Teaching Guide

8

MATHS WISE

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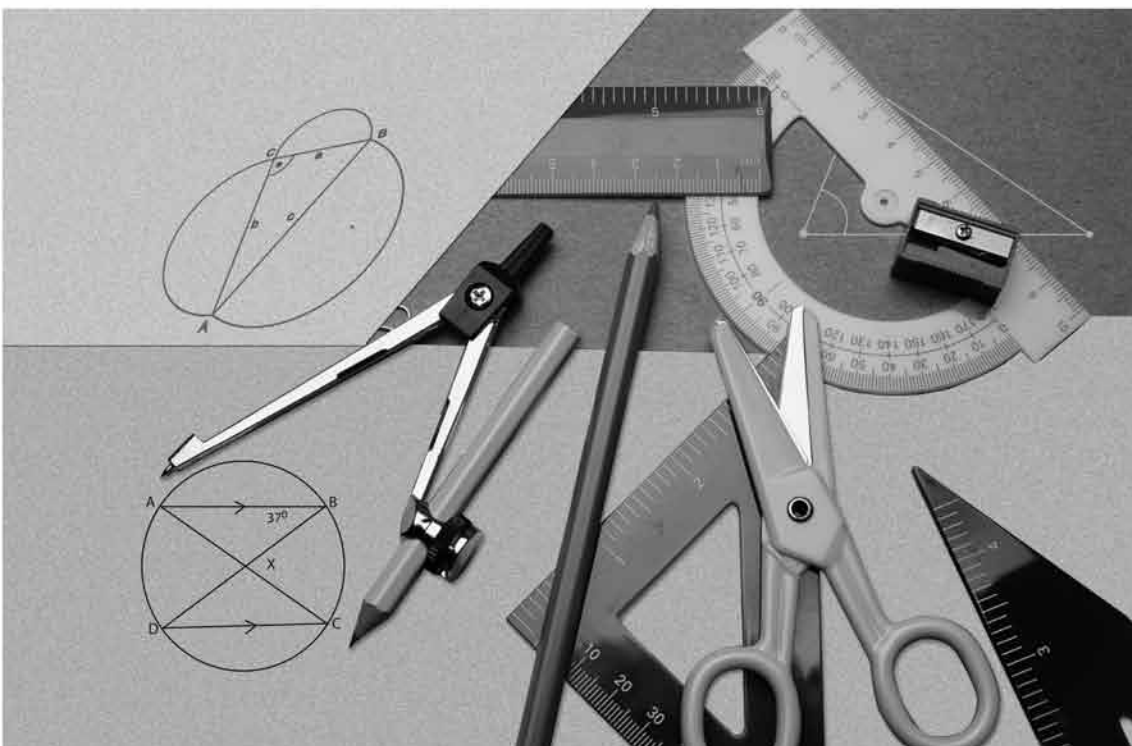

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CONTENTS

USING THIS TEACHING GUIDE

This teaching guide provides lesson plans for each unit. Each lesson starts with activities that can be completed within a specified time before the main lesson is taught. Working on starter activities help prepare the students for the more formal lessons and is an informal introduction to the topic at hand without straight away barraging them with new concepts.

While devising these activities, make sure that they can be done within a reasonable time span and that the recourses that are to be used are easily available.

Time required for completing each lesson is also given but can change depending upon the students' learning capabilities.

The guide refers to the textbook pages where necessary and exercise numbers when referring to individual work or practice session or homework.

This is not a very difficult guide to follow. Simple lesson plans have been devised with ideas for additional exercises and worksheets. Make sure that lessons from the textbook are taught well. Planning how to teach just makes it easier for the teacher to divide the course over the entire year.

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UNIT 1

SETS

lesson 1

Topic: Sets, Subsets and Power Sets, Operation on sets
Time: 3 periods

Objectives

To enable students to:

- recognize sets of
- natural numbers (N). whole numbers (W). integers (Z)
- rational numbers (Q),
- even and odd numbers
- prime and composite numbers
- differentiate between proper and improper subsets; find all possible sub-sets of a set; find and recognize Power set of a set P(S)
- perform operations on sets (two or more) union of sets, intersection of sets, complement of a set, difference of two sets and represent them by Venn diagrams
- Verify the commutative and property of Union and intersection of sets
- Verify Associative and Distribute laws, representation and verification by Venn diagrams. (three or more sets)
- State and verify De Morgan's laws
  i) $(A\cup B)' = A'\cap B'$ ii) $(A\cap B)' = A'\cup B'$

Starter activity (15 minutes)

Ask a few questions to refresh the students' memory. Following questions may be asked.

- What do you mean by the term 'set'?
- How do we define a set in technical terms?
- What does the symbol $\in$ stands for?
- What are the different ways of representing set? (tabular, descriptive, set builder)
- What does the symbol phi ($\varnothing$) means?
- What are finite and infinite sets?

Activity 1

Identify the following in the sets given below.

- pairs of overlapping sets
- pairs of disjoint sets
- pairs of equal sets
- pairs of sub-sets and super sets

When A = $\{a, b, c, \ldots\ldots z\}$, B = {1, 3, 5, 15},

C = {set of divisors of 15} D = $\{\frac{x}{x} \in n\}$

E = {set of positive even integers}

E = $\{\frac{x}{x}$ is a multiple of 5$\}$ W = {0, 1, 2, ……}

Activity 2

Give the cardinal number (number of elements) of each set.

A={1, 0, 3, 4, 6}, B={0, 1, 2,……20}

C=$\{x : x$ is a prime number and $x < 30\}$

Activity 3

Given A={1, 2, 3}, B={1}, C={1, 2}, D=$\varnothing$, E={2, 3, 1}, F={1, 3, 5}, write true or false.

i) $B \subseteq A$	ii) $A \supset C$	iii) $F \subseteq A$	iv) $A = E$
v) $A \subseteq E$	vi) $A \supset E$	vii) $C \not\subseteq A$	viii) $B \subset C$

Main lesson (50 minutes)

Write on the board, the following: A={2, 4, 8}

Ask the students to form a set using the elements of set A. They can be called in turns to the board and asked to write their answers. Next, discuss the sets written. Ask a few questions like:

{2}, {4}, {8, 2}, (4, 2}, {4, 2}, {4, 8} etc.

- Is each of the set, a sub-set of set A?
- Is it possible to find more sub-sets from the elements of set A?

Write all the possible sub-sets of set A in a certain order and ask how many sub-sets are there altogether?
{2}, {4}, {8}, {2, 4}, {2, 8}, {4, 8}, {2, 4, 8}

Another example may be given and students asked to form the sub-sets.
B={ x, y, z}
Answers will be noted. {x}, {y}, {z} and so on.
How many sub-sets of set B can be formed?

Introduce the Power set
A set of all the possible sub-sets of a given set is called the Power Set and is denoted by the symbol P(S). (Refer to textbook pages 10 and 11)

Hence from the above examples
P (A) i.e. Power set of A = {∅, {2}, {4}, {8}, {2, 4}, {2, 8}, {4,8}, {2, 4, 8}
and P(B) = {{x}, {y}, {z}........................}

Every set is an improper sub-set of itself. Recall proper and improper sub-sets.
If C={5, 7}, how many sub-sets can be formed?
P(C) = {∅, {5}, {7}, {5, 7}}
Null set is a sub-set of every set.

Number of elements of the power set will be explained.

We denote the cardinal number of number or elements of a set by n(s) so the number of elements of a power set will be denoted as n(P(s).
Explain the difference between the elements of a set and elements of a power set.
From A={2, 4, 8}
2 ∈ A and {{2}} is an element of the P(A).

Give more examples:

$x \in$ B, Y ∈ B etc. and {{x}} ∈ P(B) etc.

Formula for finding the number of elements of a power set i.e. n(P(s) will be given.

From the examples A={2, 4, 8}, B={x, y, z), C={5, 7}, we see that n(P(A) = 8, n(P(B) = 8 (each of the set A and B has three elements) and n(P(C) = 4 (C has two elements).

If we take the number of elements as K in each set then n(P(S) = 2^K
for n(P(A) =2^K=2^3=8 (A has three elements, so K=3)
Similarly, n(P(B) = 2^K=2^3=8. B also has three elements, so K=3 C but has 2 elements so. n(P(C) = 2^K = 2^2 = 4 sub-sets

How many sub-sets can be formed of a set with 4 elements, 5 elements, 1 element etc.

1. $n(P(S)) = 2^K = 2^4 = 16$ sub-sets (when the set has 4 elemets)
2. $n(P(S)) = 2^K = 2^5 = 32$ subsets
3. $n(P(S)) = 2^K = 2^1 = 2$ subsets

   If $D=\{b\}$ then $n(P(D)) = 2^K = 2^1 = 2$

Operation on sets

- Explain the following:
- The properties of sets in the examples. Draw Venn diagrams to explain.
- Union and intersection of two or more sets will be explained with the help of examples from the textbook.
- Difference of two sets.
- Complement of a set with the help of examples. Complement of set A is denoted by A^c or A'
- Representation of the union, intersection, difference and complements of sets by Venn diagram with the help of examples from the textbook.
- Commutative property of union of two sets
- Commutative property of intersection of two sets

  $A \cup B = B \cup A$ and $A \cap B = B \cap A$
- Associative property of union and intersection of two sets

  i) $A \cup (B \cup C) = (A \cup B) \cup C$ and ii) $A \cap (B \cap C) = (A \cap B) \cap C$

  Work out examples on the board to verify the law.
- Union of a set and its complement. $A=\{1, 2, 4, 8\}$, $U=\{1, 2, 3, \ldots 10\}$, $A \cup A' = U$.
- Complement of a null set is a universal set
- Complement of a universal set is a null set

  Explain Distributive law of union over intersection and intersection over union.

  i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

  ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

  Verification of the properties with the help of examples will be done (Refer textbook page 17) on the board.
- Verify De Morgan's laws giving examples.

  $U=f1, 2, 3\ldots 10\}$, $A=\{2, 3, 5, 7\}$, $B=\{1, 3, 5, 7, 9\}$

  i) $(A \cup B)' = A' \cap B'$

  ii) $A \cap B)' = A' \cup B'$

  (Refer to textbook page 18)

Explain $(A \cup B)' = U - (A \cup B)$ and $A \cap B)' = U - (A \cap B)$

Work out examples on the board with student participation.

Practice session

Worksheets will be given with questions like the following.

1. Give sets:
   U={months of the year}
   A={January, June, July}
   B={March, June, September, November}
   C={months of the year, having 31 days}

   List the elements of:

i) A'	ii) $A \cap B$	iii) $B-A$	iv) $A \cup C$
v) C'	vi) $A' \cup B'$	vii) $A' \cap B'$	

2. Find P(A) if A = {3, 5, 7}
3. Draw Venn diagrams to represent and verify the following.

i) $A \cup Q = Q \cup P$ ii) $Q - P \neq P - Q$
iii) $Q \cap P = P \cap Q$ iv) $(P \cup Q)'$ (v) $(P \cap Q)'$
P = {1, 2, 3……10}, Q={0, 2, 6, 8, 10, 12}

Individual Work

Give Exercise 1a, 1b, 1c and 1d for class practice.
More sums will be given for verification of the properties of sets.

Homework

Given the sets U={1, 2, 3……20} and A = {2, 3, 5, 7, 11, 13, 15, 17}

Verify that

i) $(A \cup B) \cup C = A \cup (B \cup C)$
ii) $(B \cap C) \cap A = B \cap (C \cap A)$
iii) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
iv) Ferify De morgan's laws for the sets A and B,
v) Find P(A) when A={a, b, c, d} and hence find n(P(A). Check with the formula $n(P(A)) = 2^K$.

Recapitulation (10 minutes)

Definitions of sets, types of sets will be revised. Also, discuss, Power set of a set P(S) and n(P(S). Revise operations of sets.

- Commutative property of union and intersection $A \cup B = B \cup A$ & $A \cap B = B \cap A$
- Associative property of union and intersection $(A \cup B) \cup C = A \cup (B \cup C)$ and,
- Distributive law of union over intersection and vice versa
  $A \cup (B \cap C = A \cup b) \cap (A \cup C)$.
- De Megan's laws
- Complement of Universal and null set.

Students will be asked to give examples. At the end of the chapter, a short test will be conducted (MCQs) and extended response questions, constructive response questions (ERQs and CRQs) will be given

Commutative law

This law states that two sets can be combined in any order. $A \cup B$ will always be equal to $B \cup A$.

Similarly $A \cap B$ and $B \cap A$ gives the same elements.

Associative law

Three sets can be combined in any order.

UNIT 2

REAL NUMBERS

Lesson 1

Topic: Irrational Numbers
Time: 1 period

Objectives

To enable students to:
- define irrational numbers (Q)
- identify rational and irrational numbers (Q′)
- define real numbers as R=Q∪Q′
- demonstrate non-terminating decimals / non-repetitive (non periodic) decimals with examples
- find the perfect square of a number and will be able to establish patterns for the squares of natural numbers
- find the square root of a natural number, common faction and decimal fraction by prime factorization and division method.
- find the square root of numbers which are not perfect squares; determine the number of digits in the square root of a perfect square
- solve real-life problems involving square roots
- identify cubes and perfect cubes
- find cube roots of a number which are perfect cubes
- recognize properties of cubes of numbers.

Starter activity

Write some sets of numbers on the board and ask the students to identify them.

A = {1, 2, 3 ....}	set of natural numbers
B = {0, 1, 2, 3 .....}	set of whole numbers
C = {− 1, −2, −3, −4 .....}	set of negative integers
D = {− −3, −2, −1, 0, 1, 2, ...}	set of integers

E = {2, 4, 6, 8 ---}   set of even numbers.

Write some fractions and ask students to recognize them.

$\frac{3}{5}, \frac{2}{7}, \frac{5}{8}, \frac{16}{11}$ a set of common fractions

.2, .46, .135, .333 .2666   decimal fractions

Which of the following numbers are terminating / recurring or non-terminating?

$\frac{1}{2} = .5, \frac{1}{8} = .125, \frac{2}{3} = .666$

What does a common fraction indicate? (ratio between numerator and denominator) and hence it is called a rational number (rational is derived from the word ratio)

Which of these are recurring/non-recurring?

.2, .36, .414141=41, .666 or .6,   1.4142135623.......

.2 (terminating), .36 (terminating), .4141 (recurring) and 1.414213

Main Lesson

Explain that irrational numbers are numbers which are neither terminating nor recurring: Rational numbers can be written in the form of $\frac{P}{q}$, where P and q are integers and $q \neq 0$,

A set of rational numbers is denoted by the letter Q. Irrational numbers cannot be written in the form $\frac{P}{q}$. A set of irrational numbers is denoted by the letter Q'. Examples of terminating, non-terminating/recurring decimals will be worked out on the board.

Explain the difference of terminating/non-terminating and non-terminating/ non-recurring decimals with examples, $\sqrt{5}$, $\sqrt{2}$ are irrational numbers. Value of π is taken as 3.14159265. More examples of rational and irrational numbers from the textbook (page 21) will be given. Worked examples will be discussed (refer page 22 of the textbook)

Introduce a real numbers set as the union of rational and irrational numbers. R denotes the set of real numbers.

$R = Q \cup Q'$

Every quantitative value can be represented by a numeral, it may be a terminating/recurring or non of these. So all the numbers are called real numbers.

Explain the number line and graphing the real, rational numbers.

(refer to pages 22 and 23 of the textbook)

Practice session

Give worksheets with questions like the following:

Convert the following rational numbers into decimal fractions and state whether they are terminating or recurring.

1. $\frac{7}{9}$, $\frac{11}{12}$, $\frac{51}{70}$, $\frac{3}{8}$, $\frac{7}{25}$
2. Express the following as rational numbers

   i) 4.5 ii) 3.05 iii) 0.0007 iv) 0.666

Individual work

Give questions 1 and 2 from Exercise 2a as class work.

Homework

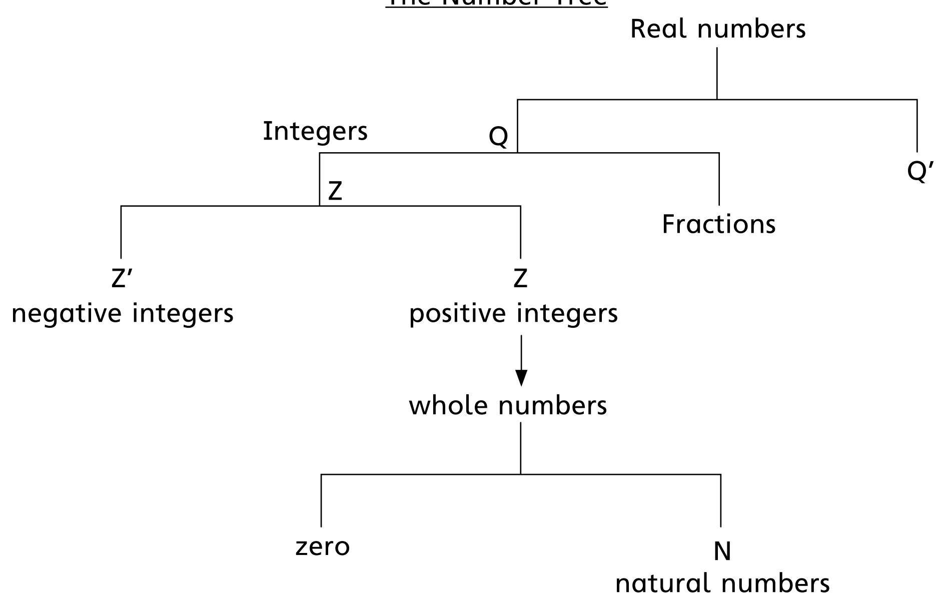
Ask the students to complete Exercise 2a, questions 3 and 4 as homework.

Recapitulation

- Difference between rational and irrational numbers will be discussed.
- Symbol or the letters used to denote rational and irrational numbers
- Set of real numbers is the union of rational and irrational numbers

  $R = Q \cup Q'$

<u>The Number Tree</u>

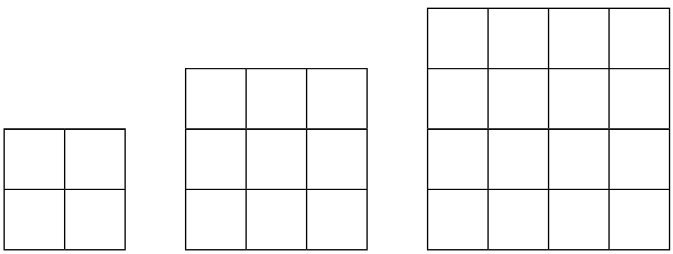


Lesson 2

Topic: Squares and cubes
Time: 3 periods.

Starter activity

Refer to textbook page 24.



2 units 3 units 4 units

Ask the following questions.

1. What is the area of the square with 2 units $2 \times 2 = 4$
2. What is the area of the square with 3 units $2 \times 3 = 9$

   4 is said to be the square of 2 and 9 is said to be the square of 3 and so on.
   Numbers like 1, 4, 9, 25, 36, ... are examples of square numbers.

Main Lesson

A square number is obtained by multiplying a number with itself.

Workout the following on the board with student participation.

$1 \times 1 = 1^2 = 1$
$2 \times 2 = 2^2 = 4$
$3 \times 3 = 3^2 = 9$
$4 \times 4 = 4^2 = 16$
$5 \times 5 = 5^2 = 25$
$6 \times 6 = 6^2 = 36$
$7 \times 7 = 7^2 = 49$
$8 \times 8 = 8^2 = 64$
$9 \times 9 = 9^2 = 81$
$10 \times 10 = 10^2 = 100$

Numbers like ∅ 1, 4, 9, 14, 25, 36, 49, 81, 100 and so on are called perfect squares because they are obtained by multiplying a number by itself.
Which of the numbers can be a perfect square? Ask the students to observe the pattern developed by squaring numbers from 1 to 10. Each of the square number has either of 0, 1, 4, 5, 6 or 9 in the one's place.

So any number having these digits in their units place can be a perfect square. For example, 361, 729, 256, 784, etc.

But numbers having the digits 2, 3, 7, 8 in their one's place are not perfect squares.
Sum of first two, three four, etc. odd numbers is a perfect square.
$1 + 3 = 4$ and $4 = 2^2$
$1 + 3 + 5 = 9$ and $9 = 3^2$
$1 + 3 + 5 = 16$ and $16 = 4^2$
Similarly other patterns for square numbers can be developed. (refer to textbook page 26)
Finding square roots of numbers will be explained with the help of examples (refer to textbook page 27)

Which number when squared, gives 4?
$2^2 = 4$, so 2 is called the square root of 4.

Similarly $3^2 = 9$, so 3 is the square root of 9.
Introduce the symbol $\sqrt{\ }$ (radical sign) for extracting the square root of a number.
So $\sqrt{4} = 2$, $\sqrt{25} = 5$.

When a number is under the radical sign it means extract the square root.
There are two ways of finding the square root. First, find the square root by factorization. To find the square root, find the factors (prime factors) of the given number.

What are the factors of 36?
Work on the board with students participation.

$36 = 2 \times 2 \times 3 \times 3$
Write the factors by pairing as squares
$36 = 2^2 \times 3^2$
$36 = 6^2$ (multiply 2 by 3)
$\therefore \sqrt{36} = 6$

2	36
2	18
3	9
3	3
	1

Similarly, workout the factors for square numbers. Students will be called in turns to perform prime factorization on the board (refer to textbook page 28).

196
$196 = 2 \times 2 \times 7 \times 7$
$= 22 \times 72$
$= 14^2$
$\therefore \sqrt{196} = 14$

2	196
2	98
7	49
7	7
	1

Is 48 a perfect square?

$48 = \overline{2 \times 2} \times \overline{2 \times 2} \times 3$

$= 2^2 \times 2^2 \times 3$

The factor 3 is occurring only once.

2	48
2	24
7	12
7	6
3	3
	1

So 48 is not a perfect square number.

Now $48 \times 3 = \overline{2 \times 2} \times \overline{2 \times 2} \times \overline{3 \times 3}$

$= 2^2 \times 2^2 \times 3^2$

$= 12^2$

$144 = 12^2$

$\therefore \sqrt{144} = 12$ (refer to textbook page 29)

Similarly, if we divide 48 by 3 we get

$48 \div 3 = \dfrac{2 \times 2 \times 2 \times 2 \times \cancel{3}}{\cancel{3}}$

$16 = \overline{2 \times 2} \times \overline{2 \times 2}$

$16 = 2^2 \times 2^2$

$16 = 4^2$

So 16 is a perfect square number.

A perfect square number can be obtained by multiplying or dividing any given number with its factor/s not appearing in pairs.

Square roots of common fractions can also be extracted by prime factorization.

example $\frac{36}{49}$

Students will be asked to solve this on the board.

$36 = 6^2$

$49 = 7^2$

$\therefore \sqrt{\frac{36}{49}} = \frac{6}{7}$

Square root of decimal fractions will be worked out on the board (refer to textbook pages 30 and 31) with student participation.

Decimal fractions with denominators 10,1000 or 100 000 cannot be perfect squares. Explain with the help of examples (refer to textbook page 31).

Method of finding square root by the division method will be explained with the help of examples (refer to textbook page 32).

To find the square root of 357604, proceed as:

$\overline{35}\ \overline{76}\ \overline{04}$

	598
5	$\overline{35}\ \overline{76}\ \overline{04}$
+5	−25
109	1076
+9	−981
1188	9504
8	−9504
	xxxx

$\therefore \sqrt{357604} = 598$

Mark off the digits in pairs from right to left.
Taking the first pair which is 35, we know that,
$1^2 = 1$, $2^2 = 4$, $3^2 = 9$, $4^2 = 16$, $5^2 = 25$
and $6^2 = 36$. So 6^2 is greater than 35. We take $5^2 = 25$.
Write 5 as the divisor and 5 as the quotient.
Subtract $5^2 = 25$ from 35.
The remainder is 10. Bring down the next pair which is 76.
Add 5 in the divisor which gives the new divisor. By trial, we find a digit for the one's place of the divisor.

$$\begin{array}{r}101\\ \times\ 1\\ \hline 101\end{array}\qquad\begin{array}{r}102\\ \times\ 2\\ \hline 204\end{array}\qquad\begin{array}{r}103\\ \times\ 3\\ \hline 309\end{array}$$

$$\begin{array}{r}104\\ \times\ 4\\ \hline 416\end{array}\qquad\begin{array}{r}105\\ \times\ 5\\ \hline 525\end{array}\qquad\begin{array}{r}106\\ \times\ 6\\ \hline 636\end{array}$$

$$\begin{array}{r}107\\ \times\ 7\\ \hline 749\end{array}\qquad\begin{array}{r}108\\ \times\ 8\\ \hline 864\end{array}\qquad\begin{array}{r}109\\ \times\ 9\\ \hline 981\end{array}$$

Put 9 in the divisors column and also in the quotient.

Add 9 for the next divisor and bring down the next pair with the remainder.

Next, we find the digit for the one's place of the new divisor.

1181×1,	118 2 × 2,	118 3 × 3,	118 4 × 4
1181	(2364)	3549	4736
118 5 × 5,	118 6 × 6,	118 7 × 7,	118 8 × 8
5925	7116	8309	9504

We see that 118 8 × 8 gives us 9504 which is our last dividend. Similarly workout the example with odd number of digits in the given number with student participation for finding the new divisor.

Example

$\hat{2}\,\overline{72}\,\overline{25}$

$$
\begin{array}{r|l}
 & \quad 165 \\
\hline
1 & \hat{2}\,\overline{72}\,\overline{25} \\
+1 & -1 \\
\hline
26 & \;172 \\
+\;\;6 & -156 \\
\hline
325 & \;\;1625 \\
5 & -1625 \\
\hline
 & \;\;\text{xxxx}
\end{array}
$$

$\therefore \sqrt{27225} = 165$

2 is the first digit, so find a number whose square is − 2.
$(1 \times 1 = 1^2 = 1)^1$
Put 1 in the divisor column and 1 in the quotient.
Subtract $1^2 = 1$ from 2.
Now the remainder is 1, bring down the next pair which is 72 and add 1 for the new divisor in divisor's column. Again by trial, find a digit for the one's place in the divisor's column. Now see that 27 × 7 = > 172, so take 26 × 6 = 156 < 172 and subtract it from 172. Put 6 in the divisor's column and 6 in the quotient. Now the remainder is 16. Bring down the next pair, (25), as the new dividend. Again by trial, find the digit for the one's place with 32. We find that 325 × 5 = 1625, so, put 5 in the divisor's column next to 32 and 5 in the quotent next to 16.

$21 \times 1 = 21$
$22 \times 2 = 44$
$23 \times 3 = 69$
$24 \times 4 = 96$
$25 \times 5 = 125$
$26 \times 6 = 156$
$27 \times 7 = 189$

$321 \times 1 = 321$
$322 \times 2 = 644$
$323 \times 3 = 969$
$324 \times 4 = 1296$
$325 \times 5 = 1625$

Finding the square root of common fractions and decimal fractions will be explained with the help of examples (refer to textbook pages 34 and 35)

Finding the square roots of numbers which are not perfect squares will be explained with the help of examples worked out on the board. (refer to textbook page 36)

Explain that square root of numbers which are not perfect squares can be extracted to a certain number of places of decimal (one place, two places, three places etc.).

Examples will be solved on the board with student participation.

Method of finding the number of digits in the square root of a number will be explained (refer to textbook page 38).

Number of digits in the square root of a number

If the number of digits in a number is even then the number of digit is in the square root will be $\frac{n}{2}$, where n is the number of digits. For example 16 is a 2-digit number, n=2 (even) $\frac{2}{2}$ = 1, the square root in a one digit number. Another example of a 16 digit number the square root will have $\frac{16}{2}$ = 8 digits.

If the number of digits of a number is odd, the square root will have n + $\frac{1}{2}$ digits.

Example

196 $\longrightarrow$ 3 digits (odd)

$\therefore \frac{n+1}{2} = \frac{3+1}{2} = \frac{4}{2} = 2$ digits

$\sqrt{196} = 14$(14 is a 2 digit number)

Estimating the square root of a number will be explained with the help of examples (refer to textbook page 38).

Explain adding or subtracting the smallest number to a given number to make a perfect square number, with the help of examples (refer to textbook page 40)

Practice session

Worksheets will be given.

Find the square root of the numbers by factorization and by the division method.

Find the number of digits in the square root of the given numbers.

Estimate the square root of the given numbers mentally.

Individual Work

Give selected questions from Exercise 2c for individual practice. Similary, give some of the word problems.

Homework

Complete Exercise 2c for homework.

UNIT 3

NUMBER SYSTEM

Topic: Number systems
Time: 4 periods

Objectives

To enable students to:

- number systems with base 2, 5, 8 and 10
- identify and explain base of a number and place values of digits
- convert a number from the decimal system to any other base (2, 5, 8)
- convert a number with base 2, 5 and 8 to the decimal system
- perform basic operations (addition, subtraction, and multiplication) in base 2, 5 and 8

Starter activity

Write some numbers on the board and ask the students to read them.
25, 73, 481, 690 etc. Ask some related questions.

1. What do you mean by 25?
   twenty and five or 2 tens and 5 ones.
2. What does the number 73 represent?
   7 tens and 3 units
3. How much is 481?
   400 + 80 + 1
4. How many different digits do we use in expressing quantities?
   ten digits i.e 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9
5. What do we call this system of writing numbers?
   decimal system or base-10 system
6. How did people count in the old days?
   In groups.
7. Is it possible to express any number using only two digits for example 0 and

1 or using 3 digits or 5 digits?
0, 1, 2; 0, 1, 2, 3, 4 etc.

Main lesson

In older civilizations, people used to count articles or keep an account of their possessions in groups of 5 (five figures in a hand) or in groups of 2s or 8s etc. They had different ways of expressing quantities.

- Mayan way of representing numbers (refer to pages 45 and 46 from the textbook) will be discussed. Similarly, other ways of expressing quantities for e.g. Egyptian or Roman can be discussed.
- Nowadays we use the base-10 or the decimal system to represent numbers or expressing quantities. In this system, we use ten digits 0, 1, 2 ... 9 and that is why it is called the base-10 system.
- Similarly, the number system based on 5 digits i.e. 0, 1, 2, 3, 4 is called the base–5 system.
- Binary number system (bi means two) is the system in which we use only two digits i.e. 0 and 1. 0 means off and 1 means on. All computers are based on this system.
- In base–8, we represent a number using any of the eight digits i.e 0, 1, 2, 3, 4, 5, 6, and 7.
- Every number system has a place value just as we have the place value in decimal or base-10 system. The basic unit is 1 in all base systems.

In the decimal system, the place values are units, tens, hundreds, thousands, etc. Explain the place value with examples, 3245 mean 3 thousands + 2 hundreds + 4 tens + 5 ones.

$(3 \times 10^3) + (2 \times 10^2) + (4 \times 10) + (5 \times 10^0)$	Multiplying the digit with the respective place value.
$(3 \times 1000) + (2 \times 100) + (4 \times 10) + 5 \times 1)$	$10^0 = 1$, any number raised to the power zero = 1

The place value is increasing by 10 times the previous value moving from right to left.
In base–5, the place value of a digit increases by 5 times the previous place (moving from right to left).
So,

.... 5^5	5^4	5^3	5^2	5^1	5^0
	↓	↓	↓	↓	↓
	625	125	25	5	1

In the binary system, the place value increases by 2 times the previous value (simply double) and in base–8, it increases 8 times the previous place.
Show the students the conversion of base–10 or decimal system to any other

base.

Lesson 1

Conversion of a number from the decimal system to base–5

The digits 1 to 4 have the same value in base–10 and base–5.

★ ______ 1

★★ ______ 2

$\boxed{★★★★★}$ ★★ 12_5

$\boxed{★★★★★}$ ★★★ 13_5

Base–10	=	Base–5	
1	=	1	
2	=	2	
3	=	3	
4	=	5	
5	=	10_5	(one group of 5 only, no units digit so ones place is zero
6	=	11_5	(one group of 5 and 1 ones)
7	=	12_5	(one group of 5 and 2 ones)
8	=	13_5	(one group of 5 and 3 ones)
9	=	14_5	how? (ask the students)
10	=	20_5	how? (ask the students)

It shows that to find the equivalent number from base–10 to base–5, we divide the number in groups of 5.

Example

34_{10} converted to base–5 will be

5	34	
5	6	4 ↑
	1	1

5 × 6 = 30, 34 − 30 = 4
4 is the remainder
5 × 1 = 5
6 − 5 = 1
1 is the remainder

34_{10} = 1145 (Write the converted number in the direction of the arrow.)

Referring to the table above, $7_{10} = 12_5$

By the division method, this can be calculated as:

5	7	
	1	2 ↑

5 × 1 = 5
7 − 5 = 2
2 is the remainder

Writing the number in the direction of the arrow, we get $7_{10} = 12_5$
Work out some more examples on the board with student participation.

Convert i) 45_{10} ii) 129 iii) 86 in base–5

To convert a base–5 number to base–10
Multiply each digit by its respective place value and add the products. Explain with examples.

a) 13_5

$5^1 \quad 5^0$

$1 \quad 3$

$= (1 \times 5) + (3 \times 1)$

$= 5 + 3$

$= 8_{10}$

b) 132_5

$5^2 \quad 5^1 \quad 5^0$

$1 \quad 3 \quad 2$

$= (1 \times 5^2) + (3 \times 5^1) + (2 \times 5^2)$

$= (1 \times 25) + (3 \times 5) + (2 \times 1)$

$= 25 + 15 + 2$

$= 42_{10}$

Work out some more examples on the board with student participation.

Individual work

Worksheets will be given with exercises like the following.

1. Convert 10, 25, 35, 113 to base–5.
2. Convert 23_5, 111_5, 204_5 to the decimal system.
3. Write true or false.
   a) $10_{10} = 10_5$
   b) $12_{10} = 22_5$
   c) $104_{10} = 44_5$
   d) $103_{10} = 403_5$
   e) $23_5 = 13_{10}$
   f) $220_5 = 52_{10}$
   g) $432_5 = 117_{10}$

Lesson 2

Addition and subtraction of base–5

1. Add $14_5 + 43_5$

$$\begin{array}{rrr} & 1 & \\ & 1 & 4_5 \\ + & 4 & 3_5 \\ \hline & 11 & 2_5 \end{array}$$

$4 + 3 = 7 = 12_5$
write 2 and carry 1
$1 + 1 + 4 = 6 = 11_5$

Recall the table format of base–10 to base–5

2.
$$\begin{array}{rrr} & 1 & \\ 1 & 3 & 4 \\ + & 4 & 2 \\ \hline 2 & 3 & 1_5 \end{array}$$

$4 + 2 = 6 = 11_5$
write 1 and carry 1
$1 + 3 + 4 = 8 = 13_5$
write 3 and carry 1
$1 + 1 = 2 = 2_5$

Work out more examples on the board with student participation.

Subtract:

1. 23_5 from 34_5

$$\begin{array}{rrr} & 3 & 4 \\ - & 2 & 3 \\ \hline & 1 & 1_5 \end{array}$$

$4 - 3 = 1$
$3 - 2 = 1$

2. 23_5 from 41_5

$$\begin{array}{rrr} & 3 & 5 \\ & \cancel{4} & 1_5 \\ - & 2 & 3_5 \\ \hline & 1 & 3_5 \end{array}$$

1 is smaller than 3 so we borrow 1 from 4 (here 1 = 5) every place is 5 times the previous so borrowing 1 mean 1 of 5 = 5
$5 + 1 = 6$
$6 - 3 = 3$
Now $3 - 2 = 1$

More examples will be worked out with student participation.

Lesson 3

Multiplication of base–5 numbers

1. $34_5 \times 21_5$

$$\begin{array}{rrrr} & & 3 & 4 \\ & \times & 2 & 1 \\ \hline & {\scriptstyle 1}\,\, & 3 & 4 \\ 1 & 2 & 3 & \\ \hline 1 & 3 & 1 & 4_5 \end{array}$$

$34 \times 1 = 34$
$34 \times 2 =$
$8 = 13_5$
$2 \times 3 = 6 + 1 = 7$
$7 = 12_5$

Convert to base–10 and check the answer
$34 = 15 + 4 = 19$
$\underline{21 = 10 + 1 = 11}$
$1314_5 = \quad = 209_{10}$
$125 + 75 + 5 + 4$
209

2. $123 \times 45_5$

Solve the above sum on the board.

Individual work

Students will be called to solve the sums on the board.

1. Find the sum of
   a) $22_5 + 40_5 + 11_5$ b) $302_5 + 123_5 + 24_5$
2. Find the difference between:
   a) 403_5 and 231_5 b) 112_5 and 11_5
3. Simplify:
   a) $224_5 + 331_5 + 342_5$ b) $423_5 - 304_5 + 233_5$
4. Find the product.
   a) $321_5 \times 12_5$ b) $14_5 \times 22_5$

Lesson 4

Conversion of decimal numbers to binary numbers

To convert a number written in decimal system to binary, divide the number by 2 (successive division).

Solve some examples on the board.

2	12	
2	6	0
2	3	0
	1	1

$2 \times 6 = 12$; $12 - 12 = 0$
0 is the remainder
$2 \times 3 = 6$, $6 - 6 = 0$
0 is the remainder
$2 \times 1 = 2$, $3 - 2 = 1$
1 is the remainder

Write the converted number in the direction of the arrow.

$12_{10} = 1100_2$

2. Convert 23_{10} to base−2

2	23	
2	11	0
2	5	0
2	2	1
	1	

$2 \times 11 = 22$
$23 - 22 = 1$ remainder
$2 \times 5 = 10$
$11 - 10 = 1$ remainder
$2 \times 2 = 4$
$5 - 4 = 1$ remainder
$2 \times 1 = 2$
$2 - 2 = 0$

$23_{10} = 10111_2$

Solve some more examples with student participation.

To convert a number from base−2 to decimal

Convert 101_2 to base−10

Multiply each digit with its respective place value of base−2

$2^2 \quad 2^1 \quad 2^2$

$$\begin{aligned} 1 \quad 0 \quad 1_2 \quad &= (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0) \\ &= (1 \times 4) + (0 \times 2) + (1 \times 1) \\ &= 4 + 0 + 1 \\ &= 5_{10} \end{aligned}$$

Convert 110_2 to decimal system

$2^2 \quad 2^1 \quad 2^0$

$$\begin{aligned} 1 \quad 1 \quad 0_2 \quad &= (1 \times 2^2) + (1 \times 2^1) + (0 \times 2^0) \\ &= (1 \times 4) + (1 \times 2) + (0 \times 1) \\ &= 4 + 2 + 0 \\ &= 6_{10} \end{aligned}$$

Solve more examples on the board with student participation.

Addition and subtraction of binary numbers will be explained on the board with examples.

Add:

1. $11_2 + 10_2$

$$\begin{array}{rccc} & & 1 & 1_2 \\ + & & 1 & 0_5 \\ \hline & 1 & 0 & 1_2 \end{array}$$

$1 + 0 = 1$ Teacher's explanation

$1 + 1 = 2 = 10_2$

2. $111_2 + 101_2$

$$\begin{array}{rcccc} & & {}^{1}1 & {}^{1}1 & 1_2 \\ + & & 1 & 0 & 1_2 \\ \hline & 1 & 1 & 0 & 0 \end{array}$$

$1 + 1 = 2 = 10_2$ Teacher's explanation

write 0 and carry 1

$1 + 1 = 2 = 10_2$

again write 0 and carry 1

$$\begin{aligned} 1 + 1 + 1 = 3 \quad &= 2 + 1 \\ &= 10_2 + 1 \\ &= 11_2 \end{aligned}$$

3. Subtract 11_2 from 101_2

$$\begin{array}{rccc} & \cancel{1} & {}^{2}0 & 1_2 \\ + & & 1 & 1 \\ \hline & & 1 & 0_2 \end{array}$$

$1 - 1 = 0$ Teacher's explanation

zero − 1 borrow 1 from the next place

$1 = 2$

$2 - 1 = 1$

Explain multiplication of binary numbers.

1. $11_2 \times 10_2$ and check the answer in the decimal system.

$$
\begin{array}{rcccc}
 & & 1^{21} & 1^{20} \\
\times & & 1 & 0_2 \\
\hline
 & & 0 & 0 \\
+ & 1 & 1 & 0 \\
\hline
 & 1 & 1 & 0_2
\end{array}
\quad = 4 + 2 + 0 = 6
\qquad
\begin{array}{r}
= 2 + 1 = 3 \\
= 2 + 0 = 2 \\
\hline
6 \\
\hline
\end{array}
$$

2. 11×11

$$
\begin{array}{rccc}
 & & 1 & 1 \\
 & \times & 1 & 1 \\
\hline
 & & 1 & 1 \\
+ & 1 & 1 & 0 \\
\hline
1 & 0 & 0 & 1
\end{array}
\qquad
\begin{array}{l}
2 + 1 = 3 \\
2 + 1 = 3 \\
\hline
9 \\
\hline
1 + 0 = 1 \\
1 + 1 = 2 = 10_2 \\
1 + 1 = 2 = 10_2 \\
8 + 0 + 0 + 1 = 9
\end{array}
$$

Individual work

Give worksheets to students for practice.

1. Convert to base–2.
   15, 27, 136
2. Find the product and check your answers.
   a) $11_2 \times 110_2$ b) $111_2 \times 10_2 \times 100_2$
3. Convert to decimal system.
   110_2, 111_2, 101_2
4. Simplify:
   a) $10_2 + 1_2 + 11_2 + 101_2$ b) $111_2 + 100_2 + 11_2$

Lesson 5

Base–8 number system

The base–8 number system consists of eight digits i.e. 0, 1, 2, 3, 4, 5, 6, and 7

Base–10 number and its equivalent base-8 number make the system clearer.

Base–10 number	=	Base 8 number
0, 1, 2, 3, 4, 5, 6, 7	=	0, 1, 2, 3, 4, 5, 6, and 7
8_{10}	=	10_8 (1 group of 8)
9	=	11_8 (1 group of 8 + 1 unit) $10_8 + 1$
10	=	12_8 (1 group of 8 + 2 units) = $10_8 + 2$
11	=	13_8 (1 group of 8 + 3 units)
12	=	14_8 (1 group of 8 + 4 units)

13 = 15_8 (1 group of 8 + 5 units)
14 = 16_8 (1 group of 8 + 6 units)
15 = 17_8 (1 group of 8 + 7 units)
16 = 20_8 (2 groups of 8) $8 + 8 = 20_8 + 0$
17 = 21_8 (1 group of 8 + 1 unit)
18 = 22_8 (2 groups of 8 + 2 units)

To convert a number given in base–10, divide it by 8 to convert it into base–8
Solve examples on the board.

1. 20_{10}

$$\begin{array}{c|c}8 & 20 \\ \hline & 2 \quad — \quad 4\end{array}$$

$20_{10} = 24_8$

$8 \times 2 = 16$
$20 - 16 = 4$ remainder

2. 39

$$\begin{array}{c|c}8 & 391 \\ \hline 8 & 48 \quad — \quad 7 \\ \hline & 6 \quad — \quad 0\end{array}$$

$39_{10} = 607_8$

To convert a base–8 number to base–10, multiply each digit with its respective place value in base–8.

Convert 17_8 to base 10.

1. $1^8 \quad 7^{80}$
$= (1 \times 8^1) + 7 \times 8^0)$
$= 8 + 7$
$= 15_{10}$

2. 243_8
$8^2 \quad 8^1 \quad 8^0$
$2 \quad 4 \quad 3$
$(2 \times 8^2) + (4 \times 8^1) + (3 \times 8^0)$
$(2 \times 64) + (4 \times 8) + (3 \times 1)$
$128 + 3^2 + 3$
163_{10}

Solve more examples with student participation.
Explain the addition, subtraction, and multiplication of base–8 numbers.

Addition of base–8 numbers

1. $23_8 + 16_8$

$$\begin{array}{rrr} & 1 & \\ & 2 & 3_8 \\ + & 1 & 6_8 \\ \hline & 4 & 1_8 \end{array}$$

$3 + 6 = 9 = 8 + 1 = (10_8 + 1)$ Teacher's explanation
$1 + 2 + 1 = 4_8$

2. $54_8 + 736_8$

$$\begin{array}{rrrr} & & 1 & \\ & 1 & 5 & 4_8 \\ + & 7 & 3 & 6_5 \\ \hline 1 & 0 & 1 & 2_8 \end{array}$$

$4 + 6 = 10 = (8 + 2) = (10_8 + 2) = 12_8$
$1 + 5 + 3 = 9 = 8 + 1 = 11_8$
$1 + 7 = 8 = 10_8$

Individual work

Give worksheets for practice.

1. Convert to base−8.
   56, 100, 205
2. Convert to base−10.
   75_8, 123_8, 405_8

Write the equivalent.

a) 18_{10} = ____________$_8$ b) 9_{10} = ____________$_8$

c) 15_8 = ____________$_{10}$ d) 11_8 = ____________$_{10}$

e) 16_8 = ____________$_{10}$ f) 21_{10} = ____________$_8$

Homework

Give selected sums from Exercise 3a.

Subtraction of base−8 numbers

1. $34_8 - 12_8$

$$\begin{array}{rcc} & 3 & 4_8 \\ - & 1 & 2_8 \\ \hline & 2 & 2_8 \end{array}$$

2. $42_8 - 16_8$

$$\begin{array}{rcc} & {\scriptstyle 3} & {\scriptstyle 8} \\ & \cancel{4} & 2_8 \\ - & 1 & 6_8 \\ \hline & 2 & 4_8 \end{array}$$

From 2 we cannot subtract 6 so borrow 1 from 4, (1 group of 8) means borrow 8

10 − 6 = 4

Multiplication of base−8 numbers

$34_8 \times 25_8$

$$\begin{array}{rccc} & & {}^{1}2 & \\ & & 3 & 4_8 \\ \times & & 2 & 5_8 \\ \hline & 2 & 1 & 4 \\ + & 7 & 0 & x \\ \hline 1 & 1 & 1 & 4_8 \\ \hline \end{array}$$

4 × 5 = 20 = 2(8)+4

2 groups of 8 and 4 keep 4 and carry 2

Now 5 × 3 = 15 + 2 = 17

17 = 2(8)+1 21_8

Now multiplying by 2

2 × 4 = 8 = 10_8

keep 0 and carry 1

2 × 3 = 6

6 + 1 = 7

Solve more examples with student participation

Recapitulation

Revise the concepts and help the students if anything is not clear.

UNIT 4 FINANCIAL ARITHMETIC

Lesson 1

Topic: Percentage, Profit and Loss, Discount
Time: 2 periods

Objectives:

To enable students to:

- understand and identify percentage, profit, loss, and discount
- solve real-life problems related to percentage profit, loss, and discount

Starter activities

Activity 1

Two days before teaching the topic, ask the students to do some shopping with their parents. When they come to the school for the lesson, they should bring with them the following:

- a simple cash memo
- bring a few discount leaflets from the super store

Ask the students to point out the (vocabulary building) terms used in the cash memo, promotional leaflets e.g. total, rate, discount, price, cost etc. Write down these terms on the board and give a quick review as they are to be used in the following lesson.

Activity 2

Students will be given activity sheets to recall their knowledge of percentage, profit, loss, selling price, cost price etc.

a) C.P. = Rs 500, S.P. = Rs 600 then Profit = Rs ?

b) I bought a book for Rs 50 and gained 50% by selling it. What is my selling price?
c) C.P. of an article is Rs 400 and S.P. = Rs 350. Find the loss%.

Discuss the answers the students give.

Main lesson

Solve examples from the textbook page 88 and 89 on the board with student participation.

Practice session

Give Exercise 4e, questions1 and 2 for practice. The students will be called in turns to the board to solve.

Individual work

Give selected questions from Exercise 4e, questions 3 to 5 for students to do individually.

Homework

Give the rest of the questions from Exercise 4e to be done as homework.

Recapitulation

Revise the terms used, cost price, selling price, profit, loss, percentage, etc.

C.P. = S.P − Profit
C.P. = S.P + loss
Profit = S.P − C.P
Loss = C.P − S.P
Profit%s = $\frac{\text{Profit}}{\text{C.P.}} \times 100$
Loss% = $\frac{\text{Loss}}{\text{C.P.}} \times 100$

Topic: Discount

Starter activity

Make some flash cards of discount offers as advertised in some newspapers or magazines and show to the class. Ask questions about the offers and what the students can understand about the offers.

Main lesson

Define and explain the meaning of discount. Discount means a reduction in price at sales on special occasions for Eid, new year festival or clearance sale etc.

Formula

Net price = marked price − discount

Discount% = $\frac{\text{discount}}{\text{marked price}} \times 100$

Refer to textbook pages 90 and 91. Examples 1, 2, and 3 from page 91 will be solved on the board.

Practice session

Give question 1 of Exercise 4f to be done on board with student participation.

Individual work

Give Exercise 4f questions 2 to 5 as class work.

Homework

Give the rest of the questions from Exercise 4f as homework.

Recapitulation

Revise the terms, discount, net price, marked price, discount % etc.

Lesson 2

Topic: Banking
Time: 1 period

Objectives

To enable students to define and understand:

- types of bank accounts
- negotiable instruments (cheques, demand drafts and pay orders)
- online banking
- transactions through ATM, Debit and Credit cards
- conversion of currencies
- calculate profit / markup
- principal amount, markup rate and period
- types of finances
- solve everyday problems related to banking and finance

Starter activity

Put up a slide show or show some pictures of the work that goes on in a bank. Next ask some questions.

- How did people save their money in the ancient times ?
- How and where do they save it now ?
- Why is it safe to keep your money in the bank?
- What can you tell about the kind of work that is done in a bank?

Main lesson

Explain what a bank is, how it functions and the facilities it provides. In the case of transactions of money (cheques and bank drafts), samples will be displayed in the classroom. Types of bank accounts will be explained. (for PLS, savings, fixed deposit, current deposit etc., please refer to page 76 of the textbook.)

Explain the concepts of online banking and convrsion of currencies. Discuss briefly, the importance of advanced computer technology. Online banking allows the customer to perform banking transactions and the payment of utility bills etc. through ATM, credit cards and from their own homes via the Internet.

Discuss and explain the following terms.

- ATM (automated teller machine)
- Credit cards
- Debit cards

Display samples of the above mentioned cards. (An organized visit to a local bank is highly recommended.) Currencies of different countries would be displayed (for example, Pakistani Rupee, $, £, Yen, Euro, etc).

Explain the conversion of currencies from foreign currency to local (Pakistani) and vice versa. Discuss the need and importance of conversion / exchange of currencies and the rate of exchange and its application. As an activity, ask the students to look up the newspaper for currency rates.

Practice session

Display a 1000 rupee note and ask the students to look for an equivalent amount in US Dollars and British Pounds (refer to pages 79 and 80 from the textbook). Solve with student participation.

Introduce and explain terms like profit, markup, principal amount, period (time).

Markup amount = Principal amount • markup rate • time period

- Profit amount = Principal amount + Markup amount
- Markup rate = $\frac{\text{Markup Amount}}{\text{Principal amount} \times \text{time}}$

For calculating period, refer to page 83 from the textbook.

Practice session

Examples on page 81 (about markup amounts, profit amounts, markup rate principal amount etc.) will be explained on the board.

Individual work

Give selected questions from Exercise 4b and 4c to be done as class work.

Homework

Give the rest of questions from Exercise 4b and 4c as homework.

Topic: Types of finances
Time: 1 period

Objectives

To enable students to define and understand:

- over draft (OD)
- running finance (RF)
- demand finance
- leasing

Main lesson

Introduce and discuss the terms as given above.

Practice session

Solve the example given on page 87 of the textbook on the board.

Individual work

Give selected questions from Exercise 4d for individual practice.

Homework

Ask students to complete Exercise 4d as homework.

Recapitulation

Revise the types of bank accounts, negotiable instruments, online banking, ATM credit and debit cards, conversion of currencies, profit markup, primcipal amount, types of finances.

Lesson 3

Topic: Insurance
Time: 1 period

Objectives

To enable students to:

- define and understand insurance
- solve real life problems related to life and vehicle insurance.

Starter activity

Two short stories or incidents can be told to explain why insurance is important.

1. a family suffering from day-to-day financial problems due to sudden death of their bread winner
2. another family not suffering from daily financial problems because the head of the family had taken a life insurance policy

Ask relevant questions based on life insurance and discuss the answers.

Main lesson

Explain the importance and need of different insurance policies (life insurance, vehicle insurance etc.) For more explanation, refer to pages 92 to 94 of the textbook.

Practice session

Solve the following questions with student participation.

1. Ali purchases a life insurance policy for Rs 2 lacs. How much he has to pay annually when the rate of premium is 2% of net amount?
2. Arif pays Rs 16000/- as annual premium for his car. What is the total amount of the car insurance policy?

Individual work

Give selected questions from Exercise 4g as class work.

Homework

Give the rest of the questions from Exercise 4g as homework.

Recapitulation

Revise the terms life insurance policy, vehicle insurance, premium, rate of premium etc.

Lesson 4

Topic: Income tax
Time: 1 period

Objectives

To enable students to:

- define and understand income tax
- differentiate between taxable and exempted income, rate of income tax and net income.

Starter activities

Display pictures of a hospital, a school and a commercial area in a city / town and ask the following questions.

1. What facilities do the government provides its citizens?
2. What does the government need to fulfil them?
3. From what resources do the government gets money to build roads, hospitals or schools?

Main lesson

Define and explain the term income tax. It is the amount imposed annually by the government on the income or net earnings of its citizens. Explain that there is always a basic amount of the net income which is tax exempted. Solve the examples from the textbook given on page 97 on the board.

Individual work

Give questions 1 and 2 of Exercise 4h to be done as practice exercises.

Homework

Give the rest of the questions from Exercise 4h as homework.

Recapitulation

Revise the terms net income, income tax, exempted income, taxable income, and running expenses etc.

UNIT 5

POLYNOMIALS

Topic: Polynomials
Time: 2 periods

Objective

To be enable students to:

- recall terms such as constant, variable, literal and algebraic expressions.

Starter activity

The students have studied about exponents, constants and variables. They will be given a worksheet to recall the terms.

Classify the following expressions into variables, constants, exponents, and coefficients.

1. 17, $2xy$, $4abc$, 102, $8a^2b^3$, p^4, $x^2 - 2x + 1$, $-7a$
2. State whether each of the following is true or false.
   a) $8ab - 2ab$ are like terms ____________
   b) $4a^2 + 3$ is a binomial ____________
   c) $3a - 4b = 7ab$ ____________
   d) $2a \times a^2 = 2a^3$ ____________
   e) $4b^3 \div 2b^3$ ____________
3. Fill in the blanks:
   a) $-9a$, $x^2 - 3x + 2$, $6a - 3$ are called ____________
   b) A constant has ____________ value
   c) A variable has ____________ value
   d) In $-b^2$ the coefficient is ____________

Main lesson

Explain to the students that in arithmetic, numbers are usually represented by figures. These figures have definite values, for example 1,2,3,5 and so on. These numbers are known as Constant.

In algebra besides numbers, we use alphabets like x, y, a, b, c to express a number. These numbers have no definite value. When x stands for numbers they may have any value assigned to them in different situations. Such numbers are called variables.

Example 1

A = $\{\frac{x}{1} \le x \le 10$ and x is a natural number$\}$

Here,'x' is a variable which denotes every number from 1 to 10 and every number of the set is a constant.

Example 2

Sana had Rs 600 and Asma has Rs $9x$. Who has more money?

We can't say that Sana has more than Asma because the value of x is unknown. However, if we take $x = 8$, then Asma has $9 \times 8 = 72$ which means she has the lesser amount. If

$x = 100$, then Asma has $9 \times 100 = 900$, so she has more amount than Sana.

Explain the term algebraic expression and its different kinds with the help of examples on the board.

Algebraic expression is a statement connecting variables and constants by operations of addition, subtraction, multiplication, and division. For example:

12, $3x^2 - 5x + 8$, $-9p$, $6x - 2 + \frac{1}{y}$ are algebraic expressions.

Explain that a polynomial is an algebraic expression where coefficients are real numbers and exponents or powers are non-negative integers,

For example, $x^3 - 5x + 4$ is a polynomial as its coefficient 1, −5 and 4 are real numbers and 3, 1, and 0 are non-negative integers.

Classification of Polynomials

Monomial: A polynomial consisting of a single term as mono means one.
Examples are: 1^1, $9x^2$, $5ab$, 0, etc.
Binomial: Consists of two terms, as bi means two.
Examples are: $7y - 8$, $p^3 + 6$, $2x - 3y$ etc.
Trinomial: Consists of 3 terms, as tri means 3.
$3a^2 + 2ab + c^2$, $a^2b^3 - 2xy + 5$ etc.
Multinomial: Consists of more than three terms.

Individual activity

Exercise 5a question 1 will be done in class

Homework

1. Fill in the blanks.
   a) $8a^2b^2c^3$ is a __________, b) $4 \times a \times b$ is a _____________
2. If a = 3, b = 5 and c = 2 then find:
   a) $5a$ b) $3a\,\frac{-8b}{2c}$ c) $5a + 2b - 6c$ d) $3a\,(2b - 6c + b^2)$

Topic: What are polynomials
Time: 40 minutes (1 period)

Objectives

To enable students to define:

- a polynomial, kinds of polynomial
- degree of a polynomial
- degree of polynomial

Recognize polynomial in one, two or more variable with various degrees.

Starter activity

Write an algebraic expression on the board and ask the following questions.

$a^3 - 5x + 7$

1. What is the degree of 'a'?
2. What is the degree or power of 'x'?
3. What is the coefficient of x?
4. What is the coefficient of a?

Main lesson

After getting the answers to the above questions, define a polynomial.
A polynomial is an algebraic expression where coefficients are real numbers and exponents are non-negative integers.
For example: $a^3 - 5x + 7$ is a polynomial as its coefficients 1, −5, and 7 are real numbers and exponents (also called powers or degree) are non-negative integers.
Explain the degree of a polynomial by the following examples.
Polynomial in one variable:

$9a + 8$	degree = 1
$4x^2 - 3x + 1$	degree = 2
$5a^6 - 3b^2 + 1$	degree = 3

The greatest power of the variable = degree of the polynomial.
Polynomials in two or more variables will be explained by giving the following examples.

$4a + 2b^2 + ab$	degree = 2 in a and b
$x^2y^2 - 4xy + 3xy^2$	degree = 4 in x and y
$4p^3q^4 + 3pq^2 + 4q^6$	degree = 7 in p and q

The degree of any term is the sum of powers of all variables in that term.

Example 1

$14x^6y^4 = 6 + 4 = 10$
Explain with the help of an example that the degree of the polynomial is the greatest sum of the powers.

Example 2

$2x^5y^4 - 4x^3y^3 + y^8\,x^2$ the degree = 10
(5+4) (3+3) (2+2)

Explain that a linear polynomial has a degree equal to 1.

Example 3

$8a + x$, $9x + 10$
A polynomial with a degree of 2 is termed a Quadratic polynomial.
A polynomial with a degree of 3 is termed a Cubic polynomial.

Individual activity

Exercise 5a questions 2, 3, 4 and 5 will be done in the class. Help the students solve these.

Homework

Students will be asked to revise the work done in the class.

Topic: Operations on polynomial, addition and subtraction
Time: 2 periods

Objective:

To be enable students to: add and subtract polynomials.

Starter activity

A few questions will be written on the board and each students will be called to solve. Rest of the students with be observing the work being done. This will help students to find mistakes and give solution or help the other students.

Example

Find the sum of $3x$ and $4y$

A worksheet will be given to each student to solve.

Find the sum of the following.

a) $3a$ and $4b$

b) $2a^2$, $3a^3$ and $4a$

c) $7a - 2ab$

d) $6x$, $-2xy$, $-11x + xy$

e) $9abc$, $2cba$, and bca

f) $8x$, $2y^2$, $7x^2$, $-5y^2$, $-2x$

Collect the worksheet and point out the mistakes of the students.

Main lesson

After pointing out the mistakes if any, in the worksheet, start explaining the topic.

Example 1

The sum of $3x$ and $4y$ will be $3x + 4y$ because they have a different base.

We can only add or subtract the polynomial when they have the same or a common base.

$3xy - 8xy$ (xy are common)

$3xy + (-8xy)$ or $(-8 + 3)xy = -5xy$

Explain and highlight the following.

When subtracting a polynomial from the other, always change the signs of the expression which is to be subtracted.

For example subtract 3a from 4b

$4b - (3a)$

$= 4b - 3a$ (base are different)

Example 2

Subtract $3a - 4b + c$ from $8a - 9b + 4c$

$(8a - 9b + 4c) - (3a - 4b + c)$

Step 1: Change the sign inside the bracket.

$8a - 9b + 4c - 3a + 4b - c$

Step 2: Collect the like terms.

$8a - 3a - 9b + 4b + 4c - c$

$= 5a - 5b + 3c$

Example 3

Find the difference between:
$3a^2b^2$ and $5a^2b^2$

difference has no sign

Therefore,
$3a^2b^2 - 5a^2b^2 = -2a^2b^2$ or $5a^2b^2 - 3a^2b^2 = 2a^2b^2$

Practice session

Write some sums on the board. Call the students turn by turn to solve the given questions on the board. Ask the rest of the class to carefully observe the solutions.

Individual activity

Give Exercise 5a questions 6 and 7 to be done in the class.

Homework

Add: $4pq - 7qr + 3rp$, $8qr - qr + 8\ ps - 8$,
$-6qr + 11rp - 2\ pq$

Subtract: $6xy - 2yz + 12$ from $7x^2y^2 - 8yz$

Topic: Multiplication and Division of Polynomials
Time: 2 periods

Objective:

To be able to multiply and divide a polynomial.

Starter activity

As the students have learnt these topics in the previous class, a worksheet maybe given to test their knowledge.

1. Cost of a book is Rs $4b - 8x$ find the cost of 3 books.
2. Write down the product of a^2 and a.
3. Write down the continued product of $b^2 \times b^2 \times b^2$.
4. Solve these.
   a) $3a^2 \times a^2 =$
   b) $7a \times 5b \times 3c =$
   c) $8a^3 \div 2a^2 =$

d) $5a^2 \times 3a^2 \times 2a^7 =$
e) $7a \times 2x \times 3xy =$
f) $18x^3y^2 \div 3xy =$

Write the correct answers on the board and ask the students to interchange their worksheets and check the answers of their peers and point out the mistakes.

Main lesson

With the help of examples, explain multiplication and division.

Example 1

Multiply $6a^2 - 4b^3$ by $7ab$

- Multiply the numeral coefficients.
- Multiply the literal coefficients.
- Add the powers.

$7ab(6a^2 - 4b^3)$
$= 6 \times 7 \times a.a^2.b - 7 \times 4 \times a.b.b^3$ (dot represents the sign of multiplication)
$= 42\ a^{1+2}b - 28\ ab^{3+1}$
$= 42a^3b - 28ab^4$

For multiplication: $- \times - = +$ $+ \times + = +$ $- \times + = -$

Example 2

$(y - 2z)(y^2 + 4yz - z^2)$
$y\ (y^2 + 4yz - z^2) \quad = y^3 + 4y^2z - yz^2$
$-2z\ (y^2 + 4\ yz - z^2) = -2y^2z - 8yz^2 + 2z^3$
$= ye + 2y^2z - 9yz^2 + 2z^3$

Simplify the like terms and write the sign of the greater value.
Note: do not add the powers while adding or subtracting.

Topic: Division of polynomials
Time: 2 periods (40 min)

Objective:

To enable students to do division of polynomials by Linear polynomial.

Starter activity

Students have already done division on polynomials in the previous class. Solve these sums with student participation on the board.

1. Divide $9a^2$ by $3a$
2. Divide $48a^2b^2$ by $12ab$
3. 72 a^3b^2c by $6ab^2c$
4. $9a^5 - 12a^2$ by $3a^2$
5. $18x^3y^2 - 27x^5y^3$ by $3x^2y^2$

Main lesson

Explain the method of division of polynomials with the help of the examples on the board.

Example 1

Divide $-11x + 2x^2 + 12$ by $x - 4$

Method

Arrange the terms in a descending order. (from greater to smaller powers of their variables leaving spaces for the missing terms)

$-11x + 2x^2 + 12$

As it not arranged in descending order, first rearrange the expression.

$2x^2 - 11x + 12$

This expression cannot be solved by the short division method.

$$
\begin{array}{r|l}
 & 2x - 3 \\
\hline
x-4 & \cancel{2x^2} - 11x + 12 \\
 & \cancel{2x^2} - 8x \\
 & - \quad + \\
\hline
 & \qquad -\cancel{3x} + \cancel{12} \\
 & \qquad -\cancel{3x} + \cancel{12} \\
 & \qquad + \qquad - \\
\hline
\end{array}
$$

Subtract, change the sign

Divide the first term of dividend by the first term of the divisor.

$\frac{2x^{2-1}}{x} = 2x$

quotient = $2x$

Multiply $x - 4$ by $2x$ $\quad 2x(x-4) = 2x^2 - 8x$

divide $-3x$ by $x - 3$ $\quad \frac{-3x}{x} = -3q$

Multiply $x - 4$ by $- 3$ $\quad -3(x - 4) = -3x+12$

Example 2

Divide $x^3 + 8$ by $x + 2$

$$
\begin{array}{r|rrrrrrr}
 & x^2 & - & 2x & + & 4 & & \\
\hline
x+2 & \cancel{x^3} & + & 00 & + & 00 & + & 8 \\
 & \cancel{x^3} & + & 2x^2 & & & & \\
 & - & & - & & & & \\
\hline
 & & - & \cancel{2x^2} & + & 00 & & \\
 & & - & \cancel{2x^2} & - & 4x & & \\
 & & + & & + & & & \\
\hline
 & & & & + & \cancel{4x} & + & \cancel{8} \\
 & & & & & \cancel{4x} & + & \cancel{8} \\
 & & & & - & & - & \\
\hline
\end{array}
$$

Quotient $= x^2 - 2x + 4$

(00 for missing terms)

$\dfrac{x^3}{x} = x^2$

$x^2\,(x + 2) = x^3 + 2x^2$

$\dfrac{-2x^{2-1}}{x} = -2x$

$-2x(x + 2) = -2x^2 - 4x$

$\dfrac{4x}{x} = 4$

$4(x + 2) = 4x + 8$

Individual activity

Exercise 5a questions 4 and 5 will be done in the class. Help the students with the questions.

Homework

Give Exercise 5a, questions 13e and 13f and 14c and 14d as homework.

Recapitulation

Review the unit and explain again where students are unclear on any concept.

UNIT 6

ALGEBRA

Topic: Algebraic formula
Time: 2 periods

Objective:

To enable students to solve $(a + b)^2$ and $(a - b)^2$ through formula.

Starter activity

The students have already learnt to find the square of an algebraic expression through formulae previously. Call a few to the board to solve the following.
Find the squares of the following.

a) $(a + b)^2$ b) $(a - b)^2$ c) $(3x - 4y)^2$
d) $(5m + 6n)^2$ e) $(204)^2$ f) $98)^2$

Help the students in recalling the steps to solve these.

Main Lesson

Explain to find the perfect square of a given expression with the help of the formula (without actual multiplication).
Establishing the formula:

$$
\begin{aligned}
(a + b)^2 &= (a + b)(a + b) \\
&= a(a + b) + b(a + b) \\
&= a^2 + \underbrace{ab + ab} + b^2 \\
&= a^2 + 2ab + b^2
\end{aligned}
$$

square of 1st term   square of 2nd term

two times the product of the 1st and 2nd terms

$(a - b)^2 = (a - b)\ (a - b)$

$(a - b)\ (a - b)$

$a(a - b) - b\ (a - b)$

$a^2 - \underbrace{ab - ab} + 2$

$a^2 - 2ab + b^2$

$(a + b)^2 = a^2 + 2ab + b^2$

$(a - b)^2 = a^2 - 2ab + b^2$

It is clear that the formula on R.H.S is the sum of three terms.
Explain the above in words as, "square of the sum of two terms is always equal to the square of the first term plus twice the product of first and second terms plus the square of the second term."

Example 1

Find the square of $2a - 3b$

$$\begin{aligned}(2a - 3b)^2 &= (2a - 3b)(2a - 3b)\\ &= a^2 - 2ab + b^2\\ &= (2a)^2 - 2(2a)(3b) + (3b)^2\\ &= 4a^2 - 12ab + 9b^2\end{aligned}$$

Example 2

$$\begin{aligned}(5x^2 + 6y^2)^2 &= (5x^2 + 6y^2)\ (5x^2 + 6y^2)\\ &= (5x^2)^2 + 2(5x^2)\ (6y^2) + 6y^2)^2\\ &= 25x^4 + 60x^2y^2 + 36y^4\end{aligned}$$

From the above examples, we can see that it is easy to find the product of expressions by using formulae.

Example 3

Find the value of $(204)^2$

$$\begin{aligned}(204)^2 &= (200 + 4)^2 \qquad \text{(split the number into two parts)}\\ & a^2 + 2ab + b^2\\ &= (200)^2 + 2(200)(4) + (4)^2\\ &= 40000 + 1600 + 16\\ &= 41616\end{aligned}$$

Example 4

Find the value of $(198)^2$

198 is nearest to 200 (split in two terms)

$(200 - 2)^2 = (198)^2$

$(200 - 2)^2$

$(200)^2 - 2(200)(2) + (2)^2$
$40000 - 800 + 4$
$= 39204$

Example 5

Find the square of 19.9

$(19.9)^2 = (20 - 9.1)^2$
$= (20)^2 - 2(20)(0.1) + (.1)^2$
$= 400 - 4.0 + .01$
$= 396.01$

Individual activity

Exercise 6a, question 1 will be given to be solved in the class.

Homework

Evaluate:

a) $(301)^2$ b) $194)^2$ c) $(997)^2$ d) $(502)^2$

Recapitulation

1. What is the formula for finding the square of a number?
2. Split 56 into two terms.
3. Split 999 into the two terms.

Topic: Algebraic Formulae
Time: 40 mins

Objective

To enable students to: understand and solve the problems with the help of the formulae.

Starter activity

Students have learnt earlier, the following formula.

i.e. $(x + y)(x - y) = x^2 - y^2$

Write a few sentences on the board to check their previous knowledge.

Find the product of the following through formulae.

1. $(a + b)(a - b) =$
2. $(2x + 3y)(2x - 3y) =$
3. $(p^2 + q^2)(p^2 - q^2) =$
4. $(a^2b^2 - c^2)(a^2b^2 + c^2) =$

Main lesson

Explain in detail, the formula on the board.

$(x + y)(x - y) = x^2 - y^2$

$x(x + y) - y\,(x + y)$

$= x^2 + \cancel{xy} - \cancel{xy} - y^2$

$= x^2 - y^2$

(product of sum and difference of two terms)

Example 2

$(3x + 4y)(3x - 4y)$

$(3x)^2 - (4y)^2$

$9x^2 - 16y^2$

Hence, the product of sum and difference of any two numbers is equal to the difference of their squares.

Example 3

Find the product of 43 and 37 with the help of the formula.

$43 = 40 + 3$ Split into two terms, the first and second

$37 = 40 - 3$ terms of both the numbers should be the same.

43×37

$= (40 + 3)(40 - 3)$

$= (40)^2 - (3)^2$

$= 1600 - 9$

$= 159.1$

Individual activity

Give Exercise 6a question. 1 (i to m) for practice in the class.

Homework

Expand each of the following by using algebraic formulae.

a) $(2p^2 - 3q^2)\,(2p^2 + 3q^2)$
b) $(a^2b^2 + c^2d^2)(a^2b^2 - c^2d^2)$
c) $(25)(15)$
d) $(32)(28)$

Topic: Algebraic formula deduction

Time: 2 periods

Objective

To enable students to solve various problems with the help of formulae.

Starter activity

Deduction will be explained to the students by the following examples.

$(x + \frac{1}{x})^2 = (x - \frac{1}{x})^2 + 4$

$(x - \frac{1}{x})^2 = (x + \frac{1}{x})^2 - 4$

Explanation

$(x - \frac{1}{x})^2 = x^2 - 2 + \frac{1}{x^2}$

$= x^2 + \frac{1}{x^2} - 2$

$(x + \frac{1}{x})^2 = x^2 + 2 + \frac{1}{x^2}$ or $x^2 + \frac{1}{x^2} + 2$

$(x^2 + \frac{1}{x^2} + 2) -2 -2$

$= (x + \frac{1}{2})^2 = -4$

If $x + \frac{1}{x} = 4$, find $x - \frac{1}{x}$

$(x - \frac{1}{x})^2 = (x + \frac{1}{x})^2 -4$

$= (4)^2 - 4$

$(x - \frac{1}{x})^2 = 16 - 4$

$(x - \frac{1}{x})^2 = 12$

$(x - \frac{1}{x}) = \sqrt{12}$

Example 2

If $x + \frac{1}{x} = 4$ find $x^2 + \frac{1}{x^2}$

$= x^2 + \frac{1}{x^2} = (x^2 + \frac{1}{x^2} + 2)-2$

$= (x + \frac{1}{x})^2 -2$

$= (4)^2 -2$

$= 16 - 2$

$= 14$

Example 3

Find $x^2 - \frac{1}{x^2}$ when $x + \frac{1}{x} = 4$

$x^2 - \frac{1}{x^2} = (x + \frac{1}{x})(x - \frac{1}{x})$

$= 4\sqrt{12}$

Practice session

Call the students to the board one by one to do the first five questions of Exercise 6a question 3.

Individual activity

Ask the students to do Exercise 6a, question 3 in their exercise books.

Homework

Exercise 6a, question 4 as homework.

Topic: Factorization
Time: 2 periods

Objectives

To enable students to factorize different types of algebraic expressions and apply them to solve different problems.

Starter activity

A worksheet will be given to the students to find the factors.

Worksheet

What are the common factors of the following numbers and expressions.

1. $a - 5a$	6. $7a - 9b + 6c$
2. $3a - 42b + 6c$	7. $48abc - 40bc$
3. 72 and 60	8. $ax + xy - xz$
4. $4xy + 6x - 20y$	9. $x - 4x + 3x$
5. $ab - ac + ad$	10. $32pq - 4pq + 8$

Write the correct answers on the board so that students can check their solutions.

Main lesson

Explain that factorization is the process in which we write an algebraic expression as a product of two or more factor.

Example 1

$ac + ad - ae$ (There are three terms in this expression and first and the last terms are not squares.)

What is common in all?
'a' is a common factor.
Divide all terms by 'a'.
$ac/a + ad/a - ae/a$
$= a(c + d - e)$

Example 2

$a^5 + a^3 + a^2$ (Find the lowest power.)
It means a^2 is common factor.
Divide all the terms by a^2
$\frac{a^{5-2}}{a^2} + \frac{a^{3-2}}{a^2} + \frac{a^{2-2}}{a^2}$ (when we divide we subtract powers)
$= a^2(a^3 + a + 1)$
a^2 is the common factor

Example 3

Factorize $ab + ac + yb + yc$

There are 4 terms given in this expression.

Is there any common factor, no.

Divide them into two groups
$(ab + ac) + (yb + yc)$
a is common: y is common
$a(b + c) + y(b + c)$
$(b + c)$ is common
$(a + y)(b + c)$
Therefore, $(a + y)$ and $(b + c)$ are the factor of the product $ab + ac + yb + yc$

Example 4

Factorize $a^2 + 6a + 9$
$a^2 + 6a + 9$
In this expression, the first and the last terms are perfect squares. This expression is a perfect square.
$\sqrt{a^2} = a$ and $\sqrt{9} = d$
Applying the formula $a^2 + 2ab + b^2$
$(a)^2 + 2(a)(3) + (3)^2$
$= (a + 3)^2$
$(a + 3)(a + 3)$ gives $a^2 + 6a + 9$

Example 5

$c^2 - 8c + 16$ (First and last terms are perfect squares.)

Apply formula $a^2 - 2ab + b^2$	$\sqrt{c^2} = c$
$= (c)^2 - 2(c)(4) + (4)^2$	$\sqrt{16} = 4$
$= (c - 4)^2$	

Example 6

Factorize $a^2 - 9$ (Here both the terms are perfect squares.)

Apply formula $(a + b)(a - b)$	$\sqrt{a^2} = a$
$= (a)^2 - (3)^2$	$\sqrt{9} = 3$
$= (a + 3)(a - 3)$	$-9 = -3 \times +3$

Individual activity

Ask the students to do Exercise 6b in their exercise books. They can work in pairs to help each other.

Homework

Factorize the following:

1. $6a^2 - 2ab + 3ac$
2. $16a^2 - 16ab + 4b^2$
3. $25a^2b^2 - 10ab + 1$
4. $25y^2 - 81z^2$
5. $ac + ad + bc + bd$
6. $49y^4 - 121z^4$

Topic: Expansion of cubes in binomials
Time: 2 periods

Objectives

To enable students to:

- recognize formula such as $(a + b)^3$ and $(a - b)^3$
- apply them to solve different problems

Starter activity

Following questions will be asked.

1.	What is the square of $5a$	$(5a)^2 = 25a^2$
2.	What is the sum of the squares of $3a$ and $5b$?	$(3a)^2 + (5b)^2 = 9a^2 + 25b^2$
3.	What is the product of a^2 and a?	$a^2 \text{ x } a = a^{2+1} = a^3$
4.	What is the continued product of a.a.a?	$a \text{ x } a \text{ x } a = a^3$
5.	What do you read a^3 as?	It is a^3
6.	Write down the cubes of 2, 3, 4, 5.	$2^3 = 8, 3^3 = 27, 4^3 = 64, 5^3 = 125$

Find the product or expand $(a + b)^3$.
$= \{(a + b)\ (a + b)\}\ (a + b)$
$\quad (a + b)^2$
$= \{a^2 + 2ab + b^2\}\ (a + b)$
$= a(a^2 + 2ab + b^2) + b(a^2 + 2ab + b^2)$
$= a^3 = 2a^2b = ab^2 + a^2b + 2ab^2 + b^3$
$= a^3 + 2a^2b + a^2b + ab^2 + 2ab^2 + b^3$
$= a^3 + 3a^2b + 3ab^2 + b^3$
Therefore, $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
i.e. cube of the sum of the binomial.

Main lesson

We have derived the formula $(a + b)^3$ by actual multiplication. We can now apply it to find the cube of any algebraic expression.

For $(a - b)^3$, through the actual multiplication we get:
$(a - b)^3 = \{(a - b)\ (a - b)\}\ (a - b)$
$= a^3 - 3a^2b + 3ab^2 - b^3$

Example 1

Expand $\quad (2x + 3y)^2$
$\quad (2x + 3y)^3 = (2x + 3y)\ (2x + 3y)\ (2x + 3y)$
Formula for $\quad (a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
$\quad = (2x)^3 + 3(2x)^2\ (3y) + 3(2x)(3y)^2 + (3y)^3$
$\quad = 8x^3 + 36x^2y + 54xy^2 + 27y^3$

Example 2

Expand $\quad (2m - 4n)^3 = (2m - 4n)\ (2m - 4n)\ (2m - 4n)$
$\quad = (2m)^3 - 3(3m)^2\ (4n) + 3(2m)\ (4n)^2 - (4n)^3$
$\quad = 8m^3 - 48m^2n + 96mn^2 - 64n^3$

Example 3

Expand $\quad (\frac{2}{a} + \frac{y}{2})^3$
$\quad = (\frac{2}{a})^3 + 3(\frac{2}{a})^2\ (\frac{y}{a}) + 3\ (\frac{2}{a})\ (\frac{y}{a})^2 + (\frac{y}{2})^3$
$\quad = \frac{8}{a^3} + 3(\frac{4}{a^2})\ (\frac{y}{f}) + 3(\frac{2}{a})\ (\frac{y^2}{4}) + \frac{y^3}{8}$
$\quad = \frac{8}{a^3} + \frac{4y}{a^2} + \frac{3y^2}{2a} + \frac{y^3}{8}$

We can write this as $= \frac{8}{a^3} + \frac{y}{8} + \frac{3y}{a}\ (\frac{2}{a} + \frac{y}{2})$ as $\frac{3y}{a}$ is a common factor of $\frac{6y}{a^2}$ and $\frac{3y^2}{2a}$

Individual activity

Exercise 6c will be done in the class. Students can work in pairs to help each other.

Homework

Expand the following:

a) $(6a - 8)^3$ b) $(7a - 5b)^3$ c) $(2x + 7y)^3$

Topic: Simultaneous equation
Time: 2 periods

Objectives

To enable students to:

- find the value of two unknowns in a problem
- recognize simultaneous linear equations in one and two variables.

Starter activity

Students of class VIII have some knowledge of linear equations in one variable. Write a few questions on the board and let students solve these one by one. The rest of the class will observe the solutions and point out the mistakes if any.

Some sample linear equations are given below:

1. $3x + 2 = 15$
2. $8x = 40$
3. $\frac{2x}{3} + 8 = 24$
4. $7x - 5 = 4x - 10$
5. $\frac{5x}{8} - 3 = \frac{2}{8x} + 12$

Main lesson

Explain to the students that if in an equation of two variables, the greatest degree of the variables is one, then the equation is called a linear equation in two variable

Example

$8x + y = 40$ is a linear equation in two variables, x and y.

Ordered pairs

Explain that if a value for x in an equation is given, a corresponding value of y can be found out.

Example

If x = 2 then the corresponding value of y would be:

$2x + y = 10$

$2 x 2 + y = 10$

$4 + y = 10$

$y = 10 - 4$

$y = 6$

Thus, $(x, y) = (2, 6)$ is an ordered pair that satisfies the equation.

Two equations necessary for two unknowns

A set of two equal or more equations is known as simultaneous equation.

Example

The following are simultaneous equations.

$2x + y = 10$

$3x - 2y = 15$

There are three methods of solving these equations.

1. Method of substitution
2. Method of equal co-efficient (elimination)
3. Method of comparison

Method of substitution

Example

Solve $x + y = 4$ (i)

$x - y = 2$ (ii)

Solution

From (i) $x = 4 - y$..... (iii)

Substituting the value of x in (ii)

$x - y = 2$

$(4 - y) - y = 2$

$= \quad 4 - y - y = 2$

$= \quad y - 2y = 2$

$= \quad -2y = 2 - 4$

$= \quad -2y = -2$

$= \quad y = -\frac{2}{2}$

$y = 1$

By substituting the value of y in equation (iii)

$x = 4 - y$

$= x = 4 - 1$

$x = 3$

Therefore, {3, 1} is the solution set.

Verification

$x + y = 4 = 3 + 1 = 4$

$x - 2 = 2 = 3 - 1 = 2$

Method of equal coefficient or elimination

In this method, the coefficient of one of the variables should be the same so that it can be eliminated.

Example

$$\begin{array}{llll} 2x + \cancel{3y} & = & 14 & \text{(i)} \\ 2x - \cancel{3y} & = & 2 & \text{(ii)} \\ \hline 4x & = & 16 & \end{array}$$

'y' has the same coefficient with + and – sign, y can be liminated

$4x = 16$

$x = \frac{\cancel{16}^{4}}{\cancel{4}} = 4$

$x = 4$

Putting the value of x in equation 1 we have

$2x + 3y = 14$

$2(4) + 3y = 14$

$8 + 3y = 14$

$3y = 14 - 8$

$3y = 6$

$y = \frac{6}{3}$

$y = 2$

Hence the solution set is {4, 2}

Verification

$2x + 3y = 14 = 2 \times 4 + 3 \times 2 = 14$ (i)

$2x - 3y = 2 \qquad 8 + 6 = 14$

$(2 \times 4) - (3 \times 2) = 2$

$8 - 6 = 2$

Method of comparison

If the coefficients of the variables are not equal

Example

$2x - 3y = 5$ i)

$3x - 4y = 6$ ii)

Multiplying equation (i) by 4 and multiplying equation (ii) by 3

i) $4(2x - 3y = 5) = 8x - 12y = 20$ (iii)

ii) $3(3x - 4y = 6) = 9x - 12y = 18$ (iv)

$$\begin{array}{lcll} 8x + \cancel{12y} & = & 20 & \text{(iii)} \\ 9x - \cancel{12y} & = & 18 & \text{(iv)} \\ - \quad + & & - & \\ \hline -x & = & 2 & \end{array}$$

y has the same coefficient in (iii) and (iv) but to eliminate we have to change the sign.

$x = \frac{2}{-1}$

$x = -2$

Substituting the value of x in equation (i) we have,

$2x - 3y = 5$

$2(-2) - 3y = 5$

$-4 - 3y = 5$

$-3y = 5 + 4$

$-3y = 9$

$y = \frac{9}{-3}$

$y = -3$

Hence the solution set is {−2, −3}

Practice session

Students will be called turn by turn to do the given questions on the board.

a) $x - y = 5$, $x + y = 7$

b) $3x + y = -4$, $2x - 7y = 5$

c) $x + y = 13$, $3x - 5y = 7$

All parts of Exercise 6d, question 1 will be done on the board by students.

Individual work

Ex 6d, question 2 will be done individually by the students.

Homework

Solve by any method.

a) $x - 6y = 33$, $7x + 4y = 1$

b) $3a + b = 7$, $2a - b = 3$

c) $4x + 3y = 41$, $3x - 4y = 12$

Topic: Real-life problems involving simultaneous equations
Time: 2 periods

Objective

To enable students to solve real-life problems.

Example

The sum of two numbers is 84. If their difference is 12, find the numbers.

Solution

Let one number be x and the other be y.
Since their sum is 84 ($x + y = 84$), the difference is 12 ($x - y = 12$)

$$\begin{array}{l} x + \cancel{y} = 84 \\ x - \cancel{y} = 12 \\ \hline 2x \quad\;\; = 96 \end{array}$$

$x = \frac{\cancel{96}^{48}}{\cancel{2}}$

$x = 48$

$x + y = 84$

$48 + y = 84$

$y = 84 - 48$

$y = 36$

The two numbers are 48 and 36.

Verification

$48 + 36 = 84$

$48 - 36 = 12$

Example 1 of sub-section 6.6 from the textbook will also be explained to the students.

Individual work

Exercise 6d, question 3 to 5 will be done in the class. Help the students in solving the problems.

Homework

Following questions will be given as homework.

1. Four times the sum of two numbers is 72 and their difference is 8.
   Find the numbers?
2. The length of a rectangle is 7 cm more than its breadth. If the perimeter is 74 cm, find the length and breadth of the rectangle.

UNIT 7

FUNDAMENTALS OF GEOMETRY

Topic: Fundamentals of geometry
Time: 2 periods

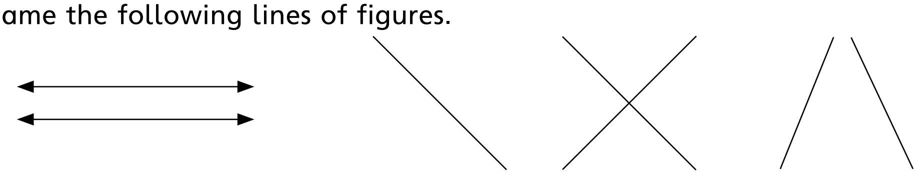
Objectives

To enable students to:

- recognize alternate angles, corresponding angles, vertical angles
- find unknown angles.

Starter activities

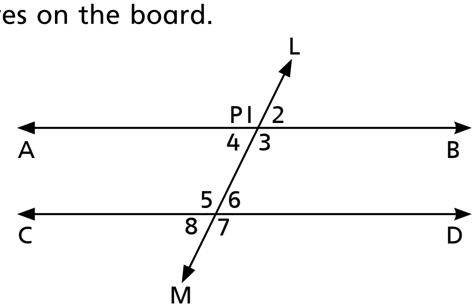
1. Name the following lines of figures.



2. Define a line, line segment, parallel lines and interesting lines.

Main lesson

Students have the concept of parallel lines. Explain the properties of parallel lines by drawing figures on the board.



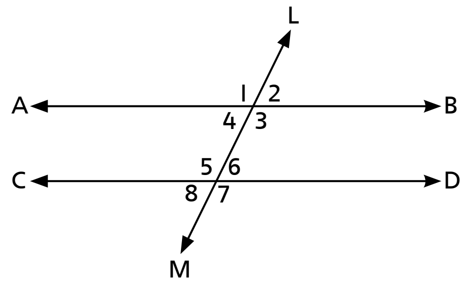
The line which intersects two parallel lines is called a Transversal.

Note that the two intersecting lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{LM}$ cannot be parallel to the third line $\overleftrightarrow{CD}$.

Properties of parallel lines

Explain to the students that if two coplanar lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are intersected by a transversal $\overleftrightarrow{LM}$ at P and Q respectively then four angles are formed at P and Q.

$\angle 1$, $\angle 2$, $\angle 7$, $\angle 8$ are exterior angles.
$\angle 3$, $\angle 4$, $\angle 5$, $\angle 6$ are interior angles.



Alternate angles

Consider $\angle 4$ and $\angle 6$.

1. $\angle 4$ and $\angle 6$ are interior angles.
2. Both $\angle 4$ and $\angle 6$ are on the opposite sides of the transversal LM.
3. They are not adjacent.

They are called alternate angles and are equal. in the figure shown, $\angle 3$ and $\angle 5$ are also alternate angles.

Corresponding angles

Now let us consider $\angle 2$ and $\angle 6$.

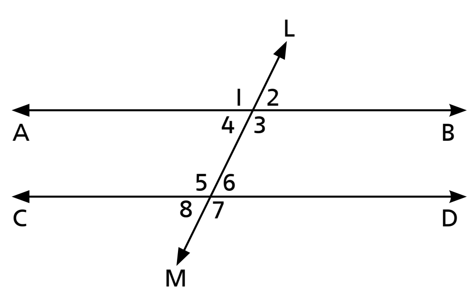
1. They are on the same side of the transversal.
2. $\angle 2$ is an exterior angle and $\angle 6$ is interior angle.
3. They are not adjacent.
   They are called corresponding angles. They are equal.
   ($\angle 2$, $\angle 6$), ($\angle 3$, $\angle 7$), ($\angle 1$, $\angle 5$), ($\angle 4$, $\angle 8$) are pairs of corresponding angles.
   We say that if $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$
   $\angle 4 = \angle 6$ and $\angle 3 = \angle 5$

Alternate angles are congruent.
In the same way $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$
$\angle 2 = \angle 6$, $\angle 3 = \angle 7$, $\angle 1 = \angle 5$, $\angle 4 = \angle 8$
Corresponding angles are congruent.

Important proposition about parallel lines:
If a transversal intersects two parallel lines, a pair of interior angles on the same side of the transversal is supplementary.

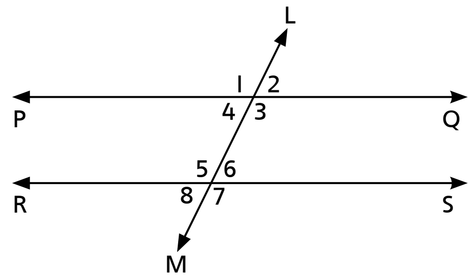
That is if $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ then $\angle 3$ and $\angle 6$ and $\angle 4$ and $\angle 5$ are supplementary angles.

$\angle 3$ and $\angle 6$ are called Allied angles. Similarly, $\angle 4$ and $\angle 5$ are also Allied angles.



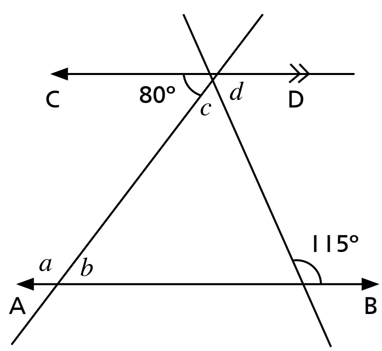
$\angle 1$ and $\angle 3$, $\angle 2$, $\angle 4$, $\angle 5$, $\angle 7$, $\angle 6$ and $\angle 8$ are the pairs of vertically opposite angles.
Vertically opposite angles are equal.

Practice session



1. If $\angle 1 = 110°$, find $\angle 3$, $\angle 6$, and $\angle 5$.
   $\angle 3 = \angle 1 = 110°$ vertically opposite angles are equal
   $\angle 6 = 180 - 110° = 70$ sum of interior angles are supplementary
   $\angle 6 = 70°$
   $\angle 5 = \angle 3 = 110°$ alternate angles
2. Write down the pairs of:
   a) corresponding angles
   b) alternate angles

Explain a few figures from Exercise 7a, so that the students can solve the rest.



$\angle b = 80°$ (alternate angles are congruent)
$\angle a = 180 - b$
$180 - 80 = 100$
$\angle a = 100°$
$\angle 80 + c = 115°$ (alternate angles 'Z' shaped)
$c = 115 - 80$
$c = 35°$
$80 + \angle c + \angle d = 180°$ (sum of the $\angle$s on a straight line = 180°)
$80 + 35 + d = 180°$
$115 + d = 180$

d = 180 − 115
d = 65
or 115 + d = 180 (sum of interior ∠s = 180°
d = 180 − 115
d = 65°

Individual activity

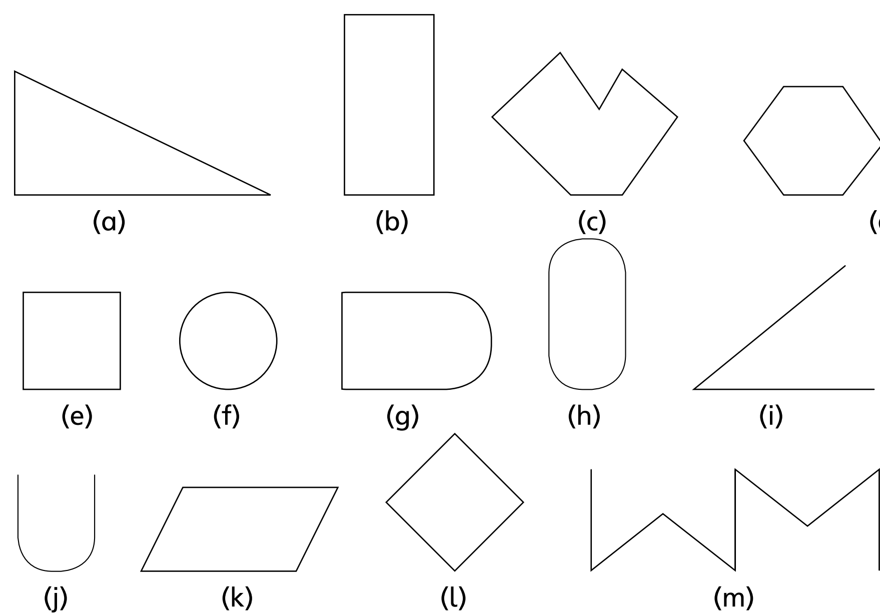
Help the students with Exercise 7a.

Topic: Polygons
Time: 2 periods

Objective: to enable students to define types of polygons

Starter activity

Draw some figures on the board and ask the students to name them.



After naming them, ask the students to write the number of line segments each of the above figure has.

1. Are all the figures made up of line segments?
2. How many vertices are there in figures c and d?
3. Does figure 'g' has 4 vertices?
4. How many vertices do a circle has?

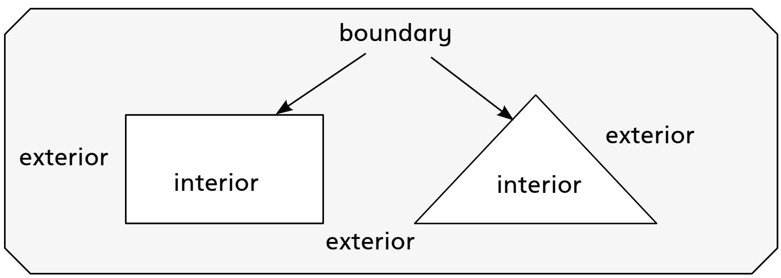
Main lesson

What is a polygon?

Let the students observe the figures drawn on the board. Some of them are bounded by three or more line segments while some are not.

A simple closed figure formed by three or more line segments is known as a polygon. In a closed figure, the beginning and ending points are the same and there are no intersection points.

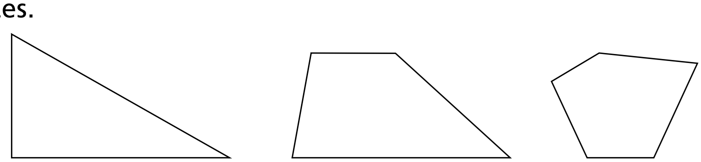
A simple closed figure divides the plane into two regions, the interior and exterior. The border of the shape is the boundary between these regions.



Explain that polygons are classified according to the number of sides they have.

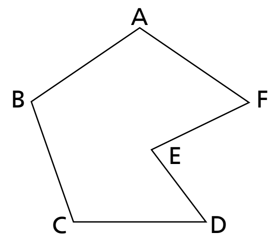
Number of sides	Name of the Polygon
3	triangle
4	quadrilateral
5	pentagon
6	hexagon
7	heptagon
8	octagon
9	nonagon
10	decagon

- A polygon has sides, vertices, adjacent sides and diagonals. The angular points of a polygon are called its vertices.
- The number of sides of a polygon is equal to the number of its vertices.
- A line which joins any two non-adjacent vertices is called a diagonal.
- A polygon is said to be convex when all its interior angles are less than two right angles.



1 2 3

Figures 1, 2, 3 are convex polygons.
If a polygon has one or more of its interior angles greater than two right angles then it is said to be a concave polygon for example.



The measure of interior angle DEF is greater than 180°.

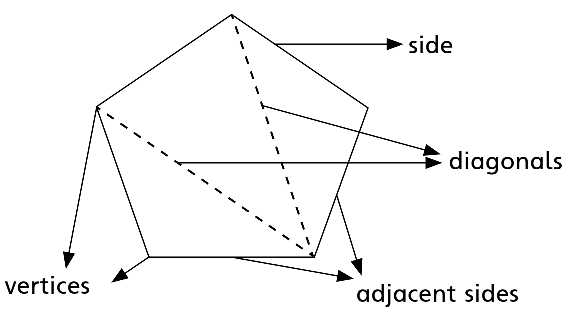
Regular polygon

If all the sides of a polygon and its angles are equal then it is called a regular polygon.

Examples are rectangle, square, equilateral triangle, pentagon, hexagon etc.

Rectangle is an equiangular polygon. Square is an equiangular as well as an equilateral polygon

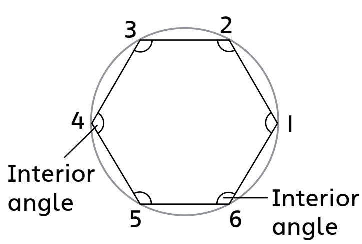
Draw a pentagon, a hexagon and an octagon on the board and ask the students to join its diagonals to the opposite vertices and count the number of triangles formed in each.



The two diagonals form 3 triangles.
Sum of each triangle = 80,3Δ = $180 \times 3 = 540°$
Interior angles add upto 540°
Each angle of the pentagon = $\frac{540}{5} = 108°$

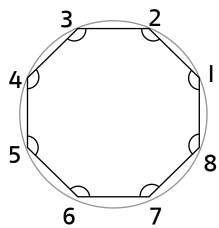
All 5 sides and 5 interior angles of a pentagon are equal.

Hexagon (six-sided polygon)



Three diagonals divide the hexagon into four triangles.
∴ the interior angles will be equal to $180 \times 4 = 720°$
Each angle of a regular hexagon will be $\frac{\cancel{720}^{120}}{\cancel{6}_{1}} = 120°$

Octagon (eight-sided polygon)



All eight sides and eight interior angles are equal.
Here the interior angles add up to 6 triangles.
$\therefore\ = 180 \times 6 = 1080$
$\therefore$ Each angle $= \dfrac{\cancel{1080}^{135}}{\cancel{8}_{1}} = 135^\circ$

Each regular polygon can fit into a circle i.e., a circle can be drawn passing through each vertex of a polygon.

Individual activity

Ask the students to draw a pentagon, hexagon and octagon on different coloured papers and then cut and paste in their exercise books and write the properties of each.

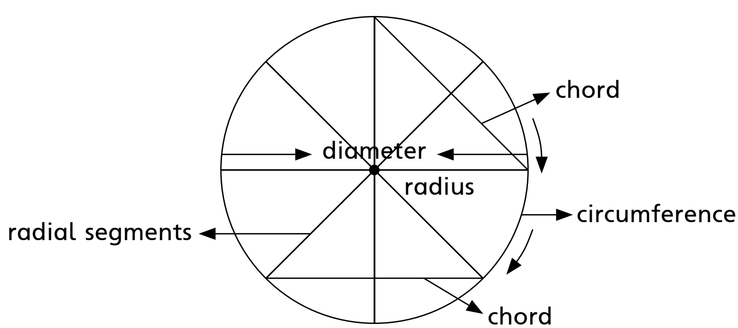
Topic: Circle
Time: one period

Objective:

to enable students to describe terms such as sector, secant, chord of a circle, cyclic points, tangent to a circle and concentric circles

Starter activity

Students have learnt about the circle previously. Ask them to draw a circle of radius 4 cm and show its diameter, radius, chord, radial segment, and circumference.



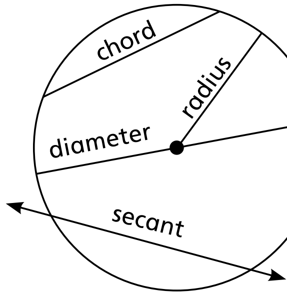
Fill in the blanks:
1. A __________ divides the circle into parts.
2. __________ the circle is called semicircle.

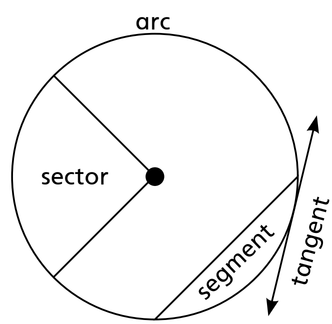
3. Outline of the circle is called __________.
4. Part of a circumference is called __________.
5. __________ is a line segment joining the two points of a circle.
6. Half the diameter is called __________.
7. The value of π (pi) = __________

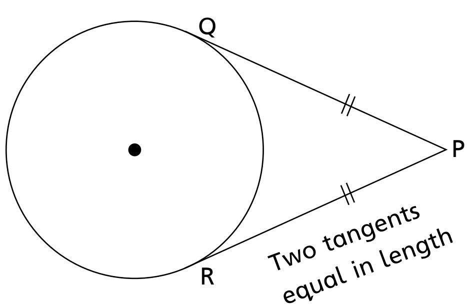
Main lesson

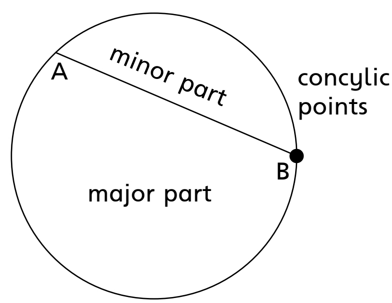
Explain to the students that apart from the parts they have labeled, there are other parts which are as important.

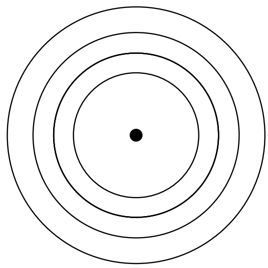
Draw a circle on the board to explain the following parts:
arc, sector, secant, tangent, concyclic points, and concentric circles



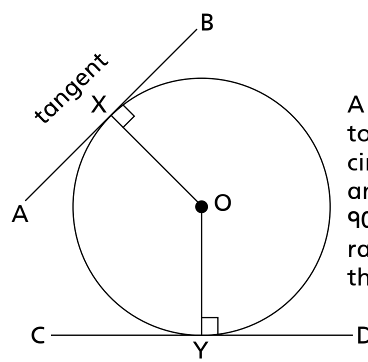








Circles with the same centre but different radii are called concentric circles.



A line that touches a circle makes an angle of 90° with the radius of the circle.

Individual activity

1. Students will be asked to copy the diagrams from the board and label them neatly and define the following:
   a. secant b. diameter c. tangent
   d. concyclic points e. concentric circle
2. Give some examples of concentric circles from real-life.

Recapitulation

Ask questions to reinforce the concepts.

1. What is a circle?
2. Which is the longest chord of a circle?
3. How many lines can be drawn from a point of circumference of a circle?

Homework

Revise the properties of a circle.

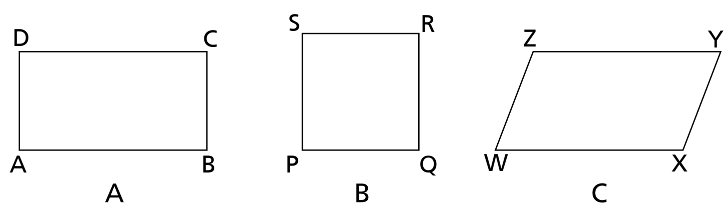
Topic: Different properties of parallelogram
Time: one period

Objective:

to enable students to demonstrate the different properties of a parallelogram

Starter activity

Draw the following figures on the board and ask the following questions.

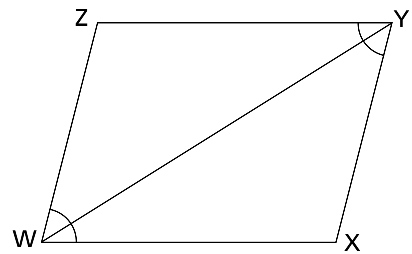


1. Name the above figures.
   rectangle, square, parallelogram
2. Which of the above figure has all four sides equal?
   square
3. Which of the above figures have opposite sides equal and parallel?
   rectangle and parallelogram
4. Which of the above figures have all angles as right angles?
   square and rectangle.

5. In what respect does a rectangle resembles a parallelogram?
   opposite sides are congruent and parallel
6. In what respect does a rectangle and parallelogram differ from each other?

Rectangle	Parallelogram
All angles are right angles, diagonals are equal and congruent	Angles are not right angles opposite angles are equal and diagonals are not congruent

Main lesson



Draw a parallelogram on the board and derive the properties of parallelogram by asking questions to the students. Explain the properties.

Explain to the students that a parallelogram is a quadrilateral with parallel opposite sides.

Derive some of the properties using the properties of parallel lines and congruence of triangles.

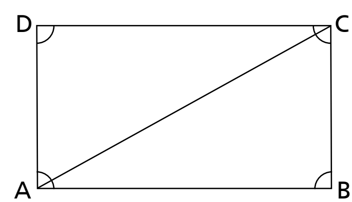
1. We start with the parallelogram WXYZ.

   Draw diagonal WY
   $\angle XYZ = \angle ZWX$
   $\therefore \angle ABC = 180^\circ - \angle BAC + \angle ACB)$
   $180^\circ - (\angle ACD + \angle DAC) - \angle ADC$
   $\therefore \angle BAD = \angle BCD$ and $\angle ABC = \angle ADC$

2. First, a diagonal AC will be drawn.
   Observe that $\angle DAC = \angle ACB$ (BC II AD)
   AC here acts as transversal; alternate angles are equal

   $\angle BAC = \angle ACD$ (alternate angles AB || CD)
   $\therefore \angle BAC + \angle DAC = \angle ACD + \angle ACB$
   $\angle BAD = \angle BCD$
   Also in Δ ABC
   $\angle ABC + \angle BAC + \angle ACB = 180^\circ$

   Also in parallelogram ABCD
   $\angle BAD + \angle ABC + \angle BCD + \angle CDA = 360^\circ$
   $2\angle BAD + \angle ABC) = 360$
   $\angle BAD + \angle ABC = 188$



Similarly, ∠BCD + ∠CDA = 180°
In Δ ABC and Δ CDA
∠BAC = ∠ACD
∠ACB = ∠DAC
AC = AC (common)
ΔABC = ΔCDA
AB = CD and BC = AD

Write the properties of a parallelogram on the board.

- opposite angles are congruent
- pairs of adjacent angles of a parallelogram are supplementary
- opposite sides are congruent
- the diagonal AC and BD bisect eachother
- a diagonal divides the parallelogram into two congruent triangles

Individual work

Students will be asked to write down the properties of parallelogram.
Question 3 of Exercise 7a will be done in the class.

UNIT 8

PRACTICAL GEOMETRY

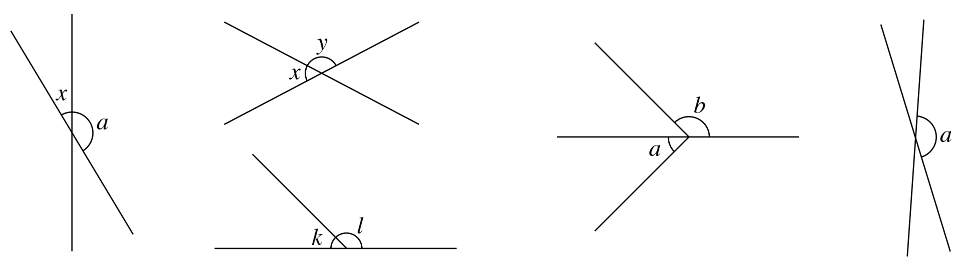
Topic: Bisection of angle between two non-parallel lines.
Time: 1 period (40 minutes)

Objective

To enable students to define and depict two non-parallel lines and find the angles between them.

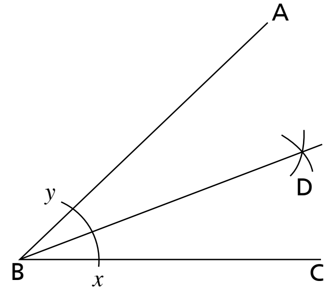
Starter activity

Draw some intersecting lines on a worksheet and ask the students to measure the angles between them.



Example

Bisection of an angle between two non-parallel lines



Steps of construction

1. Draw two non-parallel lines AB and BC so that they join at B and from an angle.
2. With B as the centre and with a convenient radius, draw an are which cuts $\overrightarrow{BC}$ $\overrightarrow{BA}$ and $\overrightarrow{BC}$ at X and Y. With X as the centre and radius more than half of XY draw an arc.
3. With Y as the centre and with the same radius, draw an arc which cuts the previous arc at D.
4. Join B and D and extend it.

Practice session

Give two or three selected questions from Exercise 8a to do in the class.
Draw the following angles and bisect them.

a) 120° b) 70° c) 45°

Homework

Draw angles of 60°, 110° and 30° and bisect them.

Topic: Construction of quadrilaterals
(square, rectangle, parallelogram, rhombus and kite
Time: 2 periods

Objective

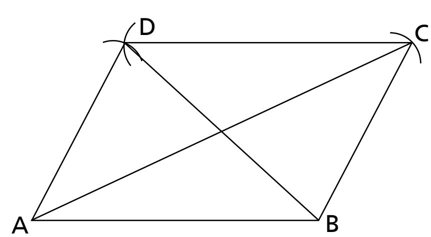
To enable students to construct and write the steps of construction

Starter activity

Draw a quadrilateral on the board and ask the students to define the following:

i) adjacent angles ii) adjacent sides iii) diagonals.

Main lesson



Explain with the help of the figure.

A simple closed two-dimensional figure bounded by four line segments is called a quadrilateral.

If A, B, C, and D are four coplanar points such that no three of them are collinear, the union of segments $\overline{AB}$, $\overline{BC}$, $\overline{CD}$ and $\overline{AD}$ is called a quadrilateral. It is denoted by $\square$ ABCD or Quad ABCD.

Vertices:

The common end points of the line segments are called its vertices. A, B, C and D are the four vertices.

Angles:

$\angle A$, $\angle B$, $\angle C$, and $\angle D$, are the four angles of $\square$ ABCD.

Opposite angles:

$\angle A$, and $\angle C$, $\angle B$, and $\angle D$, are pairs of opposite angles.

Diagonals:

The line segments joining the opposite vertices of a quadrilateral are called its diagonals. $\overline{AC}$ and $\overline{BD}$ are the diagonals of $\square$ ABCD.

Sides:

The line segments, $\overline{AB}$, $\overline{BC}$, $\overline{CD}$ and $\overline{AD}$ are called its sides.

Opposite sides:

$\overline{AD}$ and $\overline{BC}$, $\overline{AB}$ and $\overline{CD}$ are the pairs of opposite sides.

Adjacent sides:

Two sides of a quadrilateral are called adjacent if they have a common end point. It the above figure, $\overline{AB}$ and $\overline{BC}$ are adjacent sides.

Adjacent angles or (Consecutive angles):

Two angles of a quadrilateral are adjacent if they have a common arm. $\angle A$ and $\angle B$, $\angle B$ and $\angle C$, $\angle C$ and $\angle D$, $\angle D$ and $\angle A$ are the pairs of adjacent angles.

Sum of all the angles of a Quadrilateral is equal to 360°.

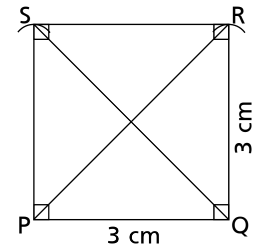
The following are the types of quadrilaterals.

1. parallelogram
2. rectangle
3. square
4. rhombus
5. trapezium

Explain the construction of a square with the help of its properties.

Example

Construct a square PQRS when mPQ = 3 cm.



Properties of a square

1. All sides are equal.
2. All angles are right angles.
3. The diagonals are equal and bisect each other.

Steps of construction

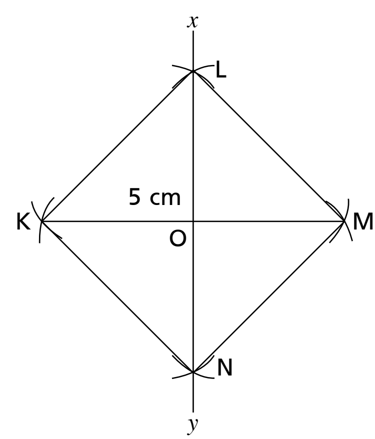
1. Draw a line segment PQ of 3 cm.
2. At P and Q, construct a right angle (90°).
3. With P and Q as the centre with radius 3 cm, draw arcs to cut PY at S and QX at R. Join R and S.

PQRS is the required square.

Construction of a square when its diagonal is given.

Construct a square KLMN when mKM = 5 cm.

First draw a plan or a rough figure.



Given KM = 5cm

KM = LN

Steps of construction

1. Draw a line segment KM of 5 cm.
2. Bisect KM with the help of a compass.
3. Join the arcs X and Y to get the midpoint O.
4. With O as the centre and the radius half of 5 cm i.e. 2.5 cm,
5. draw 2 arcs to cut OX at L and OY at N.
6. Join K to L, L to M, M to N and N to K. Measure the sides, they are equal.

KLMN is the required square.

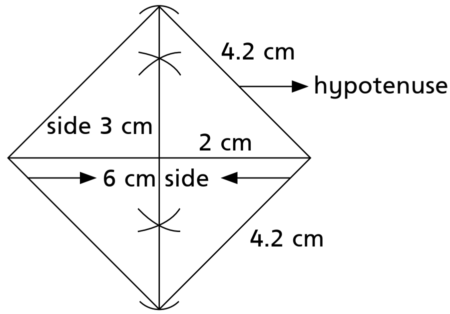
Practice session

Construct squares with diagonals as given below. Measure its sides.

a) 10 cm b) 8.4 cm

Individual activity

Construct a square where the diagonals measure 6 cm. Find its side by the Pythagoras theorem and verify it by measuring the constructed square.



Formula

$D^2 = 2 \times S^2$ D = diagonal of a square

$(6)^2 = 2 \times S^2$ S = side of a square

$36 = 25^2$

$2S^2 = 36$

$S^2 = \frac{\cancel{36}^{18}}{2}$

$S^2 = 18$

$S = \sqrt{18}$

S = 4.2 cm approximately

By the Pythagoreans Theorem for right-angled triangles, half of the diagonal $\frac{\cancel{6}^{3}}{2}$

Side of the triangle = 3 cm

$(S)^2 + (S)^2 = (hypo)^2$

$9 + 9 = 18$

length of the hypotenuse = $\sqrt{18}$

$\sqrt{18}$ = 4.2 cm approximately

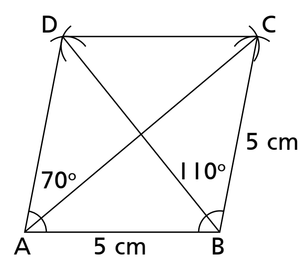
Construction of a rhombus

Case 1: When one side and angle is given

Construct a rhombus ABCD when AB = 5 cm and $\angle B = 110°$

All sides are equal.

Steps of construction



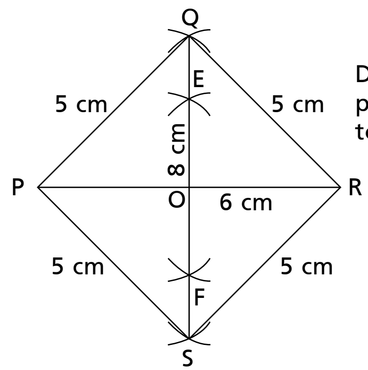
1. Draw $\overline{AB}$ = 5 cm
2. Draw angle ABx = 110°
3. With B as the centre and a radius of 5 cm, draw an arc to cut Bx at C.
4. With C as the centre and a radius of 5 cm, draw an arc.
5. With A as the centre and with the same radius, draw another arc to cut the previous arc at D. Join C to D and D to A.

   ABCD is the required rhombus.

Case 2: When the measure of two diagonals is given

Diagonals are perpendicular to each other.

Construct a rhombus PQRS, when mPR = 6 cm mQS = 8 cm



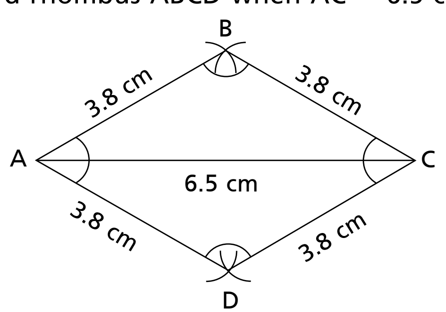
Diagonals are perpendicular to each other.

Steps of construction

1. Draw $\overline{PR}$ = 6 cm
2. Draw $\overleftrightarrow{EF}$ right bisectors of $\overline{PR}$.
3. With O as the centre and radius half of the other diagonal i.e., 4 cm, draw two arcs to cut $\overrightarrow{OE}$ at Q and $\overrightarrow{OS}$ at S.
4. Now join P to Q, Q to R, R to S, and S to P. PQRS is the required rhombus.

Case 3: When one side and diagonal are given

Construct a rhombus ABCD when $\overline{AC}$ = 6.5 cm and $\overline{AB}$ = 3.8 cm



All sides of a rhombus are equal. Diagonals bisect its interior angles.

Steps of construction

1. Draw $\overline{AC}$ = 6.5 cm.
2. With A as the centre and radius 3.8 cm, draw an arc on either side of $\overline{AC}$.
3. With C as the centre and radius 3.8 cm, draw an arc on either side of $\overline{AC}$ to cut the previous arc at B and D.
4. Join A to B, B to C, C to D, and D to A. ABCD is the required rhombus.

Individual activity

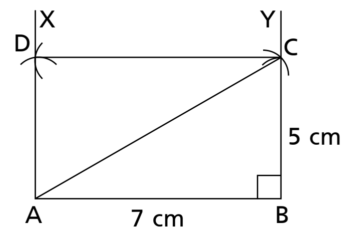
1. Construct a rhombus PQRS when mPQ = 5 cm, m $\angle$P = 70°
2. Construct a rhombus ABCD when mAC = 6 cm, BD = 4 cm
   Measure the sides for each case.

Construction of a rectangle

Explain with the help of a compass and a ruler on the board.

Case 1: Construction of a rectangle when 2 sides are given

Construct a rectangle ABCB where AB = 7 cm, mBC = 5 cm.



Steps of construction

1. Draw $\overline{AB}$ = 7 cm
2. Construct ABY = 90°
3. With radius 5 cm and with B as the centre draw an arc cutting $\overline{BY}$ at C.
4. With C as the centre and radius = 7 cm draw an arc.
5. With A as the centre and radius = 7 cm, draw another arc cutting the previous arc at D. Join BC, CD, and AD.
   ABCD is the required rectangle.

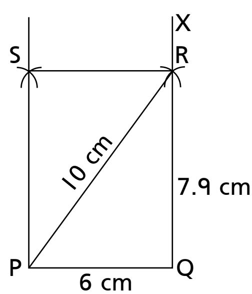
Individual activity

Construct and write the steps of construction of rectangles with the following measures.

a) 6 cm and 4.5 cm b) 5 cm and 3.5 cm

Case 2: Construction of a rectangle when the diagonal and one side is given

Construct a rectangle PQRS when $m\overline{PR}$ = 10 cm and $m\overline{PQ}$ = 6 cm and given that:



opposite sides are equal and parallel
diagonals are equal and bisect each other and they do not bisect the interior angles

Steps of construction

1. Draw PQ = 6 cm
2. Construct ∠PQX = 90°
3. With P as the centre and a radius 10 cm draw an arc to cut $\overleftrightarrow{QX}$ at R.
4. With R as the centre and radius = 6 cm (opposite sides equal) draw an arc.
5. With P as the centre and radius equal to $\overleftrightarrow{QR}$, draw another arc to cut the previous arc at S. Join RS and SP,
   PQRS is the required rectangle.

Individual activity

Construct and write the steps of construction.

1. Rectangle PQRS when $\overline{QS}$ = 8 cm and $\overline{PQ}$ = 5 cm.
2. Rectangle ABCD when $\overline{AC}$ = 10 cm, $\overline{BD}$ = 7 cm.

Topic: Construction of a parallelogram
Time: 1 period (40 minutes)

Objectives

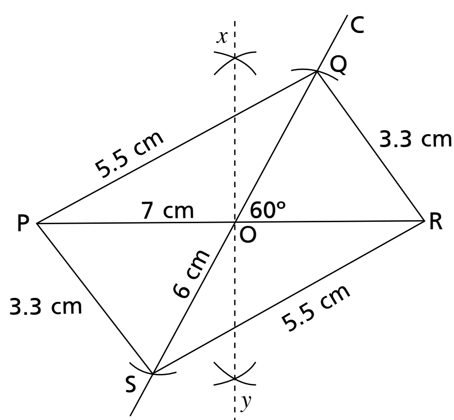
To enable students to construct a parallelogram when:

- its two diagonals and the angle between them are given
- two adjacent sides and the angle between them are given and
- to enable them to understand the properties of a parallelogram.

Explain with an example on the board.
Case 1: Construct a parallelogram PQRS where $\overline{PR}$ = 7 cm and $\overline{QS}$ = 6 cm and the included angle is 60°. Measure its sides.
The students should first know the properties of a parallelogram as then it becomes easier to follow the construction.

- Opposite sides are equal and parallel.
- Opposite angles are equal.
- Each diagonal bisects the parallelogram.

Steps of construction

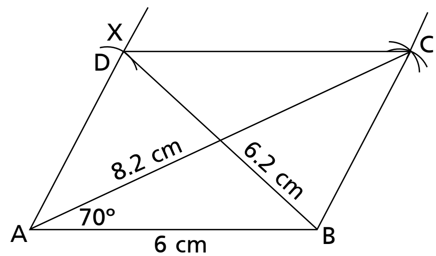


1. Draw PQ = 7 cm and draw its perpendicular bisector XY to get the midpoint O.
2. At O, make an angle COR = 60° and produce it both ways.
3. With O as the centre and a radius half of the other diagonal i.e. 3 cm, draw arcs cutting $\overrightarrow{OC}$ and $\overrightarrow{OD}$ at Q and S respectively.

4. Join QR, RS and S
5. PQRS is the required parallelogram.

Case 2: Construction of a parallelogram when two adjacent sides and the angle between them are given

Construct a parallelogram ABCD where AB = 6 cm, BC = 45 cm and ∠A = 70°. Measure the diagonals and write the steps of construction.



Steps of construction

1. Draw $\overleftarrow{AB}$ = 6 cm.
2. Draw ∠XAB = 70°
3. With A as the centre and a radius = 4.5 cm, draw an arc cutting the arm AX at D.
4. With D as the centre and a radius = 6 cm, draw an arc.
5. With B as the centre and a radius = 4.5 cm, draw another arc cutting the previous arc at C.
6. Join AB, BC, CD, and DA.
   ABCD is the required parallelogram.

Individual activity

Construct the following parallelograms and write the steps of construction.

1. ABCD, when $\overline{AB}$ = 5 cm, $\overline{BC}$ = 6 cm –B = 110°
2. KLMN, when $\overline{KL}$ = 7 cm, $\overline{KN}$ = 5.5 cm and ∠K = 65°
3. ABCD, when $\overline{AC}$ = 8 cm, $\overline{BD}$ = 6.4 cm and the included angle = 75°

Homework

Draw the following parallelograms.

1. PQRS, $\overline{PR}$ = 6 cm, $\overline{QS}$ = 8 cm and the included angle = 70°
2. EFGH when $\overline{EF}$ = 6 cm, ∠F = 115°, $\overline{FG}$ = 4 cm

Give questions from Exercise 8a as homework.

Topic: Construction of a kite
Time: 1 period (40 minutes)

Objectives

To enable students to:

- construct a kite
- differentiate between a kite and other quadrilaterals.

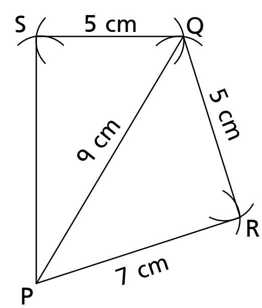
Starter activity

Show a kite and discuss its properties.

Explain the construction of a kite on the board with the help of a compass and a ruler.

Construct a kite when the two unequal sides are 5 cm and 7 cm each and one of the diagonal is 9 cm.

Sides = SQ = QR = 5 cm
Side = SP = PR = 7 cm
Diagonal PQ = 9 cm



Steps of construction

1. Draw PQ = 9 cm.
2. With P as the centre and a radius = 7 cm, draw arcs on either sides of PQ.
3. With Q as the centre and a radius 5 cm draw arcs to cut the previous arcs at R and S.
4. Join PR, RQ and QS and SP.

PRQS is the required kite.

Individual activity

Construct the kites with the following measurements and write the steps of construction.

1. diagonal 8 cm, sides = 4 cm, 7 cm
2. diagonal 9 cm, 4.5 cm, 6 cm

Topic: Construction of a triangle when the hypotenuse and one of its side are given
Time: 1 period (40 minutes)

Objective:

to enable students to construct a right-angled triangle.

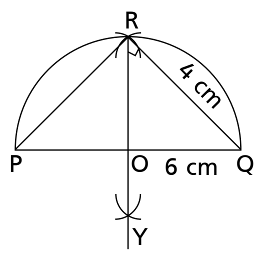
Starter activity

Draw a right-angled triangle on the board and ask the students to label the parts of a triangle.

Fill in the blanks:

1. A right-angled triangle has _________ right angles. (one, two, three)
2. The longest side of the right-angled triangle is called ___________.
3. Hypotenuse is the side which is opposite to the __________.

Construct a right-angled triangle when the measure of its hypotenuse is 6 cm and one of its side is 4 cm.



Steps of construction

1. Draw PQ = 6 cm (hypotenuse).
2. Draw a perpendicular bisector of PQ to get the mid-point O.
3. With O as the centre and a radius O to P or O to Q. Draw a semicircle.
4. With Q as the centre and a radius = 4 cm, draw an arc cutting the semicircle at R.
5. Join P to R and Q to R

PQR is the required right-angled triangle.

Individual activity

Give related questions from Exercise 8a as homework

1. Construct a right-angled triangle ABC when mAB = 5 cm and mBC (hypotenuse) = 7 cm.
2. Construct a right-angled triangle XYZ with a mYZ (hypotenuse) = 6 cm and one of its side = 3.5 cm.

Topic: Construction of a polygon
Time: 1 period

Objective:

to enable students to construct a pentagon

Starter activity

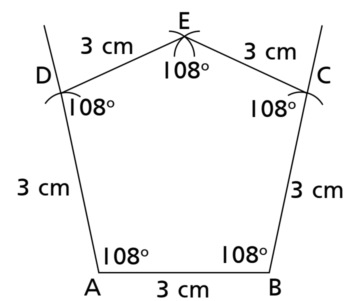
Show a pentagon to the students and ask them to examine it carefully and write the number of sides, angles, vertices.

Main lesson

Explain the construction of pentagon on the board.
In the first step we will find the interior angles of the pentagon.

$180° - \frac{\cancel{360}^{72}}{5}$

$180 - 72 = 108$ each interior angle $= 108°$



Steps of construction

1. Draw $\overline{AB} = 3$ cm
2. Draw $\angle ABX = \angle BAY = 108°$ (interior angle)
3. With A and B as the centers and with a radius = 3 cm, draw two arcs cutting $\overrightarrow{BX}$ at C and $\overrightarrow{AY}$ at D.
4. With C and D as the centers and radius = 3 cm, draw two arcs which intersect each other at E.
6. Join C to E and D to E.

ABCDE is the required pentagon.

Homework

Construct and write the steps of construction for the following pentagons.

a) 5 cm b) 3.8 cm c) 2.8 cm.

Topic: Construction of a hexagon
Time: (1 period) 40 minutes

Objective:

to enable to students to construct a hexagon

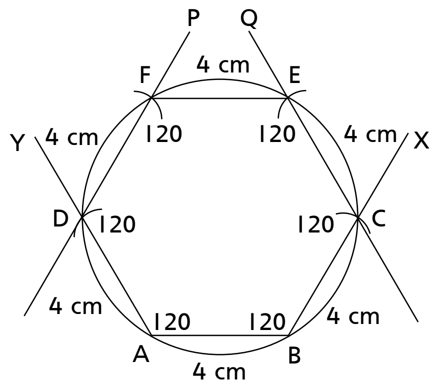
Starter activity

Draw a hexagon on the board and ask the students to write the number of sides, angles, and vertices.
Construct a hexagon with sides measuring 4 cm.

Each interior angle $= 180 - \frac{\cancel{360}^{60}}{6}$

$180 - 60 = 120°$

Each interior angle = 120°
Enterior angle = 60°



Steps of construction

1. Draw AB = 4 cm.
2. At A and B, draw ∠ABX and ∠BAX = 120°.
3. With A and B as centers and with a radius = 4 cm, draw arcs which cut $\overrightarrow{AY}$ at D and $\overrightarrow{BX}$ at C.
   each interior angle = 120°
4. On C and D, draw angles ∠BCQ and ∠ADF = 120°
5. With C and D as centers and with a radius 4 cm, draw arcs to cut CQ at E and DP at F.
6. Join AB, BC, CE, EF, FD, and DA.

ABCDEF is the required hexagon.

Individual activity

Construct the following hexagons with sides as given.

a) 3.5 cm b) 4.5 cm c) 3.8 cm

Homework

Construct a hexagon of sides as given.

a) 2.5 cm b) 3 cm

Give related questions from Exercise 8a as homework.

Recapitulation

Review the unit lessons and help the students understand any construction they are not clear about.

UNIT 9

AREA AND VOLUME

Topic: Area and Volume; Pythagoras Theorem
Time: 3 periods

Objectives

To enable students to:

- state the Pythagoras theorem
- find the sides of right-angled triangle or any triangle by applying the formula

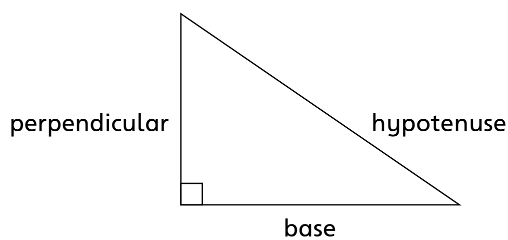
Starter activities

Activity 1

1. What is the area of the square if the sides are given as:
   a) 5 cm b) 3.4 cm c) 7 cm
2. What is the measure of the sides of a square if the area is:
   a) 49 cm^2 b) 2.5 cm^2 c) 81 cm^2
3. What is the perimeter of a square with a side of 2.5 cm?
4. Is the diagonal of a square equal to its sides?
5. How many right-angled triangles can be formed in a square when a diagonal is drawn?

Activity 2

Draw a right-angled triangle on the board and ask the students to label its elements.

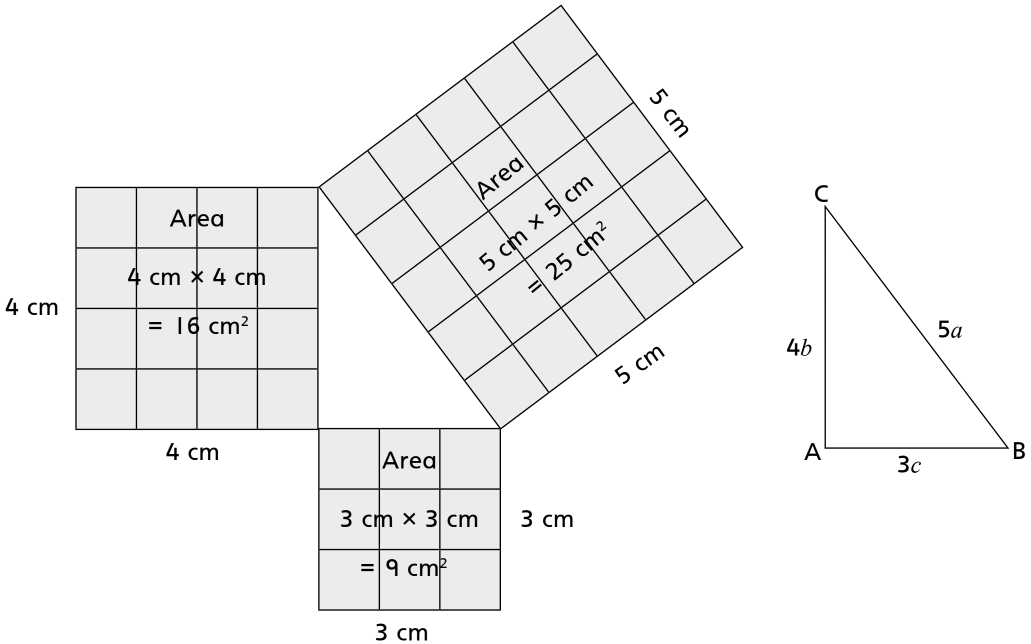


Answer the following questions.

1. Are all the sides of a right-angled triangle equal?
2. What is the longest side called?
3. Is the sum of other two sides equal to the measure of the hypotenuse?

Main lesson

Draw the following figure on the board and explain the Pythagoras theorem.



1. A right-angled triangle ABC in which mAB = 3 cm, AC 4 cm and BC = 5 cm will be constructed on the board.
2. On each side of the triangle, a square will be drawn. On side AB, a square of side 3 cm, on AC, a square of side 4 cm and, on BC, a square of side 5 cm.
3. Area of each square will be found out.
   $3 \text{ cm} \times 3 \text{ cm} = 9 \text{ cm}^2$, $4 \text{ cm} \times 4 \text{ cm} = 16 \text{ cm}^2$, $5 \text{ cm} \times 5 \text{ cm} = 25 \text{ cm}^2$
   Explain that the square formed at the hypotenuse is no greater than the squares at the other two sides.

Example 1

In a right-angled triangle ABC, find the third side if the hypotenuse c = 15 cm and side b = 9 cm.

We have to find side a.

$\therefore\ c^2 - b^2 = a^2$

(hyp) (perp) (base)

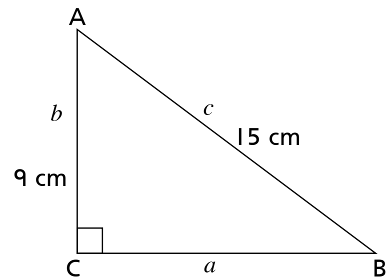
$(15)^2 - (9)^2 = (\text{base})^2$

$225 - 81$

$144 = a^2$

or $a = \sqrt{144}$

$\therefore\ a = 12$ cm



Example 2

Find c when a = 9 cm, b = 12 cm (right angle at c)

$(c)^2 = (a)^2 + (b)^2$

$(c)^2 = (9)^2 + (12)^2$

$(c)^2 = 81^2 + 144^2$

$(c)^2 = 255$

$c = \sqrt{225}$

$c = 15$ cm

Now add the area of the squares of the other two sides.

$9\text{ cm}^2 + 16\text{ cm}^2 = 25\text{ cm}^2$

From this activity we find that the area of the squares of the other two sides is equal to the area of the square at the hypotenuse which is 25 cm^2.

$9\text{ cm}^2 + 16\text{ cm}^2 = 25\text{ cm}^2$

or

$(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$

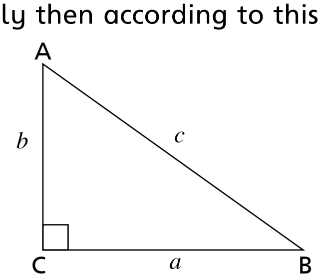
Pythagoras theorem states that ‘in any right-angled triangle, the area of the square of the hypotenuse is equal to the sum of the squares of the other two sides.’

Explain that in the right-angle triangle ABC, if we denote the opposite sides of the vertices ABC by a, b and c respectively then according to this proposition,

$c^2 = a^2 + b^2$

$a^2 = c^2 - b^2$

$b^2 = c^2 - a^2$

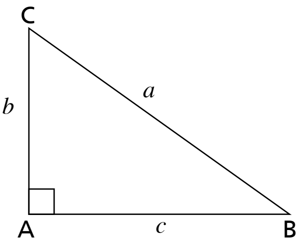


and if the right angle is at 'A' then,

$(a)^2 = (b)^2 + (c)^2$

$(c)^2 = (a)^2 - (b)^2$

$(b)^2 = (a)^2 - (c)^2$



Explain the solved examples on page 151 of the textbook.

Individual activity

Give Exercise 9a to be done individually by each student. Help them solve it.

Students can be called turn by turn to solve these on the board with the rest of the class observing.

Homework

In the right-angled triangle ABC, right-angled at C, find the third side when the other two are given.

1. If b = 16 cm and c = 20 cm find a.
2. If a = 15 cm and b = $5\sqrt{3}$ find c.

Give questions 6 and 7 of Exercise 9a as homework.

Topic: Area of a triangle by Hero's formula
Time:

Objective:

to enable students to find out the area of a quadrilateral and triangle by Hero's formula

Starter activity

Students have already learnt about triangles.

1. Find the area of a triangle if its base is 5 cm and the altitude is 8 cm.
2. What is the height of this figure?
   It is 8 cm because the altitude is the height of the figure.
3. What is the formula for finding its area?
   It is $\frac{1}{2} \times$ base $\times$ altitude

Why do we divide the base and the altitude by 2?

We have derived this formula by rectangle which is length × breadth.
The diagonal divides a rectangle into two equal triangles.

Therefore, the area of the triangle is:

$\frac{1}{2}$ × base × height (when 'h' is known)

$\frac{1}{2} \times 8 \times 5 = 20 \text{ cm}^2$

Main lesson

Find the area of a triangle whose sides are 3 cm, 4 cm and 5 cm.
What is the height of this triangle?

Only the sides are given, it is not a right-angled triangle.
Explain the area of a triangle in terms of sides by Hero's formula (Hero was a mathematician)
If altitude or height of a triangle is not known, its area can be found with the help of its three sides by using the following formula.

Hero's formula

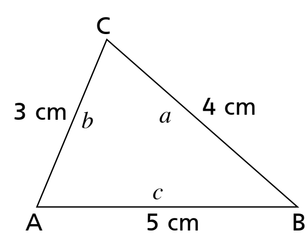
Area of a $\Delta = \sqrt{S\ (s-a)\ (s-b)\ (s-c)}$

Here a, b, and, c are the measures of the sides and S is the semi-perimeter.

$S = \frac{a + b + c}{2}$

Example 1

Let a = 4 cm, b = 3 cm, c = 5 cm
Then,
$S = 4 + 3 + \frac{2}{2} = \frac{12}{2}$
S = 6 cm



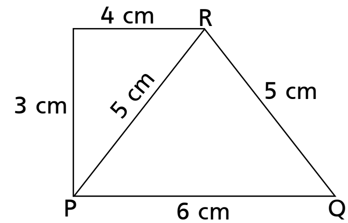
So the area of the triangle:

$$= \sqrt{6\ (6-4)\ (6-3)\ (6-5)}$$
$$= \sqrt{6\ (2)\ (3)\ (1)}$$
$$= \sqrt{36}$$
$$= 6 \text{ cm}^2$$

Area = 6 cm²

Example 2

Area of a quadrilateral when four sides and a diagonal is given:
Find the area of a quadrilateral PQRS when mPQ = 6 cm, mQR = 5 cm, mRS = 4 cm, mPS = 3 cm and mPR = 5 cm



$\overline{PR}$ the diagonal divides the quad PQRS into two triangles.
It is common to both the triangles.

Solution

First draw a plan or a rough figure to find the two triangles.
Two triangles are formed, Δ PQR and Δ PRS.

Area of Δ PQR = $S_1 = \frac{6+5+5}{2} = \frac{\cancel{16}^{\,8}}{\cancel{2}} = 8$ cm

Area of Δ PQR

$\sqrt{8\,(8-6)\,(8-5)\,(8-5)}$

$= \sqrt{8\,(2)\,(3)\,(3)}$

$= \sqrt{144}$

Area of Δ PQR = 12 cm^2
Area of quad PQRS = ΔPQR + ΔPRS
= 12 cm^2 + 6 cm^2 = 18 cm^2
Area of PQRS = 18 cm^2

Area of Δ PRS
$S_2 = \frac{5+4+3}{2} = \frac{12}{2} = 6$
Area of PRS = $\sqrt{6\,(6-5)\,(6-4)\,6-3)}$
$= \sqrt{6\,(1)\,(2)\,(3)}$
$= \sqrt{36}$
Area of Δ PRS = 6 cm^2
Δ PRS
6 cm^2 = 18 cm^2

Examples given in the textbook on page 152 should also be explained to the students.

Individual activity

Give Exercise 9b to be done in the class. Students can work in pairs.
Learning become easy in groups

Homework

Give questions 5, 6, and 7 of Exercise 9b as homework.

Recapitulation

Ask some questions so that students can recall the concepts and the terms they have learnt.

- When do we apply the Hero's formula to find the area of a triangle?
- State Pythagoras Theorem.
- What does 'S' stands for in Hero's formula?

- Which is the longest side of a right angle triangle?

Topic: Surface area and volume of a Cone and a Sphere
Time: 3 periods

Objective:

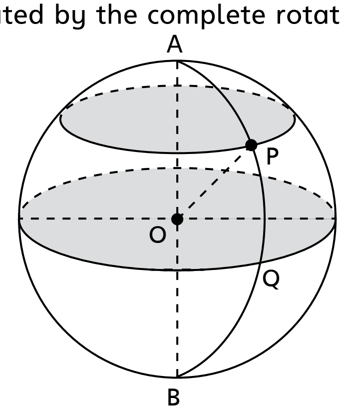
to enable students to find the surface area and volume of sphere.

Starter activity

1. Give some examples of spheres and cones that are found in real-life.
   A cricket ball, football, world globe, soccer ball, and a golf ball are some examples of a sphere.
2. Is a Rs 5 coin a sphere?
   No
3. How many faces do a sphere has?
   It does not have any fixed face.
4. Does it have a flat surface?
   It has a curved surface.

Main lesson

Explain to the students with the help of a football that a sphere is a solid figure generated by the complete rotation of a semi-circle around a fixed diameter.



The radius of the semicircle is the radius of the sphere.

OP = OA = OB

The surface area of the sphere is the surface that can be touched.
The surface area of a sphere is given by:

$$\text{Surface area} = 4\,\pi\, r^2$$

$$= 4 \times \frac{22}{\cancel{7}} \times \overset{3}{\cancel{21}} \times 21$$

$$= 5544 \text{ cm}^2$$

Volume $= \frac{4}{3} \pi r^3$

$\frac{4}{3} \times \frac{22}{7} \times 21 \times 21 \times 21$

$= 38808 \text{ cm}^3$

$\pi = \frac{22}{7}$ or

Example

Find the surface area and volume of a sphere whose radius is 21 cm.

Surface are $= 4 \pi r^2$

$= 4 \times \frac{22}{7} \times 21 \times 21$

$= 5544 \text{ cm}^2$

Volume $= \frac{4}{3} \pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times 21 \times 21 \times 21$

$= 38808 \text{ cm}^3$

Explain all the examples given in the textbook by drawing figures on the board.

Individual activity

Exercise 9c 1 part 2, 2, 5, 8, 9, 10 to be done in the class,

Homework

Give questions 13 and 15 of Exercise 9C as homework.

UNIT 10 DEMONSTRATIVE GEOMETRY

Lesson 1

Topic: Demonstrative Geometry
Time: 5 periods

Objectives

To enable the students to:
- define demonstrative geometry
- describe the basics of reasoning
- describe the types of assumptions (axioms and postulates

Starter activity

Students already know the basic concepts of geometry.
Ask the following questions:
1. Define the following:
   a point, line, line segment, ray
2. What are collinear points?
3. Are all squares rectangle?
4. Are all rectangles quadrilaterals?
5. In what respect does a rhombus resemble a square?
6. Is the distance between two parallel lines equal from the beginning to the end?
7. Is the sum of all triangles equal to 180°?
8. Are the opposite angles of a parallelogram equal? Give reasons for this?
9. What is the shape of your mathematics textbook? Give reasons for your answer?
10. What is the shape of a sharpened pencil? Give reasons for your answer.

Main lesson

Explain the importance of Geometry by giving real-life examples. It is a part of mathematics related to size, shape, angles, relative position of figures and properties of space.

Geometry is found every where in art, architecture, engineering, land surveys, astronomy, sculpture, space, nature, sports, machinery etc.

Its significance can be understood when we see the beautiful buildings or bridges. Knowledge about angles and the properties of a figure makes it easier to manage, organize, and think about engineering bridges or big skyscrapers.

When playing carom board or a billiard game, dart board, football or cricket, the ball is hit by making sense of angles to hit the ball to or when cutting some designs on a piece of paper, we can do it better by employing geometrical reasoning.

Explain the meaning of demonstrative geometry that by giving reasons one can prove or demonstrate any object or any figure while discussing its properties.

There are two types of reasoning:

- Deductive reasoning
- Inductive reasoning

Explain this by examples given in the textbook and also by real-life examples.

Deductive reasoning:

A form of reasoning in which each conclusion follows from the previous one. An argument is built by conclusions that progresses towards a final statement.

Example

Premise 1:   All humans are mortal.
Premise 2:   Razia is a human.

Conclusion: Razia is a human being, therefore Razia is a mortal.
Ask the students to give more examples.

Inductive reasoning:

It is a form of reasoning in which a conclusion is reached based on a pattern present in numerous observations.

Example:

Premise: The Sun has risen in the East every morning up until now.
Conclusion: The Sun will also rise in the East tomorrow, every day.

Ask the students to give more examples.

Explain and define Axiom, Postulates, Theorem, Converse, Corollary by giving examples.

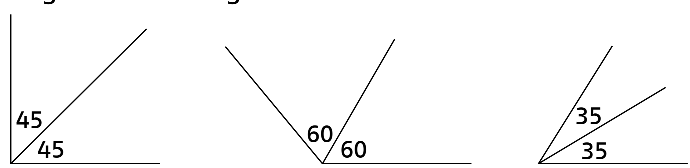
Axiom: Statements which are obviously true are called axioms. They are accepted without proof (theory dependent)

Examples

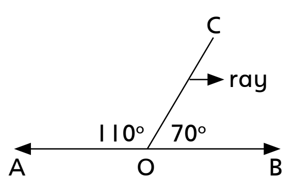
A line contains infinite number of points.
A line segment has two end points.
Things which are equal to the same things are equal to each other.
Postulate: A statement about geometry that is accepted as true without proof is known as a postulate.

Example

An angle has only one and only one bisector.

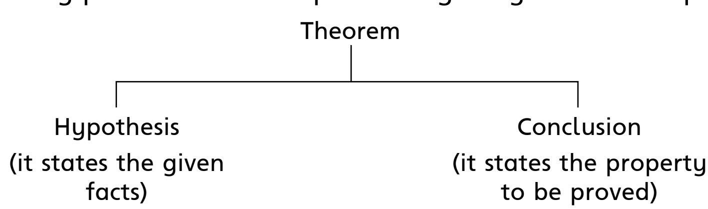


Linear pair postulate: If a ray stands on a line then the sum of two adjacent angles so formed is 180°.



adjacent angles
(110 + 70) = 180°

Theorem: A Theorem is a statement of geometrical truth which is proved by using facts already proved. It can be proved logically. It has two parts.



Converse: A Theorem in geometry whose hypothesis and conclusion are the conclusion and hypothesis of another theorem.

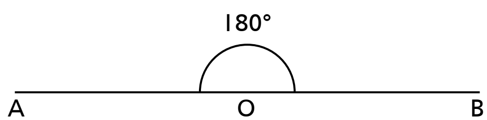
Corollary: It is a statement the truth of which is derived from a theorem.

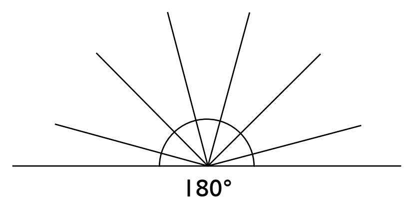
Lesson 2

Explain the theorems case by case.

Case I

Statement: If a straight line stands on another straight line the sum of measure of two angles so formed is equal to right angles.
This is an accepted truth that a straight angle forms 180°.
< AOB is a straight angle.



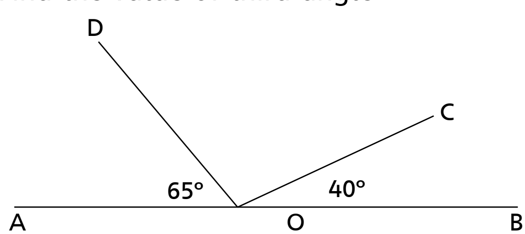


Any straight line meeting at point O will make two angles. The sum of two angles is equal to 180°.

More than two straight lines may meet at a point O and may make more than two or more angles. The sum of all angles will always be 180°.

Explain the following activity on the board.

Find the value of third angle.



If $\angle BOC = 40°$, $\angle AOD = 65°$, then find $\angle DOC$.
Let angle DOC = x

Then $65 + x + 40° = 180°$ (angles on a straight line = 180°)

$x + 105° = 180°$

$x = 180° - 105°$

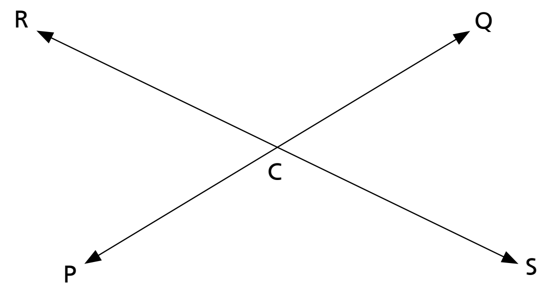
$x = 75°$

$\angle DOC = 75°$ hence proved $= 40 + 65 + 75 = 180°$

Case 2

Statement: If two lines intersect each other than the vertically opposite angles are congruent.

This is a theorem and needs to be proved. Explain it with the help of following figure.



$\overleftrightarrow{PQ}$ and $\overleftrightarrow{RS}$ are two intersecting lines. The intersecting point is C. When $\overleftrightarrow{PQ}$ and $\overleftrightarrow{RS}$ intersect each other at any point, they form four angles.

To prove:

$\angle PCR = \angle QCS$ and $\angle PCS = \angle RCQ$

Since $\overleftrightarrow{PCQ}$ is a straight line,

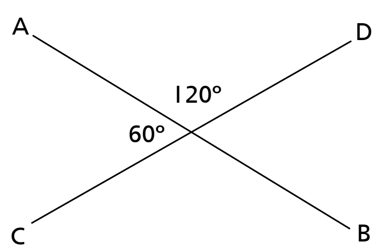
PCR + RCQ = 180° (1)

Again since, <RCS is a straight angle,

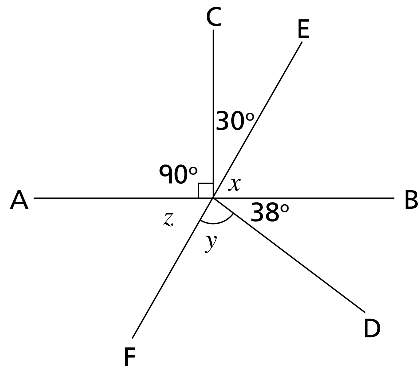
$\angle RCQ + \angle QCS = 180°$ (2)

In (1) and (2) <RCQ is common hence <PCR = QCS

Practice session



1. $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are two intersecting lines. Prove that $\angle AOD = \angle COB$.
2. Call a student to the board to perform the activity given on page 166 of the textbook. The rest of the class and the teacher can help with it.
   Find x, y and z in the given diagram, where AOB and EOF are straight lines.



Solution

$90 + 30 + x = 180°$ (adjacent angles on a straight line)
$x + 120 = 180°$
$x = 180 - 120$
$x = 66°$
$z = 60°$ (vertically opposite angles)
$z + y + 38 = 180°$ (adjacent angles on a straight line)
$60 + y + 38 = 180°$
$y + 98 = 180°$
$y = 180° - 98$
$y = 82$

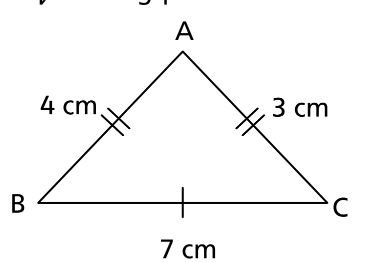
Individual activity

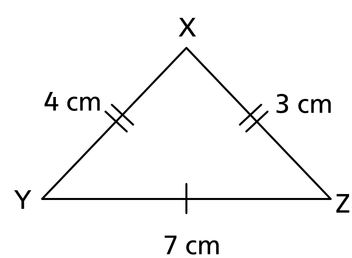
Give Exercise 10a, questions 1 and 2 to be done in the class.

Lesson 3

Case 3

In any correspondence of two triangles, if two sides and included angles of one triangle are congruent to the corresponding sides and included angle of the other, the two triangles are congruent. Explain that this is an axiom and it does not require any proof.

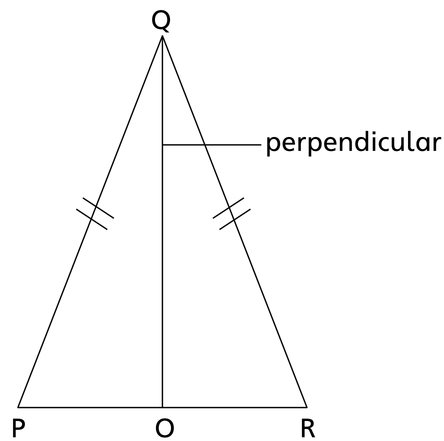




Case 4

If two side of a triangle are congruent, then the angles opposite to these sides are also congruent. This theorem can be proved with the help of the above axiom.

PQR is a triangle
given PQ = QR
To prove that $\angle QPR = \angle QRP$



PQ = QR

Solution

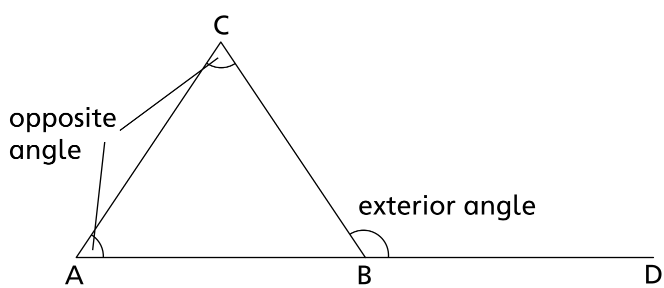
First drop a perpendicular QO from the vertex Q on PR. Note that in PQO and PQO,

⇨ PQ = RQ
⇨ QO is common
⇨ $\angle QOP = \angle QOR = 90°$
Hence $\Delta PQO = \Delta RQO$
therefore, $\angle QPO = \angle QRO$
or $\angle QPR$ = QRP

Case 5

Statement: An exterior angle of a triangle is greater in measure than either of its opposite interior angles.

Explain that it is a corollary the truth of which is derived from a theorem, that is, the sum of measures of the three angles of a triangle is 180°.



Explanation:

ABC is a triangle where AD is extended to D.
The sum of $\angle BCA + \angle CBA$ + BAC = 180° (1)
and ABD is a straight angle $\angle CBA$ + BAC = 180° (2)

In (1) and (2) CBA is common, therefore,
∠ACB + ∠CAB = ∠ABD
Since ∠CBD is the sum of two interior opposite angles hence it is always greater than either of its two opposite interior angles i.e <ACB and <CAB.

Individual work

Give Exercise 10a, question 3 to be done by the students on the board.

Homework

Ask the students to do question 3 of Exercise 10a.

Lesson 4

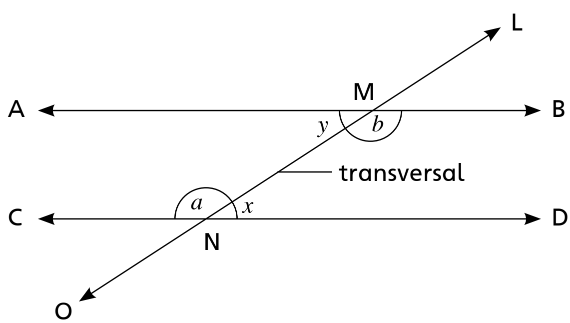
Case 6 and 7 will be explained to the students by drawing figure on the board.

Case 6

If a transversal intersects two parallel lines such that the pairs of alternate angles are congruent, then the lines are parallel.

This is the converse of the property of parallel lines that is:

If a transversal intersects two parallel lines the alternate angles so formed are congruent. (from practical geometry)



$\angle a$, $\angle b$ are alternate angles
$\angle a = \angle b$
$\angle x$, $\angle y$ are alternate angles
$\angle x = \angle y$

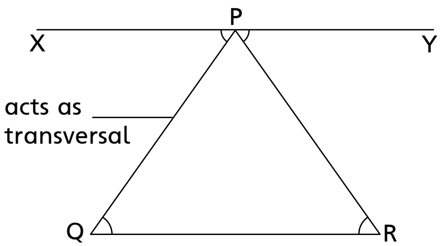
Explain that in the above figure, the transversal ∠MNO intersects $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ such that a pair of alternate angles is formed. In the above figure, ∠a and ∠b are the alternate angles.

In other words, ∠AMN = ∠MND
In order to prove that AB // CD, it is given that:
∠AMN = ∠MND
Similarly, ∠AMN = ∠LMB since they are vertically opposite angles.
∠LMB = ∠MND (corresponding angles)

From the above reasons it is proved that AB//CD.

Case 7

The sum of measures of the three angles of a traingle = 180°



$\overleftrightarrow{XY}$ and $\overleftrightarrow{QR}$ are parallel to each others

PQR is a triangle. To prove that $\angle P + \angle Q + \angle R = 180°$.

Step 1

Through P construct a line $\overleftrightarrow{XY}$ parallel to $\overleftrightarrow{QR}$.

Since $\overleftrightarrow{XY}$ and $\overleftrightarrow{QR}$ are parallel to each other PQ is a transversal.

$\angle XPQ = \angle RQP$ (1) (alternate angles) and

$\angle YPR = \angle QRP$ (2) alternate angles)

By adding (1) and (2),

$\angle XPQ + \angle YPR = \angle RQP + \angle QRP$

Step 2

Add QRP on both sides of the equation.

$\angle XPQ + \angle YPR + \angle QPR = \angle RQP + \angle QRP + \angle QPR$

$\angle XPQ + \angle YPR + \angle QPR = 180°$(angles on a straight line)

Therefore $\angle RQP + \angle QRP + \angle QPR = 180°$ or

$\angle P + \angle Q + \angle R = 180°$

Individual work

Give Exercise 10a to be done in the class.

Recapitulation

Revise the lessons one by one and explain again where the students are not clear.

UNIT 11

INTRODUCTION TO TRIGONOMETRY

Lesson 1

Topic: Trigonometry
Time: 4 periods

Objectives

To enable students to:

- define trigonometry
- define and identify trigonometric ratios
- establish trigonometric ratios, reciprocal relations between trigonometric ratios
- find trigonometric ratios of complementary angles
- find the relation between trigonometric ratios of complementary angles.
- recognize trigonometric identities and apply in solving triangles
- solve right-angled triangles applying trigonometric ratios
- define and identify angle of elevation and angle of depression
- solve real-life problems involving trigonometric ratios.

Starter activity

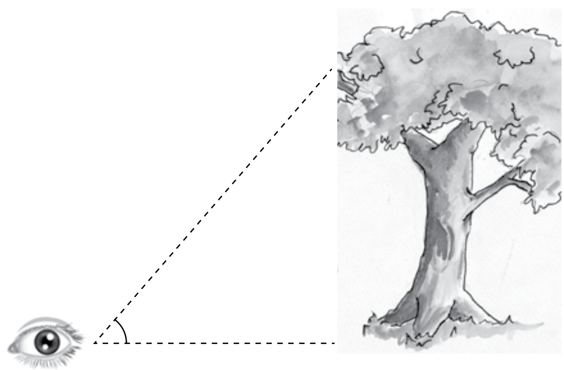
Students will be asked to:

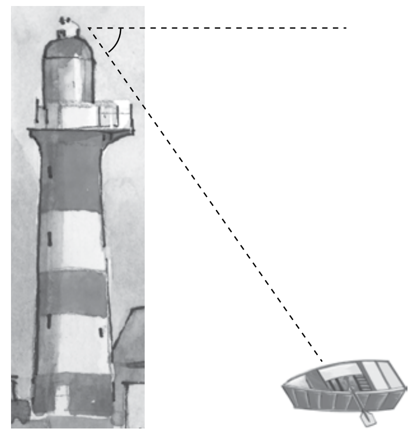
- measure the length of the board distance of the board from their seat, height of their desk etc.
- give the approximate height of the school building, (without measuring)
- wall of the classroom
- height of the flagpole
- distance of the building across the school building etc.

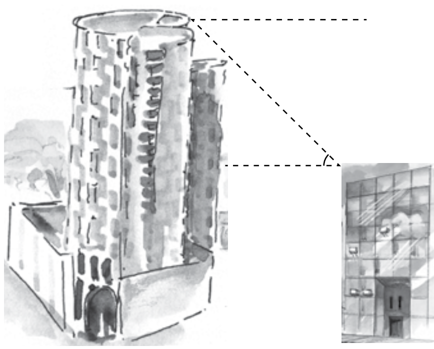
Is it possible to measure these heights and distance by a measuring tape? No.

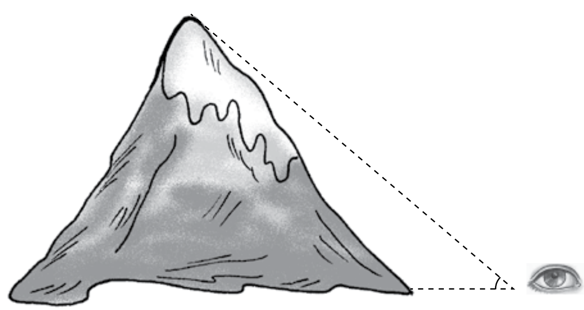
Main lesson

Display charts of indirect measurements.







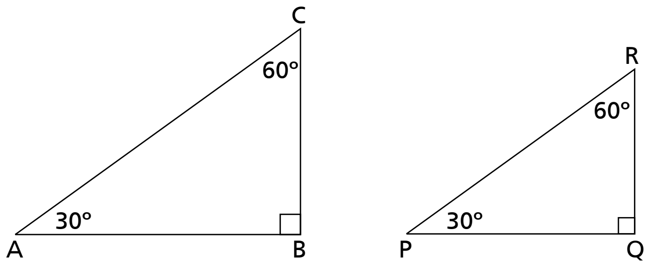


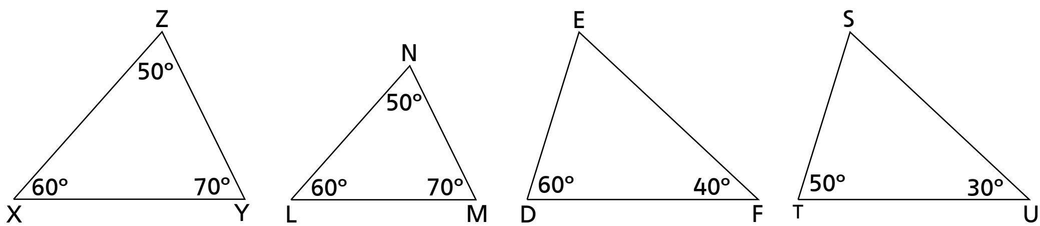
It is not possible to measure the heights of very tall buildings or the distance between two far off places. Instead, a method called indirect measurement is used.

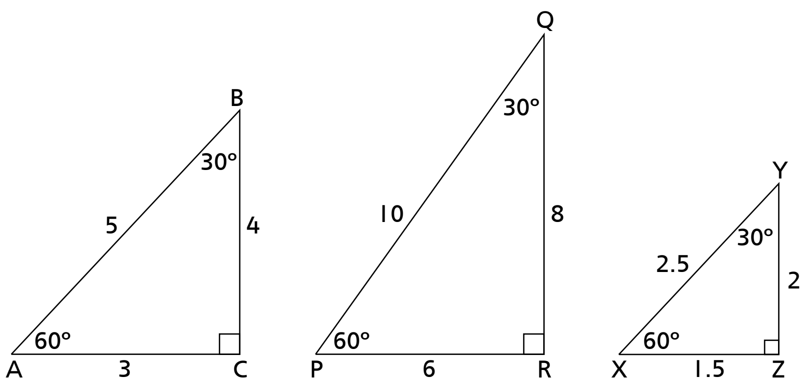
To measure indirectly, draw similar triangles. Now what is a similar triangle? Define and explain a similar triangle.

Two triangles are similar if their corresponding angles are congruent. (or equal in measure)

Give activity sheets to understand similar triangles and deduce their properties. Which of the following pairs of triangles are similar?







Ask some questions.

1. What type of triangles are ΔABC, ΔPQR and ΔXYZ?
   They are right triangles. Ask Why?
2. Are the triangles ABC, PQR and XYZ similar?
   They are congruent. Ask why?
3. Write the ratios of the sides of ΔABC, ΔPQR, and ΔXYZ.
4. Name the corresponding sides and angles.

   $\frac{m\overline{BC}}{m\overline{AB}} = \frac{4}{5}$, $\frac{m\overline{AC}}{m\overline{AB}} =$ ______, $\frac{m\overline{BC}}{m\overline{AC}} =$ ______

   $\frac{m\overline{QR}}{m\overline{PQ}} =$ ______, $\frac{m\overline{PR}}{m\overline{PQ}} =$ ______, $\frac{m\overline{QR}}{m\overline{PR}} =$ ______

$\frac{m\overline{YZ}}{m\overline{XY}} = \frac{\overline{XZ}}{\overline{XY}} =$ ______, $\frac{m\overline{YZ}}{m\overline{XZ}} =$ ______

Are the ratios of the corresponding sides equal?

6. Is $\frac{\overline{BC}}{\overline{AB}} = \frac{\overline{QR}}{\overline{PQ}} = \frac{\overline{YZ}}{\overline{XY}}$ equal?
7. What do you say about $\frac{\overline{AC}}{\overline{AB}}$, $\frac{\overline{PR}}{\overline{PQ}}$ and $\frac{\overline{YZ}}{\overline{XY}}$?
8. Does the measures of the sides of similar triangles effects the ratios between the sides? No.

Main lesson

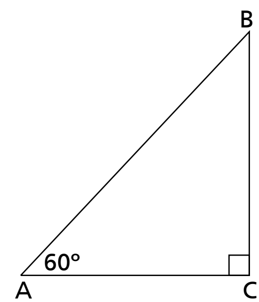
The property of similar triangles is very useful to find unknown measurements. It leads to the study of a branch of mathematics called trigonometry.

<u>Tri</u> meaning <u>three</u>, <u>gonia</u> meaning <u>sides</u>, <u>metron</u> meaning <u>measurement</u>

Literal meaning of trigonometry is measurement of triangles and the ratios between two sides of a triangle are called trigonometric ratios.

In a right-angled triangle, the two acute angles are complementary.

A + $m\angle B + m\angle C = 180°$
(sum of the measure of a Δ is $180°$)
$m\angle A + m\angle B = 180° - m\angle c$
$m\angle A + m\angle B = 180° - 90°$
$m\angle A + m\angle B = 90°$



In a right-angled triangle, the side opposite the right angle is called the hypotenuse and the other two sides are the base and perpendicular or adjacent side and opposite side.

In the right-angled triangle ABC, right-angled at C,
$m\angle A + m\angle B = 90°$ (acute angles of a right-angled triangle)
$\therefore\ m\angle B = 90° - 60° = 30°$

How many trigonometric ratios can be established between the two sides of a triangle? AB, BC and AC are the three sides.

There are six possible ratios between the two sides of a triangle.

$\frac{BC}{AB}, \frac{AC}{AB}, \frac{BC}{AC}, \frac{AB}{BC}, \frac{AB}{AC}, \frac{AC}{BC}$

Each of these ratios is given a special name with reference to an angle say for example $\angle A$.

For $\angle A$,

sine $\frac{mBC}{mAB} = \frac{BC}{AB} = \frac{\text{opposite side}}{\text{hypotenuse}}$

cosine $m\angle A = \frac{mAC}{mAB} = \frac{\text{adjacent side}}{\text{hypotenuse}}$

tangent m∠A = $\frac{mAC}{mAB}$ = $\frac{\text{opposite side}}{\text{adjacent side}}$

cotangent m∠A = $\frac{AC}{BC}$ = $\frac{\text{adjacent side}}{\text{oppositde side}}$

secant m∠A = $\frac{AB}{AC}$ = $\frac{\text{hypotenuse}}{\text{adjacent side}}$

cosecant m∠A = $\frac{AB}{BC}$ = $\frac{\text{adjacent side}}{\text{oppositde side}}$

For convenience we write sine as sin, cosine as cos, tangent as tan, cotangent as cot, secant as sec, and cosecant as cosec.

cosec m∠A i.e. $\frac{AB}{BC}$ is the reciprocal of sine m∠A i.e. $\frac{BC}{AB}$

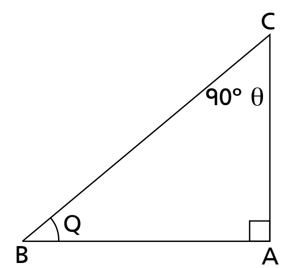
Similarly, notice that cos m∠A and sec m∠A are the reciprocals of each other. Also tan m∠A and cot m∠A are the reciprocals of each other.

So, cosec $\theta = \frac{1}{\sin\theta}$ or $\sin\theta = \frac{1}{\text{cosec}\,\theta}$

$\cos\theta = \frac{1}{\sec\theta}$ or $\sec\theta = \frac{1}{\cos\theta}$

and $\tan\theta = \frac{1}{\cot\theta}$ or $\cot\theta = \frac{1}{\tan\theta}$

Trigonometric ratios of complementary angles



In the right-angled triangle ABC,

∠A = 90°, ∠B = θ and ∠C = 90° − θ

Applying the trigonometric ratios

sin ∠c = Sin (90° − θ = $\frac{AB}{BC}$ $\frac{\text{opposite side}}{\text{hypotenuse}}$

cos ∠C = cos (90° − θ)= $\frac{AB}{BC}$

tan ∠C = tan (90° − θ) = $\frac{AB}{AC}$

cot ∠C = cot (90° − θ)= $\frac{AC}{AB}$

sin ∠B = sin

$\sin\theta = \frac{AC}{BC}$

$\cos\theta = \frac{AB}{BC}$

$\tan\theta = \frac{AC}{AB}$

sin of an angle = cos of its complement = sin 30° = cos 60°

cos of an angle = sin of its complement = cos 40° = sin 50°

tan of an angle = cot of its complement= tan 70° = cot 20°

cot of an angle = tan of its complement = cot 57° = tan 33°

sec of an angle = cosec of its complement = sec 42° = cosec = 48°

cosec of an angle = sec of its complement = cosec 29° = sec 61°

Make the students learn the trigonometric ratios of standard angles.
Ask them to make the table. (refer to textbook page 178)

θ	0°	30°	45°	60°	90°
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\sqrt{\frac{3}{2}}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	$\propto$

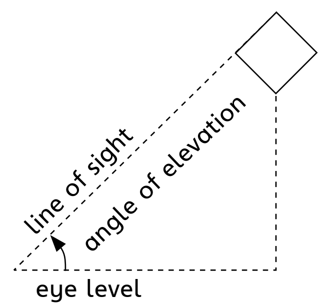
Explain finding heights using trigonometric ratios in real-life with the help of examples (refer to textbook page 180)

Lesson 2

Angles of Elevation and Depression

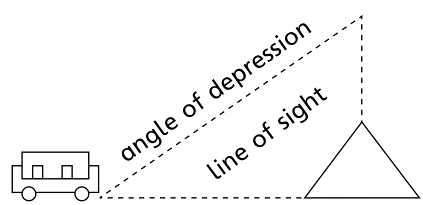
Explain the angle of elevation and angle of depression with examples.
Suppose you are looking up to view a kite. There is an angle formed between your eye level and the line of sight.
This angle is called the angle of elevation.

The angle of elevation is defined as:



When the object to be seen is at a higher level than the eye level, the angle so formed is called the angle of elevation.

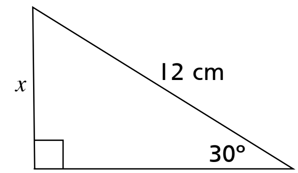
If you are standing at the top of a cliff or a hill and you are watching a car, then the angle formed between your eye level and the line of sight is called the angle of depression.



Lesson 3

Solving triangles applying trigonometric ratios

To find the unknown elements (sides or angles) of a right-angled triangle, we use trigonometric ratios.



Example 1

(refer to textbook page 181)
Explain the working on the board.
Find the length of side mark x.

sin 30° = opposite side/hypotenuse = $\frac{O}{P}$

$\therefore \frac{1}{2} = \frac{x}{2}$ $\quad (\therefore \sin 30° = \frac{1}{2})$

2x = 12 (cross multiplying)

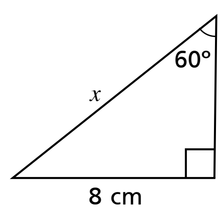
or x = $\frac{12}{2}$

x = 6 cm

Explain that the terms base and perpendicular can be replaced by adjacent and opposite side.

Example 2

Explain the example on the board.



$\sqrt{2} = 1.414$
$\sqrt{3} = 1.732$

$\sin 60° = \frac{\text{opposite side}}{\text{hypotenuse}}$

$\frac{\sqrt{3}}{2} = \frac{8}{x}$

$\sqrt{3}x = 16$ (by cross multiplication)

$x = \frac{16}{\sqrt{3}}$

$x = \frac{16}{1.73}$

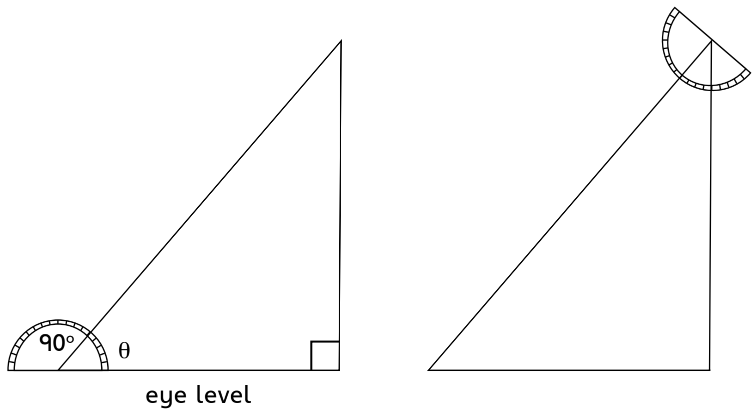
= 9.23 cm

Similarly example 3 will be worked out on the board.
We define the angle of depression as:

When the object to be seen is at a lower level than your eye level, the angle formed between your eye level and the line of sight is called the angle of depression.

Note: angles are measured by a protractor.

We can use the home-made saxton (instrument used for measuring angles of elevation and depression) for finding the angles of elevation or depression.



Explain and solve the examples given on pages 183 and 184 of the textbook.

Practice session

Give worksheets with questions like the following.

Write the values of:

sin 60° = ________

sin 30° = ________

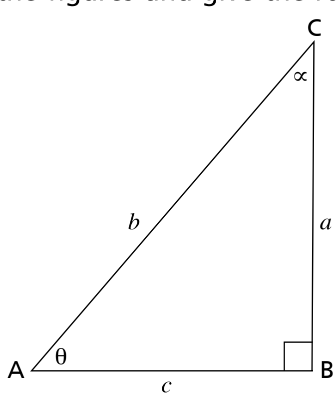
tan 45° = ________

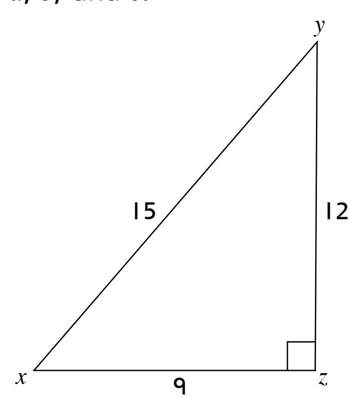
cot 60° = ________

sec 45° = ________

cos 60° = ________

Study the figures and give the ratio in terms of a, b, and c.





sin θ = ________

cos ∝ = ________

tan ∝ = ________

sec θ = ________

sin m∠x ________

cos m∠y ________

tan m$\angle y$ = __________
cot m$\angle x$ = __________
sec m$\angle y$ = __________
cos m$\angle x$ = __________
cosec $\angle y$ = __________
cosec $\angle x$ = __________

Individual work

Give Exercise 11a questions 1, 2, 5, and 8 to be done in the class.

Homework

Give the rest of the questions from Exercise 11a as homework.

Recapitulation

Trigonometry is the study of triangles. If we know three elements out of the six elements including one side, we can find the remaining three elements with the help of trigonometric ratios. Real-life problems can be solved with the help of trigonometric ratios (finding heights and distances).

UNIT 12

INFORMATION HANDLING

Lesson 1

Topic: Information handling
Time: 1 period

Objectives

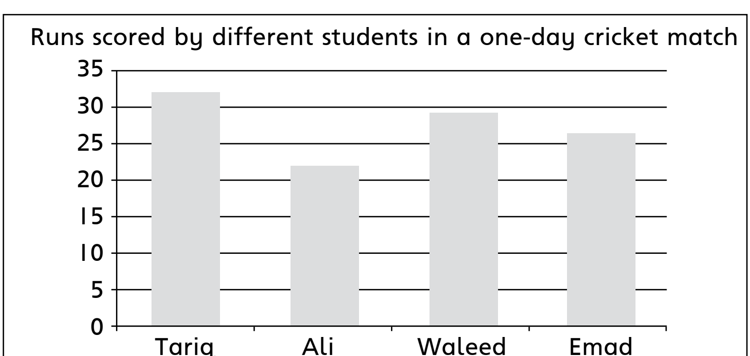
To enable students to:

- define frequency distribution
- construct frequency tables for grouped and ungrouped data
- define and construct a histogram
- define and calculate the measures of central tendency (mean, median and mode) for grouped and ungrouped data

Starter activities

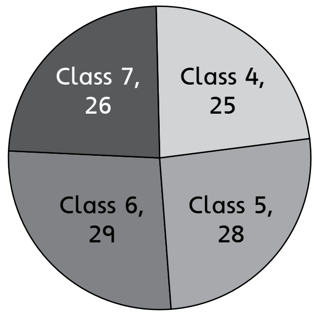
Activity 1

Display pie charts and standard bar graphs.



Ask the following questions:

- Which student has the highest score in the match?
- Which student has the lowest score in the match?
- How many runs did Waleed score in the match?



Ask the following questions:

- Which class has the highest strength?
- Which class has the lowest strength?
- What is the total number of students?

Activity 2

Give a worksheet with the following information:
25 students appeared in a test and obtained the following marks out of 100.

21 35 65 25 13 45 72 50 69 20 49 39 58 29 74
70 69 12 80 75 10 90 100 95 88

Ask the following questions to get information from the data given.

- How many students scored 50 marks?
- What was the highest score?
- How many scored 90 marks?
- How many students scored marks between 60 and 70?
- How many students scored less than 50 marks?

Compare and discuss the answers the students give.

Main lesson

Display an organized list of the above mentioned data and explain to the students the importance of organizing the data and its effective use.

Introduce frequency distribution referring to page 187 of the textbook. Solve the examples on the board. Define and explain a Histogram.
Explain how to make a histogram on the board with the help of the example on page 188 of the textbook.

Frequency distribution of ungrouped and grouped data will be explained.

- Ungrouped data: arranging the data in ascending or descending order.
- Grouped data: the data is divided into different classes or groups with a uniform class interval. The terms range, class interval, lower limit, upper limit, frequency of class interval, size of class interval will be explained with the help of examples given on page 189 of the textbook.

Practice session

1. Form a frequency distribution table from the following information.
   Note: The ungrouped data on page 196, Exercise 12a.3 of the textbook will be used.
2. Form a group frequency distribution table for the following data.
   Note: The data given on page 196, Exercise 12a.1 of the textbook will be used.
3. Draw a histogram for the following data.
   Note: Any data can be used.

Lesson 2

Topic: Measurement of Central Tendency
Time: 1 period

Explain the term, Central Tendency. Discuss its importance, methods of calculation and application in everyday life.
Mean: It is defined as the single representative value of the entire data. It is commonly known as the Average. It is calculated by adding up all the data and dividing by the total number of observations. It is denoted by $\bar{x}$.

Formula

Mean or $X = \Sigma\frac{x}{n}$

Where Σ = sum of

x = observation or value

n = no. of observations

$\overline{X} = \frac{x^2 + x^2 + x^3 \ldots}{n}$

Explain the term weighted mean as given in the textbook.
Median of ungrouped data: It is the central value of a data set, after the data has been arranged in ascending or descending order. If the number of observations is an even number, then there are two middle terms. In this case, the average of these two terms is the median.
If the number of observations is an odd number, then there will only be one

middle term and it will be the median. Explain with the help of examples given in the textbook.

Mode of ungrouped data: It is the value which occurs most frequently in a data set.

Practice session

Give worksheets comprising of questions similar to these.

1. Find the mean.
   100, 120, 125, 105, 109, 112, 120, 126, 105, 111, 113
2. Find the weighted mean of the following ages of students.
   a. 12, 13, 14, 14, 13, 12, 17, 13, 15, 14, 16, 11, 14, 10
   b. 9, 11, 10, 8, 18, 19
3. Find the median.
   a. 41, 46, 38, 37, 49, 35, 30
   b. 50, 48, 32, 55, 48, 52, 56, 60, 62, 64
4. Find the mode.
   a. 102, 135, 138, 250, 135, 102, 102, 192, 200, 240, 138
   b. 61, 68, 63, 65, 60, 61, 61, 60, 60, 48, 59, 59

Individual work

Give selected questions from Exercise 12a for class practice.

Homework

Give the rest of the questions from Exercise 12a to be done as homework.

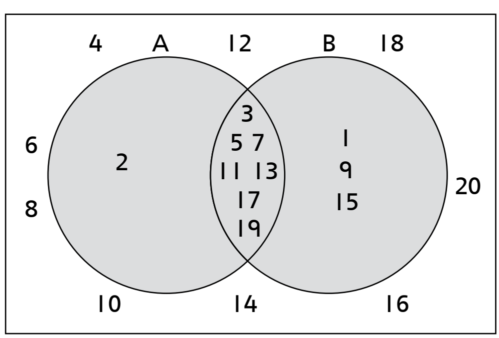
Recapitulation

Revise the formation of frequency distribution tables, the construction of histograms and the methods of calculating the mean, median and mode.

ANSWERS

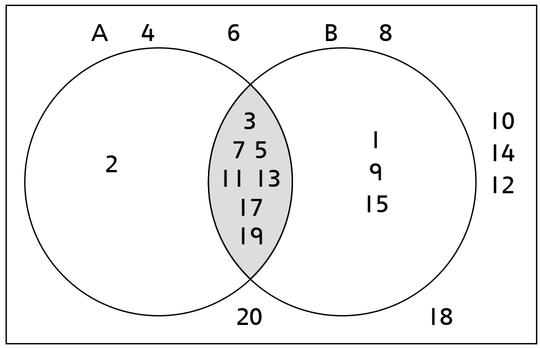
Getting Ready for Class 8

1. 56
2. 426, 486
3. 14.42 seconds
4. a) 64.38 b) 50.53 c) 29.07 d) 0.19 e) 99.10
5. Rs 306
6. a) $3a^2 + 2b^2 + 4c^2$ b) $3x^3 + x^2 + 3x + 3$
   c) -4 d) $a^{16} - b^{16}$
   e) $x + 2y - z$ f) $3a^2b^2 - 2a^2b^2bc^2 - 4abc + 8c^2$
   g) $a^2 + 0.18a + 0.36b - 0.7ab - 1.4b^2 - 0.072$
   h) $a^4 - 2a^2b^2 + b^4$ i) $6a^3$
7. a) 11 b) 4 c) 16 d) $\frac{3}{4}$ e) 5
   f) 4 g) 3 h) 8 i) 14 j) 10
8. 50
9. Rs 1000
12. Venn diagrams to be drawn here.
   a) U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20}
   A = {2, 3, 5, 7, 11, 13, 17, 19}
   B = {1, 3, 5, 7, 9, 11, 13, 15, 17, 19}
   i) A ∪ B = {1, 2, 3, 5, 7, 9, 11, 15, 17, 19}



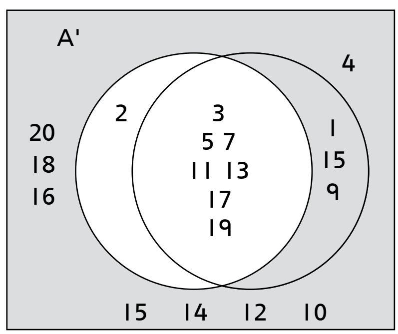
A ∪ B is shaded

ii) $A \cap B = \{3, 5, 7, 11, 13, 17, 19\}$



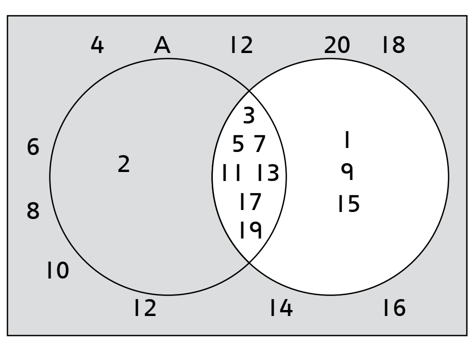
A ∩ B is shaded

iii) $A' = \{1, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20\}$



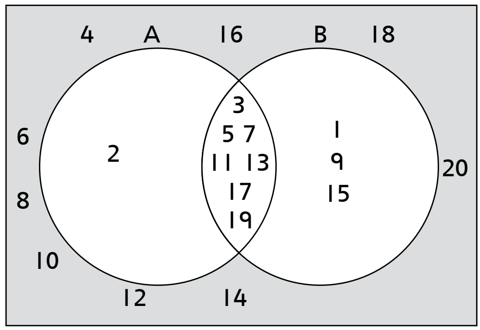
A′ is shaded

iv) $B' = \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\}$



B′ is shaded

v) $A' \cap B' = \{4, 6, 8, 10, 12, 14, 16, 18, 20\}$



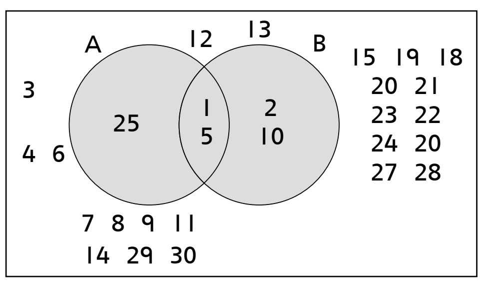
$A' \cap B'$ is shaded

b) $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30\}$

$A = \{1, 5, 25\}$

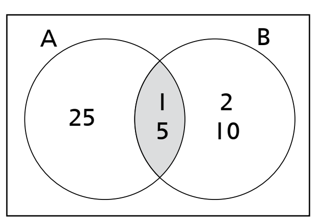
$B = \{1, 2, 5, 10\}$

i) $A \cup B = \{1, 2, 5, 10, 25\}$



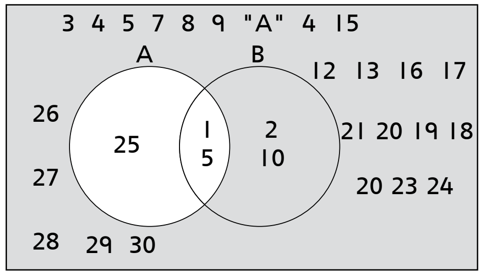
$A \cup B$ is shaded

ii) $A \cap B = \{1, 5\}$



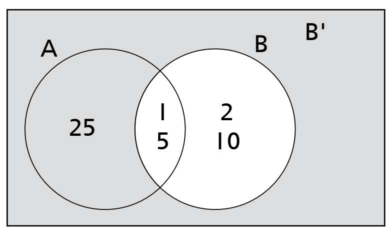
$A \cap B$

iii) A′ = {2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 26, 27, 28, 29, 30}



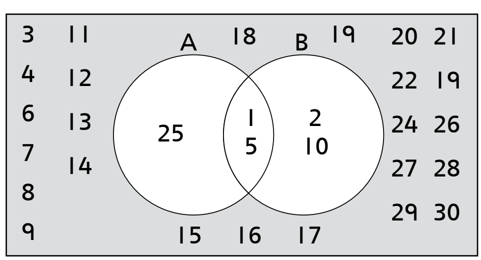
A'

iv) B′ = {3, 4, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30}



B'

v) A′ ∩ B′ = {3, 4, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 26, 27, 28, 29, 30}



A' ∩ B'

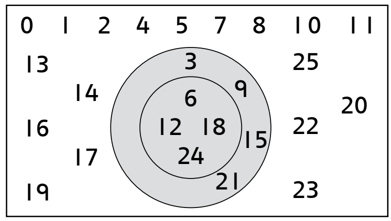
c) U = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25}

A = {3, 6, 9, 12, 15, 18, 21, 24}

B = {6, 12, 18, 24}

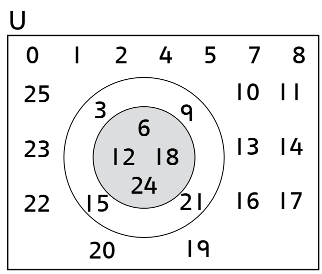
V = {7, 14, 21}

i) $A \cup B = \{3, 6, 9, 12, 15, 18, 21, 24\}$



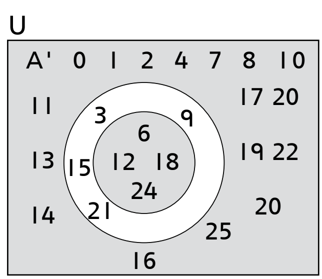
A ∪ B

ii) $A \cap B = \{6, 12, 18, 24\}$



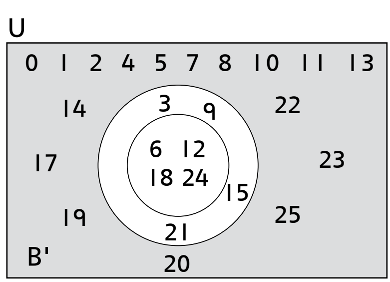
A ∩ B

iii) $A' = \{0, 1, 2, 4, 5, 7, 8, 10, 11, 13, 14, 16, 17, 19, 20, 22, 23, 25\}$



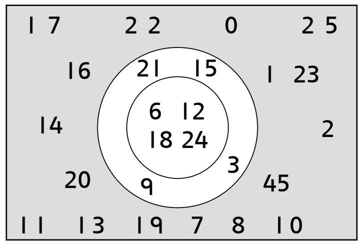
A'

iv) $B' = \{0, 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 19, 20, 21, 22, 23, 25\}$



B'

v) $A' \cap B' = \{0, 1, 2, 4, 5, 7, 8, 10, 11, 13, 14, 16, 17, 19, 20, 22, 23, 25\}$



d) U = {triangle, quadrilateral, square, rectangle, rhombus, kite, trapezium, parallelogram, pentagon, hexagon, heptagon, octagon, nonagon, decagon…}

A = {quadrilateral, square, rectangle, rhombus, kite, parallelogram, trapezium}

B = {regular polygons}

C = {squares}

i) $A \cup B$ = {all polygons}

ii) $A \cap B$ = {square, rhombus}

iii) A' = {triangle, pentagon, hexagon, heptagon, octagon, nonagon, decagon…}

iv) B' = {all irregular polygons}

v) $A' \cap B' = \varnothing$

13. 0.005 m/s

14. 1300 km/hr

15. 0.04 minutes

16. a) $\frac{1}{(45a^3)}$ b) $\frac{(a^{-3}bc^{-5})}{2}$ c) $\frac{1}{(2a)}$

d) $5p^2$ e) $\frac{195(a^2 + b)^2}{256(ab^4)(a + a^2)^3}$

17. 18 cm

18. 20 cm

19. Rs 996

20. 70.90 cm

21.

<table>
<tr><th>Values</th><th>Tally bars</th><th>Frequency</th></tr>
<tr><td>0</td><td>|||</td><td>3</td></tr>
<tr><td>1</td><td>|</td><td>1</td></tr>
<tr><td>2</td><td>||</td><td>2</td></tr>
<tr><td>3</td><td>|||</td><td>3</td></tr>
<tr><td>4</td><td>||</td><td>2</td></tr>
<tr><td>5</td><td>||||</td><td>4</td></tr>
<tr><td>6</td><td>|||</td><td>3</td></tr>
<tr><td>7</td><td>~~||||~~ |</td><td>6</td></tr>
<tr><td>8</td><td>||</td><td>2</td></tr>
</table>

22. 85, 87, 89
23. 45 m, 135 m, 270 m
24. $\frac{19}{10}$ and $\frac{7}{128}$
25. a) Area = 16.775 cm^2, Perimeter = 27.71 cm
    b) Area = 153.916 cm^2, Perimeter = 88 cm
26. 0.006 seconds
27. 20 min
28. 5 cm, equilateral
29. a) 48 cm b) 110.85 cm^2
30. 217.5 kg
31. 18:01:30
32. a) px km b) px/c hours
33. a) Rs 960 b) 75%
34. a) 1500 bricks b) Rs 150
35. 14 years
36. a) 7 and 8 b) 10 and 11
37. a) 100 m b) 970 m c) 12 cm d) 8.75 cm^2
38. 196 m^2
39. 78485.36 cm^3
40. 20
41. 6.5, 6, 7.5. No

42. 8.7: rational

−3.5: rational

$\frac{3}{4}$: rational

$\sqrt{5}$: neither

0: whole

$2 \times 7 = 14$: natural, whole, positive integer

−25: negative integer

10^2: whole, natural, positive integer, rational

43. To draw a number line

44. a) 17.997 cm b) 6.3 cm c) 154 cm^2 d) 88 cm

e) i) 78.55 m^2 ii) 157.1 m^2

45. 11.2 hours

Challenge : 244 cubes

Unit 1

Exercise 1b

1. a) 2, −2 b) $\varnothing$ (null set)
2. a) yes b) no c) no
3. a) no b) yes c) yes d) no e) no
4. Let x be an arbitrary element of A. We are going to show that $x \in \subset$. Since A $\subseteq$ B, this implies that $x \in$ B. Since B $\subseteq$ C, we see that $x \in$ C. Since $x \in$ A implies that $x \in$ C, it follows that A $\subseteq$ C.
5. a) $\{\varnothing, \{a\}\}$ b) $\{\varnothing, \{a\}, \{\{a\}\}, \{a, \{a\}\}\}$ c) $\{\varnothing, \{\varnothing\}, \{\{\varnothing\}\}, \{\varnothing, \{\varnothing\}\}\}$

Exercise 1c

1. a) {0, 1, 2, 3, 4, 5} b) {3} c) {1, 2, 4} d) {0, 5}
2. a) B $\subseteq$ A

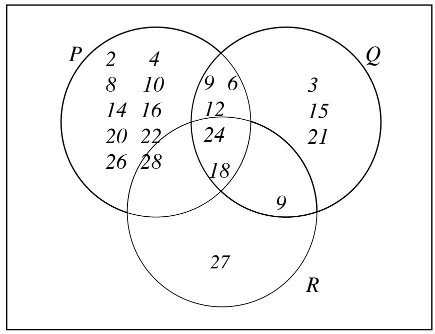
   Since A $\cup$ B = A, every element of B is also in A. But A may have more elements than B.

   b) A $\cap$ A = $\varnothing$

   A − B = A means that subtracting elements of B does not change A. Hence no elements of B are in A. In other words, sets A and B have no common elements.

Exercise 1d

3.



4. B is a subset of A. C is not a unique set.
5. a) $A \cup B = \{3, 6, 8, 9, 12\}$ $A \cap B = \{6, 9\}$
   b) $C \cup D = \{a, b, x, y, m, n, o, p\}$ $C \cap D = \varnothing$
   c) $E \cup F =$ {monkey, goat, lion, tiger} $E \cap F =$ {goat}
   d) $G \cup H = \{a, m, k, y\}$ $G \cap H = \varnothing$

Unit 2

Exercise 2a

1. rational numbers: a, b, e, f, g, i, n
   irrational numbers: c, d, h, j, k, l, m
2. a) $\frac{5}{8}$ b) $\frac{11}{30}$ c) 3 whole $\frac{7}{8}$ d) $\frac{29}{36}$
3. a) 5.3125, 5.625, 6.0625 b) 5.1875, 5.875, 6.5625
   c) 0.625, 0.75, 0.875 d) 11.2, 12, 12.8
4. a) $\sqrt{6}$, $\sqrt{8}$ b. $\sqrt{28}$, $\sqrt{31}$ c) $\sqrt{3}$, $\sqrt{5}$ d) $\sqrt{18}$, $\sqrt{21}$

Exercise 2b

1. a) T b) F c) F d) T e) T
3. a) 17424 b) 278 784
4. a and c
5. b, c, and f

Exercise 2c

1. a) 85 b) 526 c) 630 d) 737 e) $1\frac{1}{49}$
   f) $\frac{25}{82}$ g) 10.8 h) 9.5 i) 0.03 j) 0.12

2. a) 309 b) 6308 c) $\frac{36}{11}$ d) 3.80 e) 0.28
f) 4.32 g) 2.48 h) 0.81 i) 1.52
3. 2.2361, 2.4495, 2.6458
4. 0.051 a) 51 b) 5.1 c) 0.0051
5. a) 3 b) 3 c) 4 d) 4
6. 50 rows
7. 263 men
8. 128 m
9. 3589 flowers
10. 83
11. 3

Exercise 2d

1. 21
2. a) 79 507 b) 2 000 376 c) $\frac{64}{729}$ d) 0.000001 e) −15.625
3. a) 9 b) 11 c) 16
d) $\frac{-5}{7}$ e) 4.5 f) $\frac{7}{4}$
4. 169, 273
5. 36

Unit 3

Exercise 3a

1. a) $(245)_8$ b) $(33232)_5$ c) $(1101001010)_2$
d) $(10010100011)_2$ e) $(11023)_5$ f) $(84611)_8$
2. a) $(100010100)_2$ b) $(2021)_5$ c) $(551)_8$
d) $(14111)_5$ e) $(100001111)_2$ f) $(57562)_8$
3. a) $(433)_8$ b) $(3232)_5$ c) $(10111011)_2$
4. a) $(1001000001001 0)_2$ b) $(432322)_5$ c) $(57646)_8$

Unit 4

Exercise 4a

1. a) 11.25 b) 31.68 c) 13.6
d) 1.6 m e) 66 whole $\frac{2}{3}$ litres

2. 80 m
3. 29.76 kg
4. 5 hours
5. 14.9 kg
6. 859.4 sq feet
7. 2800 kg
8. Rs 14 827
9. A = Rs 30 000, B = Rs 50 000, C = Rs 70 000, D = Rs 100 000
10. P1 = Rs 20 000, P2 = Rs 50 000, Total profit = Rs 100 000

Exercise 4b

1. a) Interest = Rs 225, Amount = Rs 1725
   b) Interest = Rs 4200, Amount = Rs 39 200
   c) Interest = Rs 13 160, Amount = Rs 78 960
   d) Interest = Rs 6160, Amount = Rs 94 160
   e) Interest = Rs 4275, Amount = Rs 51 775
   f) Interest = Rs 17 550, Amount = Rs 107 550
   g) Interest = Rs 4504, Amount = Rs 60 804
2. First amount: Markup = Rs 900
   Second amount: Markup = Rs 1080
   Total amount repaid = Rs 34 980
3. Rs 1056
4. Rs 93 600
5. Rs 23 000
6. Rs 5150
7. a) Rs 7500   b) Rs 5000
8. Rs 4015

Exercise 4c

a) 2 years, 2 months, 24 days
b) 2 years, 5 months, 8 days
c) 6 years, 9 months, 16 days
d) 1 year, 3 months, 28 days

e) 2 years, 1 month, 29 days
f) 1 year, 1 month, 3 days
g) 7 months, 22 days
h) 3 years, 9 months, 15 days
i) 8 months, 28 days
j) 2 years, 11 months, 25 days

Exercise 4d

1. a) Rs 8400 b) Rs 47 600 c) Rs 4284
2. Rs 433.50
3. a) Rs 6700 b) Rs 26 800 c) 2389.67
4. Rs 6100

Exercise 4e

1. a) Rs 6.50 b) Rs 19.50 c) Rs 50 d) Rs 13
   e) Rs 1.50 f) Rs 300 g) Rs 290 h) Rs 235
   i) Rs 750 j) Rs 150 (loss)
2. a) Rs 217.40 b) Rs 26.18 c) Rs 1027.65
   d) Rs 1706.70 e) Rs 6.50 f) Rs 1014.50
   g) Rs 1357.50 h) Rs 266.70 i) Rs 3095.20 j) Rs 10096.50
3. 39.02%
4. Rs 1800
5. a) 25% b) 18.75%
6. 40%
7. 29%
8. Rs 4000
9. Rs 31.25
10. Rs 25 155

Exercise 4f

1. a) Rs 116.25; Rs 348.75
   b) Rs 100, Rs 20; Rs 380
   c) Rs 76.875, Rs 26.90; Rs 511.20
   d) Rs 35, Rs 14.175; Rs 300.83

e) Rs 22.05, Rs 7.32; Rs 285.63
f) Rs 110, Rs 29.70; Rs 960.30
g) Rs 352.50, Rs 149.81; Rs 1847.69
h) Rs 1600, Rs 640; Rs 5760
i) Rs 2053.50, Rs 359.36; Rs 4432.14
j) Rs 1375, Rs 515.625; Rs 3609.38

2. Rs 190
3. Rs 35 640
4. Rs 1900
5. 15% and 5%
6. Rs 14 364
7. 25%
8. Rs 171

Exercise 4g

1. Rs 60 000, Rs 1 000 000, Rs 300 000
2. Rs 6000
3. Rs 1 000 000, Rs 25 000
4. Rs 210 000
5. 10000-deductible is better. Rs 8000

Exercise 4h

1. Rs 170 000
2. Rs 829 000
3. Total = Rs 559 000, Taxable income = Rs 450 000
4. Rs 199 500
5. Rs 288 000

Unit 5

Exercise 5a

1. a) Monomial b) Binomial c) Binomial
   d) Trinomial e) Polynomial f) Trinomial
   g) Polynomial h) Monomial

2. a) 1 b) 2 c) 3 d) 6 e) 7 f) 8 g) 8

3. a) false b) true c) false d) true e) true

4. a) 16 b) b^2c c) $16b^2$ d) $16b$

5. a) a^2b^3 b) $\frac{2b^3}{3}$ c) $\frac{a^2}{3}$

6. a) $18x^6 + x^4 + 2x$ b) $13cz - 16ax - 8by$
   c) $8x^2 + 9x$ d) $6pq - qr$

7. $14x - 14y + 6$ units

8. a) $2a^3 - a^2 + 4a + 7$ b) $p^3 - 3p^2 - 2p - 2$

9. $3a^2 + 6ab - 3b^2$

10. $x^2 - 7$

11. $2x^2 - 5xy + 3z^2$

12. a) $21a^2 - 10a - 16$ b) $11a^2 + 5ab - 23b^2$
    c) $2x^2 + 9x - 5$ d) $\frac{1}{2}[18x^2 - 9xy - 20y^2]$

13. a) $9a - 5b + 3c$ b) $-5x^4y - 3x^2y^3 + 2x^3y^2$
    c) $x + 3$ d) $a + 9$
    e) $4p - 3q$ f) $y^4 - 2y^2 + 4$

14. a) Quotient $= 4a + 2$ Remainder $= 4$
    b) Quotient $= 3a - 4b$ Remainder $= 2b^2$
    c) Quotient $= 7x - 1$ Remainder $= -16$
    d) Quotient $= 2x + 4$ Remainder $= x$

14. $x^2 - 7x - 7$

Unit 6

Exercise 6a

1. a) $a^2 + 12a + 36$ b) $4a^2 + 16a + 16$ c) $9c^2 + 30c + 25$
   d) $16f^2 - 72f + 81$ e) $4a^2 + 12am + 9m^2$ f) $9n^2 - 36n + 36$
   g) $25p^2 - 70p + 49$ h) $36d^2 - 96d + 64$ i) $(8 + 4a)(8 - 4a)$
   j) $(12 + 4a)(12 - 4a)$ k) $(5 + 9b)(5 - 9b)$ l) $(3a + 6)(3a - 6)$
   m) $(7d - 9)(7d + 9)$

2. a) 10 404 b) 9604 c) 149 799
   d) 239 799 e) 8062 f) 14 641

3. a) $\sqrt{6}$ b) $\sqrt{2}$ c) $\sqrt{14}$ d) $8\sqrt{3}$

4. a) 0 b) 2 c) 0

Exercise 6b

1. a) $d(x + y + z)$ b) $8(2 - 3x)$ c) $3a(x + 4y)$
   d) $(6x - 5y)(4v + 3u)$ e) $(a - b)(a + 2)$ f) $(x + 3)^2$
   g) $(3a + 2)(3a - 2)$ h) $5y(x - 5z)$ i) $(k + 9)(k - 9)$
   j) $(x + 4y)(x - 4y)$

Exercise 6c

1. a) $27a^3 + 54a^2 + 36a + 8$ b) $8d^3 + 36d^2 + 54d + 27$
   c) $64c^3 + 48c^2 + 12c + 1$ d) $125 - 150m + 60m^2 - 8m^3$
   e) $1 - 15h + 75h^2 - 125h^3$ f) $343a^3 - 245a^2 + 84a - 8$
2. 198
3. 36
4. −1

Exercise 6d

1. a) $x = 6, y = 1$ b) $x = -1, y = -1$ c) $x = 9, y = 4$
   d) $x = 1, y = 3$ e) $x = 0, y = 1\frac{1}{2}$
2. a) $x = 8, y = 8$ b) $x = 2, y = -1\frac{1}{2}$ c) $x = 1, y = 2$
   d) $x = 2, y = 1$ e) $x = 2, y = -2$
3. 94 and 54
4. Table = Rs 30, Chair = Rs 100
5. Handbag = Rs 150, Briefcase = Rs 270

Unit 7

Exercise 7a

1. a) $\angle a = 70°, \angle b = 110°, \angle c = 110°, \angle d = 70°$
   b) $\angle a = 115°, \angle b = 65°, \angle c = 65°$
   c) $\angle a = 33°, \angle b = 34°, \angle c = 33°$
   d) $\angle a = 100°, \angle b = 80°, \angle c = 35°, \angle d = 65°$
   e) $\angle a = 65°, \angle b = 65°, \angle c = 115°, \angle d = 115°$
   f) $\angle a = 42°, \angle b = 46°, \angle c = 92°$
   g) $\angle a = 42°, \angle b = 48°, \angle c = 132°$
   h) $\angle a = 73°, \angle b = 107°, \angle c = 83°, \angle d = 97°$

i) $\angle x$ = 60°, $\angle y$ = 35°, $\angle z$ = 85°

j) $\angle x$ = 270°

k) $\angle a$ = 120°, $\angle b$ = 60°

l) $\angle a$ = 110°, $\angle b$ = 105°, $\angle c$ = 105°

2. a) ∠P = ∠R = 135°, ∠Q = ∠S = 45°

   b) PQ = 216 units, SP = 137 units

3. $\angle x$ = 96°, $\angle y$ = 29°, $\angle z$ = 30°
4. PA = 36 cm
5. OY = 17.9 cm

Unit 8

Exercise 8a

1. 90°
2. 128°
3. 100°
4. $3\sqrt{2}$
5. ∠B = 115°, ∠C = 65°, ∠D = 115°
6. 10 cm
7. 90°
8. 122°, 58°, 122°
9. ∠NOQ = 57°, ∠QNM = 33°, ∠QPM = 57°
10. 1260°
11. 6
12. 5
13. 202°
14. 15 cm
15. 13 cm
16. 6
17. ∠SPQ = 110°, ∠RQP = 70°, ∠SRQ = 110°, ∠PSR = 70°
18. ∠PSR = 112°, ∠SRP = 28°, ∠QRP = 40°
19. 11 sides
20. 135°, 45°, 135°

Unit 9

Exercise 9a

1. a) 19.21 b) 20.12 c) 20 cm
2. b and c are right-angled triangles
3. 12 cm
4. 11.18 cm
5. 8.66 units
6. 5 m
7. 2.24 km
8. 20 units
9. 8 m
10. 240 cm^2

Exercise 9b

1. a) 30 cm^2, 4.61 cm b) 54 cm^2, 7.2 cm
   c) 213.96 mm^2, 13.36 mm d) 1.2 cm^2, 0.9 cm
2. a) 5 cm^2 b) 1.6 cm
3. a) 14.4 mm^2 b) 6.5 mm
4. a) 110.85 cm^2 b) 0.62 cm^2
5. 816 m^2
6. 60 m
7. 41.57 cm^2

Exercise 9c

1. a) 20.95 cm^3 b) 100.57 cm^3 c) $88x^3/21$ cm^3
   d) 523.81 cm^3 e) 4190.48 cm^3 f) 268.19 cm^3
2. 261.90 cm^3
3. 368.76 cm^3
4. 414.74 cm^3
5. 2.84 cm
6. 23.87 cm
7. 3.72 cm
8. 1386 m^2

9. 7 cm
10. 792 cm^3
11. r = 21 cm, Rs 11 085
12. 12 cm
13. 4096 balls
14. 3 : 2
15. 12 cm
16. 440 balls

Unit 10

Exercise 10a

1. $\angle x$ = 22°
2. $\angle y$ = 120°
3. $\angle z$ = 105°
4. $\angle a$ = 36°
5. ∠AOB = 333°
6. ∠ABC = ∠ACB = 62°
7. ∠ABC = 74°, ∠BAC = 48°, ∠ACB = 58°
8. ∠BAC = 44°, ∠ACB = 68°, ∠ABC = 68°
9. ∠BAC = 66°, ∠ACD = 104°, ∠ACB = 76°
10. ∠BAC = 56°, ∠ACB = 62°, ∠DBC = 118°, ∠BCD = 31°, ∠BDC = 31°
11. ∠QPX = 37°, ∠RPY = 27°, ∠PXQ = 113°, ∠PYR = 113°, ∠QPR = 110°
12. ∠PQR = 40°, ∠PRQ = 70°, ∠PSR = 80°
13. ∠ACB = 75°, ∠AOC = 80°
14. ∠BOP = 93°
15. ∠ARP = 105°, ∠QRB = 105°, ∠BRP = 75°, ∠DSP = 75°
16. ∠QOR = 60°
17. ∠CPY = 110°, ∠DPX = 110°, ∠XOB = 110°, ∠XPC = 70°, ∠AOY = 110°
18. ∠ADE = 98°, ∠DBC = 98°, ∠ACB = 52°, ∠AED = 52°
19. ∠BAC = 35°, ∠ABC = 72.5°, ∠ACE = 72.5°, ∠ACB = 72.5°, ∠DCE = 72.5°
20. ∠SRQ = 75°

Unit 11

Exercise 11a

1. a) 22.5 b) 25.98 c) 8.66 d) 22.63 e) 18
2. a) 12.5 m b) 28.28 m c) 31.18 m
   d) 6 m e) 8 m f) 17.32 m
3. 57.74 m
4. 23.63 m
5. 66.04 m
6. 81.41 m
7. 80.83 m
8. 45°
9. 6.93 m
10. 550 m
11. a) 21.43 m b) 17.91 m

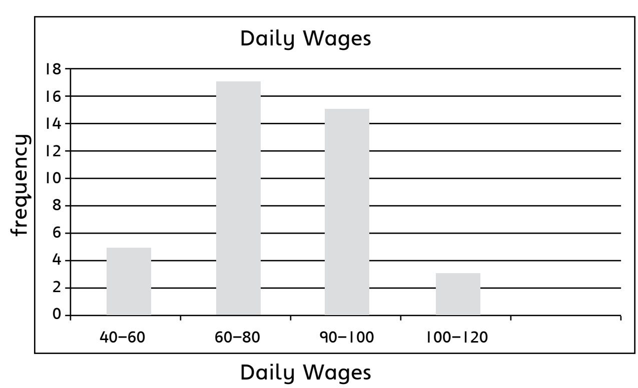
Unit 12

Exercise 12a

1. a)

Daily Wages	Frequency (Number of salesmen)
40 – 60	5
60 – 80	17
80 – 100	15
100 – 120	3

b)



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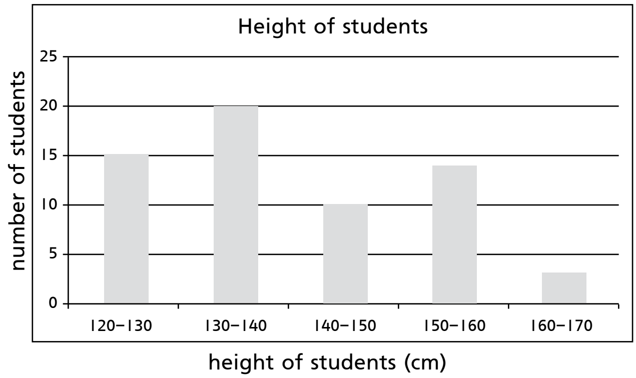

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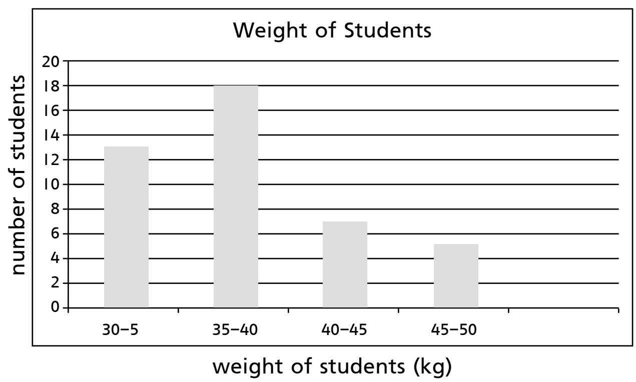
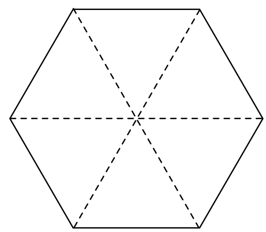


4. a) 27.4 b) 118.4 c) 252.17 d) 7.36 e) 7.8
5. 35.86
6. 27 years
7. 148.5
8. 14
9. 73.68
10. 14 years
11. a) 94 b) 13.5 c) 41 d) 5.01 e) 6
12. a) 15 and 16 (two modes) b) 47
    c) 2 and 3 (two modes) d) no mode
13. 25

14. a) 115 runs b) 102 runs c) 76 runs

15. Mode = 35, mean = 35.7

Puzzle



Notes

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