f.)

(ii) $C\hat{B}D = 180° - 85°$ (adj. $\angle$s on a str. line)

$= 95°$

$C\hat{D}B = 180° - 95° - 30°$ ($\angle$ sum of a $\triangle$)

$= 55°$

Using Sine Rule,

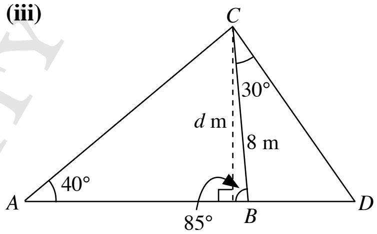
$\frac{BD}{\sin 30°} = \frac{CB}{\sin 55°}$

$BD = \frac{8}{\sin 55°} \times \sin 30°$

$= 4.883$ (to 4 s.f.)

$= 4.88$ m (to 3 s.f.)

(iii)



Using Sine Rule,

$\frac{AC}{\sin 85°} = \frac{CB}{\sin 40°}$

$\frac{AC}{\sin 85°} = \frac{8}{\sin 40°}$

$AC = \frac{8}{\sin 40°} \times \sin 85°$

$= 12.39$ m (to 4 s.f.)

Let the shortest distance be d.

$\sin 40° = \frac{d}{AC} = \frac{d}{12.39}$

$d = \sin 40° \times 12.39$

$= 7.96$ (to 3 s.f.)

$\therefore$ The shortest distance is 7.96 m.

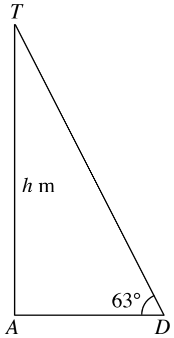
(b) Let the height of the pole be h m.

$AB + BD = 10.19 + 4.883$
$= 15.07$ m (to 4 s.f.)

$\tan 63° = \dfrac{h}{15.07}$

$h = \tan 63° \times 15.07$
$= 29.6$ (to 3 s.f.)

$\therefore$ The height of the pole is 29.6 m.



10. Let $y = \dfrac{x}{7 - x}$.

$y(7 - x) = x$
$7y - xy = x$
$x + xy = 7y$
$x(1 + y) = 7y$

$x = \dfrac{7y}{1 + y}$

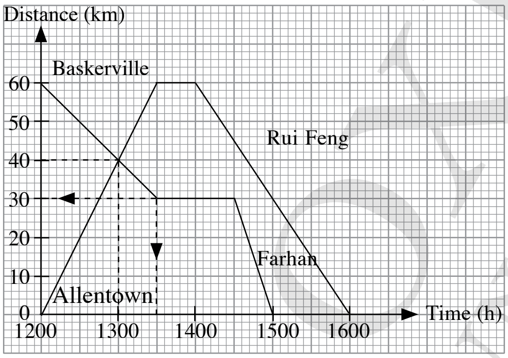
$f^{-1}(x) = \dfrac{7x}{1 + x}, x \neq -1$

$f^{-1}\left(-\dfrac{1}{2}\right) = \dfrac{7\left(-\dfrac{1}{2}\right)}{1 + \left(-\dfrac{1}{2}\right)}$
$= -7$

Part II

Section A

1.



From the graph,

(i) Farhan stopped for a rest at 13 30 and he was 30 km away from Allentown.

(ii) They passed each other at 13 00 and were 40 km away from Allentown.

(iii) For Rui Feng,

Average Speed $= \dfrac{\text{Total distance}}{\text{Total time}}$
$= \dfrac{60 + 60}{4}$
$= 30$ km/h

For Farhan,

Average Speed $= \dfrac{\text{Total distance}}{\text{Total time}}$
$= \dfrac{60}{3}$
$= 20$ km/h

2. **(i)** Smallest value of $y - 2x = -5 - 2(6)$
$= -17$

(ii) Greatest value of $x^2 - y^2 = (6)^2 - (0)^2$
$= 36$

(iii) Greatest value of $y^2 - x = (-5)^2 - \left(1\dfrac{1}{2}\right)$
$= 25 - 1\dfrac{1}{2}$
$= 23\dfrac{1}{2}$

(iv) Smallest value of $\dfrac{y}{x} = \dfrac{(-5)}{2\left(1\dfrac{1}{2}\right)}$
$= -1\dfrac{2}{3}$

3. **(i)** Volume of original cone $= \dfrac{1}{3}\pi(6)^2(20)$
$= 240\pi$ cm^3

(ii) Curved surface area of small cone : Curved surface area of original cone
$= 1^2 : 4^2$
$= 1 : 16$

(iii) $\dfrac{\text{Volume of small cone}}{\text{Volume of original cone}} = \left(\dfrac{1}{4}\right)^3$
$= \dfrac{1}{64}$

$\therefore$ Volume of remaining solid : Volume of original cone
$= 63 : 64$

(iv) Volume of remaining solid $= \dfrac{63}{64} \times 240\pi$
$= 236.25\pi$ cm^3

4. (i) $\angle XYW = \angle BYA$ (common angle)

$\angle WXY = \angle ABY$ ($AB \perp YX$, $WX \perp YX$)

$\therefore \triangle YAB$ is similar to $\triangle YWX$ (AA).

(ii) Since $\triangle YAB$ is similar to $\triangle YWX$,

$$\frac{YB}{YB + BX} = \frac{AB}{WX}$$

$$\frac{5}{5 + BX} = \frac{5}{12}$$

$$12 = 5 + BX$$

$$\therefore BX = 7 \text{ cm}$$

Similarly, $\triangle YZX$ is similar to $\triangle BAX$.

$$\frac{AB}{ZY} = \frac{BX}{YX}$$

$$\frac{5}{ZY} = \frac{7}{12}$$

$$\therefore ZY = \frac{12 \times 5}{7}$$

$$= 8\frac{4}{7} \text{ cm}$$

(iii) Since $\triangle YZX$ is similar to $\triangle BAX$,

$$\frac{AB}{ZY} = \frac{XA}{XZ} = \frac{XB}{XY} = \frac{7}{12}$$

$\therefore XA : XZ = 7 : 12$

Since $XZ = XA + AZ$,

$XA : AZ = 7 : 5$.

Section B

5. (i) Using Pythagoras' Theorem,

Length of $AB = \sqrt{9^2 - 1^2}$

$= \sqrt{80}$

$= 8.94$ cm (to 3 s.f.)

(ii) $\tan P\hat{Q}B = \dfrac{\sqrt{80}}{1}$

$P\hat{Q}B = 83.6°$ (to 1 d.p.)

6. (i) $a = -\dfrac{7}{2}$ and $b = -2$.

(ii) $y = -\dfrac{7}{2} + (x + 2)^2$

When $y = 0$,

$$-\frac{7}{2} + (x + 2)^2 = 0$$

$$(x + 2)^2 = \frac{7}{2}$$

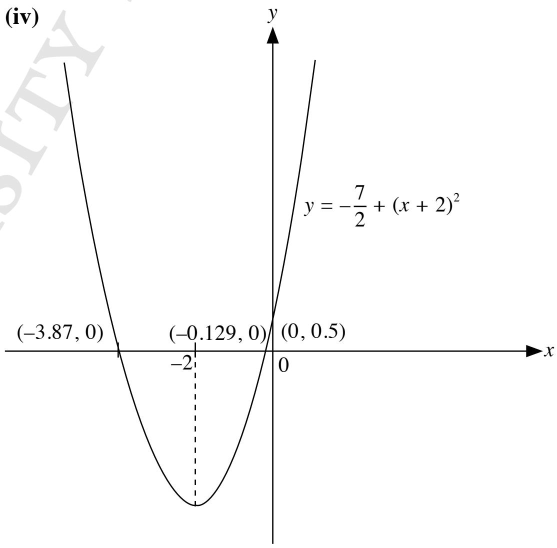
$x + 2 = -\sqrt{\dfrac{7}{2}}$ or $x + 2 = \sqrt{\dfrac{7}{2}}$

$= -3.870$ $\quad$ $= -0.1291$

$\therefore x = -3.87$ or $x = -0.129$ (to 3 s.f.)

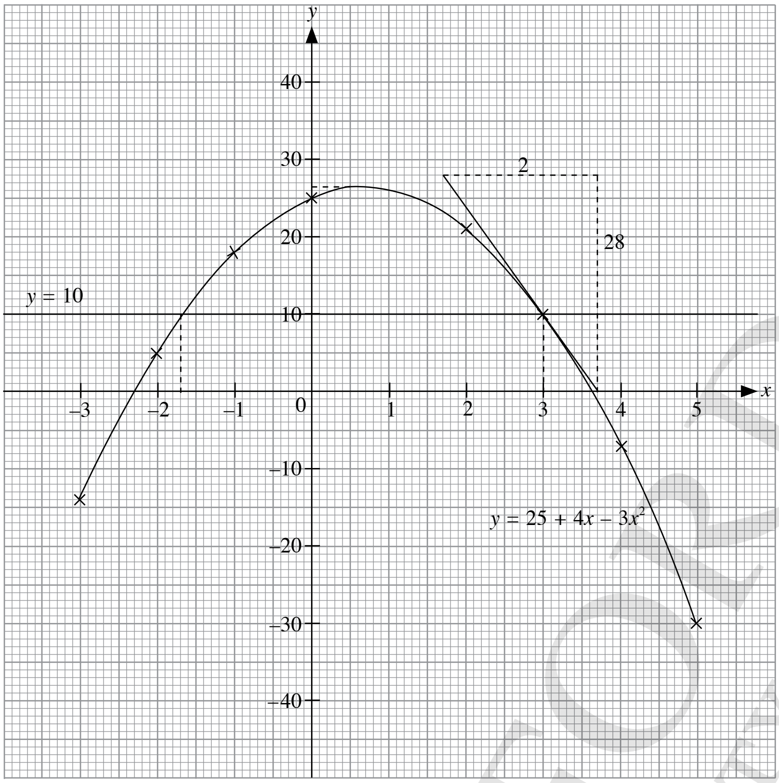
(iii) Equation of line of symmetry: $x = -2$ (x-coordinate of min. pt.)

(iv)



7.

x	–3	–2	–1	0	1	2	3	4	5
y	–14	5	18	25	26	21	10	–7	–30



(i) From the graph, the greatest value of y is 26.5.

(ii) From the graph, the solutions are –2.3 and 3.6.

(iii) From the graph, the range of values of x is $-1.7 \leqslant x \leqslant 3$.

(iv) Gradient (at $x = 3$) $= \dfrac{\text{vertical change}}{\text{horizontal change}}$

$$= -\frac{28}{2}$$

$$= -14$$

End-of-Year Examination Specimen Paper B

Part I

1. **(a)** Gradient $= \frac{7-3}{3-1}$

$= 2$

(b) Equation is $y = 2x + c$.

Since the line AB passes through the point $(1, 3)$,

$3 = 2(1) + c \Rightarrow c = 1$

$\therefore$ Equation is $y = 2x + 1$.

2. $(3k - 2)x + 5y = 17$

$5y = -(3k - 2)x + 17$

$\therefore$ Gradient $= \frac{2-3k}{5}$

$7y + (2k - 7) = 19$

$7y = -(2k - 7) + 19$

$\therefore$ Gradient $= \frac{7-2k}{7}$

$\therefore \frac{2-3k}{5} = \frac{7-2k}{7}$

$14 - 21k = 35 - 10k$

$11k = -21$

$k = -\frac{21}{11}$

$= -1\frac{10}{11}$

3. **(i)** $7(3x + 11) \geqslant 5(2x - 3) + 13(x + 5)$

$21x + 77 \geqslant 10x - 15 + 13x + 65$

$27 \geqslant 2x$

$x \leqslant 13\frac{1}{2}$

(ii) **(a)** $13\frac{1}{2}$

(b) 13

(c) 13

4. **(a)** Let s and l denote the smaller and larger pyramid respectively.

$\frac{\text{Weight}_s}{\text{Weight}_l} = \frac{\text{Volume}_s}{\text{Volume}_l} = \left(\frac{\text{Height}_s}{\text{Height}_l}\right)^3$

$\frac{270}{1250} = \left(\frac{\text{Height}_s}{\text{Height}_l}\right)^3$

$\frac{\text{Height}_s}{\text{Height}_l} = \sqrt[3]{\frac{27}{125}}$

$= \frac{3}{5}$

$\therefore$ The ratio of their heights is 3 : 5.

(b) $\frac{\text{Cost}_s}{\text{Cost}_l} = \frac{\text{Surface area}_s}{\text{Surface area}_l} = \left(\frac{\text{Height}_s}{\text{Height}_l}\right)^2$

$\frac{2.40}{\text{Cost}_l} = \left(\frac{3}{5}\right)^2$

$= \frac{9}{25}$

$\therefore \text{Cost}_l = 2.40 \times \frac{25}{9}$

$= \$6.67$ (to the nearest cent)

5. **(a)** $27^{5-8x} = 81^{2x-4}$

$(3^3)^{5-8x} = (3^4)^{2x-4}$

$\therefore 3(5 - 8x) = 4(2x - 4)$

$15 - 24x = 8x - 16$

$31 = 32x$

$x = \frac{31}{32}$

(b) $\left(\frac{1}{8}\right)^{2-7x} = 32^{x-5}$

$(2^{-3})^{2-7x} = (2^5)^{x-5}$

$\therefore -3(2 - 7x) = 5(x - 5)$

$-6 + 21x = 5x - 25$

$16x = -19$

$x = -\frac{19}{16}$

$= -1\frac{3}{16}$

6. Let $y = \frac{4}{7}x - 9$.

$\frac{4}{7}x = y + 9$

$x = \frac{7}{4}(y + 9)$

$f^{-1}(x) = \frac{7}{4}(x + 9)$

7. **(a)** **(i)** Angle of elevation $= \tan^{-1}\left(\frac{6}{6}\right)$

$= 45°$

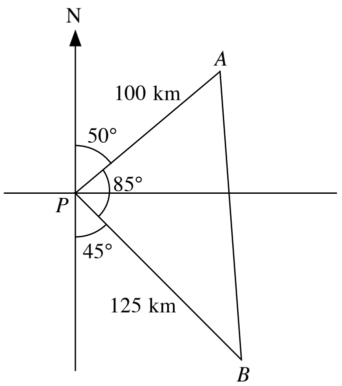
(ii) $SQ = \sqrt{8^2 + 6^2}$

$= 10$ cm

$\therefore$ Angle of elevation $= \tan^{-1}\left(\frac{6}{10}\right)$

$= 31.0°$ (to 1 d.p.)

(b)



$PA = 2.5 \times 40 = 100$ km

$PB = 2.5 \times 50 = 125$ km

$A\hat{P}B = 180° - 50° - 45°$ (adj. ∠s on a str. line)

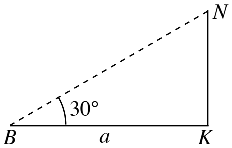
$= 85°$

Using Cosine Rule,

$AB = \sqrt{100^2 + 125^2 - 2(100)(125)\cos 85°}$

$= 153$ km (to 3 s.f.)

8.



$$\cos 30° = \frac{a}{BN}$$

$$\frac{\sqrt{3}}{2} = \frac{a}{BN}$$

$$BN = \frac{2a}{\sqrt{3}}$$

$$VN = \sqrt{(2a)^2 - \left(\frac{2a}{\sqrt{3}}\right)^2}$$

$$= \sqrt{\frac{8a}{3}}$$

$$= \frac{\sqrt{4}\sqrt{2a}}{\sqrt{3}}$$

$$= \frac{2\sqrt{2a}}{\sqrt{3}}$$

9. Distance = $(4.3 \times 365 \times 24 \times 60 \times 60 \times 300\,000)$ m

$= 4.0681 \times 10^{13}$ m (to 5 s.f.)

$= 4.068 \times 10^{10}$ km

10. $B\hat{O}T = 180° - 90° - 15°$ (∠ sum of a △, tangent ⊥ radius)

$= 75°$

$O\hat{B}P = \dfrac{180° - 75°}{2}$ (base ∠s of isos. △)

$= 52.5°$

$\therefore P\hat{B}T = 90° - 52.5°$ (tangent ⊥ radius)

$= 37.5°$

11. (a) (i) Length of arc $PRQ = 12\left(\dfrac{4\pi}{9}\right)$

$= 5\dfrac{1}{3}\pi$ cm

(ii) Area of $POQR = \dfrac{1}{2}(12)^2\left(\dfrac{4\pi}{9}\right)$

$= 32\pi$ cm^2

(b) $PQ = \sqrt{12^2 + 12^2 - 2(12)(12)\cos\left(\dfrac{4\pi}{9}\right)}$

$= 15.43$ cm (to 2 d.p.)

(c) Area of segment $= 32\pi - \dfrac{1}{2}(12)(12)\sin\left(\dfrac{4\pi}{9}\right)$

$= 29.6$ cm^2 (to 3 s.f.)

12. (i) By Sine Rule,

$$\frac{\sin C\hat{D}B}{13} = \frac{\sin 76°}{15}$$

$C\hat{D}B = \sin^{-1}\left(\dfrac{13\sin 76°}{15}\right)$ (∠ sum of a △)

$= 57.2°$ (to 1 d.p.)

(ii) $B\hat{C}D = 180° - 76° - 57.24°$

$= 46.76°$

By Sine Rule,

$$\frac{BD}{\sin 46.76°} = \frac{15}{\sin 76°}$$

$$BD = \frac{15 \times \sin 46.76°}{\sin 76°}$$

$= 11.3$ cm (to 3 s.f.)

(iii) $A\hat{B}C = \cos^{-1}\left(\dfrac{10^2 + 13^2 - 19.5^2}{2(10)(13)}\right)$

$= 115.3°$ (to 1 d.p.)

(iv) Area of $\triangle ABC = \dfrac{1}{2}(10)(3)\sin 115.33°$

$= 58.8$ cm^2 (to 3 s.f.)

Part II

Section A

1. Length of 88 000 oxygen atoms

$= 88\,000 \times 2 \times (48 \times 10^{-12})$ m

$= 8.448 \times 10^{-6}$

$= 8448 \times 10^{-9}$

$= 8450$ nm (to 3 s.f.)

2. **(a)** $\frac{V_A}{V_B} = \left(\frac{h_A}{h_B}\right)^3 = \frac{51\,200}{10\,000}$

$\frac{h_A}{h_B} = \sqrt[3]{\frac{512}{100}} = 1.7235$

$\frac{\text{Surface area}_A}{\text{Surface area}_B} = \left(\frac{h_A}{h_B}\right)^2 = \left(\sqrt[3]{\frac{512}{100}}\right)^2$

$= (1.7235)^2$

$= 2.970$

$\frac{9000}{\text{Surface area}_B} = 2.970$

$\therefore \text{Surface area}_B = \frac{9000}{2.970}$

$= 3030 \text{ cm}^2$ (to 3 s.f.)

(b) $P\hat{Q}R = 180° - 39° - 23°$ ($\angle$ sum of a $\triangle$)

$= 118°$

$\therefore \triangle PQR$ and $\triangle ZYX$ are similar.

$\frac{PQ}{ZY} = \frac{QR}{YX} = \frac{PR}{ZX}$

$\frac{8.4}{4} = \frac{PR}{8.63} = \frac{QR}{6}$

$\therefore QR = \frac{6 \times 8.4}{4}$

$= 12.6$ cm

$\therefore PR = \frac{8.4 \times 8.63}{4}$

$= 18.1$ cm (to 3 s.f.)

3. **(i)** $PT = QU$

$\sin 55° = \frac{QU}{6}$

$QU = 6 \times \sin 55°$

$= 4.91$ cm (to 3 s.f.)

$\therefore PT = 4.91$ cm

(ii) $\cos 55° = \frac{UR}{6}$

$\therefore UR = 6 \times \cos 55°$

$= 3.44$ cm (to 3 s.f.)

(iii) $P\hat{S}T = \tan^{-1}\left(\frac{4.9149}{3}\right)$

$= 58.6°$ (to 1 d.p.)

(iv) $\cos 58.6° = \frac{3}{PS}$

$PS = \frac{3}{\cos 58.6°}$

$= 5.76$ cm (to 3 s.f.)

4. **(a)** $AB = \sqrt{6^2 + 8^2}$

$= 10$ cm

$\therefore$ Radius $= 5$ cm

Area $= \pi(5)^2$

$= 25\pi \text{ cm}^2$

(b) $O\hat{A}C = \frac{1}{2}(70°)$

$= 35°$ ($\angle$ at centre $= 2 \angle$ at $\bigcirc^{ce}$)

$A\hat{C}B = 180° - 90° - 35°$

$= 55°$ ($\angle$ sum of a $\triangle$)

Section B

5. **(i)** Time taken travelling at x km/h $= \frac{280}{x}$ h

Time taken travelling at $(x - 8)$ km/h $= \frac{280}{x - 8}$ h

$\frac{280}{x-8} - \frac{280}{x} = \frac{20}{60}$

$280x - 280(x - 8) = \frac{1}{3}x(x - 8)$

$3(2240) = x^2 - 8x$

$x^2 - 8x - 6720 = 0$

(ii) $a = 1, b = -8, c = -6720$

$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1) - (6720)}}{2(1)}$

$x = \frac{8 - \sqrt{26\,944}}{2}$ or $x = \frac{8 + \sqrt{26\,944}}{2}$

$= -78.07$ (rejected) $= 86.07$ (to 2d.p.)

When $x = 86.07$, time taken $= \frac{280}{86.07}$

$= 3$ h 15 min

6. **(i)** In 1 hour, the big pipe can fill $\frac{1}{x}$ of the pool.

(ii) In 1 hour, the small pipe can fill $\frac{1}{x + 2\frac{1}{2}}$ of the pool.

(iii) In 1 hour, both pipes can fill $\frac{1}{5\frac{3}{4}}$ of the pool.

$\therefore \frac{1}{x} + \frac{1}{x + 2\frac{1}{2}} = \frac{1}{5\frac{3}{4}}$

$\frac{1}{x} + \frac{1}{x + 2\frac{1}{2}} = \frac{4}{23}$

$23\left[\left(x + 2\frac{1}{2}\right) + x\right] = 4x\left(x + 2\frac{1}{2}\right)$

$23\left(2x + 2\frac{1}{2}\right) = 4x^2 + 10x$

$46x + 57.5 = 4x^2 + 10x$

$4x^2 - 36x - 57.5 = 0$

$8x^2 - 72x - 115 = 0$ (shown)

(iv) $8x^2 - 72x - 115 = 0$

$a = 8, b = -72, c = -115$

$$x = \frac{72 \pm \sqrt{(-72) - 4(8)(-115)}}{2(8)}$$

$$x = \frac{72 - \sqrt{8864}}{16} \quad \text{or} \quad x = \frac{72 + \sqrt{8864}}{16}$$

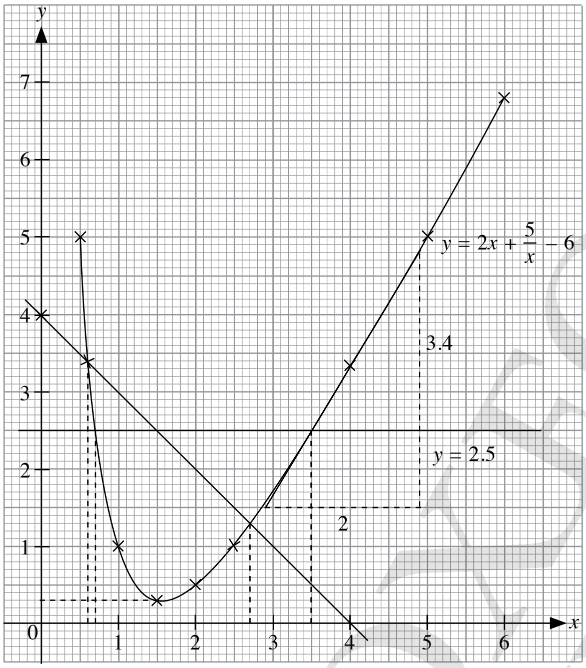
$= -1.38$ (to 2 d.p.) $\qquad = 10.38$ (to 2 d.p.)

(v) $x = -1.38$ is rejected since time > 0. The small pipe takes $(10.38 + 2.5)$ h = 12 h 53 min (to the nearest minute).

7. **(a)** $h = 2(1) + \frac{5}{1} - 6$

$= 1$

(b)



(c) From the graph, min. value of $y = 0.3$

(d) From the graph, gradient $= \frac{\text{vertical change}}{\text{horizontal change}}$

$= \frac{3.4}{2}$

$= 1.7$

(e) **(i)** $2x + \frac{5}{x} = 8.5$

$2x + \frac{5}{x} - 6 = 8.5 - 6$

$2x + \frac{5}{x} - 6 = 2.5$

$\therefore$ Draw $y = 2.5$.

From the graph, $x = 0.7$ or $x = 3.5$.

(ii) $3x + \frac{5}{x} = 10$

$3x - x + \frac{5}{x} - 6 = 10 - x - 6$

$2x + \frac{5}{x} - 6 = 4 - x$

$\therefore$ Draw $y = 4 - x$.

From the graph, $x = 0.6$ or $x = 2.7$.

NOTES

NOTES