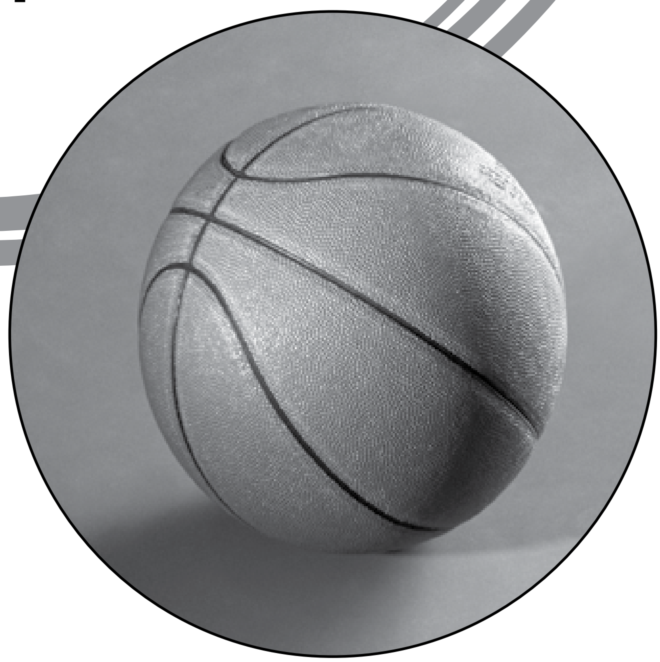
Teaching Guide

International Secondary Maths 9

COLIN WRIGLEY



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Introduction

How to use this Guide

I. Selection of work and pacing

Book 9 is designed for students of Class IX (or equivalent), i.e. they would normally be 13+ years old at the start of the academic year, and on track for entering for English medium examinations in O level or International GCSE. Accordingly, with a good class, the teacher may find that not all revision topics need to be covered in detail. Although advice is given about pacing, chapter by chapter, this should be regarded as a guide and not adhered to rigidly.

II. Integrated mathematics

This textbook exploits links between different branches of mathematics, with the sciences and with mathematical features of daily life. The links are listed in this guide and teachers are recommended to make these explicit in lessons.

III. Lesson planning

Whether or not your school has a formal reporting structure for lesson plans, there are many ways of planning lessons. In this guide, suggestions are made to assist teachers in writing their lesson plans, not only to satisfy their Heads of Department or Academic Coordinators, but also to clarify their own thinking before facing their class. The suggestions are under the following headings:

Objectives

- General objectives
- Specific objectives

These are written from the students' point of view, i.e. they describe what we hope they will learn, understand, and do, that they could not do before.

Method

Also known as strategy, procedure, methodology, or techniques, these are written from the teacher's point of view, i.e. they suggest what the teacher should do to facilitate the learning objectives of the students.

Resources

These are items needed, or desirable, for the method used by the teacher, or equipment/materials needed by the student. Normally, calculators and geometrical instruments are not listed, but are included when essential for the topic.

Assignments

Best homework material is suggested.

Vocabulary

A list of key words for the topic.

IV. Bloom's Taxonomy

In the exercises, questions are set at the lower and higher levels of Bloom's Taxonomy. Students should be challenged at the higher level of problem-solving as often as possible. A more detailed discussion of this may be found in Teaching Guides for Books 6, 7, and 8.

V. The Exercises

Each chapter contains exercises coded as follow:

A, B, C, etc. following each section of the chapter

M—a miscellaneous exercise on all sections of a chapter (if there are multiple sections)

X—a short, challenging exercise at the end of each chapter, usually involving higher levels of cognitive skills, for the more talented students only

Revision Exercises appear at regular intervals. The questions are deliberately not graded: basic factual questions are randomly mixed with quite thought-provoking ones. Students seem to find this a more interesting way to revise.

VI. Useful Sheets

The following photocopiable materials are provided:

Coordinate grid

Squared paper (9 mm)

VII. Specimen Examination Papers

These are suggested questions for summative assessment. As some students (e.g. teachers' children) may have access to the guide, it is advisable to make a few changes.

Based on chapters 1–15 (half the academic year)

- Non-calculator Paper (1 h)
- Calculator Paper ($1\frac{1}{2}$ h)

Based on chapters 1-30 (whole academic year)

- Non-calculator Paper (1 h)
- Calculator Paper ($1\frac{1}{2}$ h)

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Chapter

1 Mental Arithmetic

Despite years of practice, it is surprising how many students resort to calculators or long methods to do simple arithmetic. This chapter begins the new academic year with some consolidation work in basic numeracy.

LESSON PLANNING

Objectives

General	To multiply and divide by multiples and submultiples of 10
Specific	1. To multiply mentally a whole number or decimal by a multiple of 10 2. To multiply mentally a whole number or decimal by a submultiple of 10 3. To divide mentally a whole number or decimal by a multiple of 10 4. To divide mentally a whole number or decimal by a submultiple of 10 5. To develop speed and accuracy in mental multiplication and division as above
Pacing	3 lessons
Links	inverse operations, fractions

Method

Oral question and answer. Use the examples in the text, and similar examples to illustrate the strategies.

Emphasize the difference between mental methods and written methods. For example for 62×4:

On paper, we begin on the right: 2×4 (ones)

Mentally, it is easier to begin on the left (tens):

$$\begin{aligned} 60 \times 4 &= 240 \\ 2 \times 4 &= 8 \\ \text{Total} &= 248 \end{aligned}$$

This should be practised in class. Then move on to

62×40

62×0.4

etc. (2 digits by 1 digit)

Set EX 1A.

The division section should be introduced by asking, "What is a quotient?" The word should be known. Follow the strategies shown in the text, again with plenty of oral work and similar examples.

Set EX 1B.

EX 1M may be used to develop speed and accuracy, and provide feedback for the teacher.

List A is set and students write their personal times at the end in their exercise books. Then the answers are checked and penalty time added for errors:

Time Taken	1 min 10 s	(say)
Penalty (2 wrong)	30 s	
Total	1 min 40 s	

Allow "pair and share" discussion of mistakes. Then set List B and the challenge is to improve personal scores.

If any students find this rather easy, they can be challenged to complete EX 1X.

Resources	classroom clock with sweep seconds hand, or enough children with wristwatches or mobile phone timers
Assignments	This topic is not really suitable for homework.
Vocabulary	multiple, submultiple, product, quotient

ANSWERS

Exercises

EX 1A

1. a) $25 \times 20 = 25 \times 2 \times 10 = 50 \times 10 = 500$

   b) $1.4 \times 30 = 1.4 \times 10 \times 3 = 14 \times 3 = 42$

   c) $15 \times 0.8 = 15 \times \frac{8}{10} = \frac{120}{10} = 12$

   d) $21.3 \times 0.02 = 21.3 \times \frac{2}{100} = 0.213 \times 2 = 0.426$

	a)	b)	c)	d)
2.	680	900	1440	2240
3.	126	128	54	88
4.	13.8	23.8	40.8	14.4
5.	0.246	0.648	0.584	10.85
6.	73.2	12 000 000	0.88	186
7.	15	2.84	7.2	0.72
8.	120	51 000	10	54
9.	0.05	0.81	1015	12 400
10.	0.78	0.12	0.021	0.04

EX 1B

1. a) $4600 \div 20 = \frac{460\cancel{0}}{2 \times 1\cancel{0}} = 230$

   b) $72\,000 \div 800 = \frac{720\cancel{0}\cancel{0}}{8 \times 1\cancel{0}\cancel{0}} = 90$

   c) $8.1 \div 0.3 = \frac{8.1 \times 10}{0.3 \times 10} = \frac{81}{3} = 27$

   or $= 8.1 \div \frac{3}{10} = 8.1 \times \frac{10}{3} = \frac{81}{3} = 27$

   d) $22.44 \div 0.02 = \frac{22.44 \times 100}{0.02 \times 100} = \frac{2244}{2} = 1122$

   or $= 22.44 \div \frac{2}{100} = 22.44 \times \frac{100}{2} = \frac{2244}{2} = 1122$

2.	a) 300	b) 60	c) 70	d) 500
3.	a) 0.05	b) 20.1	c) 73	d) 3500
4.	a) 10.3	b) 13	c) 2840	d) 2.9
5.	a) 248	b) 119	c) 1110	d) 800
6.	a) 20	b) 200	c) 100	d) 1100
7.	a) 2.2	b) 19.75	c) 3.1	d) 0.03
8.	a) 23 000	b) 200	c) 160 000	d) 190
9.	a) 50	b) 15 000	c) 0.07	d) 0.007
10.	a) 2950	b) 25	c) 205.5	d) 40

EX 1M

	List A		List B
1.	19 000	1	24 600
2.	4850	2	720
3.	1.1	3	0.8
4.	6	4	2
5.	147	5	96
6.	190	6	110
7.	1.3	7	1.1
8.	2880	8	3920
9.	2000	9	2000
10.	6.3	10	4.6

EX 1X

1.	a) 1000	b) 600	c) 900	d) 100
2.	a) 0.06	b) 18	c) 10	d) 600 000
3.	a) 8000	b) 100 000 or 1 lakh	c) 304 million or 304 000 000	
	d) 40 000			

Chapter

2 Algebra Review

This chapter contains good material for the teacher to observe the quality of algebra previously learnt by the students. More emphasis than hitherto is placed upon problem solving, i.e. being able to generate equations from given information, solving them, and interpreting the solution.

LESSON PLANNING

Objectives

General	To be more confident working with algebraic expressions, equations, and formulas
Specific	1. To find the value of an expression by substitution of given values 2. To find the value of the subject of a formula by substitution of given values 3. To change the subject of a formula 4 To use the specified units when substituting in a formula 5 To construct a linear equation, or a pair of simultaneous equations, from given information 6. To solve such equations and interpret the solutions in terms of the original question
Pacing	4 lessons, 1 homework
Links	puzzles, physics, area formulas
Method	• Use the first exercise diagnostically, i.e. maximize time spent by students engaging with the material, and minimize time spent by teacher working through examples. Students learn faster by doing instead of watching. Quickly revise expressions and formulas, and the word "subject". Use examples similar to those in the text. Set EX 2A, allow discussion, allow checking of the answer page, circulate around the classroom providing hints to students in difficulty. This way you can obtain a "feel" for the general ability level. • According to feedback obtained, move on to building and solving equations with interpretation of results. Here, some board-work is advisable, giving examples. Talk through the thought process as you write it up on the board, asking rhetorical questions. For example, for the rectangle example (see text), ask, "Why do we choose the width for x? Why not the length?" Solving equations is not the end! Write on the board "ANSWER THE QUESTION!!" to remind students to use their solutions.

Assignments	Suitable homework is EX 2A, question 9 and/or EX 2B, question 7
Vocabulary	expression, formula, subject, substitute, value, equation

ANSWERS

Exercises

EX 2A

1. a) 1 b) 3 c) 15 d) 2
2. a) 5 b) −2 c) −8 d) 5
3. a) 11.25 b) 6 c) 10.5 d) −21
4. a) −11 b) 25 c) −5 d) 52
5. a) −5 b) 0 c) 0 d) −7.5
6. $u = v - at$
   a) 95 b) 42 c) 40 d) −1.5
7. $h = \dfrac{2A}{a+b}$
   a) 3 b) $2\frac{2}{3}$ c) 2 d) 9.4
8. a) 45 cm b) 43 cm c) $t = 10 - \dfrac{h}{4.5}$ or $t = \dfrac{45-h}{4.5}$
   d) 27 s
9. a) 27 m b) 135 m
10. a) $v = \dfrac{2s + at^2}{2t}$ or $v = \dfrac{s}{t} + \dfrac{at}{2}$
    b) $a = \dfrac{2(vt-s)}{t^2}$ or $a = \dfrac{2v}{t} - \dfrac{2s}{t^2}$ or $a = \dfrac{2vt - 2s}{t^2}$

EX 2B

1. $3n - 2$
   a) 4 b) 10
2. Tom \$ 6.50, Umair \$ 19.50
3. 11 cm
4. \$ 172 (red \$ 92, blue \$ 80)
5. Yasmin 16, Zaheer 8
6. 4

7. $3x - y = 10$, $2x + y = 10$, $x = 4$, $y = 2$

   A 8 m^3/h; B 2 m^3/h; C 4 m^3/h; D 6 m^3/h

8. $b + 140 = 2a$, $2a + 3b = 780$, $a = 150$, $b = 160$; 2.52 kg

9. Aisha 8, Bilal 12, Charandeep 4

10. adult: \$ 6; child: \$ 4; \$ 36

EX 2X

1. a) $f = \dfrac{uv}{u+v}$ b) $u = \dfrac{vf}{v-f}$ c) $v = \dfrac{uf}{u-f}$

2. ± 5

3. 7

Chapter

3 Pythagoras' Theorem

Chapters 1 and 2 revised some arithmetic and algebra. In this chapter we revise some geometry which first appeared in Book 8, although now harder questions are provided. A new feature is the converse of a theorem.

LESSON PLANNING

Objectives

General	To solve problems involving Pythagoras' theorem and its converse
Specific	1. To know what a hypotenuse is 2. To know Pythagoras' theorem and use it to solve right-angled triangle problems 3. To recognize and use Pythagorean triples 3, 4, 5; 5, 12, 13; 7, 24, 25 and scaled versions of them 4. To know the converse of Pythagoras' theorem and how to use it to prove that a triangle is right-angled 5. To know that some converses are not true but that the converse of Pythagoras' theorem is true
Pacing	2 lessons, 1 homework
Links	vectors, coordinates, calculator techniques

Method

Use the bullet points in the text to remind the students of the basic facts. Do a couple of examples:

(i) finding the hypotenuse, e.g. $x^2 = 3.2^2 + 4.6^2$

(ii) finding a short side, e.g. $9.3^2 = x^2 + 3.8^2$ (where making x the subject is required)

Set EX 3A, questions 1–4.

Explain "converse". Examples:

- All prime numbers after 2 are odd.
  True.
  Converse: All odd numbers after 2 are prime.
  This converse is false.
- In a rhombus opposite angles are equal.
  True.
  If opposite angles are equal, it is a rhombus.
  False.

	• However, the converse of Pythagoras' theorem is true. Use the rest of EX 3A.
Resources	calculators essential, coordinate grid (EX 3A, question 9)—photocopiable
Assignments	EX 3A, question 9 or 10

ANSWERS

Exercises

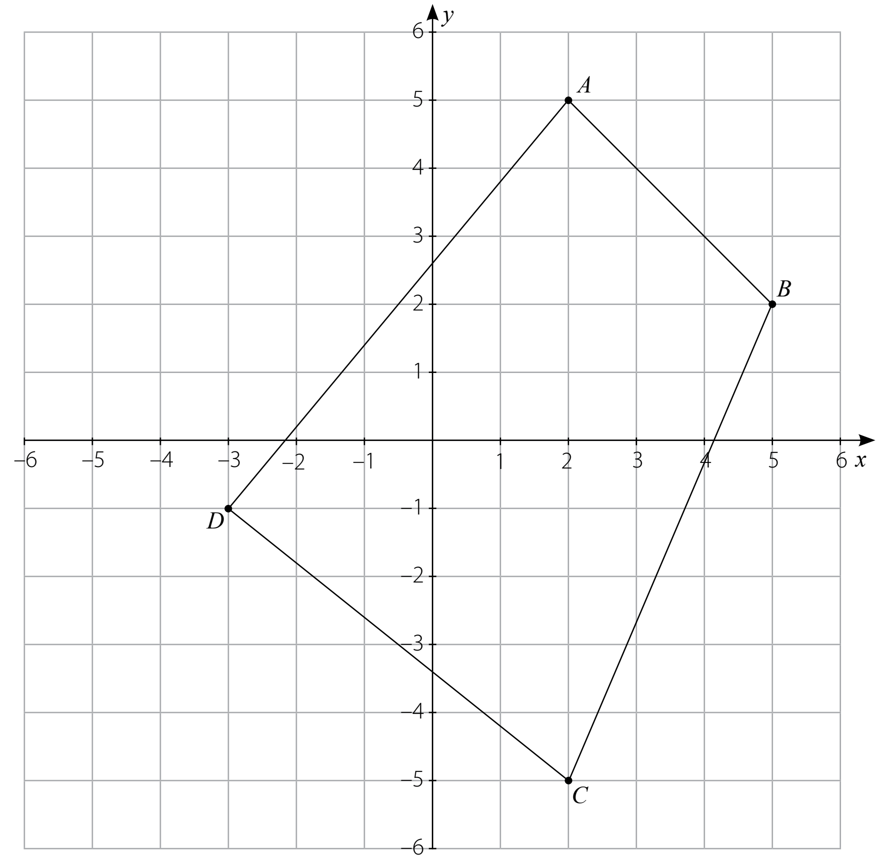
EX 3A

1. a) $3.1^2 + 5.2^2 = x^2$, $x = 6.1$ cm
   b) $3.6^2 + 2.6^2 = x^2$, $x = 4.4$ cm
   c) $7.3^2 + 9.5^2 = x^2$, $x = 12$ cm
   d) $1.6^2 + 2.8^2 = x^2$, $x = 3.2$ cm
2. a) $x^2 + 8.4^2 = 12^2$, $x = 8.6$ cm
   b) $4.2^2 + x^2 = 8.8^2$, $x = 7.7$ cm
   c) $x^2 + 6.3^2 = 8.9^2$, $x = 6.3$ cm
   d) $x^2 + 1.7^2 = 2.3^2$, $x = 1.5$ cm

3.	a) 3 cm	b) 7 cm	c) 12 cm	d) 8 cm
4.	a) 40 cm	b) 24 cm	c) 10 cm	d) 75 cm
5.	a) yes, $\angle A$	b) no	c) no	d) yes, $\angle C$
6.	a) no	b) no	c) yes, $\angle C$	d) no
7.	a) 4.2 cm	b) 8.9 cm	c) 7.2 cm	d) 6.7 cm

8. 6.83 m

9.



a) 4.2

b) 7.6

c) 6.4

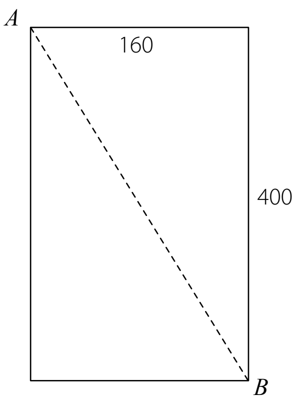
d) 7.8

10. 5.1, 3.6, 1.4, 5

EX 3X

1. a) 6 cm b) $\sqrt{24}$ cm c) $\sqrt{10}$ d) $\sqrt[4]{\frac{1}{2}}$

2.

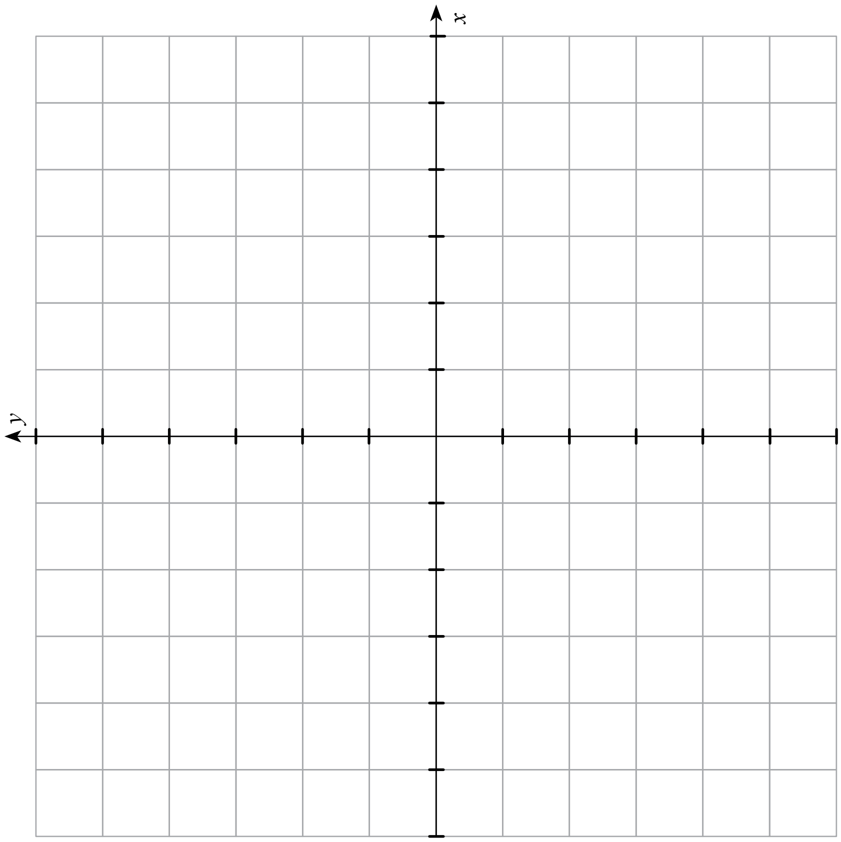


Using the net, shortest distance is shown as broken line, 4.31 m

3. 13

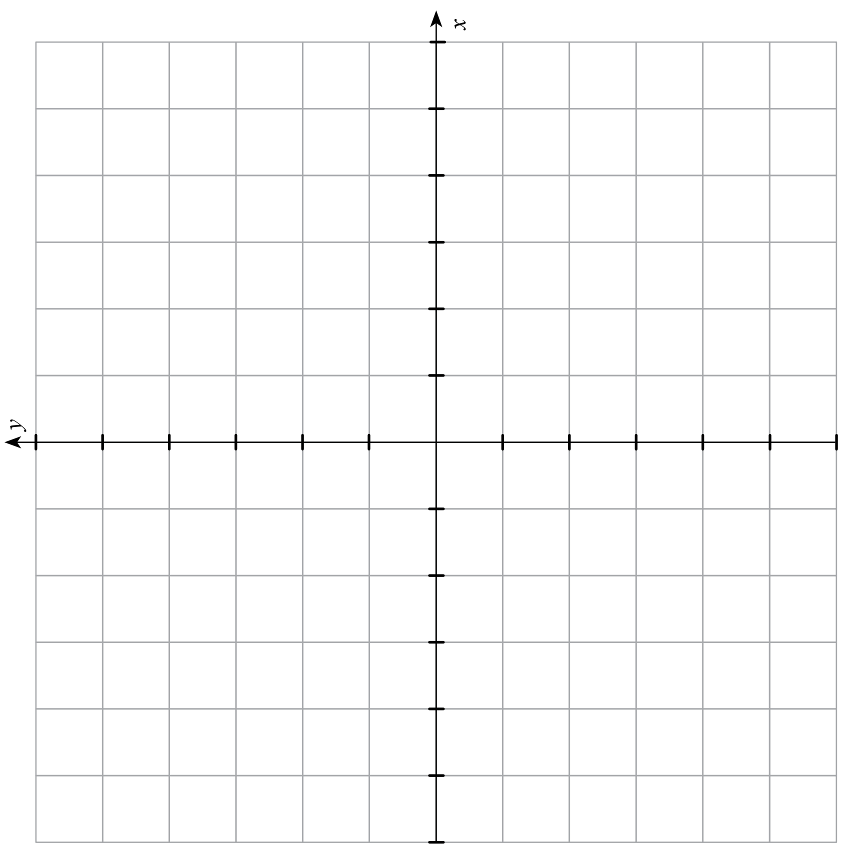
Coordinate Grid

Question No:



Coordinate Grid

Question No:



Chapter

4 Approximation and Estimation

This topic has been covered before although the significance or otherwise of zeros is defined more precisely. It is most definitely a non-calculator chapter. Students must not be permitted to use calculators to work out the answers to calculations involving estimated quantities!
[EX 4B, questions 7 and 9, and EX 4X, question 2(b), do require calculators, as stated.]

LESSON PLANNING

Objectives

General	To use rounding off rules and 1 s.f. estimation in calculations
Specific	1. To round off numbers to stated numbers of decimal places or significant figures 2. To know whether zeros are significant or not 3. To estimate calculations by prior rounding off to 1 s.f., taking special care with small divisors 4. To use estimation to 2 s.f. in cases where easy cancellation leads to simplification [5. To be aware of the upper and lower bounds of a measurement.]
Pacing	2 lessons, 1 homework
Links	area formulas, substitution, line graphs
Method	Decide whether or not your students need the reminders at the start of the chapter. A good class could tackle EX 4A immediately using the chapter introduction for reference. The estimation section may need more attention. Show 1 s.f. examples. Bring in examples where a 2 s.f. estimation enables cancelling. Specifically warn against too rough an estimate of small divisors. This is a common error. For example, 0.64 is rounded to 1 when it should be 0.6. Small differences in divisors have a large effect. This is reinforced in the text example. [The accuracy of a measurement implicit in the number of figures shown should be briefly mentioned as a precursor to lower and upper bounds, to be dealt with later. Line graphs can be used to good effect. However, this topic is not brought into the exercises.] The ≈ symbol should be used. Students may need a reminder of its meaning.

Assignments	Suitable homework EX 4B, questions 4, 5, and 7
Vocabulary	significant, leading zeros, estimate

ANSWERS

Exercises

EX 4A

1.	a)	2	b)	3	c)	1	d)	3
2.	a)	1	b)	2	c)	3	d)	3
3.	a)	8.7	b)	0.3	c)	1.8	d)	16.3
4.	a)	146.53	b)	5.65	c)	0.40	d)	6.64
5.	a)	8.960	b)	8.158	c)	0.136	d)	0.027
6.	a)	30	b)	8	c)	400	d)	0.04
7.	a)	6.4	b)	0.34	c)	0.0040	d)	290
8.	a)	3.80	b)	64.5	c)	0.452	d)	50.6
9.	a)	42.577	b)	42.6	c)	42.58	d)	40
10.	a)	$8.\underset{S}{0}$	b)	$645\underset{N}{0}$	c)	$2\underset{S}{0}3\underset{N}{0}$	d)	$\underset{N}{0}.\underset{N}{0}\underset{N}{0}18$

EX 4B

1.	a)	17	b)	0.27	c)	70	d)	9
2.	a)	1000	b)	50	c)	5	d)	50
3.	a)	300	b)	0.02	c)	62.5	d)	300
4.	a)	150	b)	1 or 0	c)	160 or 120	d)	45
5.	a)	0.02	b)	8	c)	600	d)	22.5
6.	a)	1.2	b)	0.1	c)	50	d)	31 or 32

7. Calculator:

4 a) 139 (3 s.f.) b) 1.11 (3 s.f.)

c) 127 (3 s.f.) d) 37.8 (3 s.f.)

5 a) 0.0213 (3 s.f.) b) 8.52 (3 s.f.)

c) 589 (3 s.f.) d) 20.9 (3 s.f.) Estimates are accurate to 1 s.f.

8.	a)	20 m^2	b)	21 cm^2	c)	50 cm^2	d)	12 m^2

9. 20 pieces per roll = 80 altogether; 20 per roll = 80 altogether

(4 pieces of 20 cm each left over)

10. a) 130 b) 150

EX 4X

1. a) 420 b) 60.2 c) 74 000 d) 0.006

2. a) 280 km b) 311.75 km c) 10.2 % (3 s.f.)

3. An infinite number [And it's the same infinity as the two other infinities mentioned!]

Chapter

5 Trigonometry

This branch of mathematics is introduced here for the first time. Since many students have difficulty with this topic it is taken in two stages, the remainder being in Chapter 9.

LESSON PLANNING

Objectives

General	To use trigonometrical ratios to calculate lengths in right-angled triangles; to use the $\frac{1}{2}ab\sin C$ formula for the area of a triangle
Specific	1. To identify the hypotenuse, adjacent, and opposite sides of a right-angled triangle 2. To find the correct value of a trig ratio using a calculator 3. To use trig ratios to calculate lengths of the sides of a right-angled triangle 4. To recognize when the $\frac{1}{2}ab\sin C$ formula is appropriate for calculating the area of a triangle; to use it in such cases
Pacing	4 lessons, 2 homeworks
Links	enlargements, ratios, solving equations, rounding off

Method

The strategy in the text is to build up in stages. First, we ensure that all students can identify the three sides of a right-angled triangle (with respect to a specified angle) as hypotenuse, opposite, or adjacent. This needs good board-work, with lots of examples and oral feedback from students.

When this is grasped, follow the text section on similar triangles to show that

tan 36.9° = 0.75.

Define tangent, and sine and cosine, together.

Do some calculator entry work to ensure that students can find the value of any trig X.

Two problems may arise:

- On older calculators, the angle is entered first, e.g. for sin 30° enter $\boxed{3}\boxed{0}\boxed{\text{sin}}\boxed{=}$. There are not many of these still around but you might just encounter one in your class.
- On modern calculators the angle can be set for degrees, radians or grads. A DRG button toggles these. Ensure D setting for degrees.

The students should be ready at this point to tackle EX 5A.

The next stage is to use these trig values in calculations. Use the text examples, and others similar. Avoid using cookbook recipes for manipulation of the equations, but reinforce basic algebraic techniques:

- To clear fractions, multiply through by the denominator. Sometimes this is the unknown.
- Always multiply or divide both sides by the same quantity.
- Treat the trig X as a single object.

Use EX 5B.

Follow the text to derive $\frac{1}{2}\,ab\sin C$ from $\frac{1}{2}\,bh$ area formula. Emphasize this is true for any triangle, not just right-angled ones.

Do not teach the formula; teach SAS, i.e. two sides and the included angle.

Use EX 5C. The last few questions need higher-level cognitive skills.

- Round off errors can be a nuisance. If an intermediate result is required, it may be held in full in the calculator. If not, then at least two significant figures more than the final result needs should be used.

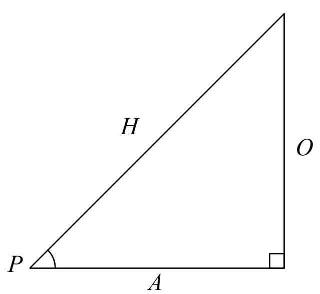
Resources	calculators, preferably the same model for the whole class (school policy request)
Assignments	Suitable for homework: EX 5B, questions 7 and 8, and EX 5C, question 8
Vocabulary	trigonometry, hypotenuse, adjacent side, opposite side, ratio, sine, cosine, tangent, SAS formula

ANSWERS

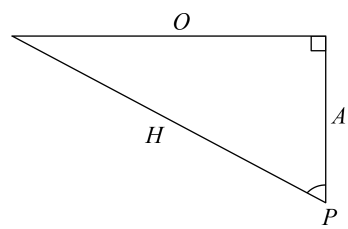
Exercises

EX 5A

1. a)



b)



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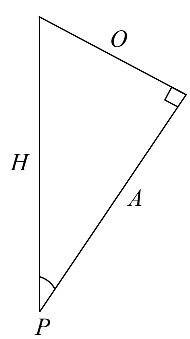
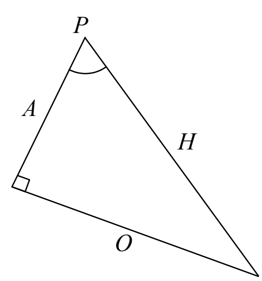
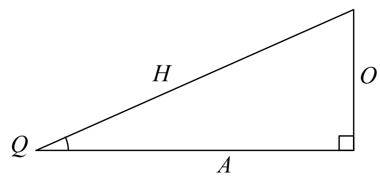


d)



2. a)



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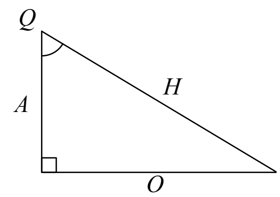
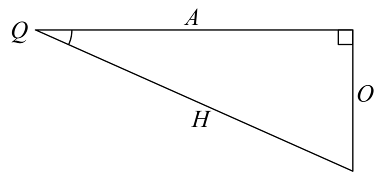
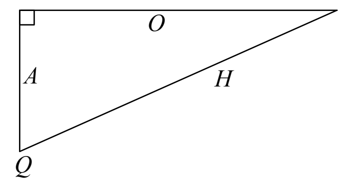


c)



d)



3. a) P b) R c) Q d) P

4. a) $\frac{5}{13}$ b) $\frac{12}{13}$ c) $\frac{5}{12}$ d) $\frac{12}{5}$

5. a) $\frac{3}{5}$ b) $\frac{3}{7}$ c) $\frac{4}{5}$ d) $\frac{7}{7.62}$

6. a) $\frac{12}{13}$ b) $\frac{7}{25}$ c) $\frac{7}{24}$ d) $\frac{24}{25}$

7. a) true b) false c) false d) true

8. a) 0.545 b) 0.225 c) 2.14 d) 0.326

9. a) 0.416 b) 0.319 c) 0.141 d) 0.0454

10. a) increases, between 0 and 1 b) decreases, between 1 and 0
    c) increases, rapidly when close to 90°

EX 5B

1. $x = 3.63$ cm, $y = 4.63$ cm
2. $p = 10.6$ cm, $q = 13.3$ cm
3. $u = 16.5$ cm, $v = 10.1$ cm
4. $x = 13.7$ cm, $y = 29.1$ cm
5. $p = 14.3$ cm, $q = 15.5$ cm
6. $u = 8.96$ cm, $v = 7.82$ cm
7. a) $x = 50.7$ cm b) $y = 52.5$ cm c) $\tan 46° = 1.0355 = \frac{y}{x}$
   d) $x^2 + y^2 = 5329.0 = 73^2$ (using 5 s.f.)
8. a) $p = 12.110$ cm (5 s.f.) or 12.1 cm (3 s.f.) b) $q = 25.8$ cm (3 s.f.)
9. $p = 20.572$ cm (5 s.f.) = 20.6 cm (3 s.f.) $q = 30.5$ cm (3 s.f.)
10. $p = 11.972$ cm (5 s.f.) = 12.0 cm (3 s.f.) $q = 10.8$ cm (3 s.f.)

EX 5C

1. a) true b) true c) false d) false

2. a) $\frac{1}{2}ac \sin P$ b) $\frac{1}{2}xy \sin P$
   c) $\frac{1}{2}rs \sin P$ d) $\frac{1}{2}mn \sin P$

3. 13.3 cm^2

4. 47.0 cm^2

5. 26.8 cm^2

6. 50.6 cm^2

7. a) 14.9 cm^2 b) 5.30 cm

8. a) 30 cm^2 b) 30 cm^2 c) Area = $9p$ d) 3.3 cm

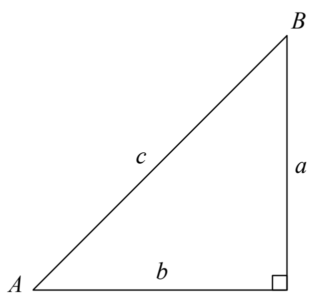
9. a) 43.3 cm^2 (3 s.f.) b) Area = $0.433\, l^2$ (3 s.f.)

10. 248 cm^2 (3 s.f.)

EX 5X

1. Equal values in each pair,

   $A + B = 90°$



$\sin A = \frac{a}{c} = \cos B$ and $A + B = 90°$

2. Each product is 1.

   $A + B = 90°$

   On the diagram, $\tan A = \frac{a}{b}$ and $\tan B = \frac{b}{a}$

   $\therefore \tan A \tan B = \frac{a}{b} \times \frac{b}{a} = 1$ and $A + B = 90°$

3. [Join each vertex to the centre to form 5 congruent triangles.]

   $\text{Area} = 5 \times \frac{1}{2} \times 10 \times 10 \times \sin 72°$

   $= 238 \text{ cm}^2$ (3 s.f.)

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Chapter

6 Integers

The introduction of this short chapter provides information for reference, most of which has been covered previously in the series.

LESSON PLANNING

Objectives

General	To calculate with integers (non-calculator)
Specific	1. To add, subtract, and multiply integers and know that the answer is an integer 2. To divide integers and know that the quotient is not necessarily an integer 3. To understand the meaning of factor and of multiple 4. To know the definition of a prime number; to obtain prime factorisation of a number expressing the factors in ascending order and using indices for repeated factors 5. To recognize square numbers and triangle numbers and solve problems involving them
Pacing	1 or 2 lessons, 1 homework
Links	sequences, substitution in algebraic expressions
Method	Start with homework: browse through the chapter introduction. Note anything puzzling. Deal with queries in class. Set EX 6A. [Strictly non-calculator, hence best done in class.] After questions 1-5 have been completed, allow checking of answers. Move on only if correct.
Assignments	EX 6A, question 10 suitable for homework. If already done in class, set a question from EX 6X.
Vocabulary	integer, positive, negative, rule of signs factors, multiples, primes, squares, triangle numbers, prime factorisation

ANSWERS

Exercises

EX 6A

1.	a) 80	b) –7	c) –112	d) 63
2.	a) 16	b) –16	c) 6	d) $\frac{1}{2}$

3. a) 15 b) 17 c) 180 d) −46

4. a) 7 b) 74 c) −15 d) 140

5. a) false b) 11, integer

6. a) 4, integer b) 3, integer

7. a) true b) false c) true d) false

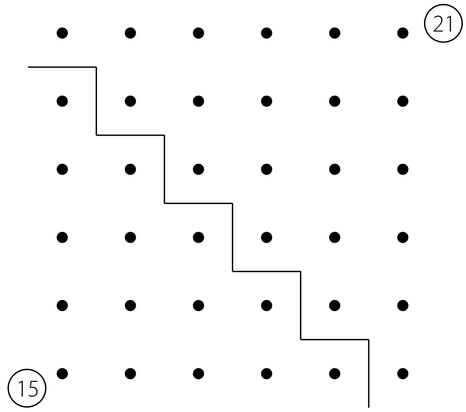
8. a) $2^2 \times 5 \times 23$ b) $3 \times 5^2 \times 7$

c) $2 \times 7 \times 11 \times 13$ d) $2^3 \times 5^2 \times 11$

9. a) 144 b) $x = -2$ or $x = 2$

10. a) 1, 3, 6, 10, 15, 21, … b) 4, 9, 16, 25, 36, … are all squares.

c)



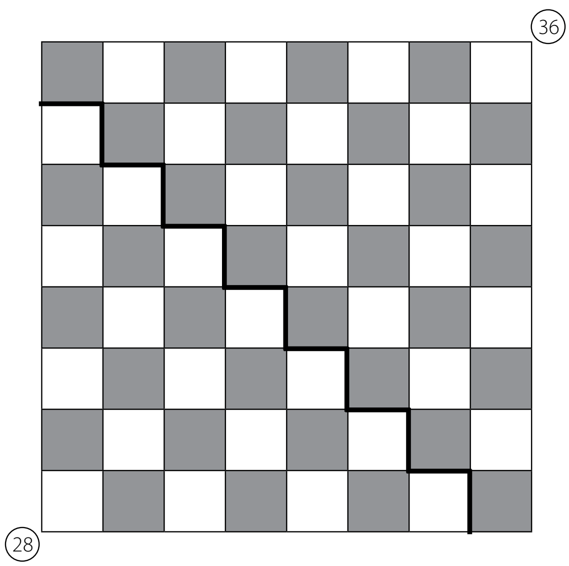
36 = 15 + 21

(Two triangles form a square.)

EX 6X

1. $2^5 \times 3^4 \times 5^3 \times 7^2$

2.



64 = 28 + 36

3. a) $\frac{9}{5}$ or 1.8 b) rational

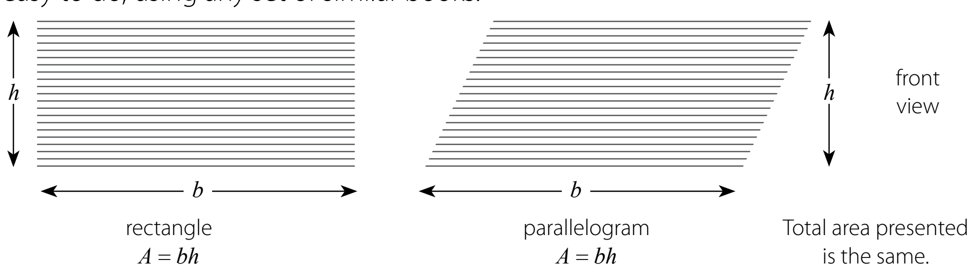
Chapter

7 Area and Perimeter

The introduction to the chapter brings together all the area formulas already studied. This provides an opportunity to show the relationships between them.

LESSON PLANNING

Objectives	
General	To be familiar with area formula for basic shapes; to solve problems involving areas and perimeters
Specific	1. To know that triangles have half the area of the surrounding rectangle or parallelogram 2. To know that a convex kite has an area of half the surrounding rectangle 3. To know that the area of a trapezium is half the area of the parallelogram formed by rotating it through 180° 4. To obtain the area of a rhombus by doubling the SAS formula of a triangle 5. To understand that the formula for the area of a concave kite is the same as that of a convex kite, i.e. half the product of its diagonals 6. To know the area and circumference formulas for a circle 7. To solve problems involving composite shapes 8. To be able to convert hectares to m^2 and vice-versa
Pacing	2 lessons, 1 homework
Links	substitution in formulas, volume of prism, Pythagoras' theorem, rotation
Method	Use the introduction to the chapter in the text to revise the area formulas shown, starting with the rectangle. It is a good opportunity to use the "pile of books" demonstration of the parallelogram formula again. It is worth repeating and so easy to do, using any set of similar books:



The idea is to show how each formula is related to another so that the correct formula has a rational basis and is not just memorized blindly.

	Go through the text examples for rhombus and concave kite, showing how areas of unknown shapes can be found by utilizing existing knowledge. Finally, revise the circle and set EX 7A. This systematically provides practice of all the formulas. Having thoroughly revised and practised all the formulas, students should be ready for more difficult questions. However, the section on units should not be omitted. Follow the text showing how all area formulas involve the product of two lengths. Also define the hectare as a land measure. Then set EX 7B.
Resources	pile of similar books for parallelogram demo (See Method.)
Assignments	EX 7B, question 9 is suitable for homework (after reminding students of the prism volume formula)
Vocabulary	perimeter, area rectangle, triangle, kite (convex and concave), parallelogram, trapezium, square, rhombus, circle hectare isosceles trapezium

ANSWERS

Exercises

EX 7A

1.	a) 4 cm	b) 10 cm	c) 12.8 cm	d) 8 cm
2.	a) 6 cm^2	b) 32 cm^2	c) 126 cm^2	d) 1920 cm^2
3.	a) 46.7 cm^2	b) 63.2 cm^2	c) 28.8 cm^2	d) 84 cm^2
4.	a) 42.5 cm^2	b) 39.1 cm^2	c) 32.5 cm^2	d) 240 cm^2
5.	a) 129 cm^2	b) 12 cm^2	c) 168 cm^2	d) 8 cm^2
6.	a) 90.2 cm^2	b) 156 cm^2	c) 49.7 cm^2	d) 83.1 cm^2
7.	a) 4070 cm^2, 226 cm	b) 17.6 cm^2, 17.2 cm		
	c) 19.6 cm^2, 17.9 cm	d) 20.3 cm^2, 18.0 cm		
8.	a) 24.0 cm^2, 26.5 cm	b) 13.4 cm^2, 21.1 cm		
	c) 24.0 cm^2, 22.4 cm	d) 137 cm^2, 53.9 cm		
9.	a) 114 cm^2, 45.1 cm	b) 83.9 cm^2, 40.0 cm		
	c) 26.0 cm^2, 25.4 cm	d) 430 cm^2, 165 cm		
10.	a) 28.6 cm^2, 25.1 cm (3 s.f.)			
	b) 50 cm^2, 28.3 cm (3 s.f.)			

EX 7B

1. 31 cm^2, 28 cm
2. 162 cm^2
3. 0.479 l
4. a) 4.752 ha b) $ 5220
5. 29.7 cm^2 (3 s.f.)
6. 3:13
7. $ 12
8. 300 m
9. 1090 m^3 (3 s.f.)
10. 52 cm^2, 36 cm

EX 7X

1. $A = \frac{7}{2}a^2$
2. a) $\pi : (4 - \pi)$ b) $1 : \frac{4 - \pi}{\pi}$, $n = 0.273$ (3 s.f.)
3. 488 cm^2 (3 s.f.), 105 cm (3 s.f.)

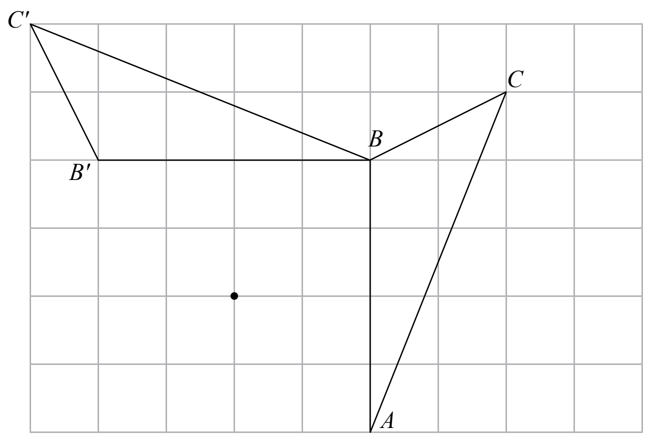
Chapter 8 Revision Exercises

ANSWERS

Exercises

EX 8A

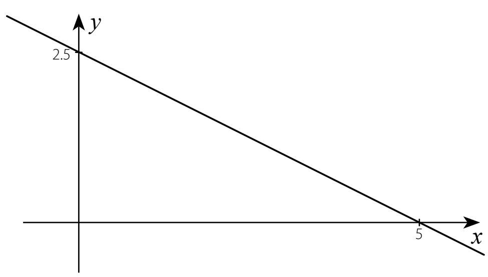
1. $x = 7.2$ cm, $y = 6.8$ cm

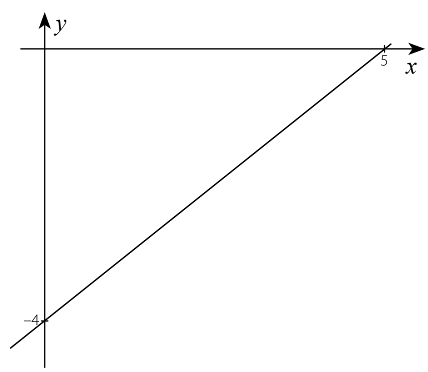
2. 

3. a) 7.6 b) 0.69 c) 0.0062 d) 560

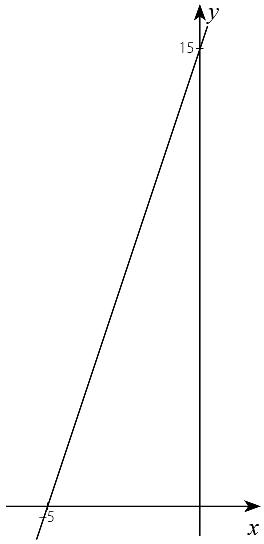
4. a) false b) true c) true d) false

5. a) $x = 5, y = 2.5$ b) $x = 5, y = -4$

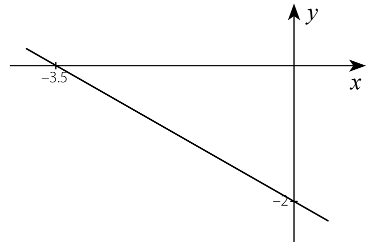




c) $x = -5, y = 15$



d) $x = -3.5, y = -2$

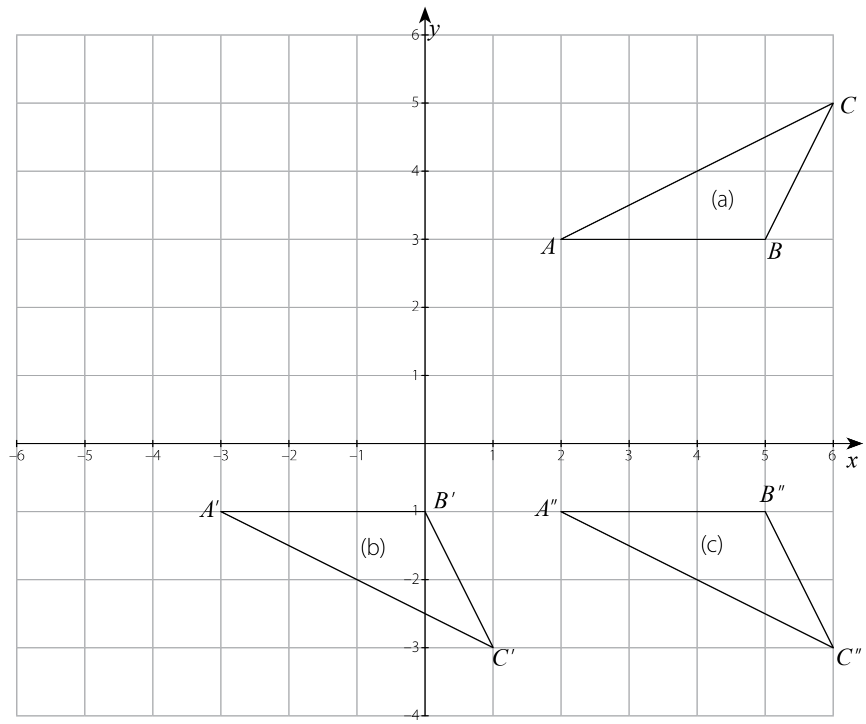


6. a) $5(x + 3y + 2z)$ b) $6(1 - 2y + 3z)$

c) not possible d) $10(5 - 6y^2)$

7. a) 20 b) 1.86 c) 9 d) 0.36

8.



d) reflection in the line $y = 1$

9. 54.7 cm^2

10. a) 5 10 20 40 80 160 320

5 10 20 40 80 160

5 10 20 40 80

It is not linear or quadratic.

b) Each term is double the previous term, starting with 5

c) 5×2^n d) 40 960

EX 8B

1. a) 125 b) 10 or 8

c) 6000 d) 544 or 390

2. a) strong negative b) zero

c) strong positive d) weak positive

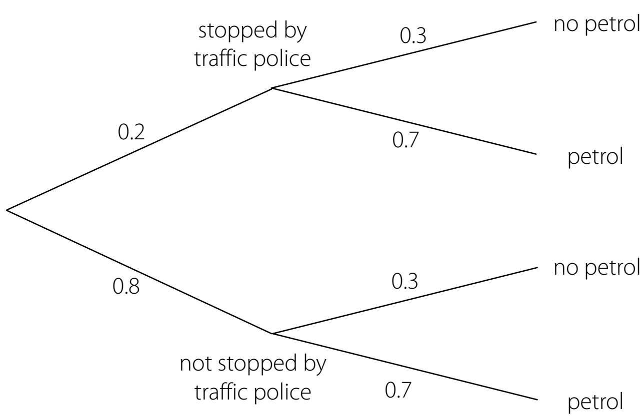
3. 47.7 cm^2, 30.2 cm

4. $x = \frac{1}{2}$ (or 0.5), $y = \frac{5}{2}$ (or 2.5)

5. a) 3.3 b) 200 c) 8000 d) 170

6. a) $u + 60$ b) $u + 60 + u = 420$, $u = 180$ Tehseen spent \$ 240, Umair \$ 180

7. a)



b) (i) 0.06

(ii) 0.56

8. $a = 3.5$ m, $b = 2.24$ m (3 s.f.), $c = 7.83$ m (3 s.f.)

9. a) true b) true c) false d) false

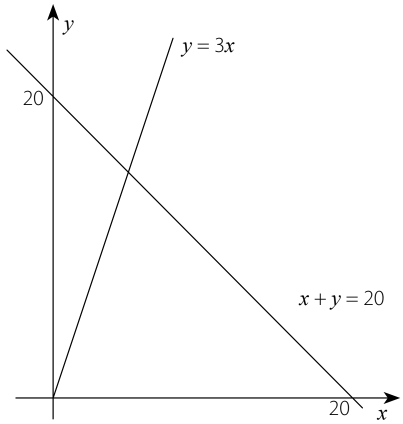
10. a) 52 b) 80 c) strong positive

d) Yes, all the points are quite close to the line of best fit.

EX 8C

1. a) $y = 3x, \quad x + y = 20$

b)



c) $x = 5, \quad y = 15$

d) at the point of intersection of the two lines

2. $f = \frac{uv}{u+v}$

a) 6.67 (3 s.f.) b) 2.53 (3 s.f.) c) −2.73 (3 s.f.) d) 1.92

3. $ 4.19 (nearest cent)

4. a) $3x^2 - 3xy$ b) $2x^2 - 2x^3 + 2x^2y$

c) $x - 3x^2 - y + 3y^2 + xy$ d) $5x^2 + 9xy - 4y^2$

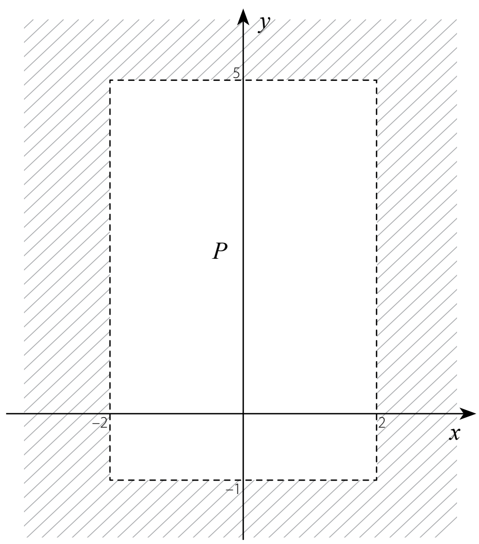
5. 1376 cm^2

6. a) 20.4 cm b) 112 cm

7. a) 34.715 (5 s.f.) b) 15.456 (5 s.f.) c) $\frac{y}{x} = 0.445$ (3 s.f.) = tan 24°

d) $x^2 + y^2 = 1444.019 = 1444$ (4 s.f.) $= 38^2$

8.



9. $2x + x + y + 2y - x = 56$ and $5x + \frac{y}{2} = 56$

$x = 10, y = 12$

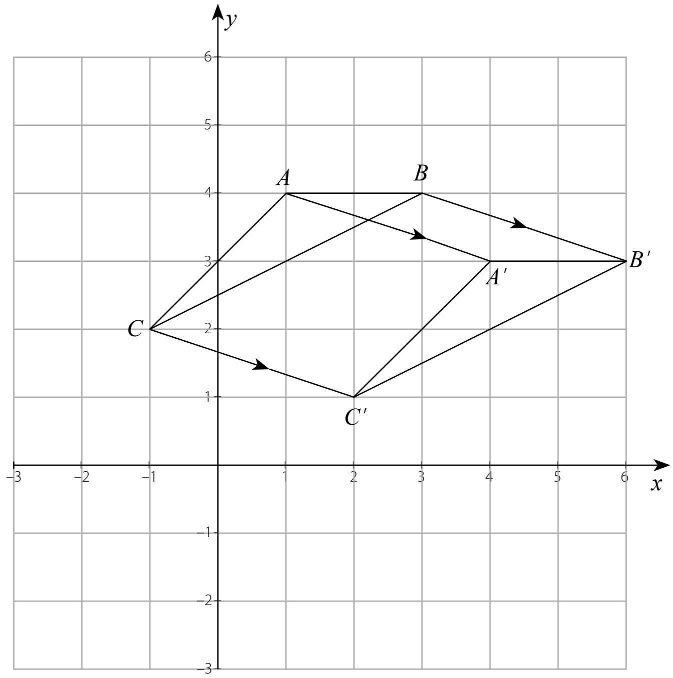
A 20, B 22, C 14, D 50, E 6 m^3/h

10. a) 45 m by 40 m b) 2400 cm^2 c) 770 m^2 d) 520 ml

EX 8D

1. $x = -5, y = -1$

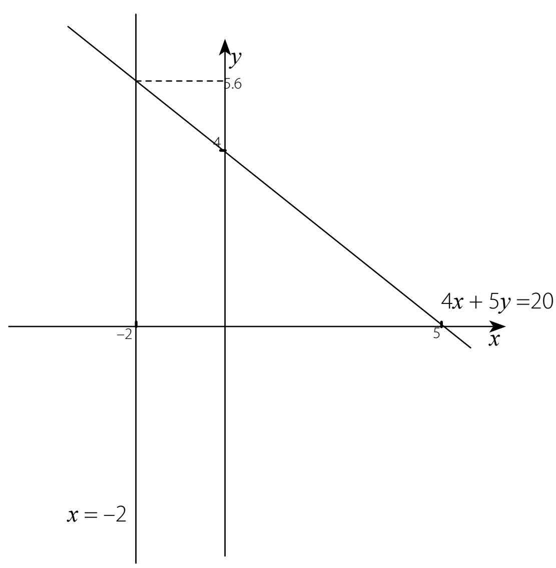
2.



b) $B'(6, 3)$

c) $C'(2, 1)$

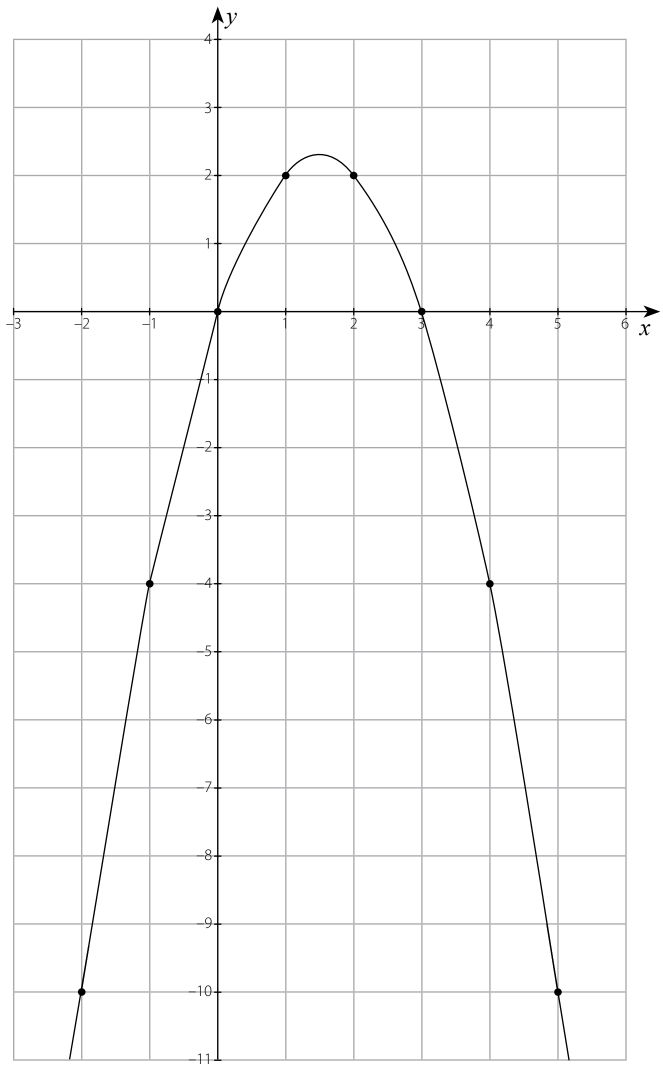
3. $x = -2, y = 5.6$



4. a)

x	-2	-1	0	1	2	3	4	5
x^2	4	1	0	1	4	9	16	25
y $\{3x$	-6	-3	0	3	6	9	12	15
$-x^2$	-4	-1	0	-1	-4	-9	-16	-25
y	-10	-4	0	2	2	0	-4	-10

b)



c) $x = 1.5$

d) (1.5, 2.25) (Use equation to calculate y accurately.)

5. a) 10.8 cm^2

b) 3.68 cm

6. a) 8.49 cm

b) 15.7 cm

c) 14.0 cm

d) 5.39 cm

7. a) $(x + 3)(x + 2y)$

b) $(1 + y)(x - 3y)$

c) $(x^2 + 1)(x + y)$

d) $(2x + 1)(x - 2y)$

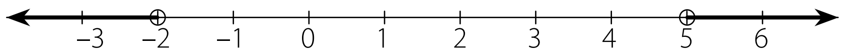
8. a) $x^2 + 4x + 5$

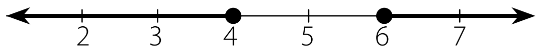
b) $2x^2 - 8x + 4$

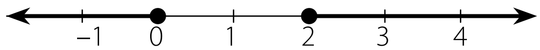
c) $x^2 - 4x - 3$

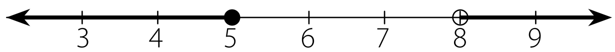
d) $5 + 3x - 2x^2$

9. a) $\frac{3}{2}$ (or 1.5) b) $-\frac{1}{4}$ (or −0.25) c) $\frac{2}{15}$ (or $0.1\dot{3}$) d) −7

10. a) 

b) 

c) 

d) 

EX 8X

2. (i) 1.591 155 195

(ii) 1.591 164 855

(iii) 1.591 384 538

(iv) 1.591 137 124

(v) 1.575

Output accuracy is never better than input accuracy. [To be safe, especially with a complicated formula, keep 2 extra s.f. throughout, rounding off only at the final answer.]

3. a)

A	$(\sin A)^2$	$(\cos A)^2$	$(\sin A)^2 + (\cos A)^2$
15	0.0670	0.9330	1.000
30	0.2500	0.7500	1.000
45	0.5000	0.5000	1.000
60	0.7500	0.2500	1.000
75	0.9330	0.0670	1.000
90	1.0000	0.0000	1.000

b) $(\sin A)^2 + (\cos A)^2 = 1$

c) This is probably true for angles in a right-angled triangle. [Mathematicians call this a conjecture.]

d) In the given triangle,

$\sin A = \frac{a}{c}$, and $\cos A = \frac{b}{c}$

$$
\begin{aligned}
(\sin A)^2 + (\cos A)^2 &= \frac{a^2}{c^2} + \frac{b^2}{c^2} \\
&= \frac{a^2 + b^2}{c^2} \\
&= \frac{c^2}{c^2} \qquad \text{because } a^2 + b^2 = c^2 \text{ (Pythag)} \\
&= 1
\end{aligned}
$$

[This is known as the trigonometric form of Pythagoras' theorem.]

Chapter

9 Problems in Trigonometry

This chapter follows on from Chapter 5 where calculations were restricted to finding lengths of sides. Here we extend to finding angles using inverse trig functions.

LESSON PLANNING

Objectives

General	To use trigonometry to solve problems, including those involving angles of elevation/depression and bearings
Specific	1. To be able to find an angle using a calculator and inverse trig functions 2. To draw angles of elevation or depression correctly on diagrams 3. To draw correct maps of journeys that are described using 3-figure bearings 4. To solve problems involving the above techniques
Pacing	3 lessons, 1 homework
Links	arrow diagrams, alternate angles

Method

Start with a speed test:

"Who can give me sin 46°?"

"Who can give me tan 15°?"

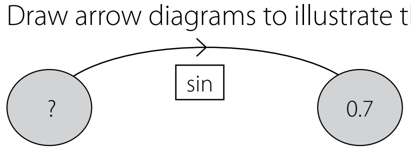
etc.

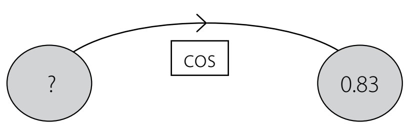
Then, proceed to questions like,

"The answer is 0.7 and it's a sine. What is the angle?"

"The cosine is 0.83, what is the angle?"

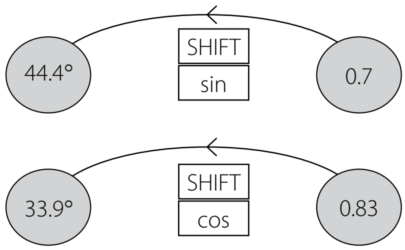
Draw arrow diagrams to illustrate these:





How to reverse the flow?

Use shift key to obtain inverse functions:



Students should to experiment until they get these answers.

- Shift trig is usually a sequence; on older calculators the two keys have to be pressed simultaneously.
- Calculators have to be set for degrees. If not, then use the DRG key to set degrees.

Check the results by finding sin 44.4° and cos 33.9°. The results are 0.7 and 0.83 respectively.

Once the key sequence is clear, move on to finding angles from a given ratio of two sides. Follow the text example and get the students to find out which key sequence works best on their own calculator.

Use EX 9A.

At this stage, the functional inverse notation trig^{-1} is not recommended although it may appear on the calculator screen.

Students should write the trig equation,

e.g. $\quad \sin A = 0.5$

then use the calculator keys to obtain A, and just write the answer

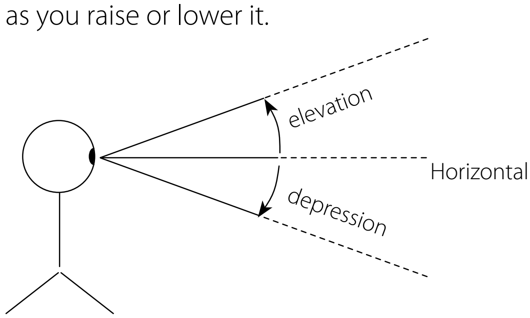
$$A = 30°$$

[Note: Although it is absolutely correct to write

$$\begin{aligned} \sin A &= 0.5 \\ A &= \sin^{-1} 0.5 \\ &= 30°, \end{aligned}$$

disconnecting sine from its angle is confusing for most students at this stage.]

Demonstrate elevation and depression by holding out your arms horizontally, holding one eye close to your arm, and then looking up or down along your arm as you raise or lower it.



Do a quick revision of bearings. (The text example is for reference only.) Set EX 9B. This has some difficult questions and students would benefit from being allowed to discuss them with a neighbour. Clear, well-labelled diagrams are essential for this topic. Insist!

Resources	calculators essential
Assignments	EX 9B, question 7 or EX 9A, question 7 suitable for homework
Vocabulary	inverse function angle of elevation, depression bearings

ANSWERS

Exercises

EX 9A

1. a) 23.6° b) 41.4° c) 74.1° d) 48.0°
2. a) 44.4° b) 73.1° c) 64.1° d) 34.4°
3. a) 24.6° b) 45.5° c) 44.3° d) 36.4°
4. a) $\sin A = \frac{3}{5}$, $\angle A = 36.9°$
   b) $\cos A = \frac{4}{5}$, $\angle A = 36.9°$
   c) $\tan A = \frac{3}{4}$, $\angle A = 36.9°$
   d) same answer by 3 methods
5. $\angle A = 65.6°$, $\angle B = 65.6°$, $\angle C = 48.9°$
6. a) $PR = 100$ cm, $\sin P = 0.28$
   b) $\angle P = 16.2602°$, $\sin P = 0.2800$
   c) same answer
7. $\angle C = 42.4°$
8. $\angle D = 53.2°$
9. $\angle B = 65.3°$
10. $\angle A = 59.0°$

EX 9B

1. a) 9.65 m (3 s.f.) b) 5.79 m (3 s.f.)
2. 8.09 m (3 s.f.)
3. 24.1 km (3 s.f.)
4. 11.9°
5. 2.77 m (3 s.f.)

6. 77.1 km (3 s.f.)

7.

Angle of elevation	Height
10°	17.64 m
30°	57.76 m
50°	119.22 m
70°	274.86 m

8. 12°

9. 80.3 km

10. a) 48.6° (3 s.f.) b) 48.6°

c) 10.58 m (4 s.f.) d) 10.58 m (4 s.f.)

EX 9X

1. 16.1°

2. 7.81 m

3. 235 km (3 s.f.)

Chapter 10 Percentages

This chapter reinforces the scale factor method of increasing and decreasing by percentage amounts, with problem solving questions in the second half.

Simple interest is rarely used in the real world of finance, but seems to have lingered on in examination syllabuses. It is an excuse for changing the subject of a formula practice.

The phrase "markup" is used occasionally as the preferred term in Islamic banking for the charge made for the time value of money. The morality or otherwise of interest is not addressed although students should see that even moderate rates of interest can be devastating for borrowers. For example,

at 15% p.a. compound, money doubles in 5 years,

i.e. $\$\,100 \times 1.15^5 = \$\,201.14$

LESSON PLANNING

Objectives

General	To solve problems involving percentage increases or decreases
Specific	1. To interpret scale factors (SF) as percentage increases or decreases, and vice-versa. 2. To use scale factor to solve problems involving more than one percentage change 3. To know the simple interest formula and be able to make any unknown the subject for the formula 4. To know the terms profit, loss, cost price, and selling price, and to be able to express these as percentages of the cost price 5. To understand compound interest as repeated applications of a scale factor (SF) 6. To solve problems involving the above
Pacing	3 lessons, 1 homework
Links	arrow diagrams
Method	The key to this topic is use of arrow diagrams and scale factors (SFs). Use plenty of diagrams. Explain SF like this example: $1.15 = \underbrace{1}_{\text{original amount}} + \underbrace{0.15}_{\text{15\% increase}}$ Show how to change back and forth from SF to % with examples on the board.

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	This is revision, so move fairly quickly onto multiple percentage changes. Use the text example and set EX 10A. Give the simple interest formula. Show how it can be rearranged to make P, R or T the subject. Explain cost price, selling price, profit, and loss. Do not dwell on compound interest, but set EX 10B.
Assignments	EX 10A, question 10 or EX 10B, question 10 are suitable for homework
Vocabulary	simple interest, profit and loss, compound interest, cost price, selling price

ANSWERS

Exercises

EX 10A

1. a) 1.26 b) 1.08 c) 0.93 d) 0.68
2. a) 46% increase b) 90% increase
   c) 19% decrease d) 40% decrease
3. a) 1.115 b) 0.985 c) 0.965 d) 1.0825
4. a) $17\frac{1}{2}$% increase b) $36\frac{1}{2}$% increase
   c) $2\frac{1}{2}$% decrease d) 28.73% decrease
5. a) 11.3% increase b) 9.14% increase
   c) 17% increase d) 4.12% decrease
6. a) 850 b) \$ 8500
   c) 120 ha d) \$ 89 000
7. $n = 45$
8. \$ 5000
9. \$ 1.36 (nearest cent)
   Each percentage increase is on the previous year's price, not the original price.
10. 8.3% (2 s.f.)
   Each quantity rate is calculated on the previous quarter's growth, not the first quarter.

EX 10B

1. a) \$ 1440 b) 1.45%
2. a) \$ 44.20 b) \$ 13.26

3. a) $ 230 b) $ 287.50 c) $ 690

4. a) 21.4% (1 d.p.) b) 7.1% (1 d.p.)

5. $ 759.63 (nearest cent)

6. a) $ 750 b) $ 19 each, $ 5 total loss

7. a) $ 128.34 b) $ 12.10 (nearest cent)

8. a) 5 years b) $ 10 000

9. 1.6%, $ 12.80

10. $ 143.94

EX 10X

1. $ 179.10 [Round off to nearest cent after each year.]

2. a) 70.62% b) 14.13% c) 15.25%

3. 8.00597 (6 s.f.) [Trial and improvement should give the answer between 8.00 and 8.01%.]

Chapter

11 Patterns and Sequences

This chapter develops higher-level cognitive skills, using some more unusual sequences as a basis for the problem solving demands. Some students will find this rather challenging.

LESSON PLANNING

Objectives	
General	To solve problems involving number sequences and design pattern sequences
Specific	1. To recognize linear, quadratic, Fibonacci and triangle number sequences by the method of differences 2. To find nth term formulas for sequences 3. To explore the number patterns in Pascal's triangle 4. To use algebra as an aid to solving sequence problems
Pacing	2 or 3 lessons: the more talented your students, the longer you should take
Links	algebraic products and common factors
Method	This is a topic hard to teach. It is rather like swimming: you have to jump in and experience the water! It is worth going through the Reminders at the start of the chapter. Pascal's triangle construction has to be shown. The rest is in the exercise. Just set EX 11A and see what the students can do. Definitely allow discussion of the work. If your students find this a real struggle, stop after question 7. Circulate and troubleshoot.
Resources	Wall poster of Pascal's triangle can be helpful.
Assignments	Not really suitable for homework. If you need one, set an investigation on Fibonacci sequences. Begin sequences with a) 1 7 ... b) 2 5 ... c) –1 2 ... d) Choose some more. Write about 8 terms, then work out the differences (3 rows). What do you notice?
Vocabulary	linear, quadratic, Fibonacci, Pascal's triangle

ANSWERS

Exercises

EX 11A

1. a) equal steps of –3, linear sequence

   b) $16 - 3n$ c) –134 d) 19th

2. a)

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1

   b) 1 2 4 8 16 32 64 128 ...

   c) Each term is double the previous term.

   d) 2^{n-1}

3. a) equal steps in 2nd differences, quadratic sequence

   b) $(n + 1)^2 - 1$, $n^2 + 2n$

   c) $n(n + 2)$

   d) 1×3 2×4 3×5 4×6 5×7 ...

4. a)

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1

b) 2 c) 6

d) The triangle numbers add up to the square of the natural number.

5. a) $1 \times 2 \quad 2 \times 3 \quad 3 \times 4 \quad 4 \times 5 \ldots$

b) 240 c) $n(n + 1)$ d) Design 20

6. 3 4 7 11 18 29

7. a) 1

b)

```
1   1
1   2   1
1   3   3   1
1   4   6   4   1
1   5   10  10  5   1
1   6   15  20  15  6   1
1   7   21  35  35  21  7   1
1   8   28  56  70  56  28  8   1
```

8. a) 1 11 121 1331 14641

b) Each term is 11 times the previous term.

c) 11^{n-1}

d) The tens digits should be carried. So the next two terms are 161051 and 1771561 which continues the sequence according to the rule.

9. a) 1

b)

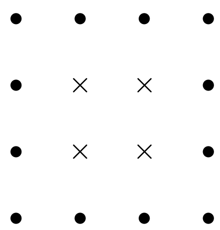
```
                1
              1   1
            1   2   1
          1   3   3   1
        1   4   6   4   1
      1   5   10  10  5   1
    1   6   15  20  15  6   1
  1   7   21  35  35  21  7   1
1   8   28  56  70  56  28  8   1
1   9   36  84  126 126 84  36  9   1
```

c) The total is just below the last term added.

d) Examples shown are $1 + 2 + 3 + 4 = 10$,

$1 + 5 + 15 = 21$ and $1 + 6 + 21 + 56 = 84$

10. a)



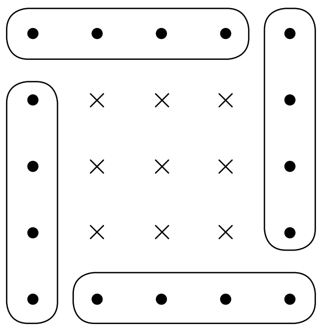
Design 4

b) $n^2 - (n - 2)^2$, $4n - 4$

c) 4 sides of n spots each $= 4n$, but each corner has been counted twice.

$\therefore$ No. of spots $= 4n - 4$

d)



Design 5

$4(n - 1)$

EX 11X

1. a) $\frac{5}{16}$ b) $\frac{21}{128}$ c) $\frac{1}{32}$ d) $\frac{35}{128}$

3. 27th term

[To solve $(n + 6)(n + 8) = 1155$, try values near $\sqrt{1155}$ because $n + 6$ and $n + 8$ are close. This soon yields $33 \times 35 = 1155$, hence $n = 27$.]

Chapter

12 Volume and Surface Area

The cube, cuboid, and cylinder are described as special cases of the prism. The sphere's volume formula cannot be proved or even demonstrated for reasonableness at this stage. Nevertheless, the r^3 in the formula, and r^2 in the surface area formula should be pointed out. In examinations, these formulas would normally be given, but it is good if students memorize them.

LESSON PLANNING

Objectives

General	To solve volume and surface area problems involving common solids and composite solids
Specific	1. To know the general volume formula for any prism and adjust it for the cube, cuboid, and cylinder 2. To use the formulas for volume and surface area of a sphere and a hemisphere 3. To solve problems of surface area by sketching the net of the solid 4. To solve volume and surface area problems for solids composed of various common solids, or with pieces removed, with good communication 5. To change the subject of mensuration formulas in order to solve problems
Pacing	3 lessons, 1 homework
Links	units, formula manipulation, and substitution

Method

Use the prism as the unifying principle to generate cube, cuboid, and cylinder volume formulas.

Follow the text for the sphere and hemisphere, explaining that more advanced mathematics is needed to prove these. However, point out that the r^3 indicates they are formulas of volume.

The composite example can be gone through but it is best left for reference. The class discussion should focus on strategy, i.e.

"What are the parts of the solid?"

"What do we need to know for each part?"

"Could we calculate the volume of each part?"

"Does the text communicate?"

Emphasize that working should clearly state what is being calculated, and not just show the figures. Set EX 12A.

For surface area show how to use nets to simplify the working. A demonstration of cutting open a cylinder is beneficial to show why the curved surface area = $2\pi rh$.

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	The sphere/hemisphere results again have to be taken on trust, but the r^2 indicates that they are area formulas. Set EX 12B. Some of these are quite difficult.
Resources	Cylinder to cut open: a used toilet roll tube is an easy option.
Assignments	EX 12A, question 4 and/or EX 12B, question 10 suitable for homework
Vocabulary	prism, cross-section cube, cuboid, cylinder sphere, hemisphere volume, surface area

ANSWERS

Exercises

EX 12A

1. 695 cm^3 (3 s.f.)
2. 11 cm
3. 2.5 m^3
4. a) 7.82 cm^2 b) 188 cm^3 (3 s.f.)
5. 10 min 8 s (nearest second)
6. 6912 cm^3
7. a) hemisphere, cylinder b) 26 800 l (3 s.f.) c) 5
8. 185 m^3 (3 s.f.)
9. a) $V = \frac{2}{3}\pi r^3 + \pi r^2 h$; $V = \pi r^2\left(\frac{2}{3}r + h\right)$ b) $h = \frac{V}{\pi r^2} - \frac{2r}{3}$
   c) i) 37.5 cm ii) 8.22 cm iii) 2.24 cm (3 s.f.)
10. $x = 2$

EX 12B

1. 184 cm^2
2. 252 cm^2
3. 2.13 m^2 (3 s.f.)
4. 5.20 l (3 s.f.)
5. 96 cm^2
6. 3820 m^2 (3 s.f.)
7. 20 m^2 [Hint: Work in metres throughout.]

8. 78.4 cm^2 (3 s.f.)

9. 3180 cm^2 (3 s.f.)

10. 180 cm^2

EX 12X

1. $h = \dfrac{A}{r(\pi + 2)}$

2. $d = \sqrt[3]{\dfrac{6V}{\pi}}$

   i) 5.76 ii) 7.26 iii) 8.31 cm (3 s.f.)

3. a) $V = 6(\pi + 2)r^3$ b) $A = 2(7\pi + 2)r^2$

Chapter

13 Equations

This chapter is a continuation of Book 8, Chapter 15 with harder questions. It is an important topic and the details need to be understood so that the students obtain mastery of the algebra techniques required.

LESSON PLANNING

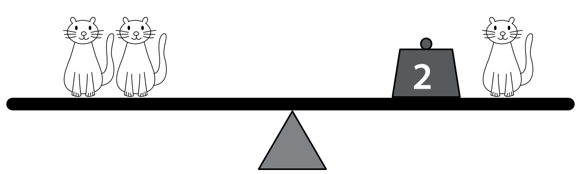
Objectives

General	To solve linear equations
Specific	1. To be able to deal with fractions or decimals in equations 2. To be able to deal with brackets in equations, especially those preceded by negative multipliers 3. To be able to deal with the intrinsic bracket in an algebraic fraction 4. To submit solutions to equations in the same form as given in the question
Pacing	2 lessons, 1 homework

Method

- The technique of scaling up to remove fractions or decimals should be shown, with examples on the board, e.g. as in the text introduction. However, if students do not see why each term has to be scaled, here are two ways to explain it:

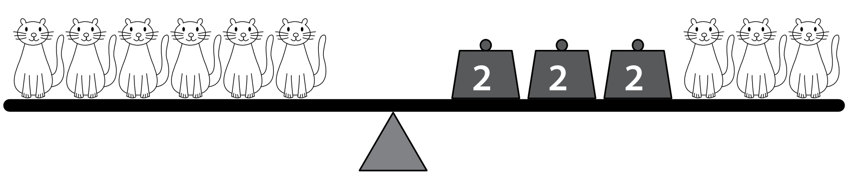
  (i) Go back to the balance:



$$2c \quad = \quad 2 + c$$

Clearly, each cat weighs 2 kg. ($c = 2$)

Scaling up by factor of 3 (say)



$$6c \quad = \quad 6 \quad + \quad 3c$$

Clearly, each cat still weighs 2 kg. ($c = 2$)

Scaling each item does not affect the solution.

(ii) Use giant brackets:

$\frac{3}{4}(x - 5) + 6 = \frac{2}{3}(x + 1)$

Scale factor 12. Each side has to be treated equally to preserve the balance.

$12\left[\frac{3}{4}(x - 5) + 6\right] = 12\left[\frac{2}{3}(x + 1)\right]$

Then remove brackets by the usual rules.

$9(x - 5) + 72 = 8(x + 1)$

This stage should be done directly, with mental cancellation. If students are forced to do tedious detail they often get lost in it.

For example, it is not recommended to write

$\overset{3}{\cancel{12}} \times \frac{3}{\cancel{4}}(x - 5) + 12 \times 6 = \overset{4}{\cancel{12}} \times \frac{2}{\cancel{3}}(x + 1)$

- A common error is to apply a scale factor inside the brackets as well, e.g. on the RHS of the above:

  $= 12\left[\frac{2}{3}(x + 1)\right]$

  $= 8(12x + 12)$ Wrong!

  To prevent this, say "bracket" and think of it as one object. For example, to scale by 5 the term $-\frac{3}{5}(x + 1)$

  say "5 times $-\frac{3}{5}$ brackets equals –3 brackets"

  Then write $-3(x + 1)$ filling the bracket.

- Another common error with negative multipliers is to forget the sign change on the second term.

  For example, $-2(3x - 2) = -6x - 4$ Wrong!

  $-2(3x - 2) = -6x + 4$ Right!

- Choosing the best scale factor to apply is usually obvious. If not, it is the LCM of the denominators. For decimals, it would be 10 or 100. Sometimes 5 will do.

  Algebraic fractions are implicitly bracketed. This is not always understood. Teach it.

  For example, $\frac{x + 1}{2}$ means $\frac{(x + 1)}{2}$

  or $\frac{1}{2}(x + 1)$

  or $0.5(x + 1)$

  The dividing line acts as a kind of bracket.

  Finally, teach students to give solutions in the form of the given equation. If the equation has decimals, give decimal solutions; if the equation has fractions, give fractional solutions.

Assignments	EX 13A, question 8 suitable for homework
Vocabulary	scale factor, solution, multiplier

ANSWERS

Exercises

EX 13A

1. a) SF = 10, $x = \frac{5}{11}$ b) SF = 12, $x = 27$

   c) SF = 30, $x = \frac{12}{5}$ d) SF = 21, $x = \frac{30}{17}$

2. a) SF = 10, $x = -2$ b) SF = 10, $x = 1.5$

   c) SF = 100, $x = 0.704$ d) SF = 100, $x = 11.7$

3. a) $x = \frac{29}{7}$ b) $x = \frac{-7}{16}$

   c) $x = \frac{-11}{5}$ d) $x = \frac{108}{67}$

4. a) $x = -0.25$ b) $x = -1.\dot{3}$

   c) $x = 2.10605$ d) $x = 1.5$

5. a) $x = 3$ b) $x = \frac{11}{19}$

   c) $x = 11$ d) $x = 9$

6. a) $x = \frac{15}{4}$ b) $x = \frac{138}{17}$

   c) $x = \frac{32}{21}$ d) $x = -25$

7. a) $x = 1, y = 2$ b) $x = 2, y = -3$

   c) $x = -4.5, y = 0.5$ d) $x = 5.1, y = 2.3$

8. a) $x = \frac{1}{2}, y = \frac{3}{2}$ b) $x = 4, y = \frac{3}{2}$

   c) $x = \frac{5}{2}, y = \frac{11}{2}$ d) $x = -0.9, y = -2.4$

9. $ 2.50

10. 5.8 tonnes, no (max load is 6 tonnes.)

EX 13X

1. $x = -1$

2. $x = \frac{-40}{51}, \quad y = \frac{35}{51}$

3. $x = 1, \quad y = -2, \quad z = 3$

Chapter

14 Calculator Practice

This chapter is the framework for practical sessions where students gain familiarity with their own calculators.

LESSON PLANNING

Objectives

General	To use the calculator efficiently for calculations involving squares, square roots, brackets, and use of memory
Specific	1. To find the correct key sequence to square a number (positive or negative) 2. To know when brackets have to be entered 3. To use the square root key to find the square roots of positive numbers (including fractions) 4. To use memory storage and recall keys for complicated calculations 5. To use 1 s.f. rounding off to estimate, before using the calculator 6. To evaluate the subject of a formula when the other unknowns are given, using a calculator
Pacing	1 lesson, I homework
Links	substitution, BODMAS
Method	Introduce the practical lesson by stating that you will not teach them anything directly—they have to puzzle it all out themselves. The aim is to learn about • squares and square roots • use of brackets • use of memory Instructions are all given in EX 14A. Divide the students into groups (with similar calculators in each group). Discussion and mutual assistance should be encouraged. The answers to questions 1–6 are all given in the exercise: the point is to find the simplest key entry sequence to obtain each answer. Written answers are required for questions 7–10.
Resources	Calculators for all (essential)
Assignments	Suitable for homework EX 14A, question 10

ANSWERS

Exercises

EX 14A

7. a) $\dfrac{50}{30-2\times 4}=\dfrac{50}{22}\approx 2,\quad 2.1$

b) $\dfrac{20\times 3-10\times 3}{2\times 5-0.8\times 4}=\dfrac{60-30}{10-3.2}=\dfrac{30}{6.8}\approx 4,\quad 2.3$

c) $(4\times 7)^2-(4\times 7)\approx 30^2-30=870,\quad 645.9$

d) $5(1+3)+(1+3)^2=20+16=36,\quad 33.8$

8. a) $\dfrac{10+20}{1\times 3}=\dfrac{30}{3}=10,\quad 7.82$

b) $\dfrac{4\times 5-2\times 1}{2\times 1-\sqrt{4\times 5}}\approx\dfrac{18}{-2}=-9,\quad -7.89$

c) $2\times 3+(2\times 3)^2=6+36=42,\quad 38.8$

d) $\dfrac{4-1}{0.3}=\dfrac{3}{0.3}=10,\quad 9.64$

9. a) $f=4.1$ b) $v=2.6$

c) $v=7.6$ d) $u=10$

10. a) $T=3.2$ b) $c=5.1$

c) $a=5.2$ d) -0.340

Chapter

15 Revision Exercises

All the questions in these exercises relate to topics covered in Chapters 1–14 of the text.

As this is the midpoint of the material for the academic year, it is advised to make maximum use of the revision exercises before moving ahead.

Two specimen examination papers, one for O level Paper 1 and the other for O level Paper 2, are included here for the first half of the year (after the answers to the exercises). In view of the fact that some students may have access to this guide for teachers, it is suggested that modifications are made.

ANSWERS

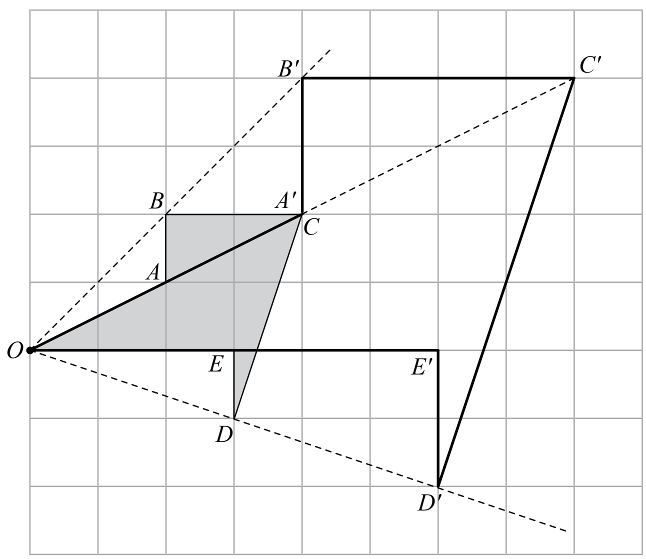
Exercises

EX 15A

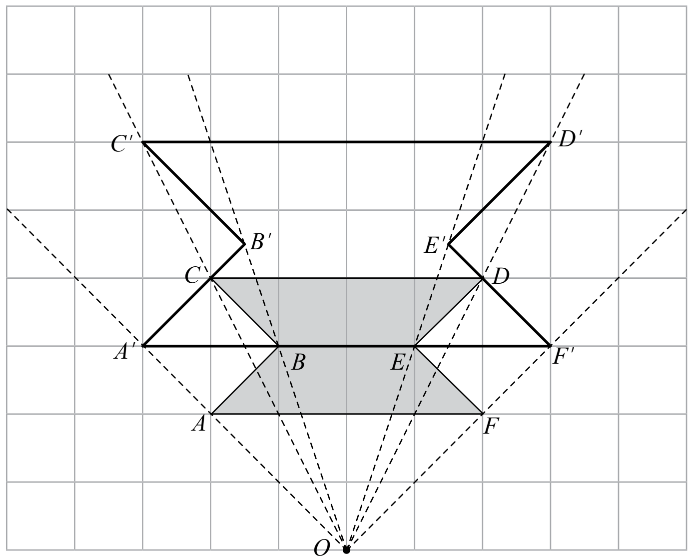
1. a) not rt$\angle$d b) $\angle B = 90°$

   c) $\angle C = 90°$ d) not rt$\angle$d

2. a)



b)



3. a) 13.2 cm b) 132 cm

4. a) $\angle A = 16.3°$ b) $AC = 12.5$ cm, $\angle A = 16.3°$ c) same

5. a) $n - 2$ b) $180\,(n - 2)$

   c) 3600° d) 183 600°

6. $u = 9.6$ cm, $v = 9.6$ cm

7. a) 55.4 b) 236 c) 78.5 d) 8330

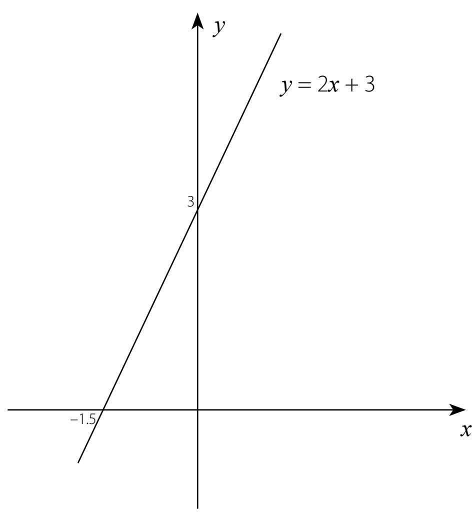
8. a) 0.81 b) 10.6 c) 700 d) 25.94

9. a) 7.07 cm b) 23.7 cm

10. a) $x = \frac{9}{8}$ b) $x = 3$ c) $x = \frac{20}{13}$ d) $x = 2$

EX 15B

1.



[all points on the line]

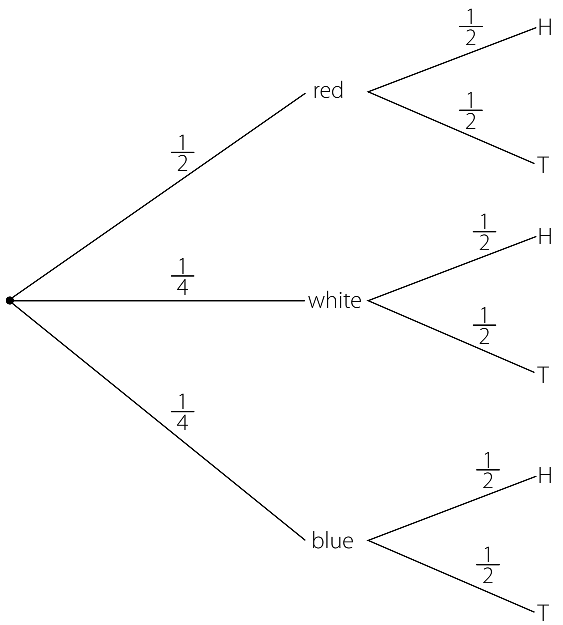
2. a) 250 b) $ 800

c) 600 ha d) $ 5000

3. $A'(-1, 0)$, $B'(2, 1)$, $C'(4, -1)$

4. a) P(red) $= \frac{1}{2}$, P(white) $= \frac{1}{4}$, P(blue) $= \frac{1}{4}$

b)



c) (i) $\frac{1}{4}$

(ii) $\frac{1}{8}$

5. a) $\frac{5}{13}$ b) $\frac{24}{7}$ c) $\frac{7}{25}$ d) $\frac{7}{25}$

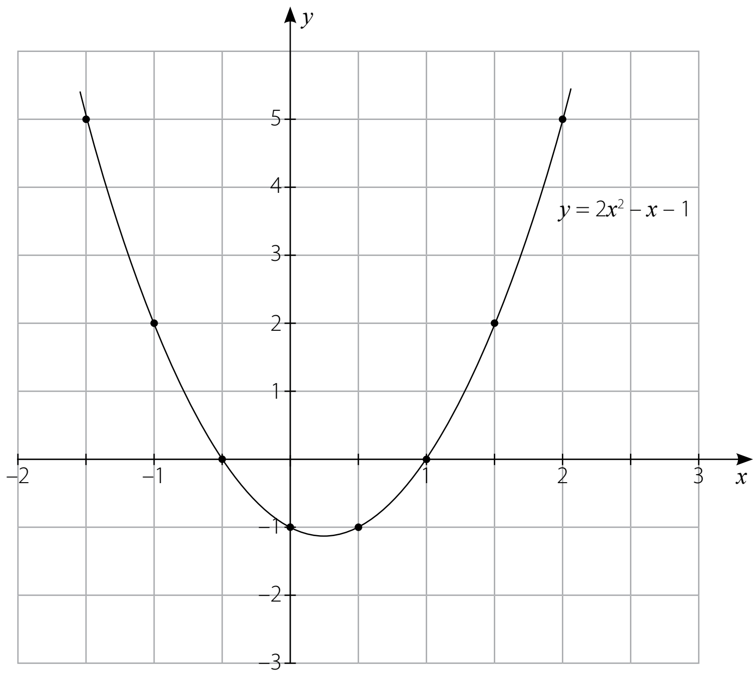
6. $x = 12$ cm, $y = 5$ cm

7. a) 27.6 b) 21 000 000 c) 0.99 d) 246

8. 4:1

9. a)

x	−1.5	−1	−0.5	0	0.5	1	1.5	2
x^2	2.25	1	0.25	0	0.25	1	2.25	4
y: $2x^2$	4.5	2	0.5	0	0.5	2	4.5	8
y: $-x$	1.5	1	0.5	0	−0.5	−1	−1.5	−2
y: -1	−1	−1	−1	−1	−1	−1	−1	−1
y	5	2	0	−1	−1	0	2	5

b) 

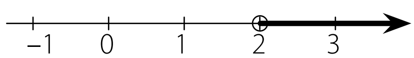
c) (0.25, −1.125) [Accept any y–value just below −1]

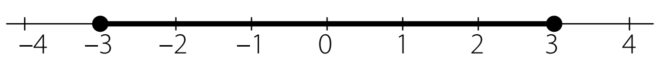
d) $x = 0.25$ e) parabola

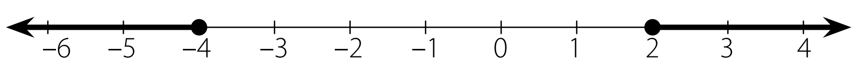
10. a) $13n + 8$ b) 6

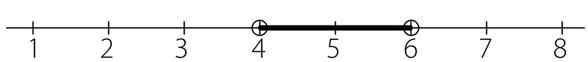
c) 6 7 13 20 33 53 86 139

EX 15C

1. a) 

b) 

c) 

d) 

2. a) 5 each, 2 remainder, natural numbers (or positive integers)

b) $\frac{1}{4}$ each, rational number

3. a) $ 2 b) $ 50 c) $ 5.94

4. a) 20 b) 20 c) 100 d) 110

5. a) 50° b) 33.9 cm^2

6. a) $t = \dfrac{v - u}{a}$ b) $t = \dfrac{2s}{u + v}$

c) $t = \sqrt{\dfrac{p}{5}}$ d) $t = 1 - \sqrt{M}$, $t = 1 + \sqrt{M}$

7. a) 11.5 b) 14.4 c) 11.5 d) 14.4

8. Veronica spent \$ 83; Wasim spent \$ 16

9. a) 168 cm^2 b) 600 cm^2

c) 31.2 cm^2 d) 29.1 cm^2

10. Prism 10 000, cuboid 4080, total 14 080 cm^3

EX 15D

1. a) $x^2 + 8x + 16$ b) $1 - 10x + 25x^2$

c) $x^2 - x - 2$ d) $2x^2 - x - 1$

2. a) $x(1 + 2y + 4x)$ b) $3y(1 - 2y + 3x)$

c) $4(1 - 3ab + 5b^2)$ d) $p(p^2 - 2p + 3)$

3. a) $a = 12$ cm b) $b = 7.5$ cm

4. 457 m^2 (3 s.f.)

5. a) 0.05 b) 7.5

c) 10 000 d) 25 [or 30]

6. a) $5(p - 2qr + 3t)$ b) $6(3 - 2y + x)$

c) not possible d) $20y(2 - 3y)$

7. 134.1 cm^2

8. a) $x = -0.2$ b) $x = 1.75$

c) $x = 0.7$ d) $x = 1.5$

9. 3, –1

10. 67.1 km

Specimen Examination Paper 1

[for the first half of the year]

Instructions	The time allowed is 1 hour.
	Electronic calculators must not be used in this paper.
	You will need pen, pencil, eraser, and ruler.
	Try to answer all the questions.
	Check your work carefully.
	The marks for each question are shown in brackets. [Max marks: 60]

1 Round off these numbers to the accuracy stated:

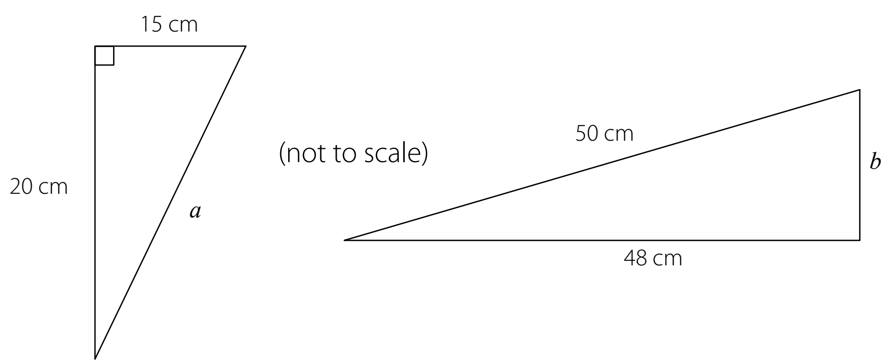
a) 4.57 to 1 d.p. b) 0.405 to 2 s.f.

c) 562 to 1 s.f. d) 0.0743 to 3 d.p. [4]

2 Solve the equation:

$\frac{x}{2} - 3 = 3x - \frac{1}{4}$ [4]

3 Write down the lengths a and b, as shown in the diagrams:



[4]

4 Solve the simultaneous equations:

$3x - 2y = 17$

$x - 5y = 36$ [4]

5 Calculate:

a) $6 + (-2)(-7)$ b) $15 - \frac{16}{-4}$

c) $\frac{18 \times -5}{-180}$ d) $[2 + (-3) - (-6)]^2$ [4]

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6 If $p = 6.5$, $q = 3.5$, and $r = 0.5$, find the value of:

a) $p + q$ b) $2p - q$

c) $\frac{q}{r}$ d) $(r - q)^2$ [4]

7 Consider the sequence 12, 7, 2, −3, −8, …

a) Use the method of differences to find out what kind of sequence it is.

b) Find a formula for the nth term.

c) What is the 40th term?

(d) What position is the term whose value is −43? [4]

8 Calculate:

a) 41×0.3 b) 11.6×0.2

c) $7200 \div 60$ d) $33.99 \div 0.03$ [4]

9. a) Rewrite the following calculation, replacing each number by its value correct to 1 s.f.:

$$\frac{0.22 \times 0.194}{0.039}$$

b) Estimate the answer. [4]

10. Work out the following calculation exactly:

$$\frac{1200 \times 0.14}{0.3 \times 2.8}$$ [4]

11. The first two terms of a Fibonacci sequence are consecutive natural numbers.

a) If the first term is n, write down a formula for the second term.

b) If the 5th term is 23, find a suitable equation and solve it to find n.

c) Write down the sequence up to the 7th term. [4]

12. Find the prime factorisation of the following. Give your answers in ascending order of the factors, and use indices.

a) 550 b) 2040 [4]

13. The price of 1 kg of apples is $\frac{4}{5}$ of the price of 1 dozen bananas. If 2.5 kg of apples and $4\frac{1}{2}$ dozen bananas cost Rs 780, find the total cost of 1 kg apples and $\frac{1}{2}$ dozen bananas. [4]

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14. The formula $H = 3.5(10 - t)$ gives the depth of water H cm in a tank, t minutes after the drain tap is opened.

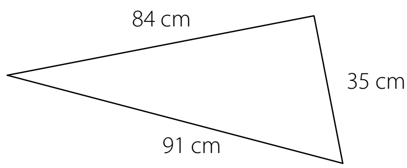
a) Write down the depth of water initially.

b) What is the depth of water 6 minutes later?

c) Rearrange the formula to make t the subject.

d) How long does it take for the depth of water to reach 7 cm? [4]

15 A triangle has sides of length shown. Is it right-angled? Give a reason for your answer.



[4]

Specimen Examination Paper 2

[for the first half of the year]

Instructions

The time allowed is $1\frac{1}{2}$ hours.
Electronic calculators may be used.
You will also need pen, pencil, eraser, and ruler.
Try to answer all the questions.
Check your work carefully.
The marks for each question are shown in brackets. [Max marks: 90]

---

1. Use a calculator to work these out, giving your answers correct to 3 s.f.:

   a) $\sqrt{\dfrac{6-2.4}{0.76}}$ b) $\dfrac{14.2^2}{21.6-3.2\times 4.2}$ [4]

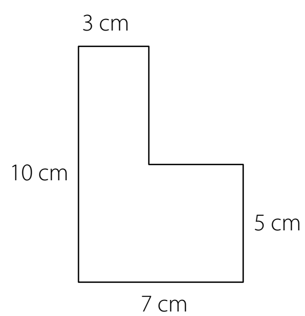
2. A rectangle has an area of 63 cm^2.

   a) If its length is 14 cm, what is its width?

   b) If its width is 1.05 cm, what is its length?

   c) If it is square, what is its length (1d.p.)? [4]

3. A prism has L-shaped cross-section as shown. If its length is 11 cm, calculate its volume.



(not to scale)

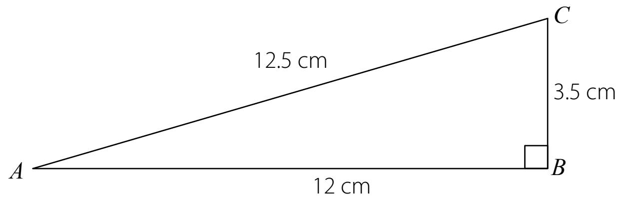
[4]

4. In the right-angled triangle shown, write down

   a) the value of sin A as a fraction

   b) the value of tan A as a decimal (1d.p.)

   c) the value of angle A in degrees (1d.p.)



(not to scale)

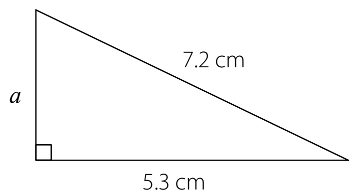
[4]

5. Write down the correct scale factor to multiply by in order to achieve the following changes:

   a) 10% increase   b) 9% decrease

   c) $7\frac{1}{2}$% increase   d) 40% decrease   [4]

6. Use Pythagoras' theorem to calculate the lengths marked a and b. (2 s.f.)



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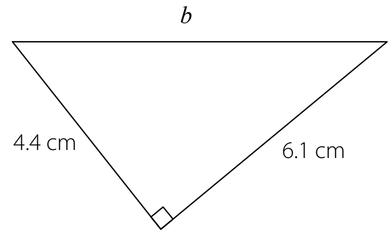
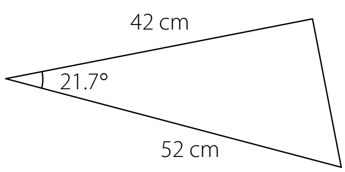


[4]

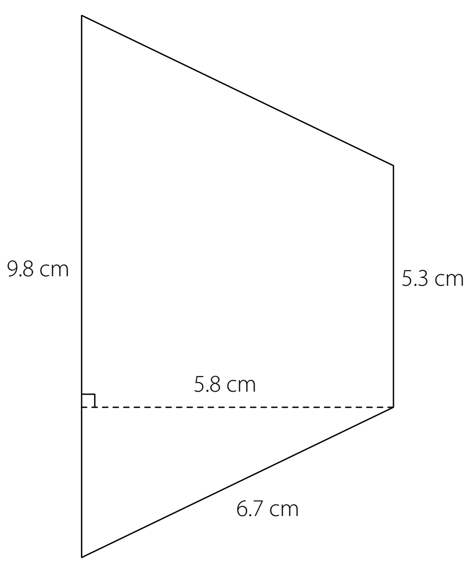
7. Calculate the area of the triangle shown, correct to 3 s.f.



(not to scale)

[4]

8. Calculate the area of the trapezium shown below, correct to 2 s.f.



(not to scale)

[4]

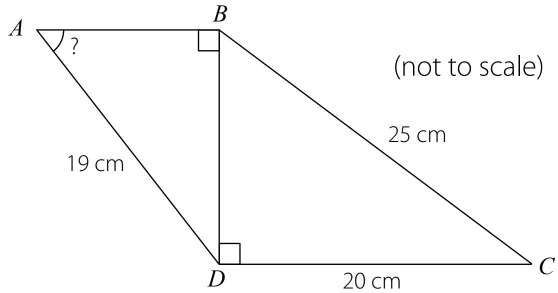
9. Calculate the original amount in each case:

a) After an increase of 6%, I have $ 4770.

b) After a decrease of 3.5%, I have $ 23 160.

c) After a price increase of 10% followed by a further price increase of 5%, the price is $ 6468. [4]

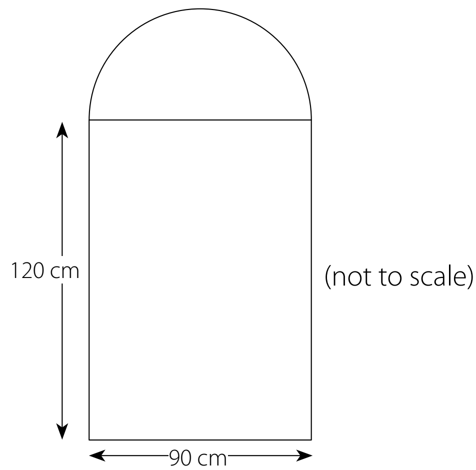
10. In the diagram, calculate the size of $\angle A$, correct to 1 d.p.



[4]

11. A window is made up of two parts: an upper semicircle and a lower rectangle, with dimensions as shown in the diagram.

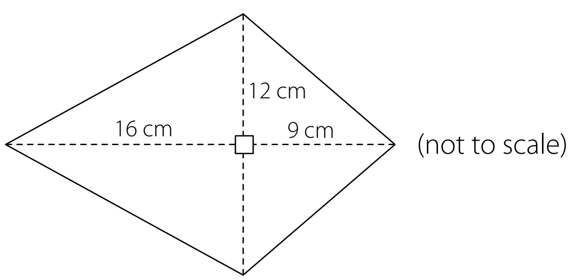
Calculate the total area of glass required, correct to 3 s.f.



[5]

12. Sara is looking straight ahead, when she lowers her eyes 25° to see a piece of litter on the ground. If the litter is 4.2 m away from her foot, how far away is it from her eyes? [1 d.p.] [5]

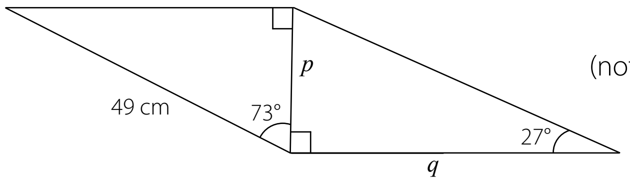
13. Calculate (a) the area, (b) the perimeter, of this kite:



[5]

14. a) Calculate the length of p, correct to 5 s.f.

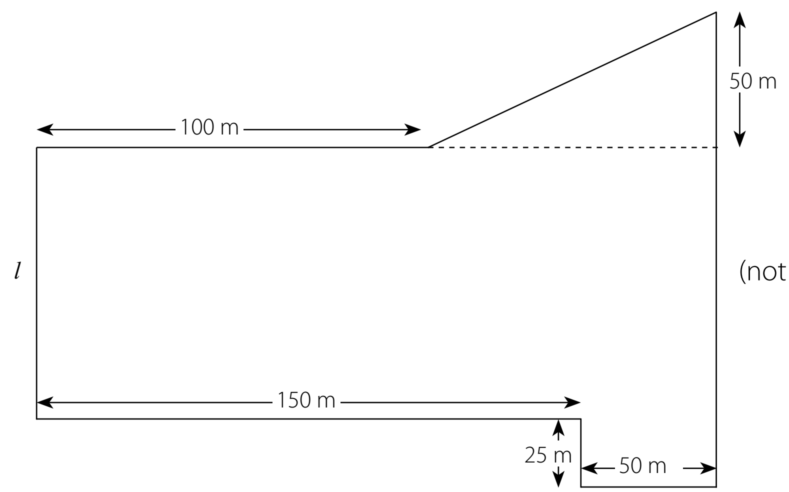
b) Hence, calculate the length of q, correct to 3 s.f.



(not to scale)

[5]

15. A farmer's field has an area of 1.225 ha. The plan view is shown in the diagram. Find the length l.



(not to scale)

[5]

16. A ship starts at a point O and travels 40 km on a bearing of 330° to reach point A. Then it travels on a bearing 090° to reach point B. From B it travels on a bearing of 240° for 50 km and returns to O. Calculate the distance AB to the nearest kilometre. [5]

17. A car dealer pays \$ 8000 for a car, hoping he can sell it for \$ 9000.

a) If he finds a customer willing to pay his asking price, what percentage profit will he make on this deal?

b) If he cannot find a willing customer and is forced to let the car go for only \$ 7500, what percentage loss does he have to bear? [5]

18. At a fairground balloons are filled with helium gas. If the balloons can be considered as spheres of diameter 16 cm, calculate the quantity of helium needed to fill 60 identical balloons. [Answer in litres, to 1 d.p.]

Note: The volume of a sphere is given by $V = \frac{4}{3}\pi r^3$. [5]

19. If $x = \frac{1+\sqrt{5}}{2+\sqrt{5}}$ calculate the value of $x + x^2 - \sqrt{x}$ correct to 3 s.f. [5]

20. a) Find the nth term formulas for the sequence:

2, 2.4, 2.8, 3.2, 3.6, …

b) What position is the term whose value is 58.4? [5]

Paper 1—answers and mark scheme:

1. a) 4.6

   b) 0.41

   c) 600

   d) 0.074 [1 each = 4]

2. $2x - 12 = 12x - 1$

   $-10x = 11$

   $x = -\frac{11}{10}$ or -1.1

   $\left[\begin{array}{ll} \text{SF4} & 2 \\ \text{ans} & 2 \end{array}\right\} = 4 \Big]$

3. $a = 25$ cm $\quad b = 14$ cm [2 each = 4]

4. $x = 5y + 36$

   subst $3(5y + 36) - 2y = 17$

   $15y + 108 - 2y = 17$

   $13y + 91 = 0$

   $y + 7 = 0$

   $y = -7$

   $x = 1$

   [subst2
   answer 1 each]

5. a) 20

   b) 19

   c) $\frac{1}{2}$ or 0.5

   d) 25 [1 each = 4]

6. a) 10

   b) 9.5

   c) 7

   d) 9 [1 each = 4]

7. a) 12 7 2 −3 −8

   −5 −5 −5 −5 linear

   b) nth term $= -5n + 17$ or $17 - 5n$

   c) −183

   d) $-5n + 17 = -43$

   $-5n = -60$

   $n = 12$ $\quad$ 12th position [1 each = 4]

8. a) 12.3

b) 2.32

c) 120

d) 1133 [1 each = 4]

9. a) $\frac{0.2 \times 0.2}{0.04}$ [(a) 1 each number = 3

b) 1 (b) 1]

10. $\frac{\overset{\overset{200}{\cancel{400}}}{\cancel{1200}} \times \cancel{14}}{3 \times \underset{2}{\cancel{28}}} = 200$ [mult. by 100 top and bottom 2

answer 2]

11. a) $n + 1$ [(a) 1

b) $5n + 3 = 23, n = 4$ (b) 2

c) 4, 5, 9, 14, 23, 37, 60 (c) 1]

12. a) $2 \times 5^2 \times 11$

b) $2^3 \times 3 \times 5 \times 17$ [2 each = 4]

13. $a = \frac{4b}{5}$ $2.5a + 4.5b = 780$ [eqns 1

Solutions $a = 96$ $b = 120$ solve 2

$a + \frac{1}{2}b = 96 + 60 = 156$ Rs 156 ans 1]

14. a) 35 cm

b) 14 cm

c) $t = 10 - \frac{H}{3.5}$

or $t = \frac{35 - H}{3.5}$

d) 8 minutes [1 each = 4]

15. Yes, Pythagorean triple 5, 12, 13 scaled 7 x [ans 1

or $84^2 + 35^2 = 91^2$ with working shown reason 3]

Paper 2—answers and mark scheme:

1. a) 2.18

   b) 24.7 [2 each = 4]

2. a) 4.5 cm [(a) 1

   b) 60 cm (b) 1

   c) 7.9 cm (c) 2]

3. Area of cross-section $= 5 \times 7 + 5 \times 3 = 50$

   or $10 \times 3 + 5 \times 4 = 50$

   Volume $= 50 \times 11 = 550 \text{ cm}^3$ [area 2, vol 2]

4. a) $\frac{7}{25}$ [(a) 1

   b) 0.3 (b) 1

   c) 16.3° (c) 2]

5. a) 1.1

   b) 0.91

   c) 1.075

   d) 0.6 [1 each = 4]

6. $a = 4.9$ cm $b = 7.5$ cm [2 each = 4]

7. Area $= \frac{1}{2} \times 42 \times 52 \times \sin 21.7° = 404 \text{ cm}^2$ [SAS formula 2, ans 2]

8. Area $= \frac{1}{2}(9.8 + 5.3)\,5.8 = 44 \text{ cm}^2$ [formula 2, ans2]

9. a) \$ 4500 [(a) 1

   b) \$ 24 000 (b) 1

   c) \$ 5600 (c) 2]

10. $BD = 15$ cm (Pythag triple 3, 4, 5 $\times 5$) [BD 1

    $\sin A = \frac{15}{19}$ sine 2

    $\angle A = 52.1°$ $\angle A$ 1]

11. Area lower rectangle $= 120 \times 90 = 10\,800 \text{ cm}^2$

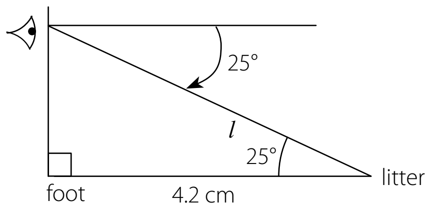
Area semicircle $= \frac{1}{2} \times \pi \times 45^2 = 3180.86 \text{ cm}^2$

Total Area $= 13\,980.86$

$= 14\,000$ (3 s.f.)

[rect 1, semi 2, total 1, 3 s.f. 1]

12.



$\cos 25° = \frac{4.2}{l}$

$l = \frac{4.2}{\cos 25}$

$= 4.6$ m

[diag 2, cos 1, l 1, ans 1]

13. a) Area $= \frac{1}{2} \times 25 \times 24 = 300 \text{ cm}^2$

b) Sides are 20 cm and 15 cm (Pythag triples)

perimeter = 70 cm

[area 2, sides 2, perimeter 1]

14. a) $\cos 73° = \frac{p}{49}$

$p = 49 \times \cos 73°$

$= 14.326$ (5 s.f.)

b) $\tan 27° = \frac{p}{q}$

$q = \frac{p}{\tan 27°}$

$= 28.1$ (3 s.f.)

[cos 1, ans 2, tan 1, ans 1]

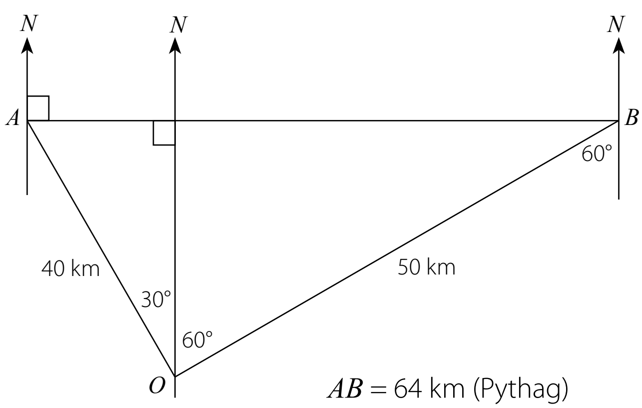
15. $1.225 \text{ ha} = 12\,250 \text{ m}^2$

$12\,250 = 200\,l + \frac{1}{2} \times 100 \times 50 + 50 \times 25$

$l = 42.5$ m

[ch to m² 1, eqn 2, ans 2]

16.



$AB = 64$ km (Pythag)

[diag 3, AB from Pythag 2]

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17. a) 12.5% $\begin{bmatrix} \text{(a) } 2 \\ \\ \text{(b) } 3 \end{bmatrix}$

b) 6.25%

18. Volume $= \frac{4}{3} \times \pi \times 8^3 \times 60 = 128\,679.6 \text{ cm}^3$ $\begin{bmatrix} \text{radius} & 1 \\ \text{formula} & 2 \\ \text{ans cm}^3 & 1 \\ \text{ans } l & 1 \end{bmatrix}$

$= 128.7\ l$

19. 0.473 [5]

20. a) $1.6 + 0.4n$ $\begin{bmatrix} \text{(a) } 2 \\ \\ \text{(b) } 3 \end{bmatrix}$

b) 142nd term

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Chapter

16 Graphs

This chapter is a collection of facts for review and consolidation previously mentioned in Book 7, Chapters 20 and 30, and in Book 8, Chapters 6 and 25. There is more practice in accurate plotting, and in recognizing the type of graph from its equation.

The second half is useful for students intending to appear for Additional Mathematics. It goes beyond most O-level/IGCSE syllabuses and may be omitted.

LESSON PLANNING

Objectives

General	To sketch and plot straight lines, parabolas and reciprocal graphs from given data
Specific	1. To use the gradient form and the intercept form of straight line equation to obtain its sketch graph 2. To recognize the equation of a parabola; and to plot an accurate graph of a quadratic function from given data 3. To recognize a reciprocal graph from its equation; to plot an accurate graph of a reciprocal function from given data [4. To plot and intercept displacement–time graphs]
Pacing	2 lessons [+1], 1 homework [+1]
Method	The Reminders in the chapter introduction in the text are mostly for reference. However, ensure that everyone can find intercepts from a straight line equation, and hence make a sketch. This is vital. Show them on the board how to cover up (i.e. set to zero) each variable in turn, as described in the text. Use the text examples, and more of a similar nature. Issue squared paper and set Ex 16A. [The topic "Functions of time" (optional) requires some input at the start. Explain that x and y are replaced by s and t, defined as in the text, and that there is a huge variety of graphs. Set Ex 16B.]
Resources	Squared paper (9 mm) is accurate enough for this topic. Photocopiable sheets are available here in the Guide.
Assignments	EX 16A, question 9 [and/or EX 16A, question 7]
Vocabulary	gradient, intercept, reciprocal, inverse reciprocals, quadratic, parabola, axis (line) of symmetry

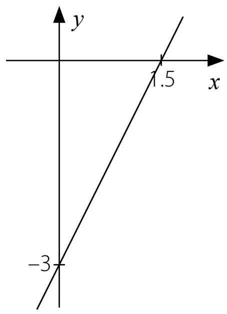
ANSWERS

Exercises

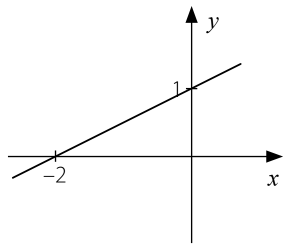
EX 16A

1.	a) Q	b) P	c) R	d) S
2.	a) S	b) R	c) P	d) Q

3. a)



b)



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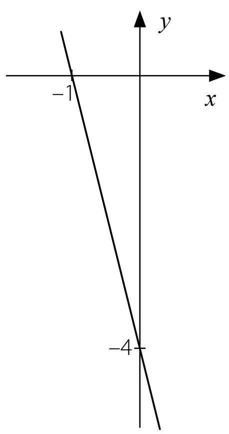
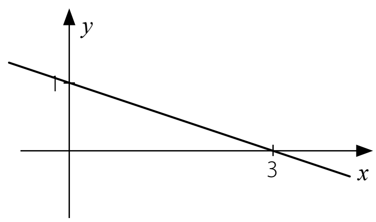
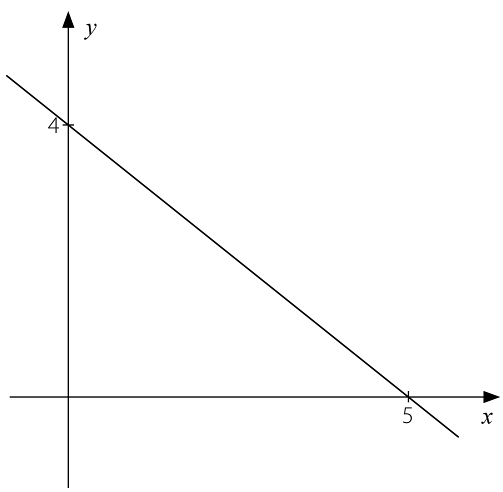


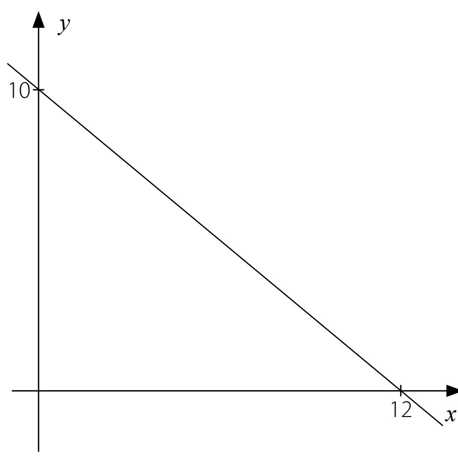
d)



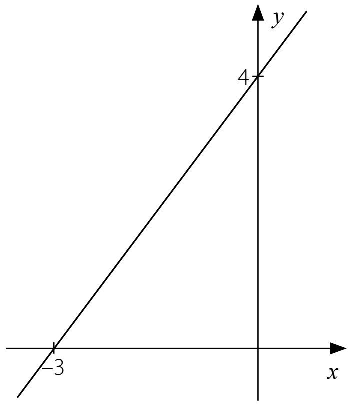
4. a)



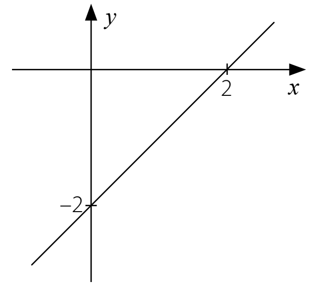
b)



c)



d)



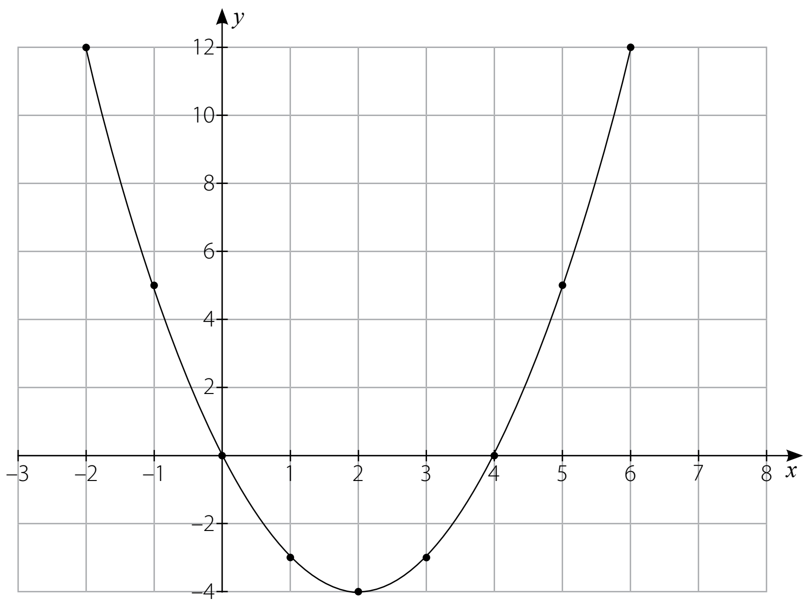
5. a) $y = 3$ b) $x = -4$

c) $y = -\frac{1}{2}x$ d) $y = 4x$

6. a)

x	–2	–1	0	1	2	3	4	5	6
$y\ \{x^2$	4	1	0	1	4	9	16	25	36
$-4x$	8	4	0	–4	–8	–12	–16	–20	–24
y	12	5	0	–3	–4	–3	0	5	12

b)



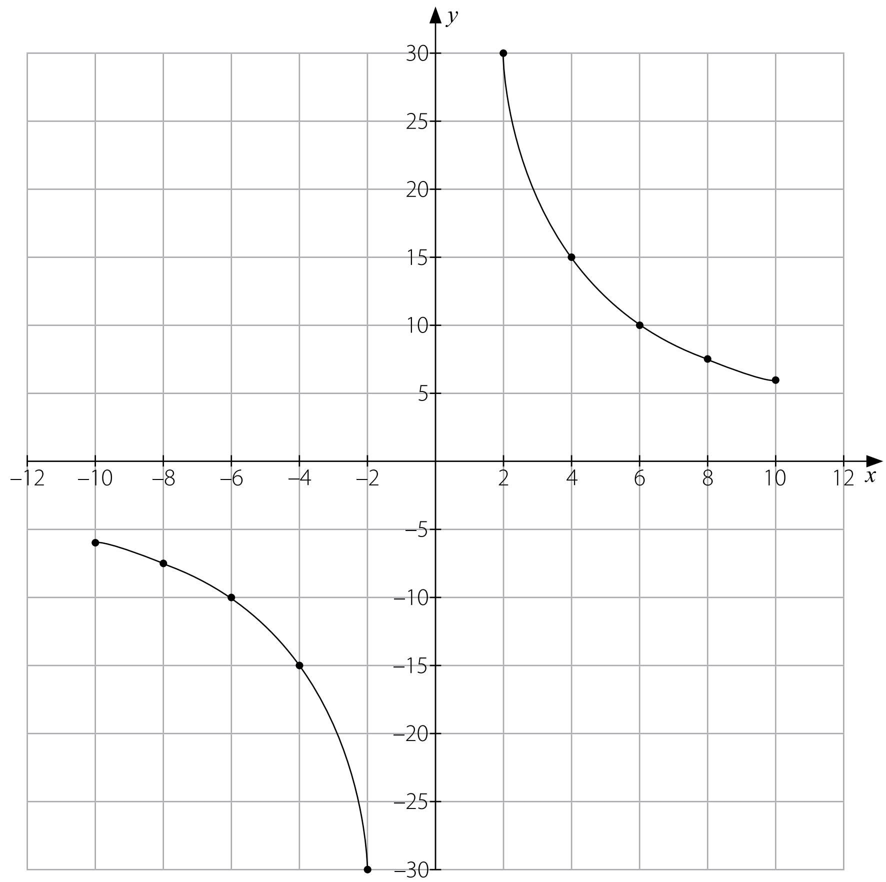
c) $x = 2$

d) $(2, -4)$

7. a)

x	–10	–8	–6	–4	–2	2	4	6	8	10
y	–6	–7.5	–10	–15	–30	30	15	10	7.5	6

b)

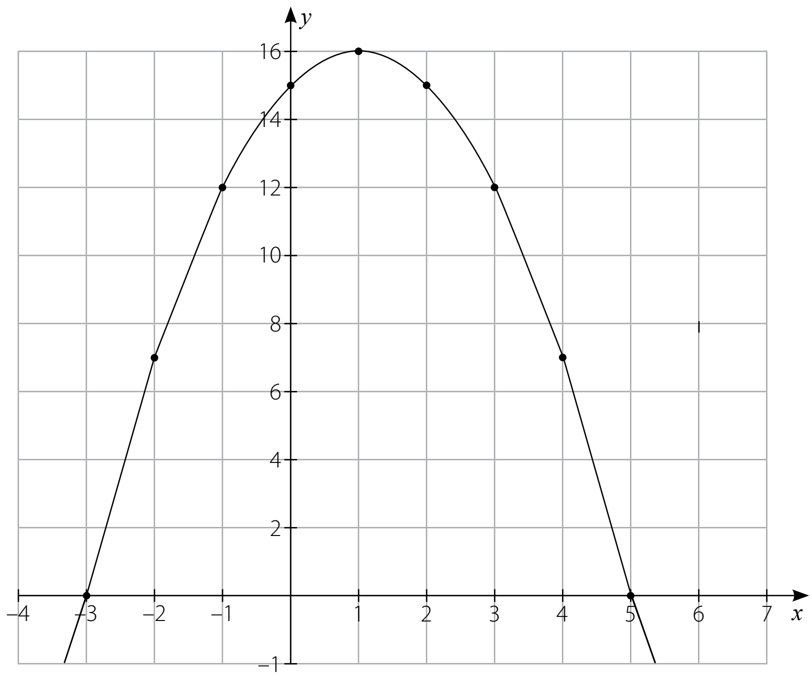


c) rotational symmetry, centre the origin, order 2

d) reciprocal

8. a)

x	−3	−2	−1	0	1	2	3	4	5
x^2	9	4	1	0	1	4	9	16	25
y: 15	15	15	15	15	15	15	15	15	15
y: $2x$	−6	−4	−2	0	2	4	6	8	10
y: $-x^2$	−9	−4	−1	0	−1	−4	−9	−16	−25
y	0	7	12	15	16	15	12	7	0



b) parabola c) (1, 16) d) $x = 1$

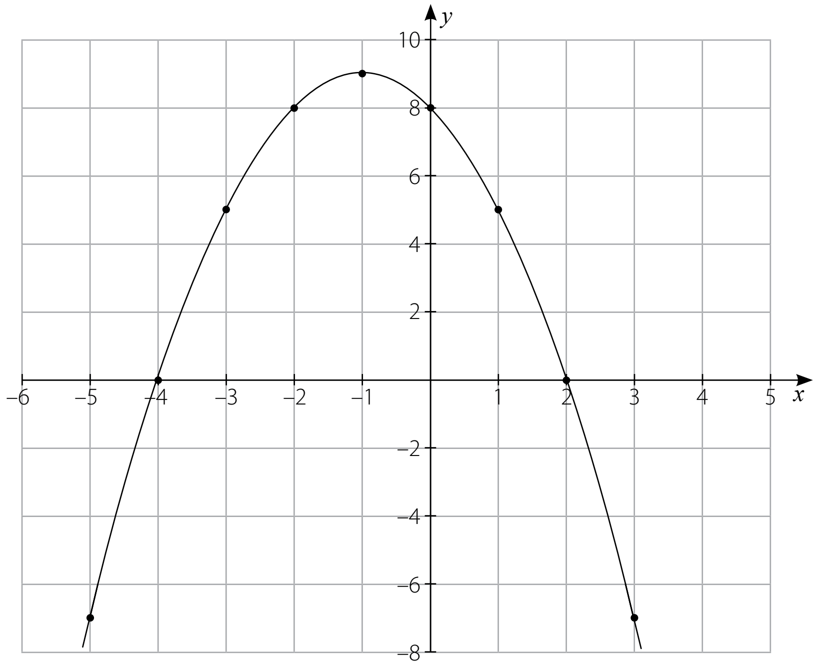
9 a) $x = 0$ or $x = 5$ b) $x = 2$ or $x = 3$

c) $x = -1$ or $x = 6$ d) $x = 2.5$

10. a) $y = -x^2 - 2x + 8$

b)

x	−5	−4	−3	−2	−1	0	1	2	3
y: $-x^2$	−25	−16	−9	−4	−1	0	−1	−4	−9
y: $-2x$	10	8	6	4	2	0	−2	−4	−6
y: $+8$	8	8	8	8	8	8	8	8	8
y	−7	0	5	8	9	8	5	0	−7

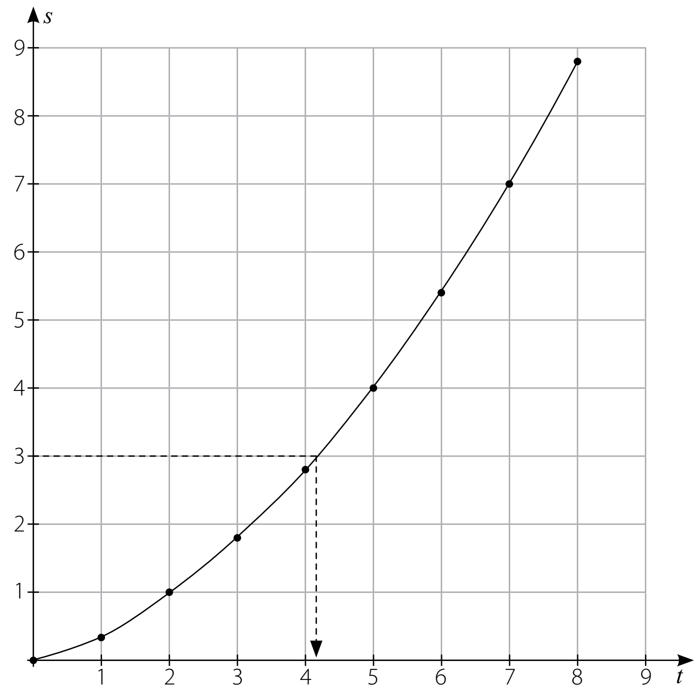


c) $x = -4$ or $x = 2$ d) $x = -3$ or $x = 1$

EX 16B

1. a) 200 m b) 12 s c) 50 m d) 12.5 m
2. a) 1 km b) standing still (resting)
   c) 4 km/h d) 3.4 km/h (1 d.p.)
3. a) 1.25 m b) 3.75 m c) level with the top of the cliff
   d) 5 m
4. a)

t	0	1	2	3	4	5	6	7	8
$t+3$	3	4	5	6	7	8	9	10	11
s	0	0.4	1.0	1.8	2.8	4.0	5.4	7.0	8.8



b) 4.2 s

5. Stage I: starting fast then slowing down

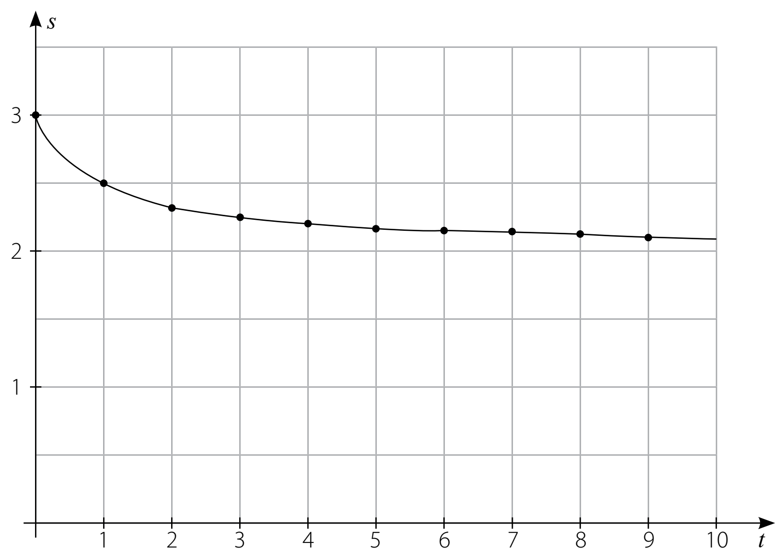
Stage II: a period at rest

Stage III: rapid acceleration

6. a)

t	0	1	2	3	4	5	6	7	8	9
$\frac{2t+3}{t+1}$	$\frac{3}{1}$	$\frac{5}{2}$	$\frac{7}{3}$	$\frac{9}{4}$	$\frac{11}{5}$	$\frac{13}{6}$	$\frac{15}{7}$	$\frac{17}{8}$	$\frac{19}{9}$	$\frac{21}{10}$
s	3.00	2.50	2.33	2.25	2.20	2.17	2.14	2.13	2.11	2.10

(to 1 d.p.)



b) Gradually slowing down

7. Stage I: running at constant speed

Stage II: resting

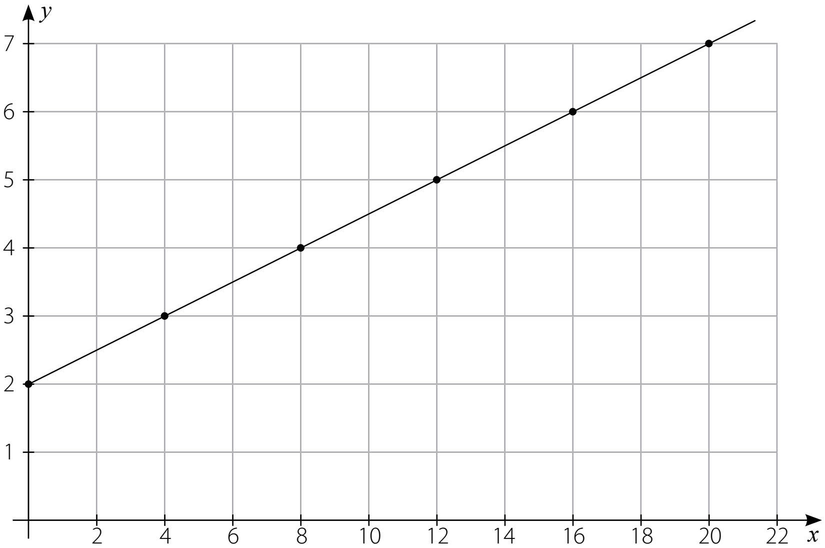
Stage III: walking at constant speed

Stage IV: resting

Stage V: running fast at constant speed

8. a) 15 m b) 25 m c) 1.1 m d) 75 m

9. (0, 2), (4, 3), (8, 4), (12, 5), (16, 6), (20, 7)

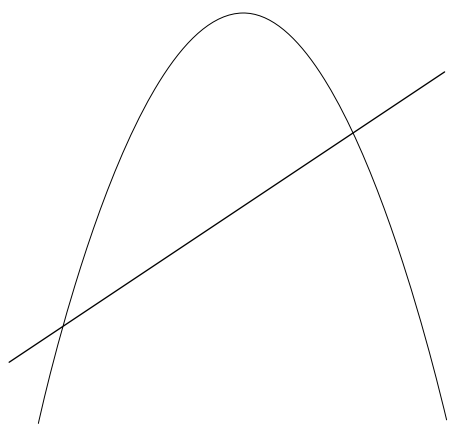


10. a) 1 m from the origin b) 6 m from the origin

c) at the origin d) 42 m to the left of the origin

EX 16X

1. Accurate graphs look like this, giving intersections where $x = -2$ and $x = 5$



2. $y = 2x + 3$

3. $x = 3, y = 4$ and $x = -4, y = -3$

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Squared paper (9 mm)

Chapter

17 Integer Indices

This chapter introduces the laws of indices and develops the meaning of zero and negative indices.

LESSON PLANNING

Objectives

General	To multiply, divide and raise to powers, numbers and algebraic expressions using the laws of indices (integer indices only)
Specific	1. To know the meaning of the terms base and index 2. To know the meaning of zero and negative integer indices 3. To be able to quote and use the laws of indices applied to numbers 4. To be able to quote and use the laws of indices applied to algebraic expressions
Pacing	2 lessons, 1 homework
Links	algebraic products and quotients

Method

Start with defining the base numbers: the word index should already be known.

Next, ensure that the plural of index is known, i.e. indices (pronounced IN-DI-SEAS). The textbook method of finding meaning for zero and negative indices follows, i.e. by repeatedly dividing, extending backwards from the positive case. Another strategy is to demonstrate the truth of the laws in the case of positive integers and then ask about special cases. For example,

$$\frac{a^m}{a^n} = a^{m-n}$$

"What if $m = n$?"

Then LHS = 1 RHS = a^{0}

$\therefore \quad a^{0} = 1$

e.g. "What if $m < n$?"

$$\frac{2^3}{2^5} = \frac{\cancel{2} \times \cancel{2} \times \cancel{2}}{\cancel{2} \times \cancel{2} \times \cancel{2} \times 2 \times 2} = \frac{1}{2^2}$$

From the law, LHS = 2^{-2} RHS = $\frac{1}{2^2}$

$$2^{-2} = \frac{1}{2^2}$$

etc.

In general $a^{-n} = \frac{1}{a^n}$

In fact, many problems can be solved in two ways, illustrating the consistency and reliability of the laws. For example,

$$2^6 \times 2^{-2} = 2^4 \qquad \text{(by adding indices)}$$

or

$$\begin{aligned} 2^6 \times 2^{-2} &= 2^6 \times \frac{1}{2^2} && \text{(using negative index meaning)} \\ &= \frac{2^6}{2^2} \\ &= 2^4 && \text{(by subtracting indices)} \end{aligned}$$

A common error is to forget that if no index is shown then it is an implicit 1. For example,

$$3^6 \times \underset{\substack{\uparrow \\ \text{(implied index of 1 here)}}}{3} = 3^7$$

Play around with the ideas on the board. For example, "What is the inverse of the inverse?":

$$\begin{aligned} \frac{1}{3^{-2}} &= 1 \div 3^{-2} \\ &= 1 \div \frac{1}{3^2} \\ &= 1 \times \frac{3^2}{1} \\ &= 3^2 \end{aligned}$$

In general, $\dfrac{1}{a^{-n}} = a^n$

Set EX 17A.

When the rules are established, move on to algebraic expressions. This should require minimal explanation only, with perhaps a couple of examples to show that numbers go first, followed by letters, preferably in alphabetical order. Use EX 17B.

Assignments	EX 17B, question 8 is a good test of correctly using the laws.
Vocabulary	base, index, indices, integer

ANSWERS

Exercises

EX 17A

1.	a) 9	b) $\frac{1}{9}$	c) $\frac{1}{8}$	d) 1
2.	a) 4^5	b) 3^7	c) 2^2	d) 5^{-2}
3.	a) 6^5	b) 7^3	c) 3^{-1}	d) 2^{-7}
4.	a) 2^{24}	b) 3^{14}	c) 9^{-6}	d) 8^8
5.	a) 4^0	b) 3^0	c) 2^0	d) 5^0

They are all 1.

6.	a) 144	b) 108	c) 24.5 or $\frac{49}{2}$	d) 4
7.	a) $n = 6$	b) $n = 51$	c) $n = -10$	d) $n = -1$
8.	a) $\frac{8}{9}$	b) 72	c) 2	d) $\frac{1}{16}$
9.	a) $n = 20$	b) $n = 7$	c) $n = -8$	d) $n = 6$
10.	a) 3^9	b) 5^0	c) 4^7	d) 7^6

EX 17B

1.	a) ab^2	b) a^4b^{-1}	c) not possible	d) $2ab^3$
2.	a) $12xy$	b) $2xy^{-1}$	c) $84x^{-2}y$	d) 12
3.	a) not possible	b) $3p^2q^3$	c) $3p^{-2}$	d) $5p^{-1}q$
4.	a) $n = 4$	b) $n = 8$	c) $n = 3$	d) $n = 4$
5.	a) x^{10}	b) y^{-3}	c) z^{-5}	d) a^2b^2
6.	a) $n = 7$	b) $n = 5$	c) $n = 4$	d) $n = 4$
7.	a) $x = -\frac{2}{3}$	b) $x = \frac{31}{16}$	c) $x = \frac{12}{25}$	d) $x = 3$
8.	a) true	b) false	c) true	d) true

9. 3^{-4}, 4^{-3}, 2^{-5}, 5^{-2}

10. 60×10^{-2}, 10×6^{-1}, $1 + 6^0$, $60 + 10$

EX 17X

1. a) $x = 7$ or $x = -7$ b) $x = 7$ or $x = -8$

   c) $x = 6$ or $x = -8$ d) $x = 6$ or $x = -9$

2. $a^{\frac{1}{2}} = \sqrt{a}$

3. True for $a < 0$; the laws fail when $a = 0$ and become meaningless.

Chapter

18 Angle Facts

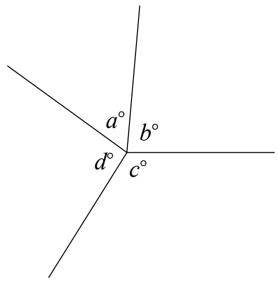
At this stage in geometry we demand much more rigour in communication. The logic is the essence; the answer is secondary. This is the beginning of formal proofs.

LESSON PLANNING

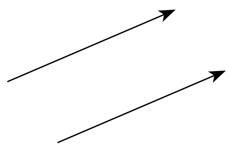
Objectives

General	To solve geometry problems by logical reasoning from basic angle facts, with step-by-step communication of the thought process
Specific	1. To be thoroughly familiar with corresponding, alternate and co-interior angles formed by parallel lines and a transversal 2. To know that angles opposite each other at a vertex are equal 3. To know that the interior angles of a triangle, and adjacent angles on a straight line, are supplementary 4. To know that the interior angles of a quadrilateral add up to 360° 5. To know that the interior angles of a polygon of n sides add up to $180(n-2)$ degrees; and that its exterior angles taken in order add up to 360° 6. To know that the exterior angle of a triangle is equal to the sum of the interior opposite angles 7. To know the meaning of supplementary and complementary angles 8. To know that the angles around a point add up to 360° 9. To know that isosceles triangles have two equal angles 10. To solve simple geometry problems using the above angles facts, stating reasons (in code) for the answers 11. To solve more complex geometry problems, correctly communicating the reason for each stage of the calculation
Pacing	3 lessons, 1 homework
Method	Most of the angle facts ought to be well known. However, now that there are so many facts, students do get confused. It is worth running through all of them.

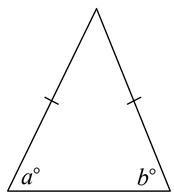
- First, start with the obvious (∠s at a pt)



$a + b + c + d = 360°$

- 

  "What do the arrows mean?"

- 

  "What do the little marks on the sides mean?"
  "Which angles are equal?"

- "What are the polygons called?"

  triangle, quadrilateral, pentagon,

  hexagon, heptagon, octagon,

  decagon, dodecagon, n-gon

Then follow the text, briefly reminding students of the facts (and reason codes). No need to dwell on them: they are there for reference if needed.

Set EX 18A. Emphasize that the reasons are the meat of the exercise.

For the harder problems in EX 18B, no new information is needed, but some guidance about how to turn a strategy into a proof may be required. The text example illustrates how to do it.

Assignments	EX 18A, questions 5 and 6 suitable for homework
Vocabulary	corresponding, alternate, vertically opposite, adjacent, interior, angle-sum, supplementary, complementary, co-interior, proof, reason

ANSWERS

Exercises

EX 18A

1.	a)	(corr ∠s)	b)	(alt ∠s)
	c)	(ext ∠ of Δ)	d)	(adj ∠s on st line)
2.	a)	(vert opp ∠s)	b)	(angle-sum of Δ)
	c)	(alt ∠s)	d)	(ext ∠ of Δ)

3. a) (int $\angle$s) b) (vert opp $\angle$s)

c) (angle-sum of quad) d) (corr $\angle$s)

4. a) (int $\angle$s) b) (adj $\angle$s on st line) c) (ext angle-sum of polygon)

d) (angle-sum of quad)

5. a) hexagon b) $180(n - 2)$ with $n = 6$ (angle-sum of polygon) = 720°

c) 360° (ext angle-sum of polygon) d) (angle-sum of Δ)

6. a) $100 + 50 + 150 + 170 + 80 + 3x + 2x = 180(7 - 2)$

$x = 70$

7. $10x = 360$ (ext angle-sum of polygon)

$x = 36$, and

$y = 180 - 36$ (adj $\angle$s on st line)

$y = 144$

8. $a = 60$ (alt $\angle$s), $b = 62$ (adj $\angle$s on st line)

$c = 58$ (alt $\angle$s) or $c = 58$ (angle-sum of Δ)

9. $a = 180 - 115 = 65$ (int $\angle$s),

$b = 65$ (vert opp $\angle$s),

$c = 65$ (corr $\angle$s, with $\angle b$) or (alt $\angle$s, with $\angle a$)

10. $a = 80$ (int $\angle$s),

$b = 110 - 90 = 20$ (ext $\angle$ of Δ)

$c = 70$ (angle-sum of Δ) or (adj $\angle$s on st line)

[Note: (angle-sum of quad) is also possible, but long.]

EX 18B

1. a) $a = 180 - (80 + 25)$ (angle-sum of Δ)

$a = 75$

$x = 75$ (corr $\angle$s)

b) $b = 25 + 80$ (ext $\angle$ of Δ)

$b = 105$

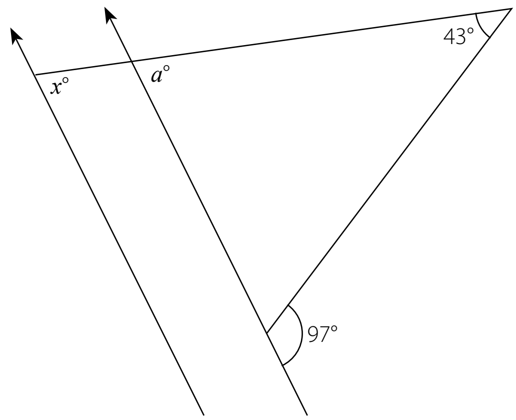
$x = 180 - 105$ (int $\angle$s)

$x = 75$

2. a) $a = 80$ (adj $\angle$s on st line) b) $b = 80$ (corr $\angle$s)

c) $b = 80$ (int $\angle$s) d) $x = 180 - 110 = 70$ (int $\angle$s)

3.

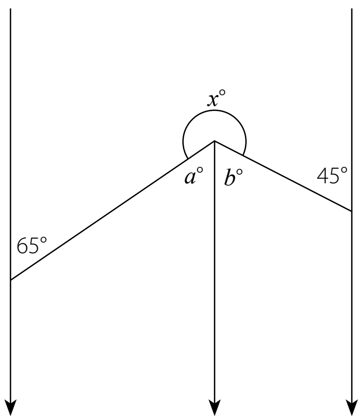


$a = 97 - 43$ (ext $\angle$ of Δ)

$a = 54$

$x = 54$ (corr $\angle$s)

4.



$a = 65$	(alt $\angle$s)
$b = 45$	(alt $\angle$s)
$x = 360 - (65 + 45) = 250$	($\angle$s at a pt)

5. $\angle BAC = 30°$ (adj $\angle$s on st line)

$\angle ACD = 30°$ (alt $\angle$s)

$\angle ADC = 180° - (120° + 30°) = 30°$ (angle-sum of Δ)

$\therefore \angle ACD = \angle ADC$

ΔACD is isosceles (base angles equal)

6. $\angle BAD = 40°$ (adj $\angle$s on st line)

$\angle ADC = 90°$ (given)

$\angle ABC = 100°$ (adj $\angle$s on st line)

$x = 360 - (40 + 90 + 100)$ (angle-sum of quad)

$x = 130$ as required

7. $90 + 2x + x + x + x = 360$ (ext angle-sum of polygon)

$x = 54$; 90°, 108°, 54°, 54°, 54°

8. a) 36 (ext angle-sum of polygon)

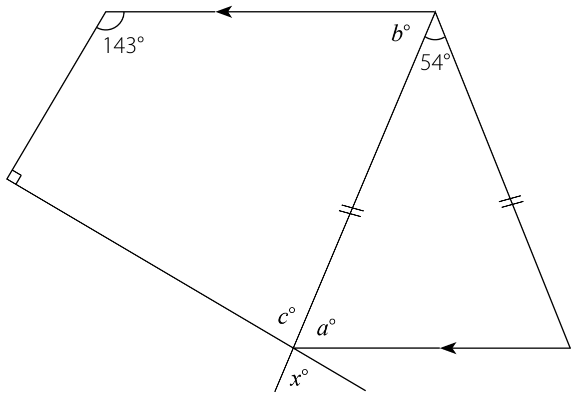
b) 170° (adj $\angle$s on st line); 6120°

c) $180\ (n - 2)$ with $n = 36$

$= 180 \times 34$

$= 6120°$ as in part (b)

9.



$a = 63$ (angle-sum of isos Δ)

$b = 63$ (alt $\angle$s)

$c = 360 - (143 + 63 + 90)$ (angle-sum of quad)

$c = 64$

$x = 64$ (vert opp $\angle$s)

10. $x + x + x = 90$ (given)

$\therefore x = 30$

In ΔADF, $\angle AFD = 60°$ (angle-sum of Δ)

$\angle EDF = 60°$ (alt $\angle$s as BE II AF)

$\angle DEF = 60°$ (isos Δ)

$\therefore y = 180 - (60 + 60)$ (angle-sum of Δ)

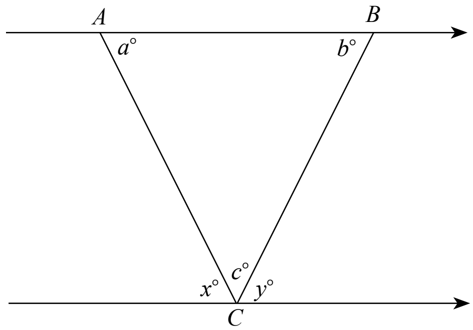
$y = 60°$

$\therefore y = 2x$ as required

[Note: There are at least two other valid methods.]

EX 18X

1.



$a = x$ (alt $\angle$s)

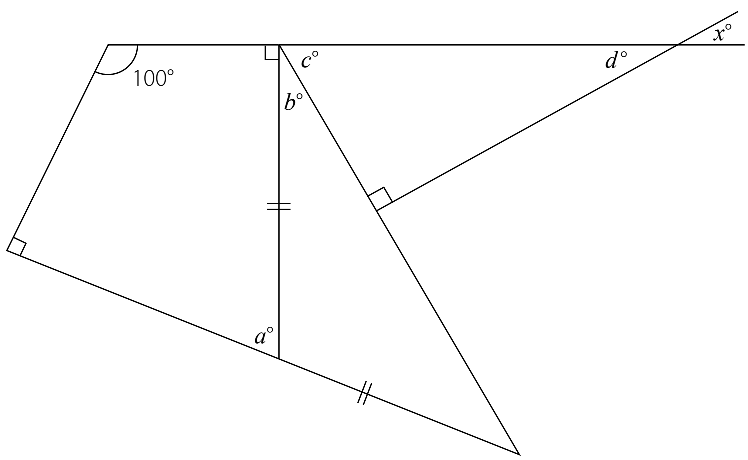
$b = y$ (alt $\angle$s)

$x + c + y = 180°$ (adj $\angle$s on st line)

Substituting from above: $a + c + b = 180$

i.e. angle-sum of Δ is 180° as required.

2.



$a = 80$	(angle-sum of quad)
$b = 40$	(ext $\angle$ of isos Δ)
$c = 50$	(adj $\angle$s on st line)
$d = 40$	(angle-sum of Δ)
$x = 40$	(vert opp $\angle$s)

3. Various methods are valid. Here is a simple one:

In ΔABD, $a + p + q + x = 180$ (angle-sum of Δ)

In ΔACD, $\underline{b + p + q + x = 180}$ (angle-sum of Δ)

Subtract: $a - b = 0$

$a = b$, as required

Chapter

19 Cumulative Frequency

Frequency tables are extended in this chapter to include a cumulative frequency column, and a new kind of graph introduced that can be used to estimate various statistics (for grouped continuous data only).

LESSON PLANNING

Objectives

General	To draw cumulative frequency curves from given data and interpret them
Specific	1. To include a CF column in a frequency table 2. To draw a CF curve from a data table, remembering to plot on top of each interval 3. To use a CF curve to estimate quartiles, medians, and percentiles 4. To use a CF curve to estimate the number of items above or below a given measurement 5. To use CF curves to determine whether or not a set of measurements is generally higher or lower than another 6. To calculate IQRs (inter-quartile range) and use them to compare the spread of two sets of data
Pacing	3 lessons, 1 homework
Method	Give a grouped set of data to view and ask "What is the mean?" "Why can't we know exactly?" "What about the median?" We can estimate it, and some other new statistics. Show how to add a CF column to a frequency table and how to plot a CF curve (on top of each interval). Follow the text examples. Then explain the meaning of quartiles, and the standard use of Q_1, Q_2 and Q_3, and how to find their positions. Use EX 19A, questions 1–3 orally, with the whole class, as examples of how to interpret a CF curve correctly. Then set the rest of EX 19A, questions 4–10. Percentiles are just a more accurate division of the data into 100 parts, appropriate for large quantities of data. Connect with the quartiles.

	Introduce IQR = $Q_3 - Q_1$ as a better measure of the spread of the data than range. Show how CF curves can be used to compare two sets of data in general. Set EX 19B.
Resources	Squared paper (9 mm squares given on page 76, are accurate enough for these exercises.)
Assignments	EX 19A, questions 9 and 10 suitable for homework
Vocabulary	grouped data, cumulative frequency, interval median, quartile, percentile distribution, spread, range, inter-quartile range

ANSWERS

Exercises

EX 19A

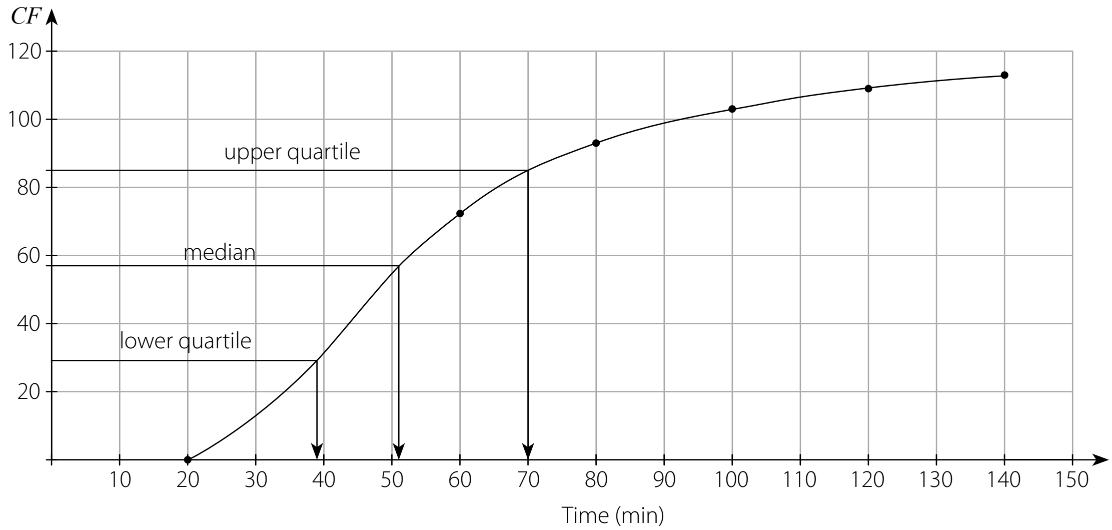
1. a) 120 b) 3.55 kg c) 3.3 kg d) 3.75 kg
2. a) 50 b) 17 c) 15 d) 93
3. a) 3.5 kg b) 4.1 kg c) 30th from the bottom
   d) 118th from the bottom or 3rd from the top
4. a) Q_1 between 29th and 30th
   Q_2 59th
   Q_3 between 88th and 89th
   b) Below Q_1: 1st to 28th = 29 babies
   Between Q_1 and Q_2: 29th to 58th = 29 babies
   Q_2 median: 59th only = 1 baby
   Between Q_2 and Q_3: 60th to 88th = 29 babies
   Over Q_3: 89th to 117th = 29 babies
   i.e. 4 groups of 29 with the median in the middle
5. Q_1: 32nd item
   Q_2: Between 63rd and 64th items
   Q_3: 95th item

Below Q_1	Q_1 to Q_2	Q_2 to Q_3	Q_3 to Q_4
1st to 31st	33rd to 63rd	64th to 94th	96th to 126th
31 items	31 items	31 items	31 items

   $4 \times 31 = 124$
   $124 + Q_1 + Q_3 = 126$ items

6. a)

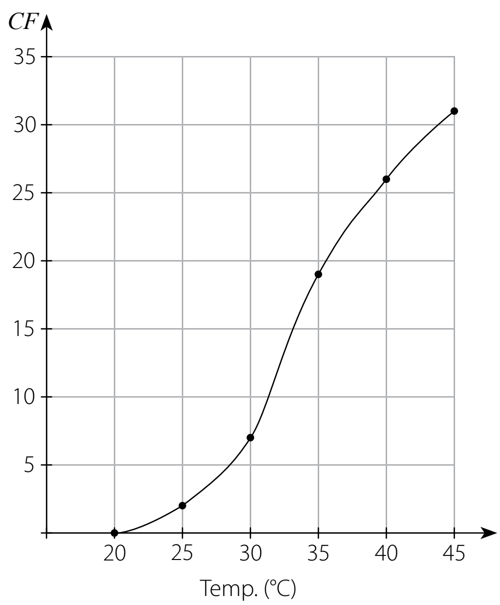
Time (min)	f	CF
20 – 40	30	30
40 – 60	42	72
60 – 80	21	93
80 – 100	10	103
100 – 120	6	109
120 – 140	4	113



b) (i) 51 min (ii) 39 min, 70 min (iii) 45 vehicles (iv) 15 vehicles

7.

Max. Temp. (°C)	f	CF
20 – 25	2	2
25 – 30	5	7
30 – 35	12	19
35 – 40	7	26
40 – 45	5	31



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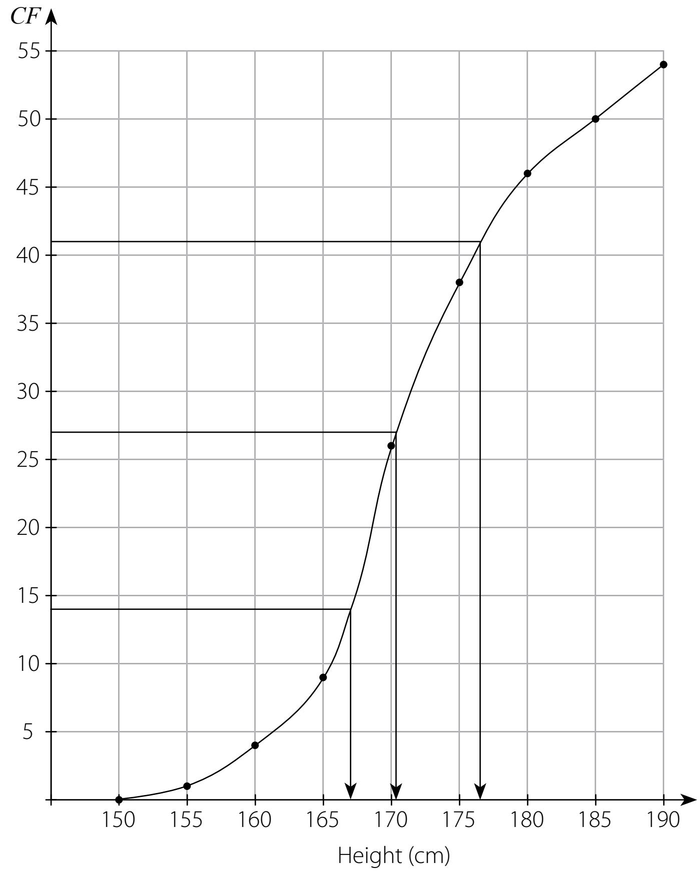
8. a) 34 °C b) 38 °C, 31 °C c) 15 days d) 20 days

9. CF column is 1 4 9 26 38 46 50 54



10. a) 171 cm b) 24 girls c) 176 cm, 167 cm d) 7th

EX 19B

1. a) false b) false c) true d) false

2. a) 40% b) 60% c) 8000 d) 11 999

3. a) 115 cm b) 17 cm

   c) 3500 children d) 7000 children

4. a) In general, the boys' weights are slightly lower than the girls'.

   b) IQR boys = 10 kg, IQR girls = 18 kg

   The weights of the girls are more spread out than those of the boys.

5. a) The girls' results are generally a little better.

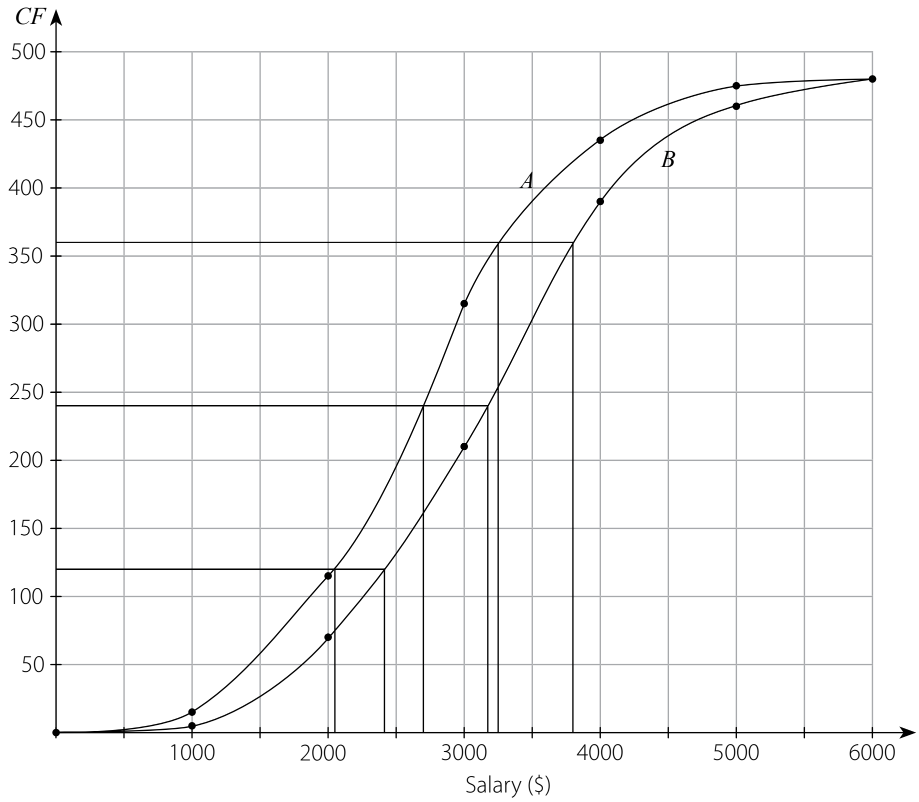
   b) IQR boys = 61%, IQR girls = 52%

   The boys' results have a greater spread than the girls'.

6. a)

Salary $ (equivalent)	Azilia		Bentina	
	f	CF	f	CF
0 – 1000	15	15	5	5
1000 – 2000	100	115	65	70
2000 – 3000	200	315	140	210
3000 – 4000	120	435	180	390
4000 – 5000	40	475	70	460
5000 – 6000	5	480	20	480

b)



c) In general, the salaries in Bentina are higher (median is higher).

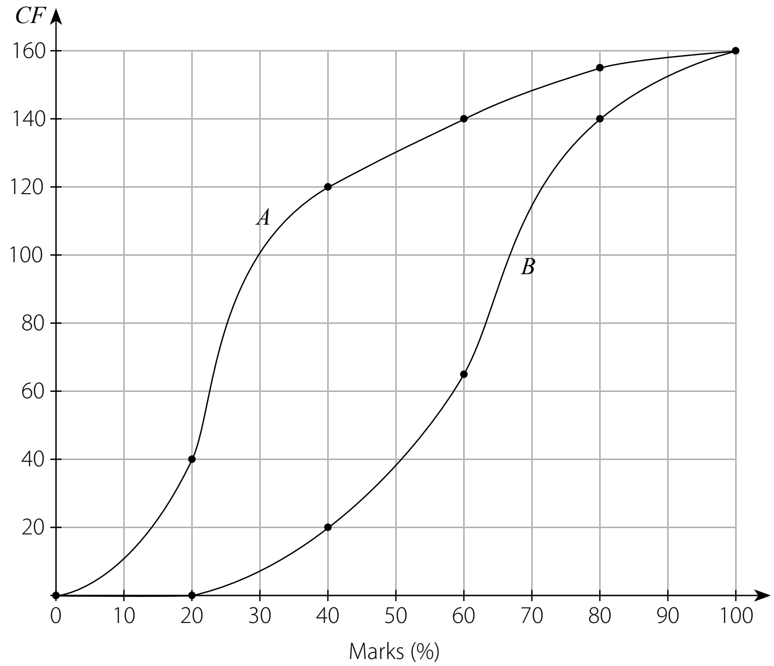
d) IQR Azilia = $ 1200, IQR Bentina = $ 1400

The salaries in Bentina are more spread out than those in Azilia.

7. a)

Marks %	Test A		Test B	
	f	CF	f	CF
0 – 20	40	40	0	0
20 – 40	80	120	20	20
40 – 60	20	140	45	65
60 – 80	15	155	75	140
80 – 100	5	160	20	160

b)



c) median A < median B

In general, Test B results were higher than Test A results.

IQR (A) = 20, IQR (B) = 20

The spread of results in both tests is about the same.

8. a) \$ 2250 b) \$ 3900

c) 79th percentile d) 67th percentile

9. a) 18% marks b) 68% marks

c) 132nd percentile d) 40th percentile

10. a) 2 s b) reduces by 1 s c) increases by 2 s

d) The spread of timings is increased by training. The answer to (c) is more significant as the range (b) is affected only by the two extreme cases.

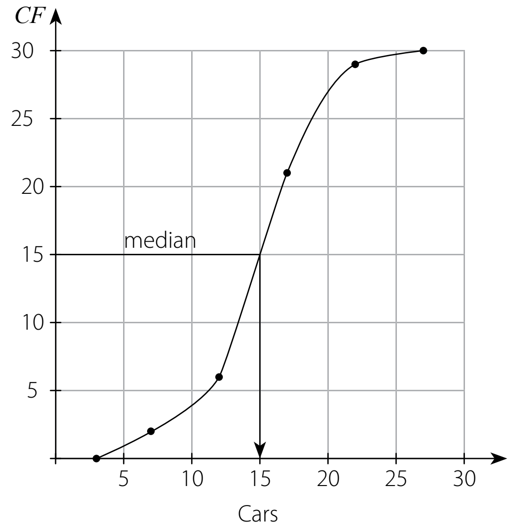
EX 19X

1. a) 54 b) 170 – 175 cm c) $\frac{1}{17}$

d) $\frac{1}{17}$ of 5cm = 0.3 cm, median = 170.3; graph answer = 171 cm (close estimate)

2. a) extend by 0.5

b)



c) 15 cars

3. a) $Q_1 = 8.25$, $Q_2 = 9.75$, $Q_3 = 10.75$

b) Q_1 is between 3rd and 4th item.

Q_2 is the 7th item.

Q_3 is between the 10th and 11th item.

c) Q_1 is the 3rd item.

Q_2 is between the 5th and 6th item.

Q_3 is the 13th item.

d) Not possible. Percentiles are meaningless for small quantities of data.

Chapter

20 Standard Form

This chapter teaches the benefits of a universal scientific notation for numbers.

LESSON PLANNING

Objectives

General	To use standard form correctly and understand the benefits of using it
Specific	1. To know that different countries have their own terms and styles for naming and writing numbers (especially very large or small) 2. To be able to write a number in standard form; to write in full a number given in standard form 3. To appreciate the accuracy and clarity of numbers expressed in standard form 4. To enter numbers in standard form into a calculator and operate on them 5. To calculate with numbers in standard form without using a calculator
Pacing	2 lessons, 1 homework
Links	laws of indices, substitution in algebraic expressions

Method

Collect together information about the methods of writing numbers, e.g. money, in different countries:

€ 2,76   £ 6.50 $ 7.$^{\underline{99}}$ Rs 4,70,000/=

Also, discuss the names used for large and small numbers, such as in the examples in the text, and maybe some others.

Scientists' work has to be read internationally, so a scientific agreement on how to write numbers has been made and is called standard form.

The decimal point can be written using either a comma or dot, so we must not use commas or dots elsewhere. We will use the decimal dot.

Define standard form as in the text.

Give examples. Show how to count right or left for large and small numbers to obtain the correct index number.

Set EX 20A, which is designed to test changing into and out of standard form, and develop appreciation of the usefulness of standard form. (It avoids numerous zeros.) EX 20A, question 10 demonstrates how the power of 10 indicates the magnitude of a number very clearly.

Show how easy it is to round off in standard form. Calculations can be more difficult. If the text method (using the distributive law) is too hard to grasp, students can always just write out the numbers in full instead.

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	Most calculators will allow standard form entry, and this may be explored. Set EX 20B, allowing calculators for this exercise.
Resources	Calculators essential for EX 20B
Assignments	EX 20B, questions 8 and 9 suitable for homework
Vocabulary	standard form, lakhs, crores

ANSWERS

Exercises

EX 20A

1. a) 7×10^4 b) 6.25×10^3
   c) 1.7×10^3 d) 4.4×10^2
2. a) 260 b) 3 850 000
   c) 507 000 000 d) 6 600 000 000
3. a) $2\,000\,000 = 2 \times 10^6$ b) $6\,000\,000\,000 = 6 \times 10^9$
   c) $500\,000 = 5 \times 10^5$ d) $720\,000\,000 = 7.2 \times 10^8$
4. a) 5×10^{-6} b) 3.6×10^{-8}
   c) 7.5×10^{-2} d) 8.15×10^{-3}
5. a) 0.045 b) 0.008 87
   c) 0.000 058 d) 0.000 000 009 91
6. a) 8.39×10^3 b) 9.71×10^4
   c) 6.137×10^{-3} d) 5.104×10^{-4}
7. a) 5×10^4 b) 1.1×10^5
   c) 2×10^5 d) 5.98×10^{24}
8. a) \$ 300 000 000 b) 31 536 000 s
   c) 8 000 000 words d) 100 000 000 000 000 cells
9. a) 0.000 000 000 025 m b) 0.000 000 003 s
   c) 0.000 000 000 753 kg d) 0.000 000 4 m
10. a) 5×10^{-2}, 3×10^2, 9×10^3
    b) 2.11×10^{-1}, 2.1×10^3, 1.2×10^4
    c) 7.45×10^{-5}, 4.57×10^{-4}, 5.47×10^{-3}
    d) 7×10^{-1}, 5×10, 1×10^5

EX 20B

1. a) 3.4×10^7 b) 4×10^3

   c) 6×10^{-9} d) 6.1×10^{-2}

2. a) 4.3×10^7 b) 7.4×10^{17}

   c) 6.7×10^{-3} d) 1.4×10^{-8}

3. a) 400 b) 0.000 002 c) 40 000 d) 0.01

4. a) 5.6×10^3 b) 2.5×10^{-4}

   c) 6.3×10 d) 4.5×10^6

5. a) 8.8×10^7 b) 3.6×10^{-4}

   c) 8.3×10 d) 1.3×10^{14}

6. a) 1.0×10^{18} b) 3.8×10^6

   c) 6.72×10^{-3} d) 9.2×10^{-3}

7. a) 2.9×10^{-2} b) 1.8×10^2

   c) 2×10^{-7} d) 1.2×10^7

8. a) 2.1×10^4 b) 2.4×10^7 c) 3.9×10^4 d) 4.9

9. a) 1.2×10^{-2} b) 2.7×10^{-1} c) 3.7×10^{-4} d) 9×10^{-4}

10. a) 1.5×10^6 b) 1×10^3

    (c) 7.5 d) 2.5×10^{-3}

EX 20X

1. a) A googol is 10^{100} or '1' followed by one hundred zeros. A googolplex is 10^{googol} or '1' followed by a googol of zeros.

   b) Google® is a play on the word 'googol', reflecting its mission to organise a seemingly infinite amount of information on the web, and to make it universally accessible and useful. The headquarters of the Google company, called the Googleplex, is in California, USA.

2. Quadrillion, quintillion, centillion, etc. These are, however, not the same in the USA and Europe. Standard form cannot be mistaken. Zillion and squillions are not actual numbers but informal language for a large, unspecified number.

3. a) $3{,}74 \times 10^3$ b) $2{,}5 \times 10^6$

   c) $1{,}5 \times 10^{-3}$ d) $2{,}771 \times 10^{-5}$

Chapter

21 Speed, Distance, and Time

In Book 8, Chapter 11, we did speed, distance, and time calculations. In this chapter we use distance–time and speed–time graphs.

LESSON PLANNING

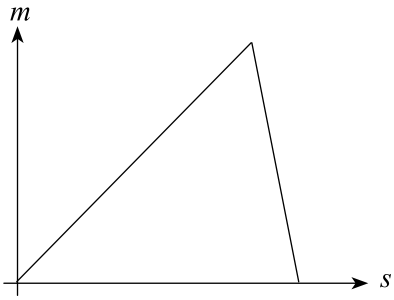
Objectives

General	To draw distance–time and speed–time graphs from given data; to interpret such graphs
Specific	1. To draw a distance–time graph from given data 2. To know that the gradient of a distance–time graph represents speed; a horizontal line represents resting 3. To know that a line sloping downwards on a distance–time graph represents movement towards the starting position 4. To read positions and times from a distance–time graph, including cases of intersecting journeys 5. To draw a speed–time graph from given data 6. To know that the gradient of a speed–time graph represents acceleration; a horizontal line represents constant speed 7. To know that a line sloping downwards on a speed–time graph represents slowing down, i.e. negative acceleration (deceleration) 8. To know that a speed–time graph tells us nothing directly about the position of an object. However, the area under the graph represents the distance it has travelled 9. To know the conversion factor to change km/h to m/s and vice-versa 10. To know that units have to be compatible when performing calculations
Pacing	5 lessons, 2 homeworks
Links	units
Method	The text starts with reminders from Book 8. Go through these. The only possible difficulty might be in manipulating decimal time on a calculator. Set EX 21A, questions 1–3 to test previous knowledge, and deal with any problems.

Explain the distance–time graph. Points to emphasize:

- Time always goes across the horizontal axis.
- The vertical axis gives the distance from the starting position, not distance travelled.

  This explains downward sloping lines, i.e. return journeys.



- Gradient indicates speed, i.e. how fast is the position changing

Talk through the two textbook examples.

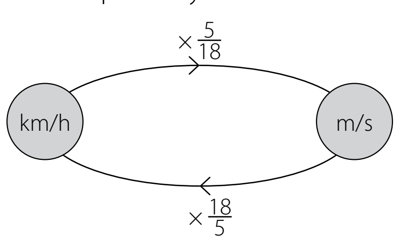
Set the rest of EX 21A, questions 4–10.

Explain how two graphs may be drawn between the same axes to obtain solutions to intersection problems. Use the text example.

Set EX 21B, questions 1–3.

Explain the features of speed-time graphs:

- The gradient represents acceleration. Units of acceleration are m/s^2 or km/h^2.
- Interpret positive, zero, and negative gradients (accelerations) as increasing speed, constant speed, and slowing down respectively
- Area (length × width) under the graph is speed × time = distance travelled
- Compatibility of units is essential. Aim to memorize speed conversion factors:



Set the rest of EX 21B, questions 4–10

Resources	Calculators essential; squared paper (9 mm)
Assignments	EX 21A, question 10; EX 21B, questions 7 and 8 suitable for homework
Vocabulary	gradient, average speed, constant (uniform) speed, acceleration

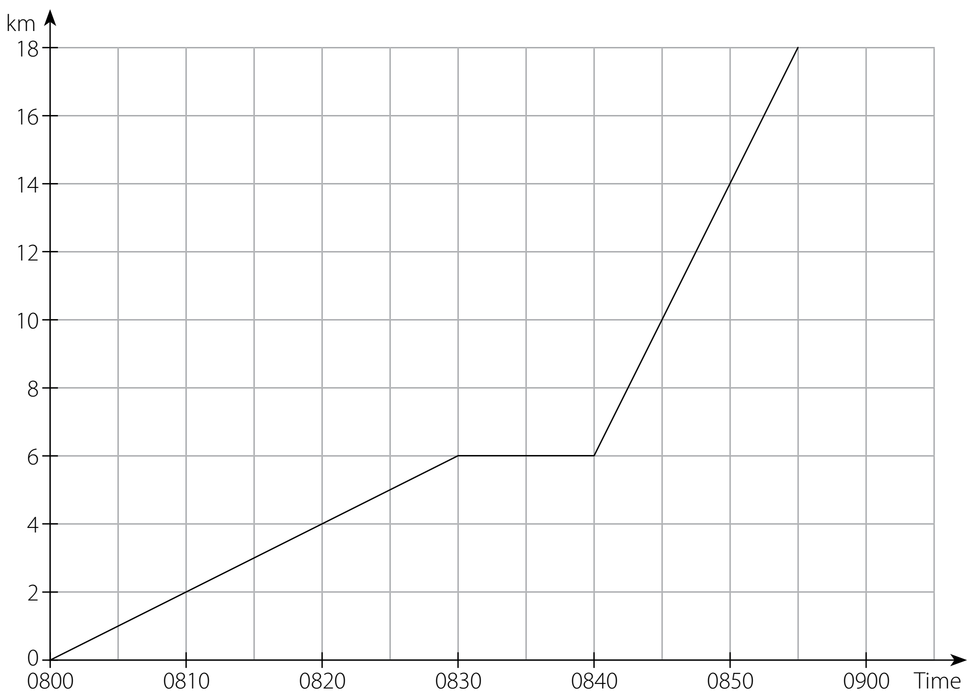
ANSWERS

Exercises

EX 21A

1.	a) 5.6 km/h	b) 11 km/h		
	c) 12 km/h	d) 600 km/h		
2.	a) 100 m	b) 420 m		
	c) 861 m (3 s.f.)	d) 233 km (3 s.f.)		
3.	a) 2 h 40 min	b) 7 min 23 s	c) 1 h 55 min	d) 28 min
4.	in heavy traffic:			
	a) S	b) R	c) Q	d) P
	On the open road:			
	a) S	b) R	c) P	d) Q
5.	a) 0655	b) 16 km, 15 min	c) before (steeper gradient)	
	d) 22 km/h			
6.	a) 16 km	b) 20 min		
	c) 48; 64 km/h	d) 34.9 km/h (3 s.f.)		

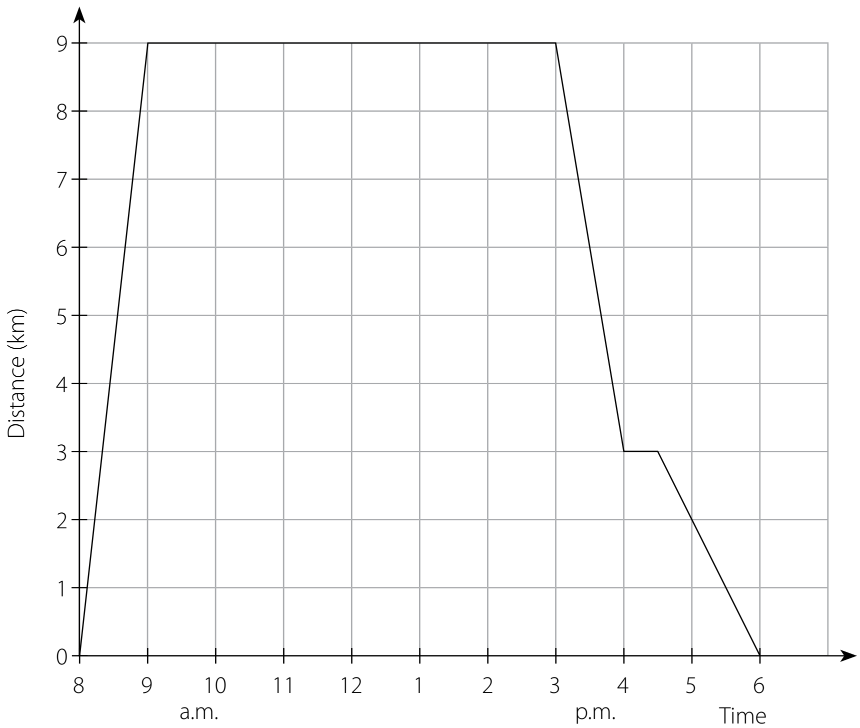
7. a)
b)



c) 48 km/h   d) 19.6 km/h (3 s.f.)

8. a)

b)



c) at the pharmacy

d) 2 km/h

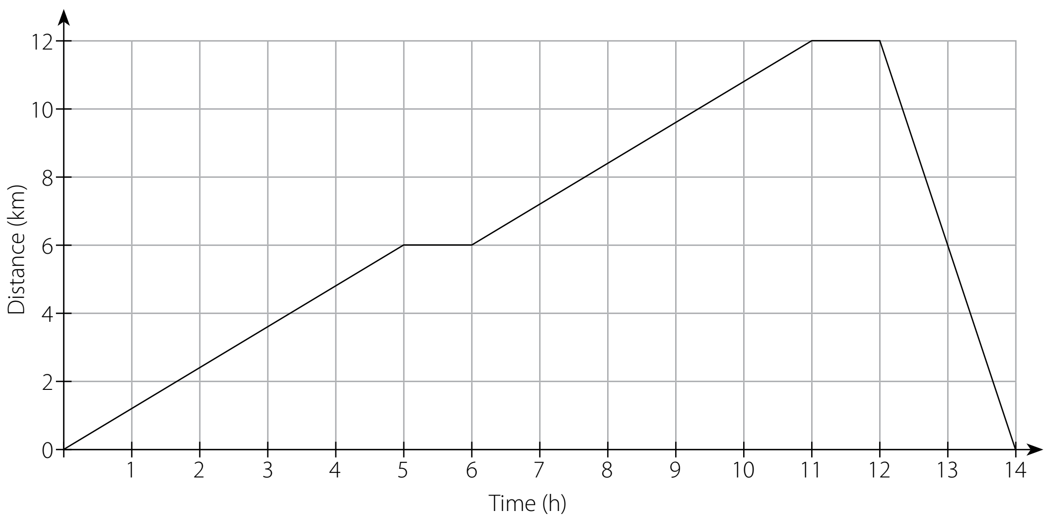
9. a) 12 km

b) 1.2 km/h

c) 3 km/h

d) 1.71 km/h

10. a)



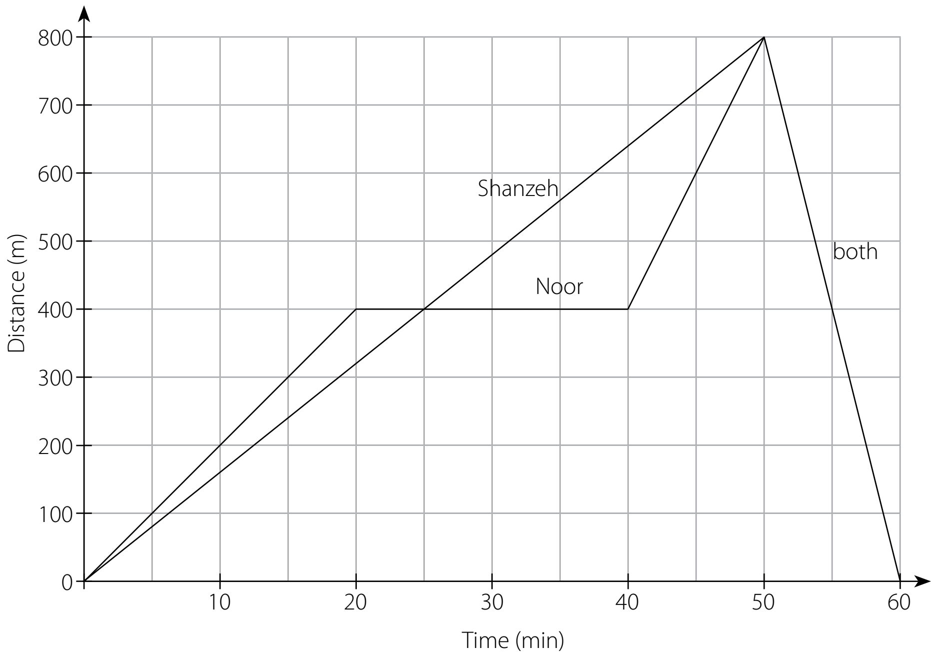
b) 9.6 km from the start (approx)

c) $9\frac{1}{4}$ h and $12\frac{1}{4}$ h from the start (approx)

d) 1.71 km/h (3 s.f.); same as the previous climber's average

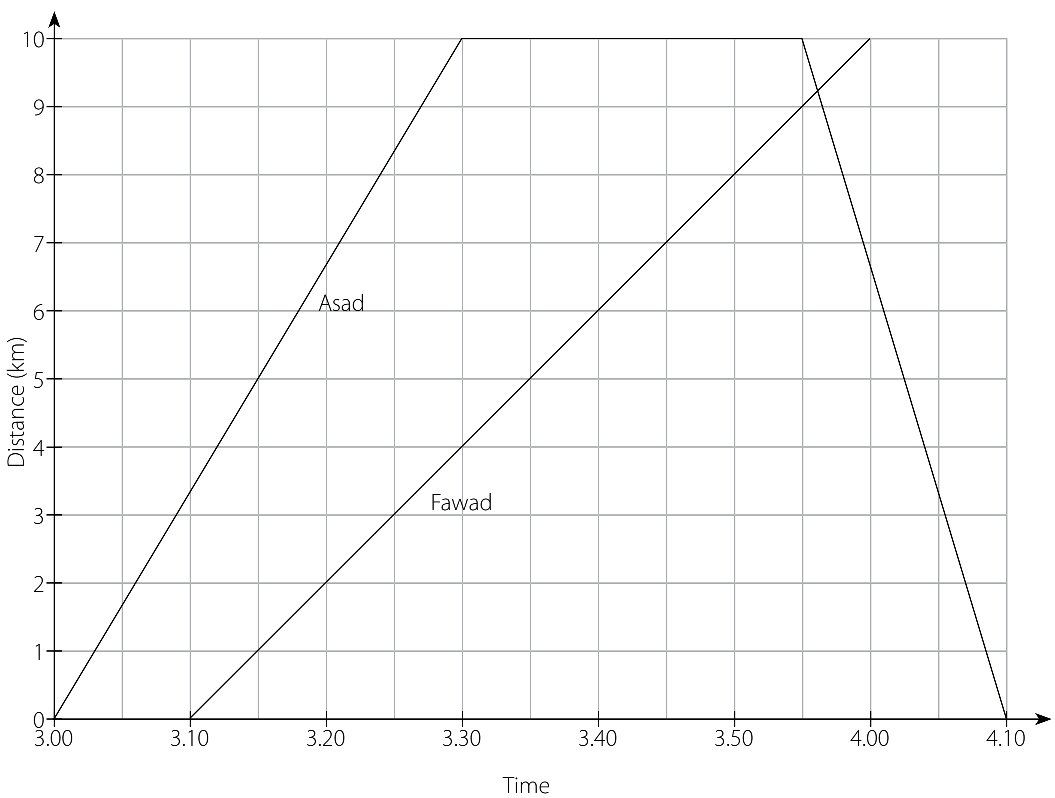
EX 21B

1. a)
b)
c)



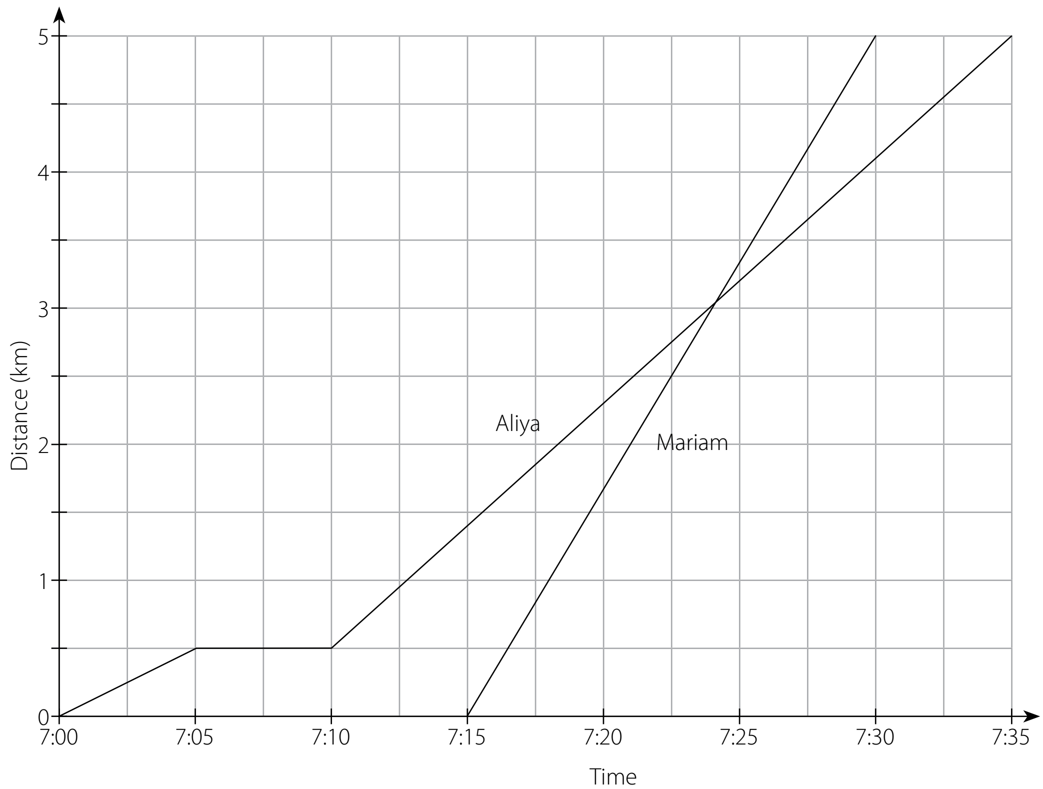
d) 25 minutes after leaving home

2. a)
b)
c)



d) 3:56 p.m. (approx)

3.



Overtakes at 7:24 a.m. (approx)

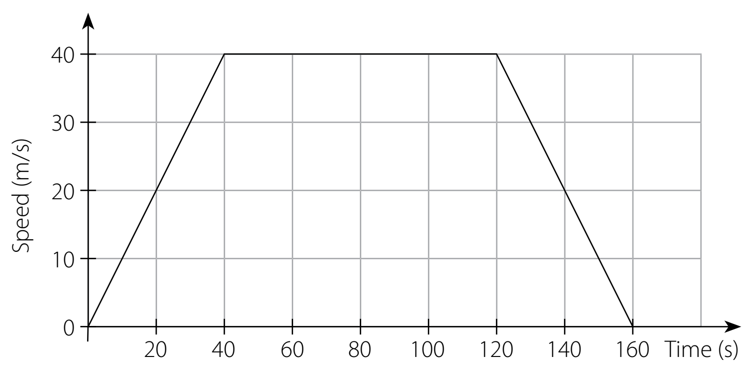
4. a) 840 s b) 13.68 km

c) 0.1 m/s^2 d) –0.025 m/s^2

5. a) constant speed of 30 km/h

b) 90 km/h^2 c) 5 km d) 62.5 km

6. a)



b) –1 m/s^2

c) 4.8 km

d) 144 km/h

7. a) 3 b) 6 min

c) 50 km/h d) 0.166 m/s^2 (3 s.f.)

8. a) −0.0772 m/s² (3 s.f.) b) 6.25 km

c) 8.75 km d) 8.62 m/s (3 s.f.)

9. a) 10:45 b) 11:15 c) 1 h d) 9 km

10. 12:29 (approx)

EX 21X

1. a) $a = \frac{v - u}{t} => v = u + at$

b) distance $= \frac{1}{2}(u + v)\,t$

2. O to t_1 first gear, rapid acceleration for short time

t_1 to t_2 second gear, longer period, acceleration less

t_2 to t_3 third gear, still longer period, acceleration again less

after t_3 fourth gear, constant speed, no acceleration

3. a) height increasing, speed decreasing to zero

b) maximum height, speed zero

c) height reducing, speed increasing (to original)

d) depends on whether the starting position is at ground level or higher (e.g. from the top of a building).

Chapter

22 Probability

This chapter makes more thought-provoking demands in the exercises and uses more formal language than before, although all the theory has been covered in Book 8, Chapters 7 and 19. Conditional probability is not mentioned here, although it is used implicitly in EX 22X. The subtleties of this are best left to a later stage. In practice, students have little difficulty multiplying probabilities across tree diagrams.

LESSON PLANNING

Objectives

General	To apply the multiplicative role for calculating the probability of the occurrence of two independent events
Specific	1. To list outcomes systematically in a possibility diagram (table) 2. To list outcomes systematically on a tree diagram 3. To solve probability problems involving two independent events using the above techniques
Pacing	1 lesson, 1 homework
Links	fractions, symmetry
Method	This is an easy chapter as all the basic facts have been covered before. With a good class the exercise may be set immediately. However, it may be necessary first to run through the Reminder list and deal with any queries raised.
Assignments	EX 22A, question 7 is a good homework question.
Vocabulary	systematically, tree diagram, fair, independent, events, outcomes, random

ANSWERS

Exercises

EX 22A

1. a)

	1	2	3	4	5
A	A1	A2	A3	A4	A5
B	B1	B2	B3	B4	B5
C	C1	C2	C3	C4	C5
D	D1	D2	D3	D4	D5
E	E1	E2	E3	E4	E5
F	F1	F2	F3	F4	F5

b) (i) $\frac{1}{30}$

(ii) $\frac{1}{6}$

(iii) $\frac{4}{5}$

(iv) 0

2. a)

		hearts		
		a	k	q
spades	A	Aa	Ak	Aq
	K	Ka	Kk	Kq
	Q	Qa	Qk	Qq
	J	Ja	Jk	Jq

b) (i) $\frac{1}{12}$

(ii) $\frac{1}{4}$

(iii) $\frac{1}{2}$

(iv) 0

3. a)

		blue					
		1	2	3	4	5	6
red	1	11	12	13	14	15	16
	2	21	22	23	24	25	26
	3	31	32	33	34	35	36
	4	41	42	43	44	45	46
	5	51	52	53	54	55	56
	6	61	62	63	64	65	66

b) (i) $\frac{1}{36}$

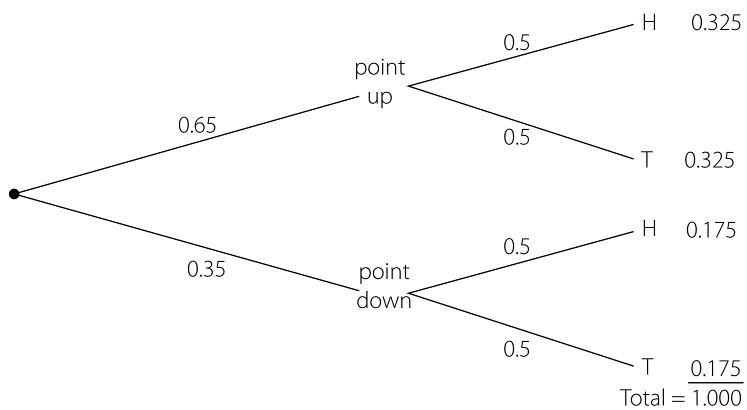
(ii) 0

(iii) $\frac{1}{2}$

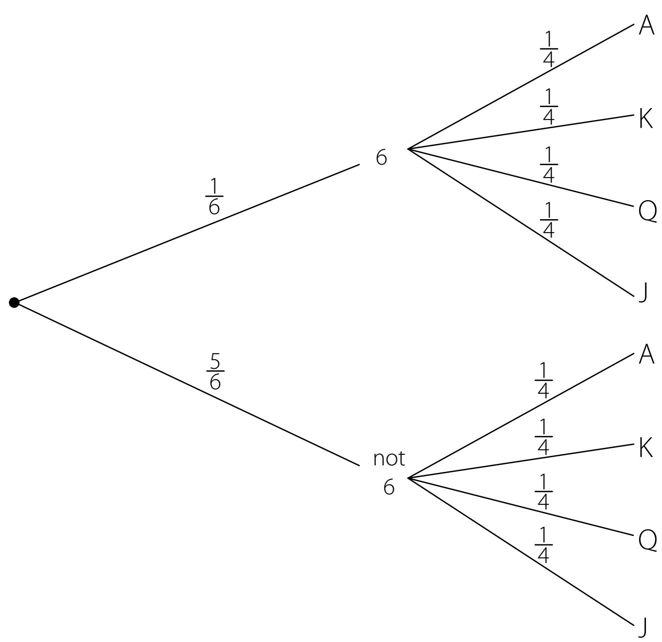
(iv) $\frac{25}{36}$

4. a) 0.65 b) yes, 0.175 c) 0.325 d) 0.175

5.

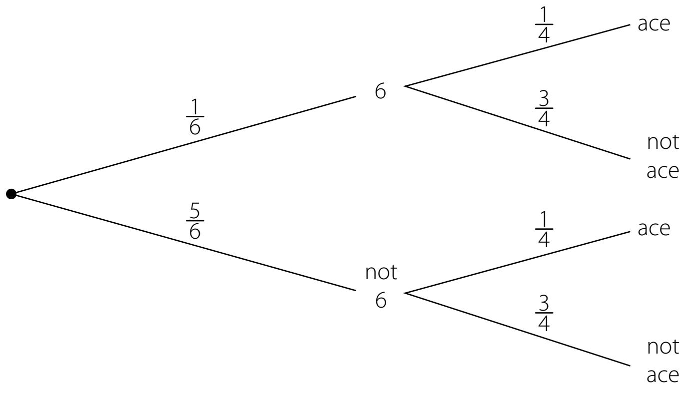


6. a)



b) (i) $\frac{1}{24}$ (ii) $\frac{5}{8}$ (iii) $\frac{1}{2}$ (iv) $\frac{1}{12}$

7. a)

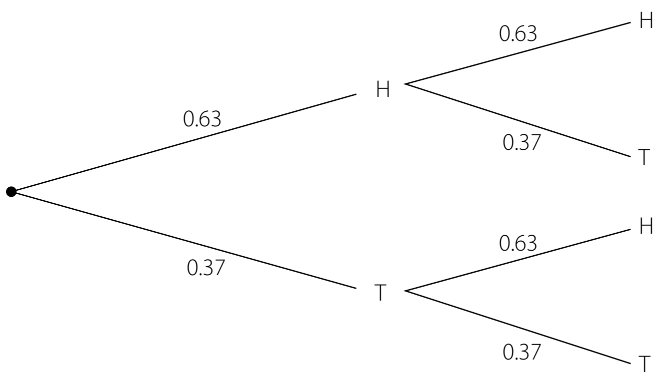


b) (i) $\frac{1}{8}$ (ii) $\frac{5}{24}$ (iii) $\frac{5}{8}$

c) $\frac{1}{24}+\frac{1}{8}+\frac{5}{24}+\frac{5}{8}=1$ d) $\frac{1}{12}$

8. a) 0.37

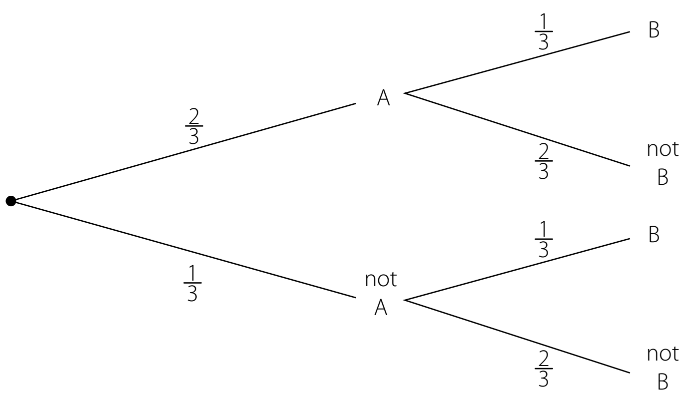
(b)



c) (i) 0.3969 (ii) 0.2331 (iii) 0.2331 (iv) 0.1369

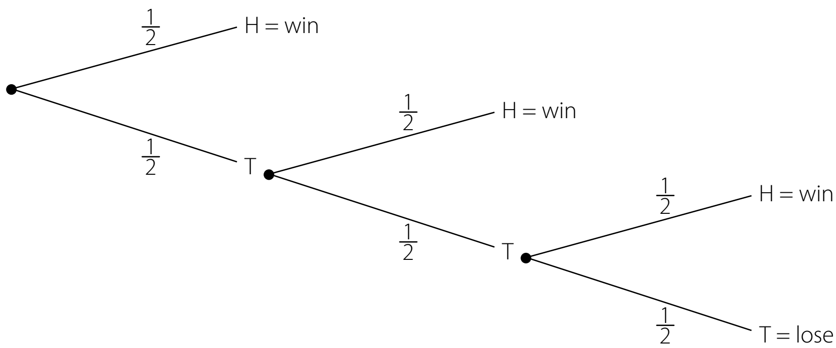
d) $0.3969 + 0.2331 + 0.2331 + 0.1369 = 1$

9.



$P(A \text{ and } B) = \frac{2}{9}$

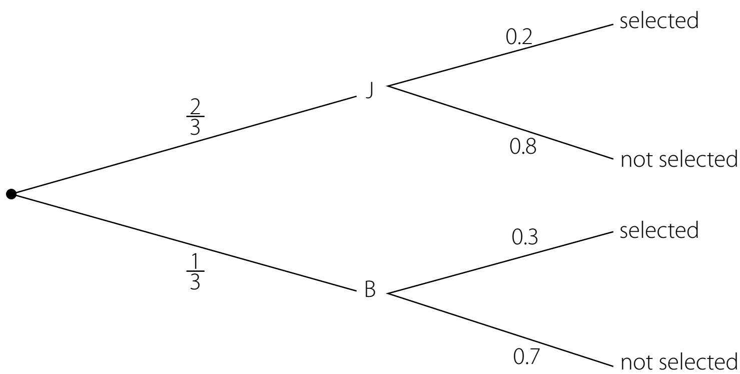
10.



(b) $\frac{7}{8}$

EX 22X

1.



P(selected) = $0.2\dot{3}$

2. $\frac{1}{17325}$

3. $\frac{1}{10000} \times \frac{1}{10000} = 0.000\,000\,01$ Assumes independence.

In practice, the two cars of the same model would likely be from the same batch/factory, so cannot be assumed to have been selected independently of each other.

Chapter

23 Revision Exercises

ANSWERS

Exercises

EX 23A

1. a) 8.49 cm^2 b) 20.6 cm^2

2. $a = 46°$ (int ∠s) $b = 46°$ (vert opp ∠s)

   $c = 46°$ (corr ∠s) with b, or (alt ∠s) with a

3. a) 2.006 b) 0.8192 c) 0.6101 d) 0.2679

4. \$ 1.74 (nearest cent); Each increase is applied to the previous year's price, not the original price.

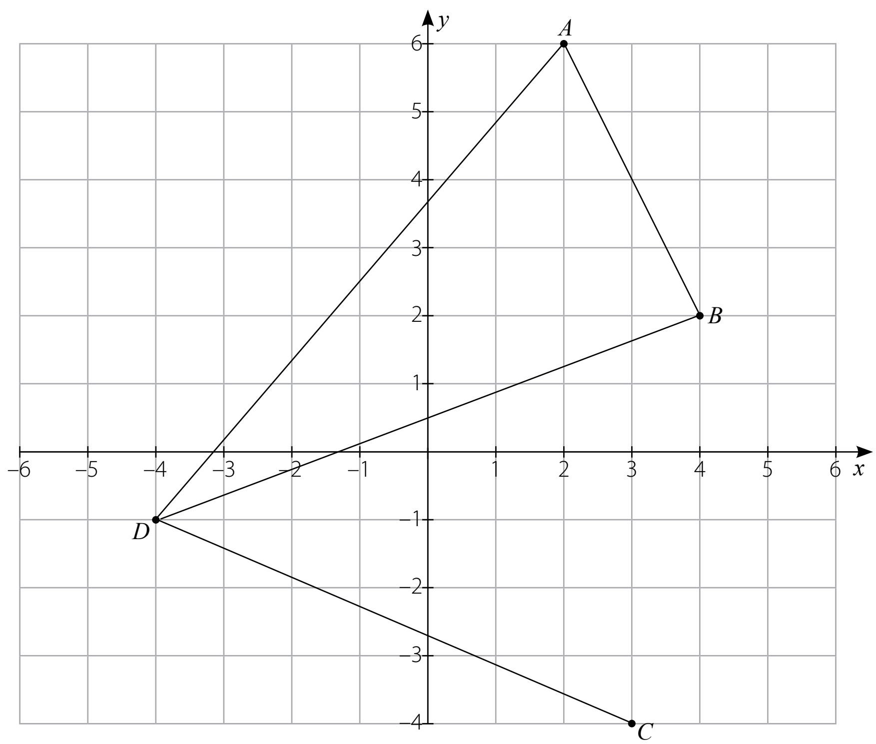
5. a) 132 cm^2; 50.5 cm (b) 25.8 cm^2; 23.0 cm

6. 4^{-4}, 3^{-5}, 5^{-3}, 6^{-2}

7. a) −15.8 b) −7

8. a) true b) false c) false d) true

9.



a) 4.5

b) 9.2

c) 8.5

d) 7.6

10. a) $n = 25$ b) $n = 9$ c) $n = -10$ d) $n = 7$

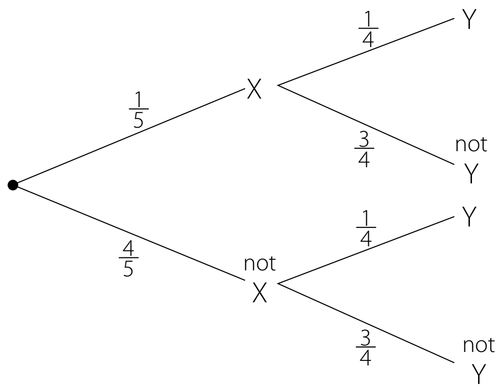
EX 23B

1. 780 km

2. $ 3.60

3. a) 41 m b) 106 m

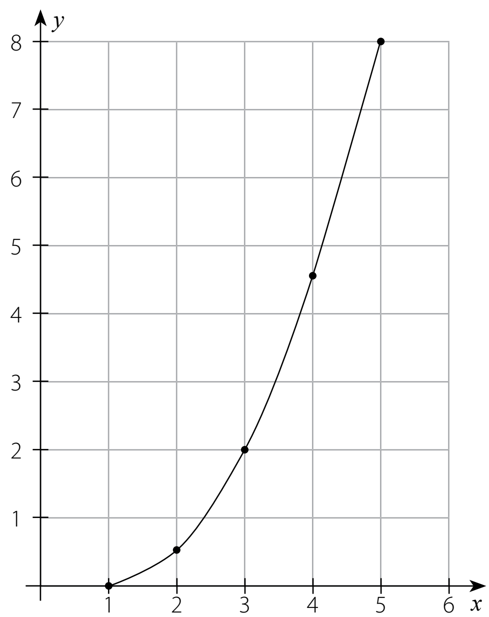
4.



P(X and Y) $= \frac{1}{20}$

5. a) (1, 0), (2, 0.5), (3, 2), (4, 4.5), (5, 8)

b)



c) accelerating upwards; curve is a parabola

d) Speed is rather slow; replacing t by $2t$ would be more realistic for a rocket. $\left(2t + 1, \frac{(2t)^2}{2}\right)$

6. a) 38.874 cm b) 51.6 cm

7. 8482 cm^2

8. a) 13:30 b) 10:45 c) $1\frac{1}{2}$ h d) 0.6 km/h

9, 59.1°

10. a) $x = 1$ or $x = 3$ b) $x = 1.25$ or 3.75 (approx)

c) $x = 2$ d) When $y = 1$

$$1 = 1 - (x - 2)^2$$
$$(x - 2)^2 = 0$$
$$x - 2 = 0$$
$$x = 2$$

EX 23C

1. 98.0 cm^2

2. a) 1200 m b) 3 min c) 8 m/s d) 13.3 m/s

3. a) 75 b) 6

4. a) $V_{\text{sphere}} = \frac{4}{3}\pi r^3$; $V_{\text{cylinder}} = \pi r^2 h$

b) $V = \frac{2}{3}\pi r^3 + \pi r^2 h = \pi r^2\left(\frac{2}{3}r + h\right)$ c) $h = \frac{3V}{\pi r^2} - \frac{2r}{3}$

d) (i) 15.8 cm (ii) 92.2 cm

5. $n = 1$, common answer = −2

6. a) 7.5×10^6 b) 1.2×10^4 c) 3.004×10^3 d) 1.6×10^{-3}

7. Altaf 42, Bismah 14, Kaleem 7

8. a) $\frac{8}{7}$ b) $-\frac{1}{2}$ c) $\frac{2}{9}$ d) −5

9. estimated 300; accurately 333, leftover 3 pieces of 2 cm length

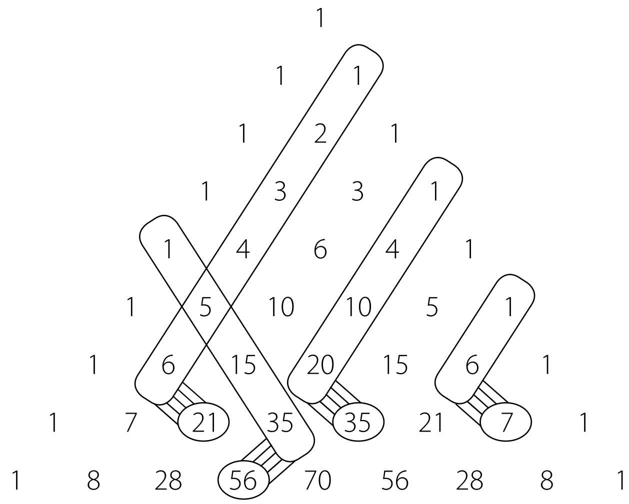
10. a) 42, 28 marks b) 30, 18 students c) 10, 15 marks

d) IX A is generally better than IX B as every percentile is higher. IX B's marks are more spread out than IX A's marks as IQR is higher.

EX 23D

1. a) 180 cm^2 b) 58.2 cm^2

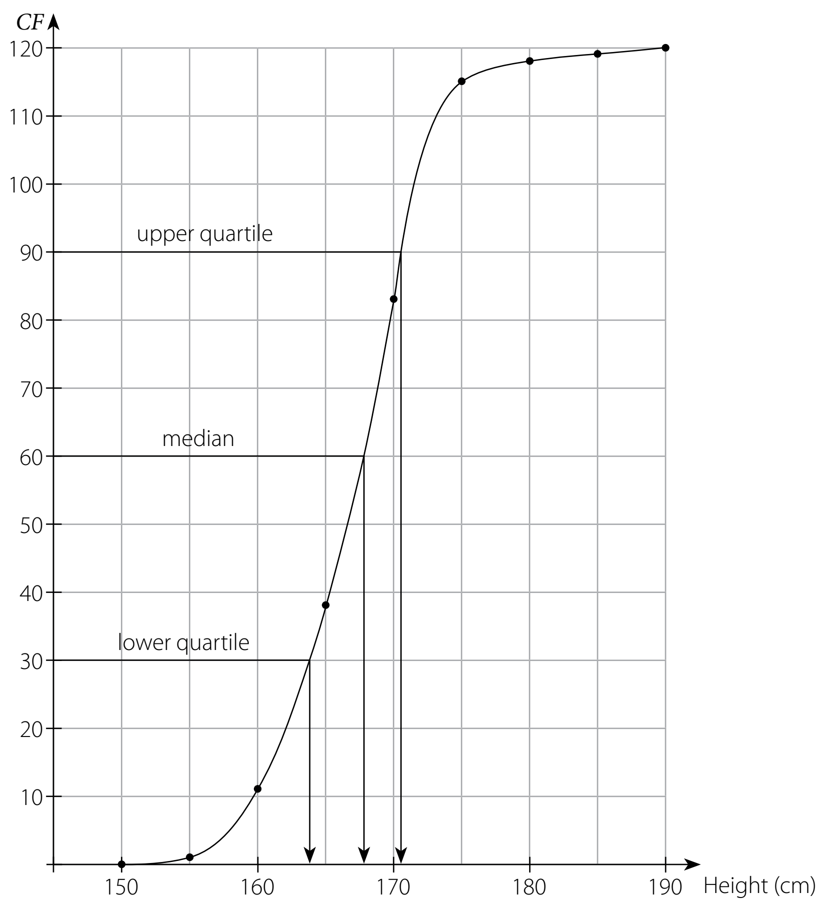
2. a) b) c) [hockey stick examples]



d) yes

3. 6170 l (nearest litre)

4. a)



b) $Q_1 = 164$, $Q_2 = 168$, $Q_3 = 171$ cm (approx) c) 7 cm

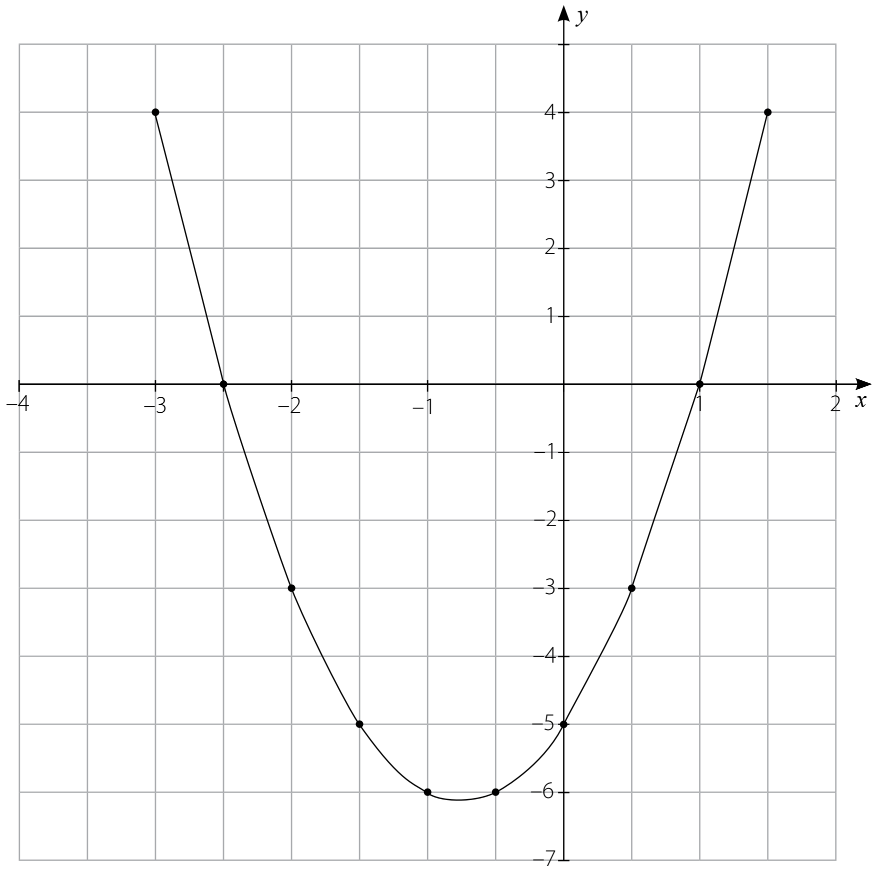
d) $P_{20} = 163$ cm (approx)

5. a) 27 b) 26.7 c) 26.660 d) 26.6597

6. a)

	x	−3	−2.5	−2	−1.5	−1	−0.5	0	0.5	1	1.5	
	x^2	9	6.25	4	2.25	1	0.25	0	0.25	1	2.25	
y	$2x^2$	18	12.5	8	4.5	2	0.5	0	0.5	2	4.5	y
	$3x$	−9	−7.5	−6	−4.5	−3	−1.5	0	1.5	3	4.5	
	−5	−5	−5	−5	−5	−5	−5	−5	−5	−5	−5	
	y	4	0	−3	−5	−6	−6	−5	−3	0	4	

b)



c) (−0.75, −6.1) approx d) $x = -0.75$ or $4x + 3 = 0$

7. $x + y = 140$ and $140 + y = 3x + \frac{y}{2}$,

$x = 60, y = 80$, exit rates are 120, 40 m^3/h

8. $a = 61$ (isos Δ), $b = 61$ (alt ∠s), $c = 97$ (angle-sum of quad), $d = 97°$ (vert opp ∠s)

9. a) \$ 35.70 b) 70 ¢

10. a) 12 cm^2 b) $3\sqrt{l^2 - 9}$ cm^2

Chapter

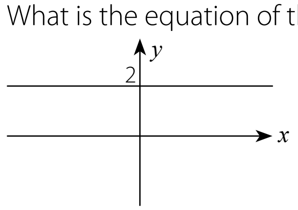
24 Straight Line Graphs

This chapter brings together features of the straight line that have been mentioned before, but demands the ability to sketch and interpret with ease. The application in scientific work of using the straight line to smooth out experimental data is mentioned. "Line of best fit" is used without explanation: more intelligent students may want to ask questions about that.

LESSON PLANNING

Objectives	
General	To know thoroughly all the basic facts about the straight line equation and its graph
Specific	1. To know and interpret the gradient form of the straight line equation; to sketch graphs using this form 2. To know and interpret the intercept form of the straight line equation; to sketch graphs from this form 3. To know the equations of horizontal and vertical lines, including the x and y-axes 4. To know the meaning of the gradient of a straight line in terms of rate of increase 5. To know that parallel lines have the same gradient, and that perpendicular gradients are negative reciprocals of each other 6. To know that scientists use straight lines to smooth out errors in experimental data
Pacing	2 lessons, 1 homework
Links	distance-time and speed-time graphs
Method	See how much is remembered from previous work by writing equations and sketches on the board, e.g.

- $y = 2x + 4$
  "What is the gradient?"
  "Can you sketch it?"
- What is the equation of the horizontal line?"



- $3x - 4y = 24$

  "Can you sketch it?"

  [Cover up method demonstrated]

etc.

Then, depending upon feedback, use the text examples to greater or lesser extent, as required. [The text information on gradients is probably best left for reference only.]

Set EX 24A

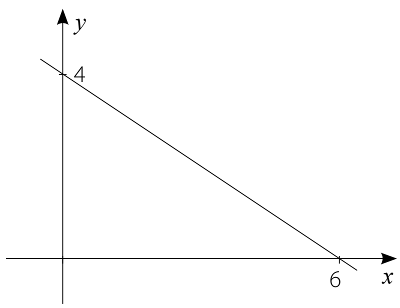
Assignment	EX 24A, question 9 or 10 for homework
Vocabulary	gradient, intercept, reciprocal, parallel, perpendicular

ANSWERS

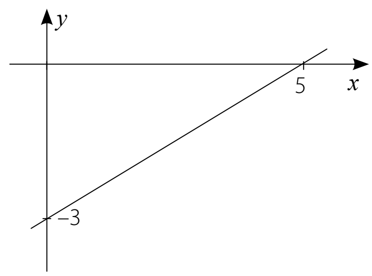
Exercises

EX 24A

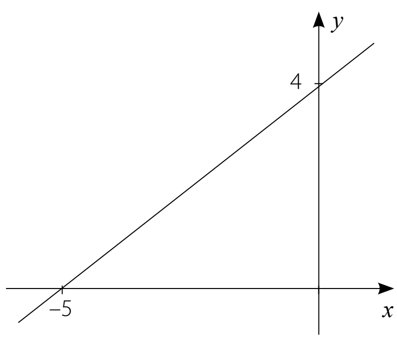
1. a)



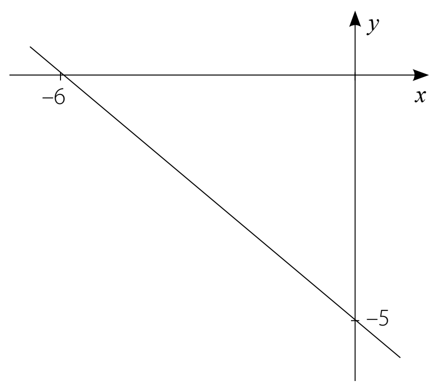
b)



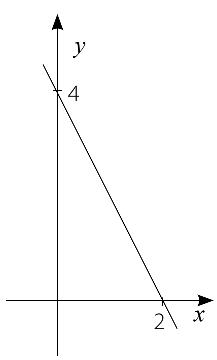
c)



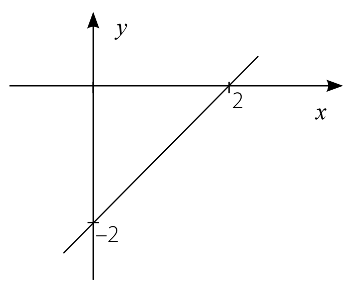
d)



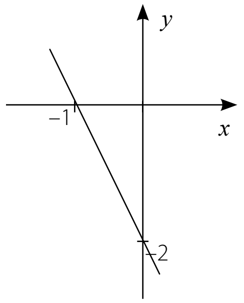
2. a)



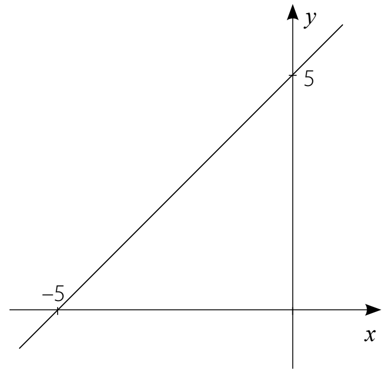
b)



c)



d)



3. a) 4 trolleys b) 12 h

c) Anne. Her graph has steeper gradient.

d) Anne 0.83, Betty 0.583 trolleys/h. So Anne earns a bonus.

4. $l_1 : y = 2$

$l_2 : y = -3$

$l_3 : x = 0$

$l_4 : x = 5$

$l_5 : x = -6$

$l_6 : y = 0$

5. a) She had difficulty reading the angles, so she might expect some results to be a little inaccurate; perhaps she had prior knowledge that led her to expect a straight line relationship; the current supplied may not have been correct; apparatus may have been faulty; she may have misread the scales owing to tiredness; etc.

b) 10 mA c) (i) $\frac{5}{8}$ (ii) 0.625 °/mA

d) $y = \frac{5}{8}x + c$ or $y = 0.625x + c$

$c = -6\frac{1}{4}$ or $c = -6.25$

$y = \frac{5}{8}x - \frac{25}{4}$ or $y = 0.625x - 6.25$

6. a) Hosepipe; the graph has steeper gradient.

b) 3 min c) (i) $d = 5t$ (ii) $d = 30$

d) $d = \frac{40}{3}t + c,\quad c = -90,\quad d = \frac{40}{3}t - 90$

7. a) $y = 2x - 7$ and $y = 2x + 1.5$ parallel

b) $y = \frac{1}{2}x - \frac{3}{5}$ and $y = \frac{1}{2}x - 7$ parallel

c) $y = 5x + \frac{1}{2}$ and $y = 4x - \frac{1}{2}$ not parallel

d) $y = \frac{1}{3}x - \frac{5}{6}$ and $y = \frac{2}{3}x + \frac{1}{6}$ not parallel

8. a) $y = -\frac{1}{2}x + 3$ and $y = 2x - 5$ perpendicular

b) $y = 3x - \frac{1}{4}$ and $y = -\frac{1}{3}x + 3$ perpendicular

c) $y = \frac{1}{2}x + \frac{5}{2}$ and $y = -2x + \frac{7}{2}$ perpendicular

d) $y = \frac{1}{5}x + 2$ and $y = 5x - 1$ not perpendicular

9. a) $y = \frac{1}{7}x + \frac{2}{7}$ and $y = \frac{1}{7}x + \frac{2}{5}$ parallel

b) $y = \frac{3}{4}x - \frac{1}{3}$ and $y = \frac{1}{4}x - \frac{1}{5}$ neither

c) $y = \frac{7}{8}x + 2$ and $y = -\frac{8}{7}x - 1$ perpendicular

d) $y = \frac{7}{2}x + \frac{5}{2}$ and $y = 19x + 1$ neither

10. a) $d = \frac{8.5}{6}$ m (better: $d = \frac{17}{12}$ m)

b) 4.96 mm (3 s.f.) c) 5 mm

d) 9.92 mm (3 s.f.); not reliable as outside the range of data obtained in the experiment

EX 24X

1. On a coordinate grid in 2 dimensions, yes. In space, no. Even perpendicular lines may possibly not intersect. [Non-parallel, non-intersecting lines in space are called skew lines.]

2. a) $y = \frac{5}{16}x - 7$ and $y = -\frac{16}{5}x + 1$ perpendicular

$\left(-\frac{480}{281}, \frac{1817}{281}\right)$

b) $y = \frac{9}{2}x + \frac{81}{10}$ and $y = \frac{9}{2}x - \frac{36}{5}$ parallel

c) $y = \frac{18}{17}x + \frac{17}{19}$ and $y = \frac{17}{19}x + \frac{18}{19}$ neither

$\left(1, \frac{35}{19}\right)$

d) $y = -0.025x + 2$ and $y = \frac{x}{0.025} - 1$ perpendicular

or $y = -\frac{1}{40}x + 2$ and $y = 40x - 1$

(0.0750, 4.00) to 3 s.f.

3. a) 3 10 23 40 62 90

7 13 17 22 28

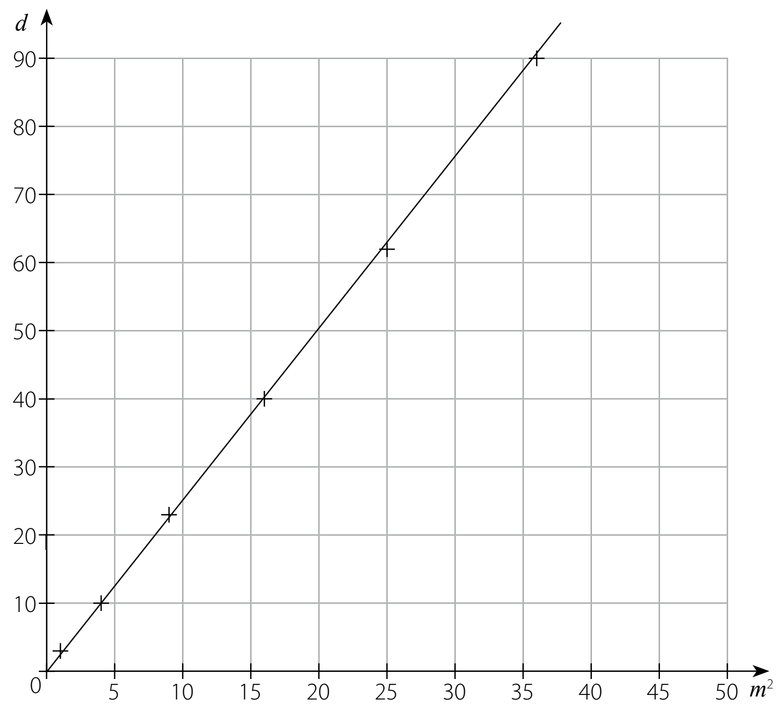
6 4 5 6 not quite equal

2nd row of (almost) equal differences => quadratic

b)

m	1	2	3	4	5	6
m^2	1	4	9	16	25	36
d	3	10	23	40	62	90

c)



$d = 2.5m^2$

[By plotting against m^2 we have "straightened out the curve", enabling the experimental errors to be eliminated.]

Chapter 25 Similarity

This chapter deals much more thoroughly with this topic raised in Book 8, Chapter 23, with alternative methods provided, and the beginning of formal proofs.

LESSON PLANNING

Objectives

General	To solve problems involving shapes that are mathematically similar, especially similar triangles
Specific	1. To know that similar figures (shapes) are the result of an enlargement followed by another transformation that preserves shapes 2. To know that corresponding angles are equal in similar figures 3. To know that corresponding sides are scaled up or down by the same scale factor in similar figures 4. To know that two pairs of equal angles in triangles is sufficient to prove similarity 5. To calculate lengths of sides in similar figures using the scale factor method and the internal ratio method 6. To recognize that sin/cos/tan are internal ratios in right-angled triangles 7. To know the connections between scale factors of length, area, and volume in similar shapes 8. To know that if SF = 1 for similar figures, then they are congruent
Pacing	4 lessons, 1 homework
Links	trigonometry, transformations, equations, ratios
Method	Remind students of corresponding angles where there are parallel lines. Use the text figure to identify some of these. (They are listed.) Now separate the shape and its image, as shown, with a little rotation. We still call the angle pairs corresponding (marked with arcs). Now seek out corresponding sides. (Some are given.) After referring to the two Reminders, set EX 25A. The last two questions of EX 25A, questions 9 and 10, lead into a discussion of how we know that shapes are similar. Clearly, all rectangles are not similar, so all angles corresponding is not sufficient. We must also have equal SFs for corresponding sides. Then deal with the special case of similar triangles where equiangular triangles are guaranteed similar. Use the trig ratio diagram in the text

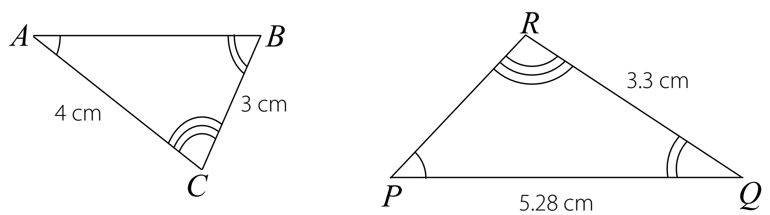
	as an example of this. Develop the internal ratio argument and show how this method is sometimes quicker/easier than using scale factors. Start the students on EX 25B. Repeat the trig ratio explanation as a special case of the internal ratio method. Also, remind students of the ratios $1:n$ length, $1:n^2$ area, $1:n^3$ volume, applicable for enlargements, and hence similar shapes. Then the rest of EX 25B should be possible.
Assignment	EX 25A, question 7 or question 9, 10
Vocabulary	enlargement, similar, corresponding angles (extended meaning), corresponding sides

ANSWERS

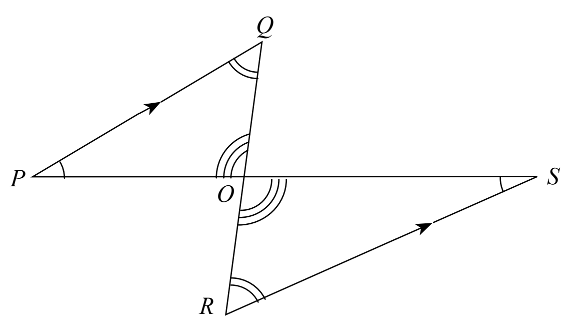
Exercises

EX 25A

1. a) 1.5 b) $x = 24$ cm
   c) $y = 17.1$ cm (3 s.f.) d) Checks
2. a) PU b) $\angle PQR$ c) 2
   d) $QR = 72$ cm, $DE = 8.1$ cm
3. SF = 1.2, $AB = 38.4$ cm, $QR = 18$ cm
4. a) $\angle QRS$ b) AD c) 1.4
   d) $x = 12$ cm, $y = 19.6$ cm, $z = 35$ cm
5. a)



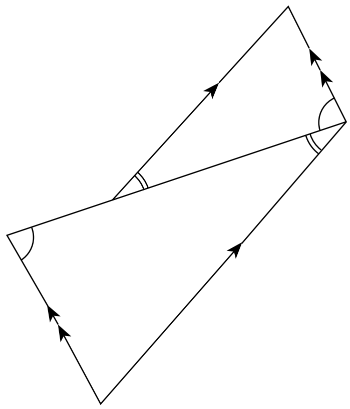
   b) QR c) 1.1 d) $AB = 4.8$ cm, $PR = 4.4$ cm
6. a) $\angle P = \angle S$ (alt $\angle$s), $\angle Q = \angle R$ (alt $\angle$s), $\angle POQ = \angle SOR$ (vert opp $\angle$s)
   b)



   c) $\frac{4}{3}$
   d) $PQ = 15$ cm, $OS = 17\frac{1}{3}$ cm

7. a) $\angle E = \angle ACB$ (alt $\angle$s), $\angle BAC = \angle ACD$ (alt $\angle$s)

b)



c) angle sum of Δ

d) $x = 15$ cm, $y = 6$ cm

8. a) 44 cm, 176 cm b) 4 c) 4 d) same

9. length SF $= \frac{4}{3} = 1.\dot{3}$, width SF $= \frac{13}{10} = 1.3$

Not quite equal, so not similar.

10. length SF $= \frac{10}{4} = 2.5$, width SF $= \frac{7}{3} = 2.\dot{3}$

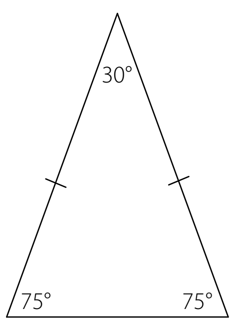
Not equal, so not similar

EX 25B

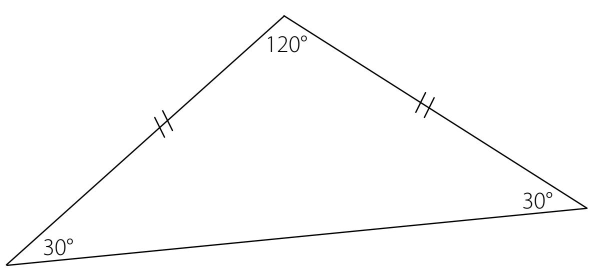
1. a) SF = 1.25, $x = 45$ cm b) $\frac{x}{50} = \frac{36}{40}$, $x = 45$ cm

c) $\frac{25}{16}$ d) 1440, 2250 cm^2, $\frac{25}{16}$

2. No, it depends on which angles are equal.



or



3. a) $n = 2.7$ b) 21.6 cm

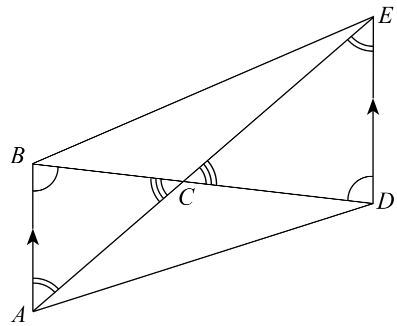
c) 80, 583.2 cm^2 d) 7.29, $2.7^2 = 7.29$ as required

4. $\frac{x}{9} = \frac{16}{6}$, $x = 24$ cm; $\frac{y}{6} = \frac{25.6}{9}$, $y = 17.1$ cm (1 d.p.)

5. $\frac{AB}{50.4} = \frac{32}{42}$, $AB = 38.4$ cm; $\frac{QR}{42} = \frac{21.6}{50.4}$, $QR = 18$ cm

6. $\frac{x}{10} = \frac{16.8}{14}$, $x = 12$ cm; $\frac{y}{14} = \frac{14}{10}$, $y = 19.6$ cm

7. a)



b) ΔABC and ΔEDC

c) To prove: ΔABC and ΔEDC are similar

Proof: $\angle BAC = \angle CED$ (alt $\angle$s)

$\angle ABC = \angle CDE$ (alt $\angle$s)

$\therefore \Delta ABC$, EDC similar (equiangular)

8. To prove: ΔABC is similar to ΔCDE

Proof: $\angle A = \angle ECD$ (alt $\angle$s)

$\angle BCA = \angle CED$ (alt $\angle$s)

$\therefore \Delta ABC$ is similar to ΔCDE (equiangular)

9. a) yes, SF = 10 b) $x = 30.4$ cm c) 3.04 m

10. 4 tonne; the real elephant is not made of model material.

EX 25X

1. 54 cm^2

2. $x = \sqrt{15}$

3. 45.4 l (3 s.f.)

Chapter

26 Equations for Solving Problems

Whereas Chapter 13 focuses on technical aspects of obtaining solutions, this short chapter provides practice in devising equations from word problems and reporting back the answer in terms of the original question. The equations are linear, simultaneous linear, and very simple quadratics (where the negative solution is rejected).

LESSON PLANNING

Objectives

General	To form equations to solve problems
Specific	1. To recognize the translation process: problem $\to$ algebra $\to$ solution (words) (equations) (words) 2. To create suitable equation(s) from a word problem 3. To solve that equation (those equations) and report the solution in terms of the stated problem
Pacing	1 lesson, 1 homework
Links	algebra, geometry, mensuration, puzzles
Method	It is not easy to teach creativity! Avoid doing too many examples on the board. Just two will do to illustrate **words** $\to$ **equations** $\to$ **solution** report process. The two text examples are best left for reference. Maximize time on task. Set EX 26A after dividing students into small collaborative groups (or pairs).
Assignments	EX 26A, question 8 is a good one to reserve for homework.
Vocabulary	linear, simultaneous, quadratic, solution, report

ANSWERS

Exercises

EX 26A

1. 9
2. $h = 4b$, $72 = \frac{1}{2}bh$, $b = 6$, $h = 24$ cm
3. $3d^2 = 192$, $d = 8$, length of fencing = 40 m
4. $1.17x - 15 = 55.20$, $x = 60$, original amount $ 60
5. First thought of –10, final answer 3
6. $\frac{1}{2}(2x)(x) \sin 30° = 50$, $x = 10$, base = 20 cm
7. $4y = x + y + 7$, $2y + 2 = x + y - 1$, $x = 8$, $y = 5$

   perimeter = 26 cm
8. $5b = 2a$, $12b + 2a = 1100$, $a = 125$, $b = 50$, 3.6 kg
9. $20 - \frac{(n + 3)}{2} + 7 = n$, $n = 17$, I first thought of 17.
10. $4.62 = 3 \times \frac{22}{7} r^2$, $r = 0.7$ m

EX 26X

1. $A(2, 4)$, $y = 2x$
2. 28 cm [Hint: Let x = half the short diagonal.]
3. 72 cm^2 [Hint: Let x = width of lawn.]

Chapter

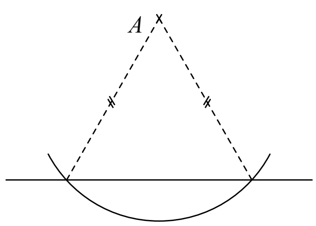
27 Loci and Constructions

This chapter provides a systematic review of Book 8, Chapter 13, with practice questions, especially on the ruler and compasses constructions.

LESSON PLANNING

Objectives

General	To construct, using ruler and compasses, standard loci and solve problems involving accurate measurement
Specific	1. To construct a triangle given SSS or SAS; to recognize that SSA may not specify a unique triangle 2. To construct the locus of points a constant distance from a fixed point 3. To construct the locus of points a fixed distance from a given line, and from a given line segment 4. To construct the locus of points equidistant from two fixed points 5. To construct the perpendicular to a line from a given external point (i.e. shortest distance) 6. To construct the locus of points equidistant from two fixed lines 7. To extend points 2 and 3 above to 3 dimensions 8. To construct diagrams of the above and take measurements to an accuracy of $\pm 1°$ or ± 1 mm
Pacing	2 lessons, 1 homework
Links	line symmetry
Method	As all the theory for this topic has been covered before, start with an oral session to see how much has been forgotten! Either go through all the text constructions systematically, or selectively. With a good class you can leave it for reference only. In explaining constructions avoid teaching a method sequence to be rote learned. The aim is to induce understanding of why such a sequence works. Symmetry is a powerful concept. For example, for the shortest distance from an external point to a line, we start by attempting to draw an isosceles triangle from the point A.



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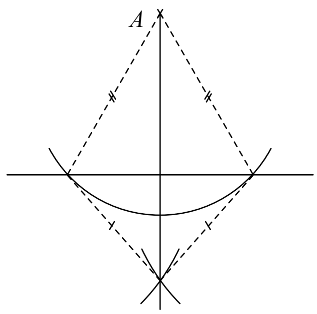
Without drawing its sides, we then proceed to form another isosceles triangle below the line by drawing equal arcs:



Again, we don't draw the sides.

The required line is the line of symmetry of both triangles (i.e. of the kite).

Symmetry can also be used effectively to explain the perpendicular bisector and angle bisector constructions.

Set EX 27A after carefully seating the students. Pair work (with discussion allowed) is advised, but choose your pairs carefully.

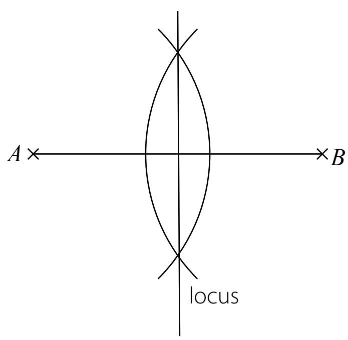
Resources	Sharp pencils (H recommended), ruler, compasses, protractor are all essential; board compasses (for demo)
Assignments	EX 27B, question 9 can be reserved for homework.
Vocabulary	locus, construction, included angle, line segment, perpendicular bisector, angle bisector, axis (line) of symmetry

ANSWERS

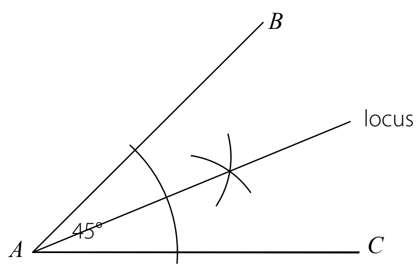
Exercises

EX 27A

1.



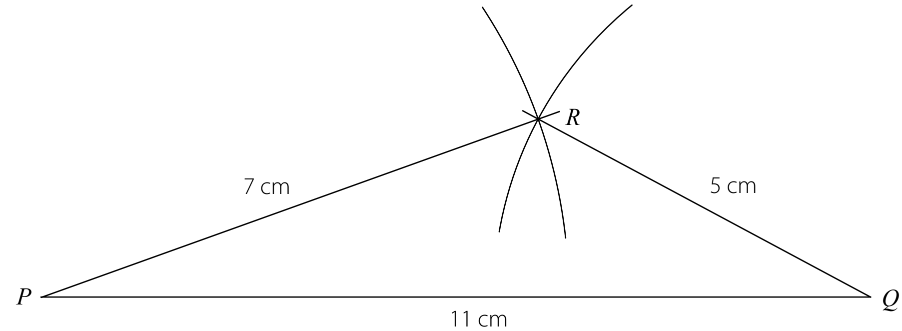
2.



3. a) parallel lines on either side of the fixed line and 12 cm from it

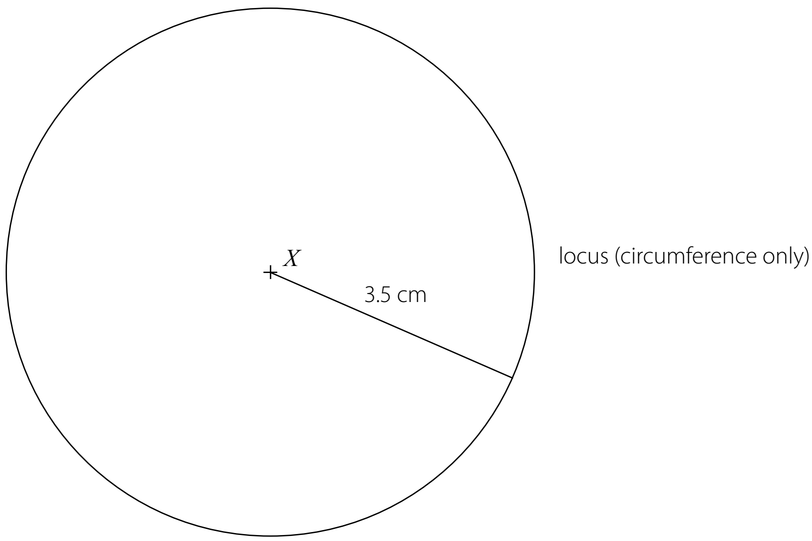
b) the surface of an infinitely long cylinder of radius 12 cm and with the given line as its axis of symmetry

4.



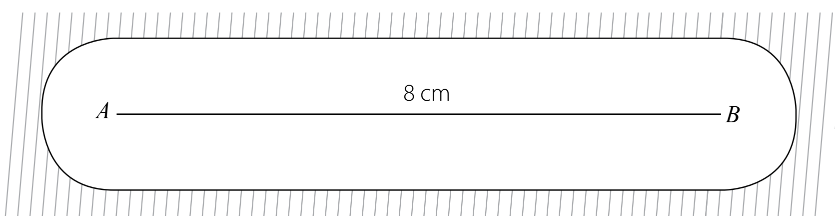
$\angle P = 20°$ [Accept ±1°]

5. a)



b) the surface of a sphere of radius 3.5 cm, centre X

6. a)

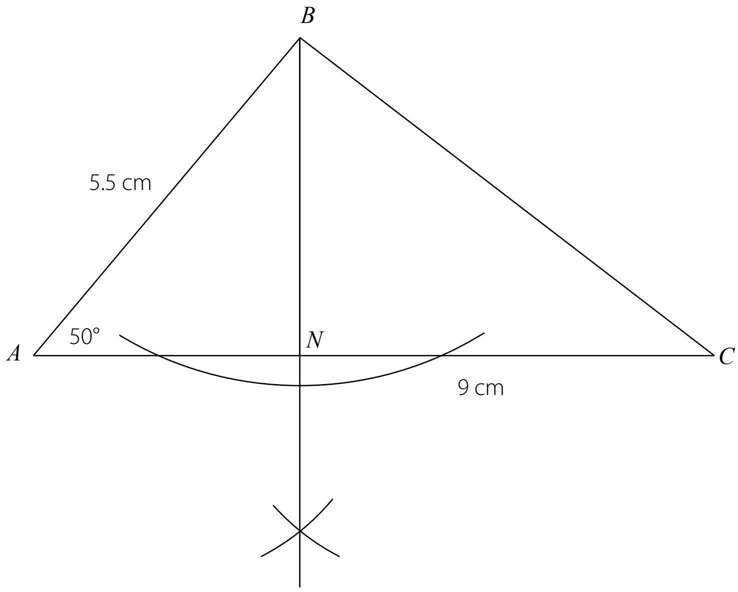


Locus is all the points inside or on the boundary.

b) Locus is all points in a composite solid formed by attaching hemispheres of radius 1 cm to each end of a cylinder of radius 1 cm and height 8 cm, such that the line segment AB lies on its axis of symmetry.

7.

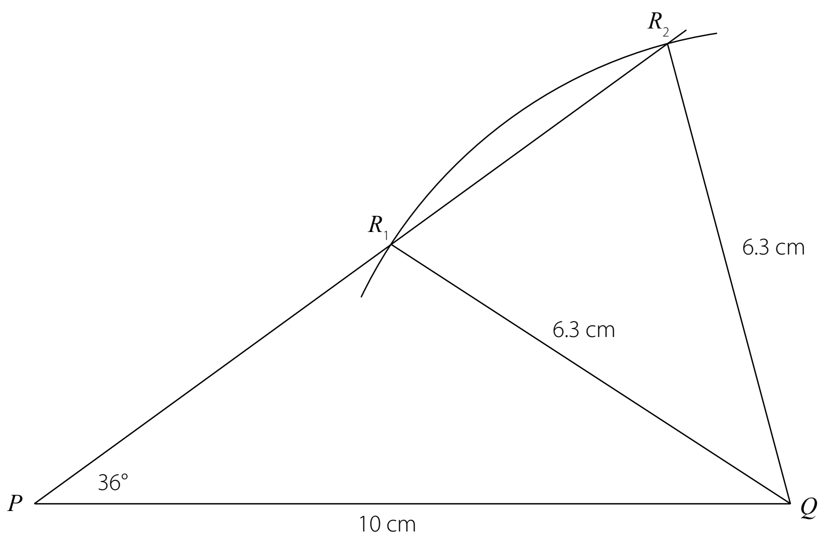
8.



$BC = 6.9$ cm $\quad BN = 4.2$ cm

[Allow ± 0.1 cm]

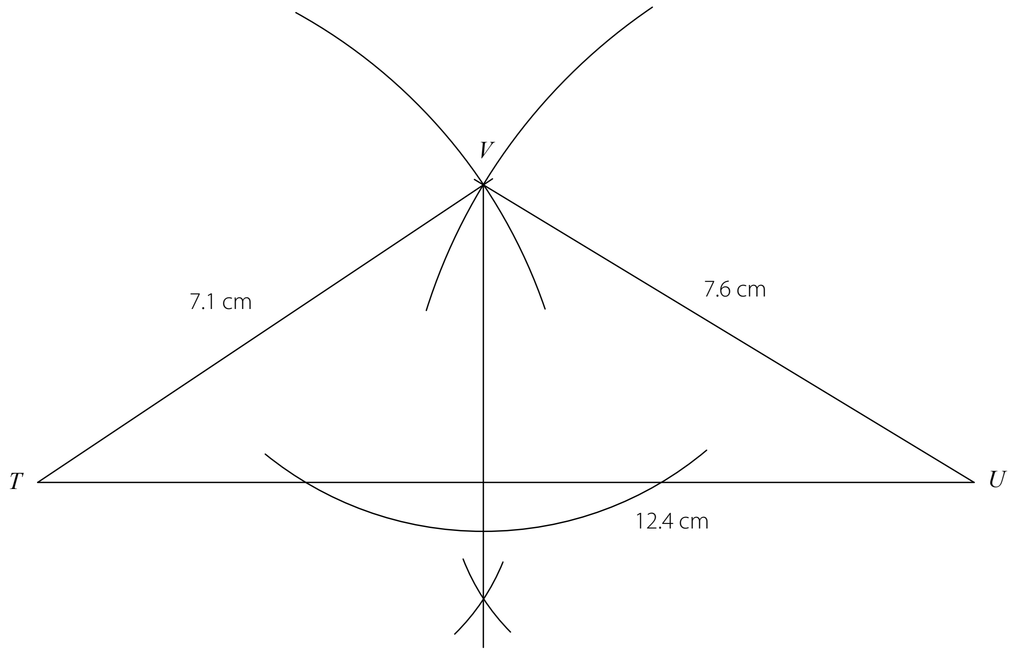
9.



$\angle PQR_1 = 33°$, $\quad \angle PQR_2 = 75°$

[Allow ± 1°]

10.

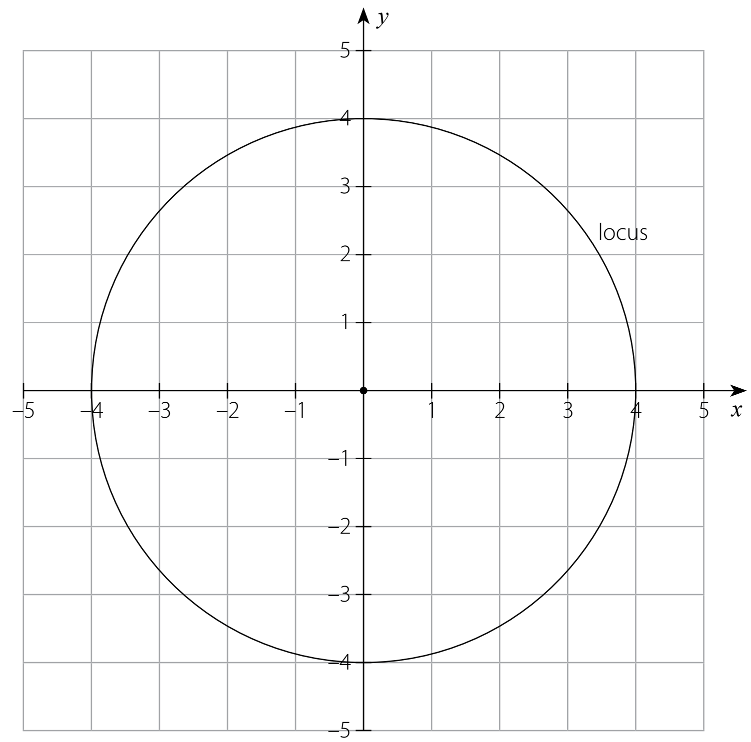


Area = 24 cm^2 (2 s.f.)

[Allow ± 1 cm^2]

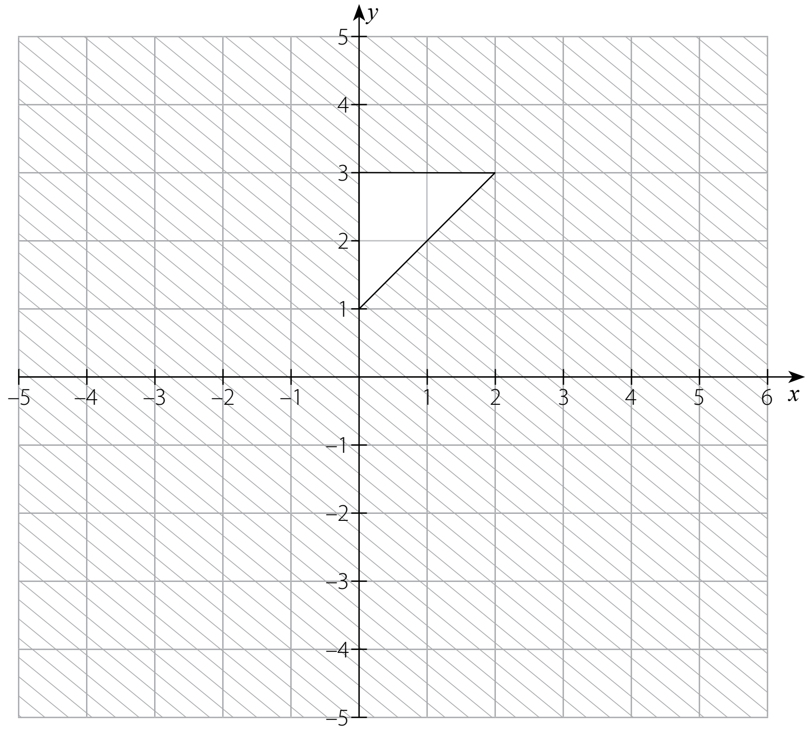
EX 27X

1.



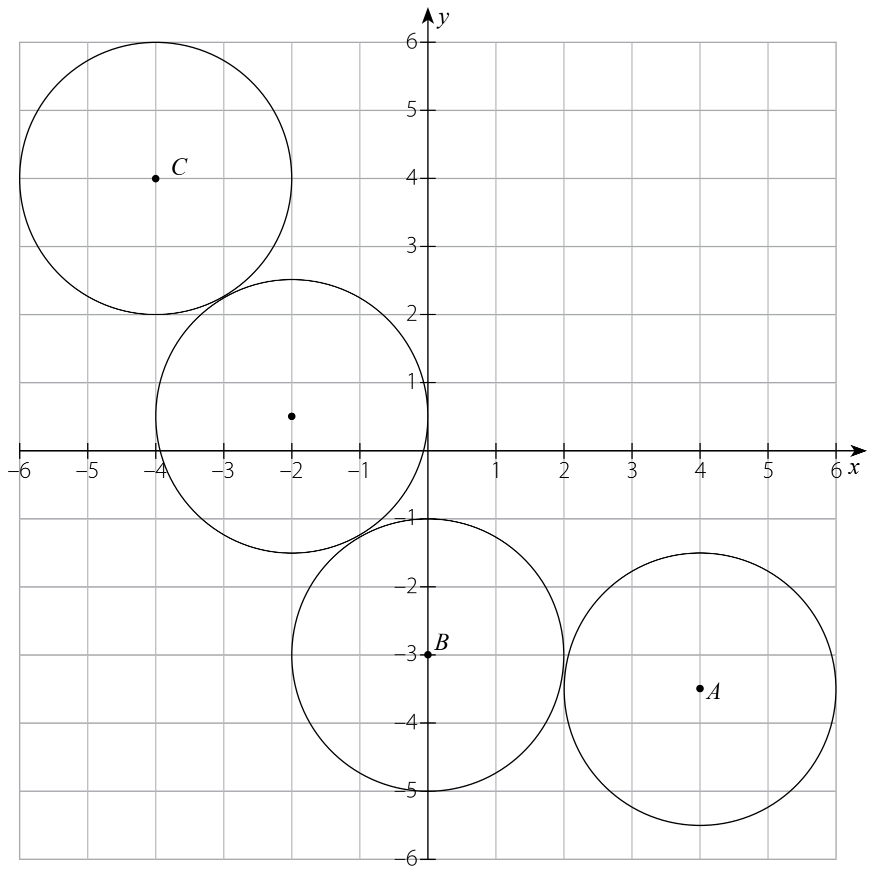
The locus is the circumference of the circle.

2.



Locus is all the points in the unshaded triangle.

3.



(−2, 0.5)

Chapter

28 Harder Factorisation

The focus in this chapter is almost entirely on the factorisation of simple trinomials because of its importance later in solving quadratic equations. It is not an easy skill to develop and requires much practice. To avoid undue complication at this stage, there are no questions here with negative term in x^2. These will be dealt with later.

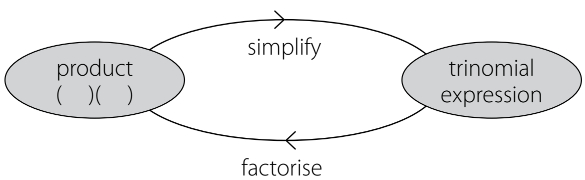
LESSON PLANNING

Objectives

General	To factorise trinomials and to recognize the difference of two squares and perfect squares identities
Specific	1. To factorise trinomials, including cases of prior extraction of a common factor 2. To recognize and use the difference of two squares factorisation 3. To recognize and use the factorisations of trinomials that are perfect squares 4. To simplify expressions by factorising and cancellation
Pacing	4 lessons, 2 homeworks

Method

Remind class of the FOIL technique for products. Factorising is reversing the process.



Follow the text examples, or similar other examples.

Unless the class is really high-powered, stop before the last example and start them on EX 28A. This exercise is graded to ensure that the method becomes embedded through practice. Every question should be attempted.

Demonstrate some special trinomials to be memorized. Using FOIL, in one case the O/I terms cancel out; in the other case they double.

Follow the text for difference of two squares and perfect square factorisations. Emphasize structure rather than formulas.

Set EX 28B, questions 1–4.

Show some examples of cancelling, stressing that we can only cancel factors. The text shows common errors in this regard.

	Set the rest of EX 28B, questions 5–10. Students finding difficulty in factorising the harder questions may be referred to the omitted example in the text, (just before EX 28A). This kind of task will result in huge differences in speed between the most- and least-able students. Make good use of EX 28X for the high flyers whilst the others catch up.
Assignments	EX 28A, questions 6 and 8, and EX 28B, questions 6 and 8 may be reserved as homework questions.
Vocabulary	factorise, product, cancel, perfect square, common factor

ANSWERS

Exercises

EX 28A

1.	a)	$(x + 1)(x + 4)$	b)	$(x + 2)(x + 5)$
	c)	$(x + 3)(x + 7)$	d)	$(x + 4)(x + 5)$
2.	a)	$(x - 4)(x + 1)$	b)	$(x - 2)(x + 1)$
	c)	$(x - 2)(x - 3)$	d)	$(x + 6)(x - 5)$
3.	a)	$(2x + 1)(x + 2)$	b)	$(2x + 3)(x + 1)$
	c)	$(2x + 1)(x + 4)$	d)	$(3x + 1)(x + 5)$
4.	a)	$(2x - 1)(x - 1)$	b)	$(2x - 1)(x + 1)$
	c)	$(2x + 5)(x - 2)$	d)	$(3x - 1)(x - 5)$
5.	a)	$(x + 1)(x + 10)$	b)	$(2x - 1)(x + 10)$
	c)	$(3x + 4)(x - 3)$	d)	$(5x - 6)(x + 1)$
6.	a)	$(3x + 2)(2x + 1)$	b)	$(4x - 1)(2x + 5)$
	c)	$(3x - 7)(3x - 1)$	d)	$(3x + 4)(5x + 2)$
7.	a)	$2(x + 7)(x - 5)$	b)	$5(x + 1)(x + 3)$
	c)	$3(x + 7)(x - 1)$	d)	$4(x + 2)(x - 9)$
8.	a)	$7(x - 8)(x + 2)$	b)	$5(x + 5)(x - 3)$
	c)	$3(x - 2)(x - 1)$	d)	$2(x + 4)(2x + 5)$
9.	a)	$(a + b)(a + 2b)$	b)	$(2a - b)(a - b)$
	c)	$(a - 3b)(a + 2b)$	d)	$(3a - b)(2a + b)$
10.	a)	$(3x - 4)(5x + 7)$	b)	$(4x - 7)(9x - 5)$
	c)	$(6x - 1)(7x + 16)$	d)	$(2x - 3)^2$

EX 28B

1. a) $x^2 - 9$ b) $x^2 - 25$ c) $4x^2 - 1$ d) $9x^2 - 16$
2. a) $(x - 4)(x + 4)$ b) $(x - 7)(x + 7)$
   c) $(2x - 3)(2x + 3)$ d) $(5x - 9)(5x + 9)$
3. a) $x^2 + 4x + 4$ b) $x^2 - 12x + 36$
   c) $4x^2 - 20x + 25$ d) $9x^2 + 6x + 1$
4. a) $(x + 5)^2$ b) $(x + 7)^2$ c) $(x - 8)^2$ d) $(x - 10)^2$
5. a) $x + 1$ b) $2x - 5$ c) $3x - 1$ d) $5x + 1$
6. a) $3(x - 2)(x + 2)$ b) $5(x - 5)(x + 5)$
   c) $2(x - 7)(x + 7)$ d) $11(x - 11)(x + 11)$
7. a) $3(x + 1)^2$ b) $5(x - 4)^2$
   c) $7(x - 1)^2$ d) $8(2x - 3)^2$
8. a) $2(x + 4)$ b) $7(x + 2)$ c) $5(x - 3)$ d) $2(x - 5)$
9. a) $x + 3$ b) $2(5x - 7)$ c) $10(2x + 11)$ d) $3(3x + 8)$
10. a) $\frac{x + 1}{x + 8}$ b) $\frac{x + 2}{x + 8}$ c) $\frac{2(x - 3)}{2x - 1}$ d) $\frac{2(x - 1)}{x + 1}$

EX 28X

1. a) $2(14x - 17y)(14x + 17y)$ b) $4xy$
   c) $10(1 - 10x)(1 + 10x)$ d) $q^2(3p - 2r)(3p + 2r)$
2. a) $1 + 4pq + p^2q^2$ b) $25p^2 - 70pqr + 49q^2r^2$
   c) $x^3 + 5x^2 + 8x + 4$ d) $8x^3 - 44x^2 + 70x - 25$
3. a) $x + 2$ b) $x + 3$ c) $1 - x$
   d) will not factorise: cannot be simplified

Chapter

29 Transformations

Again, we have a run through of previous work on this topic, but here we become serious about the full descriptions of the transformations and classifying them in terms of their effects, with a mention of future transformations to whet the appetite.

LESSON PLANNING

Objectives	
General	To give accurate, full descriptions of translations, rotations, reflections, and enlargements and to solve problems involving these transformations, including combinations of transformations
Specific	1. To give full descriptions of translations, rotations, reflections, and enlargements 2. To use the functional rotation for translations, i.e. T, R, M, E 3. To know the meaning of "corresponding" in this context 4. To apply multiple transformations in combination and use the functional notation correctly
Pacing	2 lessons, 1 homework
Links	coordinates
Method	Start with full description. "How do we describe a rotation? What do we need to know?"

After some oral work, consolidate by referring to the text summary.

Give the codes R, M, T, E.

M (reflection) is for "Mirror".

Explain that when using transformation arrows, the letters are to be taken in corresponding order. For example,

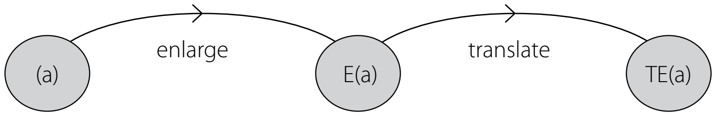
$$ABCD \to PQRS$$

means $A \to P, B \to Q, C \to R$ and $D \to S$

Explain combinations. For example,

- Reflect, then rotate, is RM
- (a) $\to$ TE (a) means shape (a) is first enlarged, then translated.

The transformation closest to the shape name is applied first. Arrow diagrams may also help:



Set EX 29A.

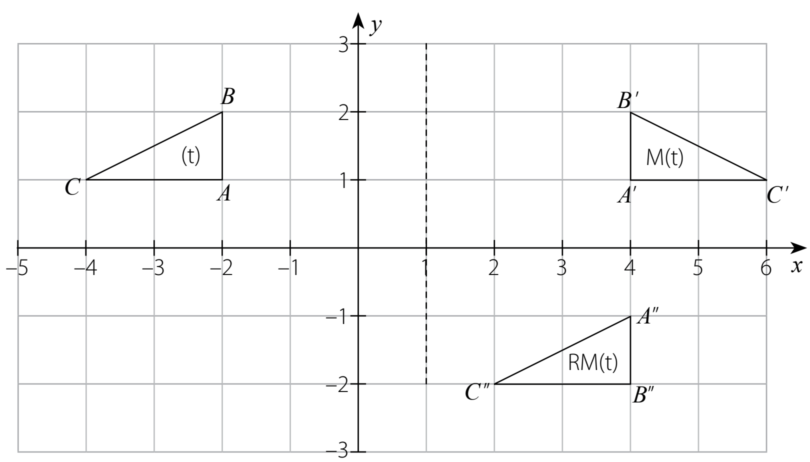
Resources	Squared paper (9 mm), coordinate grids
Assignments	EX 29A, question 9 suitable for homework (coordinate grid needed)
Vocabulary	rotation, reflection, translation, enlargement, identity transformation, description

ANSWERS

Exercises

EX 29A

1. a) translation $\begin{pmatrix} 6 \\ 2 \end{pmatrix}$ b) reflection in y-axis
   c) rotation –90° about $B(-2, 1)$
2. a) translation $\begin{pmatrix} 5 \\ -1 \end{pmatrix}$ b) reflection in $y = 3$
   c) enlargement: centre (0, 1), SF = 3 d) rotation 180° about (1.5, 3)
3. a) rotation –90° about (–1, 2) b) enlargement: centre (5, –4), SF = 2
   c) translation $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$
4. a) enlargement: centre (–7, 4), SF = 1.5 b) reflection in $x + y = 3$
   c) rotation 180° about (0, –1.5)
5. a) reflection in $y = 1.5$
   b) enlargement: centre (–8, 0), SF = 0.5 c) translation $\begin{pmatrix} 0 \\ -3 \end{pmatrix}$
6. a)
   b)
   c)

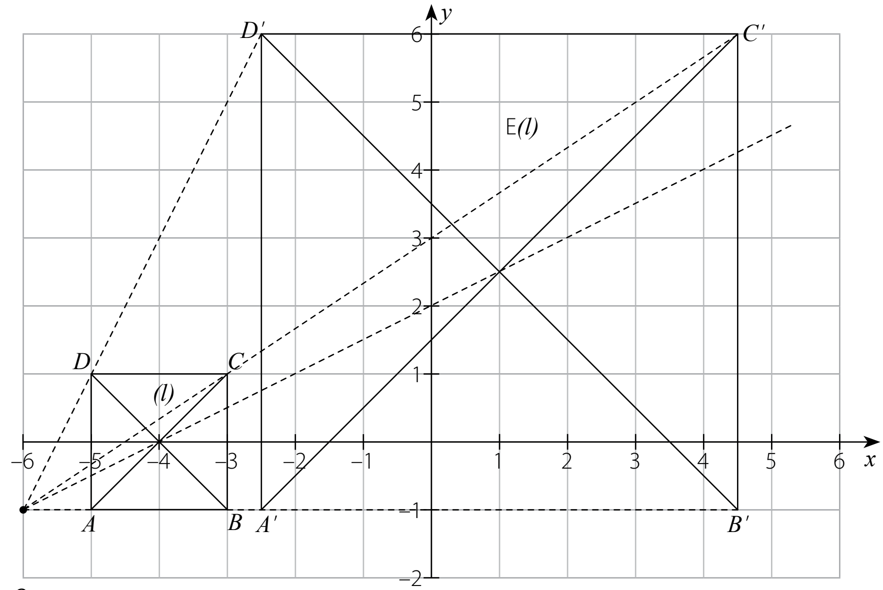


d) RM(t) takes $A \to A''$, $B \to B''$, $C \to C''$ but the translation $\begin{pmatrix} 6 \\ -3 \end{pmatrix}$ takes $A \to B''$, $B \to A''$, $C \to C''$

7. a)

b)

d)



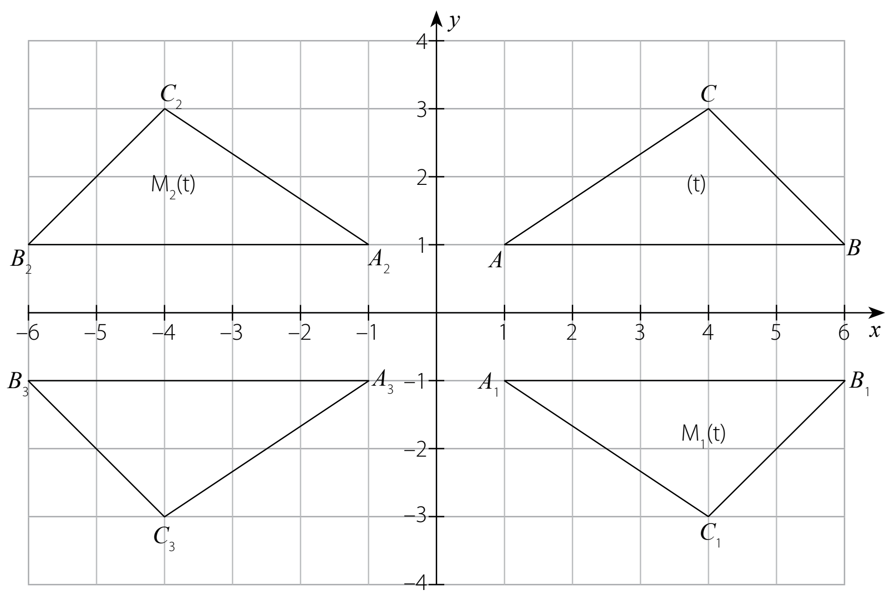
c) $\frac{2}{7}$

8. a) rotation of –60° about (1, 1)

b) 12 rotations of 30° = 360° = 1 revolution

=> return to original position

9.



$\Delta A_3B_3C_3$ is $M_1M_2(t)$ and $M_2M_1(t)$

$M_1M_2 = M_2M_1$ is true in this case.

10. a) $\begin{pmatrix} -1 \\ -6 \end{pmatrix}$ b) $\begin{pmatrix} -5 \\ -7 \end{pmatrix}$ c) $\begin{pmatrix} 6 \\ -8 \end{pmatrix}$

d) (i) 2 units parallel to the x-axis (horizontal right)

(ii) 5 units parallel to the y-axis (vertical down)

EX 29X

1. rotation of 180° about (0, 1)

2. $x' = 6 - x$, $y' = y$

3. a) 1:5 b) 1:25 c) 1:1 d) $A'(8, 1)$

Chapter

30 Revision Exercises

ANSWERS

Exercises

EX 30A

1. a) $x = -0.5$ b) $x = -17.25$

2. a) \$ 5632 b) 2.7 ha

3. a) $V_{\text{sphere}} = \frac{4}{3}\pi r^3$, $V_{\text{cylinder}} = \pi r^2 h$

   b) $V = \frac{2}{3}\pi r^3 + \pi r^2 h = \pi r^2(\frac{2}{3}r + h)$

   c) $h = \frac{V}{\pi r^2} - \frac{2r}{3}$

   d) (i) 62.3 cm (ii) 82.2 cm (3 s.f.)

4. $\angle ABF = 32°$ (alt $\angle$s)

   $\angle CBF = 42°$ (alt $\angle$s)

   $x = 360 - (32 + 42)$ (angles at a pt)

   $x = 286°$ as required

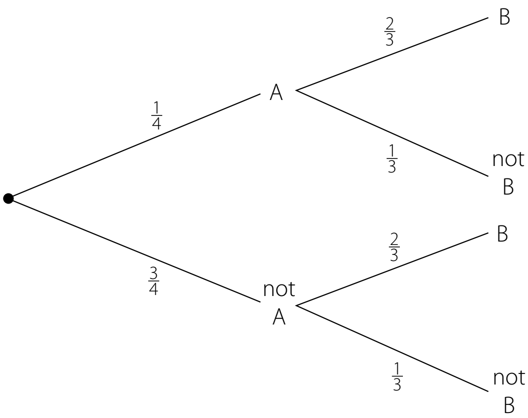
5. a) 20 b) 20 c) 10 d) 110

6. a) $\sin P = \frac{14}{50} = 0.28$, $\angle P = 16.3°$ (3 s.f.)

   b) $\cos P = \frac{48}{50} = 0.96$, $\angle P = 16.3°$ (3 s.f.)

   c) $\tan P = \frac{14}{48}$, $\angle P = 16.3°$ (3 s.f.)

   d) consistent answers

7. 

P(both)

$= \frac{1}{4} \times \frac{2}{3}$

$= \frac{1}{6}$

8. a) (ext $\angle$ of polygon) b) (angle sum of quad)

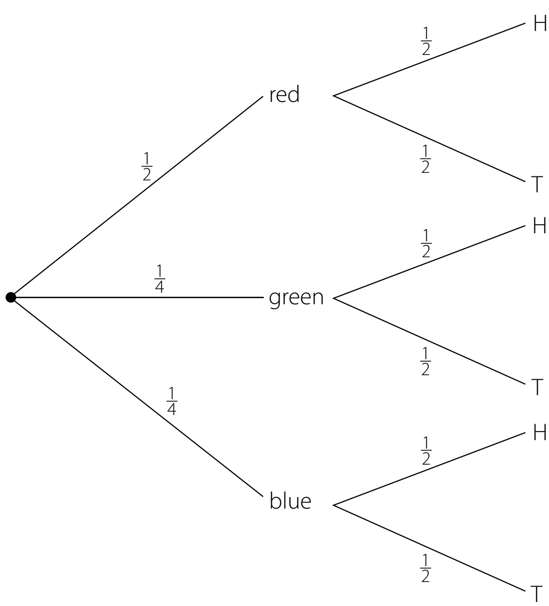
   c) (int $\angle$s) d) (adj $\angle$s on st line)

9. 2.5 l

10. \$ 1.78 (nearest cent); each percentage is on the previous year's price, so they cannot just be added.

EX 30B

1. a) P(red) $= \frac{1}{2}$, P(green) $= \frac{1}{4}$, P(blue) $= \frac{1}{4}$

   b) 

   c) (i) P(red, head) $= \frac{1}{4}$,

   (ii) P(green, tail) $= \frac{1}{8}$

2. a) $n = 7$ b) $n = 8$ c) $n = 10$ d) $n = 7$

3. a) 8.873 cm^2 b) 51.5 cm^2

4. a) 30% b) $8.\dot{3}$%

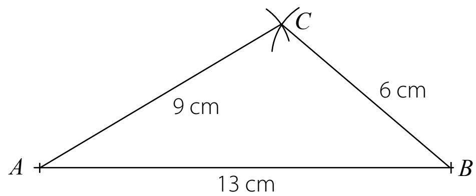
5. a) 9.82 m (3 s.f.) b) 13.8° (1 d.p.)

6. a) Reflection in line $y = x + 1$ b) Rotation $-90°$ about (6, 2)
c) Rotation 180° about (3.5, 4.5) d) Reflection in line $x = 6$

7. a) 17 min b) 16.92 km c) $0.0\dot{6}$ m/s^2 d) 0.02 m/s^2

8. a) 2^{15} b) 3^{20} c) 7^{-6} d) 8^{12}

9. $\angle A = 24°$ (nearest degree) [Allow ± 1°]



(not to scale)

10. a) $(x + 3)^2$ b) $(x + 10)^2$ c) $(x - 2)^2$ d) $(x - 5)^2$

EX 30C

1. a) 0.3 b) yes, 0.35 c) 0.15 d) 0.35

2. a) 22.6° (1 d.p.) b) 130 cm
c) 22.6° (1 d.p.) d) consistent answers

3. a) R b) Q c) S d) P

4. a) $(3x - 1)(x - 1)$ b) $(3x - 1)(x + 1)$
c) $(3x - 5)(x + 2)$ d) $(3x + 1)(x + 5)$

5. a) 5.79×10^3 b) 2.93×10^{-4}
c) 9.61×10 d) 4.42×10^6

6. $x = -1, y = 6$

7. $1.16x - 3 = 20.20, x = 20$. She had hoped to pay \$ 20.

8. a) 0.4082 b) 0.3381
c) 2.667 d) 2.084 (4 s.f.)

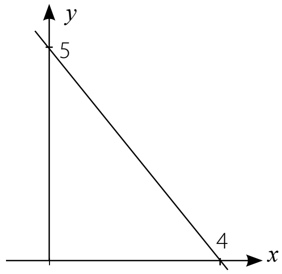
9. a) 6×10^{-6} b) 9.8×10^{-8}
c) 1.23×10^{-2} d) 9.55×10^{-3}

10. a) -16 b) 46 c) -0.8 d) -55

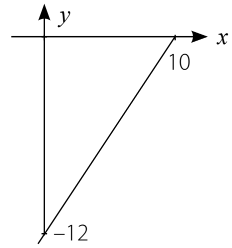
EX 30D

1. a) equiangular b) $\frac{20}{25}$, $x = 18.4$ cm

2. a)



b)



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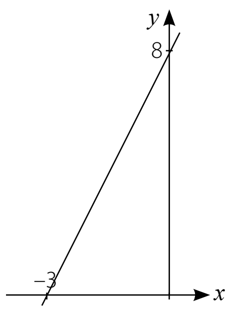
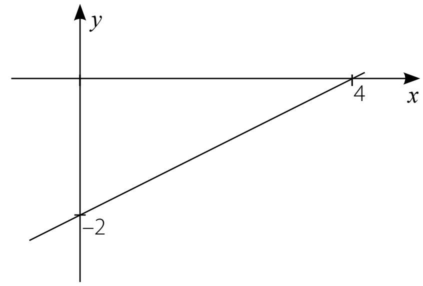
c)



d)



3. a) 18.5% increase b) 26.5% increase

   c) 6.5% decrease d) 38.28% decrease

4. a) $\angle C$ b) PS c) 2

   d) $x = 4.7$ cm, $y = 9.0$ cm, $z = 24.6$ cm

5. a) At this age the heights of the boys are generally less than the heights of the girls.

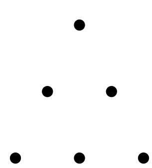
   b) The spread of boys' heights is greater than that of the girls.

   [IQR boys = 11 cm, IQR girls = 6 cm]

6. a) 1 3 6 10 15 21

   b) The numbers of dots form triangles,

   e.g. 6



   c) 1 3 6 10 15 21, same as triangle numbers

   d) 1 3 6 10 15 21

   2 3 4 5 6

   1 1 1 1

2nd row equal => formula quadratic

7. a) $y = 3$ b) $y = -4$ c) $x = 6$ d) $y = 0$

8. 642 m^2 (3 s.f.)

9. a) $Q_1 = 60$th, $Q_2 = 120$th, $Q_3 = 180$th b) $Q_1 = 63$rd, $Q_2 = 125$th, $Q_3 = 188$th

c) $Q_1 = 1050$th, $Q_2 = 2100$th, $Q_3 = 3150$th d) Q_1 = mean of 3rd and 4th items

Q_2 = mean of 6th and 7th items

Q_3 = mean of 9th and 10th items

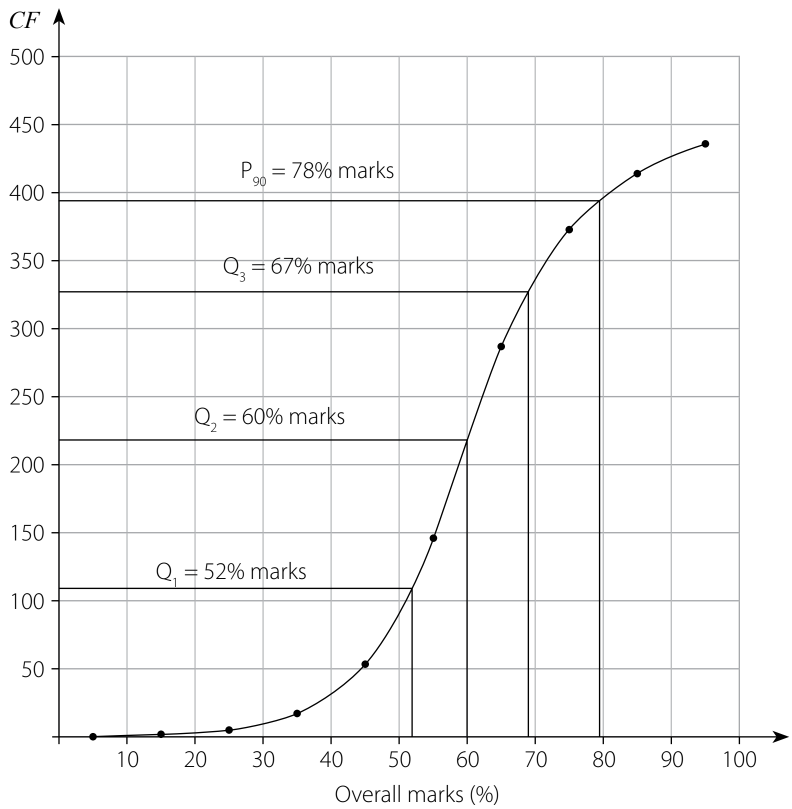
10.

Mark (upper bound)	5	15	25	35	45	55	65	75	85	95
CF	0	2	5	17	53	146	288	373	414	436

a)

b)

d)



c) IQR = 15% marks

Specimen Examination Paper 1

[based on the full year's work]

Instructions: The time allowed is 1 hour.

Electronic calculators must not be used in this paper.

You will need pen, pencil, eraser, ruler, compasses, and protractor.

Try to answer all the questions.

Check your work carefully.

The marks for each question are shown in the brackets. [Max marks: 60]

---

1. Write down the answers to the following calculations:

   a) $4800 \div 20$ b) 0.3×0.8

   c) 15.3×0.2 d) $3300 \div 1.1$ [4]

2. Round off the number 43.5875 correct to:

   a) 5 s.f. b) 1 d.p.

   c) 2 d.p. d) 1 s.f. [4]

3. Consider the sequence 20 13 6 −1 −8 −15 …

   a) By using differences (s teps) find out what kind of sequence it is.

   b) Find a formula for the nth term.

   c) What is the 20th term?

   d) What position is the term −673? [4]

4. If $A = 3 \times 10^4$ and $B = 1.2 \times 10^3$, calculate the value of each of the following, giving answers in standard form correct to 3 s.f. in each case:

   a) $A + B$ b) AB

   c) $\frac{B}{A}$ d) $2A - B$ [4]

5. Using ruler and compasses construct an accurate triangle PQR such that $PQ = 12$ cm, $PR = 7$ cm and $QR = 6.5$ cm. Measure $\angle P$ and record the result. [4]

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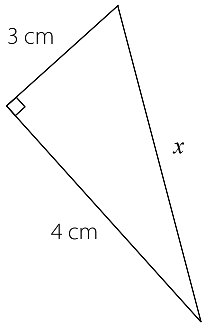
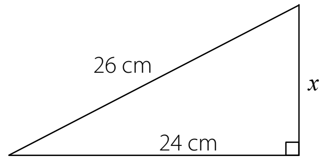
6. Find the value of x in each of these right-angled triangles:

a)

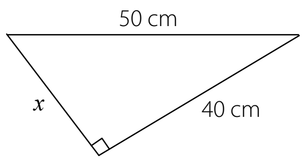


b)

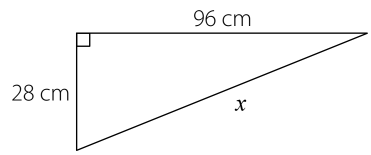


(not to scale)

c)



d)



[4]

7. Solve these equations:

a) $\frac{1}{2}(x - 1) + \frac{1}{3}(2x + 1) = 1$ [2]

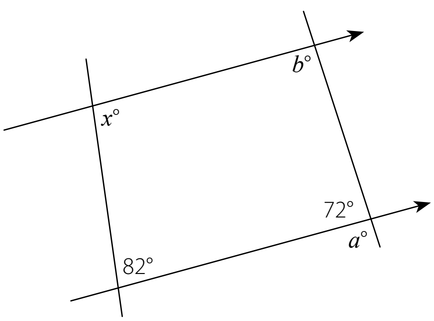
b) $0.8\,(x + 1) = 1 + 0.3\,(x - 3)$ [2]

8. Simplify the following, leaving your answers in index form:

a) $5 \times 5^2 \times 5^3$ b) $3^{-2} \times 3^5$

c) $\frac{2^{-3} \times 2^4}{2^5}$ d) $\frac{2^4 \times 2^2}{2^{-3}}$ [4]

9.



(not to scale)

In the diagram there is a pair of parallel lines (marked).

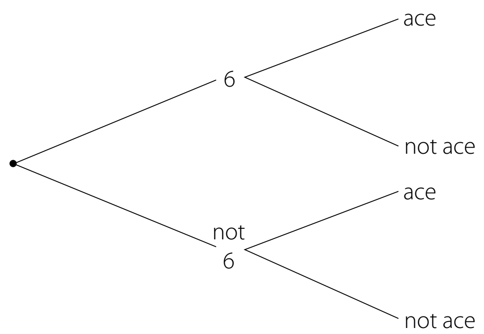
a) Give a reason that $a = 108°$.

b) Use the value of a to obtain the value of b. State the reason.

c) Without using the value of a, obtain the value of b another way, giving the reason.

d) Find the value of x, giving reason, without using a or b. [4]

10. The tree diagram shown represents the possibilities when a card is selected at random from an unknown set of cards, after rolling a fair dice.



a) Copy and complete the diagram, given that

$P(6, \text{ace}) = \frac{1}{30}$ [2]

b) Find from your diagram

(i) $P(\text{not } 6, \text{ace})$ (ii) $P(6, \text{not ace})$ [2]

11. Sketch these straight line graphs, clearly marking any points of intersection with the x and y axes.

a) $x = 5$ b) $y = 2x - 1$

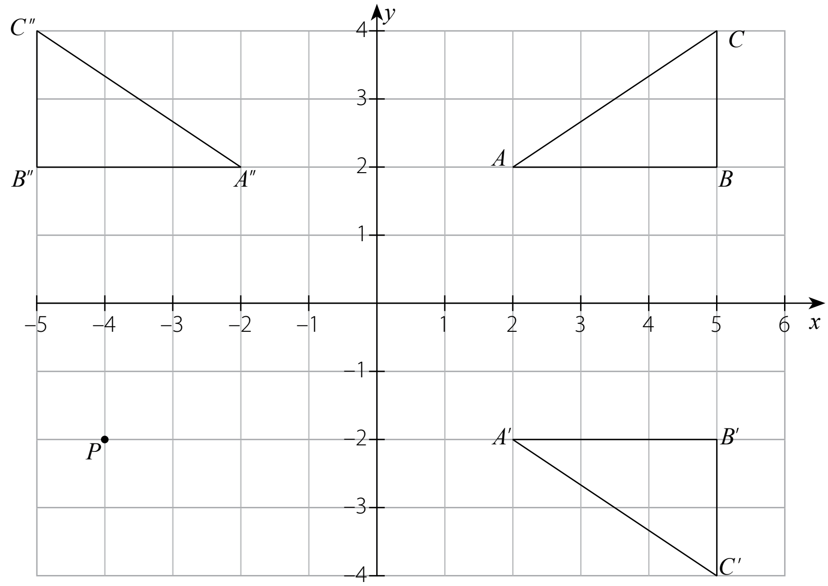
c) $7x - 5y = 35$ d) $2x + y = -8$ [4]

12. Factorise the following expressions:

a) $3x + 6xy - 9x^2y$ b) $25x^2 - 16y^2$

c) $7x^2 - 50x + 7$ d) $2x^2 - 12x + 18$ [4]

13. The diagram shows some transformations:



a) Describe fully the transformation $ABC \rightarrow A'B'C'$

b) Describe fully the transformation $A'B'C' \rightarrow A''B''C''$

c) P is (–4, –2) as shown. Imagine the $\Delta PB'C$ is drawn.
Describe fully the transformation $ABC \rightarrow PB'C$ [4]

14. Use the method of rounding off to 1 s.f. to estimate the following calculations:

a) 1.06 + 6.63 b) 0.421 – 0.078

c) 4.09×5.23 d) $10.7 \div 0.145$ [4]

15. A and B are two fixed points precisely 10 cm apart. Using ruler and compasses draw an accurate diagram of AB. Construct the locus of points that are an equal distance from A and from B. Label the locus clearly. [4]

Specimen Examination Paper 2

[based on the full year's work]

Instructions

The time allowed is $1\frac{1}{2}$ hours.
Electronic calculators may be used.
You will also need pen, pencil, eraser, and ruler.
Try to answer all the questions.
Check your work carefully.
The marks for each question are shown in the brackets. [Max marks: 90]

---

1. Use a calculator to work these out, giving your answers correct to 3 s.f.:

   a) $\dfrac{\sqrt{21}-6}{3\times 0.275}$ b) $\dfrac{32.6-12.1}{3.62\times 4.9}$ [4]

2. Calculate:

   a) The average walking speed if I can travel 17.4 km is 3 h 30 min [answer in km/h to 3 s.f.]

   b) The distance I run if I take 11.4 s at a rate of 7.8 m/s [answer to the nearest metre.]

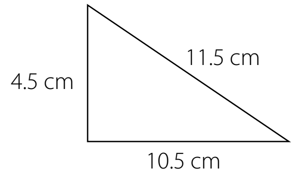
   c) The time taken on a train journey if the distance is 88 km and the average speed of the train is 62 km/h. [answer in hours and minutes, to the nearest minute.] [4]

3. a) If I sell my pet goldfish for \$ 35 having made 40% profit, what did I pay for it? [2]

   b) If I sell the fishtank for \$ 114 at 5% loss, how much (in dollars) did I actually lose? [2]

4. a) Calculate the length of the vector $\begin{pmatrix}2\\-7\end{pmatrix}$ correct to 2 s.f. [2]

   b) Prove that the triangle shown below is not right-angled.



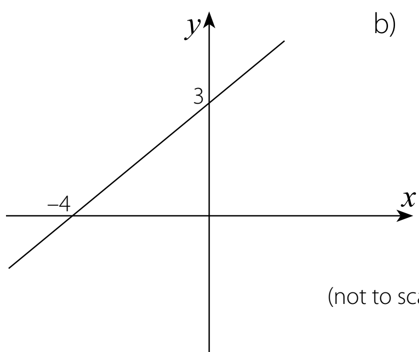
(not to scale) [2]

5. From the top of cliff 16 m above sea level a boat is observed. If the boat is 40 m from the foot of the cliff, calculate the angle of depression (1 d.p.)

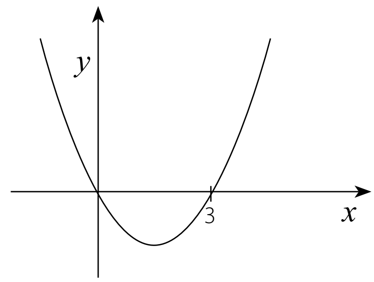
[Assume the observer is vertically above the foot of the cliff.] [4]

6. Match the graphs to their equations:

a)

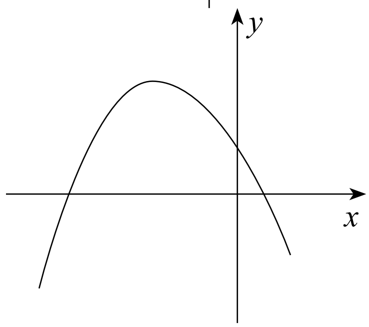


b)

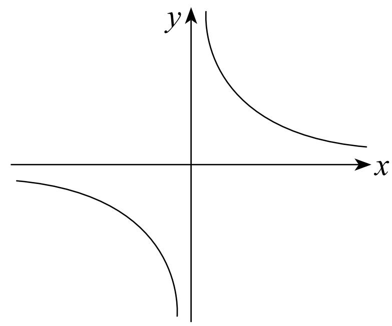


(not to scale)

c)



d)

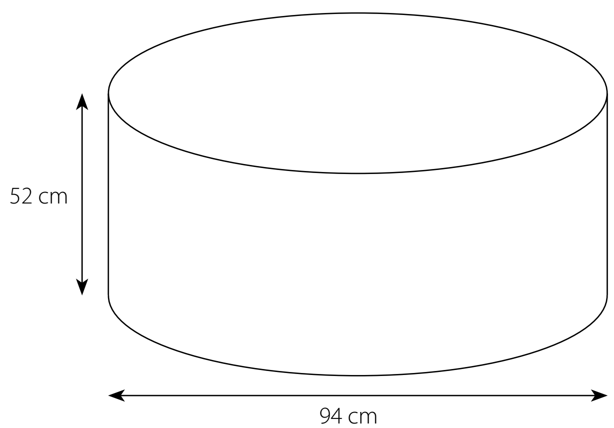


P $y = x^2 - 3x$ Q $4y = 3x + 12$

R $y = (1 - x)(x + 5)$ S $xy = 6$ [4]

7. I think of a number, add 4, multiply by 3 and subtract the result from 15. If I add another 1, I find that my final result is the same as the number I chose at the start. Use algebra to find this number. [4]

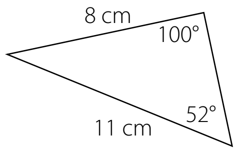
8. A cylindrical tank, open at the top, has diameter 94 cm and height 52 cm. Calculate the area of sheet metal required to construct such a tank. Answer in m^2 to 3 s.f.



(not to scale)

[4]

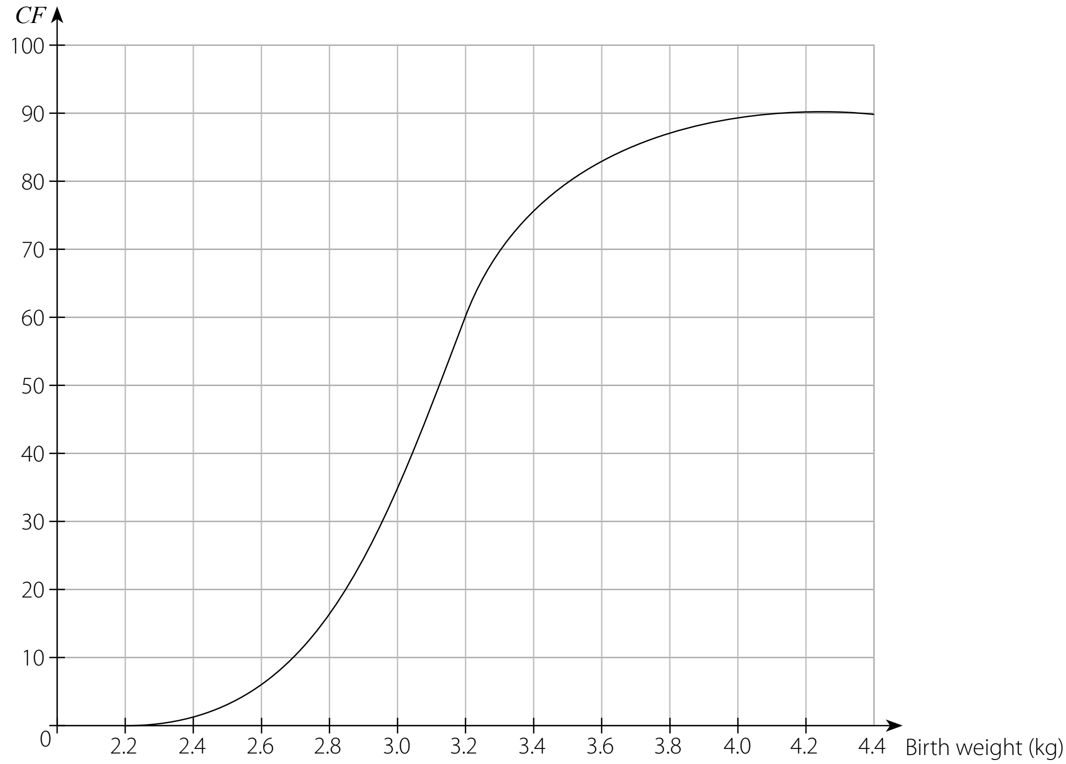
9. Calculate the area of this triangle, correct to 3 s.f.



(not to scale)

[4]

10. The cumulative frequency graph shows the weights at birth of 90 baby girls in a certain hospital.



From the graph, estimate the following:

a) the median baby weight

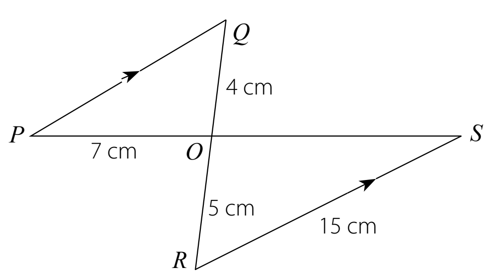
b) the number of babies born weighing over 4 kg

c) the 30th percentile

d) the interquartile range

[4]

11. In the diagram, PQ is parallel to RS.



(not to scale)

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a) Name three pairs of equal angles, giving reasons. [2]

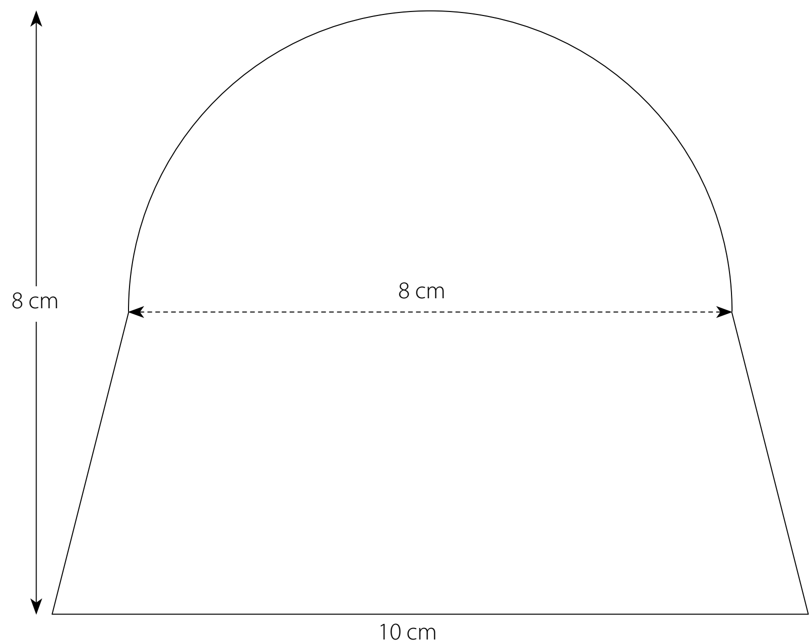
b) Calculate the lengths of PQ and OS. [3]

12. An isosceles triangle has two equal sides of 8 cm and two equal angles of 47°. Calculate (to 3 s.f.)

a) its perimeter, [2]

b) its area. [3]

13. The composite shape shown below is a trapezium and semicircle. Calculate its area.



[5]

14. A tank of water empties according to the formula

$h = 5.5\,(10 - t)$

where h cm is the depth of water remaining in the tank t minutes after the drain tap is opened.

a) What is the depth of water when the tap is opened?

b) What is the depth of water 30 seconds later?

c) Rearrange the formula to make t the subject.

d) How long does it take for the depth to go down to

(i) 11 cm? (ii) 2 cm?

[answer is minutes and seconds, to the nearest second.] [5]

15. True or false?

a) 0 is a prime number.

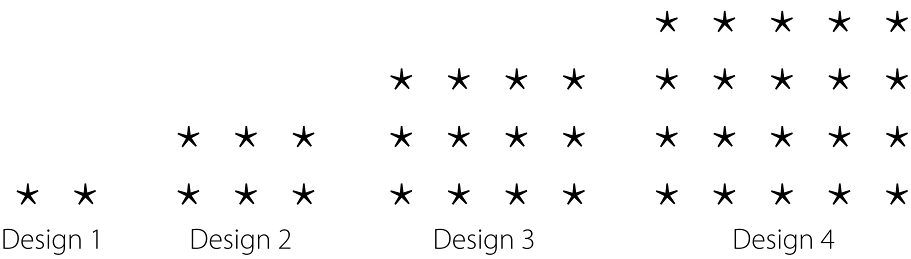
b) 1 is an odd number and is not a prime number.

c) 2 is an even number and is a prime number.

d) $-\frac{3}{4}$ is an integer.

e) 1001 is a prime number. [5]

16. Consider this sequence of designs:



a) How many rows in design n?

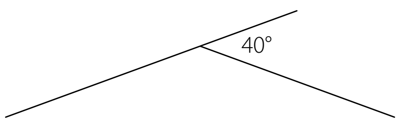
b) Write a sequence of products for the total number of stars in each design.

c) Find the nth term formula for the total number of stars in design n.

d) How many stars are in design 15?

e) In which design are there 506 stars? [5]

17. The diagram shows just one of the vertices of a regular polygon:



a) How many sides has the polygon?

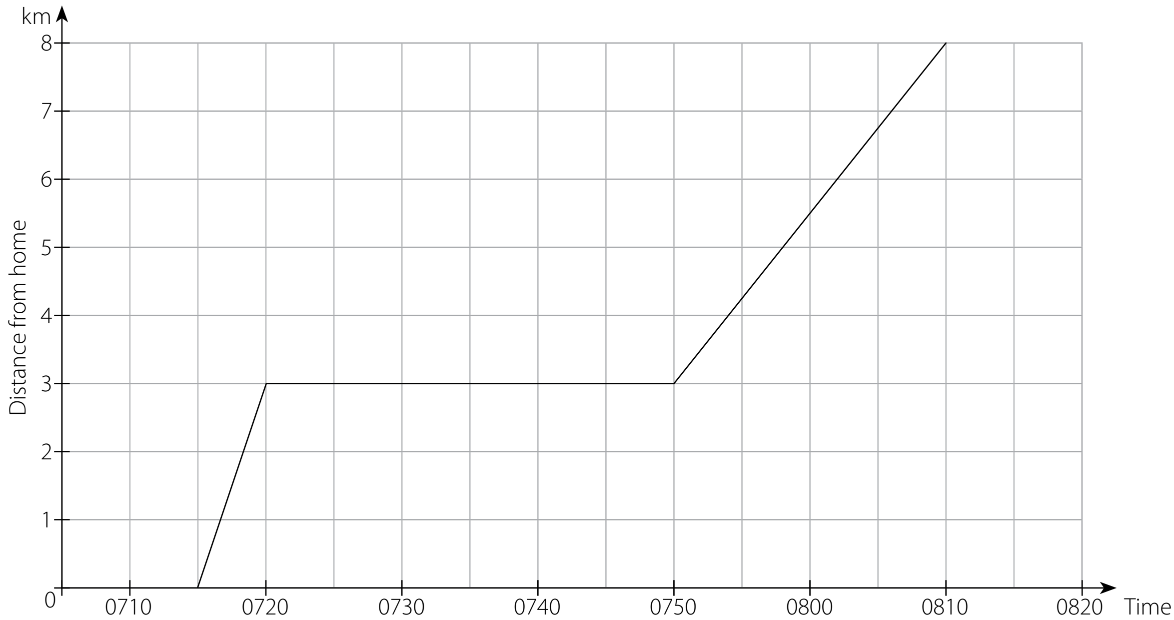
b) What is the size of each interior angle?

c) Use your answers to (a) and (b) to calculate the sum of the interior angles of this polygon.

d) State the standard formula for the sum of the interior angles of a polygon with n sides.

e) Make an equation from your answers to (c) and (d). Solve it to confirm your answer to (a). [5]

18. The graph shows Raza's journey to school. The school is 8 km from home. Raza's driver takes him first to Sohail's house. Then the two boys are taken together to school.



a) At what time does Raza leave home?

b) How far away is Sohail's house?

c) How long does Raza have to wait for Sohail?

d) Does the driver go faster before or after picking up Sohail? How do you know?

e) Calculate the average speed of the car over the whole journey, in km/h (1 d.p.). [5]

19. A ship travels 50 km on a bearing of 315°, then travels for a while on a bearing of 090°, and a final 50 km on bearing 225° which brings it back to the starting point. Calculate the total distance travelled, to the nearest km. [5]

20. Copy and complete the table of values for the curve whose equation is

$$y = 2x^2 - 4x - 3$$

x	−3	−2	−1	0	1	2	3	4	5
x^2		4							
$2x^2$							18		
$-4x$	12								−20
-3						−3			
y									

Without drawing the graph state the name of the shape of this curve, and the equation of its line of symmetry. [5]

Paper 1—answers and mark scheme:

1. a) 240

   b) 0.24

   c) 3.06

   d) 3000 [1 each = 4]

2. a) 43.588

   b) 43.6

   c) 43.59

   d) 40 [1 each = 4]

3. a) 20 13 6 −1 −8 −15

   −7 −7 −7 −7 −7

   linear

   b) n^{th} term $= -7n + 27$ (or $27 - 7n$)

   c) 20^{th} term $= -113$

   d) $-673 = -7n + 27$

   $7n = 700$

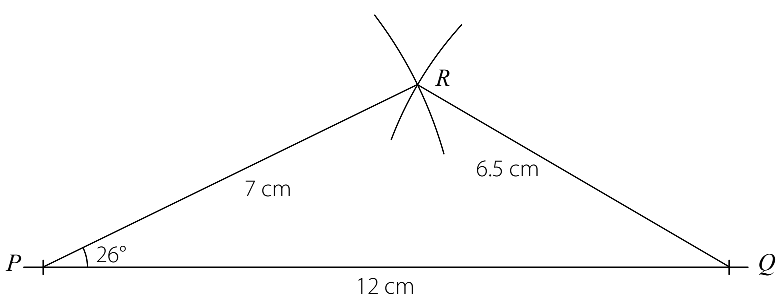
   $n = 100$ 100^{th} term [1 each = 4]

4. a) 3.12×10^4

   b) 3.6×10^7

   c) 4×10^{-2}

   d) 5.88×10^4 [1 each = 4]

5. 

   [ΔPQR labelled 1
   1 each arc 2
   $P = 26 \pm 1$ 1 } = 4]

6. a) $x = 5$ cm

   b) $x = 10$ cm

   c) $x = 30$ cm

   d) $x = 100$ cm [1 each = 4]

7. a) $3(x - 1) + 2(2x + 1) = 6$ [1

$x = 1$ 1

b) $8(x + 1) = 10 + 3(x - 3)$ 1

$x = -1.4$ (or $-\frac{7}{5}$) 1 = 4]

8. a) 5^6

b) 3^3

c) 2^{-4}

d) 2^9 [1 each = 4]

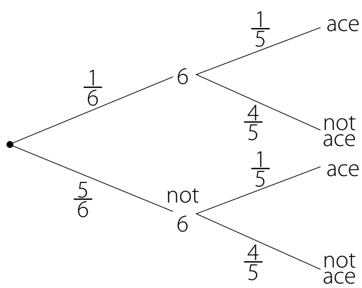
9. a) (adj ∠s on st line)

b) $b = 108$ (corr ∠s)

c) $b = 108$ (int ∠s)

d) $x = 180 - 82 = 98$ (int ∠s) [1 each = 4]

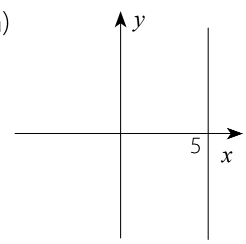
10. a)



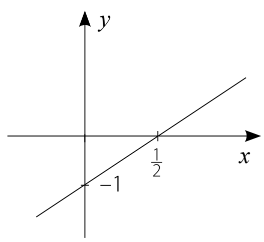
[2 (1 error = 1)]

b) (i) $\frac{1}{6}$ (ii) $\frac{2}{15}\left(\text{or } \frac{4}{30}\right)$ [1 each = 2]

11. a)



b)



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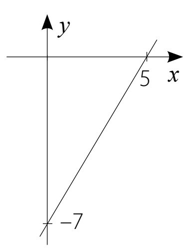
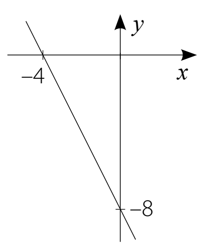
c)



d)



[1 each (must mark intersections) = 4]

12. a) $3x(1 + 2y - 3xy)$

b) $(5x - 4y)(5x + 4y)$

c) $(7x - 1)(x - 7)$

d) $2(x - 3)^2$ [1 each = 4]

13. a) reflection in the x-axis ($y = 0$) 1

b) rotation of 180° about the origin (0, 0) 1 = 4

c) enlargement: centre $C(5, 4)$, scale factor 3 2

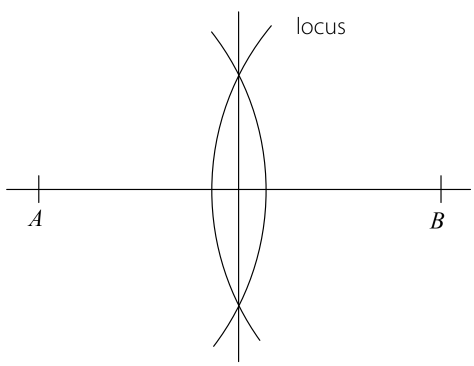
14. a) 8

b) 0.32

c) 20

d) 100 [1 each = 4]

15.



AB drawn/labelled	1	
1 each arc	2	= 4
Perp bisector locus	1	

Paper 2—answers and mark scheme:

1. a) -1.72

   b) 1.16 [2 each = 4]

2. a) 4.97 km/h

   b) 89 m

   c) 1 h 25 min

   $\left[\begin{array}{l}\text{(a) } 1\\ \text{(b) } 1\\ \text{(c) } 2\end{array}\right\} = 4\Bigg]$

3. a) \$ 25

   b) \$ 6 [2 each = 4]

4. a) $\sqrt{2^2 + 7^2}$

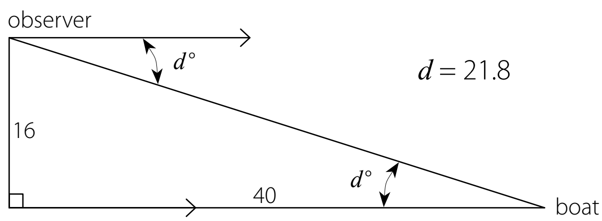
   7.3

   $\left[\begin{array}{l}1\\ 1\end{array}\right\} = 2\Bigg]$

   b) $11.5^2 = 132.25$

   $4.5^2 + 10.5^2 = 130.5$ (or equivalent)

   Pythagoras' theorem does not hold. [2]

5. 

   $d = 21.8$

   $\left[\begin{array}{ll}\text{Diag} & 2\\ \text{Ans} & 2\end{array}\right\} = 4\Bigg]$

6. a) Q

   b) P

   c) R

   d) S [1 each = 4]

7. $15 - 3(n + 4) + 1 = n$

   $n = 1$

   $\left[\begin{array}{l}2\\ 2\end{array}\right\} = 4\Bigg]$

8. $A = \pi r^2 + 2\pi rh = \pi r(r + 2h) = \pi \times 47 \times 151 \text{ cm}^2$

   $= 2.23 \text{ m}^2$

   $\left[\begin{array}{l}2\\ 2\end{array}\right\} = 4\Bigg]$

   or

   Base area $= \pi r^2 = \pi \times 47^2 = 6939.8 \text{ cm}^2$

   Curved area $= 2\pi rh = 2 \times \pi \times 47 \times 52 = 15\,356 \text{ cm}^2$

   Total surface area $= 2.23 \text{ m}^2$

   $\left[\begin{array}{l}1\\ 1\\ 2\end{array}\right\} = 4\Bigg]$

9. Third angle = 28°

Area $= \frac{1}{2} \times 8 \times 11 \times \sin 28°$ — 1

$= 20.7 \text{ cm}^2$ — 2   [1, 1, 2 = 4]

10. a) 3.1 kg

b) 2

c) 2.9 kg

d) 0.4 kg

[1 each = 4; Allow ± 0.1]

11. a) $\angle P = \angle S$ (alt $\angle$s)

$\angle Q = \angle R$ (alt $\angle$s)

$\angle POQ = \angle SOR$ (vert opp $\angle$s)

Any one of these may be replaced by (angle-sum of Δ)

[All 3 angles 2; (1 error 1)]

b) $PQ = 12$ cm, $OS = 8.75$ cm

Method: either scale factor = 1.25

or $\frac{PQ}{4} = \frac{15}{5}$ and $\frac{OS}{5} = \frac{7}{4}$

[Method 1; 1 Each answer = 3]

12. Various methods

a) 26.9 cm

b) 31.9 cm^2

[(a) 2; (b) 3 = 5]

13. Area semicircle $= \frac{1}{2}\pi 4^2 = 8\pi$ — 2

Area trapezium $= \frac{1}{2}(8 + 10)\,4 = 36$ — 2

Total area = 61.1 cm^2 (3 s.f.) — 1   [= 5]

14. a) 55 cm — 1

b) 52.25 cm — 1

c) $t = 10 - \frac{h}{5.5}$ or $t = \frac{55 - h}{5.5}$ — 1

d) (i) 8 min (ii) 9 min 38 s — 1 each   [= 5]

15. a) false

b) true

c) true

d) false

e) false

[1 each = 5]

16. a) n

b) $1 \times 2 \quad 2 \times 3 \quad 3 \times 4 \quad 4 \times 5 \quad \ldots$

c) nth term = $n(n + 1)$

d) 240

e) Design 22 [1 each = 5]

17. a) 9

b) 140°

c) 1260°

d) $180(n - 2)$

e) $1260 = 180(n - 2)$

$n = 9$ confirms (a) [1 each = 5]

18. a) 0715

b) 3 km

c) 30 min

d) before; gradient of line steeper.

e) 8.7 km/h [1 each = 5]

19.

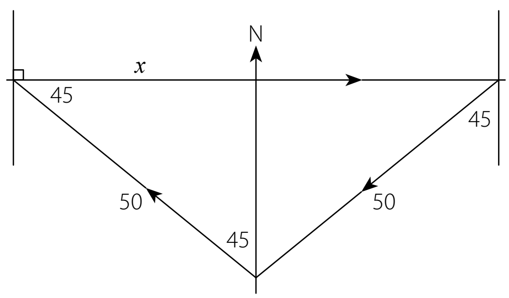


Diagram [2]

$x = 50 \sin 45°$

$2x = 100 \sin 45°$

$= 70.7$ [2]

Distance = 50 + 70.7 + 50

= 171 km (nearest km) 1]

= 5

20.

x	−3	−2	−1	0	1	2	3	4	5
x^2	9	4	1	0	1	4	9	16	25
y $2x^2$	18	8	2	0	2	8	18	32	50
$-4x$	12	8	4	0	−4	−8	−12	−16	−20 y
−3	−3	−3	−3	−3	−3	−3	−3	−3	−3
y	27	13	3	−3	−5	−3	3	13	27

Table 3 (−1 each error)

Parabola 1

Symmetry line $x = 1$ 1

= 5