



## Teaching Guide

NEW

# COUNTDOWN

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# Introduction

Welcome, users of the Countdown series. Countdown has been the choice of Mathematics teachers for many years. This Teaching Guide has been specially designed to help them teach mathematics in the best possible manner. It will serve as a reference book to streamline the teaching and learning experience in the classroom.

Teachers are entrusted with the task of providing support and motivation to their students, especially those who are at the lower end of the spectrum of abilities. In fact, their success is determined by the level of understanding demonstrated by the least able students.

Teachers regulate their efforts and develop a teaching plan that corresponds to the previous knowledge of the students and difficulty of the subject matter. The more well-thought out and comprehensive a teaching plan is, the more effective it is. This teaching guide will help teachers streamline the development of a lesson plan for each topic and guide the teacher on the level of complexity and amount of practice required for each topic. It also helps the teacher introduce effective learning tools to the students to complete their learning process.

Shazia Asad

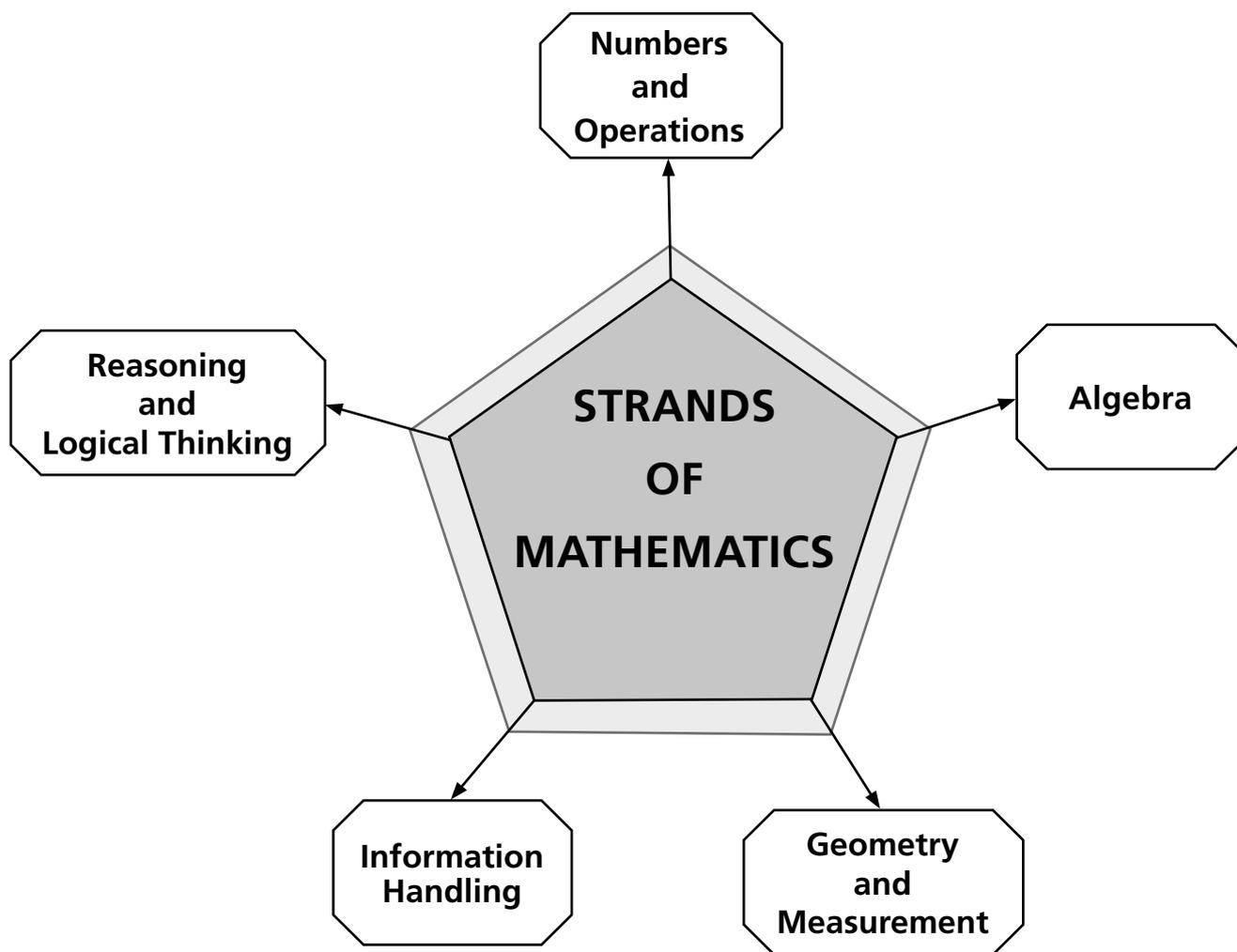


# Curriculum

## ► Strands and Benchmarks

(National Curriculum for Mathematics 2006)

The National Curriculum for Mathematics 2006 is based on these five strands:



## **Towards greater focus and coherence of a mathematical programme**

A comprehensive and coherent mathematical programme needs to allocate proportional time to all strands. A composite strand covers number, measurement and geometry, algebra, and information handling.

Each strand requires a focussed approach to avoid the pitfall of a broad general approach. If, say, an algebraic strand is approached, coherence and intertwining of concepts within the strand at all grade levels is imperative. The aims and objectives of the grades below and above should be kept in mind.

“What and how students are taught should reflect not only the topics that fall within a certain academic discipline, but also the key ideas that determine how knowledge is organised and generated within that discipline.”

William Schmidt and Richard Houang (2002)

## **Strands and Benchmarks of the Pakistan National Curriculum (2006)**

### **Strand 1: Numbers and Operations**

The students will be able to:

- identify numbers, ways of representing numbers, and effects of operations in various situations;
- compute fluently with fractions, decimals, and percentages, and
- manipulate different types of sequences and apply operations on matrices.

### **Benchmarks**

#### **Grades VI, VII, VIII**

- Identify different types of sets with notations
- Verify commutative, associative, distributive, and De Morgan’s laws with respect to union and intersection of sets and illustrate them through Venn diagrams
- Identify and compare integers, rational, and irrational numbers
- Apply basic operations on integers and rational numbers and verify commutative, associative, and distributive properties
- Arrange absolute values of integers in ascending and descending order
- Find HCF and LCM of two or more numbers using division and prime factorisation
- Convert numbers from decimal system to numbers with bases 2, 5, and 8, and vice versa
- Add, subtract, and multiply numbers with bases 2, 5, and 8
- Apply the laws of exponents to evaluate expressions
- Find square and square root, cube, and cube root of a real number
- Solve problems on ratio, proportion, profit, loss, mark-up, leasing, *zakat*, *ushr*, taxes, insurance, and money exchange

## Strand 2: Algebra

The students will be able to:

- analyse number patterns and interpret mathematical situations by manipulating algebraic expressions and relations;
- model and solve contextualised problems; and
- interpret functions, calculate rate of change of functions, integrate analytically and numerically, determine orthogonal trajectories of a family of curves, and solve non-linear equations numerically.

### Benchmarks

#### Grades VI, VII, VIII

- Identify algebraic expressions and basic algebraic formulae
- Apply the four basic operations on polynomials
- Manipulate algebraic expressions using formulae
- Formulate linear equations in one and two variables
- Solve simultaneous linear equations using different techniques

## Strand 3: Measurement and Geometry

The students will be able to:

- identify measurable attributes of objects, and construct angles and two dimensional figures;
- analyse characteristics and properties and geometric shapes and develop arguments about their geometric relationships; and
- recognise trigonometric identities, analyse conic sections, draw and interpret graphs of functions.

### Benchmarks

#### Grades VI, VII, VIII

- Draw and subdivide a line segment and an angle
- Construct a triangle (given SSS, SAS, ASA, RHS), parallelogram, and segments of a circle
- Apply properties of lines, angles, and triangles to develop arguments about their geometric relationships
- Apply appropriate formulas to calculate perimeter and area of quadrilateral, triangular, and circular regions
- Determine surface area and volume of a cube, cuboid, sphere, cylinder, and cone
- Find trigonometric ratios of acute angles and use them to solve problems based on right-angled triangles

#### **Strand 4: Handling Information**

The students will be able to collect, organise, analyse, display, and interpret data.

##### **Benchmarks**

##### **Grades VI, VII, VIII**

- Read, display, and interpret bar and pie graphs
- Collect and organise data, construct frequency tables and histograms to display data
- Find measures of central tendency (mean, median and mode)

#### **Strand 5: Reasoning and Logical Thinking**

The students will be able to:

- use patterns, known facts, properties, and relationships to analyse mathematical situations;
- examine real-life situations by identifying mathematically valid arguments and drawing conclusions to enhance their mathematical thinking.

##### **Benchmarks**

##### **Grades VI, VII, VIII**

- Find different ways of approaching a problem to develop logical thinking and explain their reasoning
- Solve problems using mathematical relationships and present results in an organised way
- Construct and communicate convincing arguments for geometric situations

## ► Syllabus Matching Grid

Unit 1: Operations on Sets	
<p><b>1.1 Sets</b></p> <p>i) Recognise set of</p> <ul style="list-style-type: none"> <li>• natural numbers (N),</li> <li>• whole numbers (W),</li> <li>• integers (Z),</li> <li>• rational numbers (Q),</li> <li>• even numbers (E),</li> <li>• odd numbers (O),</li> <li>• prime numbers (P).</li> </ul> <p>ii) Find a subset of a set.</p> <p>iii) Define proper (<math>\subset</math>) and improper (<math>\subseteq</math>) subsets of a set.</p> <p>iv) Find power set <math>P(A)</math> of a set <math>A</math>.</p>	Chapter 1
<p><b>1.2 Operations on Sets</b></p> <p>i) Verify commutative and associative laws with respect to union and intersection.</p> <p>ii) Verify the distributive laws.</p> <p>iii) State and verify De Morgan's laws.</p>	
<p><b>1.3 Venn Diagram</b></p> <p>i) Demonstrate union and intersection of three overlapping sets through Venn diagram.</p> <p>ii) Verify associative and distributive laws through Venn diagram.</p>	
Unit 2: Real Numbers	
<p><b>2.1 Irrational Numbers</b></p> <p>i) Define an irrational number.</p> <p>ii) Recognise rational and irrational numbers.</p> <p>iii) Define real numbers.</p> <p>iv) Demonstrate non-terminating /non-repeating (or non-periodic) decimals.</p>	Chapter 2
<p><b>2.2 Squares</b></p> <p>i) Find perfect square of a number.</p> <p>ii) Establish patterns for the squares of natural numbers.</p>	

<p><b>2.3 Square Roots</b></p> <p>i) Find square root of</p> <ul style="list-style-type: none"> <li>• a natural number,</li> <li>• a common fraction,</li> <li>• a decimal,</li> </ul> <p>given in perfect square form, by prime factorisation and division method.</p> <p>ii) Find square root of a number which is not a perfect square.</p> <p>iii) Use the following rule to determine the number of digits in the square root of a perfect square. Rule: Let <math>n</math> be the number of digits in the perfect square then its square root contains</p> <p style="text-align: center;"><math>\frac{n}{2}</math> digits if <math>n</math> is even, <math>\frac{n+1}{2}</math> digits if <math>n</math> is odd.</p> <p>iv) Solve real-life problems involving square roots.</p>	Chapter 2
<p><b>2.4 Cubes and Cube Roots</b></p> <p>i) Recognise cubes and perfect cubes.</p> <p>ii) Find cube roots of a number which are perfect cubes.</p> <p>iii) Recognise properties of cubes of numbers.</p>	Chapter 3
<b>Unit 3: Numbers Systems</b>	
<p><b>3.1 Number System</b></p> <p>i) Recognise base of a number system.</p> <p>ii) Define number system with base 2, 5, 8 and 10.</p> <p>iii) Explain</p> <ul style="list-style-type: none"> <li>• binary number system (system with base 2).</li> <li>• number system with base 5,</li> <li>• octal number system (system with base 8),</li> <li>• decimal number system (system with base 10).</li> </ul>	Chapter 4
<p><b>3.2 Conversions</b></p> <p>i) Convert a number from decimal system to a system with bases 2, 5 and 8, and vice versa.</p> <p>ii) Add, subtract and multiply numbers with bases 2, 5 and 8.</p> <p>iii) Add, subtract and multiply numbers with different bases.</p>	
<b>Unit 4 Financial Arithmetic</b>	
<p><b>4.1 Compound Proportion</b></p> <p>i) Define compound proportion.</p> <p>ii) Solve real-life problems involving compound proportion, partnership and inheritance.</p>	Chapter 8

<p><b>4.2 Banking</b></p> <p><b>4.2.1 Types of a Bank Account</b></p> <p>i) Define commercial bank deposits, types of a bank account (PLS savings bank account, current deposit account, PLS term deposit account and foreign currency account).</p> <p>ii) Describe negotiable instruments like cheque, demand draft and pay order.</p> <p><b>4.2.2 On-line banking</b></p> <p>iii) Explain on-line banking, transactions through ATM (Auto Teller Machine), debit card and credit card (Visa and Master).</p> <p><b>4.2.3 Conversion of Currencies</b></p> <p>iv) Convert Pakistani currency to well-known international currencies.</p> <p><b>4.2.4 Profit/Markup</b></p> <p>v) Calculate</p> <ul style="list-style-type: none"> <li>• the profit/markup,</li> <li>• the principal amount,</li> <li>• the profit/markup rate,</li> <li>• the period.</li> </ul> <p><b>4.2.5 Types of Finance</b></p> <p>vi) Explain</p> <ul style="list-style-type: none"> <li>• Overdraft (OD),</li> <li>• Running Finance (RF),</li> <li>• Demand Finance (DF),</li> <li>• Leasing.</li> </ul> <p>vii) Solve real-life problems related to banking and finance.</p>	Chapter 8
<p><b>4.3 Percentage</b></p> <p><b>4.3.1 Profit and Loss</b></p> <p>i) Find percentage profit and percentage loss.</p> <p><b>4.3.2 Discount</b></p> <p>ii) Find percentage discount.</p> <p>iii) Solve problems involving successive transactions.</p>	Chapter 7
<p><b>4.4 Insurance</b></p> <p>i) Define insurance.</p> <p>ii) Solve real-life problems regarding life and vehicle insurance.</p>	

Unit 5 Polynomials	
<b>5.1 Algebraic Expression</b> i) Recall constant, variable, literal and algebraic expression.	Chapter 12
<b>5.2 Polynomial</b> i) Define <ul style="list-style-type: none"> <li>• polynomial,</li> <li>• degree of a polynomial,</li> <li>• coefficients of a polynomial.</li> </ul> ii) Recognise polynomial in one, two and more variables. iii) Recognise polynomials of various degrees (e.g., linear, quadratic, cubic and biquadratic polynomials).	
<b>5.3 Operations on Polynomials</b> i) Add, subtract and multiply polynomials. ii) Divide a polynomial by a linear polynomial.	
Unit 6 Factorisation, Simultaneous Equations	
<b>6.1 Basic Algebraic Formulas</b> Recall the formulas: <ul style="list-style-type: none"> <li>• <math>(a + b)^2 = a^2 + 2ab + b^2</math>,</li> <li>• <math>(a - b)^2 = a^2 - 2ab + b^2</math>,</li> <li>• <math>a^2 - b^2 = (a - b)(a + b)</math>,</li> </ul> and apply them to solve problems.	Chapter 13, 14, 15, 16
<b>6.2 Factorisation</b> Factorise expressions of the following types: <ul style="list-style-type: none"> <li>• <math>ka + kb + kc</math>,</li> <li>• <math>ac + ad + bc + bd</math>,</li> <li>• <math>a^2 \pm 2ab + b^2</math>,</li> <li>• <math>a^2 - b^2</math>,</li> <li>• <math>a^2 \pm 2ab + b^2 - c^2</math></li> </ul>	
<b>6.3 Manipulation of Algebraic</b> Recognise the formulas: <ul style="list-style-type: none"> <li>• <math>(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3</math>,</li> <li>• <math>(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3</math>,</li> </ul> and apply them to solve the problems.	

<p><b>6.4 Simultaneous Linear Equations</b></p> <p>i) Recognise simultaneous linear equations in one and two variables.</p> <p>ii) Give the concept of formation of linear equation in two variables.</p> <p>iii) Know that:</p> <ul style="list-style-type: none"> <li>• a single linear equation in two unknowns is satisfied by as many pair of values as required.</li> <li>• two linear equations in two unknowns have only one solution (i.e., one pair of values).</li> </ul>	
<p><b>6.5 Solution of Simultaneous Equations</b></p> <p>i) Solve simultaneous linear equations using linear equations</p> <ul style="list-style-type: none"> <li>• method of equating the coefficients,</li> <li>• method of elimination by substitution,</li> <li>• method of cross multiplication.</li> </ul> <p>ii) Solve real-life problems involving two simultaneous linear equations in two variables.</p>	Chapter 17
<p><b>6.6 Elimination</b></p> <p>i) Eliminate a variable from two equations by:</p> <ul style="list-style-type: none"> <li>• Substitution,</li> <li>• application of formulae.</li> </ul>	
<b>Unit 7 Fundamentals of Geometry</b>	
<p><b>7.1 Parallel Lines</b></p> <p>i) Define parallel lines.</p> <p>ii) Demonstrate through figures the following properties of parallel lines.</p> <ul style="list-style-type: none"> <li>• Two lines which are parallel to the same given line are parallel to each other.</li> <li>• If three parallel lines are intersected by two transversals in such a way that the two intercepts on one transversal are equal to each other, the two intercepts on the second transversal are also equal.</li> <li>• A line through the midpoint of the side of a triangle parallel to another side bisects the third side (an application of above property).</li> </ul> <p>iii) Draw a transversal to intersect two parallel lines and demonstrate corresponding angles, alternate interior angles, vertically opposite angles and interior angles on the same side of transversal.</p> <p>iv) Describe the following relations between the pairs of angles when a transversal intersects two parallel lines.</p> <ul style="list-style-type: none"> <li>• Pairs of corresponding angles are equal.</li> <li>• Pairs of alternate interior angles are equal.</li> <li>• Pair of interior angles on the same side of transversal is supplementary, and demonstrate them through figures.</li> </ul>	* Covered in Book 7 Chapter 12

<p><b>7.2 Polygons</b></p> <p>i) Define a polygon.</p> <p>ii) Demonstrate the following properties of a parallelogram.</p> <ul style="list-style-type: none"> <li>• Opposite sides of a parallelogram are equal.</li> <li>• Opposite angles of a parallelogram are equal.</li> <li>• Diagonals of a parallelogram bisect each other.</li> </ul> <p>iii) Define regular pentagon, hexagon and octagon.</p>	<p>Chapter 19</p>
<p><b>7.3 Circle</b></p> <p>i) Demonstrate a point lying in the interior and exterior of a circle.</p> <p>ii) Describe the terms: sector, secant and chord of a circle, concyclic points, tangent to a circle and concentric circles.</p>	<p>* Covered in Book 7 Chapter 14</p>
<p><b>Unit 8 Practical Geometry</b></p>	
<p><b>8.1 Construction of Quadrilaterals</b></p> <p>i) Define and depict two converging (non-parallel) lines and find the angle between them without producing the lines.</p> <p>ii) Bisect the angle between the two converging lines without producing them.</p> <p>iii) Construct a square</p> <ul style="list-style-type: none"> <li>• when its diagonal is given,</li> <li>• when the difference between its diagonal and side is given,</li> <li>• when the sum of its diagonal and side is given,</li> </ul> <p>iv) Construct a rectangle</p> <ul style="list-style-type: none"> <li>• when two sides are given,</li> <li>• when the diagonal and a side are given,</li> </ul> <p>v) Construct a rhombus</p> <ul style="list-style-type: none"> <li>• when one side and the base angle are given,</li> <li>• when one side and a diagonal are given,</li> </ul> <p>vi) Construct a parallelogram</p> <ul style="list-style-type: none"> <li>• when two diagonals and the angle between them is given,</li> </ul> <p>vii) Construct a kite</p> <ul style="list-style-type: none"> <li>• when two unequal sides and a diagonal are given.</li> </ul> <p>viii) Construct a regular pentagon</p> <ul style="list-style-type: none"> <li>• when a side is given.</li> </ul> <p>ix) Construct a regular hexagon</p> <ul style="list-style-type: none"> <li>• when a side is given</li> </ul>	<p>Chapter 19</p>
<p><b>8.2 Construction of a Right-angled Triangle</b></p> <p>Construct a right-angled triangle</p> <ul style="list-style-type: none"> <li>• when hypotenuse and one side are given,</li> <li>• when hypotenuse and the vertical height from its vertex to the hypotenuse are given.</li> </ul>	

<b>Unit 9 Areas and Volumes</b>	
<b>9.1 Pythagoras Theorem</b> i) State the Pythagoras theorem and give its informal proof. ii) Solve right-angled triangles using Pythagoras theorem.	Chapter 22
<b>9.2 Hero's Formula</b> State and apply Hero's formula to find the areas of triangular and quadrilateral regions.	Chapter 21
<b>9.3 Surface Area and Volume</b> i) Find the surface area and volume of a sphere. ii) Find the surface area and volume of a cone. iii) Solve real-life problems involving surface area and volume of sphere and cone.	
<b>Unit 10 Demonstrative Geometry</b>	
<b>10.1 Demonstrative geometry</b> i) Define demonstrative geometry. <b>10.1.1 Reasoning</b> i) Describe the basics of reasoning. <b>10.1.2 Axioms, Postulates and Theorem</b> i) Describe the types of assumptions (axioms and postulates). ii) Describe parts of a proposition. iii) Describe the meanings of a geometrical theorem, corollary and converse of a theorem.	Chapter 20
<b>10.2 Theorems</b> Prove the following theorems along with corollaries and apply them to solve appropriate problems. i) If a straight line stands on another straight line, the sum of measures of two angles so formed is equal to two right angles. ii) If the sum of measures of two adjacent angles is equal to two right angles, the external arms of the angles are in a straight line. iii) If two lines intersect each other, then the opposite vertical angles are congruent. iv) In any correspondence of two triangles, if two sides and included angle of one triangle are congruent to the corresponding sides and included angle of the other, the two triangles are congruent. v) If two sides of a triangle are congruent, then the angles opposite to these sides are congruent. vi) An exterior angle of a triangle is greater in measure than either of its opposite interior angles.	

<p>vii) If a transversal intersects two lines such that the pair of alternate angles are congruent then the lines are parallel.</p> <p>viii) If a transversal intersects two parallel lines the alternate angles so formed are congruent.</p> <p>ix) The sum of measures of the three angles of a triangle is <math>180^\circ</math></p>	Chapter 20
<b>Unit 11 Introduction to Trigonometry</b>	
<p><b>11.1 Trigonometry</b></p> <p>i) Define trigonometry.</p>	Chapter 22
<p><b>11.2 Trigonometric Ratios of Acute Angles</b></p> <p>i) Define trigonometric ratios of an acute angle.</p> <p>ii) Find trigonometric ratios of acute angles (<math>30^\circ</math>, <math>60^\circ</math> and <math>45^\circ</math>).</p> <p>iii) Define trigonometric ratios of complementary angles.</p> <p>iv) Solve right-angled triangles using trigonometric ratios.</p> <p>v) Solve real-life problems to find heights</p>	
<b>Unit 12 Information Handling</b>	
<p><b>12.1 Frequency Distribution</b></p> <p>i) Define frequency, frequency distribution.</p> <p>ii) Construct frequency table.</p> <p>iii) Construct a histogram representing frequency table.</p>	Chapter 23
<p><b>12.2 Measures of Central Tendency</b></p> <p>i) Describe measures of central tendency.</p> <p>ii) Calculate mean (average), weighted mean, median and mode for ungrouped data.</p> <p>iii) Solve real-life problems involving mean (average), weighted mean, median and mode.</p>	



# Teaching and Learning

## ► Guiding Principles

1. Students explore mathematical ideas in ways that maintain their enjoyment of and curiosity about mathematics, help them develop depth of understanding, and reflect real-world applications.
2. All students have access to high quality mathematics programmes.
3. Mathematics learning is a lifelong process that begins and continues in the home and extends to school, community settings, and professional life.
4. Mathematics instruction both connects with other disciplines and moves toward integration of mathematical domains.
5. Working together in teams and groups enhances mathematical learning, helps students communicate effectively, and develops social and mathematical skills.
6. Mathematics assessment is a multifaceted tool that monitors student performance, improves instruction, enhances learning, and encourages student self-reflection.

## Principle 1

Students explore mathematical ideas in ways that maintain their enjoyment of and curiosity about mathematics, help them develop depth of understanding, and reflect real-world applications.

- The understanding of mathematical concepts depends not only on what is taught, but also hinges on the way the topic is taught.
- In order to plan developmentally appropriate work, it is essential for teachers to familiarise themselves with each individual student's mathematical capacity.
- Students can be encouraged to muse over their learning and express their reasoning through questions such as;
  - *How did you work through this problem?*
  - *Why did you choose this particular strategy to solve the problem?*
  - *Are there other ways? Can you think of them?*
  - *How can you be sure you have the correct solution?*
  - *Could there be more than one correct solution?*
  - *How can you convince me that your solution makes sense?*
- For effective development of mathematical understanding students should undertake tasks of inquiry, reasoning, and problem solving which are similar to real-world experiences.

- Learning is most effective when students are able to establish a connection between the activities within the classroom and real-world experiences.
- Activities, investigations, and projects which facilitate a deeper understanding of mathematics should be strongly encouraged as they promote inquiry, discovery, and mastery.
- Questions for teachers to consider when planning an investigation:
  - *Have I identified and defined the mathematical content of the investigation, activity, or project?*
  - *Have I carefully compared the network of ideas included in the curriculum with the students' knowledge?*
  - *Have I noted discrepancies, misunderstandings, and gaps in students' knowledge as well as evidence of learning?*

## Principle 2

All students have access to high quality mathematics programmes.

- Every student should be fairly represented in a classroom and be ensured access to resources.
- Students develop a sense of control of their future if a teacher is attentive to each student's ideas.

## Principle 3

Mathematics learning is a lifelong process that begins and continues in the home and extends to school, community settings, and professional life.

- The formation of mathematical ideas is a part of a natural process that accompanies pre-kindergarten students' experience of exploring the world and environment around them. Shape, size, position, and symmetry are ideas that can be understood by playing with toys that can be found in a child's playroom, for example, building blocks.
- Gathering and itemising objects such as stones, shells, toy cars, and erasers, leads to discovery of patterns and classification. At secondary level research data collection, for example, market reviews of the stock market and world economy, is an integral continued learning process. Within the environs of the classroom, projects and assignments can be set which help students relate new concepts to real-life situations.

## Principle 4

Mathematics instruction both connects with other disciplines and moves toward integration of mathematical domains.

An evaluation of maths textbooks considered two critical points. The first was, did the textbook include a variety of examples and applications at different levels so that students could proceed from simple to more complex problem-solving situations?

And the second was whether algebra and geometry were truly integrated rather than presented alternately.

- It is important to understand that students are always making connections between their mathematical understanding and other disciplines in addition to the connections with their world.
- An integrated approach to mathematics may include activities which combine sorting, measurement, estimation, and geometry. Such activities should be introduced at primary level.
- At secondary level, connections between algebra and geometry, ideas from discrete mathematics, statistics, and probability, establish connections between mathematics and life at home, at work, and in the community.

- What makes integration efforts successful is open communication between teachers. By observing each other and discussing individual students teachers improve the mathematics programme for students and support their own professional growth.

### Principle 5

Working together in teams and groups enhances mathematical learning, helps students communicate effectively, and develops social and mathematical skills.

- The Common Core of Learning suggests that teachers 'develop, test, and evaluate possible solutions'.
- Team work can be beneficial to students in many ways as it encourages them to interact with others and thus enhances self-assessment, exposes them to multiple strategies, and teaches them to be members of a collective workforce.
- Teachers should keep in mind the following considerations when dealing with a group of students:
  - *High expectations and standards should be established for all students, including those with gaps in their knowledge bases.*
  - *Students should be encouraged to achieve their highest potential in mathematics.*
  - *Students learn mathematics at different rates, and the interest of different students' in mathematics varies.*
- Support should be made available to students based on individual needs.
- Levels of mathematics and expectations should be kept high for all students.

### Principle 6

Mathematics assessment in the classroom is a multifaceted tool that monitors student performance, improves instruction, enhances learning, and encourages student self-reflection.

- An open-ended assessment facilitates multiple approaches to problems and creative expression of mathematical ideas.
- Portfolio assessments imply that teachers have worked with students to establish individual criteria for selecting work for placement in a portfolio and judging its merit.
- Using observation for assessment purposes serves as a reflection of a students' understanding of mathematics, and the strategies he/she commonly employs to solve problems and his/her learning style.

### Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Use appropriate tools strategically.
5. Attend to precision and format.
6. Express regularity in repetitive reasoning.
7. Analyse mathematical relationships and use them to solve problems.
8. Apply and extend previous understanding of operations.
9. Use properties of operations to generate equivalent expressions.
10. Investigate, process, develop, and evaluate data.

## ▶ Lesson Planning

Before starting lesson planning, it is imperative to consider teaching and the art of teaching.

### FURL

**F**irst **U**nderstand by **R**elating to day-to-day routine, and then **L**earn. It is vital for teachers to relate fine teaching to real-life situations and routine.

'R' is re-teaching and revising, which of course falls under the supplementary/continuity category. Effective teaching stems from engaging every student in the classroom. This is only possible if you have a comprehensive lesson plan.

There are three integral facets to lesson planning: curriculum, instruction, and evaluation.

#### 1. Curriculum

A syllabus should pertain to the needs of the students and objectives of the school. It should be neither over-ambitious, nor lacking. (One of the major pitfalls in school curricula arises in planning of mathematics.)

#### 2. Instructions

Any method of instruction, for example verbal explanation, material aided explanation, or teach-by-asking can be used. The method adopted by the teacher reflects his/her skills. Experience alone does not work, as the most experienced teachers sometime adopt a short-sighted approach; the same could be said for beginner teachers. The best teacher is the one who works out a plan that is customised to the needs of the students, and only such a plan can succeed in achieving the desired objectives.

#### 3. Evaluation

The evaluation process should be treated as an integral teaching tool that tells the teachers how effective they have been in their attempt to teach the topic. No evaluation is just a test of student learning; it also assesses how well a teacher has taught.

Evaluation has to be an ongoing process; during the course of study formal teaching should be interspersed with thought-provoking questions, quizzes, assignments, and classwork.

### Long-term Lesson Plan

A long-term lesson plan extends over the entire term. Generally schools have coordinators to plan the big picture in the form of Core Syllabus and Unit Studies.

Core syllabi are the topics to be covered during a term. Two things which are very important during planning are the 'Time Frame' and the 'Prerequisites' of the students.

An experienced coordinator will know the depth of the topic and the ability of the students to grasp it in the assigned time frame.

### Suggested Unit Study Format

Weeks	Dates	Months	Days	Remarks

### Short-term Lesson Planning

A short-term plan is a day-to-day lesson plan, based on the sub-topics chosen from the long-term plan.

## Features of the Teaching Guide

The Teaching Guide contains the following features. The headings through which the teachers will be led are explained as follows.



### Specific Learning Objectives

Each topic is explained clearly by the author in the textbook with detailed explanation, supported by worked examples. The guide will define and highlight the objectives of the topic. It will also outline the learning outcomes and objectives.



### Suggested Time Frame

Timing is important in each of the lesson plans. The guide will provide a suggested time frame. However, every lesson is important in shaping the behavioural and learning patterns of the students. The teacher has the discretion to either extend or shorten the time frame as required.



### Prior Knowledge and Revision

It is important to highlight any background knowledge of the topic in question. The guide will identify concepts taught earlier or, in effect, revise the prior knowledge. Revision is essential, otherwise the students may not understand the topic fully.

The initial question when planning for a topic should be how much do the students already know about the topic? If it is an introductory lesson, then a preceding topic could be touched upon, which could lead on to the new topic. In the lesson plan, the teacher can note what prior knowledge the students have of the current topic.



### Real-life Application and Activities

Today's students are very proactive. The study of any topic, if not related to practical real-life, will not excite them. Their interest can easily be stimulated if we relate the topic at hand to real-life experiences. Activities and assignments will be suggested which will do just that.

Flash cards based on the concept being taught will have more impact.



### Summary of Key Facts

Facts and rules mentioned in the text are listed for quick reference.



### Frequently Made Mistakes

It is important to be aware of students' common misunderstandings of certain concepts. If the teacher is aware of these they can be easily rectified during the lessons. Such topical misconceptions are mentioned.



## Sample Lesson Plan

Planning your work and then implementing your plan are the building blocks of teaching. Teachers adopt different teaching methods/approaches to a topic.

A sample lesson plan is provided in every chapter as a preliminary structure that can be followed. A topic is selected and a lesson plan written under the following headings:

### Topic

This is the main topic/sub-topic.

### Specific Learning Objectives

This identifies the specific learning objective/s of the sub-topic being taught in that particular lesson.

### Suggested Duration

Suggested duration is the number of periods required to cover the topic. Generally, class dynamics vary from year to year, so flexibility is important.

The teacher should draw his/her own parameters, but can adjust the teaching time depending on the receptivity of the class to that topic. Note that introduction to a new topic takes longer, but familiar topics tend to take less time.

### Key vocabulary

List of mathematical words and terms related to the topic that may need to be pre-taught.

### Method and Strategy

This suggests how you could demonstrate, discuss, and explain a topic.

The introduction to the topic can be done through starter activities and recap of previous knowledge which can be linked to the current topic.

### Resources (Optional)

This section includes everyday objects and models, exercises given in the chapter, worksheets, assignments, and projects.

### Written Assignments

Finally, written assignments can be given for practice. It should be noted that classwork should comprise sums of all levels of difficulty, and once the teacher is sure that students are capable of independent work, homework should be handed out. For continuity, alternate sums from the exercises may be done as classwork and homework.

**Supplementary Work (Optional):** A project or assignment could be given. It could involve group work or individual research to complement and build on what students have already learnt in class. The students will do the work at home and may present their findings in class.

### Evaluation

At the end of each sub-topic, practice exercises should be done. For further practice, the students can be given a practice worksheet or a comprehensive marked assessment.



# Operations on Sets



## Specific Learning Objectives

In this unit students will learn:

- how to define a subset and a power set.
- how to use Venn diagrams to define union and intersection of three sets.
- how to verify commutative and associative properties of sets.
- how to verify distributive property of union over intersection.
- how to verify De Morgan's laws.



## Suggested Time Frame

6 to 8 periods.



## Prior Knowledge and Revision

Before starting a chapter it is essential to revise the basic concepts of the topic that were introduced in previous grades. There are various ways of revising. The teacher can initiate a class discussion by asking students to describe and explain concepts they already know. As a further recall technique they can be divided into groups and the board can be divided into two columns. Have a brainstorming revision session by calling students to the board. The teach-by-asking method is a very successful way of revising and also of introducing or explaining a topic.

Students have learnt about sets in previous years and they are aware of the basic concepts of intersection, union, and subsets.

The students should revise complements, union, and intersection, as well as the signs and symbols of subsets. Union is the joining of two sets, where the common and the uncommon elements are written once. A union set is denoted by ' $\cup$ ', which is easy to remember as union begins with ' $\cup$ '. Intersection of a set includes the common elements only, which is obvious from the term itself. It is represented by ' $\cap$ '.

The complement of a set ( $A'$ ) contains the elements that are not in the universal set. For example,  $A'$  would have all the elements of the universal set that are not members of set  $A$ .

**Example:**

Universal Set = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

Set A = {1, 2, 3, 4}

Then Set A' = {5, 6, 7, 8, 9, 10} (complement of Set A)

An interesting concept: the intersection of a set with its complement will always be a null set.

In the above example:

Set A = {1, 2, 3, 4}

A' = {5, 6, 7, 8, 9, 10}

$A \cap A' = \emptyset$  or  $\{\}$

Similarly the union of a set with its complement will give the universal set, provided that set is the only set of the universal set.

Set A = {1, 2, 3, 4}

A' = {5, 6, 7, 8, 9, 10}

$A \cup A' = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A \cup A' = U$

In this chapter the concept of the power set is introduced. It tells us how to ascertain the number of subsets that can be made from a given set.

**Example:**

If  $A = \{1, 2, 3, 4\}$ , then the number of subsets formed will be:

$P(A) = 2^k$  (where  $k$  is equal to the number of elements in set  $A$ ).

$2^4 = 16$  Thus 16 subsets will be formed for Set  $A$ .

Exercise 1 contains problems related to power set and combines all previously learnt concepts of sets. It should not be difficult for students to solve the exercise once the concepts have been revised.



## Real-life Application and Activities

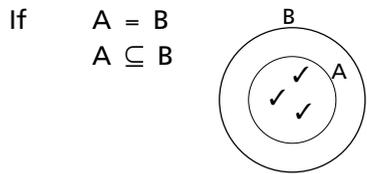
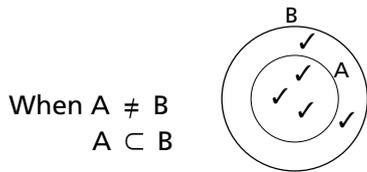
This chapter at this level becomes quite technical. The key terminology will be of significance to the teacher.

However the teacher can give a fun quiz where the students can be divided into two groups; one group prepares questions related to the topic and the other group sends its members to answer in turn. Then the activities of the groups are swapped. The group with the most points wins. The teacher can play the role of moderator.



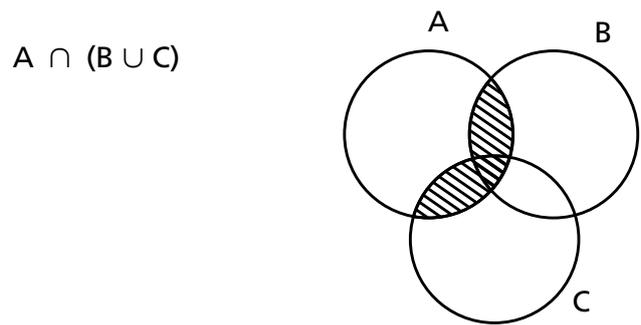
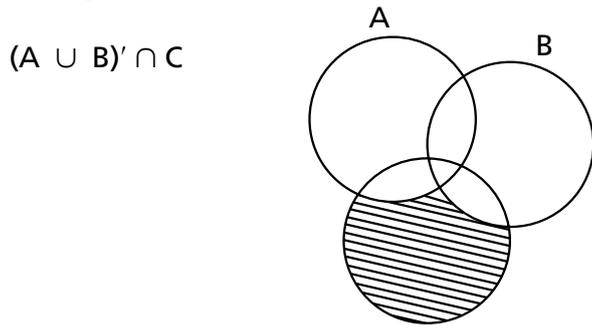
## Summary of Key Facts

- A subset is a set which is also a part of the bigger set where all the elements of the subset are in the main set.
- A subset can be a proper subset if the superset has at least one element which is not a part of the subset. Hence subset  $A$  can never be equal to subset  $B$ .



- The power set is calculated by the formula given below:  
 $2$  to the power of  $k$ , where  $k$  is the number of elements in set  $A$ .
- Union and intersection of three sets are best done with the help of Venn diagrams.

**Example:**



- The commutative property of the union of sets states that the order does not matter.
- The associative property of the union and intersection of three sets states that the order does not matter .
- The distributive property of the union of three sets states that the order of operations does not affect the result.
- De Morgan's law states that the the complement of the union of two sets is equal to the intersection of the complements of the two sets.



## Frequently Made Mistakes

Students tend to get confused with set notations involving three sets. The first thing to emphasise is that students should be confident in using the signs of unions, intersections, and complements. In order to solve a set notation they should use a Venn diagram.



# Sample Lesson Plan

## Topic

De Morgan's first law

## Suggested Duration

One period

## Key Vocabulary

Union, Intersection, Complement

## Method and Strategy

De Morgan's first law states:  $(A \cup B)' = A' \cap B'$

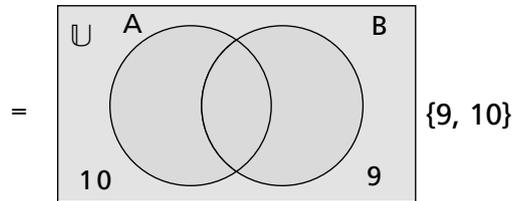
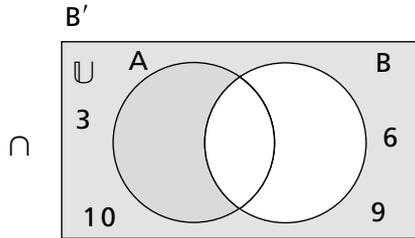
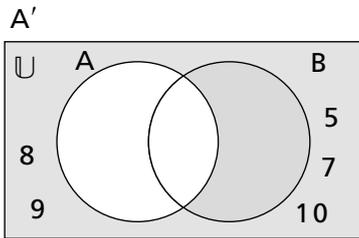
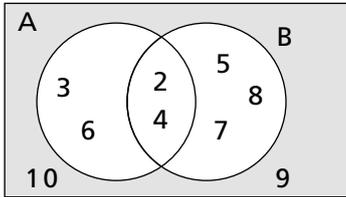
### Example:

$\cup \{2, 3, 4, 5, 6, 7, 8, 9, 10\}$ ,  $A = \{2, 3, 4, 6\}$ ,  $B = \{2, 4, 5, 7, 8\}$

Prove that  $((A \cup B)' = A' \cap B')$

To prove that  $(A \cup B)' = A' \cap B'$ , the left-hand side must be equal to the right-hand side.

$$(A \cup B)' = \{9, 10\}$$



It is important to do these sums on the board by drawing huge Venn diagrams. Using Venn diagrams to solve sets questions is very helpful.

The students can be encouraged to make cutouts of these questions on a sheet of chart paper and display their work on the soft board in the classroom. This will encourage group work and interest in the topic.

## Written Assignments

Exercise 1 Q 4 and Q 5 can be done orally before the activity of making cutouts using the above example. Later they can be done for homework.

## Evaluation

Quizzes at the beginning of each lesson will be very helpful as not only will the students revise and strengthen their understanding of key concepts, but also the teacher can use them as a tool to assess learning.

A comprehensive assessment based on multiple choice questions, and advanced conceptual questions similar to the sums in Exercise 1 can also be given.

### **After completing this chapter, students should be able to:**

- define a subset and a power set,
- use Venn diagrams to define union and intersection of three sets,
- verify the commutative and associative properties of sets,
- verify the distributive property of union over intersection, and
- verify De Morgan's laws.



# Squares and Square Roots



## Specific Learning Objectives

In this unit, students will learn:

- the properties of squares and perfect squares.
- how to find square roots of natural numbers.
- how to find square roots of decimal numbers.
- how to find square roots of vulgar fractions.



## Suggested Time Frame

5 to 6 periods



## Prior Knowledge and Revision

Squares and square roots have been introduced in previous years.

### Starter Activity

Students can revise this topic by playing a game with either a pack of cards with the picture cards removed, or the teacher can make sets of flash cards of numbers 1 to 9.

Students can work in pairs. They should shuffle their pack of cards and pick out four cards.

They then have to find the square of the number on each card and add them up and record their result. This activity can be repeated three times.

The pair with the highest total of the 3 attempts wins. The activity should not take more than five minutes.

It is a quick and enjoyable activity which will help the students to revise the squares table.



## Real-life Application and Activities

This topic may be done as a quiz on the board. The students could be divided into four groups. Each group selects one of its members to go to the board. Prompting by other members of the team may be allowed in the first two rounds, but in the final round, the team member will have to attempt the sum on his/her own.

This is a fun way to learn the steps of mathematical computation by indirect peer participation and instructions.



## Summary of Key Facts

- Irrational numbers cannot be placed as fractions as they are non-terminating and non-recurring when in decimal form.
- The square of a number is represented geometrically as a 2D shape as a square. Hence it is the product of itself.
- Square root is denoted by the radical symbol  $\sqrt{\quad}$ .
- Perfect squares and square roots can be found by the prime factorisation method.

### Example

$$\sqrt{\frac{196}{225}} = \frac{\sqrt{2 \times 2} \times \sqrt{7 \times 7}}{\sqrt{3 \times 3} \times \sqrt{5 \times 5}} = \frac{2 \times 7}{3 \times 5} = \frac{14}{15} \quad [\text{Find prime factor pairs of 196 and 225}]$$

The square root of  $\frac{196}{225}$  is  $\frac{14}{15}$ .

- The square roots of numbers that are not square numbers are found by the long division method. Finding the square root of a non-square number is done by making pairs and bringing them down. Once this is done the calculation is the same. In the division method, the first divisor and the new divisor are put together and then used for division.

### Example

$$\begin{array}{r} 18.47 \\ \hline 1 \quad | \quad 36 \quad | \quad 84 \quad | \quad 14 \quad | \quad 09 \\ + 1 \quad | \quad - 28 \quad | \quad - 24 \quad | \quad - 22 \quad | \quad - 20 \\ \hline 28 \quad | \quad 241 \quad | \quad 1714 \quad | \quad 15617 \\ + 8 \quad | \quad - 224 \quad | \quad \phantom{1714} \quad | \quad \phantom{15617} \\ \hline 364 \quad | \quad 1714 \quad | \phantom{1714} \quad | \phantom{15617} \\ + 4 \quad | \quad - 1456 \quad | \phantom{1714} \quad | \phantom{15617} \\ \hline 3687 \quad | \quad 25809 \quad | \phantom{1714} \quad | \phantom{15617} \\ \phantom{3687} \quad | \quad - 10192 \quad | \phantom{1714} \quad | \phantom{15617} \\ \hline \phantom{3687} \quad | \quad 15617 \quad | \phantom{1714} \quad | \phantom{15617} \end{array}$$

The rules of the long division method should be revised orally in class so that students know the steps. Displaying these steps on a pin-board during the duration of the chapter would benefit the students.

- To find the square roots of decimals the same process is followed but if the decimals are not evenly paired initially, zero is added to complete the pairs.

Decimals can also be converted to fractions and the square roots of the numerator and denominator found separately (either by prime factorisation or long division) and then converted back to decimals as the answer.

When finding the square root of a vulgar fraction, the denominator is converted to a perfect square by multiplying both the numerator and denominator by a common factor. Find the square root of the numerator by long division. The quotient of the square roots of the numerator and the denominator is the answer.



## Frequently Made Mistakes

When doing long division, students generally face problems in finding the clue for the next step. They should write the steps in their notebooks and the teacher should give encouragement and help wherever needed.



## Sample Lesson Plan

### Topic

Squares and square roots

### Specific Learning Objectives

Solving word problems involving squares and square roots

### Suggested Duration

One period

### Key Vocabulary

Square, Square root, Radical, Long division, Prime factorisation

### Method and Strategy

Solving word problems where the students have to apply their mathematical skills to identify which technique to apply, is difficult at every level. The students will have to decide whether to find the square or square root and then decide which method to use.

The exercise for this unit is relatively easy. The students can attempt it on their own by following the steps carefully. The word problems may have to be explained by the teacher.

#### *Example:*

The sides of two squares are 2.4 m and 1.8 m respectively. Find the side of a new square equal in area to the combined area of the two squares.

Each side of square 1 = 2.4 m

Each side of square 2 = 1.8 m

The students will have to perform multiple computations. First they will have to find the radical whose square root is required, and then the square root.

Area of square 1 =  $5.76 \text{ m}^2$

Area of square 2 =  $3.24 \text{ m}^2$

Total area =  $9 \text{ m}^2$

The side of new square =  $\sqrt{9} = 3 \text{ m}$  each

### Written Assignment

The sums on page 31 can be discussed in class and the students should then be encouraged to do them in their notebooks.

### Evaluation

A comprehensive assessment at the end of the chapter is necessary. A mental maths quiz can also be given for finding squares and square roots of numbers.

### After completing this chapter, students should be able to:

- apply the properties of squares and perfect squares,
- find square roots of natural numbers,
- find square roots of decimal numbers, and
- find square roots of vulgar fractions.



# Cubes and Cube Roots



## Specific Learning Objectives

In this unit students will learn:

- the properties of cubes and perfect cubes.
- how to find cube roots of perfect cubes.
- how to find cubes of negative integers.
- how to find cubes of rational numbers.



## Suggested Time Frame

4 to 5 periods



## Prior Knowledge and Revision

This chapter is an extension of the previous chapter on square roots. The students have learnt about square and cube numbers in earlier grades. A square number is obtained by multiplying the number by itself; cube numbers are obtained when the number is multiplied by itself three times.

An oral quiz of the first ten cube numbers can be done. Flash cards can be made and the students can come one by one, pick up a card, and give its cube.

$$1^3 = 1$$

$$2^3 = 8$$

$$3^3 = 27$$

$$4^3 = 64$$

$$5^3 = 125$$

$$6^3 = 216$$

$$7^3 = 343$$

$$8^3 = 512$$

$$9^3 = 729$$

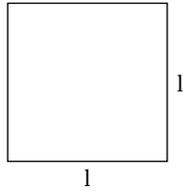
$$10^3 = 1000$$



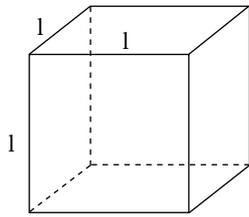
## Real-life Application and Activities

### Activity

The best way to explain cube numbers is with the use of a 3D object. A square is a 2D shape with equal dimensions; a cube has three dimensions which are equal.



A square has two basic dimensions:  $l \times b = l \times l = l^2$



A cube has 3 basic dimensions:  $l \times b \times h = l \times l \times l = l^3$

### Example

A cube with each side 3 cm has a volume of:  $3 \times 3 \times 3 = 27 \text{ cm}^3$

However, if a solid has a volume of 28 cubic centimetres, then it cannot be a cube because 28 is not a cubic number. It can be a volume of a cuboid:  $28 = 2 \times 2 \times 7$ .

The teacher should bring cubes and cuboids of different dimensions to the lesson to explain the difference.

The students can make their own presentation of card-board cutouts of cube numbers. The students can be assessed on presentation and understanding. This can be a group or individual marked assignment.

### Activity

Recall that the students learned to find LCM by the prime factorisation method in Grade 6. Finding cubes and cube roots involves a similar procedure. Now all they have to do is to make groups of threes instead of groups of twos as they did for square numbers.

### Example

Cube root of 729

3	729
3	243
3	81
3	27
3	9
3	3
	1

$$\sqrt{3 \times 3 \times 3 \times 3 \times 3 \times 3} = \sqrt{3 \times 3} = 9$$

Thus 9 is the cube root of 729.

The teacher should also explain that although there are no negative square numbers, cube numbers can be negative because when three negative signs are multiplied the result is negative.

### Example

– 8 is a cube number.

$= (-2) \times (-2) \times (-2) = -8$ . It is a cube of  $(-2)^3$ .



## Summary of Key Facts

- Perfect cubes are found when a product has three identical factors.
- The cube of a positive number is positive.
- The cube of a negative number is negative.
- The cubes of odd numbers are odd and those of even numbers are even.
- Multiple of threes cubes are also a multiple of 27.
- The cube of a number, one more than three times itself  $(3n + 1)$ , is a number of the same form.
- The cube of a number, two more than three times itself  $(3n + 2)$ , is a number of the same form.
- For any number  $n$ ,  $\sqrt[3]{n}$  is called a radical,  $n$  is the radicand, and 3 is the index of the radical.
- To find the cube root of positive and negative numbers the prime index notation method is used whereby groups of three are made and their product is the cube root.
- If  $\frac{p}{q}$  is a rational number, then  $(\frac{p}{q})^3 = \frac{p^3}{q^3}$ . This is known as the distributive property under division.



## Frequently Made Mistakes

This is a relatively easy chapter and students generally enjoy finding cubes and cube roots of numbers. Other than mathematical computation error, students generally do not make any mistakes.



## Sample Lesson Plan

### Topic

Cubes and cube roots

### Specific Learning Objectives

Making perfect cubes

### Suggested Duration

One period

### Key Vocabulary Words

Smallest possible integer, Product, Perfect cube

## Method and Strategy

"What is the smallest possible value of  $24n$ , so that it becomes a perfect cube?"

The teacher can write this question on the board. He/she can explain this concept using the 'teach-by-asking' method.

1. What is a perfect cube?
2. How then do we approach this question?
3. Do we break 24 into its prime factors?
4. Which number/numbers are not in sets of three?
5. What is missing to make sets of factors of three?
6. Is this the value of  $n$ ?
7. Is this the smallest possible value of  $n$ , whereby 24 when multiplied by it makes the number a perfect cube?

If the teaching method is enjoyed, the students should easily grasp this conceptual question.

## Written Assignment

After doing a couple of examples on the board (sums on page 41 of the textbook), students can be given questions to do independently in their notebooks.

What is the smallest integer ' $n$ ' by which the following numbers will become a perfect cube?

	Answers
1. $54n$	4
2. $84n$	882
3. $108n$	2
4. $324n$	18
5. $50n$	20

## Evaluation

Give a combined assessment of squares and cubes as this is quite a fun chapter. Questions from Exercises 2 and 3 can be used for the test. Word problems should also be given in the assessment as the students should not treat this chapter in isolation but apply it to real-life situations.

**After completing this chapter, students should be able to:**

- apply the properties of cubes and perfect cubes,
- find cube roots of perfect cubes,
- find cubes of negative integers, and
- find cubes of rational numbers.



# 4

# Number Systems



## Specific Learning Objectives

In this unit students will learn:

- about the binary system.
- how to convert:
  - base 2 numbers to base 10 numbers and vice versa.
  - base 5 numbers to base 10 numbers and vice versa.
  - base 8 numbers to base 10 numbers and vice versa.
- how to add and subtract bases 2, 5, and 8 numbers.
- how to multiply bases 2, 5, and 8 numbers.



## Suggested Time Frame

6 to 8 periods



## Prior Knowledge and Revision

This is a new topic. The way to introduce a new language of mathematics is to relate it to other notations. The teacher can point out that  $\frac{1}{4}$  can be represented as 0.25 in decimals, 25% in percentages, and 1 : 4 as a ratio.

In the decimal number system, we use digits 0 to 9. All other numbers are formed by combining these ten digits. Computers are based on the binary number system which consists of just two numbers: 0 and 1.



## Real-life Application and Activities

Since this is a new concept, an activity will be important for students' understanding.

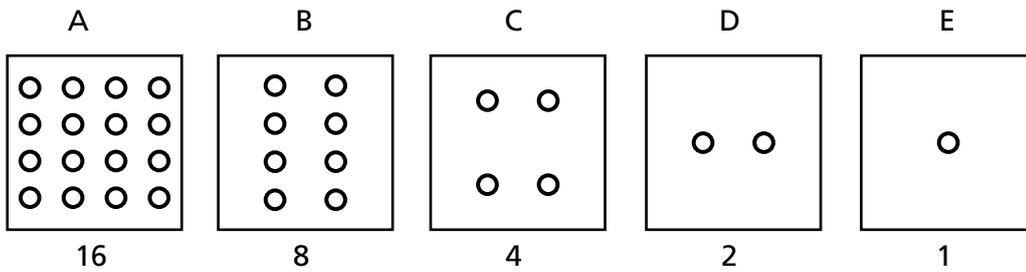
### Activity

Materials needed:

A4 size flash cards

black markers

Before the game begins, divide students into groups and demonstrate this concept with the help of flash cards.



Teach by asking.

“How many dots are on card A?”

“What do you notice about the sequence of the cards?”

“What would be the next card?”

If necessary, point out the pattern, that is each card is half of the one on its left. Card E is half of card D, card D is half of card C, and so on.

Once they recognise the pattern, ask the students to make the following sets of flash cards.

6  $\longrightarrow$  (4 dots and 2 dots cards)

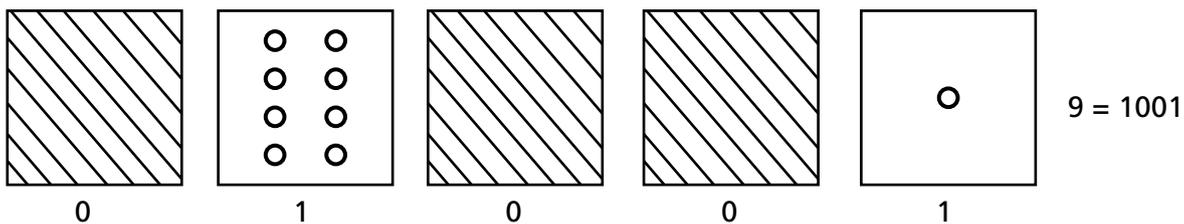
15  $\longrightarrow$  (8, 4, 2 and 1 dot cards)

21  $\longrightarrow$  (16, 4 and 1 dot cards)

Ask a volunteer to flip cards

When a binary number is not showing it is represented by zero, and when a binary number shows, it is represented by 1.

In the binary system 9 can be shown as given below:



Repeat this activity till the students are confident and then let them practice in groups.



### Summary of Key Facts

- The binary number system is a number system based on 2.
- Other number systems are the decimal number system, 8 (octal), and 5 (penta).
- In the binary system the place values of the digits are in terms of power of 2.

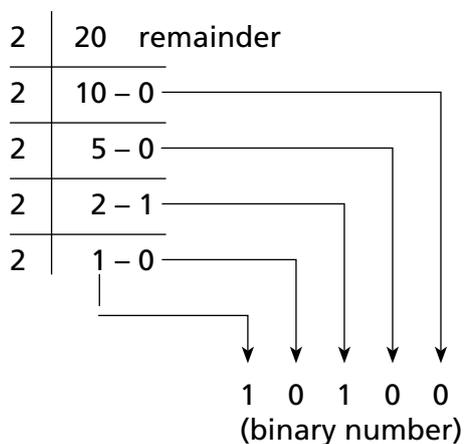
- To convert a number from the binary system to the decimal system, we follow the table below:

	Base 2 number	Base 10 number
○	0	0 $0_2 = 0_{10}$
●	1	1 $1_2 = 1_{10}$
● ○	10	2 $10_2 = 2_{10}$
● ●	11	3 $11_2 = 3_{10}$
● ○ ○	100	4 $100_2 = 4_{10}$
● ○ ●	101	5 $101_2 = 5_{10}$
● ● ○	110	6 $110_2 = 6_{10}$
● ● ●	111	7 $111_2 = 7_{10}$
● ○ ○ ○	1000	8 $1000_2 = 8_{10}$
● ○ ○ ●	1001	9 $1001_2 = 9_{10}$
● ○ ● ○	1010	10 $1010_2 = 10_{10}$

- To convert a number from the decimal to the binary number system:

### Example

Express  $20_{10}$  as a binary number.



We can use the expansion method to convert a number in any other base to the base 10 system.

- In the addition of binary numbers we follow the same method as in the decimal system, except that we carry a 2 not a 10.
- Similarly, in subtraction we borrow a 2 instead of a 10.
- In multiplication we follow the rule of multiplication which says that the product of zero and any number including itself, is zero.



### Frequently Made Mistakes

This is a completely new concept but if explained well it is quite easy to grasp.



## Sample Lesson Plan

### Topic

Binary system

### Specific Learning Objectives

Addition in the binary system

### Suggested Duration

One period

### Key Vocabulary

Binary number system, Base ten

### Method and Strategy

This can also be taught as an activity. The teacher can first demonstrate on the board, then divide the students into pairs, and practise by play.

### Activity

Formulate a set of ten questions and write them on flash cards. The students will pick a question and solve it in their notebook. The pair that finishes first with all answers correct gains a point.

For demonstration, the teacher can do the following sums on the board.

#### Example 1

Add the following: (i)  $1_{10}$  and  $1_{10}$  (ii)  $1_2$  and  $1_2$ .

Solution:

$$\begin{array}{r} \text{(i)} \quad 1_{10} \\ + 1_{10} \\ \hline 2_{10} \end{array}$$

$$\begin{array}{r} \text{(ii)} \quad 1_2 \\ + 1_2 \\ \hline 10_2 \end{array}$$

### Written Assignments

Question 5 of Exercise 4 can then be given for homework. The students would have had ample class practice when working in pairs. This also enables the students to learn from peers.

### Evaluation

A comprehensive assessment based on fill in the blanks and multiple-choice questions can be given.

### After completing this chapter, students should be able to:

- explain the binary system.
- convert:
  - base 2 number to base 10 number and vice versa,
  - base 5 number to base 10 number and vice versa,
  - base 8 number to base 10 number and vice versa,
- add and subtract bases 2, 5, and 8 numbers, and
- multiply bases 2, 5, and 8 numbers.



# Exponents and Radicals



## Specific Learning Objectives

In this unit students will learn:

- the laws of indices.
- about numbers with rational exponents.
- how to express rational numbers in radical form.
- how to express radicals as rational numbers.
- how to add and subtract radicals.
- about surds.
- how to apply the four operations on exponents and radicals.



## Suggested Time Frame

4 to 5 periods



## Prior Knowledge and Revision

Students are familiar with powers and bases. Radical form and surds will be new for them. Hence it is important that the teacher revises the concept of 'the power of'.

### *Example*

$a^n$

Where  $a$  is the base and ' $n$ ' the power.

The power or exponent will tell the number ( $n$ ) of times the base ( $a$ ) will be multiplied by itself.



## Real-life Application and Activities

### Activity

The concept of exponential notation can be re-introduced with a fun activity.

You need ' $x$ ' packs of cards with the picture cards removed.

Divide the students into groups of four.

Ask one student from each group to deal the cards.

Each student then organises his/her cards in the exponential form.

### Example

If the student has 3 fives he will write it as:  $5^3$

Next ask the students to find the product of their exponential list.

### Example

$5^3 = 5 \times 5 \times 5 = 125$ , and so on.

Now ask each student to add all the products.

Whoever gets the highest score is the winner. The activity may be timed; the winner is the one who finishes first.

This activity not only develops the students' ability to organise data but is also an indirect way of explaining what exponents / indices / power actually signify.



## Summary of Key Facts

- The base number is raised to a power or exponent.
- The laws of rational exponents in indices are as follows:
  - (i) For a positive integer  $n$ ,  $\sqrt[n]{ab} = \sqrt[n]{a} \times \sqrt[n]{b}$ ,
  - (ii) For any two rational numbers  $m$  and  $n$ ,  
 $a^m \times a^n = a^{m+n}$ ,  $a \neq 0$  (powers added when bases are same in multiplication)
  - (iii) For all rational values of  $m$  and  $n$ ,  
 $a^m \div a^n = a^{m-n}$ ,  $a \neq 0$  (powers subtracted in division of same bases)
  - (iv) For any two rational numbers  $m$  and  $n$ ,  
 $(a^m)^n = a^{mn}$ ,  $a \neq 0$  (powers multiplied when in brackets)
  - (v)  $a^0 = 1$  (any number other than zero to the power zero is 1)
  - (vi) For all rational values of  $n$ ,  
 $(ab)^n = a^n \times b^n$ ,  $a$  and  $b$  are non-zero numbers.
  - (vii) For all rational values of  $n$ ,  
 $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ ,  $a$  and  $b$  are non-zero.
- The radical of the form  $\sqrt[n]{a}$ , which cannot be reduced to a rational number, is called a surd of order  $n$ .



## Frequently Made Mistakes

Students tend to make mistakes as there is repetitive multiplication. Their oral/mental maths should be sharpened with lots of oral quizzes.



## Sample Lesson Plan

### Topic

Exponents and radicals

### Specific Learning Objective

Solving sums in of radical and index form

## Suggested Duration

One period

## Key Vocabulary

Radicals, Radicands

## Method and Strategy

The teacher should reinforce the radicals and index form concepts clearly on the board. The students should understand that evaluation can also be done for both radical and index form.

### Example

$\sqrt[3]{27}$  actually means 27 to the power of  $\frac{1}{3}$ .

Once the radical form is solved, the students are able to proceed with the operation.

### Example

$$\sqrt[3]{27} + \sqrt[2]{4} = 3^{3 \times \frac{1}{3}} + 2^{2 \times \frac{1}{2}} = 3 + 2 = 5$$

Similarly, it is important to point out that any negative fractional power becomes a positive fractional power only when the base variable of the numerator becomes the denominator, or vice versa.

### Example

$$(\sqrt{5})^{-\frac{1}{2}} \times (\sqrt{5})^{-\frac{3}{2}}$$

$$\left(\frac{1}{\sqrt{5}}\right)^{\frac{1}{2}} \times \left(\frac{1}{\sqrt{5}}\right)^{\frac{3}{2}}$$

$$\frac{1}{5^{\frac{1}{2} \times \frac{1}{2}}} \times \frac{1}{5^{\frac{1}{2} \times \frac{3}{2}}}$$

$$\frac{1}{5^{\frac{1}{4}}} \times \frac{1}{5^{\frac{3}{4}}}$$

$$\frac{1}{5^{\frac{1}{4} + \frac{3}{4}}} = \frac{1}{5^{\frac{4}{4}}} = \frac{1}{5}$$

## Written Assignments

The following sums can be done in class.

Evaluate:

1.  $\sqrt{(5)^2} + \left[ \frac{6}{5} \times \sqrt{(5)^2} \right] - \frac{15}{3}$

2.  $8 + (64)^{\frac{5}{6}}$

3.  $2^0 + 4^3$

4.  $27^{\frac{1}{3}} + 3^2$

5.  $\sqrt[3]{2^2} \times \sqrt[3]{2}$

### Answers

6

40

65

12

2

## Evaluation

This is an important topic. Regular 5 minute quizzes should be given and finally students should be tested with sums that contain all concepts taught.

### **After completing this chapter, students should be able to:**

- apply the laws of indices,
- express rational numbers in radical form,
- express radicals as rational numbers,
- add and subtract radicals,
- simplify surds, and
- apply four operations on exponents and radicals.



# Logarithms



## Specific Learning Objectives

In this unit students will learn:

- to express numbers in standard form and scientific notation.
- about logarithms.
- how to express numbers in logarithm form.
- the laws of logarithms.
- how to solve problems based on logarithms.



## Suggested Time Frame

4 to 5 periods



## Prior Knowledge and Revision

This is a new way of representing numbers.

The best way to introduce logarithms is to revise indices and scientific notation. Logarithms is a natural extension to these concepts.

Indices and scientific notation can be revised through a timed worksheet. Before handing out the revision worksheet, the basic rules of these topics should be revised. The students who finish early with correct answers gain a point.



## Real-life Application and Activities

### Activity

Understanding the key terms such as base, exponent, and the power of base ten, is critical to the use of logarithms.

To familiarise students with these terms, play the following game with them.

Materials needed: markers and A4 sized flash cards.

You need to make two sets of cards.

One set will have sums of logs written on them (at least 10 sums).

The other set will have only three cards, with base, exponent, and answer written on them.

Set 1.	Set 1.	Set 2.	Set 2.	Set 2.
$\log_2 8 = 3$	$\log_2 64 = 6$	Base	Exponent	Answer

The students are divided into pairs. One student chooses a card from set 1 and points at any of the value on the card. The other student will have to raise the card with the correct term.

For example, if one student picks up  $\log_2 64 = 6$  and points to 2, then the other student must raise the card with 'Base' written on it.

They play and swap multiple times; the one with the most correct answers wins.



## Summary of Key Facts

- In indices, numbers are denoted by a base raised to a power. For example, 1000 is denoted by 10 raised to the power 3, i.e.  $10^3$ .
- A number expressed in scientific notation is written in the form:  $A \times 10^n$ , where  $1 \leq A < 10$  and 'n' is an integer.
- In logarithmic notation,  $\log_2 64 = 6$ , 2 is called the base, 64 is called the argument, and 6 is the answer.
- The logarithm of the product of two numbers  $a$  and  $b$  is equal to the sum of their individual logarithms.  
$$\log_x ab = \log_x a + \log_x b$$
- The logarithm of the quotient of two numbers  $x$  and  $y$  is equal to the difference between their individual logarithms.  
$$\log_x a \div b = \log_x a - \log_x b$$
- The logarithm of a number  $a$  raised to a power  $b$  is equal to the product of the power  $b$  and the logarithm of the number  $a$ .  
$$\log_x ab = b \log_x a$$



## Frequently Made Mistakes

Since new terms are involved in this topic, stress should be laid upon notation through lots of practice sums and Snap games with flash cards.



# Sample Lesson Plan

## Topic

Logarithms

## Specific learning Objective

Introducing the concept of reading logs

## Suggested Duration

1 period

## Key Vocabulary

Base, Exponent, Argument

## Method and Strategy

The best way to introduce logs is to ask students to write the method in their notebooks. The following points should be shown on the board and copied in notebooks.

log base  $b$  of  $a$  is  $x$  will be written as:

$$\text{Log } b^a = x$$

What exponent is required to form a base of  $b$  to reach the value of  $a$ ?

Exponential form:  $2^6 = 64$

base	exponent	answer
2	6	64

Logarithmic form:  $\log_2 64 = 6$

log	base	answer	exponent
$\log_2$	2	64	6

## Activity

The teacher can teach logs with the tried and tested 'loop trick', which is as follows:

### The loop trick

Always draw your loop anti-clockwise

$$\text{Log}_b \overset{\curvearrowleft}{a} = x$$

$$\overset{\curvearrowleft}{b^x} = a$$

$$\text{Log}_2 64 = x$$

$$2^x = 64$$

## Activity

Another game that can be played is 'log war'.

You need to make two identical sets of cards of twenty sums of logs and shuffle them. The students are divided into pairs and the timer set for 5 minutes. The students play Snap by flipping the cards and quickly working out the values. The one who finishes first or picks out the last card and works out the sum first calls out 'Snap!'.

$\log_x 9 = 2$

log war

$\log_{10} x = 2$

## Written Assignments

The teacher should do a few sums orally and then ask the students to do questions 1 and 7 of Exercise 6 for homework.

## Evaluation

The students can be assessed while doing activities and marks assigned according to results. The activities can be done periodically during the week. A comprehensive assessment of learning can also be done after completing the unit.

### After completing this chapter, students should be able to:

- express numbers in standard form and scientific notation,
- express numbers in logarithm form,
- apply laws of logarithms, and
- solve problems based on logarithms.



# Profit, Loss, Insurance, and Taxes



## Specific Learning Objectives

In this unit students will learn:

- how to calculate percentage profit, percentage loss, and percentage discount.
- about various types of insurance: life insurance, vehicle insurance.
- about income tax.



## Suggested Time Frame

6 to 8 periods



## Prior Knowledge and Revision

Students have a good knowledge of percentages. They have done sums on percentages in earlier grades and also used them in earlier chapters. A revision activity can be done.

### Activity

The students can be divided into groups of four. They can all make their own sums on percentages and prepare a test paper. Each should contain four sums and the test papers should be swapped between the groups. The test should take ten minutes and the completed answers should be returned to the examiner group. They then check the test and grade the group and hand it back to them. This entire activity should not take more than 25 minutes.

It is a fun revision activity where the students role play teacher, examiner, and students. It is imperative that percentages are revised on the board initially.

### Role Play Activity

The teacher can create a make-believe business and explain the process in the following manner. She/he can even present it on a PowerPoint slide show.

#### 'How business work'

Raheem Brothers buy T-shirts from the suppliers. The cost of buying goods is called the **cost price**. They then sell the T-shirts to the consumer at a higher price known as the **selling price**.

The difference between the cost price and the selling price is the **profit**.

**Loss** is incurred if the seller sells at a lower price than the cost price.

To find the profit% or loss% divide the profit or loss by the cost price, and then multiply by 100%.

### Example

A shopkeeper buys toy cars in boxes of 50. He purchases 10 boxes for Rs 6000. Each toy car is then sold for Rs 15. Find his profit or loss percentage.

#### Find

- number of toy cars bought
- cost price of each toy car
- profit made
- profit percentage

$$\text{Cost price} = \frac{6000}{50} = \text{Rs } 12$$

$$\text{Profit} = \text{Rs } 15 - \text{Rs } 12 = \text{Rs } 3$$

$$\text{Profit\%} = \frac{3}{12} \times 100\% = 25\%$$



## Real-life Application and Activities

Percentages can be explained with real-life examples.

Arrange an activity as follows:

Introduce the idea of organising a school concert.

The students role-play organisers. They have to work out the **cost** of the event and then the **profit**.

To calculate the cost price:

calculate the wages of the performers and the support staff

calculate the total cost of putting on the event e.g: sound, stage, lighting, refreshments, etc.

profit projection will be based on the number of tickets sold.

Divide students into groups and ask them to work out the total cost of putting on the concert.

Groups are then asked to work out **profit projection** based on three case scenarios.

50% of the students attend the concert.

75% of the students attend the concert.

90% of the students attend the concert.

For each case they calculate the percentage profit using their cost projections and the ticket price.

Lastly, they are asked to calculate the minimum number of tickets they need to sell to break even (no loss is incurred and the cost is covered).

This is an interesting project/assignment and the students can be given two days to complete it.

Each group can be marked on this assignment.



## Summary of Key Facts

- Percentage profit or loss is calculated by dividing the profit or loss by the cost price and multiplying by 100%.
- Discount is a reduction on the cost price of the product to be sold. Its percentage can be found by dividing by the original cost price and multiplying by 100%.

- Insurance is a contract whereby the person is insured against the risk of loss to his/her property. A certain premium is paid to the insurance company monthly or yearly and in exchange he/she is insured from any loss or damage to his/her property.
- Depreciation is the loss of actual value of a product over a period of time.
- Income tax is imposed by the government. It sets the amount to be taxed and the rate at which the income is taxed.
- Exempt income is the amount upon which no tax is paid.
- Taxable income is the amount of income on which tax is paid.
- Taxes are also charged on agricultural output and business profits.



## Frequently Made Mistakes

This topic has real-life applications. Students tend to misunderstand the terms and are unable to apply the correct formula.



## Sample Lesson Plan

### Topic

Taxation

### Specific Learning Objectives

To calculate chargeable income and the tax payable on it at a given rate

### Suggested Duration

One period

### Key Vocabulary

Exempt income, Chargeable income, Taxation

### Method and Strategy

Explain this concept to student with role play.

Select a student to be the bread winner of the family role play, a wife, two kids, and a mother. Chargeable income is then calculated by an account relief given for each member of the family.

Tax is then calculated with the terms given on the board.

This activity will take 3-5 minutes but the students will understand the concept better.

Set up sums with variable incomes and reliefs and a fixed rate of taxation.

### Example

Relief:            2 children at the rate of Rs 10,000 each  
                       Member at the rate of Rs 15,000  
                       Charity of Rs 5000  
 Income:          Rs 200,000  
 Tax:                10% over and above Rs 50,000

## Written Assignment

Question 11, 12, and 13 of exercise 7 can be done as class work.

## Evaluation

This chapter involves a lot of mathematical terms and formulae. The students can be assessed on the definitions and formulae in a 10 minute quiz. A comprehensive assessment based on questions from Exercise 7 can be conducted to assess learning.

### **After completing this chapter, students should be able to:**

- calculate percentage profit, percentage loss, and percentage discount,
- calculate various types of insurance, and
- calculate various types of taxes.



# Compound Proportion and Banking



## Specific Learning Objectives

In this unit students will learn:

- to define compound proportion.
- how to solve real-life problems involving compound proportion, partnership, and inheritance
- about different types of bank accounts: current deposit account, PLS savings account, PLS term deposit, fixed deposit account, recurring deposit account, foreign currency account.
- how to calculate simple and compound interest.
- how to identify various banking instruments: cheque, demand draft, pay order.
- about bank services: ATM, debit card, credit card.
- about financial services: running finance, demand finance, lease.
- how to convert one currency to another.



## Suggested Time Frame

6 to 8 periods



## Prior Knowledge and Revision

Students are familiar with simple interest. This chapter amalgamates percentages and arithmetic into a formal teaching of banking processes.

The teacher can explain that the interweaving of all concepts taught earlier such as percentages and ratios word problems will be applied in the real-world. This real-life application will excite and challenge the students.

As a warm up, story sums of percentages and simple interest can be done on the board and the teacher can encourage student participation.



## Real-life Application and Activities

Compound interest is an extension of the concept of simple interest. This is applied by most banks.

The basic concept of compound interest is that when the tenure is of more than a year, the principal changes as the simple interest is added.

The investor gains more 'interest' as the principal amount increases every year. In simple interest, no matter how long the time period is, the principal and the interest remain fixed.

### Activity

This topic can be reinforced by organising an educational trip to a bank. A representative of the bank can explain how the banking system works. The difference and correlation between simple and compound interest, various banking terms, transactions, and how different departments work can be explained.

If a trip cannot be organised, a bank manager could be invited to give a presentation in class. This will help the students to understand:

banking terminology

departmental functions

banking calculations

The students should also be encouraged to do research and make a presentation on chart paper about the various banks in Pakistan.



### Summary of Key Facts

- There are various types of bank account.
- A current account is used when deposits and withdrawals are made frequently.
- In a PLS savings account, the profit and loss sharing is not used frequently.
- In a fixed deposit account the amount is fixed for a certain length of time.
- The formula for compound interest is given below:

$$C.I = A - P$$

$$A = P \left(1 + \frac{r}{100}\right)^t$$

where:

A is the amount

P is the principal

R is the rate

T is the time in years

[It must be explained that the rate or time can also be calculated for a given amount and the principal.]

- A cheque is a written instruction for a certain amount to be paid to the beneficiary.
- A demand draft is issued by the bank on behalf of an account holder. This can be used in any branch of the bank.
- A pay order is similar to a demand draft but can only be used to withdraw money from the same branch.
- There are various banking services.
- An automated teller machine (ATM) is a machine which allows the account holder to withdraw cash after banking hours and from wherever one has been installed.

- Debit and credit cards are plastic cards issued to account holders to make payments without using cash. Once enabled, the card transfers money from the owner's account to the recipient account without any exchange of cash.
- An overdraft facility enables the account holder to make payments regardless of insufficient funds in the account.
- Running finance is similar to an overdraft facility but it can be used for a longer period of time and the bank charges interest on the amount borrowed.
- Demand finance is an amount loaned to the account holder by the bank. The tenure can be of any length but the bank can demand the loan to be paid back at any time.
- A lease is an arrangement by which the bank pays an amount equal to the value of an account holder's asset(s) for his/her use. He/she will then repay the amount loaned and the interest in installments over a given period of time.
- The time frame is called the term, the bank is the lessor, and the customer, the lessee.
- In order to make payments abroad, the currency exchange rate issued by the regulatory bank is used. This rate is the equivalence ratio between two currencies.



## Frequently Made Mistakes

The students confuse the various banking terms. If all the terms and their definitions are displayed on a sheet of chart paper in the classroom it will be very helpful to the students. The students should be asked to note all of them in their notebooks.



## Sample Lesson Plan

### Topic

Currency conversion

### Specific Learning Objectives

Currency conversion

### Suggested Duration

One period

### Key Vocabulary

Pounds, US dollars, Rupee, Yen, Riyal, Dirham Currency, Exchange rate

### Method and Strategy

It will be exciting for the students to investigate and present various currencies of the world. The equivalence rates can be found on the business pages of news papers, or on the internet.

Once the students are familiar with the currencies of different countries, give a short quiz where the teacher names a country and the students name its currency.

Point out the fact that direct proportion is involved in currency conversion.

Currency table for a few countries:

China: yuan

USA : dollar

Canada : Canadian dollar

Australia : Australian dollar

Japan : yen

Pakistan : rupee

India : rupee

Bangladesh : taka

Thailand : bhat

The list is long and the teacher can add to it.

### **Written Assignments**

The students can copy the list and the teacher can give five sums along with the exchange rates to be calculated.

### **Evaluation**

An objective test based on true or false statements, fill in the blanks, and multiple choice questions can be given on various banking terms.

A comprehensive subjective type of test can also be given at the end of the topic.

### **After completing this chapter, students should be able to:**

- define compound proportion,
- solve real-life problems involving compound proportion, partnership, and inheritance,
- differentiate between different types of bank accounts: current account, deposit account, PLS savings account, PLS term deposit fixed deposit account, recurring deposit account, foreign currency account,
- calculate simple and compound interest,
- identify various banking instruments: cheque, demand draft, pay order,
- identify various bank services: ATM, debit card, credit card,
- differentiate between financial services provided by the banks: running finance, demand finance, lease, and
- convert one currency to another.



# 9

# Stocks and Shares



## Specific Learning Objectives

In this unit students will learn:

- about stocks, shares, and dividends.
- what nominal value and market value of shares means.
- what brokerage is in terms of buying and selling shares.



## Suggested Time Frame

5 to 6 periods



## Prior Knowledge and Revision

The students will have no prior knowledge of this topic. In fact, most of them at this age would have no idea how the stock exchange works, nor the terminology used there.

This topic will introduce them to the world of economics and will have to be related to the working of their country's stock exchange.

The students should be encouraged to read the business pages of the newspaper prior to starting this topic so that they have some understanding of the economy.



## Real-life Application and Activities

Simulation of the stock exchange can be created in the classroom.

### Activity 1

The students role-play and each could be assigned a particular role.

There can be a buyer, seller, and a broker.

The main setting can be the Karachi Stock Exchange, with newspaper clippings of stocks and shares displayed on a sheet of chart paper.

The students can play with a given amount and calculate the shares needed and their prices. The broker can calculate his commission and the buyer can create his portfolio with predictions of his gains by calculating the dividend rate.

## Activity 2

- Each group can begin with Rs 10,000.
- Each group must invest in at least 5 stocks or the teacher can set a limit of a minimum of 3 stocks and maximum of 6 stocks.
- A commission can be set for every stock transaction, e.g., 3% commission.
- The time frame for this activity could be a week.
- Each group will maintain a record of their activities and ideas. Their transaction history should also be recorded
- On the last day of the activity each group will sell their stocks and report on the value of their portfolio.
- The group with the highest gain will win.

It has to be explained that although real stocks and values will be used, no actual buying or selling will take place.

This simulation is a perfect way of helping the students understand the workings of a stock market.



## Summary of Key Facts

- A dividend is a share of a company's earnings, decided by its board of directors, to be paid to be given to the share holders.
- A holder of stock has a claim on the company's assets, which is predetermined as a certain value.
- A share certificate indicates the number of shares one holds in a particular company.
- A share has a certain market value and a nominal face value. The market value fluctuates depending on the performance of the company and the country's economy.
- A broker deals with the transactions of the buyer and the seller and charges a certain percentage of the value of the transaction as commission.



## Frequently Made Mistakes

Students may not understand the broader picture of the workings of the stock exchange. Role-play and newspaper articles will help the students to understand this better.



## Sample Lesson Plan

### Topic

Stocks and shares

### Specific learning Objectives

Calculating the dividend and brokerage commission

### Suggested Duration

One period

## Key Vocabulary

Dividend, Shares, Commission, Brokerage, Percentage

## Method and Strategy

Mr. Pervez purchased 300 shares of face value Rs 100 at Rs 800 per share. If the company pays a dividend of 40%, find his earning percent on this investment.

This question takes into account the fact that the student should understand the terms used. The teacher should first highlight the key words: purchase, face value, dividend, and percentage. The definitions and context in which they are used should be discussed.

Sifting the information and then identifying the problem are key to this topic.

Percentages will be calculated as previously taught. The students should know that when calculating the dividend the face value is used and not the market value. The dividend calculated is then multiplied by the total number of shares.

Each question is a continuation of the chapter on percentages taught in Grade 7. The students should be encouraged to use their prior knowledge in real-life situations.

## Written Assignments

Do questions 10 to 14 of Exercise 9 in class.

## Evaluation

A comprehensive assessment can be given at the end of the chapter. The students can also be assessed on their understanding of the working of the stock exchange market and their decision-making skills during the role-play activity.

## After completing this chapter, students should be able to:

- explain the terms stocks, shares, and dividends,
- differentiate between the nominal value and market value of share, and
- explain what what brokerage is in terms of buying and selling shares.



# Averages



## Specific Learning Objectives

In this unit students will learn:

- about the concept of averages: simple average, weighted average, and average speed.
  - how to find the average.
  - how to calculate the weighted average.
  - how to find the average speed.



## Suggested Time Frame

3 to 4 classes



## Prior Knowledge and Revision

This is an interesting topic where the students collect actual data from various sources. Averages is a topic that students must have already come across. The class average is often discussed when a test result is announced. The marks of all the students are added and divided by the total number of students to get the average mark.



## Real-life Application and Activities

Average is actually an everyday English word, but the students should understand the mathematical computations of averages.

A simple formula to calculate average:

$\text{sum of the quantities} / \text{number of quantities}$

The students can apply this formula easily. It is advisable not to mention frequency in place of number at this stage.

However, the teacher should be aware that averages lead to mean, median, and mode. The difference between the sum of the quantities and the total number of the values should be made clear.

## Activity

The students should be asked to collect ten different items of data whose average can be found. For example, cricket scores, heights of students, temperature of a city in the last seven days, stock market value of a certain company in a month, currency exchange rate of dollar to the rupee for a week, etc.

A fun activity can be done where each group is assigned to collect the relevant data and bring it to class. Once the data is collected, ask students to find the averages of each set in their notebooks. The teacher should remind the students to be careful about the divisor. In case of temperature of the last seven days, the students will divide by 7 and also in the case of the stock market value divide by 30.



## Summary of Key Facts

- The simple average is the sum of the quantities divided by the number of the quantities.
- The weighted average is the sum of the numbers in a data set multiplied by the weights, divided by sum of the weights.
- Average speed is the total distance covered during a journey divided by the total time of the journey.



## Frequently Made Mistakes

Students tend to make minor mistakes while adding and dividing.



## Sample Lesson Plan

### Topic

Averages

### Specific Learning Objectives

Calculating average speed

### Suggested Duration

One period

### Key Vocabulary

Time Distance, Units of speed, km/hr, M/sec

### Method and Strategy

Pervaiz drives to his office at a speed of 50 km/hr and comes back home at a speed of 40 km/hr. If the distance between his home and the office is 20 km, calculate his average speed for the entire journey.

Journey from home to office:

$$\text{Distance} = 20 \text{ km}$$

$$\text{Speed} = 50 \text{ km/hr}$$

$$S = \frac{D}{T}$$

$$\therefore T = \frac{D}{S}$$

$$\text{Time} = \frac{20}{50}$$

$$= 0.4 \text{ hours}$$

Journey: from office to home

$$\text{Distance} = 20 \text{ km}$$

$$\text{Speed} = 40 \text{ km/hr}$$

$$\text{Time} = \frac{20}{40}$$

$$= 0.5 \text{ hours}$$

$$\text{Total distance} = 20 + 20 = 40 \text{ km}$$

$$\text{Total time} = 0.4 + 0.5 = 0.9 \text{ hours}$$

$$\text{Average speed} = \frac{\text{Total distance}}{\text{Total time}}$$

$$= \frac{40}{0.9}$$

$$= 44.4 \text{ km/hr}$$

Similar sums can be done in class using real-life characters such as parents and bus drivers and their journeys.

## Written Assignments

Students can do Questions 20, 21, and 22 of Exercise 10 in their notebooks.

## Evaluation

A comprehensive assessment based on fill in the blanks, multiple choice questions, and word problems can be given to assess learning.

**After completing this chapter, students should be able to:**

- explain the concept of averages: simple average, weighted average, and average speed.
  - find the average,
  - calculate the weighted average, and
  - find the average speed.



# Matrices



## Specific Learning Objectives

In this unit students will learn:

- what a matrix is.
- about types of matrix: square matrix, rectangle matrix, row matrix, column matrix, square matrix, diagonal matrix, scalar matrix, null matrix, unit matrix.
- how to transpose a matrix.
- the conditions for two matrices to be equal.
- how to add and subtract matrices.



## Suggested Time Frame

5 to 6 periods



## Prior Knowledge and Revision

This is an entirely new topic. A matrix is yet another way of representing numbers and a unique way of applying mathematical operations. Word problems can also be solved with matrices.



## Real-life Application and Activities

A matrix is an array of numbers arranged in rows and columns. The concept of matrices can be explained in a fun way.

A matrix is usually denoted by a capital letter, for example, A or B.

The elements are placed within box-brackets in 'rows' and 'columns'. Remember, rows are horizontal and columns vertical.

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

A matrix order is written as  $A_{rc}$



## Summary of Key Facts

- The number of rows and columns of a matrix is called the order of the matrix.
- The location of an element of a matrix is denoted by writing its row and column position.

### Example

$$B = \begin{bmatrix} 6 & 4 & 16 \\ 3 & 9 & 8 \end{bmatrix}$$

$$b_{11} = 6 \text{ (element at row 1, column 1)}$$

$$b_{12} = 4 \text{ (element at row 1, column 2)}$$

$$b_{21} = 3 \text{ (element at row 2, column 1)}$$

There are 8 type of matrix.

- A 'square matrix' has an equal number of rows and columns.
- A 'rectangle matrix' has an unequal number of rows and columns.
- A 'row matrix' has only one row and can have many columns.
- A 'column matrix' has only one column and can have many rows.
- A 'diagonal matrix' has all elements except the leading diagonal, as zero.
- A 'scalar matrix' has all the elements in the leading diagonal equal.
- All elements of a 'null matrix' are zero.
- A 'unit matrix' has '1' in its leading diagonal.
- In order to transpose a matrix, we swap the rows with the columns.

### Example:

To transpose a matrix we swap the rows and columns.

We put a 'T' in top right hand corner.

$$\begin{bmatrix} \boxed{6} & \boxed{3} & \boxed{2} \\ \boxed{5} & \boxed{6} & \boxed{7} \end{bmatrix}^T = \begin{bmatrix} \boxed{6} & \boxed{5} \\ \boxed{3} & \boxed{6} \\ \boxed{2} & \boxed{7} \end{bmatrix}$$

Row  
Row  
column

- Two matrices are equal when two conditions are met: the order and the corresponding elements are equal.
- Addition and subtraction of matrices is done by making sure the order of the matrices are the same.



## Frequently Made Mistakes

This is a relatively easy chapter; however, students tend to make careless errors while adding or subtracting. Lots of oral and practice exercises should be done to avoid careless mistakes.



## Sample Lesson Plan

### Topic

Matrices: addition and subtraction of matrices

### Specific Learning Objective

Students will learn to add and subtract matrices

### Suggested Duration

One to two periods

### Key Vocabulary

Row, Column, Order

### Method and Strategy

This topic can be done in a fun way. The teacher can assign a shape to each element. Before the concept of adding and subtracting is taught, the fact that the order of matrices has to be the same, should be emphasised.

#### Addition of matrices:

Add the numbers in the same positions.

$$\begin{bmatrix} \textcircled{3} & \textcircled{2} \\ \textcircled{4} & \textcircled{5} \end{bmatrix} + \begin{bmatrix} \textcircled{4} & \textcircled{0} \\ \textcircled{1} & \textcircled{9} \end{bmatrix} = \begin{bmatrix} \textcircled{7} & \textcircled{2} \\ \textcircled{5} & \textcircled{14} \end{bmatrix}$$

These are the calculations

$$3 + 4 = 7 \quad 2 + 0 = 2$$

$$4 + 1 = 5 \quad 5 + 9 = 14$$

The two matrices must be of the same order.

#### Subtraction of matrices:

Subtract the numbers in the same positions.

$$\begin{bmatrix} \textcircled{3} & \textcircled{2} \\ \textcircled{4} & \textcircled{5} \end{bmatrix} - \begin{bmatrix} \textcircled{4} & \textcircled{0} \\ \textcircled{1} & \textcircled{9} \end{bmatrix} = \begin{bmatrix} \textcircled{-1} & \textcircled{2} \\ \textcircled{3} & \textcircled{-4} \end{bmatrix}$$

These are the calculations

$$3 - 4 = -1 \quad 2 - 0 = 2$$

$$4 - 1 = 3 \quad 5 - 9 = -4$$

## Written Assignment

Sums can be done on the board, and then ask students to do them in their notebooks.

### Answers

$$1) \begin{bmatrix} 3 & 7 \\ -2 & 5 \end{bmatrix} + \begin{bmatrix} 4 & 3 \\ 5 & 2 \end{bmatrix} \qquad \begin{bmatrix} 7 & 10 \\ 3 & 7 \end{bmatrix}$$

$$2) \begin{bmatrix} 3 & 0 \\ 5 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 7 \\ 6 & 8 \end{bmatrix} \qquad \begin{bmatrix} 3 & -7 \\ -1 & -8 \end{bmatrix}$$

$$3) \begin{bmatrix} 8 & 0 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 5 & 3 \\ 2 & -3 \end{bmatrix} \qquad \begin{bmatrix} 13 & 3 \\ 4 & -2 \end{bmatrix}$$

$$4) \begin{bmatrix} 5 & 3 \\ 2 & 1 \end{bmatrix} - \begin{bmatrix} -3 & 2 \\ 4 & -7 \end{bmatrix} \qquad \begin{bmatrix} 8 & 1 \\ -2 & 8 \end{bmatrix}$$

## Evaluation

A comprehensive assessment based on multiple choice questions, fill in the blanks, and word problems can be given. Follow the pattern of Exercise 11.

### After completing this chapter, students should be able to:

- explain what a matrix is,
- identify types of matrix: square matrix, rectangle matrix, row matrix, column matrix, square matrix, diagonal matrix, scalar matrix, null matrix, unit matrix,
- transpose a matrix,
- apply the conditions for two matrices to be equal, and
- add and subtract matrices.



# Operations on Polynomials



## Specific Learning Objectives

In this unit, students will learn:

- how to multiply polynomials.
- how to divide polynomials.



## Suggested Time Frame

4 to 5 classes



## Prior Knowledge and Revision

In Grade 7 students were taught the order, classification, addition, and subtraction of polynomials. It should be pointed out that unlike terms cannot be added or subtracted. These operations can be applied on like terms only. Rules of signs were also explained with the help of a number line.

A revision worksheet can be given. Once completed the teacher can solve the sums on the board and the students can check their partners' work. It is important that each sum is solved on the board as it will be easier for the students to do any corrections. Generally they make a mistake with signs or they tend to add the powers when adding. It should be pointed out that the powers determine whether the terms can be added or subtracted.

Following sums can be given as revision.

1. Add  $2x^2 + 3xy$  and  $4x^2 - 6xy$ .
2. What is the difference between  $10wz - 7xz$  and  $6wz - 5xz$ ?
3. What must be added to  $3x + 5y$  to get  $7x - 3y$ ?
4. Subtract  $2x^2 + 3y - z$  from  $-5x^2 + 4y - 2z$ .
5. Find the sum of  $2x + 3y + z$  and  $-5x - 7y - 3z$ .

### Answers

1.  $6x^2 - 3xy$
2.  $4x - 2xz$
3.  $4x - 8y$
4.  $-7x^2 + y - 3z$
5.  $-3x - 4y - 2z$



## Real-life Application and Activities

Explain the operations of multiplication and division. These require concrete rules, steps, and method. Addition and subtraction have different rules for signs from multiplication and division.

The following sign rules are applied in multiplication and division:

$$(+)\times(+)=(+)\quad (+)\div(+)=(+)$$

$$(+)\times(-)=(-)\quad (+)\div(-)=(-)$$

$$(-)\times(-)=(+)\quad (-)\div(-)=(+)$$

Three methods of multiplication are taught in this chapter:

1. vertical multiplication
2. horizontal multiplication
3. the FOIL method

Each method should be explained separately. It is advisable that the same sum is solved by all three methods for comparison. Students will determine the differences in the approaches by remembering the steps involved in each method given on pages 148 and 152.

### Activity

Make 20 cards of '+' sign and '-' sign each. Ask two students to come up and play Snap. The teacher will call out any operation, e.g. multiplication or division.

For example, 'multiply two positive signs'.

The students will start looking through the cards one by one.

If he/she get + sign, he/she will call out 'Snap'. Similarly the teacher may say 'divide + sign and – sign'. The student who finds the card with a – sign will says 'Snap'.

Play this game for two minutes, the student with more cards wins.

This activity can be done in pairs till the whole class understands to concept of sign.



## Summary of Key Facts

- Multiplication of polynomials can be done with unlike terms.
- The rules of signs for multiplication and division are the same.
- In the vertical method of multiplication the terms are aligned vertically and multiplication is carried out in steps.
- In horizontal multiplication, the first polynomial is split into terms and then each is individually multiplied by the second term. Like terms are then simplified.
- In the FOIL method the outer terms are multiplied, and then the inner terms, and then the last terms. The order needs to be followed in this method.
- Division requires that the first divisor term divides the first term of the dividend and then the quotient multiplies the entire divisor term. The terms are then subtracted and the process repeated until there is no remainder.



## Frequently Made Mistakes

Students generally get confused with the rules of the signs as they are different from the addition and subtraction rules. Steps and the format for multiplication and division should be written for better understanding.



## Sample Lesson Plan

### Topic

Operations on polynomials

### Specific Learning Objectives

FOIL method of multiplication

### Suggested Duration

1 period

### Key Vocabulary

FOIL, Horizontal multiplication, Polynomial

### Method and Strategy

In this lesson students will learn a new method of multiplying a binomial by a polynomial.

The teacher will write the acronym 'FOIL' on the board.

**F**irst

**O**uter

**I**nnner

**L**ast

It is advisable that the teacher uses coloured chalk or board markers when explaining this method. The teacher can use a different colour for each letter of FOIL.

### Written Assignment

Alternate sums from Exercise 12a can be done for classwork and homework. Students can also solve a sum by using two different methods for practice.

### Evaluation

The revision exercise can be used as a format for comprehensive test. Multiplication and division should be tested by specifying the required method in the question. Quizzes should be given at least three times a week in alternate lessons. This 5-minute activity helps the student and teacher identify any problems and then work on them.

### After completing this chapter, students should be able to:

- multiply polynomials, and
- divide polynomials.



# Algebraic Identities



## Specific Learning Objectives

In this unit, students will learn:

- how to find cubes of the sum of two terms.
- how to find cubes of the difference of two terms.
- how to find special products using algebraic identities.



## Suggested Time Frame

6 to 8 periods



## Prior Knowledge and Revision

Students have learnt the three identities in the earlier grade. A quick revision of these identities is extremely important as cubic identities depend on them. The students get confused over when to apply the perfect square or the difference between two squares. In most cases it should be stressed that the sums require initial factorisation and then the application of identities.

The following sums can be given as revision.

1.  $(2x + 2)^2$

2.  $(4x - y)^2$

3.  $81x^2 - 1$

4.  $(x + \frac{1}{2}y)^2$

5.  $16x^4 - y^4$

### Answers

1.  $4x^2 + 8x + 4$

2.  $16x^2 - 8xy + y^2$

3.  $(9x + 1)(9x - 1)$

4.  $x^2 + xy + y^2$

5.  $(4x^2 + y^2)(2x - y)(2x + y)$

Explain that perfect squares break up into two squares, and the product of the two terms is multiplied by 2.

The difference between two squares is identified easily as they are individually squared. They are factorised by finding the square root of the terms and then as the sum and difference.



## Real-life Application and Activities

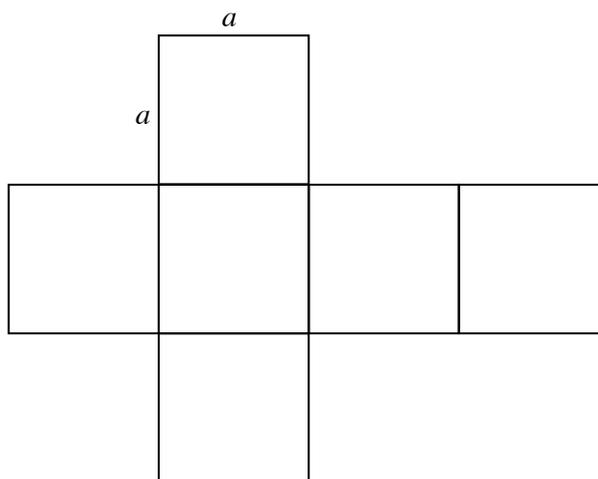
Refer to page 160 of text book where the diagrammatic/geometric explanation of the cubic identity is given.

We can convert it to an activity in class.

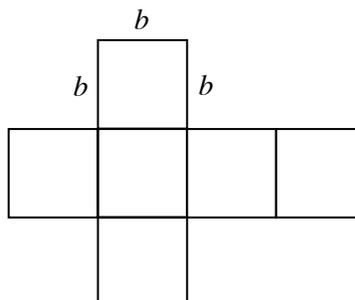
Materials required:

Chart paper with cut out of the net diagrams:

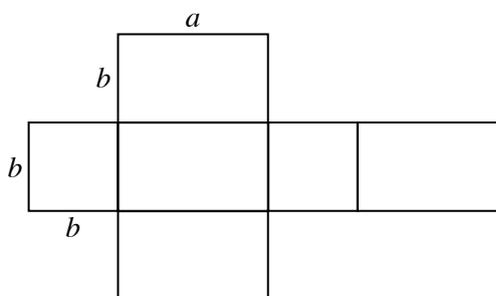
1 cube of side  $a \times a \times a$



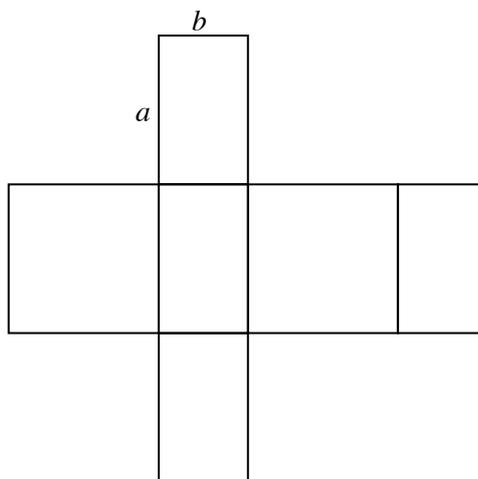
1 cube of side  $b \times b \times b$



3 cuboids of side  $a \times b \times b$



3 cuboids of side  $a \times a \times b$



Once these net diagrams have been made, join them together to make cubes and cuboids.

Ask students to write their individual volumes on each model and combine.

Next step would be to glue all the models and create a big cube of sides  $(a+b)^3$ .

This branch of algebra where the derivation is done geometrically is called geo-algebra.

This topic is conceptual and lots of examples should be explained on the board, and given to students to solve in their notebooks.

The concepts of cubic identities can be introduced by explaining that this identity is an extension from linear and quadratic functions.



## Summary of Key Facts

- Cubes of the sum of two terms,  $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$
- Cubes of the difference of two terms,  $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$
- Numeric evaluation of cubic function can be done by substituting numeric values.
- Identities can be proved using cubic functions.
- Cubic identity can be used to find the sum of three to four terms.



## Frequently Made Mistakes

The sums in this topic will require a lot of concentration as the working of each sum is quite extensive and involves many steps. When doing the sums on the board, the teacher should enunciate each step slowly and carefully. It is common for teachers to keep doing the sums on the board without explaining. If worked carefully, students will memorise, understand the process, and not miss out steps.



## Sample Lesson Plan

### Topic

Algebraic identities

### Specific Learning Objective

Resolving cubic terms into factors

### Suggested Duration

One to two periods

### Key Vocabulary

Cubic functions, Factorisation

### Method and Strategy

Students are already aware of the derivation of the identity, therefore resolving into factors would be easy.

Also students at a time to come on the board and simultaneously do three sums. The rest of the students will prompt any mistakes and also copy the working down. This way each student will get a turn on the board and peer learning will be achieved.

### Written Assignments

Exercise 13a, question 5 (parts 1 to V).

### Evaluation

Alternate sums from Exercises 13a and 13b can be given as a test to assess students' ability to use the correct identity.

### After completing this chapter, students should be able to:

- find the cubes of the sum of two terms,
- find the cubes of the difference between two terms, and
- find special products using between algebraic identities.



# Factorisation of Algebraic Expressions



## Specific Learning Objectives

In this unit, students will learn:

- how to resolve expressions into factors.
- how to factorise expressions of the form  $ax^2 + bx + c$ .



## Suggested Time Frame

5 to 6 periods



## Prior Knowledge and Revision

The students have learnt about the cubic function factorisation in the earlier chapter. This chapter takes factorisation forward. No revision is required as it is a continuation of previous learning.



## Real-life Application and Activities

By resolving into factors we derive new algebraic identities.

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Middle term break up!

In quadratic functions of  $ax^2 + bx + c$  we multiply the numeric values of 'a' and 'c' and select factors that will break up the middle term 'bx'. Hence resolving into factors by middle term break up.

### Activity

Specific learning objective: Factorising trinomials by using grouping/breakup method.

Example:

The teacher should write the trinomial  $6x^2 + x - 2$  on the board.

Teacher: Is this trinomial in the form of  $ax^2 + bx + c$ ?

If 'yes' then let us factorise.

Steps:

1. Write the values of  $a$ ,  $b$ , and  $c$ .

$$a = 6 \quad b = +1 \quad c = -2$$

2. Find ' $ac$ '.

$$6x^2 + x - 2$$

$$ac = 6 \times 2 = 12$$

3. Find two integers whose product is ' $ac$ ' and sum is ' $b$ '.

Factors of 12:  $(1 \times 12)$ ,  $(2 \times 6)$ ,  $(3 \times 4)$ .

Choose the pair  $(3, 4)$  because their product is 12 and their difference is 1.

4. Rewrite the middle term ' $bx$ ' as the sum of two terms whose coefficients are the integers found in step 3.

Hence,

$$6x^2 + x - 2$$

$$6x^2 + 4x - 3x - 2$$

5. Now let us factorise by grouping, a method that was learnt earlier.

$$6x^2 + 4x - 3x - 2$$

$$2x(3x + 2) - 1(3x + 2)$$

$$(3x + 2)(2x - 1)$$

$\therefore$  The factors of  $6x^2 + x - 2$  are  $(3x + 2)(2x - 1)$ .

Now solve these.

1.  $x^2 + 7x + 10$

2.  $x^2 - 9x + 14$

3.  $x^2 + x - 20$

4.  $x^2 - x - 12$

5.  $2x^2 + 7x + 3$

6.  $3x^2 + 7x - 6$

7.  $-x^2 - 4x + 32$

8.  $4x^2 - 6$

### Answers

1.  $(x + 2)(x + 5)$

2.  $(x - 2)(x - 7)$

3.  $(x - 4)(x + 5)$

4.  $(x + 3)(x - 4)$

5.  $(2x + 1)(x + 3)$

6.  $(3x - 2)(x + 3)$

7.  $(-x + 4)(x + 8)$

8.  $2(2x + 1)(x - 2)$



## Summary of Key Facts

The following identities are covered in this chapter:

- $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
- $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
- $ax^2 + bx + c$



## Frequently Made Mistakes

To avoid mistakes, students need to identify and then apply the correct format to find factors. Repetitive exercises and practice is a must.



## Sample Lesson Plan

### Topic

Factorisation of algebraic expressions

### Specific Learning Objectives

Resolve cubic expressions into factors

### Suggested Duration

One period

### Key Vocabulary

Cubic, Trinomial, Factoring, Grouping, Identity

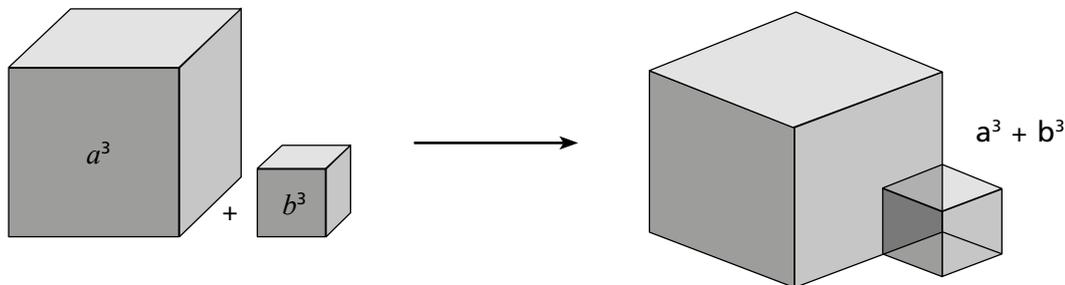
### Methodology and Strategy

Explanation of solving cubic functions can be done by the 'teach-by-asking method' and by engaging students in discussion and doing sums on board.

Derivation of algebraic identity by geometry is called 'geo-algebra'.

Deriving the  $a^3 + b^3$  identity by showing a cube of sides ' $a$ ' and another cube of sides ' $b$ ' and calculating their volumes is essentially the geometric aspect of this algebraic identity.

The students will have a better understanding of this concept.



Refer to page 168 of the textbook for further explanation of the concept.

### Written Assignments

The teacher can give the sums from Exercise 14a as classwork and for homework.

Peer checking will help them understand their mistakes better. Follow up of homework the next day with peer checking is recommended.

### Evaluation

Frequent quizzes and short tests based on algebraic identities will help students to understand the concept better.

### After completing this chapter, students should be able to:

- resolve expressions into factors, and
- factorise expressions of the form  $ax^2 + bx + c$ .



# Basic Operations on Algebraic Fractions



## Specific Learning Objectives

In this unit, students will learn:

- how to multiply algebraic fractions.
- how to divide algebraic fractions.



## Suggested Time Frame

4 to 5 classes



## Prior Knowledge and Revision

Algebraic fractions involve the same concepts as numeric fractions. It is advisable to revise a few addition and subtraction fraction sums with numerical values.



## Real-life Application and Activities

- This chapter is a culmination of factorisation. Divide students into groups of four or five and ask them to design their own test paper. The test paper format should be clearly specified as follows:
- five sums,
- the first question should be of simple reduction of algebraic fraction by factorisation,
- the second question can be of complex indices fractions where the laws of indices are applied,
- the third question can be of algebraic fractions with multiplication and division,
- the fourth and fifth questions should comprise algebraic fractions with addition and subtraction respectively.

Give students ten minutes to set the test paper.

These tests should then be handed to a different group. Students will all take the tests and then after 15 minutes, hand them back to the the group who set the paper.

The tests should then be checked and marked by the group.

This exercise is an interactive activity which can be adapted for any chapter.

The teacher should divide Exercise 15 into three sections.

The first section will involve simple factorisation of the numerator and the denominator and the terms simplified. The teacher should emphasise the three identities and middle term break up. The key to the reduction is that the students first factorise and then reduce the brackets. Once factorised, the term need to be simplified.

Before starting the lesson highlight the rules given on page 176. The students can write them in their notebooks for reference.

### Answers

$$1) \left(1 - \frac{25}{4x^2}\right) \div \left(1 - \frac{5}{2x}\right)$$

$$1) 1 + \frac{5}{2x}$$

$$2) \frac{x^2 + 2x}{x^2 + x - 2} - \frac{3x}{x + 1}$$

$$2) \frac{2x(2-x)}{(x-1)(x+1)}$$

$$3) \frac{a}{a-3} - \frac{8}{a} = 2$$

$$3) -6, 4$$

$$4) \frac{5}{6x} + \frac{6}{7x} - \frac{9}{14x} = 14$$

$$4) \frac{11}{147}$$

$$5) \frac{5}{2x-1} - \frac{4}{4x-2} - \frac{3}{6x-3} = 1$$

$$5) \frac{3}{2} \text{ or } 1\frac{1}{2}$$

$$6) \frac{a+1}{2a-8} - \frac{a+z}{12x-3a}$$

$$6) = \frac{5a+7}{6(a-4)}$$

$$7) \frac{3}{x+1} - \frac{1}{2x+2} = 5$$

$$7) -\frac{1}{2}$$

$$8) \frac{1}{4e+2f} - \frac{1}{f-2e}$$

$$8) \frac{3}{2(2e-f)}$$



### Summary of Key Facts

- To factorise a polynomial the three identities are highlighted.
- If the identities do not apply, then simply factorise.
- The next step is to check if middle term break up is applicable.
- Once the terms are factorised, they are simplified if the operation is multiplication or division.
- In the case of addition and subtraction, the denominators are converted to a common term by finding the LCM.
- The rules for finding the LCM are to take the common term once and the uncommon terms also.
- The LCM is then divided by the denominators.
- The term is then multiplied by the numerator and simplified.



## Frequently Made Mistakes

The key to solving these sums is to factorise and find the correct LCM. As students sometimes get confused, the three identities need to be revised regularly.



## Sample Lesson Plan

### Topic

Basic operations on algebraic fractions

### Specific Learning Objective

Multiply and divide algebraic fractions

### Suggested Duration

1 period

### Key Vocabulary

Factorisation, Lowest Common Multiple, Identity, Middle term, Simplification

## Method and Strategy

This chapter involves an advanced level of knowledge of algebra, involving fractions. So, if concepts of algebra taught earlier are still unclear, the students will face difficulty in handling these sums.

Exercise 15 should be divided into three parts: questions 1 to 6, questions 7 to 16, and questions 17 to 25. Each section will take a lot of time and lots of practice is required to grasp all the concepts.

It is advisable that alternate sums are done on the board by the teacher, and the remaining sums are done by the students.

Only when the students are able to tackle easier sums (questions 1 to 6), without any difficulty, should the teacher proceed to questions 7 to 16. Sums 16 onwards are quite lengthy and must be done very carefully.

### Example

$$\frac{a^2 + 7ab + 12b^2}{a^2 - b^2} \div \frac{a + 4b}{a - b}$$

Write the reciprocal of the second fraction.

$$\frac{a^2 + 7ab + 12b^2}{a^2 - b^2} \times \frac{(a - b)}{(a + 4b)}$$

Factorise each term and simplify.

$$\frac{a^2 + 4ab + 3b + 12b^2}{(a + b)(a - b)} \times \frac{(a - b)}{(a + 4b)}$$

$$\frac{(a + 4b)(a + 3b)}{(a + b)(a - b)} \times \frac{(a - b)}{(a + 4b)}$$

$$\frac{(a + 3b)}{(a + b)} \text{ Answer}$$

### Example

$$\begin{aligned} & \frac{a^2 - 5a + 6}{a^2 - 8a + 16} \\ &= \frac{a^2 - 3a - 2a + 6}{a^2 - 4a - 4a + 16} \\ &= \frac{a(a - 3) - 2(a - 3)}{a(a - 4) - 4(a - 4)} \\ &= \frac{(a - 3)(a - 2)}{(a - 4)(a - 4)} \end{aligned}$$

Factorise the numerator and denominator individually and then reduced to simplest form.

Both the expressions are of the form  $ax^2 + bx + c$ , therefore apply the rules for resolving quadratic expressions into fractions, taught in the previous chapter.

### Example

$$\frac{(x + 2)^2}{x^2 - 4} = \frac{(x + 2)(x + 2)}{(x + 2)(x - 2)} = \frac{(x + 2)}{(x - 2)}$$

## Written Assignments

Alternate sums of Exercise 15 should be done as classwork and homework.

## Evaluation

Five-minute quizzes should be given for each section to check understanding of concepts. Generally these quizzes act as a diagnostic tool to identify students' weaknesses before they proceed to a more complex stage. Assessment of learning is an important tool in teaching and learning mathematical concepts.

A comprehensive test can be given along the lines of Exercise 15 to assess students' performance.

### After completing this chapter, students should be able to:

- multiply algebraic fractions, and
- divide algebraic fractions.



# More Simple Equations



## Specific Learning Objectives

In this unit, students will learn:

- how to solve equations involving algebraic fractions.
- how to solve real-life problems involving simple equations.



## Suggested Time Frame

3 to 4 periods



## Prior Knowledge and Revision

Students should be able to transpose values in algebraic equations, where the inverse of:  
addition is subtraction,  
subtraction is addition,  
multiplication is division, and  
division is multiplication.

They know how to work out the value of an unknown variable 'x'.

A revision exercise can be done on the board, involving students in solving questions step by step.

### Answers

1.  $6x + 4 = 7$

1.  $\frac{1}{2}$

2.  $12x + 4 = 3x + 2$

2.  $-\frac{2}{9}$

3.  $5x + 7 = 2$

3.  $-1$

4.  $22 - x = 4x - 3$

4.  $5$

5.  $35 = 12x - 1$

5.  $3$



## Real-life Application and Activities

This chapter takes algebraic equations to a higher level.

The rules of transposing, learnt earlier, applies in algebraic equations. The term whose value is to be found is taken to the LHS of the equation. The other terms are transposed, the positive term as negative, and the multiplicand as divisor, and vice versa.

The basic rule is that fractions on the either side of the equation are to be cross-multiplied to get a simple algebraic equation.

If there are more than one fraction on either side of the equation, then simplify them by adding or subtracting as stated in the question.

### Example

$$\frac{17x - 5}{12x} = \frac{3}{2}$$

Multiply the term  $(17x - 5)$  by 2, and  $12x$  by 3, to form a single equation:  $2(17x - 5) = 3(12x)$

### Example

$$\frac{3}{x-1} + \frac{1}{x+1} = \frac{4}{x}$$

$$\frac{3(x+1) + 1(x-1)}{(x-1)(x+1)} = \frac{4}{x}$$

$$\frac{3x + 3 + x - 1}{(x-1)(x+1)} = \frac{4}{x}$$

$$\frac{4x + 2}{x^2 - 1} = \frac{4}{x}$$

$$x(4x + 2) = 4(x^2 - 1)$$

$$4x^2 + 2x = 4x^2 - 4$$

$$4x^2 - 4x^2 + 2x = -4$$

$$2x = -4$$

$$x = -2 \text{ Answer}$$

The rules of transposing revised earlier will play an important role in solving the equation.

Exercise 16a, questions 1 to 5 can be done in one period.

In the next lesson, point out that decimal points can be removed either by converting them to fractions or by multiplying the entire equation by multiples of 10. Questions 11 to 17 can be done alternately in class and for homework.



## Summary of Key Facts

- The unknown variable is placed on the left hand side.
- Either transpose a part from the left hand side to the right hand side by changing its sign if it is addition or subtraction, or divide or multiply.

- Problems involving real-life situations require data representation, where the required value is assigned the variable ' $x$ '.
- The unknown quantity and the given condition are expressed in an equation.
- The equation is then solved by transposing.



## Frequently Made Mistakes

Students generally make mistakes in transposing. They forget to change the sign when taking a positive term to either side of the equation.



## Sample Lesson Plan

### Topic

Algebraic equations

### Specific Learning Objective

To form equations under given conditions and solve them

### Suggested Duration

One to two periods

### Key Vocabulary

Transpose, RHS, LHS, Variable

### Method and Strategy

Forming the equations for a word problem involves thinking skills. A number of sums could be done on the board. Ask the students to break up the statement into phrases, and then form the equation. Role play can also be done where the students can enact a work problem.

The sums in Exercises 16a and 16b are progressive. Once the students have understood how to solve them, they can easily be done in class and also be given as homework.

### Example

A father is twice as old as his son now. 20 years ago the father was 4 times as old as his son. Find their present ages.

We can write this word problem algebraically.

Suppose the son's present age is ' $x$ '

$\therefore$  Father present age is  $2x$

20 years ago son's age =  $x - 20$

20 years ago father's age =  $2x - 20$

20 years ago father's age was 4 times his sons age.

$\therefore 2x - 20 = 4(x - 20)$

This algebraic equation can now be solved.

It is important that the students should have developed the ability to convert a word statements into a mathematical statement.

## Written Assignments

For questions 1 to 11 of Exercise 16b, the equations can be discussed and formed in class. Finding the solution can be given for homework. The solution of the equations in this exercise are very simple and only basic transposing is required.

## Evaluation

A comprehensive test of 20 marks can be given where 4 algebraic equations of 4 marks each can be given and one sum of 4 marks could be a word problem.

## After completing this chapter, students should be able to:

- manipulate equations involving algebraic fractions, and
- solve real-life problems.



# Simultaneous Equations



## Specific Learning Objectives

In this unit, students will learn:

- about simultaneous linear equations.
- how to solve a pair of simultaneous linear equations by elimination and by the substitution method.
- how to solve real-life problems involving simple linear equations.



## Suggested Time Frame

4 to 5 periods



## Prior Knowledge and Revision

The teacher should point out to the students that they have been solving linear equations with one unknown variable. It is time to move on and solve more complex sums where the linear equations contain two unknown variables.

Generally if the first variable is ' $x$ ', then the second unknown variable is taken to be ' $y$ '.



## Real-life Application and Activities

This concept can be taught by the 'teach-by-asking' method.

Writes  $6x + 4y = 7$  on the board.

**Teacher:** 'What is new about this equation?'

**Students:** There are two unknown variables.

**Teacher:** Good.

To solve two unknowns, we need two equations to solve simultaneously.

So, let us write another equation.

$$6x + 4y = 7 \quad (\text{i})$$

$$3x + y = 2 \quad (\text{ii})$$

What should we do now?

**Students:** Let us eliminate one of the variable.

**Teacher:** How?

**Students:** By cancelling 'y'.

**Teacher:** Correct, but how should we do that?

To eliminate 'y' we need to make:

coefficients  $\longrightarrow$  the same

signs  $\longrightarrow$  opposite.

How do we make coefficients of 'y' the same?

**Students:** By making 'y' of equation (ii) become  $4y$ .

**Teacher:** 'Great. We can do that by multiplying the entire equation by 4 and then multiplying by  $-1$  to change the signs.'

Hence:

$$6x + 4y = 7 \quad (i)$$

$$3x + y = 2 \quad (ii)$$

Multiply equation (ii) by  $-4$  and add both equations.

$$\begin{array}{r} 6x + 4y = 7 \\ -12x - 4y = -8 \\ \hline -6x = -1 \\ x = \frac{1}{6} \end{array}$$

Substitute value of  $x$  in equation (i).

$$\begin{aligned} 6 \left( \frac{1}{6} \right) + 4y &= 7 \\ 1 + 4y &= 7 \\ 4y &= 6 \\ y &= \frac{3}{2} \end{aligned}$$

Answer  $\left( \frac{1}{6}, \frac{3}{2} \right)$



## Summary of Key Facts

- The elimination method requires one of the variables to be cancelled out.
- This can be done by multiplying either one or both equations by a number to get the same coefficients.
- The signs have to be opposite to cancel the coefficient. This is possible by multiplying the entire equation by  $-1$ .
- The substitution method requires making one variable the subject of the equation and expressing it in terms of the other variable.
- We now replace the term and hence the entire equation is comprised of one variable.
- Once we get the answer of a variable by either method we substitute the value in any one equation to get the value of the other variable.



## Frequently Made Mistakes

When using elimination method the students tend to forget to multiply the entire equation and the constant. The teacher should explain that all the terms on both LHS and RHS of the equation have to be multiplied.



## Sample Lesson Plan

### Topic

Simultaneous linear equations

### Specific Learning Objective

Applying real-life conditions and forming simultaneous equations

### Suggested Duration

One period

### Key Vocabulary

Variable, Simultaneously

### Method and Strategy

When introducing real-life word problems, data representation is extremely critical.

How do you assign the variables 'x' and 'y'?

An entire lesson should be planned on formulating simultaneous linear equations and not solving them. The next lesson can be based on solutions.

### Activity

Divide the students into groups of four and write down a sum for each group on the board.

A student from each group will form the equations on the board.

The following sums can be written on the board.

1) Find the other two angles of a right-angled triangle if one angle is  $\frac{1}{3}$  of the other.

$$x + y = 90^\circ \quad \text{(i) (given)}$$

$$x = \frac{1}{3}y \quad \text{(given)}$$

$$\therefore 3x - y = 0 \quad \text{(ii)}$$

Solve equation (i) and (ii) simultaneously

$$x + y = 90^\circ$$

$$3x - y = 0$$

$$\hline 4x = 90^\circ$$

$$\therefore x = 22.5^\circ$$

Substitute the value of  $x$  in equation (i)

$$x + y = 90^\circ$$

$$22.5^\circ + y = 90^\circ$$

$$\therefore y = 67.5^\circ$$

2) The sum of two numbers is 48 and their difference is 16. Find the two numbers.

$$x + y = 48$$

$$x - y = 16$$

$$\begin{array}{r} - \quad + \quad - \\ \hline x = 32 \end{array}$$

Substitute the value of ' $x$ ' in equation (i)

$$32 + y = 48.$$

$$y = 48 - 32$$

$$y = 16$$

3) If aunt is 4 times as old as her nephew and the sum of their present ages is 50, find their present ages.

Suppose aunt's age is  $x$  and nephew's age is  $y$

$$\text{Then } x = 4y$$

$$x - 4y = 0 \quad \text{(i)}$$

$$\text{and } x + y = 50 \quad \text{(ii)}$$

$$x - 4y = 0$$

$$x + y = 50$$

$$\begin{array}{r} - \quad - \quad - \\ \hline -5y = -50 \end{array}$$

$$y = 10$$

Substitute the value of ' $y$ ' in equation (i)

$$x = 4(10)$$

$$x = 40$$

The aunt is 40 years old and the nephew 10 years old.

### Written Assignments

The equations in Questions 5 to 8 of Exercises 7 can be formed in class and solutions done as homework.

## Evaluation

Two 5-minute quizzes can be done during this lesson. The first quiz may contain two sums to be solved by the elimination method and the next two sums by the substitution method.

These quizzes will ensure that the students have understood this new concept. This chapter can be taught in three stages: finding a solution by the elimination method, finding a solution by the substitution method, and solving real-life word problems.

A comprehensive test can be given at the end of the chapter. The test should contain sums to solved by the elimination and substitution methods and word problems. Do not specify any method to solve the the word problems. The students should be encouraged to choose their own method to solve them. Generally, students prefer the elimination method to the substitution method.

### **After completing this chapter, students should be able to:**

- solve simultaneous equations by elimination and substitution method, and
- solve real-life problems involving two unknown variables.



# Symmetry



## Specific Learning Objectives

In this unit, students will learn:

- linear symmetry.
- properties of symmetrical figures.
- properties of symmetry about a bisector.



## Suggested Time Frame

2 periods



## Prior Knowledge and Revision

Students in their kindergarten and primary classes have done simple activities of cutting shapes into two equal parts and of completing a figure by replicating the other half. The Maths Flash on page 202 can be done as revision or introduction to symmetry. Symmetry can also be linked to congruency. Congruency has been covered earlier where the concept of exact replica is explained. Symmetry also involves congruent shapes on either sides of a line of symmetry.

The teacher can make a work sheet of incomplete shapes and ask students to complete the symmetrical shapes.



## Real-life Application and Activities

The teacher can bring a mirror to the lesson and ask a student to stand in front of it. Explain that the 'mirror image' is actually congruency and the mirror itself acts as a line of symmetry, separating two congruent or identical objects or images.

Students can be encouraged to bring pictures of symmetrical objects and buildings and then ask them to draw the line of symmetry.



## Summary of Key Facts

- Symmetry is when congruent shapes are seen either side of the line of symmetry.
- Vertical symmetry is when a vertical line cuts any shape into two symmetrical halves.
- The horizontal line of symmetry is a horizontal line that cuts a shape into two symmetrical halves.
- A diagonal line of symmetry is a line that cut a shape diagonally into two symmetrical halves.
- The line of symmetry is also known as axis of symmetry of any shape.
- Points of symmetry are equidistant from the axis of symmetry.
- A perpendicular bisector is a line of symmetry of a line.
- An angle bisector is equidistant from two lines forming the angle and also acts as a line of symmetry, containing points equidistant from the two lines.
- Two congruent shapes can be separated by a line of symmetry. A perpendicular bisector is drawn between the two shapes.



## Frequently Made Mistakes

Symmetry is a simple concept that has been taught in previous grades too. Students enjoy doing activities related to symmetry.



## Sample Lesson Plan

### Topic

Symmetry

### Specific Learning Objective

Properties of symmetry of shapes

### Suggested Duration

One period

### Key Vocabulary

Line of symmetry, Equidistant

### Method and Strategy

This is a fun topic and the concepts of line of symmetry and point symmetry will be taught by an activity.

### Activity

You will need: A4 paper, scissors, markers, and rulers.

Ask the students to cut out the following shapes:

rhombus

square

rectangle

parallelogram

isosceles triangle

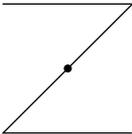
Tell them to fold the cut outs in half as many times as possible. Unfold the shape and count the lines of symmetry.

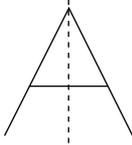
Students will then record their findings in their notebooks.

Shape	Lines of Symmetry
rhombus	2
square	4
rectangle	2
parallelogram	0
isosceles triangle	1

Students can find lines of symmetry in letters also.

For example,

i)  The letter 'Z' has no line of symmetry.

ii)  The letter 'A' has one line of symmetry.

iii)  A parallelogram has no line of symmetry.

## Written Assignments

Question 6 of Exercise 18 can be done as classwork.

## Evaluation

A short, simple test can be given to evaluate learning.

**After completing this chapter, students should be able to:**

- identify the line of symmetry,
- use properties of symmetrical figures to prove theorems, and
- draw line segments and angle bisectors.



# Practical Geometry



## Specific Learning Objectives

In this unit, students will learn:

- about lines and angles.
- to define polygons and quadrilaterals.
- how to:
  - draw parallel lines and two non-parallel lines and find the angle between them.
  - divide a line segment in given parts in the given ratio.
  - construct quadrilaterals with various conditions.
  - construct a triangle and a pentagon.
  - draw tangents to a circle at a given point on the circumference and from a point outside it.



## Suggested Time Frame

4 to 5 periods



## Prior Knowledge and Revision

Students learned the construction of a perpendicular and angle bisectors in the previous grade. It would be advisable to revise the steps on the board and then ask the students to construct two of each type in their notebooks. Revision of construction of triangles with given conditions can also be done.

The teacher should do the construction on the board using geometric instruments, as correct handling of the protractor and compasses is important.



## Real-life Application and Activities

This chapter requires students to develop skills in handling geometric instruments and remembering the steps of construction.

Once they know all the construction methods a class activity can be done by dividing students into groups. All the different constructions can be written on flash cards. Each group can pick a card and start doing the construction on A4 paper. The group that attempts the most constructions in one period wins.



## Summary of Key Facts

- The distance between two parallel lines remains constant at different points through the entire length of the lines. Two parallel lines can easily be drawn with a fixed distance between them.
- A line segment of any length can be divided into any given number of parts.
- A line segment of any length in any ratio.
- The angle between two non-parallel lines can be found without extending the lines.
- The angle between two converging lines can be bisected without producing them.
- Properties of quadrilaterals:
  - A square has all sides equal; each angle of a square equals  $90^\circ$ .
  - A rhombus has all four sides equal.
  - A rectangle has two pairs of opposite sides parallel and equal, and each angle is  $90^\circ$ .
  - A parallelogram has opposite sides parallel and equal.
  - A kite has two pairs of adjacent sides equal.
  - A trapezium has one pair of opposite sides parallel.
- A quadrilateral with two pairs of opposite sides parallel and equal is called a **parallelogram**.
- Properties of a parallelogram:
  - Opposite sides of a parallelogram are equal.
  - Opposite angles of a parallelogram are equal.
  - The diagonals of a parallelogram bisect each other.
- A quadrilateral can be constructed when:
  - the lengths of its four sides and the measure of one of the angles are given.
  - the lengths of three of its sides and the measures of two included angles are given.
  - the lengths of its four sides and a diagonal are given.
  - the lengths of three sides and two diagonals are given.
- A square can be constructed when:
  - the length of each of its side is given.
  - the diagonal is given.
  - the difference between the length of the side and diagonal is given.
  - the sum of its diagonal and side is given.
- A rectangle can be constructed when:
  - the length of two adjacent sides is given.
  - the length of one side and a diagonal are given.
- A parallelogram can be constructed when:
  - the lengths of the adjacent sides are given, and the measure of the included angle is known.
  - the diagonals and the angle between them are given.

- A rhombus can be constructed when:
  - the length of each side and the size of one angle is given
  - the length of each side and one diagonal are given
- A kite can be constructed when lengths of two side and a diagonal are given.
- A regular pentagon and hexagon can be constructed when the lengths of one side is given.
- A right-angled triangle can be constructed when the length of the hypotenuse and the vertical height from the vertex to the hypotenuse are given.  
Remember: a triangle constructed on the diameter of a circle, within a semi-circle, is also a right-angled triangle.
- The in-circle and circum-circle of a triangle can also be drawn.
- A circle is a set of points, equidistant from a fixed point on the same plane.
- A tangent to a circle is a line which meets the circle at only one point.
  - The distance of the tangent to a circle from the centre is equal to the radius of the circle.
  - A tangent to a circle is perpendicular to the radius drawn through the point of contact.
- A tangent to a circle can be drawn:
  - at a point given on the circumference.
  - from a point out side the circle.



## Frequently Made Mistakes

This chapter is mainly skills-based, and does not require much mathematical reasoning. However, emphasis should be placed on neatness and accuracy. Pencils should be well-sharp, and good compasses should be used which are neither too stiff nor too loose. Students are sometimes unable to draw perfect circum-circles and tangents. The teacher should sort out students' concerns individually.



## Sample Lesson Plan

### Topic

Geometric constructions

### Specific Learning Objective

Revision of properties of all shapes

### Suggested Duration

1 period

### Key vocabulary

Bisector, Quadrilateral, Polygon, and Circles

## Method and Strategy

Parallel lines, bisectors, and the line segments are a few key terms that the students will be dealing with in this chapter. The students will also need to revise the properties of quadrilaterals (rectangle, square, and rhombus). Since they have to do construction, they should be aware of these properties:

Square: all sides are equal and all angles are of  $90^\circ$

Rectangle: lengths and breadths are equal; all the angles are of  $90^\circ$ .

Rhombus: all sides are equal and the adjacent angles are supplementary.

This chapter not only develops the students' skills in construction, but also encourages them to link mathematical proofs and computation. The students should be involved in a 'thinking process' while dividing a line segment into ratios or while drawing an in-circle or a circum-circle. Explain that in-circle angle bisectors will give the centre of the angle. Similarly, perpendicular bisectors will need to be constructed to locate the centre of a circum-circle.

## Written Assignments

This lesson is primarily a revision class of the properties of shapes and specific construction cases. The students should be encouraged to write all the key points in their notebooks and make a table of facts.

## Evaluation

This chapter is a skills-based chapter where ability to construct a clear and precise shape is tested. Mathematical reasoning is also involved as the properties are employed to understand the given construction. There should be regular assessment of learning and understanding and their ability to draw shapes and use mathematics instruments. Exercise 19 constructions can be given as a test. For example, questions 8, 16, 17, and 20 can be given as one test. The second test can be of polygons and circles.

## After completing this chapter, students should be able to:

- define polygons and quadrilaterals,
- draw parallel lines,
- divide line segments into given parts and ratios,
- draw angle and perpendicular bisectors,
- construct quadrilaterals with different conditions,
- construct triangles and pentagons, and
- draw tangents.



# Axioms, Postulates, and Propositions



## Specific Learning Objectives

In this unit, students will learn:

- to define demonstrative geometry and reasoning.
- how to describe axioms and postulates.
- how to describe propositions or theorems.
- how to prove theorems based on lines and angles.
- how to prove theorems based on parallel lines.
- how to prove theorems based on triangles.



## Suggested Time Frame

6 to 8 periods



## Prior Knowledge and Revision

The students have learned about the properties of parallel and intersecting lines and the angles formed by them. This chapter deals with the mathematical reasoning and proofs of these facts. It is important that the teacher first does a brief recap of the basic properties of perpendicular, intersecting, and parallel lines.

The students should recall that two intersecting lines form equal vertically opposite angles.

Two parallel lines create corresponding and alternate angles.

Interior angles are not equal and add up to  $180^\circ$ .



## Real-life Application and Activities

There are nine theorems in this chapter and each theorem requires knowledge of accurate mathematical facts backed by mathematical reasoning.

Each theorem should be done on the board by the teacher and explained in detail. The students should then copy them into their notebooks.

## Activity

Students should enjoy this practical activity which will support their learning.

You will need:

2 cm diameter bamboo sticks, markers, protractors, A4 paper, and tape.

Ask the students to place the 2 bamboo sticks on the paper in the form of a cross, and to label the angles with the marker.

Keep on changing the size of the angles, and ask them to prove that the vertically opposite angles are equal by measuring the angles.

If a board protractor is not available, the students can measure the angles by tracing the angles made by the sticks on to chart paper and then measuring them.

This activity will enable the students to demonstrate their geometrical knowledge with mathematical precision.



## Summary of Key Facts

- Geometric demonstration is the proving of mathematical facts with the help of logic and measurement.
- Deductive reasoning is drawing a conclusion drawn based on given facts.
- An axiom is a fundamental statement related to numbers.
- A postulate is a fundamental statement related to a geometric figure.
- A theorem or proposition is based on axioms and postulates.
- A corollary is a statement that is deduced from a theorem.
- Theorems can be proved by mathematical and geometric reasoning.
- Two perpendicular lines form two right angles.
- Two intersecting lines will form angles adding up to  $180^\circ$ .
- Two intersecting lines form two equal, vertically opposite angles.
- If two sides of a triangle and the included angles are congruent, then the two triangles are congruent.
- The two base angles of an isosceles triangle are equal to each other.
- The exterior angle of a triangle is always greater than the sum of the opposite interior angles.
- The two alternate angles are equal, the lines forming these angles are parallel.
- When two parallel lines are intersected by a transversal, the alternate angles formed are equal.
- The sum of the sizes of all three angles of a triangle add up to  $180^\circ$ .



## Frequently Made Mistakes

The naming of lines, angles, and triangles should be accurate.

When writing mathematical proofs, geometric diagrams and reasoning should be given a lot of importance.



## Sample Lesson Plan

### Topic

Axioms and postulates

### Specific Learning Objective

To prove that two intersecting lines form supplementary angles

### Suggested Duration

1 period

### Key Vocabulary

Intersecting, Perpendicular, Supplementary

### Method and Strategy

The students are given a set of data with the help of which they have to prove a fact. This is called mathematical proof. Derivations of formulas and proving theorems are done simultaneously. The students learn to apply mathematical proofs systematically, improving their logical thinking skills. The teacher must ensure that the students do the derivation in a very systematic and organised manner.

The theorems given in the textbook should be explained in detail in the classroom.

They should be explained with diagrams on the board, and put up on the class soft-board for reference. As a further practice tool, ask the students to note them down in their notebooks.

It is important that the teacher stresses the steps of mathematical proof, labelling, and recognising the angles. It is also important to explain the reasons for the mathematic proof.

### Example

If two straight lines,  $\overleftrightarrow{AB}$  and  $\overleftrightarrow{CD}$ , intersect each other at the point O and  $m\angle AOD = 90^\circ$ , prove that the remaining angles will also be right angles.

If  $\angle AOD = 90^\circ$

then  $\angle BOC = 90^\circ$  (vertically opposite)

$\angle BOC + \angle AOC = 180^\circ$  (angles on a straight line/supplementary)

Since  $\angle BOC = 90^\circ$

$\therefore \angle AOC = 90^\circ$

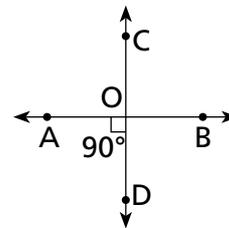
$\angle AOC = \angle BOD$  (vertically opposite angle)

Similarly  $\angle BOD = 90^\circ$

$\angle AOD = \angle BOC = \angle AOC = \angle BOD = 90^\circ$  (Proved)

$\angle AOD + \angle BOC + \angle AOC + \angle BOD = 4(90^\circ) = 360^\circ$

When proving that all angles are equal to  $90^\circ$ , it is important to mention supplementary and vertically opposite angles as reasons.



## Written Assignment

Questions 1 to 4 of Exercise 20 can be done in class.

## Evaluation

A comprehensive assessment can be given to evaluate students' understanding of theorems and their proofs. Since this is new territory, it will be advisable to give tests. The first test can cover the first four theorems and the second can cover theorems 5 to 9.

## After completing this chapter, students should be able to:

- define demonstrative geometry and reasoning,
- define axiom and postulate,
- define proposition and postulate,
- prove theorems relating to lines and angles,
- prove theorems related to parallel lines, and
- prove theorems related to triangles.



# 21

# Area and Volume



## Specific Learning Objectives

In this unit, students will learn:

- how to use Hero's formula to find the area of a triangle in terms of its sides.
- how to find volume and surface area of a right circular cone.
- how to find the volume and surface area of a sphere.



## Suggested Time Frame

4 to 5 periods.



## Prior Knowledge and Revision

In earlier classes students have learnt about the area of shapes, volume, and surface area of cubes, cuboids, and cylinders. It will be important to revise all the formulas of each shape on the board. A quick' recall quiz can be conducted and the winning group can be awarded marks that can be added to their aggregate marks as a bonus.



## Real-life Application and Activities

The students are familiar with the concepts of volume and surface area. The teacher should revise the concepts and explain the link between 2D and 3D figures. It is important to re-teach the concept of height/depth to differentiate between 2D and 3D figures.

Cubes and cuboids were studied in grade 7; the formulas could be revised and the students could be given a quiz. This will help the teacher to assess whether or not to move on with the topic.

In this chapter, cylinders, cones, and spheres have been discussed. Before the formulas are introduced, it is imperative that the teacher explains the dimensions of each shape.

**Cylinder:** this shape has radius and height.

$$\text{Volume} = \pi r^2 h$$

$$\text{Curved surface area} = 2\pi r h + \pi r^2 + \pi r^2$$

$$\text{Total surface area} = 2\pi r h + 2\pi r^2$$

**Cone:** this shape has a radius and height, but also slant height ' $l$ '.

Volume:  $\frac{1}{3} \pi r^2 h$

Curved surface area =  $\pi r l$

Total surface area =  $\pi r l + \pi r^2$

**Sphere:** this shape has a radius and a curved surface.

Volume:  $\frac{4}{3} \pi r^3$

Surface area:  $4\pi r^2$

If the students understand the formulas and can identify the dimensions, it should not pose a problem for them. Continuous revision of formulas is therefore essential.

### Activity

The students will understand better if the shapes are explained with the help of net diagrams.

To make net diagrams you will need: A4 paper, geometry instruments and markers.

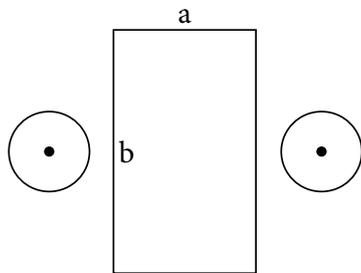
**Cylinder:** this shape is formed by folding a rectangle into a roll. The breadth of the rectangle becomes the height of the cylinder, and the length, the circumference of its base.

Net diagram of a cylinder

'a' is the height of the cylinder.

'b' is the circumference of the base of the cylinder.

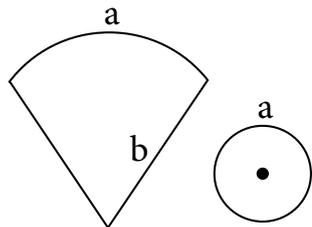
This activity can be used as a recall in order to understand other shapes better.



Net diagram of cone

'a' is the circumference of the base of the cone.

'b' is the slant height of the cone.



**Cones:** this shape is formed by folding a semicircle or any fractional part of a circle (sector), and a circle as its base.

The students should be encouraged to first cut out the 2D figure and then fold it to form the 3D figure.



## Summary of Key Facts

- Hero's formula calculates the area of a triangle by first finding the value of 's', which is the sum of all three sides divided by two. The value of 's' is then inserted in the formula:

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

where  $s$  is the semi-perimeter of a triangle:

$$s = \frac{a+b+c}{2}$$

This formula is applied when the perpendicular distance is not given.

- The surface area of a cone has two parts, the curved surface area and the total surface area.

Area of the circular base =  $\pi r^2$

Total surface area of a right circular cone = Area of the curved surface + Area of the base

$$= \pi r l + \pi r^2$$

$$= \pi r(l+r)$$

Volume of a cone is a derivative of the cylinder; it is one third the volume of a cylinder.

- The volume of a right-circular cone =  $\frac{1}{3}$  area of base  $\times$  height

$$= \frac{1}{3} \pi r^2 h$$

- Surface area of a sphere =  $4 \pi r^2$ , where  $r$  is the radius of the sphere.

- Volume of a sphere =  $\frac{4}{3} \pi r^3$ , where  $r$  is the radius of the sphere.



## Sample Lesson Plan

**Topic**

Area

### Specific Learning Objective

Finding the area of shaded regions of composite shape and Hero's formula

### Suggested Duration

One period

### Key Vocabulary

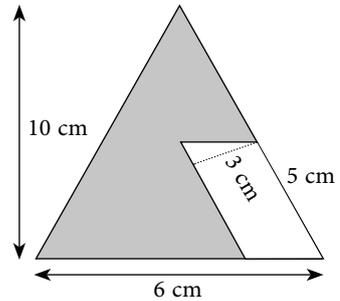
Hero's formula, Perpendicular height

### Method and Strategy

Perpendicular heights are slightly difficult for students to identify as they are sometime drawn outside the triangle. Similarly, areas of shaded regions are found by first finding the total area; then, the unshaded area is found, and subtracted from the total area.

Area of the shaded region = Area of triangle – Area of parallelogram

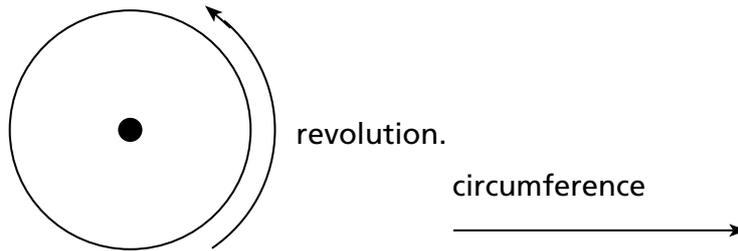
$$\begin{aligned}
 &= \left(\frac{1}{2} \times 10 \times 6\right) - (5 \times 3) \\
 &= 30 - 15 \\
 &= 15 \text{ cm}^2
 \end{aligned}$$



This chapter is an extension of the work already taught in grade 7. The teacher now has to develop the students' ability to visualise word problems and then sketch them in order to work out the values and formulas. Just as in word problems, data representation is the key to finding the solution. Similarly, diagrammatic representation is very important in chapters related to area and volume.

Students should understand that a circle's revolution covers, linearly, a distance equal to its circumference:

one revolution = circumference of the circle.



With this activity-based method, the whole class will participate enthusiastically and the least-able students will understand the concepts better.

Alternately the use of Hero's formula should be encouraged when the perpendicular height is not given. because they do not need to use Pythagoras' theorem to find it. All the three sides are added and the value of 's' calculated and substituted in the formula.

## Written Assignments

Question 3 of Exercise 21 can be done in their notebooks.

## Evaluation

Questions 8 to 13 of Exercise 21 can be done in class with the teacher and similar sums can be given as a test. Questions 1 to 8 of Exercise 21 can be given as a revision test. The teacher should make sure the concepts are revised and then the sums are given as a marked assignment.

**After completing this chapter, students should be able to:**

- use Hero's formula to calculate the area of a triangle,
- find the surface area and volume of a cone, and
- find the surface area and volume of a sphere.



# Trigonometry



## Specific Learning Objectives

In this unit, the students will learn:

- how to calculate the sides of a right-angled triangle using Pythagoras' theorem.
- how to define the trigonometric ratios of an acute angle.
- how to find the trigonometric ratios of  $30^\circ$ ,  $45^\circ$ , and  $60^\circ$ .
- how to find the trigonometric ratios of complementary angles.
- how to solve a right-angled triangle using the measures of one angle and length of one side.
- how to solve a right-angled triangle using the measures of two sides.
- how to solve real-life problems related to trigonometry.



## Suggested Time Frame

6 to 8 periods



## Prior Knowledge and Revision

Students know about right-angled triangles and their properties. They know that the remaining two angles are complementary.



## Real-life Application and Activities

Pythagoras' Theorem can be proved practically through an activity.

### Activity

You will need A4 paper, ruler, markers, glue stick, and pencils.

The activity on page 252 of the textbook can be carried out in class. Ask the students to bring the listed material to the lesson.

The students can measure and cut out the shape and glue it in their notebooks.

The mathematical proof can be done on the cut-outs and the final theorem written on the base of the cut-out in their notebooks.

## Activity

Similarly once the trigonometric ratios have been done thoroughly, examples 4, 5, and 6 on pages 254 and 255 can be done practically.

For example 4, if a ladder is available, in school it can be propped up outside the classroom and the students can go out and investigate. Alternately, if a ladder is not available, the students can stand under a staircase and the same sum can be visualised and done.

For example 5, if there are two buildings in the school, the students can be taken outside and the activity carried out. However, it would be advisable to ask the school caretaker to help by using a rope and chalk to draw right-angled triangle markings on the field.

For the activity in example 6, the teacher will be required to bring a small suitcase or any cuboid box will do. A right-angled triangle can be marked using a white board marker. The students can measure the dimensions of the suitcase and carry out the working in their notebooks.



## Summary of Key Facts

- Pythagoras' theorem states that in a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

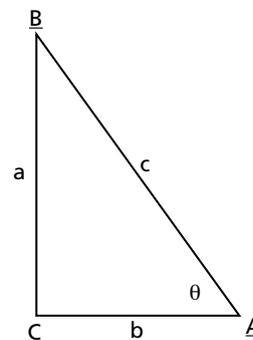
$$c^2 = a^2 + b^2$$

- The ratios of a right-angled triangle have been derived and assigned names as follows:

$$\sin \theta = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{a}{c} \quad \left(\text{Ratio } \frac{\overline{BC}}{\overline{AB}}\right)$$

$$\cos \theta = \frac{\text{base}}{\text{hypotenuse}} = \frac{b}{c} \quad \left(\text{Ratio } \frac{\overline{AC}}{\overline{AB}}\right)$$

$$\tan \theta = \frac{\text{perpendicular}}{\text{base}} = \frac{a}{b} \quad \left(\text{Ratio } \frac{\overline{BC}}{\overline{AC}}\right)$$



- The trigonometric ratios of  $30^\circ$  angles are as follows:

$$\sin 30^\circ = \frac{1}{2}$$

$$\cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$\tan 30^\circ = \frac{1}{\sqrt{3}}$$

- The trigonometric ratios of  $60^\circ$  angles are as follows:

$$\sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\cos 60^\circ = \frac{1}{2}$$

$$\tan 60^\circ = \frac{\sqrt{3}}{1} = \sqrt{3}$$

- The trigonometric ratios of  $45^\circ$  angles are as follows:

$$\sin 45^\circ = \frac{1}{\sqrt{2}}$$

$$\cos 45^\circ = \frac{1}{\sqrt{2}}$$

$$\tan 45^\circ = 1$$

- The trigonometric ratios of complementary angles are as follows:

$$\sin (90^\circ - \theta) = \cos \theta.$$

$$\cos (90^\circ - \theta) = \sin \theta.$$

$$\tan (90^\circ - \theta) = \cot \theta.$$



## Frequently Made Mistakes

Emphasise that the identification of the dimensions are very important. The position of the angle determines the perpendicular and the base. The perpendicular is always opposite to the angle and the base is adjacent to the angle. The hypotenuse is the longest side, opposite the right angle.

## Sample Lesson Plan

### Topic

Trigonometry

### Specific Learning Objective

Real-life word problems using trigonometric ratios

### Suggested Duration

1 to 2 periods

### Key Vocabulary

Sin, Cos, Tan, Hypotenuse, Perpendicular, Base

### Method and Strategy

Questions 4, 7, 8, 9, 10, 11, and 13 of Exercise 22 are based on real-life applications.

Each sum can be visualised and discussed. Students can give examples of places in school and outside their homes.

If internet is available, real-life videos of places and objects can be shown or five minute PowerPoint presentation by the teacher would help students' understanding.

Once visualised the students can then draw the diagrams of the word problems and apply the trigonometric ratio.

### **Written Assignments**

When each sum has been discussed in detail, students should do the calculations in their notebooks.

### **Evaluation**

A comprehensive assessment should be completed. Time can be assigned to do any corrections as this is also a learning process.

### **After completing this chapter, students should be able to:**

- calculate the sides of a right-angled triangle using Pythagoras' theorem,
- define the trigonometric ratios of an acute angle,
- find the trigonometric ratios of  $30^\circ$ ,  $45^\circ$ , and  $60^\circ$  angles,
- find the trigonometric ratios of complementary angles,
- solve a right-angled triangle using the size of one angle and length of one side,
- solve a right-angled triangle using the lengths of two sides, and
- solve real-life problems related to trigonometry.



# Information Handling



## Specific Learning Objectives

In this unit, the students will learn:

- how to define primary and secondary data.
- about various methods of collecting primary data.
- how to classify and tabulate data.
- about range and class interval.
- how to present data graphically: bar graphs and histograms.
- how to find mean, weight mean, median, and mode.



## Suggested Time Frame

4 to 5 periods



## Prior Knowledge and Revision

Students are familiar with this strand of mathematics. Statistical data can be represented in various ways: pictograms, line graphs, pie charts, and bar graphs.

A quick review of this representation can be done on the board where a frequency distribution table is given and the students are asked to represent the data in any two ways of their choice.



## Real-life Application and Activities

### Activity

On page 265 there is a list of questionnaires for statistical data collection and investigation.

For example, students can be given a passage to read and count each of the vowels *a, e, i, o, u*.

They can then prepare a questionnaire and collect the data from their friends.

Once the data is collected, the next step is to classify and tabulate.

The students can be taught the '*x*' and '*y*' values as the subject and its frequency respectively.

The last stage is the presentation which can be done both as a bar graph and a histogram.

This activity takes a week and at the end their presentations can be put up on the display board.

The students have learnt about statistics. It is based on the collection, organisation, and representation of data. The teacher should give them a revision worksheet of bar graphs where they interpret, read and draw bar graphs.

In this chapter, not only are bar graphs revised, but histograms are also introduced. Interestingly, histograms of grouped data are also introduced. Class intervals or class widths, frequency, and frequency distribution are terms that the students will come across in this chapter.

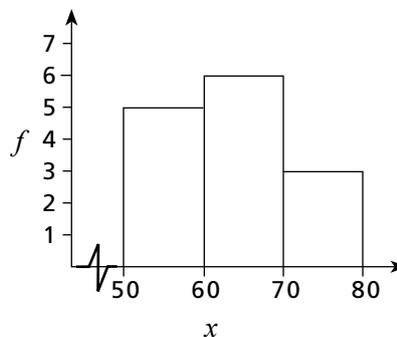
When introducing histograms, the teacher should differentiate clearly between bar graphs and histograms.

The drawing of histograms will also require clear instructions. There are no gaps between the bars of the histograms, and the scale chosen will have to be accurately represented on the graph.

Explain that the 'x' value is sometimes given in the form of groups, and that the values will be written on the graph on the sides of each bar rather than the middle (as is done in bar graphs.)

$x$	$f$
50 – 60	5
60 – 70	7
70 – 80	3

A grouped data histogram



'Averages' progresses to 'mean' at this level, as frequency has to be individually multiplied by the value of 'x' to get the ' $fx$ ' product.

### **Example**

2, 2, 2, 3, 3, 5, 6, 6, 6

$$\text{Mean} = \frac{\sum fx}{f} = \frac{(2)(3) + (3)(2) + (5)(1) + 6(3)}{9} = \frac{35}{9} = 3.88 \text{ or } 3.9$$

### **Activity**

This chapter could be used as a project topic where the students can be encouraged to collect data and then represent it on a histogram. The data collected could be:

- number of people investing in four or five types of shares.
- batting statistics of four or five famous batsmen.
- age in years.
- heights or weights of students in class.



## Summary of Key Facts

- Raw data is the first or initial collection of data that has not been organised or tabulated.
- Classification is the process of arranging data in groups or classes.
- A frequency distribution table represents a set of class intervals and their corresponding frequencies.
- Continuous frequency distribution is represented by histograms. There are no intervals or gaps on the 'x' axis.
- The measures of central tendency is the single value of the data that gives information about the central value of the data.
- There are three types of averages: mean, median, and mode.
- The mean is the average of grouped or weighted data.
- The median is the middle value of the data.
- The mode is the most frequently occurring data value.



## Frequently Made Mistakes

Students generally make mistakes in differentiating between bar graphs and histograms where the first type has intervals or gaps. This is a relatively simple chapter and students tend to enjoy the data presentation.



## Sample Lesson Plan

### Topic

Information handling

### Specific Learning Objective

Mode and median

### Suggested Duration

One period

### Key Vocabulary

Median, Mode, Frequency

### Method and Strategy

The students should understand the concept of the mean.

The teacher should then proceed to explain the other measures of central tendency.

Before explaining mode and median, the teacher should explain the most important rule of data collection: to organise the given data in ascending order.

If items of the data have to be grouped or tabulated, the first rule is to arrange them from the smallest to the greatest value.

Once this is done, the mode will be the value that occurs most frequently in a group of data.

Therefore, the 'x' value with the highest frequency is the mode.

The median is the middle value, but if the frequency is even-numbered two middle values are selected. If the frequency is odd-numbered then there is only one middle 'x' value.

This concept can be demonstrated with a small activity in class.

### **Activity**

Line up ten students according to their heights in ascending order. The teacher will point out that the measure of their heights are their 'x' value.

She/he will then remove one student at a time from the right and the left hand side. In the end, two middle height students will be left. The average of their heights will be the median.

Similarly, the teacher will add on the eleventh student and the process repeated. The students will see that only one student will be left in the middle. The middle value is the median, with equal number of values on either sides.

### **Written Assignments**

Questions 14, 15, and 16 of Exercise 23 can be done in class by the students in their notebooks.

### **Evaluation**

Since this chapter is all about presentation and tabulation, the students can be marked on their classwork and homework assignments. They should be assessed on the accuracy of their presentation and drawings.

A test comprising multiple-choice questions, true and false, and concept-based questions can be given at the end of the chapter.

### **After completing this chapter, students should be able to:**

- define primary and secondary data,
- use various methods of collecting primary data,
- classify and tabulate data,
- find range and class interval,
- present data graphically: bar graphs and histograms, and
- find the mean, weight mean, median, and mode.



# Assessment

A teacher's journey involves three stages Exposition, Practice, and Consolidation.

Exposition is the setting forth of content, and the quality and extent of the information relayed.

Practice involves problem solving, reasoning and proof, communication, representations, and correction.

Assessment is the final stage of consolidation of the process of learning.

Assessment of teaching means taking a measure of its effectiveness.

**Formative** assessment is measurement for the purpose of improving it.

**Summative** assessment is what we normally call evaluation.

An ideal and fair evaluation involves a plan that is comprehensive. It covers a broad spectrum of all aspects of mathematics. The assessment papers should test all aspects of topics taught. These can be demarcated into categories: basic, intermediate, and advanced content. The advanced content should be minimal as it tests the most able students only.

Multiple choice questions, also known as fixed choice or selected response items, required students to identify the correct answer from a given set of possible options.

Structured questions assess various aspects of students' understanding: knowledge of content and vocabulary, reasoning skills, and mathematical proofs.

All in all the teacher's assessment of students' ability must be based on classroom activity, informal assessment, and final evaluation at the end of a topic and/or the year.

# Specimen Paper

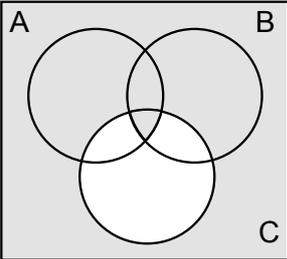
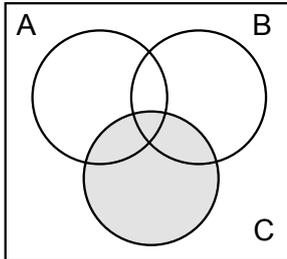
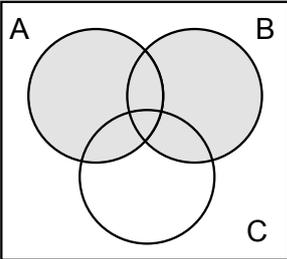
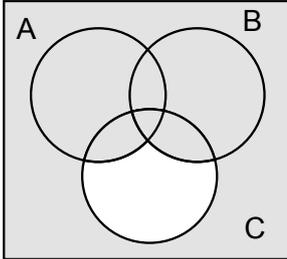
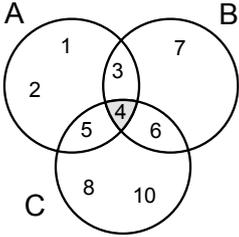
## Mathematics

### Grade 8

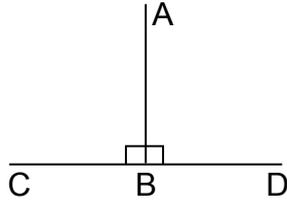
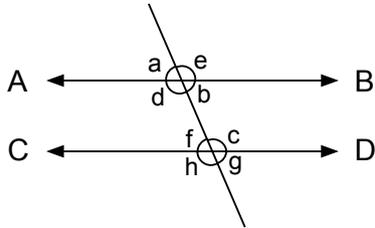
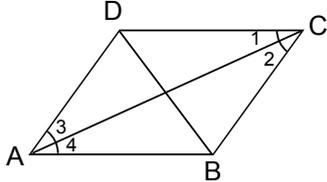
#### Section A

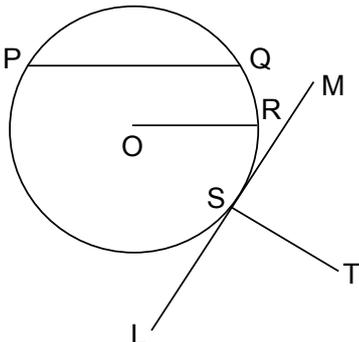
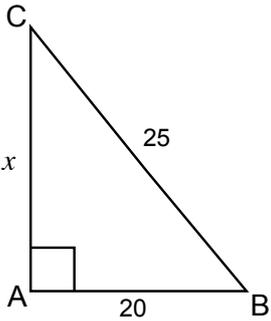
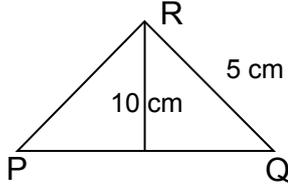
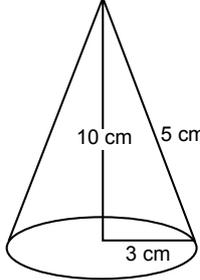
Time: 1 Hour

Total Marks: 40

<b>Q1.</b>	<p>If <math>A = \{1, 2, 3, 4\}</math>, then the proper subset of <math>A</math> is</p> <p><b>A.</b> <math>\emptyset</math></p> <p><b>B.</b> <math>\{\emptyset\}</math></p> <p><b>C.</b> <math>\{2, 3\}</math></p> <p><b>D.</b> <math>\{1, 2, 3, 4\}</math></p>	<b>Q2.</b>	<p>Which of the following is a set of composite numbers.</p> <p><b>A.</b> <math>\{2, 4, 6\}</math></p> <p><b>B.</b> <math>\{2, 3, 5\}</math></p> <p><b>C.</b> <math>\{0, 1, 2\}</math></p> <p><b>D.</b> <math>\{16, 25, 36\}</math></p>
<b>Q3.</b>	<p><math>(A \cap B) \cup C'</math></p> <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;"> <p><b>A</b></p>  </div> <div style="text-align: center;"> <p><b>B</b></p>  </div> </div> <div style="display: flex; justify-content: space-around; margin-top: 10px;"> <div style="text-align: center;"> <p><b>C</b></p>  </div> <div style="text-align: center;"> <p><b>D</b></p>  </div> </div>	<b>Q4.</b>	<div style="text-align: center;">  </div> <p>What is the value of <math>A \cap B \cap C</math>?</p> <p><b>A.</b> <math>\{4, 5, 3, 6\}</math></p> <p><b>B.</b> <math>\{4, 3\}</math></p> <p><b>C.</b> <math>\{5, 4, 6\}</math></p> <p><b>D.</b> <math>\{4\}</math></p>
<b>Q5.</b>	<p>Which of the following is correct?</p> <p><b>A.</b> <math>\frac{1}{4^3} &gt; \frac{1}{5^3}</math></p> <p><b>B.</b> <math>\frac{1}{4^3} \geq \frac{1}{5^3}</math></p> <p><b>C.</b> <math>\frac{1}{5^3} &gt; \frac{1}{4^3}</math></p> <p><b>D.</b> <math>\frac{1}{5^3} \leq \frac{1}{3^3}</math></p>	<b>Q6.</b>	<p>We can write base 10 as</p> <p><b>A.</b> <math>13_{10}</math></p> <p><b>B.</b> <math>13^{10}</math></p> <p><b>C.</b> <math>10^{13}</math></p> <p><b>D.</b> <math>10_{13}</math></p>

<b>Q7.</b>	$1_2 + 1_2$  <b>A.</b> $10_2$ <b>B.</b> $2_2$ <b>C.</b> $100_2$ <b>D.</b> $2_{10}$	<b>Q8.</b>	A machine that allows people to withdraw cash from their account without going to the bank. <b>A.</b> Debit card <b>B.</b> Automated Teller Machine <b>C.</b> Cheque <b>D.</b> Automatic Cash Machine
<b>Q9.</b>	A written instruction from the account holder to the bank for cash is a <b>A.</b> cheque <b>B.</b> pay order <b>C.</b> demand draft <b>D.</b> credit card	<b>Q10.</b>	A written agreement by which a renter can use property on rent for a specific period is called <b>A.</b> over draft. <b>B.</b> running finance. <b>C.</b> demand finance. <b>D.</b> leasing.
<b>Q11.</b>	If CP = Rs 100 and SP = Rs 90, then which of the following statement is correct? <b>A.</b> Profit of 10% <b>B.</b> Loss of 10% <b>C.</b> Loss of 90% <b>D.</b> Profit of 90%	<b>Q12.</b>	The polynomial $8x^2y^3 + 4x^2y^3 + xy^2 + x^2$ has 'n' number of terms. Find the value of n.  <b>A.</b> $n = 9$ <b>B.</b> $n = 4$ <b>C.</b> $n = 6$ <b>D.</b> $n = 5$
<b>Q13.</b>	In the polynomial $3x^4 + 5x^2 + 7x + 8$ the highest degree is <b>A.</b> 1 <b>B.</b> 2 <b>C.</b> 3 <b>D.</b> 4	<b>Q14.</b>	$2a + 8 = 7$ <b>A.</b> $-\frac{1}{2}$ <b>B.</b> $\frac{1}{2}$ <b>C.</b> 2 <b>D.</b> -2
<b>Q15.</b>	$(105)^2$ is equal to <b>A.</b> $(100)^2 + 2(100)(25) + (5)^2$ <b>B.</b> $(100)^2 + 2(100)(5) + (5)^2$ <b>C.</b> $(100)^2 + 2(10)(25) + (5)^2$ <b>D.</b> $(100)^2 + 2(10)(5) + (5)^2$	<b>Q16.</b>	Ali is 3 years older than Sara. In two years time his age will be: (take Ali's age as $x$ and Sara's age as $y$ ) <b>A.</b> $x = y + 5$ <b>B.</b> $x - 2 = y$ <b>C.</b> $x + 2 = y + 3$ <b>D.</b> $x - y = 3$

<p><b>Q17.</b></p>	<p>If <math>2x + y = 11</math> and <math>x - y = 10</math>, then <math>x</math> is equal to</p> <p>A. <math>-7</math>            B. <math>7</math>            C. <math>-\frac{1}{3}</math>            D. <math>\frac{1}{3}</math></p>	<p><b>Q18.</b></p>	<p>In the simultaneous equations <math>4x + y = 7</math> and <math>5x + 2y = 12</math>, the value of <math>x</math> is</p> <p>A. <math>\frac{2}{3}</math>            B. <math>\frac{1}{3}</math>            C. <math>\frac{3}{2}</math>            D. <math>0</math></p>
<p><b>Q19.</b></p>	<p>If <math>4z = x</math> and <math>4wz = v</math>, then elimination of 'z' by substitution method gives</p> <p>A. <math>W = \frac{v}{x}</math>            B. <math>W = \frac{x}{v}</math>            C. <math>V = 4w</math>            D. <math>V = 16wz^2</math></p>	<p><b>Q20.</b></p>	 <p>A. AB is parallel to CD.            B. AB is perpendicular CD.            C. AB is vertically opposite to CD.            D. AB and CD are not perpendicular lines.</p>
<p><b>Q21.</b></p>	<p>Sum of the interior angles of a regular pentagon is</p> <p>A. <math>90^\circ</math>            B. <math>108^\circ</math>            C. <math>540^\circ</math>            D. <math>180^\circ</math></p>	<p><b>Q22.</b></p>	 <p>If AB is parallel to CD, then which of the following statement is correct?</p> <p>A. <math>\angle a = \angle e</math>            B. <math>\angle d = \angle b</math>            C. <math>\angle b = \angle g</math>            D. <math>\angle h = \angle f</math></p>
<p><b>Q23.</b></p>	<p>Which of the following statement is correct?</p> <p>A. Any enclosed shape is a polygon.            B. Circles and squares are polygons.            C. All polygons are shapes with more than or equal to 3 sides.            D. A triangle is not a polygon.</p>	<p><b>Q24.</b></p>	<p>ABCD is a parallelogram. Which of the two angles are equal?</p>  <p>A. <math>\angle 1</math> and <math>\angle 2</math>            B. <math>\angle 3</math> and <math>\angle 4</math>            C. <math>\angle 1</math> and <math>\angle 3</math>            D. <math>\angle 1</math> and <math>\angle 4</math></p>

<p><b>Q25.</b></p>	 <p>Which of the following is a chord of a circle?</p> <p>A. <math>\overline{OR}</math>          B. <math>\overline{LM}</math>          C. <math>\overline{ST}</math>          D. <math>\overline{PQ}</math></p>	<p><b>Q26.</b></p>	 <p>The value of <math>x</math> in the above right-angled triangle is</p> <p>A. 10          B. 15          C. <math>\sqrt{45}</math>          D. 16</p>
<p><b>Q27.</b></p>	<p>The area of <math>\triangle PQR</math> is</p> <p>A. <math>25 \text{ cm}^2</math>          B. <math>15 \text{ cm}^2</math>          C. <math>30 \text{ cm}^2</math>          D. <math>10 \text{ cm}^2</math></p> 	<p><b>Q28.</b></p>	<p>The surface area of a sphere with radius 7 cm is</p> <p>A. <math>600 \text{ cm}^2</math>          B. <math>616 \text{ cm}^2</math>          C. <math>88 \text{ cm}^2</math>          D. <math>636 \text{ cm}^2</math></p>
<p><b>Q29.</b></p>	<p>A fundamental statement related to geometrical figure is</p> <p>A. a theorem.          B. a postulate.          C. an arc.          D. a corollary.</p>	<p><b>Q30.</b></p>	<p>Volume of a cone is equal to</p> <p>A. <math>\pi r(r + \ell)</math>          B. <math>\frac{1}{3}\pi r^2 h</math>          C. <math>\frac{4}{3}\pi r^3</math>          D. <math>4\pi r^2</math></p>
<p><b>Q31.</b></p>	 <p>The volume of the given cone will be</p> <p>A. <math>12\pi</math>          B. <math>40\pi</math>          C. <math>50\pi</math>          D. <math>150\pi</math></p>	<p><b>Q32.</b></p>	<p>'Every even number is divisible by 2'.          The given statement represents:</p> <p>A. a corollary.          B. an axiom.          C. a postulate.          D. a theorem.</p>

<b>Q33.</b>	An axiom is the type of assumptions which is related to  <b>A.</b> numbers. <b>B.</b> geometrical figures. <b>C.</b> corollary. <b>D.</b> angles..	<b>Q34.</b>	Cos 30° equals to  <b>A.</b> $\frac{1}{2}$ <b>B.</b> $\frac{1}{\sqrt{3}}$ <b>C.</b> $\frac{\sqrt{3}}{2}$ <b>D.</b> $\frac{2}{1}$
<b>Q35.</b>	Which of the following has value 1?  <b>A.</b> Sin 45° <b>B.</b> Cos 45° <b>C.</b> Tan 45° <b>D.</b> Sec 45°	<b>Q36.</b>	$2\sin 30^\circ + \sqrt{2}\cos 45^\circ$  <b>A.</b> $\frac{2}{\sqrt{2}}$ <b>B.</b> 2 <b>C.</b> $\frac{1}{\sqrt{2}}$ <b>D.</b> 1
<b>Q37.</b>	Sin (90° – θ) equals to  <b>A.</b> Sin θ <b>B.</b> Cos θ <b>C.</b> 1/tan θ <b>D.</b> Tan θ	<b>Q38.</b>	19, 21, 20, 18, 23, 19, 20, 18, 19, 20, 19, 0  The total frequency of the following data is  <b>A.</b> 20 <b>B.</b> 19 <b>C.</b> 11 <b>D.</b> 12
<b>Q39.</b>	Mode of 2, 4, 6, 8, 7, 8, 9, 10, 13 is  <b>A.</b> 8 <b>B.</b> 9 <b>C.</b> 10 <b>D.</b> 13	<b>Q40.</b>	$\sqrt{2}$ is  <b>A.</b> a rational number. <b>B.</b> a whole number. <b>C.</b> an irrational number. <b>D.</b> an odd number.

## Section B

Time: 2 Hours

Total Marks: 60

- 
1.  $\cup = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10,\}$  [6]  
 $A = \{1, 2, 3, 4, 6,\}$   
 $B = \{2, 4, 5, 7, 8\}$   
Prove that  $(A \cup B)' = A' \cap B'$ .

- 
2.  $\cup = \{x \mid x \in w \text{ and } | 10 \leq x \leq 30\}$  [6]  
 $A = \{x \mid z \in w \text{ and } | 12 \leq x \leq 25\}$   
 $B = \{x \mid z \in w \text{ and } | 14 \leq x \leq 27\}$   
Prove that  $(A \cap B)' = A' \cup B'$ .

- 
3. Find the value of
- (i)  $\sqrt[3]{512}$  [3]
- (ii)  $\left(\frac{1}{2}\right)^{-3}$  [3]

- 
4. Salman's salary is Rs 38000. If his relief is Rs 8000, then calculate his income tax at the rate of 5% per annum. [6]

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5. (i) A fruit seller sold 2 dozen oranges and 3 dozen bananas on Monday. On Tuesday he sold 4 dozen oranges and 1 dozen bananas . How many oranges and bananas did he sell in two days? [4]
- (ii) What is  $\begin{bmatrix} 6 & 3 \\ -4 & 7 \end{bmatrix} - \begin{bmatrix} 0 & -3 \\ 4 & 7 \end{bmatrix}$ ? [2]
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6. Sara and Ali together have Rs 1500. If Sara spends twice as much as Ali, how much does Ali spend? [6]

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7. Construct a right-angled triangle PQR, where  $\angle Q = 90^\circ$ ,  $\overline{QR} = 4$  cm and hypotenuse  $\overline{PR} = 5$  cm. Find the measure of  $\overline{QP}$ . Also write the steps of construction. [6]

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8. (i) Write  $3^3 = 27$  in logarithmic form. [1]

(ii) Write  $\log_4 256 = 4$  in exponential form. [1]

(iii) Find the value of  $x$ , if  $\log_8 x = 2$ . [2]

(iv) Find the value of  $a$ , if  $\log_a 729 = 3$ . [2]

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9. A ship is 150 m away from a lighthouse. If the angle between the top of the lighthouse and base of the ship is  $45^\circ$ , find the height of the lighthouse. [6]

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10. Draw a histogram of the following data. [6]

$0 < x \leq 10$	7
$10 < x \leq 20$	10
$20 < x \leq 30$	5
$30 < x \leq 40$	2
$40 < x \leq 50$	8

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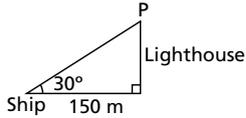
# Marking Scheme

**Marking criteria for Section A: 1 mark for each correct answer.**

1. C	2. D	3. A	4. D	5. A	6. A	7. A	8. B
9. A	10. D	11. B	12. B	13. D	14. A	15. B	16. A
17. B	18. A	19. A	20. B	21. C	22. C	23. C	24. D
25. D	26. B	27. A	28. B	29. B	30. B	31. A	32. A
33. A	34. C	35. C	36. B	37. B	38. D	39. A	40. C

**Marking criteria for Section B:**

Q.1		6 Marks	Answer
	<ul style="list-style-type: none"> <li>• For finding <math>A \cup B</math></li> <li>• For finding <math>(A \cup B)'</math></li> <li>• For finding <math>A'</math></li> <li>• For finding <math>B'</math></li> <li>• For finding <math>A' \cap B'</math></li> <li>• For correct proof</li> </ul>	1 mark 1 mark 1 mark 1 mark 1 mark 1 mark	$A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8\}$ $(A \cup B)' = \{9, 10\}$ $A' = \{5, 7, 8, 9, 10\}$ $B' = \{1, 3, 6, 9, 10\}$ $A' \cap B' = \{9, 10\}$ $(A \cup B)' = A' \cap B'$
Q.2		6 Marks	Answer
	<ul style="list-style-type: none"> <li>• For finding correct values of <math>\cup</math>, A and B</li> <li>• For finding <math>A \cap B</math></li> <li>• For finding <math>(A \cap B)'</math></li> <li>• For finding <math>A'</math></li> <li>• For finding <math>B'</math></li> <li>• For finding <math>A' \cup B'</math></li> <li>• Proved</li> </ul>	1 mark  1 mark 1 mark 1 mark 1 mark 1 mark	$\cup = \{10, 11, 12, \dots, 30\}$ $A = \{12, 13, 14, \dots, 25\}$ $B = \{14, 15, 16, \dots, 27\}$ $A \cap B = \{14, 15, 16, \dots, 25\}$ $(A \cap B)' = \{10, 11, 12, 13, 26, 27, 28, 29, 30\}$ $A' = \{10, 11, 26, 27, 28, 29, 30\}$ $B' = \{10, 11, 12, 13, 28, 29, 30\}$ $A' \cup B' = \{10, 11, 12, 13, 26, 27, 28, 29, 30\}$ $(A \cap B)' = A' \cup B'$
Q.3		6 Marks	Answer
(i)	<ul style="list-style-type: none"> <li>• For correct prime factorisation</li> <li>• For correct grouping</li> <li>• For accurate answer</li> </ul>	1 mark 1 mark 1 mark	$512 = \underline{2} \times \underline{2} \times \underline{2} \times \underline{2} \times \underline{2} \times \underline{2} \times \underline{2}$ $2 \times 2 \times 2$ 8
(ii)	<ul style="list-style-type: none"> <li>• For applying correct law of indices</li> <li>• For accurate answer</li> </ul>	2 marks 1 mark	$4^3$ 64
Q.4		6 Marks	Answer
	<ul style="list-style-type: none"> <li>• For finding the amount on which tax is to be paid</li> <li>• For finding tax at 5% per annum</li> <li>• For accurate answer</li> </ul>	3 marks 2 marks 1 mark	$38,000 - 8000 = 30,000$ 5% of 30,000 Rs 1500

Q.5		6 Marks	Answer										
(i)	<ul style="list-style-type: none"> <li>For writing information in form of a matrix</li> </ul>	2 marks	<table style="width: 100%; border: none;"> <tr> <td style="width: 50%; text-align: center;">Oranges</td> <td style="width: 50%; text-align: center;">Bananas</td> </tr> <tr> <td>Monday = <math>[2 \times 12]</math></td> <td><math>[3 \times 12]</math></td> </tr> <tr> <td>Tuesday = <math>[4 \times 12]</math></td> <td><math>[1 \times 12]</math></td> </tr> <tr> <td>Monday = <math>[24 \ 36]</math> and Tuesday = <math>[48 \ 12]</math></td> <td></td> </tr> <tr> <td colspan="2"><math>[24 \ 36] + [48 \ 12] = [24 + 48 \ 36 + 12]</math></td> </tr> </table>	Oranges	Bananas	Monday = $[2 \times 12]$	$[3 \times 12]$	Tuesday = $[4 \times 12]$	$[1 \times 12]$	Monday = $[24 \ 36]$ and Tuesday = $[48 \ 12]$		$[24 \ 36] + [48 \ 12] = [24 + 48 \ 36 + 12]$	
Oranges	Bananas												
Monday = $[2 \times 12]$	$[3 \times 12]$												
Tuesday = $[4 \times 12]$	$[1 \times 12]$												
Monday = $[24 \ 36]$ and Tuesday = $[48 \ 12]$													
$[24 \ 36] + [48 \ 12] = [24 + 48 \ 36 + 12]$													
(ii)	<ul style="list-style-type: none"> <li>For addition</li> <li>For accuracy</li> <li>For correct subtraction</li> <li>For accuracy</li> </ul>	1 mark 1 mark 1 mark 1 mark	<table style="width: 100%; border: none;"> <tr> <td style="width: 50%; text-align: center;">72 Oranges</td> <td style="width: 50%; text-align: center;">48 Bananas</td> </tr> <tr> <td colspan="2"><math>\begin{bmatrix} 6-0 &amp; 3+3 \\ -4-4 &amp; 7-7 \end{bmatrix} = \begin{bmatrix} 6 &amp; 6 \\ -8 &amp; 0 \end{bmatrix}</math></td> </tr> </table>	72 Oranges	48 Bananas	$\begin{bmatrix} 6-0 & 3+3 \\ -4-4 & 7-7 \end{bmatrix} = \begin{bmatrix} 6 & 6 \\ -8 & 0 \end{bmatrix}$							
72 Oranges	48 Bananas												
$\begin{bmatrix} 6-0 & 3+3 \\ -4-4 & 7-7 \end{bmatrix} = \begin{bmatrix} 6 & 6 \\ -8 & 0 \end{bmatrix}$													
Q.6		6 Marks	Answer										
	<ul style="list-style-type: none"> <li>For correct statement</li> <li>For solution</li> <li>For accuracy</li> </ul>	2 marks 2 marks 2 marks	$2x + x = 1500$  Rs 500										
Q.7		6 Marks	Answer										
	<ul style="list-style-type: none"> <li>For accurate drawing</li> <li>For finding <math>\overline{QP}</math></li> <li>For writing steps of construction</li> </ul>	3 marks 1 mark 2 marks	$\overline{QP} = 3 \text{ cm}$										
Q.8		6 Marks	Answer										
(i)	<ul style="list-style-type: none"> <li>For correct logarithmic form</li> </ul>	1 mark	$\log_3 27 = 3$										
(ii)	<ul style="list-style-type: none"> <li>For correct exponential form</li> </ul>	1 mark	$4^4$										
(iii)	<ul style="list-style-type: none"> <li>For correct exponential form</li> <li>For accurate answer</li> </ul>	1 mark 1 mark	$8^2 = x$ $\therefore x = 64$										
(iv)	<ul style="list-style-type: none"> <li>For writing in exponential</li> <li>For accurate answer</li> </ul>	1 mark 1 mark	$a^3 = 729$ $a^3 = 9^3 \quad \therefore a = 9$										
Q.9		6 Marks	Answer										
	<ul style="list-style-type: none"> <li>For using correct trigonometric ratio</li> <li>For applying correct values</li> <li>For accuracy</li> </ul>	2 marks 1 mark 3 marks	<p><math>\tan \theta = \frac{\text{Perpendicular}}{\text{Base}}</math></p> <p><math>\tan 45^\circ = \frac{\text{Height}}{150}</math></p> <p>Height = <math>\tan 45^\circ \times 150</math> Height = <math>1 \times 150 = 150 \text{ m}</math></p> 										
Q.10		6 Marks	Answer										
	<ul style="list-style-type: none"> <li>For choosing correct scale on both axes.</li> <li>For plotting values</li> <li>For accurate drawing of the histogram</li> </ul>	1 mark 2 marks 3 marks											