7

Teaching Guide

NEW COUNTDOWN

Second Edition

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Contents

Introduction

Welcome, users of the Countdown series. Countdown has been the choice of Mathematics teachers for many years. This Teaching Guide has been specially designed to help them teach mathematics in the best possible manner. It will serve as a reference book to streamline the teaching and learning experience in the classroom.

Teachers are entrusted with the task of providing support and motivation to their students, especially those who are at the lower end of the spectrum of abilities. In fact, their success is determined by the level of understanding demonstrated by the least able students.

Teachers regulate their efforts and develop a teaching plan that corresponds to the previous knowledge of the students and difficulty of the subject matter. The more well-thought out and comprehensive a teaching plan is, the more effective it is. This teaching guide will help teachers streamline the development of a lesson plan for each topic and guide the teacher on the level of complexity and amount of practice required for each topic. It also helps the teacher introduce effective learning tools to the students to complete their learning process.

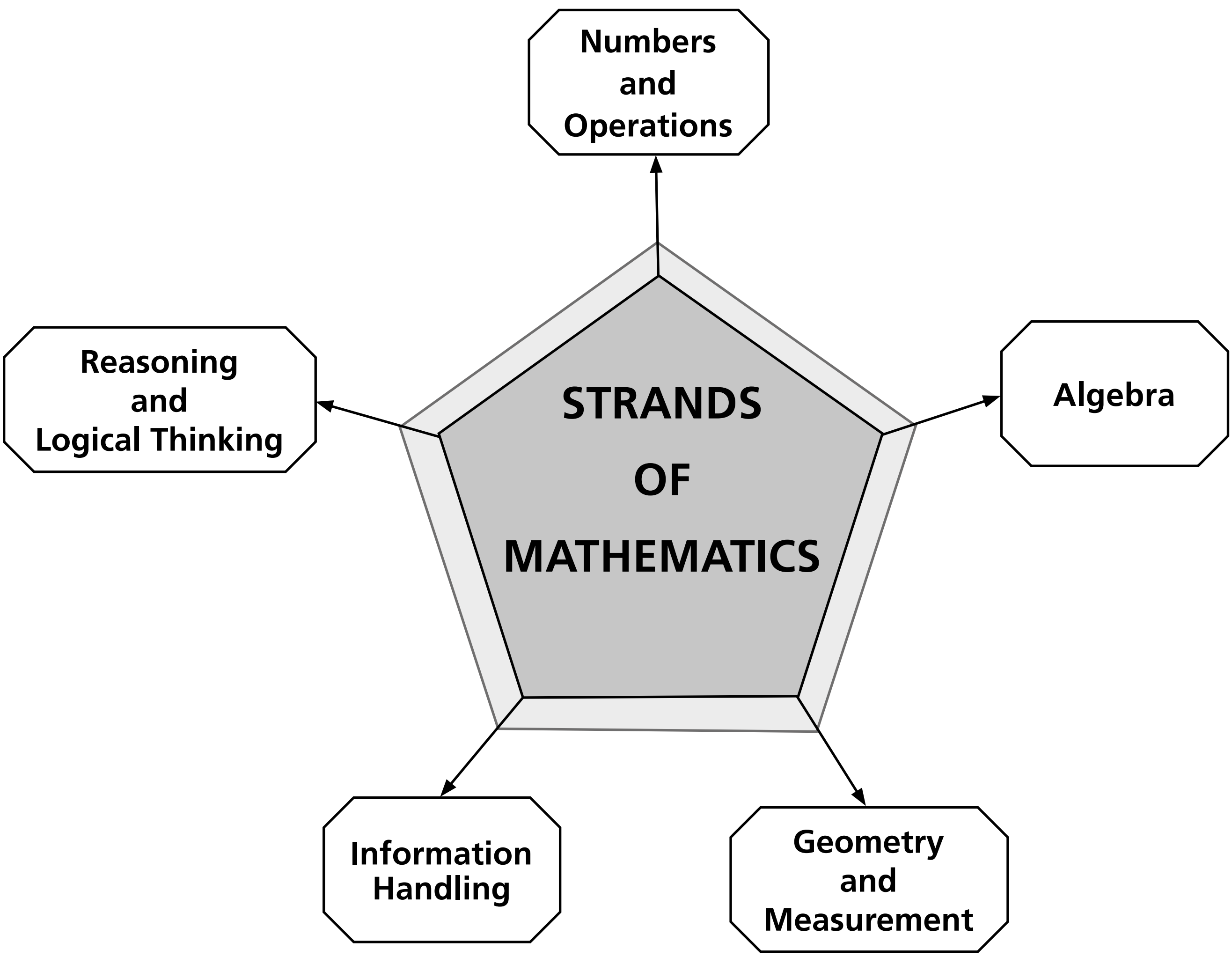
Shazia Asad

Curriculum

Strands and Benchmarks

(National Curriculum for Mathematics 2006)

The National Curriculum for Mathematics 2006 is based on these five strands:



Towards greater focus and coherence of a mathematical programme

A comprehensive and coherent mathematical programme needs to allocate proportional time to all strands. A composite strand covers number, measurement and geometry, algebra, and information handling.

Each strand requires a focussed approach to avoid the pitfall of a broad general approach. If, say, an algebraic strand is approached, coherence and intertwining of concepts within the strand at all grade levels is imperative. The aims and objectives of the grades below and above should be kept in mind.

"What and how students are taught should reflect not only the topics that fall within a certain academic discipline, but also the key ideas that determine how knowledge is organised and generated within that discipline."

William Schmidt and Richard Houang (2002)

Strands and Benchmarks of the Pakistan National Curriculum (2006)

Strand 1: Numbers and Operations

The students will be able to:

- identify numbers, ways of representing numbers, and effects of operations in various situations;
- compute fluently with fractions, decimals, and percentages, and
- manipulate different types of sequences and apply operations on matrices.

Benchmarks

Grades VI, VII, VIII

- Identify different types of sets with notations
- Verify commutative, associative, distributive, and De Morgan's laws with respect to union and intersection of sets and illustrate them through Venn diagrams
- Identify and compare integers, rational, and irrational numbers
- Apply basic operations on integers and rational numbers and verify commutative, associative, and distributive properties
- Arrange absolute values of integers in ascending and descending order
- Find HCF and LCM of two or more numbers using division and prime factorization
- Convert numbers from decimal system to numbers with bases 2, 5, and 8, and vice versa
- Add, subtract, and multiply numbers with bases 2, 5, and 8
- Apply the laws of exponents to evaluate expressions
- Find square and square root, cube, and cube root of a real number
- Solve problems on ratio, proportion, profit, loss, mark-up, leasing, *zakat*, *ushr*, taxes, insurance, and money exchange

Strand 2: Algebra

The students will be able to:

- analyse number patterns and interpret mathematical situations by manipulating algebraic expressions and relations;
- model and solve contextualised problems; and
- interpret functions, calculate rate of change of functions, integrate analytically and numerically, determine orthogonal trajectories of a family of curves, and solve non-linear equations numerically.

Benchmarks

Grades VI, VII, VIII

- Identify algebraic expressions and basic algebraic formulae
- Apply the four basic operations on polynomials
- Manipulate algebraic expressions using formulae
- Formulate linear equations in one and two variables
- Solve simultaneous linear equations using different techniques

Strand 3: Measurement and Geometry

The students will be able to:

- identify measurable attributes of objects, and construct angles and two dimensional figures;
- analyse characteristics and properties and geometric shapes and develop arguments about their geometric relationships; and
- recognise trigonometric identities, analyse conic sections, draw and interpret graphs of functions.

Benchmarks

Grades VI, VII, VIII

- Draw and subdivide a line segment and an angle
- Construct a triangle (given SSS, SAS, ASA, RHS), parallelogram, and segments of a circle
- Apply properties of lines, angles, and triangles to develop arguments about their geometric relationships
- Apply appropriate formulas to calculate perimeter and area of quadrilateral, triangular, and circular regions
- Determine surface area and volume of a cube, cuboid, sphere, cylinder, and cone
- Find trigonometric ratios of acute angles and use them to solve problems based on right-angled triangles

Strand 4: Handling Information

The students will be able to collect, organise, analyse, display, and interpret data.

Benchmarks

Grades VI, VII, VIII

- Read, display, and interpret bar and pie graphs
- Collect and organise data, construct frequency tables and histograms to display data
- Find measures of central tendency (mean, median and mode)

Strand 5: Reasoning and Logical Thinking

The students will be able to:

- use patterns, known facts, properties, and relationships to analyse mathematical situations;
- examine real-life situations by identifying mathematically valid arguments and drawing conclusions to enhance their mathematical thinking.

Benchmarks

Grades VI, VII, VIII

- Find different ways of approaching a problem to develop logical thinking and explain their reasoning
- Solve problems using mathematical relationships and present results in an organised way
- Construct and communicate convincing arguments for geometric situations

Syllabus Matching Grid

<table>
<tr><th colspan="2">Unit 1: Sets</th></tr>
<tr><td>1.1 Set
i) Express a set in
• descriptive form,
• set builder form,
• tabular form.</td><td>Chapter 1</td></tr>
<tr><td>1.2 Operations on Sets
i) Define union, intersection and difference of two sets.
ii) Find
• union of two or more sets,
• intersection of two or more sets,
• difference of two sets.
iii) Define and identify disjoint and overlapping sets.
iv) Define a universal set and complement of a set.
v) Verify different properties involving union of sets, intersection of sets, difference of sets and complement of a set, e.g., $A \cap A' = \emptyset$.</td><td>Chapter 1</td></tr>
<tr><td>1.3 Venn Diagram
i) Represent sets through Venn diagram.
ii) Perform operations of union, intersection, difference and complement on two sets A and B when
• A is subset of B,
• B is subset of A,
• A and B are disjoint sets,
• A and B are overlapping sets,
through Venn diagram.</td><td>Chapter 1</td></tr>
<tr><th colspan="2">Unit 2: Rational Numbers</th></tr>
<tr><td>2.1 Rational Numbers
i) Define a rational number as a number that can be expressed in the form $\frac{p}{q}$, where p and q are integers and $q > 0$.
ii) Represent rational numbers on number line.</td><td rowspan="2">Chapter 2</td></tr>
<tr><td>2.2 Operations on Rational Numbers
i) Add two or more rational numbers.
ii) Subtract a rational number from another.
iii) Find additive inverse of a rational number.
iv) Multiply two or more rational numbers.
v) Divide a rational number by a non-zero rational number.
vi) Find multiplicative inverse of a rational number.
vii) Find reciprocal of a rational number.</td></tr>
</table>

viii) Verify commutative property of rational numbers with respect to addition and multiplication. ix) Verify associative property of rational numbers with respect to addition and multiplication. x) Verify distributive property of rational numbers with respect to multiplication over addition/subtraction. xi) Compare two rational numbers. xii) Arrange rational numbers in ascending or descending order.	Chapter 2
Unit 3: Decimals	
3.1 Conversion of Decimals to Rational Numbers Convert decimals to rational numbers.	Chapter 3
3.2 Terminating and Non- terminating Decimals i) Define terminating decimals as decimals having a finite number of digits after the decimal point. ii) Define recurring decimals as non-terminating decimals in which a single digit or a block of digits repeats itself infinite number of times after decimal point iii) Use the following rule to find whether a given rational number is terminating or not. Rule: If the denominator of a rational number in standard form has no prime factor other than 2, 5 or 2 and 5, then and only then the rational number is a terminating decimal. iv) Express a given rational number as a decimal and indicate whether it is terminating or recurring.	
3.3 Approximate Value Get an approximate value of a number, called rounding off, to a desired number of decimal places.	Chapter 3
Unit 4: Exponents	
4.1 Exponents/Indices Identify base, exponent and value.	Chapter 5
4.2 Laws of Exponents/Indices i) Use rational numbers to deduce laws of exponents. • Product Law: when bases are same but exponents are different: $a^m \times a^n = a^{m+n}$, when bases are different but exponents are same: $a^n \times b^n = (ab)^n$, • Quotient Law: when bases are same but exponents are different: $a^m \div a^n = a^{m-n}$, when bases are different but exponents are same: $a^n \div b^n = \left(\frac{a}{b}\right)^n$,	

• Power law: $(a^m)^n = a^{mn}$. • For zero exponent: $a^0 = 1$. • For exponent as negatlve mteger: $a^{-m} = \frac{1}{a^m}$ ii) Demonstrate the concept of power of integer that is $(-a)$" when n is even or odd integer. iii) Apply laws of exponents to evaluate expressions.	Chapter 5
Unit 5: Square Root of Positive Number	
5.1 Perfect Squares i) Define a perfect square. ii) Test whether a number is a perfect square or not. iii) Identify and apply the following properties of perfect square of a number. • The square of an even number is even. • The square of an odd number is odd. • The square of a proper fraction is less than itself. • The square of a decimal less than I is smaller than the decimal.	Chapter 4
5.2 Square Roots i) Define square root of a natural number and recognise its notation. ii) Find square root, by division method and factorization method, of • natural number, • fraction, • decimal, which are perfect squares. iii) Solve real-life problems involving square roots.	Chapter 4
Unit 6: Direct and Inverse Variation	
6.1 Continued Ratio i) Define continued ratio and recall direct and inverse,proportion. ii) Solve real-life problems (involving direct and inverse proportion) using unitary method and proportion method.	Chapter 6
6.2 Time, Work and Distance i) Solve real-life problems related to time and work using proportion. ii) Find relation (i.e. speed) between time and distance. iii) Convert units of speed (kilometre per hour into metre per second and vice versa). iv) Solve variation related problems involving time and distance.	Chapter 6
Unit 7 Financial Arithmetic	
7.1 Taxes i) Explain property tax and general sales tax. ii) Solve tax-related problems.	Chapter 7

7.2 Profit and Markup i) Explain profit and markup. ii) Find the rate of profit/markup per annum. iii Solve real-life problems involving profit! markup.	Chapter 7
7.3 Zakat and U shr i) Define zakat and ushr. ii) Solve problems related to zakat and ushr.	
Unit 8: Algebraic Expressions	
8.1 Algebraic Expressions i) Define a constant as a symbol having a fixed numerical value. ii) Recall variable as a quantity which can take various numerical values. iii) Recall literal as an unknown number represented by an alphabet. iv) Recall algebraic expression as a combination of constants and variables connected by the signs of fundamental operations. v) Define polynomial as an algebraic expression in which the powers of variables are all whole numbers. vi) Identify a monomial, a binomial and a trinomial as a polynomial having one term, two terms and three terms respectively.	Chapter 8
8.2 Operations with Polynomials i) Add two or more polynomials. ii) Subtract a polynomial from another polynomial. iii) Find the product of • monomial with monomial, • monomial with binomial/trinomial, • binomials with binomial/trinomial. iv) Simplify algebraic expressions involving addition, subtraction and multiplication.	Chapter 8
8.3 Algebraic Identities Recognise and verify the algebraic identities: • $(x + a)(x + b) = x^2 + (a + b)x + ab$, • $(a + b)^2 = (a + b)(a + b) = a^2 + 2ab + b^2$, • $(a - b)^2 = (a - b)(a - b) = a^2 - 2ab + b^2$, • $a^2 - b^2 = (a - b)(a + b)$.	Chapter 9
8.4 Factorization of Algebraic Expressions i) Factorise an algebraic expression (using algebraic identities). ii) Factorise an algebraic expression (making groups).	Chapter 10

<table>
<tr><th colspan="2">Unit 9: Linear Equations</th></tr>
<tr><td>9.1 Linear Equation
i) Define a linear equation in one variable.</td><td rowspan="2">Chapter 11</td></tr>
<tr><td>9.2 Solution of Linear Equation
i) Demonstrate different techniques to solve linear equation.
ii) Solve linear equations of the type:
• $ax + b = c$,
• $\frac{ax+b}{cx+d} = \frac{m}{n}$
iii) Solve real-life problems involving linear equations.</td></tr>
<tr><th>Unit 10 Fundamentals of Geometry</th><th></th></tr>
<tr><td>10.1 Properties of Angles
i) Define adjacent, complementary and supplementary angles.
ii) Define vertically opposite angles.
iii) Calculate unknown angles involving adjacent angles, complementary angles, supplementary angles and vertically opposite angles.
iv) Calculate unknown angle of a triangle.</td><td>Chapter 12</td></tr>
<tr><td>10.2 Congruent and Similar
i) Identify congruent and similar figures.
ii) Recognise the symbol of congruency.
iii) Apply the properties for two figures to be congruent or similar.</td><td rowspan="2">Chapter 15</td></tr>
<tr><td>10.3 Congruent Triangles
Apply following properties for congruency between two triangles.
• $SSS \cong SSS$,
• $SAS \cong SAS$,
• $ASA \cong ASA$,
• $RHS \cong RHS$.</td></tr>
<tr><td>10.4 Circle
i) Describe a circle and its centre, radius, diameter, chord, arc, major and minor arcs, semicircle and segment of the circle.
ii) Draw a semicircle and demonstrate the property; the angle in a semicircle is a right angle.
iii) Draw a segment of a circle and demonstrate the property; the angles in the same segment of a circle are equal.</td><td>Chapter 14</td></tr>
<tr><th colspan="2">Unit 11: Practical Geometry</th></tr>
<tr><td>11.1 Line Segment
i) Divide a line segment into a given number of equal segments.
ii) Divide a line segment internally in a given ratio.</td><td>*Covered in Book 8</td></tr>
</table>

11.2 Triangles i) Construct a triangle when perimeter and ratio among the lengths of sides are given. ii) Construct an equilateral triangle when • base is given, • altitude is given. iii) Construct an isosceles triangle when • base and a base angle are given, • vertical angle and altitude are given, • altitude and a base angle are given.	Chapter 13
11.3 Parallelogram i) Construct a parallelogram when • two adjacent sides and their included angle are gIven, • two adjacent sides and a diagonal are given. ii) Verify practically that the sum of • measures of angles of a triangle is 180° • measures of angles of a quadrilateral is 360°	Chapter 16
Unit 12: Circumference, Area and Volume	
12.1 Circumference and Area of Circle i) Express π as the ratio between the circumference and the diameter of a circle. ii) Find the circumference of a circle using formula. iii) Find the area of a circular region using formula.	Chapter 17
12.2 Surface Area and Volume of Cylinder i) Find the surface area of a cylinder using formula. ii) Find the volume of a cylindrical region using formula. iii) Solve real-life problems involving surface area and volume of a cylinder.	Chapter 18
Unit 13: Frequency Distribution	
13.1 Frequency Distribution i) Demonstrate data presentation. ii) Define frequency distribution (i.e. frequency, lower class limit, upper class limit, class interval).	Chapter 19
13.2 Pie Graph Interpret and draw pie graph.	

Teaching and Learning

Guiding Principles

1. Students explore mathematical ideas in ways that maintain their enjoyment of and curiosity about mathematics, help them develop depth of understanding, and reflect real-world applications.
2. All students have access to high quality mathematics programmes.
3. Mathematics learning is a lifelong process that begins and continues in the home and extends to school, community settings, and professional life.
4. Mathematics instruction both connects with other disciplines and moves toward integration of mathematical domains.
5. Working together in teams and groups enhances mathematical learning, helps students communicate effectively, and develops social and mathematical skills.
6. Mathematics assessment is a multifaceted tool that monitors student performance, improves instruction, enhances learning, and encourages student self-reflection.

Principle 1

Students explore mathematical ideas in ways that maintain their enjoyment of and curiosity about mathematics, help them develop depth of understanding, and reflect real-world applications.

- The understanding of mathematical concepts depends not only on what is taught, but also hinges on the way the topic is taught.
- In order to plan developmentally appropriate work, it is essential for teachers to familiarise themselves with each individual student's mathematical capacity.
- Students can be encouraged to muse over their learning and express their reasoning through questions such as;
  – *How did you work through this problem?*
  – *Why did you choose this particular strategy to solve the problem?*
  – *Are there other ways? Can you think of them?*
  – *How can you be sure you have the correct solution?*
  – *Could there be more than one correct solution?*
  – *How can you convince me that your solution makes sense?*
- For effective development of mathematical understanding students should undertake tasks of inquiry, reasoning, and problem solving which are similar to real-world experiences.

- Learning is most effective when students are able to establish a connection between the activities within the classroom and real-world experiences.
- Activities, investigations, and projects which facilitate a deeper understanding of mathematics should be strongly encouraged as they promote inquiry, discovery, and mastery.
- Questions for teachers to consider when planning an investigation:

  – *Have I identified and defined the mathematical content of the investigation, activity, or project?*

  – *Have I carefully compared the network of ideas included in the curriculum with the students' knowledge?*

  – *Have I noted discrepancies, misunderstandings, and gaps in students' knowledge as well as evidence of learning?*

Principle 2

All students have access to high quality mathematics programmes.

- Every student should be fairly represented in a classroom and be ensured access to resources.
- Students develop a sense of control of their future if a teacher is attentive to each student's ideas.

Principle 3

Mathematics learning is a lifelong process that begins and continues in the home and extends to school, community settings, and professional life.

- The formation of mathematical ideas is a part of a natural process that accompanies prekindergarten students' experience of exploring the world and environment around them. Shape, size, position, and symmetry are ideas that can be understood by playing with toys that can be found in a child's playroom, for example, building blocks.
- Gathering and itemising objects such as stones, shells, toy cars, and erasers, leads to discovery of patterns and classification. At secondary level research data collection, for example, market reviews of the stock market and world economy, is an integral continued learning process. Within the environs of the classroom, projects and assignments can be set which help students relate new concepts to real-life situations.

Principle 4

Mathematics instruction both connects with other disciplines and moves toward integration of mathematical domains.

An evaluation of maths textbooks considered two critical points. The first was, did the textbook include a variety of examples and applications at different levels so that students could proceed from simple to more complex problem-solving situations?

And the second was whether algebra and geometry were truly integrated rather than presented alternately.

- It is important to understand that students are always making connections between their mathematical understanding and other disciplines in addition to the connections with their world.
- An integrated approach to mathematics may include activities which combine sorting, measurement, estimation, and geometry. Such activities should be introduced at primary level.
- At secondary level, connections between algebra and geometry, ideas from discrete mathematics, statistics, and probability, establish connections between mathematics and life at home, at work, and in the community.

- What makes integration efforts successful is open communication between teachers. By observing each other and discussing individual students teachers improve the mathematics programme for students and support their own professional growth.

Principle 5

Working together in teams and groups enhances mathematical learning, helps students communicate effectively, and develops social and mathematical skills.

- The Common Core of Learning suggests that teachers 'develop, test, and evaluate possible solutions'.
- Team work can be beneficial to students in many ways as it encourages them to interact with others and thus enhances self-assessment, exposes them to multiple strategies, and teaches them to be members of a collective workforce.
- Teachers should keep in mind the following considerations when dealing with a group of students:
  – *High expectations and standards should be established for all students, including those with gaps in their knowledge bases.*
  – *Students should be encouraged to achieve their highest potential in mathematics.*
  – *Students learn mathematics at different rates, and the interest of different students' in mathematics varies.*
- Support should be made available to students based on individual needs.
- Levels of mathematics and expectations should be kept high for all students.

Principle 6

Mathematics assessment in the classroom is a multifaceted tool that monitors student performance, improves instruction, enhances learning, and encourages student self-reflection.

- An open-ended assessment facilitates multiple approaches to problems and creative expression of mathematical ideas.
- Portfolio assessments imply that teachers have worked with students to establish individual criteria for selecting work for placement in a portfolio and judging its merit.
- Using observation for assessment purposes serves as a reflection of a students' understanding of mathematics, and the strategies he/she commonly employs to solve problems and his/her learning style.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Use appropriate tools strategically.
5. Attend to precision and format.
6. Express regularity in repetitive reasoning.
7. Analyse mathematical relationships and use them to solve problems.
8. Apply and extend previous understanding of operations.
9. Use properties of operations to generate equivalent expressions.
10. Investigate, process, develop, and evaluate data.

Lesson Planning

Before starting lesson planning, it is imperative to consider teaching and the art of teaching.

FURL

First **U**nderstand by **R**elating to day-to-day routine, and then **L**earn. It is vital for teachers to relate fine teaching to real-life situations and routine.
'R' is re-teaching and revising, which of course falls under the supplementary/continuity category.
Effective teaching stems from engaging every student in the classroom. This is only possible if you have a comprehensive lesson plan.
There are three integral facets to lesson planning: curriculum, instruction, and evaluation.

1. **Curriculum**

   A syllabus should pertain to the needs of the students and objectives of the school. It should be neither over-ambitious, nor lacking. (One of the major pitfalls in school curricula arises in planning of mathematics.)

2. **Instructions**

   Any method of instruction, for example verbal explanation, material aided explanation, or teach-by-asking can be used. The method adopted by the teacher reflects his/her skills. Experience alone does not work, as the most experienced teachers sometime adopt a short-sighted approach; the same could be said for beginner teachers. The best teacher is the one who works out a plan that is customised to the needs of the students, and only such a plan can succeed in achieving the desired objectives.

3. **Evaluation**

   The evaluation process should be treated as an integral teaching tool that tells the teachers how effective they have been in their attempt to teach the topic. No evaluation is just a test of student learning; it also assesses how well a teacher has taught.

   Evaluation has to be an ongoing process; during the course of study formal teaching should be interspersed with thought-provoking questions, quizzes, assignments, and classwork.

Long-term Lesson Plan

A long-term lesson plan extends over the entire term. Generally schools have coordinators to plan the big picture in the form of Core Syllabus and Unit Studies.
Core syllabi are the topics to be covered during a term. Two things which are very important during planning are the 'Time Frame' and the 'Prerequisites' of the students.
An experienced coordinator will know the depth of the topic and the ability of the students to grasp it in the assigned time frame.

Suggested Unit Study Format

Weeks	Dates	Months	Days	Remarks

Short-term Lesson Planning

A short-term plan is a day-to-day lesson plan, based on the sub-topics chosen from the long-term plan.

Features of the Teaching Guide

The Teaching Guide contains the following features. The headings through which the teachers will be led are explained as follows.

Specific Learning Objectives

Each topic is explained clearly by the author in the textbook with detailed explanation, supported by worked examples. The guide will define and highlight the objectives of the topic. It will also outline the learning outcomes and objectives.

Suggested Time Frame

Timing is important in each of the lesson plans. The guide will provide a suggested time frame. However, every lesson is important in shaping the behavioural and learning patterns of the students. The teacher has the discretion to either extend or shorten the time frame as required.

Prior Knowledge and Revision

It is important to highlight any background knowledge of the topic in question. The guide will identify concepts taught earlier or, in effect, revise the prior knowledge. Revision is essential, otherwise the students may not understand the topic fully.

The initial question when planning for a topic should be how much do the students already know about the topic? If it is an introductory lesson, then a preceding topic could be touched upon, which could lead on to the new topic. In the lesson plan, the teacher can note what prior knowledge the students have of the current topic.

Real-life Application and Activities

Today's students are very proactive. The study of any topic, if not related to practical real-life, will not excite them. Their interest can easily be stimulated if we relate the topic at hand to real-life experiences. Activities and assignments will be suggested which will do just that.

Flash cards based on the concept being taught will have more impact.

Summary of Key Facts

Facts and rules mentioned in the text are listed for quick reference.

Frequently Made Mistakes

It is important to be aware of students' common misunderstandings of certain concepts. If the teacher is aware of these they can be easily rectified during the lessons. Such topical misconceptions are mentioned.

Sample Lesson Plan

Planning your work and then implementing your plan are the building blocks of teaching.

Teachers adopt different teaching methods/approaches to a topic.

A sample lesson plan is provided in every chapter as a preliminary structure that can be followed. A topic is selected and a lesson plan written under the following headings:

Topic

This is the main topic/sub-topic.

Specific Learning Objectives

This identifies the specific learning objective/s of the sub-topic being taught in that particular lesson.

Suggested Duration

Suggested duration is the number of periods required to cover the topic. Generally, class dynamics vary from year to year, so flexibility is important.

The teacher should draw his/her own parameters, but can adjust the teaching time depending on the receptivity of the class to that topic. Note that introduction to a new topic takes longer, but familiar topics tend to take less time.

Key vocabulary

List of mathematical words and terms related to the topic that may need to be pre-taught.

Method and Strategy

This suggests how you could demonstrate, discuss, and explain a topic.

The introduction to the topic can be done through starter activities and recap of previous knowledge which can be linked to the current topic.

Resources (Optional)

This section includes everyday objects and models, exercises given in the chapter, worksheets, assignments, and projects.

Written Assignments

Finally, written assignments can be given for practice. It should be noted that classwork should comprise sums of all levels of difficulty, and once the teacher is sure that students are capable of independent work, homework should be handed out. For continuity, alternate sums from the exercises may be done as classwork and homework.

Supplementary Work (Optional): A project or assignment could be given. It could involve group work or individual research to complement and build on what students have already learnt in class.

The students will do the work at home and may present their findings in class.

Evaluation

At the end of each sub-topic, practice exercises should be done. For further practice, the students can be given a practice worksheet or a comprehensive marked assessment.

1 Operations on Sets

Specific Learning Objectives

- Operations of sets involving unions, intersections, complements, subsets, and difference of sets
- Venn diagrams
- Commutative and associative properties of sets

Suggested Time Frame

4 to 5 periods

Prior Knowledge and Revision

Students have been introduced to the concepts of sets and their descriptive and tabular notations. A quick review quiz can be conducted where the teacher writes the various types of sets from the list below and encourages the students to come up with the correct terminology.

- Empty set
- Finite and infinite sets
- Disjoint and overlapping sets
- Equivalent and equal sets
- Universal set
- Subset and superset

Real-life Application and Activities

The symbols table on page 8 needs to be highlighted in the lesson. The teacher can display the signs on chart paper on the soft board. At the beginning of every lesson, the symbols could be revised orally by looking at the chart presentation for a minute.

This will ensure that the students are well-versed in the symbols. They need to be able to read the symbols as a language of mathematics. They can feel confident when a teacher writes a set notation on the board and the students are able to explain it in words.

Example

Symbols used in sets

	Symbol	Meaning	Example
1.	=	is equal to	{1, 2, 5} = {5, 1, 2} (Sets having exactly the same members are equal sets.)
2.	≠	is not equal to	{1, 2, 5} ≠ {1, 2, 4}
3.	∈	is a member of	3 ∈ {1, 2, 3}
4.	∉	is not a member of	3 ∉ {0, 1, 2}
5.	Φ	is the empty set or the null set	{ }

Starter activity

The teacher can make a table of a fictional set of students who got A's in mathematics and English. While doing so, the students will observe that some got A's in both subjects.

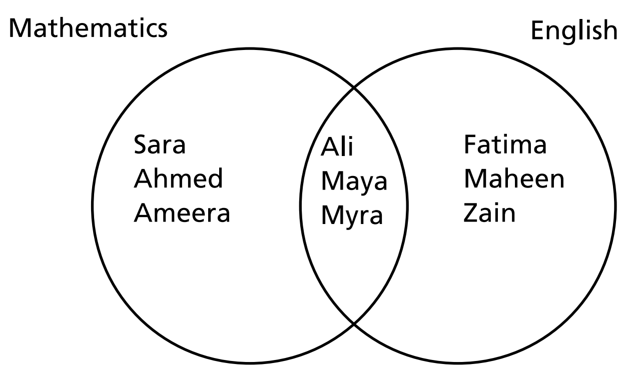
Set of students who got an A in mathematics:

{Ali, Maya, Myra, Sara, Ahmed, Ameera}

Set of students who got an A in English:

{Fatima, Maheen, Zain, Ali, Maya, Myra}

The teacher will highlight the fact that three students got an A in both subjects.



Activity

An interesting ten minute activity can be conducted by bringing two hoola hoops to class. Place them on the floor and label English and Mathematics with flash cards placed on the floor just outside the hoops. Make sure the hoops overlap.

Call out 'Begin' and the students should scramble to take their respective positions. Change the labelling of the hoops for another set of subjects and start again.

This activity will not only explain the concept of sets, but will also be helpful in the understanding of Venn diagrams and their intersections and unions.

Summary of Key Facts

- Union is the combination of two sets, where the common elements are written once. It is denoted by 'U' which is easy to remember as union begins with U.
- Intersection is the common elements only of two or more sets and this is obvious from the term itself. It is denoted by '∩'.
- If a set A is a subset of the given universal set, then the set of elements not in A is called its complement set.

Example

Universal set: {1, 2, 3, 4, 5, 6, ….,10}

Set A : {1, 2, 3, 4}

Set B : {5, 6,7,8,9,10}

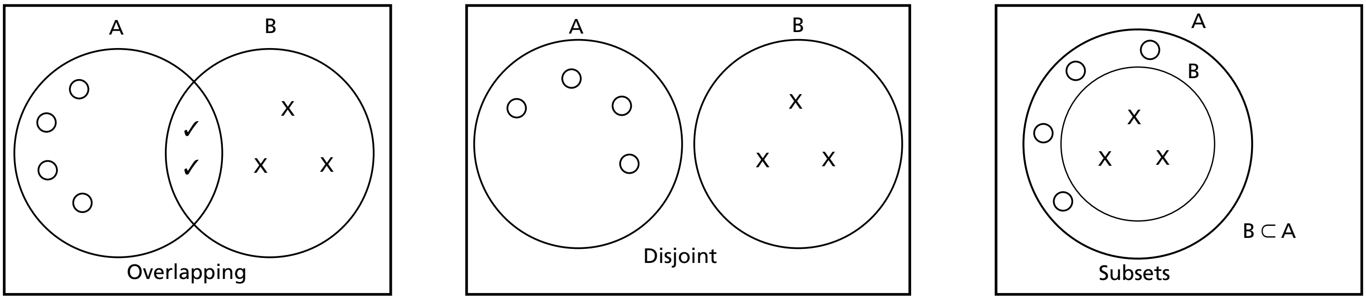
Set A' : {5, 6, 7, 8, 9, 10}

An intersecting concept to add on would be that the intersection of a complement and its set will always be a null set.

Similarly the union of a complement and its set will be the universal set.

- The difference of two sets, set A and Set B, would be the elements of Set A that are not in Set B.
- Sets can be overlapping, disjoint, or can be placed as a subset of each other.

Venn diagrams of all three types.



Changing the places of the sets during union does not alter the operation. This is the commutative property of the union of sets. That is $A \cup C = C \cup A$

Similarly, the commutative property of intersection of sets identifies the same concept in the operation of intersection. The order does not affect the result. That is $A \cap B = B \cap A$

- The associative property of union and intersection of sets highlights the fact that changing the order of a group of sets does not alter the result.
  That is $(A \cup B) \cup C = A \cup (B \cup C)$ and $(A \cap B) \cap C = A \cap (B \cap C)$

Frequently Made Mistakes

Students generally get confused with the symbols. If the suggestions stated earlier are implemented they surely will not find it difficult to decode the language of sets.

Sample Lesson Plan

Topic

Associative property

Specific Learning Objectives

Understanding the associative property of union of sets.

Suggested Duration

1 period

Key Vocabulary

Union, Venn diagram, Associative property

Method and strategy

Activity

Consider an example:

Universal set : {1, 2, 3, 4, …..,10}

Set A :{ 2, 4, 6, 8}

Set B : { 2, 3, 5, 7}

Set C : { 1, 2, 3, 4, 5}

$A \cup (B \cup C) = (A \cup B) \cup C$

Both orders will produce the following answer

{1, 2, 3, 4, 5, 6, 7, 8}.

The teacher should do similar examples on the board and prove the associative property of the union of sets. This can also be represented by a Venn diagram by shading the entire union.

Written Assignment

Questions 11, 12, 15, and 16 of Exercise 1 can be done as classwork. Five similar sums can be given for homework.

Evaluation

A marked assignment can be done in class for the entire Exercise 1 as the students progress during the course of the week.

This chapter is more presentation-based and the symbols are of utmost importance. Marks should be awarded for the correct use of symbols.

After completing this chapter, students should be able to:

- find the union, intersection, and difference set for given sets,
- draw Venn diagrams for union sets, intersection sets, and subsets, and
- prove the commutative and associative properties of sets.

2 Rational Numbers

Specific Learning Objectives

- Introduction of rational numbers
- Operations on rational numbers
- Additive and multiplicative identity
- Expressing a rational number in standard form
- Comparing rational numbers with unlike denominators

Suggested Time Frame

5 to 6 periods

Prior Knowledge and Revision

Students are already aware of natural and whole numbers as taught in earlier classes. They have been introduced to the number line and understand the laws of addition, subtraction, multiplication, and division of integers.

It would be advisable to revise the rules using a number line drawn on the board.

(+) + (+)	[Add and write a positive (+) sign in the answer.]
(–) + (–)	[Add and write a negative (–) sign in the answer.]
(+) + (–)	[Subtract and write the sign of the larger number in the answer.]
(+) × (+)	[Multiply and write a positive (+) sign in the answer.]
(–) × (–)	[Multiply and write a positive (+) sign in the answer.]
(+) × (–)	[Multiply and write a negative (–) sign in the answer.]
(+) ÷ (+)	[Divide and write a positive (+) sign in the answer.]
(–) ÷ (–)	[Divide and write a positive (+) sign in the answer.]
(+) ÷ (–)	[Divide and write a negative (–) sign in the answer.]

Real-life Application and Activities

The following activity can be done on the board as a fun game.

Divide the students into groups of three.

Write a sum.

Example:

$\frac{1}{4} \div \left(\frac{1}{2} - \frac{3}{4} + \frac{1}{4}\right)$

Ask one group to attempt the sum left to right. Ask the next group to follow the order of operation of BODMAS. See who gets the higher value and point out that order of operation matters as they end up with two different answers.

This activity will not only make the students practise together, but will also make them appreciate the significance of BODMAS. Since they will be working in groups they can help each other by pointing out any mistakes and giving the right clue if anyone is unable to grasp the concept.

Summary of Key Facts

- Rational numbers are numbers presented on the number line. They include fractions and integers. Rational numbers are numbers that can be expressed in the form $\frac{p}{q}$, where $q \neq 0$.
- Irrational numbers cannot be expressed in the form of a fraction. For example $\sqrt{2}$, $\sqrt{7}$.
- When the reciprocal of a rational number is multiplied with its number, the result is 1.
- The sum of a rational number and its additive inverse is 0.
- The commutative property of rational numbers in multiplication states that the product of two rational numbers will remain the same regardless of the order.
- The associative property of three rational numbers in multiplication states that the product remains the same regardless of the order of operation.
- Distributive property with respect to multiplication over addition states that when multiplying and adding three rational numbers, the result is the same irrespective of the order of operation.
- When a rational number is divided by a non-zero rational number, the quotient is a rational number.
- The standard form of a rational number has a positive denominator.

Frequently Made Mistakes

Students generally get confused with the terminology (or vocabulary) of rational and irrational numbers and their reciprocals. It is important that the earlier terminology (or vocabulary) and concepts of natural, whole numbers and integers are thoroughly revised before the concept of rational numbers is introduced. This is important as this chapter forms the basis of algebra. The students should recognise the significance of the order of operations and the rules of the signs.

Sample Lesson Plan

Topic

Comparing rational numbers

Specific Learning Objective

Comparing rational numbers

Suggested Duration

1 period

Key Vocabulary

Rational number, Unlike denominators

Method and Strategy

Students should understand that in order to compare rational numbers, the rational numbers should have common denominators. Ask them to rewrite the given rational numbers with positive denominators to get common denominators. To obtain common denominators the rational numbers are multiplied by the common factor. Once this is done, the numerator which is smaller is placed first and then the inequality sign is written.

Example

Write <, > or = in the box

$\frac{3}{4} \square \frac{5}{7}$

$\frac{21}{28} \boxed{>} \frac{20}{28}$

$\therefore \frac{3}{4} \boxed{>} \frac{5}{7}$

Written Assignment

Questions 3, 7, 8, and 9 of Exercise 2b can be done in class. The sums that are not completed can be given for homework.

Evaluation

An assessment can be planned along the lines of Exercise 2b. Since this chapter is technical, students can also be given a 'Fill in the blanks test'. The blanks can be based on the definitions, rules, and properties taught in this chapter.

After completing this chapter, students should be able to:

- identify rational numbers and plot them on a number line,
- add, subtract, multiply, and divide rational numbers,
- compare and order rational numbers in ascending and descending order, and
- find the reciprocal of a rational number.

3 Decimal Numbers

Specific Learning Objectives

- Terminating and non-terminating decimals
- Conversion of fractions and percentages into decimals
- Conversion of decimals into fractions and percentages
- Conversion of decimals into rational numbers
- Approximation and rounding off of decimal numbers.

Suggested Time Frame

5 to 6 periods

Prior Knowledge and Revision

Students are aware of decimals and the identification of the place value of decimals. Teachers can revise the place value of decimals by mentioning tenths, hundredths, and thousandths as the first, second, and third decimal places. Operations in decimals should also be revised as students can do story sums containing all the four operations of finding the sum, difference, product, and division of decimals.

Revision of place value of decimals can be called ‘pinning the decimal point’, along the lines of pinning the donkey's tail game. It is a short five-minute activity where the students can scramble into groups and the teacher divides the board into as many columns as the number of groups. The teacher writes 5 sums in each column and each group sends a volunteer. It can become a rowdy game as the students are allowed to help their volunteers. The group that finishes all five sums first and correctly gains a point.

The sums on the board can be as follows:

1) 7643, 3 hundredths, so the student will place the decimal point after 6.
   76.43
2) 807945, 5 thousandths, the decimal point will be placed after 7.
   807.945

Real-life Application and Activities

Decimals are associated with money. The teacher can ask for newspaper clippings where the growth rate or national reserves of the country are mentioned in decimals. Similarly, money conversions have decimal points.

Example

Dollars can be converted to rupees according to the conversion rate.

A list of currencies and their conversions could be shared in class by the teacher.

When explaining terminating and non-terminating or recurring decimals, the literal meanings of the words could be explained. Terminating means to end; therefore terminating decimals have decimal places that are fixed and complete. Recurring decimals have decimal places that keep on repeating indefinitely. These decimal places can go on to infinity and the recurrence can sometimes be in groups or sequences of numbers.

Summary of Key Facts

- Fractions can be converted to decimals by the long division method where a decimal point is introduced and a zero is added. The alternate method is to convert to an equivalent fraction with a denominator with multiples of ten. When doing so, the factor identified to convert the denominator to a multiple of ten is also multiplied with the numerator to create equivalence. The number of decimal places depends on the number of zeroes in the denominator.
- Conversion of percentages into decimals is quite simple as the percentage itself is a denominator of 100, which is easily converted to a fraction by creating two decimal places.
- When converting a decimal into a percentage, the decimal is first converted into a fraction with a denominator with multiples of 10 and then multiplied by 100.
- When converting decimals into rational numbers, the fraction has to be reduced or simplified. Generally, the fraction with a denominator of multiples of ten is easily reduced or simplified.
- Terminating decimals have a finite number of digits after the decimal point.
- When an infinite number of digits occur, after the decimal point they are called non-terminating, or recurring, decimals.
- Approximate value or rounding off has a set of simple rules. The place value that needs to be rounded off is circled. If the number after the circled digit is 5 or greater than 5, then the value of the circled digit is increased by 1. If the number after the circled digit is less than 5, then the value remains the same.

Frequently Made Mistakes

When doing long division, students sometimes get confused about when to introduce the decimal point and hence the zero to the dividend. A lot of practice questions in long division to create decimals needs to be done in class, on the board initially and then in students' notebooks.

Sample Lesson Plan

Topic

Rounding off and approximation

Specific Learning Objectives

Rounding off of decimals

Suggested Duration

1 period

Key Vocabulary

Rounding off, Approximation, Decimal places, Decimal point

Method and Strategy

A number that needs to be rounded off has to be circled. If the number to its right is 5 or more, then the circled digit is increased by 1 and the rest is deleted.

Example

Round off 28.89 to the nearest whole number.

28.89

↑

This rounding off can also be explained by using a number line. Whenever a decimal has to be rounded off to the nearest whole number, a number line comes in handy.

A number line can be made on the floor in the corner of the class room and kept during the duration of this chapter. Coloured electrical tape can be taped to the ground to form the number line and the numbers can be placed as flash cards that can be replaced according to the demands of the sums or number sets. The gradings or the dashes on the number line can also be made semi-permanent by making the markings with a different coloured electrical tape.

Written Assignments

Questions 12, 13, and 14 of Exercise 3 can be done in class. Similar sums can be given for homework.

Evaluation

A comprehensive test can be conducted where learning of all concepts taught can be assessed. Story sums involving rounding off and conversions can be asked. This will develop critical thinking skills.

After completing this chapter, students should be able to:

- convert fractions and percentages to decimal numbers, and vice versa,
- identify terminating and non-terminating decimal numbers, and
- round off decimal numbers to the nearest whole number and the required decimal place.

4 Squares and Square Roots

Specific Learning Objectives

- Introduction of the concept of perfect squares and square roots
- Derivation of the square root of a perfect square
- Derivation of positive square root by prime factorization
- Derivation of positive square root by division

Suggested Time Frame

4 to 5 periods

Prior Knowledge and Revision

Students are aware of square numbers and the area of a square. The correlation of the fact that a square number and area of a square are the same is important. The teacher can hold a quick one minute quiz where he/she calls out a number and the students multiply it by itself and state its square.

A game of snap can also be played.

Make a set of 20 flash cards with the squares of 1 to 10 written on each card twice. Cards are shuffled and distributed between two students; when one of them calls out snap as he/she gets the same square number, he/she needs to call out the number it is a square of. This can be played by all students in turn and can be a five minute fun time with the rest cheering.

The multiplication rule of two negative numbers can be recalled and it can be pointed out that when two negative numbers are multiplied, we get a positive square.

Example

$13 \times 13 = 169$

$(-13) \times (-13) = 169$

The fact that all concepts converge and build up to form new concepts has to be recognised by the teacher. This is imperative in order to create a network of mathematical concepts amongst students.

Real-life Application and Activities

Although this is an entirely computation-based chapter, you can create a game so that students can learn the steps of the prime factorization and division methods faster.

You will require a white board, different coloured board markers, a stop watch, and flash cards.

The students select a square number from the flash cards. They also pick out the option of prime factorization or division method. The student goes to the board, solves the sum, and his/her finishing time in the case of a correct answer is written in a column at the side of the board. All students take turns till the whole class has had a turn. This activity will involve the whole class while each sum is being solved. This will result in a lot of practice as students will all follow each sum done on the board and will be encouraged to point out any mistakes. This will also quicken their mathematical computation.

Summary of Key Facts

- A perfect square can be expressed as the product of two integers of equal value.
- The area of a square is a square number.
- When determining the positive square root of numbers by the prime factorization method, the number is divided by its prime factors till it comes to a 1. The prime factors are grouped in pairs and one factor from each pair is taken and the product is calculated. This is the square root of the square number.
- When determining the positive square root by the division method, the number is paired by putting bars on each set starting from the unit. For each set, the largest number whose square is contained in each pair, is used as a dividend. The whole process of division is thus followed by subtracting the square from the pair and the cycle continued with the second dividend. For the second divisor, the original quotient is added to itself. Find out the new divisor and place it next to the sum found earlier. Multiply the new divisor with the entire set and place the product under the dividend and subtract. Repeat this process till the dividend becomes zero.
- Whenever a square root is found, always check by multiplying it by itself to see if it comes to the given square number.

Frequently Made Mistakes

This is an entirely mathematical concept with steps to be learnt. Students sometimes make mistakes if they don’t remember the steps. Therefore they should focus on learning the steps. The steps of the division and prime factorization methods, along with a worked example, can be displayed on chart paper for the students' perusal during the course of the week.

Sample Lesson Plan

Topic

Squares and square roots

Specific Learning Objectives

Finding the smallest whole number to be multiplied to make a number a square number.

Suggested Duration

1 period.

Key Vocabulary

Square numbers, Prime factorization, Exponents, Power

Method and Strategy

In order to find the number which will make the number a complete square, the students first have to revise the prime factorization method. The exponential form representation is important.

Example

$120 = 2^3 \times 3 \times 5$

2, 3, and 5 need to be introduced to make complete pairs.

Hence 30 is the smallest number to be multiplied to 120 to make it a perfect square.

This is a difficult and conceptual topic. A lot of sums should be done on the board and then, once the teacher feels that the students can work independently, they can do sums in their exercise notebooks.

Written Assignment

Sums like Q# 12 of Exercise 4b can be done in class and the rest can be given for homework.

Find the smallest whole number by which the following numbers can be multiplied to make them a perfect square.

1)	120	3)	66	5)	260	7)	95	9)	45
2)	325	4)	35	6)	180	8)	21	10)	500

Evaluation

As mentioned earlier, this is an extremely conceptual chapter with steps of mathematical computation. An oral quiz on the steps of the division method can be given twice or thrice at the beginning of each lesson. A written quiz can also be given for the steps of division method. A comprehensive test including word problems should be given at the end of the chapter. Assessment of learning during the course of the topic is important and can be implemented in the form of five minute quizzes.

After completing this chapter, students should be able to:

- state the square roots of the first 12 perfect squares,
- perform tests for perfect squares,
- find the square of a number, and
- derive the square root of a positive integer by prime factorization and division.

5 Exponents

Specific Learning Objectives

- Concept of exponential notation
- Expressing rational numbers in exponential notation
- Laws governing the powers or exponents in multiplication and division

Suggested Time Frame

4 to 5 periods

Prior Knowledge and Revision

The teacher explains the repetitive multiplication of a number by itself and links it to squares and cubes of a number. The term 'powers' is already known by the students. They also know the following terms: variable, power, constant, product, and quotient.

Students have been doing prime factorization and expressing numbers in their index form.

Real-life Application and Activities

The concept of exponential notation can be reintroduced with a fun activity.

You need 2 or 3 decks of cards.

Remove the aces, kings, queens, jacks, and jokers.

Divide the students into groups of four. Ask one student from each group to deal the cards. The packs can be shuffled and divided equally among the groups.

They then divide and organise their cards in exponential order.

For example, if the student has 3 fives he will write it as $5^{3.}$

Next, ask the students to find the product of their exponential list.

Example

$5^3 = 5 \times 5 \times 5 = 125$, and so on.

Now ask them to add all the products.

The group with the highest score is the winner.

You can time them using a stopwatch and you will have another winner in terms of timings.

This activity not only develops their mental skills and ability to organise data, but also practises the terminologies of powers/exponents/index.

Summary of Key Facts

- 2 to the power of five means that 2 is repetitively multiplied with itself five times, where 2 is the base and 5 is the power or exponent.
- When a negative base is raised to an even power, the product is always positive.
- Similarly, the negative base of an odd number power stays negative.
- Reciprocals of rational numbers with powers remain the same, only the base is swapped from numerator to the denominator or vice versa.

Example

- The 6 Laws of Exponents are as follows:

Law I: $\boxed{x^m \times x^n = x^{m+n}}$

Law II: $\boxed{x^n \times y^n = (x \times y)^n}$ (where y is also a non-zero rational number)

Law III: $\boxed{x^m \div x^n = x^{m-n}}$

Law IV: $\boxed{x^n \div y^n = (\frac{x}{y})^n}$ (where y is also a non-zero rational number)

Law V: $\boxed{(x^m)^n = x^{mn}}$

Law VI: $\boxed{x^0 = 1}$ (if x is any non-zero rational number)

Frequently Made Mistakes

Students invariably do not apply the laws while doing the operation of multiplication and division and tend to resort to cancelling in division and counting the bases in multiplication.

Also, the reciprocal laws have to be explained meticulously. Students tend to put the negative power of the base to the exponents. They tend to get confused when dealing with negative bases.

Sample Lesson PlanTopic

Laws of exponents

Specific Learning Objectives

Two laws of exponents will be introduced.

$x^m \times x^n = x^{m+n}$

$x^m \div x^n = x^{m-n}$

Suggested Duration

1 period

Key Vocabulary

Product, Quotient, Power, Exponent, Base, Variable

Method and Strategy

When explaining the two laws, the teacher should manually break up the bases and show that when multiplying we actually end up adding the powers.

Similarly, the teacher should show division on the board, where the bases are cancelled, which is actually the difference of the powers of the two bases.

A small activity can be done with sweets.

Collect ten sweets of the same type and arrange them in a line. Tell the students that the sweets are to the power of ten.

If the sweets are put in two sets: four together and six together which becomes ten when added.

Exponentially,

$S^4 \times S^6 = S^{10}$

Similarly, in division, the two sets of sweets can be put on a fractional bar and divided.

This activity will help the students understand the laws easily.

Written Assignments

Mixed sums of multiplication and division can be given to be done in students' notebooks.

Ask the students to highlight the laws on a separate page of their notebooks for reference.

Evaluation

Exercise 5 and the revision exercise on page 71 can be given as an assessment. Sums should be mixed and chosen from these exercises in such a way that the students are assessed on their recall and association skills related to the laws.

After completing this chapter, students should be able to:

- identify the base and power of a number,
- express numbers in exponential form, and
- apply the product, quotient, and power laws of exponents to solve questions.

6 Direct and Inverse Variation

Specific Learning Objectives

- Direct and inverse variations
- Continued ratio
- Unitary and proportion method
- Ratio problems solving three quantities

Suggested Time Frame

4 to 5 periods

Prior Knowledge and Revision

This chapter is a continuation of the topic of ratios. The teacher should conduct a recall session in which rules of ratio are revised. The facts to be revised are:

- Ratios are always expressed in their simplest form.
- Quantities are in the same units.
- Ratios are placed in the following order new : old

Examples

a. 4 : 8
   1 : 2

b.

400 g	:	1000 kg
400 g	:	1,000,000
4	:	10,000
1 g	:	2500 kg

Real-life Application and Activities

Real-life examples on pages 73 and 74 can be discussed in class and a brainstorming session can be conducted.

Example

- Cost and number of apples
- Speed of the car and time
- Amount of food and the days it will last
- Number of pipes filling up a tank and the time taken

With the help of these examples, students should be encouraged to explore parity. If one quantity increases, the other also increases. Sometimes if one quantity increases, the other quantity or value decreases.

The difference between direct and inverse proportion should be explained through real-life examples and applications. Only when the students are able to distinguish between the two, should the teacher proceed.

Summary of Key Facts

- Direct variation occurs when both quantities increase or decrease simultaneously. The methods to solve them are the unitary and proportional methods.
- In the proportion method, the data is set and cross multiplication occurs.
- In inverse variation, one quantity increases and the other decreases.
- In inverse variation, once the data is set, horizontal multiplication takes place.
- Continued ratio is an expression of three ratios. Two sets of ratios are combined into one linear ratio. The common quantity becomes the lowest common multiple and the ratios are combined.

Example

red: green

2 : 3

green : blue

5 : 7

2 : 3 (multiply by 5)

5 : 7 (multiply by 3)

Hence,

10 : 15 and 15 : 21

Therefore,

10 : 15 : 21

- Dividing an amount into proportional parts is generally done in the form of a word problem. Each proportion is made into a fraction. Each fractional value is then multiplied to get its proportional part.

Frequently Made Mistakes

This is an easy chapter and the students enjoy it as long as they can differentiate between direct and inverse proportion.

Sample Lesson Plan

Topic

Direct and inverse variation

Specific Learning Objective

Direct and inverse variation

Suggested Duration

1 period

Key Vocabulary

Direct variation, Inverse variation, Proportion

Method and Strategy

Real-life examples of inverse proportions should be given in class. The teacher should highlight the fact that if one quantity increases, the other decreases.

A very logical deduction is that if the speed is greater, the car will take less time to finish a journey.

Activity

A simple activity can be done in class, in which two identical toy cars are brought into the lesson and are pushed with different forces to travel a given distance. The students will record the times on the stop watch and see that more force results in less time, and vice versa.

Once the students have decided the proportion, whether direct or inverse, the teacher should then explain the method. When the operation is inverse, horizontal multiplication is done.

Written Assignment

Questions 4 to 13 of Exercise 6 should be given together so that students can distinguish between direct and inverse variation and then carry out the mathematical computation.

Evaluation

A quiz should be given after each concept so that the teacher can assess whether to move on to the next concept or reinforce earlier learning. Quizzes are assessment of learning which are very beneficial.

A comprehensive assessment can be given along the lines of Exercise 6 and students, can be evaluated.

After completing this chapter, students should be able to:

- identify problems involving direct variation, inverse variation, and continued ratio,
- apply the unitary and proportion methods to solve ratio problems, and
- solve problems involving continued ratio.

7 Financial Arithmetic

Specific Learning Objectives

The concepts of:

- discount
- profit and loss percent
- taxation, property tax, and general sales tax
- simple interest
- zakat and ushr

Suggested Time Frame

At least 10 periods

Prior Knowledge and Revision

Students studied percentages in Grade 6. Financial transactions are the application of percentages in real-life scenarios. The teacher should first have a revision worksheet prepared for the lesson in which the concepts of percentages are revised. The teacher should not embark on this new chapter until the students have thoroughly revised and revisited the concepts.

Real-life Application and Activities

Profit and loss is a real-life application. Students can be shown newspaper clippings of sale advertisements and taught how to calculate the discount from the marked price.

Students can be encouraged to create their own business plan and present it in class.

Activity

A class outing can be organised to a manufacturing unit e.g. a shoe factory.

The manager could be asked to present a simple break-down of the production costs to the students.

Overhead costs = Rs x

Material costs = Rs y

Labour = Rs w

Total costs = Rs z

Sale price = Rs v

Profit = $v - z$

Profit % = $\frac{v - z}{z} \times 100$

It should be highlighted that the above values are for one shoe or per unit. It should also be pointed out that the sale price has to be higher than the cost price to make a profit.

Activity

The teacher can ask parents to take their child to a discount store. They can write an essay on their experience and findings.

Example

Item 1

Marked price: RS 400

Discount : 30%

$= \frac{30}{100} \times 400$

Discount = Rs 120

Sale price after discount = 400 – 120 = Rs 280

They can make a list of various items in this format, but it is important that they visit a sale shop and have a hands-on experience.

Activity

Students should be asked to prepare a mock report of assets and savings, e.g. gold jewellery, savings, where they calculate their value and then work out the *zakat* on the assets.

All the activities could be recorded on chart paper and displayed in class for all to view. In this way mathematics can be made interesting and relevant.

Summary of Key Facts

- A reduction made on the marked price is called discount.
- Profit is incurred when selling price > cost price.
- Loss is incurred when the cost price > selling price.
- Overheads are included when calculating cost price.
- Tax paid as a percentage of the value of property to the government is called property tax.
- The tax paid as a percentage of the selling price by the buyer to the seller is called general sales tax.
- Interest is incurred when a principal amount is either invested or borrowed at a certain percentage of interest rate over a period of time.

- The amount is the money paid after a principal amount is invested and it accrues a simple interest. Hence the amount is the sum of the principal amount and the simple interest accrued over a specified period of time.
- *Zakat* is an obligatory tax paid at the end of each year by every Muslim at a rate of 2.5% of the total value of his/her savings and assets.
- *Ushr* is the tax paid by a Muslim on his/her agricultural assets, levied at the rate of 10% on agricultural output if irrigated by natural sources, and 5% if irrigated by artificial means.

Frequently Made Mistakes

Students usually think that if a discount is given, there will be a loss. This is not true as the discount is given on the marked price to which some percentage of profit has already been added.

Sample Lesson Plan

Topic

Simple interest

Specific Learning Objective

Calculating simple interest

Suggested Duration

1 period

Key Vocabulary Words

Principal, Rate, Per annum, Interest, Amount

Method and Strategy

Formula for calculating simple interest:

$$\text{Simple interest} = \frac{\text{Principal} \times \text{Rate} \times \text{Time}}{100}$$

where,

P is the principal amount,

R is the rate per annum or year, and

T is the time in years.

Amount = principal + simple interest.

If the interest is given and alternate values are asked, the formula has to be manipulated.

$$\text{If } SI = \frac{PRT}{100}$$

$$\text{Then } T = \frac{SI \times 100}{PR}$$

$$R = \frac{SI \times 100}{PT}$$

Written Assignments

Selected sums from Exercise 7c can be done in class and the rest for homework.

Worksheets with sums with the names of real banks and students' names can be made and handed out as assignments.

Example

Erum's father, Mr Ayub invests Rs 100,000 in the National Bank of Pakistan for a year at an interest rate of 4%. Calculate the amount he would receive from the bank at the end of the year. Also if tax is deducted by the government at the rate of 1% of his total interest, calculate the tax paid by Mr Ayub and his net total taking.

Evaluation

This chapter is very comprehensive and critical. Before a test of financial arithmetic is taken, an assessment of learning should be carried out after completion of every topic (at the end).

After completing this chapter, students should be able to:

- find the cost price, selling price, profit or loss percent on a transaction,
- calculate simple discount when given the price and discount rates,
- compute property tax and general sales tax,
- calculate simple interest when given the principal, time, and interest rate, and
- calculate *zakat* and *ushr* by applying the correct rates.

8 Algebraic Expressions

Specific Learning Objectives

- Terminology of algebraic expressions, comprising variables, coefficients, and constants
- Order of algebraic terms
- The four operations of addition, subtraction, multiplication, and division of algebraic terms

Suggested Time Frame

6 to 8 periods

Prior Knowledge and Revision

The students have a basic knowledge of algebra. They should to recall that it is a branch of mathematics where quantities are expressed in letters and variables, and unknown values are figured out by forming algebraic expressions and equations.

The teacher can have a recap session in class by writing sums on the board and eliciting responses from the students who have to say the algebraic expressions out loud.

Example

1) Saima's age after 5 years $= s + 7$
2) The cost of x apples if one costs 50 cents $= 50x$
3) The weight of a full truck with bricks is w and the weight of empty truck is z:
   the weight of the bricks $= w - z$.

Also explain to the students that the power notation studied earlier is linked with algebra.

Example

2 to the power of 2 is the value of 4.

$\therefore 2^2 = 4$

$y \times y \times y = y^3$

$w \times w \times w \times w \times w = w^5$ (w to the power or exponent of 5, where w is the base variable)

Real-life Application and Activities

Algebra should be considered as a language of mathematics where the unknown value is denoted by a letter. The rules of operations that were taught in class 6 can be revised again by playing a game with flash cards.

Each child is given a set of flash cards with signs of plus and minus written separately on each card.

The teacher calls out the operation of addition, subtraction, multiplication, and division.

The child picks up two flash cards, look at the signs and figures out the sign of the result of the product, sum, or difference.

The student can be timed or the teacher can give 30 seconds and they have to write as many operations and answers as possible in their notebooks.

This encourages them to be quick with their laws and operations.

Summary of Key Facts

- A variable is an unknown number denoted by a letter.
- A number placed before the variable is a coefficient.
- A number with a fixed numerical value is a constant.
- An algebraic expression consists of terms connected by either of the operations.
- A polynomial comprises more than one term.
- When arranging polynomial terms, descending order is followed. The power of the variable decides the value of the term: the larger the power, the larger the term.
- In subtraction, the term to be subtracted has all its signs switched.
- In multiplication, powers with the same base are added.
- In division, same base powers are subtracted.
- When multiplying one algebraic expression by another expression, each term of the first expression is multiplied by each term of the second expression.

The sign rules of addition and subtraction and the rules of multiplication and division should be revised before simplifying polynomials.

Frequently Made Mistakes

Students' most common mistake is placing the incorrect sign while applying the four operations. Teachers need to be extremely careful when explaining the sign concept.

Sample Lesson Plan

Topic

Division of polynomials

Specific Learning Objectives

Division of polynomials

Suggested Duration

1 period

Key vocabulary

Polynomial, Coefficient, Power, Exponent

Method and Strategy

Students should be introduced to this operation by relating it to regular division. The only difference is that all terms are divided or cancelled by the divisor term but only the powers of the same base variable are subtracted.

Lots of practice worksheets on this operation should be given and the rules should be revised.

(+) and (+) = +

(+) and (–) = –

(–) and (–) = +

Written Assignments

Questions 8, 9, and 10 of Exercise 8b can be done in class in the students' notebooks. They can complete the exercise at home.

Evaluation

This chapter is relatively easy and a comprehensive assessment by using sums from Exercise 8a and 8b can be given.

Board quizzes can also be done for all the operations.

After completing this chapter, students should be able to:

- identify and solve algebraic expressions,
- arrange polynomials in increasing and decreasing order, and
- add, subtract, multiply, and divide given algebraic expressions.

9 Algebraic Indentities

Specific Learning Objectives

- Introduction of the sum and difference of a perfect square
- The algebraic identity of the difference of two squares
- The numerical application of these identities to solve questions

Suggested Time Frame

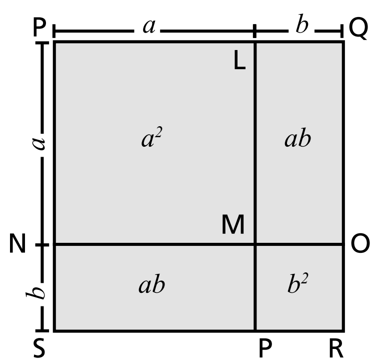
6 to 7 periods

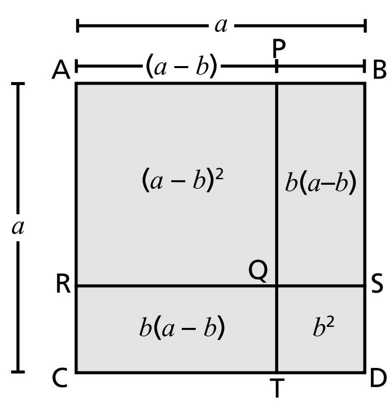
Prior Knowledge and Revision

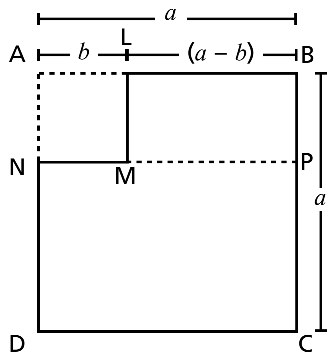
Students have not studied algebraic identities before. This topic is an extension of their understanding of the rules and laws governing algebraic polynomials.

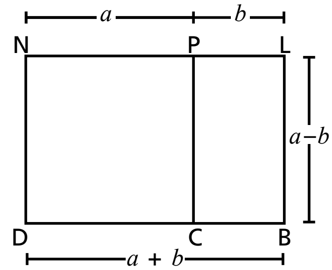
Real-life Application and Activities

The identities can be explained geometrically through the areas of rectangles and squares.









The teacher should make cutouts of these diagrams on chart paper and then explain them by proving the identities geometrically. The sides of the squares and rectangles should be denoted by variables 'a' and 'b'.

Summary of Key Facts

- The sum of the square is the sum of the squares of 'a' and 'b' and twice the product of both.

$$(a + b)^2 = a^2 + 2ab + b^2$$

- The difference of the square is the sum of the squares of 'a' and 'b' and twice the product of both with a minus sign.

$$(a - b)^2 = a^2 - 2ab + b^2$$

- The product of the sum and difference of two variables is the difference of the two squares.

$$(a + b)(a - b) = a^2 - b^2$$

When solving numerical values of a term using identities the term is first broken down into one of the three identities and then expanded into a numeric algebraic identity to calculate the values.

Frequently Made Mistakes

When solving the first and the second identity, the students get confused with the 2ab expression. It is to be highlighted that this product has a minus sign in the difference of the whole square identity. The fact that b squared can never have a minus sign should also be pointed out.

$$(-b)^2 = (-b) \times (-b) = +b^2$$

Sample Lesson Plan

Topic

Algebraic identities

Specific learning Objectives

Solving arithmetic expressions using the first algebraic identity

Suggested Duration

1 period

Key Vocabulary

The sum of a perfect square, Numerical value, Algebraic identity

Method and Strategy

When explaining the fact that arithmetic expressions do not have to be evaluated manually with arithmetic operations, it should be pointed out that algebraic identity is a quicker way.

Example

105×105

The students can be told that the product can be found arithmetically but this will take longer. Also highlight the fact that the question mentions the use of algebraic identity to evaluate.

$105 \times 105 = 105^2$

Using the first algebraic identity:

$a^2 + 2ab + b^2$

$= (100 + 5)^2$

$= (100)^2 + 2\,(100)\,(5) + (5)^2$

$= 10000 + 1000 + 25$

$= 11025$

Written Assignments

Sums similar to Q# 7 of Exercise 9 can be given as a classwork assignment once it has been explained.

Example

1) 206×206
2) 505×505
3) 101×101
4) 702×702
5) 4001×4001

Evaluation

Assessment of learning will play a key role during this chapter. Short, five minute quizzes should be given after each identity and concept taught. This will inform the teacher whether to proceed or not. It is important for the teacher to understand that algebra is a relatively easy and enjoyable branch of Mathematics but the identities are a critical step in this area. The concept of identities and their application is very important. The students should be given enough mini-tests to ensure that they are well versed in each identity before a comprehensive assessment including all concepts is given.

Revision Exercise on pages 142 and 143 can be used as comprehensive test for this chapter.

After completing this chapter, students should be able to:

- derive the algebraic identities related to the square of a sum and the difference of two terms and the product of a sum and difference of two terms $(a + b)(a - b)$, and
- solve questions by applying the correct algebraic identities.

10 Factorization of Algebraic Expressions

Specific Learning Objectives

- Introduction of factorization of algebraic expressions
- Introduction of factorization of expressions using algebraic identities
- Introduction of factorization by making groups

Suggested Time Frame

4 to 5 periods

Prior Knowledge and Revision

Students have just studied algebraic identities and this chapter is a continuation of its application. It is also an extension and build-up of the concepts of identities. The teacher should not have any issues with this chapter as it is a progression of the earlier topic.

Real-life Application and Activities

The teacher should be aware of the complexities of this chapter and should display the geometric cutouts of the identities on the soft board during the week. The three identities should also be written on chart paper and displayed on the soft board. This is latent learning where the students are encouraged to use it for reference while doing sums and in this way the identities become embedded in their minds.

Summary of Key Facts

- A polynomial with multiple terms has a common factor in terms of a number or a variable.
- An expression which can be written as the difference of two squares can be factorised as their sum and difference. While doing that, students first look for a common factor or term.
- When there are multiple terms and the factors are not obvious, they are generally rearranged and regrouped so that a common factor is obvious.

Frequently Made Mistakes

The fact that sometimes the sums need to be factorised multiple times is a bit challenging for students. It is important that the teacher highlights that checking whether a common factor can be found should be done before proceeding with the factorization using identities.

Sample Lesson Plan

Topic

Factorization of algebraic expressions

Specific Learning Objective

Multiple factorization

Suggested Duration

1 period

Key Vocabulary

Factorization, Difference of two squares, Common factor

Method and Strategy

The first and foremost rule to teach is to check for common numbers as factors and then common variables. The common variables and numbers have to be the smallest for them to be a factor for all terms.

Example

$4xy + 6x^2y + 10xyz$

The common factors that are variables are x and y. 2 is the common number factor for all. Hence $2xy$ is the common term.

$2xy(2 + 3x + 5z)$

Furthermore, for multiple factorization, the difference of two squares will be applied after the common factorization.

Example

$27x^3 - 3x$

$3x(9x^2 - 1)$

$3x(3x + 1)(3x - 1)$

$3x$ is the common factor.

Written Assignment

Selected sums from Exercise 10a and 10b can be done in class once the example provided above has been done on the board.

A worksheet can be provided as a homework assignment.

Example

			Answers
1)	$90x^2y^2 - 10$	1)	$10\ (3xy + 1)\ (3xy - 1)$
2)	$2x^2 + 10x + 50$	2)	$2\ (x + 5)^2$.
3)	$49 - 4a^2$	3)	$(7 + 2a)\ (7 - 2a)$
4)	$81a^4 - 1$	4)	$(9a^2 + 1)\ (3a + 1)\ (3a - 1)$
5)	$(x - 4)^2 - 25$	5)	$(x + 1)\ (x - 9)$

Evaluation

At the beginning of each lesson there should be a two-minute recap test of the concepts taught in the previous lesson. Peer checking can be done and the sums solved on the board. Conducting this activity throughout this chapter will not take not more than five minutes in every lesson and will ensure that there are no gaps in the understanding and application of the concepts.

After completing this chapter, students should be able to:

- factorise algebraic expressions by applying algebraic identities, and
- factorise algebraic expressions by making groups.

11 Simple Equations

Specific Learning Objectives

- The concept of algebraic equations
- Axioms of solving algebraic equations
- Procedure for solving an algebraic equation
- Forming, or constructing, an algebraic equation

Suggested Time Frame

6 to 7 periods

Prior Knowledge and Revision

Students have been forming algebraic expressions out of sentences and statements. The algebraic equation has been introduced in previous lessons. The concept of transposing should be revised. The key properties should also be done on the board.

The fact that 'LHS = RHS' should be revised.

An algebraic equation is a combination of terms that are conjoined by an 'equal to' sign stating that the variables and numbers on each sides are equal. This a perfect way of finding the value of an unknown variable. The formation of such mathematical statements is fundamental to problem solving in arithmetic and geometry. It has to be highlighted that algebra is not an isolated branch of mathematics, but the structural base of mathematics.

Real-life Application and Activities

There are different types of equations: linear equations, equations with brackets, and equations with denominators. Each type of equation should be taught separately in a different lesson. A mixed exercise such as Exercise 11a can then be given.

Word problems can be converted to real-life situations by substituting the names of the students in the questions. Similarly, real-life word problems can be written on the board and can be role played.

Example

If Ali has two brothers in real-life, the teacher can make a word problem knowing that they all like to play cricket.

Question

If Ali scored 53 runs and his brother Amir scored 23 runs and the collective score of all three brothers was 212 , how many runs did Umar, the third brother, score?

Solution

$$
\begin{aligned}
53 + 23 + x &= 212 \\
x + 76 &= 212 \\
x &= 212 - 76 \\
x &= 136
\end{aligned}
$$

Umar scored 136 runs in the cricket match.

Summary of Key Facts

- A (+) plus sign term transposes to the other side as a minus sign.
- A coefficient in the denominator will go on the other side as a numerator.
- All variable terms are collected on the LHS and the constants are moved to the RHS.
- Simplify equations by opening brackets. Multiply the term outside the bracket by all the terms inside. This is called expansion in algebra.
- If there are fractional terms, then the LCM should be found to simplify the terms. The LCM which is in the denominator is then transposed on the other side of the equation.
- To solve word problems, the first step is to convert each phrase into a mathematical expression where the unknown value is substituted with a variable.
- The process of solving the equation is the same as to find the value of the unknown variable.

Frequently Made Mistakes

Students who are weak at algebraic rules will have difficulty in transposing and solving the equation. If that is the case, the teacher can revisit the number line concept and explain how the rules of algebraic signs are derived.

Sample Lesson Plan

Topic

Algebraic equations

Specific Learning Objectives

Solving equations with fractional terms

Suggested Duration

1 period

Key Vocabulary

Denominators, LCM, Transpose

Method and Strategy

Example

Solve $\frac{3x+1}{16} + \frac{2x-3}{7} = \frac{x+3}{8} + \frac{3x-1}{14}$.

Solution:

$$\frac{3x+1}{16} + \frac{2x-3}{7} = \frac{x+3}{8} + \frac{3x-1}{14}$$

$$\frac{3x+1}{16} - \frac{x+3}{8} = \frac{3-1}{14} - \frac{2x-3}{7}$$

$$\frac{3x+1-2(x+3)}{16} = \frac{3x-1-2(2x-3)}{14}$$

$$\frac{3x+1-2x-6}{16} = \frac{3x-1-4x+6}{14}$$

$$\frac{x-5}{16} = \frac{-x+5}{14}$$

$$14(x-5) = 16(5-x)$$

$$14x - 70 = 80 - 16x$$

$$30x = 150$$

$$x = 5$$

This is the most complex form of solving an equation at this level. The concept of transposing comes right at the end. Initially the LCM is found and the numerators are multiplied by the number found by the division of the LCM and the denominator. When doing so, the students have to be very careful of the minus sign as all signs will change when multiplied by a negative number. The denominators are then cross-multiplied once the LHS and RHS both have single terms, keeping in mind the rules of transposing, and the equation is then solved.

These types of sums should be done on the board and students can take turns to solve them. A lot of practice worksheets should be handed out for homework and classwork assignments.

Written Assignment

All even-numbered sums of Exercise 11a can be done in class and the odd-numbered sums can be given for homework. This will ensure that sums of all levels of difficulty are done in class and then parallel sums are given for homework.

Evaluation

This is quite a comprehensive chapter and two assessments can be given. One can involve all types of equations and the other can be completely based on word problems where the students will be expected to form the equation and then solve it.

After completing this chapter, students should be able to:

- solve algebraic equations by applying relevant axioms and identities.

12 Lines and Angles

Specific Learning Objectives

- Concepts of perpendicular and parallel lines
- Constructing perpendicular and parallel lines
- Property of a transversal and its construction

Suggested Time Frame

2 to 3 periods

Prior Knowledge and Revision

Students are well aware of the geometry strand of mathematics. It is important to recall the correct use of the geometric instruments.

- A protractor is used to construct angles.
- A pair of compasses is used to construct line segments.

Recognition and definitions of lines, rays, and line segments should also be quickly revisited in class. While revising the three types, the difference between the three with regard to the end point should be emphasised.

Real-life Application and Activities

Activity

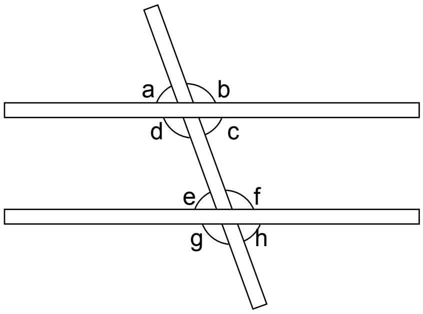
A fun way to explain angles formed by the transversal on parallel lines would be with an activity.

Every student will need:

A4 paper, three straws, a pair of scissors, a glue stick, and a protractor.

Ask each student to paste two straws onto the A4 paper parallel to each other and the third as a transversal.

Label the angles formed as: a, b, c, d, e, f, g, and h.
Then compare and measure them.

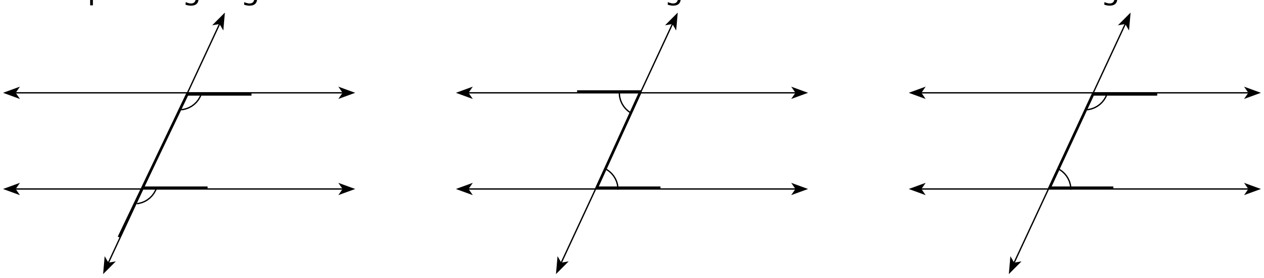


Pairs adding up to 180°: a + b, b + c, c + d, d + a, e + f, f + h, g + e, and g + h
Corresponding ∠s: b and f, c and h, a and e, d and g (These are equal pairs.)
Alternate ∠s: c and e, d and f (These are equal to each other.)
Interior ∠s: c and f, d and e (These are not equal but add up to 180°.)

Summary of Key Facts

- Perpendicular lines are two lines that intersect each other at a right angle.
- Two lines that have a constant distance between them and therefore never meet are parallel lines.
- When parallel lines are cut by a transversal, alternate, corresponding, and interior angles are formed.

Corresponding angles form an F. Alternate angles form a Z. Interior angles form a U.



- Adjacent angles share a common vertex.
- Adjacent angles that add up to 90° are called complementary angles.
- Adjacent angles that add up to 180° are called supplementary angles.
- Two non adjacent angles formed by two intersecting lines are equal and are called vertically opposite angles.

Frequently Made Mistakes

Students sometimes confuse interior angles with corresponding and alternate angles. Interior angles add up to 180° but alternate and corresponding angles are equal to each other.

Sample Lesson Plan

Topic

Lines and angles

Specific learning objectives

Construction of perpendicular lines

Suggested duration

1 period

Key vocabulary

Perpendicular

Method and Strategy

This activity can be made fun by doing it on chart paper.

The steps of construction can be written on the board. Two students are chosen and each is given a sheet of chart paper.

They are timed for five minutes after the sums have been written on the board. The pair that manages to finish the most constructions accurately are the winners.

Steps of constructions:

- Let $\overline{AB}$ be the given line. Take a point N on the line $\overline{AB}$ and a point P at a certain distance from the line.
- Place the set square in such a way that it is at a right angle to the base line.
- Slide the ruler and the set square in such a way that the pencil marks the points P and N of the measured distance.
- With a pencil, draw a line along the side of the set square. $\overline{PN}$ is perpendicular to $\overline{AB}$.

Written Assignments

Once the activity is done, a few construction sums can be done in students' notebooks. The steps of construction can be written as bullet points in their notebooks for quick referral at any later time.

Evaluation

This chapter is a fun chapter and the students can be marked on their group activities and assignments.

After completing this chapter, students should be able to:

- construct perpendicular and parallel lines,
- describe the properties of perpendicular and parallel lines,
- identify adjacent, complementary, and supplementary angles, and
- find the unknown angle in a triangle when two angles are given.

13 Geometrical Constructions

Specific Learning Objectives

- Constructing perpendicular line bisectors
- Constructing an angle equal to a given angle
- Constructing an angle twice the size of a given angle
- Constructing angles of measurement 30°, 90°, 120°, 135°, 150°, and 165°
- Constructing triangles with three sides
- Constructing triangles with two sides and the included angle given
- Constructing triangles with two given angles and the length opposite to the angles
- Constructing a right-angled triangle given the hypotenuse and one side
- Constructing equilateral and isosceles triangles

Suggested Time Frame

6 to 8 periods

Prior Knowledge and Revision

This chapter involves the use of geometric instruments. Prior to beginning this chapter, the students should revise the key words, bisection, equilateral, and isosceles triangles. The properties of types of angles and triangles are also important.

Board geometric instruments should be used to teach the students the correct handling of the instruments.

Real-life Application and Activities

The teacher can stimulate the interest of the students by informing them that we can role-play by acting as architects and can plan designs using the geometric instruments.

Henceforth, all work is hands-on and as the teacher explains the steps of construction, the students should write them in their notebooks and construct figures accordingly.

Summary of Key Facts

- To construct a perpendicular bisector of line segments, the compasses should measure more than half the length of the line. Draw two arcs above and below the line segment that cut each other, and join them to get a bisector.
- To construct angle bisectors, the compasses should measure less than half the length of the lines. Draw two arcs between the arms of the angle that intersect each other. Draw a line through the point of intersection and the vertex of the angle.
- To construct a 60° angle, draw the base line and with a suitable radius, draw an arc that cuts the base line. With the same radius, draw another arc which cuts the previous arc. Extend the line from the end point to the point of intersection of the arcs.
- An angle of 30° is constructed by bisecting the constructed 60° angle.
- To construct a 90° angle, follow the steps of construction given in the book.
- An angle of 45° is constructed by bisecting the constructed 90° angle.
- To construct a 120° angle, draw two 60° angles side-by-side.
- To construct an angle of 150°, draw a 90° angle and a 60° angle side-by-side.
- To construct a 165° angle, draw a 150° angle and an adjacent 30° angle which is bisected to get a 15° angle. Hence 150° + 15° = 165°.
- A triangle can be constructed in many ways depending on the conditions given.

  We can construct triangles if:
  - all three sides are given.
  - two sides and an included angle are given,
  - one side and two base angles are given,
  - the lengths of one side and the hypotenuse of a right-angled triangle are given,
  - the perimeter and ratio between the lengths of sides are given,
  - the altitude of an equilateral triangle is given,
  - the vertical angle and the altitude of an isosceles triangle are given,
  - the base angle and the altitude of an isosceles triangle are given,

Frequently Made Mistakes

Students should be made aware that the correct use of geometric instruments for example how to hold and place the instruments, is important to produce accurate drawings.

Sample Lesson Plan

Topic

Geometric constructions

Specific Learning Objectives

Constructing triangles with the ratio of the sides and perimeter given

Suggested Duration

1 period

Key vocabulary

Ratios, Perimeter, Compasses

Method and Strategy

To construct a triangle with a given set of ratios, first revise the concept of proportional ratios.

Example:

Triangle ABC has sides in the ratio of 1 : 4 : 5.

If perimeter is 30 cm, then the sides will be calculated as:

$\overline{AB} = \frac{1}{10} \times 30 = 3$ cm

$\overline{BC} = \frac{4}{10} \times 30 = 12$ cm

$\overline{AC} = \frac{5}{10} \times 30 = 15$ cm

Draw $\overline{AC}$, the longest side, as the base line. With A and C as centres draw two arcs with radius 3 cm and 12 cm respectively, cutting each other at B.

Join B to A and C.

It should be pointed out that sometimes mathematical computations are done before proceeding with construction of the triangle. Similarly, to construct a triangle with given altitude or vertical angle, mathematical working will be required.

Written Assignment

Practice sums should be given in class and the teacher should approach each child individually to help them use the geometric instruments along with helping with the mathematical concepts.

Evaluation

Marked assignments should be given in class and homework periodically before taking a comprehensive assessment of this chapter.

The test should have a choice of options of different cases of constructions and at least 5 sums of constructions should be given for the duration of a one-period test.

After completing this chapter, students should be able to:

- bisect a given line segment using a ruler and a pair of compasses,
- construct a line perpendicular to a given line,
- draw angles of a required size with a protractor,
- construct angles measuring 60°, 90°, 120°, 135°, 150°, and 165°,
- bisect 60°, 90°, 120°, 135°, 150°, and 165° angles, and
- construct a triangle when different measurements of sides and angles are given.

14 Circles

Specific Learning Objectives

- The properties of circles
- Constructing circles, semicircles, and segments

Suggested Time Frame

2 to 3 periods.

Prior Knowledge and Revision

Parts of a circle have been taught earlier; however, the difference between a chord, diameter, and, radius should be explained with the help of diagrams.

The radius is the distance from the centre to the circumference of the circle, whereas the diameter is the measure of the circle across, passing through the centre.

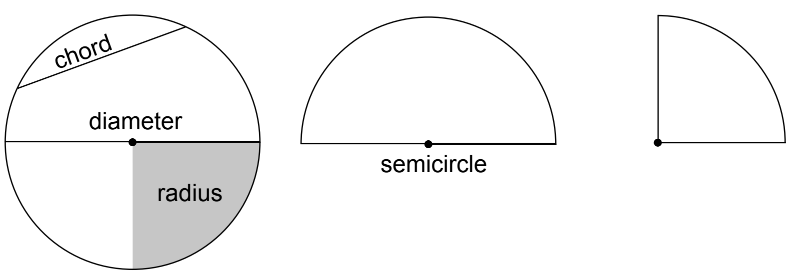
The radius touches the circumference of a circle at one point, while the diameter touches it at two points.

The radius and diameter are constant values.

A chord touches the circle at two points but does not pass through the centre.

A semicircle is half a circle, subtended (meeting at two points) by a diameter. A quadrant is a quarter of a circle subtended by two radii.

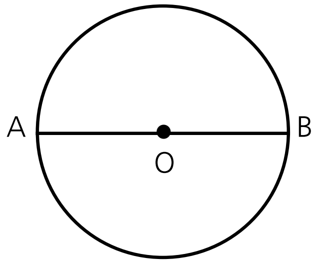
The circumference of a circle is its perimeter and the circular measure of its boundary.

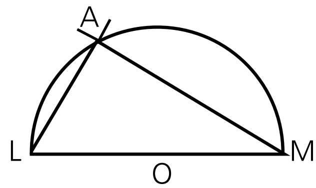


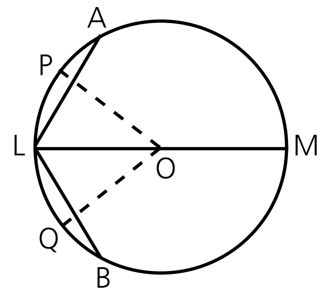
Real-life Application and Activities

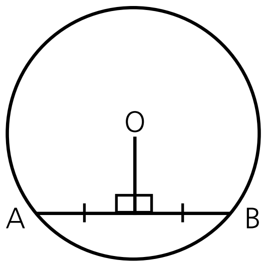
Construction of circles and semicircles is relatively easy as only the use of compasses is required and the students need to get the value of the radius on the compasses and the circle or semicircle can be drawn.

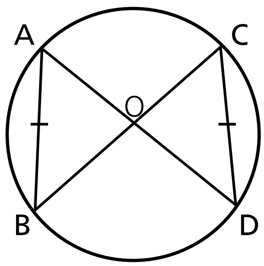
The theorems of circles are extremely critical and these can only be explained if done practically.

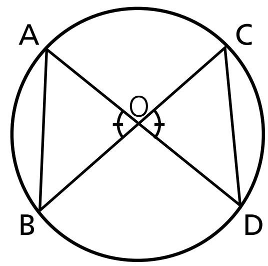












Activity

You will need chart paper, drawing pins, and thread.

Cut out a big circle with the width of the chart paper as the diameter.

Put the drawing pins at the end point of the diameter. Put a thread around the drawing pins to make a loop. Pull the thread and pin it opposite the diameter on the semicircle.

Measure the angle formed on the circumference with a protractor or a set square: it will be 90º.

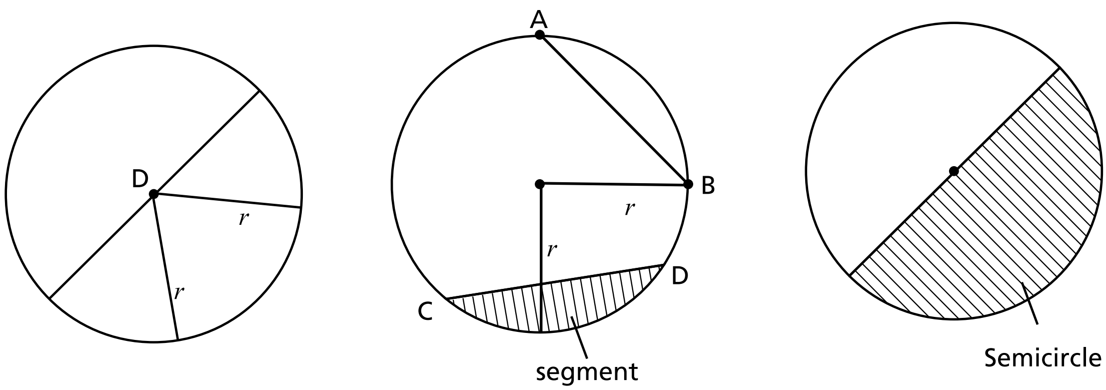
Similarly, on the same chart paper loop a thread around the two drawing pins and pin it on the circle at two points on the circumference, this time to create a chord and not a diameter. Measure the distance of the chord from the centre and use the same measurement to tie another chord on the other side of the centre of the circle. Measure the length of the threads forming the chord: they will be equal.

By this method all theorems can be proved. The students write the theorem on the chart paper. Help students to prove all the theorems practically.

Summary of Key Facts

Elements of a circle

A circle is traced on a plane by a point moving in such a way that its distance from another fixed point on the plane is always constant.



Centre:

The centre of a circle is the fixed point on the plane from which the distance of the moving point is always constant.

Circumference:

The circumference of a circle is the boundary or **perimeter** of the circle.

Radius:

The distance between the centre and a point on the circumference is called the radius. It is denoted by 'r'.

Sector:

The area between two radii is called a sector.

Minor sector:

If the angle between the two radii is less than 180°, then the sector is a minor sector.

Major sector:

If the angle between the two radii is greater than 180°, then the sector is a major sector.

Diameter:

The diameter is the distance between two points on a circumference along a straight line that passes through the centre. The diameter is denoted by 'D'. It is equal to twice the radius i.e. D = 2r.

Arc:

Any part of the circumference or perimeter of a circle is known as an arc of the circle. The curved segment AB is an arc of the circle.

Chord:

A line segment that joins the endpoints of an arc is called a chord. The line segment $\overline{AB}$ is a chord of the circle. The diameter can be defined as a chord that passes through the centre of the circle. The diameter is the longest chord of a circle.

Segment:

A segment of a circle is the area enclosed between an arc and the corresponding chord.

Major arc:

When a circle is divided into two parts by a chord, the arc that forms the larger part is called the major arc.

Minor arc:

When a circle is divided into two parts by a chord, the arc that forms the smaller part is called the minor arc.

Major segment:

When a circle is divided into two parts by a chord, the larger segment formed is called the major segment.

Minor segment:

When a circle is divided into two parts by a chord, the smaller segment formed is called the minor segment.

Semicircle:

A semicircle is one half of a circle formed when a circle is divided by the diameter.

- Concentric circles are circles with a common centre but different radii.
- Equal chords are equidistant from the centre, and vice versa.
- A perpendicular line drawn from the centre to the chord bisects the chord, and vice versa.
- Equal chords subtend equal angles at the centre and vice versa.
- The angle subtended by the diameter of a circle at the circumference of the circle is a right angle.

Frequently Made Mistakes

Students even at a higher grade mix up the concepts of chords and diameters. This causes further confusion later on while working on the theorems.

Sample Lesson Plan

Topic

Circles

Specific Learning Objective

The properties of the circle:

- equal chords are equidistant from the centre,
- the perpendicular line from the centre bisects the chord, and
- equal chords subtend equal angles at the circumference.

Suggested Duration

1 period

Key Vocabulary

Chords, Equidistant, Subtend, Perpendicular and Bisect.

Method and Strategy

The activity stated earlier can be shown to the students to revise the theorems. However, for the theorems to be more effective in application, a lot of practice sums should be done.

Written Assignments

Exercise 14b should be done in class on the board and then given for homework.

Evaluation

A comprehensive assessment on this chapter should be given. It should be pointed out to students that inaccuracy in construction will result in the loss of marks.

After completing this chapter, students should be able to:

- describe a circle in terms of its elements,
- construct circles, semicircles, and segments using a pair of compasses, and
- demonstrate the properties of a circle.

15 Congruence and Similarity

Specific Learning Objectives

- Properties of congruent triangles
- Properties of similar triangles
- Applying properties of similarity and congruence

Suggested Time Frame

6 to 8 periods.

Prior Knowledge and Revision

Students are aware of triangles and other polygons; in this chapter a new concept of congruence and similarity is introduced.

The teacher should brainstorm with the students and prompt and explain the meanings of congruence and similarity.

Congruence: exact same (equal) size, angles, faces etc.

Similarity: same shape but different sizes

The students may come up with geometric instruments of the same brand that are exactly the same and link it with congruence, and of different brands that are similar in relation to sizes.

Real-life Application and Activities

The teacher should explain congruence with real-life examples. Various examples are apartments in building complexes, pots and pans of the exact same size, and Lego blocks. Due to the fact that they are exactly equal in all aspects of measurement, they tend to look like clones. This is a helpful analogy to create and make a list of real-life congruence.

Similarity is best explained with the example of the Russian dolls that fit one inside the other. Though they are of different sizes, they fit inside each other as the curves/angles are the same. The teacher can bring in clay plant pots that are similar and of different sizes, and show that the measurements of length are proportional but the angular aspect remains exactly the same.

Similarity is related to enlargement and magnification by a scale factor. Another real-life example could be enlarged pictures on a computer printing each picture in various sizes.

Students should be encouraged to make a table/chart presentation of similar and congruent objects in real-life.

Summary of Key Facts

- Congruency of triangles is defined by four properties: SSS, SAS, ASA and RHS.
- Similarity is when all corresponding angles are equal and corresponding sides are in proportional ratios.

Frequently Made Mistakes

The difference between similar and congruent figures must be explained clearly and students must learn the properties. They tend to jumble up the proof of similarity with that of congruence.

Sample Lesson Plan

Topic

Congruence

Specific Learning Objectives

The cases of RHS and SAS and their application

Suggested Duration

1 period

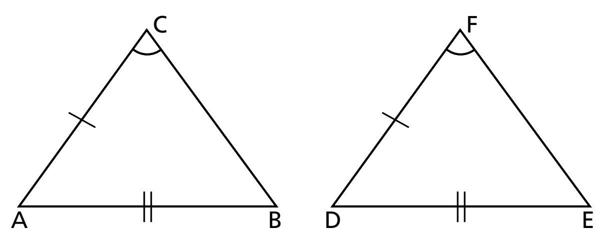
Key Vocabulary

Hypotenuse, Adjacent and included angle

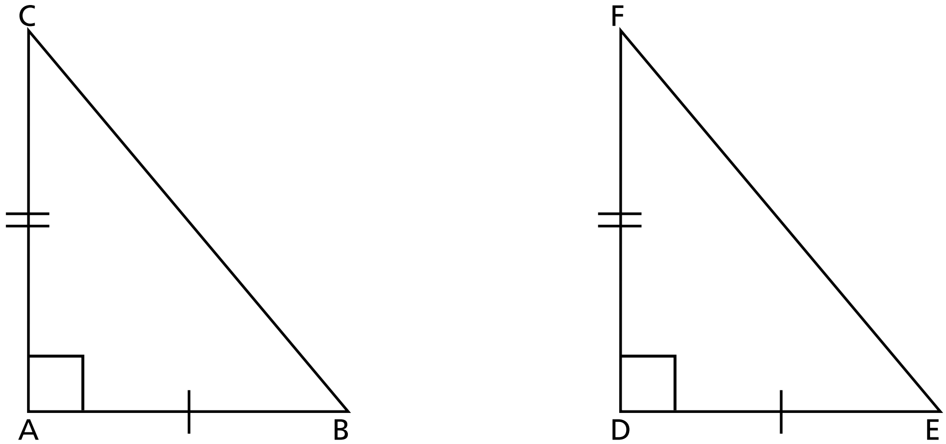
Method and Strategy

Students have already been introduced to the cases of SSS and SAS. It should be made clear to them that when a right-angled triangle has a side and hypotenuse congruent, the case becomes RHS. However, to prove congruency with the included angle should be between the congruent sides. Invariably, a triangle with two sides and one angle which is not in between the two congruent sides makes the case null and void. Similarly, in the case of RHS, two right-angled triangles need not be congruent if two angles and a side, or a right angle with its arms congruent is given. Then, the cases would become ASA and SAS respectively.

Example:



Included $\angle A$ and $\angle D$ are not given, therefore triangles are not congruent.



✗ Not RHS
✓ SAS

Written Assignment

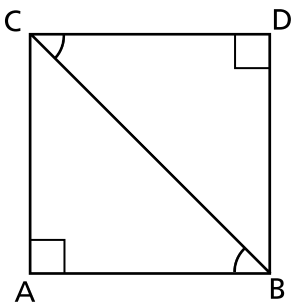
Practice sums from Exercise 15 in the chapter can be done in class and the rest can be given for homework.

The followings sum can be given in class as a quiz.

1. State the case of congruency if congruent.

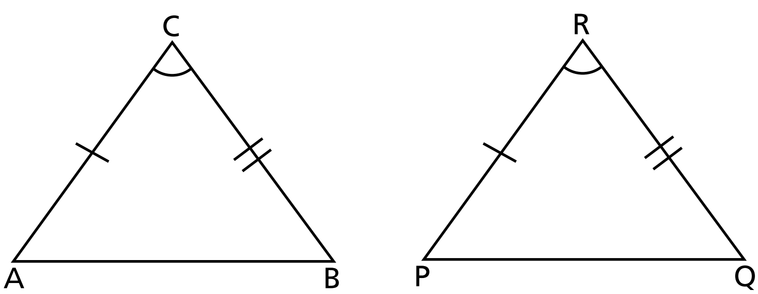
Answers

(a)



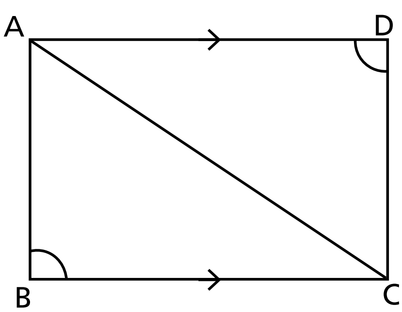
Yes the triangles are congruent.
Property: AAS

(b)



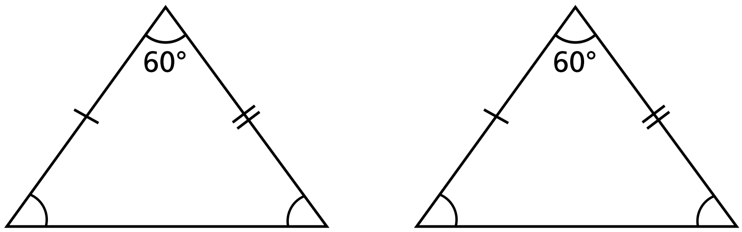
Yes the triangles are congruent.
Property: SAS

(c)



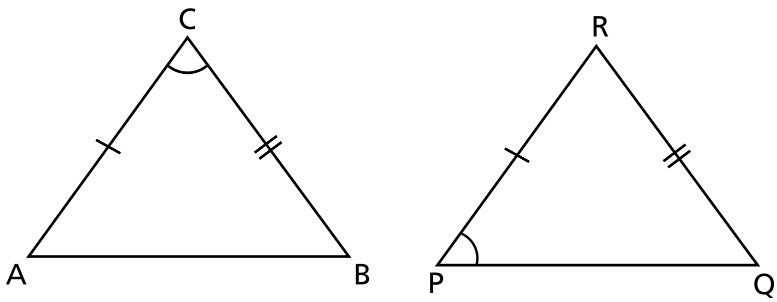
Yes the triangles are congruent.
Property: AAS or SSS

(d)



Yes the triangles are congruent.
Property: SAS

(e)



The triangles are not congruent.
No property is satisfied.

Evaluation

A comprehensive assessment should be given at the end of the topic but in between short quizzes on the board could be conducted towards the end of each lesson to check students' understanding. This chapter introduces an entirely new concept and stage-by-stage assessment is necessary.

After completing this chapter, students should be able to:

- identify congruent figures,
- identify similar figures,
- establish congruence and similarity between geometric figures, and
- test for the congruency of two triangles using the SSS, ASA, and RHS properties of congruency of triangles.

16 Quadrilaterals

Specific Learning Objectives

- Properties of quadrilaterals: parallelograms, rhombuses, rectangles, and squares
- Constructing a parallelogram

Suggested Time Frame

5 to 6 periods

Prior Knowledge and Revision

The concept of shapes made by 3 or more line segments has already been introduced to the students. A brainstorming session on identification of various shapes can be done on the board. After the identification of various quadrilaterals, the teacher should prompt the students to identify the properties of each shape.

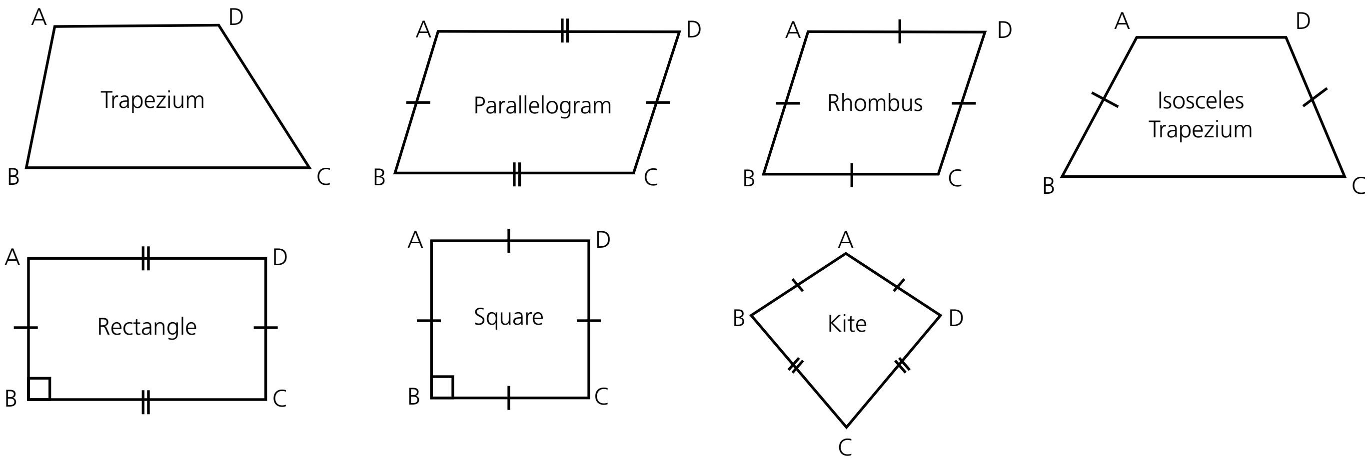
Example

A rhombus has all sides equal but it is not a square. Why? (The angles are not right angles.)

A parallelogram has length and breadth but it is not a rectangle. Why? (There are no right angles at the vertices.)

A kite is an unusual quadrilateral with equal sides adjacent to each other (two small adjacent sides equal and two longer adjacent sides equal).

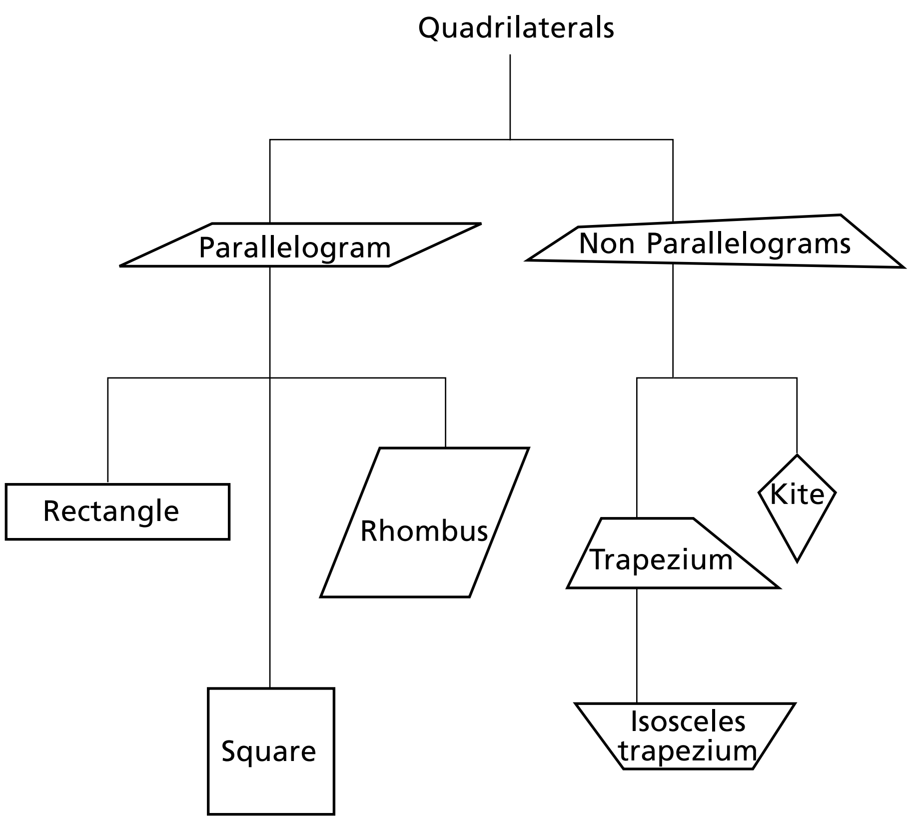
A trapezium is different from a parallelogram. Give two properties supporting the statement (only one set of parallel lines and the parallel lines are not equal in length).



Real-life Application and Activities

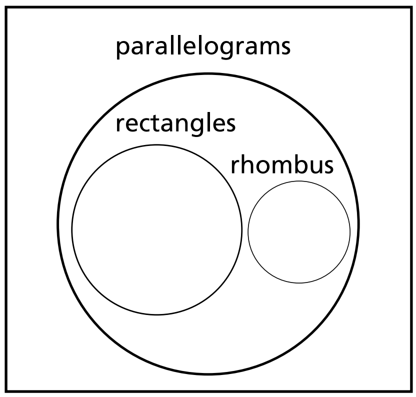
To reinforce knowledge of the properties, the students can be divided into groups and each group can be assigned a quadrilateral. The group then makes a cut-out of the shape assigned from chart paper and give a minute-long presentation on the properties of the assigned quadrilateral.

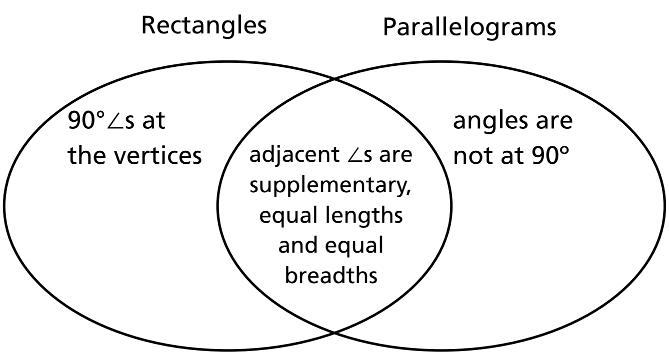
Summary of Key Facts

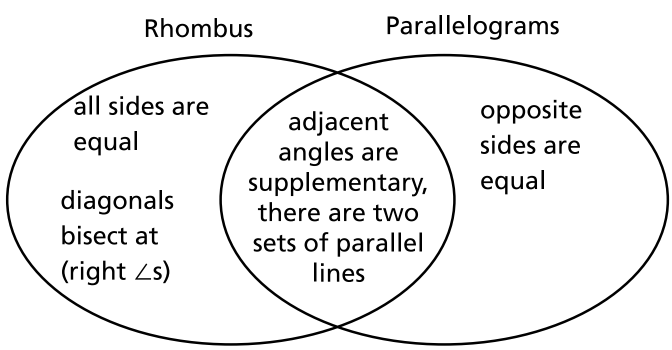


- A **parallelogram** is a quadrilateral in which the opposite sides are parallel.
- Opposite angles of a parallelogram are congruent.
- A **rhombus** is a parallelogram in which all four sides are equal.
- An additional property of a rhombus not found in a parallelogram is that its diagonals bisect at right angles.
- A **rectangle** is a parallelogram in which all the angles are right angles.
- A **square** is a rectangle with four equal sides.
- A **trapezium** is a quadrilateral with only two sides parallel.
- An **isosceles trapezium** is a trapezium in which the non-parallel sides are equal.
- A **kite** is a quadrilateral in which the two pairs of adjacent sides are equal (in general, the opposite sides are not parallel or equal).
- The tree diagram above explains the relationship between the different polygons.
- The sum of the angles in a quadrilateral is 360°.

Quadrilateral







Frequently Made Mistakes

Students find the relationship between the shapes of quadrilaterals a bit challenging. If the shapes are taught in a way where the overlapping properties are first pointed out and then the additional properties which differentiate one shape from the other are explained it will help the students immensely. Venn diagrams of similarities and differences will also help.

Sample Lesson Plan

Topic

Constructing a parallelogram

Specific Learning Objectives

Constructing a parallelogram

Suggested Duration

1 period

Key Vocabulary

Adjacent, Included angle

Method and Strategy

When doing constructions, the instruments have to be in sound condition. Often the protractor readings are not clear or have been erased. Similarly the compasses is loose which results in inaccurate measurements.

Case: when two adjacent sides and included angle are given

There are two ways of approaching this case. If the included angle is given and we know that adjacent angles are supplementary, the second angle can be calculated and both angles can then be drawn on the base line segment. Arcs of measurement equal to the breadth are then made on the arms of the angles and the parallelogram is completed.

The second approach is mentioned in the text book on page 201. The need for calculating the second angle does not arise as the arc measurements of the length and breadth gives us the third vertex, and subsequently the fourth vertex.

Written Assignments

5 sums of construction can be given to the student to do in class and then a set of another 5 sums can be given for homework.

The worked examples in the textbook can also be done in their notebooks.

Evaluation

Exercise 16 is a comprehensive exercise on the concepts of this chapter with multiple choice questions. An assessment along the lines of this exercise can be given.

After completing this chapter, students should be able to:

- identify the different types of quadrilaterals,
- identify the properties of parallelograms, rectangles, squares, and rhombuses, and
- construct parallelograms and rhombuses.

17 Perimeter and Area of Geometric Figures

Specific Learning Objectives

- Calculating the perimeter of a parallelogram, rectangle, square, triangle and trapezium
- Calculating the area of a parallelogram, rectangle, square, triangle, trapezium, rhombus
- Introducing tessellations
- Calculating the perimeter and area of a circle

Suggested Time Frame

6 to 8 periods

Prior Knowledge and Revision

This chapter is a continuation of concepts taught in the earlier grades. The students are aware of the concepts of area and perimeter, so no formal introduction is needed.

Revision of shapes and calculating the area and perimeter of composite shapes made up of squares and rectangles can be done.

Activity

The teacher can make the revision fun by bringing cut-outs on chart papers and dividing the class into groups and asking them to calculate the area and perimeter of the cut-outs. These cut-outs can be put on the floor and the groups can work on the floor. This activity should not take more than five minutes. The calculations can be done in their exercise notebooks.

Shapes are everywhere; architecture involves spatial geometry, and the construction of a house involves calculation of materials required, areas, etc. Even something as relatively simple as making a wooden table or cupboard requires knowledge of the concepts taught in this chapter. If there is an in-house handy man or carpenter in school, he can be invited to the lesson to explain the dimensions and material requirements of making a desk.

Students should not be given the formulae as mathematical computations alone. They need to understand the derivation to appreciate the real-life application.

Activity

Things you will need:

2 sheets of coloured chart paper, thick markers, ruler, and a pair of scissors

Ask the students to draw a parallelogram of the same size as the chart paper. You can help them draw the parallel lines. Help the students to use the protractor to draw the interior angles. Label the shape.

They will see the shape can be divided into two congruent triangles. The concept of congruency can be applied here.

Now show the derivation on the board and explain that the triangles are two congruent shapes so the ½ of the formula is cancelled and we end up with $b \times h$.

Real-life Application and Activities

The value of pi (π) can easily be explained with an interesting hands-on activity.

Activity

You will need: a 1 metre length of yarn, (any thick thread will also do), a marker, different everyday objects that are circular e.g. a CD, circular plate, circular sharpener, a 30 cm ruler, and play dough or any adhesive.

Fasten the yarn **around** the circular object with the play dough so it stays in position.

Measure the yarn and record the length in centimetre.

Now place the yarn **across**, making sure it passes through the centre.

Measure and record the length.

Ask the students to calculate the value: around/across.

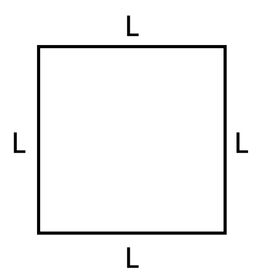
They should come to a value close to 3.142.

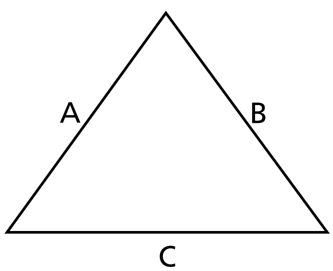
They should repeat this process with two more circular objects of different sizes.

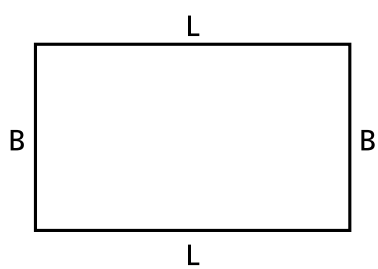
The teacher should then point out the constant value of π that it is 3.142 for all circles.

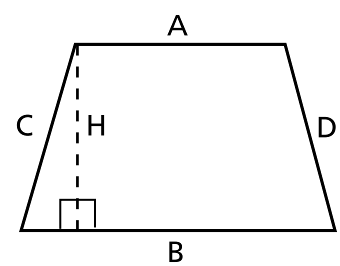
Once the formula for the circumference is introduced he/she will relate **around** to the circumference and **across** to the diameter.

Summary of Key Facts









- The distance around a 2D shape is the perimeter.
  Perimeter of a rectangle $= 2(l+b)$
  Perimeter of a square $= 4l$
  Perimeter of a parallelogram $= 2(l+b)$
  Perimeter of a triangle $= a + b + c$
  Perimeter of a trapezium $= a + b + c + d$
  Perimeter (circumference) of a circle = $2\pi r$ or πd
- The area of a 2D shape is the number of square units that the figure covers.
  Area of a rectangle $= l \times b$

  Area of a square $= l \times l = l^2$

  Area of a parallelogram $= b \times h$

It should be pointed out that the height of the parallelogram is critical in calculating the area of the shape.

Area of a triangle $= \frac{1}{2}(b \times h)$

Area of a trapezium $= \frac{1}{2}(a + b) \times h$ (a and b or parellal sides)

Area of a rhombus = $\frac{1}{2}(d_1 \times d_2)$ (d_1 and d_2 or the two diagonal)

Area of a circle = πr^3

- The concept of altitude used in parallelograms, triangles, and trapeziums should be made clear. It is the perpendicular distance to the base of the shape. Incidentally, it is also the shortest distance between the two sides.
- To find the areas that are borders the concepts of external area and internal area should be made clear. Once these have been found, the areas are subtracted to get the area of the borders.
- Tessellations are repetitive patterns of the same shape. To tessellate is to repeat a pattern in such a way that no gaps in the area are created.

Frequently Made Mistakes

Students generally get confused with the identification of the altitude and perpendicular lines. The earlier chapter on this can be revised to obtain correct values which are substituted in the formulae.

Sample Lesson Plan

Topic

Tesselation

Specific Learning Objectives

The concept of tessellation

Suggested Duration

1 period

Key Vocabulary

Tessellation or tessellate, Polygon, Pattern

Method and Strategy

Real-life examples of tessellations can be given.

Example

Honeycomb in a beehive is made up of hexagons.

Tiles on the floor are repetitive patterns of the same shape.

Activity

A hands-on activity can be done in class.

Each student requires sheets of coloured paper, marker, ruler, glue stick and a pair of scissors.

A set of flash cards can be made with the name of a different shape written on each flash card. For example, pentagon, parallelogram, equilateral triangle, square, rectangle etc.

Each student is asked to pick a flash card and then asked to make 10 or 12 cut-outs of the shape. They are then glued side by side in his/her notebook, to check whether the shapes tessellate or not.

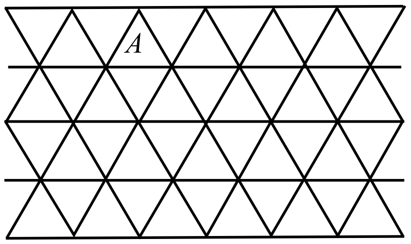
This activity will be fun and the concept of tessellation will be very clear.

Written Assignment

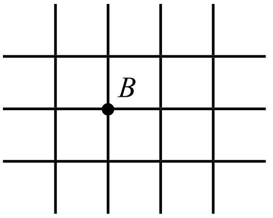
An assignment can be given where the students are asked to draw tessellations of five shapes.

Example

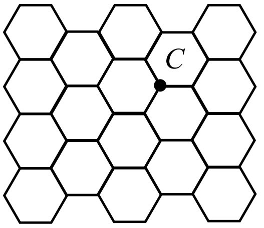
Tessellations of:



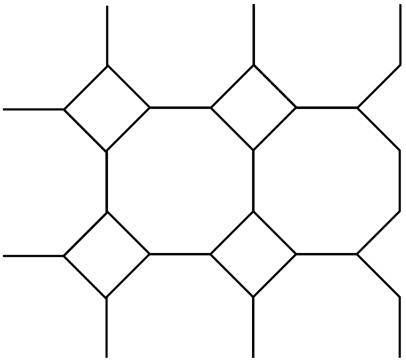
A tessellation formed by equilateral triangles



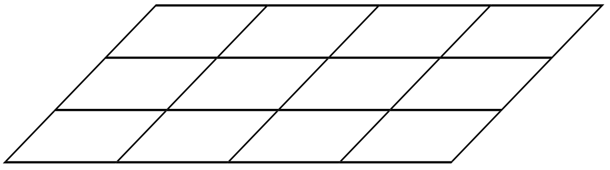
A tessellation formed by squares



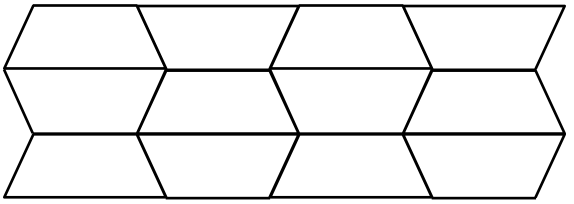
A tessellation formed by regular hexagons



A tessellation formed by squares and regular octagons



A tessellation formed by parallelograms



A tessellation formed by isosceles trapeziums

Evaluation

A comprehensive test along the lines of Exercises 17a and 17b can be given on the completion of the chapter.

After completing this chapter, students should be able to:

- calculate the perimeter of a parallelogram, rectangle, square, and triangle by applying the relevant formulae,
- calculate the area of a parallelogram, rectangle, square, triangle, and trapezium by applying the relevant formulae,
- calculate the circumference of a circle when the diameter or radius is given,
- calculate the area of a circle by applying the formula, and
- identify tessellated patterns in the environment.

18 Volume and Surface Area

Specific Learning Objectives

- Calculating the surface area of a cube, cuboid and cylinder
- Calculating the volume of a cube, cuboid, and cylinder

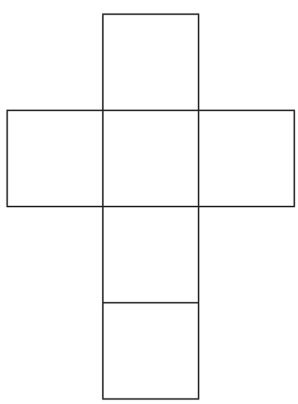
Prior Knowledge and Revision

Students calculated the surface area and volume of cubes and cuboids in the previous grade.

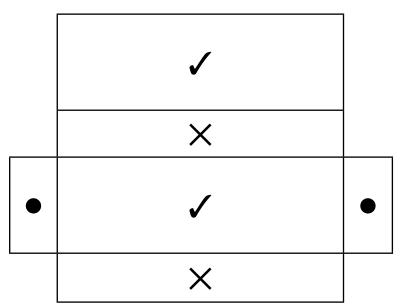
A brief revision at the beginning of the lesson can be done where the faces and edges of a cube and a cuboid are identified and the surface area and volume formulae are revised.

Activity

Net diagrams of a cube and cuboid can be photocopied and handed to the students. They can cut them out and tape the edges to create a 3D shape out of a 2D cut-out. This will highlight the fact that a 2D shape can be converted into a 3D shape that will have a volume.



Net diagram of a cube
Since all the sides are equal, the faces are all equal in area and dimensions.



Net diagram of a cuboid
Since the dimensions are different, 2 faces each have the same dimensions and same area.

Real-life Application and Activities

Activity

The relationship between volume and base area of cubes, cuboids, and cylinders can be explained. The basic formula for volume is:

Volume = base area × height

The shaded region in each diagram below is the base.

Therefore the volume of each shape can now be easily calculated.

i) Volume of a cuboid = base area × height

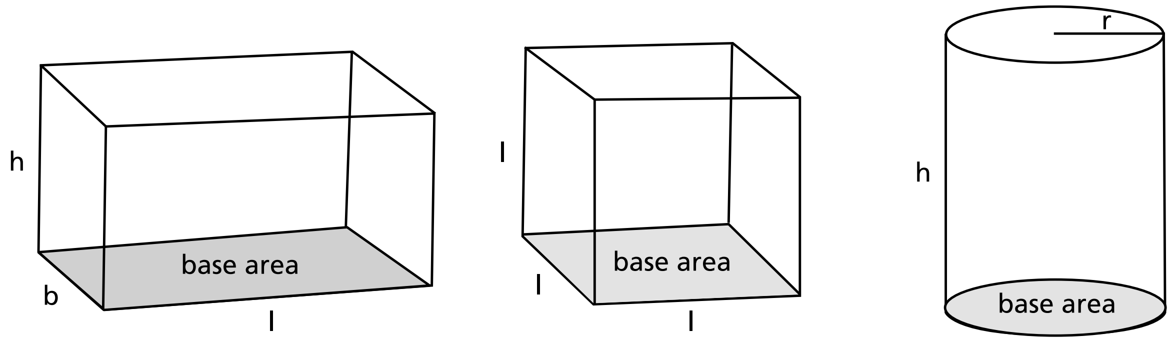
$V = (l \times b) \times h$ (base is a rectangle, therefore the base area = $l \times b$)

ii) Volume of a cube = base area × height

$V = (l \times l) \times l = l^3$ (base is a square, therefore the base area = $l \times l$)

iii) Volume of a cylinder = base area × height

$V = (\pi r^2)h$ (base is a circle, therefore the base area = πr^2)



Summary of Key Facts

- 1 cubic metre = 1000 litres
- 1 litre = 1000 cm^3
- Volume of a cube = l^3
- Total surface area of a cube = $6l^2$
- Volume of a cuboid = length × breadth × height
- Total surface area of a cuboid = $2(l \times b) + 2(b \times h) + 2(l \times h)$

Frequently Made Mistakes

The identification of dimensions when applying the formula is very important. The students tend to put in the value of the diameter instead of the radius. Similar mistakes also occur in the case of cubes and cuboids.

The hands-on activity of the net diagram will ensure that the concepts of the dimensions and their shapes are clear.

Sample Lesson Plan

Topic

Volume and surface area of a cylinder

Specific Learning Objectives

Calculating the surface area of a cylinder

Suggested Duration

1 period

Key Vocabulary

Radius, Circumference, Height, Curved surface area, Total surface area

Method and Strategy

Activity

The most effective way of teaching the formula of the surface area of a cylinder is to take a piece of paper and show the class the length and breadth of the paper.

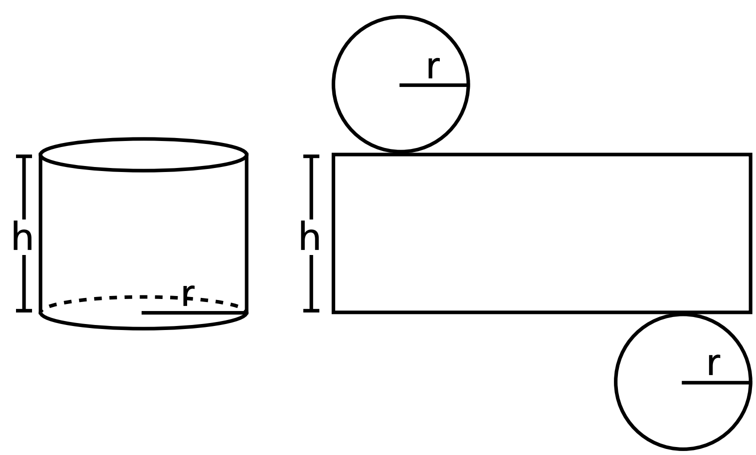
Highlight the fact that the rectangular paper is actually the curved surface area of the cylinder as it folds to form a cylinder.

Put two circle cut-outs on the top and bottom of the paper cylinder. Make sure that the circumference of the two circles is equal to the length of the rectangle; only then they will be placed perfectly.

This fact can be pointed out to the students.

Curved surface area of a cylinder = $2\pi r \times h$

Total surface area of a closed cylinder = $2\pi rh + 2\pi r^2$



Written Assignment

Questions 6 and 7 from Exercise 18b can be done in notebooks. They should note the formulae down with markers in their notebooks before they proceed to do the sums.

5 sums of finding total and curved surface areas of cylinders can be given for homework.

Evaluation

This is an extremely conceptual topic. Quizzes to find the area or volume of any one shape should be given at the beginning of each lesson. This way the concepts will be further enhanced as the chapter progresses. A comprehensive assessment covering all concepts should be given once the students are confident. The revision exercise on pages 236 to 237 can be used to assess mensuration.

After completing this chapter, students should be able to:

- calculate the volume of a cuboid, cube, and right circular cylinder by applying the formulae,
- calculate the surface area of a cuboid, cube, and right circular cylinder by applying the formulae.

19 Information Handling

Specific Learning Objectives

- The importance of data presentation
- Difference between ungrouped and grouped data
- Concepts of class interval and frequency, range of data, lower and upper class limit
- Constructing a frequency distribution table
- Reading bar charts and pie charts
- Constructing bar charts

Suggested Time Frame

4 to 5 periods

Prior Knowledge and Revision

Students are aware of line graphs and bar graphs. The teacher can give a Power Point presentation and show colourful slides of various bar graphs and line graphs. A list of questions can be read out and the students can answer by looking at the slides. This activity will not only help them revise the concepts, but will also add variety to mathematics lessons..

Real-life Application and Activities

Students are aware of simple distribution of data where frequency is not mentioned. The facts that the data is now grouped and the quantity is within a range have to be explained clearly. The steps of converting raw data into grouped data and that of constructing a bar graph, have to be explained clearly and highlighted as a soft board presentation.

Students should be encouraged to draw on chart paper representations of a bar graph. They can work in groups. This will enhance their understanding as they will benefit from peer cooperation.

Summary of Key Facts

- Data in its raw form is called ungrouped data.
- Tally marks consist of vertical lines with the fifth line drawn diagonally through the 4 vertical lines. This gives a bundle of five.
- A frequency distribution table consists of class intervals and their corresponding frequencies.
- The difference between the greatest and smallest data values is called the range of the data.
- A pie chart represents the distribution in the form of sectors of a circle.

Frequently Made Mistakes

Students often make mistakes adding frequency. In the case of a pie chart, if the angles of the sector are to be calculated, then they should check that their sum is 360°, as angles at a point add up to 360°. Similarly care should be taken while calculating percentages as their total should be 100%.

Sample Lesson Plan

Topic

Pie charts

Specific Learning Objectives

To calculate the value of the angles of a pie chart

Suggested Duration

1 period

Key Vocabulary

Sectors, Circles, Frequency distribution, Pie charts

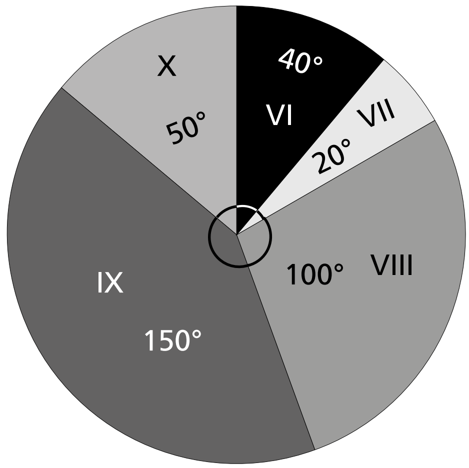
Method and Strategy

A pie chart represents information in a circle. Each distribution is represented by a sector. All the sectors together form one complete circle. The angle of each sector is calculated arithmetically and it should be pointed out that since angles at a point add up to 360°, the angles of all sectors should also add up to 360°.

Any distribution can be presented in the form of a bar graph or a pie chart.

Example

The number of students in different classes of a school who like to play hockey are given below. Draw a pie chart to represent the same.



Grade VI: $\frac{4}{36} \times 360° = 40°$

Grade VII: $\frac{2}{36} \times 360° = 20°$

Grade VIII: $\frac{10}{36} \times 360° = 100°$

Grade IX: $\frac{15}{36} \times 360° = 150°$

Grade X: $\frac{5}{36} \times 360° = 50°$

To check : 40° + 20° + 100° + 150° + 50° = 360°

It should be explained that in order to calculate the values of the angles of a pie chart, we take the frequency of the subject over the total to create a fraction and then multiply by 360° as it is going to be a fraction of a full circle.

Angle of a sector $= \frac{\text{frequency}}{\text{total frequency}} \times 360°$

Written Assignment

Questions 6 to 10 of Exercise 19 can all be used to convert the data given into pie chart data. The teacher should explain and do a couple on the board and ask for the rest to be done in classwork notebooks.

Only when the students are clear about the calculations of 'how' and 'why' should the teacher ask the students to bring protractors to the next lesson and proceed to teach the construction of pie charts.

Evaluation

This is a presentation-based chapter. Marks can be awarded on assignments and classwork involving bar graphs and pie charts.

After completing this chapter, students should be able to:

- explain the importance of presenting data clearly and accurately,
- group data in the form of a frequency distribution table,
- determine class size by using data range and number of classes,
- read bar charts and pie charts, and
- construct bar charts for given data.

Assessment

A teacher's journey involves three stages Exposition, Practice, and Consolidation.

Exposition is the setting forth of content, and the quality and extent of the information relayed.

Practice involves problem solving, reasoning and proof, communication, representations, and correction.

Assessment is the final stage of consolidation of the process of learning. Assessment of teaching means taking a measure of it effectiveness. **Formative** assessment is measurement for the purpose of improving it. **Summative** assessment is what we normally call evaluation.

An ideal and fair evaluation involves a plan that is comprehensive. It covers a broad spectrum of all aspects of mathematics. The assessment papers should test all aspects of topics thought. These can be demarcated into categories: basic, intermediate, and advanced content. The advanced content should be minimal as it tests the most able students only.

Multiple choice questions, also known as fixed choice or selected response items, require students to identify the correct answer from a given set of possible options.

Structured questions assess various aspects of students' understanding: knowledge of content and vocabulary, reasoning skills, and mathematical proofs.

All in all the teaching's assessment of students' ability must be based on classroom activity, informal assessment, and final evaluation at the end of a topic and/or the year.

Specimen Paper
Mathematics
Grade 7

Section A

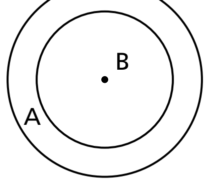
Time: 1 hour **Total marks: 40**

1.	A set containing the common elements of A and B is formed by **A.** adding set **B.** union of sets **C.** intersection of set **D.** difference of two sets	5.	Which of the following is a non-terminating decimal? **A.** $\frac{3}{6}$ **B.** $\frac{1}{8}$ **C.** $\frac{2}{5}$ **D.** $\frac{11}{17}$
2.	If A = {1, 2, 3, ….., 50} and B = {1, 3, 5, ….. 49}, what is A∪B? **A.** Set A **B.** Set B **C.** A – B **D.** B – A	6.	$\frac{25}{4}$ is same as **A.** 625 **B.** 0.625 **C.** 6.25 **D.** 62.5
3.	Which property is represented by? $\left(\frac{a}{b} \times \frac{c}{d}\right) \times \frac{g}{h} = \frac{a}{b} \times \left(\frac{c}{d} \times \frac{g}{h}\right)$ **A.** commutative property of addition **B.** associative property with respect to multiplication **C.** distributive with respect to multiplication **D.** BODMAS rule	7.	What must be added to 1999.061 to make 2000? **A.** 0.0939 **B.** 0.939 **C.** 1.061 **D.** 1000
4.	Which number should be subtracted from $-\frac{9}{4}$ to get $-\frac{3}{16}$? **A.** $\frac{6}{16}$ **B.** $-\frac{39}{16}$ **C.** $-\frac{33}{16}$ **D.** $\frac{39}{16}$	8.	The square root of 10 × 10 × 10 × 10 is **A.** 100 **B.** 1000 **C.** 10000 **D.** 10

9.	If 36 boys stand making equal number of rows and columns, how many boys are in each row? **A.** 6 **B.** 18 **C.** 9 **D.** 4	15.	A decrease in selling price is a **A.** loss **B.** tax **C.** profit **D.** % discount
10.	0.49 is the square of **A.** 7 **B.** 0.07 **C.** 0.7 **D.** 4.9	16.	The cost price of an object is Rs 4500 and the selling price is Rs 4050. Calculate the profit or loss per cent. **A.** 45% gain **B.** 70% loss **C.** 10 % loss **D.** 10% gain
11.	$\frac{64}{125}$ is same as **A.** $\frac{2^5}{5^3}$ **B.** $\frac{(2^2)^3}{5^3}$ **C.** $\frac{4^3}{5^2}$ **D.** $\frac{2^4}{125}$	17.	The interest gained on Rs 25,000 at the rate of 10% for 3 years is **A.** Rs 7500 **B.** Rs 32,500 **C.** Rs 750 **D.** Rs 750,00
12.	If Ali makes 50 baskets in 5 days. How many basket will he make in 3 days? **A.** 250 **B.** 15 **C.** 30 **D.** 150	18.	A person pays Rs 36,000 as income tax at the rate of 12% per year. What is his income? **A.** Rs 72,000 **B.** Rs 4,320 **C.** Rs 300,000 **D.** Rs 30,000
13.	If a : b = 4 : 7 and b : c = 7 : 9 what is a : b : c? **A.** 4 : 14 : 9 **B.** 4 : 7 : 9 **C.** 4 : 1 : 9 **D.** 1 : 7 : 9	19.	$P + \frac{P \times R \times T}{100} =$ **A.** Simple interest **B.** Rate **C.** Amount **D.** Income tax
14.	A car travels 110 km in 2 hours. How far will it travel in 3 hours? **A.** 220 km **B.** 165 km **C.** 330 km **D.** 73.2 km	20.	Simplify $9x(-x)^2$ **A.** $-9x^3$ **B.** $9x^3$ **C.** $9x^2$ **D.** $9x$

21.	Three sides of a triangle are $(x + 5)$ m, $(x + 8)$ m and $(x - 3)$ m. What will be its perimeter? **A.** $(2x + 10)$ m **B.** $(3x + 10)$ m **C.** $(3x + 16)$ m **D.** $(10x)$ m	26.	The value of $a^2 + 2a^2 - a = 2$, if **A.** $a = -1$ **B.** $a = 1$ **C.** $a = 0$ **D.** $a = 2$
22.	How much should be added to $x + 9$ to make it $3x - 4$? **A.** $-2x + 13$ **B.** $4x + 13$ **C.** $4x + 5$ **D.** $2x - 13$	27.	If $50x = (45)^2 - (35)^2$ then the value of x is **A.** 2 **B.** 8 **C.** 400 **D.** 16
23.	$9x^2 - 25y^2$ expressed as product of two terms will be **A.** $(3x - 5y)(3x + 5y)$ **B.** $(3x)^2 (5y)^2$ **C.** $(9x - 25y)(9x + 25y)$ **D.** $(9x)(25y)$	28.	Which of the following set of angles is both supplementary and vertically opposite? **A.** 45°, 35° **B.** 100°, 80° **C.** 90°, 90° **D.** 35°, 75°
24.	If the area of a square is $16x^2 - 56xy + 49y^2$, then the length of each sides is **A.** $4x + 7y$ **B.** $16x - 49y$ **C.** $4x - 7y$ **D.** $8x - 7y$	29.	Which of the following angle is an obtuse angle? **A.** $\frac{2}{9}$ of 90° **B.** $\frac{2}{9}$ of 180° **C.** $\frac{5}{6}$ of 180° **D.** $\frac{7}{10}$ of 90°
25.	Factorise $81 - 9p^2$. **A.** $(9 - 3p)(9 - 3p)$ **B.** $9(3 - p)(3 + p)$ **C.** $3(3 - p)(3 + p)$ **D.** $(9 - 3p)$	30.	The sum of interior angles of a triangle is **A.** 90° **B.** 180° **C.** 180° **D.** 120°
31.	Which of the following is an isosceles triangle? **A.** m $\overline{AB}$ = 5 cm m $\overline{BC}$ = 3 cm m $\overline{AC}$ = 3 cm **B.** m $\overline{AB}$ = 7 cm m $\overline{BC}$ = 6 cm m $\overline{AC}$ = 5 cm **C.** m $\overline{AB}$ = 8 cm m $\overline{BC}$ = 8 cm m $\overline{AC}$ = 8 cm **D.** m $\overline{AB}$ = 5 cm m $\overline{BC}$ = 4 cm m $\overline{AC}$ = 7 cm	36.	The sum of interior angles of a quadrilateral is **A.** 180° **B.** 270° **C.** 630° **D.** 360°

32. If A and B are two concentric circles as shown in the figure below, then



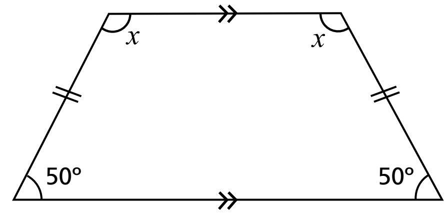
A. diameter of circle A > diameter of circle B
B. diameter of circle A < diameter of circle B
C. diameter of circle A is twice the diameter of circle B
D. diameter of circles A and B are equal

33. The boundary of a circle is called
A. major sector
B. circumference
C. major arc
D. minor arc

34. Two figures are similar, if
A. they have same shapes
B. they are of same size
C. they have same angles with different sides
D. they have same angles with sides in same ratio

35. Two triangles are congruent if they have
A. same angles
B. same angles and same sides
C. same sides
D. equal angles and unequal sides

37. The value of x in the given isosceles trapezium is
A. 50°
B. 90°
C. 130°
D. 120°



38. If the radius of a circle is 14 cm, then its area is
A. 44 cm^2
B. 616 cm^2
C. 154 cm^2
D. 160 cm^2

39. The radius of a cylinder is 7 cm and its height is 10 cm. Find the volume of the cylinder.
$\left\{\text{take } \pi = \frac{22}{7}\right\}$
A. 1540 cm^2
B. 154 cm^2
C. 15.40 cm^2
D. 1.540 cm^2

40. The difference between the greatest and smallest value in a data is the
A. upper limit
B. frequency
C. range of the data
D. lower limit

Section B

Time: 2 hours **Total marks: 60**

Q.1 (i) Simplify $\left(-\frac{7}{5} \times \frac{3}{14}\right) + \left(\frac{2}{3} \times \frac{-3}{10}\right) - \left(-\frac{8}{7} \times \frac{-21}{4}\right)$ [5]

(ii) If $\mathbb{U} = \{0, 1, 2, 3, 4, 5, 6\}$,
$A = \{1, 3, 5\}$,
$B = \{0, 1, 2, 3\}$,
$C = \{2, 4, 6\}$,
find
(a) $A \cup C$ (b) $B \cap C$ (c) B' [3]

(iii) Find the positive square root of 181476.
[4]
[Total marks 12]

Q.2 (i) Express $\frac{64}{216}$ as power of rational numbers. [2]

(ii) Express $\left[\left(-\frac{5}{7}\right)^{-2}\right]^{-1}$ with negative exponents. [2]

(iii) 260 students in a hostel have food for 25 days. If 10 students leave the hostel, how long will the food last for the remaining students? [5]

(iv) Asim's property is worth Rs 4,000,000. If the rate of property tax is 5% per year, how much property tax will he pay in 2 years? [3]

[Total marks 12]

Q.3 Simplify:

(i) $4(x-5) + 3(x-8) - (x-10)$ [3]

(ii) $49p^2 + 112pq + 64q^2$ [3]

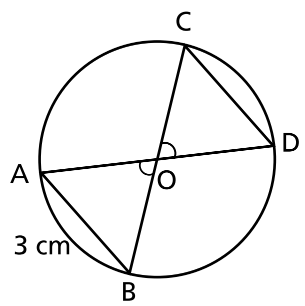
(iii) Resolve into factors $25x^2 - 81y^2$ [3]

(iv) when 8 is subtracted from 5 times a number, the result is 37. Find the number.
[3]
[Total marks 12]

Q.4 (i) Construct the following isosceles triangle.

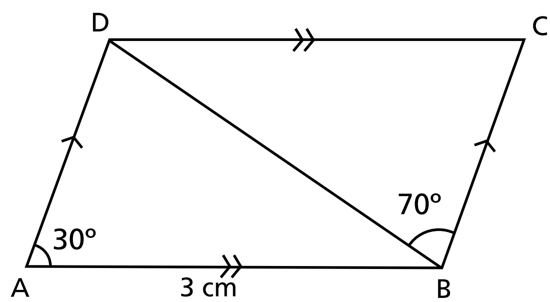
m $\overline{AB}$ = 5 cm m $\angle$ A = 55° [5]

(ii) $\overline{AB}$ and $\overline{CD}$ are two chords of a circle with centre O.
If m $\overline{AB}$ = 3 cm, m $\angle$ $\overline{AOB}$ = m $\angle$ COD, find m $\overline{CD}$. [3]



(iii) In the given parallelogram find [4]

(a) m $\overline{DC}$
(b) m $\angle$ ADB
(c) m $\angle$ DCB
(d) m $\angle$ ABD

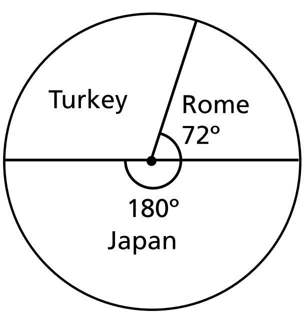


[Total marks 12]

Q.5 (i) The diameter of a circular hall is 14 m. Find the cost of flooring the hall at the rate of Rs 400 per square metre. $\left[\text{Take } \pi = \frac{22}{7}\right]$ [4]

(ii) Find the volume and surface area of a solid cylinder whose radius is 21 cm and height in 45 cm. [5]

(iii) In a a survey 500 people were asked the name of their favourite country. The pie chart shows the name of the country.



(a) Find the angle of sector of Turkey. [1]
(b) How many people like Japan? [1]
(c) What percentage of people like Rome? [1]

[Total marks 12]

Marking Scheme

Marking criteria for Section A: 1 mark for each correct answer.

Answers

1. C	2. A	3. B	4. C	5. D	6. C	7. B	8. A
9. A	10. C	11. B	12. C	13. B	14. B	15. A	16. C
17. A	18. C	19. C	20. B	21. B	22. D	23. A	24. C
25. B	26. B	27. D	28. C	29. C	30. B	31. A	32. A
33. B	34. D	35. B	36. D	37. C	38. B	39. A	40. C

Marking criteria for Section B

Q.1		**12 Marks**	**Answer**
	(i) • Simplification within brackets [3] • Simplification with LCM [1] • Accuracy [1]	5 marks	$-6\frac{1}{2}$
	(ii) • Correct concept of ∪, ∩ and complement of a set and accurate answer [1] • Correct [1] • Accurate answer [1]	3 marks	(a) {1, 2, 3, 4, 5, 6,} (b) {2} (c) {4, 5, 6}
	(iii) • Correct procedure of prime factorization and pairing [2] • Correct use of radical sign [1] • Correct answer [1]	4 marks	426
Q.2		12 Marks	**Answer**
	(i) • For finding correct powers [2]	2 marks	$(\frac{4}{6})^3$
	(ii) • Accurate answer [2]	2 marks	$(-\frac{7}{5})^{-2}$
	(iii) • Identifying the proportion (inverse) [1] • Correct equation [3] • Correct answers [1]	5 marks	26 days
	(iv) • Method / formula [1] • Calculation [1] • Accuracy in answer [1]	3 marks	Rs 400,000

Q.3		12 Marks	Answer
	(i) • Opening of the brackets [1] • Collecting terms and simplification [1] • Correct answer [1]	3 marks	$6x - 34$
	(ii) • Identify the whole square formula [1] • Accurate application [1] • Correct answer [1]	3 marks	$(7p + 8q)\ (7p + 8q)$
	(iii) • Identifying the identity [1] • Correct application [1] • Accuracy in answer [1]	3 marks	$(5x - 9y)\ (5x + 9y)$
	(iv) • Forming correct equation [1] • Solution of equation [1] • Correct Answer [1]	3 marks	$x = 9$
Q.4		**12 marks**	**Answer**
	(i) • Know that in an isosceles triangle two sides and two angles are equal [1] • Construction [2] • Accurate measurement [2]	5 marks	
	(ii) • Use of 3rd property of circle [1] • Application and reasoning [2]	3 marks	3 cm
	(iii) • Correct reasoning and correct answer in each case [4]	4 marks	(a) 3cm (b) 70° (c) 30° (d) 80°
Q.5		**12 marks**	**Answer**
	(i) • For finding area of circle (using correct formula) [2] • Calculation of cost [1] • Accurate answer [1]	4 marks	Rs 61600
	(ii) • Correct application of formula of volume [1] • Accurate answer [1] • Correct application of formula of surface area [1] • Correct calculation and accuracy [2]	5 marks	V = 62370cm^3 SA = 8712cm^3
	(iii) • Correct usage of concepts in each case [3]	3 marks	(a) 108° (b) 250 (c) 20%