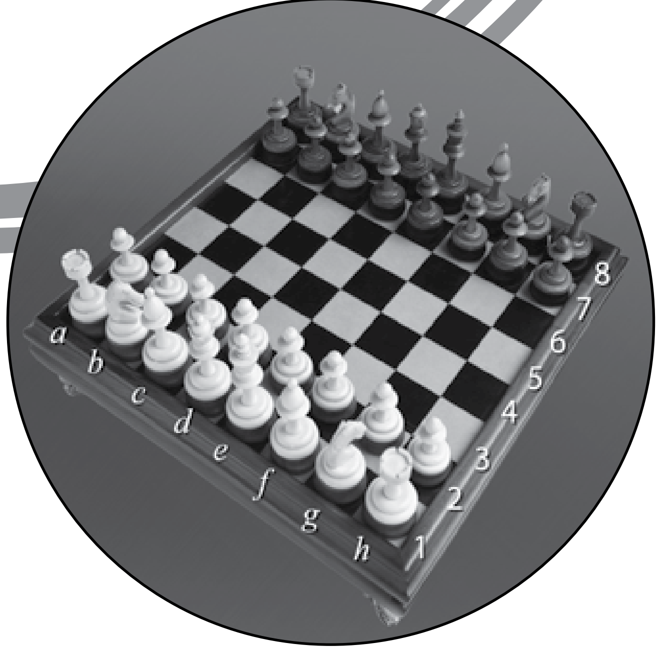
Teaching Guide

International Secondary Maths 11

COLIN **WRIGLEY**



OXFORD
UNIVERSITY PRESS

Introduction

How to use this Guide

I. Selection of work and pacing

Book 11 is designed for students of class XI (or equivalent), i.e. they would normally be 15+ years old at the start of the academic year, and intending to appear for O level or IGCSE examinations at the end of the year. Although there are significant new topics presented, there is also much that is revision and teachers should use their professional judgement about how much of that is needed. The chapter-by-chapter recommendations regarding pacing should be used as a rough guide only.

II. Integrated mathematics

This textbook series deliberately exploits links between the different branches of mathematics. It lists such links chapter by chapter and teachers are advised to make these explicit in lessons.

III. Lesson planning

This guide does not attempt to provide perfect lesson plans. Schools have their own requirements, and good teachers will always experiment. The intention of the guide notes is to assist teachers at the planning stage, and to make a few pedagogical suggestions. The headings used in the guide are as follows:

Objectives

General - an overview/summary.

Specific - detailed learning objectives, stated from the students' point of view

Pacing and links also come under this heading. (See I and II.)

Method

Here there are ideas of the teaching strategies to use, written from the teacher's point of view.

Resources

Lists of any vital equipment, worksheets, special paper or other items useful for the lesson.

Assignments

Suitable homework assignments suggested

Vocabulary

Key words essential for understanding

IV. Bloom's Taxonomy

A detailed discussion of this may be found in the Teaching Guides 6, 7, and 8. It should always be a teacher's goal to challenge students to use higher level problem-solving skills, and to avoid teaching "recipes".

V. The Exercises

The exercises follow a pattern:

Exercise A, B, etc. follow each section of a chapter

Exercises X are challenging questions, beyond the main course, for students of higher ability in order to provoke curiosity and enable them to see ahead just a little.

Revision Exercises These are banks of questions on all previous topics. They are not graded: there is a mixture of routine practice and thought-provoking material. Non-calculator questions have been separated from questions where calculators may be required.

Specimen Examination Papers, a feature of earlier texts in the series, are not provided in this guide. Teachers are advised to use readily available past papers of the relevant syllabus as a model for internal school examinations at this stage.

VI. Useful sheets

Graph paper (photocopiable) is provided in this guide, of a size that is more useful than large sheets.

Contents

Chapter

1 Vectors

Although vectors have been introduced previously (Book 8, Chapter 17), they have been used only as a way of describing translations. In this chapter we move to a more formal treatment leading to problem solving using vector algebra.

LESSON PLANNING

Objectives

General	To use vectors (in two dimensions) related to geometrical diagrams, with the correct notation
Specific	1. To understand that vectors represent translations, or any other concept that has magnitude and direction 2. To know how to state the components of a vector when it is drawn on a squared grid; to distinguish between the components of a vector and the coordinates of a point 3. To recognise and use scalar multiples of vectors, their geometrical representation, and the different notation for vectors and scalars 4. to understand the triangle law of combination of vectors; to use it in problems 5. To know that the negative of a vector reverses its direction and changes the signs of both of its components; to use this fact in problems 6. To use Pythagoras' theorem to obtain the length of any vector, with correct notation 7. To distinguish between free vectors and position vectors; to use equal and parallel free vectors in problems 8. To use position vectors to find the vector between two given points 9. To switch between fractions of a vector and ratios of line segments as required to solve problems 10. To use simple geometric properties of shapes and parallel lines to assist in solving problems of vector geometry
Pacing	4 lessons, 2 homeworks
Links	Transformations (translations), coordinates, basic geometry especially of quadrilaterals

Method

- Although vectors in component form describing translations have been used before, this whole topic is quite unlike any previous mathematics in a number of respects. It cannot be rushed. Be patient. Explain clearly the vector notation (underlining) to distinguish arrows from numbers (scalars). Vector geometry is a "geometry of shifting".

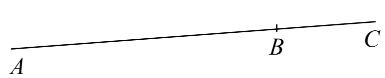
- Components are quite easy. Use the text examples orally and invent similar. This is easily grasped (and is revision).
- Scalar multiples are also quite easy to explain. Diagrams are essential here. It is helpful if you have a ready-made panel of squares on your whiteboard/ blackboard.
- The combination law follows. This is a vital fact of vector algebra that must be clearly assimilated. The text indicates one approach, i.e. travelling. For example you could say that if you start here (A), go to the moon (B), and back to here (C, on the other side of the classroom), the combined vector is just $\overrightarrow{AB}$. This emphasizes that it is the net shift, not the distance travelled, that is relevant in vector combination. It also confirms that vector notation is essential:

  $$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$$

  but $\quad AB + BC > AC$

  Follow the notes (1) to (4) in the text.
- Negative vectors can be introduced by asking the students to guess what **–a** or $-\overrightarrow{AB}$ could mean. The definition is quite intuitive. Again, follow the text and show how negatives can be used to combine vectors "pointing the wrong way".
- For length of a vector, again elicit solutions from a specific example, e.g. "How long is $\begin{pmatrix}4\\3\end{pmatrix}$?" Do not just quote a formula at the start. The formula in the text may be given once the fact that Pythagoras is always applicable becomes apparent.

  Use EX 1A. There is a lot of theory covered, so allow and encourage students to look back at the text definitions and examples.
- In problem solving, students will need to be aware that the usual geometry conventions (arrows on parallel lines, etc.) are usually not followed in vector geometry. The question needs to be read carefully to identify useful information. The geometric properties of quadrilaterals are often especially useful. The text examples are worth using to demonstrate this.
- Position vectors are to be defined as vectors attached to the origin of coordinates. When the "tail" is attached the "nose" point has the same coordinates as the components of the vector. Ability to switch between points and position vector can be practised. Students are often careless about notation: a point is not a vector.
- Revise ratios and fractions. It is simple work but mistakes are often made. For example,

  If $AB:BC = 5:2$

  then $AB = \frac{5}{7}$ of AC

  and $BC = \frac{2}{7}$ of AC

  Students need to be able to switch from ratios to fractions and vice-versa.

  For the vector context follow the text.

	Set EX 1B which uses all the above systematically. In EX 1X for fast-working students, question 2 and question 3 should be solvable by the majority.	
Resources	If possible, obtain a blackboard/whiteboard with permanent squares. Every mathematics classroom should have one.	
Assignments	Easy homework: EX 1A, questions 1–6. Also EX 1B, questions 1–5	
Vocabulary	vector, component, coordinate, scalar, free vector, position vector	

ANSWERS

Exercises

EX 1A

1. $\mathbf{a} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$ $\mathbf{b} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$ $\mathbf{c} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ $\mathbf{d} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$

   $\mathbf{e} = \begin{pmatrix} 0 \\ -3 \end{pmatrix}$ $\mathbf{f} = \begin{pmatrix} -4 \\ -3 \end{pmatrix}$ $\mathbf{g} = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$ $\mathbf{h} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$

   $\mathbf{i} = \begin{pmatrix} -6 \\ 0 \end{pmatrix}$

2. a) $\overrightarrow{AB} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$ b) $\overrightarrow{BC} = \begin{pmatrix} 10 \\ -6 \end{pmatrix}$

   c) $\overrightarrow{BM} = \begin{pmatrix} 5 \\ -3 \end{pmatrix}$ d) $\overrightarrow{AM} = \begin{pmatrix} 6 \\ 1 \end{pmatrix}$

3. a) true b) true c) true d) false

4. $k = -1, \quad l = 3, \quad m = 4, \quad n = 2$

5. a) $\overrightarrow{BD} = \mathbf{u} - \mathbf{v}$ b) $\overrightarrow{BC} = \mathbf{u} - \mathbf{v} + \mathbf{w}$

   c) $\overrightarrow{AC} = \mathbf{u} + \mathbf{w}$ d) $\overrightarrow{CA} = -\mathbf{u} - \mathbf{w}$

6. a) $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ b) $\begin{pmatrix} 2 \\ -5 \end{pmatrix}$ c) $\begin{pmatrix} -1 \\ 5 \end{pmatrix}$ d) $\begin{pmatrix} 3 \\ -8 \end{pmatrix}$

7. $k = -1, \quad n = 6$

8. a) $\begin{pmatrix} 0 \\ 5 \end{pmatrix}$ b) $\begin{pmatrix} -5 \\ 5 \end{pmatrix}$ c) $\begin{pmatrix} 4 \\ -3 \end{pmatrix}$ d) $\begin{pmatrix} 0 \\ 5 \end{pmatrix}$

9. a) $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$ b) $\overrightarrow{AP} = \frac{1}{3}(\mathbf{b} - \mathbf{a})$

   c) $\overrightarrow{OP} = \frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$ d) $\overrightarrow{OQ} = \frac{1}{3}\mathbf{a} + \frac{2}{3}\mathbf{b}$

10. a) $\mathbf{a} - 2\mathbf{b} = \begin{pmatrix} 9 \\ -10 \end{pmatrix}$ b) $n = 52$ c) $t = -\frac{1}{2}, \quad s = -2$

EX 1B

1. a) $\mathbf{w} = -\mathbf{u} - \mathbf{v}$ b) $\mathbf{w} = \mathbf{u} + \mathbf{v}$

   c) $\mathbf{w} = 2\mathbf{u} - \mathbf{v}$ d) $\mathbf{w} = \frac{3}{2}\mathbf{u} - \mathbf{v}$

2. a) $\overrightarrow{AC} = \begin{pmatrix} 10 \\ -1 \end{pmatrix}$ b) $\overrightarrow{BD} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$

   c) $|\overrightarrow{AC}| = \sqrt{101}$ d) $|\overrightarrow{BD}| = \sqrt{13}$

3. a) $\mathbf{a} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$ b) $B(5, 3)$ c) $\overrightarrow{AB} = \begin{pmatrix} 6 \\ 1 \end{pmatrix}$ d) $D(-4, 0)$

4. a) $\overrightarrow{AB} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$ b) $\overrightarrow{AC} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$

   c) $\overrightarrow{CB} = \begin{pmatrix} -2 \\ -1 \end{pmatrix}$ d) $|\overrightarrow{CA}| = \sqrt{10}$

5. $B(6, 2.5)$

6. a) $\overrightarrow{AB} = \begin{pmatrix} 2 \\ -6 \end{pmatrix}$ b) $\overrightarrow{AC} = \begin{pmatrix} 4 \\ -12 \end{pmatrix}$

   c) $\mathbf{c} = \begin{pmatrix} 6 \\ -7 \end{pmatrix}$ d) $D(12, -3)$

7. a) $\overrightarrow{AP} = \begin{pmatrix} -1 \\ -3 \end{pmatrix}$ b) $\overrightarrow{AB} = \begin{pmatrix} -3 \\ -9 \end{pmatrix}$

   c) $k = 3$ d) $AP : PB = 1 : 2$

8. a) $\overrightarrow{AG} = \frac{2}{7}\overrightarrow{AB} = \frac{2}{7}(\mathbf{b} - \mathbf{a})$

$$\mathbf{g} = \overrightarrow{OG} = \overrightarrow{OA} + \overrightarrow{AG}$$
$$= \mathbf{a} + \frac{2}{7}(\mathbf{b} - \mathbf{a})$$
$$= \frac{5}{7}\mathbf{a} + \frac{2}{7}\mathbf{b}$$ as required

   b) $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$ $\overrightarrow{AG} = \mathbf{g} - \mathbf{a}$

$$= \frac{4}{9}\mathbf{a} + \frac{5}{9}\mathbf{b} - \mathbf{a}$$
$$= \frac{-5}{9}\mathbf{a} + \frac{5}{9}\mathbf{b}$$
$$= \frac{5}{9}\overrightarrow{AB}$$

$\therefore AG : GB = 5 : 4$ as required

9. a) $\mathbf{b} = \mathbf{a} + \mathbf{c}$ b) $\mathbf{f} = \mathbf{c} - \mathbf{a}$

   c) $\mathbf{m} = \frac{1}{2}(\mathbf{a} + \mathbf{c})$ d) $\mathbf{n} = -\frac{1}{2}\mathbf{a} + \frac{3}{2}\mathbf{c}$

10. a) $\overrightarrow{MN} = \frac{-3}{5}\mathbf{a} + \frac{1}{2}\mathbf{b}$ b) $\mathbf{r} = \frac{7}{10}\mathbf{a} + \frac{3}{4}\mathbf{b}$

EX 1X

1. $\overrightarrow{AB} = \mathbf{b} - \mathbf{a} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} - \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} b_1 - a_1 \\ b_2 - a_2 \end{pmatrix}$

gdt $AB = \dfrac{b_2 - a_2}{b_1 - a_1}$

Equation AB is $y = \left(\dfrac{b_2 - a_2}{b_1 - a_1}\right)x + c$

At $A(a_1, a_2)$: $\quad a_2 = \left(\dfrac{b_2 - a_2}{b_1 - a_1}\right)a_1 + c$

$$\therefore c = a_2 - \left(\frac{b_2 - a_2}{b_1 - a_1}\right)a_1$$

Substituting for c, equation AB is:

$$y = \left(\frac{b_2 - a_2}{b_1 - a_1}\right)x + a_2 - \left(\frac{b_2 - a_2}{b_1 - a_1}\right)a_1$$

Mult. by $(b_1 - a_1)$;

$$(b_1 - a_1)y = (b_2 - a_2)x + a_2(b_1 - a_1) - (b_2 - a_2)a_1$$

$$(a_2 - b_2)x - (a_1 - b_1)y = a_2b_1 - a_1a_2 - a_1b_2 + a_1a_2$$

$$(a_2 - b_2)x - (a_1 - b_1)y = a_2b_1 - a_1b_2 \quad \text{as required}$$

2. $\overrightarrow{OP} = 2\mathbf{a}, \qquad \mathbf{q} = 4\mathbf{b}$

$\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$

$\overrightarrow{BR} = 3(\mathbf{b} - \mathbf{a})$

$\mathbf{r} = \overrightarrow{OB} + \overrightarrow{BR}$

$= \mathbf{b} + 3(\mathbf{b} - \mathbf{a})$

$= 4\mathbf{b} - 3\mathbf{a}$

$\overrightarrow{RQ} = \mathbf{q} - \mathbf{r}$

$= 4\mathbf{b} - (4\mathbf{b} - 3\mathbf{a})$

$= 3\mathbf{a}$

$= \frac{3}{2}\overrightarrow{OP}$

RQ is parallel to OP and $RQ = \frac{3}{2}OP$.

$\therefore$ $OPQR$ is a trapezium and $OP : RQ = 2 : 3$.

3. a) $\angle OAB = \angle DAE$ (vert opp $\angle$s)

$OA = AD$ (given)

$\angle OBA = \angle AED$ (alt $\angle$s)

ΔAOB is congruent to ΔADE (AA corr S)

b) $\mathbf{e} = 2\mathbf{a} - \mathbf{b}$

c) X is at the centre of the parallelogram, i.e. at the intersection of its diagonals OC and BD.

d) $\overrightarrow{XE} = \mathbf{a} - \frac{3}{2}\mathbf{b}$.

OXFORD
UNIVERSITY PRESS

Chapter

2 Transformations of the Plane

This chapter extends the concept of transformation of a shape to transformation of the whole x–y plane. Emphasis is placed on those transformations that can be represented by a square matrix in preparation for later work.

LESSON PLANNING

Objectives	
General	To recognise standard transformations of the plane from diagrams; to describe them fully; to represent them using mapping statements
Specific	1. To give accurate full descriptions of reflections, rotations, translations, and enlargements 2. To use functional notation correctly, i.e. M,R,T, and E, individually and in combinations 3. To write and interpret general mapping statements for transformations of the plane 4. To identify invariant points or lines under the various transformations of the plane
Pacing	4 lessons, 2 homeworks
Links	Coordinates, mappings
Method	• Quickly revise transformations already taught. Ensure that the students know how to describe each transformation fully and know their symbols: M,R,T, and E. • The new word to introduce is **invariant**. This is related to transforming the whole plane, not just one shape. Go through M,R,T, and E transformations finding invariant lines and points when the plane is transformed. • Mapping statement examples may be given for simple cases, e.g. enlargement with negative scale factor. • Set EX 2A without too much prior explanation. The text may be used for reference rather than worked through in detail. The key concept at this stage is the transformation of the whole plane: shapes drawn on the plane illustrate the transformation, but all points on the plane are transformed, not just the shape, unless of course they are invariant.
Rresources	Squared paper or graph paper (photocopiable sheets available in this guide)
Assignments	Suitable homework EX 2A, questions 5 and 6
Vocabulary	transformation, invariant, mapping statement

OXFORD UNIVERSITY PRESS

ANSWERS

Exercises

EX 2A

1. a) translation $\begin{pmatrix} -1 \\ 7 \end{pmatrix}$ b) rotation +90° about (0, 0)

   c) enlargement, scale factor −2, centre (0, 0) d) reflection about the line $x = -4$

2. a) none b) (0, 0) c) (0, 0) d) $x = -4$

3. a) reflection in line $y = x$ b) rotation −90° about (−2, 4)

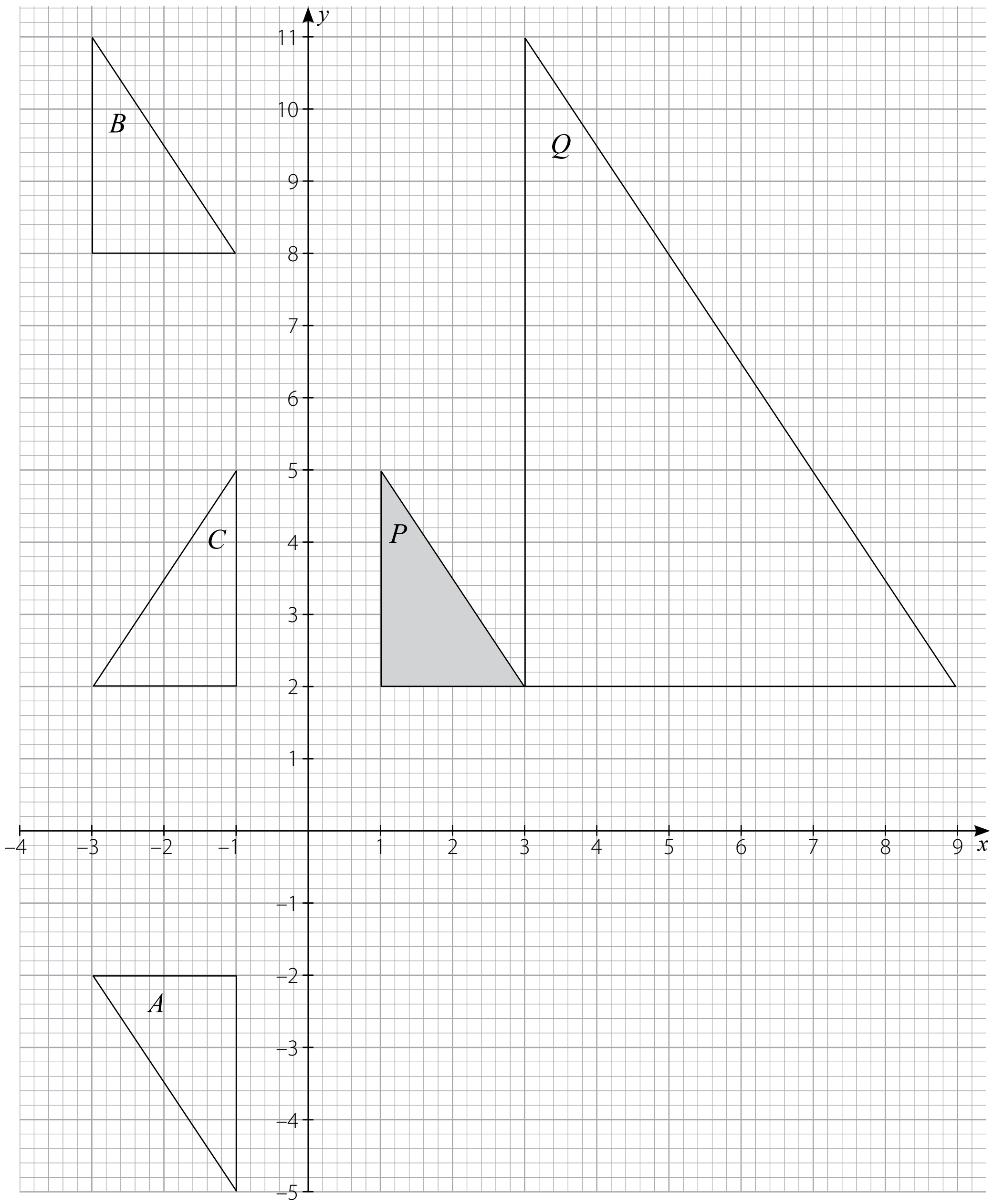
   c) translation $\begin{pmatrix} 6 \\ -4 \end{pmatrix}$ d) enlargement, scale factor 1.5, centre (−7, −2)

4. a) $y = x$ b) (−2, 4) c) none d) (−7, −2)

5. a)

   b)

   c)



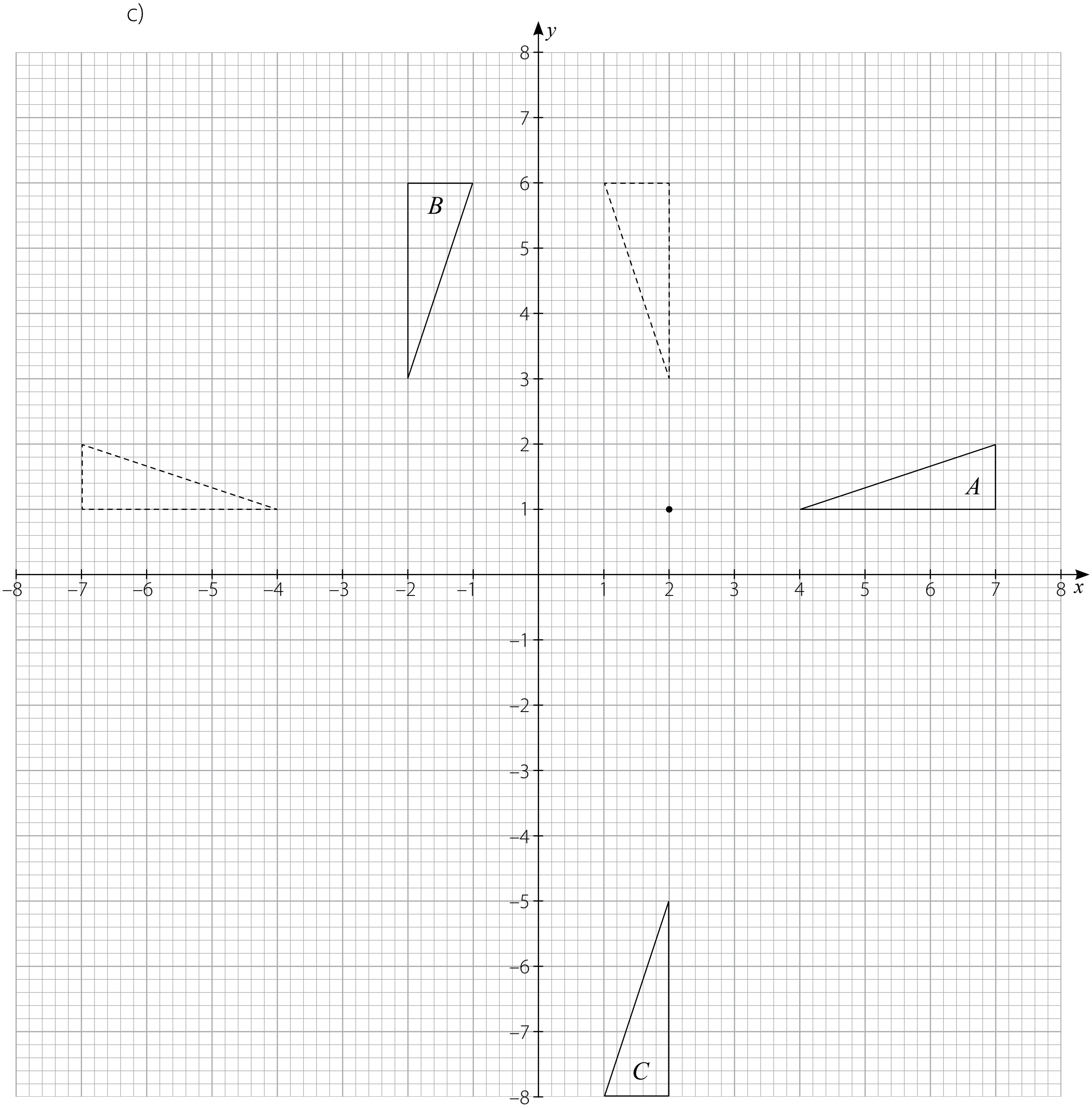
d) enlargement, scale factor 3, centre (0, 2)

6. a) (0, 0) b) none c) y-axis d) (0, 2)

7. a)

b)

c)



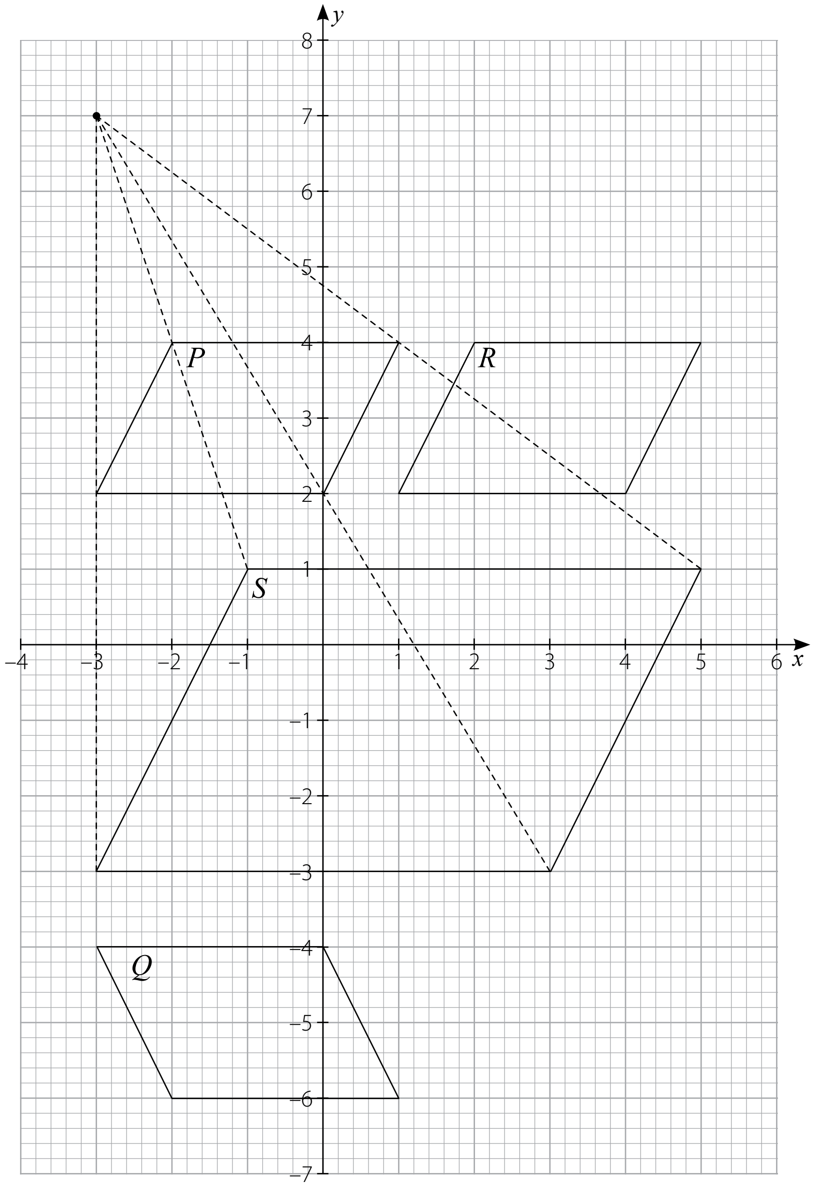
d) rotation of 180° about (0, −1)

8. a)

b)

c)

d)



9. $x' = x + 1, \quad y' = -y - 2$

10. $x' = x, \quad y' = -y$

EX 2X

1. a) S b) P c) Q d) R

2. a) $\mathbf{i} \to \mathbf{i}, \quad \mathbf{j} \to -\mathbf{j}$ b) $\mathbf{i} \to \mathbf{j}, \quad \mathbf{j} \to -\mathbf{i}$

c) $\mathbf{i} \to 2\mathbf{i}, \quad \mathbf{j} \to 2\mathbf{j}$ d) $\mathbf{i} \to -\mathbf{i}, \quad \mathbf{j} \to \mathbf{j}$

3. a) rotation of $\pm 120°$ about the origin

b) $n = 2$

Squared paper (2 mm)

Chapter

3 Functions

This chapter introduces more formal ideas about functions with particular emphasis on algebraic functions.

LESSON PLANNING

Objectives

General	To solve simple problems involving algebraic functions, their inverses, and composites
Specific	1. To interpret both methods of specifying an algebraic function 2. To find the image of a number under a specified function 3. To find a general statement specifying the inverse of a given function 4. To know that the variable in a function specification may be changed at will; to use this fact to find inverse functions and composite functions 5. To draw and interpret graphs of functions 6. To solve equations involving functions
Pacing	4 lessons, 2 homeworks
Links	General algebraic techniques, arrow diagrams, transformations, trigonometric functions, graphs

Method

- "What are letters used for in mathematics?" Begin with this open-ended question and see what responses you obtain, e.g.

numbers	(algebra)
vectors	(underlined, for vector algebra)
sin, cos, tan	(trigonometry)
M,E,R,T	(transformations)
n(A)	(set language)

  Some of these are functions, i.e. they describe an action/process/procedure—something done.

  Algebraic functions do something to numbers.

  Then follow the text example.

  Give other examples, lead up to how we write functions correctly, both methods.

Take time over the notation. There is a difference between $3(x + 2)$ and $f(x + 2)$ but they look similar.

$f(x + 2) \neq fx + 2f$

if f is a function.

- Follow the text examples 1 to 3, and make up similar.
- Set EX 3A, questions 1–5.
- Arrow diagrams are really helpful for explaining the concept of inverse functions. The variable change causes much confusion. Students need to grasp that x in $f(x)$ and x in $f^{-1}(x)$ are "not the same x". The text method of changing one of them immediately is highly recommended. This seems to work better than changing it at the end, which seems like cheating.
- Use Examples 1 and 2 given under Dummy Variables and similar.
- Set EX 3B, questions 6–10.
- Do not move on to composites until students are confident with inverses. Then arrow diagrams are again a helpful aid to understanding. The order of application of $fg(x)$ is analogous to the rule used for combining transformations. This link should be exploited.
- Example 1 given under composite functions should be sufficient to go through in detail. Leave Examples 2, 3, 4 and the note on graphs for reference.
- Set EX 3B.

Resources	Graph paper (photocopiable) for EX 3B, question 6
Assignments	Suitable homework EX 3A, questions 9 and 10 and EX 3B, questions 7 and 8
Vocabulary	functions, maps, image inverse functions, composite function dummy variable

ANSWERS

Exercises

EX 3A

1.	a) 1	b) −0.4	c) −2	d) 4
2.	a) 26	b) 37	c) 96	d) 0
3.	a) $f(n) = 3n - 2$	b) $f(y) = 3y - 2$		
	c) $f(t) = 3t - 2$	d) $f(x) = 3x - 2$		
4.	a) $x = 6$	b) $x = -1$	c) $x = 1$	d) $x = -29$
5.	a) $x = 4$	b) $x = \frac{70}{3}$	c) $x = \pm 1$	d) $x = 1.5$
6.	a) −6	b) 2		
	c) $f^{-1}(x) = \frac{x+5}{3}$	d) $g^{-1}(x) = \frac{1-x}{4}$		

7. a) 6 b) 1.5

c) $f^{-1}(x) = \dfrac{2x}{x-5}$ d) $g^{-1}(x) = \dfrac{5x}{x-2}$

8. a) 2 b) −14 c) 0 d) $1 - \dfrac{x}{2}$

9. a) 7 b) $x = 8$ c) $f^{-1}(x) = \dfrac{3x+7}{2}$ d) $\dfrac{1}{2}$

10. a) $\dfrac{-11}{4}$ b) $f^{-1}(x) = \dfrac{x}{2}$, $g^{-1}(x) = \dfrac{x+1}{3}$, $h^{-1}(x) = \dfrac{1}{x}$

c) $\dfrac{1}{2}$ d) $x = 1$

EX 3B

1. a) $fg(x) = 2x + 3$ b) $fg(3) = 9$

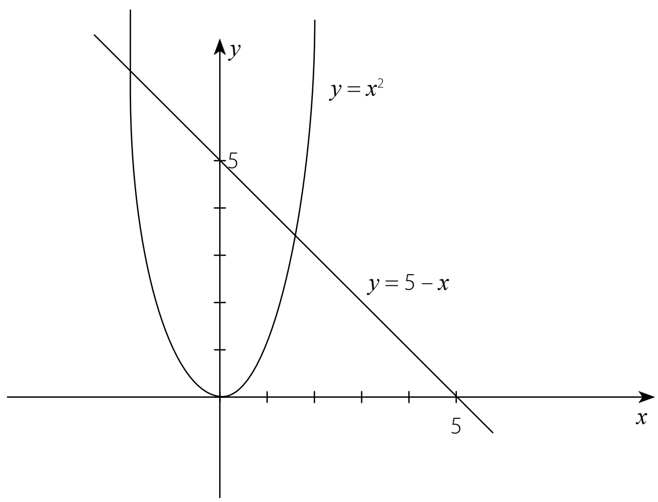
c) $gf(x) = 2x + 6$ d) $gf(3) = 12$

2. a) $fg(x) = 2x^2 + 1$ b) $gf(x) = (2x + 1)^2$

c) $f^2(x) = 4x + 3$ d) $f^2(5) = 23$

3. a) $fg(1) = -2$ b) $gf(2) = 1$ c) $f^2(3) = 7$ d) $g^2(4) = 4$

4. a)



b) $x = 1.79$, $x = -2.79$ (3 s.f.)

5. a) $f^2(-1) = 11$ b) $gf(x) = \dfrac{x^2+5}{2}$

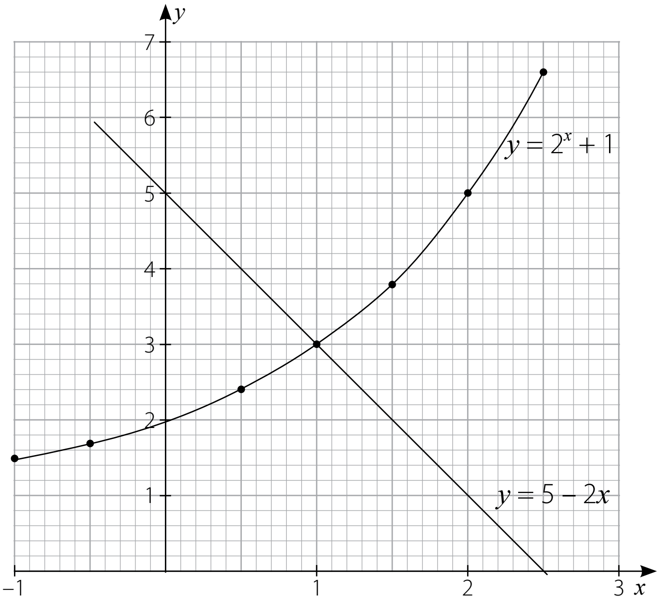
c) $g^{-1}(x) = 2x - 3$ d) $fg^{-1}(x) = 4x^2 - 6x + 11$

6. a)

x	−1	−0.5	0	0.5	1.0	1.5	2.0	2.5
2^x	0.5	0.71	1	1.41	2	2.83	4	5.66
+1	1	1	1	1	1	1	1	1
$g(x)$	1.50	1.71	2.00	2.41	3.00	3.83	5.00	6.66

b)

c)



d) i) $x = 1.6$ (1 d.p.)

ii) $x = 2.5$

iii) $x = 1$

7. a) $g^{-1}(x) = \frac{x+1}{4}$ b) $h^2(x) = x$ c) $gh(0.2) = 3$ d) $x = -3$

8. a) $h(-2) = 2$ b) $gh(4) = \frac{1}{3}$

c) $f^{-1}(x) = \frac{x-1}{3}$ d) $g^{-1}(x) = \frac{7-x}{x}$

9. $A = \frac{1}{2}, B = -2$

10. $A = 5, B = \frac{2}{3}$

EX 3X

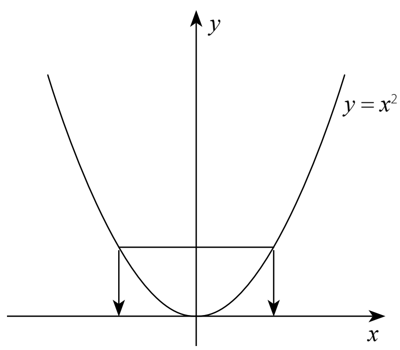
1. a) 717 b) $x = 3, \quad x = 4, \quad x = 5$

c) $x = 4.5$ d) $x = -1, -2, -3$

2. a) $fgh(x) = \frac{7-2x}{5}$ b) $fhg(x) = \frac{11x-1}{x-1}$

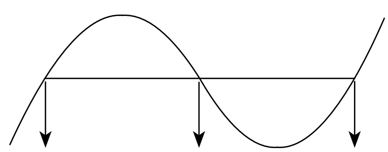
c) $ghf(x) = \frac{-2x}{5}$ d) $hgf(x) = \frac{10x+5}{2x}$

3. i)



Each value of y has two possible associated values of x. The function is not reversible.

ii)



Parts of this function have 2 or 3 possible values of x associated with the y value. The function is not reversible.

OXFORD UNIVERSITY PRESS

Chapter

4 Equations and Expressions

This chapter provides considerable practice in algebraic techniques. Not all students may need to attempt all of it.

LESSON PLANNING

Objectives

General	To be expert in solving linear and quadratic equations and manipulating algebraic fractions
Specific	1. To solve word problems leading to linear or quadratic equations 2. To solve simultaneous linear equations by elimination or substitution, selecting the more appropriate method 3. To deal with equations involving fractions or decimals, including algebraic fractions 4. To remove brackets correctly, especially when there are negative multipliers 5. To solve quadratic equations by factorisation, and by formula with accurate data entry into calculators 6. To combine two algebraic fractions into a single fraction 7. To apply the cross-multiplication technique, and to know when it is not applicable
Pacing	At least 4 lessons, with 2 homeworks
Links	Graphs, use of calculator
Method	• Use the material diagnostically, i.e. set work, and circulate, trouble-shooting, after the briefest of introductions. This is the last opportunity in this course to sort out any difficulties with algebra. It may be helpful to use the Reminders section in the text initially, giving examples, but at this stage time is best spent in practice. Set EX 4A. • The section covering combining algebraic fractions and cross-multiplication needs some care. A lot of errors are made with algebraic fractions. Do plenty of examples similar to Examples 4 and 5 in the text—it is easy to make them up. Cross-multiplication is a "recipe" and the problem with recipes is that students can misapply them. If must be hammered home that it works only when fraction = fraction Set EX 4B.

OXFORD UNIVERSITY PRESS

Resources	Calculators essential
Assignments	Any of the exercises are suitable for homework. One strategy is to omit parts d) in class and set those for homework, e.g. EX 4A, questions 1–5 parts d) only, or EX 4B, questions 1–5 parts d) only.
Vocabulary	equation, expression algebraic fraction cross-multiplication

Note: The method of completing the square for solving quadratic equations is explained as a postscript to the chapter. It is recommended that this be used only for the most able students.

ANSWERS

Exercises

EX 4A

1. a) $x = 1, \quad y = -2$ b) $x = -2, \quad y = 5$
   c) $x = 3, \quad y = -4$ d) $x = 4, \quad y = 5$
2. a) $x = \frac{-10}{43}$ b) $x = \frac{21}{20}$ c) $x = 2$ d) $x = 0.5$
3. a) $x = \frac{8}{7}$ b) $x = \frac{19}{3}$ c) $x = \frac{-31}{14}$ d) $x = 3$
4. a) $x = 1, \quad x = 17$ b) $x = \frac{1}{3}, \quad x = \frac{1}{2}$
   c) $x = 2, \quad x = -13$ d) $x = -1, \quad x = -5$
5. a) $x = 2.18, \quad x = 0.153$ b) $x = 4.56, \quad x = 0.438$
   c) $x = 3.64, \quad x = 0.137$ d) $x = 0.175, \quad x = -1.43$
6. a) $x = \pm 4.61$ b) $x^2 = 3^2 + 3.5^2, \quad x = \pm 4.61$
   c) 4.61 cm (3 s.f.) d) A length cannot be negative.
7. 17 apples, 19 bananas
8. a) 24.49 m by 48.99 m b) 10 m by 120 m
   c) 15 m by 80 m d) 24 m by 50 m
9. a) $x = \frac{117}{31}$ b) $x = \frac{91}{55}$
   c) $x = \frac{-57}{58}$ d) $x = 2.57, \quad x = -0.907$ (3 s.f.)
10. a) $x = -1, \quad y = -2$ b) $x = 0, \quad y = -2$
    c) $x = \frac{13}{5}, \quad y = \frac{-2}{5}$ d) $x = \frac{12}{5}, \quad y = \frac{-8}{5}$

EX 4B

1. a) $\frac{5x+7}{x^2+3x+2}$ b) $\frac{3x+5}{x^2+x}$

   c) $\frac{6x+1}{2x^2+x}$ d) $\frac{-x+11}{x^2-4x-5}$

2. a) $t=2$ b) $v=-2, \quad v=-3$

   c) $x=3$ d) $y=3, \quad y=-5$

3. a) $\frac{-1}{12x}$ b) $\frac{5y-9}{y^2-3y}$ c) $\frac{13}{10p}$ d) $\frac{-1}{14q}$

4. a) $\frac{7x+4}{x^2+2x}$ b) $\frac{x+8}{x^2+x-2}$

   c) $\frac{4x+5}{x^2+13x+30}$ d) $\frac{7x-11}{x^2-4x-5}$

5. a) $\frac{1}{x^2+x-2}$ b) $\frac{-8x+9}{x^2-3x}$

   c) $\frac{-3x^2+8x+4}{4x^2+8x}$ d) $\frac{5x+14}{x^2+6x+8}$

6. Multiply through by scale factor below, and simplify:

   a) $x(x-1)$ b) $x(x+2)$ c) $x(2x-1)$ d) $3x$

7. a) (2.1, 7.1) and (−7.1, −2.1)

   b) (7.3, 0.3) and (−0.3, −7.3)

   c) (1.4, 1.8) and (−0.4, −1.8)

   d) (0.6, 1.7) and (−0.6, −1.7)

8. a) $x \le 14$ b) $x > 6.25$ c) $x < 0.7$ d) $x \ge 6$

9. a) $x=0.5, \quad x=-1$ b) $x=1, \quad x=-1.5$

10. a) $x=1$ b) $x=0$

EX 4C

1. a) $y=3x-1$ b) $y=-4x+3$

   c) $y=\frac{2}{3}x-14$ d) $y=-x+11$

2. a) $y=-2x+8$ b) $y=\frac{1}{3}x+4$

   c) $y=3x+5$ d) $y=-6x-20$

3. a) $y=5x+2$ b) $y=7x+18$ c) $y=\frac{8}{5}x-\frac{9}{5}$ d) $y=\frac{-9}{2}x$

4. a) $y=-x-4$ b) $y=-9x+47$

   c) $y=17x+71$ d) $y=-3x+12$

5. a) $y = -5x + 25$ b) $y = -5x + 51$

c) $y = -5x + 77$ d) $y = -5x + 103$

6. a) $y = \frac{5}{4}x - \frac{15}{4}$ b) $y = \frac{4}{3}x + \frac{38}{3}$ c) $y = 9x + 56$ d) $y = x - 4$

7. gdt $AB = 1.5$, gdt $CD = 1.5$, $\therefore AB \| CD$

8. a) gdt $RP = \frac{-1}{7}$, gdt $PQ = 7$

b) 90° c) $O\,(1, 2)$ d) $y = \frac{3}{4}x + \frac{5}{4}$

9. trapezium [gdt AB = gdt CD]

10. a) $y = -6x + 7$ b) $\left(\frac{54}{37}, \frac{-65}{37}\right)$

EX 4X

1. Apply scale factor $x(x + 1)(x + 2)$ and simplify.

2. a) $x = 6, \quad x = -4$ b) $x = 8, \quad x = -4$

c) $x = 10, \quad x = -4$ d) $\frac{8}{x-4} = \frac{x-4}{8}, \quad x = 12, \quad x = -4$

3. a) (0.5, –4) and (2, –1) b) (–1, –3.5) and $\left(\frac{7}{6}, 3\right)$

Chapter

5 Matrices

Matrices are introduced here with reasonably rigorous definitions and formality. Once students gain freedom in the arithmetic and algebra of matrices, applications are brought in, especially the use of 2×2 square matrices to represent transformations of the plane.

LESSON PLANNING

Objectives

General	To represent and interpret data expressed in matrix form; to manipulate matrices; to solve problems involving 2×2 matrices representing transformations of the plane
Specific	1. To know the definition of a matrix and its order 2. To add and subtract matrices and scalar multiples of matrices 3. To multiply two matrices; to know when such multiplication is possible; to be aware of the non-commutative nature of matrix products 4. To find the determinant and inverse of a 2×2 square matrix; to know when an inverse is not possible 5. To know that the product of a matrix and its inverse is the identity matrix; to use this fact to solve simple matrix equations 6. To represent data expressed in rectangular tables of values as matrices; to interpret such matrices and the products of such matrices 7. To use 2×2 square matrices to represent transformations of the plane; to use matrices to calculate the coordinates of points on a diagram under transformation 8. To use the unit vectors in the x and y directions to find the transformation represented by a given matrix 9. To use the mapping statement of a general point under transformation as an aid to finding the matrix that represents it
Pacing	5 lessons, 2 homeworks, maybe more
Links	vectors, transformations
Method	• Start by finding examples of data presented in rectangular array—there are many. Introduce matrices as mathematical objects inspired by this fact. Follow the text, which gives vectors as a simple matrix example, defines order, and establishes the capital letter notation.

OXFORD
UNIVERSITY PRESS

Some students are confused by rows and columns.

Rows go across.	(x direction)
Columns go up/down.	(y direction)

x comes before y in the order.

rows × columns

Pronunciation may need to be given:

matrix	"may - tricks"
matrices	"may - tri - seas"

Proceed, following the text, to define addition, subtraction, and scalar multiples. Mention the zero matrix. (It is not in the text but should be mentioned.)

$\mathbf{A} - \mathbf{A} = \mathbf{Z}$ "a box of zeroes"

At this point, use EX 5A, questions 1–5.

- For products of matrices, establish from the start that we do not just multiply corresponding entries: we have a somewhat bizarre definition that provides more practical uses for matrices. (more later)

  Follow the text. The non-commutative nature of matrix products is important. We have to take care to write **AB** or **BA** : they are not interchangeable. Follow the text method to determine whether a product is even possible.

  Explain how to calculate the product elements. The text method **twist and slide** is highly recommended. Students need to practise the techniques.

  Set EX 5A, questions 4 and 5.

- Division of matrices may be contemplated. It cannot be done, but we can multiply by the inverse.

  Analogy with numbers: $5 \div 3 = 5 \times \frac{1}{3} = 5 \times 3^{-1}$

  (Multiplying by an inverse is equivalent to division)

  What is the inverse of a matrix?

  Move bizarre work! They don't all have inverses.

  Follow the text for finding determinants and inverses, and establish the properties of the identity matrix.

  Here is another routine technique that requires practice.

  Set EX 5A, questions 6 and 7.

- Matrix algebra follows, with very simple equations.

  Use text examples on identity square matrix, pre-multiplication and post-multiplication, and applications of matrices, and set EX 5A, questions 8 and 9.

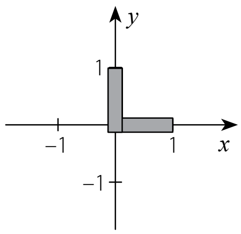
- EX 5A, question 10 is just commonsense, but gives practice in close reading.
- The real meat of this chapter, the major application of matrices to represent transformation of the plane, follows. This is a heavy dose of theory. It is recommended to do it all in one lesson, so it connects together. Use all five text examples.

  The common transformations are listed for reference but memorization is not recommended. Students should be able to describe fully a transformation from its matrix and vice-versa by using the methods described.

Time spent on this section will depend upon how well students have grasped the Chapter 2 work on transformations.

Set EX 5B. Students should work through systematically. Be available for individual trouble-shooting. This is not an easy topic.

Resources When explaining the unit vector method two rulers on the board can be helpful.



The rulers can be moved according to the given matrix columns to illustrate the type of transformation.

Assignments Possible homework EX 5A, question 9; EX 5B, questions 9 and 10.

Vocabulary Matrix, matrices, order (of a matrix)
determinant, inverse (of a matrix)
identity matrix, zero matrix
pre-multiply, post-multiply
transformations: reflection, rotation, enlargement, invariant, unit vector

ANSWERS

Exercises

EX 5A

1. a) 2×1 b) 2×2 c) 1×4 d) 3×2

2. a) $\begin{pmatrix} 5 \\ 0 \end{pmatrix}$ b) not possible c) $\begin{pmatrix} 3 & 1 & -1 \\ 4 & -1 & 13 \end{pmatrix}$ d) $\begin{pmatrix} 4 \\ 3 \\ 14 \end{pmatrix}$

3. a) $\begin{pmatrix} 2 & 6 \\ 14 & -4 \end{pmatrix}$ b) $\begin{pmatrix} 0 & \frac{1}{2} \\ 1 & 2 \end{pmatrix}$

   c) $\begin{pmatrix} 1 & 1 \\ 3 & -10 \end{pmatrix}$ d) $\begin{pmatrix} 1 & 12 \\ 28 & -11 \end{pmatrix}$

4. a) $\begin{pmatrix} 2 \\ -2 \end{pmatrix}$ b) not possible

   c) $\begin{pmatrix} -15 & 28 & 23 \\ -15 & 30 & 20 \end{pmatrix}$ d) $\begin{pmatrix} 7 & -41 \\ -28 & 17 \end{pmatrix}$

5. a) not possible b) $\begin{pmatrix} 4 & 20 \\ -6 & -3 \end{pmatrix}$

   c) $\begin{pmatrix} 60 & 0 & -38 \\ -20 & 0 & 36 \end{pmatrix}$ d) $\begin{pmatrix} -27 & -88 & -52 & 17 \\ 12 & 48 & 30 & -12 \end{pmatrix}$

OXFORD
UNIVERSITY PRESS

6. a) $|\mathbf{A}| = 2$, $\mathbf{A}^{-1} = \begin{pmatrix} 1.5 & 0.5 \\ 3.5 & 1.5 \end{pmatrix}$ b) $|\mathbf{B}| = 7$, $\mathbf{B}^{-1} = \begin{pmatrix} \frac{1}{7} & 0 \\ \frac{-8}{7} & 1 \end{pmatrix}$

c) $|\mathbf{C}| = -1$, $\mathbf{C}^{-1} = \begin{pmatrix} 5 & 4 \\ -1 & -1 \end{pmatrix}$ d) $|\mathbf{D}| = -2$, $\mathbf{D}^{-1} = \begin{pmatrix} 0.5 & -1.5 \\ 1 & -4 \end{pmatrix}$

7. $\mathbf{E}^{-1} = \begin{pmatrix} \frac{1}{7} & 0 \\ \frac{6}{7} & -1 \end{pmatrix}$ $\mathbf{F}^{-1} = \begin{pmatrix} -0.5 & -1.5 \\ 1.5 & 3.5 \end{pmatrix}$

$|\mathbf{G}|$ −0, no inverse $\mathbf{H}^{-1} = \begin{pmatrix} 0.5 & 0.5 \\ 1 & 0 \end{pmatrix}$

8. a) $\mathbf{X} = \begin{pmatrix} 4 & 0 \\ -8 & 18 \end{pmatrix}$ b) $\mathbf{X} = \begin{pmatrix} 4 & 0 \\ -4 & 11 \end{pmatrix}$

c) $\mathbf{X} = \begin{pmatrix} -0.5 & 0 \\ 2 & -4 \end{pmatrix}$ d) $\mathbf{X} = \begin{pmatrix} -1 & 0 \\ -4 & 6 \end{pmatrix}$

9. a) $\mathbf{A}^{-1} = \begin{pmatrix} -1 & -2 \\ -2 & -3 \end{pmatrix}$ b) $\mathbf{P} = \begin{pmatrix} 0 & -3 \\ -1 & -3 \end{pmatrix}$

c) $\mathbf{Q} = \begin{pmatrix} -12 & 5 \\ -21 & 9 \end{pmatrix}$ d) $\mathbf{R} = \begin{pmatrix} -23 & 12 \\ -40 & 21 \end{pmatrix}$

10. a) $\mathbf{PW} = \begin{pmatrix} 117 \\ 152 \end{pmatrix}$; total points scores of the divers; Babar won.

b) $\mathbf{PS} = (5400 \quad 5295 \quad 3680)$; total sale value of all the shoes, sandals, and boots, respectively

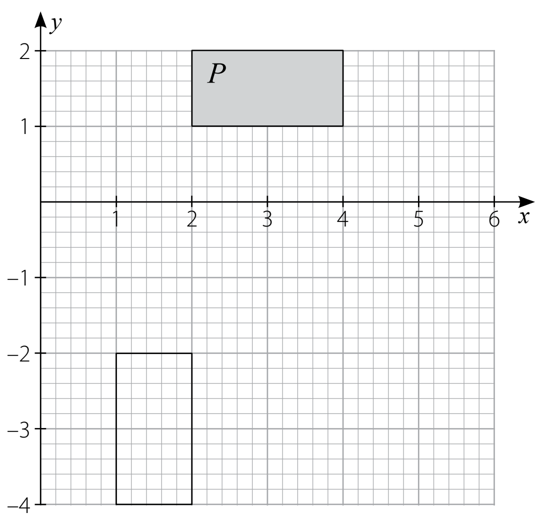
EX 5B

1. a) reflection in the y-axis b) y-axis c) $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

d) $\mathbf{A}^2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \mathbf{I}$; reflection repeated returns all points to their original positions.

2. a) (1, −2), (1, −4), (2, −4), (2, −2)

b)



c) rotation of −90° about the origin d) origin

3. a) reflection in the x-axis b) rotation of 180° about the origin

   c) enlargement, centre the origin, scale factor 5 d) rotation of +90° about the origin

4. a) reflection in the line $y = x$ b) reflection in the line $y = -x$

   c) rotation of −90° about the origin d) reflection in the y-axis

5. a) $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ b) $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

   c) $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ d) $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

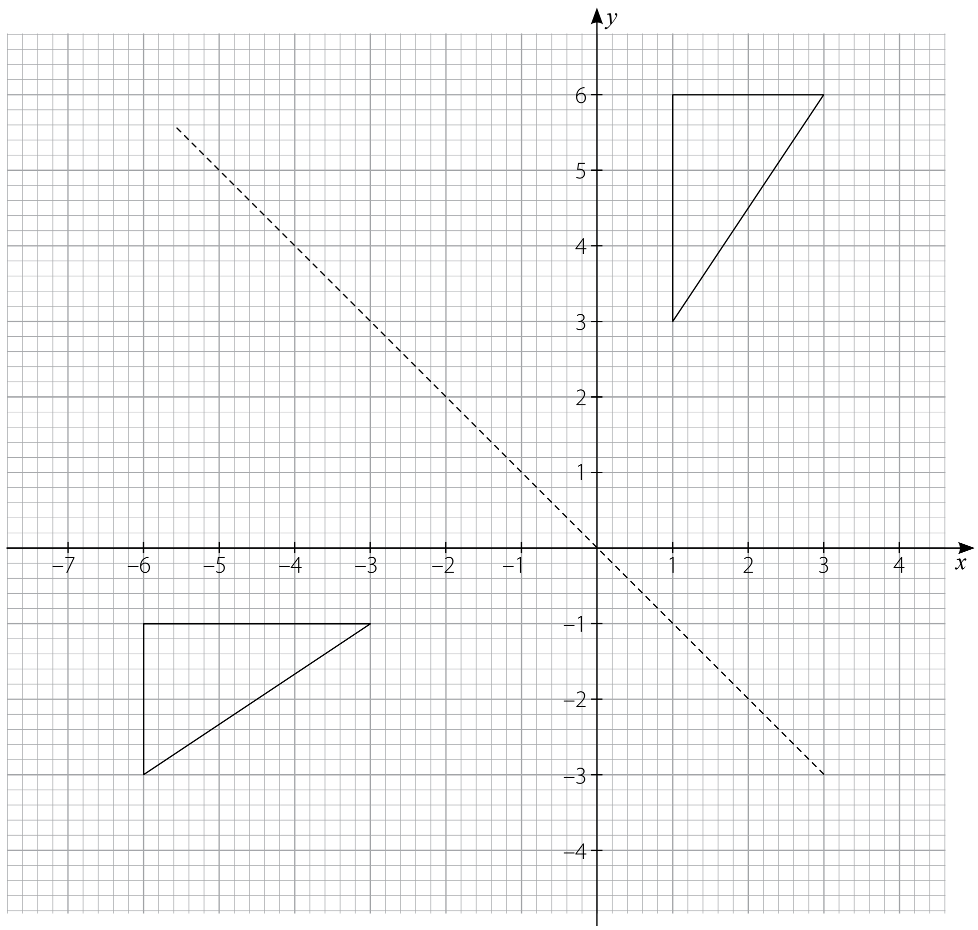
6. a) (−1, 1), (−5, 1), (−5, 3) b) (−2, −2), (−10, −2), (−10, −6)

   c) (1, 1), (1, 5), (3, 5) d) (−1, −1), (−5, −1), (−5, −3)

7. a) enlargement, centre the origin, scale factor 3 b) $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$

8. a) $\begin{pmatrix} -3 & -6 & -6 \\ -1 & -1 & -3 \end{pmatrix}$

   b)



   c) reflection in the line $y = -x$

9. a) rotation about the origin, −90° b) $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

10. a) reflection in the x-axis

b) i) (−1, −2), (−1, −3), (−3, −3)

ii) reflection in the y-axis iii) y-axis

EX 5X

1. a) $\mathbf{AI} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \mathbf{A}$

$\mathbf{IA} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \mathbf{A}$

$\mathbf{AI} = \mathbf{IA} = \mathbf{A}$ as required

b) $|\mathbf{A}| = ad - bc$

$$\mathbf{A}^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix} = \begin{pmatrix} \frac{d}{ad-bc} & \frac{-b}{ad-bc} \\ \frac{-c}{ad-bc} & \frac{a}{ad-bc} \end{pmatrix}$$

$$\mathbf{A}\mathbf{A}^{-1} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} \frac{d}{ad-bc} & \frac{-b}{ad-bc} \\ \frac{-c}{ad-bc} & \frac{a}{ad-bc} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{ad-bc}{ad-bc} & \frac{-ab+ab}{ad-bc} \\ \frac{cd-cd}{ad-bc} & \frac{-bc+ad}{ad-bc} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \mathbf{I}$$

$$\mathbf{A}^{-1}\mathbf{A} = \begin{pmatrix} \frac{d}{ad-bc} & \frac{-b}{ad-bc} \\ \frac{-c}{ad-bc} & \frac{a}{ad-bc} \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$= \begin{pmatrix} \frac{ad-bc}{ad-bc} & \frac{bd-bd}{ad-bc} \\ \frac{-ac+ac}{ad-bc} & \frac{-bc+ad}{ad-bc} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \mathbf{I}$$

$\therefore \mathbf{AA}^{-1} = \mathbf{A}^{-1}\mathbf{A} = \mathbf{I}$ as required

2. a) no change b) no change

c) 4 times greater d) 9 times greater

3. MATHS

ROCKS [Decode using inverse; message uses A = 1, B = 2, etc]

Chapter

6 Revision Exercises

[Non-calculator]

ANSWERS

Exercises

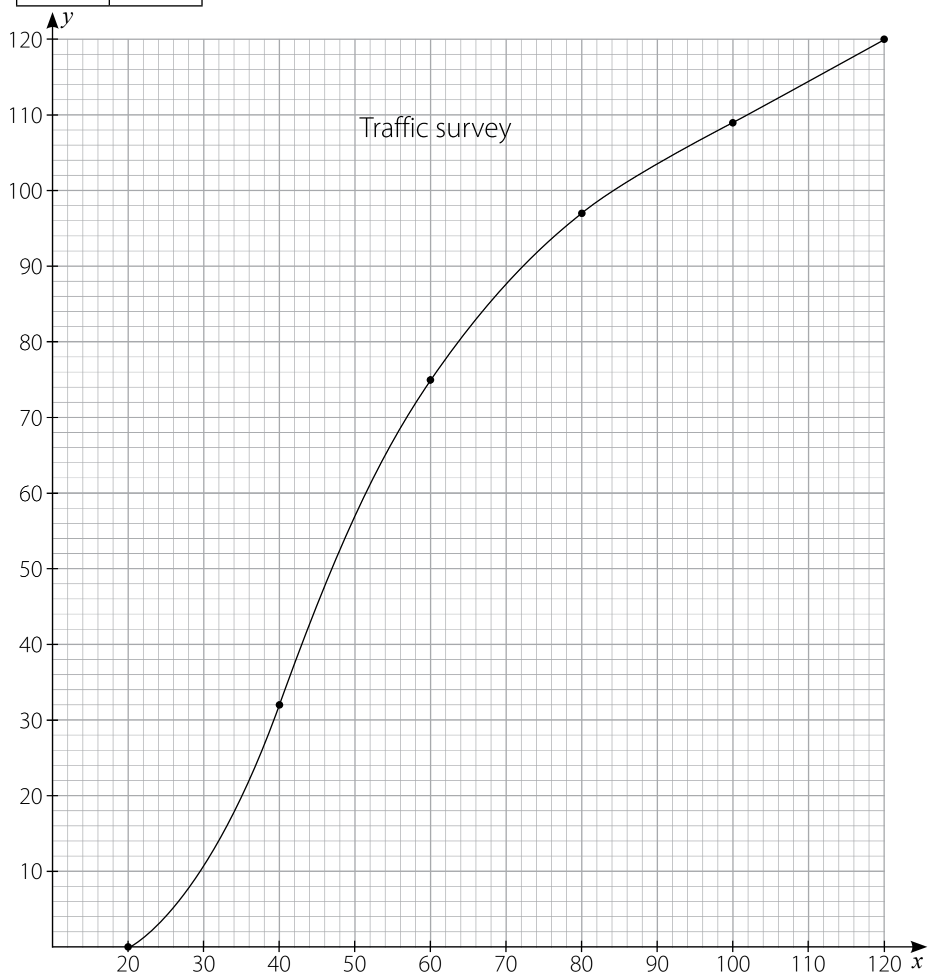
EX 6A

1. a) 0.05 b) 10 c) 100 000 d) 20

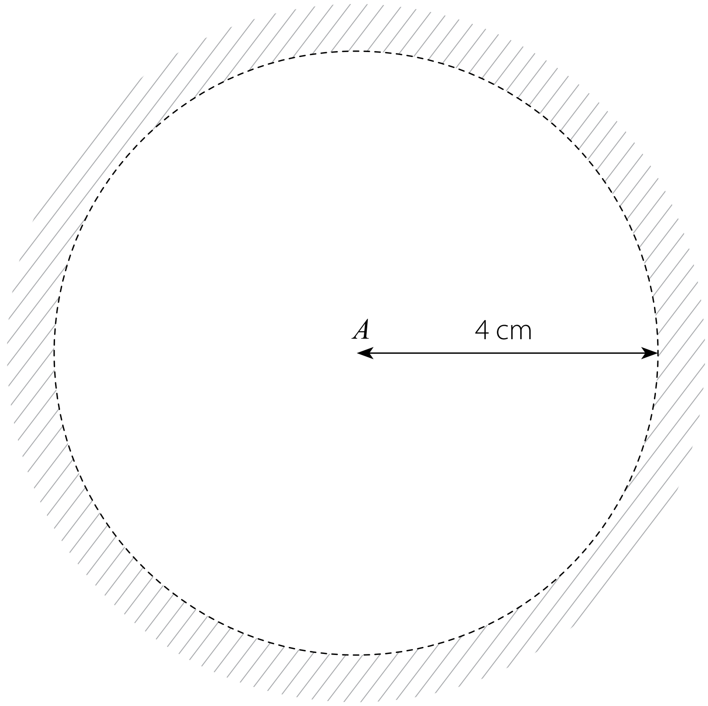
2. a)

f	CF
32	32
43	75
22	97
12	109
11	120

b) 16 vehicles



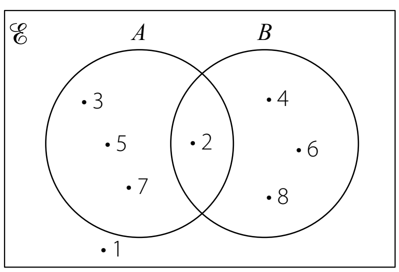
3. a)



b) the region of points within a sphere of radius 4 cm, centre B

4. a) $A = \{2, 3, 5, 7\}$, $B = \{2, 4, 6, 8\}$

b)



c) $A \cap B' = \{3, 5, 7\}$ d) $n(A \cup B)' = 1$

5. a) $n \propto \frac{1}{m}$ b) $n = \frac{1200}{m}$ b) 120

6. $x = 70, y = 40$

7. a) 4 b) 9 c) 4 d) infinite

8. a) $\mathbf{u} + \mathbf{v}$ b) $\mathbf{u} + \mathbf{v} + \mathbf{w}$ c) $\mathbf{v} + \mathbf{w}$ d) $-\mathbf{w} - \mathbf{v}$

9. a) 2.64 b) 0.378 c) 16.64 d) 35.65

10. a) true b) false

EX 6B

1. a) $\frac{12}{17}$ b) $\frac{12}{5}$ c) $\frac{12}{13}$ d) $\frac{5}{13}$

2. In general the boys' results were worse than the girls' (lower median). However, there is a greater spread in the boys' results. (IQR = 60.2 boys, IQR = 58.4 girls)

3. a) $x - 2$ b) $2x - 3$ c) $x + 4$ d) $2x + 3$

4. a) $x = \pm 5$ b) $x = \pm 8$ c) $x = \pm\frac{7}{3}$ d) $x = \pm\frac{7}{4}$

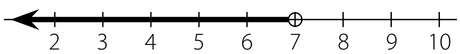
5. a) 4 b) 2 c) $2\frac{1}{2}$ d) 0.05

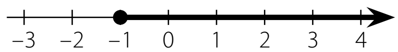
6. lines 1 and 3 ; lines 2 and 4

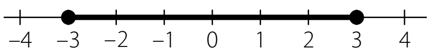
7. a) 13 b) 1 c) 4 d) 3

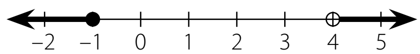
8. a) false b) true c) true d) false

9. $a = 26, b = 64, c = 32, d = 32, e = 148, f = 58$

10. a) 

b) 

c) 

d) 

EX 6C

1. a) $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$ b) $B(5, 3)$

2. a) 100 b) 11.8 c) −8 d) 122

3. a) $1 \times 3,\quad 2 \times 4,\quad 3 \times 6, \ldots$ b) 440

c) $n(n + 2)$ d) design 100

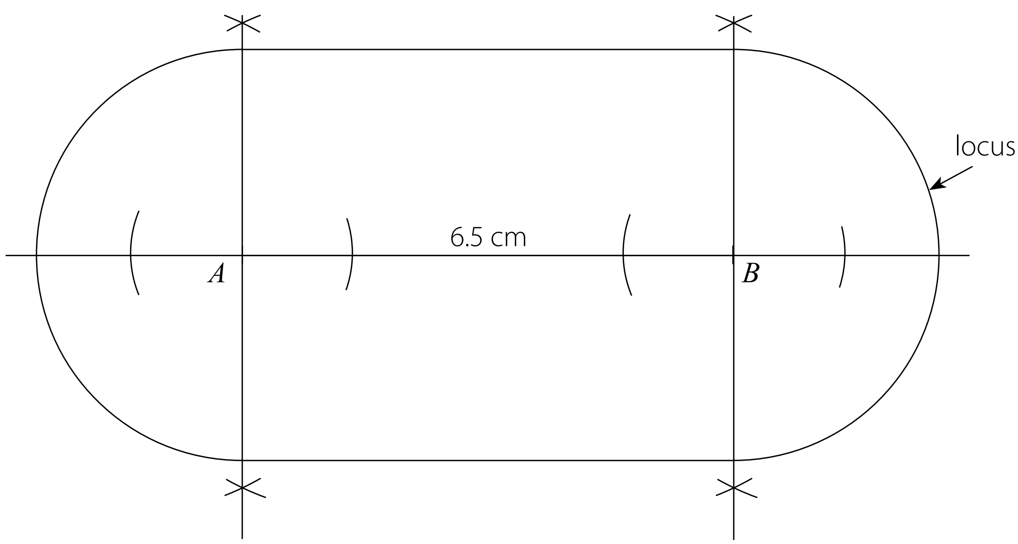
4. a) x^{10} b) x^{-3}

c) $\frac{1}{3}x^{-2}$ or $\frac{1}{3x^2}$ d) $\frac{1}{4}x^{-4}$ or $\frac{1}{4x^4}$

5. 3

6. a) $-\frac{2}{7}$ b) $-\frac{7}{2}$ c) $\frac{7}{2}$ d) $-\frac{2}{7}$

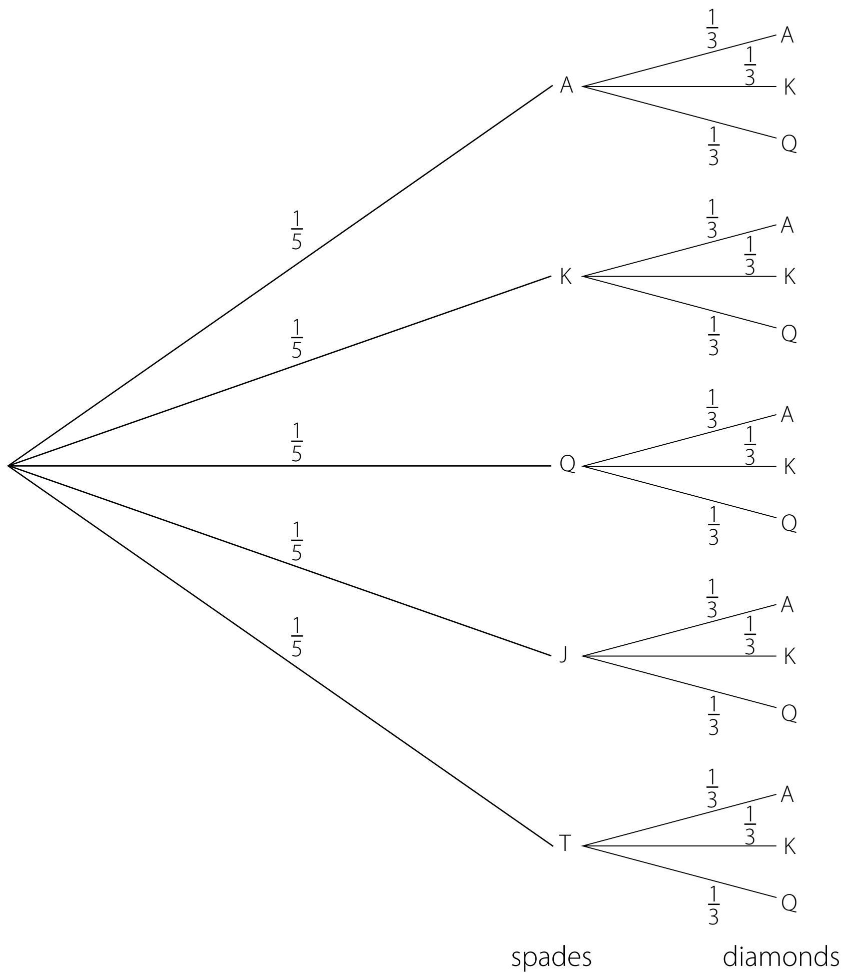
7.



8. a) 17.5% increase b) 12.5% increase

c) 7% decrease d) 12.5% decrease

9. a)



b) i) $\frac{1}{15}$ ii) $\frac{2}{15}$ iii) $\frac{4}{5}$

10. $x = 116$, $y = 60$

EX 6D

1. $x = 62$ (∠ at centre)

$y = 118$ (opp ∠s of cyclic quad)

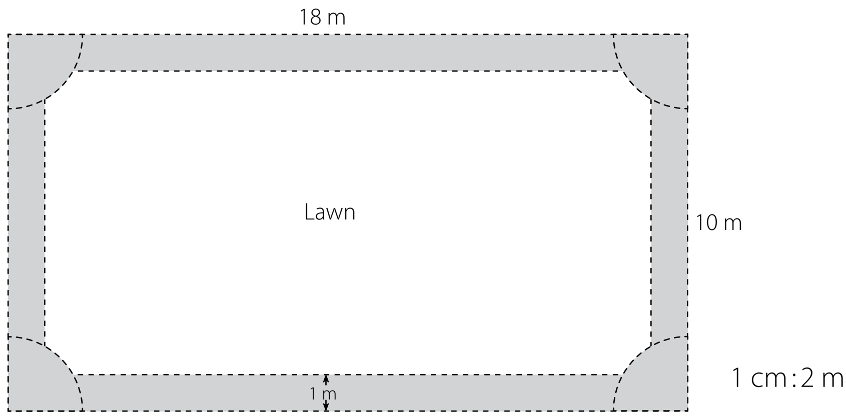
2. a) −3 b) $\frac{3}{5}$ c) $-\frac{2}{3}$ d) 0

3. a) $p \propto \frac{1}{q}$ b) $p = \frac{c}{q}$ c) $p = 24$ d) $q = -4$

4. a) $\frac{x^2 - 5x + 1}{x^2 + x - 6}$ b) $\frac{4x + 9}{x^2 + 3x}$

c) $\frac{-3x^2 + 11x + 10}{4x^2 + 8x}$ d) $\frac{2x^2 + 17x + 37}{x^2 + 8x + 15}$

5.



6. a) 2 b) $4x^2 + 5$ c) $\frac{x-5}{2}$ d) 50

7. a) $8x^{12}$ b) $\frac{1}{16}x^{-10}$ or $\frac{1}{16x^{10}}$

c) $25x^4$ d) $\frac{1}{9}x^{-12}$ or $\frac{1}{9x^{12}}$

8. a) 62 kg b) 13 kg c) 3000 d) 3000

9. 17

10. a) $x = \frac{11}{5}$ b) $x = \frac{5}{7}$

EX 6E

1. a) 0.0541 b) 8780

c) 0.000 85 d) 630 000 000

2. At the time of the survey the boys were generally taller (higher median and quartiles) although the spread of heights was greater for the boys. (IQR 8 cm boys, 6 cm girls)

3. a) rotation of –90° about the origin

b) (0, 0) is invariant. c) $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

4. a) true b) true c) false d) false

5. $\angle CBD = 20°$ (adj $\angle$s on st line)

$\angle BDE = 20°$ (alt $\angle$s)

$\angle BED = 20°$ (angle-sum of Δ)

Since $\angle BDE = \angle BED$

ΔBDE is isosceles. (base angles equal)

6. a) 4.7×10^7 b) 5.3×10^{-1}

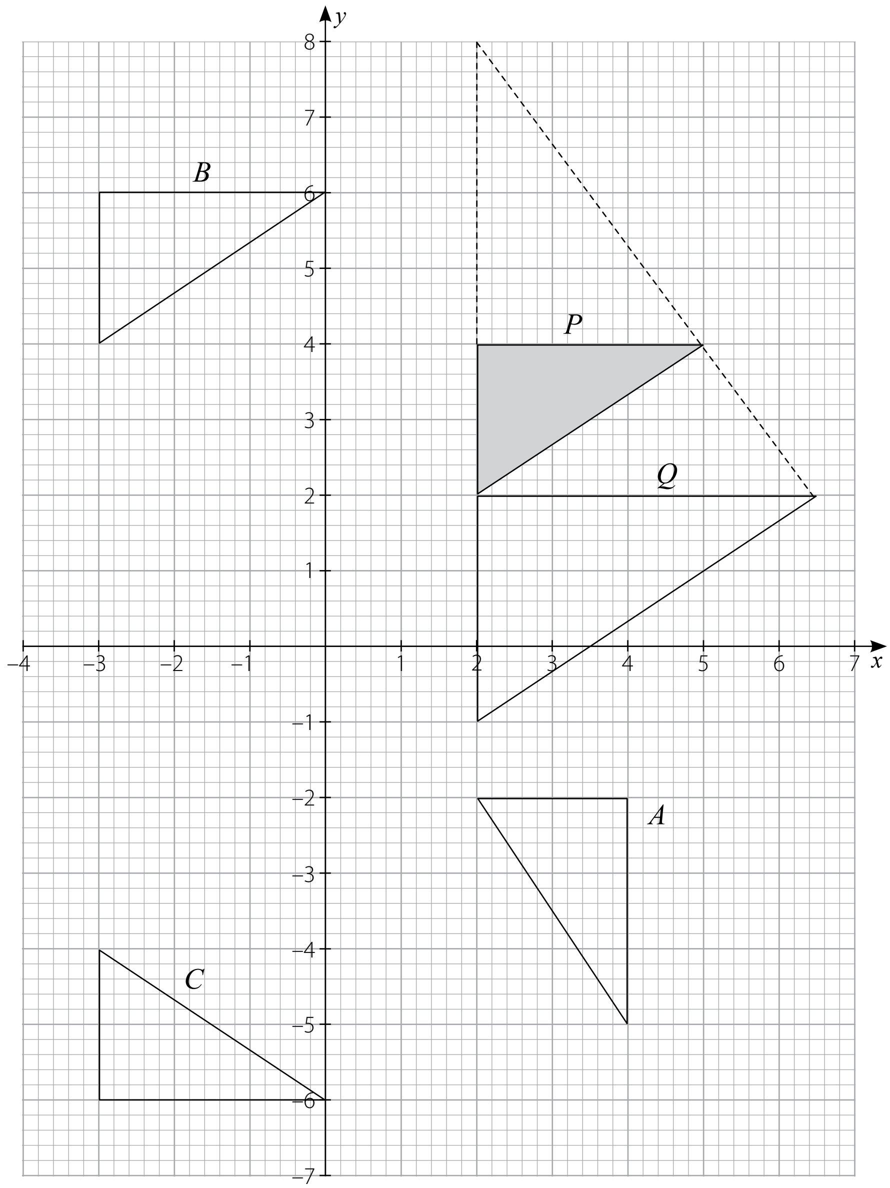
c) 8.0×10^{-3} d) 7.1×10^7

7. a) 342 b) 1910 c) 110 d) 600

8. a)

b)

c)



d) enlargement scale factor 1.5, centre (2, 8)

9. a) $y = 3$ b) $x = -5$

c) $y = -\frac{x}{3}$ d) $y = 5x + 3$

10. a) $\begin{pmatrix} 1 & 0 \\ 0 & 2.5 \end{pmatrix}$ b) $\begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix}$

c) $\begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$ d) $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

Chapter

7 Revision Exercises

ANSWERS

Exercises

EX 7A

1. a) hosepipe; greater gradient

   b) i) $h = \frac{10}{3}t$ ii) $h = 20$ iii) $h = 10t - 60$

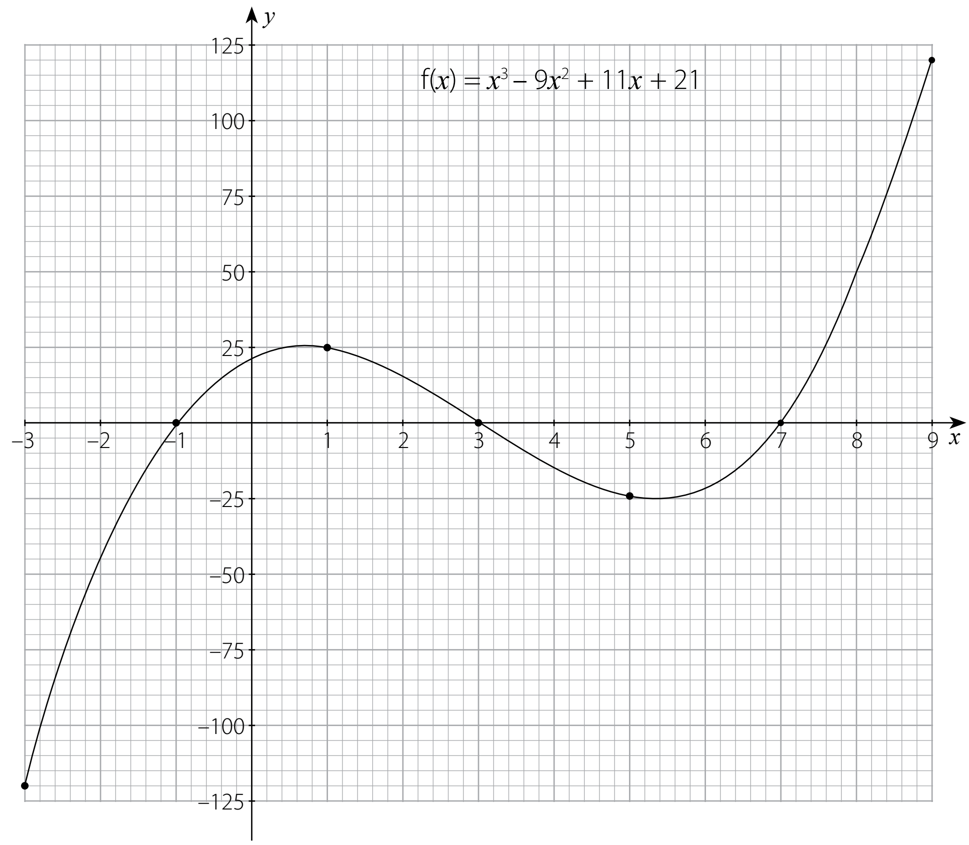
   c) 0.00056 m/s (2 s.f.)

2. a)

x	-3	-1	1	3	5	7	9
x^2	9	1	1	9	25	49	81
x^3	-27	-1	1	27	125	343	729
$-9x^2$	-81	-9	-9	-81	-225	-441	-729
$+11x$	-33	-11	11	33	55	77	99
$+21$	21	21	21	21	21	21	21
y	-120	0	24	0	-24	0	120

OXFORD UNIVERSITY PRESS

b)



c) i) $x = -1, 3, 7$ ii) $x = 8.1$

3. a) 6570 cm^2 b) 7230 cm^2

4. $\angle P = 31.5°$, $\angle R = 46.5°$, $p = 33.1$ cm

5. 745 cm^2 (3 s.f.)

6. a) \$ 550 b) \$ 800

7. $a = 44.6$ cm, $b = 166$ cm

8. a) −60.5 b) $f^{-1}(x) = \frac{x+3}{5}$ c) 0.68

9. Many proofs possible. The following are all true:

$QP = QR$ (equal tangents)

QS is common (given)

$\angle PQS = \angle RQS$ (symmetry property)

$\angle QPS = \angle QRS = 90°$ (radius $\perp$ tangent)

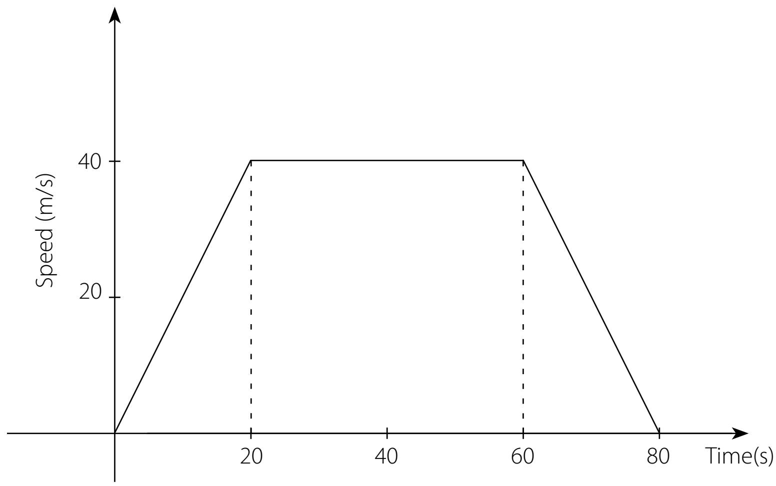
$PS = RS$ (radii)

Selecting three of these makes congruence possible by (SSS), (SAS), (RHS) or (AA corr S)

10. a) $x = 27.7$ b) $y = 15.9$

EX 7B

1. a)



b) -2 m/s^2 c) 144 km/h d) 2.4 km

2. a) 0.04 b) 0.32 c) 0.36

3. 2100 cm^2

4. $x = 7, y = 11.5$ (1 d.p.)

5. a) no b) no c) yes d) yes

6. 143 km

7. $x = 9.5$ cm, $y = 32$ cm, $z = 57.6$ cm

8. $a = 3.2$ (perp through centre P bisects the chord)

   $b = 4.5$ (equal chords are equidistant from centre P)

   $c = 90$ (bisector of chord through centre Q is perp.)

   $d = 4$ (chords equidistant from centre Q are equal.)

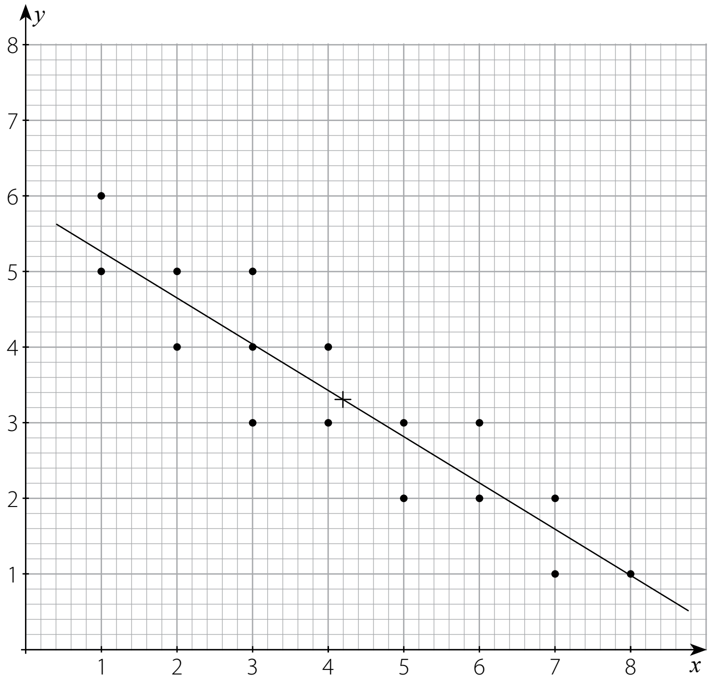
9. a) (ext angle-sum of polygon)

   b) (adj $\angle$s on st line)

   c) (angle-sum of quad)

   d) (int $\angle$s)

10. a)
b)



Mean is (4.2, 3.3)

c) $y = 4.4$ [Allow ±0.1]   d) strong negative

EX 7C

1. a) $x = \frac{3-y}{2}$ b) $x = \frac{2p}{a+b}$ c) $x = \pm\sqrt{\frac{q}{7}}$ d) $x = 1 \pm n$

2. a) $x = 1.3, x = 0.3$ b) $x = \pm\, 0.7$

   c) $x = \pm\, 1.2$ d) $x = 0.8, x = 0.4$

3. a) equiangular   b) 6.5

4. $a = 90, b = 7, c = 6.2$

5. a) 1.3×10^{17} b) 1.9×10^{-1} c) 4.2×10^{-2} d) 1.6×10^{-2}

6. \$ 344 000

7. a) not possible   b) $\begin{pmatrix} 27 & -4 \\ -18 & 3 \end{pmatrix}$

   c) $\begin{pmatrix} 4 & 8 & 24 \\ -2 & -1 & -6 \end{pmatrix}$ d) $\begin{pmatrix} -6 & -32 & -28 & 26 \\ 30 & 28 & 20 & 2 \end{pmatrix}$

8. 79 cm

9. $x = -0.4, y = 0.9$

10. a) $RQ = 8.76$ cm   b) $\angle R = 73.3°$ c) Area = 34.6 cm^2

EX 7D

1. 3:1

2. $11.415 \le d < 11.425$

3. a) 0.471 m^2 b) 1.27 m c) $40.3°$

4. a) 6 cm b) $\frac{6}{6.5}$, 22.6°

c) $\frac{6}{2.5}$, 67.4° d) $\angle B$ and $\angle C$ are complementary.

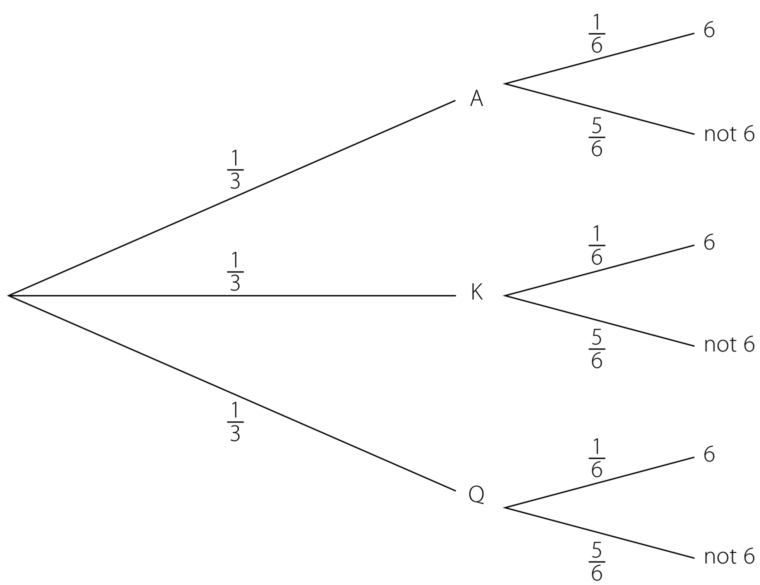
5. a) 80 cm^2 b) 386 cm^2 (3 s.f.)

6. $x = 90, y = 68, z = 44$

7. a) \$ 320 b) 4% c) \$ 98

8. a) $\angle B = 93.1°$ b) $\angle E = 104.2°$

9. a)



b) i) $\frac{1}{18}$ ii) $\frac{2}{3}$ iii) $\frac{1}{3}$

10. a) $p = 39.6$ cm b) $q = 45.1$ cm

EX 7E

1. a) $\frac{7}{25}$ b) $\frac{5}{13}$ c) $\frac{12}{5}$ d) $\frac{24}{25}$

2. a) 0730 b) 10 min

c) 24 km/h d) 13.7 km/h

3. a) 243 b) 0.125 c) 0.001 98 d) 1

4. a) equiangular b) $OS = 14$ cm, $PQ = 13.6$ cm

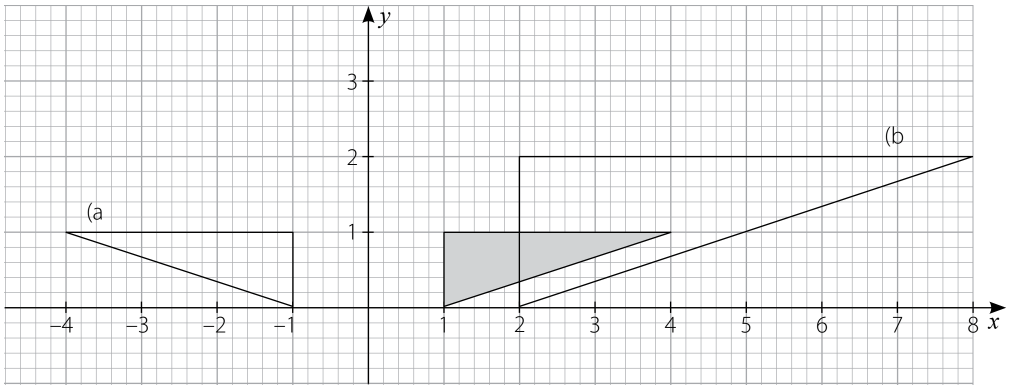
5. a) 0.184 m^3 b) \$ 56.30

6. $11^2 = a^2 + (x + 1)^2$, $9^2 = a^2 + x^2$, $x = 19.5$

7. a) $\overrightarrow{AB} = \begin{pmatrix} 3 \\ -8 \end{pmatrix}$ b) $\overrightarrow{AC} = \begin{pmatrix} 6 \\ -16 \end{pmatrix}$

c) $C(9, -10)$ d) $D(15, -5)$

8.



9. a) $x = 11$ b) $x = 0$ c) $x = 0.5$ d) $x = 5$

10. a) $\begin{pmatrix} 0.5 & 0.5 \\ 2.5 & 3.5 \end{pmatrix}$ b) $\begin{pmatrix} 2.5 & 3.5 \\ 9.5 & 12.5 \end{pmatrix}$

c) $\begin{pmatrix} 2 & 2 \\ 14 & 13 \end{pmatrix}$ d) $\begin{pmatrix} 10 & -1 \\ -5 & 4 \end{pmatrix}$

Chapter

8 Trigonometric Functions

This chapter is not strictly necessary at this stage. However, it provides an opportunity to give meaning to trig ratios of angles not possible in right-angled triangles, and to show that these functions have far wider applications. Having said that, most of the exercise is to reinforce the sine rule, cosine rule and the two area formulas of a triangle.

LESSON PLANNING

Objectives

General	To provide a context for trig ratios of obtuse angles and introduce graphs of these functions
Specific	1. To understand that sine, cosine, and tangent can be defined for any angle 2. To solve simple trig equations where the solutions are acute or obtuse 3. To know that the trig functions have graphs that are infinite and possess symmetries (details not required) 4. To solve problems of trigonometry involving sine rule, cosine rule, area formulas, and Pythagoras' theorem 5. To be aware that a triangle can have only one possible obtuse angle and that it would be opposite the longest side (if it exists at all) 6. To know that the cosine distinguishes between acute and obtuse angles (by sign) but the sine does not
Pacing	3 lessons, 1 homework
Links	Functions and their inverses
Method	• Go through Reminders 1, 2 and 3 in the text, making clear that when we use the sine rule and cosine rule with obtuse angles, we just blindly use the calculator values, although trig $x°$ was initially defined only for the acute angles in a right-angled triangle. This kind of extension of an idea has been met before. Ask! Elicit indices, developed from positive to negative and rational, or the number system itself, from $\mathbb{N}$ to $\mathbb{Z}$ to $\mathbb{Q}$. • The graphs may be developed by plotting the acute angle values and then using symmetry to make a more useful function. The plotting can be a class activity. Plot on the board at 15° intervals, getting students to call out (rounded off) values. [See also Assignments.] Students may be able to recognise the value of a wave function for radio, sound, vibrations, etc.

- From the graphs, focus on acute and obtuse angles, because that is immediately relevant. Show how the sine curve is ambiguous (sin 30 = sin 150, etc.), but that the cosine curve distinguishes by sign. This explains why the cosine rule is safe, but the sine rule has to be handled with care.
- Use the text examples and similar.

  Set EX 8A. Most of this revises standard techniques.

Resources	Calculators essential
Assignments	Draw the graph of $y = \sin x°$ for $0 \leq x \leq 720$ at intervals of 15° (prior to lesson) EX 8A, questions 9 and 10
Vocabulary	trig functions

ANSWERS

Exercises

EX 8A

1.	a) $x = 40$	b) $x = 20$	c) $x = 30$	d) $x = 10$
2.	a) $x = 129.1$	b) $x = 78.0$	c) $x = 47.7, x = 132.3$	d) $x = 63.3$
3.	a) $x = 42.3$	b) $x = 144.7$	c) $x = 40$	d) $x = 80$
4.	a) 36.3°	b) 4.08 km	c) 058°	d) 80.9°
5.	a) 27.0°	b) 12.2 cm		
6.	a) 52.3 m	b) 75.0 m	c) 1750 m^2	d) 24.55 m
7.	a) 76.1 cm	b) 50.2 cm	c) 2140 cm^2	d) 64.9 cm

8. a) $QS^2 = 1.2^2 + 0.5^2 = 1.69 \Rightarrow QS = 1.3$ as required

   b) $\angle Q$, because it is opposite the longest side.

   c) 90.9° d) 6 m^3

9. a) $AC = 91.580$ m (5 s.f.), area = 2980 m^2 b) 95.4 m

10. a) 30° b) 16.1 cm c) 10 cm

    d) 68.3°, acute because $30^2 > 20^2 + AC^2$, or from cosine rule.

EX 8X

1. $a = 16.6$ cm, $b = 16.0$ cm, $c = 13.8$ cm

2. a) $x = 30$, $x = 150$ b) $x = 78.5$, $x = -78.5$

   c) $x = 71.6$, $x = 251.6$ d) $x = -45$, $x = -135$

3. Proof: We have $a^2 = b^2 + c^2 - 2bc\cos A$ ——— 1

and $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ ——— 2

From 2, $b = a\dfrac{\sin B}{\sin A}$ and $c = \dfrac{a\sin C}{\sin A}$

Substituting in 1:

$$a^2 = \left(a\frac{\sin B}{\sin A}\right)^2 + \left(a\frac{\sin C}{\sin A}\right)^2 - 2\left(a\frac{\sin B}{\sin A}\right)\times\left(a\frac{\sin C}{\sin A}\right)\cos A$$

Multiplying by $(\sin A)^2$:

$$a^2(\sin A)^2 = a^2(\sin B)^2 + a^2(\sin C)^2 - 2a^2\sin B\sin C\cos A$$

Dividing by a^2:

$(\sin A)^2 = (\sin B)^2 + (\sin C)^2 - 2\sin B\sin C\cos A$ as required

Chapter

9 Miscellaneous Calculations and Proofs

As Important examinations approach, this chapter gives some attention to the importance of communication. Clarity in explanation is good mathematics, and it has the added benefit of assisting students to maximize marks in examinations.

LESSON PLANNING

Objectives

General	To practise giving clear, concise explanations of calculations; to provide reasoned arguments in simple proofs
Specific	1. To specify calculations on paper before working them out in order to assist the reader to understand the methods used 2. To define any terms not given in the question 3. To reject solutions when the numbers are not of the specified type 4. To solve simple linear inequalities 5. To construct equations from given information and simplify them 6. To use reason codes to explain calculations and proofs in geometry 7. To use a proof style when prompted by the phrase "Show that ..." 8. To write an acceptable sentence when prompted by the word the "explain ..."
Pacing	3 or 4 lessons
Links	Many previous topics in all branches of mathematics

Method

- Students will be looking for examination assistance at this point in the year. Although much of this may have been taught before, it is a time when students may be more receptive to honing their skills in being kind to examiners!

  "Is mathematics a language?" Opening question of class discussion. Elicit some features of language. For example,

  $$\underset{\text{subject}}{y} \; \underset{\text{verb}}{=} \; \underbrace{2x + 1}_{\text{predicate}}$$

  (sentences)

  This leads on to the need to communicate clearly with anyone reading your work.

  "Examiners cannot guess your thoughts. You must communicate your thought process".

  Write the keyword **communicate** on the board during this discussion.

OXFORD UNIVERSITY PRESS

It is also good mathematics. Communicating clearly clarifies your own thinking too, and avoids guessing and confusion.

Examples 1 and 2 in the text are models: one of arithmetic, one of algebra. Go through these. Explain inequalities, especially the negative multiplier feature. Use examples 3 and 4. Students who do not see why $-x > 9$ becomes $x < -9$ may use test data to work out what is happening.

		$-x > 9$				
Test $x =$	-12	-11	-10	10	11	12
	true	true	true	false	false	false
		$x < -9$				

- Set EX 9A. These are calculations. Emphasize the communication aspect. Many of these are very simple questions so that students can concentrate on their explanatory skills.
- For proofs, explain that really rigorous proofs are necessary in higher mathematics, but at this stage we just expect some reasoning prompted by "Show that ...."

  Go through the examples: three very different types of problems to illustrate the variety of responses required.

  Point out that the final line is always exactly what had to be proved.

  Set EX 9B and 9C. Model answers are provided in the answers section in the text, but challenge students to write out full and complete answers without looking there first.

  Set EX 9D.

Assignments	EX 9A and 9C are not suitable for homework as full model answers are given.
Vocabulary	communicate prove, show that, explain

ANSWERS

Exercises

EX 9A

1. a) $\$ \frac{6.64}{0.83}$ b) $ 8

2. lower bound = $83.5 \times 56.5 = 4717.75$ cm^2

   upper bound = $84.5 \times 57.5 = 4858.75$ cm^2

3. Let the width of the glass be x cm, and the cost \$ C.

$$\begin{aligned} \text{Then} \quad C &\propto x^2 \\ C &= kx^2 \\ \text{When } x = 30, \quad C &= 90 \\ 90 &= k(900) \end{aligned}$$

$$k = 0.1$$

$$C = 0.1x^2$$

When $x = 40$,

$$C = 0.1 \times 40^2$$

$$= 160$$

The cost is \$ 160.

4. Petrol angle $= \dfrac{480}{720} \times 360 = 240°$

Diesel angle $= \dfrac{90}{720} \times 360 = 45°$

LPG angle $= \dfrac{148}{720} \times 360 = 74°$

Electricity angle $= \dfrac{2}{720} \times 360 = 1°$

5. Let n be my original number.

Then $\sqrt{n}\,\sqrt{n+7} = 12$

Square: $n(n + 7) = 144$

$n^2 + 7n - 144 = 0$

Factorise: $(n + 16)(n - 9) = 0$

$n = -16$ or $n = 9$

Since $\sqrt{n}$ exists, we reject $n = -16$.

The original number is 9.

6. a) $\dfrac{2x-1}{3} < 3$

$2x - 1 < 9$

$2x < 10$

$\underline{x < 5}$

b) $\dfrac{x}{2} < x + 1$

$x < 2(x + 1)$

$x < 2x + 2$

$-x < 2$ [or, $-2 < x$

$\underline{x > -2}$ $x > -2$]

c) $\dfrac{2x-5}{4} \le 6$

$2x - 5 \le 24$

$2x \le 29$

$\underline{x \le 14.5}$

d) $\dfrac{1-x}{2} \ge 3$

$1 - x \ge 6$

$-x \ge 5$ [or, $-5 \ge x$

$x \le -5$ $x \le -5$]

7. Sequence A nth term = $3n - 2$

Sequence B nth term = $-n + 5$

Sequence C nth term = $(3n - 2)(-n + 5)$

$= \underline{-3n^2 + 17n - 10}$

9th term = $-3(81) + 17(9) - 10 = -100$

$$\left[\begin{array}{l} \text{or, use the factorised form} \\ \quad (27 - 2)(-9 + 5) \\ = \quad 25 \times (-4) \\ = \quad -100 \end{array}\right]$$

8. $\angle BED = 64°$ (isos Δ)

$a = 180 - 2 \times 64$ (angle-sum of Δ)

$\underline{a = 52}$

$\underline{b = 52}$ (alt $\angle$s)

9. $\underline{a = 128}$ (int $\angle$s)

Sum of angles in pentagon $= 180(5 - 2) = 540°$

$52 + a + 3b = 540$

$180 + 3b = 540$

$3b = 360$

$\underline{b = 120}$

10. Ratio of volumes large : small

= 250 : 16

= 125 : 8

Ratio of lengths = 5 : 2

Ratio of areas = 25 : 4

= 6.25 : 1

Surface area of larger glass $= 31 \times 6.25$

$= 193.75 \text{ cm}^2$

EX 9B

1. a) Q b) S c) R d) P

2.	a) \$ 90	b) 350 km		
	c) 150 litres	d) 3h 10 min		
3.	a) JPY 17 734.05	b) GBP 140.50		
	c) \$ 281.16	d) \$ 6.79		

4. equilateral (All the sides are now 7.5 cm.)

5. 3 3.3 3.63 3.993 4.3923

6.	a) \$ 230.80	b) \$ 45.20	c) Rs 4578	
7.	a) AED 5.24	b) JPY 32.09		
	c) \$ 1.42	d) JPY 3324.10 (to 2 d.p.)		
8.	a) 3 : 2	b) 4 : 5	c) 8 : 7	d) 10 : 9
9.	a) a ar ar^2 ar^3 ar^4		b) ar^{n-1}	c) 128 000

10. a) \$ 50

b) Rs 3000 approx; Rs 2970 exactly

c) 110. The value of \$ 1 in rupees

EX 9C

1. P(first ball red) $= \frac{n}{10}$

This leaves 9 balls in the bag, of which $n - 1$ are red.

P(second ball red) $= \frac{n-1}{9}$

These are independent events.

P(two consecutive reds) $= \frac{n}{10} \times \frac{n-1}{9} = \frac{1}{15}$

$\therefore \quad \frac{n(n-1)}{90} = \frac{1}{15}$

Mult by 90: $\quad n(n-1) = 6$

$\underline{n^2 - n - 6 = 0}$ as required

Factorising $\quad (n-3)(n+2) = 0$

$n = 3$ or $n = -2$ (reject as $n > 0$)

<u>There are 3 red balls in the bag at the start.</u> as required

2. nth term $= 3n + 4$

If 200 is a term in the sequence, then

$$3n + 4 = 200$$

$$3n = 196$$

$$n = 65.33$$

But n must be a positive integer.

Hence, 200 is not a term in the sequence.

3. The red sector is $\frac{1}{4}$ of the pie chart circle.

$$\frac{1}{4} \text{ of } 200 = 50$$

$\therefore$ 50 people stated red. as required

4. a) $\angle AOB = \frac{360}{8} = 45°$ as the octagon is regular.

Area sector $AOB = \frac{1}{2}(6)^2 \sin 45°$ (SAS formula)

$= 18 \sin 45°$

Total area of octagon $= 8 \times 18 \sin 45° = 101.82$

$= \underline{102 \text{ cm}^2 \text{ (3 s.f.)}}$ as required

b) Area of circle $= \pi(6)^2$

$= 113.1 \text{ cm}^2$

Shaded area = area of circle − area of octagon

$= 113.1 - 101.8 = 11.3$

$\approx \underline{11 \text{ cm}^2}$ as required

5. Scratch card, $P(6) = \frac{3}{21} = \frac{1}{7}$

Dice, $P(6) = \frac{1}{6}$

$\frac{1}{7} < \frac{1}{6}$, so Maliha is more likely to win.

6. $a + 1 = 2(b + 1)$

$a + 1 = 2b + 2$

$\underline{a = 2b + 1}$ as required ——— (1)

$a + 9 = 1.5(b + 9)$

$2a + 18 = 3(b + 9)$

$2a + 18 = 3b + 27$

$\underline{2a - 3b = 9}$ as required ——— (2)

Substituting from (1) into (2):

$2(2b + 1) - 3b = 9$

$4b + 2 - 3b = 9$

$b = 7$

Sub in 1,

$a = 15$

Adil is 15 and Bashir is 7 years old now.

7. $V = \frac{2}{3}\pi r^3$ —— 1

$A = 2\pi r^2$ —— 2

From 1: $3V = 2\pi r^3$

$3V = (2\pi r^2)r$

Sub from 2: $\underline{3V = Ar}$ as required

8. As the polygon is regular, exterior angles are equal.

ext $\angle = \frac{360}{18} = 20°$ (ext angle sum of polygon)

interior $\angle = 180 - 20$ (adj $\angle$s on st line)

$= 160°$

$\therefore$ Sum of interior angles $= 18 \times 160$

$= 2880°$ as required

[OR Sum of interior angles $= 180(18 - 2)$

$= 180 \times 16$

$= \underline{2880°}$]

9. a) LHS $= \frac{3\frac{1}{2} + \frac{1}{4}}{\frac{1}{7} + \frac{1}{8}}$

$= \frac{3\frac{3}{4}}{\frac{15}{56}}$

$= \frac{\cancel{15}}{4} \times \frac{56}{\cancel{15}}$

$= 14$

= RHS as required

b) LHS $= \left(4 - \frac{1}{4}\right)\left(\frac{3}{5} - \frac{1}{15}\right)$

$= \left(3\frac{3}{4}\right)\left(\frac{8}{15}\right)$

$= \frac{\cancel{15}}{4} \times \frac{8}{\cancel{15}}$

= 2

= RHS as required

10. $$\frac{2400}{x} = \frac{1500}{x+2} + 150$$

$2400(x + 2) = 1500 + 150x(x + 2)$

$240x + 480 = 150x + 15x^2 + 30x$

$0 = 15x^2 - 60x - 480$

$0 = x^2 - 4x - 32$

$\underline{x^2 - 4x - 32 = 0}$ as required

$(x - 8)(x + 4) = 0$

$x = 8$ or $x = -4$ (reject as –ve)

The speeds of the cars are 8 m/s and 10 m/s, as required

EX 9D

1. 1
2. 4 years
3. 35 years
4. $k = 0.8$, $q = 1.65888$
5. a) $450\,000 = k(1.0618)^{1947}$
   and $y = k(1.0618)^{2013}$

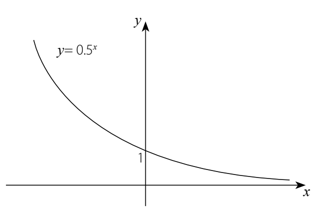
   Dividing gives
   $y = 450\,000 \times 1.0618^{66} \approx 23$ million as required

   b) over 65 million
6. \$16 324
7. $a = 2, k = 1$; 1 2 4 8 16 32 64 128
8. a) 1.25 °C b) 80 °C
9. $x = 3$
10. a)

x	–4	–3	–2	–1	0	1	2	3	4
y	16	8	4	2	1	0.5	0.25	0.13	0.06

(2 d.p)

b)



c) $y = 0.5^x = \left(\frac{1}{2}\right)^x = (2^{-1})^x = 2^{-x}$

d) reflection in the y-axis

EX 9X

1. Let the lower integer be n.

Then $n(n + 1) + 2(n + n + 1) = 376$

$n^2 + n + 4n + 2 = 376$

$n^2 + 5n - 374 = 0$

$(n - 17)(n + 22) = 0$

$n = 17$ or $n = -22$

The integers are 17 and 18, or −22 and −21 as required

2. A conjecture is something that seems to be true but has not yet been proved. A famous conjecture is Fermat's Last Theorem. He stated that he had a proof that $a^n = b^n + c^n$ is not possible for positive integers if $n > 2$. But then he died in 1665 before showing anyone his proof. For hundreds of years, mathematicians tried to prove it and failed, but nobody could find an example to show that it was false. Eventually, Andrew Wiles prove it in 1995 to great acclaim.

3. The flaw lies in the cancellation of $(a - b)$. Since $a = b$ the value of this factor is zero. Cancellation of zero is not allowed, but the algebra disguises the zero.

e.g. $3 \times 0 = 4 \times 0$ is true

but $3 = 4$ is false.

We cannot divide through by a zero.

Chapter

10 Surds

Surds are introduced as a subset of the irrationals, and a formal approach is avoided. Their role as exact solutions is highlighted in various familiar contexts. The text material may go somewhat beyond immediate examination requirements and teachers may wish to be selective.

LESSON PLANNING

Objectives

General	To use surds in familiar types of problems where exact answers are required
Specific	1. To know the definition of a surd 2. To recognise that the requirement for an "exact answer" demands a whole number or surd in response 3. To rationalise the denominator of a fraction involving surds 4. To estimate values of expressions involving surds 5. To locate a given surd between two consecutive integers
Pacing	1 lesson, 1 homework
Links	vectors, estimation, 3-D geometry, prime factorisation, difference of two squares, similar figures, Pythagoras' theorem

Method

The main purpose of this chapter is to handle surds in various contexts. The number of listed links is indicative of the variety.

First define a surd and establish basic operational rules in a common sense kind of way, using factors, e.g.

$$
\begin{array}{lll}
 & 25 & = 5^2 \\
\therefore & \sqrt{25} & = 5 \\
 & 100 & = 4 \times 25 \\
 & & = 2^2 \times 5^2 \\
\therefore & \sqrt{100} & = 2 \times 5 = 10 \quad \text{taking "half the factors"} \\
\text{But} & 75 & = 3 \times 25 \\
 & & = 3 \times 5^2 \\
\therefore & \sqrt{75} & = \sqrt{3} \times 5 = 5\sqrt{3} \\
\text{Similarly,} & 45 & = 9 \times 5 \\
 & & = 3^2 \times 5 \\
\text{So} & \sqrt{45} & = 3\sqrt{5} \quad \text{and similar examples.}
\end{array}
$$

Then follow the text for squaring surds and "rationalising the denominator". It is an opportunity to revise number types: naturals, integers, rationals, and irrationals. Also, to revise the difference of two squares identity. $A^2 - B^2 = (A + B)(A - B)$.

Set EX 10A fairly quickly. Students should be prompted to expect surds to appear in a variety of contexts.

Assignments	EX 10A, questions 9 and 10 suitable for homework
Vocabulary	surd, rationalise, exact length (or magnitude), prime factorisation, difference of two squares factorisation

ANSWERS

Exercises

EX 10A

1. a) $\sqrt{13}$ b) $\sqrt{17}$ c) $\sqrt{20}$ d) $\sqrt{61}$
2. $\dfrac{30 - 4^2}{\sqrt{100}} = 1.4$
3. $\sqrt{152}$ cm
4. a) $\sqrt{37}$ b) $\sqrt{53}$ c) $\sqrt{2}$
5. a) $M(1, -1)$ b) $\mathbf{m} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ $|\mathbf{m}| = \sqrt{1^2 + (-1)^2} = \sqrt{2}$ as required
   c) $n = 5$
6. a) 8 b) 9 c) 5 d) 10
7. a) 0.3780 b) 0.6076 c) 0.5486 d) 0.6458
8. a) $2 < \sqrt{5} < 3$ b) $3 < \sqrt{10} < 4$
   c) $3 < \sqrt{15} < 4$ d) $12 < \sqrt{150} < 13$
9. a) $300 = 2^2 \times 3 \times 5^2$ b) $2\sqrt{75}, 5\sqrt{12}, 10\sqrt{3}$ c) 13
10. a) $AC = 2$ cm b) $DE = 5\sqrt{3}$ cm, $EF = \sqrt{87}$ cm
    c) $81 < 87 < 100$
    $\Rightarrow 9^2 < 87 < 10^2$
    $\Rightarrow 9 < \sqrt{87} < 10$ as required

EX 10B

1. a) true b) false c) true d) true
2. a) false b) false c) false d) true
3. a) false b) false c) false d) true
4. a) false b) false c) true d) true

5. a) false b) true c) true d) true

6. $\mathbb{N}$

7. All false; no real solutions

8. a) 10 b) 1

   c) 3430 d) $42\frac{2}{3}$ (or $\frac{128}{3}$)

9. LHS = $(\sqrt{2} + \sqrt{3})(\sqrt{2} - \sqrt{3}) + 1$

   $= \sqrt{2}^2 - \sqrt{3}^2 + 1$ using diff of two squares

   $= 2 - 3 + 1$

   $= 0$

   = RHS, as required

10. $9^3 - 9^2$

    $= 9^2(9 - 1)$

    $= 9^2(8)$ which is a multiple of 8, as required

EX 10X

1. a) −0.707 b) 0.172 c) 4.414 d) 1.121

2. $x = \sqrt{2}$; $y = \sqrt{3}$

3. $x = 4\sqrt{2}$; $y = 3$

Chapter

11 Revision Exercises

[Non-calculator]

ANSWERS

Exercises

EX 11A

1. a) $4n - 2$ b) design 25

2. a) 0.3015 b) 0.4317 c) 0.4633 d) 12.6332

3. a) $x \le \frac{9}{7}$ b) $x > \frac{15}{4}$ c) $x < \frac{9}{8}$ d) $x \ge 5$

4. $k = -4, n = 1$

5. a) $x - 2$

   b) $LB = MC = DN = AK = 2$ (given)

   $BM = NC = DK = AL = x - 2$ from (a)

   $\angle A = \angle B = \angle C = \angle D = 90°$ (given square)

   $\therefore$ shaded triangles congruent (SAS)

   c) total area $= x^2$ area of one shaded triangle $= \frac{1}{2} \times 2(x - 2) = x - 2$

   shaded area $= 4(x - 2)$

   $\therefore$ unshaded area $=$ total area $-$ shaded area

   $= x^2 - 4(x - 2)$

   $= \underline{x^2 - 4x + 8}$ as required

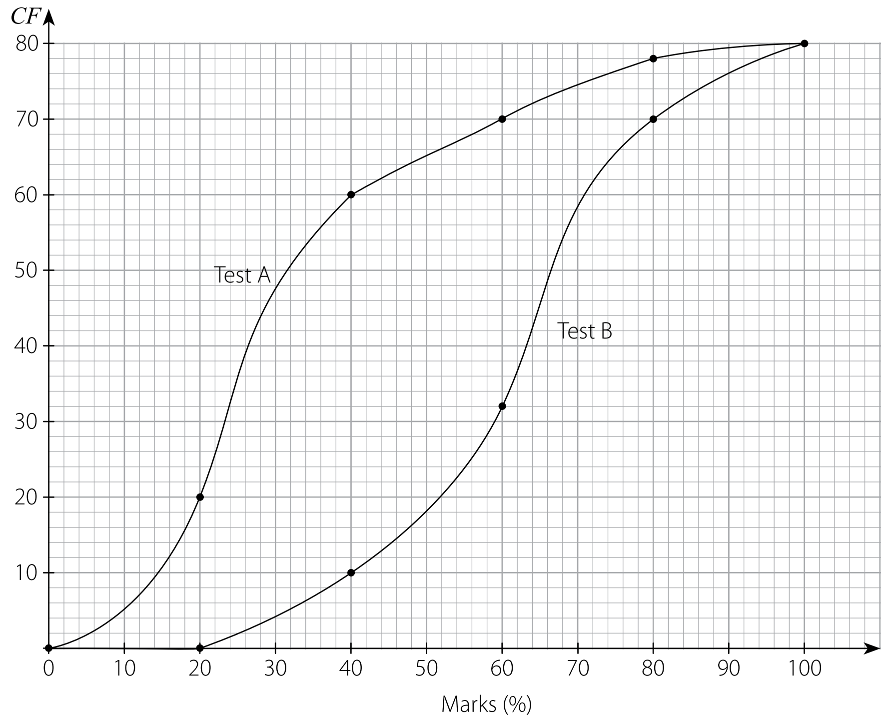
   d) $x = 6$

6. 50 cm^2

7. a) 33 km/h b) 250 g c) 14 min 2.16 s d) 3.48 m

8. a)

TEST B CF
0
10
32
70
80

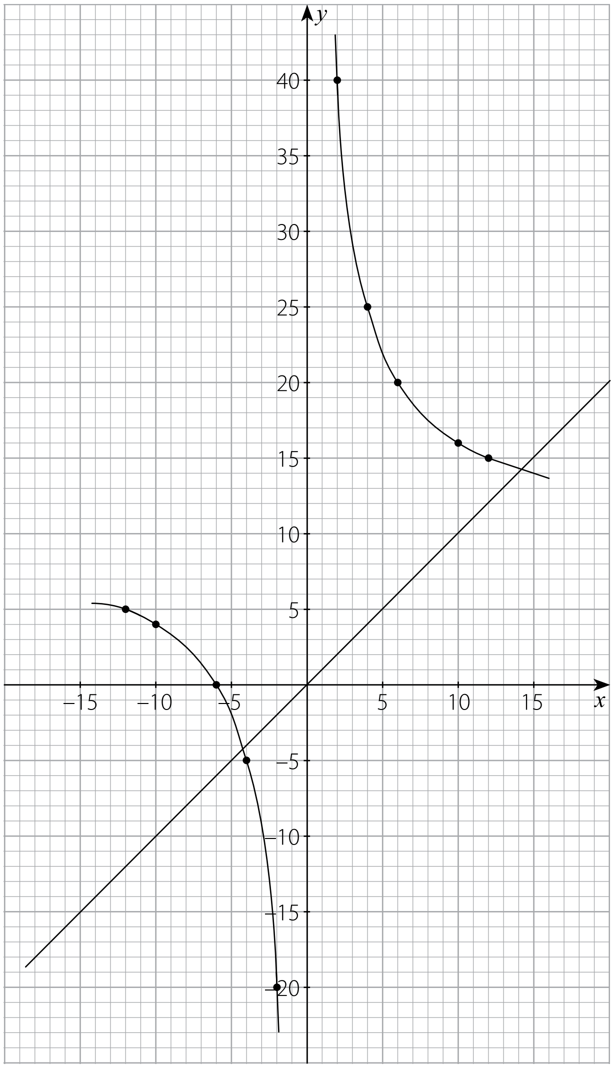


b) Test B results were better than test A's results in general (higher median and quartiles); the spread of results is unchanged. (same IQR of 32 marks)

9. 90 ml

10. a)

x	−12	−10	−6	−4	−2	2	4	6	10	12
$\frac{60}{x}$	−5	−6	−10	−15	−30	30	15	10	6	5
10	10	10	10	10	10	10	10	10	10	10
y	5	4	0	−5	−20	40	25	20	16	15



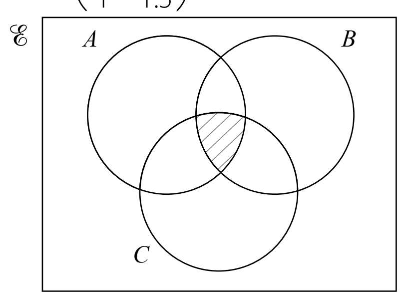
b) rotational symmetry of order 2 about (0, 10) c) $x = -4, \; x = 14$

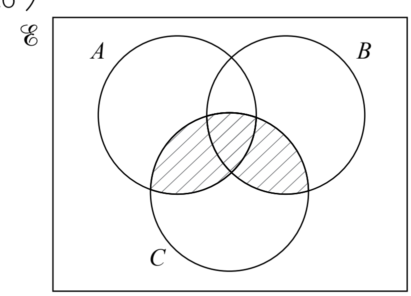
EX 11B

1. a) $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ b) $\begin{pmatrix} 9 \\ -6 \end{pmatrix}$ c) $k = \frac{1}{3}$ d) $\sqrt{205}$

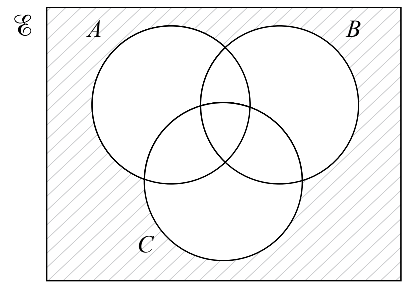
2. $\mathbf{A}^{-1} = \begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix}$ $\mathbf{B}^{-1}$ not possible

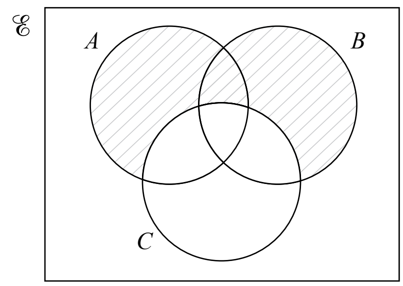
$\mathbf{C}^{-1} = \begin{pmatrix} 1 & 1 \\ 1 & 1.5 \end{pmatrix}$ $\mathbf{D}^{-1} = \begin{pmatrix} -0.2 & -0.1 \\ 0.2 & 0.6 \end{pmatrix}$

3. 



$A \cap B \cap C$ $\qquad\qquad$ $(A \cup B) \cap C$

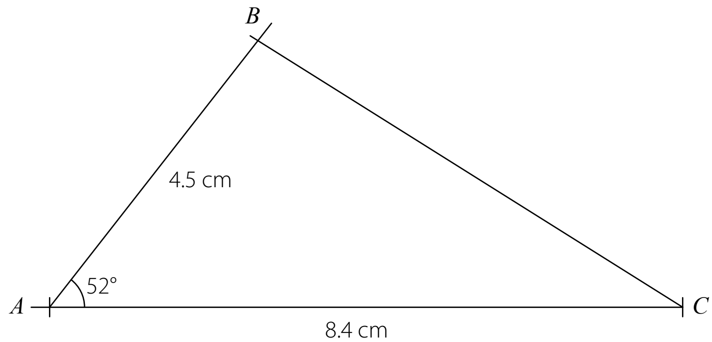




$(A \cup B \cup C)'$ $\qquad\qquad$ $(A \cup B) \cap C'$

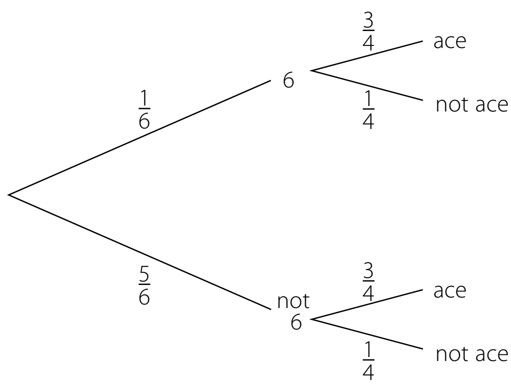
4. The weights of the boys are greater than the girls in general (higher median); the boys' weights are more diverse. (IQR of 21 kg compared with 16 kg for the girls.)

5.



$BC = 6.7$ cm (± 0.1)

6. a)



b) P(6, not ace) $= \frac{1}{24}$ $\quad$ c) P(not 6, ace) $= \frac{5}{8}$ $\quad$ d) P(not 6, not ace) $= \frac{5}{24}$

7. a) $2(x - 6)(x - 4)$ $\quad$ b) $3(x - 7)(x + 1)$ $\quad$ c) $4(x + 1)^2$ $\quad$ d) $5(x - 3)^2$

8. a) rotation of 180° about the origin $\quad$ b) (0, 0)

c) $P(x, y) \rightarrow P'(-x, -y)$ $\quad$ d) $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

9. a) 80° (opp ∠s of cyclic quad) $\quad$ b) 40° (opp ∠s of cyclic quad)

c) 60° (angle-sum of Δ) $\quad$ d) 120° (opp ∠s of cyclic quad)

10. a) 4 5 6 b) 4.5 6.75 10.125

EX 11C

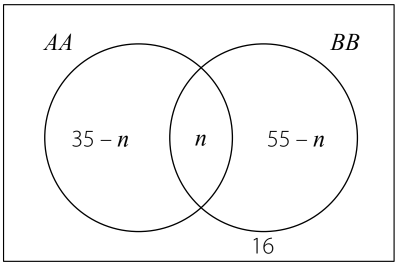
1. a) 3 stations b) 2 min c) 40 km/h

d) area under slowing phase $= \frac{1}{2}$(2 min)(40 km/h)

$= \frac{1}{2} \times \frac{2}{60} \times 40$ km

$= \frac{2}{3}$ km as required

2. a)



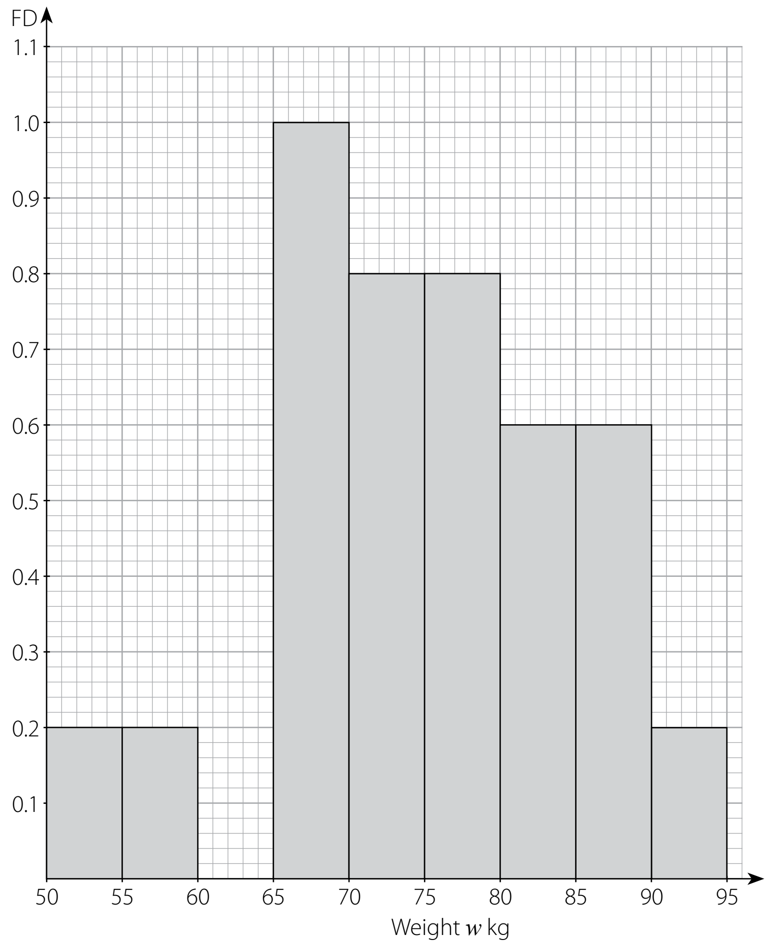
6 people visited both.

b) 0.29

3. a) $x \propto y$ b) $x = 105y$ c) \$ 200 d) Rs 2100

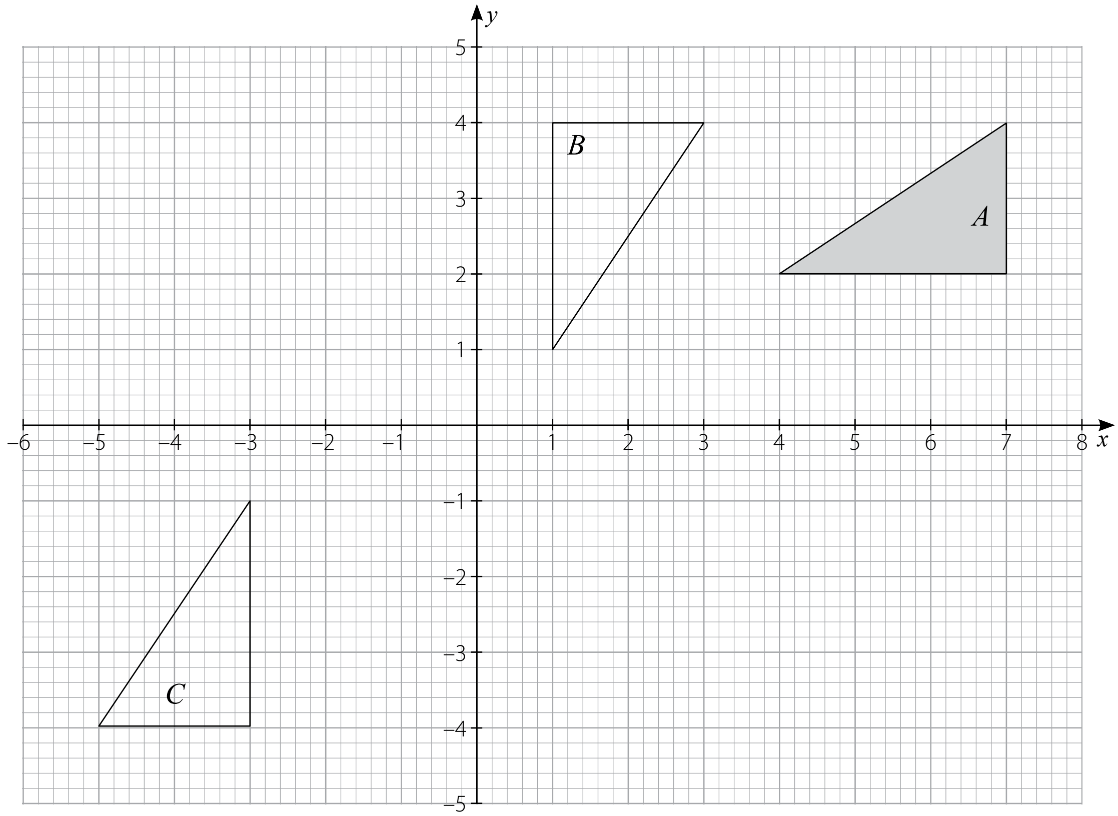
4.

Weight w kg	f	Frequency Density
$50 < w \leq 55$	1	0.2
$55 < w \leq 60$	1	0.2
$60 < w \leq 65$	0	0
$65 < w \leq 70$	5	1.0
$70 < w \leq 75$	4	0.8
$75 < w \leq 80$	4	0.8
$80 < w \leq 85$	3	0.6
$85 < w \leq 90$	3	0.6
$90 < w \leq 95$	1	0.2


FD
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
50
55
60
65
70
75
80
85
90
95
Weight w kg

5. a)

b)



c) rotation 180° about (–1, 0)

6. a) base circumference $= \text{curved sector boundary}$

$= \frac{3}{4} \times 2\pi(8)$

$= 12\pi$ cm, as required

b) $r = 6$ c) $\sqrt{28}$ cm

d) volume $= \frac{1}{3}\pi r^2 h$ $= \frac{1}{3}\pi 6^2\sqrt{28}$

$= \frac{1}{\cancel{3}}\pi \overset{12}{\cancel{36}}\sqrt{7 \times 4}$

$= 24\sqrt{7}\pi$ cm^3, as required

7. a) $h(-2) = 8$, $hg(6) = -1$ b) $f^{-1}(x) = \frac{x-4}{3}$

c) $x = 5$ d) $x = -1, x = 3$

8. a) 1 b) $y = x + 4$ c) $\begin{pmatrix} 10 \\ 10 \end{pmatrix}$ d) $\sqrt{200}$

9. $3x + x + x + x + 90 = 360$ (ext angle-sum of polygon)

$x = 45$

90°, 135°, 135°, 135°, 45° interior angles

10. a) 11 b) 22 c) $\frac{11}{42}$ d) $\frac{11}{31}$

EX 11D

1. a) $n = 3$ b) $n = -1$ c) $n = -\frac{3}{4}$ d) $n = -\frac{1}{6}$

2. a) -1 b) -8

   c) $f^{-1}(x) = \frac{4x}{x-3}$ d) $fg^{-1}(3) = 5.4$

3. 560 cm^2

4. a) 288 b) $\frac{1}{12}$ c) 24 d) 112

5. a) $\frac{48}{x}$ b) $\frac{48}{x-1}$

   c) $\frac{48}{x-1} - \frac{48}{x} = 4$

   $48x - 48(x-1) = 4x(x-1)$

   $\cancel{48x} - \cancel{48x} + 48 = 4x^2 - 4x$

   $12 = x^2 - x$

   $x^2 - x - 12 = 0$ as required

   d) 12 km/h

6. a) $u_n = 3n - 1$ b) $v_n = -2n + 12$ c) 501st term

7. $PR = \sqrt{52}$ cm, area = 12 cm^2

8. a) 3.01×10^{-2} b) 1.69×10^{2} c) 2×10^{-10} d) 1.7×10^{5}

9. a) $g^{-1}(x) = \frac{x+1}{3}$ b) $h^2(x) = x$ c) $hg\left(\frac{2}{3}\right) = 1$ d) $x = -4$

10. a) $25\sqrt{3} \text{ cm}^2$

    b) $\angle MPC = \angle MQC = 90°$ (symmetry)

    $\angle QCP = 60°$ (equilateral Δ)

    $\therefore \angle QMP = 120°$ (angle-sum of quad)

    $PMQC$ is cyclic. (opp $\angle$s supplementary)

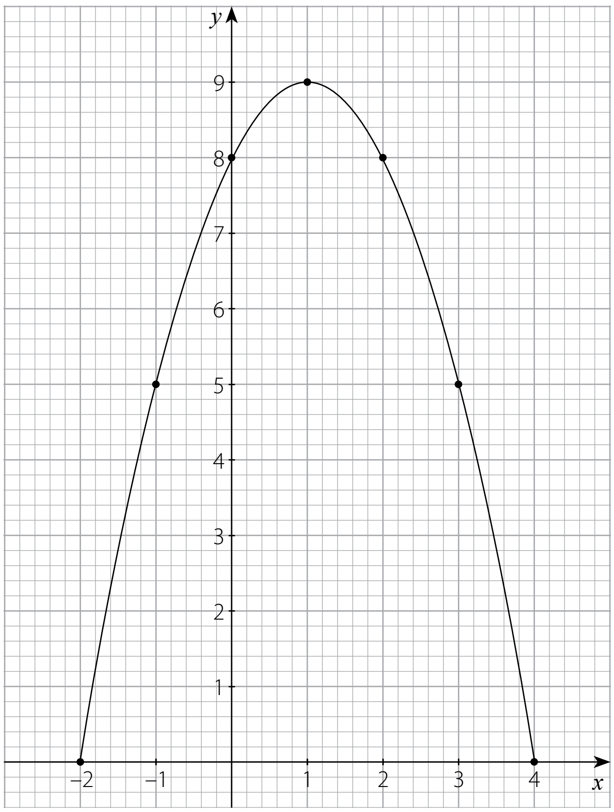
EX 11E

1. a) $x = -9, \quad x = -8$ b) $x = 5, \quad x = -3$ c) $x = \pm 4.5$ d) $x = \frac{1}{4}$

2. a)

x	-2	-1	0	1	2	3	4
x^2	4	1	0	1	4	9	16
8	8	8	8	8	8	8	8
$2x$	-4	-2	0	2	4	6	8
$-x^2$	-4	-1	0	-1	-4	-9	-16
y	0	5	8	9	8	5	0

b)



c) symmetry line $x = 1$ d) $x = 3.4, x = -1.4$

3. a) 4 6 8 10 $2n + 2$

b) 4 7 10 13 $3n + 1$

c) $n - 1$

d) $(3n + 1)(2n + 2) = 2(3n + 1)(n + 1)$

multiple of 2 $\Rightarrow$ even

4. $V = \frac{1}{3}\pi r^2 h$ ① $C = 2\pi r$ ②

$6V = 6 \times \frac{1}{3}\pi r^2 h$ from ①

$= 2\pi r^2 h$

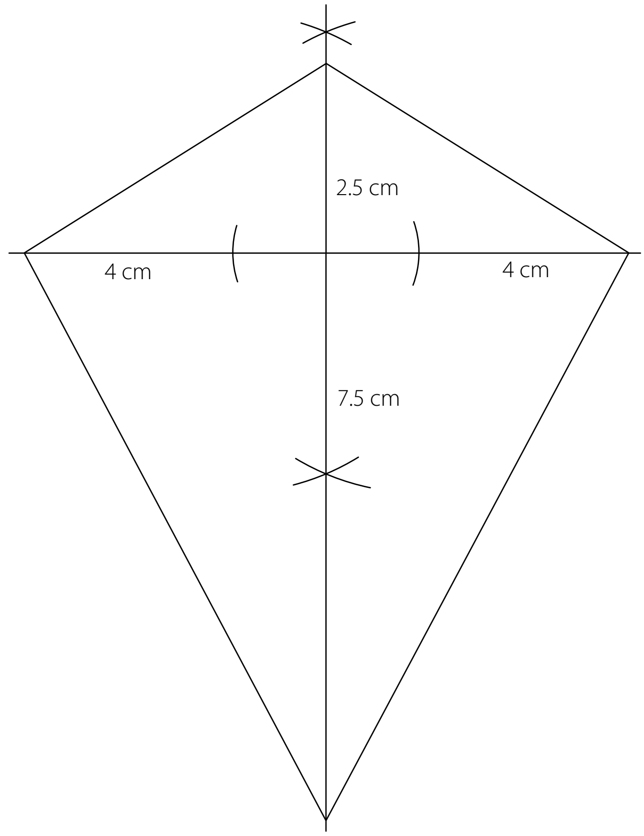
$= (2\pi r)(rh)$

$= Crh$ from ②   as required

5. $OM \neq ON$

6. 8

7.



8. a) $\angle Q = 100°$ (int $\angle$s)

b) $\angle SRT = 100°$ (corr $\angle$s)

$\angle T = 48°$ (angle-sum of Δ)

c) parallelogram

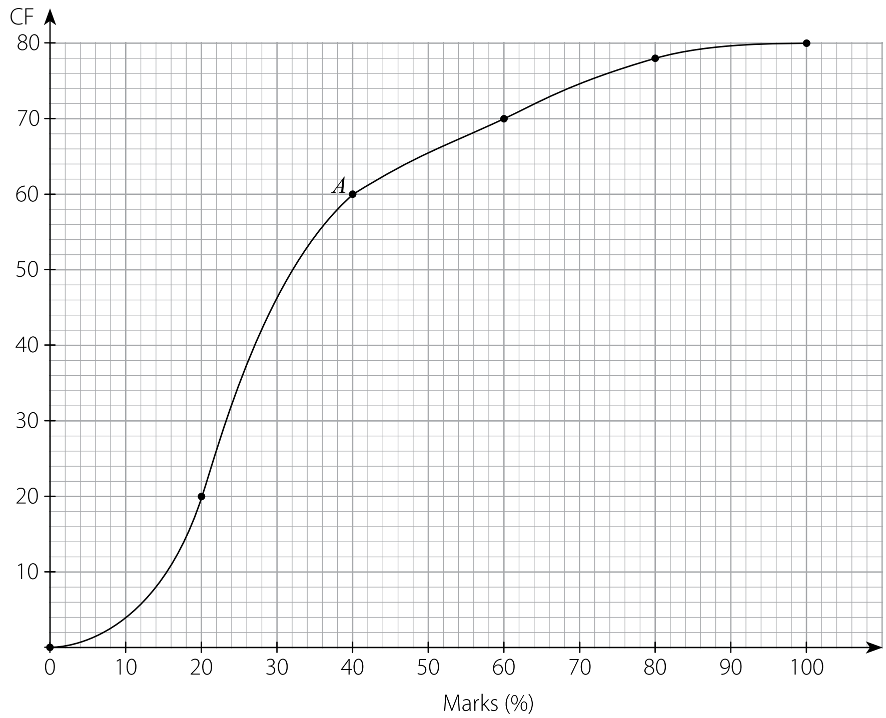
d) trapezium

9. a) $x = -\frac{-3}{2}$ b) $x = \frac{7}{8}$ c) $x = \frac{4}{9}$ d) $x = -2$

10. a) (2, 3) b) (6, 3) c) (–4.5, –1) d) (–1, 2)

EX 11A, question 8 (worksheet)

Marks	Test A		Test B	
m %	f	CF	f	CF
$0 < m \leq 20$	20	20	0	
$0 < m \leq 40$	40	60	10	
$0 < m \leq 60$	10	70	22	
$0 < m \leq 80$	8	78	38	
$0 < m \leq 100$	2	80	10	

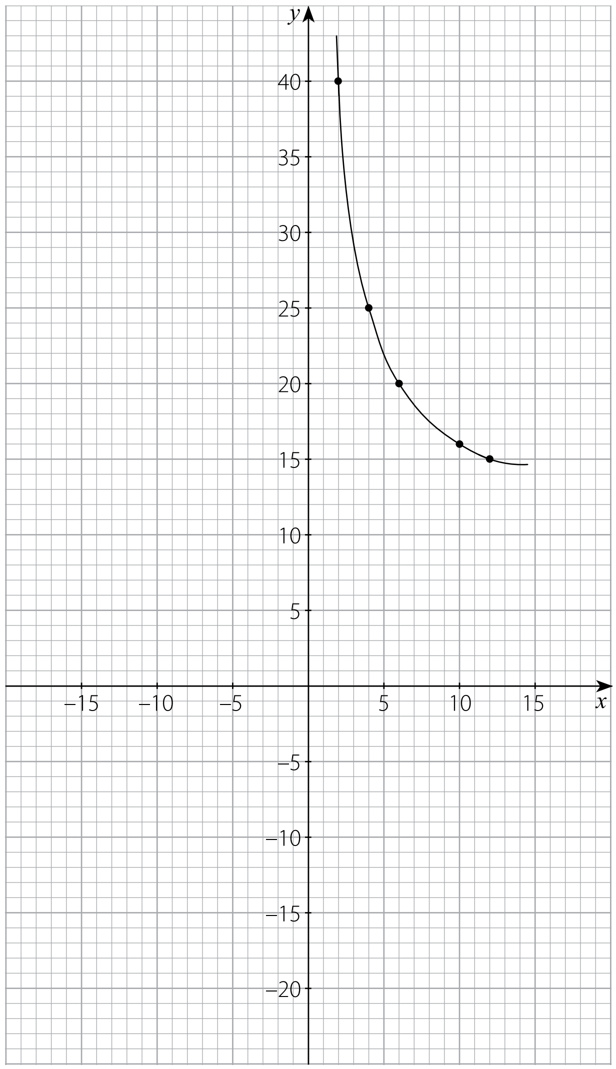


Comparison Statement:

EX 11A, question 10 (worksheet)

a)

	x	−12	−10	−6	−4	−2	2	4	6	10	12
y	$\frac{60}{x}$	−5			−15		30	15	10	6	5
	10	10		10			10	10	10	10	10
	y						40	25	20	16	15



b) Symmetry

____

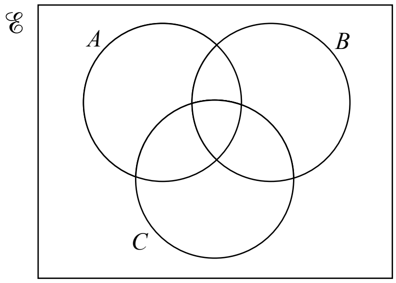
____

____

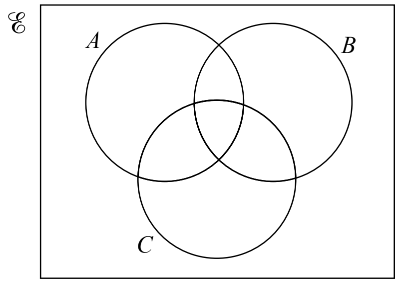
c) The solutions of $x^2 - 10x - 60 = 0$ are

____________ and ____________ (nearest whole number)

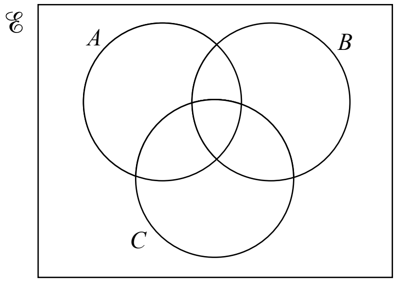
EX 11B, question 3 (worksheet)



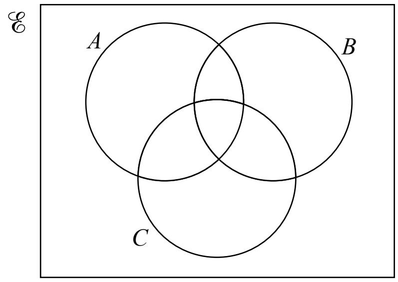
$A \cap B \cap C$



$(A \cup B) \cap C$



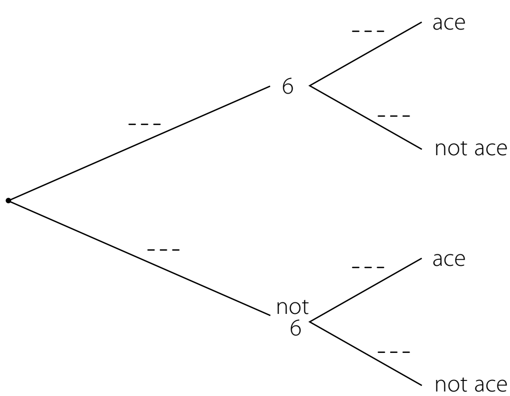
$(A \cup B \cup C)'$



$(A \cup B) \cap C'$

EX 11B, question 6 (worksheet)

a)



b) P(6, not ace) =

c) P(not 6, ace) =

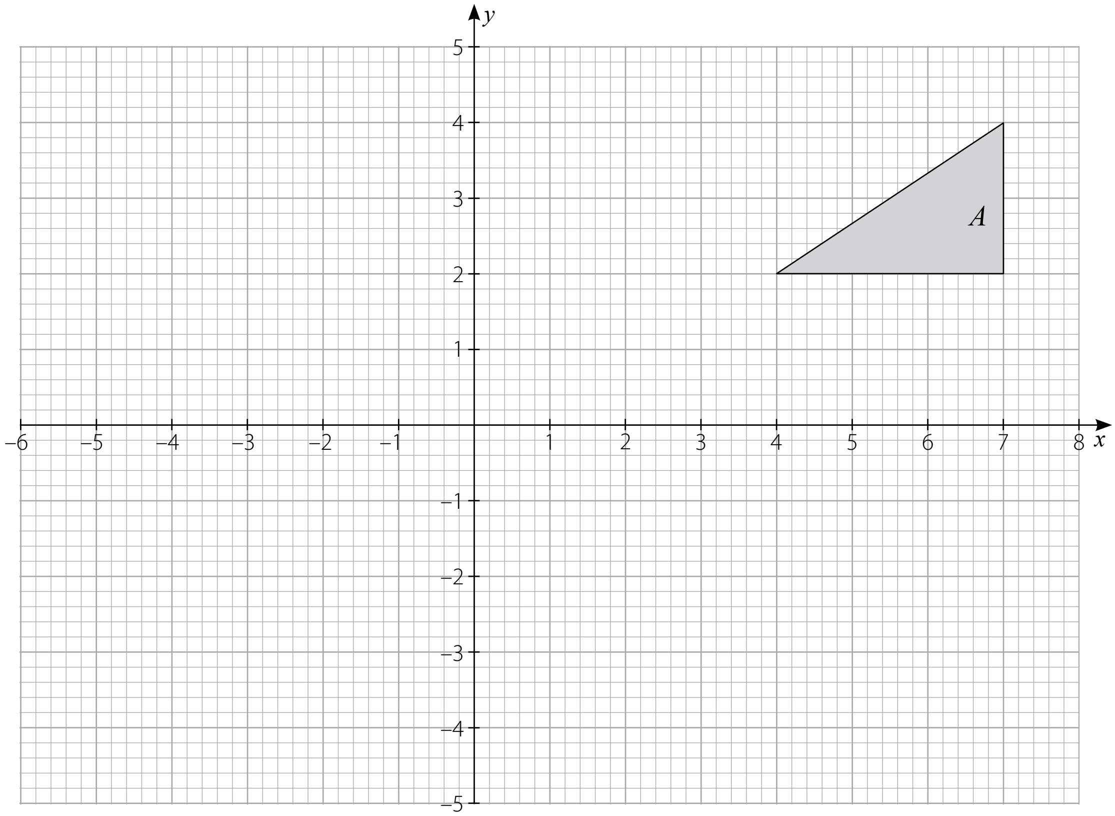
d) P(not 6, not ace) =

[Use fractions throughout.]

EX 11C, question 5 (worksheet)

a)

b)



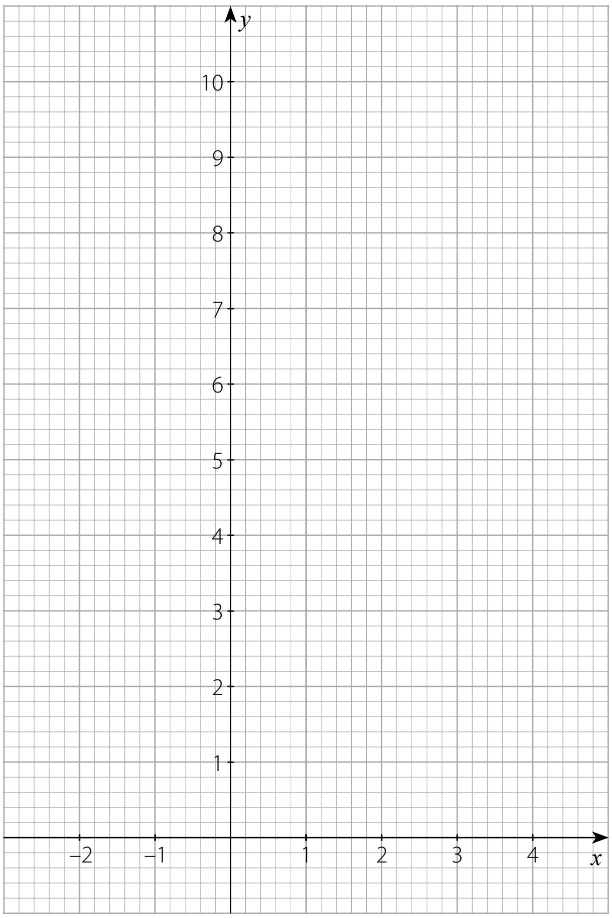
c) $C \rightarrow B$ by the transformation

EX 11E, question 2 (worksheet)

a)

x	-2	-1	0	1	2	3	4
x^2		1		1		9	
8				8			8
$2x$	-4			2			
$-x^2$			0	-1	-4		
y				9			

b)



c) The line of symmetry is ____________________

d) Solutions are

$x =$ __________

and $x =$ _______

Chapter 12 Revision Exercises

ANSWERS

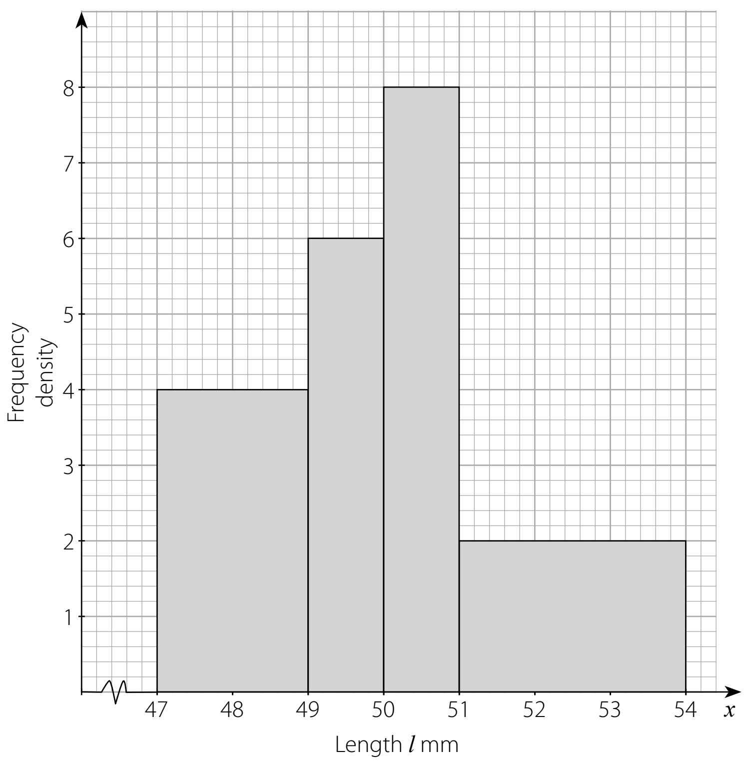
Exercises

EX 12A

1. a)

Length l mm	Frequency	Frequency density
$47 < l \le 49$	8	4
$49 < l \le 50$	6	6
$50 < l \le 51$	8	8
$51 < l \le 54$	6	2
	28	

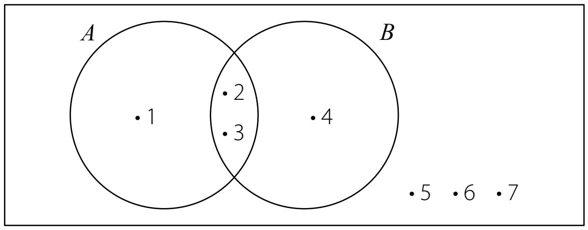
b)



c) 54 mm, grouped data d) 50.0 mm

2. a) $\begin{pmatrix} 6 \\ 0 \end{pmatrix}$ b) $\begin{pmatrix} -5 \\ 6 \end{pmatrix}$ c) $\begin{pmatrix} 4 \\ -1 \end{pmatrix}$ d) $\begin{pmatrix} 1 \\ -11 \end{pmatrix}$

3. a)



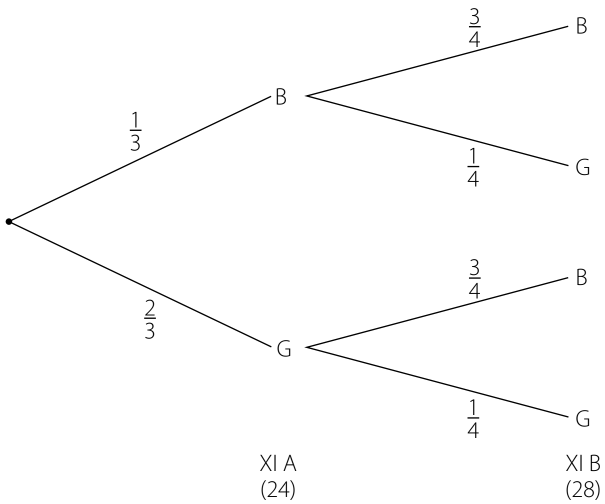
b) i) $A' = [4, 5, 6, 7\}$

ii) $A \cap B' = \{1\}$

iii) $B \cup A' = \{2, 3, 4, 5, 6, 7\}$

iv) $(A \cup B)' = \{5, 6, 7\}$

4.



a) $\frac{1}{4}$ b) $\frac{3}{4}$ c) $\frac{7}{12}$ d) $\frac{1}{12}$

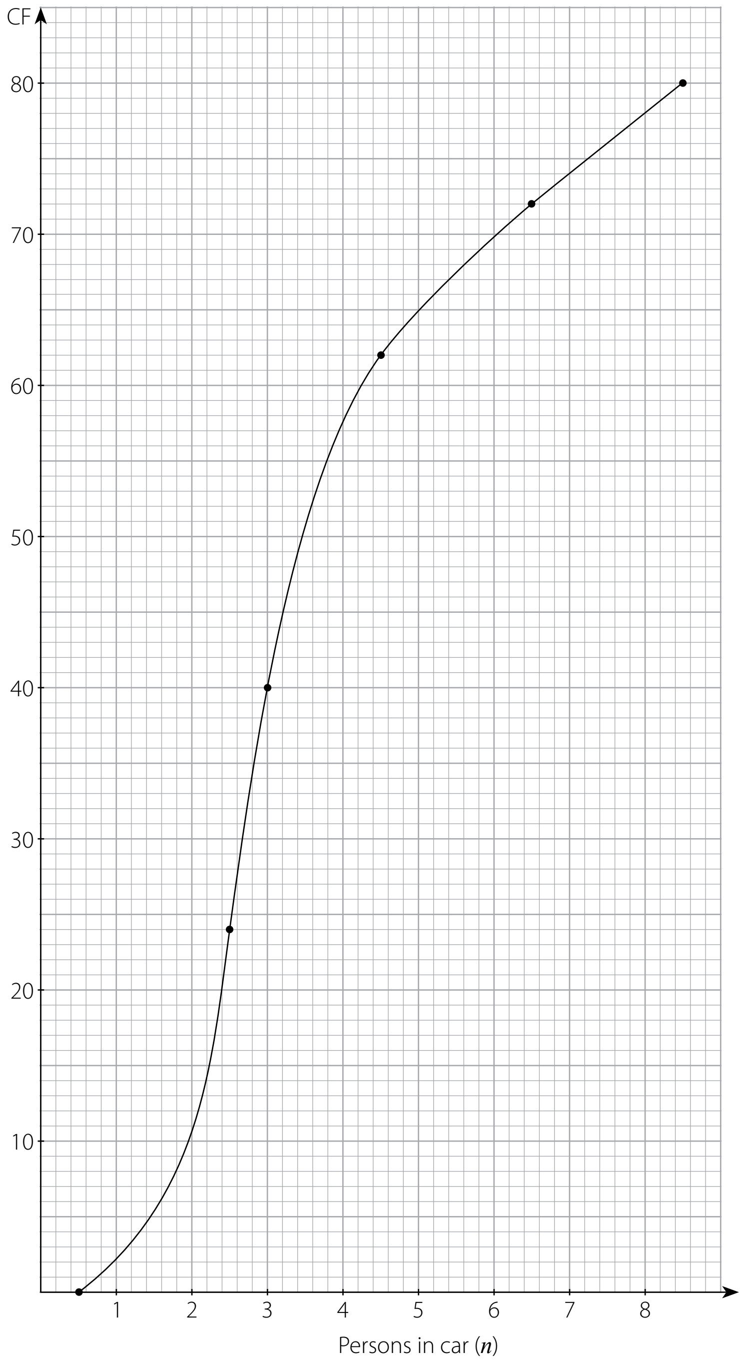
5. a) $n = \frac{11}{2}$ b) $n = \frac{10}{3}$ c) $n = -3$ d) $n = \frac{-7}{2}$

6. $12.25 \leq t < 12.35$

7. a)

Persons in car (n)	Frequency	CF
$0.5 \leq n < 2.5$	24	24
$2.5 \leq n < 4.5$	38	62
$4.5 \leq n < 6.5$	10	72
$6.5 \leq n < 8.5$	8	80

b)



c) median estimate 3.0 d) mean estimate 3.6

8. a) $t \propto \frac{1}{v}$ b) $t = \frac{2100}{v}$ c) 5 min d) 8.75 m/s

9. a) 8 b) –5 c) 0 d) $x - 4 - \frac{x}{4}$

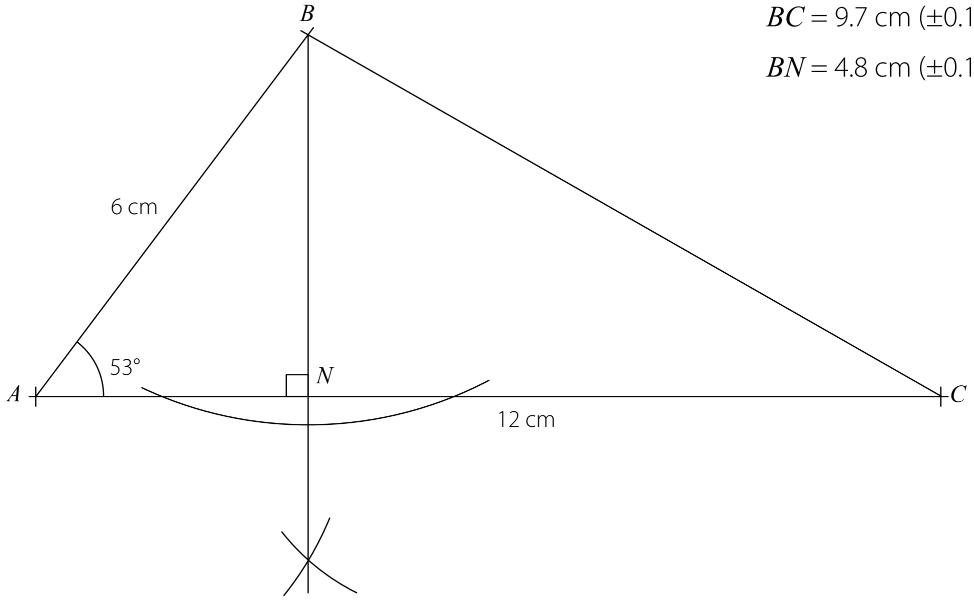
10. 140 cm

EX 12B

1. 48 cm long, 24 cm wide, 12 cm high

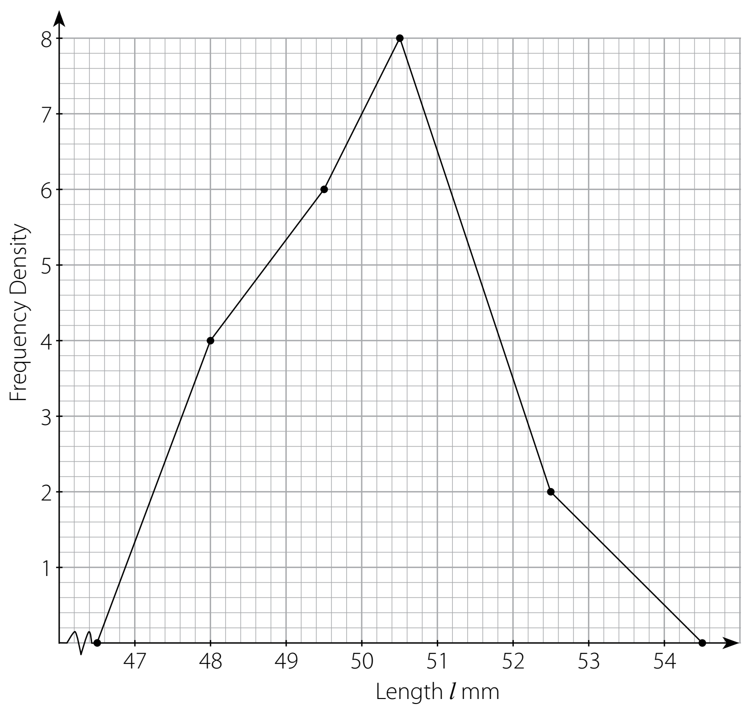
2. a) $BC = 9.7$ cm (±0.1)

   b) $BN = 4.8$ cm (±0.1)



   c) 28.8 cm^2 d) 28.8 cm^2 (3 s.f.)   using SAS formula.

3.



4. \$ 577.50

5. 267 km (3 s.f.)

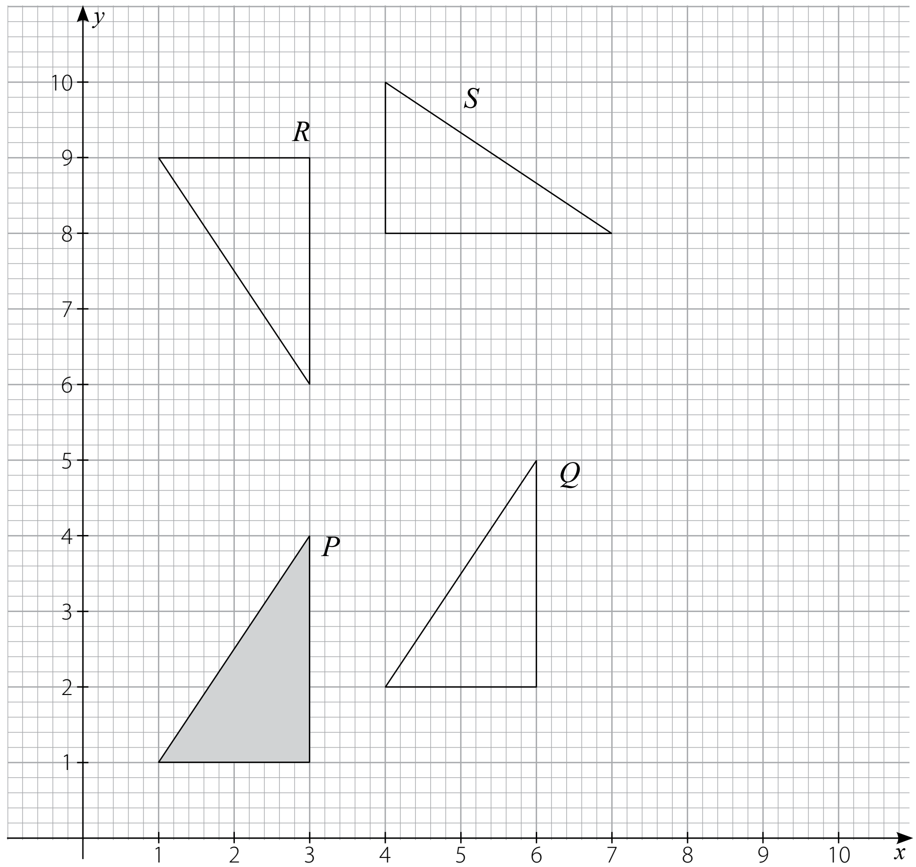
6. a) $x = 2$ (nearest integer)   b) $x = 3.63$ (2 d.p.)

   c) $74.5 \le x < 75.5$ d) $4.355 \le x < 4.365$

7. a) 0.76 b) 0.06 c) 0.44 d) 0.56

8. a)

b)



c) translation $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$

d) $P \to Q$ none
$P \to R$ $y = 5$ invariant
$P \to S$ (7, 4) invariant

9. 40.7°

10. \$ 8689.68

EX 12C

1. a) 35 °C b) 32 C° and 39 °C

c) 13 days d) 18 days [±1]

2. a) $3(x + 1)$ b) $4(x + 2)$ c) $5(x + 1)$ d) $5(x + 4)$

3. 1435 beads

4. a) 1.89 b) 1.16 c) 3.76 d) 1.87

5. Sum of interior angles of n-gon $= 180(n - 2)$

$3960 = 180(n - 2)$

$22 = n - 2$

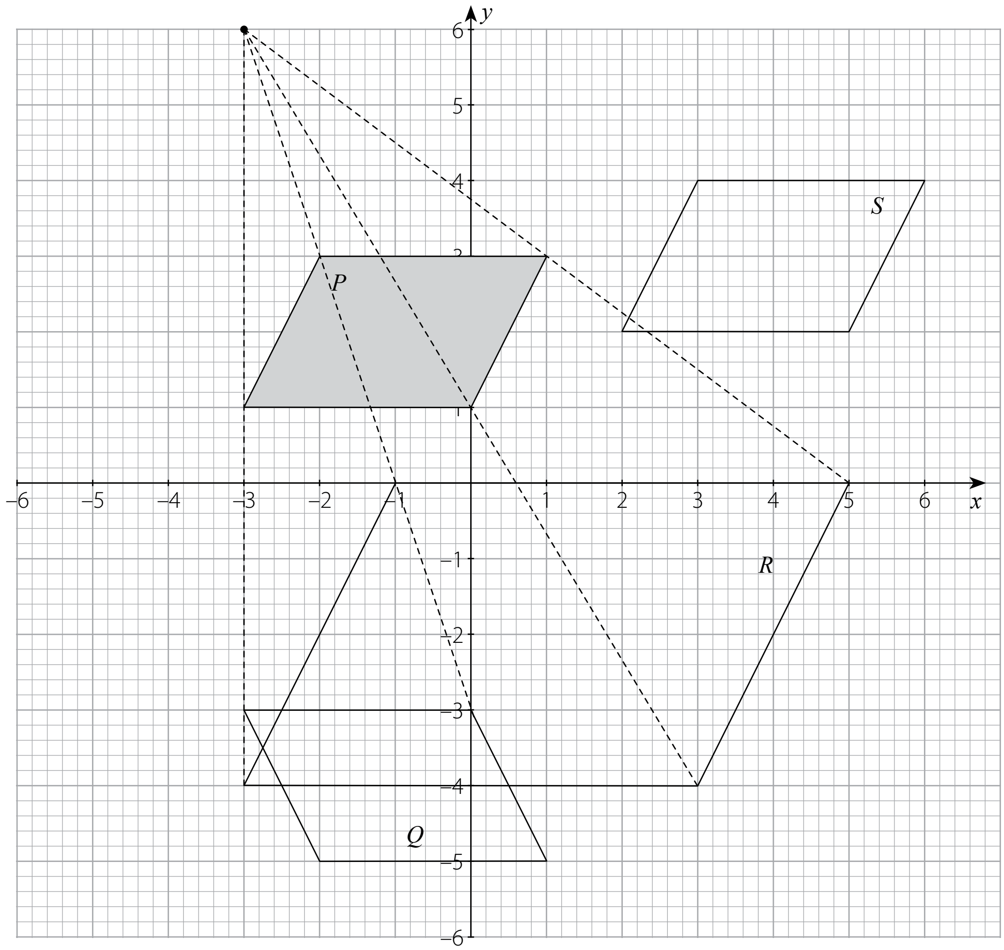
$n = 24$ as required

6. 6 cm

7. a)

b)

c)



d) rotation of 180° about (1, –0.5)

8. $x = 2.5$

9. $\angle QTR = 65°$ (isos Δ)

$x = 50$ (angle-sum of Δ)

$y = 50$ (alt ∠s)

10. $x = 10$

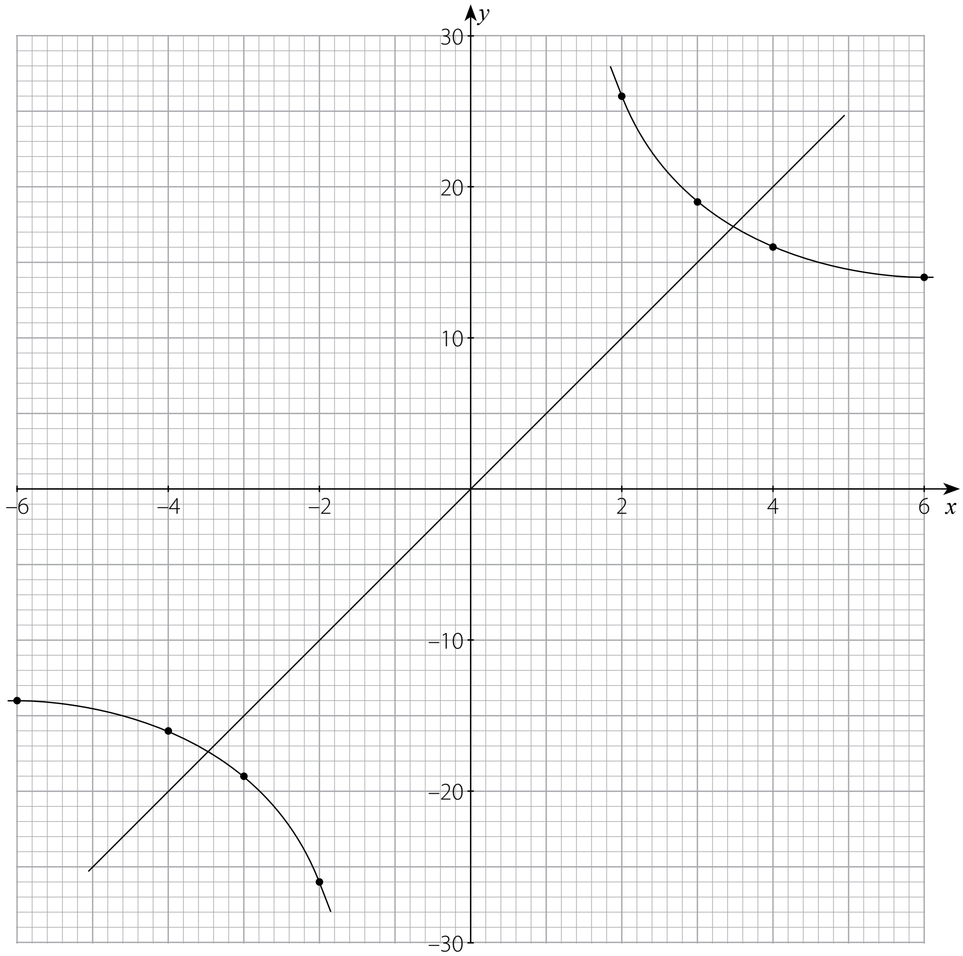
EX 12D

1. a)

x	–6	–4	–3	–2	2	3	4	6
$\frac{48}{x}$	–8	–12	–16	–24	24	16	12	8
y	–14	–16	–19	–26	26	19	16	14

b)

c)



c) intersect at $x = \pm 3.5$

d) $\left.\begin{aligned} y &= \frac{48}{x} + x \\ y &= 5x \end{aligned}\right\}$

intersect when $\frac{48}{x} + x = 5x$

$$\frac{48}{x} = 4x$$

$$48 = 4x^2$$

$$12 = x^2$$

$$x = \pm\sqrt{12}$$ as required.

2. a) $x + 22$ b) $x + 28$

c) $(x + 22) + (x + 28) = 180$ (opp ∠s of cyclic quad)

$$2x + 50 = 180$$

$$2x = 130$$

$$x = 65$$ as required.

d) 65°, 115°, 87°, 93°

3. $\angle C = 111.1°$

4. a) 5:1 b) Ayesha \$ 55 000, Bilal \$ 11 000

c) \$ 65 505.88 d) 2750 shares

5. a) $x = 1, x = -\frac{2}{3}$ b) $x = 1.18, x = -0.847$

6. a) i) $3n + 1$ ii) n^2 iii) 4^{n-1}

b) i) 61 ii) 400 iii) 2.75×10^{11} (3 s.f.)

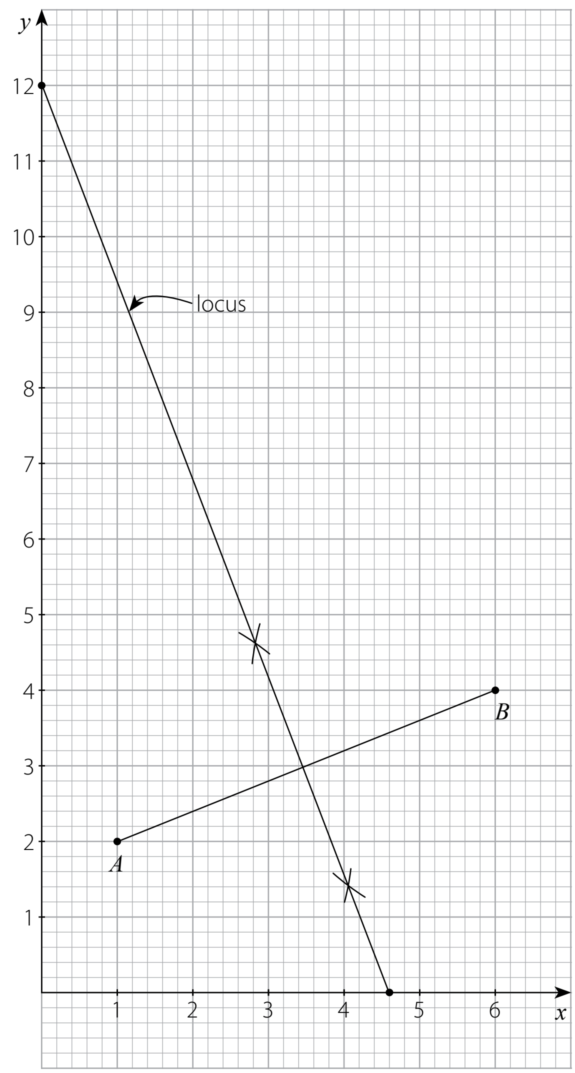
7. a) 30 sides (ext angle-sum of polygon)

b) 5040 c) $180(30 - 2) = 5040$

8. a) 86 400 b) 6 000 000 000 000 000 000 000 000

c) 3 470 000 d) 299 800 000

9.

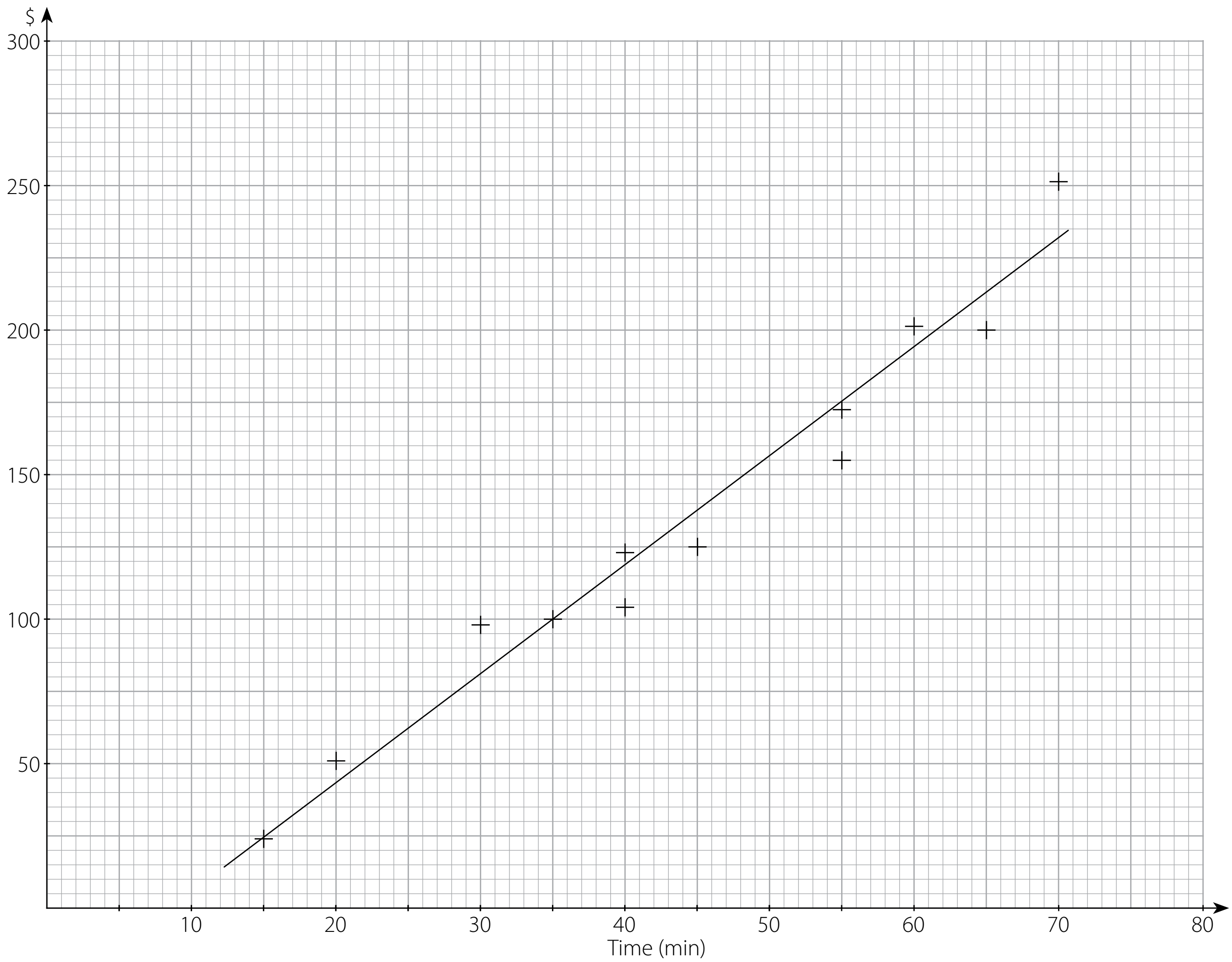


10. 27.2°

EX 12E

1. $A = \frac{3}{4}, \quad B = -\frac{1}{4}$

2. a) c)



b) strong positive correlation

d) \$ 143 (±2)

3. 73.7 km (3 s.f.)

4. a) 4^{-5} 3^{-6} 5^{-4} 6^{-3} b) $70+\sqrt{6}$ $7^{\sqrt{6}}$ $70\sqrt{6}$ $\frac{7000}{\sqrt{6}}$

5. a) yes b) no c) yes d) no

6. 30 cm^2

7. a) $2 \times 3^2 \times 5^2$ b) $15\sqrt{2}, 3\sqrt{50}, 5\sqrt{18}$ c) 53

8. a) $\mathbf{g} = \overrightarrow{OG}$

$= \overrightarrow{OA} + \overrightarrow{AC}$

$= \mathbf{a} + \frac{1}{5}\overrightarrow{AB}$

$= \mathbf{a} + \frac{1}{5}(\mathbf{b} - \mathbf{a})$

$= \mathbf{a} + \frac{1}{5}\mathbf{b} - \frac{1}{5}\mathbf{a}$

$= \frac{4}{5}\mathbf{a} + \frac{1}{5}\mathbf{b}$ as required.

b) $\overrightarrow{AG} = \mathbf{g} - \mathbf{a}$

$= \left(\frac{3}{4}\mathbf{a} + \frac{1}{4}\mathbf{b}\right) - \mathbf{a}$

$= \frac{1}{4}\mathbf{b} - \frac{1}{4}\mathbf{a}$

$= \frac{1}{4}(\mathbf{b} - \mathbf{a})$

$= \frac{1}{4}\overrightarrow{\mathrm{AB}}$

$\therefore \overrightarrow{GB} = \frac{3}{4}\overrightarrow{AB}$

$AG:AB$

$= \frac{1}{4}:\frac{3}{4}$

$= 1:3$ as required.

9. a) $x \geq -\frac{-11}{5}$ b) $x > \frac{3}{2}$ c) $x < -\frac{56}{15}$ d) $x \geq 14$

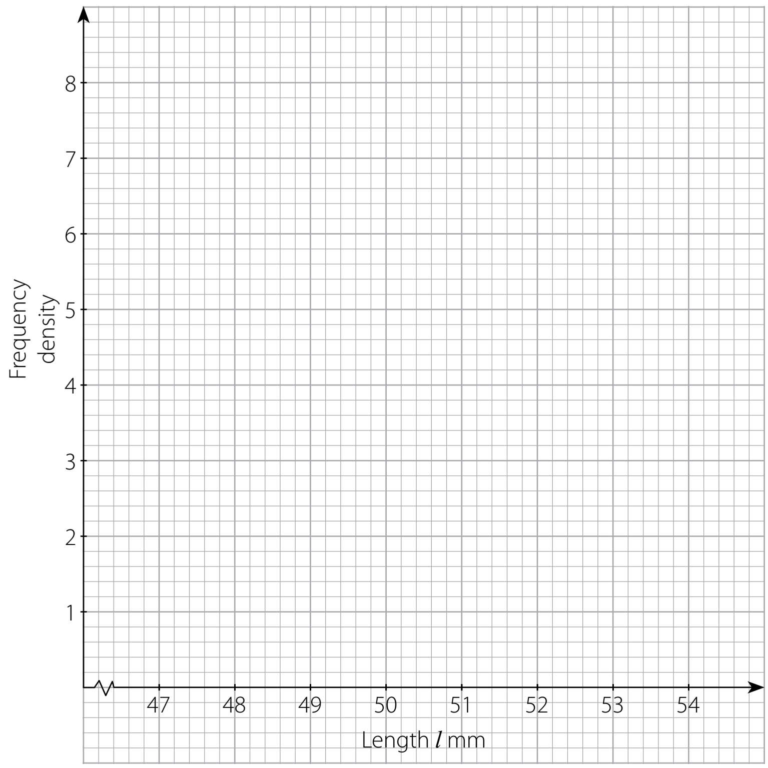
10. a) $\begin{pmatrix} 3 & 18 \\ 0 & -6 \end{pmatrix}$ b) $\begin{pmatrix} 5 & 6 \\ 0 & 2 \end{pmatrix}$ c) $\begin{pmatrix} -1 & -9 \\ 0 & 3.5 \end{pmatrix}$ d) $\begin{pmatrix} 4 & 6 \\ 0 & 1 \end{pmatrix}$

EX 12A, question 1 (worksheet)

a)

Length l mm	Frequency	Frequency density
$47 < l \leq 49$	8	4
$49 < l \leq 50$	6	
$50 < l \leq 51$	8	
$51 < l \leq 54$		
Total	28	

b)



c) Upper bound ____________________

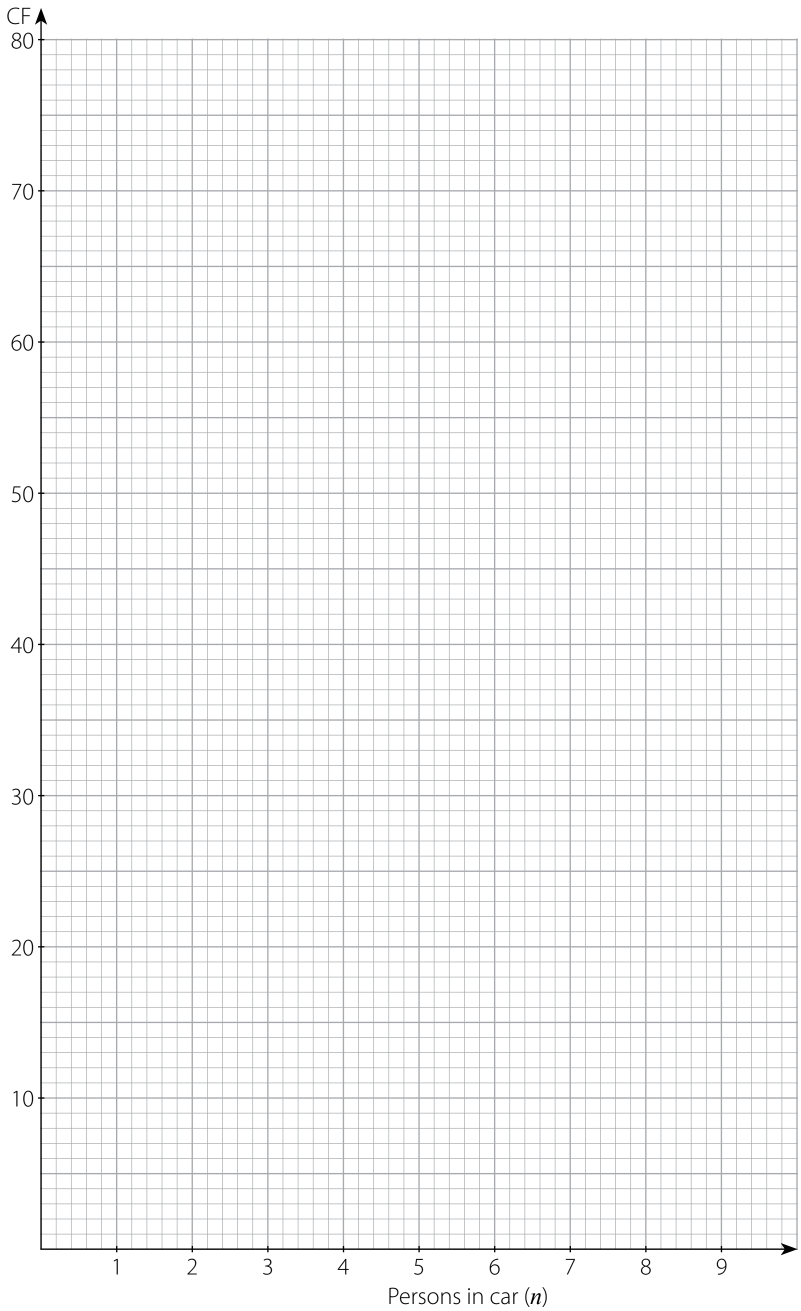
d) Mean of l (estimate) ____________________

EX 12A, question 7 (worksheet)

a)

Persons in car (n)	Frequency	CF
$0.5 \le n < 2.5$	24	
$2.5 \le n < 4.5$	38	
	10	
	8	80

b)

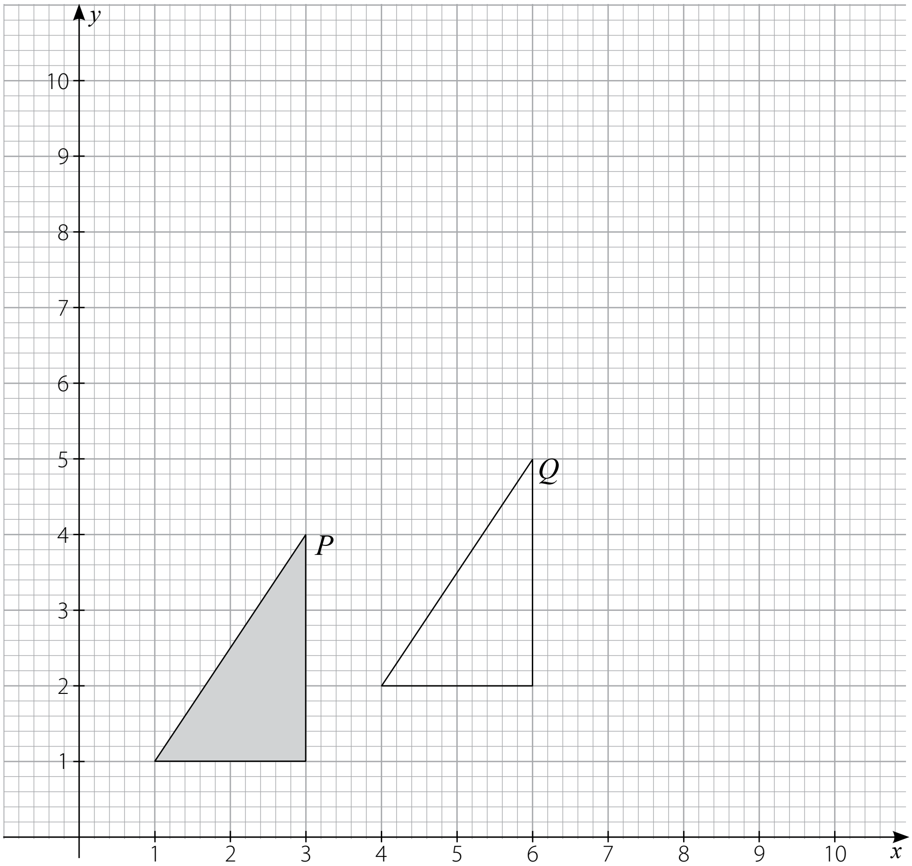


c) Median estimate: ____________ d) Mean estimate: ____________

EX 12B, question 8 (worksheet)

a)

b)



c) $P \rightarrow Q$ is the transformation

d) The invariant points and lines of the following transformations are

$P \rightarrow Q$

$P \rightarrow R$

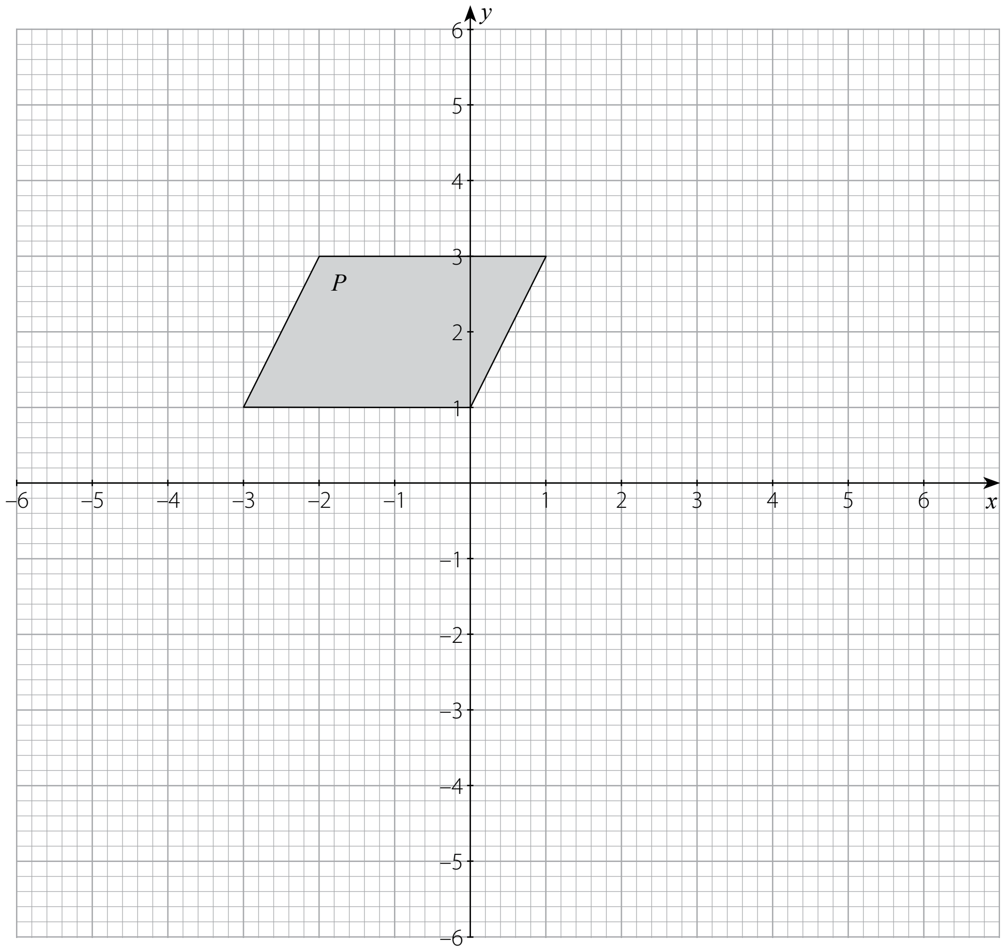
$P \rightarrow S$

EX 12C, question 7 (worksheet)

a)

b)

c)



d) The transformation $S \rightarrow Q$ is ______________________

____

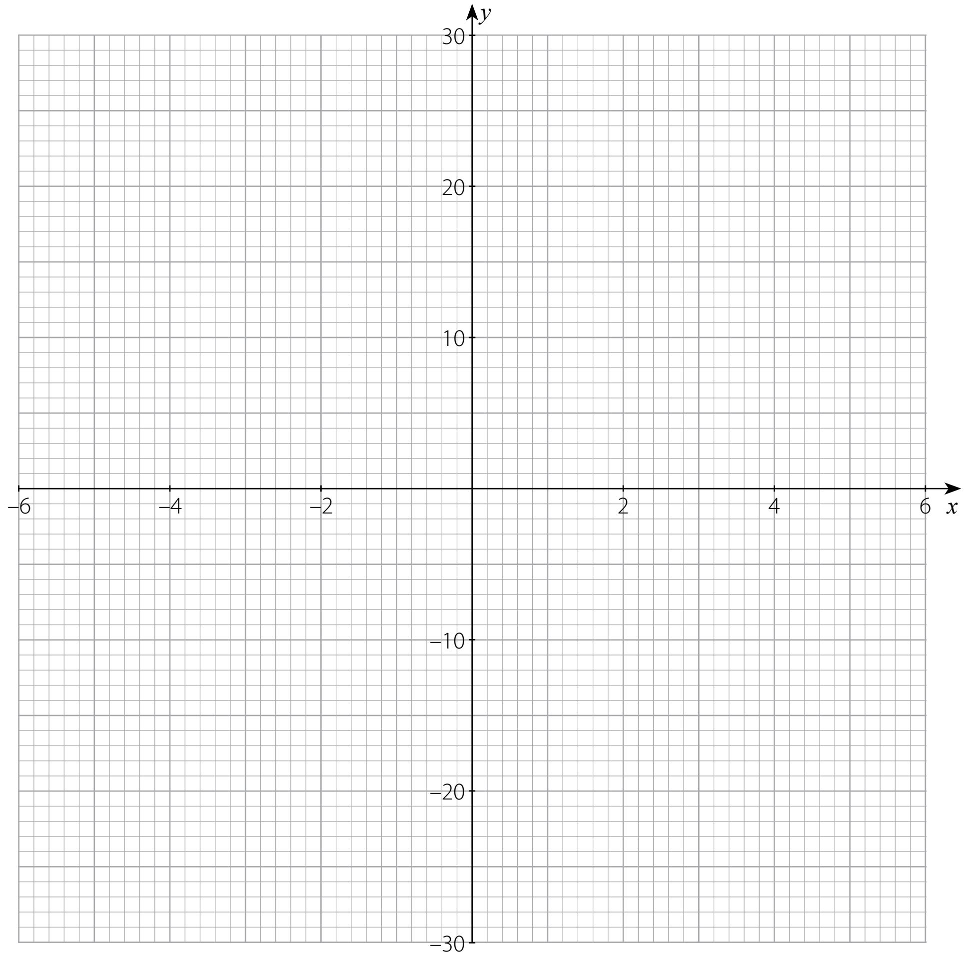
____

EX 12D, question 1 (worksheet)

a)

x	-6	-4	-3	-2	2	3	4	6
$\frac{48}{x}$	-8			-24		16		
y	-14					19		

b)



c) Intersection values

$x =$ ____________ and $x =$ ____________ (1 d.p.)

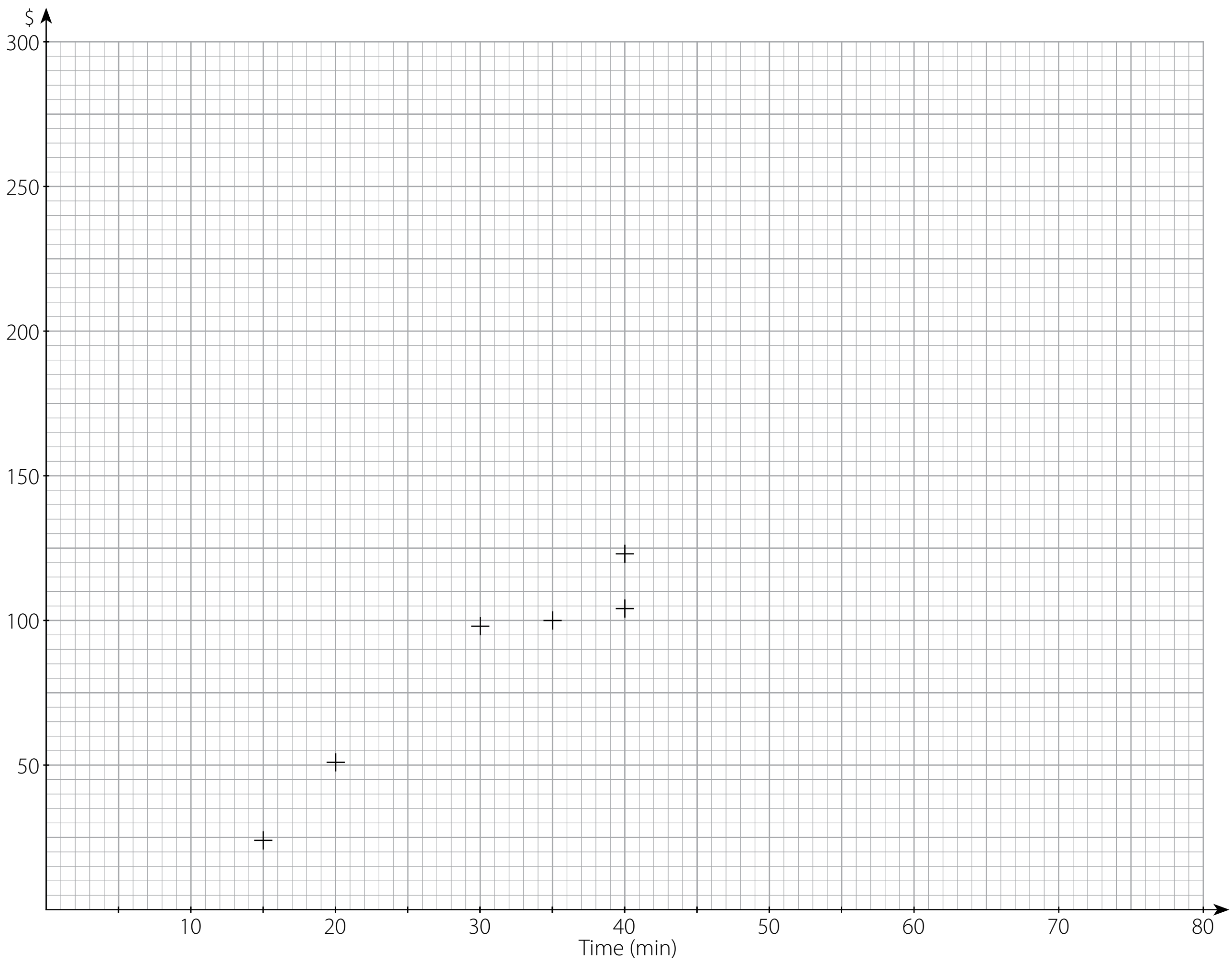
d)

EX 12E, question 2 (worksheet)

Time spent (min)	15	20	30	35	40	40	45	55	55	60	65	70
Money spent ($)	24	51	98	100	104	123	125	155	172	206	200	252

a)

c)



b) The correlation shown is ______________________

d) Estimated customer spending is $ __________