Teaching Guide

International Secondary Maths 10

COLIN WRIGLEY

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Introduction

How to use this Guide

I. Selection of work and pacing

Book 10 is designed for students of Class X (or equivalent), i.e. they would normally be 14+ years old at the start of the academic year, and intending to appear for English medium examinations in O Level or International GCSE in due time. The teacher may find that some exercises can be completed very quickly, being largely revision, but knowledge of the students is a professional attribute and individual judgement is called for. The chapter-by-chapter advice given here on pacing should be regarded only as a rough guide.

II. Integrated mathematics

This textbook series deliberately exploits links between the different branches of mathematics, with other academic subjects and with everyday life. Teachers are advised to make such links explicit in lessons: this guide lists such links for each chapter.

III. Lesson planning

There are many ways of planning lessons, and schools have their own requirements for formally recording these plans. This guide aims to assist teachers to construct their own lesson plans, not just to satisfy their managers but also to focus their thinking about how best to design lessons as learning opportunities for their students. The headings used in this guide are as follows:

Objectives

- General objectives: an overview
- Specific objectives: What we hope the students will learn/understand/apply/practice, as a result of the lessons. These are written from the students' point of view.

Pacing and Links come under this heading. (See I and II.)

Method

Here we suggest the strategies, procedures, techniques, etc. to be used by the teacher to facilitate the objectives, i.e. these are written from the teacher's point of view.

Resources

Lists of equipment/materials needed by the teacher or students. The textbook, calculators and geometrical instruments are not normally listed unless vital for the work of the chapter.

Assignments

Advice on how to use the exercises is given, especially the suitability for homework or classwork.

Vocabulary

A list of key words, the meanings of which are essential to understanding the topic.

IV. Bloom's Taxonomy

In common with all previous books in the series, the exercises contain questions at all levels of Bloom's taxonomy. Students should be challenged to tackle problem-solving questions at the higher levels as often as possible. A more detailed discussion of this may be found in the Teaching Guides of Books 6, 7, and 8.

V. The Exercises

The exercises follow a pattern:

Exercises A, B, C, etc. follow each section in a chapter.

Exercises M are miscellaneous, at the end of chapters with multiple sections.

Exercises X are challenging questions beyond the main course, for students of above average ability, intended to kindle interest and curiosity.

Revision Exercises appear at regular intervals. Questions are not graded. It is believed that a mixture of thought-provoking questions and routine practice questions provides a more interesting revision diet.

VI. Useful Sheets

This guide contains photocopiable materials. (See contents.)

VII. Specimen Examination Papers

Examination questions are suggested to cover the course at the half-way point and end of the academic year, with marking schemes. As some students may have access to this guide, it is advisable to use these papers as a model rather than copying them unaltered.

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Contents

Chapter

1 Sets and Venn Diagrams

This topic used to be far more prominent in school textbooks but has been downgraded in recent years despite its power and relevance. At university level the concept of the set is fundamental, even more basic than definitions of number. The whole mathematical edifice is built upon it. Nevertheless, examinations at school level limit it to only a minor place.

LESSON PLANNING

Objectives

General	To use Venn diagrams to solve simple logic problems
Specific	1. To define sets by listing and describing using set-builder notations 2. To represent a locus as a set of points 3. To know the membership and non-membership symbols and use them correctly 4. To use the symbols for empty set, union, and intersection correctly 5. To draw Venn diagrams from given information 6. To list the complement of a set given the universal set 7. To understand the meaning of a subset and use subset symbols correctly 8. To count the number of members of a region on a Venn diagram, using **n** notation correctly 9. To solve logic problems using Venn diagrams
Pacing	Probably 4 lessons, 1 homework, as there is a lot of new material to digest
Links	Locus, number line graphs, coordinates, HCF (intersection), LCM (union), forming linear equations and solving
Method	• Connect with locus as a set of points, previously introduced. Explain that the set is basic to mathematics, even more important than numbers, and that sets can be of anything. "How do we know what is contained in the set?" Explain the importance of defining clearly. For example, {chairs} is not well-defined: we do not know which chairs are in or out. • Go through the definition options given in the text, giving more examples and eliciting examples from students. Explain the correct mathematical notation as you go, especially $\in$ and $\notin$. • A lot of fun can be had with the empty set, thinking up ludicrous impossibilities, e.g. $\varnothing$ = {green cats with 100 legs},

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but also mathematical ones,

e.g. $\varnothing$ = {multiples of 10 less than 10}

$\varnothing$ = {the two largest numbers}

- Follow the text for union and intersection, again giving the notation, and explain universal set and complement, with more examples in addition to those in the text.
- Use EX 1A. Allow sufficient time for this to be completed before moving on.
- The subset is a fairly easy concept to grasp from a Venn diagram. Follow the text, but give additional examples.

  Move onto overlapping of three sets and correctly naming the regions. Do this before introducing n, the number of a set.
- Use EX 1B, questions 1–7.
- Take time on the problems. As in the text example, it is usually best to start at the centre (triple overlap) and work outwards.

  Make it clear that Venn diagrams with numbers of members in the regions should not have spots. When there are spots, each spot represents a specific member (which is labelled); when there are no spots, the numbers indicate the number of members in each region.
- Use EX 1B, questions 8–10.

Assignments	Possible homework: EX 1A, questions 9 and 10 or EX 1B, questions 6 and 7. EX 1B, questions 8–10 should be done in class. EX 1X is provided for those students who get ahead of the class.
Vocabulary	set, list, describe, such that locus member (element), empty set, union, intersection Venn diagrams, universal set, complement of a set subset, number of a set

ANSWERS

Exercises

EX 1A

1. a) $A = \{x, y, z\}$ b) B = [three names]

   c) C = {Tuesday, Thursday}

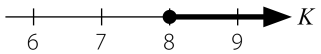
   d) $D = \{6, 9, 12\}$

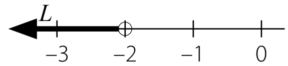
2. a) $E = \{1, 3, 5\}$

   b) $F = \{5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95\}$

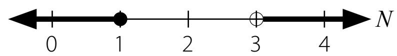
   c) $G = \{-6, -5, -4, -3, -2, -1\}$

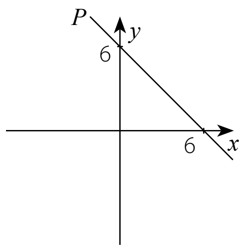
   d) $H = \{-1, 0, 1, 2, 3, 4\}$

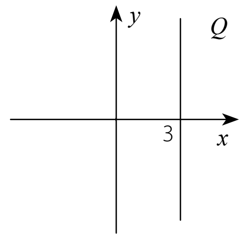
3. a) 

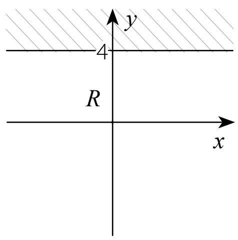
b) 

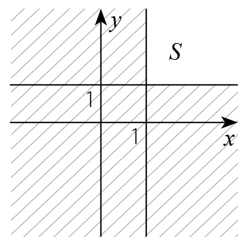
c) 

d) 

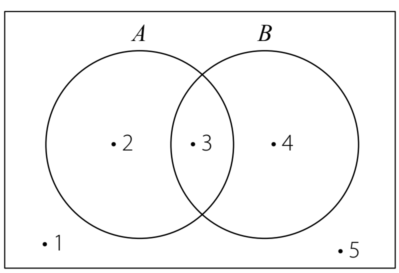
4. a) 

b) 

c) 

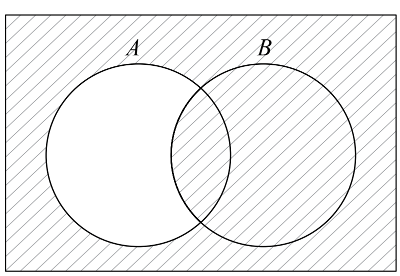
d) 

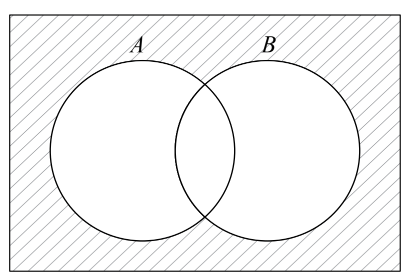
5. a) true	b) false	c) true	d) false
6. a) {1, 2, 3, 4, 5}	b) {3, 4, 5, 6, 7}	c) {3}	d) ∅
7. a) {2, 3, 4}	b) {1, 2, 5}	c) {1, 4, 5}	d) {3}

8. 

9. a) $\{a, b, g\}$ b) $\{b\}$

c) $\{a, b, c, d, e, g\}$ d) $\{a, b, f, g\}$

10. a) 

b) 

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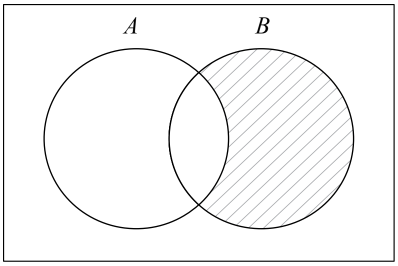

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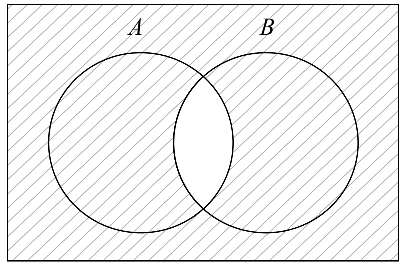
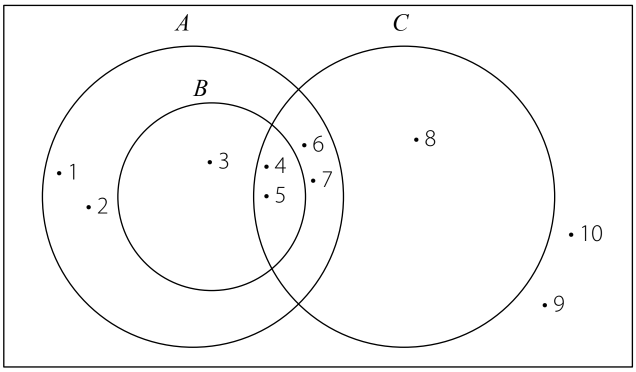


EX 1B

1. a) true b) false c) true d) true

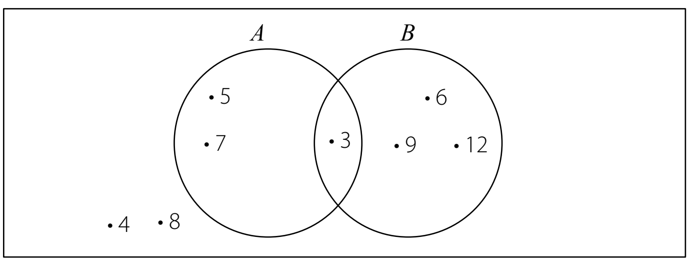
2.



3. a) 4 b) 2 c) 1 d) 0

4. a) $A = \{3, 5, 7\}$ $B = \{3, 6, 9, 12\}$

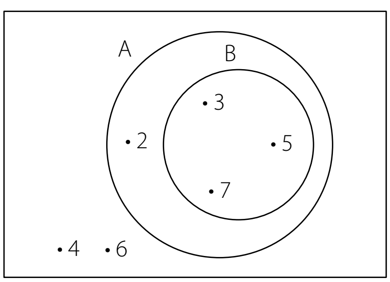
b)



c) {3, 4, 5, 7, 8} d) 3

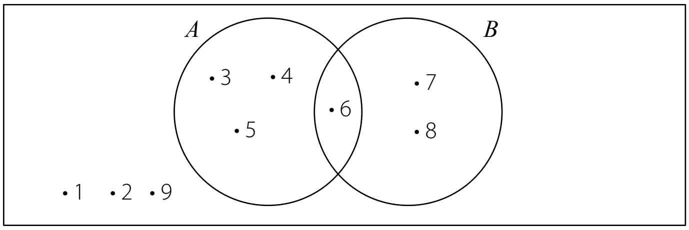
5. a) $A = \{2, 3, 5, 7\}$ $B = \{3, 5, 7\}$

b) $\mathscr{E}$



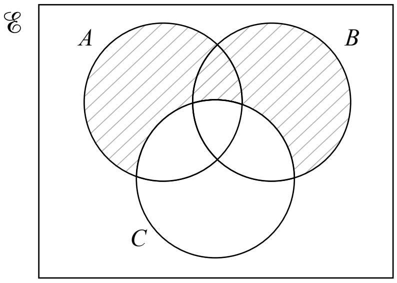
c) {2} d) 5

6. a)

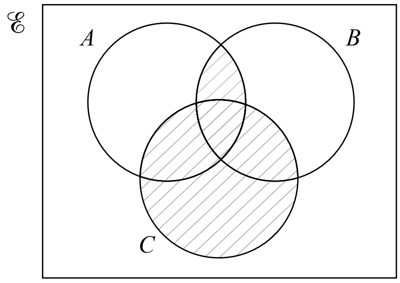


b) {1, 2, 9}   c) 8   d) {7, 8}

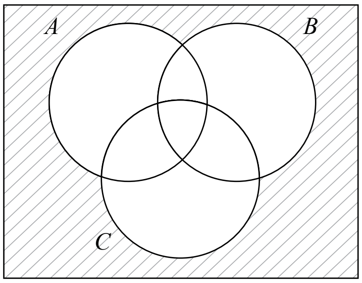
7. a)



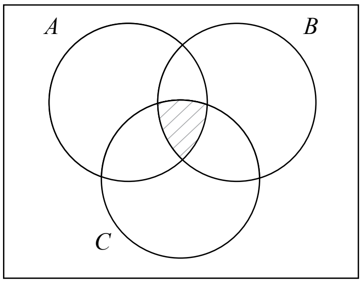
b)



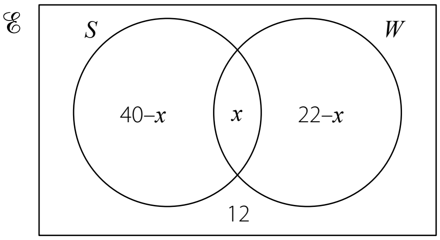
c)



d)



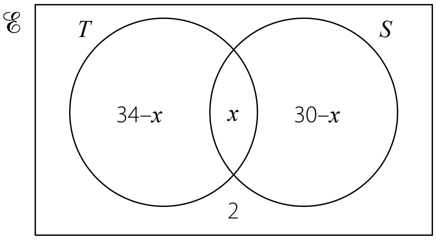
8.



$x = 2$

2 people used both.

9.



$x = 18$

18 students between 150–160 cm

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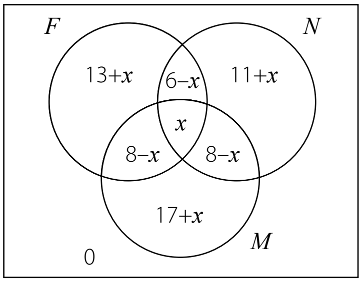
10. 

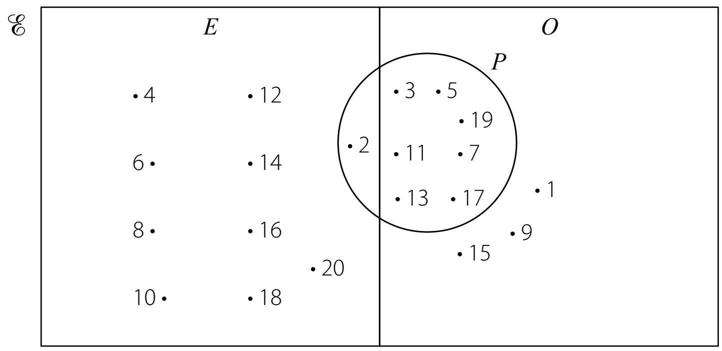
$x = 1$

Only 1 student chose all three.

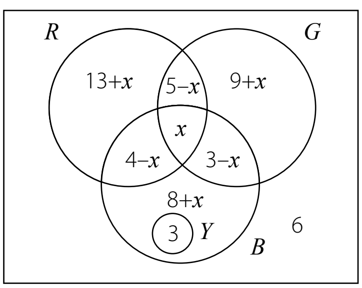
EX 1X

1. a) $\{-2, 2\}$ b) $\{(0, 2), (1, 2.5), (2, 3), (3, 3.5), (4, 4)\}$

c) {H, I, N, O, S, X, Z} d) $\{(2, 4), (3, 9)\}$

2. a) 

b) 1 c) $\{1, 9, 15\}$ d) $\varnothing$

3. 

$x = 1$

$\therefore$1 person liked red, green, and blue.

Chapter

2 Probability

Most of the techniques and problems used in this chapter have been met before. It is, however, an opportunity to use set notation and Venn diagrams as visual assistance to understanding the probability calculations for two or more events.

The reason for multiplying probabilities of independent events has not really been given before. The possibility space does explain it. The two dimensions are independent and the areas (found by multiplying lengths by widths) represent the various probabilities.

LESSON PLANNING

Objectives

General	To understand how to recognize and deal with independent and mutually exclusive events in probability calculations
Specific	1. To define mutually exclusive events using set language and Venn diagrams 2. To know that the complement of a set can represent the non-occurence of an event 3. To know the definition of independent events and how they can be represented on a possibility space 4 To calculate correctly probabilties of independent and mutually exclusive events 5 To use tree diagrams to calculate probabilities of sequences of events
Pacing	1 lesson, 1 homework, as this is mostly revision with extra rigour demanded
Method	Very simple examples may be shown for finding the probability of independent events. For example, P(A) = 0.2, P(B) = 0.3 P(A and B) = P(A) × P(B) = 0.2 × 0.3 = 0.06 P(B and not A) = P(B) × P(not A) = 0.3 × 0.8 = 0.24 P(A and not B) = P(A) × P(not B) = 0.2 × 0.7 = 0.14 P(not A and not B) = P(not A) × P(not B) = 0.8 × 0.7 = 0.56 Total probability (all 4 mutually exclusive combinations) = 0.06 + 0.24 + 0.14 + 0.56 = 1

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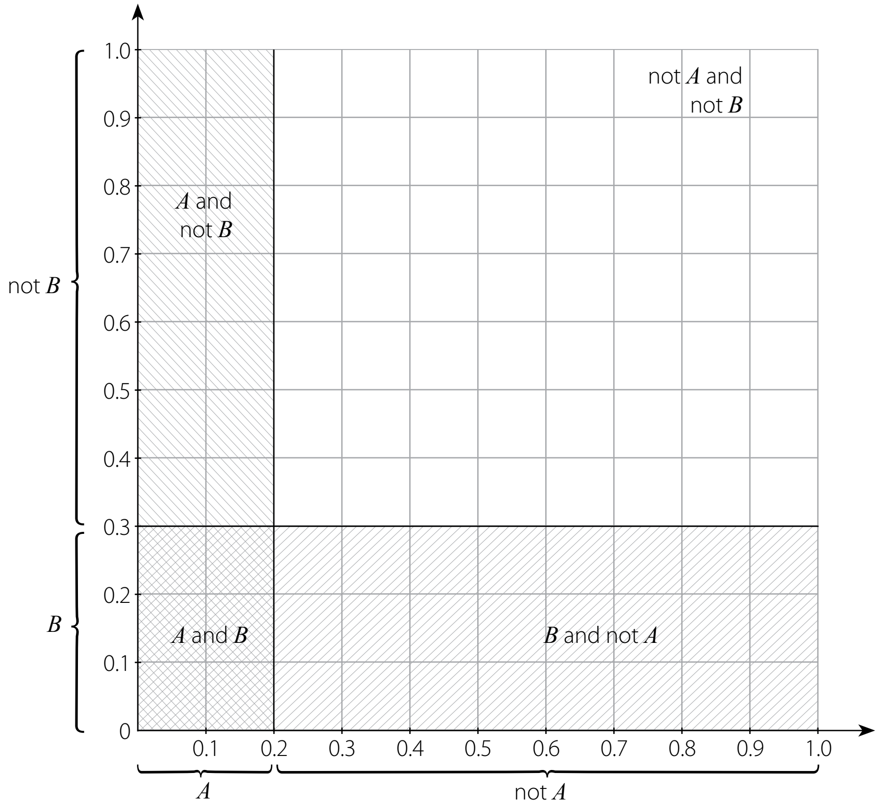
This example can be illustrated by the possibility space given below:



Examples given of independent events may cause some reasoned dissent. If a dice produces a 6, surely it is less likely to give 6 on the next throw? Such matters have been debated by philosophers before. Pascal, amongst others, argued that the tendency of a fair dice to give $\frac{1}{6}$ of each face number in the long run does not remove the independence of each individual throw.

It is harder to argue that a single throw of a set of dice together can be regarded as a set of independent events. Surely, the dice interfere with each other when rolled? This is, of course, true, but the numerous collisions and cause-effect interferences can be successfully modelled as independent random events. A good class may be interested in the concept of mathematical modelling. Examples of successful use, such as car accident insurance, can be given.

Point out the need for precision in mathematics, and using the correct mathematical terminology. Then define "mutually exclusive" and follow the text, with diagrams. Similarly, for independence. How you handle this depends a lot on the students: a group of high-flyers could get into the philosophical issues mentioned at the start of this chapter above; others will be content with knowing how to calculate the probabilities correctly. However, the possibility space example above should be demonstrated to all as previously no justification has been given for multiplying probabilities.

Use EX 2A. This should give little trouble if Chapter 1 has been completed thoroughly.

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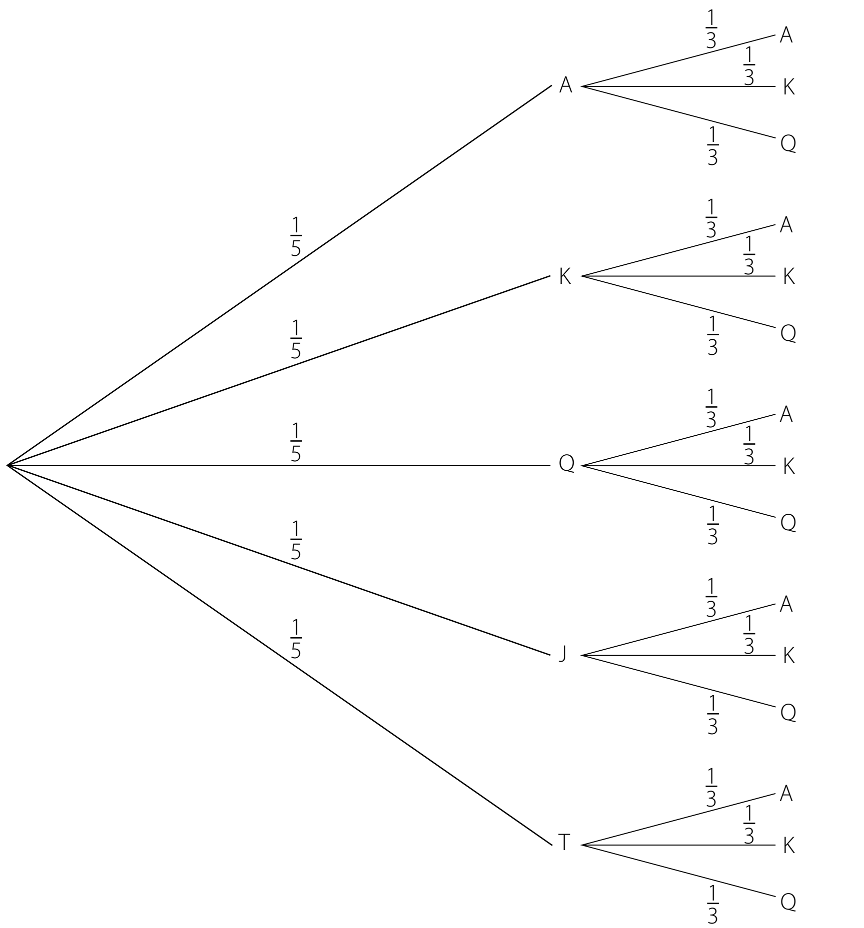
Assignments	Reserve EX 2A, questions 7 and 10 for homework
Vocabulary	mutually exclusive, independent possibility space, tree diagram

ANSWERS

Exercises

EX 2A

1. a) $\frac{2}{7}$ b) $\frac{1}{7}$ c) $\frac{3}{7}$ d) 0
2. a) $\frac{1}{25}$ b) $\frac{1}{125}$ c) $\frac{4}{125}$ d) $\frac{16}{125}$
3. a) 40

   b) i) $\frac{4}{5}$ ii) $\frac{7}{40}$ iii) $\frac{1}{20}$ iv) $\frac{19}{40}$
4. a) $\frac{1}{8}$ b) $\frac{1}{168}$ c) $\frac{1}{84}$ d) $\frac{49}{64}$
5. 

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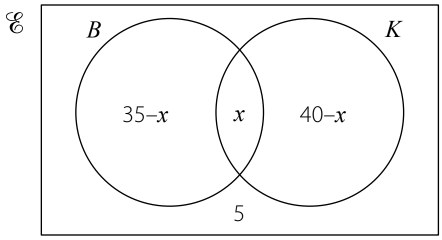
a) $\frac{1}{5}$ b) $\frac{2}{15}$ c) $\frac{4}{5}$ d) $\frac{13}{15}$

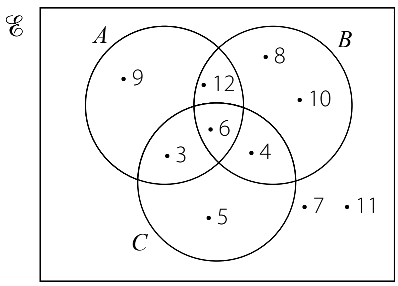
6. a) 0.0001 b) 0.19

c) 0.531 441 d) 0.468 559

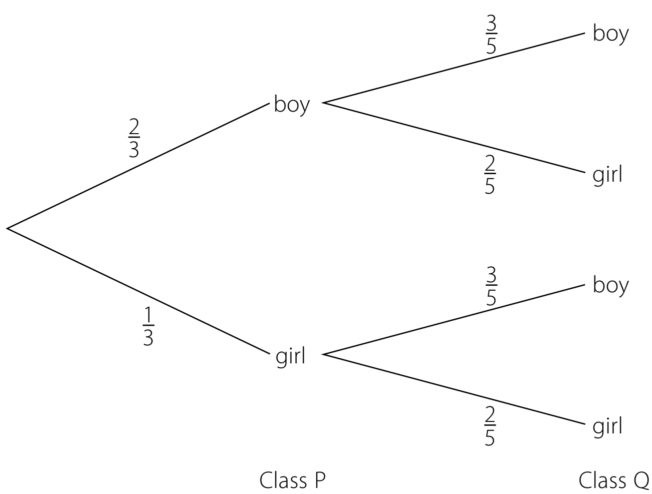
7. a) 

$x = 30$. 30 young people visited both.

b) 0.1

8. a) 

b) 0.2 c) 0.3 d) 0.2

9. 

a) $\frac{2}{5}$ b) $\frac{3}{5}$ c) $\frac{7}{15}$ d) $\frac{4}{15}$

10. a) $\frac{1}{243}$ b) $\frac{211}{243}$

EX 2X

1. 17 [P(at least one red) = $1 - 0.96^n$. Find n by trial and improvement.]

2. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

3. All true. [Infinity is very strange!]

Chapter

3 Indices

This chapter revises the laws of indices, previously used with both positive and negative integers, and extends their meaning to encompass rationals. It is an example of what mathematicians do all the time, i.e. take simple cases and attempt to generalise.

LESSON PLANNING

Objectives

General	To manipulate algebraic expressions and solve equations involving the laws of indices with integer and/or rational indices
Specific	1. To be confident in using the laws of indices with integer indices 2. To understand the meaning of roots and that the laws can apply with rational indices 3. To simplify expressions and solve equations involving the same base 4. To simplify expressions involving different bases
Pacing	Quite fast if the previous work on integer indices has been done well [See Book 9, Chapter 17] 1 lesson, 1 homework should be sufficient.

Method

- Start with natural indices, e.g.

  $5^3 = 5 \times 5 \times 5$

  "How many factors of 5?" Ans 3

  3 is the index number.

  "How about 5^{-2}?"

  Can't have −2 factors!

  Remind students how we gave it meaning:

  $5^3 = 5 \times 5 \times 5$

  ÷5: $5^2 = 5 \times 5$

  ÷5: $5^1 = 5$

  ÷5: $5^0 = 1$

  ÷5: $5^{-1} = \frac{1}{5}$

  ÷5: $5^{-2} = \frac{1}{25}$ or $\frac{1}{5 \times 5}$ or $\frac{1}{5^2}$

  etc.

  The meaning has been developed/extended.

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Now we develop further, to rationals (fractions and decimals).

Follow the text, revising the laws, then showing how index $\frac{1}{2}$ implies square root, etc.

Give plenty of examples of using the laws in combination, in addition to the text examples.

Use EX 3A, questions 1–6.

- Show how equations can be solved when both sides can be expressed on the same base.

Complete EX 3A, questions 7–10.

Assignments	Reserve EX 3A, questions 6 and 9 for homework
Vocabulary	Index, indices, integers, rationals square root, cube root, fourth root base

ANSWERS

Exercises

EX 3A

1.	a) $5^{1/2}$	b) $6^{1/3}$	c) $7^{-1/2}$	d) $24^{1/4}$
2.	a) 7	b) $\frac{1}{6}$	c) $\frac{1}{2}$	d) 2
3.	a) 4	b) 4	c) $\frac{1}{25}$	d) 8
4.	a) $\frac{3y^2}{x}$	b) $\frac{x^2y^3}{3}$	c) $\frac{8}{y^3}$	d) $\frac{y}{7x^2}$
5.	a) $8x^9$	b) $\frac{1}{9x^4}$	c) $25x^6$	d) $\frac{1}{49x^{14}}$
6.	a) 24	b) 63	c) 1	d) 121
7.	a) $n = 11$	b) $n = 2$	c) $n = \frac{1}{21}$	d) $n = \frac{1}{6}$
8.	a) $n = 5.5$	b) $n = \frac{7}{3}$	c) $n = \frac{1}{7}$	d) $n = -\frac{9}{2}$
9.	a) $m = 0$	b) $m = 0$	c) $m = 0$	d) $m = 0$
10.	a) $p = 0.5$	b) $p = \frac{5}{3}$	c) $p = \frac{1}{6}$	d) $p = 0$

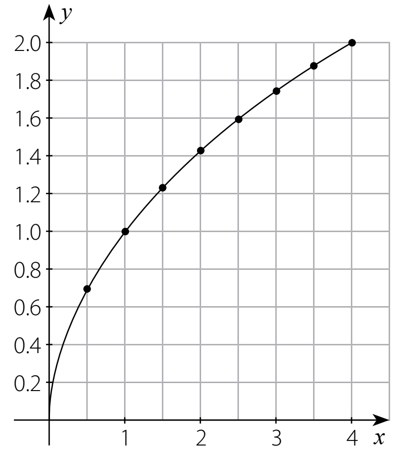
EX 3X

1. $x = 0$ or $x = 1$ [Things are not so simple when the base is unknown.]

2.

x	0	0.5	1.0	1.5	2	2.5	3	3.5	4
y	0	0.71	1.00	1.22	1.41	1.58	1.73	1.87	2.00

(2 d.p.)



$y = x^{0.5}$

$y^2 = (x^{0.5})^2 = x$

Parabola [reflection of $y = x^2$ in the line $y = x$]

3. Yes; yes; 9.735 171 039; 9.738 517 742; 9.750 858 08;
$5^{\sqrt{2}}$ lies between $5^{1.414}$ and $5^{1.415}$ since $1.414 < \sqrt{2} < 1.415$. Since $\sqrt{2}$ cannot be expressed as an exact decimal, neither can $5^{\sqrt{2}}$. The calculator values are approximations. Similarly with 5^{π}. As $5^{3.141} < 5^{\pi} < 5^{3.142}$, it cannot be written as an exact decimal.

Chapter

4 Quadratic Equations: Factorisation Method

Students need a lot of practice to become fluent in factorising, introduced in Chapter 28 of Book 9. Here we use factorisation to solve those quadratic equations amenable to this method.

LESSON PLANNING

Objectives

General	To solve factorisable quadratic equations
Specific	1. To recognize that if the product of two factors is zero, then at least one of the factors is zero; to understand that this fact is the basis of the method of solution 2. To use the method of factorisation to solve quadratic equations 3. To expect two solutions of a quadratic equation in the solution set (one may be repeated) 4. To construct a quadratic equation from given information in order to solve a problem
Pacing	1 or 2 lessons, 1 homework
Links	Factorisation of trinomials

Method

Start by firing off a few easy mental questions orally, like multiplication tables, e.g. 6×9, 7×8, etc.

Then say, "The product is 12, what are the two factors?"

Many answers! More answers if rationals are allowed. Move to, "The answer is zero, what are the two factors?"

Best answer is that one factor must be zero, the other can be anything.

Now write $(x-2)(x-3)=0$

"Is the LHS a product of two factors?" Yes.

So either $x-2=0$ or $x-3=0$

$x=2$ $\quad$ $x=3$

Now expand the brackets and write it above the product, i.e. it looks like this:

$$x^2-5x+6=0$$

$$(x-2)(x-3)=0$$

Either $x-2=0$ or $x-3=0$

$x=2$ $\quad$ $x=3$

"You have just solved a quadratic equation."

Two solutions? Check in the top line.

$x = 2$; $\quad 2^2 - 5(2) + 6 = 0$ $\qquad$ True

$x = 3$; $\quad 3^2 - 5(3) + 6 = 0$ $\qquad$ True

Now go through some of the text examples.

Use EX 4A.

[Note: For the problems in questions 8 and 9, the negative solutions have to be rejected because lengths in a diagram must be positive. This is a general feature of mathematical models: the results have to be interpreted realistically.]

Assignments	Reserve all the parts (d) in questions 1–7 for homework.
Vocabulary	quadratic equation, factorise, factorisable

ANSWERS

Exercises

EX 4A

1. a) $x = -4, x = -1$ b) $x = -4, x = -5$
   c) $x = 2, x = 3$ d) $x = -1, x = -1.5$
2. a) $x = 1, x = \frac{1}{2}$ b) $x = 5, x = \frac{1}{3}$
   c) $x = 3, x = -\frac{4}{3}$ d) $x = \frac{1}{4}, x = -\frac{5}{2}$
3. a) $x = -7, x = 5$ b) $x = 9, x = -2$
   c) $x = 8, x = -2$ d) $x = -4, x = -\frac{5}{2}$
4. a) $x = -5, x = -2$ b) $x = 4, x = -1$
   c) $x = -6, x = 5$ d) $x = \frac{1}{2}, x = -1$
5. a) $x = 3, x = -3$ b) $x = 7, x = -7$
   c) $x = \frac{3}{2}, x = -\frac{3}{2}$ d) $x = \frac{9}{5}, x = -\frac{9}{5}$
6. a) $x = 6$ b) $x = \frac{1}{2}$ c) $x = \frac{5}{2}$ d) $x = 10$
7. a) $x = -7, x = -1$ b) $x = 7, x = -2$
   c) $x = \frac{7}{3}, x = -\frac{7}{3}$ d) $x = \frac{5}{3}$
8. $x(x + 1) = 25 + x$, $\quad x = 5$ or $x = -5$(reject); $\quad$ length 6 m, width 5 m, area 30 m^2
9. $25 = \frac{1}{2}(x)(2x)$, $\quad x = 5$ or $x = -5$(reject); $\quad$ base 5 cm
10. $8800 = x(300 - 2x)$, $\quad x = 40$ or $x = 110$;
    220 m by 40 m $\quad$ or $\quad$ 110 m by 80 m

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EX 4X

1. a) 5th term = 12, 4th term = 6, 5th = 2 × 4th term

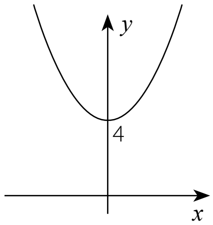
   b) $n^2 - 3n + 2 = 0$ for $n = 1$ or $n = 2$

   c) 3rd term = 5, 6th term = 20

   Hence 1st + 2nd + 3rd + 4th + 5th = 20 = 6th term

2. $x(x + 2) = 3(2x - 1)$; $x = 1,\ y = 3$ or $x = 3,\ y = 15$

3. a)



   b) There is no point on the curve where $y = 0$.

   c) $x^2 + 4 = 0$ doest not factorise

   $x^2 = -4$

   $x = \pm\sqrt{-4}$ not a real number

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Chapter

5 Straight Lines

The gradient and intercept forms of the straight line equation should be well established. [See Book 9, Chapter 24.] In this chapter, the focus is on perpendicular lines, gradients, and mid-points.

LESSON PLANNING

Objectives

General	To use the property of the gradients of perpendicular lines and of the coordinates of the midpoint of a line segment in coordinate geometry
Specific	1. To write down the gradient of a given vector 2. To write down the gradient of a line perpendicular to a line with given gradient 3. To know that the product of the gradients of perpendicular lines is –1 4. To write down the components of a vector half the length of a given vector 5. To write down the coordinates of the midpoint of a line segment with given end points 6. To solve simple problems using the above facts
Pacing	1 lesson, 1 homework
Links	Vectors

Method

- First check whether the students know the two forms of the straight line equations:

  $y = mx + c$ gradient form

  $ax + by = c$ intercept form

  Then check that they remember how to draw a vector from its components. Draw some examples. Find their gradients.

  Note that reversing the direction of a vector does not change its gradient, e.g.

  if $\mathbf{v} = \begin{pmatrix} 6 \\ -1 \end{pmatrix}$ then $-\mathbf{v} = \begin{pmatrix} -6 \\ 1 \end{pmatrix}$

  and gdt $\mathbf{v} = \frac{-1}{6}$ also gdt $(-\mathbf{v}) = \frac{-1}{6}$

  Then follow the text up to the product of the gradients of perpendicular lines, and the example following.

  Use EX 5A, questions 1–6.

- Not everyone will follow the vector argument in the text for finding midpoints. Some students will be happy to "average the endpoints", but they should at least see the justification for the method. Diagrams of specific examples can be quite convincing.

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	Use EX 5A, questions 7–10.
Assignments	Reserve parts (d) in questions 1–9 for homework, or the whole of question 10
Vocabulary	gradient, perpendicular, midpoint, line segment vector, component

ANSWERS

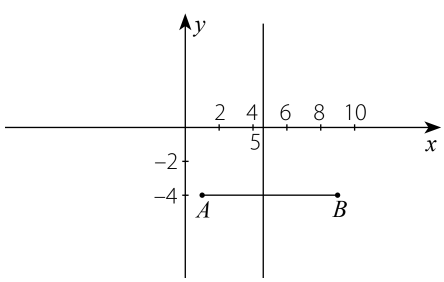
Exercises

EX 5A

1.	a) -5	b) 3	c) $-\frac{2}{3}$	d) 0
2.	a) $\frac{4}{3}$	b) $\frac{5}{2}$	c) $-\frac{1}{6}$	d) 3
3.	a) no	b) yes	c) yes	d) no
4.	a) $-\frac{1}{2}$	b) $-\frac{10}{3}$	c) 1	d) -2
5.	a) $-\frac{3}{5}$	b) $-\frac{5}{3}$	c) $\frac{5}{3}$	d) $-\frac{3}{5}$
6.	a) $x + 3y = 9$	b) $7x + 4y = 20$	c) $3x + 5y = -24$	d) $3y = 4x$
7.	a) $(1.5, 4.5)$	b) $(2, 3.5)$	c) $(5.5, 4)$	d) $(3, -1)$
8.	a) true	b) false	c) true	d) false
9.	a) $x + y = 5$	b) $y = x - 6$		
	c) $x + y = 12$	d) $3y - x = 4$		
10.	a) $M(4, 6)$	b) $N(0, 1.5)$		
	c) $\frac{9}{8}$	d) $8y - 9x = 12$		

EX 5X

1. $y = -1.25x + 1.4$
2. $x = 5$



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3. $\text{Gdt } PQ = \frac{2-1}{8-1} = \frac{1}{7}$

$\text{Gdt } PR = \frac{5-1}{4-1} = \frac{4}{3}$

$\text{Gdt } RQ = \frac{5-2}{4-8} = \frac{3}{-4}$

$\text{Gdt } PR \times \text{Gdt } RQ = \frac{4}{3} \times \frac{3}{-4} = -1$

$\therefore \quad PR \perp RQ$

$\therefore \quad \Delta PQR$ right-angled at R.

$\overrightarrow{PR} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \qquad |\overrightarrow{PR}| = 5$ (Pythagoras)

$\overrightarrow{RQ} = \begin{pmatrix} -4 \\ 3 \end{pmatrix} \qquad |\overrightarrow{RQ}| = 5$ (Pythagoras)

$\text{Area} = \frac{1}{2} \times PR \times RQ$

$= \frac{1}{2} \times 5 \times 5$

$= 12.5 \text{ unit}^2$

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Chapter

6 Revision Exercises

These exercises cover material from the first five chapters of this book, but most of it is from Book 9. At this early stage in the academic year the exercises provide information to the teacher about how well previous work has been learnt.

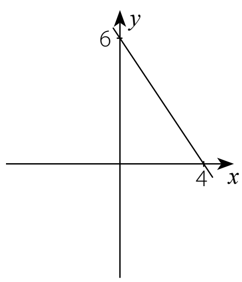
One way to use them is to set selected questions for homework whilst continuing with the new work in class.

ANSWERS

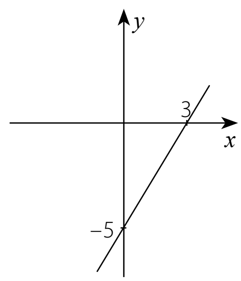
Exercises

EX 6A

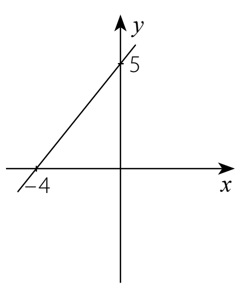
1. a)



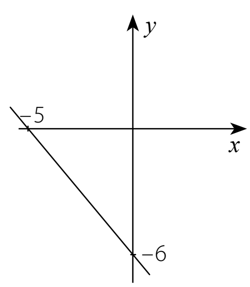
b)



c)



d)



2. a) $x = \frac{1}{7}$ b) $x = -4$

3. a) true   b) false   c) true   d) true

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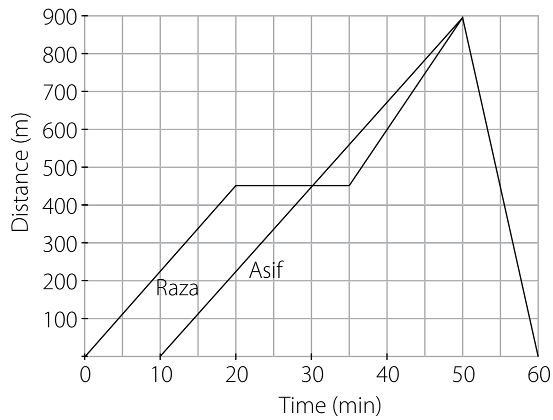
4. a)
b)
c)



d) 30 min after Raza left home

5. a) 60 m b) 20 s c) 15 m d) 3.75 m

6. a) 81 b) $\frac{1}{16}$ c) $\frac{1}{27}$ d) 1

7. a) Q b) R c) P d) S

8. 4

9. a) a^2b^2 b) a^5b c) $\frac{5ab}{6c}$ d) $5ab^2c$

10. a) $P = \{a, b, c, d, e\}$ b) individual answers

c) R = {Monday, Tuesday, Wednesday, Thursday, Friday}

d) $S = \{11, 13, 17, 19\}$

EX 6B

1. a) translation $\begin{pmatrix} -7 \\ 2 \end{pmatrix}$ b) rotation 90° about A

c) reflection in the line $x = 1$

2. a) $x = 10.6$ cm (3 s.f.) b) $x = 23.7$ cm (3 s.f.)

3. a) 1010 cm^2 (3 s.f.) b) 76.1 cm^2 (3 s.f.)

4. a) 3×10^3 b) 3×10^5 c) 3×10^6 d) 3×10^7

5. a) -2 b) $\frac{1}{3}$ c) $-\frac{2}{5}$ d) infinite

6. a) 120 b) 3.35 kg c) 5 d) 0.35 kg

7. a) 5.88 km/h b) 10 km/h

c) 13.1 km/h (3 s.f.) d) 648 km/h (3 s.f.)

8. a) $x = -\frac{26}{63}$ b) $x = -\frac{194}{17}$

9. a) $\frac{4}{3}$ b) 16 cm c) Both 3 : 1 d) $1{:}\frac{16}{9}$

10. a) $x^2 - 16$ b) $x^2 - 6.25$ c) $9x^2 - 1$ d) $16x^2 - 25$

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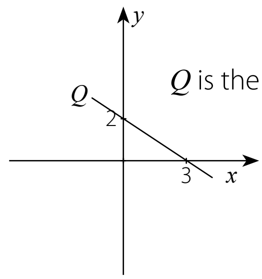
EX 6C

1.



Q is the line.

2. a) $x = -4, x = -2$ b) $x = -6, x = -4$

   c) $x = -3, x = 10$ d) $x = -\frac{1}{2}, x = 5$

3. a) $x^2 - 4x + 4$ b) $1 - 6x + 9x^2$

   c) $x^2 + x - 12$ d) $5x^2 + 4x - 1$

4. a) (corr $\angle$s) b) (alt $\angle$s)

   c) (ext $\angle$ of Δ) d) (adj $\angle$s on st line)

5. 286 cm^2 (3 s.f.)

6. a) 0.4 b) yes c) 0.3 d) 0.2

7. a)

	1	2	3	4	5	6
A	A1	A2	A3	A4	A5	A6
B	B1	B2	B3	B4	B5	B6
C	C1	C2	C3	C4	C5	C6
D	D1	D2	D3	D4	D5	D6
E	E1	E2	E3	E4	E5	E6
F	F1	F2	F3	F4	F5	F6
G	G1	G2	G3	G4	G5	G6

   b) i) $\frac{1}{42}$ ii) $\frac{1}{7}$ iii) $\frac{5}{6}$ iv) 0

8. a) no b) yes, $\angle Q$ c) no d) yes, $\angle Q$

9. a) $6^{\frac{1}{2}}$ b) $7^{\frac{1}{3}}$ c) $2^{-\frac{1}{2}}$ d) $25^{\frac{1}{4}}$ (or $5^{\frac{1}{2}}$)

10. a) $(x + 7)(x + 1)$ b) $(x + 8)(x + 2)$

    c) $(x - 4)(x - 3)$ d) $(x + 5)(x + 6)$

EX 6D

1. a) 1.5 b) 39 cm c) 27.2 cm d) 27.2 cm

3. a) false b) false c) true d) false

4. a) P(red) = 0.6, P(green) = P(blue) = 0.2

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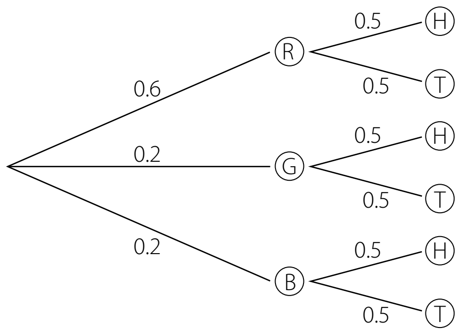
b)



c) 0.3

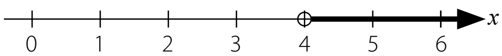
d) 0.1

5. a) 4.68×10^6 (3 s.f.) b) 5.3×10^5 (2 s.f.)

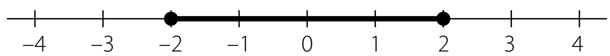
c) 5×10^{-3} (1 s.f.) d) 7.1×10^{-4} (2 s.f.)

6. a) $\frac{1}{8}$ b) $\frac{1}{4}$ c) $\frac{3}{8}$ d) 0

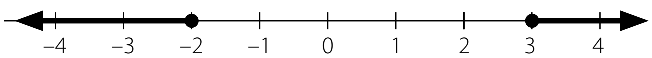
7. a)



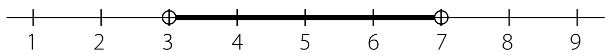
b)



c)

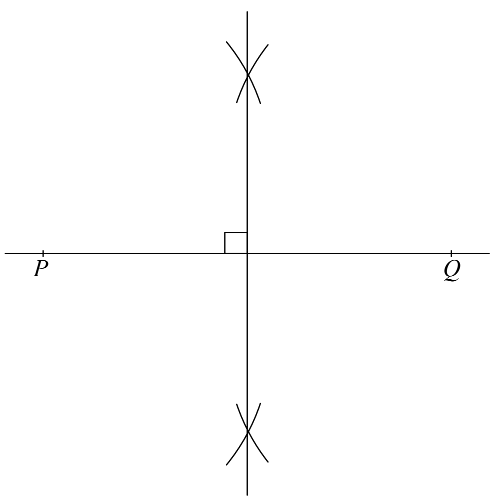


d)



8. 47.7 km (3 s.f.)

9.



The locus is the perpendicular bisector.

10. a) $a = 180 - (30 + 58)$ (angle-sum of Δ)

$= 92$

But $x = a$ (corr $\angle$s)

$x = 92$

b) $b = 30 + 58$ (ext $\angle$ of Δ)

$= 88$

But $x + b = 180$ (int $\angle$s)

$x + 88 = 180$

$x = 92$

Chapter

7 Graphs

This chapter revises standard curves and moves on to more unusual graphs with more complicated equations.

LESSON PLANNING

Objectives

General	To know the shapes of the graphs of standard functions and to use them to solve problems
Specific	1. To recognise the shape of the graphs of linear, quadratic, cubic, reciprocal, inverse square, and exponential functions from the structure of their equations 2. To make a table of values for a given equation in order to draw an accurate graph 3. To draw the graphs of curves whose equations combine elements of two standard types 4. To estimate the gradient of a curve at any given point by drawing a tangent 5. To know that gradient measures rate of change 6. To solve problems using a graph by drawing a suitable straight line
Pacing	5 lessons, 2 homeworks. The exercises should not be rushed.
Links	Gradient of a line; algebraic substitution; inequalities; trigonometrical tangents
Method	• Follow the text through the systematic review. The cubic has only been mentioned briefly since Book 8, so will need some attention. • The inverse square function can be introduced as a variety of reciprocial graph, with more powerful effect (because of the squaring). The case of negative c can be disregarded. • Exponential functions are new. For the first time the variable x is an index. Emphasize how rapidly this makes y increase. (The case of base < 1 may be disregarded.) • Revise making a table of values. For reciprocal and inverse square graphs, division by zero is not possible (splitting the curve into two parts). When building up the function, use brackets to signify the lines to combine for y. This helps to avoid errors. Use EX 7A.

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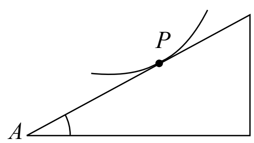
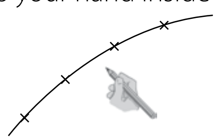
- Harder functions may be graphed. Use the text example showing how some features of both parts of the equation show up on the graph. The only extension in graphing is that tables of values become a little more complicated: the process is exactly the same.
- For problem solving, start with the equation to be solved and manipulate it to obtain the given graph equation's right hand side (equivalent to y). This determines the line to be drawn.
- For gradients, draw any curve on the board and move a ruler along it showing the gradient at each point. It is fairly intuitive that the gradient of the tangent line (ruler) is the gradient of the curve.
- The word tangent for a touching line is new [Latin tangere = to touch]. However, the gradient calculation $\frac{\text{opposite}}{\text{adjacent}}$ is the trigonometrical tangent of the angle with the horizontal, so there is a connection between these two uses of tangent.



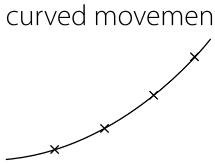
tan A = gradient of curve at P

- Warn students to read horizontal and vertical distances according to the scales on the axes: they may be different.
- **Hints** for drawing curves:

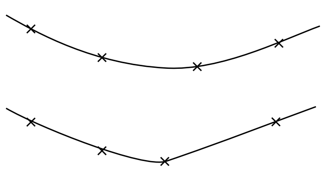
  Keep your hand inside the curve, e.g.



- However, for this, turn the paper upside down so your hand follows a natural curved movement.



- Aim to draw through 2 or 3 points at a time. This helps to avoid "corners" appearing on curves:



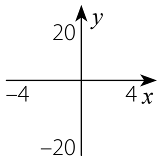
right

wrong

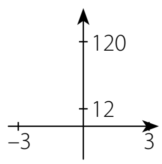
- **Hints** for drawing tangents are given in the text and should be explicitly taught.

- **Hint** for preparing the axes for a graph:

  After making the table, do a rough sketch showing the range of x and, y e.g.



  This shows that the origin should be in the centre of the graph paper. Scales can be chosen to enable the limits of each axis to be reached, e.g.



  Clearly, here there is no need for negative y and the x-axis can be positioned at the foot of the graph-paper.

  Scales should be chosen so that the divisions represent

  1, 2, 5

  10, 20, 50

  0.1, 0.2, 0.5

  etc.

  Unusual scales (such as 3s) may fit the paper better but are hard to read accurately.

  Use EX 7B.

Resources	Photocopiable graphs for the questions in EX 7B are available at the end of this chapter. Standard 2 mm graph paper is also photocopiable. (1 mm graph paper is not recommended.)
Assignments	Suggested questions for homework EX 7A, question 8 and EX 7B, question 8.
Vocabulary	linear, quadratic, cubic, reciprocal, inverse square, exponential, exponent, base gradient of curve, tangent, rate of change

ANSWERS

Exercises

EX 7A

1.	a) Q	b) R	c) S	d) P
2.	a) Q	b) P	c) S	d) R
3.	a) R	b) Q	c) P	d) S

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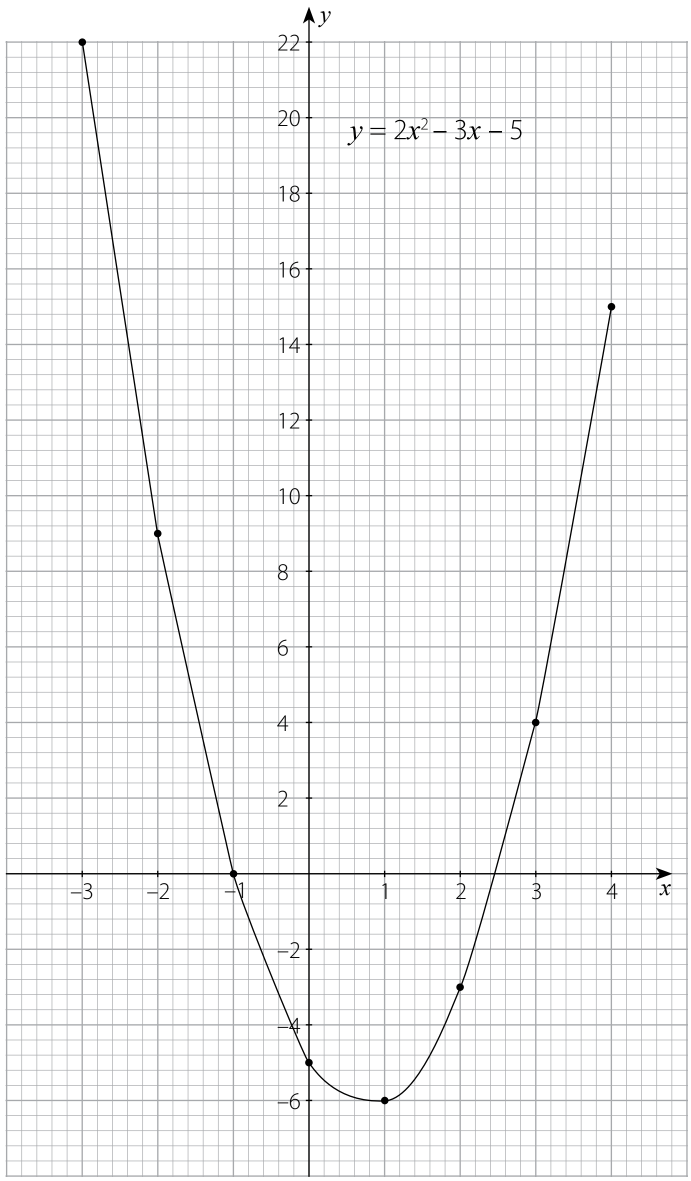
4. a)

x	-3	-2	-1	0	1	2	3	4
x^2	9	4	1	0	1	4	9	16
$2x^2$	18	8	2	0	2	8	18	32
$-3x$	9	6	3	0	-3	-6	-9	-12
-5	-5	-5	-5	-5	-5	-5	-5	-5
y	22	9	0	-5	-6	-3	4	15

b)



c) $x = 0.75$ d) $x = -1, x = 2.5$

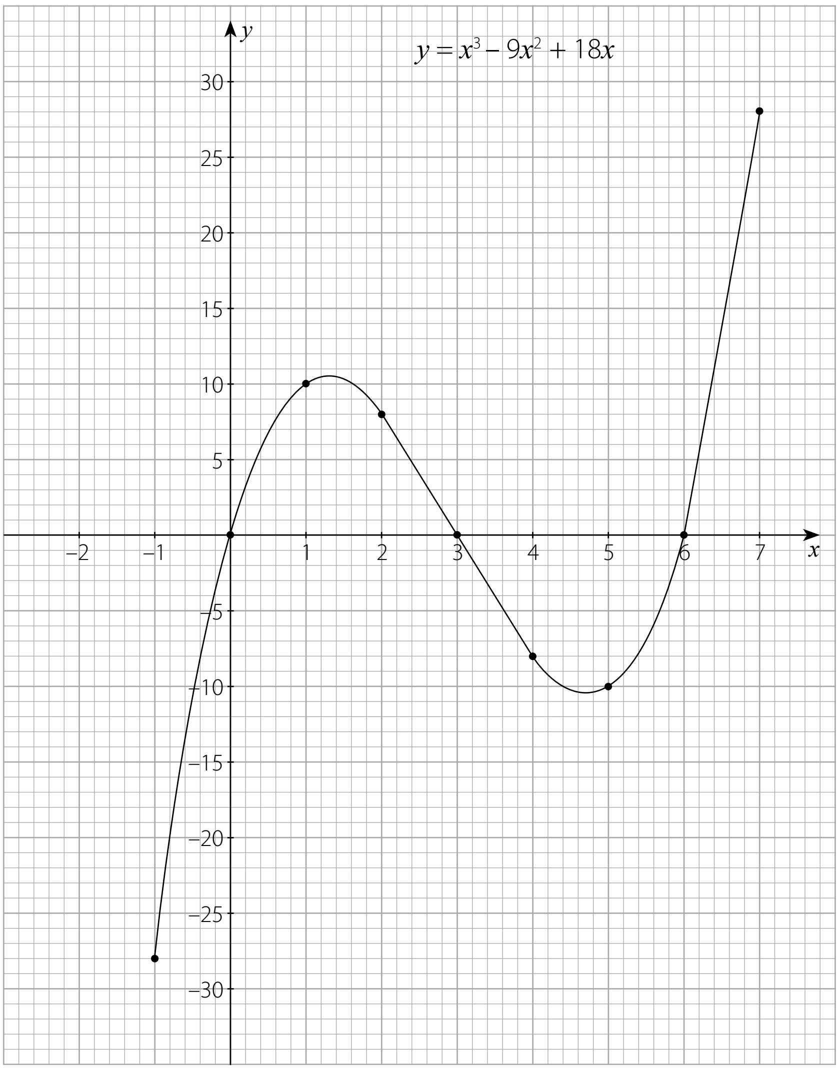
5. a) $x = 0, x = 1.5$ b) $x = -1.3, x = 2.8$

c) $x = -1.4, x = 2.9$ d) $x = -1.1, x = 2.6$

6. a)

x	−1	0	1	2	3	4	5	6	7
x^2	1	0	1	4	9	16	25	36	49
x^3	−1	0	1	8	27	64	125	216	343
$-9x^2$	−9	0	−9	−36	−81	−144	−225	−324	−441
$+18x$	−18	0	18	36	54	72	90	108	126
y	−28	0	10	8	0	−8	−10	0	28

b)

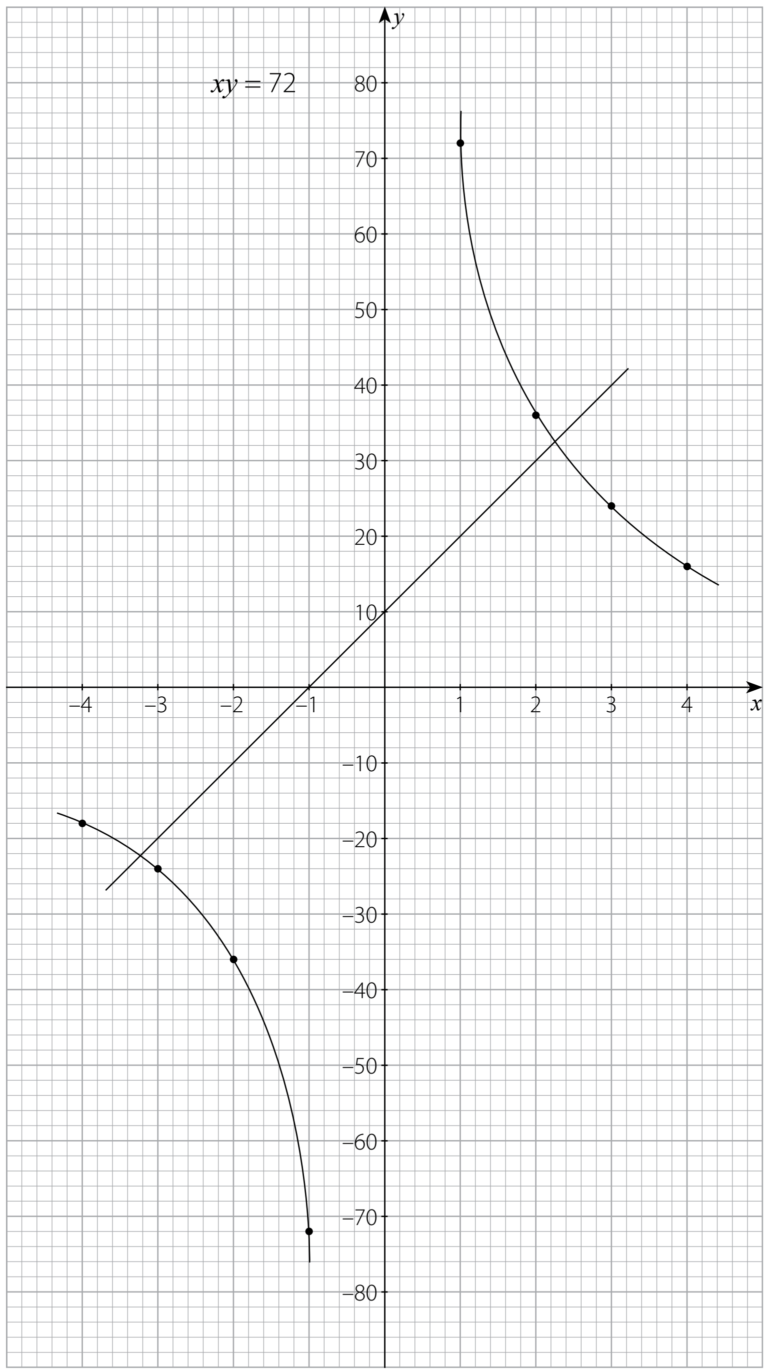


c) $x = 0, x = 3, x = 6$ d) $x = 0.6, x = 2, x = 6.4$

Chapter **7 Graphs**

7. a)

c)



b) rotational symmetry about the origin, order 2

d) $x = -3.2, x = 2.2$

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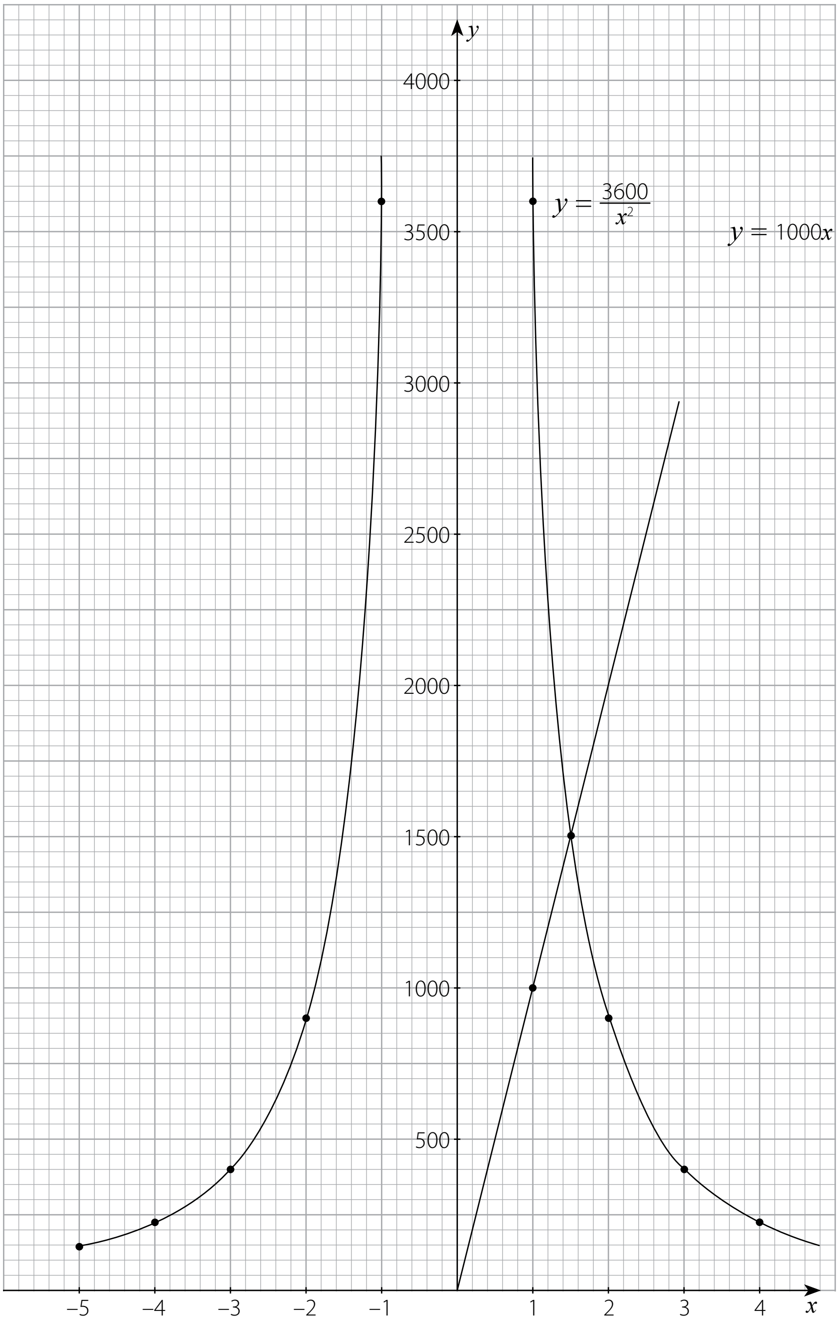
8. a)

x	-5	-4	-3	-2	-1	0	1	2	3	4
x^2	25	16	9	4	1	0	1	4	9	16
$y = \frac{3600}{x^2}$	144	225	400	900	3600	–	3600	900	400	225

b)

c)



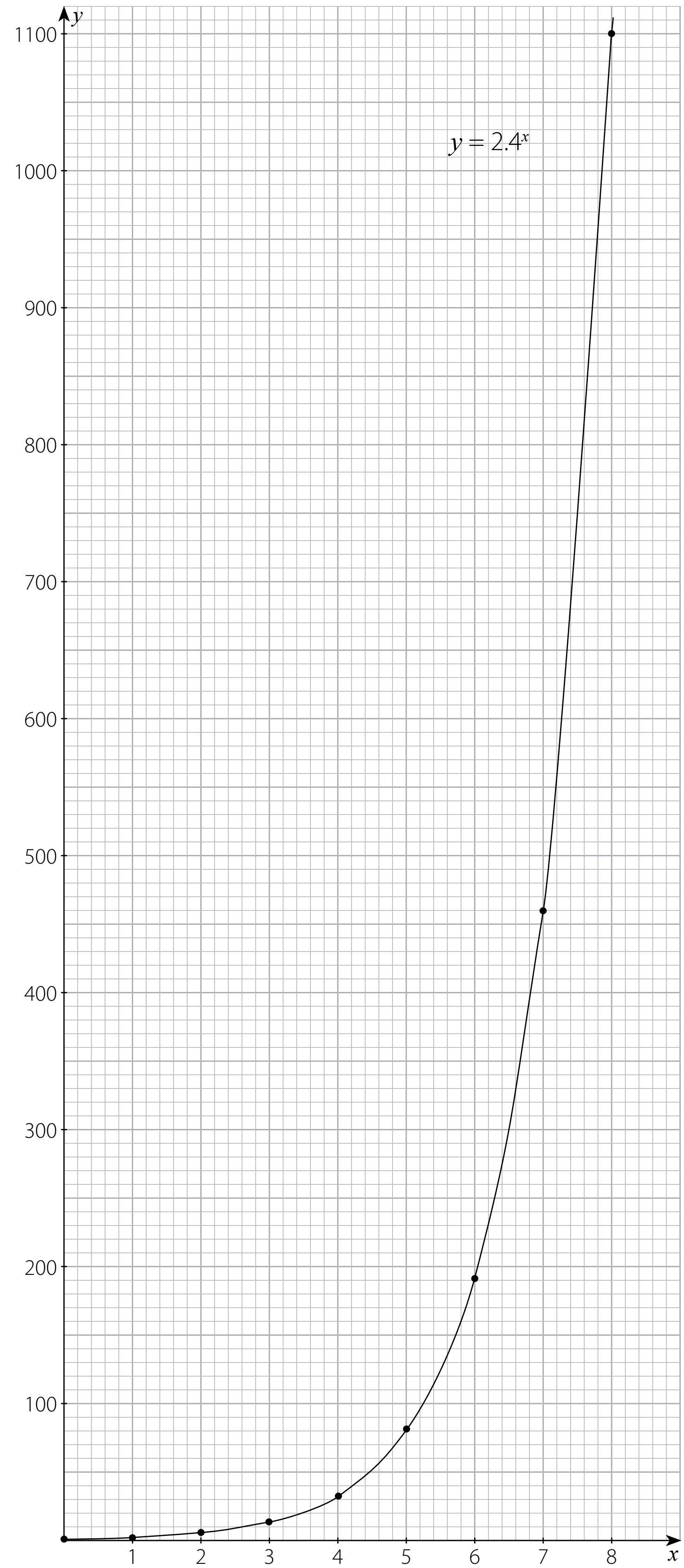
d) 1.5

9. a)

x	0	1	2	3	4	5	6	7	8
$y = 2.4^x$	1	2	6	14	33	80	191	459	1101

(rounded)

b)



c) 300 (nearest 10)

d) 7.1 (1 d.p.)

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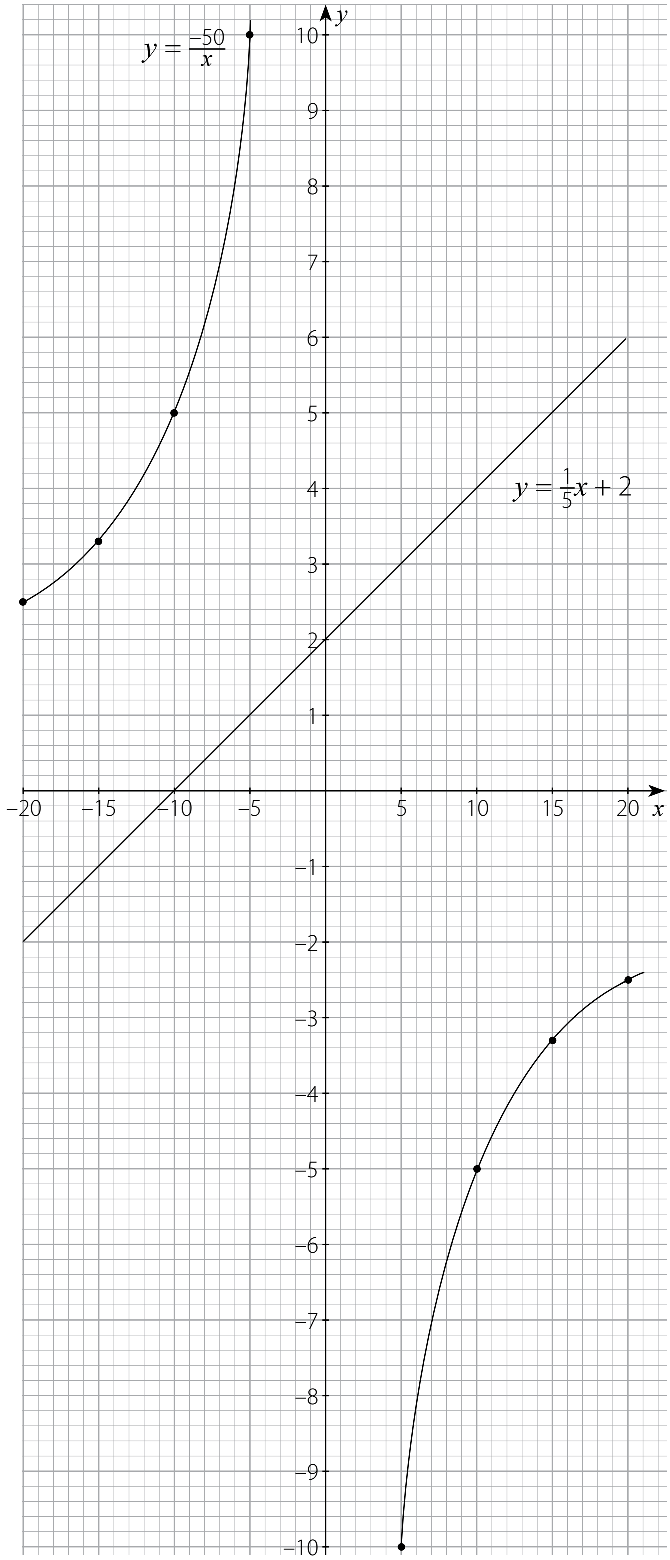
10. a) $y = \frac{-50}{x}$ reciprocal graph

b)

x	−20	−15	−10	−5	0	5	10	15	20
$y = \frac{-50}{x}$	2.5	3.3	5	10		−10	−5	−3.3	−2.5

c)



d) $y = \frac{1}{5}x + 2$

intersects

$xy + 50 = 0$

where

$x(\frac{1}{5}x + 2) + 50 = 0$

$x^2 + 10x + 250 = 0$

But from the graph, there is no intersection. The equation has no real solution.

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EX 7B

1. a) $y = x$ b) $y = 2x$ c) $y = x + 1$ d) $y = -3x$

2. a) -16 b) -0.9 c) 0 d) $-6, 1.5$

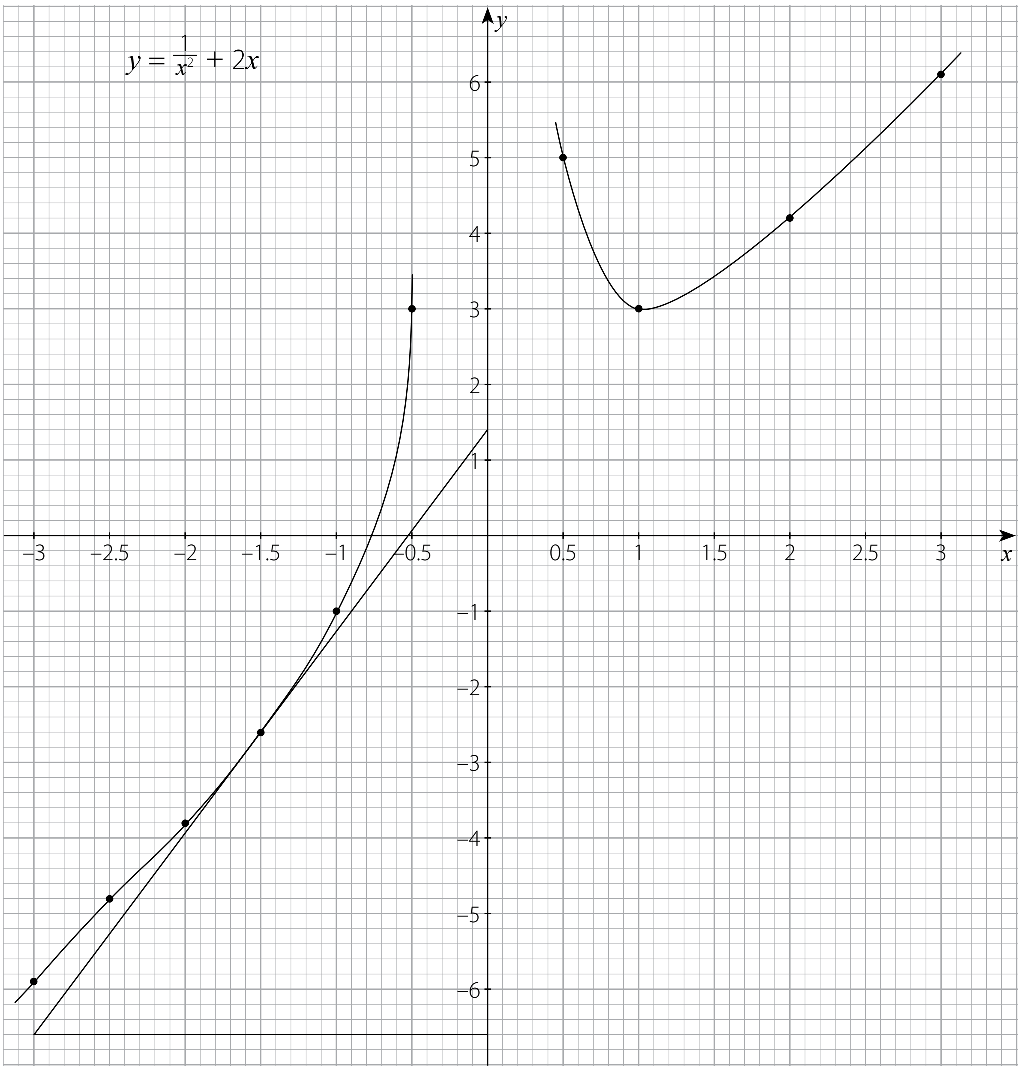
3. a) $y = 20x$ b) $x = 1.8$ c) 8.3 d) 0.7

4. a) $x = 2.7$ b) 2.6

5. a)

	x	-3	-2.5	-2	-1.5	-1	-0.5	
	x^2	9	6.25	4	2.25	1	0.25	
y	$\frac{1}{x^2}$	0.11	0.16	0.25	0.44	1	4	y
	$2x$	-6	-5	-4	-3	-2	-1	
	y	-5.9	-4.8	-3.8	-2.6	-1	3	

b)



c) 2.6 d) $y = \frac{1}{2}x + 1$

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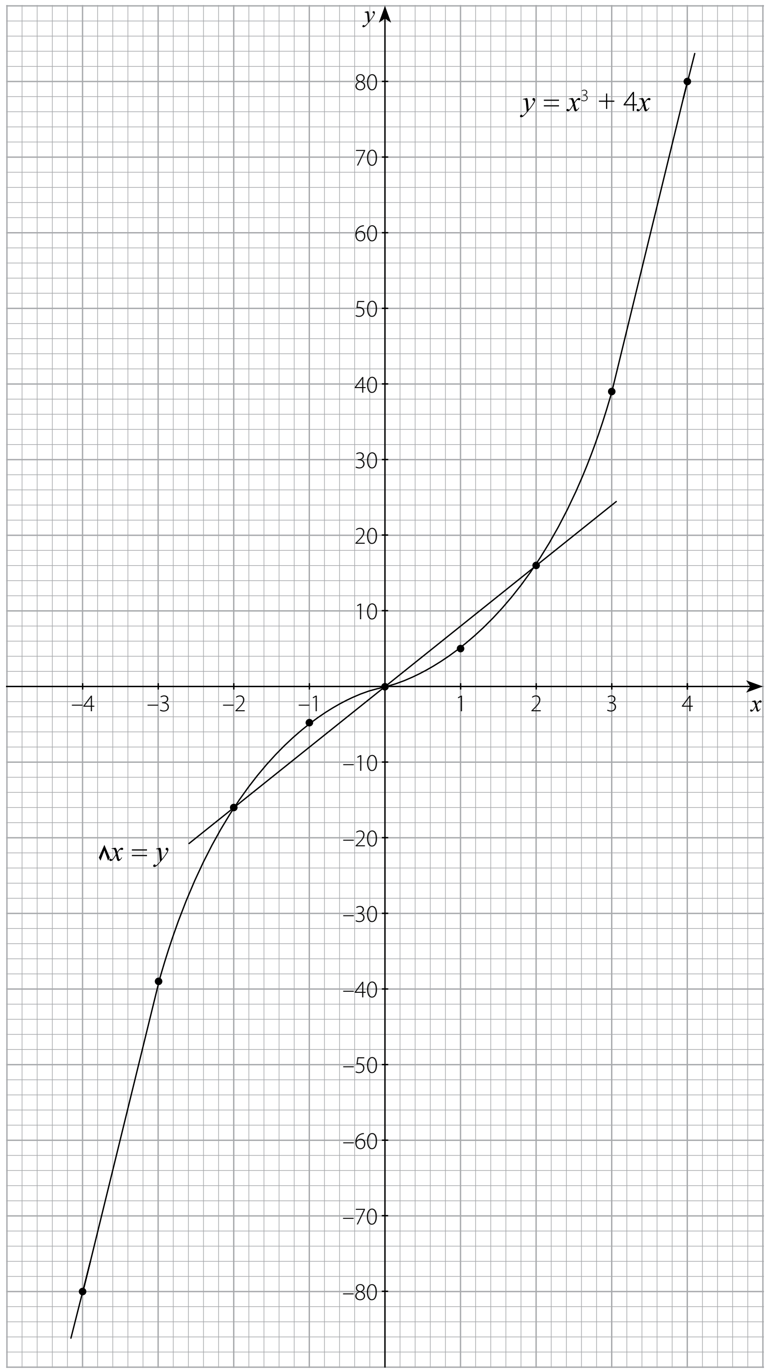
6. a)

x	1	2	3	4
x^3	1	8	27	64
$4x$	4	8	12	16
y	5	16	39	80

b)



c) 0

d) (Draw $y = 8x$) $x = -2, 0, 2$

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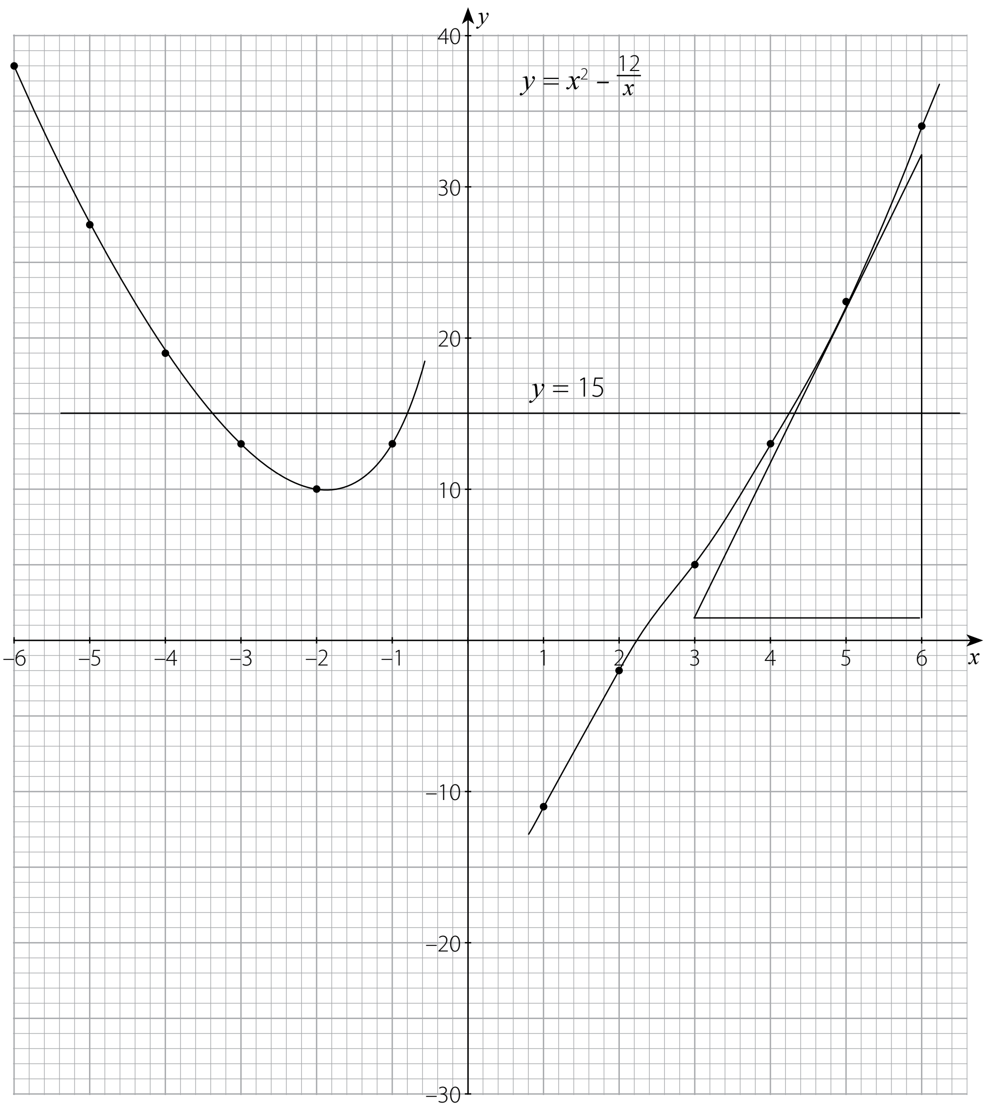
7. a)

x	−6	−5	−4	−3	−2	−1
x^2	36	25	16	9	4	1
$-\frac{12}{x}$	2	2.4	3	4	6	12
y	38	27.4	19	13	10	13

b)

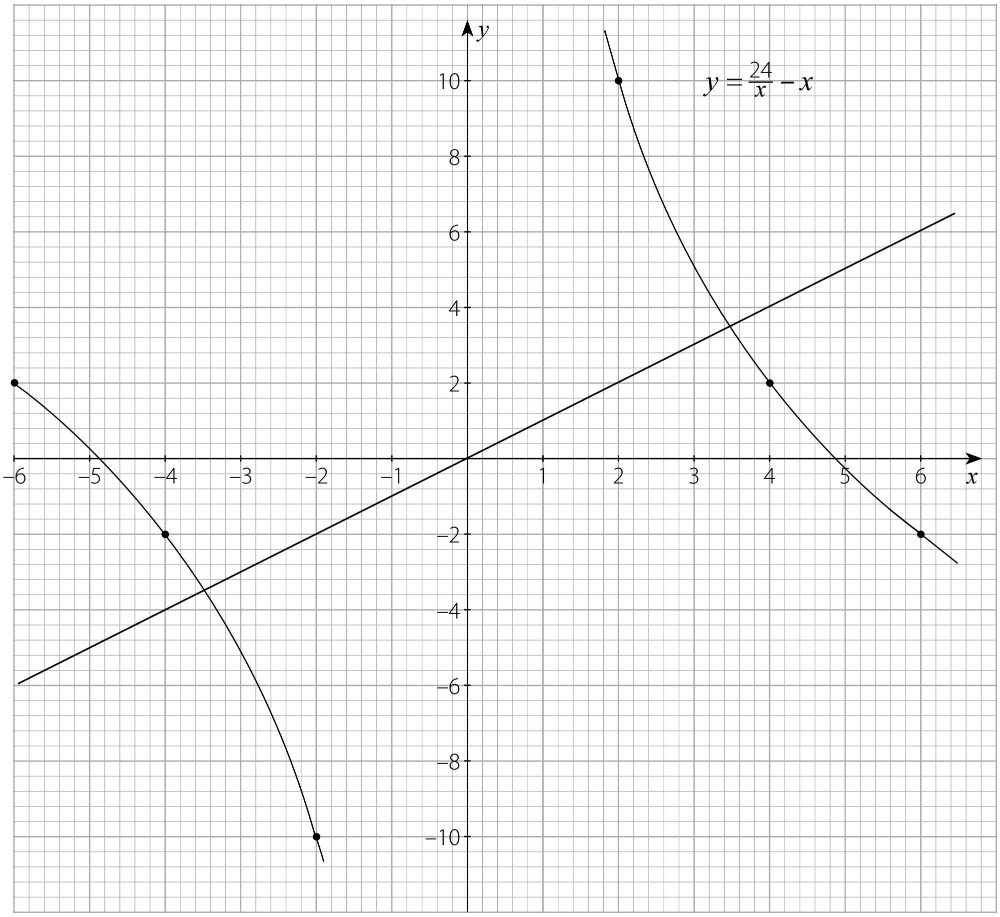


c) 10 d) $y = 15$ cuts curve 3 times

8. a)

x	−6	−4	−2	0	2	4	6
$\frac{24}{x}$	−4	−6	−12	–	12	6	4
$-x$	6	4	2	0	−2	−4	−6
y	2	−2	−10	–	10	2	−2

b)



c) ± 3.5

d) When $y = \frac{24}{x} - x$ and $y = x$ interact

$$\frac{24}{x} - x = x$$

$$\frac{24}{x} = 2x$$

$$x^2 = 12$$

$$x = \pm\sqrt{12}$$

9. a)

x	−1	0	1	2	3	4	5
2^x	0.5	1	2	4	8	16	32
−1	−1	−1	−1	−1	−1	−1	−1
y	−0.5	0	1	3	7	15	31

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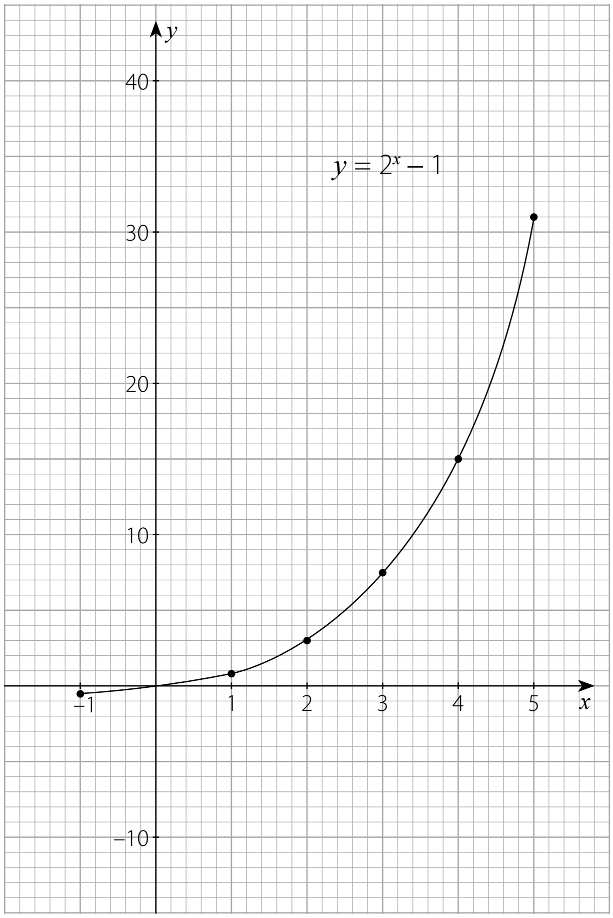
b)



c) $2^{x-1} = 3$

$\frac{2^x}{2} = 3$

$2^x = 6$

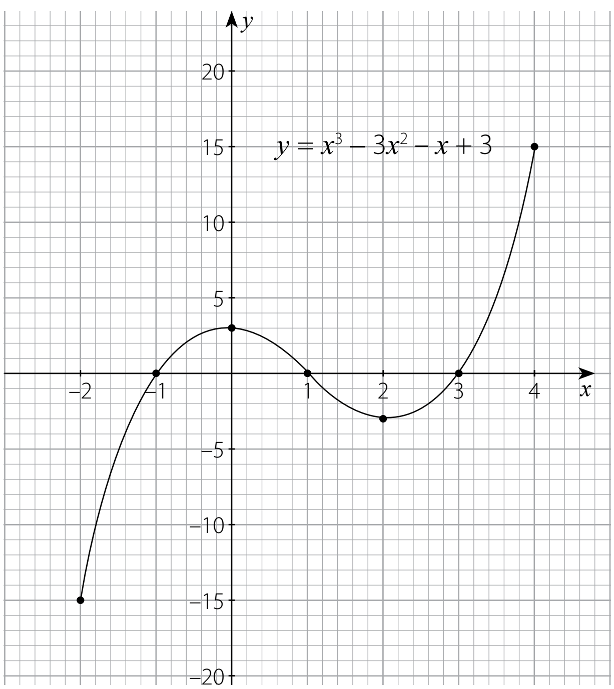
$2^x - 1 = 5$

$\therefore y = 5$

$x = 2.6$

d) increases rapidly

10. a)



b) $k < -3$ or $k > 3$

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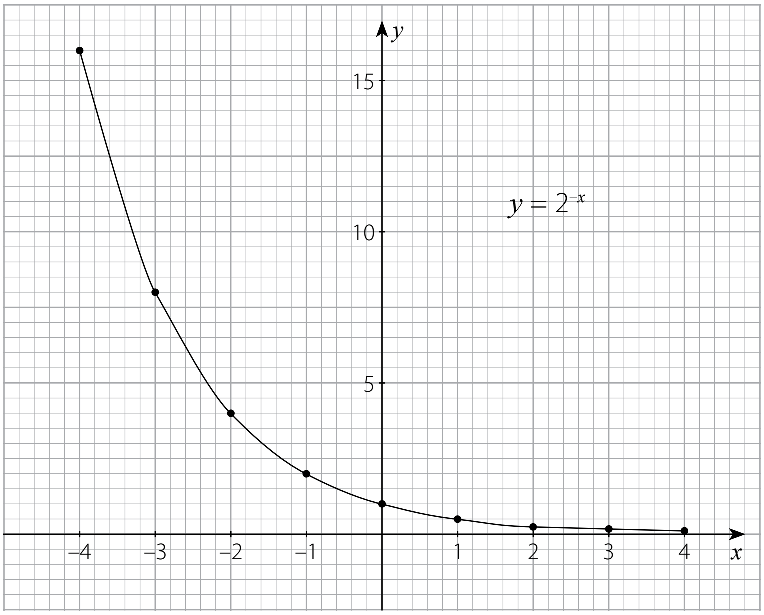
EX 7X

1. a)

x	-4	-3	-2	-1	0	1	2	3	4
$y = 2^{-x}$	16	8	4	2	1	0.5	0.25	0.13	0.06

(approx.)

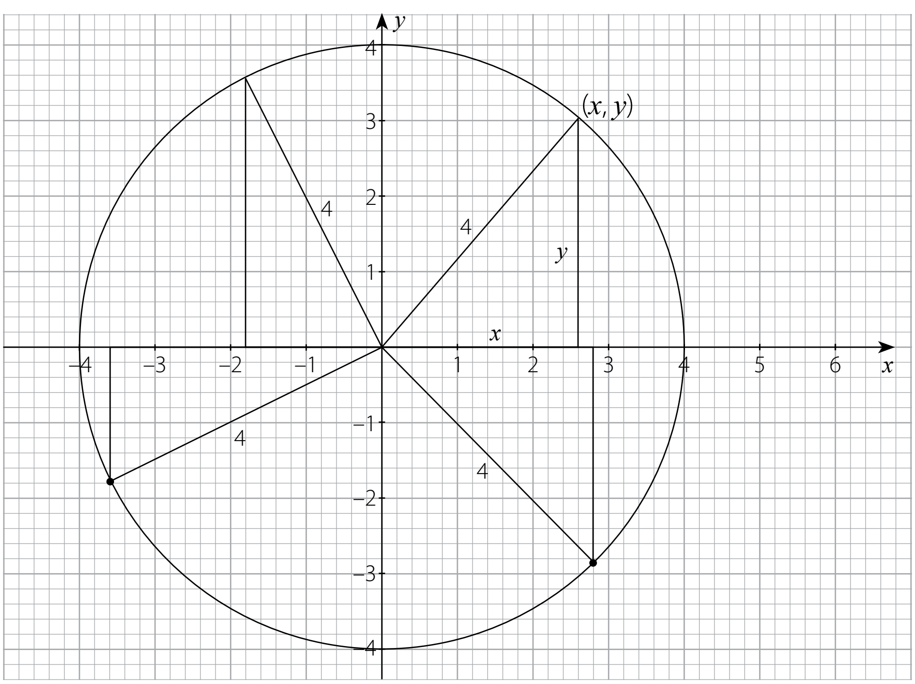
b)



Reflection of $y = 2^x$ in the y-axis

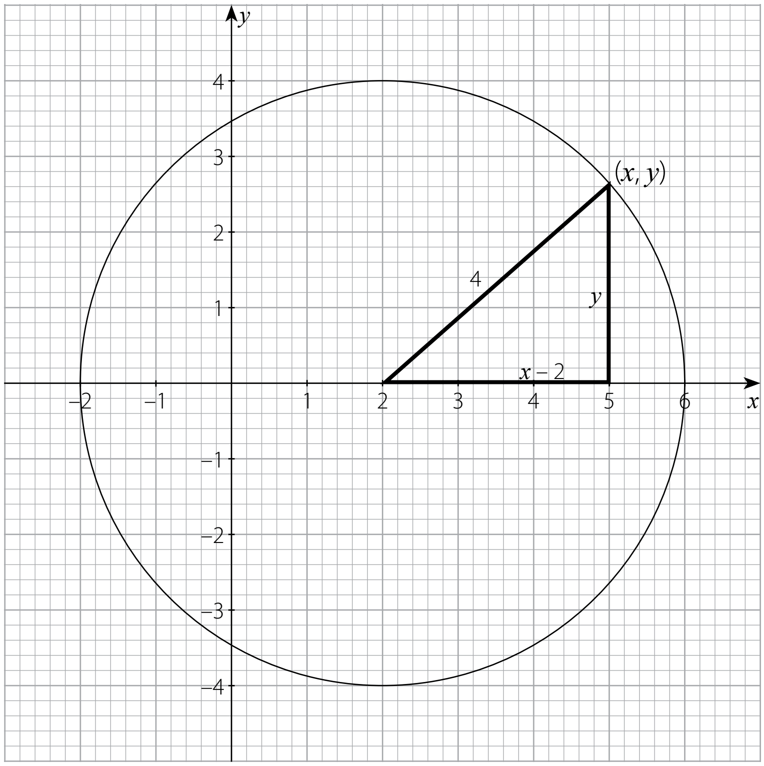
Radioactive materials decay according to negative exponential laws.

2. $x^2 + y^2 = r^2$ (Pythag)



$x^2 + y^2 = 4^2$

$x^2 + y^2 = 16$



$(x-2)^2 + y^2 = 4^2$

$x^2 - 4x + 4 + y^2 = 16$

$x^2 + y^2 - 4x - 12 = 0$

General circle equation: $(x-a)^2 + (y-b)^2 = r^2$

3.

	x	−3	−2	−1	0	1	2	3	
	x^2	9	4	1	0	1	4	9	
y	x^4	81	16	1	0	1	16	81	y
	$-4x^2$	−36	-16	−4	0	−4	−16	−36	
	y	45	0	−3	0	−3	0	45	

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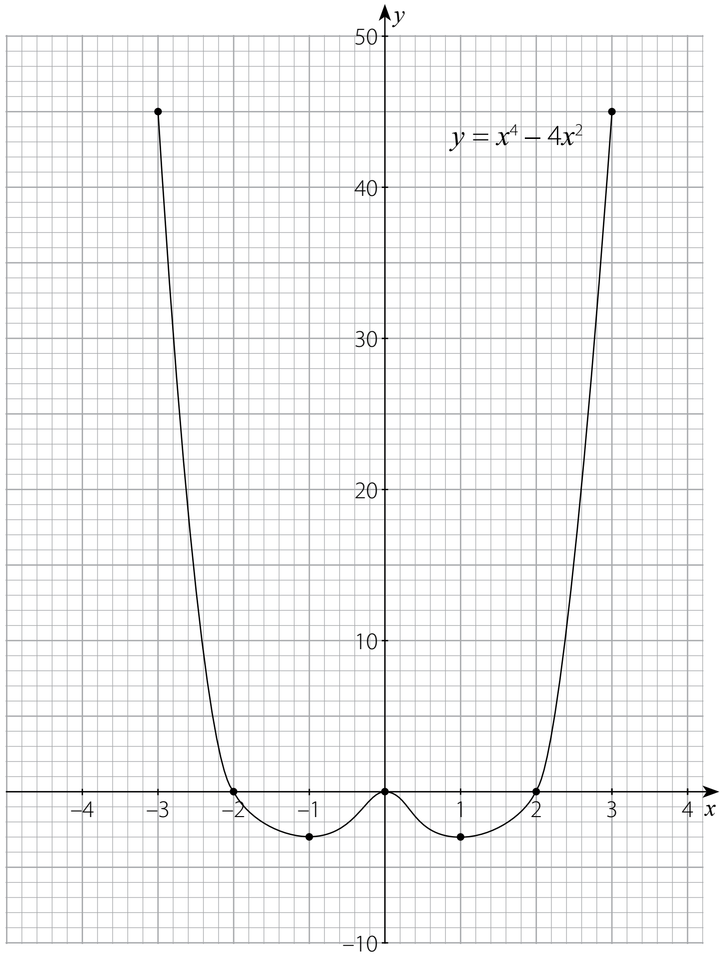

$-3 < k < 0$

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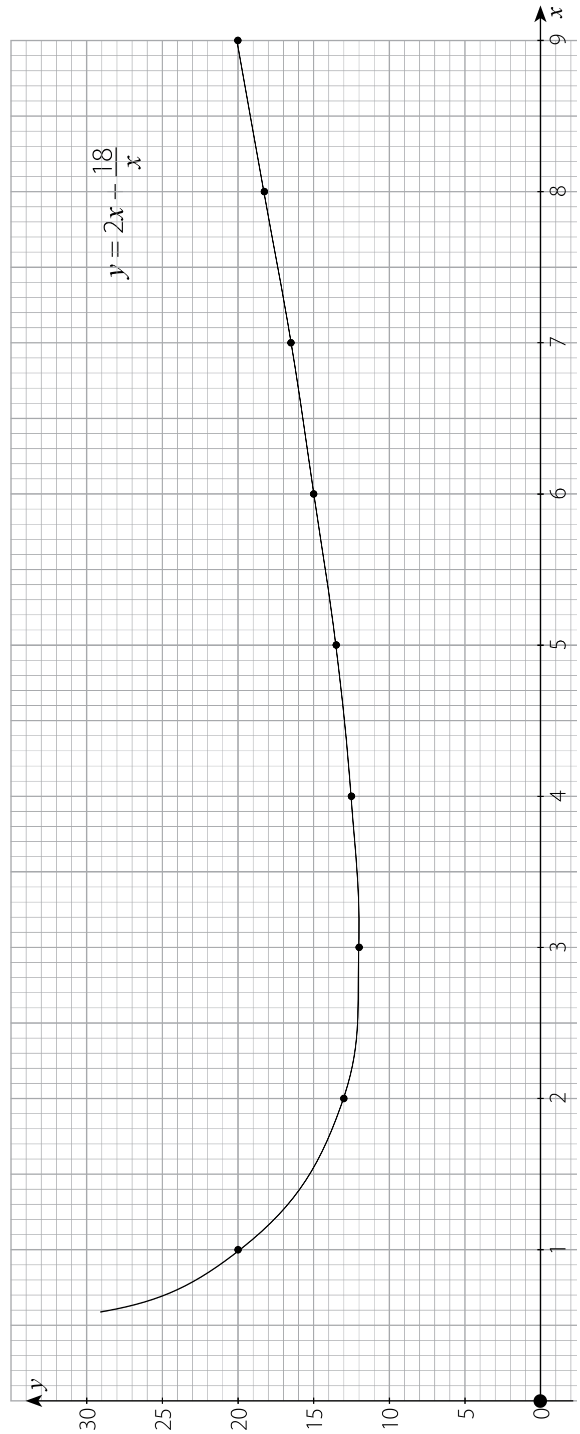
EX 7B, question 2 (worksheet)



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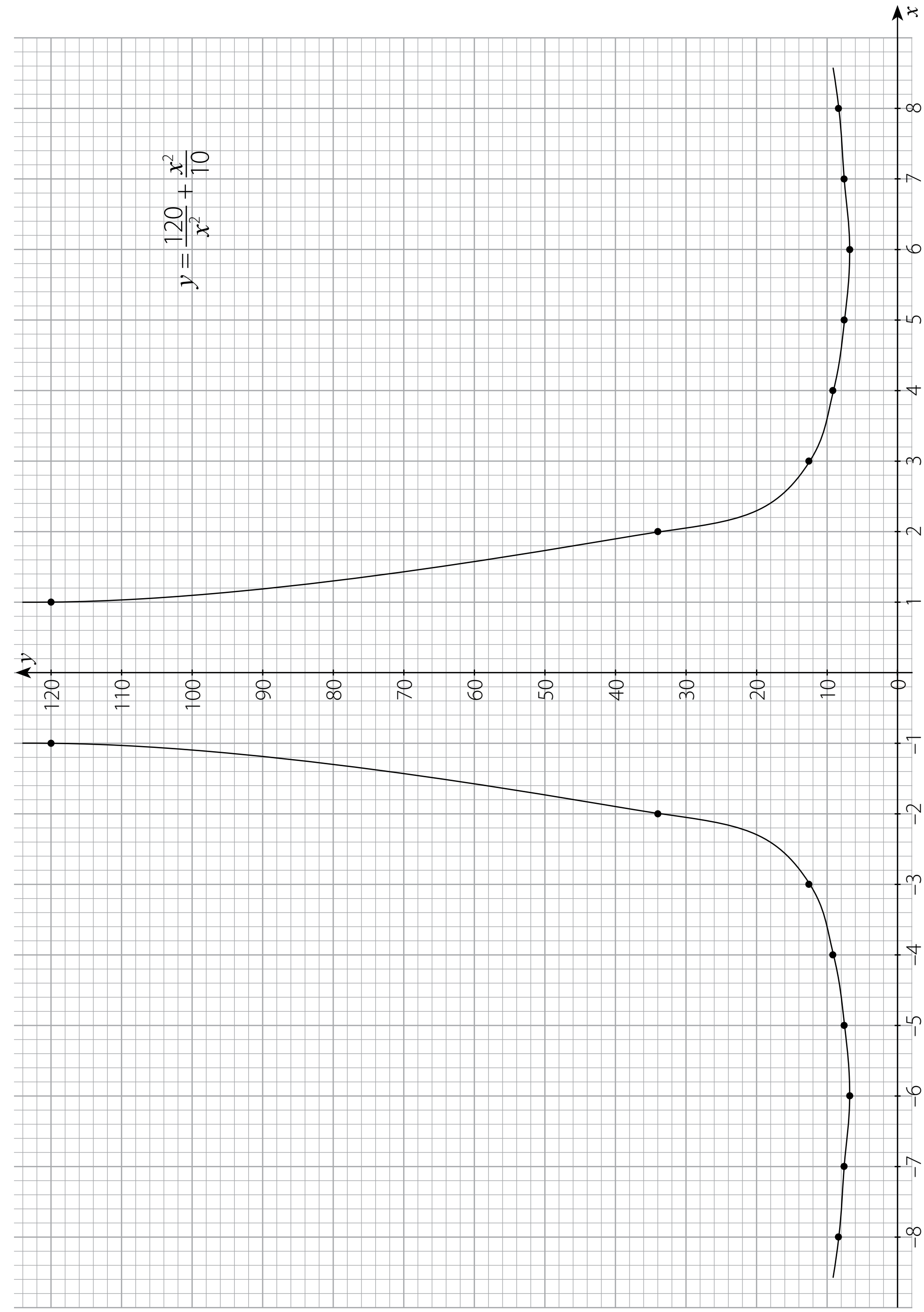
EX 7B, question 3 (worksheet)



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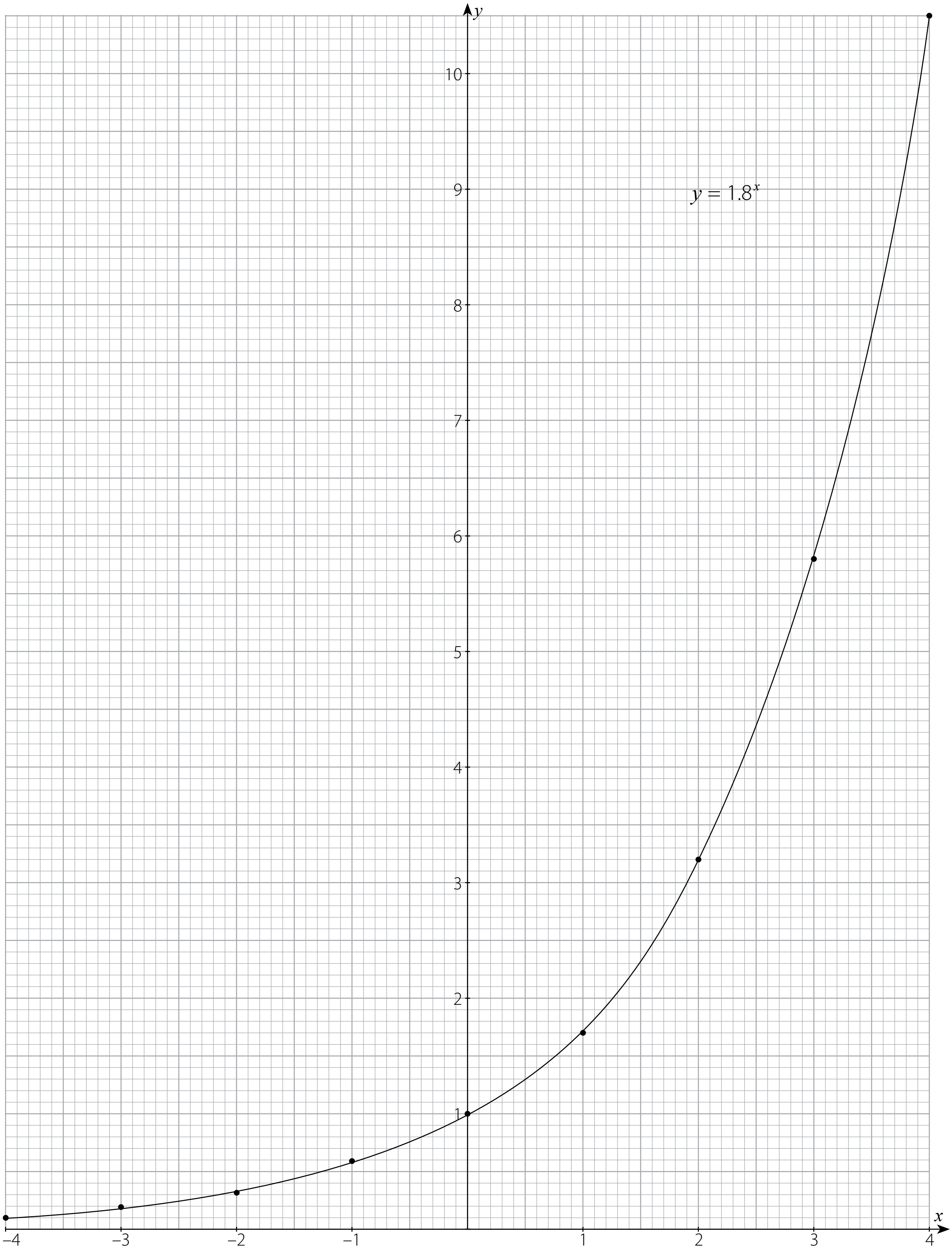
EX 7B, question 4 (worksheet)



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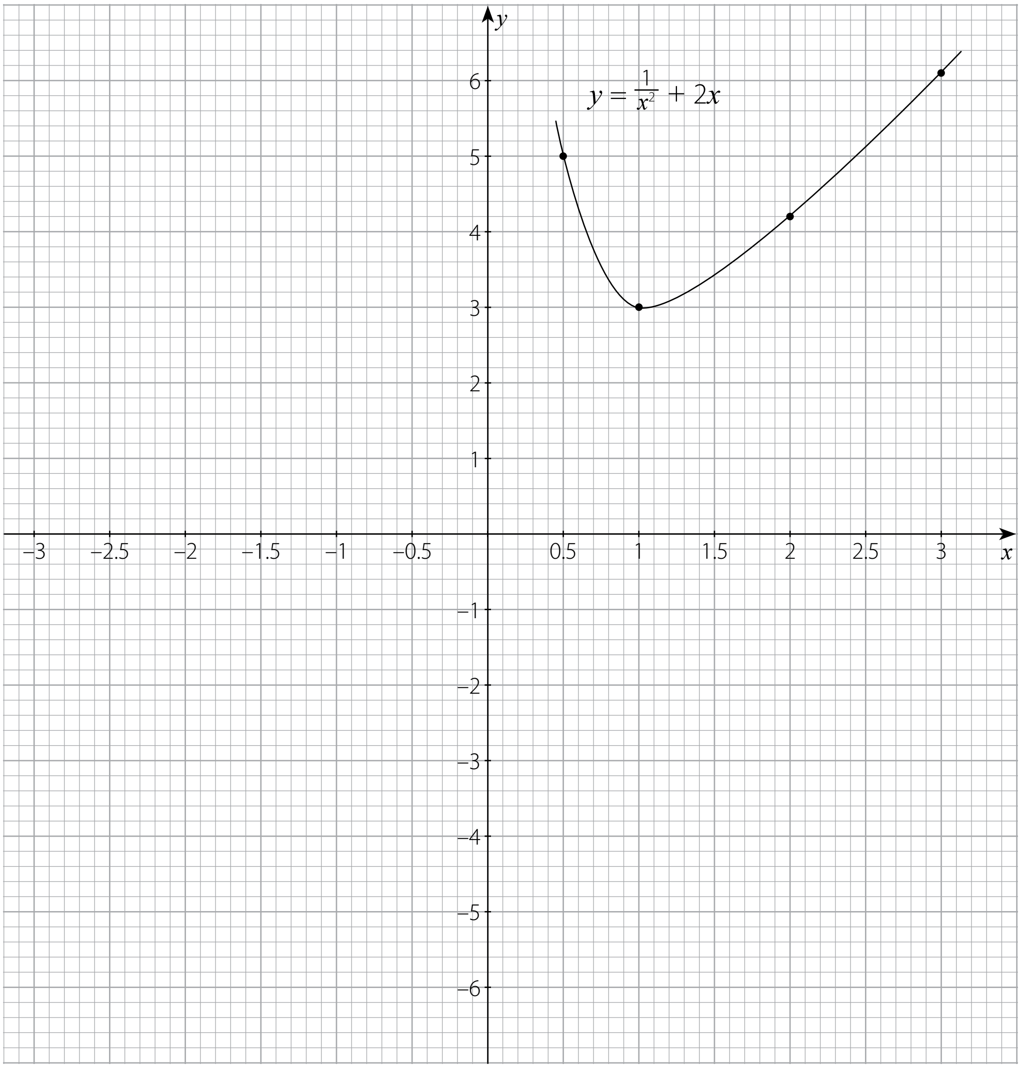
EX 7B, question 5 (worksheet)



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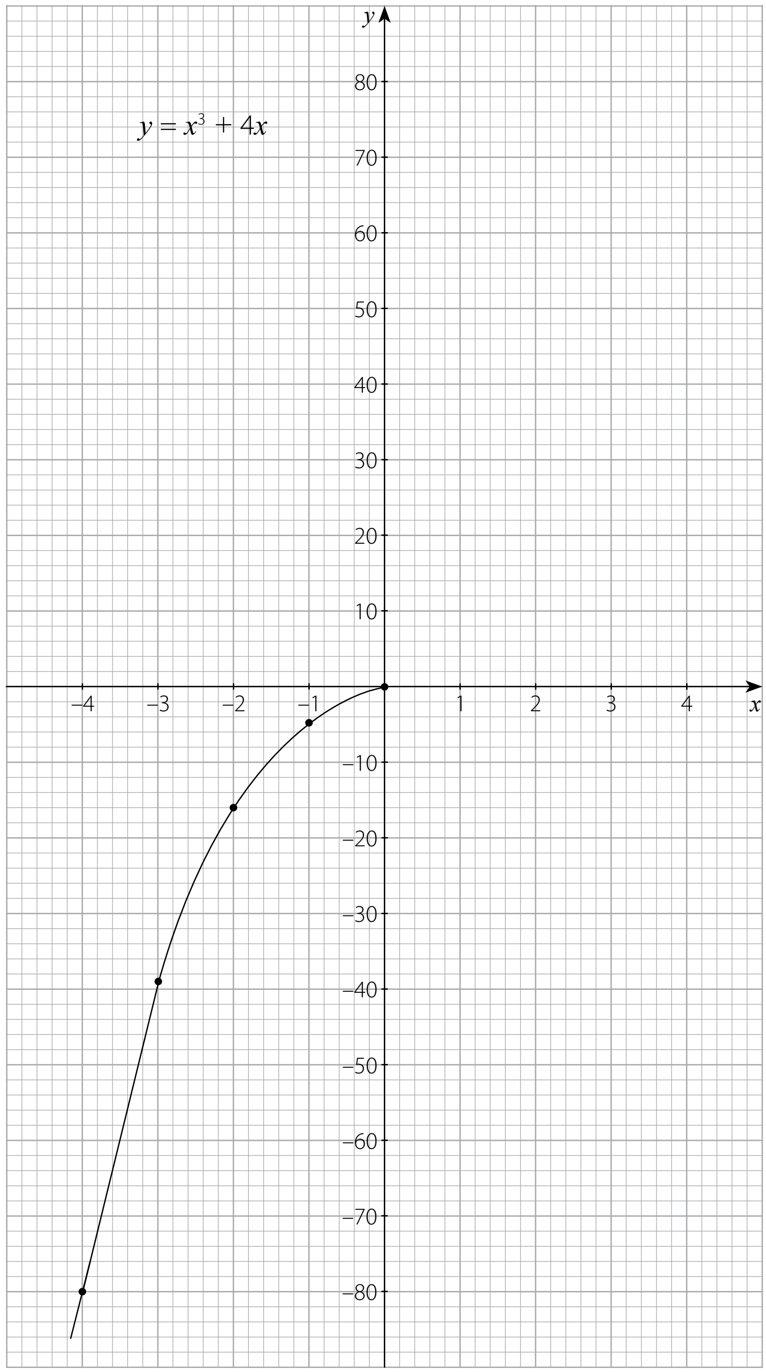
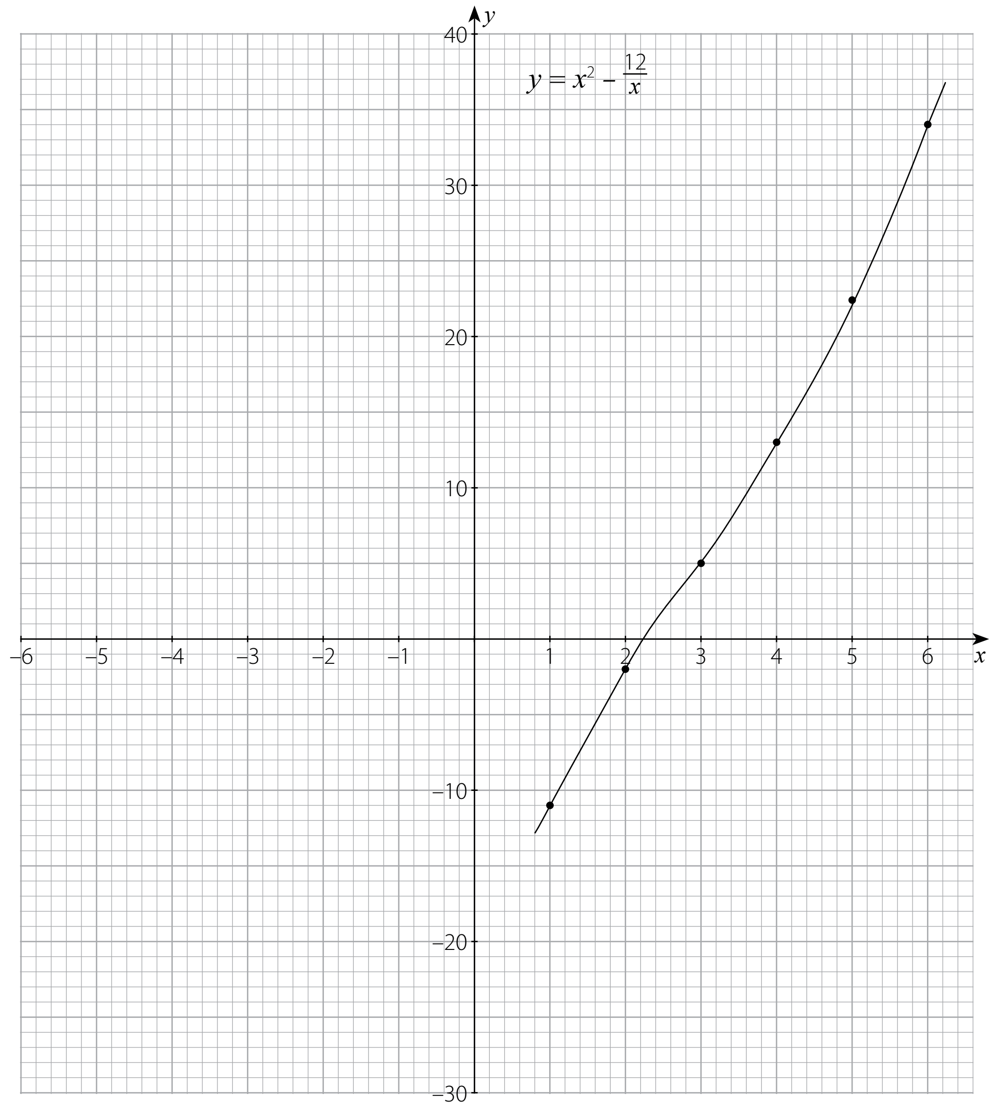
EX 7B, question 6 (worksheet)



EX 7B, question 7 (worksheet)



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Graph paper (2 mm)

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Chapter

8 Variation

This topic presents more of a language issue than a mathematical one, but recent graphical work is used to illuminate it by providing a visual image of what is happening algebraically.

LESSON PLANNING

Objectives

General	To use the language and symbol of variation correctly, relate them to graphs and their equations, and use them to solve problems
Specific	1. To relate two quantities that vary directly or inversely using correct notation 2. To determine the appropriate equation from a statement of variation 3. To know the general shape of the graphs of quantities directly or inversely varying 4. To be able to find the constant of variation using given data 5. To solve problems involving variation
Pacing	1 lesson, 1 homework
Links	gradient of a line, proportion, graphs
Method	• Start with a simple straight line graph through the origin e.g. $y = 3x$ Make a table of values. "What is the ratio of $x:y$?" Ans 1:3 "What is the gradient of the line?" Ans 3 Then introduce a new phrase "y varies directly as x" and its symbol $y \propto x$ Follow the text. Move on to inverse variation (i.e. indirect proportion). The squares, both direct and inverse, are a fairly easy extension. • Show how to obtain an equation from a variation statement, with a constant to be determined from given data. Work some examples, in addition to those in the text, showing how to state clearly what is being done. Use EX 8A.
Assignments	Suitable for homework EX 8A, question 7 and/or question 10
Vocabulary	variation, varies directly as, varies inversely as, proportion, direct, indirect constant of variation

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ANSWERS

Exercises

EX 8A

1. a) $v \propto t$ b) $p \propto \frac{1}{q}$ c) $s \propto x^2$ d) $F \propto \frac{1}{d^2}$

2. a) $v = 7t$ b) $p = \frac{8}{q}$ c) $s = 1000x^2$ d) $F = \frac{150}{d^2}$

3. a) i) 17.5 ii) 24.5

   b) i) 0.2 ii) 0.2

   c) i) 10 ii) 0.1 assuming $x > 0$

   d) i) 9.375 ii) 5.5 (2 s.f.) assuming $d > 0$

4. a) y varies inversely as x. b) y varies directly as x.

   c) y varies inversely as the square of x.

   d) y varies directly as the square of x.

5. a) $c \propto n$; $c = 1.5n$ b) $t \propto \frac{1}{n}$; $t = \frac{32}{n}$

   c) $n \propto \frac{1}{l}$; $n = \frac{5000}{l}$ d) $p \propto q$; $p = 4q$

6. a) direct square b) inverse square

   c) direct d) inverse square

7. a) $d \propto r$ b) $d = \frac{r}{120}$ c) \$ 150 d) Rs 540

8. a) $t \propto \frac{1}{v}$; $t = \frac{c}{v}$ b) $t = \frac{2400}{v}$

   c) 2 min 30 s d) The distance travelled in metres

9. 160 mR

10. a) 10 b) 17.28

    c) 14.4 d) 8.33 (3 s.f.)

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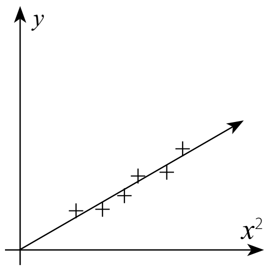
EX 8X

1. We have $y = cx^2$ (c constant) as our hypothesis.

   By plotting y against x^2 we expect to obtain a straight line of gradient c, if our hypothesis is correct.



- This enables the inevitable experimental errors to be averaged out easily by drawing the line of best fit.
- The gradient of a line may be calculated easily, thus giving the value of the constant, and hence the equation of the relationship between x and y.
- If a straight line does not appear from this experiment, then the hypothesis is wrong.

2. $F \propto \dfrac{m_1 m_2}{d^2}$ and $F = \dfrac{Gm_1 m_2}{d^2}$

   G is the constant of gravitation. For force measured in newtons, mass in kg, and distance in metres, the value of G is 6.673×10^{-11}.

3. s varies partly directly as t and partly directly as the square of t.

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Chapter

9 Quadratic Equations: Formula Method

This chapter follows on from Chapter 4 where the factorisation method of solution is explained. It is presented as a method of finding solutions to all quadratic equations that have real solutions.

LESSON PLANNING

Objectives

General	To know the formula for solution of a quadratic equation, and to use it correctly
Specific	1. To be aware that not all quadratic equations can be factorised, and that some of these do not have solutions 2. To know the formula for the solution of the general quadratic equation $ax^2 + bx + c = 0$ and to observe its verification 3. To apply the formula to solving given equations, with correct substitution of values and use of calculator 4. To solve problems requiring use of the formula and/or factorisation
Pacing	2 lessons, 2 homeworks
Links	factorisation, perfect squares, parabolic curves

Method

Show, using the examples below, that not all easy-looking quadratic equations can be factorised, e.g.

$x^2 + x + 1 = 0$

$x^2 + 2x + 5 = 0$

$2x^2 - 7x - 3 = 0$ (nearly, but not quite!)

Also, use the text example.

- Quote the formula. It has to be memorised.
  A little old-fashioned class chorus might be relevant here!
  Insist on finishing "all over $2a$" to pre-empt the common error of $x = -b \pm \frac{\sqrt{b^2 - 4ac}}{2a}$
- Follow the text to demonstrate that the formula is just a re-arrangement of the equation. The algebra may be too difficult for some, but it should be shown so that the formula is not seen as a magic recipe but is derived logically. [At this stage very few would be able to follow the classic derivation by completing the square, but it is given, with hints, as a question in Exercise 9X.]
- Concentrate then on using the formula efficiently.

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Hint 1:

Write $a =$ $b =$ $c =$ at the start.

Hint 2: For the $4ac$, use brackets. This helps to avoid sign errors, e.g.

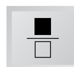
$a = 1,\quad b = -2,\quad c = -3$

$x = \frac{2 \pm \sqrt{4 - 4(1)(-3)}}{2}$

etc.

Hint 3: When using calculators there is no need for surd simplification. Just enter the values immediately. For example,

$x = \frac{-3 \pm \sqrt{48}}{4}$

Either enter as a fraction using 

or, enter $(-3 + \sqrt{48}) \div 4$

Then edit to change the + to – for the second solution.

- Use EX 9A. The questions are graded in order of difficulty. From question 4, the equations may have to be re-arranged to obtain a, b, c correctly.
- Show that factorisable equations are also solvable by formula, but that it usually takes longer. Use the text example and others (e.g. from EX 4A). EX 9B explores this theme and leads on to a few problems requiring construction of equations and choice of method of solution.

Assignments	As EX 9A is graded, a good class could be set parts (a) and (c) only, with (b) and (d) reserved for homework (2 sessions). EX 9B is best done in class working in pairs.
Vocabulary	plus or minus (±) integers, rational numbers

ANSWERS

Exercises

EX 9A

1. a) $x = 3.41, x = 0.586$ b) $x = 0.449, x = -4.22$

   c) $x = 0.772, x = -7.77$ d) $x = 7.77, x = -0.772$

2. a) $x = 4.83, x = -0.828$ b) $x = 0.828, x = -4.83$

   c) $x = 10.4, x = -1.35$ d) $x = -1.30, x = -7.70$

3. a) $x = 1.27, x = -2.77$ b) $x = 2.77, x = -1.27$

   c) $x = 2.28, x = 0.219$ d) $x = 3.39, x = -0.886$

4. a) $x = 1.28, x = -0.781$ b) $x = 1.35, x = -1.85$

   c) $x = -2.22, x = 0.225$ d) $x = 3.16, x = -0.158$

5. a) $x = 1.22, x = -0.549$ b) $x = 1.43, x = 0.232$

   c) $x = 2.18, x = 0.153$ d) $x = 2.22, x = 0.451$

6. a) $x = 1.39, x = 0.360$ b) $x = \pm 0.866$
   c) $x = \pm 1.66$ d) $x = 1.79, x = -1.54$
7. a) $x = -0.710, x = -1.69$ b) $x = 0.458, x = -3.06$
   c) $x = 3.44, x = -0.640$ d) $x = 0.0652, x = -3.07$
8. a) $x = 0.456, x = -0.313$ b) $x = 0.313, x = -0.456$
   c) $x = 0.277, x = -0.902$ d) $x = 1.06, x = -0.838$
9. a) $x = 1.24, x = -0.404$ b) $x = 0.551, x = -1.45$
   c) $x = 0.814, x = 0.102$ d) $x = 0.576, x = -1.04$
10. a) $x = 1.71, x = 0.293$ b) $x = 0.843, x = -0.593$
   c) $x = 2.46, x = -3.66$ d) $x = 1.11, x = 0.323$

EX 9B

1. a) $x = -1, x = -4$ b) $(x + 1)(x + 4)$
2. a) $x = 9, x = -2$ b) same
3. a) $x = \frac{7}{4}, x = \frac{5}{3}$ b) $x = \frac{7}{4}, x = \frac{5}{3}$
   c) same d) equally difficult?
4. a) $x = -5, x = -2$ b) $x = \pm\frac{1}{2}$
   c) $x = -1.70, x = -5.30$ d) $x = \pm 1.12$
5. a) $x = 2, x = 1.3$ b) $(x - 2)(x - 1.3) = 0$ c) $x = 2, x = 1.3$
   d) same solutions, because the equation is the same after dividing through by 10.
6. a) $4x^2 - 20x + 25$ b) $x = 2.5$ c) $x = 2.5$
   d) same solution, because the equation is the same after dividing through by 2.
7. a) (5.45, 10.9) and (0.551, 1.10) (3 s.f.)
   b) (1.62, 2.62) and (–0.618, 0.382) (3 s.f.)
   c) (2.24, 0.24) and (–2.24, –4.24) (3 s.f.) d) (2, 4)
8. $x(x + 2) = x + 10$, $x = 2.70$ (reject negative solution)
   width = 2.70 cm, length = 4.70 cm, area = 12.7 cm^2 (3 s.f.)
9. $x = 2, x = -1$
10. $x^2 + (x + 7)^2 = (x + 9)^2$ (Pythag),
   $x = 8$ (reject negative solution), diagonal = 17 cm

EX 9X

1. $x = 0, x = \frac{1}{2}, x = 3$

2. a) will not factorise

   b) Obtain $\sqrt{-3}$ in the formula.

   The equation has no real solution.

   The parabola $y = x^2 - x + 1$ lies above the x-axis,

   i.e. it is not possible for $y = 0$

3. $\left(x + \frac{b}{2a}\right)^2 = x^2 + 2(x)\left(\frac{b}{2a}\right) + \left(\frac{b}{2a}\right)^2$

$$= x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}$$

We have $ax^2 + bx + c = 0$ as our general equation

Divide by a $\quad x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

$$\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} + \frac{c}{a} = 0 \quad \text{using initial result}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$$

$$= \frac{b^2 - 4ac}{4a^2}$$

Take sq. roots: $\quad x + \frac{b}{2a} = \frac{\pm\sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

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Chapter

10 Symmetry Properties of Circles

This is the first of two chapters on circle geometry (continued in Chapter 13). Symmetry is a powerful argument in geometry and is used here to maximum advantage.

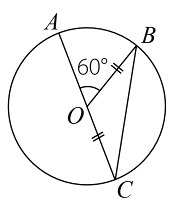
LESSON PLANNING

Objectives

General	To calculate lengths and angles on diagrams involving the chord and tangent properties of circles
Specific	1. To use the terms **radius**, **diameter**, **circumference**, **segment**, **sector**, **arc**, **chord**, **tangent**, and **cyclic quadrilateral** correctly 2. To do geometrical calculations using the property that equal chords are equidistant from the centre; and that the perpendicular bisector of a chord passes through the centre 3. To do geometrical calculations using the property that a tangent is perpendicular to the radius at the point of contact; and that tangents from an external point are equal
Pacing	4 lessons, 2 homeworks
Links	Angle facts, especially those related to triangles and parallelograms, Pythagoras' theorem, trigonometric ratios

Method

- Quickly revise angle-sum of a triangle, exterior angle of a triangle, and angle-sum of a quadrilateral. Show that in circle geometry there are a lot of isosceles triangles formed with equal radii as special cases of the above. For example



$\angle B = \angle C$
because ΔOAB is isosceles.
$\angle B = 30°$, $\angle C = 30°$
(ext $\angle$ of Δ)

Look out for the equal radii!

- Use the text definitions for an oral session. Students will know most of these already, but segment and sector need to be carefully distinguished. The cyclic quadrilateral is mentioned. The new terms are given now although they will not be used until chapter 13, when they can be repeated.
- Use symmetry as a concept to justify what appear to be commonsense facts about circles. Start with a line of symmetry (through the centre, obviously).

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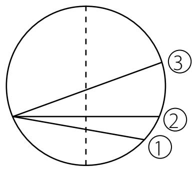
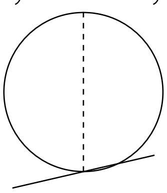


Now, by placing a ruler (representing a chord) show that in positions ① and ③ the chord is divided unequally. Only in ② is symmetry preserved. This implies that it is the perpendicular bisector of the chord that is the line of symmetry.

- Now revert to the text, and follow the argument for two chords. Use the text examples to demonstrate calculations. Follow through with the section on perpendicular bisectors, concentrating now on the text examples.

  Use EX 10A. Reasons for each stage are not required unless specifically asked for. However, verbalising the logic is a good practice: students should not just write an answer "because it looks right". They should be able to articulate reasons.

- Tangents are introduced in the text as the limit of a moving chord. Symmetry may be invoked to show the impossibility of a tangent in the situation illustrated: the "tangent" cuts the circle twice unless it is absolutely symmetrically positioned, perpendicular to the line of symmetry (diameter).

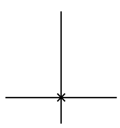
  

  Then follow the text for two tangent situations, and work through the examples. Always emphasize symmetry.

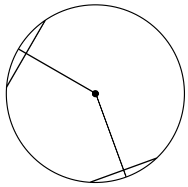
  Use EX 10B.

Resources

A simple piece of equipment may be construed from two thin sticks and two elastic bands:



This may be used on a blackboard circle to demonstrate the equal chords equidistant from centre properly. It is a dynamic demonstration of the rotational symmetry.



Assignments

Homework: EX 10A, questions 8–10 and EX 10B, questions 5 and 6

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Vocabulary	circumference, diameter, radius, arc, tangent segment, sector, cyclic quadrilateral equidistant, point of contact perpendicular bisector, midpoint line of symmetry

ANSWERS

Exercises

EX 10A

1. a) equidistant

   b) equal length

   c) $x = 4, y = 8$

   d) cyclic quadrilateral

2. $p = 3, \quad q = 3, \quad r = 90, \quad s = 8$

3. $x = 90, \quad y = 58$

4. $x = 22, \quad y = z = 68$

5. $\angle OCB = 90°, \quad \angle ADC = 61°$

6. $p = 2, \quad q = 4, \quad r = 9, \quad s = 18$

7. $ON \neq OM$

8. $x = 90, \quad y = 60, \quad z = 7$

9. $x = 40, \quad y = 90, \quad z = 9;$ cyclic quadrilateral

10. a) $MP \neq NP$

    b) $\angle MBP \neq \angle NBP$

    c) $\angle PAM = \angle PBM$

    d) P is the centre of the circle.

EX 10B

1. $x = 90, \quad y = 66, \quad z = 24$

2. $x = 90, \quad y = 115, \quad z = 25$

3. $x = 105, \quad y = 75, \quad z = 15$

4. 39.2 cm (1 d.p.)

5. $a = 28, \quad b = 62, \quad c = d = 31, \quad e = 149, \quad f = 59$

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6. $x = 7, \quad y = 24$

7. a) $\angle UAC = \angle VBC = 90°$

   b) AU is parallel to BV

   c) $\angle AUC = 75°$ (corr $\angle$s)

   d) $\angle UVD = 105°$

8. $x = 30$

9. a) 81°

   b) 73.9 cm (1 d.p.)

10. $\angle AOC = 116°$

EX 10X

1. a) 055° b) 325° c) 235° d) 020°

2. 17 cm

3. 3.3 cm

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Chapter

11 Bounds

This is quite an important topic in terms of general numeracy, although it does not appear extensively in examination papers.

LESSON PLANNING

Objectives

General	To calculate using lower and upper bounds of given measurements
Specific	1. To find the lower and upper bounds of a given measurement 2. To write the range of possible values of a measurement as an interval using inequalities 3. To solve problems involving bounds 4. To be aware of inaccuracy caused by premature rounding off 5. To avoid spurious accuracy by rounding off the numbers displayed on a calculator sensibly
Pacing	2 lessons, 2 homeworks
Links	substitution in formulas

Method

- Class discussion, "What does 8 mean?"
  "What does 8 cm mean?"
  "Is 8.0 cm the same, or not?"

  leading to s.f./d.p. levels of accuracy in measurement, and expressing the set of possible readings as an interval.
- Do board examples using whole numbers and 1 d.p. number, such as the first four examples in the chapter.

  Hint: When finding bounds it is easier to find the upper bound first.

  For example,

  - for measurement of 4.7
    upper bound = 4.75 by writing 5 in the next decimal place
    lower bound = 4.65 by symmetry
  - for measurement of 16.00
    upper bound = 16.005 by writing extra 5
    lower bound = 15.995 by symmetry

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- Students should be clear that
  16 implies nearest whole number
  16.0 implies accurate to 1 d.p.
  16.00 implies accurate to 2 d.p., etc.
- With large whole numbers ending in zeros the accuracy level has to be given because the zeros may be place fillers, i.e. not significant.
  Use EX 11A.
- The second section requires more thoughtful responses where bounds are used in problems.
  Pair and share technique works well here.
  Set EX 11B and circulate, offering hints. Insist on students checking answers as they work, so that any difficulties can be tackled immediately.
  The two examples given under "Calculations" after EX 11A are for reference. The spurious accuracy issue is best dealt with when it arises, although EX 11B, question 7 addresses it specifically,.

Assignments	Homeworks: EX 11A, questions 6, 8, and 10 and EX 11B, questions 4, 8, and 9
Vocabulary	lower bound, upper bound analogue, digital

ANSWERS

Exercises

EX 11A

1. a) 23 385 km b) 27 405 km

2.

Ayesha	146.5 cm, 147.5 cm
Bilal	130.5 cm, 131.5 cm
Cyrus	149.5cm, 150.5 cm
Leena	151.5 cm, 152.5 cm

3.

zinc oxide	32.35 g $\le m <$ 32.45 g
potassium sulphate	28.55 g $\le m <$ 28.65 g
sodium bicarbonate	27.45 g $\le m <$ 27.55 g
Iron filings	45.95 g $\le m <$ 46.05 g

4. a) 64 550 000, 64 650 000 b) 64 595 000, 64 605 000
   c) 64 599 500, 64 600 500 d) 64 599 950, 64 600 050

5. a) 130.7 cm^2, 134.8 cm^2 (4 s.f.) b) 40.53 cm, 41.15 cm (4 s.f.)

6. 58.295 m $\le d <$ 58.305 m

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7. 41.75 m/s, 30.75 m/s

8. a) $m = 4$ (nearest whole number)

   b) $m = 2.68$ (2 d.p. or 3 s.f.)

   c) $m = 150$ (nearest 10)

   d) $m = 70\,000$ (nearest 1000)

9. a) 984.2 m^2 (4 s.f.)

   b) 974.2 m^2 (4 s.f.)

10. a) 197.3 cm^2 (4 s.f.)

    b) 203.2 cm^2 (4 s.f.)

EX 11B

1.	a)	R	b)	S				
2.	0.827, 0.807 (3 s.f.)							
3.	13.2, 14.4 (1 d.p.)							
4.	11.14 cm, 7.87 cm (2 d.p.)							
5.	$A = 42.9, B = 41.6$							
6.	\$ 35 400							
7.	a)	1 min 3.76 s	b)	473.6 m	c)	353 g	d)	43 km/h
8.	a)	63.75	b)	56.7525				
	c)	56.0750 (4 d.p.)	d)	56.0075 (4 d.p.)				
9.	a)	1.96	b)	2.0705 (4 d.p.)				
	c)	2.0820 (4 d.p.)	d)	2.0820 (4 d.p.)				
10.	a)	14 cm (2 s.f.)	b)	67.2 cm				
	c)	$64.94\text{ cm}^2 \le A < 66.585\text{ cm}^2$						
	d)	Premature rounding is wildly inaccurate. In this case answer (b) is not within the possible interval of values of A.						

EX 11X

1. 7.897 cm, 3.957 cm (4 s.f.)
2. 13.84 cm, 14.01 cm (4 s.f.)
3. 0.2857 (4 d.p.)

[Note: For questions 1 and 3 you have to consider all the possible options.]

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Chapter

12 Revision Exercises

These questions cover the topics in the first 11 chapters of this book, with a few Book 9 questions.

There are two questions where students will need to be provided with graphs to work on. The photocopiable sheets are available here.

ANSWERS

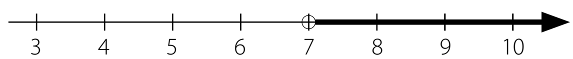
Exercises

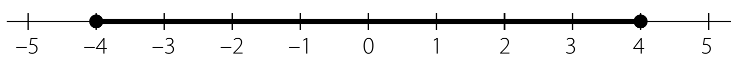
EX 12A

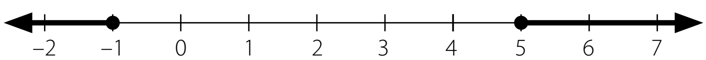
1. a) $\frac{1}{16}$ b) $\frac{1}{64}$ c) $\frac{9}{256}$ d) $\frac{27}{256}$

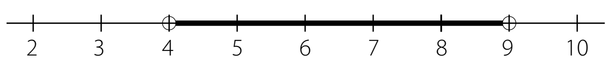
2. a) 1810 cm^2 (3 s.f.) b) 43.2 cm^2 (3 s.f.)

3. a) $x = 5.70$, $x = -0.70$ (2 d.p.) b) $x = -1.35$, $x = -6.65$ (2 d.p.)

4. a) 

b) 

c) 

d) 

5. a) $x \propto y$ [or $y \propto x$] b) $x = \frac{y}{115}$ [or $y = 115x$]

c) $ 200 d) Rs 575

6. $a = 6.5, b = 8, c = 90, d = 16$

7. a) (−1, 8) b) (−0.5, 2.0)

8. a) 8 b) $\frac{1}{9}$ c) $\frac{1}{3}$ d) 2

9. a) $\angle PAB = \angle QBC = 90°$ (tan $\perp$ radius)

$\therefore AP \parallel BQ$ (corr $\angle$s)

b) 18°

10. a) 19.10 b) −2.30

c) 89.99 (4 s.f.) d) 0.7840 (4 s.f.)

EX 12B

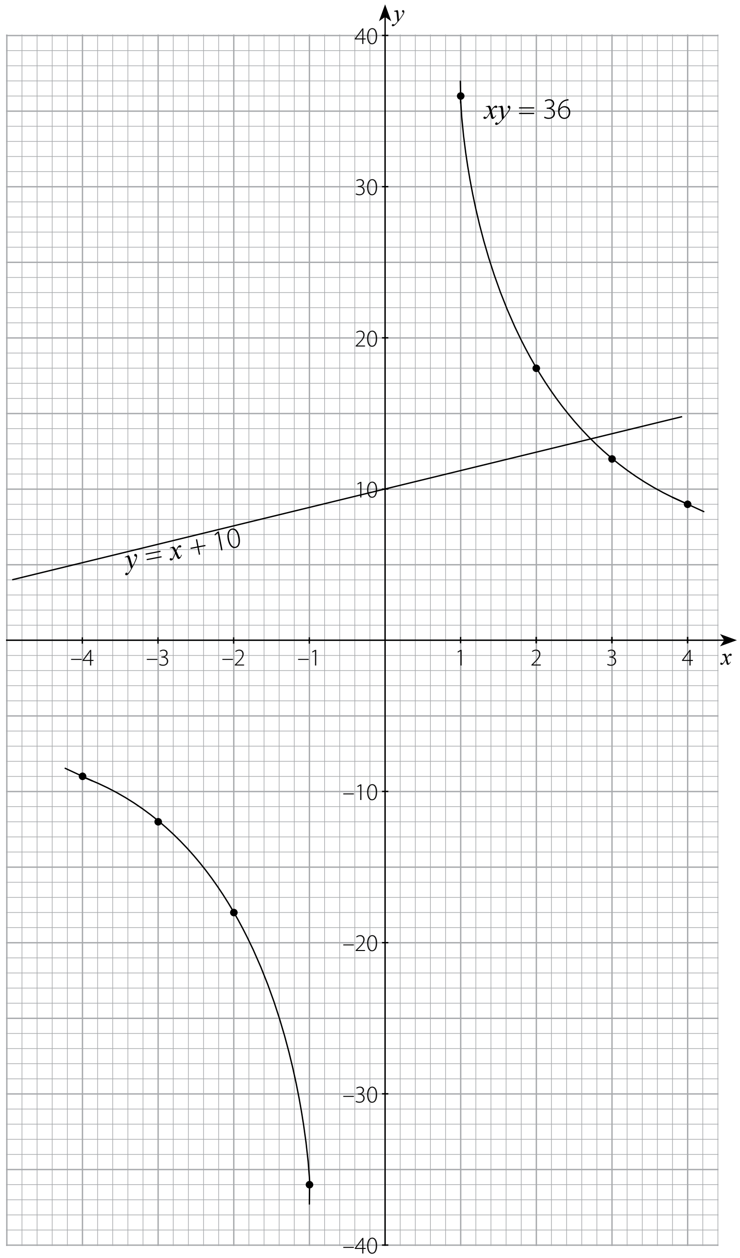
1. a) 14.24 b) 14.24 c) −421 d) −19.69

2. a) $h = \dfrac{3V}{\pi r^2}$ b) $h = \dfrac{2A}{a+b}$

3. a)

x	−4	−3	−2	−1	0	1	2	3	4
$y = \frac{36}{x}$	−9	−12	−18	−36		36	18	12	9

b)



c) rotational symmetry order 2 about the origin. d) $x \approx 2.8$

4. a) $x = -7, x = 1.5$ b) same

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5. a) $P = \{1, 3\}$ b) $Q = \{4, 8, 12, 16, 20, 24, 28, 32, 36\}$

c) $R = \{-10, -9, -8, -7, -6, -5, -4, -3, -2, -1\}$ d) $S = \{-2, -1, 0, 1, 2, 3\}$

6. a) R b) Q c) S d) P

7. a) $x = 1.5, x = -1$ b) $x = 1, x = \frac{5}{3}$

8. a) {0, 2, 3, 4, 5, 6} b) {0, 1, 2, 6}

c) {0, 2, 6} d) {0, 1, 2, 3, 5, 6}

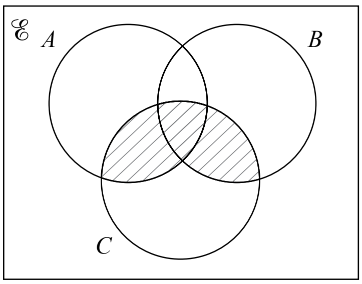
9. spurious accuracy a) 829.8 m b) 8848 m

10. a) $x = 0.09, \quad x = -1.92$ b) $x = 4.05, \quad x = -0.05$ (2 d.p.)

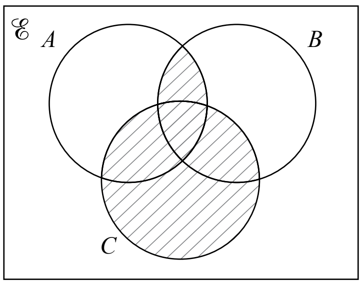
EX 12C

1. a) P b) R c) Q d) S

2. a)

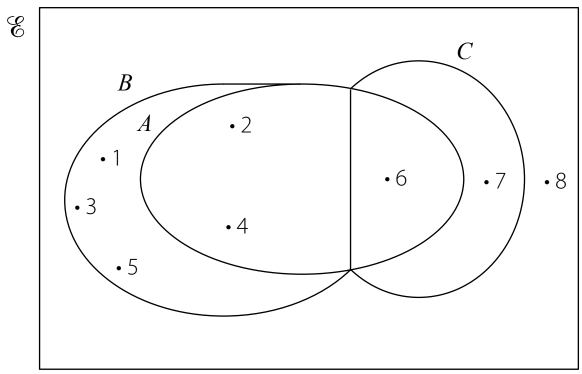


b)



3. a) 10 s b) 16 m c) 10.24 m d) 4 m

4.



5. a) 5 km/h b) 10 km/h

c) 18.9 km/h d) 727 km/h (3 s.f.)

6. $x = 90, y = 64, z = 52$

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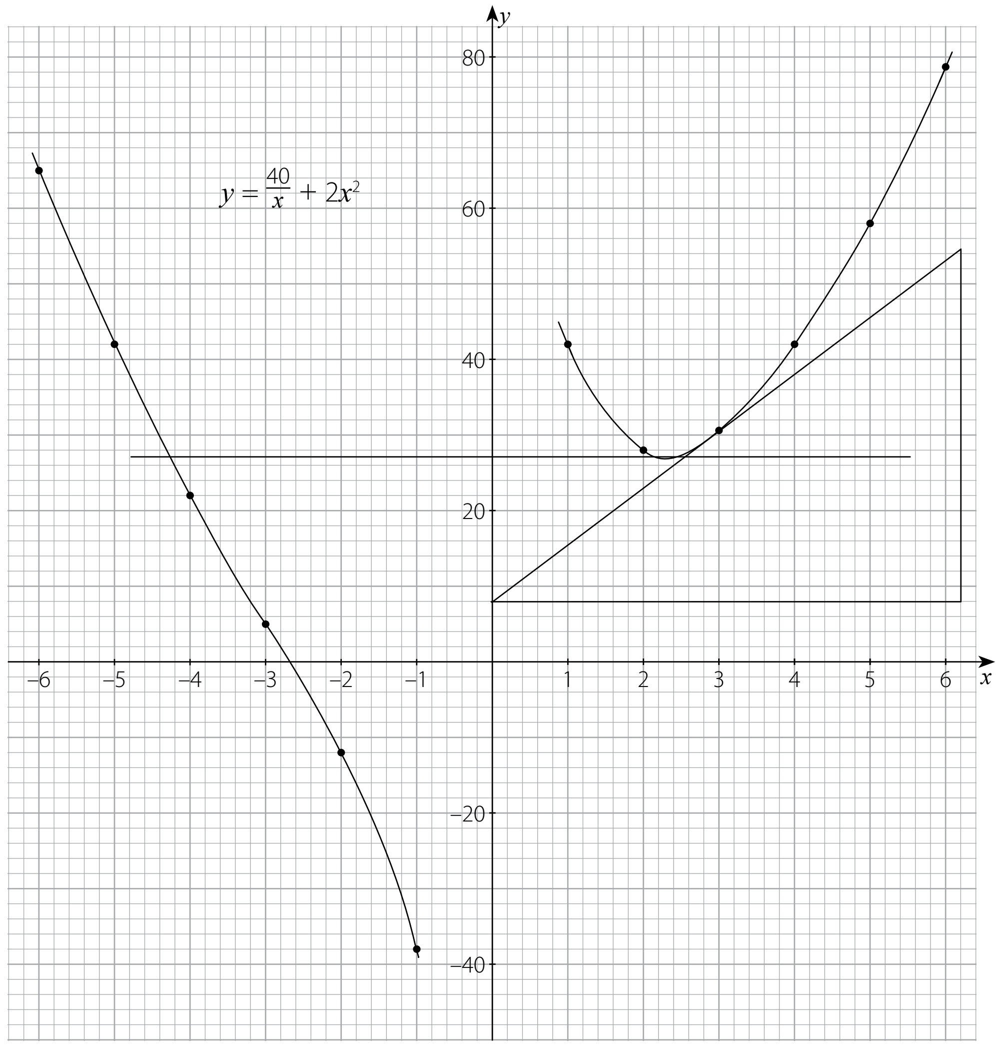
7. a)

x	1	2	3	4	5	6
x^2	1	4	9	16	25	36
$\frac{40}{x}$	40	20	13.3	10	8	6.7
$2x^2$	2	8	18	32	50	72
y	42	28	31.3	42	58	78.7

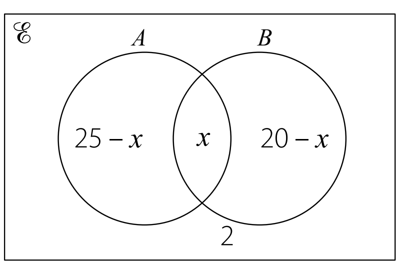
b)



c) gdt ≈ 8 d) $k_{\text{min}} = 28$

8. a) 33.4 cm^2 b) 33.2 cm^2 (3 s.f.)

9. a)



$x = 7$

7 young people owned both these types of phone.

b) 0.45

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10. $AB \neq BC$

EX 12D

1. a) $x = -3, x = -5$ b) $x = -\frac{1}{3}, x = 5$
2. a) 6 b) −2 c) −8 d) 0
3. a) $-\frac{5}{3}$ b) 7 c) $-\frac{1}{4}$ d) $\frac{3}{2}$ (or 1.5)
4. a) yes b) 0.35 c) 0.15 d) 0.35
5. a) $n = 3$ b) $n = 2$ c) $n = \frac{1}{10}$ d) $n = \frac{1}{30}$
6. −0.75
7. 589 cm (or 5.89 m)
8. a) $y \propto \frac{1}{x^2}$, $y = \frac{2430}{x^2}$
   b) $y \propto \sqrt{x}$, $y = 10\sqrt{x}$
9. a) $x = \frac{1}{2}, x = -17$ b) same
10. a) $x = \pm\frac{5}{4}$ b) $x = \frac{7}{2}$

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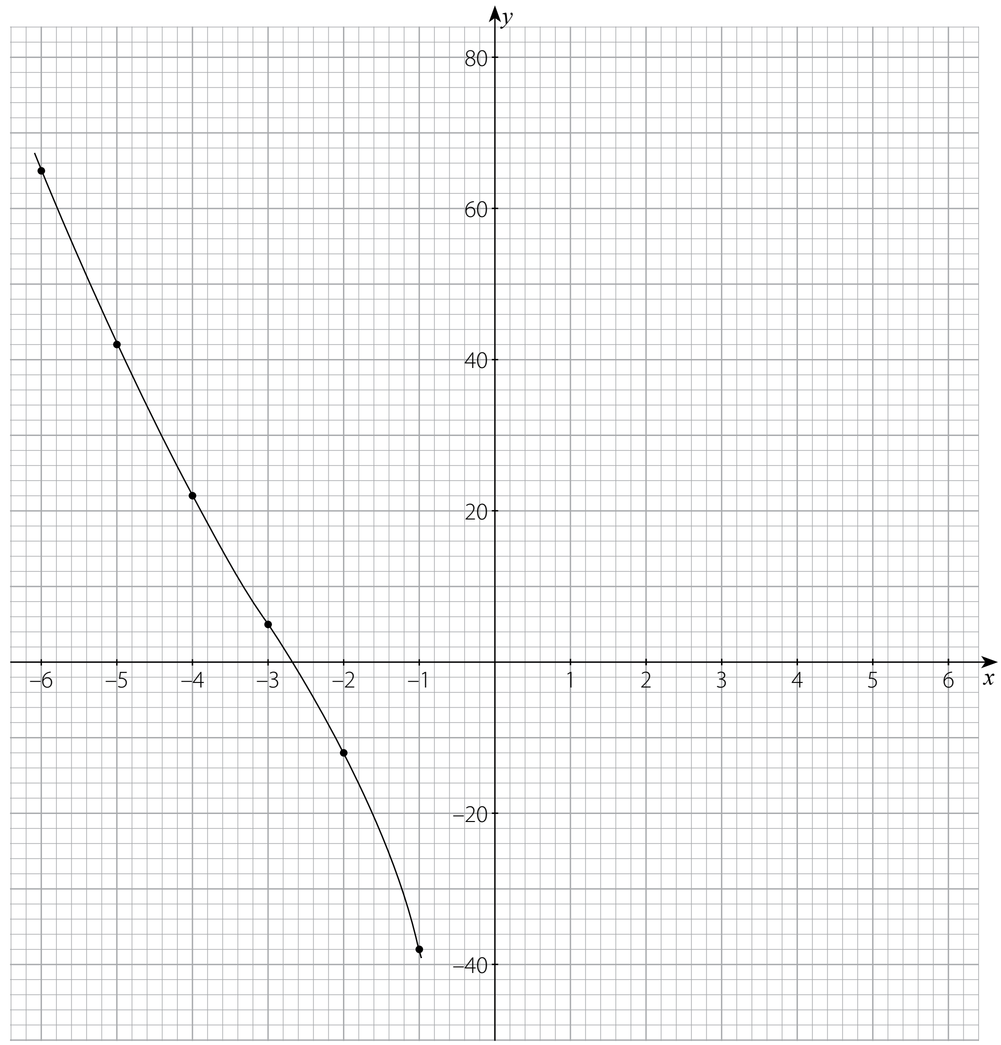
EX 12C, question 7 (worksheet)



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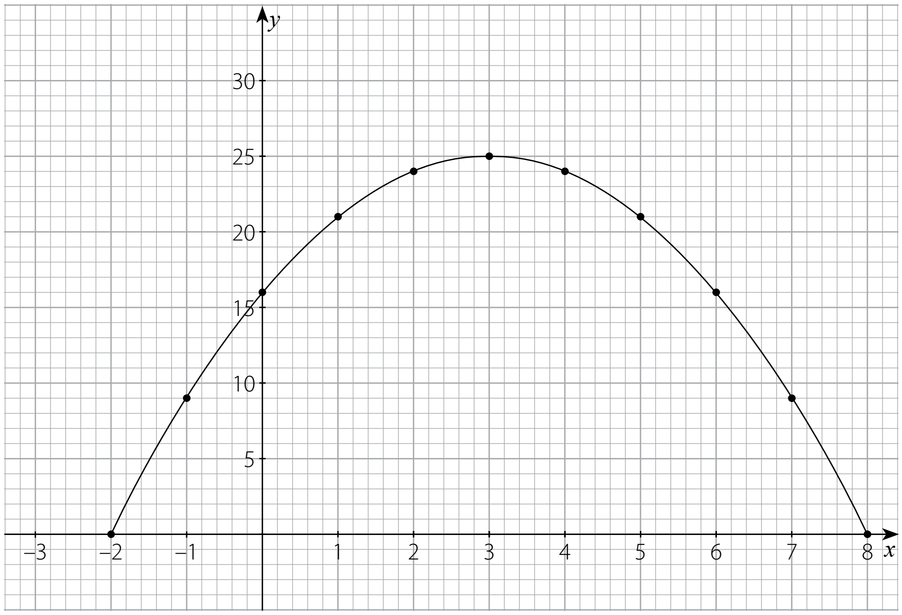
EX 12D, question 2 (worksheet)



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Specimen Examination Paper 1

[for the first half of the year]

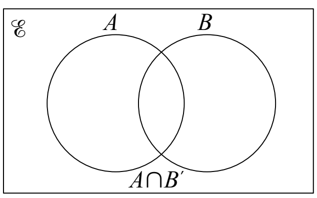
Instructions
The time allowed is 1 h 30 min.
Electronic calculators must not be used.
You will also need pen, pencil, eraser, and ruler.
Try to answer all the questions.
Check your work carefully.
The marks for each question are shown in brackets. [Max marks: 65]

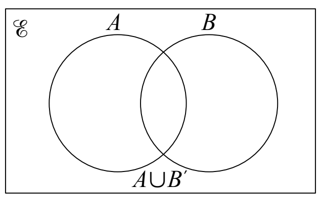
---

1 A school auditorium has 820 seats. If 60% of the seats are occupied, calculate the number of empty seats.

Answer .... [1]

2 Shade the required region on each Venn diagram:





[2]

3 Three dice are rolled together. The dice are fair. Each one has 2 red faces, 1 white face, and 3 blue faces. Calculate the probability that at least one of the dice shows red.

Answer .... [2]

4 a) Find the value of m when $5^m \times 5^3 = 5^9$.

Answer (a) $m =$ .... [1]

b) Find the value of n when $3^n \div 3^4 = \sqrt{3}$.

Answer (a) $n =$ .... [1]

5 Solve the quadratic equation $2x^2 - 11x - 6 = 0$.

Answer $x =$ ...., $x =$ .... [2]

6 Given the points $A(4, -4)$, $B(12, 6)$, $C(2, -8)$ and $D(1, 0)$, and that M is the midpoint of AB, and N is the midpoint of CD, find the coordinates of the midpoint of MN.

Answer (.........., ...) [1]

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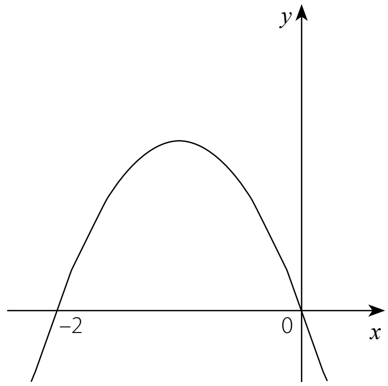
7.



In the sketch the parabola has equation

$Ax = x^2 - y$

Find the value of the constant A and rewrite the equation with y as the subject.

Answer $A =$ .... [1]

$y =$ .... [1]

8 A rocket is orbiting the Earth at a constant speed of 8 km/s.

a) Calculate how far it travels in 1 hour.

b) Write this distance in standard form

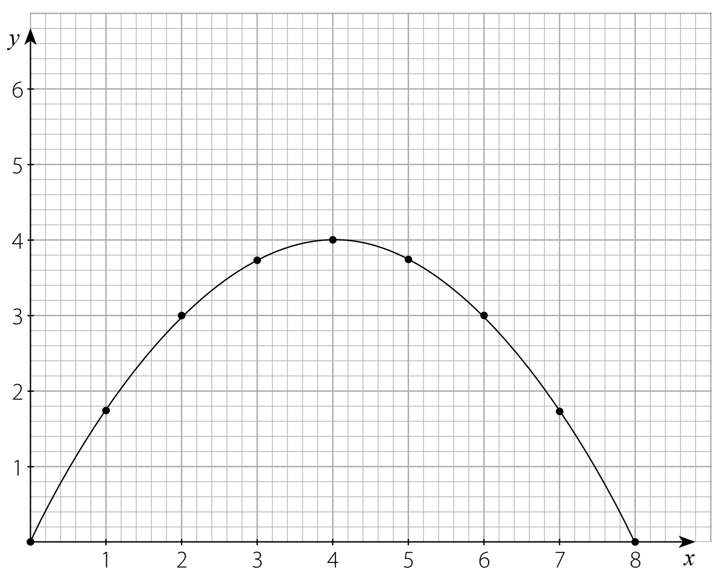
Answer (a) .... [1]

(b) .... [1]

9. Find the length of the straight line from $P(-1, 1)$ to $Q(4, 13)$.

Answer .... [2]

10.



By adding lines to the diagram estimate the gradient of the curve:

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a) where $x = 1.5$, and

b) where $x = 7$

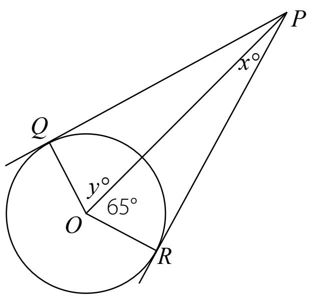
Answer (a) .... [1]

(b) .... [1]

11. The time period, T seconds, of a simple pendulum varies directly as the square root of its length, l metres. If a 4 m pendulum has a period of 4 s, calculate the period of a pendulum 2.25 m in length.

Answer .... [2]

12. In the diagram, O is the centre of the circle. PQ and PR are tangents. Find the values of x and y.



Answer x .... [1]

y .... [1]

13. a) A field has an area of 4900 m^2. Calculate its area in cm^2.

Answer (a) .... cm^2 [1]

b) A map of the field is drawn to a scale of 1:50 000. If the actual distance between two points on the field is 50 m, how far apart are the two points on the map in cm?

Answer (b) .... cm [1]

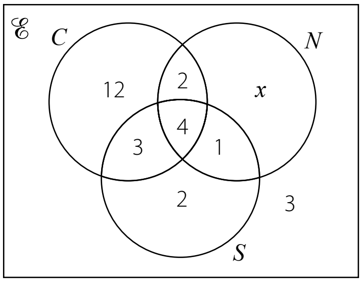
14. A class of 32 students were asked which kind of bread they liked.

C = {students who like chapati}

N = {students who like naan}

S = {students who like sliced bread}

This Venn diagram shows the results:



a) Find the value of x.

Answer (a) $x =$ .... [1]

b) One of the students is randomly selected. Find the probability that this student

i) likes chapati but not naan,

Answer (b) (i) .... [1]

ii) likes only two kinds of bread

Answer (b) (ii) .... [1]

c) A student is chosen at random from those who like chapati. Find the probability that this student also likes sliced bread.

Answer (c) .... [1]

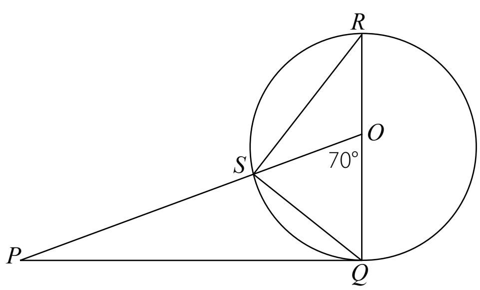
15. In the diagram, O is the centre of the circle.

PQ is a tangent.

PSO is a straight line.

RQ is a diameter.

$\angle SOR = 70°$.



Calculate the size of

a) $\angle OPQ$

Answer (a) $\angle OPQ$ .... [1]

b) $\angle SRO$

Answer (b) $\angle SRO$ .... [1]

c) $\angle PSQ$

Answer (c) $\angle PSQ$ .... [1]

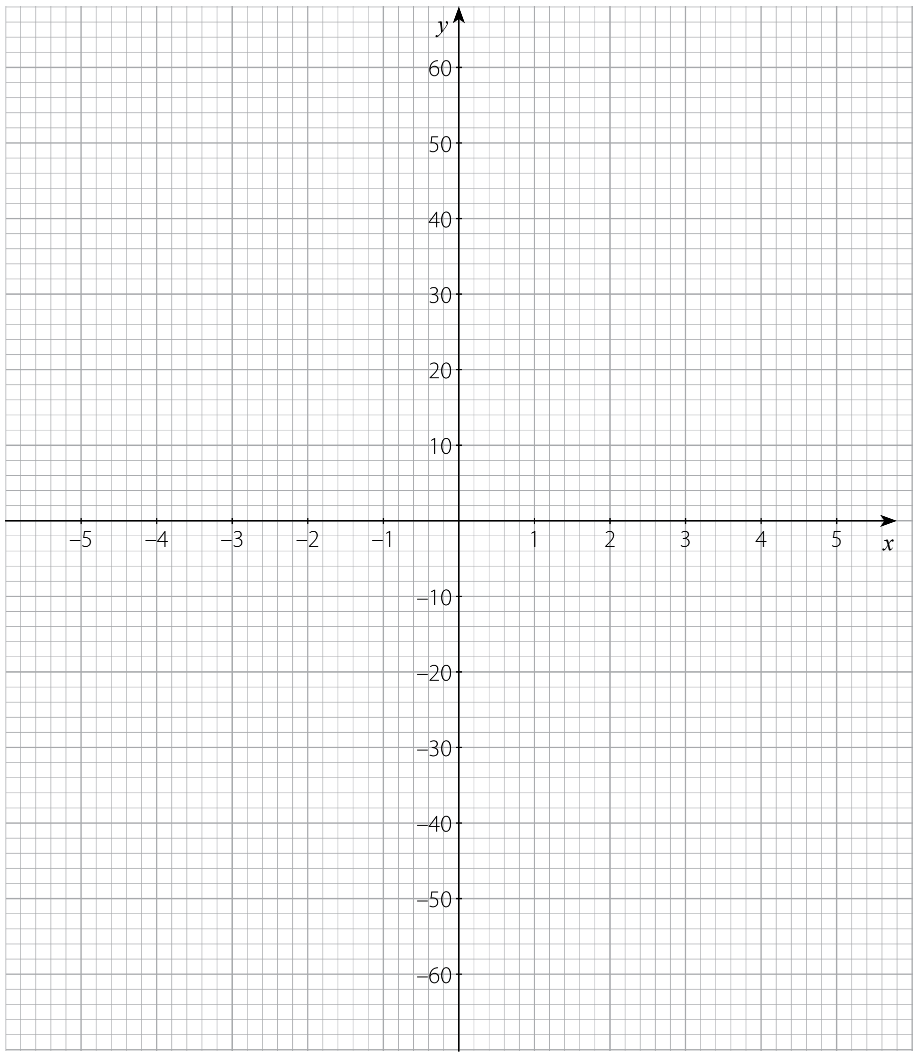
16. a) Complete the table of values for $y = \dfrac{60}{x}$

x	−5	−4	−3	−2	−1	1	2	3	4	5
y			−20				30			12

[2]

b) On the grid draw the graph whose equation is $y = \dfrac{60}{x}$

for $-5 \le x \le 5$ $(x \ne 0)$ [3]



c) Use your graph to solve the equation $\dfrac{60}{x} = 40$.

Answer (c) $x =$ .... [1]

d) (i) On the grid, draw the line joining (5, 0) to (0, –50). [1]

(ii) What is the gradient of a perpendicular to this line?

Answer .... [1]

17. a) Evaluate:

(i) $64^{1/2}$

(ii) $49^{-1/2}$

Answer (a) (i) .... [1]

(ii) .... [1]

b) Simplify

(i) $\frac{3x^2y^4}{6xy}$

Answer (b) (i) .... [1]

(ii) $\left(\frac{x^8y^4}{27x^2y}\right)^{1/3}$

Answer (b) (ii) .... [1]

(iii) $(32x^{10})^{-3/5}$

Answer (b) (iii) .... [1]

c) Show that

$72(2^{-3} + 3^{-2}) = 17$

Write down all the steps of your working.

[2]

d) Work out $(5 \times 10^7)^2$ giving your answer in standard form.

Answer (d) .... [1]

18. The nth term formula of a sequence is $n^2 - 13n + 35$.

a) Find the value of the first two terms.

Answer (a) 1st term .... [1]

2nd term .... [1]

b) There are two terms with the value 5.

Form an equation and solve it to find the position in the sequence of these two terms.

Answer (b) .... and .... [2]

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c) Another sequence has nth term formula $n^2 - 2n - c$. The 5th term of this sequence is equal to the 5th term of the original sequence.
Find the value of c and the value of the common term.

Answer (c) c .... [1]

5th term .... [1]

19. y varies inversely as x^2, and $x = 2$ when $y = 1$.

a) Find the equation connecting x and y.

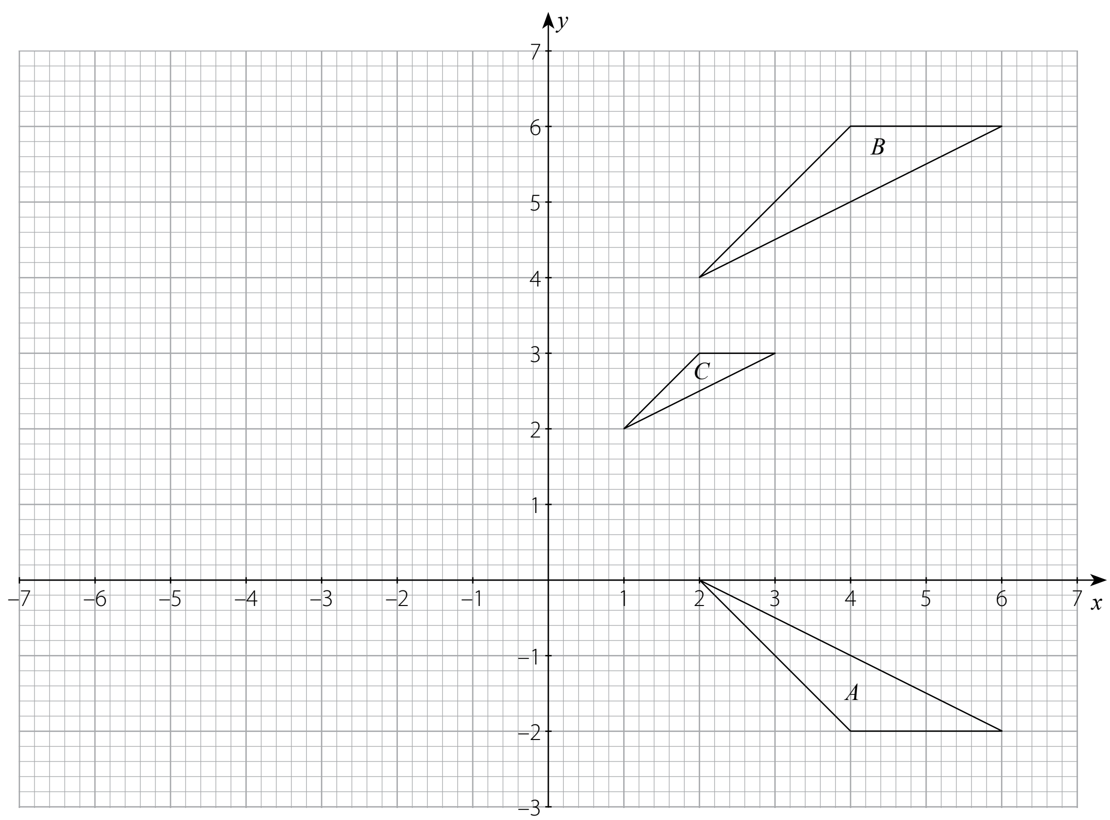
Answer (a) .... [2]

b) Show that for $\frac{2}{5} \leq x \leq \frac{2}{3}$ the value of y lies in the interval $A \leq y \leq B$, where A and B are constants to be found.

Answer (b) $A =$ [2]

$B =$ [2]

20. The diagram shows triangles A, B and C.



a) Describe fully the single transformation that maps triangle A to B.

Answer ....

.... [2]

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b) Describe fully the single transformation that maps triangle C to B.

Answer ....

.... [2]

c) The mapping of triangle $A \longrightarrow$ triangle D is a 180° rotation about the point (0, 2). Draw triangle D on the grid. [2}

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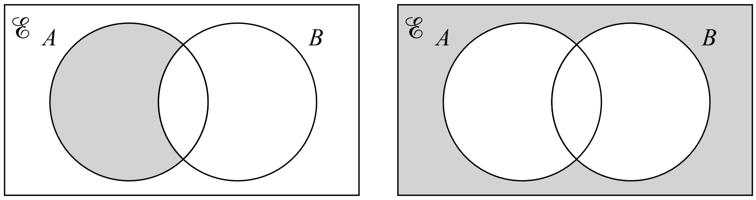
Paper 1—answers and mark scheme:

1. 328 [1, any method]

2. 

[1 each = 2]

3. P(no reds) $= \left(\frac{4}{6}\right)^3 = \left(\frac{2}{3}\right)^3 = \frac{8}{27}$

P(at least 1 red) $= 1 - \frac{8}{27} = \frac{19}{27}$

[1 Method, 1 Answer = 2]

4. a) $m = 6$

b) $n = 4.5$

[1, 1 = 2]

5. $2x^2 - 11x - 6 = 0$

$(2x + 1)(x - 6) = 0$

$x = -0.5, x = 6$

[1 factors, 1 Answers = 2]

6. $M(8, 1), N(1.5, -4)$

midpoint $(4.25, -1.5)$ [1}

7. $Ax = x^2 - y$

$A(-2) = (-2)^2 - 0$ at $(-2, 0)$

$A = -2$

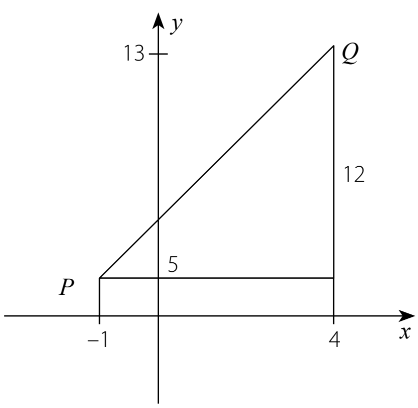
$y = x^2 + 2x$

[1, 1 = 2]

8. a) 28 800 km

b) 2.88×10^4 km

[1, 1 = 2]

9. 

[Method 1, Ans 1 = 2]

10. a) gdt = 1.25 (Allow 1 to 1.5) — 1

b) gdt = −1.5 (Allow −1.75 to −1.25) — 1 = 2

11. $T \propto \sqrt{l}$

$T = k\sqrt{l}$

$l = T = 4;\ 4 = k\sqrt{4} \Rightarrow k = 2$ — 1

$T = 2\sqrt{l}$ = 2

$l = 2.25 = 2\frac{1}{4} = \frac{9}{4};\quad T = 2\sqrt{\frac{9}{4}} = 2 \times \frac{3}{2} = 3$ s — 1

12. $x = 25°$ — 1

$y = 65°$ — 1 = 2

13. a) 49 000 000 cm^2 — 1

b) 0.1 cm — 1 = 2

14. a) $12 + 2 + x + 3 + 4 + 1 + 2 + 3 = 32$

$x = 5$ — 1

b) (i) $\frac{15}{32}$ — 1

(ii) $\frac{6}{32}$ or $\frac{3}{16}$ — 1 = 4

c) $\frac{7}{21}$ or $\frac{1}{3}$ — 1

15. a) $\angle OPQ = 20°$ — 1

b) $\angle SRO = 35°$ — 1 = 3

c) $\angle PSQ = 125°$ — 1

16. a)

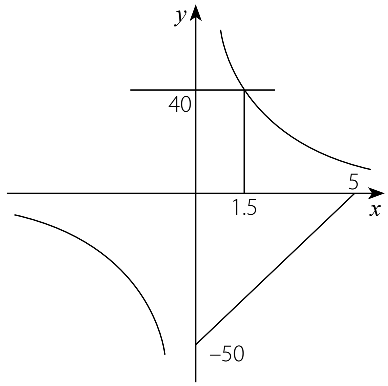
x	−5	−4	−3	−2	−1	1	2	3	4	5
y	−12	−15	−20	−30	−60	60	10	20	15	12

2 all correct

1 if one error/omission = 2

0 otherwise

b)



1 general shape, 1 accuracy, 1 smooth curve = 3

c) $x = 1.5$ [1]

d) (i)

(ii) gdt $= \frac{-50}{5} = -10$, perp gdt $= \frac{-1}{10}$ (or -0.1)

line correct 1, gdt 1 = 2

17. a) (i) 8

(ii) $\frac{1}{7}$

1, 1 = 2

b) (i) $\frac{xy^3}{2}$ or $\frac{1}{2}xy^3$

(ii) $\frac{x^2y}{3}$ or $\frac{1}{3}x^2y$

(iii) $\frac{1}{8x^6}$ or $\frac{1}{8}x^{-6}$

1, 1, 1 = 3

c) $72(2^{-3} + 3^{-2})$

$= 72\left(\frac{1}{8} + \frac{1}{9}\right)$

$= 72\left(\frac{9+8}{72}\right)$

$= 72 \times \frac{17}{72}$

$= 17$ [2, with valid working]

d) $(5 \times 10^7)^2 = 25 \times 10^{14}$

$= 2.5 \times 10^{15}$ [1]

18. a) $t_1 = 1^2 - 13(1) + 35 = 1 - 13 + 35 = 23$

$t_2 = 2^2 - 13(2) + 35 = 4 - 26 + 35 = 13$

1, 1 = 2

b) $n^2 - 13n + 35 = 5$

$n^2 - 13n + 30 = 0$

$(n - 10)(n - 3) = 0$

$n = 10, n = 3$ position 3rd and 10th

1 equation, 1 answer = 2

c) $\cancel{5^2} - 13(5) + 35 = \cancel{5^2} - 2(5) - c$

$-30 = -10 - c$

$c = +20$ 1

$t_5 = -5$ 1 $= 2$

19. a) $y \propto \dfrac{1}{x^2}$

$y = \dfrac{k}{x^2}$

$x = 2, y = 1; \quad 1 = \dfrac{k}{4} \Rightarrow k = 4$ 1

$y = \dfrac{4}{x^2}$ 1 $= 2$

b) When $x = \dfrac{2}{5}, y = \dfrac{4}{\left(\frac{2}{5}\right)^2} = 4 \times \dfrac{25}{4} = 25$ 1 method

When $x = \dfrac{2}{3}, y = \dfrac{4}{\left(\frac{2}{3}\right)^2} = 4 \times \dfrac{9}{4} = 9$ 1 method

$9 \leq y \leq 25$ $= 4$

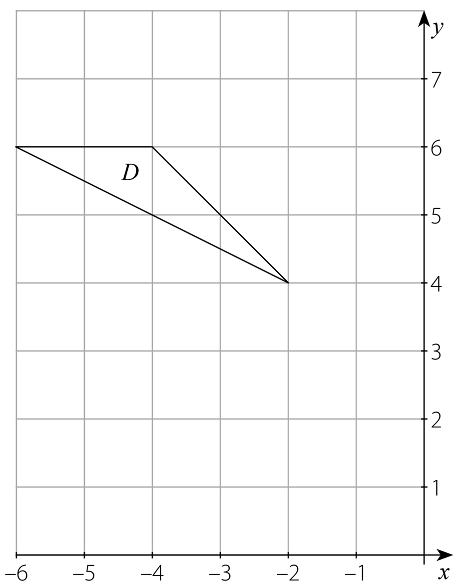
$A = 9$ 1 answer

$B = 25$ 1 answer

20. a) reflection in the line $y = 2$

b) enlargement SF 2, Centre (0, 0)

c)



1 reflection

1 line equation

1 enlargement $= 6$

1 SF & centre

2 drawing on grid

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Specimen Examination Paper 2

[for the first half of the year]

Instructions The time allowed is 2 hours.
Electronic calculators should be used.
You will also need pen, pencil, eraser, ruler, and protractor.
Try to answer all the questions.
Check your work carefully.
The marks for each question are shown in brackets. [MAX 85]

1 Solve the equation $3x^2 + 2x - 14 = 0$ giving your solutions correct to 2 decimal places.

Answer .... [4]

2 A rectangular metal sheet is 30.6 cm long and 24.8 cm wide, measured to the nearest 0.1 cm. Calculate the lower and upper bounds of the area of the metal.

Answer: lower bound = ... cm^2 [3]

upper bound = ... cm^2 [3]

3 Rearrange these formulas to make y the subject in each case:

a) $xy + 5 = x$

Answer (a) y .... [2]

b) $2x + 5 = \sqrt{\frac{y}{3}}$

Answer (b) $y =$ .... [2]

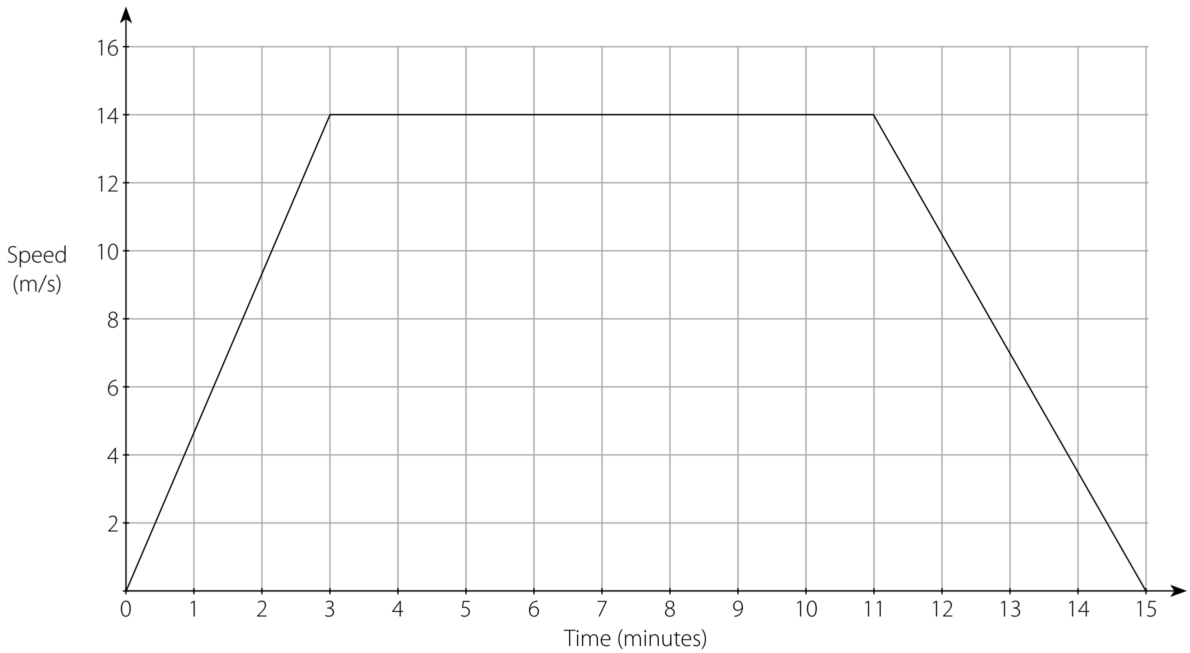
c) $P = \frac{3}{4}(x + y)$

Answer (c) $y =$ .... [3]

d) $\frac{1-y}{1+y} = \frac{x}{2}$

Answer (d) $y =$ .... [3]

4 The diagram shows the speed-time graph of a train as it travels between two stations. The train accelerates for 3 minutes, maintains constant speed for 8 minutes, then slows down to a stop.



a) Write down the time in seconds that the train travels whilst maintaining constant maximum speed.

Answer .... s [1]

b) Find the acceleration of the train (in m/s^2) during the first 3 minutes of its journey, correct to 2 d.p.

Answer .... m/s^2 [3]

c) Calculate the distance (in metres) between the two stations.

Answer .... m [3]

5 Myera, Rania and Sameera are given money in the ratio 11 : 6 : 9.
Myera receives $ 44.

a) Calculate the total amount.

Answer (a) $ .... [2]

b) Myera spends 36% of her $ 44 on some stationery items.
Calculate how much she has left.

Answer (b) $ .... [2]

c) Sameera spends $ 33 of her share on a pair of shoes. She bought them in a sale at 25% discount. Calculate the original price of the shoes.

Answer (c) $ .... [3]

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d) Rania puts $ 20 of her share in the bank. The bank offers compound interest of 1.5% per annum. Calculate the amount Rania will have in the bank after two years if she leaves her money there for the whole two year period. Give your answer to the nearest cent.

Answer (d) $ .... [3]

6 $\mathscr{E} = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$A = \{x : x \text{ is even}\}$

$B = \{x : x \text{ is prime}\}$

$C = \{x : x^2 - 7x + 10 = 0\}$

a) Write down the value of n(B).

Answer (a) n(B) = ... [1]

b) List the two elements of set C by solving the equation.

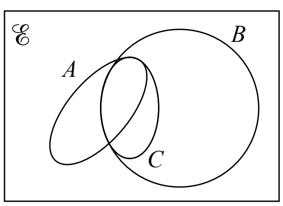
Answer (b) C = {...................., ...} [2]

c) List the elements of A and of B.

Answer (c) A = { .... }

B = { .... } [2]

d) Complete the Venn diagram by writing all the elements of $\mathscr{E}$ in their correct regions.



[3]

e) Complete the following:

i) $B \cap C'$ = {....................} [1]

ii) $4 \in$ .... [1]

iii) n$(A \cup B)'$ = ... [1]

7. A trapezium has parallel sides of length x cm and $(x + 4)$ cm. The distance between the parallels is $(x - 1)$ cm. The area of the trapezium is 90 cm^2.

a) Write down a formula for the area of this trapezium in terms of x.

Answer (a) 90 = ... [2]

b) Show that $x^2 + x - 92 = 0$

Answer (b) .... [3]

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c) Solve the equation $x^2 + x - 92 = 0$, giving solutions correct to 1 decimal place.

Answer (c) $x =$ ...., $x =$ .... [3]

d) Calculate the length of the longest parallel side of the trapezium

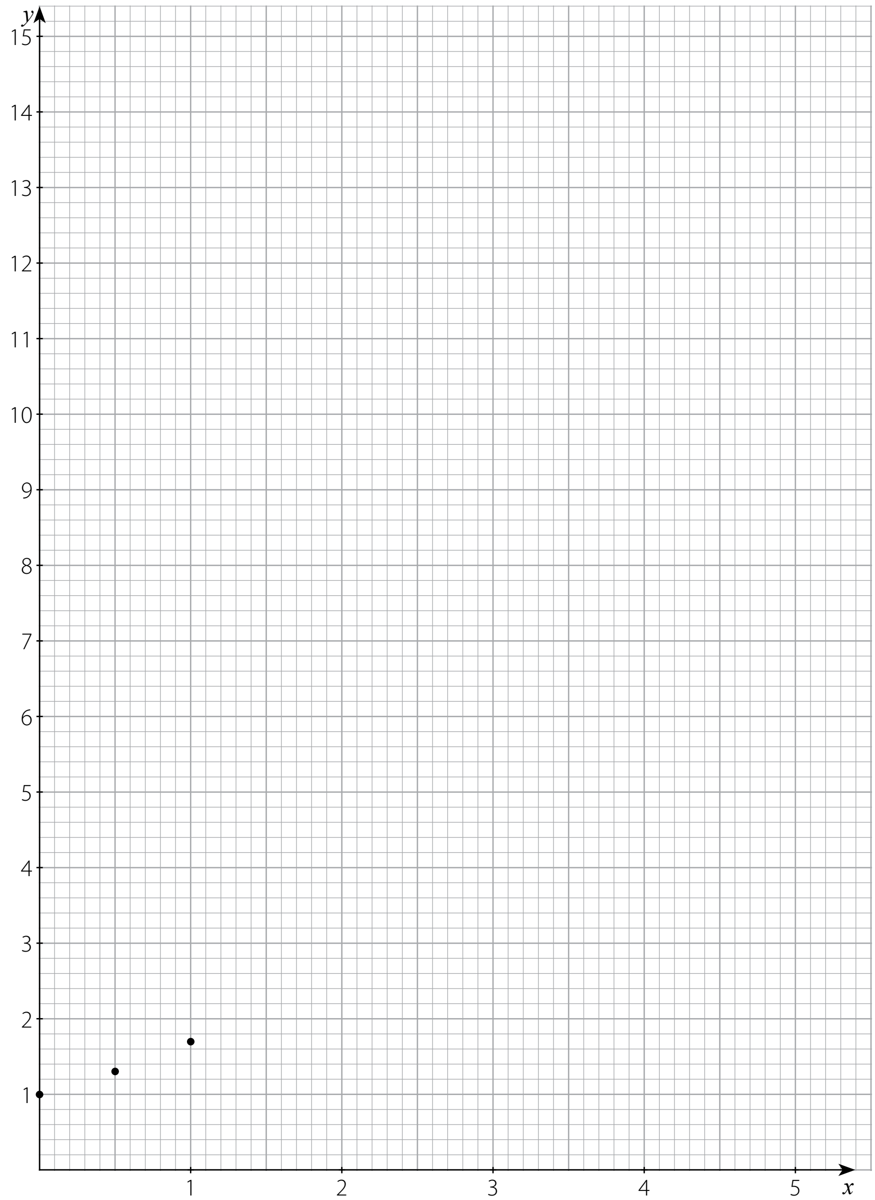
Answer (d) .... cm [1]

8 a) Complete the table of values for the equation $y = 1.7^x$ for $0 \le x \le 5$.
Round off to 1 d.p.

x	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
y	1	1.3	1.7				4.9			10.9	14.2

[3]

b) On the grid, draw the graph of $y = 1.7^x$ for $0 \le x \le 5$. The first 3 points have been plotted already.



[3]

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c) i) On the grid, draw the line $y = 12$. [1]

ii) Use your graph to estimate the solution to the equation $1.7^x = 12$.

Answer (c) (ii) $x =$ .... (1 d.p.) [1]

d) Estimate the gradient of the graph where $x = 3.5$ by drawing a suitable tangent.

Answer (d) .... [2]

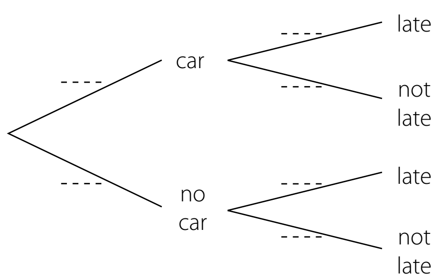
9. Give all your answers as fractions in this question:

Mahnoor travels to school by car with a probability of $\frac{9}{10}$.

If she travels by car, the probability that she is late is only $\frac{1}{12}$.

If she does not use the car, the probability that she is late is $\frac{1}{3}$.

a) Complete the tree diagram:



[3]

b) Find the probability that Mahnoor

i) uses the car and is not late

Answer (b) (i) .... [1]

ii) does not use the car and is late

Answer (b) (ii) .... [1]

iii) is not late whether she uses the car or not

Answer (b) (iii) .... [2]

c) If Mahnoor uses the car and is late, the probability of her getting a late mark is $\frac{19}{20}$. Calculate the probability that none of these three happen, i.e. she does not use the car, is not late, and does not get a late mark.

Answer (c) .... [2]

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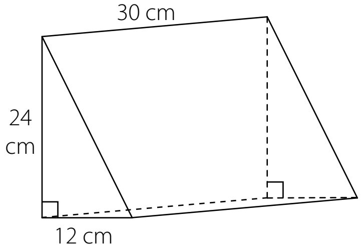
10.



The diagram shows a solid plastic prism with a right-angled triangle as cross-section.

a) Calculate the volume of the prism

Answer (a) .... cm^3 [3]

b) Calculate the length of the diagonal of the sloping face, correct to 2 decimal places.

Answer (b) .... cm (2 d.p.) [3]

c) The prism is melted and the plastic is used to make small plastic balls of diamater 1 cm. How many complete balls can be made from this plastic material?

Answer (c) .... balls [4]

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Paper 2—answers and mark scheme:

1. $3x^2 + 2x - 14 = 0$

   $a = 3, b = 2, c = -14$

   $$x = \frac{-2 \pm \sqrt{4 - 4(3)(-14)}}{6}$$

   $$= \frac{-2 \pm \sqrt{172}}{6}$$ 2

   $= 1.85$ or -2.52 (2 d.p.) 2

   = 4

2. lower $= 30.55 \times 24.75 = 756.1125\ \text{cm}^2$ 3

   upper $= 30.65 \times 24.85 = 761.6525\ \text{cm}^2$ 3

   = 6

3. a) $y = \frac{x-5}{x}$ or $y = 1 - \frac{5}{x}$ 2

   b) $y = 3(2x + 5)^2$ 2

   c) $y = \frac{4P}{3} - x$ or $y = \frac{4P - 3x}{3}$ 3

   d) $y = \frac{2-x}{2+x}$ 3

   = 10

4. a) 480s 1

   b) $\frac{14}{3 \times 60} = 0.08\ m/s^2$ (2 d.p.) 3

   c) $y = \frac{1}{2}(8 + 15) \times 60 \times 14 = 9660m$ 3

   = 7

5. a) \$ 104 2

   b) \$ 28.16 2

   c) \$ 44 3

   d) \$ 20.60 3

   = 10

6. a) 4 1

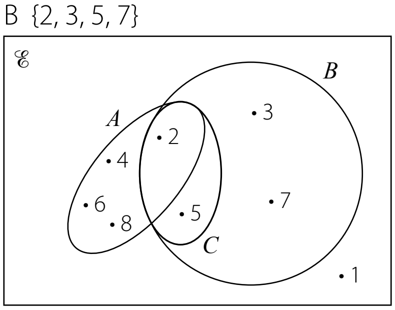
   b) C = {2, 5} 2

   c) A = {2, 4, 6, 8} 2

   B {2, 3, 5, 7}

   d) 3

   deduct 1 for each error or omission

   = 8

e) (i) $\{3, 7\}$ 1

(ii) A 1

(iii) 1 1

$= 3$

7. a) $90 = \frac{1}{2}(2x + 4)(x - 1)$ or equivalent 2

b) $90 = (x + 2)(x - 1)$ or $180 = (2x + 4)(x - 1)$

$90 = x^2 + x - 2$ $\quad$ $180 = 2x^2 + 2x - 4$ $\quad$ 3 Method

$0 = x^2 + x - 92$ etc.

c) $x = \dfrac{-1 \pm \sqrt{1 - 4(1)(-92)}}{2}$

$\dfrac{-1 \pm \sqrt{369}}{2}$ 1

$x = 9.1, x = -10.1$ (1 d.p.) 1 $\quad = 2$

d) 13.1 cm (1 d.p.) 1

$= 9$

8. a)

x				1.5	2	2.5		3.5	4		
y				2.2	2.9	3.8		6.4	8.4		

t

[3 less 1 each error]

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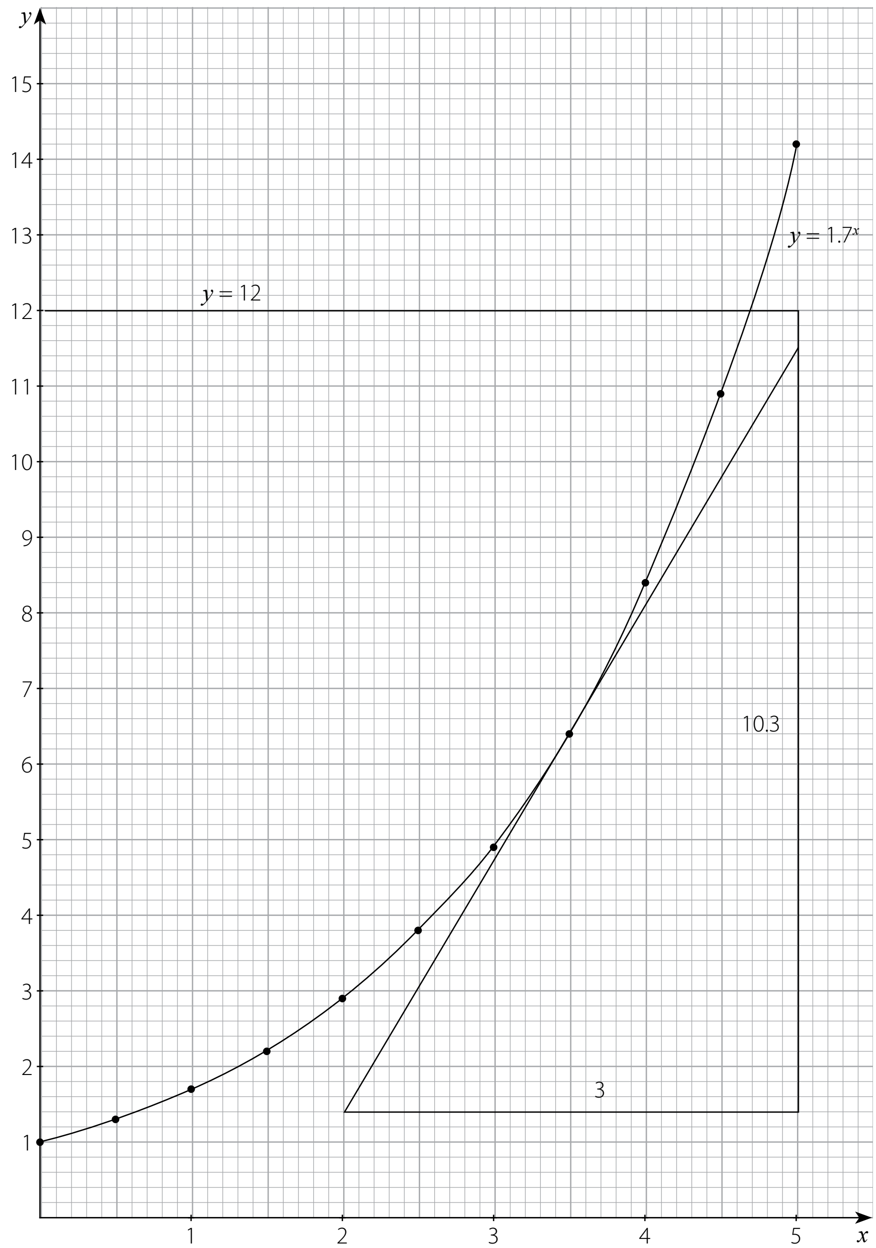
b)



[3 correct plot + smooth curve]

c) (i) line $y = 12$ drawn [1]

(ii) $x = 4.7$ [1]

d) 3.4 (Accept ± 0.3) if tangent shown [2]

[= 4]

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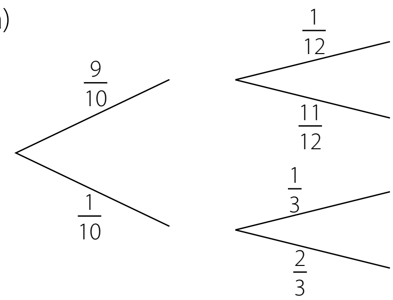
9. a)



$\left[\frac{1}{2}\text{ each (Round down)}\right] = 3$

b) (i) $\frac{9}{10} \times \frac{11}{12} = \frac{33}{40}$ — 1

(ii) $\frac{1}{10} \times \frac{1}{3} = \frac{1}{30}$ — 1

(iii) $\frac{33}{40} + \frac{1}{10} \times \frac{2}{3} = \frac{107}{120}$ — Method 1, Ans 1 = 2

[b) total = 4]

c) P(car, late, mark) $= \frac{9}{10} \times \frac{1}{12} \times \frac{19}{20} = \frac{57}{800}$

P(none of these) $= 1 - \frac{57}{800} = \frac{743}{800}$

[Method 1, Ans 1] = 2

10. a) Volume $= \frac{1}{2} \times 12 \times 24 \times 30 = 4320\text{ cm}^3$

[Method 2, Ans 1] = 3

b) Diagonal$^2 = 30^2 +$ hypotenuse^2

Hypotenuse$^2 = 24^2 + 12^2 = 720$

Diagonal$^2 = 900 + 720$

Diagonal $= \sqrt{1620} = 40.25$ cm (2 d.p.)

[Method 2, Ans 1] = 3

c) Volume of 1 sphere $= \frac{4}{3}\pi(0.5)^3 = 0.5236$

Number of balls $= \frac{4320}{0.5236} = 8250$ (rounded down)

[2 method, 1 answer, 1 rounding down] = 4

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Chapter

13 Angle Properties of Circles

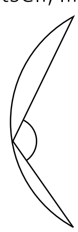
This chapter is a sequel to Chapter 10 on the symmetry properties of the circle. Together, the symmetry and angle properties are used as the basic facts for proving simple theorems and doing calculations of lengths and angles.

LESSON PLANNING

Objectives	
General	To do geometrical calculations and simple formal proofs based upon the angle properties of the circle
Specific	1. To identify the angle subtended by a major or minor arc at the centre of a circle or at its circumference 2. To know and to use the property that the angle at the centre is twice the angle at the circumference when subtended by the same arc 3. To know and to use the properties that angles in the same segment are equal; and that the angle in a semicircle is a right-angle 4. To know and to use the properties of cyclic quadrilaterals: opposite angles supplementary; exterior angle equal to interior opposite angle 5. To solve problems involving calculations or proofs using the circle properties in conjunction with other basic geometric facts
Pacing	3 lessons, 1 homework
Links	Symmetry properties of the circle, geometry of triangles, quadrilaterals and parallel lines.

Method

- Start by explaining the meaning of "subtends". Follow the text, and its examples. It is also relevant to mention that an arc never subtends an angle onto itself, like this:



(Such an angle would be subtended by the major arc completing the circle.)

Follow this with the angle at the centre property. Proof of the simple case is given in the text.

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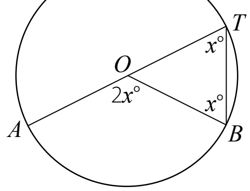
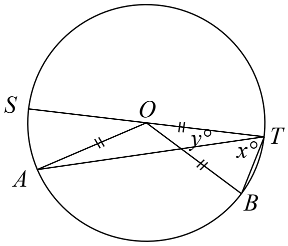
The case when AOT is a straight line can also be given:



ΔOTB is isosceles. (equal radii)

The two equal angles marked $x°$, and the exterior angle marked $2x°$, follow immediately.

If T is allowed to move closer to B, it is more difficult. We draw a diameter through OT to S:



Let $\angle ATB = x$ and $\angle OTA = y$

Then $\angle SOA = 2y$ (ext $\angle$ of isos Δ OAT) and

$\angle SOB = 2x + 2y$ (ext $\angle$ of isos Δ OTB)

$$\begin{aligned} \angle AOB &= \angle SOB - \angle SOA \\ &= 2x + 2y - 2y \\ &= 2x \end{aligned}$$

$\therefore \angle AOB = 2 \times \angle ATB$

The angle at the centre property is verified in this case. Not all students will follow the proofs completely, but it is good if they see them done, however briefly, so they see that the property is not just an experimental result.

- It follows immediately that angles in the same segment are equal, because they are each equal to half the angle at the centre subtended by the same arc. "Same segment" will require explanation: angles are located in the opposite segment to the one containing the arc subtending them. Follow the text diagram. Also the special case of the angle in a semicircle.

  Examples, such as those in the text, need to be demonstrated.

  Use EX 13A.
- Provided the subtending arcs of angles has been thoroughly understood, the cyclic quadrilateral properties are easy to prove, as shown in the text. This leads straight into EX 13B.
- In EX 13M, other geometric properties are required in conjunction with symmetry properties and angle properties of the circle. The questions give fewer indications of the method of solution and usually need a number of logical steps. All students should tackle this exercise.
- In the text, arcs are assumed to be minor unless otherwise stated.

  For example, arc AB means the minor arc AB.

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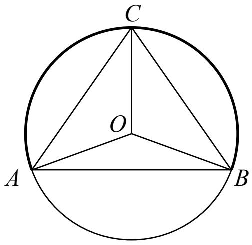
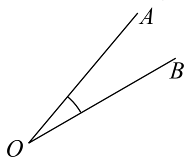
A major arc has to be explicitly stated, in one of two ways:



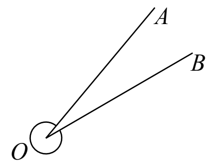
major arc AB or arc ACB

This is analogous to angle notation.

When we write $\angle AOB$ we assume it is acute or obtuse. For a reflex angle we must explicitly write reflex $\angle AOB$



$\angle AOB$



reflex $\angle AOB$

Assignments	suitable for homework: EX 13A, questions 8–10 EX 13B, questions 6, 7, and 9; EX 13M, questions 8–10
Vocabulary	subtends, arc, minor arc, major arc, circumference, segment, semicircle cyclic quadrilateral, supplementary, produced

ANSWERS

Exercises

EX 13A

1. a) arc AB b) arc AE
   c) arc BC d) arc AE
2. a) arc AB b) arc AB c) 50° d) 25°
3. a) major arc AC (or arc ABC)
   b) major arc AC (or arc ABC) c) 100° d) 80°
4. a) 90° b) 30° c) 30° d) 60°
5. a) arc PR b) arc PR
   c) 100° d) ext $\angle$ of Δ
6. $x = 35$, $y = 15$, $z = 15$
7. $x = 30$, $x = 15$, $z = 15$
8. a) major arc AD (or arc $ABCD$)

	b) major arc AD	(or arc $ABCD$)		
	c) 105°	d) 90°		
9.	$a = 40, b = 70, c = 25, d = 65$			
10.	$a = 50, b = 50, c = 96, d = 48$			

EX 13B

1.	a) 70°	b) 140°	c) 20°	
2.	a) 65°	b) 115°	c) 40°	
3.	$x = 45, y = 100$			
4.	$x = 49, y = 131$			
5.	$x = 124, y = 68$			
6.	$x = 30, y = 120$			
7.	a) 55°	b) 45°	c) 80°	d) 100°
8.	a) $x + 35$	b) $x + 33$	c) $(x + 35) + (x + 33) = 180, x = 56$	
	d) $\angle DAB = 91°, \angle ABC = 56°, \angle BCD = 89°, \angle ADC = 124°$			
9.	a) 65°	b) 130°	c) 65°	d) 115°
10.	a) $x = 4$	b) 40°		

EX 13M

1.	37°			
2.	7.5 cm (1 d.p.)			
3.	a) 16°	b) 40°		
4.	a) 44°	b) 80°	c) 36°	d) 126°
5.	$x = 33, y = 114$			
6.	a) 29°	b) 122°		
7.	74°			
8.	17°			
9.	$x = 38, y = 52$			
10.	a) 60°	b) 60°	c) 30°	d) 46°
	e) 46°	f) 104°		

EX 13X

1. $\angle ACB = 90°$ (∠ in semicircle)

   $\angle BDE = 90°$ (given)

   $\therefore \angle ACB = \angle BDE$

   $\therefore BDEC$ is cyclic (ext ∠ of cyclic quad)

2. Using equal tangents from each external point P, Q and R,

   we have $a + b = 12$

   $a + c = 11$

   $b + c = 13$

   Solving, by substitution, gives $a = 5$, $b = 7$, $c = 6$ cm

3. Join O to the point of contact of AB with smaller circle, forming a right angle at the midpoint of AB.

   Then, use Pythagoras. $AB = 24$ cm

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Chapter

14 Areas and Perimeters

This chapter revises areas of standard shapes, especially the less well known, and introduces area and perimeter of sectors of circles.

LESSON PLANNING

Objectives

General	To solve problems involving area and perimeter of standard shapes, including arc lengths and areas of sectors of circles
Specific	1. To know and use the formulas for calculating areas of triangles, quadrilaterals and circles 2. To know and use the formulas for calculating arc lengths and areas of sectors of circles 3. To solve problems involving the perimeters of standard shapes and composite shapes, including volumes of prisms with a composite shape for cross-section
Pacing	3 lessons, I homework
Links	Pythagorean triples, trig ratios

Method

- It should not be necessary to teach the basics again. The triangle and quadrilateral formulas and the way they relate to each other was given in Book 9, Chapter 7. However, it may be necessary to spend a little time on kites and rhombuses, and the SAS formula for triangles. EX 14A, questions 1–5 is revision, and could be omitted or used diagnostically.
- Question 5 is a bridge to the new work. Where half, quarter, and a third of circle are intuitive, we extend this idea to any fraction. Follow the text for arc length and area of sector. Clearly, these are in direct proportion to the angle at the centre.

  "If we double the angle, what happens to the arc length?"

  "If we double the angle, what happens to the area of the sector?"

  $$\frac{\text{angle at centre}}{360} = \frac{\text{arc length}}{2\pi r} = \frac{\text{area of sector}}{\pi r^2}$$

  These fractions (or ratios) are equal.

  Angle, arc, and area vary directly as each other.

  Complete the rest of EX 14A, questions 6–10.
- Composite shapes is a simple enough concept to grasp. Two examples in the text are given; one by addition, the other by subtraction. Other examples can be given.

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Insist on clear explanations from students:

"Communicate your thought process."

The example given in "Composite shapes" is really quite difficult at this stage. Some easier examples are recommended first. Remind students of the volume of a prism, before setting EX 14B.

Resources	Blackboard (or whiteboard) compasses
Assignments	EX 14A, questions 1–5 or EX 14B, questions 7, and 9
Vocabulary	arc, sector, direct proportion, vary directly as

ANSWERS

Exercises

EX 14A

1.	a)	16.5 cm^2	b)	21.1 cm^2	c)	17.8 cm^2	d)	6 cm^2
2.	a)	48 cm^2; 28 cm	b)	160 cm^2; 60 cm				
3.	a)	49 cm^2	b)	15 cm^2	c)	18 cm^2	d)	45 cm^2
4.	a)	120 cm^2; 52 cm	b)	336 cm^2; 100 cm				
	c)	49 cm^2; 56 cm	d)	128 cm^2; 64 cm				
5.	a)	95.0 cm^2	b)	113 cm^2	c)	50.3 cm^2	d)	16.8 cm^2
6.	a)	368 cm^2	b)	185 cm^2	c)	431 cm^2	d)	257 cm^2
7.	a)	34.6 cm	b)	26.7 cm	c)	12.6 cm	d)	8.38 cm
8.	a)	40.8 cm	b)	16.1 cm	c)	20.5 cm	d)	51.5 cm
9.	a)	178 cm	b)	182 cm				
10.	a)	28.6°; 81 cm^2	b)	52.1° ; 220 cm^2				

EX 14B

1.	a)	27.5 cm	b)	26 cm^2				
2.	5.46 m (3 s.f.)							
3.	34.4 cm^2							
4.	a)	96 cm^2	b)	73.7°	c)	250 cm^2	d)	1:2.6
5.	a)	628 cm^2 (3 s.f.)	b)	6280 cm^2 (3 s.f.)				
	c)	20.9 cm (3 s.f.)	d)	329 cm (3 s.f.)				
6.	1360 cm^2							
7.	106 cm							

8. $ 201.60

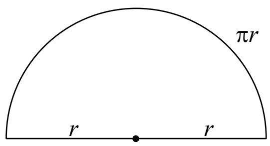
9. 1267 mm^3

10. a) 12.6 cm^2 b) 8 cm^2 c) 66.3 cm^2 d) 50.3 cm

EX 14X

1. P = arc + diameter

$P = \pi r + 2r$



$P = r(\pi + 2)$

$r = \dfrac{P}{\pi + 2}$

Area, $A = \dfrac{1}{2}\pi r^2$

$= \dfrac{1}{2} \times \pi \times \left(\dfrac{P}{\pi + 2}\right)^2$ substituting for r

$= \dfrac{\pi}{2}\left(\dfrac{P}{\pi + 2}\right)^2$ as required.

2. length of wire $= 3 \times 10 = 30$ cm

circumference $= 2\pi r = 30$

$r = \dfrac{15}{\pi}$

Area, $A = \pi r^2$

$A = \pi\left(\dfrac{15}{\pi}\right)^2$

$A = \dfrac{225}{\pi}$ cm^2 as required.

3. perimeter, $P = \dfrac{60}{360} \times 2\pi r + 2r$

$= \dfrac{\pi r}{3} + 2r$

$\therefore\ 3P = \pi r + 6r$

$= r(\pi + 6)$

$\therefore\ r = \dfrac{3P}{\pi + 6}$ as required.

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Chapter

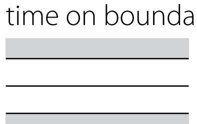
15 Loci and Constructions

This chapter attempts more significant work on the topic than previously. Set notation is used extensively in describing loci, and problems of multiple loci are solved. Some of these may involve scale drawing, compass points, or 3-figure bearings.

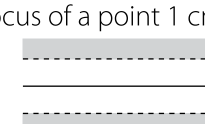
LESSON PLANNING

Objectives

General	To solve problems using loci and standard geometrical constructions by accurate drawing
Specific	1. To construct a triangle given three sides 2. To construct other simple shapes from given information 3. To construct the locus of a point less than, equal to, or greater than a given distance from a fixed point 4. To illustrate the locus of a point less than, equal to, or greater than a given distance from a fixed line or line segment 5. To construct the locus of a point equidistant from two fixed points 6. To construct the shortest distance from a given point to a fixed line 7. To construct the locus of a point equidistant from two fixed intersecting lines 8. To describe the locus of a point using set notation, including cases of inequalities 9. To use the conventional notation of shading unwanted regions, and indicating unwanted boundary lines by broken lines 10. To solve problems by accurate drawing, to scale if necessary
Pacing	4 lessons, 2 homeworks
Links	Geometry of triangles and quadrilaterals, straight line equation, inequalities
Method	• Depending on the level of the students, the Reminders section at the start of the chapter may be run through slowly or quickly. It is worth spending a little time on boundaries, e.g. for the locus of a point 1 cm from a line:



not more than 1 cm
less than or equal to 1 cm
i.e. boundary acceptable



up to 1 cm
less than 1 cm
i.e. boundary not acceptable

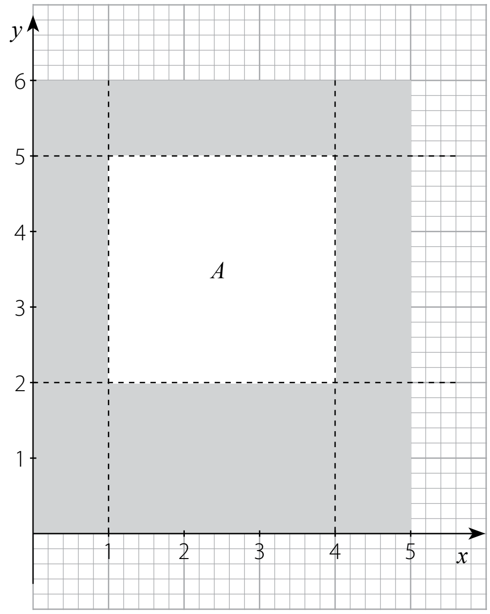
Shading out unwanted regions is also standard and should be made explcit.

EX 15A, questions 1–7 may be used.

- Set notation, introduced in Chapter 1, is used here to indicate loci: lines and regions on graphs. Students need to know their straight line equations and inequalities. Unpack the notation, e.g.

$A = \{(x, y) : \underbrace{1 < x < 4}_{x \text{ is strictly between 1 and 4}}, \underbrace{2 < y < 5}_{y \text{ is strictly between 2 and 5}}\}$

points such that



The steps are:

- Draw the grid.
- Draw the broken boundaries of x.
- Draw the broken boundaries of y.
- Shade out unwanted regions.
- Label the locus A.

Remind students also how to code inclusion/exclusion at end point of a line, e.g.

Set rest of EX 15B, questions 8–10.

- Sloping lines with loci on one side or the other do need careful explanation.

The position of the line is the first step and by now should not be difficult. Decide whether to draw it continuous or broken. Now which side is the acceptable region?

Students tend to guess, but it is not always obvious.

"What do they do when looking for oil?" - Do a test drill.

Use the origin for a test drill.

This is the basis of the text example.

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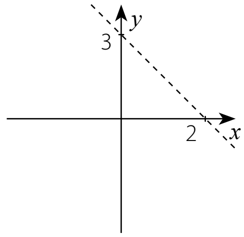
Here is another example: $3x + 2y < 6$

- Sketch the line $3x + 2y = 6$, using broken line because equality is excluded.



- Test drill at the origin (0, 0): $3(0) + 2(0) < 6$

$$< 6$$

True.

This is on the acceptable side of the line.

Shade out the other side.

- But, good technique to check the other side.

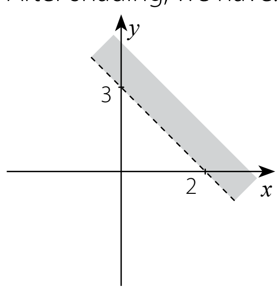
Choose any simple point there, (2, 3) say

$$3(2) + 2(3) < 6$$
$$6 + 6 < 6$$
$$12 < 6$$

False.

This confirms that the point is in the unacceptable region.

- After shading, we have:



locus unshaded region

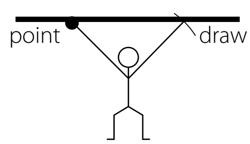
Set EX 15B, questions 1–4.

- The rest of EX 15B, questions 6–10 should not require further explanation.

Resources

Geometrical instruments (for students)

Board compasses (for teacher). If not available the teacher can become an effective pair of compasses by using two straight arms: one finger becomes the point, the other hand holding chalk or marker becomes the pencil. Holding arms at fixed angle the teacher can inscribe arcs on the board. It takes practice but is effective!



Assignments

EX 15A, questions 1–4 using the Reminders for reference without any explanation may be suitable homework for an excellent class; otherwise EX 15A, questions 5–7.

Also EX 15B, questions 9 and 10.

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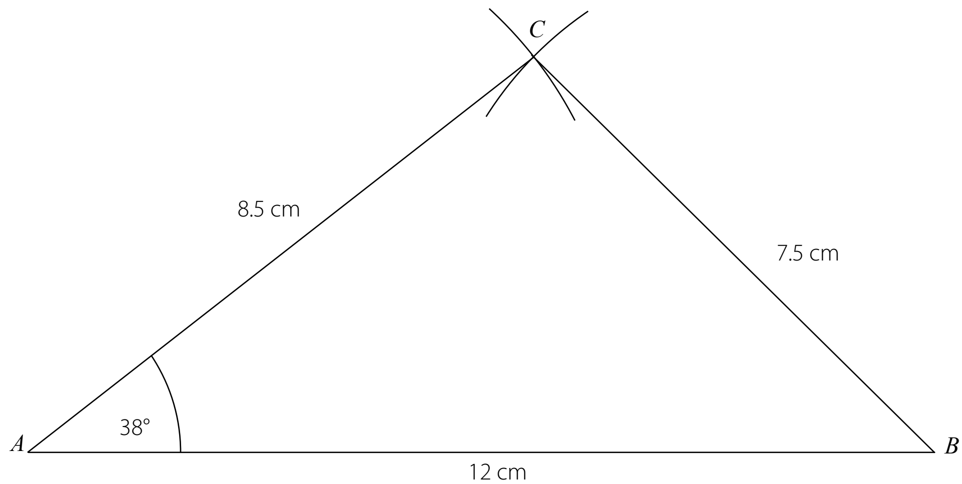
Vocabulary	construct, draw a diagram, locus

ANSWERS

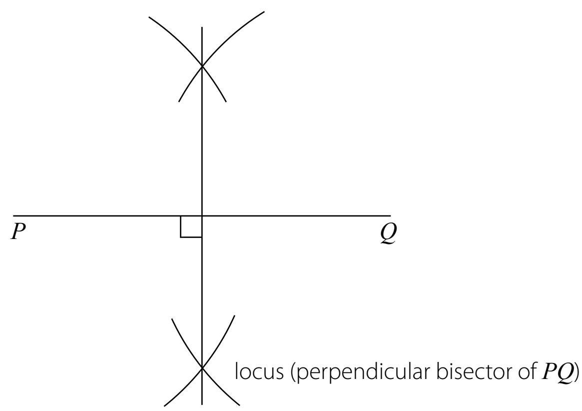
Exercises

EX 15A

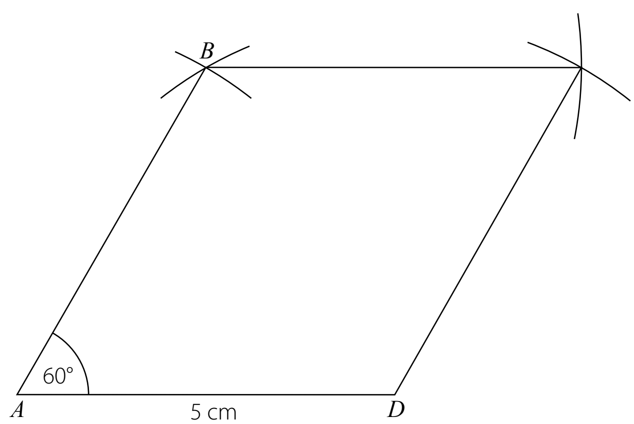
1.



2.



3.



[Treat as two equilateral triangles.]

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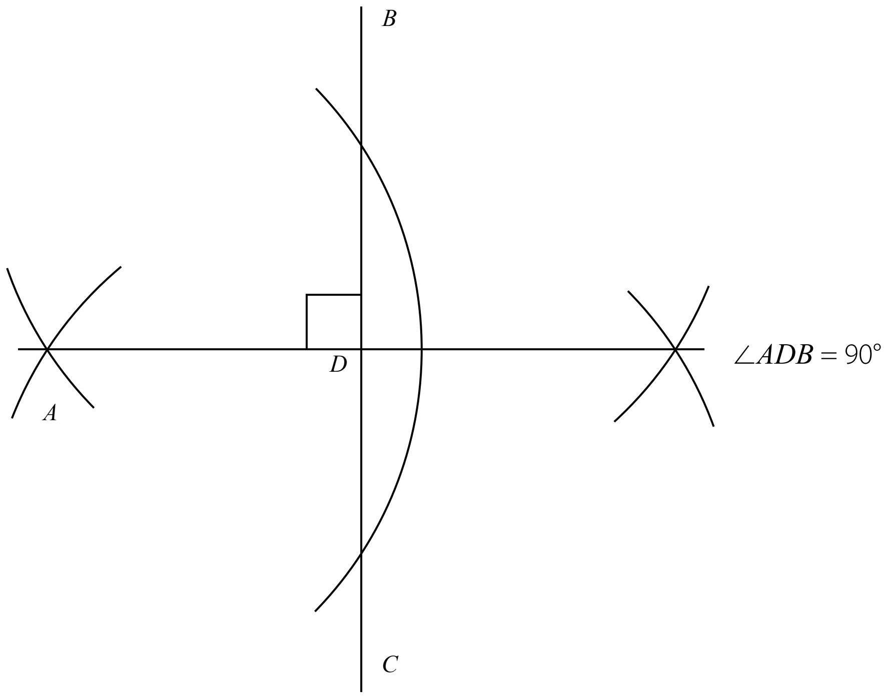
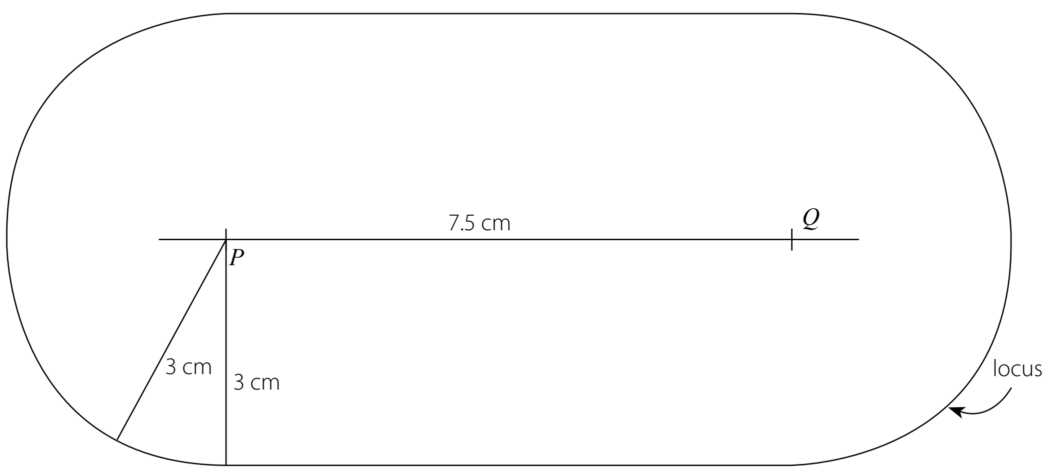
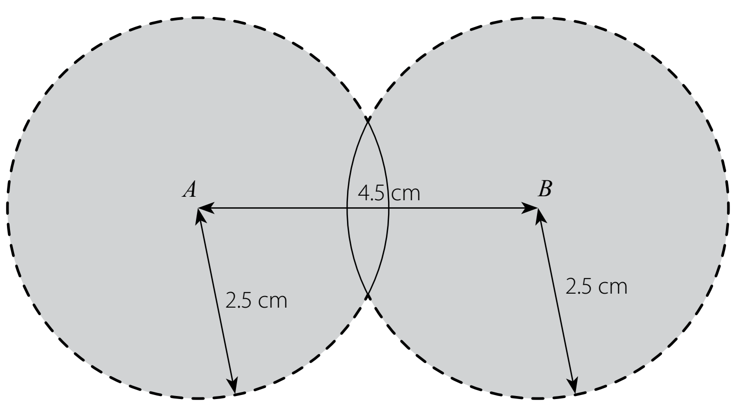
4.



5.



6.



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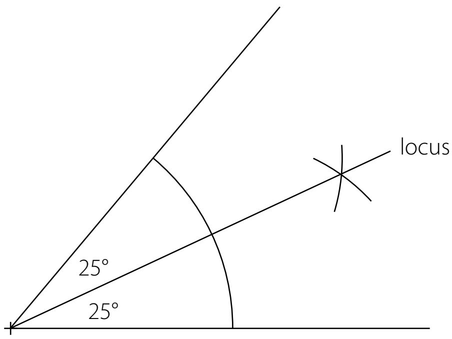
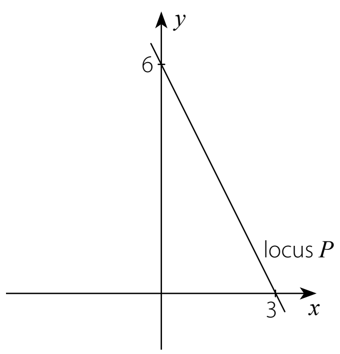
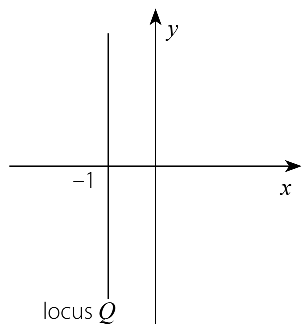
7.



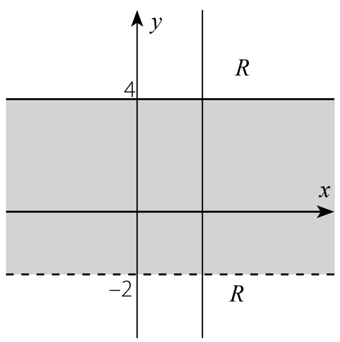
8. a)



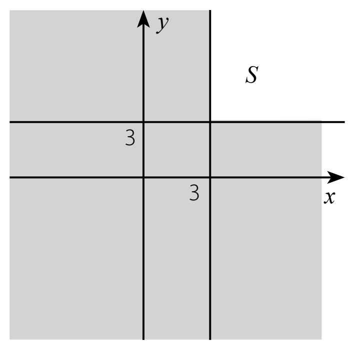
b)



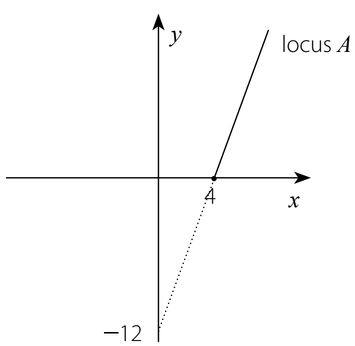
c)



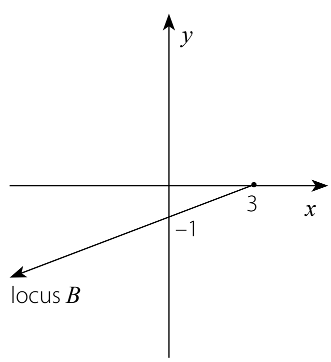
d)



9. a)



b)



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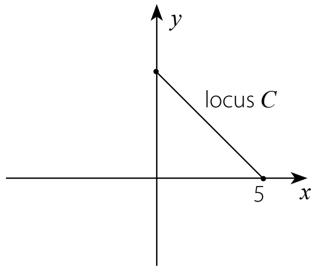
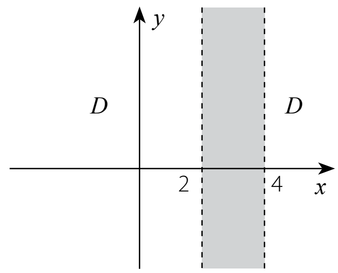
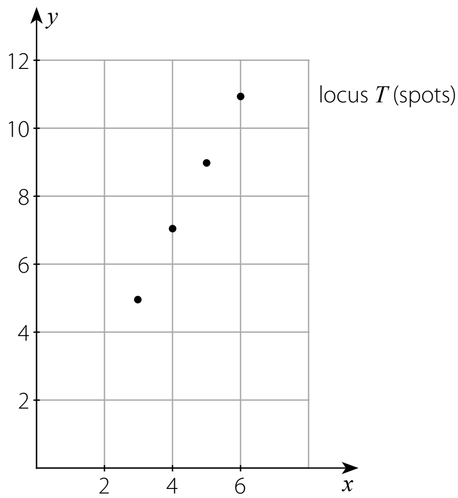
c)


y
locus C
5
x

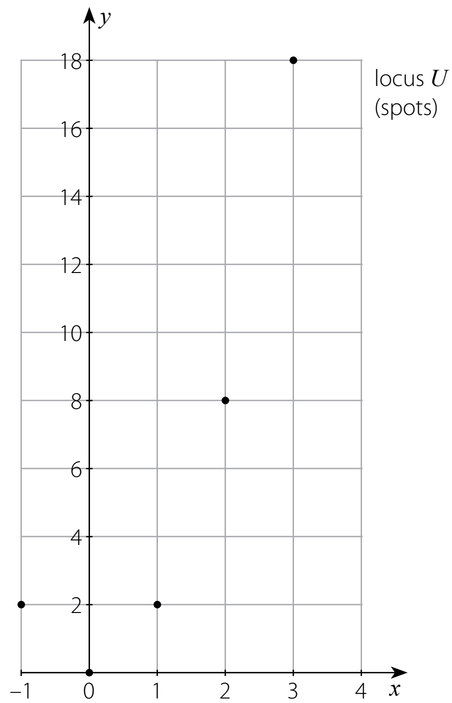
d)


y
D
D
2
4
x

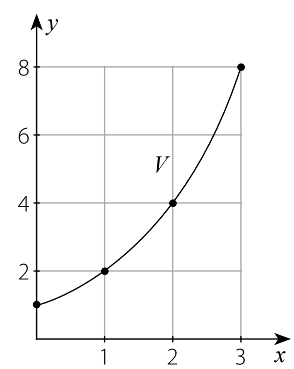
10. a)


y
12
10
8
6
4
2
2
4
6
x
locus T (spots)

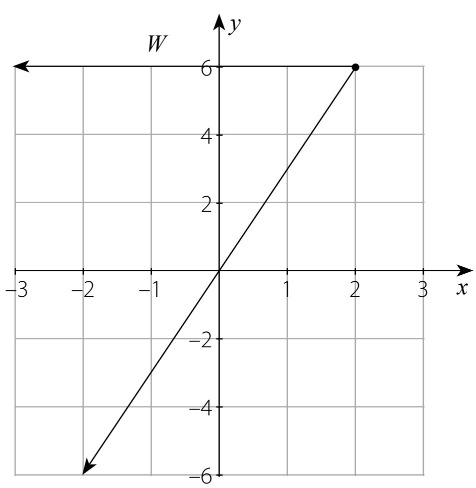
b)


y
18
16
14
12
10
8
6
4
2
–1
0
1
2
3
4
x
locus U
(spots)

c)

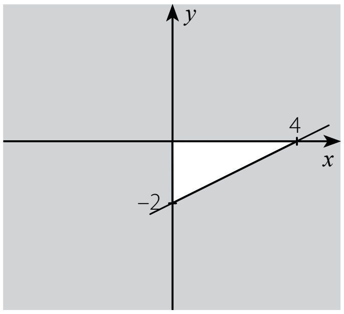

y
8
6
4
2
V
1
2
3
x

d)

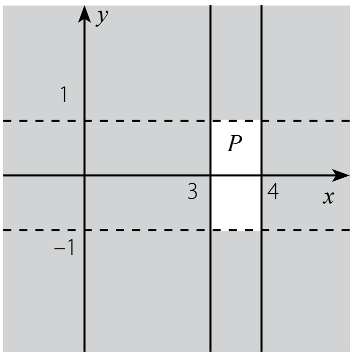

y
W
6
4
2
–2
–4
–6
–3
–2
–1
1
2
3
x

EX 15B

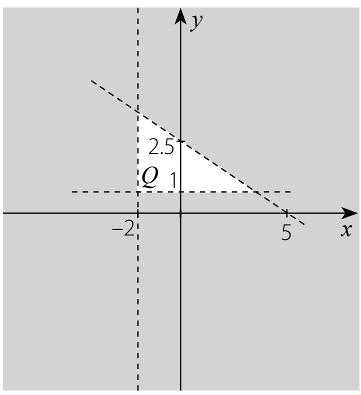
1.



2.

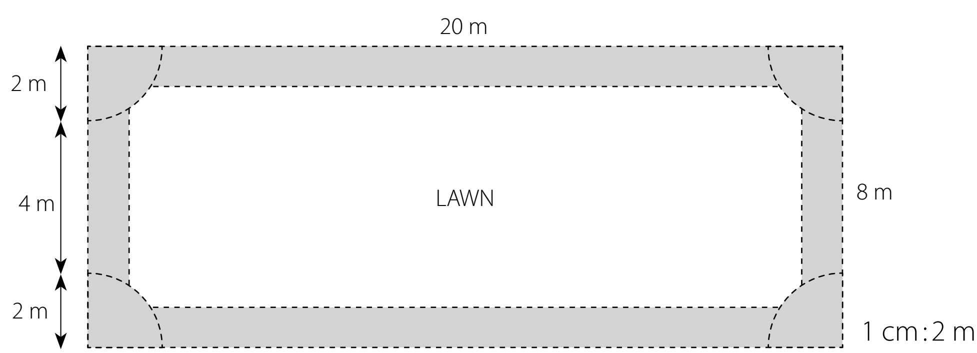


3.



4. $T = \{(x, y) : 5y \leq 3x + 12,\ 5y < -3x - 12,\ x < 6\}$

5.



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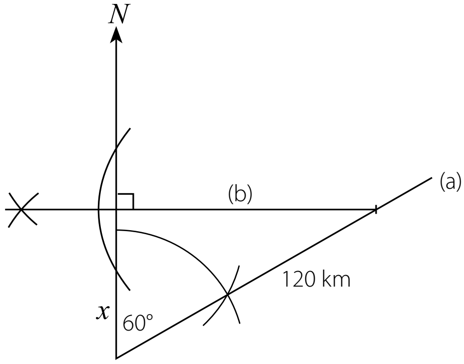
6.

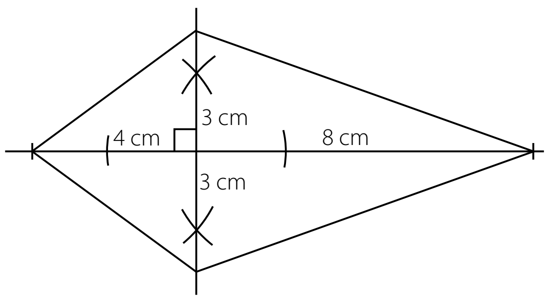


1 cm : 20 km

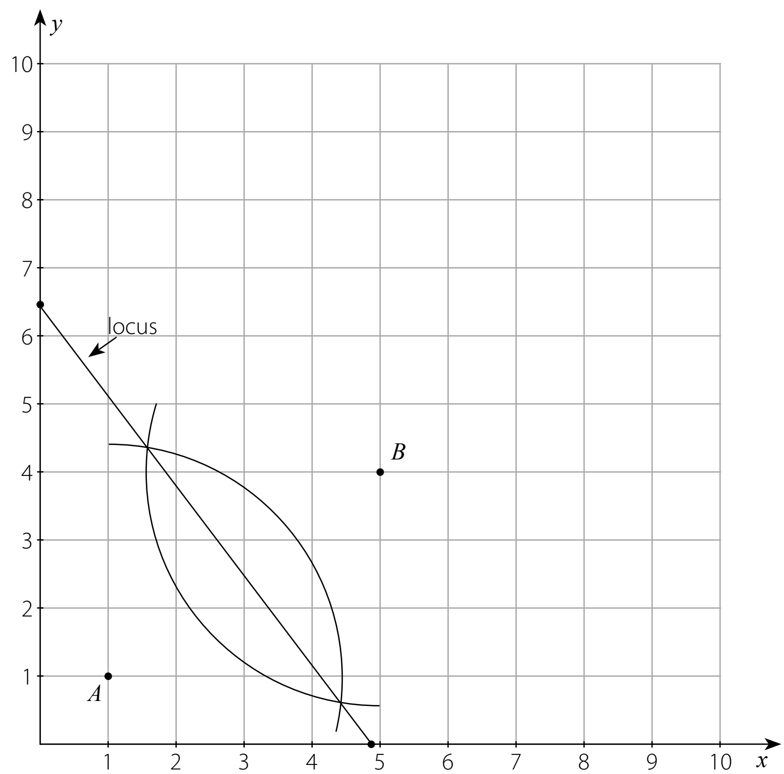
c) $x = 60$ km

d) $\cos 60° = \dfrac{x}{120}$ $x = 60$ km

7.



8.



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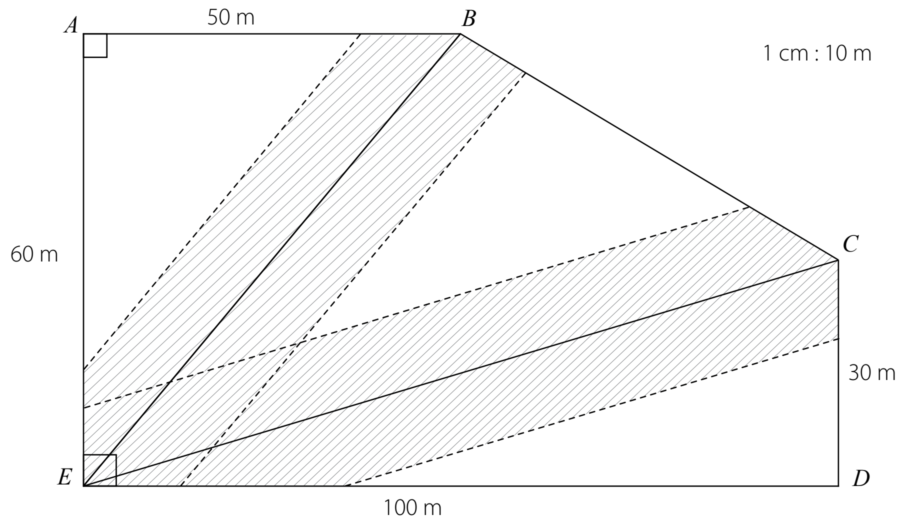
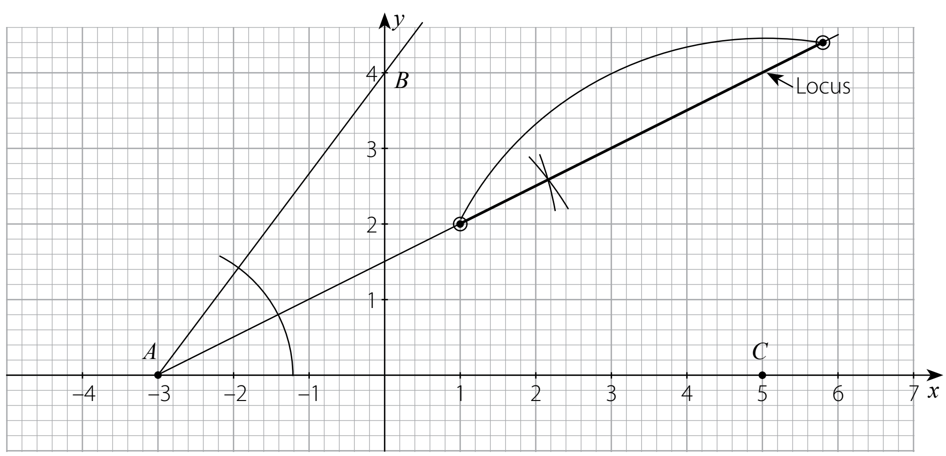
9. a)
b)
c)



d) 1643 m^2 [1620 to 1670 acceptable]

10.



EX 15X

1. $R = \{(x, y) : 2y \leq 7x + 22,\ 7x + y < 25,\ -3 < y \leq 4\}$

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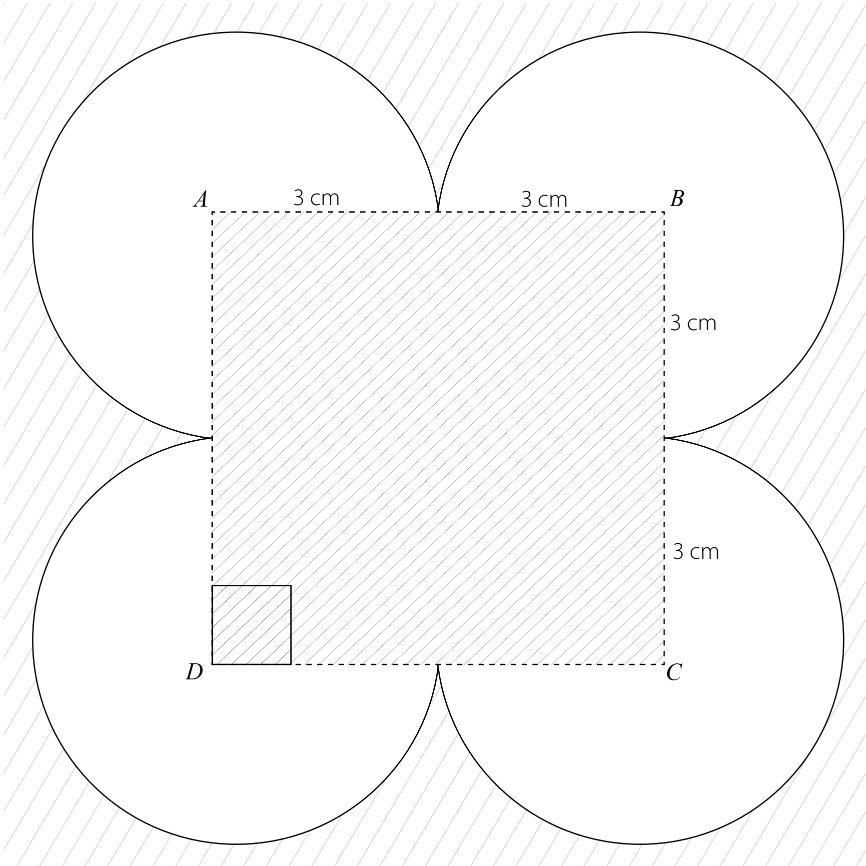
2.



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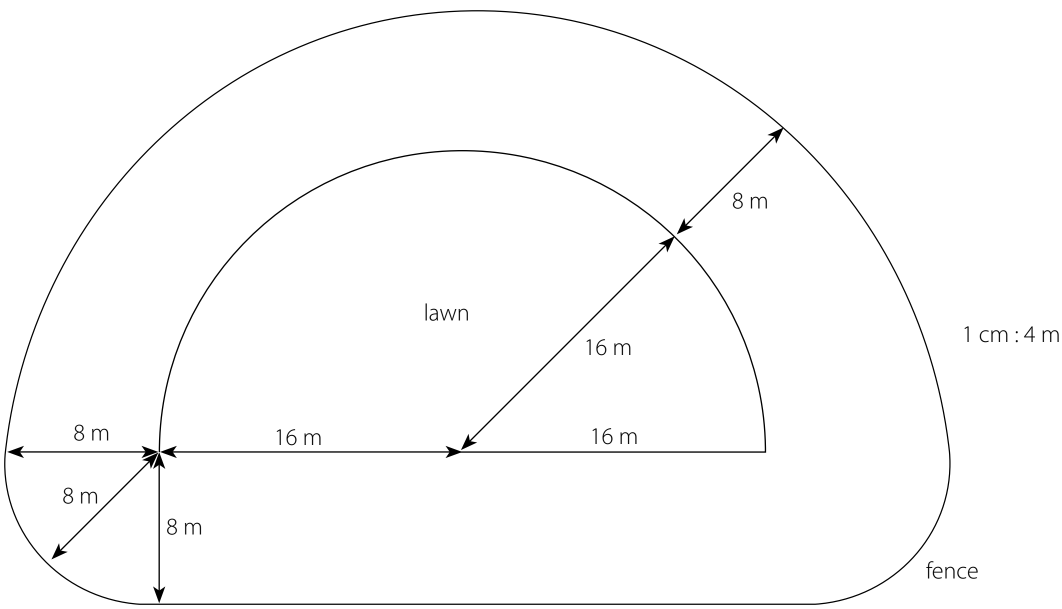
3.



Chapter

16 Trigonometry Review

Trigonometry was introduced in Book 9, Chapters 3, 5 and 9. Having revised the basics this chapter uses trigonometry to solve problems in 3-dimensions. At this stage all the relevant triangles are right-angled. The sine rule and cosine rule are dealt with later.

LESSON PLANNING

Objectives

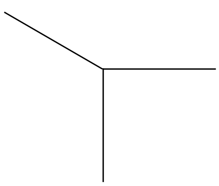
General	To use trigonometry to solve problems in 3-dimensions
Specific	1. To calculate angles and lengths on right-angled triangles using trigonometry 2. To know the meaning of the terms angle of elevation and angle of depression 3. To solve problems that involve maps and bearings or compass directions with trigonometry 4. To solve problems in 3-dimensions by identifying right-angled triangles in the arrangement
Pacing	3 lessons, 1 homework
Links	Bearings

Method

Either review the basics in detail, or use the text introduction for reference, or somewhere in between, according to your assessment of your students' prior knowledge. EX 16A, questions 1–5 is just basic calculator work using trig functions. Questions 6–10 are problems using trig functions, Pythagoras, and area formulas—still fairly basic, straightforward but essential.

The second part is more difficult because it requires students to visualise situations in 3-dimensions.

Emphasize that right-angles in drawings may not appear to be right-angles. The upper corner of a classroom can be a simple visual aid:



There are three right angles there,.
but they appear as approx 120°.

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	Follow the text. Example 1 shows that a horizontal N-S line may well be drawn diagonally across the page. Example 2 shows that extracting the relevant triangle and drawing it again, without 3-D distortion, often clarifies the calculation required. Set EX 16B.
Resources	Calculators essential equipment for this topic
Assignments	EX 16A, questions 1–5 used diagnostically (without prior explanation), or EX 16B, questions 7–9 are suitable for homework.
Vocabulary	SOHCAHTOA elevation, depression space diagonal

ANSWERS

Exercises

EX 16A

1. a) 0.8290 b) 0.3007 c) 0.2679 d) 0.4081
2. a) 46.9° b) 84.6° c) 81.3° d) 34.8°
3. a) 5.13 b) 2.37 c) 41.2 d) 35.7
4. a) 27.8 b) 17.5 c) 120 d) 46.9
5. a) 32.0 b) 10.7 c) 15.2 d) 47.7
6. $x = 14$, $y = 69.0$ (3 s.f.)
7. a) 115 cm^2 b) 8.55 cm
8. 185 km (3 s.f.)
9. 9.5° (1 d.p.)
10. 21.2 m

EX 16B

1. a) 70.7 cm b) 35.3°
2. a) 3.99 cm b) 18.4°
3. a) 1776.5 m^2

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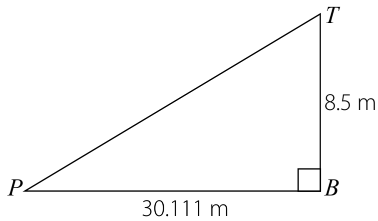
b)



c) 15.8°

4. a) 8.4853 cm b) 4.2426 cm c) 7.35 cm d) 54.7°

5. a) 2.25 cm^2 b) 10.0°

6. a) 0.812 m b) 21.1°

7. 181 m (3 s.f.)

8. 17.3 cm (3 s.f.)

9. 63.6° (1 d.p.)

10. 0.2° (1 d.p.)

EX 16X

1. a) $x\sqrt{14}$ b) 15.5° (1 d.p.)

2. 24.3° (2 s.f.)

3. 73.9° (1 d.p.)

Chapter

17 3-Dimensional Shapes

The volume and surface area formulas of basic solids are reviewed in this chapter and the symmetry of solids explored for the first time.

LESSON PLANNING

Objectives

General	To use formulas for the volume and surface area of solids to solve problems; to recognize axes and planes of symmetry
Specific	1. To know that the cube, cuboid, and cyclinder are special cases of the prism, when calculating volumes 2. To know that cones and pyramids belong to the same family of solids, when calculating volumes 3. To use nets to calculate surface areas of solids 4. To use the volume and surface area formulas of the sphere and hemisphere 5. To solve problems involving composite solids, or solids transformed into another shape 6. To recognize planes and axes of symmetry of simple solids
Pacing	3 lessons, 1 homework
Links	Changing the subject of a formula
Method	• The briefest review of the volume formulas is best done by emphasizing three families of shapes: prisms, pyramids, and spheres. The special prisms' formulas are of a similar structure; similarly, the cone may be considered a special case of the pyramid. The sphere is a separate category whose formula has to be just quoted. Point out to students the dimensions of the volume formulas: that they involve the product of three lengths. The surface area formulas, however, have only two dimensions, i.e. they involve the product of only 2 lengths. Not all students will follow the argument in the text for the πrl curved surface area of a cone, but they ought to see it done. A demonstration with a large thin cardboard sector is helpful. Set EX 17A as soon as possible. Allow students to refer back to the text for formulas. • For planes and axes of symmetry, use models, preferably large ones. Use a piece of card to indicate where the planes cut into the solids. Use two fingers to point to the entry and exit points of axes.

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See if the students can make the following connections:

i) A prism has a plane of symmetry for each line of symmetry of the cross-section, plus an extra one perpendicular to its length at the midpoint.

ii) A prism has an axis of symmetry through the midpoint of its length parallel to each line of symmetry of the cross-section, plus an extra one through the centre of the cross-section parallel to its length.

iii) Pyramids and cones have planes of symmetry through the apex and lines of symmetry of the base.

iv) Pyramids and cones have one axis of symmetry through the apex, of order equal to the order of rotational symmetry of the base.

These connections need not be rigorously specified in words: it is sufficient if they can be seen on a model and used to find the planes and axes systematically.

Set EX 17B.

Resources	• Models, the larger the better • Cardboard sector, to demonstrate that it is the net of a cone. Cut out from a cardboard circle a sector subtending about 120°. • Strong hollow cone and cylinder with the same height and base radius, and clean sand or beads. Show that the cylinder needs 3 cones full of sand or beads to fill it.
Assignments	EX 17A, questions 8–10, suitable for homework
Vocabulary	pyramid, cone apex, slant height plane of symmetry, axis of rotational symmetry

ANSWERS

Exercises

EX 17A

1.	a) 168 cm^3	b) 754 cm^3	c) 165 cm^3	d) 73.0 cm^3
2.	a) 29.3 cm^3	b) 24.0 cm^3	c) 12.0 cm^3	d) 156 cm^3
3.	a) 181 cm^2	b) 201 cm^2	c) 113 cm^2	d) 1020 m^2
4.	a) 15.811 cm	b) 416 cm^2	c) 500 cm^3	
5.	1014 cm^2			
6.	a) 353 000 cm^3	b) 5890 cm^2		
7.	a) major arc = $2\pi r$	b) 5 cm	c) 3.3166 cm	d) 86.8 cm^3
8.	a) 955.04 cm^3	b) 0.52 360 cm^3	c) 1641 beads	
9.	151 mm			

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10. $ 117.13 (nearest cent)

EX 17B

1. 5 planes, 1 axis order 4, 2 axes order 2
2. 4 planes, 1 axis order 3, 3 axes order 2
3. 2 planes, 1 axis order 2
4. 4 planes, 1 axis order 4
5. An infinite number of planes, 1 axis of infinite order
6. An infinite number of planes, 1 axis of infinite order
7. 6 planes, 1 axis order 6
8. 4 planes, 1 axis order 4
9. An infinite number of planes, 1 axis of infinite order
10. 1 plane, 1 axis of order 2

EX 17X

1. 38
2. 1:0.62 (2 s.f.)
3. 3 planes, 1 axis of order 3

Chapter

18 Revision Exercises

ANSWERS

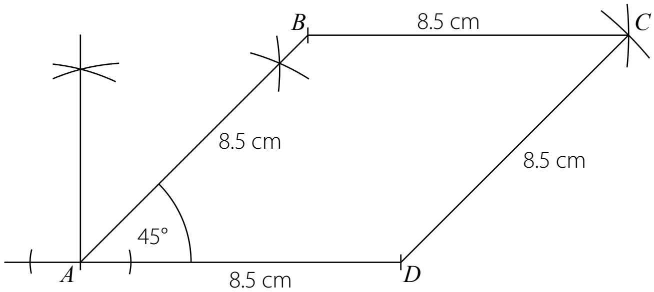
Exercises

EX 18A

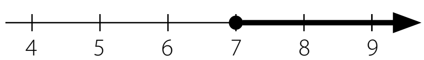
1. a) true b) false c) false d) true

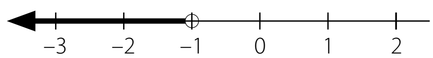
2. a) 30.6 cm^2 b) 53.6 cm^2

3.

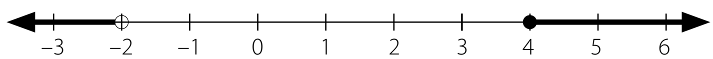


4. a) R b) S c) P d) Q

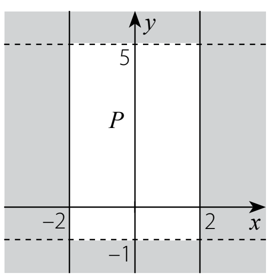
5. a) 

b) 

c) 

d) 

6.



7. a) rotation of + 90° about (−2, 0) b) enlargement: SF = 2, centre (−2, −4)

c) translation $\begin{pmatrix} 4 \\ 1 \end{pmatrix}$

8. a) 35

b) i) $\frac{28}{35}$ ii) $\frac{6}{35}$ iii) $\frac{3}{35}$ iv) $\frac{19}{35}$

9. 163 m^2

10. a) 36.2 cm^2 b) 136 cm^2

EX 18B

1. a) $x = 1.10, x = -2.43$ b) $x = -1.10, x = 2.43$
2. a) false b) true c) false d) false
3. a) 10 b) 17 c) 7 d) 5
4. a) 6 000 000; 6×10^6 b) 3 000 000 000; 3×10^9
   c) 400 000; 4×10^5 d) 50 000 000; 5×10^7
5. a) no b) yes c) yes d) no
6. a) $x = -7, x = 6$ b) $x = -3, x = 9$
7. a) $x = 3.33$ b) $x = 51.3$
8. a) 9 b) 8 c) $\frac{1}{5}$ d) 3
9. −0.51, 0.16

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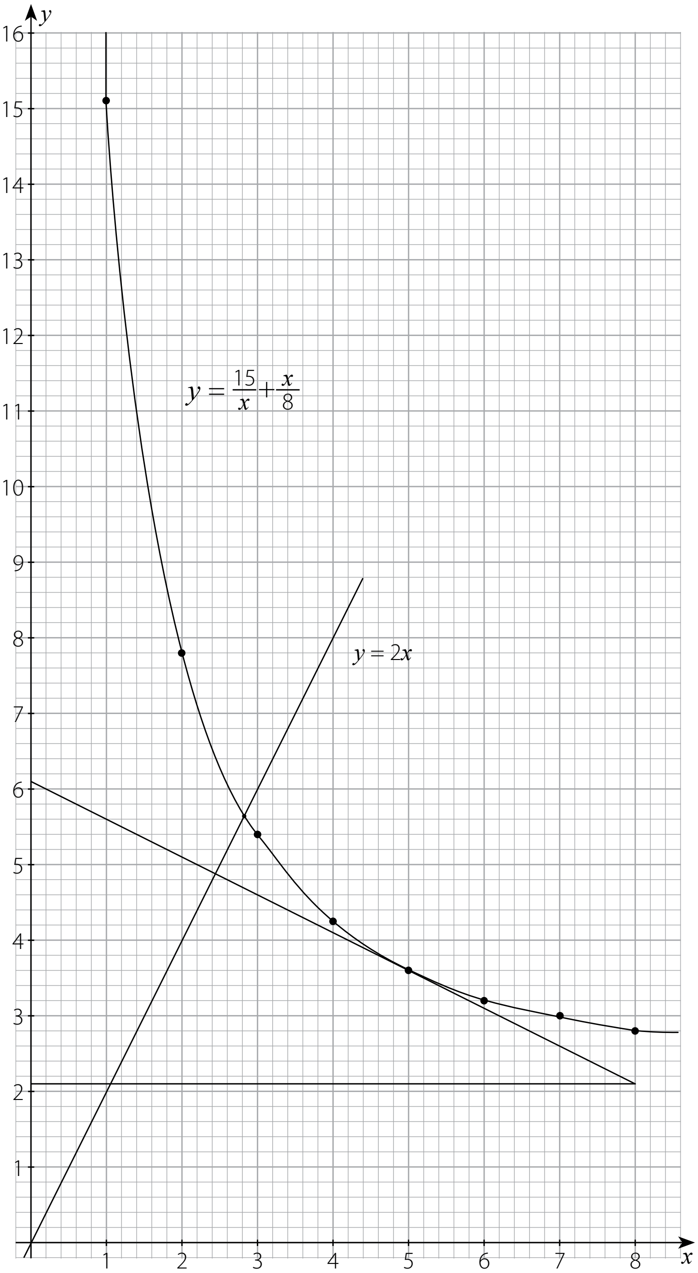
10. a)
c)



intersect at (2.8, 5.6)

gdt = −0.5

[readings rounded to 1 d.p.]

b) $y = \frac{15}{x} + \frac{x}{8}$ and $y = 2x$

Substituting ,

$2x = \frac{15}{x} + \frac{x}{8}$

$8x(2x) = 8(15) + x(x)$

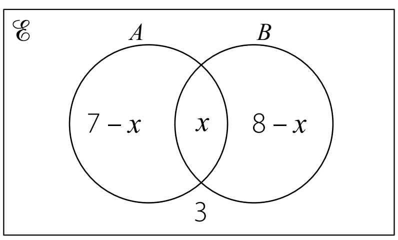
$16x^2 = 120 + x^2$

$15x^2 = 120$

$x^2 = 8$

$x = \pm\sqrt{8}$ as required.

EX 18C

1. a) $x^2 - 6x + 9$ b) $1 - 10x + 25x^2$
   c) $x^2 + 2x - 15$ d) $15x^2 + 4x - 3$
2. a) 12^3 b) 5^{-6} c) 4^{-3} d) 2^2
3. $x = 75, y = 15, z = 75$
4. 20.8 cm
5. a) $y \propto \frac{1}{x}$ b) $y = \frac{c}{x}$ c) $c = 55$ d) $y = 5.5$
6. a) $x = \frac{7}{2}, x = -\frac{11}{6}$ b) same solutions [factors are $(2x - 7)(6x + 11) = 0$]
7. 

   $x = 4$

   4 girls ate both.
8. a) 3.77 cm b) 12.5 cm
9. a) 42.75 cm^2 b) 100 cm^2
10. a) M is the midpoint of AB and O is the centre of the circle.
    $\therefore OM \perp AB$ (symmetry) b) $y = 48$

EX 18D

1. $x = 19, y = 109, z = 45$
2. a) $x = 5.7, x = -0.7$ b) $x = -0.3, x = -3.2$
3. a) major arc BD or arc BAD b) major arc BD or arc BAD
   c) 125° d) 55° (opp $\angle$s of cyclic quad)

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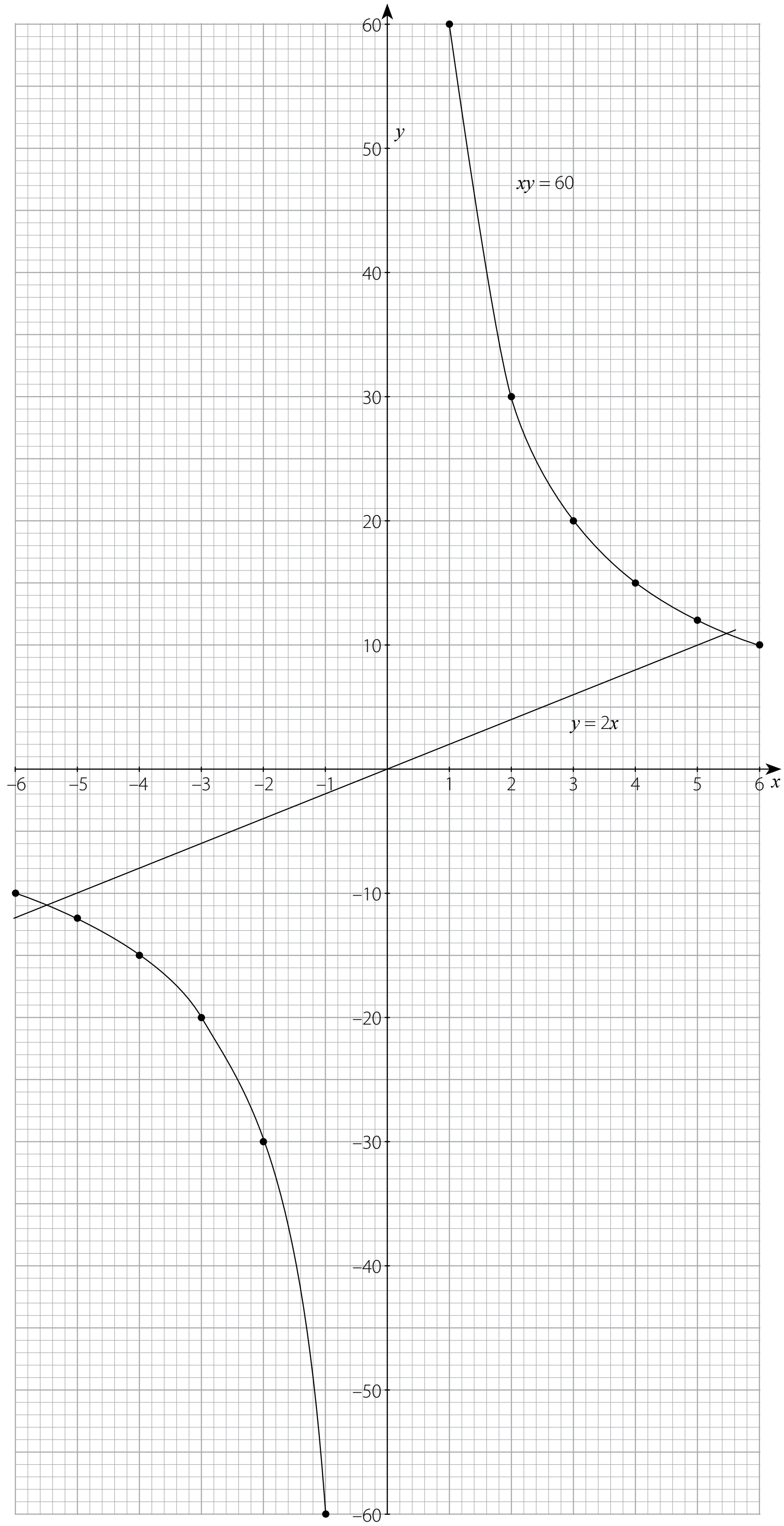
4. a)

x	−6	−5	−4	−3	−2	−1	0	1	2	3	4	5	6
y	−10	−12	−15	−20	−30	−60		60	30	20	15	12	10

B)



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c) rotational symmetry of order 2 about the origin

d) $x = \pm 5.5$ (1 d.p.)

$xy = 60$ and $y = 2x$

$x(2x) = 60$

$2x^2 = 60$

$x^2 = 30$

$x = \pm\sqrt{30}$ as required

5. a) R b) P

c) Q There are 6 planes joining opposite edges, and 3 planes halfway between opposite faces and parallel to them—9 altogether. There are 3 axes joining centres of opposite faces, 4 axes joining opposite vertices (space diagonals) and 6 joining the midpoints of opposite edges—13 altogether.

d) S

6. $x = 55, y = 105$

7. a) Q b) S c) R d) P

8. a) $-\frac{5}{4}$ b) 9 c) $-\frac{1}{3}$ d) $-\frac{4}{3}$

9. a) 364 cm^2 b) 71.1 cm

10.

Juniper	$1.15 \leq l < 1.25$
Pine	$1.45 \leq l < 1.55$
Oak	$2.35 \leq l < 2.45$
Cedar	$2.55 \leq l < 2.65$

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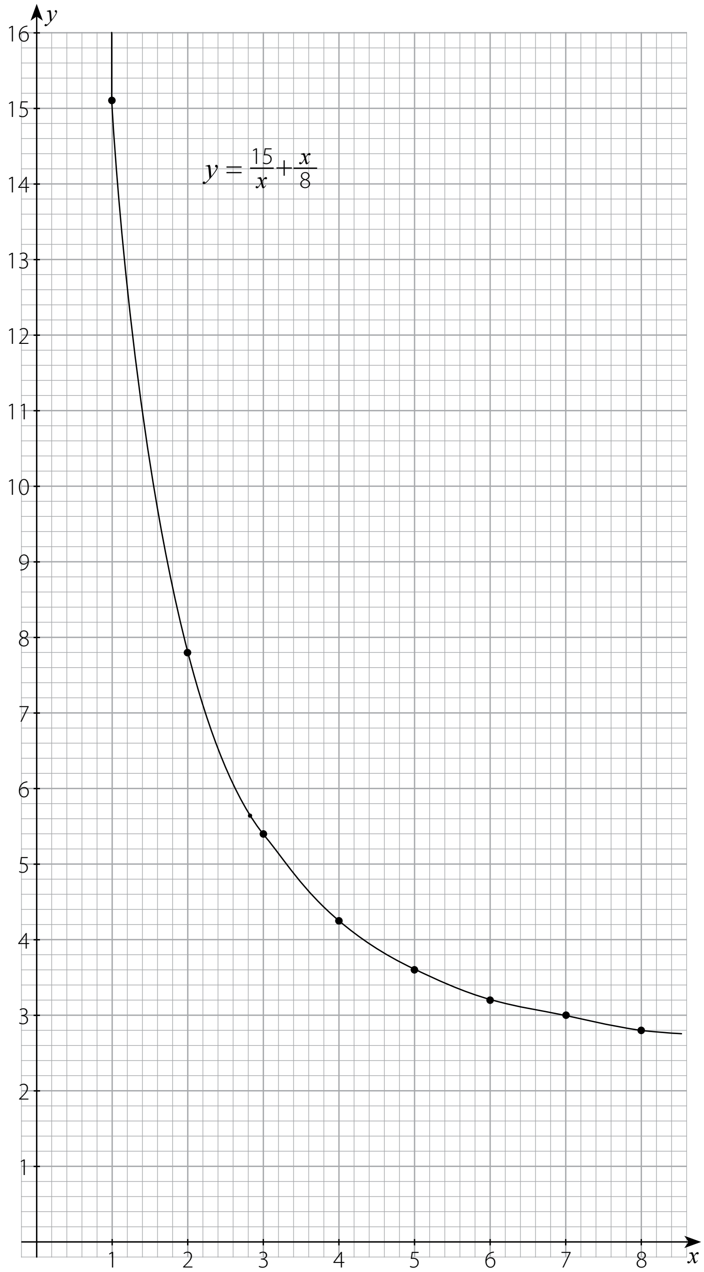
EX 18B, question 10 (worksheet)



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Chapter

19 The Cosine Rule

The cosine rule is introduced in this chapter as a generalisation of Pythagoras' theorem for triangles that are not right-angled.

LESSON PLANNING

Objectives

General	To use the cosine rule to find sides and angles in triangles from given data
Specific	1. To understand the structure of the cosine rule as an extension of Pythagoras' theorem with an adjustment 2. To know that cosines of obtuse angles are negative, and that $-ab \cos C$ (or equivalent) is therefore positive in such cases 3. To use the cosine rule to calculate the third side of a triangle given two sides and the included angle 4. To use the cosine rule to calculate the size of an angle given three sides of a triangle 5. To solve problems involving scalene triangles requiring the cosine rule for solution
Pacing	2 lessons, 1 homework
Links	Pythagoras' theorem, expanding brackets, efficient use of calculator

Method

- Explain the need for solving triangles that are not right-angled. Follow the text, showing that a side may be larger or shorter than the hypotenuse in the case of a right-angled triangle.

  Quote the cosine rule. (Leave the proof until later.)

  Focus on the adjustment term $-2ab \cos C$.

  For acute C the term needs to remain negative,

  i.e. $\cos C$ must be positive.

  For obtuse C the term needs to be made positive,

  i.e. $\cos C$ needs to be negative.

  Call out some angles and show that this is built in to the definition of cosine and stored in the calculator. For example,

  cos 20°, cos 40°, cos 60°, cos 80°, cos 90°

  Then, push the boundary:

  cos 100°, cos 150°, etc. Defined negative.

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Having established the reasonableness of the rule, show how to use it, using the text examples 1–3 and similar other examples. Emphasize the SAS structure so that students are not locked into a specific formulation of the rule.

Set EX 19A, questions 1–3.

- Recap the basic principles, and then go through the proof for the acute angle case in the text. Students should see that it can be proved, even if they do not follow the proof in detail. Go through text Examples 4 and 5 dealing thoroughly with the obtuse-angled case and set the rest of EX 19A, questions 4–10.

Assignments	EX 19A, questions 5, 7, and 9 are suitable for homework
Vocabulary	cosine rule, acute, obtuse

ANSWERS

Exercises

EX 19A

1. a) $c^2 = 12^2 + 11^2 - 2(12)(11) \cos 100°$ b) $p^2 = 18^2 + 16^2 - 2(18)(16) \cos 82°$

   c) $8^2 = 12^2 + 9^2 - 2(12)(9) \cos X$ d) $15^2 = 16^2 + 18^2 - 2(16)(18) \cos S$

2. a) A should be B.

   $6.5^2 = 5^2 + 4^2 - 2(5)(4) \cos B$

   b) x and 21 should be interchanged.

   $x^2 = 21^2 + 19^2 - 2(21)(19) \cos 25°$

   c) final term incorrect (confused with area formula)

   $r^2 = 8.5^2 + 5^2 - 2(8.5)(5) \cos R$

   d) wrong angle used; should be $\angle D = 120°$.

   $d^2 = 16^2 + 13^2 - 2(16)(13) \cos 120°$

3. a) $x = 11.7$ cm b) $x = 7.08$ cm

   c) $x = 19.2$ cm d) $x = 20.7$ cm

4. a) $\angle B = 78.8°$ b) $\angle Q = 58.6°$

   c) $\angle T = 40.8°$ d) $\angle K = 37.4°$

5. a) $a = 18.7$ cm b) $l = 26.5$ m

   c) $q = 6.97$ cm d) $f = 103$ cm

6. a) $\angle B = 128.8°$ b) $\angle E = 146.4°$

   c) $\angle Q = 133.1°$ d) $\angle U = 122.5°$

7. a) $\angle A = 90°$

   b) $a^2 = b^2 + c^2 - 2bc \cos A$, but when $A = 90°$, $\cos A = 0$, and it becomes Pythagoras' theorem.

   c) 8.21 cm

   d) same answer, using $\sin 20 = \frac{x}{12}$ (x is half the base)

8. 56 km

9. 137 km

10. 108 km

EX 19X

1. In the right triangle: $a^2 = x^2 + y^2$ (Pythag) ——— 1

   In the large right angled triangle

   $$c^2 = (b + x)^2 + y^2 \quad \text{(Pythag)}$$

   $$c^2 = b^2 + 2bx + x^2 + y^2 \quad \text{——— 2}$$

   Substituting from 1 into 2

   $$c^2 = b^2 + 2bx + a^2 \quad \text{——— 3}$$

   In the right triangle: $\cos (180 - C) = \frac{x}{a}$

   $$x = a \cos (180 - C)$$

   $$x = -a \cos C \quad \text{——— 4}$$

   Substituting 4 into 3

   $$c^2 = b^2 + 2b(-a \cos C) + a^2$$

   $$c^2 = a^2 + b^2 - 2ab \cos C \quad \text{as required.}$$

2. 13.6 cm, 6.40 cm

3. 44.4 cm (3 s.f.)

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Chapter

20 Congruence and Similarity

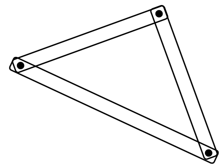
Congruence and similarity are defined by using transformations. In this chapter we focus on triangles and the minimum requirements for proving congruence and similarity. It provides another opportunity to develop a mathematically sound proof style.

LESSON PLANNING

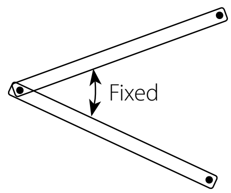
Objectives

General	To recognize the conditions for similar and congruent figures (especially triangles) and to do relevant calculations and proofs
Specific	1. To know that similar figures have the same shape; that they are equiangular 2. To know that corresponding sides in similar figures are scaled up or down by the same scale factor; to use this fact in calculations 3. To understand that internal ratios of corresponding sides are preserved in similar figures; to use this fact in calculations 4. To know that two pairs of equal angles is sufficient to prove similarity in triangles; to write out simple proofs 5. To use in calculations the area and volume (mass) scale factors' relationship to the length scale factor in similar figures in 2 and 3 dimensions 6. To understand the meaning of congruence; to prove congruence of two figures from given data 7. To know and use the four possible sets of requirements for congruent triangles in calculations and proofs
Pacing	3 lessons, 1 homework
Links	transformations; geometry of parallel lines; circle geometry
Method	• The similarity work has been covered previously in Book 9, Chapter 25 and kept alive in Revision Exercises, so it may be sufficient to set EX 20A immediately, using the chapter introduction Similarity Reminders for reference only, and Examples 1, 2 and 3. However, for some students it may be necessary to work through this in more detail. • Congruence is a simpler notion but the four sets of conditions for proving congruent triangles need to be explained, i.e. SSS, SAS, AA corr S, and RHS. These conditions are the same as the minimum requirements to specify a triangle.

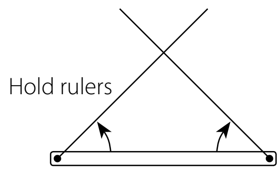
Use 3 cardboard strips to demonstrate:



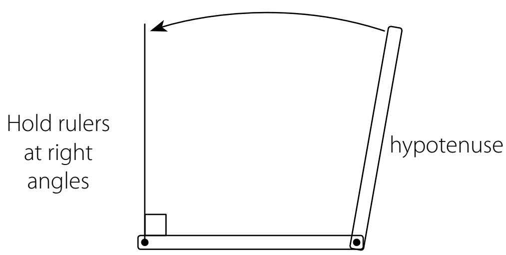
SSS - The three sides fix the shape—angles cannot be changed.



SAS - The third side can only go in one position



AA corr S - The given side and two angles fix the third vertex.



RHS - Start with a side and a right angle. Then move the hypotenuse to close the triangle.

Use example preceding EX 20B to show how to write out a proof.

Set EX 20B.

Resources	Cardboard strips for demonstration (see method) Suitable lengths are 20, 25, and 30 cm. Punch holes near the ends for fastening with paper clips.
Assignments	Suitable for homework, EX 20A questions 4, 7, 10
Vocabulary	common, figure (meaning a shape), equiangular

ANSWERS

Exercises

EX 20A

1. $x = 13.5, \quad y = 9.87$

2. $x = 6.25$ cm, $\quad y = 8$ cm

3. $x = 24.5, \quad y = 8$ cm

4. a) $\angle A = \angle C$ (alt $\angle$s)
$\angle B = \angle D$ (alt $\angle$s)
$\therefore$equiangular

b) $AB = 14$ cm
$OC = 17.5$ cm

5. $x = 20$

6. a) $\angle AEB = \angle ADC$ (corr $\angle$s)
$\angle ABE = \angle ACD$ (corr $\angle$s)
$\therefore$equiangular

b) $x = \sqrt{12}$ cm

7. 14.4 cm^2

8. 35 g

9. 39.2°

10. a) $\angle FAB = \angle EFC$ (corr $\angle$s)
$\angle ABC = \angle FCD$ (corr $\angle$s)
$\angle BCF = \angle CDE$ (corr $\angle$s)
$\angle AFC = \angle FED$ (corr $\angle$s)
$\therefore$equiangular

b) internal ratio
$\frac{AB}{FC} = \frac{2}{4} = \frac{1}{2}$
but $\frac{FC}{ED} = \frac{4}{6} = \frac{2}{3}$
$\frac{AB}{FC} \neq \frac{FC}{ED}$
$\therefore$not similar

EX 20B

1. Corresponding angles are 90° (given),
$\therefore$equiangular
$AB = FE$ (opp sides of rectangle) $= BC$ (given)
$BE = CD$ (opp sides of rectangle)
$FE = BC$ (given) $= ED$ (opp sides of rectangle)
$AF = BE$ (opp sides of rectangle)
$\therefore$corresponding sides are equal,
$\therefore$rectangle $ABEF$ is congruent to rectangle $BCDE$, as required.

2. $\angle ABD = \angle BDC$ (alt $\angle$s)
$\angle ADB = \angle DBC$ (alt $\angle$s)
$\therefore$equangular
Also BD is common,
$\therefore \Delta ABD$ is congruent to ΔCDB, as required.

3. $x = 50, y = 9.14$

4. $x = 11.06, y = 10.57$

5. $\angle BAC = \angle DAC$ (given)

   $AB = AD$ (given)

   AC is common,

   ΔABC is congruent to ΔADC (SAS),

   $\therefore BC = CD$, as required.

6. $AB = CB$ (symmetry)

   $OA = OC$ (radii)

   OB is common,

   $\therefore \Delta ABO$ is congruent to ΔCBO

   or $\angle OAB = \Delta OCB = 90°$ (tan $\perp$ rad)

   giving congruence (RHS) or (SAS).

7. a) $AB = CD = 13$ cm (given)

      $AD = CB = 10$ cm (given)

      DB is common,

      $\therefore \Delta ABD$ is congruent to ΔCDB (SSS).

   b) Hence $\angle ABD = \angle CDB$

      $AB \parallel CD$ (alt $\angle$s)

8. $AB = BC$ (given)

   $\angle A = \angle C = 90°$ (given)

   BD is common,

   $\therefore \Delta ABD$ is congruent to ΔCBD (RHS).

   $\therefore \angle ADB = \angle CDB$, as required.

9. a) $\angle ABC = \angle CFE$ (alt $\angle$s)

      $\angle BCG = \angle DCF$ (vert opp $\angle$s)

      $\angle AGC = \angle CDE$ (alt $\angle$s)

      $\therefore \angle A = \angle E$ (angle-sum of quad)

      $\therefore$equiangular, as required.

   b) $GC = CD$ for congruence.

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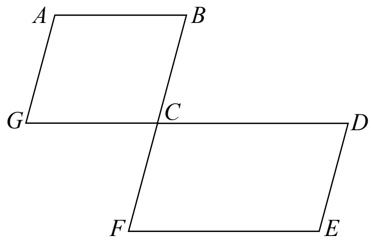
c) 

equiangular but not congruent

d) Since $BC = CF$, SF =1

But $GC \neq CD$, SF $\neq$ 1

Different SF applied to corresponding sides,

$\therefore$not similar

10. a) $AD = CD$, (sides of square)

$\angle A = \angle C$, 90° (vertices of square)

$AP = RC$ (given)

$\therefore \Delta APD$ is congruent to ΔCRD (SAS).

b) Hence, $DP = DR$

DQ is common,

$\angle DQP = \angle DQR = 90°$ (given)

$\therefore \Delta DQP$ is congruent to ΔDQR (RHS).

EX 20X

1. $AD = BC$ (given)

$\angle A = \angle C = 90°$ ($\angle$ in semicircle)

BD is common,

$\therefore \Delta ABD$ is congruent to ΔCDB (RHS).

2. $EC = BH$ (space diagonal of cuboid)

BE is common,

$BC = EH$ (opp edges of cuboid)

$\therefore \Delta BCE$ is congruent to ΔHEB.

3. 26.0 cm

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Chapter

21 Presentation of Data

After a brief overview of some basic charts, diagrams, and graphs, the new material in this chapter is about dealing with unequal intervals when data is grouped.

LESSON PLANNING

Objectives

General	To interpret correctly information presented in bar charts, bar line graphs, pie charts, scatters diagrams, histograms (equal and unequal class intervals) and frequency polygons; to draw such graphs correctly from given data
Specific	1. To draw accurate bar charts and bar line graphs, horizontal and vertical, with correct labelling, etc; to interpret such graphs 2. To calculate the angles subtended at the centre of a pie chart correctly from given data; to interpret the meaning of a given pie chart 3. To understand correlation and how it is illustrated in scatter diagrams; to draw a line of best fit and use it to make predictions 4. To represent continuous and/or grouped data correctly on a horizontal scale in frequency diagrams of various types 5. To calculate the frequency densities of class intervals in order to draw a histogram (equal and unequal intervals); to interpret a given histogram 6. To draw a frequency polygon from given data (equal and unequal intervals) including completion of the polygon by taking the graph down to the baseline on the left and right; to interpret from a given frequency polygon
Pacing	3 lessons, 1 homework (graphs take time to draw well)
Links	All previous work on data handling in Books 6–9

Method

- It is a good time to go back to basics:

  "What is a graph?" (Not the x, y relationship type)

  "Graphs of data/information—why do we need them?"

  Answer: The visual picture is easier to grasp than lists of numbers. They help us to interpret data.

  Therefore, graphs must be fair: we need to be careful about how we present these images.

  Now, follow the text, establishing that

  - bar charts/line graphs show differences.
  - pie charts show proportions.

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- scatter diagrams show correlation. (how well things go together)
- frequency diagrams show how often something occurs at (or between) a certain measurement.

In each, the visual impact should be emphasized.

- This sets the scene for dealing with unequal intervals. In bar charts, the visual impact is the area of each bar, but if they are of equal width, then their heights illustrate the values, because height is proportional to area.

  But if they are of unequal width, we have to adjust the height so that the areas (visual impact) give a fair illustration. The method of calculating frequency density, etc. can then be followed from the text example.

- Frequency polygons, following the text, can be introduced as simplified histograms. If intervals are equal, then the vertical scale can be frequency; if intervals are unequal, the vertical scale should be frequency density.

  In practice, the latter is unusual.

  Taking the polygon down to the baseline is explained briefly in the text. Not all syllabuses expect this at this stage and it may be disregarded. It does, however, give a reason for calling such graphs polygons: without the extra points on the baseline the graph is the just a zigzag line.

  Use EX 21A. Give students sufficient time to draw neatly labelled, accurate graphs.

Resources	It is quite helpful to have some poster-size graphs of various types prepared in advance. They could be on classroom display boards as models of good presentation of data, or just held up in the lesson. Graph paper (2 mm)—photocopiable sheets available in the guide
Assignments	EX 21A, questions 8 and 9 provide a good test of understanding, suitable for homework
Vocabulary	bar chart, line graph, pie chart, scatter diagram ("scattergram") sector, correlation, frequency, frequency density, class interval

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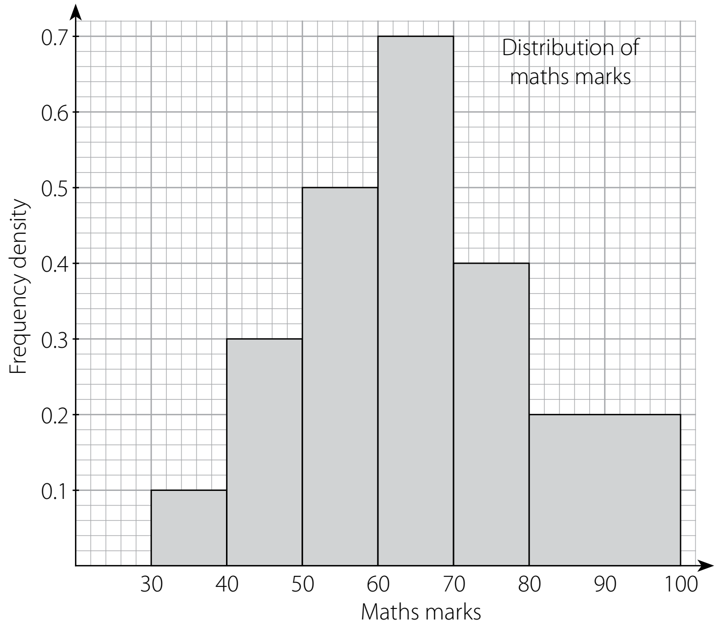
ANSWERS

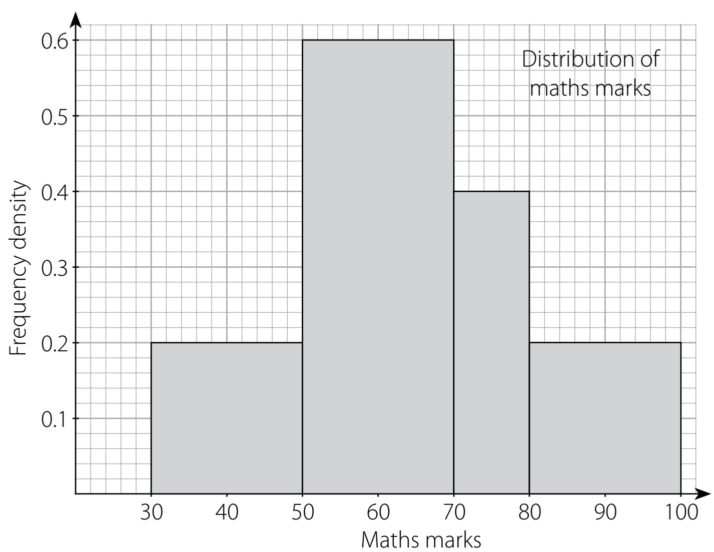
Exercises

EX 21A

1. a) No title; horizontal scale not in equal steps; frequency of third column (50 to 60) should be 5 (not 6).

   b) $60 \le x < 70$ marks

2. 

3. a) 

   b) $50 < x \le 70$ marks

4. Veg sector too small, cheese sector too large. Correct angles are 60° and 120°.

5. a) Mathematics, Physics, Biology b) English lit

   c) 50 marks d) Chemistry and Urdu

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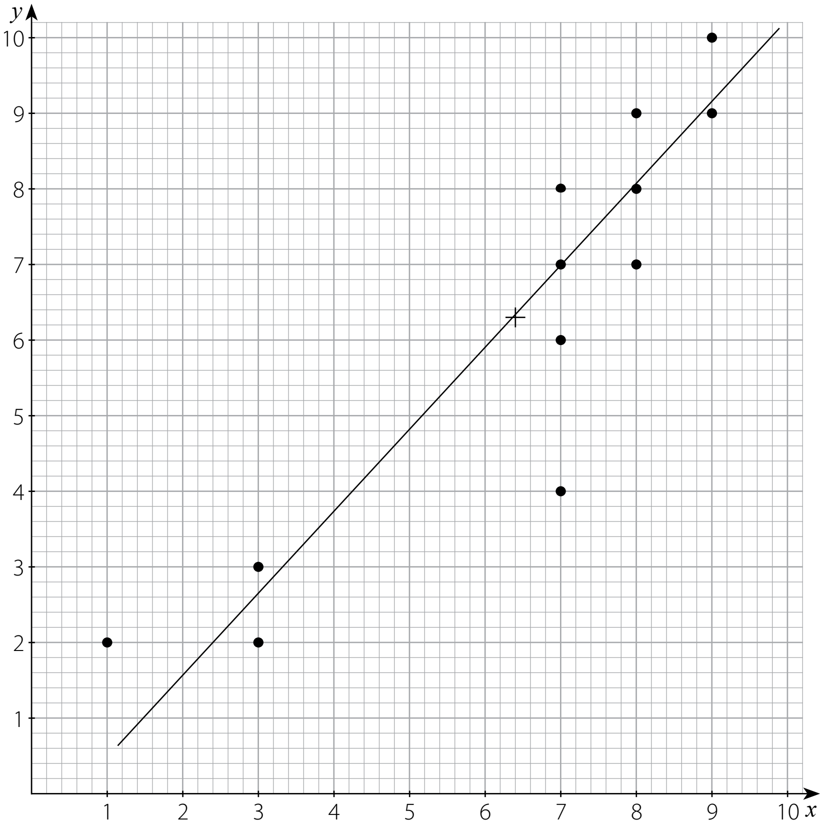
6. a)

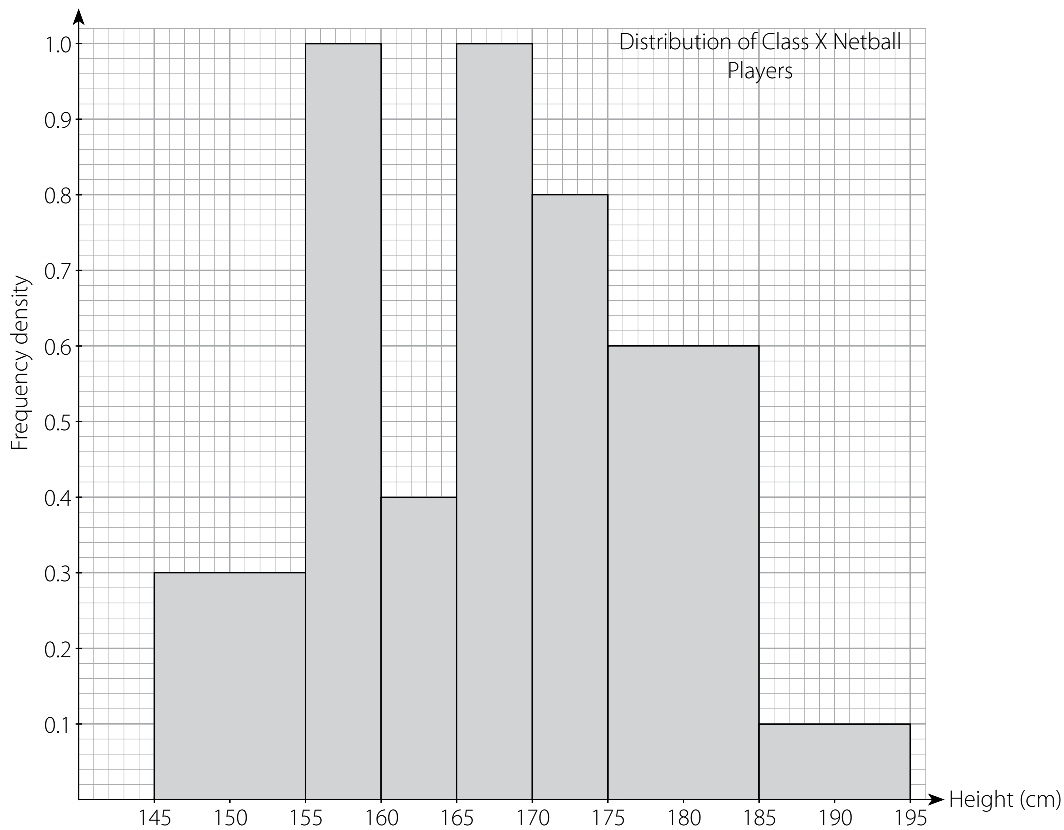


b) (6.4, 6.3)

c) 4.8

d) Strong positive

7.

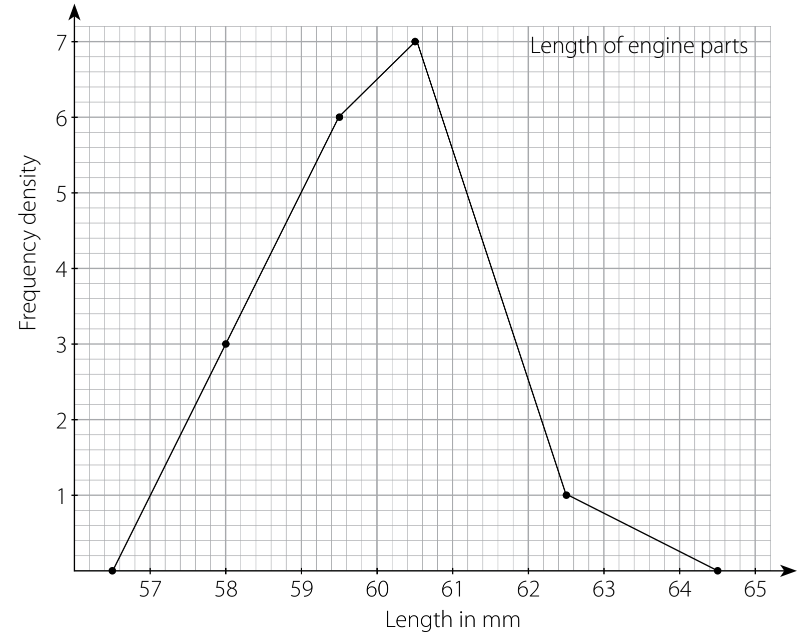


8. a) 6 b) 22

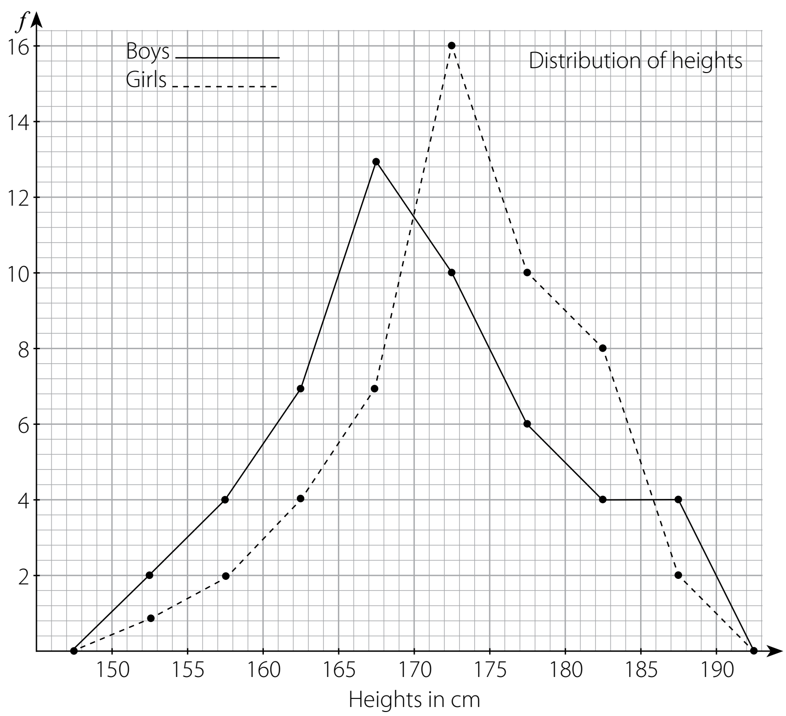
c) 7; value of smallest and largest not given

d) 59.8 mm e) $60 \le l < 61$ mm

9.



10. a)

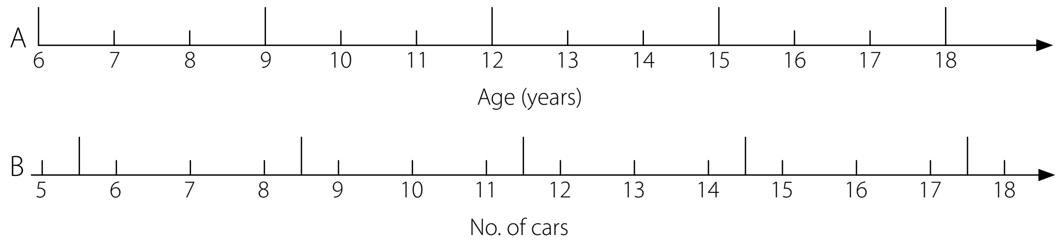


b) Boys $165 < h \le 170$; Girls $170 < h \le 175$

c) The girls are generally taller than the boys in this group and the spread of heights is approximately the same. [Girls graph is to the right of the boys' and average (modal class interval) is higher; ranges (upper bounds) are equal.]

EX 21X

1. 3.3 cm

2. 

3. $\left(\frac{2}{50} \times \frac{1}{49}\right)^2 = 6.66 \times 10^{-7}$

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Chapter

22 The Sine Rule

This chapter completes the techniques required for solution of triangles, by introducing the sine rule.

LESSON PLANNING

Objectives

General	To apply the sine rule where appropriate in the solution of triangles
Specific	1. To write a sine rule formula for any triangle with specified vertices 2. To know that the sine rule can be proved 3. To apply the sine rule to triangles with given data to calculate unknown sides and/or angles, dealing correctly with any obtuse angles 4. To avoid use of the sine rule in right-angled triangles and isosceles triangles 5. To be aware that if given data is SSS or SAS then the sine rule cannot be used immediately: the cosine rule is appropriate first, and then the sine rule can be used to find the remaining unknowns 6. To use the angle-sum of a triangle property when solving triangles 7. To use the sine rule in the solution of problems involving angles of elevation and depression and 3-figure bearings
Pacing	3 lessons, 1 homework
Links	cosine rule trig ratios, circle geometry

Method

- Recap knowledge of triangles: "What do you know about triangles?" Elicit angle-sum, ext $\angle$properties.

  "What if it is right-angled?" Elicit Pythagoras and trig ratios.

  "What if it is isosceles?" Base angles, line of symmetry, two congruent right-angled triangles.

  "What if it is scalene?" Elicit cosine rule.

  "When can it be used?" Cases of SAS or SSS.

  "What if given information is different?" Need for new rule. Follow the text examples 1 and 2, or similar.

  Give some examples of $\triangle PQR$ or $\triangle STU$, etc. and get students to state the sine rule using given letters, i.e. emphasize the structure of the rule (as we did in Chapter 19 for the cosine rule).

  Now jump to "Obtuse angles" in the text and explain how to deal with them, using the given example.

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- Once facility in use has been established, students should witness a proof. The circumcircle method in the text is easy to understand and connects with students' recent learning of Circle Geometry in Chapter 13. The SAS formula for area, already recently revised in Chapter 14, may be briefly given again: it is in the text for reference. It is worth pointing out that the SAS term in the cosine rule is $-\frac{1}{2}bc\cos A$ whereas the area formula is $\frac{1}{2}bc\sin A$.

  EX 22B is about solving triangles systematically in questions 1–6 and in questions 7–10 problems are set needing sine rule to solve (with or without cosine rule).

  EX 22X, question 2 would be a useful task for students intending to appear for Additional Mathematics.

Resources	calculator (essential), graph paper (EX 22X, question 2 only)
Assignments	EX 22A, questions 8–10 or EX 22B, questions 5–7 suitable for homework
Vocabulary	sine rule, circumcircle, inverse sine

ANSWERS

Exercises

EX 22A

1. a) $\dfrac{a}{\sin 36^\circ} = \dfrac{12}{\sin B} = \dfrac{c}{\sin 46^\circ}$

   b) $\dfrac{17}{\sin 100^\circ} = \dfrac{q}{\sin Q} = \dfrac{10}{\sin R}$

   c) $\dfrac{12}{\sin R} = \dfrac{s}{\sin 65^\circ} = \dfrac{11}{\sin T}$

   d) $\dfrac{19}{\sin 25^\circ} = \dfrac{28}{\sin K} = \dfrac{l}{\sin L}$

2. a) $\angle B = 98^\circ$ b) $a = 7.12$ cm c) $c = 8.72$ cm

3. a) $\angle R = 35.4^\circ$ b) $\angle Q = 44.6^\circ$ c) $q = 12.1$ cm

4. a) Information supplied is SAS; too many unknowns in sine rule equations

   b) Use cosine rule c) 59.8 cm^2

5. a) $\angle K = 141.5^\circ$ b) $\angle L = 13.5^\circ$ c) $l = 10.5$ cm

6. $\angle C = 57.2^\circ$, $\angle A = 29.8$, $a = 28.4$ cm

7. $\angle G = 23.9^\circ$, $\angle F = 68.1^\circ$, $f = 68.7$ cm

8. $\angle M = 14^\circ$, $l = 236$ cm, $n = 239$ cm (3 s.f.)

9. $x = 107$, $y = 148$

10. $AB = 100$ m $\angle ACB = 145.2$

EX 22B

[Answers given to 3 s.f. or 1 d.p. (angles) unless exact.]

1. a) $x = 13.1$ b) $y = 33.0$
2. a) $x = 75, y = 6.21$ b) $z = 100, w = 27.4$
3. $\angle Q = 141.3°$, $\angle R = 14.7°, r = 8.11$ cm
4. $u = 7.55$m $\angle S = 53.4°, \angle T = 66.6°$
5. $\angle W = 70.5°, \angle X = 50.5°, \angle V = 59.0°$
6. a) $BC = 9.08$ cm b) 52.9 cm^2
7. a) 127 km (3 s.f.) b) 299°
8. 221 km
9. 39.7 m
10. 115.0°, 87.0 cm^2

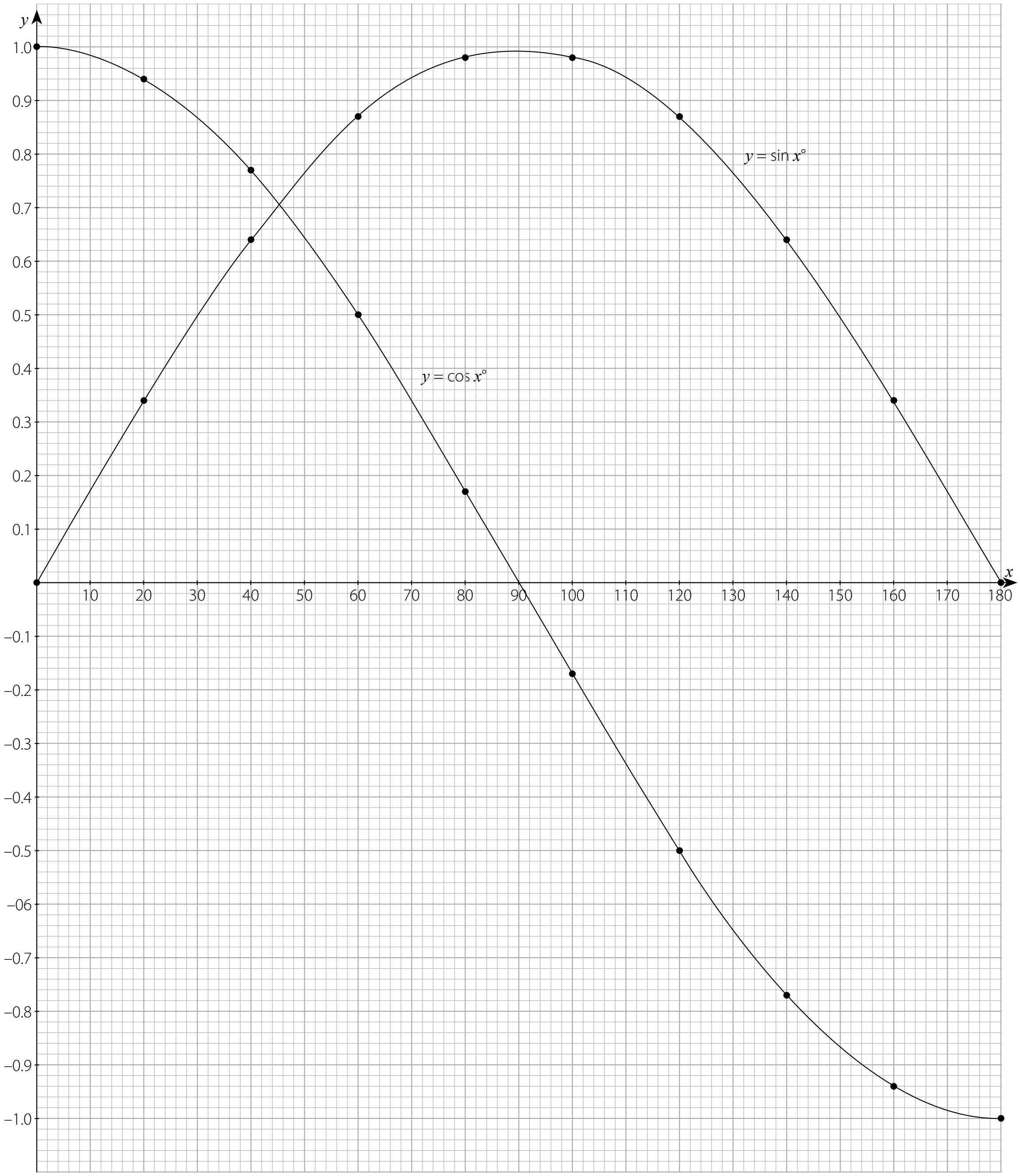
EX 22X

1. 38.4 m

2. a)

x	0	20	40	60	80	100	120	140	160	180
$y = \sin x°$	0	0.34	0.64	0.87	0.98	0.98	0.87	0.64	0.34	0
$y = \cos x°$	1	0.94	0.77	0.50	0.17	−0.17	−0.50	−0.77	−0.94	−1

b)



c) line symmetry about the line $x = 90$

d) rotational symmetry (order 2) about the point (90,0)

3. 352°

Chapter

23 Cumulative Frequency

This chapter revises CF tables and graphs. In Book 9, Chapter 19, where the topic was introduced in detail, the given data was always on a continuous scale. Here we also deal with fitting discrete data to a continuous scale, including cases of continuous measurements that have been rounded off.

LESSON PLANNING

Objectives

General	To draw and use cumulative frequency tables and diagrams to estimate medians, quartiles, and percentiles; to interpret from given CF curves
Specific	1. To draw CF curves correctly, plotting on top of each class interval and including a point for zero frequency 2. To fit discrete data to a continuous scale by using upper and lower bounds (with age a special case) 3. To use CF curves to estimate medians, quartiles, and percentiles 4. To compare distributions by use of the median and inter-quartile range as evidence
Pacing	2 lessons, 1 homework
Links	upper and lower bounds

Method

This is largely a revision chapter, so teachers have to adapt to the needs of their students. General points to make about CF curves are:

- They are not used for small data sets. The technique is relevant to large quantities of data, inevitably grouped.
- Because of the grouping we do not know, and cannot find, medians, and quartiles, etc. We must estimate. The CF curve is a way of facilitating the estimation.
- Medians, quartiles, and percentiles tell us the score **below** which a certain proportion of the distribution lies. That is why we plot on top of each class interval.

This leads naturally into focussing on class intervals: how to write them using inequalities; how to deal with discrete data by extending to the upper and lower bounds. Follow the text examples, and similar.

[As regards age as a measure, it is thought-provoking and therefore of educational value. However, at this stage it is not essential and may be omitted if teachers feel that it would confuse their students. Age appears in EX 23A, question 10 and EX 23X, questions 1 and 2 only.]

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	Use EX 23A.
Resources	Flexicurves for drawing the graphs: the S-shaped CF curve is quite hard to draw freehand.
Assignments	Suggested homework EX 23A, questions 7 and 8
Vocabulary	median, lower quartile, upper quartile, percentile, cumulative frequency (CF) class interval, continuous, discrete

ANSWERS

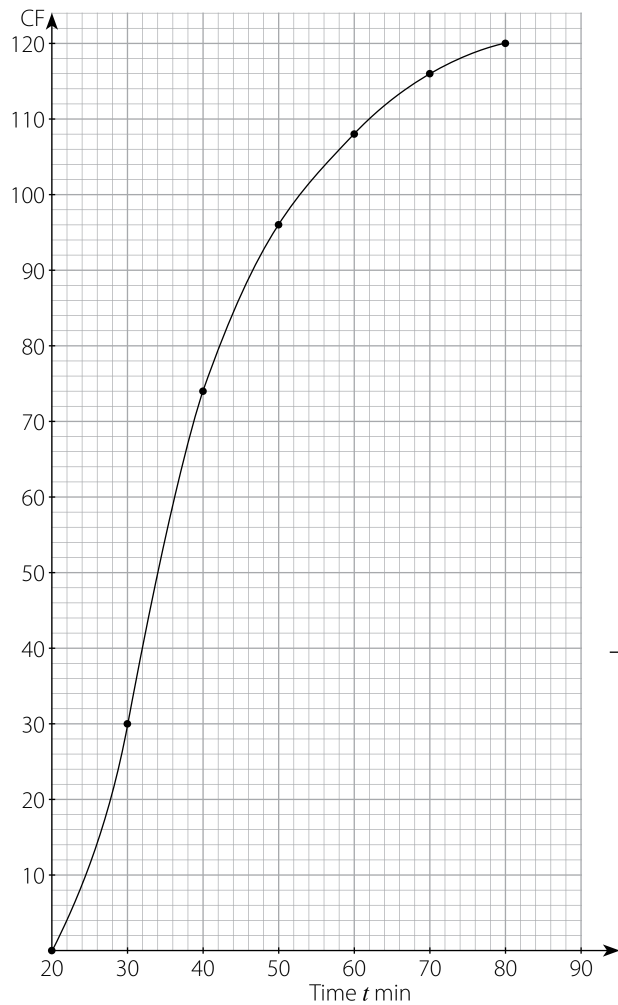
Exercises

EX 23A

1. a)

Time t min to office	f	CF
$20 \le t < 30$	30	30
$30 \le t < 40$	44	74
$40 \le t < 50$	22	96
$50 \le t < 60$	12	108
$60 \le t < 70$	8	116
$70 \le t < 80$	4	120

b)



c) 38 min

d) 32 workers

2. a) $Q_3 = 46$ min, $Q_1 = 30$ min b) IQR = 16 min

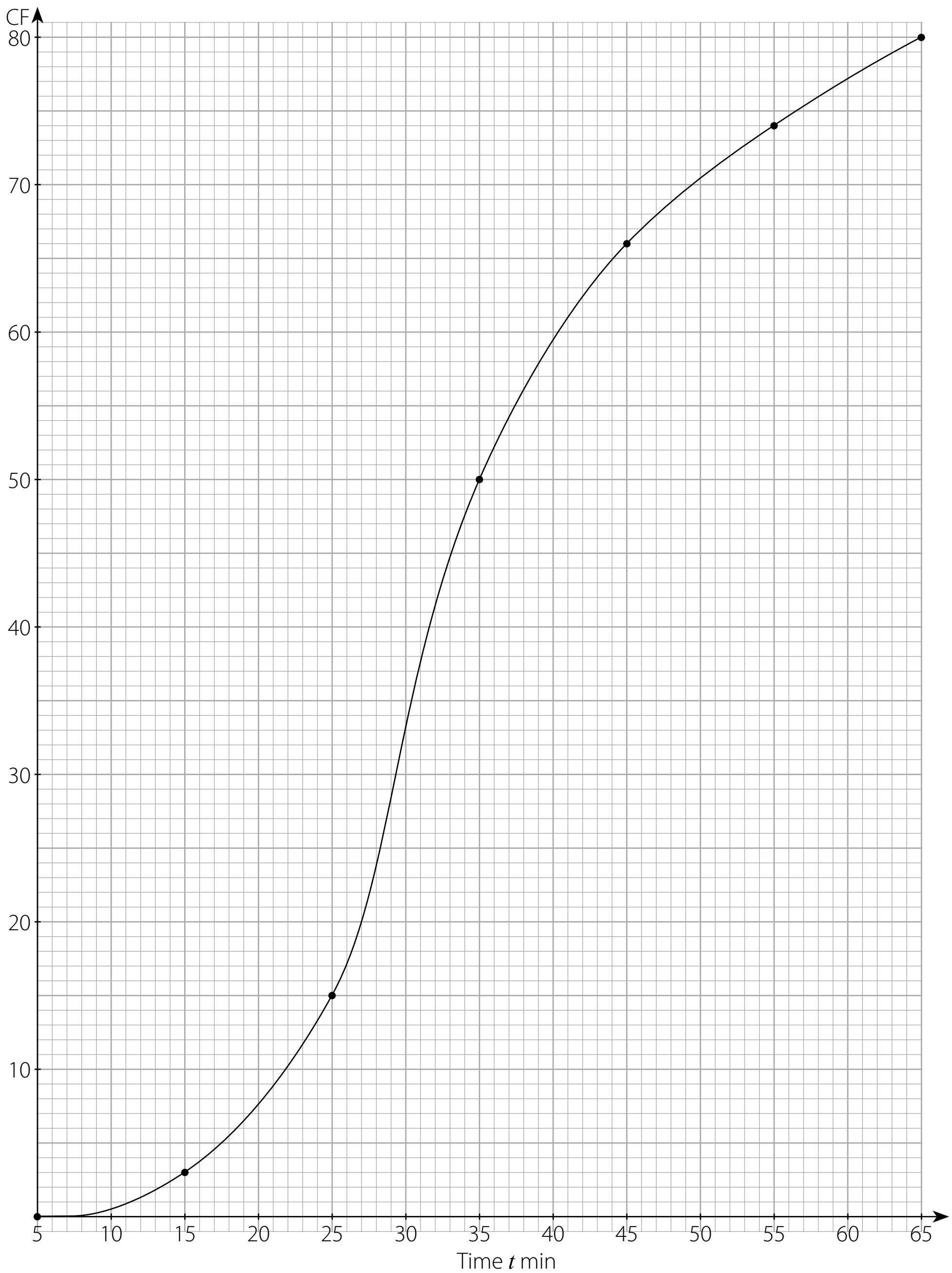
c) The middle 60 workers' times were within 16 minutes of each other.

d) 73rd percentile

3. a)

Time to office (t min)	f	CF
$5 \le t < 15$	3	3
$15 \le t < 25$	12	15
$25 \le t < 35$	35	50
$35 \le t < 45$	16	66
$45 \le t < 55$	8	74
$55 \le t < 65$	6	80

b)



c) IQR = 13 min

d) 83rd percentile

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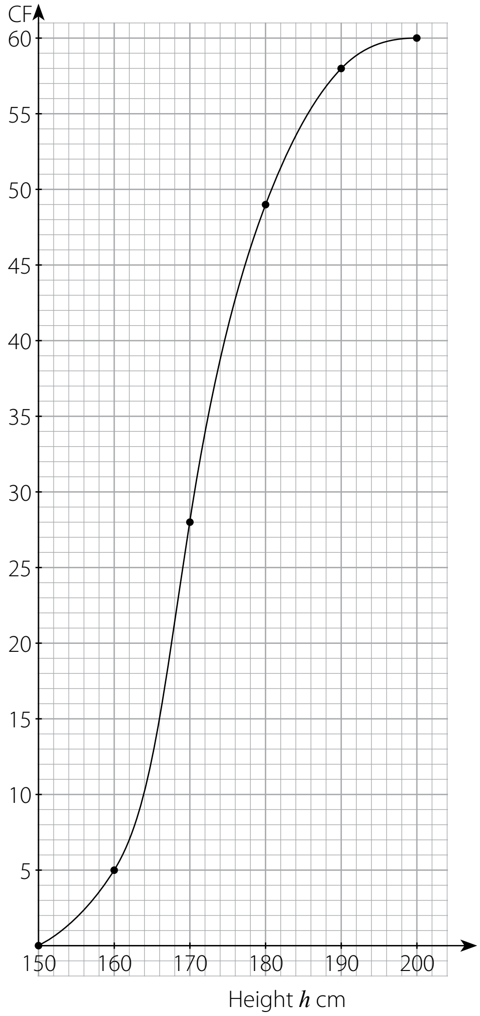
4. a) false b) false c) true d) false

5. a) 114 cm b) 18 cm

c) 5000 children d) 5000 children

6. a)



b) 171 cm

c) 11 cm

d) 55th position

e) $160 \leq h < 170$ cm

7. a) Test A 28; Test B 63 b) Test A 20: Test B 22

c) The results of test B were higher in general (median); the spread of Test B is slightly wider (IQR).

d)

% marks (m)	Test A (f)	Test B (f)
$0 \leq m < 20$	10	0
$20 \leq m < 40$	20	5
$40 \leq m < 60$	5	11
$60 \leq m < 80$	4	19
$80 \leq m < 100$	1	5
Totals	40	40

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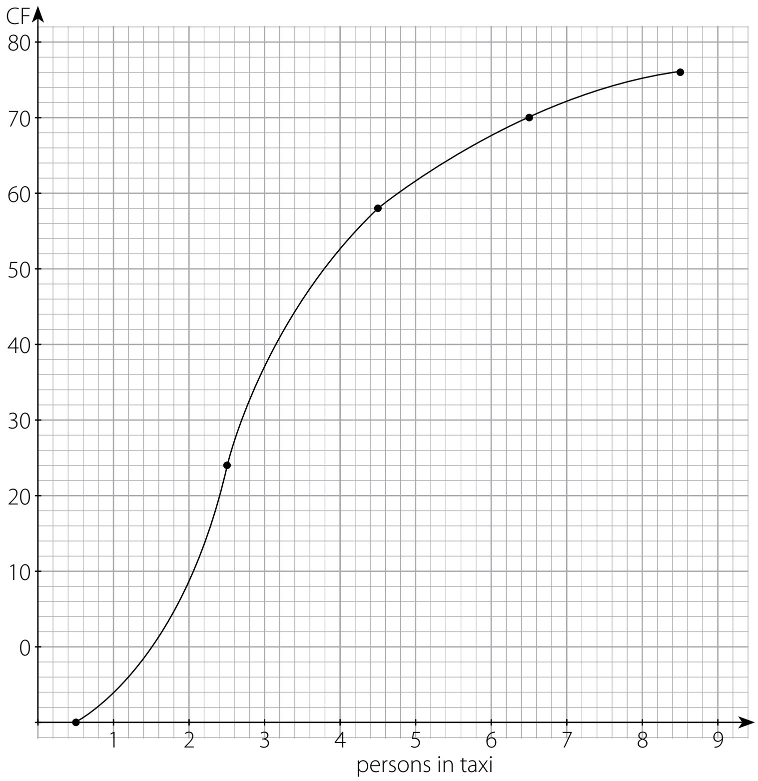
8. a)

Persons in taxi (n)	f	CF
$0.5 \le n < 2.5$	22	22
$2.5 \le n < 4.5$	36	58
$4.5 \le n < 6.5$	12	70
$6.5 \le n < 8.5$	6	76

b)



c) 3.2 persons

d) No. A taxi can take 4 or 5 persons comfortably.

9. The weights of the girls are generally a little higher than the boys' (all quartiles); the spread of the boys' weights is a little wider than the girls' (IQR)

10. a)

Age A years of passing test	f	CF
$11 \le A < 12$	3	3
$12 \le A < 13$	12	15
$13 \le A < 14$	20	35
$14 \le A < 15$	10	45
$15 \le A < 16$	3	48
$16 \le A < 17$	2	50

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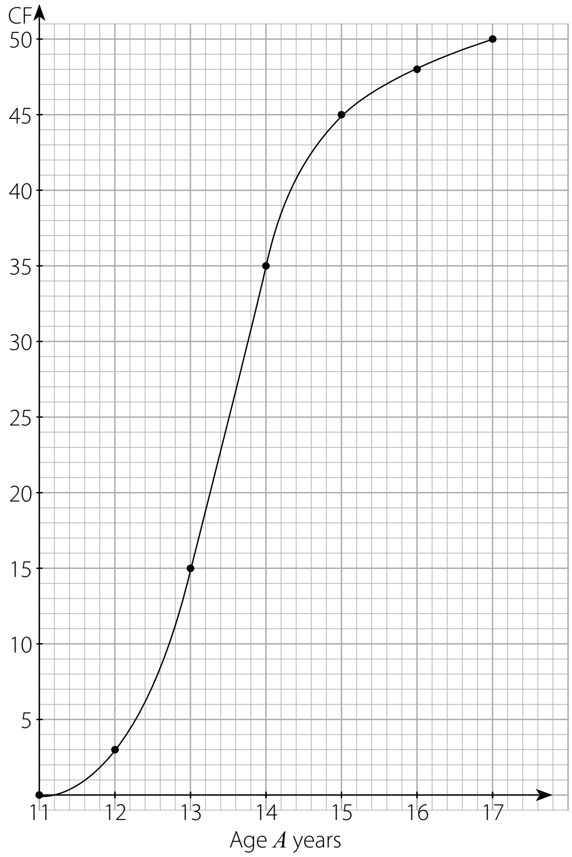
b) 

c) 14.3 y (or 14 y 4 months)

d) 7 boys

EX 23X

1. 12 y 10 months

2. Mean = 13.6 (1 d.p.); mode = 13 y (i.e. $13 \le A < 14$)

   median = 13.5 (from graph or calculation)

   Mean > median and mode.

   Usually this means a positive skew, i.e. there are quite a few high scores at the top end influencing the mean but not the median. In this case, however, the difference is so small that the distribution is almost symmetrical.

3. We have Q_2 equal for A and B

   But for Q_1: $A > B$

   and for Q_3: $A < B$

   $\therefore$ IQR for $A <$ IQR for B

   The scores of A and B are generally about the same as their medians are equal. However, test B scores are more spread out; test A scores cluster closer to the median.

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Chapter

24 Revision Exercises

ANSWERS

Exercises

EX 24A

1. a) $x = 3.4$ b) 1.7

2. a) $\angle B = 65.3°$ b) $\angle Q = 95.7°$

3. a) $x = 0.768, x = -0.434$ b) $x = 1.85, x = -0.180$

4. $R = \{(x, y) : x < 5, 7y \geq 3x + 6, 7y > -3x - 6\}$ or equivalent

5. a) 19.5% increase b) 32.5% increase
   c) 5% decrease d) 16.25% decrease

6. a) 0.007 76 b) 0.000 085 c) 0.000 000 001 19 d) 0.05

7. $x = 24, y = z = 66$

8. a) $\angle QPA = \angle ARS$ (alt $\angle$s) $\angle PQA = \angle ASR$(alt $\angle$s)
   $\angle PAQ = \angle RAS$ (vert opp $\angle$s) or (angle-sum of Δ)
   $\therefore$ ΔPAQ and ΔRAS are equiangular. Similar, as required.
   b) $PQ = 8.1$ cm, $AR = 14$ cm

9. a) $x = 5$, $x = -2$ b) $x = -1.44$, $x = -5.56$

10. a) Q b) R c) S d) P

EX 24B

1. 17.68, 16.32 cm

2. a) $QS = 11.3$ cm b) 60.5°

3. 4800 altogether; Cheese 1200, Chicken 1600, Veg 800

4. $\angle BED = 35°$ (alt $\angle$s) $\angle DEG = 47°$ (alt $\angle$s)
   $\angle BEG = 35 + 47 = 82°$
   $\therefore$ reflex $\angle BEG = 360 - 82 = 278°$ as required.

5. $x = 50$ ($\angle$ at centre), $y = 130$ (opp $\angle$s of cyclic quad)

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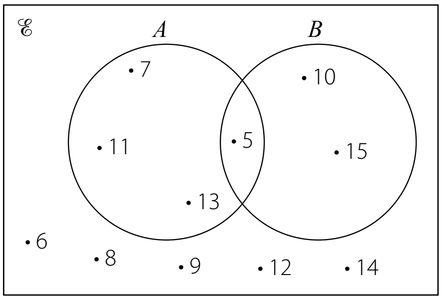
6. a) $A = \{5, 7, 11, 13\}$ $B = \{5, 10, 15\}$

b)



c) $A \cup B' = \{5, 6, 7, 8, 9, 11, 12, 13, 14\}$

d) $n(A' \cap B) = 2$

7. $AC = 15.2$ cm

8. a)

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1

b) The diagonal lines below the outer layer of 1 s.

c) The diagonal lines below the natural number sequences.

d) 285

9. $x = 12.2$, $y = 22.7$ (3 s.f.)

10. a) $\frac{1}{10}$ b) $\frac{1}{360}$ c) $\frac{1}{240}$ d) $\frac{81}{100}$

EX 24C

1. a) 600 cm^2, 100 cm b) 76.8 cm^2 (3 s.f.), 36.4 cm

2. a) 540 s b) 8.82 km

c) 0.233 m/s^2 (3 s.f.) d) –0.117 m/s^2 (3 s.f.)

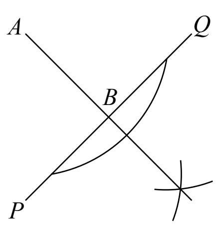
3. a) $\frac{4x}{y^3}$ b) $\frac{3y^2}{2x^2}$

4. a) $\angle FEB = 53°$ b) $\angle EAD = 93°$

c) $\angle EFD = 40°$ d) $\angle BCD = 130°$

5. a) 1 axis, 4 planes b) 1 axis, infinite number of planes

6. a)



$\angle PBA = 90°$

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7. a) $x = -7, \quad x = 6$ b) $x = \frac{1}{3}, \quad x = -1$

8. a) false b) false c) true d) true

9. a) 8.54×10^3 b) 4.51×10^{-2} c) 3.11×10^6 d) 4.60×10

10 \$ 60

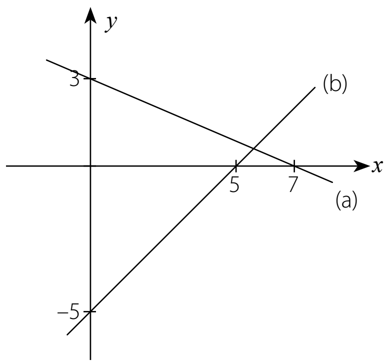
EX 24D

1. a) $\frac{4}{5}, \ \frac{4}{5}$ b) $\frac{3}{5}, \ \frac{3}{5}$ c) $\frac{4}{3}, \ \frac{3}{4}$

d) $\sin x° = \cos y, \quad \cos x° = \sin y°, \quad \tan x° = \frac{1}{\tan y°}, \quad \tan y° = \frac{1}{\tan x°}$

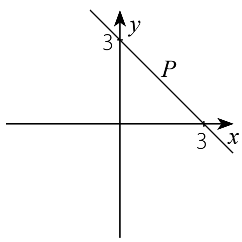
$(\sin x°)^2 + (\cos x°)^2 = 1, \quad (\sin y°)^2 + (\cos y°)^2 = 1$

2.

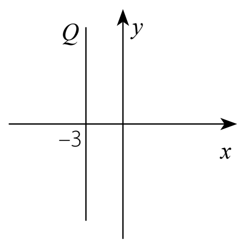


3. 2.98 m^3

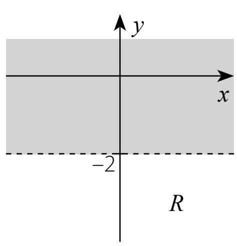
4. a)



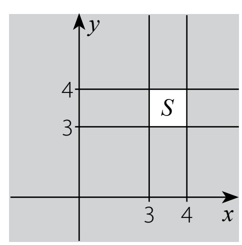
b)



c)



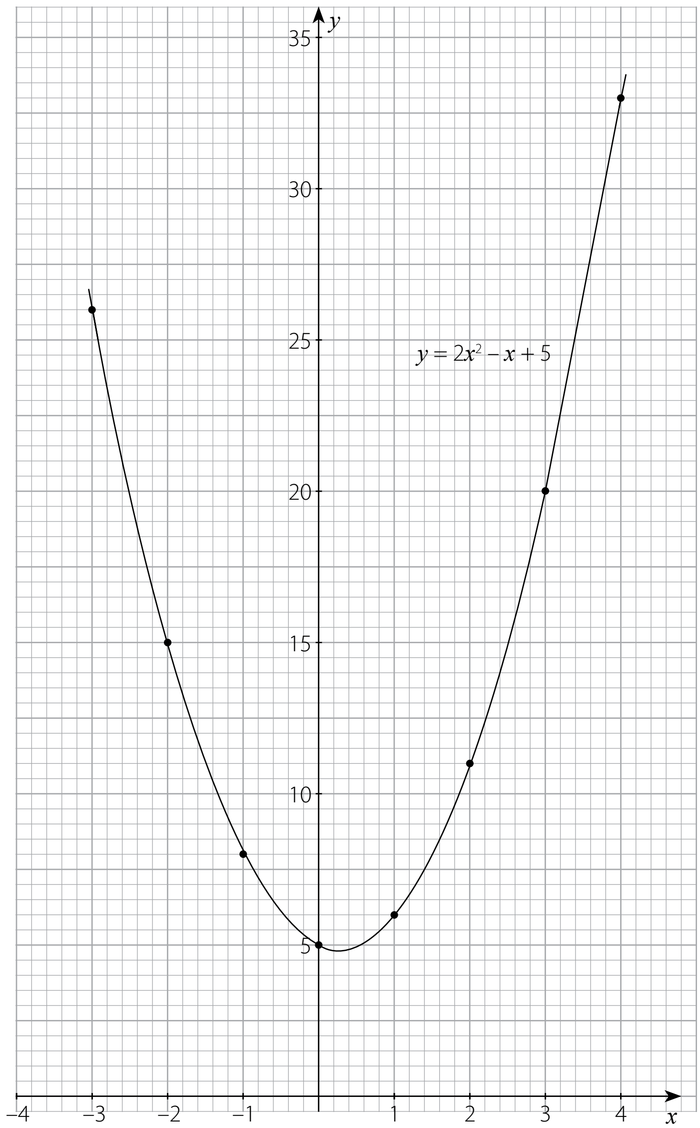
d)



5. a)

x	−3	−2	−1	0	1	2	3	4
x^2	9	4	1	0	1	4	9	16
$2x^2$	18	8	2	0	2	8	18	32
$-x$	3	2	1	0	−1	−2	−3	−4
$+5$	5	5	5	5	5	5	5	5
y	26	15	8	5	6	11	20	33

b)



c) $x = 0.3$ (1 d.p.) d) $x = -0.8, x = 1.3$ (1d.p.)

6. a) int $\angle$s) b) (adj $\angle$s on st line)

c) (angle-sum of quad) d) (ext angle-sum of polygon)

7. 59.4 cm

8. $x = 13.2$ cm, $y = 9.5$ cm

9. a) y varies directly as x. b) y varies directly as the square of x.

c) y varies inversely as x. d) y varies inversely as the square of x.

10. $l = 9.25$ (3 s.f.) $\angle M = 55.6°$ (1 d.p.), $\angle N = 66.4°$

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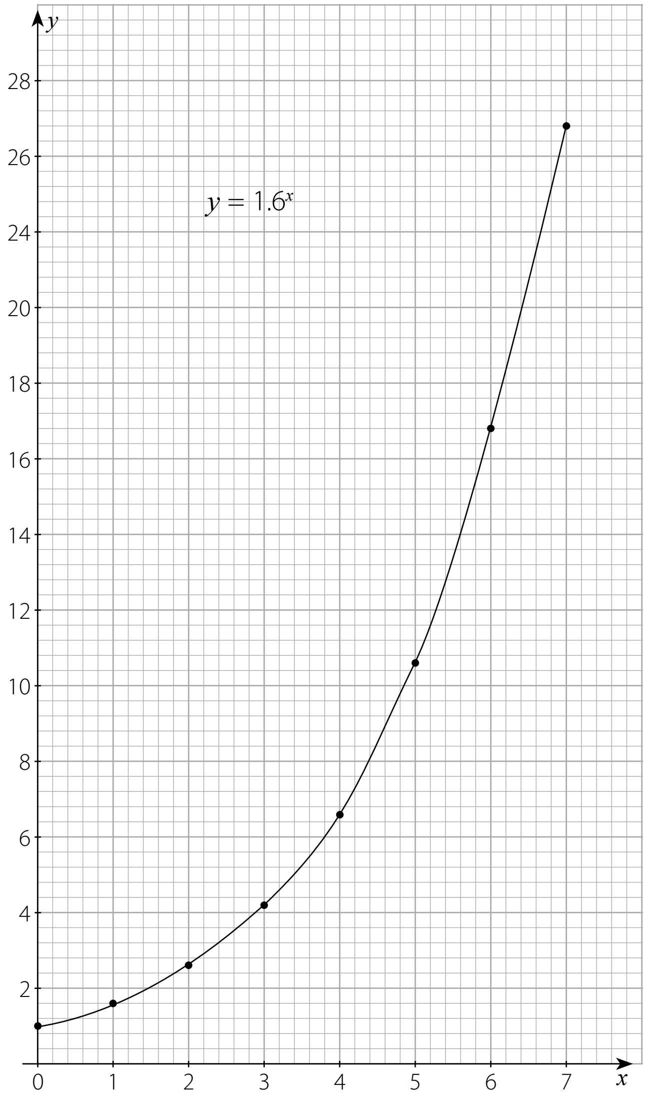
EX 24A, question 1 (worksheet)



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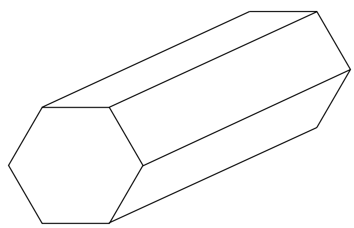
Specimen Examination Paper 1

[end of academic year]

Instructions **Calculators are not allowed.**
Geometrical drawing instruments are required.
Attempt all the questions.
The marks for each question are shown in square brackets.
Time Allowed: $1\frac{1}{2}$ hours [Max. marks: 85]

1. Write down the number of axes of symmetry and planes of symmetry of each of the following solids:

   a) a prism with a regular hexagon as cross-section



   b) a pyramid with a regular hexagon for its base and 6 congruent isosceles triangles for its other faces [4]

2. a) If y varies directly as x squared, and $y = 20$ when $x = 2$, find the value of y when $x = 3$.

   b) If y varies inversely as x, and $y = 20$ when $x = 2$, find the value of x when $y = 5$ [4]

3. Use the laws of indices to find the value of n in each case:

   a) $6^n \div 6^5 = \sqrt{6}$ b) $7^n \times 7^2 = \sqrt[3]{7}$

   c) $25^n \times 5^n = \frac{1}{5}$ d) $32^{n/5} = 4$ [4]

4. A survey of 200 students was carried out to find out how long they took to travel to school in the morning. The grouped data results of the survey were as follows:

Time t min to school	f	CF
$10 \le t < 20$	50	50
$20 \le t < 30$	73	
$30 \le t < 40$	37	160
$40 \le t < 50$	20	
$50 \le t < 60$	13	
$60 \le t < 70$	7	200

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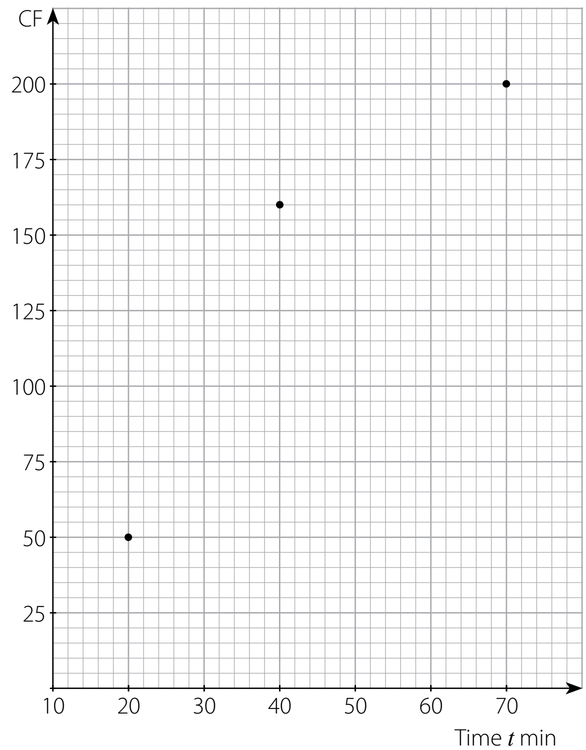
a) Complete the cumulative frequency column on the table.

b) Plot the remaining four points on the graph and draw the CF curve.

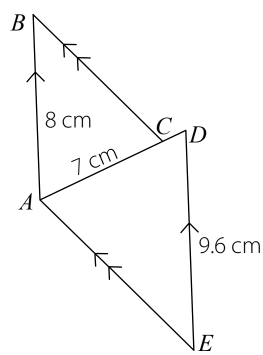
c) From your graph estimate the median time to school.

d) From your graph estimate how many students took longer than 45 minutes to reach school.



[5]

5. In the diagram, $AB = 8$ cm, $AC = 7$ cm, $DE = 9.6$ cm. AB is parallel to ED, and BC is parallel to AE.



[not to scale]

a) Prove that ΔABC is similar to ΔDEA. b) Find the length of CD. [5]

6. If $p = 4 \times 10^3$, $q = 1.2 \times 10^{-3}$, and $r = 3 \times 10^{-2}$ calculate the following, giving your answers in standard form:

a) $pr + 100$ b) $\frac{q}{r}$ c) $\frac{q}{p}$ d) $q + r$ [5]

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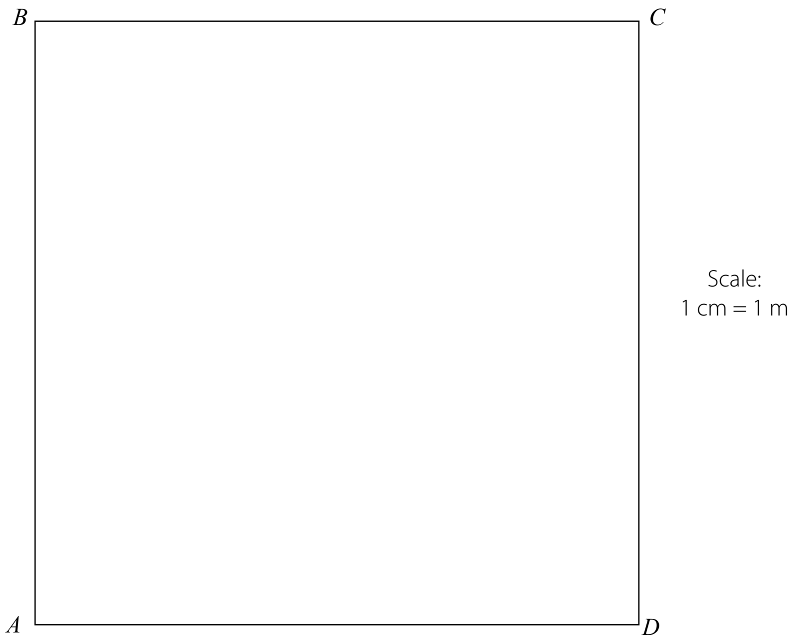
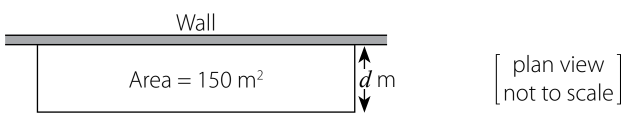
7. The diagram is a map of a square garden. There is a tree at the midpoint of AB on the boundary. There is also an underground drain running from the midpoint of BC to the midpoint of AD. AB is of length 8 m. Plants will not grow well under the shade of the tree nor near the drain. The gardener decides not to plant within a radius of 4 m of the tree nor within a distance of 2 m from the drain. Illustrate, by shading unwanted regions, the locus of points suitable for planting.



[5]

8. A farmer wishes to construct a rectangular pen against a wall to fence in his animals. The area he requires is 150m². The length of the pen is 47 m longer than the width d metres.



Use an equation to work out the total length of fencing required to make this pen. [6]

9. If A is (–3, –1), B(1, –3), C(4, 7) and D(4, –1):

a) Find M, the midpoint of AB. b) Find N, the midpoint of CD.

c) Find the gradient of MN. d) Find the equation of MN. [6]

10. If $p = -1$, $q = 2$ and $r = -3$, find

a) the value of $(p^2 - q^2 + r^2)^2$ b) the value of x when $x(x + p) = -r - x + 1$ [6]

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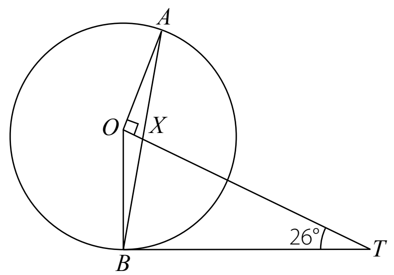
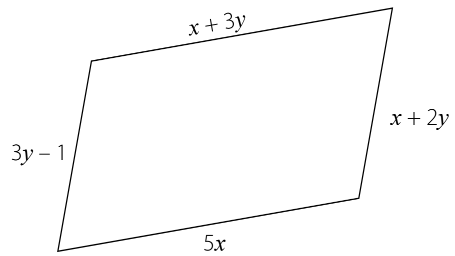
11. In the diagram, O is the centre of the circle and BT is a tangent. OT and AB intersect at x, and $\angle AOT$ is a right-angle. Find the size of $\angle OAB$.



[6]

12. The diagram is a parallelogram whose sides are of the lengths shown as algebraic expressions. Use algebra to find the perimeter of the parallelogram.



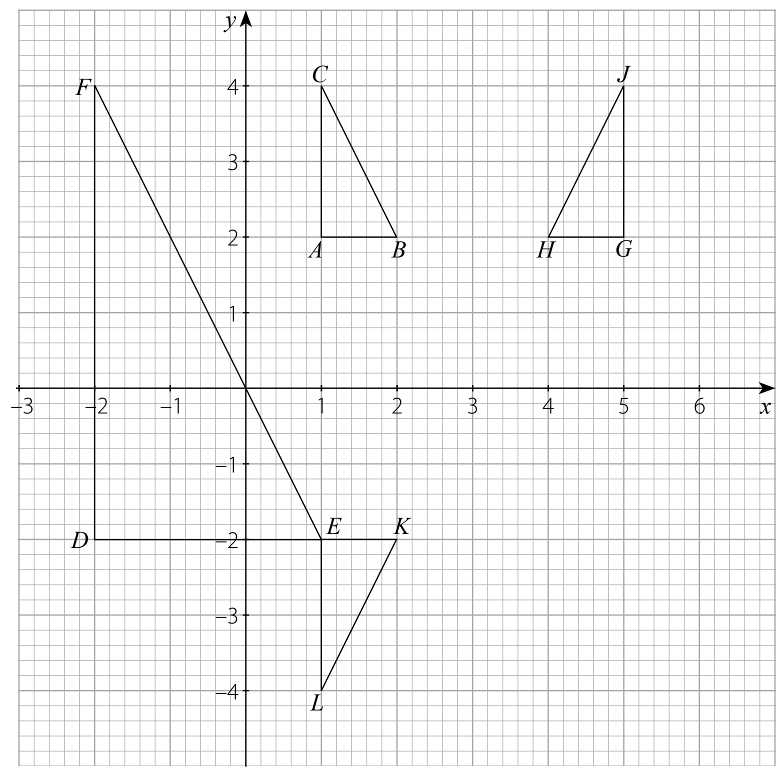
[not to scale]

[7]

13. Describe fully the transformations of the triangles shown in the diagram:

a) $ABC \rightarrow DEF$ b) $ABC \rightarrow GHJ$

c) $GHJ \rightarrow EKL$



[7]

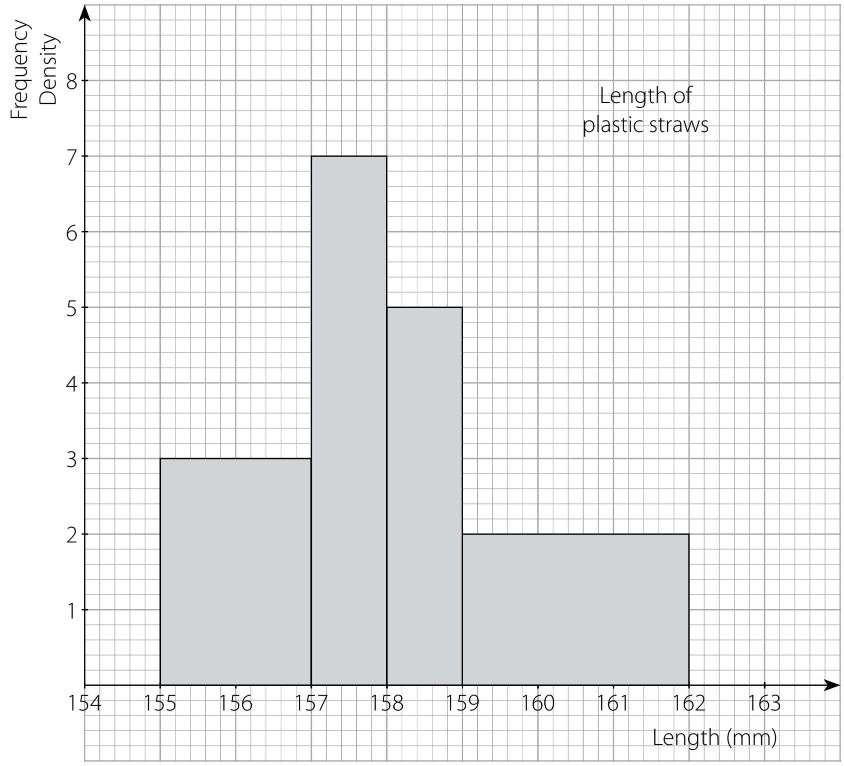
14. $\mathscr{E}$ = {people who attended a fitness centre in one day)

P = {people who used the swimming pool}

M = {people who used the machines}

If n(ξ) = 80, n(P) = 45, n(M) = 55 and n($P \cup M$)′ = 10 use a Venn diagram to find the value of n($P \cap M$). [7]

15. The diagram is a histogram illustrating the distribution of the lengths of plastic straws made by a machine in a factory.



a) How many straws were between 159 mm and 162 mm in length?

b) How many straws were measured altogether in compiling this data?

c) Draw a frequency polygon of this information by constructing it over the histogram provided.

d) Without working it out, show how you could estimate the mean length of straw. [8]

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Specimen Paper 1—answers and guide to marking:

1. a) 7 axes, 7 planes b) 1 axis, 6 planes [1each = 4]

2. a) $y = kx^2$

 $y = 45$

 b) $y = \frac{k}{x}$ or $xy = k$

 $x = 8$

 [1, 1, 1, 1 = 4]

3. a) $n = \frac{11}{2}$ or $5\frac{1}{2}$

 b) $n = \frac{-5}{3}$ or $-1\frac{2}{3}$

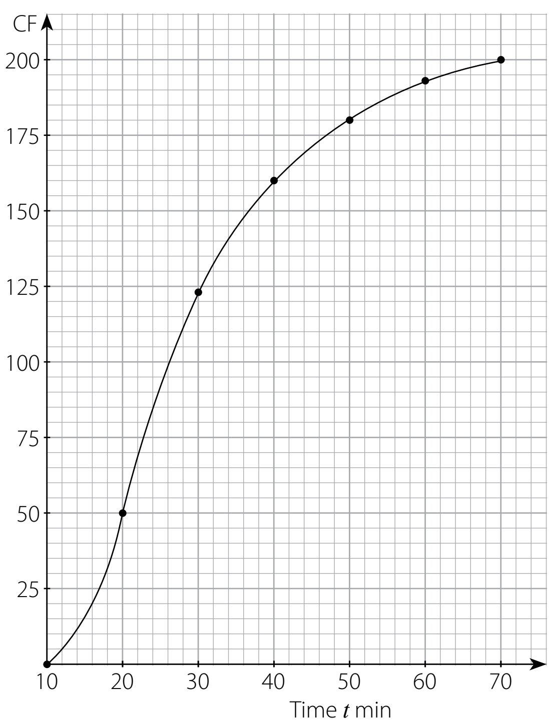
 c) $n = \frac{-1}{3}$

 d) $n = 2$ [1each = 4]

4. a)

CF
50
123
160
180
193
200

 b)



(a) 1
(b) 1 curve
 1 zero pt
(c) 1 =5
(d) 1
Allow ± 1
for (c) and (d)

 c) median 26

 d) 28 students

5. a) $\angle BAC = \angle ADC$ (alt $\angle$s) 1

$\angle BCA = \angle CAE$ (alt $\angle$s) 1

$\angle ABC = \angle DEA$ (angle–sum of Δ) =5

$\therefore \Delta\, BAC =$ is similar to $\Delta\, DEA$ (equiangular) 1

b) $CD = 1.4$ cm 2

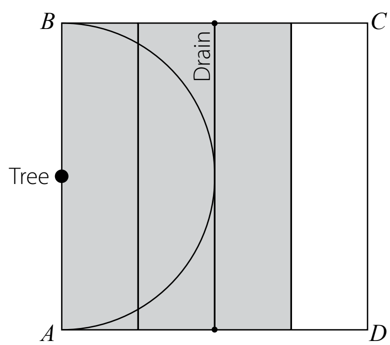
6. a) 1.048×10^{2} 1

b) 4×10^{-2} 1

c) 3×10^{-7} 1 =5

d) 3.12×10^{-2} 2

7.



2 semicircle around tree
2 lines parallel to drain =5
1 shading

8. $d(d + 47) = 150$ 2 equation

$d^2 + 47\,d = 150$

$d^2 + 47\,d - 150 = 0$

$(d - 3)(d + 50) = 0$ =6

$d = 3$ or $d = -50$ (reject) 2 solution

Fencing $= 3 + 3 + 50 = 56$ m 2

9. a) $M(-1, -2)$ 1

b) $N(4, 3)$ 1

c) $gdt\ MN = 1$ 2 =6

d) $y = x - 1$ 2

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10.	a)	$((-1)^2 - (2)^2 + (-3)^2)^2$		
		$= (1 - 4 + 9)^2$	1	
		$= 6^2$	1	=3
		$= 36$	1	
	b)	$x(x - 1) = 3 - x + 1$	1	
		$x^2 - x = 4 - x$		=3
		$x^2 = 4$	1	
		$x = \pm 2$	1	
11.		$\angle OBT = 90$ (tan $\perp$ rad)	2	
		$\therefore \angle BOT = 64°$ (angle-sum of Δ)	1	=6
		$\therefore \angle AOB = 90 + 64 = 154°$	1	
		$\therefore \angle OAB = 13°$ (angle-sum of isos Δ)	2	
12.		$x + 3y = 5x$ and $3y - 1 = x + 2y$	2 equations	
		$3y = 4x$ $\quad\quad y - 1 = x$		
		$3y = 4(y - 1)$	1 substitution	
		$3y = 4y - 4$		=7
		$4 = y$ $\quad\quad x = 3$	2 solutions	
		Sides are 15 cm and 11 cm		
		Perimeter = 52 cm	2	
13.	a)	Enlargement, SF 3, Centre (2.5, 4)	2	
	b)	Reflection in line $x = 3$	2	=7
	c)	Rotation 180° about (3, 0)	3	

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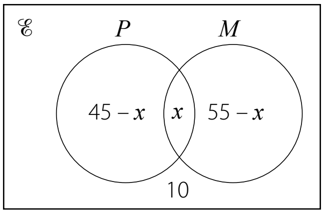
14.



2

Let $x = n(P \cap M)$ 1

$(45 - x) + x + (55 - x) + 10 = 80$ 2

$110 - x = 80$

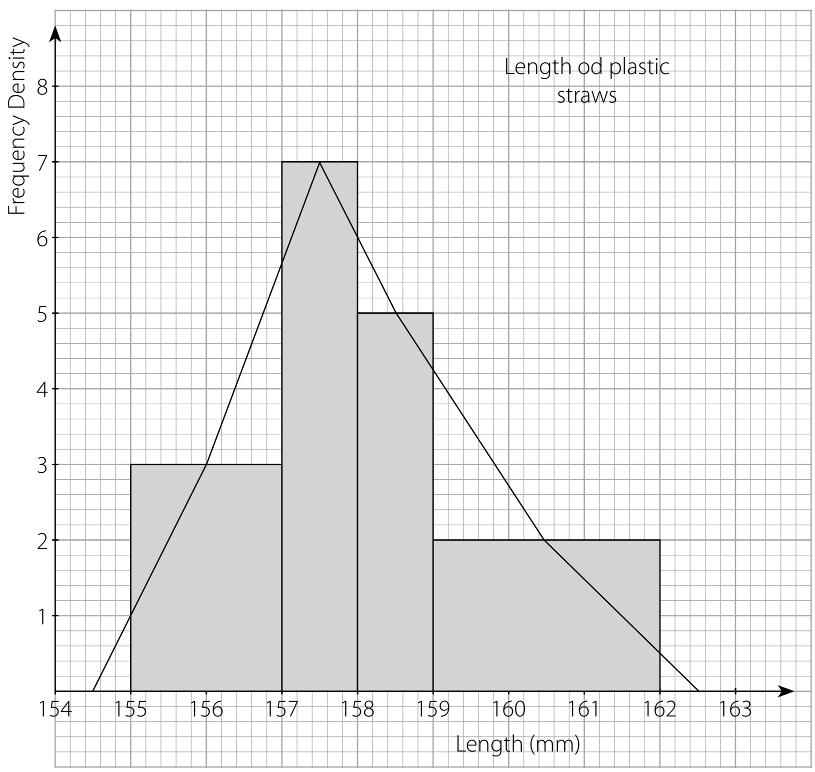
$x = 30$ 2

=7

15. a) 6 2

b) 24 2

c)



1 mid points

1 zero points

d) $\dfrac{156 \times 3 + 157.5 \times 7 + 158.5 \times 5 + 160.5 \times 2}{24}$ 2

= 8

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Specimen Examination Paper 2

[End of academic year]

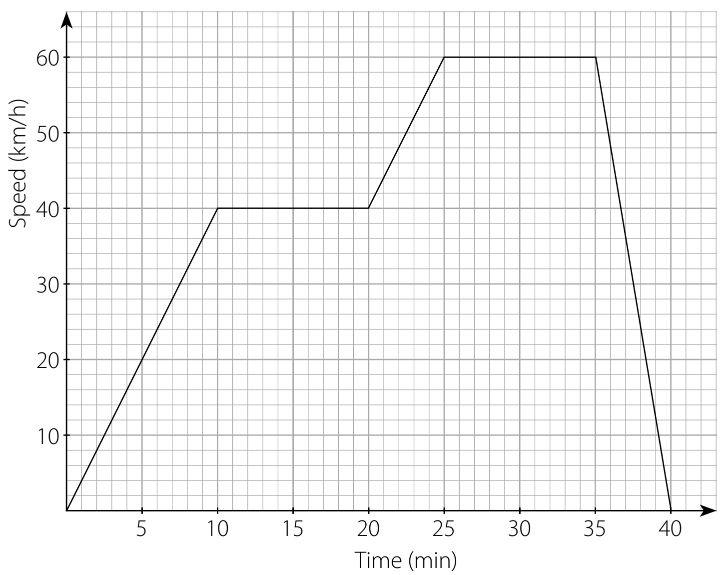
Instructions Calculators are required.
Attempt all the questions.
The marks for each question are shown in square brackets.
Time allowed: 2 hours [Max. marks: 115]

1. In a sale, the price of jewellery is reduced by 15%. Later, to clear the stock, a further discount of 25% is allowed on the sale price.

   a) Calculate the final price to be paid for ear-rings originally priced at $ 140.

   b) Calculate the original price of a necklace sold for $ 107.10. [5]

2. The diagram shows the speed–time graph of a bus travelling between two bus stops.



   a) Describe the motion of the bus after 15 min.

   b) Calculate the acceleration of the bus during the first 10 minutes, in m/s^2, correct to 3 s.f.

   c) State the maximum speed of the bus.

   d) Calculate the total distance travelled in km, correct to 1 d.p. [6]

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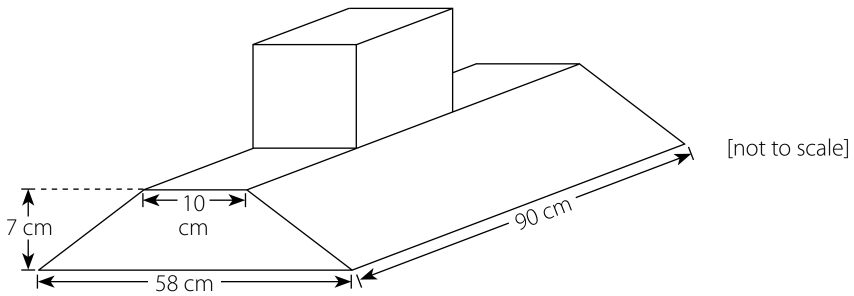
3. A composite solid is made up of a cube of side 10 cm placed symmetrically on a prism with an isosceles trapezium for cross-section, with dimensions as shown. Calculate (a) the total volume (b) the total surface area, of the composite solid. (3 s.f.)



[6]

4. The upper bound of the area of a circle is $U\pi$. Its lower bound is $L\pi$. If the diameter of the circle is 12.2 cm (correct to 1 d.p.) find the values of U and L. [6]

5. Find the coordinates of the points of intersection of the line $y = 2x$ with the parabola $y = x^2 - 5x + 1$, giving your answers correct to 1 d.p. [8]

6. A card is selected at random from a set of the ace, king and queen of spades. Another card is selected at random from a set of ace, king, queen and jack of diamonds. Draw a tree diagram to illustrate this situation. Use your diagram to calculate the probabilities that

a) both cards are kings

b) there is an ace-king combination

c) the jack is not chosen

d) there is an ace or jack in the pair selected. [Give all answers as fractions in lowest terms.] [8]

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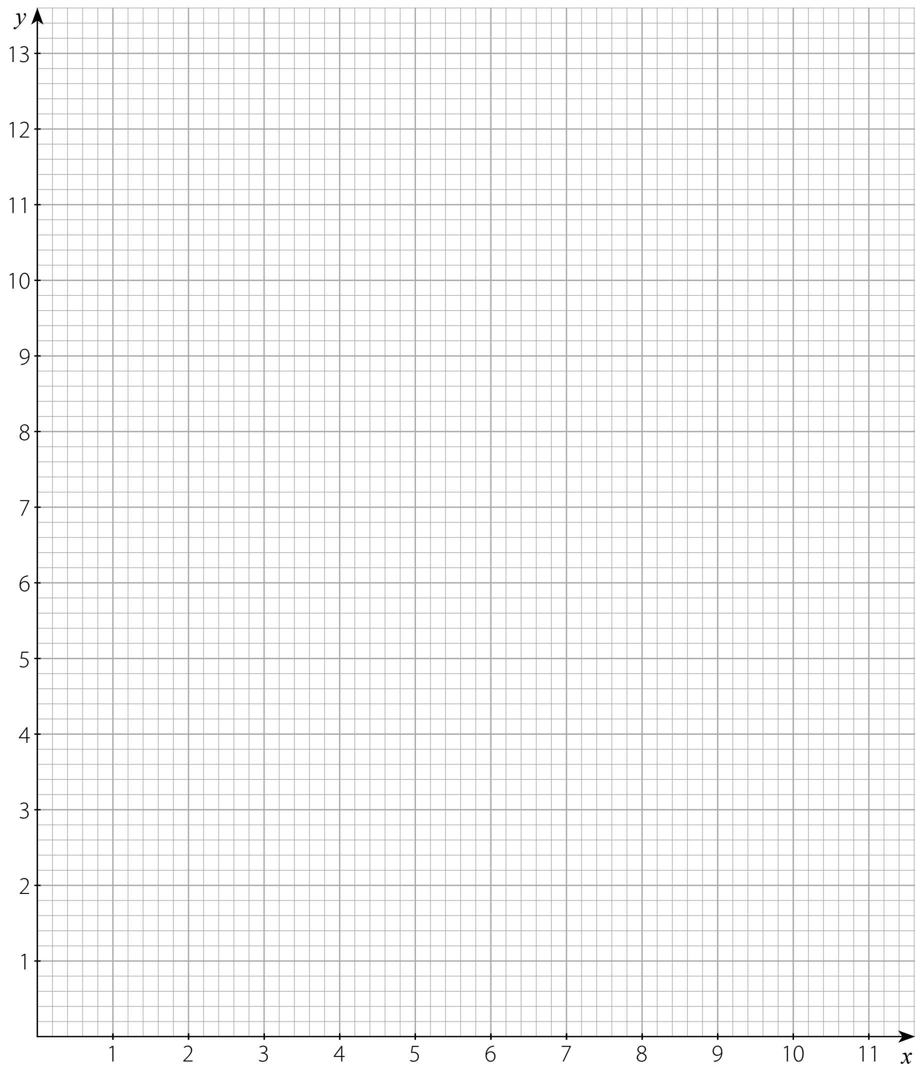
7. a) On the graph plot a scatter diagram of the following data:

x	1	4	5	8	9	9	9	9	10	10	10	11
y	3	4	4	5	7	9	9	9	10	11	12	13

b) Calculate the mean of x and of y and plot the mean point on the graph.

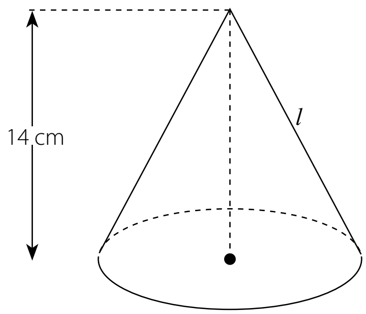
c) Draw the line of best fit through the mean point. Estimate the value of y if $x = 6$.

d) Describe the correlation between x and y.



[8]

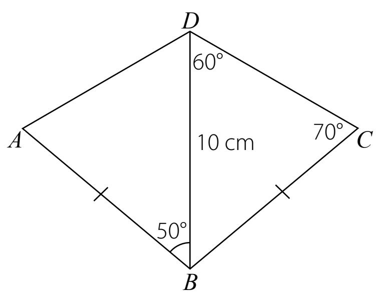
8. The diagram shows a right circular cone (i.e. its apex is vertically above the centre of the base) of height 14 cm and volume 250 cm^3.



a) Calculate the base radius (3 s.f.) $[V = \frac{1}{3}\pi r^2 h]$

b) calculate the semi vertical angle (nearest degree), i.e. the angle between the axis of symmetry and the slant height l. [8]

9.



In the diagram, $AB = BC$, $BD = 10$ cm, $\angle ABD = 50°$, $\angle BDC = 60°$, and $\angle BCD = 70°$.

a) Prove that $\triangle ABD$ is congruent to $\triangle CBD$.

b) Calculate the length of AB. (3 s.f.)

c) Calculate the area of the quadrilateral $ABCD$. (3 s.f.) [8]

10. A ship sails 52 km on a bearing of 215°, then due East for 70 km. It then turns and sails straight back to its starting point. Find the bearing for this final part of its journey. [8]

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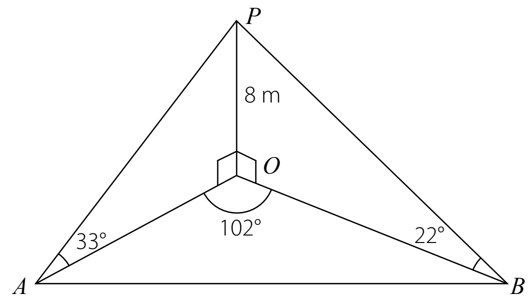
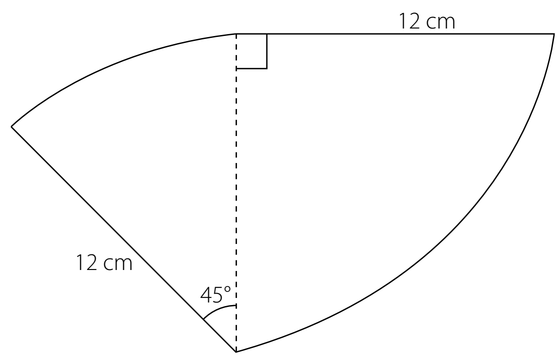
11. $\triangle AOP$ is a triangle on level horizontal ground. OP is a vertical post. The top of the post P is observed from A with an angle of elevation of 33°, and from B with an angle of elevation of 22°. P is 8 m above O. $\angle AOB = 102°$.



Find:

a) the length of AB (3 s.f.) b) the size $\angle OAB$ and of $\angle OBA$ (1 d.p.) [8]

12. The diagram illustrates two sectors of a circle of radius 12 cm joined together. The composite shape has area A cm^2 and perimeter P cm.



a) Show that $P = 24 + 9\pi$ b) Show that $A = 54\pi$

c) Show that $A = 6\,(p - 24)$ [8]

13. a) A fair dice is rolled once.

Event A is obtaining an even number.

Event B is obtaining a prime number.

State whether A and B are independent or mutually exclusive events, or both of these, or neither.

b) The dice is rolled twice. Find the probability of obtaining a prime number both times.

c) The dice is rolled three times. Find the probability of obtaining an even number all three times.

d) The dice is rolled five times. Find the probability of obtaining neither an even nor a prime number on all five occasions, giving your answer in standard form. (3 s.f.) [8]

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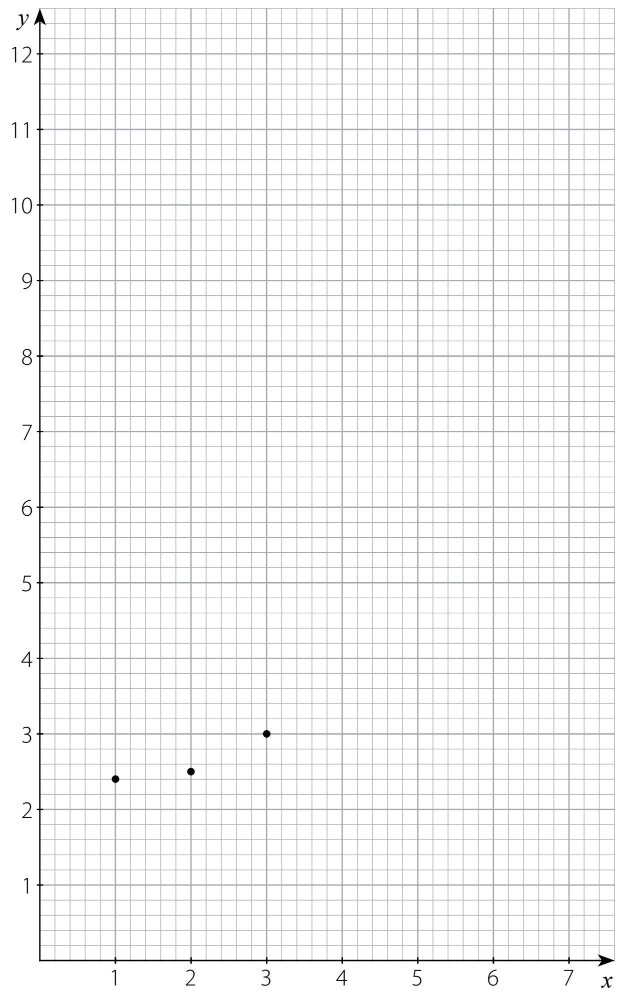
14. If $2^m \times 4^{3/2} = 8^{m-5}$ and $\frac{\sqrt{3^5}}{3^n} = 9^{n+2}$, show that $2(n + m) = 17$. [10]

15. The diagram shows part of the graph whose equation is $y = 1.4^x + \frac{1}{x}$ for $1 \le x \le 7$.

a) Complete the table of values, rounding off to 1 d.p. throughout.

x	1	2	3	4	5	6	7
1.4^x	1.4	2.0	2.7				
$\frac{1}{x}$	1.0	0.5	0.3				
y	2.4	2.5	3.0				

b) Plot the remaining points and draw the curve.



c) By drawing a tangent estimate the gradient of the curve where $x = 4.5$.

d) Use your graph to estimate a solution to the equation $1.4^x = 8 - \frac{1}{x}$ [10]

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Specimen Paper 2 Answers and mark scheme

1. a) $ 89.25 — 2
   b) $ 168 — 3 } = 5 any method

2. a) uniform speed of 40 km/h (or constant speed) — 1
   b) 0.0185 m/s^2 — 2
   c) 60 km/h — 1
   d) 26.7 km — 2 } = 6

3. a) $V_{\text{cube}} = 1000 \text{ cm}^3$ — 1

   Area of cross-section of prism $= \frac{1}{2}(10 + 58)7 = 238 \text{ cm}^2$

   $V_{\text{prim}} = 238 \times 90 = 21420 \text{ cm}^3$ — 1

   total volume $= 22\,420 \text{ cm}^3$ — 1

   b) slant height = 7 cm (Pythag) — 1

   $$\begin{aligned}\text{Area} &= 58 \times 90 + 2(7 \times 90)\\ &\quad + 2(10 \times 40) + 2 \times 238\\ &\quad + 5 \times 100\\ &= 190\,406\,480 \text{ cm}^2\\ &= 190\,000\,000 \text{ cm}^2 \text{ (3 s.f.)} \quad [\text{or } 1900 \text{ m}^2]\end{aligned}$$ — 2 } = 6

4. $12.15 \le d \le 12.25$ — 1

   $\therefore \quad 6.075 \le r \le 6.125$ — 1

   $\therefore \quad 36.905625 \le r^2 \le 37.515625$

   $A = \pi r^2$

   $U = 37.515625$ — 2

   $L = 36.905625$ — 2 } = 6

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5. Substitute $2x = x^2 - 5x + 1$ — 1

$0 = x^2 - 7x + 1$ — 1

$x = \dfrac{7 \pm \sqrt{49 - 4(1)(1)}}{2}$ — 1

$= \dfrac{7 \pm \sqrt{45}}{2}$

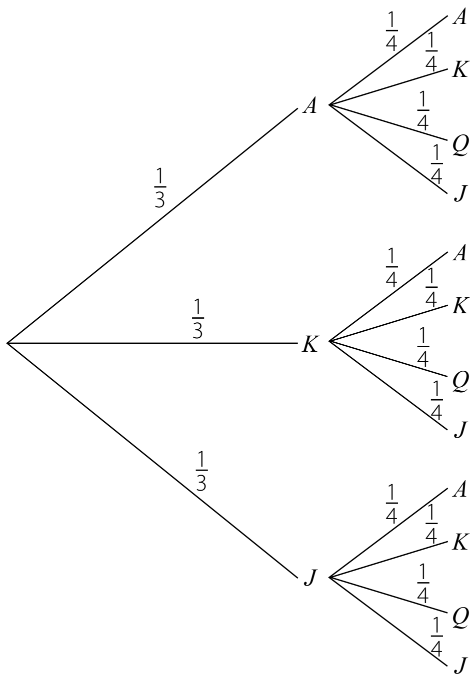
$= 6.854$ or 0.146 — 1 each

$y = 13.71 \qquad 0.292$ — 1 each

Intersect at (6.9, 13.7) and (0.1, 0.3) — 1

= 8

6.



2 diagram

a) $\dfrac{1}{3} \times \dfrac{1}{4} = \dfrac{1}{12}$ — 1

b) $\dfrac{1}{3} \times \dfrac{1}{4} + \dfrac{1}{3} \times \dfrac{1}{4} = \dfrac{1}{6}$ — 1

c) $\dfrac{1}{3} \times \dfrac{3}{4} \times 3 = \dfrac{3}{4}$ — 2

[or $1 - \dfrac{1}{3} \times \dfrac{1}{4}$ x3]

d) $\dfrac{1}{3} \times 1 + \dfrac{1}{3} \times \dfrac{1}{4} + \dfrac{1}{3} \times \dfrac{1}{4} = \dfrac{1}{2}$ — 2

= 8

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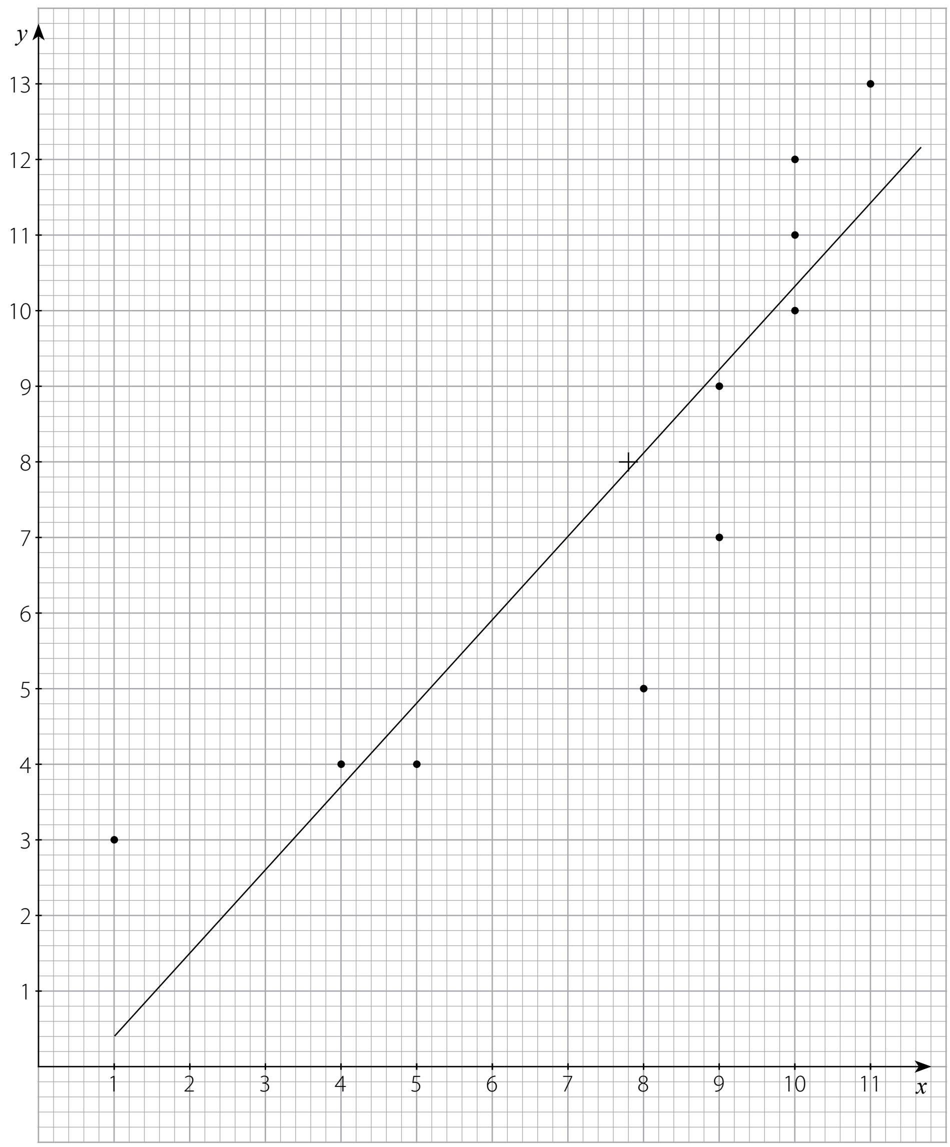
7. a) Mean (7.9, 8.0)

b) 5.8 [Allow ± 0.4]

c) Positive

2 graph points
2 line
2 mean
1 estimate
1 correlation
= 8



8. a) $250 = \frac{1}{3}\pi r^2 14$

$r = \sqrt{\frac{250 \times 3}{14\pi}}$

$= 4.13$ cm

b) $\tan^{-1}\left(\frac{r}{14}\right) = 16°$

2
1
2
3
= 8

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9. a) $AB = CB$ (given) 1

$\angle DBC = 50°$ (angle-sum of Δ) $= \angle DBA$ 1

DB is common. 1

$\therefore$ ΔABD is congruent to ΔCBD (SAS) 1

b) Sine rule: $\dfrac{BC}{\sin 60°} = \dfrac{10}{\sin 70°}$

$BC = 9.22$ cm

$AB = 9.22$ cm 2

c) Area $\Delta = \dfrac{1}{2}\ AB \times BD \times \sin 50°$

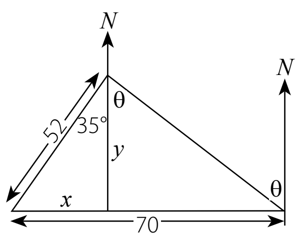
$\therefore$ Area $ABCD = AB \times BD$ x $\sin 50°$

$= 9.22 \times 10 \sin 50°$

$= 70.6\text{ cm}^2$ 2

= 8

10.



2 diag

$x = 52 \sin 35° = 29.826$ 1

$y = 52\cos 35° = 42.596$ 1

$\tan \theta = \dfrac{70 - x}{y}$

$= \dfrac{40.174}{42.596}$ 2

$\theta = 43.3°$ 1

Bearing is $360 - \theta = 317°$ 1

= 8

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11. a) $\tan 33° = \dfrac{8}{AO}$ and $\tan 22° = \dfrac{8}{BO}$

$AO = \dfrac{8}{\tan 33°} = 12.319 \qquad BO = \dfrac{8}{\tan 22°} = 19.801$ 2

Cosine rule:

$AB^2 = 12.319^2 + 19.801^2 - 2(12.319)(19.801)\cos 102°$ 2

$= 543.84 - 101.43 = 645.27$

$AB = 25.402$

$= 25.4$ m (3 s.f.) 1 = 8

b) Sine rule:

$\dfrac{OB}{\sin \angle OAB} = \dfrac{AB}{\sin 102°}$ or, find $OA = 12.3189$ etc. 1

$\dfrac{19.801}{\sin \angle OAB} = \dfrac{25.402}{\sin 102°}$

$\angle OAB = 49.7°$ 1

$\angle OBA = 28.3°$ 1

12. a) $P = \dfrac{45}{360} \times 2\pi(12) + 12 + \dfrac{90}{360} \times 2\pi(12) + 12$ 2

$P = 24 + 2\pi(12)\left(\dfrac{1}{8} + \dfrac{1}{4}\right)$

$P = 24 + 24\pi\left(\dfrac{3}{8}\right)$

$P = 24 + 9\pi$ as required. 1

b) $A = \dfrac{1}{4}\pi(12)^2 + \dfrac{1}{8}\pi(12)^2$ 2 = 8

$= \left(\dfrac{1}{4} + \dfrac{1}{8}\right)\pi(144)$

$= 54\pi$ as required. 1

c) $A = 54\pi$

$A = 6 \times 9\pi$

$= 6 \times (P - 24)$ as $P = 24 + 9\pi$

$A = 6(P - 24)$ as required [or equivalent] 2

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13. a) neither 2

b) $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$ 2

c) $\left(\frac{1}{2}\right)^3 = \frac{1}{8}$ 2

d) $\left(\frac{1}{6}\right)^5 = 1.29 \times 10^{-4}$ 2

= 8

14. $2^m \times (2^2)^{\frac{3}{2}} = (2^3)^{m-5}$ 2

$2^m \times 2^3 = 2^{3m-15}$

$m + 3 = 3m - 15$

$18 = 2m$

$m = 9$ 2

and $\dfrac{3^{\frac{5}{2}}}{3^n} = (3^2)^{n+2}$ 2

$3^{\frac{5}{2}-n} = 3^{2n+4}$

$\frac{5}{2} - n = 2n + 4$

= 10

$\frac{5}{2} - 4 = 3n$

$\frac{-3}{2} = 3n$

$n = \frac{-1}{2}$ 2

$2(n + m) = 2(\frac{-1}{2} + 9)$

$= 2 \times 8\frac{1}{2}$

$= 17$ as required. 2

15. a)

x	1	2	3	4	5	6	7
1.4^x				3.8	5.4	7.5	10.5
$\frac{1}{x}$				0.3	0.2	0.2	0.1
y				4.1	5.6	7.7	10.6

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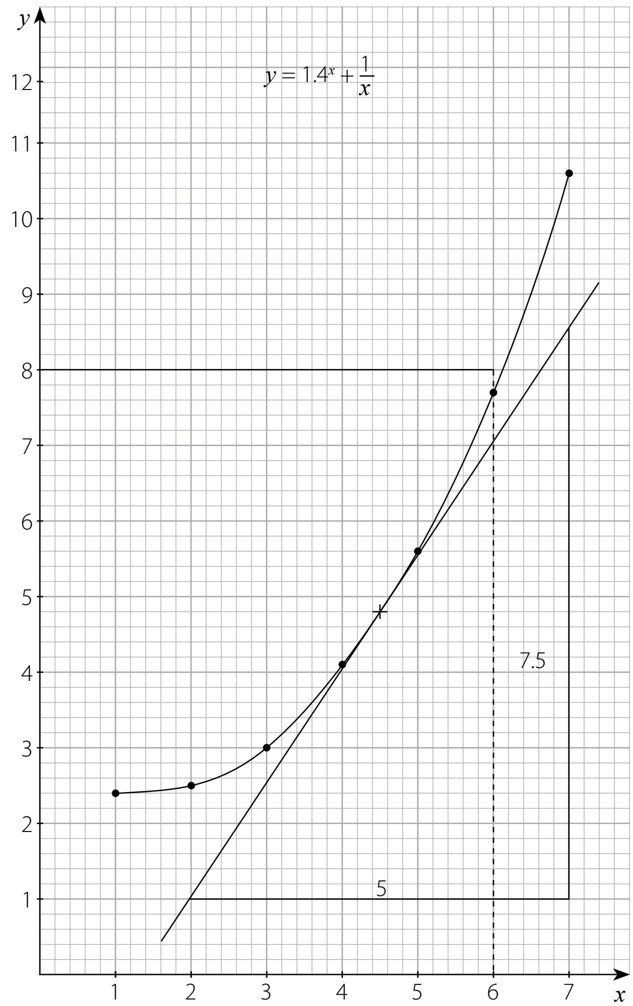
b)



$\left.\begin{array}{l} \text{4 table} \\ \text{2 curve} \\ \text{2 gradient} \\ \text{2 estimate} \end{array}\right\} = 10$

c) 1.5 [allow ± 0.1]

d) $x = 6.1$ [allow ± 0.1]

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NOTES

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